Test 2 Info, Math 330

Definitions to know: Suppose $D \subseteq \mathbb{R}$ and $f : D \to \mathbb{R}$.

1. We say f is **continous at** $x_0 \in D$ iff...
   (a) sequential def $\forall \{x_n\} \subseteq D$, if $\lim_{n\to\infty} x_n = x_0$, then $\lim_{n\to\infty} f(x_n) = f(x_0)$.
   (b) $\epsilon - \delta$ def $\forall \epsilon > 0$, $\exists \delta > 0$ such that $\forall x \in D$, if $|x - x_0| < \delta$, then $|f(x) - f(x_0)| < \epsilon$.

remark: we say f is **continuous on** D or simply f is **continuous** iff $\forall x_0 \in D$, the function f is continuous at x_0.

2. We say f is **uniformly continuous on** D iff...
   (a) sequential def
   $\forall \{u_n\}, \{v_n\} \subseteq D$, if $\lim_{n\to\infty} (u_n - v_n) = 0$, then $\lim_{n\to\infty} (f(u_n) - f(v_n)) = 0$.
   (b) $\epsilon - \delta$ def
   $\forall \epsilon > 0$, $\exists \delta > 0$ such that $\forall x_0, x \in D$, if $|x - x_0| < \delta$, then $|f(x) - f(x_0)| < \epsilon$.
3. We say that $x_0 \in br$ is a **limit point of** D iff $\exists x_n \subseteq D \backslash \{x_0\}$ such that $\lim_{n\to\infty} x_n = x_0$.
4. Suppose x_0 is a limit point of D and $L \in \mathbb{R}$. We say the **limit of** f **as** x **approaches** x_0 **is** L and write $\lim_{x\to x_0} f(x) = L$ iff...
   (a) sequential def $\forall \{x_n\} \subseteq D \backslash \{x_0\}$, if $\lim_{n\to\infty} x_n = x_0$, then $\lim_{n\to\infty} f(x_n) = L$.
   (b) $\epsilon - \delta$ def $\forall \epsilon > 0$, $\exists \delta > 0$ such that $\forall x \in D$, if $0 < |x - x_0| < \delta$, then $|f(x) - L| < \epsilon$.

Results to know:

1. Algebra of Continuous Functions: 3.4, 3.5, 3.6
2. Extreme Value Theorem (3.9): Suppose $f : [a, b] \to \mathbb{R}$ is continuous. Then f attains maximum and minimum values.
   I.e. $\exists x_1, x_2 \in [a, b]$ such that $\forall x \in [a, b]$, we have $f(x_1) \leq f(x) \leq f(x_2)$
3. Intermediate Value Theorem(3.11): Suppose $f[a, b] \to \mathbb{R}$ is continuous. If c is strictly between $f(a)$ and $f(b)$, then $\exists x_0 \in \mathbb{R}$ such that
   $$a < x_0 < b \quad \text{and} \quad f(x_0) = c$$
4. Theorem 3.17. Suppose $f : [a, b] \to \mathbb{R}$. If f is continuous, then f is uniformly continuous.