Bestimmtes Integral HÜ

Bsp 6.25a)

$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$\int_0^3 1\,dx$$

$$U_n = \Delta x \sum_{i=0}^{n-1} 1 = \frac{3}{n} \cdot \sum_{i=0}^{n-1} 1 = 3$$

$$O_n = \Delta x \sum_{i=1}^{n} 1 = \frac{3}{n} \sum_{i=1}^{n} 1 = 3$$

$$\lim_{n \to \infty} 3 = 3 = U_n$$

$$\lim_{n \to \infty} 3 = 3 = O_n$$

$$\int_0^3 1\,dx = \lim_{n \to \infty} U_n = \lim_{n \to \infty} O_n = 2$$

6.25b)

$$\Delta x = \frac{4-0}{n} = \frac{4}{n}$$

$$\int_0^4 2x\,dx$$

$$U_n = \Delta x \cdot \sum_{i=0}^{n-1} f(x_i) = \frac{4}{n} \cdot \sum_{i=0}^{n-1} 2 \cdot \left(0 + \frac{4}{n} \cdot i\right)$$

$$U_n = \frac{4}{n} \sum_{i=0}^{n-1} \frac{8}{n} i = \frac{32}{n^2} \cdot \frac{(n-1) \cdot n}{2} = \frac{32n - 32}{2n}$$

$$O_n = \frac{4}{n} \cdot \sum_{i=1}^{n} 2 \cdot \left(0 + \frac{4}{n} \cdot i\right) = \frac{32}{n^2} \sum_{i=1}^{n} i = \frac{32n + 32}{2n}$$

$$\lim_{n \to \infty} U_n = \lim_{n \to \infty} \frac{32n - 32}{2n} = \frac{n\left(32 - \frac{32}{n}\right)}{2n} = 16$$

$$\lim_{n \to \infty} O_n = \lim_{n \to \infty} \frac{32n + 32}{2n} = \frac{n\left(32 + \frac{32}{n}\right)}{2n} = 16$$

$$\int_0^4 2x\,dx = \lim_{n \to \infty} U_n = \lim_{n \to \infty} O_n = 16$$