HW5
Math 537 Ordinary Differential Equations
Due Nov 13, 2020

Student Name: ______________________________ **ID** ______________

1: [25 points] Consider the following second-order ordinary differential equations (ODEs) for nonlinear pendulum oscillations:

$$\frac{d^2\theta}{dt^2} + \epsilon\frac{d\theta}{dt} + sin(\theta) = 0. \tag{1.1}$$

Applying Taylor series expansions, Eq. (1) can be simplified into one of the following systems:

$$\frac{d^2\theta}{dt^2} + \theta = 0. \tag{1.2}$$

$$\frac{d^2\theta}{dt^2} + \epsilon\frac{d\theta}{dt} + \theta = 0. \tag{1.3}$$

$$\frac{d^2\theta}{dt^2} + \left(\theta - \frac{\theta^3}{6}\right) = 0. \tag{1.4}$$

(a) [21 points] Perform a linear stability analysis in each of Eqs. (1.2)-(1.4).

(b) [4 points] Discuss the concept of structural stability using results in (1a).

2: [35 points] Consider the following system:

$$\frac{d^2x}{dt^2} - \alpha x = e^{\beta t}. \tag{2.1}$$

Complete the following problems with $(\alpha, \beta) = (1, -1)$ and $(\alpha, \beta) = (-1, -1)$.

(a) [10 points] Solve Eqs. (2.1) for the solutions.

(b) [5 points] Convert Eqs. (2.1) into an autonomous linear system which consists of three first-order differential equations.

(c) [15 points] Solve for the eigenvalues and eigenvectors of the autonomous systems in (2b).

(d) [5 points] Compare the results in (2a) and (2c).

3: [40 points] Consider the following coupled harmonic oscillator (as shown in Fig. 1):

$$\frac{d^2x_1}{dt^2} = -k_1x_1 + k_2(x_2 - x_1), \tag{3.1}$$

$$\frac{d^2x_2}{dt^2} = -k_2(x_2 - x_1). \tag{3.2}$$

Let $k_1 = 4X_c^2$ and $k_2 = X_c^2$ (and $m_1 = m_2 = 1$).

(a) [5 points] Convert the above equations into a linear system with first-order differential equations.

(b) [15 points] Find the eigenvalues and eigenvectors.

(c) [20 points] Find the general solutions.

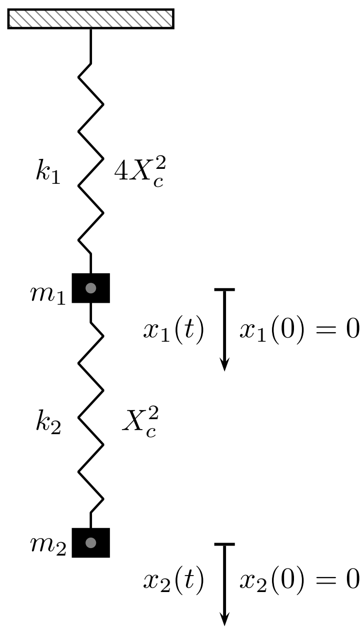

Figure 1: Coupled spring/mass system