Math 543, Spring 2020
Midterm #1, Due Friday April 10 at 11:59pm.

Submission: UPLOAD ALL PAGES TO GRADESCOPE, attach extra page(s) AFTER the 10 numbered pages

Tools: Pen/Pencil/Eraser/Paper.
Rules: This is a take-home midterm; see below...

Problem	Pts Possible	Pts Scored
1(a)	5	
1(b)-i	5	
1(b)-ii	5	
1(b)-iii	5	
1(b)-iv	5	
1(c)-i	5	
1(c)-ii	5	
1(c)-iii	5	
2(a)	5	
2(b)	5	
2(c)	5	
2(d)	5	
2(e)	5	
2(f)-i	5	
2(f)-ii	5	
2(f)-iii	5	
2(f)-iv	5	
3(a)	5	
3(b)	5	
3(c)	5	
3(d)	5	
4	5	
5	0	
Total	110	

I, _____, pledge that this exam is ***completely my own work,*** and that I did not take, borrow or steal any portions from any other person, including *"Questionable Cousin Chegg."* Any and all references I used are clearly cited in my solutions. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

_____
Signature

Rules: This midterm is open-book, open-notes. You cannot consult any other human. If you refer to results from books (including the class text), research papers, or the web (other than the class web page(s)) carefully cite your source(s).

1. ***The Relative Condition Number*** —

(a) (5 pts.) For the function $f : X \to Y$, define the relative condition number $\kappa(x)$.

(b) i. (5 pts.) What is the smallest (in magnitude) real perturbation $\epsilon_{\mathbb{R}} \in \mathbb{R}\backslash\{0\}$?

ii. (5 pts.) What is the smallest (in magnitude) integer perturbation $\epsilon_{\mathbb{Z}} \in \mathbb{Z}\backslash\{0\}$?

iii. (5 pts.) What is the smallest (in magnitude) IEEE 754-1985 64-bit floating point $\mathbb{F}_{64}$ perturbation $\epsilon_{\mathbb{F}_{64}} \in \mathbb{F}_{64}\backslash\{0\}$?

iv. (5 pts.) What is the smallest (in magnitude) IEEE 754-1985 128-bit floating point $\mathbb{F}_{128}$ perturbation $\epsilon_{\mathbb{F}_{128}} \in \mathbb{F}_{128}\backslash\{0\}$?

(c) i. (5 pts.) For the specific function $f(x) = \begin{cases} -55, & x < 0.5 \\ 55, & x \geq 0.5 \end{cases}$ where $x \in \mathbb{R}$ is a ***real variable*** in the interval $[0, 1]$, what is the relative condition number $\kappa(x)$, for all values of x (*i.e.* as a function of x)?

ii. (5 pts.) For the specific function $f(x) = \begin{cases} -55, & x < 0.5 \\ 55, & x \geq 0.5 \end{cases}$ where $x \in \mathbb{F}_{64}$ is a ***64-bit floating point variable*** in the interval $[0, 1]$, what is the relative condition number $\kappa(x)$, for all values of x (*i.e.* as a function of x)?

iii. (5 pts.) For the specific function $f(x) = \begin{cases} -55, & x \leq 9,989,224 \\ 55, & x \geq 9,989,225 \end{cases}$ where $x \in \mathbb{Z}$ is an ***integer variable***, what is the relative condition number $\kappa(x)$, for all values of x (*i.e.* as a function of x)?

2. ***Least Squares Problems*** — Derive the matrix least squares problem for fitting a data set $\{y(t_i), t_i\}_{i=1,\ldots,m}$ by

   (a) (5 pts.) a constant,

   (b) (5 pts.) a straight line $z(t) = a + bt$.

   (c) (5 pts.) What is the solution in case (a)?

   (d) (5 pts.) Explain why, in general, solving the least squares problem using the normal equations is not such a good idea.

   (e) (5 pts.) Suggest one alternative approach, what are its advantages and disadvantages?

(f) Download the file

[clickable]

https://github.com/CSSEGISandData/COVID-19/raw/master/csse_covid_19_data/csse_covid_19_time_series/time_series_covid19_confirmed_global.csv

[/clickable]

Extract the data for the US (it's a single line starting with ",US,37.0902,-95.7129," followed by the Confirmed Cases (since 1/22 until your download date).

i. (5 pts.) Plot the data on a log-scale

[Attach plot at the end of the exam]

ii. (5 pts.) Identify period of exponential growth (a period where the growth looks linear on the log scale)

Specify day range:

iii. (5 pts.) Find the best log-linear (that is linear fit to the $\log_{10}$ of the data,) and plot that fit on the same plot.

[Attach plot at the end of the exam]

iv. (5 pts.) Use the model to extrapolate 7 days (add to the plot); what is the count?

Specify extrapolated case count:

[Attach plot at the end of the exam]

[Attach code at the end of the exam]

Note: For most meaningful results, plot everything in ONE plot.

3. ***Stability and Backward Stability*** — Let $f(x)$ denote the exact solution to the exact problem, let $\tilde{f}(x)$ denote the solution computed by an algorithm.

   (a) (5 pts.) Complete the statement: *"We say that an algorithm $\tilde{f}$ for a problem $f(x)$ is* stable *if for each $x \in X$*

   [ ] ."

   for some $\tilde{x}$ with

   [ ] ."

   (b) (5 pts.) Complete the statement: *" We say that an algorithm $\tilde{f}$ for a problem $f(x)$ is* backward stable *if for each $x \in X$*

   [ ] ."

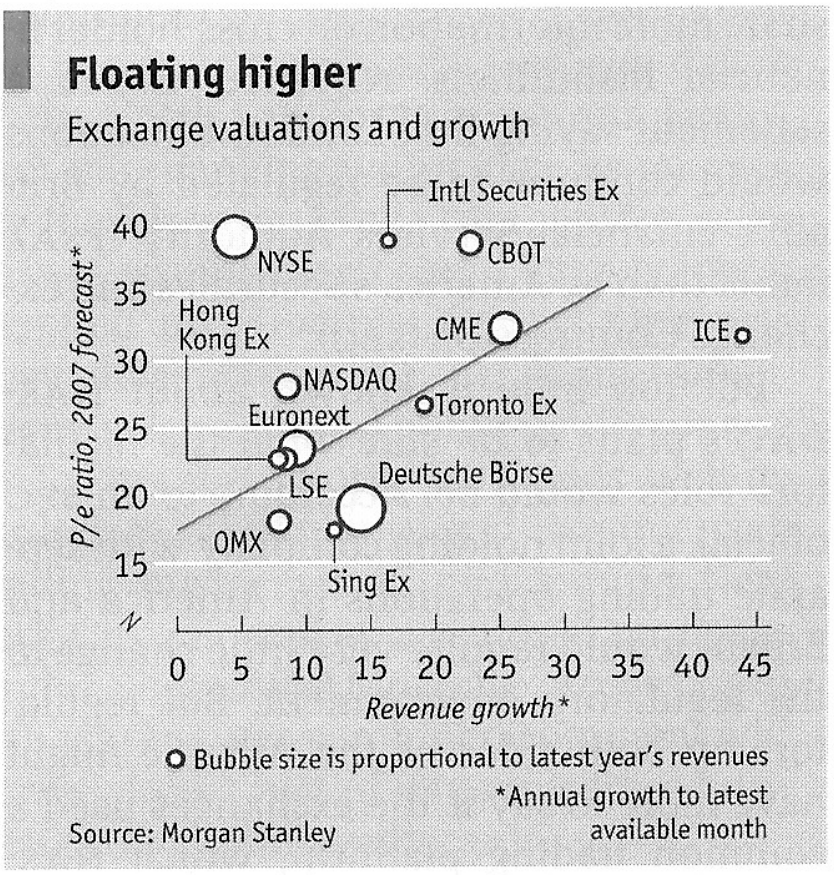
   for some $\tilde{x}$ with

   [ ] ."

(c) (5 pts.) Show that the obvious algorithm for solving (for x) the 1-by-1 system $rx = b$ is backward stable if executed in matlab on a modern-day computer. Explain your notation and any results / definitions / axioms you are invoking.

(d) (5 pts.) Explain how backward stability, the condition number, and the available "computational precision" limit how well we can solve a problem numerically. (You may state a theorem, but it is not necessary (but highly recommended)).

4. (5 pts.) The following figure was published in *The Economist* (March 18, 2006).



The sloped line is most likely a least squares fit (of some kind). Do you have any comments? (Do NOT use more space than this page and the back! Do not feel compelled to fill the space; sometimes less is more!)

5. (0 pts.) ***Conditioning, Stability, and Accuracy of the US Presidential Election*** — Use your toolbox from ***** this class ***** to comment on the election process. You may choose to limit the scope to a single-state scenario. Clearly define the concepts you need for your discussion. (If you are completely unfamiliar with the US Presidential Election, you may comment on ***any*** democratic election process (with more than one candidate), just make sure you clearly define the rules of the chosen process.)

    - This question is **not due**, but still worth thinking about on a dark and stormy night.