MT on MONDAY

- 4 Q's
- 3 hrs open til midnight

Review

HW-1: Q1 ball Q

$h$ - height
$a$ - acceleration due to gravity
$m$ - mass
$v_0$ - initial velocity

$$h = a^x m^y v_0^z$$

$$[L] = \left[\frac{L}{T^2}\right]^x \cdot [M]^y \cdot \left[\frac{L}{T}\right]^z = L^{x+z} M^y T^{-2x-z}$$

RHS LHS

$$\left.\begin{array}{r} x+z = 1 \\ y = 0 \\ -2x-z = 0 \end{array}\right\} \quad x=1,\ y=0,\ z=2$$

$$\therefore\ h = a^{-1} m^0 v_0^2$$

$$\Rightarrow h \sim \frac{v_0^2}{a}$$

HW1: Q2.1

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right)$$

NON-DIM

$y = [y]y^*$
$t = [t]t^*$

$$\frac{dy}{dt} = \frac{\cancel{[y]}}{[t]}\frac{dy^*}{dt^*} = r\cancel{[y]}y^*\left(1 - \frac{[y]y^*}{k}\right)$$

$$\frac{dy^*}{dt^*} = [t]ry^*\left[1 - \frac{[y]y^*}{K}\right]$$

$$\Rightarrow [t] = \frac{1}{r}, \quad [y] = k$$

$$\therefore \frac{dy^*}{dt^*} = ry^*\left[1 - y^*\right]$$

HW1: Q2.2

$$\frac{dy}{dt} = sy(a-y)(y-b) \rightarrow \frac{[M]}{[T]} = \frac{1}{[T][M]^2}[M][M][M]$$

$y = [y]y^*$
$t = [t]t^*$

$$\frac{dy}{dt} = \frac{\cancel{[y]}}{[t]}\frac{dy^*}{dt^*} = s\cancel{[y]}y^*(a - [y]y^*)([y]y^* - b)$$

$$\frac{dy^*}{dt^*} = [t]s\,y^*(a - [y]y^*)([y]y^* - b)$$

$[y] = a$

$$\frac{dy^*}{dt^*} = s[t]y^*(a - ay^*)(ay^* - b)$$

$$= s[t]y^* \cdot a(1 - y^*)a(y^* - b/a)$$

$$= s[t]\, a^2\, y^*(1-y^*)(y^*-\alpha), \quad \alpha = b/a$$

$$[t] = \frac{1}{sa^2} = \frac{1}{\frac{1}{TM^2}\cdot M^2} = [T]$$

$$\Rightarrow \frac{dy^*}{dt^*} = y^*(1-y^*)(y^*-\alpha), \quad \alpha = b/a$$

HW1: Q3: $$\frac{dN}{dt} = r_B N\left(1-\frac{N}{K_B}\right) - B\frac{N^2}{A^2+N} \qquad (1)$$

$$\Downarrow$$

$$\frac{du}{d\tau} = ru\left(1-\frac{u}{q}\right) - \underbrace{\frac{u^2}{1+u^2}}_{\text{Hint} \to \text{start here no constants}} \qquad (2)$$

$$N = [N]N^*, \quad t = [t]t^*$$

$$\text{Eq}(1) \Rightarrow \frac{dN^*}{dt^*} = r_B[t]\,N^*\left[1-\frac{[N]N^*}{K_B}\right] - \frac{B[t][N]N^{*2}}{A^2\left(1+\frac{[N]^2N^{*2}}{A^2}\right)}$$

$$[N] = A, \quad [t] = \frac{A^2}{BA} = \frac{A}{B}$$

$$\frac{dN^*}{dt^*} = \underbrace{r_B\cdot\frac{A}{B}}\cdot N^*\left[1-\underbrace{\frac{AN^*}{K_B}}\right] - \frac{N^{*2}}{1+N^{*2}}$$

$$N^* = u, \quad r = r_B\frac{A}{B}, \quad q = \frac{K_B}{A}$$

$$\Rightarrow \frac{du}{d\tau} = ru\left(1-\frac{u}{q}\right) - \frac{u^2}{1+u^2}$$

HW1: Q4.1

$$\ddot{\theta} = \alpha\dot{\theta} + \beta \sin \gamma\theta = 0$$
$$\theta(0) = 0$$
$$\dot{\theta}(0) = 1$$

$$\alpha; \gamma = O(1), \quad \beta >> 1$$

$$t = [t]t^*$$

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{1}{[t]}\frac{d\theta}{dt^*}, \qquad \ddot{\theta} = \frac{1}{[t]^2}\frac{d^2\theta}{dt^{*2}}$$

$$\frac{1}{[t]^2}\frac{d^2\theta}{dt^{*2}} = \alpha \cdot \frac{1}{[t]}\frac{d\theta}{dt^*} + \beta\sin(\gamma\theta)$$

$$\Rightarrow \frac{d^2\theta}{dt^{*2}} = \alpha[t]\frac{d\theta}{dt^*} + \beta[t]^2\sin(\gamma\theta)$$

$$\dot{\theta}(0) = \frac{1}{[t]}\frac{d\theta}{dt^*}(0) = 1$$

$$\frac{d\theta}{dt^*}(0) = [t] \cdot 1 \quad \checkmark$$

$$[t] = \frac{1}{\sqrt{\beta}}$$

$$\Rightarrow \frac{d^2\theta}{dt^{*2}} = \frac{\alpha}{\sqrt{\beta}} \cdot \frac{d\theta}{dt^*} + \sin(\gamma\theta)$$

$$\eta = \frac{\alpha}{\sqrt{\beta}}$$

$$\theta(0) = 0$$

$$\frac{d\theta}{dt^*}(0) = \frac{1}{\sqrt{\beta}}\alpha \qquad \text{order less than } 1$$

HW1: Q.4.2 $\rightarrow$ similar

HW1: Q4.3

$$\alpha \sim \beta\gamma \sim \frac{1}{\gamma} >> 1$$

$$t = [t]\, t^*, \quad \theta = [\theta]\, \theta^*$$

$\Rightarrow$ eqn $$\frac{d^2\theta^*}{dt^{*2}} + \alpha[\theta]\frac{d\theta^*}{dt} + \beta\frac{[t]^2}{[\theta]}\sin(\gamma\theta^*[\theta]) = 0$$

$$\theta^*(0) = \frac{1}{[\theta]}\theta(0) = 0$$

$$\frac{d\theta^*}{dt^*} = \frac{1}{[\theta]}[t]\frac{d\theta}{dt}(0) = \frac{[t]}{[\theta]} \quad \checkmark$$

$$[\theta] = \frac{1}{\gamma}, \quad [t] = \frac{1}{\alpha}$$

$$\Rightarrow \frac{d^2\theta^*}{dt^{*2}} + \frac{d\theta^*}{dt^*} + \frac{\beta\gamma}{\alpha^2}\cdot\sin(\theta^*) = 0$$

$$\eta = \frac{\beta\gamma}{\alpha^2} \sim \underbrace{\frac{\beta\gamma}{\alpha}}_{O(1)}\cdot\underbrace{\frac{1}{\alpha}}_{\alpha>>1} << O(1)$$

HW1: Q5

$$\frac{dy}{dx} = D\frac{d^2y}{dt^2} + \gamma y^3$$

$$\Rightarrow \frac{dy}{dx} = \frac{[M]}{[L]}.$$

$$\Rightarrow D\frac{d^2y}{dx^2} = D\cdot\frac{[M]}{[L]^2} \sim \frac{M}{[T]}$$

$$D \sim \frac{[L]^2}{[T]}$$

$$\Rightarrow \gamma y^3 = \gamma[M]^3 \sim \frac{[M]}{[T]}$$

$$\gamma \sim \frac{1}{[M]^2[T]}$$

Q5.2 $\Rightarrow$ regular ND

$$\Downarrow$$

$$\frac{dy^*}{dt^*} = \alpha\frac{d^2y^*}{dx^{*2}} + \beta y^{*3}$$

$$\alpha = \frac{D[t]}{[x]^2}, \quad \beta = \gamma[t]y_0^2$$

for large diffusion relatively,

$$\alpha >> \beta$$

$$\frac{D[t]}{[x]^2} >> \gamma[t]y_0^2$$

$$\Rightarrow \frac{D}{\gamma[x]^2y_0^2} >> 1$$

$$\Rightarrow [X] << 1 \quad \left(\text{ie } \sqrt{\frac{\gamma y_0^2}{D}}\right)$$

for small diffusion

$$\alpha << \beta$$

~~so~~

$$\frac{D}{r[X]^2 y_0^2} << 1$$

$$\Rightarrow \quad [X] >> 1$$

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HW2: Q6

$$\begin{tikzcd} {} \arrow[r, "rMe^{-\varphi rM}"] & \boxed{I} \arrow[r, "\gamma I"] \arrow[d, "\delta I"] & \boxed{M} \arrow[d, "\mu M"] \\ & {} & {} \end{tikzcd}$$

$$\frac{dI}{dt} = rMe^{-\varphi r M} - \delta I - \gamma I$$

$$\frac{dM}{dt} = \gamma I - \mu M$$

Q6.2 W/ control

$$F = \frac{dI}{dt} = (1-\theta) r M e^{-\varphi \gamma M (1-\theta)} - \delta I - \gamma I$$

$$G = \frac{dM}{dt} = \gamma I - \mu M$$

$$\left.\begin{aligned}\frac{dI}{dt} &= 0\\ \frac{dM}{dt} &= 0\end{aligned}\right\}$$

$\Rightarrow$ (1) $I = 0,\ M = 0 \qquad E_0 = (0,0)$

$$E^* = \begin{cases} \text{(2)}\ I^* = -\ln\left(\dfrac{\delta + \gamma\left(\frac{M}{r}\right)}{r(1-\theta)}\right)\dfrac{\mu}{\psi\gamma}\left(\dfrac{1}{r(1-\theta)}\right) \\[2ex] M^* = -\ln\left(\dfrac{(\delta+\gamma)\left(\frac{\mu}{\gamma}\right)}{r(1-\theta)}\right)\left(\dfrac{1}{\psi r(1-\theta)}\right)\end{cases}$$

Stability:

Jacobian $\quad J = \begin{bmatrix} \dfrac{\partial F}{\partial I} & \dfrac{\partial F}{\partial M} \\[2ex] \dfrac{\partial G}{\partial I} & \dfrac{\partial G}{\partial M}\end{bmatrix}$

$$J = \begin{bmatrix} -\delta-\gamma & e^{-\psi r(1-\theta)M}\left[(1-\theta)r - m\psi r^2(1-\theta)^2\right] \\ \gamma & -\mu \end{bmatrix}$$

At $E_0 = (0,0)$, $\quad I_0 = 0,\ M = 0$

$$J\big|_{E_0} = \begin{bmatrix} -\delta-\gamma & (1-\theta)r \\ \gamma & -\mu\end{bmatrix}$$

$\mathrm{tr}J = -\delta - \gamma - \mu < 0$

$\det J =$

$E_0$ is stable $\det J > 0$
unstable $\det J < 0$

$$\det J = \mu(\delta+\gamma) - \gamma r(1-\theta) > 0$$

for stable $E_0$ (extinction)

$$\Rightarrow \quad \theta > 1 - \frac{\mu(\delta+\gamma)}{\gamma r} \quad \text{for control}$$

otherwise unstable

- same thing for $I^*, M^* =$

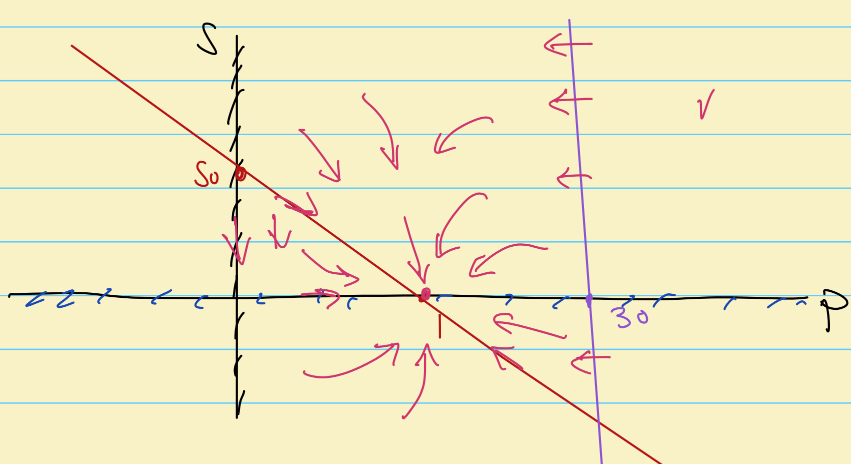
HW2: Q4

$$\left.\begin{aligned} \frac{dP}{dt} &= P(0.5 - 0.5P - 0.01S) \\ \frac{dS}{dt} &= S(-0.3 + 0.01P) \end{aligned}\right\} = 0$$

$$\frac{dP}{dt} = 0 \Rightarrow P = 0, \quad 0.5 - 0.5P - 0.01S = 0$$
$$S = 50 - 50P$$

$$\frac{dS}{dt} = 0 \Rightarrow S = 0, \quad -0.3 + 0.01P = 0$$
$$P = 30$$

plot 4 lines for bifurc.



HW2: Q5 Bifurcation diagram