Quiz-3
Math 537 Ordinary Differential Equations
Due 9:00AM Wednesday, October 7, 2020

Student Name: ______________________________ **ID** ____________

Goal: Based on Eq. (1.3) of the Mid-Term Part A, we will derive a second-order ODE with its solution as a hyperbolic secant squared function ($sech^2$). The second-order ODE is mathematically identical to the Korteweg-de Vries (KdV) equation in a traveling-wave coordinate. The solution is known as a solitary wave or logistic distribution. It also represents the solution (I) of the simplified SIR model under the condition of "weak outbreak".

Total points: 30

1: [30 points] In the Mid-term Part A, we have completed the following:

(*) Consider the following logistic equation:

$$\frac{df}{dt} = f(1-f). \qquad (MT-1.2)$$

Introduce a new dependent variable (g) to transform Eq. (MT-1.2) into the following ODE:

$$\frac{dg}{dt} = \frac{1}{4} - g^2. \qquad (MT-1.3)$$

(*) Express the solutions of Eqs. (MT-1.2) and (MT-1.3) in terms of the sigmoid and hyperbolic tangent functions, respectively.

Here, by defining $Z = dg/dt$, please derive the following ODE from Eq. (MT-1.3):

$$\frac{d^2Z}{dt^2} - Z + 6Z^2 = 0, \qquad (1.1)$$

which can be written using a new time variable (τ) as follows:

$$\frac{d^2Z}{d\tau^2} - Z/2 + 3Z^2 = 0. \qquad (1.2)$$

Eq. (1.2) is mathematically identical to the KdV equation in the traveling-wave coordinate (Shen 2020, IJBC, in press).