¡b¿ Stephen Giang, ¡br¿ WeBWorK @ Dept of Mathematics and Statistics @ SDSU ¡/b¿ ¡br¿ WeBWorK problems. WeBWorK assignment Laplace due 05/02/2020 at 04:00am PDT.

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1. (2 pts)

(1) Set up an integral for finding the Laplace transform of $f(t) = t + 14$.

$F(s) = \mathcal{L}\{f(t)\} = \int_A^B$ ________

where $A =$ ___ and $B =$ ___.

(If one limit is ∞, then type 'infinity'.)

(2) Find the antiderivative (with constant term 0) corresponding to the previous part.

____

(3) Evaluate appropriate limits to compute the Laplace transform of $f(t)$:

$F(s) = \mathcal{L}\{f(t)\} =$ ________

(4) Where does the Laplace transform you found exist? In other words, what is the domain of $F(s)$?

____

Answer(s) submitted:

- `e^(-st)(t+ 14)dt`
- `0`
- `infinity`
- `(t*e^(-st))/(-s) - (e^(-st))/(s^2) - (14e^(-st))/(s)`
- `1/(s^2) + 14/s`
- `s > 0`

(correct)
Correct Answers:

- `(t+14) * e^{-s*t} * dt`
- `0`
- `INFINITY`
- `-t/s*e^(-s*t) - 1/(s^2)*e^(-s*t) + -14/s*e^(-s*t)`
- `1/(s^2) - -14/s`
- `s > 0`

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2. (1 pt) Consider $f(t) = e^{(t-2)^2}$.

(1) The function $f(t)$ is

- A. continuous on $0 \le t < \infty$.
- B. discontinuous but piecewise continuous on $0 \le t < \infty$.
- C. neither.

(2) Is $f(t)$ exponentially bounded on $0 \le t < \infty$? $\boxed{?}$

(3) Does the Laplace transform of $f(t)$ exist (on some domain)? $\boxed{?}$

Answer(s) submitted:

- `A`
- `no`
- `no`

(correct)
Correct Answers:

- `A`
- `no`
- `no`

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3. (3 pts)

(1) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = 8e^{-7t} + 7t + 8e^{10t}$, defined on the interval $t \ge 0$.

$F(s) = \mathcal{L}\left\{8e^{-7t} + 7t + 8e^{10t}\right\} =$ ________________

(2) For what values of s does the Laplace transform exist?

____

Answer(s) submitted:

- `(8)/(s+7) + (7)/(s^2) + (8)/(s-10)`
- `s > 10`

(correct)
Correct Answers:

- `8/(s+7)+7/(s^2)+8/(s-10)`
- `s > 10`

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4. (2 pts) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = -\left(\frac{9}{s^2} + \frac{1}{s+6}\right)$.

$f(t) = \mathcal{L}^{-1}\left\{-\left(\frac{9}{s^2} + \frac{1}{s+6}\right)\right\} =$ ________________

Answer(s) submitted:

- `-9t -e^(-6t)`

(correct)
Correct Answers:

- `-[9*t+e^(-6*t)]`

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5. (2 pts) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \frac{2s}{s^2 - 36}$.

$f(t) = \mathcal{L}^{-1}\left\{\frac{2s}{s^2-36}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s+6} + \frac{1}{s-6}\right\} =$ ________

Answer(s) submitted:

- `e^(-6t) + e^(6t)`

(correct)

Correct Answers:

- `e^(-6*t)+e^(6*t)`

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6. (2 pts)

(1) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = 8 + \sin(2t)$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\{8 + \sin(2t)\} =$ ______

(2) For what values of s does the Laplace transform exist?

______

Answer(s) submitted:

- `8/(s) + 2/(s^2 + 4)`
- `s > 0`

(correct)

Correct Answers:

- `8/s+2/(s^2+4)`
- `s > 0`

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7. (2 pts)

(1) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = \sin^2(wt)$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\left\{\sin^2(wt)\right\} =$ ______

Hint: Use a double-angle trigonometric identity.

(2) For what values of s does the Laplace transform exist?

______

Answer(s) submitted:

- `(1)/(2s) + (-1)/(2)(s)/(s^2 + 4w^2)`
- `s > 0`

(correct)

Correct Answers:

- `1/(2*s)-0.5*s/(s^2+4*w^2)`
- `s > 0`

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8. (2 pts) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \frac{24}{s^4} - \frac{3}{s}$.

$f(t) = \mathcal{L}^{-1}\left\{\frac{24}{s^4} - \frac{3}{s}\right\} =$ ______

Answer(s) submitted:

- `4t^3 - 3`

(correct)

Correct Answers:

- `4*t^3-3`

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9. (2 pts) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = -\frac{6s+5}{s^2+36}$.

$f(t) = \mathcal{L}^{-1}\left\{-\frac{6s+5}{s^2+36}\right\} =$ ______

Answer(s) submitted:

- `-6cos(6t) - (5sin(6t))/(6)`

(correct)

Correct Answers:

- `-[6*cos(6*t)+0.833333*sin(6*t)]`

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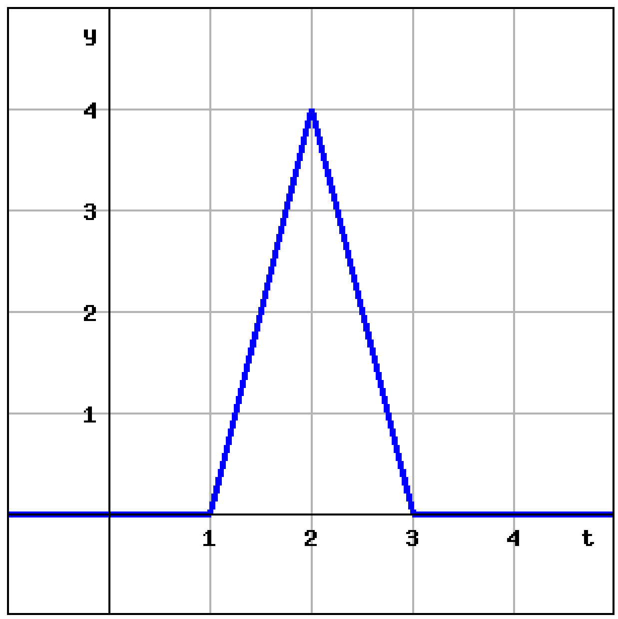
10. (2 pts)

The graph of $f(t)$ is given in the figure. Represent $f(t)$ using a combination of Heaviside step functions. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$f(t) =$ ______



Graph of $y = f(t)$

Answer(s) submitted:

- `4(t-1)(h(t-1) - h(t-2)) - 4(t-3)(h(t-2) - h(t-3))`

(correct)

Correct Answers:

- `4*(t-1)*[h(t-1)-h(t-2)]-4*(t-3)*[h(t-2)-h(t-3)]`

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11. (2 pts) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = e^{t-6}h(t-6)$, defined on the interval $t \geq 0$. The $h(t-a)$ is the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$F(s) = \mathcal{L}\left\{e^{t-6}h(t-6)\right\} =$ ______

Answer(s) submitted:

- `(e^(-6s))/(s-1)`

(correct)
Correct Answers:

- `e^(-6*s)/(s-1)`

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12. (2 pts) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = e^t\cos(4t)$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\left\{e^t\cos(4t)\right\} =$ ________________
Answer(s) submitted:

- `(s-1) / ((s-1)^2 + 16)`

(correct)
Correct Answers:

- `(s-1)/[(s-1)^2+16]`

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13. (2 pts) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{4s-18}{s^2-8s+17}$.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{4s-18}{s^2-8s+17}\right\} =$ ________________
Answer(s) submitted:

- `4e^(4t)cos(t) - 2e^(4t)sin(t)`

(correct)
Correct Answers:

- `4*e^(4*t)*cos(t)-2*e^(4*t)*sin(t)`

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14. (2 pts) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{e^{-2s}(2s-7)}{s^2+25}$. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{e^{-2s}(2s-7)}{s^2+25}\right\} =$ ________________
Answer(s) submitted:

- `h(t-2)(2cos(5(t-2)) - (7)/(5)sin(5(t-2)))`

(correct)
Correct Answers:

- `2*cos(5*(t-2))*h(t-2)-1.4*sin(5*(t-2))*h(t-2)`

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15. (2 pts) Consider the function

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < 6\pi \\ \sin(t-6\pi) & \text{if } 6\pi \leq t. \end{cases}$$

(1) Use the graph of this function to write it in terms of the Heaviside function. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$f(t) =$ ________________

(2) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$.

$F(s) = \mathcal{L}\{f(t)\} =$ ________________
Answer(s) submitted:

- `h(t -6pi)sin(t - 6pi)`
- `(e^(-6pis))/(s^2 + 1)`

(correct)
Correct Answers:

- `h(t-6*pi)*sin(t-6*pi)`
- `e^(-6*pi*s)/(s^2+1)`

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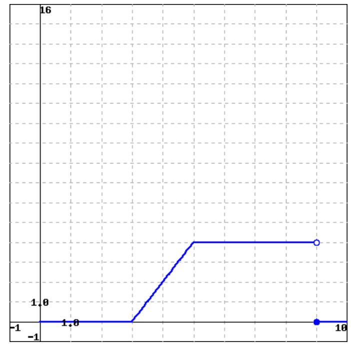
16. (3 pts) The graph of $f(t)$ is given below:



(Click on graph to enlarge)

(1) Represent $f(t)$ using a combination of Heaviside step functions. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$f(t) =$ ________________

(2) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ for $s \neq 0$.

$F(s) = \mathcal{L}\{f(t)\} =$ ________________
Answer(s) submitted:

- `2(t-3) (h(t-3) - h(t-5))+ 4(h(t-5) - h(t-9))`
- `(2e^(-3s))/(s^2) + (-2e^(-5s))/(s^2) + (-4e^(-9s))/(s)`

(correct)
Correct Answers:

- `2*(t-3)*[h(t-3)-h(t-5)]+4*[h(t-5)-h(t-9)]`
- `2*e^(-3*s)/(s^2)-2*e^(-5*s)/(s^2)-4*e^(-5*s)/s+4*[e^(-5*s)`

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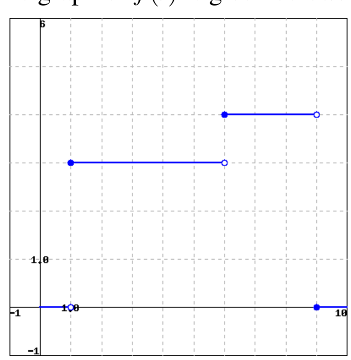
17. (2 pts) The graph of $f(t)$ is given below:



(Click on graph to enlarge)

(1) Represent $f(t)$ using a combination of Heaviside step functions. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$f(t) =$ ______________________

(2) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ for $s \neq 0$.

$F(s) = \mathcal{L}\{f(t)\} =$ ______________________

Answer(s) submitted:

- `3( h(t-1) - h(t-6) ) + 4( h(t-6) - h(t-9))`
- `(3e^(-s))/(s) + (e^(-6s))/(s) + (-4e^(-9s))/(s)`

(correct)

Correct Answers:

- `3*[h(t-1)-h(t-6)]+4*[h(t-6)-h(t-9)]`
- `3*[e^(-s)/s-e^(-6*s)/s]+4*[e^(-6*s)/s-e^(-9*s)/s]`

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18. (4 pts) Consider the initial value problem

$$y' + 3y = \begin{cases} 0 & \text{if } 0 \le t < 1 \\ 12 & \text{if } 1 \le t < 5 \\ 0 & \text{if } 5 \le t < \infty, \end{cases} \qquad y(0) = 8.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

______________ = ______________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ______________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$y(t) =$ ______________________

In your written HW, write the complete details on how you solved this problem with Laplace Transforms.

Answer(s) submitted:

- `sY(s) - 8 + 3Y(s)`
- `(12)/(s)(e^(-s) - e^(-5s))`
- `((12)/(s)(e^(-s) - e^(-5s)) + 8) / (s+3)`
- `4h(t-1)(1 - e^(-3(t-1))) - 4h(t-5)(1 - e^(-3(t-5)))+ 8e^(-3t)`

(correct)

Correct Answers:

- `s*Y(s)-8+3*Y(s)`
- `12*[e^(-s)/s-e^(-5*s)/s]`
- `(12*[e^(-s)/s-e^(-5*s)/s]+8)/(s+3)`
- `4*[h(t-1)-h(t-1)*e^[-3*(t-1)]-h(t-5)+h(t-5)*e^[-3*(t-5)]]+`

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19. (2 pts) Consider the rational function

$$F(s) = \frac{s^3 - 3}{(s^2+7)^2(s+10)^2}.$$

Select ALL terms below that occur in the general form of the complete partial fraction decomposition of $F(s)$. The capital letters A, B, C, . . . , L denote constants.

- A. $\dfrac{G}{s+10}$
- B. $\dfrac{J}{(s+10)^2}$
- C. $\dfrac{Ks+L}{(s+10)^2}$
- D. $\dfrac{D}{(s^2+7)^2}$
- E. $\dfrac{Bs+C}{s^2+7}$
- F. $\dfrac{Hs+I}{s+10}$
- G. $\dfrac{Es+F}{(s^2+7)^2}$
- H. $\dfrac{A}{s^2+7}$

Answer(s) submitted:

- `( A, B, E, G )`

(correct)

Correct Answers:

- `ABEG`

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20. (3 pts) Consider the function $F(s) = \dfrac{3s-8}{s^2-5s+6}$.

(1) Find the partial fraction decomposition of $F(s)$:

$$\frac{3s-8}{s^2-5s+6} = \underline{\qquad\qquad} + \underline{\qquad\qquad}$$

(2) Find the inverse Laplace transform of $F(s)$.

$f(t) = \mathcal{L}^{-1}\{F(s)\} =$ ______________________

Answer(s) submitted:

- `1 / (s-3)`
- `2/(s-2)`
- `e^(3t) + 2e^(2t)`

(correct)

Correct Answers:

- `1/(s-3)`
- `2/(s-2)`
- `1*e^(3*t) + 2*e^(2*t)`

21. (3 pts) Consider the function $F(s) = \dfrac{6s^2+5s+2}{s^3+s}$.

(1) Find the partial fraction decomposition of $F(s)$:

$$\frac{6s^2+5s+2}{s^3+s} = \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_$$

(2) Find the inverse Laplace transform of $F(s)$.

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Answer(s) submitted:

- `2/ s`
- `(4s + 5)/ (s^2 + 1)`
- `2 + 4cos(t)  + 5sin(t)`

(correct)
Correct Answers:

- `(4*s+5)/(s^2+1)`
- `2/s`
- `4*cos(1*t) + 5/1*sin(1*t) + 2`

22. (3 pts) Consider the initial value problem

$$y' + 3y = 45t, \qquad y(0) = 7.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

$$\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_$$

(2) Solve your equation for $Y(s)$.

$$Y(s) = \mathcal{L}\{y(t)\} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$$y(t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Answer(s) submitted:

- `sY(s) - 7 + 3Y(s)`
- `45/s^2`
- `45/((s^2)(s+3)) + 7/(s+3)`
- `-5 + 15t + 12e^(-3t)`

(correct)
Correct Answers:

- `s*Y(s)-7+3*Y(s)`
- `45/(s^2)`
- `45/[s^2*(s+3)]+7/(s+3)`
- `15*t-5+12*e^(-3*t)`

23. (4 pts) Consider the initial value problem

$$y'' + 16y = 64t, \qquad y(0) = 8, \quad y'(0) = 2.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

$$\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_$$

(2) Solve your equation for $Y(s)$.

$$Y(s) = \mathcal{L}\{y(t)\} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$$y(t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

In your written HW, write the complete details on how you solved this problem with Laplace Transforms.

Answer(s) submitted:

- `(s^2 + 16)Y(s) -2(4s+1)`
- `64/s^2`
- `(8s)/(s^2 + 16) + (2)/(s^2 + 16) + (64)/((s^2)(s^2 + 16))`
- `8cos(4t) -(1)/(2)sin(4t) + 4t`

(correct)
Correct Answers:

- `s^2*Y(s)-8*s-2+16*Y(s)`
- `64/(s^2)`
- `64/[s^2*(s^2+16)]+(8*s+2)/(s^2+16)`
- `4*t-sin(4*t)+8*cos(4*t)+0.5*sin(4*t)`

24. (4 pts) Consider the initial value problem

$$y'' + 25y = \cos(5t), \qquad y(0) = 6, \quad y'(0) = 9.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

$$\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_$$

(2) Solve your equation for $Y(s)$.

$$Y(s) = \mathcal{L}\{y(t)\} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$$y(t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

In your written HW, write the complete details on how you solved this problem with Laplace Transforms.

Answer(s) submitted:

- `(s^2 +25)Y(s) - (6s + 9)`
- `s/(s^2 + 25)`
- `(6s)/(s^2 + 25) + (9)/(s^2 + 25) + (s)/(s^2 + 25)^2`
- `6cos(5t) + (9/5)(sin(5t)) + (t/10)(sin(5t))`

(correct)

Correct Answers:

- `s^2*Y(s)-6*s-9+25*Y(s)`
- `s/(s^2+25)`
- `s/[(s^2+25)^2]+(6*s+9)/(s^2+25)`
- `t/10*sin(5*t)+6*cos(5*t)+1.8*sin(5*t)`

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25. (4 pts) Consider the initial value problem

$$y'' + 16y = g(t), \qquad y(0) = 0, \quad y'(0) = 0,$$

where $g(t) = \begin{cases} t & \text{if } 0 \le t < 3 \\ 0 & \text{if } 3 \le t < \infty. \end{cases}$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

$\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_$

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} = \_\_\_\_\_\_\_\_\_\_$

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$y(t) = \_\_\_\_\_\_\_\_\_\_$

In your written HW, write the complete details on how you solved this problem with Laplace Transforms.

Answer(s) submitted:

- `(s^2 + 16)Y(s)`
- `(1/s^2) - e^(-3s)((1/s^2) + (3/s))`
- `(1/((s^2)((s^2 + 16)))) - e^(-3s)((1/s^2) + (3/s)) / ((s^2 + 16))`
- `(t/16) - (sin(4t) / (16*4)) - (h(t-3)(t-3)/16) + (h(t-3) sin(4(t-3)) / (16*4)) - (3 h(t-3) / 16) + (3h(t-3)cos(4(t-3)),`

(correct)

Correct Answers:

- `s^2*Y(s)+16*Y(s)`
- `1/(s^2)-e^(-3*s)/(s^2)-3*e^(-3*s)/s`
- `1/[s^2*(s^2+16)]-e^(-3*s)/[s^2*(s^2+16)]-3*e^(-3*s)/[s*(s^2+16)]`
- `0.0625*[t-0.25*sin(4*t)-(t-3)*h(t-3)+0.25*sin(4*(t-3))*h(t-3)]+(-0.1875*[h(t-3)-h(t-3)*cos(4*(t-3))])`

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26. (2 pts) Evaluate the following:

(1) $\int_{-1}^{6} (8 + e^{-2t})\,\delta(t-2)\,dt = \_\_\_\_\_\_$

(2) $\int_{-1}^{6} (8 + e^{-2t})\,\delta(t-9)\,dt = \_\_\_\_\_\_$

(3) $\int_{-1}^{6} (8 + e^{-2t})\,\delta(t)\,dt = \_\_\_\_\_\_$

Answer(s) submitted:

- `8 + e^(-4)`
- `0`
- `9`

(correct)

Correct Answers:

- `8.01832`
- `0`
- `9`

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27. (4 pts) Consider the following initial value problem, in which an input of large amplitude and short duration has been idealized as a delta function.

$$y' + y = 2 + \delta(t-3), \qquad y(0) = 0.$$

(1) Find the Laplace transform of the solution.

$Y(s) = \mathcal{L}\{y(t)\} = \_\_\_\_\_\_\_\_\_\_$

(2) Obtain the solution $y(t)$. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$y(t) = \_\_\_\_\_\_\_\_\_\_$

(3) Express the solution as a piecewise-defined function and think about what happens to the graph of the solution at $t = 3$.

$$y(t) = \begin{cases} \_\_\_\_\_\_ & \text{if } 0 \le t < 3, \\ \_\_\_\_\_\_ & \text{if } 3 \le t \le \infty. \end{cases}$$

Answer(s) submitted:

- `2/(s(s+1)) + (e^(-3s))/(s+1)`
- `2 - 2e^{-t} + h(t-3)e^{-(t-3)}`
- `2 - 2e^{-t}`
- `2 - 2e^{-t} + e^{-(t-3)}`

(correct)

Correct Answers:

- `2/[s*(s+1)]+e^(-3*s)/(s+1)`
- `2*[1-e^(-t)]+h(t-3)*e^[-(t-3)]`
- `2*[1-e^(-t)]`
- `2*[1-e^(-t)]+e^[-(t-3)]`

28. (4 pts) Consider the following initial value problem, in which an input of large amplitude and short duration has been idealized as a delta function.

$$y'' - 2y' = \delta(t-4), \qquad y(0) = 7, \quad y'(0) = 0.$$

(1) Find the Laplace transform of the solution.

$Y(s) = \mathcal{L}\{y(t)\} =$ ______

(2) Obtain the solution $y(t)$. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$y(t) =$ ______

(3) Express the solution as a piecewise-defined function and think about what happens to the graph of the solution at $t = 4$.

$$y(t) = \begin{cases} \_\_\_\_\_\_ & \text{if } 0 \le t < 4, \\ \_\_\_\_\_\_ & \text{if } 4 \le t \le \infty. \end{cases}$$

Answer(s) submitted:

- `(7s-14)/(s^2 - 2s) + (e^(-4s))/(s^2 - 2s)`
- `7 + ((e^(2(t-4))/2) - (1/2))h(t-4)`
- `7`
- `7 + ((e^(2(t-4))/2) - (1/2))`

(correct)

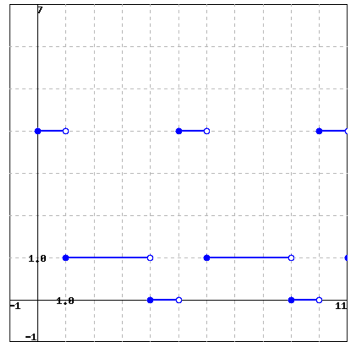
Correct Answers:

- `0.5*e^(-4*s)*[1/(s-2)-1/s]+7/s`
- `0.5*h(t-4)*e^[2*(t-4)]-0.5*h(t-4)+7`
- `7`
- `6.5+0.5*e^[2*(t-4)]`

29. (3 pts) Our theorem for a periodic function $f(t)$ with period T states:

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \cdot \left(\int_0^T e^{-st} f(t)dt \right)$$

Find the Laplace transform of the periodic function $f(t)$ whose graph is given below.



(Click on graph to enlarge)

$$F(s) = \mathcal{L}\{f(t)\} = \_\_\_\_\_\_ \cdot \left(\int_0^1 \_\_\_\_\_\_ + \int_1^4 \_\_\_\_\_\_ + \int_4^5 \_\_\_\_\_\_ \right)$$

$=$ ______

Answer(s) submitted:

- `1/(1-e^(-5s))`
- `4e^(-st)dt`
- `e^(-st)dt`
- `0dt`
- `(-3e^(-s) - e^(-4s) + 4)/(s(1-e^(-5s)))`

(correct)

Correct Answers:

- `1/(1 - e^(-5*s))`
- `4*e^(-s*t)*dt`
- `e^(-s*t)*dt`
- `0`
- `[4-4*e^(-s)-e^(-4*s)+e^(-s)]/(s*[1-e^(-5*s)])`