Hypothesis Testing Cheat Sheet

Introduction

Hypothesis testing in statistics is a way for us to test the results of an experience or survey to see if the results are meaningful.

Key Concepts

- **The null hypothesis H_o:** a default hypothesis that a quantity to be measured is zero (null).
- **The alternative hypothesis H_a:** a position that states something is happening, a new theory is preferred instead of an old one (null).
- **Test statistic:** is the tool we use to decide whether or not to reject the null hypothesis.
- **P-value:** is the probability of getting a sample statistic at least as extreme as the observed value.
- **Critical value:** is the standard score that separates the rejection region from the rest of a given curve.

- **Type I error:** incorrectly rejecting a true null hypothesis (false negative).
- **Type II error:** incorrectly failing to reject an untrue null hypothesis (false positive).

Decision	**H_o is actually true**	**H_o is actually false**
Reject H_o	Type I Error P = α	Correct decision
Fail to reject H_o	Correct decision	Type II Error P = β

Five Steps in Hypothesis testing

1. Specify the Null Hypothesis H_o.
2. Specify the Alternative Hypothesis H_a.
3. Set the Significant Level α.
4. Calculate the **test statistic** and corresponding **p-value**.
5. Interpret the results to accept or reject the null hypothesis.

Z-test

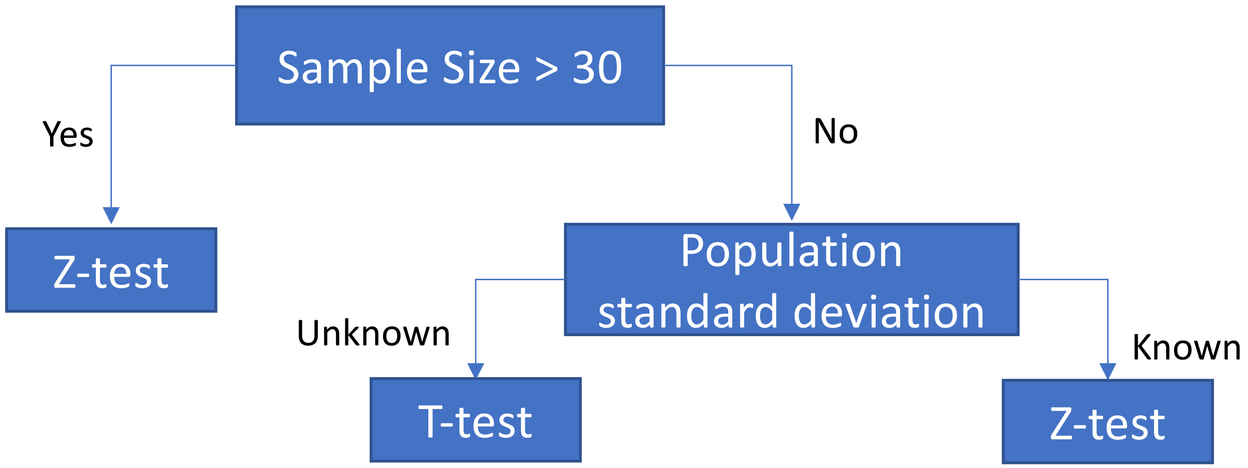
Types	Description	Degrees of Freedom	Z statistic
One-Sample	Test whether a sample mean is different from the population mean	n-1	$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$ where $\bar{x}$ is sample mean μ is population mean σ is population standard deviation n is sample size
Two-Sample	Test whether the means of two populations are significantly different from one another	$(n_1 - 1)$ +$(n_2 - 1)$	$$z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{(\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2})}}$$ where $\overline{x_1} - \overline{x_2}$ is the difference between sample mean $\mu_1 - \mu_2$ is the difference between population mean σ_1, σ_2 are population standard deviation n_1, n_2 are sample size

T-test

Types	Description	Degrees of Freedom	T statistic
One-Sample	Test whether a sample mean is different from the population mean	n-1	$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$ where $\bar{x}$ is sample mean μ is population mean s is sample standard deviation n is sample size
Two-Sample	Test whether the means of two populations are significantly different from one another	$(n_1 - 1)$ +$(n_2 - 1)$	$$t = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{(\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2})}}$$ where $\overline{x_1} - \overline{x_2}$ is the difference between sample mean $\mu_1 - \mu_2$ is the difference between population mean s_1, s_2 are sample standard deviation n_1, n_2 are sample size

Deciding between T-test and Z-test

Use Z-test when	**Use T-test when**
Sample size is big	Sample size is small (less than 30)
Standard deviation of population is known	Standard deviation of population is unknow



Note: If the standard deviation of the population is unknown, but the sample size is greater than or equal to 30, then the assumption of the sample variance equaling the population variance is made while using the z-test.

Chi-Square test

Purpose	Description	Degrees of Freedom	Chi-Square statistic
Goodness of fit	Checks whether an observed pattern of data fits some given distribution	Number of categories in the distribution - 1	$\chi^2_{df} = \Sigma \frac{(O_i - E_i)^2}{E_i}$ where O is the observed value E is the expected value
Test of independence	Determines if there is a significant relationship between two categorical variables	(r−1) +(c−1)	$\chi^2 = \Sigma \frac{(O_{r,c} - E_{r,c})^2}{E_{r,c}}$ where $O_{r,c}$ is the observed frequency count at level *r* of Variable A and level *c* of Variable B. $E_{r,c}$ is the expected frequency count at level *r* of Variable A and level *c* of Variable B.

F test and Analysis of Variance (ANOVA)

Source of Variance	Sums of Squares (SS)	Degrees of Freedom (df)	Mean Squares (MS)	F statistic
Between Treatments	SSB = $\sum n_j(\bar{X}_j - \bar{X})^2$	k-1	$MSB = \frac{SSB}{k-1}$	$F = \frac{MSB}{MSE}$
Error (or Residual)	SSE = $\sum\sum(X - \bar{X}_j)^2$	N-k	$MSE = \frac{SSE}{N-k}$	
Total	SST = $\sum\sum(X - \bar{X})^2$	N-1		

where

- X = individual observation
- $\bar{X}_j$ = sample mean of the j^{th} treatment (or group)
- $\bar{X}$= overall sample mean
- k = number of treatments or independent comparison groups
- N = total number of observation or sample size