Homework 2
Numerical Matrix Analysis
Math 543
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Exercise 4.1: Determine the SVD's of the following matrices.

$$a.\begin{bmatrix}3 & 0\\ 0 & -2\end{bmatrix} \qquad b.\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix} \qquad c.\begin{bmatrix}0 & 2\\ 0 & 0\\ 0 & 0\end{bmatrix} \qquad d.\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix} \qquad e.\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

Solution Exercise 4.1:

$$a.\begin{bmatrix}3 & 0\\ 0 & -2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}^*$$

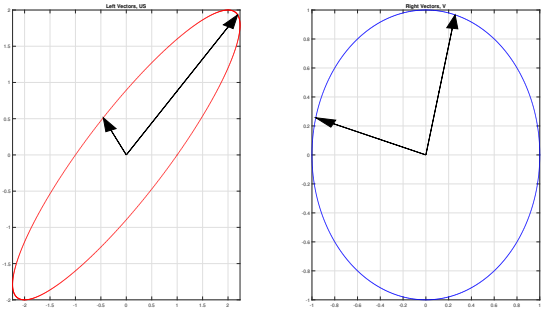
$$b.\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}^*$$

$$c.\begin{bmatrix}0 & 2\\ 0 & 0\\ 0 & 0\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}^*$$

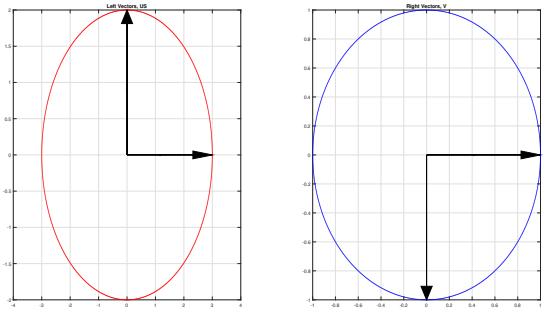
$$d.\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1.4142 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}.7071 & -.7071\\ .7071 & .7071\end{bmatrix}^*$$

$$e.\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} = \begin{bmatrix}-.7071 & -.7071\\ -.7071 & .7071\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}-.7071 & .7071\\ -.7071 & -.7071\end{bmatrix}^*$$

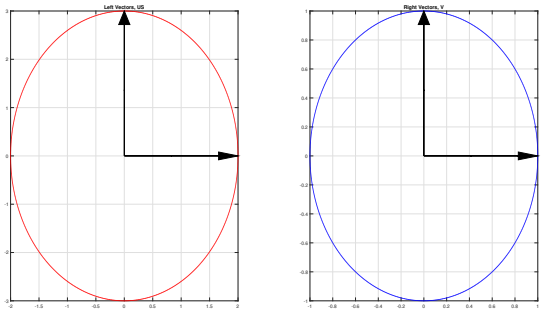
Exercise 4.3: Write a MATLAB program (see Lecture 9) which, given a real 2 x 2 matrix $\mathbb{A}$, plots the right singular vectors v_1 and v_2 in the unit circle and also the left singular vectors u_1 and u_2 in the appropriate ellipse, as in Figure 4.1. Apply your program to the matrix (3.7) and also to the 2 x 2 matrices of Exercise 4.1. —— Matrix 3.7: $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$



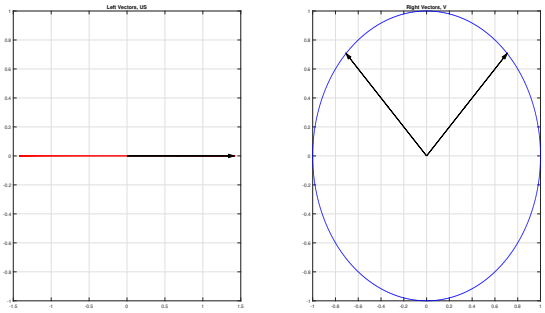
(a) Matrix 3.7



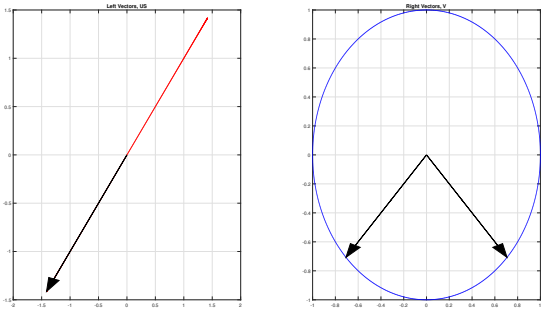
(b) Matrix (a)



(c) Matrix (b)



(d) Matrix (d)



(e) Matrix (e)