Homework 4

In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
import random

import warnings
warnings.filterwarnings('ignore')
```

Problem 0

Write a summary of all important formulas and definitions from these eigenvalue notes.

Characteristic Polynomial (2×2)

$$P_A(\lambda) = det(A - \lambda I_2) = \lambda^2 - tr(A)\lambda + det(A)$$

Discriminant of A:

$$D = [tr(A)]^2 - 4det(A)$$

- If $D > 0$, 2 Real and Distinct Eigenvalues
- If $D < 0$, 1 Pair of Complex Conjugate Eigenvalues
- If $D = 0$, 2 Real and Equal Eigenvalues

Characteristic Polynomial ($n \times n$)

$$P_A(\lambda) = det(A - \lambda I_n)$$

- Easy to solve when A is a triagular matrix, such that the product of the diagonal entries is the Characteristic Polynomial.

Problem 1

Reproduce Fig 5 of the article "Chaos in the cubic mapping".

Note: the procedure on how to produce bifurcation diagrams, in addition of what I explained in class, it is briefly explained in the first paragraph in p18 of our textbook.

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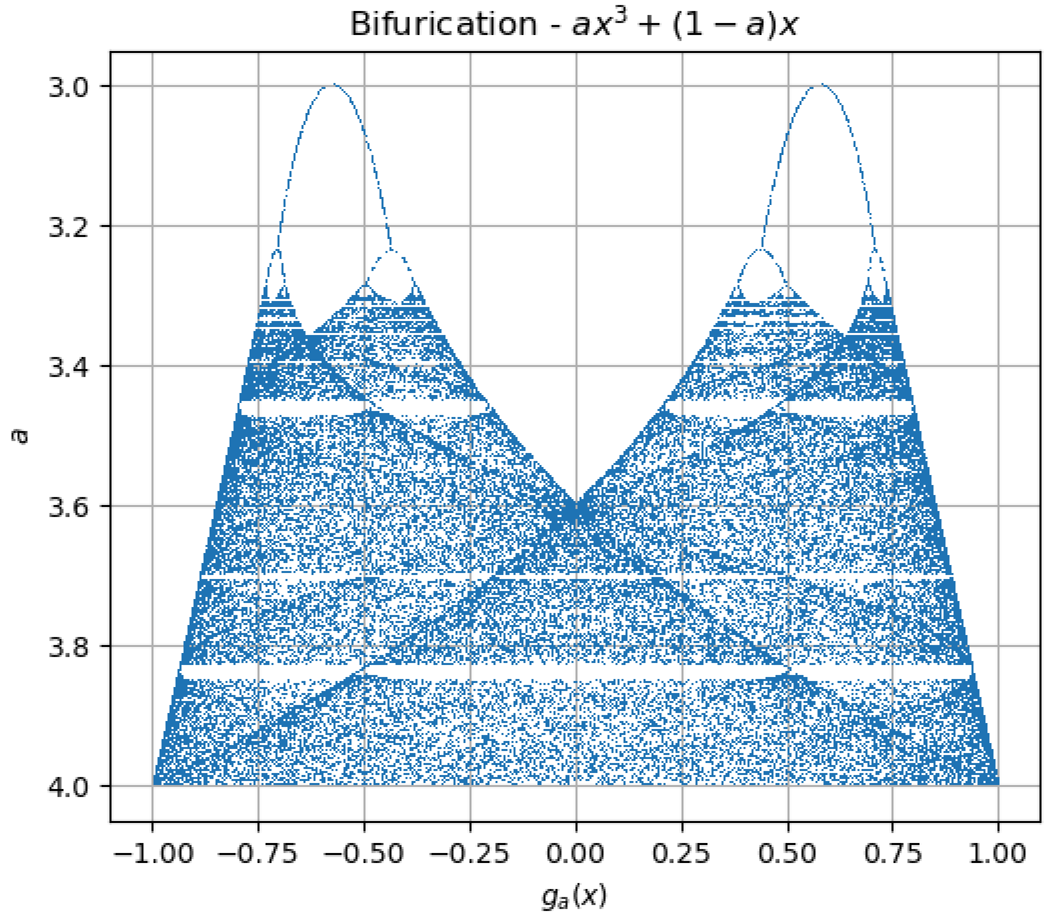
In [2]:
```
def plot_bifurication(a_ranges, f, title, filename, swap_x_a=False):
    fig, ax = plt.subplots(ncols=len(a_ranges), figsize=(6 * len(a_ranges),
    if len(a_ranges) == 1:
```

```python
        ax = [ax]
    for axis, a_vals in zip(ax, a_ranges):
        x_vals = []
        plot_a_vals = []
        for a in a_vals:
            x = random.random()
            for _ in range(int(1e3)):
                x = f(a, x)
            for _ in range(int(1e2)):
                x = f(a, x)
                x_vals.append(x)
                plot_a_vals.append(a)
        if swap_x_a:
            axis.plot(x_vals, plot_a_vals, ',')
            axis.invert_yaxis()
        else:
            axis.plot(plot_a_vals, x_vals, ',')

        axis.set_xlabel('$g_a(x)$')
        axis.set_ylabel('$a$')
        axis.set_title('Bifurication - ' + title)
        axis.grid()

    plt.savefig(filename + '.png')

    return fig
```

In [3]:

```python
a = plot_bifurication([np.linspace(3, 4, int(1e3))], lambda a,x: a*x**3 + (1
```



Problem 2

Reproduce Fig. 2.3 (p51 of our textbook). Note that Fig. 2.3.b is with $b = -0.3$ and $a = 1.5$ (and not $a = 1.4$)

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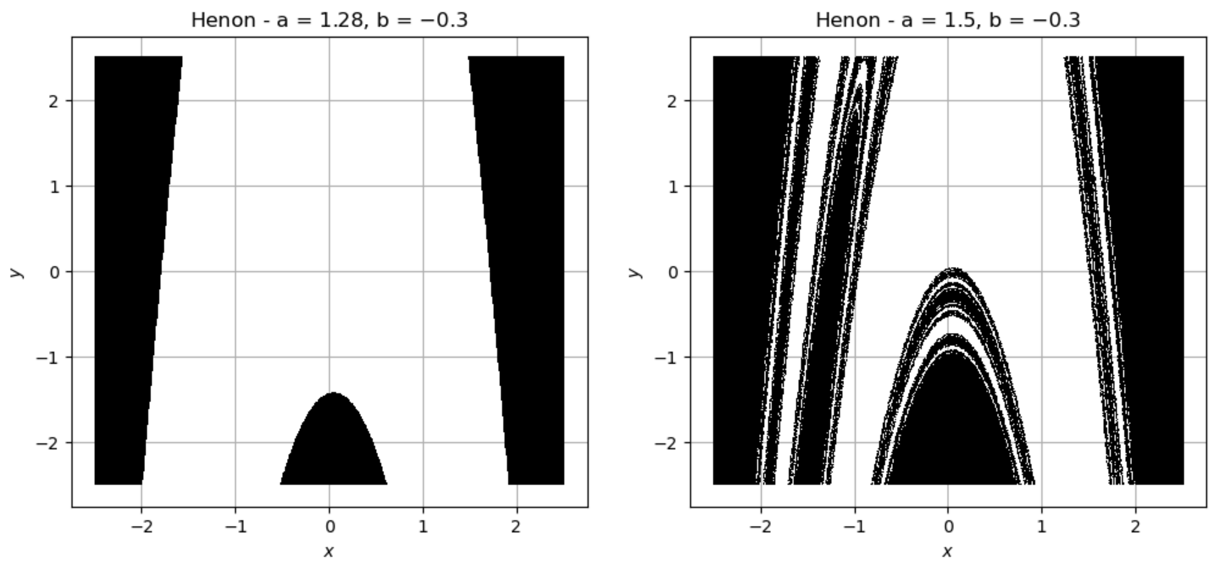
In [4]:
```
def plot_henon_analysis(a_range, b_range, x_range, y_range, num_iters, filer
    fig, ax = plt.subplots(ncols=len(a_range), figsize=(6 * len(a_range), 5)

    if len(a_range) == 1:
        ax = [ax]

    for (a, b, axis) in zip(a_range, b_range, ax):
        diverged_points = []
        for x0 in x_range:
            for y0 in y_range:
                x = x0
                y = y0
                try:
                    for _ in range(num_iters):
                        prev_x = x
                        prev_y = y
                        x = a - prev_x**2 + b*prev_y
                        y = prev_x
```

```
                        if np.isinf(x) or np.isnan(x) or np.isinf(y) or np.i
                            raise Exception("invalid number")
                except (RuntimeError, Exception):
                    diverged_points.append((x0, y0))

        diverged_x = [x[0] for x in diverged_points]
        diverged_y = [x[1] for x in diverged_points]

        axis.plot(diverged_x, diverged_y, 'k,')
        axis.set_xlabel('$x$')
        axis.set_ylabel('$y$')
        axis.set_title('Henon - a = $' + str(a) + '$, b = $' + str(b) + '$')
        axis.grid()

    plt.savefig(filename + '.png')

    return fig
```

In [5]:
```
a = plot_henon_analysis([1.28, 1.5], [-0.3, -0.3], np.linspace(-2.5, 2.5, 70
```



Problem 3

Reproduce Fig. 2.10 (p61 of our textbook).

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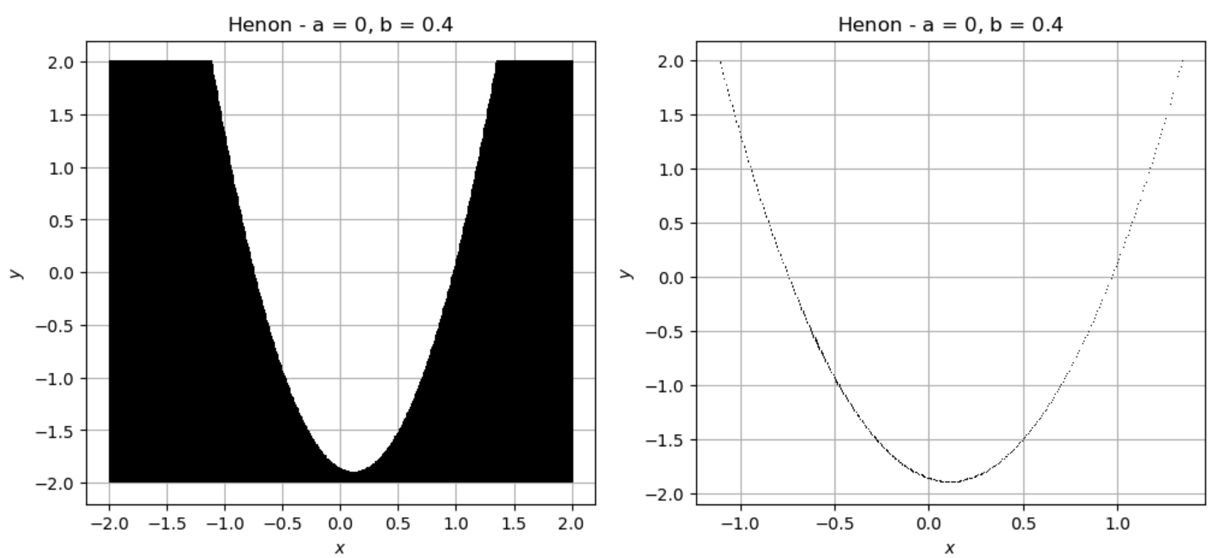
In [6]:
```
def plot_henon_analysis_3(a, b, x_range, y_range, fixed_point, tol, num_iter
    fig, ax = plt.subplots(ncols=2, figsize=(12, 5))

    converged_points = []
    diverged_points = []
    for x0 in x_range:
        for y0 in y_range:
            x = x0
            y = y0
            try:
                for _ in range(num_iters):
```

```
                    prev_x = x
                    prev_y = y
                    x = a - prev_x**2 + b*prev_y
                    y = prev_x
                    if np.isinf(x) or np.isnan(x) or np.isinf(y) or np.isnan
                        raise Exception("invalid number")
                    if np.abs(x - fixed_point[0]) < tol and np.abs(y - fixed
                        converged_points.append((x0, y0))
            except (RuntimeError, Exception):
                diverged_points.append((x0, y0))

    diverged_x = [x[0] for x in diverged_points]
    diverged_y = [x[1] for x in diverged_points]
    converged_x = [x[0] for x in converged_points]
    converged_y = [x[1] for x in converged_points]

    ax[0].plot(diverged_x, diverged_y, 'k,')
    ax[1].plot(converged_x, converged_y, 'k,')
    ax[1].plot([fixed_point[0]], [fixed_point[1]], 'k,')
    for i in range(len(ax)):
        ax[i].set_xlabel('$x$')
        ax[i].set_ylabel('$y$')
        ax[i].set_title('Henon - a = $' + str(a) + '$, b = $' + str(b) + '$'
        ax[i].grid()

    plt.savefig(filename + '.png')

    return fig
```

In [7]:
```
a = plot_henon_analysis_3(0, 0.4, np.linspace(-2, 2, 500), np.linspace(-2, 2
```



Problem 4

Reproduce Fig. 2.11 (p61 of our textbook).

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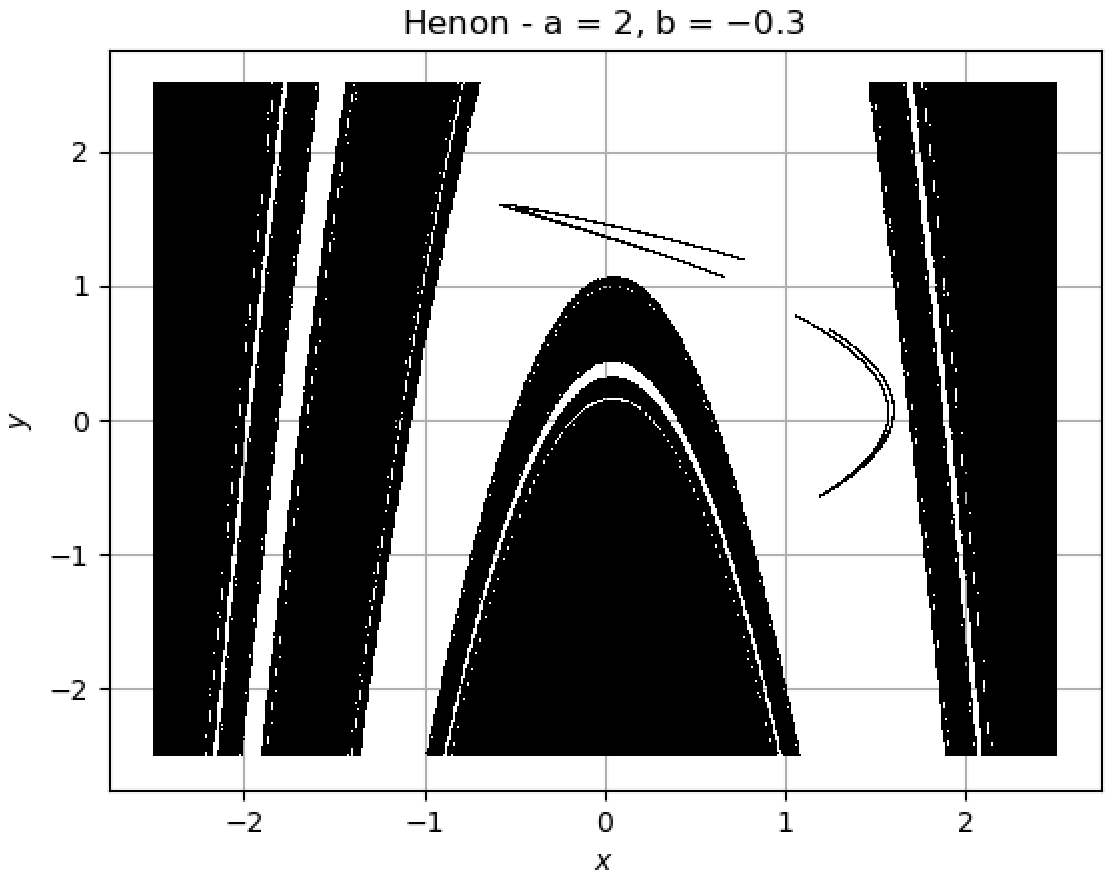
```
In [8]: def plot_henon_analysis_4(a, b, x_range, y_range, num_iters, filename):
            fig, ax = plt.subplots()

            converged_points = []
            diverged_points = []
            for x0 in x_range:
                for y0 in y_range:
                    x = x0
                    y = y0
                    try:
                        for _ in range(num_iters):
                            prev_x = x
                            prev_y = y
                            x = a - prev_x**2 + b*prev_y
                            y = prev_x
                            if np.isinf(x) or np.isnan(x) or np.isinf(y) or np.isnar
                                raise Exception("invalid number")
                        converged_points.append((x, y))
                    except (RuntimeError, Exception):
                        diverged_points.append((x0, y0))

            diverged_x = [x[0] for x in diverged_points]
            diverged_y = [x[1] for x in diverged_points]
            converged_x = [x[0] for x in converged_points]
            converged_y = [x[1] for x in converged_points]

            ax.plot(diverged_x, diverged_y, 'k,')
            ax.plot(converged_x, converged_y, 'k,')
            ax.set_xlabel('$x$')
            ax.set_ylabel('$y$')
            ax.set_title('Henon - a = $' + str(a) + '$, b = $' + str(b) + '$')
            ax.grid()

            plt.savefig(filename + '.png')

            return fig
```

```
In [9]: a = plot_henon_analysis_4(2, -0.3, np.linspace(-2.5, 2.5, 700), np.linspace(
```


Henon - a = 2, b = −0.3
y
x
2
1
0
−1
−2
−2
−1
0
1
2