

Quiz-2
Math 537 Ordinary Differential Equations
Due: 9:00AM Wednesday, September 2, 2020

Student Name: _____ **ID** _____

Total points: 20 points

1: [4 points] Introduce a new time variable τ to convert the following ODE

$$\frac{dy}{dt} = \sigma y$$

into

$$\frac{dy}{d\tau} = y.$$

2: [6 points] Consider the following Logistic equation

$$\frac{dy}{dt} = \alpha y - \beta y^2.$$

Covert the above ODE into the following ODE

$$\frac{dz}{d\tau} = z - z^2.$$

by introducing a new time variable τ and a new time-dependent variable z .
Find τ and z .

3: [10 points] Consider the improper integral

$$\int_{-1}^1 \frac{1}{x} dx. \quad (3)$$

(a) Verify whether the following derivations are correct.

The above has the following two parts:

$$\int_{-1}^0 \frac{1}{x} dx + \int_0^1 \frac{1}{x} dx.$$

By introducing $y = -x$ for the first part, we have

$$\int_1^0 \frac{1}{y} dy + \int_0^1 \frac{1}{x} dx,$$

which becomes

$$- \int_0^1 \frac{1}{y} dy + \int_0^1 \frac{1}{x} dx = 0.$$

[2 points]

(b) Represent Eq. (3) as follows:

$$\lim_{\epsilon \rightarrow 0} \int_{-1}^{-\epsilon} \frac{1}{x} dx + \lim_{\epsilon \rightarrow 0} \int_{\epsilon}^1 \frac{1}{x} dx.$$

Complete the above integrals.

[2.5 points]

(c) Represent Eq. (3) as follows:

$$\lim_{\epsilon \rightarrow 0} \int_{-1}^{-2\epsilon} \frac{1}{x} dx + \lim_{\epsilon \rightarrow 0} \int_{\epsilon}^1 \frac{1}{x} dx.$$

Complete the above integrals.

[2.5 points]

(d) Compare the answers in (b) and (c) to provide justifications to your analysis in (a). [3 points]