HW4
Math 537 Ordinary Differential Equations
Due Oct 30, 2020

Student Name: ______________________________ **ID** ______________

1: [15 points] Consider the Lorenz model:

$$\frac{dX}{dt} = -\sigma X + \sigma Y, \tag{1.1}$$

$$\frac{dY}{dt} = -XZ + rX - Y, \tag{1.2}$$

$$\frac{dZ}{dt} = XY - \beta Z. \tag{1.3}$$

(a) Find the Jacobian matrix at the trivial critical point $(X, Y, Z) = (0, 0, 0)$. [5 points]

(b) Choose $\sigma = 10$. Perform a (linear) stability analysis in r, β-space using the matrix in (a). [10 points]

[Hint: Describe the regions where the Jacobian matrix has real and/or complex eigenvalues.]

2: [20 points] Consider the non-dissipative Lorenz model:

$$\frac{dX}{dt} = \sigma Y, \tag{2.1}$$

$$\frac{dY}{dt} = -XZ + rX, \tag{2.2}$$

$$\frac{dZ}{dt} = XY. \tag{2.3}$$

(a) Find critical points. [5 points]

(b) Find the Jacobian matrix at critical points(s). [5 points]

(c) Perform a linear stability analysis at each of the critical points. [10 points]

3: [35 points] Consider the following harmonic oscillators:

$$\frac{d^2x_1}{dt^2} = -k_1x_1, \tag{3.1}$$

$$\frac{d^2x_2}{dt^2} = -k_2x_2. \tag{3.2}$$

Let $k_1 = 4\omega_1^2$ and $k_2 = \omega_2^2$.

(a) Convert the above equations into a linear system with four first-order differential equations. Find the matrix A that represents the 4D system. [5 points]

(b) Find the eigenvalues and eigenvectors of A in the 4-D phase space. [15 points]

(c) Find the linear map T using (b) and compute $T^{-1}AT$. [15 points]

4: [30 points] Consider the following matrix:

$$A = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

(a) Find the eigenvector(s) and generalized eigenvector(s) associated with the matrix A. [15 points.]

(b) Construct a linear map T using the eigenvector(s) and generalized eigenvector(s) in (a) and compute $T^{-1}AT$. [15 points.]