Problem set 11

Professor McNamara

Exercises to hand in: 12.7, 12.8, 12.9, 12.22, 12.23, 12.24

12.7 Guessing regular differencing from an ACF

(no dataset)

- Series A
- Series B
- Series C
- Series D
- Series E

12.8 Guessing seasonal differencing from an ACF

(no dataset)

- Series A
- Series B
- Series C
- Series D
- Series E

12.9 Matching time series plot to its ACF

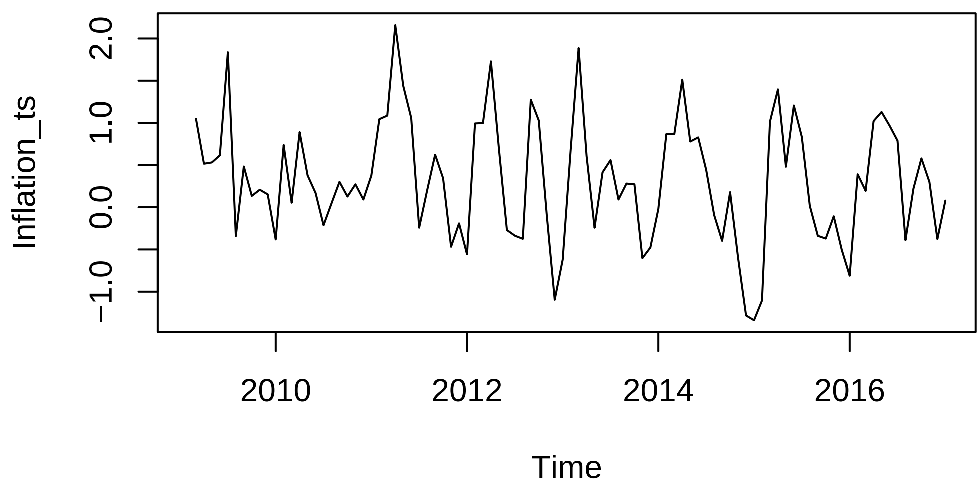
(no dataset)

- Series A goes with Series
- Series B goes with Series
- Series C goes with Series
- Series D goes with Series
- Series E goes with Series

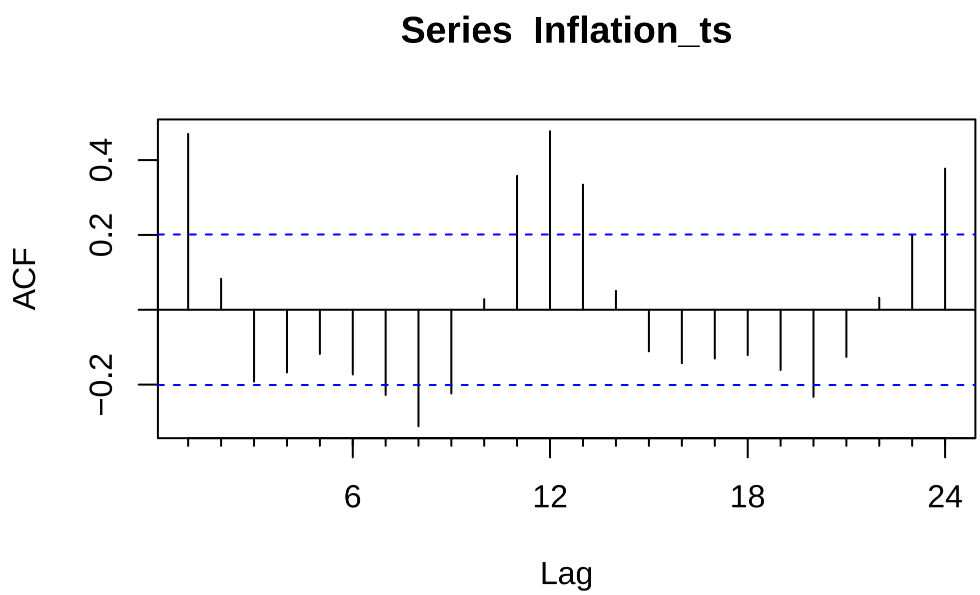
12.22 Consumer Price Index: ordinary first differences

```
data("Inflation")
Inflation <- Inflation %>%
  mutate(diff = CPI - lag(CPI))
```

```
Inflation_ts <- ts(Inflation$diff, start = c(2009, 2), frequency = 12)
plot(Inflation_ts)
```




```
Acf(Inflation_ts)
```


Series Inflation_ts
ACF
0.4
0.2
−0.2
6
12
18
24
Lag

12.23 Consumer Price Index: autoregressive models

a. Fit a first-order autoregressive model to the differences of the CPI series (i.e., an ARIMA(1,1,0) model). Check if the constant term is important to keep in the model. If not, drop it and refit. Write down the fitted ARIMA(1,1,0) model.

```
m1 <- Arima(Inflation$CPI, order = c(1,1,0), include.constant = TRUE)
m1
```

```
Series: Inflation$CPI
ARIMA(1,1,0) with drift

Coefficients:
         ar1   drift
      0.4709  0.3233
s.e.  0.0902  0.1192

sigma^2 = 0.3937:  log likelihood = -89.58
AIC=185.16   AICc=185.42   BIC=192.82
```

```
coeftest(m1)
```

```

z test of coefficients:

      Estimate Std. Error z value  Pr(>|z|)
ar1   0.470944   0.090186  5.2219 1.771e-07 ***
drift 0.323348   0.119220  2.7122  0.006684 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The constant is significant.

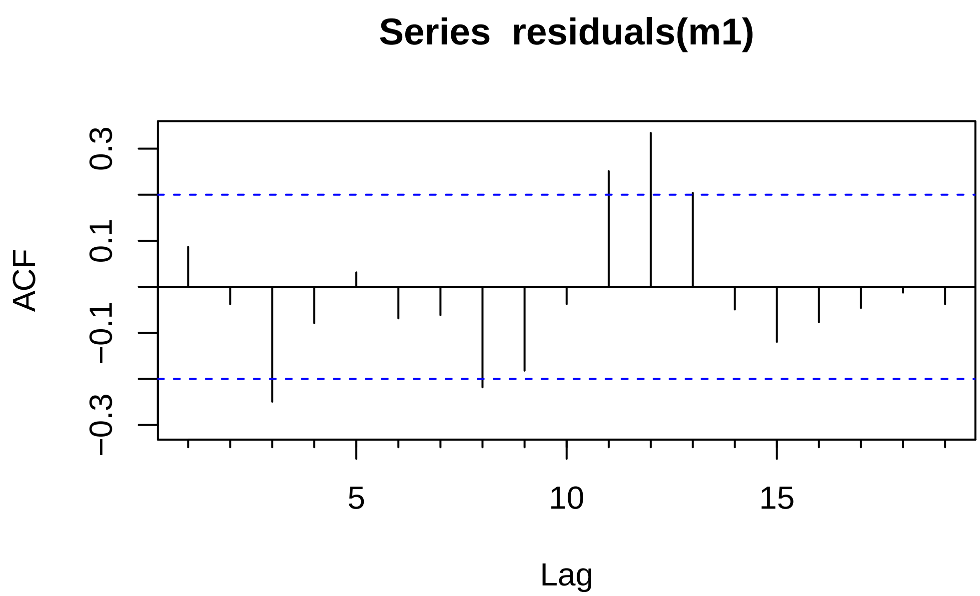
$$\hat{y}_t = 0.17 + 1.47 \cdot y_{t-1} - 0.47 \cdot y_{t-2}$$

b. Is the first-order autoregressive term needed in this model? Justify your answer.

Since the p-value is significant, the first-order autoregressive term is needed in this model.

c. Comment on anything interesting you see in the ACF of the residuals for this model.

```
Acf(residuals(m1))
```



d. Now add a second autoregressive term to run an ARIMA(2,1,0) for the CPI series. Is the second-order autoregressive term helpful in the model?

```
m2 <- Arima(Inflation$CPI, order = c(2,1,0), include.constant = TRUE)
m2
```

```
Series: Inflation$CPI
ARIMA(2,1,0) with drift

Coefficients:
         ar1      ar2   drift
      0.5552  -0.1777  0.3234
s.e.  0.1009   0.1010  0.1003

sigma^2 = 0.3852:  log likelihood = -88.06
AIC=184.11   AICc=184.56   BIC=194.33
```

```r
coeftest(m2)
```

```
z test of coefficients:

      Estimate Std. Error z value  Pr(>|z|)    
ar1    0.55522    0.10094  5.5006 3.786e-08 ***
ar2   -0.17770    0.10095 -1.7602  0.078372 .  
drift  0.32343    0.10027  3.2254  0.001258 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Since the second order autoregressive term has a p-value greater than 0.05, it is not a significant term to help the model.

12.24 Consumer Price Index: moving average models

a. [Note correction!] Fit a first-order moving average model to the differences of the CPI series (i.e., an ARIMA(0,1,1) model). Check if the constant term is important to keep in the model. If not, drop it and refit. Write down the fitted ARIMA(0,1,1) model.

```r
# hint: since this is a moving average model, you don't have to do the arithmetic for the
m3 <- Arima(Inflation$CPI, order = c(0,1,1), include.constant = TRUE)
m3
```

```
Series: Inflation$CPI
ARIMA(0,1,1) with drift

Coefficients:
         ma1   drift
      0.4278  0.3223
s.e.  0.0778  0.0915

sigma^2 = 0.4016:  log likelihood = -90.49
AIC=186.99   AICc=187.25   BIC=194.65
```

```r
coeftest(m3)
```

```
z test of coefficients:

      Estimate Std. Error z value  Pr(>|z|)
ma1   0.427811   0.077798  5.4990  3.82e-08 ***
drift 0.322258   0.091508  3.5216 0.0004289 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
0.3223 * (1-0.4278)
```

```
[1] 0.1844201
```

Since the p-value is less than 0.05, the constant term is significant.

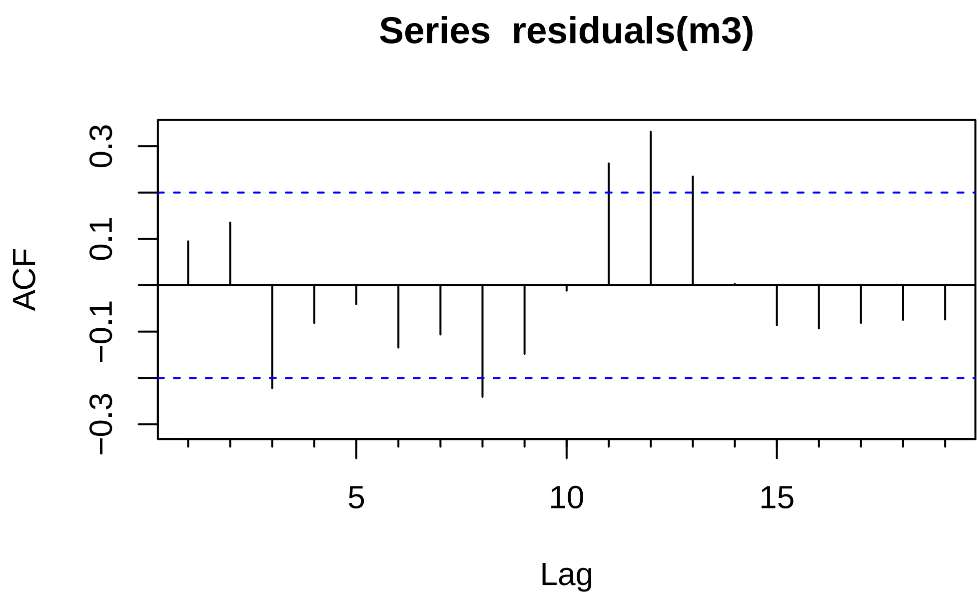
$$\hat{y}_t = 0.32 + 0.18 \cdot y_{t-1} + 0.43 \cdot \hat{\epsilon}_{t-1}$$

b. Is the first-order moving average term needed in this model? Justify your answer.

Since the p-value of the first-order moving average term is less than 0.05, it is significant and needed in the model.

c. Comment on anything interesting you see in the ACF of the residuals for this model.

```
Acf(residuals(m3))
```



The ACF plot looks stationary.

d. Now add a second moving term to run an ARIMA(0,1,2) for the CPI series. Is the second-order moving average term helpful in the model?

```
m4 <- Arima(Inflation$CPI, order = c(0,1,2), include.constant = TRUE)
m4
```

```
Series: Inflation$CPI
ARIMA(0,1,2) with drift
```

```
Coefficients:
         ma1     ma2   drift
      0.5794  0.3444  0.3254
s.e.  0.0969  0.1137  0.1179

sigma^2 = 0.3739:  log likelihood = -86.72
AIC=181.43   AICc=181.88   BIC=191.65
```

```r
coeftest(m4)
```

```
z test of coefficients:

      Estimate Std. Error z value  Pr(>|z|)    
ma1   0.579443   0.096943  5.9772 2.271e-09 ***
ma2   0.344422   0.113709  3.0290  0.002454 ** 
drift 0.325427   0.117908  2.7600  0.005780 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Since the second-order moving average term has a p-value less than 0.05, it is significant and is therefore helpful in the model.