Homework 7
Numerical Matrix Analysis
Math 543
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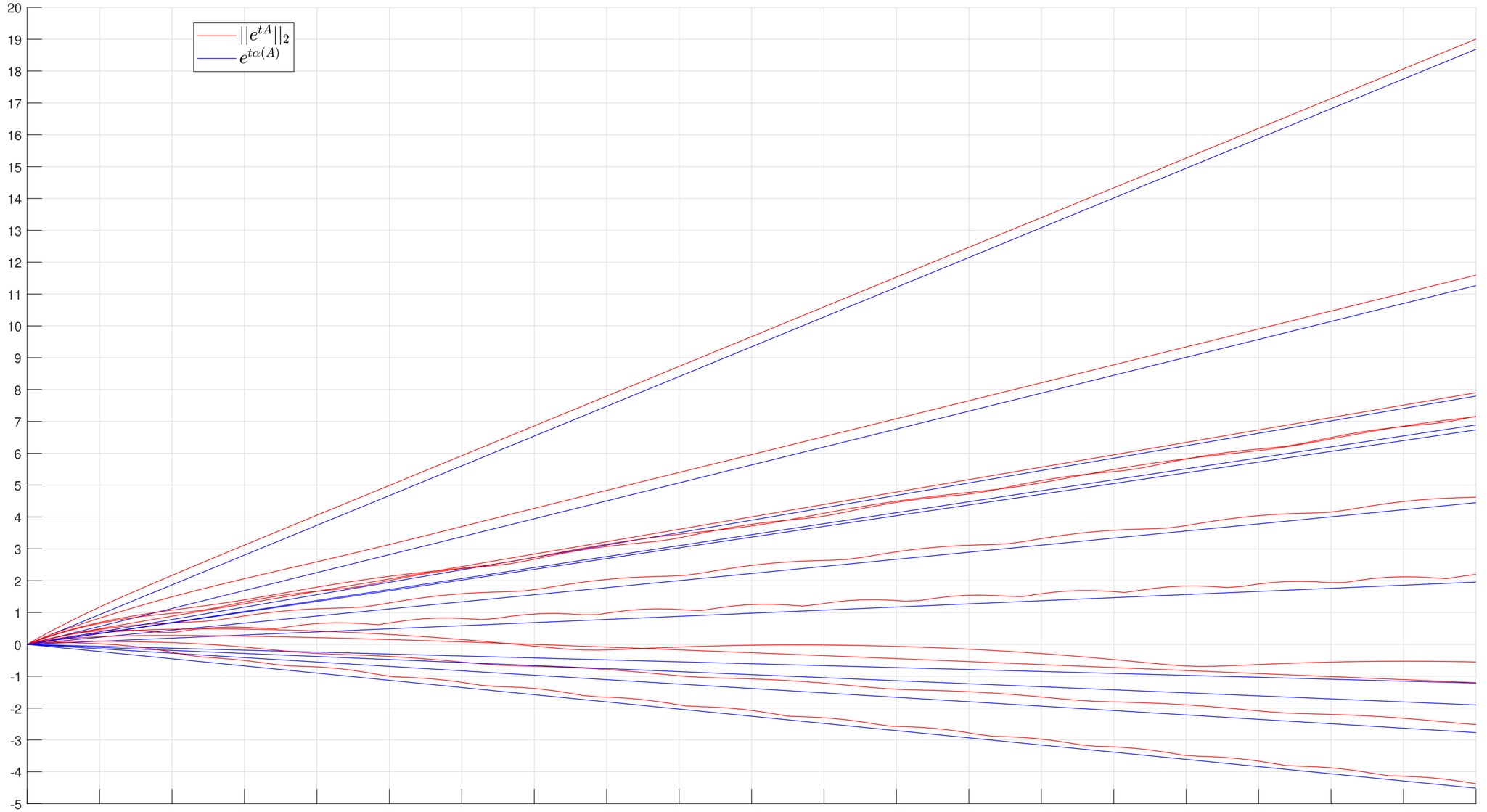
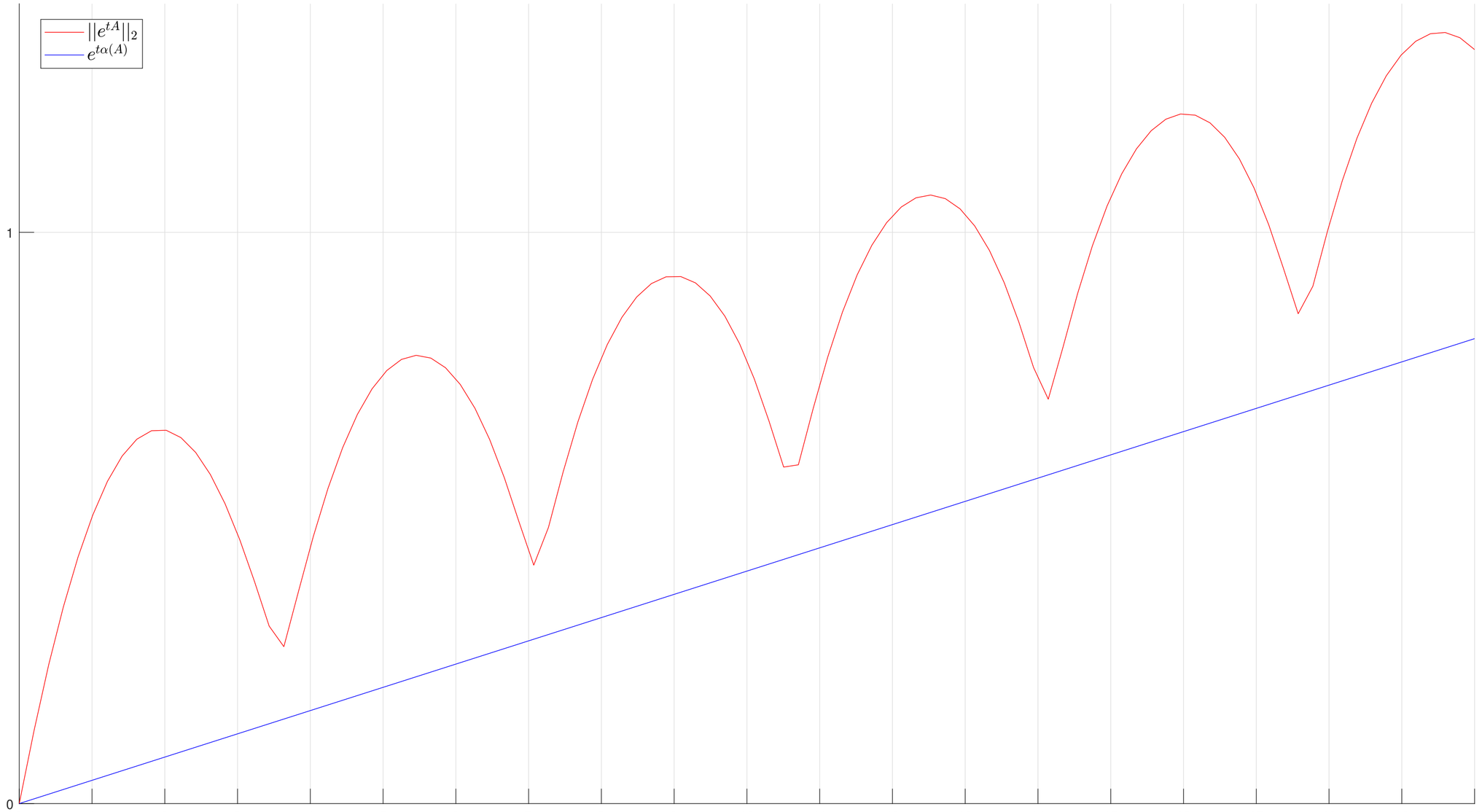
Problem 24.3:

Results I am able to see is that the two functions, $||e^{tA}||_2, e^{t\alpha(A)}$, are almost identical on a log scale. However, we see that $||e^{tA}||_2$ either looks like an exponential or an oscillating solution. I am also able to see that for matrices, A, with lesser real eigenvalues, the more likely it was to be oscillating as $t \to \infty$. For matrices with all complex eigenvalues, it would look more and more like the straight line $e^{t\alpha A}$. On page (4-5), you can see an example of an oscillating function with only 2/10 real eigenvalues.

```
clear
figure(101)
clf
hold off

grid on
hold on

I = eye(10);
t = linspace(0,20,100);

for i = 0 : 0
    A = randn(10) - 2*I;

    yvals = zeros(length(t),1);
    yvals1 = zeros(length(t),1);
    specAbsc = max(real(eig(A)));

    for j = 0 : length(t) - 1
        yvals(j + 1) = norm(expm(t(j + 1)*A));
        yvals1(j + 1) = exp(t(j + 1)*specAbsc);
    end

    t = transpose(linspace(0,20,100));
    logYvals = log10(yvals);
    logYvals1 = log10(yvals1);

    plot(t, logYvals,'r');
    plot(t,logYvals1,'b');
end

xticks(0:20);
yticks(-5:1:20);
legend('$||e^{tA}||_2$','$e^{t\alpha (A)}$','interpreter','latex','fontsize',↙
14,'location','best');
```


$||e^{tA}||_2$
$e^{t\alpha(A)}$
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

```
>> A

A =

   -1.7457    0.3221   -0.5516    0.8821    0.1418   -1.1420   -1.7996   -0.8844↙
-0.5175   -0.6954
    0.1248   -1.4255    1.8738   -0.8611   -1.4499   -0.7530   -0.6561   -0.6560↙
0.5465    0.6876
    0.0876   -0.5027   -1.5018    0.1804   -0.3922    0.0300    2.2252   -1.1895↙
0.2932    0.9751
    1.1375    1.8070    0.8347   -2.2442   -0.9302   -0.4893    0.3171    0.0063↙
0.9902    0.1322
    0.0766    1.0365   -0.7648    2.0082   -2.9916    3.3458    1.2406    0.9877↙
0.7224    0.2420
    1.1205   -1.5516    0.8572    0.1928   -0.2409   -2.0673    0.5891    0.7215↙
-0.9788    1.4233
   -0.3232   -1.1251    1.1612    0.6067    0.0416    0.4274   -3.3676   -1.9677↙
-0.0691    1.2617
   -0.3675   -1.9000   -0.6747   -1.3227    1.5213   -0.5738    1.1454   -2.4807↙
0.1643   -0.8619
    0.0028   -0.4320   -0.4463    1.0808   -1.3086    0.4574    0.7140   -1.8885↙
-3.8106   -0.9471
    0.5219   -1.1422   -0.4722   -0.7845   -1.7596   -0.1840   -1.4193    1.8427↙
1.5516    0.6217

>> eig(A)

ans =

  -0.1977 + 2.6983i
  -0.1977 - 2.6983i
   0.0937 + 0.8954i
   0.0937 - 0.8954i
  -2.5320 + 0.0000i
  -4.3455 + 0.0000i
  -3.4773 + 2.2164i
  -3.4773 - 2.2164i
  -3.4866 + 1.7456i
  -3.4866 - 1.7456i

>>
```


$||e^{tA}||_2$
$e^{t\alpha(A)}$
0
1
0
1
2
3
4
5
6
7
8
9
10
11
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14
15
16
17
18
19
20

```
function A = HouseToHessen (A)

m = size(A);
for k = 1 : (m - 2)
    x = A((k + 1): m, k);
    e1 = zeros(size(x));
    e1(1) = 1;
    vk = sign(x(1)) * norm(x)*e1 + x;
    vk = vk / norm(vk);
    A((k + 1): m, k : m) = A((k + 1): m, k : m) - 2*vk*(transpose(vk) *A((k + 1): m, k :↙
m));
    A(1 : m, (k + 1) : m) = A(1 : m, (k + 1) : m) - 2*(A(1 : m, (k + 1) : m) * vk) *↙
transpose(vk);
end
```

```
>> A = rand(5)

A =

    0.9725    0.7272    0.1193    0.1973    0.9633
    0.2860    0.4008    0.9785    0.8629    0.2696
    0.5990    0.3106    0.5183    0.2679    0.2657
    0.8522    0.6538    0.5078    0.5387    0.3100
    0.2448    0.3918    0.3250    0.5515    0.5573

>> HouseToHessen(A)

ans =

    0.9725   -0.6170    0.9446   -0.4602    0.1583
   -1.1076    1.5834   -0.5296   -0.0991   -0.2880
         0   -1.1230    0.0181   -0.0271    0.1702
         0   -0.0000   -0.2786    0.2760    0.1631
   -0.0000    0.0000    0.0000   -0.1321    0.1376

>> hess(A)

ans =

    0.9725   -0.6170    0.9446   -0.4602    0.1583
   -1.1076    1.5834   -0.5296   -0.0991   -0.2880
         0   -1.1230    0.0181   -0.0271    0.1702
         0         0   -0.2786    0.2760    0.1631
         0         0         0   -0.1321    0.1376

>>
```

```
function [lambdam, vm] = RayleighIteration (A)

m = size(A,1);
v = zeros(m, m);
v(:,1) = rand(m,1);
v(:,1) = v(:,1) / norm(v(:,1));
lambda = zeros(m,1);
lambda(1) = transpose(v(:,1)) * A * v(:,1);
k = 1;
I = eye(m);

while k < m
    k = k + 1;
    w = linsolve(A - lambda(k-1)*I, v(:,(k-1)));
    v(:,k) = w / norm(w);
    lambda(k) = transpose(v(:,k)) * A * v(:,k);
end

vm = v(:,m);
lambdam = lambda(m);
```

```
>> A = rand(5)

A =

    0.3782    0.4976    0.8808    0.3787    0.6171
    0.2458    0.4278    0.1600    0.9255    0.1479
    0.6762    0.5678    0.0892    0.0570    0.0290
    0.2613    0.6130    0.3131    0.9940    0.5725
    0.9615    0.8070    0.6074    0.6670    0.0300

>> [a,b] = RayleighIteration (A)

a =

    2.3788


b =

    0.4711
    0.3741
    0.2520
    0.5339
    0.5381

>> [c,d] = eig (A)

c =

    0.4711    0.6450   -0.6730   -0.5855   -0.0155
    0.3741   -0.5022    0.2064    0.7094    0.4111
    0.2520    0.3116    0.4333    0.1387   -0.5307
    0.5339   -0.3424   -0.2269   -0.1318   -0.3454
    0.5381    0.3426    0.5149   -0.3425    0.6556


d =

    2.3788         0         0         0         0
         0    0.5431         0         0         0
         0         0   -0.6860         0         0
         0         0         0    0.0129         0
         0         0         0         0   -0.3297

>>
```