Homework 1
Math Modeling
Math 636
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Problem 1: A ball is thrown directly upward from the surface of the Earth. Assuming that the maximum height reached is a monomial function of the acceleration due to gravity, the mass of the ball and the initial velocity, use dimensional analysis to approximate the expression for the maximum height reached.

Problem 2: Non-dimensionalize the following equations:

(a)

$$\frac{dy}{dt} = ry(1 - \frac{y}{K}), \text{ where } r \text{ and } K \text{ are constant}$$

(b)

$$\frac{dy}{dt} = sy(a - y)(y - b), \text{ where } s,\ a,\ b \text{ are constants}$$

Problem 3: For a given model equation

$$\frac{dN(t)}{dt} = r_B N(t)\left[1 - \frac{N(t)}{K_B}\right] - B\frac{N(t)^2}{A^2 + N(t)^2}$$

perform dimensional analysis to reduce the equation to the form

$$\frac{du}{dr} = ru\left(1 - \frac{u}{q}\right) - \frac{u^2}{1+u^2}$$

Problem 4: For the damp pendulum equation

$$\ddot{\theta} + \alpha\dot{\theta} + \beta \sin\gamma\theta = 0, \quad \theta(0) = 0, \quad \dot{\theta}(0) = 1$$

find suitable rescaling for the following cases:

(a)

$$\alpha, \gamma = \mathcal{O}, \quad \beta >> 1$$

(b)

$$\alpha, \beta = \mathcal{O}, \quad \gamma >> 1$$

(c)

$$\alpha \sim \beta\gamma \sim 1/\gamma >> 1$$

Problem 5: Consider the following model equation:

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} + \gamma y^3$$

$$y(x,0) = 0, \quad 0 < x < \infty$$

$$y(0,t) = y_0, \quad t > 0, \quad y(\infty,t) = 0, \quad t > 0$$

(a) Write the dimensions of each variable, parameter, and term in the equation.

(b) Non-dimensionalize the equation.

(c) Analyze the non-dimensionalized parameters to find out what parameters to choose to make (relative) small diffusion or (relative) large diffusion.

Problem 6: According to the radioactive decay law, the per capita decay rate of the amount $A(t)$ of C^{14} (Carbon 14) is $-\lambda$. Suppose that an archeologist excavates a bone and measures its content for C^{14}. If the result is 25% of the carbon present in bones of a living organism, what can be said about the age of the bone? The half-life of C^{14} is 5730 years