

# AI1110: Probability and Random Variables

## Assignment 10

Vishal Vijay Devadiga (CS21BTECH11061)

June 17, 2022

# Outline

## 1 Question

## 2 Solution

- Data given
- Solution for 1st Question
- Solution for 2nd Question

## Question

If  $\bar{x}$  is the sample mean and  $\bar{v}$  is the sample variance of  $x_i$ , if the random variables  $x_i$  have the same mean  $E\{x_i\} = \eta$  and variance  $\sigma_i^2 = \sigma^2$  then prove:

$$E\{\bar{x}\} = \eta \quad (1)$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \quad (2)$$

# Data given

Given,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (3)$$

$$\bar{v} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (4)$$

Also,

$$E\{x_i\} = \eta \quad (5)$$

$$\sigma_i^2 = \sigma^2 \quad (6)$$

# Solution

By linearity of expected value and by (3),(5)

$$E\{\bar{x}\} = \frac{1}{n} \sum_{i=1}^n E\{x_i\} \quad (7)$$

$$= \frac{1}{n} \sum_{i=1}^n \eta \quad (8)$$

$$= \eta \quad (9)$$

# Solution

By (4), and (6)

$$\sigma_{\bar{x}}^2 = \frac{1}{n^2} \sum_{i=1}^n \sigma_i^2 \quad (10)$$

$$= \frac{1}{n^2} \sum_{i=1}^n \sigma^2 \quad (11)$$

$$= \frac{\sigma^2}{n} \quad (12)$$