E Q U A T I O N S Alastair Horn

1. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Supposing that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = f(y),$$

find an equation for $y(x)$ in terms of f.

2. Solve the differential equation for y in terms of x:

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x \frac{\mathrm{d}y}{\mathrm{d}x} - y = 0$$

3. (2009/10 British Mathematical Olympiad Round 1)
Find all functions $f : \mathbb{R} \to \mathbb{R}$, which satisfy the equation $f(x)f(y) = f(x+y) + xy$ for all real numbers x and y.

4. (2008/9 British Mathematical Olympiad Round 2) Find all functions $f : \mathbb{R} \to \mathbb{R}$, which satisfy the equation

$$f(x^3) + f(y^3) = (x+y)(f(x^2) + f(y^2) - f(xy))$$

for all real numbers x and y.