Numerical Methods (Period 1, 2022/2023)

Week 4 Tutorial: Interpolation

§4.1 Polynomial Interpolation

Plot the polynomial interpolation through the following sets of data points. Use Python's module `numpy.polynomial` for evaluating polynomials, and for polynomial interpolation. See document in Canvas.

(a).

i	0	1	2	3	4
x_i	4	6	7	8	9
y_i	4	6	10	0	-6

(b).

i	0	1	2	3	4	5	6	7	8	9
x_i	2.1	3.4	5.7	6.2	8.1	12.4	13.0	13.5	14.2	15.0
y_i	-2.0	-5.2	1.3	4.6	-0.3	5.7	8.2	-2.5	3.9	0.0

§4.2 Errors

Exercise 1 from Computer Exercises 4.2 (page 187 ed 7, 163 ed 6).

§4.3 Differentiation

Find the derivative for the following functions at the points indicated

(a). $f(x) = \cos(x)$ at $x = 0$.

(b). $f(x) = \arctan(x)$ at $x = 1$.

Use the following derivative-formulas.

(i). $f'(x) \approx \frac{1}{h}\big(f(x+h) - f(x-h)\big)$.

(ii). $f'(x) \approx \frac{1}{4h}\big(f(x+3h) - f(x-h)\big)$.

(iii). $f'(x) \approx \frac{1}{2h}\big(4f(x+h) - 3f(x) - f(x+2h)\big)$.

Experiment with several h values (e.g. $0.1, 0.05, 0.01, \ldots$), and compare with the exact derivative values.