Lagrange's Interpolation

What is Interpolation?

Interpolation is a method of finding new data points within the range of a discrete set of known data points (Source Wiki). In other words interpolation is the technique to estimate the value of a mathematical function, for any intermediate value of the independent variable.

For example, in the given table we're given 4 set of discrete data points, for an unknown function f(x) :

i	1	2	3	4
x_i	0	1	2	5
$y_i = f_i(x)$	2	3	12	147

How to find?

Here we can apply the Lagrange's interpolation formula to get our solution.

The Lagrange's Interpolation formula:

If, y = f(x) takes the values y0, y1, … , yn corresponding to x = x0, x1 , … , xn then,

$$f(x) = \frac{(x-x_2)(x-x_3)...(x-x_n)}{(x_1-x_2)(x_1-x_3)...(x_1-x_n)}y_1 + \frac{(x-x_1)(x-x_3)...(x-x_n)}{(x_2-x_1)(x_2-x_3)...(x_2-x_n)}y_2 + ... + \frac{(x-x_1)(x-x_2)...(x-x_n-1)}{(x_n-x_1)(x_n-x_2)...(x_n-x_n-1)}y_n$$

This method is preferred over its counterparts like Newton's method because it is applicable even for unequally spaced values of x.

We can use interpolation techniques to find an intermediate data point say at x = 3.

```cpp
// C++ program for implementation of Lagrange's Interpolation
#include<bits/stdc++.h>
using namespace std;

// To represent a data point corresponding to x and y = f(x)
struct Data
{
    int x, y;
};

// function to interpolate the given data points using Lagrange's formula
// xi corresponds to the new data point whose value is to be obtained
// n represents the number of known data points
double interpolate(Data f[], int xi, int n)
{
    double result = 0; // Initialize result

    for (int i=0; i<n; i++)
    {
        // Compute individual terms of above formula
        double term = f[i].y;
        for (int j=0;j<n;j++)
        {
            if (j!=i)
                term = term*(xi - f[j].x)/double(f[i].x - f[j].x);
        }

        // Add current term to result
        result += term;
    }

    return result;
}

// driver function to check the program
int main()
{
    // creating an array of 4 known data points
    Data f[] = {{0,2}, {1,3}, {2,12}, {5,147}};

    // Using the interpolate function to obtain a data point
    // corresponding to x=3
    cout << "Value of f(3) is : " << interpolate(f, 3, 5);

    return 0;
}
```

Output:

```
Value of f(3) is : 35
```

Complexity:

The time complexity of the above solution is $O(n^2)$ and auxiliary space is O(1).

References:

https://en.wikipedia.org/wiki/Lagrange_polynomial

Higher Engineering Mathematics , Dr. B.S. Grewal

https://mat.iitm.ac.in/home/sryedida/public_html/caimna/interpolation/lagrange.html

This article is contributed by Ashutosh Kumar. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

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Number of times the largest perfect square number can be subtracted from N

Length of the longest substring with no consecutive same letters

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Find the longest sub-string which is prefix, suffix and also present inside the string

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Shankhadeep Das • 7 months ago

Hello,
I think the range of the terms should be
[x1, x2, x3 .... xn] for x and
[y1, y2, y3 .... yn] for y,

for the formula provided here.
Thank you.

Reply •

Utkarsha Gaumat • 3 years ago

Is there a way to do interpolation accurately in lesser time complexity

Reply •

Ashutosh Kumar ➚ Utkarsha Gaumat • 3 years ago

We can use linear interpolation to reduce the time complexity ,however that would reduce the accuracy. So basically, its a Complexity Vs Accuracy trade off.
As per my knowledge this is the best one in comparison with the Newton's method or Gaussian method.

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