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1. (4 pts) Consider the following model for the populations of rabbits and wolves (where R is the population of rabbits and W is the population of wolves).

$$\frac{dR}{dt} = 0.1R(1-0.001R) - 0.00234375RW$$
$$\frac{dW}{dt} = -0.08W + 0.00032RW$$

Find all the equilibrium solutions:
(a) In the absence of wolves, the population of rabbits approaches ________.
(b) In the absence of rabbits, the population of wolves approaches ________.
(c) If both wolves and rabbits are present, their populations approach $r =$ ________ and $w =$ ________.

Answer(s) submitted:

- .1/.0001
- 0
- .08/.00032
- (.1 - .0001(.08/.00032))/(.00234375)

(correct)

Correct Answers:

- 1000
- 0
- 250
- 32

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2. (15 pts) Consider the model given by the equations:

$$\frac{dX}{dt} = X(0.22 - 0.039X - 0.0154Y),$$
$$\frac{dY}{dt} = Y(0.41 - 0.025Y - 0.0846X)$$

a. In your written HW, give a brief explanation of each species' ecological behavior. Describe each term on the right hand side of the differential equations.

b. Determine all possible equilibria.

Extinction: $(X_e, Y_e) =$ (___, ___).

Only Species X: $(X_e, Y_e) =$ (___, ___).

Only Species Y: $(X_e, Y_e) =$ (___, ___).

Coexistence: $(X_e, Y_e) =$ (___, ___).

c. In your written HW assignment, create a graph of the phase plane. Show all equilibria and draw the nullclines. Introduce arrows to show representative directions of the trajectories.

d. Perform a linear stability analysis, giving eigenvalues and eigenvectors at each equilibrium. (If you have complex eigenvalues, then always list the one with negative imaginary part first). For your answers on the eigenvectors, you will either make one of the components equal to 1 or will be given which component of the eigenvector is 1. Classify the equilibria according to one of the following: Stable Node, Unstable Node, Saddle Node, Center, Stable Spiral, Unstable Spiral, or None of the Above.

Linearization at Extinction equilibrium: (List the smallest eigenvalue first).
$\lambda_1 =$ ___, Eigenvector, $\xi_1 =$ [___, ___]
$\lambda_2 =$ ___, Eigenvector, $\xi_2 =$ [___, ___]
Type of Equilibrium = ______.

Linearization at Only Species X: (List the smallest eigenvalue first).
$\lambda_1 =$ ___, Eigenvector, $\xi_1 =$ [___, ___]
$\lambda_2 =$ ___, Eigenvector, $\xi_2 =$ [1, ___]
Type of Equilibrium = ______.

Linearization at Only Species Y: (List the smallest eigenvalue first).
$\lambda_1 =$ ___, Eigenvector, $\xi_1 =$ [___, ___]
$\lambda_2 =$ ___, Eigenvector, $\xi_2 =$ [___, 1]
Type of Equilibrium = ______.

Linearization at Coexistence Equilibrium: (List the smallest eigenvalue first).
$\lambda_1 =$ ___, Eigenvector, $\xi_1 =$ [1, ___]
$\lambda_2 =$ ___, Eigenvector, $\xi_2 =$ [1, ___]
Type of Equilibrium = ______.

e. Does this model exhibit 'Competitive Exclusion' or 'Coexistence' of the two species? __________.

Answer(s) submitted:

- 0
- 0
- .22/.039
- 0
- 0
- .41/.025
- 2.4829
- 7.9978
- .22
- 1
- 0
- .41
- 0

- 1
- Unstable Node
- -.22
- 1
- 0
- -0.0672
- -0.1528/0.0869
- Stable Node
- -.41
- 0
- 1
- -.03256
- 0.3774/-1.3874
- Stable Node
- -0.3173
- 0.6766 / 0.1174
- 0.0205
- -0.1173 / 0.0382
- Saddle Node
- Competitive Exclusion

(correct)

Correct Answers:

- 0
- 0
- 5.64102564102564
- 0
- 0
- 16.4
- 2.48291849682772
- 7.99780380673499
- 0.22
- 1
- 0
- 0.41
- 0
- 1
- UNSTABLE NODE
- -0.22
- 1
- 0
- -0.0672307692307692
- -1.75855962219599
- STABLE NODE
- -0.41
- 0
- 1
- -0.03256
- -0.272040592746353
- STABLE NODE
- -0.317296662455682
- 5.7657023053919
- 0.020517745911026
- -3.06906233602466
- SADDLE NODE
- COMPETITIVE EXCLUSION

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3. (15 pts) Consider the model given by the equations:

$$\frac{dX}{dt} = X(0.7 - 6.7X + 5.9Y),$$

$$\frac{dY}{dt} = Y(0.8 - 5.1Y + 3.9X)$$

a. In your written HW, give a brief explanation of each species' ecological behavior. Describe each term on the right hand side of the differential equations.

b. Determine all possible equilibria.

Extinction: $(X_e, Y_e) = (\_\_\_, \_\_\_)$.

Only Species X: $(X_e, Y_e) = (\_\_\_, \_\_\_)$.

Only Species Y: $(X_e, Y_e) = (\_\_\_, \_\_\_)$.

Coexistence: $(X_e, Y_e) = (\_\_\_, \_\_\_)$.

c. In your written HW assignment, create a graph of the phase plane. Show all equilibria and draw the nullclines. Introduce arrows to show representative directions of the trajectories.

d. Perform a linear stability analysis, giving eigenvalues and eigenvectors at each equilibrium. Order your list the eigenvectors in increasing value (with complex listed with negative imaginary first). For your answers on the eigenvectors, you will either make one of the components equal to 1 or will be given which component of the eigenvector is 1. Classify the equilibria according to one of the following: Stable Node, Unstable Node, Saddle Node, Center, Stable Spiral, Unstable Spiral, or None of the Above.

Linearization at Extinction equilibrium:
$\lambda_1 = \_\_\_$, Eigenvector, $\xi_1 = [\_\_\_, \_\_\_]$
$\lambda_2 = \_\_\_$, Eigenvector, $\xi_2 = [\_\_\_, \_\_\_]$
Type of Equilibrium = ____.

Linearization at Only Species X:
$\lambda_1 = \_\_\_$, Eigenvector, $\xi_1 = [\_\_\_, \_\_\_]$
$\lambda_2 = \_\_\_$, Eigenvector, $\xi_2 = [1, \_\_\_]$
Type of Equilibrium = ____.

Linearization at Only Species Y:
$\lambda_1 = \_\_\_$, Eigenvector, $\xi_1 = [\_\_\_, \_\_\_]$
$\lambda_2 = \_\_\_$, Eigenvector, $\xi_2 = [\_\_\_, 1]$
Type of Equilibrium = ____.

Linearization at Coexistence Equilibrium:
$\lambda_1 = \_\_\_$, Eigenvector, $\xi_1 = [1, \_\_\_]$
$\lambda_2 = \_\_\_$, Eigenvector, $\xi_2 = [1, \_\_\_]$
Type of Equilibrium = ____.

e. In your written HW, discuss what happens ultimately for this biological system according to the mathematical model.

Answer(s) submitted:

- 0
- 0
- .7/6.7

- 0
- 0
- .8/5.1
- 0.7428
- 0.7249
- .7
- 1
- 0
- .8
- 0
- 1
- Unstable Node
- -0.7000
- 1
- 0
- 1.2075
- -1.9075 / -0.6164
- Saddle Node
- -0.8000
- 0
- 1
- 1.6255
- 2.4255 / 0.6118
- Saddle Node
- -7.9144
- 2.9378 / -4.3825
- -0.7592
- -4.2174 / -4.3825
- Stable Node

(correct)
Correct Answers:

- 0
- 0
- 0.104477611940299
- 0
- 0
- 0.156862745098039
- 0.742831541218638
- 0.724910394265233
- 0.7
- 1
- 0
- 0.8
- 0
- 1
- UNSTABLE NODE
- -0.7
- 1
- 0
- 1.20746268656716
- 3.09443099273608
- SADDLE NODE
- -0.8
- 0
- 1
- 1.62549019607843
- 3.96474358974359
- SADDLE NODE
- -7.91473326345321
- -0.670307767580663
- -0.759281073464355
- 0.962348412834287
- STABLE NODE