Assignment

October 4, 2023

Exercise 1.

A defendant is accused of having fathered a child as a result of a rape. The DNA profiles of the defendant, mother, and child are available (see Table below). The Likelihood Ratio $LR_{1,2}$ of the genetic evidence for the hypotheses

H_1: The defendant is the father of the child,

H_2: The defendant is unrelated to the child,

is very high, thus providing strong evidence for paternity of the defendant.

Locus	Mother	Child	Defendant	$LR_{1,2}$	$LR_{3,2}$	$LR_{1,3}$
CSF1PO	10, 14	10, 15	14, 15	4.56		
D2S1338	17, 17	17, 24	17, 24	4.26		
D3S1358	14, 16	14, 17	17, 18	2.36		
D5S818	11, 13	12, 13	11, 12	2.83		
D7S820	11, 12	11, 12	11, 12	2.92		
D8S1179	10, 14	10, 15	14, 15	4.56		
D13S317	8, 13	12, 13	12, 12	3.24		
D16S539	9, 10	9,9	9, 12	4.81		
D18S51	13, 14	14, 18	13, 18	5.45		
D19S433	14, 14	14, 14	14, 14	2.93		
D21S11	29, 29	29, 30	30, 33.2	2.15		
FGA	22, 24	24, 24	22, 24	3.63		
TH01	9.3, 9.3	9.3, 9.3	7, 9.3	1.64		
TPOX	8,8	8,8	8,8	1.84		
vWA	15, 18	15, 16	16, 16	4.96		
All				50218439		

The defendant claims that he is innocent, but that a brother of his is the actual father of the child. We do not have DNA profiles for the defendant's brother(s). We formulate a third hypothesis

H_3: The defendant's brother is the father of the child.

(1) Give the algebraic formula for $LR_{1,2}$ on the loci CSF1PO, D7S820, D19S433.

(2) Give the algebraic formula for $LR_{3,2}$ on the same loci.

(3) Compute $LR_{3,2}$ on these three loci.

(4) Compute $LR_{1,3}$ on these loci.

(5) Can you calculate the probabilities $P(H_i \mid E)$ in light of the considered genetic evidence E? If yes, do so; if not, argue why not and what can be said about them.

Exercise 2.

Child B is a child of mother A. For B's father there are two hypotheses: H_1 stating that a brother of A is the father of B, and H_2 stating that the father of B is unrelated to A.
(a) Draw the pedigrees corresponding to these hypotheses.
(b) Let, on an autosomal locus, the genotypes of A and B be written as (a^{pat}, a^{mat}) and (b^{pat}, b^{mat}), with a^{pat} being the allele A inherited from her father and a^{mat} the allele she inherited from her mother; and analogous for B. For hypothesis 1, assuming the pedigree founders are all outbred (i.e., have inbreeding coefficient equal to zero), calculate the probability for each constellation of IBD between the four alleles of A and B, using notation as in Figure 1.
(c) Suppose the observed genotypes are $g_A = (a, b), g_B = (a, a)$. Compute the LR for H_1 versus H_2.
(d) Suppose the observed genotypes are $g_A = (a, a), g_B = (a, b)$. Compute the LR for H_1 versus H_2.

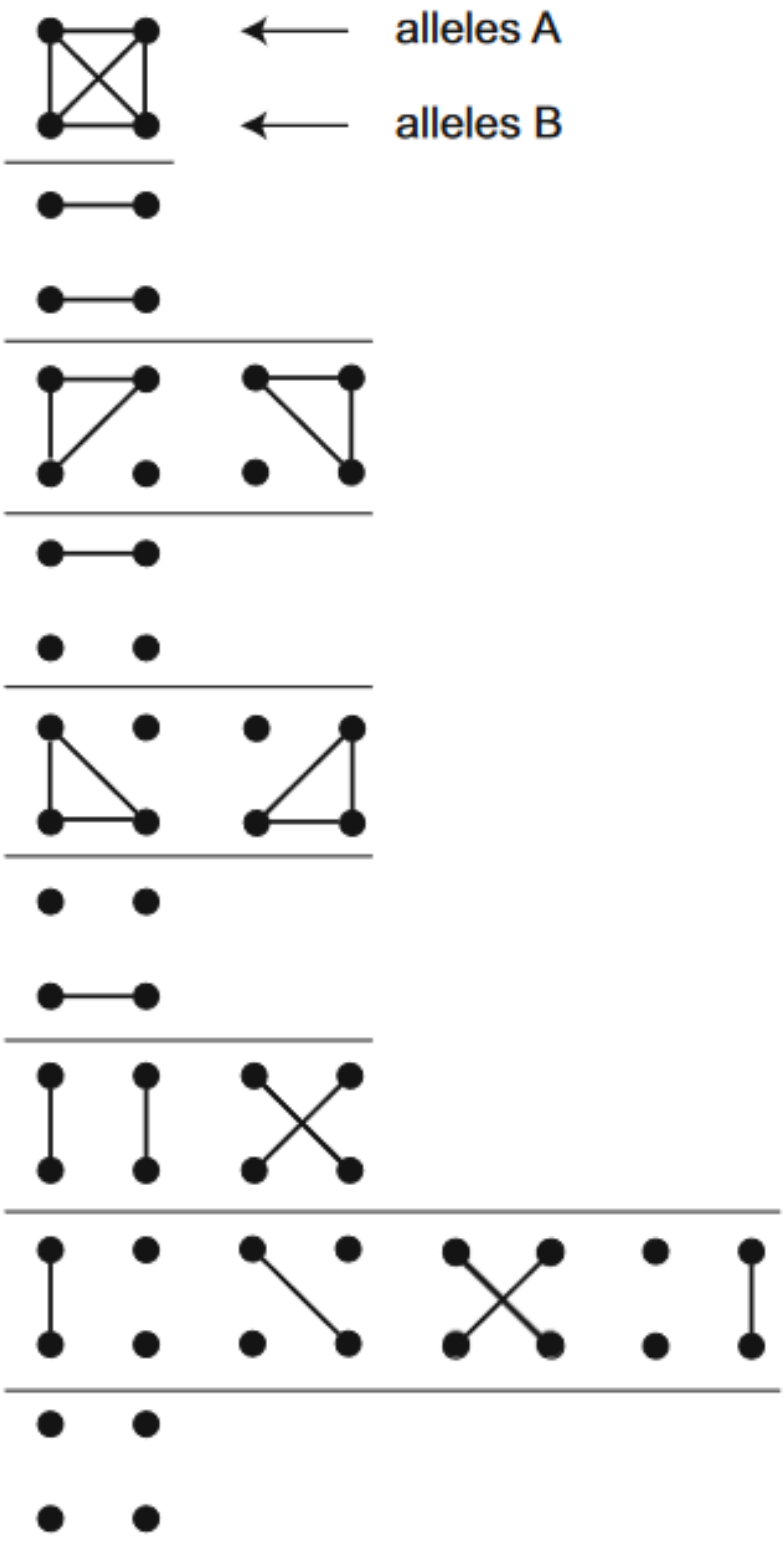

Figure 1: Possible IBD between A and B. The top row represents the alleles of A and the bottom row those of B. In each constellation, one column corresponds to the paternally inherited allelels and the other column the maternally inherited alleles