CS 413 Homework 1

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Problem 1

False. Consider the set of two men $M = \{m_1, m_2\}$ and the set of two women $W = \{w_1, w_2\}$ with the following set of preference lists:

m_1 prefers w_1 to w_2

m_2 prefers w_2 to w_1

w_1 prefers m_2 to m_1

w_2 prefers m_1 to m_2

In this set of preference lists there does not exist a pair $(m, w) \in M \times W$ such that m ranks w first and w ranks m first. Hence no such pair could occur in any matching including a stable one.

Problem 2

True. Suppose not. That is suppose that for a set $M = \{m_1, m_2, ..., m_n\}$ of n men and a set $W = \{w_1, w_2, ..., w_n\}$ of n women where there exists a man $m \in M$ and a woman $w \in W$ such that m and w are first on each other's preference lists, there exists a stable matching $S \subseteq M \times W$ such that the pair $(m, w) \notin S$. Since S is a stable matching S is perfect and there is no instability with respect to S. Since S is perfect it must be that m and w have some other partners w' and m' respectively so that $(m, w') \in S$ and $(m', w) \in S$. But now by definition of m and w, m prefers w to w' and w prefers m to m' and so (m, w) is an instability with respect to S. It follows that there is no instability with respect to S and there is an instability with respect to S which is a contradiction.

Problem 3

There exists a set of TV shows and associated ratings such that there does no exist a stable pair of schedules.

	Slot 1	**Slot 2**
Network A	show a_1: 2	show a_2: 4
Network B	show b_1: 3	show b_2: 1

There is no stable pair of schedules for the above set of TV shows and associated ratings because Network A can switch show a_1 and a_2 and win 2 prime time slots instead of 1.