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Vehicle Dynamics Terminology

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This memo defines specialized terms applicable to VS Math Models from Mechanical Simulation in products such as BikeSim, CarSim, and TruckSim. The terminology is used throughout VS documentation and applies to the naming of parameters, output variables, import variables, and state variables. When applicable, the definitions draw mainly from two sources:

1. ISO 8855, **Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary** (2010)
2. SAE Surface Vehicle Recommended Practice J670 **Vehicle Dynamics Terminology** (2008)

Terms defined in this memo are shown in italics. Within each subsection, terms are defined in alphabetical order. **Vectors** are always shown in bold typeface. As in other VS documentation, `Courier` font is used for symbols and names of computer variables and keywords.

Machine-Generated Documents

BikeSim, CarSim, and TruckSim provide machine-generated text documentation for listing all available output variables, import variables, and state variables. These can be viewed from the **Run Control** screen of a VS Browser using the **View** button in the lower-right corner, with the type of documentation selected with an adjacent drop-down control (Figure 1). These files are also used to select output variables on the **Plot Setup** screen and **I/O Channel** screens.

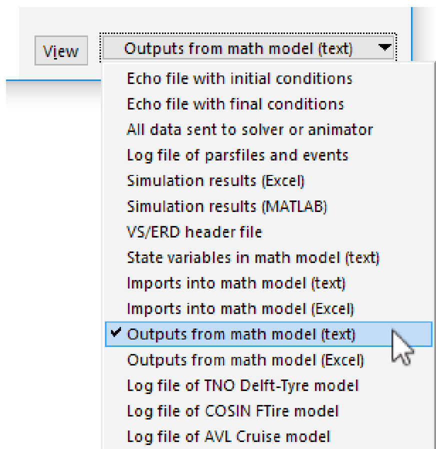

Figure 1. Viewing machine-generated documentation from the Run Control screen.

Axis Directions

Some of the VS documentation uses a convention from multibody dynamics in which unit vectors that indicate the direction of an axis are indicated with a lower-case bold letter indicating a *reference frame* with x-y-z subscripts (e.g., the axis directions for the sprung mass are written $\mathbf{s}_X$, $\mathbf{s}_Y$, and $\mathbf{s}_Z$).

In the machine-generated text files, a reference frame is identified by a capital letter (e.g., S for a sprung mass), and the directions of the three axes of the coordinate system are written with lower-case letters enclosed by square braces (e.g., `[sx]` is the direction of the X axis of reference frame S, $\mathbf{s}_X$). Table 1 lists the types of axis directions that are used in the machine-generated text files, along with the corresponding ISO/SAE names, when applicable.

Table 1. Axis directions as the appear in machine-generated text files.

Text symbols	ISO/SAE symbols	Axis system
`[nx]`, `[ny]`, `[nz]`	$\mathbf{X}_E$, $\mathbf{Y}_E$, $\mathbf{Z}_E$	N: Earth-fixed axis system, Newtonian reference
`[x]`, `[y]`, `[z]`	**X**, **Y**, **Z**	Intermediate axis system
`[sx]`, `[sy]`, `[sz]`	$\mathbf{X}_V$, $\mathbf{Y}_V$, $\mathbf{Z}_V$	S: vehicle sprung mass
`[kx]`, `[ky]`, `[kz]`	—	Motion sensor fixed in body such as sprung mass
`[rx]`, `[ry]`, `[rz]`	—	Road reference frame
`[wx]`, `[wy]`, `[wz]`	—	W: wheel knuckle rigid body (non-spinning)

Angles and Angular Rates

Euler angles for a body in a multibody system are angles used in sequence to unambiguously define the orientation of the body in 3D space. For example, the Euler angles for a sprung mass in a VS Math model are yaw, pitch, and roll. Starting with the body axes aligned with `[nx]`, `[ny]`, and `[nz]`, the body is first rotated about the Z axis (yaw), then the new rotated Y axis (pitch), then the new rotated X axis (Euler roll). The third angle is between the final orientation of the body and a virtual orientation (before the third rotation) that does not physically exist. Hence, the third Euler angle cannot be measured directly in a physical system.

Some of the output variables are components of motion vectors such as velocity, acceleration, angular velocity, etc. The direction associated with a component is usually mentioned in the description (e.g., `[y]`).

Angular velocity variables typically begin with AV and angular acceleration variables typically begin with AA. Be aware that the angular velocity variables are not necessarily the derivatives of similarly named angular variables in the model. For example, AVx is the component of the roll angular velocity vector with respect to the sprung mass X axis; this is usually not equal to AV_R, the derivative of the sprung mass roll angle.

The word *spin* is used only for a moving part with a spin axis, and identifies the angular speed or acceleration of the body rotating about the spin axis.

Axis Systems and Coordinate Systems

Multibody vehicle dynamics models are typically generated using right-handed *axis systems* and *coordinate systems*. The SAE, ISO, and VS vehicle axis orientation has X pointing forward, Z pointing up, and Y pointing to the left-hand side of the vehicle.

> **Note** SAE J670 also includes vehicle axis systems where Z points down. The Z-down convention is never used in VS Math Models.

Unit vectors that indicate the direction of an axis are represented with capital letters in J670 and ISO 8855. In this document, axis names are shown with capital letters (e.g., Z axis), and the associated direction vectors are shown with capital bold letters (e.g., **Z**). As noted earlier, the plain-text machine-generated documents depict unit vectors with square braces, e.g., `[nz]`.

Definitions of Terms

Axis system — a set of three orthogonal X, Y, and Z axes. In a right-handed system, $\mathbf{Z} = \mathbf{X} \times \mathbf{Y}$. (Here, **X**, **Y**, and **Z** refer to an arbitrary orthogonal axis system.)

Coordinate system — a numbering convention used to assign a unique ordered trio of numbers to each point in a *reference frame*. A typical rectangular coordinate system consists of an *axis system* plus an origin point.

Earth-fixed axis system ($\mathbf{X}_E$, $\mathbf{Y}_E$, $\mathbf{Z}_E$) — right-handed orthogonal *axis system* that serves as the global *inertial reference* frame. The $\mathbf{Z}_E$ direction is parallel to the gravity vector. $\mathbf{Z}_E$ points up in the VehicleSim and ISO systems. These directions usually match those of the *inertial (Newtonian) reference*.

The convention in VS example datasets is that $\mathbf{X}_E$ points east and $\mathbf{Y}_E$ points north (when curvature of the earth is negligible in the definitions of east and north).

VS Math Model documentation files designate these directions as `[nx]`, `[ny]`, and `[nz]`, referring to the inertial reference N.

Earth-fixed coordinate system — *coordinate system* based on the *earth-fixed axis system*. The origin normally lies somewhere on the Earth's surface.

Inertial (Newtonian) reference — a reference frame (N or E) that is assumed to have zero *acceleration* (both *linear* and *angular*) and zero *angular velocity*.

Intermediate axis system (**X**, **Y**, **Z**) — right-handed orthogonal *axis system* whose **Z** axis is parallel to $\mathbf{Z}_E$, and whose **Y** axis is perpendicular to both $\mathbf{Z}_E$ and $\mathbf{X}_V$. This axis system can be obtained by rotating the Earth-fixed axis system about $\mathbf{Z}_E$ by the vehicle *yaw angle* (ψ). VS machine-generated documentation files designate the **X** and **Y** directions as `[x]` and `[y]`.

Reference frame — a geometric environment in which points remain fixed with respect to each other at all times. Each *axis system* is tied to a single reference frame. A reference frame may have multiple axis systems, but in most cases there is a one-to-one correspondence. Reference frames and axis systems are identified in machine-generated documentation with capital letters such as N (inertial reference) and S (sprung mass).

Ground axis system ($\mathbf{X}_G$, $\mathbf{Y}_G$, $\mathbf{Z}_G$) — right-handed orthogonal *axis system* whose $\mathbf{Z}_G$ axis is normal to the ground, at the *center of tire contact*, and whose $\mathbf{X}_G$ axis is perpendicular to the wheel *spin axis* ($\mathbf{Y}_W$). For uneven ground, a different ground axis system can exist for each tire. The VS tire documentation uses the multibody convention to identify these directions as $\mathbf{g}_X$, $\mathbf{g}_Y$, and $\mathbf{g}_Z$.

Road axis system ($\mathbf{X}_R$, $\mathbf{Y}_R$, $\mathbf{Z}_R$) — right-handed orthogonal *axis system* whose $\mathbf{Z}_R$ axis is normal to the ground, at a point located such that the $\mathbf{Z}_R$ axis passes through the vehicle *aerodynamic reference point*. The $\mathbf{X}_R$ axis is perpendicular to $\mathbf{Z}_R$ and $\mathbf{Y}_V$, and $\mathbf{Y}_R$ is perpendicular to $\mathbf{X}_R$ and $\mathbf{Z}_R$. These axis directions are used in VS Math Models to define roll and pitch relative to the road, for use on surfaces with significant inclination. VS Math Model documentation files designate these directions as `[rx]`, `[ry]`, and `[rz]`.

Ground plane — a reference plane tangent to the ground surface at the tire *contact center*. For uneven ground, a different ground plane can exist for each tire.

Vehicle axis system ($\mathbf{X}_V$, $\mathbf{Y}_V$, $\mathbf{Z}_V$) — right-handed orthogonal *axis system* fixed in the *vehicle reference frame*. The $\mathbf{X}_V$ axis is primarily horizontal in the *vehicle plane of symmetry* and points forward. The $\mathbf{Z}_V$ axis is vertical and the $\mathbf{Y}_V$ axis is lateral. The directions should coincide with the *earth-fixed axis system* when the vehicle is upright and aligned with the $\mathbf{X}_V$ axis parallel to the $\mathbf{X}_E$ axis.

VS machine-generated documentation files designate the $\mathbf{X}_V$, $\mathbf{Y}_V$, $\mathbf{Z}_V$ directions as `[sx]`, `[sy]`, and `[sz]`, referring to the sprung mass rigid body S.

Vehicle plane of symmetry — the lateral center plane of the vehicle. The $\mathbf{Y}_V$ direction is normal to this plane.

Vehicle reference frame — reference frame associated with the vehicle body. It is typically defined to coincide with the undeformed body of the vehicle body structure. In a VS Math Model, this is the sprung mass.

Wheel axis system ($\mathbf{X}_W$, $\mathbf{Y}_W$, $\mathbf{Z}_W$) — right-handed orthogonal *axis system* whose $\mathbf{Y}_W$ axis is parallel with the *spin axis* of the wheel and whose $\mathbf{X}_W$ axis is perpendicular to $\mathbf{Z}_G$. The VS tire documentation uses the multibody convention to identify these directions as $\mathbf{w}_X$, $\mathbf{w}_Y$, and $\mathbf{w}_Z$.

The multibody model has a non-spinning wheel knuckle. The axes fixed in the rigid body do not match the ISO/SAE wheel axis system. The rigid-body axes are used to define force and moments applied by the tire(s) to the wheel body origin, and are written in machine-generated documentation as `[wx]`, `[wy]`, and `[wz]`.

Mathematical Definitions

All of the ISO/SAE coordinate systems and axes are built from five reference directions:

1. vertical, as defined by the direction of the gravity vector ($-\mathbf{Z}_E$ or `-[nz]`) ,
2. the X axis of the *vehicle reference frame* ($\mathbf{X}_V$ or `[sx]`),
3. the Y (spin) axis of a wheel of interest ($\mathbf{Y}_W$),
4. the direction normal to the ground at the *center of tire contact* ($\mathbf{Z}_G$), and
5. the direction normal to the ground at a point located such that vertical axis ($\mathbf{Z}_R$) passes through the vehicle *aerodynamic reference point*.

Table 2 defines the intermediate, wheel, and ground axis systems in terms of $\mathbf{X}_E$, $\mathbf{Y}_E$ and $\mathbf{Z}_E$, $\mathbf{X}_V$, $\mathbf{Y}_V$ and $\mathbf{Z}_V$, $\mathbf{Y}_W$, and $\mathbf{Z}_G$. Two equivalent definitions are provided for the $\mathbf{X}$ and $\mathbf{Y}$ directions (*intermediate axis system*).

Vectors

A multibody model is described with many vector quantities. However, output variables may only be scalars. Most outputs are defined mathematically as vector dot products between a vector quantity (e.g., *angular velocity*) and a unit vector (e.g., `[sx]`).

Motion Vectors

Acceleration vector (linear) — time derivative of the *velocity vector* of a point.

Angular acceleration vector — time derivative of the *angular velocity vector* of a *reference frame*.

Table 2. Axis system definitions.

Name	X Direction	Y Direction	Z Direction
Earth $\mathbf{X}_E, \mathbf{Y}_E, \mathbf{Z}_E$	$\mathbf{X}_E$	$\mathbf{Y}_E$	$\mathbf{Z}_E$
Vehicle $\mathbf{X}_V, \mathbf{Y}_V, \mathbf{Z}_V$	$\mathbf{X}_V$	$\mathbf{Y}_V$	$\mathbf{Z}_V$
Intermediate $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ ($\mathbf{Z} = \mathbf{Z}_E$)	$\frac{(\mathbf{Z}_E \times \mathbf{X}_V) \times \mathbf{Z}_E}{\lvert(\mathbf{Z}_E \times \mathbf{X}_V) \times \mathbf{Z}_E\rvert}$ or $\mathbf{X}_E \cos(\psi) + \mathbf{Y}_E \sin(\psi)$	$\frac{\mathbf{Z}_E \times \mathbf{X}_V}{\lvert\mathbf{Z}_E \times \mathbf{X}_V\rvert}$ or $\mathbf{Y}_V \cos(\psi) - \mathbf{X}_E \sin(\psi)$	$\mathbf{Z}_E$
Wheel $\mathbf{X}_W, \mathbf{Y}_W, \mathbf{Z}_W$	$\frac{\mathbf{Y}_W \times \mathbf{Z}_R}{\lvert\mathbf{Y}_W \times \mathbf{Z}_R\rvert}$	$\mathbf{Y}_W$	$\frac{(\mathbf{Y}_W \times \mathbf{Z}_R) \times \mathbf{Y}_W}{\lvert(\mathbf{Y}_W \times \mathbf{Z}_R) \times \mathbf{Y}_W\rvert}$
Ground $\mathbf{X}_G, \mathbf{Y}_G, \mathbf{Z}_G$ ($\mathbf{X}_G = \mathbf{X}_W$)	$\frac{\mathbf{Y}_W \times \mathbf{Z}_R}{\lvert\mathbf{Y}_W \times \mathbf{Z}_R\rvert}$	$\frac{\mathbf{Z}_R \times (\mathbf{Y}_W \times \mathbf{Z}_R)}{\lvert\mathbf{Z}_R \times (\mathbf{Y}_W \times \mathbf{Z}_R)\rvert}$	$\mathbf{Z}_G$

Angular velocity vector — vector describing the absolute 3D angular velocity of a *reference frame* with respect to the *inertial reference*. Formally, angular velocity is a quantity that satisfies the equation:

$$\dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}$$

where $\mathbf{r}$ is a vector fixed in a reference frame and $\boldsymbol{\omega}$ is the angular velocity vector of this reference frame.

Position vector — *vector* describing the position of one point relative to a reference point. Unless specified otherwise, the reference point is the origin of the *earth-fixed coordinate system.*

Vector — object that has a direction in 3D space and a magnitude. The existence and meaning of a vector are not dependent on the choice of coordinate or axis system.

Velocity vector — time derivative of the *position vector* of a point.

Resultant Force and Moment Vectors

All actions on a body that would cause it to accelerate in translation or rotation if not opposed by other actions can be combined into a single *resultant force vector* and a single *resultant moment vector* about a point at which the resultant force vector is applied. (Although a moment vector applies to an entire rigid body, the resultant moment vector depends on the location of the specific point associated with the resultant force.) Resultant force and moment vectors are used to describe actions on the vehicle due to the ground, the air, and impacts with other objects.

Entire vehicle

Dimensions and Coordinates

Coordinates in the **X**, **Y**, and **Z** directions of a coordinate system are usually indicated with symbols that begin with the letter X, Y, and Z. Coordinates can have positive or negative values.

Lengths are used for positive distances between two objects, and are usually represented in a VS Math Model with a symbol beginning with a capital L.

Components of vectors

Forces, moments, and motion vectors for the entire vehicle are commonly decomposed into three rotational and three translational terms. The following adjectives are used.

Lateral — **Y** component of force or translational motion vector.

Longitudinal — **X** component of force or translational motion vector.

Pitch — **Y** component of moment or rotational motion vector.

Roll — **X** component of moment or rotational motion vector.

Vertical — **Z** component of force or translational motion vector.

Yaw — **Z** component of moment or rotational motion vector.

Points

C.G. (Center of gravity) — a point in the *vehicle reference frame* that coincides with the center of mass of the entire vehicle when the suspensions are in equilibrium and the vehicle is resting on a flat level surface.

Aerodynamic reference point — a point in the *vehicle reference frame* that is used to define aerodynamic effects. It can be set to any location in VS Math Models. SAE specifies that this point is at the intersection of the *vehicle plane of symmetry* and the ground plane, mid-way between the front and rear axles, when the suspensions are in equilibrium and the vehicle is resting on a flat level surface.

Vertical position Z — $\mathbf{Z}_E$ coordinate of the *C.G.*

X position X — $\mathbf{X}_E$ coordinate of the *C.G.*

Y position Y — $\mathbf{Y}_E$ coordinate of the *C.G.*

Translational Motion

Lateral acceleration A_y — **Y** component of *acceleration vector* of the *C.G.*

Lateral velocity V_y — **Y** component of *velocity vector* of the *C.G.*

Longitudinal acceleration A_x— **X** component of *acceleration vector* of the *C.G.*

Longitudinal velocity V_x — **X** component of *velocity vector* of the *C.G.*

Vertical acceleration A_z — **Z** component of *acceleration vector* of the *C.G.*

Vertical velocity V_z — **Z** component of *velocity vector* of the *C.G.*

Angles

Aerodynamic sideslip angle β_{aero} — angle from **X** to velocity vector of air relative to the *vehicle reference frame*.

Euler angles (ψ, θ, ϕ) — sequence of consecutive rotations about $\mathbf{Z}_E$ (yaw), **Y** (pitch), and $\mathbf{X}_V$ (roll) axes to convert from the *earth-fixed axis system* to the *vehicle axis system*. Note that ϕ is not identical to *roll* (ϕ_V). The relationship is: $\phi = \sin^{-1}(\sin(\phi_V)/\cos(\theta))$

Pitch θ — angle from **X** to $\mathbf{X}_V$, about **Y**. θ can be calculated using a vector dot product: $\theta = -\sin^{-1}(\mathbf{X}_V \bullet \mathbf{Z}_E)$

Roll ϕ_V — angle from $\mathbf{X}_E \times \mathbf{Y}_E$ plane to $\mathbf{Y}_V$, about **X.** Roll can be calculated using a vector dot product: $\phi_V = \sin^{-1}(\mathbf{Y}_V \bullet \mathbf{Z}_E)$. It can also be calculated from the Euler angles θ and ϕ:

$$\phi_V = \sin^{-1}(\cos(\theta)\ \sin(\phi)).$$

Sideslip angle β — angle from the **X** to the projection of the C.G. velocity vector onto the $\mathbf{X} \times \mathbf{Y}$ plane, about **Z**.

Sideslip can be calculated from the lateral velocity V_y and longitudinal velocity V_x.

$$\beta = \tan^{-1}(V_y/V_x)$$

Yaw ψ — angle from $\mathbf{X}_E$ to **X**, about **Z**. The convention in VS Math Models is that yaw is zero when the vehicle X axis aligns with the global X axis.

The convention in VS example datasets is that the global X axis points east; this convention implies that yaw is zero when the vehicle is pointing east, 90° when pointing north, etc.

Angular Velocity and Acceleration

Pitch acceleration α_y — **Y** component of *angular acceleration vector* of *vehicle reference frame*.

Pitch velocity ω_y — **Y** component of *angular velocity vector* of *vehicle reference frame*.

Roll acceleration α_x — **X** component of *angular acceleration vector* of the *vehicle reference frame*.

Roll velocity ω_x — **X** component of *angular velocity vector* of *vehicle reference frame*.

Yaw acceleration α_z — **Z** component of *angular acceleration vector* of the *vehicle reference frame*.

Yaw velocity ω_z — **Z** component of vehicle *angular velocity vector* of the *vehicle reference frame*.

Aerodynamic Forces and Moments

Forces and moments acting from the air on the vehicle are summed into a single resultant aerodynamic force vector, and a single resultant aerodynamic moment vector taken about the *aerodynamic reference point*.

Aerodynamic lateral force F_{yaero} — **Y** component of aerodynamic resultant force.

Aerodynamic longitudinal force F_{xaero} — **X** component of aerodynamic resultant force.

Aerodynamic pitch moment M_{yaero} — **Y** component of aerodynamic resultant moment.

Aerodynamic roll moment M_{xaero} — **X** component of aerodynamic resultant moment.

Aerodynamic vertical force F_{zaero} — **Z** component of aerodynamic resultant force.

Aerodynamic yaw moment M_{zaero} — **Z** component of aerodynamic resultant moment.

Sprung Mass

As with other coordinate systems, the axes of the sprung mass coordinate system are oriented with the X axis forward, the Z axis up, and the Y axis pointing to the left side of the vehicle. The location of the origin is not rigorously defined in VS Math Models, to allow compatibility with a variety of conventions and existing datasets. However, it is common to locate the origin in a X-Z plane that splits the sprung mass laterally (the plane of symmetry), and in a Y-Z plane containing the spin axis of the front wheel.

Heights of points of interest that are not fixed in the sprung mass (e.g., wheel centers) are specified relative to the origin of the sprung mass when the vehicle is in a *design load condition*. Given that the physical location and orientation of the sprung mass relative to the ground depends on the tire sizes and the loads carried by the vehicle, coordinates of points in other bodies (e.g., wheel centers) will change with load conditions.

Design load condition — a load condition used to define vertical locations of various points in the vehicle relative to the origin of the sprung mass coordinate system when the sprung mass pitch and roll angles are zero. The specific conditions that define the design load tend to vary between companies so there is not single definition associated with VehicleSim models. However, it is important that all heights apply for the same load condition.

For more information about design loads, please see the Technical Memo *The Design Load Condition in CarSim and TruckSim*.

Suspensions and steering

Axles and Suspensions

Axle — a wheel or a group of wheels and associated components with the same longitudinal position on the vehicle. There may or may not be an actual physical axle that connects the wheels on the left side of the vehicle to those on the right.

Independent suspension — a suspension in which vertical movement of a wheel on one side of the axle does not cause noticeable movement of the other wheel if the anti-roll bar is disconnected.

Solid-axle suspension — a suspension that has an actual axle or linkage system that causes the non-spinning parts of both wheels to move together as one body.

Suspension — a mechanical system that allows the wheels to move vertically relative to the vehicle body, while controlling positions and angles of the wheels relative to the ground. For *solid-axle suspensions*, the term *suspension* applies to the entire system for the axle. For *independent suspensions*, the term *suspension* refers to one side.

Size and Weight

Track L_{TK} — SAE and ISO define track as the measured distance between the *centers of tire contact* for one axle. In case of dual wheels, the midpoints of the *centers of tire contact* for each side are used. Track typically varies slightly with suspension *jounce* for *independent* suspensions.

VS Math Models define track as the distance between the wheel centers in the *design load condition*. (This definition is independent of the tire size and properties.)

Unsprung weight — portion of weight supported by a tire that is considered to move with the wheel. This usually includes a portion of the weight of the suspension elements.

Wheelbase — SAE and ISO define wheelbase as the measured distance between the *centers of tire contact* for two axles on one side of the vehicle.

VS Math Models do not include wheelbase as a parameter; they use the distance the wheel center is behind the origin of the sprung mass. (The X coordinate of the nominal wheel center location is nearly always a negative number; thus, the VS convention defines parameters whose values are nearly always positive.)

Kinematics

Camber — outward angular lean of wheel relative to *vehicle reference frame*: angle from $\mathbf{Z}_V$ to the $\mathbf{X}_W \times \mathbf{Z}_W$ plane. In VS Math Models, camber is defined as $\sin^{-1}(\mathbf{Y}_W \cdot \mathbf{Z}_V)$ for the left wheel and with the opposite sign for the right wheel such that outward lean relative to $\mathbf{Z}_V$ is always positive.

The symmetric sign convention for camber is convenient for describing certain kinematical and compliance relationships for both sides of the vehicle.

Damper mechanical advantage R_d — ratio of damper compression per unit of wheel jounce. This ratio is usually less than unity.

Driver steer δ_d — portion of *steer* due to steering wheel angle, with no forces or moments applied by the ground to the tires, and with no suspension movement.

Jounce — vertical movement of wheel or axle relative to the *vehicle reference frame*. Jounce is positive for compressive movement (wheel moving up relative to the body). Downward movement of the wheel relative to the body is *rebound*. There is no standard definition of zero jounce.

Roll center — imaginary point in the $\mathbf{Y}_V \times \mathbf{Z}_V$ plane containing the two *wheel centers* of an axle, at which a lateral force applied to the vehicle body is reacted without producing a *suspension roll angle*. An alternate definition is that the roll center is the intersection of the two lines shown in Figure 2.

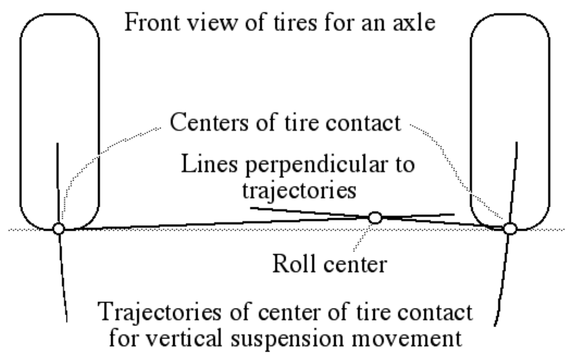

Figure 2. Roll center.

Spring mechanical advantage R_s — ratio of spring compression per unit of wheel jounce. This ratio is usually less than unity.

Steer δ — angle from $\mathbf{X}$ to $\mathbf{X}_W$, about $\mathbf{Z}$.

Suspension roll angle — angle from line joining the *wheel centers* of an axle to the $\mathbf{X}_V \times \mathbf{Y}_V$ plane in the *vehicle reference frame*.

Toe — inward steer of wheel relative to the *vehicle reference frame*: angle from $\mathbf{X}$ to $\mathbf{X}_W$ with the sign set such that inward steer is positive. The symmetric sign convention is convenient for describing certain kinematical and compliance relationships for both sides of the vehicle.

Forces and Moments

Auxiliary roll moment M_{aux} — the *suspension roll moment* minus the moments due to the *suspension forces* from the two sides. A positive moment causes positive vehicle roll.

Damping force F_d — compressive force applied to the vehicle body by a damper.

Spring force F_s — compressive force applied to the vehicle body by a suspension spring.

Suspension roll moment M_{roll} — total static roll moment applied to *sprung mass* due to *suspension roll angle*. A positive moment causes positive vehicle roll.

Tires and wheels

Kinematics

Center of tire contact — point at intersection of a line passing through the wheel center and the ground, where the line is parallel with $\mathbf{Z}_W$. The *center of tire contact* is not necessarily at the center of the physical tire contact patch.

Inclination γ — lean (angle) of wheel relative to *ground plane*: angle from $\mathbf{Z}_G$ to $\mathbf{Z}_W$, about $\mathbf{X}_G$.

Longitudinal slip — the ratio: $\frac{\omega - \omega_0}{\omega_0}$

where ω is the *angular velocity* of the wheel about its *spin axis* and ω_o is the free rolling angular velocity of the wheel that would be measured at zero *slip angle* and zero *inclination*. (ω_o is the longitudinal velocity of the wheel center, divided by the effective circumference of the tire at that speed and load condition.) (SAE, ISO)[13]

Slip angle α — angle from $\mathbf{X}_G$ to the *velocity vector* of the *center of tire contact*, about $\mathbf{Z}_G$.

Spin axis $\mathbf{Y}_W$ — axis of rotation of wheel about spindle.

Wheel center — intersection of *spin axis* and *wheel plane*.

Wheel plane — central plane of wheel, normal to the *spin axis*.

Forces and Moments

Forces and moments acting from the ground on the tire are summed into a single resultant force vector and a single resultant moment vector taken about the *center of tire contact.*

Aligning moment M_z — $\mathbf{Z}_G$ component of ground resultant moment.

Overturning moment M_x — $\mathbf{X}_G$ component of ground resultant moment.

Rolling moment M_y — $\mathbf{Y}_G$ component of ground resultant moment.

Driving moment— $\mathbf{Y}_W$ component of moment applied by the vehicle to the wheel about the spin axis.

Lateral tire force F_y — $\mathbf{Y}_G$ component of ground resultant force.

Longitudinal tire force F_x — $\mathbf{X}_G$ component of ground resultant force.

Vertical tire force F_z — $\mathbf{Z}_G$ component of ground resultant force.

Roads and Paths

A VS Reference Path is a continuous line that exists in a horizontal plane with continuity in position and gradient. That is, there are no sharp corners. Paths are used in VS Math Models to define 2D coordinate systems that are alternatives to the global X and Y coordinates used in vehicle dynamics. The path coordinates are station S (distance along the path) and lateral coordinate L (distance a point is from the path, measured on a line that intersects the point and the path, and is perpendicular to the path at the point of intersection (Figure 3).

Heading — angular direction of the path at a designated *station S*. In VS Math Models, heading and *yaw* use the same convention: a vehicle oriented to follow a path has a yaw angle equal to the path heading. The convention in VS examples is that a heading of zero means the path is pointing due east.

Heading (relative) — yaw of a vehicle relative to the instant heading of a path. When heading is used to describe a vehicle or moving object relative to a path, the relative heading is the yaw of the object minus the instant heading of the path at the station where the object is located.

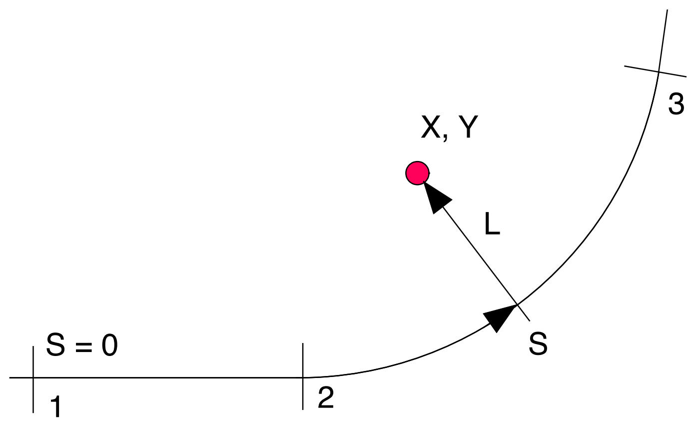

Figure 3. S and L are the coordinates of a reference path.

Lateral coordinate, L — distance a point is from the path, measured on a line that intersects the point and the path, and is perpendicular to the path at the point of intersection (Figure 3). The default name of a lateral coordinate variable is *L*.

Looped path — a *reference path* that loops. *Station* on a looped path covers a range limited by the length of one lap of the path.

Reference path — a continuous line that exists in a horizontal plane with continuity in position and gradient. If a path is not looped, is extends infinitely.

Station, S — coordinate used to define any point along the path. The default name of a station variable is *S*. The range of S for a non-looped path is ±∞. For a *looped path*, station is limited to a range that matches the length of one lap of the path.