Homework 6
Algebraic Coding Theory
Math 525
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Problem 1:

(a) **Exercise 3.3.3:** Find a generator matrix in standard form for a Hamming code of length 15, then encode the message 11111100000.

Notice the code length 15, can be written as $2^r - 1$ with $r = 4$. So we can write a Parity Check Matrix with dimension 15×4.

Notice we can write Parity Check Matrix as the binary representation of 1 - 15 and then convert it into standard form:

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \implies H = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From this we get the Generator Matrix is:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Finally to encode the message $u = 11111100000$, we get that $uG = 111111000000100$

(b) **Exercise 3.3.6:** Show that each of the following is a parity check matrix for a Hamming code of length 7, and that the codes are both equivalent to the one in Example 3.3.1.

Notice both of them have codewords of length 3 because $7 = 2^3 - 1$. They share the same codewords so they are both parity check matrices for a Hamming code of length 7.

(c) **Exercise 3.3.8:** No, because H have 2 identical rows 0110.

(d) **Exercise 3.3.9:** Show that the Hamming code of length $2^r - 1$ for r = 2 is a trivial code.

Notice the length = 3, and $r = 2$, such that

$$H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

From this we get any codeword $u = uG$, so thus the trivial code.

Problem 2: **Exercise 3.3.4:** Construct an SDA for a Hamming code of length 7, and use it to decode the following words: 1101011. Note: Do not use the SDA for decoding; instead, use the method presented on slide # 6 of Section 3.3 (the latter is much less effort).

Notice the following:

$$H = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

So we have that $r = 1101011 = c + e_i$. So we get that

$$syn(1101011) = (c + e_i) \cdot H_3 = e_i \cdot H_3 = \text{ith row of } H_3$$

Notice that $rH = 110$ which is the 6th row. Thus we get $c = 1101011 + 0000010 = 1101001$

Problem 3:

(a) **Exercise 3.4.3:** Find generating and parity check matrices for an extended Hamming code of length 8.

$$H = \left[\begin{array}{ccc|c} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad G = \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

(b) **Exercise 3.4.4:** Construct an SDA for an extended Hamming code of length 8, and use it to decode the following words: 10101010

Notice that we have 2^4 cosets.

Coset Leader u	$syn(u)$
00000000	0000
10000000	0011
01000000	0101
00100000	0111
00010000	1001
00001000	1011
00000100	1101
00000010	1111
00000001	0001
00000000	
00000000	
00000000	
00000000	
00000000	
00000000	
00000000	

(c) **Exercise 3.4.5:** Show that an extended Hamming code of length 8 is a self-dual code (that is, show $C = C^{\perp}$).