9te Hausübung am 07.11.2023

Beispiele) 4.27a)b), 4.28a)b), 4.30c), 4.33c), 4.35a) 1-3) 4.55a)b)

4.27a)

$-5 \cdot \frac{dy}{dx} + 4y = 0 \iff \frac{dy}{y} = \frac{4}{5}x \Rightarrow \int \frac{1}{y}\,dy = \int \frac{4}{5}\,dx$

$\ln|y| = \frac{4}{5}x + C \;|\; e^{\square}$

$y_h(x) = e^{\frac{4}{5}x} \cdot C$

4.27b) $8\frac{ds}{dt} - 3s = 0 \iff \frac{ds}{s} = \frac{3}{8} \Rightarrow \int \frac{ds}{s} = \int \frac{3}{8}\,dt$

$\ln|s| = \frac{3}{8}t + C \;|\; e^{\square}$

$s_h(t) = e^{\frac{3}{8}t} \cdot C$

4.28a)

$3y' + 2x \cdot y = 0 \iff 3\frac{dy}{dx} + 2 \cdot x \cdot y = 0 \Rightarrow 3\frac{dy}{dx} = -2xy \;|:y\;|\cdot dx\;|:3$

$\frac{dy}{y} = -\frac{2x}{3} \Rightarrow \int \frac{1}{y}\,dy = \int -\frac{2}{3}x\,dx$

$\ln|y| = -\frac{1}{3}x^2 + C \;|\; e^{\square}$

$y_h(x) = e^{-\frac{1}{3}x^2} \cdot C$

4.28b) $2y' - 3x^2 \cdot y = 0 \iff 2 \cdot \frac{dy}{dx} - 3x^2 \cdot y = 0$

$\frac{dy}{y} = +\frac{3}{2}x^2 dx \Rightarrow \int \frac{dy}{y} = +\frac{3}{2}\int x^2\,dx$

$\ln|y| = +\frac{1}{2} \cdot x^3 + C \;|\; e^{\square}$

$y_h(x) = e^{\frac{x^3}{2}} \cdot C$

4.30c) $3x \cdot y' - y = 0 \iff 3x \cdot \frac{dy}{dx} - y = 0 \;|+y\;|\cdot dx\;|:y\;|:3x$ $\quad y(1) = 3$

$3\int \frac{dy}{y} = \int \frac{1}{x} \Rightarrow 3\ln(|y|) = \ln|x| + c \;|\; e^{\square} \;|:3$

$\ln|y| = \ln|x^{\frac{1}{3}}| + \frac{1}{3}c \;|\; e^{\square}$

$y_h(x) = \sqrt[3]{x} \cdot C$

$3 = \sqrt[3]{1} \cdot C \iff \underline{C = 3}$

$y_h = \sqrt[3]{x} \cdot 3$

4.33c) $R \cdot C \cdot y' + y = 0 \quad R \cdot C\frac{dy}{dx} + y = 0 \iff \frac{dy}{y} = -\frac{1}{RC}\,dt$

$\int \frac{1}{y}\,dy = -\int \frac{1}{RC}\,dt$

$\ln|y| = -\frac{1}{RC} \cdot t + C \;|\; e^{\square}$

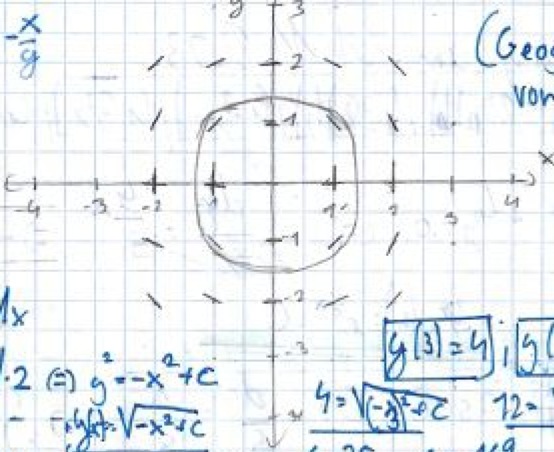
$y_h(t) = e^{-\frac{1}{RC} \cdot t} \cdot C$

4.35) $y \cdot y' + x = 0 \iff y' = \frac{-x}{y}$

(Geogebra-Lösung vorhanden)

1)

x \ y	-2	-1	0	1	2
-2	-1	-2	\	2	1
-1	$-\frac{1}{2}$	-1	\	1	$\frac{1}{2}$
0	\	\	\	\	\
1	$\frac{1}{2}$	1	\	-1	$-\frac{1}{2}$
2	1	2	\	-2	-1



2) $y \cdot \frac{dy}{dx} = -x \iff y \cdot dy = -x \cdot dx$

$\int y \, dy = \int -x \, dx \Rightarrow \frac{y^2}{2} = \frac{-x^2}{2} + C \,/\cdot 2 \iff y^2 = -x^2 + C$

$y(x) = \sqrt{-x^2+C}$

$\boxed{y(3)=4}$; $\boxed{y(5)=12}$

$4 = \sqrt{(-3)^2 + C}$ $\quad$ $12 = \sqrt{(5)^2 + C}$

$\underline{C = 25}$ $\quad$ $\underline{C = 169}$ $\quad$ $\underline{y^2 + x^2 = 25}$

$y_1(x) = \sqrt{-x^2+25}$ $\quad$ $y_2(x) = \sqrt{-x^2+169}$

3) $y^2 + x^2 = C$. Die umgeformte allg. Lösung entspricht der Kreisgleichung in Mittelpunktslage, daher ist die Grafik konzentrisch.

4.55 a) b)

a) $y' + 2y = 3e^{x}$ $\quad$ $y' + 2y = 0$ $\quad$ $\frac{dy}{dx} = -2y \iff \frac{dy}{y} = -2\,dx$

$\int \frac{1}{y} \, dy = \int -2 \, dx \iff \ln|y| = -2x + C \,/ e^{\square}$ $\quad$ $y_h(x) = e^{-2x} \cdot C$

$y(x) = e^{-2x} \cdot C(x)$

$u = e^{-2x}$ $\quad$ $u' = -2e^{-2x}$

$v = C(x)$ $\quad$ $v' = C'(x)$

$y'(x) = -2e^{-2x} \cdot C(x) + e^{-2x} \cdot C'(x)$

$\cancel{-2e^{-2x} \cdot C(x)} + e^{-2x} \cdot C'(x) + \cancel{2e^{-2x} \cdot C(x)} = 3e^{x}$

$C'(x)$ auf eine Seite

$C'(x) = 3e^{3x}$

$\int C'(x) \, dx = \int 3e^{3x} \, dx$

$C(x) = \frac{3}{3} e^{3x}$

$y(x) = y_h(x) + y_p(x) = \underline{\underline{e^{-2x} \cdot C + e^{x}}}$

4.55 b) $y' - 5y = e^{5x}$ $\quad$ $y' - 5y = 0 \iff \frac{dy}{y} = 5 \cdot dx \iff \int \frac{dy}{y} = \int 5 \, dx$

$\ln|y| = 5x + C \,/ e^{\square}$ $\quad$ $\underline{\underline{y_h(x) = e^{5x} \cdot C}}$ $\quad$ $y(x) = e^{5x} \cdot C(x)$

$y'(x) = 5 \cdot e^{5x} \cdot C(x) + C'(x) \cdot e^{5x}$

$\cancel{5 \cdot e^{5x} \cdot C(x)} + C'(x) \cdot e^{5x} \cancel{- 5 \cdot e^{5x} \cdot C} = e^{5x} \;|\; : e^{5x}$

$C'(x) = 1 \iff \int C'(x) \, dx = \int 1 \, dx$

$C(x) = x$ $\quad$ $y_p(x) = x \cdot e^{5x}$

$y(x) = y_h(x) + y_p(x) = \underline{\underline{e^{5x} \cdot C + x \cdot e^{5x}}}$

Steigungsfeld = Richtungsfeld(-(x / y), 25, 0.3, -14, -14, 14, 14)

NummerischesIntegral = Ortslinie(-(x / y), (3, 4))

NummerischesIntegral2 = Ortslinie(-(x / y), (5, 12))

Eingabe...