Stephen Giang **MATH Summer 2001**

Homework Set PowerSer due 05/08/2020 at 04:00am PDT

This first set (set 0) is designed to acquaint you with using WeBWorK. **Your score on this set will not be counted toward your final grade**

You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer.

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1. (3 pts) Let $T_5(x)$ be the fifth degree Taylor polynomial of the function $f(x) = \cos(0.7x)$ at $a = 0$.

A. Find $T_5(x)$. (Enter a function.)

$T_5(x) =$ ______

B. Find the largest integer k such that for all x for which $|x| < 1$ the Taylor polynomial $T_5(x)$ approximates $f(x)$ with error less than $\frac{1}{10^k}$.

$k =$ ______

Answer(s) submitted:

- `(1/(1)) + (-49/(200)) (x^2) +(2401/(240000)) (x^4)`
- `3`

(correct)

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2. (2 pts) Match each of the Maclaurin series with the function it represents.

___1. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$

___2. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

___3. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

___4. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

A. e^x
B. $\arctan(x)$
C. $\sin(x)$
D. $\cos(x)$

Answer(s) submitted:

- `b`
- `c`
- `d`
- `a`

(correct)

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3. (3 pts) Write the Taylor series for $f(x) = e^x$ about $x = 2$ as $\sum_{n=0}^{\infty} c_n (x-2)^n$.

Find the first five coefficients.

$c_0 =$ ______
$c_1 =$ ______
$c_2 =$ ______
$c_3 =$ ______
$c_4 =$ ______

Answer(s) submitted:

- `e^2`
- `e^2`
- `e^2 / 2`
- `e^2 / 6`
- `e^2 / 24`

(correct)

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4. (4 pts) Write the Maclaurin series for $f(x) = 7x^2 e^{-3x}$ as $\sum_{n=0}^{\infty} c_n x^n$.

Find the first six coefficients.

$c_0 =$ ______
$c_1 =$ ______
$c_2 =$ ______
$c_3 =$ ______
$c_4 =$ ______
$c_5 =$ ______

Answer(s) submitted:

- `0`
- `0`
- `14/2`
- `-126/6`
- `756/24`
- `-3780/120`

(correct)

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5. (7 pts) Solve the initial value problem

$$y'' + 1xy' - 4y = 0, \, y(0) = 9, \, y'(0) = 0.$$

$y =$ ______

Answer(s) submitted:

- `9 + 18x^2 + 3x^4`

(correct)

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6. (7 pts) Assume that y is a solution of the differential equation

$$y'' + (4x-2)y' + 3y = 0\,.$$

If y is written as a power series

$$y = \sum_{n=0}^{\infty} c_n x^n ,$$

then its coefficients c_n are related by the equation

$c_{n+2} =$ ________ c_{n+1} + ________ c_n .

Answer(s) submitted:

- `(2n + 2)/((n+2)(n+1))`
- `-(4n+3)/((n+2)(n+1))`

(correct)

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7. (8 pts) Use power series to solve the initial-value problem

$$(x^2-3)y''+8xy'+6y=0\ , \qquad y(0)=1\ , \qquad y'(0)=0\ .$$

Answer: $y = \sum_{n=0}^{\infty}$ ________ $x^{2n} + \sum_{n=0}^{\infty}$ ________ x^{2n+1} .

Answer(s) submitted:

- `(n+3)!/(2*3^(n+1)(n+1)! )`
- `0`

(correct)

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8. (6 pts) Assume that y is the solution of the initial-value problem

$$y'-2y = \begin{cases} \frac{5\sin x}{x} & x \neq 0 \\ 5 & x = 0 \end{cases} , \qquad y(0)=1\ .$$

If y is written as a power series

$$y = \sum_{n=0}^{\infty} c_n x^n ,$$

then the first few terms are
________ + ________ x + ________ x^2 + ________ x^3 + ________ x^4 .

Note: You do not have to find a general expression for c_n. Just find the coefficients one by one.

Answer(s) submitted:

- `1`
- `7`
- `7`
- `79/18`
- `((0) + (79/9))/4`

(correct)

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9. (7 pts) Find two linearly independent solutions of $y''+5xy=0$ of the form

$y_1 = 1+a_3x^3+a_6x^6+\cdots$

$y_2 = x+b_4x^4+b_7x^7+\cdots$

Enter the first few coefficients:

$a_3 =$ ________
$a_6 =$ ________

$b_4 =$ ________
$b_7 =$ ________

Answer(s) submitted:

- `-5/6`
- `5/36`
- `-5/12`
- `25/(7*6*4*3)`

(correct)

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10. (8 pts) Solve the initial value problem

$$(4+x^2)y''+3y=0,\ y(0)=0,\ y'(0)=12.$$

If the solution is

$y = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5 + c_6x^6 + c_7x^7 + \cdots$,

enter the following coefficients:

$c_0 =$ ________
$c_1 =$ ________
$c_2 =$ ________
$c_3 =$ ________
$c_4 =$ ________
$c_5 =$ ________
$c_6 =$ ________
$c_7 =$ ________

Answer(s) submitted:

- `0`
- `12`
- `0`
- `-12/8`
- `0`
- `27/160`
- `0`
- `-207/8960`

(correct)

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11. (7 pts) Use power series to solve the initial-value problem

$$y''+4xy'+4y=0\ , \qquad y(0)=1\ , \qquad y'(0)=0\ .$$

Answer: $y = \sum_{n=0}^{\infty}$ ________ $x^{2n} + \sum_{n=0}^{\infty}$ ________ x^{2n+1} .

Answer(s) submitted:

- `(-2)^n / n!`
- `0`

(correct)