

# New Metrics for the Reliability of Approximate and Probabilistic Adders

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## Keywords

Adders, Probabilistic Logic, Measurement, Vectors, Logic Gates, Integrated Circuit Reliability, Energy Efficiency, Adders, Inexact Computing, Reliability, Error Masking, Approximate Logic, Imprecise Arithmetic, Mean Error Distance, Normalized Error Distance, Power

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## Abstract

Addition is a fundamental function in arithmetic operation; several adder designs have been proposed for implementations in inexact computing. These adders show different operational profiles; some of them are approximate in nature while others rely on probabilistic features of nanoscale circuits. However, there has been a lack of appropriate metrics to evaluate the efficacy of various inexact designs. In this paper, new metrics are proposed for evaluating the reliability as well as the power efficiency of approximate and probabilistic adders. Reliability is analyzed using the so-called sequential probability transition matrices (SPTMs). Error distance (ED) is initially defined as the arithmetic distance between an erroneous output and the correct output for a given input. The mean error distance (MED) and normalized error distance (NED) are then proposed as unified figures that consider the averaging effect of multiple inputs and the normalization of multiple-bit adders. It is shown that the MED is an effective metric for measuring the implementation accuracy of a multiple-bit adder and that the NED is a nearly invariant metric independent of the size of an adder. The MED is, therefore, useful in assessing the effectiveness of an approximate or probabilistic adder implementation, while the NED is useful in characterizing the reliability of a specific design. Since inexact adders are often used for saving power, the product of power and NED is further utilized for evaluating the tradeoffs between power consumption and precision. Although illustrated using adders, the proposed metrics are potentially useful in assessing other arithmetic circuit designs for applications of inexact computing.

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