Sept 30, 2024

☒ Fitting Differential Equation model:

Step-1: Consider data (t_i, Z_i)

Step-2: List parameters (include initial conditions)

$\vec{\theta}$: the total number of parameter

$$\frac{d\vec{y}}{dt} = \vec{f}(\vec{y}, \underset{\sim}{\vec{\theta}})$$

Step-3: Among θ, decide parameters to be fixed (known values from literature survey, data, experiments, etc.), and remaining to be estimated.

$$\theta = \theta_f + \underset{\sim}{\theta_e} \quad (\text{fixed} + \text{estimated})$$

Step-4: Iterative and simultaneous algorithm

Initial guess for θ_e^0

$\Downarrow$

solve model: $\frac{d\vec{y}}{dt} = f(\vec{y}, \theta_f, \theta_e)$

$\Downarrow$

Calculate values from the model $(\underbrace{\text{Model}_i(\theta_e)})$ corresponding to data available $(\underset{\sim}{Z_i})$

$\Downarrow$

Sum of squared residuals

$$SSR = \sum_{i=1}^{n} \left(Z_i - \text{Model}_i(\theta_e)\right)^2$$

$\Downarrow$

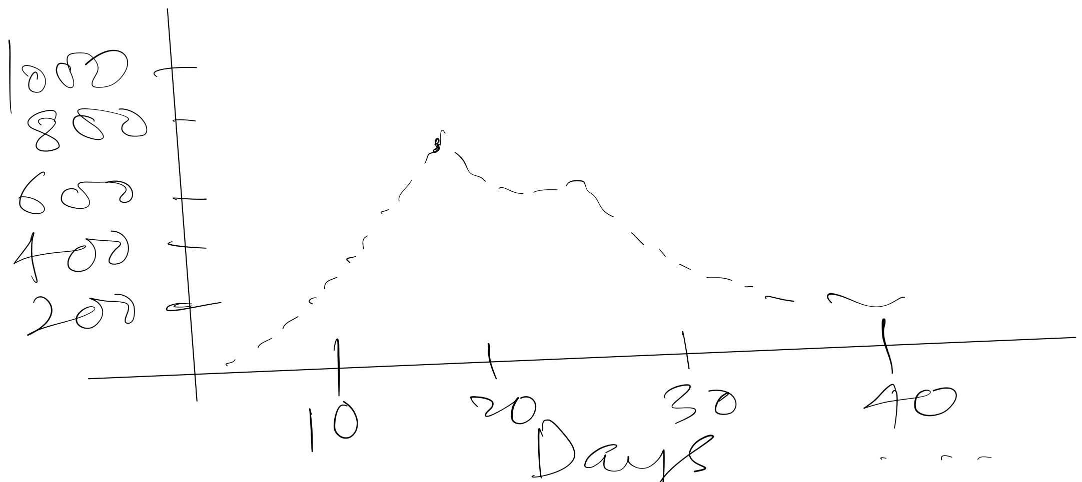
Identify parameters θ_e that minimize SSR:

$$\min_{\theta_e} \sum_{i=1}^{n} \left(Z_i - \text{Model}_i\right)^2$$

☑ Example (case study) : 2009 H1N1 Influenza Outbreak

Step-1: Consider data (Washington State University)

t_i (Days)	Z_i (# of infected individuals)
1	⋮
2	
⋮	



Step-2: list parameters (including initial conditions)

$$
\begin{tikzcd}
S \arrow[r, "aSI"] \arrow[d, "bS"] & I \arrow[r, dash, "dI"] & R \\
P \arrow[ur, "cPI"'] & &
\end{tikzcd}
$$

$$\frac{dS}{dt} = -aSI - bS$$

$$\frac{dP}{dt} = bS - cPI$$

$$\frac{dI}{dt} = aSI + cPI - \underline{\underline{dI}}$$

$$\frac{dR}{dt} = dI$$

$\vec{\theta}$: $a, b, c, d, S_0, P_0, I_0, R_0$

<u>Step-3:</u> $\theta = \theta_f + \theta_e$ (fixed + estimated)

θ_f : $d = \frac{1}{6}$, $S_0 = 18223$, $P_0 = 0$, $I_0 = 11$, $R_0 = 0$

θ_e : a, b, c.

<u>Step-4:</u> Iterative and simultaneous algorithm.

Initial guess for θ_e^0 $\left[a = 5.45e-6; \; b = 0.2; \; c = 3.74e-7 \right]$

$\Downarrow$

$\Downarrow$

[MATLAB: fminsearch:]

Matlab CODE:

Files:

- MainFitting.m
- SSR.m
- Deqns.m
- DataHIN1.dat

Estimated parameters:

$a = 5.1056e{-5}$

$b = 0.12227,$

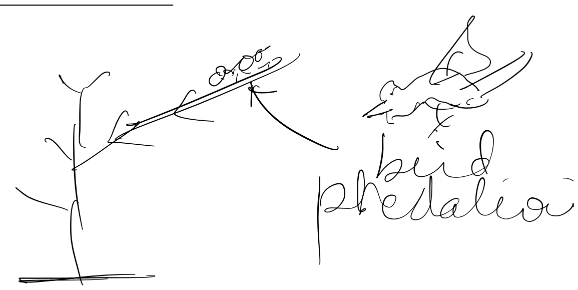
$c = 1.8758e{-6},$

Graphical Techniques for Global Dynamics:

Case study: spruce Budworm (Pest/Insect Control)

- Insect native to North America
- destructive to forest (spruce/balsam fir)
- periodic outbreak
- Goal: to control.

Food Chain:



bird
↓
budworm
↓
trees

Model for the budworm:

- number of birds is fixed
- trees population remains constant.

Growth (logistic)

$\boxed{N(t)}$ → predation by birds

$N(t)$: the population size of budworm

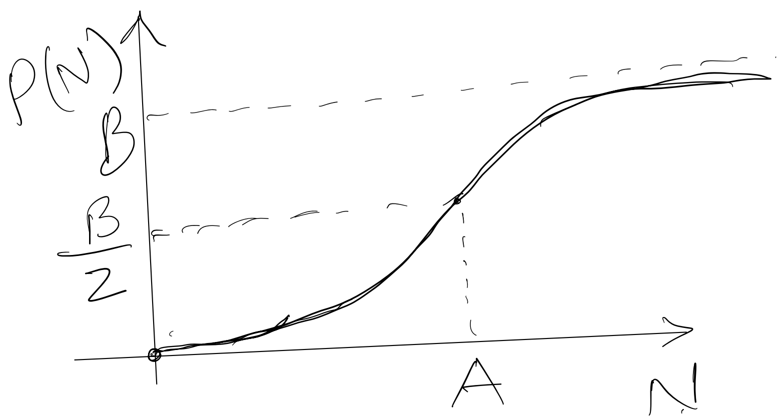
$$\frac{dN}{dt} = \underbrace{r_B N\left(1 - \frac{N}{K_B}\right)}_{\text{logistic growth}} - \underbrace{P(N(t))}_{\text{predation by birds}}$$

- Pattern/feature of the predation
  - birds are lazy

$$P(N) << 1 \text{ if } \underbrace{N << N_0}_{\text{critical value switching}}$$

  - "eat like a bird"

$\exists$ a saturation level



$$\Rightarrow P(N) = \frac{BN^2}{A^2+N^2}$$

B: ~ # of birds (max. predation)

A: The value of N corresponding to half saturation $\left(\frac{B}{2}\right)$.

Model:

$$\boxed{\frac{dN(t)}{dt} = r_B N(t)\left[1 - \frac{N(t)}{K_B}\right] - B\frac{N(t)^2}{A^2+N(t)^2}}$$

Dimensional analysis: (HW-1)

$$\Rightarrow \frac{du}{d\tau} = \underbrace{ru\left(1-\frac{u}{q}\right) - \frac{u^2}{1+u^2}}_{} = f(u)$$

$r = \frac{B r_B}{A}$, $q = \frac{K_B}{A}$