Assignment 3, Math 330

Name:

Due Thursday, October 3. Feel free to work with others on this and other homework assignments.

1. Use the $N - \epsilon$ verification of convergence for the sequence $\left\{ \dfrac{7n^2}{n^2 - n} \right\}_{n=2}^{\infty}$.

2. Prove the following corollary to our Boundedness Lemma. You can use the lemma we proved or just mimic its proof – it's easier to use it in my opinion.
   Suppose that $\lim\limits_{n\to\infty} a_n = a \neq c$. Prove $\exists \beta > 0$ and $N \in \mathbb{N}$ such that $\forall n \in \mathbb{N}$, if $n \geq N$, then $\beta < |a_n - c|$.

3. Suppose that $\lim\limits_{n\to\infty} a_n = a$. Prove that $\lim\limits_{n\to\infty} a_n^2 = a^2$.
   Use an $N - \epsilon$ verification rather than Theorem 2.13 Product Property for Limits – in fact, look at our proof in class for Theorem 2.13 and try to mimic that proof.

4. Closed sets and not...
   (a) Show that the set $[20, \infty)$ is closed.
   (b) Show that the set $\bigcup\limits_{n=1}^{\infty} \left[\frac{1}{n}, 1\right]$ is not closed.

5. Prove that $\lim\limits_{n\to\infty} (\sqrt{n+1} - \sqrt{n}) = 0$. Who says rationalizing numerators is not important? Not me, never.

6. They call this extra credit.
   * Suppose that $0 < r < 1$. Prove that $\exists N \in \mathbb{N}$ such that $\forall n > N$ we have
   $$\frac{(n+1)r^{n+1}}{nr^n} < 1.$$
   ** Suppose that $0 < r < 1$. Prove that $\lim\limits_{n\to\infty} nr^n = 0$