Lecture 20

October 22, 2021 3:10 PM

Basic (Standard) Control Problem

Stability:

$$E(s) = \frac{1}{1+CG} R(s) + \frac{-G}{1+CG} D(s)$$

$$V(s) = \frac{C}{1+CG} R(s) + \frac{1}{1+CG} D(s)$$

For stability, we need GG to be BIBO stable

↳ Poles of

$$\left\{ \underbrace{\frac{1}{1+CG}}_{1}, \underbrace{\frac{G}{1+CG}}_{2}, \underbrace{\frac{C}{1+CG}}_{3} \right\} \text{ in OLHP}$$

Thus, we need zeroes of $1+CG$ in OLHP

↳ Q: Are there other poles showing up in 2, 3?

Perhaps, due to pole / zero cancellations

Option 1 A zero of C gets cancelled by a pole of G

Option 2 vice versa

Option 1

$$C(s) = (s-p)\, C'(s) \qquad G(s) = \frac{1}{s-p} G'(s)$$

The poles of 1 are the zeroes of $1+CG$

The poles of ① are the zeroes of $1+CG$

↳ = zeroes of $1+C'G'$

$$\frac{G}{1+CG} = \frac{G'(s-p)}{1+C'G'} = \frac{G'}{(s-p)(1+C'G')}$$

Pole/Zero Cancellation at $s=p$ shows up at a pole of ③

Option 2

$C(s) = \frac{C'(s)}{s-p}$ $G(s) = (s-p)\,G'(s)$

$$\frac{C}{1+CG} = \frac{C'(s)/(s-p)}{1+C'G'} = \frac{C'(s)}{(s-p)(1+C'G')}$$

Pole/Zero cancellation at $s=p$ shows up!

Conclusion:

The poles of the G4 TFs are:

$\{$ zeroes of $1+CG \} \cup \{$ any pole/zeroes that are cancelled in $CG \}$

For stability of the G4 we need

1. zeroes of $1+C(s)G(s)$ in OLHP
2. Any cancellations in $C(s)G(s)$ must occur in OLHP

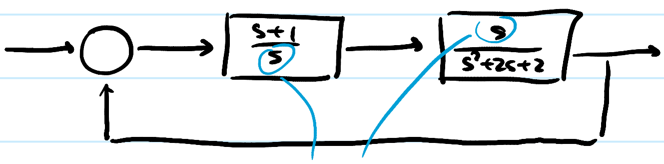
THM1

↳ The G4 are BIBO stable if the roots

THM1

↳ The GH are BIBO stable if the roots of $1+CG$ are in OLHP and there are no illegal cancellations in CG

(pole/zero cancellations in CRHP $\{s \in \mathbb{C}, \mathrm{Re}(s) \geq 0\}$



illegal cancellation!

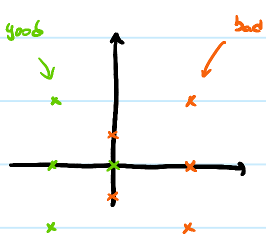
Stop w/ stability for now

↳ Move to disturbance rejection, tracking performance

Fundamental Tool: FVT

if $\underbrace{\lim_{t \to \infty} f(t) = f(\infty) \text{ exists}}$, then $f(\infty) = \lim_{s \to 0} sF(s)$

Q: when is this true?



A: when the following condition (holds

↳ $F(s)$ has poles with real part ≤ 0 and at most one pole with real part $= 0$ (at the origin

Use Routh!

$$Y(s) = \frac{s+3}{s(\underbrace{s^3 + 2s^2 + 5s + 1}))}$$

pole at 0 are these in OLHP?

$\Longrightarrow$

s^3	1	5	0
s^2	2	1	0
s	9/2	0	0
1	1	0	0

$\therefore$ Condition C holds, can use FVT

$$y(\infty) = \lim_{s \to 0} sY(s) = \frac{3}{1} = 3$$

Tracking Performance (set $d(t) \equiv 0$)

Assume G, C are BIBO stable

↳ roots of $1+CG$ in OLHP

Q: Find conditions under which $e(t) \to 0$ when $r(t) \in \mathcal{R}$

Assume (for now) that $r(t)$ is a polynomial of degree $\leq k-1$

$$r(t) = c_0 + c_1 t + \dots + c_{k-1} t^{k-1} \iff R(s) = \frac{N_R(s)}{s^k}$$

$$\deg(N_R) \leq k-2$$

$$E(s) = \frac{1}{1+CG} \frac{N_R}{s^k}$$

In order for $e(t) \to 0$ as $t \to \infty$, we need all poles of $E(s)$ to be in OLHP

↳ so all singularities at $s=0$ coming from $R(s)$ must be cancelled out

$$C = \frac{N_C}{P_C} \qquad G = \frac{N_G}{P_G}$$

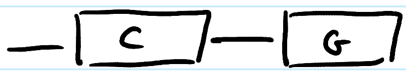
By assumption, the roots of $1+CG$ are in OLHP roots of $1+CG$ = roots of $1+\frac{N_C N_G}{D_C D_G}$

= roots of $D_C D_G + N_C N_G$

$$E(s) = \frac{1}{1+\frac{N_C N_G}{D_C D_G}} \cdot \frac{N_R}{s^k} = \frac{D_C D_G}{N_C N_G + D_C D_G} \cdot \frac{N_R}{s^k}$$

$e(t) \to 0$ iff $D_C D_G$ contain k zeroes at $s=0$

iff the product CG has at least k poles at 0



What if CG has $\ell \leq k-1$ poles at 0?

Then $$E(s) = \frac{(D_C' D_G')s^\ell}{N_C N_G + D_C D_G} \frac{N_R}{s^k} = \frac{}{s^{k-\ell}(N_C N_G + D_C D_G)}$$

E will have $k-\ell$ poles at 0

If $\ell = k-1$ $e(\infty)$ exists but is nonzero

If $\ell < k-1$ $e(\infty)$ does not exist $e(t) \to \infty$