---

1. (2 pts) Solve the following initial value problem. (Note this problem may use techniques from previous sections.)

$$\frac{dy}{dt} = 3t^2 y, \qquad y(0) = 8.$$

$y(t) =$ ____________________

Answer(s) submitted:

- `8e^(t^3)`

(correct)
Correct Answers:

- `8*exp(t^3)`

---

2. (2 pts) Solve the following initial value problem. (Note this problem may use techniques from previous sections.)

$$\frac{dy}{dt} = (2t+4)e^{-y}, \qquad y(0) = 16.$$

$y(t) =$ ____________________

Answer(s) submitted:

- `ln(t^2 +4t + e^16)`

(correct)
Correct Answers:

- `ln(1*t^2+4*t+exp(16))`

---

3. (2 pts) Find a solution to $\frac{dy}{dx} = xy + 9x + 5y + 45$. If necessary, use k to denote an arbitrary constant.

____

Answer(s) submitted:

- `y = ke^( (x^2 / 2) + 5x ) - 9`

(correct)
Correct Answers:

- `y = k * e^(x^2/2 + 5 x) - 9`

---

4. (2 pts) Solve the following initial value problem. (Note this problem may use techniques from previous sections.)

$$\frac{dy}{dt} = 24\cos(8t)y^2, \qquad y(0) = 6.$$

$y(t) =$ ____________________

Answer(s) submitted:

- `-1 / ( 3sin(8t) - (1/6) )`

(correct)
Correct Answers:

- `1/((1/6)-3*sin(8*t))`

---

5. (2 pts) Solve the following initial value problem. (Note this problem may use techniques from previous sections.)

$$\frac{dy}{dt} = \frac{(10-4t)}{2y}, \qquad y(0) = 4.$$

$y(t) =$ ____________________

Answer(s) submitted:

- `sqrt(10t - 2t^2 + 16)`

(correct)
Correct Answers:

- `sqrt(10*t-2*t^2+16)`

---

6. (2 pts) Solve the following initial value problem. (Note this problem may use techniques from previous sections.)

$$\frac{dy}{dt} = 3t^2 + 4, \qquad y(0) = 1.$$

$y(t) =$ ____________________

Answer(s) submitted:

- `t^3 + 4t + 1`

(correct)
Correct Answers:

- `t^3+4*t+1`

7. (2 pts) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{x}{49y}$.

(1) Find an implicit solution and put your answer in the following form:
__________ = constant.

(2) Find the equation of the solution through the point $(x,y) = (-7,1)$.
____

(3) Find the equation of the solution through the point $(x,y) = (0,-2)$. Your answer should be of the form $y = f(x)$.
____

Answer(s) submitted:

- `y^2- (x^2 / 49 )`
- `y = -x / 7`
- `y = -sqrt (4 + (x^2 / 49 ))`

(correct)

Correct Answers:

- `y^2-(x/7)^2`
- `x+7*y = 0`
- `y = -sqrt( (x/7)^2 + 4 )`

---

8. (2 pts) Solve the following initial value problem. (Note this problem may use techniques from previous sections.)

$$(1+y)\frac{dy}{dt} = 6t, \qquad y(2) = 6.$$

$y(t) =$ ____________________

Answer(s) submitted:

- `sqrt(6t^2 + 25) - 1`

(correct)

Correct Answers:

- `-1+sqrt(6*t^2+25)`

---

9. (2 pts) Find the general solution to

$$(t^2+16)y' + 2ty = t^2(t^2+16).$$

Use C to denote the arbitrary constant in your answer (and begin your answer $y =$).

____

Answer(s) submitted:

- `y = ((t^5 / 5) + (16t^3 / 3) + C) / (t^2 + 16)`

(correct)

Correct Answers:

- `y = ( t^5/5 + 16 t^3/3 + C ) / (t^2 + 16)`

---

10. (2 pts) Find the solution of the differential equation

$$(\ln(y))^7 \frac{dy}{dx} = x^7 y$$

which satisfies the initial condition $y(1) = e^2$.
$y =$ _______________.

Answer(s) submitted:

- `e^(x^8 + 2^8 - 1)^(1/8)`

(correct)

Correct Answers:

- `e^((x^8+2^8-1)^0.125)`

---

11. (2 pts) Find a family of solutions to the differential equation

$$(x^2 - xy)dx + xdy = 0$$

(To enter the answer in the form below you may have to rearrange the equation so that the constant is by itself on one side of the equation.) Then the solution in implicit form is the set of points (x,y) where

$F(x,y) =$ ____________________ = constant

Answer(s) submitted:

- `(y -x - 1) / e^x`

(correct)

Correct Answers:

- `e^(-x) (y-x-1)`

---

12. (2 pts) Find the function $y(t)$ that satisfies the differential equation

$$\frac{dy}{dt} - 2ty = -9t^2 e^{t^2}$$

and the condition $y(0) = -1$.
$y(t) =$ ____________________.

Answer(s) submitted:

- `(-3*t^3 - 1)*exp(t^2)`

(correct)

Correct Answers:

- `(-3*t^3 + -1)*2.71828182845905^(t^2)`

---

13. (2 pts) a. Denote by $L(t)$ the length of a fish (cm) at time t and assume that the fish grows according to the von Bertalanffy equation

$$\frac{dL}{dt} = k(43 - L(t)) \quad L(0) = 1.$$

If 4 months later the fish is 6 cm, then determine the value of k
$k =$ _______ months^{-1}
and the solution $L(t) =$ __________________ cm.

b. Find the length of the fish when $t = 12$.
$L(12) =$ _______ cm
Also, find the asymptotic length of the fish (as $t \to \infty$).

Limiting length = _______ cm

Answer(s) submitted:

- `ln( (6 - 43) / (-42) ) / -4`
- `-42e^( (ln( (6 - 43) / (-42) ) / -4) * -t) + 43`
- `-42e^( (ln( (6 - 43) / (-42) ) / -4) * -12) + 43`
- `43`

(correct)

Correct Answers:

- `0.031687926409786`
- `43-(43-1)*exp(-0.031687926409786*t)`
- `14.2851473922902`
- `43`

---

14. (2 pts) a. Suppose that an initially clean lake ($c(0) = 0$) with a constant volume of 1000000 m^3 has one stream flowing in at $f = 3000$ m^3/day. This stream is found to contain a pollutant at a concentration of $Q = 10$ ppb. There is another stream flowing in at $f = 3000$ m^3/day, and this stream is found to contain the same pollutant at a concentration of $Q = 8$ ppb. The lake is well-mixed and water leaves at the same rate as it flows in from the two feeding streams. Set up the differential equation for the concentration of the pollutant in the lake.

$\frac{dc}{dt} =$ ____________________

(Write this differential equation in the form $\frac{dc}{dt} = A - Bc$ Solve this differential equation.

$c(t) =$ ____________________ ppb.

b. Determine how long until the lake has a concentration of 8 ppb of the pollutant. $t =$ _______ days.

Find the limiting concentration of the pollutant.

Limiting concentration = _______

Answer(s) submitted:

- `(3000*(10+8)/10^6) - (3000*(2c)/10^6)`
- `-9e^(-6t/10^3) + 9`
- `ln( (8-9) / (-9) ) * (10^3 / -6)`
- `9`

(correct)

Correct Answers:

- `0.054-0.006*c`
- `9*(1-exp(-0.006*t))`
- `366.204096222703`
- `9`

---

15. (6 pts) Most of the Western European countries are having a dramatic decline in their growth rate to the point where their populations will actually begin to decline early in this century. Consider the case of Austria. Its population was 6.94 million in 1950, 7.46 million in 1970, and 7.71 million in 1990.

a. Use the nonautonomous Malthusian growth model given by

$$\frac{dP}{dt} = (b - at)P, \qquad P(0) = 6.94.$$

Let t be the number of years after 1950, then solve this differential equation.

$P(t) =$ ______________________,

where this solution includes the parameters a and b along with the independent variable t.

Use the population data for Austria to find the constants a and b.

$a =$ _______

$b =$ _______

b. The population for Austria was 8.13 million in 2000. Use the model above to estimate the population of Austria, then compute the percent error from the actual census data.

Predicted population in 2000 = _______ (million)

Percent Error = _______

c. When does the model predict that Austria will have its largest population (value of t) and what is that population?

Time of Max Population = _______ (years afer 1950).

Maximum population = _______ (million)

Answer(s) submitted:

- `(347*e^(-t*(a*t - 2*b)/2))/50`
- `0.0000982271655000001`
- `0.00459495364500000`
- `(347*e^0.1069637254)/50`
- `-5.000202275`
- `46.77884801`
- `7.727420395`

(correct)

Correct Answers:

- `6.94*exp(b*t-a*t^2/2)`
- `9.82271658434691E-05`
- `0.00459495364328251`
- `7.72348355055613`
- `-5.00020233018286`
- `46.7788478251001`
- `7.72742039260013`

---

16. (4 pts) a. A population of yeast is growing according to a Malthusian growth model. Suppose that it satisfies the initial value problem

$$\frac{dY}{dt} = 0.02Y, \qquad Y(0) = 4500,$$

where t is in hours. Solve this differential equation and determine how long it takes for this population to double.

$Y(t) =$ ______________________

Doubling time = _______ hr.

b. Because of competition from another organism in the broth, the yeast has dwindling supplies of food for growth. An approximate model with a time varying growth rate from this competition is given by the following:

$$\frac{dY}{dt} = (0.02 - 0.002t)Y, \qquad Y(0) = 4500.$$

Solve this differential equation.
$Y(t) =$ ________________

c. Find the maximum of this population and when this occurs.
Maximum population = ________
Time of Max Population = ________ hr.
Also, determine when the population returns to 4500.
Time when population is 4500 again = ________ hr.
You should sketch a graph for this population.

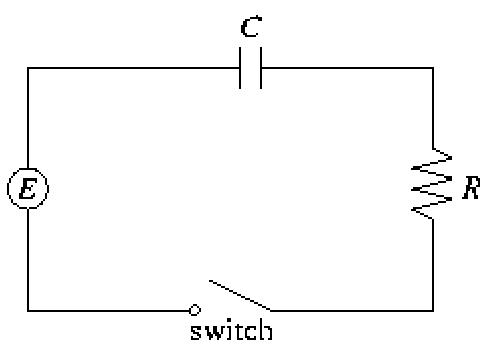
Answer(s) submitted:

- `4500e^(.02t)`
- `ln(2)/.02`
- `4500e^(.02t - (.002t^2 / 2))`
- `4973.269`
- `10`
- `20`

(correct)

Correct Answers:

- `4500*exp(0.02*t)`
- `34.6573590279973`
- `4500*exp(0.02*t-0.001*t^2)`
- `4973.26913134041`
- `10`
- `20`



17. (3 pts)

The figure above shows a circuit containing an electromotive force, a capacitor with a capacitance of C farads (F), and a resistor with a resistance of R ohms Ω. The voltage drop across the capacitor is Q/C, where Q is the charge (in coulombs), so in this case Kirchhoff's Law gives

$$RI + \frac{Q}{C} = E(t).$$

Since $I = \frac{dQ}{dt}$, we have

$$R\frac{dQ}{dt} + \frac{1}{C}Q = E(t).$$

Suppose the resistance is 20Ω, the capacitance is 0.2F, a battery gives a constant voltage of 60V, and the initial charge is $Q(0) = 0$C.

Find the charge and the current at time t.
$Q(t) =$ ________________,
$I(t) =$ ________________.

Answer(s) submitted:

- `-12e^(-t/4) + 12`
- `(-1/4)(-12e^(-t/4) + 12 - 12)`

(correct)

Correct Answers:

- `60*0.2*(1-2.71828182845905**(-t/20/0.2))`
- `60/20*2.71828182845905**(-t/20/0.2)`

---

18. (10 pts) Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

The growth of fish has been shown to satisfy a model given by the von Bertalanffy equation:

$$L(t) = L_\infty(1 - e^{-bt}),$$

where L_∞ and b are constants that fit the data. Modeling from before has shown that there is often an allometric model relating the weight and length of different animals. A model relating the weight of a fish as a function of its length is given by

$$W(L) = kL^a,$$

where k and a are constants that fit the data.

a. Below are growth data for the Albacore (*Thunnus alalunga*) [1].

Age (yr)	Length (m)	Age (yr)	Length (m)
1	0.4	6	1.09
2	0.6	7	1.13
3	0.84	8	1.15
4	0.97	9	1.16
5	1.02	10	1.17

Find the least squares best fit of the data to the von Bertalanffy equation above. Give the values of the constants L_∞ and b and write the model with these constants. Include the value of the least sum of squares error fitting the data.

$L_\infty =$ ________ m
$b =$ ________
$L(t) =$ ________________ m
$SSE =$ ________

Find the L-intercept and the horizontal asymptote for the length of the Albacore.

L-intercept = _______ m
Horizontal Asymptote $L =$ _______ m

Give the model prediction at age 4 and 10 and find the percent error at each of these ages from the actual data given:

Length at age 4 = _______ m
Percent Error at 4 = _______
Length at age 10 = _______ m
Percent Error at 10 = _______

b. In your written HW, create a graph with the data and the von Bertalanffy model for $t \in [0, 15]$. Create a short paragraph that briefly describes the rate of growth of this fish from the graph and what the maximum size of this fish can be. Include how well the model simulates the data.

c. Below are data on the length and weight for the Albacore [2].

Length (m)	Weight (kg)	Length (m)	Weight (kg)
0.55	3	0.92	15
0.62	6	1.01	21
0.71	7	1.09	26
0.77	10	1.14	34
0.86	14	1.21	39

Use MatLab to find an allometric model of the form above. Give the value of the constants k and a (to at least 5 significant figures) and write the model with these constants. Graph the data and the model.

$k =$ _______
$a =$ _______
$W(L) =$ _______________ kg
$SSE =$ _______

Give the model prediction for the weight of the Albacore with lengths 0.71 and 1.09 and find the percent error at each of these lengths from the actual data given:

Weight at length 0.71 = _______ kg
Percent Error at 0.71 = _______
Weight at length 1.09 = _______ kg
Percent Error at 1.09 = _______

Use information about maximum length of a Albacore to estimate the maximum weight that a Albacore obtains:

Maximum weight = _______ kg

d. In your written HW, create a graph with the data and the allometric model found above. Create a short paragraph that briefly describes this graph and describe how well the model simulates the data.

e. Create a composite function to give the weight of the Albacore as a function of its age, $W(t)$.

$W(t) =$ _______________ kg

Find the intercepts and any asymptotes for $W(t)$.

W-intercept = _______ kg
Horizontal Asymptote $W =$ _______ kg

Find the derivative of $W(t)$ using the chain rule. Determine the weight and rate of change in weight at $t = 7$.

$W'(t) =$ _______________ kg/yr
$W(7) =$ _______ kg
$W'(7) =$ _______ kg/yr
Also, compute the second derivative, then determine when this second derivative is zero, t_p. From this information, find at what age the Albacore are increasing their weight the most and determine what that weight gain is.

Point of Inflection, $t_p =$ _______ yr
$W(t_p) =$ _______ kg
$W'(t_p) =$ _______ kg/yr

f. In your written HW, create a graph the weight of a Albacore as it ages, $W(t)$. Also, create a graph of the derivative, $W'(t)$. Write a short paragraph describing these graphs. Include a discussion explaining the significance of the point of inflection in the first graph and how it is reflected in the second graph. Summarize your modeling efforts in this lab and briefly discuss the strengths and weaknesses of these models.

3

[1] M. G. Hinton. Status of Blue Marlin in the Pacific Ocean. Website accessed 1/04.
[2] J. H. Uchiyama and T. K. Kazama. Updated Weight-on-length relationaships for pelagic fishes caught in the central north Pacific Ocean and bottomfishes from the Northwestern Hawaiian Islands, www.nmfs.hawaii.edu/adminrpts/PIFSC Admin Rep 03-01.pdf, (accessed 1/04)

Answer(s) submitted:

- 1.204415785
- 0.384651137
- 1.204415785(1 - exp(-0.384651137 * t))
- 0.003480572
- 0
- 1.204415785
- 0.9458512396
- -2.48956293
- 1.178696566
- 0.7432962
- 21.2319
- 3.0589
- 21.2319 * (L)^3.0589
- 13.90984273
- 7.447371577
- 6.3910225
- 27.63584668
- 6.2917180
- 37.50385478
- 21.2319 * (1.204415785(1 - exp(-0.384651137 * t)))^3.0589
- 0
- 37.50385478
- 30.08829639*(1.204415785 - 1.204415785*exp(-0.384651137*t))^2.0589*exp(-0.384651137*t)
- 30.26488788
- 2.586164382
- 2.906673779
- 11.17272610
- 6.384930855

(correct)
Correct Answers:

- 1.20441282326544
- 0.384646229942635
- 1.20441282326544*(1-exp(-0.384646229942635*t))
- 0.00348057221795095
- 0
- 1.20441282326544
- 0.945843838546718
- -2.4903259230188
- 1.17869240544655
- 0.742940636456945
- 21.2319240429943
- 3.05886830457459
- 21.2319240429943*L^3.05886830457459
- 13.9104939443232
- 7.4474608548456
- 6.39229792636577
- 27.6358024882107
- 6.29154803157947
- 37.503394063913
- 21.2319240429943*(1.20441282326544*(1-exp(-0.384646229942635*t)))^3.05886830457459
- 0
- 37.503394063913
- 3.05886830457459*0.384646229942635*exp(-0.384646229942635*t)21.2319240429943*(1.20441282326544*(1-exp(-0.38464622994263
- 30.2643523562949
- 2.58615411858448
- 2.9066839225635
- 11.1725570456236
- 6.3847849258905

---

19. (10 pts) Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

For many years, lead (Pb) was an additive to paint used to reduce molds and improve adhesion. Lead was also a gas additive used to improve combustion and reduce the knocking of car engines under stress. These sources have created a major problem of lead-laden dust, especially in the inner city. Lead is problematic in the neural development of small children. Small children are exposed to lead through the dust ingested by normal hand-to-mouth play activities and from breathing the lead-laden dust. Once the lead enters the body it does not leave the body (or leaves very slowly). This lead builds up in the children's bodies and has been linked to developmental problems with their nervous system. Scientists have discovered that lead concentrations as low as 10 mg/dl in the blood results in developmental toxicity. (**Lead in Children**)

a. The exposure of lead for children begins very low (since small babies hardly move), then increase to maximum during the early years from crawling and hand-to-mouth activities. As the child increases in height, he or she moves further away from the primary contaminated source and drops many of the hand-to-mouth activities, which lowers exposure. Assume that the weighted activity that exposes a boy to lead as a function of the age, t, in years satisfies the differential equation:

$$\frac{dA}{dt} = -kA + be^{-qt}, \qquad A(0) = 0,$$

where $A(t)$ is the weighted activity time of exposure in hours per day. Suppose that the values of the parameters are $k = 0.33$ (yr^{-1}), $b = 5$ (hr/day/yr), and $q = 0.56$ (yr^{-1}). Solve this differential equation. Find the weighted activity time of exposure at ages 3, 4.5, and 11.

$A(t) =$ ______________________ .
Activity at 3, $A(3) =$ ______ hr/day.
Activity at 4.5, $A(4.5) =$ ______ hr/day.
Activity at 11, $A(11) =$ ______ hr/day.

Find the maximum level of activity exposing the boy to lead and the age at which this occurs.

Age of Maximum Activity $t_{max} =$ ______ yr.
Maximum Activity Time of Exposure $A(t_{max}) =$ ______ hr/day.

b. In your written HW, create a graph for the weighted activity time of exposure, $A(t)$, for $t \in [0, 12]$. Briefly discuss if this graph reasonably models lead exposure for young children based on your understanding of child behavior and where the lead persists.

c. The lead enters the boy's body proportional to his weighted activity time, $A(t)$, and is assumed to not leave following exposure. This suggests that the total amount of lead,

$P(t)$, in his body (in μg) will satisfy the following differential equation:

$$\frac{dP}{dt} = KA(t), \qquad P(0) = 0,$$

where $A(t)$ is the solution obtained from Part a and $K = 460$ μg day/hour of play/yr. Find the solution $P(t)$ and determine the amount of lead in the body at ages 3, 4.5, and 11.

$P(t) =$ ________________ .
Amt of Pb at 3, $P(3) =$_______ μg.
Amt of Pb at 4.5, $P(4.5) =$_______ μg.
Amt of Pb at 11, $P(11) =$_______ μg.

d. In your written HW, graph this solution for accumulation of lead in the boy for $t \in [0,12]$. Briefly discuss this graph and explain how well this differential equation models the accumulation of lead in a child.

e. Suppose that the von Bertalanffy equation of growth provides an approximation for the weight gain of a child. Assume that the boy grows according to the initial value problem,

$$\frac{dw}{dt} = r(83 - w), \qquad w(0) = 3.2,$$

where $r = 0.066$. Find the solution $w(t)$. Based on this model, determine the weight of the boy at ages 3, 4.5, and 11. What would be the maximum weight of this boy for large values of t? (Note that the equation for $w(t)$ loses accuracy significantly through the teenage years.)

$w(t) =$ ________________ .
Weight of boy at 3, $w(3) =$_______ kg.
Weight of boy at 4.5, $w(4.5) =$_______ kg.
Weight of boy at 11, $w(11) =$_______ kg.
Maximum Weight of boy, $w_{max} =$_______ kg.

e. In your written HW, graph this solution for the weight of the boy for $t \in [0,12]$. Briefly discuss this graph and explain how well this differential equation models the weight of a child.

f. Assume that this lead is uniformly distributed throughout the body. If the concentration of lead in the blood (in μg/dl), $c(t)$, satisfies the equation,

$$c(t) = \frac{0.1P(t)}{w(t)},$$

Find the concentration of lead in the blood of the boy at ages 3, 4.5, and 11. What would be the maximum concentration of lead in the blood of this boy and at what age does this occur.

Concentration of lead in boy at 3, $c(3) =$_______ μg/dl.
Concentration of lead in boy at 4.5, $c(4.5) =$_______ μg/dl.
Concentration of lead in boy at 11, $c(11) =$_______ μg/dl.
Maximum Concentration of lead in boy, $c_{max} =$_______ μg/dl.
Age of Maximum Concentration $t_{max} =$_______ yr.

g. In your written HW, graph this solution for the concentration of lead in the boy for $t \in [0,12]$. Briefly discuss this graph and explain how well this differential equation models the concentration of lead in a child. Check the website given above or any other sources and write a brief paragraph describing the health risks that the boy modeled above might encounter.

Answer(s) submitted:

- `-(500*exp(-(14*t)/25))/23 + (500*exp(-(33*t)/100))/23`
- `-(500*exp(-42/25))/23 + (500*exp(-99/100))/23`
- `3.174842042`
- `-(500*exp(-154/25))/23 + (500*exp(-363/100))/23`
- `2.299322301`
- `4.180705651`
- `-(1000000*exp(-(33*t)/100))/33 + (125000*exp(-(14*t)/25))/`
- `-(1000000*exp(-99/100))/33 + (125000*exp(-42/25))/7 + 2875`
- `7018.958848`
- `-(1000000*exp(-363/100))/33 + (125000*exp(-154/25))/7 + 28`
- `83 - (399*exp(-(33*t)/500))/5`
- `83 - (399*exp(-99/500))/5`
- `23.70508781`
- `83 - (399*exp(-363/500))/5`
- `83`
- `0.1*(-(1000000*exp(-99/100))/33 + (125000*exp(-42/25))/7 +`
- `29.60950368`
- `0.1*(-(1000000*exp(-363/100))/33 + (125000*exp(-154/25))/7`
- `30.33656218`
- `5.783690455`

(correct)

Correct Answers:

- `21.7391304347826*(exp(-0.33*t)-exp(-0.56*t))`
- `4.02614597788339`
- `3.17484204188992`
- `0.530520242112205`
- `2.29932230116813`
- `4.18070565093315`
- `10000*(exp(-0.56*t)/0.56-exp(-0.33*t)/0.33)+12445.88744588`
- `4514.09443293613`
- `7018.95884461706`
- `11680.0855146478`
- `83 + (3.2 - 83)*exp(-0.066*t)`
- `17.5344857196011`
- `23.7050878135172`
- `44.3895291798543`
- `83`
- `25.7440936969712`
- `29.6095036636594`
- `26.3127042130212`
- `30.3365621674902`
- `5.78369045442277`