MATH 693A Advanced Numerical Methods: Computational Optimization

Dr. George

Homework 2
Due October 3rd

1. (70 points) Write a Program that implements the dogleg method. Choose B_k to be the exact Hessian. Apply it to solve Rosenbrock's function

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

Use an initial trust region radius of 1. Set maximum trust region radius to 300.
Use the initial point: $\boldsymbol{x}_0 = [-1.2, 1]$ and then try another point $\boldsymbol{x}_0 = [2.8, 4]$. Do the following for each of the initial points.

Additional information for the Dogleg Algorithm:

a. Use $\| \nabla f(\boldsymbol{x}_k) \| < 10^{-8}$ as the stopping criteria for your optimization algorithm.

b. State the total number of iterations obtained in your optimization algorithm.

c. Plot the objective function $f(\boldsymbol{x})$. On the same figure, plot the $\boldsymbol{x}_k$ values at the different iterates of your optimization algorithm.

d. Plot the size of the objective function as a function of the iteration number. Use semi-log plot.

e. You should hand in (i) your code (ii) the first 6 and last 6 values of $\boldsymbol{x}_k$ obtained from your program.

f. Determine the minimizer of the Rosenbrock function $\boldsymbol{x}^*$.

g. Describe how you computed τ^* in the dogleg algorithm.

2. (5 points) Experiment with the update rule for the trust region by changing the constants in Algorithm 4.1 in the text Numerical Optimization by Nocedal and Wright 2006. State what you experimented with and discuss your observations.

3. (25 points) Write a Program that implements the Cauchy Point. Choose B_k to be the exact Hessian. Apply it to solve Rosenbrock's function

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

Use an initial trust region radius of 1. Set maximum trust region radius to 300.
Use the initial point: $\boldsymbol{x}_0 = [-1.2, 1]$ and then try another point $\boldsymbol{x}_0 = [2.8, 4]$. Do the following for each of the initial points.

Additional information for the Cauchy Point Algorithm:

a. Use $\| \nabla f(\boldsymbol{x}_k) \| < 10^{-8}$ as the stopping criteria for your optimization algorithm.

b. State the total number of iterations obtained in your optimization algorithm.

c. Plot the objective function $f(\boldsymbol{x})$. On the same figure, plot the $\boldsymbol{x}_k$ values at the different iterates of your optimization algorithm.

d. Plot the size of the objective function as a function of the iteration number. Use semi-log plot.

e. You should hand in (i) your code (ii) the first 6 and last 6 values of $\boldsymbol{x}_k$ obtained from your program.

f. Determine the minimizer of the Rosenbrock function $\boldsymbol{x}^*$.