1. (1 pt) Before you begin testing WeBWorK, please change your password and update your email address.
Your E-mail address is very important since many announcements/updates/info will be sent via e-mail.

To change your password and email address, use the Main Menu of the WebWork page.

When you change your password, WebWork responds with a statement in green. The last three words in that statement are "... password ______."
Fill in the 3 words of the blank above [do not use a period at the end].

The same item from the Main Menu allows you to add your email address.

When you have added your email, WebWork responds with a sentence in green. What is that sentence? [Do not forget the period at the end of the phrase]

______

Answer(s) submitted:

- `has been changed`
- `Your email address has been changed.`

(correct)
Correct Answers:

- `HAS BEEN CHANGED`
- `YOUR EMAIL ADDRESS HAS BEEN CHANGED.`

2. (1 pt) A population of bacteria begins with 900000. The population is growing according to the Malthusian growth equation given by

$$P'(t) = 0.012P(t),$$

where t is in minutes. Give the solution to this differential equation.
$P(t) =$ ______
Find how long it takes for this population to double.
Doubles in ______ min.
Find the population after $t = 120$.
$P(120) =$ ______.

Answer(s) submitted:

- `900000e^(.012t)`
- `ln(2)/.012`
- `900000e^(.012*120)`

(correct)
Correct Answers:

- `900000*exp(0.012*t)`
- `57.7622650466621`
- `3798626.2352969`

3. (1 pt) A radioactive substance with a half-life of 58 days decays according to the differential equation:

$$\frac{dR}{dt} = -kR.$$

Suppose that initially there are 55 mg of the substance. Find the rate constant
$k =$ ______ (days $^{-1}$).
Determine the amount of the substance after 30 days. $R(30) =$ ______.

Answer(s) submitted:

- `ln(1/2) / -58`
- `55e^(30ln(1/2) / 58)`

(correct)
Correct Answers:

- `0.0119508134579301`
- `38.4288616018132`

4. (1 pt) A population is growing according to the Malthusian growth equation given by

$$\frac{dP}{dt} = rP(t),$$

where t is in years with $P(0) = 800$. Suppose the population doubles every 6.5 years. Find the rate constant
$r =$ ______ (yr $^{-1}$).
Determine the population after 35 years. $P(35) =$ ______.

Answer(s) submitted:

- `ln(2) / 6.5`
- `800e^(35ln(2) / 6.5)`

(correct)
Correct Answers:

- `0.106638027778453`
- `33421.0994613711`

5. (2 pts)
Matching list example

Match the differential equation with its appropriate solution. You should verify your choice.
(The c is an arbitrary constant.)

___1. $\frac{dy}{dt} = 2y$
___2. $\frac{dy}{dt} = 2ty$
___3. $\frac{dy}{dt} = 1 - 2t$
___4. $\frac{dy}{dt} = -y$
___5. $\frac{dy}{dt} = 1 - y$

A. $y(t) = t - t^2 + c$
B. $y(t) = ce^{t^2}$
C. $y(t) = ce^{-t}$
D. $y(t) = ce^{2t}$
E. $y(t) = ce^{-t} + 1$

Answer(s) submitted:

- D
- B
- A
- C
- E

(correct)
Correct Answers:

- D
- B
- A
- C
- E

6. (3 pts) Below are several second order differential equations. Each differential equation has two independent solutions, which are listed amongst the solutions below. Enter the two letters corresponding to the two solutions to the differential equations. (Letters may be repeated in your choices.)
You should verify your choices.
(The c is an arbitrary constant.))

___1. $y'' + 4y = 0$
___2. $y'' + 2y' + 2y = 0$
___3. $y'' - 4y = 0$
___4. $y'' - y' - 2y = 0$

A. $y(t) = ce^{2t}$
B. $y(t) = c\cos(2t)$
C. $y(t) = ce^{-t}$
D. $y(t) = ce^{-t}\cos(t)$
E. $y(t) = ce^{-2t}$
F. $y(t) = c\sin(2t)$
G. $y(t) = ce^{t}$
H. $y(t) = ce^{-t}\sin(t)$

Answer(s) submitted:

- BF
- DH
- AE
- AC

(correct)
Correct Answers:

- BF
- DH
- AE
- AC

7. (2 pts) A cylindrical bucket has a hole in the bottom. If $h(t)$ is the height of the water at any time t in hours, then the differential equation describing this leaky bucket is given by the equation:

$$\frac{dh(t)}{dt} = -6\sqrt{h(t)}.$$

If initially, there are 4 inches of water in the bucket $(h(0) = 4)$. What is the solution to this differential equation?

A. $h(t) = 4 - 6t^2$
B. $h(t) = (2 - 3t)^2$
C. $h(t) = \sqrt{16 - 2t}$
D. $h(t) = (3 - 3t)^2$

Enter the letter corresponding to the correct answer: _____
Verify that this is the solution to this differential equation and satisfies the initial condition.
Determine when the bucket empties.
Empties in _______ hours.
You should sketch a graph of the solution.

Answer(s) submitted:

- B
- 2/3

(correct)
Correct Answers:

- B
- 0.666666666666667

8. (2 pts) It can be helpful to classify a differential equation, so that we can predict the techniques that might help us to find a function which solves the equation. Two classifications are the **order of the equation** – (what is the highest number of derivatives involved) and whether or not the equation is **linear** .
Linearity is important because the structure of the the family of solutions to a linear equation is fairly simple. Linear equations can usually be solved completely and explicitly.

Determine whether or not each equation is linear:

$\boxed{?}$ 1. $y'' - y + t^2 = 0$
$\boxed{?}$ 2. $t^2 \frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin t$
$\boxed{?}$ 3. $(1 + y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$

$\boxed{?}$ 4. $\frac{dy}{dt} + ty^2 = 0$

Answer(s) submitted:

- `2Linear`
- `2Linear`
- `2Nonlinear`
- `1Nonlinear`

(correct)

Correct Answers:

- `2LINEAR`
- `2LINEAR`
- `2NONLINEAR`
- `1NONLINEAR`

---

9. (2 pts) Solve the following differential equation with the given initial condition:

$$\frac{dz}{dt} = 0.9z - 12, \quad z(0) = 14.$$

$z(t) =$ ______________________

Answer(s) submitted:

- `(.6e^(.9t) + 12) / .9`

(correct)

Correct Answers:

- `(14-12/0.9)*exp(0.9*t)+12/0.9`

---

10. (1 pt) Solve the following differential equation with the given initial condition:

$$\frac{dy}{dt} = 0.54y, \quad y(2) = 20.$$

$y(t) =$ ______________________

Answer(s) submitted:

- `(20/(e^(.54*2)))e^(.54t)`

(correct)

Correct Answers:

- `20*exp(0.54*(t-2))`

---

11. (2 pts) Solve the following differential equation with the given initial condition:

$$\frac{dr}{dt} = 6 - \frac{r}{6}, \quad r(2) = 18.$$

$r(t) =$ ______________________

Answer(s) submitted:

- `(-18/e^(-1/3))e^(t/-6) + 36`

(correct)

Correct Answers:

- `(18-6*6)*exp(-(t-2)/6)+6*6`

---

12. (3 pts) You have just boiled a new batch of broth for your important culture of E. coli, so it is at 100°C. You have it sitting in a room that is at 21 °C, and you find 5 minutes later that it's cooled to 95 °C. You want to innoculate the culture when it reaches 40 °C. You are interested in knowing if you can safely go off to exercise while the broth cools.

a. Let $T(t)$ be the temperature of the broth. Assume that the broth satisfies Newton's Law of Cooling. Find the cooling constant k.

$k =$ ______ min^{-1}

Give the solution to this problem (using the value of k that you found).

$T(t) =$ ____________________ °C.

b. Find how long it will be until you need to innoculate the broth with your culture.

$t_i =$ ______ min

Answer(s) submitted:

- `ln( (95 - 21) / (100 - 21) ) / -5`
- `21 + (100 - 21) e ^ (-ln( (95 - 21) / (100 - 21) ) t/ -5)`
- `ln ( (40-21) / (100-21) ) / (ln( (95 - 21) / (100 - 21) )`

(correct)

Correct Answers:

- `0.0130765518525703`
- `(100-21)*exp(-0.0130765518525703*t)+21`
- `108.97436031812`

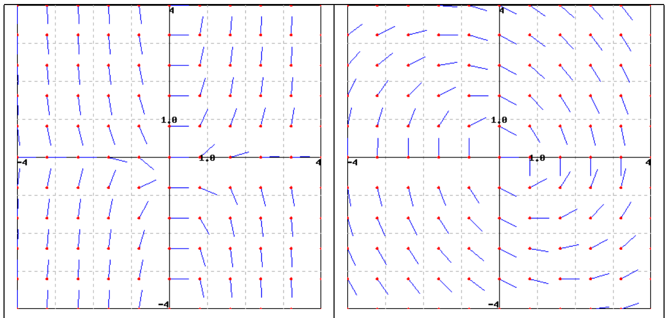
---

13. (3 pts) Match the following equations with their direction field. Clicking on each picture will give you an enlarged view. While you can probably solve this problem by guessing, it is useful to try to predict characteristics of the direction field and then match them to the picture.

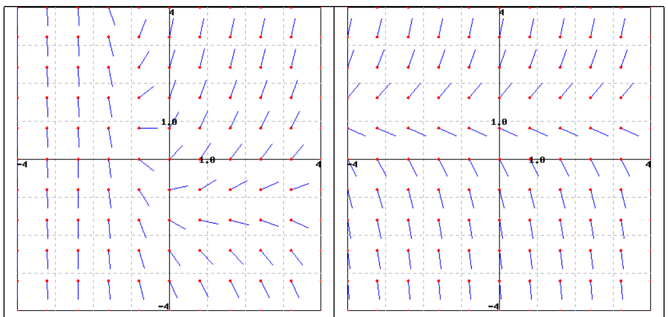
Here are some handy characteristics to start with – you will develop more as you practice.

A. Set y equal to zero and look at how the derivative behaves along the x-axis.
B. Do the same for the y-axis by setting x equal to 0
C. Consider the curve in the plane defined by setting $y' = 0$ – this should correspond to the points in the picture where the slope is zero.
D. Setting y' equal to a constant other than zero gives the curve of points where the slope is that constant. These are called isoclines, and can be used to construct the direction field picture by hand.

___1. $y' = -\frac{(2x+y)}{(2y)}$
___2. $y' = y + xe^{-x} + 1$
___3. $y' = 2xy + 2xe^{-x^2}$
___4. $y' = 2y - 2$



A	B



C	D

Answer(s) submitted:

- B
- C
- A
- D

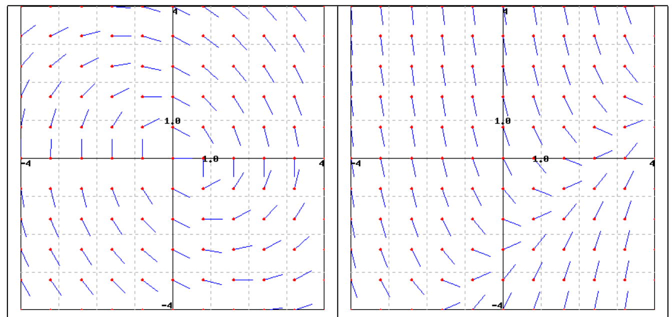
(correct)
Correct Answers:

- B
- C
- A
- D

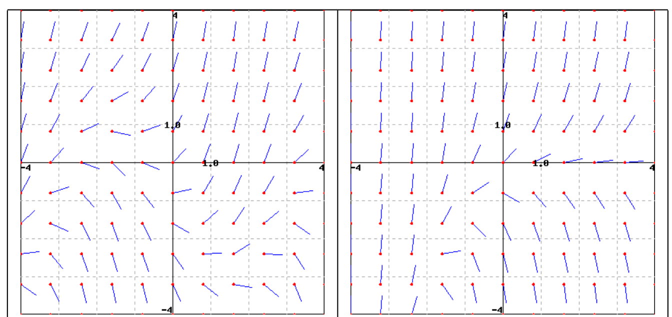
14. (3 pts) Match the following equations with their direction field. Clicking on each picture will give you an enlarged view. While you can probably solve this problem by guessing, it is useful to try to predict characteristics of the direction field and then match them to the picture. Here are some handy characteristics to start with – you will develop more as you practice.

A. Set y equal to zero and look at how the derivative behaves along the x-axis.
B. Do the same for the y-axis by setting x equal to 0
C. Consider the curve in the plane defined by setting $y' = 0$ – this should correspond to the points in the picture where the slope is zero.
D. Setting y' equal to a constant other than zero gives the curve of points where the slope is that constant. These are called isoclines, and can be used to construct the direction field picture by hand.

___1. $y' = -2 + x - y$
___2. $y' = 2\sin(x) + 1 + y$
___3. $y' = -\dfrac{(2x+y)}{(2y)}$
___4. $y' = e^{-x} + 2y$



A	B



C	D

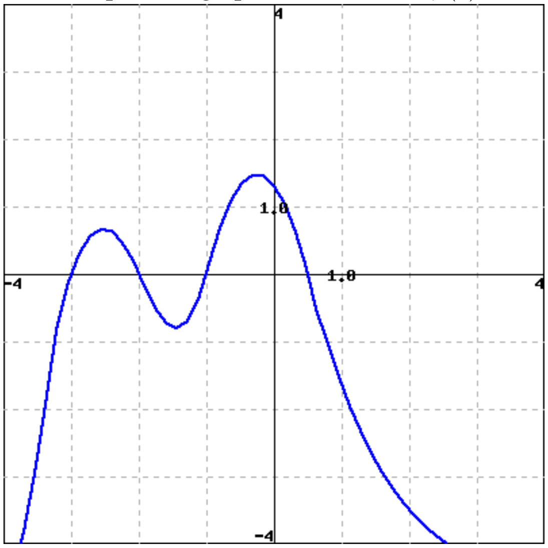
Answer(s) submitted:

- B
- C
- A
- D

(correct)
Correct Answers:

- B
- C
- A
- D

15. (3 pts) The graph of the function $f(x)$ is



(the horizontal axis is x.)
Consider the differential equation $x'(t) = f(x(t))$.

List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable.

_____ ?
_____ ?
_____ ?
_____ ?

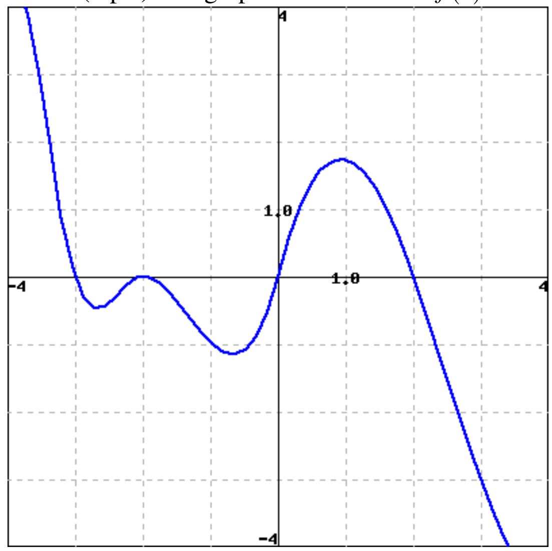
Answer(s) submitted:

- -3
- unstable
- -2
- stable
- -1
- unstable
- .5
- stable

(correct)
Correct Answers:

- -3
- UNSTABLE
- -2
- STABLE
- -1
- UNSTABLE
- 0.5
- STABLE

16. (3 pts) The graph of the function $f(x)$ is



(the horizontal axis is x.)
Given the differential equation $x'(t) = f(x(t))$.
List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable.

_____ ?
_____ ?
_____ ?
_____ ?

Answer(s) submitted:

- -3
- stable
- -2
- semi-stable
- 0
- unstable
- 2
- stable

(correct)
Correct Answers:

- -3
- STABLE
- -2
- SEMI-STABLE
- 0
- UNSTABLE
- 2
- STABLE

17. (4 pts) Given the differential equation $x'(t) = -x^4 + 7x^3 + 0x^2 - 36x + 0$.
List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable. (It helps to sketch the graph. You may want to use Maple to graph this function and find where the function is zero.)

_____ $\boxed{?}$
_____ $\boxed{?}$
_____ $\boxed{?}$
_____ $\boxed{?}$

Answer(s) submitted:

- `-2`
- `unstable`
- `0`
- `stable`
- `3`
- `unstable`
- `6`
- `stable`

(correct)

Correct Answers:

- `-2`
- `UNSTABLE`
- `0`
- `STABLE`
- `3`
- `UNSTABLE`
- `6`
- `STABLE`

---

18. (5 pts) Because of the accuracy of WebWork, you should use 5 or 6 significant figures on all problems.

a. Now consider these functions, $f(x)$ and $g(x)$.

$$f(x) = -2x - 3.1 \quad \text{and} \quad g(x) = 0.9 + 2.6x - 2.1x^2 - x^3.$$

Find the x and y-intercepts and the slope for the line.
x-intercept_____ y-intercept_____
Slope = _____

For the cubic equation, find the x and y-intercepts. (Be sure to order your x-intercepts A, B, and C with $A < B < C$.)
x-intercepts _____, _____, and _____
y-intercept_____

Find the points of intersection between $f(x)$ and $g(x)$. (Be sure to order your x-values A, B, and C with $A < B < C$.)
First point of intersection = (_____,_____).
Second point of intersection = (_____,_____).
Third point of intersection = (_____,_____).

b. In your written HW, use MatLab to graph these functions. Choose a domain such that the graph clearly shows all the points you identified above.

c. Consider the following linear and rational functions:

$$f(x) = x - 4.1 \quad \text{and} \quad g(x) = \frac{2.1x}{x^2 + x - 6}.$$

Find the x and y-intercepts and the slope for the line.
x-intercept = _____ y-intercept = _____
Slope = _____

For the rational function, $g(x)$, find the x and y-intercepts.

x-intercept = _____
y-intercept = _____

Find the vertical asymptotes for $g(x)$. (Be sure to order your x-values x_1 and x_2 with $x_1 < x_2$.)

Vertical Asymptotes at $x_1 =$_____
Vertical Asymptotes at $x_2 =$_____

Find the horizontal asymptote for $g(x)$.

Horizontal Asymptote at $y =$_____

Find the points of intersection between $f(x)$ and $g(x)$. It will be useful to create a graph in Maple and use Maple to help you find all points of intersection. Give both the x and y values at these points of intersection. (Be sure to order your x-values A, B, and C with $A < B < C$.)

First point of intersection = (_____,_____).
Second point of intersection = (_____,_____).
Third point of intersection = (_____,_____).

d. In your written HW, use MatLab to create a graph of these functions for $x \in [-10, 10]$ with the range restricted so that $y \in [-10, 10]$. The graph needs adequate points to be smooth, and you are to insert dotted or dashed lines for all vertical and horizontal asymptotes to show them clearly. Briefly discuss how you found your vertical and horizontal asymptotes. Describe how you found your points of intersection.

6 2

Answer(s) submitted:

- `-1.55`
- `-3.1`
- `-2`
- `-2.891534`
- `-.2882543`
- `1.079788`
- `.9`
- `-3.155957`
- `3.2119144`
- `-.7154866`
- `-1.669027`
- `1.7714438`
- `-6.642888`
- `4.1`
- `-4.1`
- `1`
- `0`
- `0`

- -3
- 2
- 0
- -3.177095
- -7.277095
- 1.686805
- -2.413195
- 4.5902896
- .49028963

(correct)
Correct Answers:

- -1.55
- -3.1
- -2
- -2.89153373535284
- -0.288254268771741
- 1.07978800412458
- 0.9
- -3.15595718961225
- 3.2119143792245
- -0.715486608604858
- -1.66902678279028
- 1.77144379821711
- -6.64288759643422
- 4.1
- -4.1
- 1
- 0
- 0
- -3
- 2
- 0
- -3.17709460034813
- -7.27709460034812
- 1.68680496945918
- -2.41319503054082
- 4.59028963088895
- 0.49028963088895

---

19. (8 pts) Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

a. The normal body temperature for cats varies over a narrow range of temperatures. One cat has a history of a body temperature of 38.4°C. One night this cat is hit by a car at some time during the night. The dead cat is discovered at 7 AM, and a young scientist wants to determine the time of death of the poor cat. He measures the temperature of the cat and finds that the body temperature of the cat is 20.6°C ($H(0) = 20.6$). For this problem, $t = 0$ corresponds to 7 AM. The early morning temperature is found to be 14.5°C. Newton's Law of Cooling with a constant environmental temperature ($T_e = 14.5$) gives the differential equation:

$$\frac{dH}{dt} = -k_1(H - 14.5), \qquad H(0) = 20.6$$

where $H(t)$ is the body temperature of the cat and k_1 is a kinetic constant of cooling. Find the solution to this differential equation with its initial condition (including the cooling constant k_1 written as 'k1').

$H(t) =$ ______________ .

If one hour later the temperature of the body is found to be 18.3°C ($H(1) = 18.3$), then determine the value of the constant of cooling, k_1, in the differential equation.

$k_1 =$ _______ .

Find the time t_d when the death occurs and give the time on the clock.

$t_d =$ _______ .

Time on the clock = _______ : _______ AM,
where the minutes are a decimal value with at least 4 significant figures.

b. Since it is early in the morning, the temperature has been decreasing for some length of time rather than remaining constant. Suppose that a more accurate cooling law is given by the differential equation

$$\frac{dH}{dt} = -k_2(H - (14.5 - 0.3t)), \qquad H(0) = 20.6$$

where $H(t)$ is the body temperature of the cat and k_2 is a kinetic constant of cooling. Find the solution to this differential equation with its initial condition (including the cooling constant k_2 written as 'k2').

$H(t) =$ ______________ .

Determine the value of the constant of cooling, k_2, in this differential equation.

$k_2 =$ _______ .

Find the time t_d when the death occurs assuming this linear environmental temperature and give the time on the clock.

$t_d =$ _______ .

Time on the clock = _______ : _______ AM,
where the minutes are a decimal value with at least 4 significant figures.

c. In the trigonometric section, we found that the daily temperature is often well approximated by a trigonometric function. Suppose that the environmental temperature is approximated by the function:

$$T_e(t) = 15.7 - 1.8\cos(\omega(t+3)),$$

where $t = 0$ is 7 AM and $\omega = 0.2618$. This temperature function gives the environmental temperature at 7 AM as:

Temperature at 7 AM = _______ .

The minimum temperature has what value and occurs what time during the night.

Minimum temperature = _______ .

Clock Time of Min Temp = _______ : _______ AM, where the minutes are a decimal value with at least 4 significant figures.

If the environmental temperature follows the trigonometric function given by $T_e(t)$, then a more accurate cooling law is given by the differential equation:

$$\frac{dH}{dt} = -k_3(H - T_e(t)), \qquad H(0) = 20.6$$

where $H(t)$ is the body temperature of the cat and k_3 is a kinetic constant of cooling. Find the solution to this differential equation using Maple, but don't bother to write this solution as you will see that it is rather long and messy. Still you can use similar techniques to the ones above to determine the value of the constant of cooling, k_3, in this differential equation.

$k_3 =$ ______ .

Find the time t_d when the death occurs assuming this trigonometric environmental temperature and give the time on the clock.

$t_d =$ ______ .

Time on the clock = ______ : ______ AM,

where the minutes are a decimal value with at least 4 significant figures.

d. In your written HW, create a graph (MatLab) showing the three different environmental temperatures for $t \in [-7, 2]$ (from midnight to 9 AM). Compare and contrast these graphs. Discuss how each of these environmental temperature graphs coincide with your understanding of daily temperature during this period of time.

On a different graph, show the three different environmental temperatures and add the body temperature of the cat for $t \in [-7, 2]$ (from midnight to 9 AM). Be sure to make the body temperature of the cat equal to 38.4°C up until the time of death. Include data points for the body temperature at 7 and 8 AM. Briefly, discuss the differences between the predictions and how accurate you believe these predictions to be. Do the more complicated environmental temperature approximations predict a significantly different time of death?

Answer(s) submitted:

- `14.5 + ( 20.6 - 14.5 )e^(-k1t)`
- `-ln( ( 18.3 - 14.5 ) / ( 20.6 - 14.5 ) )`
- `ln( ( 38.4 -14.5 ) / ( 20.6 -14.5 ) ) / ln( ( 18.3 - 14.5`
- `4`
- `06.8808`
- `-(3*t)/10 + 29/2 + 3/(10*k2) + e^(-k2*t)*(61/10 - 3/(10*k2`
- `0.4578058067`
- `-3.081761100`
- `3`
- `55.094334`
- `14.42721013`
- `13.9`
- `4`
- `00`
- `0.4857621801`
- `-2.727641236`
- `4`
- `16.34152584`

(correct)

Correct Answers:

- `14.5 + (20.6 - 14.5)*exp(-k1*t)`
- `0.473287704446926`
- `-2.88532677888601`
- `4`
- `6.88039326683967`
- `14.5-0.3*t+0.3/k2-exp(-k2*t)*(k2*14.5+0.3-20.6*k2)/k2`
- `0.457805799661172`
- `-3.08176114881187`
- `3`
- `55.0943310712879`
- `14.4272101314798`
- `13.9`
- `4`
- `0`
- `0.485762176943347`
- `-2.72764125394784`
- `4`
- `16.3415247631295`