Chapter 6

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Lemma 6.1.a. $\forall x \in \mathbb{R},\ x \neq 3 \implies x^2 - 2x + 3 \neq 0.$

Proof. Let x be any real number such that $x \neq 3$. We must show that $x^2 - 2x + 3 \neq 0$. Assume that it is equal to 0 and derive a contradiction. Completing the square of $x^2 - 2x + 3 = 0$ we obtain

$$\begin{aligned} x^2 - 2x + 3 &= 0 \\ (x^2 - 2x + 3) + 1 &= 0 + 1 \\ (x^2 - 2x + 1) + 3 &= 1 \\ x^2 - 2x + 1 &= -2 \\ (x-1)^2 &= -2 \end{aligned}$$

which is a contradiction since $\forall y \in \mathbb{R},\ y^2 \geq 0$. □

Lemma 6.1.b. $\exists x \in \mathbb{C},\ x \neq 3 \wedge x^2 - 2x + 3 = 0.$

Proof. When dealing with complex numbers the prior proposition does not hold. To show this we show that the negation is true. Let $x = 1 + i\sqrt{2}$. This is not equal to 3 because the definition of equality on complex numbers is equal real and imaginary parts and $1 \neq 3$ and $\sqrt{2} \neq 0$. Next we verify that $x^2 - 2x + 3 = 0$.

$$\begin{aligned} x^2 - 2x + 3 &= 0 \\ (1 + i\sqrt{2})^2 - 2(1 + i\sqrt{2}) + 3 &= 0 \\ (1 + i\sqrt{2}) \cdot (1 + i\sqrt{2}) - 2 - i \cdot 2\sqrt{2} + 3 &= 0 \\ 1 + i \cdot 2\sqrt{2} + (i\sqrt{2})^2 - i \cdot 2\sqrt{2} + 1 &= 0 \\ (1 + 1) + (i \cdot 2\sqrt{2} - i \cdot 2\sqrt{2}) + (-1 \cdot 2) &= 0 \\ 2 + 0 - 2 &= 0 \\ 0 &= 0 \end{aligned}$$

□

Lemma 6.1. *This is Lemma 6.2.*

Proof. □

Lemma 6.2. *This is Lemma 6.3.*

Proof. □