

PROBLEM SET 0

CODE APPENDIX

1 Part 0: Logit Inclusive Value

Define the (log-sum-exp) inclusive value

$$\text{IV}(x) = \log\left(\sum_{i=1}^N e^{x_i}\right), \quad x \in \mathbb{R}^N.$$

Let $S = \sum_{j=1}^N e^{x_j}$ and $p_i = e^{x_i}/S$. Then

$$\nabla \text{IV}(x) = p \quad \text{and} \quad \nabla^2 \text{IV}(x) = \text{diag}(p) - pp^\top.$$

For any $v \in \mathbb{R}^N$,

$$v^\top \nabla^2 \text{IV}(x) v = \sum_{i=1}^N p_i v_i^2 - \left(\sum_{i=1}^N p_i v_i\right)^2 = \text{Var}_p(V) \geq 0,$$

where V takes value v_i with probability p_i . Hence $\nabla^2 \text{IV}(x)$ is positive semidefinite and IV is convex.