Today 11/14 • Review Problems

• Derivative Definition / Intro Stuff

Example: To help w/ HW 5 ①

Suppose $f : [a, \infty) \to \mathbb{R}$.

Suppose $f_1 : [a,b] \to \mathbb{R}$ by $f_1(x) = f(x)$ is uniformly continuous

Suppose $f_2 : [b, \infty) \to \mathbb{R}$ by $f_2(x) = f(x)$ is uniformly continuous.

Then f is uniformly continuous.

proof: Let $\varepsilon > 0$, Choose $\delta_1, \delta_2 > 0$ s.t.

$\forall x, y \in [a,b]$, if $|x-y| < \delta_1$, then $|f_1(x) - f_1(y)| < \varepsilon/2$

and $\forall x, y \in [b, \infty)$, if $|x-y| < \delta_2$, then $|f_2(x) - f_2(y)| < \varepsilon/2$.

Let $\delta = \min\{\delta_1, \delta_2\} > 0$. Suppose $x, y \in [a, \infty)$ with

$x < y$ and $|x-y| < \delta$.

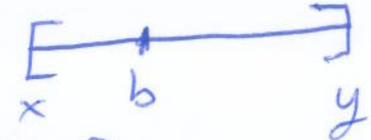
Case 1: Suppose $x, y \in [a,b]$. Since $|x-y| < \delta \le \delta_1$,

$$|f_1(x) - f_1(y)| = |f(x) - f(y)| < \tfrac{\varepsilon}{2} < \varepsilon.$$

Case 2: Suppose $x, y \in [b, \infty)$ — similar to case 1.

Case 3: Suppose $x \in [a,b]$ and $y \in [b, \infty)$.

So $x \le b \le y$.



Notice $b - x \le y - x < \delta \le \delta_1$

and $y - b \le y - x < \delta \le \delta_2$.

From earlier, $|f_1(b) - f_1(x)| = |f(b) - f(x)| < \varepsilon/2$

and $|f_2(y) - f_2(b)| = |f(y) - f(b)| < \varepsilon/2$.

So
$$\begin{aligned}|f(y) - f(x)| &= |f(y) - f(b) + f(b) - f(x)| \\ &\le |f(y) - f(b)| + |f(b) - f(x)| \\ &< \varepsilon/2 + \varepsilon/2 = \varepsilon. \qquad \boxed{\checkmark}\end{aligned}$$

4.1 Derivative definition & basic rules & results.

Suppose $f : D \to \mathbb{R}$. Let $x_0 \in D$, then a <u>neighborhood</u> of x_0 is an interval st. $a < x_0 < b$.
$(a,b) \subseteq D$.

Suppose x_0 has a neighborhood in D.

We say f is <u>differentiable</u> at x_0 iff

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \text{ exists.}$$

we write $f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$ when the limit exists.