Homework 5
Abstract Algebra
Math 320
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Section 2.2 Problem 14: Solve the following equations:

a) $x^2 + x = [0]$ in $\mathbb{Z}_5$

b) $x^2 + x = [0]$ in $\mathbb{Z}_6$

If p is prime, prove that solutions of the equation below are [0] and [p-1]

c) $x^2 + x = [0]$ in $\mathbb{Z}_p$

Solution (a): x = 0, 4

x	x^2 + x
0	[0][0] + [0] = [0]
1	[1][1] + [1] = [3]
2	[2][2] + [2] = [1]
3	[3][3] + [3] = [2]
4	[4][4] + [4] = [0]

Solution (b): x = 0, 2, 3, 5

x	x^2 + x
0	[0][0] + [0] = [0]
1	[1][1] + [1] = [3]
2	[2][2] + [2] = [0]
3	[3][3] + [3] = [0]
4	[4][4] + [4] = [2]
5	[5][5] + [5] = [0]

Solution (c) Let p be prime.

$$
\begin{aligned}
&x^2 + x = [0] \in \mathbb{Z}_p \\
&[x(x+1)] = [0] \\
&[(0)(0+1)] = [0] \\
&[(p-1)(p-1+p)] = [p] = [0]
\end{aligned}
$$

For $x^2 + x = [0] \in \mathbb{Z}_n$, the solutions will be 0, n-1, and $\{q \in \mathbb{Z}^+ | q(q+1) = kn \ \forall k \in \mathbb{Z}\}$. Because prime numbers don't have any factors, except itself and 1, the only solutions would be 0 and n-1.

Section 2.3 Problem 1: Find all Units in

a) $\mathbb{Z}_7$ b) $\mathbb{Z}_8$ c) $\mathbb{Z}_9$ d) $\mathbb{Z}_{10}$

Solution

a) $1, 2, 3, 4, 5, 6$ b) $1, 3, 5, 7$ c) $1, 4, 5, 7, 8$ d) $1, 3, 7, 9$

Section 2.3 Problem 2: Find all Zero Divisors in

a) $\mathbb{Z}_7$ b) $\mathbb{Z}_8$ c) $\mathbb{Z}_9$ d) $\mathbb{Z}_{10}$

Solution

a) none b) $2, 4$ c) 3 d) $2, 5$

Section 2.3 Problem 6: If n is composite, prove that there is at least one zero divisor in $\mathbb{Z}_n$.

Solution
Let n be composite, so let $0 < q < n$, be a factor of n, so that $n = qr$. So

$$[0] = [n] = [qr] = [q][r]$$

Thus their exists some $0 < q < n$, that when multiplied with an nonzero number, r, qr = 0.