MATH 537, Fall 2020

Ordinary Differential Equations

Lecture #17

Mini Project

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Mini Project: An Extension of Problem 4

- A mini project is to extend the problem 4

Mid Term (30%)

- Part A: take-home problems, 15% Sep. 30 (W)
- Part B: 9:00-9:50 am, 15%, Oct 2 (F)

4: [20 points] Show off Your Skills and and Knowledge.

(a) [7 points] Design your problem using the skills and knowledge that have been discussed in the textbook or lectures.

(b) [7 points] Discuss why your problem is unique, as compared to the above problems and/or problems in homework (1-2).

(c) [6 points] Solve the problem.

0.2 * 15% = 3%

Mini Project as Part of the Final Exam (30%)

- Part A: take-home problems, 15% Dec 11 (F)
- Part B: 8:00-10:00 am, 20% Dec 14 (M)

A mini project is:

- optional,
- part of Part A of the final exam,
- worth of 20, 40 or 60 points (equivalent to 1, 2, or 3 problems in Part A of the final exam (to be determined by Nov. 18, Wed)

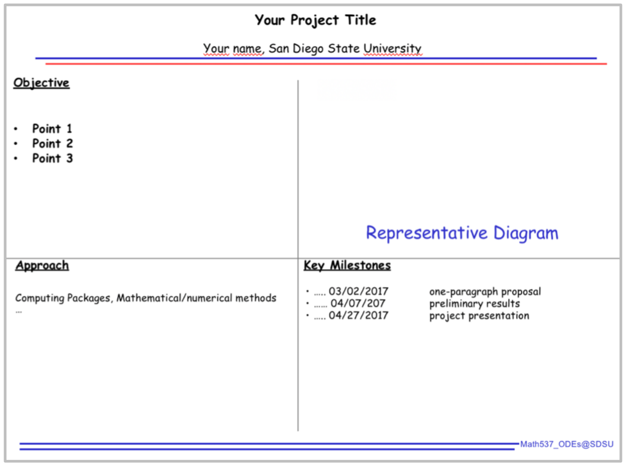
Tasks to finish a mini project include:

1. Compile what you have done into a mini report (4 pages)
2. Complete a QuadChart (TBD below)

How to Start

- Find a topic after studying or performing a review
- Digest and compile related materials
- Complete an initial QuadChart as an outline
- Complete a brief report and Finalize the QuardChart

➢ Find the hanger



QuadChart @gradescope (pdf) & canvas/supp (ppt)

Your Project Title

Your name, San Diego State University

Objective

- Point 1
- Point 2
- Point 3

Representative Diagram

Approach

Computing Packages, Mathematical/numerical methods
…

Key Milestones

- ….. 03/02/2017 one-paragraph proposal
- …… 04/07/207 preliminary results
- ….. 04/27/2017 project presentation

Math537_ODEs@SDSU

"Initial" QuadChart Due on Nov. 16 (Monday)

@gradescope

Math537-Fall-2020 | Fall 2020

DESCRIPTION

Edit your course description on the Course Settings page.

THINGS TO DO

- Finish grading Quiz I.
- Respond to 1 outstanding regrade request for Quiz 2.
- Finish grading MT Part A.

ACTIVE ASSIGNMENTS	RELEASED	DUE (PDT)	SUBMISSIONS	% GRADED	PUBLISHED	REGRADES
Initial Quad Chart	OCT 14	NOV 16 AT 11:59PM	0	0%		ON

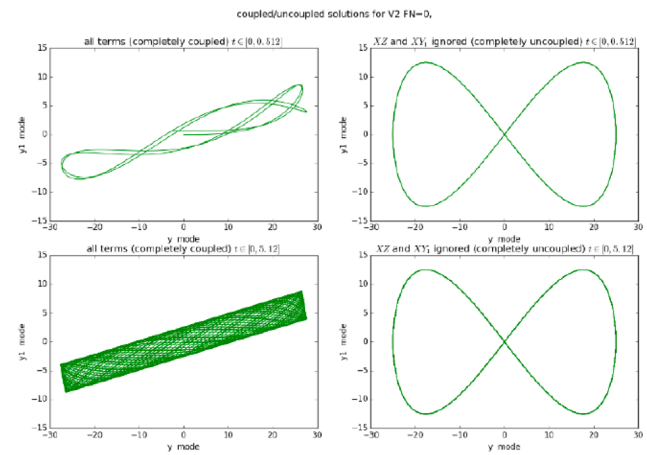
A Sample of QuadChart:

On quasi-periodic solutions associated with the extended nonlinear feedback loop of the five-dimensional nondissipative Lorenz model

Sara Faghih-Naini, San Diego State University

Objective

- **Analysis and comparison of 5D NLM**
- **Fundamental role of nonlinear terms in producing quasi-periodic solutions**
- **Collective role of the nonlinear feedback loop and heating term in producing quasi-periodic solutions**



Y-Y1-plots of periodic/quasiperiodic solutions for tend=0.512 (top) and tend=5.12 (bottom) of coupled (left) and uncoupled (right) LL 5D NLM

Approach

- **Performing analytical, symbolic (python scipy) and numerical (Python ODE solver (odeint)) analysis of 5D NLM**
- **Analyzing quasiperiodic solutions of 5D NLM**
- **Comparing to solutions of 5D NLM, locally linearized 5D NLM, 3D NLM, …**
- **Performing frequency analysis using Python fft package**
- **Visualizing quasiperiodic solutions using Python visualization packages**

Key Milestones

- **09/30/2016: solve for eigenvalues of 5D NLM (analytical and symbolic computation)**
- **11/01/2016: verify results, plots for comparison of 3D NLM, LL 3D NLM FN=0 and FN=1, 3D-eigenvaluesolution (analytical and symbolic); plots for comparison of 5D NLM, LL 5D NLM FN=0 and FN=1, 5D-eigenvaluesolution (analytical and symbolic)**
- **11/20/2016: finish frequency analysis**
- **11/22/2016: create slides for presentation**
- **11/30/2016: finish oral presentation**
- **12/06/2016: draft of paper**

An Outcome: Published Paper

Featured Articles: Most-Read Articles of 2018 (IJBC)

International Journal of Bifurcation and Chaos, Vol. 28, No. 6 (2018) 1850072 (20 pages)

DOI: 10.1142/S0218127418500724

Quasi-Periodic Orbits in the Five-Dimensional Nondissipative Lorenz Model: The Role of the Extended Nonlinear Feedback Loop

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Received January 28, 2018; Revised April 11, 2018

A Phd student in Germany; published 4 papers with me

An Outcome: Published Paper

Chaos, Solitons and Fractals 125 (2019) 1–12

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

A recurrence analysis of chaotic and non-chaotic solutions within a generalized nine-dimensional Lorenz model

Tiffany Reyes, Bo-Wen Shen*

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very rewarding salary in a private company; published 4 papers with me

An Invited Talk in 2020

You are also invited to subscribe to attend the web conference and follow the 3 day program along with the important Keynote, Plenary and Invited contributions from:

James A. Yorke, University of Maryland, USA on "The equations of nature and the nature of equations" with Sana Jahedi

Harold M Hastings, Bard College at Simon's Rock, USA on "Vector difference equations, Gerschgorin's theorem, and design of multi-networks to reduce spread of epidemics" with Tai Young-Taft

Vladimir L. Kalashnikov, Sapienza Universita di Roma, Italy on "Spatiotemporal Turbulence in a Multimode Fiber Laser" with Stefano Wabnitz

Shunji Kawamoto, Osaka Prefecture University, Japan on "Chaos identification of a colliding constraint body on a moving belt"

George Savvidy, National Centre for Scientific Research "Demokritos", Athens, Greece on "Maximally Chaotic Dynamical Systems"

A. Shvets, National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine on "Generalizing of Attractor Notion for Spherical Pendulum Systems" with S. Donetskyi

Bo-Wen Shen, San Diego State University, USA on "Applying a Kernel PCA Method to Reveal Coexisting Attractors within a Generalized Lorenz Model" with Jialin Cui

Tatiana F. Filippova, Krasovskii Institute of Mathematics and Mechanics, Ekaterinburg, Russia on "Interacting Populations: Dynamics and Viability in Bounded Domains under Uncertainty"

Marek Lampart, VSB-Technical University of Ostrava, IT4Innovations, Czech Republic on "Chaos identification of a colliding constraint body on a moving belt" with Jaroslav Zapoměl

J.P. Lebacque, UGE (University Gustave Eiffel) COSYS GRETTIA, France on "Chaotic behavior of dynamical systems associated with dynamic traffic assignment in transportation" with M.M. Khoshyaran

Evelina V. Prozorova, St. Petersburg State University, Russia on "Mechanism of Formation of Fluctuation Phenomena"

C H Skiadas, ManLab, TUC, Greece on "How the unsolved problem of finding the Healthy Life Expectancy (HLE) in the far past was resolved: The case of Sweden (1751-2016) with forecasts to 2060 and comparisons with HALE"

A Phd student with TA support in CS of NCSU; published 2 papers with me

A Final Report for the Mini Project

A final report with 4 pages, due on Dec 11 (F), should include:

1. An abstract (~300 words)
2. A QuadChart
3. Discussions
4. A summary (600-1000 words)

Potential Topics

1. Materials outlined in the “one slide summary”
2. Materials (HW, MT, Quiz) related the SIR model
3. Projects you are working
4. Your problem 4 in MT
5. Topics you feel interested

Criteria: Find a topic you have passion to complete

One Slide Summary

1st order	2nd order	eigenvalue problem
$y' = \alpha y - \beta y^{\mathbf{2}}$ (logistic eq.)	$x'' + \beta x' + \alpha x = 0$	$x' = ax + by$ $y' = cx + dy$
$y' = \alpha y - \beta y^3$	$x' = y$ $y' = -\alpha x - \beta y$	$X' = AX$ $AX = \lambda X$ $X = \begin{pmatrix} x \\ y \end{pmatrix}; A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

	nonlinear	a system of ODEs	
Non-dissipative Lorenz model, NLS, Duffing Eq.	$\boxed{x'' - \alpha x + \beta x^{3} = 0}$ (DE-sech)	$x' = y \equiv F$ $y' = \alpha x - \beta \mathrm{x}^3 \equiv G$	$JX = \lambda X$ $J = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix}_{X_c}$
KdV	$x'' - \alpha x + \beta x^{\mathbf{2}} = 0$ (DE-sech²)	$x' = y \equiv F$ $y' = \alpha x - \beta \mathrm{x}^2 \equiv G$	• mathematical analysis … • stability analysis….

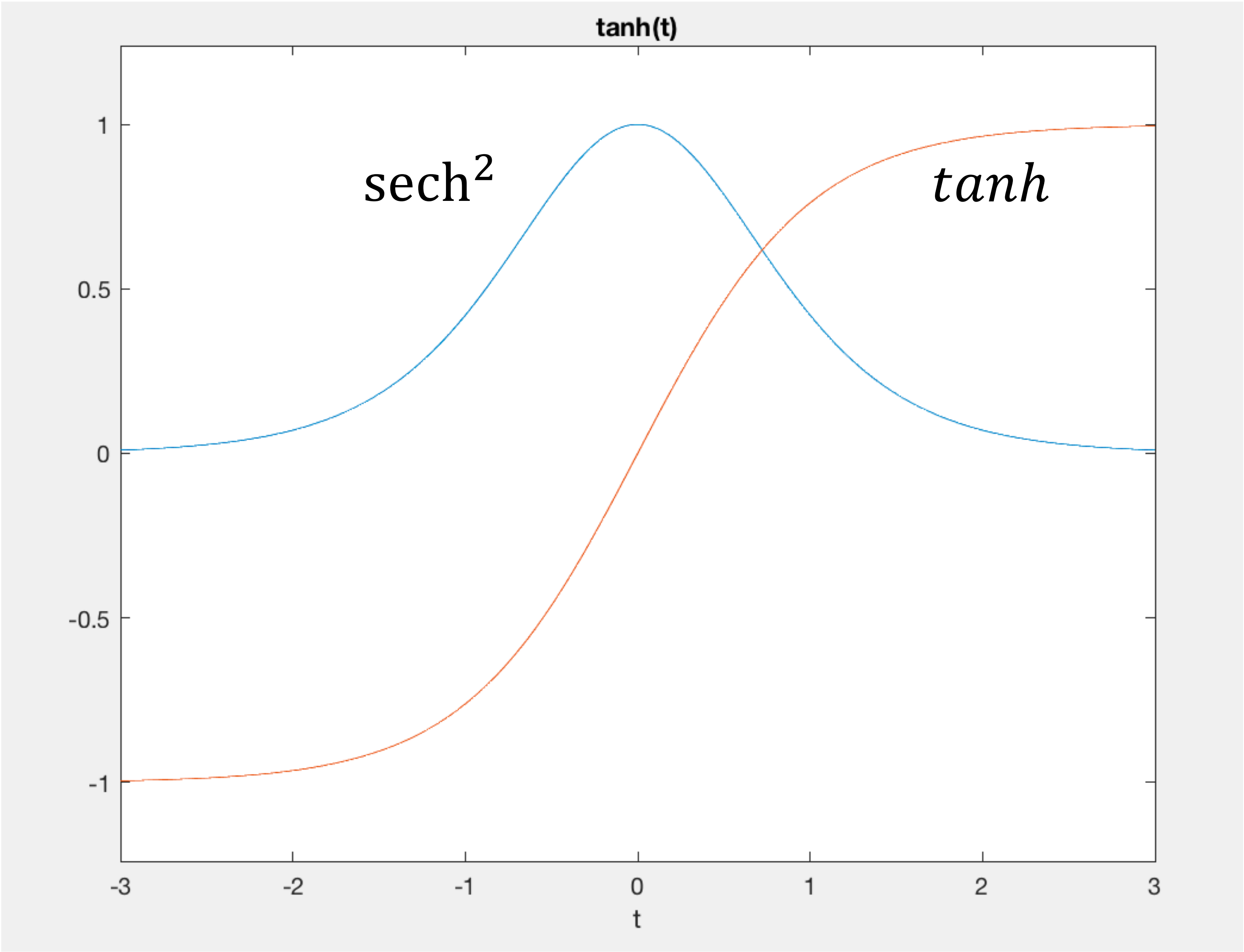
Adding Meats with Important Concepts

Asymptotic Matching	Homoclinic Orbits
Asymptotic Series	Jacobian Matrix & Linearization
A System of ODEs	Logistic Eq.
Bifurcation	Lorenz Model
Boundary Layer Theory	Perturbation Theory/Method
Critical Points	Quasi-periodicity
Conjugacy	Sigmoid function
DE-sech (e.g., Duffing Eq. & Nonlinear Schrodinger Eq.)	Solitary Wave
DE-sech^2 (e.g., KdV Eq.)	Stability Analysis (source, sink, saddle)
Eigenvalue Problem	Variational Eq.
Homeomorphism	WKB Analysis (oscillatory vs. exponential solutions)

Topic 2: A Simplified SIR

The SIR Model or KdV Eq.	Fundamental Models	Solutions
The Equation for R' $\frac{dR}{dt} = \nu\left(N - R - S(0)e^{-\frac{\beta}{N\nu}(R(t)-R(0))}\right)$		
A simplified Eq. for "weak outbreak" $\frac{dR}{dt} = \nu\,(N - R - S(0)(1 - x + x^2/2))$ $x = \frac{\beta}{N\nu}(R(t) - R(0))$ $e^{-x} \approx 1 - x + x^2/2$ MT Part A	The Logistic Equation $\frac{df}{dt} = f(1-f)$	$sigmoid$
	$\frac{dg}{dt} = \frac{1}{4} - g^2$	$tanh$
The Korteweg-de Vries Equation $\frac{d^2J}{d\xi^2} + 3J^2 - cJ = 0$ $c = 1/2$ Quiz 3	$\frac{d^2Z}{d\tau^2} + 3Z^2 - Z/2 = 0$ $Z = dg/dt$, $\tau = \sqrt{2}t$	$sech^2$

tanh vs. sech



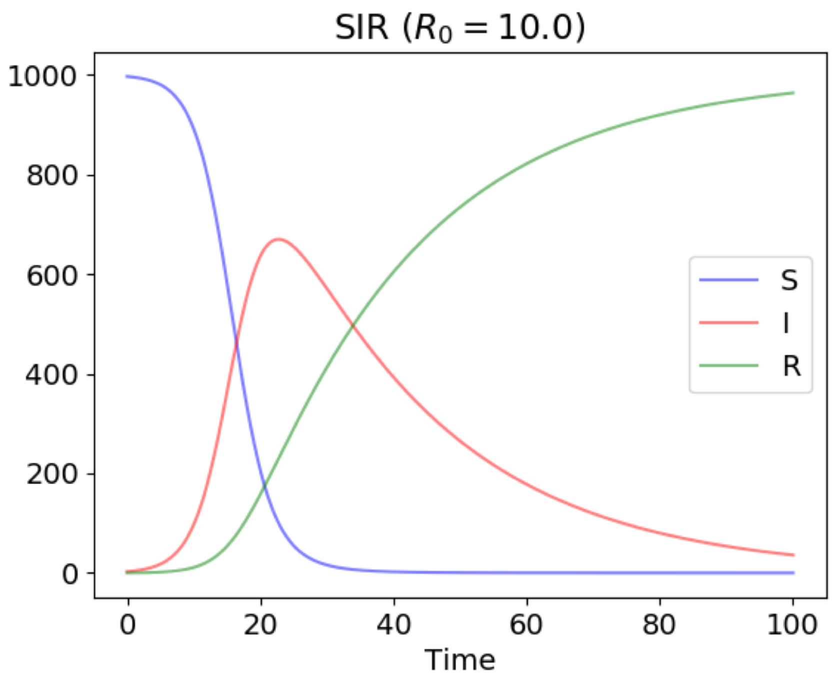
"tanh" and "sech"

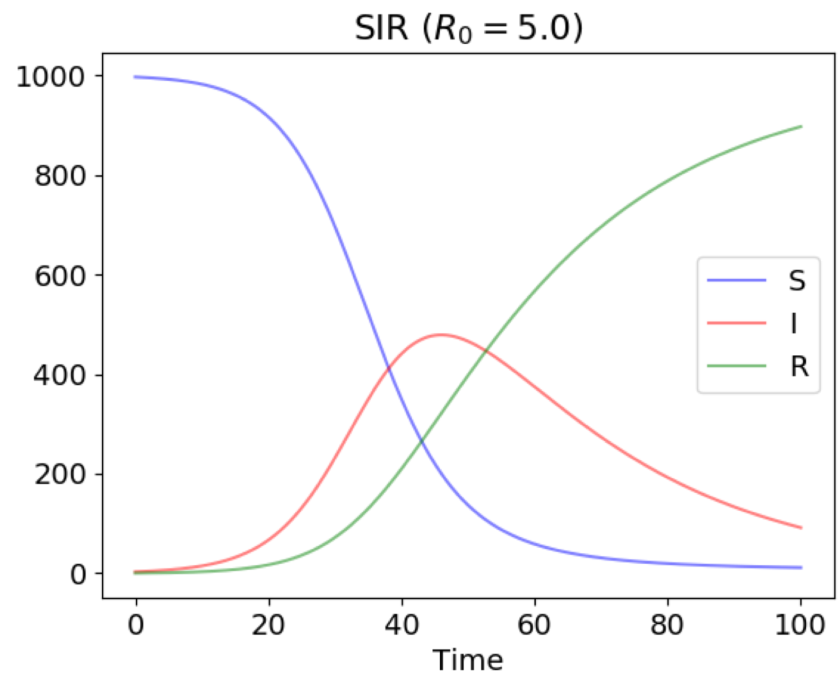
Supp

$R_0 = \frac{\beta}{\nu}$: basic reproduction #, or contact #.

$\beta > 0$: infection rate
(transmission rate, or transmission coefficients);

$\nu > 0$: recovery rate;





$I \approx \mathrm{sech}^2$

$R \approx tanh$

4: [25 points] Consider the following SIR epidemic model (Kermack and McKendrick, 1927):

$$\frac{dS}{dt} = -\frac{\beta}{N} SI, \tag{4.1}$$

$$\frac{dI}{dt} = \frac{\beta}{N} SI - \nu I, \tag{4.2}$$

$$\frac{dR}{dt} = \nu I. \tag{4.3}$$

Here, S, I, and R denote susceptible, infected, and recovered individuals, respectively. Three parameters, $\beta > 0$, $\nu > 0$, and $N > 0$, represent a transmission rate, a recovery rate, and a fixed population ($N = S + I + R$), respectively. Complete the following:

(a) [5 points] Find critical points.

(b) [8 points] Compute the Jacobian matrix.

(c) [12 points] Perform a linear stability analysis for critical points.

(a) $I = 0$

(b1)

$$J = \begin{pmatrix} -\frac{\beta}{N} I & -\frac{\beta}{N} S & 0 \\ \frac{\beta}{N} I & \frac{\beta}{N} S - \nu & 0 \\ 0 & \nu & 0 \end{pmatrix}$$

(b2) the 2D Jacobian matrix at the critical point(s):

$$J_{2D} = \begin{pmatrix} 0 & -\frac{\beta}{N} S \\ 0 & \frac{\beta}{N} S - \nu \end{pmatrix}$$

(c1) $\lambda_1 = 0$ and $\lambda_2 = \frac{\beta}{N} S - \nu$.

(c2) $\lambda_2 > 0$ when $S > \frac{N\nu}{\beta}$.

(c3) $\lambda_2 < 0$ when $S < \frac{N\nu}{\beta}$.

Research Approaches: Certainty vs. Reality

“As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.”

-Albert Einstein

Reductionism vs. Complexity

Additional Help

A Sample Problem in HW (from Math542, Fall 2019)

3: [25 points] Provide a brief report on the progress of your project. [Points will be given after the completion of the final project presentation.]

(a) Please state what has been done during the past two weeks and what will be performed in the next two weeks. (you may want to provide the following: an introduction of the backgrounds about your study, your methodology, required computing skills that have been tested, etc.)

(b) Provide any preliminary results (e.g., figures, mathematical equations, references/web sites, etc).

(c) Please document any scientific, mathematical, numerical, computing, and/or technical issues.

(d) If you have a revised plan or a mitigation plan (to resolve the issues in (2c)), please state it.