HW6
Math 537 Ordinary Differential Equations
Due Dec 04, 2020

Student Name: _____ **ID** _____

1: [15 points] Compute the Picard iterations for the initial value problem:

$$\frac{dy}{dt} = ay, \quad y(t=0) = 1.$$

2: [15 points] Consider the following second-order homogeneous nonlinear differential equation:

$$\frac{d^2X}{dt^2} + h(X, \frac{dX}{dt}) + g(X) = 0.$$

Let $E = \frac{1}{2}\left(\frac{dX}{dt}\right)^2 + \int g(X)dX$.

(a) [5 points] Show that $\frac{dE}{dt} = -h\frac{dX}{dt}$.

(b) [10 points] Consider the Van der Pol equation:

$$\frac{d^2X}{dt^2} + \mu(X^2 - 1)\frac{dX}{dt} + X = 0.$$

Discuss the conditions under which $\frac{dE}{dt}$ is positive (and negative).

3: [35 points] Consider the following second-order differential equation

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = y,$$

which has an irregular singular point at ∞. Apply the substitution $y = e^{S(x)}$ to show that the leading behavior of $y(x)$ is given by

$$y(x) \sim cx^{-1/4}e^{2x^{1/2}}, \quad x \to +\infty,$$

here c is a constant.

4: [35 points] Consider a boundary-layer problem with the following second-order linear differential equation:

$$\epsilon \frac{d^2y}{dx^2} + (1+\epsilon)\frac{dy}{dx} + y = 0,$$

$$y(0) = 0 \text{ and } y(1) = 1.$$

(a) [10 points] Solve for the exact solution.

(b) [10 points] Plot the solution for $\epsilon = 0.01, 0.05$ and 0.1. [plots using computer coldes are preferred.]

(c) [15 points] Determine the inner and outer limit of the solution.

5: [35 points] This part is for students who are working on their mini projects. Provide a brief report on the progress of your mini project. [Points will be given after the completion of the final project report.]

(a) Document the sections of Introduction and Methodology (e.g., numerical schemes, data sets, etc.).

(b) Provide at least one figure as (part of) preliminary results.

(c) Please document any scientific, mathematical, numerical, computing, and/or technical issues.

(d) If you have a revised plan or a mitigation plan (to resolve the issues in (5c)), please state it.