{Lecture-1}

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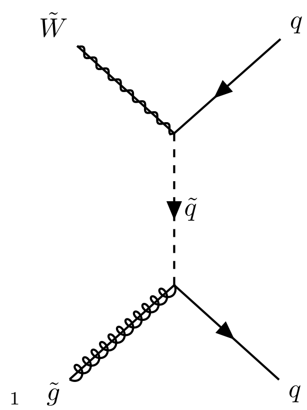

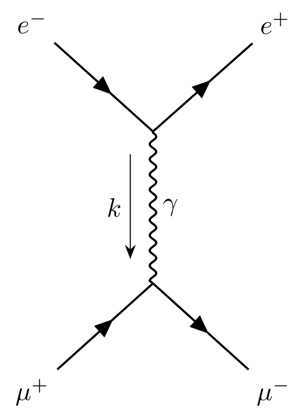

What is more important is that you are willing to work hard;
to devote the time needed;
to ask questions when things don't make sense!

Contents

1 Introduction

1.1 Purpose of Discussion

We will show how photons arise from the quantization of the electromagnetic field and how massive, charged particles such as electrons arise from the quantization of matter fields.There are electron fields, but also quark fields, neutrino fields, gluon fields, W and Z-boson fields, Higgs fields and a whole slew of others

1.2 Why We Need QFT

There is an unavoidable contraction between classical physics and modern physics(Local Interaction & Action at a distance).To be more specific, the locality give us motivation to investigate two problems:

Answer 1 The combination of quantum mechanics and special relativity implies that particle number is not conserved

1.2.1 A simple & interesting physical problem

According to uncertainty principle, particles restricted inside a box satisfy:

$$\Delta p \geq \hbar/L \tag{1}$$
$$\Delta E \geq \hbar c/L \tag{2}$$

Following above equation, when the variation of energy reaches $\Delta E = 2mc^2$, there will be unavoidable new particles appear. If we still obey the non-relativistic QM, the only explanation is the particle number is not conserved!

Answer 2 All particles of the same type are the same Is is unbelievable that the proton from the beginning of the universe and that detected on the earth are exactly the same.One explanation that might be offered is that there's a sea of proton "stuff" filling the universe and when we make a proton we somehow dip our hand into this stuff and from it mould a proton

1.3 What is QFT

The rules for quantizing a field are no different from that transforming CM into QM,thus the basic degrees of freedom in quantum field theory are operator valued functions of space and time. This means that we are dealing with an infinite number of degrees of freedom

2 Units

What we are familiar is SI units, however natural units are generally applied in cosmology and high-energy physics.

$$c = \hbar = \epsilon_\circ = k_B = 1 \tag{3}$$

- $c = 2.9979 \times 10^8$m/s
- $\hbar = 1.0546 \times 10^{-34}$Js

- $\epsilon_o = 8.8542 \times 10^{-12}\mathrm{A}^2\mathrm{s}^4\mathrm{kg}^{-1}\mathrm{m}^{-3}$
- $k_B =$ Boltzmann constant $= 1.3806 \times 10^{-23}\mathrm{JK}^{-1}$

A simple formula to convert numerical value between different units:

$$^{2}\left(\mathrm{kg}^{\alpha}\mathrm{m}^{\beta}\mathrm{s}^{\gamma}\right) \rightarrow (E)^{\alpha-\beta-\gamma}\hbar^{\beta+\gamma}c^{\beta-2\alpha} \tag{4}$$

2.0.1 Plank Mass

Let us examine the Einstein Equations with cosmological constant

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

In natural units, the above equation becomes:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{5}$$

If we define the planck mass as $m_p = \sqrt{\frac{\hbar \times c}{G}} = 2.7164 \times 10^{-8}kg$, then we can calculate $m_p = 1.2209 \times 10^{19}\mathrm{GeV}$ **Calculate It!**

3 Classical Field Review

Concepts When constructing the Lagrangian for a field distribution(discussing only scalar & non-interacting field), the spacetime coordinates are just index, comparing to the individual particles. The dynamical variable can be treated as $\varphi(\mathbf{r}, t)$, given a matter distribution: $\rho = \rho(\mathbf{r}, t)$, or we can also treat the field variables as a map:$\varphi : \mathbf{R}^4 \rightarrow \mathbf{R}$, in a typical spacetime domain:

$$\mathcal{R} = \{(t, x, y, z) | t_1 \leq t \leq t_2, -\infty < x, y, z < \infty\} \tag{6}$$

Thus we are dealing with a system with an infinite number of degrees of freedom, compared to CM. For convenience, we can treat the field quantity defined on spacetime as a function on $4 - D$ manifold.

3.1 Electromagnetic Field Description

From what we are really used to write "Maxwell Equations":

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \tag{7}$$

$$\vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J} \tag{8}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{9}$$

$$\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \tag{10}$$

The above equation based on the Gaussian Units:

- $\vec{E} \longrightarrow \frac{1}{\sqrt{4\pi\epsilon_0}}\vec{E}$, $\vec{B} \longrightarrow \sqrt{\frac{\mu_0}{4\pi}}\vec{B}$ $\rho \longrightarrow \sqrt{4\pi\epsilon_0}\rho$, $\vec{J} \longrightarrow \sqrt{4\pi\epsilon_0}\vec{J}$

[2] E can arbitrarily chosen energy unit, popular as GeV

As a result, we can write the Maxwell Equations as:

$$^{3}\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A} \tag{11}$$

3.2 Lagrangian

Similar in the CM, the Lagrangian & Hamiltonian are important to catch the system dynamics. In Field theory, the Lagrangian can more or less treated as the hypersurface integral:

$$L(t) = \int d^3x \mathcal{L}\left(\phi_a, \partial_\mu \phi_a\right) \tag{12}$$

The action is just:

$$S = \int_{t_1}^{t_2} dt \int d^3x \mathcal{L} = \int d^4x \mathcal{L} \tag{13}$$

We notice that the constructed Lagrangian is only related to first derivative with respect to spacetime coordinates. But in practice, it is more common to find relationship between higher derivatives

3.3 Variational Principle

In physics, fundamental theories always arise from a variational principle.Ultimately this stems from their roots as quantum mechanical systems, as can be seen from Feynman's path integral formalism.
In a typical spacetime domain, via the Lagrangian we construct the action:$S = S[\varphi]$ which is a functional of field φ. Like the procedure in particle circumstances,the variational principle goes as follows:
Consider any family of functions, labeled by a parameter λ, which includes some given function λ_0 at $\lambda = 0$,we can think of φ_λ as defining a curve in the manifold which passes through the point φ_0,getting the action:

$$S(\lambda) := S\left[\varphi_\lambda\right] \tag{14}$$

the critical point $\lambda = 0$ is just the same as in real number space, (just manifold space), and the turning points of the functional $S[\varphi]$ correspond to functions on spacetime which solve the Euler-Lagrangian equations.
Let us consider a curve that passes through a putative critical point φ (say $\lambda = 0$) then:

$$\varphi_\lambda : \lambda = 0, \varphi_0 = \varphi. \tag{15}$$

If $\varphi_{\lambda=0} \equiv \varphi$ is a critical point in the "field manifold",then:

$$\delta S := \left(\frac{dS(\lambda)}{d\lambda}\right)_{\lambda=0} = 0 \tag{16}$$

$$\delta \varphi := \left(\frac{d\varphi(\lambda)}{d\lambda}\right)_{\lambda=0} \tag{17}$$

- δS the first variation of the action.
- $\delta \varphi$ the variation of φ.

[3] $A^\mu(\vec{x}, t) = (\phi, \vec{A})$

In the general case, we can reduce the variation of the action based on the Lagrangian density:

$$\delta S[\varphi]=\int_{\mathcal{R}} d^4xF(x)\delta\varphi(x), F(x)\equiv\frac{\delta S}{\delta\varphi(x)} \tag{18}$$

The general variational principle tells us that $\frac{\delta S}{\delta\varphi}=0$ can be obtained.

3.4 Euler-Lagrangian Equations

Generally, what we discuss is the local field theory(global theory is deep and different)

3.4.1 Example

Take the KG field as an example,the time-dependent total Lagrangian on a hypersurface and total Lagrangian for KG field is:

$$L=\int_{\mathbf{R}^3} d^3x\frac{1}{2}\left(\varphi_{,t}^2-(\nabla\varphi)^2-m^2\varphi^2\right) \tag{19}$$

$$S[\varphi]=\int_{t_1}^{t_2} dtL \tag{20}$$

The variation of Lagrangian is:

$$\begin{aligned}\delta\mathcal{L}&=\varphi_{,t}\delta\varphi_{,t}-\nabla\varphi\cdot\nabla\delta\varphi-m^2\varphi\delta\varphi\\&=\left(-\varphi_{,tt}+\nabla^2\varphi-m^2\varphi\right)\delta\varphi+\frac{\partial}{\partial x^\alpha}V^\alpha\end{aligned} \tag{21}$$

where: $V^0=\varphi_{,t}\delta\varphi,\quad V^i=-(\nabla\varphi)^i\delta\varphi$.When taking the boundary conditions into account, the last term in (21) usually vanish in the process of integral.

3.4.2 General Case

For any formula like $F\left(x,\varphi,\varphi_\alpha\right)$, we give the definition of total derivative with respect to some coordinates:

$$D_\alpha F\left(x,\varphi,\varphi_\alpha\right)=\frac{\partial F}{\partial x^\alpha}+\frac{\partial F}{\partial\varphi}\varphi_\alpha+\frac{\partial F}{\partial\varphi_\beta}\varphi_{\alpha\beta} \tag{22}$$

Following the procedure, we define the variation of the Lagrangian density as:

$$\delta\mathcal{L}:=\frac{\partial\mathcal{L}}{\partial\varphi}\delta\varphi+\frac{\partial\mathcal{L}}{\partial\varphi_\alpha}\delta\varphi_\alpha \tag{23}$$

then, we can rewrite the expression as:

$$\delta\mathcal{L}=\left(\frac{\partial\mathcal{L}}{\partial\varphi}-D_\alpha\frac{\partial\mathcal{L}}{\partial\varphi_\alpha}\right)\delta\varphi+D_\alpha V^\alpha, V^\alpha=\frac{\partial\mathcal{L}}{\partial\varphi_\alpha}\delta\varphi \tag{24}$$