DIAGNOSTICS & SUBSET SELECTION

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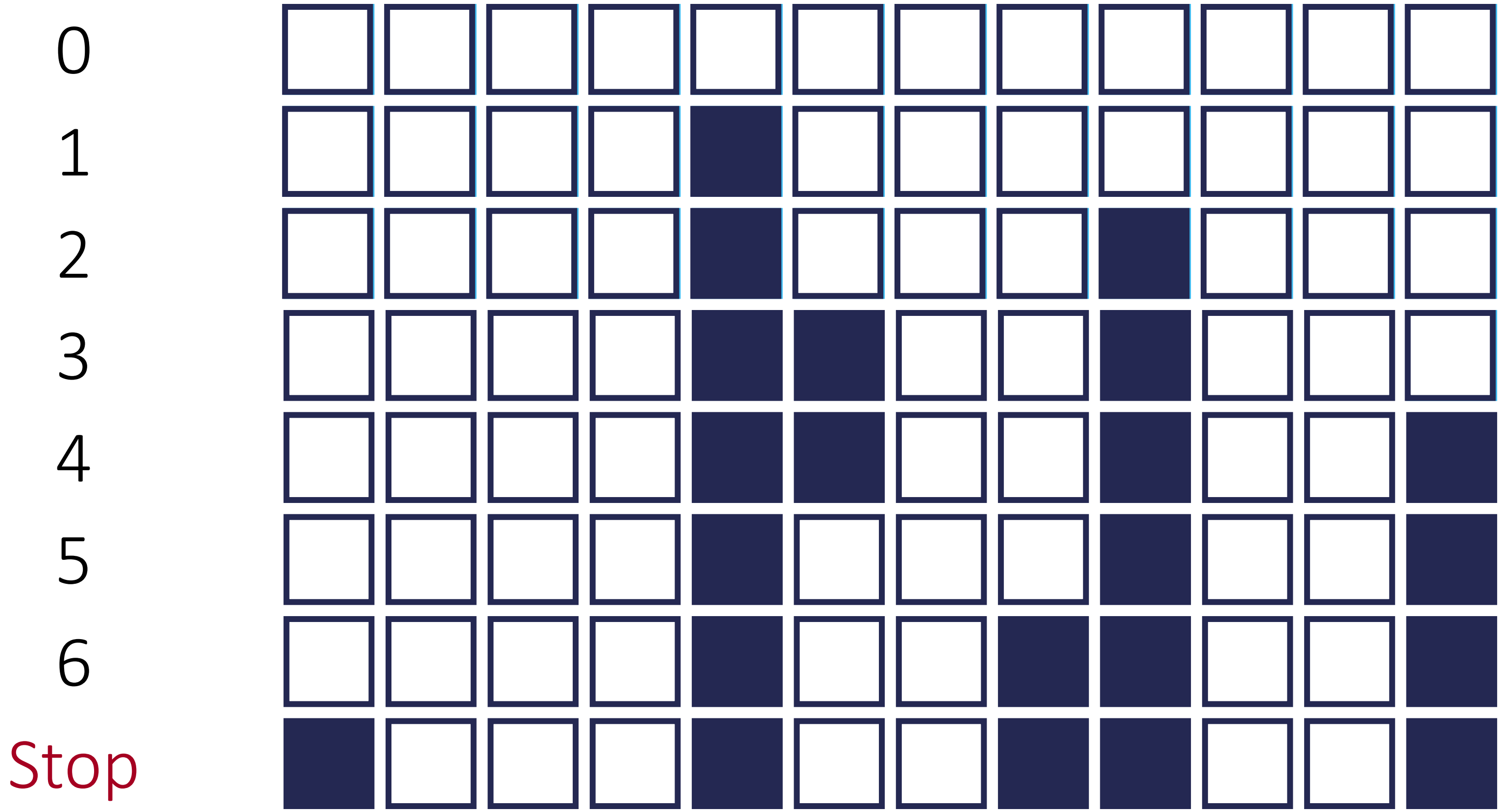
SUBSET SELECTION METHODS

Ames Real Estate Data

- 2930 homes in Ames, Iowa in the early 2000's.
- Physical attributes of homes along with sales price of home.

Stepwise Selection



Stepwise Selection

```
full.model <- glm(Bonus ~ Gr_Liv_Area + factor(House_Style) + Garage_Area +
                          Fireplaces + factor(Full_Bath) + factor(Half_Bath) +
                          Lot_Area + factor(Central_Air) + Second_Flr_SF +
                          TotRms_AbvGrd + First_Flr_SF,
                  data = train, family = binomial(link = "logit"))

empty.model <- glm(Bonus ~ 1,
                   data = train, family = binomial(link = "logit"))

step.model <- step(empty.model,
                   scope = list(lower=formula(empty.model),
                                upper=formula(full.model)),
                   direction = "both")
```

Stepwise Selection

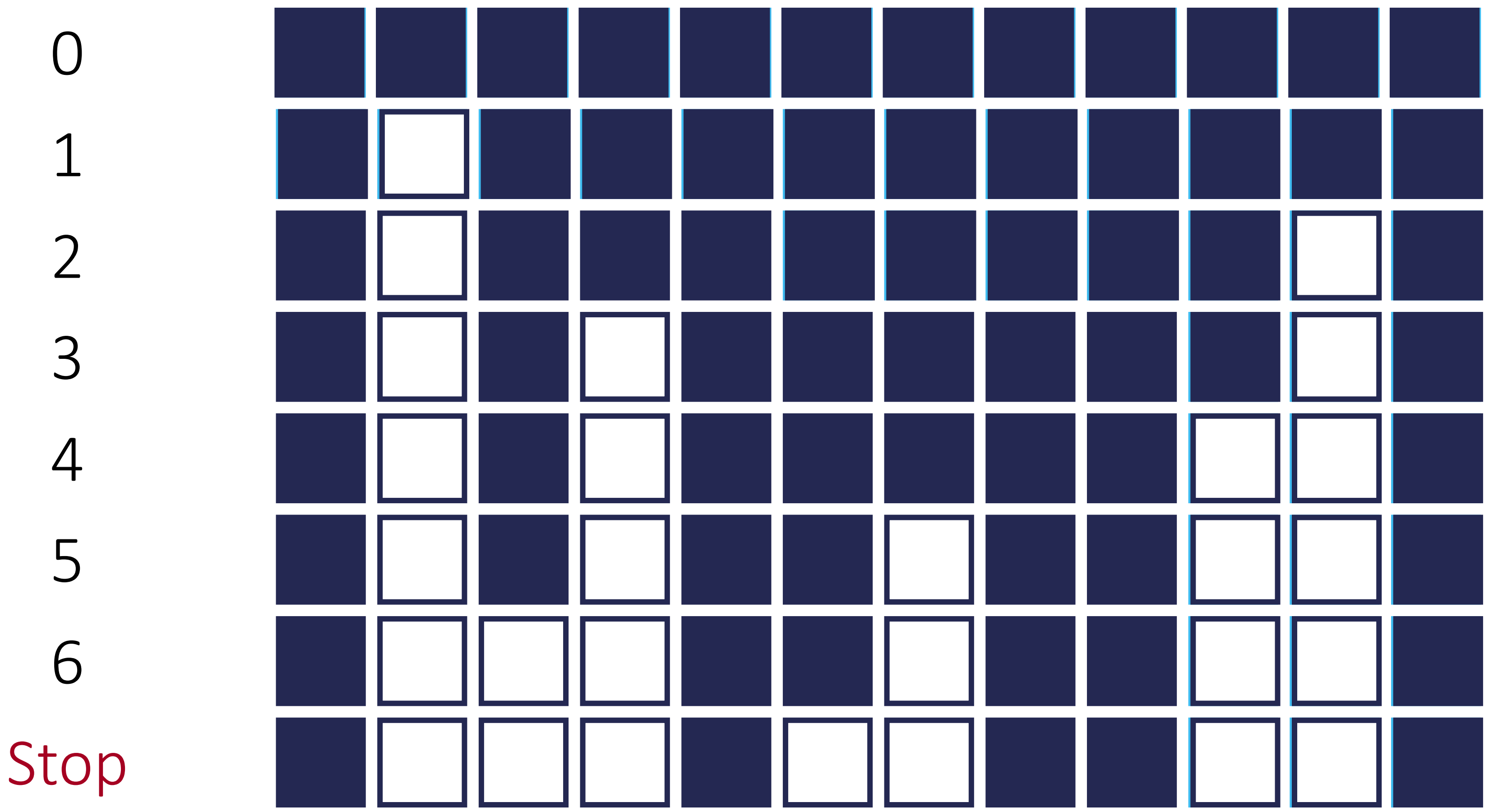
```
Start:  AIC=2777.81
Bonus ~ 1

                        Df Deviance    AIC
+ factor(Full_Bath)      4   1911.5 1921.5
+ Gr_Liv_Area            1   1926.4 1930.4
+ Garage_Area            1   2135.4 2139.4
+ First_Flr_SF           1   2294.1 2298.1
+ Fireplaces             1   2423.7 2427.7
+ TotRms_AbvGrd          1   2449.7 2453.7
+ factor(House_Style)    7   2542.1 2558.1
+ factor(Half_Bath)      2   2608.1 2614.1
+ Lot_Area               1   2621.9 2625.9
+ Second_Flr_SF          1   2631.8 2635.8
+ factor(Central_Air)    1   2654.3 2658.3
<none>                       2775.8 2777.8
```

⋮

Stepwise Selection

```
Coefficients:
                                  Estimate Std. Error z value Pr(>|z|)    
(Intercept)                     -1.041e+01  1.537e+00  -6.775 1.24e-11 ***
factor(Full_Bath)1              -6.860e-01  1.325e+00  -0.518  0.60469    
factor(Full_Bath)2               1.894e+00  1.341e+00   1.412  0.15785    
factor(Full_Bath)3               4.152e+00  1.610e+00   2.579  0.00991 ** 
factor(Full_Bath)4              -1.261e+00  2.493e+00  -0.506  0.61305    
Garage_Area                      3.583e-03  5.187e-04   6.907 4.96e-12 ***
Fireplaces                       9.142e-01  1.272e-01   7.186 6.67e-13 ***
Gr_Liv_Area                      3.827e-03  4.033e-04   9.488  < 2e-16 ***
factor(House_Style)One_and_Half_Unf -8.941e+00  3.682e+02  -0.024  0.98063    
factor(House_Style)One_Story     2.396e+00  3.285e-01   7.295 2.99e-13 ***
factor(House_Style)SFoyer        1.760e+00  6.382e-01   2.757  0.00583 ** 
factor(House_Style)SLvl          1.105e+00  4.530e-01   2.438  0.01476 *  
factor(House_Style)Two_and_Half_Fin -4.855e-01  6.945e+00  -0.070  0.94427    
factor(House_Style)Two_and_Half_Unf  8.329e-01  8.891e-01   0.937  0.34890    
factor(House_Style)Two_Story     9.801e-01  3.380e-01   2.900  0.00373 ** 
factor(Half_Bath)1               1.195e+00  2.153e-01   5.553 2.81e-08 ***
factor(Half_Bath)2              -1.301e-01  8.053e-01  -0.162  0.87163    
TotRms_AbvGrd                   -4.322e-01  8.128e-02  -5.317 1.05e-07 ***
factor(Central_Air)Y             1.620e+00  5.866e-01   2.762  0.00575 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Backward Elimination



Backward Selection

```
full.model <- glm(Bonus ~ Gr_Liv_Area + factor(House_Style) + Garage_Area +
                          Fireplaces + factor(Full_Bath) + factor(Half_Bath) +
                          Lot_Area + factor(Central_Air) + Second_Flr_SF +
                          TotRms_AbvGrd + First_Flr_SF,
                  data = train, family = binomial(link = "logit"))

back.model <- step(full.model, direction = "backward")
```

Backward Selection

```
Coefficients:
                                   Estimate Std. Error z value Pr(>|z|)    
(Intercept)                      -1.028e+01  1.541e+00  -6.673 2.51e-11 ***
factor(House_Style)One_and_Half_Unf -9.208e+00  3.686e+02  -0.025  0.98007    
factor(House_Style)One_Story      2.062e+00  4.945e-01   4.171 3.04e-05 ***
factor(House_Style)SFoyer         1.464e+00  7.213e-01   2.030  0.04234 *  
factor(House_Style)SLvl           9.390e-01  4.891e-01   1.920  0.05489 .  
factor(House_Style)Two_and_Half_Fin  1.085e+00  6.908e+00   0.157  0.87524    
factor(House_Style)Two_and_Half_Unf  8.376e-01  8.904e-01   0.941  0.34687    
factor(House_Style)Two_Story      1.010e+00  3.498e-01   2.886  0.00390 ** 
Garage_Area                       3.499e-03  5.210e-04   6.716 1.87e-11 ***
Fireplaces                        8.965e-01  1.279e-01   7.010 2.39e-12 ***
factor(Full_Bath)1               -6.540e-01  1.330e+00  -0.492  0.62302    
factor(Full_Bath)2                1.930e+00  1.347e+00   1.433  0.15196    
factor(Full_Bath)3                4.355e+00  1.618e+00   2.691  0.00712 ** 
factor(Full_Bath)4               -1.073e+00  2.436e+00  -0.440  0.65971    
factor(Half_Bath)1                1.228e+00  2.215e-01   5.545 2.94e-08 ***
factor(Half_Bath)2               -6.069e-02  8.103e-01  -0.075  0.94030    
factor(Central_Air)Y              1.590e+00  5.909e-01   2.690  0.00715 ** 
Second_Flr_SF                     3.466e-03  6.632e-04   5.226 1.73e-07 ***
TotRms_AbvGrd                    -4.339e-01  8.142e-02  -5.329 9.86e-08 ***
First_Flr_SF                      4.011e-03  4.351e-04   9.220  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Comparison of Backward with Stepwise

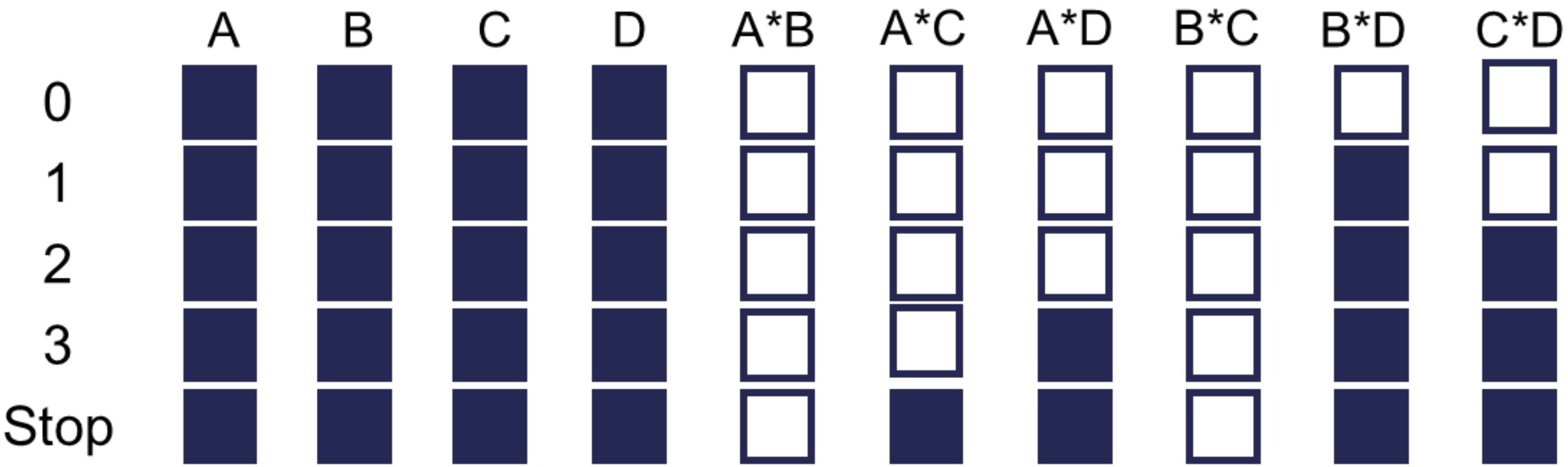
Stepwise Selection Variables

- Full Bath
- Garage Area
- Fireplaces
- Greater Living Area
- House Style
- Half Bath
- Total Rooms (Above Ground)
- Central Air

Backward Selection Variables

- Full Bath
- Garage Area
- Fireplaces
- House Style
- Half Bath
- Total Rooms (Above Ground)
- Central Air
- 1st Floor Sqft
- 2nd Floor Sqft

Interactions with Forward Selection



P-VALUE VS. AIC/BIC METRICS

P-value vs. AIC Selection

- Lot of attention being given to p-values and how other selection techniques are better.
- Some of these are **the same as** p-values…

$$AIC = -2\log(L) + 2p$$

P-value vs. AIC Selection

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Model better with lower AIC…

$$AIC_{p+1} < AIC_p$$

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Model better with lower AIC…

$$AIC_{p+1} < AIC_p$$

$$-2\log\left(L_{p+1}\right) + 2(p+1) < -2\log\left(L_p\right) + 2(p)$$

$$2 < 2\left(\log\left(L_{p+1}\right) - \log\left(L_p\right)\right)$$

P-value vs. AIC Selection

- Lot of attention being given to p-values and how other selection techniques are better.
- Some of these are **the same as** p-values…

$$AIC = -2\log(L) + 2p$$

Model better with lower AIC…

$$AIC_{p+1} < AIC_p$$

$$-2\log(L_{p+1}) + 2(p+1) < -2\log(L_p) + 2(p)$$

$$2 < \boxed{2(\log(L_{p+1}) - \log(L_p))}$$

LRT!

P-value vs. AIC Selection

- Lot of attention being given to p-values and how other selection techniques are better.
- Some of these are **the same as** p-values…

$$AIC = -2\log(L) + 2p$$

Model better with lower AIC…

$$AIC_{p+1} < AIC_p$$

$$-2\log\left(L_{p+1}\right) + 2(p+1) < -2\log\left(L_p\right) + 2(p)$$

$$2 < \chi_1^2$$

P-value vs. AIC Selection

- Lot of attention being given to p-values and how other selection techniques are better.
- Some of these are **the same as** p-values…

$$AIC = -2\log(L) + 2p$$

Model better with lower AIC…

$$AIC_{p+1} < AIC_p$$

$$-2\log\left(L_{p+1}\right) + 2(p+1) < -2\log\left(L_p\right) + 2(p)$$

$$2 < \chi_1^2$$

Model better with variable p-value below sig. level…

$$1 - P(\chi_1^2 > 2) = 0.1573$$

P-value vs. BIC Selection

- Lot of attention being given to p-values and how other selection techniques are better.
- Some of these are **the same as** p-values…

$$BIC = -2\log(L) + p\times\log(n)$$

Model better with lower BIC…

$$BIC_{p+1} < BIC_p$$

$$-2\log\left(L_{p+1}\right) + \log(n)\,(p+1) < -2\log\left(L_p\right) + \log(n)\,(p)$$

$$\log(n) < \chi_1^2$$

Model better with variable p-value below sig. level…

$$1 - P(\chi_1^2 > \log(n)) = \cdots$$

P-value vs. BIC Selection

- For our Ames housing data set, BIC selection is the same as the p-value selection with the following alpha:

$$1 - P(\chi_1^2 > \log(n)) = 1 - P(\chi_1^2 > \log(2051)) = 0.0057$$

- Lot of attention being given to p-values and how other selection techniques are better.
- Attention **should** be on significance level (α), **not** on p-value.
- **DON'T ALWAYS USE 0.05!**

?

GOODNESS-OF-FIT

OPTIONAL: SELF-PACED STUDY

Calibration

- **Calibration** measures how well predicted probabilities agree with actual frequency counts of outcomes.
- Helps detect bias!
  - Are predictions systematically too low or too high?

Calibration Curve

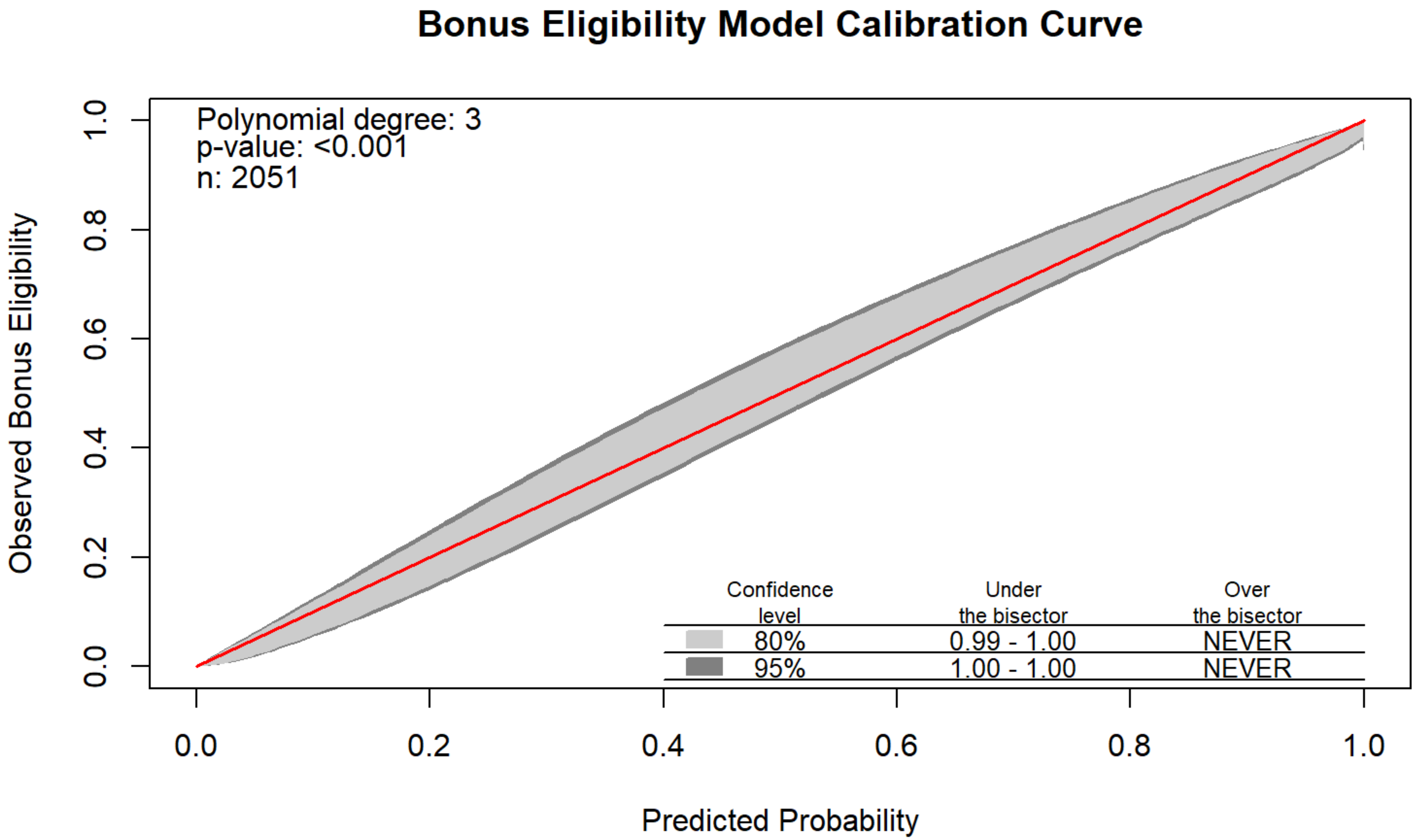
- Curve **above** 45° line indicates the model is predicting **lower** probabilities than actually observed.
- Curve **below** 45° line indicates the model is predicting **higher** probabilities than actually observed.

- Caveat:
  - Calibration depends on the observed proportion of events in the data, so models will likely have poor calibration on out-of-sample data.
  - Best used for goodness-of-fit in training, not on validation.

Calibration Curve

```
library(givitiR)

cali.curve <- givitiCalibrationBelt(o = train$Bonus,
                                    e = predict(logit.model, type = "response"),
                                    devel = "internal", maxDeg = 5)

plot(cali.curve, main = "Bonus Eligibility Model Calibration Curve",
                 xlab = "Predicted Probability",
                 ylab = "Observed Bonus Eligibility")
```

Calibration Curve



DIAGNOSTICS

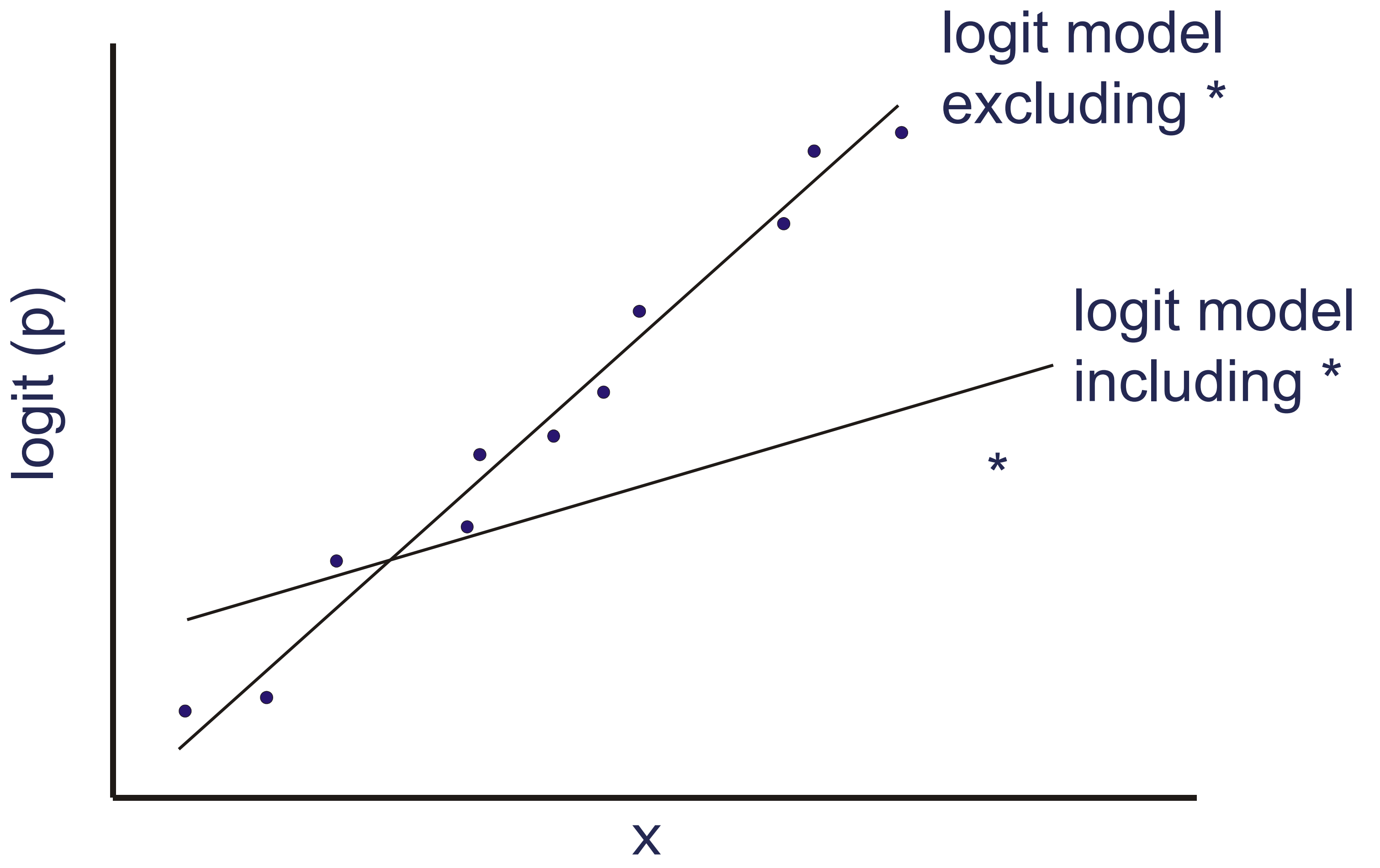
Residuals?

- Linear regression residuals have properties useful for model diagnostics.
- What is a residual in a binary response model?
- Many types of “residuals” in binary response model setting, just not as intuitive.
  - Deviance residuals
  - Partial residuals
  - Pearson residuals
  - Etc.

Deviance

- Model is a summary of a data set.
- The **saturated** model fits the data perfectly, but isn't really a useful summary.
- **Deviance** is a measure of how far our fitted model is from the saturated model – essentially our "error."
- Logistic regression minimizes the sum of squared **deviances**!
- Deviance residuals tell us how much each observation reduces the deviance.

Influence Statistics



Influence Statistics

- DIFDEV
  - Measures change in deviance with deletion of the observation.
- DFBETAS
  - Measure standardized change in each parameter estimate with deletion of observation.
- Cook's D
  - Measures the overall impact to the coefficients in the model.

Diagnostics

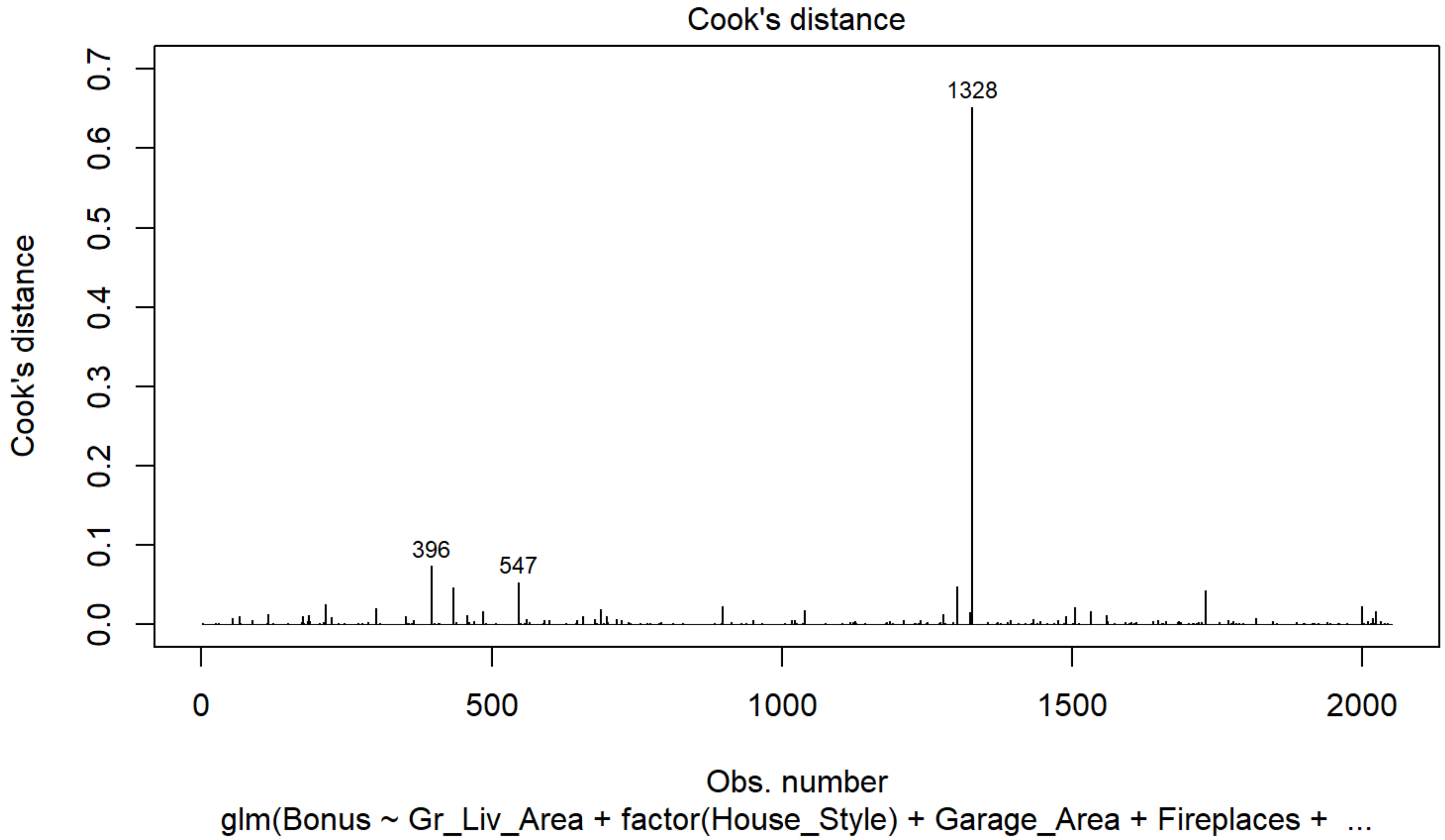
```
library(car)

logit.model <- glm(Bonus ~ Gr_Liv_Area + factor(House_Style) + Garage_Area +
                           Fireplaces + factor(Full_Bath) + Lot_Area +
                           factor(Central_Air) + TotRms_AbvGrd +
                           Gr_Liv_Area:Fireplaces,
                   data = train, family = binomial(link = "logit"))
```

```
influence.measures(logit.model)$infmat
```

Prints out metrics previously listed for every observation!
Output not shown here.

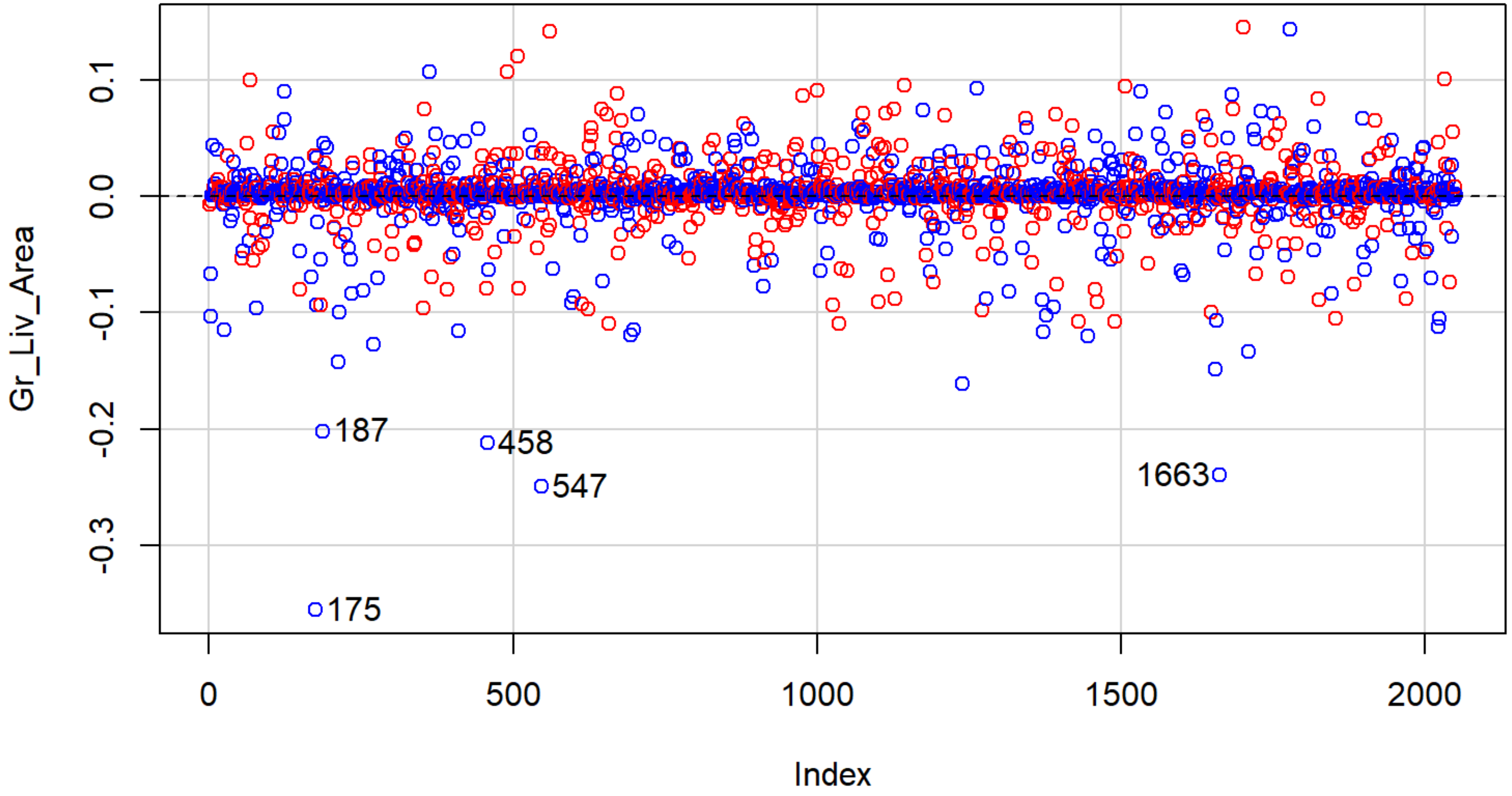
Diagnostics

```
plot(logit.model, 4)
```



Diagnostics

```
dfbetasPlots(logit.model, terms = "Gr_Liv_Area", id.n = 5,
             col = ifelse(logit.model$y == 1, "red", "blue"))
```



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