Bsp's]

26. Schulübung am 30.11.2022

Bsp] $f(x) = \frac{2x^2+5}{3x+1}$ $u = 2x^2+5$ $u' = 4x$ $v = 3x+1$ $v' = 3$ $f'(x) = \frac{u' \cdot v - v' \cdot u}{v^2}$

$f'(x) = \frac{4x \cdot (3x+1) - 3 \cdot (2x^2+5)}{(3x+1)^2} = \frac{12x^2+4x-(6x^2-15)}{} = \frac{6x^2+4x-15}{(3x+1)^2}$

4.121 c)

$f(x) = \frac{x+e^x}{x^2-e^x}$ $u = x+e^x$ $v = x^2-e^x$ $u' = e^x$ $v' = 2x-e^x$

$f'(x) = \frac{e^x \cdot (x^2-e^x) - (2x-e^x) \cdot (x+e^x)}{(x^2-e^x)} = \frac{e^x x^2 - \cancel{e^{2x}} - (2x^2+2xe^x - e^x x - \cancel{e^{2x}})}{}$

$\frac{e^x x^2 - x^2 - 2xe^x + e^x x}{(x^2-e^x)}$

4.122 d)

$y = \frac{\ln(x)}{2x}$ $u = \ln(x)$ $u' = \frac{1}{x}$ $v = 2x$ $v' = 2$

$f'(x) = \frac{2 - 2 \cdot \ln(x)}{(2x)^2} = \frac{1-\ln(x)}{2x^2}$

b) $y = \frac{x}{\ln(x)}$ $u = x$ $u' = 0$ $v = \ln(x)$ $v' = \frac{1}{x}$

$f'(x) = \frac{\ln(x) - 1}{\ln^2(x)}$

c) $y = \frac{1-\ln(x)}{x^6}$ $u = 1-\ln(x)$ $u' = -\frac{1}{x}$ $v = x^6$ $v' = 6x^5$

$f'(x) \frac{-\frac{1}{x} \cdot x^6 - 6x^5 \cdot 1 - \ln(x)}{(x^6)^2} = \frac{x^5(-1-6+\ln(x))}{x^{12}}$

$f'(x) = \frac{6 \cdot \ln(x) - 7}{x^7}$

4.126 c) , 4.128 h)

4.126 c) $y = \frac{\sin(x) - \cos(x)}{\sin(x) \cdot \cos(x)}$ $u = \sin(x) - \cos(x)$ $u' = \cos(x) + \sin(x)$ $v = \sin(x) \cdot \cos(x)$ $v' = \cos(x) \cdot \sin(x)$

$a = \sin(x)$ $a' = \cos(x)$

$b = \cos(x)$ $b' = -\sin(x)$

$v' = \cos^2(x) - \sin^2(x)$

$f'(x) = \frac{(\cos(x)+\sin(x)) \cdot \sin(x) \cdot \cos(x) +}{}$

$f'(x) = \frac{(\cos(x)+\sin(x)) \cdot \sin(x) \cdot \cos(x) - (\cos^2(x)-\sin^2(x)) \cdot (\sin(x) - \cos(x))}{\sin^2(x) \cdot \cos^2(x)}$

$\sin(x) \cdot \cos^2(x) + \sin^2(x) \cdot \cos(x)$

[illegible]

$$\cancel{x = r\cdot\left(s+\frac{r+s}{r-s}\right), \quad \frac{dx}{dr} = ? \quad \frac{dx}{ds}} \; ?$$

$$\frac{(\cos(x)+\sin(x))\cdot\sin(x)\cdot\cos(x)-(\cos^2(x)-\sin^2(x))(\sin(x)-\cos(x))}{\sin^2(x)\cdot\cos^2(x)}$$

Fertig?

$$\frac{\cancel{\cos^2(x)\cdot\sin(x)}+\sin^2(x)\cdot\cos(x)-(\cancel{\cos^2(x)\cdot\sin(x)}-\cos^3(x)-\sin^3(x)+\sin^2(x)\cdot\cos(x))}{\sin^2(x)\cdot\cos^2(x)}$$

$$f'(x)=\frac{\cos^3(x)+\sin^3(x)}{\sin^2(x)\cdot\cos^2(x)}$$

4.128b) $x = r\cdot\left(s+\frac{r+s}{r-s}\right)$ $\quad \frac{dx}{dr} = ?$

$$x = r\cdot\left(\frac{s\cdot(r-s)+(r+s)}{r-s}\right) \qquad \frac{dx}{ds}=?$$

$$x = \frac{r^2s-rs^2+r^2+rs}{r-s}$$

$u = r^2s - rs^2 + r^2 + rs$

$u' = 2rs - s^2 + 2r + s$

$$\frac{dx}{dr} = \frac{(2rs-s^2+2r+s)\cdot(r-s)-r^2s+rs^2-r^2-rs}{(r-s)^2}$$

$v = (r-s)$

$v' = -1$

$$\frac{dx}{dr} = \frac{2r^2s-2rs^2-s^2r+s^3+2r^2-2rs+sr-s^2-r^2s+rs^2-r^2-rs}{(r-s)^2}$$

$$= \frac{s^3+r^2s-s^2-2rs^2-2rs+r^2}{(r-s)^2}$$

$\frac{dx}{ds}$ für $u = r^2s - rs^2 + r^2 + rs$

$u' = r^2 - 2rs + r$

$v = r-s \qquad v' = -1$

$$\frac{(r^2-2rs+r)\cdot(r-s)-((r^2s-rs^2+r^2+rs))\cdot -1}{(r-s)^2}$$

$$\frac{r^3-\cancel{r^2s}-2r^2s+\cancel{2rs^2}+\cancel{r^2}-(rs+\cancel{r^2s}-\cancel{rs^2}+\cancel{r^2}+rs)}{(r-s)^2} = \frac{r^3-2r^2s+rs^2+2r^2}{(r-s)^2}$$