MATH 525
Section 1.11 - Error-detecting codes

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Let v be the sent codeword, e the error pattern, and w the received word. Then:

$$w = v + e.$$

We say that code C detects the error pattern e if and only if $v + e \notin C$ for all $v \in C$.

Example

Let $C = \{00000, 10101, 00111, 11100\}$. Determine whether C detects each of the error patterns: $e = 10101$, $e = 01010$, and $e = 11011$.

Definition

The (minimum) distance of a block code C is defined as:

$$\begin{aligned} d(C) &= \min\{d(u, v) \mid u, v \in C, u \neq v\} \\ &= \min\{\mathrm{wt}(u + v) \mid u, v \in C, u \neq v\}. \end{aligned}$$

Theorem

If $d(C) = d$, then C detects all non-zero error patterns of weight $d - 1$ or less. Moreover, there is at least one error pattern of weight d which C will not detect.

Definition

A code C is said to be a t-error-detecting code if it detects all error patterns of weight t or less and it does not detect at least one error pattern of weight $t + 1$.

For example, $C = \{000, 111\}$ detects all error patterns of weight two or less.

Remarks: Two alternative ways for determining the error patterns that C will detect:

1. The IMLD table can be used to determine the error patterns that a code C will detect: Regard an element e in the first column as an error pattern. Then C detects e if and only if no codeword appears in the row led by e (excluding the last entry).
2. C *does not* detect e if and only if $v + e = u$ where $u, v \in C$.
   Thus, C *does not* detect e if and only if $e = v + u$ where $u, v \in C$. After determining all the error patterns that C does not detect, the remaining error patterns will be detected by C.