Lecture 21

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THM1: conditions for GH to be BIBO stable
↳ no illegal cancellations in CG
↳ zeroes of 1+CG in OCHP

spec(b): Tracking

Assume $R(s) = \frac{N_R(s)}{s^k}$, GH are BIBO stable

Then $e(\infty) \to 0$ iff $C \cdot G$ has k poles at 0

Terminology: A TF has type $l \in \mathbb{N}$ if it has exactly l poles at 0

Proposition 1: Suppose $R(s) = \frac{N_R(s)}{s^k}$, $d(t) \equiv 0$, ← No disturbance
and that GH are BIBO stable

Then $e(t) \to 0$ as $t \to \infty$ iff CG has Type k
↳ Type k : CG has k poles at zero

If $C \cdot G$ has type $k-1$ then $e(\infty)$ is nonzero, finite
If CG has type $k-2$ then $e(\infty)$ is unbounded

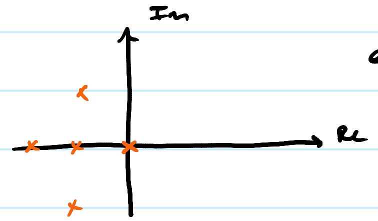
If:
$C \cdot G$ has type k, then $C \cdot G = \frac{C' \cdot G'}{s^k}$

$$E(s) = \frac{1}{1 + C(s)G(s)} \cdot R(s) = \frac{1}{1 + \frac{C'(s)G'(s)}{s^k}} \cdot \frac{N_R}{s^k} = \frac{N_R(s)}{\underbrace{s^k + C'(s)G'(s)}}$$

roots coincide with those
of 1+CG (which are in OCHP)

$C \cdot G$ has type $k-1$, then $CG = \frac{C'G'}{s^{k-1}}$

$$E(s) = \dots = \frac{N_R}{\underset{\text{pole at zero}}{s}\underbrace{(s^{k-1} + C'G')}_{\text{poles in OCHP}}}$$



$e(\infty) \neq 0$
↳ finite

Use FVT:

↳ $e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{\cancel{s} N_R}{\cancel{s}(s^{k-1} + C'G')}$

If CG has type $k-1=0$, CG has no poles at 0, $CG = C'G'$

$$e(\infty) = \begin{cases} \frac{N_R(0)}{1+C'(0)G'(0)} = \frac{N_R(0)}{1+C(0)G(0)}, & k=1 \\ \frac{N_R(0)}{C'(0)G'(0)}, & k>1 \end{cases}$$

Terminology:

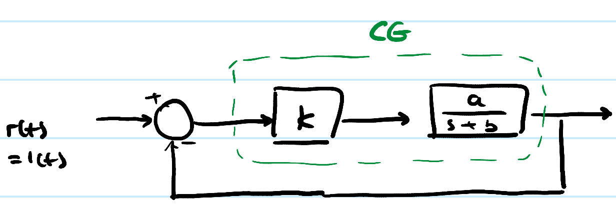
↳ If CG has no poles at zero (type 0)
  ↳ The number $C \cdot G$ is denoted K_p and called the position constant. Also called DC gain

↳ If $C \cdot G$ is type 1
  ↳ The number $\lim_{s\to 0} sC(s)G(s) = K_v$ (velocity constant)

↳ If CG is type 2
  ↳ The number $\lim_{s\to 0} s^2 C(s)G(s) = K_a$ (acceleration constant)

e.g. Speed Control of PMDC motor (simplified)

$G(s) = \frac{a}{s+b}$, $a, b > 0$



$R(s) = 1/s$
pole at zero

↳ need $C \cdot G$ to be type 1

But $C \cdot G$ is type 0

If the G4 are BIBO stable, then $e(\infty) = \frac{1}{1+k_p}$

$k_p = k\frac{a}{b}$ $\qquad e(\infty) = \frac{1}{1+\frac{ka}{b}} \to 0$ as $k \to \infty$

Need CG to have type 1, so $C(s)$ MUST have type $= 1$

$C(s) = \frac{c_1 + c_2 s}{s} = c_1 \cdot 1/s + c_2$ $\qquad e \rightarrow \boxed{C(s)} \rightarrow u$

This is called a PI Controller! ← Proportional - Integral

$$u(t) = \underbrace{c_1 \int e(\tau)\, d\tau}_{\text{Integral}} + \underbrace{c_2 e(t)}_{\text{Proportional}}$$

Check Stability (Prop 1 says if G4 BIBO stable, then $e(t) \to 0$ as $t \to \infty$)

Check THM 1

↳ are there illegal cancellations in CG?

↳ No, G has no zeroes

↳ are the roots of $1 + CG = 1 + \underbrace{\frac{c_1 + c_2 s}{s}}_{C} \underbrace{\frac{a}{s+b}}_{b}$ in OLHP?

|| roots of $s^2 + (b + ac_2)s + ac_1 = 0$?

↳ Yes if $c_1, c_2 > 0$