Coq for natural language semantics day 3: FraCoq and some other goodies[1]

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[1]Parts of this talk are based on Jean-Philippe Bernardy's tutorial at the GF summer school, Riga 2017

Brief summary of the talk

- Present a type-theoretical framework for formal semantics leveraging two well-studied tools
  - Grammatical Framework (GF, [Ranta(2011)])
  - Coq
- Providing a compositional resource semantics for GF
  - A tutorial on FraCoq
- Evaluation on the FraCaS test suite
- State-of-the-art results
- Other goodies
  - Some Type Theory with Records (TTR)
  - Some Montagovian Generative Lexicon
  - Some neo-Davidsonian semantics
  - Co-predication and individuation

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Structure the talk

- Brief intro to the systems used, GF and Coq
- Presenting the FraCoq system
  - We concentrate on the most linguistically relevant features and also the features relevant for the FraCaS
- Evaluation against the FraCaS test suite
  - Some brief remarks about the FraCaS and NLI platforms
    - Results and comparison with previous approaches
    - The issue of automation
- Conclusions and Future work

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Background: Grammatical Framework plus a proof-assistant

- Grammatical Framework (GF, [Ranta(2011)])

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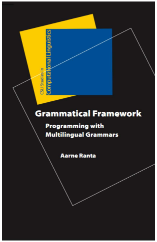
Background: Grammatical Framework plus a proof-assistant

- Grammatical Framework (GF, [Ranta(2011)])
  - Chalmers based
  - Programming language for multilingual applications



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Background: Grammatical Framework (GF)

- Involves an abstract syntax, comprised of:
  - A number of syntactic categories
  - A number of syntactic construction functions, which provide the means to compose basic syntactic categories into more complex ones
    - ★ $AdjCN{:}AP \rightarrow CN \rightarrow CN$ (appending an adjectival phrase to a common noun and obtaining a new common noun)
- GF comes with a library of mappings from abstract syntax to concrete
  - These mappings can be inverted by GF, thus offering parsers from natural text into abstract syntax
  - We use the parse trees constructed by [Ljungloöf and Siverbo(2011)] thereby avoiding any syntactic ambiguity (GF FraCaS treebank).

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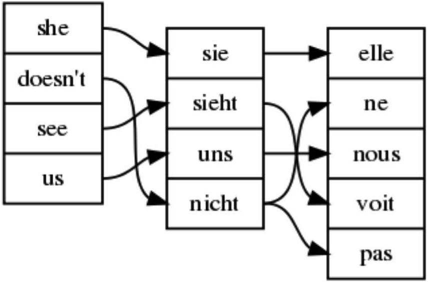
Background: Grammatical Framework

- One abstract syntax (set of rules), lots of concrete ones (linearizations of the rules)



Background: Grammatical Framework

- Precise symbolic parser
  - Mildly context-sensitive expressive power (equivalent to a parallel multiple CFG)
    - At least a version of GF (there is also unrestricted GF)
    - Have a look at Ljunglöf's thesis [Ljunglöf(2004)] if you want to know more

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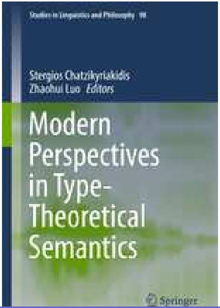
Background: Type-Theoretical Semantics (you've heard all this before!)

- We use the type of logics that have been traditionally dubbed as constructive
  - Initiated by the work of Martin-Löf [Martin-Löf(1975), **?**]
  - In linguistics this types of logics go back to Ranta's seminal work [Ranta(1994)] or even earlier to [Sundholm(1986)]
    - ★ More recent approaches can be found as well. Please see [Chatzikyriakidis and Luo(2017)] for a collection of papers on constructive type theories for NL semantics



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Background: Type Theoretical Semantics

- Main features of MTTs
  - Type many-sortedness.
  - Dependent sum and product types
    - Σ-types, often written $\sum_{x:A} B[x]$ and which have product types $A \times B$ as a special case when B does not depend on x.
    - Dependent product, Π-types, often written $(\prod_{x:A} B[x])$, and which have arrow-types $A \to B$ as a special case
    - They generalize universal quantification and function types and they offer type polymorphism
  - Proof-theoretical specification and support for effective reasoning.
    - Most powerful proof assistants implement MTTs (e.g. Coq, Agda)

Background: Coq (some more repetition!)

- Proof assistant based on the calculus of inductive constructions (extension of CoC, see [Paulin-Mohring(2015)])
  - Arguably one of the leading proof assistants
    - a proof of the four-color theorem [Gonthier(2008)]
    - a proof of the odd order theorem [Gonthier et al.(2013)Gonthier, Asperti, Avigad, Bertot, Cohen, Garillot, L
    - developing CompCert, a formally verified compiler for C [Leroy(2013)]
    - One of the assistants used in the Univalent Foundations project (Homotopy Type Theory, [Program(2013)])

Background: Coq

- Important features used
  - Π types
    - in Coq: $\prod_{x:A} B[x]$ is written `forall (x:A), B` or (simply `A` $\to$ `B` when `B` does not depend on `x`)
  - Record types
    - Generalization of Σ-types and are encoded as (trivial) inductive types with a single constructor.
    - $\Sigma x{:}A.B(x)$ can be expressed as a dependent record type in Coq:

```
Record AB:Type:=mkAB{x:> A;P:B x}
```

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The FraCoq system

- We use Ljünglof's FraCaS treebank and take these trees to their semantic counterparts
- The structure of the semantic representation
  1. Every GF syntactic category C is mapped to a Coq *Set*, noted $[\![C]\!]$.
  2. GF Functional types are mapped compositionally : $[\![A \to B]\!] = [\![A]\!] \to [\![B]\!]$
  3. Every GF syntactic construction function $f{:}X$ is mapped to a function $[\![f]\!]$ such that $[\![f]\!] : [\![X]\!]$.
  4. GF function applications are mapped compositionally: $[\![t(u)]\!] = [\![t]\!]([\![u]\!])$.

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From GF to Coq in practice

- Note the following toy GF grammar (taken from Bernardy's tutorial on the GF summer school, August 2017):

```
abstract Grammar = {
flags startcat = S ;
cat
S ; Cl ; NP ; VP ; AP ; CN ; PN ; Det ; N ; A ; V ; V2 ;
AdA ;  Pol ; Conj ;
data
UseCl   : Pol -> Cl -> S ; PredVP  : NP -> VP -> Cl ;
ComplV2 : V2 -> NP -> VP ; DetCN   : Det -> CN -> NP ;
ModCN   : AP -> CN -> CN ; CompAP  : AP -> VP ;
AdAP    : AdA -> AP -> AP ; ConjS   : Conj -> S  -> S  ->
ConjNP  : Conj -> NP -> NP -> NP ; UseV     : V -> VP ;
UsePN   : PN -> NP ; UseN     : N -> CN ;
UseA    : A -> AP ; some_Det, every_Det : Det ;
i_NP, you_NP : NP ; very_AdA : AdA ;
Pos, Neg : Pol ; and_Conj, or_Conj : Conj ;
```

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From GF to Coq in practice

```
}
abstract Test = Grammar ** {

fun
man_N, woman_N, house_N, tree_N : N ;
big_A, small_A, green_A : A ;
walk_V, arrive_V : V ;
love_V2, please_V2 : V2 ;
john_PN, mary_PN : PN;

} ;
```

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From GF to Coq in practice

- A trivial way of doing this

```
Parameter S: Type. Parameter Cl: Type. Parameter VP: Type.
Parameter PN: Type. Parameter NP: Type. Parameter AP: Type.
Parameter A: Type. Parameter CN: Type. Parameter Det: Type.
Parameter N: Type. Parameter V: Type. Parameter V2: Type.
Parameter AdA: Type. Parameter Pol: Type. Parameter Conj: Type
...
Parameter man_N:  N. Parameter woman_N: N .
Parameter house_N:  N. Parameter tree_N: N .
...
Parameter john_PN: PN . Parameter mary_PN: PN.
```

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From GF to Coq in practice

- This is of course quite trivial but, now, for every abstract syntax expression in GF, there is a well-typed expression in Coq
  - You can even do some reasoning :)

```
Theorem thm0 : UseCl Pos (PredVP (UsePN john_PN) walk_V) ->
UseCl Pos (PredVP (UsePN john_PN) walk_V).
intro H. exact H. Qed.
```

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From GF to Coq in practice

- Getting more proper semantics step by step

```
Definition S     : Type := Prop .
Definition Cl    : Type := Prop .
Definition Pol   : Type := Prop -> Prop .
Definition Pos : Pol := fun p => p.
Definition Neg : Pol := fun p => not p.
Definition UseCl : Pol -> Cl -> S :=
fun pol c => pol c.Definition S     : Type := Prop .
```

- We can get clausal negation yay!

From GF to Coq in practice

- Quantifiers, PN and VP

```
Parameter object : Type.
Definition VP    : Type :== object -> Prop.
Definition V     : Type := object -> Prop.
Definition UseV  : V -> VP := fun v => v.
Definition PredVP  : NP -> VP -> Cl := fun np vp => vp np.
Definition NP: Type := VP -> Prop .
Definition UsePN: PN -> NP := fun pn vp => vp pn.
Definition PredVP: NP -> VP -> Cl := fun np vp => np vp.
Definition everyoneNP : NP := fun vp => forall x, vp x.
```

- Getting better, we can do some elementary quantification now!

From GF to Coq in practice

- Introducing quantifiers like *all* and *some* needs a definition of quantifier
  - This we do with *Det*, a function from *CNs* to *NPs*.
  - We define *all* and *some*

```
Definition CN    : Type := PN -> Prop .
Definition N    : Type := CN .
Definition Det    : Type := CN -> NP .
Definition DetCN : Det -> CN -> NP := fun det cn => det cn.
Definition every_Det : Det := fun cn vp =>
forall x, cn x -> vp x.
Definition some_Det : Det := fun cn vp =>
exists x, cn x /\ vp x.
```

  - You get the picture!
  - Let us now get fine-grained and see FraCoq!

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The FraCoq system

- Sentences
  - As we said, we interpret sentences as propositions: $[\![S]\!] = Prop$.
  - To verify that P entails H, we prove the proposition $[\![P]\!] \rightarrow [\![H]\!]$.

  ```
  Definition S := Prop.
  ```

- Common Nouns
  - Predicates over an abstract object type

  ```
  Parameter object : Set.
  Definition CN := object->Prop.
  ```

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The FraCoq system

- Verb phrases
  - Parameterize over the *noun* of the subject (using Π types)

```
Definition VP := forall (subjectClass : CN)
object -> Prop.
```

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The FraCoq system

- Adjectives
  - Functions from cn to cn (predicates to predicates)

    ```
    Definition A := CN -> CN.
    ```

  - Different classes of adjectives are captured using coercions (subtyping). All special classes of adjectives are subtypes of A.

    ```
    Definition IntersectiveA := object -> Prop.
    Definition wkIntersectiveA : IntersectiveA -> A
    := fun a cn (x:object) => a x /\ cn x.
    Coercion wkIntersectiveA : IntersectiveA >-> A.
    ```

  - Provision is made for intersective, subsective, privative and non-committal adjectives
- intersective adjectives are then declared as follows:

```
Parameter green_A : IntersectiveA.
```

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The FraCoq system

- Another example: privative adjectives

```
Inductive PrivativeA : Type :=
mkPrivativeA : ((object -> Prop) -> (object -> Prop)) ->
PrivativeA.
Definition wkPrivativeA : PrivativeA -> A
:= fun aa cn (x:object) => let (a) :=
aa in a cn x /\ not (cn x).
Coercion wkPrivativeA : PrivativeA >-> A.
Definition NonCommitalA := A.
```

- Again adjectives are declared as: $fake_N$:$Privative_A$
- For non commitals, it suffices to declare them as A.

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The FraCoq system

- Adverbs
  - Similar method to adjectives but instead the modification is on verbal predicates
  - The adverb cases in the FraCaS are all veridical and covariant.
  - We define such a subclass *VeridicalAdv* and declare it as a coercion *Adv*
    - ★ Adverbs of type *VeridicalAdv* are also of type *Adv*

```
Definition VeridicalAdv :=
{ adv : (object -> Prop) -> (object -> Prop)
& prod (forall (x : object) (v : object -> Prop), (adv v) x
 -> v x) (forall (v w : object -> Prop),
(forall x, v x -> w x) -> forall (x : object),
adv v x -> adv w x)
}.
Definition WkVeridical : VeridicalAdv -> Adv
:= fun adv => projT1 adv.
Coercion WkVeridical : VeridicalAdv >-> Adv.
Parameter on_time_Adv : VeridicalAdv .
```

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The FraCoq system

- Noun Phrases and Predeterminers
  - A clean definition of NPs as functions from predicates to truth values will not work
    - ★ Problem with GF's abstract syntax: existence of pre-determiners which include cases like *most*, *all* among others and are defined as functions from NPs to NPs
    - ★ In general, the category includes elements that are naturally interpreted as GQs
    - ★ Solution: Remember the components of NPs (number, quantifier and common noun)
    - ★ Pre-determiners then are able to substistute the quantifier part with the appropriate quantifier
    - ★ This has to be done, otherwise pre-determiners introduce a dummy indefinite in these cases

The FraCoq system

- Predeterminers

```
Definition all_Predet : Predet
:= fun np => let (num,qIGNORED,cn) := np
in mkNP num all_Quant cn.

Definition at_least_Predet : Predet
:= fun np => let (num,qIGNORED,cn) := np
in mkNP num (fun num cn vp => interpAtLeast num (CARD
(fun x => cn x /\ vp cn x))) cn.
```

- For *all_Predet* the number is ignored and the quantifier part is substituted with a universal

The FraCoq system

- The function *interpAtLeast* is an interpretation function of number for *at least*

```
Fixpoint interpAtLeast (num:Num) (x:nat) :=
match num with
| singular => x >= 1
| plural   => x >= SOME
| unknownNum => True
| moreThan n => interpAtLeast n x
| cardinal n => x >= n
end.
```

- Thus, in the case of *at least 3* what we would get is a situation where the cardinality of x is equal or more than 3.
- CARD a context-dependent abstract function which turns a predicate into a natural number is used to get the correct semantics
- Other predeterminers are determined accordingly

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The FraCoq system

- Generalized Quantifiers
  - They turn a number and a common noun into a noun-phrase (which we call *NP*0).

    ```
    Definition Quant := Num -> CN -> NP0.
    ```

  - Some quantifiers ignore the number and are given usual definitions (e.g. *some* or *all*), whereas others make essential use of number (e.g. *at most*)
    - ★ In the latter case, the function `CARD` is used

The FraCoq system

- Quantifiers *some*

```
Definition someSg_Det : Det:= (singular, fun num P Q=>
exists x,  P x /\ Q P x ).
```

- Number ignored here

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The FraCoq system: Quantifiers

- Quantifier *at most*

  ```
  Definition atMost_quant : Quant
  := fun num cn vp => interpAtMost num
  (CARD (fun x => cn x /\ vp cn x))
  ```

- Essential use of number
- *interpAtMost* checks that the given number is less than the given cardinality

The FraCoq system: Cardinalities

- CARD is a context-dependent abstract function which turns a predicate into a natural number.
  - Common-sense axioms of set cardinality, such as monotonicity are provided

    ```
    Variable CARD_monotonous : forall a b:CN,
    (forall x, a x -> b x) -> CARD a <= CARD b.
    ```

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The FraCoq system

- The definite article
  - Checks for plural noun phrases
    - ★ If found, then universal quantification
    - ★ If not, it looks up the object of discourse in an abstract *environment*, which is a function which turns a common noun into an object

```
Definition DefArt:Quant:= fun (num : Num) (P:CN)=> fun Q:VP
=> match num with plural => (forall x, P x -> Q P x)
/\ Q P (environment P) /\ P (environment P) |
_ => Q P (environment P) /\ P (environment P) end.
```

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The FraCoq system

- Prepositions
  - Takes a NP and returns a higher order predicate (1)
  - Covariant and veridical (2 and 3)

```
Definition NP1 := (object -> Prop) ->Prop.
Inductive Prep : Type :=
mkPrep : forall
(prep : NP1 -> (object -> Prop) -> (object -> Prop)), (*1*)
(forall (prepArg : NP1) (v : object -> Prop)
(subject : object), prep prepArg v subject -> v subject) ->
(forall (prepArg : NP1) (v w : object -> Prop), (*2*)
(forall x, v x -> w x) -> forall x, prep prepArg v x ->
prep prepArg w x)  -> Prep. (*3*)
```

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Evaluation: The FraCaS test suite

- A test suite for NLI
  [Cooper et al.(1996)Cooper, Crouch, van Eijck, Fox, van Genabith, Jasp
  - 346 NLI examples in the form of one or more premises followed by a question along with an answer to that question
  - Three potential answers
    - YES: The declarative sentence formed out of the question follows from the premises
    - NO: The declarative sentence does not follow from the premises
    - UNK: The declarative sentence neither follows nor does not follow fro the premises

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Evaluation: The FraCaS test suite

(1) A Swede won the Nobel Prize.
Every Swede is Scandinavian.
Did a Scandinavian win the Nobel prize? [Yes, FraCas 049]

(2) No delegate finished the report on time..
Did any Scandinavian delegate finish the report on time? [No, FraCaS 070]

(3) A Scandinavian won the Nobel Prize.
Every Swede is Scandinavian.
Did a Swede win the Nobel prize? [UNK, FraCaS 065]

Evaluation: The FraCaS test suite

- The FraCaS has considerable weaknesses
  - Small size
  - Artificial nature of the examples
- However, it covers a lot of phenomena associated with NLI
- It is still a very good suite to test logical approaches
  - And it is actually the one (or one of the suites) used in these approaches!

Evaluation

- Evaluation against 5 sections of the FraCaS
  - Total of 174 examples
  - Excluded sections where a lot of context-dependency has to be taken into consideration (e.g. the section on ellipsis)
    - Note that no one has ever made a full run of the suite
- YES: a proof can be constructed from the premises to the hypothesis
- NO: a proof of the negated hypothesis can be constructed
- UNK: otherwise

Evaluation

- The following table presents the results (Ours) as well as a comparison with the approach in Mineshima et al. [Mineshima et al.(2015)Mineshima, Martınez-Gómez, Miyao, and Bekki] Bos [Bos(2008)] and Abzianidze [Abzianidze()]

	Section	# examples	Ours	MINE	Nut	Langpro
1	Quantifiers	75	.96	.77	.53	.93 (44)
2	Plurals	33	.76	.67	.52	.73 (24)
3	Adjectives	22	.95	.68	.32	.73 (12)
4	Comparatives	31	.56	.48	.45	-
5	Attitudes	13	.85	.77	.46	.92 (9)
6	Total	174 (181)	0.83	0.69	0.50	0.85

- Our approach outerperforms Mineshema et al. by 13 percentage points.
- The approach by Abzianidze has an accuracy of 0.85 without involving the comparative section. If this section is taken out, our system's accuracy rises to 0.88

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Error Analysis

- Improvement over earlier approaches. Still, there were a couple of difficulties
  - Comparatives: cases that needed one to provide adequate semantics for *more* but also to take care of ellipsis

(4) ITEL won more orders than APCOM.
ITEL won some orders.
Did ITEL win some orders? [Yes, FraCaS 233]

  - Definite Plurals: Universal reading was captured. Cases of existential readings were not

(5) The inhabitants of Cambridge voted for a Labour MP.
very inhabitant of Cambridge voted for a Labour MP.
Did every inhabitant of Cambridge vote for a Labour MP?
[UNK, FraCaS 094]

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Automation

- So far, our proofs are not automated
  - A couple of steps (usually very few) to reach a proof
  - Earlier approaches using Coq (e.g. Chatzikyriakidis 2014 and Mineshima et al. 2015) use Coq's tactical language LTac to define automated macros of actions
    - ★ This is not difficult to do in our case as well
    - ★ Just go through all the proof tactics or observe the tactics that are used in the proofs to create a macro that will automate the proofs
    - ★ The question remains: can that macro of tactics generalize outside the suite?
    - ★ Answer: only to a limited extent, i.e. when exactly the same set of tactics yields a proof
    - ★ For this reason, we have not automated proof search to obtain the results presented in this paper, even though this can be done easily

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Automation

- Automating would also make an unprincipled use of higher-order logic (HOL)
  - No algorithm which can decide if a proposition has a proof or not
    - We must use heuristics both to search for proofs and to decide when to give up searching
- Most problems have either obvious proofs or obviously lack a proof (fortunately)
  - Due to its heuristic nature the proof search necessarily contains a human component
    - Problematic to make a statement about the suitability of FraCoq outside FraCas
    - Small dataset and lack of separation between a development and a test set does not help the situation either
    - Related shortcoming: specialized semantics for specific lexical entries

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Future Work

- Address the issue of automation
  - Define a decidable fragment of the logic and only work within such fragment
    - Possible to concisely characterize how the approach generalises
  - Train a neural network on a body of freely available proofs on the net and see whether it can generalize to automatically provide the proof tactics for the cases interested
- Improvement at the GF level: make the abstract syntax more compatible with compositional semantics
  - For example, do something about problematic syntactic categories like pre-determiners or cases where the syntax makes it impossible to recover elliptical fragments
- Extend into the whole suite (first attempt to do anaphora using monads on the way!)

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Conclusions

- We have connected two well-defined systems based on type-theory
  - GF and Coq
  - Providing resource semantics for GF
- The issue of generalization remains a shortcoming
  - It is possible to achieve very precise semantics for specific domains
    - Our system outerperforms previous logical systems w.r.t accuracy
- Useful in performing inference tasks on controlled natural language domains
- Hybrid NLI systems

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Conclusions

- The system can be found here: https://github.com/GU-CLASP/FraCoq

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