Stephen Giang

1) (20) Tall building, height shown. Ball 1: thrown down, at $v_0 = 1\ m/s$. Ball 2: thrown upward, at speed shown. Do they collide before either one hits ground? If so: find height of collision. If not: find speed of each ball, as it hits ground

2) (15) Object moving along x-axis; v_x vs t graph shown; several points labeled.

a) direction object is moving at point 2?
b) at which points does it reverse direction?
c) at which points ~~does~~ is magnitude of acceleration largest?
d) at which points is mag of accel. smallest?

(Just give answers. No explanations or work needed or wanted)

3) (20) Object moving in xy-plane: $x = 2t^3 - 5t$, $y(t)$ as shown, gives x, y in meters for t in seconds.

At time t shown, find:

a) speed
b) angle of velocity with $+x$-axis
c) magnitude of acceleration
d) speeding up, slowing down, or neither (explain reasoning for (d))

4) (15) Cannon ball, launched from edge of very high cliff. Along its trajectory, at least one point has the property shown. Find smallest possible value launch angle θ could be. Show all work.

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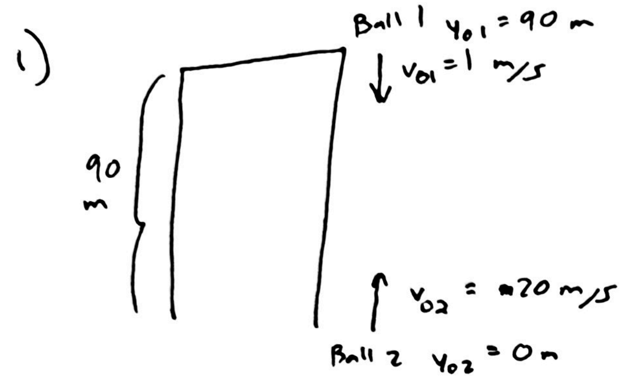
1)



work for Ball 1

Because of ball being thrown down, all accel, velocity, position in terms of y-coordinate.

$a_{y_1} = -g = -9.8$

$v_{y_1} = v_{oy_1} + a_{y_1} t$

$= -1 + -9.8t$

$y_1 = y_{01} + v_{oy_1} t + \frac{1}{2} a_{y_1} t^2$

$= 90 - t - 4.9t^2$

work for Ball 2

$a_{y_2} = -g = -9.8$

$v_{y_2} = v_{oy_2} + a_{y_2} t$

$= 20 + -9.8t$

$y_2 = y_{02} + v_{oy_2} t + \frac{1}{2} a_{y_2} t^2$

$= 0 + 20t - 4.9t^2$

collision case

$y_1 = 90 - t - 4.9t^2 = 20t - 4.9t^2 = y_2$

$90 = 21t$

$\frac{90}{21} = t$

$\boxed{4.2857\text{ sec} = t}$

$\boxed{\text{At } t = 4.2857 \text{ secs, the balls collide}}$

2)

a) right

b) Point 4, Point 1

c) Point 6, Point 2

d) Point 1, Point 4

3) $x(t) = 2t^3 - 5t$ $\quad v_x(t) = \frac{d}{dt}x(t) = 6t^2 - 5$

$y(t) = 6 - t^4$ $\quad v_y(t) = \frac{d}{dt}y(t) = -4t^3$

$a_x(t) = \frac{d}{dt}v_x(t) = 12t$

$a_y(t) = \frac{d}{dt}v_y(t) = -12t^2$

a) speed: $\sqrt{v_x^2 + v_y^2}$

$$v(t) = \sqrt{v_x(t)^2 + v_y(t)^2}$$

$$v(.5) = \sqrt{v_x(.5)^2 + v_y(.5)^2}$$

$$= \sqrt{(6(.5)^2 - 5)^2 + (-4(.5)^3)^2}$$

$$= \sqrt{(-3.5)^2 + (-.5)^2}$$

$$= \boxed{3.5355 \text{ m/s}}$$

b) angle of velocity with +x-axis: $\theta = \tan^{-1}\left(\frac{v_y(t)}{v_x(t)}\right)$

$$\theta(.5) = \tan^{-1}\left(\frac{v_y(.5)}{v_x(.5)}\right) = \cancel{\tan^{-1}\left(\frac{-3.5}{-.5}\right)} = \tan^{-1}\left(\frac{-.5}{-3.5}\right) = \cancel{8.1301^\circ}$$

$= 8.1301^\circ$

Bc $v_x < 0$, $v_y < 0$,
$\theta = 8.1301 + 180 = 188.1301^\circ$

c) magnitude of accel: $\sqrt{a_x^2 + a_y^2}$

$$|a(t)| = \sqrt{a_x(t)^2 + a_y(t)^2}$$

$$|a(.5)| = \sqrt{a_x(.5)^2 + a_y(.5)^2}$$

$$= \sqrt{(12(.5))^2 + (-12(.5)^2)^2}$$

$$= \sqrt{(6)^2 + (-3)^2} = \boxed{6.7082 \ \frac{m}{s^2}}$$

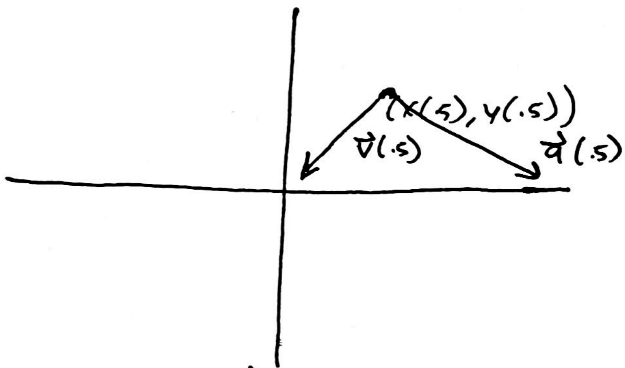
3) d) ~~Speeding up :~~

~~Acceleration is positive~~

~~acceleration is positive when velocity is positive~~
~~acceleration is negative when velocity is negative.~~

~~v(.5) =~~ Notice diagram

$\boxed{\text{neither}}$



Because the acceleration vector is not directly opposite of the velocity vector, it doesn't show slowing down. It also isn't aligned with the velocity vector, so not speeding up. So I conclude neither.

4) ~~$\theta(t) = \tan^{-1}\left(\frac{v_y(t)}{v_x(t)}\right)$~~

$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

~~$v_y(t) = v_0 \sin\theta$ $x_0(t)$~~

$$\theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right)$$

$a_x = 0.$ $\qquad a_y = -g = -9.8.$

$v_x = v_{0x}$ $\qquad v_y = v_{0y} + a_y t = v_{0y} + -9.8t$

$x = v_{0x} t + x_0$ $\qquad y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$

$= y_0 + v_{0y} t + -4.9t^2$

So I know $0^\circ \le \theta \le 90^\circ$.