

AMA 4205 UNIT NAME: CALCULUS II

ASSIGNMENT II

BSIT/BTME/BTIT/BSSC/BMCS

The assignment

Work in groups of four and members must be different from those you worked with in assignment I

[every member to compile 2 questions only]

Due date; 5/12/2023

1. By letting $x = a \cosh \theta$ evaluate $\int \sqrt{x^2 - a} dx$
2. Evaluate $\int \frac{t^2}{\sqrt{25-t^2}} dt$
3. Find the volume of solid of revolution formed by revolving the region bounded by $y = x - x^2$ and the x -axis $0 \leq x \leq 1$ about y -axis
4. Evaluate $\int \frac{x^2+12x+12}{x^3-4x} dx$
5. Determine the area of a region bounded by curves $x = y^2 + 1$ and $x = y + 3$
6. Find the arc length of the graph of $y = \ln(\cos x)$ from $x = 0$ and $x = \frac{\pi}{4}$
7. The marginal cost of producing some, x bolts is given by $\frac{dC}{dx} = 4x + \frac{4x}{\sqrt{16+x^2}}$.
Determine the cost C as a function of x given that the cost of producing 100 such bolts is Kshs.50,040 hence calculate the cost of producing 450 bolts.
8. Show that the height above the ground of an object thrown vertically upward from a point S_0 feet above the ground with an initial velocity v_0 feet per second is given by $h(t) = -16t^2 + v_0 t + S_0$ (take $g = \frac{32ft}{s^2}$)

Further revision questions

- a) Use the **substitution** $x = \sec \theta$, find the exact value of $\int_{\sqrt{2}}^2 (x^2 - 1)^{\frac{-3}{2}} dx$
- b) Use **integration by parts** to find the exact value of $\int_0^{\ln 2} x^2 e^x dx$
- c) Find the constants A , B and C such that $\frac{x^2+5}{x+2} = Ax + B + \frac{C}{x+2}$ hence find the integral $\int \frac{x^2+5}{x+2} dx$
- d) Find $\int \frac{3}{\sqrt{-4x^2-4x+8}} dx$

- e) Use trapezoidal rule with $h = 0.5$ to **determine** the approximate area R (shown in figure 1 below) under the curve of the function $y = \sqrt{2^x + 1}$ for $0 \leq x \leq 3$

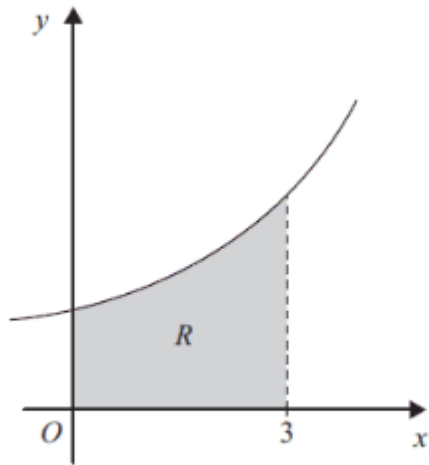
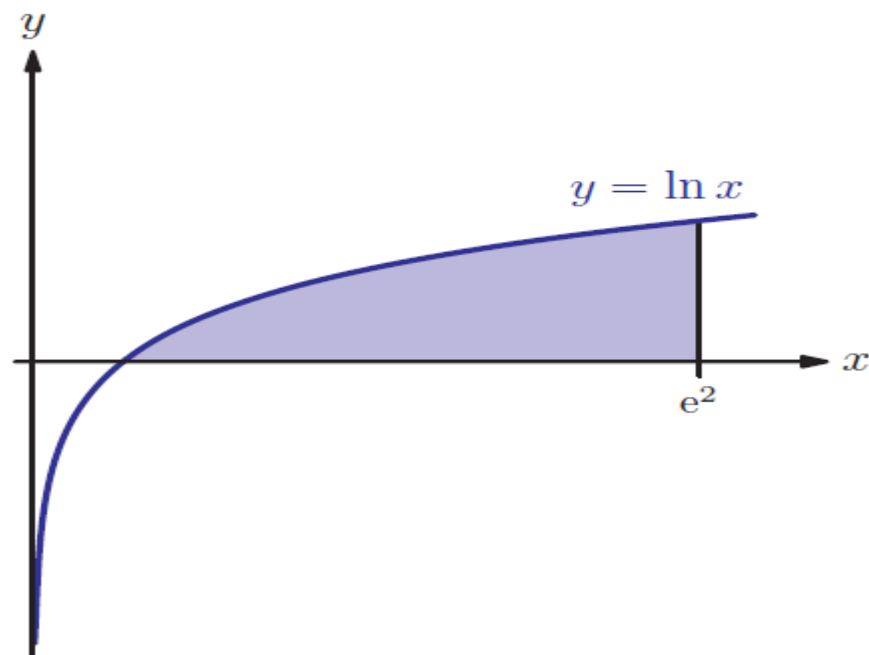


Figure 1

- f) Find $\int \frac{x+1}{\sqrt{1-x^2}} dx$
- g) The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 - 8x + 5$ and the curve passes through the point $(1, -4)$. **Find** the equation of the curve
- h) In this question you shall calculate the volumes of revolution
- I) The graph of $y = \sin 2x$, $0 \leq x \leq \frac{\pi}{2}$ is rotated 360° around the x-axis. **Find** the volume of the solid generated, in terms of π
 - II) Find the **volume of revolution** generated when the shaded region in figure 2 is rotated π radians around the y-axis



- i) **Determine** the arc length of $y = 7(6 + x)^{\frac{1}{2}}$, $189 \leq y \leq 875$

- j) **Show** that $\sin^3 x = \sin x - \cos^2 x \sin x$ hence find $\int \sin^3 x dx$
- k) The velocity (ms^{-1}) of a car at time t seconds after passing a flag is modelled by $v = 17 - 4t$, for $0 \leq t \leq 5$. Find the acceleration of the car and the maximum displacement of the car from the flag?
- l) Work out the indefinite integral given by $\int \sinh^2 x dx$
- m) Use reduction formula to find $\int \cos^5 x dx$
- n) In this question you shall solve a simple harmonic problem
- A mass attached to a spring is free to oscillate, with angular velocity ω , in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time $t = 0$. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0 . [Hint : Start with the equation $\mathbf{x} = a \cos(\omega t + \theta)$ and note that the initial velocity is negative.] [5 marks]
 - From figure three below derive the equation of the oscillating mass [6 marks]

