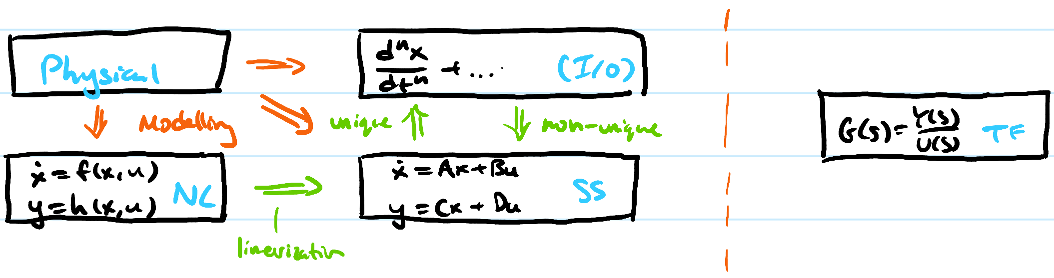
Lecture 10

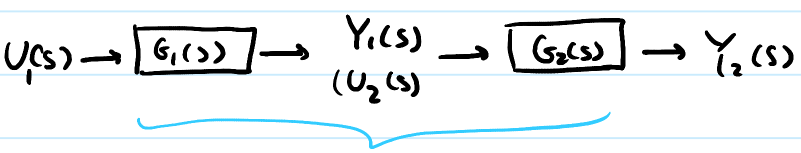
September 30, 2021 12:10 PM

Control Systems Models



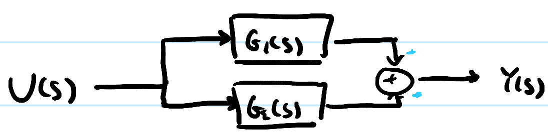
Block Diagrams

Cascade Connection



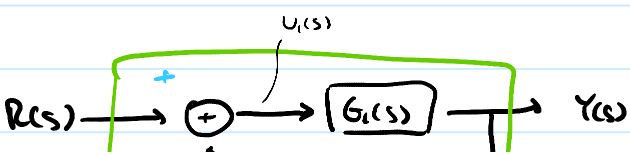
$$G(s) = G_1(s)G_2(s)$$

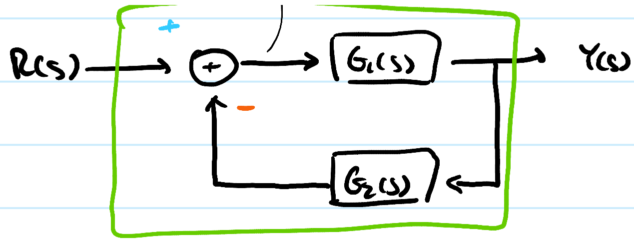
Parallel Interconnection



$$G(s) = G_1(s) + G_2(s)$$

Feedback Interconnect





Closed Loop System

$$Y(s) = G_1(s)\,U_1(s)$$

$$U_1(s) = R(s) - G_2(s)\,Y(s)$$

$$Y(s) = G(s)\left(R(s) - G_2(s)Y(s)\right)$$

$$\Downarrow$$

$$Y(s) = \frac{G_1(s)R(s)}{1 + G_1(s)G_2(s)}$$

$$\boxed{\text{TF: } \frac{Y(s)}{R(s)} = G(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}}$$

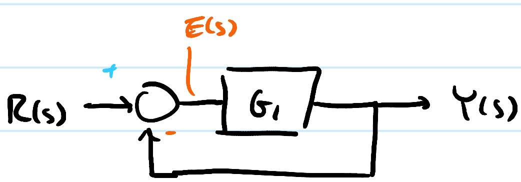
$$\begin{aligned} U_1(s) &= R(s) - G_2(s)Y(s) \\ &= R(s) - G_2(s)\frac{G_1(s)}{1 + G_1(s)G_2(s)}R(s) \\ &= \frac{1}{1 + G_1(s)G_2(s)}R(s) \end{aligned}$$

$$\boxed{U_1(s) = \frac{1}{1 + G_1(s)G_2(s)}R(s)}$$

Special Case

↳ $G_2(s) = 1$ (Unity Feedback)

$$Y(s) = \frac{G_1(s)}{1 + G_1(s)}R(s)$$



$$\begin{aligned} E(s) &= R(s) - Y(s) \quad \text{(Tracking Error)} \\ &= \frac{1}{1 + G(s)}R(s) \end{aligned}$$

Time Response

↳ output signal $y(t)$ in response to an input signal $u(t)$

Objective: Predict salient qualitative properties of $y(t)$ from $Y(s)$ without inverting $\mathcal{L}$ and with minimal computation

e.g. PMDC motor (speed control mode)

$$G(s) = \frac{K_t/(L_a J)}{s^2 + (b/J + R_a/L_a)s + (K_t K_e + bR_a)/(L_a J)} = \frac{1}{s^2+2s+2}$$

suitable choice of constants

$$Y(s) = G(s)\,U(s) \qquad u(t) = V_0\,1(t) \qquad U(s) = \frac{V_0}{s}$$

speed   voltage

Q: predict $y(t)$ qualitatively w/ minimal computation

$$Y(s) = \frac{V_0}{s(s^2+2s+2)} = \frac{V_0}{s(\underbrace{(s+1)^2+1}_{-1\pm i})}$$

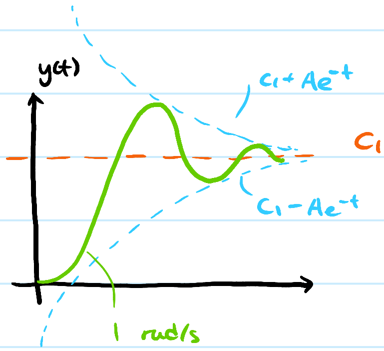
$$(s+a)^2 + b^2 = 0 \iff s = -a \pm ib$$

$$Y(s) = \frac{c_1}{s} + \frac{c_2 s + c_3}{(s+1)^2+1} = \frac{c_1}{s} + \frac{s}{(s+1)^2+1}$$

$$\frac{s}{(s+1)^2+1} = \underbrace{\frac{s+1}{(s+1)^2+1}}_{\mathcal{L}^{-1} = e^{-t}\cos(t)} - \underbrace{\frac{1}{(s+1)^2+1}}_{\mathcal{L}^{-1} = e^{-t}\sin(t)}$$

$$y(t) = c_1 1(t) + \underbrace{e^{-t}(\cos(t) - \sin(t))}_{Ae^{-t}(\sin(t+\phi))} 1(t)$$

$y(t) = C_1 + Ae^{-t}(\sin(t+\phi))$



$y(0) = 0$

$\dot{y}(0) = 0$