1. (2 pts) Solve the system using matrices (row operations)

$$\begin{cases} 6x - 4y + 5z = -27 \\ 5x + 4y - 2z = 18 \\ 5x - 3y + 6z = -27 \end{cases}$$

$x =$ ___
$y =$ ___
$z =$ ___

Answer(s) submitted:

- `0`
- `3`
- `-3`

(correct)

Correct Answers:

- `0`
- `3`
- `-3`

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2. (4 pts)

For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

1.

$$\begin{cases} 9x + 4y = 0 \\ -3x - 8y = 0 \end{cases}$$

- A. Unique solution: $x = 0,\ y = 0$
- B. Unique solution: $x = 8,\ y = 9$
- C. No solutions
- D. Infinitely many solutions
- E. Unique solution: $x = 13,\ y = -11$
- F. None of the above

2.

$$\begin{cases} 3x + 8y = -31 \\ -4x - 7y = 23 \end{cases}$$

- A. No solutions
- B. Unique solution: $x = -5,\ y = 3$
- C. Infinitely many solutions
- D. Unique solution: $x = 0,\ y = 0$
- E. Unique solution: $x = 3,\ y = -5$
- F. None of the above

3.

$$\begin{cases} 3x + 3y = -6 \\ -6x - 6y = 13 \end{cases}$$

- A. Unique solution: $x = -6,\ y = 13$
- B. Unique solution: $x = 13,\ y = -6$
- C. Infinitely many solutions
- D. Unique solution: $x = 0,\ y = 0$
- E. No solutions
- F. None of the above

4.

$$\begin{cases} -2x + 5y = 30 \\ 6x - 15y = -90 \end{cases}$$

- A. No solutions
- B. Unique solution: $x = 0,\ y = 0$
- C. Unique solution: $x = 30,\ y = -90$
- D. Infinitely many solutions
- E. Unique solution: $x = -15,\ y = 0$
- F. None of the above

Answer(s) submitted:

- `A`
- `E`
- `E`
- `D`

(correct)

Correct Answers:

- `A`
- `E`
- `E`
- `D`

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3. (2 pts) Perform one step of row reduction, in order to calculate the values for x and y by back substitution. Then calculate the values for x and for y. Also calculate the determinant of the original matrix.

You can let webwork do much of the calculation for you if you want (e.g. enter 45-(56/76)(-3) instead of calculating the value out). You can also use the preview feature in order to make sure that you have used the correct syntax in entering the answer.

[Note– since the determinant is unchanged by row reduction it will be easier to calculate the determinant of the row reduced matrix.]

$$\begin{bmatrix} 5 & 7 \\ 14 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 7 \\ 0 & \_\_\_\_ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \text{-1} \\ \_\_\_\_ \end{bmatrix}$$

$x =$ ______
$y =$ ______
det = ______

Answer(s) submitted:

- `-108/5`
- `29/5`
- `(3 - 2(29/108) ) / (14)`
- `-29/108`
- `-108`

(correct)

Correct Answers:

- `-21.6`
- `5.8`
- `0.175925925925926`

- `-0.268518518518519`
- `-108`

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4. (1 pt) Determine the value of h such that the matrix is the augmented matrix of a linear system with infinitely many solutions.

$$\left[\begin{array}{cc|c} 4 & -7 & 6 \\ 16 & h & 24 \end{array}\right]$$

$h =$ _____
Answer(s) submitted:

- `-28`

(correct)
Correct Answers:

- `-28`

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5. (2 pts) Find a and b such that

$$\begin{bmatrix} 19 \\ 27 \\ 33 \end{bmatrix} = a \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} + b \begin{bmatrix} 7 \\ 6 \\ 9 \end{bmatrix}.$$

$a =$ _____
$b =$ _____
Answer(s) submitted:

- `5`
- `2`

(correct)
Correct Answers:

- `5`
- `2`

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6. (1 pt) Find the characteristic polynomial of the matrix
$A = \begin{bmatrix} -2 & -10 \\ -2 & 8 \end{bmatrix}$
$p(x) =$ __________.
Answer(s) submitted:

- `x^2 -6x - 36`

(correct)
Correct Answers:

- `x**2  - 6 * x  - 36`

---

7. (2 pts) Find the eigenvalues of the matrix $A = \begin{bmatrix} 2 & -6 \\ 3 & -7 \end{bmatrix}$
The smaller eigenvalue is $\lambda_1 =$ ___.
The bigger eigenvalue is $\lambda_2 =$ ___.

Answer(s) submitted:

- `-4`
- `-1`

(correct)
Correct Answers:

- `-4`
- `-1`

---

8. (2 pts) The matrix $B = \begin{bmatrix} 5 & -9 & -3 \\ 0 & -6 & -7 \\ 0 & 0 & -2 \end{bmatrix}$
has three distinct eigenvalues, $\lambda_1 < \lambda_2 < \lambda_3$,
where $\lambda_1 =$ ___, $\lambda_2 =$ ___, and $\lambda_3 =$ ___.
Answer(s) submitted:

- `-6`
- `-2`
- `5`

(correct)
Correct Answers:

- `-6`
- `-2`
- `5`

---

9. (2 pts) The matrix $C = \begin{bmatrix} -9 & 14 & 35 \\ 28 & -23 & -70 \\ -14 & 14 & 40 \end{bmatrix}$
has two distinct eigenvalues, $\lambda_1 < \lambda_2$:
$\lambda_1 =$ ___ has multiplicity ___, and
$\lambda_2 =$ ___ has multiplicity ___.
Answer(s) submitted:

- `-2`
- `1`
- `5`
- `2`

(correct)
Correct Answers:

- `-2`
- `1`
- `5`
- `2`

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10. (2 pts) The matrix $A = \begin{bmatrix} -7 & k \\ 7 & -5 \end{bmatrix}$
has two distinct real eigenvalues if and only if $k >$ _____.
Answer(s) submitted:

- `-1/7`

(correct)
Correct Answers:

- `-0.142857142857143`

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11. (2 pts) Suppose that the trace of a 2×2 matrix A is $\mathrm{tr}(A) = 9$, and the determinant is $\det(A) = 20$. Find the eigenvalues of A.
smaller eigenvalue = ___,
larger eigenvalue = ___.
Answer(s) submitted:

- `4`
- `5`

(correct)
Correct Answers:

- `4`
- `5`

12. (2 pts) For which value of k does the matrix
$A = \begin{bmatrix} 8 & k \\ -2 & 2 \end{bmatrix}$
have one real eigenvalue of multiplicity 2?
$k =$ ________.

Answer(s) submitted:

- `36/8`

(correct)

Correct Answers:

- `4.5`

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13. (2 pts) Given that $v_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are eigenvectors of the matrix $A = \begin{bmatrix} -5 & 6 \\ -4 & 5 \end{bmatrix}$ determine the corresponding eigenvalues.
$\lambda_1 =$ ___.
$\lambda_2 =$ ___.

Answer(s) submitted:

- `-1`
- `1`

(correct)

Correct Answers:

- `-1`
- `1`

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14. (2 pts) Find the eigenvalues of the matrix $\begin{bmatrix} -6 & -3 \\ 0 & -3 \end{bmatrix}$.

Smaller eigenvalue = _____
Associated eigenvector = _______________

Larger eigenvalue = _____
Associated eigenvector = _______________

Note: vectors are entered with ”angle brackets”, such as ¡1,2¿ or ¡0, -4¿.

Answer(s) submitted:

- `-6`
- `<1, 0>`
- `-3`
- `<-1, 1>`

(correct)

Correct Answers:

- `-6`
- `<1,0>`
- `-3`
- `<-1,1>`

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15. (2 pts) The matrix $A = \begin{bmatrix} 4 & -2 \\ 2 & 0 \end{bmatrix}$
has one eigenvalue of multiplicity 2. Find this eigenvalue and the dimenstion of the eigenspace.
eigenvalue = _____,
dimension of the eigenspace = _____.

Answer(s) submitted:

- `2`
- `1`

(correct)

Correct Answers:

- `2`
- `1`