Math-524-Spring-2020 Midterm 1

Stephen Giang

TOTAL POINTS

275 / 300

QUESTION 1

Are the Subsets also Subspaces? 100 pts

1.1 Is the subset Sa also a Subspace? 25 / 25

✓ **- 0 pts** **Correct**

1.2 Is the subset Sb also a Subspace? 25 / 25

✓ **- 0 pts** **Correct**

1.3 Is the subset Sc also a Subspace? 25 / 25

✓ **- 0 pts** **Correct / Acceptable**

1.4 Is the subset Sd also a Subspace? 25 / 25

✓ **- 0 pts** **Correct**

QUESTION 2

2 Linear Independence, Proof 100 / 100

✓ **- 0 pts** **Correct: Three key elements (1) Linear combination $$\sum a_kv_k =0$$, must show coefficients are all zero; (2) Transform the lin.combo ((keep in mind $$T(0)=0$$ on the right-hand-side)) and use linearity to get $$\sum a_kT(v_k)=0$$; (3) since the $$T(v)$$-vectors are linearly independent, the coefficients must be zero.**

QUESTION 3

The Matrix of a Linear Transformation L(V,W) 100 pts

3.1 3(a) M(T) 75 / 75

✓ **- 0 pts** **Correct**

1 Just the coefficients: $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 &0 &6 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

3.2 3(b) is M(T) invertible 0 / 25

✓ **- 25 pts** **Blank / Incorrect conclusion / Unclear what properties and/or what matrix is being used to come to the conclusion (whether it is correct or not)**

Math 524, Spring 2020
Midterm #1, In-Class

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Tools: Pencil/Eraser/Paper/TEXTBOOK.
Rules: This is an in-class midterm; see below:

I, Stephen Giang, pledge that this exam is **completely my own work**, and that I did not take, copy, borrow or steal any portions from any other person; furthermore, I did not knowingly let anyone else take, copy, or borrow any portions of my exam. Further, I pledge to abide by the rules set out below. I understand that if I violate this honesty pledge, (*i*) I will get ZERO POINTS on this exam; *(ii)* I will get reported to The SDSU Center for Student Rights and Responsibilities; and *(iii)* I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

Signature (REQUIRED for credit)

Rules:

- This is a 75-minute exam.
- This midterm is **OPEN-BOOK (Sheldon Axler, *"Linear Algebra Done Right"*)**, closed-notes, no phones, no calculators, no slide-rules allowed.
- No communications / internet enabled devices allowed.
  - **PDF-book allowed — in full-page mode on internet-disabled device.**
- Write solutions/answers on the attached sheets, and HAND IN the entire packet.
- Note: there should be lots of space to write your solutions, do not feel the need to fill it all...
- Present your solutions using standard notation in an easy-to-read format. It is your job to convince the grader you did the problem correctly, not the grader's job to decipher cryptic messages scribbled in the margin! *Your answers MUST logically follow from your calculations in order to be considered! (**"Miracle solutions"** $\Rightarrow$ **zero points.**)*
- The exam will be graded and returned as soon as possible. NO GRADING CORRECTIONS WILL BE CONSIDERED ONCE YOU REMOVE THE EXAM FROM THE LECTURE HALL / PROFESSOR'S OFFICE.

Problem	Pts Possible	Pts Scored
1	100	
2	100	
3	100	
Total	300	

- You MUST stay for at least 20 minutes. (Draw a unicorn on the back if you have too much time on your hands!)

$1\cdot 3 + 3(-1) = 0.$

$(1, 3, -1)$ $2\cdot 6$ $12 + 3(-2) \neq 0.$

1. For each of the following subsets of $\mathbb{F}^3$, determine whether it is a subspace of $\mathbb{F}^3$:

(Significance: Understanding of basic building blocks).

Yes (a) (25 pts.) $S_a = \{(z_1, z_2, z_3) \in \mathbb{F}^3 : z_1 + 2z_2 + 3z_3 = 0\}$

No. (b) (25 pts.) $S_b = \{(z_1, z_2, z_3) \in \mathbb{F}^3 : z_1 + 2z_2 + 3z_3 = 4\}$

Yes (c) (25 pts.) $S_c = \{(z_1, z_2, z_3) \in \mathbb{F}^3 : z_1 = 3z_3\}$

No. (d) (25 pts.) $S_d = \{(z_1, z_2, z_3) \in \mathbb{F}^3 : z_1 * z_2 + 3z_3 = 0\}$

$\forall a, b, c, d, e, f, \lambda^{\neq 0} \in \mathbb{F}$

a) Test: $(0,0,0) \in S_a$

so $0 + 20 + 30 = 0$ so $0 \in S_a$ ✓.

let $(a, b, c) \in S_a$ so $a + 2b + 3c = 0$

$(d, e, f) \in S_a$ $d + 2e + 3f = 0.$

Test: $(a+d, b+e, c+f) \in S_a$

$(a+d) + 2(b+e) + 3(c+f) = a + 2b + 3c + d + 2e + 3f$

$= 0 + 0$

$= 0$

so $(a+d, b+e, c+f) \in S_a$ so closed under add. ✓.

Test: ~~(ad, be, cf)~~

$\lambda(a,b,c) = (\lambda a, \lambda b, \lambda c) \in S_a$ let $(a,b,c) \in S_a$ so $a + 2b + 3c = 0.$

~~let $(a,b,c) \in S_a$~~

$\lambda(a+2b+3c)$

$\lambda(0) = 0$ $\lambda a + 2\lambda b + 3\lambda c$

$= \lambda(a + 2b + 3c)$

$= \lambda(0) = 0$

so $(\lambda a, \lambda b, \lambda c) \in S_a$ so closed under scalar mult. ✓.

$\lambda(a,b,c) \in S_a$

Bc, S_a contains 0, closed under (+) & scalar (x), then $S_a \subseteq \mathbb{F}^3$

Whenever you rely on a specific definition or theorem from the book, carefully specify which one (by name, or n.nn-reference). Always be clear on what properties you are checking, what is satisfied, and what is not.

b.) Let $(0,0,0) \in S_b$

so $0 + 2(0) + 3(0) \neq 4$. ~~...~~ so False

$\boxed{\text{By contradiction, } 0 \notin S_b \text{ so } S_b \not\subseteq \mathbb{F}^3}$

c.) ~~Test~~: $(0,0,0) \in S_c$ $\qquad \forall a,b,c,d,e,f,\lambda^{\neq 0} \in \mathbb{F}$.

$0 = 3(0)$ ✓ so $0 \in S_c$ ✓.

Let $(a,b,c) \in S_c$ so $a = 3c$ so $a - 3c = 0$

$(d,e,f) \in S_c$ so $d = 3f$. so $3f - d = 0$.

Test: $(a+d, b+e, c+f) \in S_c$

$a+d = 3(c+f)$ $\qquad$ so $(a+b, b+e, c+f) \in S_c$ ✓.

$a+d = 3c + 3f$. $\qquad$ so closed under add.

$a - 3c = 3f - d$

$0 = 0$

Test: $(\lambda a, \lambda b, \lambda c) \in S_c$. $\qquad$ Let $(a,b,c) \in S_c$

so $a = 3c$.

$a - 3c = 0$.

$\lambda a - \lambda 3c$

$= \lambda(a - 3c)$

$= \lambda(0)$

$= 0$

so $\lambda a - 3\lambda c = 0$. so closed under scallar (x).

$\boxed{\text{Bc } S_c \text{ contains } 0 \text{ \& is closed under } (+) \text{ \& scallar } (x) \text{ then } S_c \subseteq \mathbb{F}^3}$

d) Test: $(\lambda a, \lambda b, \lambda c) \in S_d$ $\qquad$ Let $(a,b,c) \in S_d$ $\qquad \forall a,b,c,\lambda^{\neq 0} \in \mathbb{F}$

so $ab + 3c = 0$.

$\lambda a \lambda b + 3\lambda c$

$= \lambda^2 ab + 3\lambda x$

$\neq \lambda(ab + 3c)$ so not closed under scalar (x).

$\boxed{\text{Bc not closed under scalar } (x), \; S_d \not\subseteq \mathbb{F}^3}$

2. (100 pts) Suppose $T \in \mathcal{L}(V, W)$ and $v_1, \dots, v_m$ is a list of vectors in V such that $T(v_1), \dots, T(v_m)$ is a linearly independent list in W. Prove that $v_1, \dots, v_m$ is linearly independent.

(Significance: Foundational Concept — Linear Independence)

Let $T \in \mathcal{L}(V,W)$ w/ $v_1, \dots, v_m \in V$.

Let $Tv_1, \dots, Tv_m$ be lin. indep $\in W$.

Bc $Tv_1, \dots, Tv_m$ is lin indep: $0 = a_1 Tv_1 + \dots + a_m Tv_m$. $\forall a_1 \dots a_m = 0$

$= T(a_1 v_1 + \dots + a_m v_m)$

Bc the only way for $a_1 Tv_1 + \dots + a_m Tv_m = 0$ is if $\forall a_1 \dots a_m = 0$, then the only way for $T(a_1 v_1 + \dots + a_m v_m) = 0$ is $\forall a_1 \dots a_m = 0$.

so $a_1 v_1 + \dots + a_m v_m$ has to equal 0 only if $\forall a_1, \dots a_m = 0$ bc $T(0) = 0$ for all Linear Maps $T \in \mathcal{L}(V,W)$ from Th. 3.11

meaning $\boxed{v_1 \dots v_m \text{ is lin indep.}}$

Whenever you rely on a specific definition or theorem from the book, carefully specify which one (by name, or n.nn-reference). Always be clear on what properties you are checking, what is satisfied, and what is not.

$x^3 \to 3x^2 \to 6x \to 6.$

$T(1) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

3. Let $T \in \mathcal{L}(\mathcal{P}_3(\mathbb{R}), \mathbb{R}^{2\times 2})$ be defined by

$$T(p) = \begin{bmatrix} p(0) & p''(0) \\ p'''(0) & p'(0) \end{bmatrix}$$

$T(x) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$T(x^3) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Let $\mathfrak{B}(\mathcal{P}_3(\mathbb{R})) = \{1, x, x^2, x^3\}$ be the (standard basis for $\mathcal{P}_3(\mathbb{R})$); below, select a single basis $\mathfrak{B}(\mathbb{R}^{2\times 2})$ for $\mathbb{R}^{2\times 2}$, and then

(75 pts) compute $\mathcal{M}(T, \mathfrak{B}(\mathcal{P}_3(\mathbb{R})), \mathfrak{B}(\mathbb{R}^{2\times 2}))$;

(25 pts) based on the properties of $\mathcal{M}(T, \mathfrak{B}(\mathcal{P}_3(\mathbb{R})), \mathfrak{B}(\mathbb{R}^{2\times 2}))$ (or some other alternative), is the map T invertible?

$T(x^2) = 2\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Basis candidates (circle your choice):

- $\mathfrak{B}_1(\mathbb{R}^{2\times 2}) = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$
- $\mathfrak{B}_2(\mathbb{R}^{2\times 2}) = \left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \right\}$ $3x$
- $\mathfrak{B}_3(\mathbb{R}^{2\times 2}) = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$

(Significance: Linear Transformations, Bases, Matrices)

Scratch

Compute $M(T, B(P_3(\mathbb{R})), B_1(\mathbb{R}^{2\times 2}))$

$$M(T, B(P_3(\mathbb{R})), B_1(\mathbb{R}^{2\times 2})) = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & 2\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & 6\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & 1\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$$

Whenever you rely on a specific definition or theorem from the book, carefully specify which one (by name, or n.nn-reference). Always be clear on what properties you are checking, what is satisfied, and what is not.

a)

$$M\left(T,\ B(P_3(\mathbb{R})),\ B(\mathbb{R}^{2\times 2})\right)$$

$$= \begin{array}{c} \\ \begin{bmatrix}1&0\\0&0\end{bmatrix} \\ \begin{bmatrix}0&1\\0&0\end{bmatrix} \\ \begin{bmatrix}0&0\\1&0\end{bmatrix} \\ \begin{bmatrix}0&0\\0&1\end{bmatrix} \end{array} \begin{array}{c} \begin{array}{cccc} 1 & x & x^2 & x^3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad :)$$

b) $M\left(T, B(P_3(\mathbb{R})), B(\mathbb{R}^{2\times 2})\right)$ is not invertible

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