Section 4.5

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October 13, 2020

Compute $\lfloor x \rfloor$ and $\lceil x \rceil$ for each of the values of x in 1-4.

Problem 1 and Solution

$x = 37.999$

$37 < 37.999 < 38$; hence $\lfloor 37.999 \rfloor = 37$ and $\lceil 37.999 \rceil = 40$.

Problem 2 and Solution

$x = \frac{17}{4} = 4.25$

$4 < 4.25 < 5$; hence $\lfloor 4.25 \rfloor = 4$ and $\lceil 4.25 \rceil = 5$.

Problem 3 and Solution

$x = -14.00001$

$-15 < -14.00001 < -14$; hence $\lfloor -14.00001 \rfloor = -14$ and $\lceil -14.00001 \rceil = -14$.

Problem 4 and Solution

$x = -\frac{32}{5} = -6.4$

$-7 < -6.4 < -6$; hence $\lfloor -6.4 \rfloor = -7$ and $\lceil -6.4 \rceil = -6$.

Problem 5

Use the floor notation to express 259 *div* 11 and 259 *mod* 11.

Solution

$259\ div\ 11 = \left\lfloor \frac{259}{11} \right\rfloor, \quad 259\ mod\ 11 = 259 - 11 \cdot \left\lfloor \frac{259}{11} \right\rfloor$

Problem 6

If k is an integer, what is $\lceil k \rceil$? Why?

Solution

Since k is an integer, $k - 1 < k \leq k$. It follows that $\lceil k \rceil = k$.

Problem 7

If k is an integer, what is $\left\lceil k + \frac{1}{2} \right\rceil$? Why?

Solution

Since k is an integer, $k < k + \frac{1}{2} < k + 1$. It follows that $\left\lceil k + \frac{1}{2} \right\rceil = k + 1$.

Problem 8

Seven pounds of raw material are needed to manufacture each unit of a certain product. Express the number of units that can be produced from n pounds of raw material using either the floor or the ceiling notation. Which notation is more appropriate?

Solution

Floor notation is more appropriate. With floor notation the number of units that can be produced from n pounds of raw material is $\left\lfloor \frac{n}{7} \right\rfloor$. With ceiling notation two formulas are required. In the case that $\frac{n}{7}$ is an integer the number of units that can be produced from n pounds of raw material is $\left\lceil \frac{n}{7} \right\rceil$. In the case that $\frac{n}{7}$ is not an integer the the number of units that can be produced from n pounds of raw material is $\left\lceil \frac{n}{7} - 1 \right\rceil$.

Problem 9

Boxes, each capable of holding 36 units, are used to ship a product from the manufacturer to a wholesaler. Express the number of boxes that would be required to ship n pounds of raw material using either the floor or the ceiling notation. Which notation is more appropriate?

Solution

The ceiling notation is more appropriate. With the ceiling notation $\left\lceil \frac{n}{36} \right\rceil$ boxes would be required. With the floor notation two formulas are required. In the case that $\frac{n}{36}$ is an integer the number of boxes required would be $\left\lfloor \frac{n}{36} \right\rfloor$. In the case that $\frac{n}{36}$ is not an integer the number of boxes required would be $\left\lfloor \frac{n}{36} + 1 \right\rfloor$.

Problem 10

If 0 = Sunday, 1 = Monday, 2 = Tuesday,...,6 = Saturday, then January 1 of year n occurs on the day of the week given by the following formula:

$$\left(n+\left\lfloor\frac{n-1}{4}\right\rfloor-\left\lfloor\frac{n-1}{100}\right\rfloor+\left\lfloor\frac{n-1}{400}\right\rfloor\right)\ mod\ 7.$$

(a) User this formula to find January 1 of

  (i) 2050

  (ii) 2100

  (iii) the year of your birth - 1997

(b) Interpret the different components of this formula.

Solution

(a) January 1 of

  (i) $\left(2050+\left\lfloor\frac{2050-1}{4}\right\rfloor-\left\lfloor\frac{2050-1}{100}\right\rfloor+\left\lfloor\frac{2050-1}{400}\right\rfloor\right)\ mod\ 7=6=$ Saturday.

  (ii) $\left(2100+\left\lfloor\frac{2100-1}{4}\right\rfloor-\left\lfloor\frac{2100-1}{100}\right\rfloor+\left\lfloor\frac{2100-1}{400}\right\rfloor\right)\ mod\ 7=5=$ Thursday.

  (iii) $\left(1997+\left\lfloor\frac{1997-1}{4}\right\rfloor-\left\lfloor\frac{1997-1}{100}\right\rfloor+\left\lfloor\frac{1997-1}{400}\right\rfloor\right)\ mod\ 7=3=$ Wednesday.

(b) Suppose that every year has exactly 365 days(this is false because of leap years). We all know from experience that the days of the week always proceed from Sunday through Saturday in the same order and then reset back to Sunday and repeat. It follows that if today is a certain day of the week Sunday through Saturday then n weeks from now will be the same day of the week. However 52 weeks from a certain day is only 364 days from the present day. Since a year is 365 days in total then one year from any day of the week will end up being the next day. This explains the n portion of the formula. Every 4 years is a leap year which means that a year has 366 days instead of 365. Thus every 4 years it will take one extra day to reach January 1 and this explains the $\left\lfloor\frac{n-1}{4}\right\rfloor$ portion of the formula. Lastly a leap year is skipped once per century unless that century is a multiple of 400. This explains the $-\left\lfloor\frac{n-1}{100}\right\rfloor+\left\lfloor\frac{n-1}{400}\right\rfloor$ portion of the formula.

Problem 11

State a necessary and sufficient condition for the floor of a real number to equal that number.

Solution

$\forall x\in\mathbb{R}$, $\lfloor x\rfloor=x \iff x\in\mathbb{Z}$. In other words for all real numbers x, $\lfloor x\rfloor=x$ if and only if x is an integer.

Problem 12

Prove that if n is any even integer, then $\lfloor \frac{n}{2} \rfloor = \frac{n}{2}$.

Theorem: If n is any even integer, then $\lfloor \frac{n}{2} \rfloor = \frac{n}{2}$.

Proof. Let n be any even integer. Then $n = 2k$ for some integer k.

$$\left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{2k}{2} \right\rfloor = \lfloor k \rfloor = k$$

because k is an integer and $k \leq k < k + 1$. Since $k = \frac{n}{2}$ and $k = \lfloor \frac{n}{2} \rfloor$ it follows that $\frac{n}{2} = \lfloor \frac{n}{2} \rfloor$. $\square$

Problem 13

Suppose that n and d are integers and $d \neq 0$. Prove each of the following.

(a) If $d \mid n$, then $n = \lfloor \frac{n}{d} \rfloor \cdot d$.

(b) If $n = \lfloor \frac{n}{d} \rfloor \cdot d$, then $d \mid n$.

(c) Use the floor notation to state a necessary and sufficient condition for an integer n to be divisible by an integer d.

Solution

(a) *Theorem*: If $d \mid n$, then $n = \lfloor \frac{n}{d} \rfloor \cdot d$.

Proof. Let n and d be integers such that $d \neq 0$. Further, suppose that $d \mid n$. Then $n = dk$ for some integer k.

$$\left\lfloor \frac{n}{d} \right\rfloor \cdot d = \left\lfloor \frac{dk}{d} \right\rfloor \cdot d = \lfloor k \rfloor \cdot d$$

Since k is an integer and $k \leq k < k+1$, $\lfloor k \rfloor = k$. It follows from this that $\lfloor \frac{n}{d} \rfloor \cdot d = dk$. Since $n = dk$ and $\lfloor \frac{n}{d} \rfloor \cdot d = dk$, $n = \lfloor \frac{n}{d} \rfloor \cdot d$. $\square$

(b) *Theorem*: If $n = \lfloor \frac{n}{d} \rfloor \cdot d$, then $d \mid n$.

Proof. Let n and d be integers such that $d \neq 0$. Further, suppose that $k = \lfloor \frac{n}{d} \rfloor$. It follows from the definition of floor that k is an integer. It follows from this that $n = dk$. Thus $d \mid n$. $\square$

(c) It follows from part(a) and part(b) above that $d \mid n \iff n = \lfloor \frac{n}{d} \rfloor \cdot d$.

Problem 14

Determine if for all real numbers x and y, $\lfloor x-y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor$.

Counterexample: Let $x = 2$ and let $y = 1.2$. Then x and y are real numbers and $\lfloor x-y \rfloor = \lfloor 2-1.2 \rfloor = \lfloor .8 \rfloor = 0$. But $\lfloor x \rfloor - \lfloor y \rfloor = \lfloor 2 \rfloor - \lfloor 1.2 \rfloor = 2 - 1 = 1 \neq 0$.

Problem 15

Prove that for all real numbers x, $\lfloor x-1 \rfloor = \lfloor x \rfloor - 1$.

Theorem: For all real numbers x, $\lfloor x-1 \rfloor = \lfloor x \rfloor - 1$.

Proof. Let x be an real number and let $n = \lfloor x \rfloor$. Then n is an integer and $n \leq x < n+1$. Subtracting 1 from all parts of the inequality gives

$$n - 1 \leq x - 1 < n$$

It follows that $n - 1 = \lfloor x-1 \rfloor$. Since $n = \lfloor x \rfloor$, $n - 1 = \lfloor x \rfloor - 1$, but $n - 1 = \lfloor x-1 \rfloor$ and so $\lfloor x-1 \rfloor = \lfloor x \rfloor - 1$. □

Problem 16

Determine if for all real numbers x, $\lfloor x^2 \rfloor = \lfloor x \rfloor^2$.

Counterexample: Let $x = 1.5$. Then x is a real number and $\lfloor x^2 \rfloor = \lfloor 2.25 \rfloor = 2 \neq \lfloor x \rfloor^2 = \lfloor 1.5 \rfloor^2 = \lfloor 1 \rfloor^2 = 1$.

Problem 17

Prove that for all integers n,

$$\left\lfloor \frac{n}{3} \right\rfloor = \begin{cases} \frac{n}{3} & \text{if } n \ mod \ 3 = 0 \\ \frac{n-1}{3} & \text{if } n \ mod \ 3 = 1 \\ \frac{n-2}{3} & \text{if } n \ mod \ 3 = 2 \end{cases}$$

Proof. Let n be any integer.

Case 1: If $n \ mod \ 3 = 0$ then $n = 3q + 0 = 3q$ for some integer q.

$$\left\lfloor \frac{n}{3} \right\rfloor = \left\lfloor \frac{3q}{3} \right\rfloor = \lfloor q \rfloor = q = \frac{n}{3}$$

Case 2: If $n \ mod \ 3 = 1$ then $n = 3q + 1$ for some integer q.

$$\left\lfloor \frac{n}{3} \right\rfloor = \left\lfloor \frac{3q+1}{3} \right\rfloor = \left\lfloor \frac{3q}{3} + \frac{1}{3} \right\rfloor = \left\lfloor q + \frac{1}{3} \right\rfloor = q = \frac{n-1}{3}$$

Case 3: If $n \bmod 3 = 2$ then $n = 3q + 2$ for some integer q.

$$\left\lfloor \frac{n}{3} \right\rfloor = \left\lfloor \frac{3q+2}{3} \right\rfloor = \left\lfloor \frac{3q}{3} + \frac{2}{3} \right\rfloor = \left\lfloor q + \frac{2}{3} \right\rfloor = q = \frac{n-2}{3}$$

$\square$

Problem 18

Determine if for all real numbers x and y, $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$.

Counterexample: Let $x = 1.5$ and let $y = 1.5$. Then x and y are real numbers and $\lceil x + y \rceil = \lceil 1.5 + 1.5 \rceil = \lceil 3 \rceil = 3 \neq \lceil x \rceil + \lceil y \rceil = \lceil 1.5 \rceil + \lceil 1.5 \rceil = 1 + 1 = 2$.

Problem 19

Prove that for all real numbers x, $\lceil x - 1 \rceil = \lceil x \rceil - 1$.

Theorem: For all real numbers x, $\lceil x - 1 \rceil = \lceil x \rceil - 1$.

Proof. Let x be any real number and let n be an integer such that $n = \lceil x - 1 \rceil$. Then $n - 1 < x - 1 \leq n$. Add 1 to all parts of the inequality to obtain

$$n < x \leq n + 1$$

It follows that $\lceil x \rceil = n + 1$. Subtracting 1 from both sides of the equation gives $\lceil x \rceil - 1 = n$. But $n = \lceil x - 1 \rceil$. Thus $\lceil x - 1 \rceil = \lceil x \rceil - 1$. $\square$

Problem 20

Determine if for all real numbers x and y, $\lceil xy \rceil = \lceil x \rceil \cdot \lceil y \rceil$.

Counterexample: Let $x = .5$ and let $y = 2$. Then x and y are real numbers and $\lceil xy \rceil = \lceil (.5)(2) \rceil = \lceil 1 \rceil = 1 \neq \lceil x \rceil \cdot \lceil y \rceil = \lceil .5 \rceil \lceil 2 \rceil = 1 \cdot 2 = 2$.

Problem 21

Prove that for all odd integers n, $\left\lceil \frac{n}{2} \right\rceil = \frac{n+1}{2}$.

Theorem: For all odd integers n, $\left\lceil \frac{n}{2} \right\rceil = \frac{n+1}{2}$.

Proof. Let n be an odd integer. Then $n = 2k + 1$ for some integer k.

$$\begin{aligned}\left\lceil \frac{n}{2} \right\rceil &= \left\lceil \frac{2k+1}{2} \right\rceil \\ &= \left\lceil \frac{2k}{2} + \frac{1}{2} \right\rceil \\ &= \left\lceil k + \frac{1}{2} \right\rceil \\ &= k + 1, \qquad k < k + 1 \leq k + 1\end{aligned}$$

Solving the definition of n for k gives $k = \frac{n-1}{2}$. By substitution

$$\begin{aligned}k + 1 &= \frac{n-1}{2} + 1 \\ &= \frac{n-1}{2} + \frac{2}{2} \\ &= \frac{n+1}{2}\end{aligned}$$

Since $\lceil \frac{n}{2} \rceil = k + 1$ and $k + 1 = \frac{n+1}{2}$, $\lceil \frac{n}{2} \rceil = \frac{n+1}{2}$. $\square$

Problem 22

Determine if for all real numbers x and y, $\lceil xy \rceil = \lceil x \rceil \cdot \lfloor y \rfloor$.

Counterexample: Let $x = .5$ and let $y = 2$. Then x and y are real numbers and $\lceil xy \rceil = \lceil (.5)(2) \rceil = \lceil 1 \rceil = 1 \neq \lceil x \rceil \cdot \lceil y \rceil = \lceil .5 \rceil \cdot \lceil 2 \rceil = 1 \cdot 2 = 2$.

Problem 23

Prove that for any real number x, if x is not an integer, then $\lfloor x \rfloor + \lfloor -x \rfloor = -1$.

Theorem: For all real numbers x, if x is not an integer, then $\lfloor x \rfloor + \lfloor -x \rfloor = -1$.

Proof. Let x be a real number such that x is not an integer. Let $n = \lfloor x \rfloor$. It follows from the definition of floor and because x is not an integer that $n < x < n + 1$. Multiply all parts of the inequality by -1 to obtain

$$-n > -x > -n - 1$$

$$-n - 1 < -x < -n$$

It follows that $\lfloor -x \rfloor = -n - 1$. Thus $\lfloor x \rfloor + \lfloor -x \rfloor = n + (-n - 1) = -1$. $\square$

Problem 24

Prove that for any integer m and any real number x, if x, is not an integer, then $\lfloor x \rfloor + \lfloor m - x \rfloor = m - 1$.

Theorem: For all integers m and real numbers x, if x is not an integer, then $\lfloor x \rfloor + \lfloor m - x \rfloor = m - 1$.

Proof. Let m be an integer and let x be a real number such that x is not an integer. Let $n = \lfloor x \rfloor$. It follows from the definition of floor and since x is not an integer that

$$n < x < n + 1$$

$$-n > -x > -n - 1$$

$$m - n > m - x > m - n - 1$$

$$m - n - 1 < m - x < m - n$$

It follows that $\lfloor m - x \rfloor = m - n - 1$. Thus

$$\lfloor x \rfloor + \lfloor m - x \rfloor = n + (m - n - 1) = m - 1$$

$\square$

Problem 25

Prove that for all real numbers x, $\lfloor \lfloor x/2 \rfloor /2 \rfloor = \lfloor x/4 \rfloor$.

Theorem: For all real numbers x, $\lfloor \lfloor x/2 \rfloor /2 \rfloor = \lfloor x/4 \rfloor$.

Proof. Let x be any real number and let $n = \lfloor x/2 \rfloor$. Then $n \leq \frac{x}{2} < n + 1$.

Case 1 (n is even): If n is even then $n = 2k$ for some integer k.

$$2k \leq \frac{x}{2} < 2k + 1$$

$$k \leq \frac{x}{4} < \frac{2k + 1}{2}$$

$$k \leq \frac{x}{4} < k + \frac{1}{2}$$

$$k \leq \frac{x}{4} < k + 1$$

It follows that $k = \left\lfloor \frac{x}{4} \right\rfloor$. Since $n = \left\lfloor \frac{x}{2} \right\rfloor$ and $n = 2k$, $k = \frac{n}{2} = \lfloor x/2 \rfloor /2$. Since k is an integer it follows that $\lfloor k \rfloor = k$. Thus $\lfloor \lfloor x/2 \rfloor /2 \rfloor = \lfloor x/4 \rfloor$.

Case 2 (n is odd): If n is odd then $n = 2k + 1$ for some integer k.

$$2k + 1 \leq \frac{x}{2} < 2k + 2$$

$$\frac{2k+1}{2} \leq \frac{x}{4} < \frac{2(k+1)}{2}$$

$$k + \frac{1}{2} \leq \frac{x}{4} < k+1$$

$$k \leq \frac{x}{4} < k+1$$

It follows that $k = \left\lfloor \frac{x}{4} \right\rfloor$. Since $n = 2k+1$, $\frac{n}{2} = k + \frac{1}{2}$. Thus $\left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor k + \frac{1}{2} \right\rfloor = k$. Since $n = \left\lfloor \frac{x}{2} \right\rfloor$, it follows that $\left\lfloor \frac{n}{2} \right\rfloor = \lfloor \lfloor x/2 \rfloor /2 \rfloor = k = \lfloor x/4 \rfloor$. Thus $\lfloor \lfloor x/2 \rfloor /2 \rfloor = \lfloor x/4 \rfloor$.

Since $\lfloor \lfloor x/2 \rfloor /2 \rfloor = \lfloor x/4 \rfloor$ in the case that $\lfloor x/2 \rfloor$ is even and in the case that $\lfloor x/2 \rfloor$ is odd and since the floor of any real number is an integer and since all integers are either even or odd we can conclude that for all real numbers x, $\lfloor \lfloor x/2 \rfloor /2 \rfloor = \lfloor x/4 \rfloor$. □

Problem 26

Prove that for all real numbers x, if $x - \lfloor x \rfloor < \frac{1}{2}$ then $\lfloor 2x \rfloor = 2 \lfloor x \rfloor$.

Theorem: For all real numbers x, if $x - \lfloor x \rfloor < \frac{1}{2}$ then $\lfloor 2x \rfloor = 2 \lfloor x \rfloor$.

Proof. Let x be any real number such that $x - \lfloor x \rfloor < \frac{1}{2}$.

$$2x - 2 \lfloor x \rfloor < 1$$

$$2x < 2 \lfloor x \rfloor + 1$$

It follows from the definition of floor that $\lfloor x \rfloor \leq x$. Thus $2 \lfloor x \rfloor \leq 2x$. Combining the inequalities gives

$$2 \lfloor x \rfloor \leq 2x < 2 \lfloor x \rfloor + 1$$

It now follows from the definition of floor that $\lfloor 2x \rfloor = 2 \lfloor x \rfloor$. □

Problem 27

Prove that for all real numbers x, if $x - \lfloor x \rfloor \geq \frac{1}{2}$ then $\lfloor 2x \rfloor = 2 \lfloor x \rfloor + 1$.

Theorem: For all real numbers x, if $x - \lfloor x \rfloor \geq \frac{1}{2}$ then $\lfloor 2x \rfloor = 2 \lfloor x \rfloor + 1$.

Proof. Let x be any real number such that $x - \lfloor x \rfloor \geq \frac{1}{2}$.

$$2x - 2 \lfloor x \rfloor \geq 1$$

$$2x \geq 2 \lfloor x \rfloor + 1$$

$$2 \lfloor x \rfloor + 1 \leq 2x \quad (1)$$

Let n be an integer such that $n = \lfloor x \rfloor$. Then $x < n + 1$. Multiply both sides of the inequality by 2 to obtain $2x < 2n + 2$. Since $n = \lfloor x \rfloor$ it follows that

$$2x < 2\lfloor x \rfloor + 2 \quad (2)$$

Now combine inequalities (1) and (2) to obtain

$$2\lfloor x \rfloor + 1 \leq 2x < 2\lfloor x \rfloor + 2$$

It now follows that $\lfloor 2x \rfloor = 2\lfloor x \rfloor + 1$. □

Problem 28

Prove that for any odd integer n,

$$\left\lfloor \frac{n^2}{4} \right\rfloor = \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)$$

Proof. Let n be any odd integer. Then $n = 2k + 1$ for some integer k.

$$\begin{aligned}
\left\lfloor \frac{n^2}{4} \right\rfloor &= \left\lfloor \frac{(2k+1)^2}{4} \right\rfloor \\
&= \left\lfloor \frac{4k^2 + 4k + 1}{4} \right\rfloor \\
&= \left\lfloor \frac{4k^2}{4} + \frac{4k}{4} + \frac{1}{4} \right\rfloor \\
&= \left\lfloor k^2 + k + \frac{1}{4} \right\rfloor
\end{aligned}$$

It follows from closure under multiplication and addition that k^2+k is an integer. Since $k^2 + k$ is an integer and

$$k^2 + k \leq k^2 + k + \frac{1}{4} < k^2 + k + 1$$

it follows from the definition of floor that $k^2 + k = \left\lfloor k^2 + k + \frac{1}{4} \right\rfloor$. Thus $\left\lfloor \frac{n^2}{4} \right\rfloor = k^2 + k$. Solving the definition of n for k gives $k = \frac{n-1}{2}$. By substitution,

$$\begin{aligned}
\left\lfloor \frac{n^2}{4} \right\rfloor = k^2 + k &= \left(\frac{n-1}{2}\right)^2 + \frac{n-1}{2} \\
&= \left(\frac{n-1}{2}\right)\left(\frac{n-1}{2} + 1\right) \\
&= \left(\frac{n-1}{2}\right)\left(\frac{n-1}{2} + \frac{2}{2}\right) \\
&= \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)
\end{aligned}$$

□

Problem 29

Prove that for any odd integer n,

$$\left\lceil \frac{n^2}{4} \right\rceil = \frac{n^2+3}{4}$$

Proof. Let n be any off integer. Then $n = 2k+1$ for some integer k.

$$\begin{aligned}
\left\lceil \frac{n^2}{4} \right\rceil &= \left\lceil \frac{(2k+1)^2}{4} \right\rceil \\
&= \left\lceil \frac{4k^2+4k+1}{4} \right\rceil \\
&= \left\lceil k^2+k+\frac{1}{4} \right\rceil \\
&= k^2+k+1
\end{aligned}$$

Solving the definition of n for k gives $k = \frac{n-1}{2}$. By substitution,

$$\begin{aligned}
\left\lceil \frac{n^2}{4} \right\rceil = k^2+k+1 &= \left(\frac{n-1}{2}\right)^2 + \frac{n-1}{2} + 1 \\
&= \left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+1\right)+1 \\
&= \left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+\frac{2}{2}\right)+1 \\
&= \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)+1 \\
&= \frac{n^2-1}{4}+1 \\
&= \frac{n^2-1}{4}+\frac{4}{4} \\
&= \frac{n^2+3}{4}
\end{aligned}$$

□

Problem 30

Find the mistake in the following proof that $\lfloor n/2 \rfloor = (n-1)/2$ if n is an odd integer.

proof: Suppose n is any odd integer. Then $n = 2k+1$ for some integer k. Consequently,

$$\left\lfloor \frac{2k+1}{2} \right\rfloor = \frac{(2k+1)-1}{2} = \frac{2k}{2} = k$$

But $n = 2k+1$. Solving for k gives $k = (n-1)/2$. Hence, by substitution, $\lfloor n/2 \rfloor = (n-1)/2$.

Solution

This proof is flawed because it exhibits circular reasoning. It assumes what is to be proved namely that $\lfloor n/2 \rfloor = (n-1)/2$. Although true, as presented in the proof there is no basis for the equality.