Markov Chain Monte Carlo Diagnostics

For ignoring the MCMC diagnostics, he was sentenced to the Markov chain gang.

Additional Topics

Topics

BSTS

- Introduction
- State space models
- Fundamentals of BSTS
- Example (Level and Trend)
- Seasonality
- Example (Seasonality)

Change-point

- Introduction
- Techniques
- Examples

BTST

BSTS

Uses the Bayesian setting (distributions on parameters) and state space (think Exponential Smoothing models) together.

Was developed by Google around 2013

Easy to implement and run in R

R documentation: https://cran.r-project.org/web/packages/bsts/bsts.pdf

State Space models: Recall Holt-Winters Additive ESM

$$\hat{Y}_{t+h} = L_t + hT_t + S_{t-p+h}$$

$$L_t = \theta\left(Y_t - S_{t-p}\right) + (1-\theta)(L_{t-1} + T_{t-1})$$

$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1}$$

$$S_t = \delta(Y_t - L_{t-1} - T_{t-1}) + (1-\delta)S_{t-p}$$

Recall Holt-Winters Additive ESM

$$\hat{Y}_{t+h} = L_t + hT_t + S_{t-p+h}$$

$$L_t = \theta\left(Y_t - S_{t-p}\right) + (1-\theta)(L_{t-1} + T_{t-1})$$
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$$S_t = \delta(Y_t - L_{t-1} - T_{t-1}) + (1-\delta)S_{t-p}$$

Each of these components evolve over time….

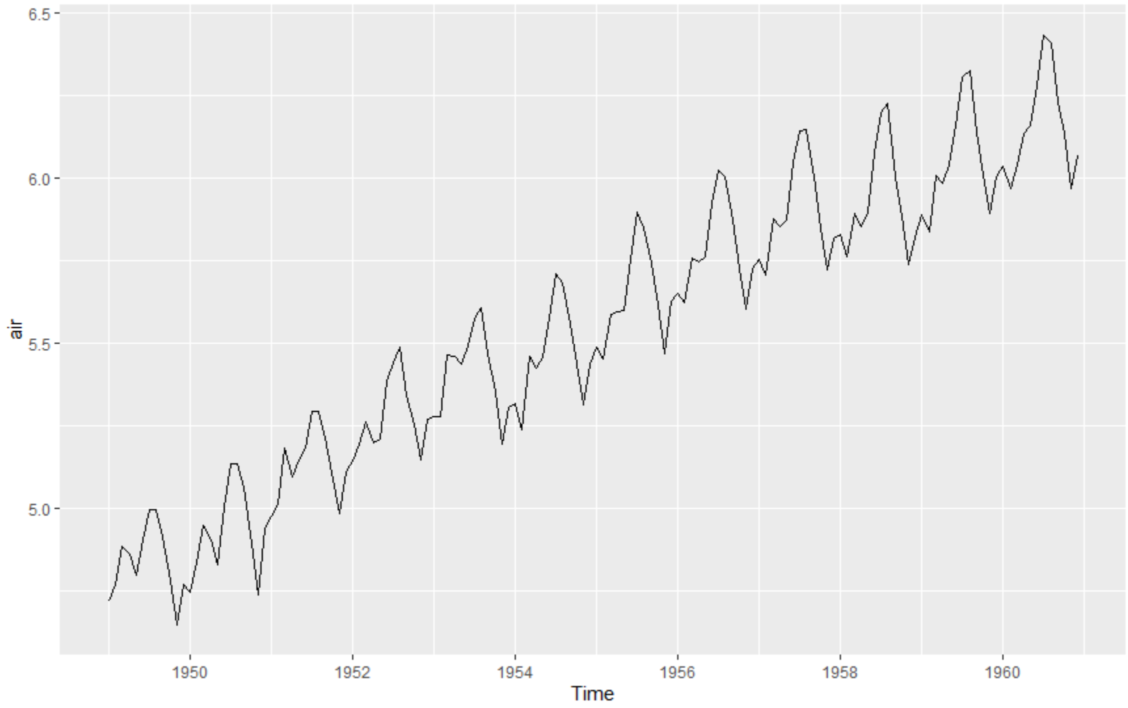
Simple "level" model

$$y_t = \underbrace{\mu_t}_{\text{Level}} + \epsilon_t$$

$\mu_{t+1} = \mu_t + u_t$ Level (u_t is error….assumes Normal distribution)

Example (airline passengers…older data)

Use the older airline passenger data (we will use the Log(passenger) as our time series)

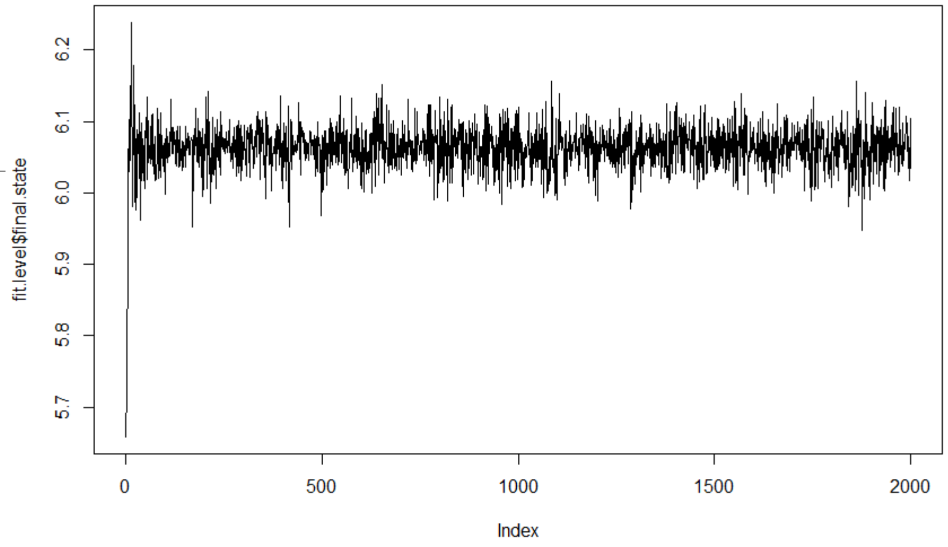


Fitting a level BSTS

```
library(bsts)
library(tidyverse)
air.bsts=airline$LogPsngr
```

```
model_components=list()
```

```
model_components <- AddLocalLevel(model_components, y = air.bsts)
fit.level=bsts(air.bsts, model_components, niter = 2000)
plot(fit.level$final.state,type='l')
```



Fitting a level BSTS

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air.bsts=airline$LogPsngr
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model_components=list()
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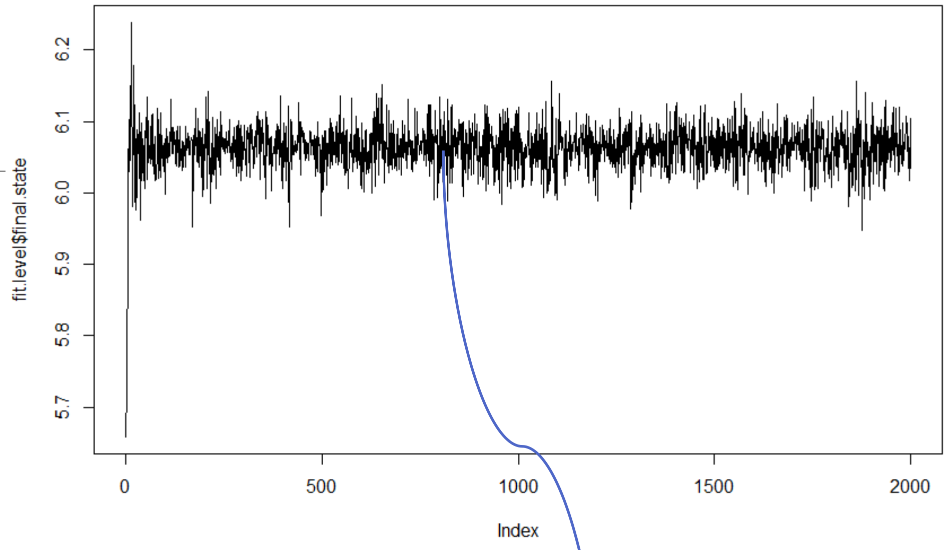
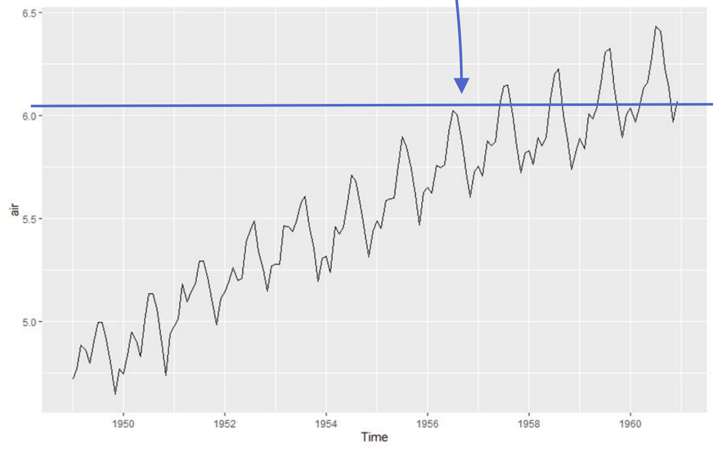
model_components <- AddLocalLevel(model_components, y = air.bsts)
fit.level=bsts(air.bsts, model_components, niter = 2000)
plot(fit.level$final.state,type='l')
```



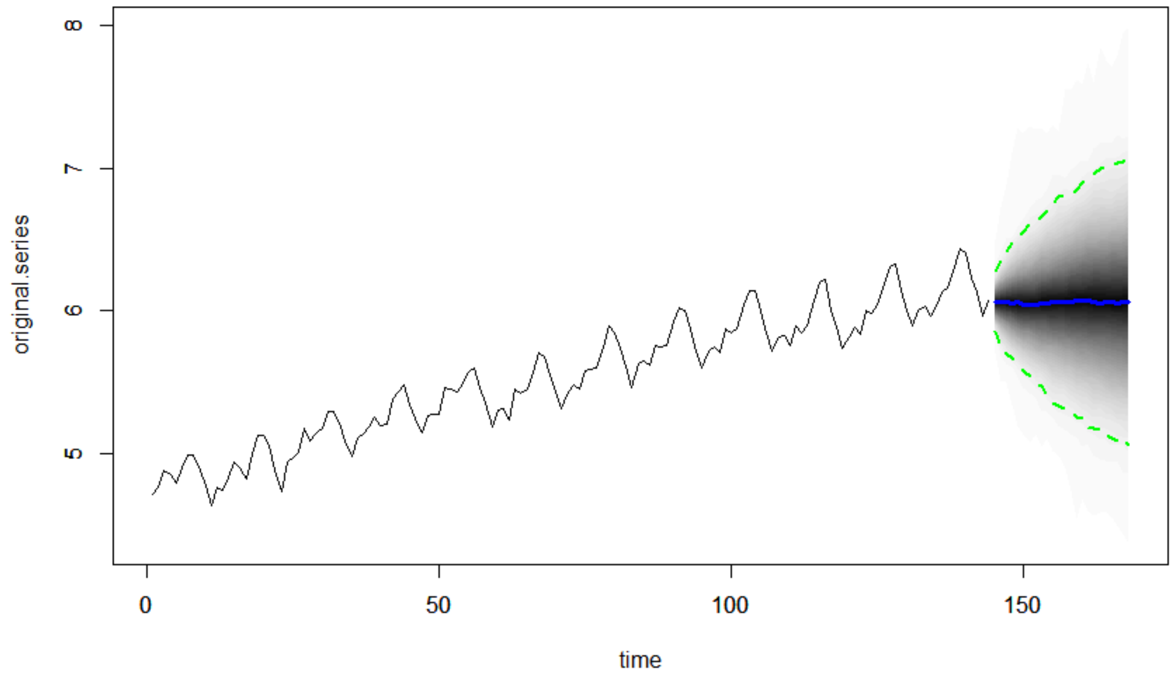


Making forecasts

```
pred.level<-predict(fit.level,burn = 200,horizon = 24)
plot(pred.level)
```

```
> pred.level$mean
 6.060904 6.058905 6.059396 6.057654
6.057066 6.052285 6.053708 6.053631
6.057384 6.059317 6.059745 6.061799
6.063273 6.064737 6.064800 6.067155
6.070143 6.066728 6.066637 6.062760
6.060922 6.056811 6.056758 6.058400
```

Can also ask for median and interval



Trend BSTS

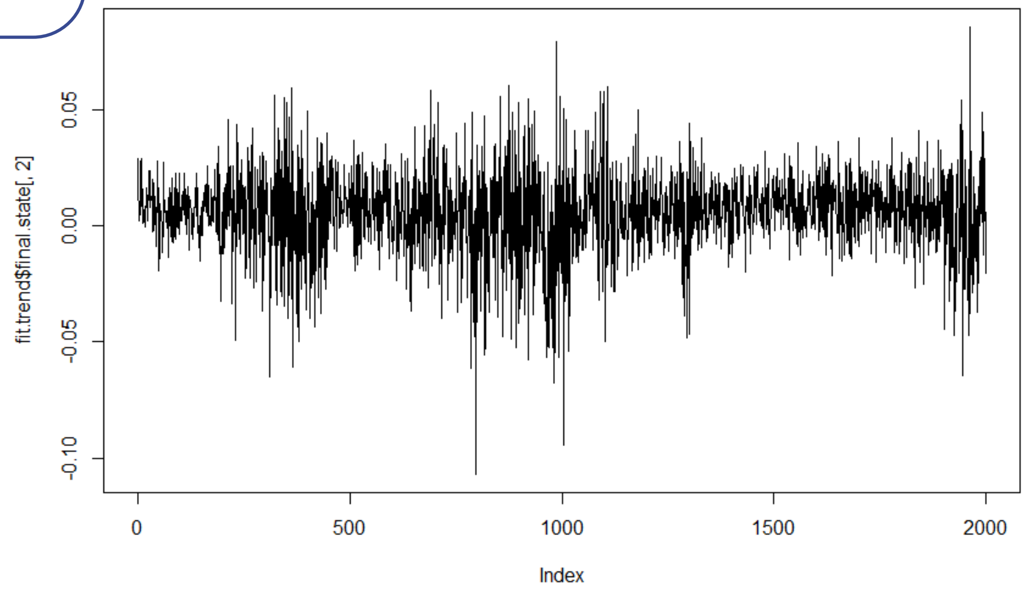
$$y_t = \underbrace{\mu_t}_{\text{Level/Trend}} + \epsilon_t$$

$$\left.\begin{aligned} \mu_{t+1} &= \mu_t + \delta_t + u_t \\ \delta_{t+1} &= \delta_t + v_t \end{aligned}\right\} \text{Level and Trend } (u_t \text{ and } v_t \text{ are error})$$

Fit a trend BSTS

```
model_components=list()
model_components=AddLocalLinearTrend(model_components,
                                     y = air.bsts)
fit.trend=bsts(air.bsts, model_components, niter = 2000)
plot(fit.trend$final.state[,2],type='l')
```

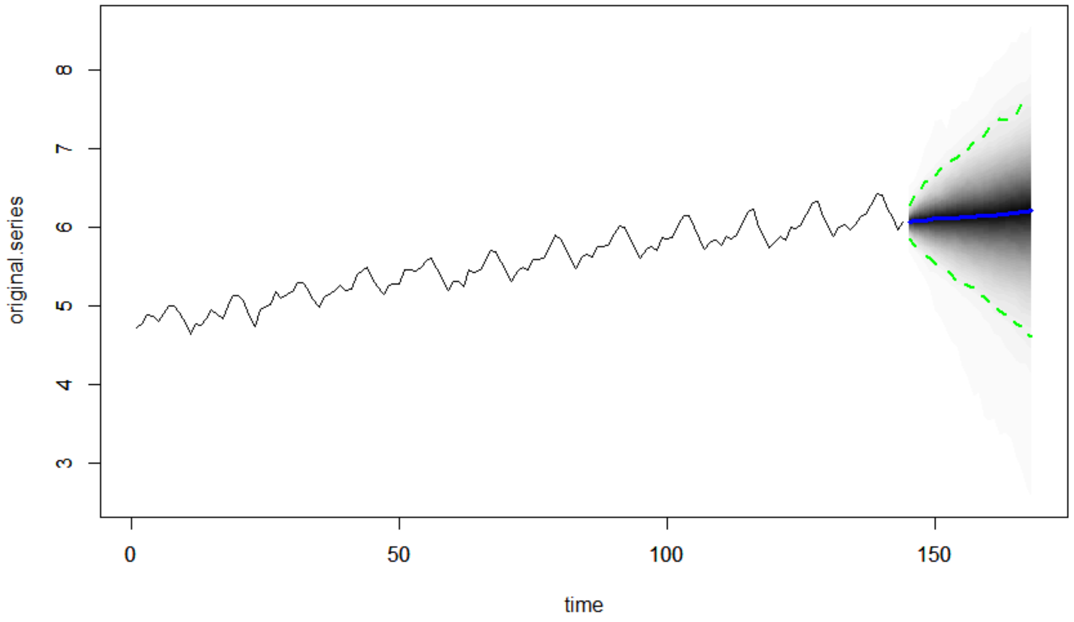
```
> head(fit.trend$final.state)
         [,1]        [,2]
[1,] 6.270559 0.028903721
[2,] 6.188468 0.015266010
[3,] 6.176237 0.006683758
[4,] 6.144252 0.002293991
[5,] 6.260150 0.005248778
[6,] 6.229388 0.005921211
```



Plot forecasts

```
pred.trend<-predict(fit.trend,burn = 500,horizon = 24)
plot(pred.trend)
```

```
> pred.trend$median
 6.069794 6.080965 6.084830 6.088169 6.094706
6.107259 6.112713 6.111001 6.114586 6.120687
6.129326 6.123488 6.131942 6.145842 6.141558
6.147235 6.145968 6.165901 6.164742 6.176605
6.187887 6.196770 6.202536 6.215285
```



Seasonal BSTS

$$y_t = \underbrace{\mu_t}_{\text{Level}} + \underbrace{\tau_t}_{\text{Season}} + \epsilon_t$$

$$\left.\begin{aligned} \mu_{t+1} &= \mu_t + \delta_t + u_t \\ \delta_{t+1} &= \delta_t + v_t \end{aligned}\right\}$$

Level and Trend (u_t and v_t are error)

$$\tau_{t+1} = -\sum \tau_t + w_t$$

Seasonality (w_t is the error term)

How to model Seasonality

Dummy variables

Trigonometric Functions

Dummy variables

Basically, you are doing a linear regression!!! Let's say we have monthly data (i.e. we need 11 dummy variables)…

$$x_1 = \begin{cases} 1 & if\ January \\ 0 & otherwise \end{cases}$$

$$x_2 = \begin{cases} 1 & if\ February \\ 0 & otherwise \end{cases}$$

$$\vdots$$

$$x_{11} = \begin{cases} 1 & if\ November \\ 0 & otherwise \end{cases}$$

Dummy variables

Basically, you are doing a linear regression!!! Let's say we have monthly data (i.e. we need 11 dummy variables)…

$$Y_t = 38.5 + 2.9X_1 + 5.95X_2 + 6.7X_3 + 3.9124356X_4 - 0.91X_5 - 3.8X_6 - 2.27X_7 + 1.49X_8 + 3.1X_9 + 1.4X_{10} - 0.53X_{11}$$

$$x_1 = \begin{cases} 1 & if\ January \\ 0 & otherwise \end{cases}$$

$$x_2 = \begin{cases} 1 & if\ February \\ 0 & otherwise \end{cases}$$

•

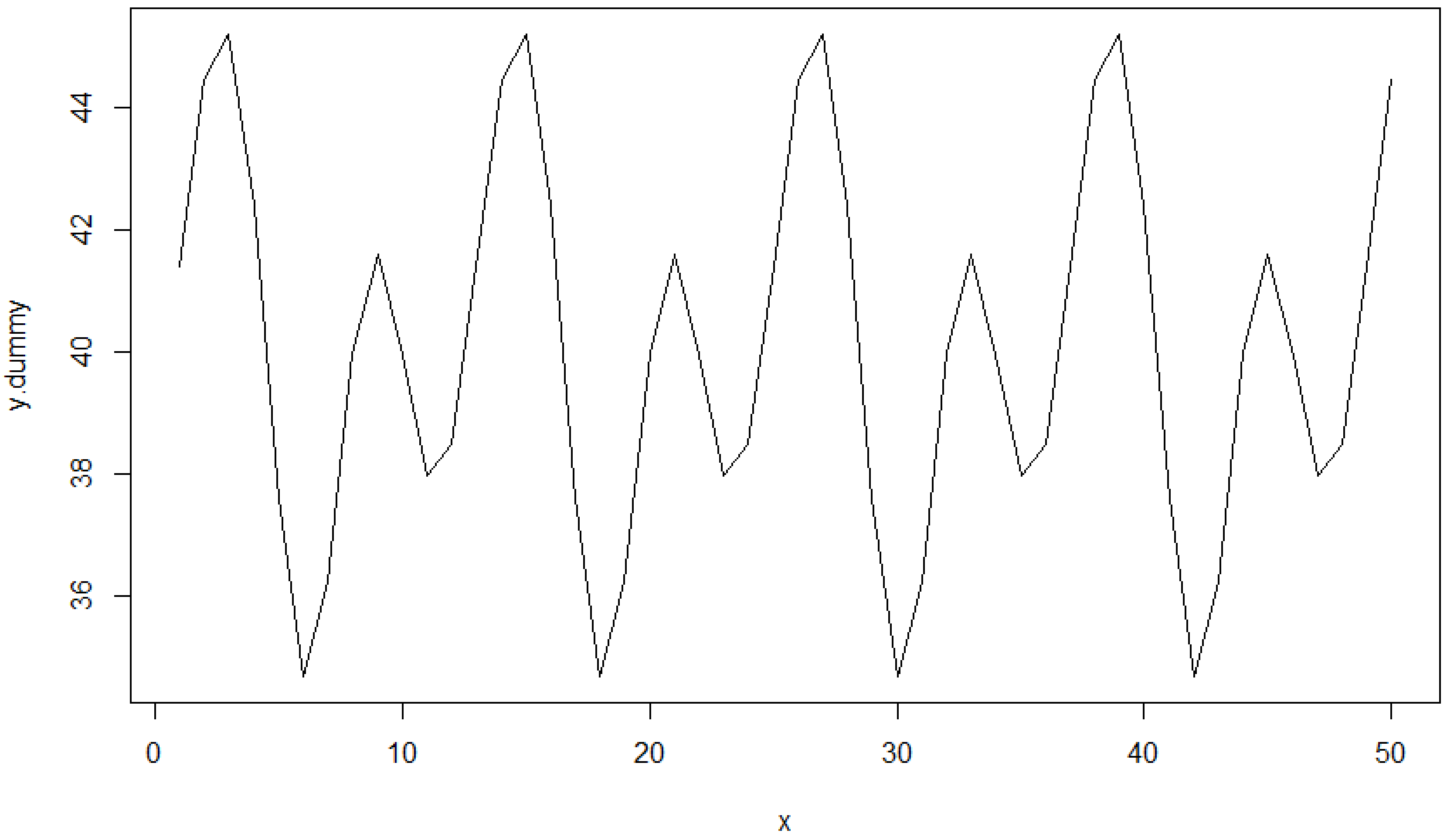
•

•

$$x_{11} = \begin{cases} 1 & if\ November \\ 0 & otherwise \end{cases}$$

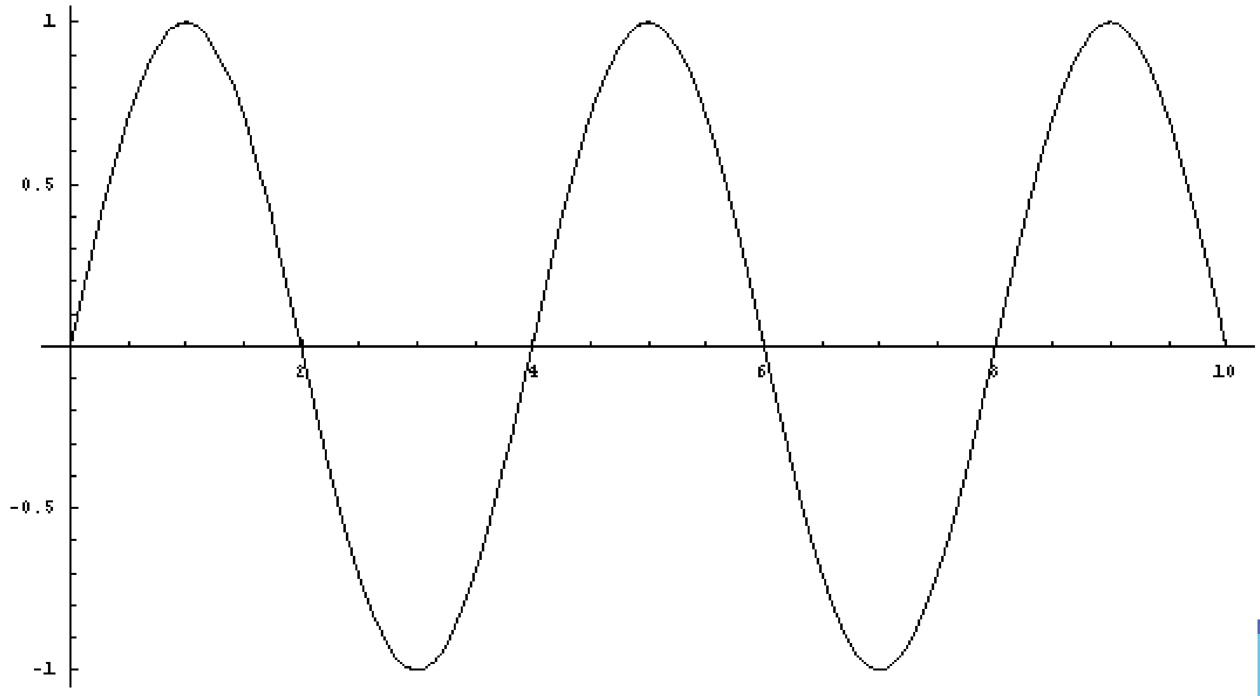
```
x.dummy[1:3,]
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11]
[1,]    1    0    0    0    0    0    0    0    0     0     0
[2,]    0    1    0    0    0    0    0    0    0     0     0
[3,]    0    0    1    0    0    0    0    0    0     0     0
```

```
y.dummy=38.5+x.dummy%*%c(2.9052559,5.9516660,6.7000000,
3.9124356,-0.9052559,-3.8000000,-
2.2660254,1.4875644,3.1000000 ,1.4483340,-0.5339746)
```


y.dummy
36
38
40
42
44
0
10
20
30
40
50
x

Trigonometric Functions

- Trigonometric functions in mathematics have a cyclical pattern.
- Use trigonometric functions, such as sine and cosine, to model seasonality.



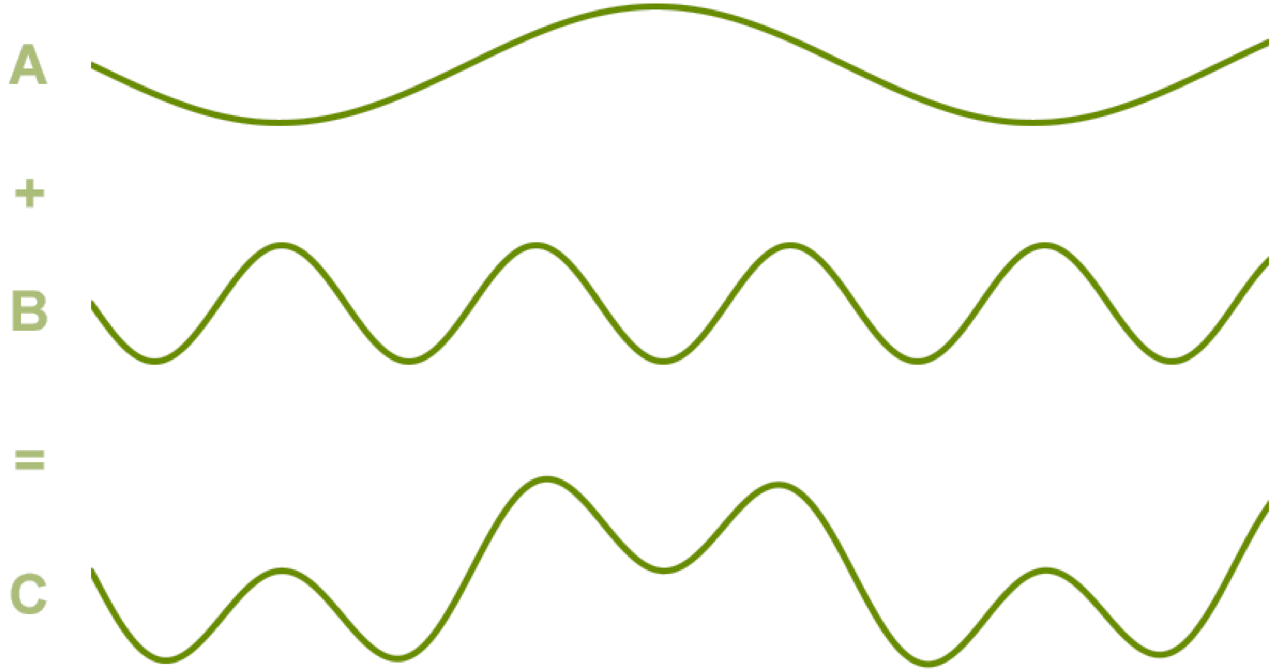
Trigonometric Regression

$$X_t = \sin\left(\frac{2\pi t}{S}\right)$$

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t$$

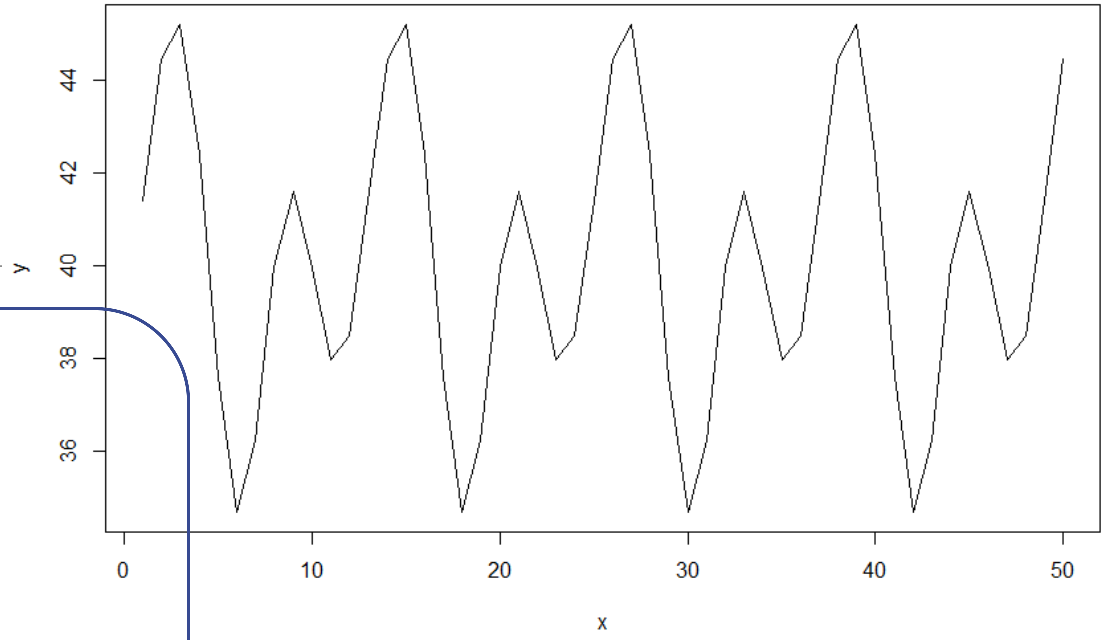
Trigonometric Regression

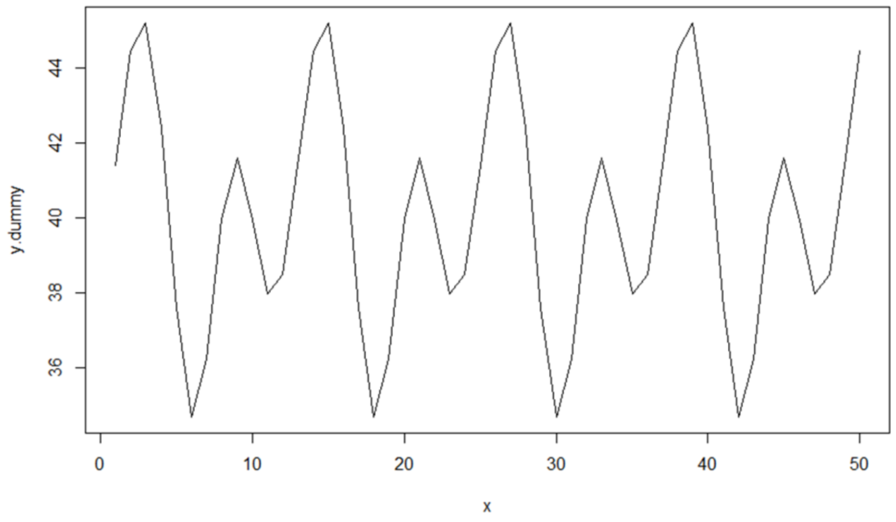
- You don't have to limit yourselves to only one sine or cosine variable.
- Mixing sine and cosine functions might better fit your data (Fourier analysis).



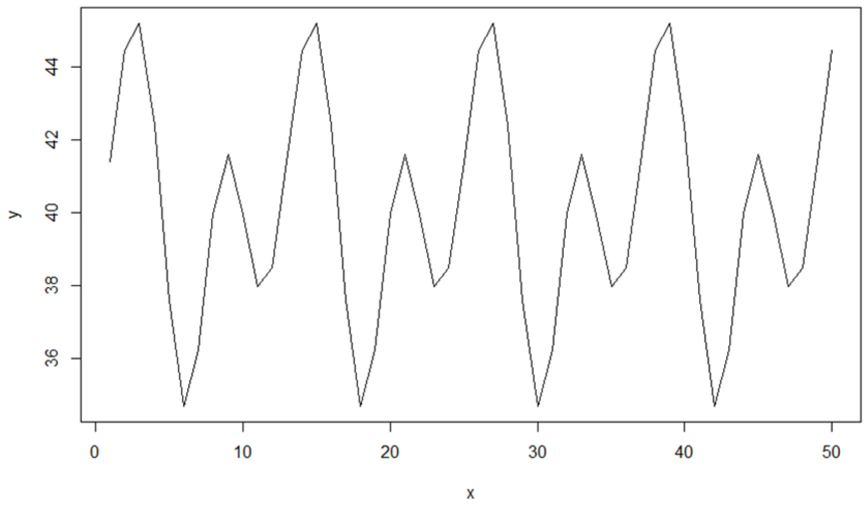
For example:

```
> x=seq(1,50,by=1)
> sin1=sin(1*2*pi*x/12)
> cos1=cos(1*2*pi*x/12)
> sin2=sin(2*2*pi*x/12)
> cos2=cos(2*2*pi*x/12)
> sin3=sin(3*2*pi*x/12)
> cos3=cos(3*2*pi*x/12)
> y=40+2*sin1+1.6*cos1+0.6*sin2-3.4*cos2+0.2*sin3+0.3*cos3
> plot(x,y,type='l')
```





y.dummy
x
0
10
20
30
40
50
36
38
40
42
44

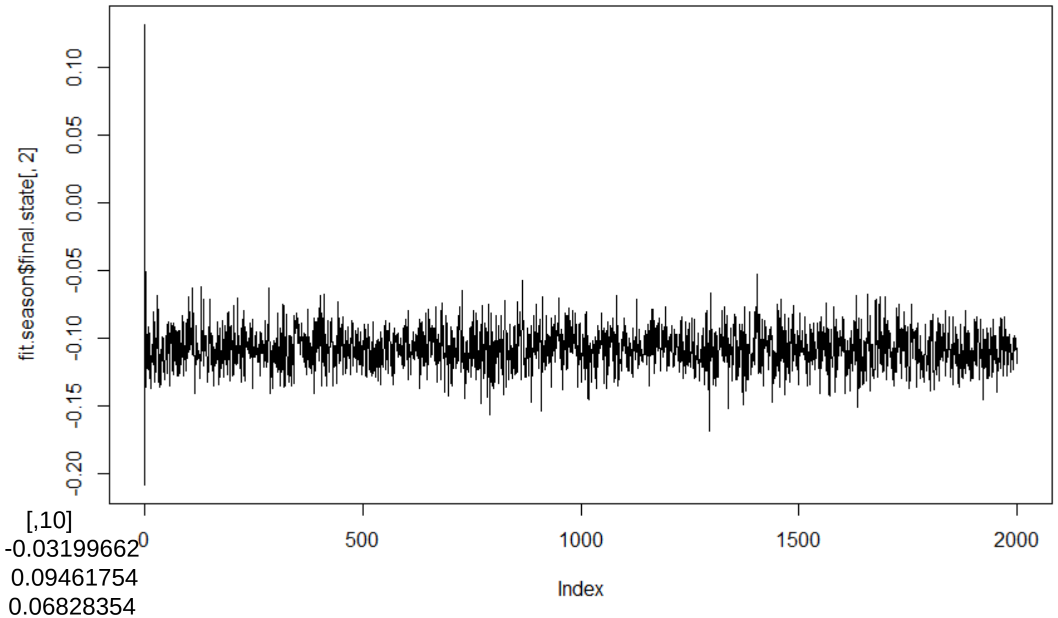


y
x
0
10
20
30
40
50
36
38
40
42
44

Seasonal (with Dummy and Level)

```
model_components=list()
model_components = AddLocalLevel(model_components,
                                 y = air.bsts)
model_components=AddSeasonal(model_components, y
= air.bsts, nseasons   = 12)

fit.season=bsts(air.bsts, model_components, niter = 2000)
plot(fit.season$final.state[,2],type='l')
```

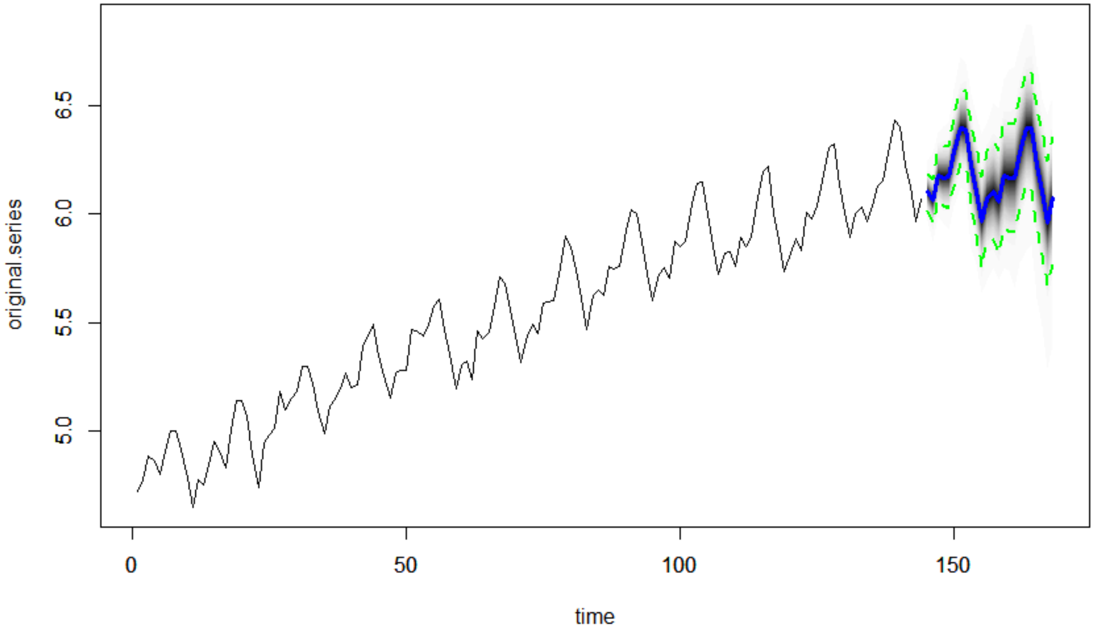



```
> head(fit.season$final.state)
         [,1]        [,2]       [,3]        [,4]         [,5]       [,6]       [,7]       [,8]         [,9]       [,10]
[1,] 5.611838  0.13122061 -0.2187225 -0.11985584 -0.009119432 0.18958485 0.11092930 0.20312464 -0.073750644 -0.03199662
[2,] 5.775567 -0.20784084 -0.2606871 -0.01023421  0.039516655 0.08946901 0.28148379 0.21886119 -0.072926289  0.09461754
[3,] 5.839541 -0.05099446 -0.1414026 -0.19565411  0.081198015 0.29500666 0.04275373 0.08099343  0.034689610  0.06828354
[4,] 5.847991 -0.08382377 -0.2623133 -0.20962298  0.230671767 0.20365406 0.18841063 0.12364207 -0.073111699 -0.02250975
[5,] 6.029962 -0.13583900 -0.1897727 -0.04038770  0.146914790 0.20151903 0.35220462 0.01275095  0.001114029 -0.03964137
[6,] 6.095697 -0.10922017 -0.2577005 -0.06908177  0.077977186 0.18769086 0.26883741 0.14330611 -0.023051989  0.04296503
             [,11]       [,12]
[1,] -0.1377430382 -0.20602724
[2,]  0.0093251931 -0.05982110
[3,]  0.0063418976 -0.05214867
[4,]  0.0804886616 -0.15250747
[5,] -0.0253000053 -0.05157723
[6,] -0.0004721743 -0.15021407
```

1 Level + 11 Dummy variables

Forecast(with dummy)

```
pred.season<-predict(fit.season,burn = 500,horizon = 24)
plot(pred.season)
```

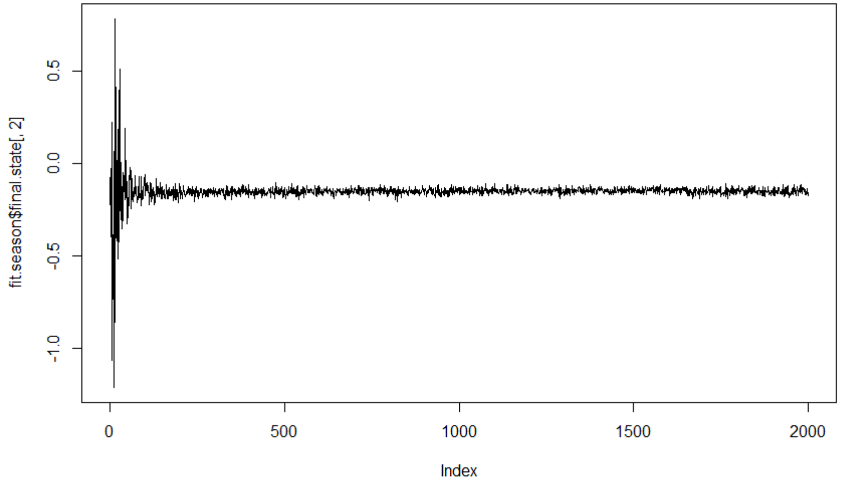



```
> pred.season$mean
 [1] 6.104505 6.061527 6.182944 6.172317 6.173189 6.286979 6.401651 6.394572
6.226572 6.108766 5.964014 6.070397 6.105952  6.060050 6.179451 6.169520 6.167948
6.283719 6.400189 6.392691 6.224080 6.106716 5.961663 6.069726
```

Seasonal (Trig with level)

```
model_components=list()
model_components = AddLocalLevel(model_components,
                                 y = air.bsts)
model_components=AddTrig(model_components, y =
air.bsts,
                         period  = 12,frequencies = 1:3)

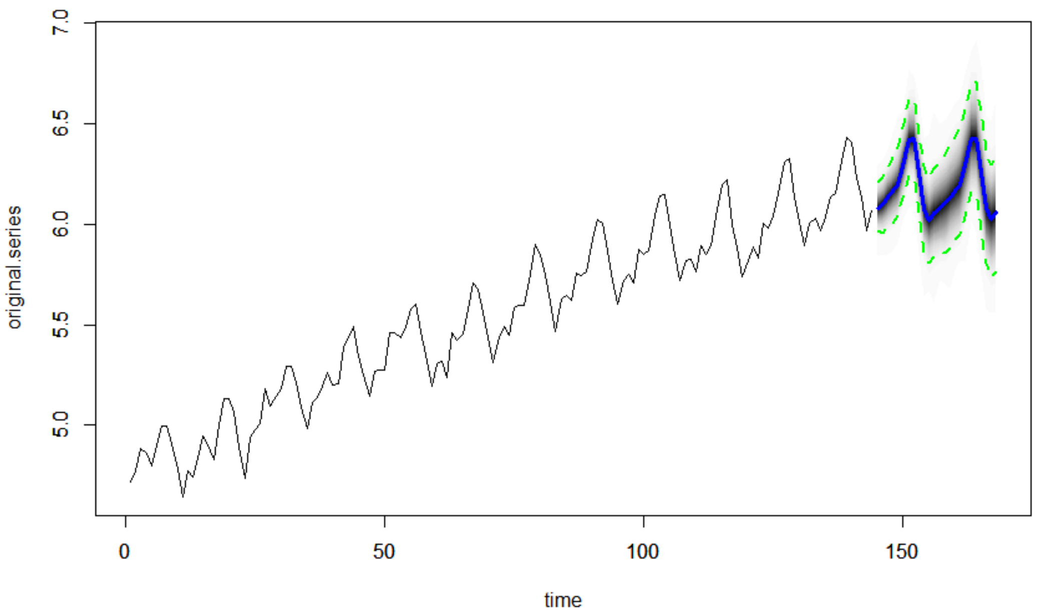
fit.season=bsts(air.bsts, model_components, niter = 2000)
plot(fit.season$final.state[,2],type='l')
```




```
> head(fit.season$final.state)
         [,1]        [,2]       [,3]        [,4]        [,5]      [,6]       [,7]
[1,] 5.579081 -0.14760642 -0.5717763  0.01409946 -0.82855883 0.3028314  0.1562200
[2,] 5.601328 -0.07600871 -0.4598823  0.14894884  0.20325190 1.0879606  0.1213216
[3,] 5.574211 -0.37114738 -0.2070764 -0.35487942 -1.35175170 0.5059916 -0.2237004
[4,] 5.614885 -0.39524983 -1.1895666  0.04371342  0.09413661 0.5763212  0.3782019
```

Forecast (Trig)

```
pred.season<-predict(fit.season,burn = 500,horizon = 24)
plot(pred.season)
```




```
> pred.season$interval
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]
2.5%	5.965759	5.958060	5.984099	6.002435	6.041751	6.126399	6.241519	6.235748	6.056018	5.864842	5.808555	5.838206
97.5%	6.208353	6.235114	6.278911	6.324366	6.380711	6.484603	6.621303	6.623532	6.464349	6.287322	6.231317	6.276204

	[,13]	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]	[,21]	[,22]	[,23]	[,24]
2.5%	5.854577	5.859481	5.883375	5.926380	5.952490	6.034250	6.161035	6.149160	5.988632	5.806161	5.740073	5.762272
97.5%	6.306913	6.345058	6.393916	6.430722	6.483931	6.569421	6.701481	6.709872	6.553636	6.353762	6.293718	6.340721

Can also fit other X variables…

$$y_t = \underbrace{\mu_t}_{\text{Level}} + \underbrace{\tau_t}_{\text{Season}} + \underbrace{\beta^T x_t}_{\text{“X” Variables}} + \epsilon_t$$

$$\left.\begin{aligned} \mu_{t+1} &= \mu_t + \delta_t + u_t \\ \delta_{t+1} &= \delta_t + v_t \end{aligned}\right\}$$

Level and Trend (u_t and v_t are error)

$$\tau_{t+1} = -\sum \tau_t + w_t$$

Seasonality (w_t is the error term)

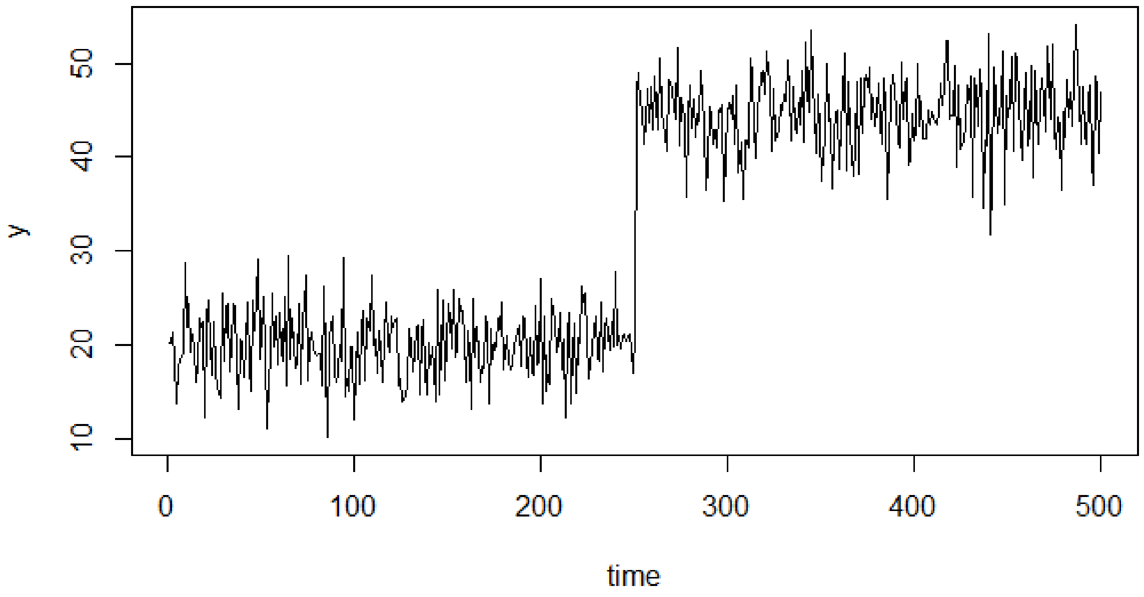
Change-point

A change-point is a point in your data in which the “signal” changes.

CHANGE-POINT

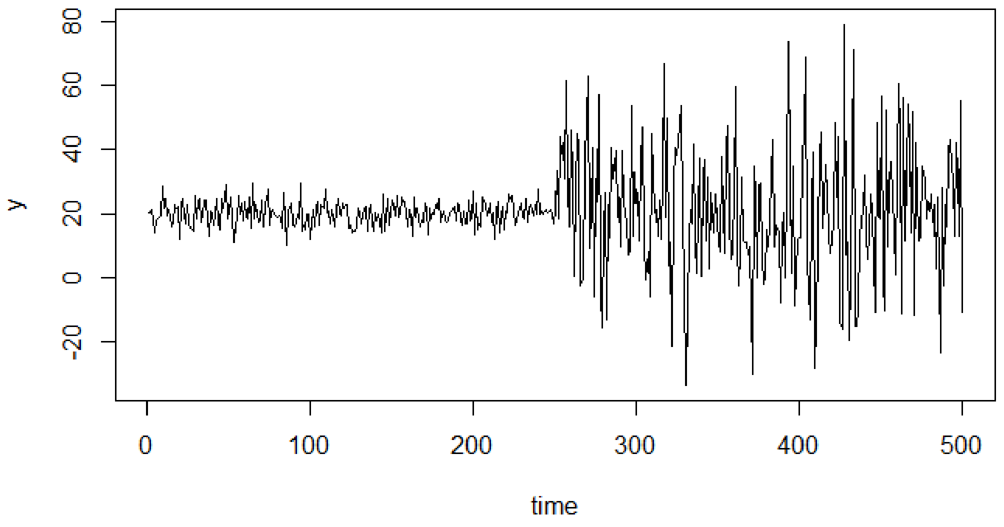
Change-point

A change-point is a point in your data in which the “signal” changes. It can be a shift in means:



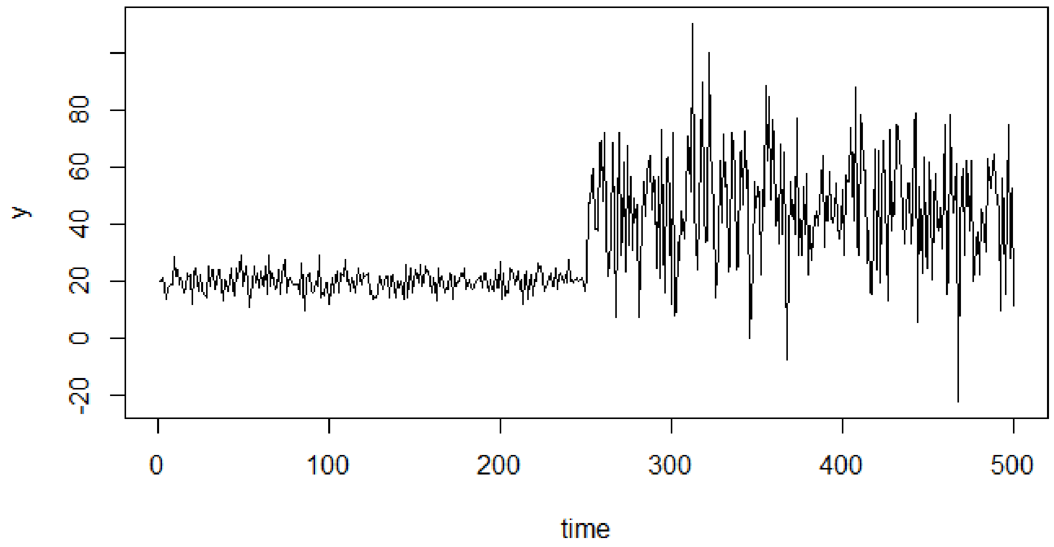
Change-point

A change-point is a point in your data in which the “signal” changes. It can be a change in variance:



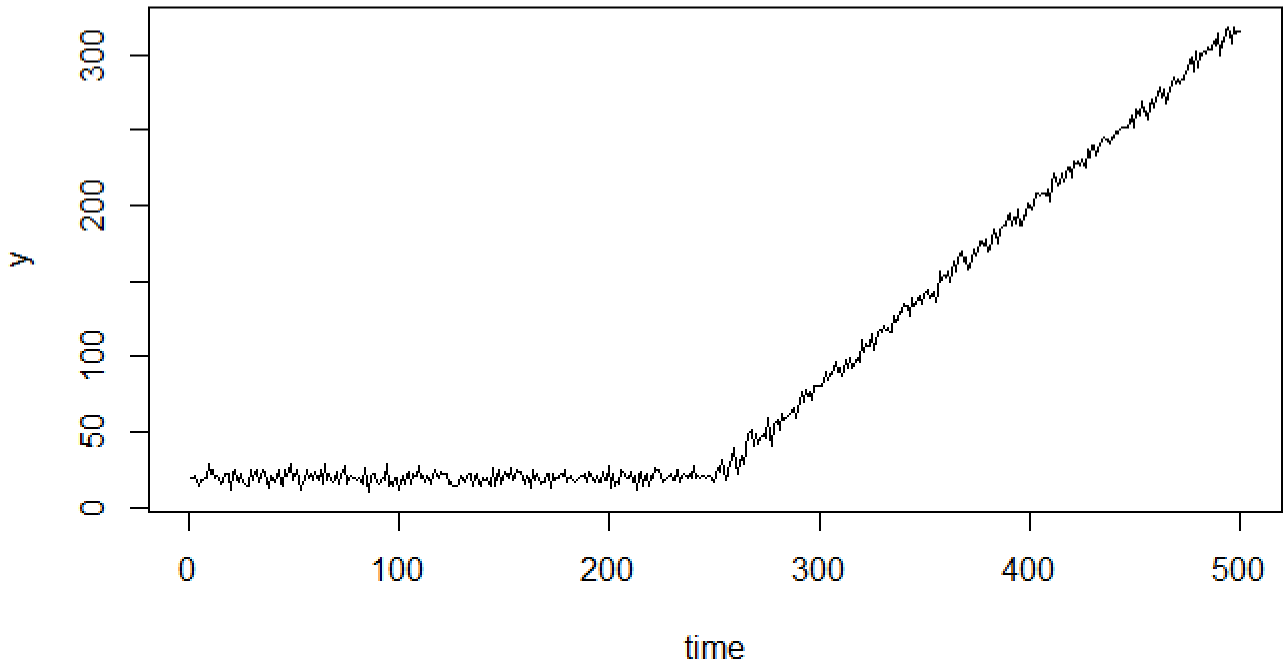
Change-point

A change-point is a point in your data in which the “signal” changes. It can be a change in variance AND a shift in means:



Change-point

A change-point is a point in your data in which the “signal” changes. It can be a change in pattern (going from a level to a trend):



Change-point

And sooooo many more!!!

Can be challenging to find change-points (they can happen in many different ways AND there can be more than one!)

An overview of change point packages in R

Source: `vignettes/packages.Rmd`

OBS: I have yet to review these packages: `not`, `breakfast`, `IDetect`, `trendsegmentR`, `mosum`, `ChangepointTesting`, `changepoint.mv`, `changepointsHD`, `changepointsVar`, `InspectChangepoint`, `breakpoint`, `segmentr`, `Segmentor3IsBack`, `trendsegmentR`, `BayesPiecewiseICAR`, `BayesPieceHazSelect`.

There are a lot of change point packages out there already, so why `mcp`? Here are my (probably biased) thoughts about this. I compiled some tables, summarising change point packages (.xlsx file here). I will demonstrate each of these packages in an applied example below to discuss their merits and shortcomings. I recommend this nice overview of the methodologies used in many of these packages.

Of the packages reviewed here, I think `segmented`, and `EnvCpt` are good if the data fits what they can model, and `mcp` is a more capable and general-purpose package at the cost of speed. You can see the immediate roadmap for `mcp` at the GitHub issues tracker.

Modeling options

How much flexibility does the package allow for modeling the system, you're studying? A clear difference here is between packages that allow you to *specify* the number of change points, and packages which infer the number of change points *automatically* (using some criteria), captured in the `N` column below.

Package		Segment models					Change points					Other
Name	version	Formulas	GLM	Time series	Survival	Multiv.	N	Mean	Random	Variance	AR	Share pars
mcp	0.1.0	specify: all	yes	AR(N)	planned	no	specify	yes	yes	yes	yes	yes
segmented	1.0.0	specify: slope	yes	ARIMA(N)	yes	no	both	yes	no	no	no	no
strucchange::breakpoints	1.5.2	specify: all	no	no	no	no	specify	yes	no	no	no	no
ecp	3.1.2	auto: ?	no	?	no	yes	auto	yes	no	no	no	no
bcp	4.0.3	auto: slope + int	no	no	no	yes	auto	yes	no	no	no	no
changepoint	2.2.2	auto: int	no	no	no	no	auto	yes	no	yes	no	no
changepoint.np	1.0.1	auto: int?	no	no	no	no	auto	yes	no	?	no	no
strucchange::Fstats	1.5.2	specify: all	no	no	no	no	only 1	yes	no	no	no	no
TSMCP	1.0.0	auto: ?	no	yes	no	yes	auto	yes	no	no	no	no
cpm	2.2.0	auto: int	(yes)	no	no	no	auto	yes	no	yes	no	no
EnvCpt	1.1.1	specify: (select)	no	AR(2)	no	no	auto	yes	no	yes	no	no
wbsts	2.0.0	auto: ?	no	yes	no	no	auto	yes	no	no	no	no

From the author's of mcp

Methods

Focus on the change-point package

Changepoint package:

- Can detect multiple change-points (do NOT need to specify number of change-points)
- Uses the PELT algorithm (“Pruned Exact Linear Time” by Killick, Fearnhead, and Eckley 2012)
- Combines the binary segmentation algorithm with the segment neighborhood algorithm for an efficient, exact test (minimizes a cost function with a penalty)
- Can handle multiple change-points in mean and/or variances

Data sets

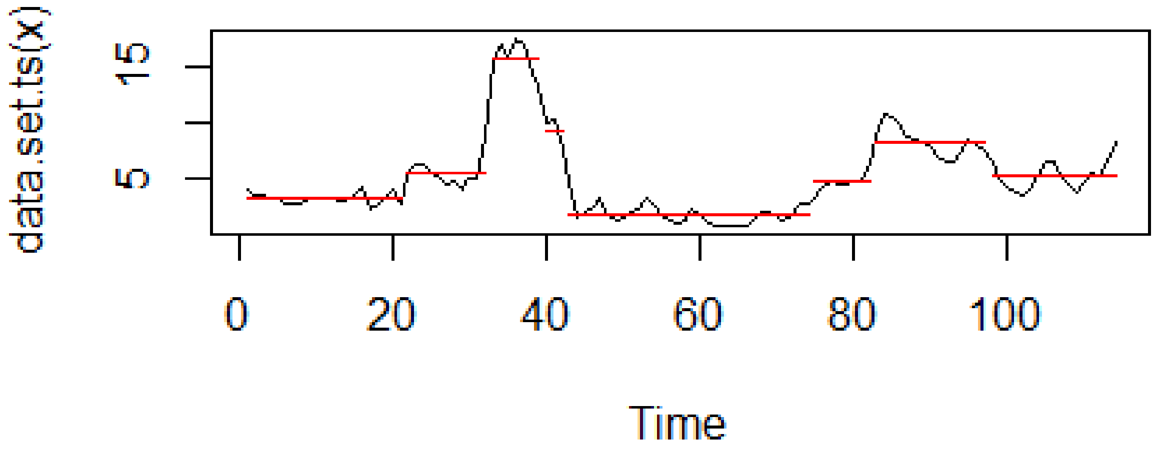
We will be using the Netherlands unemployment data set (use percent unemployed…yearly data): many thanks to Dr. LaBarr for this data set

Change-point algorithm

```
library(changepoint)
cp.n <- cpt.mean(netherlands$Percentage.Unemployed, method = “PELT",
Q = 10)
plot(cp.n)
summary(cp.n)
```

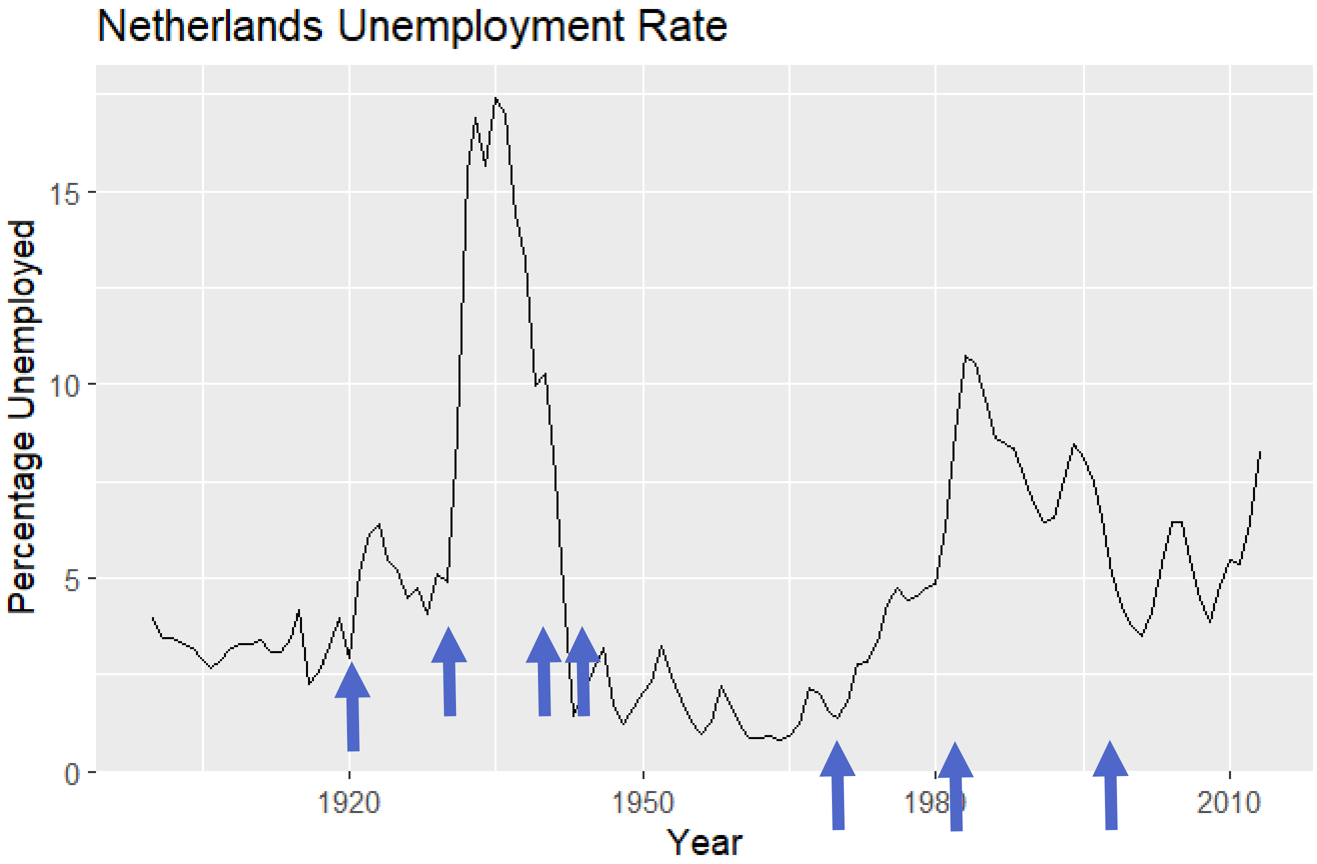
Edited R output

Created Using changepoint version 2.2.4
Changepoint type   : Change in mean
Method of analysis   : PELT
Test Statistic : Normal
Type of penalty   : MBIC with value, 14.2086
Minimum Segment Length : 1
Maximum no. of cpts   : Inf
Changepoint Locations : 21 32 39 42 74 82 97



Changepoints on graph

netherlands$Year[c(21, 32, 39, 42, 74, 82, 97)]
[1] 1920 1931 1938 1941 1973 1981 1996



Questions?