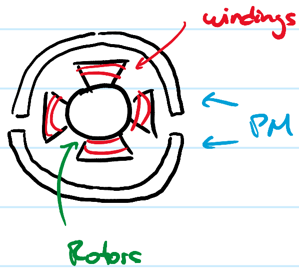
Lecture 3

September 14, 2021 3:10 PM

Non-Linear State Space Control

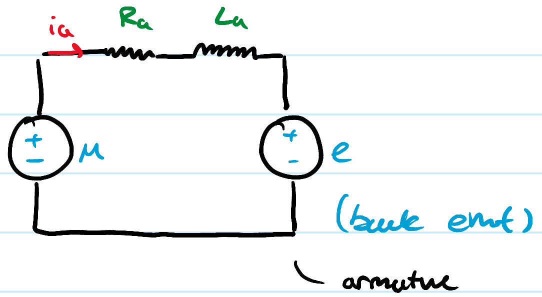
e.g. Permanent Magnet DC (PMDC)
↳ position and speed control



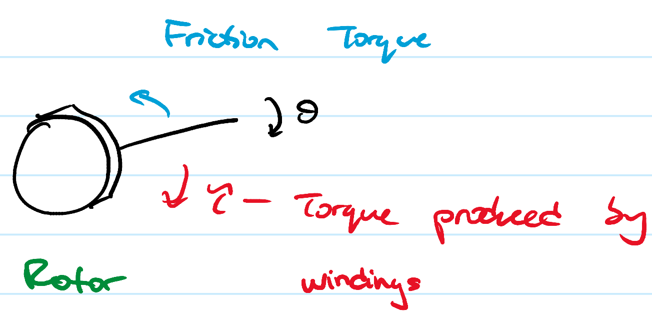
As the rotor rotates, windings move in a magnetic field (produced by PM's)

① Faraday's Law: Back EMF voltage induced in the windings

② Lorentz's Law: Current in the windings produces a force (torque) due to B field from PM's



Electrical



Rotor

$e = \underbrace{k_e}_{\text{back emf constant}} \dot{\theta}$

$\tau = \underbrace{k_t}_{\text{motor torque constant}} i_a$

Armature:

$$L_a \frac{di_a}{dt} + R_a i_a + \underbrace{e}_{k_e \dot{\theta}} = u$$

Rotor

$$I\ddot{\theta} = -\tau_{friction} + \underbrace{\tau}_{k_t i_a}$$

← Coupled →

assume $\tau_{friction} = b\dot{\theta}$ (viscous friction)

Choose some states:

$x_1 = \theta \quad , \; x_2 = \dot{\theta} \quad , \; x_3 = i_a$

$\dot{x}_1 = \dot{\theta} = x_2$

$\dot{x}_2 = \ddot{\theta} = \frac{1}{I}(-b\dot{\theta} + k_t i_a) = -\frac{b}{I}x_2 + \frac{k_t}{I}x_3$

$\dot{x}_3 = \frac{di_a}{dt} = -\frac{R_a}{L_a}i_a - \frac{k_e}{L_a}\dot{\theta} + \frac{1}{L_a}u = -\frac{R_a}{L_a}x_3 - \frac{k_e}{L_a}x_2 + \frac{1}{L_a}u$

$y = \theta = x_1$ (position control)

$y = \dot{\theta} = x_2$ (speed control)

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -b/I & k_t/I \\ 0 & -k_e/L_a & -R_a/L_a \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1/L_a \end{bmatrix} u$$

$y = [1 \quad 0 \quad 0]x$ or $y = [0 \quad 1 \quad 0]x$

LTI in state space model!

Summary:

Ex1

$\dot{v} = -\frac{B}{M}v + \frac{1}{M}u$ (neglecting disturbance)

$y = v$

LTI in both I/O and state space

Ex2

$LC\frac{d^2y}{dt^2} + RL\frac{dy}{dt} + y = u$ LTI I/O Model

$$\dot{x} = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} u$$

LTI state space model

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

Ex3

$$\dot{x}_1 = x_2$$

(neglecting disturbances)

$$\dot{x}_2 = -\frac{mgl}{I}\sin(x_1) + \frac{1}{I}u$$

$$y = x$$

Nonlinear State Space Model

Control Systems Models

↳ Non-Linear Time Invariant State Space Models

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad \dot{x}_i = f_i(x_1, \dots, x_n, u_1, \dots, u_m) \quad i = 1, \dots, n$$

$$y_j = h_j(x_1, \dots, x_n, u_1, \dots, u_m) \quad j = 1, \dots, p$$

$$\dot{x} = f(x, u) \qquad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad u = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$$

$$y = h(x, u)$$

$$f = \begin{bmatrix} f_1(\,) \\ \vdots \\ f_n(\,) \end{bmatrix} \qquad h = \begin{bmatrix} h_1(\,) \\ \vdots \\ h_p(\,) \end{bmatrix}$$

↳ LTI state space models

$$\dot{x}_i = a_{i1}x_1 + \dots + a_{in}x_n + b_{i1}u_1 + \dots + b_{im}u_m \qquad i = 1, \dots, n$$

$$y_j = c_{j1}x_1 + \dots + c_{jn}x_n + d_{j1}u_1 + \dots + d_{jm}u_m$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

↳ LTI Input / Output (I/O) Models

1 input (u) — 1 output (y)

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m u}{dt^m} + \dots + b_0 u$$