

Aug 28, 2024

Chapter 1: Mathematical Modeling - Basics

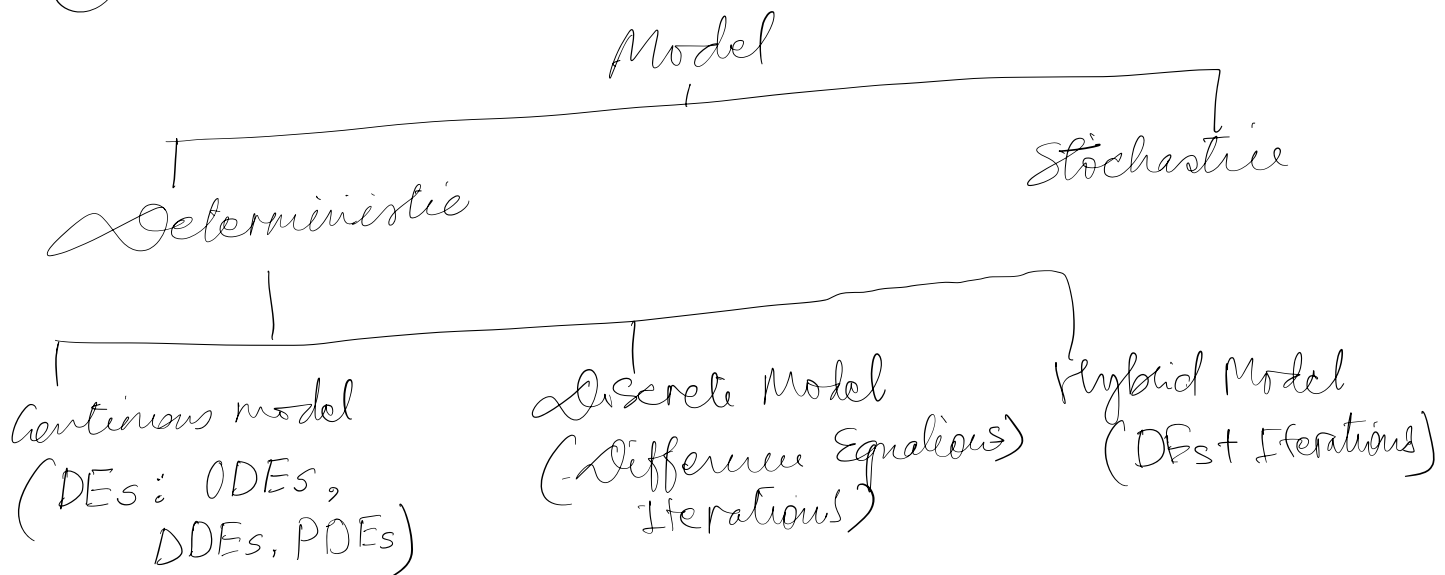
⊗ Modeling:

- No established way of teaching; difficult to teach
- learn by experience; there are no set of rules, and an understanding of the 'right' way to model.
- can be learnt by familiarities with a wealth of examples.
- highly interdisciplinary.

⊗ Model (Mathematical):

- a representative of a process (phenomena)
- not a mirror-reflection
- usually takes the form of equations
- SUCCESS \equiv the balance of mathematical feasibility and real-life reality.

⊗ Model Classification:



Purpose of Modeling:

- Explain observation (phenomena)
- Answer some specific question
- Prediction / Validation or invalidation of assumption
- Motivate mathematical theorem

(*) Formulation of model:

- Based on laws/rules well established (physical sciences)

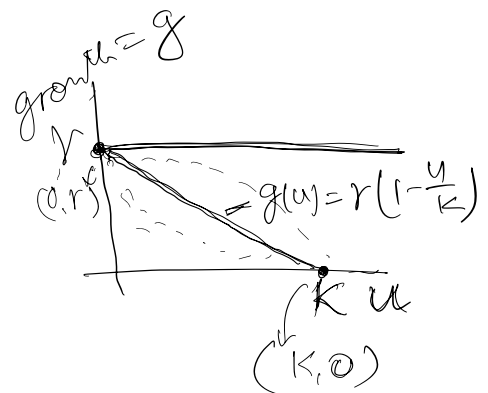
$$\boxed{\text{Newton's law: } \ddot{x} = \frac{F}{m}}$$

- Based on philosophy / phenomenological observation (assumption)

(*) Exponential model: $\frac{du}{dt} = ru$ (Bacteria growth)

(*) Logistic model:

$$\frac{du}{dt} = ru \left(1 - \frac{u}{K}\right)$$



(*) Model Validation:

- Can explain observations
- Prediction matches observation
- Data fitting (Explain the data) - with reasonable parameters
- Limitation \Rightarrow leading to new model.

⑧ Procedure of Modeling:

- Identify the problem (real world)
- Identify variables (dependent, independent)
 - important compartments.
 - dimension analysis, scaling, parameter identification
- Formulate the model
 - idealization, assumption/simplification
 - model selection
- Model analysis
 - theoretical (qualitative)
 - numerical (quantitative)
- Validation
 - comparison with experiment/observation
- Prediction/application \Rightarrow modification, development, generalization
 - [Interpretation of these mathematical results for the user community]

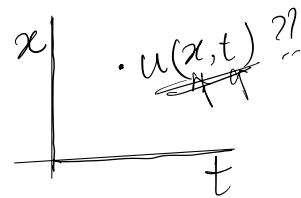
NOTE: Every model has its limitation in applicability !!!

"No model is good. Some models are useful."

Example: Traffic flow

- Determine the impact of the behaviors of drivers on the traffic flow
⇒ education, policy making according to law.
- Predict the traffic flow
⇒ { optimal design of traffic light
speed limit
highway entrance/exit.
- + Specifically, what is velocity field?

↑ x | single lane
|
| velocity at a
| location at a
| time



- Identify variables
 - density ✓
 - flow
 - speed/velocity
 - size of the road
 - weather

• Identify relationship

$$\left. \begin{aligned} \text{velocity} &= F(\text{density}) \\ \text{velocity} &= f(\text{position, time}) \end{aligned} \right\}$$