

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be continuous. Supposing that

$$\frac{d^2y}{dx^2} = f(y),$$

find an equation for $y(x)$ in terms of f .

2. Solve the differential equation for y in terms of x :

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$$

3. (2009/10 British Mathematical Olympiad Round 1)

Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$, which satisfy the equation $f(x)f(y) = f(x+y) + xy$ for all real numbers x and y .

4. (2008/9 British Mathematical Olympiad Round 2)

Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$, which satisfy the equation

$$f(x^3) + f(y^3) = (x+y)(f(x^2) + f(y^2) - f(xy))$$

for all real numbers x and y .