Lecture 14

October 8, 2021 3:10 PM

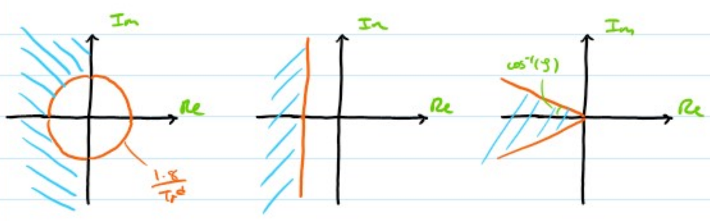
Recall the setup:

$$\frac{1}{s} \longrightarrow \boxed{\frac{a}{s^2+2s\zeta\omega_n+\omega_n^2}} \longrightarrow Y(s)$$

$$T_r \doteq \frac{1.8}{\omega_n}$$

$$\zeta = \frac{-\ln(\%OS)}{\sqrt{\pi^2+\ln^2(\%OS)}} \qquad T_s \doteq \frac{4}{\zeta\omega_n}$$

$$\%OS = \exp\left(-\zeta\pi/\sqrt{1-\zeta^2}\right)$$



$T_r \le T_r^d$ $\qquad$ $T_s \le T_s^d$ $\qquad$ $\%OS \le \%OS^d$

e.g.

Proportional control design for speed control of PMDC

↳ can we improve performance using closed loop control?

Open Loop (No Control)

$G(s) = \frac{1}{s^2+2s+2}$ $\qquad$ $U(s) = 1/s$

$V_0 \longrightarrow \boxed{\text{PMDC}} \longrightarrow y(t)$

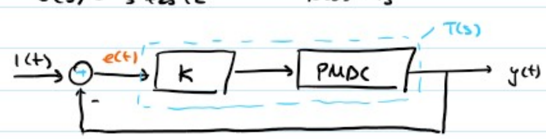
$s^2+2\zeta\omega_n s+\omega_n$

$\omega_n = \sqrt{2}$ $\qquad$ $\zeta = \frac{1}{\sqrt{2}} \in (0,1)$

$\boxed{T_r = \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{2}} \doteq 1.27\text{ s}}$

Closed Loop Control

$G(s) = \frac{1}{s^2+2s+2}$ $\qquad$ $R(s) = 1/s$



$E(s) = \frac{1}{1+T(s)} \cdot R(s)$ $\qquad$ $T(s) = K \cdot \frac{1}{s^2+2s+2}$

$E(s) = \frac{s^2+2s+2}{(s^2+2s+2+K)s}$ ← Examine using FVT

FVT: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

$e(\infty) = \frac{2}{K+2}$ $\qquad$ $e \to 0$ as $K \to \infty$ (good!)

$$Y(s) = \frac{T(s)}{1+T(s)}R(s) = \frac{K/(s^2+2s+2)}{1+K/(s^2+2s+2)} \cdot 1/s$$

$$= \frac{K}{s(s^2+2s+\underbrace{2+K}_{\omega_n=\sqrt{2+K}})} \quad \text{compare to} \quad \frac{a}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$\zeta = \frac{1}{\sqrt{2+K}}$

$\boxed{T_r} \doteq \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{K+2}} \xrightarrow[K\to\infty]{} \boxed{0}$ $\qquad$ (good!)

$T_r = \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{2}} \doteq 1.27\ s$

$T_s = \frac{4}{\zeta\omega_n} = 4\ s$

$\%OS = e^{-\pi} = 4\%$

$T_r \doteq \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{k+2}} \xrightarrow[k\to\infty]{} 0$ (good!)

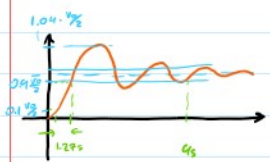
$T_s = \frac{4}{\zeta\omega_n} = 4\ s$ (same as before :( )

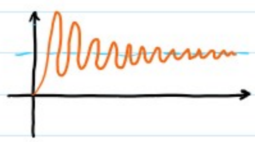
%OS : $\zeta \to 0$ as $k \to \infty$ (bad!!!)

↳ %OS → 100% as $k \to \infty$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = \sqrt{k+2}\,\underbrace{\sqrt{1-\frac{1}{k+2}}}_{\sqrt{\frac{k+1}{k+2}}} = \sqrt{k+1}$

$\omega_d \to \infty$ as $k \to \infty$

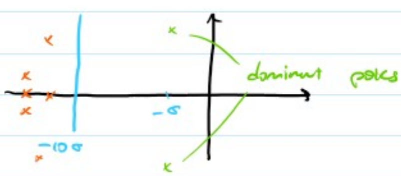




P control inadequate!!!

Relax Conditions

1. G(s) only has 2 complex conj poles
   - ↳ Can have more poles, as long as their real poles are much more negative than the real part of the 2 c.c. poles
   - ↳ real components of secondary poles at least 10 times more negative than dominant poles



2. G(s) has no zeroes
   - ↳ same story as for poles
   - ↳ zeroes in RHP change the sign of $y(\infty)$
     - ↳ called nonminimum phase