

**Slide #11.** Let  $r(x) = x^9 + x^7 + x^4 + x^2 + 1$ . Then

$$r(\beta) = \beta^9 + \beta^7 + \beta^4 + \beta^2 + 1.$$

Adding the above elements, we obtain:

$$\begin{array}{rcl}
 0101 & \leftarrow & \beta^9 \\
 1101 & \leftarrow & \beta^7 \\
 1100 & \leftarrow & \beta^4 \\
 + & & \\
 0010 & \leftarrow & \beta^2 \\
 1000 & \leftarrow & 1 \\
 \hline
 1110 & \xrightarrow{\text{red}} & \beta^{10}
 \end{array}$$

Therefore,  $r(\beta) = \beta^{10}$ .

Next,

$$\begin{aligned}
 r(\beta^3) &= \beta^{27} + \beta^{21} + \beta^{12} + \beta^6 + 1 \\
 &= \beta^{12} + \beta^6 + \beta^{12} + \beta^6 + 1 \\
 &= 1
 \end{aligned}$$

since  $\beta^{12} + \beta^{12} = 0$  and  $\beta^6 + \beta^6 = 0$ .