Mechanical Simulation
755 Phoenix Drive, Ann Arbor MI, 48108, USA
Phone: 734 668-2930 • Fax: 734 668-2877 • Email: info@carsim.com

Trailer Backing Controller

Backing a trailer is a typical skill required for a commercial driver license (CDL). For example, the U. S. state of Michigan advises applicants to be prepared for straight line backing, offset backing, and alley dock maneuvers during the vehicle control skills test[1]. These maneuvers can be simulated in CarSim and TruckSim using the trailer backing controller (TBC) included with the built-in closed-loop driver model.

Specifically, the vehicle to be controlled is required to be a tractor-trailer which can be represented in the idealized form of Figure 1. The front wheel of this bicycle model is assumed to model a single, steered front axle, with axle steer defined as the average steer at the ground of the left and right wheels. The rear wheel and the trailer wheel can each be considered to come from a single axle, a tandem, or a tridem, with all wheels lumped into their average position.

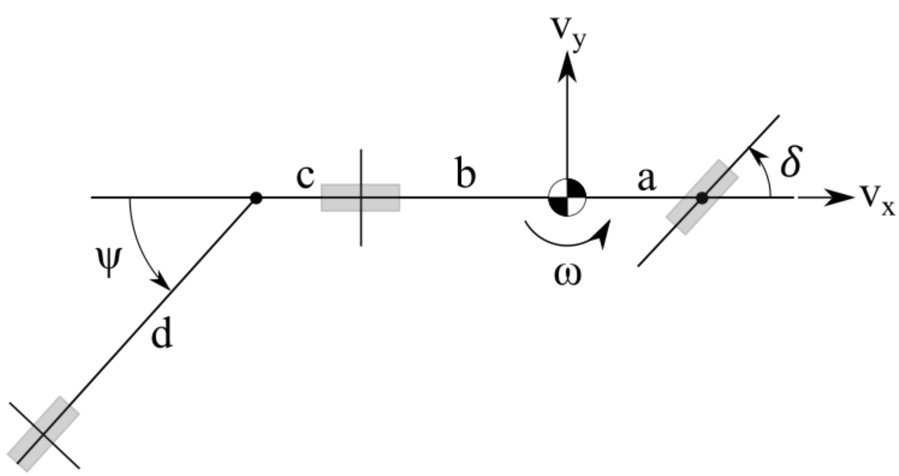

Figure 1. The idealized tractor-trailer assumed for the trailer backing controller.

The vehicle parameters needed for the TBC are given by three dimensions:

- $a + b$, the distance between the tractor's front axle and the tractor's rear axle.
- c, the distance between the tractor's rear axle and the hitch.
- d, the distance between the hitch and the trailer's rear axle.

1 Michigan Commercial Driver License Manual, Michigan Department of State, 2005.

The parameters a, b, and d are all assumed positive, whereas the parameter c can be positive, negative, or zero. (The positive sense of c is drawn in Figure 1.) It is not necessary to enter these parameters separately for the TBC; they are calculated automatically from the vehicle data.

Using the Trailer Backing Controller

The TBC is activated with the `INSTALL_DM_TBC` VS Command and is available to use if `OPT_DM` is 3 (single preview point mode) and the vehicle is a supported type.

When the TBC is installed and available, the Driver Model section of the Echo file (Figure 2) shows the reference model parameters as `TBC_L_TRACTOR` $(a + b)$, `TBC_L_HITCH` (c), and `TBC_L_TRAILER` (d).

```
1081 OPT_DM                 3 ! Driver model option: 0 -> no driver model; 1 -> use linear
1082                          ! dynamic model and 10 preview points; 2 -> same as 1, but
1083                          ! with no rear steer effect (legacy); 3 -> use geometry and a
1084                          ! single preview point [I]
1085 OPT_DRIVER_ACTION      1 ! [D] Use steer from driver model (Steer_DM) when OPT_DM > 0?
1086                          ! 1 -> use Steer_DM, 0 -> ignore calculated Steer_DM [I]
1087 OPT_STR_BY_TRQ         0 ! Control by steering wheel torque? 0 -> no, 1 -> yes [I]
1088 A_SW_MAX_DM          720 ; deg ! Limit steering wheel angle for DM
1089 AV_SW_MAX_DM        1200 ; deg/s ! Limit steering wheel rate for DM
1090 VLOW_DM               10 ; km/h ! Minimum speed for preview dist = V*TPREV
1091 ! XREF_DM          -3261 ; mm ! CALC -- Local X coordinate of DM reference point
1092 XREF_DM_F              0 ; mm ! [D] X distance of DM ref. point in front of axle 1
1093 XREF_DM_R              0 ; mm ! [D] X distance of DM ref. point in front of rear axle
1094 YREF_DM                0 ; mm ! [D] Local Y coordinate of DM ref. point
1095 OPT_DM_AUX             0 ! [D] Activate auxiliary path follower? 0 -> no, 1 -> yes [I]
1096
1097 INSTALL_DM_OUTPUTS       ! VS Command to install XYZ outputs for DM preview point(s)
1098 ! NPREVIEW             0 ! No. of installed preview sensors for external DM (read only)
1099
1100 INSTALL_DM_TBC           ! VS Command to install trailer backing controller. This
1101                          ! calculates steering wheel angle when OPT_DM = 3, the
1102                          ! vehicle is reversing, and MODEL_LAYOUT is a supported type.
1103 ! TBC is available and uses the following data:
1104 TBC_GAIN              10 ; 1/s ! Trailer backing controller gain
1105 ! TBC_L_TRACTOR  3261 ; mm ! CALC -- Tractor length for TBC reference model
1106 ! TBC_L_HITCH    1039 ; mm ! CALC -- Hitch length for TBC reference model
1107 ! TBC_L_TRAILER  2864 ; mm ! CALC -- Trailer length for TBC reference model
```

Figure 2. The Driver Model section of an Echo file showing the TBC installed and available.

The Echo file also reports if a TBC is installed but not available. What this means is that while the TBC is installed, the `OPT_DM` setting is not the single preview point mode, or the vehicle is not a supported type (for example, a tractor without a trailer).

The TBC works by using steering wheel angle to control the hitch articulation angle such that it matches the hitch angle needed for the trailer to follow the desired path. This is a proportional controller using feedback linearization with one dedicated tuning parameter, `TBC_GAIN` (line 1104, Figure 2). Typical values of `TBC_GAIN` are between 1 and 10 s^{-1}.

The remaining parameters affecting the TBC's behavior are shared with the single preview point driver model (preview time, maximum steering wheel angle, etc.). If needed, the preview time can be given as a configurable function to have different preview times for traveling forward vs. backing the trailer.

All the necessary settings can be managed from the closed-loop driver model screen, using the custom settings yellow field to activate the TBC (Figure 3).

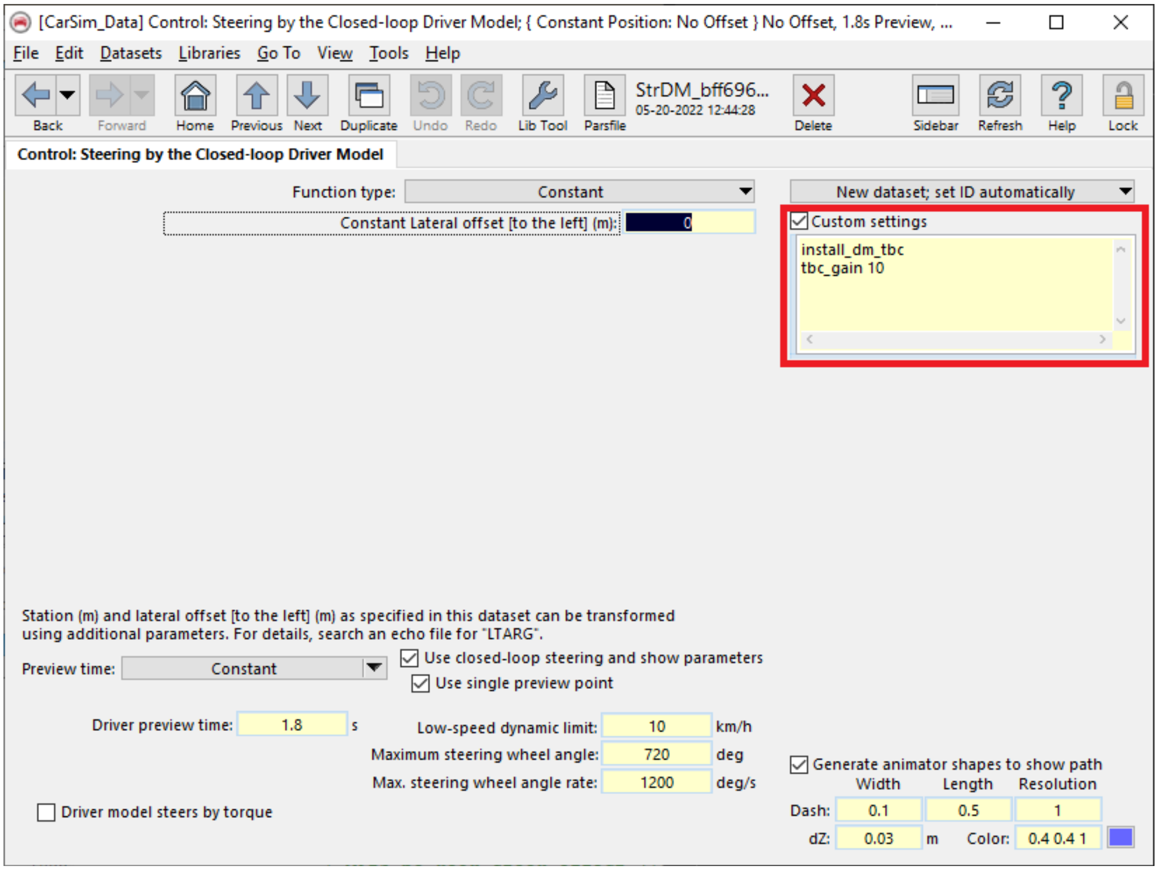

Figure 3. Using the TBC from the closed-loop driver model library screen.

The hitch angle needed for the trailer to follow the path is available using the output variable `Art_H_DM`. If needed, this hitch angle used by the controller can be modified or replaced using the import variable `IMP_ART_H_DM`. The imported value will not affect the output variable `Art_H_DM`, which will always reflect the internally calculated value; consider defining a new output if needed. Outputs such as the steering wheel angle calculated by the controller, `StrCtlDM`, are also shared between the standard single preview point mode and the TBC mode, depending on if the vehicle is driving forward or reversing.

> **Note** When the TBC is used with multiple vehicles within a single solver, the TBC parameters and variables will be indexed by vehicle, which is the indexing used by the closed-loop driver model. On the other hand, hitch parameters and variables are indexed by the hitch number. This means that, for example, the output variable `Art_H_DM_2` could represent the desired hitch angle for the trailer of vehicle #2, corresponding to the output `Art_H3` representing the actual hitch angle for the trailer of vehicle #2 (assuming this trailer is attached using hitch #3).

Figure 4 shows an example simulation in which the vehicle backs up to locate the trailer into a marked target area. Figure 5 shows the hitch angle requested by the TBC for this scenario, along with the actual hitch angle from the simulation.

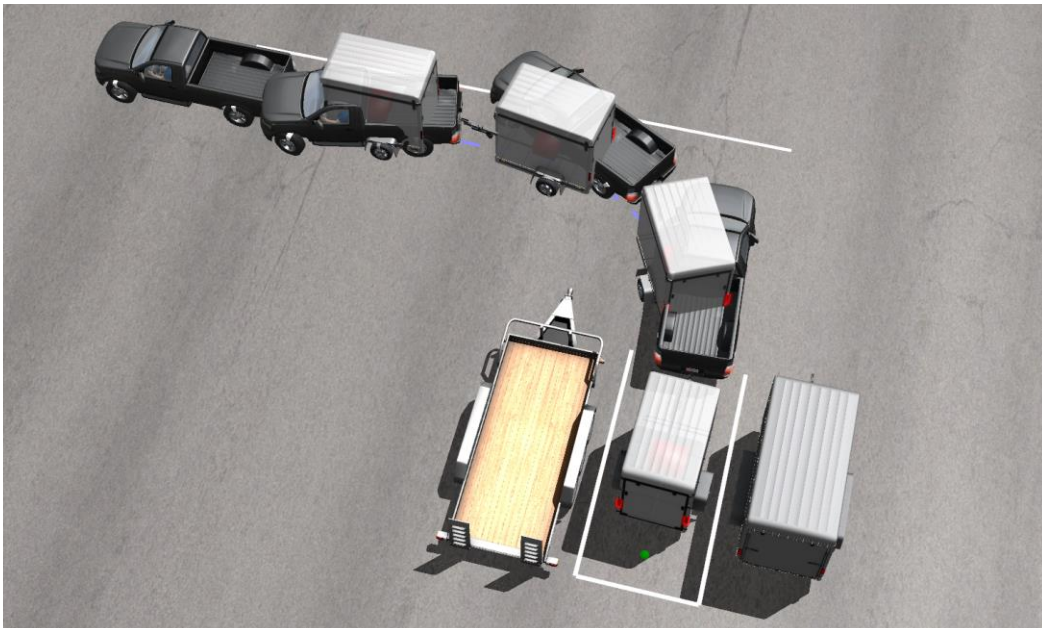

Figure 4. CarSim, `TBC_GAIN`=10 s^{-1}, midsize pickup, one-axle rental trailer, alley dock maneuver at -5 km/h with 18-ft turning radius.

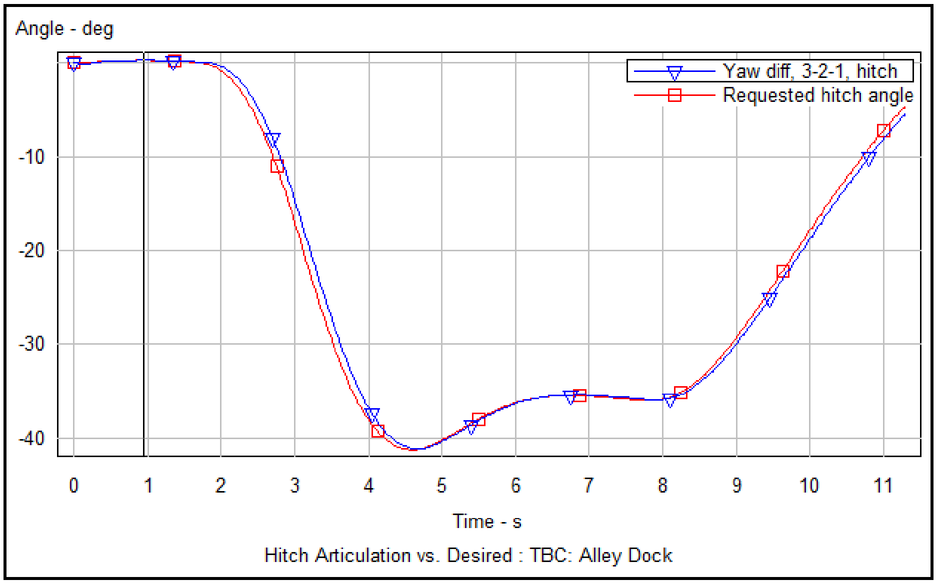

Figure 5. TBC performance for alley dock maneuver of Figure 4.

Figure 6 shows part of a maneuver in which the vehicle drives forward from the space on the left (empty in the figure) and then reverses to locate the trailer in an adjacent space on the right. Figure 7 shows plots of the corresponding requested and actual hitch angles.

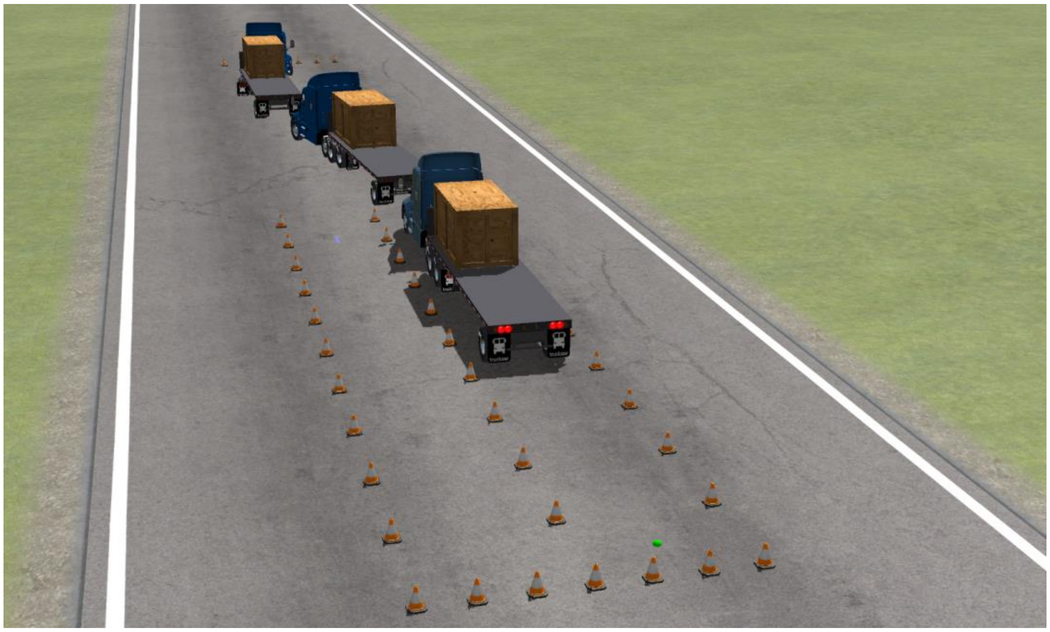

Figure 6. Offset backing maneuver, TruckSim, `TBC_GAIN`*=3* s^{-1}*, -5 km/h.*

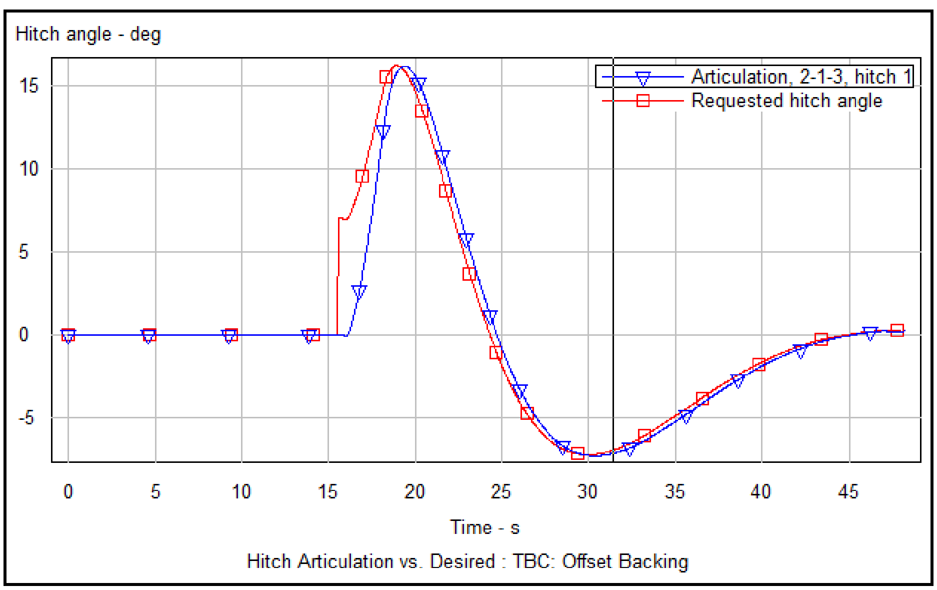

Figure 7. TBC performance for offset backing maneuver of Figure 6.

For precise, accurate placement of the trailer, especially for long trailers, we have found that making small lateral target offsets to the intended path helps tune the placement of the trailer within the parking space. Often this is not needed, and the trailer stays in the center of the space (Figure 4 and Figure 6).

Equations for the Trailer Backing Controller

Calculating the Desired Hitch Angle

The desired hitch articulation is calculated using the single preview point described in the *Driver Controls* document under the subsection entitled *Steer Control Using Geometry and a Single Preview Point*, as well as the geometry of the simplified tractor-trailer.

In particular, the TBC assumes the rear tire of the tractor as well as the trailer tire to be rolling without slipping (Figure 8). Consequently, the lines normal to the tires imply an instant center of rotation. The desired hitch angle ψ_d is such that the preview point, which is known, lies on the trajectory of the trailer tire. In other words, the distance between the trailer tire and the instant center, r, is equal to the distance between the preview point and the instant center, s.

In the coordinate system of Figure 8, the trailer tire is located by

$$x = -d\, cos\, \psi_d$$

$$y = -d\, sin\, \psi_d.$$

The line normal to the tractor rear tire is given by the equation

$$x = c.$$

The line normal to the trailer rear tire can be written as

$$y = -x\, cot\, \psi_d - d\, csc\, \psi_d.$$

The instant center is given by the intersection of the two lines; specifically, the instant center has coordinates

$$x = c$$

$$y = -c\, cot\, \psi_d - d\, csc\, \psi_d.$$

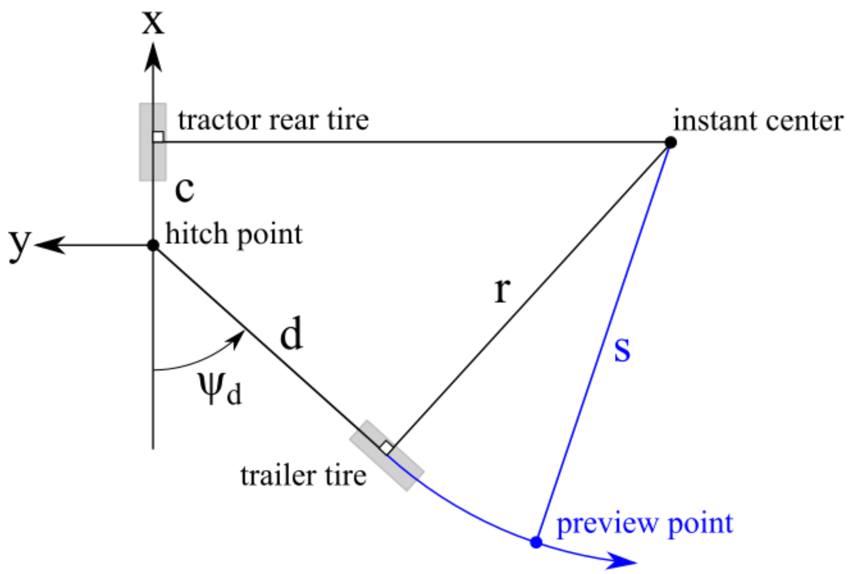

Figure 8. Relationship between the hitch angle and the desired path.

The distance between the trailer tire and the instant center is given by

$$r^2 = (-d\,cos\,\psi_d - c)^2 + (-d\,sin\,\psi_d + c\,cot\,\psi_d + d\,csc\,\psi_d)^2,$$

while the distance between the preview point (x, y) and the instant center is given by

$$s^2 = (x - c)^2 + (y + c\,cot\,\psi_d + d\,csc\,\psi_d)^2.$$

Setting $r^2 = s^2$ and simplifying results in the equation

$$c\,cot\,\psi_d + d\,csc\,\psi_d = \frac{2cx - x^2 - y^2 - d^2}{2y}.$$

This equation is solved for the desired hitch angle ψ_d. In the case where the preview point is straight behind, $y = 0$, the desired hitch angle is simply zero. When the hitch is centered on the rear axle of the tractor, $c = 0$, the solution is easily found. The general case, when $c \neq 0$ and $y \neq 0$, is more complicated and can have multiple solutions; the solution giving the smallest path radius is selected.

Relationship Between Front Steer and Hitch Angle

The relationship between front steer and hitch angle is considered using the idealized vehicle model of Figure 1 (page 1). The vehicle kinematics are given by

- v_x, the forward velocity of the tractor's center of mass.
- v_y, the lateral velocity of the tractor's center of mass.
- ω, the angular velocity of the tractor.
- δ, the steer angle of the front axle.
- ψ, the articulation angle of the hitch.

These kinematics variables are defined to be consistent with CarSim and TruckSim vehicle models. The forward speed of the tractor, v_x, is assumed to be constant, negative, and low in magnitude (e.g., −5 km/h).

The velocity of the front tire of the tractor may be written as

$$v_f = \left(v_x, v_y + a\omega\right).$$

Similarly, the tractor's rear tire has velocity

$$v_r = \left(v_x, v_y - b\omega\right).$$

The velocity of the trailer tire is

$$v_t = \left(v_x + \omega d\,sin\,\psi + \dot{\psi} d\,sin\,\psi\,, v_y - \omega(b + c + d\,cos\,\psi) - \dot{\psi} d\,cos\,\psi\right).$$

For the rear tire to be rolling without slipping,

$$v_r = (v_x, 0),$$

implying $v_y = b\omega$. The front tire will roll without slipping provided its velocity is parallel to the steered wheel; another way to write this is that

$$v_f \cdot (-\,sin\,\delta\,, cos\,\delta) = 0.$$

This equation gives $tan\, \delta = \left(v_y + a\omega\right)/v_x$, which can be rewritten as

$$\omega = \frac{v_x\, tan\, \delta}{a + b}.$$

Thus, the angular velocity of the tractor is given by the longitudinal speed, wheelbase, and steer.

For the trailer tire to be rolling without slipping,

$$v_t \cdot (-sin\, \psi, cos\, \psi) = 0.$$

By expansion and substitution, the hitch angle dynamics are given by

$$\dot{\psi} = \frac{-v_x}{d}\left(sin\, \psi + \frac{c\, cos\, \psi + d}{a + b} tan\, \delta\right).$$

This gives the articulation rate of the trailer from the longitudinal speed, wheelbase, hitch/trailer geometry, steer, and hitch angle. While the hitch articulation angle is expected to be large, the steer input at the front axle is likely much smaller. Consequently, the small angle approximation $tan\, \delta \approx \delta$ is used in the final the equation of motion:

$$\dot{\psi} = -\frac{v_x}{d} sin\, \psi - \frac{v_x(c\, cos\, \psi + d)}{d(a + b)} \delta.$$

While this equation is for the simplified vehicle model of Figure 1, rather than the full CarSim or TruckSim vehicle, this knowledge is helpful for designing the controller.

Controlling Hitch Angle with Front Steer

The chosen form for the equation of motion for the simplified model is conducive to *feedback linearization*. Given there are no specific requirements on steer input δ other than that it produces the desired hitch angle, consider a new control variable v, where

$$\delta = -\frac{d(a + b)}{v_x(c\, cos\, \psi + d)}\left(v + \frac{v_x}{d} sin\, \psi\right).$$

This expression may be substituted into the equation of motion, producing a much simpler, linear equation of motion:

$$\dot{\psi} = v.$$

The transfer function may be found by taking the Laplace transform:

$$s\Psi(s) - \psi(0) = V(s).$$

Assuming that the initial hitch angle is zero, the transfer function of the linearized model is

$$G(s) \coloneqq \frac{\Psi(s)}{V(s)} = \frac{1}{s}.$$

This system is controlled using the proportional controller of Figure 9, which contains one new parameter K, the proportional control gain. The transfer function of the overall system is

$$H(s) \coloneqq \frac{\Psi(s)}{\Psi_d(s)} = \frac{KG(s)}{1 + KG(s)}.$$

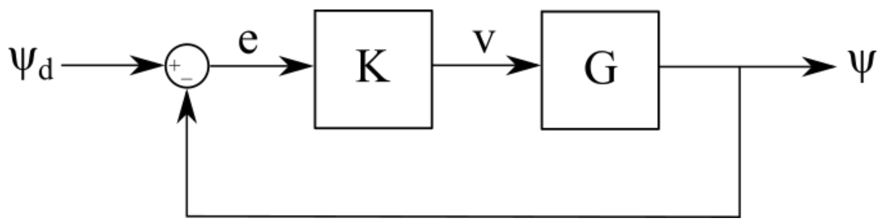

Figure 9. Controller block diagram. "e" is the error between the desired and actual hitch angles.

The system transfer function may be rewritten as

$$H(s) = \frac{K}{s + K}$$

which implies the root of the characteristic equation is $-K$. Consequently, the system is stable for positive values of K.

The step response of the system is

$$\Psi_s = \frac{H}{s} = \frac{K}{s(s + K)} = \frac{1}{s} - \frac{1}{s + K}.$$

Using the inverse Laplace transform, the step response in the time domain is

$$\psi_s = 1 - e^{-Kt}.$$

Various choices of K are compared in Figure 10. In each case, the hitch angle eventually matches that desired (1 rad), with K establishing the time constant of the response. One can show that the time constant $\tau = 1/K$.

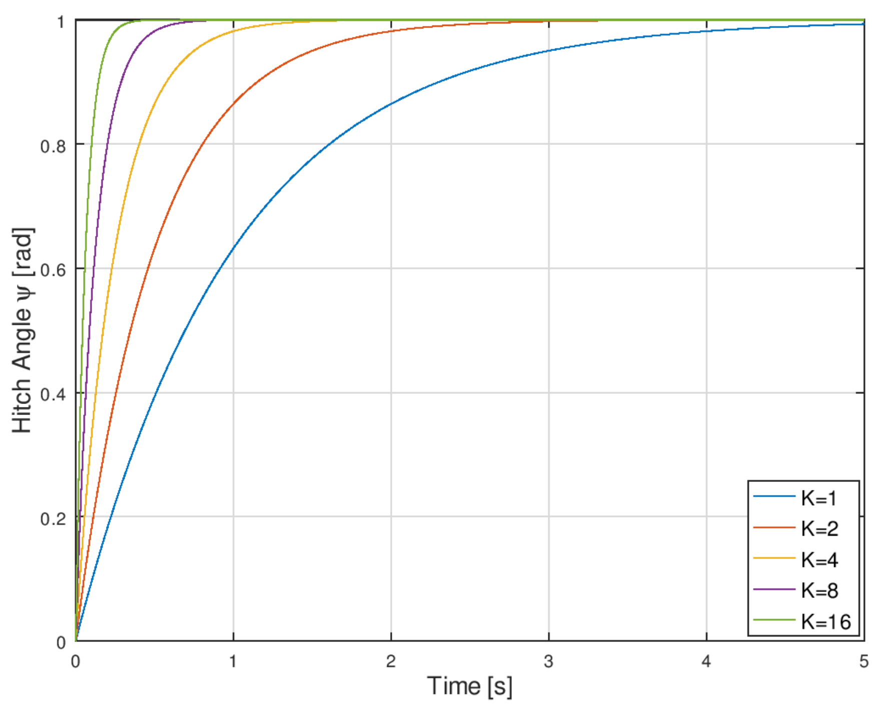

Figure 10. The effect of proportional gain K on the step response of the system. K has units of s^{-1}.

Putting all of this together, the steer input which tracks the desired hitch angle is given by

$$\delta = -\frac{d(a+b)}{v_x(c\cos\psi + d)}\left(K(\psi_d - \psi) + \frac{v_x}{d}\sin\psi\right).$$

This is valid for the simplified vehicle model of Figure 1, and can be used as input to a CarSim or TruckSim vehicle provided the detailed model is reasonably represented by the simple model (hence the restrictions on when the TBC is available even though it is installed). The input to the CarSim or TruckSim vehicle's handwheel is then given by

$$\delta_{sw} = i_s\delta,$$

where i_s is the tractor's steering ratio, as seen by the output variable `RstrA_1`. This handwheel angle value is subject to the magnitude and velocity limits per the driver model parameters `A_SW_MAX_DM` and `AV_SW_MAX_DM`, which are settable from the closed-loop driver model screen (Figure 3).