Lecture 19

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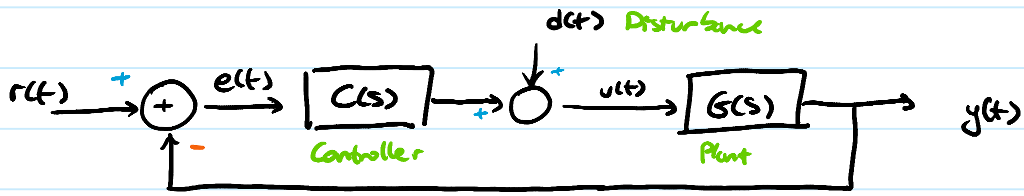
Recap:

THM 3. If $\dot{x} = Ax$ is asympt. stable, then $\forall (B,C,D)$, the TF $G(s) = C(sI-A)^{-1}B + D$ is BIBO stable

Routh Criterion: an algorithm that counts (# roots) of a real polynomial with real part > 0

Basic (Standard) Control Problem



$r(t)$: Reference (Input) Signal

$e(t) = y(t) - r(t)$: Tracking Error

$d(t)$: Disturbance

$u(t)$: Control Input

$y(t)$: Output Signal

e.g. Servo Motor

Load unknown: how much torque needed?

$r(t)$: desired position

$e(t)$: delta between desired and actual

$d(t)$: load torque/motion on motor shaft

$u(t)$: signal to motor

$y(t)$: output angle of servo

Assumption:

↳ $R(s)$ and $D(s)$ are rational, strictly proper

↳ poles on the imag axis

↳ poles of R, D fixed and given

↳ zeroes and gain of R, D arbitrary

$$R(s) = \frac{N_R(s)}{(s-p_1)(s-p_2)\cdots} \qquad D(s) = \frac{N_D(s)}{(s-q_1)(s-q_2)\cdots}$$

p_i, q_i on the imag axis and given

N_R, N_D are arbitrary poly of degree

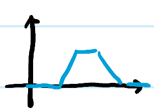
$\le \deg(\underbrace{D_R}_{\text{Denom of R}})-1$, $\le \deg(\underbrace{D_D}_{\text{Denom of D}})-1$ respectively

Ex1 : $r(t) = c$ $\quad$ c arbitrary

↳ or $d(t)$

$r(t) = c \implies R(s) = \frac{c}{s}$ $\qquad$ $c = N_R(s)$

Ex2 : $r(t) = c_1 + c_2 t$ $\quad$ c_1, c_2 arbitrary



$r(t) = c_1 + c_2 t \implies R(s) = \frac{c_1}{s} + \frac{c_2}{s^2}$

c_1, c_2 $\qquad$ $N_R(s)$ = arbitrary 1st order

↳ arbitrary

Ex3 : $d(t) = A\sin(\omega t + \phi)$ $\qquad \omega$ fixed

A, ϕ arbitrary

$$D(s) = \frac{\text{1st order poly}}{s^2 + \omega^2}$$

Ex4 : $d(t) = A\sin(\omega t + \phi) + C$ $\qquad \omega$ fixed

A, ϕ, C arbitrary

$$D(s) = \frac{\text{2nd order poly}}{s(s^2 + \omega^2)}$$

THE BASIC CONTROL PROBLEM

Let $\mathcal{R} = \{$ class of $r(t)$ whose $\mathcal{L}$ has desired fixed poles $\}$

$\mathcal{D} = \{$ class of disturbances whose $\mathcal{L}$ has desired fixed poles $\}$

Design a TF $C(s)$ meeting these three specs:

a) Stability for any bounded $(r(t), d(t))$, want $(e(t), u(t))$ to be bounded

b) Asymptotic Tracking Assume $d(t) \equiv 0$ then $\forall\, r(t) \in \mathcal{R}$, $e(t) \to 0$

c) Disturbance Rejection Assume $r(t) \equiv 0$ then $\forall\, d(t) \in \mathcal{D}$, $e(t) \to 0$

RMK:

by superposition, if (b)(c) hold, then

$\forall\, r(t) \in \mathcal{R}$, $d(t) \in \mathcal{D}$, $e(t) \to 0$

Requirement a) — Stability

$\swarrow$ set $R(s)=0$

$$E(s) = \frac{1}{1+CG} R(s) + \frac{-G}{1+CG} D(s)$$

$$U(s) = \frac{C}{1+CG} R(s) + \frac{1}{1+CG} D(s)$$

$\uparrow$ set $R(s)=0$

$$\left.\begin{aligned} E(s) &= \frac{1}{1+CG} R(s) + \frac{-G}{1+CG} D(s) \\ U(s) &= \frac{C}{1+CG} R(s) + \frac{1}{1+CG} D(s) \end{aligned}\right\} \text{gang of four TF's}$$

Req. A implies (BIBO stability) implies

$$\underbrace{\frac{1}{1+CG}}_{TF1}, \quad \underbrace{\frac{C}{1+CG}}_{TF2}, \quad \underbrace{\frac{G}{1+CG}}_{TF3}$$ are simultaneously BIBO stable

i.e. poles of TF 1, 2, 3 must be in OLHP

Poles of 1 are zeroes of $1 + C(s)G(s)$

Poles of 2

e.g.

$C = 1/s \quad G = \frac{s+2}{s+1} \quad \frac{1/s}{1+\frac{1}{s}\frac{s+2}{s+1}} = \frac{1}{s+\frac{s+2}{s+1}} = \frac{s+1}{s(s+1)+(s+2)}$