Give light and people will find the way. –Ella Baker

Relationship Between AR and MA

- The best part about AR models and MA models is that they are the same thing – approximately.
- In certain situations (stationarity), AR models can be represented as an infinite MA model.
- In certain situations (invertible), MA models can be represented as an infinite AR model.

ARMA Model

- There is nothing to limit both an AR process and an MA process to be in the model simultaneously.
- These “mixed” models are typically used to help reduce the number of parameters needed for good estimation in the model.
- For example, the most basic model with only one lag of each piece – the ARMA(1,1) model.

$$Y_t = \omega + \phi Y_{t-1} + \theta e_{t-1} + e_t$$

Notation

- ARMA(*p*,*q*) is used to denote mixture models….*p* indicates the number of autoregressive terms and *q* represents the number of moving average terms
- For example, ARMA(2,3) is the following model:

$$Y_t = \omega + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3} + e_t$$

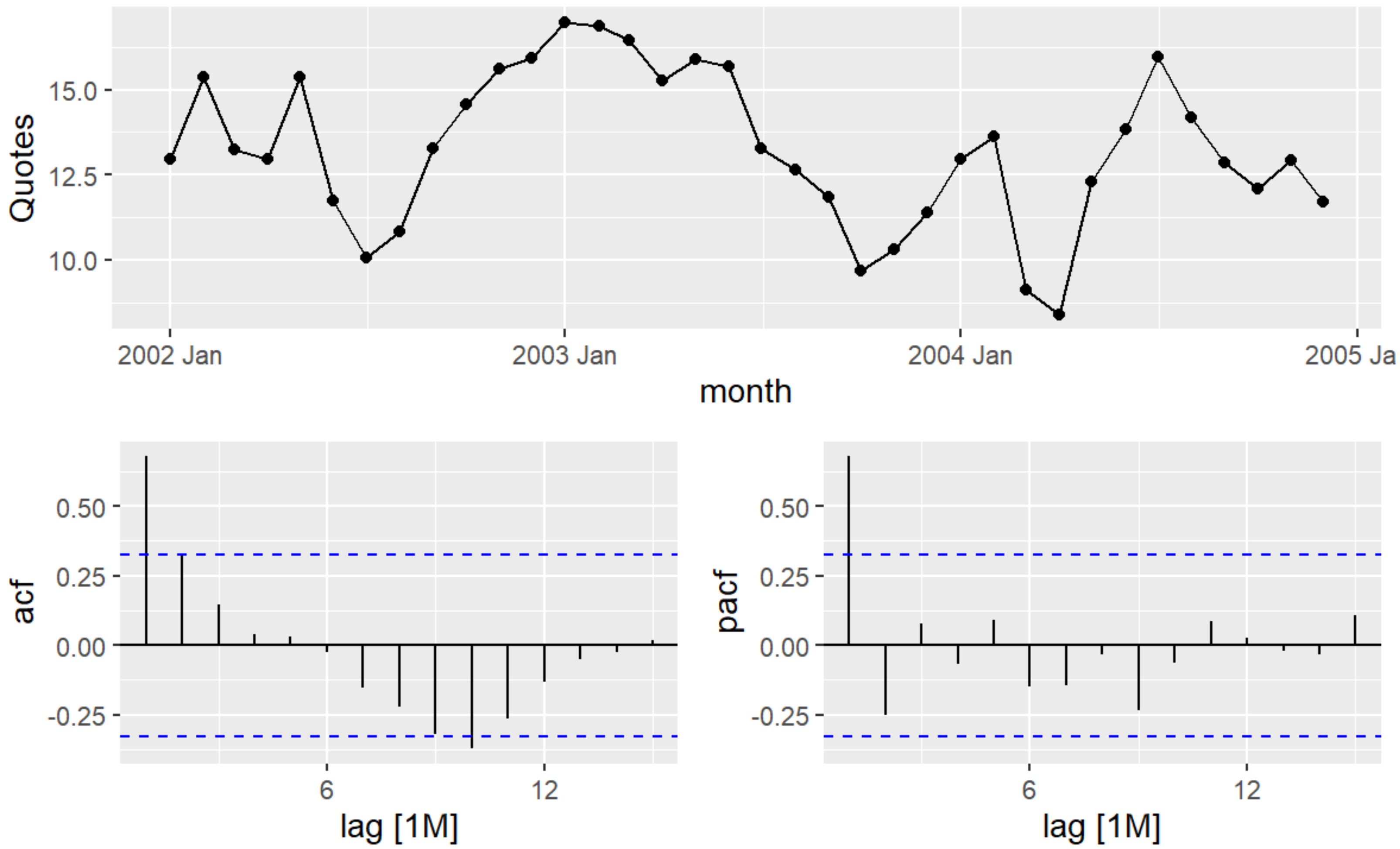
- If the above model (on right hand side) is done on the first differences, then we say this is an ARIMA(2,1,3)

Correlation graphs

- Although correlation graphs can potentially help us, they become very complicated with these mixed models.
- There are some important things to note:
  - Characteristics from both are in the correlation functions.
  - All of the functions tail off exponentially as the lags increase.
  - We tend to look for “spikes” to help us “guess” best model…spikes in ACF would potentially indicate MA terms and spikes in the PACF would potentially indicate AR terms
- Let’s try a couple….

Recall the Quotes data set:

```
Quotes_train %>% gg_tsdisplay(Quotes,plot_type = 'partial')
```



Try some models:

```
quotes_model <-Quotes_train %>%  model(
  ar1 = ARIMA(Quotes ~ pdq(1,0,0) + PDQ(0,0,0)),
  ma1 = ARIMA(Quotes ~ pdq(0,0,1) + PDQ(0,0,0)),
  search1 = ARIMA(Quotes),
  search2 = ARIMA(Quotes,stepwise = F))
quotes_model2<-as.data.frame(quotes_model)
t(quotes_model2)
```

OUTPUT:

```
ar1       ARIMA(1,0,0) w/ mean
ma1       ARIMA(0,0,1) w/ mean
search1 ARIMA(1,0,1) w/ mean
search2 ARIMA(1,0,1) w/ mean
```

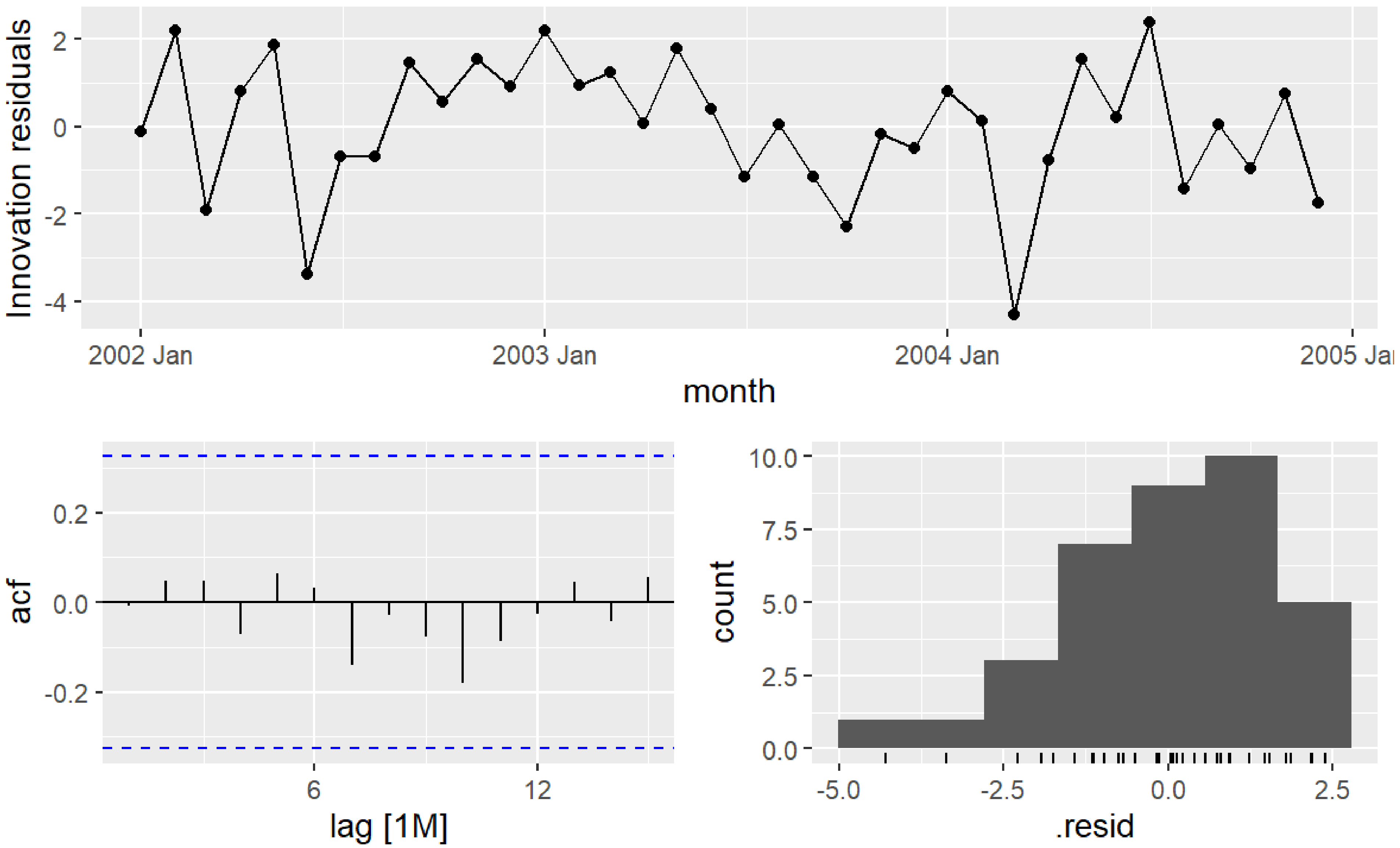
Take a look at models:

```
glance(quotes_model) %>% arrange(AICc) %>% select(.model:BIC)
```

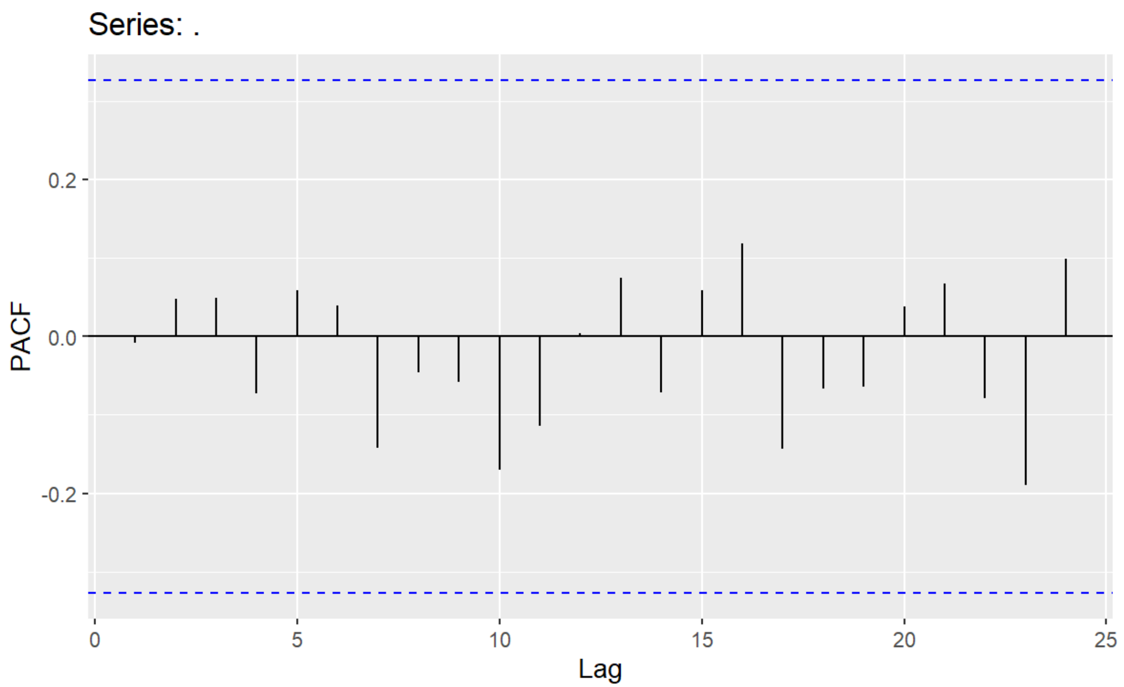
.model <chr>	sigma2 <dbl>	log_lik <dbl>	AIC <dbl>	AICc <dbl>	BIC <dbl>
search1	2.542801	-66.78471	141.5694	142.8597	147.9035
search2	2.542801	-66.78471	141.5694	142.8597	147.9035
ma1	2.750904	-68.68142	143.3628	144.1128	148.1134
ar1	2.785908	-68.79466	143.5893	144.3393	148.3399

Clearly, ARMA(1,1) looks best…let's see if we get white noise…

quotes_model %>% select(search1) %>% gg_tsresiduals()



```
quotes_model %>% select(search1) %>% residuals() %>% ggPacf()
```

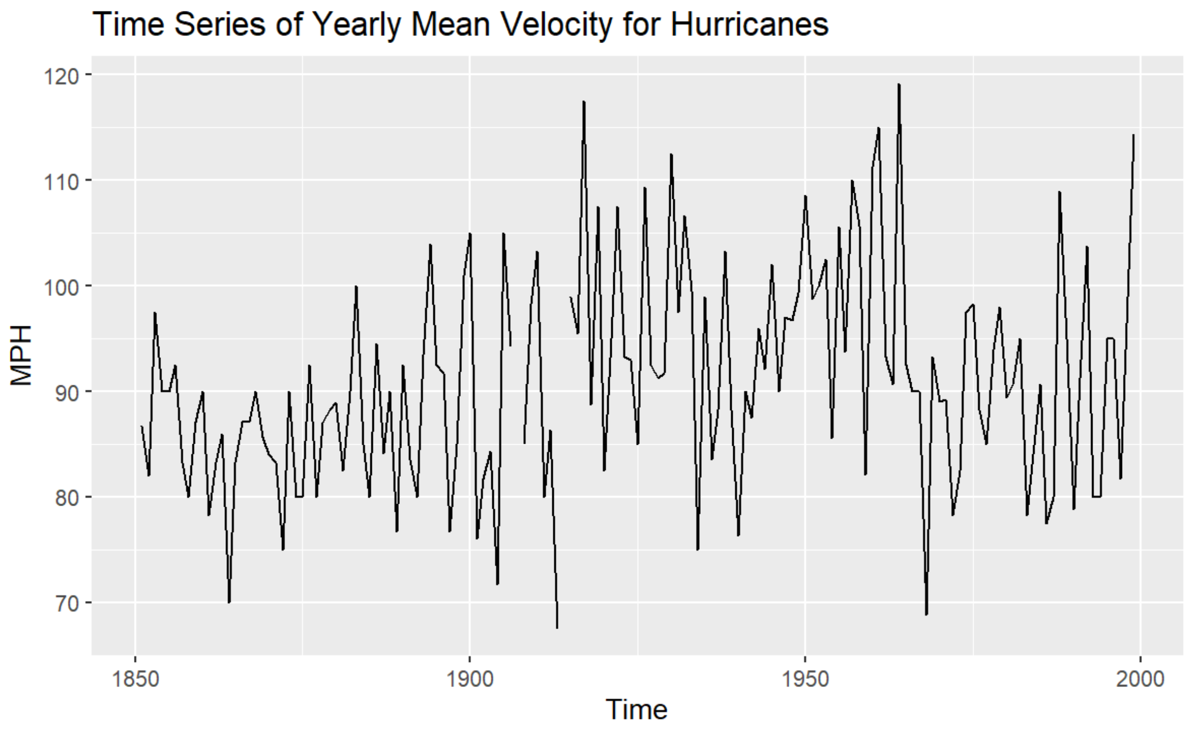



```
augment(quotes_model) %>% filter(.model=='search1') %>% features(.innov,ljung_box, lag=10, dof = 2)
```

.model <chr>	lb_stat <dbl>	lb_pvalue <dbl>
search1	3.596194	0.8915972

Second example: Hurricane (mean maximum velocity)

```
Hurricane.ts<- hurricane %>% as_tsibble(index=Year)
autoplot(Hurricane.ts,MeanVMax)+labs(title="Time Series of Yearly Mean Velocity for Hurricanes", x="Time",
y="MPH")
Hurricane.ts %>% features(MeanVMax,unitroot_ndiffs)
```



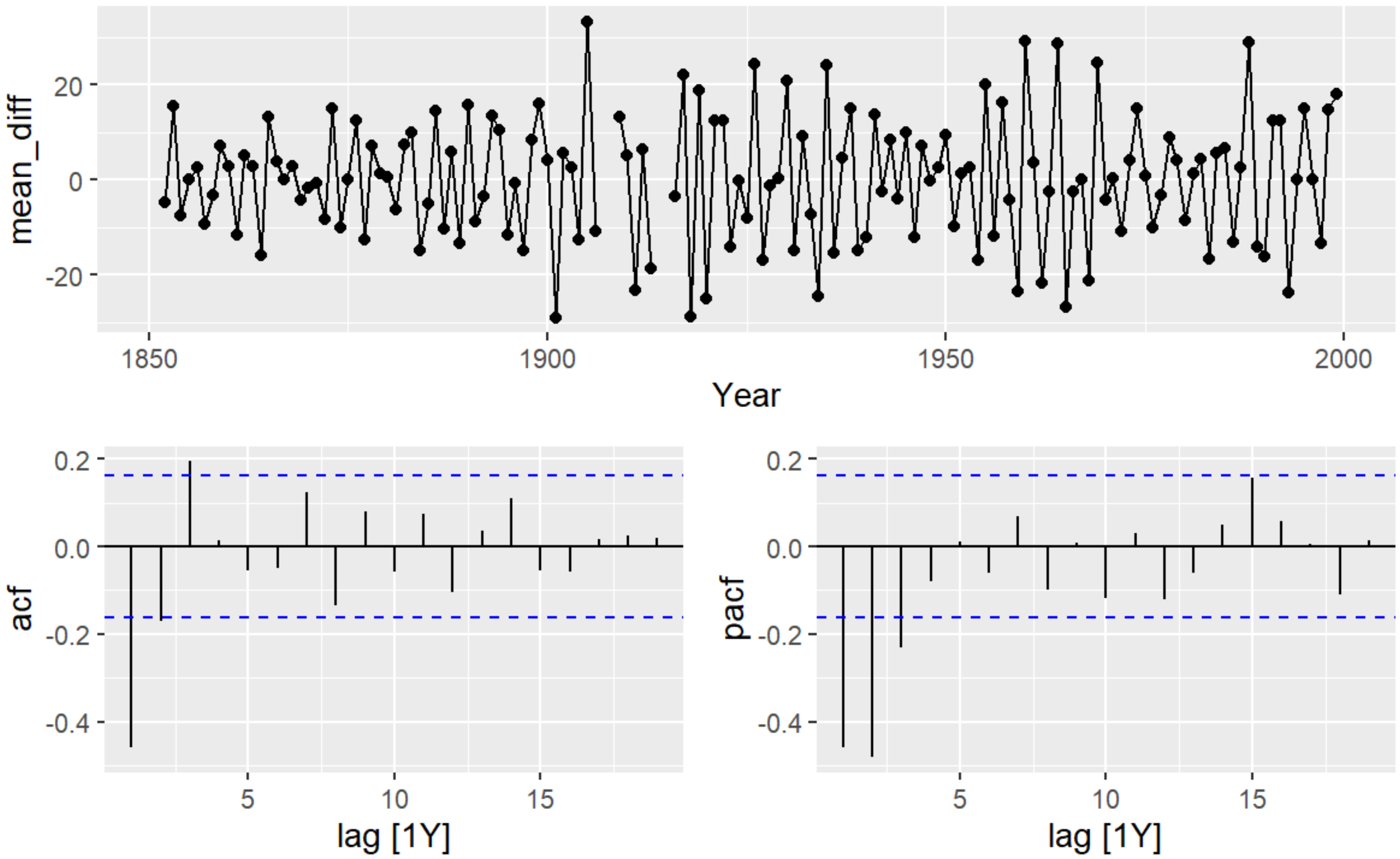
A tibble: 1 × 1

ndiffs <int>
1

1 row

Need to look at differences:

```
Hurricane_train %>% gg_tsdisplay(mean_diff,plot_type = 'partial')
```

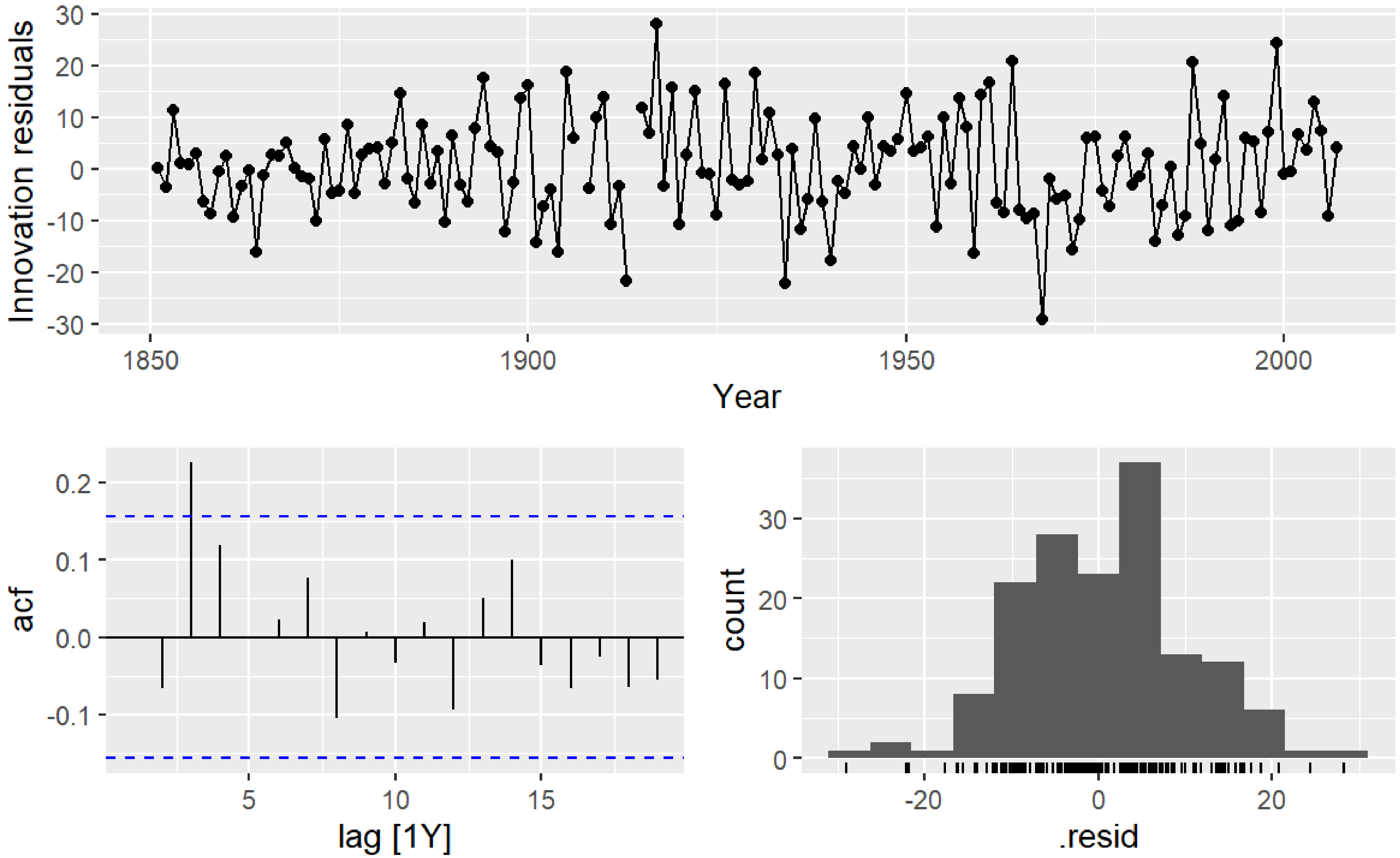


```
hurr_model <-Hurricane_train %>%  model(ar3 = ARIMA(MeanVMax ~ 0 + pdq(3,1,0) + PDQ(0,0,0)),
      ma2 = ARIMA(MeanVMax ~ 0 + pdq(0,1,2) + PDQ(0,1,0)),
      arima32 = ARIMA(MeanVMax~0 + pdq(3,1,2) + PDQ(0,0,0)),
      search1 = ARIMA(MeanVMax),
      search2 = ARIMA(MeanVMax,stepwise = F))
hurr_model2<-as.data.frame(hurr_model)
t(hurr_model2)
glance(hurr_model) %>% arrange(AICc) %>% select(.model:BIC)
```

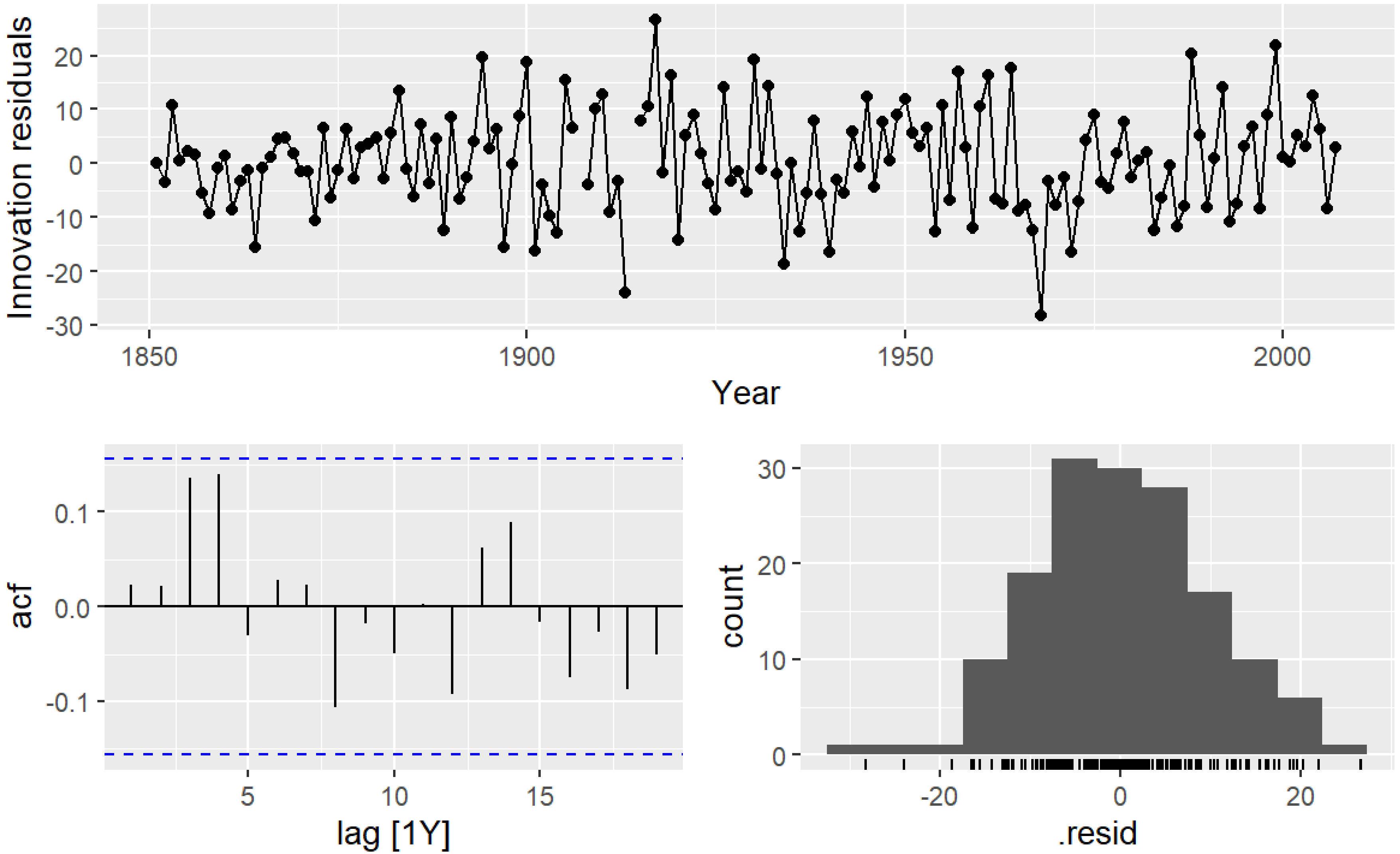
ar3 ARIMA(3,1,0)
ma2 ARIMA(0,1,2)
arima32 ARIMA(3,1,2)
search1 ARIMA(0,0,2) w/ mean
search2 ARIMA(0,0,2) w/ mean

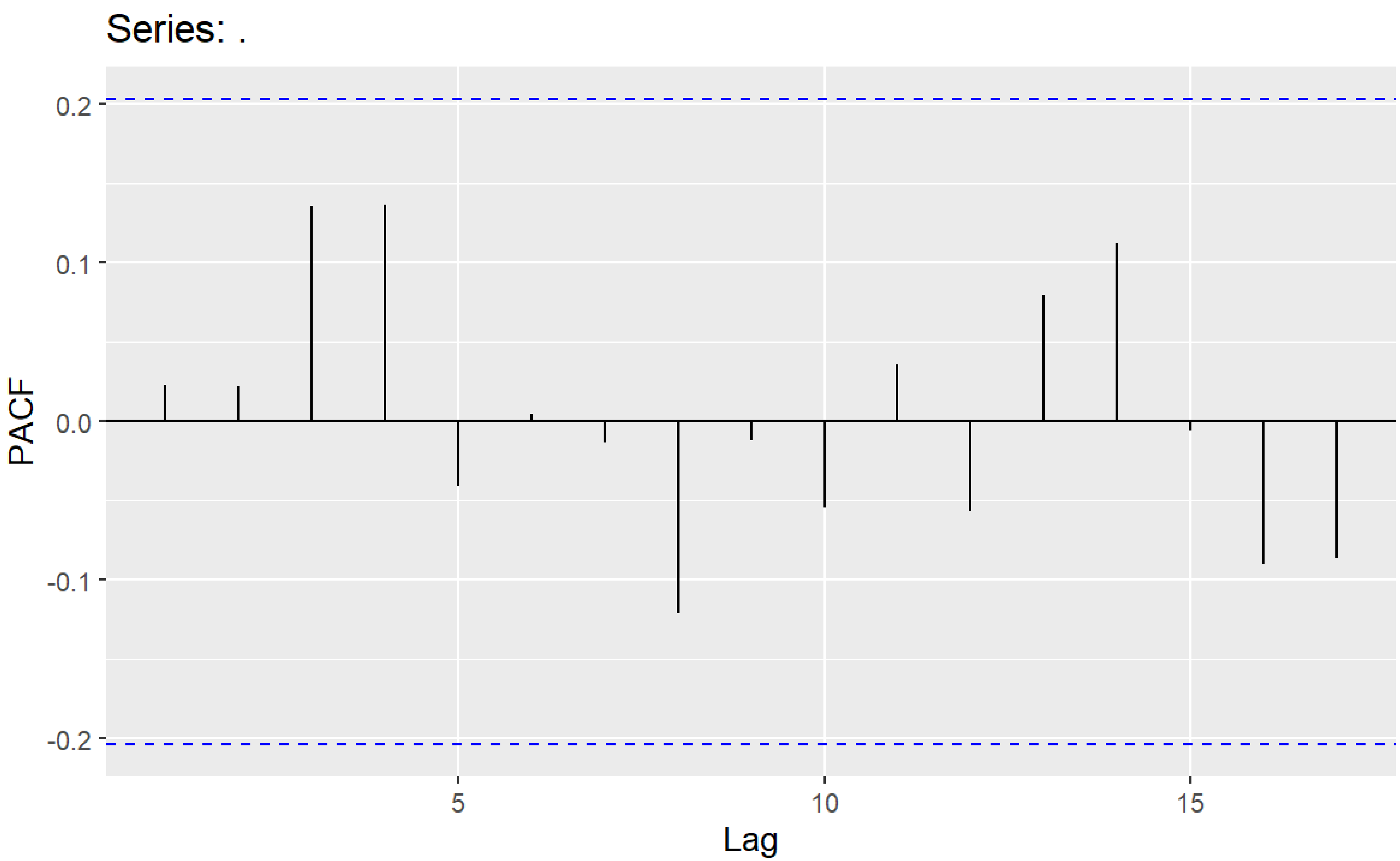
.model <chr>	sigma2 <dbl>	log_lik <dbl>	AIC <dbl>	AICc <dbl>	BIC <dbl>
arima32	94.60665	-538.9024	1089.805	1090.401	1107.788
ma2	97.36121	-542.3252	1090.650	1090.817	1099.642
search2	93.05631	-540.1616	1094.323	1095.118	1115.351
search1	96.28303	-543.9478	1095.896	1096.173	1107.911
ar3	107.88596	-549.0493	1106.099	1106.378	1118.088

MA(2)


Innovation residuals
30
20
10
0
-10
-20
-30
1850
1900
1950
2000
Year
acf
0.2
0.1
0.0
-0.1
5
10
15
lag [1Y]
count
30
20
10
0
-20
0
20
.resid

ARIMA(3,1,2)


Innovation residuals
20
10
0
-10
-20
-30
1850
1900
1950
2000
Year
acf
0.1
0.0
-0.1
5
10
15
lag [1Y]
count
30
20
10
0
-20
0
20
.resid


Series: .
PACF
0.2
0.1
0.0
-0.1
-0.2
5
10
15
Lag

White noise test:

```
augment(hurr_model) %>% filter(.model=='arima32') %>% features(.innov,ljung_box, lag=10, dof = 5)
```

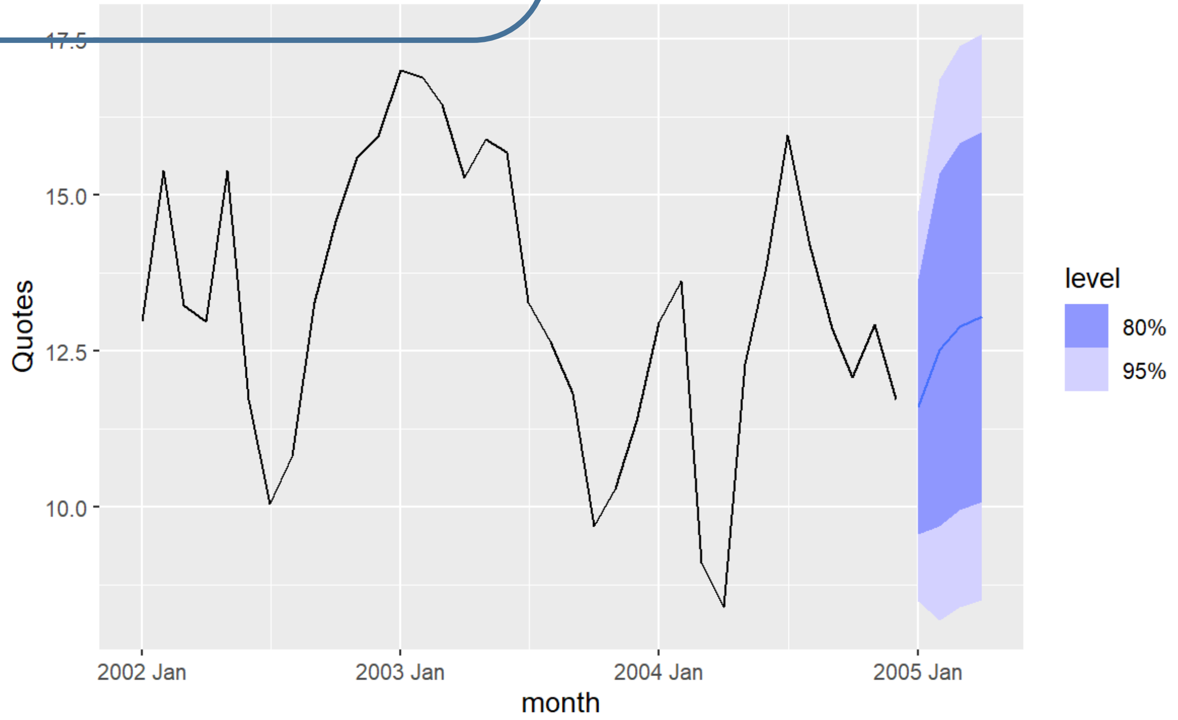
.model <chr>	lb_stat <dbl>	lb_pvalue <dbl>
arima32	5.151377	0.3976869

FORECASTING

Quotes data set:

```
quotes_model %>% select(search1) %>% fabletools::forecast(h=4) %>%
autoplot(Quotes_train)
quotes_for<-quotes_model %>% select(search1) %>%
fabletools::forecast(h=4)
quotes_resid<-Quotes$Quotes[37:40]-quotes_for$.mean
MAPE<-mean(abs(quotes_resid/Quotes$Quotes[37:40]))
MAE<-mean(abs(quotes_resid))
```

MAPE
0.2330841
MAE
3.9527

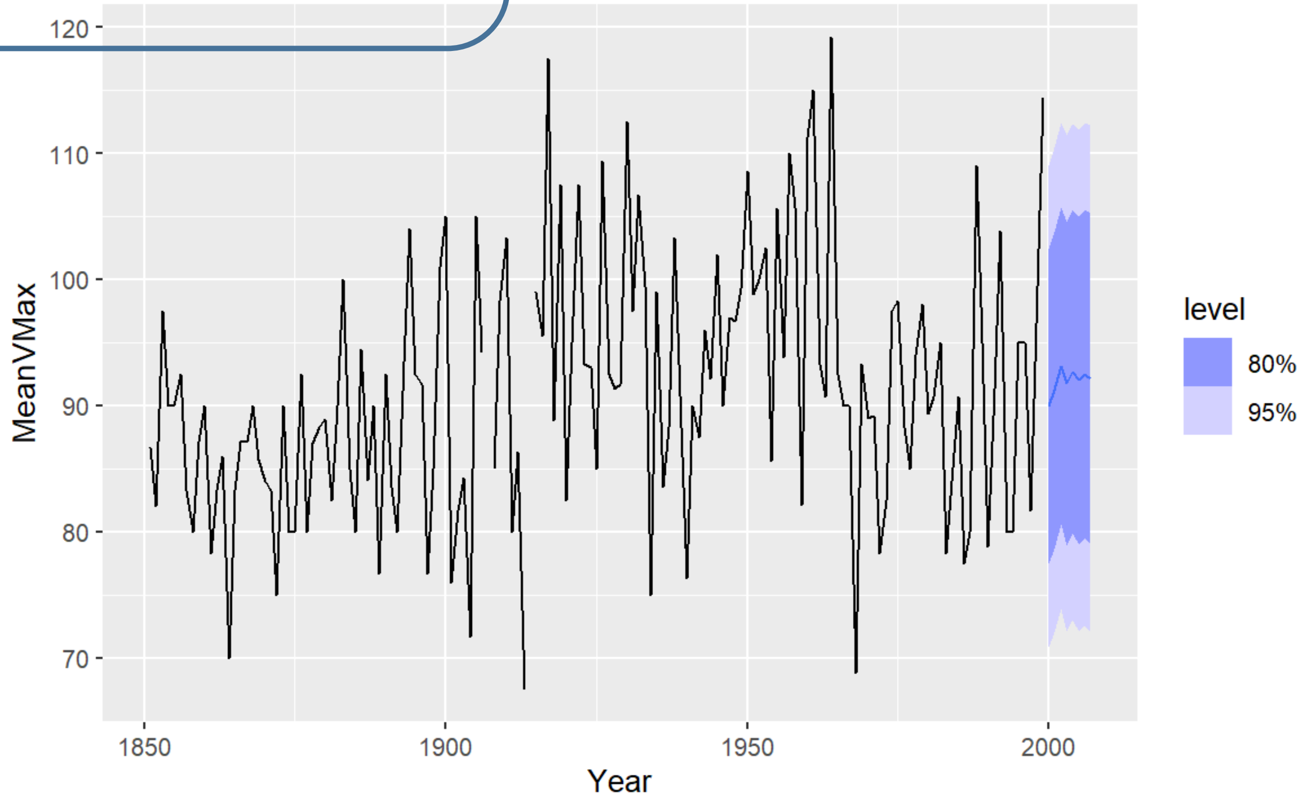


Hurricane data set:

```
hurr_model %>% select(arima32) %>% fabletools::forecast(h=8) %>%
autoplot(Hurricane_train)
hurr_for<-hurr_model %>% select(arima32) %>% fabletools::forecast(h=8)
hurr_resid<-hurricane$MeanVMax[150:157]-hurr_for$.mean
MAPE<-mean(abs(hurr_resid/hurricane$MeanVMax[150:157]))
MAE<-mean(abs(hurr_resid))
```

MAPE
0.060676
MAE
5.98627

NOTE: It forecasts the **actual values**….NOT the differenced value



Questions?