Exam 2
Algebraic Coding Theory
Math 525
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Problem 2: Let C be the linear code with parity-check matrix

$$H = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

(a) The minimum distance d of C.

Let $i, j \in \{0, 1, 2, 3, ..., 9, 10\}$, and let h_i represent the rows of parity-check matrix, H.

Notice the following:

Suppose we had a minimum distance of 1, then we would have a row $h_i = 0\ 0\ 0\ 0$, which is not the case.

Suppose we had a minimum distance of 2, then we would have two rows such that $h_i + h_j = \vec{0}$. However, the only way for this to be true is for $h_i = h_j$, which again is not the case.

So notice that $h_4 + h_6 + h_7 = \vec{0}$. **Thus we get a minimum distance of $d = 3$.**

(b) Determine a codeword in C of weight d, where d denotes the value you found in part (a).

Notice the codeword in C, $v = 01010001$