Final
Algebraic Coding Theory
Math 525
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Problem 2: Before starting this problem, you will need to obtain two words of length four, namely,s_1 and s_3, as follows. If the last three letters in your last name are one of:

$$s_1 = 0011 \qquad s_3 = 1011$$

Consider the field GF(2^4) constructed from $1 + x + x^4$, see Table 5.1, p. 114. Let C_{15} be the BCH code of length 15 with generator polynomial $g(x) = m_1(x) \cdot m_3(x)$ where $m_1(x)$ and $m_3(x)$ are the minimal polynomials of β and β^3, respectively, with β a primitive element of GF(2^4),exactly as in Table 5.1. Suppose messages are encoded using C_{15} and a certain received vector r has syndrome equal to $[s_1, s_3]$. Determine the location of the errors (if any) in r. Note: Each location is an integer in [0..14].

(1) $s = [s_1, s_3] = [0011, 1011] = [\beta^6, \beta^{13}]$

(2) $s_1 \neq 0$ and $s_3 \neq s_1^3$

$$\begin{aligned} x^2 + s_1 x + \left(\frac{s^3}{s_1} + s_1^2\right) &= 0 \\ x^2 + \beta^6 x + \left(\beta^7 + \beta^{12}\right) &= 0 \\ x^2 + \beta^6 x + \beta^2 &= 0 \end{aligned}$$

Notice the following:

$$\beta^6 = \beta^7 + \beta^{10} \text{ and } \beta^7 \cdot \beta^{10} = \beta^{17} = \beta^2$$

So $e(x) = x^2 + x^3$

(3) So locations are 3 and 4