Quiz-4
Math 537 Ordinary Differential Equations
Due 9:00AM Wednesday, October 21, 2020

Student Name: _____ **ID** _____

Goal: (1) Understand the relationship between the Logistic (differential) equation and the Logistic map; (2) Understand how higher derivative tests may help analyze the stability of critical points.

Total points: 50

1: [25 points] Consider the Logistic equation:

$$\frac{dX}{dt} = rX(1-X). \tag{1.1}$$

(a) Assume a time step $\triangle t$ and apply the Euler method to derive a discrete equation where X_{n+1} can be computed from X_n.

(b) Introduce a new variable Y and transform the above discrete equation into the following equation:

$$Y_{n+1} = \rho Y_n(1-Y_n). \tag{1.2}$$

Express Y_n in terms of X_n and find ρ.

Eq. (1.2) is called the Logistic map that possesses chaotic solutions for large values of ρ.

2: [25 points] Consider the general first-order ODE:

$$x' = f(x). \tag{2}$$

When both f and f' are zero at the critical point, the stability is determined by the sign of the first non-vanishing higher derivatives. Apply Taylor series expansions and provide simple functions $f(x)$ to illustrate the following:

(a) If the first non-vanishing higher derivative is even (e.g., f"), the point is a saddle point, attracting on one side but repelling on the other.

(b) If that derivative is odd, it follows the same sign rules as f'.