Section 4.2

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Problem 1

Write $-\frac{35}{6}$ as a ratio of two integers.

Solution

Let $a = -35$ and let $b = 6$. Then a and b are integers and $\frac{a}{b} = -\frac{35}{6}$.

Problem 2

Write 4.6037 as a ratio of two integers.

Solution

Let $a = 46037$ and let $b = 10000$. Then a and b are integers and $\frac{a}{b} = \frac{46037}{10000} = 4.6037$.

Problem 3

Write $\frac{4}{5} + \frac{2}{9}$ as a ratio of two integers.

Solution

$$\frac{4}{5} + \frac{2}{9} = \frac{36}{45} + \frac{10}{45} = \frac{46}{45}$$

Let $a = 46$ and let $b = 45$. Then a and b are both integers and $\frac{a}{b} = \frac{46}{45} = \frac{4}{5} + \frac{2}{9}$.

Problem 4

Write .37373737... as a ratio of two integers.

Solution

$$x = .37373737...$$
$$100x = 37.37373737...$$
$$100x - x = 37.37373737... - .37373737... = 37$$
$$100x - x = 99x$$
$$99x = 37$$
$$x = \frac{37}{99}$$

Let $a = 37$ and let $b = 99$. Then a and b are integers and $\frac{a}{b} = \frac{37}{99} = .37373737...$

Problem 5

Write .57575757... as a ratio of two integers.

Solution

$$x = .57575757...$$
$$100x = 57.57575757...$$
$$100x - x = 57.57575757... - .57575757... = 57$$
$$100x - x = 99x$$
$$99x = 57$$
$$x = \frac{57}{99}$$

Let $a = 57$ and let $b = 99$. Then a and b are integers and $\frac{a}{b} = \frac{57}{99} = ..57575757...$

Problem 6

Write 320.5492492492 as a ration of two integers.

Solution

$$x = 320.5492492492...$$
$$10,000x = 3,205,492.492492...$$
$$10x = 3205.492492492...$$
$$10,000x - 10x = 3,205,492.492492... - 3,205.492492492... = 3,202,287$$
$$10,000x - 10x = 9,990x$$
$$9,990x = 3,202,287$$
$$x = \frac{3,202,287}{9,990}$$

Let $a = 3,202,287$ and let $b = 9,990$. Then a and b are both integers and $\frac{a}{b} = \frac{3,202,287}{9,990} = 320.5492492492...$

Problem 7

Write 52.4672167216721... as a ratio of two integers.

Solution

$$x = 52.4672167216721...$$
$$100,000x = 5,246,721.67216721...$$
$$10x = 524.672167216721...$$
$$100,000x - 10x = 5,246,197 = 99,990x$$
$$x = \frac{5,246,197}{99,990}$$

Let $a = 5,246,197$ and let $b = 99,990$. Then a and b are integers and $\frac{a}{b} = \frac{5,246,197}{99,990} = 52.4672167216721...$

Problem 8

The zero product property says that if a product of two real numbers is 0, then one of the numbers must be 0.

(i) Write this property formally using quantifiers and variables.

(ii) Write the contrapositive of your answer to part 1.

(iii) Write an informal version for your answer to part 2.

Solution

(i) $\forall a, b \in R \; ab = 0 \implies a = 0$ or $b = 0$

(ii) $\forall a, b \in R \; a \neq 0$ and $b \neq 0 \implies ab \neq 0$

(iii) If any two integers are both not equal to 0 then their product is not equal to 0.

Problem 9

Assume that a and b are both integers and that $a \neq 0$ and $b \neq 0$. Explain why $(b - a)/(ab^2)$ must be a rational number.

Solution

$(b-a)/(ab^2)$ must be a rational number because the numerator is a difference of integers and is therefore an integer and the denominator is a product or integers and is therefore an integer. Furthermore neither a nor b is 0 and therefore the denominator is not 0. Therefore the expression must be a rational number.

Problem 10

Assume that m and n are both integers and that $n \neq 0$. Explain why $(5m + 12n)/(4n)$ must be a rational number.

Solution

$(5m + 12n)/(4n)$ must be a rational number as the numerator is a sum of two products of integers and is therefore an integer and the denominator is an integer as it is a product of integers. Furthermore, the denominator is not equal to 0 as $n \neq 0$ and $4 \neq 0$. Therefore the expression must be a rational number.

Problem 11

Prove that every integer is a rational number

Theorem: Every even integer is a rational number.

Proof. Let n be any integer. Then $\frac{n}{1} = n$. Because this is a ratio of integers n is a rational number. □

Problem 12

Fill in the blanks in the following proof that the square of any rational number is rational.

Theorem: Prove that the square of any rational number is rational.

Proof. Suppose that r is a rational number. By definition of rational, $r = \frac{a}{b}$ for some integers a and b with $b \neq 0$. By substitution,

$$r^2 = (\frac{a}{b})^2 = \frac{a^2}{b^2}.$$

Since a and b are both integers, so are the products a^2 and b^2. Also $b^2 \neq 0$ by the zero product property. Hence r^2 is a ratio of two integers with a nonzero denominator, and so r^2 is rational by definition of rational. □

Problem 13

Consider the statement: The negative of any rational number is rational.

(i) Write the statement formally using a quantifier and a variable.

(ii) Determine whether the statement is true or false and justify your answer.

Solution

(i) $\forall r \in Q$, $-r$ is rational.

(ii) The statement is true as the negative of an integer is an integer. Furthermore the denominator is not 0 as the negative of an integer can only be 0 if the integer is 0 by the zero product property.

Problem 14

Consider the statement: The cube of any rational number is a rational number

(i) Write the statement formally using a quantifier and a variable.

(ii) Determine whether the statement is true or false and justify your answer.

Solution

(i) $\forall r \in Q$, r^3 is rational.

(ii) The statement is true as the cube of an integer is an integer. Furthermore the denominator is not 0 as the cube of an integer can only be 0 if the integer is 0 by the zero product property.

Problem 15

Determine if the product of any two rational numbers is a rational number.

Theorem: The product of any two rational numbers is a rational number.

Proof. Let a and b be rational numbers. Then $a = \frac{c}{d}$ and $b = \frac{e}{f}$ for some integers c, d, e, f with $d \neq 0$ and $f \neq 0$.

$$ab = \frac{c}{d} * \frac{e}{f} = \frac{ce}{df}$$

It follows from closure under multiplication that both the numerator and denominator are integers. It follows from the zero product property that the denominator is not 0. It follows from the definition of rational that ab is rational. □

Problem 16

Determine if the quotient of any two rational numbers is a rational number.

Counterexample: Let $a = 0$ and let $b = 1$. Then a and b are both rational numbers but $\frac{b}{a} = \frac{1}{0}$ is not a rational number as $\frac{1}{0}$ is not a number.

Problem 17

Determine if the difference of any two rational numbers is a rational number.

Theorem: The difference of any two rational numbers is rational.

Proof. Let a and b be rational numbers. Then $a = \frac{c}{d}$ and $b = \frac{e}{f}$ for some integers c, d, e, f with $d \neq 0$ and $f \neq 0$.

$$a - b = \frac{c}{d} - \frac{e}{f} = \frac{fc - de}{df}$$

It follows from closure under multiplication and subtraction that the numerator and the denominator are both integers. It follows from the zero product property that the denominator is not 0. Since the numerator and denominator are both integers and the denominator is not 0 the expression is a rational number. □

Problem 18

Determine if $\frac{r+s}{2}$ is rational for all rational numbers r and s.

Theorem: $\frac{r+s}{2}$ is rational for all rational numbers r and s.

Proof. Let r and s be rational numbers. Then $r = \frac{c}{d}$ and $s = \frac{e}{f}$ for some integers c, d, e, f with $d \neq 0$ and $f \neq 0$.

$$r + s = \frac{c}{d} + \frac{e}{f} = \frac{fc + de}{fd}$$

$$\frac{r+s}{2} = \frac{fc + de}{2fd}$$

It follows from closure under addition and multiplication that the numerator and denominator are integers. It follows from the zero product property that the $2fd$ is not 0. It follows from the definition of rational that $\frac{r+s}{2}$ is rational. □

Problem 19

Determine if for all real numbers a and b, if $a < b$ then $a < \frac{a+b}{2} < b$.

Theorem: For all real numbers a and b, if $a < b$ then $a < \frac{a+b}{2} < b$.

Proof. Let a and b be any real numbers such that $a < b$. If $a < b$ then

$$a + a < a + b$$

$$2a < a + b$$

$$a < \frac{a+b}{2}$$

If $a < b$ then

$$a + b < b + b$$
$$a + b < 2b$$
$$\frac{a+b}{2} < b$$

It follows that $a < \frac{a+b}{2} < b$. $\square$

Problem 20

Determine whether given any two rational numbers r and s with $r < s$, there is another rational number between r and s.

Theorem: Given any two rational numbers r and s with $r < s$, there is another rational number between r and s.

Proof. Let r and s be any rational numbers. It follows from problem 18 that $\frac{r+s}{2}$ is rational. If r and s are rational numbers then r and s are real numbers. It follows from problem 19 that $r < \frac{r+s}{2} < s$. If $r < \frac{r+s}{2} < s$ and $\frac{r+s}{2}$ is a rational number then there is another rational number between r and s. $\square$

Problem 21

True or false? If m is any even integer and n is any odd integer, then $m^2 + 3n$ is odd. Explain.

Solution

True. m^2 is even as the product of any two even integers is even. $3n$ is odd as n is odd and 3 is odd and the product of any two odd integers is odd. It follows then from the fact that the sum of any odd integer and any even integer is an odd integer that $m^2 + 3n$ is odd.

Problem 22

True or false? If a is any odd integer, then $a^2 + a$ is even. Explain.

Solution

True. a^2 is odd as the product of any two odd integers is odd. a is odd by definition. It follows then from the fact that the sum of any two odd integers is even that $a^2 + a$ is even.

Problem 23

True or false? If k is any even integer and m is any odd integer, then $(k+2)^2 - (m-1)^2$ is even. Explain.

Solution

$k+2$ is even as the sum of any two even integers is even. $(k+2)^2$ is even as the product of any two even integers is even. $m-1$ is even as the difference of any two odd integers is even. $(m-1)^2$ is even as the product of any two even integers is even. It follows then from the fact that the difference of any two even integers is even that $(k+2)^2-(m-1)^2$ is even.

Problem 24

Prove that for any rational numbers r and s, $2r+3s$ is rational.

Corollary: For any rational numbers r and s, $2r+3s$ is rational.

Proof. Let r and s be any rational numbers. By problem 11 every integer is a rational number. Thus 2 and 3 are rational numbers as 2 and 3 are integers. By problem 15 the product of any two rational numbers is rational. Thus $2r$ and $3s$ are rational as they are the product of two rational numbers. By theorem 4.2.2 the sum of any two rational numbers is rational. Thus $2r+3s$ is a rational number as it is the sum of two rational numbers. □

Problem 25

Prove that if r is any rational number, then $3r^2-2r+4$ is rational.

Corollary: If r is any rational number, then $3r^2-2r+4$ is rational.

Proof. Let r be any rational number. By problem 15 which states the the product of any two rational numbers is rational and by problem 11 which states that every integer is a rational number, $3r^2$ and $2r$ and 4 are rational numbers. By theorem 4.2.2 which states that the sum of any two rational numbers is rational and by problem 17 which states that the difference of any two rational numbers is rational $3r^2-2r+4$ is rational. □

Problem 26

Prove that for any rational number s, $5s^3+8s^2-7$ is rational.

Corollary: For any rational number s, $5s^3+8s^2-7$ is rational.

Proof. Let s be any rational number. By problem 15 and by problem 11, $5s^3$, $8s^2$, and 7 are all rational. By theorem 4.2.2 and by problem 17, $5s^3+8s^2-7$ is rational. □

Problem 27

It is a fact that if n is a nonnegative integer, then

$$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^n} = \frac{1 - \frac{1}{2^{n+1}}}{1 - \frac{1}{2}}$$

Is the right hand side of this equation rational? If so, express it as a ratio of two integers.

Solution

Proof. Let n be a nonnegative integer. Then n is an integer. It follows from closure under addition and multiplication that 2^{n+1} is an integer. It follows from the previous sentence and from the definition of rational number that $\frac{1}{2^{n+1}}$ is a rational number. It follows from problem 17 that $1 - \frac{1}{2^{n+1}}$ is a rational number. It follows from the definition of rational, problem 17, and problem 11 that $1 - \frac{1}{2}$ is rational. It follows from the theorem that the quotient of any two rational numbers is rational provided the the denominator is not 0 that

$$\frac{1 - \frac{1}{2^{n+1}}}{1 - \frac{1}{2}}$$

is rational. $\square$

By simple algebra the right side of the equation can be expressed as

$$\frac{2^{(n+1)} - 1}{2^n}$$

Problem 28

Suppose a, b, c, and d are integers and $a \neq c$. Suppose also that x is a a real number that satisfies the equation

$$\frac{ax + b}{cx + d} = 1.$$

Must x be rational? If so, express x as a ratio of two integers.

Solution

$$\begin{aligned} ax + b &= cx + d \\ ax + b - cx - d &= 0 \\ x(a - c) &= d - b \\ x &= \frac{d - b}{a - c} \end{aligned}$$

It follows from closure under addition that $d - b$ and $a - c$ are integers. Furthermore, $a - c \neq 0$ as $a \neq c$. It follows from the definition of rational that x is rational.

Problem 29

Suppose that a, b, and c are integers and x, y, and z are nonzero real numbers that satisfy the following equations:

$$\frac{xy}{x+y} = a \qquad \frac{xz}{x+z} = b \qquad \frac{yz}{y+z} = c$$

Is x rational? If so, express it as a ratio of two integers.

Solution

First solve equation 1 for y. By basic algebra you get

$$y = \frac{xa}{x-a}$$

Next solve equation 2 for z. By basic algebra you get

$$z = \frac{xb}{x-b}$$

Now replace y and z in equation 3 with their equivalents above

$$\frac{yz}{y+z} = \frac{\frac{xa}{x-a} \cdot \frac{xb}{x-b}}{\frac{xa}{x-a} + \frac{xb}{x-b}} = c$$

Solve the equation above for x. By basic algebra you get

$$x = \frac{2abc}{ac - ab + bc}$$

It follows from closure under multiplication that $2abc$ is an integer. It follows from closure under multiplication, addition, and subtraction that $ac - ab + bc$ is an integer. Thus x is rational as long as $ac - ab + bc \neq 0$.

Problem 30

Prove that if one solution for a quadratic equation of the form $x^2 + bx + c = 0$ is rational(where b and c are rational), then the other solution is also rational.

Theorem: If one solution for a quadratic equation of the form $x^2 + bx + c = 0$ is rational(where b and c are rational), then the other solution is also rational.

Proof. Let r and s be the solutions to the equation $x^2 + bx + c = 0$ and let r be known to be rational as stated in the problem. Then

$$(x - r)(x - s) = x^2 + bx + c$$

$$x^2 - xr - xs + rs = x^2 + x(-r - s) + rs$$

$$x^2 + bx + c = x^2 + x(-r - s) + rs$$

It follows that $(-r - s) = b$. Thus $s = -r - b$. It follows from problem 13 which states that the negative of any rational number is rational and from problem 17 which states that the difference of any two rational numbers is rational that $-r - b$ is rational and thus s must be rational. $\square$

Problem 31

Prove that if a real number c satisfies a polynomial equation of the form

$$r_3x^3 + r_2x^2 + r_1x + r_0 = 0,$$

where r_0, r_1, r_2 and r_3 are rational numbers, then c satisfies an equation of the form

$$n_3x^3 + n_2x^2 + n_1x + n_0 = 0,$$

where $n_0, n_1, n_2,$ and n_3 are integers.

Proof. Let r_0, r_1, r_2 and r_3 be rational numbers. Then $r_0 = \frac{a_0}{b_0}, r_1 = \frac{a_1}{b_1}, r_2 = \frac{a_2}{b_2}$, and $r_3 = \frac{a_3}{b_3}$ for some integers $a_0, a_1, a_2,$ and a_3 and some nonzero integers $b_0, b_1, b_2,$ and b_3. Let c be a real number that satisfies the equation

$$r_3x^3 + r_2x^2 + r_1x + r_0 = 0$$

By substitution

$$\frac{a_3}{b_3} \cdot c^3 + \frac{a_2}{b_2} \cdot c^2 + \frac{a_1}{b_1} \cdot c + \frac{a_0}{b_0} = 0$$

Obtain a common denominator

$$\frac{a_3b_2b_1b_0}{b_3b_2b_1b_0} \cdot c^3 + \frac{a_2b_3b_1b_0}{b_3b_2b_1b_0} \cdot c^2 + \frac{a_1b_3b_2b_0}{b_3b_2b_1b_0} \cdot c + \frac{a_0b_3b_2b_1}{b_3b_2b_1b_0} = 0$$

Multiply both sides by $b_3b_2b_1b_0$

$$a_3b_2b_1b_0 \cdot c^3 + a_2b_3b_1b_0 \cdot c^2 + a_1b_3b_2b_0 \cdot c + a_0b_3b_2b_1 = 0$$

Let $n_3 = a_3b_2b_1b_0$, $n_2 = a_2b_3b_1b_0$, $n_1 = a_1b_3b_2b_0$, and $n_0 = a_0b_3b_2b_1$. It follows from closure under multiplication that $n_3, n_2, n_1,$ and n_0 are integers. Thus c satisfies the equation

$$n_3x^3 + n_2x^2 + n_1x + n_0 = 0,$$

where $n_0, n_1, n_2,$ and n_3 are integers. $\square$

Problem 32

Prove that for all real numbers c if c is a root of a polynomial with rational coefficients, then c is a root of a polynomial with integer coefficients.

Proof. Let c be any real number such that c is a root of a polynomial with rational coefficients. If c is a root of a polynomial with rational coefficients then

$$p(c) = \sum_{i=0}^{n} r_i c^i = \sum_{i=0}^{n} \frac{a_i}{b_i} c^i = 0$$

where for all $i, r_i = \frac{a_i}{b_i}$ for some integers a_i and b_i with $b_i \neq 0$. Let v be the product of all b_i. Then $v = b_n b_{n-1} ... b_2 b_1 b_0$. Multiply both sides of the polynomial by v to get

$$v \cdot p(c) = \sum_{i=0}^{n} v \cdot \frac{a_i}{b_i} c^i = 0$$

Clearly for all i, b_i divides v. It follows from the previous sentence and closure under multiplication that $v \cdot \frac{a_i}{b_i}$ is an integer for all i. Let that integer be m_i. Then

$$v \cdot p(c) = \sum_{i=0}^{n} m_i c^i = 0$$

Thus c is a root of a polynomial with integer coefficients. □

Problem 33

When expressions of the form $(x - r)(x - s)$ are multiplied out, a quadratic polynomial is obtained. For instance, $(x - 2)(x - (-7)) = (x - 2)(x + 7) = x^2 + 5x - 14$.

(i) What can be said about the coefficients of the polynomial obtained by multiplying out $(x-r)(x-s)$ when both r and s are odd integers? When both r and s are even integers? When one of r and s is even and the other is odd?

(ii) It follows from part (i) that $x^2 - 1253x + 255$ cannot be written as a product of two polynomials with integer coefficients. Explain why this is so.

Solution

(i) First note that $(x - r)(x - s) = x^2 - x(r + s) + rs$. Three cases arise:

  (i) *Both r and s odd*: It follows from the fact that the sum of any two odd integers is even and the product of any two odd integers is odd that $x^2 - x(r + s) + rs = x^2 - even \cdot x + odd$.

(ii) *Both r and s even*: If follows from the fact that the sum of any two even integers is even and the product of any two even integers is even that $x^2 - x(r+s) + rs = x^2 - even \cdot x + even$.

(iii) *r even and s odd or r odd and s even*: If follows from the fact that the sum of any odd integer and any even integer is odd and the product of any odd integer and any even integer is even that $x^2 - x(r+s) + rs = x^2 - odd \cdot x + even$.

(ii) $x^2 - 1253x + 255$ cannot be written as a product of two polynomials with integer coefficients as it takes the form $x^2 - odd \cdot x + odd$. This is not among the possible forms that a polynomial that can be written as a product of two polynomials with integer coefficients can take as demonstrated above.

Problem 34

Observe that $(x-r)(x-s)(x-t) = x^3 - (r+s+t)x^2 + (rs+rt+st)x - rst$.

(i) Derive a result for cubic polynomials similar to the result in part (i) of exercise 33 for quadratic polynomials.

(ii) Can $x^3 + 7x^2 - 8x - 27$ be written as a product of three polynomials with integer coefficients? Explain.

Solution

(i) 8 cases arise:

(1) r even, s even, t even: $x^3 - even \cdot x^2 + even \cdot x - even$.

(2) r even, s even, t odd: $x^3 - odd \cdot x^2 + even \cdot x - even$.

(3) r even, s odd, t even: $x^3 - odd \cdot x^2 + even \cdot x - even$.

(4) r even, s odd, t odd: $x^3 - even \cdot x^2 + odd \cdot x - even$.

(5) r odd, s even, t even: $x^3 - odd \cdot x^2 + even \cdot x - even$.

(6) r odd, s even, t odd: $x^3 - even \cdot x^2 + odd \cdot x - even$.

(7) r odd, s odd, t even: $x^3 - even \cdot x^2 + odd \cdot x - even$.

(8) r odd, s odd, t odd: $x^3 - odd \cdot x^2 + odd \cdot x - odd$.

(ii) It follows that $x^3 + 7x^2 - 8x - 27$ cannot be written as a product of three polynomials with integer coefficients as it takes the form $x^3 + odd \cdot x^2 - even \cdot x - odd$ and this form is not a possibility for a polynomial that can be written as a product of three polynomials with integer coefficients as demonstrated above.

Problem 35

Find the mistake in the "proof" that the sum of any two rational numbers is rational.

"proof": Any two rational numbers produce a rational number when added together. So if r and s are particular but arbitrarily chosen rational numbers, then $r + s$ is rational.

Solution

This proof assumes what is to be proved but gives no justification that it is true.

Problem 36

Find the mistake in the "proof" that the sum of any two rational numbers is rational.

"proof": Let rational numbers $r = \frac{1}{4}$ and $s = \frac{1}{2}$ be given. Then $r+s = \frac{1}{4}+\frac{1}{2} = \frac{3}{4}$, which is a rational number. This is what was to be shown.

Solution

This proof merely demonstrates the existence of one case in which the sum of two rational numbers is rational. It does not shows that the sum of any two rational numbers is rational.

Problem 37

Find the mistake in the "proof" that the sum of any two rational numbers is rational.

"proof": Suppose r and s are rational numbers. By definition of rational, $r = \frac{a}{b}$ for some integers a and b with $b \neq 0$. Then

$$r + s = \frac{a}{b} + \frac{a}{b} = \frac{2a}{b}.$$

Let $p = 2a$. Then p is an integer since it is a product of integers. Hence $r+s = \frac{p}{b}$, where p and b are integers and $b \neq 0$. Thus $r + s$ is a rational number number by definition of rational. This is what was to be shown.

Solution

This proof only shows that the sum of two rational numbers which are identical is a rational number. This proof does not prove that the sum of any two rational numbers including rational numbers that are different is rational.

Problem 38

Find the mistake in the "proof" that the sum of any two rational numbers is rational.

"proof": Suppose that r and s are rational numbers. Then $r = \frac{a}{b}$ and $c = \frac{b}{c}$ for some integers a, b, c, and d with $b \neq 0$ and $d \neq 0$. Then

$$r + s = \frac{a}{b} + \frac{b}{c}$$

But this is a sum of two fractions, which is a fraction. So $r + s$ is a rational number since a rational number is a fraction.

Solution

This proof makes a converse error. A statement that is true is rational $\implies$ fraction. The proof states fraction $\implies$ rational. This is a false statement and is an example of the converse error. The reason that this is a false statement is that rational numbers are fractions but more specifically they are fractions of integers. The proof does not demonstrate that both the numerator and denominator of the fraction are integers and that the denominator is not 0.

Problem 39

Find the mistake in the "proof" that the sum of any two rational numbers is rational.

"proof": Suppose that r and s are rational numbers. If $r + s$ is a rational number then by definition of rational $r + s = \frac{a}{b}$ for some integers a and b with $b \neq 0$. Also since r and s are rational, $r = \frac{i}{j}$ and $s = \frac{m}{n}$ for some integers, i, j, m, and n with $j \neq 0$ and $n \neq 0$. It follows that

$$r + s = \frac{i}{j} + \frac{m}{n} = \frac{a}{b}$$

which is a quotient of integers with a nonzero denominator. Hence it is a rational number. This is what was to be shown.

Solution

This proof assumed what was to be proven that $r + s$ is rational is true. The proof then set $r + s$ to be equal to some rational number and then set $r + s$ equal to the result that they already proclaimed true without evidence. This is an example of circular reasoning. In other words you cant use the supposition that $r + s$ is rational as evidence that $r + s$ is rational.