HW3
Math 537 Ordinary Differential Equations
Due Oct 16, 2020

Student Name: _____ **ID** _____

1: [35 points] Consider the following system:

$$X' = AX, \tag{1.1}$$

where

$$A = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix} \text{ and } X = \begin{pmatrix} x \\ y \end{pmatrix}.$$

(a) [5 points] Solve for eigenvalue(s) and eigenvector(s).

(b) [10 points] Construct T using the results from problem (1a) and calculate $T^{-1}AT$.

(c) [5 points] Let $X = TY$. Show

$$Y' = (T^{-1}AT)Y. \tag{1.2}$$

Here Y is a column vector and its transpose is defined as $Y^T = (u, w)$.

(d) [10 points] Solve Eq. (1.2) for Y.

(e) [5 points] Find the solution X to Eq. (1.1).

2: [30 points] Consider the following set of differential equations:

$$\frac{dx}{dt} = y,$$

$$\frac{dy}{dt} = -\omega^2 x - by,$$

here both b and ω are real.

(a) [15 points] Find the conditions under which the system is hyperbolic.

(b) [15 points] Discuss whether the system has a saddle point.

3: [35 points] Consider the following two differential equations

$$x'' + ax' + bx = 0,$$

$$x'' + cx' + dx = 0.$$

Show that the two systems are topologically conjugate when a, b, c and d are positive.