Math 524, Spring 2020
Midterm #2, Take-Home,

Tools: Pencil/Eraser/Paper/TEXTBOOK.
Rules: This is a take-home midterm; see below:

RedID v 2020.03.26.t	First Letter of Last Name

I, ________________________, pledge that this exam is **completely my own work**, and that I did not take, copy, borrow or steal any portions from any other person; furthermore, I did not knowingly let anyone else take, copy, or borrow any portions of my exam. Further, I pledge to abide by the rules set out below. I understand that if I violate this honesty pledge, (*i*) I will get ZERO POINTS on this exam; *(ii)* I will get reported to The SDSU Center for Student Rights and Responsibilities; and *(iii)* I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

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Signature (REQUIRED for credit)

Rules:

- **Due 4/11/2020, 11:59pm, UPLOAD ALL PAGES TO GRADESCOPE, attach extra page(s) AFTER the 8 numbered pages**
- This midterm is **OPEN-BOOK (Sheldon Axler, *"Linear Algebra Done Right"*)**, open-notes, open-wikipedia, but **CLOSED CHEGG** (or other organized do-my-homework-sites); it the exam is uploaded (all, or in-part) to such a website, I may post a new version (making this version VOID).
- You cannot consult any Human / Primate / Extra-Terrestrial Alien / Artificial Intelligence; Dolphins are OK.
- If you refer to results from books (other than the class text), research papers, or the web (other than the class web page(s)) carefully cite your source(s).
- Present your solutions using standard notation in an easy-to-read format. It is your job to convince the grader you did the problem correctly, not the grader's job to decipher cryptic messages scribbled in the margin! *Your answers MUST logically follow from your calculations in order to be considered! (**"Miracle solutions" ⇒ zero points.**)*
- The exam will be graded and returned (via Gradescope) as soon as possible.

Problem	Pts Possible	Pts Scored
1	100	
2	100	
3	100	
Total	300	

1. Let $T \in \mathcal{L}(\mathbb{F}^3)$ be defined by $T(z_1, z_2, z_3) = (2z_2, 0, 5z_3)$

   (a) (50 pts.) Find all eigenvalues and eigenspaces of T

   (b) (10 pts.) What is range(T) — "find a basis for range(T)":

   (c) (10 pts.) What is null(T) — "find a basis for null(T)":

   (d) (10 pts.) Is $\mathbb{F}^3 = \text{null}(T) \oplus \text{range}(T)$?

   (e) (10 pts.) Is $\mathbb{F}^3 = E(\lambda_1, T) \oplus \cdots \oplus E(\lambda_m, T)$?

   (f) (10 pts.) Is T diagonalizable? Why/Why not?

   (g) Bonus round: let $S = T^2$ (T from above)

      i. (5 pts.) Find all eigenvalues and eigenspaces of S
      ii. (1 pts.) What is range(S) — "find a basis for range(S)":
      iii. (1 pts.) What is null(S) — "find a basis for null(S)":
      iv. (1 pts.) Is $\mathbb{F}^3 = \text{null}(S) \oplus \text{range}(S)$?
      v. (1 pts.) Is $\mathbb{F}^3 = E(\lambda_1, S) \oplus \cdots \oplus E(\lambda_m, S)$?
      vi. (1 pts.) Is S diagonalizable? Why/Why not?

Whenever you rely on a specific definition or theorem from the book, carefully specify which one (by name, or `n.nn`-reference). Always be clear on what properties you are checking, what is satisfied, and what is not.

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2. (100 pts.) Consider $(x_1, \ldots, x_n) \in \mathbb{R}^n$, where $x_\ell > 0$, $\forall \ell \in \{1, \ldots, n\}$; find a lower bound for

$$\left(\sum_{k=1}^{n} x_k\right)\left(\sum_{k=1}^{n} \frac{1}{x_k}\right)$$

(Significance: Inner Products and Norms).

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3. (100 pts.) Consider the inner product $\langle p, q \rangle = \displaystyle\int_0^1 p(x)q(x)\,dx$ for $p, q \in \mathcal{P}(\mathbb{R})$. On $\mathcal{P}_2(\mathbb{R})$ our friends Gram & Schmidt kindly provide an orthonormal basis:

$$\left\{ u_1(x) = 1, \quad u_2(x) = \sqrt{3}(-1+2x), \quad u_3(x) = \sqrt{5}(1-6x+6x^2) \right\}$$

Find a polynomial $q \in \mathcal{P}_2(\mathbb{R})$ so that $\forall p \in \mathcal{P}_2(\mathbb{R})$:

$$p\left(\frac{1}{2}\right) = \int_0^1 p(x)q(x)\,dx$$

(Significance: Inner Product spaces).

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