13) 1b) 7) 20) 6a)

13)

$$P := \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$hyperbel(x) := \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \xrightarrow{solve, y} \begin{bmatrix} \frac{-\sqrt{a^2 \cdot b^2 \cdot x^2 - a^4 \cdot b^2}}{a^2} \\ \frac{\sqrt{a^2 \cdot b^2 \cdot x^2 - a^4 \cdot b^2}}{a^2} \end{bmatrix}$$

$$y_{aysmptote}(x) := \frac{1}{2} \cdot x$$

$$kreis(x) := x^2 + y^2 = r^2 \xrightarrow{solve, y} \begin{bmatrix} \sqrt{-x^2 + r^2} \\ -\sqrt{-x^2 + r^2} \end{bmatrix}$$

$$r := kreis(-3)_0 = 1 \xrightarrow{solve, r} \begin{bmatrix} \sqrt{10} \\ -\sqrt{10} \end{bmatrix}$$

$$hyperbel(x) := \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \xrightarrow{solve, y} \begin{bmatrix} \frac{-\sqrt{a^2 \cdot b^2 \cdot x^2 - a^4 \cdot b^2}}{a^2} \\ \frac{\sqrt{a^2 \cdot b^2 \cdot x^2 - a^4 \cdot b^2}}{a^2} \end{bmatrix}$$

Lösung Kreis:

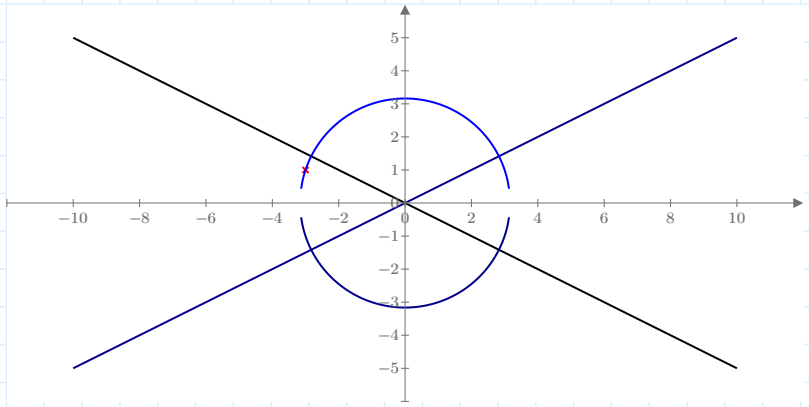
$$kreis(x) := x^2 + y^2 = 10 \xrightarrow{solve, y} \begin{bmatrix} \sqrt{-x^2 + 10} \\ -\sqrt{-x^2 + 10} \end{bmatrix}$$

Lösung Hyperbel:

$$[a \;\; b] := \begin{bmatrix} \frac{b}{a} = \frac{1}{2} \\ \frac{(-3)^2}{a^2} - \frac{1^2}{b^2} = 1 \end{bmatrix} \begin{matrix} solve, a, b \\ assume, a > 0 \\ assume, b > 0 \end{matrix} \rightarrow \begin{bmatrix} \sqrt{5} & \frac{\sqrt{5}}{2} \end{bmatrix}$$

$$hyperbel := \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow \frac{4 \cdot y^2}{5} + \frac{x^2}{5} = 1$$

$$hyperbel := x^2 - 4 \cdot y^2 = 5$$



$y_{aysmptote}(x)$

$-y_{aysmptote}(x)$

P_1

$kreis(x)_0$

$kreis(x)_1$

x x P_0 x

7)

$ellipse(x) := 4 \cdot x^2 + 9 \cdot y^2 = 36 \xrightarrow{solve,y} \begin{bmatrix} \frac{\sqrt{-(36 \cdot x^2) + 324}}{9} \\ \frac{-\sqrt{-(36 \cdot x^2) + 324}}{9} \end{bmatrix}$

$a := \sqrt{\frac{36}{4}} \to 3 \qquad b := \sqrt{\frac{36}{9}} \to 2$

$g(x) := 4\ x + 3\ y = 12 \xrightarrow{solve,y} \frac{-(4 \cdot x) + 12}{3}$

$ellipse(x)_0 = g(x) \xrightarrow{solve,x} \begin{bmatrix} \frac{9}{5} \\ 3 \end{bmatrix}$

$g\left(\frac{9}{5}\right) \to \frac{8}{5} \qquad g(3) \to 0$

clear (k, d)

$S_1 := \begin{bmatrix} \frac{9}{5} \\ g\left(\frac{9}{5}\right) \end{bmatrix} \to \begin{bmatrix} \frac{9}{5} \\ \frac{8}{5} \end{bmatrix} \qquad S_2 := \begin{bmatrix} 3 \\ g(3) \end{bmatrix} \to \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$f(x) := x + 2 \cdot y = 5 \xrightarrow{solve,y} \frac{-x + 5}{2}$

Tangente 1:

$x_t := 3$

Tangente 2:

$t(x) := k \cdot x + d$

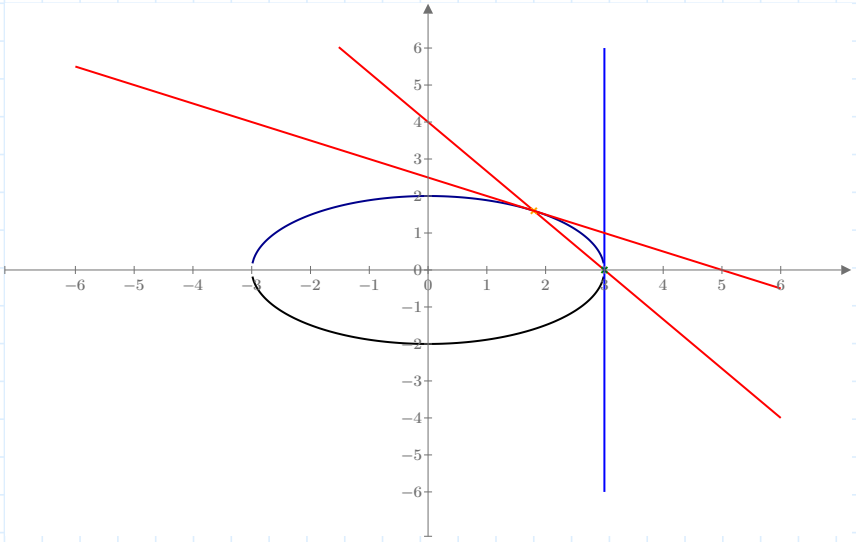
$k'(x) := \frac{d}{dx} ellipse(x)_0 \rightarrow \frac{-(2 \cdot x)}{3 \cdot \sqrt{-x^2 + 9}}$

$t'(x) := \frac{d}{dx} t(x) \rightarrow k$

$k := k'\left(\frac{9}{5}\right) \rightarrow -\frac{1}{2}$

$d := 3^2 \cdot k^2 + 2^2 = d^2 \xrightarrow{simplify} \frac{25}{4} = d^2 \xrightarrow{solve, d} \begin{bmatrix} \frac{5}{2} \\ -\frac{5}{2} \end{bmatrix}$

$t(x) := k \cdot x + d_0 \xrightarrow{simplify} \frac{-x + 5}{2}$



$ellipse(x)_0$

$ellipse(x)_1$

$g(x)$

S_{1_1}

S_{2_1}

y

$t(x)$

x x x S_{1_0} S_{2_0} 3

x

1b)

$g(x) := -x + 3 \quad y - 8 = 0 \xrightarrow{solve,\,y} \frac{x+8}{3}$

$\mathbf{clear}\,(d)$

$kreis(x) := (x-0.5)^2 + (y-2.5)^2 = \left(\sqrt{2.5}\right)^2 \xrightarrow{solve,\,y} \begin{bmatrix} (-1.0\cdot x^2 + x + 2.25)^{0.5} + 2.5 \\ -1.0\cdot(-1.0\cdot x^2 + x + 2.25)^{0.5} + 2.5 \end{bmatrix}$

$k_x := kreis(x)_0 = g(x) \xrightarrow{solve,\,x} 1.8696938456699068589 \quad clear(a, t_1, t_2, b, c, d, k, d)$

$k_{x1} := kreis(x)_1 = g(x) \xrightarrow{solve,\,x} -1.0696938456699068589$

$kreis'(x) := \frac{d}{dx} kreis(x)_0 \rightarrow \frac{-1.0\cdot x + 0.5}{(-1.0\cdot x^2 + x + 2.25)^{0.5}}$

$kreis_1'(x) := \frac{d}{dx} kreis(x)_0 \rightarrow \frac{-1.0\cdot x + 0.5}{(-1.0\cdot x^2 + x + 2.25)^{0.5}}$

$k_t := kreis'(k_x) \rightarrow -1.734013676289095869$

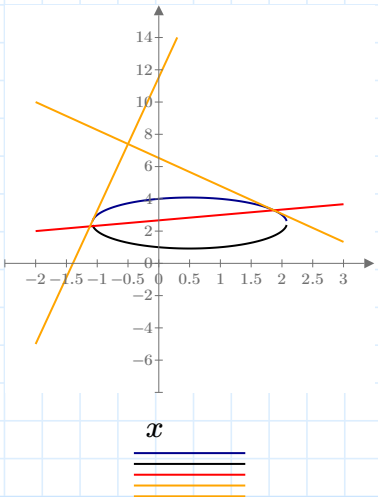
$k_{t1} := kreis_1'(k_{x1}) \rightarrow 8.2659863237109041242$

$d := \left(\sqrt{2.5}\right)^2 \cdot \left(k_t^{\,2} + 1\right) = (k_t\cdot 0.5 - 2.5 + d)^2 \xrightarrow{solve,\,d} \begin{bmatrix} 0.20204102886728760727 \\ 6.5319726474218082617 \end{bmatrix}$

$d1 := \left(\sqrt{2.5}\right)^2 \cdot \left(k_{t1}^{\,2} + 1\right) = (k_{t1}\cdot 0.5 - 2.5 + d1)^2 \xrightarrow{solve,\,d1} \begin{bmatrix} -14.797958971132712379 \\ 11.531972647421808255 \end{bmatrix}$

$t_1(x) := k_t\cdot x + d_1 \rightarrow -1.734013676289095869\cdot x + 6.5319726474218082617$

$t_2(x) := k_{t1}\cdot x + d1_1 \rightarrow 8.2659863237109041242\cdot x + 11.531972647421808255$



$kreis(x)_0$

$kreis(x)_1$

$g(x)$

$t_1(x)$

$t_2(x)$

$$\text{clear}(a, b, c, d, ellipse, e, P)$$

6a)

$$F_1 := \begin{bmatrix} -8 \\ 0 \end{bmatrix} \quad F_2 := \begin{bmatrix} 8 \\ 0 \end{bmatrix} \quad P := \begin{bmatrix} -6 \\ 4.8 \end{bmatrix}$$

$$ellipse(x) := \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \xrightarrow{solve, y} \begin{bmatrix} \frac{\sqrt{-(a^2 \cdot b^2 \cdot x^2) + a^4 \cdot b^2}}{a^2} \\ \frac{-\sqrt{-(a^2 \cdot b^2 \cdot x^2) + a^4 \cdot b^2}}{a^2} \end{bmatrix}$$

$$[a \;\; b] := \begin{bmatrix} \frac{(-6)^2}{a^2} + \frac{48^2}{10^2 \;\; b^2} = 1 \\ a^2 - b^2 = 8^2 \end{bmatrix} \begin{array}{l} solve, a, b \\ assume, a = real \\ assume, a > 0 \\ assume, b > 0 \end{array} \longrightarrow [10 \;\; 6]$$

$$ellipse(x) := \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow \frac{y^2}{36} + \frac{x^2}{100} = 1$$

20)

$$parabel(x) := y^2 = 4 \cdot x \xrightarrow{solve, y} \begin{bmatrix} \sqrt{4 \cdot x} \\ -\sqrt{4 \cdot x} \end{bmatrix}$$

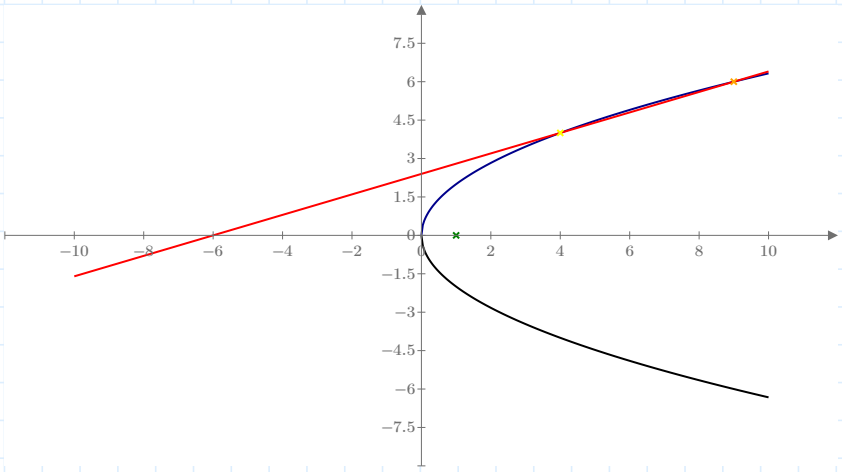
$$g(x) := 2 \cdot x - 5 \cdot y = -12 \xrightarrow{solve, y} \frac{2 \cdot x + 12}{5}$$

$$p := 2 \;\; p = 4 \xrightarrow{solve, p} 2$$

$$parabel(x)_0 = g(x) \xrightarrow{solve, x} \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$S_1 := \begin{bmatrix} 4 \\ g(4) \end{bmatrix} \rightarrow \begin{bmatrix} 4 \\ 4 \end{bmatrix} \qquad S_2 := \begin{bmatrix} 9 \\ g(9) \end{bmatrix} \rightarrow \begin{bmatrix} 9 \\ 6 \end{bmatrix} \qquad F_1 := \begin{bmatrix} p \\ 2 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$a := S_1 + S_2 \rightarrow \begin{bmatrix} 13 \\ 10 \end{bmatrix} \qquad b := S_2 + F_1 \rightarrow \begin{bmatrix} 10 \\ 6 \end{bmatrix} \qquad c := F_1 + S_1 \rightarrow \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$



$parabel(x)_0$

$parabel(x)_1$

$g(x)$

S_{1_1}

S_{2_1}

F_{1_1}

x x x

S_{1_0} S_{2_0} F_{1_0}