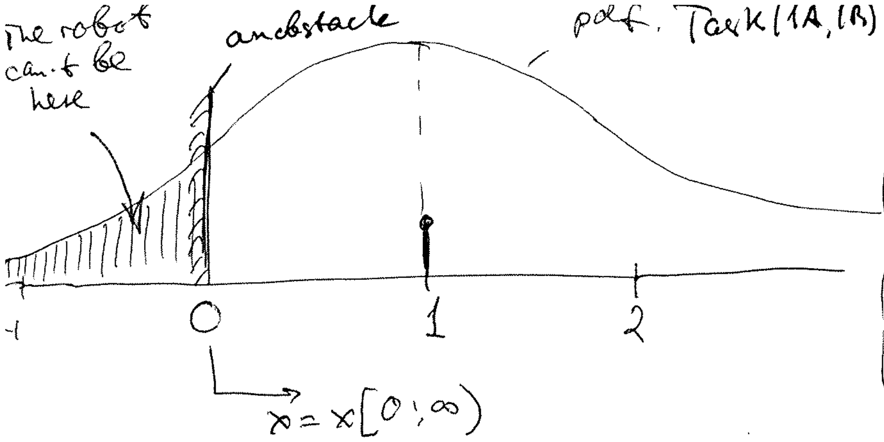
Task(1A,1B)



cdf returns a probability of random X is less than or equal to determined x.
We set x as 0. So, cdf(0) is needed to obtain a probability of the collision.

$\Rightarrow$ the probability of the collision a $\sim 0,16$.

Task(1C:1E)

$p(z|x) = \mathcal{N}(z; x, \sigma^2)$

$\sigma^2 = 0,2$

Bayes' theorem:

$$p(x|z) = \frac{p(z|x)\cdot p(x)}{p(z)}$$

?

The robot is ~1 meter away from the obstacle.
It's sensor ~~is depe~~ observation data is dependent on the robot's position. That is why $p(z|x) = \mathcal{N}(z; x, \sigma^2)$
As a result, we have an observation $z = 0,75$ $\Rightarrow$ $p(z|x) = \mathcal{N}(z; 0.75, 0,2)$ „likelyhood function"

Joint pdf: $p(x,y) = \underbrace{p(z|x)}_{\text{likelihood function}} \cdot \underbrace{p(x)}_{\text{prior pdf}}$

Task 2A:

I $\mu_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$\Sigma_x = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Conclusions:
1) „x" and „y" are uncorrelated
2) variance for „y" is higher than one for „x"
$\Rightarrow$ that is why the iso-contours are elongated along the axis „y".

II $\mu_x = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$
$\Sigma_x = \begin{bmatrix} 3 & -0,4 \\ -0,4 & 2 \end{bmatrix}$

Conclusions:
1) „x" and „y" are correlated, negative correlated
2) Iso-contours are elongated along the axis, where „x" reducing leads to „y" reducing.
$\rightarrow$ provided on a figure

III $\mu_x = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$
$\Sigma_x = \begin{bmatrix} 9.1 & 6 \\ 6 & 4 \end{bmatrix}$

Conclusions
1) „x" and „y" are positive correlated.
2) Cross-covariance values are much higher that in II case $\Rightarrow$ iso-contours are much more elongated along the axis, where increasing of „x" leads to increasing of „y"

Task 2C:

When random samples number increases, an iso-contour for the estimated Gaussian parameters becomes closer to the initial one. It occurs because the random samples ~~distribution~~ becomes ~~more~~ like more ~~ident~~ „independent and „identically distributed", because of their amount. As a result an error between iso-contours reduces. Sample mean and sample covariance are not real ones. They are just estimated. ~~But~~ However, larger the random samples number, better the accuracy.

Task 3_B:

$$\begin{bmatrix} x \\ y \end{bmatrix}_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_{t-1} + \begin{bmatrix} \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix}_t + \begin{bmatrix} \eta_x \\ \eta_y \end{bmatrix}_t$$

$$E\left\{\begin{bmatrix} x \\ y \end{bmatrix}_t\right\} = E\left\{\begin{bmatrix} x \\ y \end{bmatrix}_{t-1}\right\} + \begin{bmatrix} \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \underbrace{\begin{bmatrix} V_x \\ V_y \end{bmatrix}_t}_{\text{commanded values}} + E\left\{\begin{bmatrix} \eta_x \\ \eta_y \end{bmatrix}_t\right\}$$

→ const for t=t.

$$\Sigma\left\{\begin{bmatrix} x \\ y \end{bmatrix}_t\right\} = \Sigma\left\{\begin{bmatrix} x \\ y \end{bmatrix}_{t-1}\right\} + 0 + \Sigma\left\{\begin{bmatrix} \eta_x \\ \eta_y \end{bmatrix}_t\right\}$$

because $\begin{bmatrix} \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}\begin{bmatrix} v_x \\ v_y \end{bmatrix}_t = E\left\{\begin{bmatrix} \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}\begin{bmatrix} v_x \\ v_y \end{bmatrix}_t\right\}$

So, for each $t = t+1$ we can repeat equations above.

Task 3_D:

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_t = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_{t-1} + \begin{bmatrix} \cos\theta \cdot \Delta t & 0 \\ \sin\theta \cdot \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} V \\ \omega \end{bmatrix}}_{} + \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_\theta \end{bmatrix}$$

3 Dimensions, non-linear!!

Linearization:

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_t = \begin{bmatrix} x_{t-1} + V \cdot \cos\theta_{t-1} \cdot \Delta t \\ y_{t-1} + V \cdot \sin\theta_{t-1} \cdot \Delta t \\ \theta_{t-1} + \omega \cdot \Delta t \end{bmatrix}$$

Jacobian:

$$J = \left. \begin{bmatrix} 1 & 0 & -\sin\theta_{t-1} \cdot V \cdot \Delta t \\ 0 & 1 & \cos\theta_{t-1} \cdot V \cdot \Delta t \\ 0 & 0 & 1 \end{bmatrix} \right|_{\mu_{\theta_{t-1}}}$$

$$\Rightarrow E\left\{\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_t\right\} = E\left\{\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_{t-1}\right\} + E\left\{\begin{bmatrix} \cos\theta \cdot \Delta t & 0 \\ \sin\theta \cdot \Delta t & \cdot 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} V \\ \omega \end{bmatrix}\right\} + E\left\{\begin{bmatrix} \eta_x \\ \eta_y \\ \eta_\theta \end{bmatrix}\right\}$$

$$\Sigma\left\{\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_t\right\} = J \cdot \Sigma_{\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_{t-1}} \cdot J^T + \Sigma_{\begin{bmatrix} \eta_x \\ \eta_y \\ \eta_\theta \end{bmatrix}}$$

Task 3_E:

Both cases of noise in a state and an action states are rather similar to each other, but different. The robot moves forward and rotates.