Numerical Matrix Analysis

Lecture Notes #1 — Introduction

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San Diego State University
San Diego, CA 92182-7720

http://terminus.sdsu.edu/

Spring 2020

Outline

Academic Life — MSc

- KTH MSc. Engineering Physics, Royal Institute of Technology (KTH), Stockholm, Sweden. Thesis Advisers: Michael Benedicks, Department of Mathematics KTH, and Erik Aurell, Stockholm University, Department of Mathematics. Thesis Topic: *"A Renormalization Technique for Families with Flat Maxima."*

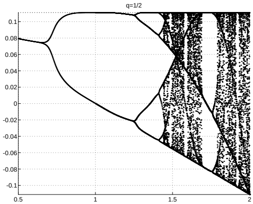

Figure: Bifurcation diagram for the family $f_{a,\frac{1}{2}}$ [Blomgren-1994]

Academic Life

PhD

- UCLA PhD. UCLA Department of Mathematics. Adviser: Tony F. Chan. PDE-Based Methods for Image Processing. Thesis title: *"Total Variation Methods for Restoration of Vector Valued Images."*

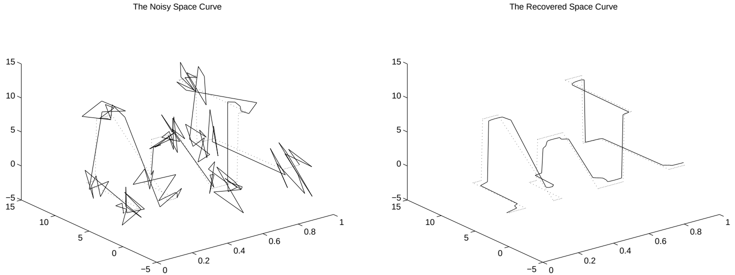

Figure: The noisy (SNR $= 4.62\,$dB), and recovered space curves. Notice how the edges are recovered. [BLOMGREN-1998]

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Academic Life — Postdoc

- Research Associate. Stanford University, Department of Mathematics. Main Focus: Time Reversal and Imaging in Random Media (with George Papanicolaou, *et. al.*)

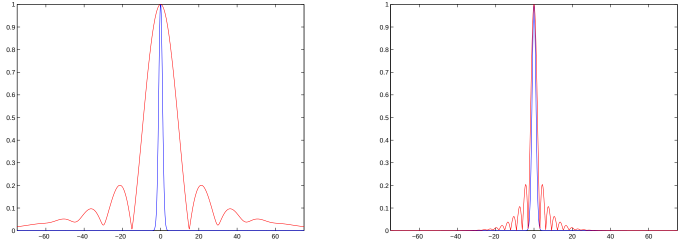


Figure: Comparison of the theoretical formula for a medium with $L = 600\,m$, $a_e = 195\,m$, $\gamma = 2.12 \times 10^{-5}\,m^{-1}$. [LEFT] shows a homogeneous medium, $\gamma = 0$, with $a = 40\,m$ TRM (in red / wide Fresnel zone), and a random medium with $\gamma = 2.12 \times 10^{-5}$ (in blue). [RIGHT] shows $\gamma = 0$, with $a = a_e = 195\,m$ (in red), and $\gamma = 2.12 \times 10^{-5}$, with $a = 40\,m$ (in blue). The match confirms the validity of [the theory]. [BLOMGREN-PAPANICOLAOU-ZHAO-2002]

Academic Life — Professor

- San Diego State University Professor, SDSU, Department of Mathematics and Statistics. Projects: Computational Combustion, Biomedical Imaging (Mitochondrial Structures, Heartcell Contractility, Skin/Prostate Cancer Classification), carbon sequestration, compressed sensing.

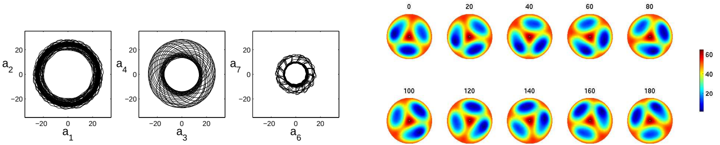


Figure: [Left] Phase-space projections produced by the time coefficients of the POD decomposition of the rotating pattern shown in [Right]. [Blomgren-Gasner-Palacios-2005]

Primary Research Interests — Current | High Performance Computing

Development of algorithms achieving near-optimal GPU utilization, with applications to Computational PDEs, Computational Linear Algebra, and Computational Optimization.

Project #1: Fast Multipole Method for *Waves over Vortices*, w/Chris Curtis & Daniel Matteson.

Project #2: ???, w/*???* & ???

i9-7980XE, ≈ \$1,699
18 Cores, 36 Threads.

RTX 2080 Ti, ≈ \$1,199
4352 CUDA Cores

Fun Times... $\Leftrightarrow$ Endurance Sports

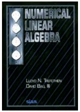
- Triathlons:
  - (13) Ironman distance $(2.4 + 112 + 26.2)$ — [PR] 11:48:57
  - (16) Half Ironman distance — 5:14:20
- Running
  - (1) 100k Race (62.1 miles) — 15:37:46 (15:05/mi)
  - (1) Trail Double-marathon (52 miles) — 10:59:00 (12:32/mi)
  - (5) Trail 50-mile races — 9:08:46 (10:59/mi)
  - (8) Trail 50k (31 mile) races — 5:20:57 (10:20/mi)
  - (16) Road/Trail Marathons — 3:26:19 (7:52/mi)
  - (30) Road/Trail Half Marathons — 1:35:00 (7:15/mi)

Contact Information

Office	GMCS-587
Email	blomgren.peter@gmail.com
Web	http://terminus.sdsu.edu/SDSU/Math543/ https://canvas.sdsu.edu/
Office Hours	TuTh 11:00am – 11:50am, 2:00pm – 2:50pm, or by appointment.

Math 543: Literature

“Required” —



Numerical Linear Algebra, Lloyd N. Trefethen and David Bau, III, Society for Industrial and Applied Mathematics (SIAM), 1997. ISBN 978-0-898713-61-9.

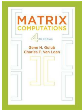
“Required” — (Supplemental)

Class notes and class web-page.

Math 543: Literature

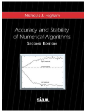
Everything You Ever Wanted to Know, but Were Afraid to Ask...

"Optional" — (Classic, Comprehensive Reference)



Matrix Computations, 4th Edition, Gene H. Golub and Charles F. Van Loan, Johns Hopkins University Press, 2012. ISBN-13 978-1421407944.

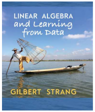
"Optional" — (Comprehensive Reference)



Accuracy and Stability of Numerical Algorithms, Second Edition, Nicholas J. Higham, Society for Industrial and Applied Mathematics (SIAM), 2002. ISBN 978-0-898715-21-7.

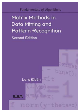
Math 543: Literature

“Inspiration for the Road Ahead” —



Linear Algebra and Learning From Data, Gilbert Strang, Society for Industrial and Applied Mathematics (SIAM), 2019. ISBN 978-069219638-0.

“Considered... but, no.” —



Matrix Methods in Data Mining and Pattern Recognition, Second Edition, Lars Eldén, Society for Industrial and Applied Mathematics (SIAM), 2019. ISBN: 978-1-611975-85-7.

Math 543: Introduction — Grading etc.

$33\frac{1}{3}\%$ `Homework:` both theoretical, and implementation (programming) — Recommended languages: Python, Matlab, C/C++, or Fortran; however anything goes: 6510 assembler, Java, M\$-D$^\flat$, Haskell...

$33\frac{1}{3}\%$ `Midterm:` [$\frac{1}{2}$ Take-Home, and $\frac{1}{2}$ In-Class].

$33\frac{1}{3}\%$ `Final or Final Project:` [Complete details TBA].

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- Most class attendance is “OPTIONAL” — Homework and announcements will be posted on the class web page. If/when you attend class:

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  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.

Expectations and Procedures, II

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- The instructor will make special arrangements for students with documented learning disabilities and will `try` to make accommodations for other unforeseen circumstances, *e.g.* illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. **Please contact the instructor EARLY regarding special circumstances.**

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- Students are expected **and encouraged** to ask questions in class!
- Students are expected **and encouraged** to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!

Expectations and Procedures, II $\frac{I}{II}$

Late HW Policy

- Assignments accepted up to 7 days after original deadline, with a 10% penalty.
- Further extensions will only be granted in extreme, well-documented, circumstances.

Expectations and Procedures, III

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Expectations and Procedures, III

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- Missed final exam: Don't miss the final! Contact the instructor ASAP or a grade of incomplete or F will be assigned.
- **Academic honesty**: submit your own work — but feel free to discuss homework with other students in the class!

Honesty Pledges, I

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- Work missing the honesty pledge **may not be graded!**

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Math 543: Computer Resources

You need access to a computing environment in which to write your code; — you may want to use any combination of Matlab (for quick prototyping and short homework assignments) and other languages; emphe.h. Python, C/C++ or Fortran (or something completely different, like Java or M$-D$^\flat$).

Free C/C++ (gcc) and Fortran (f77) compilers are available for Linux/UNIX.

SDSU students can download a copy of matlab from

https://www.mathworks.com/academia/tah-portal/san-diego-state-university-1108597.html

[Licensing Subject to Change With Minimal Notice]

Math 543: Introduction — What you should know already

Prerequisite: `Math 340`

340 ⇒ **Programming in Mathematics**

- Introduction to programming in mathematics. Modeling, prob-lem solving, visualization..

Prerequisite: `Math 254` **or** `Math 342A` **or** `AE 280`

254 ∩ 342A ⇒ **Basic Linear Algebra**

- Vectors, Matrices, Eigenvalues and Eigenvectors

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \qquad A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \dots & \vec{a}_n \end{bmatrix}$$

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Math 543: Introduction — Why???

Solution of linear systems and eigenvalue problems show up in many applications in applied & computational mathematics / sciences / engineering.

Although we probably know about Gaussian Elimination for solving for

$$\vec{x} = A^{-1}\vec{b}$$

in infinite precision, finding this solution (or at least a good approximation thereof) in **finite precision** (*i.e.* on a computer) is sometimes a challenge — especially if we need the solution fast.

Math 543: Introduction — Why???

The computational complexity (number of operations needed) for Gaussian Elimination is $\mathcal{O}(n^3)$, which is quite slow as n grows "large."

Applications (sources of Numerical Linear Algebra problems):

- Solution of partial differential equations (PDEs)
- Optimization (Operations Research)
- Model Analysis and Fitting (Least Squares)
- Image Processing
- Protein Folding
- DNA sequencing, etc. etc. etc.
- Data Science

Math 543: Introduction — What we will learn

$$\mathbf{A}\tilde{\mathbf{x}} = \tilde{\mathbf{b}}, \qquad \mathbf{A}\tilde{\mathbf{x}} = \lambda\tilde{\mathbf{x}}, \qquad \mathbf{Q}^T\mathbf{A}\mathbf{Q} = \mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$$

- QR-Factorization / Least Squares
- The SVD
- Conditioning and Stability
- Gaussian Elimination, Pivoting
  - $\Rightarrow$ LU- and Cholesky-factorization
- Eigenvalue Problems
- Iterative Methods
  - $\Rightarrow$ Arnoldi, GMRES, Lanczos, Conjugate Gradients

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