Day 5

Deep Generative Models

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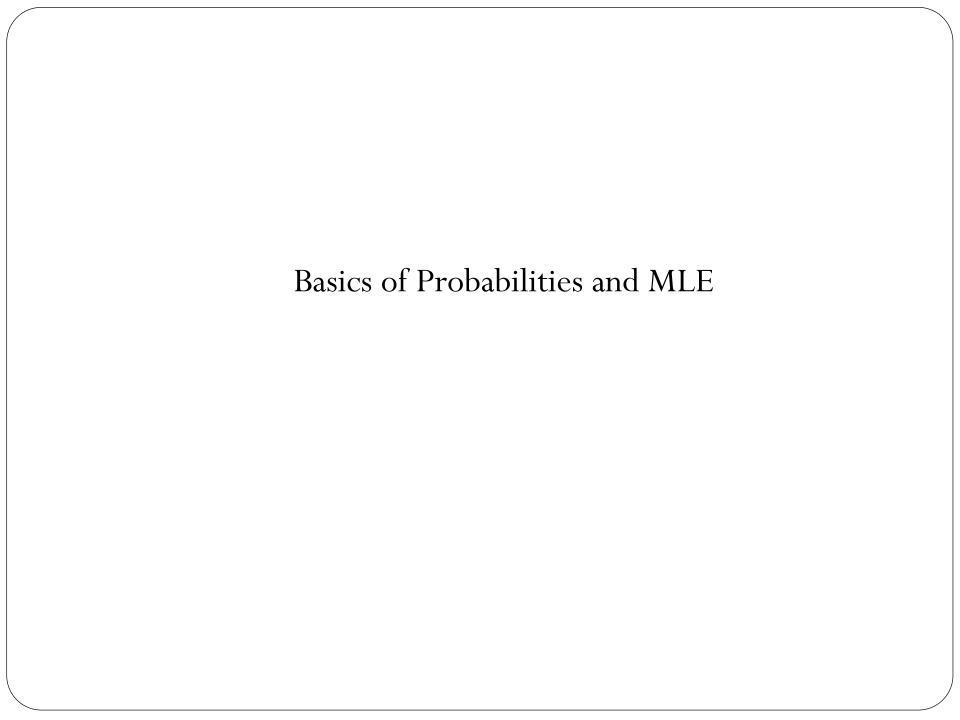
Tsinghua Deep Learning Summer School, Beijing, 2018

A bit about the Instructor

- ♦ Jun Zhu, Professor, Depart. of Computer Science & Technology. I received my Ph.D. in DCST of Tsinghua University in 2009. My research interests include statistical machine learning, Bayesian nonparametrics, and data mining
- ♦ I did post-doc at the Machine Learning Department in CMU with Prof. Eric P. Xing. Before that I was invited to visit CMU for twice. I was also invited to visit Stanford for joint research (with Prof. Li Fei-Fei)
- 2015: Adjunct Professor at CMU
- ♦ Published 100+ research papers on the top-tier ML conferences and journals, including JMLR, TPAMI, ICML, NIPS, etc.
- Served as Area Chairs for ICML, NIPS, UAI, AAAI, IJCAI; Associate Editor for PAMI, AI Journal
- ♦ Research is supported by National 973, NSFC, "Tsinghua 221 Basic Research Plan for Young Talents".
- ♦ Homepage: http://bigml.cs.tsinghua.edu.cn/~jun

Outline

- Review of Probability and Statistics
 - MLE
- Generative Models
- Deep Generative Models
 - VAE
 - GAN
- ZhuSuan
 - Programming library
- Applications



Independence

Independent random variables:

$$P(X,Y) = P(X)P(Y)$$
$$P(X|Y) = P(X)$$



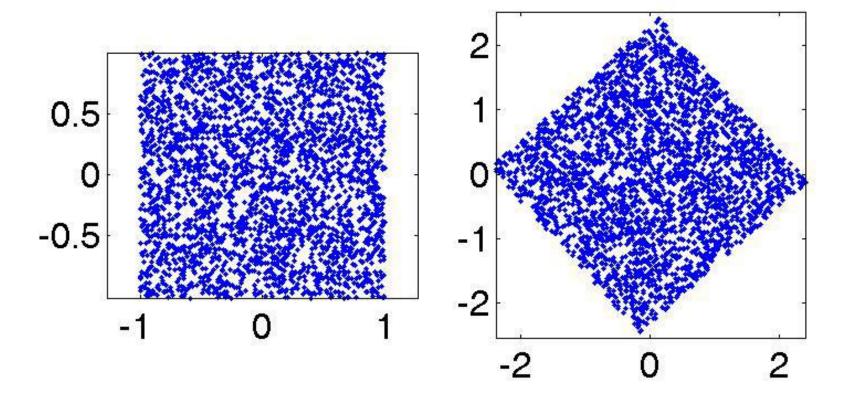


- Y and X don't contain information about each other
 Observing Y doesn't help predicting X
 Observing X doesn't help predicting Y
- Examples:
 - Independent:
 - winning on roulette this week and next week
 - Dependent:
 - Russian roulette





Dependent / Independent?

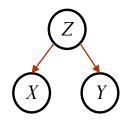


Conditional Independence

Conditionally independent:

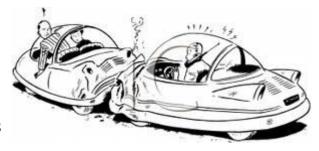
$$P(X,Y|Z) = P(X|Z)P(Y|Z)$$

knowing Z makes X and Y independent



Examples:

London taxi drivers: A survey has pointed out a positive and significant correlation between the number of accidents and wearing coats. They concluded that coats could hinder movements of drivers and be the cause of accidents. A new law was prepared to prohibit drivers from wearing coats when driving.



Finally another study pointed out that people wear coats when it rains...



Maximum Likelihood Estimation (MLE)

Flipping a Coin

- What's the probability that a coin will fall with a head up (if flipped)?
- Let us flip it a few times to estimate the probability



The estimated probability is: 3/5 "frequency of heads"

Questions:





The estimated probability is: 3/5 "frequency of heads"

- Why frequency of heads?
- How good is this estimation?

Question (1)

- Why frequency of heads?
 - Frequency of heads is exactly the Maximum Likelihood
 Estimator for this problem
 - MLE has nice properties (interpretation, statistical guarantees, simple)

MLE for Bernoulli Distribution

Data,
$$D=$$

$$D=\{X_i\}_{i=1}^n,\ X_i\in\{\mathrm{H},\mathrm{T}\}$$

$$P(Head) = \theta$$
 $P(Tail) = 1 - \theta$

- Flips are i.i.d:
 - Independent events that are identically distributed according to Bernoulli distribution
- lacktriangle MLE: choose eta that maximizes the probability of observed data

Maximum Likelihood Estimation (MLE)

lacktriangle MLE: choose eta that maximizes the probability of observed data

$$\begin{split} \hat{\theta}_{MLE} &= \arg\max_{\theta} P(D|\theta) \\ &= \arg\max_{\theta} \prod_{i=1}^{n} P(X_i|\theta) \quad \text{Independent draws} \\ &= \arg\max_{\theta} \prod_{i:X_i=H} \theta \prod_{i:X_i=T} (1-\theta) \quad \text{Identically distributed} \\ &= \arg\max_{\theta} \theta^{N_H} (1-\theta)^{N_T} \end{split}$$

Maximum Likelihood Estimation (MLE)

lacktriangle MLE: choose eta that maximizes the probability of observed data

$$\hat{\theta}_{MLE} = \arg \max_{\theta} P(D|\theta)$$

$$= \arg \max_{\theta} \theta^{N_H} (1 - \theta)^{N_T}$$

Solution?

$$\hat{\theta}_{MLE} = \frac{N_H}{N_H + N_T}$$

• Exactly the "Frequency of heads"

Question (2)

• How good is the MLE estimation?

$$\hat{\theta}_{MLE} = \frac{N_H}{N_H + N_T}$$

□ Is it biased?

How many flips do I need?

♦ I flipped the coins 5 times: 3 heads, 2 tails

$$\hat{\theta}_{MLE} = \frac{3}{5}$$

• What if I flipped 30 heads and 20 tails?

$$\hat{\theta}_{MLE} = \frac{30}{50}$$

• Which estimator should we trust more?

A Simple Bound

 \bullet Let θ^* be the true parameter. For *n* data points, and

$$\hat{\theta}_{MLE} = \frac{N_H}{N_H + N_T}$$

 \bullet Then, for any $\epsilon>0$, we have the Hoeffding's Inequality:

$$P(|\hat{\theta} - \theta^{\star}| \ge \epsilon) \le 2e^{-2n\epsilon^2}$$

Probably Approximately Correct (PAC) Learning

- \bullet I want to know the coin parameter θ , within ϵ =0.1 error with probability at least 1- δ (e.g., 0.95)
- How many flips do I need?

$$P(|\hat{\theta} - \theta^*| \ge \epsilon) \le 2e^{-2n\epsilon^2} \le \delta$$

Sample complexity:

$$n \ge \frac{\ln(2/\delta)}{2\epsilon^2}$$

Deep Generative Models (DGMs)

Why generative models?

"What I cannot create, I do not understand."

-Richard Feynman

Why generative models?

- Leverage unlabeled datasets, which are often much larger than labeled ones
 - Unsupervised learning
 - Semi-supervised learning
- Conditional generative models
 - □ Speech synthesis: Text ⇒ Speech
 - Machine Translation: French ⇒ English

- A simple unigram language model
 - Observations (e.g., bag-of-words)

$$\mathbf{x} = \{x_1, \dots, x_d\}$$

Racing Thompson: an Efficient Algorithm for Thompson Sampling with Non-conjugate Priors

Anonymous Author(s)

Affiliation Address email

Abstract

Thompson sampling has impressive empirical performance for many multi-armed bandit problems. But current algorithms for Thompson sampling only work for the case of conjugate priors since these algorithms require to infer the posterior, which is often computationally intractable when the prior is not conjugate. In this paper, we propose a novel algorithm for Thompson sampling which only requires to draw samples from a tractable distribution, so our algorithm is efficient even when the prior is non-conjugate. To do this, we reformulate Thompson sampling as an optimization problem via the Gumbel-Max trick. After that we construct a set of random variables and our goal is to identify the one with highest mean. Finally, we solve it with techniques in best arm identification.



In multi-armed bandit (MAB) problems [20], an agent chooses an action (in the literature of MAB, an action is also named as an arm.) from an action set repeatedly, and the environment returns a reward as a response to the chosen action. The agent's goal is to maximize the cumulative reward over a period of time. In MAB, a reward distribution is associated with each arm to characterize the uncertainty of the reward. One key issue for MAB and many on-line learning problems [3] is to well-balance the exploitation-exploration tradeoff, that is, the tradeoff between choosing the action that has already yielded greatest rewards and the action that is relatively unexplored.



Term	D1	D2
game	1	0
decision	0	0
theory	2	0
probability	0	3
analysis	0	2

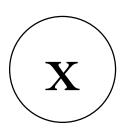
- A simple unigram language model
 - Observations (e.g., bag-of-words)

$$\mathbf{x} = \{x_1, \dots, x_d\}$$

Joint distribution (likelihood)

$$p(\mathbf{x};\theta) = \prod p(x_i;\theta)$$

Graphical representation (parameters ignored)



- Learn a simple generative model
 - Given a set of observations

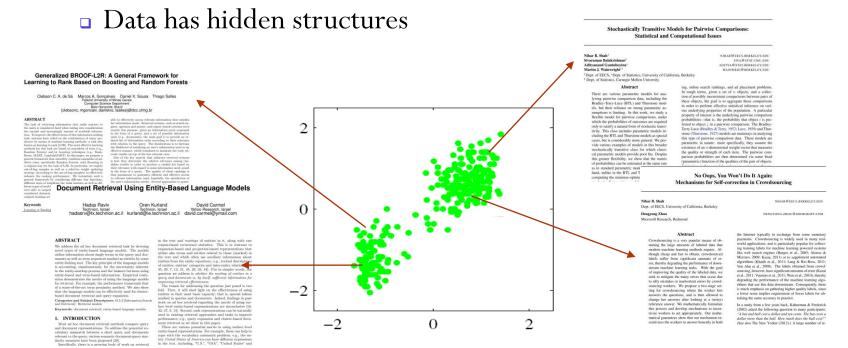
$$X = \{x_1, \dots, x_N\}$$

Maximize the log-likelihood

$$\max_{\theta} \log p(X; \theta) = \sum_{i} \log p(x_i; \theta)$$

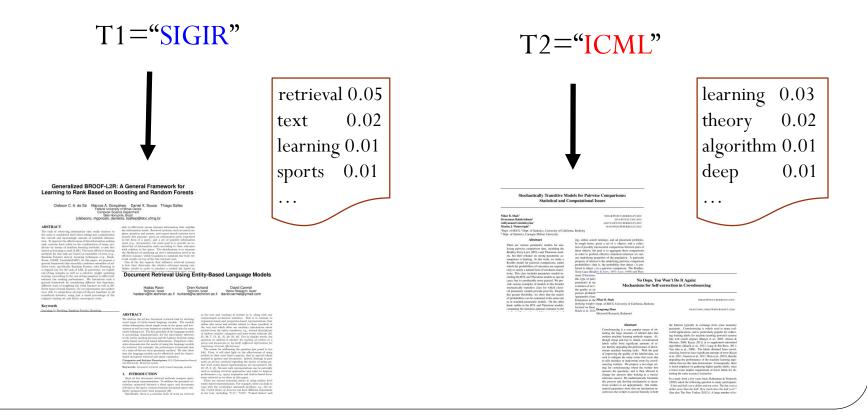
- Simple closed-form solutions:
 - count frequency for discrete or empirical mean/variance for Gaussian distribution

A fully-observed model is not sufficient

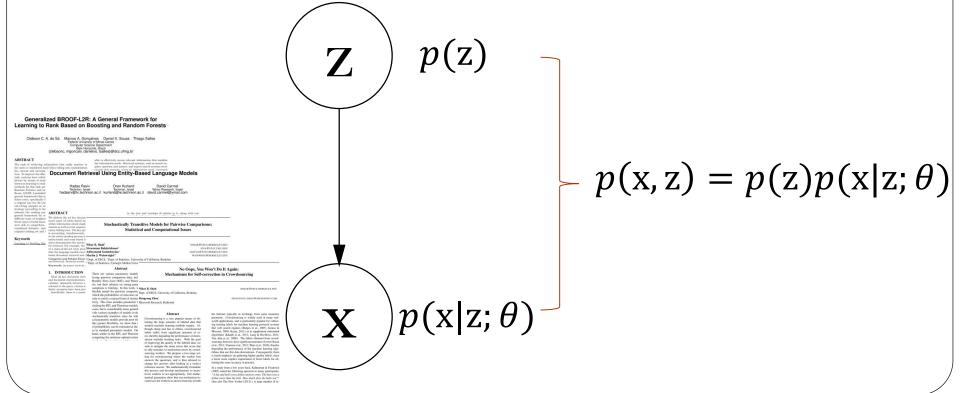


A simple distribution is not sufficient

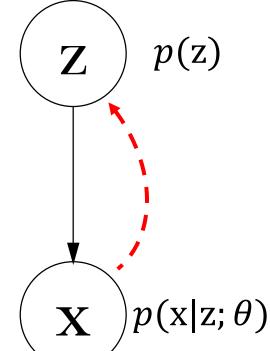
- Mixture model --- a simple generative model with hidden factors
 - Separate the data into different groups



- Mixture model --- a simple generative model with hidden factors
 - Graphical model representation



- Mixture model --- a simple generative model with hidden factors
 - □ Infer the latent Z:



Bayes' Rule:

$$p(z|x) = \frac{p(x, z)}{p(x)}$$

$$\propto p(z)p(x|z; \theta)$$

No Oops, You Won't Do It Again: Mechanisms for Self-correction in Crowdsourcing

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Abstract

Crowdocorting is a very popular means of detaining the large amounts of labeled data than modern machine learning methods require. All the contract of the latest support of the contract labeles suffer from significant amounts of eror, thereby degrading the performance of down stream machine learning tasks. With the goa seek to mitigate the many errors that occur due to sally mistakes or insulventum errors that occur due souring workers. We propose a tro-stage set and support of the contracting where the worker first fig for condocuting where the worker first change her answers after looking at a (noisy) change her answers after looking at a (noisy) efference answer. We undemantically formulas this process and develop mechanism to incomtantial guarantees show that our mechanism to martial guarantees show that our mechanism to martial guarantees show that our mechanism to the latent hypidally in exchange from some mountary parament. Consolutioning is shelf you find in many realworld applications, and is particularly popular for collecing turning latels for machine learning powered systems like web search engines (Blugges et al., 2005; Alonos & Marzan, 2009; Kazu, 2011 to on supplement automated algorithms (Eshalt et al., 2011; Lang & Rob Robert and Algorithms (Eshalt et al., 2011; Lang & Rob Robert and search and algorithms (Eshalt et al., 2011; Marzan & Consolution), and search and algorithms (Eshalt et al., 2011; Music et al., 2010; thereby depending the performance of the muchine learning algorithms that use this data downstraum. Consequently, there a lower noise intellect presuments of the weight their for each a lower noise intellect presuments of the weight their for each a lower noise intellect presuments of the weight their for each

taining the same accuracy in practice.

In a study from a few years back, Kahneman & Frederick (2002) asked the following question to many participants: "A bat and ball cost a dollar and ten cents. The bat costs a dollar more than the ball. How much does the ball cost? (See also The New Yorker (2012).) A large number of re-

- Mixture model --- a simple generative model with hidden factors
 - EM algorithm to learn the unknown language models

E-step: Infer the hidden Z

Generalized BROOF-L2R: A General Framework for Learning to Rank Based on Boosting and Random Forests

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Growth C.A. drift M. Marco

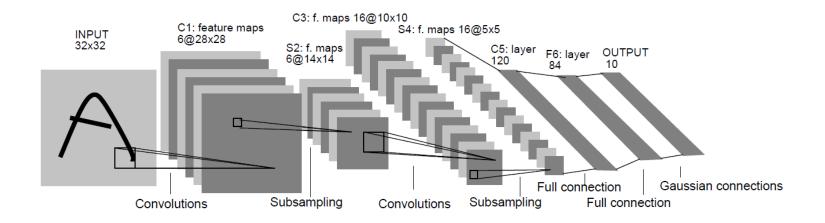
M-step: Update the parameters

retrieval 0.05 text 0.02 learning 0.01 sports 0.01

learning 0.03 theory 0.02 algorithm 0.01 deep 0.01

Discriminative Deep Learning

Learn a deep NN to map an input to output



- Gradient back-propagation
- Dropout
- Activation functions:
 - rectified linear

Generative Modeling

Have training examples

$$x \sim p_{data}(x)$$

• Want a model that can draw samples:

$$x' \sim p_{\text{model}}(x)$$

• where $p_{\text{model}}(x) \approx p_{\text{data}}(x)$

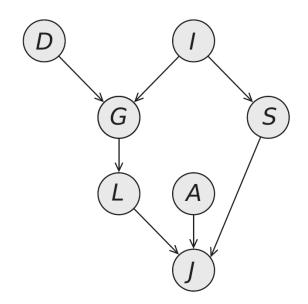
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Two ways to build deep generative models

- Traditional one
 - Hierarchical Bayesian methods
- More modern one
 - Deep generative models

Hierarchical Bayesian Modeling

Build a hierarchy throng distributions in analytical forms



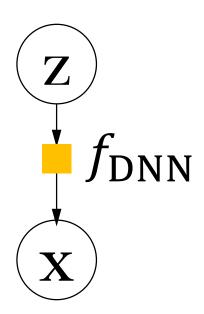
$$P(D, I, G, S, L, A, J) = P(D)P(I)P(G|D, I)P(S|I) \cdot P(A) P(L|G)P(J|L, A, S)$$

Simple, Local Factors: a conditional probability distribution

Deep Generative Models

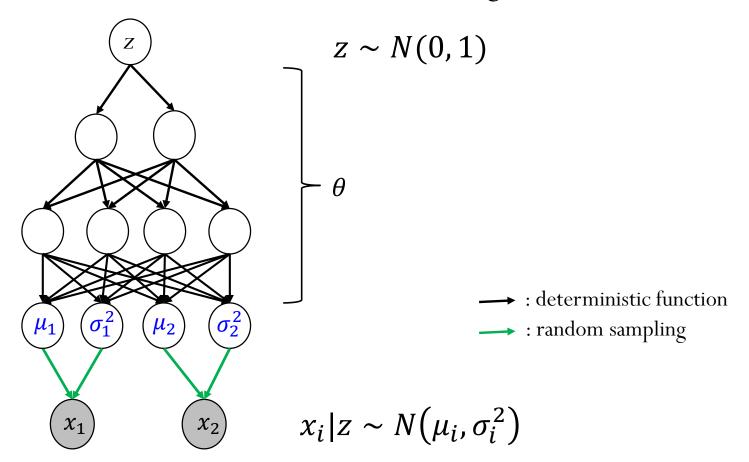
More flexible by using differential function mapping between random variables

DGMs learn a function transform with deep neural networks



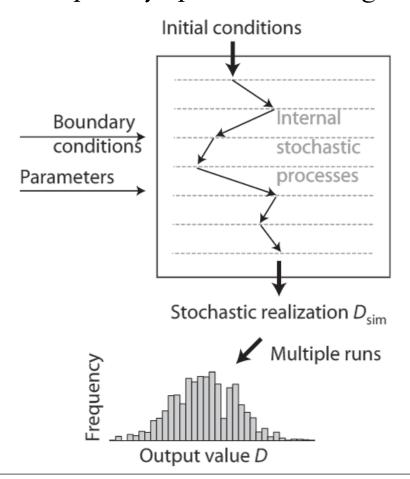
An example with MLP

- ♦ 1D latent variable z; 2D observation x
- ♦ Idea: NN + Gaussian (or Bernoulli) with a diagonal covariance



Implicit Deep Generative Models

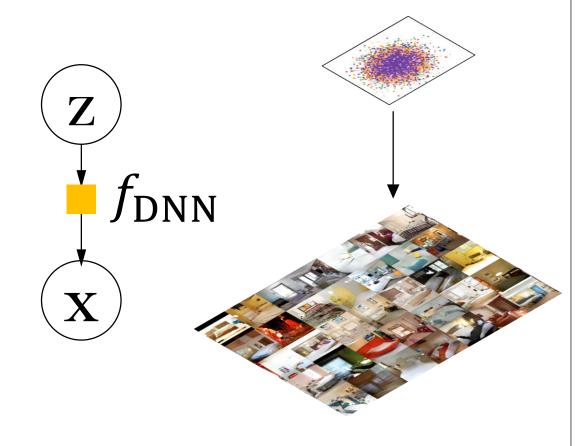
• Generate data with a stochastic process whose likelihood function is not explicitly specified (Hartig et al., 2011)



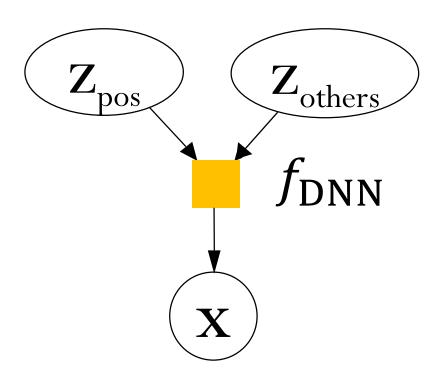
Deep Generative Models

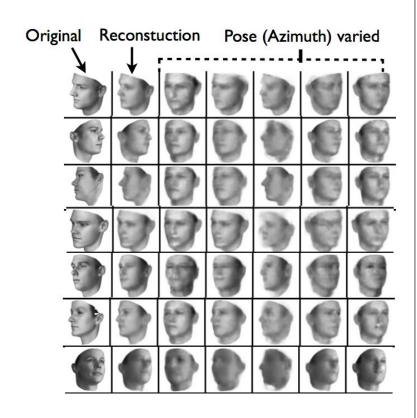
[Image Generation: Generative Adversarial Nets, Goodfellow13 & Radford15]





Deep Generative Models

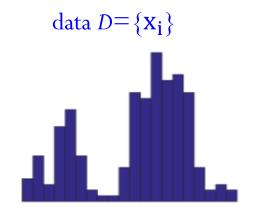


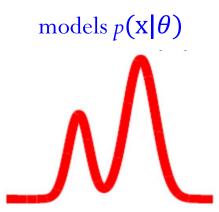


[Image Understanding: Variational Autoencoders, Kingma13 & Tejas15 & Eslami16]

Learning Deep Generative Models

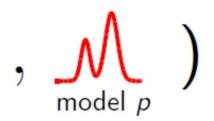
◆ Given a set D of unlabeled samples, learn the unknown parameters (or a distribution)





Find a model that minimizes





Learning Deep Generative Models

Maximum likelihood estimation (MLE):

$$\hat{\theta} = \operatorname{argmax} p(D|\theta)$$

- has an explicit likelihood model
- Minimax objective (e.g., GAN)
 - □ A two-player game to reach equilibrium
 - □ A three-player game for semi-supervised learning (NIPS'17)
- Moment-matching:
 - □ draw samples from p: $\widehat{D} = \{y_i\}_{i=1}^M$, where $y_i \sim p(x|\theta)$
 - □ Kernel MMD (NIPS'16):
 - rich enough to distinguish any two distributions in certain RKHS
 - □ PMD (NIPS'17)

Variational Bayes

Consider the log-likelihood of a single example

$$\log p(x; \theta) = \log \int p(z, x; \theta) dz$$

- Log-integral/sum is annoying to handle directly
- \bullet Derive a variational lower bound $L(\theta, \phi, \mathbf{x})$

$$\log p(\mathbf{x}; \theta) = L(\theta, \phi, \mathbf{x}) + \mathrm{KL}(q(\mathbf{z}|\mathbf{x}; \phi) || p(\mathbf{z}|\mathbf{x}; \theta))$$

$$L(\theta, \phi, \mathbf{x}) = \mathbf{E}_{q(\mathbf{z}|\mathbf{x};\phi)}[\log p(\mathbf{z}, \mathbf{x}; \theta) - \log q(\mathbf{z}|\mathbf{x}; \phi)]$$

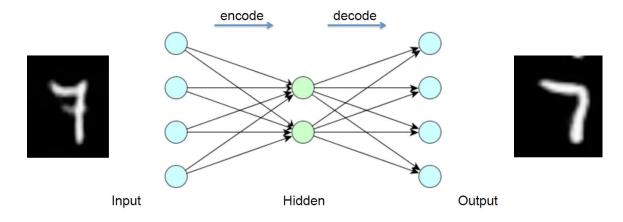
$$= \mathbf{E}_{q(\mathbf{z}|\mathbf{x};\phi)}[\log p(\mathbf{x}|\mathbf{z}; \theta) + \log p(\mathbf{z}; \theta) - \log q(\mathbf{z}|\mathbf{x}; \phi)]$$

$$= \mathbf{E}_{q(\mathbf{z}|\mathbf{x};\phi)}[\log p(\mathbf{x}|\mathbf{z}; \theta)] - \mathrm{KL}(q(\mathbf{z}|\mathbf{x}; \phi)||p(\mathbf{z}; \theta))$$

reconstruction term

prior regularization

Recap: Auto-Encoder

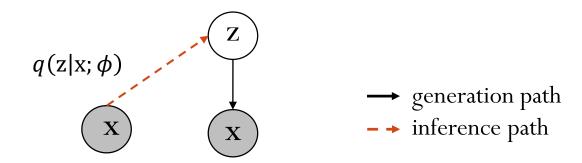


- \bullet Encoder: h = s(Wx + b)
- ♦ Decoder: x' = s(W'h + b')
- Training: minimize the reconstruction error (e.g., square loss, cross-entropy loss)
- Denoising AE: randomly corrupted inputs are restored to learn more robust features

Auto-Encoding Variational Bayes (AEVB)

What's unique in AEVB is that the variational distribution is parameterized by a deep neural network

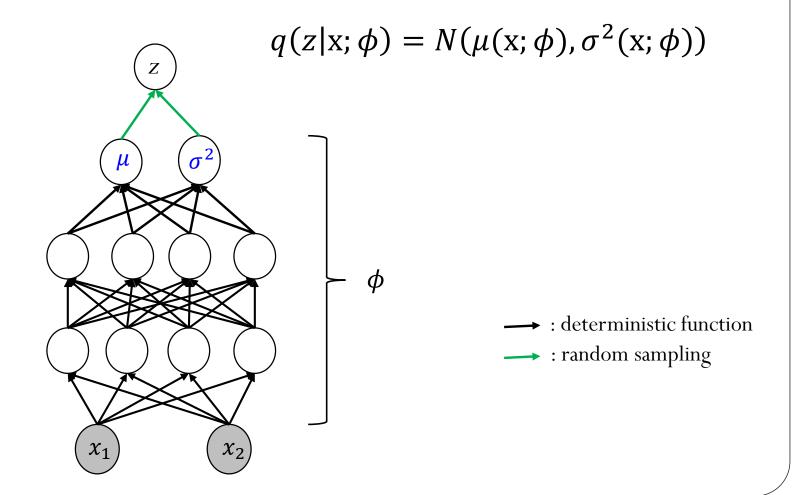
$$q(\mathbf{z}|\mathbf{x};\phi) \approx p(\mathbf{z}|\mathbf{x};\theta)$$



- □ We call it an inference (recognition, encoder) network or a Q-network
- All the parameters are learned jointly via SGD with variance reduction

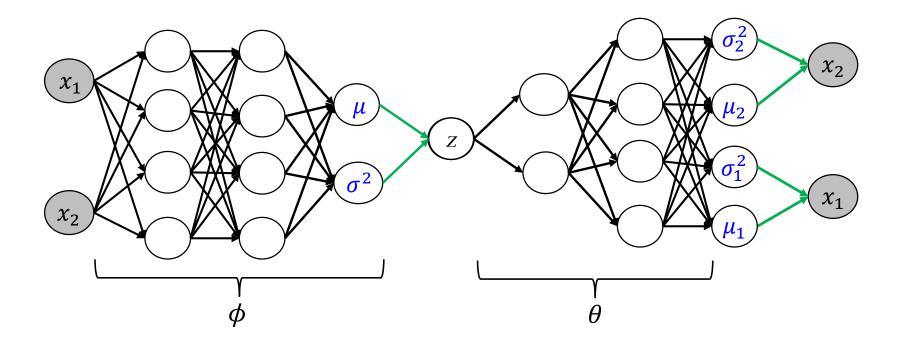
The Encoder Network

♠ A feedforward NN + Gaussian



The Complete Auto-encoder

• The Q-P network architecture:



→ : deterministic function

→ : random sampling

Stochastic Variational Inference

Variational lower-bound for a single example

$$L(\theta, \phi, \mathbf{x}) = \mathbf{E}_{q(\mathbf{z}|\mathbf{x}; \phi)}[\log p(\mathbf{x}|\mathbf{z}; \theta)] - \mathrm{KL}(q(\mathbf{z}|\mathbf{x}; \phi) || p(\mathbf{z}; \theta))$$

Variational lower-bound for a set of examples

$$L(\theta, \phi, D) = \sum_{i} \mathbf{E}_{q(\mathbf{z}_{i}|\mathbf{x}_{i};\phi)} [\log p(\mathbf{x}_{i}|\mathbf{z}_{i};\theta)] - \mathrm{KL}(q(\mathbf{z}_{i}|\mathbf{x}_{i};\phi)||p(\mathbf{z}_{i};\theta))$$

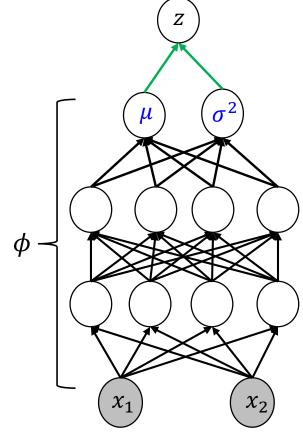
- Use stochastic gradient methods to handle large datasets
- Random mini-batch
 - for each i, infer the posterior $q(\mathbf{z}_i|\mathbf{x}_i;\boldsymbol{\phi})$; As we parameterize as a neural network, this in fact optimizes $\boldsymbol{\phi}$
- However, calculating the expectation and its gradients is non-trivial, often intractable

• Use N(0,1) as prior for z; $q(z|x;\phi)$ is Gaussian with parameters $(\mu(x;\phi),\sigma^2(x;\phi))$ determined by NN

■ The KL-divergence

$$-\text{KL}(q(z|x;\phi)||p(z;\theta)) = \frac{1}{2}(1 + \log \sigma^2 - \mu^2 - \sigma^2)$$

Homework: finish the derivation



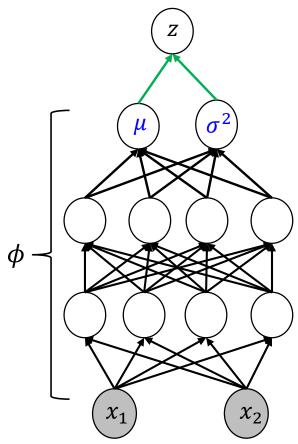
- Use N(0,1) as prior for z; $q(z|x;\phi)$ is Gaussian with parameters $(\mu(x;\phi),\sigma^2(x;\phi))$ determined by NN
 - The expected log-likelihood

$$\mathbf{E}_{q(\mathbf{z}|\mathbf{x};\boldsymbol{\phi})}[\log p(\mathbf{x}|\mathbf{z};\boldsymbol{\theta})]$$

If the likelihood is Gaussian

$$-\log p(x_i|z_i) = \sum_{j} \frac{1}{2} \log \sigma_j^2 + \frac{(x_{ij} - \mu_{xi})^2}{2\sigma_j^2}$$

■ The expectation is still hard to compute because of nonlinearity functions



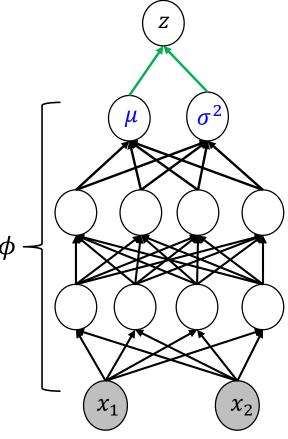
- Use N(0,1) as prior for z; $q(z|x;\phi)$ is Gaussian with parameters $(\mu(x;\phi),\sigma^2(x;\phi))$ determined by NN
 - The expected log-likelihood

$$\mathbf{E}_{q(\mathbf{z}|\mathbf{x};\boldsymbol{\phi})}[\log p(\mathbf{x}|\mathbf{z};\boldsymbol{\theta})]$$

Approximate via Monte Carlo methods

$$\mathbf{E}_{q(\mathbf{z}|\mathbf{x};\boldsymbol{\phi})}[\log p(\mathbf{x}|\mathbf{z};\boldsymbol{\theta})] \approx \frac{1}{L} \sum_{k} \log p(\mathbf{x}|\mathbf{z}^{(k)})$$
$$z^{(k)} \sim q(z|\mathbf{x};\boldsymbol{\phi})$$

An unbiased estimator



♦ The KL-regularization term (closed-form):

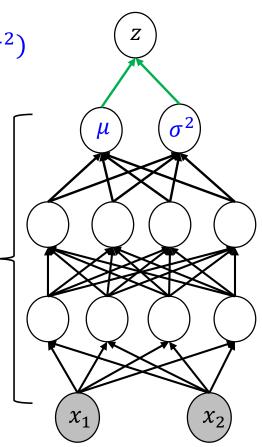
$$-\mathrm{KL}(q(z|\mathbf{x};\boldsymbol{\phi})\big\|p(z;\boldsymbol{\theta})\big) = \frac{1}{2}(1 + \log\sigma^2 - \mu^2 - \sigma^2)$$

- Easy to calculate gradient
- The expected log-likelihood term (MC estimate)

$$\mathbf{E}_{q(\mathbf{z}|\mathbf{x};\phi)}[\log p(\mathbf{x}|\mathbf{z};\theta)] \approx \frac{1}{L} \sum_{k} \log p(\mathbf{x}|\mathbf{z}^{(k)}) \phi$$

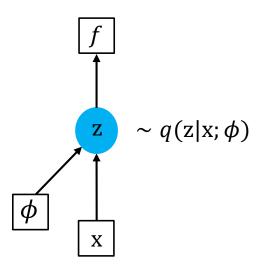
$$z^{(k)} \sim q(z|\mathbf{x}; \boldsymbol{\phi})$$

- Gradient needs back-propagation!
- However, $\mathbf{Z}^{(k)}$ is a random variable, we can't take gradient over a randomly drawn number



Reparameterization Trick

Backpropagation not possible through random sampling



: deterministic node

$$z^{(k)} \sim N(\mu(\mathbf{x}, \phi), \sigma^2(\mathbf{x}, \phi))$$

 $q(\mathbf{z}|\mathbf{x}; \boldsymbol{\phi})$

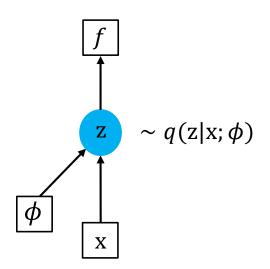
$$\{-1.5, -0.5, 0.3, 0.6, 1.5, \dots\}$$

Cannot back-propagate through a randomly drawn number

Reparameterization Trick

: random node

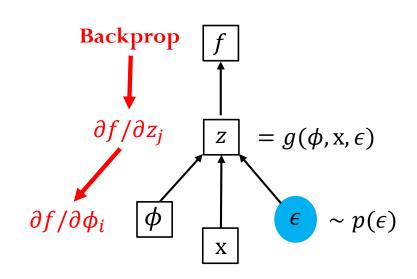
Backpropagation not possible through random sampling



: deterministic node

$$z^{(k)} \sim N(\mu(x, \phi), \sigma^2(x, \phi))$$

Cannot back-propagate through a randomly drawn number



$$\epsilon^{(k)} \sim N(0,1)$$

$$z^{(k)} = \mu(x,\phi) + \sigma(x,\phi) \cdot \epsilon^{(k)}$$

Z has the same distribution, but now can back-prop Separate into a deterministic part and noise

The General Form

The VAE bound

$$L(\theta, \phi, \mathbf{x}) = \mathbf{E}_{q(\mathbf{z}|\mathbf{x}; \phi)} [\log p(\mathbf{z}, \mathbf{x}; \theta) - \log q(\mathbf{z}|\mathbf{x}; \phi)]$$
$$= \mathbf{E}_{q(\mathbf{z}|\mathbf{x}; \phi)} \left[\log \frac{p(\mathbf{z}, \mathbf{x}; \theta)}{q(\mathbf{z}|\mathbf{x}; \phi)} \right]$$

Monte Carlo estimate:

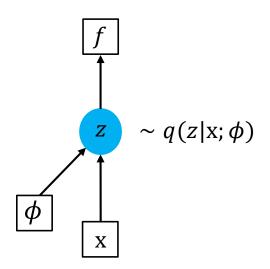
$$L(\theta, \phi, \mathbf{x}) \approx \frac{1}{L} \sum_{k} \log \frac{p(\mathbf{z}^{(k)}, \mathbf{x}; \theta)}{q(\mathbf{z}^{(k)} | \mathbf{x}; \phi)}$$
$$z^{(k)} \sim q(z | \mathbf{x}; \phi)$$

 Again, we cannot back-prop through the randomly drawn numbers

Reparameterization Trick

: random node

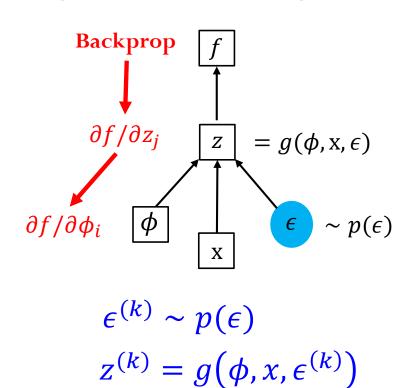
Backpropagation not possible through random sampling



: deterministic node

$$z^{(k)} \sim q(z|\mathbf{x}; \phi)$$

Cannot back-propagate through a randomly drawn number



Z has the same distribution, but now can back-prop Separate into a deterministic part and noise

Reparam-Trick Summary

The VAE bound

$$L(\theta, \phi, \mathbf{x}) = \mathbf{E}_{q(\mathbf{z}|\mathbf{x}; \phi)} \left| \log \frac{p(\mathbf{z}, \mathbf{x}; \theta)}{q(\mathbf{z}|\mathbf{x}; \phi)} \right|$$

Reparameterized as

$$L(\theta, \phi, \mathbf{x}) = \mathbf{E}_{p(\epsilon)} \left[\log \frac{p(g(\mathbf{x}, \epsilon, \phi), \mathbf{x}; \theta)}{q(g(\mathbf{x}, \epsilon, \phi) | \mathbf{x}; \phi)} \right]$$

- ullet where $oldsymbol{\epsilon}$ is a simple distribution (e.g., standard normal) and $oldsymbol{g}$ is a deep NN
- The gradients are

$$\nabla_{\theta} L(\theta, \phi, \mathbf{x}) = \mathbf{E}_{p(\epsilon)} \left[\nabla_{\theta} \log \frac{p(g(\mathbf{x}, \epsilon, \phi), \mathbf{x}; \theta)}{q(g(\mathbf{x}, \epsilon, \phi) | \mathbf{x}; \phi)} \right]$$

- Back-prop is applied over the deep NN
- lacktriangle Similar for $oldsymbol{\phi}$

Importance Weighted Auto-Encoder (IWAE)

♦ The VAE lower bound of log-likelihood

$$L(\theta, \phi, \mathbf{x}) = \mathbf{E}_{q(\mathbf{z}|\mathbf{x}; \phi)} \left| \log \frac{p(\mathbf{z}, \mathbf{x}; \theta)}{q(\mathbf{z}|\mathbf{x}; \phi)} \right|$$

A better variational lower bound (IWAE)

$$L_K(\theta, \phi, \mathbf{x}) = \mathbf{E}_{q(\mathbf{z}|\mathbf{x}; \phi)} \left[\log \left(\frac{1}{K} \sum_{k=1:K} \frac{p(\mathbf{z}^{(k)}, \mathbf{x}; \theta)}{q(\mathbf{z}^{(k)}|\mathbf{x}; \phi)} \right) \right]$$

where
$$z^{(k)} \sim q(z|x;\phi)$$

- This is a lower-bound of the log-likelihood
- When K=1, recovers the VAE bound
- When $K = \infty$, recovers the log-likelihood
- A monotonic sequence:

$$L_K(\theta, \phi, \mathbf{x}) \le L_{K+1}(\theta, \phi, \mathbf{x}), \quad \forall \theta, \phi, \mathbf{x}$$

[Burda et al., arXiv, 2015]

Reparametrization Trick

The IWAE bound:

$$L_K(\theta, \phi, \mathbf{x}) = \mathbf{E}_{q(\mathbf{z}|\mathbf{x};\phi)} \left[\log \left(\frac{1}{K} \sum_{k=1:K} w(\mathbf{z}^{(k)}, \mathbf{x}; \theta) \right) \right]$$

where
$$z^{(k)} \sim q(z|\mathbf{x}; \phi)$$
 $w(\mathbf{z}^{(k)}, \mathbf{x}; \theta, \phi) = \frac{p(\mathbf{z}^{(k)}, \mathbf{x}; \theta)}{q(\mathbf{z}^{(k)}|\mathbf{x}; \phi)}$

Reparameterization form:

$$L_K(\theta, \phi, \mathbf{x}) = \mathbf{E}_{p(\epsilon)} \left[\log \left(\frac{1}{K} \sum_{k=1:K} w(g(\epsilon^{(k)}, \mathbf{x}, \phi), \mathbf{x}; \theta) \right) \right]$$
where $\epsilon^{(k)} \sim p(\epsilon)$

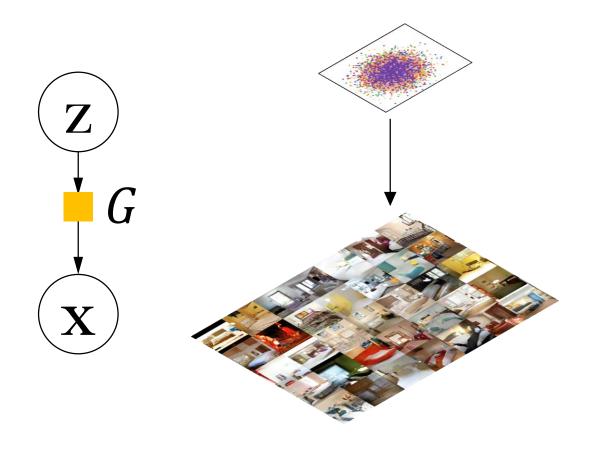
The gradient can be calculated as in VAE

Generative Adversarial Networks (GAN)

- A game between two players:
 - A discriminator D
 - □ A generator *G*
- D tries to discriminate between:
 - A sample from the data distribution.
 - □ And a sample from the generator *G*.
- *G* tries to "trick" *D* by generating samples that are hard for *D* to distinguish from data.

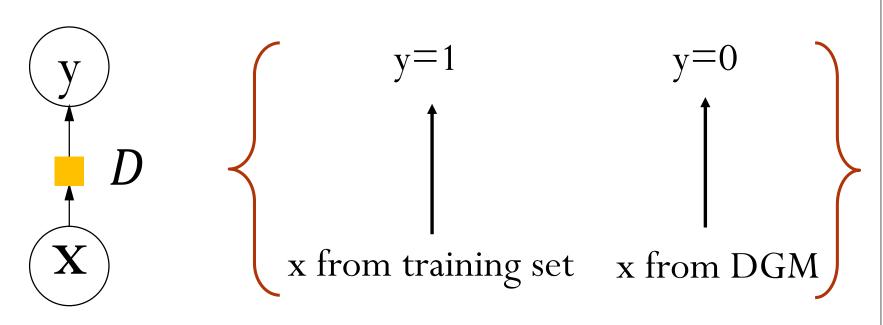
GAN – architecture

- ♦ The generator-network *G* is a DGM
 - It generates samples from random noise



GAN – architecture

- lacktriangle The discriminator-network D is a binary classifier
 - It aims to assign the correct label to both training samples and the samples from G



This is a supervised learning task (binary classification)!

GAN - objective

- lacktriangle The discriminator-network D is a binary classifier
 - It aims to assign the correct label to both training samples and the samples from G
- Maximum likelihood estimation (MLE) is the natural choice!

$$\max_{D} \mathbf{E}_{p_{\text{data}}(\mathbf{x})}[\log D(\mathbf{x})] + \mathbf{E}_{p(\mathbf{z})}[\log (1 - D(G(\mathbf{z})))]$$

$$\mathbf{x} \text{ from training set} \qquad G(\mathbf{z}) \text{ from generator}$$

$$D(\mathbf{x}) = p(\mathbf{y} = 1 \mid \mathbf{x})$$

aka. cross-entropy loss minimization

GAN - objective

- lacktriangle The generator aims to fool the discriminator D
 - Generated samples should be identified as "real" by D
 - Maximize the likelihood of being real:

$$\max_{G} \mathbf{E}_{p(\mathbf{z})} \left[\log \left(D(G(\mathbf{z})) \right) \right]$$

G(z) is a sample from generator

Or minimize the likelihood of being fake:

$$\min_{G} \mathbf{E}_{p(\mathbf{z})} \left[\log \left(1 - D(G(\mathbf{z})) \right) \right]$$

GAN – objective

Minimax objective function

$$\min_{G} \max_{D} \mathbf{E}_{p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbf{E}_{p(\mathbf{z})} \left[\log \left(1 - D(G(\mathbf{z})) \right) \right]$$

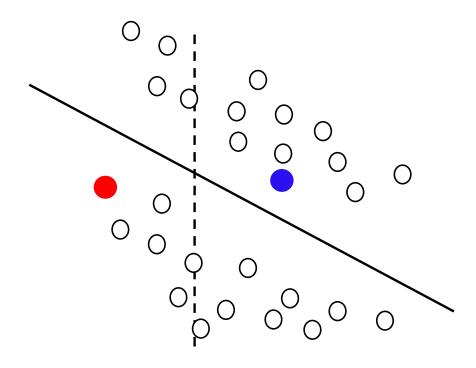
• Optimal strategy of the discriminator for any $p_{\text{model}}(x)$ is

$$D(x) = \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_{\text{model}}(x)}$$

- Assume infinite data, infinite model capacity, direct updating generator's distribution
 - Unique global optimum
 - Optimum corresponds to data distribution

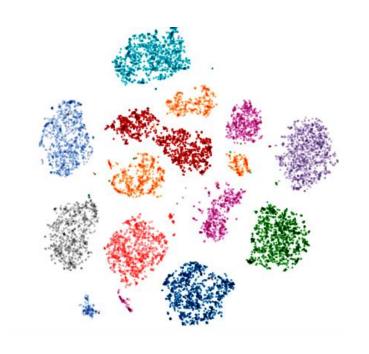
Semi-supervised Learning

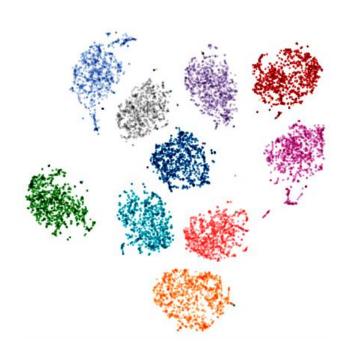
A toy example



Representation Matters

◆ t-SNE embedding of learned representations by different DGM models on CIFAR10





Triple Generative Adversarial Nets

- A minimax game for semi-supervised learning
 - GAN is for unsupervised learning
 - □ We aim to learn the joint distribution $p_{\text{model}}(x, y) = p_{\text{data}}(x, y)$
- We need three players
 - factorization form with conditionals

$$p(x,y) = p(x)p(y|x)$$

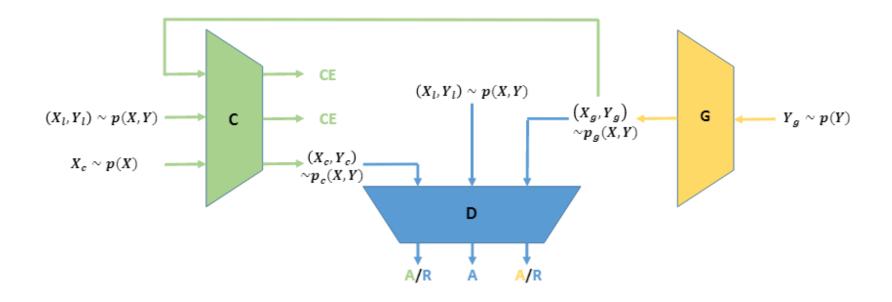
$$= p(y)p(x|y)$$
A classifier
$$= p(y)p(x|y)$$
A class-conditional generator

 $p_{\text{model}}(\mathbf{x}) = p_{\text{data}}(\mathbf{x})$

- \Box Two generators to generate (x,y)
- $lue{}$ A discriminator to distinguish fake (x,y)

Triple-GAN

♦ The network architecture



- Both C and G are generators
- D is the discriminator
- CE: cross-entropy loss for learning classifier

[Li et al., NIPS 2017]

A minimax game

The optimization problem

$$\min_{C,G} \max_{D} U(C,G,D) = E_{p}[\log D(x,y)] + \alpha E_{p_{c}}[\log(1-D(x,y))] + (1-\alpha)E_{p_{g}}[\log(1-D(x,y))]$$

- \Box The hyper-parameter *a* is often set at 1/2
- The standard supervised loss can be incorporated

$$\min_{C,G} \max_{D} \tilde{U}(C,G,D) = U(C,G,D) + E_{\rho}[-\log p_{c}(y|x)]$$

Major theoretical results

Theorem

The equilibrium of $\tilde{U}(C,G,D)$ is achieved if and only if $p(x,y)=p_g(x,y)=p_c(x,y)$ with $D_{C,G}^*(x,y)=\frac{1}{2}$ and the optimum value is $-\log 4$.

Lemma

For any fixed C and G, the optimal discriminator D is:

$$D_{C,G}^*(x,y) = \frac{p(x,y)}{p(x,y) + p_{\alpha}(x,y)},$$

where $p_{\alpha}(x, y) := (1 - \alpha)p_{g}(x, y) + \alpha p_{c}(x, y)$.

Some Practical Tricks for SSL

- Pseudo discriminative loss: using $(x, y) \sim p_g(x, y)$ as labeled data to train C
 - Explicit loss, equivalent to $KL(p_g(x,y)||p_c(x,y))$
 - Complementary to the implicit regularization by D
- Collapsing to the empirical distribution p(x, y)
 - Sample $(x, y) \sim p_c(x, y)$ as true data for D
 - Biased solution: target shifting towards $p_c(x, y)$
- Unlabeled data loss on C
 - Confidence (Springenberg [2015])
 - Consistence (Laine and Aila [2016])

Some Results

Semi-supervised classification

Table 1: Error rates (%) on partially labeled MNIST, SHVN and CIFAR10 datasets. The results with † are trained with more than 500,000 extra unlabeled data on SVHN.

Algorithm	MNIST $n = 100$	SVHN $n = 1000$	CIFAR10 $n = 4000$
M1+M2 [11]	$3.33 (\pm 0.14)$	$36.02 (\pm 0.10)$	
<i>VAT</i> [18]	2.33		24.63
Ladder [23]	$1.06 (\pm 0.37)$		$20.40 (\pm 0.47)$
Conv-Ladder [23]	$0.89 \ (\pm 0.50)$		
ADGM [17]	$0.96 (\pm 0.02)$	22.86 [†]	
SDGM [17]	$1.32 (\pm 0.07)$	$16.61(\pm 0.24)^{\dagger}$	
MMCVA [15]	$1.24~(\pm 0.54)$	4.95 (±0.18) [†]	
CatGAN [26]	$1.39 (\pm 0.28)$		19.58 (±0.58)
Improved-GAN [25]	$0.93 (\pm 0.07)$	$8.11 (\pm 1.3)$	$18.63 \ (\pm 2.32)$
<i>ALI</i> [5]		7.3	18.3
Triple-GAN (ours)	0.91 (±0.58)	$5.77(\pm 0.17)$	16.99 (±0.36)

Some Results

Class-conditional generation



Disentangle class and style

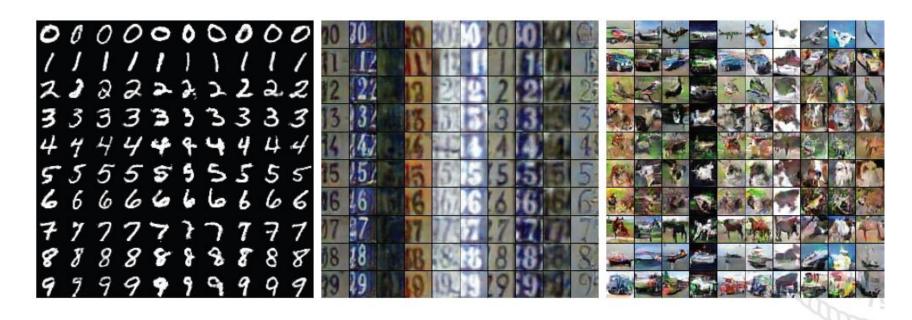


Figure: Same y for each row. Same z for each column.

Latent space interpolation on MNIST

Latent space interpolation on SVHN



Latent space interpolation on CIFAR10

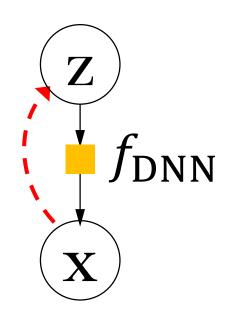


ZhuSuan

http://zhusuan.readthedocs.io

Bayesian inference

$$p(\mathbf{z}|\mathbf{x}) = \frac{p(\mathbf{x}, \mathbf{z})}{p(\mathbf{x})}$$



Given Disease, what is the Cause?
Given Object, what are the Components?
Given Docs, what are the Topics?

Find Cause of Disease
Extract Topics from Docs
Identify Objects from Images
Recognize Words in Speeches

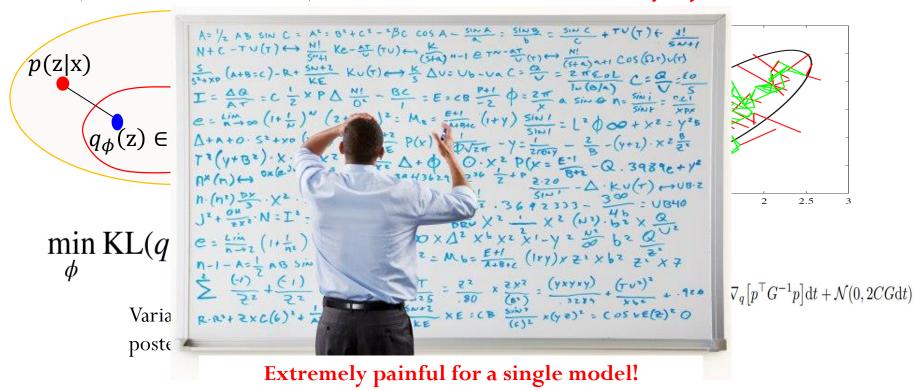
Inference in Old Days

Variational Inference

MCMC

(Too much math!!!)

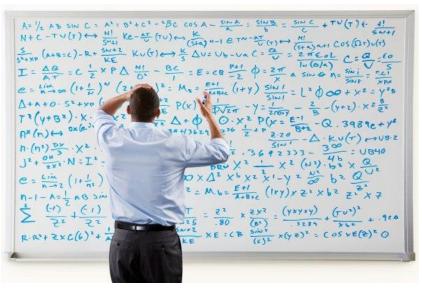
(Many dynamics!!!)





Inference in ZhuSuan

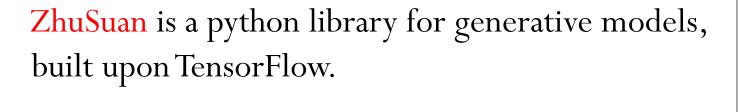
Turn painful math deviations into Easy and Intuitive (Probabilistic) Programming

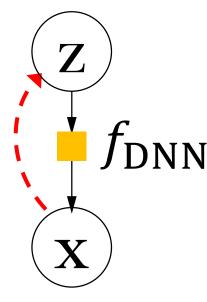






ZhuSuan



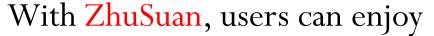


Unlike existing DL libraries, which are mainly for supervised tasks, ZhuSuan is featured for:

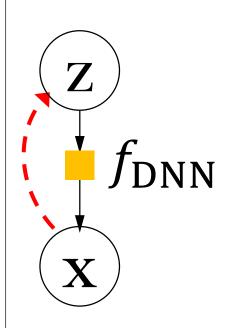
- its deep root into Bayesian Inference
- supporting various kinds of generative models: traditional hierarchical Bayesian models & recent deep generative models.



ZhuSuan



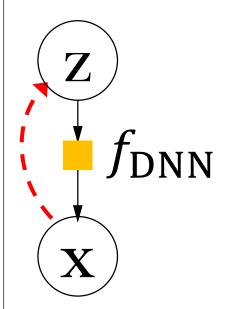
- powerful fitting and multi-GPU training of deep learning
- while at the same time they can use generative models to
 - ✓ model the complex world
 - exploit unlabeled data
 - ✓ deal with uncertainty by performing principled Bayesian inference
 - ✓ generate new samples





Model Primitives: BayesianNet

- A DAG representing a Bayesian Network
- Two types of nodes:
 - Deterministic nodes: Can be composed of any Tensorflow operations.
 - Stochastic nodes: Use Stochastic Tensor's from ZhuSuan's library.
- Start a BayesianNet environment

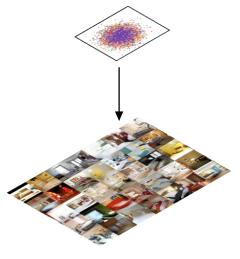


```
import zhusuan as zs
with zs.BayesianNet() as model:
    # build the model
```



Example: Variational Autoencoders

ZHUSUAN



```
Z
f_{DNN}: 2L MLP
```

```
egin{aligned} z &\sim 	ext{N}(z|0,I) \ x_{logits} &= f_{NN}(z) \ x &\sim 	ext{Bernoulli}(x|	ext{sigmoid}(x_{logits})) \end{aligned}
```

import tensorflow as tf
from tensorflow.contrib import layers
import zhusuan as zs

```
with zs.BayesianNet() as model:
    z_mean = tf.zeros([n, n_z])
    z_logstd = tf.zeros([n, n_z])
    z = zs.Normal('z', z_mean, z_logstd)
    h = layers.fully_connected(z, 500)
    x_logits = layers.fully_connected(
        h, n_x, activation_fn=None)
    x = zs.Bernoulli('x', x_logits)
```



Variational Inference in ZhuSuan

```
with zs.BayesianNet() as variational:
    # build variational ...
qz_samples, log_qz = variational.query(
    'z', outputs=True, local_log_prob=True)
```

Build variational posterior as BayesianNet

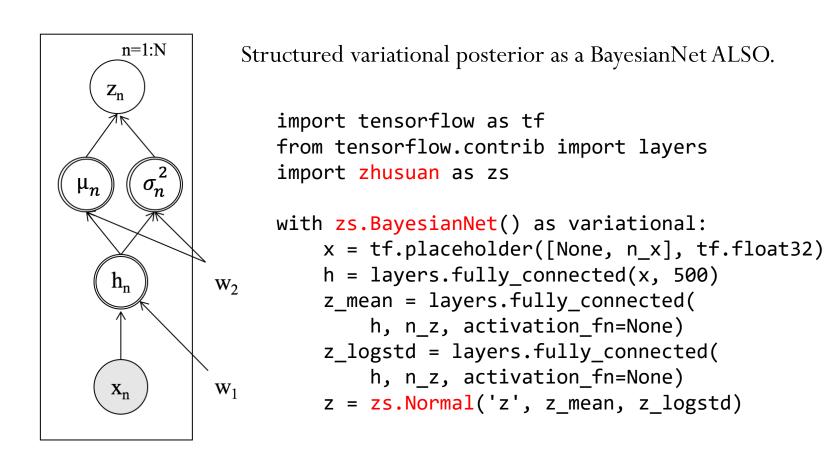
Call variational objectives

```
optimizer = tf.train.AdamOptimizer(learning_rate=0.001)
run_op = optimizer.minimize(-lower_bound)
With tf.Session() as sess:
    for iter in range(iters):
        sess.run(run_op)
```

Run gradient descent!



Example: Variational Autoencoders ZHUSUAN





Variational Inference Algorithms

ZhuSuan supports a broad class of variational objectives, ranging from widely used evidence lower bounds to recent state-of-arts.

Works for continuous latent variables

- ZS.SgVb: Stochastic gradient variational Bayes.
- ZS.iwae: Importance weighted lower bounds.

Works for both continuous and discrete latent variables

- **zs.nvil**: Variance reduced score function estimator/REINFORCE.
- **ZS.Vimco**: Variance reduced multi-sample score function estimator.

*This is like optimization algorithms (SGD, momentum, Adam, etc.) in deep learning software. Users need not dive into the technical details of these algorithms because ZhuSuan provides easy-to-use APIs for users to directly try on their generative models.



like an optimizer!

HMC like a TensorFlow optimizer

```
# like creating the variable to optimize over.
                                                     Create the variable to store
z = tf.Variable(0.)
                                                     samples
# like optimizer = tf.train.AdamOptimizer(...)
                                                     Initialize HMC
hmc = zs.HMC(step size=1e-3, n leapfrogs=10)
# like optimize op = optimizer.minimize(...)
                                                     Call sample() method to
sample op, hmc info = hmc.sample(
                                                     return a sample operation
  log joint, observed={'x': x}, latent={'z': z})
with tf.Session() as sess:
    for iter in range(iters):
        # like sess.run(optimize op)
                                                     Run the sample operation
```

= sess.run(sample op)

Applications: ZhuSuan as a Research Platform



ZhuSuan is featured for both Bayesian Statistics and Deep Learning. State-of-the-Art models can be found in ZhuSuan's examples.

- Bayesian Logistic Regression
- Bayesian Neural Nets for Multivariate Regression
- ♦ (Convolutional) Variational Autoencoders (VAE)
- Semi-supervised learning for images with VAEs
- Deep Sigmoid Belief Networks
- Generative Adversarial Networks (GAN)
- Gaussian processes (GPs)
- Topic Models
- More to come ...



ZhuSuan: GitHub Page

📮 thu-ml / zhusuan	Watch 107 ★ Star 1,103				
⟨⟩ Code ① Issues 5 ② Pull requests 4 ② Projects 0					
Branch: master ▼ zhusuan / docs / index.rst					
Jiaxin Change README to markdown for displaying the logo.	ZhuSuan				
1 contributor	ZhuSuan is a python library for Generative Models , built upon Tensorflow. Unlike existing deep learning libraries, v				
nc lines (C2 also) 2 40 KB	mainly designed for supervised tasks, ZhuSuan is featured for its deep root into Bayesian Inference, thus supporting kinds of generative models: both the traditional hierarchical Bayesian models and recent deep generative models.				

Welcome to ZhuSuan

ZhuSuan is a python probabilistic programming library for advantages of Bayesian methods and deep learning. ZhuSu which are mainly designed for deterministic neural network primitives and algorithms for building probabilistic models algorithms include:

- Variational inference with programmable variational po (SGVB, REINFORCE, VIMCO, etc.).
- Importance sampling for learning and evaluating mode, with programmable proposals.
- Hamiltonian Monte Carlo (HMC) with parallel chains, and optional automatic parameter tuning.

earning libraries, which are ce, thus supporting various enerative models.

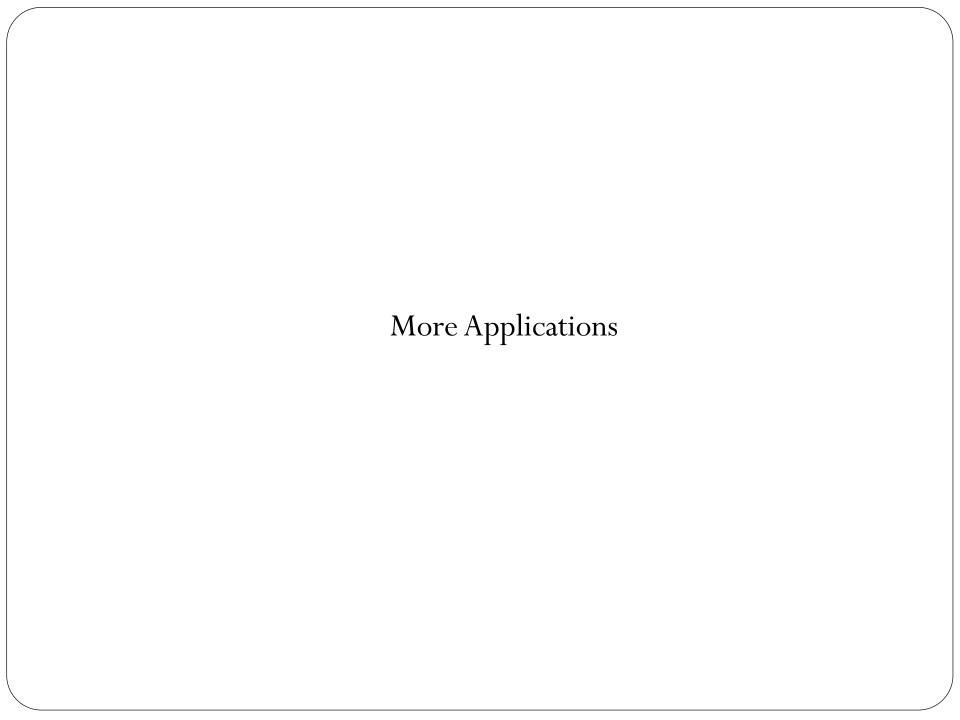
With ZhuSuan, users can enjoy powerful fitting and multi-GPU training of deep learning, while at the same time they can use generative models to model the complex world, exploit unlabeled data and deal with uncertainty by performing principled Bayesian inference.

[™] Supported Inference

(Stochastic) Variational Inference (VI & SVI)

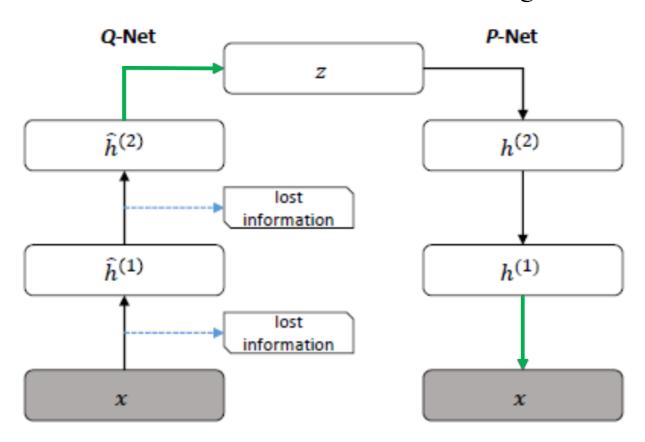
- Kinds of variational posteriors we support:
 - o Mean-field posterior: Fully-factorized.
 - o Structured posterior: With user specified dependencies.
- Variational objectives we support:
 - o SGVB: Stochastic gradient variational Bayes
 - IWAE: Importance weighted objectives
 - o NVIL: Score function estimator with variance reduction
 - o VIMCO: Multi-sample score function estimator with variance



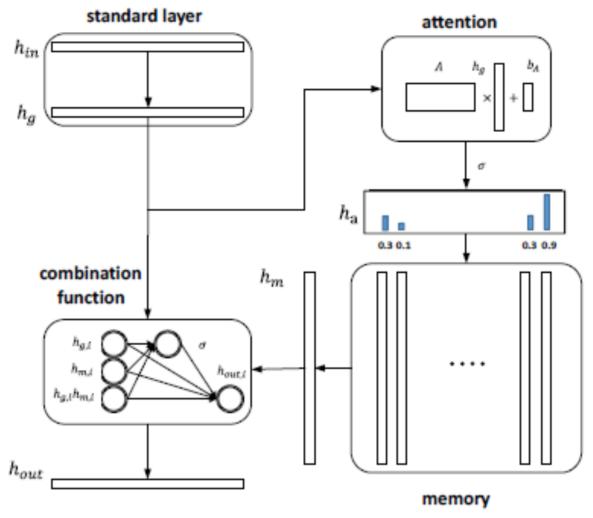


Symmetric Q-P Network

Problem: detail information is lost during abstraction



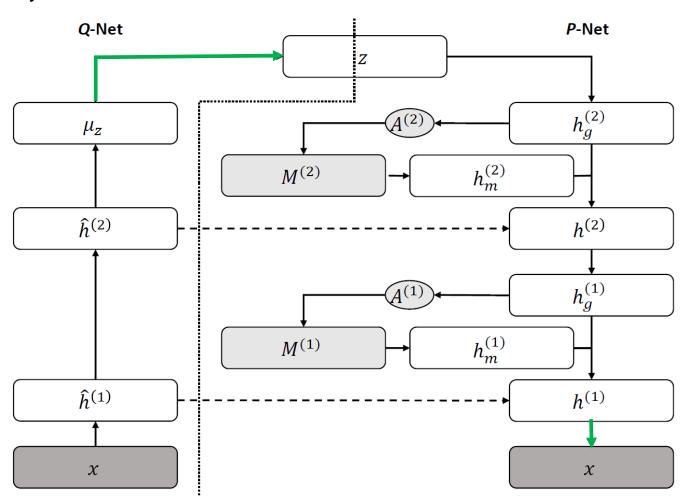
A Layer with Memory and Attention



[Li, et al., ICML 2016]

A Stacked Deep Model with Memory

Asymmetric architecture

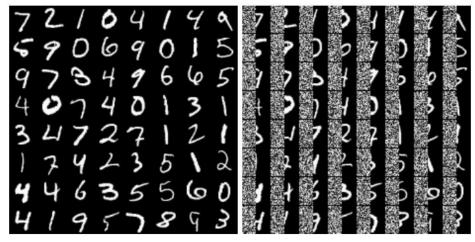


Density estimation

Models	MNIST	OCR-LETTERS		
VAE	-85.69	-30.09		
MEM-VAE(ours)	-84.41	-29.09		
IWAE-5	-84.43	-28.69		
MEM-IWAE-5(ours)	-83.26	-27.65		
IWAE-50	-83.58	-27.60		
MEM-IWAE-50(ours)	-82.84	-26.90		

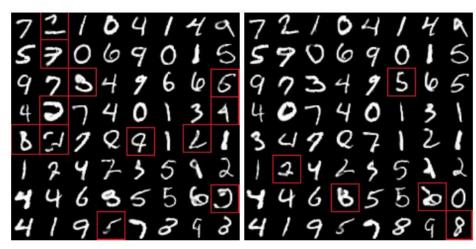
- Better than symmetric VAE networks
- Comparable with state-of-the-art with much fewer parameters

Missing Value Imputation



(a) Data

(b) Noisy data



(c) Results of VAE

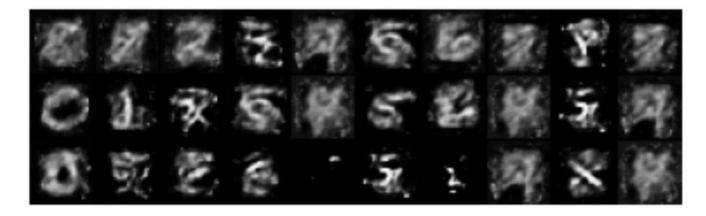
(d) Results of MEM-VAE

Learnt Memory Slots

Average preference over classes of the first 3 slots:

"0"	"1"	"2"	"3"	"4"	"5"	"6"	"7"	"8"	"9"
0.27	0.82	0.33	0.11	0.34	0.15	0.49	0.27	0.09	0.28
0.24	0.09	0.06	0.11	0.30	0.13	0.12	0.27	0.09	0.21
0.18	0.05	0.06	0.11	0.07	0.07	0.05	0.11	0.09	0.18

Corresponding images:



References

- Ningma, D. P. and Welling, M. Auto-encoding variational Bayes. In ICLR, 2013.
- Rezende, D. J., Mohamed, S., and Wierstra, D. Stochastic backpropagation and approximate inference in deep generative models. In ICML, 2014.
- Burda, Y., Grosse, R., and Salakhutdinov, R. Importance weighted autoencoders. In arXiv:1509.00519, 2015.
- Soodfellow, I. J., Abadie, J. P., Mirza, M., Xu, B., Farley, D. W., S.ozair, Courville, A., and Bengio, Y. Generative adversarial nets. In NIPS, 2014
- Bengio, Y., Thlhodeau-Laufer, E., Alain, G., and Yosinski, J. Deep generative stochastic networks trainable by backprop. In ICML, 2014.
- Li, C., Zhu, J., and Zhang, B. Learning to generate with memory. In ICML, 2016

Thanks!



ZhuSuan: A Library for Bayesian Deep Learning. J. Shi, J. Chen, J. Zhu, S. Sun, Y. Luo, Y. Gu, Y. Zhou. arXiv preprint, arXiv:1709.05870, 2017

Online Documents: http://zhusuan.readthedocs.io/