Homework 1

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In [34]:
```
import matplotlib.pyplot as plt
import numpy as np
```

Cobweb Plot

The way I went about plotting the cobweb plot was to first graph $y = x$ and the given function $y = f(x)$. Then to plot the dashed lines, or the cobwebs, I placed the starting coordinate to be $($$x_1$ (input_value), $0)$. I then drew a vertical line until it went up to $(x_1, f(x_1))$. Then I drew a horizontal line from $(x_1, f(x_1))$ to $(f(x_1), f(x_1))$. I then repeated doing that for n, number of iterations.

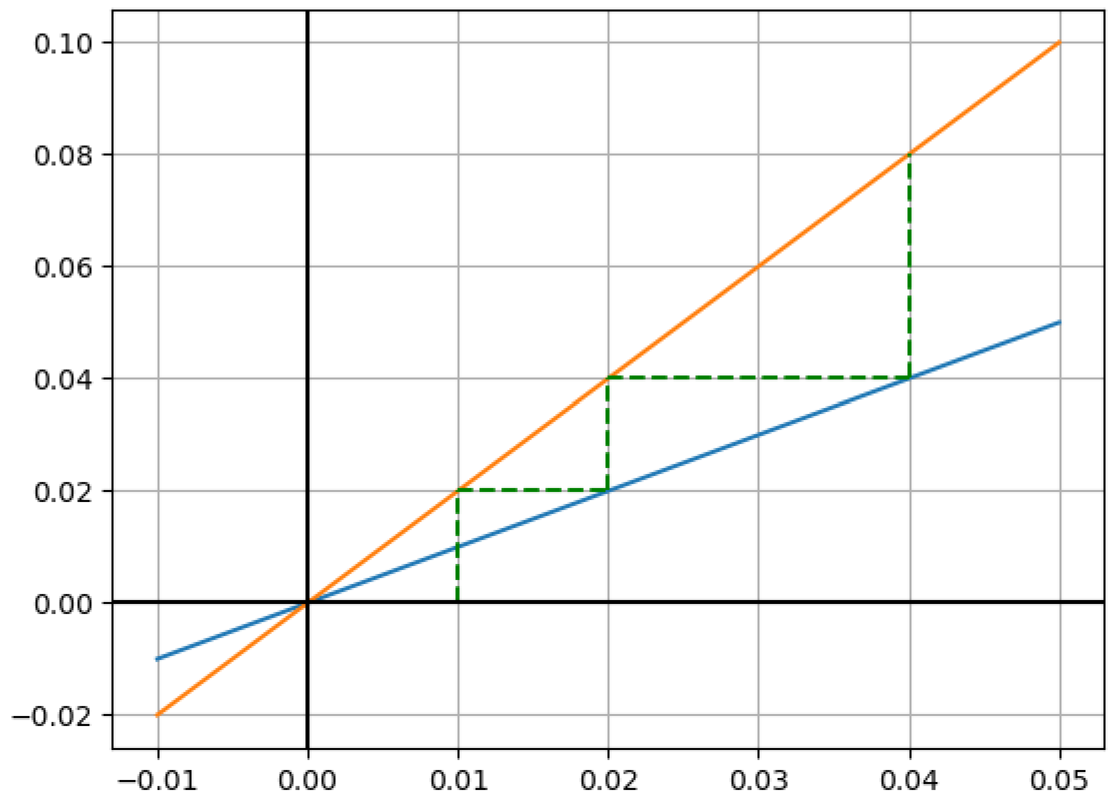
Ex: $\Big((f(x_1), f(x_1)) \rightarrow (f(x_1), f^2(x_1))\Big), \Big((f(x_1), f^2(x_1)) \rightarrow (f^2(x_1), f^2(x_1))\Big)$

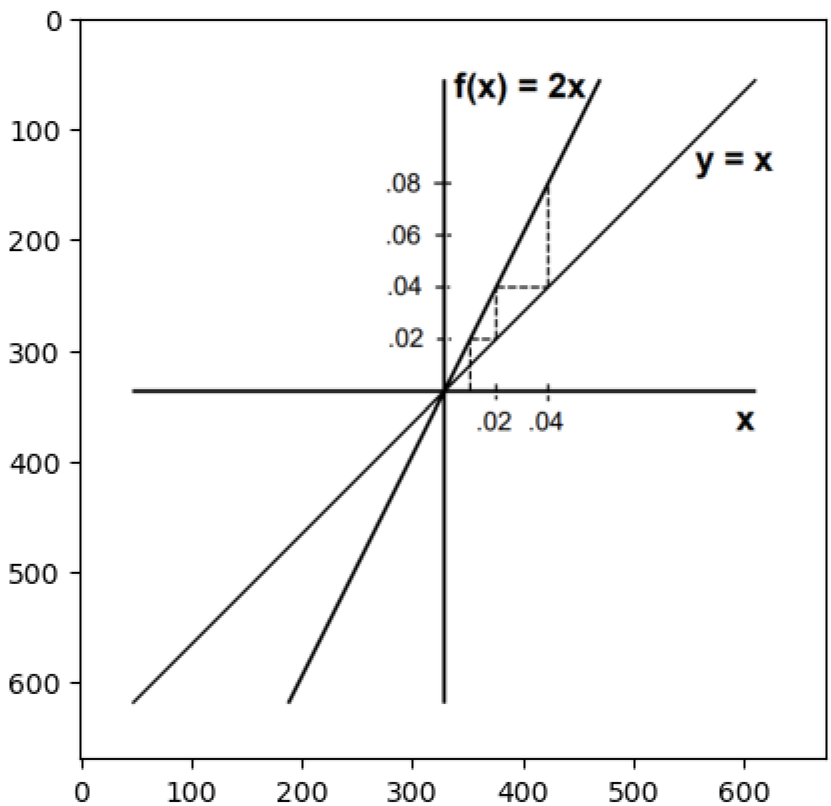
In [35]:
```
def cobweb_plot(input_vals, f, x_range, num_iters):
    x_vals = np.linspace(x_range[0], x_range[1], 10_000)
    y_vals = x_vals
    f_vals = f(x_vals)

    fig, ax = plt.subplots()

    # Plot f and Diagonal
    ax.plot(x_vals, y_vals, '-')
    ax.plot(x_vals, f_vals, '-')
    ax.grid()

    for input_val in input_vals:
        vert_bottom_y = 0
        horz_left_x = input_val
        for i in range(num_iters):
            #Vertical Line
            ax.vlines(horz_left_x, vert_bottom_y, f(horz_left_x),
                      colors='green', linestyles='dashed')
            #Horizontal Lines
            if i != num_iters - 1:
                ax.hlines(f(horz_left_x), horz_left_x, f(horz_left_x),
                          colors='green', linestyles='dashed')
            #Update Starting Values
            vert_bottom_y = f(horz_left_x)
            horz_left_x = f(horz_left_x)

    #Axes
    ax.axhline(0, color='black', linewidth=1.5)
    ax.axvline(0, color='black', linewidth=1.5)

    return ax
```

Figure 1.1

In [36]:
```
cobweb_plot([0.01], lambda x: 2 * x, (-0.01, 0.05), 3)
plt.figure()
plt.imshow(plt.imread('textbook-1.1.png'))
```

Out[36]: <matplotlib.image.AxesImage at 0x7fbdd0d86a10>





Notice we can calulate the fixed point of this equation by solving the equation:

$$2x = x \qquad \rightarrow \qquad x = 0$$

Notice the cobwebs are being repulsed by the fixed point $x^* = 0$.

This is because the general rule for cobweb plots is the following:

If $x_1 > 0$ and $f(x_1) > x_1$, then the next mapping of x_2 will be to the right of x_1 and thus move away from the fixed point $x^* = 0$. Thus making this fixed point a repulsor, or a source.

If $x_1 < 0$ and $f(x_1) < x_1$, then the next mapping of x_2 will be to the left of x_1 and thus move away from the fixed point $x^* = 0$. Thus making this fixed point a repulsor, or a source.

If $x_1 > 0$ and $f(x_1) < x_1$, then the next mapping of x_2 will be to the left of x_1 and thus move towards the fixed point $x^* = 0$. Thus making this fixed point an attractor, or a sink.

If $x_1 < 0$ and $f(x_1) > x_1$, then the next mapping of x_2 will be to the right of x_1 and thus move towards the fixed point $x^* = 0$. Thus making this fixed point an attractor, or a sink.

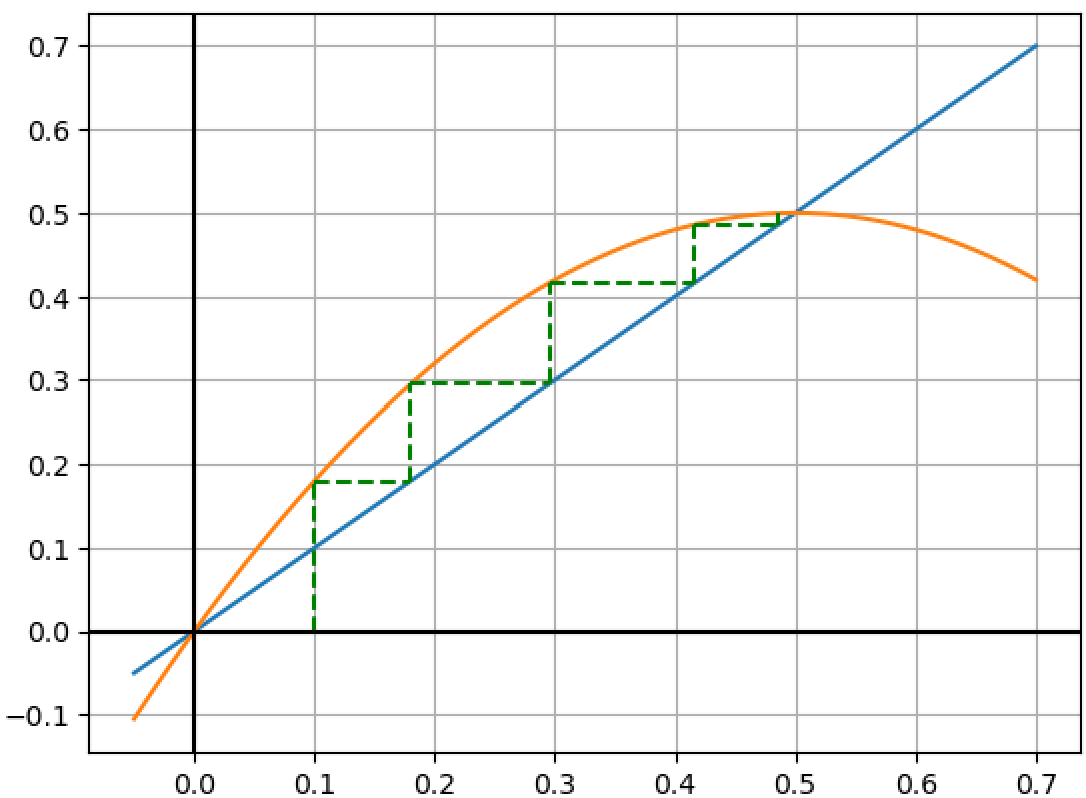
Figure 1.2

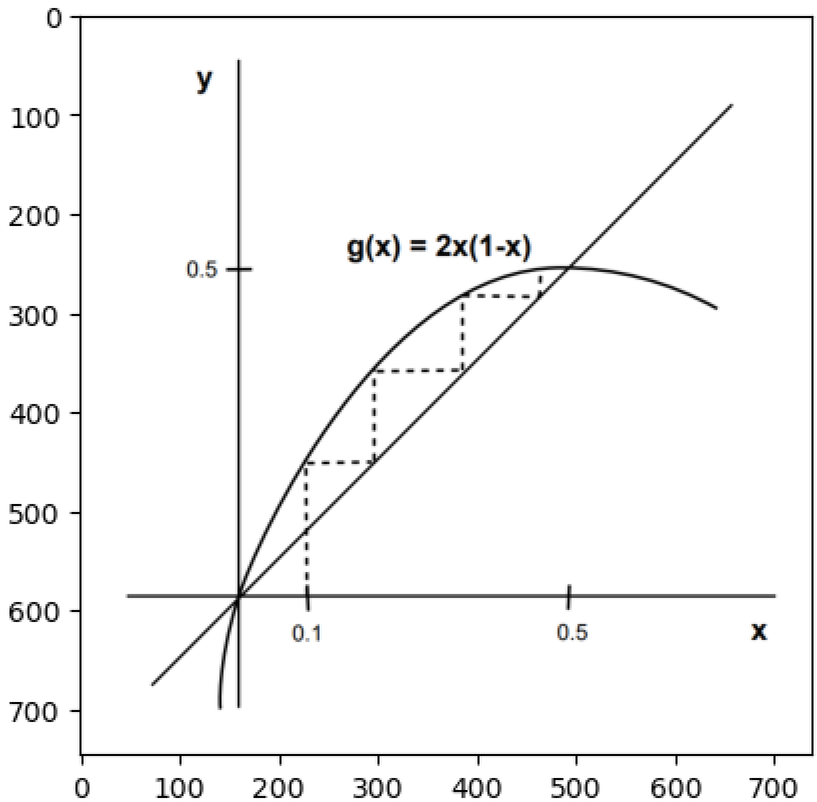
In [37]:

```
cobweb_plot([0.1], lambda x: 2 * x * (1 - x), (-.05, 0.7), 5)
plt.figure()
plt.imshow(plt.imread('textbook-1.2.png'))
```

Out[37]:

```
<matplotlib.image.AxesImage at 0x7fbdd0c9c390>
```





Notice we can calulate the fixed point of this equation by solving the equation:

$$2x(1-x) = x \qquad \rightarrow \qquad x = 0, 0.5$$

Using the same rules as the above explanation, we can see the following for different values of x_1:

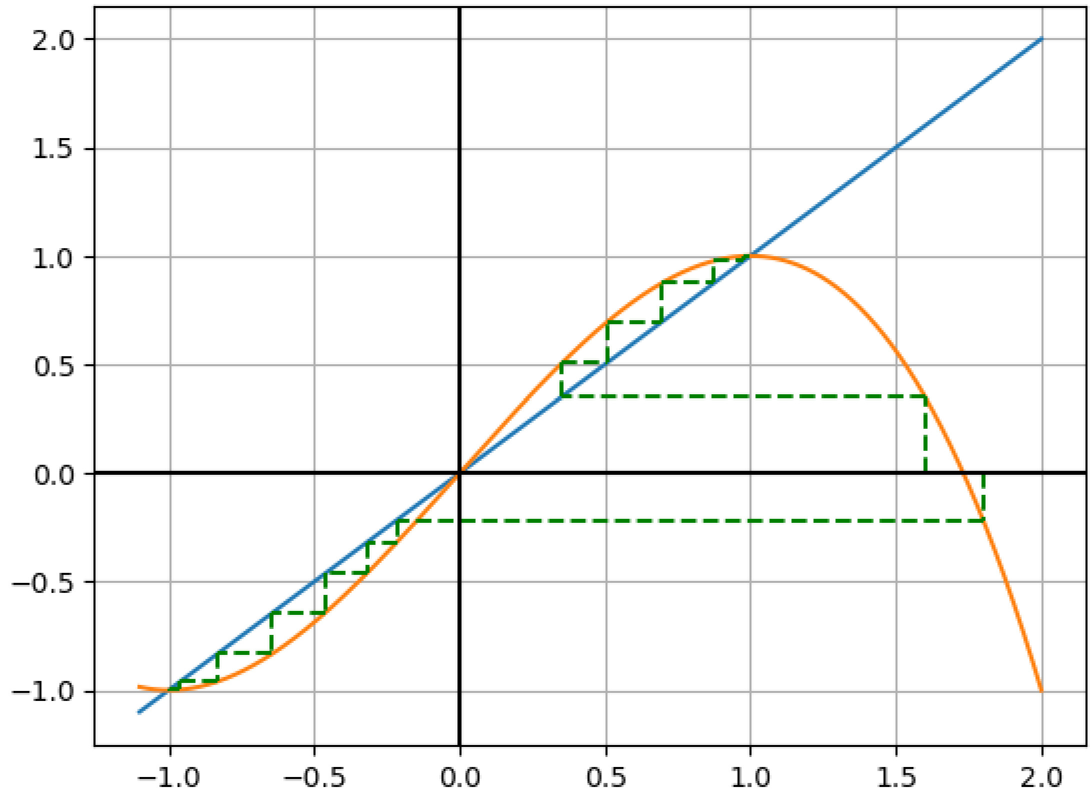
By choosing an $0 < x_1 < 0.5$, we get $f(x_1) > x_1$ which shows that the cobwebs will always move in the right and upward direction. Thus making the fixed point, $x^* = 0.5$ a sink and the fixed point, $x^* = 0$ a source.

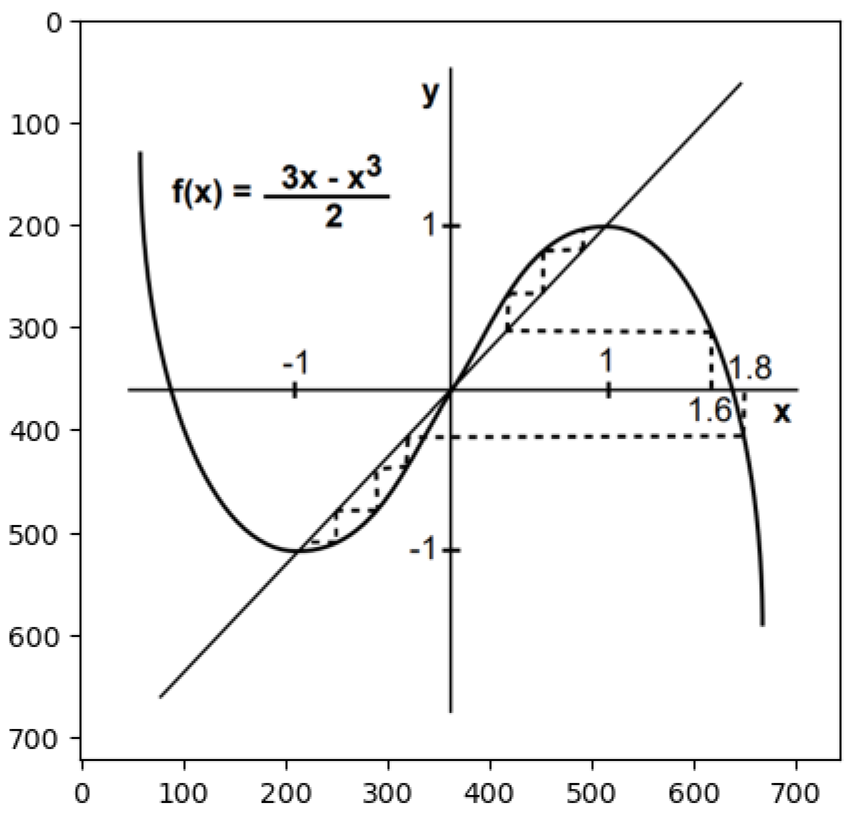
By choosing an $0.5 < x_1 < 1$, we get $0 < f(x_1) < 0.5 < x_1$ which shows that the cobwebs will map itself to the earlier case, and thus have similar results.

Figure 1.3

In [38]:
```
cobweb_plot([1.6, 1.8], lambda x: ((3 * x) - (x ** 3) ) / 2, (-1.1, 2), 8)
plt.figure()
plt.imshow(plt.imread('textbook-1.3.png'))
```

Out[38]:
```
<matplotlib.image.AxesImage at 0x7fbdd0a09110>
```





Notice we can calulate the fixed point of this equation by solving the equation:

$$\frac{3x - x^3}{2} = x \qquad \rightarrow \qquad x = 0, 1, -1$$

By choosing x_1 to be on the right or left of the root $x = 0, \pm\sqrt{3}$, it dictates where the cobwebs get mapped to either the left or right side of the y-axis.

Choosing $x_1 = 1.6 < \sqrt{3}$, we got that $0 < f(x_1) < x_1$, which gave us very similar results as in figure 1.2

Choosing $x_1 = 1.8 > \sqrt{3}$, we got that $f(x_1) < 0$, which gave us very similar results as in figure 1.2 but in the opposite direction

Notice the cobwebs converges towards the fixed point $x = 1$. When the function lies above the diagonal, this means that the cobwebs will keep moving in the upward and rightward direction leading in the direction of the fixed point $x^* = 1$, a sink and away from the fixed point $x^* = 0$, a source.

Notice the cobwebs formed by $x_2 = 1.8$ converges towards the fixed point $x^* = -1$. Note that starting on the right of the root mapped the cobwebs to the negative side of the x-axis leading to the part of the function that lies below the diagonal. When the function lies below the diagonal, this means that the cobwebs will keep moving in the downward and leftward direction leading in the direction of the fixed point $x^* = -1$, a sink and away from the fixed point $x^* = 0$, a source.