HW 6, Math 330

Due Tuesday, December 10

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable at $x_0 = 0$. Let $a, c \in \mathbb{R}\backslash\{0\}$. Prove that
$$\lim_{x\to 0} \frac{f(ax) - f(0)}{cx} = \frac{a}{c} f'(0).$$

2. Suppose that $h : \mathbb{R} \to \mathbb{R}$ is bounded – notice you are not assuming $h$ is differentiable. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 h(x)$. Prove that $f'(0) = 0$.

3. Prove that there is exactly one solution to the equation $x^3 + 2x^2 - 10 = 0$ on the interval (1,2).

   Review Problems

4. Suppose that $D$ is dense in $\mathbb{R}$ and that $f : \mathbb{R} \to \mathbb{R}$ is continuous. Prove that if $\forall x \in D$ we have $f(x) = 10$, then $\forall x \in \mathbb{R}$ we have $f(x) = 10$.

5. Prove that the function $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = \dfrac{12}{x^2 + 2}$ attains a maximum value and does not attain a minimum value.

6. Use the $\epsilon - \delta$ definition of limit convergence to show that
$$\lim_{x\to 1} \frac{3x - 4}{x - 2} = 1$$