Math336 Midterm Exam by Dr. Samuel Shen

October 21, 2020, Wed, 10AM, due 11:59PM on the second day/Thur/Oct 22

Due to COVID, this is an open-book exam. You can use google to search for any helpful information. You can use a calculator, computer, or experiment. However, you CANNOT ask anyone to help you. Problems #1, 2 and 3 are the hand-written part. Start each problem on a new page. When done, scan all the pages into pdf. Problem #4 is for R programming. Save your R results using `Compile Report...` *function in RStudio to produce a pdf file, or copying-and-pasting your code and output to a WORD file and converting it into pdf. Finally, you combine the hand-written part together with the pdf file for R to form your final pdf file for submission. Save and submit your R file too. Therefore, you will submit two files to BB: a single pdf file and a single R file.*

Student Name:

Student ID Number:

1. $[27\ points]$
   (a) Figure 1 shows a simple pendulum with mass m , string length l , and the Earth gravitational acceleration g. Use the dimensional analysis method to determine the period τ of the pendulum as a function of m, l, g, determined up to a constant α, i.e., $\tau = \alpha m^a l^b g^c$.

   (b) Use the conservation law of energy to find an approximate value of α in Part (a) under the condition of $\sin x \approx x$ when x is close to be zero.

   (c) If the string length is increased to $1.02l$ due to expansion in a higher temperature environment, and its corresponding period is denoted by τ_2. Express τ_2 in terms of τ found in Part (a) of this problem.

   (d) Given that $l = 75$ [cm] and $g = 9.8$ [m/s], calculate τ with unit in second. Write down your steps. You can use a calculator or R to do the calculation. You do not need to submit your R code for this problem even if you use R here.

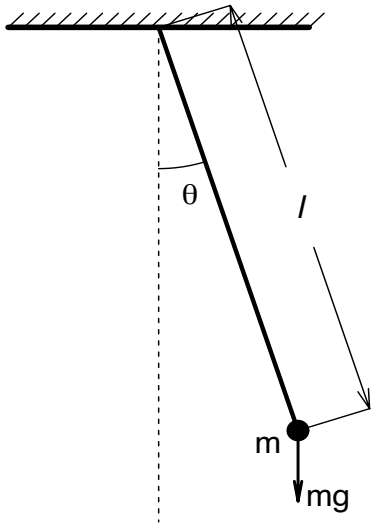

Figure 1 Simple pendulum of mass m and length l under the action of Earth's gravitational force.

2. [20 *points*] The SVD result of a matrix A is below

```
svdA=svd(A)

svdA$d
[1] 2.0 1.0

svdA$u
     [,1] [,2]
[1,]    0    1
[2,]    1    0
svdA$v
     [,1] [,2]
[1,]  0.0    1
[2,]  0.4    0
[3,]  0.9    0
```

(a) Write down three matrices U, D, V in the SVD formula $A = UDV'$ where V' denotes the transpose matrix of V.

(b) Use the SVD formula $A = UDV'$ to approximately recover the original matrix A by hand calculation for the multiplication of the three matrices. Show your work and steps. You can use a calculator to do your number multiplication, but you still need to show your work. You may use R to verify your solution, but that is not required.

3. [20 *points*] (a) Derive a formula for the monthly mortgage payment x, expressed in terms of the principal amount P, monthly interest rate r, and total number of months of the loan n. Show your work and the detailed steps. The answer for this step is a formula.

(b) Given the data: The principal amount (i.e., the total loan) is $P = \$250,000$, the annual interest rate is 3.6% (converted into the monthly rate 0.3%), and the loan is to be paid off in 30 years (equivalent to 360 months). Use the above derived formula and the data to compute the monthly mortgage payment x by a calculator or R. The answer should be an amount of money per month. You do not need to submit the R code for this problem.

(c) If the annual rate is reduced to 2.9% in the above data, what is the monthly mortgage payment?

(d) If the principal is increased to $P = \$270,000$, the annual rate is 2.9%, and the loan period is still 30 years, what is the the monthly mortgage payment now?

4. [33 *points*] **The R programming part**

(i) Use R to solve the following linear equations for x, y, z:

$$\begin{cases} -x + 2.9y + z = 1 \\ -1.5x - y + z = 2.1 \\ 2.2x + y - 4z = 0 \end{cases}$$

Copy your R solution result to your R code as comment lines after #.

(ii) Figure 2 shows the history of the global average December mean temperature anomalies. Use R and the dataset `EarthTemperatureData.tex` or `EarthTemperatureData.csv` downloadable from BB's Assignment/Midterm block (or from the instructor's email) to plot a similar figure but for **June** and with the following requirements.

(a) Replace "Samuel Shen" and "December" in the main title by your name and June.

(b) Change the curve's color from black to purple and use `lwd=5`.

(c) Compute the linear trend of the June temperature anomalies for the entire time span from 1850 to 2015.

(d) Change the linear trend line's color from black to blue, and use `lwd=3` for the trend line.

(e) Change the text "December trend = 0.52 deg C/century" to "June trend = ?? deg /Century", and use the trend calculated from Step c) in the position "??".

(f) Save your plot as a png file with the filename as "first2letters-of-your-last-name-temp.png". If your `Compile Report ...` is successful, you do not need to do this figure saving step.

(g) Plot the histogram of the June temperature anomalies from 1850 to 2015. The title of the figure is "Histogram of the June Temperature Anomalies." The x-label is "Temperature anomalies [deg C]." Save this figure as the above step (f) but with a file name "first2letters-of-your-last-name-histogram.png".

(h) Save all your work as a pdf file and combine all your work for this exam into a single pdf file.

(i) Submit both pdf and R files into BB.

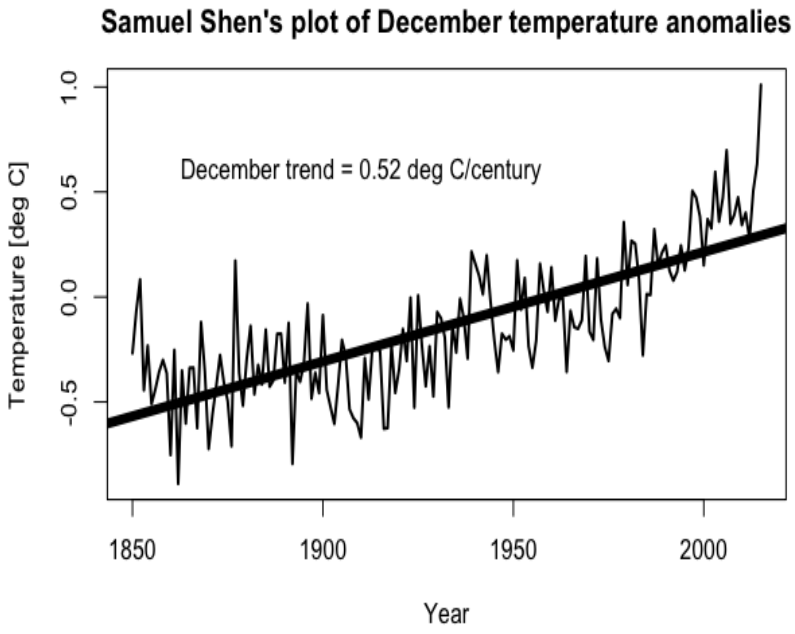

Figure 2 Global average December mean global average surface air temperature anomalies from 1850-2015.