Sept 16, 2024

⊛ Biochemical reaction (contd.)

Furthermore, the concentration of complex changes fast so that it gives quasi-steady state (hypothesis)

$$\frac{dC}{dt} = 0 \Rightarrow k_1 S(E_0 - C) - (k_{-1} + k_2) C = 0$$

$$\Rightarrow C = \frac{k_1 S E_0}{k_{-1} + k_2 + k_1 S} = \frac{S E_0}{k_m + S}$$

where $k_m = \dfrac{k_{-1} + k_2}{k_1}$

$$\therefore \frac{dS}{dt} = -k_1 S\left(E_0 - \frac{S E_0}{k_m + S}\right) + k_{-1} \cdot \frac{S E_0}{k_m + S}$$

$$= \frac{-k_2 E_0 S}{k_m + S} = -\frac{V_m S}{k_m + S}$$

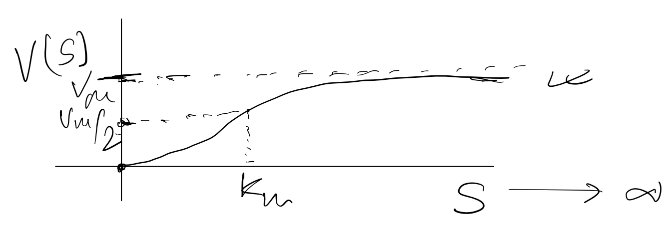
$$\therefore \boxed{\frac{dS}{dt} = -\frac{V_m S}{k_m + S}}$$

$$S \Rightarrow C \Rightarrow E, P$$

Note: $V = \dfrac{dP}{dt} = \dfrac{d}{dt}(S_0 - S - C) = -\dfrac{dS}{dt}$

$$\therefore V = \frac{V_m S}{k_m + S} \quad \text{(velocity of reaction)}$$

This is known as the Michaelis-Menten rate equation. V_m, K_m are called Michaelis-Menten constant.



(4) Basic Epidemiological Model (SI, SIS, SIR)

- SI-Model

$$\boxed{S} \xrightarrow{f(S,I)} \boxed{I}$$

$f(S,I)$: rate of new infections per unit time

$$\left.\begin{aligned} \frac{dS}{dt} &= \underbrace{-f(S,I)} \\ \frac{dI}{dt} &= f(S,I) \end{aligned}\right\} \quad N = S + I$$

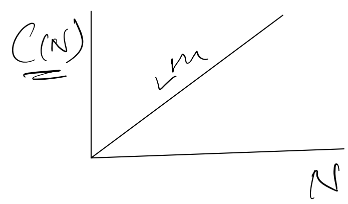
$f(S,I) = \lambda(I) S$; λ = force of infection

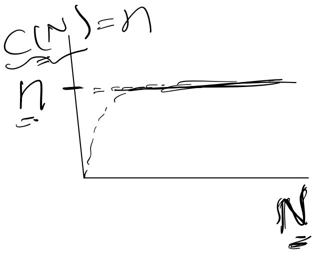
$$\lambda(I) = \underbrace{C(N)}_{\text{contact rate}} \cdot \underbrace{\frac{I}{N}} \cdot p$$

p → probability of transmission per contact

$\frac{I}{N}$ → probability that contact is with infected

$$C(N) = \begin{cases} \text{density dependent, } mN \\ \Rightarrow \lambda(I) = m\cancel{N} \cdot \frac{I}{\cancel{N}} \cdot p = \beta I \\ \text{frequency dependent, } n \\ \Rightarrow \lambda(I) = n \cdot \frac{I}{N} \cdot p = \beta \frac{I}{N}, \end{cases}$$





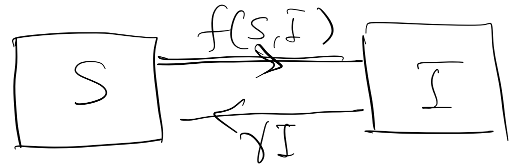
$$\Rightarrow \begin{cases} \frac{dS}{dt} = -\beta I S \\ \frac{dI}{dt} = \beta I S \end{cases}$$

$$\Rightarrow \frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} = 0 \Rightarrow N = \text{constant}$$

$$\Rightarrow \frac{dI}{dt} = \beta I(N-I) \Rightarrow \frac{dI}{dt} = \underbrace{(\beta N)}_{r} I \left(1 - \frac{I}{\underbrace{N}_{K}}\right)$$

$\Rightarrow$ Infected population follows logistic growth with growth rate $r = \beta N$ and carrying capacity $\underline{\underline{K = N}}$

- <u>SIS-Model</u>



$f(S,I)$: rate of new infection per unit time

γ: per capita rate of infected getting recovered and becoming susceptible immediately.

$$\begin{cases} \dfrac{dS}{dt} = -f(S,I) + \gamma I \\ \dfrac{dI}{dt} = f(S,I) - \gamma I \end{cases}$$

$$\begin{cases} \dfrac{dS}{dt} = -\beta S I + \gamma I \\ \dfrac{dI}{dt} = \underbrace{\beta S I - \gamma I} \end{cases}$$

$\dfrac{dN}{dt} = \dfrac{dS}{dt} + \dfrac{dI}{dt} = 0 \Rightarrow \underline{\underline{N}}$ - constant

$$\Rightarrow \frac{dI}{dt} = \beta I(N-I) - \gamma I = I\left(\beta N - \beta I - \gamma\right)$$

$$= \underbrace{(\beta N - \gamma)}_{r} I \left(1 - \frac{I}{\underbrace{N - \frac{\gamma}{\beta}}_{K}}\right)$$

$\Rightarrow$ Infected population follows logistic growth with growth rate $r = \beta N - \gamma$ and carrying capacity $K = N - \dfrac{\gamma}{\beta}$.

- SIR – Epidemic model

$$\boxed{S} \xrightarrow{f(S,I)} \boxed{I} \xrightarrow{\gamma I} \boxed{R}$$

Susciptible Infected recovered/removed

$f(S,I)$: rate of new infection per unit time

γ : per capita rate of infected getting recovered becoming immune.

$$\begin{cases} \dfrac{dS}{dt} = -\beta I S \\ \dfrac{dI}{dt} = \beta I S - \gamma I \\ \dfrac{dR}{dt} = \gamma I \end{cases}$$

$$\Rightarrow \frac{dN}{dt} = 0 \Rightarrow N = S + I + R = \text{constant}$$

$$\begin{cases} \dfrac{dS}{dt} = -\beta I S \\ \dfrac{dI}{dt} = \beta I S - \gamma I \end{cases}$$

Dealing with Mathematical Models (Ordinary Differential Equations).

Mathematical Model : Equations

Parameter Estimation
- Literature Survey
- Data Fitting.

Quantitative Solution
- Closed form solution
- Numerical Solution

Qualitative Analysis
- Equilibria Analysis
- Stability Analysis
- Bifurcation Analysis.

- Understanding
- Prediction
- Control.