Notebook
Ordinary Differential Equations
Math 537
Stephen Giang

08/24/20 - Lecture 1a

1. Homework due every other Friday starting September 11

08/26/20 - Lecture 1b

1. Second solution for Constant-Coefficient Repeated

$$\begin{aligned}&\lim_{r\to r_1}\frac{e^{rx}-e^{r_1x}}{r-r_1}\\&=\lim_{r\to r_1}\frac{xe^{rx}}{1}\\&=xe^{r_1x}\end{aligned}$$

   When using L'Hospitals, we take the derivative in respect to r.

08/28/20 - Lecture 2

1. System with $f=f(x;a)$ that is not a function of time is called an autonomous system
2. $f(x;a)-a$ is a parameter
3. $x'=f(x_c;a)=0$, is a critical point or equilibrium point
4. A space using dependent variables such as x and t is called a phase space.
5. A 1-D phase space is called a phase space
6.

$$\begin{aligned}\frac{dx}{dt}&=f(x)\\f(x_c)&=0\\\frac{dx}{dt}&=f(x)=f(x_c)+f'(x_c)(x-x_c)+...\\\frac{dx}{dt}&=f(x)=f'(x_c)(x-x_c)+...\approx ax\\x-x_c&=c_0e^{f'(x_c)t}\\f'(x_c)&=a\end{aligned}$$

7. So we can see how $f'(x_c)=a$ will help determine the behavior of the solutions. (Sink (Stable) or Source (Unstable))
8. Bifurcation - "significant" change in structure when "a" varies

9. Whiteboard Ex:

$$\begin{aligned} x' &= ax \\ \int_{-1}^{1} \frac{dx}{x} &= \int_{0}^{T} a\,dt \\ 0 &= aT \end{aligned}$$

10. Logistics Eq:

$$\begin{aligned} x' &= a(x - x^2) \\ \frac{dx}{x(1-x)} &= a\,dt \\ \left(\frac{1}{x} + \frac{1}{1-x}\right) dx &= a\,dt \\ \ln(x) - \ln(1-x) &= at + C \\ \ln\left(\frac{x}{1-x}\right) &= at + C \\ x &= C_0 e^{at}(1-x) \\ (1 + C_0 e^{at})x &= C_0 e^{at} \\ x &= \frac{C_0 e^{at}}{1 + C_0 e^{at}} \end{aligned}$$

08/31/20 - Lecture 2

1.