

Slide #11.**Example.** $p(x) = x^2 + x + 1$. Then:

$$p(\beta) = \beta^2 + \beta + 1 = 0010 + 0100 + 1000 = 1110 = \beta^{10}.$$

$$p(\beta^5) = \beta^{10} + \beta^5 + 1 = 1110 + 0110 + 1000 = 0000 = 0.$$

The word representations of the powers of β are found on slide #10.

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Slide #14.**Example:** Minimal polynomial of $\alpha = \beta^3$.

Step 1:

$$\begin{aligned} \beta^3, (\beta^3)^2 = \beta^6, (\beta^6)^2 = \beta^{12}, (\beta^{12})^2 = \beta^{24} = \beta^9, \\ (\beta^9)^2 = \beta^{18} = \beta^3. \end{aligned}$$

Step 2:

$$\begin{aligned} m_\alpha(x) &= \underbrace{(x + \beta^3) \cdot (x + \beta^6)} \cdot \underbrace{(x + \beta^9) \cdot (x + \beta^{12})} \\ &= (x^2 + (\beta^3 + \beta^6)x + \beta^9) \cdot (x^2 + (\beta^9 + \beta^{12})x + \beta^{21}) \\ &= (x^2 + \beta^2x + \beta^9) \cdot (x^2 + \beta^8x + \beta^6) \\ &= x^4 + (\beta^8 + \beta^2)x^3 + (\beta^6 + \beta^9 + \beta^{10})x^2 \\ &\quad + (\beta^8 + \beta^2)x + 1 \\ &= x^4 + x^3 + x^2 + x + 1. \end{aligned}$$

Slide #14.

Example: Minimal polynomial of $\alpha = \beta^5$.

Step 1:

$$\beta^5, (\beta^5)^2 = \beta^{10}, (\beta^{10})^2 = \beta^{20} = \beta^5.$$

Step 2:

$$\begin{aligned} m_\alpha(x) &= (x + \beta^5) \cdot (x + \beta^{10}) \\ &= (x^2 + (\beta^5 + \beta^{10})x + \beta^{15}) \\ &= x^2 + x + 1. \end{aligned}$$