Stefano Sperti
September 9, 2025

CHICAGO BOOTH
The University of Chicago Booth School of Business

Problem set 0
Code Appendix

---

1 Part 0: Logit Inclusive Value

Define the (log-sum-exp) inclusive value

$$\mathrm{IV}(x) \;=\; \log\!\left(\sum_{i=1}^{N} e^{x_i}\right), \qquad x \in \mathbb{R}^N.$$

Let $S = \sum_{j=1}^{N} e^{x_j}$ and $p_i = e^{x_i}/S$. Then

$$\nabla \mathrm{IV}(x) = p \quad \text{and} \quad \nabla^2 \mathrm{IV}(x) = \mathrm{diag}(p) - pp^\top.$$

For any $v \in \mathbb{R}^N$,

$$v^\top \nabla^2 \mathrm{IV}(x)\, v = \sum_{i=1}^{N} p_i v_i^2 - \Big(\sum_{i=1}^{N} p_i v_i\Big)^2 = \mathrm{Var}_p(V) \;\geq\; 0,$$

where V takes value v_i with probability p_i. Hence $\nabla^2 \mathrm{IV}(x)$ is positive semidefinite and IV is convex.

2 Part 2: Numerical integration

2.7

The following tables present the results obtained from the code.

Method	Value	Abs. error	Rel. error	N points
QuadGK, True Value	0.555939	-	-	—
MC 200	0.541772	0.014167	2.548265	200
MC 400	0.543203	0.012736	2.290979	400
GH 4	0.555916	2.3e-5	0.004225	4
GH 8	0.555939	0.0	1.1e-5	8
GH 12	0.555939	0.0	0.0	12

Table 1: 1D integration results: value, error, and number of points.

Method	Value	Abs. error	Rel. error	N points
QuadGK, True Value	0.725886	-	-	-
MC 200	0.703327	0.022559	3.107809	200
MC 400	0.711449	0.014437	1.988858	400

Table 2: 2D integration results: value, error, and number of points.