PROBLEM SET 4

Problem 1. Consider the subset

$$S = \{11000, 00011, 01110\}$$

of K^5.

(a) Find the code C generated by S (i.e., list all of its codewords).
(b) Find $C^\perp$, the dual code of C (i.e., list all of its codewords).

Problem 2. Consider the set

$$S = \{110011, 010100, 001101, 100111\}$$

of words in K^6.

(a) Find a generator matrix G, in RREF, for the code $C = \langle S \rangle$. What is $\dim C$?
(b) From the matrix G above, find a parity-check matrix H for C.
(c) Use H to determine the distance of C.

Problem 3. Let

$$M = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

be a matrix with entries in K and let C be the code generated by it.

(a) Find a systematic encoding matrix G for C.
(b) Use G to encode the information vector $(u_0, u_1, u_2, u_3) \in K^4$.
(c) Find the dimension of C and $C^\perp$. Find the number of codewords in C and $C^\perp$.
(d) Find a parity-check matrix H for C.
(e) From H, conclude that $C = C^\perp$ in this case.

Problem 4. Exercises 2.6.5, 2.6.6, 2.6.10, and 2.6.13 on pp. 43–45.

Problem 5. Exercises 2.7.4, 2.7.5, 2.7.9–2.7.11 on pp. 46–48.

Problem 6. Exercises 2.8.11, 2.8.12, and 2.8.14 on p. 52.

Problem 7. Exercises 2.9.4 and 2.9.5 on p. 53.

Problem 8. Exercises 2.10.6–2.10.8 and 8 on p. 56.

Problem 9. Exercise 2.11.8–2.11.10 on p. 60.

Problem 10. Exercises 2.11.16 and 2.11.19–2.11.21 on pp. 61 and 62.

Problem 11. Exercise 2.12.2 on p. 63.