Math-538

Chap 1: ONE DIM. MAPS

x_0: Initial pop. (1 u)
- cycle, tic of clock: 1 day

$x_0 = 1$.

1 day: $x_1 = f(x_0) = 2 \cdot x_0 = 2 \cdot 1 = 2$
2 days: $x_2 = f(x_1) = 2 \cdot x_1 = 2 \cdot 2 = 4$
n-th day: $x_{n+1} = f(x_n) = 2x_n$
recursively: $x_n = 2^n \cdot x_0$
? what happens @ $n \to \infty$?

$x_0 > 0$: $\to$ $x_\infty = \lim_{n\to\infty} x_n = \infty$

Def: Deterministic Dyn. Syst.
→ deterministic rule that prescribes the future, from a set of possible states, from the past.

Deterministic: future is UNIQUELY determined by past.

- Random: we will not consider → random/stochastic Dyn. Syst.
- Discrete time Dyn. Syst.
- continuous time: Physics, classical Mech → Newton's law

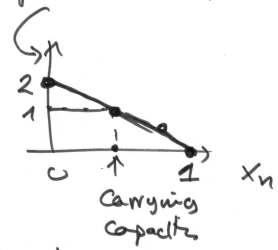
1.1 1D Maps

Notation:
$$x_{n+1} = f(x_n) = f(f(x_{n-1})) = \vdots = f(f\cdots(x_0)) = f^{n+1}(x_0)$$
$$\therefore \quad x_n = f^n(x_0)$$
(x_0: Initial condition (IC))

- Improve Sect. model.
$$x_{n+1} = 2x_n$$
(2: growth rate)

- logistic or Malthusian growth.

grow rate = grow rate (x_n)



$f(x)$
↓
$g(x) = \underbrace{2(1-x)}_{\text{growth}} x$

Summary:
if $x_0 \in (0,1) \Rightarrow x_\infty \to 1/2$
if $x_0 \in \{0,1\} \Rightarrow x_\infty = 0$
if $x_0 \in (-\infty,0) \cup (1,+\infty) \Rightarrow x_\infty = -\infty$

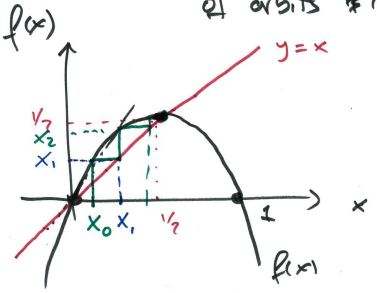
- growth rate @ $x=0$ is "a"

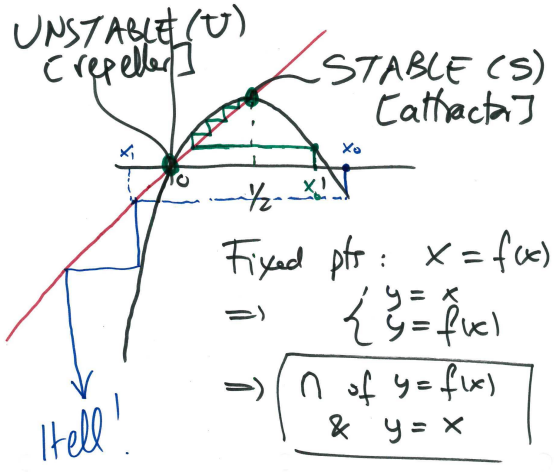
$\Rightarrow$ $\boxed{g_a(x) = ax(1-x)}$ LOGISTIC MAP

Def: FIXED PT: x^*
$$x^* = f(x^*)$$
to Find f. pts. you solve:
$$\boxed{f(x) = x}$$
For our example $a = 2$
$$2x(1-x) = x$$
$x_1^* = 0$ is f. pt.

If $x \neq x_1^* \Rightarrow 2(1-x) = 1$
$\Rightarrow 1 - x = 1/2 \Rightarrow x = 1 - 1/2$
$\therefore$ $x_2^* = 1/2$ is f. pt

1.2 Cobweb plot: graph. representation of orbits in 1D maps

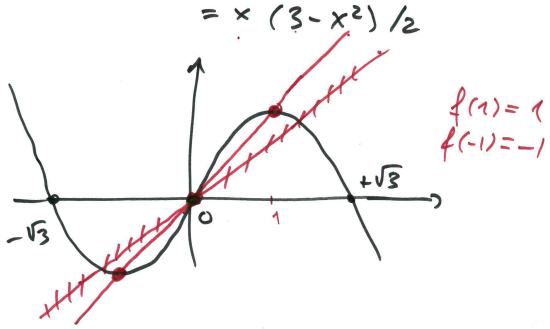




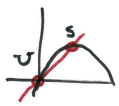
Fixed pts: $x = f(x)$
$$\Rightarrow \begin{cases} y = x \\ y = f(x) \end{cases}$$
$\Rightarrow$ $\boxed{\cap \text{ of } y = f(x) \text{ \& } y = x}$

Power of Cobweb:

Ex: $f(x) = (3x - x^3)/2$
$= x(3 - x^2)/2$

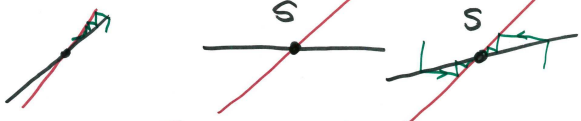


Local Stability

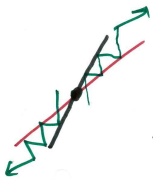


$p = f(p)$

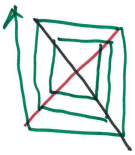
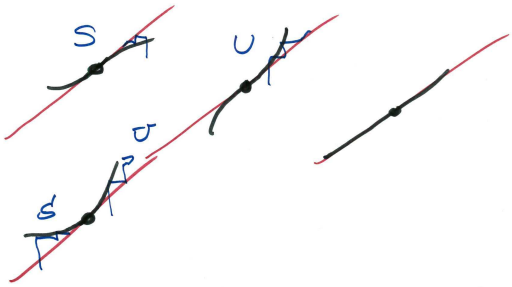
Zoom: S



U



$\rightarrow f'(x^*) < 1$
$\Rightarrow$ S

$\rightarrow f'(x^*) > 1$
$\Rightarrow$ U

$f'(x^*) << -1$