L15 Occupancy Grid Mapping (ProbRob Ch9)

* Why do we want maps?

Pose SLAM does not require landmarks either by marginalizing them (Schur) or considering PC Alignment and loop closure

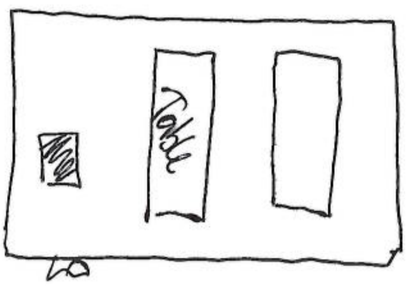
How to calculate correctly loop closure observations?

scan - scan

$\boxed{\begin{array}{c}\text{scan - map}\\ \text{map - map}\end{array}}$

Several PC's can be grouped in a "mini" map
↳ less noisy

- For planning purposes, a map contains information regarding free traversable space.



a map of the class sketched.

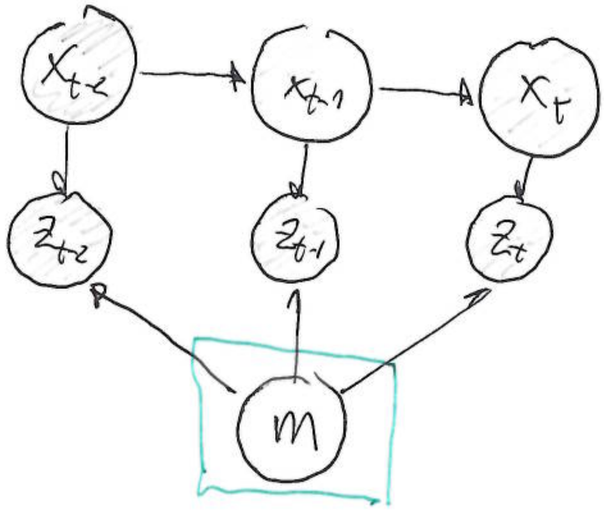
Information on obstacles, or objects or semantics + obst.

* How to compose a Map?

1/ Poses are Known by whatever means

2/ After solving SLAM, poses are best estimated ($\approx$ Known)

Mapping $\longrightarrow$ $p(m \mid Z, x)$



Inference to estimate the map.

(more general, but more memory)

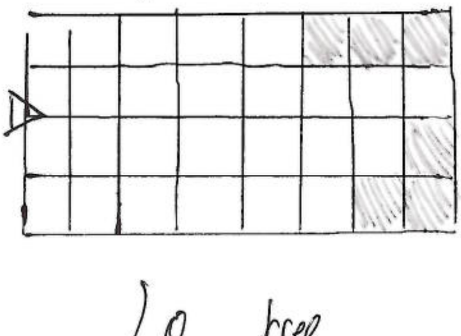
Explicit map:
- lines
- objects
- surfaces (meshes)

Implicit map:
- Point Cloud
- grid
- voxels
- octree
- Signed Distance Function (SDF)
- TSDF

* Occupancy Grid as a Map

Ex:

grid



Previously landmarks were obtained from features in a map. $\Rightarrow$ Abstraction of the map.

$$m_i = \begin{cases} 0 & \text{free} \\ 1 & \text{occupied} \end{cases}$$

Binary state

$m = \{m_i\}_{6\times 5}$ set of cells = map

↳ the combination of possible maps is huge

Ex: 7×4 grid are $2^{28} \sim 268$ M

a map of 100×100 are $2^{10.000}$ → Intractable.

We need some approximations:

1) $p(m \mid Z, X) = \prod_i p(m_i \mid Z, X)$ Cells are independent.

2) Inverse model $p(m \mid z_t)$

we have used the likelihood function $p(z_t \mid m)$ Too large space

where it was easier to describe distribution as causes (Bayes)

* Cells as random variables

$$m_i = \begin{cases} 0 & \text{free} \\ 1 & \text{occupied} \end{cases}$$

cells are Binary random vars.

$$p(m_i) = \begin{cases} 0 & \text{free} \\ 1 & \text{occ.} \\ 0.5 & \text{no knowledge} \end{cases}$$

Log odds (ProbRob p. 94)

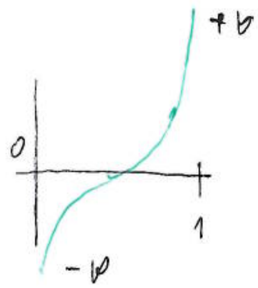
$$\frac{p(x)}{p(\bar{x})} = \frac{p(x)}{1 - p(x)}$$

An alternative way of using pdf's when the state is binary.

$$\Rightarrow \quad l(x) = \log \frac{p(x)}{1-p(x)} \qquad (\text{log odds})$$

r.v x, $x \in [0, 1]$ (Binary)

r.v l, $l \in (-\infty, \infty)$



$$p(x) = 1 - \frac{1}{1 + \exp(l(x))} \qquad (\exists \text{ inverse})$$

* Binary Bayes filter for cells

$$bel(m) = p(m \mid Z, X) = \prod p(m_i \mid Z, X)$$

1 cell:

$$p(m_i \mid z_{1:t}, X) = \frac{p(z_t \mid m_i, z_{1:t-1}, X)\, p(m_i \mid z_{1:t-1}, x_{0:t})}{p(z_t \mid z_{1:t-1}, X)}$$

(Markov)
$$= \frac{\boxed{p(z_t \mid m_i, x_t)}\, p(m_i \mid z_{1:t-1}, x_{0:t-1})}{p(z_t \mid z_{1:t-1}, X)} \qquad (1)$$

x_t pose is not required.

even for 1 cell this h.H is harder to compute than the inverse model.

$$p(z_t \mid m_i, x_t) = \frac{\overbrace{p(m_i \mid z_t, x_t)}\, p(z_t \mid x_t)}{p(m \mid x_t)}$$

this we can calculate.

substituting in (1)

$$p(m_i \mid z_{1:t}, X) = \frac{p(m_i \mid z_t, x_t)\,\cancel{p(z_t \mid x_t)}}{\underbrace{p(m_i \mid x_t)}_{p(m_i) \text{ for simplification}}} \cdot \frac{p(m_i \mid z_{1:t-1}, x_{0:t-1})}{\underbrace{\cancel{p(z_t \mid z_{1:t-1}, X)}}_{\text{Markov}}}$$

Equivalently for the negated $\bar{m}_i$:

$$p(\bar{m}_i \mid z_{1:t}, X) = \frac{p(\bar{m}_i \mid z_t, x_t)\,\cancel{p(z_t \mid x_t)}}{p(\bar{m}_i)} \cdot \frac{p(\bar{m}_i \mid z_{1:t-1}, x_{0:t-1})}{\cancel{p(z_t \mid z_{1:t-1}, X)}}$$

We are interested in the log odds (quotient):

$$\frac{p(m_i \mid Z, X)}{p(\bar{m}_i \mid Z, X)} = \frac{p(m_i \mid z_t, x_t)\,p(m_i \mid z_{1:t-1}, x_{0:t-1})}{p(m_i)} \cdot \frac{p(\bar{m}_i)}{p(\bar{m}_i \mid z_t, x_t)\,p(\bar{m}_i \mid z_{1:t-1}, X}$$

$$= \underbrace{\frac{p(\bar{m}_i)}{p(m_i)}}_{\text{Prior}} \cdot \underbrace{\frac{p(m_i \mid z_t, x_t)}{p(\bar{m}_i \mid z_t, x_t)}}_{\text{current update}} \cdot \underbrace{\frac{p(m_i \mid z_{1:t-1}, x_{0:t-1})}{p(\bar{m}_i \mid z_{1:t-1}, x_{0:t-1})}}_{\text{recursive term.}}$$

$$l_{t,i} = l(m_i \mid Z, X) = \log\left(\frac{p(m_i \mid Z, X)}{p(\bar{m}_i \mid Z, X)}\right) = \ldots$$

$$= \underbrace{\log \frac{p(\bar{m}_i)}{p(m_i)}}_{l_0 = -\log \frac{p(\bar{m}_i)}{p(m_i)}} + \underbrace{\log \frac{p(m_i \mid z_t, x_t)}{p(\bar{m}_i \mid z_t, x_t)}}_{\text{Inverse sensor model}} + \underbrace{\log \frac{p(m_i \mid z_{1:t-1}, x_{0:t-1})}{p(\bar{m}_i \mid z_{1:t-1}, x_{0:t-1})}}_{l_{t-1,i}}$$

$$= \boxed{l_{t-1,i} + \text{inv\_sensor\_model}(m_i, z_t, x_t) - l_0}$$

$$p(m_i \mid Z, X) = 1 - \frac{1}{1 + \exp(l_{t,i})}$$

Alg: Occuppancy Grid Mapping

OGM($\{l_{t-1,i}\}$, x_t, z_t):

for all cells m_i:

  if $m_i \subset$ 'perceptual field of z_t':

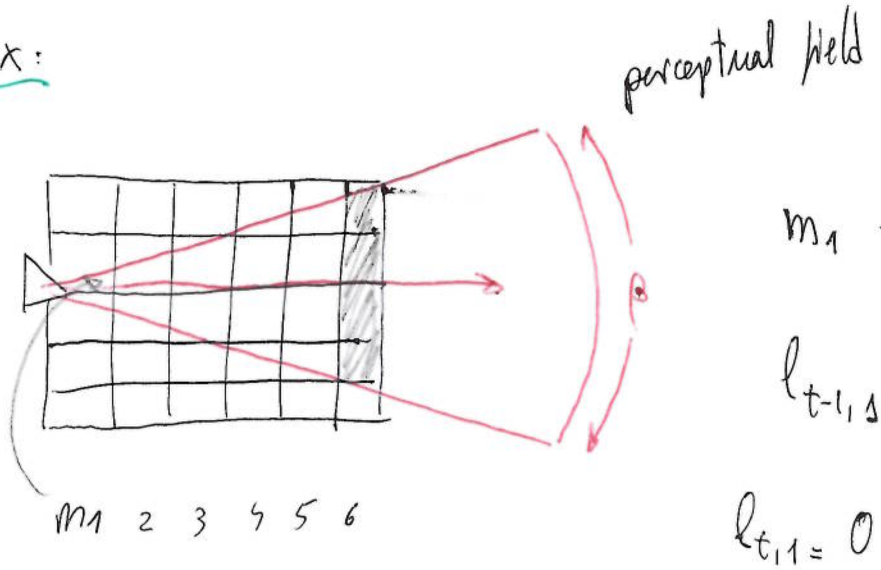
    $l_{t,i} = l_{t-1,i} + \text{inv\_sensor\_model}(m_i, z_t, x_t) - l_0$

  else

    $l_{t,i} = l_{t-1,i}$

return $\{l_{t,i}\}$

Ex:



(inv. model)

$m_1 \longrightarrow l_{free}$

$l_{t-1,1:6} = 0$

$l_{t,1} = 0 + l_{free} - l_0 = -100$

$l_{t,6} = 0 + l_{occ} - l_0 = 100$

(more in ProbRob 288)

the inverse models returns 'occuppied' or 'free' for each cell by raytracing up to the observation. For undetermined results returns l_0.