Instructions: Please submit your exam on Canvas. While completing the exam, you may refer to **only** the course notes and the recommended course text for formulas.

Start time: 2:00pm
Stop time: 2:55pm

1. (15 points) Given the following partial differential equation:

$$\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \qquad 0 < x < a, \qquad 0 < y < b, \qquad t > 0,$$

with **boundary conditions**

$$u(0, y, t) = 0, \qquad u(a, y, t) = 0, \qquad 0 < y < b, \qquad t > 0,$$

$$u(x, 0, t) = 0, \qquad u(x, b, t) = 0, \qquad 0 < x < a, \qquad t > 0,$$

and **initial conditions**

$$u(x, y, 0) = f(x, y), \qquad 0 < x < a, \qquad 0 < y < b.$$

a. If we separate the variables by letting $u = \phi(x)g(y)h(t)$, we obtain the following three ODEs:

$$\begin{gathered} \phi'' + \mu^2\phi = 0, \quad \phi(0) = 0, \quad \phi(a) = 0, \\ g'' + \nu^2 g = 0, \quad g(0) = 0, \quad g(b) = 0, \\ h' + c^2(\mu^2 + \nu^2)h = 0. \end{gathered}$$

Solve the three ODEs and obtain the **product** solution for u.

b. **Extra credit** (5 points) : Show that if we assume $u = \phi(x)g(y)h(t)$, then the seperation of variables method yields:

$$\begin{gathered} \phi'' + \mu^2\phi = 0, \quad \phi(0) = 0, \quad \phi(a) = 0, \\ g'' + \nu^2 g = 0, \quad g(0) = 0, \quad g(b) = 0, \\ h' + c^2(\mu^2 + \nu^2)h = 0. \end{gathered}$$