Assignment 1

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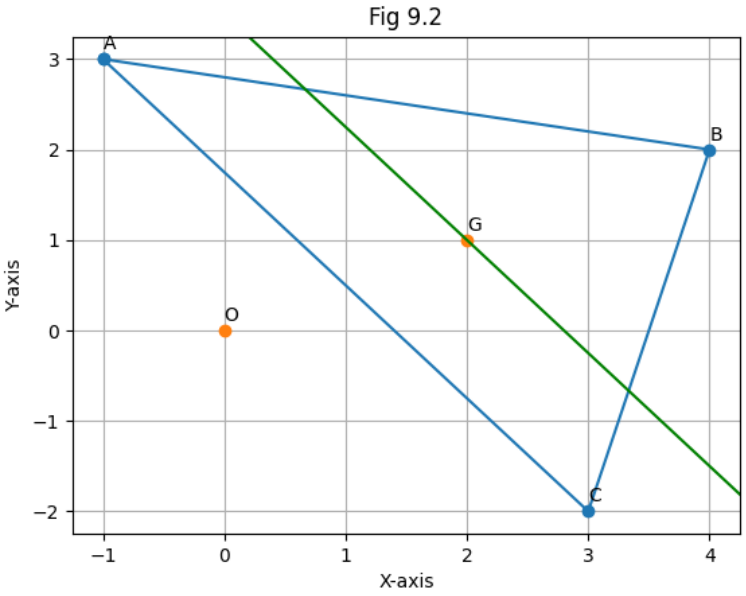
Question:

A(-1, 3), B(4,2) and C(3,-2) are the vertices of a triangle.

a) Find the coordinates of the centroid G of the triangle

b) Find the equation of the line through G and parallel to AC.

Solution:



1) Let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ be the points vectors OA,OB,OC respectively, where O is the origin. Thus,

$$\mathbf{A} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{1}$$

Using centroid formula, the desired point vector $\mathbf{G}$ is given by:

$$\mathbf{G} = \frac{1}{3}(\mathbf{A} + \mathbf{B} + \mathbf{C}) \tag{2}$$

$$= \frac{1}{3}(\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}) \tag{3}$$

$$= \frac{1}{3}\begin{pmatrix} 6 \\ 3 \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{5}$$

$\mathbf{G}$ is the point vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

2) Let L be the line that passes through G such that $L \parallel AC$ Then, L can be expressed as $\mathbf{G} + k\hat{AC}$

$$\hat{AC} = \frac{\mathbf{C} - \mathbf{A}}{|\mathbf{C} - \mathbf{A}|} \tag{6}$$

$$= \frac{\begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix}\right|} \tag{7}$$

$$= \frac{\begin{pmatrix} 4 \\ -5 \end{pmatrix}}{\left|\begin{pmatrix} 4 \\ -5 \end{pmatrix}\right|} \tag{8}$$

$$= \frac{\begin{pmatrix} 4 \\ -5 \end{pmatrix}}{\sqrt{41}} \tag{9}$$

$$L = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \frac{k}{\sqrt{41}}\begin{pmatrix} 4 \\ -5 \end{pmatrix} \tag{10}$$

Thus, Line L is $\begin{pmatrix} 2 \\ 1 \end{pmatrix} + m\begin{pmatrix} 4 \\ -5 \end{pmatrix}$. Slope of line L is $\frac{-5}{4}$, obtained by vector form. The normal equation of the line can be found by finding the y intercept of the line. For y-intercept, $\begin{pmatrix} 0 \\ y \end{pmatrix}$ satisfies the vector form of line.

$$\begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} 2 + 4m \\ 1 - 5m \end{pmatrix} \tag{11}$$

$$2 + 4m = 0 \tag{12}$$

$$m = \frac{-1}{2} \tag{13}$$

$$y = 1 - 5m \tag{14}$$

$$y = 1 - 5 \times \frac{-1}{2} \tag{15}$$

$$y = \frac{7}{2} \tag{16}$$

Thus, line L is $y = \frac{-5}{4}x + \frac{7}{2}$