1. (4 pts)

(1) Find a particular solution to the nonhomogeneous differential equation $y'' + 4y' + 5y = -10x + e^{-x}$.

$y_p =$ ______________________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ ______________________

(3) Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ ______________________

Answer(s) submitted:

- `- 2x + (8/5) + (1/2)e^(-x)`
- `c1 e^(-2x) cos(x) + c2 e^(-2x) sin(x)`
- `c1 e^(-2x) cos(x) + c2 e^(-2x) sin(x) - 2x + (8/5) + (1/2)e^(-x)`

(correct)

Correct Answers:

- `-4*-2/5 + -2 x + 0.5 e^(-x) + a e^(-2x) cos(x) + b e^(-2x) sin(x)`
- `c1 e^(-2x) cos(x) + c2 e^(-2x) sin(x)`
- `-4*-2/5 + -2 x + 0.5 e^(-x) + c1 e^(-2x) cos(x) + c2 e^(-2x) sin(x)`

2. (2 pts) Consider the differential equation

$$y'' + \alpha y' + \beta y = t + e^{6t}.$$

Suppose the form of the particular solution to this differential equation as prescribed by the method of undetermined coefficients is

$$y_p(t) = A_1 t^2 + A_0 t + B_0 t e^{6t}.$$

Determine the constants α and β.

$\alpha =$ _____

$\beta =$ _____

Answer(s) submitted:

- `-6`
- `0`

(correct)

Correct Answers:

- `-6`
- `0`

3. (3 pts) Consider the initial value problem

$$y'' - 16y = e^{-t}, \quad y(0) = 1, \quad y'(0) = y_0'.$$

Suppose we know that $y(t) \to 0$ as $t \to \infty$. Determine the solution and the unknown initial condition.

$y(t) =$ ______________

$y'(0) =$ _____

Answer(s) submitted:

- `(0) e^(4t) + (1+(1/15)) e^(-4t) - (1/15)e^(-t)`
- `-4(1+(1/15)) + (1/15)`

(correct)

Correct Answers:

- `(-16 / -15) e^(-4 t) + e^(-t) / -15`
- `-4.2`

4. (3 pts)

(1) Find a particular solution to the nonhomogeneous differential equation $y'' + 8y' - 20y = e^{3x}$.

$y_p =$ ______________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ ______________

(3) Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ ______________

Answer(s) submitted:

- `(1/13)e^(3x)`
- `c1e^(2x) + c2e^(-10x)`
- `c1e^(2x) + c2e^(-10x) + (1/13)e^(3x)`

(correct)

Correct Answers:

- `1/13 e^(3 x) + a e^(2 x) + b e^(-10 x)`
- `c1 e^(2 x) + c2 e^(-10 x)`
- `1/13 e^(3 x) + c1 e^(2 x) + c2 e^(-10 x)`

5. (3 pts)

(1) Find a particular solution to the nonhomogeneous differential equation $y'' - 4y' + 4y = e^{2x}$.

$y_p =$ ____________________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants and enter them as c1 and c2.

$y_h =$ ____________________

(3) Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ ____________________

Answer(s) submitted:

- `(1/2)x^2 e^(2x)`
- `e^(2x)(c1 + c2x)`
- `e^(2x)(c1 + c2x) + (1/2)x^2 e^(2x)`

(correct)

Correct Answers:

- `1/2 x^2 e^(2 x) + (a x + b) e^(2 x)`
- `(c1 x + c2) e^(2 x)`
- `1/2 x^2 e^(2 x) + (c1 x + c2) e^(2 x)`

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6. (3 pts)

(1) Find a particular solution to the nonhomogeneous differential equation $y'' + 25y = \cos(5x) + \sin(5x)$.

$y_p =$ ______________________________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants. Enter c_1 as c1 and c_2 as c2.

$y_h =$ ______________________________

(3) Find the solution to the original nonhomogeneous differential equation satisfying the initial conditions $y(0) = 8$ and $y'(0) = 2$.

$y =$ ______________________________

Answer(s) submitted:

- `(x/10)(-cos(5x) + sin(5x))`
- `c1 cos(5x) + c2 sin(5x)`
- `8cos(5x) + (21/50) sin(5x) + (x/10)(-cos(5x) + sin(5x))`

(correct)

Correct Answers:

- `-1/10 x cos(5 x) + 1/10 x sin(5 x) + a cos(5 x) + b sin(5 x)`
- `c1 cos(5 x) + c2 sin(5 x)`
- `-1/10 x cos(5 x) + 1/10 x sin(5 x) + 8 cos(5 x) + 21/50 si`

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7. (4 pts) A 10 kilogram object suspended from the end of a vertically hanging spring stretches the spring 9.8 centimeters. At time $t = 0$, the resulting mass-spring system is disturbed from its rest state by the force $F(t) = 100\cos(10t)$. The force $F(t)$ is expressed in Newtons and is positive in the downward direction, and time is measured in seconds.

(1) Determine the spring constant k.
$k =$ _____ Newtons / meter

(2) Formulate the initial value problem for $y(t)$, where $y(t)$ is the displacement of the object from its equilibrium rest state, measured positive in the downward direction. (Give your answer in terms of y, y', y'', t.

Differential equation: ____________________

Initial conditions: $y(0) =$ ___ and $y'(0) =$ ___

(3) Solve the initial value problem for $y(t)$.
$y(t) =$ ____________________

(4) Plot the solution and determine the maximum excursion from equilibrium made by the object on the time interval $0 \le t < \infty$. If there is no such maximum, enter *NONE* .
maximum excursion = __________ meters

Answer(s) submitted:

- `1000`
- `10y'' + 1000y = 100cos(10t)`
- `0`
- `0`
- `sin(10*t)*t/2`
- `NONE`

(correct)

Correct Answers:

- `10 * 9.8 / 0.098`
- `10*y''+1000*y = 100*cos(10*t)`
- `0`
- `0`
- `10/20 t sin(10 t)`
- `NONE`

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8. (3 pts) Consider the initial value problem

$$my'' + cy' + ky = F(t), \quad y(0) = 0, \quad y'(0) = 0$$

modeling the motion of a spring-mass-dashpot system initially at rest and subjected to an applied force $F(t)$, where the unit of force is the Newton (N). Assume that $m = 2$ kilograms, $c = 8$ kilograms per second, $k = 80$ Newtons per meter, and $F(t) = 100\cos(8t)$ Newtons.

a. Solve the initial value problem.

$y(t) =$

b. Determine the long-term behavior of the system. Is $\lim_{t\to\infty} y(t) = 0$? If it is, enter zero. If not, enter a function that approximates $y(t)$ for very large positive values of t.

For very large positive values of t, $y(t) \approx$ ______

Answer(s) submitted:

- `-(13*exp(-2*t)*sin(6*t))/12 + (3*exp(-2*t)*cos(6*t))/4 + sin(8*t) - (3*cos(8*t))/4`
- `sin(8*t) - (3*cos(8*t))/4`

(correct)

Correct Answers:

- `0.75 e^(-2t) cos(6t) + -1.08333 e^(-2t) sin(6t) + -0.75 cos(8t) + 1 sin(8t)`
- `-0.75 cos(8t) + 1 sin(8t)`

---

9. (3 pts) Find the solution of

$$y'' + 7y' + 10y = 36e^{1t}$$

with $y(0) = 4$ and $y'(0) = 1$.

$y =$

In your written HW, write a complete solution with details on how you found the general solution.

Answer(s) submitted:

- `-e^(-5t) + 3e^(-2t) + 2e^t`

(correct)

Correct Answers:

- `( 2 ) * exp(t) + (1 - 2) *exp((-7/2 - 3/2)*t) + (1 + 2) *exp((-7/2 + 3/2)*t)`

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10. (3 pts) Use the method of undetermined coefficients to find one solution of

$$y'' - 10y' + 22y = 8e^{8t}.$$

$y =$ ______

(It doesn't matter which specific solution you find for this problem.)

Answer(s) submitted:

- `(8/6)e^(8t)`

(correct)

Correct Answers:

- `( 4/3 ) * exp((8)*t) + c*e^(6.73205080756888*t) +d*e^(3.26794919243112*t)`

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11. (3 pts) Use the method of undetermined coefficients to find one solution of

$$y'' - 6y' + 57y = 64e^{3t}\cos(7t) + 96e^{3t}\sin(7t) + 3e^{2t}.$$

(It doesn't matter which specific solution you find for this problem.)

$y =$

Answer(s) submitted:

- `-64exp(3*t)*cos(7*t)-96exp(3*t)*sin(7*t)+(3/49)exp(2*t)`

(correct)

Correct Answers:

- `(-64 )*exp((3)*t)*cos(7 *t) + (-96 )*exp((3)*t)*sin(7 *t)`

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12. (3 pts) Use the method of undetermined coefficients to find one solution of

$$y'' + 2y' + 2y = (10t + 7)\, e^{-t}\cos(t) + (11t + 25)\, e^{-t}\sin(t).$$

(It doesn't matter which specific solution you find for this problem.)

$y =$

Answer(s) submitted:

- `((-11/4)*t^2 + (-10)*t + (0))*exp(-t)*cos(t) + ((10/4)*t^2`

(correct)

Correct Answers:

- `(-11/4 )*t*t*exp(-t) *cos(t) + (-10 )*t*exp(-t) *cos(t)`

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13. (3 pts) Find a particular solution to the differential equation

$$-9y'' + 6y' - 1y = -2t^2 + 2t + 3e^{4t}.$$

$y_p =$ ______

Answer(s) submitted:

- `2*t^2 + 22*t + 96 + (-3/121)*exp(4*t)`

(correct)

Correct Answers:

- `2*(t**2) + 22 *(t) + 96 + -0.0247933884297521 exp(4*t) + a`

14. (3 pts) Find a particular solution to

$$y'' + 16y = -16\sin(4t).$$

$y_p =$ ______________________________

In your written HW, write a complete solution with details on how you found the general solution.

Answer(s) submitted:

- `2*t*cos(4*t)`

(correct)

Correct Answers:

- `2 * t * cos(4*t)  + a*sin(4*t) + b*cos(4*t)`

15. (3 pts) Find a particular solution to

$$y'' - 10y' + 25y = -10.5e^{5t}.$$

$y_p =$ ______________________________

In your written HW, write a complete solution with details on how you found the general solution.

Answer(s) submitted:

- `(-10.5/2)e^(5t)t^2`

(correct)

Correct Answers:

- `(-10.5/2)*(t**2)*exp(5*t) + a*e^(5*t) + b*t*e**(5*t)`

16. (3 pts) Find a particular solution to the differential equation

$$y'' - 7y' + 12y = -288t^3.$$

$y_p =$ ______________________________

In your written HW, write a complete solution with details on how you found the general solution.

Answer(s) submitted:

- `-24*t^3 - 42*t^2 - 37*t - 14.5833`

(correct)

Correct Answers:

- `-24*(t**3) + -42 *(t**2) + -37*t + -14.5833333333333 + a*e^(3*t) + b*(e^(4*t)) + 0*t*e**(3*t))`

17. (3 pts) Find a particular solution to

$$y'' + 5y' + 6y = -2te^{3t}.$$

$y_p =$ ______________________________

Answer(s) submitted:

- `(-2/30)*t*exp(3*t) + (11/(15*30))*exp(3*t)`

(correct)

Correct Answers:

- `(-0.0666666666666667 * t + 0.0244444444444444) * ((2.71828182845905)**(3*t))  + a*e^(-2*t) + b*e**(-3*t)`

18. (3 pts) Consider the following initial value problem:

$$t^2y'' - 4ty' + 6y = 0, \qquad y(1) = 4, \quad y'(1) = 10.$$

Solve this initial value problem.

$y(t) =$ ______________________________.

Answer(s) submitted:

- `2t^3 + 2t^2`

(correct)

Correct Answers:

- `2*t**3 + 2*t**2`

19. (3 pts) Consider the following initial value problem:

$$t^2y'' - 1ty' - 8y = 0, \qquad y(1) = 6, \quad y'(1) = 12.$$

Solve this initial value problem.

$y(t) =$ ______________________________.

Answer(s) submitted:

- `4t^4 + 2t^(-2)`

(correct)

Correct Answers:

- `2*t**-2 + 4*t**4`

20. (3 pts) Consider the following initial value problem:

$$4t^2y'' - 8ty' + 9y = 0, \qquad y(1) = 6, \quad y'(1) = 11.$$

Solve this initial value problem.

$y(t) =$ ______________________________.

Answer(s) submitted:

- `(6 + 2ln(t))t^(3/2)`

(correct)

Correct Answers:

- `t**1.5*(6 + 2*ln(t))`

21. (3 pts) Find y as a function of x if

$$x^2y'' + 13xy' + 36y = x^6,$$

$y(1) = -8,\ \ y'(1) = -2.$

$y =$ ____________________________________

In your written HW, write a complete solution with details on how you found the general solution.

Answer(s) submitted:

- `-1153/(144*x^6) - (601*ln(x))/(12*x^6) + x^6/144`

(correct)

Correct Answers:

- `x**-6 * (-8.00694444444444  - 50.0833333333333 * ln x) + 0.00694444444444444 x^6`

22. (3 pts)

Find a particular solution to

$$y'' - 4y' + 4y = \frac{16.5e^{2t}}{t^2+1}\ .$$

$y_p =$ ____________________________

In your written HW, write a complete solution with details on how you found the general solution.

Answer(s) submitted:

- `-8.25ln(t^2 + 1)e^(2t) + 16.5arctan(t)te^(2t)`

(correct)

Correct Answers:

- `(16.5*exp(2*t))(-ln(1+t^2)/2+t atan(t)) + a*e^(2*t) + b*t*`

23. (3 pts) Find a particular solution to $y'' + 4y = 8\sec(2t)$.

$y_p =$ ____________________________

Answer(s) submitted:

- `-2ln(|sec(2t)|)cos(2t) + 4tsin(2t)`

(correct)

Correct Answers:

- `8 * [2^(-2)cos(2*t)ln( abs(cos(2*t))) + t*2^(-1)*sin(2 t)]`