Stevan Vlajić

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2) $\sum_{n=1}^{\infty} \frac{1}{(n+1)\cdot(n+2)\cdot n} \Rightarrow \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2}$

$\left(\frac{1}{n} - \frac{1}{n-1} + \frac{1}{n-1} - \frac{1}{n+2}\right)$ ← Trick nutzen

$\frac{1}{(n+1)\cdot(n+2)\cdot n} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2} \quad \Big| \cdot (n+1)\cdot(n+2)\cdot n$

$1 = A\cdot(n+1)(n+2) + B(n+2)\cdot n + C\cdot(n+1)\cdot n$

$1 = An^2 + 2nA + An + 2A + Bn^2 + 2Bn + Cn^2 + Cn$

$n^2\cdot(A+B+C) + n(3A+2B+C) + 2A = 1$

$2A = 1 \Rightarrow \underline{\underline{A = \frac{1}{2}}}$

$3A + 2B + C = 0$

$3\cdot\frac{1}{2} + 2\cdot(-1) + C = 0$

$C = \frac{1}{2}$

$2B + C = -1{,}5 \quad | \; -2B$

$C = -1{,}5 - 2B$

$\sum_{n=1}^{\infty} \frac{1}{n\cdot(n+1)(n+2)} = \underbrace{\frac{0{,}5}{n} - \frac{1}{n+1} + \frac{0{,}5}{n+2}}$

$\frac{1}{2}\cdot\left(\left(\frac{1}{n} - \frac{1}{n+1}\right) + \left(\frac{1}{n+2} - \frac{1}{n+1}\right)\right)$

$\frac{1}{2} + B - 1{,}5 - 2B = 0 \qquad \underline{B = -1}$

$s_1 = \frac{1}{2}\cdot\left(\left(1 - \frac{1}{2}\right) + \frac{1}{3} - \frac{1}{2}\right)$

$s_2 = \frac{1}{2}\cdot\left(\left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{2}\right)\right) + \frac{1}{2}\left(\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{4} - \frac{1}{3}\right)\right)$

$s_3 = \frac{1}{2}\cdot\left(\left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{2}\right)\right) + \frac{1}{2}\cdot\left(\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{4} - \frac{1}{3}\right)\right) + \frac{1}{2}\cdot\left(\frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{4}\right)$

$s_n = \frac{1}{2}\cdot\left(\left(1 - \frac{1}{2}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{2}}\right)\right) + \frac{1}{2}\cdot\left(\left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{3}}\right)\right) + \frac{1}{2}\cdot\left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \frac{1}{5} - \frac{1}{4}\right) + \ldots + \frac{1}{2}\cdot\left(\frac{1}{n} - \frac{1}{n+1} + \frac{1}{n+2} - \frac{1}{n+1}\right)$

$= \frac{1}{4}\cdot + \frac{1}{2}\cdot\left(\frac{1}{n+2} - \frac{1}{n+1}\right)$

$s_n = \frac{1}{4}$

$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1}{4} + \frac{1}{2}\left(\frac{1}{n+2} - \frac{1}{n-1}\right) \to 0$

$\frac{1}{n+2} \to 0 \qquad \frac{1}{n-1} \to 0$

Somit ist die Reihe konvergent und deren Summe beträgt $\frac{1}{4}$.

3) $\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right) = \ln(2) + \ln\left(\frac{3}{2}\right) \ldots$

$s_1 = \ln(1+1)$

$s_2 = \ln(1+1) + \ln\left(\frac{3}{2}\right)$

$s_3 = \ln(2) + \ln\left(\frac{3}{2}\right) + \ln\left(1 + \frac{1}{3}\right) + \ln\left(1 + \frac{1}{4}\right)$

$\ln\left(2\cdot\frac{3}{2}\cdot\frac{4}{3}\right) = \ln(4)$

$s_4 = \ln\left(2\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\right) = \ln(5)$

$s_5 = \ln\left(2\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\right) = \underline{\ln(6)}$

$\vdots$

$s_n = \ln(n+1)$

$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \ln(n+1) \to \infty \Rightarrow$ divergent