Kapitel 5.2.2 Beispiel 1a, 2a, 5a, 6a)
Kapitel 5.3.3 Beispiel 1)

Beispiel 1a)

$$X := \begin{bmatrix} 140 \\ 160 \\ 180 \\ 190 \\ 200 \end{bmatrix} \qquad Y := \begin{bmatrix} 10 \\ 20 \\ 45 \\ 57 \\ 70 \end{bmatrix}$$

$$n := \mathrm{rows}(X) \rightarrow 5$$

$$a_{lin}(k,d) := \sum_{i=0}^{n-1} \left(k \cdot X_i + d - Y_i\right)^2$$

$$a_{k\_lin}(k,d) := \frac{d}{dk} a_{lin}(k,d) \rightarrow 307400 \cdot k + (1740 \cdot d - 75060)$$

$$a_{d\_lin}(k,d) := \frac{d}{dd} a_{lin}(k,d) \rightarrow 1740 \cdot k + (10 \cdot d - 404)$$

$$[k \;\; d] := \begin{bmatrix} a_{k\_lin}(k,d) = 0 \\ a_{d\_lin}(k,d) = 0 \end{bmatrix} \xrightarrow{solve,k,d} \begin{bmatrix} \frac{1191}{1160} & -\frac{553}{4} \end{bmatrix}$$

$$a_{lin}(x) := k \cdot x + d \rightarrow \frac{1191 \cdot x}{1160} - \frac{553}{4}$$

$$a_{quad}(a,b,t) := \sum_{i=0}^{n-1} \left(a \cdot {X_i}^2 + b \cdot X_i + t - Y_i\right)^2$$

$$a_{a\_quad}(a,b,t) := \frac{d}{da} a_{quad}(a,b,t) \rightarrow 307400 \cdot t + 55062000 \cdot b + (9984980000 \cdot a - 14047400)$$

$$a_{b\_quad}(a,b,t) := \frac{d}{db} a_{quad}(a,b,t) \rightarrow 1740 \cdot t + 307400 \cdot b + (55062000 \cdot a - 75060)$$

$$a_{t\_quad}(a,b,t) := \frac{d}{dt} a_{quad}(a,b,t) \rightarrow 10 \cdot t + 1740 \cdot b + (307400 \cdot a - 404)$$

$$[a \;\; b \;\; t] := \begin{bmatrix} a_{a\_quad}(a,b,t) = 0 \\ a_{b\_quad}(a,b,t) = 0 \\ a_{t\_quad}(a,b,t) = 0 \end{bmatrix} \xrightarrow{solve,a,b,t} \begin{bmatrix} \frac{347}{37520} & -\frac{39609}{18760} & \frac{57912}{469} \end{bmatrix}$$

$$a_{quad}(x) := a \cdot x + b \cdot x + t \rightarrow -\frac{78871 \cdot x}{37520} + \frac{57912}{469}$$

Beispiel 2a)

$$\text{clear}\left(a, b, k, n, c, d, x, t, X, Y, a_{quad}, a_{lin}, a_{a\_quad}, a_{b\_quad}, a_{k\_lin}, a_{d\_lin}, a_{t\_quad}\right)$$

$$X := \begin{bmatrix} 140 \\ 160 \\ 180 \\ 190 \\ 200 \end{bmatrix} \qquad Y := \begin{bmatrix} 70 \\ 55 \\ 34 \\ 22 \\ 10 \end{bmatrix}$$

$$n := \text{rows}(X) \rightarrow 5$$

linear)

$$n_{lin}(k, d) := \sum_{i=0}^{n-1} \left(k \cdot X_i + d - Y_i\right)^2$$

$$n_{k\_lin}(k, d) := \frac{d}{dk} n_{lin}(k, d) \rightarrow 307400 \cdot k + (1740 \cdot d - 61800)$$

$$n_{d\_lin}(k, d) := \frac{d}{dd} n_{lin}(k, d) \rightarrow 1740 \cdot k + (10 \cdot d - 382)$$

$$\begin{bmatrix} k & d \end{bmatrix} := \begin{bmatrix} n_{k\_lin}(k, d) = 0 \\ n_{d\_lin}(k, d) = 0 \end{bmatrix} \xrightarrow{solve, k, d} \begin{bmatrix} -\frac{1167}{1160} & \frac{853}{4} \end{bmatrix}$$

$$n(x) := k \cdot x + d \rightarrow -\frac{1167 \cdot x}{1160} + \frac{853}{4}$$

quadratisch)

$$\text{clear}(n)$$

$$n := \text{rows}(X) \rightarrow 5$$

$$n_{quad}(a, b, c) := \sum_{i=0}^{n-1} \left(a \cdot {X_i}^2 + b \cdot X_i + c - Y_i\right)^2$$

$$n_{a\_quad}(a, b, c) := \frac{d}{da} n_{quad}(a, b, c) \rightarrow 307400 \cdot c + 55062000 \cdot b + (9984980000 \cdot a - 10151600)$$

$$n_{b\_quad}(a, b, c) := \frac{d}{db} n_{quad}(a, b, c) \rightarrow 1740 \cdot c + 307400 \cdot b + (55062000 \cdot a - 61800)$$

$$n_{c\_quad}(a,b,c) := \frac{d}{dc} n_{quad}(a,b,c) \rightarrow 10 \cdot c + 1740 \cdot b + (307400 \cdot a - 382)$$

$$[a \;\; b \;\; c] := \begin{bmatrix} n_{a\_quad}(a,b,c) = 0 \\ n_{b\_quad}(a,b,c) = 0 \\ n_{c\_quad}(a,b,c) = 0 \end{bmatrix} \xrightarrow{solve,a,b,c} \begin{bmatrix} -\frac{1041}{187600} & \frac{16449}{18760} & \frac{52727}{938} \end{bmatrix}$$

$$\boxed{n_{quad}}(x) := a \cdot x^2 + b \cdot x + c \rightarrow -\frac{1041 \cdot x^2}{187600} + \frac{16449 \cdot x}{18760} + \frac{52727}{938}$$

5a)

clear $(Y, X, k, d, a, b, c, n_{quad}, n_{lin}, n_{k\_lin}, n_{d\_lin}, n_{a\_quad}, n_{b\_quad}, n_{c\_quad}, n)$

clear $(a, b, k, n, c, d, x, t, X, Y, a_{quad}, a_{lin}, a_{a\_quad}, a_{b\_quad}, a_{k\_lin}, a_{d\_lin}, a_{t\_quad})$

linear)

$$X := \begin{bmatrix} 60 \\ 70 \\ 80 \\ 90 \\ 100 \end{bmatrix} \qquad Y := \begin{bmatrix} 5800 \\ 6300 \\ 7215 \\ 8327 \\ 9120 \end{bmatrix}$$

$$n := \mathrm{rows}(X) \rightarrow 5$$

$$a_{lin}(k,d) := \sum_{i=0}^{n-1} \left(k \cdot X_i + d - Y_i\right)^2$$

$$a_{k\_lin}(k,d) := \frac{d}{dk} a_{lin}(k,d) \rightarrow 66000 \cdot k + (800 \cdot d - 6055260)$$

$$a_{d\_lin}(k,d) := \frac{d}{dd} a_{lin}(k,d) \rightarrow 800 \cdot k + (10 \cdot d - 73524)$$

$$[k \;\; d] := \begin{bmatrix} a_{k\_lin}(k,d) = 0 \\ a_{d\_lin}(k,d) = 0 \end{bmatrix} \xrightarrow{solve,k,d} \begin{bmatrix} \frac{8667}{100} & \frac{2094}{5} \end{bmatrix}$$

$$\boxed{a_{lin}}(x) := k \cdot x + d \rightarrow \frac{8667 \cdot x}{100} + \frac{2094}{5}$$

quadratisch

$\mathbf{clear}\left(Y, X, k, d, a, b, c, n_{quad}, n_{lin}, n_{k\_lin}, n_{d\_lin}, n_{a\_quad}, n_{b\_quad}, n_{c\_quad}, n\right)$

$\mathbf{clear}\left(a, b, k, n, c, d, x, t, X, Y, a_{quad}, a_{lin}, a_{a\_quad}, a_{b\_quad}, a_{k\_lin}, a_{d\_lin}, a_{t\_quad}\right)$

$$X := \begin{bmatrix} 60 \\ 70 \\ 80 \\ 90 \\ 100 \end{bmatrix} \qquad Y := \begin{bmatrix} 5800 \\ 6300 \\ 7215 \\ 8327 \\ 9120 \end{bmatrix}$$

$n := \mathrm{rows}(X) \rightarrow 5$

$$a_{quad}(a, b, c) := \sum_{i=0}^{n-1} \left(a \cdot X_i^{\,2} + b \cdot X_i + c - Y_i\right)^2$$

$$a_{a\_quad}(a, b, c) := \frac{d}{da} a_{quad}(a, b, c) \rightarrow 66000 \cdot c + 5600000 \cdot b + (487080000 \cdot a - 513149400)$$

$$a_{b\_quad}(a, b, c) := \frac{d}{db} a_{quad}(a, b, c) \rightarrow 800 \cdot c + 66000 \cdot b + (5600000 \cdot a - 6055260)$$

$$a_{c\_quad}(a, b, c) := \frac{d}{dc} a_{quad}(a, b, c) \rightarrow 10 \cdot c + 800 \cdot b + (66000 \cdot a - 73524)$$

$$\begin{bmatrix} a & b & c \end{bmatrix} := \begin{bmatrix} a_{a\_quad}(a, b, c) = 0 \\ a_{b\_quad}(a, b, c) = 0 \\ a_{c\_quad}(a, b, c) = 0 \end{bmatrix} \xrightarrow{solve, a, b, c} \begin{bmatrix} \frac{783}{1400} & -\frac{1971}{700} & \frac{136023}{35} \end{bmatrix}$$

$$a_{quad}(x) := a \cdot x^2 + b \cdot x + c \rightarrow \frac{783 \cdot x^2}{1400} + \left(\frac{136023}{35} - \frac{1971 \cdot x}{700}\right)$$

kubisch

$\mathbf{clear}\left(Y, X, k, d, a, b, c, n_{quad}, n_{lin}, n_{k\_lin}, n_{d\_lin}, n_{a\_quad}, n_{b\_quad}, n_{c\_quad}, n\right)$

$\mathbf{clear}\left(a, b, k, n, c, d, x, t, X, Y, a_{quad}, a_{lin}, a_{a\_quad}, a_{b\_quad}, a_{k\_lin}, a_{d\_lin}, a_{t\_quad}\right)$

$$X := \begin{bmatrix} 60 \\ 70 \\ 80 \\ 90 \\ 100 \end{bmatrix} \qquad Y := \begin{bmatrix} 5800 \\ 6300 \\ 7215 \\ 8327 \\ 9120 \end{bmatrix}$$

$n := \mathrm{rows}(X) \rightarrow 5$

$$a_{kub}(a,b,c,d) := \sum_{i=0}^{n-1} \left(a \cdot X_i^3 + b \cdot X_i^2 + c \cdot X_i + d - Y_i\right)^2$$

$$a_{a\_kub}(a,b,c,d) := \frac{d}{da} a_{kub}(a,b,c,d) \rightarrow 5600000 \cdot d + 487080000 \cdot c + 43280000000 \cdot b + (391578000000$$

$$a_{b\_kub}(a,b,c,d) := \frac{d}{db} a_{kub}(a,b,c,d) \rightarrow 66000 \cdot d + 5600000 \cdot c + 487080000 \cdot b + (43280000000 \cdot a - 513$$

$$a_{c\_kub}(a,b,c,d) := \frac{d}{dc} a_{kub}(a,b,c,d) \rightarrow 800 \cdot d + 66000 \cdot c + 5600000 \cdot b + (487080000 \cdot a - 6055260)$$

$$a_{d\_kub}(a,b,c,d) := \frac{d}{dd} a_{kub}(a,b,c,d) \rightarrow 10 \cdot d + 800 \cdot c + 66000 \cdot b + (5600000 \cdot a - 73524)$$

$$\begin{bmatrix} a & b & c & d \end{bmatrix} := \begin{bmatrix} a_{a\_kub}(a,b,c,d) = 0 \\ a_{b\_kub}(a,b,c,d) = 0 \\ a_{c\_kub}(a,b,c,d) = 0 \\ a_{d\_kub}(a,b,c,d) = 0 \end{bmatrix} \xrightarrow{solve,a,b,c,d} \begin{bmatrix} -\frac{367}{6000} & \frac{4267}{280} & -\frac{121424}{105} & \frac{1173899}{35} \end{bmatrix}$$

$$a_{kub}(x) := a \cdot x^3 + b \cdot x^2 + c \cdot x + d \rightarrow -\frac{367 \cdot x^3}{6000} + \frac{4267 \cdot x^2}{280} + \left(\frac{1173899}{35} - \frac{121424 \cdot x}{105}\right)$$

6a)

clear $(Y, X, k, d, a, b, c, n_{quad}, n_{lin}, n_{k\_lin}, n_{d\_lin}, n_{a\_quad}, n_{b\_quad}, n_{c\_quad}, n)$

clear $(a, b, k, n, c, d, x, t, X, Y, a_{quad}, a_{lin}, a_{a\_quad}, a_{b\_quad}, a_{k\_lin}, a_{d\_lin}, a_{t\_quad})$

$$X := \begin{bmatrix} 60 \\ 70 \\ 80 \\ 90 \\ 100 \end{bmatrix} \qquad Y := \begin{bmatrix} 9600 \\ 8402 \\ 7045 \\ 6345 \\ 5125 \end{bmatrix}$$

$$n := \text{rows}(X) \rightarrow 5$$

$$n_{lin}(k,d) := \sum_{i=0}^{n-1} \left(k \cdot X_i + d - Y_i\right)^2$$

$$n_{k\_lin}(k,d) := \frac{d}{dk} n_{lin}(k,d) \rightarrow 66000 \cdot k + (800 \cdot d - 5622580)$$

$$n_{d\_lin}(k,d) := \frac{d}{dd} n_{lin}(k,d) \rightarrow 800 \cdot k + (10 \cdot d - 73034)$$

$$[k \;\; d] := \begin{bmatrix} n_{k\_lin}(k,d) = 0 \\ n_{d\_lin}(k,d) = 0 \end{bmatrix} \xrightarrow{solve, k, d} \begin{bmatrix} -\frac{11007}{100} & 16109 \end{bmatrix}$$

$$\boxed{n_{lin}}(x) := k \cdot x + d \rightarrow -\frac{11007 \cdot x}{100} + 16109$$

$$\mathbf{clear}\left(Y, X, k, d, a, b, c, n_{quad}, n_{lin}, n_{k\_lin}, n_{d\_lin}, n_{a\_quad}, n_{b\_quad}, n_{c\_quad}, n\right)$$

$$\mathbf{clear}\left(a, b, k, n, c, d, x, t, X, Y, a_{quad}, a_{lin}, a_{a\_quad}, a_{b\_quad}, a_{k\_lin}, a_{d\_lin}, a_{t\_quad}\right)$$

$$X := \begin{bmatrix} 60 \\ 70 \\ 80 \\ 90 \\ 100 \end{bmatrix} \qquad Y := \begin{bmatrix} 9600 \\ 8402 \\ 7045 \\ 6345 \\ 5125 \end{bmatrix}$$

$$n := \mathrm{rows}(X) \rightarrow 5$$

$$n_{quad}(a,b,c) := \sum_{i=0}^{n-1} \left(a \cdot X_i^{\,2} + b \cdot X_i + c - Y_i\right)^2$$

$$n_{a\_quad}(a,b,c) := \frac{\mathrm{d}}{\mathrm{d}a} n_{quad}(a,b,c) \rightarrow 66000 \cdot c + 5600000 \cdot b + (487080000 \cdot a - 446924600)$$

$$n_{b\_quad}(a,b,c) := \frac{\mathrm{d}}{\mathrm{d}b} n_{quad}(a,b,c) \rightarrow 800 \cdot c + 66000 \cdot b + (5600000 \cdot a - 5622580)$$

$$n_{c\_quad}(a,b,c) := \frac{\mathrm{d}}{\mathrm{d}c} n_{quad}(a,b,c) \rightarrow 10 \cdot c + 800 \cdot b + (66000 \cdot a - 73034)$$

$$[a \;\; b \;\; c] := \begin{bmatrix} n_{a\_quad}(a,b,c) = 0 \\ n_{b\_quad}(a,b,c) = 0 \\ n_{c\_quad}(a,b,c) = 0 \end{bmatrix} \xrightarrow{solve, a, b, c} \begin{bmatrix} \frac{613}{1400} & -\frac{126089}{700} & \frac{131766}{7} \end{bmatrix}$$

$$\boxed{n_{quad}}(x) := a \cdot x^2 + b \cdot x + c \rightarrow \frac{613 \cdot x^2}{1400} + \left(\frac{131766}{7} - \frac{126089 \cdot x}{700}\right)$$

$$\mathbf{clear}\left(Y, X, k, d, a, b, c, n_{quad}, n_{lin}, n_{k\_lin}, n_{d\_lin}, n_{a\_quad}, n_{b\_quad}, n_{c\_quad}, n\right)$$

$$\mathbf{clear}\left(a, b, k, n, c, d, x, t, X, Y, a_{quad}, a_{lin}, a_{a\_quad}, a_{b\_quad}, a_{k\_lin}, a_{d\_lin}, a_{t\_quad}\right)$$

Kapitel 5.3.3 Beispiel 1)

a)

$K(x) := 0.001 \cdot x^3 - 0.1 \cdot x^2 + 4 \cdot x + 50$

$p := 3$

$E(x) := p \cdot x \rightarrow 3 \cdot x$

$G(x) := E(x) - K(x) \rightarrow -0.001 \cdot x^3 + 0.1 \cdot x^2 - 1.0 \cdot x - 50.0$

b)

Gewinngrenzen:

$$P := G(x) = 0 \xrightarrow[\text{float}]{\text{solve}, x} \begin{bmatrix} 79.516028913777038068 \\ -16.844954207003932543 \\ 37.328925293226894475 \end{bmatrix}$$

$gewinn\_grenze_x := \text{floor}\left(P_0\right) \rightarrow 79$

$break\_even_x := \text{floor}\left(P_2\right) \rightarrow 37$

$gewinn\_grenze_y := G\left(P_0\right) \rightarrow -\left(0.12089807135279035527 \cdot 10^{-17}\right)$

$break\_even_y := G\left(P_2\right) \rightarrow 0.76232965252887030516 \cdot 10^{-20}$

Die Gewinngrenze liegt bei einer Stückzahl von 79 und der Break-Even-Point bei 37 Stück.

c)

max Gewinn:

$$G'(x) := \frac{d}{dx} G(x) \rightarrow -0.003 \cdot x^2 + 0.2 \cdot x - 1.0$$

$$G_{max} := G'(x) = 0 \xrightarrow{solve, x} \begin{bmatrix} 5.4446657821974817341 \\ 61.222000884469184933 \end{bmatrix}$$

$$G_{y\_max} := G\left(G_{max_1}\right) \rightarrow 34.123112486952065451$$

max Erlös:

$$E'(x) := \frac{d}{dx} E(x) \rightarrow 3$$

$$E_{x\_max} := E'(x) = 0 \rightarrow 0$$

$$E_{y\_max} := E\left(E_{x\_max}\right) \rightarrow 0$$

Der maximale Gewinn liegt bei 61.22 GE und der maximale Erlös liegt bei 0 GE.

Skizze: