Lecture 2

September 14, 2021 3:10 PM

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Lecture 2

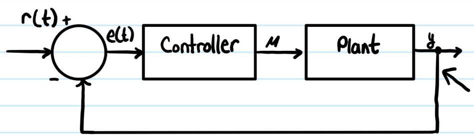
September 10, 2021 3:06 PM

Last Time:
- ↳ Course Organization
- ↳ Introductory examples → gymnast robot, snake robot

Today:
- ↳ The basic feedback loop
- ↳ Examples of control systems

Short-term objective: understand what are the different mathematical representations of control systems & adopt one of them for this course

The Basic Feedback Control Loop



"Plant" contains actuators, an actual system we want to control, & sensors

$u(t)$ – control input, the decision variable

$y(t)$ – output, the variable that we measure using sensors, and also the variable(s) that we want to control

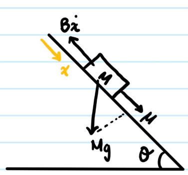
$r(t)$ → the reference signal. Want to ensure that

$$y(t) \xrightarrow[t\to\infty]{} r(t)$$

$e(t) := r(t) - y(t)$, the tracking error

Want to ensure that $e(t) \xrightarrow[t\to\infty]{} 0$

Example: Cruise Control Of A Car

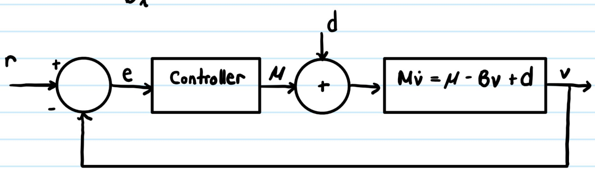


u – force impacted by the engine

$$M\ddot{x} = u - B\dot{x} + \underbrace{Mg\sin\theta}_{d}$$

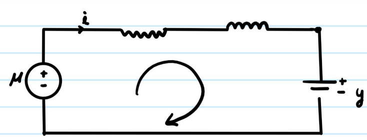
Problem: to make $u(t) := \dot{x}(t)$ converge to a desired value converge to a desired value

Viscous Factor: force $\propto$ speed
$= B\dot{x}$



$$M\ddot{x} = u - B\dot{x} + d \iff M\dot{v} = u - Bv + d$$

Example 2: Voltage ~~Current~~ Control of An RLC Circuit



Problem: design $u(t)$ such that $y(t) \xrightarrow[t \to \infty]{} C\sin(\omega t + \phi)$

KVL: $u - Ri - L\frac{di}{dt} - y = 0$

KCL: $\boxed{i = C\dot{y}}$

$$u - RC\dot{y} - LC\ddot{y} - y = 0$$

$$\boxed{LC\ddot{y} + RC\dot{y} + y = u} \quad (1)$$ Input - Output model

Choose a state, (x_1, x_2), with $x_1 := y$, $x_2 := i$

$$\dot{x}_1 = \dot{y} = \frac{1}{C} i = \frac{1}{C} x_2$$

$$\dot{x}_2 = \frac{di}{dt} = \frac{1}{L} y - \frac{R}{L} i + \frac{1}{L} u = -\frac{1}{L} x_1 - \frac{R}{L} x_2 + \frac{1}{L} u$$

$$\dot{x}_1 = \frac{1}{C} x_2$$

$$\dot{x}_2 = -\frac{1}{L} x_1 - \frac{R}{L} x_2 + \frac{1}{L} u \qquad x := \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$y = x_1$$

$$\boxed{\begin{aligned} \dot{x} &= \overbrace{\begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}}^{A} x + \overbrace{\begin{bmatrix} 0 \\ 1/L \end{bmatrix}}^{B} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}} \quad (2)$$ State - space representation

(2) is a special case of:

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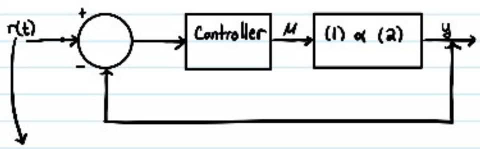
$$\dot{x} = Ax + B\mu$$
$$y = Cx + D\mu$$

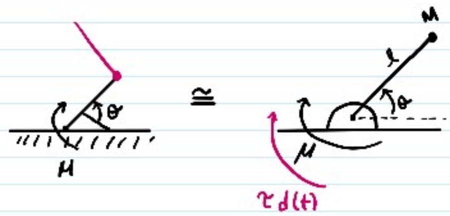
general state-space representation



$C\sin(\omega t + \phi)$

Example 3: Stance control of a ==plastic bag== prosthetic leg

Problem: regulate θ to $\pi/2$



$$I\ddot{\theta} = -Mgl\sin\theta$$
$$= -\mu - \tau_d(t)$$

moment of inertia around pivot

Choose A State: $x_1 = \theta$
$x_2 = \dot{\theta}$

Northern State Space Model

Nonlinear State Space Method

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \ddot{\theta} = -\frac{Mgl}{I}\sin(x_1) - \frac{1}{I}\mu - \frac{1}{I}\tau_d(t) \\ y &= x_1 \end{aligned}}$$