Stephen Giang **math-337-mahaffy**
Assignment Exact-EU due 02/21/2020 at 03:00am PST

1. (2 pts) The following differential equation is exact.
Find a function $F(x,y)$ whose level curves are solutions to the differential equation

$$ydy - xdx = 0$$

$F(x,y) =$ ______

In your written HW create a graph showing representative level curves with several different constants, C. Include C values that are positive, negative, and zero.

Answer(s) submitted:

- `-x^2 / 2 + y^2 / 2`

(correct)

Correct Answers:

- `2 x^2 - 2 y^2`

2. (2 pts) Use the "mixed partials" check to see if the following differential equation is exact.
If it is exact find a function $F(x,y)$ whose level curves are solutions to the differential equation

$$(3x^3 - y)dx + (-(x+2y))dy = 0$$

?

$F(x,y) =$ ______

Answer(s) submitted:

- `exact`
- `3x^4/4 - xy - y^2`

(correct)

Correct Answers:

- `EXACT`
- `( 3/4) x^(4)  +-1 xy + ( -2/2)y^2`

3. (2 pts) Solve the following initial value problem.

$$y - 9\cos(t) + (t+2)\frac{dy}{dt} = 0, \qquad y(0) = 19.$$

$y(t) =$ ______

Answer(s) submitted:

- `(-9sin(t) - 38)/(-t-2)`

(correct)

Correct Answers:

- `(9*sin(t) + 2*19)/(t + 2)`

4. (2 pts) Use the "mixed partials" check to see if the following differential equation is exact.
If it is exact find a function $F(x,y)$ whose level curves are solutions to the differential equation

$$(-4xy^2 - 4y)dx + (-4x^2y - 4x)dy = 0$$

?

$F(x,y) =$ ______

Answer(s) submitted:

- `exact`
- `-4y^2x^2/2 - 4xy`

(correct)

Correct Answers:

- `EXACT`
- `-4/2 x^2y^2  + -4 x y`

5. (2 pts) Solve the following initial value problem.

$$2y^2 - 11e^{3t}\frac{dy}{dt} = 0, \qquad y(0) = 5.$$

$y(t) =$ ______

Answer(s) submitted:

- `165*exp(t)^3/(23*exp(t)^3 + 10)`

(correct)

Correct Answers:

- `(3*11*5)/(2*5*exp(-3*t) + 3*11 - 2*5)`

6. (2 pts) Use the "mixed partials" check to see if the following differential equation is exact.
If it is exact find a function $F(x,y)$ whose level curves are solutions to the differential equation

$$(-e^x\sin(y) + y)dx + (x - e^x\cos(y))dy = 0$$

?

$F(x,y) =$ ______

Answer(s) submitted:

- `exact`
- `xy - exp(x)siny`

(correct)

Correct Answers:

- `EXACT`
- `-1 e^x sin(y) + 1 x y`

7. (2 pts) A Bernoulli differential equation is one of the form

$$\frac{dy}{dx}+P(x)y=Q(x)y^n.$$

Observe that, if $n=0$ or 1, the Bernoulli equation is linear. For other values of n, the substitution $u=y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx}+(1-n)P(x)u=(1-n)Q(x).$$

Use an appropriate substitution to solve the equation

$$xy'+y=-9xy^2,$$

and find the solution that satisfies $y(1)=-3$.

$y(x)=$ ____________________.

Answer(s) submitted:

- `1/(9xlnx - x/3)`

(correct)

Correct Answers:

- `1/(9*x*ln(x)+x/-3)`

---

8. (2 pts) A Bernoulli differential equation is one of the form

$$\frac{dy}{dx}+P(x)y=Q(x)y^n \qquad (*)$$

Observe that, if $n=0$ or 1, the Bernoulli equation is linear. For other values of n, the substitution $u=y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx}+(1-n)P(x)u=(1-n)Q(x).$$

Consider the initial value problem

$$xy'+y=2xy^2, \quad y(1)=8.$$

(a) This differential equation can be written in the form $(*)$ with
$P(x)=$ _____,
$Q(x)=$ _____, and
$n=$ _____.

(b) The substitution $u=$ _____ will transform it into the linear equation
$\frac{du}{dx}+$ _____ $u=$ _____.

(c) Using the substitution in part (b), we rewrite the initial condition in terms of x and u:
$u(1)=$ _____.

(d) Now solve the linear equation in part (b). and find the solution that satisfies the initial condition in part (c).
$u(x)=$ ____________________.

(e) Finally, solve for y.
$y(x)=$ ____________________.

Answer(s) submitted:

- `1/x`
- `2`
- `2`
- `y^(-1)`
- `-1/x`
- `-2`
- `1/8`
- `-2xlnx + x/8`
- `1 / (-2xlnx + x/8)`

(correct)

Correct Answers:

- `1/x`
- `2`
- `2`
- `1/y`
- `- 1/x`
- `-2`
- `0.125`
- `-2*x*ln(x)+x/8`
- `1/(-2*x*ln(x)+x/8)`

---

9. (2 pts) A Bernoulli differential equation is one of the form

$$\frac{dy}{dx}+P(x)y=Q(x)y^n.$$

Observe that, if $n=0$ or 1, the Bernoulli equation is linear. For other values of n, the substitution $u=y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx}+(1-n)P(x)u=(1-n)Q(x).$$

Use an appropriate substitution to solve the equation

$$y'-\frac{6}{x}y=\frac{y^4}{x^3},$$

and find the solution that satisfies $y(1)=1$.

$y(x)=$ ____________________.

Answer(s) submitted:

- `(1 / ( (-3 / (16x^2) ) + (19 / 16x^-18) ) )^(1/3)`

(correct)

Correct Answers:

- `( (- 3)/(16*x^{2}) + 1.1875/x^{18} )^(- 1/3)`

10. (2 pts) Suppose $y' = f(x,y) = \dfrac{xy}{\cos(x)}$.

(1) $\dfrac{\partial f}{\partial y} =$ ______________

(2) Since the function $f(x,y)$ is $\boxed{?}$ at the point $(0,0)$, the partial derivative $\frac{\partial f}{\partial y}$ $\boxed{?}$ and is $\boxed{?}$ at and near the point $(0,0)$, the solution to $y' = f(x,y)$ $\boxed{?}$ near $y(0) = 0$

Answer(s) submitted:

- `x / cosx`
- `continuous`
- `exists`
- `continuous`
- `exists and is unique`

(correct)

Correct Answers:

- `x*cos(x)/([cos(x)]^2)`
- `continuous`
- `exists`
- `continuous`
- `exists and is unique`