## SOLUTIONS/HINTS TO PROBLEM SET 6

**Problem 1.** Exercise 3.3.3: Since the generator matrix was required to be in standard form, we use define the parity-check matrix $H$ as

$$H = \left[\begin{array}{cccc} 0&0&1&1\\ 0&1&0&1\\ 0&1&1&0\\ 0&1&1&1\\ 1&0&0&1\\ 1&0&1&0\\ 1&0&1&1\\ 1&1&0&0\\ 1&1&0&1\\ 1&1&1&0\\ 1&1&1&1 \\ \hline 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{array}\right]$$

Then

$$G = \left[\begin{array}{cccccccccccc|cccc} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1\\ 0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&1&1&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&0&1&1&1\\ 0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&1\\ 0&0&0&0&0&1&0&0&0&0&0&0&1&0&1&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&1&0&1&1\\ 0&0&0&0&0&0&0&1&0&0&0&0&1&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&1&1&0&1\\ 0&0&0&0&0&0&0&0&0&1&0&0&1&1&1&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1 \end{array}\right].$$

The message $u = 11111100000$ is encoded as $u \cdot G = 111111000000100$.

Exercise 3.3.8: No, since $H$ has two identical rows.

Exercise 3.3.9: The generator matrix of the code is $G = [111]$. The code generated by $G$ is the repetition code of length 3.

**Problem 3.** Exercise 3.4.3

$$H = \left[\begin{array}{ccc|c} 0&1&1&1\\ 1&0&1&1\\ 1&1&0&1\\ 1&1&1&1\\ 1&0&0&1\\ 0&1&0&1\\ 0&0&1&1 \\ \hline 0&0&0&1 \end{array}\right], \qquad G = \left[\begin{array}{ccccccc|c} 1&0&0&0&0&1&1&1\\ 0&1&0&0&1&0&1&1\\ 0&0&1&0&1&1&0&1\\ 0&0&0&1&1&1&1&0 \end{array}\right].$$

Exercise 3.4.4 The SDA must have $2^4$ coset leaders.

Exercise 3.4.5 Let $\boldsymbol{g}_i$ denote the $i$th row of $G$ for $i = 1, 2, 3, 4$ (as above). Then $\boldsymbol{g}_i \cdot \boldsymbol{g}_j = 0$ for all $i, j \in \{1, 2, 3, 4\}$. This means that $C$ is contained in $C^\perp$. However, since $|C| = |C^\perp| = 2^4$, we conclude that $C = C^\perp$.

Exercise 3.4.6 See Section 3.4, slide #4.

Exercise 3.4.7 Answered in the textbook.