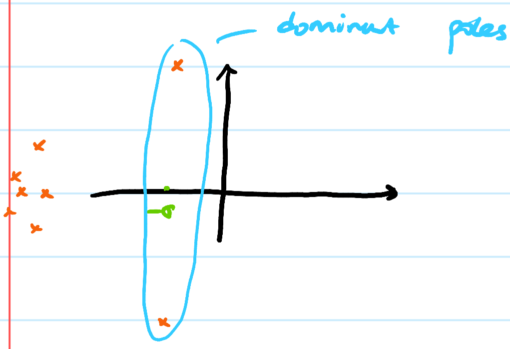
Lecture 15

October 12, 2021 3:08 PM

Quiz:

↳ Content from Lec 1 – 12

Stability of LTI Systems



Lagrange

↳ derived method of writing equations of motion

↳ developed on Newton's work

↳ Mechanic Analytiz

Lyapunov

2 Notions of Stability

1. Internal Stability

↳ $\dot{x} = Ax + B\cancel{u}$ ← set $u=0$

$\cancel{y = Cx + Du}$ ← discard output

↳ $\dot{x} = Ax$ (Internal Stability)

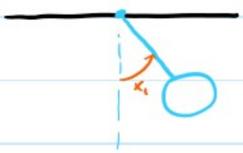
2. I/O Stability

↳ $u(t)$

$y = Cx + Du$

} only consider $u(t), y(t)$

eg. Crane Model



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{g}{\ell}\sin(x_1) - \frac{1}{\ell}\cos(x_1)\,u$$

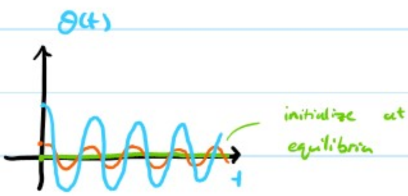
Internal Stability set $u=0$

↳ stability: 2 equilibria

↳ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (down) stable

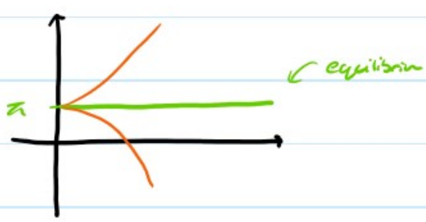
↳ $\begin{bmatrix} \pi \\ 0 \end{bmatrix}$ (up) unstable

Any perturbation from a stable equilibrium should tend towards stable



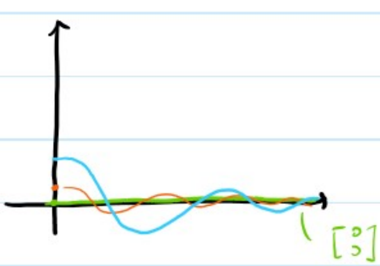
1. If we start at equilibria, we stay near the eq
2. The closer we start to equilibria, the closer we stay to eq.

} stable



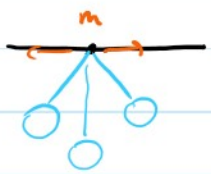
No matter how close to $\begin{bmatrix} \pi \\ 0 \end{bmatrix}$ we stay, we move away from $\begin{bmatrix} \pi \\ 0 \end{bmatrix}$ } unstable

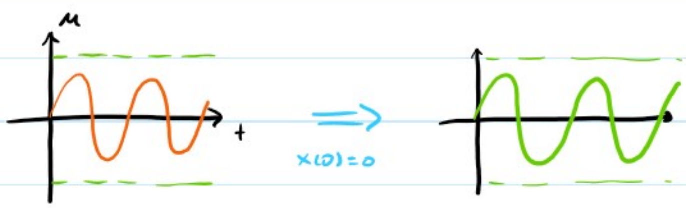
Suppose friction occurs



Asymptotically stable: if we start near $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, we stay near $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ AND we converge to $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Input Output Stability



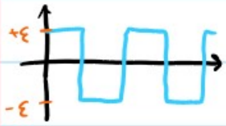


Bounded - Input Bounded - Output Stable
(BIBO stable)

The pendulum without friction is NOT BIBO stable

Mechanical energy of system: $E = \frac{1}{2} ml\, x_2^2 - mgl\cos(x_1)$

Pick $\varepsilon > 0$, set $u(t) = -\varepsilon\, \mathrm{sign}(\frac{1}{2}\cos(x_1))$



$\frac{dE}{dt} = l\,|x_2\cos(x_1)| \geq 0 \implies E(t) \to \infty$

$\theta(t)$ unbound

Definition of Stability (Internal Stability)

↳ $\dot{x} = Ax + Bu$

$y = Cx + Du$

Discard y, set $u = 0$

$\boxed{\dot{x} = Ax}$ $\Rightarrow$ 0 is an equilibrium

(1)

Sys (1) is stable

$\forall x_0 \in \mathbb{R}^n$, the solution $x(t)$ of $\dot{x} = Ax$

with init. cond. $x(0) = x_0$ is bounded

↳ "Bounded" means : $\exists M > 0$ s.t. $\forall t \geq 0$

$\|x(t)\| := (x(t)^T x(t))^2 \leq M$

Sys(1) is asymptotically stable if $\forall x_0 \in \mathbb{R}^n$,

the solution $x(t)$ with init. cond $x(0) = x_0$

tends to the zero vector as $t \to \infty$

Sys(1) is unstable if it is NOT stable

Definition of Stability (Input / Output Stable)

$\dot{x} = Ax + Bu$ — (2) $\qquad$ $Y(s) = G(s)\,U(s)$ — (3)

$y = Cx + Du$

Sys (2) with $x(0) = 0$ or (3) is BIBO stable if for any bounded input $u(t)$, the corresponding output $y(t)$ is also bounded

Sys (2) or (3) is BIBO Unstable if there exists a bounded input giving an unbounded output