1. (5 pts) For this problem use the Method of Undetermined Coefficients to solve this problem. In your written answer be sure to show all of the steps for obtaining your answer.

(1) Find a particular solution to the nonhomogeneous differential equation

$$y'' - 5y' = 40e^{5x} - 100x$$

$y_p =$ ______________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ ______________

(3) Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ ______________

Answer(s) submitted:

- `8xe^{5x} + 10x^2 + 4x`
- `c1 + c2e^{5x}`
- `c1 + c2e^{5x} + 8xe^{5x} + 10x^2 + 4x`

(correct)

2. (5 pts) For this problem use the Variation of Parameters method to solve this problem. In your written answer be sure to show all of the steps for obtaining your answer.

(1) Find a particular solution to the nonhomogeneous differential equation

$$y'' + 16y = 12\sec^2(4x)$$

$y_p =$ ______________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ ______________

(3) Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ ______________

Answer(s) submitted:

- `(3/4)(-1 + sin(4x)ln|tan(4x) + sec(4x)|)`
- `c1cos(4x) + c2sin(4x)`
- `c1cos(4x) + c2sin(4x) + (3/4)(-1 + sin(4x)ln(tan(4x) + sec`

(correct)

3. (5 pts) For this problem use the Variation of Parameters method to solve this problem. In your written answer be sure to show all of the steps for obtaining your answer.

(1) Find a particular solution to the nonhomogeneous differential equation

$$x^2y'' + xy' - 4y = 12x^{-2}$$

$y_p =$ ______________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ ______________

(3) Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ ______________

Answer(s) submitted:

- `(-3/(4x^2)) + (-3ln(x)/x^2)`
- `c1x^2 + c2x^{-2}`
- `c1x^2 + c2x^{-2} + (-3/(4x^2)) + (-3ln(x)/x^2)`

(correct)

4. (5 pts) This problem uses the Method of Undetermined Coefficients to analyze and find the solution. You are given the form of the particular solution that must be used to solve this problem, which allows one to solve for unknown constants in the differential equation. Subsequently, you are asked to find the general solution to this problem. In your written answer be sure to show all of the steps for obtaining your answer.

(1) Consider the following nonhomogeneous differential equation, which contains unknown constants α and β:

$$y^{''} + \alpha y^{'} + \beta y = 324x^2 - 42\sin(3x)$$

Suppose the form of the particular solution to this differential equation as prescribed by the method of undetermined coefficients satisfies:

$$y_p(x) = A_2x^2 + A_1x + A_0 + B_1x\cos(3x) + C_1x\sin(3x)$$

Determine the constants α and β.

$\alpha =$ _____
$\beta =$ _____

(2) With these constants α and β, find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ ____________________

(3) With the form of the particular solution above use the Method of Undetermined Coefficients to find the unknown coefficients, A_2, A_1, A_0, B_1, and C_1, thus, finding the general solution to the original nonhomogeneous differential equation.

$A_2 =$ _____
$A_1 =$ _____
$A_0 =$ _____
$B_1 =$ _____
$C_1 =$ _____

Answer(s) submitted:

- `1`
- `3`
- `c1cos(3x) + c2sin(3x)`
- `36`
- `0`
- `-8`
- `7`
- `0`

(correct)

5. (3 pts) This problem examines the Method of Undetermined Coefficients. Consider the following nonhomogeneous differential equation:

$$y'' + 6y' + 9y = 4xe^{-3x}\sin(3x) - 7x^2e^{-3x}$$

(1) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ ____________________

(2) In your written answer, give the form of the particular solution that you would guess when using the Method of Undetermined Coefficients.

Answer(s) submitted:

- `c1e^{-3x} + c2xe^{-3x}`

(correct)

6. (6 pts) Consider the initial value problem

$$y'' - 4y' + 4y = 32e^{-2t}, \qquad y(0) = 5, \quad y'(0) = 5.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

____________________ = ____________________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ______________________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ______________________________________

(4) In your written work be sure to show all the steps you took to obtain your answers above. Include your partial fractions decomposition (PFD) and the expression for $Y(s)$ that allows you to easily invert this to $y(t)$. (All elements need to come from our Laplace Table.)

Answer(s) submitted:

- `(s^2 - 4s + 4)Y(s) - (5s-15)`
- `32/(s+2)`

- `((5s-15) / (s-2)^2) + 32/((s+2)(s-2)^2)`
- `3e^{2t} + 3e^{2t}t + 2e^{-2t}`

(correct)

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7. (7 pts) Consider the initial value problem

$$y'' + 6y' + 25y = g(t), \qquad y(0) = 4, \quad y'(0) = 0,$$

where $g(t) = \begin{cases} 0 & \text{if } 0 \le t < 5 \\ 80e^{-3(t-5)} & \text{if } 5 \le t < \infty. \end{cases}$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

$$\underline{\hspace{5cm}} = \underline{\hspace{5cm}}$$

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ______________________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$. Use $h(t-a)$ for the Heaviside function shifted a units horizontally. (Class notes have $u_a(t) = h(t-a)$.)

$y(t) =$ ______________________________________

(4) In your written work be sure to show all the steps you took to obtain your answers above. Include your partial fractions decomposition (PFD) and the expression for $Y(s)$ that allows you to easily invert this to $y(t)$. (All elements need to come from our Laplace Table.)

Answer(s) submitted:

- `(s^2 + 6s + 25)Y(s) - (4s + 24)`
- `{80e^{-5s}}/{s+3}`
- `{4s + 24}/{(s+3)^2 + 16} + {80e^{-5s}}/{(s+3)(s^2 + 6s + 25)}`
- `4e^{-3t}cos(4t) + 3e^{-3t}sin(4t) + 5e^{-3(t-5)}h(t-5) -5e^{-3(t-5)}cos(4(t-5))h(t-5)`

(correct)

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8. (5 pts) This problem extends our Laplace Table with two new entries using the theory of Laplace transforms.

(1) The first part of this problem has you use the definition of the Laplace transform to find of te^{iat} and te^{-iat}, so you evaluate the improper integrals:

$$\mathcal{L}\left\{te^{iat}\right\} = \int_0^\infty e^{-st}te^{iat}\,dt \quad \text{and} \quad \mathcal{L}\left\{te^{-iat}\right\} = \int_0^\infty e^{-st}te^{-iat}\,dt.$$

In your written work show both your integration by parts and the evaluation of the limits of these improper integrals.

(2) You are reminded that the trig functions are readily defined by complex exponentials. In particular,

$$\sin(at) = \frac{e^{iat} - e^{-iat}}{2i} \quad \text{and} \quad \cos(at) = \frac{e^{iat} + e^{-iat}}{2}.$$

In your written work use the linearity of the integral and the Laplace transform with your results from Part a to find the following Laplace transforms (you are not performing any integration here!):

$$\mathcal{L}\{t\sin(at)\} \quad \text{and} \quad \mathcal{L}\{t\cos(at)\}$$

Find a common denominator for each of these to simplify your results for easy use in the last part of this problem.

(3) Consider the initial value problem:

$$y'' + 4y' + 20y = 32e^{-2t}\cos(4t), \qquad y(0) = 20, \quad y'(0) = 0.$$

Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you solve for $Y(s)$).

$$\underline{\hspace{5cm}} = \underline{\hspace{5cm}}$$

Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ______________________________

Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$, using the new Table entries that you created at the beginning of this problem.

$y(t) =$ ______________________________________

In your written work be sure to show all the steps you took to obtain your answers above. Include any partial fractions decomposition (PFD) and the expression for $Y(s)$ that allows you to easily invert this to $y(t)$. (All elements need to come from our Laplace Table or the elements created earlier in this problem.)

Answer(s) submitted:

- `(s^2 + 4s + 20)Y(s) - (20s + 80)`
- `{32(s+2)}/{(s+2)^2 + 16}`
- `{20(s+2)}/{(s+2)^2 + 16} + {40}/{(s+2)^2 + 16} + {32(s+2)}`
- `20e^{-2t}cos(4t) + 10e^{-2t}sin(4t) + 4te^{-2t}sin(4t)`

(correct)