

# Assignment 1

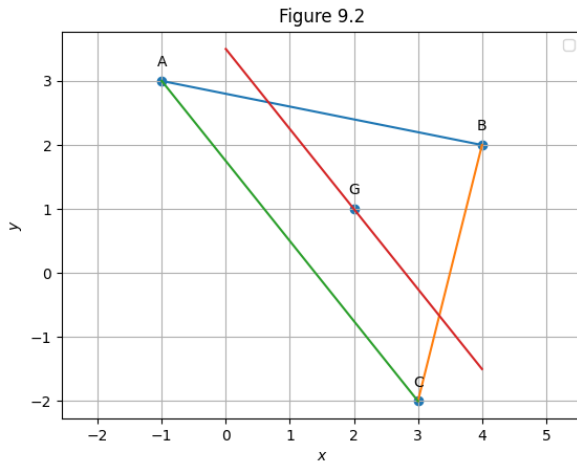
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## Question:

A(-1, 3), B(4,2) and C(3,-2) are the vertices of a triangle.

- Find the coordinates of the centroid G of the triangle
- Find the equation of the line through G and parallel to AC.

## Solution:



- Let  $\mathbf{A}, \mathbf{B}, \mathbf{C}$  be the points vectors.

$$\mathbf{A} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (1)$$

Using centroid formula, the desired point vector  $\mathbf{G}$  is given by:

$$\mathbf{G} = \frac{1}{3}(\mathbf{A} + \mathbf{B} + \mathbf{C}) \quad (2)$$

$$= \frac{1}{3} \left( \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} \right) \quad (3)$$

$$= \frac{1}{3} \begin{pmatrix} 6 \\ 3 \end{pmatrix} \quad (4)$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (5)$$

$\mathbf{G}$  is the point vector  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

- Let  $L$  be the line that passes through  $\mathbf{G}$  such that  $L \parallel AC$ . The direction vector of  $AC$ ,  $\mathbf{m}$ , is given by,

$$\mathbf{m} = \mathbf{A} - \mathbf{C} \quad (6)$$

$$= \begin{pmatrix} -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (7)$$

$$= \begin{pmatrix} -4 \\ 5 \end{pmatrix} \quad (8)$$

Normal vector of the line is  $\mathbf{n}$ , such that

$$\mathbf{m}^T \mathbf{n} = 0 \quad (9)$$

$$\Rightarrow \begin{pmatrix} -4 & 5 \end{pmatrix} \mathbf{n} = 0 \quad (10)$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad (11)$$

$$\Rightarrow \mathbf{n}^T = \begin{pmatrix} 5 & 4 \end{pmatrix} \quad (12)$$

The normal equation of the line  $L$  is given by,

$$\mathbf{n}^T (\mathbf{x} - \mathbf{G}) = 0 \quad (13)$$

$$\Rightarrow \begin{pmatrix} 5 & 4 \end{pmatrix} \left( \mathbf{x} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right) = 0 \quad (14)$$

$$\Rightarrow \begin{pmatrix} 5 & 4 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 5 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 0 \quad (15)$$

$$\Rightarrow \begin{pmatrix} 5 & 4 \end{pmatrix} \mathbf{x} = 14 \quad (16)$$

Thus, line  $L \equiv \begin{pmatrix} 5 & 4 \end{pmatrix} \mathbf{x} = 14$