On the Dual Nature of Chaos and Order in Weather and Climate: New Insights and Opportunities Within a Generalized Lorenz Model

By

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06 November 2020

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On the Dual Nature of Chaos and Order in Weather

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RESEARCH ARTICLE | 28 SEPTEMBER 2020

Is Weather Chaotic? Coexistence of Chaos and Order within a Generalized Lorenz Model

Bo-Wen Shen; Roger A. Pielke, Sr.; Xubin Zeng; Jong-Jin Baik; Sara Faghih-Naini; Jialin Cui; Robert Atlas

Bull. Amer. Meteor. Soc. 1–28.

https://doi.org/10.1175/BAMS-D-19-0165.1

By revealing two kinds of attractor coexistence within Lorenz models, we suggest that the entirety of weather possesses a dual nature of chaos and order with distinct predictability.

Milestones toward the Revised View

Years	Major Milestones	References
2004 -2006	• Deployed the highest resolution global weather/climate model • Produced remarkable hurricane track and intensity predictions (highlighted by American Geophyisical Union (AGU) and Science Magazine)	Lin, Shen et. al. (03) Atlas et al. (05); Shen et al. (06a,b)
2007 -2010	• Produced remarkable 7-30 day predictions (featured by UMCP and NASA as well as Dr. Anthes of NCAR in 2011)	Shen et al. (10 a, b) Shen et al. (11)
2011	• Published MMF results for 30-day simulations • Completed the first draft using the newly developed 5D Lorenz model that produces negative feedback in order to suppress chaos (published in 2014)	Shen et al. (11) Shen (14, the Journal of the Atmospheric Sciences (JAS))
2012-13	• Developed the multiscale analysis package	Shen et al. (12, 13)
2014 -2016	• Published a paper using a 10-year multiscale analysis • Developed the 6D, 7D, 9D Lorenz models • Developed various types of Lorenz models to reveal periodic, homoclinic, quasi-periodic orbits	Shen (15, 18); Wu and Shen (16); Shen et al. (17); Faghih-Naini and Shen (18)

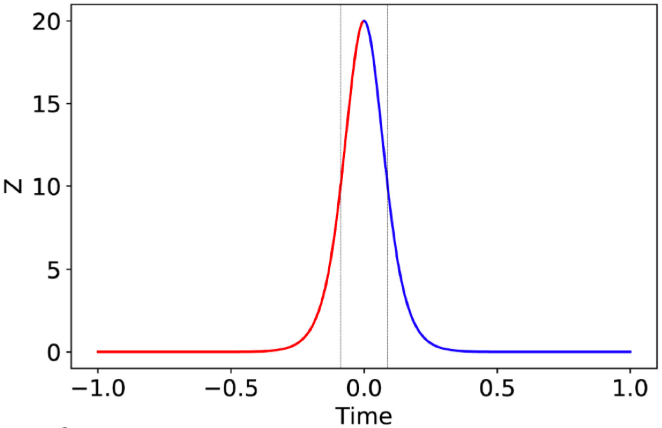
Milestones toward the Revised View

Years	Major Milestones	References
2017 -2019	• Completed an initial BAMS draft in summer 2017 • Submitted the first BAMS draft in spring 2018 • Presented coexisting attractors and the generalized Lorenz model at Chaos 2018 conferences in 2018 • Submitted and published papers that discuss coexisting attractors using the generalized Lorenz model in 2019 • Resubmitted the second BAMS draft in spring 2019 • Completed and published **a review article** in summer 2019 that summarizes work using the global model and Lorenz models • Presented the revised view on the duality in weather at the IHP meeting in Paris	Shen et al. (19) Shen (19a, b) Reyes and Shen (19)
2020	• Presented results at the Chaos 2020 conference • Re-wrote a new, concise BAMS InBox draft in June • Published the BAMS paper on-line in Sep, 2020 • Published a paper showing mathematical universalities among the non-dissipative Lorenz model, KdV eq., nonlinear Schrodinger eq., and the Pedlosky model	Shen et al. (2020a, BAMS) Shen et al. (2020b, conference article) Shen (2020, IJBC)

More than 100 page Responses

Our responses provided a list of regular weather systems that display dynamics comparable to the regular solutions within the theoretical models:

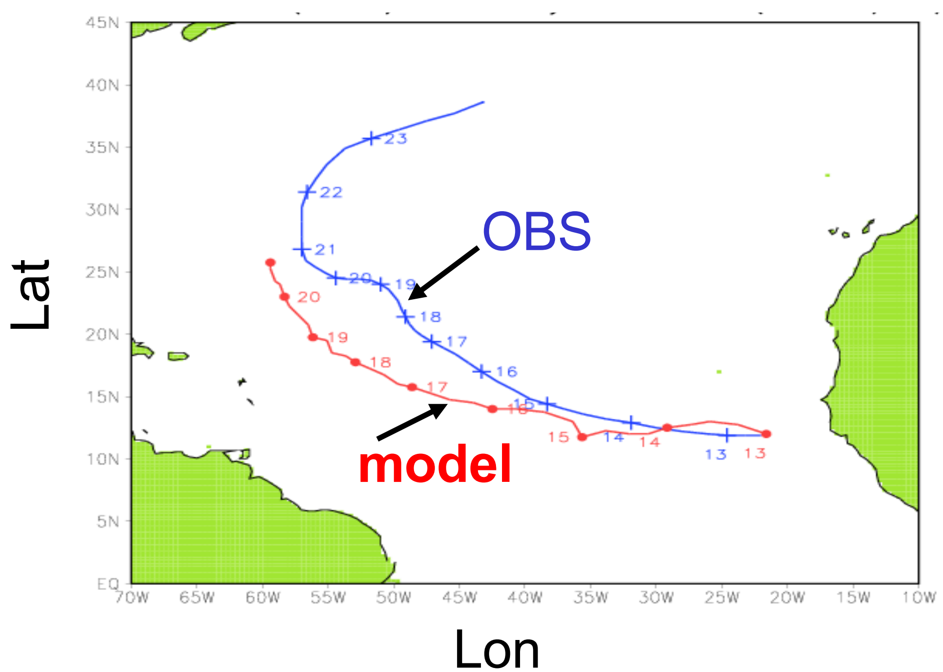
➢African Easterly Waves (AEWs)	nonlinear oscillatory orbits
➢Atmospheric Blocking	stable steady solution
➢QBO (quasi biennial oscillation)	nonlinear oscillatory orbits
➢40-day low frequency variability:	limit cycle (or torus)
➢Vortex Shedding	limit cycle (or torus)
➢Roll Clouds	solitary waves



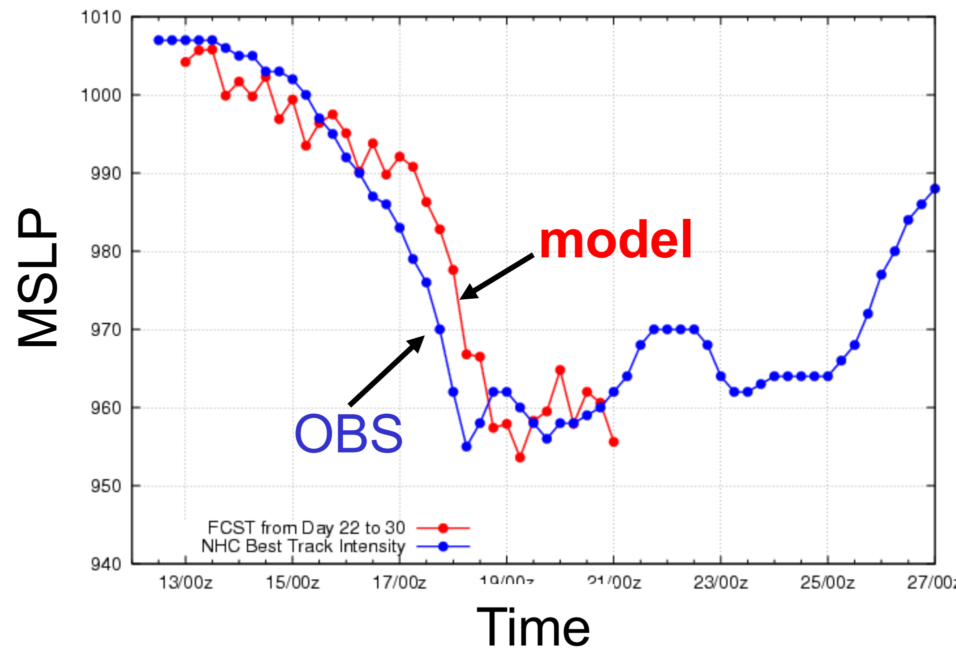
$\mathrm{sech}^2()$ *as* a solution of the non-dissipative Lorenz model

Early Studies: Promising 30-day Simulations

Track Forecast



Intensity Forecast

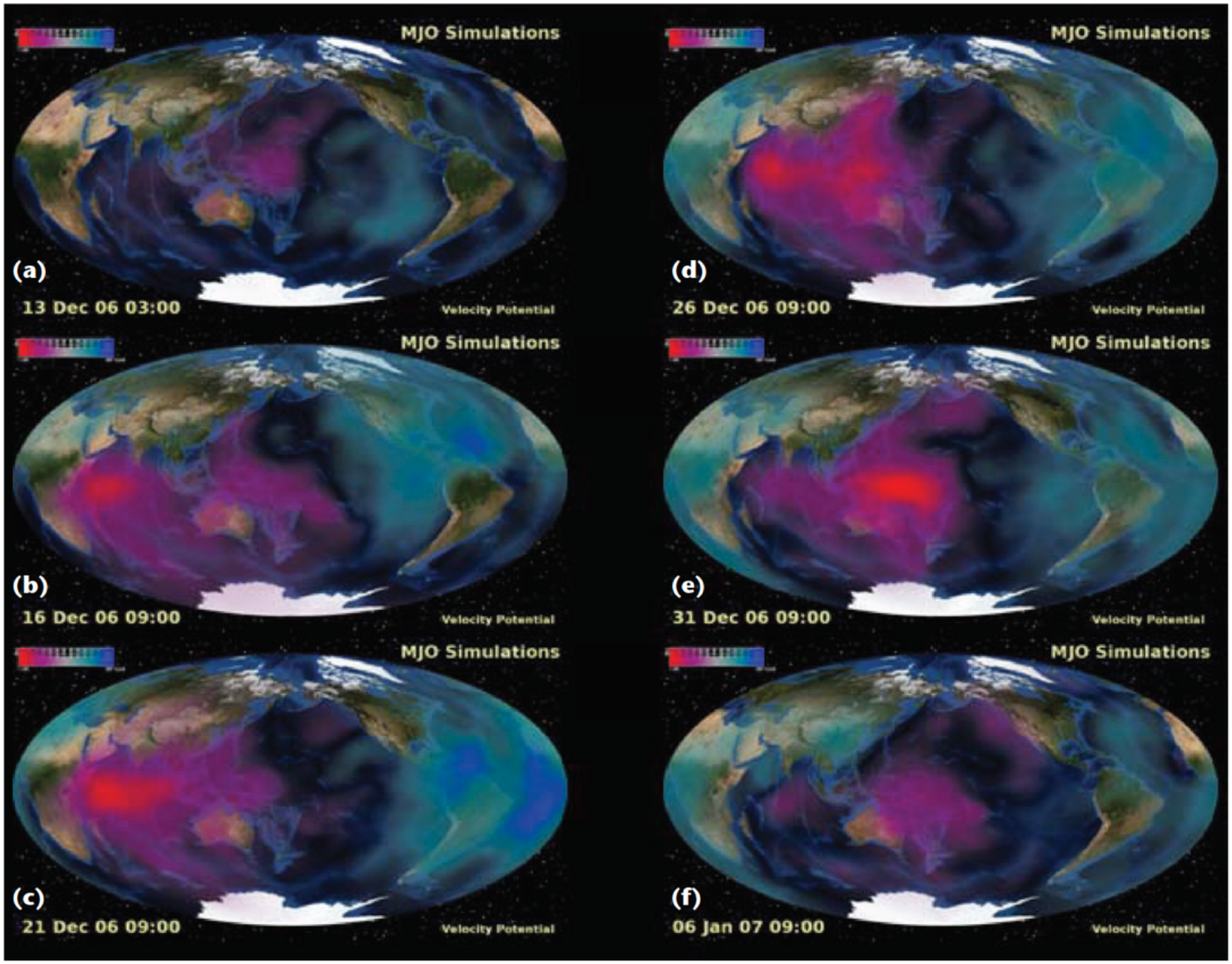


Day 30 Day 22 Day 22 Day 30

Hurricane Helene: 12-24 September, 2006

(Shen et al, 2010, GRL; Shen, 2019b, Geosciences)

Promising 30 Day MJO Simulation



A 30-day MMF simulation of the Madden-Julian Oscillation (MJO) initialized at 0000 UTC 13 December 2006, as shown by the 200-hecto Pascal (hPa) velocity potential (Shen et al., 2011, CiSE)

Goals and Approaches

Our goals include addressing the following questions:

- Can global models have skill for extended-range (15-30 day) numerical weather prediction? Why?
- Is weather chaotic?

A large scientific research cycle should consist of Modeling, Observation, Analysis, Synthesizing, Theorizing (MOAST):

- Modeling,
- Observation,
- Analysis,
- Synthesizing,
- Theorizing.

- CAMVis,
- Global 30-days Simulations,
- TRMM and QuikSCAT

- 10-year Analysis with the PEEMD;
- Analysis with RQA and KPCA

- A Revised View on the Dual Nature of Chaos and Order in Weather

- A Generalized Lorenz Model

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By revealing two kinds of attractor coexistence within Lorenz models, we suggest that the entirety of weather possesses a dual nature of chaos and order with distinct predictability.

The two kinds of attractor existence may be enabled and/or modulated by the following two mechanisms:

(1) the aggregated negative feedback of small-scale convective processes,
(2) the large-scale time varying forcing (heating).

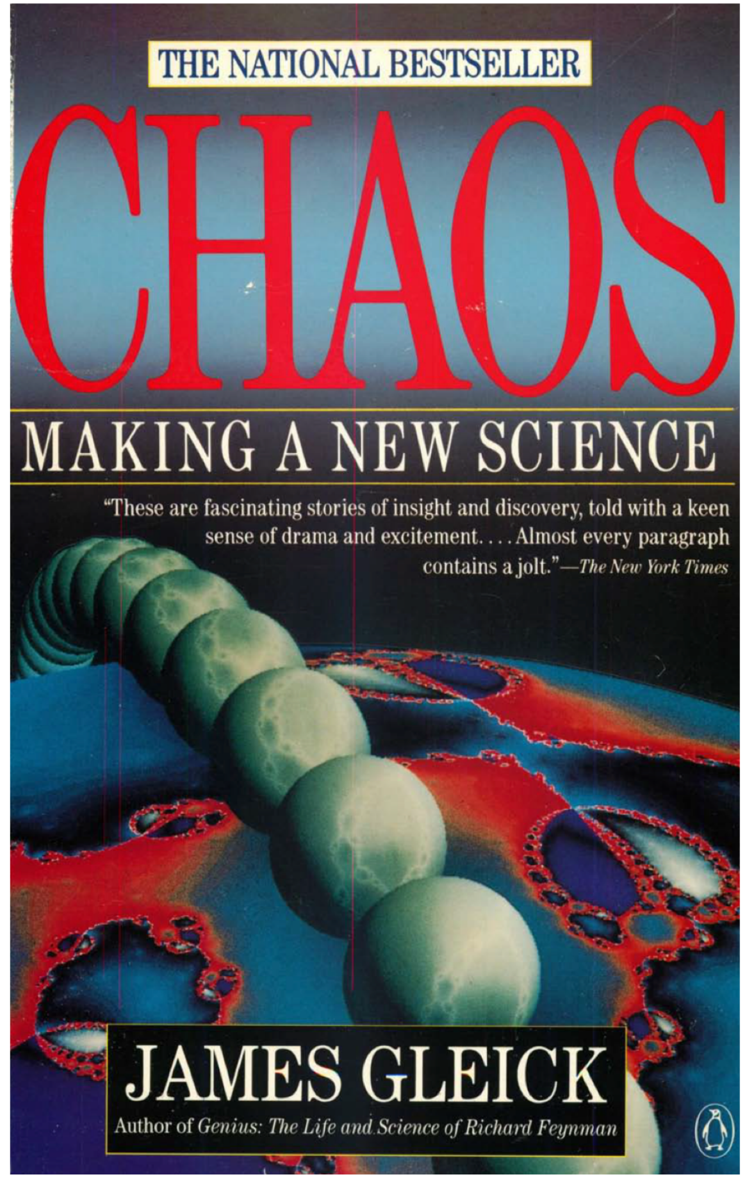
Chaos: Making a New Science (Gleick, 1987)

Gleick, James. "National Book Awards - 1987". *Chaos: Making a New Science.* National Book Foundation.

Contents

A study by Prof. Lorenz of MIT in 1963

Chaos and the Butterfly Effect



- Lorenz (1963) and Lorenz (1972) are major studies regarding Lorenz's butterfly effects ("*The Essence of Chaos*" by Lorenz in 1993)

BE1 (Lorenz, 1963)

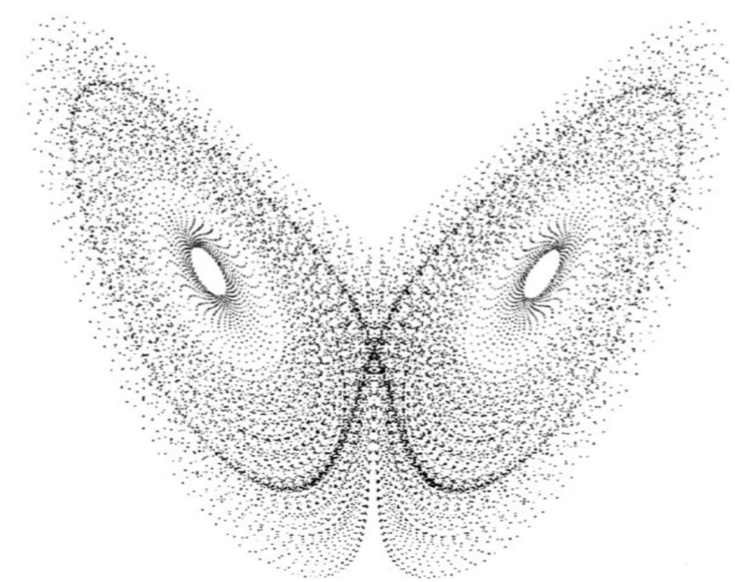


Figure 2. The butterfly.

BE2 (Lorenz, 1972/1969)

APPENDIX 1

The Butterfly Effect

THE FOLLOWING is the text of a talk that I presented in a session devoted to the Global Atmospheric Research Program, at the 139th meeting of the American Association for the Advancement of Science, in Washington, D.C., on December 29, 1972, as prepared for press release. It was never published, and it is presented here in its original form.

Predictability: Does the Flap of a Butterfly's Wings in Brazil Set off a Tornado in Texas?

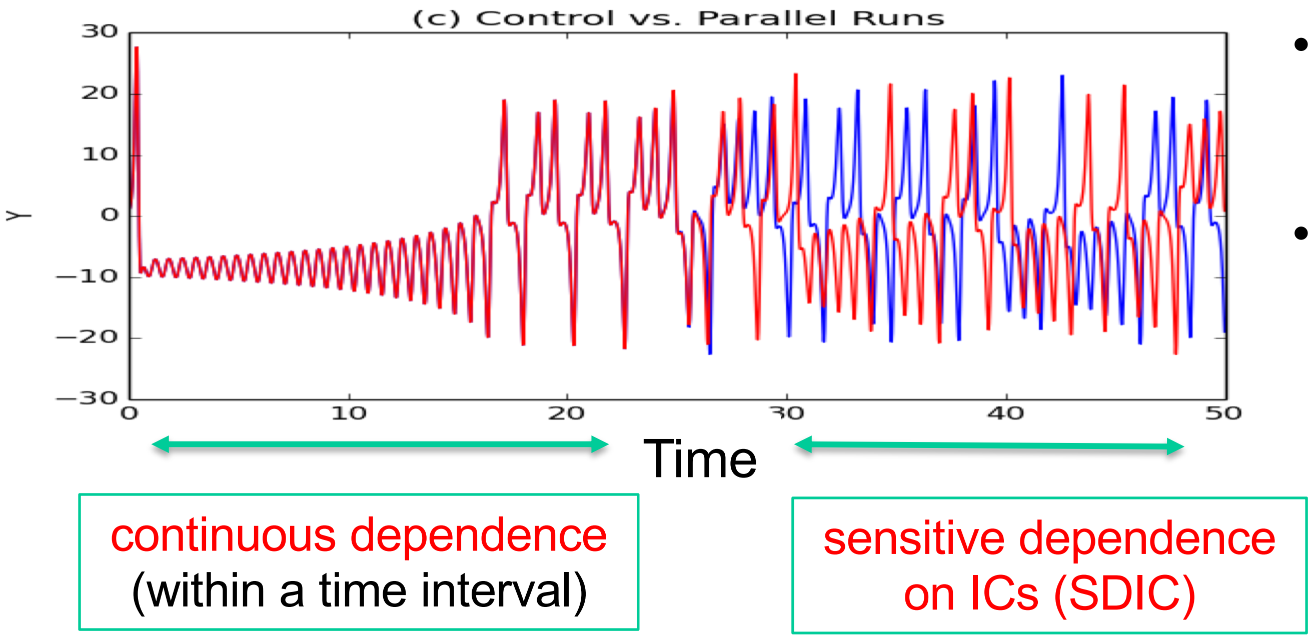
- The title of Lorenz (1972), which includes the word "butterfly", was given by session chair Philip Merilees.
- Major results in Lorenz (1972) may come from Lorenz's study in 1969 (e.g., Rotunno and Snyder, 2008; Durran and Gingrich, 2014; Palmer et al. 2014).

Butterfly Effect of the First and Second Kind

Two kinds of butterfly effects can be identified as follows (Lorenz, 1963, 1972):

1. The butterfly effect of the first kind (BE1):

   Indicating *sensitive dependence on initial conditions* (Lorenz, 1963).



- control run (blue): $(X, Y, Z) = (0,1,0)$
- parallel run (red): $(X, Y, Z) = (0,1 + \epsilon, 0)$, $\epsilon = 1e - 10$.

2. The butterfly effect of the second kind (BE2):

   a *metaphor (or symbol) for indicating that small perturbations can create a large-scale organized system* (Lorenz, 1972/1969).

The Lorenz (1963) Model (3DLM)

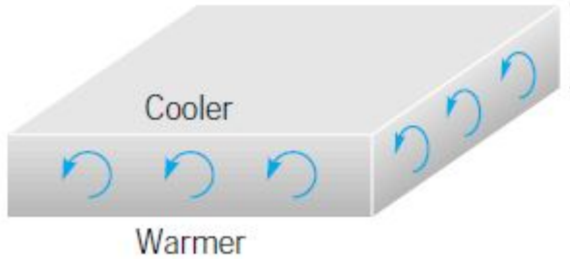
The classical Lorenz model (Lorenz, 1963) with three variables and three parameters, referred to as the 3DLM, is written as follows:

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$$

$$\frac{dY}{d\tau} = -XZ + rX - Y,$$

$$\frac{dZ}{d\tau} = XY - bZ.$$

- r – Rayleigh number: (Ra/Rc)
  a dimensionless measure of the temperature difference between the top and bottom surfaces of a liquid; proportional to effective force on a fluid;
- σ – Prandtl number: (ν/κ)
  the ratio of the kinetic viscosity (κ, momentum diffusivity) to the thermal diffusivity (ν);
- b – Physical proportion: ($4/(1+a^2)$), b = 8/3;
- a – a=l/m, the ratio of the vertical height, h, of the fluid layer to the horizontal size of the convection rolls. b = 8/3; l = aπ/H and m = π/H.

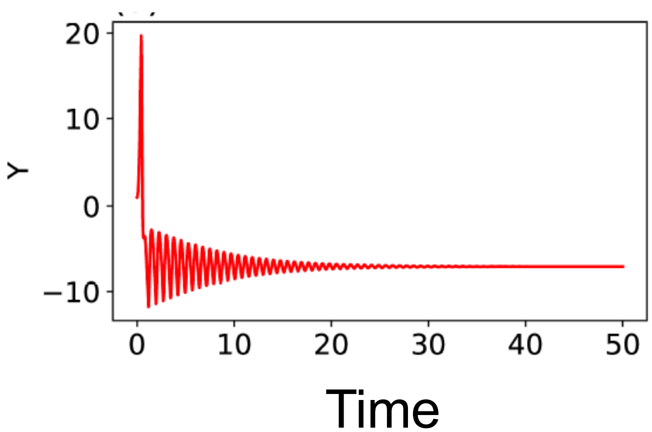


- Note that X, Y, and Z represent the amplitudes of Fourier modes for the streamfunction and temperature.
- A phase space (or state space) is defined using the state variables X, Y and Z as coordinates. The dimension of the phase space is determined by the number of variables.
- A trajectory or orbit is defined by time varying components within the phase space, also known as a solution.
- Two nonlinear terms form a nonlinear feedback loop (NFL).

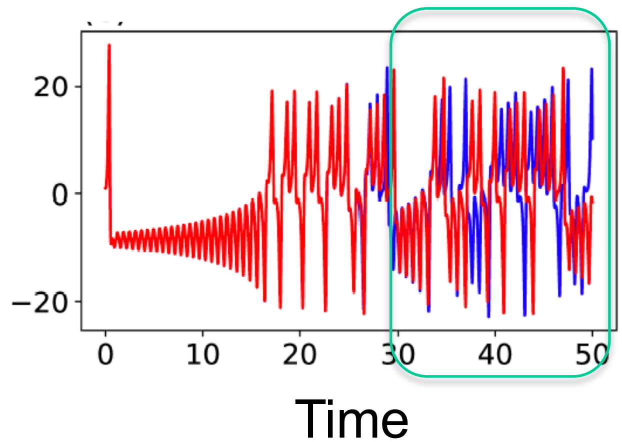
Three Kinds of Attractors and Butterfly Effect

- Depending on the relative strength of heating, as compared to $r_c = 24.74$ and $R_c = 313$, three types of solutions within the 3DLM are:
  - ✓ control run (blue): $(X, Y, Z) = (0,1, \quad 0)$
  - ✓ parallel run (red): $(X, Y, Z) = (0,1 + \epsilon, 0)$, $\epsilon = 1e - 10$

A steady-state solution
$(r < r_c = 24.74)$

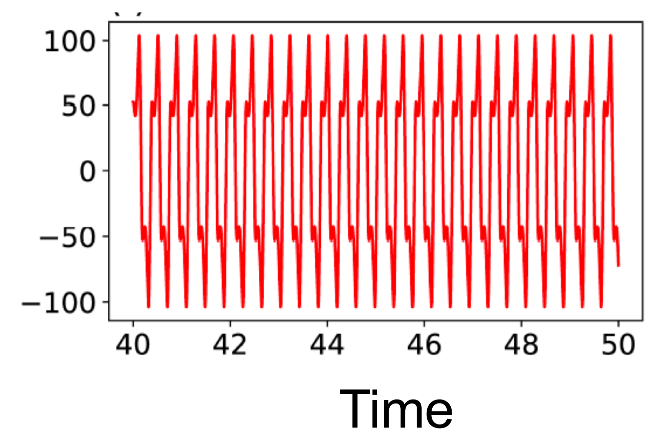


A chaotic solution
$(r_c < \mathrm{r} < R_c)$



sensitive dependence

A limit cycle
$(313 = R_c < r)$



The LC, except for its phase, has no long term memory regarding ICs.

- Butterfly effect of the first kind (BE1)
- appearing within a finite range of parameters

Three Attractors Within the 3DLM

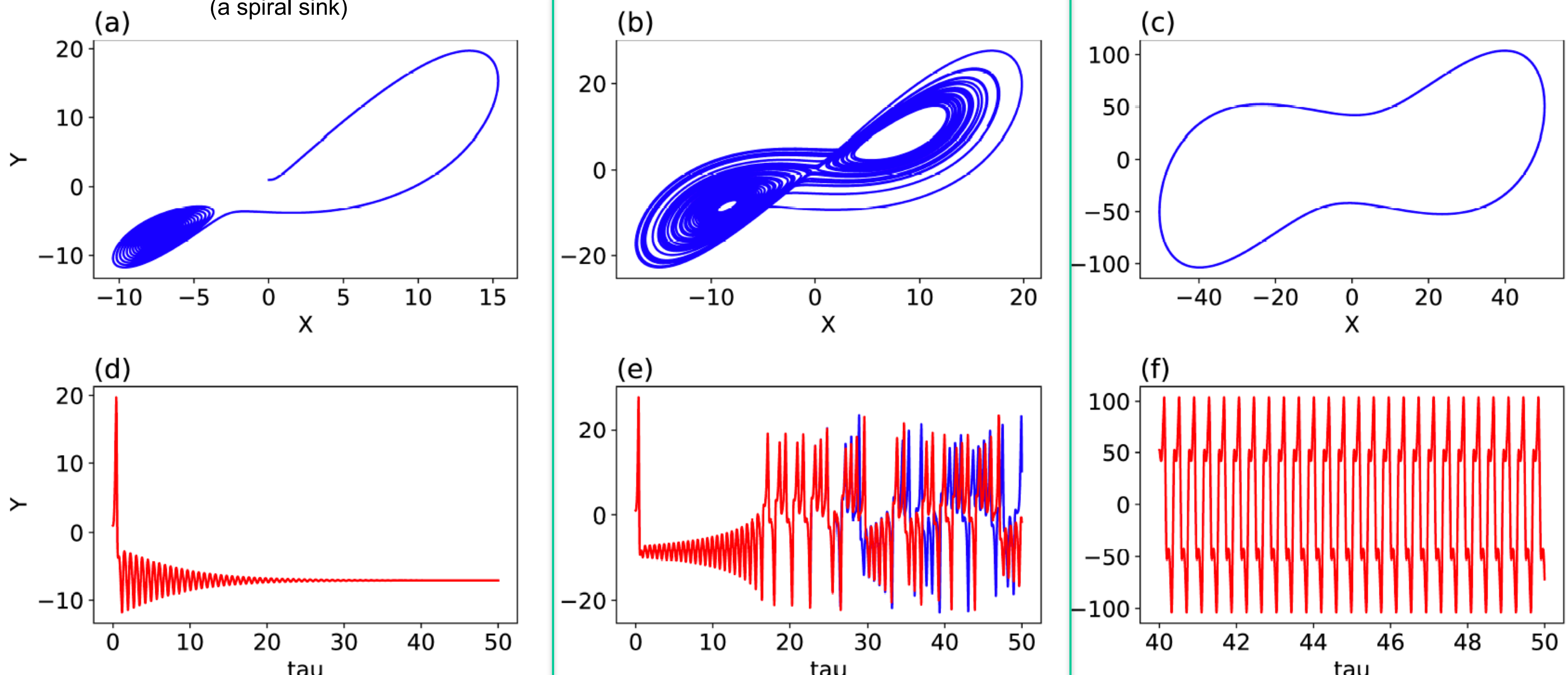
$$24.74 = r_c < \mathrm{r} < R_c = 313$$

A point attractor

(a spiral sink)

A chaotic attractor

A periodic attractor

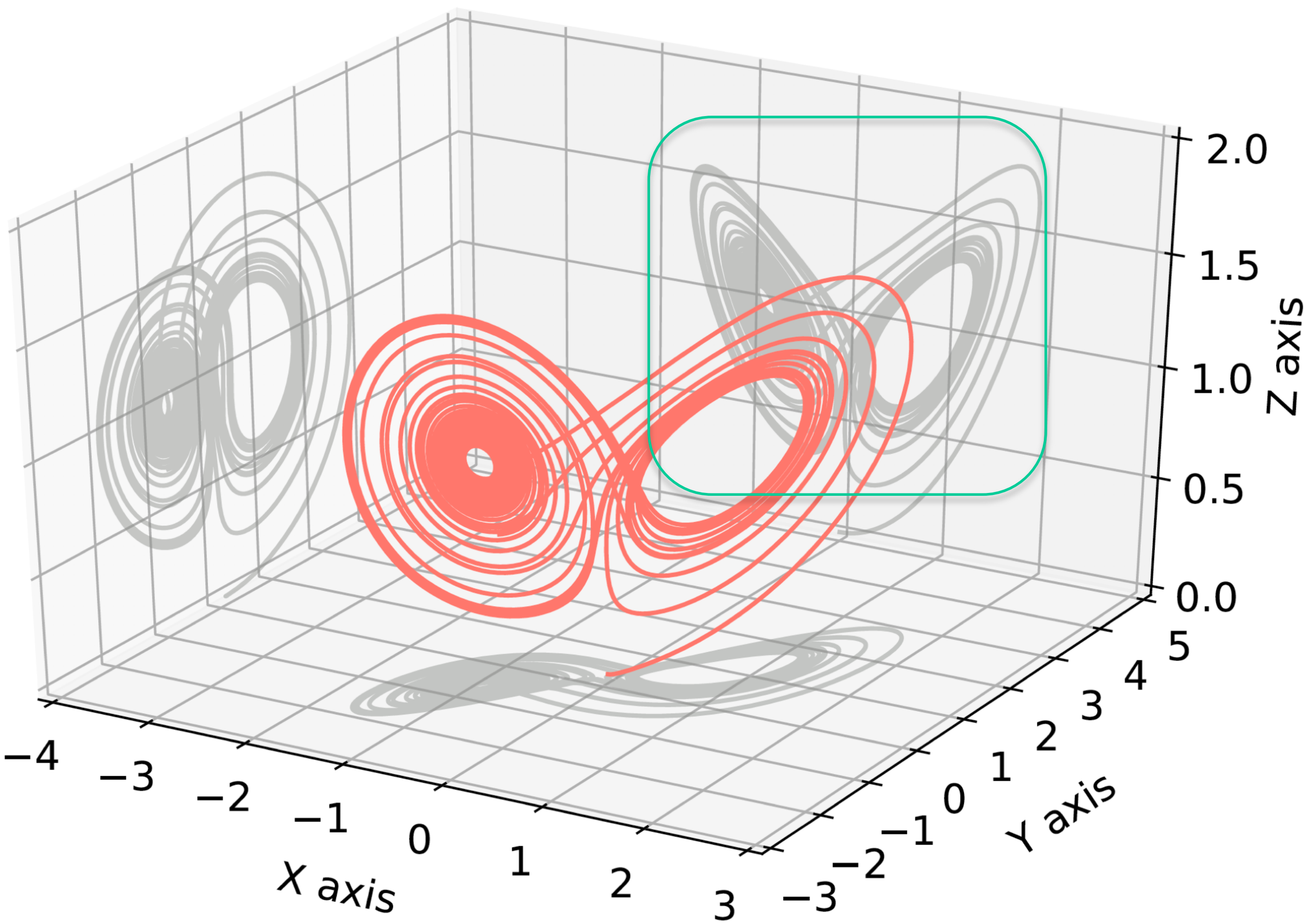


A steady-state solution with a small r

A chaotic solution with a moderate r

A limit cycle with a large r

A Butterfly Pattern in Chaotic Orbits



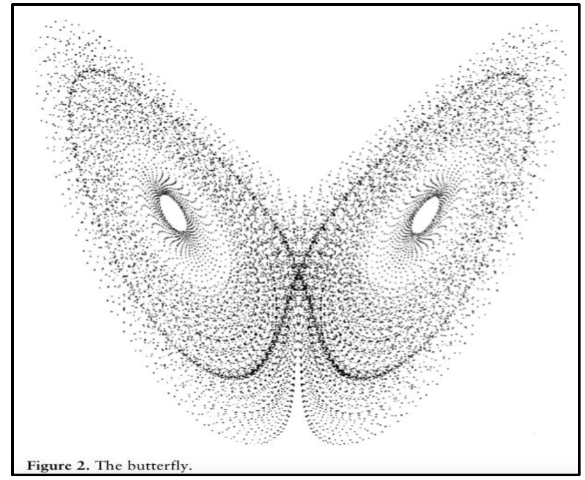
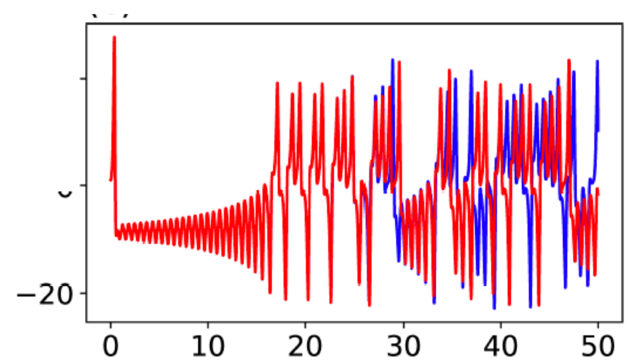


Figure 2. The butterfly.

Lorenz (1993)

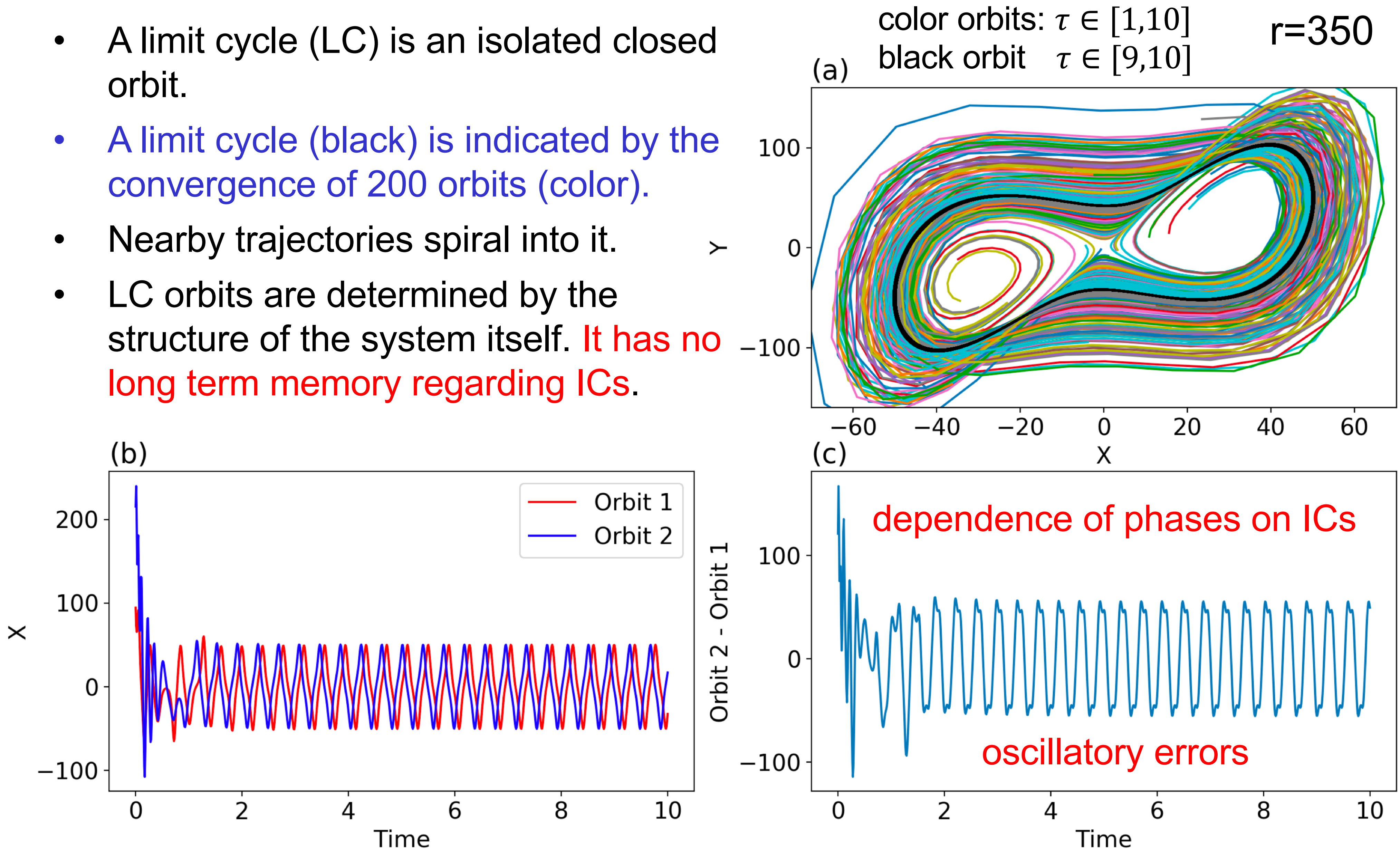
A chaotic solution

$(r_c < \mathrm{r} < R_c)$



Limit Cycle: An Isolated Closed Orbit

- A limit cycle (LC) is an isolated closed orbit.
- A limit cycle (black) is indicated by the convergence of 200 orbits (color).
- Nearby trajectories spiral into it.
- LC orbits are determined by the structure of the system itself. It has no long term memory regarding ICs.



Impact of Initial Tiny Perturbations Within the 3DLM

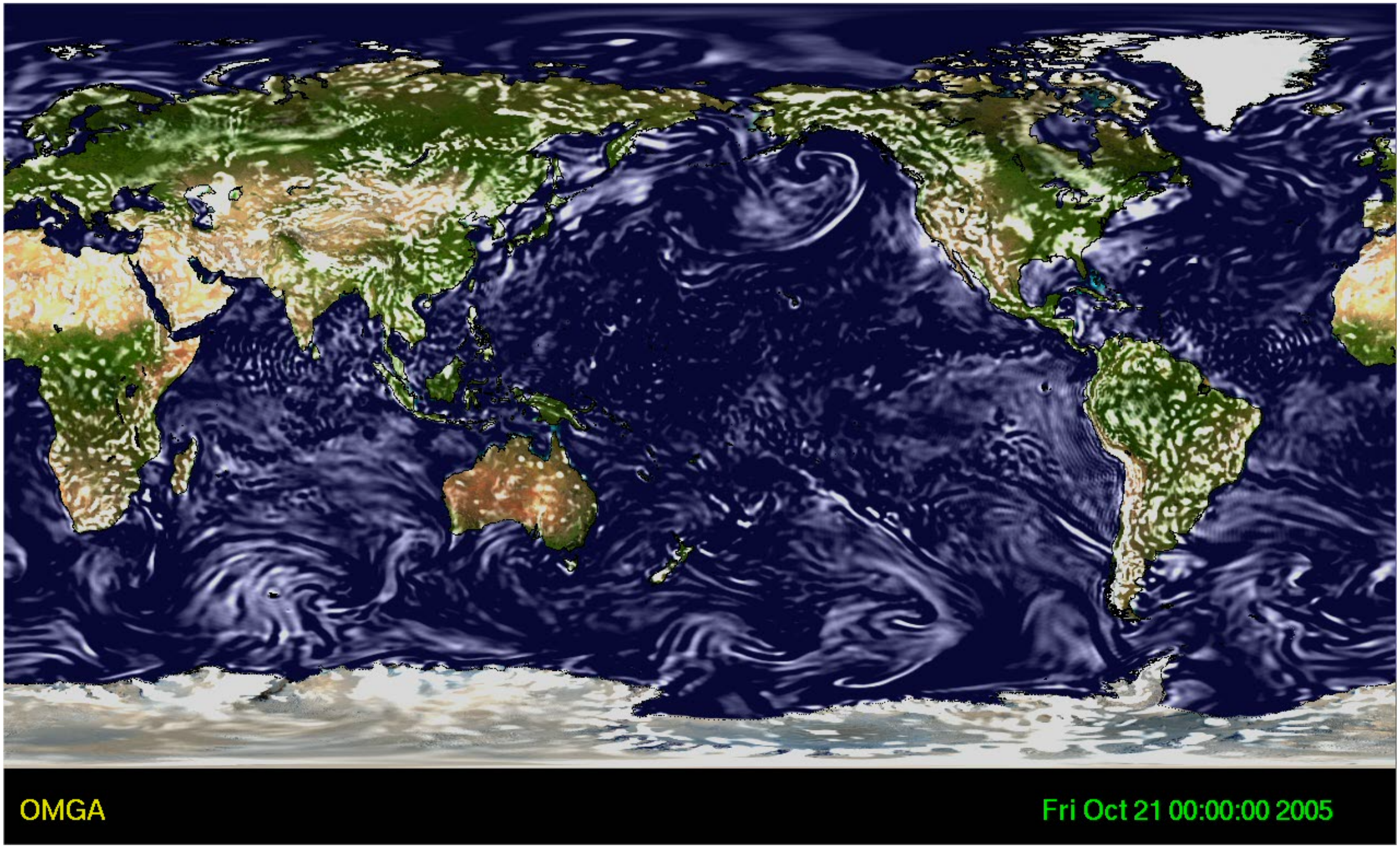
- Steady state or nonlinear periodic solutions have no (long-term) memory regarding their initial tiny perturbations
  - initial tiny perturbations completely dissipate

- Chaotic solutions display a sensitive dependence on initial conditions
  - initial tiny perturbations do not dissipate (before making a large impact)

- 3DLM: within the chaotic solutions, any tiny perturbation can cause large impacts. *Does BE1 always appear?*

- We may ask what kind of impact tiny perturbations may introduce in real world models

Concurrent Visualizations: Butterfly Effects?



A selected frame from a global animation of the vertical velocity in pressure coordinates from a run initialized at 0000 UTC 21 October 2005. The corresponding animation is available as a google document: http://bit.ly/2GS2flD. The animation displays dissipation of the initial noise associated with an imbalance between the model and the initial conditions (Shen, 2019b)

The Lorenz Model and A Generalized Lorenz

The Lorenz 1963 Model
(3DLM, Lorenz, 1963)
Three Dimension

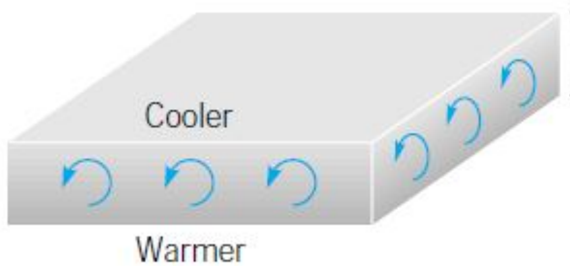
$$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$$

$$\frac{dY}{d\tau} = -XZ + rX - Y,$$

$$\frac{dZ}{d\tau} = XY - bZ.$$

primary scale modes

- σ – Prandtl number
- r – Rayleigh number
- b – Physical proportion



A Generalized Lorenz Model
(GLM, Shen, 2019a)
Any Odd Dimension

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$$

$$\frac{dY}{d\tau} = -XZ + rX - Y,$$

$$\frac{dZ}{d\tau} = XY - XY_1 - bZ,$$

an extension of the nonlinear feedback loop

smaller scale modes

$$\frac{dY_j}{d\tau} = jXZ_{j-1} - (j+1)XZ_j - d_{j-1}Y_j, \quad j \in [1, N],$$

$$\frac{dZ_j}{d\tau} = (j+1)XY_j - (j+1)XY_{j+1} - \beta_j Z_j, \quad j \in [1, N],$$

$$N = \frac{M-3}{2}; \quad d_{j-1} = \frac{(2j+1)^2 + a^2}{1+a^2}; \quad \beta_j = (j+1)^2 b.$$

Major Features of the GLM

As discussed in Shen (2019a) and Shen et al. (2019), the GLM with many M modes possesses the following features:

(1) any odd number of M greater than three; a conservative system in the dissipationless limit;

(2) three types of solutions (that also appear within the 3DLM);

(3) energy transfer across scales by the nonlinear feedback loop (NFL);

(4) slow and fast variables across various scales;

(5) aggregated negative feedback;

(6) increased temporal complexities of solutions associated with additional (incommensurate) frequencies that are introduced by the extension of the NFL (i.e., spatial mode-mode interactions);

(7) hierarchical scale dependence;

(8) two kinds of attractor coexistence;

- The 1st kind of Coexistence for Chaotic and Steady-state Solutions,
- The 2nd kind of Coexistence for Limit Cycle and Steady-state Solutions.

Aggregated Negative Feedback

model	r_c	heating terms	solutions	references
3DLM	24.74	rX	steady, chaotic, or LC	Lorenz (1963)
3D-NLM	n/a	rX	periodic	Shen (2018)
5DLM	42.9	rX	steady, chaotic, or LC/LT	Shen (2014a,2015a,b)
5D-NLM	n/a	rX	quasi-periodic	Faghih-Naini and Shen (2018)
6DLM	41.1	rX, rX_1	steady or chaotic	Shen (2015a,b)
7DLM	116.9	rX	steady, chaotic or LC/LT	Shen (2016, 2017)
7D-NLM	n/a	rX	quasi-periodic	Shen and Faghih-Naini (2017)
8DLM	103.4	rX, rX_1	steady or chaotic	Shen (2017)
9DLM	102.9	rX, rX_1, rX_2	steady or chaotic	Shen (2017)
9DLMr	679.8	rX	steady, chaotic, or LC/LT	Shen (2019a)

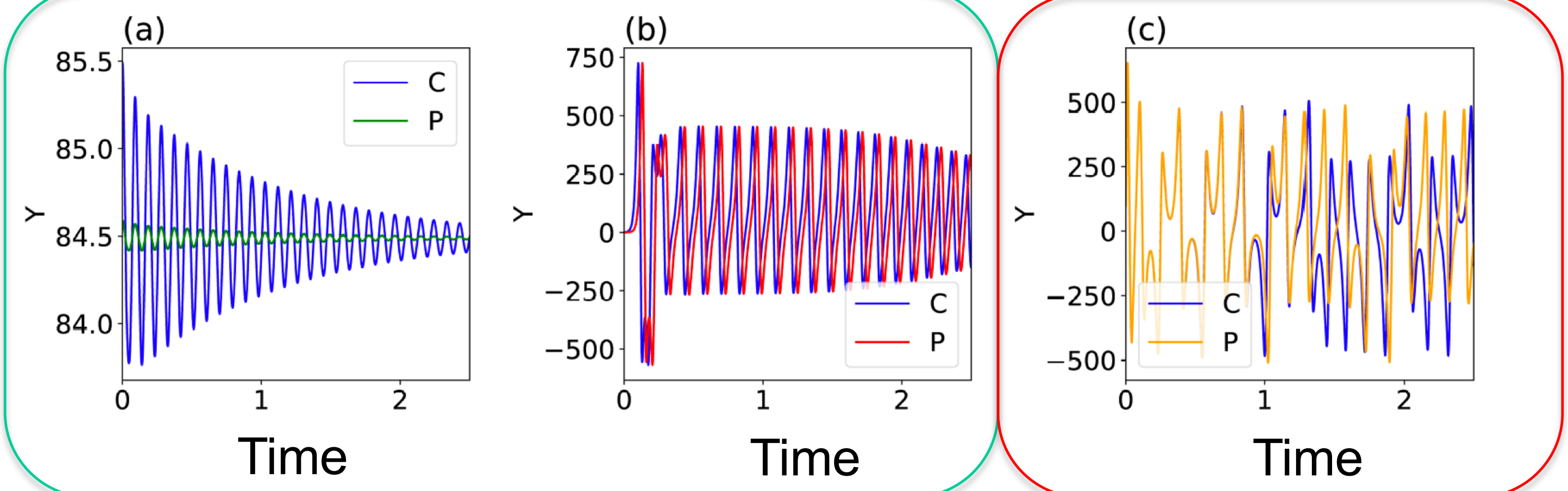
rc: a critical value of the Raleigh parameter for the onset of chaos; LC: limit cycle; LT: limit torus

- Higher-dimensional LMs require larger heating parameters for the onset of chaos, indicating aggregated negative feedback (Shen, 2019a).
- The aggregated negative feedback may change the stability of non-trivial critical points, leading to attractor coexistence.

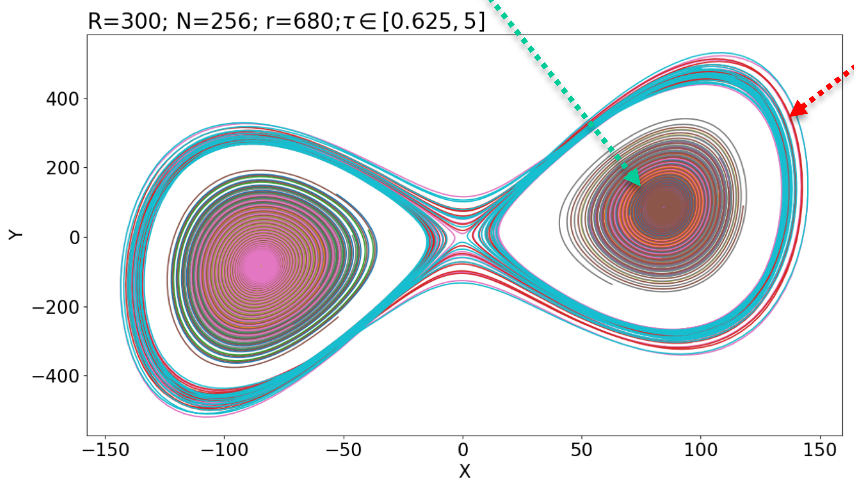
The First Kind of Attractor Coexistence

- The 1st kind of attractor coexistence within the 9DLM indicates final state sensitivity to ICs.

non-chaotic orbits | chaotic orbit



r=680



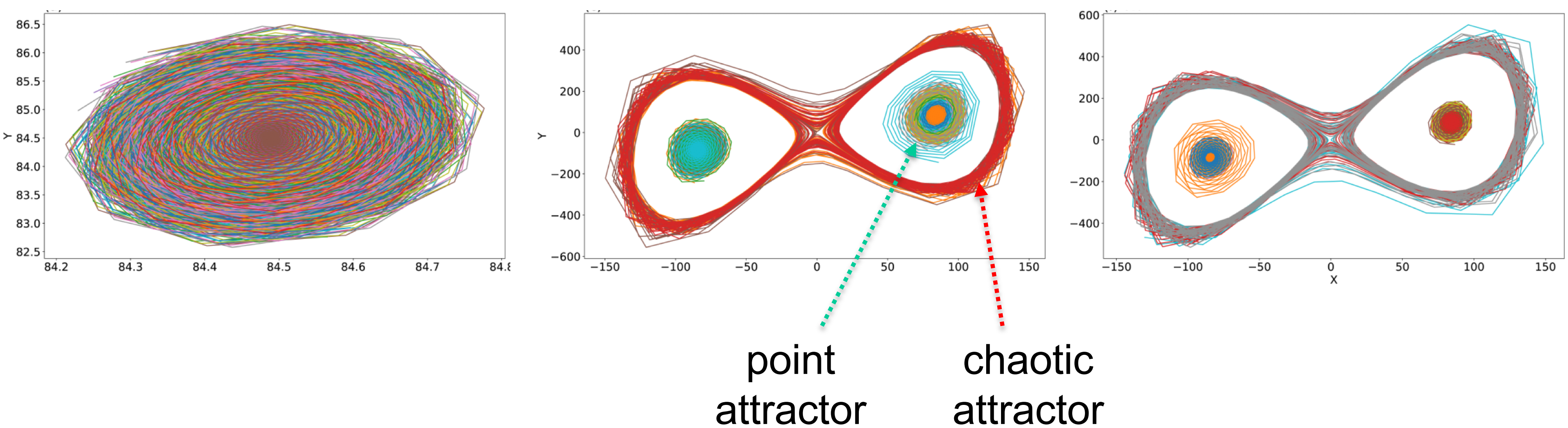
The coexistence of chaotic and non-chaotic orbits displays distinct regions of attraction.

(Note that the figure displays 256 orbits with different ICs)

Ensemble Runs for Revealing Attractor Coexistence

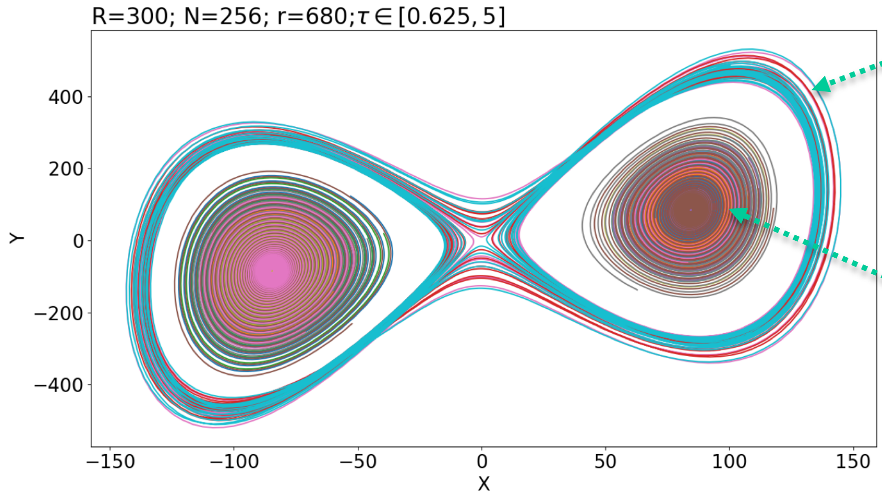
- Ensemble runs with N members were performed to reveal attractor coexistence.
- Generated as Gaussian random variables, initial conditions are distributed over a hypersphere with a center at the non-trivial critical point.
- The “radius” of the hypersphere, R, represents the spatial extent of the initial conditions within the phase space.

N=4096 and R=5 | N=512 and R=200 | N=64 and R=500



Two Kinds of Dependence on ICs

- The 9DLM with attractor coexistence displays two kinds of data dependence.



a chaotic attractor displays:

- sensitive dependence on ICs
- i.e., BE1

a point attractor displays:

- insensitivity to ICs
- i.e., no BE1

- The role of initial tiny perturbations is different within the 3D- and 9D-LM.

3DLM	9DLM
always important	important or unimportant
BE1 always appears	BE1 may or may not appear

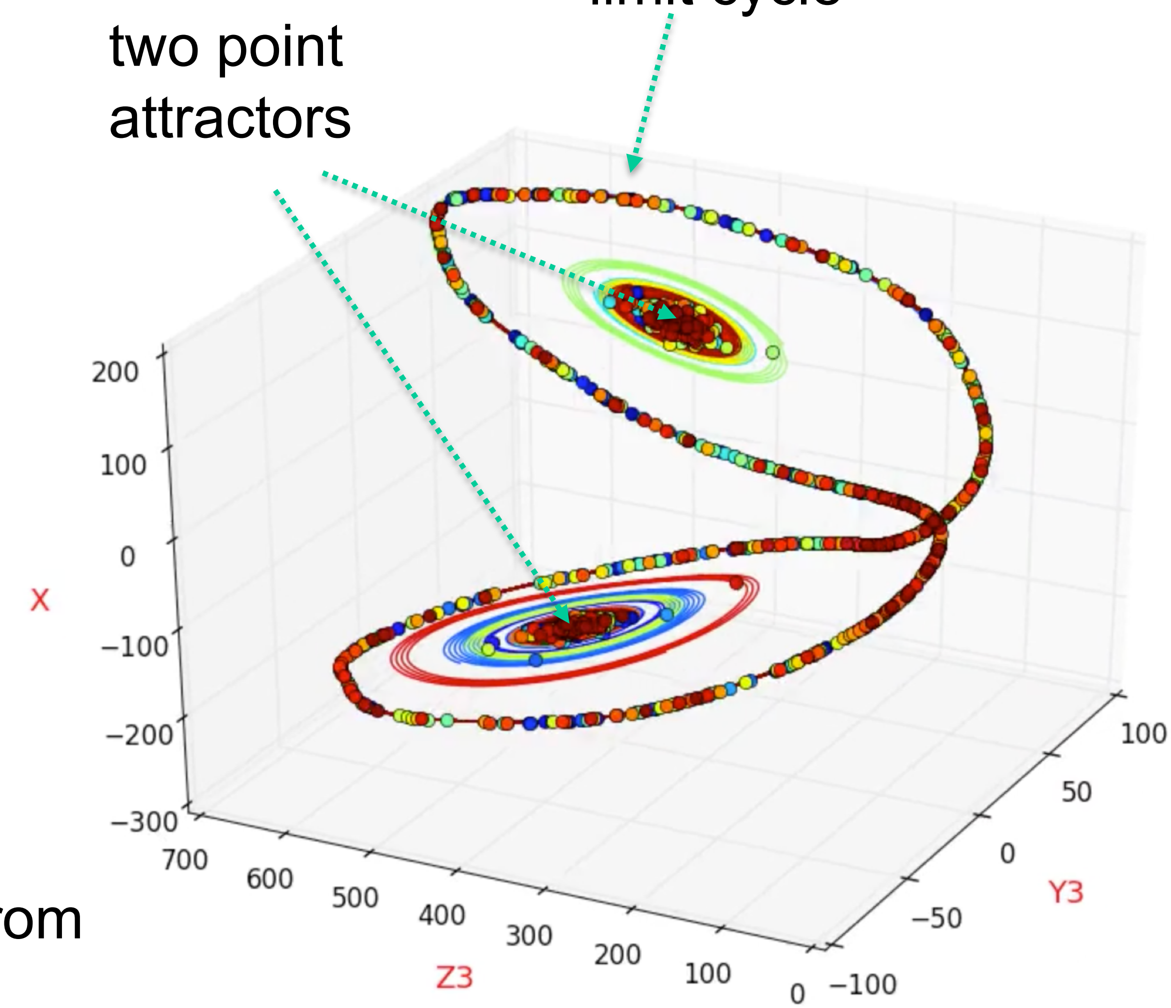
- As a result, the 9DLM is more realistic, as compared to the 3DLM. The BE1 does not always appear.

The Second Kind of Attractor Coexistence

- For the 2nd kind of attractor coexistence, a limit cycle (LC) that is an isolated closed orbit coexists with point attractors.

r=1600

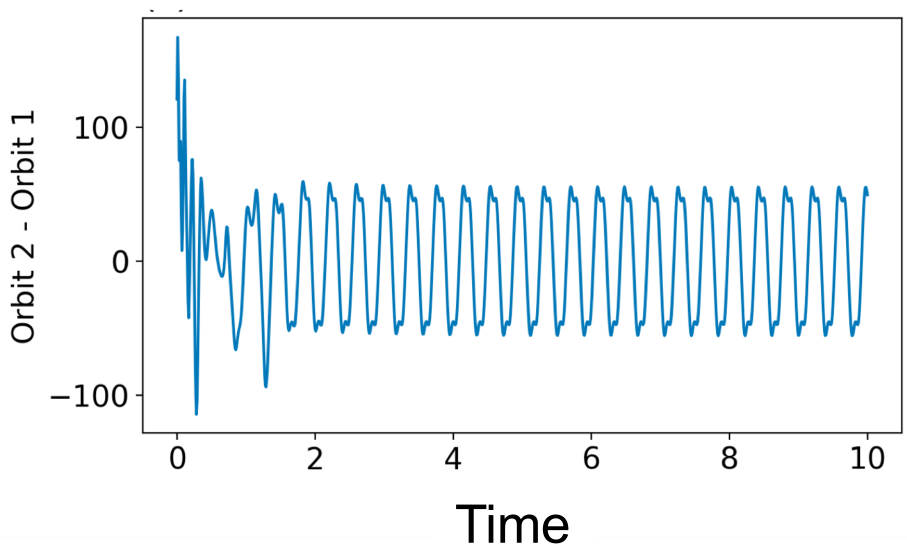
- Time evolution of 2,048 orbits in the X-Y_3-Z_3 space using the 9DLM.
- The total simulation time is τ = 3.5.
- Transient orbits are only kept for the last 0.25 time units, i.e. for the time interval of [max (0, T-0.25), T] at a given time T.

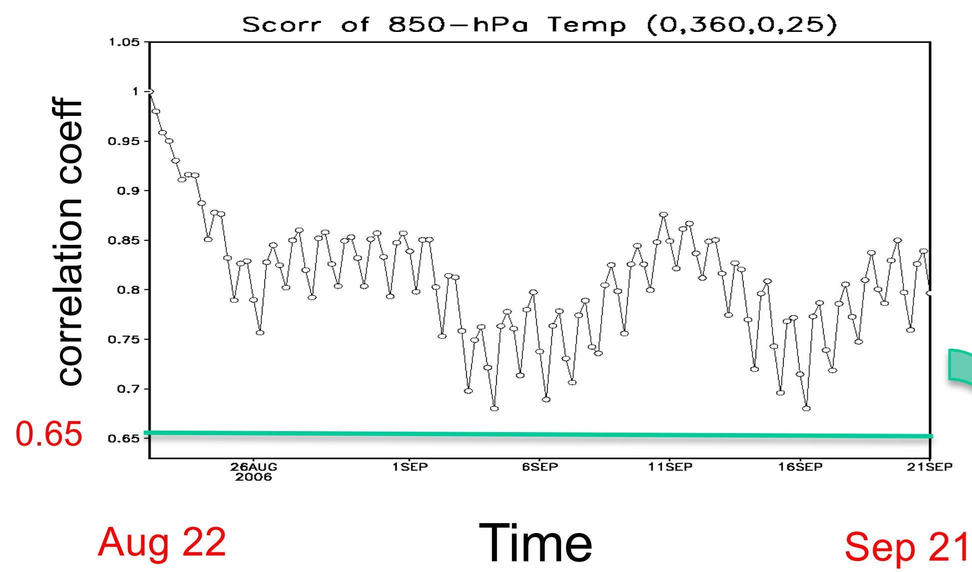


- The animation is available from https://goo.gl/sMhoUb.

A Hypothetical Mechanism for the Predictability of the 30-day Runs

Limit Cycle	African Easterly Waves (AEWs)
oscillatory errors	oscillatory correlation coefficients



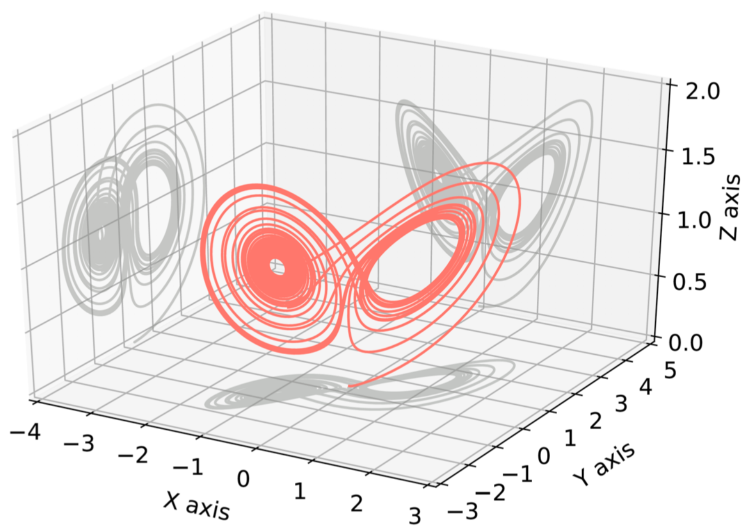


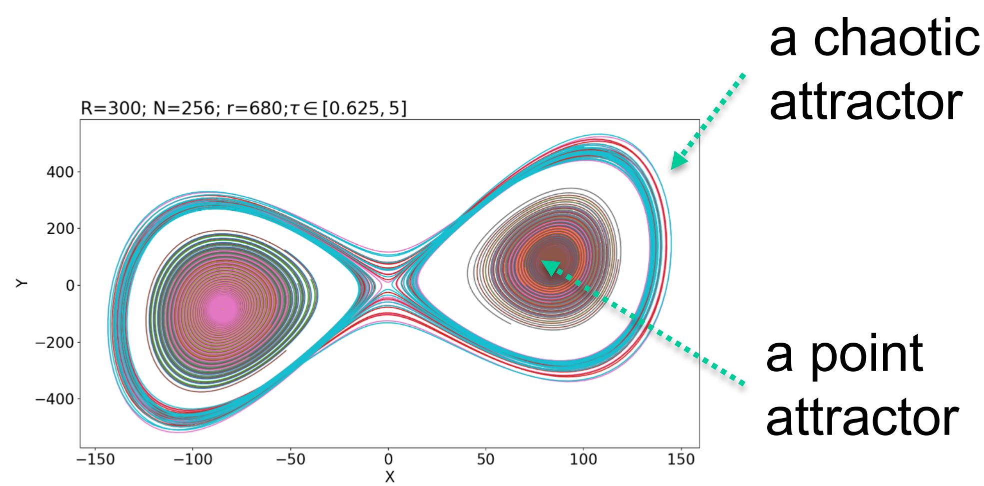
strong heating + nonlinearity	during summer (JAS)

- The realistic simulation of Hurricane Helene (2006) from Day 22 to 30 became possible as a result of the realistic simulation of the
  1. periodicity (or recurrence) of AEWs and
  2. downscaling process of the 4th AEW.

Monostability vs. Multistability

The Lorenz Model	The Generalized Lorenz Model
Single (Type) Attractors*	Coexisting Attractors
Monostability	Multistability
Unstable critical points	Coexisting stable and unstable critical points
SDIC**	SDIC or no SDIC
*coexisting attractors appear within a small range of r, $24.06 < r < 24.74$ (Yorke & Yorke, 1979)	**SDIC: sensitive dependence on initial conditions





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https://doi.org/10.1175/BAMS-D-19-0165.1

By revealing two kinds of attractor coexistence within Lorenz models, we suggest that the entirety of weather possesses a dual nature of chaos and order with distinct predictability.

The two kinds of attractor existence may be enabled and/or modulated by the following two mechanisms:

(1) the aggregated negative feedback of small-scale convective processes,
(2) the large-scale time varying forcing (heating).

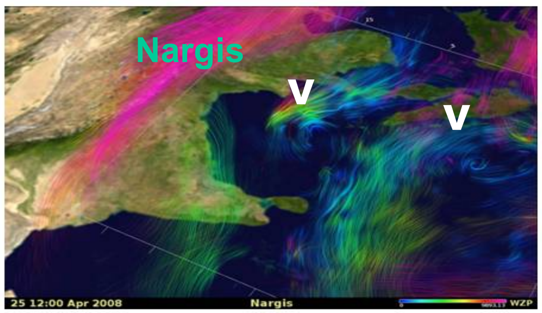
A Conceptual Multiscale Model in 2010

Large Scale — Medium Scale — Small Scale

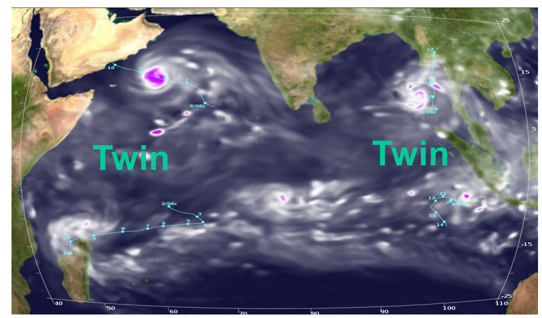
Tropical Waves ⟷ PEEMD ⟷ Tropical Cyclone Formation ⟷ SAT ⟷ small-scale processes

Downscaling Processes (modulation)

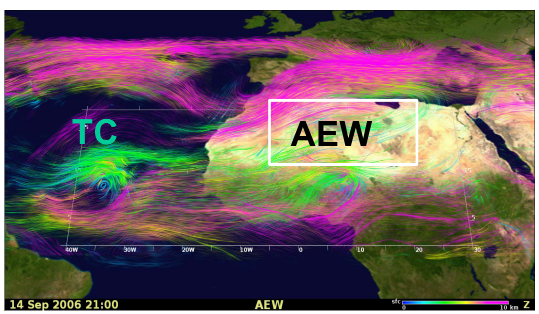
Upscaling Prcoesses (feedback)



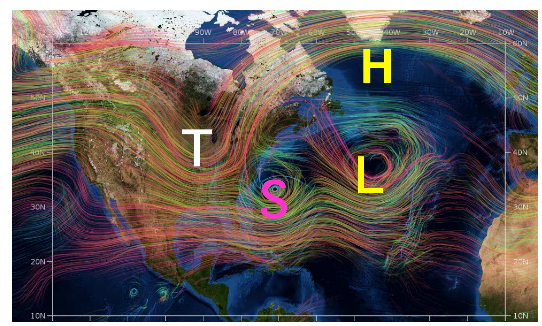
Nargis and an ER Wave (Shen et al. 2010a)



Twin TCs and a MRG Wave (Shen et al., 2012)



Helene and an AEW (Shen et al., 2010b)



Sandy and Multiscale Systems (Shen et al., 2013)

Why Downscale Processes? Modulations

- To what extent can large-scale flows (e.g., AEWs) determine the timing and location of TC genesis? (e.g., downscaling)

➔ Why Downscaling Processes?

- Hierarchical Scale Dependence (e.g., 7DLM)
- Attractor Coexistence (e.g., 9DLM)
  - BE1 does not always appear in the models
  - Some tiny perturbations may dissipate
  - Small-scale dissipative systems should possess deterministic predictability
- Comparable Downscale and Upscale Transfer (Lorenz, 1969)

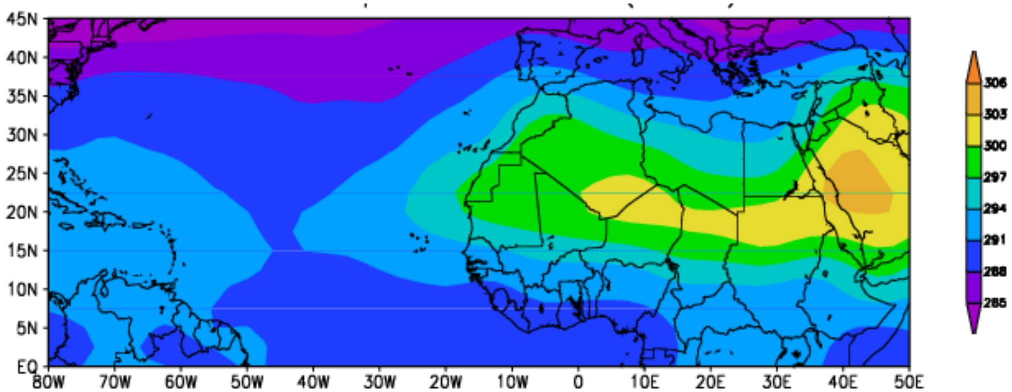
Five AEWs in 30-day Simulations

- Initial conditions: at 00zz Aug 22, 2006

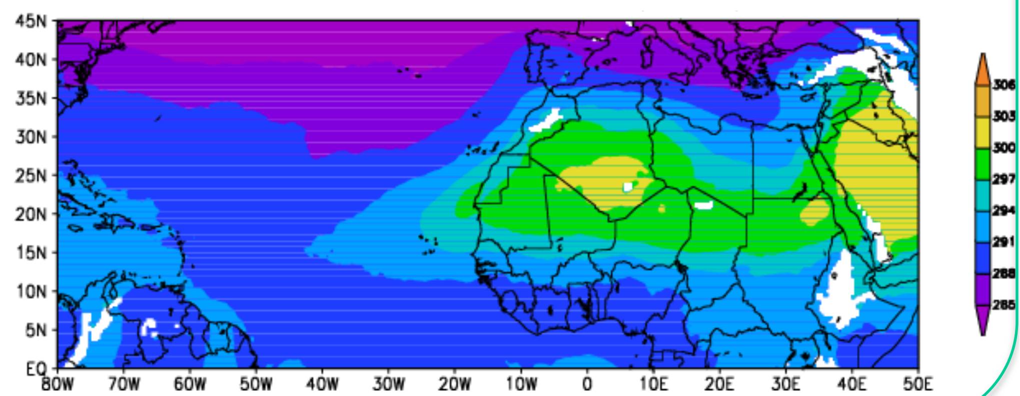
Shen, B.-W. et al., 2010b:

realistic simulation of ”environmental flows”

NCEP Reanalysis (Ave T of 850-hpa)

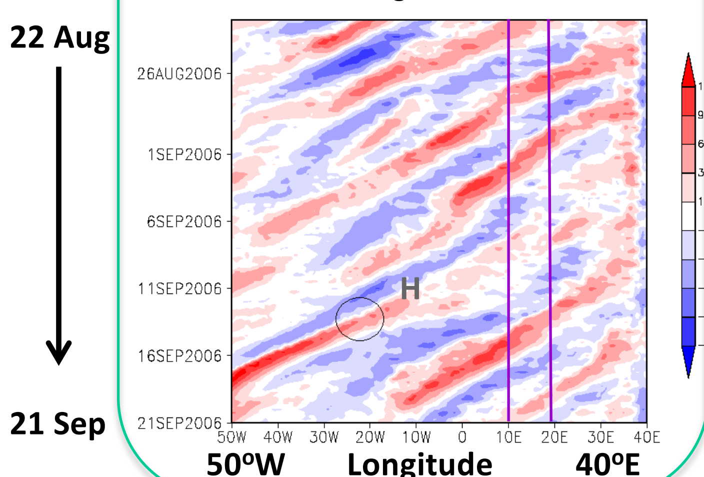


Model Simulations (Ave T of 850-hpa)



multiple AEWs

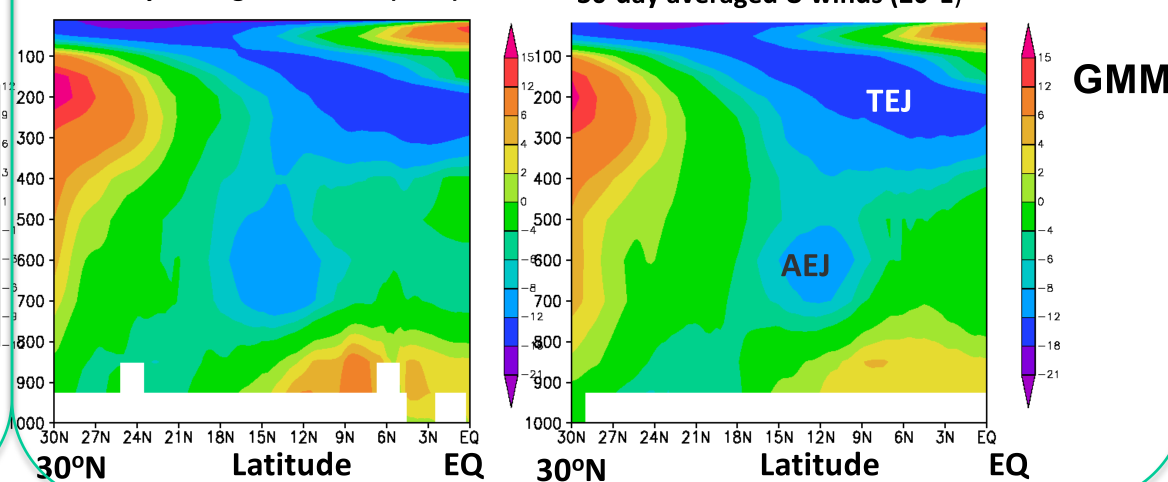
V winds averaged over 5°-20°N



AEJ (African Easterly Jet)

30-day averaged U winds (10°E)

30-day averaged U winds (20°E)



Hilbert Huang Transform (HHT): EMD

PEEMD: parallel EEMD (Shen et al., 2016; 2017; Wu and Shen, 2016)

EEMD: ensemble empirical mode decomposition (Wu and Huang, 2009)

EMD: empirical mode decomposition

HHT = EMD + Hilbert Transform (Huang et al., 1998)

Raw data $\xrightarrow{\textbf{FFT}}$ **Fourier Wave Modes**

- Linear
- Orthogonal

Raw data $\xrightarrow{\textbf{EMD}}$ **Intrinsic Mode Functions (IMFs)**

- each IMF consists of oscillatory components with comparable frequencies
- A trend mode: non-oscillatory components = raw data – sum of IMFs
  ➔ representing the environmental flow

EMD as a Filter Bank

- Each of the IMFs contains signals at comparable scales.

The right figure displays the first 9 IMFs for the Gaussian White Noises with 2^{20} (1 million) points, showing the characteristics of the bank filters (i.e., a dyadic filter).

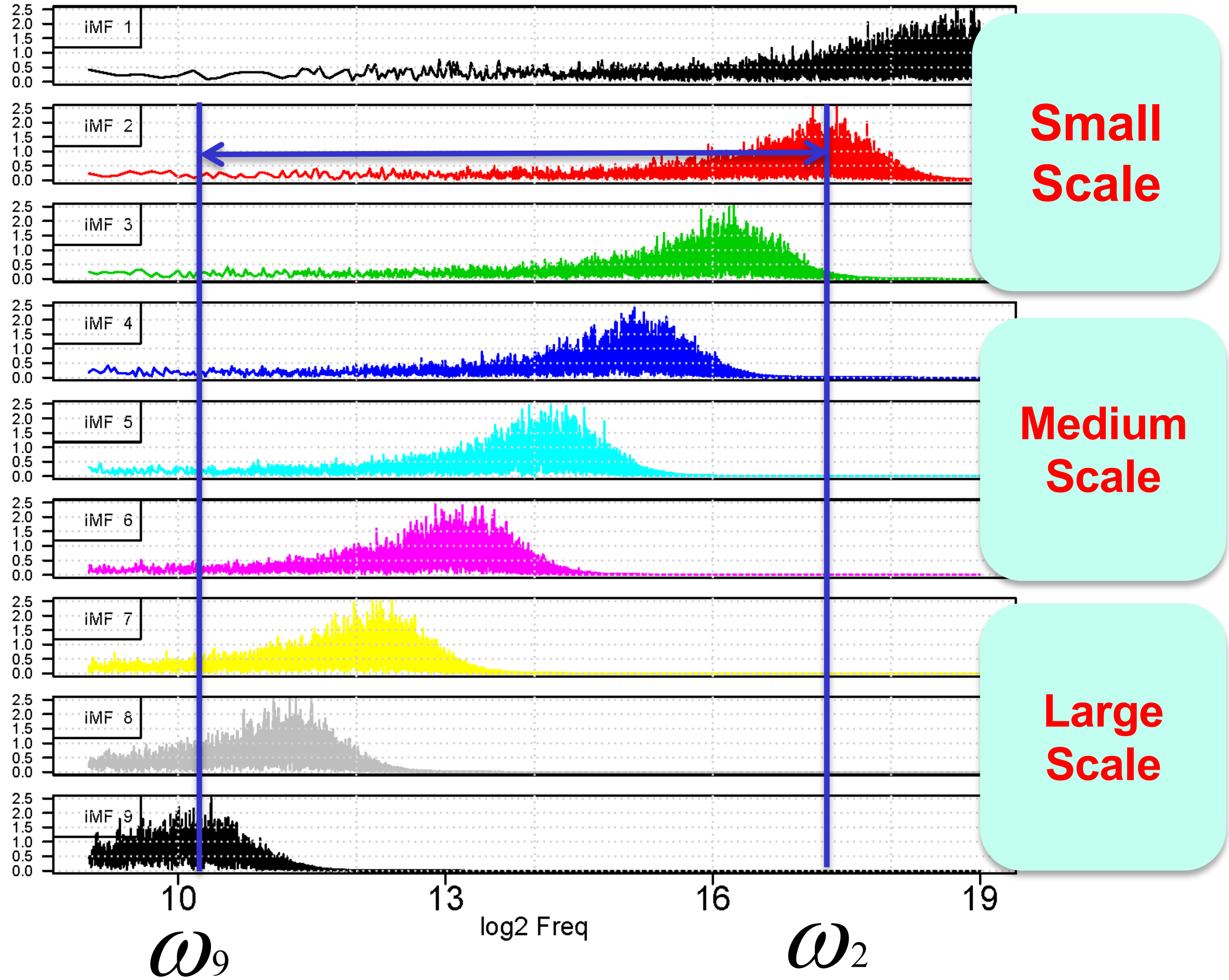
Assume T and ω (=1/T) to be the period and frequency, respectively, we have

$$\log_2(\omega) = -\log_2(T)$$

$$\log_2(T_{n+1}) - \log_2(T_n) = 1$$

$$\boxed{T_{n+1}/T_n = 2}$$

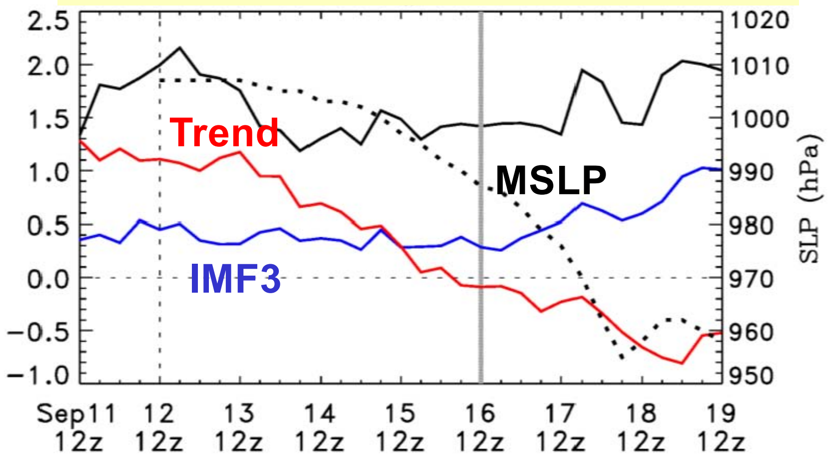
which indicates a doubling of the mean period.



See details in Wu et al. (2004); Reproduced by Shen et al. (2012b).

A 10-year Multiscale Analysis with the PEEMD

Downscale Transfer (Helene)



In the mid-east Main Development Region (MDR, 7º-20º N and 60º W - 15º E.)

- 42 TD/TSs appeared in association with AEWs,
- 25 of these TDs/TSs eventually turned into hurricanes,
- 13 hurricanes showed the features of downscaling processes.

Physical interpretations of decomposed components

- ❖ Uy in the trend mode indicates the magnitude of shear of the basic state, a potential indicator of (barotropic) shear instability.
- ❖ Uy in the IMF3 indicates the strength of a TC or an AEW (as vorticity).

Approach using the PEEMD:

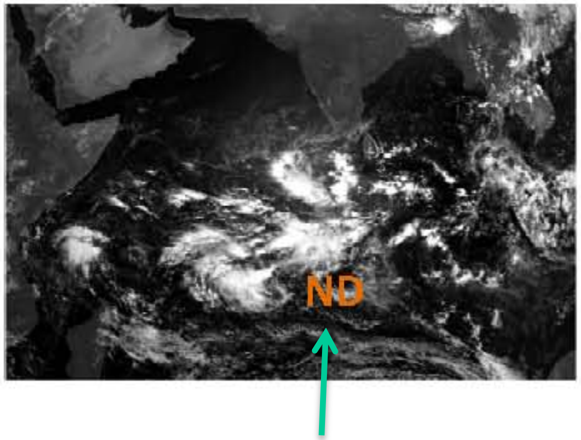
1. Decompose U (and V) winds at 700 hPa into a set of IMFs
2. Calculate the wind shear, Uy (and Vx), in each of the IMFs
3. Calculate spatial average Uy (and Vx) as a function of time

- Wu, Y.-L.,and B.-W. Shen, **2016:** An Evaluation of the Parallel Ensemble Empirical Mode Decomposition Method in Revealing the Role of Downscaling Processes Associated with African Easterly Waves in Tropical Cyclone Genesis. J. Atmos. Oceanic Technol. 33, 1611-1628, DOI: 10.1175/JTECH-D-15-0257.1.

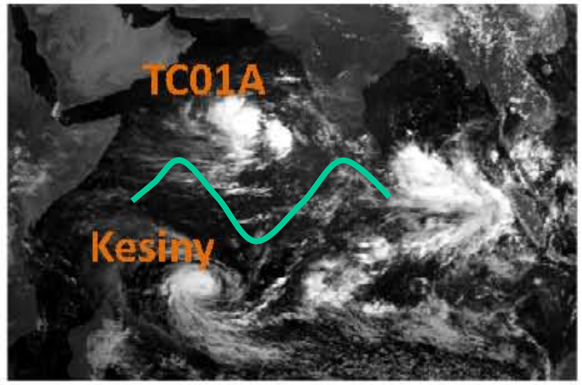
Observations: Twin Tropical Cyclones

MOAST

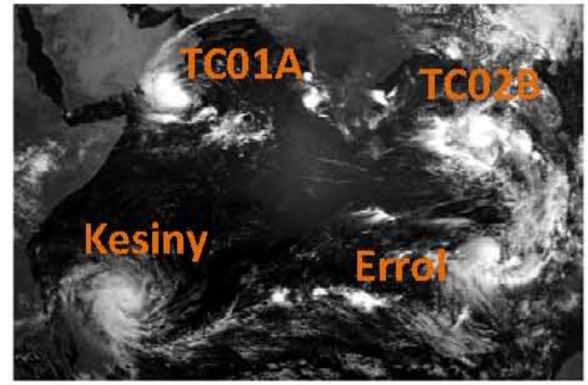
(a) 0630 UTC 1 May 2002



(b) 0000 UTC 6 May 2002

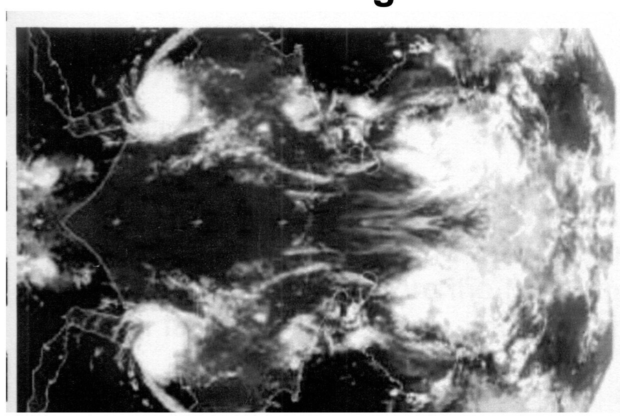


(c) 0000 UTC 9 May 2002



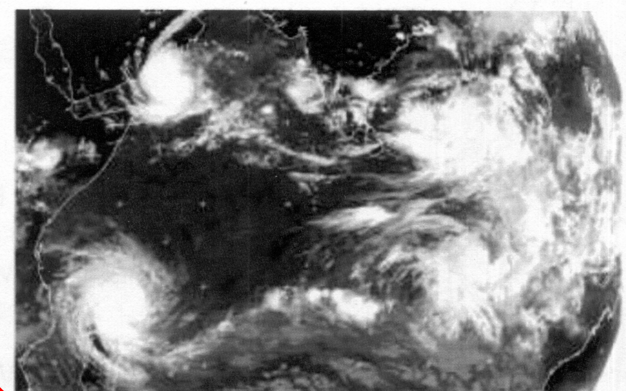
EQ

South mirroring north



EQ

0900 UTC May 9 2002



Three convective systems are:

1. ND, (non-developing),
2. Kesiny (05/03/06z – 05/11/18z), and
3. TC01A (05/06/18z – 05/10/12z).

- Next, we will show the performance of the model in predicting the above TCs.
- We also reveal the association with the TCs with a large-scale system (MJO).

Early Results: Genesis of Six TCs

Init at 05/01

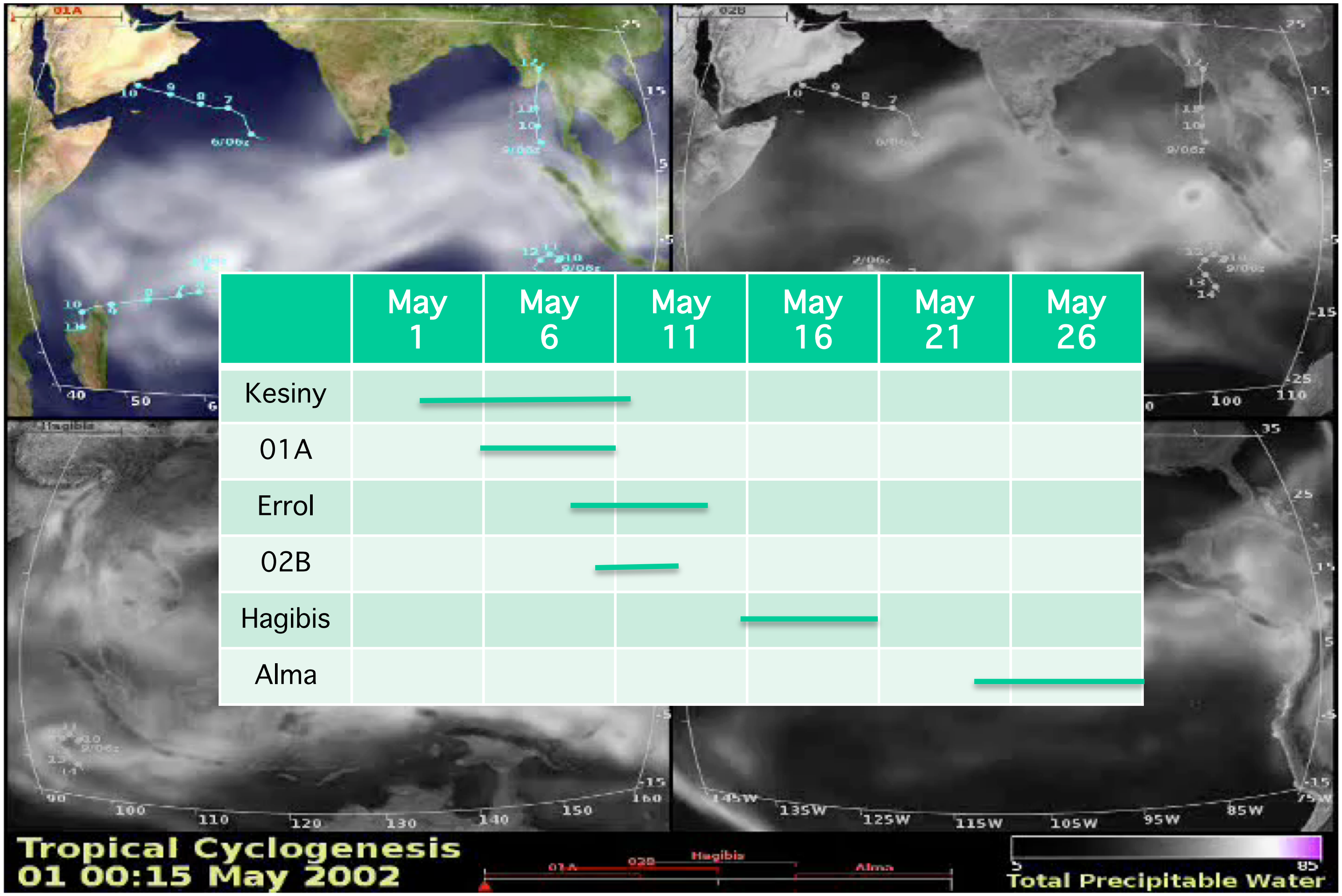
TC01A
(May 6-10)

Kesiny
(May 3-11)

Init at 05/11

Typhoon Hagibis
(May 15-21)

Best tracks (observations) indicated by blue lines:



	May 1	May 6	May 11	May 16	May 21	May 26
Kesiny						
01A						
Errol						
02B						
Hagibis						
Alma						

Init at 05/06

TC02B
(May 10-12)

Errol
(May 9-14)

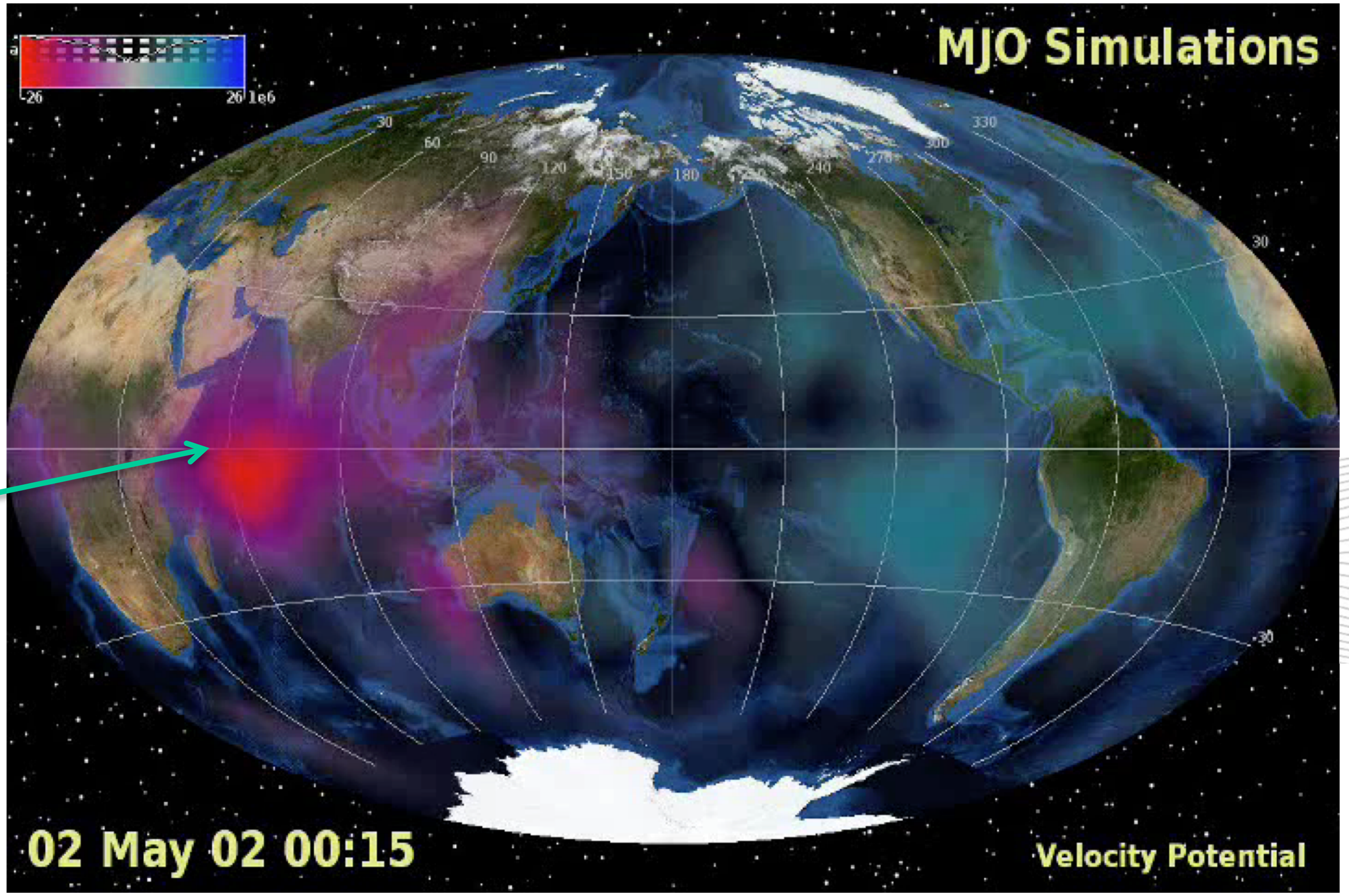
Init at 05/22

Hurricane Alma
(May 24- June 1)

``Although some aspects of the transformation of atmospheric disturbances into tropical cyclones are relatively well understood, the general problem of tropical cyclogenesis remains in large measure, one of the greatest mysteries of the tropical atmosphere." – Kerry Emanuel, The Divine Wind (2005)

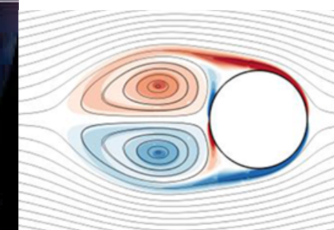
Modulations of TCs by an MJO in May 2002

200mb
Velocity
Potential



Indian
Ocean

Moving
cylinder



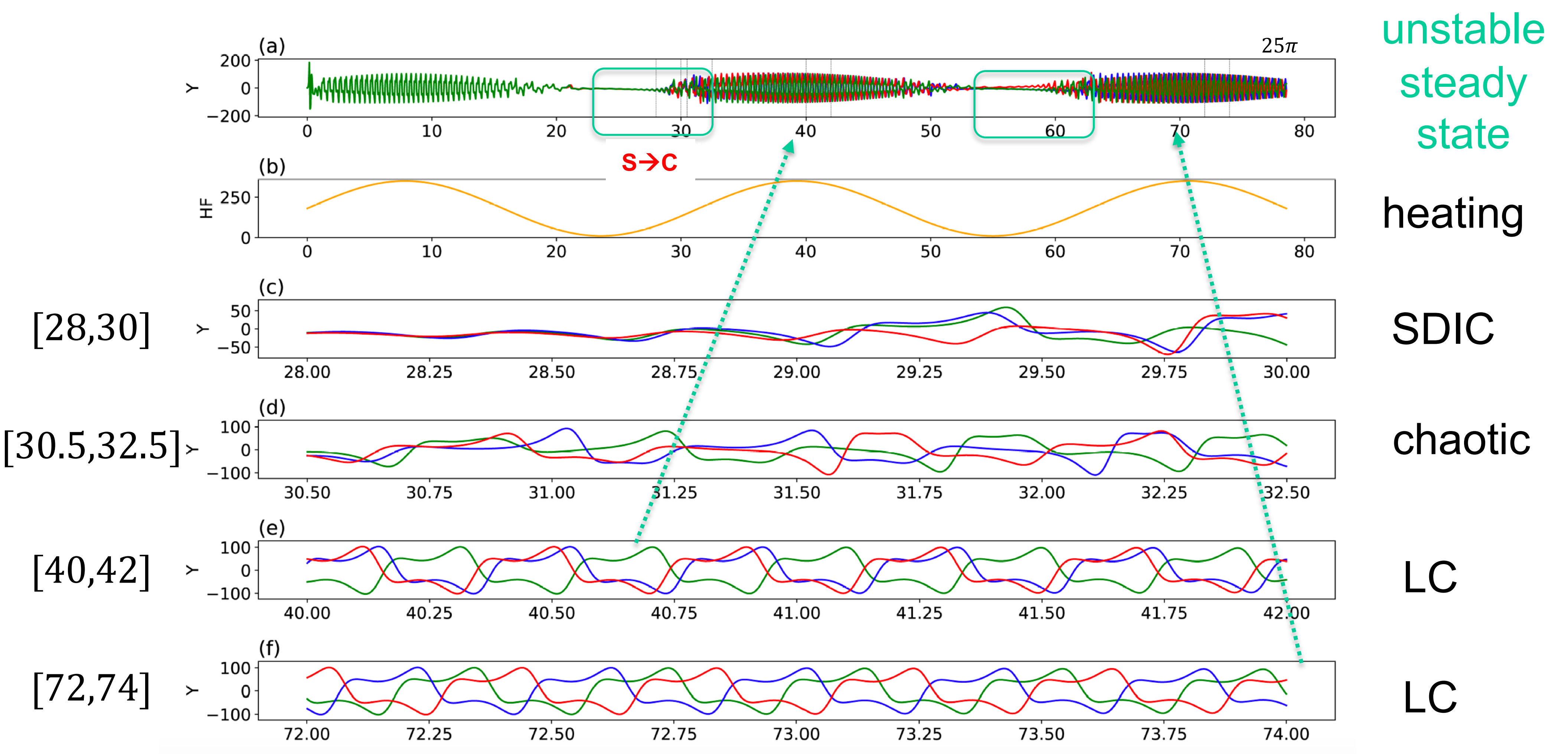
The MJO, also referred to as the 30-60 day or 40-50 day oscillation, turns out to be the main intra-annual fluctuation that explains weather variations in the tropics. <u>The MJO affects the entire tropical troposphere</u> but is most evident in the Indian and western Pacific Oceans.

Complexities: Time Varying Parameters

- Three types of solutions can be found within the 3DLM.
- Two kinds of attractor coexistence are documented within the 9DLM. The first kind of attractor coexistence consists of chaotic and steady-state solutions, while the second kind of attractor coexistence includes limit cycle and steady state solutions.
- Additionally, coexisting two periodic solutions were documented using the 9DLM with r = 1120 (e.g., Shen 2019a).
- As a result, when system parameters change at a large time scale (e.g., at climate time scales), different kinds of attractor coexistence may alternatively or concurrently appear, leading to complexities that better resemble real weather and climate.

The Alternative Appearance of Solutions

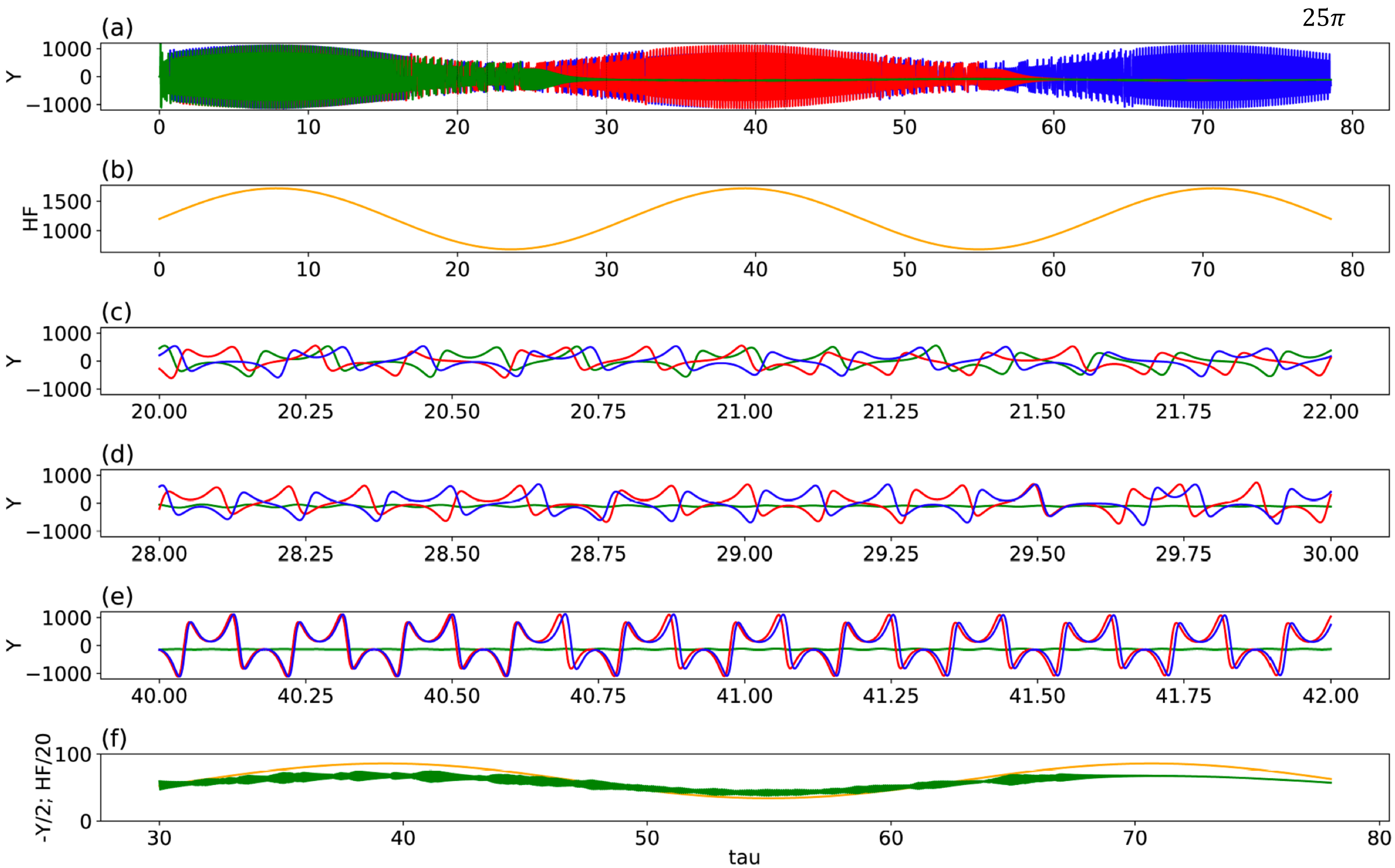
- Three initial, nearby trajectories evolve with a periodic heating function.
- The 3DLM displays one type of solutions at a given time.
- It shows a transition from a stable to unstable steady state solution (S→C).
- SDIC: sensitive dependence on IC
- $r(\tau) = 180 + 170\sin\left(\frac{\tau}{5}\right)$; $r \in [10,350]$



The Alternative and Concurrent Appearance

- Three initial, nearby trajectories evolve with a periodic heating function.
- The GLM may display two types of solutions at a given time.
- It does not have a transition from a stable to chaotic orbit.

- $r(\tau) = 1200 + 520\sin\left(\frac{\tau}{5}\right); \quad r \in [680,1720]$



[20,22] chaotic

[28,30] 1^{st} kind

[40,42] 2^{nd} kind

[30,78] large-scale stable steady

heating

Mathematical Universality: 3DLM, KdV, and NLS

A 3D non-dissipative LM	Solutions	Other Equations
$$\frac{d^2X}{d\tau^2} - \left(\sigma r + \frac{C_1}{C_0}\right)X + \frac{X^3}{2} = 0$$	cn()	Duffing Eq. $X'' + \delta X' + \alpha X + \beta X^3 = \gamma\cos(\omega\tau)$ $\delta = 0 \ and \ \gamma = 0$
DE-sech $$\left(\frac{dX}{d\tau}\right)^2 - \sigma r X^2 + \frac{1}{4}X^4 = 0$$	$sech()$	Nonlinear Schrodinger (NLS) Eq. $(h')^2 + \delta h^2 + \frac{\gamma}{2}h^4 = E = 0$ $\delta < 0 \ and \ \gamma > 0$
DE-sech^2 $$\frac{d^2Z}{d\varsigma^2} + 3Z^2 - 4rZ = 0$$ $\varsigma = \sqrt{\sigma}\tau$	$sech^2()$	Korteweg-de Vries (KdV) Eq. $u_t + 6uu_x + u_{xxx} = 0$ $u = f(x - ct)$ $f'' + 3f^2 - cf = 0$ (with c = 4r)

$$X' = \pm\sqrt{\sigma r}\, X\sqrt{1 - \frac{X^2}{4\sigma r}}$$

$sigmoid$

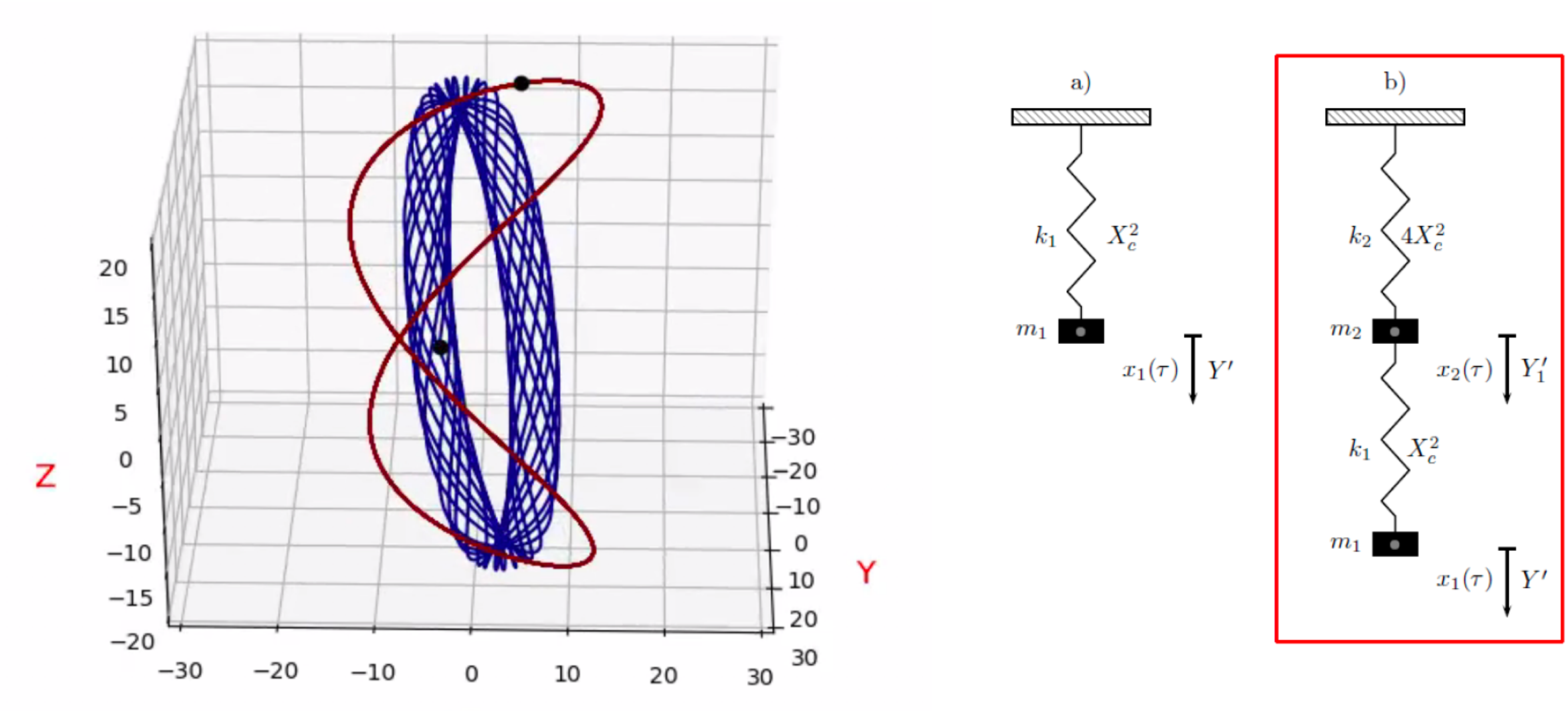
$E = X^2$

The Lorenz and the Pedlosky Models

Model	The Lorenz Model	The Pedlosky Model	References
Dynamics of the PDEs	Rayleigh-Benard Convection	Finite-Amplitude Baroclinic Wave	
dissipative or viscid	$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$ $\frac{dY}{d\tau} = -XZ + rX - Y,$ $\frac{dZ}{d\tau} = XY - bZ$	$\frac{d^2R}{d\tau^2} + \alpha\eta\frac{dR}{d\tau} - R + R(R^2 - D) = 0$ $\frac{dD}{d\tau} + \eta D + \beta\eta R^2 = 0$	Eq. 6.9 of P71; Eq. 2.11a, b of PF80
	$r = \frac{1+\sigma}{\sigma}$	$R^2 = \frac{X^2}{2}; D = \frac{1}{2}X^2 - \sigma Z$ $\alpha = \frac{1+\sigma}{b}; \beta = \frac{2\sigma - b}{b}; \eta = b$	
non-dissipative or inviscid	$\frac{d^2X}{d\tau^2} - (\sigma r + \frac{C_1}{C_o})X + \frac{X^3}{2} = 0$	$\frac{d^2R}{d\tau^2} - (1 + D_o)R + R^3 = 0$ $\frac{dD}{d\tau} = 0$	Eq. 6.8 of P70
	$\left(\frac{dX}{d\tau}\right)^2 - (\sigma r + \frac{C_1}{C_o})X^2 + \frac{X^4}{4} = 0$	$(\frac{dR}{d\tau})^2 - (1 + D_o)R^2 + \frac{R^4}{2} = C_3$	Eq. 3.7 of P72
	C_1 and C_2 are constants	D_o and C_3 constants	

Table: Mathematical Similarities in the Lorenz and Pedlosky models. The 3DLM and Pedlosky71 refer to the Eqs. 25-27 of Lorenz 1963 and Eq. 6.9 of Pedlosky 1971, respectively. Pedlosky (1970), Pedlosky (1971), Pedlosky (1972), and Pedlosky and Frenzen (1980) are denoted as P70, P71 P72 and PF80, respectively.

Quasi-periodicity via (Physical) Spatial Interactions



(left) A closed orbit (red) and torus with dense orbit (blue) obtained from the 3D and 5D non-dissipative Lorenz Models (NLMs), respectively. (right) The locally linear 5D NLM is equivalent to the mathematical model of two coupled springs (Faghih-Naini and Shen, 2018)

Opportunities for Students' Projects

- Quasi-periodic orbits in high dimensional space
  - their transition to high-dimensional chaos
  - the role of the nonlinear feedback loop (NFL) and its extension
- Multiple time scales within the GLM
  - slow vs. fast variables
  - in relation to non-autonomous systems
- Recurrence plot (RP) analysis and kernel PCA (K-PCA)
  - transition from oscillatory growing (decaying) orbits to chaotic orbits
  - "detection" of attractor coexistence with the RP and K-PCA (e.g., classification of chaotic and non-chaotic orbits)
- The 3D non-dissipative LM (3D-NLM) and homoclinic orbits
  - the 3D-NLM vs. KdV and NLS equations
  - homoclinic orbits in high-dimensional (non-dissipative) Lorenz models
- The Lorenz error growth model & computational chaos
- Various Lorenz models (1963, 1969, 1984, 1996, 2005)
- Simple turbulence and spatiotemporal chaos models
  - Leith's model (Leith, 1971; Leith and Krainchnan, 1972)
  - the Benney, KdV, and Kuramoto-Savashinsky equations.

Concluding Remarks

1. We have developed the generalized Lorenz model with the major feature of two kinds of attractor coexistence (Shen 2019a, b). The BE1 does not always appear (Shen et al. 2019).
2. Two kinds of attractor coexistence are enabled and/or modulated by (1) the aggregated negative feedback of small-scale convective processes and (2) the large-scale time varying forcing (heating).
3. We propose the revised view on the dual nature of chaos and order in weather and climate (Shen et al. 2019; 2020)
4. The refined view on the dual nature of weather is neither too optimistic nor pessimistic as compared to the Laplacian view of deterministic predictability that is unlimited and the Lorenz view of deterministic chaos with finite predictability.
5. The refined view may unify our theoretical understanding of different predictability within various types of solutions of Lorenz models and recent numerical simulations of advanced global weather/climate models that can simulate large-scale tropical systems (e.g., African easterly waves and Madden-Julian Oscillations) beyond two weeks (Shen 2010b, 2011; Shen 2019b).

”A Paradigm Shift” in Predictability Study

- *``As with Poincare and Birkhoff, everything centers around periodic solutions” (Lorenz, 1993).*

- After Lorenz (1963, 1972), Prof. Lorenz and chaos researchers focused on the existence of non-periodic solutions and their complexities.

- Based on the concept of attractor coexistence within the original and generalized Lorenz models (Shen, 2019a), we (Shen et al., 2020a, b) propose a revised view that focus on the duality of chaos and order.

- An effective detection and classification of chaotic and non-chaotic processes may help extend the lead time of predictions.

Future Tasks

- Detect (Nonlinear) Oscillatory Signals by
  - applying the Parallel Ensemble Empirical Mode Decomposition (PEEMD) to decompose data into oscillatory modes and non-oscillatory trend mode (Shen et al. 2017; Wu and Shen, 2016);
  - performing the Recurrence Analysis (Reyes and Shen, 2019a,b);
  - performing the Kernel Principle Component Analysis for classification of solutions (Cui and Shen, 2019, under revision).
  - [Apply the above to analyze MJO signals and compare results with those using the analysis of Real-time Multivariate MJO (RMM) Index]
- Improve the Simulations of Nonlinear Oscillatory Signals (e.g., limit cycle or quasi-periodic orbits) by
  - reducing numerical dissipations to avoid computational chaos (e.g., the Logistic equation vs. the Logistic map; Lorenz, 1989)
  - examining the potential impact of increased resolutions and newly added components on the generation of new incommensurate or commensurate frequencies, leading to quasi-periodic solutions.

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Thank you!