PROBLEM SET 1

NO DUE DATE

Problem 1. Exercises 1.2.1, 1.2.2, 1.2.3 on p. 3.

Problem 2. Exercises 1.3.4, 1.3.5, 1.3.6, 1.3.7, 1.3.8, 1.4.1 on pp. 5–6.

Problem 3. Let C be the code of all words of length 3. Can this code detect any errors?

Problem 4. Let C be code

$$C = \{00000, 00100, 00101, 10010, 00110\}.$$

Determine whether C detects the error patterns: (a) 01000, (b) 00001, and (c) 10001.

Problem 5. For any $n \in \mathbb{N}$, let C be the code obtained by adding a parity-check digit to K^n. Which types of error patterns can C detect? As in the textbook, $K = \{0, 1\}$.

Problem 6. Let C be the code of all words of length 4 having an even number of ones.

(a) If the word 0010 is received, what codeword(s) was(were) most likely sent?
(b) If the word 0110 is received, what codeword(s) was(were) most likely sent?

Problem 7. Exercises 1.6.2 and 1.6.5—1.6.9 on pp. 8–10.

Problem 8. Let $u, v \in K^n$. Prove that $\mathrm{wt}(u+v) \leq \mathrm{wt}(u) + \mathrm{wt}(v)$. *Hint*: Use the triangle inequality.

Problem 9.

(a) Let $u = (u_1, \dots, u_n)$ and $v = (v_1, \dots, v_n)$ be vectors in K^n. Define

$$u * v := (u_1 v_1, \dots, u_n v_n),$$

and then prove that

$$\mathrm{wt}(u+v) = \mathrm{wt}(u) + \mathrm{wt}(v) - 2 \cdot \mathrm{wt}(u * v).$$

Hint: By direct inspection, prove the assertion assuming $n = 1$. Then carefully generalize it for any n.

(b) Conclude that $\mathrm{wt}(u+v)$ is even if and only if either both $\mathrm{wt}(u)$ and $\mathrm{wt}(v)$ are even or both $\mathrm{wt}(u)$ and $\mathrm{wt}(v)$ are odd.

Problem 10. Read Exercises 1.7.1–1.7.3 on p. 11.

Problem 11. Exercises 1.8.1–1.8.2 on p. 12.