

Quadratic Formula

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$$\begin{aligned}ax^2 + bx + c &= 0 \\x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \\x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2 &= 0 \\ \left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2 &= 0 \\ \left(x + \frac{b}{2a}\right)^2 &= -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \left(x + \frac{b}{2a}\right)^2 \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{-c}{a} + \frac{b^2}{4a^2} \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{-4ac}{4a^2} + \frac{b^2}{4a^2} \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ \left(x + \frac{b}{2a}\right) &= \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\ \left(x + \frac{b}{2a}\right) &= \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\end{aligned}$$

Q.E.D