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Assignment PhasePort due 02/07/2020 at 05:00pm PST

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1. (2 pts) Consider the differential equation given by

$$\frac{dy}{dt} = 10 - 0.2y, \qquad y(0) = 10.$$

a. Solve this differential equation,
$y(t) =$ _____

b. Find the equilibrium for this equation.
Equilibrium is $y_e =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

c. From the collection of phase portraits shown, find the letter (A-H) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are unstable and full circles represent stable.) _____
See the **Phase Portraits**

Answer(s) submitted:

- `50-40*exp(-(1/5)*t)`
- `50`
- `Stable`
- `E`

(correct)

Correct Answers:

- `50+(10-50)*exp(-0.2*t)`
- `50`
- `STABLE`
- `E`

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2. (2 pts) Consider the differential equation given by

$$\frac{dy}{dt} = 0.1y - 12, \qquad y(0) = 135.$$

a. Solve this differential equation,
$y(t) =$ _____

b. Find the equilibrium for this equation.
Equilibrium is $y_e =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

c. From the collection of phase portraits shown, find the letter (A-H) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are unstable and full circles represent stable.) _____
See the **Phase Portraits**

Answer(s) submitted:

- `120+15*exp((1/10)*t)`
- `120`
- `Unstable`
- `H`

(correct)

Correct Answers:

- `120+(135-120)*exp(0.1*t)`
- `120`
- `UNSTABLE`
- `H`

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3. (2 pts) Consider the differential equation given by

$$\frac{dy}{dt} = 0.3y(4 - y^2).$$

a. Find the equilibria for this equation.
($y_{1e} < y_{2e} < y_{3e}$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{3e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

b. From the collection of phase portraits shown, find the letter (A-H) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are unstable and full circles represent stable.) _____
See the **Phase Portraits**

Answer(s) submitted:

- `-2`
- `Stable`
- `0`
- `Unstable`
- `2`
- `Stable`
- `A`

(correct)

Correct Answers:

- `-2`
- `STABLE`
- `0`

- UNSTABLE
- 2
- STABLE
- A

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4. (2 pts) Consider the differential equation given by

$$\frac{dy}{dt} = 0.1y\left(1 - \frac{y}{20}\right).$$

a. Find the equilibria for this equation.
($y_{1e} < y_{2e}$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

b. From the collection of phase portraits shown, find the letter (A-H) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are unstable and full circles represent stable.) _____
See the **Phase Portraits**
Answer(s) submitted:

- 0
- Unstable
- 20
- Stable
- d

(correct)
Correct Answers:

- 0
- UNSTABLE
- 20
- STABLE
- D

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5. (2 pts) Consider the differential equation given by

$$\frac{dy}{dt} = 0.8 - 0.2y - 0.1y^2.$$

a. Find the equilibria for this equation.
($y_{1e} < y_{2e}$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

b. From the collection of phase portraits shown, find the letter (A-H) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are unstable and full circles represent stable.) _____
See the **Phase Portraits**
Answer(s) submitted:

- -4
- Unstable
- 2
- Stable
- B

(correct)
Correct Answers:

- -4
- UNSTABLE
- 2
- STABLE
- B

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6. (2 pts) Consider the differential equation given by

$$\frac{dy}{dt} = -\frac{0.2y}{1+y^2}.$$

a. Find the equilibrium for this equation.
The equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

b. From the collection of phase portraits shown, find the letter (A-H) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are unstable and full circles represent stable.) _____
See the **Phase Portraits**
Answer(s) submitted:

- 0
- Stable
- g

(correct)
Correct Answers:

- 0
- STABLE
- G

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7. (2 pts) Consider the differential equation given by

$$\frac{dy}{dt} = 0.1y^2 - 4y.$$

a. Find the equilibria for this equation.
($y_{1e} < y_{2e}$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

b. From the collection of phase portraits shown, find the letter (A-H) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are unstable and full circles represent stable.) _____
See the **Phase Portraits**

Answer(s) submitted:

- 0
- Stable
- 40
- Unstable
- f

(correct)

Correct Answers:

- 0
- STABLE
- 40
- UNSTABLE
- F

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8. (2 pts) Consider the differential equation given by

$$\frac{dy}{dt} = \cos(y).$$

a. Find the equilibria for this equation.
($-5 < y_{1e} < y_{2e} < y_{3e} < y_{4e} < 5$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{3e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{4e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

b. From the collection of phase portraits shown, find the letter (A-H) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are unstable and full circles represent stable.) _____
See the **Phase Portraits**

Answer(s) submitted:

- -3pi/2
- Stable
- -pi/2
- Unstable
- pi/2
- Stable
- 3pi/2
- Unstable
- c

(correct)

Correct Answers:

- -4.712388981
- STABLE
- -1.570796327
- UNSTABLE
- 1.570796327
- STABLE
- 4.712388981
- UNSTABLE
- C

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9. (3 pts) (Subcritical Pitchfork bifurcation) Consider the following differential equation

$$\frac{dy}{dt} = y^3 - \alpha y.$$

a. Let $\alpha = 4$. Find all the equilibria for this equation.
($y_{1e} < y_{2e} < y_{3e}$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{3e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

b. From the collection of phase portraits shown, find the letter (A-F) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are Unstable and full circles represent stable.) _____
Phase Portraits - Bifurcations

c. Let $\alpha = -4$. Find the equilibrium for this equation.
The equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

d. From the collection of phase portraits shown, find the letter (A-F) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are Unstable and full circles represent stable.) _____
Phase Portraits - Bifurcations

e. You should determine what are the differences in behavior of these two cases. What value of α results in the change between these two behaviors?
$\alpha =$ _____ .

Answer(s) submitted:

- -2
- Unstable
- 0
- Stable
- 2
- Unstable
- c
- 0
- Unstable
- e
- 0

(correct)
Correct Answers:

- -2
- UNSTABLE
- 0
- STABLE
- 2
- UNSTABLE
- C
- 0
- UNSTABLE
- E
- 0

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10. (3 pts) (Transcritical bifurcation) Consider the following differential equation

$$\frac{dy}{dt} = \alpha y - y^2.$$

a. Let $\alpha = 3$. Find all the equilibria for this equation.
($y_{1e} < y_{2e}$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

b. From the collection of phase portraits shown, find the letter (A-F) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are Unstable and full circles represent stable.) _____
Phase Portraits - Bifurcations

c. Let $\alpha = -3$. Find the equilibrium for this equation.
($y_{1e} < y_{2e}$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

d. From the collection of phase portraits shown, find the letter (A-F) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are Unstable and full circles represent stable.) _____
Phase Portraits - Bifurcations

e. You should determine what are the differences in behavior of these two cases. What value of α results in the change between these two behaviors?
$\alpha =$ _____ .

Answer(s) submitted:

- 0
- Unstable
- 3
- Stable
- f
- -3
- Unstable
- 0
- Stable
- a
- 0

(correct)
Correct Answers:

- 0
- UNSTABLE
- 3
- STABLE
- F
- -3
- UNSTABLE
- 0
- STABLE
- A
- 0

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11. (3 pts) (Saddle-node or Blue sky bifurcation) Consider the following differential equation

$$\frac{dy}{dt} = \alpha - y^2.$$

a. Let $\alpha = 4$. Find all the equilibria for this equation. If none exist, then type "None" in all blanks.
($y_{1e} < y_{2e}$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

b. From the collection of phase portraits shown, find the letter (A-F) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are Unstable and full circles represent stable.) _____
Phase Portraits - Bifurcations

c. Let $\alpha = -4$. Find the equilibrium for this equation. If none exist, then type "None" in all blanks.
($y_{1e} < y_{2e}$)
One equilibrium is $y_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $y_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____

d. From the collection of phase portraits shown, find the letter (A-F) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are Unstable and full circles represent stable.) _____
Phase Portraits - Bifurcations

e. You should determine what are the differences in behavior of these two cases. What value of α results in the change between these two behaviors?
$\alpha =$ _____ .

Answer(s) submitted:

- -2
- Unstable
- 2
- Stable
- b
- none
- none
- none
- none
- d
- 0

(correct)
Correct Answers:

- -2
- UNSTABLE
- 2
- STABLE
- B
- NONE
- NONE
- NONE
- NONE
- D
- 0

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12. (3 pts) (Allee effect) Suppose that a population, $P(t)$ (in thousands), is given by the model

$$\frac{dP}{dt} = P\left(4 - 0.01(P-50)^2\right).$$

a. Find all the equilibria for this equation.
($P_{1e} < P_{2e} < P_{3e}$)
One equilibrium is $P_{1e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $P_{2e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
Another equilibrium is $P_{3e} =$ _____ .
The stability of the equilibrium point is Stable or Unstable _____
You should sketch the graph of the phase portrait.

b. Find the carrying capacity for this particular population.
Carrying capacity = _____
Determine the critical threshhold number of animals required to avoid extinction.
Threshhold value = _____

c. From the collection of phase portraits shown, find the letter (A-D) corresponding to the appropriate phase portrait for this differential equation. (Note open circles are Unstable and full circles represent stable.) _____
Phase Portraits - Allee Effect

Answer(s) submitted:

- 0
- Stable
- 30
- Unstable
- 70
- Stable
- 70
- 30
- d

(correct)
Correct Answers:

- 0
- STABLE
- 30
- UNSTABLE
- 70
- STABLE
- 70
- 30
- D