Consider the quantum state:  $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$ 

100 qubits are prepared in this quantum state and measured.

In those 100 measurements:

- 1. How many times would you expect to measure a |0)?
- 2. How many times would you expect to measure a  $|1\rangle$ ?

Consider the quantum state:  $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ 

3. Select values for  $\alpha$  and  $\beta$  such that it is more likely that a  $|0\rangle$  will be measured than  $|1\rangle$ .

$$\alpha = \frac{1}{\sqrt{5}}$$

$$\alpha = \frac{2}{\sqrt{5}}$$

$$\alpha = \frac{2}{\sqrt{5}}$$

$$\alpha = \frac{1}{\sqrt{5}}$$

$$\alpha = \frac{1}{\sqrt{5}}$$

$$\alpha = \frac{3}{\sqrt{5}}$$

$$\alpha = \frac{3}{\sqrt{5}}$$

$$\beta = \frac{4}{\sqrt{5}}$$

$$\beta = \frac{2}{\sqrt{5}}$$

$$\beta = \frac{2}{\sqrt{5}}$$

Consider the quantum state :  $\frac{1}{\sqrt{2}} \mid \! 0 \rangle + \frac{1}{\sqrt{2}} \mid \! 1 \rangle$ 

- 4. What is the probability of measuring a |1)?

  - a.  $\frac{1}{2}$  b.  $\frac{1}{4}$  c.  $\frac{1}{5}$

- 5. The notation used to describe the quantum state above is called
  - vector notation
- *b* bra-ket notation

c. standard basis notation

- 6. Which of the following describes the same quantum state?

- $c. \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

none of the above

Consider the quantum state :  $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ 

7. What is the probability of measuring a |0)?

a. 
$$\frac{1}{2}$$
 b.  $\frac{1}{4}$  c.  $\frac{3}{4}$ 

b. 
$$\frac{1}{4}$$

$$c. \frac{3}{4}$$

$$d. \frac{\sqrt{3}}{4}$$

8. What is the probability of measuring a  $|1\rangle$ ?

a. 
$$\frac{1}{2}$$
 b.  $\frac{1}{4}$  c.  $\frac{3}{4}$ 

b. 
$$\frac{1}{4}$$

$$c. \frac{3}{4}$$

$$d. \frac{\sqrt{3}}{4}$$

9. Which of the following describes the same quantum state?

$$a. \frac{1}{2} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$

$$b. \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$a. \ \frac{1}{2} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$
  $b. \ \begin{vmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{vmatrix}$   $c. \ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ 

$$d. \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}$$

Consider the quantum state :  $\begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}$ 

$$\begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}$$

10. What is the probability of measuring a |0)?

$$a. \frac{1}{\sqrt{5}}$$
  $b. \frac{4}{5}$   $c. 0.2$ 

d. 0.4

11. What is the probability of measuring a  $|1\rangle$ ?

$$a. \frac{1}{\sqrt{5}}$$
  $b. \frac{4}{5}$   $c. 0.2$ 

d. 0.4

12. Which of the following describes the same quantum state?

a. 
$$\frac{1}{\sqrt{5}} |0\rangle + \frac{2}{\sqrt{5}} |1\rangle$$
 b.  $0.8|0\rangle + 0.2|1\rangle$ 

$$c.\frac{1}{\sqrt{5}}\begin{bmatrix}2\\1\end{bmatrix}$$

 $c.\frac{1}{\sqrt{5}}\begin{vmatrix} 2\\1 \end{vmatrix}$   $d.\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ 

Notation, Single Qubit Math HW

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Consider the quantum state :  $\begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$ 

$$\begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

13. What is the probability of measuring a |0)?

$$t. \ \frac{1}{3}$$

a. 
$$\frac{1}{3}$$
 b.  $\frac{1}{\sqrt{3}}$  c.  $\frac{2}{3}$ 

$$c. \frac{2}{3}$$

$$e.\sqrt{3}$$

14. What is the probability of measuring a  $|1\rangle$ ?

a. 
$$\frac{1}{3}$$
 b.  $\frac{1}{\sqrt{3}}$  c.  $\frac{2}{3}$ 

$$\frac{1}{\sqrt{3}}$$

$$c. \frac{2}{3}$$

$$e.\sqrt{3}$$

15. Which of the following describes the same quantum state?

a. 
$$\frac{\sqrt{2}}{\sqrt{5}}|0\rangle + \frac{1}{2}|$$

$$b. \ \frac{1}{\sqrt{3}} \begin{bmatrix} \sqrt{2} \\ 1 \end{bmatrix}$$

$$a. \frac{\sqrt{2}}{\sqrt{3}}|0\rangle + \frac{1}{3}|1\rangle$$
  $b. \frac{1}{\sqrt{3}}\begin{bmatrix} \sqrt{2} \\ 1 \end{bmatrix}$   $c. \frac{1}{\sqrt{3}}|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}|1\rangle$ 

$$\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} \end{bmatrix}$$

Consider the quantum state :



16. What is this state in bra-ket notation?

$$a. -\frac{1}{2}|0
angle + \frac{1}{2}|1
angle$$

$$b. \frac{1}{2}|0\rangle - \frac{1}{2}|1\rangle$$

$$C. \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$d. -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$a._{-rac{1}{2}|0
angle + rac{1}{2}|1
angle} \quad b._{rac{1}{2}|0
angle - rac{1}{2}|1
angle} \quad c._{rac{1}{\sqrt{2}}|0
angle - rac{1}{\sqrt{2}}|1
angle} \quad d._{-rac{1}{\sqrt{2}}|0
angle + rac{1}{\sqrt{2}}|1
angle} \quad e._{-rac{1}{\sqrt{2}}|0
angle - rac{1}{\sqrt{2}}|1
angle}$$

17. What is this state in vector notation?

$$a.\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$
  $b.\begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$   $C.\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ 

$$b. \left[ \frac{-\frac{1}{2}}{\frac{1}{2}} \right]$$

$$C. \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$d. \begin{bmatrix} -rac{1}{\sqrt{2}} \\ rac{1}{\sqrt{2}} \end{bmatrix}$$

$$e.\begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

- 18. What is the probability of measuring |1)?
  - *a*. 25%

*h* 50%

0%

100%

Consider a quantum state in which the probability of measuring a |0\) is 10\%.

16. What is the probability of measuring a |1)?

$$a. \frac{1}{\sqrt{10}}$$
  $b. \frac{9}{10}$   $c. \frac{3}{\sqrt{10}}$   $d. \frac{1}{10}$ 

$$b. \frac{9}{10}$$

$$c. \frac{3}{\sqrt{10}}$$

$$d. \frac{1}{10}$$

Given that  $\alpha|0\rangle+\beta|1\rangle$  and the probability above, what is the value of  $\alpha$ ?

a. 
$$\frac{9}{10}$$

$$0. \frac{1}{\sqrt{10}}$$

a. 
$$\frac{9}{10}$$
 b.  $\frac{1}{\sqrt{10}}$  c.  $\frac{3}{\sqrt{10}}$  d.  $\frac{1}{10}$ 

$$l. \frac{1}{10}$$

Given that  $\alpha|0\rangle + \beta|1\rangle$  and the probability above, what is the value of  $\beta$ ?

$$a. \frac{3}{\sqrt{10}}$$
  $b. \frac{\sqrt{3}}{4}$   $c. \frac{9}{10}$   $d. \frac{1}{\sqrt{10}}$ 

$$b. \frac{\sqrt{3}}{4}$$

$$c.\frac{9}{10}$$

$$J. \frac{1}{\sqrt{10}}$$

- 19. (True / False) 0.5|0
  angle + .5|1
  angle is a valid possible quantum state.
- 20. (True / False)  $0.9|0\rangle + .1|1\rangle$  is a valid possible quantum state.
- 21. (True / False)  $\frac{1}{2} |0\rangle + \frac{\sqrt{3}}{2} |1\rangle$  is not a valid possible quantum state.
- 22. (True / False)  $\frac{1}{2} \left| \frac{1}{\sqrt{3}} \right|$  is a valid possible quantum state.
- 23. (True / False)  $\begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$  is not a valid possible quantum state.
- 24. (True / False)  $\begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{\end{bmatrix}}$  and  $\frac{1}{2} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$  describe the same quantum state.

$$egin{bmatrix} 1 & 3 \ 5 & 7 \end{bmatrix} egin{bmatrix} 1 \ 2 \end{bmatrix} = egin{bmatrix} ? \ ? \end{bmatrix}$$

a. 
$$\begin{bmatrix} 7 \\ 19 \end{bmatrix}$$

b. |

24

 $^{\mathrm{c.}}\left|_{14}
ight|$ 

d.  $\begin{bmatrix} \mathbf{3} \\ 70 \end{bmatrix}$ 

$$egin{bmatrix} 1 & 3 \ 5 & 7 \end{bmatrix} egin{bmatrix} 2 \ 1 \end{bmatrix} = egin{bmatrix} ? \ ? \end{bmatrix}$$

a. 
$$\begin{bmatrix} 8 \\ 12 \end{bmatrix}$$

b. | 13

c.  $\begin{bmatrix} 0 \\ 35 \end{bmatrix}$ 

 $d. \begin{bmatrix} 5 \\ 17 \end{bmatrix}$ 

$$egin{bmatrix} 4 & 5 \ 6 & 7 \end{bmatrix} egin{bmatrix} 2 \ 3 \end{bmatrix} = egin{bmatrix} ? \ ? \end{bmatrix}$$

$$\operatorname{a.} \left| egin{array}{c} 40 \ 126 \end{array} 
ight|$$

b.  $\begin{bmatrix} 23 \\ 33 \end{bmatrix}$ 

c.  $\begin{vmatrix} 18 \\ 39 \end{vmatrix}$ 

 $d. \begin{bmatrix} 11 \\ 16 \end{bmatrix}$ 

$$egin{bmatrix} 1 & 3 \ 4 & 2 \end{bmatrix} egin{bmatrix} 6 \ 5 \end{bmatrix} = egin{bmatrix} ? \ ? \end{bmatrix}$$

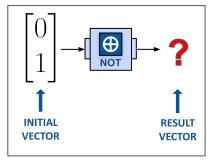
a. 
$$\begin{bmatrix} 24 \\ 30 \end{bmatrix}$$

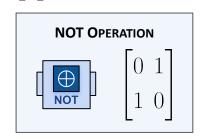
b. | 11

c.  $\begin{bmatrix} 10 \\ 40 \end{bmatrix}$ 

 $\mathrm{d.} egin{bmatrix} 21 \ 34 \end{bmatrix}$ 

The NOT Operator is applied to an initial vector:





What will the result be in vector notation?

$$a. \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$b.$$
  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 

$$c.$$
  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ 

d. Cannot be determined

What will the result be in bra-ket notation?

$$a. |0\rangle$$

$$b. |1\rangle$$

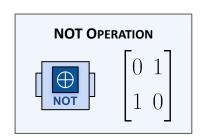
$$c. \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

d. Cannot be determined

The NOT Operator is applied to an initial vector:  $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$  .

$$\frac{1}{\sqrt{2}} \mid 0 \rangle + \frac{1}{\sqrt{2}} \mid 1 \rangle \longrightarrow ?$$

$$\uparrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad$$



What will the result be in bra-ket notation?

$$a. \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$

b. 
$$\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

$$a. \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$
  $b. \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$   $c. \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$   $d.$  Cannot be determined

What will the result be in vector notation?

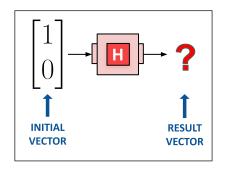
$$a. \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

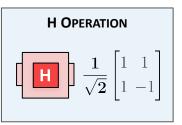
$$b.$$
  $\left[\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]$ 

$$C.$$
  $\begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$ 

d. Cannot be determined

The H operation is applied to an initial vector:





What will the result be in vector notation?

$$a. \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$b.\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$c. \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$d.$$
  $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$ 

What will the result be in bra-ket notation?

a. 
$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
 b.  $\frac{2}{\sqrt{5}}|0\rangle + \frac{1}{\sqrt{5}}|1\rangle$ 

$$b. \frac{2}{\sqrt{5}} |0\rangle + \frac{1}{\sqrt{5}} |1\rangle$$

$$C. \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

$$c. \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$
  $d. \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$