Experimenting unified differential evolution

Tosch Thomas
UHA (University of High Alsace)
Mulhouse, France
thomas.tosch@uha.fr

Abstract—This article displays experimentation results for the unified differential evolution algorithm. Multiple algorithms are used to measure performances of the unified Differential Evolution method, including Sphere, Rosenbrock, Griewank and Rastrigin. Initial results displayed no superiority in terms of performances. It displayed that fine tuning is a required step. The experimentation took place on a course held by Dr. Idoumghar. During the course, we took as assignment to produce test results over the unified Differential Evolution algorithm.

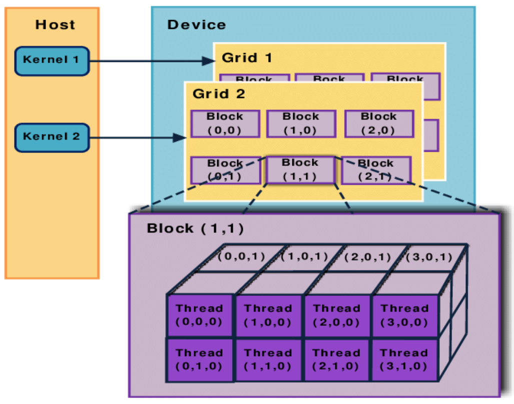
Keywords—CUDA, Compute Unified Device Architecture, DE, Differential Evolution, GPU, Graphics Processing Unit, Massively Parallel Computing

I. Introduction

In this article we are going to compare multiple implementations experimentation results for the unified differential evolution algorithm. We will be using multiple algorithms to measure performances of the implementations, including Sphere, Rosenbrock, Griewank and Rastrigin. From a source code made by L. Idoumghar for a course over parallel programming, we implemented the unified mutation strategy and the previously mentioned functions. First we'll be describing how the Cuda language can be used to improve performances.

II. GPU Computing with CUDA

On contemporary NVIDIA GPUs, NVIDIA's CUDA technology provides a parallel computing architecture in which hundreds of threads are clustered into numerous blocks. Each grid has a number of blocks that share control logic, instruction cache, low-latency shared memory, registers, and other resources. With a high delay, all grids share global memory. The number of grids and global memory size varies by GPU type and brand. However, the number of blocks, shared memory size, and register count in each grids are all determined by the GPU's computational capability.



III. Differential Evolution

In evolutionary computation, differential evolution is a simple yet powerful field.

A. Overview of DE

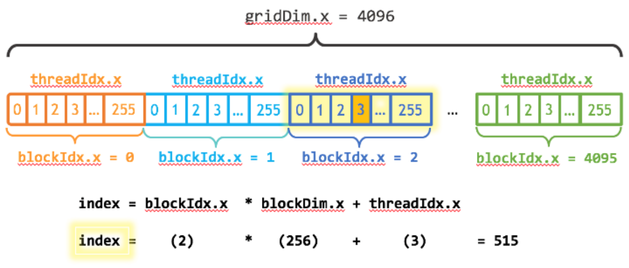
```
1:  Generate initial population $P^0 = \{\vec{x}_1^0, \vec{x}_2^0, \ldots, \vec{x}_N^0\}$
2:  Let $t = 0$
3:  repeat
4:  for each individual $\vec{x}_i^t$ in the population $P^t$ do
5:  Generate three random integers $r_1$, $r_2$ and
6:    $r_3 \in \{1,2,\ldots,N\} \setminus i$, with $r_1 \neq r_2 \neq r_3$
7:  Generate a random integer $j_{rand} \in \{1,2,\ldots,D\}$
8:  for each parameter $j$ do
9:      $u_{i,j}^{t+1} = \begin{cases} x_{r_3,j}^t + F \times (x_{r_1,j}^t - x_{r_2,j}^t), & \text{if } (rand \le CR \,\|\, j = rand[1,D]) \\ x_{i,j}^t, & \text{otherwise} \end{cases}$
10:   end for
11:   Replace $\vec{x}_i^t$ with the child $\vec{u}_i^{t+1}$ in the population $P^{t+1}$,
12:   if $\vec{u}_i^{t+1}$ is better, otherwise $\vec{x}_i^t$ is retained
13:   end for
14:    $t = t + 1$
15:   until the termination condition is achieved
```

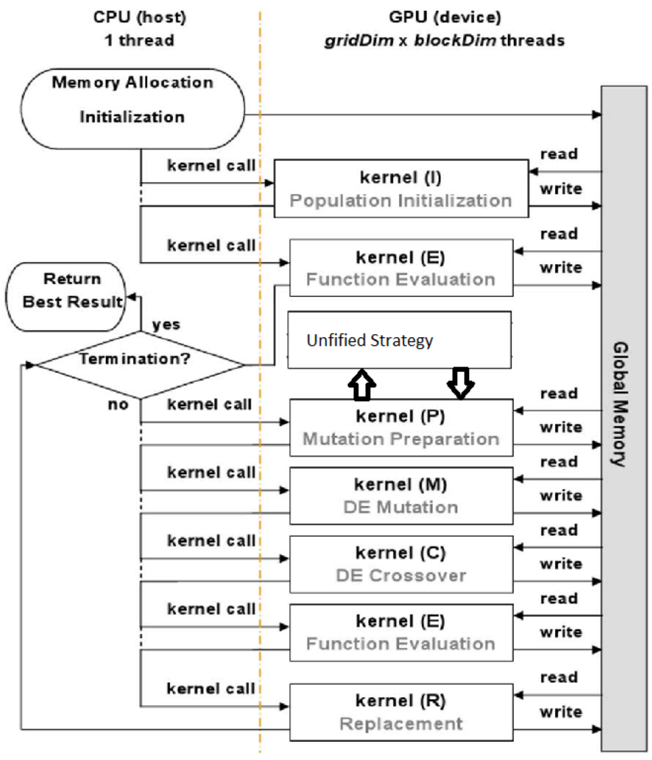
DE uses three major operations, namely mutation, crossover, and replacement, to evolve a population of candidate solutions in search of global optima. Each proposed solution's quality is assessed in accordance with particular aim functions First, within the population, a size-fixed population is randomly initialized. Then, in order, each population member, or target vector, goes through three operations. A Mutation where a base vector is constructed from the population to determine the mutation's reference point. The vector difference of randomly selected population members omitting the target vector is then scaled and added to the base vector to create a mutant vector. Different mutation strategies can be used depending on multiple parameter as described in the unified DE. The Crossover is used to construct a trial vector, a particular probability (CR) is applied between the above-generated mutant vector and the target vector under consideration. And then the Replacement where if the trial vector is of higher quality than the target vector under consideration, it will take its place in the next generation's population. Otherwise, the target vector will persist in the next generation's population. These three processes are used to update the population iteratively until particular termination criteria are met, such as reaching a pre-specified maximum number of function 2 evaluations.

B. CUDA-based DE implementation

To improve performance of the differential evolution kernel, we are using a C++ implementation of the Cuda framework. Cuda is a GPU compiled sub-language of C++ that allows to write instructions headed for the GPU to process. In theory, this allows to improve speed of loop-based algorithms in comparison to their sequential implementations. Cuda allows easy Threads management by letting the user set manually the amount of Threads that should run the code in parallel. Standard GPUs like the Nvidia GTX 980 used in the computer this article in written on has up to 2048 cuda cores, allowing to run in parallel many operations. With Google Colab's P100 GPUs can process even more operations with 3584 cuda cores. To manage data with operations than the amount of cuda cores, we can take advantage of Blocks. Blocks allow to to prepare the GPU to receive the amount of data of our operations. If we had 1 million operations, we would split them over 128 or 256 threads and take the upper value of the division. See following cuda implementation.

The implementation of our algorithm will now run over 256 threads until every operations have been done. To access the current iteration from our function, we have access to multiple built-ins from cuda. 3 blockIdx give us access on the current Block iteration. With blockDim we have access to the size of our blocks, which was set to 256 in our previous exemple.
The threadIdx value provides the current thread the code is currently running in. Doing a simple calculation we can get the current iteration of our parallel implementation.





IV. Experimental Results

The unified cudaDE's performances are assessed and compared to the sequential DE implementation (sDE) in terms of computing time using four numerical problems of 10D, 50D, and 100D, where "D" denotes the dimension. The effects of several algorithmic population sizes (P50, P100, P500, and P1000) on computing time efficiency are investigated

A. *Experimental Setup*

Experiments are conducted on a colab node equipped with an Intel(R) Xeon(R) at 2.30GHz and a Nvidia P100 GPU with 16GB of GDDR5 global memory. The P100 has a capacity of 9.3 TFLOPS and 1.32GHz of GPU memory clock. Our development environment is made with an

```
int blockSize = 256;
int numBlocks = (N + blockSize - 1) / blockSize;
evolutionKernel<<<numBlocks, blockSize>>>(N, x, y);
```

Ubuntu 20.04 operating system. 5 Our DE implementations in comparison, i.e. sDE, cudaDe, are based on the "unified" algorithm with parameters set as: CR = 0.8, 1 = 0.25, F2 = 0.25, F3 = 0.2, F4 = 0.2. Population sizes are set to 50, 100, 500 and 1000 respectively for each test case. Each DE implementation under a specified parameter setting is run 25 times for each test problem of a specific dimension, starting from different random states each time, whereas the two implementations share the same initial random state for each each run. The algorithm ends when the maximum number of function evaluations is reached, which is set to 10'000 times the problem dimension divided by the population size of the problem, for example, for a 10D problem with a population size of 50, the maximum number of function evaluations is 2000 ((10'000 * 10) / 50).

B. *Test Problems*

We choose four numerical problems for testing:

F_1: Shifted sphere function

$$f_1(x) = \sum_{i=1}^{D} z_i^2, z = x - o$$

F_2: Shifted Rosenbrock's function

$$f_2(x) = \sum_{i=1}^{D-1} \left(100 * (z_i^2 - z_{i+1})^2 + (z_i + 1)^2\right), z = x - o$$

F_3: Shifted Griewank's function

$$f_3(x) = \sum_{i=1}^{D} \frac{z_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1, z = x - o$$

F_4: Shifted Rastrigin's function

$$f_4(x) = \sum_{i=1}^{D} \left(z_i^2 - 10 * \cos(2\pi z_i) + 10\right), z = x - o$$

Multiple threads are used to calculate the goal function values, with each thread processing 6 function components that correspond to one or more elements of one or more population members.

V. Results

Using four numerical problems with varied issue dimensions and algorithmic population sizes, we analyze and compare the computational time efficiency of two DE implementations, sDE and cudaDE.

		P50	P100	P500	P1000
F1	Best EFV	-450 (1.72633e-05)	-450 (6.10352e-06)	-450 (0)	-450 (0.000156564)
	Time (sec)	107.549 (379.604)	18.3662 (0.110718)	16.0359 (0.146722)	20.705 (0.152736)
F2	Best EFV	390.066 (0.268385)	390.038 (0.0798391)	390.467 (0.153961)	397.106 (1.89901)
	Time (sec)	46.1683 (0.187814)	27.02 (0.106834)	18.7858 (0.184944)	21.9927 (0.254022)
F3	Best EFV	-180 (0.00145861)	-180 (3.00563e-05)	-179.968 (0.0114503)	-179.58 (0.0850493)
	Time (sec)	52.9977 (0.0956197)	30.5071 (0.0944979)	19.5963 (0.131352)	21.8657 (0.154719)
F4	Best EFV	-330 (2.72958e-05	-330 (1.36479e-05)	-329.751 (0.0951895)	-327.077 (0.537065)
	Time (sec)	48.8638 (0.0985188)	28.6398 (0.10667)	19.5193 (0.155161)	22.1646 (0.167134)

uDE 10 dimensions

		P50	P100	P500	P1000
F1	Best EFV	-450 (8.23409e-05)	-450 (7.22178e-05)	-450 (8.18873e-05)	-449.969 (0.00463213)
	Time (sec)	489.254 (1.35994)	291.19 (0.795224)	278.225 (0.34185)	296.061 (0.33632)
F2	Best EFV	398.396 (8.684)	391.399 (0.969263)	402.461 (2.92022)	777.548 (50.5921)
	Time (sec)	947.084 (2.2686)	517.377 (1.24267)	335.035 (0.579582)	315.189 (0.509783)
F3	Best EFV	-180 (2.76348e-05)	-180 (2.20065e-05)	-179.993 (0.00155314)	-178.9 (0.0118595)
	Time (sec)	1045.58 (2.39125)	576.956 (1.08437)	354.695 (0.278562)	337.632 (0.443143)
F4	Best EFV	-324.548 (1.44348)	-325.864 (1.29518)	-318.442 (0.859305)	-289.257 (2.89001)
	Time (sec)	974.584 (3.46262)	547.475 (1.18178)	346.441 (0.274041)	335.168 (0.464686)

uDE 50 dimensions

		P50	P100	P500	P1000
F1	Best EFV	-450 (4.22864e-05)	-450 (3.76246e-05)	-450 (7.27318e-05)	-449.884 (0.0144555)
	Time (sec)	1760.79 (6.12097)	1066.28 (2.63083)	1044.45 (0.603709)	1075.89 (0.455502)
F2	Best EFV	419.009 (20.5124)	397.199 (3.52763)	454.351 (13.4546)	1976.69 (121.402)
	Time (sec)	3597.74 (8.89897)	1997.63 (3.65667)	1263.34 (0.471276)	1205.76 (0.580355)
F3	Best EFV	-180 (1.58574e-05)	-180 (1.67152e-05)	-179.99 (0.00177534)	-178.609 (0.0428756)
	Time (sec)	3927.31 (8.73395)	2212.07 (2.93232)	1340.37 (0.448757)	1275.16 (0.465865)
F4	Best EFV	-315.596 (2.1557)	-319.122 (1.445)	-298.954 (1.95028)	-227.196 (3.09385)
	Time (sec)	3624.83 (12.3424)	2056.47 (4.35575)	1301.48 (0.561792)	10244.64 (0.479688)

uDE 100 dimensions

A. *Sequential Defferential Evolution*

An sequential implementation is used to compare performances of cuda DE. This implementation was worked on from the code base of the cuda DE. But with lack of time and understanding of the differential evolution algorithm, ended with an incomplete sequential implementation. The code ran correctly but there was no results showing up, implying that an error was occurring in the code base.

		P50	P100	P500	P1000
F1	Best EFV	-428.6508281269648 (7.31186516634)	-416.43723716835007 (7.46248125135)	-175.0586331270339 (20.2930189353)	-166.9323406377331 (13.5892907423)
	Time (sec)	398.79486084 (6.83218782717)	199.997520447 (3.23541555549)	35.9956741333 (0.884660652481)	25.1176261902 (0.484521369386)
F2	Best EFV	38584.96078865421 (31637.9479773)	84486.8229425085 (32527.4361176)	2280198.9614914614 (248689.528058)	288863.6979022934 (319602.872968)
	Time (sec)	1073.96501541 (12.1330055544)	531.776008606 (7.62798668248)	101.960744858 (1.94230040713)	56.6309928894 (1.18222359242)
F3	Best EFV	-121.09398131120842 (32.4645464267)	-67.56285417444157 (19.3275561292)	760.2936879165917 (53.886265316)	772.9666406909193 (59.8010458207)
	Time (sec)	709.627275467 (10.8141683129)	353.507890701 (5.3105627211)	63.2189464569 (1.31167483864)	38.2913208008 (0.798946033336)
F4	Best EFV	-79.62374001008924 (37.4225709068)	-28.686004559949474 (24.4262365985)	373.7213316873689 (27.6586666932)	378.27843529284206 (22.3217039518)
	Time (sec)	1895.06689072 (12.9376656385)	947.151832581 (9.19065972991)	183.579511642 (3.25836988147)	100.666484833 (2.01054807819)

sDE 10 dimension

		P50	P100	P500	P1000
F1	Best EFV	-348.9110403621618 (31.4656810881)	-354.49249296454144 (17.9253908187)	152.87353824761124 (40.0193066207)	178.537593761033 (22.3186303876)
	Time (sec)	1296.62432671 (15.8790633798)	634.355812073 (5.5717792015)	122.545833588 (2.21842706967)	52.1974754333 (0.841795613206)
F2	Best EFV	449459.62218015397 (218826.968605)	385565.012507683 (132657.41586)	6239045.567351831 (640584.533488)	6217069.830197088 (680142.024476)
	Time (sec)	4128.26592445 (18.4313183884)	2086.10423088 (24.3454005862)	409.544181824 (9.39154906858)	188.517560959 (4.40024027043)
F3	Best EFV	200.77752524586992 (122.691968744)	145.09060398217264 (71.6982633971)	1911.0011404535212 (131.522924153)	1963.5468026935428 (96.9543675611)
	Time (sec)	2447.00567245 (27.4126138768)	1211.04588509 (12.885313635)	228.145046234 (1.70541642007)	94.4014263153 (0.962482381004)
F4	Best EFV	392.8674974204457 (62.815359619)	375.49432920232937 (57.6501011994)	1172.907069598961 (43.3628425427)	1199.8340422544854 (38.789967046)
	Time (sec)	6941.89335823 (11.9393969051)	3467.06618309 (4.62479523109)	689.184627533 (2.10331041466)	333.077316284 (1.23201251792)

sDE 50 dimension

The code used to produce the sequential results was provided by Lhassane Idoumghar and was modified to run the same setup as the Cuda implementation. In comparison, we can see overhaul that our uDE implementation beats the sequential implementation. For the first function (F1, Shifted Sphere), performances seam to decrease with populations size. At P50, we can see close results. The sequential implementation increase in distance from the solution faster and faster when the population increases, this could be explained by the mutation strategy difference. Our sequential implementation is not using the unified mutation strategy, which could explain the result difference. On the contrary, it seems that the run time of the sequential implementation keeps being

		P50	P100	P500	P1000
F1	Best EFV	-348.9110403621618 (31.4656810881)	-354.49249296454144 (17.9253908187)	152.87353824761124 (40.0193066207)	178.537593761033 (22.3186303876)
	Time (sec)	1296.62432671 (15.8790633798)	634.355812073 (5.5717792015)	122.545833588 (2.21842706967)	52.1974754333 (0.841795613206)
F2	Best EFV	449459.62218015397 (218826.968605)	385565.012507683 (132657.41586)	6239045.567351831 (640584.533488)	6217069.830197088 (680142.024476)
	Time (sec)	4128.26592445 (18.4313183884)	2086.10423088 (24.3454005862)	409.544181824 (9.39154906858)	188.517560959 (4.40024027043)
F3	Best EFV	200.77752524586992 (122.691968744)	145.09060398217264 (71.6982633971)	1911.0011404535212 (131.522924153)	1963.5468026935428 (96.9543675611)
	Time (sec)	2447.00567245 (27.4126138768)	1211.04588509 (12.885313635)	228.145046234 (1.70541642007)	94.4014263153 (0.962482381004)
F4	Best EFV	392.8674974204457 (62.815359619)	375.49432920232937 (57.6501011994)	1172.907069598961 (43.3628425427)	1199.8340422544854 (38.789967046)
	Time (sec)	6941.89335823 (11.9393969051)	3467.06618309 (4.62479523109)	689.184627533 (2.10331041466)	333.077316284 (1.23201251792)

sDE 100 dimension

below the uDE implementation. This could be explained by different factors, including the power comparaison between the GPU running the Cuda implementation being weaker than the CPU running the sequential implementation. It could also be explained by the initial kernel initialisation taking an initial setup time for the uDE implementation to be running.

B. *Comparaison with non-unified Cuda DE*

Optimisation of our Cuda implementation was based on an article on the topic of the DE implementation Qin et al., 2012. This article shows result from their implementation and we can see way better results across all setups. This could be explained by further optimisations on the implementation. The cross rate which decides when to mutate a generation was set to 0.8 both in the cited article and our implementation.

Moving this value proved to change the results but tweaking it to different values didn’t improve enough the results to conclude that our implementation was better.

VI. Conclusion

Implementing the Cuda Unified Differential Algorithm was an interesting first hand experimentation over parallel programming. We learned how to solve large problems with the help of GPU computing. In the process of learning the uDE implementation we got to face limit in term of resources. Google Colab provides access to high performance GPU at low cost. The issue for us was that Google Colab is not always available and reserved resources are not always up to our implementation requirements. Personal Computers GPU’s are currently a scarce resource and are very expensive, making it very hard to run our implementation locally.

References

[1] Horrigue, L., Ghodhbane, R., Saidani, T., & Atri, M. (2018). Gpu acceleration of image processing algorithm based on matlab cuda. 18, 9.

[2] Qin, K., Raimondo, F., Forbes, F., & Ong, Y. (2012). An improved cuda-based implementation of differential evolution on gpu. GECCO’12 - Proceedings of the 14th International Conference on Genetic and Evolutionary Computation. https : / / doi . org / 10 . 1145 / 2330163.2330301

[3] Ao, Youyun & Chi, Hongqin. (2010). An Adaptive Differential Evolution Algorithm to Solve Constrained Optimization Problems in Engineering Design. Engineering. 02. 10.4236/eng.2010.21009.

[4] Horrigue, Layla & Ghodhbane, Refka & Saidani, Taoufik & Atri, Mohamed. (2018). GPU Acceleration of Image Processing Algorithm Based on MATLAB CUDA. 18. 9.