Exam 2
Algebraic Coding Theory
Math 525
Stephen Giang RedID: 823184070

Problem 3: Let C be the linear code with parity-check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Construct a standard decoding array (SDA) for C. Hint: Let $syn(w)$ denote the syndrome (with respect to C) of the word $w \in K^8$. Then:

$$\begin{cases} \text{syn}(01000001) = \text{syn}(10000010) = \text{syn}(00110000) = \text{syn}(00001100) \\ \text{syn}(01010000) = \text{syn}(10000100) = \text{syn}(00100001) = \text{syn}(00001010) \\ \text{syn}(00000110) = \text{syn}(00010001) = \text{syn}(01100000) = \text{syn}(10001000) \\ \text{syn}(10010000) = \text{syn}(01000100) = \text{syn}(00001001) = \text{syn}(00100010) \\ \text{syn}(01001000) = \text{syn}(10100000) = \text{syn}(00010010) = \text{syn}(00000101) \\ \text{syn}(00010100) = \text{syn}(00101000) = \text{syn}(11000000) = \text{syn}(00000011) \\ \text{syn}(10000001) = \text{syn}(01000010) = \text{syn}(00011000) = \text{syn}(00100100) \end{cases}$$

Coset Leader u	syn(u) uH
01000001	1100
01010000	1010
00000110	0110
10010000	1001
01001000	0101
00010100	0011
10000001	1111

(b) Suppose $w = 10111000$ is received. Find the closest codeword(s) in C to w.

Notice:

$$wH = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

Notice that the syndrome 1010 refers to the coset leader $u = 01010000$. So we find the closet codeword in C to w from $\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{u} = \mathbf{10111000} + \mathbf{01010000} = \mathbf{11101000}$

(c) Calculate $\theta_p(C)$, i.e., the probability that if a codeword $v \in C$ is sent over a BSC of reliability $p > \frac{1}{2}$, then IMLD will correctly conclude that v was sent.

Notice that we have $n = 8$ and the weight of each coset leader is $wt(u) = 2$. Also notice that there are 7 coset leaders of minimum weight 2. Thus we have that:

$$\theta_p(C) = 7p^{8-2}(1-p)^2 = 7p^6(1-p)^2$$