submitted_assignment3_part1

January 7, 2021

```
[1]: version = "v1.8.100820"
```

1 Assignment 3 Part 1: Single Time Series Forecasting (50 pts)

In this assignment, we're going to practise forecasting a single time series.

```
[2]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline

from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters()

# Suppress warnings
import warnings
from statsmodels.tools.sm_exceptions import ValueWarning
warnings.simplefilter("ignore", ValueWarning)
```

We will explore the same time series about **daily new COVID-19 cases globally** as we had in **Assignment 2 Part 1**. In order not to reinvent the wheel, let's import the load_data function you wrote previously.

```
[3]: # Copy and paste the function you wrote in Assignment 2 Part 1 here and import

→ any libraries necessary

# We have tried a more elegant solution by using

# from ipynb.fs.defs.assignment2_part1 import load_data

# but it doesn't work with the autograder...

def load_data():
    daily_new_cases = None

covid_df = pd.read_csv('assets/time_series_covid19_confirmed_global.csv')

# print(covid_df.head())

# try melting
```

```
covid_melty = pd.melt(covid_df, id_vars = ['Country/Region', 'Province/
     →State'], value_vars = covid_df.columns[4:],
                          var_name = 'Date', value_name = 'Cumulative_Cases')
         print(covid melty.tail(5))
        #try to group by date, then later take the diff of each column before...
       covid_groupdate = covid_melty.groupby('Date').sum().reset_index()
         print(covid_groupdate.head())
        #Create a column that converts every Date string into a pd.DatetimeIndex,
    →then set this as the index and drop the old
        #Date column
       covid_groupdate['Date_Time'] = pd.to_datetime(covid_groupdate['Date'])
       covid_groupdate.sort_values(by='Date_Time',inplace = True)
       covid_groupdate.drop('Date', axis = 1, inplace = True)
       covid_groupdate.set_index('Date_Time', inplace = True)
         covid_groupdate.head()
        #Take the difference between every day its respective next day.
        #Rename the column to New cases as calculated be .diff()
       \#Drop\ NA\ rows. The top row after running .diff() is always NA\ as\ there\ was_{\sqcup}
    →no day before it
        covid_new_cases = covid_groupdate.diff()
        covid_new_cases.rename(columns = {'Cumulative_Cases':'New_Cases'},__
    →inplace=True)
        covid_new_cases.dropna(inplace=True)
          covid_new_cases
       daily_new_cases = covid_new_cases['New_Cases']
       return daily_new_cases
[4]: # Sanity checks to make sure you have imported the correct function - no points
    \rightarrow awarded
   stu_ans = load_data()
   assert isinstance(stu_ans, pd.Series), "QO: Your function should return a pd.
    ⇔Series. "
   assert len(stu ans) == 212, "Q0: The length of the series returned is incorrect.
   assert isinstance(stu_ans.index, pd.DatetimeIndex), "Q0: The index of your_
    ⇔series must be a pd.DatetimeIndex. "
```

```
assert (("2020-01-23" \leq stu_ans.index) & (stu_ans.index \leq "2020-08-21")).
 →all(), "Q0: The index of your series contains an incorrect time range. "
assert not stu_ans.isna().any(), "QO: Your series contains NaN values. "
assert np.issubdtype(stu_ans.dtype, np.floating), "QO: Your series should have_
→a float dtype. "
del stu_ans
```

Question 1: Stationarity Tests (20 pts)

Let's first try to understand whether our time series is stationary or not. Recall that a stationary time series has stable statistics, such as constant means and variances, over time. A non-stationary time series would not be very interesting to study, as it is essentially equivalent to a white noise, carrying little information.

1.1.1 Question 1a (15 pts)

One way of determining stationarity is to calculate some summary statistics. A rolling mean and a rolling standard deviation are the mean and the standard deviation over a rolling window of values. They both have the same length as the original time series. For a rolling window of size k, the j-th component of the rolling mean μ and the rolling standard deviation σ is precisely defined

$$\mu_{j} = \frac{1}{\min(k, j)} \sum_{i=\max(1, i-k+1)}^{j} x_{i} = \begin{cases} \frac{1}{j} \sum_{i=1}^{j} x_{i} & \text{if } j \leq k \\ \frac{1}{k} \sum_{i=i-k+1}^{j} x_{i} & \text{if } j > k \end{cases}$$
 (1)

$$\mu_{j} = \frac{1}{\min(k,j)} \sum_{i=\max(1,j-k+1)}^{j} x_{i} = \begin{cases} \frac{1}{j} \sum_{i=1}^{j} x_{i} & \text{if } j \leq k \\ \frac{1}{k} \sum_{i=j-k+1}^{j} x_{i} & \text{if } j > k \end{cases}$$

$$\sigma_{j} = \sqrt{\frac{1}{\min(k,j)}} \sum_{i=\max(1,j-k+1)}^{j} (x_{i} - \mu_{j})^{2} = \begin{cases} \sqrt{\frac{1}{j} \sum_{i=1}^{j} (x_{i} - \mu_{j})^{2}} & \text{if } j \leq k \\ \sqrt{\frac{1}{k} \sum_{i=j-k+1}^{j} (x_{i} - \mu_{j})^{2}} & \text{if } j > k \end{cases}$$

$$(2)$$

where $i \geq 1$.

Complete the function below that takes as input a time series and that calculates the rolling mean and the rolling standard deviation of the input time series. The size of the rolling window is governed by the argument wd_size.

This function should return a tuple of length 2, whose first component is the rolling mean as a np.ndarray and whose last component is the rolling standard deviation as a np.ndarray.

```
[5]: import math
    def calc rolling stats(ser, wd size=7):
        Takes in a series and returns the rolling mean and the rolling std for a_{\sqcup}
     →window of size wd size
        11 11 11
        # YOUR CODE HERE
        rolling_mean = []
        rolling_std = []
```

```
for j in range(1,len(ser)+1):
    if j < wd_size:</pre>
        short_wd = ser[:j]
        #Get rolling mean for window size short_wd
        accum_vals_mean = []
        for i in range(len(short wd)):
            accum_vals_mean.append(short_wd[i])
        new_val_mean = sum(accum_vals_mean)/len(short_wd)
        rolling_mean.append(new_val_mean)
        #Get rolling std for windown size short_wd
        accum_vals_std = []
        for i in range(len(short_wd)):
            accum_vals_std.append((short_wd[i]-new_val_mean)**2)
        new_val_std = math.sqrt(sum(accum_vals_std)/len(short_wd))
        rolling_std.append(new_val_std)
    else:
        full_wd = ser[j-wd_size:j]
        #Get rolling mean for full window sieze full_wd
        accum_vals_mean = []
        for i in range(len(full_wd)):
            accum_vals_mean.append(full_wd[i])
        new_val_mean = sum(accum_vals_mean)/len(full_wd)
        rolling_mean.append(new_val_mean)
        #Get rolling std for windown size full_wd
        accum_vals_std = []
        for i in range(len(full_wd)):
            accum_vals_std.append((full_wd[i]-new_val_mean)**2)
        new_val_std = math.sqrt(sum(accum_vals_std)/len(full_wd))
        rolling_std.append(new_val_std)
```

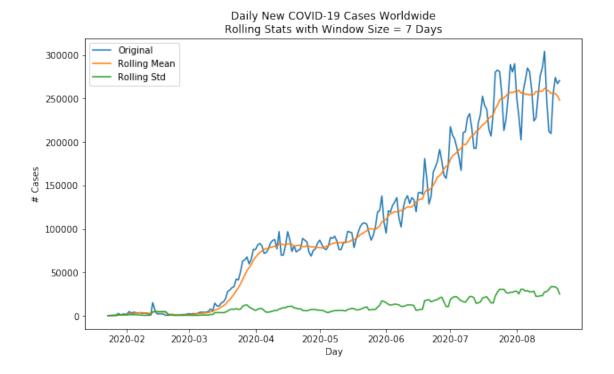
```
rolling_mean, rolling_std = np.asarray(rolling_mean), np.
     →asarray(rolling_std)
       return rolling_mean, rolling_std
[6]: # Autograder tests
   stu_ser, wd_size = load_data(), 7
   stu_ans = calc_rolling_stats(stu_ser, wd_size)
   assert isinstance(stu_ans, tuple), "Q1a: Your function should return a tuple. "
   assert len(stu_ans) == 2, "Q1a: The length of the tuple returned is incorrect. "
   assert isinstance(stu ans[0], np.ndarray), "Q1a: Please return the rolling mean"
    →as np.ndarray. "
   assert isinstance(stu_ans[1], np.ndarray), "Q1a: Please return the rolling stdu
    →as np.ndarray. "
   assert len(stu_ans[0]) == len(stu_ser), "Q1a: Your rolling mean should be of⊔
    →the same length as your data. "
   assert len(stu_ans[1]) == len(stu_ser), "Q1a: Your rolling std should be of the_
    ⇒same length as your data. "
   assert np.issubdtype(stu_ans[0].dtype, np.floating), "Q1a: Your rolling mean⊔
    ⇒should have a float dtype. "
   assert np.issubdtype(stu_ans[1].dtype, np.floating), "Q1a: Your rolling std⊔
    ⇒should have a float dtype. "
   # Some hidden tests
   del stu_ans, stu_ser, wd_size
```

Let's plot and see the rolling statistics together with the original time series. Is our time series stationary? Why or why not?

```
[7]: # Let's plot and see the rolling statistics

ser, wd_size = load_data(), 7
rolling_mean, rolling_std = calc_rolling_stats(ser, wd_size)

fig, ax = plt.subplots(figsize=(10, 6))
ax.plot(ser, label="Original")
ax.plot(pd.Series(rolling_mean, index=ser.index), label="Rolling Mean")
```



1.1.2 **Question 1b (5 pts)**

Now let's see whether the *log return* of our time series is stationary. Complete the function below that computes the log return of a given time series and that returns the result as a pd.Series like the following:

```
2020-08-19 0.072750
2020-08-20 -0.026456
2020-08-21 0.013266
Length: 211, dtype: float64
```

where * the index of the series is a pd.DatetimeIndex; * the values of the series are the log returns; and * the series doesn't contain any NaN values.

This function should return a pd. Series, whose index is a pd. DatetimeIndex.

```
[8]: def calc_log_ret(ser):
        11 11 11
        Takes in a series and computes the log return
          log_ret = ser.apply(lambda x: (np.log(x) - np.log(x - 1)) if x != ser[0])
       log_ret = pd.Series([np.log(ser[i]) - np.log(ser[i-1]) for i in_
    →range(1,len(ser))], index = ser.index[1:])
       print(log_ret)
        # YOUR CODE HERE
       return log_ret
[9]: # Autograder tests
   stu_ser = load_data()
   stu_ans = calc_log_ret(stu_ser)
   assert isinstance(stu_ans, pd.Series), "Q1b: Your function should return a pd.

→Series. "
   assert len(stu_ans) == len(stu_ser) - 1, "Q1b: The length of the series⊔
    ⇔returned should be one less than that of your data. "
   assert isinstance(stu ans.index, pd.DatetimeIndex), "Q1b: The index of your ...
    ⇒series must be a pd.DatetimeIndex. "
   assert (("2020-01-24" <= stu ans.index) & (stu ans.index <= "2020-08-21")).
    →all(), "Q1b: The index of your series contains an incorrect time range. "
   assert not stu ans.isna().any(), "Q1b: Your series contains NaN values."
   assert np.issubdtype(stu_ans.dtype, np.floating), "Q1b: Your series should have⊔
    →a float dtype. "
    # Some hidden tests
   del stu_ans, stu_ser
   Date_Time
```

This time let's plot and see the rolling statistics together with the log returns. Are the log returns of our time series stationary? Why or why not?

```
[10]: # Let's plot and see the rolling statistics

log_ret, wd_size = calc_log_ret(load_data()), 7

rolling_mean, rolling_std = calc_rolling_stats(log_ret, wd_size)

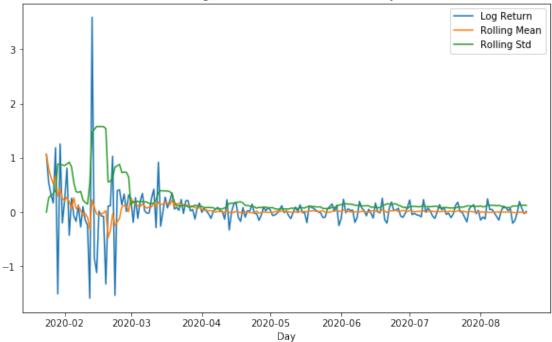
fig, ax = plt.subplots(figsize=(10, 6))
ax.plot(log_ret, label="Log Return")
ax.plot(pd.Series(rolling_mean, index=log_ret.index), label="Rolling Mean")
ax.plot(pd.Series(rolling_std, index=log_ret.index), label="Rolling Std")

ax.set_xlabel("Day")
ax.set_title("Log Return of Daily New COVID-19 Cases Worldwide\n" + f"Rolling_\to\setminus \setminus Stats with Window Size = {wd_size} Days")
ax.legend()

del fig, ax, log_ret, wd_size, rolling_mean, rolling_std
```

```
Date_Time
2020-01-24
             1.064362
2020-01-25
             0.541027
2020-01-26
             0.327449
2020-01-27
             0.167841
2020-01-28
             1.186893
2020-08-17
            -0.013336
2020-08-18
             0.196096
2020-08-19
             0.072750
2020-08-20
            -0.026456
2020-08-21
             0.013266
Length: 211, dtype: float64
```





Yet another way of determining stationarity would be to use a statistical test, such as the Augmented Dickey-Fuller unit root test. The null hypothesis is usually that the time series is non-stationary. A *p*-value less than 0.05 would lead to the conclusion that the time series is stationary, although some scientists have rised up against this magic numer!

```
[11]: # An example of performing an Augmented Dickey-Fuller unit root test

from statsmodels.tsa.stattools import adfuller

_, pval, *_ = adfuller(load_data())
print(f"p-value: {pval}")

del adfuller, pval
```

p-value: 0.67658525115441

1.2 Question 2: Autocorrelations (10 pts)

Observations in a time series are often not isolated but rather correlated. That is, there might be a correlation between an observation y_t and another observation y_{t-k} that is k time steps (or lags) earlier. (Partial) autocorrelations precisely capture this idea.

1.2.1 Question 2a (5 pts)

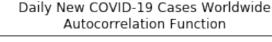
Let's see a plot of the ACF.

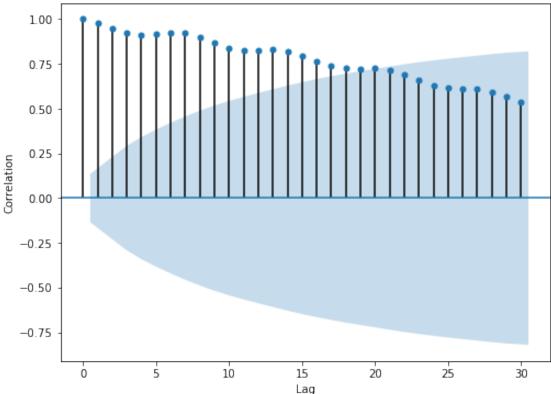
[14]: from statsmodels.graphics.tsaplots import plot_acf

Complete the function below to calculate the **Autocorrelation Function (ACF)** of the input time series, with the maximum number of lags to consider specified by the parameter max_lag. You may use the acf function from the statsmodels library.

This function should return a np.ndarray of length max_lag + 1.

```
[12]: from statsmodels.tsa.stattools import acf, pacf
     def calc_acf(ser, max_lag):
         Takes a series and calculates the ACF
         # YOUR CODE HERE
         ans_acf = acf(ser, nlags = max_lag)
         print(len(ans_acf))
         print(ans_acf)
         return ans_acf
[13]: # Autograder tests
     stu_ser, max_lag = load_data(), 30
     stu_ans = calc_acf(stu_ser, max_lag)
     assert isinstance(stu_ans, np.ndarray), "Q2a: Your function should return a np.
     →ndarray. "
     assert len(stu_ans) == max_lag + 1, "Q2a: The length of the ACF returned is⊔
     →incorrect. "
     assert np.issubdtype(stu_ans.dtype, np.floating), "Q2a: Your np.ndarray should⊔
     ⇔have a float dtype. "
     # Some hidden tests
     del stu_ans, stu_ser, max_lag
    31
                0.97535964 0.94724559 0.92385736 0.91386313 0.9167115
    Γ1.
     0.92474083 0.92029816 0.89570334 0.86531167 0.83986903 0.82678187
     0.82650348 0.82935306 0.82125569 0.79719604 0.76575177 0.74163109
     0.72568749 0.72337754 0.72382142 0.71434956 0.68860387 0.65799162
     0.63121293 0.61553004 0.61048868 0.60786787 0.59472779 0.56872414
     0.538462771
```





1.2.2 **Question 2b (5 pts)**

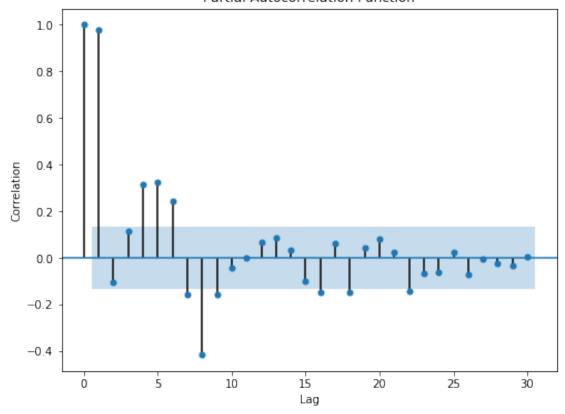
Complete the function below to calculate the **Partial Autocorrelation Function (PACF)** of the input time series, with the maximum number of lags to consider specified by the parameter max_lag. You may use the pacf function from the statsmodels library.

This function should return a np.ndarray of length max_lag + 1.

```
[15]: def calc_pacf(ser, max_lag):
    """
```

Let's see a plot of the PACF.

Daily New COVID-19 Cases Worldwide Partial Autocorrelation Function



1.3 Question 3: ARMA on Log Returns (10 pts)

Complete the function below that fits an ARMA(p,q) model on the **log return** of an input series. Your function should return a multi-day forecast in the original data space (i.e., the number of daily new cases globally) starting from 2020-08-22. For example, if num_forecasts=20, your function should return a pd.Series similar to

```
2020-08-22 239936.746954

2020-08-23 237307.407386

2020-08-24 240073.408295

...

2020-09-08 279778.977067

2020-09-09 307210.157343

2020-09-10 305203.431533
```

Freq: D, Name: predicted_mean, dtype: float64

where * the index of the series is a pd.DatetimeIndex; * the values of the series are the forecasted daily new cases; and * the series doesn't contain any NaN values.

This question is graded on the Root Mean Sqaure Error (RMSE) of your forecasts. You have complete freedom in how you'd like to implement the function, but one recommended API to

use is the ARIMA class from the statsmodels library. Why do we recommend ARIMA, when the question actually asks for a ARMA(p, q) model? Hopefully you'll find it out while working on the implementation!

This function should return a pd.Series of length num_forecasts, whose index is a

```
pd.DatetimeIndex.
 []:
[31]: from statsmodels.tsa.arima.model import ARIMA
     def arma_log_ret(ser, p, q, num_forecasts):
         Takes a series and fits an ARMA(p, q) model on log return.
         Returns a number of forecasts as specified by num forecasts.
         # YOUR CODE HERE
         model = ARIMA(ser, order = (p, 0, q))
         fit = model.fit()
         forecasts = fit.forecast(steps = num_forecasts)
         return forecasts
[32]: test3 = arma_log_ret(load_data(), 7, 7, 20)
    /opt/conda/lib/python3.7/site-packages/statsmodels/base/model.py:568:
    ConvergenceWarning: Maximum Likelihood optimization failed to converge. Check
    mle_retvals
```

ConvergenceWarning)

```
[32]: 2020-08-22
                   241043.375514
     2020-08-23
                   206304.507850
     2020-08-24
                   208371.721607
     2020-08-25
                   241506.101457
     2020-08-26
                   260975.128654
     2020-08-27
                   271461.938620
     2020-08-28
                   269353.195719
     2020-08-29
                   241718.980512
     2020-08-30
                   205894.069500
     2020-08-31
                   209840.019282
     2020-09-01
                   239518.611886
     2020-09-02
                   260496.912092
     2020-09-03
                   270234.552301
     2020-09-04
                   269478.643678
     2020-09-05
                   240921.633802
     2020-09-06
                   206845.999158
     2020-09-07
                   210068.445739
     2020-09-08
                   238568.245742
     2020-09-09
                   259263.322474
```

2020-09-10 269587.394470 Freq: D, Name: predicted mean, dtype: float64

```
[33]: # Autograder tests
     stu_ser = load_data()
     p, q, num\_forecasts = 7, 7, 20
     stu_ans = arma_log_ret(stu_ser, p, q, num_forecasts)
     assert isinstance(stu ans, pd.Series), "Q3: Your function should return a pd.
     ⇔Series. "
     assert len(stu_ans) == num_forecasts, "Q3: The length of the series returned is_
     ⇒incorrect. "
     assert isinstance(stu ans.index, pd.DatetimeIndex), "Q3: The index of your |
     ⇔series must be a pd.DatetimeIndex. "
     assert (("2020-08-22" \leq stu_ans.index) & (stu_ans.index \leq "2020-09-10")).
     →all(), "Q3: The index of your series contains an incorrect time range. "
     assert not stu_ans.isna().any(), "Q3: Your series contains NaN values. "
     assert np.issubdtype(stu_ans.dtype, np.floating), "Q3: Your series should have_
     →a float dtype. "
     # Some hidden tests
     del stu_ser, stu_ans, p, q, num_forecasts
```

/opt/conda/lib/python3.7/site-packages/statsmodels/base/model.py:568:
ConvergenceWarning: Maximum Likelihood optimization failed to converge. Check
mle_retvals
 ConvergenceWarning)

Now let's plot and compare the original time series, your forecasts and the ground-truth values of your forecasts.

```
[34]: ser = load_data()
    p, q, num_forecasts = 7, 7, 20

forecasts = arma_log_ret(ser, p, q, num_forecasts)
    actual = pd.read_pickle("assets/actual.pkl")
    rmse = np.sqrt(np.mean((actual - forecasts) ** 2))

fig, ax = plt.subplots(figsize=(10, 6))
    ax.plot(ser, label="Original")
    ax.plot(ser[-1:].append(forecasts), label="Forecasted")
    ax.plot(ser[-1:].append(actual), label="Actual")

ax.set_xlabel("Day")
```

```
ax.set_title("Daily New COVID-19 Cases Worldwide\n" + f"A {len(forecasts)}-day

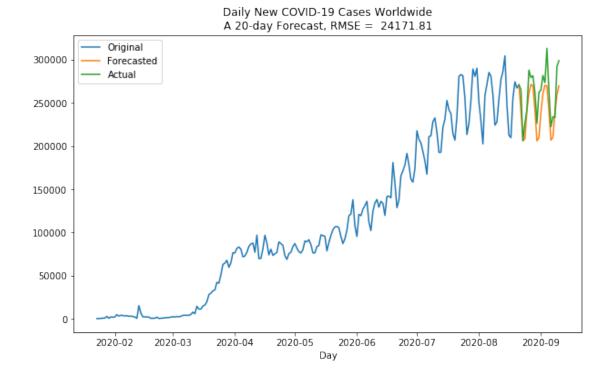
→Forecast, RMSE = {rmse: .2f}")

ax.legend()

del fig, ax, ser, p, q, num_forecasts, forecasts, actual
```

/opt/conda/lib/python3.7/site-packages/statsmodels/base/model.py:568:
ConvergenceWarning: Maximum Likelihood optimization failed to converge. Check
mle_retvals

ConvergenceWarning)



1.4 Question 4: ARMA on First-order Differences (10 pts)

Complete the function below that fits an ARMA(p,q) model on the **first-order differences** of an input series. Your function should return a multi-day forecast in the original data space (i.e., the number of daily new cases globally) starting from 2020-08-22. For example, if num_forecasts=20, your function should return a pd. Series similar to

| 2020-08-22 | 242994.084820 |
|------------|---------------|
| 2020-08-23 | 205194.792913 |
| 2020-08-24 | 201803.644029 |
| | |
| 2020-09-08 | 214574.419936 |

```
2020-09-09 243506.281330
2020-09-10 253847.751339
Freq: D, Name: predicted_mean, dtype: float64
```

where * the index of the series is a pd.DatetimeIndex; * the values of the series are the forecasted daily new cases; and * the series doesn't contain any NaN values.

This question is graded on the Root Mean Sqaure Error (RMSE) of your forecasts. You have complete freedom in how you'd like to implement the function, but one recommended API to use is the ARIMA class from the statsmodels library. Why do we recommend ARIMA, when the question actually asks for a ARMA(p,q) model? Again, hopefully you'll find it out while working on the implementation!

This function should return a pd.Series of length num_forecasts, whose index is a pd.DatetimeIndex.

```
[35]: from statsmodels.tsa.arima.model import ARIMA
     def arma_first_diff(ser, p, q, num_forecasts):
         Takes a series and fits an ARMA(p, q) model on first-order diff.
         Returns a number of forecasts as specified by num_forecasts.
         model = ARIMA(ser, order = (p, 0, q))
         fit = model.fit()
         forecasts = fit.forecast(steps = num forecasts)
         return forecasts
[36]: # Autograder tests
     stu_ser = load_data()
     p, q, num_forecasts = 7, 7, 20
     stu_ans = arma_first_diff(stu_ser, p, q, num_forecasts)
     assert isinstance(stu_ans, pd.Series), "Q4: Your function should return a pd.
     →Series. "
     assert len(stu_ans) == num_forecasts, "Q4: The length of the series returned is_
      →incorrect. "
     assert isinstance(stu_ans.index, pd.DatetimeIndex), "Q4: The index of youru
     ⇒series must be a pd.DatetimeIndex. "
     assert (("2020-08-22" <= stu_ans.index) & (stu_ans.index <= "2020-09-10")).
     →all(), "Q4: The index of your series contains an incorrect time range. "
     assert not stu ans.isna().any(), "Q4: Your series contains NaN values."
     assert np.issubdtype(stu_ans.dtype, np.floating), "Q4: Your series should have⊔
     →a float dtype. "
     # Some hidden tests
```

```
del stu_ser, stu_ans, p, q, num_forecasts
```

/opt/conda/lib/python3.7/site-packages/statsmodels/base/model.py:568:
ConvergenceWarning: Maximum Likelihood optimization failed to converge. Check
mle_retvals
 ConvergenceWarning)

Now let's plot and compare the original time series, your forecasts and the ground-truth values of your forecasts. How does this compare with the one trained on log returns?

/opt/conda/lib/python3.7/site-packages/statsmodels/base/model.py:568: ConvergenceWarning: Maximum Likelihood optimization failed to converge. Check mle_retvals

ConvergenceWarning)



