

The diagram shows a solid cylinder standing on a horizontal circular base with centre O and radius 4 units. Points A, B and C lie on the circumference of the base such that AB is a diameter and angle $BOC = 90^{\circ}$. Points P, Q and R lie on the upper surface of the cylinder vertically above A, B and C respectively. The height of the cylinder is 12 units. The mid-point of CR is M and N lies on BQ with BN = 4 units.

Unit vectors \mathbf{i} and \mathbf{j} are parallel to OB and OC respectively and the unit vector \mathbf{k} is vertically upwards.

Evaluate $\overrightarrow{PN} \cdot \overrightarrow{PM}$ and hence find angle MPN .	[7]

