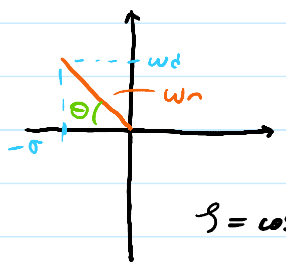
Lecture 12

October 5, 2021 3:06 PM

Idea for time response



$$Y(s) = \frac{c_1}{s+p} + \frac{as+b}{(s+\sigma)^2+\omega_d^2} + \dots +$$

$$(\sigma, \omega_d) \Longleftrightarrow (\zeta, \omega_n)$$

damping ratio natural frequency

$\zeta = \cos(\theta)$

$$\sigma = \zeta \omega_n$$

$$\omega_d = \sqrt{\omega_n^2 - \zeta^2\omega_n^2} = \omega_n\sqrt{1-\zeta^2}$$

$$\zeta = \frac{\sigma}{\sqrt{\sigma^2+\omega_d^2}} \qquad \omega_n = \sqrt{\sigma^2+\omega_d^2}$$

$$\frac{as+b}{(s+\sigma)^2+\omega_d^2} \Longleftrightarrow \frac{as+b}{s^2+2\zeta\omega_n s+\omega_n^2}$$

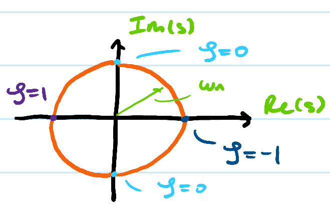
e.g.

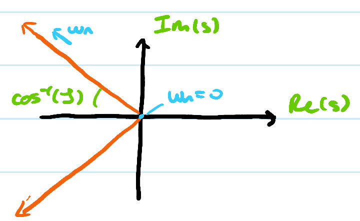
$$Y(s) = \frac{*}{s^2+2s+2} \qquad \omega_n^2 = 2 \qquad \omega_n = \sqrt{2}$$

$$\zeta\omega_n = 1 \qquad \zeta = 1/\omega_n = 1/\sqrt{2}$$

Ensure $\zeta \in [-1, 1]$

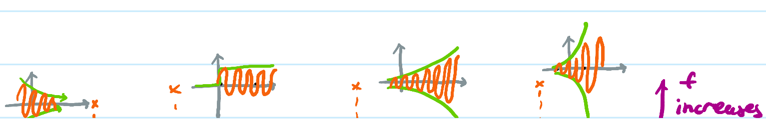
↳ otherwise poles are real





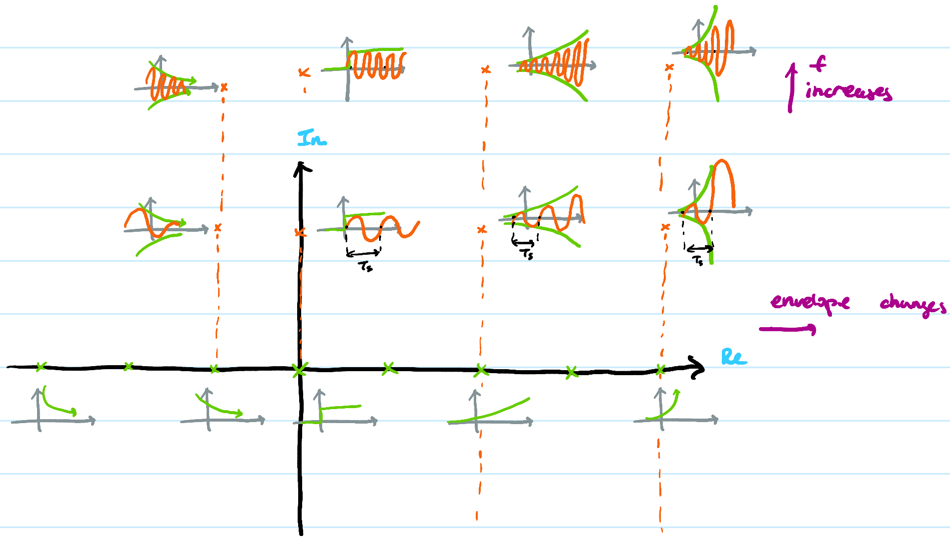
ω_n constant

ζ = constant $\in (0,1)$



Moving horizontally:

↳ envelope changes

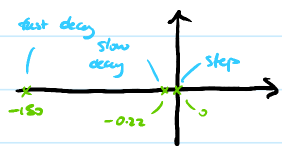


Moving horizontally:
↳ envelope changes
↳ frequency remains constant

Moving vertically
↳ envelope remains same
↳ frequency increases

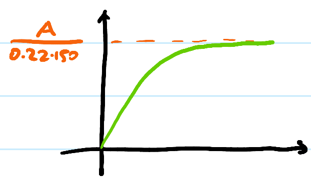
e.g. PMDC from lab 1

$$Y(s) = \frac{A}{s(s+0.22)(s+150)}$$

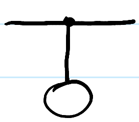


$$y(t) = (c_1 + c_2 e^{-0.22t} + c_3 e^{-150t})1(t)$$

FVT: $y(\infty) = \lim_{s\to 0} sY(s) = \frac{A}{0.22\cdot 150}$



e.g. Crane linearized at $\bar{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad \bar{u} = 0$



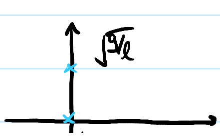
$\tilde{x} = x - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad \tilde{u} = u - 0$ $\quad \tilde{y} = y - 0$

$$\dot{\tilde{x}} = \begin{bmatrix} 0 & 1 \\ -g/\ell & 0 \end{bmatrix}\tilde{x} + \begin{bmatrix} 0 \\ -1/\ell \end{bmatrix}\tilde{u}$$

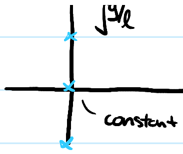
$$\tilde{y} = \begin{bmatrix} 1 & 0 \end{bmatrix}\tilde{x}$$

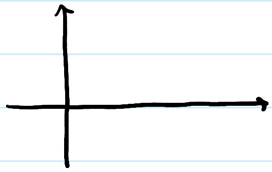
$\tilde{u}(t) = 1(t)$ $\qquad \tilde{Y}(s) = \frac{-1/\ell}{s^2 + g/\ell}\frac{1}{s}$

$$y(t) = [c_1 + A\sin(\sqrt{g/\ell}\,t + \phi)]1(t)$$



$\text{Res}(Y(s)e^{st}, s=0)$





$$\mathrm{Res}\left(Y(s)e^{st},\ s=0\right)$$

$$= \mathrm{Res}\left(\frac{-1/l}{s^2+g/l}\,\frac{1}{s}\,e^{st},\ s=0\right)$$

$$\frac{-1/l}{g/l} = -1/g$$