L06: EKF and localization

* Summary of L05

Transition function $x_t = g(x_{t-1}, u_t)$

discrete-time model obtained by integrating $\dot{x} = f(x, u)$ (NL)

Probabilistic model: - Add noise to the state/action space

- Linearize $g(\cdot)$

$p(x_t | x_{t-1}, u_t)$ - covariance transformation: $x_t \sim N(g(\mu_{t-1}, u_t), G_t \Sigma G_t^T + R)$

RBT 2D: $\qquad {}^0p = {}^gT_e \cdot {}^ep = {}^gT_a \, {}^aT_e \cdot {}^ep$

Chain of transformations

Observation function $\rightarrow$ Landmarks $\quad m_i = [m_{ix}, m_{jy}]^T$

$$z = h(x, m_i) = \begin{bmatrix} r \\ \phi \\ s \end{bmatrix} \quad \text{range, bearing, (appearance)}$$

(feature from sensor data)

$$p(z|x) \sim N(z; h(\mu_x, m), \Sigma_z)$$

* Kalman filter: Linear systems + Gaussian priors

$$\left.\begin{array}{ll} \text{I} & \bar{\mu}_t = A_t \mu_t + B_t \cdot u_t \\ \text{II} & \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{array}\right\} \text{prediction (marginalize)}$$

$$\left.\begin{array}{ll} \text{III} & K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q)^{-1} \\ \text{IV} & \mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t) \\ \text{V} & \Sigma_t = (I - K_t C_t) \bar{\Sigma}_t \end{array}\right\} \text{correction (conditioning)}$$

- Motion model: first order Taylor expansion

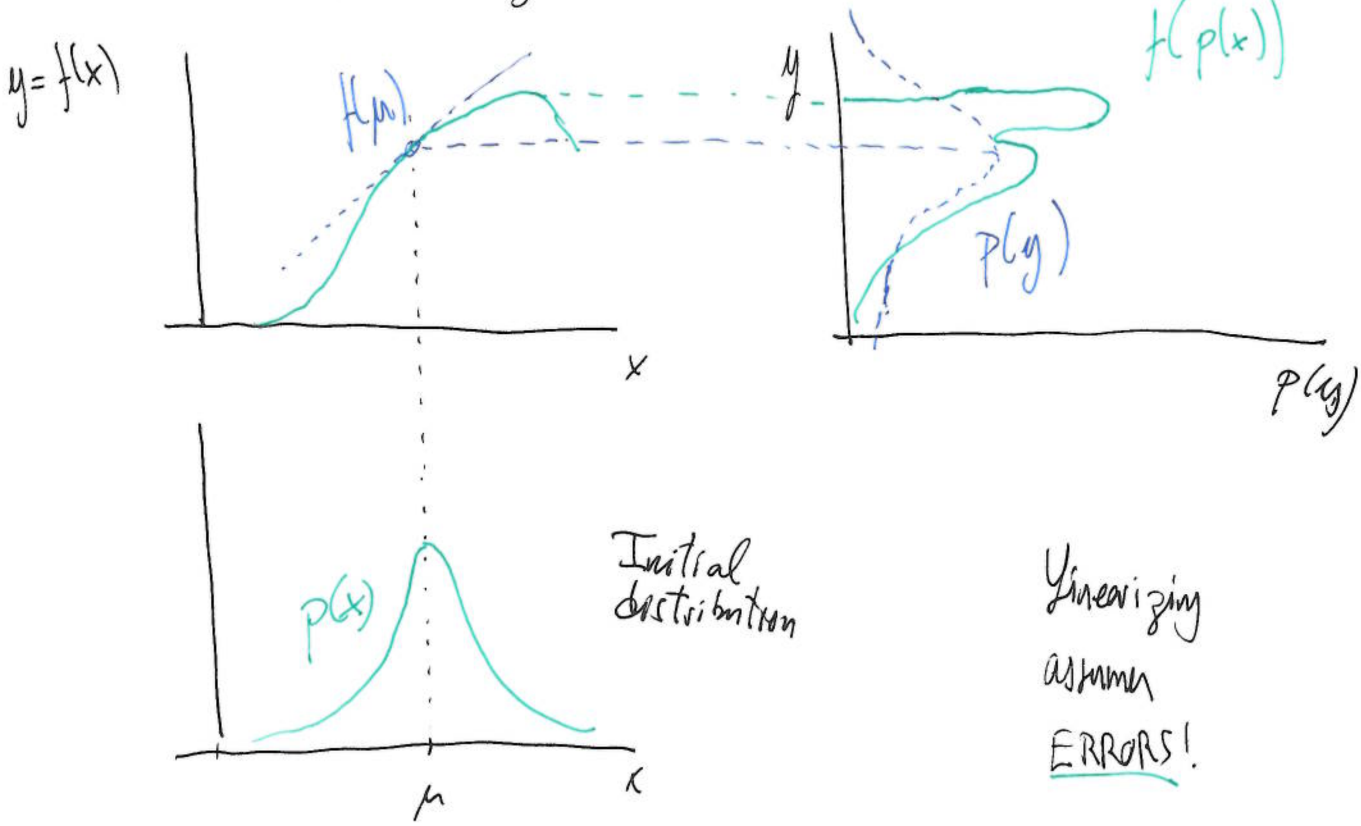
$$x_t = g(x_{t-1}, u_t, \varepsilon_t) \simeq g(\mu_{t-1}, u_t) + \underbrace{\frac{\partial g}{\partial x_{t-1}}\Big|_{\mu_{t-1}}}_{G_t} (x_{t-1} - \mu_{t-1}) + \varepsilon_t$$

L05 discussed on how to model $g(\cdot)$ for different systems and how to obtain the probabilistic Model.

- Sensor model: we observe features of landmarks (L06)

$$z_t = h(x_t, \eta_t) \simeq h(\mu_t) + \underbrace{\frac{\partial h}{\partial x_t}\Big|_{\mu_t}}_{H_t} (x_t - \mu_t) + \eta_t$$

- Intuition on linearization



Linearizing assumes ERRORS!

6.3

* Extended Kalman Filter

Inputs: $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$

1: $\bar{\mu}_t = g(\mu_{t-1}, u_t)$

2: $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$

3: $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$

4: $\mu_t = \bar{\mu}_t + K_t (\underbrace{z_t - h(\bar{\mu}_t)}_{\Delta z})$ $\quad \Delta z$ (Innovation vector)

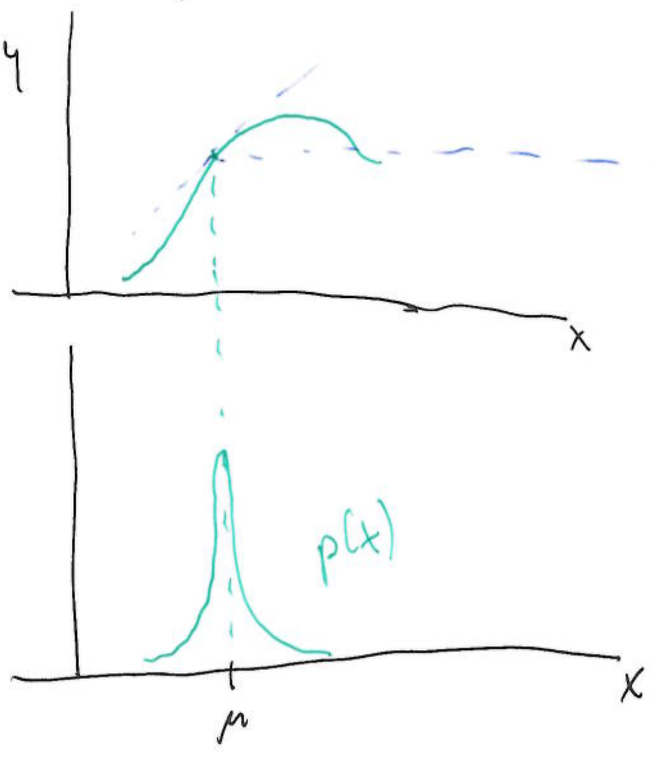
5: $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$

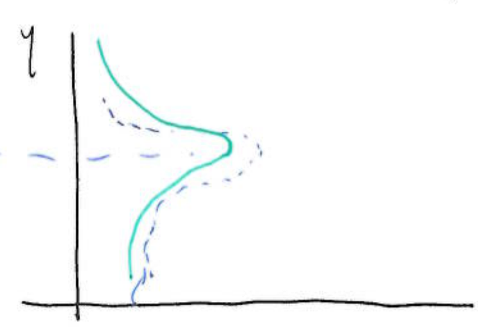
return μ_t, Σ_t $\quad (\mathcal{N}(\mu_t, \Sigma_t))$

Properties

- EKF is very efficient $O(k^{2.4} + n^2)$ (as KF)
- Not optimal, but in practice works well

Depends on the non-linearities (some are more problematic)





Compact initial distribution reduces the error because we are "near" the linearization point

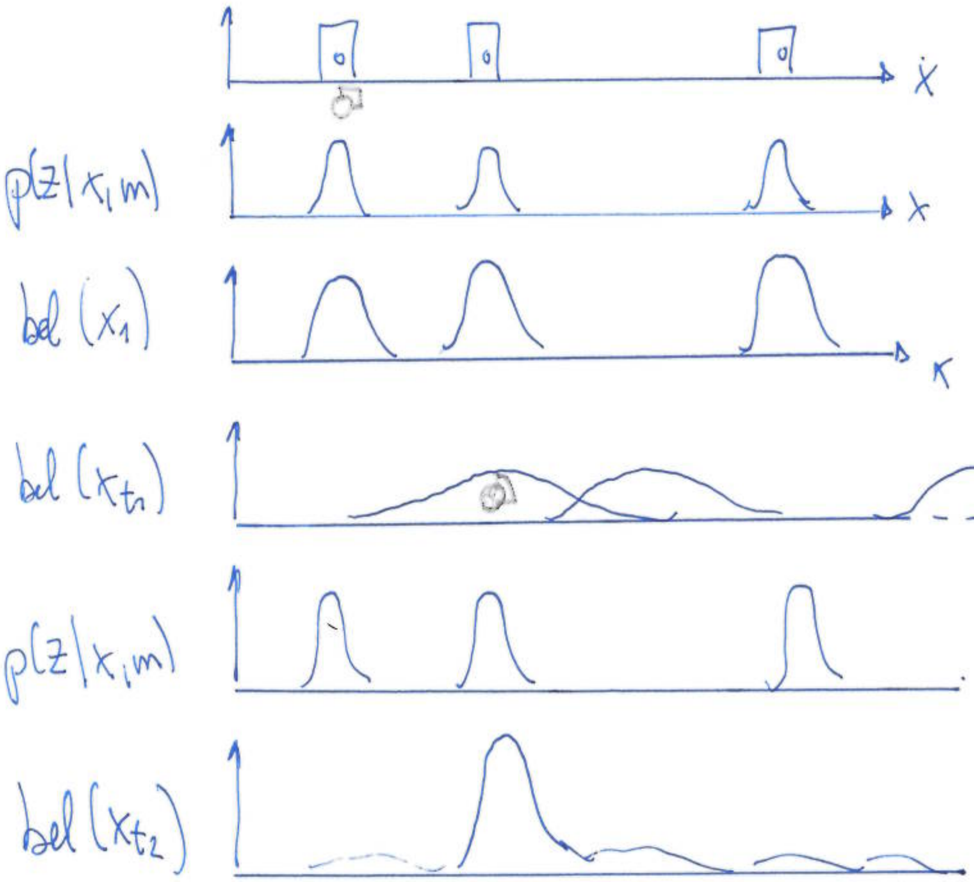
$(O(\|\Delta x\|))$

* Markov localization : directly uses Bayes filter.

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}, m)\, bel(x_{t-1})\, dx_{t-1}$$

$$bel(x_t) = \eta \cdot p(z_t \mid x_t, m)\, \overline{bel}(x_t)$$

Ex: 1d 3 doors problem.



any of the three doors could have been detected

Only propagation

Again a door is detected

Given the previous bel the robot is better localiz[ed]

* Localization problems (taxonomy)

- local , (position tracking) VS global - x_0 unknown
  given x_0 - Kidnapped problem
  x_t can change at any time.

- Static vs Dynamic (moving furniture, doors, snow, ...
- Passive vs Active (exploration, belief planning)
- Single-robot vs Multi-robot

* EKF localization

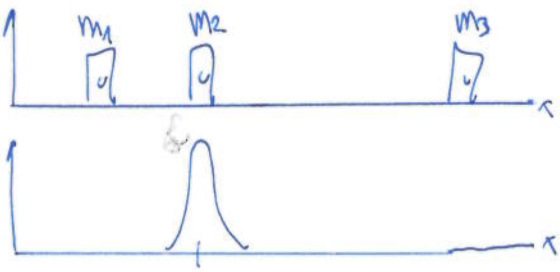
$p(z|x)$

but Gaussian are unimodal! (on 3 doors we used a multi-mod

we need to solve the data association problem landmark - observed

Ex:

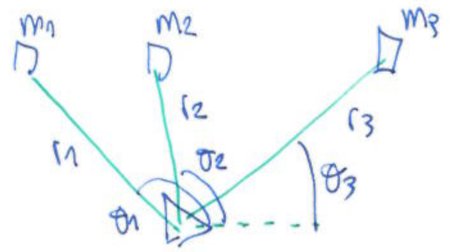
$p(z|x, m, c_j)$



We will assume known correspondences

$c_j = j$ (from landmark m_j)

Ex: 2D



3 observations of each landmark

$$p(z|x, m, c) = \prod_{j=1}^{3} p(z_j|x, m, c_j)$$

Algorithm: EKF localization with known correspondences

Inputs: $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, m$ (Prob Rob 204)

(205) 1: $G_t = \dfrac{\partial g(x_{t-1}, u_t)}{\partial x_{t-1}}$, $V_t = \dfrac{\partial g(x_{t-1}, u_t)}{\partial u_t}$, $M_t^{arc} = \begin{bmatrix} \alpha_1 v_t^2 + \alpha_2 w_t^2 & 0 \\ 0 & \alpha_3 \cdot v_t^2 + \alpha_4 w \end{bmatrix}$

(arc circular model ↑)

2: $\bar{\mu}_t = g(\mu_{t-1}, u_t)$ (I)

3: $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + \underbrace{V_t M_t V_t^T}_{R_t}$ (II)

4: $Q_t = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}$ Range and bearing observation cov. noise.
Known correspondences $\Rightarrow$ $\sigma_s^2 = 0$ (eliminated)

5: for $\{ i \mid z_t^i = [r_t^i, \phi_t^i]^T \}$:

6: $$\hat{z}_t^i = \begin{bmatrix} \overbrace{\sqrt{(m_{j,x} - \bar{\mu}_{t,x})^2 + (m_{j,y} - \mu_{t,y})^2}}^{=\sqrt{q}} \\ \operatorname{atan2}(m_{j,y} - \bar{\mu}_{t,y}, m_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{bmatrix} \quad \text{(L06)}$$

7: $$H_t^i = \left.\frac{\partial h(x_t)}{\partial x_t}\right|_{\bar{\mu}_t} = \begin{bmatrix} -\frac{(m_{j,x} - \bar{\mu}_{t,x})}{\sqrt{q}} & -\frac{(m_{j,y} - \mu_{t,y})}{\sqrt{q}} & 0 \\ \frac{m_{j,y} - \bar{\mu}_{t,y}}{q} & -\frac{(m_{j,x} - \bar{\mu}_{t,x})}{q} & -1 \end{bmatrix}_{2\times 3}$$

8: $S_t^i = H_t^i \bar{\Sigma}_t (H_t^i)^T + Q_t$

9: $K_t^i = \bar{\Sigma}_t (H_t^i)^T (S_t^i)^{-1}$

10: $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$ innovation vector for z_t^i

11: $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$

12: endfor

13: $\mu_t = \bar{\mu}_t$ $\left(\mu_t = \bar{\mu}_t + \sum_i K_t^i (z_t^i - \hat{z}^i)\right)$

14: $\Sigma_t = \bar{\Sigma}_t$

15: $p_{z_t} = \prod_i \eta \cdot \exp\left\{ -\frac{1}{2} (z_t^i - \hat{z}_t^i)^T (S_t^i)^{-1} (z_t^i - \hat{z}_t^i) \right\}$

Q: Why are we updating the prediction belief $\overline{bel}(x_t)$ I times? Because we assume $\{z_t^i\}$ are independent.

$$bel(x_t) = p(z \mid x, m, c) \cdot \overline{bel}(x_t)$$

$$= \prod p(z^i \mid x, m, c)\, \overline{bel}(x_t)$$

$$= p(z^1 \mid x, m, c) \cdot$$

$$p(z^2 \mid x, m, c)$$

$$\vdots$$

$$p(z^I \mid x, m, c)\, \overline{bel}(x_t)$$

summation of $K_t^i(z^i - \hat{z}^i)$

recursive conditioning

$\prod^{I-1} p(\,)N(\,)$

conditioning a joint Gaussian.