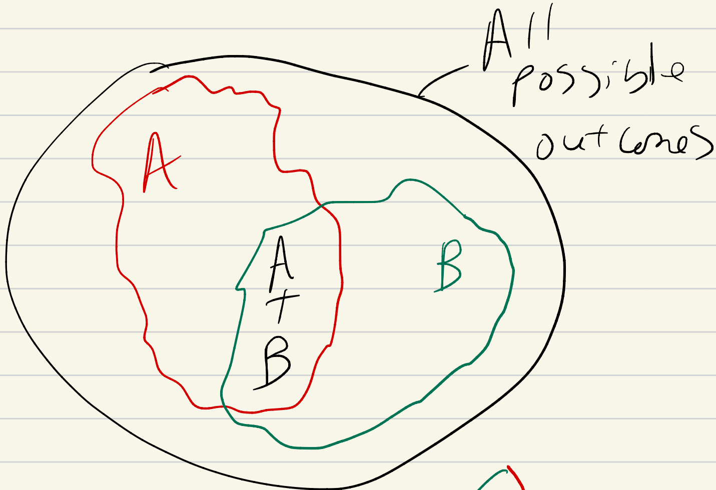
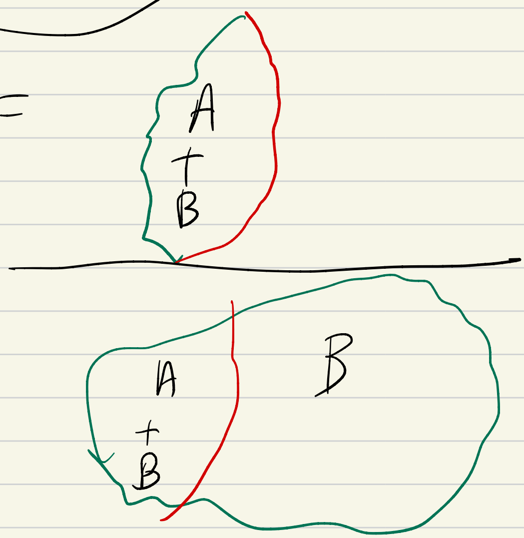
Conditional Probability

We call Pr(A|B) the probability of A conditional on B.



$Pr(A|B) =$

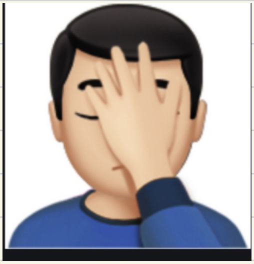


Baye's Rule

Formal Definition $$Pr(A|B) = \frac{Pr(B|A) \cdot Pr(A)}{Pr(B)}$$

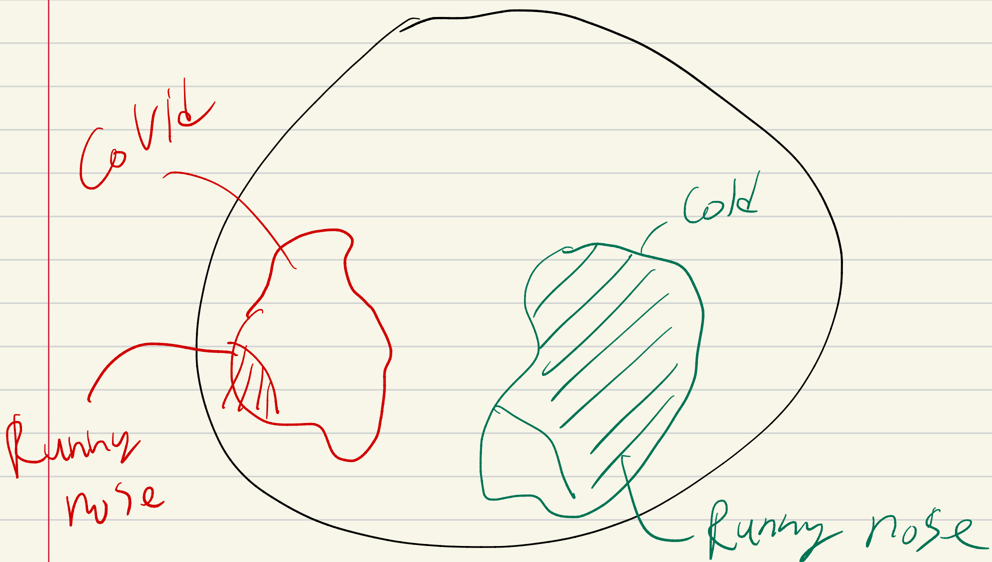
so...

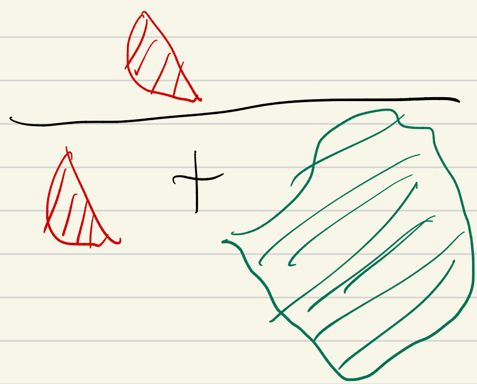
If you know the things on the right you can figure out the things on the left.

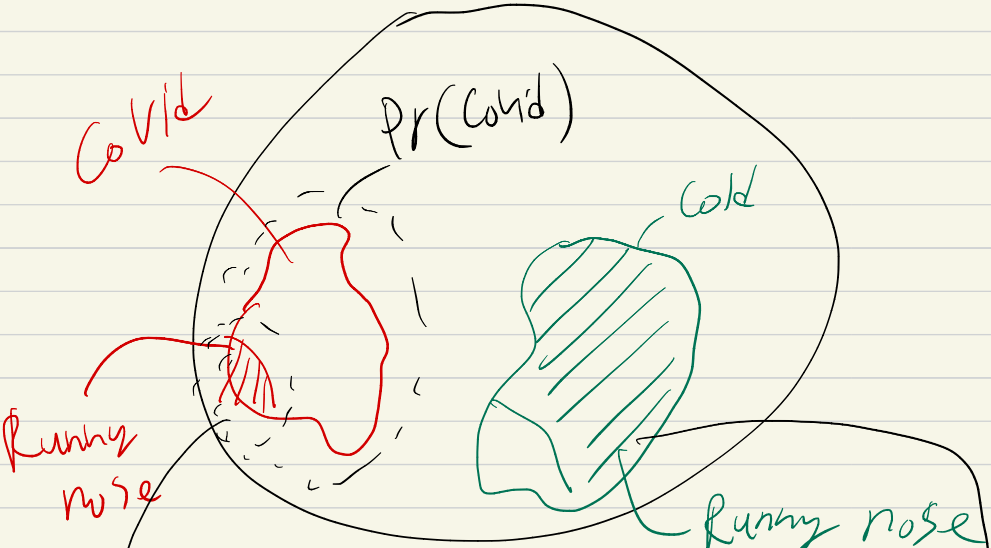


Baye's Rule can help us out when we have counts of observations.

Example: You know how frequently some types of symptoms occur if you have a cold virus and if you have coronavirus. Given that you have a runny nose, what is the probability you have COVID?



$\Pr(\text{covid} \mid \text{Runny nose}) =$ 



$$\# \text{ Covid w/RN} = Pr(RN \mid Covid) \cdot Pr(Covid)$$

$$\# \text{ Cold w/RN} = Pr(RN \mid Cold) \cdot Pr(Cold)$$

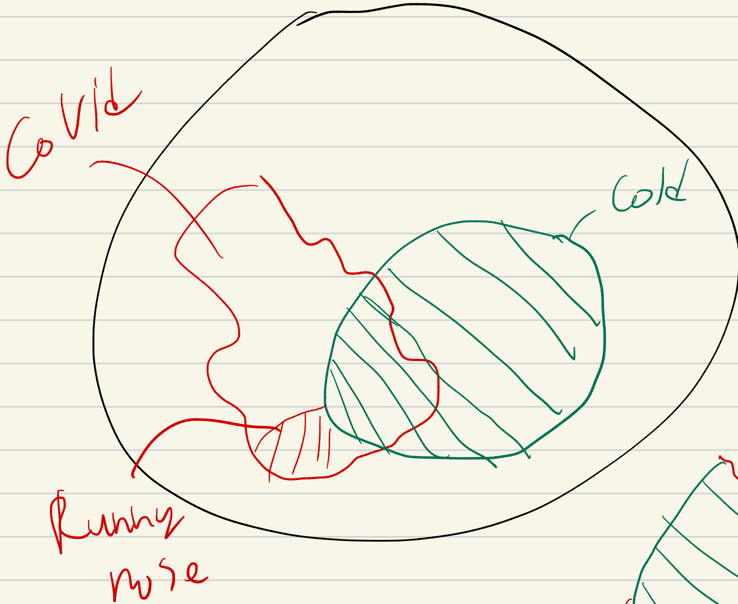
$$Pr\left(\text{Covid} \mid \begin{matrix}\text{Runny}\\ \text{Nose}\end{matrix}\right) = \frac{Pr(RN \mid \text{Covid})\,Pr(\text{Covid})}{Pr(RN \mid \text{Covid}) \cdot Pr(\text{Covid}) + Pr(RN \mid \text{Cold})\,Pr(\text{Cold})}$$

e.g. $$\frac{X}{X+Y}$$

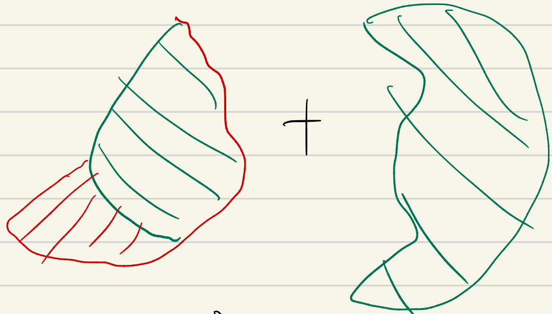
So if $Y \uparrow$ the value goes down.

To the Break out Rooms!!

Redo with possibility that you have both COVID and a cold, but always have symptoms with a cold. Assume chance of getting each is independent.



$$Pr\left(\text{Covid} \,\middle|\, \text{Runny nose}\right) = \frac{\text{[image 2]}}{\text{[image 3]}}$$



$$= \frac{Pr(\text{Covid}) \cdot \left(Pr(\text{Cold}) \cdot 1 + (1 - Pr(\text{Cold})) \cdot Pr(RN \mid \text{Cold})\right)}{Pr(\text{Covid}) \cdot \left(Pr(\text{Cold}) \cdot 1 + (1 - Pr(\text{Cold})) \cdot Pr(RN \mid \text{Cold})\right) + Pr(\text{Cold})\left(1 - Pr(\text{Covid})\right)}$$

How does this relate to Bayesian data analysis?.

$d \equiv$ data

$p \equiv$ parameters

$$Pr(p \mid d) = \frac{Pr(d \mid p) \cdot Pr(p)}{Pr(d)}$$

What is Bayesian data analysis ?

1. Use Bayesian updating to create posterior probability for all hypotheses

2. A set of tools to approximate complicated likelihood functions

3. A set of tools to compare posterior distributions.