
MATH 537

Ordinary Differential Equations

Fall 2020

Lecture #1a

Instructor: Dr. Bo-Wen Shen

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URL: <http://bwshen.sdsu.edu>

Department of Mathematics and Statistics
San Diego State University

Outline

1. Zoom basic features
2. Introduction
 - my professional experiences
 - a personal view on education
3. Syllabus
 - information for textbook and homework/exams
 - a big picture of course materials
 - grading Policy
 - HWs, quizzes, exams
4. An illustration for a Quiz on GradeScope

Join the Meeting with Zoom

- The information for joining a meeting for math252 is provided below:

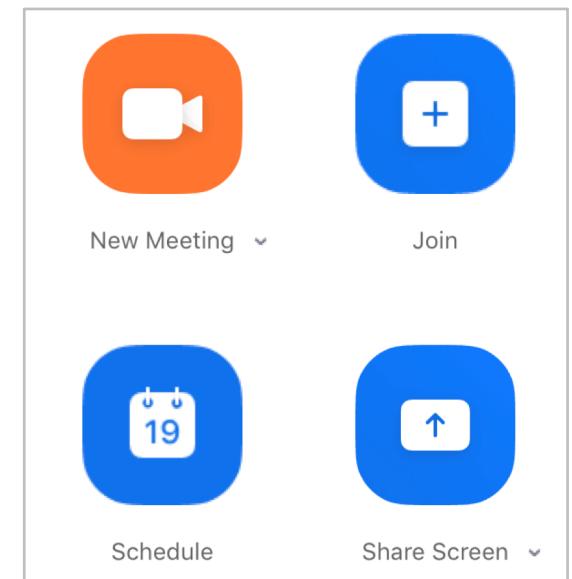
<https://SDSU.zoom.us/j/8718196242>

Meeting ID: 871 819 6242

- You may click on the above link,

or

- You can click on “Join” and type the above ID.



While at the Zoom Meeting

After joining the meeting, you will see the following:

- a large window showing the power point slide, and
 - a small window(bottom right) showing what my computer camera sees (e.g., a white board or another screen)

During the online Lectures.....

Unless stated otherwise, we use the following terms to refer the two different windows:

1. Power point window (or simply power point **slides**)
2. Camera window (or **whiteboard** window)

In some special conditions, I may share the entire desktop, which will be referred to as:

3. Desktop window (or simply my desktop)

Student Tips for Participating in Online Learning



Helpful tips for students success online

- Make sure you're muted when not talking
- Be yourself and respect others
- Ask questions using chat
- Use reactions to engage with your class
- Think before you write
- Utilize the raise your hand feature if wanting to ask a question live
- Set up an intentional space where the class is going to happen

zoom

Student Tips for Participating in Online Learning | March 2020

Zoom Attendee Features: “Participants”

Please do the following:

- click on “Participants”
- then click on “yes”

The screenshot shows the Zoom control bar at the bottom of the screen. The 'Participants' button is highlighted with a green box. The main window displays the 'Participants (2)' list. Each participant has a small profile icon, the name, and two red control icons. Below the list is a row of interactive buttons: 'raise hand' (blue hand icon), 'yes' (green circle with checkmark, highlighted with a green box), 'no' (red X icon), 'go slower' (left arrow icon), 'go faster' (right arrow icon), and 'more' (three dots icon). At the bottom right is a red 'End' button. The 'Participants' section shows two entries: 'Bo-Wen Shen (me)' and 'Bo-Wen Shen (Host)'. Both have the same profile icon and red control icons.

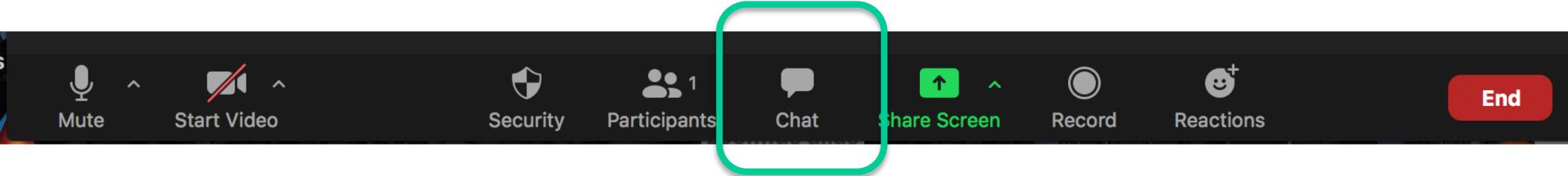
Participant	Profile Icon	Control Icons
Bo-Wen Shen (me)	BS	[red]
Bo-Wen Shen (Host)	BS	[red]

Zoom Attendee Features: “Chat”



Please do the following:

1. Switch to the window showing my whiteboard
2. Find the number and send it via "chat"



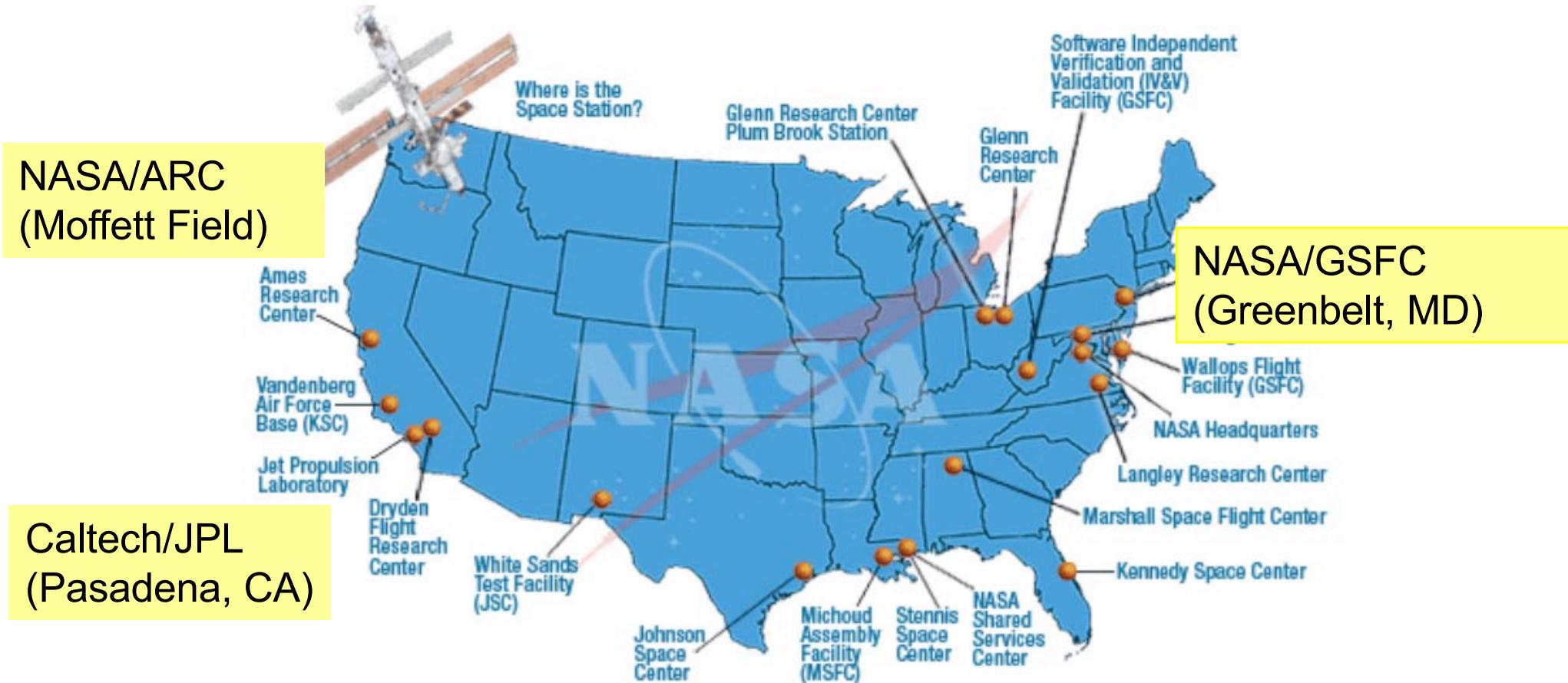
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4. A brief review for elementary ODEs

Employment

- 2014/Aug - Present: Associate Professor, SDSU
- 2012-2014: Associate Research Scientist, UMCP and NASA/GSFC
- 2006-2012: Assistant Research Scientist, UMCP and NASA/GSFC
- 1999-2006: Senior Software Engineer (Research Scientist), Science Application International Corporation (SAIC) and NASA/GSFC
- 1998-1999: Research Scientist, North Carolina State University (NCSU)
- 1995-1998: Research Assistant (part time), NCSU
- 1994-1995: Research Assistant, National Central University (NCU), Taiwan
- 1992-1994: Meteorological Officer (Unix System Application Developer and Administrator, military service), Weather Center of Weather Wing, Taiwan
- 1990-1992: Teaching and Research Assistant (part time), NCU, Taiwan

NASA Centers



GSFC: Goddard Space Flight Center

ARC: Ames Research Center

JPL: Jet Propulsion Laboratory

Dream, Hope, and Reality

It is difficult to say what is impossible, for the **dream of yesterday is the **hope** of today and the **reality** of tomorrow.**

--- Robert H. Goddard

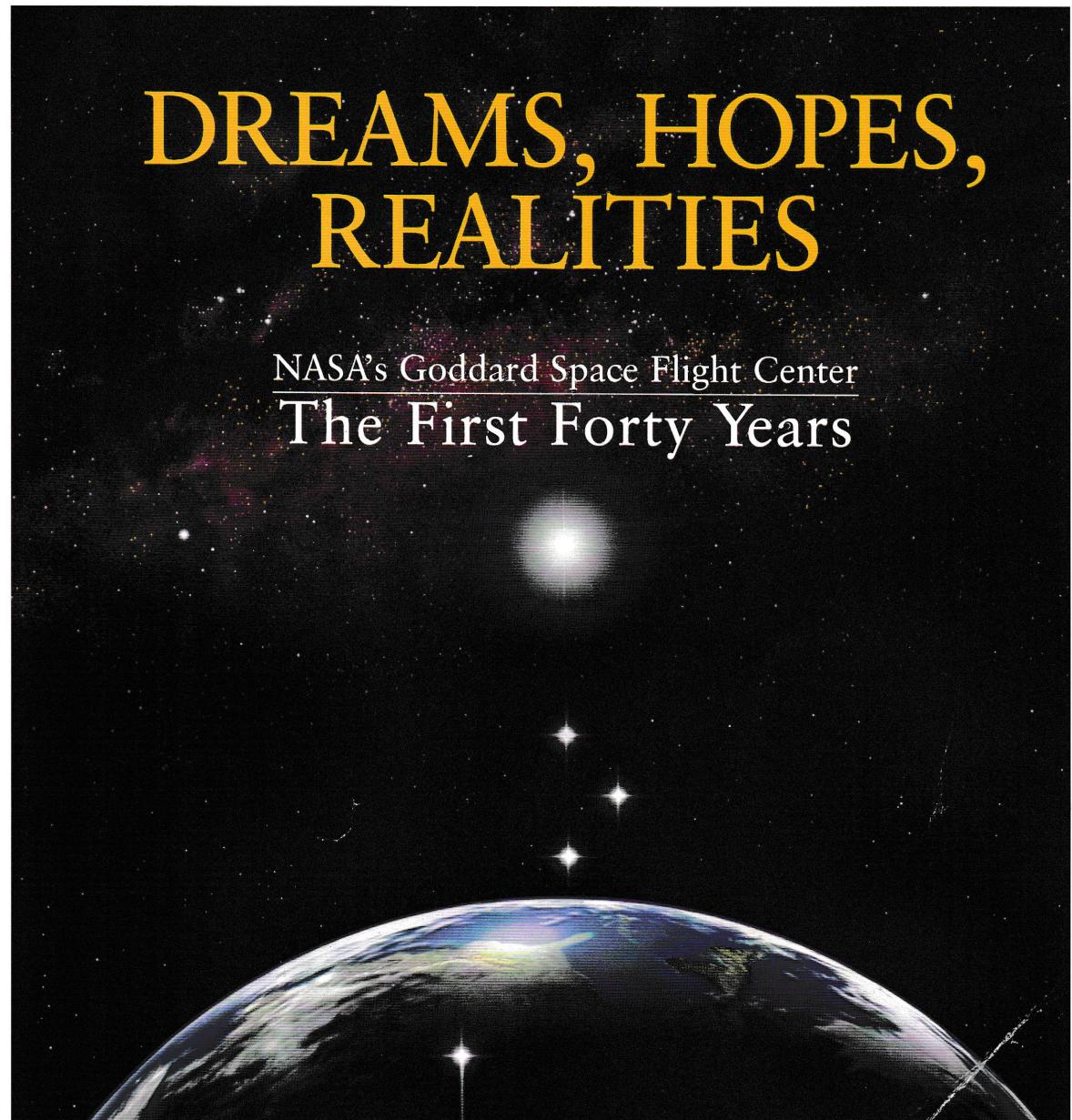
- What are the differences?
- How can one move from a current stage into the next stage?

Students' efforts and needs include:

- Hard work,
- Guidance, direction,
- Opportunities,
- Connection, etc.

A major role of a faculty:

- act as a bridge between course work and real world applications as well as jobs.



Areas of Interests

1. High-Resolution Global Climate/Weather **Modeling** to understand

- the predictability of high-impact tropical weather (e.g., Hurricanes);
- tropical cyclone (hurricane) genesis and movement;
- the impacts of physics parameterizations (e.g., cumulus parameterizations) on hurricane simulations;

2. High-end computing and Large-scale **Scientific computing** (parallel, distributed, and grid computing and parallel I/O; Unix/Linux system and network programming).

3. Numerical methods for linear and nonlinear data analysis (e.g., **Hilbert Huang Transform**, Empirical Model Decomposition);

4. Nonlinear Dynamics (e.g., **Chaos** and Mountain Meteorology)

- High performance computing
- Climate modeling,
- Weather modeling,
- Chaos/Predictability Study
- “Big Data Technology”

Petascale Computing (10^{15})

Chapter 2

(2007)

Petascale Computing: Impact on Future NASA Missions

Rupak Biswas

NASA Ames Research Center

Michael Aftosmis

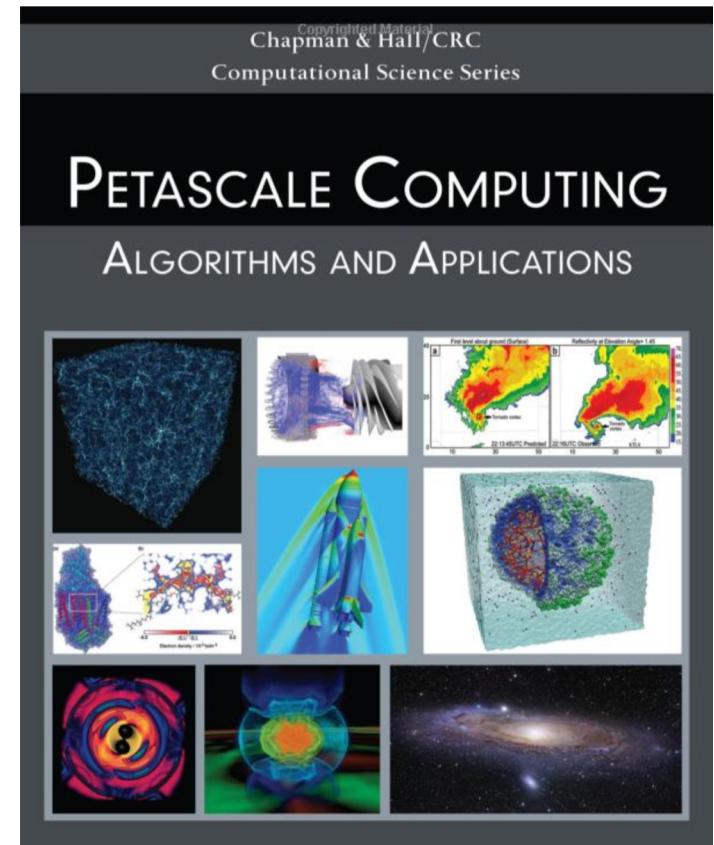
NASA Ames Research Center

Cetin Kiris

NASA Ames Research Center

Bo-Wen Shen

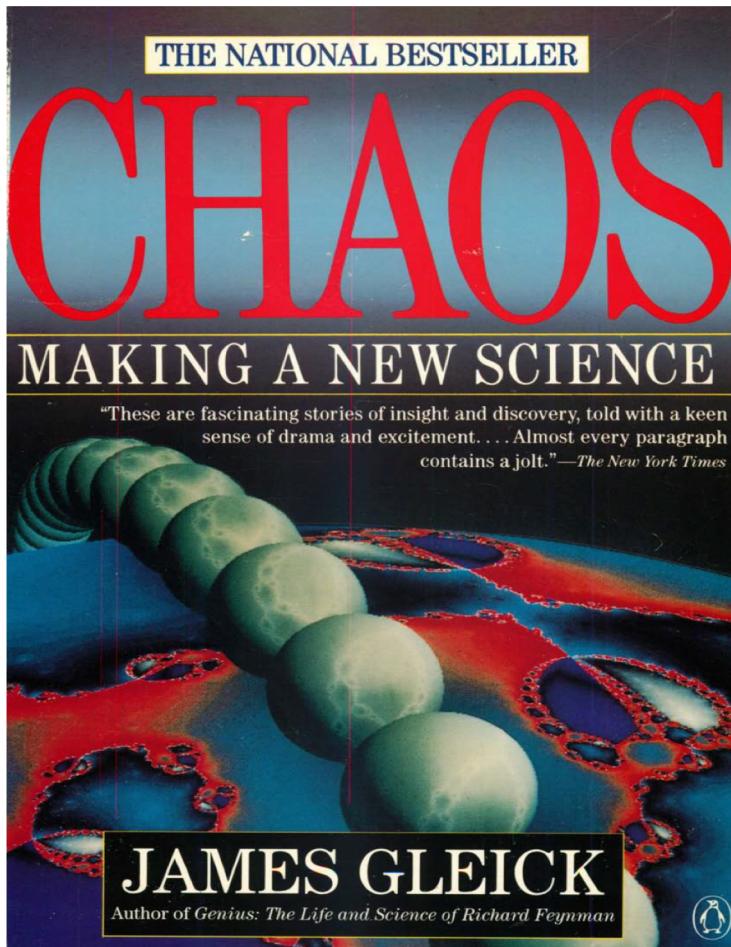
NASA Goddard Space Flight Center



Pleiades Supercomputer (as of June 2015)

- R_{\max} of 4,089 teraflops (LINPACK); R_{peak} of 4,970 teraflops
- **185,344 cores** in total; Intel Xeon processors, Nehalem, Westmere, Sandy Bridge, Ivy Bridge
- **663 TB** memory; 3.1 PB disk space; Largest InfiniBand network.

Chaos: Making a New Science



Contents

Prologue

1

The Butterfly Effect

9

Edward Lorenz and his toy weather. The computer misbehaves. Long-range forecasting is doomed. Order masquerading as randomness. A world of nonlinearity. "We completely missed the point."

Chaos, the Butterfly Effect, and Lorenz Model

The Lorenz Model and A Generalized Lorenz

The Lorenz 1963 Model
(3DLM, Lorenz, 1963)
Three Dimension

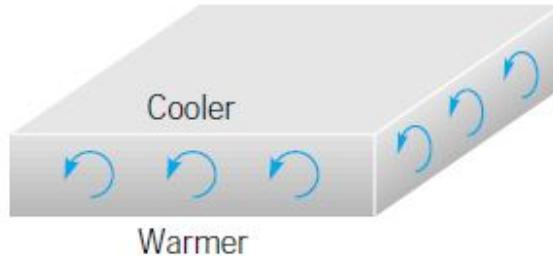
$$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$$

$$\frac{dY}{d\tau} = -XZ + rX - Y,$$

$$\frac{dZ}{d\tau} = XY - bZ.$$

primary scale modes

- σ – Prandtl number
- r – Rayleigh number
- b – Physical proportion



A Generalized Lorenz Model
(GLM, Shen, 2019a)
Any Odd Dimension

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$$

$$\frac{dY}{d\tau} = -XZ + rX - Y,$$

$$\frac{dZ}{d\tau} = XY - XY_1 - bZ,$$

an extension of the nonlinear feedback loop

$$\frac{dY_j}{d\tau} = jXZ_{j-1} - (j+1)XZ_j - d_{j-1}Y_j, \quad j \in [1, N],$$

$$\frac{dZ_j}{d\tau} = (j+1)XY_j - (j+1)XY_{j+1} - \beta_j Z_j, \quad j \in [1, N],$$

$$N = \frac{M-3}{2}; \quad d_{j-1} = \frac{(2j+1)^2 + a^2}{1+a^2}; \quad \beta_j = (j+1)^2 b.$$

International Journal of Bifurcation and Chaos

As of August 22, 2020

The screenshot shows the homepage of the International Journal of Bifurcation and Chaos on the World Scientific website. The top navigation bar includes links for Subject, Journals, Books, Major Reference Works, Partner With Us, Open Access, About Us, Current Issue, Available Issues, Search, My Cart, and Sign in. The main content area features a section for 'MOST READ ARTICLES' with six entries, each with a title, authors, volume information, and a 'Show More' link.

MOST READ ARTICLES

Article Title	Authors	Volume	Authors	Volume	Authors
HIDDEN ATTRACTORS IN DYNAMICAL SYSTEMS. FROM HIDDEN OSCILLATIONS IN HILBERT-KOLMOGOROV, AIZERMAN,	G. A. LEONOV and N. V. KUZNETSOV	Vol. 23, No. 01	Yury Kolokolov and Anna Monovskaya	Vol. 29, No. 05	Yongqing Fu, Yanan Li, Lin Zhang and Xingyuan Li
Quasi-Periodic Orbits in the Five-Dimensional Nondissipative Lorenz Model: The Role of the Extended	Sara Faghhih-Naini and Bo-Wen Shen	Vol. 28, No. 06	Bo-Wen Shen	Vol. 29, No. 03	Y. Charles Li and Hong Yang
Guess-Work and Reasonings on Centennial Evolution of Surface Air Temperature in Russia. Part V: Stability					
The DPSK Signal Noncoherent Demodulation Receiver Based on the Duffing Oscillators Array					

Show More

Is Weather Chaotic? Coexistence of Chaos and Order Within a Generalized Lorenz Model

by

Bo-Wen Shen^{1*}, Roger A. Pielke Sr.², Xubin Zeng³, Jong-Jin Baik⁴,
Tiffany Reyes¹, Sara Faghih-Naini⁵, Robert Atlas⁶, and Jialin Cui¹

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²CIRES, University of Colorado at Boulder, USA

³The University of Arizona, USA

⁴Seoul National University, South Korea

⁵Friedrich-Alexander University Erlangen-Nuremberg, Germany

⁶AOML, National Oceanic and Atmospheric Administration, USA

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Big Data, Data Assimilation and Uncertainty Quantification

Institut Henri Poincaré (IHP), Paris, France

12-15 November 2019

"A Paradigm Shift" in Predictability Study

- ``As with *Poincare* and *Birkhoff*, everything centers around *periodic solutions*'' (Lorenz, 1993).
- After Lorenz (1963, 1972), Prof. *Lorenz* and chaos advocates focused on the existence of *non-periodic solutions* and their complexities.
- Based on the concept of *attractor coexistence* within the original and generalized Lorenz models (Shen, 2019a), we (Shen et al., 2019) propose a revised view that focus on *the duality of chaos and order*.

Web Site: <http://bwshen.sdsu.edu>



PROFESSIONAL PROFILES

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⁶University of Bayreuth and Friedrich-Alexander University Erlangen-Nürnberg, Germany

⁷AMS Annual Meeting

Boston Convention and Exhibition Center, Boston, MA

12–16 January 2020



Biography

Education and Employment

Honors and Awards

Research News and Highlights

Research Publications

Research Visualizations

Teaching

*Research Interests with Selected Publications

Research Gate



Review

On the Predictability of 30-Day Global Mesoscale Simulations of African Easterly Waves during Summer 2006: A View with the Generalized Lorenz Model

Bo-Wen Shen
Department of Mathematics and Statistics, San Diego State University, 5500 Campanile Drive, San Diego, CA 92182, USA; bshen@mail.sdsu.edu
Received: 23 April 2019; Accepted: 11 June 2019; Published: 26 June 2019

Is Weather Chaotic? Coexistence of Chaos and Order Within a Generalized Lorenz Model

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Big Data, Data Assimilation and Uncertainty Quantification
Institut Henri Poincaré (IHP), Paris, France
12–15 November 2019

Coexistence of Chaotic and Non-chaotic Orbits in a New Nine-Dimensional Lorenz Model

B.-W. Shen, T. Reyes and S. Faghini-Naini

Department of Mathematics and Statistics,
San Diego State University, 5500 Campanile Drive, San Diego, CA 92182, USA
e-mail: bshen@mail.sdsu.edu
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Open Access Article 01:1281–14



A recurrence analysis of chaotic and non-chaotic solutions within a generalized nine-dimensional Lorenz model

Tiffany Reyes, Bo-Wen Shen
Department of Mathematics and Statistics, San Diego State University, 5500 Campanile Drive, San Diego, CA 92182, USA

A Generalized Lorenz Model (GLM)

The GLM is derived based on extensions of the NFL that can provide negative nonlinear feedback to stabilize solutions. The GLM is written as follows:

$$\begin{aligned} \text{primary scale modes} & \left[\begin{aligned} \frac{dX}{dt} &= -\sigma X + \sigma Y, \\ \frac{dY}{dt} &= -XZ + rX - Y, \\ \frac{dZ}{dt} &= XY - XY - bZ, \end{aligned} \right] \quad \text{3DLM} \\ & \left. \begin{aligned} \frac{dY_j}{dt} &= jXZ_{j-1} - (j+1)XZ_j - d_{j-1}Y_j, \\ \frac{dZ_j}{dt} &= (j+1)XY_j - (j+1)XY_{j+1} - \beta_j Z_j, \end{aligned} \right. \quad j \in [1, N], \end{aligned}$$

$$\begin{aligned} \text{smaller scale modes} & \left. \begin{aligned} \frac{dY_j}{dt} &= jXZ_{j-1} - (j+1)XZ_j - d_{j-1}Y_j, \\ \frac{dZ_j}{dt} &= (j+1)XY_j - (j+1)XY_{j+1} - \beta_j Z_j, \end{aligned} \right. \quad j \in [1, N], \end{aligned}$$

- The “backbone” of the linearized NFL is analogous to the spring of the above system.



PETASCALE COMPUTING ALGORITHMS AND APPLICATIONS

Parallel Implementation of the Ensemble Empirical Mode Decomposition and Its Application for Earth Science Data Analysis

Volume 10, Number 1, March 2019, pp. 1–16

DOI: 10.1142/S1793043319500001

ISSN: 1793-0433 (print) 1793-0441 (electronic)

Journal homepage: <http://www.worldscientific.com/worldscinet/petascalc>

Edited by DAVID A. BADER

Computing

On periodic solutions in the non-dissipative Lorenz model: the role of the nonlinear feedback loop

Volume 98, Number 1, March 2019, pp. 1–14

DOI: 10.1142/S0219025718500001

ISSN: 0219-0257 (print) 1793-6659 (electronic)

Journal homepage: <http://www.worldscientific.com/worldscinet/computing>

Edited by ROBERT A. PIELKE SR.

On periodic solutions in the non-dissipative Lorenz model: the role of the nonlinear feedback loop

Volume 98, Number 1, March 2019, pp. 1–14

DOI: 10.1142/S0219025718500001

ISSN: 0219-0257 (print) 1793-6659 (electronic)

Journal homepage: <http://www.worldscientific.com/worldscinet/computing>

Edited by ROBERT A. PIELKE SR.

Using High-End Computers to Model Our Climate and Weather

National Aeronautics and Space Administration

Volume 30, Number 3, March 2019, pp. 1–14

DOI: 10.1142/S0219025718500001

ISSN: 0219-0257 (print) 1793-6659 (electronic)

Journal homepage: <http://www.worldscientific.com/worldscinet/computing>

Edited by ROBERT A. PIELKE SR.

Science

Volume 355, Number 6324, February 2017, pp. 1–14

DOI: 10.1126/science.1253000

ISSN: 0278-4272 (print) 1095-9203 (electronic)

Journal homepage: <http://www.worldscientific.com/worldscinet/computing>

Edited by ROBERT A. PIELKE SR.

Hurricane Research Division

NASA/Goddard Space Flight Center and Meteorological Laboratory

Volume 355, Number 6324, February 2017, pp. 1–14

DOI: 10.1126/science.1253000

ISSN: 0278-4272 (print) 1095-9203 (electronic)

Journal homepage: <http://www.worldscientific.com/worldscinet/computing>

Edited by ROBERT A. PIELKE SR.

超級電腦與颶風預測

Volume 355, Number 6324, February 2017, pp. 1–14

DOI: 10.1126/science.1253000

ISSN: 0278-4272 (print) 1095-9203 (electronic)

Journal homepage: <http://www.worldscientific.com/worldscinet/computing>

Edited by ROBERT A. PIELKE SR.

Books Studied Recently (Since 2015)



A Personal View on Education

1. Give a man a fish and you feed him for a day;
2. Teach a man to fish and you feed him for a lifetime;

Learn skills and make efforts to earn your credits

3. Guide a man to think/plan, so he can learn new skills in the rapidly changing world.

Understand future "needs" and learn critical skills effectively

Short-term goals:

Daily life, "tips" for quizzes and exams; good grades; good GPA

Long-term goals:

higher degree; good jobs; good career; happy life;

Teaching at SDSU Since 2014

- Math151 Calculus II
- Math252 Calculus III
- Math531 Partial Differential Equations) (PDEs)
- Math537 Ordinary Differential Equations (ODEs)
- Math542 Computational ODEs (CODEs)
- Math596 High-Performance Computing for Applied Math

- Formulate
- Solve
- Interpret



- Find the hanger



-
-
- Find the hanger and holder to organize your stuffs

What? How?

Example: Chapter 14 of Calc III (1 of 2)

“Goal”:

One Formula Summary for Sections 14.4 – 14.8

- 14.1: Functions of Several Variables
 - 14.2: Limits and Continuity
 - 14.3: Partial Derivatives
- a) 14.4: Tangent Plane and Linear Approximation
 - b) 14.5: The Chain Rule
 - c) 14.6: Directional Derivatives (and the gradient vector)
 - d) 14.7: Maximum and Minimum Values
 - e) 14.8: Lagrange Multipliers

Example: Chapter 14 of Calc III (2 of 2)

One Formula Summary for Sections 14.4 – 14.8

$$(14.4) \quad df = f_x dx + f_y dy = \nabla f \cdot d\vec{r} \quad \text{total differential}$$

$$(14.5) \quad \frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt} = \nabla f \cdot \frac{d\vec{r}}{dt} \quad \begin{matrix} \text{chain rule with} \\ x = x(t) \text{ and } y = y(t) \end{matrix}$$

$$(14.6) \quad D_u f = (f_x, f_y) \cdot (a, b) = \nabla f \cdot \frac{d\vec{r}}{|d\vec{r}|} \quad \begin{matrix} \text{directional derivative} \\ \vec{u} = \frac{\Delta \vec{r}}{|\Delta \vec{r}|} \end{matrix}$$

$$(14.7) \quad \nabla f = 0 \quad \text{max and min}$$

$$(14.8) \quad \nabla f = \lambda \nabla g \quad \text{Lagrange multiplier}$$

Clothes Hanger and Holder



➤organize your stuffs

SDSU Flexible Course Design Summer Institute



Training Wheel and Bicycle Trailer

Your choice?
Why?

(A)



(B)



(C)



(D)



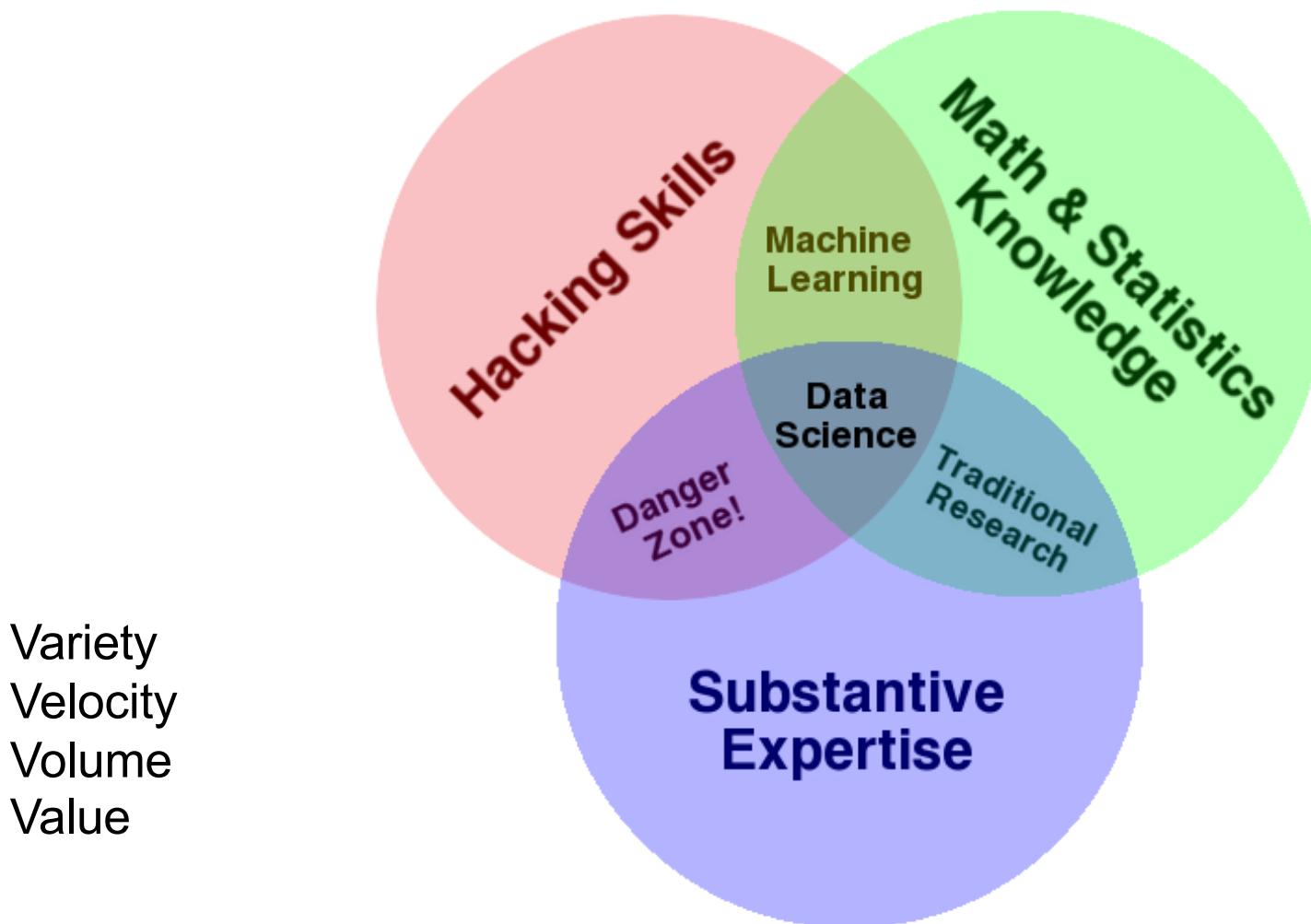
Austin Shaw-O'Leary of Falmouth celebrates a win in Amateur Sport Bike on Sunday. (Jenn Harrie photo)

A Balance

Freedom vs. Protection
Flexibility vs. Constraint

New Opportunities: Equations vs. Data

The Data Science View Diagram



<http://drewconway.com/zia/2013/3/26/the-data-science-venn-diagram>

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Syllabus

(Last updated: 2020/08/21)

Math 537: Ordinary Differential Equations

Instructor: Dr. Bo-Wen Shen

Office: GMCS 569
Ph: 619-594-5962

Email: sdsu.math537.shen@gmail.com

Days, Times and Locations

Lecture: MWF, 9:00-9:50 am

Office Hours: 1:30-2:30pm MWF or by appointment (@GMCS 569)

Text:

- Hirsch, M.W., S. Smale, and R.L. Devaney (**HSD**), 2013: Differential Equations, Dynamical Systems and an Introduction to Chaos, 3rd ed. ISBN: 0-12-349703-5 (**Chapters 1-6 & 7**)
- Bender, C. M., and S. A. Orszag, (**BO**) 2010: *Advanced Mathematical Methods for Scientists and Engineers*. Springer-Verlag, 593 pp. ISBN 978-1-4419-3187-0. (**selected sections in Chapters 3, 7, 9, and 10**).

Topics



Prerequisites: Math 337 (ODEs) with minimum grade of C

Topics covered in this course

1. First Order Equations
2. Planar Linear Systems
3. Phase Portraits for Planar Systems
4. Classification of Planar Systems
5. Higher Dimensional Linear Algebra
6. Higher Dimensional Linear Systems
7. Nonlinear Systems
8. Asymptotic Series and Local Analysis
9. Perturbation Series
10. Boundary Layer Theory
11. WKB Theory

Hirsch, Smale, and Devaney
([HSD](#))

~ 9.5 weeks

“bridge”

Bender and Orszag
([BO](#))

~4 weeks

Computing: You are encouraged to apply mathematics software, such as Matlab, R or Python for plotting.

Topics

COURSE OUTLINE

Chapter	Sections	Remarks
1: First Order Equations		~ 1.5 weeks
	1.1 The Simplest Example	
	1.2 The Logistic Population Model	
	1.3 Constant Harvesting and Bifurcations	
	1.4 Periodic Harvesting and Periodic Solutions	
	1.5 Computing the Poincaré Map	
2: Planar Linear Systems		~ 1.5 weeks
	2.1 Second-Order Differential Equations	
	2.2 Planar Systems	
	2.3 Preliminaries from Algebra	
	2.4 Planar Linear Systems	
	2.5 Eigenvalues and Eigenvectors	
	2.6 Solving Linear Systems	
	2.7 The Linearity Principle	

Topics

3: Phase Portraits for Planar Systems		~ 1 week
	3.1 Real Distinct Eigenvalues	
	3.2 Complex Eigenvalues	
	3.3 Repeated Eigenvalues	
	3.4 Changing Coordinates	
4. Classification of Planar Systems		1.5 lectures
	4.1 The Trace-Determinant Plane	
	4.2 Dynamical Classification	
5. Higher Dimensional Linear Algebra		~ 2 weeks
	5.1 Preliminaries from Linear Algebra	
	5.2 Eigenvalues and Eigenvectors	
	5.3 Complex Eigenvalues	
	5.4 Bases and Subspaces	
	5.5 Repeated Eigenvalues	
	5.6 Genericity	

6. Higher Dimensional Linear Systems		~ 1.5 weeks
	6.1 Distinct Eigenvalues	
	6.2 Harmonic Oscillators	
	6.3 Repeated Eigenvalues	
	6.4 The Exponential of a Matrix	
	6.5 Nonautonomous Linear Systems	
7. Nonlinear Systems		~ 1 week
	7.1 Dynamical Systems	
	7.2 The Existence and Uniqueness Theorem	
	7.3 Continuous Dependence of Solutions	
	7.4 Linearization and The Variational Equation	

Topics

8. Asymptotic Series and Local Analysis (Chapter 3 of Bender and Orszag)		~ 1 week
	8.1 Classification of Singular Points	
	8.2 Local Behavior Near Ordinary Points	Taylor Series
	8.3 Local Series Expansion About Regular Singular Points	Frobenius Method
	8.4 Local Behavior at Irregular Singular Points	Asymptotic series; asymptotic relations; controlling factor
	8.5 Local Analysis of Inhomogeneous Linear Eqs.	
	8.6 Asymptotic Relations (for Oscillatory Functions)	
	8.7 Asymptotic Series	

Topics

9. Perturbation Series (Chapter 7 of Bender and Orszag)		~ 1 week
	9.1 Perturbation Series	
	9.2 Regular and Singular Perturbation Theory	
	9.3 Asymptotic Matching	
10. Boundary Layer Theory (Chapter 9 of Bender and Orszag)		~ 1 week
	10.1 Introduction to Boundary Layer Theory	
	10.2 Mathematical Structure of Boundary Layer	
	10.3 Higher Order Boundary Layer Theory and Internal Boundary Layers*	
11. WKB Theory (Chapter 10 of Bender and Orszag)		~ 1 week
	11.1 Introduction to WKB Theory (WKB expansion)	
	11.2 Conditions for Validity of the WKB Approximation	
	11.3 Patched and Matched Asymptotic Approximations*	

*Optional: these lectures will be presented subject to time availability.



One Slide Summary

1 st order	2 nd order	eigenvalue problem
$y' = \alpha y - \beta y^2$ (logistic eq.)	$x'' + \beta x' + \alpha x = 0$	$x' = ax + by$ $y' = cx + dy$
$y' = \alpha y - \beta y^3$	$x' = y$ $y' = -\alpha x - \beta y$	$X' = AX$ $AX = \lambda X$ $X = \begin{pmatrix} x \\ y \end{pmatrix}; A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

nonlinear	a system of ODEs	
$x'' - \alpha x + \beta x^3 = 0$ (DE-sech)	$x' = y \equiv F$ $y' = \alpha x - \beta x^3 \equiv G$	$JX = \lambda X$ $J = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix}_{X_c}$
$x'' - \alpha x + \beta x^2 = 0$ (DE-sech ²)	$x' = y \equiv F$ $y' = \alpha x - \beta x^2 \equiv G$	

Important Concepts and/or ODEs



Asymptotic Matching	Homoclinic Orbits
Asymptotic Series	Jacobian Matrix & Linearization
A System of ODEs	Logistic Eq.
Bifurcation	Lorenz Model
Boundary Layer Theory	Perturbation Theory/Method
Critical Points	Quasi-periodicity
Conjugacy	Sigmoid function
DE-sech (e.g., Duffing Eq. & Nonlinear Schrodinger Eq.)	Solitary Wave
DE-sech ² (e.g., KdV Eq.)	Stability Analysis (source, sink, saddle)
Eigenvalue Problem	Variational Eq.
Homeomorphism	WKB Analysis (oscillatory vs. exponential solutions)

DE-sech → Non-dissipative Lorenz Model

$$x'' - \alpha x + \beta x^3 = 0$$

$$\begin{aligned}x' &= \sigma y \\ \sigma y' &= \alpha x - \beta x^3\end{aligned}$$

$$\begin{aligned}x' &= \sigma y \\ y' &= \frac{\alpha}{\sigma} x - \frac{\beta}{\sigma} x^3\end{aligned}$$

$$\begin{aligned}r &= \frac{\alpha}{\sigma} \\ x' &= \sigma y \\ y' &= rx - \frac{\beta}{\sigma} x^2 x\end{aligned}$$

$$\begin{aligned}x' &= \sigma y \\ y' &= rx - xz \\ z &= \frac{\beta}{\sigma} x^2\end{aligned}$$

$$\begin{aligned}x' &= \sigma y \\ y' &= rx - xz \\ z' &= 2\beta xy\end{aligned}$$

Non-dissipative Lorenz Model

$$\begin{aligned}x' &= \sigma y & -\sigma x \\ y' &= rx - xz & -y \\ z' &= 2\beta xy & -bz\end{aligned}$$

Lorenz Model

Grading Policy



- **Grading Policy:** The final grades for this section will be determined as follows:
 - Homework 30%
 - Quizzes 5%
 - Midterm (Sep. 30 & Oct. 2) 30%
 - Part A: take-home, due 11:59 pm Sep. 30 15%
 - Part B: exam, 9:00-9:50 am Oct. 2 15%
 - Final Exam (Dec. 11&14) 35%
 - Part A: take-home, due 11:59 pm Dec 11 15%
 - Part B: exam, 8:00-10:00 am Dec 14 20%
 - Total----- 100%
- **Assignments:** Bi-weekly will be due at 11:50 pm **Friday**
- Quiz I: Due at 9:00 am Aug 26, 2020
- HW1: Posted on Aug. 28 (Fri); Due on September 11 (Fri).

Grade

- **Course Grade:** You will be guaranteed the following grades as given by your percentage score on the homework, pop quizzes, midterms, and final exam.

A 90%

B 80%

C 70%

D 60%

A- [89%, 90%)

B+ [85%, 89%)

C+ [75%, 79%)

D+ [65%, 69%)

B [80%, 85%)

C [70%, 75%)

D [60%, 65%)

B- [79%, 80%)

C- [69%, 70%)

D- [59%, 60%)

- **Class Attendance:** The students are required to attend all the classes. The class attendance will be taken randomly in lectures. **Those who attend every lecture will receive a 2% bonus.**

Biweekly Homework (30%)

Due Dates

- HW1: Sep 11 (F)
- HW2: Sep 25 (F)
- HW3: Oct 16 (F)
- HW4: Oct 30 (F)
- HW5: Nov 13 (F)
- HW6: Dec 4 (F)

- Late submission:
 - submitted within one week after the deadline
 - a deduction of 10 points for late submission

Take-home Quizzes (5%)

- About 5~6 take-home quizzes
 - Each is worth 20-35 points
 - Students who earn 140 points will receive a full credit of 5% for “Quizzes”
-
- Quiz1: Aug 26 (W) (for testing GradeScore)
 - Quiz2: Sep 2 (W)
-
- Late submission:
 - submitted within one week after the deadline
 - a deduction of 5 points for late submission

Mid Term (30%)

- Part A: take-home problems, 15% Sep. 30 (W)
- Part B: 9:00-9:50 am, 15%, Oct 2 (F)

Students need to submit their Part-B work (to GradeScope) by 10:00 am on Oct 2.

Final Exam(30%)

- Part A: take-home problems, 15% Dec 11 (F)
- Part B: 8:00-10:00 am, 20% Dec 14 (M)

Students need to submit their Part-B work (to GradeScope) by 10:10 am on Dec 14.

— **MWF/MW Classes**

Class Meeting Start Time	Examination Date	Exam Hour
0700 MWF	Monday, Dec. 14	0600-0800
0800 MWF	Friday, Dec. 11	0800-1000
0900 MWF	Monday, Dec. 14	0800-1000

https://registrar.sdsu.edu/calendars/final_exam_schedule/fall_2020_final_exam_schedule

Important Dates

Quiz1: Aug 26 (W)
 Quiz2: Sep 2 (W)
 HW1: Sep 11 (F)
 HW2: Sep 25 (F)
Part A: Sep 30 (W)
Part B: Oct 2 (F)

Quiz: Oct 7 (W)
 HW3: Oct 16 (F)
 HW4: Oct 30 (F)
 HW5: Nov 13 (F)
 Quiz: Nov 25 (W)
 HW6: Dec 4 (F)
Part A: Dec 11 (F)
Part B: Dec 14 (M)

August						
Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					
3:○ 11:● 18:● 25:○						

September						
Su	Mo	Tu	We	Th	Fr	Sa
						1
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			
2:○ 10:● 17:● 23:○						

October						
Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
1:○ 9:● 16:● 23:○ 31:○						

November						
Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					
8:● 15:● 21:○ 30:○						

December						
Su	Mo	Tu	We	Th	Fr	Sa
						1
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		
7:● 14:● 21:○ 29:○						

GradeScope

- Create an account using your SDSU email address @gradescope
- Login into your account @gradescope

The screenshot shows the GradeScope dashboard. At the top, there's a browser header with tabs for 'Dashboard | Gradescope' and '+'. Below it is a toolbar with icons for 'Apps', 'SDSU', 'Getting Started', 'Apple Accessories...', 'Logout', 'Power & Cables -...', 'Topological mixin...', and 'Phys. Rev. E 49, 1...'. On the left, a sidebar titled 'Your Courses' displays a welcome message: 'Welcome to Gradescope! Click on one of your courses to the right, or on the Account menu below.' It lists 'Fall 2020' courses: 'Math537-Fall-2020' (Ordinary Differential Equations) with '2 assignments'. To the right, a teal button says '+ Create a new course'. A red dashed box highlights the '+ Create a new course' button.

GradeScope

- Create an account using your SDSU email address @gradescope
- Login into your account @gradescope
- Find Quiz 1 under the course entitled Math537-Fall-2020

The screenshot shows the GradeScope interface for the course Math537-Fall-2020. On the left, there's a sidebar with navigation links: gradescope, Math537-Fall-2020 (selected), Ordinary Differential Equations, Dashboard, Regrade Requests, INSTRUCTORS, and Bo-Wen Shen. The main area displays the course title Math537-Fall-2020 and Fall 2020. Below this, a table lists a single quiz entry:

NAME	STATUS	RELEASED	DUE (PDT) ▾
Quiz I	No Submission	AUG 20	4 days, 14 hours left AUG 26 AT 9:00AM LATE DUE DATE: SEP 02 AT 9:00AM

Download Quizzes/HW and Submit Your Work

Assign Questions and Pages

Quiz-1 | Assign Questions and Pages

SUBMITTED AT: AUGUST 20, 7:10 PM

Select questions and pages to indicate where your responses are located. Use **esc** to deselect all items and hold **shift** to select multiple questions.



Question Outline

Select pages to assign to Question 3.

TITLE	POINTS
1 1st-order ODEs	10.0 pts
P1 x	
2 2nd-order ODEs	10.0 pts
P2 x	
3 Euler-Cauchy Eq.	10.0 pts
P3 x	
4 Summary	5.0 pts

HW-01
Math 537 Ordinary Differential Equations
Due: 8:00AM Wednesday, August 26, 2020

Student Name: _____ ID: _____

Goal: The following problems are selected to help students review elementary differential equations.
Total points: 35
Notes: (i) Homework "Solutions" will be due date on Wednesday are selective (questions marked with a star) and the related submission is optional. (ii) Please cite references properly.

1: [10 points] Consider the following first-order ODEs:
(a) Separable ODEs;
(b) Linear ODEs;
(c) Exact ODEs;
(d) Bernoulli Equations.
Please provide one example for each of the above ODEs and discuss the corresponding solutions.

2: [10 points] Consider the following homogeneous linear 2nd-order ODEs with constant coefficients:
 $y'' + ay' + by = 0$,
(i) where a and b are constant. Please discuss three types of solutions based on the so-called characteristic equation.
(ii) Please discuss three types of solutions.

3: [10 points] Consider the following Euler-Cauchy equation:
 $x^2y'' + axy' - by = 0$,
(i) where a and b are constant.
(ii) Introduce a new independent variable (i), $x = e^t$, to convert the above Euler-Cauchy equation into a second-order ODE with constant coefficients (i.e., in the form of Eq. 2).

4: [5 points] Provide a brief summary on what has been completed for this assignment.

A Brief Summary with 4 Slides

Topics



Prerequisites: Math 337 (ODEs) with minimum grade of C

Topics covered in this course

1. First Order Equations
2. Planar Linear Systems
3. Phase Portraits for Planar Systems
4. Classification of Planar Systems
5. Higher Dimensional Linear Algebra
6. Higher Dimensional Linear Systems
7. Nonlinear Systems

Hirsch, Smale, and Devaney
([HSD](#))

~ 9.5 weeks

8. Asymptotic Series and Local Analysis
9. Perturbation Series
10. Boundary Layer Theory
11. WKB Theory

Bender and Orszag
([BO](#))

~4 weeks

Computing: You are encouraged to apply mathematics software, such as Matlab, R or Python for plotting.

Grading Policy



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 - Quizzes 5%
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 - Part B: exam, 9:00-9:50 am Oct. 2 15%
 - Final Exam (Dec. 11&14) 35%
 - Part A: take-home, due 11:59 pm Dec 11 15%
 - Part B: exam, 8:00-10:00 am Dec 14 20%
 - Total----- 100%
- **Assignments:** Bi-weekly will be due at 11:50 pm **Friday**
- Quiz I: Due at 9:00 am Aug 26, 2020
- HW1: Posted on Aug. 28 (Fri); Due on September 11 (Fri).



One Slide Summary

1 st order	2 nd order	eigenvalue problem
$y' = \alpha y - \beta y^2$ (logistic eq.)	$x'' + \beta x' + \alpha x = 0$	$x' = ax + by$ $y' = cx + dy$
$y' = \alpha y - \beta y^3$	$x' = y$ $y' = -\alpha x - \beta y$	$X' = AX$ $AX = \lambda X$ $X = \begin{pmatrix} x \\ y \end{pmatrix}; A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

nonlinear	a system of ODEs	
$x'' - \alpha x + \beta x^3 = 0$ (DE-sech)	$x' = y \equiv F$ $y' = \alpha x - \beta x^3 \equiv G$	$JX = \lambda X$ $J = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix}_{X_c}$
$x'' - \alpha x + \beta x^2 = 0$ (DE-sech ²)	$x' = y \equiv F$ $y' = \alpha x - \beta x^2 \equiv G$	

Important Concepts and/or ODEs



Asymptotic Matching	Homoclinic Orbits
Asymptotic Series	Jacobian Matrix & Linearization
A System of ODEs	Logistic Eq.
Bifurcation	Lorenz Model
Boundary Layer Theory	Perturbation Theory/Method
Critical Points	Quasi-periodicity
Conjugacy	Sigmoid function
DE-sech (e.g., Duffing Eq. & Nonlinear Schrodinger Eq.)	Solitary Wave
DE-sech ² (e.g., KdV Eq.)	Stability Analysis (source, sink, saddle)
Eigenvalue Problem	Variational Eq.
Homeomorphism	WKB Analysis (oscillatory vs. exponential solutions)

Take-home Quiz-I

Quiz-1
Math 537 Ordinary Differential Equations
Due 9:00AM Wednesday, August 26, 2020



Student Name: _____ ID _____

➤ Find the hanger

Goal: The following problems are selected to help students review fundamental ordinary differential equations.

Total points: 35

1: [10 points] Consider the following **first-order** ODEs:

- (a) Separable ODEs;
- (b) Linear ODEs;
- (c) Exact ODEs;
- (d) Bernoulli Equations.

Please provide one example for each of the above ODEs and discuss the corresponding solutions.

Take-home Quiz-I

2: [10 points] Consider the following homogeneous linear 2nd-order ODEs with constant coefficients:

$$y'' + ay' + by = 0, \quad (2)$$

where a and b are constant. Please discuss three types of solutions based on the so-called characteristic equation.

Take-home Quiz-I

3: [10 points] Consider the following Euler-Cauchy equation:

$$x^2y'' + axy' + by = 0, \quad (3)$$

where a and b are constant.

- (a) Please discuss three types of solutions.
- (b) Introduce a new independent variable (t), $x = e^t$, to convert the above Euler-Cauchy equation into a second-order ODE with constant coefficients (i.e., in the form of Eq. 2).

Take-home Quiz-I

4: [5 points] Provide a brief summary on what has been completed in this assignment.



- Find the hanger

Homework #0: Optional

Due at 11:59 PM on Sep. 11 (Friday) [HW1 is also due on the same day.]

Introduction email/video (optional)

- Introduce yourself using a video or an email with photos.
Send to sdsu.math537.shen@gmail.com
- Your major
- Your Grade of Math337
- Expected grade
- Your career goal (higher degrees)
- Efforts to be made (hours/week) to achieve your goal

Note: Will read your emails first and then provide feedbacks by Sep. 20.