

Homework 1
Numerical Matrix Analysis
Math 543
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$$\begin{array}{ccccccc}
 \text{Spring 1, } k_{12}, l_{12} & & \text{Spring 2, } k_{23}, l_{23} & & \text{Spring 3, } k_{34}, l_{34} & & (1) \\
 m_1 \text{ ---} & m_2 \text{ ---} & m_3 \text{ ---} & m_4 & & & (2) \\
 f_1 => & <= f_1 & f_2 => & <= f_2 & f_3 => & <= f_3 & (3)
 \end{array}$$

$$\text{Let } f_1 = k_{12}(-x_1 + x_2 - l_{12}) \quad (4)$$

$$\text{Let } f_2 = k_{23}(-x_2 + x_3 - l_{23}) \quad (5)$$

$$\text{Let } f_3 = k_{34}(-x_3 + x_4 - l_{34}) \quad (6)$$

$$F_1 = -k_{12}x_1 + k_{12}x_2 - k_{12}l_{12} \quad (7)$$

$$F_2 = k_{12}x_1 + (-k_{12} - k_{23})x_2 + k_{23}x_3 + k_{12}l_{12} - k_{23}l_{23} \quad (8)$$

$$F_3 = k_{23}x_2 + (-k_{23} - k_{34})x_3 + k_{34}x_4 + k_{23}l_{23} - k_{34}l_{34} \quad (9)$$

$$F_4 = k_{34}x_3 - k_{34}x_4 + k_{34}l_{34} \quad (10)$$

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix} = \begin{pmatrix} -k_{12} & k_{12} & 0 & 0 \\ k_{12} & -k_{12} - k_{23} & k_{23} & 0 \\ 0 & k_{23} & -k_{23} - k_{34} & k_{34} \\ 0 & 0 & k_{34} & -k_{34} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} -k_{12}l_{12} \\ k_{12}l_{12} - k_{23}l_{23} \\ k_{23}l_{23} - k_{34}l_{34} \\ k_{34}l_{34} \end{pmatrix}$$

\mathbf{F} is Newtons and \mathbf{x} is meters, so \mathbf{K} is, or the dimensions of the entries of \mathbf{K} are of $\frac{\text{Newtons}}{\text{Meters}}$.

The $\det(\mathbf{K})$ is of $\frac{\text{Newtons}^4}{\text{Meters}}$. I know this because the determinant would be a product of the diagonal, which is made of 4 entries.

\mathbf{K}' is 10^{-1} of the meters of \mathbf{K} , 10^{-3} of the grams of \mathbf{K} , and the same as the seconds of \mathbf{K} . The dimensions of the $\det(\mathbf{K})$ is the same as the $\det(\mathbf{K}')$ because the dimensions of the matrix doesn't change when multiplied by scalar multiples.