First Day Assignment, Math 330

Name:

Please turn this in on Tuesday, September 3. Feel free to work with others on this and other homework assignments.

1. Compute the following. Think about the rules you use in the computations – do you know *why* those rules work?

    (a) $\lim\limits_{x \to 2} \dfrac{x^2 + 4x}{2x}$

    (b) $\lim\limits_{n \to \infty} \dfrac{\sin(n)}{n}$

    (c) $\frac{d}{dx}[e^{3x}(x^2 + 1)]$

    (d) $\frac{d}{dx}\left[\sqrt{\dfrac{4x}{x^2 + 1}}\right]$

2. Write the negation of the following statements. Prove that the statements below here are false by showing the negation is true.

    (a) For all $n \in \mathbb{N}$ we have $3n - 7 > 0$ or $n \le 1$.

    (b) For all $n \in \mathbb{N}$, if n is odd, then $2n$ is even and n^2 is even.

3. Prove the following statements.

    (a) For all $n \in \mathbb{Z}$, if n is even, then n^2 is even.

    (b) For all $n \in \mathbb{Z}$, if n^2 is even, then n is even.

    (c) For all $\epsilon > 0$ and $x \in \mathbb{R}$, if $|x - 1| < \frac{\epsilon}{3}$, then $|(3x + 4) - 7| < \epsilon$.

    (d) (Induction) For all $n \in \mathbb{N}$,

    $$\sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6}$$