# PROBLEM SET 5

**Problem 1.** Exercises 3.1.5 and 3.1.6 on p. 66.

**Problem 2.** Exercises 3.1.10 and 3.1.11 on pp. 67–68.

**Problem 3.** Use the Hamming bound to determine the maximum dimension $k$ a linear code of length $n$ and distance $d$ can have when: (a) $n = 8, d = 3$, (b) $n = 7, d = 3$, (c) $n = 15, d = 3$, (d) $n = 23, d = 7$. Show your calculations.

**Problem 4.**

(a) Determine the largest $M$ for which you can guarantee the existence of a linear code of size $M$, length $n = 10$, and distance $d = 5$.

(b) Find an upper bound for the size of a linear code with length $n = 10$ and distance $d = 5$.

(c) Is there a perfect code with $n = 10$ and $d = 5$?

**Problem 5.** Use the Gilbert-Varshamov bound to determine the *smallest* $n$ for which there exists a code of length $n$ and rate $1/3$ that can correct 2 errors.

**Problem 6.** Exercises 3.1.18–21 and 20 on p. 70.

**Problem 7.** Consider an $(n, k)$ linear code $C$ whose generator matrix $G$ contains no zero column. Arrange all the codewords of $C$ as rows of a $2^k$-by-$n$ array.

(a) Show that each column of the array consists of $2^{k-1}$ zeroes and $2^{k-1}$ ones.

(b) Conclude that

$$d(C) \leq \frac{n \cdot 2^{k-1}}{2^k - 1}.$$

**Problem 8.** Exercises 3.2.5 and 3.2.6 on p. 72.