MATH 525
Section 1.9 - Maximum Likelihood Decoding

September 2, 2020

Suppose the sender and the receiver agree on a code C and assume the receiver receives $w \in K^n$. The decoder can proceed as follows:

1. Complete Maximum Likelihood Decoding (CMLD): Let $v \in C$. If $d(v, w) < d(v_1, w) \forall v_1 \in C, v_1 \neq v$, then decode w as v. If there is more than one codeword closest to w, select one of them arbitrarily and conclude that it was the sent codeword.
2. Incomplete Maximum Likelihood Decoding (IMLD): Let $v \in C$. If $d(v, w) < d(v_1, w) \forall v_1 \in C, v_1 \neq v$, then decode w as v. If there is more than one codeword closest to w, request retransmission.

Recall that $w = v + e$ where w is the received word, v is the sent codeword, and e is the error pattern. Thus,

$$d(v, w) = \mathrm{wt}(v + w) = \mathrm{wt}(e).$$

In conclusion, *the decoder's strategy is to decode w into the codeword v which yields the error pattern of smallest weight.*

Example

Let $C = \{000, 001, 010, 011\}$. Construct an IMLD table for it.

Received	Error Pattern				Decode
w	w+000	w+001	w+010	w+011	v
000					
100					
010					
001					
110					
101					
011					
111					

* indicates the error pattern of smallest weight (in its row).
Remark: Should there be a tie, the decoder asks for retransmission, see Example 1.9.4, pp. 14–15.