Math 532: Homework 8
Due 11/06/19
Everyone turns in an individual copy.

Note, as gone over in class, by a homotopy between continuous closed paths $C_1(t)$ and $C_2(t)$ where $C_j : [t_0, t_f] \rightarrow D$, $C_j(t_0) = C_j(t_f)$, we mean there exists a continuous function $H : [0,1] \times [t_0, t_f] \rightarrow D$ such that

$$H(0,t) = C_1(t), \; H(1,t) = C_2(t), \; H(s,t_0) = H(s,t_f).$$

We say a given domain D is *simply connected* if every closed path is homotopic to some point $z_0 \in D$.

1. (2pts each) 4.57.1
2. (5pts) 4.57.10
3. (5pts) 5.61.2
4. (5pts) 5.61.4
5. (5pts) 5.61.9
6. (5 pts) A domain D is called convex if for all $z, w \in D$, the line segment $\lambda z + (1-\lambda)w \in A$ for $\lambda \in [0,1]$. Show that any convex domain is simply connected.
7. (5 pts) Show that any disc $D_R(z_0)$, i.e. all those $z \in \mathbb{C}$ such that

   $$|z - z_0| \leq R$$

   is a convex set.
8. (5 pts) A domain $D \subset \mathbb{C}$ is called *star-shaped* if there is a point $z_* \in D$ such that for all $z \in D$, the line segment joining z and z_* is in D. Show that D is simply connected.