L12 Incremental SAM and Pose SLAM

* Summary of L11 $\sqrt{SAM}$ (Dellaert '2006)

  - Cholesky $A^TA = \Lambda = R^TR$ (R upper triang.)

    solve $\left.\begin{array}{l} Ry = A^Tb \\ R^T\delta = y \end{array}\right\}$ backsubstitution + forward - substitution

  - QR: $Q^TA = \begin{bmatrix} R \\ 0 \end{bmatrix}$, solve $R\delta = d$

  - Schur Λ_m diagonal $\Rightarrow$ easy to invert

    Eliminates all dependencies from landmarks + solve Cholesky

Importance of the order in the graph (colperm, coland)

* Incremental square root factorization (iSAM Kaess'2008)

As new observations are available, update A, R without recalculating everything.

QR factorization incrementally

$$A = QR \quad , \quad R = Q^T A$$

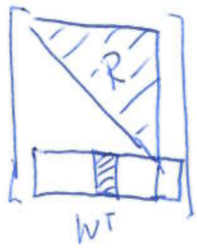
New obs: $\qquad b = \begin{bmatrix} d \\ e \end{bmatrix}$

$$\begin{bmatrix} Q^T & \\ & 1 \end{bmatrix} \begin{bmatrix} A \\ W^T \end{bmatrix} = \begin{bmatrix} R \\ W^T \end{bmatrix} \quad , \quad \begin{bmatrix} d \\ \gamma \end{bmatrix}$$

accordingly update the vector d.

odom: $\quad W^T = [\qquad G_i^{i-1} \quad -I \qquad]$ (sparse)

obs: $\quad W^T = [\quad H^i \qquad\qquad J^i \quad]$

$R' =$ 

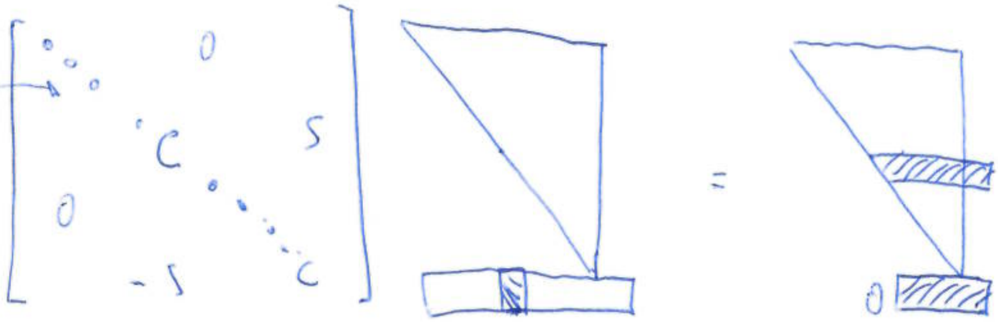
$\to$ ↑ new odometry we increase the dimensions

Givens rotations

$$\Phi = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \qquad \text{such as} \qquad \Phi \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} r & s \\ 0 & t \end{bmatrix}$$

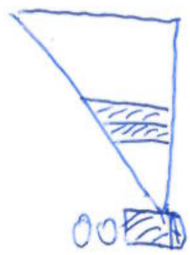
A sequence of Givens rotations produce the QR decomposition

1:



Φ . $\Phi Q^T = Q'^T$ is being updated.

We keep applying Givens below the diagonal until we get an upper triangular matrix



iSAM algorithm:

1: New information w^T, update $\begin{bmatrix} R \\ w^T \end{bmatrix}$

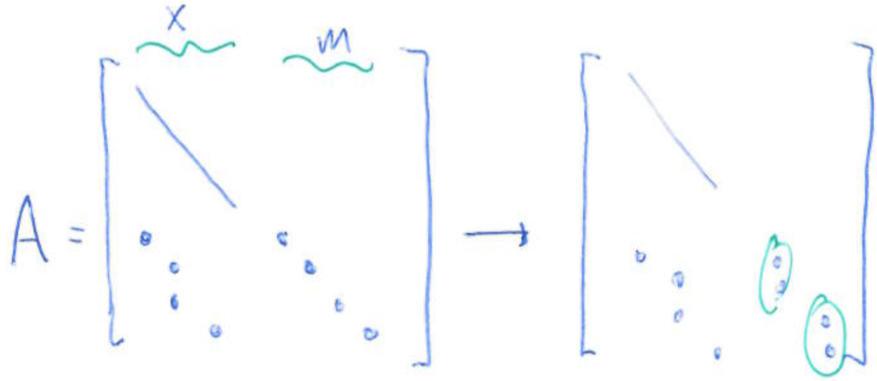
2: Givens rotations until R' upper triangular. Update d'

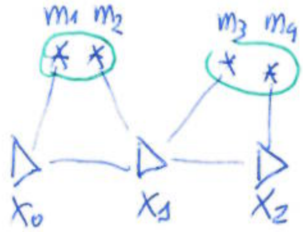
3: Solve $R' \delta = d'$

Discussion

- It is only possible for some time, eventually we need recalculate R, A
- The ordering is important for next incremental poses. May produce fill-ins → iSAM2 (Kaess 2011) and the Bayes tree graph.

* Data Association in SAM





Grouping landmarks, undoing wrong correspondences, etc. implies a change on A. (EKF "must" filter correct c_t)

Grouping landmarks by a likelihood test + greedy but tractable.

$$\Delta_{j,k} = \begin{bmatrix} m_k - m_j \\ m_j - m_k \end{bmatrix}$$

$p(a,b)$ a: k is j, b: j is k

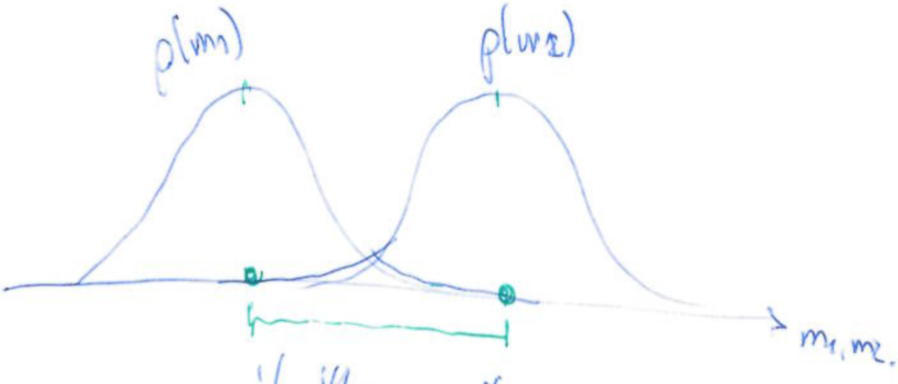
$$\|\Delta_{j,k}\|^2_{\Sigma_{j,k}} < \chi^2_{d,\alpha}$$

($d = 4$, α = confidence interval)

other alternatives might work, like conditioning the joint domain. Open problem.

→ We need the posterior covariance Σ_t, and from there marginalize everything except j, k landmarks.

Ex. 1d



if they are the same landmark Δ_m should be small.

* Covariance in $\sqrt{SAM}$

$\Sigma = \Lambda^{-1}$ (Λ is sparse but inverting is not efficient)

idea: No need to invert Λ, we have R.

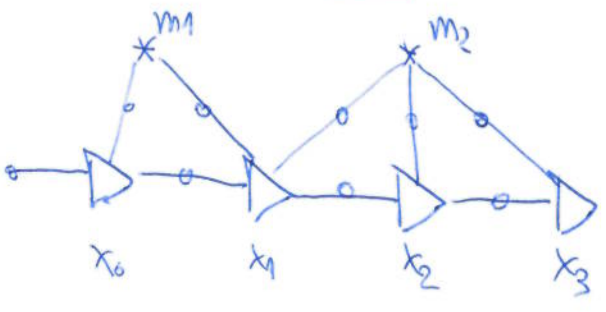
$$\Lambda = A^T A = R^T R = \Sigma^{-1}$$

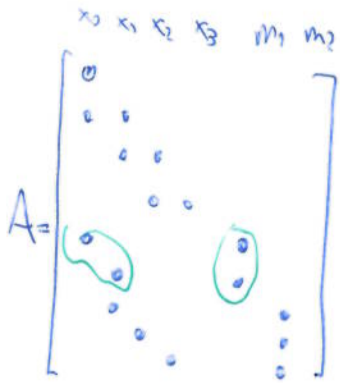
$$\Rightarrow R^T R \cdot \Sigma = I$$

$$\begin{cases} R^T \cdot Y = I \\ R \cdot \Sigma = Y \end{cases}$$

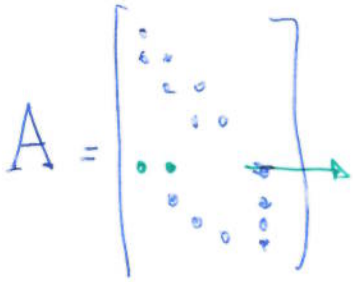
2 back substitutions, now of a matrix (set of vectors)

* Landmark elimination

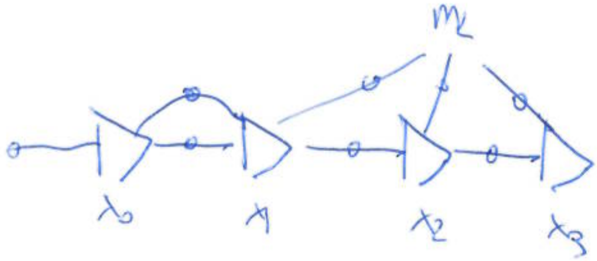




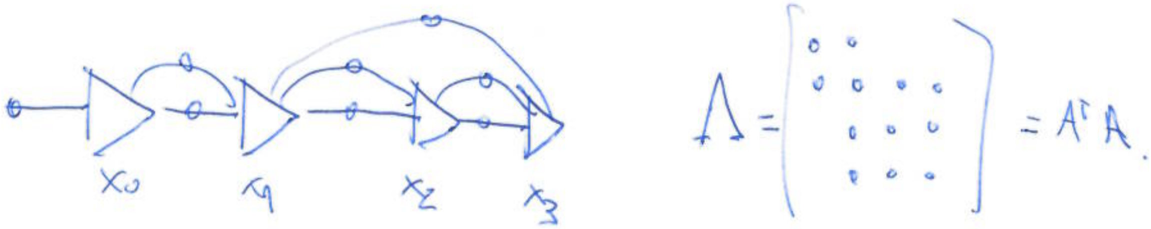
Eliminate m_1, will ~~...~~ add more factors to substitute the previous factors to m_1.



new "observation" relating x_0, x_1 through the removed m_1



eliminate m_2



all landmarks eliminated, but new (equivalent) factors have appeared to express the same relations

* Relation to the Schur complement.

$$\Lambda = A^T A = \begin{bmatrix} \Lambda_x & \Lambda_{xm} \\ \Lambda_{mx} & \Lambda_m \end{bmatrix} \qquad \delta = \begin{bmatrix} \boxed{\delta_x} \\ \delta_m \end{bmatrix}, \qquad A^T b = \begin{bmatrix} \boxed{b_x} \\ b_m \end{bmatrix}$$

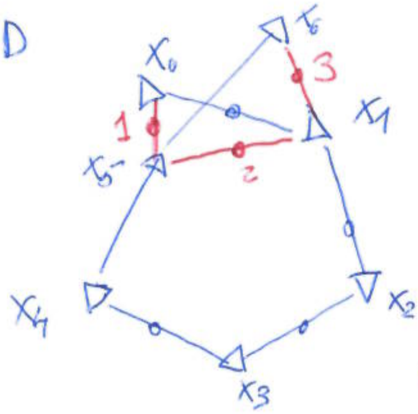
$$\left(\Lambda_x - \Lambda_{xm} \Lambda_m^{-1} \Lambda_{mx}\right) \delta_x = b_x - \Lambda_{xm} \Lambda_m^{-1} b_m$$

the Schur complement is equivalent to eliminate (marginalize) all landmarks in the information matrix. These new factors are a new way to express the marginalization.

12.7

* **Pose SLAM** Only poses are estimated

Ex: 2D



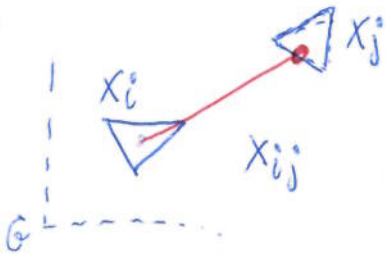
factors (observations):

- Odometry
- Relative pose observations (aka loop closure)

$$z^k = h(i,j) = \begin{bmatrix} \overbrace{R_i^T (x_j - x_i)}^{\Delta p \text{ in global frame}} \\ \theta_j - \theta_i \end{bmatrix}$$

(2D Pose)

"from pose i we observe pose j"



x_{ij} XYT param.

$$A = \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{ccccccc} \bullet & & & & & & \\ \bullet & \bullet & & & & & \\ & \bullet & \bullet & & & & \\ & & \bullet & \bullet & & & \\ & & & \bullet & \bullet & & \\ & & & & \bullet & \bullet & \\ & & & & & \bullet & \bullet \\ \bullet & & & & & \bullet & \\ & \bullet & & & & \bullet & \\ & \bullet & & & & & \bullet \end{array}\right] \end{array} \begin{array}{l} \left.\vphantom{\begin{array}{c}a\\a\\a\\a\\a\\a\\a\end{array}}\right\} \text{Odom} \\ \left.\vphantom{\begin{array}{c}a\\a\\a\end{array}}\right\} \text{Observations} \end{array}$$

2D Pose Jacobians

$$H_k^j = \begin{bmatrix} R_i^T & 0 \\ 0 & 1 \end{bmatrix} \qquad H_k^i = \left[\begin{array}{c|c} -R_i^T & \begin{matrix} -s\Delta x + c\Delta y \\ -c\Delta x - s\Delta y \end{matrix} \\ 0 & -1 \end{array}\right]$$

$\partial x_i \; \partial y_i \qquad \partial / \partial \theta_i$

1/ Pose SLAM obtained after marginalizing landmarks (not practical)

2/ Virtual observation between poses $\longrightarrow$ Registration problem (L14)