Yunhong Bao PS 6

3.54 $E[|X|+4] = \frac{1}{3}\times(2+4)+\frac{1}{3}\times 4+\frac{1}{3}\times 5$ $\quad V[|X|+4] = \frac{1}{3}\times(6-5)^2+\frac{1}{3}\times(4-5)^2+\frac{1}{3}\times(5-5)^2$

$= 5$ $\quad = \frac{2}{3}$

3.67 $M_{a+bX}(t) = E[e^{t(a+bX)}] = E[e^{ta}\cdot e^{tbX}] = e^{ta}\cdot E[e^{tbX}] = e^{ta}\cdot E[e^{btX}] = e^{ta}\cdot M_X(bt)$

3.71 $M_X(t) = \cosh t = \frac{e^{t}+e^{-t}}{2}$ $\quad \mu_r = \frac{d^r m}{dt^r}\Big|_{t=0}\ \frac{d^r}{dt^r}\left(\frac{e^t}{2}\right)+\frac{d^r}{dt^r}\left(\frac{e^{-t}}{2}\right) = \frac{1}{2}+\left(\frac{1}{2}\right)(-1)^r$

As the MGF is written as a sum and $M(t) = E[e^{tx}]$, the radom variable is discrete.

the support is $\{1, -1\}$ $\quad f(x) = \begin{cases} \frac{1}{2} & x=1 \\ \frac{1}{2} & x=-1 \end{cases}$

3.74 $E[X(3-X)] = \frac{1}{3}\times(0\times 3)+\frac{2}{3}\times(3\times 0) = 0$

3.78 (a) $\int_0^4 f(x)dx = 1 \Rightarrow \int_0^4 cx\,dx = 1$ $\quad \frac{cx^2}{2}\Big|_0^4 = 1$ $\quad 8c=1$ $\quad c=\frac{1}{8}$

(b) $P(X>1) = \int_1^4 f(x)dx = \int_1^4 \frac{x}{8}dx = \frac{x^2}{16}\Big|_1^4 = \frac{15}{16}$

(c) $E[X] = \int_0^4 xf(x)dx = \int_0^4 \frac{x^2}{8}dx = \frac{x^3}{24}\Big|_0^4 = \frac{16}{6} = \frac{8}{3}$

3.81 The moment generating function is written as a sum. So the random variable is discrete. And the support is $\{4, 7, 9\}$

The PMF is $\begin{cases} 0.2 & x=4 \\ 0.7 & x=7 \\ 0.1 & x=9 \end{cases}$ $\quad$ So $P(X=7) = 0.7$

3.105 Mean $= \int_0^1 x f(x)dx = \int_0^1 \theta x^{\theta} dx = \frac{\theta}{\theta+1}x^{\theta+1}\Big|_0^1 = \frac{\theta}{\theta+1}$

3.117 (a) $E[(X-\pi)^3] = E[X^3-3X^2\pi+3X\pi^2-\pi^3] = 0-3\pi(5)+3\pi^2(2)-\pi^3 = -\pi^3+6\pi^2-15\pi$

(b) $V[17-4X] = V[-4X] = E[16X^2]-(E[-4X])^2 = 16E[X^2]-(E[-4X])^2 = 80-64 = 16$

(c) $V[X^2] = E[X^4]-(E[X^2])^2 = 30-25 = 5$

3.133 (a) $\int_0^1 f(x)dx = 1 \Rightarrow \int_0^1 c(x^2+2x)dx = 1$ $\quad \int_0^1 c(x^2+2x)dx = \left[\frac{c}{3}x^3+cx^2\right]_0^1 = \frac{4}{3}c = 1$ $\quad c=\frac{3}{4}$

(b) $E[X] = \int_0^1 x f(x) = \int_0^1\left(\frac{3}{4}x^3+\frac{3}{2}x^2\right)dx = \left[\frac{3}{16}x^4+\frac{1}{2}x^3\right]_0^1 = \frac{11}{16}$

3.135 The PMF is $\begin{cases} \frac{1}{2} & x=c \\ \frac{1}{2} & x=-c \end{cases}$

$V[X] = \frac{1}{2}c^2+\frac{1}{2}c^2 = c^2 = 1$ $\quad$ So $c = 1$