Homework 7
Algebraic Coding Theory
Math 525
Stephen Giang RedID: 823184070

Problem 1:

(a) **Exercise 3.5.1:** Show that the word of all ones is in C_{24}. Deduce that C_{24} contains no words of weight 20

Notice $H^T = [I\ B]$ and let $v = (1, 1, 1, ..., 1, 1)$ be the word of all ones. We can see that $vH = 0$, which means that $v \in C_{24}$

Let there be a codeword $c \in C_{24}$ with weight 20. Then we get that $v + c \in C_{24}$ has weight 4. This is a contradiction as the minimum distance of C_{24} is 8.

(b) **Exercise 3.5.2:** Prove fact (4) about C_{24}.

Notice we get that $GH = B + B = 0$. Also G has rank 12 so it is in fact a generator matrix.

(c) **Exercise 3.5.3:** Prove fact (5) about C_{24}.

Notice that a generator matrix for $C^{\perp}$ is $[I\ B^T]$. Notice that $B = B^T$ such that the generator matrix is $[I\ B]$. Because C and $C^{\perp}$ share the same generator matrix, they are the same.

Problem 2:

(a) **Exercise 3.6.5:** The code is C_{24}. Find the most likely error pattern if possible, for each of the following received words w.

$w = 111000000000011011011011$. Notice the following:

$$s = wH = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Notice the weight of s is 3. Thus we get

$$u = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) **Exercise 3.6.6:** Find the most likely error pattern for any word with the given syndromes.

$s_1 = 010010000000, s_2 = 011111010000$

Because $wt(s_1) \leq 3$, we get $u = 0100100000000$

Problem 3:

(a) **Exercise 3.7.3:** Decode each of the following received words that were encoded using C_{23}.

$r = 10101110000010101011011$

Let $ri = 101011100000101010110110$.

From this we get $s_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$, which has weight of 4. So we must find s_2, which is $\begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$, which has a weight of 8.

Because $s_1 + b_i$ and $s_2 + b_i$ are never less than or equal to 2, the algorithm requests for retransmission.

(b) **Exercise 3.7.5:** Find the reliability of C_{23} transmitted over a BSC of probability p.

Notice the coset leaders:

- There is 1 coset leader of weight 0
- There is $\binom{23}{1} = 23$ coset leaders of weight 1
- There is $\binom{23}{2} = 253$ coset leaders of weight 2
- There is $\binom{23}{3} = 1771$ coset leaders of weight 3

By Th 3.2.8, we get that all these coset leaders are unique such that

$$\theta(p) = p^{23} + 23p^{22}(1-p) + 253p^{21}(1-p)^2 + 1771p^{20}(1-p)^3$$

(c) **Exercise 3.7.7:** See solution guide

(d) **Exercise 3.7.8:** See solution guide