---

1. (3 pts) Consider the following initial value problem:

$$\frac{dy}{dt} = 0.3y, \qquad y(0) = 45.$$

Solve this initial value problem.
$y(t) =$ ____________________
The value of the solution at $t = 1$ is
$y(1) =$ ______
Use Euler's method to approximate the solution $y(1)$ using a stepsize of $h = 0.2$ for $t \in [0,1]$.
$y(0.2) \simeq y_1 =$ ______
$y(0.4) \simeq y_2 =$ ______
$y(0.6) \simeq y_3 =$ ______
$y(0.8) \simeq y_4 =$ ______
$y(1) \simeq y_5 =$ ______

Compute the error between the actual solution and the approximate solution using Euler's method at $t = 1$.
Percent Error = ______

Answer(s) submitted:

- `45e^(.3t)`
- `45e^(.3)`
- `47.7`
- `50.562`
- `53.59572`
- `56.8114632`
- `60.22015099`
- `(100*(60.22015099 - 45e^(.3)) / (45e^(.3)))`

(correct)
Correct Answers:

- `45*exp(0.3*t)`
- `60.7436463409201`
- `47.7`
- `50.562`
- `53.59572`
- `56.8114632`
- `60.220150992`
- `-0.861810873160378`

---

2. (3 pts) Consider the following initial value problem:

$$\frac{dy}{dt} = 0.5y - 1, \qquad y(0) = 44.$$

Solve this initial value problem.
$y(t) =$ ____________________
The value of the solution at $t = 1$ is
$y(1) =$ ______
Use Euler's method to approximate the solution $y(1)$ using a stepsize of $h = 0.2$ for $t \in [0,1]$.
$y(0.2) \simeq y_1 =$ ______
$y(0.4) \simeq y_2 =$ ______
$y(0.6) \simeq y_3 =$ ______
$y(0.8) \simeq y_4 =$ ______
$y(1) \simeq y_5 =$ ______

Compute the error between the actual solution and the approximate solution using Euler's method at $t = 1$.
Percent Error = ______

Answer(s) submitted:

- `2 + 42e^(.5t)`
- `2 + 42e^(.5)`
- `48.2`
- `52.82`
- `57.902`
- `63.4922`
- `69.64142`
- `((69.64142 - 2 - 42e^(.5)) / (2 + 42e^(.5)))*100`

(correct)
Correct Answers:

- `(44-1/0.5)*exp(0.5*t)+1/0.5`
- `71.2462933694054`
- `48.2`
- `52.82`
- `57.902`
- `63.4922`
- `69.64142`
- `-2.25257103704224`

---

3. (3 pts) Consider the following initial value problem:

$$\frac{dy}{dt} = 3t - 0.2y, \qquad y(0) = 15.$$

a. Find the solution to this initial value problem.
$y(t) =$ ________________________

Find the solution at $t = 1$.
$y(1) =$ ______

b. Use Euler's method to approximate the solution $y(1)$ using a stepsize of $h = 0.2$ for $t \in [0,1]$.
$y(0.2) \simeq y_1 =$ ______
$y(0.4) \simeq y_2 =$ ______
$y(0.6) \simeq y_3 =$ ______
$y(0.8) \simeq y_4 =$ ______
$y(1) \simeq y_5 =$ ______

Compute the error between the actual solution and the approximate solution using Euler's method at $t = 1$.

Percent Error = ______

c. Use the Improved Euler's method to approximate the solution $y(1)$ using a stepsize of $h = 0.2$ for $t \in [0,1]$.
$y(0.2) \simeq y_1 =$ ______
$y(0.4) \simeq y_2 =$ ______
$y(0.6) \simeq y_3 =$ ______
$y(0.8) \simeq y_4 =$ ______
$y(1) \simeq y_5 =$ ______

Compute the error between the actual solution and the approximate solution using the Improved Euler's method.
Percent Error = ______

Answer(s) submitted:

- `15t - (15*5) + (15*6)e^(-t/5)`
- `15 - (15*5) + (15*6)e^(-1/5)`
- `14.4`
- `13.944`
- `13.62624`
- `13.4411904`
- `13.38354278`
- `((13.38354278 - (15 - (15*5) + (15*6)e^(-1/5)))/(15 - (15*5) + (15*6)e^(-1/5)))*100`
- `14.472`
- `14.0822976`
- `13.82547153`
- `13.69631305`
- `13.68981758`
- `((13.68981758 - (15 - (15*5) + (15*6)e^(-1/5)))/(15 - (15*5) + (15*6)e^(-1/5)))*100`

(correct)
Correct Answers:

- `15*t - 75 +90*exp(-0.2*t)`
- `13.6857677770184`
- `14.4`
- `13.944`
- `13.62624`
- `13.4411904`
- `13.383542784`
- `-2.20831595232733`
- `14.472`
- `14.0822976`
- `13.82547153408`
- `13.6963130499441`
- `13.6898175783863`
- `0.0295913348368249`

---

4. (3 pts) A population of animals that includes emigration satisfies the differential equation

$$\frac{dP}{dt} = 0.55P - 20, \qquad P(0) = 620.$$

a. Solve this differential equation.
$P(t) =$ ______________________
The value of the solution at $t = 1$ is
$P(1) =$ ______

b. Use Euler's method with $h = 0.25$ to approximate the solution at $t = 1$.
$P(0.25) \simeq P_1 =$ ______
$P(0.5) \simeq P_2 =$ ______
$P(0.75) \simeq P_3 =$ ______
$P(1) \simeq P_4 =$ ______

Find the percent error between the actual solution and this approximate solution at $t = 1$.

Percent Error = ______

Answer(s) submitted:

- `(20/.55) + (620 - (20/.55))exp(.55t)`
- `(20/.55) + (620 - (20/.55))exp(.55)`
- `700.25`
- `791.534375`
- `895.3703516`
- `1013.483775`
- `100 * (1013.483775 - ((20/.55) + (620 - (20/.55))exp(.55))`

(correct)
Correct Answers:

- `(620-20/0.55)*exp(0.55*t)+20/0.55`
- `1047.95312497352`
- `700.25`
- `791.534375`
- `895.3703515625`
- `1013.48377490234`
- `-3.28920724121545`

---

5. (4 pts) Radioactive elements are often the products of the decay of another radioactive element. A differential equation describing this situation is given by the following:

$$\frac{dR}{dt} = -0.25R + 3e^{-0.05t}, \qquad R(0) = 70,$$

where t is in years.

a. Use Euler's method with a stepsize of $h = 0.5$ to find the approximate solution at $t = 3$.
$R(0.5) \simeq R_1 =$ ______
$R(1) \simeq R_2 =$ ______
$R(1.5) \simeq R_3 =$ ______
$R(2) \simeq R_4 =$ ______
$R(2.5) \simeq R_5 =$ ______
$R(3) \simeq R_6 =$ ______

b. Find the actual solution to this problem.

$R(t) =$ ______________________

Evaluate the solution at $t = 3$.
$R(3) =$ ______
Use this solution to find the percent error of Euler's method at $t = 3$.
Percent Error = ______

Answer(s) submitted:

- `62.75`
- `56.36921487`
- `50.74990715`
- `45.79778398`
- `41.43031711`
- `37.57527283`
- `15e^(-t/20) + 55e^(-t/4)`
- `15e^(-3/20) + 55e^(-3/4)`
- `100 *(37.57527283 - (15e^(-3/20) + 55e^(-3/4))) / (15e^(-3/20) + 55e^(-3/4))`

(correct)

Correct Answers:

- `62.75`
- `56.3692148680425`
- `50.7499071462883`
- `45.7977839824951`
- `41.4303171117371`
- `37.5752728266469`
- `15*exp(-0.05*t) + 55*exp(-0.25*t)`
- `38.8907800471317`
- `-3.38256835910862`

---

6. (4 pts) The temperature of an object is initially $55°C$. If it is in a room where the temperature, $T_e(t)$, is slowly decreasing with $T_e(t) = 23 - t$, then using Newton's Law of Cooling, the temperature of the object satisfies the differential equation

$$\frac{dT}{dt} = -0.2(T - (23 - t)),$$

where t is in hours.

Find the solution to this initial value problem.

$T(t) =$ ______________________________

Find the temperature at $t = 2$.

$T(2) =$ _______

Use Euler's method with $h = 0.5$ to approximate the solution at $t = 2$.

$T(0.5) \simeq T_1 =$ _______

$T(1) \simeq T_2 =$ _______

$T(1.5) \simeq T_3 =$ _______

$T(2) \simeq T_4 =$ _______

Find the percent error between the actual solution and this approximate solution at $t = 2$.

Percent Error = _______

Answer(s) submitted:

- `-t + 28 + 27*exp(-t/5)`
- `26 + 27*exp(-2/5)`
- `51.8000`
- `48.8700`
- `46.1830`
- `43.7147`
- `100*(43.7147 - (26 + 27*exp(-2/5)))/(26 + 27*exp(-2/5))`

(correct)

Correct Answers:

- `28 - t - -27*exp(-0.2*t)`
- `44.0986412429623`
- `51.8`
- `48.87`
- `46.183`
- `43.7147`
- `-0.870641888594542`

---

7. (7 pts) The body temperature of a particular animal is normally 37.5°C. Suppose this animal is hit by a car at midnight ($t = 0$), and the environmental temperature is approximately 22°C. From Newton's Law of Cooling, the temperature of the roadkill satisfies the differential equation

$$\frac{dT}{dt} = -0.2(T - 22),$$

where t is in hours.

a. Solve this differential equation.

$T(t) =$ ________________________

Find the temperature of the body at 2 AM, i.e., find the value of the solution at $t = 2$ is

$T(2) =$ _______

b. Use Euler's Method with $h = 0.5$ to approximate the temperature at $t = 2$.

$T(0.5) \simeq T_1 =$ _______

$T(1) \simeq T_2 =$ _______

$T(1.5) \simeq T_3 =$ _______

$T(2) \simeq T_4 =$ _______

Find the percent error between the actual solution and this approximate solution at $t = 2$.

Percent Error = _______

c. Suppose that the temperature is actually dropping about 0.5°C/hr, then the differential equation describing the temperature of the roadkill is

$$\frac{dT}{dt} = -0.2(T - (22 - 0.45t)).$$

Solve this differential equation.

$T(t) =$ ______________________________

Find the temperature of the body at 2 AM, i.e., find the value of the solution at $t = 2$ is

$T(2) =$ _______

d. Use Euler's Method with $h = 0.5$ to approximate the temperature at $t = 2$ for the modified differential equation.
$T(0.5) \simeq T_1 =$ ______
$T(1) \simeq T_2 =$ ______
$T(1.5) \simeq T_3 =$ ______
$T(2) \simeq T_4 =$ ______

Find the percent error between the actual solution and this approximate solution at $t = 2$.
Percent Error = ______

Answer(s) submitted:

- `22 + (31*exp(-t/5))/2`
- `22 + (31*exp(-2/5))/2`
- `35.9500`
- `34.5550`
- `33.2995`
- `32.1696`
- `100*(32.1696 - (22 + (31*exp(-2/5))/2))/(22 + (31*exp(-2/5))/2)`
- `-(9*t)/20 + 97/4 + (53*exp(-t/5))/4`
- `467/20 + (53*exp(-2/5))/4`
- `35.9500`
- `34.5325`
- `33.2342`
- `32.0433`
- `100*(32.0433 - (467/20 + (53*exp(-2/5))/4))/(467/20 + (53*exp(-2/5))/4)`

(correct)
Correct Answers:

- `15.5*exp(-0.2*t)+22`
- `32.3899607135524`
- `35.95`
- `34.555`
- `33.2995`
- `32.16955`
- `-0.68049083326053`
- `24.25 - 0.45*t + (37.5 - 24.25)*exp(-0.2*t)`
- `32.2317406099722`
- `35.95`
- `34.5325`
- `33.23425`
- `32.043325`
- `-0.584565420317172`

---

8. (13 pts) Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

This problem asks you to examine two numerical methods for solving differential equations. The techniques are Euler's and Improved Euler's methods.

a. Consider the differential equation

$$\frac{dy}{dt} = \frac{2}{6}y^2, \qquad y(0) = 1.$$

Find the solution to this differential equation.
$y(t) =$ ______________

The solution $y(t)$ is defined on the interval $t \in (-\infty, A)$, with a vertical asymptote at
$A =$ ______

b. This part of the problem examines the Euler's and Improved Euler's methods to numerically solve the differential equation in Part a. We begin by using Euler's and Improved Euler's method with $h = 0.1$ for $t \in [0, 3]$. Find the exact solution and approximate solutions at $t = 1.5, 2.1, 2.9$. Also determine the percent error at each of these times for the approximate solutions.

At $t = 1.5$ with $h = 0.1$
Actual solution, $y(1.5) =$ ______
Euler's solution, $y_E(1.5) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(1.5) =$ ______
Percent Error (Improved Euler) = ______

At $t = 2.1$ with $h = 0.1$
Actual solution, $y(2.1) =$ ______
Euler's solution, $y_E(2.1) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(2.1) =$ ______
Percent Error (Improved Euler) = ______

At $t = 2.9$ with $h = 0.1$
Actual solution, $y(2.9) =$ ______
Euler's solution, $y_E(2.9) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(2.9) =$ ______
Percent Error (Improved Euler) = ______

Next you repeat the process above with $h = 0.05$.

At $t = 1.5$ with $h = 0.05$
Euler's solution, $y_E(1.5) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(1.5) =$ ______
Percent Error (Improved Euler) = ______

At $t = 2.1$ with $h = 0.05$
Euler's solution, $y_E(2.1) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(2.1) =$ ______
Percent Error (Improved Euler) = ______

At $t = 2.9$ with $h = 0.05$
Euler's solution, $y_E(2.9) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(2.9) =$ ______
Percent Error (Improved Euler) = ______

c. In your written HW, create one graph showing the true solution and the approximate solutions with Euler's method and stepsizes of $h = 0.1$ and $h = 0.05$. Create a separate graph with the true solution and the approximate solutions with Improved Euler's method and stepsizes of $h = 0.1$ and $h = 0.05$. Write a brief discussion about how varying the stepsize affects the convergence of the approximate solutions to the true solution. Use the errors you computed in your discussion. Also, write a brief discussion of how the Euler's and Improved Euler's methods compare. Does your error analysis show that decreasing the stepsize or changing methods does better at approximating the true solution?

d. Pollution in lakes can affect the growth rate of an organism. Suppose that an organism satisfies a time-varying Malthusian growth model with growth declining linearly with time. The differential equation for this model is given by:

$$\frac{dP}{dt} = (1.58 - 0.72t)P, \qquad P(0) = 400.$$

Find the solution to this differential equation using techniques from class or Maple's dsolve.

$P(t) =$ ______

Find when the solution has a maximum and what that maximum population is.

$t_{max} =$ ______
$P(t_{max}) =$ ______

e. Next we numerically simulation the population model above. We apply the Euler's and Improved Euler's methods to this problem with stepsizes of $h = 0.2$ and $h = 0.1$ for $t \in [0,6]$. Find the exact solution and approximate solutions at $t = 1, 3, 5$. Also determine the percent error at each of these times for the approximate solutions.

At $t = 1$ with $h = 0.2$
Actual solution, $y(1) =$ ______
Euler's solution, $y_E(1) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(1) =$ ______
Percent Error (Improved Euler) = ______

At $t = 3$ with $h = 0.2$
Actual solution, $y(3) =$ ______
Euler's solution, $y_E(3) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(3) =$ ______
Percent Error (Improved Euler) = ______

At $t = 5$ with $h = 0.2$
Actual solution, $y(5) =$ ______
Euler's solution, $y_E(5) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(5) =$ ______
Percent Error (Improved Euler) = ______

Next you repeat the process above with $h = 0.1$.

At $t = 1$ with $h = 0.1$
Euler's solution, $y_E(1) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(1) =$ ______
Percent Error (Improved Euler) = ______

At $t = 3$ with $h = 0.1$
Euler's solution, $y_E(3) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(3) =$ ______
Percent Error (Improved Euler) = ______

At $t = 5$ with $h = 0.1$
Euler's solution, $y_E(5) =$ ______
Percent Error (Euler) = ______
Improved Euler's solution, $y_{IE}(5) =$ ______
Percent Error (Improved Euler) = ______

f. In your written HW, create a graph showing the true solution of this population model and the approximate solutions with Euler's and Improved Euler's methods with a stepsize of $h = 0.2$. Write a brief discussion about the two different methods and how they track the true solution. Do the solutions give a good approximation for the maximum population and when it occurs? Also, briefly write how varying stepsize affects convergence to the true solution. Does your error analysis show that decreasing the stepsize or changing methods does better at approximating the true solution?

Answer(s) submitted:

- `-3/(t - 3)`
- `3`
- `2`
- `1.91850725946564`
- `100*(1.91850725946564 - 2)/(2)`
- `1.99789491619805`
- `100*(1.99789491619805- 2)/(2)`
- `3+(1/3)`
- `2.98150733903026`
- `100*(2.98150733903026 - (3+(1/3)))/(3+(1/3))`
- `3.32008413197625`
- `100*(3.32008413197625- (3+(1/3)))/(3+(1/3))`
- `30`
- `9.55521210720124`
- `100*(9.55521210720124 - 30)/(30)`
- `23.8615755215395`
- `100*(23.8615755215395- 30)/(30)`
- `1.95672067690644`

- 100*(1.95672067690644 - 2)/(2)
- 1.99945875216790
- 100*(1.99945875216790- 2)/(2)
- 3.13705049061557
- 100*(3.13705049061557- (3+(1/3)))/(3+(1/3))
- 3.32987078490925
- 100*(3.32987078490925- (3+(1/3)))/(3+(1/3))
- 13.3314294069612
- 100*(13.3314294069612- 30)/(30)
- 27.5914858207063
- 100*(27.5914858207063- 30)/(30)
- 400*exp(-t*(-79 + 18*t)/50)
- 2.194444444
- 2264.423903
- 400*exp(61/50)
- 1258.96934372839
- 100*(1258.96934372839 - 400*exp(61/50))/(400*exp(61/50))
- 1339.77399566630
- 100*(1339.77399566630 - 400*exp(61/50))/(400*exp(61/50))
- 400*exp(3/2)
- 1847.59651463126
- 100*(1847.59651463126 - (400*exp(3/2)))/(400*exp(3/2))
- 1769.38673639331
- 100*(1769.38673639331- (400*exp(3/2)))/(400*exp(3/2))
- 400*exp(-11/10)
- 103.808154503199
- 100*(103.808154503199 - (400*exp(-11/10)))/(400*exp(-11/10))
- 137.785211779067
- 100*(137.785211779067- (400*exp(-11/10)))/(400*exp(-11/10))
- 1303.66982663939
- 100*(1303.66982663939- 400*exp(61/50))/(400*exp(61/50))
- 1350.76656513980
- 100*(1350.76656513980- 400*exp(61/50))/(400*exp(61/50))
- 1817.47990813463
- 100*(1817.47990813463- (400*exp(3/2)))/(400*exp(3/2))
- 1786.59567943421
- 100*(1786.59567943421- (400*exp(3/2)))/(400*exp(3/2))
- 119.094963057523
- 100*(119.094963057523- (400*exp(-11/10)))/(400*exp(-11/10))
- 134.118733279369
- 100*(134.118733279369- (400*exp(-11/10)))/(400*exp(-11/10))

(correct)
Correct Answers:

- 1/(1 - 0.333333333333333*t)
- 3
- 2
- 1.91850725946564
- -4.0746370267178
- 1.99789491619805
- -0.10525419009737
- 3.33333333333333
- 2.98150733903026
- -10.5547798290922
- 3.32008413197625
- -0.397476040712519
- 29.9999999999999
- 9.55521210720124
- -68.1492929759958
- 23.8615755215395
- -20.4614149282015
- 1.95672067690644
- -2.16396615467778
- 1.9994587521679
- -0.027062391605015
- 3.13705049061557
- -5.88848528153289
- 3.32987078490924
- -0.10387645272262
- 13.3314294069612
- -55.5619019767957
- 27.5914858207062
- -8.02838059764556
- 400*exp(1.58*t-0.36*t^2)
- 2.19444444444444
- 2264.4239032607
- 1354.87509344853
- 1258.96934372839
- -7.07856762471248
- 1339.7739956663
- -1.11457490474634
- 1792.67562813523
- 1847.59651463126
- 3.06362654983837
- 1769.38673639331
- -1.29911353601318
- 133.148433479232
- 103.808154503199
- -22.0357673082268
- 137.785211779067
- 3.4824129572342
- 1303.66982663939
- -3.77933486686295
- 1350.7665651398
- -0.303240374599851
- 1817.47990813463
- 1.38364574215818
- 1786.59567943421
- -0.339154981837963
- 119.094963057523
- -10.5547395898584
- 134.118733279369
- 0.728735423153535

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9. (9 pts) Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

Several of you are considering careers in medicine and biotechnology. Drug therapy and dose response is very important in the treatment of many diseases, particularly cancer. Since cancer cells are very similar to your normal body cells, their destruction relies on very toxic drugs. There are some very fine lines in certain cancer treatments between an ineffective dose, one that destroys the cancer, and one that is toxic to all cells in the body. At the base of many of the calculations for these treatments are simple mathematical models for drug uptake and elimination.

a. The simplest situation calls for an injection of the drug into the body. In this case, the differential equation describing the amount of drug in the body is given by:

$$\frac{dA}{dt} = -kA, \qquad A(0) = A_0,$$

where A_0 is the amount of drug injected and k depends on how the drug is metabolized and excreted from the body. Suppose that $A_0 = 20$ μg of a particular drug is injected into the body, and that it has been determined that the half-life of the drug in this patient is 21 days. Solve this differential equation and find the value of k.

$k =$ ______ .
Solution of the differential equation, using the value of k.
$A(t) =$ ____________________ .
Determine how long the drug is effective, if it has been determined that the patient must have 3.8 μg in his body.

Effective for $t_e =$ ______ days.
b. With new materials being developed, the drug can be inserted into polymers that slowly decay and release the drug into the body (See **Norplant**). This delivery system can prevent large toxic doses in the body and maintain the drug level for longer at theurapeutic doses. A differential equation that describes type of drug delivery system is given by

$$\frac{dA}{dt} = re^{-qt} - kA, \qquad A(0) = 0,$$

where $r = 3$ μg/day and $q = 0.15$ (day^{-1}). (It can be shown with integration that if $r/q = A_0$, then this is the same amount of drug as delivered in Part a.) Solve this differential equation.

$A(t) =$ ____________________ .
Over what time period (if any) is this therapy effective. Is this time period longer or shorter than your answer from Part a?

Effective for $t_e =$ ______ days.
LONGER or SHORTER ____
Find the time, t_{max}, where the maximum occurs and the value of $A(t_{max})$.
$t_{max} =$ ______
$A(t_{max}) =$ ______
c. In your written HW, create a single graph that shows both solutions for 100 days ($t \in [0,100]$.) Briefly, describe the graphs of these treatments and what are the significant differences. Which treatment do you consider to be superior and why?
d. For this part of the problem, we want to find the numerical solution of the differential equation in Part b, using the Improved Euler's method of the previous lab. Take a stepsize of $h = 0.5$ on the differential equation describing the drug delivery system with polymers and use the Improved Euler's method to simulate the differential equation for $t \in [0,100]$. Below enter the solutions at times t = 24, 46 and 98 for the model found in Part b, the solution found using Improved Euler's Method and the percent error between these two values.

For time t = 24:
Actual Solution ______ μg
Improved Eulers ______ μg
Percent Error ______

For time t = 46:
Actual Solution ______ μg
Improved Eulers ______ μg
Percent Error ______

For time t = 98:
Actual Solution ______ μg
Improved Eulers ______ μg
Percent Error ______
e. In your written HW, create a graph of the numerical solution using the Improved Euler's method with the actual solution found in Part b. Does this numerical solution adequately represent the actual solution of the differential equation?
Answer(s) submitted:

- `ln(1/2)/-21`
- `20exp(ln(1/2)(-t)/-21)`
- `50.31450220`
- `1260*exp(-(3*t)/20)/(-63 + 20*ln(2)) - 1260*2^(-t/21)/(-63`
- `57.80882271 - 1.443857165`
- `LONGER`
- `12.94022279`
- `-641.1270809/(-63 + 20*ln(2))`
- `1260*exp(-18/5)/(-63 + 20*ln(2)) - 315*2^(6/7)/(-63 + 20*l`
- `10.9145023807222`
- `100*(10.9145023807222 - (1260*exp(-18/5)/(-63 + 20*ln(2))`
- `1260*exp(-69/10)/(-63 + 20*ln(2)) - 315*2^(17/21)/(2*(-63`
- `5.59342287534425`
- `100*( 5.59342287534425 -(1260*exp(-69/10)/(-63 + 20*ln(2))`
- `1260*exp(-147/10)/(-63 + 20*ln(2)) - 315*2^(1/3)/(8*(-63 +`
- `1.00995498486485`
- `100*( 1.00995498486485 - (1260*exp(-147/10)/(-63 + 20*ln(2`

(correct)
Correct Answers:

- `0.0330070085980926`
- `20*exp(-0.0330070085980926*t)`
- `50.3145022029539`
- `25.6425616958033*(exp(-0.0330070085980926*t)-exp(-0.15*t))`
- `56.3649655413476`
- `LONGER`
- `12.9402227883757`
- `13.0477307363971`
- `10.9118872315044`
- `10.9145023807222`
- `0.0239660579542179`
- `5.59190731577887`
- `5.59342287534425`

- `0.0271027304244736`
- `1.00960201322704`
- `1.00995498486485`
- `0.0349614633476882`

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10. (10 pts) Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

One important issue in environmental health is being able to maintain air quality in workplaces. It has been been shown that extended exposure to carbon monoxide as low as 0.00012 can be harmful.

a. Consider a room with a volume, $V = 1700$ m^3, containing machinery that produces carbon monoxide (CO) at a rate $Q(t) = 0.005$ m^3/hr. Assume that ventillation brings fresh air into the room (assume constant volume and constant pressure) where it mixes completely, then exhausts at a rate of $f = 10$ m^3/hr. If $c(t)$ is the concentration of CO in the room at any time, then the differential equation describing this situation is given by (use 'c' for the variable for concentration).

$\frac{dc(t)}{dt} =$ ______________________

If the room is initially free of CO, so $c(0) = 0$, then solve this differential equation.

$c(t) =$ ______________________

Find how long it takes until the air becomes unhealthy (exceeds 0.00012).

Air Unhealthy when $t =$ _______

Eventually (limit as t tends to infinity), what will be the level of CO in this room?

Limiting concentration = _______

b. In your written HW, graph the solution for 48 hours. Briefly describe how you created the differential equation for this model and the techniques used to solve this linear differential equation. Include a description of the solution that you found.

c. The equilibrium concentration in the room is found by setting the right hand side of the differential equation equal to zero. Assuming that $Q(t)$ and V are fixed at the levels in Part a, then find the minimum flow rate of fresh air f_c such that the equilibrium concentration is 0.00012.

Critical flow rate $f_c =$ _______

d. In this part we assume that the machinery is producing CO in a cyclical manner. In this case the peak production of CO matches the rate given in Part a, but falls to zero each day. The machinery produces CO on a daily cycle, so has a period of 24 hours. A function that describes the release of CO is given by

$$Q(t) = 0.0025(1 + \sin(\omega t)).$$

Find the value of ω based on the daily cycling of the machine.

$\omega =$ _______

With the same flow rate (constant ventillation), $f = 10$, and volume, $V = 1700$, from Part a, write a new differential equation describing the concentration of CO in the room at any time (use 'c' for the variable for concentration).

$\frac{dc(t)}{dt} =$ ______________________________

then solve the new differential equation (with $c(0) = 0$) using the Improved Euler's method with $h = 0.5$ and $t \in [0, 200]$. Find the value of ths approximate solution at $t = 31, 53, 82, 118$.

$c(31) \approx$ _______
$c(53) \approx$ _______
$c(82) \approx$ _______
$c(118) \approx$ _______

From the output of the Improved Euler's method, find the first time when the air quality exceeds safe levels. (Note that it is possible that this time will exceed $t = 200$.)

Unsafe Air when $t_u =$ _______

e. In your written HW, graph the Improved Euler's solution with $h = 0.5$ and $t \in [0, 200]$. Write a brief description of the behavior of the approximate solution from the Improved Euler's solution. Compare the model for the cyclic production of CO in Part d to the model for constant CO production in Part a.

Answer(s) submitted:

- `(.005/1700) - (10c/1700)`
- `50000000007/100000000000000 - (50000000007*exp(-t/170))/10`
- `46.65426375`
- `50000000007/100000000000000`
- `.005/.00012`
- `2*pi/24`
- `((.0025(1+sin(2*pi*t/24)))/1700) - (10c/1700)`
- `.00004791803368297250`
- `6.973718681933690 * 10^-5`
- `1.040469992831640 * 10^-4`
- `1.230031713264610 * 10^-4`
- `104`

(correct)

Correct Answers:

- `-(10/1700)*(c-0.005/10)`
- `(0.005/10)*(1 - exp(-10*t/1700))`
- `46.6542637692993`
- `0.0005`
- `41.6666666666667`
- `0.261799387833333`
- `-(10/1700)*(c-(0.0025/10)*(1 + sin(0.261799387833333*t)))`
- `4.79180336878438E-05`
- `6.97371868288284E-05`
- `0.000104046999289611`
- `0.00012300317131588`
- `104`