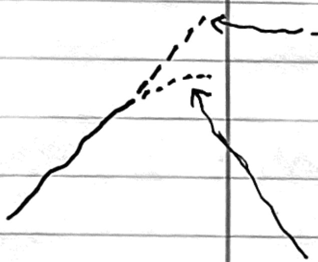
Time Series Review

- What is it ment by trend & Season?
  - Trend can be thought of as a ~~[illegible]~~ line that tries to fit the data. Long term direction ~~of~~ of the data.
  - Seasonality is the repeating patterns in the data. Seasonality occurs at regular intervals.
- Decomposition: trend + seasonality + Remainder/error
  - STL v.s. Classical decomposition: STL allows the seasonal componet to change over time (Non-fixed). STL is Robust to outliers, Allows Changing effects for trend too.
  - additive decomposition: $Y_t = T_t + S_t + R_t$
  - multiplicitive decomposition: $Y_t = T_t \cdot S_t \cdot R_t \equiv \log S_t + \log T_t + \log R_t$
  - Seasonaly adjusted decomposition:
    - additive $\quad Y_t - S_t = T_t + R_t$
    - Multiplicitive $\quad Y_t / S_t = T_t + R_t$
- The different types of Exponential Smoothing Models (ESM):
  - Simple Exponential Smoothing

    $\hat{Y}_{t+1} = L_t$

    1 param $\quad L_t = \alpha Y_t + (1-\alpha) L_{t-1}$



  - Holt/Linear

    $\hat{Y}_{t+h} = L_t + hT_t$

    2 params

    $$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$
    $$T_t = \beta(L_t - L_{t-1}) + (1-\beta) T_{t-1}$$

  - Damped Trend

    $\hat{Y}_{t+h} = L_t + \sum_{i=1} \phi^i T_t$

    3 params

    $$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + \phi T_{t-1})$$
    $$T_t = \beta(L_t - L_{t-1}) + (1-\beta)\phi T_{t-1}$$

  - Holt-Winters
    - Additive:

      $\hat{Y}_{t+h} = L_t + hT_t + S_{t-p+h}$
    - Multiplicitive:

      $\hat{Y}_{t+h} = (L_t + hT_t) S_{t-p+h}$

- Accuracy Statistics for time series models:
  - (MAPE) Mean Absolute Percent Error
    - Cons: overweights of over-predictions
    - Actual of Ø ($Y_t \neq \emptyset$)
  - (MAE) Mean Absolute Error
    - Cons: · not Scale invariant
  - (RMSE) Square Root of Square Error
    - Cons: · Overweight of larger errors
    - · ~~Divide by Ø, Still asymmetric~~ Not Scale invariant
  - AIC : balanced, overfits on small samples
  - AICc : Prefered for small sample situations
  - BIC : biggest penalty, prefers simpler models.

- What is the difference between Accuracy & G.O.F. Statistics?
  - G.O.F. Statistics are calculated on the training data
  - Accuracy Statistics are calculated on data the model has not seen before.

- Random Walk:

$$Y_t = Y_{t-1} + e_t$$

RW with Drift

$$Y_t = c + Y_{t-1} + e_t$$

  - To fix this we can: Difference.

- How to Identify a Stationary time Series:
  - ~~[illegible]~~
  - Constant Mean & Variance & correlation structure
  - We take differences to make a ts stationary. or transform the data
  - We can use the Unit Root Test to identify Stationarity. or KPSS

- White Noise: $Y_t = \varepsilon_t$
  - White noise is ~~[illegible]~~ what we want after modeling.
  - White noise has a normal distributed error terms.
  - To tell if we have white noise:
    - ACF & PACF has no significant "spikes"
    - Ljung-Box test
      - H_0: No significant Autocorrelation ← What we want.
      - H_a: Significant Autocorrelation

White noise is tested on the residuals

$e \equiv \varepsilon$

- (AR) AutoRegressive Model: $Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t$
  - Forecast based soley on past values of Y_t
  - PACF plot gives us our p for our AR(p)
- (MA) Moving Average Model:
  - Forecast based soley on past error values
  - ACF plot gives us or q for our MA(q)

~~(PACF) Partial Auto~~ $Y_t = c + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_q \varepsilon_{t-q}$

- ~~ACF can help us determine~~ ~~time~~
- (ACF) AutoCorrelation Function: $\rho_k = Corr(Y_t, Y_{t-k})$
  - Autocorrelation is the correlation between two sets of observations that are seperated by K points in time.
  - ACF(1) Implies two consecutive time points are related. Note: the relationship can be either positive or negative, we don't know from ACF.
- (PACF) Partial Autocorrelation Function:
  - Partial Autocorrelation is the correlation between two sets of obs that are seperated by K points in time, after adjusting for all previous autocorrelations.
- ARMA & ARIMA Models:

(p,q) (p,d,q) P=AR terms q=MA terms d=differcing

  - ESM focused on trend & seasonality, ARIMA/ARMA on Autocorrelations.
  - To use ARMA/ARIMA we must have the following:
    - Stationary data

$$ARMA(2,3) \equiv Y_t = w + \underbrace{\phi_1 Y_{t-1} + \phi_2 Y_{t-2}}_{AR(2)} + \underbrace{\theta \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3}}_{MA(3)} + \varepsilon_t$$

ARIMA(2,1,3) is an ARMA(2,3) done on the 1st differences.

- How to make trending data Stationary:
  - differencing
  - ARIMAX: fits a linear model with time and then fitting an ARMA to the Residuals.