AI1110: Probability and Random Variables

Assignment 8

Vishal Vijay Devadiga (CS21BTECH11061)

May 30, 2022

Outline

Question

A fair die is rolled five times. Find the probability that one shows twice. three shows twice, and six shows once.

Theory

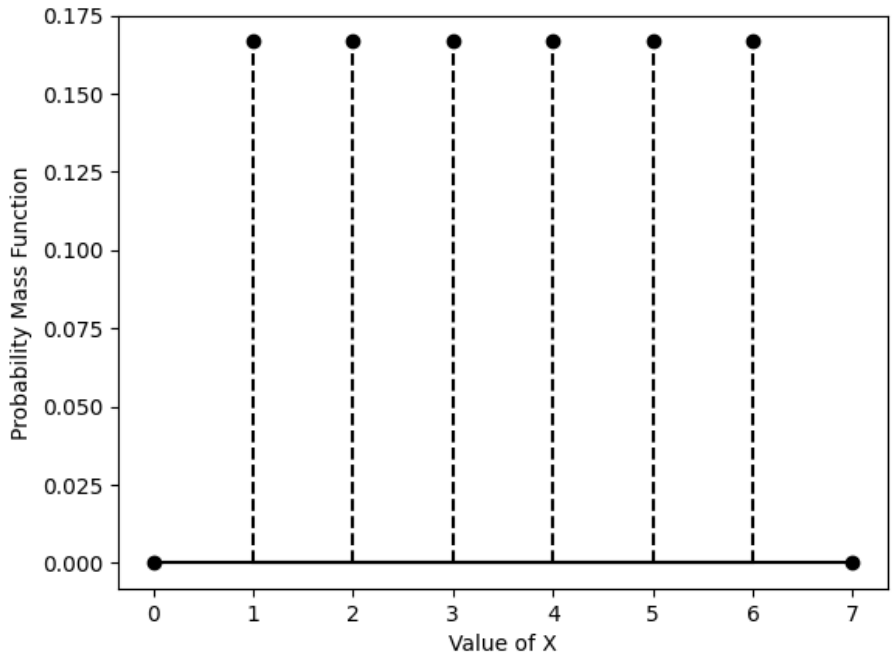
For a Bernoulli trial, with events $A_1, A_2, \ldots A_r$, and $\Pr(A_i) = p_i$ where $\sum_{i=1}^{r} p_i = 1$,
If the experiment is repeated n times where we denote by $p_n(k_l, k_2, \ldots, k_r)$, the probability of the event A_i occurs k_i times in any order where $\sum_{i=1}^{r} k_i = n$, then

$$p_n(k_l, k_2, \ldots, k_r) = \frac{n!}{k_1! k_2! \ldots k_r!} p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r} \tag{1}$$

PMF

Let the random variable X denote the number that appears on rolling the die. The PMF is given by:

$$\Pr(X = k) = \begin{cases} \frac{1}{6}, & 1 \geq k \geq 6 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$



Result

By (1) and (2),

$$p_5(2,0,2,0,0,1) = \frac{5!}{2!2!1!}\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^1 \tag{3}$$

$$= \frac{120}{4}\left(\frac{1}{6}\right)^5 \tag{4}$$

$$= \frac{5}{6^4} \tag{5}$$

By (5), the probability that one shows twice. three shows twice, and six shows once is $\frac{5}{6^4}$.