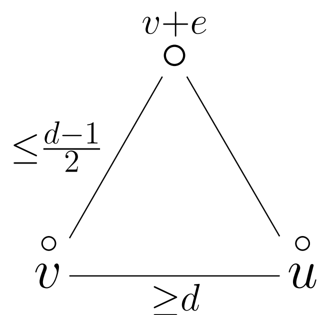
Slide #1.
Assume that d is odd so that $\lfloor\frac{d-1}{2}\rfloor = \frac{d-1}{2}$.



In order to prove that the code corrects all error patterns of weight $\leq \frac{d-1}{2}$, we must show that

$$d(v, v{+}e) < d(v{+}e, u) \quad \text{for any } e \text{ such that } \mathrm{wt}(e) \leq \frac{d-1}{2}.$$

Key idea. Apply the triangle inequality to estimate $d(v+e, u)$:

$$\begin{aligned} d(v+e, u) &\geq d(v, u) - d(v, v+e) \\ &\geq d - \frac{d-1}{2} \\ &= \frac{d+1}{2}. \end{aligned}$$

- Now we show that there exists at least one error pattern of weight equal to $\frac{d-1}{2} + 1 = \frac{d+1}{2}$ which the code does not correct.
- Let v and u be codewords such that $d(v,u) = d$. Let $e' = v + u$, so $\mathrm{wt}(e') = \mathrm{wt}(v+u) = d$.
- If d is odd, change $\frac{d-1}{2}$ of the 1s in e into 0s. Denote the obtained word by e. Note that $\mathrm{wt}(e) = d - \frac{d-1}{2} = \frac{d+1}{2}$.
- **Remark**: If d is even, we change $\frac{d}{2}$ of the 1s in e into 0s.
- **Claim**: The code does not correct e. Indeed,

  $$d(v, v+e) = \mathrm{wt}(v+v+e) = \mathrm{wt}(e) = \frac{d+1}{2}.$$

  On the other hand,

  $$d(v+e, u) = \mathrm{wt}(u+v+e) = \mathrm{wt}(e'+e) = \frac{d-1}{2}.$$

  Thus, $d(v, v+e) > d(v+e, u)$.