Assignment 1

Vishal Vijay Devadiga (CS21BTECH11061)

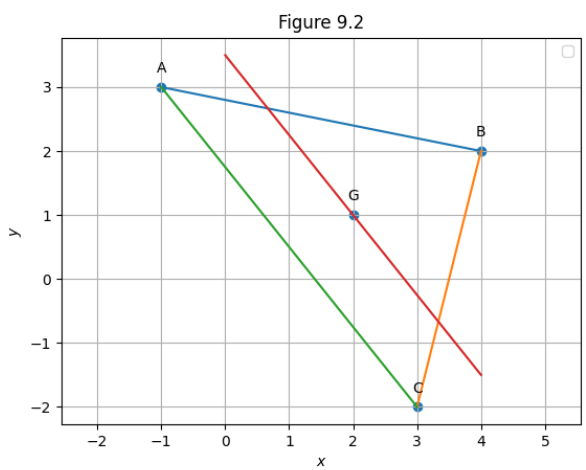
Question:

A(-1, 3), B(4,2) and C(3,-2) are the vertices of a triangle.

a) Find the coordinates of the centroid G of the triangle

b) Find the equation of the line through G and parallel to AC.

Solution:



1) Let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ be the points vectors.

$$\mathbf{A} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{1}$$

Using centroid formula,the desired point vector $\mathbf{G}$ is given by:

$$\mathbf{G} = \frac{1}{3}(\mathbf{A} + \mathbf{B} + \mathbf{C}) \tag{2}$$
$$= \frac{1}{3}\left(\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}\right) \tag{3}$$
$$= \frac{1}{3}\begin{pmatrix} 6 \\ 3 \end{pmatrix} \tag{4}$$
$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{5}$$

$\mathbf{G}$ is the point vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

2) Let L be the line that passes through $\mathbf{G}$ such that $L \parallel AC$ The direction vector of AC, $\mathbf{m}$, is given by,

$$\mathbf{m} = \mathbf{A} - \mathbf{C} \tag{6}$$
$$= \begin{pmatrix} -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{7}$$
$$= \begin{pmatrix} -4 \\ 5 \end{pmatrix} \tag{8}$$

Normal vector of the line is $\mathbf{n}$, such that

$$\mathbf{m}^\top \mathbf{n} = 0 \tag{9}$$
$$\implies \begin{pmatrix} -4 & 5 \end{pmatrix} \mathbf{n} = 0 \tag{10}$$
$$\implies \mathbf{n} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{11}$$
$$\implies \mathbf{n}^\top = \begin{pmatrix} 5 & 4 \end{pmatrix} \tag{12}$$

The normal equation of the line L is given by,

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{G}) = 0 \tag{13}$$
$$\implies \begin{pmatrix} 5 & 4 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 2 \\ 1 \end{pmatrix}\right) = 0 \tag{14}$$
$$\implies \begin{pmatrix} 5 & 4 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 5 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 0 \tag{15}$$
$$\implies \begin{pmatrix} 5 & 4 \end{pmatrix} \mathbf{x} = 14 \tag{16}$$

Thus, line L $\equiv \begin{pmatrix} 5 & 4 \end{pmatrix} \mathbf{x} = 14$