

Assignment 1

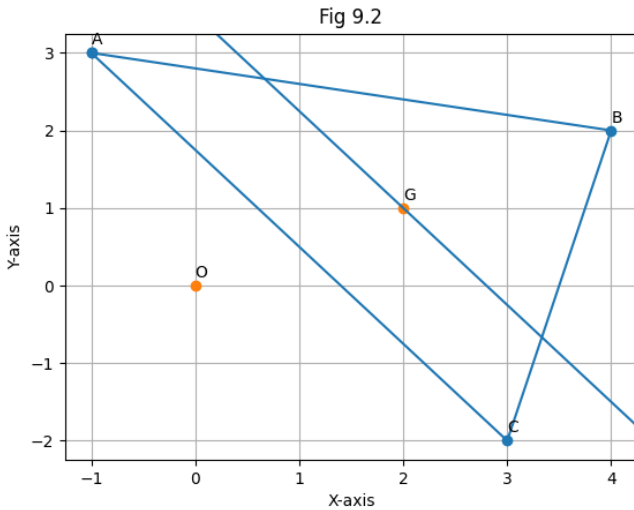
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Question:

A(-1, 3), B(4,2) and C(3,-2) are the vertices of a triangle.

- Find the coordinates of the centroid G of the triangle
- Find the equation of the line through G and parallel to AC.

Solution:



\mathbf{G} is the point vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

- Let L be the line that passes through G such that $L \parallel AC$. Then, L can be expressed as $\mathbf{G} + k\hat{AC}$

$$\hat{AC} = \frac{\mathbf{C} - \mathbf{A}}{|\mathbf{C} - \mathbf{A}|} \quad (6)$$

$$= \frac{\begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right|} \quad (7)$$

$$= \frac{\begin{pmatrix} 4 \\ -5 \end{pmatrix}}{\left| \begin{pmatrix} 4 \\ -5 \end{pmatrix} \right|} \quad (8)$$

$$= \frac{\begin{pmatrix} 4 \\ -5 \end{pmatrix}}{\sqrt{41}} \quad (9)$$

$$L = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \frac{k}{\sqrt{41}} \begin{pmatrix} 4 \\ -5 \end{pmatrix} \quad (10)$$

Thus, Line L is $\begin{pmatrix} 2 \\ 1 \end{pmatrix} + m \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

- Let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ be the points vectors $\mathbf{OA}, \mathbf{OB}, \mathbf{OC}$ respectively, where O is the origin. Thus,

$$\mathbf{A} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (1)$$

Using centroid formula, the desired point vector \mathbf{G} is given by:

$$\mathbf{G} = \frac{1}{3}(\mathbf{A} + \mathbf{B} + \mathbf{C}) \quad (2)$$

$$= \frac{1}{3} \left(\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} \right) \quad (3)$$

$$= \frac{1}{3} \begin{pmatrix} 6 \\ 3 \end{pmatrix} \quad (4)$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (5)$$