FINAL PROJECT PROPOSAL CHECKPOINT ONE

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1. Proposal

For the final term project in the Decision Analytics course, our group plans to examine a case study problem focused on minimizing transportation and distribution costs for a brewery. This problem, which is presented in Chapter 8.3 of "Business Optimization Using Mathematical Programming," underscores the applicability of the decision analytics topics we've covered in this course to the manufacturing industry (Kallrath 2021). To solve this problem, we will need to formulate a sophisticated and detailed model to represent the ways in which the breweries, commodity types, packaging units, and demand points all connect. This formulation will also need to take into account a broad array of constraints described in the problem, including constraints focused on brewing capacity, consumer demand, packaging unit capacity, and more. We will collaborate on this linear programming problem via a Jupyter Notebook available to all group members on Github, and we'll consolidate our findings into a final report and video presentation once we've finished our analysis. For the unabridged description of this problem, please feel free to refer to the full problem text provided in Appendix A.

2. What is your topic and what have you done thus far?

The topic of our project is a minimum-cost flow problem applied through the "Distribution Planning for a Brewery" problem described above and in Appendix A. So far, we have created a rough outline of the problem's flow and identified possible research papers to incorporate into our literature review.

3. What are you planning to do next?

The group will collaborate to find a solution for this problem. A goal is to have a potential solution by the next checkpoint. We will continue our research of additional academic references for the literature review section.

The group meets weekly on Thursdays. We will update each other on our individual contributions to the project's progress.

4. How can the instructor help (or what questions do we have for the instructor)?

Can we take as given the precise problem sizes, configurations, and costs contained within the brewery.xlsx file that was made available from the Springer Collection or should we augment them in some way and/or create scenarios? While this problem can be solved via Discrete Event Simulation, since we are given variables such as transportation and distribution costs based on known demand and capacity, the problem also appears solvable with linear programming or network flow, would solving the problem in this way be appropriate?

References

Daskin, Mark S. 2013. *Network and Discrete Location: Models, Algorithms, and Applications*. 2nd ed. Somerset: John Wiley & Sons, Incorporated. https://doi.org/10.1002/9781118537015.

Kallrath, Josef. 2021. *Business Optimization Using Mathematical Programming: An Introduction with Case Studies and Solutions in Various Algebraic Modeling Languages.* Vol. 307. Cham: Springer International Publishing AG.

Snyder, Lawrence V., Mark S. Daskin, and Chung-Piaw Teo. 2007. "The Stochastic Location Model with Risk Pooling." *European Journal of Operational Research* 179, no. 3: 1221–38. https://doi.org/10.1016/j.ejor.2005.03.076.

Appendix A – Full Text of Term Project Problem

This appendix presents the full text of the problem as originally written in chapter 8.3 of "Business Optimization Using Mathematical Programming" (Kallrath 2021). This is the problem that our team plans to solve for our term project.

8.3 Distribution Planning for a Brewery

In the UK, brewing companies brew two major different types of beer: ale and lager. This is also the case for our specific brewer who wanted to optimize the business using mathematical programming. Each type of beer comes in several varieties (liquid types), distinguished by taste, color, flavor, and strength and several other characteristics.

Most beers are sold in four major containers: cans, bottles, barrels, and "cellar tanks." When packaged in the latter form, the beer is delivered in bulk to a tank at the retail outlet (usually a public house) and then served to the customer straight from the tank. Not all liquid types go into each container, and in fact the actual number of commodities (i.e., liquid types in specific containers) is significantly smaller than the maximum number theoretically possible.

When a particular liquid is brewed at one of the breweries, it is taken to a packaging unit to be placed in a container. Sometimes this journey is very short if, for instance, the packaging unit is sited adjacent to the brewery and connected to it by pipeline. But since packaging units are expensive it is sometimes necessary to transport liquid in bulk from the brewery to a packaging unit of the correct type.

Each brewery belonging to the company has a separate limited capacity for lager and for ale, but the capacity within these two sub-categories may be freely allocated to any of the liquid types. A packaging unit can only put liquids into one type of container.

After liquids have been put into containers (and have become a commodity), they are then transported to customer zones (demand points). These are typically depots owned by the company, or the premises owned by wholesalers or supermarkets.

8.3.1 *Dimensions, Indices, Data, and Variables*

The parameters or dimensions of the model and the indices are summarized below

$$
\begin{aligned}
&b \in \{1, \ldots, N^B\} \text{ , the set of breweries}\\
&c \in \{1, \ldots, N^C\} \text{ , the set of commodity types}\\
&d \in \{1, \ldots, N^D\} \text{ , the set of demand points}\\
&l \in \{1, \ldots, N^L\} \text{ , the set of liquid types, } N^L = N^{AL} + N^{LA}\\
&p \in \{1, \ldots, N^P\} \text{ , the set of packaging units}\\
&t \in \{1, \ldots, N^T\} \text{ , the set of containers (packing type),}
\end{aligned}
$$

where N^{AL} and N^{LA} denote the numbers of ales and lagers.

The company wished to consider the data as being driven by the commodities, that is, the combinations of liquid type in a particular container. A partial list might be

Commodity number	Liquid	Container
...	...	...
16	B23L	Barrels
17	B23L	Half pint bottles
18	G26A	Pint bottles
...	...	...

Data were collected and assembled into the following list:

C^{TBP}_{pb}	the unit transport cost from brewery b to packaging unit p
C^{TPD}_{tdp}	the unit transport cost of container type t from packaging unit p to demand point d
R^L_c	the liquid required by commodity c
R^C_c	the container type required by commodity c
C^B_{bl}	the brewing capacity at brewery b for liquid l
C^P_p	the total packaging capacity at p
D_{cd}	the final demand for commodity c at demand point d
I^{BL}_{lb}	1if brewery b can brew liquid l
N^{BL}_l	the number of breweries that can brew liquid l
B^U_b	the maximum amount that can be brewed at b
B^L_b	the minimum amount that can be brewed at b
P^U_p	the maximum amount that can be packed at p
P^L_p	the minimum amount that can be packed at p.

The continuous, non-negative decision variables needed for the problem are as follows:

b_{lb},	the amount of liquid l brewed at brewery b
s_{lp},	the throughput of liquid l through packaging unit p
l_{blp},	the amount of liquid l sent from brewery b to packaging unit p
p_{cp},	the amount of commodity c packed at packaging unit p
x_{pcd},	the quantity of commodity c sent from packaging unit p to demand point d.

Note that in order to reduce the number of variables in the model listed below the variables are only defined if certain logical conditions on data and indices are fulfilled. In addition we need the binary variables $\beta_{lb}, \delta_{lp} \in \{0, 1\}$ defined as

$$\beta_{lb} := \begin{cases} 1 \text{ , if any liquid } l \text{ is brewed at brewery } b \\ 0 \text{ , otherwise} \end{cases} \tag{8.3.1}$$

and

$$\delta_{lp} := \begin{cases} 1 \text{ , if any liquid } l \text{ is packaged at packaging unit } p \\ 0 \text{ , otherwise.} \end{cases} \tag{8.3.2}$$

Note that the β_{lb} variables are only defined if brewery b can brew liquid l and there is more than one brewery that can brew the liquid. The δ_{lp} are only defined if the capacity of the packaging unit p is non-zero.

Explicitly allowing for zero capacity at a packaging unit may seem peculiar, but one of the aims of the exercise was to consider the possible closure of one or more of the packaging units. Different scenarios could easily be considered by setting $C_p^P = 0$ for a packing unit that was to be closed.

Note that liquid 1 denotes *Ale*, while liquid 2 is *Lager*.

8.3.2 Objective Function

The objective function to be minimized was the total cost of transporting liquids from breweries to packaging units, plus the total cost of distributing commodities from packaging units to demand points.

$$\sum_{p=1}^{N^P} \sum_{b=1}^{N^B} \sum_{l=1}^{N^L} C_{pb}^{TBP} l_{blp} + \sum_{p=1}^{N^P} \sum_{c=1}^{N^C} \sum_{d=1}^{N^D} C_{\tau dp}^{TPD} x_{pcd} \quad , \quad \tau = R_c^C . \tag{8.3.3}$$

It would be easy to incorporate different brewery packaging costs for each brewery and packing unit, but these were not thought to be important in the initial stages of the study.

8.3.3 *Constraints*

The following constraints were operative.

Liquid balance at each brewery:

$$\sum_{p=1}^{N^P} l_{blp} = b_{lb} \quad , \quad \forall l \in \{1, 2, ..., N^L\} \quad , \quad \forall b \in \{1, 2, ..., N^B\}, \tag{8.3.4}$$

which ensures that for each liquid/brewery combination, all of it is sent to some packaging unit. The constraint

$$\sum_{c \text{ containing } l} p_{cp} = s_{lp} \quad , \quad \forall l \in \{1, 2, ..., N^L\} \quad , \quad \forall p \in \{1, 2, ..., N^P\} \tag{8.3.5}$$

defines the throughput of liquid l through packaging unit p as the total amount packaged of all commodities containing liquid l.

The total amount of ale brewed at a brewery is less than or equal to its ale capacity, and correspondingly for lager. Hence,

$$\sum_{p} l_{b,ale,p} \leq C^B_{b,ale} \quad , \quad \forall b \in \{1, 2, ..., N^B\} \tag{8.3.6}$$

$$\sum_{p} l_{b,lager,p} \leq C^B_{b,lager} \quad , \quad \forall b \in \{1, 2, ..., N^B\}. \tag{8.3.7}$$

Conservation of liquid going into the packaging units and being packed means that

$$\sum_{c \text{ requiring } l} p_{cp} = \sum_{b=1}^{N^B} l_{blp} \quad , \quad \forall l \in \{1, 2, ..., N^L\} \quad , \quad \forall p \in \{1, 2, ..., N^P\}. \tag{8.3.8}$$

The capacity limitation at the packaging unit means that

$$\sum_{c=1}^{N^C} p_{cp} \leq C^P_p \quad , \quad \forall p \in \{1, 2, ..., N^P\}. \tag{8.3.9}$$

Conservation of each commodity leaving each packaging unit gives us the equations

$$\sum_{d=1}^{N^D} x_{pcd} = p_{cp} \quad , \quad \forall p \in \{1, 2, ..., N^P\} \quad , \quad \forall c \in \{1, 2, ..., N^C\}. \tag{8.3.10}$$

Satisfying the demands for each commodity at each demand point exactly yields

$$\sum_{p=1}^{N^P} x_{pcd} = D_{cd} \quad , \quad \forall c \in \{1, 2, ..., N^C\} \quad , \quad \forall d \in \{1, 2, ..., N^D\}. \tag{8.3.11}$$

The throughput, s_{lp}, of liquid l through packaging unit p is zero if δ_{lp} is zero (forced by (8.3.12), otherwise (forced by (8.3.13)) it must be greater than the minimum packaging level for any commodity and less than the maximum packaging capacity

$$s_{lp} \leq P_p^U \delta_{lp} \quad , \quad \forall l \in \{1, 2, ..., N^L\} \quad , \quad \forall p \in \{1, 2, ..., N^P\} \tag{8.3.12}$$

$$s_{lp} \geq P_p^L \delta_{lp} \quad , \quad \forall l \in \{1, 2, ..., N^L\} \quad , \quad \forall p \in \{1, 2, ..., N^P\}. \tag{8.3.13}$$

This is a common formulation device. As the δ_{lp} variables are binary (0 or 1), then if the binary variable is 0 the continuous variable is zero, while if it is 1 then the continuous variable is constrained to be less than the maximum value P_p^U and greater than the sensible minimum value P_p^L. (These types of constraints occur frequently enough for the semi-continuous variable construct to have been devised specially. For the sake of simplicity they are not used here. But see Sect. 9.4.5 for the benefits of such variables in B&B.)

Finally, we need constraints similar to (8.3.12) and (8.3.13) above, but applying to the brewing of liquids at breweries. These constraints only apply if there is more than one brewery that can brew a particular liquid.

$$b_{lb} \leq B_l^U \beta_{lb} \quad \forall l \in \{1, 2, ..., N^L\}, b \in \{1, 2, ..., N^B\} \mid \; N_l^{BL} > 1 \tag{8.3.14}$$

$$b_{lb} \geq B_b^L \beta_{lb} \quad \forall l \in \{1, 2, ..., N^L\}, b \in \{1, 2, ..., N^B\} \mid \; N_b^{BL} > 1. \tag{8.3.15}$$

So if a β_{lb} is 0, the amount brewed is 0. But if a β_{lb} is 1, then the amount brewed must be at least the smallest sensible quantity to brew at the brewery but not more than the maximum amount that can be brewed by any brewery. We must also declare β_{lb} and δ_{lp} as binary variables; they can only take the value 0 or 1.

The data for most of the tables were held in ranges in a `Lotus 1-2-3` spreadsheet and imported by the `DISKDATA -l` command of `mp-model`. A useful approach is employed by the first `DISKDATA -l` command that fills up a table called `SIZES` from a spreadsheet range `SIZES`. The spreadsheet software is, in 2020, somewhat outdated. Currently, the majority uses Excel, but who knows what will be used in 15 years. Note that the `DISKDATA` command for reading text data is supported by `mp-model`'s successor `FICO Xpress Mosel`.

8.3.4 Running the Model

The brewery case study is contained in MCOL under the problem name *brewery*. Most of the data were assembled into the single `Lotus 1-2-3` spreadsheet file *brewdata.wk3*, but some were held in data files. Unfortunately, already after 20 years, this software seems not be used anymore — our data are lost. This should serve as a good warning and advise for the future. Only ASCII data have a reasonably long life time. Everything else depends strongly on market changes. The life time of software seems to be very limited — and even if the software still exists, backward compatibility is not guaranteed either.