current for any combination of ratios and impedances probably should not exceed ten percent of the full-load rated current of the smaller unit.

More than two transformers may of course be paralleled, and the division of load may be calculated from an extended equivalent circuit similar to the one in Fig. 45.

44. Three-Phase Transformer Banks

The same considerations apply for the parallel operation of symmetrical three-phase transformer banks as have been outlined for single-phase transformers. In addition it is necessary to make sure that polarity and phase-shift between high-voltage and low-voltage terminals are similar for the parallel units. A single-phase equivalent circuit may be set up on a line-to-neutral basis to represent one phase of a balanced three-phase bank, using the theory of symmetrical components.

When three-phase transformer banks having any considerable degree of dissymmetry among the three phases are to be analyzed, it is necessary either to set up a complete three-phase equivalent circuit, or to interconnect equivalent sequence networks in a manner to represent the unbalanced portion of the circuit according to the rules of symmetrical components.

45. Three-Winding Transformers

Currents flowing in the individual windings of parallel three-winding banks can be determined by solving an equivalent circuit, such as that shown in Fig. 46. The

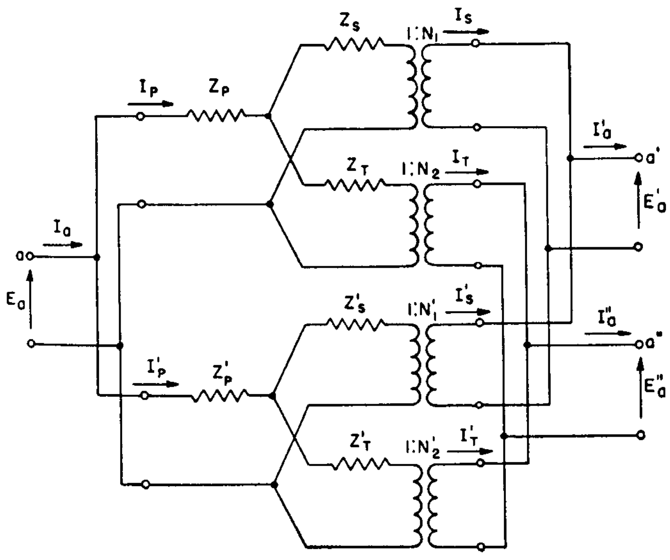

Fig. 46—Equivalent circuit for parallel connection of single-phase three-winding transformers.

terminal loads, as well as winding ratios and impedances, affect the division of currents among the windings of a three-winding transformer, so all these factors must be known before a solution is attempted.

46. Three-Winding Transformer in Parallel With Two-Winding Transformer

The equivalent circuit for a three-winding transformer paralleled with a two-winding transformer is given in Fig. 47.

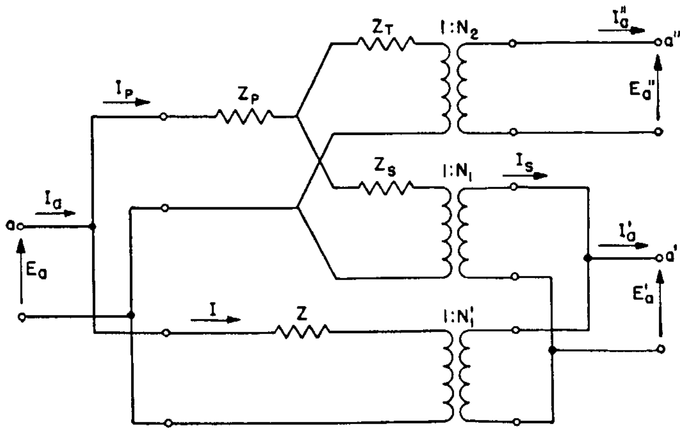

Fig. 47—Equivalent circuit for a single-phase three-winding transformer paralleled with a two-winding unit.

Division of currents may be calculated from this circuit, if the load currents I_a' and I_a'' are assumed.

Parallel operation of two such transformers is not usually satisfactory, since a change in tertiary load will alter the distribution of load between the other two windings. If the impedances are proportioned to divide the load properly for one load condition, the load division between transformers at some other loading is likely to be unsatisfactory. An exception is the case wherein the a″ circuit of Fig. 47 represents a delta tertiary winding in a three-phase bank, with no load connected to the tertiary; in this instance the transformers can be made to divide currents similarly at all loads.

It is possible to design a three-winding transformer so that the load taken from the tertiary winding does not seriously affect load division between the paralleled windings of the two transformers. If the impedance Z_P is made equal to zero, then current division at the a' terminals will be determined by Z_S and Z only, and this impedance ratio will remain independent of tertiary loading. It is difficult to obtain zero as the value for Z_P, particularly if this winding is of high voltage; however, values near zero can be obtained with special design at increased cost. Such a design may result in a value of Z_T which is undesirable for other reasons.

XVIII. TRANSFORMER PRICES

47. Two-Winding Type OA Transformers

Estimating prices for Type OA, oil-immersed, self-cooled, 60-cycle, two-winding transformers are given in Fig. 48. The estimating prices per kva are based on net prices as of December 1, 1949. As prices change frequently, the curves should be used principally for comparing the prices of different voltage classes, comparing banks made of single-phase and three-phase units, etc.

If the insulation level of the low-voltage winding is 15 kv, or higher, the prices in Fig. 48 should be corrected in accordance with Table 16. Price additions are also required when the rating of either the high- or low-voltage winding is 1000 volts and below.

Transformers designed for star connection of the high-voltage winding may be built with a lower insulation level

Table 16—Additions to be Made to Prices in Fig. 48 when Low-Voltage Winding Insulation Level is 15 kv or Higher

Low-Voltage Winding		Price Addition in Percent									
		Single-Phase Equivalent 55 C kva Self-Cooled Rating					3-Phase Equivalent 55 C kva Self-Cooled Rating				
Insulation Class KV	Basic Impulse Levels-kv	501 to 1800	1801 to 3500	3501 to 7000	7001 to 13500	13501 and above	501 to 3600	3601 to 7000	7001 to 14000	14001 to 27000	27001 and above
15	110	3½%	1½%	0%	0%	0%	3½%	1½%	0%	0%	0%
25	150	7	4	3	2	1	7	4	3	2	1
34.5	200	10	7	6	5	4	10	7	6	5	4
46	250	14	11	10	9	8	14	11	10	9	8
69	350	21	18	17	16	15	21	18	17	16	15
92	450	29	26	24	23	21	29	26	24	23	21
115	550	37	34	32	30	28	37	34	32	30	28
138	650	..	42	39	36	34	..	42	39	36	34
161	750	..	..	46	44	41	..	..	46	44	41

at the neutral end than at the line end of the winding. Table 17 summarizes the possible savings in cost with these designs. Reference should be made to section 16 for a discussion of the minimum insulation level that should be used at the transformer neutral.

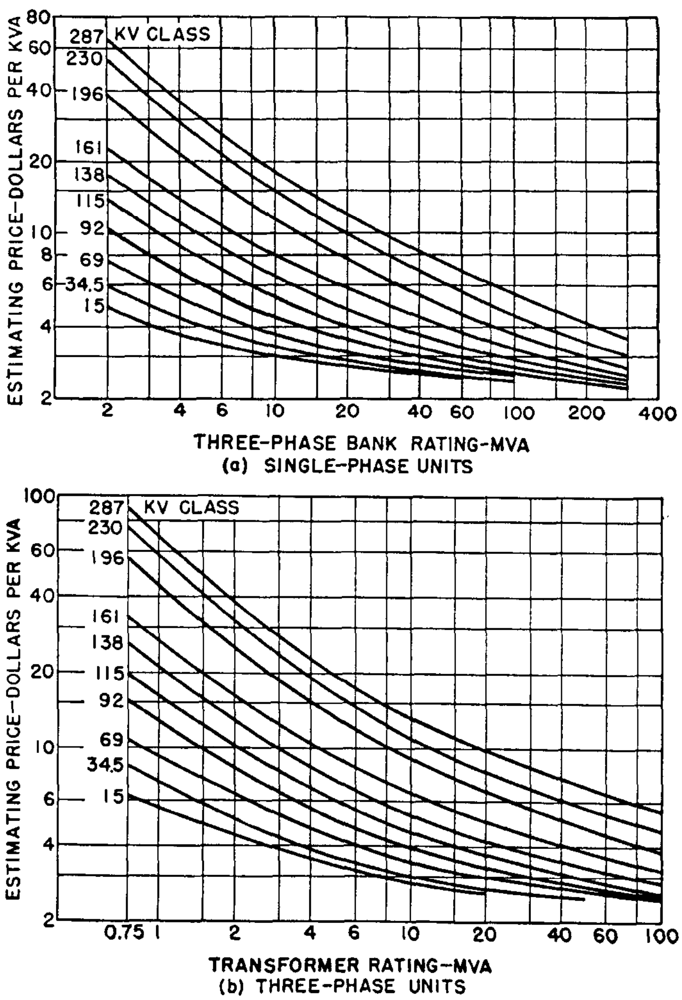

Fig. 48—Curve for estimating prices of oil-immersed, 60-cycle, two-winding, type OA power transformers.

48. Multi-Winding Units

If a multi-winding transformer is designed for simultaneous operation of all windings at their rated capacities, the price of the unit can be estimated from the curves given for two-winding transformers by using an equivalent

Table 17—Price Reduction for Grounded Neutral Service

Winding Insulation Class at Line End	Insulation* Class at Neutral End	Price Reduction Percent
69	15	0
92	15	3.0
92	25–69	1.5
115	15	5.0
115	25–69	2.5
115	92	1.0
138	15	6.0
138	25–46	5.0
138	69–92	3.0
138	115	1.5
161	15	7.0
161	25–46	5.5
161	69–92	3.5
161	115–138	2.0
196	15	9.0
196	25–46	7.5
196	69–115	4.5
196	138–161	2.5
230	15	10.0
230	25–69	7.5
230	92–138	5.0
230	161	3.0
287	15	12.0
287	25–69	9.0
287	92–138	5.0
287	161–196	3.0

*Reference should be made to section 16 for a discussion of minimum permissible neutral insulation levels.

two-winding capacity equal to the sum of the rated capacities of the various windings divided by two. If a multi-winding transformer is not designed for simultaneous operation of all windings at their rated capacities, the price of the unit can be estimated from the curves given for two-winding transformers, using an equivalent two-winding capacity equal to

$$\text{Equivalent} = A + \tfrac{3}{4}(B - A) \qquad (79)$$

Where $A = \frac{1}{2}$(Sum of the simultaneous loadings).
$B = \frac{1}{2}$(Sum of the maximum rated capacities of the various windings).

In addition, 5 percent must be added for three-winding transformers; 7.5 percent for four-winding transformers; and 10 percent for five-winding transformers.

49. Estimating Prices for Other Types of Cooling

Table 18 is a summary of the approximate cost of three-phase power transformers employing auxiliary cooling systems. All cost figures are expressed in per unit of OA transformer cost, where the OA rating used to determine the base cost is equal to the highest rating of the force-cooled or specially-cooled unit. The kva ratings listed in the second column of Table 18 are the highest ratings of forced-cooled units; for example, the kva rating listed for OA/FA/FOA transformers is the FOA value.

TABLE 18—RELATIVE COST OF THREE-PHASE TRANSFORMERS WITH SPECIAL COOLING

Each cost is in per unit, based on the cost of an OA transformer having a rating equal to the maximum of the special unit being considered[(c)]

Type[(a)]	Three-Phase Bank Rating MVA[(b)]	Insulation Class—KV								
		15	34.5	69	92	115	138	161	196	230
OA/FA	1	1.08	1.07	1.05	1.05	1.05	1.06	1.08	1.06	1.07
	2	1.00	1.01	1.01	1.02	1.02	1.04	1.05	1.05	1.05
	5	0.92	0.95	0.95	0.96	0.97	0.99	0.99	0.99	1.00
	10	0.90	0.91	0.92	0.93	0.93	0.95	0.95	0.96	0.96
	20	0.88	0.88	0.90	0.90	0.90	0.92	0.91	0.92	0.93
	50		0.87	0.87	0.88	0.89	0.91	0.91	0.92	0.92
	100			0.87	0.88	0.89	0.90	0.90	0.91	0.91
OA/FA/FOA	20	0.74	0.75	0.77	0.78	0.79	0.81	0.80	0.83	0.83
	50		0.73	0.73	0.75	0.77	0.80	0.81	0.81	0.82
	100			0.72	0.74	0.75	0.76	0.78	0.81	0.80
FOA	20	0.66	0.67	0.71	0.73	0.75	0.77	0.78	0.81	0.82
	50		0.64	0.68	0.70	0.71	0.75	0.79	0.81	0.82
	100			0.66	0.67	0.67	0.68	0.71	0.75	0.75
OW	2	1.05	1.03	0.99	0.99	1.03	1.07	1.02	0.97	0.91
	5	0.91	0.92	0.92	0.92	0.93	0.97	0.96	0.93	0.91
	10	0.85	0.84	0.90	0.88	0.90	0.89	0.93	0.91	0.90
	20	0.82	0.84	0.87	0.88	0.89	0.88	0.91	0.90	0.90
	50		0.87	0.86	0.85	0.85	0.84	0.88	0.87	0.87
	100			0.85	0.85	0.85	0.82	0.82	0.85	0.81
FOW	20	0.60	0.61	0.65	0.67	0.69	0.71	0.71	0.74	0.75
	50		0.59	0.62	0.64	0.65	0.69	0.73	0.71	0.72
	100			0.60	0.61	0.62	0.62	0.65	0.69	0.69

(a) OA/FA—Oil-Immersed Self-Cooled/Forced-Air-Cooled.
OA/FA/FAO—Triple-Rated, Self-Cooled/Forced-Air-Cooled/Forced-Oil-Cooled.
FOA—Forced-Oil-Cooled with Forced-Air-Coolers.
OW—Oil-Immersed Water Cooled.
FOW—Forced-Oil Cooled with Water Coolers.
(b) The MVA ratings tabulated for OA/FA and OA/FA/FOA units are the FA and the FOA ratings respectively.
(c) Example: The cost of a 15 kv OA/FA three-phase unit rated 10 000 kva (FA) is equal to 0.90 times the cost of a 15 kv OA three-phase unit rated 10 000 kva.

XIX. REACTORS

50. Application of Current-Limiting Reactors

Current-limiting reactors are inductance coils used to limit current during fault conditions, and to perform this function it is essential that magnetic saturation at high current does not reduce the coil reactance. If fault current is more than about three times rated full load current, an iron core reactor designed to have essentially constant magnetic permeability proves overly expensive, therefore air core coils having constant inductance are generally used for current-limiting applications. A reactor whose inductance increased with current magnitude would be most effective for limiting fault current, but this characteristic has not been practically attained.

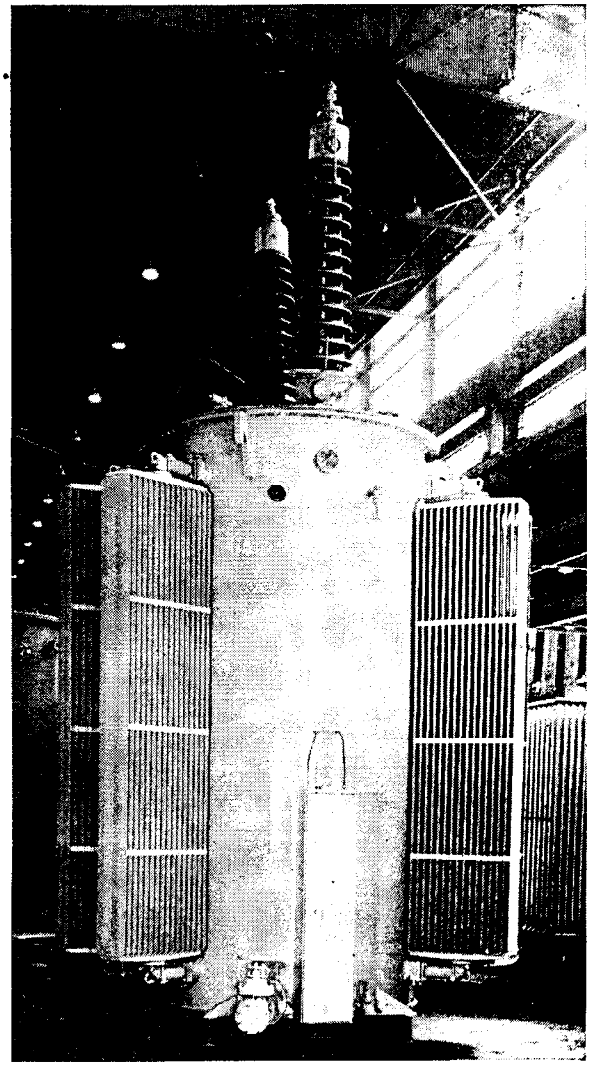

Fig. 49—Oil-immersed air-core reactor.

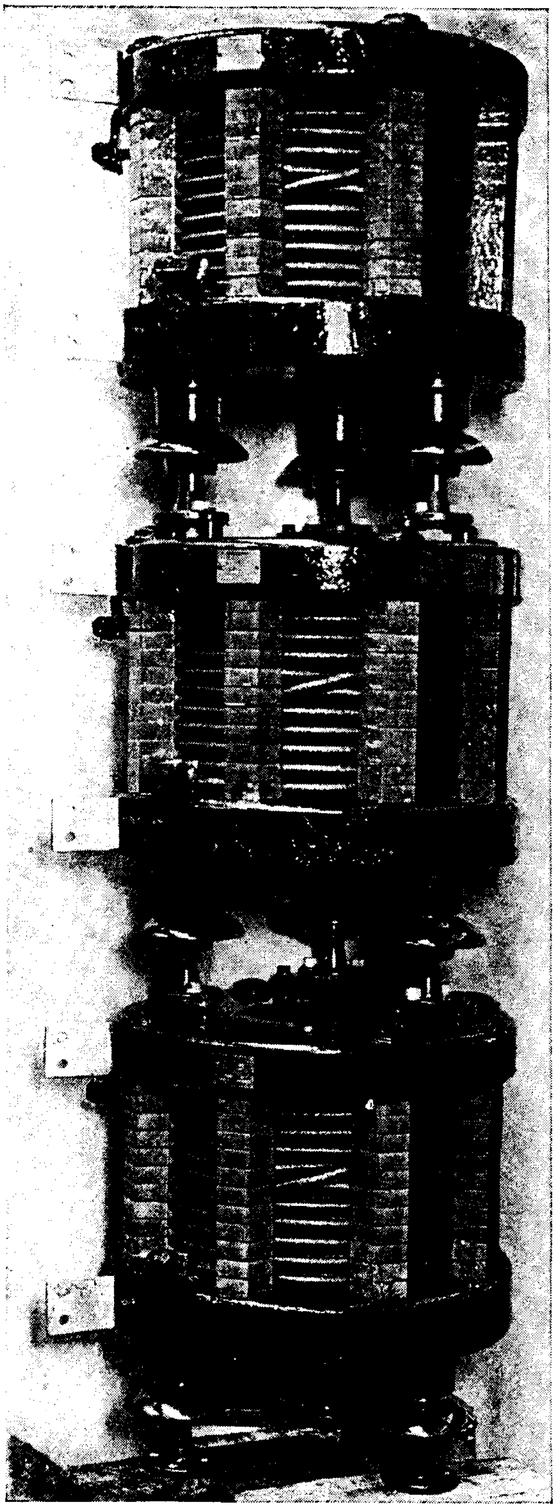

Fig. 50—Dry-type air-core reactor.

Air core reactors are of two general types, oil-immersed (Fig. 49) and dry-type (Fig. 50). Oil-immersed reactors can be cooled by any of the means commonly applied to power transformers. Dry-type reactors are usually cooled by natural ventilation but can also be designed with forced-air and heat-exchanger auxiliaries where space is at a premium.

Oil-immersed reactors can be applied to a circuit of any voltage level, for either indoor or outdoor installation. The advantages of oil-immersed reactors also include:

1. A high factor of safety against flashover.
2. No magnetic field outside the tank to cause heating or magnetic forces in adjacent reactors or metal structures during short-circuits.
3. High thermal capacity.

Dry-type reactors depend upon the surrounding air for insulation and cooling. Because of the required clearances and construction details necessary to minimize corona, these reactors are limited to 34.5 kv as a maximum insulation class. Free circulation of air must be maintained to provide satisfactory heat transfer. These coils should not be surrounded with closed circuits of conducting material because the mutual inductance may be sufficient to produce destructive forces when short-circuit current flows in the coil. Structures such as I-beams, channels, plates, and other metallic members, either exposed or hidden, should also be kept at a distance from the reactor even though they do not form closed circuits. A side clearance equal to one-third the outside diameter of the coil, and an end clearance of one-half the outside diameter of the coil will produce a temperature rise less than 40 C in ordinary magnetic steel. For the same size members, brass will have about the same rise, aluminum about one and one half times, and manganese steel about one-third the rise for ordinary magnetic steel. Reinforcing rods less than three-fourths inch in diameter which do not form a complete electrical circuit are not included in these limitations, because the insulation clearances from the reactor should be sufficient to avoid undue heating in such small metal parts.

In order to avoid excessive floor loading due to magnetic forces between reactors the spacing recommended by the manufacturer should be observed. Sometimes this spacing can be reduced by use of bracing insulators between units or using stronger supporting insulators and increasing the strength of the floor. This should always be checked with the manufacturer since bracing increases the natural period of vibration and may greatly increase the forces to be resisted by the building floors or walls.

51. Reactor Standards

The standard insulation tests for current-limiting reactors are summarized in Table 19.

Dry-type current-limiting reactors are built with Class B insulation and have an observable temperature rise by resistance of 80 C with normal continuous full-load current. Dry-type and oil-immersed current-limiting reactors are designed mechanically and thermally for not more than 33⅓ times (3 percent reactive drop) normal full-load current for five seconds under short-circuit conditions.

52. Determination of Reactor Characteristics

When specifying a current-limiting reactor, information should be included on the following:

1. Indoor or outdoor service.
2. Dry- or oil-immersed type.
3. Single-phase or three-phase reactor.
4. Ohms reactance.

TABLE 19—STANDARD DIELECTRIC TESTS FOR CURRENT-LIMITING REACTORS

Insulation Class kv (a)	Low Frequency Tests(b)		Impulse Tests (Oil Type)(c)		
	Oil Type kv rms	Dry Type (c) kv rms	Chopped Wave Voltage kv crest	Chopped Wave Min. Time to Flash-over in μs	Full Wave kv crest
1.2	12	12	54	1.5	45
2.5	17	25	69	1.5	60
5.0	21	30	88	1.6	75
8.66	29	40	110	1.8	95
15.0	36	60	130	2.0	110
23.0	60	85	175	3.0	150
34.5	80	115	230	3.0	200
46.0	105		290	3.0	250
69.0	160		400	3.0	350
92.0	210		520	3.0	450
115.0	260		630	3.0	550
138.0	310		750	3.0	650
161.0	365		865	3.0	750
196.0	425		1035	3.0	900
230.0	485		1210	3.0	1050
287.0	590		1500	3.0	1300
345.0	690		1785	3.0	1550

Notes:
(a) Intermediate voltage ratings are placed in the next higher insulation class unless specified otherwise.
(b) Turn-to-turn tests are made by applying these low-frequency test voltages, at a suitable frequency, across the reactor terminals: dry-type reactors for outdoor service require a turn-to-turn test voltage one-third greater than the tabulated values.
(c) No standard impulse tests have been established for dry-type current-limiting reactors.

5. Continuous current rating, amperes.
6. Reactor rating in kva.
7. Voltage class.
8. Circuit characteristics:
   (a) Single-phase or three-phase.
   (b) Frequency.
   (c) Line-to-line voltage.
   (d) Type of circuit conductors.

Standardization of current ratings and ohmic reactances for current-limiting reactors is not yet completed, but semi-standard values are available and should be used where feasible in the preparation of reactor specifications.

53. Reactor Prices

The estimating prices included in this section should be used for comparative purposes only because reactor prices are subject to change from time to time.

Estimating prices for single-phase, 60-cycle, dry-type current-limiting reactors are given in Fig. 51 for kva ratings between 10 and 5000. Reactors for use in 1201 to 13 800 volt circuits may be estimated from the curve labeled "15 kv and below." The prices given apply to single-phase reactors with current ratings between 300 and 600 amperes. For current ratings below 300 amperes, price additions must be made in accordance with Table 20. When the current rating exceeds 600 amperes make a price addition of one percent for each 100 amperes, or fraction thereof, above 600 amperes.

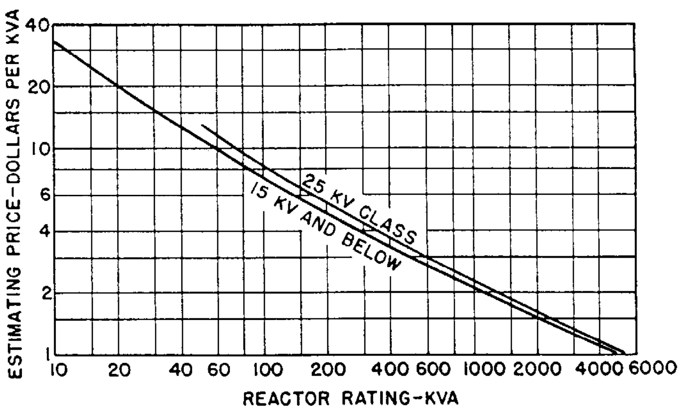

Fig. 51—Curve for estimating prices of single-phase, 60-cycle, dry-type current-limiting reactors.

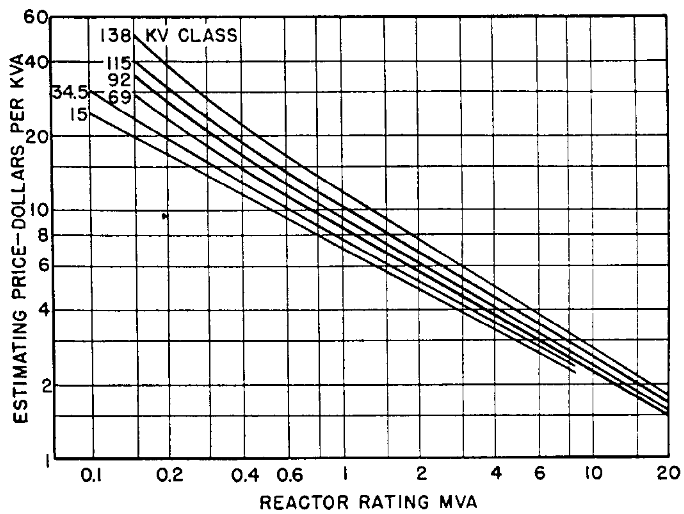

Fig. 52—Curve for estimating prices of single-phase, 60-cycle, oil-immersed current-limiting reactors.

Estimating prices for single-phase, 60-cycle, oil-immersed current-limiting reactors are given in Fig. 52 for insulation classes between 15 and 138 kv. For current ratings above 800 amperes make a price addition of two percent for each 100 amperes, or fraction thereof, above 800 amperes.

Estimating prices for 60-cycle, oil-immersed, self-cooled shunt reactors may be estimated by adding 10 percent to the prices given in Fig. 48 (a) for two-winding transformers.

TABLE 20—PRICE ADDITIONS FOR DRY-TYPE REACTORS RATED BELOW 300 AMPERES

Current Rating Amperes	Price Addition Percent
250–299	5
200–249	10
150–199	15
125–149	22
100–124	29
75– 99	36
50– 74	43

XX. EQUIVALENT CIRCUITS FOR SINGLE PHASE TRANSFORMERS

Representation of a transformer by an equivalent circuit is a commonly used method for determining its performance as a circuit element in complex power and distribution networks. Without the simplifications offered by the use of such equivalent circuits the handling of transformers with their complex array of leakage and mutual impedances would be a formidable problem.

For the purposes of calculating short circuit currents, voltage regulation, and stability of a power system, the normal magnetizing current required by transformers is neglected. Thus Figs. 2(c), (d), or (e), as the choice may be, will adequately represent a two-winding transformer for calculation purposes.

For three-, four-, and in general multi-winding transformers, an equivalent network can be always determined that will consist only of simple impedances (mutual impedances eliminated) and accurately represent the transformer as a circuit element. The impedances which can be most readily determined by test or by calculation are those between transformer windings taken two at a time, with other windings considered idle; therefore the impedances in an equivalent circuit can well be expressed in terms of these actual impedances between the transformer windings taken two at a time.

The number of independent impedances required in an equivalent circuit to represent a multi-winding transformer shall be, in general, equal to the number of all possible different combinations of the windings taken two at a time. Thus, one equivalent impedance is required to represent a two-winding transformer, three branch impedances for a three-winding transformer, and six independent branch impedances to represent a four-winding transformer.

Equivalent circuits for the two-winding transformer and auto-transformer are presented in sections 1 and 27, respectively. The following sections discuss the equivalent circuits for three-winding and four-winding transformers.

54. Equivalent Circuits for Three-Winding Transformer

The equivalent circuit for a transformer having three windings on the same core is shown in Fig. 53, where the magnetizing branches have been omitted. The number of turns in the P, S, and T windings are n_1, n_2, and n_3, respectively. The equivalent circuit is shown in Fig. 53 (b) with all impedance in ohms on the P winding voltage base and with ideal transformers included to preserve actual voltage and current relationships between the P, S, and T windings. On the P winding voltage base:

$$\begin{aligned} Z_P &= \tfrac{1}{2}\left(Z_{PS}+Z_{PT}-\frac{1}{N_1^2}Z_{ST}\right) \\ Z_S &= \tfrac{1}{2}\left(\frac{1}{N_1^2}Z_{ST}+Z_{PS}-Z_{PT}\right) \\ Z_T &= \tfrac{1}{2}\left(Z_{PT}+\frac{1}{N_1^2}Z_{ST}-Z_{PS}\right) \\ N_1 &= \frac{n_2}{n_1} \\ N_2 &= \frac{n_3}{n_1} \end{aligned} \tag{80}$$

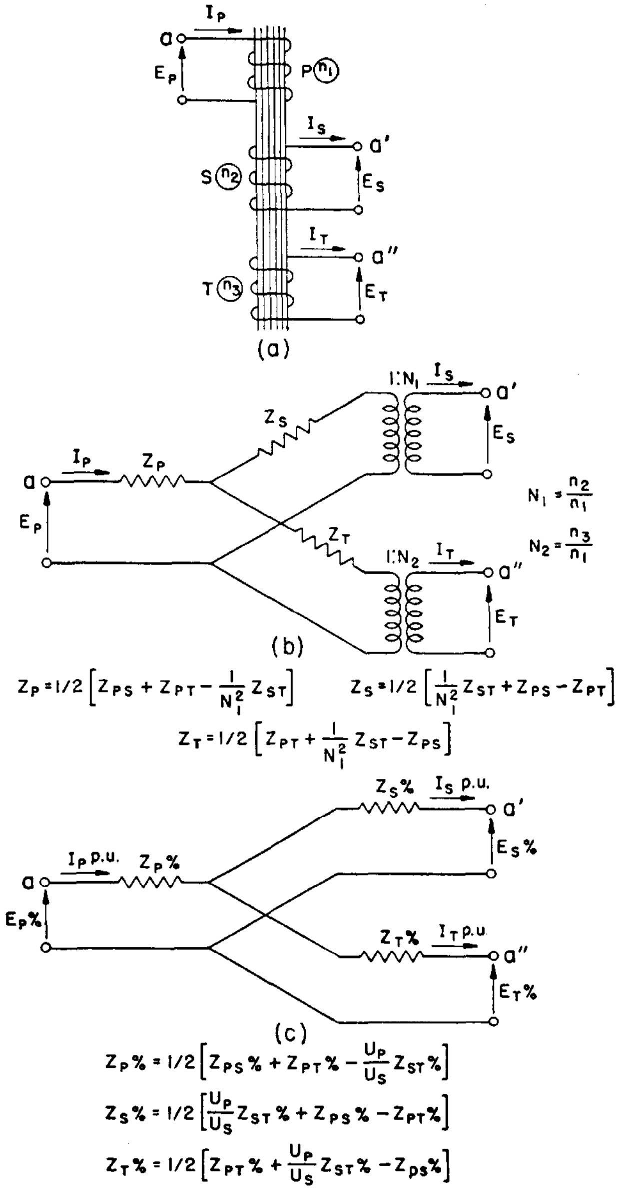

Fig. 53—Three-winding transformer.

(a) winding diagram.
(b) equivalent circuit in ohms.
(c) equivalent circuit in percent.

Note that Z_P and Z_S as defined and used here differ from Z_P and Z_S in Eq. 10. The equivalent circuit expressed in percent is given in Fig. 53 (c) with all impedances referred to the kva of the P winding.

$$\begin{aligned} Z_P\% &= \tfrac{1}{2}\left(Z_{PS}\%+Z_{PT}\%-\frac{U_P}{U_S}Z_{ST}\%\right) \\ Z_S\% &= \tfrac{1}{2}\left(\frac{U_P}{U_S}Z_{ST}\%+Z_{PS}\%-Z_{PT}\%\right) \\ Z_T\% &= \tfrac{1}{2}\left(Z_{PT}\%+\frac{U_P}{U_S}Z_{ST}\%-Z_{PS}\%\right) \end{aligned} \tag{81}$$

The quantities can be expressed in percent on any arbitrary kva base, U_C, by multiplying each impedance by the ratio $\frac{U_C}{U_P}$. The notation used is defined as follows:

U_P = kva of the P winding.
U_S = kva of the S winding.
U_T = kva of the T winding.

Z_{PS} = leakage impedance between the P and S windings as measured in ohms on the P winding with the S winding short-circuited and the T winding open-circuited.

$Z_{PS}\%$ = leakage impedance between the P and S windings, with the T winding open-circuited, expressed in percent on the kva and voltage of the P winding.

Z_{PT} = leakage impedance between the P and T windings as measured in ohms on the P winding with the T winding short-circuited and the S winding open-circuited.

$Z_{PT}\%$ = leakage impedance between the P and T windings, with the S winding open-circuited, expressed in percent on the kva and voltage of the P winding.

Z_{ST} = leakage impedance between the S and T windings as measured in ohms on the S winding with the T winding short-circuited and the P winding open-circuited.

$Z_{ST}\%$ = leakage impedance between the S and T windings, with the P winding open-circuited, expressed in percent on the kva and voltage of the S winding.

The equations given in Fig. 53 (b) and Fig. 53 (c) for Z_P, $Z_P\%$, etc., are derived from the relationships:

$$\begin{aligned} &Z_{PS}=Z_P+Z_S && Z_{PS}\%=Z_P\%+Z_S\% \\ &Z_{PT}=Z_P+Z_T && Z_{PT}\%=Z_P\%+Z_T\% \\ &Z_{ST}=N_1^2(Z_S+Z_T) && Z_{ST}\%=\frac{U_S}{U_P}(Z_S\%+Z_T\%) \end{aligned} \tag{82}$$

also

$$\begin{aligned} Z_P&=R_P+jX_P \\ Z_{PS}&=R_{PS}+jX_{PS}=R_P+R_S+j(X_P+X_S) \\ Z_{PS}\%&=R_{PS}\%+jX_{PS}\% \text{ etc.,} \end{aligned} \tag{83}$$

where X_{PS} is the leakage reactance between the P and S windings (with T open-circuited); and R_{PS} is the total effective resistance between the P and S windings, as measured in ohms on the P winding with S short-circuited and T open-circuited. $R_{PS}\%$ and $X_{PS}\%$ are the respective quantities expressed in percent on the kva and voltage of the P winding.

The equivalent circuits completely represent the actual transformer as far as leakage impedances, mutual effects between windings, and losses are concerned (except exciting currents and no load losses). It is possible for one of the three legs of the equivalent circuit to be zero or negative.

55. Equivalent Circuits for Four-Winding Transformer

The equivalent circuit representing four windings on the same core, shown in Fig. 54 (a), is given in Fig. 54 (b) using ohmic quantities. This form is due to Starr[11,12] and

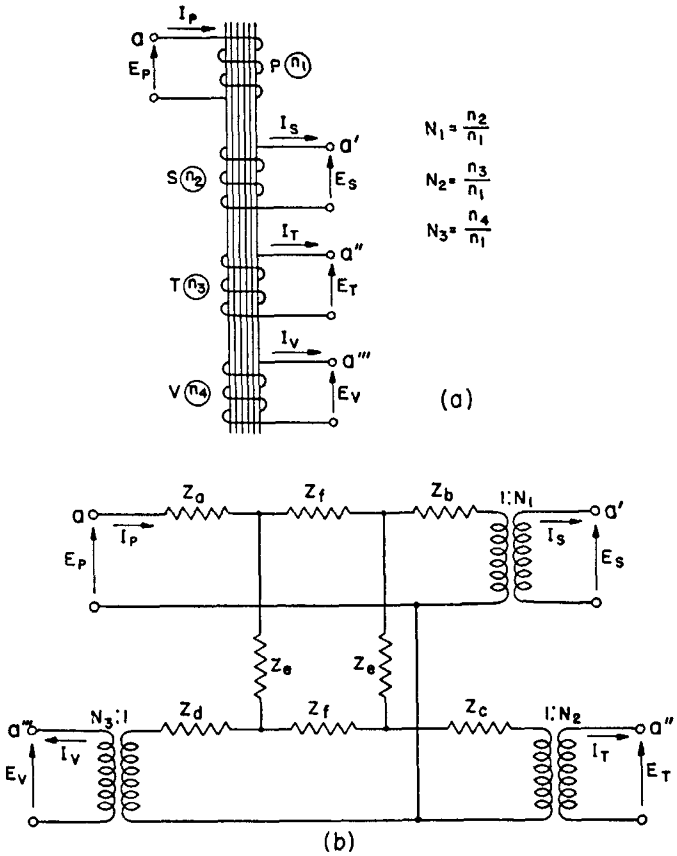

Fig. 54—Four-winding transformer.

(a) winding diagram.
(b) equivalent circuit.

here again the magnetizing branches are omitted. The branches of the equivalent circuit are related to the leakage impedances between pairs of windings as follows:

$$\begin{aligned} Z_a&=\tfrac{1}{2}\left(Z_{PS}+Z_{PV}-\frac{1}{N_1^2}Z_{SV}-K\right) \\ Z_b&=\tfrac{1}{2}\left(Z_{PS}+\frac{1}{N_1^2}Z_{ST}-Z_{PT}-K\right) \\ Z_c&=\tfrac{1}{2}\left(\frac{1}{N_1^2}Z_{ST}+\frac{1}{N_2^2}Z_{TV}-\frac{1}{N_1^2}Z_{SV}-K\right) \\ Z_d&=\tfrac{1}{2}\left(\frac{1}{N_2^2}Z_{TV}+Z_{PV}-Z_{PT}-K\right) \\ Z_e&=\sqrt{K_1K_2}+K_1 \\ Z_f&=\sqrt{K_1K_2}+K_2 \end{aligned} \tag{84}$$

where, $$K=\sqrt{K_1K_2}=\frac{Z_eZ_f}{Z_e+Z_f}$$

$$K_1=Z_{PT}+\frac{1}{N_1^2}Z_{SV}-Z_{PS}-\frac{1}{N_2^2}Z_{TV}$$

$$K_2=Z_{PT}+\frac{1}{N_1^2}Z_{SV}-Z_{PV}-\frac{1}{N_1^2}Z_{ST}$$

The windings will ordinarily be taken in the order that makes K_1 and K_2 positive so that Z_e and Z_f will be positive. The leakage impedances are defined as before; for example, Z_{PS} is the leakage impedance between the P

and S windings as measured in ohms on the P winding with the S winding short-circuited and with the T and V windings open-circuited. The equivalent circuit in percent has the same form as Fig. 54 (b), omitting the ideal transformers.

$$Z_a\% = \tfrac{1}{2}\left(Z_{PS}\% + Z_{PV}\% - \frac{U_P}{U_S}Z_{SV}\% - K\%\right), \text{ etc.}$$

$$K_1\% = Z_{PT}\% + \frac{U_P}{U_S}Z_{SV}\% - Z_{PS}\% - \frac{U_P}{U_T}Z_{TV}\%, \text{ etc.} \quad (85)$$

Similar equations, derived from Eq. (84), apply for the other quantities in the equivalent circuit.

XXI. SEQUENCE IMPEDANCE CHARACTERISTICS OF THREE-PHASE TRANSFORMER BANKS

56. Sequence Equivalent Circuits

The impedance of three-phase transformer banks to positive-, negative-, and zero-sequence currents, and the sequence equivalent circuits, are given in the Appendix, under Equivalent Circuits for Power and Regulating Transformers. The equivalent circuits were developed by Hobson and Lewis[2,13]. The same notation as defined in the early part of this chapter is used to denote leakage impedances in ohms and in percent.

The impedance to negative-sequence currents is always equal to the impedance to positive sequence currents, and the equivalent circuits are similar except that the phase shift, if any is involved, will always be of the same magnitude for both positive- and negative-sequence voltages and currents but in opposite directions. Thus, if the phase shift is $+\alpha$ degrees for positive-sequence, the phase shift for negative-sequence quantities will be $-\alpha$ degrees.

The impedance of a three-phase bank of two-winding transformers to the flow of zero-sequence currents is equal to the positive-sequence impedance for three-phase shell-form units (or for a bank made up of three single-phase units) if the bank is star-star with both star points grounded. If the bank is connected star-delta, with the star point grounded, the zero-sequence impedance viewed from the star-connected terminals for shell-form units, or banks of three single-phase units, is equal to the positive-sequence impedance; the zero-sequence impedance viewed from the delta-connected terminals is infinite.

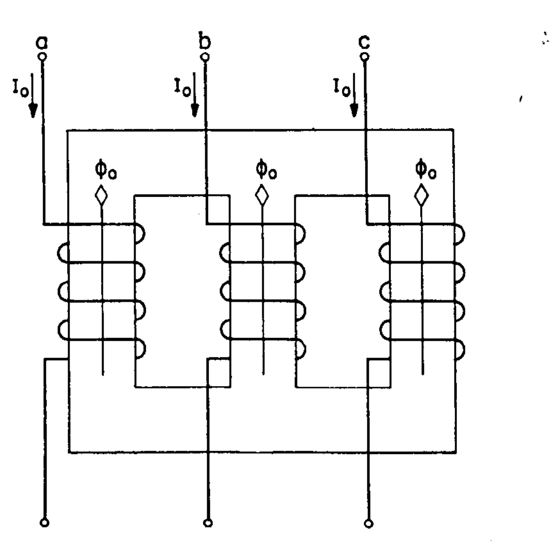

Fig. 55—Zero-sequence exciting currents and fluxes in a three-phase core-form transformer.

The impedance to the flow of zero-sequence currents in three-phase core-form units is generally lower than the positive-sequence impedance. Figure 55 illustrates that there is no return for the zero-sequence exciting flux in such a unit, except in the insulating medium, or in the tank and metallic connections other than the core. The flux linkages with the zero-sequence exciting currents are therefore low, and the exciting impedance to zero-sequence currents correspondingly low. Although the exciting impedance to positive-sequence currents may be several thousand percent, the exciting impedance to zero-sequence currents in a three-phase core-form unit will lie in the range from 30 to 300 percent, the higher values applying to the largest power transformers. Low exciting impedance under zero-sequence conditions is reflected in some reduction in the through impedances to zero-sequence current flow. A star-star grounded, three-phase, two-winding unit of the core-form, or a star-star grounded autotransformer of the three-phase core form acts, because of this characteristic, as if it had a tertiary winding of relatively high reactance. In small core-form units this characteristic is particularly effective and can be utilized to replace a tertiary winding for neutral stabilization and third harmonic excitation.

The zero-sequence exciting impedance is affected by the magnitude of excitation voltage, and it is also affected by tank construction. For example, the zero-sequence exciting impedance of a 4000-kva, 66 000-2400-volt unit was measured to be 84 percent at normal voltage before the core was placed in the tank; it was measured to be 36 percent at normal voltage after the core and coils were placed in the tank. In this case the tank saturated but acted as a short-circuited secondary winding around the transformer, tending to limit the area of the flux return path to that between tank and windings. The zero-sequence exciting impedance is measured by connecting the three windings in parallel and applying a single-phase voltage to the paralleled windings.

The zero-sequence exciting impedance of three-phase core-form units is generally much lower than the positive-sequence exciting impedance, and much lower than the zero-sequence exciting impedance of three-phase shell-form units or three single-phase units. For this reason it is necessary to consider the zero-sequence exciting impedance in deriving the zero-sequence impedance characteristics for certain connections involving core-form units. The exciting impedance to zero-sequence current has been denoted by Z_{SE}, Z_{PE}, etc., where the first subscript refers to the winding on which the zero-sequence exciting impedance is measured in ohms. Following the same notation, $Z_{SE}\%$ is the exciting impedance of the S winding to zero-sequence currents expressed in percent on the kva of the S winding. The number of branches required to define an equivalent circuit of three-phase two- or multi-winding transformers is the same in general as has been described for single phase transformers. A notable exception to this will exist in the formulation of the zero-sequence impedance of core form transformers with grounded neutral. In this case an extra impedance branch must be provided in the equivalent circuit, this branch being always short-circuited to the neutral bus, and having a value dependent upon the zero-sequence excitation impedances of the windings as well as the grounding impedance in the transformer neutral. If the three-phase bank connections are unsymmetrical as in the case of the open-delta connection, mutual coupling will exist between the sequence networks.

57. Derivation of Equivalent Circuits

In the derivation of equivalent circuits for three-phase transformers and banks made up of three single-phase transformers, it is convenient to represent each winding of the transformer by a leakage impedance and one winding of an ideal transformer. This method may be used in the development of circuits for two- and three-winding transformers.

Two magnetically-coupled windings of a single-phase transformer having n_1 and n_2 turns, respectively, are shown schematically in Fig. 56(a). The customary equivalent circuit used to represent such a single-phase transformer is shown in Fig. 56(b) in which Z_A and Z_B are components of the transformer leakage impedance, with a more or less arbitrary division of the leakage impedance between Z_A and Z_B. Z_M is the so-called "magnetizing shunt branch." Since the numerical value of Z_M is very large compared to Z_A and Z_B, for most calculations Fig. 56(b) is approximated by Fig. 56(c) where Z_M is considered infinite. Either of these circuits has serious deficiencies as a device representing the actual transformer; the voltage and current transformation effected by transformer action is not represented in the equivalent circuit, and the circuit terminals a and a' are not insulated from each other as in the actual transformer. These disadvantages are evidenced particularly when analyzing transformer circuits wherein several windings or phases are interconnected. To overcome these deficiencies it is expedient to use the equivalent circuit shown in Fig. 56(d) which combines the circuit of Fig. 56(b) with an ideal transformer. The ideal transformer is defined as having infinite exciting impedance (zero exciting current) and zero leakage impedance, and serves to transform voltage and current without impedance drop or power loss; the ideal transformer thus restores actual voltage and current relationships at the terminals a and a'. The circuit of Fig. 56(e) is obtained from Fig. 56(d) by converting the impedance Z_B to the E_a' voltage base (by multiplying Z_B by the square of the voltage ratio). This process may be thought of as "sliding the ideal transformer through" the impedance Z_B. If the exciting, or no load, current may be neglected (Z_M considered as infinite) the circuit of Fig. 56(e) becomes Fig. 56(f).

Finally, if Z_M is considered infinite, the circuit of Fig. 56(f) becomes Fig. 56(g), in which the two parts of the leakage impedance, Z_A and Z_B, combine into the complete leakage impedance Z_{PS}, where

$$Z_{PS}=Z_A+Z_B \qquad (86)$$

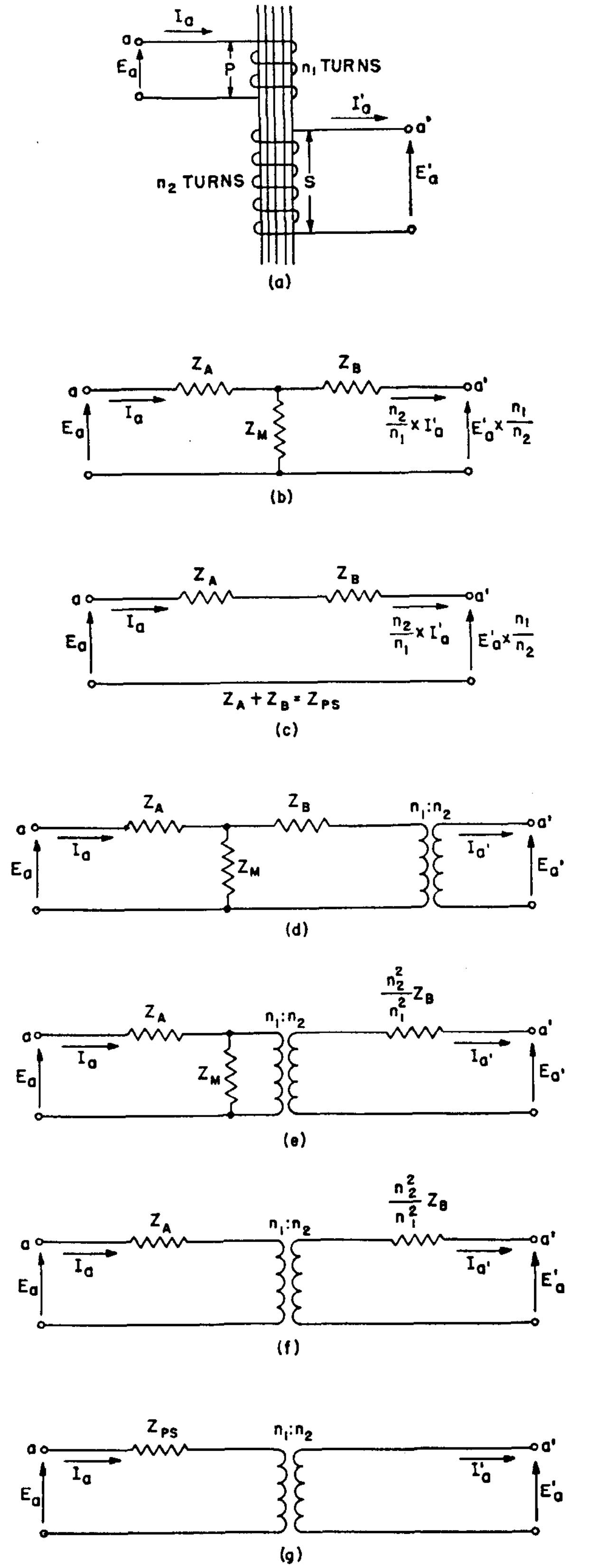

Fig. 56—Steps in the derivation of the equivalent circuit of a two-winding transformer.

In most developments the circuit of Fig. 56(g) will be found most convenient, although in some cases it becomes desirable to have part of the leakage impedance associated with each winding, and the circuit of Fig. 56(f) may be used.

To be perfectly definite, Z_{PS} is understood to mean the leakage impedance, as measured in ohms, with the S winding short circuited, and voltage applied to the P winding. When the test is reversed, with voltage applied to the S winding, and the P winding short circuited, the impedance is denoted by Z_{SP}. It is obvious from the development given that, when Z_M may be considered infinite,

$$Z_{SP}=\frac{n_2^2}{n_1^2}Z_{PS}. \tag{87}$$

58. Derivation of Equivalent Circuit for Star-Delta Bank

In Fig. 57 each transformer winding is represented by an impedance and one winding of an ideal transformer, the transformer having n_1 turns in the P winding and n_2 turns in the S winding. The windings shown in parallel are assumed to be on the same magnetic core. The voltages e_p and e_s represent the voltages across the P and S windings of the ideal transformers.

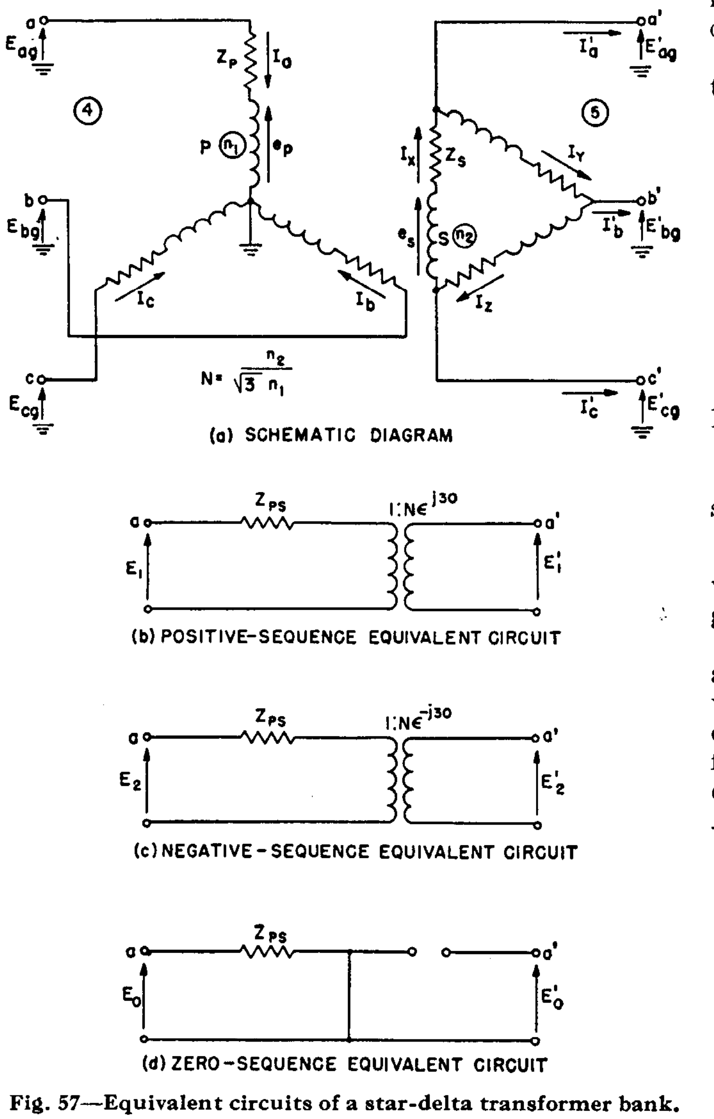

Fig. 57—Equivalent circuits of a star-delta transformer bank.

Assuming positive-sequence voltages E_{ag}, E_{bg}, and E_{cg} applied to the terminals abc, and a three-phase short-circuit at the $a'b'c'$ terminals, the following relations can be written:

$$\begin{aligned}
&E'_{ag}=E'_{bg}=E'_{cg}=0 && n_2I_x=n_1I_a\\
&e_s=I_xZ_s && I_x=\frac{n_1}{n_2}I_a\\
&e_p=\frac{n_1}{n_2}e_s=\frac{n_1}{n_2}I_xZ_S=\left(\frac{n_1}{n_2}\right)^2I_aZ_S\\
&E_{ag}=e_p+I_aZ_P=I_a\left[Z_P+\left(\frac{n_1}{n_2}\right)^2Z_S\right]
\end{aligned} \tag{88}$$

Designating the circuits connected to the abc and $a'b'c'$ terminals as circuits 4 and 5, respectively,

$$Z_{45}=\frac{E_{ag}}{I_a}=Z_P+\left(\frac{n_1}{n_2}\right)^2Z_S=Z_{PS} \tag{89}$$

Z_{45} is defined as the impedance between circuits 4 and 5 in ohms on the circuit 4 voltage base. Z_{PS} is the impedance between the P and S windings as measured by applying voltage to the P winding with the S winding short-circuited.

With positive-sequence voltages applied to the abc terminals and the $a'b'c'$ terminals open circuited,

$$\begin{aligned}
&E_{bg}=a^2E_{ag} && E'_{bg}=a^2E'_{ag}\\
&E_{cg}=aE_{ag} && E'_{cg}=aE'_{ag}\\
&e_s=\frac{n_2}{n_1}e_p=E'_{ag}-E'_{cg}\\
&E_{ag}=e_p=\frac{n_1}{n_2}(E'_{ag}-aE'_{ag})\\
&\quad=\frac{n_1}{n_2}E'_{ag}(1-a)=\sqrt{3}\frac{n_1}{n_2}E'_{ag}\epsilon^{-j30}.
\end{aligned} \tag{90}$$

Letting $N=\frac{n_2}{\sqrt{3}n_1}$, $E'_{ag}=NE_{ag}\epsilon^{j30}$.

As positive-sequence quantities were used in this analysis, the final equation can be expressed as follows:

$$E_1'=NE_1\epsilon^{j30}, \tag{91}$$

where E_1' and E_1 are the positive-sequence voltages to ground at the transformer terminals.

The above relations show that the line-to-ground voltages on the delta side lead the corresponding star-side voltages by 30 degrees, which must be considered in a complete positive-sequence equivalent circuit for the transformer. A consideration of Eqs. (88) will show that the currents I_a', I_b' and I_c' also lead the currents I_a, I_b and I_c by 30 degrees.

$$\begin{aligned}
&I_x=\frac{n_1}{n_2}I_a && I_y=\frac{n_1}{n_2}I_b=\frac{n_1}{n_2}a^2I_a\\
&I_a'=I_x-I_y=\frac{n_1}{n_2}(I_a-a^2I_a)\\
&\quad=\frac{I_a}{N}\epsilon^{j30}.\\
&I_1'=\frac{I_1}{N}\epsilon^{j30}.
\end{aligned} \tag{92}$$

The complete positive-sequence circuit in Fig. 57(b) therefore includes the impedance Z_{PS} and an ideal transformer having a turns ratio N and a 30-degree phase shift.

A similar analysis, made with negative-sequence voltages and currents, would show that

$$I'_2 = \frac{I_2}{N}\epsilon^{-j30}. \tag{93}$$

$$E'_2 = NE_2\epsilon^{-j30}. \tag{94}$$

The positive- and negative-sequence circuits are therefore identical excepting for the direction of the phase shifts introduced by the star-delta transformation.

The zero-sequence circuit is derived by applying a set of zero-sequence voltages to the *abc* terminals. In this case

$$E_{ag} = E_{bg} = E_{cg} = E_0$$

$$I_a = I_b = I_c = I_0$$

$$E_{ag} = e_p + Z_P I_a.$$

$e_s - I_x Z_S = 0$ because no zero-sequence voltage can be present between line terminals. (95)

$$I_x = \frac{n_1}{n_2} I_a$$

$$e_p = \frac{n_1}{n_2} e_s = \left(\frac{n_1}{n_2}\right)^2 I_a Z_S$$

$$E_{ag} = I_a\left[\left(\frac{n_1}{n_2}\right)^2 Z_S + Z_P\right] = I_a Z_{PS}$$

$Z_0 = \frac{E_0}{I_0} = \frac{E_{ag}}{I_a} = Z_{PS}$, which is the same impedance as was obtained with positive-sequence voltages and currents. (96)

If zero-sequence voltages are applied to the $a'b'c'$ terminals, no current can flow because no return circuit is present. The zero-sequence impedance of the transformer bank is therefore infinite as viewed from the delta side.

59. Derivation of Equivalent Circuit for Autotransformer with Delta Tertiary

The basic impedances of an autotransformer with a delta tertiary may be defined in terms of the leakage impedances between pairs of windings, with the third winding open circuited. The impedance between the primary and secondary, or common and series, windings of the transformer in Fig. 58(a) may be obtained by applying a voltage across the P winding with the S winding short circuited, and the T winding open circuited. Referring to Fig. 59,

$$e_s = \left(\frac{1}{n_1}\right) I Z_S \qquad e_p = \frac{e_s}{n_1} = \frac{I Z_S}{n_1^2}$$

$$E = e_p + I Z_P$$

$$= I\left(\frac{Z_S}{n_1^2} + Z_P\right) \tag{97}$$

$$Z_{PS} = E/I = \frac{Z_S}{n_1^2} + Z_P.$$

Similar relations can be derived for the impedances

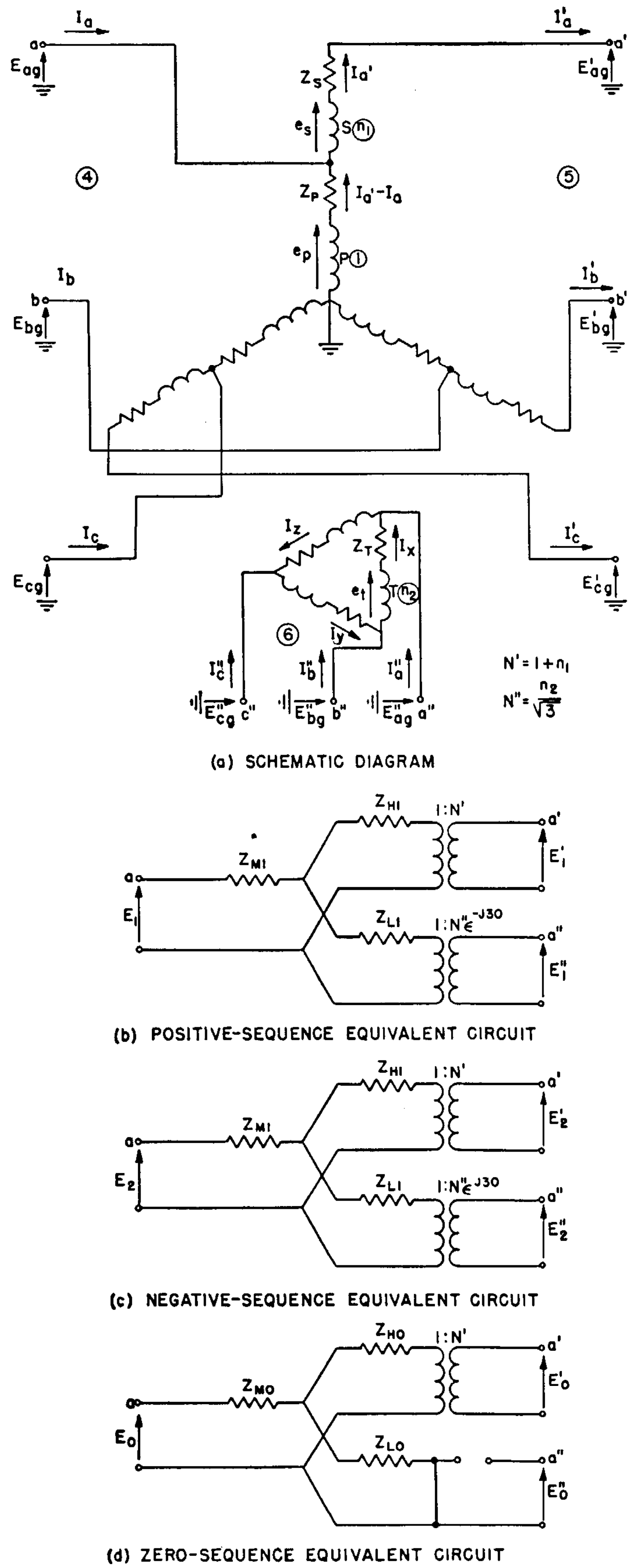

Fig. 58—Equivalent circuits of a three-winding autotransformer.

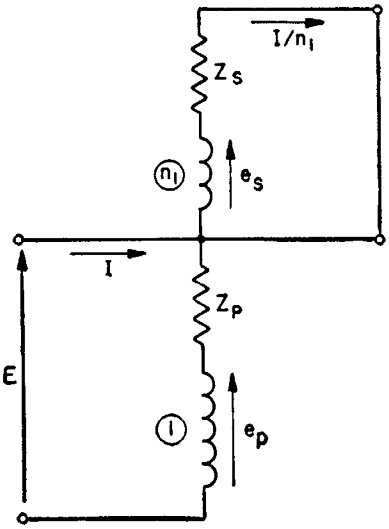

Fig. 59—Representation of the primary- to secondary-winding impedance of an autotransformer.

between the P and T, and S and T windings, resulting in the set of equations

$$\begin{aligned} Z_{PS} &= \frac{Z_S}{n_1^2} + Z_P \\ Z_{PT} &= \frac{Z_T}{n_2^2} + Z_P \\ Z_{ST} &= \left(\frac{n_1}{n_2}\right)^2 Z_T + Z_S. \end{aligned} \tag{98}$$

These equations can be solved for the individual winding impedances Z_P, Z_S and Z_T.

$$\begin{aligned} Z_P &= \tfrac{1}{2}\left[Z_{PS} + Z_{PT} - \frac{Z_{ST}}{n_1^2}\right] \\ Z_S &= \tfrac{1}{2}[Z_{ST} + n_1^2 Z_{PS} - n_1^2 Z_{PT}] \\ Z_T &= \tfrac{1}{2}\left[\left(\frac{n_2}{n_1}\right)^2 Z_{ST} + n_2^2 Z_{PT} - n_2^2 Z_{PS}\right]. \end{aligned} \tag{99}$$

The impedances among circuits 4, 5 and 6 can be derived in terms of the impedances between windings, using the same procedure as employed in the derivation of the impedances of the star-delta bank in section 58.

With positive-sequence voltages applied to terminals abc, terminals $a'b'c'$ short circuited and terminals $a''b''c''$ open circuited, the following relations can be written:

$$E'_{ag} = E'_{bg} = E'_{cg} = 0 \qquad e_s = n_1 e_p$$

$$I'_a - I_a + n_1 I'_a = 0 \qquad I'_a = \frac{I_a}{1+n_1}$$

$$e_p + e_s - (I_a' - I_a)Z_P - I_a' Z_S = 0.$$

Eliminating e_s and I'_a from the above equation:

$$e_p(1+n_1) = \frac{I_a}{1+n_1}(Z_P + Z_S) - I_a Z_P$$

$$e_p = \frac{I_a}{(1+n_1)^2}(Z_P + Z_S) - \frac{I_a Z_P}{1+n_1} \tag{100}$$

$$\begin{aligned} E_{ag} &= e_p - (I_a' - I_a)Z_P \\ &= e_p + I_a Z_P\left(1 - \frac{1}{1+n_1}\right) \\ &= I_a \times \frac{Z_S + n_1^2 Z_P}{(1+n_1)^2} \\ Z_{45} &= \frac{E_{ag}}{I_a} = \frac{n_1^2}{(1+n_1)^2}\left[Z_P + \frac{Z_S}{n_1^2}\right] \\ &= \frac{n_1^2}{(1+n_1)^2} \times Z_{PS}. \end{aligned} \tag{101}$$

Representing the circuit transformation ratio $(1+n_1)$ by N',

$$Z_{45} = \left(\frac{N'-1}{N'}\right)^2 \times Z_{PS} \tag{102}$$

The impedance between circuits 4 and 6 may be obtained by applying positive-sequence voltages to terminals abc, with terminals $a'b'c'$ open and $a''b''c''$ shorted circuited. In this case:

$$\begin{aligned} e_t &= I_x Z_T \qquad\qquad I_x = I_a/n_2 \\ e_p &= \frac{e_t}{n_2} = \frac{I_a Z_T}{n_2^2} \\ E_{ag} &= e_p + I_a Z_P \\ &= I_a\left[\frac{Z_T}{n_2^2} + Z_P\right] \\ Z_{46} &= \frac{E_{ag}}{I_a} = \frac{Z_T}{n_2^2} + Z_P = Z_{PT}. \end{aligned} \tag{103}$$

With positive-sequence voltages applied to terminals $a'b'c'$, terminals abc open and terminals $a''b''c''$ short circuited,

$$\begin{aligned} e_t &= I_x Z_T \qquad\qquad I_x = -\frac{1+n_1}{n_2} I_a' \\ e_p + e_s &= \frac{1+n_1}{n_2} e_t = -\left(\frac{1+n_1}{n_2}\right)^2 I_a' Z_T \\ E'_{ag} &= e_p + e_s - I'_a(Z_P + Z_S) \\ &= -I'_a\left[\left(\frac{1+n_1}{n_2}\right)^2 Z_T + Z_P + Z_S\right] \\ Z_{56} &= \left(\frac{1+n_1}{n_2}\right)^2 Z_T + Z_P + Z_S. \end{aligned} \tag{104}$$

Expressing Z_P, Z_S and Z_T in terms of impedances between windings as given in Eq. (99):

$$Z_{56} = (1+n_1)Z_{PT} + \left(\frac{1+n_1}{n_1}\right) Z_{ST} - n_1 Z_{PS}. \tag{105}$$

The above equation is the impedance between circuits 5 and 6 in ohms on the circuit 5 voltage base. As Z_{45} and Z_{46} are ohmic impedances on the circuit 4 base, it is convenient to express the circuit 5 to circuit 6 impedance on the same base. Dividing by $(1+n_1)^2$,

$$\frac{Z_{56}}{(1+n_1)^2} = \frac{Z_{PT}}{1+n_1} + \frac{Z_{ST}}{n_1(1+n_1)} - \frac{n_1}{(1+n_1)^2} Z_{PS}$$

$$\frac{Z_{56}}{(N')^2} = \frac{1}{N'} \times Z_{PT} + \frac{Z_{ST}}{N'(N'-1)} - \frac{N'-1}{N'^2} \times Z_{PS} \tag{106}$$

The transformer can be represented by the positive-sequence equivalent circuit in Fig. 58(b). The relations between the impedances in the equivalent circuit and the impedances between circuits can be expressed as follows:

$$\begin{aligned} &Z_{M1}+Z_{H1}=Z_{45}\\ &Z_{M1}+Z_{L1}=Z_{46}\\ &Z_{H1}+Z_{L1}=\frac{Z_{56}}{(N')^2}\end{aligned}\tag{107}$$

$$\begin{aligned} &Z_{H1}=\tfrac{1}{2}\left[Z_{45}+\frac{Z_{56}}{(N')^2}-Z_{46}\right]\\ &Z_{M1}=\tfrac{1}{2}\left[Z_{45}+Z_{46}-\frac{Z_{56}}{(N')^2}\right]\\ &Z_{L1}=\tfrac{1}{2}\left[Z_{46}+\frac{Z_{56}}{(N')^2}-Z_{45}\right]\end{aligned}\tag{108}$$

$$\begin{aligned} &Z_{H1}=\frac{N'-1}{2N'}\left[\frac{N'-2}{N'}Z_{PS}+\frac{Z_{ST}}{(N'-1)^2}-Z_{PT}\right]\\ &Z_{M1}=\frac{N'-1}{2N'}\left[Z_{PS}+Z_{PT}-\frac{Z_{ST}}{(N'-1)^2}\right]\\ &Z_{L1}=\frac{N'-1}{2N'}\left[\frac{N'+1}{N'-1}Z_{PT}+\frac{Z_{ST}}{(N'-1)^2}-Z_{PS}\right]\end{aligned}\tag{109}$$

$$\begin{aligned} &Z_{45}=\left(\frac{N'-1}{N'}\right)^2 Z_{PS}\\ &Z_{46}=Z_{PT}\\ &Z_{56}=N'Z_{PT}+\frac{N'}{N'-1}Z_{ST}-(N'-1)Z_{PS}\end{aligned}\tag{110}$$

$$\begin{aligned} &Z_{PS}=\left(\frac{N'}{N'-1}\right)^2 Z_{45}\\ &Z_{PT}=Z_{46}\\ &Z_{ST}=(N'-1)\left[\frac{Z_{56}}{N'}+\frac{N'}{N'-1}Z_{45}-Z_{46}\right]\end{aligned}\tag{111}$$

In the above equations Z_{H1}, Z_{M1}, Z_{L1}, Z_{45} and Z_{46} are in ohms on the circuit 4 (*abc* terminals) voltage base. Z_{56} is in ohms on the circuit 5 ($a'b'c'$ terminals) voltage base. Z_{PS} and Z_{PT} are in ohms on the P winding voltage base and Z_{ST} is in ohms on the S winding voltage base. N' is defined as $1+n_1$, which is the ratio of line-to-line or line-to-neutral voltages between circuit 5 ($a'b'c'$ terminals) and circuit 4 (*abc* terminals).

The phase shifts between circuit voltages can be determined by applying positive-sequence voltages to terminals *abc* with the other two circuits open circuited. Under these conditions,

$$E_{ag}=e_p \qquad E'_{ag}=e_p+e_s$$

$E'_{ag}=(1+n_1)\,E_{ag}=N'E_{ag}$, which shows that the one ideal transformer has an N' ratio but no phase shift.

$$e_t=E_{ag}''-E_{bg}''=E_{ag}''(1-a^2)$$

$$e_t=n_2e_p=n_2E_{ag}$$

$$E_{ag}''=\frac{n_2}{1-a^2}E_{ag}=\frac{n_2}{\sqrt{3}}E_{ag}\epsilon^{-j30}\tag{112}$$

Defining $\frac{n_2}{\sqrt{3}}$ as N'',

$$E_{ag}''=N''E_{ag}\epsilon^{-j30}.$$

The second ideal transformer therefore has an N'' turns ratio and a 30 degree phase shift.

Negative-Sequence Circuit—A similar analysis made with negative-sequence voltages would show that the impedances in the equivalent circuit are the same as in the positive-sequence circuit, and that the terminal voltages are related as follows:

$$\begin{aligned} &E_2'=N'E_2\\ &E_2''=N''E_2\epsilon^{+j30}.\end{aligned}\tag{113}$$

The positive- and negative-sequence circuits are therefore identical excepting for the direction of the phase shift introduced by the star-delta transformation.

Zero-sequence circuit—The zero-sequence characteristics of the transformer can be obtained as follows:

1. Apply zero-sequence voltages to terminals *abc* with terminals $a'b'c'$ connected to ground and the delta opened. This permits evaluation of the zero-sequence impedance between circuit 4 and circuit 5.
2. Apply zero-sequence voltages to terminals *abc* with the delta closed and terminals $a'b'c'$ open circuited.
3. Apply zero-sequence voltages to terminals $a'b'c'$ with the delta closed and terminals *abc* open circuited.

The general procedure in writing the necessary equations is similar to that followed in the positive-sequence analysis given above, and the zero-sequence analysis in section 57. It will be found that the zero-sequence impedances in the equivalent circuit shown in Fig. 58(d) are the same as the positive-sequence quantities, that is,

$$\begin{aligned} &Z_{H0}=Z_{H1}\\ &Z_{M0}=Z_{M1}\\ &Z_{L0}=Z_{L1}\end{aligned}\tag{114}$$

If the neutral of the autotransformer is ungrounded, the zero-sequence equivalent circuit is altered considerably as shown in Fig. 60. In this case zero-sequence current flows

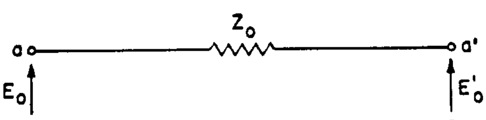

Fig. 60—Zero-sequence equivalent circuit of an ungrounded three-winding autotransformer.

between terminals *abc* and $a'b'c'$ without transformation. Current in the S winding is balanced by circulating currents in the tertiary, with no current flow in the P winding. The zero-sequence impedance is therefore determined by the leakage impedance between the S and T windings. Applying zero sequence voltages to the *abc* terminals, with the $a'b'c'$ terminals connected to ground and the tertiary closed,

$$I_a=I_a'=-\frac{n_2}{n_1}I_x$$

$$e_t=I_xZ_T$$

$$e_s=\frac{n_1}{n_2}e_t=-\left(\frac{n_1}{n_2}\right)^2 I_a'Z_T$$

$$E_{ag} = I_a' Z_S - e_s$$
$$= I_a'\left[Z_S + \left(\frac{n_1}{n_2}\right)^2 Z_T\right]$$
$$Z_0 = \frac{E_{ag}}{I_a} = \frac{E_{ag}}{I_a'} = Z_S + \left(\frac{n_1}{n_2}\right)^2 Z_T = Z_{ST}$$
$$= (N'-1)\left[\frac{Z_{56}}{N'} + \frac{N'}{N'-1} Z_{45} - Z_{46}\right] \tag{115}$$

Percent Quantities—The manufacturer normally expresses transformer impedances in percent on a kva base corresponding to the rated kva of the circuits involved. These percent values can be converted to ohms by the familiar relation

$$Z = \frac{10Z\%E^2}{\text{kva}}, \text{ where} \tag{116}$$

Z = impedance in ohms.
$Z\%$ = impedance in percent.
kva = 3-phase kva rating of circuit.
E = line-to-line circuit voltage in kv.

Using the nomenclature employed in the derivations,

$$Z_{45} = \frac{10Z_{45}\%\ E_4^2}{U_4}, \text{ where}$$

E_4 = line-to-line voltage, in kv, of circuit 4.
U_4 = three-phase kva rating of circuit 4.
$Z_{45}\%$ = impedance between circuits 4 and 5 in per cent on kva rating of circuit 4.
Z_{45} = impedance between circuits 4 and 5 in ohms on the circuit 4 voltage base.

Similar relations can be written for the other impedances involved.

It should be noted that the impedances, as used in this chapter and in the Appendix, are expressed in terms of the voltage or kva rating of the circuit or winding denoted by the first subscript. For example Z_{45} is in ohms on the circuit 4 voltage base, whereas Z_{54} would be in ohms on the circuit 5 voltage base. These impedances can be converted from one circuit base to another as follows:

$$Z_{54} = \left(\frac{E_5}{E_4}\right)^2 Z_{45}$$
$$Z_{54}\% = \frac{U_5}{U_4} Z_{45}\% \tag{117}$$

The equivalent circuits can be based directly on percent quantities as shown in Table 7 of the Appendix. Considering the autotransformer with delta tertiary (case D-1 in Table 7), the equivalent circuit impedances can be obtained from the impedances between circuits as follows:

$$Z_{H1}\% = \tfrac{1}{2}\left[\frac{U_4}{U_5} Z_{56}\% + Z_{45}\% - Z_{46}\%\right]$$
$$Z_{M1}\% = \tfrac{1}{2}\left[Z_{45}\% + Z_{46}\% - \frac{U_4}{U_5} Z_{56}\%\right] \tag{118}$$
$$Z_{L1}\% = \tfrac{1}{2}\left[Z_{46}\% + \frac{U_4}{U_5} Z_{56}\% - Z_{45}\%\right]$$

The resulting impedances will all be in percent on the circuit 4 kva base.

REFERENCES

1. Electric Circuits—Theory and Applications, by O. G. C. Dahl (a book) Vol. 1, p. 34, McGraw-Hill Book Company, Inc., New York.
2. Regulating Transformers in Power-System Analysis, by J. E. Hobson and W. A. Lewis, *A.I.E.E. Transactions*, Vol. 58, 1939, p. 874.
3. Fundamental Concepts of Synchronous Machine Reactances, by B. R. Prentice, *A.I.E. E. Transactions*, Vol. 56, 1937, pp. 1–22 of Supplement.
4. Simplified Computation of Voltage Regulation with Four Winding Transformers, by R. D. Evans, *Electrical Engineering*, October 1939, p. 420.
5. Surge Proof Transformers, by H. V. Putman, *A.I.E.E. Transactions*, September 1932, pp. 579–584 and discussion, pp. 584–600.
6. American Standards for Transformers, Regulators, and Reactors. American Standards Association, ASA C57, 1948.
7. Loading Transformers by Copper Temperature, by H. V. Putman and W. M. Dann *A.I.E.E. Transactions*, Vol. 58, 1939, pp. 504–509.
8. Equivalent Circuit Impedance of Regulating Transformers, by J. E. Clem, *A.I.E.E. Transactions*, Vol. 58, 1939, pp. 871–873.
9. Theory of Abnormal Line to Neutral Transformer Voltages, by C. W. LaPierre, *A.I.E.E. Transactions*, Vol. 50, March 1931, pp. 328–342.
10. Standards for Transformers NEMA Publication No. 48–132, September 1948.
11. An Equivalent Circuit for the Four-Winding Transformer, by F. M. Starr, *General Electric Review*, March 1933, Vol. 36, pp. 150–152.
12. Transformer Engineering, by L. F. Blume, et al, (a book), John Wiley and Sons (1938).
13. Equivalent Circuits for Power and Regulating Transformers, by J. E. Hobson and W. A. Lewis, *Electric Journal* Preprint, January 1939.
14. J. and P. Transformer Book, by Stigant, 6th Edition, 1935, Johnson and Phillips, London.

CHAPTER 6

MACHINE CHARACTERISTICS

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BEFORE the growth of the public utilities into their present enormous proportions with large generating stations and connecting tie lines, machine performance was largely judged in terms of the steady-state characteristics. The emergence of the stability problem gave rise to the analysis of the transient characteristics of machines and was largely responsible for our present knowledge of machine theory. A further contributing urge was the need for more accurate determination of short-circuit currents for the application of relays and circuit breakers.

The variable character of the air gap of the conventional salient-pole synchronous generator, motor, and condenser with its concentrated field windings requires that their analysis follow a different line from that for machines such as induction motors, which have a uniform air gap and distributed windings. Blondel originally attacked this problem by resolving the armature mmf's and fluxes into two components, one in line with the axis of the poles and the other in quadrature thereto. When the study of the transients associated with system stability was undertaken this conception was quickly recognized as an invaluable tool[1,2]. Since that time the method has been extended by subsequent investigators,[3–9] notably Doherty and Nickle, who introduced into the industry several new constants, such as transient reactance and subtransient reactance to describe machine performance under transient conditions.

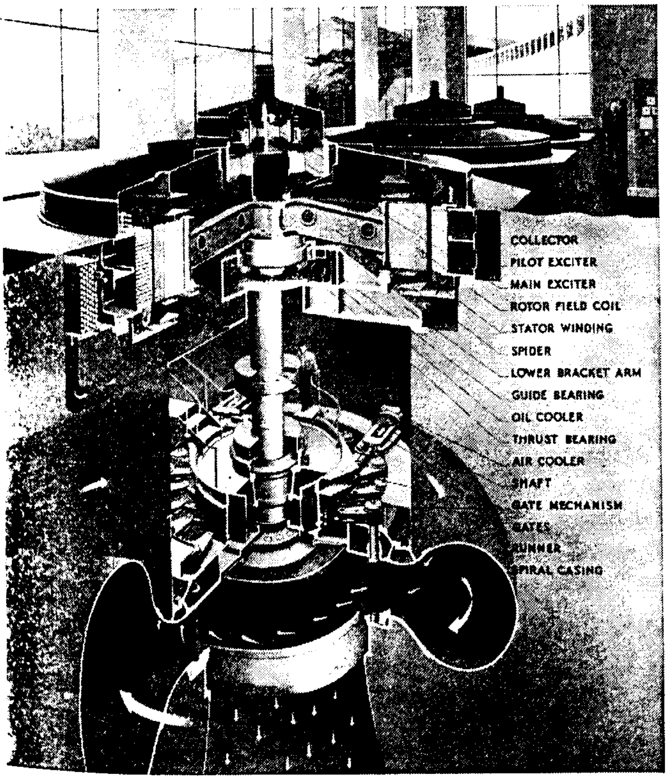


Fig. 1—Cut-away view of umbrella-type waterwheel generator.

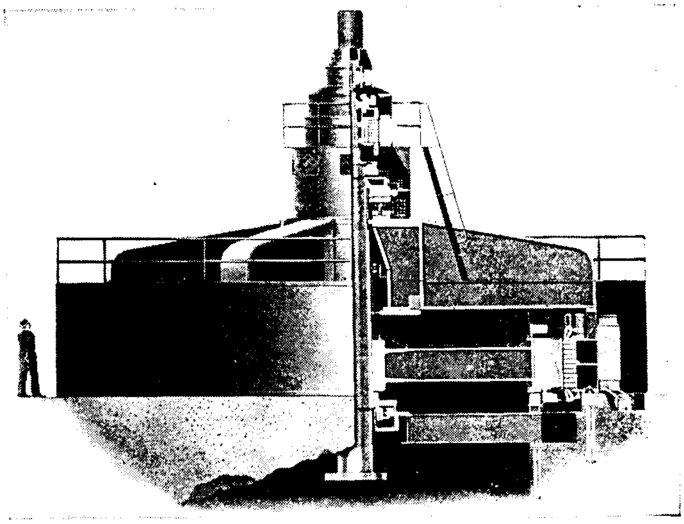


Fig. 2—Cut-away view of conventional waterwheel generator.

This chapter treats of the characteristics of synchronous and induction machines in the light of the development of the past twenty-five years. It will consider steady-state and transient conditions for both salient pole and cylindrical rotor machines under both balanced and unbalanced conditions. There follows a discussion of the characteristics of induction motors under such transient conditions as might contribute to the short-circuit current of a system and might influence the choice of a circuit breaker.

I. STEADY-STATE CHARACTERISTICS OF SYNCHRONOUS MACHINES

The two general types of synchronous machines are the cylindrical rotor machine or turbine generator which has an essentially uniform air gap and the salient-pole generator. Figs. 1 to 5 illustrate the outward appearances and cross-sectional views of typical modern machines.

Typical saturation curves for a hydrogen-cooled turbine generator, a waterwheel-generator and a synchronous condenser are shown in Figs. 6, 7, and 8 respectively.

Because of the necessity of matching the speed of waterwheel-generators to the requirements of the waterwheels it is difficult to standardize units of this type. However,

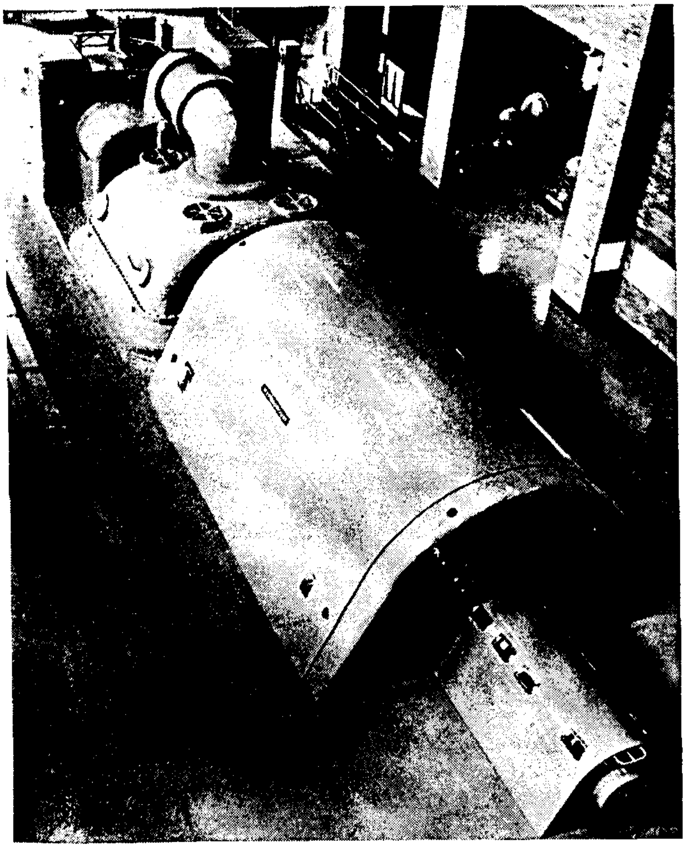


Fig. 3—Steam turbine generator installed at the Acme Station of the Toledo Edison Company, 90 000 kw, 85-percent power factor, 85-percent *SCR*., 13 800 volt, 3-phase, 60-cycle.

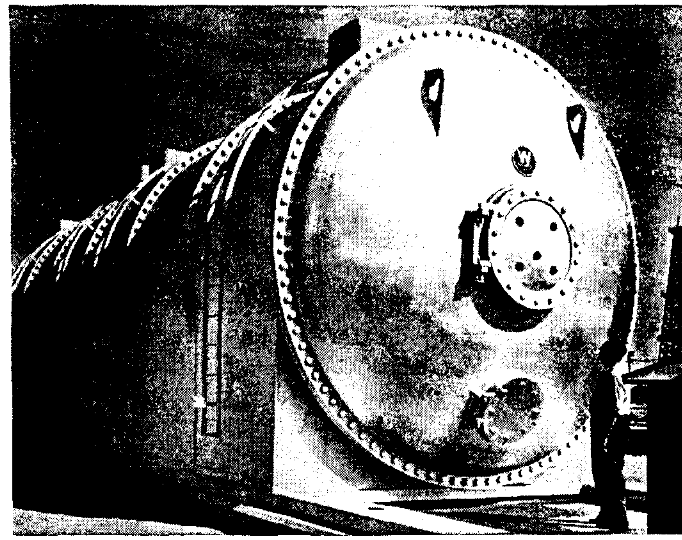


Fig. 5—Hydrogen-cooled frequency changer set installed on the system of the City of Los Angeles, 60 000 kva; 600 rpm; 50 cycle—11 500 volts; 60 cycles—13 200 volts.

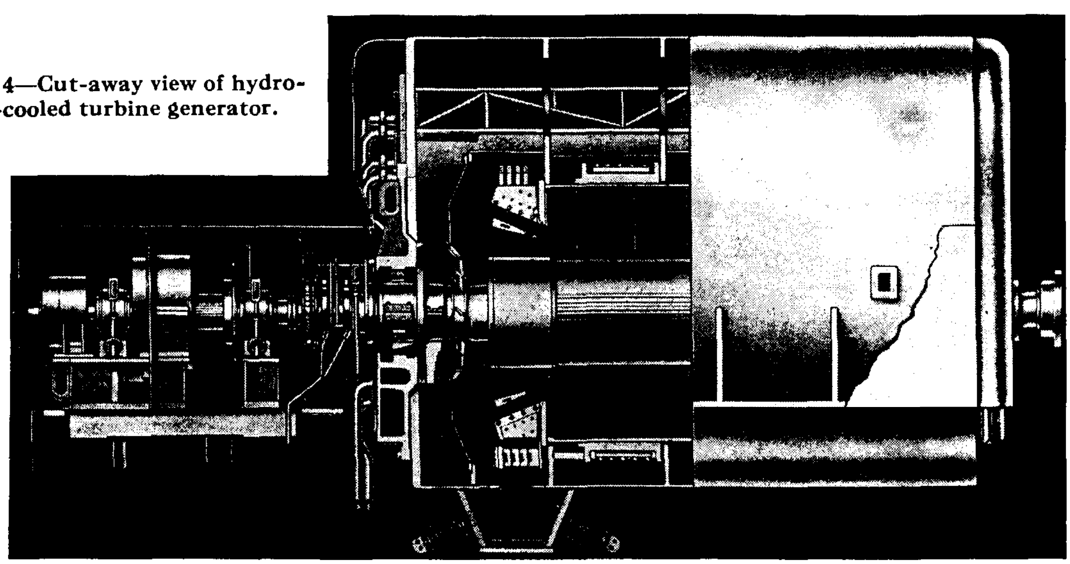
great strides have been made with large 3600-rpm condensing steam turbine-generators. These find their greatest application in the electric utility industry. Table 1 of Chap. 1 gives some of the specifications[20] for these machines.

The concept of per-unit quantity is valuable in comparing the characteristics of machines of different capacities and voltages. However, care must be exercised in the case of generators to use the same reference value for field current. Depending upon the application, either the field current for rated voltage in the air gap or the actual field current for rated voltage, including saturation, is used.

1. Unsaturated Cylindrical-Rotor Machine Under Steady-State Conditions

The vector diagram of Fig. 9 is the well-known diagram of a cylindrical-rotor machine. Consistent with the policy of this book, familiarity with this diagram is assumed. Let it suffice merely to indicate the significance of the quantities. The vectors e_t and i represent the terminal voltage to neutral and armature current, respectively. Upon adding the armature resistance drop, ri, and armature leakage reactance drop, $x_l i$ to e_t, the vector e_l is obtained, which represents the voltage developed by the air-gap flux Φ, which leads e_l by 90 degrees. This flux represents the net flux in the air gap. To produce this flux a field current, I_t, is required. The current I_t can be taken from the no-load

Fig. 4—Cut-away view of hydrogen-cooled turbine generator.



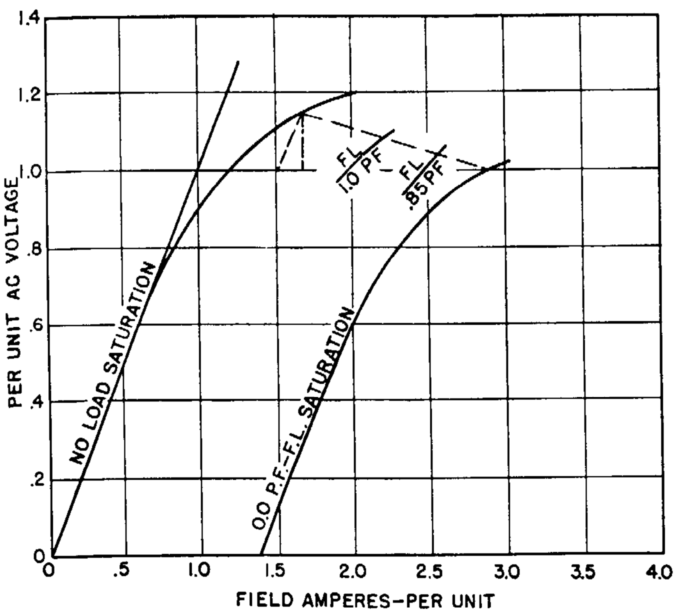


Fig. 6—Saturation curves for typical hydrogen-cooled turbine generator.

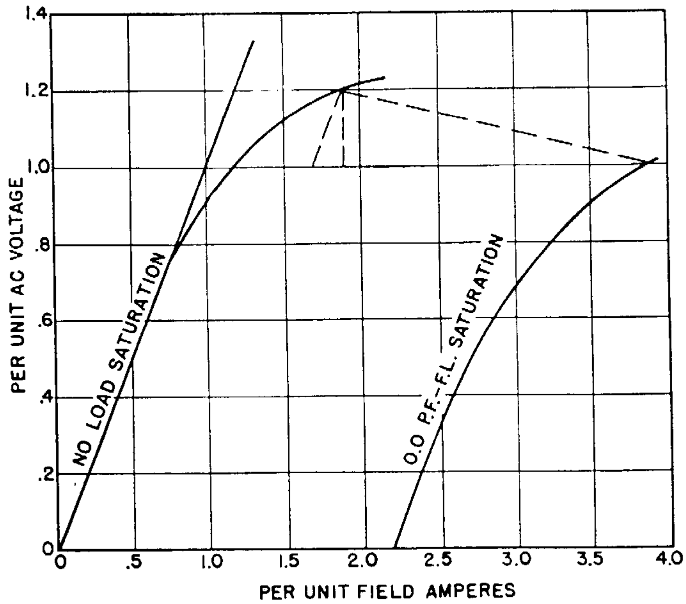


Fig. 8—Saturation curves for typical hydrogen-cooled condenser.

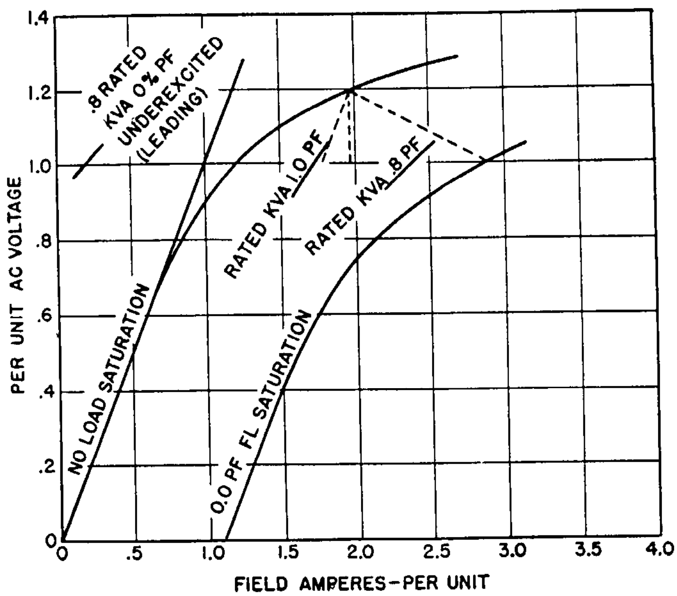


Fig. 7—Saturation curves for typical waterwheel generator.

saturation curve of Fig. 10 as being the current required to produce e_l. But, the armature current produces an mmf by its so-called armature reaction, which is in time phase with it and in terms of the field can be expressed as Ai. To produce the net mmf represented by the current, I_t, the field current must be of such magnitude and the field structure must adjust itself to such position as to equal I_f. In other words, I_f has now such position and magnitude that I_f and Ai added in vectorial sense equals I_t. The triangle OAB, formed by drawing AB perpendicular to i or Ai and OB perpendicular to OC, is similar to the triangle ODC; OB has the same proportionality to OC and AB to Ai as e_l has to I_t. Neglecting saturation, OB, designated as e_i, is thus the open-circuit voltage corresponding to the field current I_f; it is the voltage taken from the air-gap line of the no-load saturation curve for the abscissa corresponding to I_f. The side AB of the triangle, since it is proportional to Ai and consequently proportional to the armature current, can be viewed as a fictitious reactance drop. It is called the drop of armature reactance and is designated $x_a i$. The reactance drops $x_l i$ and $x_a i$ can be combined into a single term called the synchronous reactance drop and there results

$$x_d = x_l + x_a \tag{1}$$

It follows from the foregoing that the internal voltage, e_i, is equal to the vector sum of e_t, ri and $j\,x_d i$. The field current, I_f, can be determined for any condition of loading (neglecting saturation, of course) by merely calculating e_i and taking I_f from the air-gap line of Fig. 10.

At no load the axis of the field winding, the line OC, leads the terminal voltage by 90 degrees. At zero power-factor, the vector diagram reduces to that shown in Fig. 11, which shows that, except for the effect of the resistance drop, the foregoing statement would still be true. As ri is only about one or two percent in practical machines, the statement

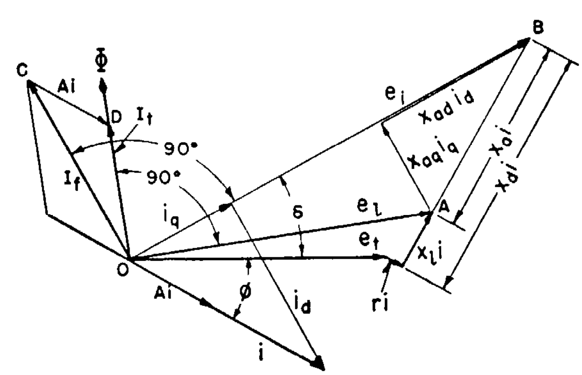


Fig. 9—Vector diagram of cylindrical-rotor machine.

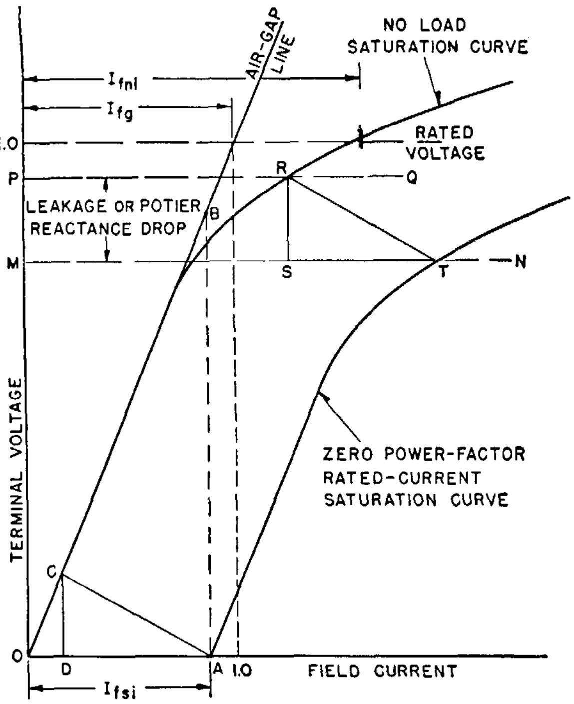


Fig. 10—No-load and full-load zero power-factor characteristics of a generator.

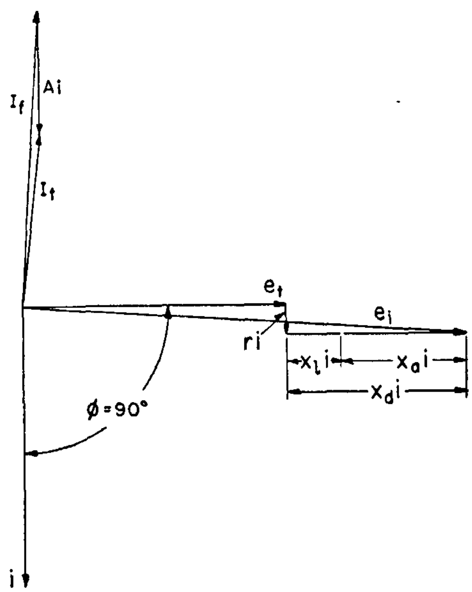


Fig. 11—Vector diagram of cylindrical-rotor generator at zero power-factor.

can be accepted as true for all practical purposes. However, as the real load is applied to the machine the angle δ increases from zero and the lead of OC ahead of e_t increases from 90 degrees to 90 degrees plus δ. The angle δ is a real angle; it can be measured without much difficulty.

It is convenient for some purposes to resolve the reactions within the machine into two components, one along the axis of the field winding and the other in quadrature thereto. In Fig. 9, the armature current is divided into the two components, i_d, and, i_q, in which the subscripts are significant of their respective components. When this is done, it can be seen that $x_a i$ can likewise be thought of as arising from the two components of i in the form of $x_{ad} i_d$ and $x_{aq} i_q$, respectively, in leading quadrature to i_d and i_q. In the case of a cylindrical rotor machine, x_{ad} and x_{aq} are both equal to x_a but a case will soon be developed for which they are not equal.

The synchronous reactance, x_d, can be obtained most conveniently from the no-load curve and the full-load zero power-factor curve. In Fig. 10 OA is the field current required to circulate full-load current under short-circuit conditions, the terminal voltage being zero. In this case all of the internal voltage (the ri drop can be neglected justifiably) must be consumed as synchronous reactance drop ($x_d i$) within the machine. If there were no saturation, the internal voltage can be determined by simply reading the terminal voltage when the short-circuit is removed, maintaining the field current constant meanwhile. This voltage would in Fig. 10 be equal to AB. Thus the unsaturated synchronous reactance per phase is equal to the phase-to-neutral voltage AB divided by the rated current. When the saturation curve is expressed in per unit or percent it is equal to AB; but where expressed in generator-terminal voltage and field amperes, it is equal to $\frac{I_{fsi}}{I_{fg}}(100)$ in percent or $\frac{I_{fsi}}{I_{fg}}$ in per unit.

2. Unsaturated Salient-Pole Machine Under Steady-State Conditions

Given the proper constants, the performance of an unsaturated salient-pole machine at zero power-factor is the same as for a uniform air-gap machine. For other power-factors, conditions are different. A vector diagram for such machines is shown in Fig. 12. As before e_t and i are the

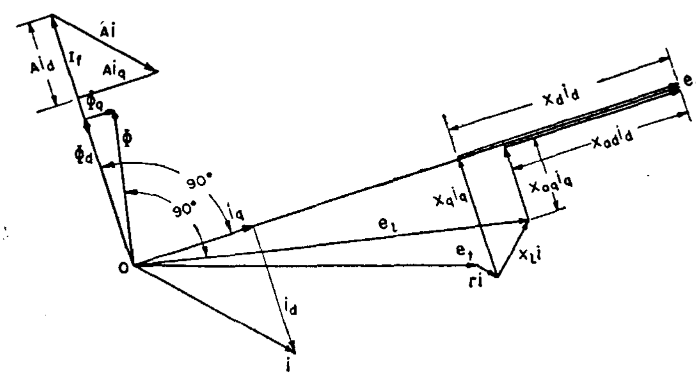


Fig. 12—Vector diagram of salient-pole machine.

terminal voltage to neutral and the armature current, respectively, and e_l is the "voltage behind the leakage reactance drop." The flux Φ is required to produce e_l. This flux can be resolved into two components Φ_d and Φ_q. The flux Φ_d is produced by I_f and Ai_d, the direct-axis component of Ai, and Φ_q is produced by Ai_q, the quadrature-axis component of Ai. Here the similarity ceases. Because of the saliency effect, the proportionality between the mmf's and their resultant fluxes is not the same in the two axes. When saturation effects are neglected Φ_d can be regarded as made up of a component produced by I_f acting

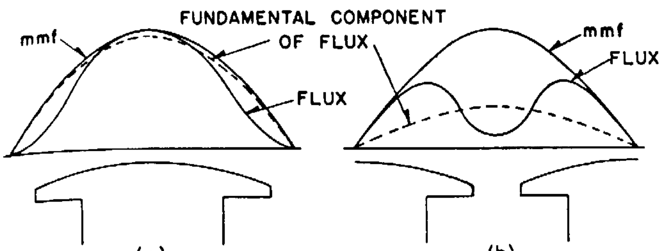


Fig. 13—Flux resulting from a sinusoidal mmf in

(a) direct axis,
(b) quadrature axis.

alone and a component produced by Ai_d. The component produced by I_f can be regarded as producing the internal voltage e_d. The mmf produced by Ai_d has a general sinusoidal distribution in the direct axis as shown by Fig. 13(a). The resultant flux because of the variable reluctance of the air gap has the general shape indicated. It is the sinusoidal component of this flux that is effective in producing the $x_{ad}\, i_d$ drop shown in Fig. 12. In the quadrature axis, the component of mmf is likewise sinusoidal in nature as shown in Fig. 13b, and gives rise to the distorted flux form. In proportion to the mmf the sinusoidal component of flux is much less than for the direct axis. The effect of this component is reflected in the $x_{aq}\, i_q$ drop of Fig. 12. In general x_{aq} is much smaller than x_{ad}.

The armature resistance and leakage reactance drops can also be resolved into its two components in the two axes much as $x_a i$ of Fig. 9 was resolved. When this is done the internal voltage e_d can be obtained by merely adding ri_d and ri_q and then $j\, x_q\, i_q$ and $j\, x_d\, i_d$ to the terminal voltage e_t. The notation e_d is used to differentiate the internal voltage in this development from that used with the cylindrical rotor machine theory.

Another form of the vector diagram of the machine is presented in Fig. 14, which shows much better the relation between those quantities that are most useful for calculation purposes. If from B the line BP of length $x_q i$ is drawn perpendicular to i, then since angle CBP is equal to $\phi+\delta$, the distance BC is equal to $x_q i$ cos $(\phi+\delta)$, or $x_q i_q$. By comparing this line with the corresponding line in Fig. 12, it can be seen that the point P determines the angle δ. This relation provides an easy construction for the determination of the angle δ having given the terminal voltage, the armature current, and the power-factor angle, ϕ. Further,

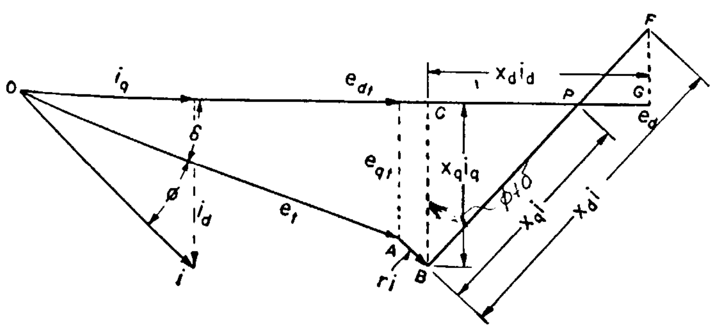


Fig. 14—Determination of internal angle, δ, and excitation of an unsaturated salient-pole machine when loading is known.

the projection of BF upon OG is equal to $x_d i_d$ so that OG becomes equal to e_d, the fictitious internal voltage, which is proportional to I_f.

The armature resistance is usually negligible in determining either the angle δ or the excitation and for this case

$$e_t \sin \delta = x_q i_q = x_q i \cos(\phi+\delta) \tag{2}$$

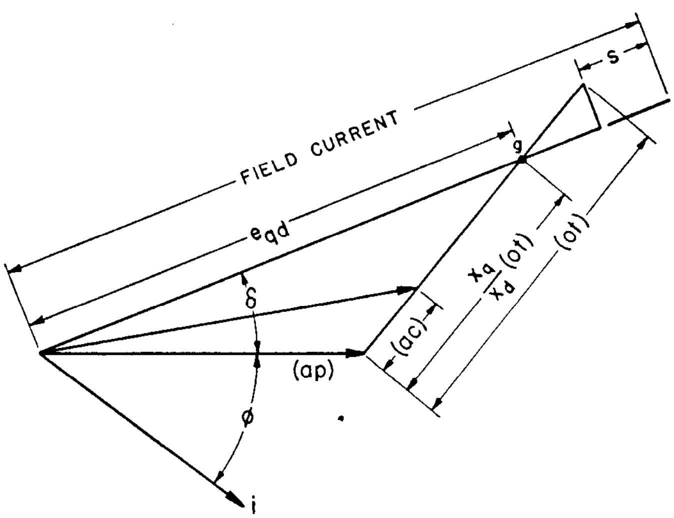
Upon expanding the last term and solving for δ

$$\tan \delta = \frac{x_q i \cos \phi}{e_t + x_q i \sin \phi} \tag{3}$$

From Fig. 14, the internal voltage

$$e_d = e_t \cos \delta + x_d i \sin (\phi+\delta) \tag{4}$$

The unsaturated synchronous reactance, x_d, can be determined from the no-load and full-load zero power-factor curves just as for the machine with uniform air gap. The quadrature-axis synchronous reactance is not obtained so



FOR SIGNIFICANCE OF QUANTITIES IN PARENTHESES REFER TO FIG.18

Fig. 15—Determination of internal angle, δ, and excitation of a saturated salient pole machine when loading is known.

easily but fortunately there is not as much need for this quantity. It can be determined from a test involving the determination of the angular displacement of the rotor as real load is applied to the machine and the use of Eq. (2), which gives

$$x_q = \frac{e_t \sin \delta}{i \cos (\phi+\delta)} \tag{5}$$

or it can be determined by means of a slip test. The slip test is described in the A.I.E.E. Test Code for Synchronous Machines[10] of 1945 for a determination of x_d. The test for the determination of x_q is identical except that the minimum ratio of armature voltage to armature current is used.

3. Saturation in Steady-State Conditions

Short-circuit ratio is a term used to give a measure of the relative strengths of the field and armature ampere turns. It is defined as the ratio of the field current required to produce rated armature voltage at no load to the field current required to circulate rated armature current with the armature short-circuited. In Fig. 10 the SCR is equal to $\frac{I_{fnl}}{I_{fsi}}$. When no saturation is present it is simply the reciprocal of the synchronous impedance, x_d.

It is impossible to specify the best specific *SCR* for a given system. In the past it has been the practice in Europe to use somewhat smaller *SCR*'s than was the practice in this country. In recent years, however, the trend in this country has been toward smaller values. The Preferred Standards for Large 3600-rpm Condensing Steam Turbine-Generators[20] specifies *SCR* of 0.8.

The desire for smaller *SCR*'s springs from the fact that the cost is smaller with smaller *SCR*. On the other hand, static stability is not as good with smaller *SCR*. Regulation is also worse but both of these effects are alleviated in part by automatic voltage regulators. For most economical design a high *SCR* machine usually has a lower x_d'. Therefore, both because of its lower x_d' and higher WR^2 a high *SCR* has a higher transient stability. This is not usually a significant factor particularly in condensing turbine applications, because transient stability is not of great importance in the systems in which they are installed. It may be quite important for hydro-generators; the Boulder Dam machines, for example, are designed for *SCR*'s of 2.4 and 2.74.

The effects of saturation arise primarily in the determination of regulation. Tests indicate that for practical purposes both the cylindrical rotor and the salient-pole machine can be treated similarly. Consideration will be given first to the characteristics for zero-power-factor loading. Fig. 11 shows that for zero power-factor, the *ri* drop of the machine is in quadrature to the terminal voltage and internal drop and can have little effect upon regulation. It will therefore be neglected entirely.

The determination of the rated-current zero-power-factor curve can be developed as follows. Take any terminal voltage such as *MN* of Fig. 10. The voltage behind leakage reactance is obtained by adding to this voltage the leakage reactance drop, *SR*, which gives the line *PQ*. The distance *PR* then gives the field current necessary for magnetizing purposes. In addition, however, field current is required to overcome the demagnetizing effect of the armature current. This mmf is represented on the curve by the distance *ST*, giving *MT* as the field current required to produce the terminal voltage *OM* with rated current in the armature. Other points on the rated-current zero-power-factor curve can be obtained by merely moving the triangle *RST* along the no-load saturation curve.

Upon sliding the triangle *RST* down to the base line, it can be seen that the total field current required to circulate rated current at short circuit which is represented by the point *A*, can be resolved into the current *OD* necessary to overcome leakage reactance drop and the current *DA* required to overcome demagnetizing effects. Neither leakage reactance nor the field equivalent of armature current are definite quantities in the sense that they can be measured separately. They may be calculated but their values are dependent upon the assumptions made for the calculations. Synchronous reactance, x_d, is a definite quantity and is equal to the distance *AO* expressed in either per unit or percent. When either x_l or x_a is assumed, then the other becomes determinable from Eq. (1) or from the triangle just discussed.

The foregoing analysis is not strictly correct, as it neglects certain changes in saturation in the pole structure. The leakage from pole to pole varies approximately proportional to the field current and the point *T* was determined upon the basis that this leakage was proportional to the field current *MS*. The increased field leakage at the higher excitation produces greater saturation in the field poles and this in turn increases the mmf required to force the flux through the pole. The net effect is to increase the field current over that determined by the method just discussed causing the two curves to separate more at the higher voltages.

The concept of the determination of the curve of rated current at zero-power-factor by the method just described is valuable and in an attempt to retain the advantages of this method the concept of *Potier reactance*, x_p, is introduced. The Potier reactance is the reactance that, used in a triangle of the general type described, will just fit between the two curves at rated voltage. It can be determined from test curves, see Fig. 16, by drawing *DE* equal

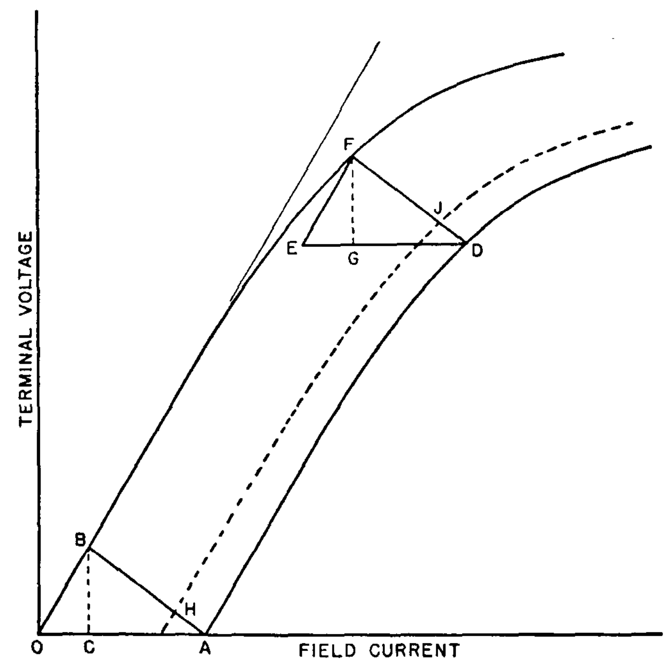


Fig. 16—Zero power-factor characteristics of generator.

to *OA* and then *EF* parallel to *OB*. The distance *FG* is then the Potier reactance drop. Potier reactance is thus a fictitious reactance that gives accurate results for only one point, the point for which it is determined. For most machines it is sufficiently accurate to use the one value obtained at rated voltage and rated current. Potier reactance decreases with increased saturation. Sterling Beckwith[19] proposed several approximations of Potier reactance, the two simplest are:

$$x_p = x_l + 0.63(x_d' - x_l)$$

and

$$x_p = 0.8\, x_d'.$$

For other loads at zero-power-factor, the conventional method is to divide the lines BA and FD of Fig. 16 in proportion to the armature current. Thus for three-fourths rated current the regulation curve would be the line HJ in which BH and FJ are three-fourths of BA and FD, respectively.

For power-factors other than zero, several methods are available to determine the regulation. They all give surprisingly close results, particularly at lagging power-factors. The problem may take either of two forms; the determination of the terminal voltage when the load current, load power-factor, and excitation are given, or the determination of the excitation when the load current, load power-factor, and terminal voltage are given. The resistance drop is so small that it is usually neglected.

(a) Adjusted Synchronous Reactance Method*—This method utilizes the no-load and the rated-current zero-power-factor curves. To obtain the excitation at any other power-factor for rated current, an arbitrary excitation is chosen such as OC of Fig. 17. The no-load voltage

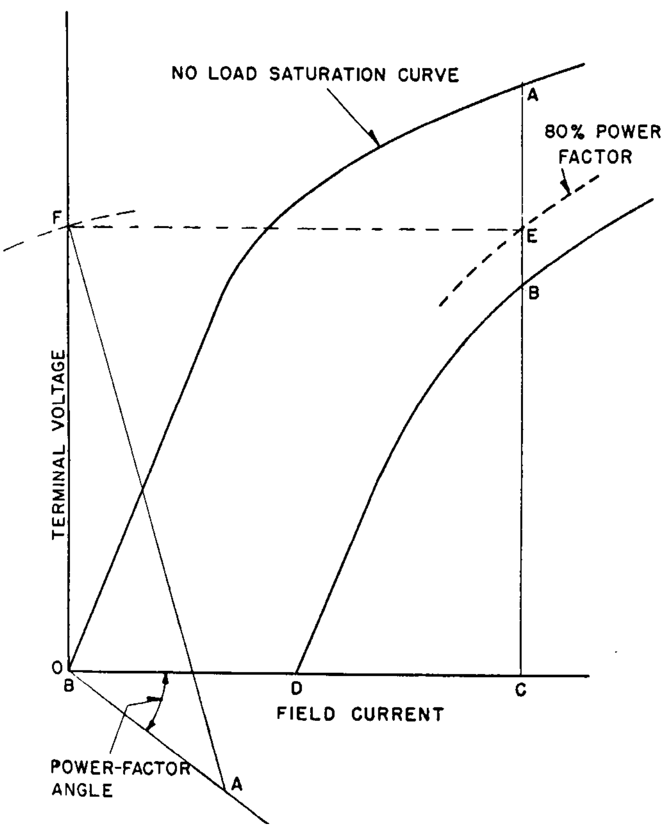


Fig. 17—Determination of regulation curves for power-factors other than zero by the "adjusted synchronous reactance method."

CA is then regarded as an internal voltage and the distance AB as an internal drop of pure reactance, which is laid off in proper relation with the terminal voltage as indicated by the power-factor of the load. The construction is as follows: The adjusted synchronous reactance drop AB is laid off to make an angle with the X-axis equal to the power-factor angle. A line equal to the distance AC is then scribed from the point A until it intercepts the Y-axis at

*Described as Method (c) Para. 1.540 in Reference 10.

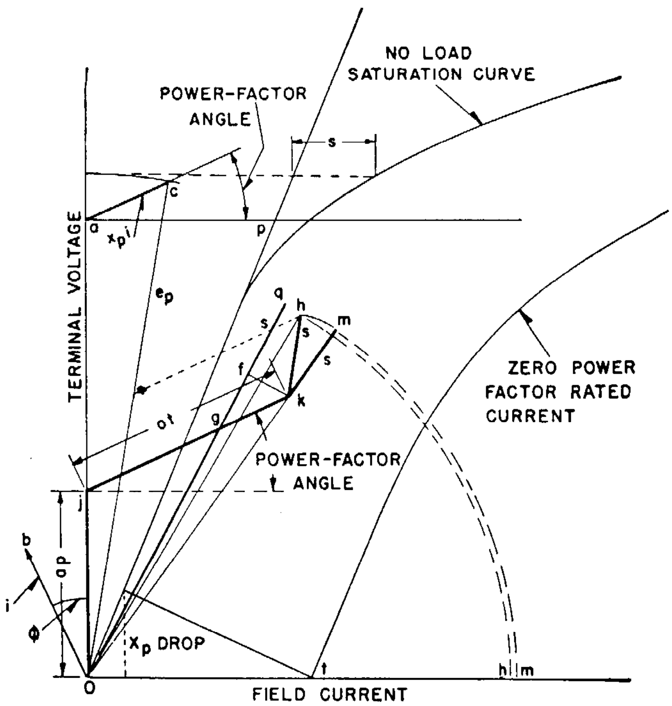


Fig. 18—Determination of excitation, including the effects of saturation.

the point F. The vertical distance OF is then the terminal voltage for the particular excitation. Following this procedure another excitation is chosen and the construction repeated from which the dotted line is obtained. The intersection of the line with the normal voltage gives the excitation for the desired power-factor at rated load. If the machine is not operating at rated current, the zero-power-factor curve corresponding to the particular current should be used.

(b) General Method—For lack of a better name this method has been called the "General Method." It is based upon the assumption that saturation is included by reading the excitation requirements from the no-load saturation curve for a voltage equal to the voltage behind the Potier reactance drop.

The method is described in Fig. 19 with all terms expressed in per unit. The voltage, e_p, is the Potier internal voltage or the voltage behind the Potier reactance drop.

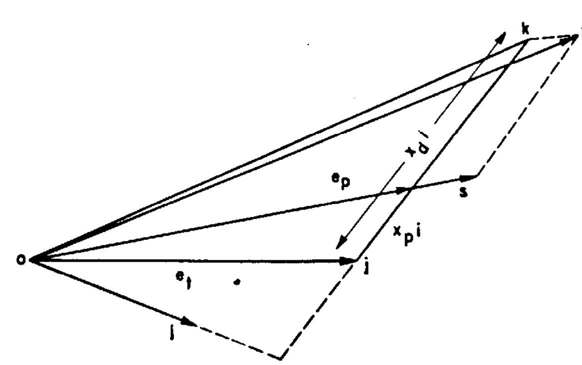


Fig. 19—Determination of field current for round rotor machine with saturation included by adding s in phase with e_p.

The distance jk represents the synchronous reactance drop, $x_d i$. If there were no saturation the synchronous internal voltage would be Ok. When using per unit quantities throughout this is also equivalent to the field current. This method includes the effect of saturation by simply adding s_1 the increment in field current for this voltage in excess of that required for no saturation, to Ok in phase with e_p, giving as a result, Oh. When per unit quantities are not used the construction is a little more complicated. It involves the construction of e_p separately so that s can be obtained in terms of field current. This quantity is then added to the diagram for no saturation in terms of the field current. In Fig. 18, first lay off from the terminal voltage, Oa, and then the x_p drop ac at an angle with the horizontal equal to the power-factor angle. Oc then represents e_p. By scribing this back to the ordinate and reading horizontally, the excitation corresponding to this voltage is obtained. The effect of saturation is introduced by the distance s. The field current required if there were no saturation is obtained by the construction Oj and jk where Oj represents the excitation, ap, required to produce the terminal voltage at no load and jk the excitation, ot, for the synchronous reactance drop, read from the abscissa. These vectors correspond to e_t and jk, respectively, in Fig. 19 except that they are in terms of field current. If kh, equal to the saturation factor, s, is added along a line parallel to Oc, the total excitation Oh is obtained.

(c) Round Rotor Potier Voltage Method*—This method is the same as (b) except that the effect of saturation s, in Fig. 18 is, for the sake of simplicity laid off along Ok, making om the desired excitation. As can be seen, there is little difference between those two methods. This method gives the best overall results, especially at leading power factors. The particular name of this method was assigned to distinguish it from the next method.

(d) Two-Reaction Potier Voltage Method—This method is similar to that of (c), except that the two-reaction method of construction shown in Fig. 14 is used to determine the excitation before including the saturation factor s. Fig. 15 shows the entire construction, For the sake of comparison with other methods, the construction is also shown in Fig. 18. The construction is the same as (c) except that the line Oq is made to pass through the point g instead of k. This arises because x_q is smaller than x_d.

4. Reactive Power Capacity

The capacity of a synchronous machine to deliver reactive power is dependent upon the real power that it delivers. Two limitations from the heating standpoint are recognized: (1) that due to the armature, and (2) that due to the field. Figure 20 shows the reactive power capability of a standardized 3600-rpm steam turbine-generator. Real power is plotted as abscissa and reactive power as ordinate. All the curves are arcs of circles. The line centering about the origin represents the limit imposed by the condition of constant armature current whereas the other arc by constant field current. With regard to the latter, the generator can be likened to a simple transmission line of pure reactance, x_d, with the receiver voltage held at a constant value, e_t, the terminal voltage of the generator, and with the

*Described as Method (a) Para. 1.520 in Reference 10.

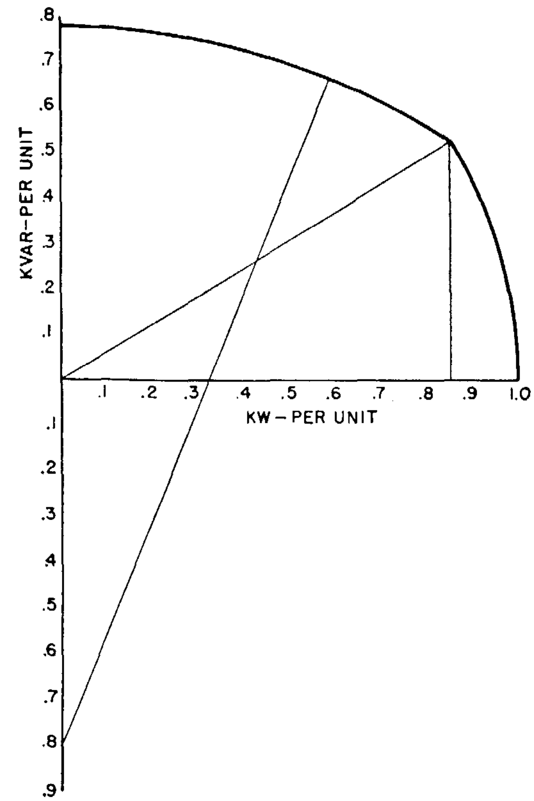


Fig. 20—Reactive power capacity of steam turbine generator 20 000 kw, 23 529 kva, 0.85 p.f., 0.8 *SCR*, at 0.5 psig hydrogen.

sending voltage held at a constant value e_d. As shown in Chaps. 9 and 10 the power circle of a line of such characteristics has its center in the negative reactive axis at $\frac{e_t^2}{x_d}$ and its internal voltage, e_d, must be such that its radius, $\frac{e_d e_t}{x_d}$, passes through the point of rated real power and rated reactive power. Actually, however, the center is usually located at a point equal to (SCR) times (rated kva). This is to take care of saturation effects. Since, however, with no saturation $\frac{1}{x_d}$ is equal to SCR, it can be seen that for this condition both relations reduce to an equivalence.

The leading kvar capacity (underexcited) of air-cooled condensers is usually about 50 percent of the lagging kvar capacity but for hydrogen-cooled condensers about 42 percent.

II. THREE-PHASE SHORT CIRCUIT

In addition to its steady-state performance, the action of a machine under short-circuit conditions is important. The presence of paths for flow of eddy currents as provided by the solid core in turbine generators and by the damper windings in some salient-pole machines makes the treatment of these machines, from a practical viewpoint, less complicated than that for salient-pole machines without damper windings. For this reason the three-phase short-circuit of these types of machines will be discussed first. Armature resistance will be neglected except as it influences decrement factors.

5. Three-phase Short-Circuit of Machines with Current Paths in Field Structures

Consideration will be given to a simultaneous short-circuit on all phases while the machine is operating at no-load normal voltage without a voltage regulator.

The general nature of the currents that appear is shown in Fig. 21. They can be divided into two parts:

a. An alternating component in the armature and associated with it an unidirectional component in the field. These two components decay or decrease together with the same time constants. The alternating armature component can be regarded as being produced by its associated unidirectional component in the field. All phase components of the alternating current are essentially the same except that they are displaced 120 electrical degrees.*

b. An unidirectional component in the armature and an alternating component in the field or in the damper windings. In this case, likewise, the alternating current in the field winding can be regarded as produced by the unidirectional component in the armature.

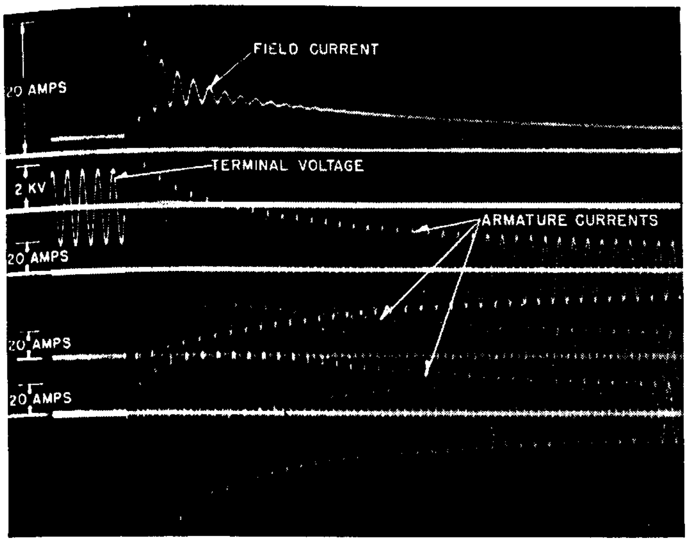


Fig. 21—Three-phase short circuit in salient-pole machine with damper windings.

6. Alternating Component of Armature Current

This component can in turn be resolved into several components, the r.m.s. values of which are shown in Fig. 22. They are:

a. The steady-state component
b. The transient component
c. The subtransient component

Each of these components will be discussed separately.

Steady-State Components—The steady-state component, as its name implies, is the current finally attained. Because of the demagnetizing effect of the large short-circuit current, the flux density within the machine decreases below a point where saturation is present. Saturation is important only as it affects the field current necessary to produce normal voltage at no load. The steady-state value of short-circuit current is thus equal to the line-to-neutral voltage read from the air-gap line for the value of field current required to produce normal voltage divided by the synchronous reactance in ohms.

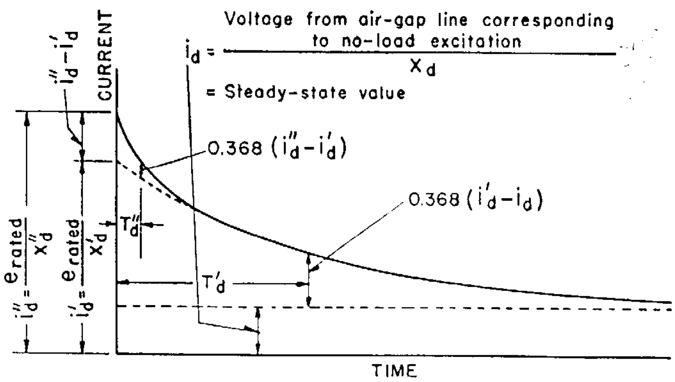


Fig. 22—Symmetrical component of armature short-circuit current (three-phase short circuit from no-load rated voltage). Values are rms.

Transient Component—If the excess of the symmetrical component of armature currents over the steady-state component be plotted on semi-log paper, it can be seen that this excess, except for the first few cycles, is an exponential function of time (the points lie in a straight line). Extending this straight line back to zero time and adding the steady-state component, the so-called *transient component*, i_d', or armature current is obtained. This component is defined through a new reactance, called the *transient reactance* by means of the expression

$$i_d' = \frac{e_{\text{rated}}}{x'_d}$$

The manner in which this quantity is related to the exponential and steady-state terms is shown in Fig. 22.

In discussing this component, the presence of the damper-winding currents of salient-pole machines and rotor eddy currents of turbine generators can, for the moment, be neglected. Before short-circuit occurs the flux associated with the field windings can be broken up into two components (see Fig. 23), a component Φ that crosses the air gap and a component Φ_l, a leakage flux that can be regarded as linking all of the field winding. Actually, of course, the leakage flux varies from the base of the pole to the pole tip. The flux Φ_l is so weighted that it produces the same linkage with all the field turns as the actual leakage flux produces with the actual turns. It is approximately proportional to the instantaneous value of the field current I_f. The total flux linkages with the field winding are then those produced by the flux $(\Phi+\Phi_l)$. As the field structure rotates, a balanced alternating voltage and current of normal frequency are produced in the armature. Because the armature resistance is relatively small, its circuit can be regarded as having a power-factor of zero. The symmetrical current thus produced develops an mmf that rotates synchronously and has a purely demagnetizing, as contrasted with cross magnetizing, effect on the field fluxes.

It is a well-known fact that for the flux linkages with a

*The machine used in this case was a salient-pole machine. As will be seen later, such machines also contain a second harmonic component of current. This type of machine was chosen to show more clearly the presence of field and damper currents.

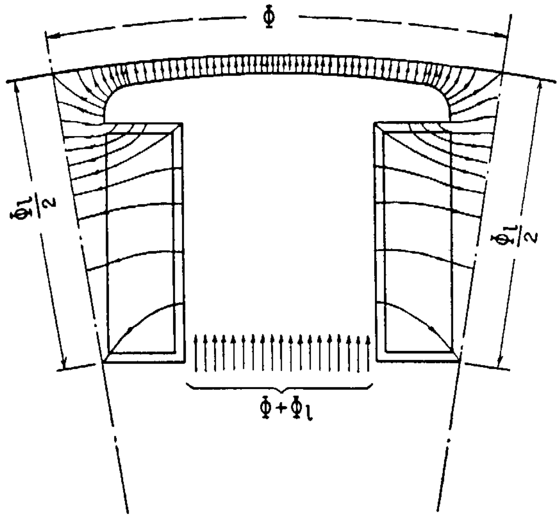


Fig. 23—Air-gap and leakage fluxes at no load.

circuit to change instantly, an infinitely large voltage is necessary and the assumption is justified that, for the transition period from the no load open-circuited condition to the short-circuited condition, the flux linkages with the field winding can be regarded as constant. This is equivalent to saying that the flux $(\Phi+\Phi_l)$ remains constant. In order that this flux remain constant in the presence of the demagnetizing effect of the armature current, it is necessary that the field current I_f increase to overcome the demagnetizing effect of the armature current. If I_f increases then Φ_l, which is proportional to it, must likewise increase. It follows then that Φ must decrease. Consideration of the steady-state conditions has shown that the air-gap voltage, e_l, is proportional to the air-gap flux Φ. The armature current for short-circuit conditions is equal to $\frac{e_l}{x_l}$. If Φ and consequently e_l had remained constant during the transition period, then the transient component of short-circuit current would be merely the no-load voltage before the short-circuit divided by the leakage reactance and the transient reactance would be equal to the armature leakage reactance x_l. However, as just shown, the air-gap flux decreases and, therefore, the armature current is less. It follows then that the transient reactance must be greater than the armature leakage reactance. It is a reactance that includes the effect of the increased field leakage occasioned by the increase in field current.

Under steady-state conditions with no saturation, the armature current can be viewed as produced by a fictitious internal voltage equal to $x_d\,i_d$ whose magnitude is picked from the air-gap line of the no-load saturation curve for the particular field current. At the first instant of short-circuit, the increased armature current, i_d', can likewise be viewed as being produced by a fictitious internal voltage behind synchronous reactance, whose magnitude is $x_d i_d'$ or $x_d\frac{e_{\text{rated}}}{x_d'}$, if the short-circuit be from rated voltage, no load. This voltage provides a means for determining the initial value of the unidirectional component of field current by picking off the value of I_f on the air-gap line of the no-load saturation curve corresponding to this voltage. If it were possible to increase the exciter voltage instantaneously to an amount that would produce this steady-state field current, then this component of short-circuit current would remain sustained. It is important to grasp the significance of this truth. There is always a constant proportionality between the alternating current in the armature and the unidirectional (often called direct-current) component of current in the field winding, whether the operating condition be steady-state or transitory.

The initial value of armature current, as stated, gradually decreases to the steady-state and the induced current in the field winding likewise decreases to its steady-state magnitude. The increments of both follow an exponential curve having the same time constant. Attention will next be given to considerations affecting this time constant.

If a constant direct voltage is suddenly applied to the field of a machine with the armature open-circuited, the current builds up exponentially just as for any circuit having resistance and inductance in series. The mathematical expression of this relation is:

$$I_f=\frac{e_x}{r_f}\left[1-\epsilon^{-\frac{t}{T'_{do}}}\right] \qquad (6)$$

in which

e_x is the exciter voltage.
r_f is the resistance of the field winding in ohms.
T'_{do} is the open-circuit transient time constant of the machine or of the circuit in question in seconds.
t is time in seconds.

The time constant is equal to the inductance of the field winding divided by its resistance. In the case of the short-circuited machine, it was shown that at the first instant the flux linkages with the field winding remain the same as for the open-circuit condition, but that the direct component of field current increases to $\frac{x_d}{x_d'}$ times the open-circuit value before short-circuit. Since inductance is defined as the flux linkages per unit current, it follows then that the inductance of the field circuit under short-circuit must equal $\frac{x_d'}{x_d}$ times that for the open-circuit condition. The short-circuit transient time constant, that is, the time constant that determines the rate of decay of the transient component of current must then equal

$$T_d'=\frac{x_d'}{x_d}T'_{do} \text{ in seconds}$$

The component of armature current that decays with this time constant can then be expressed by

$$(i_d'-i_d)\,\epsilon^{-\frac{t}{T_d'}}$$

When t is equal to T_d' the magnitude of the component has decreased to ϵ^{-1} or 0.368 times its initial value. This instant is indicated in Fig. 22.

Subtransient Component—In the presence of damper windings or other paths for eddy currents as in turbine generators, the air-gap flux at the first instant of short-circuit is prevented from changing to any great extent. This results both from their close proximity to the air gap and from the fact that their leakage is much smaller than that of the field winding. Consequently, the initial short-circuit currents of such machines are greater. If this excess of the symmetrical component of armature currents over the transient component is plotted on semi-log paper, the straight line thus formed can be projected back to zero time. This zero-time value when added to the transient component gives the subtransient current, i_d''. This *subtransient current* is defined by the *subtransient reactance* in the expression

$$i_d''=\frac{e_{\text{rated}}}{x_d''}$$

The subtransient reactance approaches the armature leakage differing from that quantity only by the leakage of the damper windings.

Since the excess of the armature currents represented by the subtransient components over the transient components are sustained only by the damper winding currents, it would be expected that their decrement would be determined by that of the damper winding. Since the copper section of this winding is so much smaller than that of the field winding, it is found that the short-circuit subtransient time constant, T_d'', is very small, being about 0.05 second instead of the order of seconds as is characteristic of the transient component. The component of armature current that decays with this time constant is $(i_d''-i_d')$ and can be expressed as a function of time as

$$(i_d''-i_d')\epsilon^{-\frac{t}{T_d''}}$$

Thus the time in seconds for this component to decrease to 0.368 times its initial value gives T_d'' as indicated in Fig. 22.

Tests on machines without damper windings show that because of saturation effects, the short-circuit current even in this case can be resolved into a slow transient component and a much faster subtransient component. The influence of current magnitudes as reflected by saturation upon the transient and subtransient reactance is discussed in more detail under the general heading of Saturation.

7. Total Alternating Component of Armature Current

The total armature current consists of the steady-state value and the two components that decay with time constants T_d' and T_d''. It can be expressed by the following equation

$$i_{ac}=(i_d''-i_d')\epsilon^{-\frac{t}{T_d''}}+(i_d'-i_d)\epsilon^{-\frac{t}{T_d'}}+i_d \tag{7}$$

The quantities are all expressed as rms values and are equal but displaced 120 electrical degrees in the three phases.

8. Unidirectional Component of Armature Current

To this point consideration has been given to flux linkages with the field winding only. The requirement that these linkages remain constant at transition periods determined the alternating component of armature current. Since these components in the three phases have a phase displacement of 120 degrees with respect to each other, only one can equal zero at a time. Therefore at times of three-phase short-circuits, the alternating component of current in at least two and probably all three phases must change from zero to some finite value. Since the armature circuits are inductive, it follows that their currents cannot change instantly from zero to a finite value. The "theorem of constant flux linkages" must apply to each phase separately. The application of this theorem thus gives rise to an unidirectional component of current in each phase equal and of negative value to the instantaneous values of the alternating component at the instant of short circuit. In this manner the armature currents are made continuous as shown in Fig. 24. Each of the unidirectional components

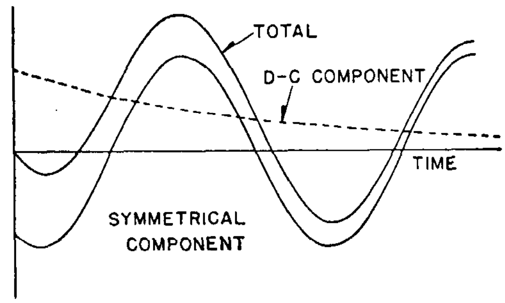


Fig. 24—The inclusion of a d-c component of armature current whose existence is necessary to make the armature current continuous at the instant of short circuit.

in the three phases decays exponentially with a time constant T_a, called the armature short-circuit time constant. The magnitude of this time constant is dependent upon the ratio of the inductance to resistance in the armature circuit. As will be shown the negative-sequence reactance, x_2, of the machine is a sort of average reactance of the armature with the field winding short-circuited, so that it is the reactance to use in determining T_a. There exists then the relation

$$T_a=\frac{x_2}{2\pi f r_a}\text{in seconds} \tag{8}$$

in which r_a is the d-c resistance of the armature. The quantity $2\pi f$ merely converts the reactance to an inductance.

The maximum magnitude which the unidirectional

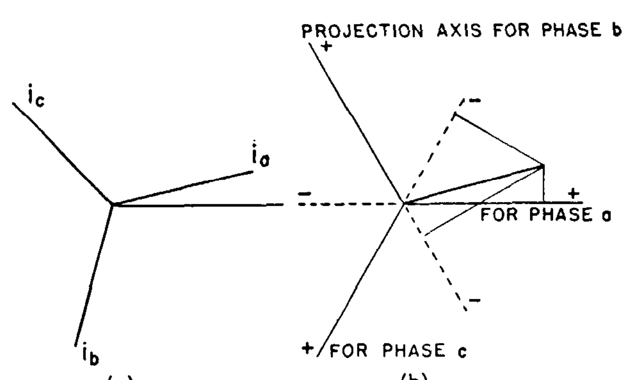


Fig. 25—Representation of instantaneous currents of a three phase system. (a) Three separate vectors projected on x-axis, (b) Single vector projected on three axes.

component can attain is equal to the maximum of the alternating component. Therefore,

$$i_{dc(max.)} = \sqrt{2}\frac{e_{rated}}{x_d''} \quad (9)$$

A symmetrical three-phase set of currents can be represented as the projection of three equal-spaced and equal length vectors upon a stationary reference, say the real axis. They can also be represented as the projection, as it rotates, of one vector upon three stationary axes, spaced 120 degrees. These axes can conveniently be taken as shown in Fig. 25, as the horizontal-axis and two axes having a 120-degree relation therewith. Since the initial magnitude of the unidirectional components are the negatives of the instantaneous values of the alternating components at zero time, then the unidirectional components can be represented as the projection of a single vector onto the three equal-spaced axes. This fact is used at times to determine the maximum magnitude which the unidirectional component can attain. By its use it is unnecessary to await a test in which the maximum happens to occur. This method is in error, however, for machines in which x_q'' and x_d'' are radically different.

9. Total RMS Armature Current

The rms armature current at any instant is

$$\sqrt{i_{dc}^2+i_{ac}^2}$$

The minimum current thus occurs in the phase in which the unidirectional component is zero and the maximum occurs when the unidirectional component is a maximum, that is, when maximum dissymmetry occurs. Since the maximum value that the unidirectional component can attain is $\sqrt{2}\frac{e_{rated}}{x_d''}$, then

$$i_{rms\ (max)} = \sqrt{\left[\frac{\sqrt{2}e_{rated}}{x_d''}\right]^2 + \left[\frac{e_{rated}}{x_d''}\right]^2} = \sqrt{3}\frac{e_{rated}}{x_d''} \quad (10)$$

Of course, a rms value as its name implies, is an average quantity and is usually taken over a cycle or half cycle of time. The foregoing expression assumes that both the alternating and the unidirectional components do not decrease, because of the natural decrement, during the first cycle. In reality the decrement is usually sufficient to make the effect noticeable. In applying circuit breakers it is usual to use a factor 1.6 instead of $\sqrt{3}$. This factor includes a small decrement.

10. Effect of External Impedance

If the short-circuit occurs through an external impedance $r_{ext}+j\, x_{ext}$, and r_{ext} is not too large, their effect can be introduced by merely increasing the armature constants by these amounts. Thus the components of short-circuit current become

$$i_d'' = \frac{e_{rated}}{x_d''+x_{ext}} \quad (11)$$

$$i_d' = \frac{e_{rated}}{x_d'+x_{ext}} \quad (12)$$

$$i_d = \frac{e_{\text{airgap at no load}}}{x_d+x_{ext}} \quad (13)$$

The short-circuit time constant is affected in a similar manner

$$T_d' = \frac{x_d'+x_{ext}}{x_d+x_{ext}} T_{do}' \text{ in seconds} \quad (14)$$

For the armature time constant, the external reactance must be added to the negative-sequence reactance of the machine and the external resistance to the armature resistance of the machine. The expression then becomes

$$T_a = \frac{x_2+x_{ext}}{2\pi f(r_a+r_{ext})} \text{ in seconds} \quad (15)$$

Because of the much lower ratio of reactance to resistance in external portions of circuits, such as transformers or transmission lines, in the vast majority of cases T_a for faults out in the system is so small as to justify neglecting the unidirectional component of current.

11. Short Circuit from Loaded Conditions

The more usual case met in practice is that of a short-circuit on machines operating under loaded conditions. As before, the short-circuit current in the armature can be divided into two components, a symmetrical alternating component, and a unidirectional component.

Alternating Component—The alternating component in turn can be resolved into three components: (1) steady state, (2) transient, and (3) subtransient. Each of these components will be discussed individually.

The load on the machine affects the *steady-state component* only as it influences the field current before the short circuit. The field current can be determined by any of the methods discussed under the heading of "Steady-State Conditions." Saturation will be more important than for the no-load condition. The steady-state short-circuit current is then equal to the line-to-neutral voltage read from the air-gap line for the field current obtained for the loaded condition divided by x_d.

In the discussion of the determination of the *transient component* from the no-load condition, it was stated that the quantity that remained constant during the transition period from one circuit condition to another, is the flux linkages with the field winding. For the short-circuit from loaded conditions this same quantity can be used as a basis for analysis. Consideration will be given first to a load before short circuit whose power factor is zero, lagging, and whose current is i_{dL}. The flux linkages before short circuit will be determined by a superposition method, obtaining first the linkages with the field winding for zero armature current and any terminal voltage and then the flux linkages with armature current, i_{dL}, and zero terminal voltage. The total flux linkages is the sum of the two values so obtained.

Let ψ_1 be the flux linkages with the field winding at no-load at rated voltage. For any other terminal voltage such as e_t, the flux linkages ψ will be equal to

$$\frac{e_t}{e_{rated}}\psi_1 \quad (16)$$

By definition the transient reactance of a machine is equal to the reactance which, divided into the line-to-neutral rated voltage, gives the transient component of short-circuit current at no-load normal voltage. If this short-circuit current is designated as $i_d'_1$, then

$$i_d'_1 = \frac{e_{\text{rated}}}{x_d'} \tag{17}$$

At the instant of short-circuit from no-load at rated voltage, the flux linkages with the field winding, ψ_1, remain constant. The demagnetizing effect of the armature current is overcome by an increase in the field current. Thus the armature current $i_d'_1$ with its associated field current which is always proportional to it, can be regarded as producing the flux linkages ψ_1 with the field winding. For any other armature current, i_d', assuming always that the armature is short-circuited, the flux linkages with the field winding are equal to $\frac{i_d'}{i_d'_1}\psi_1$. Combined with Eq. (17), $i_d'_1$ can be eliminated giving $\psi = i_d' \frac{x_d'\psi_1}{e_{\text{rated}}}$. While this expression was derived from considerations applying only to the instant of transition, its application is more general. The only necessary considerations that must be satisfied are that the armature be short-circuited and that the field current contain a component of current to overcome the demagnetizing effect of the armature current. But these conditions are always satisfied even under steady-state conditions of short circuit, so, in general, it is permissible to replace i_d' in this expression by i_{dL}. The flux linkages with the field winding for the steady-state short-circuit condition thus become $i_{dL}\frac{x_d'\psi_1}{e_{\text{rated}}}$.

By application of the superposition theorem, the total flux linkages with the field winding can then be regarded as the sum of the flux linkages produced by the terminal voltage, namely $\frac{e_t}{e_{\text{rated}}}\psi_1$ and those by the armature current with zero terminal voltage, namely $i_{dL}\frac{x_d'\psi_1}{e_{\text{rated}}}$. *If the armature current lags the voltage by 90 degrees*, then the linkages are directly additive, and there results for the flux linkages with the field

$$\psi = \frac{e_t}{e_{\text{rated}}}\psi_1 + i_{dL}\frac{x_d'\psi_1}{e_{\text{rated}}}$$

$$= (e_t + x_d' i_{dL})\frac{\psi_1}{e_{\text{rated}}}. \tag{18}$$

Since the flux linkages with the field winding produced by a unit of current i_d under short-circuit conditions is equal to $\frac{x_d'\psi_1}{e_{\text{rated}}}$ then the transient component of short-circuit current i' can be determined by dividing these linkages into the total flux linkages just determined. This gives

$$i' = \frac{\psi}{\frac{x_d'\psi_1}{e_{\text{rated}}}} = \frac{e_t + x_d' i_{dL}}{x_d'} \tag{19}$$

The numerator of this quantity can be regarded as an internal voltage, e_d', which is equal to the terminal voltage plus a transient reactance drop produced by the load current.

When the power factor of the loads considered is other than zero lagging, the vector sense of current and terminal voltage must be introduced. This can be accomplished by computing e_d' for the operating condition in the same manner that e_d was determined in Fig. 14, except that x_d should be replaced by x_d'. The voltage e_d' should then replace $e_t + x_d' i_{dL}$ in (19). However, for nearly all practical purposes it is sufficiently accurate to replace e_d' by the amplitude of a quantity e_i', which is usually referred to as the voltage behind transient reactance to distinguish it from similar internal voltages for which leakage, synchronous or subtransient reactance is used. The construction for this quantity is shown in Fig. 26 and to assist in the ready evaluation of the amplitude the curves in Fig. 27 are provided. The transient component of short-circuit current is then

$$i' = \frac{e_i'}{x_d'} \tag{20}$$

The *subtransient component* of short-circuit current is obtained in a manner similar to the transient component except that the subtransient reactance is used in the calculation of the internal voltage e_i''. For loads of zero-power-factor lagging the subtransient reactance drop, $x_d'' i_{dL}$,

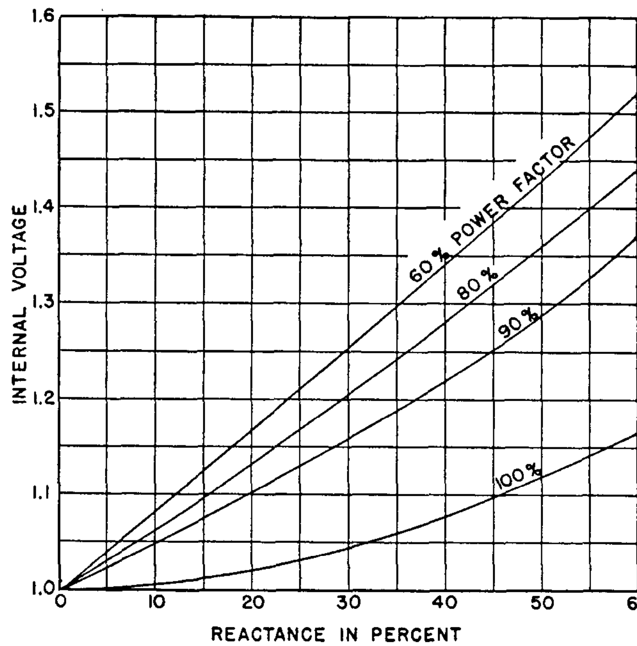


Fig. 27—Machine internal voltage as a function of reactance. Full-load rated voltage.

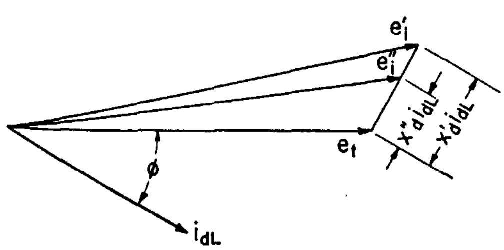


Fig. 26—Construction for the determination of internal voltages e_i' and e_i''.

caused by the armature current is directly additive to the terminal voltage and for zero-power-factor leading directly subtractive. For other power-factors e_i'' can be obtained from Fig. 27 by using x_d''. The subtransient component of short-circuit current is then

$$i''=\frac{e_i''}{x_d''} \tag{21}$$

Unidirectional Component—In the three-phase short-circuit from no load, the unidirectional component of current was introduced to prevent a non-continuous transition of the instantaneous value of current from the no-load to the short-circuit condition. The unidirectional current performs a similar role for the short-circuit from loaded condition. Before the short-circuit the armature current is equal to i_{dL} and has some position with reference to e_t such as shown in Fig. 28. The subtransient component, i'', lags e_i'' by ninety degrees so i'' and i_{dL} will be determined with respect to each other. The $\sqrt{2}$ times the vector difference between these two quantities (since they are rms magnitudes) gives the unidirectional component necessary to produce smooth transition. The magnitude of this quantity varies between this amplitude and zero depending upon the point in the cycle at which short-circuit occurs.

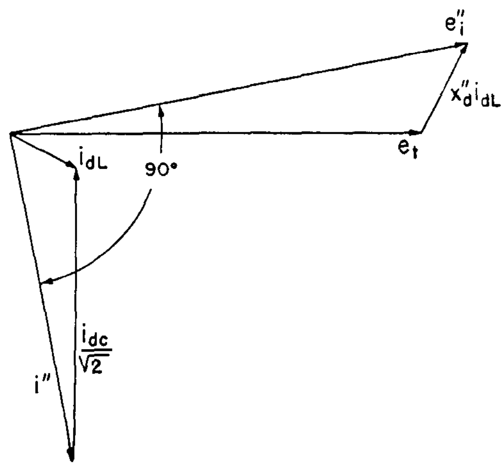


Fig. 28—Showing that i_{dc} for a short circuit from load is equal to the negative of $\sqrt{2}$ times the difference between i'' and i.

Other Considerations—Time constants are not influenced by the nature of loading preceding the short-circuit. Total rms currents can be determined by the relations already given.

12. Three-Phase Short Circuit of Salient-Pole Machine without Damper Windings

For most applications it is sufficiently accurate to treat the salient-pole machine without damper windings just as other machines. It must be recognized, however, that this is only an approximate solution. Among other complications, in reality a strong second harmonic is present in the armature current. Doherty and Nickle[6] have developed expressions for the armature currents for a three-phase short circuit from no load. These are given below.

$$\begin{aligned} i_a=&\frac{x_d-x_d'}{x_dx_d'}e_t\epsilon^{-\frac{t}{T_d'}}\cos(2\pi ft+\alpha)+\frac{e_t}{x_d}\cos(2\pi ft+\alpha)\\ &-\frac{x_q-x_d'}{2x_d'x_q}e_t\epsilon^{-\frac{t}{T_a}}\cos(4\pi ft+\alpha)-\frac{x_q+x_d'}{2x_d'x_q}e_t\epsilon^{-\frac{t}{T_a}}\cos\alpha \end{aligned} \tag{22}$$

$$\begin{aligned} i_b=&\frac{x_d-x_d'}{x_dx_d'}e_t\epsilon^{-\frac{t}{T_d'}}\cos(2\pi ft+\alpha-120°)\\ &+\frac{e_t}{x_d}\cos(2\pi ft+\alpha-120°)\\ &-\frac{x_q-x_d'}{2x_d'x_q}e_t\epsilon^{-\frac{t}{T_a}}\cos(4\pi ft+\alpha-120°)\\ &-\frac{x_q+x_d'}{2x_d'x_q}e_t\epsilon^{-\frac{t}{T_a}}\cos(\alpha+120°) \end{aligned} \tag{23}$$

$$\begin{aligned} i_c=&\frac{x_d-x_d'}{x_dx_d'}e_t\epsilon^{-\frac{t}{T_d'}}\cos(2\pi ft+\alpha+120°)\\ &+\frac{e_t}{x_d}\cos(2\pi ft+\alpha+120°)\\ &-\frac{x_q-x_d'}{2x_d'x_q}e_t\epsilon^{-\frac{t}{T_a}}\cos(4\pi ft+\alpha+120°)\\ &-\frac{x_q+x_d'}{2x_d'x_q}e_t\epsilon^{-\frac{t}{T_a}}\cos(\alpha-120°). \end{aligned} \tag{24}$$

Where

e_t = Terminal voltage before short-circuit.

$$T_d'=\frac{r^2+x_d'x_q}{r^2+x_dx_q}T_{do}'. \tag{25}$$

$$T_a=\frac{2x_d'x_q}{r(x_d'+x_q)}. \tag{26}$$

α = Angle which indicates point on wave at which short-circuit occurs.

The instantaneous field current, I_d, is

$$I_d=\frac{x_d-x_d'}{x_d'}I_f\left[\epsilon^{-\frac{t}{T_d'}}-\epsilon^{-\frac{t}{T_a}}\cos 2\pi ft\right]+I_f \tag{27}$$

Where

I_f = Initial value of field current.

III. UNBALANCED CONDITIONS

13. Phase Currents for Unbalanced Short Circuits

As explained in the chapter relating to Symmetrical Components, the unbalanced operating conditions of a rotating machine can for most purposes be described in terms of three characteristic constants: the positive-sequence impedance, the negative-sequence impedance, and the zero-sequence impedance. The short-circuit currents can be resolved, as before, into the steady-state, transient, and subtransient components. The difference between these components decreases exponentially as before. The components of armature current and the time constants for the different kinds of short-circuits are given below for short-circuits at the terminals of the machine.

For three-phase short-circuit:

$$i''=\frac{e_i''}{x_d''}\qquad i'=\frac{e_i'}{x_d'}\qquad i=\frac{e_i}{x_d}\qquad T_d'=\frac{x_d'}{x_d}T_{do}' \tag{28}$$

For terminal-to-terminal short circuit, the a-c components of the phase currents are given by

$$i''=\frac{\sqrt{3}e_i''}{x_d''+x_2} \qquad i'=\frac{\sqrt{3}e_i'}{x_d'+x_2}$$
$$i=\frac{\sqrt{3}e_i}{x_d+x_2} \qquad T_d'=\frac{x_d'+x_2}{x_d+x_2}T_{do}' \qquad (29)$$

in which x_2 is the negative-sequence impedance of the machine

For terminal-to-neutral short circuit, the a-c components of the phase currents are given by

$$i''=\frac{3e_i''}{x_d''+x_2+x_0} \qquad i'=\frac{3e_i'}{x_d'+x_2+x_0}$$
$$i=\frac{3e_i}{x_d+x_2+x_0} \qquad T_d'=\frac{x_d'+x_2+x_0}{x_d+x_2+x_0}T_{do}' \qquad (30)$$

in which x_0 is the zero-sequence impedance of the machine. The subtransient time constant, T_d'', does not change significantly with different conditions and, therefore, the single value is used for all conditions. The unidirectional components and the rms values are determined just as described under the general subject of "Short Circuit from Load." The above values of e_i, e_i' and e_i'' will naturally be those values corresponding to the particular load condition.

The ratio of the phase currents for terminal-to-neutral to three-phase short circuits can be obtained from Eq's (30) and (28). Thus, for the phase currents

$$\frac{\text{Terminal-to-neutral short circuit}}{\text{Three-phase short circuit}}=\frac{3x_d''}{x_d''+x_2+x_0}$$

The negative-sequence impedance, x_2, is usually equal to x_d'', but for many machines x_0 is less than x_d''. For these cases, the terminal-to-neutral short-circuit current is greater than the three-phase short-circuit current. The generator standards require that the machine be braced only for currents equal to the three-phase values. In order that the terminal-to-neutral current not exceed the three-phase current a reactor should be placed in the neutral of the machine of such value as to bring the zero-sequence impedance of the circuit equal to x_d''. Thus, the neutral reactor, x_n, should be

$$x_n \geq \frac{1}{3}(x_d''-x_0)$$

14. Negative-Sequence Reactance

The negative-sequence impedance of a machine is the impedance offered by that machine to the flow of negative-sequence current. A set of negative-sequence currents in the armature creates in the air gap a magnetic field that rotates at synchronous speed in a direction opposite to that of the normal motion of the field structure. Currents of double frequency are thereby established in the field, and in the damper winding if the machine has one. The imaginary component of the impedance is called the negative-sequence reactance and the real component the negative-sequence resistance. These will be discussed separately, in the order mentioned.

If a single-phase voltage is applied across two terminals of a salient-pole machine without dampers while its rotor is stationary, the resulting current is dependent upon the position of the rotor with respect to the pulsating field set up by the armature current. If the axis of the short-circuited field winding lines up with the axis of pulsating field then the current is large and if the rotor is moved through 90 electrical degrees then the current is much smaller. The first position corresponds to the case of a transformer in which the secondary winding is short-circuited, the field winding in this case corresponding to the secondary winding of the transformer. This is the position in which the subtransient reactance, x_d'', is determined. It is equal to one-half of the voltage from terminal-to-terminal divided by the current. For the second position the field winding is in quadrature to the pulsating field and consequently no current flows in the field winding. The armature current is then determined by the magnetizing characteristics of the air gap in the quadrature axis. The subtransient reactance, x_q'', is determined when the field is in this position and is equal to one-half the quotient of the voltage divided by the current. The reactance for intermediate positions varies between these two amounts in accordance with the curve shown in Fig. 29.

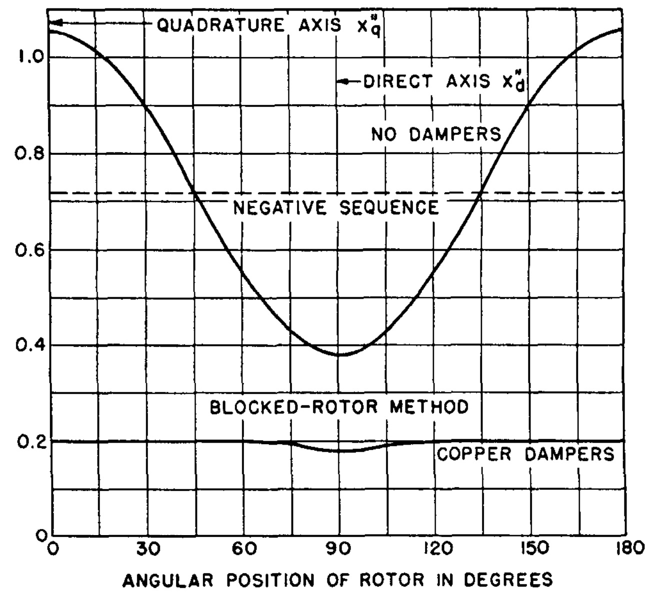


Fig. 29—Relation between subtransient and negative-sequence reactance.

When a set of negative-sequence currents is made to flow through the armature with the field short-circuited and rotating in its normal direction, then the field winding takes different positions successively as the armature field rotates with respect to it. The nature of the impedances in the two extreme positions, that is, where the field winding lines up with the magnetic field and where it is in quadrature with it, should be somewhat the same as x_d'' and x_q'', the only signnificant difference being the fact that, in the determination of x_d'' and x_q'', currents of normal frequency were induced in the field, whereas, in the negative-sequence case the currents are of twice normal frequency. One would expect therefore that the negative-sequence reactance x_2 is some sort of a mean between x_d'' and x_q'', and such is the case. According to the AIEE test code,[10] the definition of negative-sequence reactance is equal to "the ratio of the fundamental component of reactive armature voltage, due to the fundamental negative-sequence component of armature current, to this component of armature current at rated frequency." A rigorous interpretation of this definition results in x_2 equal to the arithmetic mean $\frac{x_q''+x_d''}{2}$. However, several different definitions can be given for x_2. That this is possible is dependent largely upon the fact that when a sinusoidal set of negative-sequence voltages is applied to the armature the currents will not be sinusoidal. Conversely if the currents are sinusoidal the voltages will not be.

In Table 1 are shown expressions[11] for x_2 based upon different definitions. This table is based on a machine without damper windings for which x_q'' is equal to x_q, and x_d'' is equal to x_d'. In this table

$$b=\frac{\sqrt{x_q}-\sqrt{x_d'}}{\sqrt{x_q}+\sqrt{x_d'}}$$

For each test condition it is possible to establish definitions based on whether fundamental or root-mean-square currents are specified. For example, in the first definition if the fundamental component of armature current is used in calculating x_2 then the expression in the first column should be used, but if the root-mean-square figure of the resultant current is used then the expression in the second column should be used.

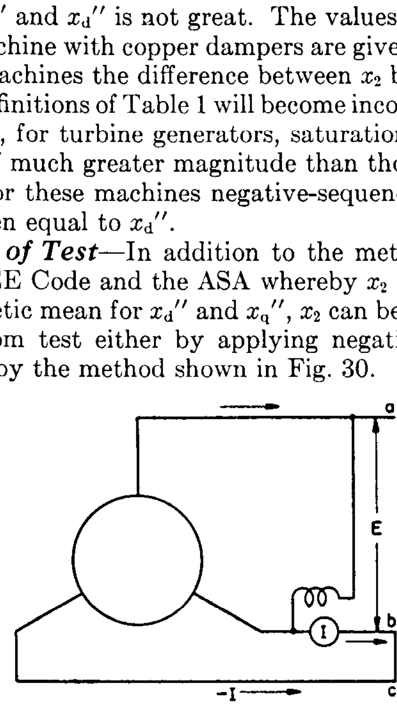
In order to orient one's self as to the relative importance of the different expressions, figures have been inserted in the expressions given in Table 1 for a typical machine having the constants $x_d'=35\%$, $x_q=70\%$, and $x_d=100\%$. The magnitudes are tabulated in the righthand columns of Table 1. From the standpoint of practical application, the negative-sequence reactance that would result in the proper root-mean-square current for method (3) would appear to be the most important. However, the method of test to determine this quantity involves a sudden short-circuit and from this standpoint proves rather inconvenient. On the other hand, the figure for x_2 obtained from the use of the root-mean-square values in a sustained single-phase short-circuit current [method (4)], is nearly equal to this quantity. When the resistance is negligible this negative-sequence reactance is equal to

$$x_2=\frac{\sqrt{3}E}{I}-x_d \tag{31}$$

where I equals the root-mean-square armature current in the short-circuited phase; and E equals the root-mean-square open-circuit voltage between terminals before the short-circuit is applied or the no-load voltage corresponding to the field current at which I is read.

In general, the same arguments can be applied to other types of machines such as turbine generators and salient-pole machines with damper windings when the parameters x_d'' and x_q'' are used. For such machines the difference between x_q'' and x_d'' is not great. The values for x_q'' and x_d'' of a machine with copper dampers are given in Fig. 29. For such machines the difference between x_2 based on the different definitions of Table 1 will become inconsequential. In addition, for turbine generators, saturation introduces variables of much greater magnitude than those just considered. For these machines negative-sequence reactance can be taken equal to x_d''.

Method of Test—In addition to the method implied by the AIEE Code and the ASA whereby x_2 is defined as the arithmetic mean for x_d'' and x_q'', x_2 can be determined directly from test either by applying negative-sequence voltage or by the method shown in Fig. 30.



$I_a=0 \quad I_b=I \quad I_c=-I$ $\qquad I_{a2}=\frac{1}{3}(0+a^2I-aI)=\frac{a^2-a}{3}I$

$E_A=0 \quad E_B=E \quad E_C=-E$ $\qquad E_{A2}=\frac{1}{3}(0+a^2E-aE)=\frac{a^2-a}{3}E$

$E_{a2}=j\frac{E_{A2}}{\sqrt{3}}=\frac{j(a^2-a)}{3\sqrt{3}}E$ $\qquad z_2=\frac{E_{a2}}{I_{a2}}=\frac{jE}{I\sqrt{3}}$

If $\phi=\cos^{-1}\frac{P}{EI}$, where P=wattmeter reading,

then, $z_2=\frac{E}{\sqrt{3}I}(\sin\phi+j\cos\phi)=r_2+jx_2$

Fig. 30—Determination of the negative-sequence impedance of symmetrically-wound machines.

TABLE 1—DEFINITIONS OF NEGATIVE-SEQUENCE REACTANCE

Definition	Analytical Expressions: Fundamental	Analytical Expressions: Root-Mean-Square	Numerical Values $x_d'=35\%$ $x_q=70\%$ $x_d=100\%$: Fundamental	Numerical Values: Root-Mean-Square
(1) Application of sinusoidal negative-sequence voltage	$\frac{2x_d'x_q}{x_q+x_d'}$	$\frac{\sqrt{2}x_d'x_q}{\sqrt{x_q^2+x_d'^2}}$	47	44
(2) Application of sinusoidal negative-sequence current	$\frac{x_q+x_d'}{2}$	$\frac{1}{2}\sqrt{(x_q+x_d')^2+9(x_q-x_d')^2}$	53	74
(3) Initial symmetrical component of sudden single-phase short-circuit current	$\sqrt{x_d'x_q}$	$x_d'\,\sqrt{1-b^2}-1)+\sqrt{x_d'x_q}\,\sqrt{1-b^2}$	50	48
(4) Sustained single-phase short-circuit current	$\sqrt{x_d'x_q}$	$x_d\,(\sqrt{1-b^2}-1)+\sqrt{x_d'x_q}\,\sqrt{1-b^2}$	50	47
(5) Same as (4) with 50% external reactance			51	50
(6) A.I.E.E. and A.S.A.	$\frac{x_q+x_d'}{2}$		53	

With the machine driven at rated speed, and with a single-phase short-circuit applied between two of its terminals (neutral excluded) the sustained armature current and the voltage between the terminal of the free phase and either of the short-circuited phases are measured. The reading of a single-phase wattmeter with its current coil in the short-circuited phases and with the above mentioned voltage across its potential coil is also recorded. The negative-sequence *impedance* equals the ratio of the voltage to the current so measured, divided by 1.73. The negative-sequence *reactance* equals this impedance multiplied by the ratio of power to the product of voltage and current.

15. Negative-Sequence Resistance

The power associated with the negative-sequence current can be expressed as a resistance times the square of the current. This resistance is designated the negative-sequence resistance. For a machine without damper windings the only source of loss is in the armature and field resistances, eddy currents, and iron loss. The copper loss in the armature and field is small as is also the iron and eddy loss in the armature, but the iron and eddy loss in the rotor may be considerable. Copper damper windings provide a lower impedance path for the eddy currents and hinder the penetration of flux into the pole structure. The relatively low resistance of this path results in a smaller negative-sequence resistance than if the flux were permitted to penetrate into the rotor. For higher resistance damper windings the negative-sequence resistance increases to a point beyond which the larger resistance diminishes the current in the rotor circuits sufficiently to decrease the loss.

Induction-Motor Diagram—The nature of the negative-sequence resistance is best visualized by analyzing the phenomena occurring in induction motors. In Fig. 31

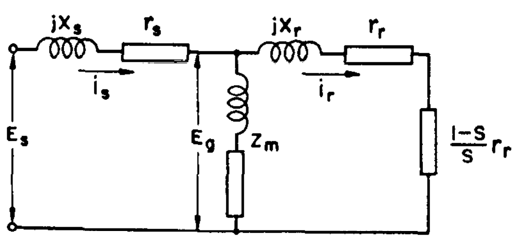


Fig. 31—Equivalent circuit of induction motor.

is given the usual equivalent circuit of an induction motor in which

r_s = stator resistance.
x_s = stator-leakage reactance at rated frequency.
r_r = rotor resistance.
x_r = rotor-leakage reactance at rated frequency.
z_m = shunt impedance to include the effect of magnetizing current and no-load losses.
E_s = applied voltage.
I_s = stator current.
I_r = rotor current.
s = slip.

The justification for this diagram is shown briefly as follows: The air-gap flux created by the currents I_s and I_r induces the voltage E_g in the stator and sE_g in the rotor. In the rotor the impedance drop is

$$r_rI_r+jsx_rI_r \qquad (32)$$

since the reactance varies with the frequency of the currents in the rotor. The rotor current is therefore determined by the equation

$$sE_g=r_rI_r+jsx_rI_r$$

or

$$E_g=\frac{r_r}{s}I_r+jx_rI_r \qquad (33)$$

It follows from this equation that the rotor circuit can be completely represented by placing a circuit of impedance $\frac{r_r}{s}+jx_r$ across the voltage E_g. The total power absorbed by $\frac{r_r}{s}$ must be the sum of the rotor losses and the useful shaft power, so that, resolving $\frac{r_r}{s}$ into the resistances r_r and $\frac{1-s}{s}r_r$, the power absorbed by r_r represents the rotor copper loss. The power absorbed by $\frac{1-s}{s}r_r$ represents the useful shaft power.

Neglecting r_s and the real part of z_m, the only real power is that concerned in the rotor circuit. Assume that the induction motor drives a direct-current generator. At small slips the electrical input into the stator is equal to the copper loss, i.e., the $I^2_rr_r$ of the rotor plus the shaft load. With the rotor locked, the shaft load is zero, and the total electrical input into the stator is equal to the rotor copper loss. At 200-percent slip, i.e., with the rotor turning at synchronous speed in the reverse direction, the copper loss is $I_r^2r_r$, the electrical input into the stator is $\frac{I_r^2r_r}{2}$, and the shaft load $\frac{1-2}{2}r_rI_r^2$ or $\frac{-I_r^2r_r}{2}$. A negative shaft load signifies that the direct-current machine instead of functioning as a generator is now a motor. Physically that is just what would be expected, for as the slip increases from zero the shaft power increases to a maximum and then decreases to zero for 100-percent slip. A further increase in slip necessitates motion in the opposite direction, which requires a driving torque. At 200-percent slip the electrical input into the stator is equal to the mechanical input through the shaft; half of the copper loss is supplied from the stator and half through the shaft. This is the condition obtaining with respect to the negative-sequence in which the rotor is rotating at a slip of 200 percent relative to the synchronously rotating negative-sequence field in the stator. Half of the machine loss associated with the negative-sequence current is supplied from the stator and half by shaft torque through the rotor.

The factors of fundamental importance are the power supplied to the stator and the power supplied to the shaft, which can always be determined by solving the equivalent circuit involving the stator and rotor constants and the magnetizing-current constants. A more convenient device,

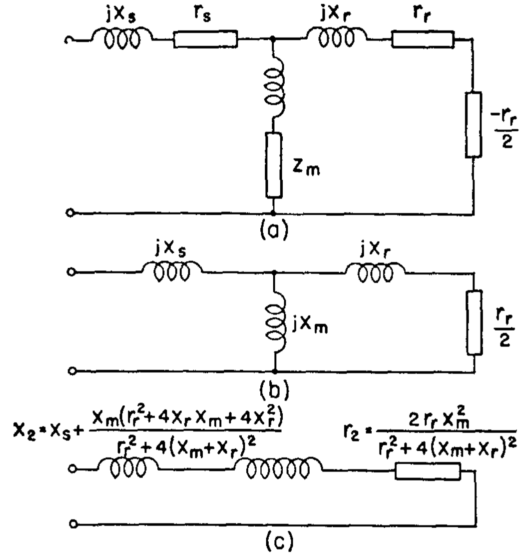


Fig. 32—Development of negative-sequence resistance and reactance from equivalent circuit of induction motor. (a) Negative-sequence diagram for induction motor; (b) neglecting armature and no load losses; (c) simplified network-negative-sequence resistance and reactance.

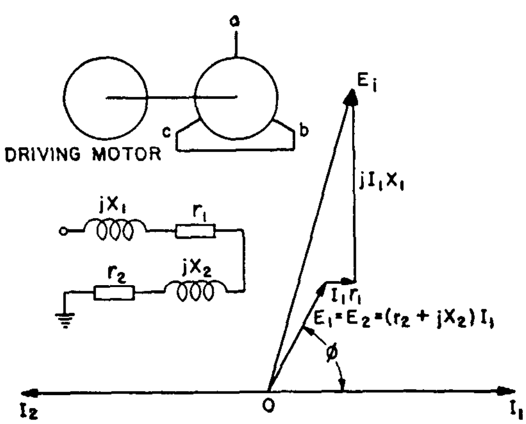
since s is constant and equal to 2 for the negative-sequence, is to reduce the equivalent network to a simple series impedance as shown in Fig. 32 (c). The components of this impedance will be called the negative-sequence resistance r_2, and the negative-sequence reactance x_2. The current flowing through the negative-sequence impedance is the current flowing through the stator of the machine, and the power loss in r_2 is equal to the loss supplied from the stator of the machine and the equal loss supplied through the shaft.

The total electrical effect of the negative-sequence resistance in system analysis problems is obtained by inserting the negative-sequence resistance in the negative-sequence network and solving the network in the usual manner. All three of the sequence currents are thus affected to some extent by a change in the negative-sequence resistance. The total electrical output of a generator, not including the shaft torque developed by negative-sequence current, is equal to the total terminal power output plus the losses in the machine. However, the negative- and zero-sequence power outputs are merely the negative of their losses. In other words, their losses are supplied by power flowing into the machine from the system. Therefore, the contribution of the negative- and zero-sequences to the electrical output is zero. The total electrical output reduces then to that of the positive-sequence and to include the positive-sequence armature-resistance loss it is necessary only to use the positive-sequence internal voltage in the calculations. Or viewed differently, since there are no internal generated voltages of the negative- or zero-sequence, the corresponding internal power must be zero. In addition to this electrical output, which produces a torque tending to decelerate the rotor, there also exists the negative-sequence shaft power supplied through the rotor. It was shown that this power tending to decelerate the rotor is numerically equal to the negative-sequence power supplied to the stator, which, in turn is equal to the loss absorbed by the negative-sequence resistance. Therefore the total decelerating power is equal to the positive-sequence power output plus the loss in the negative-sequence resistance.

The assumption was made that the stator resistance and the losses in the magnetizing branch were neglected. For greater refinements, the stator resistance and the losses in the magnetizing branch can be taken into consideration by substituting them in the equivalent circuit and reducing that circuit to simple series resistance and reactance, wherein the resistance becomes the negative-sequence resistance and the reactance the negative-sequence reactance. The ratio of the negative-sequence shaft power to the loss in the negative-sequence resistance is then equal to the ratio of the power loss in $\frac{r_r}{2}$ for unit negative-sequence current in the stator to r_2. This ratio can be obtained easily by test by measuring the shaft torque and the negative-sequence input when negative-sequence voltages only are applied to the stator.

While this analysis has premised induction-motor construction, the conclusions can also be applied to synchronous machines.

Method of Test—While r_2 and x_2 can be determined by applying negative-sequence voltage from another source of supply to the armature, the following method has the advantage that the machine supplies its own negative-sequence voltage. Two terminals of the machine under test are short-circuited and the machine driven at rated frequency by means of a direct-current motor. The equivalent circuit and vector diagram for this connection are shown in Fig. 33. The positive-sequence power per phase at the terminals is equal to the product of $\bar{E}_1$ and $\bar{I}_1$ and the cosine of the angle ϕ. This power is positive. However, the negative-sequence power output per phase is equal to the product of $\bar{E}_2$, $\bar{I}_2$, and the cosine of the angle between E_2 and I_2, and since $I_2 = -I_1$, and $E_1 = E_2$, the negative-



$$r_2 = \frac{3(P_s - P_{F+W})}{2I^2}$$

in which

P_s = shaft input.
P_{F+W} = friction and windage loss.

Fig. 33—Negative-sequence resistance of a synchronous machine.

sequence power output is the negative of the positive-sequence power output, which, of course, must follow since the output of the machine is zero. A negative output is equivalent to a positive input. This input is equal to $r_2\bar{I}_2^2$ per phase. Therefore, the positive-sequence terminal output per phase is $r_2\bar{I}_2^2$, and adding to this the copper loss due to I_1, gives the total shaft power due to the positive-sequence as $3(r_2\bar{I}_2^2+r_1\bar{I}_1^2)$.

Now from Fig. 32(a), if z_m be neglected, the negative-sequence input per phase is equal to

$$\left(r_r+r_s-\frac{r_r}{2}\right)\bar{I}_2^2 \text{ or} \left(\frac{r_r}{2}+r_s\right)\bar{I}_2^2,$$

from which it follows that

$$r_2=\frac{r_r}{2}+r_s. \tag{34}$$

As shown previously the negative-sequence shaft power per phase is equal to $\frac{r_r}{2}\bar{I}_2^2$, which on substituting $\frac{r_r}{2}$ from (34) reduces to $(r_2-r_s)\bar{I}_2^2$. But since $r_s=r_1$, the expression for the negative-sequence shaft power per phase can also be written $(r_2-r_1)\bar{I}_2^2$. Incidentally, from this the rotor losses are equal to $2(r_2-r_1)\bar{I}_2^2$. Therefore the total shaft input into the alternating-current machine is equal to $3[r_2\bar{I}_2^2+r_1\bar{I}_1^2+(r_2-r_1)\bar{I}_2^2]$ and, since $\bar{I}_1=\bar{I}_2$, reduces to $6r_2\bar{I}_2^2$. Including the effect of friction and windage, $P_{(F+W)}$, and calling P_s the total input into the alternating-current machine from the driving tool,

$$r_2=\frac{P_s-P_{(F+W)}}{6\bar{I}_2^2} \tag{35}$$

and, since $\bar{I}_2=\frac{I}{\sqrt{3}}$ where $\bar{I}$ is the actual measured phase current,

$$r_2=\frac{[P_s-P_{(F+W)}]}{2\bar{I}^2} \tag{36}$$

The foregoing neglects the effects of saturation. Tests on salient pole machines with and without dampers verify the fact that the loss varies as the square of the negative-sequence currents. The loss for turbine generators, on the other hand, varies as the 1.8 power of current.

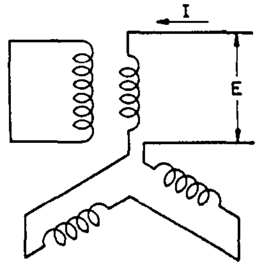
16. Zero-Sequence Impedance

The zero-sequence impedance is the impedance offered to the flow of unit zero-sequence current, i.e., the voltage drop across any one phase (star-connected) for unit current in each of the phases. The machine must, of course, be star-connected for otherwise no zero-sequence current can flow.

The zero-sequence reactance of synchronous machines is quite variable and depends largely upon pitch and breadth factors. In general, however, the figures are much smaller than those of positive and negative sequences. The nature of the reactance is suggested by considering that, if the armature windings were infinitely distributed so that each phase produced a sinusoidal distribution of the mmf, then the mmfs produced by the equal instantaneous currents of the three phases cancel each other and produce zero field and consequently zero reactance except for slot and end-connection fluxes. The departure from this ideal condition introduced by chording and the breadth of the phase belt determines the zero-sequence reactance.

The zero-sequence resistance is equal to, or somewhat larger than, the positive-sequence resistance. In general, however, it is neglected in most calculations.

Method of Test—The most convenient method for test of zero-sequence impedance is to connect the three phases together, as shown in Fig. 34, with the field short-circuited. This connection insures equal distribution of current between the three phases. For this reason it is preferable to connecting the three phases in parallel. The zero-sequence impedance is then equal to $Z_0=\frac{E}{3I}$ as indicated in the illustration.



Rotor at synchronous speed
(or blocked)
Zero-sequence impedance,
$z_0=\frac{E}{3I}$

Fig. 34—Connection for measuring zero-sequence impedance.

IV. PER UNIT SYSTEM

The performance of a whole line of apparatus, regardless of size, can often be expressed by a single set of constants when those constants are expressed in percentages. By this is meant that the loss will be a certain percentage of its kilowatt rating, its regulation a certain percentage of its voltage rating, etc. The advantage of this method of representation extends to a better comparison of performance of machines of different rating. A 100-volt drop in a transmission line has no significance until the voltage base is given, whereas, as a percentage drop would have much significance.

A disadvantage of the percentage system is the confusion that results from the multiplication of percentage quantities. Thus, a 20-percent current flowing through a 40-percent reactance would by simple multiplication give 800 which at times is erroneously considered as 800-percent voltage drop, whereas, the correct answer is an 8-percent voltage drop.

The per unit system[4] of designation is advanced as possessing all the advantages of the percentage system but avoids this last mentioned disadvantage. In this system the rating quantity is regarded as unity. Any other amount of the quantity is expressed as a fraction of the rated amount. It is the same as the percentage system except that unity is used as a base instead of 100. The foregoing multiplication example would in the per unit system be expressed as follows: A 0.20 per unit current flowing through a 0.40 per unit reactance produces an 0.08 per unit voltage drop, which is correct.

A further advantage of the percentage and per unit systems lies in the elimination of troublesome coefficients. However, this is not an unmixed blessing as a definite disadvantage of the use of the per unit system lies in the loss of the dimensional check.

V. POWER EXPRESSIONS

It is frequently necessary to know the manner in which the power output of a machine varies with its excitation and internal angle. A particular application of this knowledge is the stability problem. Several simple cases will be considered.

17. Machine Connected through Reactance to Infinite Bus and also Shunt Reactance across its Terminals, Resistance of Machine Neglected

The schematic diagram for this case is shown in Fig. 35(a), which also shows the significance of the various symbols to be used in this discussion. The reactances x_s, x_c, and the one indicated by the dotted lines represent the branches of an equivalent π circuit, for which the resistance components are neglected. For the purposes of determining the power output of the generator the reactance shown dotted can be neglected. The vector diagram which applies is Fig. 35(b). The total machine current is equal to i,

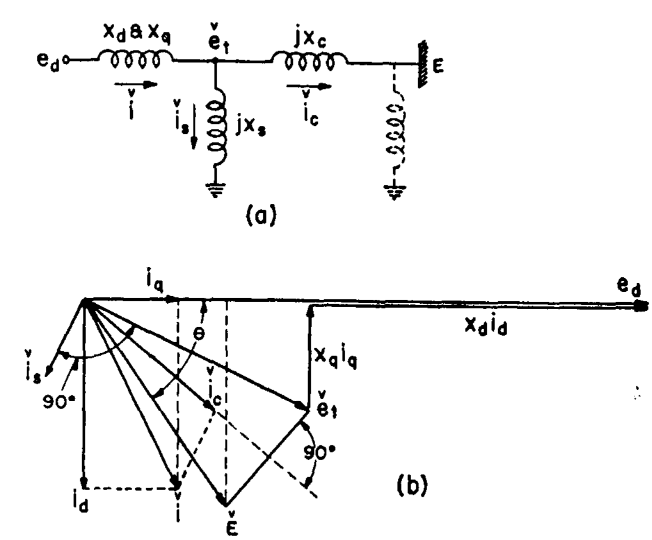


Fig. 35—Machine connected to infinite bus through a reactance.

which from the internal and external currents one can obtain*

$$i_q - ji_d = \check{i}_s + \check{i}_c$$

And inserting the equivalents of $\check{i}_s$ and $\check{i}_c$

$$i_q - ji_d = \frac{\check{e}_t}{jx_s} + \frac{\check{e}_t - \check{E}}{jx_c}$$

$$= -j\left(\frac{x_s+x_c}{x_s x_c}\right)\check{e}_t + j\frac{\check{E}}{x_c} \tag{37}$$

But $\qquad \check{e}_t = e_d - x_d i_d - jx_q i_q$

and $\qquad \check{E} = E\cos\theta - jE\sin\theta$

Upon substituting $\check{e}_t$ and E in equation (37), there results

$$i_q - ji_d = -j\left(\frac{x_s+x_c}{x_s x_c}\right)[e_d - x_d i_d - jx_q i_q] + j\frac{1}{x_c}[E\cos\theta - jE\sin\theta] \tag{38}$$

Equating reals

$$i_q = -\frac{x_q(x_s+x_c)}{x_s x_c} i_q + \frac{E\sin\theta}{x_c}$$

$$= \frac{x_s E\sin\theta}{x_s x_c + x_q x_s + x_q x_c} \tag{39}$$

And equating imaginaries

$$i_d = \frac{x_s+x_c}{x_s x_c}[e_d - x_d i_d] - \frac{1}{x_c}E\cos\theta$$

$$= \frac{(x_s+x_c)e_d - x_s E\cos\theta}{x_s x_c + x_d x_s + x_d x_c} \tag{40}$$

The power output, P, is equal to the sum of the products of the in-phase components of armature current and terminal voltage, namely

$$P = i_q(e_d - x_d i_d) + i_d(x_q i_q)$$
$$= e_d i_q + (x_q - x_d) i_d i_q$$
$$= [e_d + (x_q - x_d) i_d] i_q \tag{41}$$

The power is then obtained by calculating i_q and i_d from (39) and (40) and inserting into (41). If E and e_d are expressed in terms of rms volts to neutral and reactances in ohms per phase, then the above expression gives the power in watts per phase; but if the emf's are expressed in terms of the phase-to-phase volts the expression gives total power. On the other hand, if all quantities are expressed in p.u. then the power is also expressed in p.u. where unity is equal to the kva rating of the machine. If e_d' rather than e_d is known then e_d should be replaced by e_d' and x_d by x_d' wherever they appear in Eqs. (40) and (41).

For the special case of a machine with cylindrical rotor in which $x_q = x_d$, the expression reduces immediately to

$$P = e_d i_q$$
$$= \frac{x_s E e_d \sin\theta}{x_s x_c + x_q x_s + x_q x_c} \tag{42}$$

Another interesting special case is that for which the shunt reactance is not present or $x_s = \infty$. Then

$$P = \left[e_d + (x_q - x_d)\frac{e_d - E\cos\theta}{x_c + x_d}\right]\frac{E\sin\theta}{x_c + x_q}$$
$$= \frac{e_d E\sin\theta}{x_c + x_d} + \frac{(x_d - x_q)E^2\sin 2\theta}{2(x_d + x_c)(x_q + x_c)} \tag{43}$$

And if $x_c = 0$ and $x_s = \infty$, then

$$P = \frac{e_d E\sin\theta}{x_d} + \frac{(x_d - x_q)E^2\sin 2\theta}{2x_d x_q} \tag{44}$$

*The symbol caret over a quantity indicates a phasor quantity.

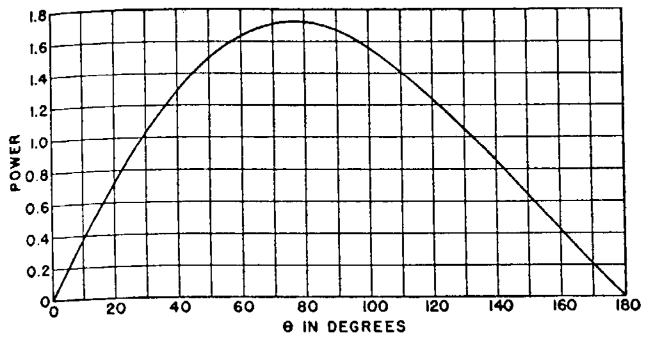


Fig. 36—Power-angle diagram of a salient-pole machine—excitation determined to develop rated kva at 80-percent power factor. $x_d=1.15$; $x_q=0.75$.

In Fig. 36 is shown a power-angle diagram of a salient-pole machine whose excitation is determined by loading at full kva at 80-percent power factor.

An expression frequently used to determine the maximum pull-out of turbine generators is the following

$$\text{Pull-out in kw}=\frac{OC}{OD}(\text{rating of generator in kva})$$

where OC is the field current for the particular operating condition and OD is the field current for the rated-current zero-power factor curve for zero terminal voltage (see Fig. 17). This expression is based upon the maintenance of rated terminal voltage up to the point of pull-out. At pull-out the angle δ of Fig. 15 is equal to 90 degrees. Since the extent of saturation is measured by the voltage behind the Potier reactance drop, it can be seen from Fig. 15 that for δ equal to 90 degrees this voltage is less than rated voltage, and that therefore little saturation is present. From Eq. (44) since $x_d=x_q$ and $\theta=90$ degrees, the pull-out is $\frac{e_dE}{x_d}$. But e_d is proportional to OC on the air-gap line and x_d is likewise proportional to OD on the air-gap line.

Examination of Eq. (44) shows that even if the excitation is zero ($e_d=0$) the power-angle curve is not equal to zero, but equal to $\frac{(x_d-x_q)E^2\sin 2\theta}{2x_dx_q}$. This results from the effects of saliency. Note that it disappears for uniform air-gap machines for which $x_d=x_q$. Advantage is sometimes taken of this relation in the case of synchronous condensers to obtain a somewhat greater capability in the leading (under-excited) kva range. With some excitation systems (see Chap. 7, Excitation Systems) it is possible to obtain negative excitation. The excitation voltage, e_d, in Eq. (44) can be somewhat negative without producing an unstable power-angle diagram. By this device the leading kva range can be increased as much as 15 or 20 percent.

18. Inclusion of Machine Resistance or External Resistance

If the machine is connected to an infinite bus through a resistance and reactance circuit, the external resistance and reactance can be lumped with the internal resistance and reactance and the following analysis used. The vector diagram for this case is shown in Fig. 37 for which

$$e_t\sin\theta+ri_d-x_qi_q=0 \tag{45}$$

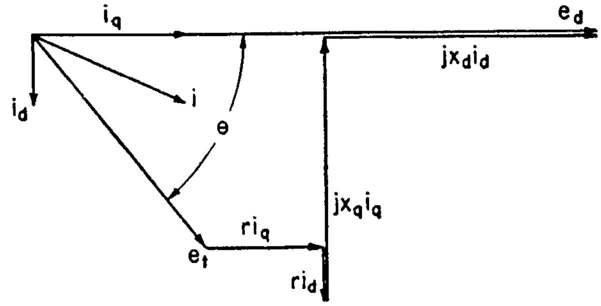


Fig. 37—Vector diagram of salient-pole machine including effect of series resistance.

$$e_t\cos\theta+ri_q+x_di_d-e_d=0 \tag{46}$$

From (45)

$$i_q=\frac{1}{x_q}(e_t\sin\theta+ri_d) \tag{47}$$

Substituting (47) into (46)

$$e_t\cos\theta+\frac{r}{x_q}e_t\sin\theta+\frac{r^2}{x_q}i_d+x_di_d-e_d=0$$

from which

$$i_d=\frac{1}{r^2+x_dx_q}\Big[x_qe_d-re_t\sin\theta-x_qe_t\cos\theta\Big] \tag{48}$$

and substituting in (47)

$$i_q=\frac{1}{r^2+x_dx_q}\Big[re_d+x_de_t\sin\theta-re_t\cos\theta\Big] \tag{49}$$

The power *output*, P, is equal to the sum of the products of the in-phase components of i and e_t, or

$$P=i_qe_t\cos\theta+i_de_t\sin\theta \tag{50}$$

Upon substituting (48) and (49) this reduces to

$$P=\frac{e_t}{r^2+x_dx_q}\Big[e_d(r\cos\theta+x_q\sin\theta)+\frac{x_d-x_q}{2}e_t\sin 2\theta-re_t\Big] \tag{51}$$

The power *input* into the machine is equal to P plus ri^2. The expression for this quantity does not simplify and it is better to calculate it through the intermediate step of evaluating ri^2, which is equal to $r(i_d^2+i_q^2)$.

The foregoing expressions apply to the steady-state conditions. In stability problems it is necessary to determine the average power from instant to instant. In general for this purpose it is permissible to neglect both the unidirectional component of currents and the subtransient component of the alternating current, leaving only the transient component. These latter are determined by the instantaneous value of e_d'. It follows then that the power expressions are simply those derived for the steady-state condition with e_d replaced by e_d' and x_d by x_d'.

VI. EFFECT OF CHANGE IN EXCITATION

Field forcing in certain industrial applications and considerations of system stability require that the voltage increase in response to a sudden need. This increase is brought about automatically either by means of the same control that produced the increase in load or through the use of a voltage regulator. It is necessary, therefore, to be able to predetermine the effect of an increase in exciter voltage upon the output of the synchronous machine. In general, significant changes in exciter voltage never require less than about one-tenth of a second to bring about the change. By the time this effect has been felt through the synchronous machine, which has a time constant of about a second, it will be found that the result is always slow when compared to the subtransient and unidirectional components of the transients associated with the change. In other words, variations in exciter voltage are reflected only in the transient components. As an example, suppose it is desired to calculate the armature current of a machine for a three-phase short-circuit while it is operating at no load with a voltage regulator set for rated voltage.

Immediately after the inception of the short circuit there is a slight lag in the regulator until its contacts and relays close. The exciter voltage (and voltage across the field of the main machine) then rises as shown in the upper curve of Fig. 38. The bottom curve refers to the armature cur-

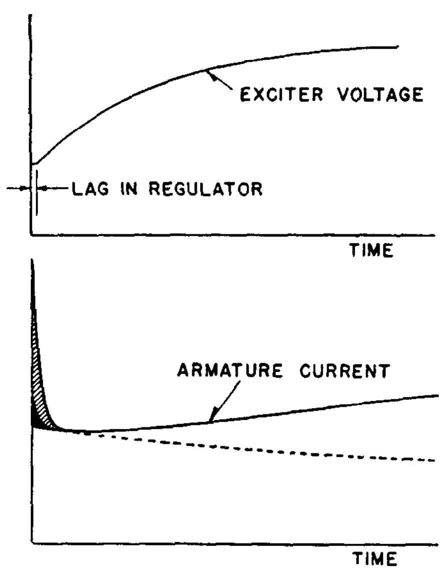


Fig. 38—Illustration showing relative importance of different components of armature short-circuit current and response of transient component to the exciter voltage.

rent, the dotted line showing the nature of transient component if there were no regulator, the exciter voltage remaining constant. The line immediately above shows how the transient component changes as a result of the change in exciter voltage. To approximately the same scale, the cross-hatched area shows the increment in current caused by subtransient effects. The blackened area shows how the unidirectional component would contribute its effect. This component is quite variable and for a short-circuit on the line might be entirely completed in a cycle or less. In any event regardless of its magnitude it can be merely added to the transient and subtransient component. It is independent of the exciter voltage.

19. Fundamental Equation

Being restricted to the transient component, the effect of exciter response can then be defined entirely by effects in the field circuit. The beauty of the per unit system is exemplified in the analysis of this problem. In p.u. the differential equation for the field circuit takes the following form

$$e_x = e_d + T'_{do}\frac{de_d'}{dt} \tag{52}$$

In this equation e_x represents the exciter voltage or the voltage across the field if there is no external field resistor in the field circuit. The unit of e_x is that voltage required to circulate such field current as to produce rated voltage at no load on the air-gap line of the machine. The term e_d is the synchronous internal voltage necessary to produce the instantaneous value of armature current for the given armature circuit regardless of what it may be. Its unit is rated voltage. It is synonymous with field current when unit field current is that field current necessary to produce rated voltage at no load on the air-gap line. It will be seen then that the use of e_d is merely a convenient way of specifying the instantaneous field current during the transient conditions; it is the field current necessary to produce the armature current existent at that instant. As shown previously, e_d', is proportional to the flux linkages with the field winding. It is the quantity that, during the transition period from one circuit condition to another, remains constant. The foregoing equation has its counterpart in the more familiar forms

$$e_x = Ri + N(10^{-8})\frac{d\phi}{dt} \tag{53}$$

or

$$e_x = Ri + L\frac{di}{dt}. \tag{54}$$

To familiarize the reader with (52), suppose that normal exciter voltage is suddenly applied to the field winding at no load. Since the armature is open-circuited e_d' and e_d are equal and the equation can be written

$$e_x = e_d + T'_{do}\frac{de_d}{dt} \tag{55}$$

When steady-state conditions are finally attained $\frac{de_d}{dt}$ is equal to zero and $e_d = e_x$. This states that since $e_x = 1.0$, e_d must also equal 1.0, that is, the excitation is equal to the normal no-load voltage. It will attain this value exponentially with a time constant T'_{do}.

Another example. Suppose the synchronous machine to be short-circuited from no-load and to be operating without a regulator. At any instant the armature current, i, is equal to e_d'/x_d'. But since e_d, which can be regarded as the instantaneous field current required to produce i, is equal to $x_d\ i$, then eliminating i between these equations

$$e_d = \frac{x_d}{x_d'}e_d' \tag{56}$$

Then equation (52) takes the form

$$1 = \frac{x_d}{x_d'}e_d' + T'_{do}\frac{de_d'}{dt}$$

or if it is to be expressed in terms of armature current

$$1=x_d i+T'_{do}x_d'\frac{di}{dt}. \tag{57}$$

The sustained magnitude of i, is then

$$i=\frac{1}{x_d}$$

The initial magnitude of i, since e_d' remains constant during the transition and is initially equal to 1, is

$$i=\frac{1}{x_d'}$$

The homogeneous equation for (57) is

$$0=x_d i+T'_{do}x_d'\frac{di}{dt}$$

or

$$0=i+T'_{do}\frac{x_d'}{x_d}\frac{di}{dt} \tag{58}$$

Thus i changes from $\frac{1}{x_d'}$ to $\frac{1}{x_d}$ exponentially with a time constant equal to $\frac{x_d'}{x_d}T'_{do}$.

In all problems involving a transition from one circuit condition to another the one quantity (when subtransient effects are neglected and the time constant in the quadrature axis is zero) that remains constant within the machine is the flux linkages with the field winding, which in turn is reflected in the quantity e_d'. It is necessary, therefore, to calculate e_d' for the circuit condition preceding the transition. All the discussion of the following cases assumes that this point is understood and that e_d' is known for the beginning of the transient period.

Several cases will now be discussed.

20. Three-phase Short Circuit of Unsaturated Machine

The problem is to determine the transient component of short-circuit current in response to the exciter voltage given in Fig. 39. This is most quickly and conveniently found by a graphical method, which, for want of a better name, has been called by the author "the follow-up method." It is a method that can be applied to any problem involving a simple time constant.

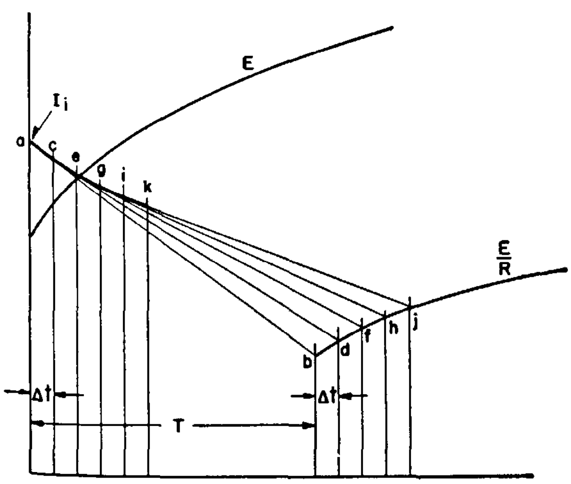


Fig. 39—Illustration of "Follow-up Method."

To demonstrate the method, consider a simple resistance-inductance circuit to which the voltage, E, is applied. Let the differential equation for this circuit be

$$E=RI+L\frac{dI}{dt} \tag{59}$$

where the symbols have the customary significance. Dividing through by R, there results

$$\frac{E}{R}=I+\frac{L}{R}\frac{dI}{dt} \tag{60}$$

The coefficient of $\frac{dI}{dt}$ is called the time constant of the circuit and will be designated by T, giving

$$\frac{E}{R}=I+T\frac{dI}{dt}$$

$$\frac{dI}{dt}=\frac{\frac{E}{R}-I}{T} \tag{61}$$

In this expression $\frac{E}{R}$ is the steady-state current that I approaches for the instantaneous value of E. I is the instantaneous magnitude of current. If the current at any instant is plotted by the point a (Fig. 40) and the corresponding value of $\frac{E}{R}$ for that instant is plotted as the point b (Fig. 40) displaced horizontally by a time T,

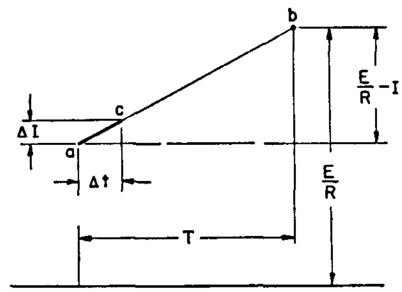


Fig. 40—Construction derivation of "Follow-up Method."

then the vertical distance between a and b gives the numerator of (61) and the horizontal distance the denominator. The slope of the line between a and b is equal to $\frac{dI}{dt}$. If an interval of time Δt is chosen following the instant under consideration and E is assumed constant over the interval then the change in I during the interval, ΔI, is equal to $\frac{dI}{dt}\Delta t$. The final value of current for the interval is then given by the point c. If $\frac{E}{R}$ at an instant Δt later is then plotted and the line drawn from c then the value for ΔI for the second interval is obtained. Following such procedure it is possible to construct the complete curve for I. The construction is illustrated in Fig. 39, in which the curve marked E is the instantaneous magnitude of E from time $t=0$. Plot $\frac{E}{R}$ displaced to the right a time T. Let I_i be the initial value of I at $t=0$. Divide the time into intervals of length Δt. Draw the line ab, then cd, ef, etc. The accuracy will be greater the smaller the intervals and can be increased somewhat for a given element width by using $T-\frac{\Delta t}{2}$ instead of T for the distance by which the steady-state curve which I tends to approach, is offset horizontally.

Now returning to the problem in hand. The differential equation governing the case is given by (52). The exciter voltage e_x is assumed given and expressed in p.u. For a three-phase short circuit at the terminals of the machine e_d is equal to $x_d i$ and $e_d'=x_d'i$. Therefore Eq. 52 becomes

$$e_x=x_d i+x_d' T'_{do}\frac{di}{dt} \tag{62}$$

Dividing through by x_d

$$\frac{e_x}{x_d}=i+\frac{x_d'}{x_d}T'_{do}\frac{di}{dt} \tag{63}$$

The construction dictated by this equation and the follow-up method is shown in Fig. 41. $\frac{e_x}{x_d}$ is plotted against

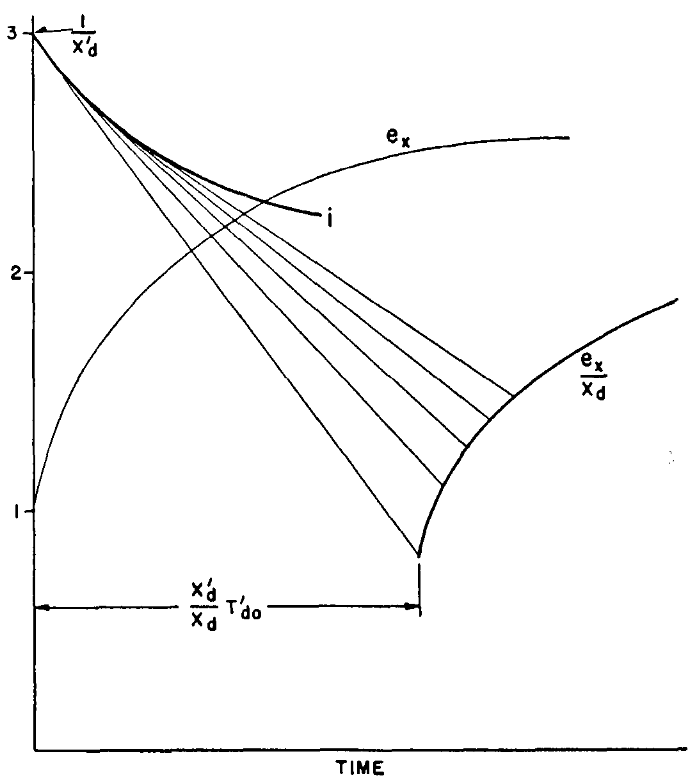


Fig. 41—Transient component of short-circuit current, i′, as influenced by excitation.

time, its zero being displaced an interval $\frac{x_d'}{x_d}T'_{do}$ from reference zero. The initial value of i is determined through e_d' which was 1.0 at $t=0$. This makes the initial amount of $i=\frac{1}{x_d'}$. Starting from this value the actual magnitude of i is obtained as a function of time.

21. Unsaturated Machine Connected to Infinite Bus

As stated previously the subtransient and unidirectional components of current are not of importance in the stability problem. For this application it is desirable to determine how e_d' varies as this influences the power output of the machine and in turn dictates the degree of acceleration or deceleration of the rotor. The circuit shown in Fig. 35(a) is typical of a setup that might be used for an analytical study to determine the effect of exciter response in increasing stability limits. Another case of considerable importance is the action of a generator when a heavy load, such as a large induction motor, is connected suddenly across its terminals or across the line to which is is connected. In starting the motor the line voltage may drop an excessive amount. The problem might be to determine the amount to which this condition could be ameliorated by an appropriate excitation system. Since reactive kva is more important than the real power in determining regulation, the motor can be represented as a reactor and the circuit in Fig. 35(a) utilized. Having determined the manner in which e_d' varies, the power in the case of the stability problem and the terminal voltage $(e_d'-x_d'i)$ in the case of the voltage problem, can be calculated easily. Equation (52) must be used again to determine the manner in which e_d' varies in response to changes in exciter voltage and phase position of the rotor with respect to the infinite bus. The instantaneous armature current can be found in terms of the rotor angle θ and e_d' by replacing e_d and x_d of Eq. (40) by e_d' and x_d', respectively, giving

$$i_d=\frac{(x_s+x_c)e_d'-x_sE\cos\theta}{x_sx_c+x_d'x_s+x_d'x_c} \tag{64}$$

The synchronous internal voltage, e_d, is equal at any instant to

$$e_d=e_d'+(x_d-x_d')i_d \tag{65}$$

and upon substituting (64)

$$e_d=e_d'+(x_d-x_d')\frac{(x_s+x_c)e_d'-x_sE\cos\theta}{x_sx_c+x_d'x_s+x_d'x_c}$$
$$=\frac{(x_sx_c+x_dx_s+x_dx_c)e_d'-x_s(x_d-x_d')E\cos\theta}{x_sx_c+x_d'x_s+x_d'x_c}$$

Substituting this expression in (52), there results.

$$e_x=\frac{x_sx_c+x_dx_s+x_dx_c}{x_sx_c+x_d'x_s+x_d'x_c}e_d'-\frac{x_s(x_d-x_d')}{x_sx_c+x_d'x_s+x_d'x_c}\mathrm{E}\cos\theta+T'_{do}\frac{de_d'}{dt} \tag{66}$$

which can be converted to

$$\frac{T_d'}{T'_{do}}e_x+\frac{x_s(x_d-x_d')}{x_sx_c+x_dx_s+x_dx_c}E\cos\theta=e_d'+T_d'\frac{de_d'}{dt} \tag{67}$$

in which

$$T_d'=\frac{x_sx_c+x_d'x_s+x_d'x_c}{x_sx_c+x_dx_s+x_dx_c}T_{do}' \tag{68}$$

The time constant T_d' is the short-circuit transient time constant.

If θ were constant or if its motion as a function of time were known then the whole left-hand side could be plotted (displaced by the time T_d') and treated by the follow-up method as the quantity that e_d' tends to approach. Unfortunately θ is not in general known beforehand, and it is necessary to calculate θ simultaneously in small increments in a simultaneous solution of e_d' and θ. The magnitude of θ is determined by the electro-mechanical considerations discussed in the chapter dealing with System Stability. In solving for e_d' a progressive plot of the left-hand side can be made or (67) can be transformed to the following form

$$\frac{de_d'}{dt}=\frac{1}{T_d'}\left[\frac{T_d'}{T_{do}'}e_x+\frac{x_s(x_d-x_d')}{x_sx_c+x_dx_s+x_dx_c}E\cos\theta-e_d'\right] \quad (69)$$

and the increment calculated from the equation

$$\Delta e_d'=\frac{de_d'}{dt}\Delta t \quad (70)$$

A shunt resistance-reactance load such as an induction motor is not much more difficult to solve numerically but the expressions become too involved for analytical solution. It is necessary only to calculate i_d in terms of e_d' and θ just as was done before and then follow the same steps as used for the reactance load.

22. Unsaturated Machine Connected to Resistance-Reactance Load

A case not too laborious to carry through analytically is that for which a resistance-reactance load is suddenly applied to a synchronous machine. Let r_{ext} and x_{ext} be the external resistance and reactance. The addition of a subscript t to machine constants indicates the addition of r_{ext} or x_{ext} to the respective quantity. The equations of Sec. 17 then apply to this case, if e_t in the equations is made equal to zero and x_d replaced by x_{dt}, etc.

Following the same procedure as previously, there results from Eq. (48) when e_d and x_d are replaced by e_d' and x_{dt}' and e_t is equal to zero.

$$i_d=\frac{x_{qt}}{r_t^2+x_{dt}'x_{qt}}e_d' \quad (71)$$

The field current or its equivalent, the synchronous internal voltage, is then

$$\begin{aligned} e_d&=e_d'+(x_{dt}-x_{dt}')i_d\\ &=e_d'+(x_{dt}-x_{dt}')\frac{x_{qt}}{r_t^2+x_{dt}'x_{qt}}e_d'\\ &=\frac{x_{dt}x_{qt}+r_t^2}{x_{dt}'x_{qt}+r_t^2}e_d' \end{aligned} \quad (72)$$

Substituting this expression in (52) there results that

$$e_x=\frac{x_{dt}x_{qt}+r_t^2}{x_{dt}'x_{qt}+r_t^2}e_d'+T_{do}'\frac{de_d'}{dt} \quad (73)$$

which can be converted to

$$\frac{T_d'}{T_{do}'}e_x=e_d'+T_d'\frac{de_d'}{dt} \quad (74)$$

in which

$$T_d'=\frac{x_{dt}'x_{qt}+r_t^2}{x_{dt}x_{qt}+r_t^2}T_{do}'. \quad (75)$$

From this point the follow-up method can be used as before. After e_d' is determined as a function of time any other quantity such as terminal voltage can be obtained readily.

23. Saturation

In analyzing transient phenomenon of machines in the unsaturated condition, the theory was built around the concept of the transient internal voltage, e_d', a quantity evaluated by using the transient reactance, x_d'. In the presence of saturation it was found that for steady-state conditions by the introduction of the Potier reactance, x_p (see Sec. 3) the proper regulation was obtained at full load zero power-factor. The use of x_p and e_p also resulted in satisfactory regulation for other power-factors. In extending the analysis into the realm of transient phenomenon, e_p will continue to be used as a base from which to introduce additional mmf into the field circuit to take care of saturation effects. The treatment will follow quite closely the same assumptions as were used in determining the steady-state regulation according to the Two-Reaction Potier Voltage method of Sec. 3(d).

With this assumption the fundamental Eq. (52) for the field circuit becomes

$$e_x=e_d+(s\text{ due to }e_p)+T_{do}'\frac{de'_d}{dt} \quad (76)$$

As before e_d represents, neglecting saturation, the voltage behind the synchronous reactance of the machine or what is equivalent the field current required to produce the instantaneous e_d', including the demagnetizing effect of the instantaneous armature current. The total field current is obtained by adding s to e_d. In some cases it is found simpler to convert all of the right hand side to the single variable e_p but in others it is simpler to retain the variable in the form of e_d'. Two applications of this equation will be discussed.

Machine Connected to Infinite Bus—The circuit shown in Fig. 35(a) is the one under discussion and for which Eq. (66) applies for the unsaturated condition. This equation can be expanded to include saturation, in accordance with Eq. (76), to the following

$$e_x=\frac{x_sx_c+x_dx_s+x_dx_c}{x_sx_c+x_d'x_s+x_d'x_c}e_d'-\frac{x_s(x_d-x_d')}{x_sx_c+x_d'x_s+x_d'x_c}E\cos\theta$$
$$+(s\text{ due to }e_p)+T_{do}'\frac{de_d'}{dt}. \quad (77)$$

This can be converted to

$$\frac{de_d'}{dt}=\frac{e_x-(s\text{ due to }e_p)}{T_{do}'}+\frac{x_s(x_d-x_d')\,E\cos\theta}{(x_sx_c+x_dx_s+x_dx_c)T_d'}-\frac{e_d'}{T_d'} \quad (78)$$

in which T_d' is defined from Eq. (68). Before (78) can be used it will be necessary to determine e_p in terms of e_d'.

The components of current, i_q and i_d, can be determined from (39) and (40) by replacing e_d, by e_d' and x_d by x_d'. Thus

$$i_q=\frac{x_sE\sin\theta}{x_sx_c+x_qx_s+x_qx_c} \quad (79)$$

$$i_d=\frac{(x_s+x_c)e_d'-x_sE\cos\theta}{x_sx_c+x_d'x_s+x_d'x_c}. \quad (80)$$

The direct-axis component of e_p is equal to

$$e_{pd}=e_d'-(x_d'-x_p)i_d$$
$$=\frac{x_sx_c+x_px_s+x_px_c}{x_sx_c+x_d'x_s+x_d'x_c}e_d'+\frac{x_s(x_d'-x_p)E\cos\theta}{x_sx_c+x_d'x_s+x_d'x_c} \quad (81)$$

and the quadrature-axis component of e_p is

$$e_{pq}=(x_q-x_p)i_q$$
$$=\frac{x_s(x_q-x_p)E\sin\theta}{x_sx_c+x_qx_s+x_qx_c} \quad (82)$$

The amplitude of e_p is then equal to

$$e_p=\sqrt{e_{pd}^2+e_{pq}^2} \quad (83)$$

While this quantity does not simplify greatly, it does not appear so formidable after numerical values are inserted. e_p can thus be calculated for any instantaneous value of e_d' and the s corresponding thereto substituted in Eq. (78). Equation (78) provides a means for computing increments of change in e_d' for use in step-by-step solution. Thus

$$\Delta e_d'=\frac{de_d'}{dt}\Delta t \quad (84)$$

As s becomes small and saturation effects disappear, the solution relapses into the same type as used when saturation is negligible (Eq. 66), for which the follow-up method is frequently applicable.

The relations just developed are useful in estimating the extent to which e_d' varies in system stability problems. Fig. 42 shows the results of calculations on a system in which a generator is connected to a large network, represented as an infinite bus, through a reactance equal to $j0.6$.

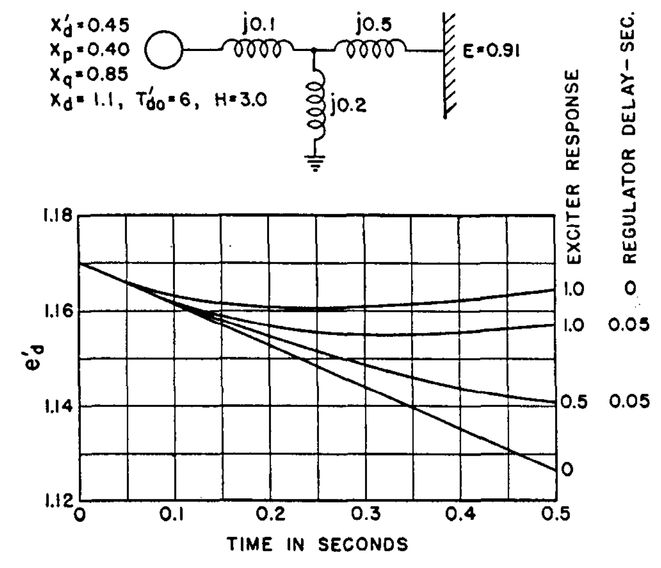


Fig. 42—Effect of rate of response upon e_d' as a line-to-line fault represented by the three-phase shunt load j0.2 is applied to generator which had been operating at 90 percent power-factor. 20 percent of air-gap mmf required for iron at rated voltage.

A line-to-line fault is assumed applied to the connecting transmission line on the high tension bus at the generating end which is equivalent to a three-phase short circuit through a reactance of $j0.2$ ohms. The curves justify the assumption that is usually made in stability studies that where quick response excitation is installed, e_d' may be regarded as constant.

Machine Connected to Resistance-Reactance Load—This case is the same as that considered in Sec. 22 except that saturation effects are to be included. Upon including the saturation term s into Eq. (74) there results that

$$\frac{T_d'}{T_{do}'}\left[e_x-(s\text{ due to }e_p)\right]=e_d'+T_d'\frac{de_d'}{dt} \quad (85)$$

in which

$$T_d'=\frac{x_{dt}'x_{qt}+r_t^2}{x_{dt}x_{qt}+r_t^2}T_{do}'. \quad (86)$$

It is well to recall again that this analysis neglects subtransient effects and assumes that the time constant in the quadrature axis is zero. If in Eqs. (48) and (49) e_t is made equal to zero, e_d is replaced by e_d' and the corresponding changes in reactance associated with e_d' are made, and in addition the subscripts are changed to indicate total reactances, Then

$$i_d=\frac{x_{qt}}{x_{dt}'x_{qt}+r_t^2}e_d' \quad (87)$$

$$i_q=\frac{r_t}{x_{dt}'x_{qt}+r_t^2}e_d'. \quad (88)$$

The total current is then

$$i=\frac{\sqrt{x_{qt}^2+r_t^2}}{x_{dt}'x_{qt}+r_t^2}e_d'. \quad (89)$$

The voltage e_p is

$$e_p=i\sqrt{x_{pt}^2+r_t^2}$$
$$=\frac{\sqrt{(x_{pt}^2+r_t^2)(x_{qt}^2+r_t^2)}}{x_{dt}'x_{qt}+r_t^2}e_d' \quad (90)$$

Upon substituting e_d' from (90) into (85) and using (86) also there results that

$$\frac{\sqrt{(x_{pt}^2+r_t^2)(x_{qt}^2+r_t^2)}}{x_{dt}x_{qt}+r_t^2}e_x-\frac{\sqrt{(x_{pt}^2+r_t^2)(x_{qt}^2+r_t^2)}}{x_{dt}x_{qt}+r_t^2}(s\text{ due to }e_p)$$
$$=e_p+T_d'\frac{de_p}{dt} \quad (91)$$

As can be seen from Fig. 43 the solution of this equation lends itself well to the follow-up method. On the right-hand side the assumed exciter response curve, e_x, is plotted as a function of time. Multiplying this quantity by the coefficient of e_x, the term $e_{p\infty}$ is obtained. This is the value e_p tends to attain if there were no saturation effects. As in the follow-up method, the zero of time from which the instantaneous curve of e_p is drawn, is displaced to the left an amount T_d' minus half the interval of time chosen in the step-by-step solution. Along the ordinate of e_p a curve s_1 equal to the second term is plotted in which s is obtained from the no-load saturation curve shown in (b). For any instantaneous value of e_p, s_1 is plotted downward from $e_{p\infty}$ as the construction progresses. So starting from the initial value of e_p, of which more will be said later, a construction line is drawn to a point for which s_1 was the value corresponding to the initial value of e_p. For the second interval

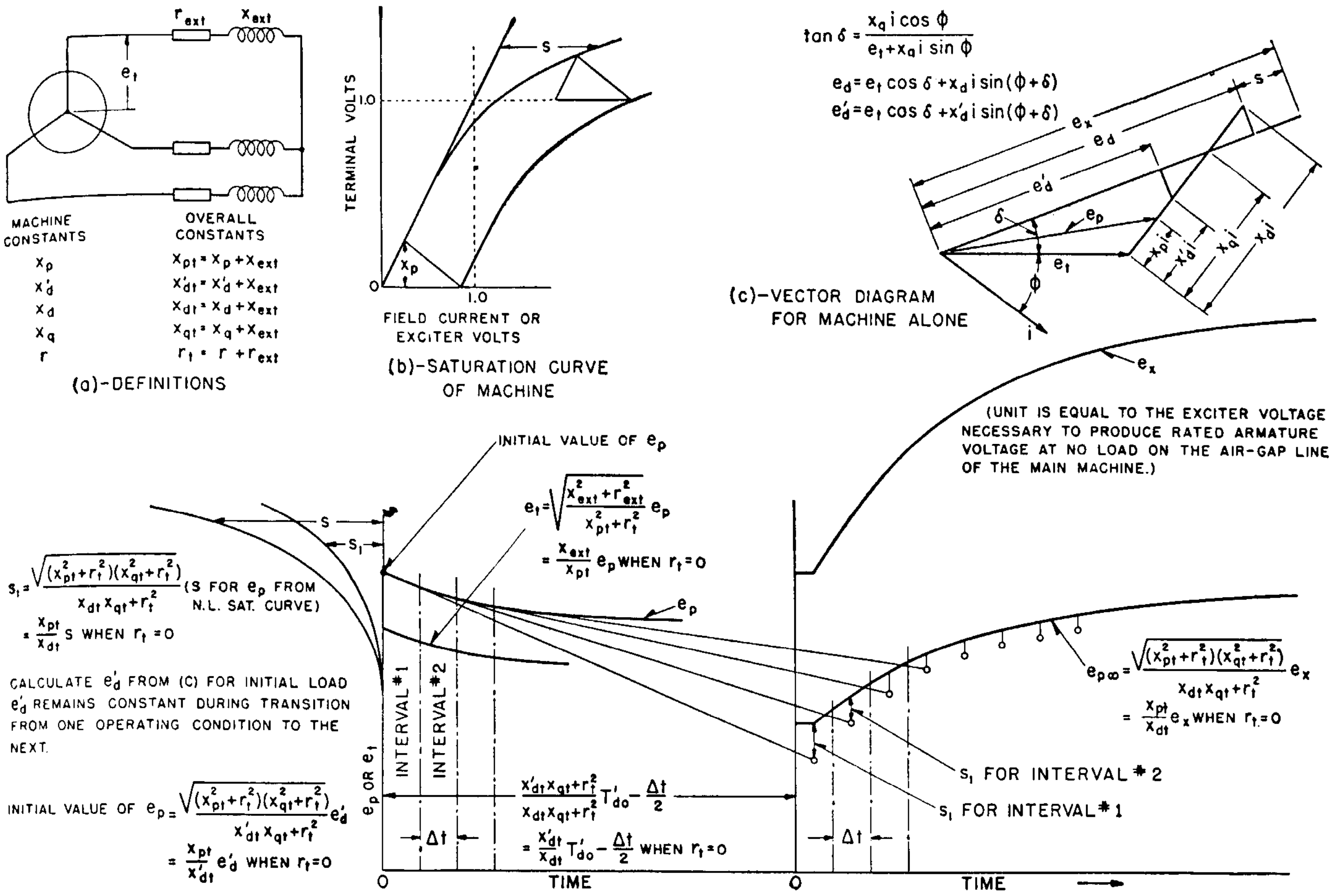


Fig. 43—Graphical determination of terminal voltage as polyphase series resistances, r_{ext} and reactances x_{ext} are suddenly applied.

s_1 is taken for the value of e_p at the end of the first interval or, to be slightly more accurate for an estimated average value of e_p for the second interval. And so the construction proceeds.

By the same reasoning whereby e_p was obtained in Eq. (90) the terminal voltage e_t can likewise be obtained, giving

$$e_t = i\sqrt{x^2_{ext} + r^2_{ext}}$$
$$= \frac{\sqrt{(x^2_{qt} + r^2_t)(x^2_{ext} + r^2_{ext})}}{x_d{}'_t x_{qt} + r^2_t} e_d'$$

and substituting e_d' from (90)

$$e_t = \sqrt{\frac{x^2_{ext} + r^2_{ext}}{x^2_{pt} + r^2_t}}\, e_p \qquad (92)$$

This permits of the calculation of e_t from e_p after the construction has been completed.

During the transition from one operating condition to the next, only e_d' remains constant; e_p changes. It is essential therefore that e_d' be computed for the initial operating condition. The conventional construction shown in Fig. 43(c) can be used. This determines the initial value of e_d' for the new operating condition from which the initial value of e_p can be computed by Eq. (90).

Common cases for which these calculations apply are the determination of regulation for loads suddenly applied to a generator. Instances in which this can occur are the sudden disconnection of a loaded generator from the bus throwing its load upon the remaining units or the starting of an induction motor by direct connection to a generator. For the latter case, if the capacity of the induction motor is a significant fraction of the kva of the generator, a severe drop in voltage results. Thus a 500-hp motor thrown on a 3300-kva generator produces an instantaneous drop in voltage of the order of 13 percent. The effective impedance of the induction motor varies with slip and to be rigorous this variation should be taken into consideration. It is usually sufficiently accurate to use the blocked rotor reactance for the motor impedance up to the speed corresponding to maximum torque in calculating the factor which determines $e_{p\infty}$ in terms of e_x. Beyond the slip corresponding to maximum torque, the effective impedance varies rapidly to the running impedance. Simultaneously with the increase in impedance the lagging kva likewise drops off which results in a considerable rise in voltage. This effect is clearly shown in Fig. 44 taken from some tests made by Anderson and Monteith.[20] As running speed is approached the generator voltage rises, the excitation being too high for the particular loading. To form a better idea of the magnitudes involved in such calculations, Fig. 45 shows curves of terminal voltage as an induction motor equal in horse power to 20 percent of the kva of a generator is suddenly thrown upon an unloaded generator for differ-

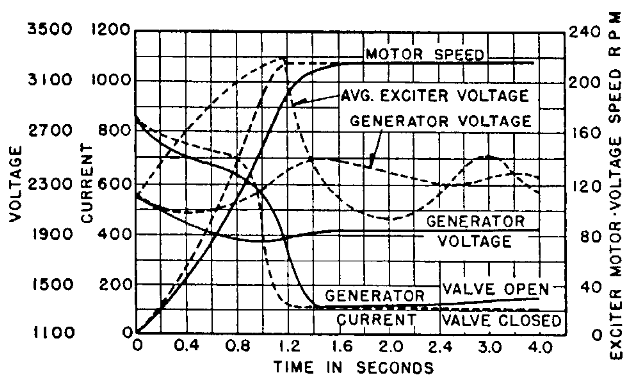


Fig. 44—Performance of 3333 kva, 0.6 power-factor, 3600 rpm, 1.7 short-circuit ratio generator as a single 500-hp induction-motor pump is started. Induction-motor starting torque equal to full-load torque and pull-out torque equal to 2.8 full-load torque. Full lines represent operation with fixed excitation and dotted lines under regulator control.

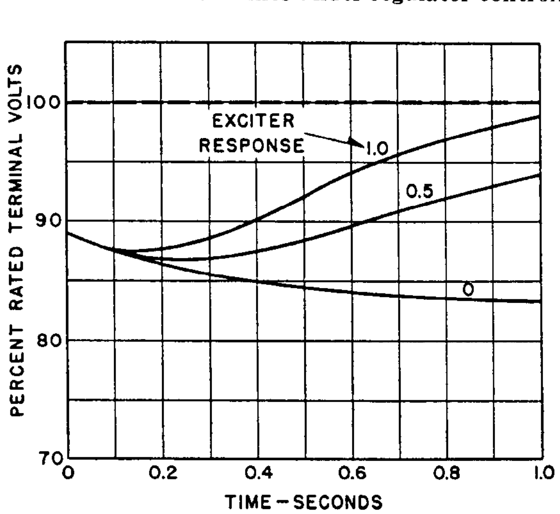


Fig. 45—Terminal voltage of a 500 kva, 80-percent power-factor engine-type generator ($x_d = 1.16$, $x_q = 0.59$, $x_d' = 0.30$, 13 percent saturation) as a 100-hp induction motor is connected.

ent rates of response of the exciter. Ordinarily one is primarily interested in the minimum voltage attained during the accelerating period and so the calculations have been carried out to only 1.0 second. The curves show conditions for constant excitation and for exciters with 0.5 and 1.0 ratios, respectively.

24. Drop in Terminal Voltage with Suddenly-Applied Loads

When a relatively large motor is connected to a generator, the terminal voltage may decrease to such an extent as to cause undervoltage release devices to operate or to stall the motor. This situation arises particularly in connection with the starting of large motors on power-house auxiliary generators. The best single criterion to describe this effect when the generator is equipped with a regulator to control the excitation is the maximum drop. The previous section describes a method whereby this quantity can be calculated. However, the problem arises so frequently that Harder and Cheek[22,23] have analyzed the problem generally and have plotted the results in curve form.

The analysis has been carried out for both self-excited and separately-excited exciters. The results for the former are plotted in Fig. 46, and for the latter in Fig. 47. These curves are plotted in terms of the four parameters: (1) magnitude of load change (2) $X'_{d\,sat}$ (3) T'_{do}, and (4) rate of exciter response, R. The response is defined in the chapter on Excitation Systems. It is shown by Harder and Cheek[22] that variations in x_q, saturation factor of the generator and power factor between zero and 60 percent have little effect upon the maximum drop. The assumed value of x_d for these calculations was 120 percent. An accurate figure for maximum voltage drop can be obtained for values of x_d other than 120 percent by first expressing reactances and the applied load on a new kva base, such that x_d on the new base is 120 percent, and then applying the curves. For example, suppose a load of 1500 kva (expressed at full voltage) of low power factor is to be applied to a 3000-kva generator having 30-percent transient reactance and 150-percent synchronous reactance. Suppose that the generator time constant is 4.0 seconds and the exciter has a nominal response of 1.0. To determine the drop, express the transient reactance and the applied load on the kilovolt-ampere base upon which x_d is 120 percent. The base in this case will be $3000 \times 120/150 = 2400$ kva. On this base the transient reactance x_d' is $30 \times 2400/3000 =$ 24 percent, and the applied load is $1500/2400 = 62.5$ percent. If the exciter is self-excited then from the curves of Fig. 46, the maximum voltage drop is 15 percent for 62.5-percent load applied to a generator having 24-percent transient reactance, a time constant of 4.0 seconds, and an exciter of 1.0 nominal response. This same maximum drop would be obtained with the machine and load under consideration.

The initial load on a generator influences the voltage drop when additional load is suddenly applied. As shown in Fig. 48, a static or constant-impedance initial load reduces the voltage drop caused by suddenly applied load. However, a load that draws additional current as voltage decreases may increase the voltage drop. Such loads will be referred to as "dynamic" loads. For example, a running induction motor may drop slightly in speed during the voltage dip so that it actually draws an increased current and thereby increases the maximum voltage drop. The dynamic initial load curve of Fig. 48 is based on an initial load that draws constant kilowatts and power factor as the voltage varies.

VII. CONSTANTS FOR USE IN STABILITY PROBLEMS

The stability problem involves the study of the electromechanical oscillations inherent in power systems. A fundamental factor in this problem is the manner in which the power output of the generator varies as the position of its rotor changes with respect to some reference voltage. The natural period of power systems is about one second. Because of the series resistance external to the machine, the time constant of the unidirectional component of armature current is usually so small as to be negligible in

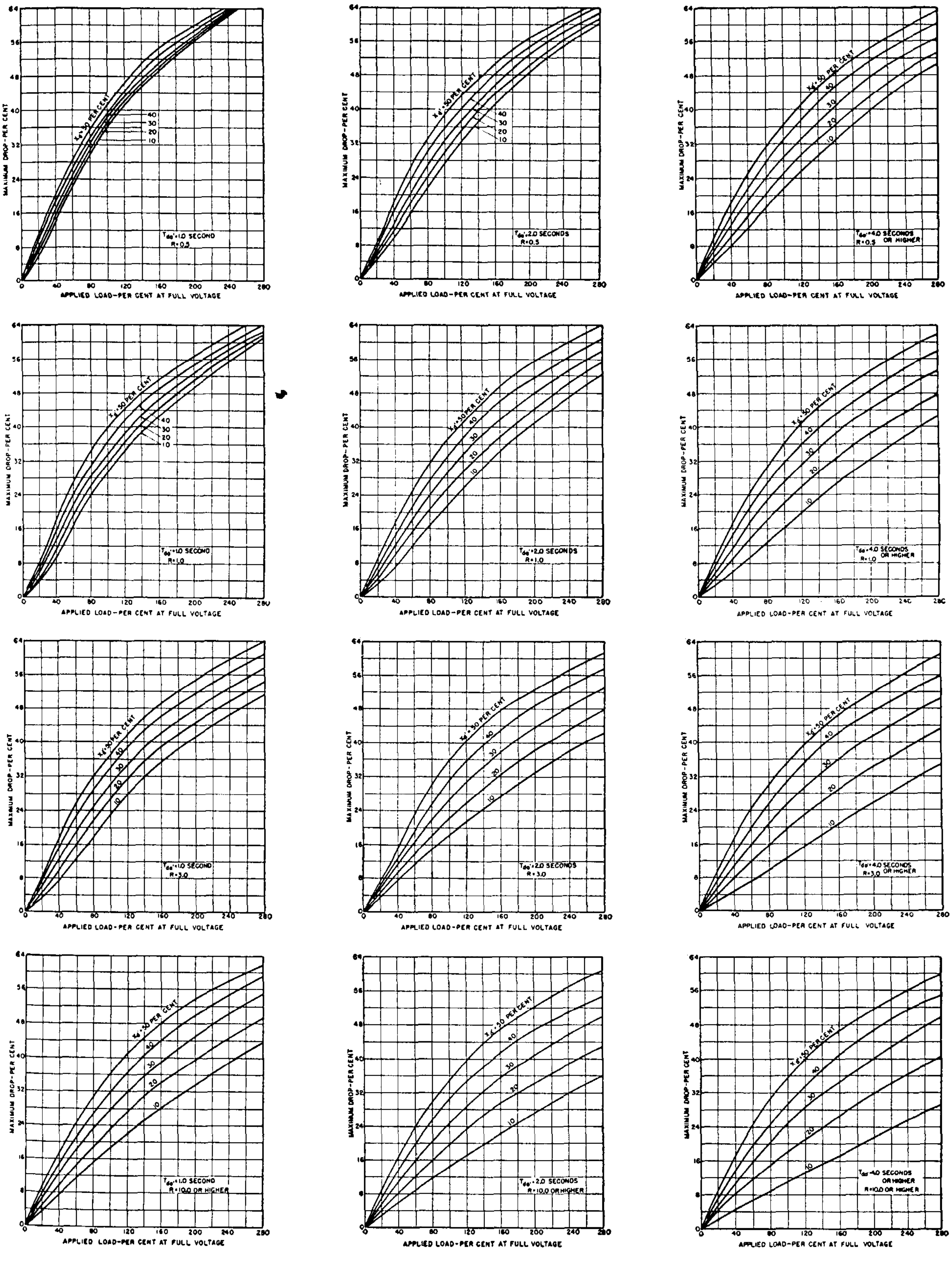


Fig. 46—Maximum voltage drop of a synchronous machine WITH SELF-EXCITED EXCITER as affected by (a) magnitude of load change, (b) $x'_{d\ sat}$, (c) T'_{do} and (d) rate of exciter response. x_d' on curves refer to saturated or rated-voltage value. Assumptions used in calculations: $x_d' = 1.07\ x_d'{}_{sat}$; $x_d = 1.20$; $x_q = 0.75$; no-load saturation curve/air gap line normal voltage = 1.2; time lag of regulator = 0.05 second; added load is constant impedance of 0.35 pf.; initial load zero.

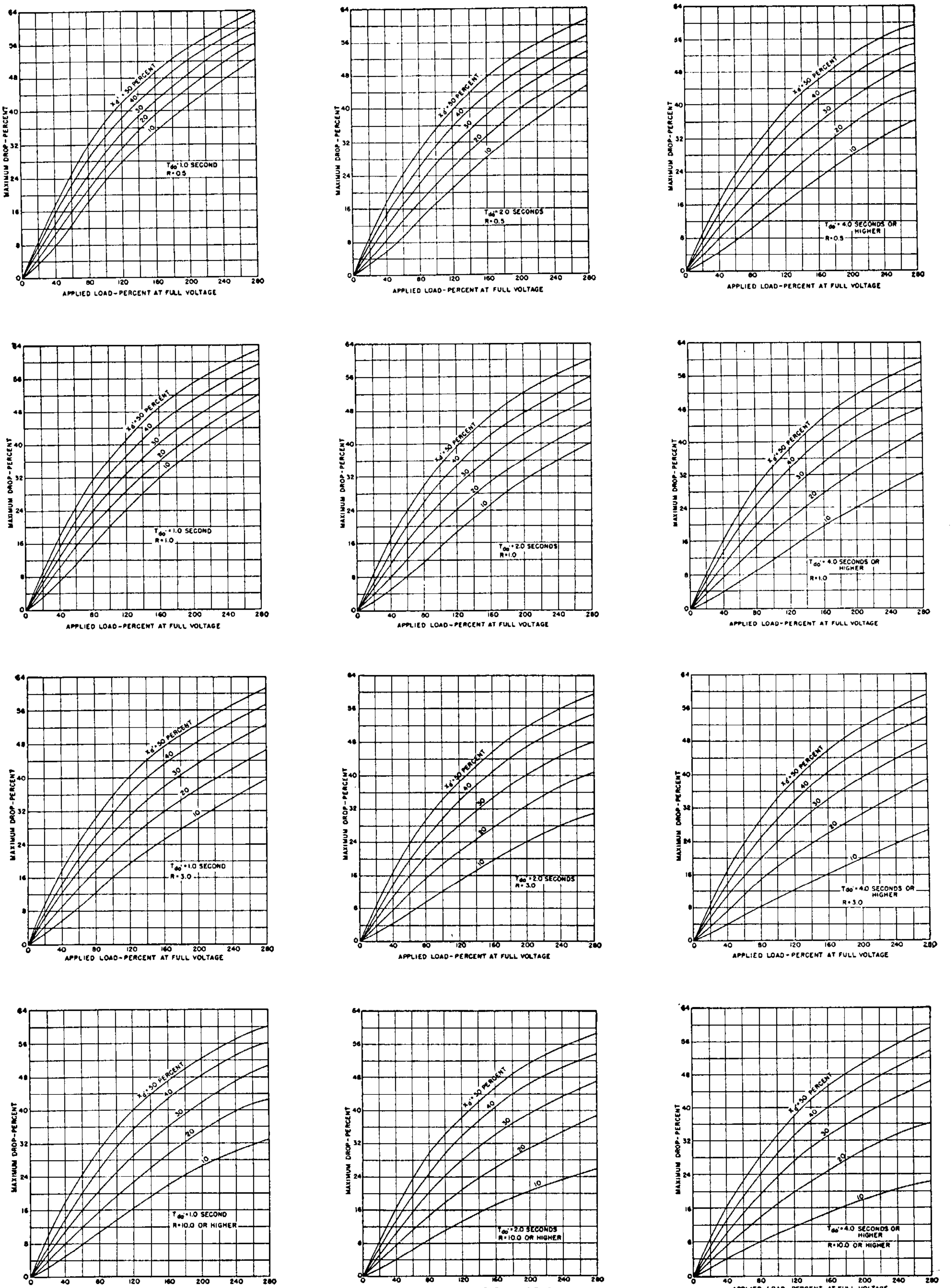


Fig. 47—Maximum voltage drop of a synchronous machine WITH SEPARATELY-EXCITED EXCITER as affected by (a) magnitude of load change, (b) $x'_{d\ sat.}$ (c) T_{do}' and (d) rate of exciter response.

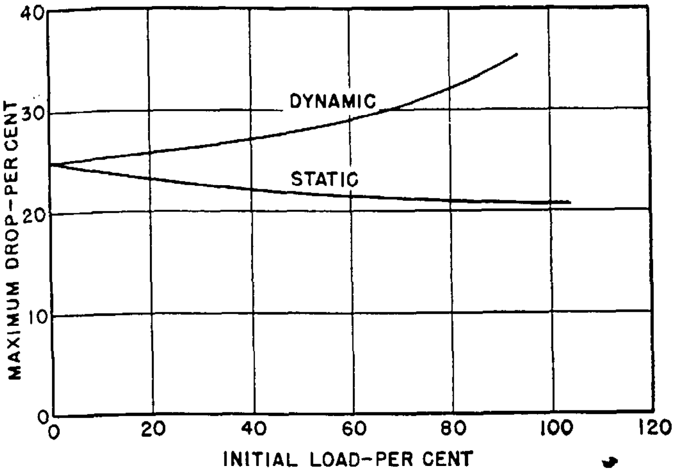


Fig. 48—Effect of type (whether dynamic or static) and initial load, assumed at 0.80 power factor, upon the maximum voltage drop when 100-percent low-power-factor load is suddenly applied to an a-c generator.

comparison with this natural period. The subtransient component is likewise so small that its effects can be neglected. There remains then only the transient components, those components associated with the time constants of the field winding, that are important.

25. Representation of Machine

The transient stability problem is primarily concerned with the power-angle relations during system swings following a disturbance. Because of the dissymmetry of the two axes, it is necessary theoretically to take this dissymmetry into consideration. However, in most cases an impedance is in series externally to the machine so that the difference in reactances in the two axes becomes a smaller proportion of the total reactance. The results of calculations presented in Chap. 13 show that for most practical purposes it is sufficiently accurate to *represent the unsymmetrical machine with a symmetrical machine having the same* x_d'.

In spite of the close agreement of salient-pole with cylindrical-rotor results, a few cases arise for which it is necessary to use salient-pole theory. Relations for calculating the power output have been given in Secs. 16 and 17 and for computing the change in internal voltages in Sec. 22(a). It is shown in the latter section that if the exciter is of the quick-response type, the *voltage* e_d' can, for all practical purposes, be regarded as constant. Methods for the inclusion of these factors into the stability calculations have also been treated in Chap. 13.

A knowledge of the *inertia constant, H*, is a requisite for the determination of the acceleration and deceleration of the rotor. It represents the stored energy per kva and can be computed from the moment of inertia and speed by the following expression

$$H=\frac{0.231\,WR^2(\text{rpm})^2 10^{-6}}{\text{kva}} \qquad (93)$$

where H = Inertia constant in kw-sec. per kva.
WR^2 = Moment of inertia in lb-ft^2.

Further consideration of this constant is given in Part XIII of this chapter.

26. Network Calculator Studies

For most problems the synchronous machine can be represented by its transient reactance and a voltage equal to that behind transient reactance. For the rare case for which salient-pole theory is required, the following procedure can be followed. It is impossible to set up the two reactances in the two axes by a single reactor, but if the reactance, x_q, is used and a new voltage, e_{qd}, introduced as representing the internal voltage, both position of the rotor and the variations in e_d' can be carried through quite simply.

Fig. 49 shows a vector diagram similar to Fig. 14 in which e_{qd} is included. This voltage is laid off along e_d and e_d' and terminates at the point a. The reading of power at e_{qd} is the same as the actual output of the machine. As the exciter voltage changes e_d' and e_{qd} likewise change.

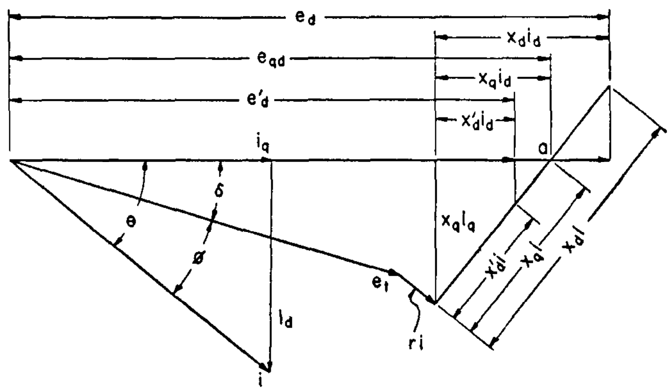


Fig. 49—Construction of e_{qd} for network calculator studies.

The incremental changes in e_{qd} can be obtained as follows. From Fig. 49 it is evident that at any instant

$$e_{qd}=e_d'+(x_q-x_d')i_d \qquad (94)$$

From Eq. (52)

$$\frac{de_d'}{dt}=\frac{1}{T'_{do}}(e_x-e_d)$$

and

$$\Delta e_d'=\frac{de_d'}{dt}\Delta t=\frac{1}{T'_{do}}(e_x-e_d)\Delta t \qquad (95)$$

where $\Delta e_d'$ is the increment of e_d' in the increment of time Δt. From Fig. 49 there results also that

$$e_d=e_{qd}+(x_d-x_q)i_d \qquad (96)$$

so that

$$\Delta e_d'=\frac{1}{T'_{do}}\Big[e_x-e_{qd}-(x_d-x_q)i_d\Big]\Delta t. \qquad (97)$$

In network calculator studies of system stability, e_x, e_{qd}, and i_d are known at any instant. From Eq. (94) it is evident that the increment of e_{qd} is equal to the increment in e_d'. Thus

$$\Delta e_{qd}=\frac{1}{T'_{do}}\Big[e_x-e_{qd}-(x_d-x_q)i_d\Big]\Delta t \qquad (98)$$

This method can be applied regardless of the number of machines involved in the study.

To obtain the initial value of e_{qd}, calculate e_d' from the steady-state conditions before the disturbance. e_d' is the quantity which remains constant during the instant representing the change from one operating condition to another. The proper e_{qd} is obtained by changing the magnitude of e_{qd} until Eq. (94) is satisfied.

To include the effect of saturation, break the reactance x_q, which represents the machine, into two components x_p and (x_q-x_p), the latter being next to the voltage e_{qd}. The voltage at the junction of these two reactances is e_p, the voltage behind x_p. The effect of saturation will be included by adding the saturation factor s taken from the no-load saturation curve (see Fig. 17) for e_p, to the excitation obtained by neglecting saturation. This corresponds to method (*d*) of Sec. 3 for steady-state conditions. Eq. (98) then becomes

$$\Delta e_{qd}=\frac{1}{T'_{do}}\left[e_x-e_{qd}-(x_d-x_q)i_d-s\right]\Delta t. \qquad (99)$$

27. Armature Resistance

For most stability studies the loss associated with the resistance of the armature is so small as to be negligible. The exception to this rule is the case for which a fault occurs near the terminals of a generator.

The losses in an a-c generator during a three-phase short circuit can be large enough to affect significantly the rate at which the rotor changes angular position. This is of particular importance for stability studies. Two of the most important factors determining this effect are the location of the fault and the value of the negative-sequence resistance. The latter is difficult of evaluation particularly for turbo-generators—the type of machine in which the effect is greatest. One must rely almost entirely upon calculations, which are extremely complicated. For a-c board studies of system stability it is convenient to represent the machine losses by means of a resistance placed in series in the armature. The value of this resistance should be chosen so that its loss, with the reactance of the machine represented by x_d', be equivalent to that of the machine under actual conditions. An approximate evaluation of this equivalent resistance will be developed for a turbo-generator.

Let the initial value of the subtransient component of short-circuit current be designated, i''. The components of the unidirectional current have a *maximum* value $\sqrt{2}i''$ and are related in the three phases in a manner as discussed in Sec. 8. The sum of the unidirectional components in all three phases produce an essentially sinusoidal wave of *mmf* that is stationary with respect to the armature. This stationary *mmf* develops a flux that in turn generates currents having a frequency of 60 cps in the rotor. This effect is similar to that produced by negative-sequence currents in the armature except that the latter produce a sinusoidal *mmf* wave that rotates at a speed corresponding to 60 cps in a direction opposite to the rotation of the shaft and ultimately generates circulating currents in the rotor having a frequency of 120 cps. The magnitudes of the *mmf* waves in the two cases are equal for the same *crest* values of unidirectional and negative-sequence currents. The crest value of the negative-sequence current, i_2, is $\sqrt{2}i_2$ and the crest value of i'' is $\frac{\sqrt{2}}{x_d''}$.

In the case of negative-sequence currents, part of the loss is supplied by the shaft and part is supplied through the armature. The loss associated with the circulating currents in the rotor as developed in Section 15 is *approximately* equal to $2(r_2-r_1)i_2^2$. Assuming for the moment that the loss varies as the square of the current and neglecting the differences due to the frequencies in the two cases, the loss for the unidirectional components of current is

$$\left(\frac{\frac{\sqrt{2}}{x_d''}}{\sqrt{2}i_2}\right)^2 2(r_2-r_1)i_2^2 \text{ or } \frac{2(r_2-r_1)}{(x_d'')^2}.$$

Actually, however, the loss varies more nearly as the 1.8 power of the current so that the expression becomes

$$\frac{2(r_2-r_1)}{(x_d'')^{1.8}}.$$

Now considering the effect of frequencies. Since the depth of current penetration varies inversely as the square root of the frequency, the resistance varies directly as the square root of the frequency. The loss for the unidirectional component is then

$$\frac{2(r_2-r_1)}{\sqrt{2}(x_d'')^{1.8}} \text{ or } \frac{\sqrt{2}(r_2-r_1)}{(x_d'')^{1.8}}. \qquad (101)$$

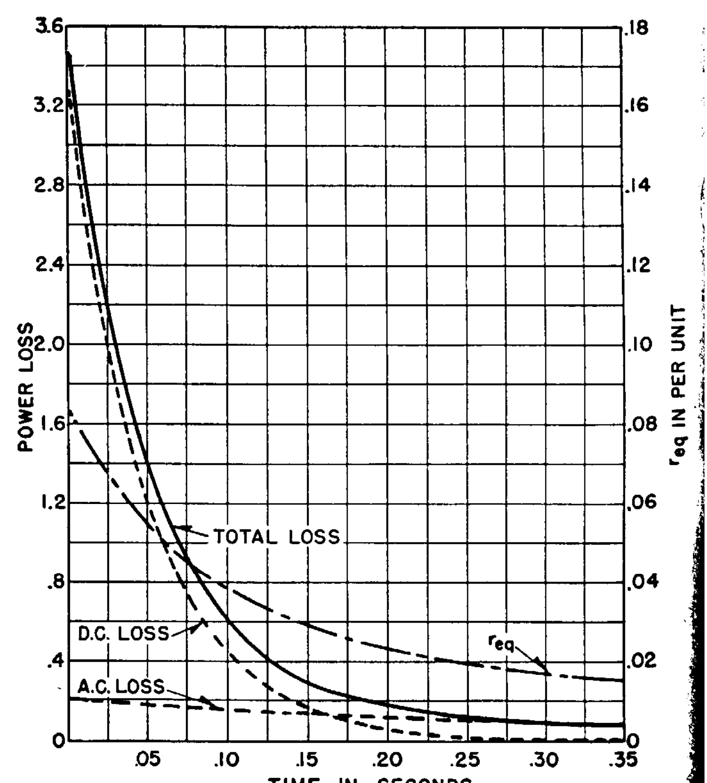


Fig. 50—Development of r_{eq} of a turbo-generator for the condition of a three-phase short circuit across the terminals of the machine for various duration of the short circuit.

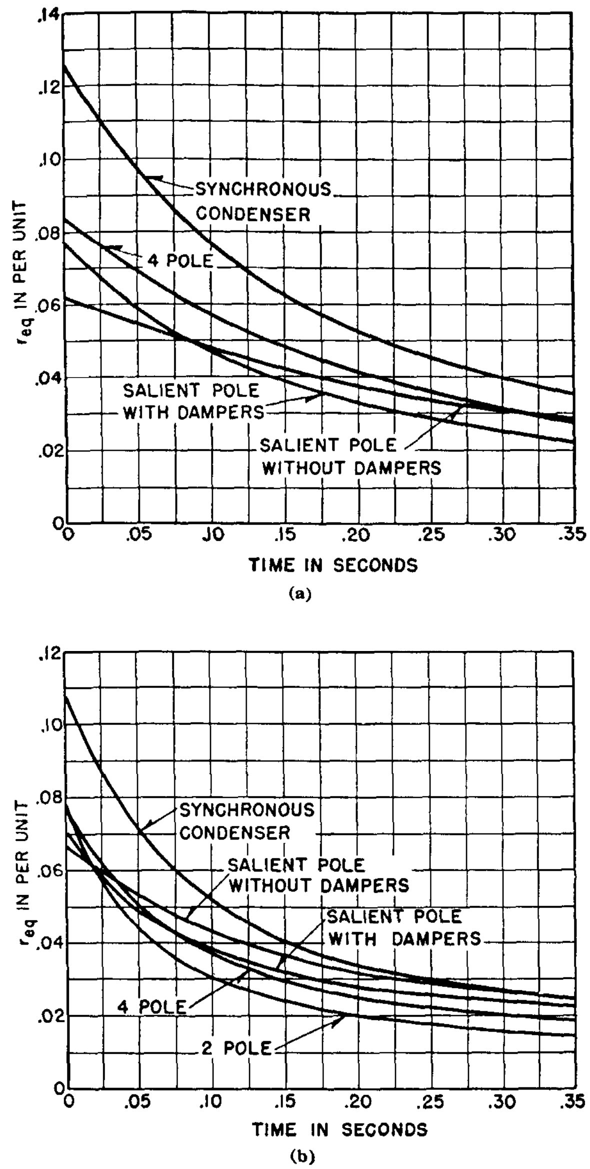


Fig. 51—Typical equivalent resistance, r_{eq}, for different types of machines.

(a) for three-phase short circuit across the terminals used
(b) for three-phase short circuit across the terminals of a series-connected transformer of 10 percent impedance.

Since the unidirectional current decreases exponentially with a time constant T_a, the loss as a function of time is

$$\frac{\sqrt{2}(r_2-r_1)\epsilon^{-\frac{1.8t}{T_a}}}{(x_d'')^{1.8}}. \tag{102}$$

In addition to the losses associated with the unidirectional current, the load losses as reflected by r_1 can also be significant for a three-phase fault across the terminals. Neglecting the sub-transient component, the a-c component of short-circuit for a three-phase short circuit from no-load is

$$\left[\left(\frac{1}{x_d'}-\frac{1}{x_d}\right)\epsilon^{-\frac{t}{T_d'}}+\frac{1}{x_d}\right]. \tag{103}$$

The loss associated with this current is

$$r_1\left[\left(\frac{1}{x_d'}-\frac{1}{x_d}\right)\epsilon^{-\frac{t}{T_d'}}+\frac{1}{x_d}\right]^2. \tag{104}$$

To form an idea of the order of magnitudes of these losses, let

$x_d''=0.09$. $T_a=0.09$.
$x_d'=0.15$. $T_d'=0.6$.
$x_d=1.25$.
$r_2=0.035$.
$r_1=0.005$.

The results of the calculations are shown in Fig. 50. The upper dashed curve is the loss associated with the unidirectional component and the lower dashed curve the load losses. The full line represents the total losses. The current flowing in the generator as represented on the board is constant and equal to $\frac{1}{x_d'}$. The equivalent resistance, r_{eq}, to be inserted in series with x_d' must be such that the integrated loss over any interval must be the same as that in Fig. 50. The dot-dash curve in Fig. 50 gives the values of r_{eq} obtained by this method.

Figure 51 gives similar values of r_{eq} for other types of machines. The curves in Fig. 51(a) were calculated for short circuits at the terminals of the machines, those in Fig. 51(b) are for three-phase short circuits across the terminals of a transformer connected in series with the machine.

VIII. UNBALANCED SHORT CIRCUITS ON MACHINES WITHOUT DAMPER WINDINGS

Because of the dissymmetry of salient-pole machines without damper windings, the armature currents at times of three-phase short-circuits, as shown in Sec. 12, contain second-harmonic components. For unsymmetrical short-circuits, such as from terminal-to-terminal, the wave forms of currents and voltages become even more complex. Both odd and even harmonics are present.

28. Terminal-to-Terminal Short Circuit

In particular consider a salient-pole machine in which saturation is neglected and which is operating at no load to which a short-circuit is suddenly applied across two terminals. The short-circuit current[5] in these phases is then

$$i=\frac{\sqrt{3}I_f[\sin(2\pi ft+\phi_0)-\sin\phi_0]}{(x_q+x_d')+(x_q-x_d')\cos 2(2\pi ft+\phi_0)} \tag{105}$$

in which ϕ_0 indicates the phase position during the cycle at which the short-circuit occurred.

It will be observed that this can be resolved into two components

$$\text{First: } \frac{\sqrt{3}\,I_f\sin(2\pi ft+\phi_0)}{(x_q+x_d')+(x_q-x_d')\cos 2\,(2\pi ft+\phi_0)} \tag{106}$$

$$\text{Second: } \frac{\sqrt{3}\,I_f\sin\phi_0}{(x_q+x_d')+(x_q-x_d')\cos 2\,(2\pi ft+\phi_0)} \tag{107}$$

The first component is shown in Fig. 52(a) for a typical machine and consists of odd harmonics only. The second

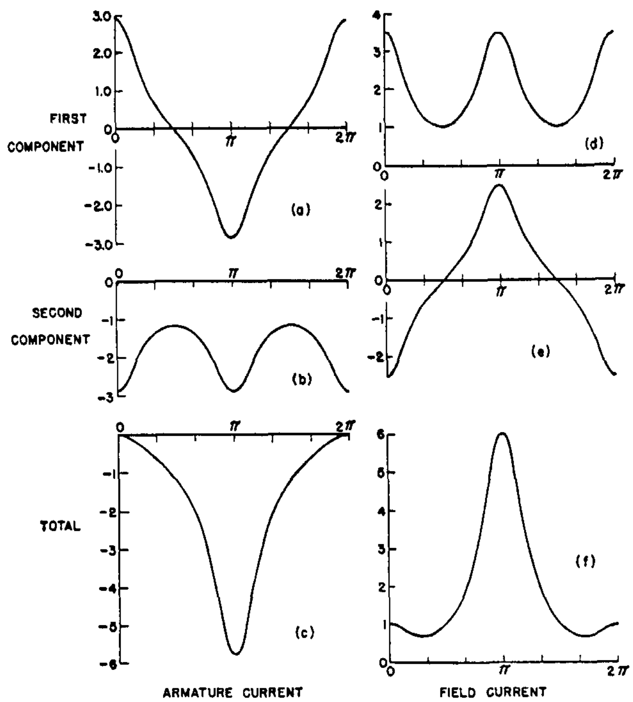


Fig. 52—Armature current and field current in a synchronous machine when a terminal-to-terminal short circuit is suddenly applied.

$x_d'=0.30$ $x_d=1.1$ $x_q=0.75$ $\phi_o=90°$

component is shown in Fig. 52(b) for $\phi_0=+90°$ and consists of even harmonics only. The latter component is dependent upon the instant during the cycle at which the short-circuit occurs and may vary anywhere between the values given and the negative of those values in accordance with the coefficient, sin ϕ_0. Figure 52(c) gives the total current, the sum of Figs. 52(a) and 52(b).

The units chosen are the p.u. in which for the machine operating at no-load at rated circuit voltage I_f would be equal to 1.0 and in this case the current i is given in terms of *crest* magnitude of rated phase current.

The components of armature current shown in Figs. 52(a) and 52(b) have associated with them the field currents shown in Figs. 52(d) and 52(e), respectively, the former consisting only of even harmonics and the latter only of odd harmonics. In Fig. 52(f) is shown the total field current. The average magnitude of this current is equal to

$$\frac{x_d+\sqrt{x_q x_d'}}{x_d'+\sqrt{x_q x_d'}} I_f.$$

The odd-harmonic component of field current and its associated even harmonic in the armature decay to zero with time. The even harmonics of the field and their associated odd harmonics of armature current decay to constant, steady-state amounts. Their initial values are in excess of their steady-state magnitudes by the amount the average of I_f is in excess of its steady-state amount, I_f. The steady-state value of i is then equal to the initial amount of the odd-harmonic component multiplied by

$$\frac{x_d'+\sqrt{x_q x_d'}}{x_d+\sqrt{x_q x_d'}}.$$

Thus

$$i_{\text{steady-state}}=\sqrt{3} I_f \frac{x_d'+\sqrt{x_q x_d'}}{x_d+\sqrt{x_q x_d'}} \cdot \frac{\sin(2\pi f t+\phi_0)}{[(x_q+x_d')+(x_q-x_d')\cos 2(2\pi f t-\phi)]}. \tag{108}$$

With the assistance of Fig. 52 it will be seen from Eq. (105) that the maximum amount of the odd harmonic component is equal to $\frac{\sqrt{3}\, I_f}{2x_d'}$. The maximum value of the total current is dependent upon the instant during the cycle at which short-circuit occurs and reaches a maximum of $\frac{\sqrt{3}\, I_f}{x_d'}$.

Assuming no decrement for either the odd or even harmonics

$$i_{\text{rms (even)}}=\frac{\sqrt{3}}{2}\frac{I_f \sin\phi_0}{\sqrt{x_q x_d'}}\sqrt{\frac{1+b^2}{1-b^2}} \tag{109}$$

$$i_{\text{rms (odd)}}=\sqrt{3}\frac{I_f}{x_d'+\sqrt{x_q x_d'}}\frac{1}{\sqrt{1-b^2}} \tag{110}$$

$$b=-\frac{\sqrt{x_q/x_d'}-1}{\sqrt{x_q/x_d'}+1}. \tag{111}$$

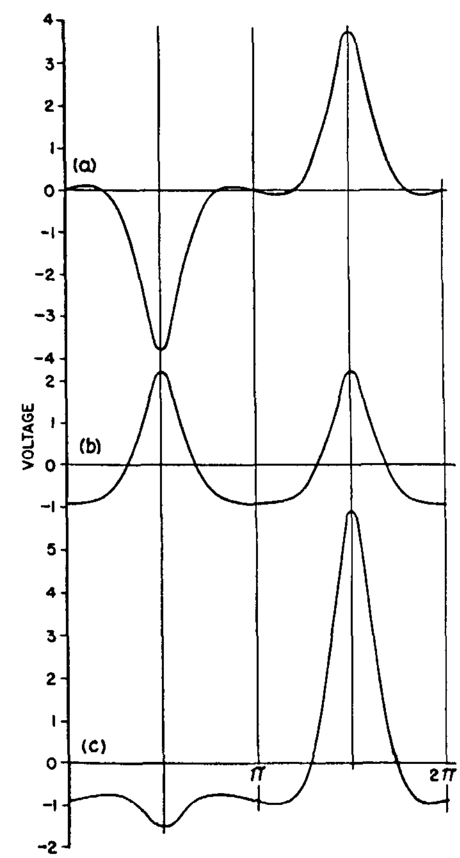


Fig. 53—Wave form of voltage across terminals of a water-wheel generator without damper windings for a terminal-to-terminal short circuit from no-load. $x_q/x_d'=2.5$.

(a) Initial value of odd harmonic component (decays slowly);
(b) initial value of even harmonic component for sin $\phi_0=1$ (decays rapidly). Its magnitude varies between that given and its negative depending upon the point during the cycle at which short circuit occurs. It may be zero.
(c) Total initial value for sin $\phi_0=1$

The rms total current is equal to the square root of the sum of the squares of those components. It must be remembered that the unit of current is the *crest of rated terminal current*. When expressed in terms of the *rated rms current* the above figures must be multiplied by $\sqrt{2}$.

The voltage from the short-circuited terminals to the free terminal, neglecting decrements, is equal to

$$\begin{aligned} e_a - e_b = e_{ab} = &-3I_f K[\sin(2\pi ft+\phi_0)+3b\sin 3(2\pi ft+\phi_0) \\ &+5b^2\sin 5(2\pi ft+\phi_0)+\ldots] \\ &+3I_f\sin\phi[2b\cos 2(2\pi ft+\phi_0) \\ &+4b^2\cos 4(2\pi ft+\phi)+\ldots] \end{aligned} \tag{112}$$

in which

$$K=\frac{\sqrt{x_q/x_d'}}{\sqrt{x_q/x_d'}+1} \tag{113}$$

and b has its previous significance.

Like the short-circuit current this voltage can likewise be resolved into two components that together with the total voltage are plotted in Fig. 53. The maximum possible voltage, that which occurs when $\sin\phi_0$ is equal to unity, is

$$e_{ab(\text{maximum for max. flux linkages})}=\frac{3}{2}I_f\left(2\frac{x_q}{x_d'}-1\right) \tag{114}$$

When $\sin\phi_0=0$, the even harmonic component is equal to zero and for this case the maximum voltage is

$$e_{ab(\text{maximum for minimum flux linkages})}=\frac{3}{2}I_f\frac{x_q}{x_d'} \tag{115}$$

The corresponding line-to-neutral voltages for the terminal-to-terminal short-circuit are ⅔ of the above figures. In all of these expressions the *crest* value of rated line-to-neutral voltage has been used as a base. When the *rms* figure is used, the above quantity must be multiplied by $\sqrt{2}$.

For a terminal-to-neutral short circuit, neglecting decrements, the short-circuit current is

$$i=\frac{3I_f[\cos(2\pi ft+\phi_0)-\cos\phi_0]}{(x_d'+x_q+x_0)+(x_d'-x_q)\cos 2(2\pi ft+\phi_0)} \tag{116}$$

29. Unsymmetrical Short Circuits Under Capacitive Loading

When a salient-pole machine without damper windings is loaded by a highly capacitive load,[12,13] there is danger,

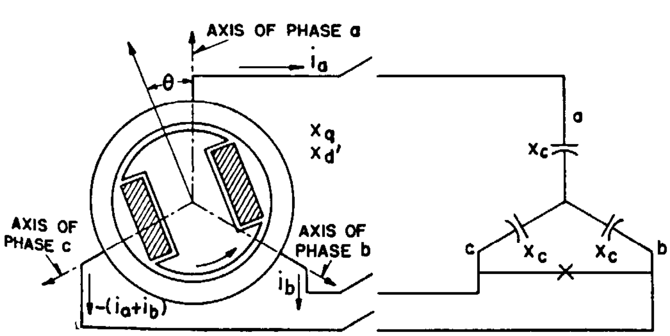


Fig. 54—Schematic diagram of a three-phase, salient-pole alternator to which a three-phase bank of capacitors and a terminal-to-terminal short circuit are applied simultaneously.

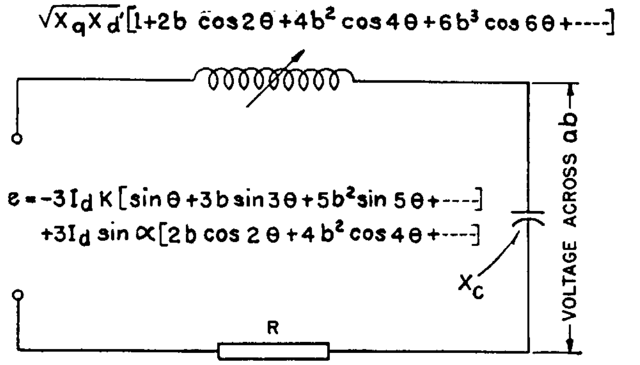


Fig. 55—Equivalent circuit to which Fig. 54 may be reduced.

$$b=-\frac{\sqrt{x_q/x_d'}-1}{\sqrt{x_q/x_d'}+1} \qquad k=\frac{\sqrt{x_q/x_d'}}{\sqrt{x_q/x_d'}+1}$$

at times of unbalanced short circuit, that resonance occur between the reactance of the machine and the load with the possibility that dangerously high voltages might result. Considering a purely capacitive load such as an unloaded transmission line, the schematic diagram is shown in Fig. 54 and the equivalent circuit in Fig. 55 for the condition of a terminal-to-terminal short circuit. The emf applied to the circuit is equal to the open-circuit voltage for the same short-circuit condition. The oscillographic results of tests made on a particular machine as terminal-to-terminal short circuits are applied for different amounts of connected capacitance are shown in Fig. 56. Resonance will occur near points for which the quantity $\frac{x_c}{\sqrt{x_d'x_q}}=n^2$, where n represents the integers 1, 2, 3, etc., and also the order of the harmonic. The nature of this resonance phenomenon is illustrated more clearly by the curve of Fig. 57, in which is plotted the maximum voltage during short-circuit in per unit.

To orient one's self with regard to the length of line involved in these considerations, the figure in miles which appears below each oscillogram of Fig. 56 represents approximately the length of single-circuit 66- or 220-kv transmission line that, with a generator having the characteristics of the one used in the test, is required to satisfy the given value of $x_c/\sqrt{x_d'x_q}$. These figures were arrived at by assuming a generator capacity of 25 000, 75 000, and 200 000 kva for 66-, 132-, and 220-kv lines, respectively. For smaller machines the length will decrease in proportion.

The possibility of the existence of such resonant conditions can be determined for other types of loads and other types of faults by setting up the network for the system and replacing the machine by the reactance $\sqrt{x_q x_d'}$. This circuit should be set up for the positive-, negative-, and zero-sequence networks and the networks connected in accordance with the rules of symmetrical components. Any condition for which the impedance as viewed from the machine is zero or very small should be avoided.

Since the danger of these high voltages arises from the dissymmetry of the machines, it can be eliminated effectively by the installation of damper windings. Fig. 58 presents oscillographic evidence of the voltages existing for machines equipped with different types of dampers as

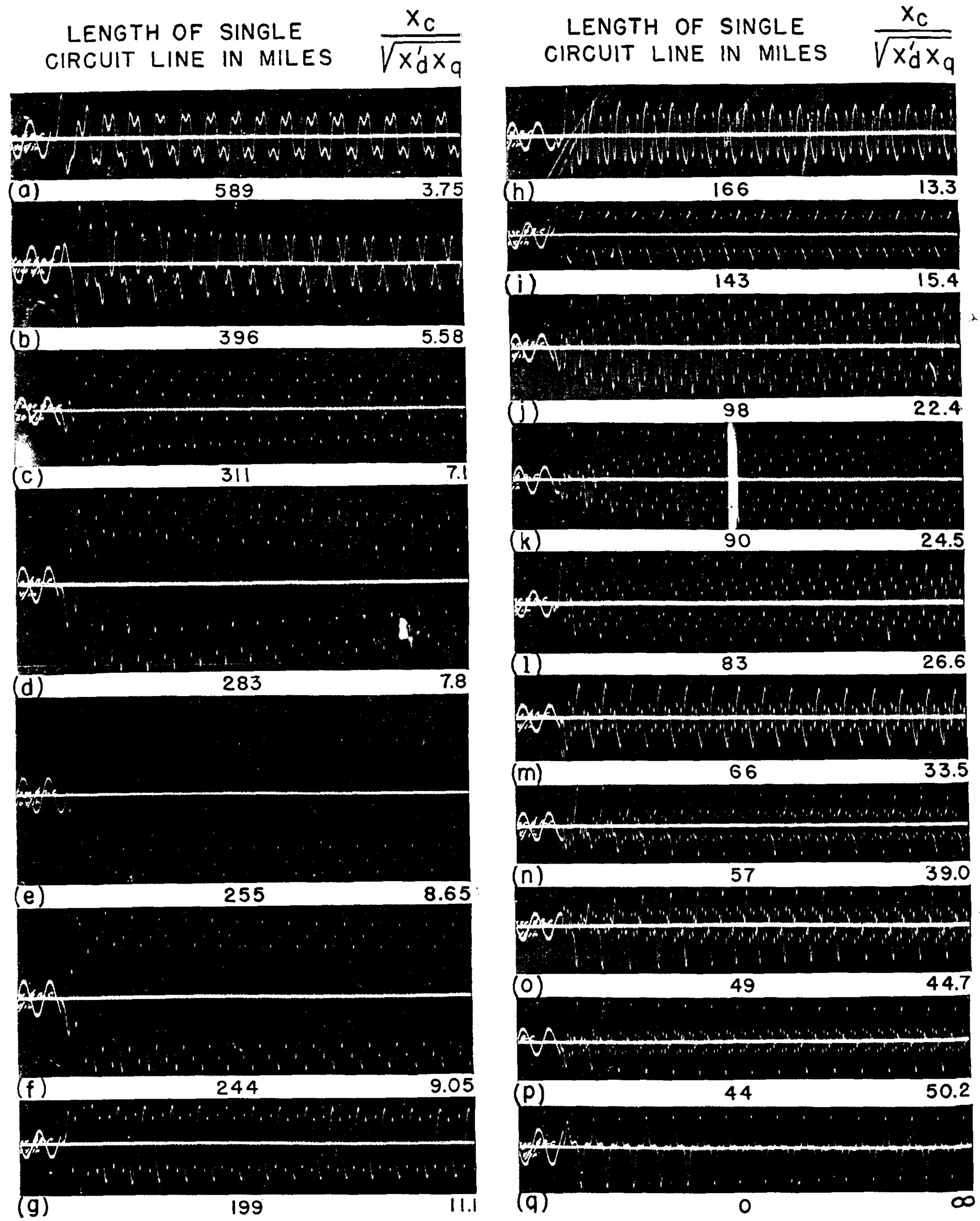


Fig. 56—Effect upon the terminal voltage of varying the shunt capacitive reactance when a terminal-to-terminal short circuit is applied to a machine without damper windings.

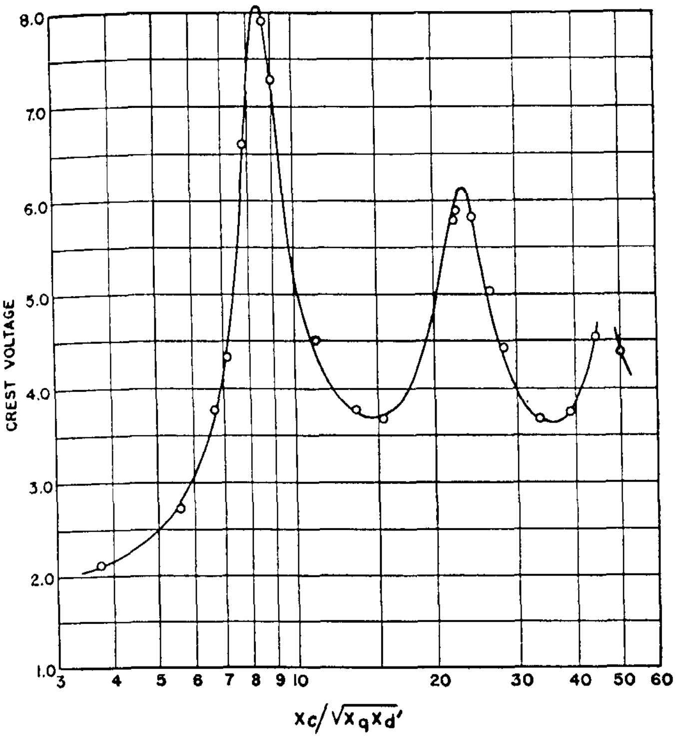


Fig. 57—Experimental values of crest voltages (twelfth cycle) from terminal a to b when switch in Fig. 54 is closed. Unit of voltage is crest of terminal-to-terminal voltage before short circuit. $x_q/x_d' = 2.2$. Machine without damper winding.

terminal-to-terminal short circuits and capacitive reactances are applied simultaneously. While a continuous or connected damper winding is most effective, a non-connected damper winding having a ratio of $\frac{x_q''}{x_d''}$ equal to at least 1.35 will be found adequate for practically all purposes.

IX. DAMPER WINDINGS

The addition of copper damper windings to machines effectively simplifies the characteristics of the machines as viewed externally in that harmonic effects are largely eliminated. However, the addition of other possible circuits for current flow complicates the internal calculations. The influence of dampers can in most cases be evaluated in terms of their effect[14] upon the subtransient reactances in the two axes.

30. Types of Damper Windings

Damper windings are of several general types.

Connected Dampers—These consist essentially of windings similar to a squirrel-cage or an induction motor. They are continuous between poles as shown in Fig. 59 in which (a) shows the connection between poles for a slow-speed machine and (b) shows the additional bracing required in the form of an end ring for higher speed machines. In this type of damper, x_q'' and x_d'' have nearly the same magnitudes.

Non-connected Dampers—The dampers in each pole face are independent from those in adjacent poles, as shown in Fig. 60. They are somewhat cheaper than connected dampers but at the expense of no longer being able to make x_q'' and x_d'' equal.

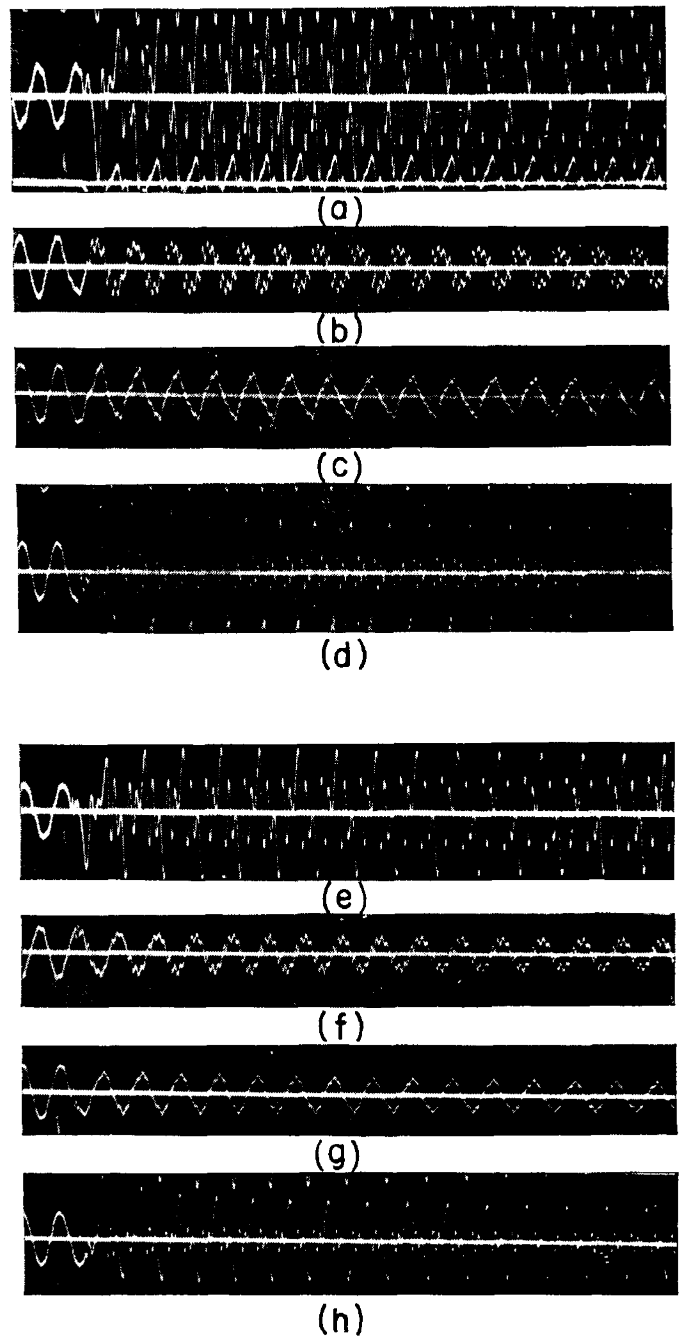


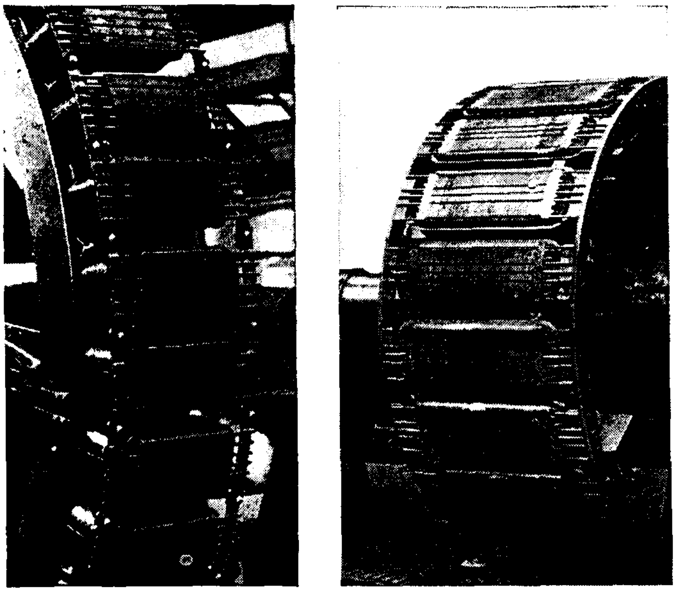
Fig. 58—Effect of damper windings.

Terminal-to-terminal short circuit:

(a) No dampers.
(b) Connected copper damper.
(c) Connected high resistance damper.
(d) Non-Connected copper damper.

Terminal-to-neutral short circuit:

(e) No damper.
(f) Connected copper damper.
(g) Connected high resistance damper.
(h) Non-connected copper damper.



(a) (b)

Fig. 59—Connected damper windings:

(a) Slow-speed machine.
(b) High-speed machine.

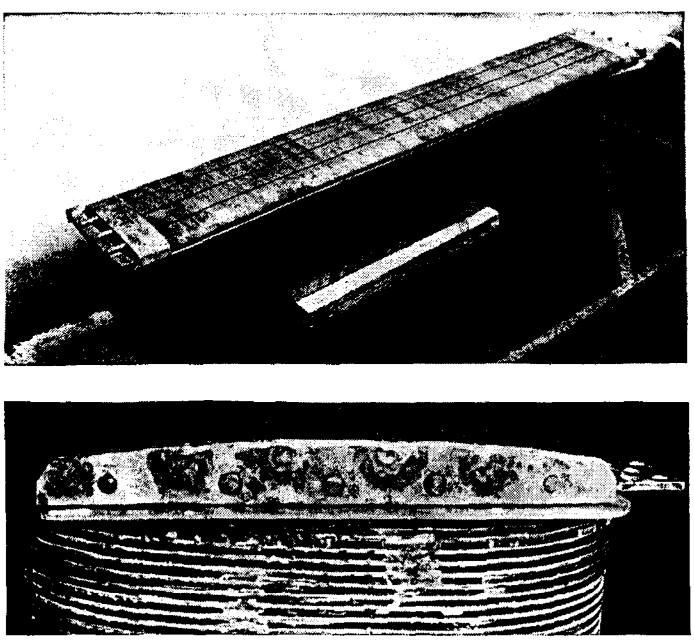


Fig. 60—Two types of non-connected damper windings.

Special Dampers—In this classification fall such dampers as double-deck windings, which are in effect a double winding, one of high resistance and low reactance and the other of low resistance and high reactance. The principal uses of this type are in motors where the combination provides better starting characteristics. At low speeds the high reactance of the low-resistance winding forces the current to flow through the high-resistance winding, which produces a high torque. At higher speeds the low-resistance winding becomes effective. Another type of special winding is one that is insulated from the iron and connected in series to slip rings. By connecting a variable resistor externally to the slip rings the starting characteristics can be varied at will.

The general characteristics of damper windings will be discussed under the following heads.

31. Balancing Action and Elimination of Voltage Distortion

One of the earliest needs for damper windings arose from the use of single-phase generators and, later, phase balancers. Such machines if unequipped with damper windings have characteristics which resemble closely those of a three-phase machine without damper windings when a single-phase load is drawn from it. Voltage distortion similar to that discussed under unbalanced short-circuits occurs. In addition, if this condition persists the currents that flow in the body of the pole pieces, produce excessive heating. The addition of damper windings provides a low-resistance path for the flow of these currents and prevents both wave distortion and excessive heating. Because of the steady character of the load, damper windings in single-phase machines and phase balancers must be heavier than those in three-phase machines.

The best criteria of a polyphase machine to carry unbalanced load are its negative-sequence reactance and resistance. The former reflects its ability to prevent unbalancing of the voltage and the latter its ability to carry the negative-sequence current without undue heating of the rotor. These properties are particularly important for such fluctuating loads as electric furnaces. Not only do the dampers reduce voltage unbalance but also reduce wave form distortion.

32. Negative-Sequence Reactance and Resistance

As discussed previously the negative-sequence reactance and resistance of a machine are both affected by the damper windings. Table 2 shows the effect of different types of windings upon a 100-kva generator[12] and Table 3 upon a 5000-kva synchronous condenser.[14] Both of these tables represent test results.

TABLE 2—CONSTANTS OF A SYNCHRONOUS GENERATOR AS AFFECTED BY TYPE OF DAMPER WINDING (100 KVA, 2300 VOLTS, 25.2 AMPERES)

Type	x_d'	x_q	$\sqrt{x_d'x_q}$	$\frac{x_q}{x_d'}$	r_{ad}''	r_{aq}''
No damper....	0.260	0.577	0.388	2.22	0.028	0.105
	x_d''	x_q''	$\sqrt{x_d''x_q''}$	$\frac{x_q''}{x_d''}$		
Connected Copper.....	0.157	0.146	0.151	0.93	0.036	0.047
Connected Everdur....	0.171	0.157	0.164	0.92	0.063	0.111
Non-connected Copper.....	0.154	0.390	0.245	2.53	0.037	0.113

33. Damping Effect

In the early days when prime movers consisted mostly of reciprocating engines the pulsating character of the

TABLE 3—CONSTANTS OF A SYNCHRONOUS CONDENSER AS AFFECTED BY TYPE OF DAMPER WINDING (5000 KVA, 4000 VOLTS, 721 AMPERES)

Type	r_2		$x_2 = \frac{1}{2}(x_d'' + x_q'')$	
	Test	Calculated	Test	Calculated
No damper	0.045	0.040	0.75	0.69
Connected copper	0.026	0.029	0.195	0.215
Connected brass	0.045	0.044	0.195	0.215
Connected Everdur	0.12	0.125	0.20	0.215

torque made parallel operation difficult. This was successfully solved by damper windings in that the damper winding absorbed the energy of oscillation between machines and prevented the oscillations from becoming cumulative. More recently in consideration of the stability problem low-resistance damper windings have been advocated for the same reason. While a low-resistance damper winding will decrease the number of electro-mechanical oscillations following a disturbance this effect in itself is not important[14] in increasing the amount of power that can be transmitted over the system.

The general influence of damper windings, their negative-sequence resistance and reactance, and also their purely damping action, upon the stability problem, is discussed in more detail in Chap. 13.

34. Other Considerations Affecting Damper Windings

Synchronous generators feeding loads through transmission lines having a high ratio of resistance to reactance tend to set up spontaneous hunting.[15] This tendency is greater at light loads than at heavy loads, the criterion at which it tends to disappear being when the angle between the transient internal voltage and the load voltage equals the impedance angle of the connecting impedance. There need not be any periodic impulse, such as the pulsating torque of a compressor, to initiate this phenomenon but it may very well aggravate the condition. Damper windings are very effective in suppressing such inherent hunting conditions and also alleviate hunting produced by periodic impulses, although the latter phenomenon is usually eliminated by altering the natural frequency of the system by changing the fly wheel effect of the generator or motor or both. Synchronous motors connected through high resistance lines or cables also develop spontaneous hunting but not so frequently as they are always provided with a damper winding.

Series capacitors in decreasing the effective series reactance increase the ratio of resistance to reactance and thus tend to increase the likelihood of spontaneous hunting.

In general, where beneficial effects can accrue with the use of damper windings, the benefits are greater for connected or continuous dampers than for non-connected dampers. Mechanically there is no choice as both types can be made equally reliable. The non-connected winding lends itself somewhat easier to the removal of a pole but not to sufficient extent to constitute a consideration in the choice of type to install. A ratio of x_q'' to x_d'' as low as about 1.35 can be obtained with non-connected and 1.1 with connected dampers. Damper windings for which this ratio is greater than 1.35 and less than 1.35 add 2 and 3 percent, respectively, to the price of the machine. In consideration of the many complicated problems involved in the selection of a damper winding it would appear, in view of the low increase in cost of the connected damper, that if any damper winding is thought necessary, the connected type should be used.

X. SELF-EXCITATION OF SYNCHRONOUS MACHINES

When a synchronous machine is used to charge an unloaded transmission line whose charging kva is equal approximately to the kva of the machine, the machine may become self-excited and the voltage rise beyond control. The conditions that must be satisfied for this phenomenon to occur are made manifest by determining the machine characteristics for a constant inductive reactive load.

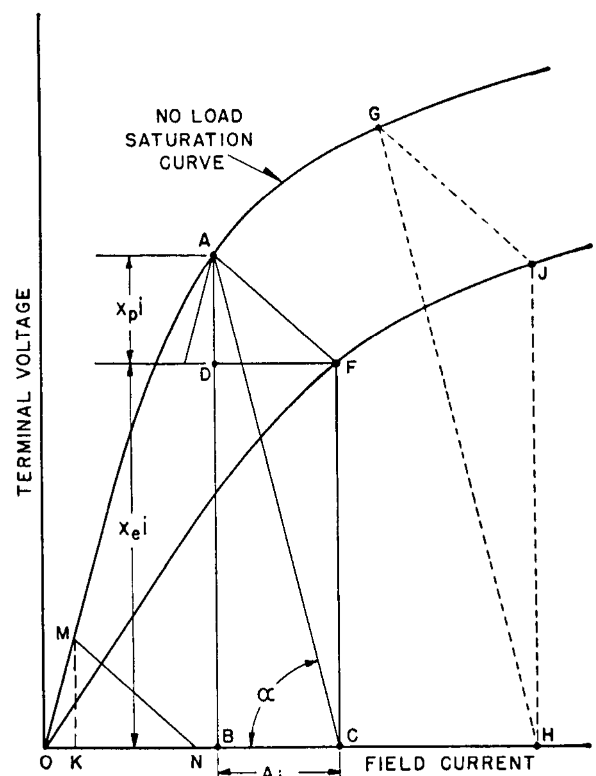


Fig. 61—Construction of regulation curves for induction loading.

In Fig. 61 the line OAG represents the no-load saturation curve. Suppose the machine is loaded with a three-phase reactor equal to x_e ohms per phase. To determine the regulation curve for this impedance, that is, a curve of terminal voltage plotted against field current, proceed as follows: Choose an armature current such that $x_e i$*, the terminal voltage, is approximately rated voltage. This voltage is given by the distance BD in Fig. 61. By adding

*In this discussion, the terminal voltage is regarded as the terminal-to-neutral value. When terminal-to-terminal voltage is used the voltage drops considered will have to be multiplied by $\sqrt{3}$.

to this distance the x_pi drop, DA, the voltage behind Potier reactance denoted by the point A is obtained. The magnetizing current to produce this voltage is given by the distance OB. In addition to this, however, the field current Ai is required to overcome the demagnetizing effect of the armature current. For normal current, Ai is the distance KN in the Potier triangle, OMN. In conclusion, to produce the terminal voltage F, the field current OC is necessary. The triangle BAC is a sort of Potier triangle, in which the Potier reactance is replaced by a reactance equal to (x_p+x_e). Thus by drawing any line HG parallel to CA and GJ parallel to AF, the intersection with the vertical from H determines the terminal voltage for the excitation H.

When the load consists of balanced capacitors having a reactance x_c in which x_c is greater than x_p, the impedance as viewed from the voltage behind Potier reactance is capacitive and the armature current is magnetizing instead of demagnetizing. This case can be treated in a manner similar to that for an inductive-reactance load with some modifications as is shown in Fig. 62. In this figure the distance CF represents the terminal voltage produced by the external drop x_ci. Since the current leads the terminal voltage by ninety degrees the voltage behind Potier reactance for the assumed armature current is found by sub-

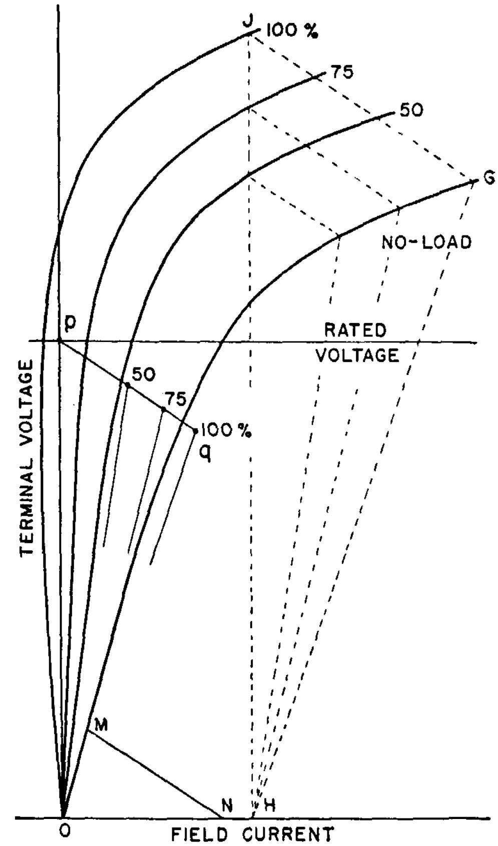


Fig. 63—Regulation curves for constant capacitive load of such values as to give the loads at rated voltage indicated on the curves. HG parallel to 0q. Point q represents excitation and internal voltage, neglecting saturation, to produce rated terminal voltage with 100-percent capacitive current.

tracting the drop x_pi giving the distance CD or BA. To produce this voltage the magnetizing current OB is required but since the armature current is magnetizing to the extent of Ai, the actual field current necessary is only OC. This determines F as a point in the regulation curve. For other field currents such as the point H, draw HG parallel to CA until it intercepts the no-load saturation curve at G. Then draw GJ parallel to AF. The intersection with the vertical from H determines the point J.

Fig. 63 depicts the regulation curves for different sizes of capacitors. The number assigned to each curve represents the percent kva delivered at rated voltage.

The angle α in Fig. 62 is equal to $\tan^{-1}\frac{(x_c-x_p)i}{Ai}$. At zero excitation it can be seen that if this angle is sufficiently small, intersection with the no-load saturation curve is possible, but as α increases a point is finally reached at which intersection is impossible and the solution fails. This signifies that when this point is reached self-excitation does not occur. This critical condition occurs when the slope $\frac{(x_c-x_p)i}{Ai}$ equals the slope of the no-load saturation curve. In discussing the significance of x_d use was made of Fig. 10, where it was pointed out that DA is the current necessary to overcome the demagnetizing effect, Ai, of the armature current. The distance AB is the synchronous

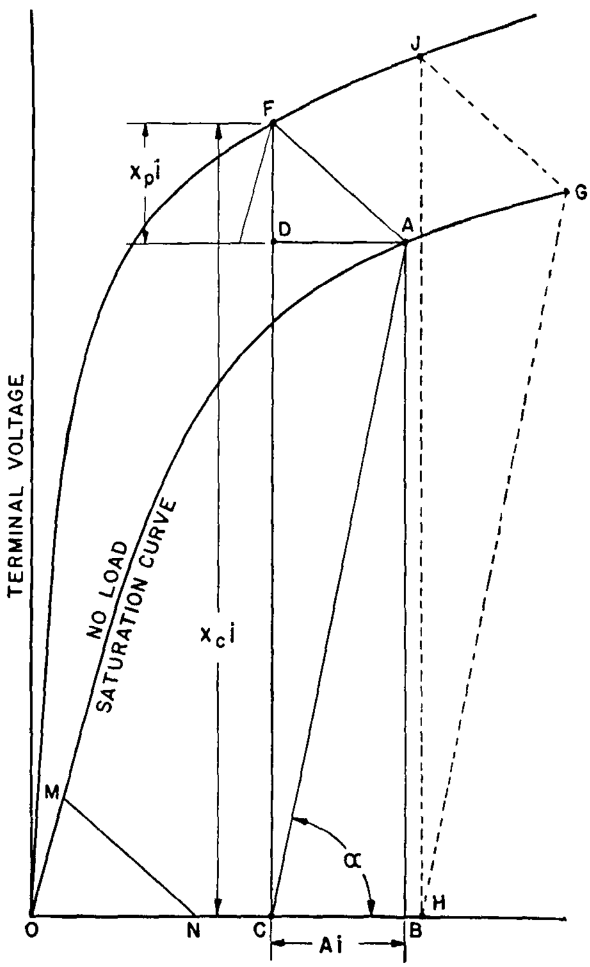


Fig. 62—Construction of regulation curves for capacitive loading.

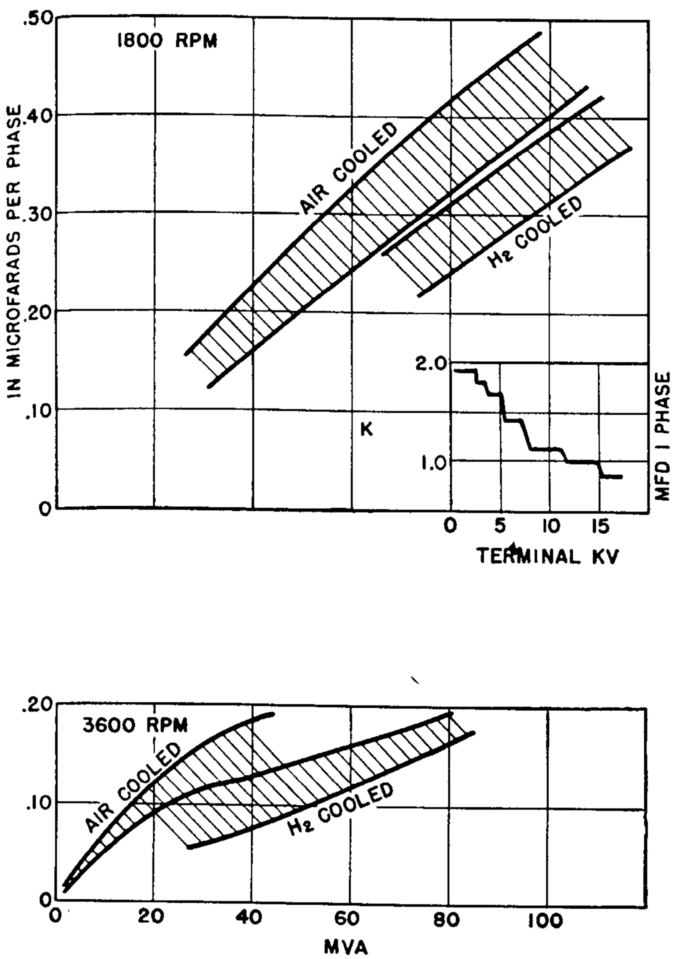


Fig. 64—Capacitance to ground of TURBINE-GENERATOR windings for 13 200-volt machines in microfarads per phase. For other voltages multiply by factor K in insert.

reactance drop $x_d i$ and DC the Potier reactance drop. Thus the slope of the no-load saturation curve is equal to $\frac{x_d i - x_p i}{Ai}$. The condition for self excitation is then that

$$\frac{(x_c - x_p)i}{Ai} < \frac{(x_d - x_p)i}{Ai}$$

or

$$x_c < x_d \tag{117}$$

Stated otherwise, the machine will become self-excited if the kva of the machine as defined by $\frac{E^2}{x_d}$ is less than the charging kva of the line $\frac{E^2}{x_c}$. Since x_d is, except for special cases, of the order of 120 percent, danger may threaten when the charging kva requirements of the line exceed approximately 80 percent of the kva of the machine.

XI. CAPACITANCE OF MACHINE WINDINGS

A knowledge of the capacitance to ground of machine windings is necessary for several reasons, among which are:

(a) Grounding of Generators. This is discussed in considerable detail in the chapter on Grounding. The capacitance to ground of the windings must be known so that the associated resistance can be selected.

(b) System Grounding. The capacitance must be known so that the contribution of this element to the ground current can be determined for single line-to-ground faults. The contribution to the fault current for this condition is equal to $\sqrt{3}\ 2\pi f C_0 E \times 10^{-6}$ where f is the system frequency, E the line-to-line voltage and C_0 the capacitance per phase in microfarads.

(c) System Recovery Voltage. The capacitance of the rotating machines may be an important element in the determination of the system recovery voltage. It is cus-

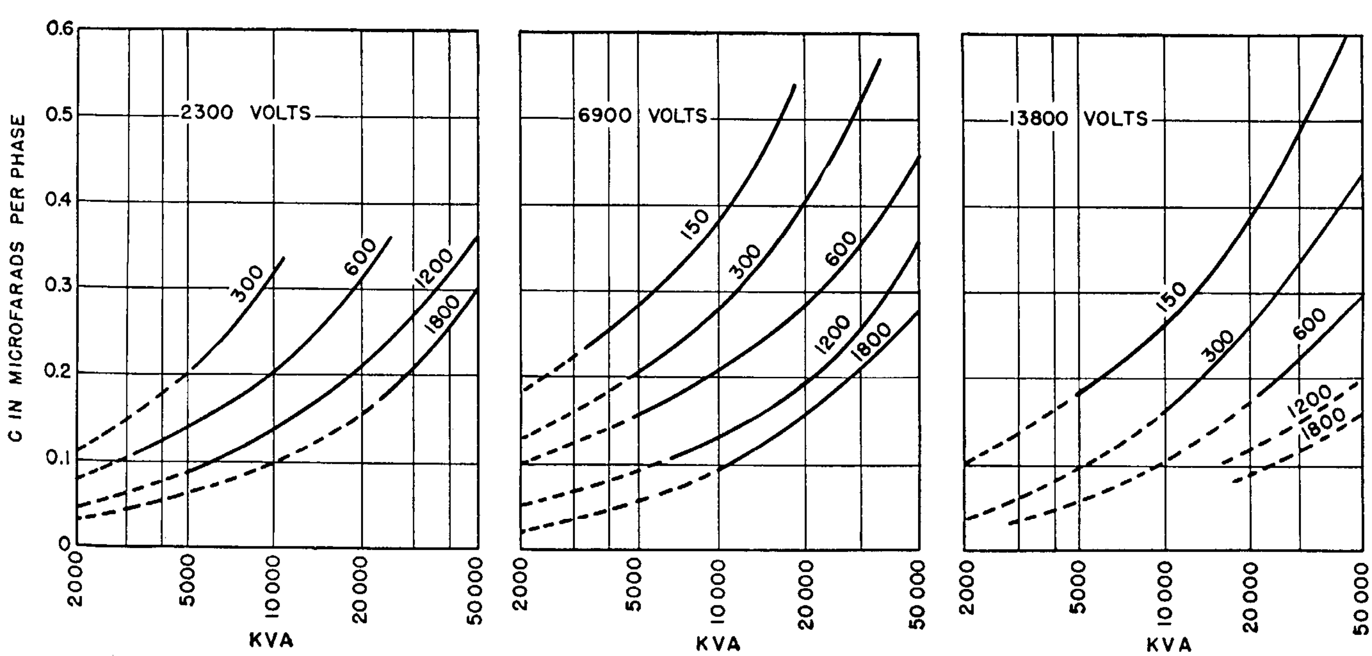


Fig. 65—Capacitance to ground of SALIENT-POLE GENERATORS AND MOTORS in microfarads per phase.

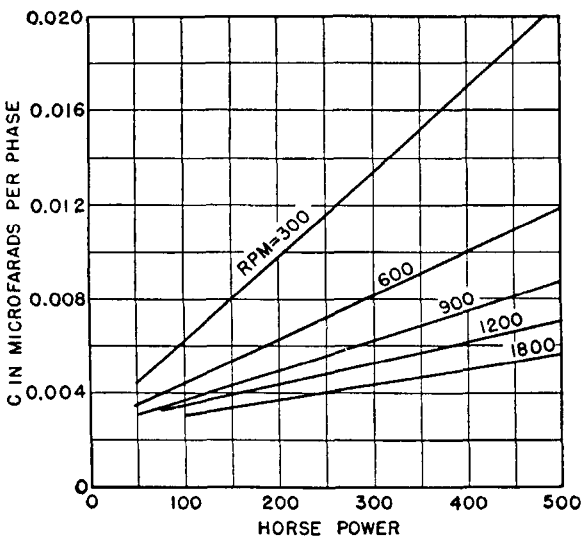


Fig. 66—Capacitance to ground of 2300-volt SYNCHRONOUS MOTORS in microfarads per phase to ground. For voltages between 2300 and 6600, the capacitance will not vary more than ±15 percent from the values for 2300 volt.

tomary to represent the machine capacitance in this work by placing one-half of the total capacitance to ground at the machine terminal. For details of this type of calculation refer to the chapter on Power-System Voltages and Currents During Abnormal Conditions.

(d) Charging Kva. In testing the insulation of machines, particularly in the field, it is sometimes necessary to know the approximate charging kva of the windings so that a transformer of sufficiently high rating can be provided beforehand to do the job. This is required either for normal routine testing, for testing at time of installation or for testing after rewinding. The charging kva per phase is equal to $2\pi f C_0 E^2 \times 10^{-6}$ where C_0 is the

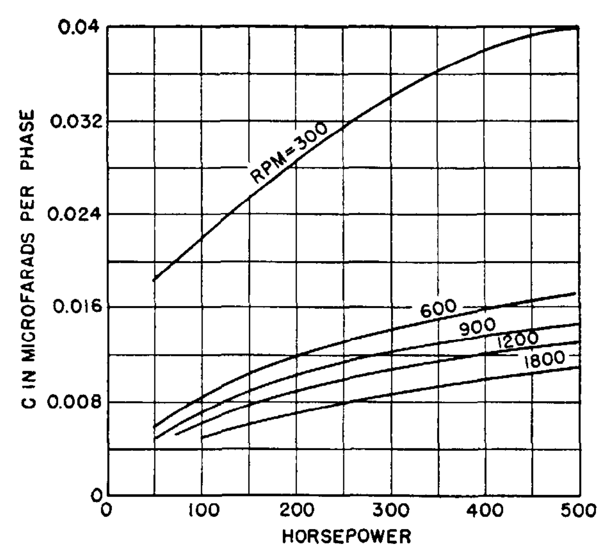


Fig. 67—Capacitance to ground of 2300-volt INDUCTION MOTORS in microfarads per phase. For voltages between 2300 and 6600, the capacitance will not vary more than ±15 percent from the values for 2300 volts.

capacitance per phase to ground in microfarads and E is the applied voltage from winding to ground.

Figures 64 to 67 provide basic data calculated for Westinghouse turbine generators and salient-pole generators and motors. The generator data was obtained from reference 23 and the motor data from some unpublished material of Dr. E. L. Harder. This information should be typical of other machines to within about ± 50 percent. In general, it should be borne in mind that these characteristics vary greatly between machines of different designs. Fortunately, however, not very great accuracy is required for the applications cited above.

XII. NATURAL FREQUENCY OF SYNCHRONOUS MACHINE CONNECTED TO INFINITE BUS

A synchronous machine connected to an infinite bus possesses a natural period of oscillation which is given in the ASA C50—1943 Rotating Electrical Machinery Standards as

$$f_n = \frac{35\,200}{(\text{rpm})}\sqrt{\frac{P_r \times f}{WR^2}} \text{ cycles per minute} \qquad (113)$$

where P_r is the synchronizing power in kw per degree displacement,

f is the system frequency.

When given an angular displacement, the machine oscillates with this frequency and finally subsides unless subjected to periodic impulse of proper magnitude. It is not within the scope of this work to discuss this subject in its entirety, but merely to derive the above expression.

If an incremental displacement $\Delta\theta$ be assumed, the corresponding synchronizing power is

$$\Delta P = P_r \Delta\theta \text{ in kw} \qquad (114)$$

and $\Delta\theta$ is in degrees. From the Stability Chapter it can be seen that the acceleration of the rotor is

$$\alpha = \frac{180\, f}{\text{kva } H}\Delta P \text{ in deg/sec}^2$$

$$= \frac{\pi f}{(\text{kva})\, H}\Delta P \text{ in rad/sec}^2 \qquad (115)$$

where the kva refers to the rating of the machine and H the inertia constant. Substituting H from Eq. (93)

$$\alpha = \frac{\pi 10^6}{0.231}\;\frac{f}{(\text{WR}^2)(\text{rpm})^2}\Delta P \text{ in rad/sec}^2 \qquad (116)$$

and substituting ΔP from Eq. (114)

$$\alpha = \frac{\pi 10^6}{0.231}\;\frac{fP_r}{(WR^2)(\text{rpm})^2}\Delta\theta \qquad (117)$$

$$= -K\Delta\theta. \qquad (118)$$

$$K = -\frac{\pi 10^6}{0.231}\;\frac{fP_r}{(WR^2)(\text{rpm})^2}. \qquad (119)$$

The sign of P_r is actually negative as an increment in $\Delta\theta$ produces a torque which tends to return the machine to the operating angle. Thus, K is positive. Now

$$\alpha = \frac{d^2(\Delta\theta)}{dt^2} = -K\Delta\theta. \qquad (120)$$

Further, let

$$\Delta\theta = A \sin 2\pi f_n t \tag{121}$$

then substituting this relation into Eq. (120)

$$-(2\pi f_n)^2 A \sin 2\pi f_n t = -KA \sin 2\pi f_n t$$

from which

$$f_n = \frac{\sqrt{K}}{2\pi}.$$

Substituting K from Eq. (119)

$$f_n = \frac{587}{\text{rpm}}\sqrt{\frac{fP_r}{WR^2}} \text{ cycles per sec} \tag{122}$$

which converts to Eq. (113).

XIII. TYPICAL CONSTANTS AND COSTS

Both the voltage and the current at which a machine operates affect certain of the principal constants through the variability of the permeability of the iron. In this sense, these so-called constants are not in reality constant. Consider the transient reactance, x_d'. If three-phase short-circuits are applied to a machine from no load, the reactances so obtained vary with the excitation. Two of these quantities have been given special designations. Thus the reactance obtained when the excitation is such as to produce rated voltage at no load before the short-circuit is called the "rated-voltage reactance" and the reactance obtained when the excitation is reduced so as to produce from no load a transient component of the short-circuit current equal to rated value is called the "rated-current reactance."

A knowledge of these two values of x_d' is not sufficient for all applications for which x_d' is required. The rated-current x_d', because of lower excitation, lends itself more readily for determination from test. The rated-voltage x_d' is that required for short-circuit studies. Saturation within the machine is a minimum for the former and a maximum for the latter. The rated voltage value is sometimes called the "saturated value" and is the value usually given by the designer. Certain applications, such as stability studies, demand a quantity determined under conditions for which the terminal voltage is near rated voltage and the armature current is likewise near its rated current. Fig. 68 obtained from data presented by Kilgore[16] shows how the reactances of typical machines of different classes vary if three-phase short-circuits were applied from rated voltage no load, the current being altered by introducing different external reactances in the armature circuits. The rated-current figure is used as a base for all the curves. The particular reactance on the curves for rated current is the one that would have greatest utility for stability and regulation problems. No specific name has been assigned to this quantity.

Similar considerations apply to the subtransient reactances, with this difference, that the rated-current reactance x_d'' is obtained from the same test as that for which the rated-current reactance of x_d' was obtained. In this case rated current refers to the transient component and not the subtransient component of current. Fig. 69 shows how

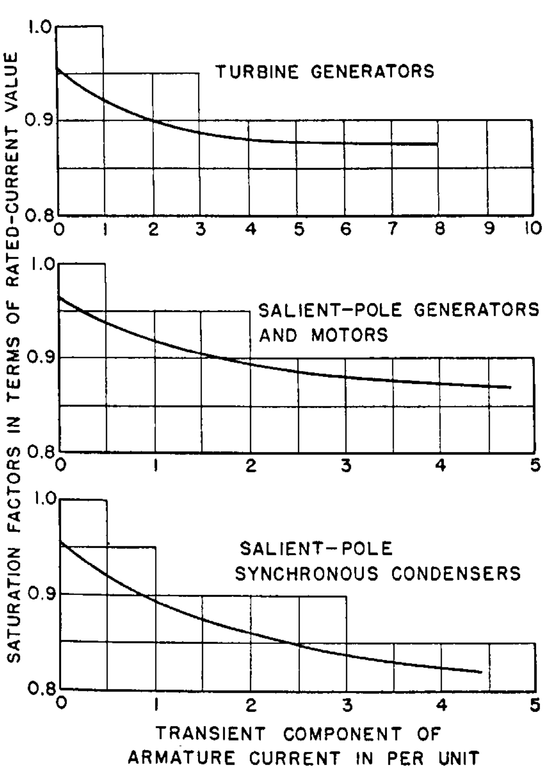


Fig. 68—Saturation factors for transient reactance. Three-phase short circuits from rated voltage no load. Current limited by series reactance.

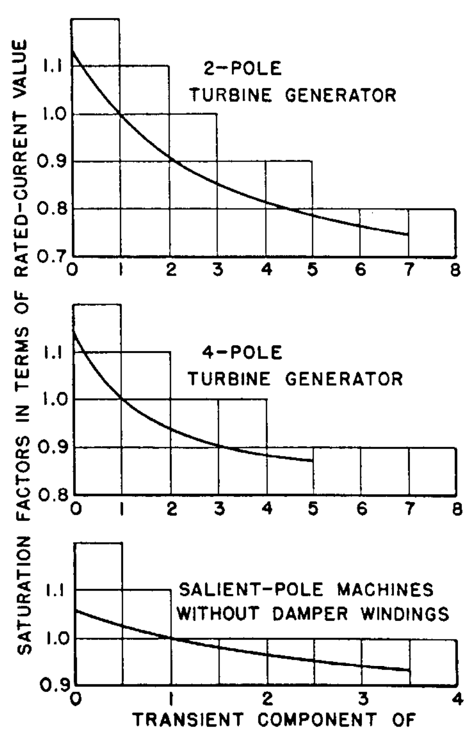


Fig. 69—Saturation factors for subtransient reactance. "Rated current" value used as base. All reactances from three-phase short circuits without external reactance. Saturation factors for salient-pole machine with damper winding is equal to unity.

x_d'' varies with the transient component of current, all points being obtained from three-phase short-circuits with no external reactance, the current being altered by the excitation before the short-circuit.

In general, it is unnecessary to make this distinction for the negative-sequence reactance. The *AIEE* code[10] suggests determination of x_2 by means of the method discussed

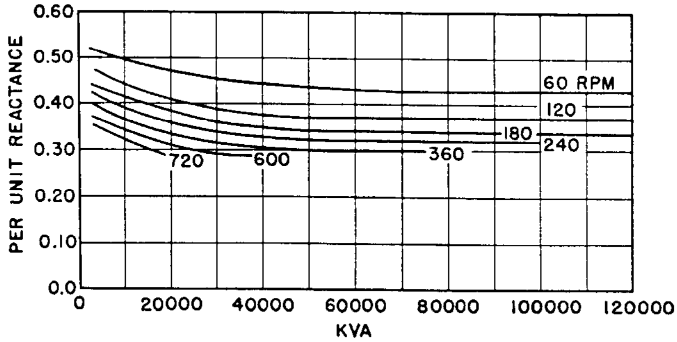


Fig. 70—Normal unsaturated transient reactance (x_{du}') for waterwheel generators.

under Negative-Sequence Reactance, the current during the sustained terminal-to-terminal short-circuit being limited to the rated current.

The normal value of x'_{du} designed into waterwheel generators varies with the kva capacity and speed. These values are plotted in curve form in Fig. 70. To obtain lower values than those indicated usually involves an increased cost.

The angular relations within the machine are determined to a large extent by x_q. The variation, by test, of x_q for several salient-pole machines[12,17] is shown in Fig. 71.

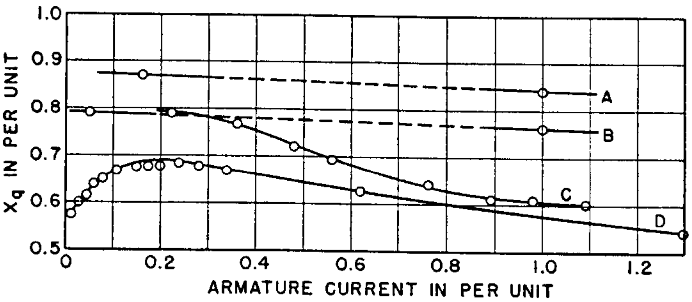


Fig. 71—x_q for salient-pole machines.

A = 7500 kva generator without damper winding.
B = 750 kva generator without damper winding.
C = 331 kva motor with damper winding removed.
D = 100 kva generator with damper winding.

The zero-sequence reactance, as evidenced by Fig. 72 taken from Wright's paper,[17] is not affected to any great extent in the region for which it has greatest use.

For practical purposes the effect of saturation upon the open-circuit transient time constant $T_d'{}_o$ and the subtransient short-circuit time constant T_d'' can be neglected. In general, T_d' varies [17] in the same manner as x_d', so that the relation $T_d' = \dfrac{x_d'}{x_d} T_d'{}_o$ is still maintained. Because of

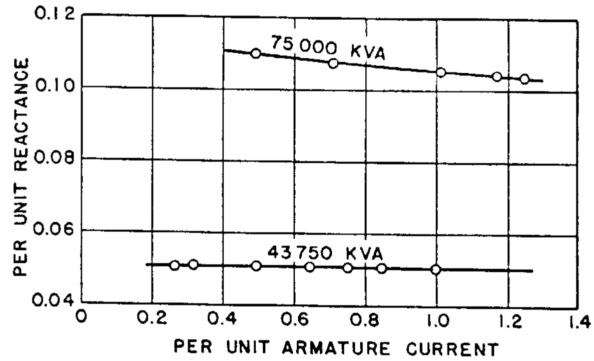


Fig. 72—Variation of x_o for turbine generators.

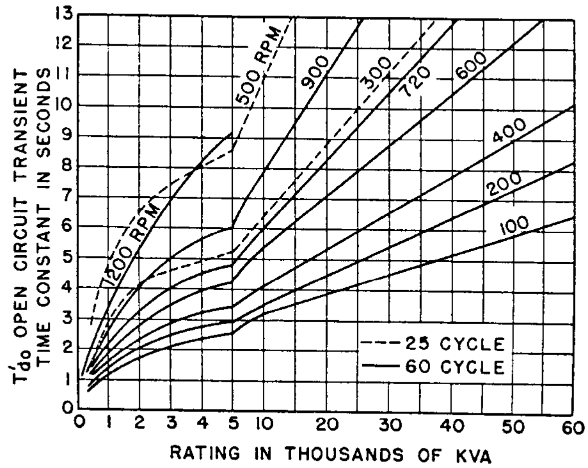


Fig. 73—Open-circuit transient time constants of a-c generators and motors.

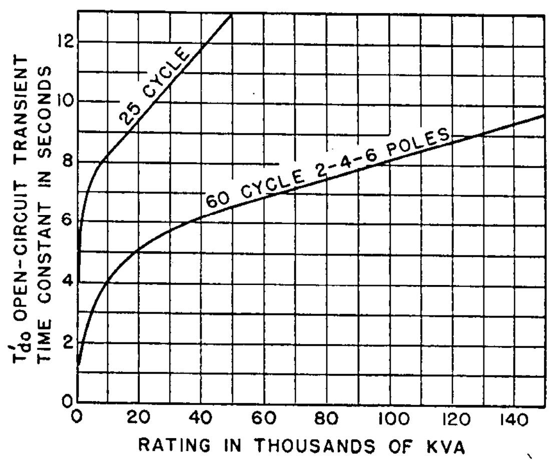


Fig. 74—Open-circuit transient time constants of turbine generators.

the wide variation of $T_d'{}_o$ with the size of the unit the curves of Figs. 73 and 74 taken from a paper by Hahn and Wagner,[18] are also included.

Table 4 gives both the range of typical constants that are characteristic of normal designs and also an average that can be used for general purposes when the specific value of a particular machine is not known. The negative-sequence resistance is that obtained at a negative-sequence current equal to rated current. It must be kept in mind that the loss associated therewith varies as the second power of i_2 for salient-pole machines either with or without damper windings and as the 1.8 power of i_2 for turbine generators. Column (9) in Table 4 refers to the a-c resistance, r_1, (which includes the effect of load losses) and column (10) the d-c resistance, r_a.

The inertia constant, H, which is discussed in Chap. 13 is likewise given in Table 4. The general variation of H of turbogenerators and the corresponding figures for waterwheel generators are given in Fig. 75. The effect upon H

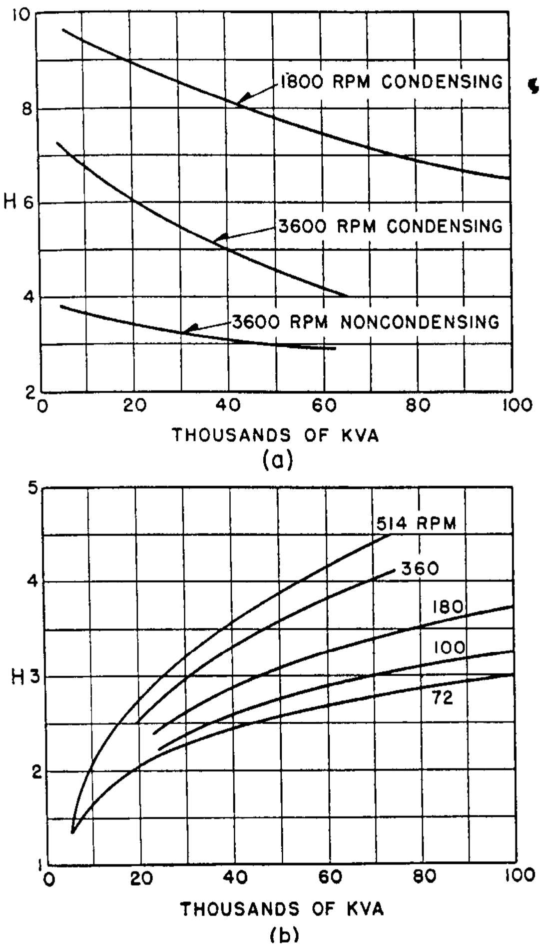


Fig. 75—Inertia constants.

(a) Large turbine generators, turbine included.
(b) Large vertical type waterwheel generators, including allowance of 15 percent for waterwheels.

of increasing the short-circuit ratio and changing the power-factor is given in Fig. 76. The WR^2 represented by the curves of Figs. 75 and 76 are those obtained from a normally designed machine in which no particular effort has been made to obtain abnormally high H. When magnitudes of WR^2 in excess of these are desired a more expensive machine results. The additional cost of the additional WR^2 is about proportional as shown in Fig. 77.

The cost per kva of water-wheel generators depends upon its kva and speed. The extent of this variation is shown in Fig. 78. Machines of higher short-circuit ratio or power-factor are more expensive in the proportion shown

TABLE 4—TYPICAL CONSTANTS OF THREE-PHASE SYNCHRONOUS MACHINES

Reactances are per unit, time constants are in seconds. Values below the line give the normal range of values, while those above give an average value

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	x_d (unsat)	x_q rated current	x_d' rated voltage	x_d'' rated voltage	x_2 rated current	(*) x_0 rated current	x_p	(†) r_2	(‡) r_1	(‡) r_a	T_{d0}'	T_d'	T_d''	T_a	H
2-Pole turbine generators	1.20 / 0.95-1.45	1.16 / 0.92-1.42	0.15 / 0.12-0.21	0.09 / 0.07-0.14	$=x_d''$	0.03 / 0.01-0.08	0.10 / 0.07-0.17	0.025-0.04	0.004-0.011	0.001-0.007	5.0 / See Fig. 74	0.6 / See Fig. 74	0.035 / 0.02-0.05	0.13 / 0.04-0.24	See fig. 75(a)
4-Pole turbine generators	1.20 / 1.00-1.45	1.16 / 0.92-1.42	0.23 / 0.20-0.28	0.14 / 0.12-0.17	$=x_d''$	0.08 / 0.015-0.14	0.17 / 0.12-0.24	0.03-0.045	0.003-0.008	0.001-0.005	8.0 / See Fig. 74	1.0 / See Fig. 74	0.035 / 0.02-0.05	0.20 / 0.15-0.35	See Fig. 75(a)
Salient-pole generators and motors (with dampers)	1.25 / 0.60-1.50	0.70 / 0.40-0.80	0.30 / 0.20-0.50(*)	0.20 / 0.13-0.32(*)	0.20 / 0.13-0.32(*)	0.18 / 0.03-0.23	0.28 / 0.17-0.40	0.012-0.020	0.005-0.020	0.003-0.015	See Fig. 73 3.0-5.0 / 1.5-10	1.5 / 0.5-3.3	0.035 / 0.01-0.05	0.15 / 0.03-0.25	See Fig. 75(b)
Salient-pole generators (without dampers)	1.25 / 0.60-1.50	0.70 / 0.40-0.80	0.30 / 0.20-0.50(*)	0.30 / 0.20-0.50(*)	0.48 / 0.35-0.65	0.19 / 0.03-0.24	0.28 / 0.17-0.40	0.03-0.045	0.005-0.020	0.003-0.015	See Fig. 73 3.0-5.0 / 1.5-10	1.5 / 0.5-3.3	—	0.30 / 0.10-0.50	See Fig. 75(b)
Condensers air cooled	1.85 / 1.25-2.20	1.15 / 0.95-1.30	0.40 / 0.30-0.50	0.27 / 0.19-0.30	0.26 / 0.18-0.40	0.12 / 0.025-0.15	0.25 / 0.20-0.35	0.025-0.07	0.0065 / 0.004-0.009	0.0035 / 0.0025-0.008	9.0 / 6.0-14.0	2.0 / 1.2-2.8	0.035 / 0.02-0.04	0.17 / 0.1-0.3	Large 2.4 Small 1.0
Condensers hydrogen cooled at ½ psi kva rating	2.20 / 1.50-2.65	1.35 / 1.10-1.55	0.48 / 0.36-0.60	0.32 / 0.23-0.36	0.31 / 0.22-0.48	0.14 / 0.030-0.18	0.27 / 0.22-0.37	0.025-0.07	0.0065 / 0.005-0.009	0.0035 / 0.0025-0.005	9.0 / 6.0-14.0	2.0 / 1.2-2.8	0.035 / 0.02-0.04	0.20 / 0.15-0.3	Large 2.0 Small 1.10

(*) High speed units tend to have low reactance and low speed units high reactance.
(*) X_0 varies so critically with armature winding pitch that an average value can hardly be given. Variation is from 0.1 to 0.7 of x_d''. Low limit is for ⅔ pitch windings.
(†) r_2 varies with damper resistance.
(‡) r_1 and r_a vary with machine rating, limiting values given are for about 50,000 kva and 500 kva.

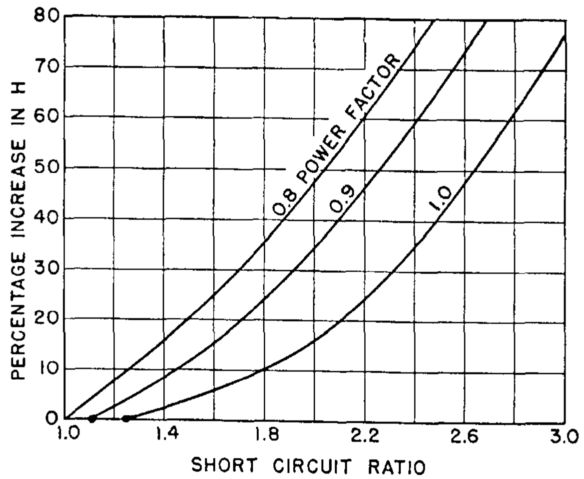


Fig. 76—Effect of short-circuit ratio upon H.

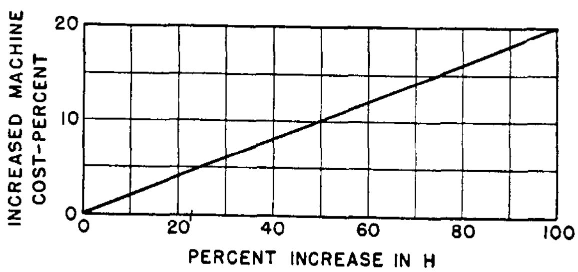


Fig. 77—Effect of increasing H above the normal values given by Fig. 75.

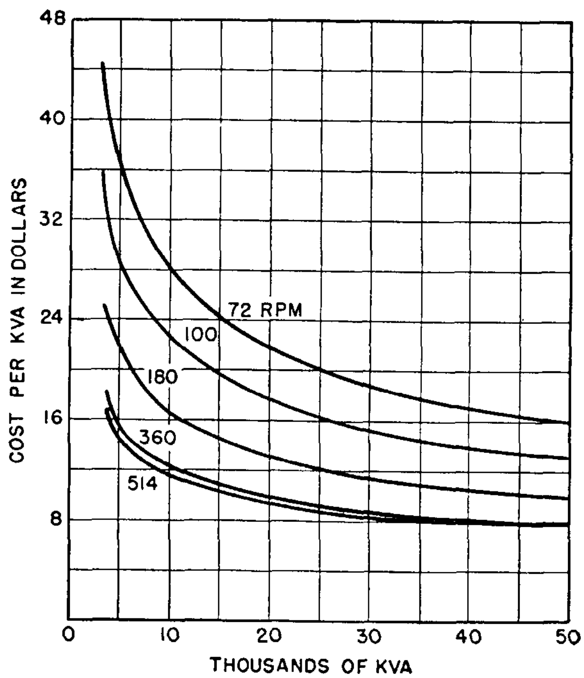


Fig. 78—Cost of waterwheel generators including direct-connected exciters only.

(0.8 power-factor and 1.0 short circuit ratio)
(0.9 power-factor and 1.1 short circuit ratio)
(1.0 power-factor and 1.25 short circuit ratio)

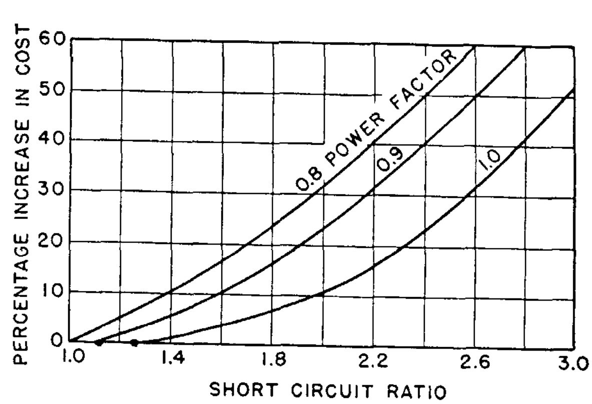


Fig. 79—Effect of short-circuit ratio upon cost (Normal 1.0 short-circuit ratio and 0.8 power-factor used as base).

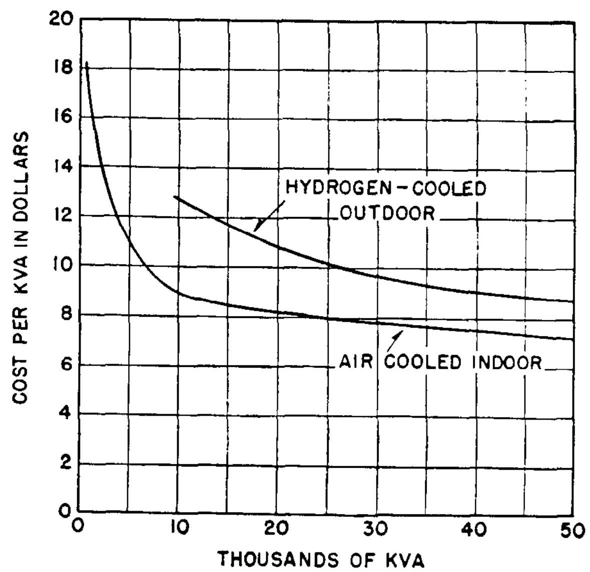


Fig. 80—Cost of synchronous condensers including exciter and autotransformer.

in Fig. 79. Naturally these figures will vary from year to year with the cost of materials and labor.

The condenser cost per kva including the exciter, pilot exciter, and auto-transformer is plotted in Fig. 80. The exciter kw varies with the size of the unit, ranging from 1.2, 0.7, and 0.32 percent for a 1000, 5000, and 50 000-kva unit, respectively.

The cost of normal exciters for water-wheel generators varies from 7 to 13 percent of the cost of the generator alone for slow speeds, and from 2.5 to 6 percent for high speeds. The larger figures apply for units of about 3000 kva and the smaller figures for machines of about 50 000 kva. Direct-connected pilot exciters cost approximately 30 percent of that of the exciter.

XIV. INDUCTION MOTORS

The equivalent circuit of the induction motor is shown in Fig. 31. The loss in the resistor $\frac{1-s}{s}r_r$ represents the shaft power and since the circuit is on a per phase basis, the total shaft power is thus

$$\text{Total shaft power} = \frac{1-s}{s}(3r_r i_r^2) \text{ in watts} \tag{123}$$

$$= \frac{1}{746}\,\frac{1-s}{s}(3r_r i_r^2) \text{ in hp.} \tag{124}$$

The rotor copper loss is $(3r_r i_r^2)$. Therefore, neglecting other losses, the efficiency is:

$$\text{Efficiency} = \frac{\text{total shaft power}}{\text{total shaft power} + \text{rotor copper loss}}$$

$$= \frac{\dfrac{1-s}{s}}{\dfrac{1-s}{s}+1} = 1-s. \tag{125}$$

Thus, the efficiency decreases with increasing slip. For 10 percent slip the efficiency is 90 percent, for 90 percent slip the efficiency is 10 percent. Similarly, the rotor copper loss is directly proportional to slip; being 10 percent for 10 percent slip and 90 percent for 90 percent slip.

The total shaft power can also be expressed in terms of torques. Thus,

$$\text{Total shaft horse power} = \frac{2\pi}{33\,000}(T_{\text{in lb ft}})(\text{rpm})_{\text{syn.}}(1-s). \tag{126}$$

Equating (124) and (126), the torque is

$$T = 7.04\frac{1}{(\text{rpm})_{\text{syn.}}}\;\frac{(3r_r i_r^2)_{\text{in watts}}}{s_{\text{in per unit}}}\text{lb ft.} \tag{127}$$

The equivalent circuit of Fig. 31 can be simplified considerably by shifting the magnetizing branch to directly across the terminals. The resultant approximate circuit is shown in Fig. 81. This approximation permits of relatively simple determination of i_r, so that Eq. (127) becomes

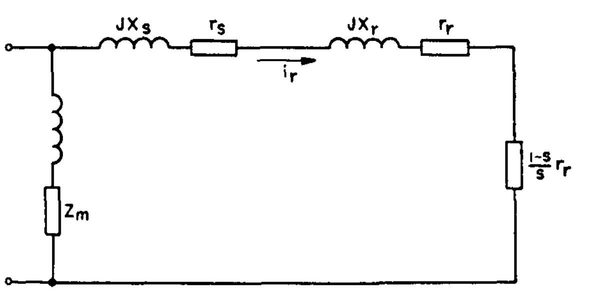


Fig. 81—Approximate equivalent circuit of induction motor.

$$T = \frac{7.04E^2}{(\text{rpm})_{\text{syn.}}}\;3\;\frac{\dfrac{r_r}{s}}{\left(r_s+\dfrac{r_r}{s}\right)^2+(x_s+x_r)^2}\text{lb ft.} \tag{128}$$

Most transients involving induction motors fall within one of two categories; first, those in which the machine is disconnected from the source of power and, second, those in which the machine remains connected to the source of power. In the first case the transient is determined largely by changes in magnetization and may be quite long. An example of this case is the phenomena that occurs during the interval between the transfer of power-house auxiliaries from one source to another. In the second case, the transient is determined by reactions involving both the stator and rotor and the duration is quite short. Examples, of this case, are the sudden energization of an induction motor or sudden short circuit across its terminals.

35 Contribution to System Short-Circuit Current

In the calculation of system short circuits only synchronous machines are usually considered but in special cases where induction machines constitute a large proportion of the load, their contribution to the short-circuit current even if its duration is only a few cycles may be large enough to influence the choice of the breaker from the standpoint of its short-time rating, that is, the maximum rms current the breaker can carry for any time, however small.

As a first approximation the short-circuit current supplied by an induction motor can be resolved into an alternating and a unidirectional component much like that for a synchronous machine. The initial rms magnitude of the alternating component is equal to the terminal voltage to neutral divided by the blocked rotor impedance per phase. The time constants are namely,
for the alternating component,

$$\frac{(\text{blocked rotor reactance per phase in ohms})}{2\pi\,(\text{rotor resistance per phase in ohms})}\;\text{in cycles.}$$

for the unidirectional component,

$$\frac{(\text{blocked rotor reactance per phase in ohms})}{2\pi\,(\text{stator resistance per phase in ohms})}\;\text{in cycles.}$$

Fig. 82 shows the short-circuit current of a 25-horse-power, 550-volt squirrel-cage motor. The dotted line in the upper

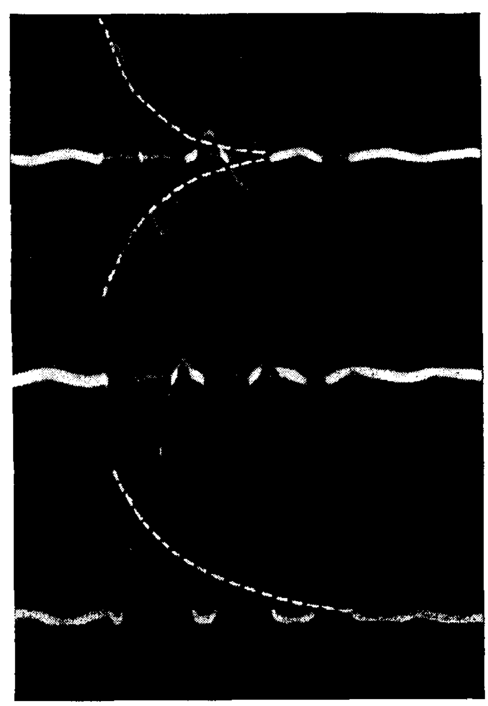


Fig. 82—Short-circuit currents in armature of squirrel-cage induction motor.

curve indicates the computed value of the envelope of the alternating component of short-circuit current. The amplitude shows a substantial check but the computed time constant was low. This can probably be attributed to using the a-c resistance of the rotor rather than the d-c resistance. The dotted line in the lower curve is the computed value of the unidirectional component which checks quite well.

Wound-rotor motors, operated with a substantial amount of external resistance, will have such small time constants that their contribution to the short-circuit can be neglected.

36. Electro-Mechanical Starting Transient

Fig. 31 shows the conventional diagram of an induction motor. In the present discussion the per unit system of units will continue to be used, in which unit current is the current necessary to develop the rated power at the rated voltage. The unit of both power and reactive volt-amperes will be the rated kva of the motor and *not* the rated power either in kilowatts or horse power. This convention is consistent with the choice of units for the impedances. At rated slip the volt-amperes input into the stator must be equal to unity but the power absorbed in the resistor $\frac{1-s}{s}r_r$ will be less than unity and will be equal numerically to the ratio of the rated power of the motor to the rated kva. The unit of shaft torque requires special comment. The shaft power can be expressed as

$$\text{Shaft Power in kw} = \text{kva}_{\text{rated}}\, I_r^2 r_r \frac{1-s}{s}. \qquad (129)$$

In terms of torque the shaft power is equal to

$$\text{Shaft Power in kw} = 0.746\frac{T \text{ in lb ft } 2\pi(\text{rpm})_{\text{synch.}}(1-s)}{33\,000}. \qquad (130)$$

Equating, there results that

$$T \text{ in lb ft} = \frac{33\,000}{2\pi(0.746)(\text{rpm})_{\text{synch.}}}\text{kva}_{\text{rated}}\, I_r^2\frac{r_r}{s}. \qquad (131)$$

If unit torque be defined as that torque required to produce a shaft power equal to rated kva at synchronous speed, then from (130), the unit of torque is

$$\frac{33\,000}{2\pi(0.746)(\text{rpm})_{\text{synch.}}}\text{kva}_{\text{rated}}$$

and equation (131) in per unit becomes

$$T \text{ in p.u.} = I_r^2\frac{r_r}{s}. \qquad (132)$$

For the purpose of determining the nature of electro-mechanical transients upon starting a motor from rest, the first step involves the calculation of the shaft torque as a function of the speed. Either the conventional method of the circle diagram or expression (132) can be used. In using the latter method it is only necessary to solve the network of Fig. (31) and substitute the solution of I_r therefrom into Eq. (132). A solution of a typical motor is shown in Fig. 83. For most motors the magnetizing branch can be neglected, for which case the torque expression becomes

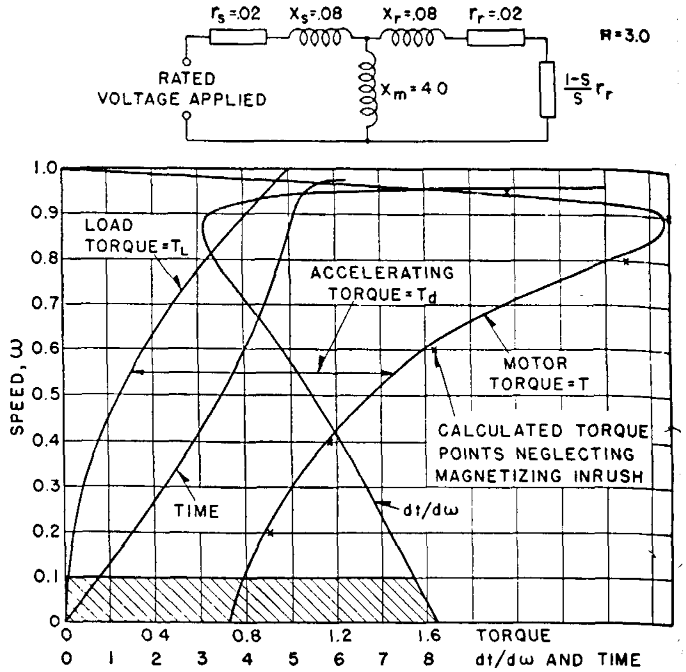


Fig. 83—Illustrating calculation of speed-time curve of an induction motor upon application of full voltage.

$$T \text{ in per unit} = e_t^2\frac{\frac{r_r}{s}}{(x_s+x_r)^2+\left(r_s+\frac{r_r}{s}\right)^2} \qquad (133)$$

The crosses close to the torque curve in Fig. 83 were computed by this expression.

In Fig. 83 is also shown the torque requirements of a particular load such as a blower. Upon applying voltage to the motor the difference between the torque developed by the motor and that required by the load is the torque available for acceleration of the rotor. To convert to acceleration it is convenient to introduce a constant, H, which is equal to the stored energy in kw-sec. per kva of rating at synchronous speed. H may be computed by means of Eq. (93). WR^2 must, of course, include the WR^2 of the connected load.

Suppose that one per unit torque is applied to the motor which means that at synchronous speed the power input into acceleration of the rotor is equal to rated kva, and suppose further that the rotor is brought to synchronous speed in one second. During this interval the acceleration is constant (1 per unit) and the power input increases linearly with time so that at the end of one second the stored energy of rotation is (½ kva) in kw-sec. Thus 1 per unit of torque produces 1 per unit of acceleration if the inertia is such that ½ kva of stored energy is produced in one second. From this it can be seen that if the inertia is such that at synchronous speed the stored energy is H, then to develop this energy in one second, the same acceleration but a torque $2H$ times as great is required. Therefore, there results that

$$\alpha = \frac{T-T_L}{2H} \qquad (134)$$

Acceleration can be expressed as $\frac{d\omega}{dt}$ and its reciprocal as $\frac{dt}{d\omega}$. Thus from (134)

$$\frac{dt}{d\omega}=\frac{2H}{T-T_L} \tag{135}$$

This function is likewise plotted in Fig. 83. The utility of this form of the expression may be seen at once from the fact that $\frac{dt}{d\omega}$ is known as a function of ω and the time to reach any value of ω can be determined by a simple integration. Thus

$$t=\int\left(\frac{dt}{d\omega}\right)d\omega \tag{136}$$

By summing up areas (such as indicated by the shaded portion) in a vertical direction, the time to reach any speed is obtained. The curve of time so obtained is plotted in Fig. 83.

The following formula can be used to form an approximate idea of the time required to accelerate a motor, whose load varies as the square or cube of the speed, to half speed

$$\text{Time to half speed}=\frac{H(x_s+x_r)^2}{r_r e^2_t}\text{ in seconds} \tag{137}$$

All of the above units must be expressed in per unit. Remember also that x_s should include any external reactance in the stator back to the point where the voltage may be regarded as constant and e_t should be that constant voltage.

37. Residual Voltage

If an induction motor is disconnected from its supply, it rotates for some time, the rate of deceleration being determined by the inertia of its own rotor and the inertia of the load and also by the nature of the load. Because of the inductance of the rotor, flux is entrapped and voltage appears at the open terminals of the machine. If the voltage source is reapplied when the source voltage and residual voltage of the motor are out of phase, currents exceeding starting values may be obtained.

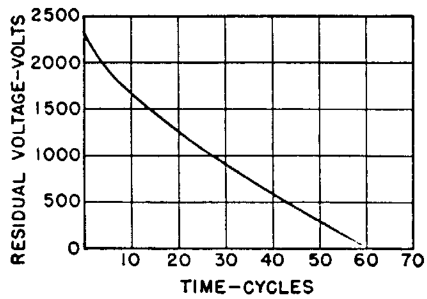


Fig. 84—Decay of residual voltage[25] of a group of power house auxiliary motors.

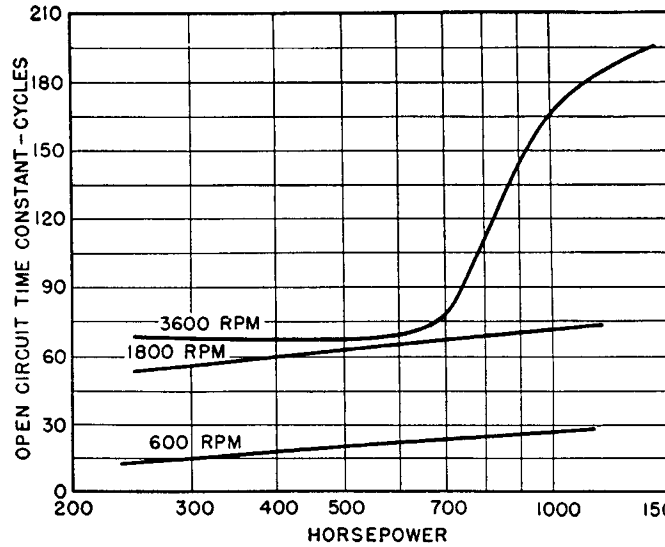


Fig. 85—Typical time constants for 2300-volt squirrel cage induction motors.

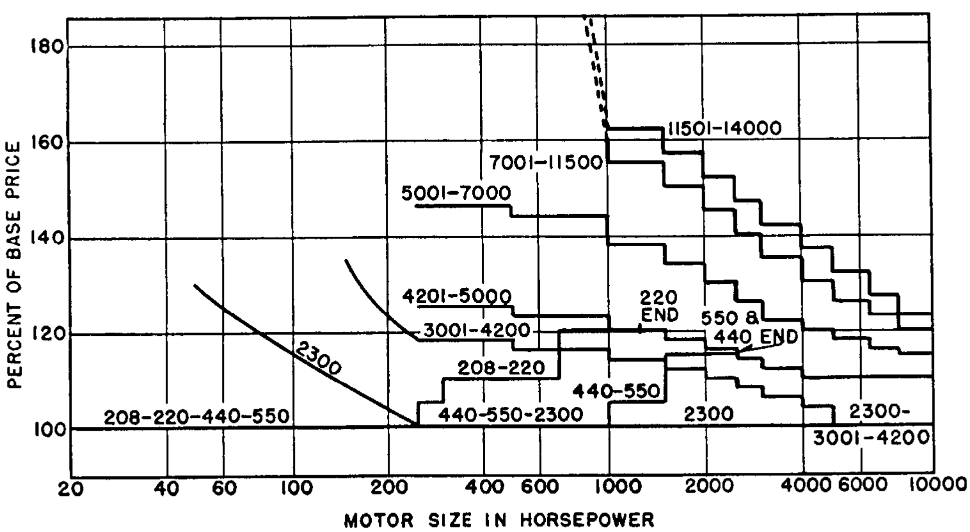


Fig. 86—Approximate variation of price with voltage and horsepower of squirrel-cage induction motors. These values apply approximately for 8 poles or less for 60-cycle motors. Most economical used as base price.

Figure 84 shows the decay of a group of power-house auxiliary motors[25]. The group had a total rating of 2500 kw of which the largest was 1250 hp. This curve includes not only the effect of magnetic decay but the reduction in voltage due to decrease in speed. The open-circuit time constant for individual 2300-volt machines is given in Fig. 85. There is a great variance in this constant between different designs but these curves give an idea of the magnitude for squirrel-cage induction machines.

38. Cost of Induction Motors

The price of induction motors of a given rating varies with the voltage. As the rating increases the most economical voltage also increases. To form a basis of judgment of the effect of voltage upon size the curve in Fig. 86 was prepared.

REFERENCES

1. Power System Transients, by V. Bush and R. D. Booth, *A.I.E.E. Transactions*, Vol. 44, February 1925, pp. 80–97.
2. Further Studies of Transmission Stability, by R. D. Evans and C. F. Wagner, *A.I.E.E. Transactions*, Vol. 45, 1926, pp. 51–80.
3. Synchronous Machines—I and II—An extension of Blondel's Two Reaction-Theory—Steady-State Power Angle Characteristics, by R. E. Doherty and C. A. Nickle, *A.I.E.E. Transactions*, Vol. 45, 1926, pp. 912–942.
4. Synchronous Machines—III. Torque Angle Characteristics Under Transient Conditions, by R. E. Doherty and C. A. Nickle, *A.I.E.E. Transactions*, Vol. 46, 1927, pp. 1–14.
5. Synchronous Machines, IV, by R. E. Doherty and C. A. Nickle, *A.I.E.E. Transactions*, Vol. 47, No. 2, April 1928, p. 457.
6. Synchronous Machines, V. Three-Phase Short Circuit Synchronous Machines, by R. E. Doherty and C. A. Nickle, *A.I.E.E. Transactions*, Vol. 49, April 1930, p. 700.
7. Definition of an Ideal Synchronous Machine and Formula for the Armature Flux Linkages, by R. H. Park, *General Electric Review*, June 1928, pp. 332–334.
8. Two-Reaction Theory of Synchronous Machines—I, by R. H. Park, *A.I.E.E. Transactions*, Vol. 48, No. 2, July 1929, p. 716.
9. Two-Reaction Theory of Synchronous Machines, II, by R. H. Park, *A.I.E.E. Transactions*, Vol. 52, June 1933, p. 352.
10. *A.I.E.E. Test Code for Synchronous Machines*. A.I.E.E. Publication No. 503, June 1945.
11. Discussion, by C. F. Wagner, *A.I.E.E. Transactions*, July 1937, p. 904.
12. Unsymmetrical Short-Circuits in Water-Wheel Generators Under Capacitive Loading, by C. F. Wagner, *A.I.E.E. Transactions*, November 1937, pp. 1385–1395.
13. Overvoltages on Water-Wheel Generators, by C. F. Wagner, *The Electric Journal*, August 1938, p. 321 and September 1938, p. 351.
14. Damper Windings for Water-Wheel Generators, by C. F. Wagner, *A.I.E.E. Transactions*, Vol. 50, March 1931, pp. 140–151.
15. Effect of Armature Resistance Upon Hunting of Synchronous Machines, by C. F. Wagner, *A.I.E.E. Transactions*, Vol. 49, July 1930, pp. 1011–1024.
16. Effects of Saturation on Machine Reactances, by L. A. Kilgore, *A.I.E.E. Transactions*, Vol. 54, 1935, pp. 545–550.
17. Determination of Synchronous Machine Constants by Test, by S. H. Wright, *A.I.E.E. Transactions*, Vol. 50, 1931, pp. 1331–1350.
18. Standard Decrement Curves, by W. C. Hahn and C. F. Wagner, *A.I.E.E. Transactions*, 1932, pp. 353–361.
19. Approximating Potier Reactance, by Sterling Beckwith, *A.I.E.E. Transactions*, July 1937, p. 813.
20. Auxiliary Power at Richmond Station, by J. W. Anderson and A. C. Monteith, *A.I.E.E. Transactions*, 1927, p. 827.
21. Preferred Standards for Large 3600-RPM 3-Phase 60-Cycle Condensing Steam Turbine-Generators, AIEE Standards Nos. 601 and 602, May 1949.
22. Regulation of A-C Generators With Suddenly Applied Loads, by E. L. Harder and R. C. Cheek, *A.I.E.E. Transactions*, Vol. 63, 1944, pp. 310–318.
23. Regulation of A-C Generators with Suddenly Applied Loads—II, by E. L. Harder and R. C. Cheek, *A.I.E.E. Transactions*, 1950.
24. Practical Calculation of Circuit Transient Recovery Voltages, by J. A. Adams, W. F. Skeats, R. C. Van Sickle and T. G. A. Sillers, *A.I.E.E. Transactions*, Vol. 61, 1942, pp. 771–778.
25. Bus Transfer Tests on 2300-Volt Station Auxiliary System, by A. A. Johnson and H. A. Thompson, presented before AIEE Winter Meeting, Jan. 1950.

CHAPTER 7

EXCITATION SYSTEMS

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PRIOR to 1920 relatively little difficulty was encountered in the operation of electrical systems, and operating engineers had little concern about system stability. As the loads grew and systems expanded, it became necessary to operate synchronous machines in parallel, and difficulties encountered were not well understood. In certain areas it became necessary to locate generating stations some distance from the load centers, which involved the transmission of power over long distances. It soon became apparent that system stability was of vital importance in these cases and also in the operation of large interconnected systems.

In 1922, a group of engineers undertook solution of the stability problem to determine the factors involved that most affected the ability of a system to transfer power from one point to another. The results of these studies were presented before the AIEE in a group of papers* in 1924, and it was pointed out that the synchronous machine excitation systems are an important factor in the problem of determining the time variation of angle, voltage, and power quantities during transient disturbances. E. B. Shand stressed the theoretical possibility of increasing the steady-state power that could be transmitted over transmission lines through the use of a generator voltage regulator and an excitation system with a high degree of response so that operation in the region of dynamic stability would be possible. It was not recommended that this region of dynamic stability be considered for normal operation, but that it be considered additional margin in determining permissible power transfer.

Improvement of the excitation systems, therefore, appeared to be at least one method of increasing the stability limits of systems and preventing the separations occurring during transient conditions. Greater interest in the design of excitation systems and their component parts developed, and exciters with higher speeds of response and faster, more accurate generator voltage regulators were soon introduced to the industry.

Early excitation systems were of many different forms depending principally upon whether the main generators were small or large in rating and whether the installation was a steam or hydroelectric station. The two broad classifications were those using a common excitation bus and those using an individual exciter for each main generator. The common exciter bus was generally energized by several exciters driven by motors, turbines, steam engines, waterwheels, or combinations of these to provide a main and emergency drive. Standby exciter capacity was provided in the common-bus system by a battery floated on the bus. It usually had sufficient capacity to carry the excitation requirements of the entire station for at least an hour.

Motor or turbine drive was also used in the individual-exciter system, but it was not long before it was realized that direct-connection of the exciter to the generator shaft offered an excellent answer to the many problems encountered with separately-driven exciters and this system grew rapidly in popularity. The standby excitation source was usually a spare exciter, either motor- or turbine-driven, and in case of trouble with the main exciter, transfer was accomplished manually.

Pilot exciters had not been used up to that time. The exciters were invariably self-excited. In the common-bus system without a floating battery, the bus was operated at constant voltage supplied by compound-wound d-c generators. Thus, practically constant voltage was obtained on the bus and control of the individual a-c generator field voltage was accomplished by using a variable rheostat in each field as shown in Fig. 1. When a standby battery was

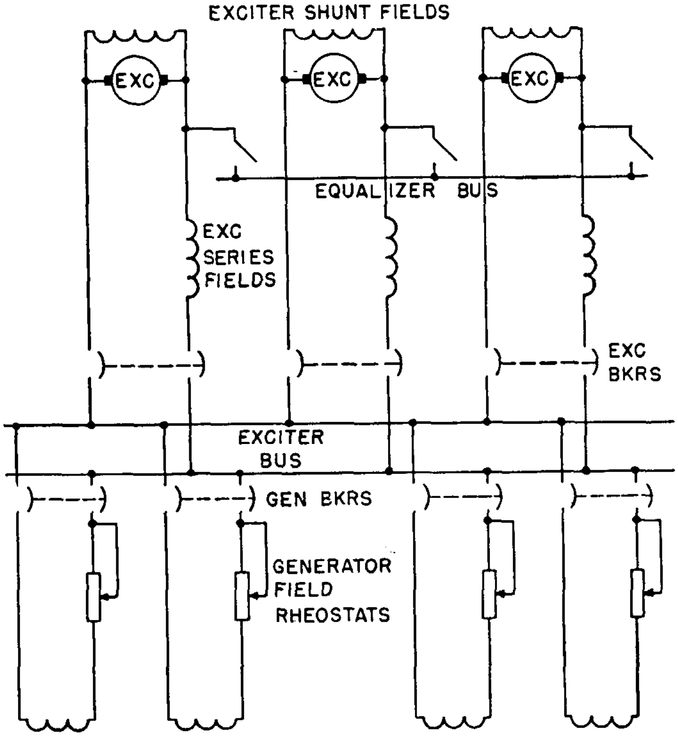


Fig. 1—Common-exciter-bus excitation system using flat-compounded exciters and a-c generator field rheostats.

A.I.E.E. Transactions, Vol. 43, 1924, pp. 16–103.

floated on the common bus, however, the exciters were shunt-wound to prevent polarity reversal by reversal of the series-field current. The a-c generator-field rheostat required in the common-bus system was a large and bulky device, which had considerable loss and required a great deal of maintenance. Control of voltage was under hand-regulation.

In the individual-exciter system, the exciter was a shunt-wound machine with field control enabling it to operate as a variable-voltage source. The exciter usually operated at voltages between 30 and 100 percent, lower field voltages being obtained with a generator-field rheostat so that the exciter could operate slightly saturated and be stable.

The generator voltage regulators in use at that time were predominantly of the continuously-vibrating type. The fact that these regulators were not suitable for use with the new exciters with fast response and high ceiling voltages prompted the development of new types of regulators.*

In the past 25 years, there have been many developments in excitation-system design and practices. There is an unceasing search among designers and users alike to find ways of improving excitation-system performance through use of various types of d-c generators, electronic converters, and better controlling devices. The ultimate aim is to achieve an ideal in rate of response, simplicity, reliability, accuracy, sensitivity, etc. The achievement of all of these ideals simultaneously is a difficult problem.

A review of the common excitation systems in use at the present time is presented in this chapter. The design and characteristics of each of the component parts are discussed, along with the methods of combining these parts to form an excitation system having the most desirable features. Methods of calculating and analyzing excitation system performance are also included.

I. DEFINITIONS

In discussing excitation systems, a number of terms are used, the meaning of which may not be entirely clear. The following definitions are proposed for inclusion in the new edition of the American Standards Association, Publication C42, "Definitions of Electrical Terms".

Excitation System—An excitation system is the source of field current for the excitation of a principal electric machine, including means for its control.

An excitation system, therefore, includes all of the equipment required to supply field current to excite a principal electric machine, which may be an a-c or d-c machine, and any equipment provided to regulate or control the amount of field current delivered.

Exciter Ceiling Voltage—Exciter ceiling voltage is the maximum voltage that may be attained by an exciter with specified conditions of load. For rotating exciters ceiling should be determined at rated speed and specified field temperature.

Nominal Exciter Ceiling Voltage—Nominal exciter ceiling voltage is the ceiling voltage of an exciter loaded with a resistor having an ohmic value equal to the resistance of the field winding to be excited. This resistance shall be determined at a temperature of:

(a) 75C for field windings designed to operate at rating with a temperature rise of 60C or less
(b) 100C for field windings designed to operate at rating with a temperature rise greater than 60C.

For rotating exciters the temperature of the exciter field winding should be considered to be 75C.

Rated-Load Field Voltage—Rated-load field voltage is the voltage required across the terminals of the field winding of an electric machine under rated continuous load conditions with the field winding at:

(a) 75C for field windings designed to operate at rating with a temperature rise of 60C or less
(b) 100C for field windings designed to operate at rating with a temperature rise greater than 60C.

No-Load Field Voltage—No-load field voltage is the voltage required across the terminals of the field winding of an electric machine under conditions of no load, rated speed and terminal voltage, and with the field winding at 25C.

In the definitions of rated-load and no-load field voltage, the terminals of the field winding are considered to be such that the brush drop is included in the voltage in the case of an a-c synchronous machine having slip rings.

Excitation System Stability—Excitation system stability is the ability of the excitation system to control the field voltage of the principal machine so that transient changes in the regulated voltage are effectively suppressed and sustained oscillations in the regulated voltage are not produced by the excitation system during steady-load conditions or following a change to a new steady-load condition.

Exciter Response—Exciter response is the rate of increase or decrease of the exciter voltage when a change in this voltage is demanded.

Main-Exciter Response Ratio—The main-exciter response ratio is the numerical value obtained when the response, in volts per second, is divided by the rated-load field voltage; which response, if maintained constant, would develop, in one-half second, the same excitation voltage-time area as attained by the actual exciter. The response is determined with no load on the exciter, with the exciter voltage initially equal to the rated-load field voltage, and then suddenly establishing circuit conditions which would be used to obtain nominal exciter ceiling voltage.

Note: For a rotating exciter, response should be determined at rated speed. This definition does not apply to main exciters having one or more series fields or to electronic exciters.

In using the per-unit system of designating exciter voltages, several choices are available from which to choose the unit.

First, the rated voltage of the exciter would appear to be the fundamental basis, but for system analysis it has very little utility.

Second, for specification purposes it has become standard through the adoption by the AIEE and ASA to use the rated-load field voltage as unity. It should be noted that rated-load field voltage is the voltage formerly referred to as "nominal slip-ring" or "nominal collector-ring" voltage.

Third, the exciter voltage necessary to circulate the field current required to produce rated voltage on the air-gap line of the main machine. For analytical purposes this is the one most generally used and is the one used in the analytical work in Chap. 6. Under steady-state conditions,

*A symposium of papers on excitation systems was presented before the AIEE in 1920 and gives details of equipment and practices in use at that time. See *AIEE Transactions*, Vol. 39, Part II, 1920, pp. 1551–1637.

no saturation, and using this definition, exciter voltage, field current and synchronous internal voltage become equal.

Fourth, the slip-ring voltage necessary to produce rated voltage at no load or no-load field voltage is sometimes, but rather infrequently used. This definition includes the small amount of saturation present within the machine at no load.

Exciters for turbine generators of less than 10 000 kilowatts capacity are rated at 125 volts, and those for larger units are generally rated 250 volts. Some of the large units placed in service recently have exciters rated 375 volts. The vast majority of exciters in use with all types of synchronous machines greater than 10 000 kilowatts in capacity are rated 250 volts. On this rating the rated-load field voltage is of the order of 200 volts or 80 percent of the exciter rating. The exciter voltage required to produce the field current in the main machine corresponding to rated voltage on the air-gap line is usually about 90 volts or 36 percent of the exciter rating. Using this value as 1.0 per unit exciter voltage, the rated-load field voltage is approximately 2.2 per unit.

The nominal exciter ceiling voltage is defined above and can be interpreted as being the maximum voltage the exciter attains with all of the field-circuit resistance under control of the voltage regulator short circuited. On a 250-volt exciter, the ceiling voltage is usually about 300 to 330 volts, which is 120 to 132 percent of the exciter rated voltage, or 3.3 to 3.7 per unit. The relative values of these quantities are shown graphically in Fig. 2.

The construction of the response line in accordance with the definition for determining main-exciter response ratio is also included in Fig. 2. The curve *aed* is the actual voltage-time curve of the exciter as determined under the specified conditions. Beginning at the rated-load field voltage, point *a*, the straight line *ac* is drawn so that the area under it, *abc*, during the one-half second interval from zero time is equal to the area under the actual voltage-time curve, *abde*, during the same interval. The response used in determining response ratio is the slope of the line *ac* in volts per second;

$$\frac{100 \text{ volts}}{0.5 \text{ second}} = 200 \text{ volts per second.}$$

The rated-load field voltage is 200 volts, and the response ratio, obtained by dividing the response by the rated-load field voltage, is 1.0. The work can also be done by expressing the voltages as per-unit values.

The half-second interval is chosen because it corresponds approximately to one-half period of the natural electromechanical oscillation of the average power system. It is the time during which the exciter must become active if it is to be effective in assisting to maintain system stability.

II. MAIN EXCITERS

The main exciter is a source of field current for the principal electric machine. Thus, any d-c machine that might be used to serve this purpose can be called a main exciter. Seldom are storage batteries used as main exciters. With a main generator of any appreciable size, the difficulties encountered in finding room for the battery, in maintaining the charge, and in keeping the battery in good operating condition are such as to make it impractical. Many other types of d-c machines have been developed

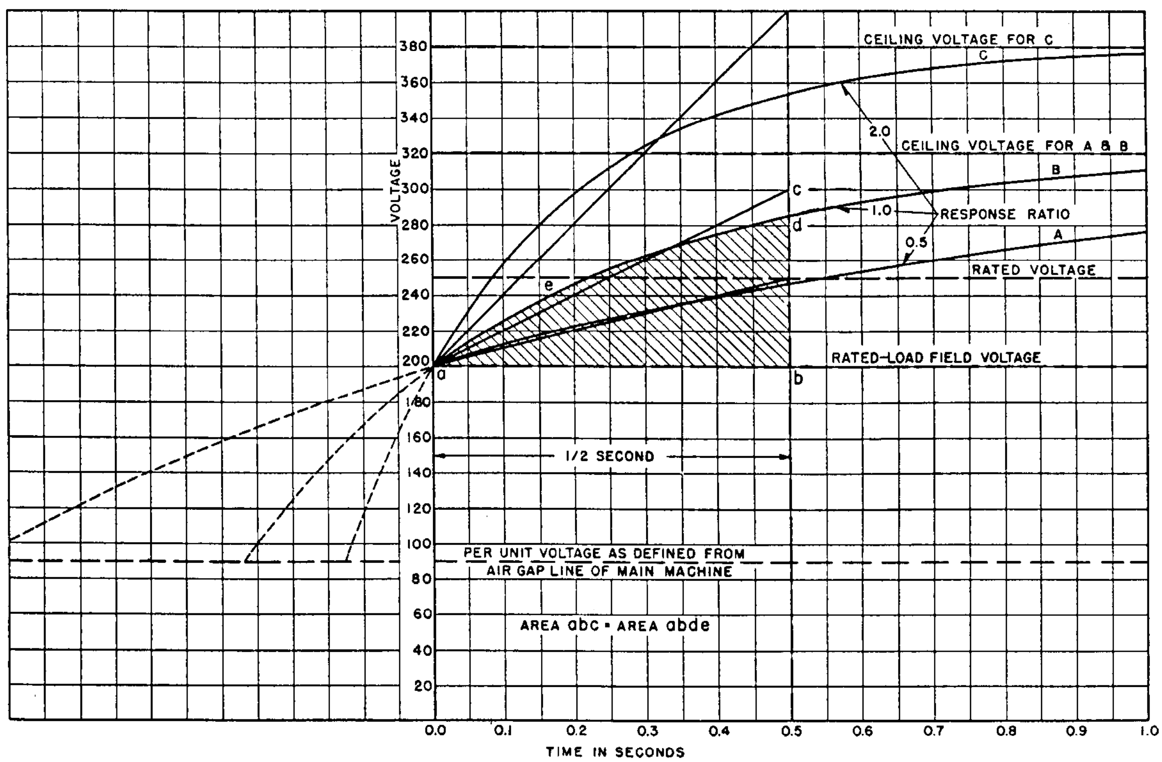


Fig. 2—Construction for determining main-exciter response ratio showing relative values of important quantities for 250-volt main exciter.

to a high degree of specialization for use as main exciters that offer many operating and maintenance advantages over a battery.

Main exciters, in general, can be grouped into two classifications; i.e., rotating and non-rotating d-c machines. The most common form of rotating main exciter is the more or less conventional d-c generator. The term "conventional" is used with reservation since a d-c generator built for the purpose of supplying excitation for a synchronous machine has incorporated in it many features to improve reliability and reduce maintenance not found on d-c generators used for other purposes. Aside from these special features, the theory of operation is the same as the conventional d-c generator. A new form of rotating exciter that has made its appearance in recent years is the main-exciter Rototrol. The Rototrol or rotating amplifier is very different in its operation from the conventional main exciter. The major static or non-rotating form of main exciter is the electronic exciter.

Each of these d-c machines, in regard to its application as a main exciter, is discussed in detail in the sections that follow.

1. Prime Movers for Main Exciters

Rotating main exciters are of either the direct-connected type or the separately-driven type. A direct-connected main exciter is one coupled directly to the shaft of the main generator and rotates at the same speed. A modification is the geared or shaft-driven exciter, driven through a gear by the shaft of the main generator. Problems of gear maintenance are introduced, but this enables the two machines to operate at different speeds. A separately-driven main exciter is usually driven by a motor, the complete unit being called an exciter m-g set, or it can be driven by some other form of prime mover such as a steam turbine or a hydraulic turbine.

Loss of excitation of an a-c generator generally means that it must be removed from service. Hence a reliable source of excitation is essential. If the main exciter should stop running while the main generator is still capable of operating, blame for the resultant outage would be placed on the main exciter. Considerable expense, therefore, can be justified to provide a reliable source of power to drive the main exciter. The type of drive accepted as reliable depends upon the type of synchronous machine being excited; that is, whether it be a generator or a synchronous condenser.

Exciter M-G Set—The exciter m-g set can be driven by a synchronous or induction motor. Direct-current motors have been used in some cases. The synchronous motor drive is undesirable, because of the possibility of transient disturbances on the motor supply system causing instability. Induction motors are ordinarily applied where the exciter m-g set is used. In any event, the motor must be specially designed to drive the main exciter through any form of system disturbance.

Power supply for the motor is, of course, important. The exciter m-g set might be classed as an essential auxiliary for operation of the generator, and may receive its power from the auxiliary power-supply system. Most essential auxiliaries have a dual power supply comprising a normal and an emergency supply, and automatic quick-transfer to the emergency supply is provided in case of failure of the normal supply. In some cases, dual prime movers are used such as a motor and a steam turbine, the turbine taking over the drive when the motor power supply fails. The driving motor can be connected directly to the main generator terminals through an appropriate transformer. It is then subject to voltage disturbances on the main system.

The motor is apt to be subjected to voltage disturbances regardless of the source of its power supply, and it is necessary to construct the m-g set so that it can withstand these disturbances without affecting the excitation of the main a-c generator. The inertia constant of the m-g set and the pull-out torque of the motor must be high enough to assure that the speed of the set does not change appreciably or the motor stall during momentary voltage dips. The response ratio and ceiling voltage of the exciter must take into consideration any speed change that may occur. In arriving at values for these various factors, it is necessary that some time interval and voltage condition for the system disturbance be chosen. A common requirement is that the exciter m-g set be capable of delivering maximum forcing excitation to the generator field during a system disturbance when the motor voltage is 70 percent of normal for a period of one-half second. Based on this criterion, characteristics of the exciter m-g set have become fairly well standardized as follows:

Inertia constant of the entire m-g set, $H=5.0$.
Pull-out torque of driving motor, $P_{max}=500$ percent.
Response ratio of main exciter when operating at rated speed, $R=2.0$.
Nominal exciter ceiling voltage when operating at rated speed, $E_{max}=160$ percent.

When an exciter m-g set is used with a synchronous condenser, the logical source of power for the motor is the system that energizes the condenser. In this respect, the use of exciter m-g sets with synchronous condensers does not involve many complications.

Direct-Connected Exciter—The most reliable prime mover for the main exciter is the same prime mover that drives the a-c machine being excited. This was realized many years ago when main exciters were first coupled to the shafts of the generators. The reliability of this form of drive is obvious and no elaboration is necessary. However, in the case of high-speed turbine generators, early installations experienced trouble in operation of the d-c exciters at high speeds. These difficulties have been completely overcome by adequate design of the exciter, special features being included for operation at 3600 rpm. Direct connection of the main exciter is widely accepted in the utility industry.

2. Conventional Main Exciters

Conventional main exciters, in general, can be classified according to their method of excitation, being either self-excited or separately-excited. In the former the field winding or windings are connected across the terminals of the machine through variable resistors and in the latter the field windings with their resistors are connected to a source

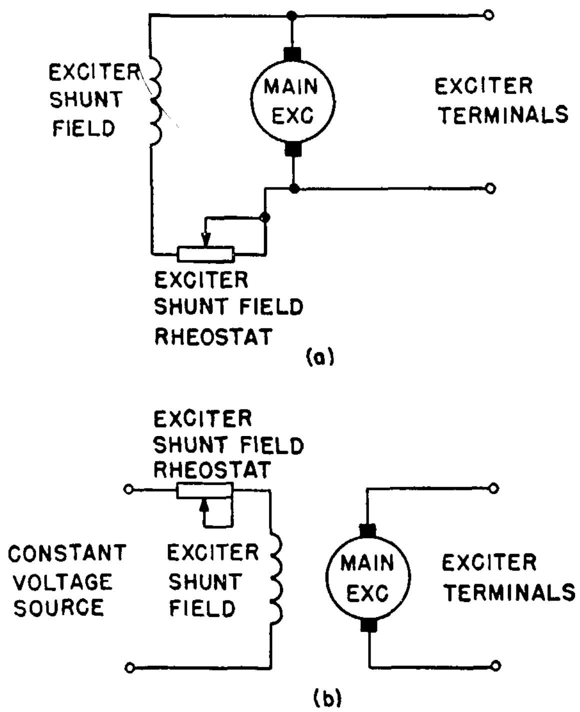


Fig. 3—Two common forms of shunt-excited main exciters.

(a) self-excited.
(b) separately-excited.

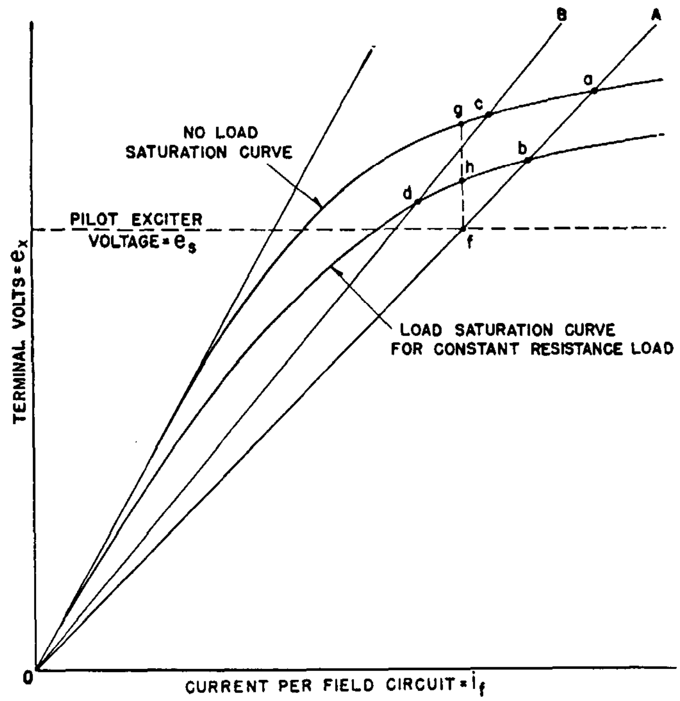


Fig. 4—Steady-state operating points for unloaded and loaded self-excited and separately-excited machines.

of essentially constant voltage such as a small auxiliary flat-compounded exciter, called a pilot exciter. The basic connections of these two forms of main exciter are shown in Figs. 3(a) and (b).

The curve *oca* in Fig. 4 represents the no-load saturation curve of a conventional d-c generator that might be used as a main exciter. An examination of the curve reveals that for values of voltage less than approximately 75 percent of rated armature voltage substantially all of the field current is expended in forcing magnetic flux across the air gap of the machine. In this region the voltage output is directly proportional to the field current, and a line drawn coinciding with the straight portion of the curve is called the *air-gap* line. Above the straight-line portion of the curve, the voltage output is no longer proportional to the field current, and a given percentage increase in voltage output requires a greater percentage increase in the field current. Under this condition, the machine is saturated and a greater proportion of the field ampere-turns are used in forcing flux through the magnetic circuit.

The field windings of the main exciter are frequently divided into two or more parallel circuits and in the present discussion the field current is always referred to as the current in one of the parallel circuits. For either the self- or separately-excited exciter, the terminal voltage is varied by simply changing the resistance of the field circuit. The field resistance line OA in Fig. 4 is drawn so that its slope is equal to the resistance of the field, that is, the ordinate at any point divided by the field current is the total resistance of one circuit of the field winding. At no-load, the intersection of the no-load saturation curve with the line OA determines the operating point, namely a. For the particular constant-resistance load for which the line *odb* represents the saturation characteristic, the operating point is likewise the intersection with OA, namely b. If some resistance is inserted in the field circuit so that its resistance line is changed to OB, then the operating point is c for the no-load condition and d for the constant-resistance load condition. In this manner of changing the exciter field resistance, any exciter voltage within limits can be obtained.

Should the field resistance be increased so that the resistance line coincides with the air-gap line, the output voltage theoretically can establish itself at any value between zero and the point where the no-load saturation curve begins to bend away from the air-gap line. Operation in this region is unstable unless some artificial means of stabilizing is provided.

On the other hand, if the machine were separately-excited by a pilot exciter, the field current is determined by the intersection of the resistance line with the pilot-exciter voltage line. Thus in Fig. 4, for the resistance line OA and the constant pilot-exciter voltage e_s, the field current of the exciter is determined by the intersection at f, and the terminal voltages for no-load and constant-resistance load are at points g and h, respectively.

3. Calculation of Response of Conventional Main Exciters

It will be observed that the definition of exciter response is based upon the no-load voltage build-up curve. This may differ in several essential points from the load condition which will be discussed later. For the present, the response will be calculated for the no-load condition and will be applied to a self-excited machine.

If

e_x = terminal voltage of the exciter and also the voltage across its field circuit

i_f = field current per circuit in amperes

r_f = total resistance of each field circuit in ohms

ψ = flux linkages per circuit of the field winding in 10^{-8} lines-turns

then there exists for the field circuits the following equation:

$$e_x = r_f i_f + \frac{d\psi}{dt} \tag{1}$$

where each term is expressed in volts. This expression can be rewritten in the following form

$$\frac{d\psi}{dt} = e_x - r_f i_f \tag{2}$$

The flux linkages, ψ, can be regarded as made up of two components; first, those produced by the useful flux in the air gap and, second, those produced by the leakage fluxes. The first component is proportional to the no-load terminal voltage as this is the flux which produces that voltage. The designer can give the useful flux at any particular voltage or it can be obtained from the design constants of the machine. Multiplying this flux in 10^{-8} lines by the turns, N, linked by the flux, which is equal to the number of turns per pole times the number of poles per circuit, gives the total linkages due to this component. These linkages may be designated as $k_u e_x$, where, to be specific with respect to the particular voltage concerned, we may write

$$k_u = \frac{\begin{pmatrix}\text{total useful flux linkages}\\ \text{per pole at rated voltage}\end{pmatrix}\begin{pmatrix}\text{number of poles}\\ \text{per circuit}\end{pmatrix}}{\text{rated voltage}} \tag{3}$$

The leakage component is more complex as not all of the leakage flux cuts all of the turns. If there were no saturation effects in the pole pieces and yoke, the leakage fluxes would be proportional to the field current. If, however, the leakage fluxes are specified at some definite current such as that required to produce rated voltage at no load, then the leakage at higher currents will be less than proportional to the current and at lower currents will be more than proportional to that at the specified point. Inasmuch as the leakage flux is only about 10 percent of the useful flux, considerable error is permissible in the leakage component without affecting the result significantly. The leakage flux may be said to contribute the flux linkages $k_l i_f$ to the total. The coefficient k_l can be defined by requesting from the designer both the flux linkages per pole at rated voltage due to the useful flux and the total flux linkages per pole at rated voltage. The coefficient k_l is then

$$k_l = \frac{\begin{pmatrix}\text{Total } \psi & & \psi \text{ per pole due}\\ \text{per pole at} & - & \text{to useful flux}\\ \text{rated voltage} & & \text{at rated voltage}\end{pmatrix}\begin{pmatrix}\text{Number}\\ \text{of poles}\\ \text{per circuit}\end{pmatrix}}{i_f \text{ at rated voltage}} \tag{4}$$

The total flux linkages per circuit are then

$$\psi = k_u e_x + k_l i_f \tag{5}$$

These quantities are illustrated in Fig. 5.

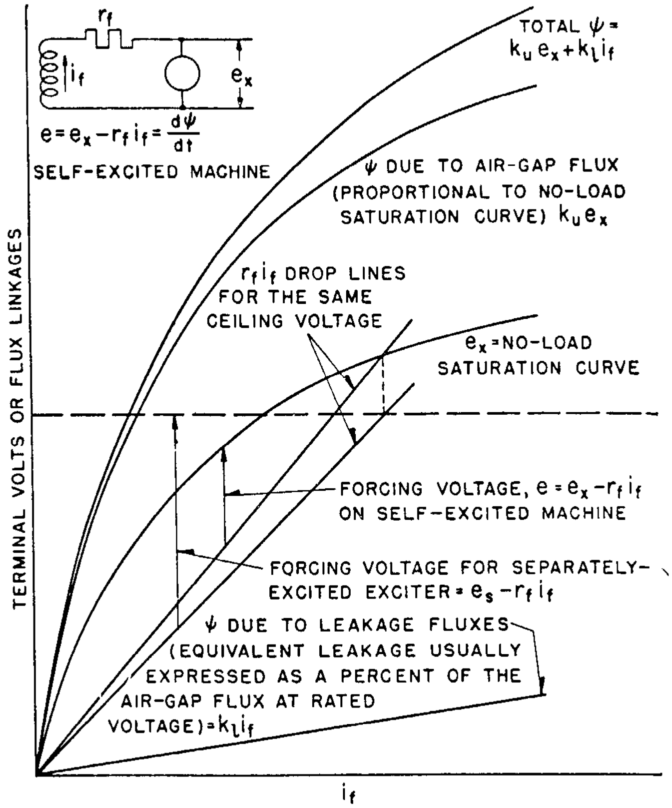


Fig. 5—Forcing voltages and flux linkages concerned in calculating response.

Equation (2) states that the time rate of rise of ψ is proportional at any instant to a forcing voltage which is equal to the vertical distance between the terminal-voltage curve and the straight-line curve of resistance drop at any given field current. It shows that the flux within the machine will increase so long as $(e_x - r_f i_f)$ is positive, that is, until the point of intersection of the two curves, as shown in Fig. 5, is attained. Beyond this point $(e_x - r_f i_f)$ becomes negative. If, for any reason, the flux within the machine extends beyond this point, it will decrease.

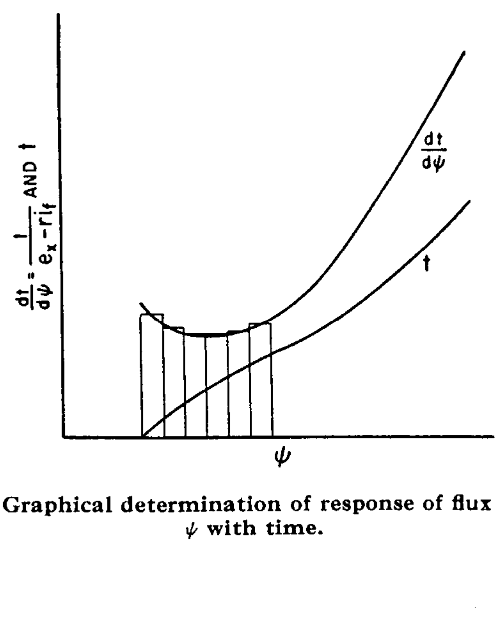


Fig. 6—Graphical determination of response of flux linkages ψ with time.

In other words, the intersection is a stable operating point.

Equation (2) can be transformed to

$$dt=\frac{d\psi}{e_x-r_f i_f} \tag{6}$$

from which

$$t=\int_0^t dt=\int_{\psi_{\text{initial}}}^{\psi}\frac{d\psi}{e_x-r_f i_f} \tag{7}$$

By choosing particular values of i_f from Fig. 5, it is possible to plot ψ as a function of $\frac{1}{e_x-r_f i_f}$ or $\frac{dt}{d\psi}$ shown in Fig. 6. From Eq. (7) it can be seen that t can be obtained as a function of ψ by simply obtaining the area of the vertical strata of increments, starting from ψ corresponding to the starting value of e_x. After ψ is obtained, e_x can be plotted as a function of time by taking corresponding points from Fig. 5. The simplest method for obtaining the area is to divide the region into a large number of increments and then sum them progressively on a recording adding machine.

If the machine is separately excited, the variable terminal voltage e_x in the expression for the forcing voltage should be replaced by the voltage e_s of the pilot exciter and the forcing voltage then becomes $(e_s-r_f i_f)$, which is illustrated in Fig. 5. The difference in these forcing voltages shows why separately-excited exciters are usually faster in response.

When systematized, it is found that this calculation is quite simple, as will be illustrated by an example. Let it be desired to determine the exciter response for the separately-excited machine whose characteristics are given in Fig. 7. In Table 1, columns (1) and (2), tabulate the terminal voltage and field currents from Fig. 7. Columns (3) and (4) are simply steps in the determination of the total ψ of column (5). Columns (6) and (7) are likewise steps in the determination of $\frac{dt}{d\psi}$ of column (8). From this point a choice may be made of two procedures. If the graphical method is used, plot the value of $\frac{dt}{d\psi}$ from column (8) as ordinate against the value of ψ from column (5) as abscissa

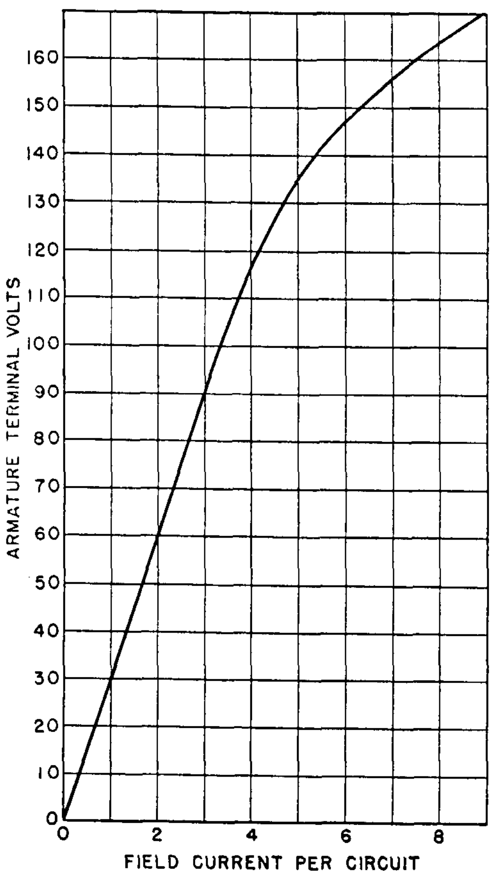


Fig. 7—Example for calculation of response of exciter.

167 kw, 125 volts, 1200 rpm, 6 poles
Separately excited—$e_s=125$ volts
Three circuits—two poles per circuit
Ceiling voltage—165 volts.
i_f at ceiling voltage = 8.16 amperes per circuit
Resistance per circuit = 15.3 ohms
Two field windings = 6.8 ohms
External resistance per circuit = 8.5 ohms
Total external resistance = 2.8 ohms
ψ per pole at 125 volts due to useful flux = 18
Total ψ per pole at 125 volts = 20.3

$k_u=\frac{18\times 2}{125}=0.288$ $\qquad k_1=\frac{(20.3-18)2}{4.4}=1.05$

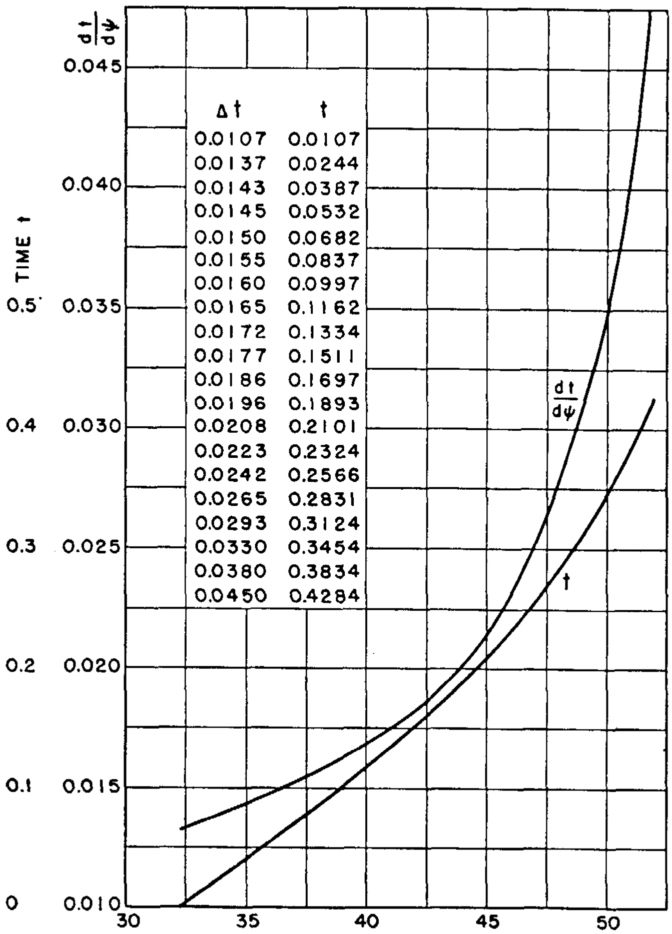


Fig. 8—Auxiliary curves for calculation of response for example given in Fig. 7.

TABLE 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
e_x	i_f	$k_u e_x$	$k_1 i_f$	ψ	$r_f i_f$	$e_s - r_f i_f$	$\frac{1}{e_s - r_f i_f} = \frac{dt}{d\psi}$	$\Delta\psi$	Mean $\frac{dt}{d\psi}$	Δt	t in sec.
		0.288×(1)	1.05×(2)	(3)+(4)	15.3×(2)	125−(6)	rec. of (7)	from (5)	from (8)	(9)×(10)	Σ(11)
100	3.30	28.8	3.5	32.3	50.5	74.5	0.0134	...	......	......	0
110	3.70	31.7	3.9	35.6	56.6	68.4	0.0146	3.3	0.014	0.0462	0.0462
120	4.14	34.5	4.3	38.8	63.4	61.6	0.0162	3.2	0.0154	0.0493	0.0955
130	4.66	37.5	4.9	42.4	71.3	53.7	0.0186	3.6	0.0174	0.0626	0.1581
140	5.34	40.3	5.6	45.9	81.7	43.3	0.0231	3.5	0.0208	0.0728	0.2309
150	6.26	43.2	6.6	49.8	95.8	29.2	0.0342	3.9	0.0287	0.112	0.343
155	6.80	44.6	7.1	51.7	104.0	21.0	0.0476	1.9	0.0409	0.078	0.421
160	7.46	46.1	7.8	53.9	114.0	11.0	0.091	2.2	0.0693	0.152	0.573

giving the curve shown in Fig. 8. Time can then be determined by integrating this curve. One method of doing this is by means of the table constituting the insert of this figure. This is found by dividing ψ into increments of unit width, except for the first element for which $\Delta\psi$ is only 0.8. This is done to obtain convenient divisions. Increments of time Δt are enumerated in the first column. The second column represents time, the summation of the Δt column. On the other hand, the same integration can be accomplished in tabular form. Continuing in Table 1, column (9), the difference of successive values of ψ from column (5), constitutes the base of increments of area of curve $\frac{dt}{d\psi}$ in Fig. 8. Likewise, column (10), the mean of successive values of column (8), constitutes the mean of elementary areas. The product of these two values tabulated in column (11) is the increment of time. Column (12) is merely a progressive summation of (11) and gives actual time. By plotting column (1) against column (12), the response curve is obtained.

For higher speeds of response, the eddy currents produced in the solid yokes can retard the buildup of the flux. The extent to which this is effective is given by the curve

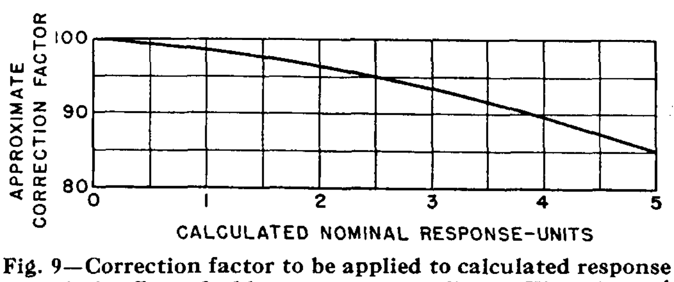


Fig. 9—Correction factor to be applied to calculated response to include effect of eddy currents, according to W. A. Lewis.[1]

in Fig. 9 by W. A. Lewis[1]. This curve supplies a correction to be applied to calculated responses.

Separately-excited exciters are usually, but not necessarily, faster in response than self-excited exciters. They do, however, have other advantages, such as being more stable at low voltages, voltages at which self-excited exciters may have a tendency to creep. Improvement in speed of response can be obtained by two general methods; (1) decreasing the time constant of the field circuit, and (2) increasing the pilot-exciter voltage in the case of separately-excited exciters or the ceiling voltage in the case of self-excited exciters. The former is usually accomplished by paralleling the field circuits placing at the same time resistors in series to limit the current. Thus, if the parallels are doubled, the number of poles and likewise ψ per circuit are halved. It is necessary to add more resistance to the external circuit so that the resistance per circuit remains the same. In Eq. (7) the only change is that ψ is one-half and, therefore, the terminal voltage rises twice as fast.

4. Calculation of Response Under Loaded Conditions

Most of the cases for which the exciter response is desired are concerned with sudden changes, such as short circuits, in the armature circuit of the synchronous machine. Associated with these changes one usually finds that the field current of the alternator has increased a considerable amount, perhaps in excess of the armature current rating of the exciter. Because of the high inductance of the field circuit of the synchronous machine, the armature current of the exciter can usually be regarded as remaining substantially constant at this increased value during the period for which the response is desired.

When current flows in the armature, the phenomenon of armature reaction must be taken into consideration except for those machines that have a compensating winding. The function of the compensating winding, which is wound into the pole face of the field winding, is to annul the effect of the cross-magnetizing mmf of armature reaction. However, for machines without compensating windings, the mmf of armature reaction produces an mmf that varies linearly from the center of the pole piece, one side being positive and the other side negative. This effect is shown in Fig. 10 (a) in which MN represents the maximum magnetizing mmf at one pole edge and PQ represents the maximum demagnetizing mmf at the other pole edge. Fig. 10 (b) represents a section of the no-load saturation curve in which O represents the generated voltage on the vertical co-ordinate and the field mmf on the horizontal co-ordinate. If A and C are so laid off that OA and OC equal MN and PQ, respectively, from Fig. 10 (a), then because of the linearity of QN of Fig. 10 (a), the abscissa of Fig. 10 (b) between CA represents the mmf distribution along the pole face. Further, since the generated voltages are propor-

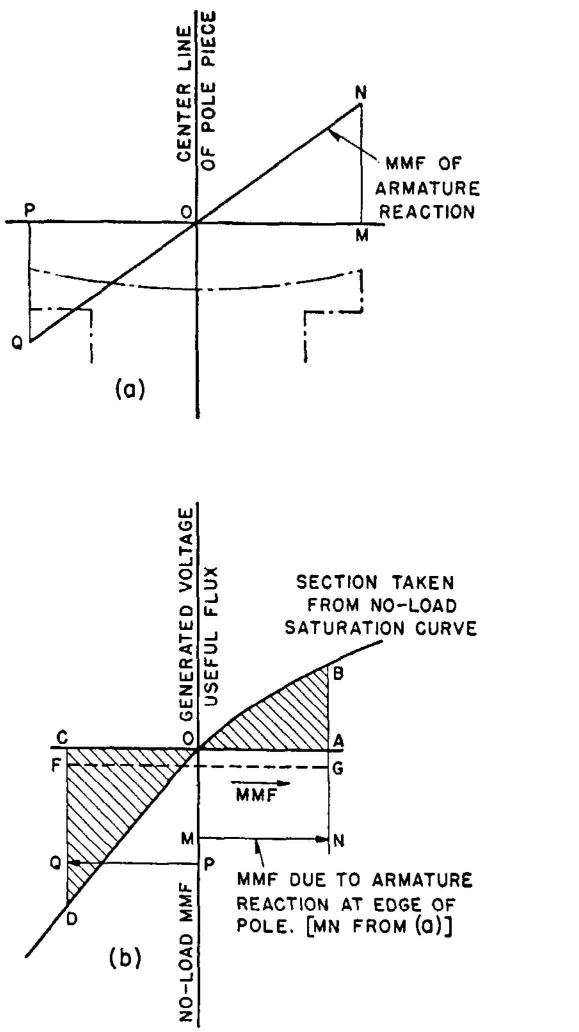


Fig. 10—Effect of armature reaction in reducing total flux across gap. (a) Shows distribution of armature mmf; (b) Section of no-load saturation curve.

tional to the air-gap fluxes, the section of no-load saturation curve shows the effect of the superposed armature mmf upon the density of air-gap flux across the pole. The higher mmf does not increase the flux on the right-hand side as much as the lower mmf decreases the flux on the left-hand side. As a result, the total flux and consequently the generated voltage are decreased from the value indicated by CA to that indicated by FG, which is obtained by integrating the area under the curve DOB and drawing FG so that the two triangular areas are equal. The extent to which the average flux or voltage is decreased can be indicated by a "distortion curve," such as shown by the dotted curve of Fig. 11. This effect is most pronounced in the region of the knee of the saturation curve as at both higher and lower field currents, there is a tendency to add on the one side of the pole just as much flux as is subtracted on the other. The terminal voltage is reduced still further by the armature resistance and brush drops, resulting in a load saturation curve for constant current, such as shown in Fig. 11.

From this same curve it can be seen that for a given field resistance line, the forcing voltage $(e_x - r_f i_f)$ for a self-excited

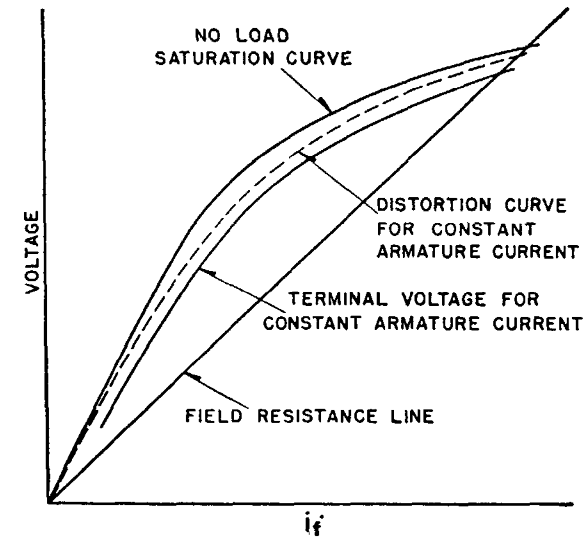


Fig. 11—Load saturation curves for exciter assuming constant armature current.

machine is very much smaller under load than under no load. In calculating the flux linkages in accordance with Eq. (5), the distortion curve should be used for e_x. Except for these two changes, the load response can be calculated in the same manner as the no load response.

For separately-excited exciters, the forcing voltage remains unaltered by the loading on the machine as it is independent of the terminal voltage. The armature resistance can be regarded as part of that of the main field winding. There remains only the distortion effect to consider which amounts to only several percent. For machines with compensating windings, this effect is negligible.

5. Effect of Differential Fields on Response

Differential windings are provided to reduce the exciter voltage to residual magnitude or below. They consist of a small number of turns wound on each pole, so connected that the mmf produced thereby is opposite to that of the main windings. Fig. 12 (a) shows schematically such an arrangement. If the differential windings are not opened when the regulator contacts close to produce field forcing, the differential circuit reduces the response of the exciter. The extent to which this is effective may be calculated as follows: Let

a = number of parallel paths in the main winding.
b = number of parallel paths in the differential winding.
c = number of turns per pole of the main winding.
d = number of turns per pole of the differential winding.
N = total number of poles of exciter.
i_m = current per circuit of main winding.
i_d = current per circuit of differential winding.

The resistors R_m and R_d in series with the combined main and differential windings, respectively, may be included in the calculation by increasing the actual resistances in each of the main and differential circuits by aR_m and bR_d, respectively. With these increases the resistances of each of the main and differential circuits will be designated by the

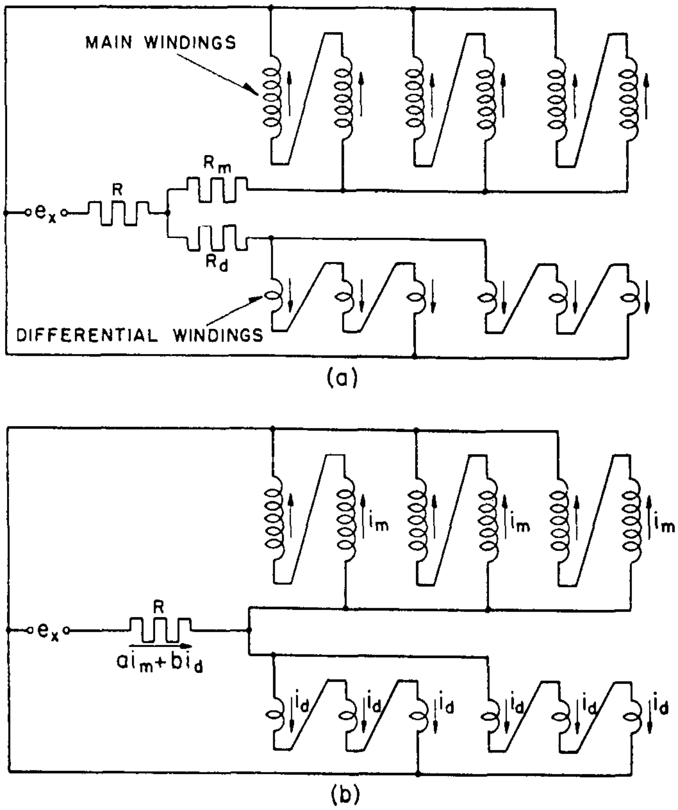


Fig. 12—Schematic diagram for main and differential windings.

symbols r_m and r_d, respectively. Referring to Fig. 12 (b) the following equations can be written

$$e_x=R(ai_m+bi_d)+r_m i_m+\frac{d\psi}{dt} \tag{8}$$

$$e_x=R(ai_m+bi_d)+r_d i_d+\frac{d\psi_d}{dt} \tag{9}$$

in which ψ and ψ_d are the flux linkages in each of the two respective circuits.

If all the field flux cuts all turns, then

$$\psi=\frac{N}{a}c\times(\text{flux per pole in } 10^{-8} \text{ lines})$$

$$\psi_d=\frac{N}{b}d\times(\text{flux per pole in } 10^{-8} \text{ lines})$$

or

$$\psi_d=\frac{ad\psi}{bc}. \tag{10}$$

If it be assumed that the two windings be replaced by another winding having the same number of turns and circuit connections as the main windings, then the instantaneous mmf of this winding is the same as that of the combination if its current, i, is

$$i=i_m-\frac{d}{c}i_d$$

from which

$$i_m=i+\frac{d}{c}i_d \tag{11}$$

If (10) and (11) are inserted in (8) and (9), then

$$e_x=(Ra+r_m)(i+\frac{d}{c}i_d)+Rbi_d+\frac{d\psi}{dt} \tag{12}$$

$$e_x=Ra(i+\frac{d}{c}i_d)+(Rb+r_d)i_d+\frac{ad}{bc}\frac{d\psi}{dt} \tag{13}$$

By multiplying (13) by $\frac{bc}{ad}$, $\frac{d\psi}{dt}$ can be eliminated by subtracting from (12). The current i_d can then be solved in terms of i. Upon substituting the expression for i_d into (12) there is finally obtained that

$$\frac{1-\frac{d}{c}\frac{r_m}{r_d}}{A}e_x=\frac{1+R\left(\frac{a}{r_m}+\frac{b}{r_d}\right)}{A}r_m i+\frac{d\psi}{dt} \tag{14}$$

in which

$$A=1-\frac{ad^2}{bc^2}\frac{1}{r_d}\left[r_m+\left(\frac{b^2c^2}{ad^2}-a\right)R\right] \tag{15}$$

Equation 14 shows that the ordinary flux-linkage curve for the exciter and conventional method of calculation can be used if the coefficient of i be used as the resistance of each circuit, i be the current read from the saturation curve, and the voltage across each circuit be multiplied by the coefficient of e_x. In other words, the calculations should be carried out as though the differential winding were not present, except that instead of using the expression $(e_x-r_f i_f)$ to determine the forcing voltage, e_x should be multiplied by $\left(1-\frac{d}{c}\frac{r_m}{r_d}\right)\Big/A$, and r_f by $\left[1+R\left(\frac{a}{r_m}+\frac{b}{r_d}\right)\right]\Big/A$.

6. Three-Field Main Exciter

The three-field main exciter shown schematically in Fig. 13 is of conventional construction so far as mechanical details and armature winding are concerned, but it is built with three electrically independent shunt fields. Field 1 is connected in series with a variable resistance across the

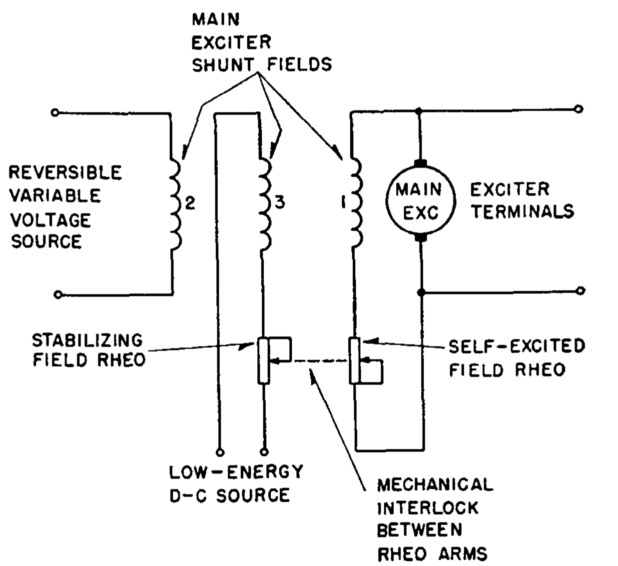


Fig. 13—Schematic diagram of three-field main exciter. Field 1 is self-excited and provides base excitation, field 2 is a separately-excited controlling field, and field 3 is a small-capacity battery-excited stabilizing field.

main terminals of the exciter and operates in the manner of the self-excited field discussed in Sec. 1. Field 1 provides the base excitation for the machine. Field 3 is a small separately-excited shunt field that obtains its energy from a station battery or any other source of substantially constant d-c voltage. It is capable of supplying 5 to 10 percent of the normal total excitation requirements of the main exciter, and its purpose is to provide exciter stability at low voltage output under hand control. Field 3 is used only when the exciter speed of response or range of voltage output makes it desirable. Field 2 is a shunt field that is excited from a reversible variable-voltage d-c source under control of a voltage regulator. This field also provides for stability of the exciter when the voltage regulation is under control of the voltage regulator.

Fields 1 and 3 have rheostats in their energizing circuits. These are usually motor-operated under manual control. The rheostat arms are mechanically connected together so that resistance is added in one field circuit as it is removed from the other. Thus, when the self-energized shunt field is carrying a high excitation current, the separately-excited field 3 carries a negligible current. The combined effect of fields 1 and 3 is shown in Fig. 14 and can be explained by assuming that the current in field 2 is zero. When the field rheostat is adjusted to give a voltage output greater than that represented by the distance Oc, all excitation is supplied by field 1, and the relation between the exciter terminal voltage and the total field ampere-turns is represented by the line ab. Operation in this region is the same as a self-excited exciter. If the resistance in the circuit of field 1 were increased to give a value of ampere-turns less than Od in Fig. 14, and if field 1 were the only field excited, the machine would be unstable as pointed out in Sec. 1.

To obtain a terminal voltage less than Oc, such as Of, the resistance in the self-excited field circuit would be increased to reduce the ampere-turns produced by that field to Oj. These ampere-turns would cause a generated voltage equal to Oh. However, at the same time the current in field 1 is reduced, the current in field 3 is increased, and the generated voltage due to field 3 being energized is represented by hf. The ampere-turns of the two fields and the generated voltages add so that the distance Of is the total terminal voltage. Since the current in field 3 is controlled by the amount of current in field 1 through the mechanical coupling of the field-rheostat arms, the total terminal voltage can be plotted as a function of the ampere-turns in field 1 alone and is represented by the curve $ekab$ in Fig. 14. If the field-resistance characteristic of the self-excited field is plotted on the same curve, there will always be a positive point of intersection between the resistance line and the saturation curve $ekab$ and stable operation can be obtained for any voltage greater than Oe. The voltage represented by Oe is usually less than 10 percent of the rated voltage of the exciter. Operation at smaller values would not ordinarily be necessary except in the case of a synchronous-condenser exciter. Smaller terminal voltages are obtained by holding the current in the self-excited field to zero and reducing the current in separately-excited field 3. Exciter polarity can be reversed by reversing both field circuits when the currents are zero and building up

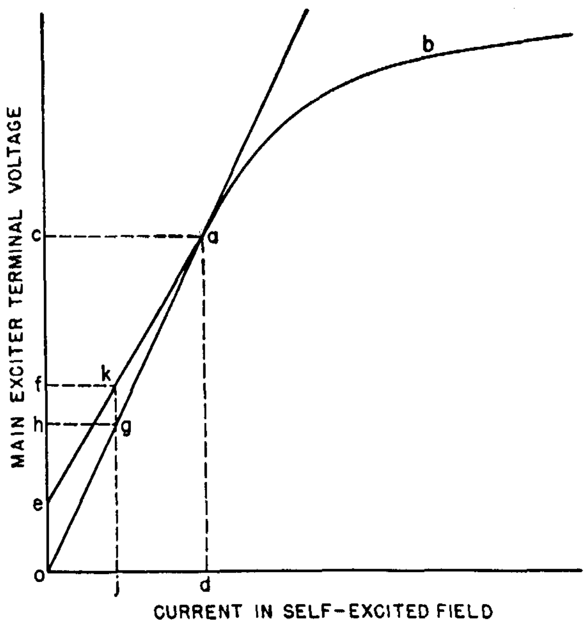


Fig. 14—Equivalent no-load saturation curve of three-field main exciter showing effect of stabilizing field 3. Field 2 is open-circuited.

in the opposite direction. Thus, manual control of voltage is possible over the complete range necessary.

When the voltage of the main exciter is under the control of a voltage regulator that varies the magnitude and polarity of voltage applied to the separately-excited field 2, the manually-operated field rheostat in field 1 circuit is set to provide some base amount of excitation. This setting is determined by the operator, but is generally high enough to supply sufficient field current to the a-c generator field to maintain steady-state stability. The current in field 3 is usually negligible with such a setting of the rheostat when the generator is carrying any load. The polarity and magnitude of the voltage applied to field 2 are then regulated so that the flux produced by field 2 either aids or opposes the flux produced by the base excitation in field 1, thus, either increasing or decreasing the exciter terminal voltage. Since the effect of field 1 is that of a conventional self-excited machine, a small amount of energy input to field 2 can control the output voltage over a wide range. The operation of the three-field main exciter is made stable by separate means for the two conditions of operation: by a separately-excited stabilizing field under manual control, and by the voltage regulator controlling the input to field 2 under regulator control.

The three-field main exciter has an advantage over the single-field separately-excited main exciter described in Sec. 1 in that control of the exciter terminal voltage is not completely lost if any trouble should occur in the separately-excited field circuit. The trouble might involve the variable-voltage source for field 2 or the voltage regulator that controls it, but even though the current in the field should become zero, the exciter will continue operating at a terminal voltage determined by the setting of the rheostat in the self-energized field circuit. The only effect on the a-c generator would be a change in its internal voltage which would cause a change in reactive loading of the machine. Under similar circumstances of failure with the single-field exciter, the source of excitation for the a-c generator field would be lost and a shut-down of the unit would be necessary.

7. Calculation of Response of Three-Field Main Exciter

A method of calculating the response of a single-field exciter is given in Sec. 2. The method uses step-by-step integration to take into account the saturated condition of the exciter. If additional fields are present, damping currents flow in those fields during voltage changes. Their effect is to reduce the rate of change of flux in the exciter iron paths. The following analysis presents a means of replacing the assembly of several fields with one equivalent field so that the response can be calculated.

The specific fields involved in the three-field main exciter are the self-excited field 1, the battery-excited field 3, and the separately-excited field 2 as shown in Fig. 13. The three fields are wound to form a single element to be mounted on the field pole, so that the mutual coupling is high and can be assumed to be 100 percent with small error. Also, the same leakage coefficient can be applied to each of the fields. In the following symbols the subscript indicates the particular field to which the symbol applies. Thus, N_1 is the *turns per pole of field 1*, N_2 the *turns per pole of field 2*, etc.

P = Number of poles, assumed to be connected in series.
N = Number of turns per pole in the field winding.
ϕ' = Total useful flux per pole in Maxwells times 10^8.
ϕ_0 = Initial useful flux per pole in Maxwells times 10^8.
ϕ = Change in flux per pole $= \phi' - \phi_0$.
i' = Total amperes in field circuit.
i_0 = Initial amperes in field circuit.
i = Change in amperes in field winding $= i' - i_0$.
L = Inductance of field winding in Henrys.
K = Flux proportionality constant $= \dfrac{\text{Maxwells} \times 10^8 \text{ per pole}}{\text{Ampere turns per pole}}$
λ = Flux leakage factor $= 1 + \dfrac{\text{leakage flux}}{\text{useful flux}}$
c = Voltage proportionality constant $= \dfrac{\text{terminal volts}}{\text{Maxwells} \times 10^8 \text{ per pole}}$
R = Resistance of the complete field circuit, ohms.
t = Time constant of complete field circuit, seconds.
E_t' = Terminal voltage applied to field 1.
E_{t0} = Initial value of terminal voltage.
E_t = Change in terminal voltage $= E_t' - E_{t0}$.
E_2' = Voltage applied to field 2.
E_{20} = Initial value of voltage applied to field 2.
E_2 = Change in voltage applied to field 2.
E_3' = Fixed voltage applied to field 3.
p = Differential operator $\dfrac{d}{dt}$.

The initial or steady-state value of total useful flux per pole is

$$\phi_0 = K(N_1 i_{10} + N_2 i_{20} + N_3 i_{30}). \tag{16}$$

When the field currents are changed to force an increase in terminal voltage, the total useful flux at any later instant of time is

$$\phi' = K(N_1 i_1' + N_2 i_2' + N_3 i_3'). \tag{17}$$

The change in total flux per pole is the difference between these two values,

$$\phi = \phi' - \phi_0 = K(N_1 i_1 + N_2 i_2 + N_3 i_3). \tag{18}$$

The basic formula for the self-inductance of any of the field circuits is

$$L = \frac{N\phi 10^{-8}}{i} \text{henrys},$$

and since the flux is expressed as Maxwells per pole times 10^8, the self-inductance of the circuit of field 1 becomes

$$L_1 = \frac{P\phi N_1 \lambda}{i_1} = PKN_1^2\lambda. \tag{19}$$

The time constant of the field circuit is the total self-inductance divided by the total resistance,

$$t_1 = \frac{L_1}{R_1} = \frac{PKN_1^2\lambda}{R_1}. \tag{20}$$

Equations similar to Eq. (19) can be written for self-inductances L_2 and L_3 and similar to Eq. (20) for time constants t_2 and t_3.

The voltage applied to each of the field circuits is absorbed in Ri drop in the circuit resistance and $N\dfrac{d\phi}{dt}$ drop in the circuit inductance. The voltage equations at any instant of time are

$$E_t' = c\phi' = R_1 i_1' + N_1\lambda P p\phi' \tag{21}$$
$$E_2' = R_2 i_2' + N_2\lambda P p\phi' \tag{22}$$
$$E_3' = R_3 i_3' + N_3\lambda P p\phi'. \tag{23}$$

During the initial steady-state conditions, when the total useful flux is constant and $p\phi_0 = 0$,

$$E_{t0} = c\phi_0 = R_1 i_{10} + N_1\lambda P p\phi_0 \tag{24}$$
$$E_{20} = R_2 i_{20} + N_2\lambda P p\phi_0 \tag{25}$$
$$E_{30} = R_3 i_{30} + N_3\lambda P p\phi_0. \tag{26}$$

Subtracting the two sets of voltage equations, a set in terms of changes from steady-state conditions is obtained. Since the voltage E_3' is supplied from a constant-potential source, $E_3' - E_{30} = 0$.

$$c\phi = R_1 i_1 + N_1\lambda P p\phi \tag{27}$$
$$E_2 = R_2 i_2 + N_2\lambda P p\phi \tag{28}$$
$$0 = R_3 i_3 + N_3\lambda P p\phi \tag{29}$$

If Eqs. (27), (28), and (29) are multiplied by $\dfrac{KN_1}{R_1}$, $\dfrac{KN_2}{R_2}$, and $\dfrac{KN_3}{R_3}$, respectively, and added, the result obtained after substituting from Eqs. (18) and (20) is

$$\frac{t_1}{PN_1\lambda}c\phi + \frac{t_2}{PN_2\lambda}E_2 = \phi + (t_1 + t_2 + t_3)p\phi. \tag{30}$$

Rearranging the terms in Eq. (30);

$$\frac{t_2}{PN_2\lambda}E_2 = \left[\left(1 - \frac{ct_1}{PN_1\lambda}\right) + (t_1 + t_2 + t_3)p\right]\phi. \tag{31}$$

When solved, Eq. (31) expresses ϕ and hence the terminal voltage as a function of time if saturation and the consequent change in constants are neglected.

The three fields on the exciter can be assumed to be replaced with a single equivalent self-excited field as shown in Fig. 15. The quantities referring to the equivalent field are designated by the subscript e. The field has applied to it a voltage equal to the terminal voltage $c\phi'$ plus an equivalent voltage E_e' supplied by the regulator. During steady-state conditions,

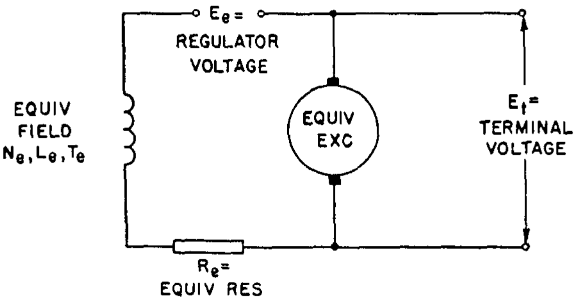


Fig. 15—Self-excited single-field equivalent of three-field main exciter.

$$E_{e0}+c\phi_0=R_e i_{e0}+N_e\lambda P\rho\phi_0. \tag{32}$$

At any instant of time,

$$E_e'+c\phi'=R_e i_e'+N_e\lambda P\rho\phi'. \tag{33}$$

Subtracting Eq. (32) from (33)

$$E_e+c\phi=R_e i_e+N_e\lambda P\rho\phi. \tag{34}$$

Using the relations

$$\phi=KN_e i_e \tag{35}$$

$$L_e=\frac{P\phi N_e\lambda}{i_e}=PKN_e^2\lambda \tag{36}$$

$$t_e=\frac{PKN_e^2\lambda}{R_e}. \tag{37}$$

Eq. (34) reduces to

$$\frac{t_e}{PN_e\lambda}E_e=\left[\left(1-\frac{ct_e}{PN_e\lambda}\right)+t_e\rho\right]\phi. \tag{38}$$

Equation (38) is of the same form as Eq. (31), and by comparing similar terms, it is derived that

$$t_e=t_1+t_2+t_3 \tag{39}$$

$$N_e=\frac{N_1}{t_1}t_e. \tag{40}$$

The self-inductance of the equivalent field is given by Eq. (36), and the resistance is

$$R_e=\frac{L_e}{t_e}. \tag{41}$$

The applied regulator voltage is

$$E_e=\frac{t_2N_e}{t_eN_2}E_2.$$

Eliminating $\frac{N_e}{t_e}$ by using Eq. (40)

$$E_e=\frac{t_2N_1}{t_1N_2}E_2. \tag{42}$$

Equations (38) and (31) can be solved only if saturation is neglected. However, for a small interval of time, it can be assumed that the machine constants do not change, and the change in flux calculated by either equation will be the same. If at the end of the first time interval, the machine constants are appropriately adjusted to new values applicable to the next small interval of time, the flux change can be calculated for the second interval and will be the same by either equation. Thus, the flux rise calculated from the equation for the single equivalent field by using the normal step-by-step methods that take into account saturation will be the same as the actual flux rise with the assembly of several fields. The various time constants for the machine in the unsaturated condition may be used to determine the constants of the equivalent field.

The above equations can be generalized to the case of a machine having any number of the three types of fields considered. Letting t_r, E_r and N_r refer to all coils to which regulator voltages are applied, and t_s and N_s refer to all coils which are self excited, Eq. (31) in the general form becomes

$$\sum\frac{t_rE_r}{PN_r\lambda}=\left[\left(1-c\sum\frac{t_s}{PN_s\lambda}\right)+\rho\Sigma t\right]\phi \tag{43}$$

where Σt = sum of time constants of coils of all types. The sum of the time constants should also include a value for the frame slab, which acts as a short-circuited turn, and eddy currents in the slab cause a delay in the flux rise. For d-c machines of the size used as main exciters, the frame-slab time constant may approach 0.2 second.

The constants of the equivalent self-excited field are determined from the following:

$$t_e=\Sigma t \tag{44}$$

$$N_e=\frac{t_e}{\sum\frac{t_s}{N_s}} \tag{45}$$

L_e is determined by Eq. (36)

$$R_e=\frac{L_e}{t_e} \tag{46}$$

and the regulator voltage to be applied

$$E_e=\frac{N_e}{t_e}\sum\frac{t_r}{N_r}E_r. \tag{47}$$

If no self-excited fields are present in the machine, the only requirements to be satisfied are given by Eqs. (44) and (47). Any value of N_e can be used provided the appropriate value of R_e is calculated from Eqs. (36) and (46). When no self-excited fields are present, the equivalent field is not self-excited and has applied to it only the regulator voltage.

If no regulator-controlled fields are present, the requirements to be met are given by Eqs. (44), (45), (46), and

(47), and the equivalent field is a self-excited field with no regulator voltage applied.

Using this equivalent single-field representation of the multiple-field main exciter, the voltage response can be calculated by the step-by-step method of Sec. 2. The voltage E is determined by the source of voltage under regulator control. For example, if the regulated field is a self-excited field, the voltage E becomes equal to the exciter terminal voltage at each instant of time.

8. Main-Exciter Rototrol

The most recent development in the field of rotating main exciters is the adaptation of the *Rototrol rotating amplifier* as a main exciter. Any generator is in fact a "rotating amplifier" in that a small amount of energy input to the field is amplified to a large energy output at the generator terminals. However, the name rotating amplifier has been specifically applied to a form of rotating machine possessing an unusually large amplification factor. In such machines, the change in input energy to the field is a small fraction of the resulting change in energy output of the armature. In the ordinary d-c generator, the change in field energy required to produce 100-percent change in output energy is usually within the range of 1 percent to 3 percent of the machine rating. Thus, the amplification factor might be between 30 and 100. In the case of the Rototrol, the amplification factor can exceed 10^6 depending upon the design of the machine.

The main-exciter Rototrol is not adaptable at present to use with generators operating at less than 1200 rpm. The principal field of application is with 3600-rpm turbine generators. The two-stage main-exciter Rototrol can be built with sufficient capacity to supply the excitation requirements of the largest 3600-rpm generator, but when used with 1800- or 1200-rpm generators, the maximum rating of generator is restricted. In any event, the Rototrol is direct-connected to the generator shaft.

The slower the speed of a generator, the larger the physical size. For a given voltage output, the reduction in speed is compensated by an increase in the total flux, requiring a larger volume of iron to maintain the same flux density.

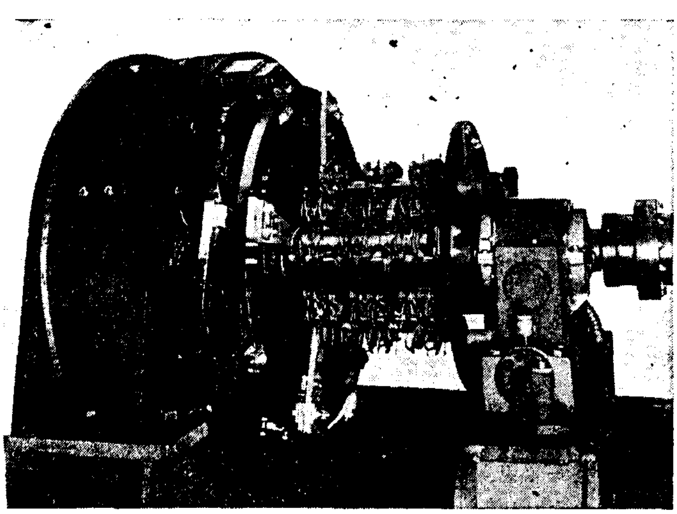


Fig. 16—A 210-kw, 250-volt, 4-pole main-exciter Rototrol for direct-connection to generator shaft at 3600 rpm.

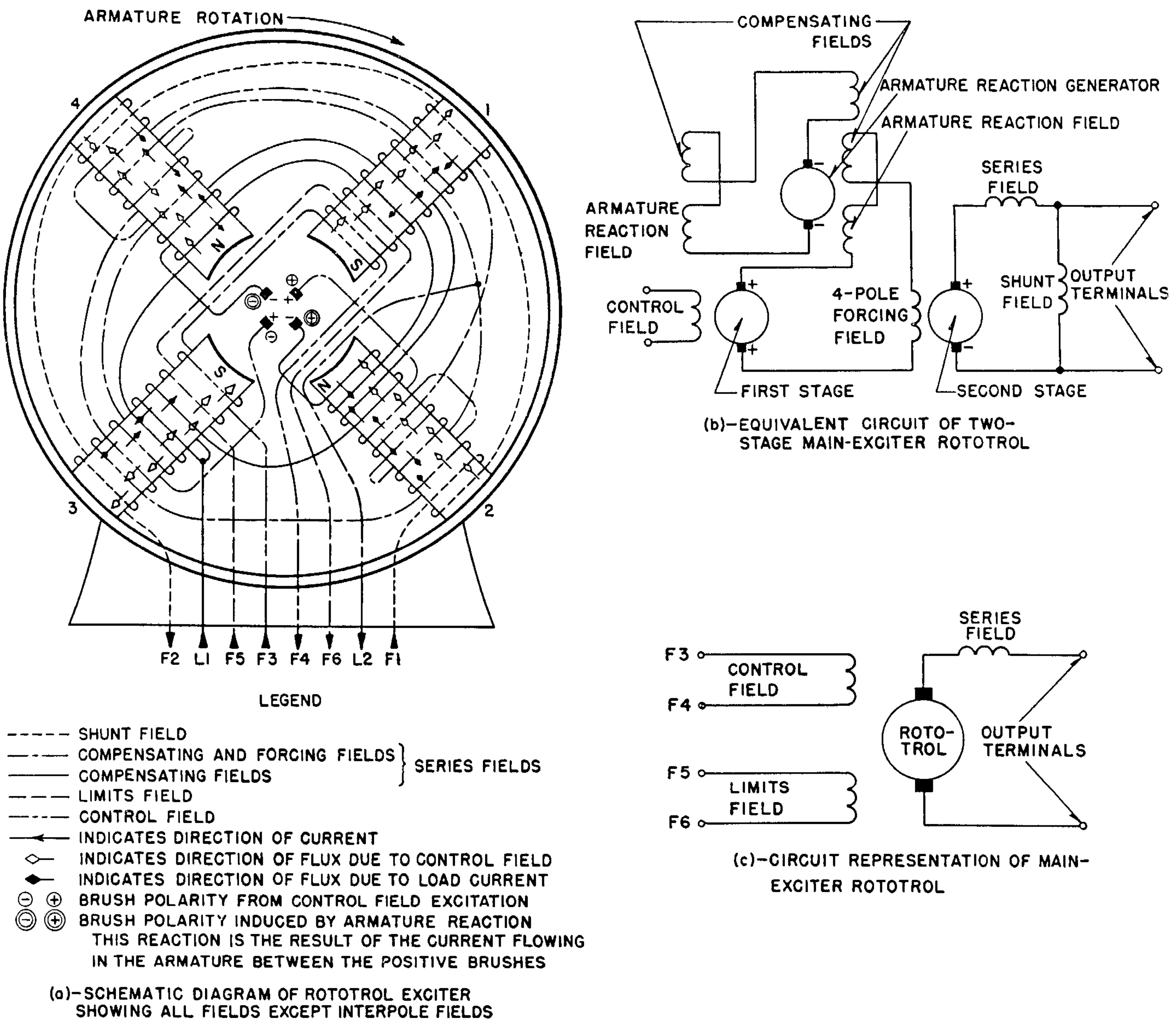
The excitation requirements, therefore, are greater for slow-speed generators. The main-exciter Rototrol has not been built in capacities large enough to supply the excitation requirements of large slow-speed a-c generators. Furthermore, as the Rototrol rated speed is decreased, its excitation requirements also increase and a larger controlling energy is required. The combination of these factors has largely restricted the use of the main-exciter Rototrol to direct-connection with 3600-rpm turbine generators.

A 210-kw, 250-volt, 3600-rpm main-exciter Rototrol is illustrated in Fig. 16, and to all outward appearances it is a conventional type of d-c machine. The mechanical details such as the enclosure, brush holders, commutator, etc., are of conventional 3600-rpm exciter construction, but the electrical connections are quite different. The armature winding is of the lap form but has no cross connections, and there are a number of specially-connected field windings to provide the high amplification factor.

A detailed discussion of the theory of operation of the Rototrol is beyond the scope of this chapter, and can be found in the References. The discussion here will be confined to a description of the operating principle as it applies to use of the Rototrol in excitation systems.

A schematic diagram of the main-exciter Rototrol is shown in Fig. 17 (a), and the equivalent schematic diagram is shown in Fig. 17 (b). The Rototrol can be built with one or more stages of amplification, and the main exciter Rototrol is of the two-stage type. The field connected between terminals F3-F4 is called the control field, and windings appear on only the two south poles, 1 and 3. The circuit between terminals F5-F6 energizes a field similar to the control field, and it also appears on only the two south poles. This field operates in the same manner as the control field in controlling the Rototrol terminal voltage but it is called the limits field. The control field is energized by the voltage regulator and normally has control of the voltage output. However, the limits field is energized by devices that restrict the maximum or minimum voltage output, so that the limits field can, under certain conditions, overcome the effect of the control field. The output terminals are L1-L2, and it should be noted that the circuit between the brushes of like polarity energizes additional field windings that are compensating and forcing fields and also serve as series fields. The windings energized by the circuit between terminals F1-F2 are shunt-field windings used for tuning purposes as discussed later. As far as external circuits are concerned, the main-exciter Rototrol can be represented as shown in Fig. 17 (c): the control field is energized by some exciter-voltage controlling device, the limits field is energized by a device for limiting the maximum or minimum output or both, and the line terminals supply voltage to the load in series with the series field.

The operation of a conventional self-excited d-c generator is unstable when the field-resistance line coincides with the air-gap line of the saturation curve as shown in Sec. 1. Although this characteristic is undesirable in the self-excited generator, it is an important part of the Rototrol principle. Reasoning identical to that in Sec. 1 can be applied to a series-excited generator where the self-excited winding is in series with the load and both the load and the field can be considered as a shunt across the armature.



(a)—SCHEMATIC DIAGRAM OF ROTOTROL EXCITER SHOWING ALL FIELDS EXCEPT INTERPOLE FIELDS

(b)—EQUIVALENT CIRCUIT OF TWO-STAGE MAIN-EXCITER ROTOTROL

(c)—CIRCUIT REPRESENTATION OF MAIN-EXCITER ROTOTROL

Fig. 17—Two-stage main-exciter Rototrol, complete schematic diagram and equivalent representations.

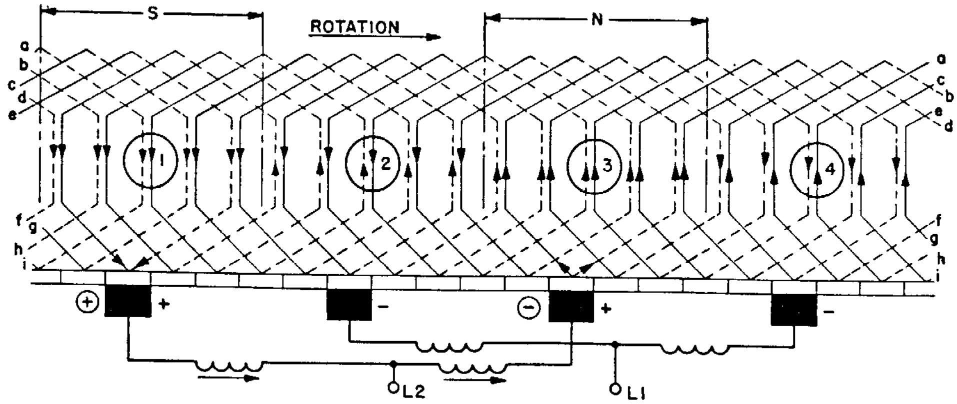
The series-field current then is directly proportional to the armature voltage in the same way as the shunt-field current in the self-excited shunt-wound machine.

The Rototrol is operated on the straight portion of its saturation curve and the adjustments necessary to meet this condition are termed tuning of the Rototrol. This is usually done by adjusting the resistance of the load or an adjustable resistance in series with the load, but can also be done by varying the air gap between the field poles and the rotor surface, which shifts the position of the air-gap line. Thus, the series-field circuit is tuned so that the resistance line of the circuit coincides with the air-gap line. Exact coincidence of the resistance line with the air-gap line cannot always be obtained by these two means so a small-capacity shunt field is provided to serve as a vernier adjustment. The resistance of the shunt-field circuit is adjusted to change the position of the terminal voltage-series-field current relation to tune the machine perfectly.

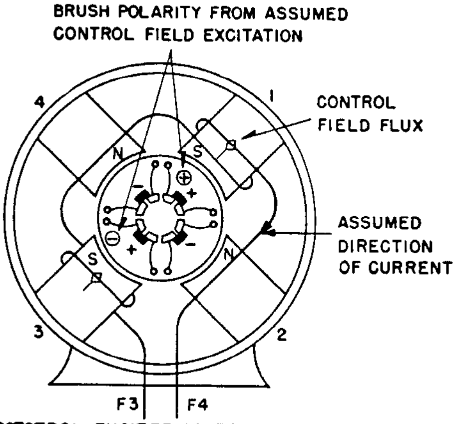
It is particularly significant that under steady-state conditions, the self-excited field of the Rototrol furnishes all of the ampere-turns required to generate the terminal voltage. However, the control field forces the change in ampere-turns required to stabilize the machine or to change and establish the terminal voltage required for a new load condition. The ampere-turns of the self-excited field and those of the control and limits fields are superimposed, and the algebraic sum of the ampere-turns on all of the Rototrol fields determines the terminal voltage.

9. Operating Principle of the Main-Exciter Rototrol

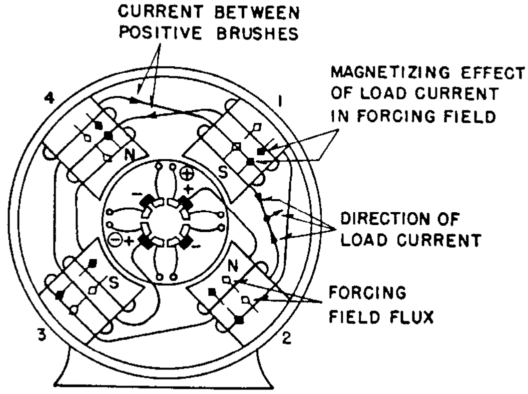
The fundamental principle by which a small amount of energy in the control field forces a large change in Rototrol output is that of unbalancing the ampere-turns on two poles of like polarity; in this case, two south poles. A current in a given direction in the control field will weaken



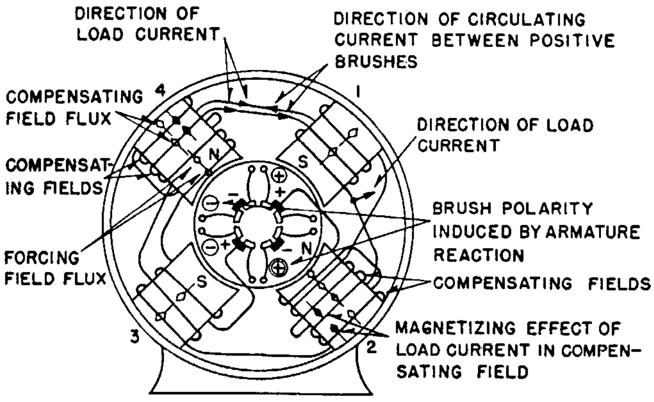
(a) SIMPLIFIED WINDING-DEVELOPMENT DIAGRAM OF TWO-STAGE ROTOTROL SHOWING CURRENT FLOW WITH CONTROL FIELD ENERGIZED



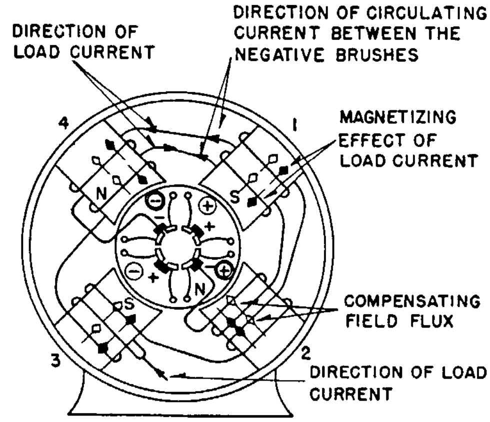
(b) – ROTOTROL EXCITER CONTROL FIELD WINDING



(c) ROTOTROL EXCITER FORCING FIELDS CONNECTED IN SERIES BETWEEN THE POSITIVE BRUSHES



(d) – ROTOTROL EXCITER FORCING AND COMPENSATING FIELDS CONNECTED IN SERIES BETWEEN THE POSITIVE BRUSHES



(e) ROTOTROL EXCITER COMPENSATING FIELDS CONNECTED IN SERIES BETWEEN THE NEGATIVE BRUSHES

—◆ FLUX DUE TO LOAD CURRENT
—◇ FLUX DUE TO CONTROL FIELD BEING ENERGIZED

Fig. 18—Principle of operation of two-stage Rototrol.

one south pole and strengthen the other, and by virtue of the form of the armature winding, causes a difference in polarity between two brushes of like polarity. Current-direction arrows and corresponding flux-direction arrows are shown in Fig. 17 (a), and the operation can be understood best by describing the sequence of events for a given operating condition.

A current is shown flowing in the control field in Fig. 18 (b). The current is in a direction to cause an increase in the terminal voltage of the Rototrol and produces fluxes as shown by the flux arrows to strengthen south pole 1 and weaken south pole 3. Reversing the polarity of the voltage applied to the control field would reverse the effect and cause a decrease in terminal voltage. The resulting unbalance of the south-pole fluxes causes a phenomenon that is suppressed in the usual d-c generator; and that is the unbalance of voltage generated in the armature when the magnetic flux densities in the field poles are unequal. The effect of the unbalanced south poles on the armature winding can be analyzed by assuming the unbalanced fluxes are the only ones present in the machine.

The winding-development diagram of Fig. 18 (a) is drawn for the control-field flux in the direction shown in Fig. 18 (b). So far as the control-field flux is concerned, pole 1 is a south pole and pole 3 is a north pole; thus, the flux direction under pole 1 is out of the paper and under pole 3 is into the paper in Fig. 18 (a). For clockwise armature rotation, the conductor moves under poles 1, 2, 3, and 4 in that order, so the current directions in the armature conductors are as shown. The result is that the positive brush under pole 1 is raised to a higher potential than the positive brush under pole 3. The relative polarities of the two positive brushes are, therefore, as indicated by the encircled polarity marks. Further analysis shows that the positive brush of higher potential is always under the south control-field pole for the conditions of Fig. 18.

The potential difference between the two positive brushes is used to energize another special field called the forcing field, as shown in Fig. 18 (c). For control-field current in the direction shown, the fluxes produced by the forcing-field windings are in a direction to increase the flux densities in all four poles as shown by the open-headed flux arrows, which is in the direction to increase the terminal voltage of the machine. With the opposite control-field polarity, the forcing-field mmf's decrease the flux densities.

The forcing-field current also flows through the armature winding as shown in Fig. 18 (a). The two conductors in a common slot under poles 2 and 4 carry currents in opposing directions. The conductors under poles 1 and 3, however, carry currents in a common direction. Thus, an armature reaction is developed which is in the direction to weaken north pole 2 and strengthen north pole 4. The effect is similar to that caused by current flow in the control field, except that the unbalance in generated voltage appears between the two negative brushes with polarities as shown by the encircled marks in Fig. 18 (d). The resulting current flow between the two negative brushes would cause an armature reaction in opposition to the control field, greatly reducing its effectiveness if compensation were not provided in some way. The compensating windings in series with the forcing fields in Fig. 18 (d) oppose the armature reaction caused by current between the positive brushes, holding to a minimum the voltage difference between the negative brushes and minimizing the armature reaction that would oppose the control field.

A group of compensating fields are also connected in series in the circuit between the negative brushes, and serve a purpose similar to that of the compensating fields between the positive brushes. These are shown in Fig. 18 (e).

All of these currents and fluxes are summarized in Fig. 17 (a), which shows all of the field windings and the current and flux arrows for the assumed condition. Tracing the circuit of the load current reveals that the load current must flow through the forcing and compensating fields. The coils are wound on the field poles in such a direction that the load current cancels so far as any magnetizing effect is concerned, while the magnetizing effects of the unbalance currents add. This is verified in the circuits of Figs. 18 (c), (d), and (e).

In addition to the field windings described above, a set of commutating-pole windings are included in the Rototrol. These windings produce the proper mmf in the commutating poles to assist commutation of the current in the armature.

The overall effect of current in the control field is shown in Fig. 17 (b), the equivalent circuit of the two-stage main-exciter Rototrol. The Rototrol is represented as three separate generators; two of them are two-pole machines and the third is a four-pole machine. The difference in potential between the two positive brushes caused by current in the control field is represented as a two-pole generator excited by the control field and is the first stage of amplification in the Rototrol. The output of this machine is fed into the field of the four-pole generator which is the second stage of amplification. The four-pole field windings are the forcing fields of the Rototrol. Current flowing in the first-stage machine sets up an armature reaction represented by a two-pole armature-reaction generator. The armature reaction is represented by a field exciting this generator and the compensation for armature reaction between the positive brushes is another field on this same machine. The mmf's produced in the armature-reaction and compensating fields are in opposition.

The armature reaction establishes a potential difference between the negative brushes as shown, and the current flowing between these brushes energizes additional compensating windings on all four poles. Two of these windings appear as compensating windings on the armature-reaction generator since they further compensate for the armature reaction produced by the current between the positive brushes. The remaining two compensating windings compensate for the armature reaction caused by the current flowing between the negative brushes, this armature reaction being in opposition to the control field exciting the first stage.

10. Series-Field Effect in Main-Exciter Rototrol

The definition of main-exciter response ratio given in Part I does not apply to main exciters having series fields. Thus, the response ratio of the main-exciter Rototrol cannot be stated in the conventional manner. As stated in Sec. 7, the series field of the Rototrol supplies all of the ampere-turns necessary to generate the terminal voltage under steady-state conditions. The response-ratio definition also states that the test for voltage response should be made under conditions of no load on the exciter, which would seriously hamper the rate of voltage build-up in the Rototrol, because there would be no mmf produced by the series field.

As shown in Chap. 6, Part II, a short circuit at the terminals of an a-c generator induces a large direct current in the generator field winding. The induced current is in the same direction as the current already flowing in the field circuit and serves to maintain constant flux linkages with the field winding. This occurs when the generator voltage is low, and if the induced current were sustained at its initial value, the internal voltage of the generator would be at a high value when the fault is removed. The function of a quick-response excitation system is to increase the exciter voltage as rapidly as possible under such conditions, in order to keep the field current at as high a value as possible. The same effect takes place, although to a smaller extent, when a load is suddenly applied to the generator terminals. Removal of a fault or sudden reduction of the load causes an induced current in the opposite direction due to removal of the armature demagnetizing effect. Thus, a current of appropriate magnitude is induced in the field winding of an a-c generator when there is any change in the terminal conditions, but this current cannot be sustained by conventional main exciters because their voltage cannot ordinarily be increased fast enough.

The main-exciter Rototrol benefits directly from this induced current through its series-field winding and immediately increases the mmf produced by that winding.

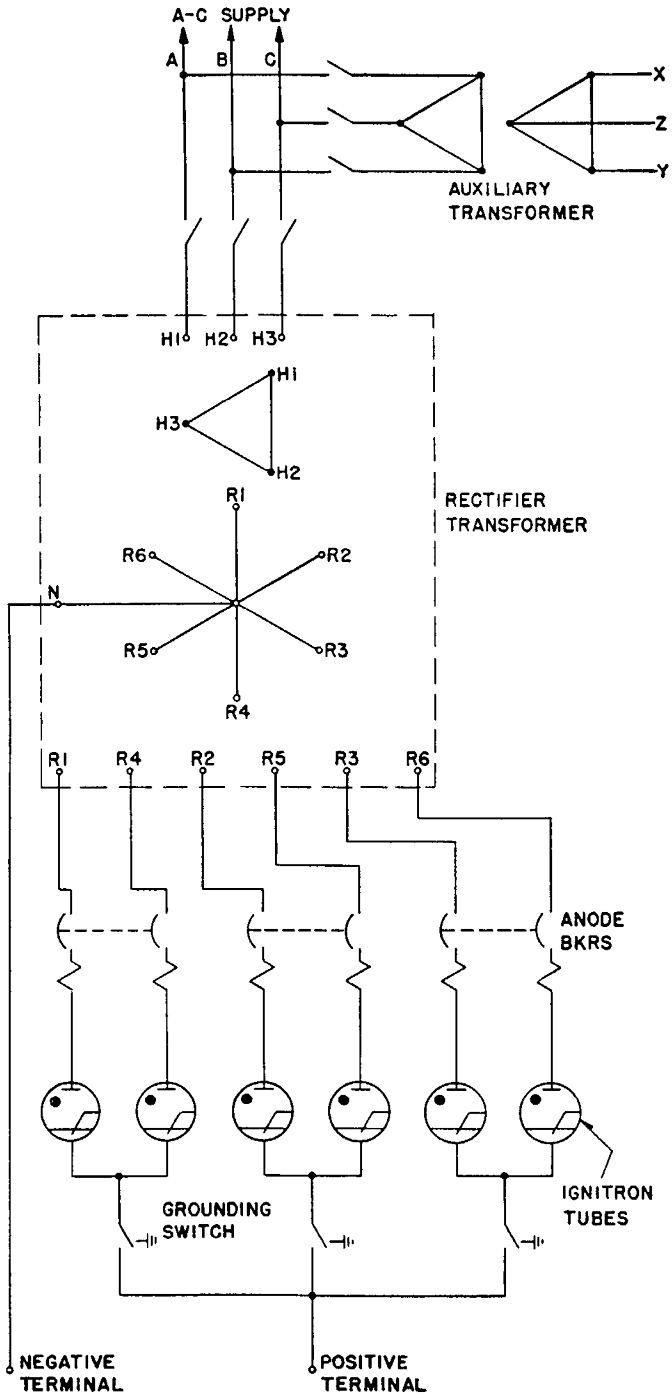


Fig. 19—Simplified circuit of electronic main exciter supplied from the a-c generator terminals through a rectifier transformer.

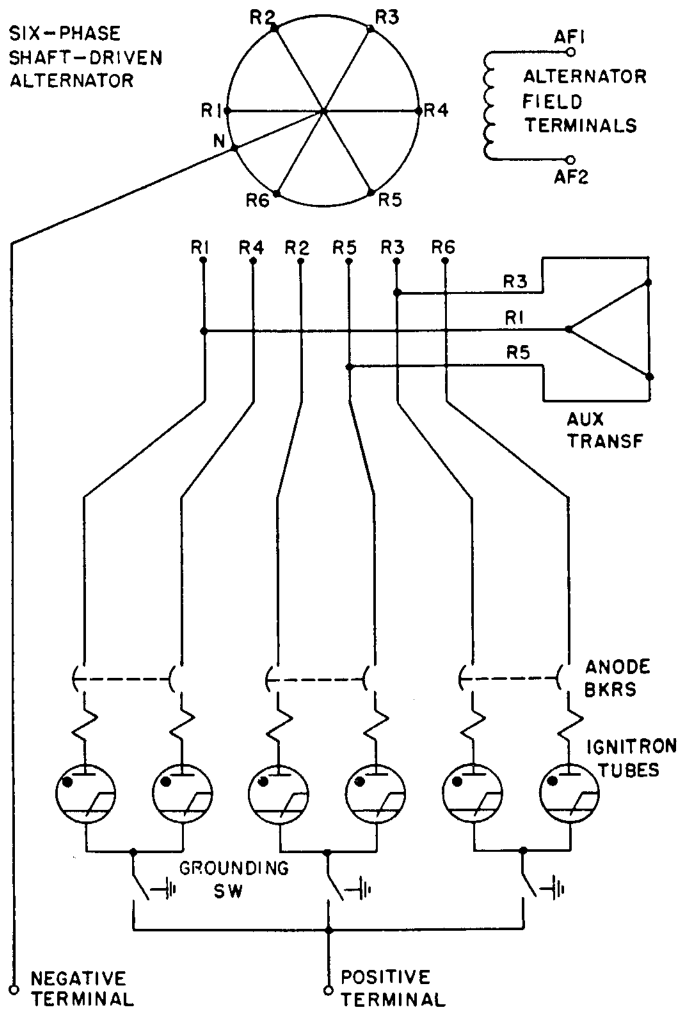


Fig. 20—Simplified circuit of electronic main exciter supplied from a six-phase alternator direct-connected to the main generator shaft.

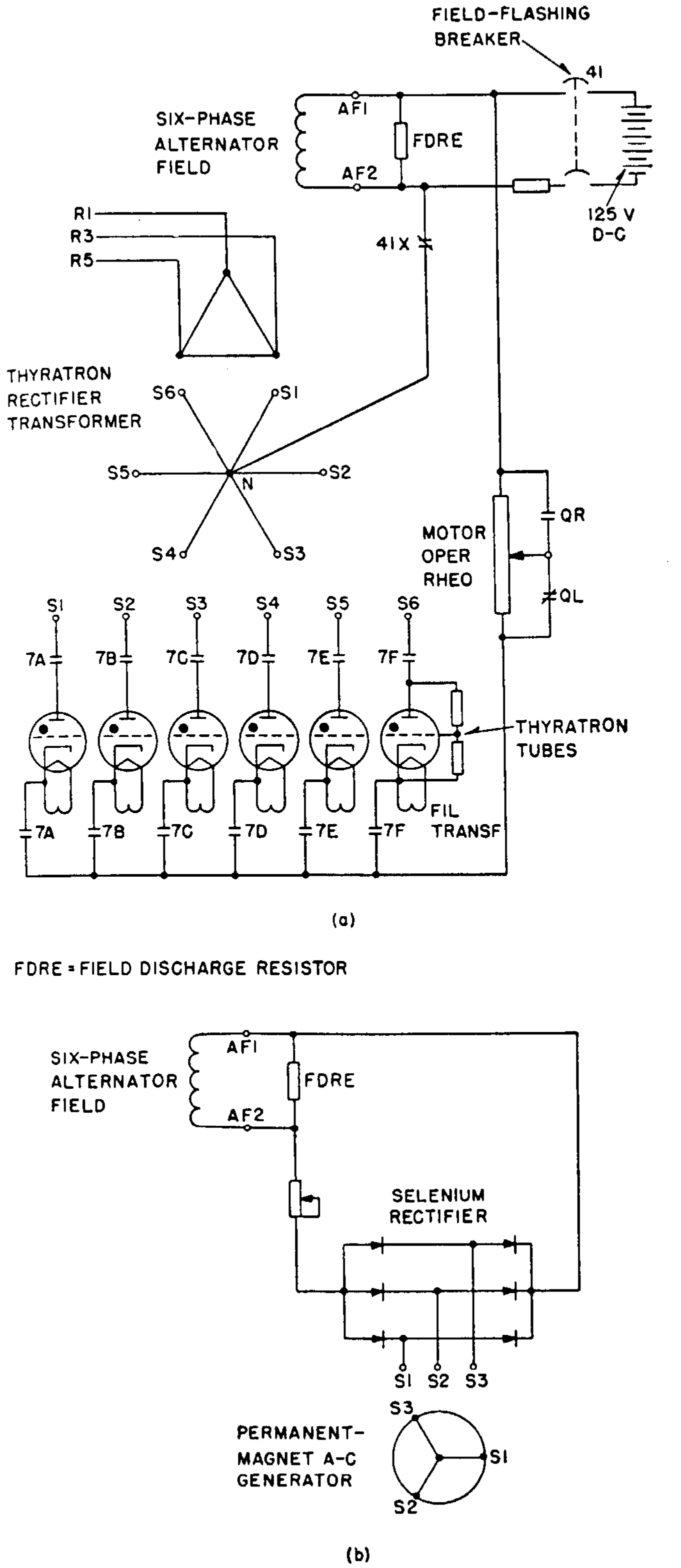


Fig. 21—Two methods of supplying excitation for the six-phase alternator.

(a) Self-excitation using a thyratron rectifier supplied from six-phase alternator terminals through a rectifier transformer. A voltage regulator is used to hold the alternator field voltage approximately constant. The battery is used to flash the alternator field to start operation.

(b) Separate-excitation using a three-phase permanent-magnet generator and dry-type rectifiers.

The Rototrol terminal voltage is raised to a value that can sustain the induced current. If the induced current is caused by a short circuit, it gradually decays in magnitude, and the Rototrol voltage follows the decay in current. The result is that the Rototrol terminal voltage follows a magnitude dependent largely upon the induced current in the generator field winding, and it cannot be duplicated in a voltage-response test with the exciter unloaded.

The series-field effect in the Rototrol is a desirable phenomenon in improving the response of the excitation system and in aiding to maintain system stability. It enables the main exciter to anticipate the change in a-c generator excitation voltage required. As the series-field mmf is following the induced current, the voltage regulator delivers energy to the control field to increase further the Rototrol terminal voltage. There is some time delay before the control-field current is effective in changing the terminal voltage, whereas the series-field effect is substantially instantaneous.

11. Electronic Main Exciters

Power rectifiers of the ignitron type have been used for many years in industrial applications and have given reliable and efficient performance. Their use as main exciters for a-c synchronous machines has been limited, principally because they cost more than a conventional main exciter. The electronic main exciter, however, offers advantages over rotating types.

An electronic exciter consists essentially of a power rectifier fed from an a-c source of power and provided with the necessary control, protective, and regulating equipment. The coordination of these component parts presents problems that must be solved in meeting the excitation requirements of a large a-c generator.

The output of a rectifier is only as reliable as the source of a-c input power. Thus, this a-c source might be considered a part of the rectifier, and so far as service as an excitation source is concerned, it must be reliable. Three sources have been used in operating installations:

1. A-c power for the rectifier taken directly from the terminals of the a-c generator being excited.
2. A-c power taken from a separate a-c supply that is essentially independent of the a-c generator terminals.
3. A-c power taken from a separate generator which supplies power to the rectifier only, and which has as its prime mover the same turbine that drives the main a-c generator.

In the first of these, the electronic main exciter is self-excited, since its power supply is taken from its own output, and in the second and third forms, it is separately-excited.

When power for the rectifier is supplied by a high-voltage source such as the generator terminals, a rectifier transformer must be used to reduce the voltage to the proper magnitude for the rectifier. The transformer is connected delta on the high-voltage side and six-phase star on the secondary side. No transformer is required when the six-phase shaft-driven generator is used as a power source, since the generator can be designed for the proper voltage.

A simplified circuit diagram of an electronic exciter and

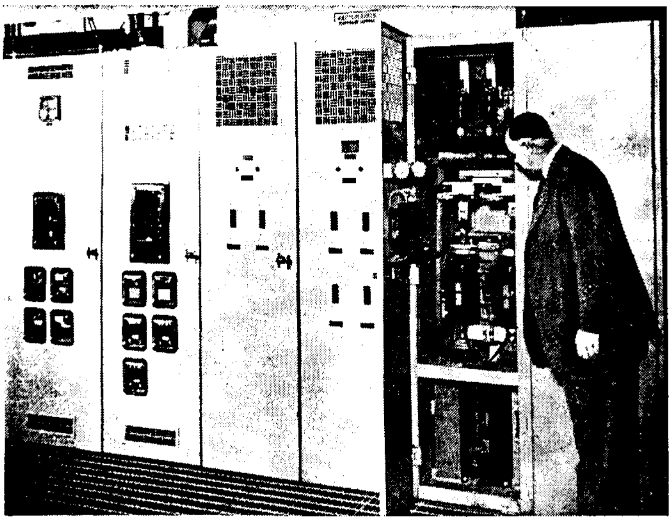
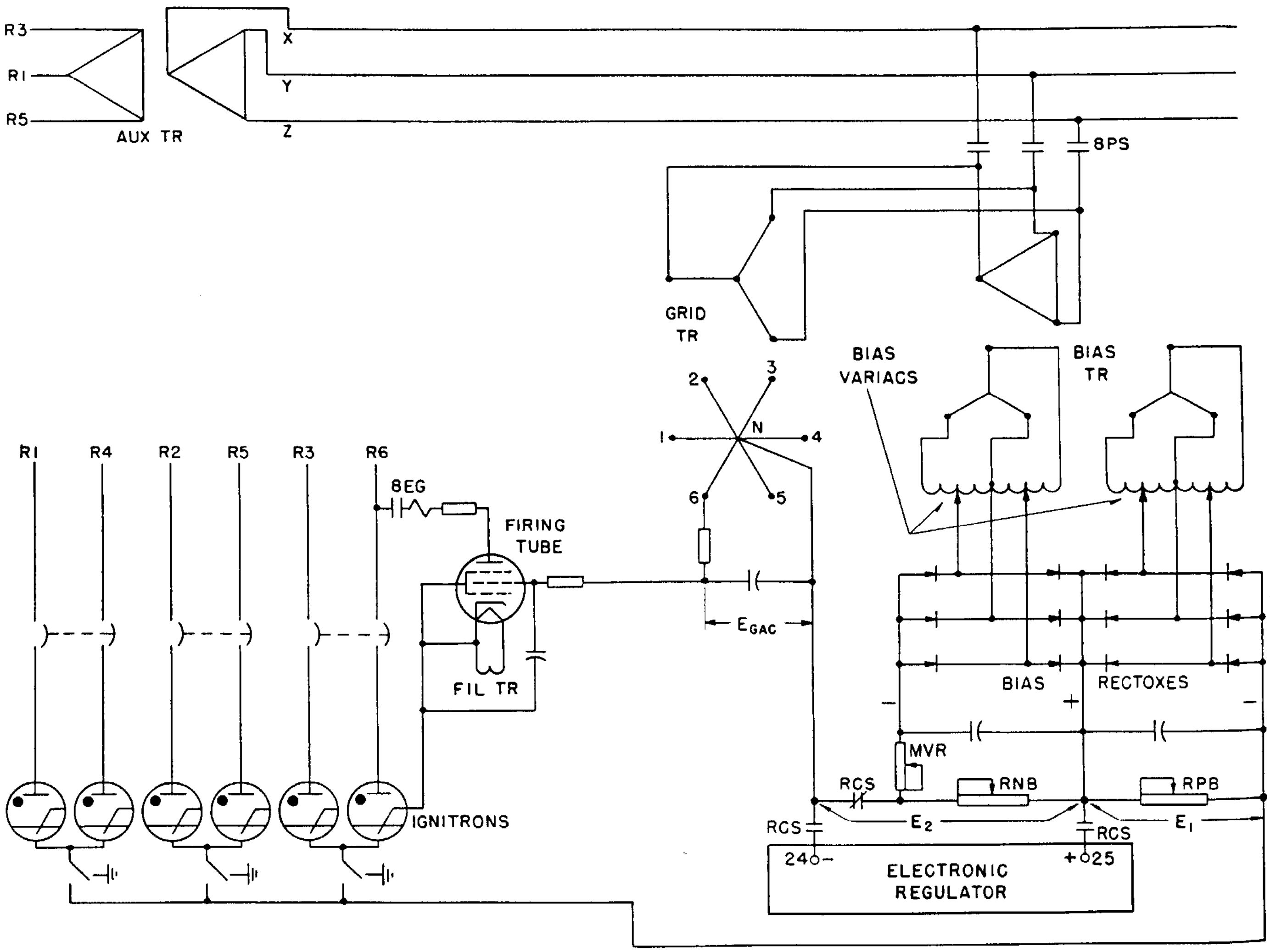


Fig. 22—Installation photograph of electronic main exciter.

rectifier transformer is shown in Fig. 19. The delta primary of the transformer can be energized from the terminals of the main a-c generator, from the plant auxiliary power supply, or from some other independent source. The rectifier comprises three groups of two ignitron tubes each, the two tubes of each group being connected to diametrically opposite phases of the six-phase transformer secondary through a two-pole, high-speed anode circuit breaker. Thus if a breaker is opened, both tubes of a group are deenergized. Each pole of the anode breaker is equipped with a reverse-current trip attachment and the breaker is automatically reclosed. If an ignitron arc-back should occur, the breaker is automatically opened at high-speed and reclosed when the arc-back has been cleared. Should a second arc-back occur within a short time, the anode breaker again opens and locks in the open position to permit inspection of the unit.

The simplified circuit diagram of the electronic exciter supplied from a six-phase alternator is shown in Fig. 20.



RCS–REGULATOR CONTROL SWITCH IN POSITION FOR MANUAL CONTROL

Fig. 23—Method of controlling release of the thyratron firing tube to regulate the main-exciter voltage. The firing control circuit for ignitron tube 6 is shown.

So far as the main-exciter rectifier is concerned, the details of the circuit are the same as Fig. 19. A complication is introduced, however, since it is necessary to provide for excitation of the six-phase alternator. Two methods of accomplishing this are shown in Fig. 21. In the method of Fig. 21(a), the excitation is provided through a six-phase thyratron rectifier, which receives its power input from the same source used to supply the main-exciter rectifier. A permanent-magnet a-c generator is used as the power supply in Fig. 21(b). It consists of high-quality permanent magnets mounted on the same shaft with the main a-c generator to serve as the rotor and a conventional three-phase armature winding on the stator. The output of the permanent-magnet generator is rectified by a three-phase bridge-type selenium rectifier and fed directly into the field of the six-phase alternator. While the shaft-driven generator in Fig. 20 is shown with six phases, it can be a standard three-phase unit in which case a rectifier transformer would be required to convert the ignitron rectifier input to six-phase.

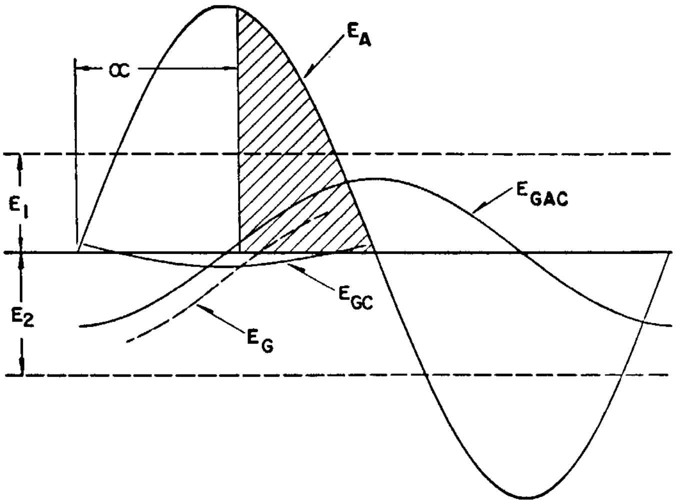
Each group of two ignitron tubes with its anode breaker, cathode-disconnecting switch, firing tubes and associated control circuit is located in one of three individual compartments of the main rectifier cubicle as shown in Fig. 22.

Ignitron Firing Circuit and D-C Voltage Control —The firing circuit for each ignitron tube is of the anode-firing type as shown in Fig. 23. A thyratron tube is connected in parallel with the ignitron through its igniter. The thyratron is made conductive when its anode voltage is positive with respect to its cathode and its grid is released. Current then passes through the ignitron igniter which initiates a cathode spot and fires the ignitron. If the ignitron should fail to conduct for any reason, the thyratron attempts to carry the load current but is removed from the circuit by the thyratron anode breaker.

The magnitude of the output voltage of the electronic exciter is varied by controlling the point on its anode voltage wave at which the ignitron tube is made conductive. This point is determined by releasing the control grid of the firing thyratron, which is controlled by a sine-wave grid transformer, a Rectox supplying a fixed positive bias, a Rectox supplying variable negative bias for manual control, and an electronic regulator supplying variable negative bias for automatic control. The circuits of these devices are shown in Fig. 23.

The grid circuit of the thyratron firing tube can be traced from the cathode of the thyratron through the ignitron to rheostats *RPB* and *RNB* and through the grid transformer to the control grid of the thyratron. The voltage E_1 appearing across rheostat *RPB* is a positive grid bias, while the voltage E_2 appearing across *RNB* is a negative grid bias. The sine-wave voltage E_{GAC} impressed on the grid of the thyratron is delayed almost 90 degrees from the anode voltage and is connected in series with the positive and negative biases. These voltages are shown in Fig. 24.

Rheostats *RPB* and *RNB* are initially adjusted to give the desired values of positive and negative grid-bias voltages. Manual control of the exciter voltage is obtained by changing the setting of rheostat *MVR* which varies the negative bias. The bias voltages E_1, E_2 and E_{GAC} add to



E_A–FIRING TUBE ANODE VOLTAGE
E_{GC}–CRITICAL GRID VOLTAGE OF FIRING TUBE
E_{GAC}–PHASE SHIFTED A-C GRID BIAS VOLTAGE
E_1–FIXED POSITIVE GRID BIAS
E_2–VARIABLE NEGATIVE GRID BIAS
E_G–TOTAL GRID BIAS VOLTAGE
α–ANGLE OF GRID DELAY

Fig. 24—Control grid voltages applied to thyratron firing tube.

give a total grid-bias voltage represented by E_G and varying the negative bias determines the point at which the total grid voltage becomes more positive than the critical grid voltage E_{GC} of the firing tube releasing the tube for conduction. The ignitron is then made conductive by current in the igniter and remains conductive for the remainder of the positive half-cycle of anode voltage. The angle α in Fig. 24 is defined as the angle of grid delay.

The use of a positive and negative grid bias in this manner provides for a wide range of control of the angle of grid delay, and consequently, for a wide range of control of the exciter output voltage. When the exciter voltage is under control of the automatic electronic regulator, the manually-controlled negative bias E_2 is replaced by a variable negative bias voltage from the regulator.

12. Electronic Exciter Application Problems

Modern a-c generators have proven their capability of continuous operation over long periods without being shut down for maintenance. It is necessary, therefore, that main exciters and excitation systems be capable of similar operation and that wearing parts be replaceable without requiring shutdown or even unloading. The ignitron and thyratron tubes in the electronic exciter are subject to deterioration and eventual failure and replacement, and it is essential that such a failure and consequent replacement be sustained without interfering with excitation of the a-c generator.

In its usual form, the electronic main exciter is designed so that it can supply full excitation requirements continuously with two of the six ignitron tubes out of service. With all six tubes in service, the capacity is approximately 150 percent of the requirements. Furthermore, the overload capacity of the ignitron tubes is such that the rectifier can supply full excitation for a short time with only two of the six tubes in service. Should a tube failure occur, the ignitron anode breaker, grounding switch, and firing-tube anode breaker are opened enabling replacement of the ignitron or firing thyratron of any group without disturbing the continuous operation of the remaining two tube groups.

For the electronic exciter to be completely reliable, it must be provided with a reliable source of a-c power. When self-excited from the terminals of the main a-c generator, the input to the rectifier is subject to voltage changes during system disturbances. Thus during nearby faults on the system when it is desirable to increase the generator excitation as much as possible, the rectifier voltage output may be low due to the low a-c voltage. To compensate for the low voltage, the rectifier can be designed for a voltage output much higher than that required during normal operation; that is, the rectifier may be designed to produce normal ceiling voltage when the a-c input voltage is 75 percent of normal. Under normal load conditions the voltage is reduced to that required by control of the firing point. This method of compensation requires a larger rectifier transformer and means that the firing is delayed longer during normal operation.

When separate-excitation is used to supply power to the rectifier, the input is no longer subject to variation during disturbances on the main system. It is possible that a disturbance in the system supplying power to the rectifier may cause a disturbance in the excitation of the a-c generator and a consequent disturbance on the main system. This is overcome by making the rectifier power supply as reliable as possible. Since the same philosophy applies to the system used to supply the powerhouse auxiliaries, this system can be used to supply the rectifier. The shaft-driven three- or six-phase alternator, however, offers the most reliable solution. It is also possible to use duplicate supply with automatic changeover during disturbances in the normal supply, but this is not justified normally.

13. Response of the Electronic Main Exciter

The ignitron rectifier has the ability to increase or decrease its voltage output with substantially no time delay. Compared with the rate of voltage build-up of other types of d-c machines, it might be considered instantaneous. If the response ratio of the electronic exciter were expressed in accord with the definition given in Part I, it would convey a false impression. The line *Oa* in Fig. 25 represents the actual voltage response of the electronic exciter. The line *ab* represents the ceiling voltage. The line *Oc* is drawn so that the area *Ocd* is equal to the area *Oabd* under the actual response curve during the 0.5-second interval. According to the definition, the rate of response is the slope of the line *Oc*, which implies that the exciter voltage has not reached its ceiling value at the end of a 0.5-second interval.

If the distance *Oa* is set equal to 1.0 per unit, then the distance *dc* almost equals 2.0 per unit. The rate of voltage build-up is *dc* divided by 0.5 second or 4.0 per unit per second. The actual time required for the voltage to increase from *O* to *a* is much less than 0.1 second, and therefore, the actual rate of voltage increase exceeds 10 per unit per second.

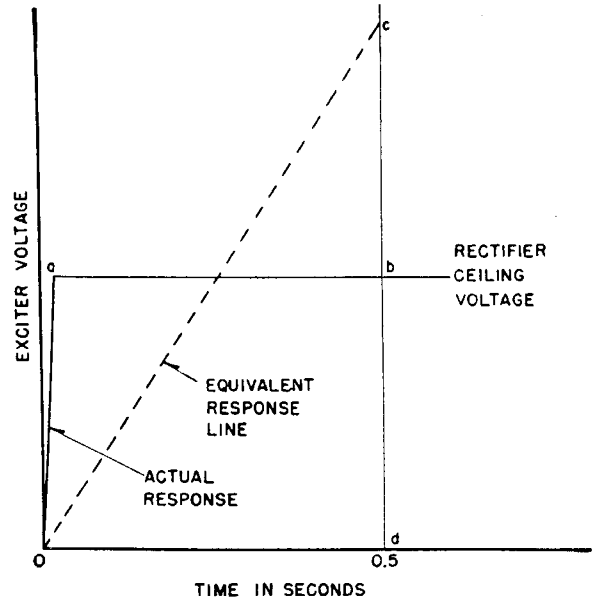


Fig. 25—Response of the electronic main exciter.

III. PILOT EXCITERS

When the main exciter of an a-c synchronous machine is separately-excited, the d-c machine which supplies the separate excitation is called a pilot exciter. A main exciter can be supplied with excitation from more than one source, as is the three-field main exciter, which has a self-excited field and two separately-excited fields, but the sources of separate excitation are still considered as pilot exciters.

Older excitation systems used a storage battery as a pilot exciter, but maintenance problems soon prompted its replacement with rotating types of d-c machines. Two general classifications of pilot exciters are constant-voltage and variable-voltage types. The constant-voltage type is used where control of the main exciter voltage output is by a rheostat in the exciter's separately-excited field circuit, and the variable-voltage type is used where the pilot-exciter voltage must vary to give variable voltage on the exciter field.

14. Compound-Wound Pilot Exciter

The most common form of constant-voltage pilot exciter is the compound-wound d-c generator. The circuit diagram

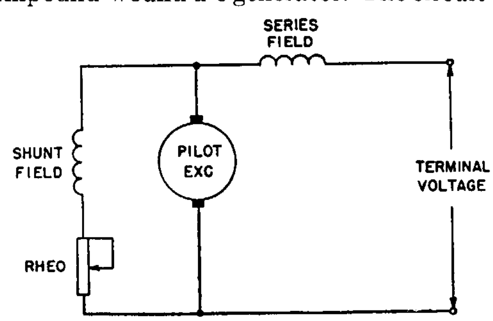


Fig. 26—Compound-wound conventional pilot exciter.

is shown in Fig. 26. The pilot exciter is invariably a 125-volt machine with a self-excited shunt field and a series-excited field, adjusted to give substantially flat-compounding. Thus, regardless of the load on the pilot exciter, the magnitude of its terminal voltage is practically constant.

The compound-wound pilot exciter is normally mounted on the shaft of the main exciter, and where the main exciter is direct-connected, the a-c generator, main exciter, and pilot exciter all rotate at the same speed. A rheostat, either under the control of a voltage regulator or under manual control, is connected in series with the output circuit of the pilot exciter to regulate the voltage applied to the field of the main exciter.

15. Rototrol Pilot Exciter

The Rototrol, described in Sec. 8 as a main exciter, is also used as a variable-voltage pilot exciter. Depending upon the excitation requirements of the main exciter, the Rototrol pilot exciter may be of either one or two stages of amplification. Generally, when the main exciter and Rototrol pilot exciter are direct-connected to the generator shaft and operating at 3600 rpm, the pilot exciter has a single stage of amplification. When the pilot exciter is operated at a speed lower than 3600 rpm, such as 1800 or 1200 rpm, it is of the two-stage type.

The single-stage Rototrol is a stabilized series-excited d-c generator as shown in Fig. 27. The control field is a

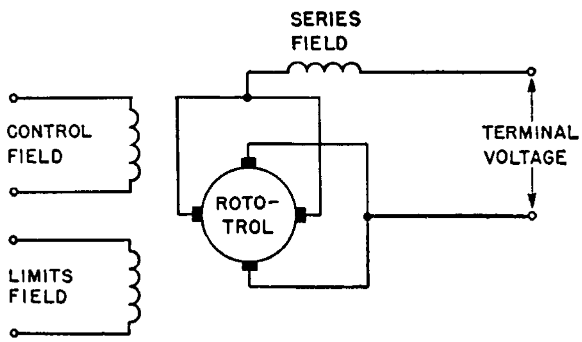


Fig. 27—Equivalent circuit of single-stage Rototrol pilot exciter.

separately-excited shunt field. The principal difference between this and a conventional series-excited d-c generator is the fact that the Rototrol is operated in the unsaturated region, that is, on the air-gap line. Under steady-state conditions, the sustaining series field supplies practically all of the ampere-turns required to maintain the Rototrol terminal voltage. The input to the control field acts as a stabilizing force to hold the voltage at any point on the straight-line portion of the saturation curve.

IV. GENERATOR EXCITATION SYSTEMS

In the ten-year period following 1935, two basic types of generator voltage regulators filled substantially all needs of the electrical industry. These were the indirect-acting exciter-rheostatic regulator and the direct-acting rheostatic regulator. Excitation systems are now in the midst of a period of changes by reason of progress in the development of regulating and excitation systems. Efforts have been directed particularly toward the development of more reliable, more accurate, more sensitive, and quicker-acting systems. Consequently, there are now many different excitation systems in use, each filling a specific need of the industry.

The preceding sections have discussed the various types of main and pilot exciters in use at present. The remainder of the chapter will be a comprehensive discussion of the application of these d-c machines in excitation systems in conjunction with various types of generator voltage regulators.

Four types of voltage regulators are being used to control the excitation of synchronous machines:

1. Direct-acting rheostatic type
2. Indirect-acting exciter-rheostatic type
3. Impedance-network or static-network type
4. Electronic type.

Each of these are described in their application in various types of excitation systems in the order named.

16. The Direct-Acting Rheostatic Regulator

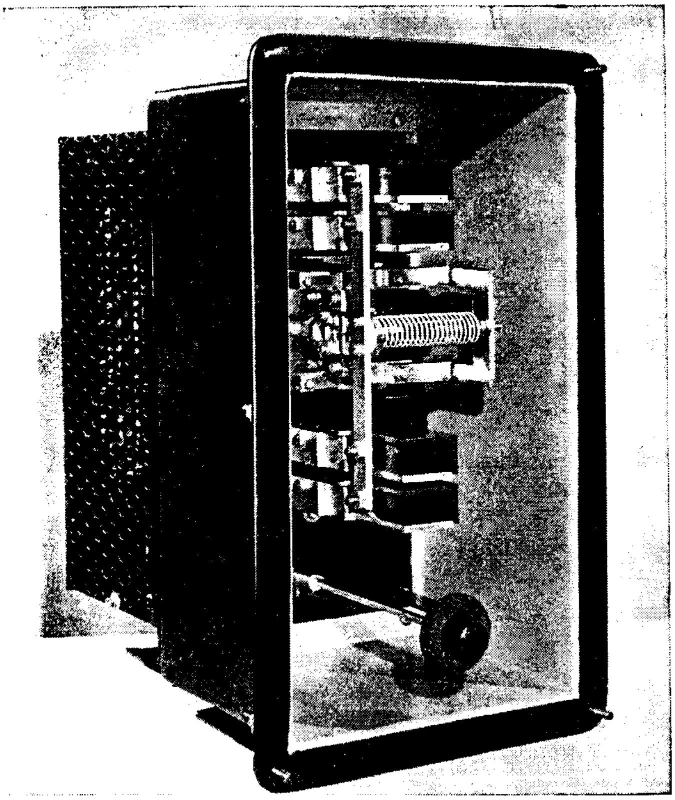
The Silverstat generator voltage regulator is a common and widely used form of the direct- and quick-acting rheostatic type of regulator. It is specifically designed for the automatic voltage control of small and medium size generators. For generators rated above 100 kva, the Silverstat or SRA regulator is available in five sizes, the largest being used with generators as large as 25 000 kva. A typical SRA regulator of medium size is shown in Fig. 28 (a).

The direct-acting rheostatic type of regulator controls the voltage by the regulator element varying directly the regulating resistance in the main exciter field circuit. The different sizes of SRA regulators are suitable for the automatic voltage control of constant-speed, one-, two- or three-phase a-c generators excited by individual *self-excited* exciters. The exciter must be designed for shunt-field control and self-excited operation, with its minimum operating voltage not less than 30 percent of its rated voltage. Each regulator is designed for and limited to the control of one exciter.

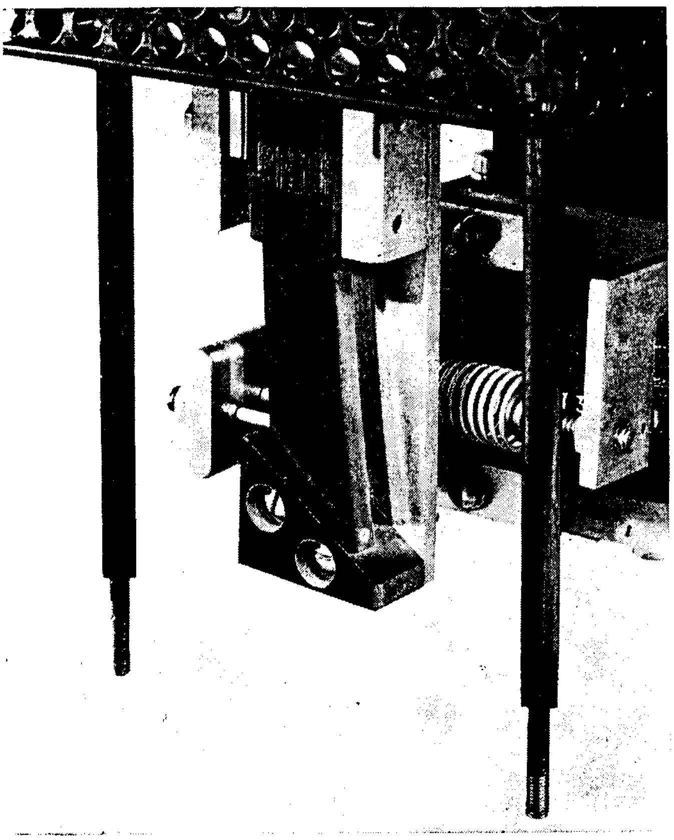
Where a-c generators are operated in parallel and are within the range of application of this regulator, the practice is to provide each generator with an individual exciter, with the exciters operated non-parallel. Each generator and its exciter is provided with an individual regulator and suitable cross-current compensation provided between the regulators.

Sensitivity—The sensitivity of a generator voltage regulator is the band or zone of voltage, expressed as a percentage of the normal value of regulated voltage, within which the regulator holds the voltage with steady or gradually changing load conditions. This does not mean that the regulated voltage does not vary outside of the sensitivity zone, but does mean that when the regulated voltage varies more than the percentage sensitivity from the regulator setting due to sudden changes in load or other system disturbances, the regulator immediately applies corrective action to restore the voltage to the sensitivity zone.

Regulator sensitivity must not be confused with overall



(a)



(b)

Fig. 28—(a) SRA-4 Silverstat generator voltage regulator. (b) Silver-button assembly of Silverstat regulator.

regulation, which involves not only regulator sensitivity but also the time constants of the machines and the character and magnitude of the voltage changes. The magnitude and rate of load change determine how far the voltage deviates outside of the regulator sensitivity zone, and the time constants of the machines chiefly determine the time required to restore the voltage to the sensitivity zone. For these reasons only sensitivity can be specified so far as the voltage regulator is concerned and not overall regulation, which involves factors over which the regulator has no control.

The rated sensitivity of the SRA voltage regulators depends on the size of the regulator. The SRA-1 and SRA-2, the two smaller sizes, have rated sensitivities of plus or minus 2½ and 1½ percent, respectively. The larger SRA-3, SRA-4 and SRA-5 regulators are rated at plus or minus ½ of 1 percent sensitivity.

17. Operation of the Direct-Acting Rheostatic Regulator

The silver-button assembly, Fig. 28 (b), provides the means for changing the resistance in the exciter shunt-field circuit under control of the regulator. This basic assembly consists of a group of spring-mounted silver buttons so arranged that the buttons are separated from each other

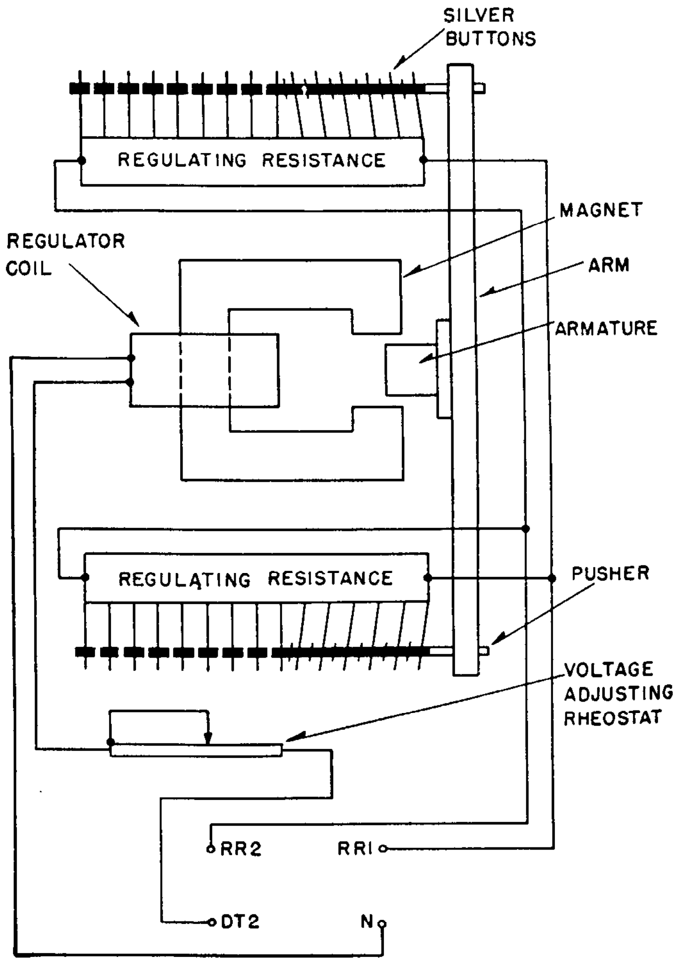


Fig. 29—Schematic internal diagram of SRA-3 Silverstat regulator.

normally, but can be closed or opened in sequence by a suitable driver having a travel of a fraction of an inch. The springs or leaves that carry the silver buttons are insulated from each other and each leaf is connected to a tap on a resistance element as shown in Fig. 29. Varying amounts of the resistance are short circuited by closing of the silver-button contacts. One or more of these basic elements are used in regulators of different sizes, four being used in the SRA-4 regulator illustrated in Fig. 28 (a).

The control element of the regulator is a d-c operated device. A spring-mounted armature is centered in the air gap of the electromagnet as shown in Fig. 29. In regulating a-c voltage, a full-wave rectox rectifier is used to convert the a-c to d-c for energizing the control element.

A typical excitation system under control of an SRA regulator is shown schematically in Fig. 30. The regulating

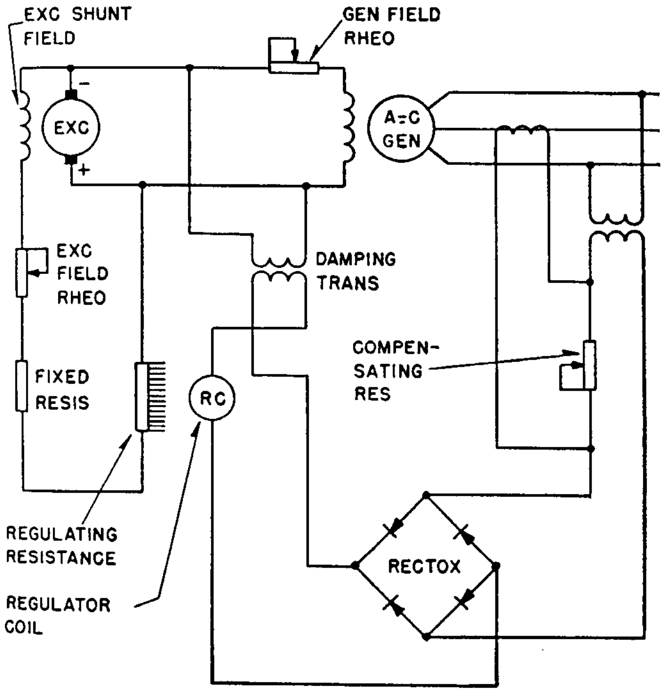


Fig. 30—Self-excited main exciter controlled by Silverstat regulator. The compensating resistance is used to provide cross-current compensation during parallel operation of a-c generators or to provide line-drop compensation.

resistance is connected directly in the exciter shunt-field circuit. At one end of the travel of the moving arm, all of the silver buttons are apart from each other, placing maximum resistance in the field circuit. At the other end of the travel, the buttons are closed and the resistance is short circuited. The moving arm can hold the resistance at any intermediate value and, since the travel is short, all the resistance can be inserted or removed from the field circuit quickly. The speed of operation of the regulating element depends upon the magnitude and rate of change of the operating force. With a sudden drop in a-c voltage of 10 to 12 percent, the time required for the regulator to remove all resistance from the exciter shunt-field circuit is approximately 0.05 second or 3 cycles on a 60-cycle basis.

The regulating action of the SRA regulator is that of a semi-static device that operates only when a correction in voltage is necessary. For a given value of regulated voltage and load on the machine being regulated there is a corresponding value of regulating resistance required in the field circuit; and a corresponding position of the moving arm and silver buttons that gives this value of resistance. Under such conditions the magnetic pull on the moving arm is balanced against the spring pull at that position of its travel. When there is a change in load on the machine being regulated, a corresponding change in voltage results, and the voltage is restored to its correct value by the moving arm and silver buttons taking a new position. Since the pressure on silver contacts determines the resistance of the contacts, an infinite number of steps of regulating resistance are obtained. If the required value of exciter field resistance should lie between two of the tapped points of the regulating resistance, the pressure of the silver contacts changes to provide the correct intermediate value of resistance.

The fixed resistance in the exciter field circuit in Fig. 30 is used when it is desired to limit the exciter shunt-field current when the maximum or ceiling current is such as to interfere with the best performance of the voltage regulating equipment. The exciter shunt-field rheostat and the generator field rheostat are provided primarily for control of the generator excitation when the regulator is not in service. Excitation current in the generator field can be regulated by changing the exciter output voltage or by holding the exciter voltage constant and changing the generator field resistance. When the voltage regulator is in

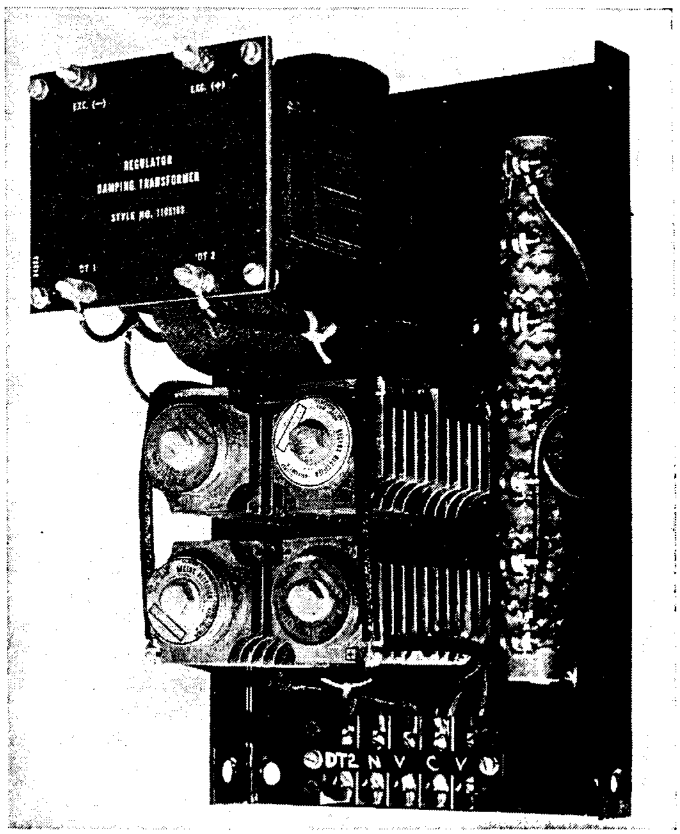


Fig. 31—Silverstat regulator damping transformer and rectox rectifier assembly.

operation and controlling the generator voltage, the exciter shunt-field and generator field rheostats are ordinarily turned to the "all out" position so that the regulator has full control of the excitation voltage.

Damping—To stabilize the regulated voltage and prevent excessive swinging under various conditions of excitation change, a damping effect is introduced into the regulator coil circuit by means of a damping transformer as shown in Fig. 30. The damping transformer is illustrated in Fig. 31. The use of this device eliminates the need for dashpots or similar mechanical anti-hunting devices.

The damping transformer is of a special type having a small air gap in the laminated-iron magnetic circuit. One winding is connected across the field of the generator whose voltage is being regulated, and the other winding is connected in series with the voltage regulator coil. When there is a change in excitation voltage as a result of the regulating action of the regulator, energy is transferred by induction from one winding to the other of the damping transformer. This energy introduced into the circuit of the regulator coil acts by reason of its direction, magnitude, and time relation to electrically damp excessive action of the moving arm, preventing the moving arm from carrying too far the change in regulating resistance and consequent change in generator excitation. Since the damping transformer operates only when the excitation of the generator is changing, it has no effect when the regulated voltage is steady and the regulator is balanced.

Parallel Operation—As is true with most generator voltage regulators, the SRA regulator can control only one exciter at a time. Where several a-c generators operate in parallel and all the generators are excited from one common exciter, a single Silverstat regulator can be used, provided the exciter is of a size that is within the range of application of this type of regulator. However, where a-c generators operate in parallel, the usual practice is to provide each one with an individual exciter controlled by an individual regulator. This scheme of operation requires that the exciters be operated non-parallel, and it is necessary to supply a means of assuring proper division of reactive kva between the generators. The division of the kilowatt load among paralleled a-c generators is dependent upon the power input to each generator and is controlled by the governor of its prime mover. Thus the division of kilowatt load is practically independent of the generator excitation. However, changes in the field excitation of paralleled a-c generators do affect the reactive kva or wattless component of the output, and the division of the reactive kva is directly affected by the operation of the voltage regulators.

Thus, wattless current circulates between the paralleled a-c generators unless some provision is made whereby the generators are caused to properly divide the reactive kva. This is accomplished by means of cross-current compensation, which functions to cause each generator to shirk wattless current by means of a slight droop in the regulated voltage with increase in the wattless component of current. The effect of the small droop required is usually negligible under operating conditions as found in actual practice.

For three-phase a-c generators with the SRA regulator, the compensation is obtained by a standard current transformer connected in one lead of each generator being regulated as shown in Fig. 30. The current transformer is connected to an adjustable resistance in the a-c supply circuit to the regulator operating element. The adjustable resistance permits adjustment of the compensation to suit the application. The current transformer is connected in one generator lead, while the potential transformer that operates the regulator is connected to the other two leads. Thus the phase relationship is such that for lagging reactive kva, the voltage drop across the compensating resistance adds to the a-c voltage energizing the regulator and subtracts in the case of leading reactive kva. This action tends to cause the regulator to lower excitation for lagging reactive kva and raise excitation for leading reactive kva. In this manner each generator tends to shirk reactive kva, and the wattless power is automatically divided in proportion among the paralleled a-c generators.

In many applications, reactance in the form of power transformers, bus reactors, etc., exists between paralleled a-c generators. If each generator is excited by an individual exciter under control of an individual voltage regulator, and if the reactance is such as to cause from four to six percent reactive drop between the two generators, then stable operation and proper division of the wattless component can usually be obtained without using cross-current compensation between the regulators. This is because the reactance produces an effect similar to that obtained where cross-current compensation is used.

18. Indirect-Acting Exciter-Rheostatic Regulator

In recent years the increase in capacity of generating units, the extension of transmission systems, and the interconnection of established systems, have reached a point where quick-response excitation is valuable for improving stability under fault conditions and large load changes. On applications of this kind the type BJ regulator is particularly adapted to the control of a-c machines employing quick-response excitation. The BJ regulator is of the indirect-acting exciter-rheostatic type for the automatic control of medium and large size a-c generators.

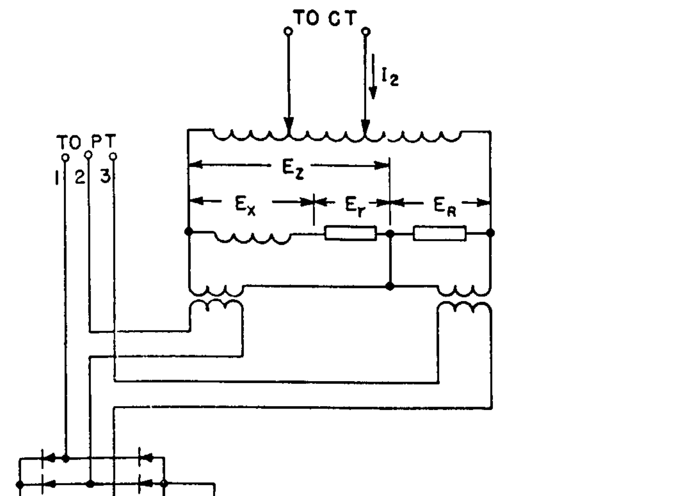
The indirect-acting exciter-rheostatic type of generator voltage regulator controls the voltage of an a-c machine by varying the resistance in the field circuit of the exciter that excites the a-c machine. The exciter is preferably separately-excited from a pilot exciter or other source. If the exciter is self-excited, its minimum operating voltage must not be less than 30 percent of its rated voltage if stable operation is to be obtained. When lower voltages are necessary, the main exciter must be separately-excited.

A schematic wiring diagram of the BJ generator voltage regulator and its auxiliary contactors is shown in Fig. 32. This diagram in conjunction with the simplified schematic of Fig. 33 is used to describe the operation of the device.

The main control element of the regulator is energized from two single-phase potential transformers connected to the a-c machine leads. Two sets of contacts are on the moving lever arm of the regulator element shown in Fig. 32, namely, the normal-response contacts *R-L* and the quick-response contacts *AR-AL*. The normal-response contacts control the rheostat motor contactors *NR* and *NL*, to raise or lower the a-c machine voltage, respectively. The quick-response *AR* and *AL* contacts control the high-speed contactors QR and QL, which are the "field forcing up" and "field forcing down" contactors, respectively. When contactor QR in Fig. 33 is closed, all external resistance is shorted out of the main-exciter field circuit, and when QL is opened by energizing its coil, a block of resistance is inserted in the field circuit.

Normal Response—When the a-c voltage is normal, the regulator lever arm is balanced and in this position neither the normal-response contacts R-L nor the quick-response contacts AR-AL are closed. Should the a-c voltage fall below normal by a small amount, depending upon the sensitivity setting of the regulator, the normal-response contact R will close, energizing the rheostat motor control contactor NR. The contacts NR energize the rheostat motor which then turns the rheostat in a direction to remove resistance from the exciter field circuit, thereby increasing the voltage applied to the exciter field.

The rheostat-motor control contactor NR has three contacts that close in independent circuits simultaneously. The one circuit is that just described which operates the rheostat motor. The second is the circuit of the anti-hunting winding NH of the regulator main control element and the third set of contacts complete a timing-condenser circuit. The anti-hunt device operates to increase the gap distance between the contact faces of the regulator contacts R and L, thereby opening the circuit at the R contacts. This change in position of the R contact is equivalent to changing the regulator setting to a lower voltage so far as the raise contacts are concerned, and to a higher voltage so far as the lower contacts are concerned. Where the deviation from normal voltage is small and within the recalibration effect of the anti-hunt device, the immediate



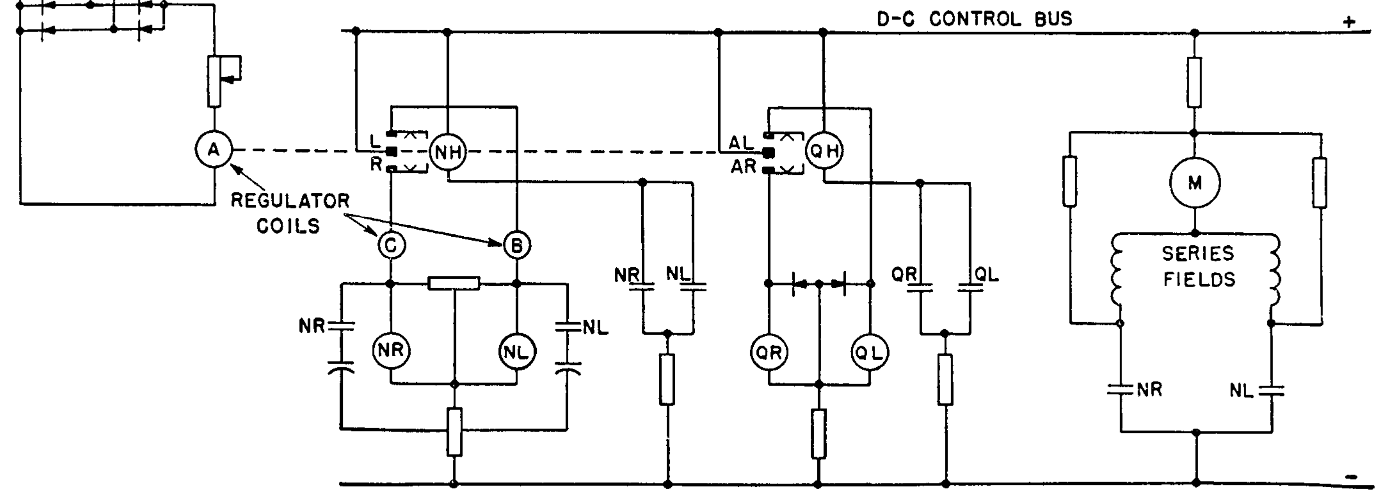


Fig. 32—Schematic diagram of the BJ regulator controlling the voltage of a separately-excited main exciter.

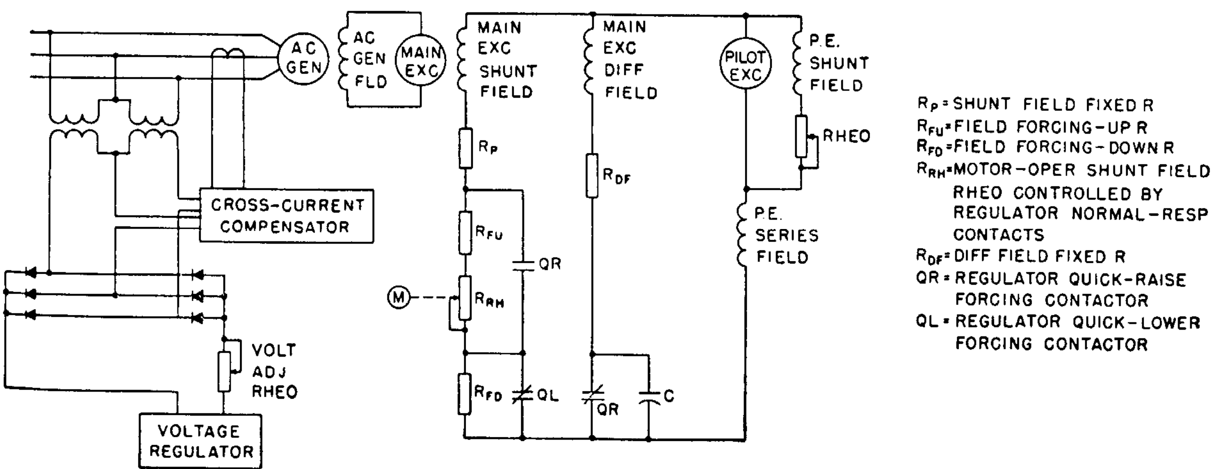


Fig. 33—Main-exciter circuits under control of BJ regulator in Fig. 32.

result of the closing of the contacts on contactor NR is to cause the opening of the regulator R contact, which in turn opens the circuit to the coil of contactor NR, to stop the motor of the exciter field rheostat and thus stop the rheostat moving arm. However, contactor NR does not immediately open due to a time-delay circuit around its coil that maintains the coil voltage. Thus the rheostat arm is permitted to move a definite distance, for example, from one button to the next on the rheostat faceplate, and at the end of its time delay, contactor NR opens to stop the rheostat motor and deenergize the anti-hunt device.

After the rheostat motor stops, it is desirable to provide some time delay to allow the a-c machine voltage to reach its final value. Such delay is obtained by a dashpot on the anti-hunt device that prevents the regulator contacts from immediately returning to their normal position. After this time delay has expired and the contacts have returned to their normal position, the normal response contact R again closes if the a-c voltage has not returned to normal. This starts another cycle of operation such as just described and these cycles continue until the normal value of regulated voltage is established.

Where the original voltage deviation is large enough the regulator contacts remain closed continuously even though the anti-hunt device changes the contact setting. In this case the regulator arm is caused to follow the change in contact position made by the anti-hunt device, and the R contact and the contactor NR remain closed. This causes the rheostat motor to run continuously until the a-c voltage is within the zone for which the anti-hunt device is set, at which time the notching action takes place to bring the voltage to normal.

By means of the continuous or notching action of the rheostat, dependent upon the magnitude of the voltage change, time is allowed for the a-c voltage to come to rest between each voltage correction as the voltage approaches its normal value. The action of the dashpot is also such that the time required for the contacts to remake is longer as the lever arm approaches the normal voltage position. This results in a decreased motor speed as the rheostat arm moves nearer to its new position, preventing overshooting of the rheostat position and bringing the a-c voltage to normal in a minimum length of time.

When the a-c voltage rises above the regulated value, an action similar to that described for low voltage takes place, except that the regulator contact L closes energizing the rheostat motor control contactor NL, which operates the rheostat motor in a direction to increase the resistance in the exciter field circuit.

Quick Response—When a large drop in voltage occurs, such as might be caused by a large block of load being thrown on the system or by a fault, the normal-response contacts R on the regulator close, followed by closing of the quick-response contacts AR. Contacts AR close the circuit to the high-speed field-forcing-up contactor QR, which short circuits all of the external resistance in the exciter field circuit, applying full exciter voltage to the field circuit. This causes the a-c machine voltage to start to return to normal very rapidly by forcing action.

When the field-forcing-up contactor QR closes, an auxiliary contact on this contactor closes at the same time in the circuit of the anti-hunt device QH, which operates to spread the AR and AL contacts in the same manner as described for the NH device and the R and L contacts. Therefore, if the deviation from normal voltage is within the recalibration effect of the QH anti-hunt device, the field-forcing-up contactor closes and opens rapidly while the rheostat arm approaches the required new position. If the deviation from normal voltage is greater than the recalibrated setting of QH anti-hunt device, the field-forcing-up contactor closes and remains closed until the a-c voltage is brought within the recalibrated setting.

As the a-c voltage comes within the setting of the AR contacts and they no longer close, the normal response contacts R take control and by notching the rheostat, return the a-c voltage to normal. Since the rheostat moves at maximum speed while the quick-response contacts are closed, it takes only a minimum of additional movement after the normal-response contacts take control to return the voltage to normal.

When the main exciter has a differential field as shown in Fig. 33, a contact in the QR contactor opens the differential-field circuit. In this way, the damping effect of the differential field in slowing the exciter response is removed.

19. Sensitivity of the BJ Regulator

The rated sensitivity of the BJ generator voltage regulator is plus or minus ½ of one percent. The sensitivity is adjusted by varying the spacing between the regulator contacts R and L. The quick-response contacts are set to a wider spacing than the normal-response contacts so that larger deviations from normal voltage are required to close them. The usual range of settings of the quick-response contacts is from plus or minus 2½ percent to plus or minus 10 percent, the setting depending somewhat on the setting of the normal-response contacts and upon the operating conditions of the particular installation.

The main coil of the control element in Fig. 32 consists of a voltage winding energized by a d-c voltage, rectified from the three-phase a-c source being regulated. Thus, the coil is energized by a voltage equal to the average of the phase voltages and the regulator holds this average voltage within the rated sensitivity zone. The level of the regulated voltage is set by adjustment of the voltage-adjusting rheostat; resistance being added in series with the regulator voltage coil to increase the level of the regulated voltage, and resistance being removed to decrease the level of the regulated voltage. The normal range of adjustment is approximately plus or minus 10 percent from the normal generator voltage.

20. Cross-Current Compensation with BJ Regulator

When cross-current compensation is required to give the voltage regulator a drooping characteristic, one compensator and one current transformer are required, connected as shown in Fig. 32. The compensator is designed to supply a compensating voltage in two phases of the three-phase regulator potential circuit. This insures applying a balanced three-phase voltage to the regulator element, which would not be the case if only one leg was compensated.

The vector diagram of the compensating circuit is shown

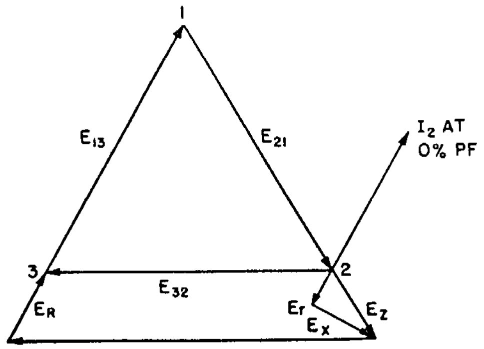


Fig. 34—Vector diagram of cross-current compensation used with BJ regulator. Circuit shown in Fig. 32.

in Fig. 34, the potential transformer secondary voltages being represented by E_{21}, E_{32} and E_{13}. The current applied to the autotransformer of the compensator in Fig. 32 is taken from the secondary of a current transformer in phase 2 of the a-c circuit. Two compensating voltages are produced; one between terminals $X1$-$X2$ designated as E_Z on the vector diagram and the other between terminals $Y1$-$Y2$ designated as E_R on the vector diagram. Voltages E_R and E_Z are 120 degrees apart in time phase and, therefore, can be added to a three-phase set of voltages without unbalancing it.

The vector diagram shows E_Z and E_R for zero power factor, under which condition maximum compensation is obtained. As the power factor approaches unity, these voltage vectors swing through an arc of 90 degrees and give zero compensation at 100 percent power factor. At zero power factor, vectors E_Z and E_R add directly to vectors E_{21} and E_{13}, respectively. For power factors greater than zero, only a proportionate component of these voltages E_Z and E_R add directly to voltages E_{21} and E_{13}. The addition of these compensating voltages to the line voltages as the load increases or the power factor changes gives the regulator element a high voltage indication resulting in a reduction or droop in regulated voltage. Usually the compensator should cause from four to six percent droop in voltage at zero power factor full load on the a-c generator.

21. Line-Drop Compensation with BJ Regulator

The wide use of interconnected power systems has eliminated to a large extent the need for line-drop compensation. However, it is sometimes desirable to regulate for a constant voltage to be maintained at some point on the system external to or distant from the station where the a-c machine and its regulator are located. The principle by which this is accomplished is shown by the circuit and vector diagrams of Fig. 35.

The voltage regulator is to maintain the voltage E_2 constant. If it were possible to supply the regulator with pilot wires so that it could measure the voltage at the load center, the regulator could adjust the excitation of the generator to maintain E_2 constant. Since in actual practice it is impractical to use pilot wires, the regulator potential winding is energized from the generator bus voltage E_1, and the two components XI and RI are subtracted from it artificially by the compensation. The resultant voltage E_2 is then supplied to the regulator. If the components XI

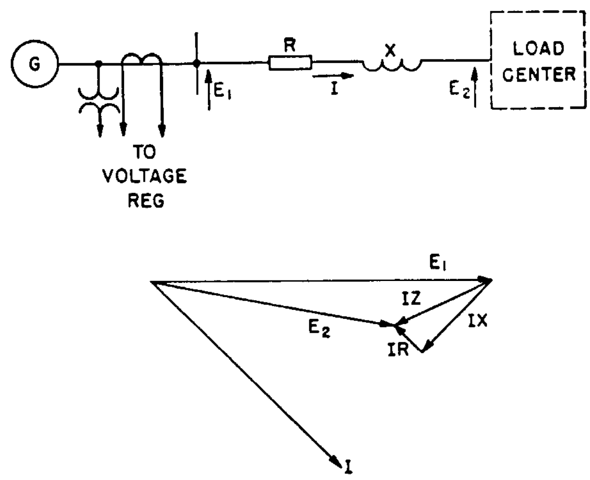


Fig. 35—Principle of line-drop compensation.

and RI are proportioned to and in phase with the corresponding values of line reactance and resistance voltage drops, the regulator controls the voltage as if it were connected by pilot wires to the load center.

In general, since the reactance component XI of the line predominates, it is necessary to compensate mainly for this component of the line drop, the resistance component RI having a relatively small effect.

Parallel operation of a-c generators, each under the control of a voltage regulator, requires a droop in regulated voltage with an increase in wattless load. On the other hand, reactance line-drop compensation requires a rising characteristic for the regulated voltage with an increasing wattless load. In order to compensate for reactive cross current between machines and for complete line drop when machines are operating in parallel in the same station, three current transformers and two compensators with suitable auxiliary equipment must be used for each machine. In any event, the XI line-drop compensation must never exceed the XI cross-current compensation; i.e., there must be a net droop in regulated voltage with increase in wattless load.

Complete line-drop compensation is not always necessary, and a simple compromise solution is available to provide approximate line-drop compensation and reactive-droop compensation. The RI-drop compensation is set to approach the XI drop of the line for some average power factor. When the RI-drop compensation is so set, the XI-drop compensation can be adjusted independently to provide the required cross-current compensation, and there is no interference between the two compensators.

22. Synchronous Condenser Excitation with BJ Regulator

The type BJ generator voltage regulator can also be used to control the excitation of a synchronous condenser. The circuit is essentially the same as that shown in Figs. 32 and 33.

When the excitation of a synchronous condenser is increased above a certain value, the condenser furnishes a lagging (overexcited) current to the system thereby causing the voltage to rise. In a similar manner, decreasing the excitation lowers the voltage. Thus, when a generator voltage regulator is applied to a synchronous condenser, it regulates the line voltage to a constant value by varying the excitation of the condenser, provided the condenser has sufficient corrective rkva capacity.

It is often necessary that the condenser furnish leading (underexcited) rkva as well as lagging (overexcited) rkva, and it is necessary to reduce the excitation to an extremely low value. Where the minimum value is less than 30 percent of the main exciter rated voltage, it is necessary to use a separately-excited main exciter. In many cases it is necessary to reverse the excitation voltage to obtain full leading rkva capacity from the condenser. This is accomplished by the differential field in the conventional main exciter, and by reversing the pilot exciter voltage in the case of the Rototrol pilot exciter.

In the operation of a synchronous condenser under abnormal conditions, a situation may occur where the condenser does not have sufficient corrective rkva capacity to handle all, or the most severe, system requirements. At such a time, the regulator in trying to hold the line voltage overexcites the condenser, causing it to carry excessive current and become overheated. To protect against this condition, a current-limiting device is used to limit the maximum excitation voltage to a level that does not cause damage due to continuous overloading of the condenser.

When the BJ regulator is used to control the excitation of a synchronous condenser, a time-delay current-limiting device is used. The equipment is designed to recognize two conditions; first, the case of a slowly rising load current to a predetermined limiting or unsafe value, and second, a sudden increase in load current such as might be caused by a system fault.

Protection against overcurrent is provided by a current-operated device having its operating coil energized by the line current and having its main contacts connected in series with the main control contacts of the voltage regulator. If the synchronous condenser load is gradually increased, the current-limiting contact in series with the *R* contact of the regulator opens the "raise" control circuit and prevents any further increase in excitation. At the same time, a second contact of the current-limiting device energizes the "lower" control circuit of the regulator, causing the excitation and load current to be reduced to the safe limiting value. This protection against a gradual increase in load operates in the normal-response *R-L* circuits of the voltage regulator.

In the case of a sudden increase in load current, an instantaneous overcurrent relay set to pick-up at a higher value of current than the current-limiting device closes its contacts. One set of contacts initiates a timing cycle, and the other set deenergizes an auxiliary relay. Deenergizing the auxiliary relay allows the contacts of the voltage-regulating element to remain in control for the time setting of the timing relay, thus permitting the use of both normal- and quick-response excitation for stability purposes under fault conditions.

Control of the excitation is automatically returned to the voltage-regulator control element when the overload disappears. Should the decreasing overload remain for a time below the setting of the instantaneous overcurrent relay but within the setting of the current-limiting element, the latter maintains control to prevent increase in excitation.

23. Impedance-Type Voltage Regulator

The excitation system shown in Fig. 36 employs a main-exciter Rototrol to supply excitation to the a-c generator. With the high degree of amplification obtainable with a Rototrol, the energy requirements of the control field are sufficiently small that they can be supplied by instrument transformers. The intelligence transmitted to the control field of the Rototrol as a function of the generator terminal voltage is determined by the voltage-regulator potential unit, voltage adjusting unit, and automatic control unit. These voltage-regulator devices consist entirely of imped-

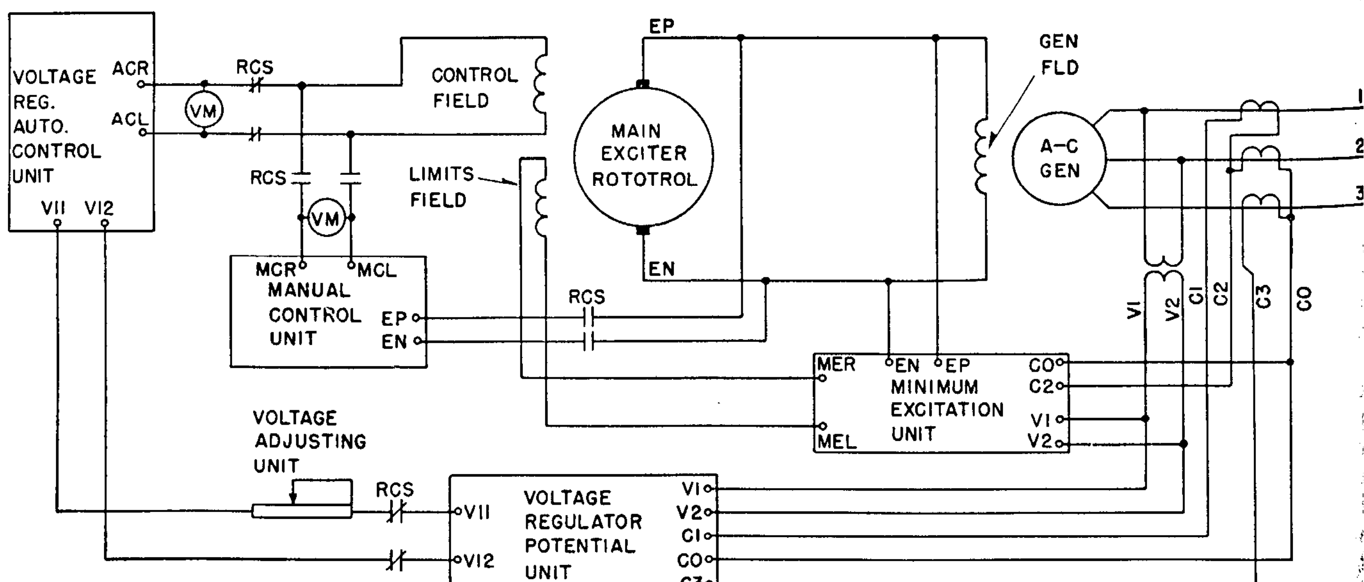


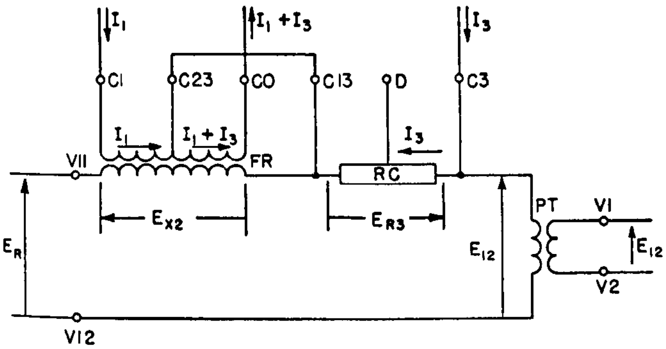
Fig. 36—Block diagram of the impedance-type voltage regulator as used in a main-exciter Rototrol excitation system.

ance elements and from this consideration the combination of devices in Fig. 36 is referred to as an impedance-type or static-type voltage regulator.

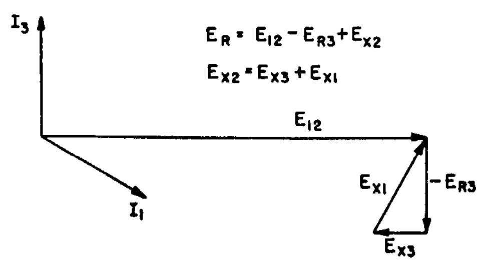
The voltage regulator potential unit is energized by the generator line-to-line voltage and the currents of two phases. Its output is a single-phase a-c voltage, applied to the series connection of the voltage adjusting unit and the automatic control unit. The automatic control unit is a voltage-sensitive device, the output of which is a d-c voltage. The polarity and magnitude of this d-c voltage are determined solely by the magnitude of the impressed a-c voltage. The output of the automatic control unit is the control signal that energizes the control field of the main-exciter Rototrol.

When the generator output voltage is exactly at the regulated value, the output voltage of the automatic control unit is zero. If the generator voltage increases above the regulated value, the d-c output voltage is in the direction to decrease the excitation voltage, working through the Rototrol exciter. When the generator voltage falls below the selected value, the d-c output voltage of the automatic control unit is in the direction to increase the a-c generator excitation.

When the voltage regulator is not in service, manual control of the a-c generator excitation is by means of the manual control unit. To guarantee synchronous machine steady-state stability, that is, insure adequate excitation for all kilowatt loads, a minimum excitation unit is used. The minimum excitation unit used with the Rototrol excitation systems is of a form that provides a variable minimum limit depending on the kilowatt load.



(a)



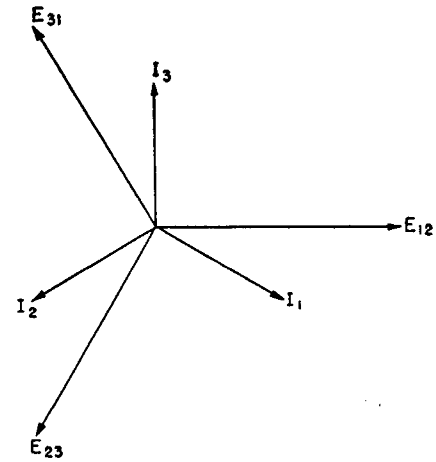
(b)

Fig. 37—Impedance-type regulator potential unit.

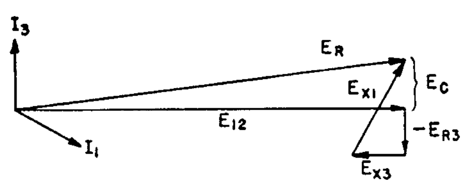
(a) Schematic diagram.
(b) Vector diagram.

Potential Unit—The voltage-regulator potential unit, shown schematically in Fig. 37, consists of a potential transformer, a filter reactor and a set of resistors. The output voltage of the potential unit is directly proportional to the positive-sequence component of the generator terminal voltage, and therefore, the voltage regulator is not affected by generator voltage unbalance and regulates to constant positive-sequence voltage. The circuit is a negative-sequence voltage-segregating filter so connected that the negative-sequence voltage is subtracted from the line voltage which, in the absence of a zero-sequence component, yields positive-sequence voltage.

The primary of the filter or mutual reactor is energized by the phase 1 and 3 current transformers. The flux produced thereby induces a voltage in the secondary winding which is added vectorially to the phase-3 drop in the

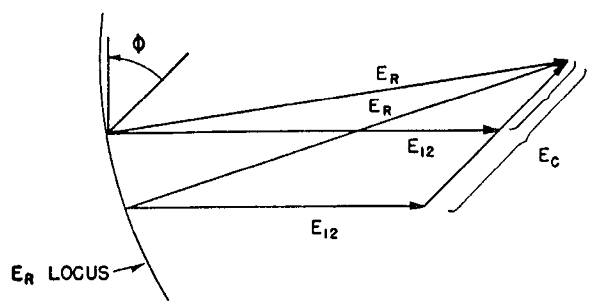


(a)



$E_R = E_{12} + E_C$

(b)



$E_R = E_{12} + E_C$

(c)

Fig. 38—Vector diagrams showing how cross-current compensation is obtained with the potential unit of the impedance-type regulator.

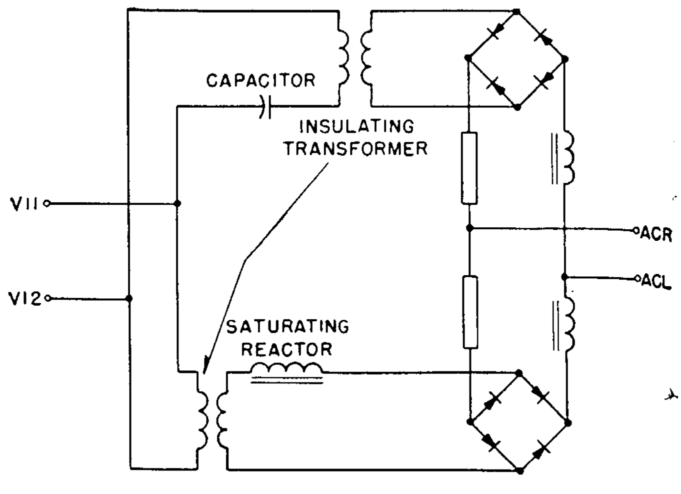
resistor *RC*, the sum being proportional to the negative-sequence voltage at the generator terminals. This negative-sequence voltage is the component of the three-phase voltage that represents the unbalance in voltage resulting from load unbalance. It is subtracted vectorially from the generator voltage to give the desired positive-sequence voltage across the terminals V11 and V12.

The potential unit can also provide compensation for parallel operation of a-c generators when each machine is equipped with a voltage regulator. Reactive-droop compensation is obtained by adjustment of the resistance *RC* in the potential unit in Fig. 37. The vector relations of the generator line currents and terminal voltages are shown in Fig. 38 (a). If the ohmic value of the resistor *RC* is 100 percent, the voltage equation of the circuit and the vector diagram are those shown in Fig. 37. If the ohmic value of *RC* is reduced to 50 percent, the vector diagram becomes that shown in Fig. 38 (b). E_R and E_{12} no longer are identical, although for unity power factor their difference in magnitude is of negligible proportion. The difference vector E_C can appropriately be called the reactive-droop compensator voltage. Assuming a given lagging power factor generator load, the vector diagram of Fig. 38 (c) shows how the generator terminal voltage E_{12} must vary for the automatic control-unit input voltage E_R to remain constant. As the generator load increases, E_C also increases and E_{12} must decrease, since E_R remains constant in magnitude. Thus the generator voltage is given a drooping characteristic with increase in lagging power factor load.

Voltage Adjusting Unit—The voltage adjusting unit in Fig. 36 is a rheostat that enables the operator to set the a-c generator regulated voltage at any value within a band of plus or minus 10 percent of the rated generator voltage. By means of the voltage adjusting unit, the resistance between the generator terminals and terminals V11 and V12 of the automatic control unit can be changed, causing a directly proportional change in voltage drop in the circuit. The drop requires a change in a-c generator voltage to produce the regulator balance-point voltage across the terminals V11 and V12.

Automatic Control Unit—The automatic control unit is the voltage-sensitive element of the impedance-type voltage regulator. It measures the voltage to be regulated and delivers energy to the main-exciter control field only when necessary. The voltage-sensitive circuit in Fig. 39 consists essentially of two parallel-circuit branches; one containing a capacitor and the other a saturating reactor. The voltage-current characteristic curves of the capacitor and saturating reactor are shown in Fig. 39 (b). The curve of the reactor indicates that its current increases more rapidly than voltage, and the currents through the two branches of the circuit are equal at only one value of voltage where the characteristics intersect. This point of intersection is called the balance point of the two impedances. The operation of the voltage regulator depends upon the fact that when the voltage increases above this point, the current in the reactor is greater than the current in the capacitor. When the voltage decreases below the balance point, the capacitor current is the greater.

The output of the reactor circuit and the output of the capacitor circuit are rectified by single-phase full-wave dry-type rectifiers, which are connected with additive relation in series through a resistor and smoothing reactors. The control field of the Rototrol is connected between a mid-tap on the resistor and the opposite side of the rectifier circuit. When the applied voltage is at the balance point and the capacitor and reactor currents are equal in magnitude, the output currents of the rectifiers are equal and circulate between the rectifiers. Under this condition there is no potential difference between the terminals ACR-ACL of the Rototrol control field and no current flows in the field. Should the a-c voltage become low, however, the rectified current of the capacitor circuit is large compared with that of the reactor circuit raising the potential of terminal ACR above that of ACL and causing current to flow in the control field in a direction to increase the excitation voltage and



(a)

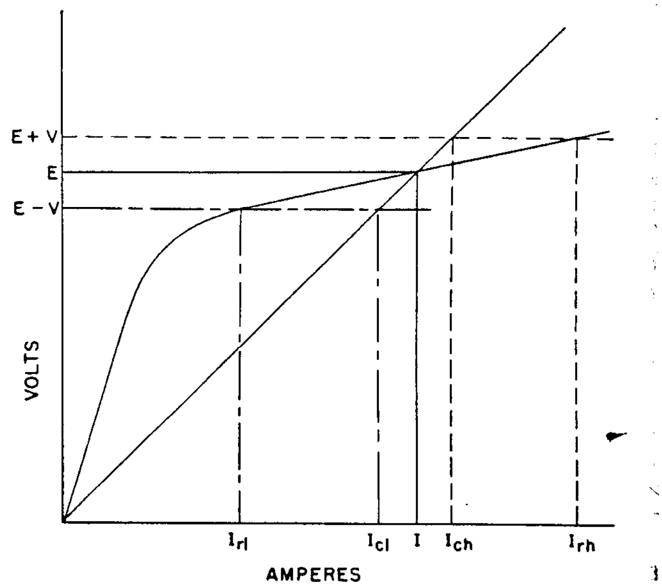


Fig. 39—Impedance-type regulator automatic control unit.

(a) Circuit diagram.

(b) Intersecting impedance characteristics of saturating reactor and capacitor.

raise the a-c voltage. For an increase in a-c voltage, the direction of current flow in the control field would be reversed causing a reduction in excitation voltage. Thus with normal a-c voltage applied to the automatic control unit, the control-field current is nearly zero and any deviation in a-c voltage causes a corrective current to flow in the control field.

The current in the control field of the Rototrol is directly proportional to the horizontal difference between the capacitor and saturating reactor volt-ampere characteristics in Fig. 39 (b). Examination of the curves shows that the control-field current is approximately proportional to the

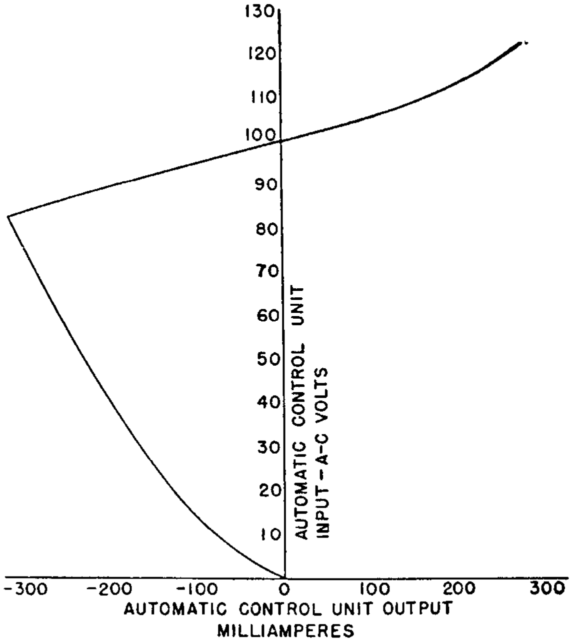


Fig. 40—Typical output curve of automatic control unit as function of a-c voltage.

change in a-c voltage for small changes. The control-field current as a function of the a-c voltage applied to the automatic control unit is shown in Fig. 40. Maximum current in the direction to raise the Rototrol terminal voltage occurs when the a-c voltage is approximately 85 percent of the balance-point voltage. The small current output of the automatic control unit is sufficient to control the Rototrol output over the entire range of the Rototrol capability.

Minimum Excitation Unit—Like other units of the impedance-type voltage regulator, the minimum-excitation unit normally used is comprised of impedance elements.

The minimum-excitation unit establishes a minimum point or limit below which the excitation of the a-c generator cannot be lowered. The minimum point can be a fixed limit or a variable limit. On machines that carry considerable real or kilowatt load it usually is desirable to make the minimum limit vary approximately directly proportional to the kilowatt load, thereby maintaining a margin of excitation current above that at which the machine would pull out of synchronism. Since the main-exciter Rototrol is limited to use with 3600-rpm turbine generators, the minimum excitation unit is of the variable-limit type.

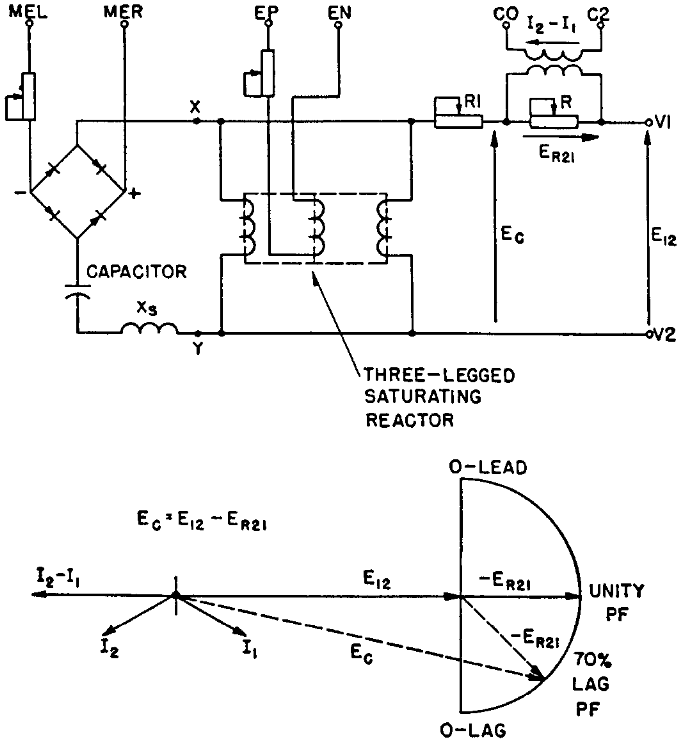


Fig. 41—Schematic diagram of the impedance-type minimum excitation unit and vector diagram showing how variable minimum limit is obtained.

The schematic diagram and vector diagram of the minimum-excitation unit is shown in Fig. 41. A saturable reactor with coils on the three legs of a B-shaped core is used. The two outside legs are connected in parallel, such that at any given instant, both windings produce an a-c flux in the same direction through the center leg of the core. The winding on the center leg is the d-c control coil. The d-c current in this winding controls the saturation of the iron core, thereby controlling the inductance and reactance of the two outer a-c windings. When the d-c control current is low, saturation of the core is slight, and the reactance of the a-c coils is high; and when the d-c current is high, the core has a higher degree of saturation and the reactance of the a-c windings is low.

The center-leg winding is energized by the main-exciter Rototrol output voltage as shown in Fig. 36. When the a-c generator is operating at normal voltage and the excitation voltage is normal, the current in the reactor control winding is relatively high, and consequently the reactance of the a-c windings is low. A substantial amount of a-c current is allowed to flow through the reactor windings under this condition. The relatively high a-c current through resistor *R1* causes a large voltage drop such that the a-c voltage appearing across *X-Y* is relatively small. When the voltage is low across the series circuit composed of the saturating reactor, capacitor and rectifier, current in the series circuit is substantially zero. However, because of the impedance characteristic of this series circuit, there is a voltage at which the series-circuit current begins to increase rapidly with small increases in voltage.

If for some reason system conditions should cause the voltage regulator to introduce current into the control field of the Rototrol to reduce the excitation voltage, the current in the reactor control winding is also reduced. The reactance of the a-c windings increases, and the current through resistor *R1* is reduced, causing less voltage drop in the circuit and increasing the voltage across *X-Y*. If the voltage across *X-Y* rises to the conducting point of the series circuit, a-c current increases sharply in this circuit, and this current rectified is supplied to the minimum excitation control field of the Rototrol exciter. The minimum excitation control field is the limits field in Fig. 36. The direct current supplied to the minimum excitation control field is in the direction to raise the excitation voltage, and the minimum excitation unit thus begins to regulate for a preset minimum excitation voltage to keep the circuit of the unit balanced. When system conditions cause the automatic control unit to increase the excitation above that provided by the minimum excitation unit, the regulator again takes control and holds the voltage for which it is adjusted.

The variable minimum excitation limit is obtained by the compensating circuit shown in the left-hand portion of Fig. 41. The voltage E_{12} across terminals V1-V2 is held constant by the automatic control unit under balanced load conditions. A compensating voltage that is a function of line currents I_2-I_1 is added vectorially to E_{12} such that the a-c voltage applied to the saturating reactor is equal to E_C. The currents I_1 and I_2 in the vector diagram of Fig. 41 are drawn for the unity power factor condition and the resulting magnitude of E_C is represented by the vector drawn with a solid line. If the magnitudes of the line currents are held constant and the power factor changed to 70 percent lagging, the voltage E_{R21} is shifted such that the magnitude of E_C becomes that represented by the dotted vector. Thus, the magnitude of the voltage E_C is dependent on the magnitude of the in-phase component of the line current, and hence varies with the kilowatt load on the generator. The locus of the magnitude of E_C for a particular magnitude of current at various power factors is represented by the semi-circle as shown. Therefore, since the voltage input to the saturating reactor is a function of the kilowatt load, the voltage across *X-Y* applied to the series circuit also varies with kilowatt load. The minimum excitation limit becomes a variable quantity dependent upon the kilowatt load of the generator.

The individual and combined volt-ampere characteristics of the saturating reactor, capacitor and resistance (equivalent resistance of the reactor, rectifier and load) are shown in Fig. 42 (a). As the voltage across *X-Y* is increased, the combined characteristic shows that the circuit conducts practically no current until the voltage E_1 is reached. The current then undergoes a large increase to the value I_1. When the volt-ampere characteristic of the resistor *R1* is included, the combined characteristic is modified to that shown in Fig. 42 (b). The sudden large increase in current shown when voltage E_1 is reached in Fig. 42 (a) is eliminated, but the current increases rapidly and linearly with increase in voltage in the range above E_1. The practical operating range of the unit is determined by the intersection of the capacitive reactance line X_C with the saturating reactor line X_S. Two ratings of minimum-excitation units are available; one giving an operating range

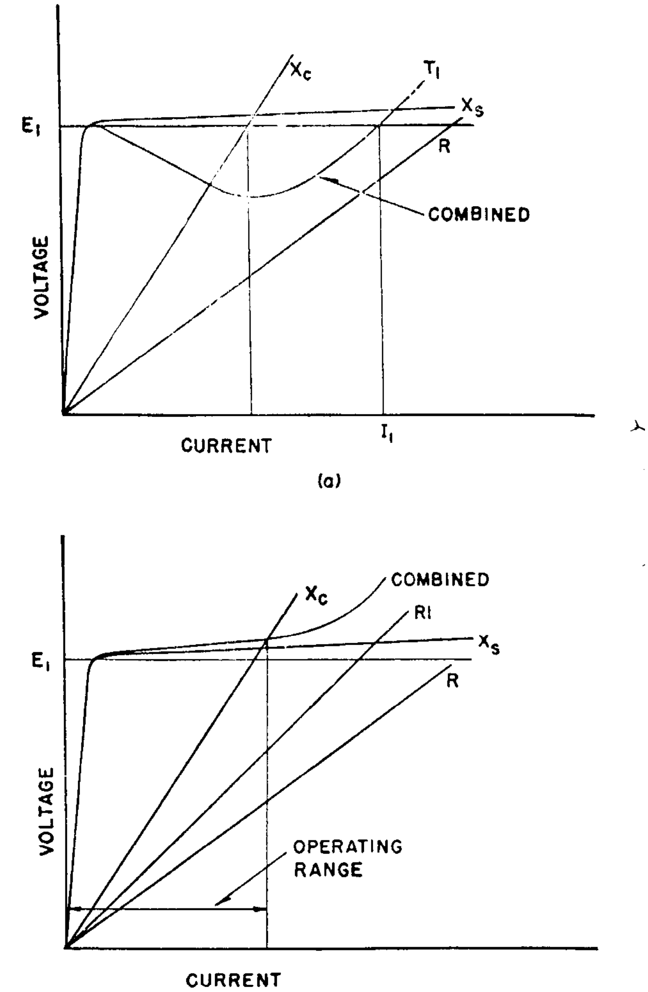


Fig. 42—Volt-ampere characteristics of individual components of minimum excitation unit and combined volt-ampere characteristic.

(a) Effect of *R1* omitted.
(b) Effect of *R1* included.

of 0–300 milliamperes, and the other giving an operating range of 0–750 milliamperes. The unit having the larger operating range is used with the main-exciter Rototrol.

Manual Control Unit—The manual control unit used with the main-exciter Rototrol excitation system of Fig. 36 is a bridge-type circuit as shown in Fig. 43. Such a circuit is required to reverse the direction of current in the control field as required to raise or lower the Rototrol voltage. In addition, the unit is a d-c voltage regulator in itself, maintaining essentially constant main-exciter voltage and constant a-c generator voltage for a given load.

The bridge circuit consists of two fixed resistors, a potentiometer and two selenium rectifiers connected as shown. The main exciter terminal voltage is applied across two terminals of the bridge and the control field of the Rototrol is connected across the other two terminals. The exciter terminal voltage is adjusted by changing the position of the potentiometer. The selenium rectifiers form the controlling element of the bridge circuit since the voltage drop in this leg of the bridge is practically independent of

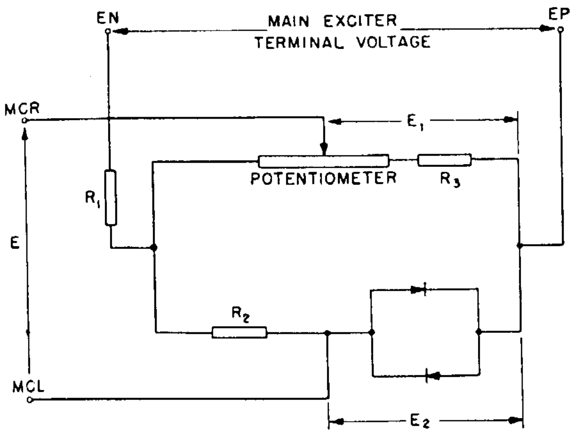


Fig. 43—Schematic diagram of the impedance-type regulator manual control unit.

the current through the rectifiers, and will remain substantially constant. Thus the voltage E_2 in Fig. 43 can be considered constant.

For a given setting of the potentiometer, the bridge circuit is balanced when the voltage E_1 is equal to E_2 and under this condition there is no current in the Rototrol control field. If the main exciter voltage should increase for any reason, the current through the bridge increases, which increases the voltage drop E_1 so that MCR is positive with respect to MCL. Current then flows in the control field in a direction to reduce the exciter voltage until the bridge circuit is again balanced. For a drop in exciter voltage, the control field current would be in the raise direction. Thus, the a-c voltage may be adjusted for any value from zero to maximum, and the manual control unit holds the excitation voltage constant.

24. Main-Exciter Rototrol Generator Excitation System

The Rototrol with its two stages of amplification can be built with large power output capabilities while the control field energy requirements are sufficiently small to be supplied by instrument transformers. Also, since the Rototrol is a high-speed machine with air-gap dimensions the same as any other form of d-c machine, it can be direct-connected to the shaft of a turbine generator. The direct-connected main-exciter Rototrol is a step toward simplification of turbine generator construction, operation and maintenance by completely eliminating the pilot exciter. The circuit of the main-exciter Rototrol excitation system is that shown in Fig. 36.

The effect on the main-exciter Rototrol of induced field current caused by changes in generator load was discussed in Sec. 10. Evidence of the importance of this effect and illustration of the comparative performance of the main-exciter Rototrol excitation system is given in Fig. 44. The solid line shows the time variation of the a-c generator voltage under control of an impedance-type regulator and a main-exciter Rototrol, and the dashed-line curve shows the variation under control of an indirect-acting exciter-rheostatic type of regulator and a conventional main exciter with 0.5 response ratio. In each case, a three-phase reactance load was suddenly applied to the generator to

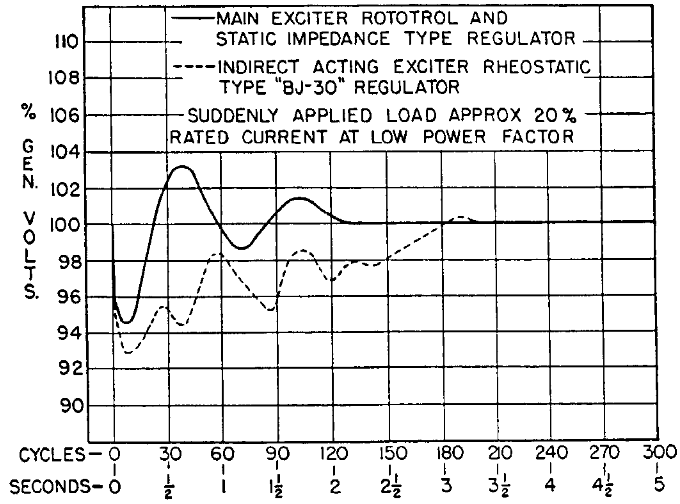


Fig. 44—Voltage-recovery performance of main-exciter Rototrol excitation system compared with performance of conventional main-exciter system under control of BJ regulator. Approximately 20 percent of generator rated amperes at 0 percent lagging power factor added at zero time.

cause approximately 20 percent of rated generator amperes to flow in the circuit. The rapid recovery of the voltage under control of the impedance-type regulator and main-exciter Rototrol is an important factor in maintaining system stability, particularly during the period of overshoot when the generator voltage is greater than 100 percent.

The main-exciter Rototrol excitation system has the advantage of a voltage regulator without moving parts, without contactors, and requiring no large motor-operated main-exciter field rheostat. The overall performance of the system shows marked improvement in voltage dip and recovery time when compared with a conventional main-exciter excitation system. The system also eliminates the use of any pilot exciter.

25. Rototrol Pilot Exciter with Single-Field Main Exciter

The simplest form of an excitation system using a Rototrol pilot exciter is shown in Fig. 45. When the speed of rotation of the main a-c generator is 1200, 1800 or 3600 rpm, the main exciter and Rototrol pilot exciter can be direct-connected to the generator shaft. A second possibility is to have the main exciter mounted on the shaft of the a-c generator and the Rototrol separately-driven by a small motor, the m-g set having sufficient inertia to carry through system disturbances without appreciable speed change. This arrangement might be used where the generator speed is less than 1200 rpm. A third arrangement is to have the main exciter and the Rototrol pilot exciter driven by a motor and operating at 1200 or 1800 rpm. The latter arrangement is applicable with a generator of any speed.

In the conventional excitation system, the pilot exciter is a constant-voltage generator. The Rototrol pilot exciter is a variable voltage pilot exciter and the method of operating the excitation system of Fig. 45 is essentially no different than the operation of conventional exciter-rheostatic systems, except that no regulator-controlled, motor-operated exciter-field rheostat is used. Variable voltage is supplied to the main-exciter field by the Rototrol pilot exciter, which is connected directly to the field and is under the control of the voltage regulator automatic control unit or the manual control unit.

The voltage regulator potential unit, voltage adjusting unit, automatic control unit and the manual control unit are those described in Sec. 23.

The Rototrol pilot exciter used in this excitation system can provide either one or two stages of amplification, depending on the energy requirements of the main-exciter shunt field. The Rototrol can easily be constructed to provide rates of response and ceiling voltage equal to or in excess of those obtained with conventional d-c machines.

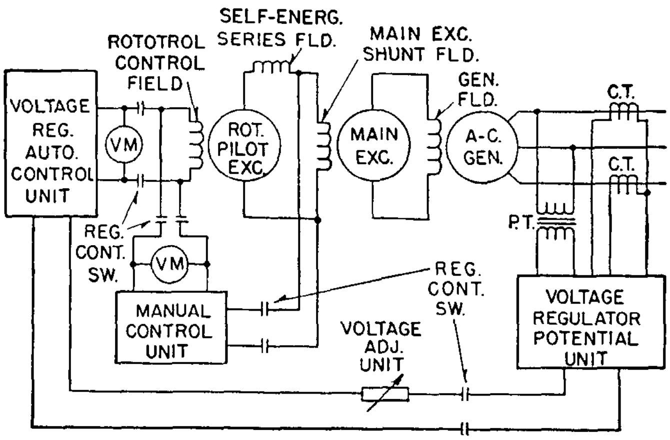


Fig. 45—Excitation system with Rototrol pilot-exciter and single-field main exciter controlled by impedance-type regulator.

The excitation system shown in Fig. 45, therefore, provides performance characteristics at least equal to those obtained with conventional excitation systems.

The Rototrol pilot exciter in Fig. 45 supplies all the excitation requirements of the main exciter. In this respect this system is identical with exciter-rheostatic systems using pilot exciters. The essential advantage is the elimination of the comparatively complicated exciter-rheostatic regulator with its moving parts and elimination of the motor-operated main-exciter field rheostat. As is the case with the exciter-rheostatic excitation system, loss of the pilot exciter through a short circuit or open circuit causes loss of excitation on the a-c generator.

26. Rototrol Buck-Boost Pilot Exciter

The buck-boost Rototrol excitation system using a two- or three-field main exciter, as shown in Fig. 46, offers a number of advantages over the single-field main exciter system described in Sec. 25. In the system of Fig. 46, the Rototrol pilot exciter operates in a different manner from that in Fig. 45.

The operation of the three-field main exciter was described in Sec. 6. The Rototrol buck-boost pilot exciter supplies the proper voltage to field 2 of the main exciter to control the output voltage. Briefly, the excitation provided by field 1 is set by the operator to give a base excitation in the main exciter, and the excitation provided by field 2 adds to or subtracts from this base excitation to vary the output voltage. Thus the Rototrol must be capable of bucking or boosting the main exciter base excitation to give the necessary range of main exciter voltage. The Rototrol-excited field of the main exciter also acts as a stabilizing field under regulator control.

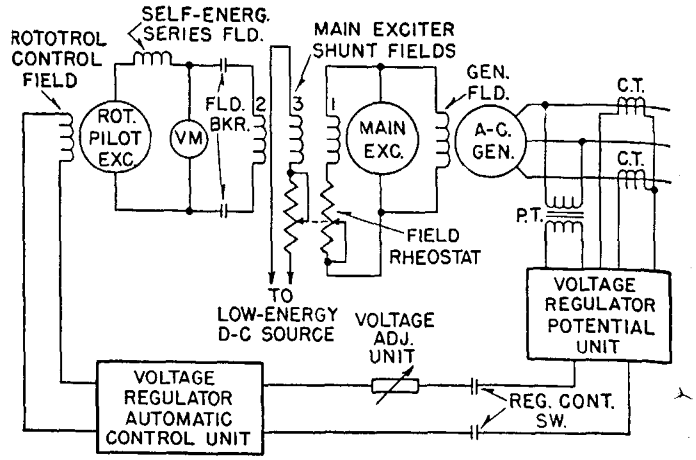


Fig. 46—Excitation system with Rototrol buck-boost pilot exciter and three-field main exciter.

All of the voltage regulator component parts in Fig. 46 are those described in Sec. 23. The manual control unit is not required, since manual control is obtained by operating the main exciter as a self-excited exciter with a stabilizing field, and voltage control is by means of the shunt-field rheostat.

Since the main exciter base excitation is supplied by the self-excited field, complete excitation is not lost or is the continuity of the load disturbed upon the occurrence of any trouble in the Rototrol buck-boost pilot exciter circuits or in any part of the impedance-type voltage regulator elements. Even in the event of a short circuit or open circuit in the pilot exciter output circuit, the preset base excitation remains rheostat controlled and undisturbed. If a circuit failure occurs when the a-c generator is carrying a load other than that used to determine the rheostat

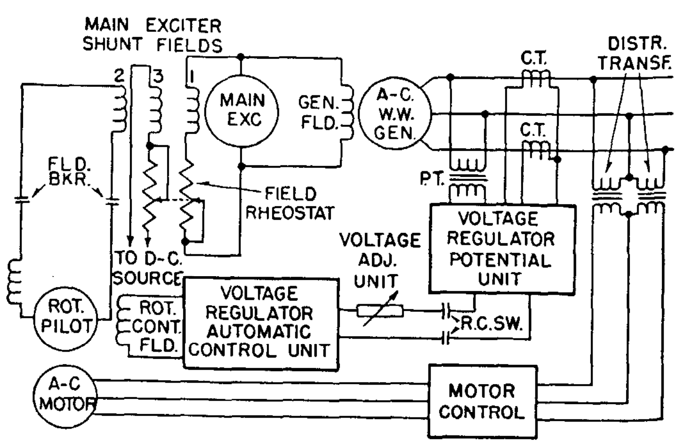


Fig. 47—Excitation system for hydroelectric generator with motor-driven Rototrol buck-boost pilot exciter and three-field main exciter.

setting, the a-c generator continues to carry its kilowatt load, but at a different power factor.

27. Rototrol Excitation for Hydroelectric Generators

It is impractical to direct-connect the Rototrol to the shaft of a waterwheel generator, because of the multiplicity of speeds and sizes involved. The Rototrol pilot exciter, therefore, must be driven by a small motor, introducing the problem of a reliable power supply for the driving motor. The three-field main exciter and Rototrol buck-boost pilot exciter excitation system of Fig. 46 is readily adaptable to use with slow-speed generators and is shown in Fig. 47.

During start-up of the generator when no outside source of supply is available for driving the Rototrol motor-generator set, the main exciter is operated as a self-excited machine and provides excitation for the main generator. As soon as a-c voltage is available, the Rototrol can be started and the voltage regulator placed in service, or operation can be continued under hand control with the operator controlling excitation with the self-excited shunt field rheostat.

Under short-circuit conditions on the a-c system, the excitation system must be capable of supplying full excitation to the generator field. With the system shown in Fig. 47, this is accomplished by building sufficient inertia into the Rototrol m-g set to carry it through such disturbances with very little change in speed even under severe forcing conditions.

28. Rototrol Excitation for Synchronous Condensers

The Rototrol excitation system for synchronous condensers is similar to that shown in Fig. 47 for waterwheel generators. However, in the usual case, the main exciter for the condenser is also motor-driven so that the motor supply circuit has to be modified to supply sufficient power for the motor-generator set. Electrically, the circuit is the same as Fig. 47, but the main and pilot exciters are normally on the same shaft and driven by a large motor.

Where some form of current limit is desired as discussed in Sec. 22, a static current-limit device can be used with the Rototrol excitation system. The circuit of the current-limit unit is similar to that of the minimum excitation unit shown in Fig. 41. The rectified a-c load current of the condenser is used to energize the center or control winding of the three-legged reactor, and to control the magnitude

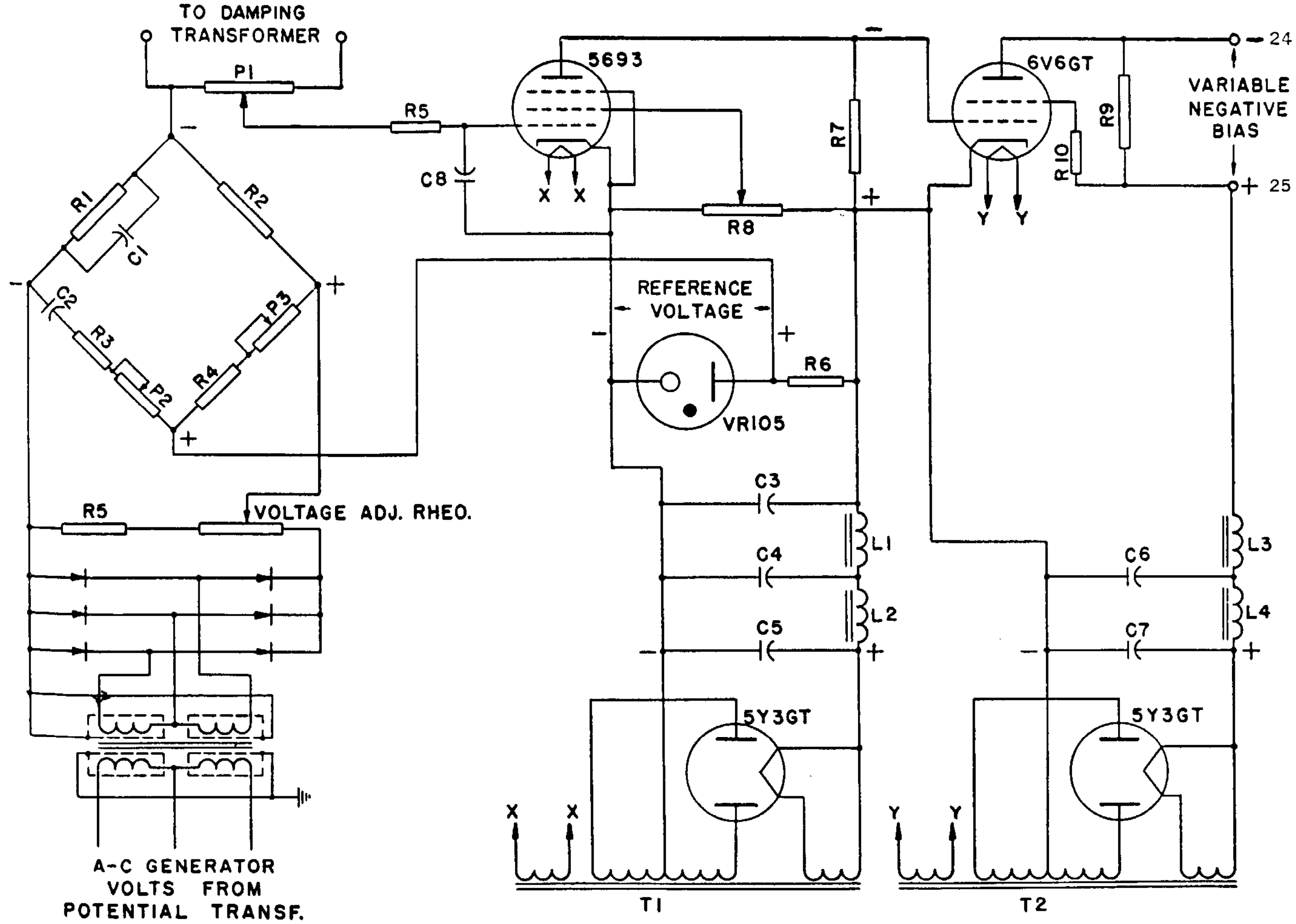


Fig. 48—Schematic diagram of the electronic generator voltage regulator. The variable output appearing across terminals 24 and 25 can be used to control the firing point of an electronic main exciter or can be adapted to control a Rototrol excitation system.

of the voltage applied to the series reactor-capacitor-rectifier circuit. At a certain magnitude of a-c current, the series circuit begins conducting a rapidly increasing current, which is applied to the limits field of the Rototrol. The current in the limits field is in the direction to lower the excitation voltage. Should the control field be conducting current in the raise direction, the combined effect of the two fields is such that the excitation voltage is held constant at the limiting value. Time delay can be provided in the limiting circuit to enable full forcing of the condenser excitation during transient overloads.

29. Electronic-Type Voltage Regulator

Electronic-type voltage regulators are available in many different forms, a typical one being shown in Fig. 48. This particular regulator is used with the electronic main exciter in Fig. 23, but it can be modified for use with Rototrol excitation systems.

A d-c voltage proportional to the average three-phase a-c generator voltage is obtained from a three-phase bridge-type rectifier, the output of which is applied to a voltage-adjusting rheostat and a modified Wein bridge-type filter. The bridge, comprising resistors *R1*, *R2*, *R3* and *R4*, capacitors *C1* and *C2* and potentiometers *P2* and *P3*, filters the 360-cycle ripple voltage in the d-c output of the rectifier. Thus, the output of the bridge circuit, which is the input to the regulator, is a smooth d-c voltage. The bridge-type filter provides a high degree of filtering without adding unduly long time constants to the regulator input circuit.

The generator voltage regulator consists of two d-c amplifiers and a reference voltage. Regulation is obtained by comparing the rectified generator terminal voltage with the reference voltage. The first d-c amplifier is a high-gain voltage amplifier using a 5693 tube, which is an industrial-type tube with characteristics the same as a type 6SJ7 tube. The output of the voltage amplifier is fed into a power amplifier using a 6V6GT tube. The high-gain voltage stage gives the regulator its high degree of sensitivity and the power amplifier supplies the variable negative bias voltage for controlling the thyratron firing tubes in Fig. 23.

A full-wave rectifier (5Y3GT tube) is used to supply the plate voltage of the 5693 tube. The rectified output of transformer *T1* is fed into a two-section condenser input filter giving a smooth d-c voltage with polarities as indicated. The d-c reference voltage is obtained from the voltage drop across a type VR-105 voltage regulator tube connected in series with resistor *R6* across the d-c power supply. The reference voltage is also a smooth d-c voltage that remains constant for wide variations of supply voltage.

The rectified generator voltage is connected differentially with the reference voltage and applied to the grid circuit of the 5693 tube. This circuit can be traced from the grid of the tube through the grid resistor *R5* to the negative side of the rectified generator voltage; from the positive side of the rectified generator voltage to the positive side of the reference voltage; and from the negative side of the reference voltage to the cathode of the 5693 tube. The amplified voltage from the 5693 tube appears across the load resistor *R7* with polarities as shown and this voltage drop is applied to the grid of the 6V6GT tube. The grid circuit of the 6V6GT tube can be traced from the grid through resistor *R7* to the cathode. The variable negative d-c voltage output of the regulator is obtained across the load resistor *R9* of the 6V6GT tube and applied to the grid circuits of the thyratron firing tubes in Fig. 23.

Under balanced conditions when the a-c generator voltage is equal to the regulated value, the grid of the 5693 tube is established at a particular bias voltage depending on the magnitudes of the reference voltage and the rectified a-c voltage. This grid bias establishes the current in the 5693 tube and the drop across *R7*, which in turn establishes the grid bias of the 6V6GT tube. Current in the 6V6GT tube is thus fixed, as is the drop across load resistor *R9*. The voltage output is constant as long as the a-c generator voltage is equal to the regulated value.

Should the a-c generator voltage increase above the normal value, the differential connection of the rectified generator voltage and the reference voltage makes the grid bias of the 5693 tube more negative than previously, which reduces the current in the tube and in resistor *R7*. The lower voltage drop across *R7* reduces the negative bias voltage on the grid of the 6V6GT tube and causes an increase in current through the tube and load resistor *R9*. Thus, the negative voltage output across terminals 24 and 25 is increased. Reference to Fig. 24 shows that the increase in negative-bias voltage on the thyratron firing tubes causes an increase in the angle of grid delay, which reduces the main-exciter voltage. In a similar manner, low a-c voltage causes the grid bias of the 5693 tube to be less negative than previously, which causes a reduction in the voltage across terminals 24 and 25 and a consequent reduction in the thyratron firing tube angle of grid delay.

REFERENCES

1. Quick-Response Excitation, by W. A. Lewis, *The Electric Journal*, Vol. 31, August 1934, pp. 308–312.
2. Determining the Ratio of Exciter Response, by A. van Niekerk, *The Electric Journal*, Vol. 31, September 1934, pp. 361–364.
3. The Exciter-Rheostatic Regulator, by A. G. Gower, Jr., *The Electric Journal*, Vol. 32, February 1935, pp. 73–75.
4. The Generator Rheostatic Regulator, by A. G. Gower, Jr., *The Electric Journal*, Vol. 32, April 1935, pp. 143–144.
5. Recent Developments in Generator Voltage Regulators, by C. R. Hanna, K. A. Oplinger and C. E. Valentine, *A.I.E.E. Transactions*, Vol. 58, 1939, pp. 838–844.
6. Static Voltage Regulator for Rototrol Exciter, by E. L. Harder and C. E. Valentine, *A.I.E.E. Transactions*, Vol. 64, 1945, pp. 601–606.
7. The Multistage Rototrol, by M. M. Liwschitz, *A.I.E.E. Transactions*, Vol. 66, 1947, pp. 564–568.
8. Two-Stage Rototrol for Low-Energy Regulating Systems, by A. W. Kimball, *A.I.E.E. Transactions*, Vol. 66, 1947, pp. 1507–1511.
9. Rototrol Excitation Systems, by J. E. Barkle and C. E. Valentine, *A.I.E.E. Transactions*, Vol. 67, 1948, pp. 529–534.
10. Main Exciter Rototrol Excitation for Turbine Generators, by C. Lynn and C. E. Valentine, *A.I.E.E. Transactions*, Vol. 67, 1948, pp. 535–539.

CHAPTER 8

APPLICATION OF CAPACITORS TO POWER SYSTEMS

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I. SHUNT CAPACITOR FUNDAMENTALS

THE function of a shunt capacitor applied as a single unit or in groups of units is to supply lagging kilovars to the system at the point where they are connected. A shunt capacitor has the same effect as an overexcited synchronous condenser, generator or motor. It supplies the kind of kilovars or current to counteract the out-of-phase component of current required by an induction motor as illustrated in Fig. 1.

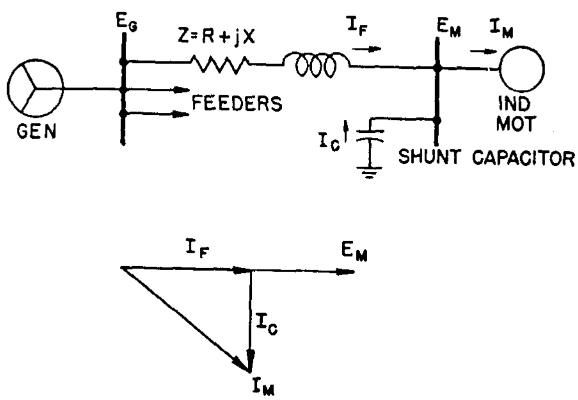

Fig. 1—Shunt Capacitors supplying kvar required by an induction motor.

Shunt capacitors applied on the load end of a circuit supplying a load of lagging power factor have several effects, one or more of which may be the reason for the application:

1. Reduces lagging component of circuit current.
2. Increases voltage level at the load.
3. Improves voltage regulation if the capacitor units are properly switched.
4. Reduces I^2R power loss in the system because of reduction in current.
5. Reduces I^2X kilovar loss in the system because of reduction in current.
6. Increases power factor of the source generators.
7. Decreases kva loading on the source generators and circuits to relieve an overloaded condition or release capacity for additional load growth.
8. By reducing kva load on the source generators additional kilowatt loading may be placed on the generators if turbine capacity is available.
9. To reduce demand kva where power is purchased. Correction to 100 percent power factor may be economical in some cases.
10. Reduces investment in system facilities per kilowatt of load supplied.

The shunt capacitor affects all electrical equipment and circuits on the source side of where they are installed. If the capacitor kvar is small, say ten percent of the circuit rating, it is usually sufficient to make an analysis on the circuit involved for the application. However, where the capacitor kvar is large, its effect on each part of the system back to and including the source should be considered.

In determining the amount of shunt capacitor kvar required, it must be recognized that a voltage rise increases the lagging kvar in the exciting currents of transformer and motors. Thus, to get the desired correction some additional capacitor kvar may be required above that based on initial conditions without capacitors. If the load includes synchronous motors, it may be desirable, if possible, to increase the field currents to these motors.

Shunt capacitors are applied in groups ranging from one capacitor unit of 15 kvar to large banks of these standard units totaling as much as 20 000 kvar. Many small banks of 45 kvar to 360 kva are installed on distribution circuits. Banks of 520 kvar to about 3000 kvar are common on distribution substations of moderate size. Larger banks of 5000, 10 000 and 15 000 kvar are in service in a number of larger substations. Usual voltage ratings of capacitor banks start at 2400 volts and range upward for groups of capacitors connected in series for 46 kv. Consideration is being given to voltages up to and including 138 kv. This is feasible provided the bank is sufficiently large in kvar.

1. History

Shunt capacitors were first applied for power-factor correction about 1914. Their use, however, was limited during the next twenty years because of high cost per kvar, large size and weight. Prior to 1932 all capacitors employed oil as the dielectric. At about this time the introduction of chlorinated aromatic hydrocarbon impregnating compounds (askarels) and other advances in the capacitor construction brought about sharp reductions in size and weight. As shown by Fig. 2 the present weight per kvar is less than 5 pounds compared with over 20 pounds in 1925.

Before 1937 practically all capacitors were installed indoors in industrial plants. Extensive utility use started after the appearance of outdoor units, which eliminated steel housings and other accessories. By 1939 capacitor costs had been reduced almost proportionately with weight and they had been proved in service. Starting in 1939 and continuing to the present, capacitor use has increased phenomenally year by year, as shown in Fig. 2. The acceptance of capacitors has been due to the following:

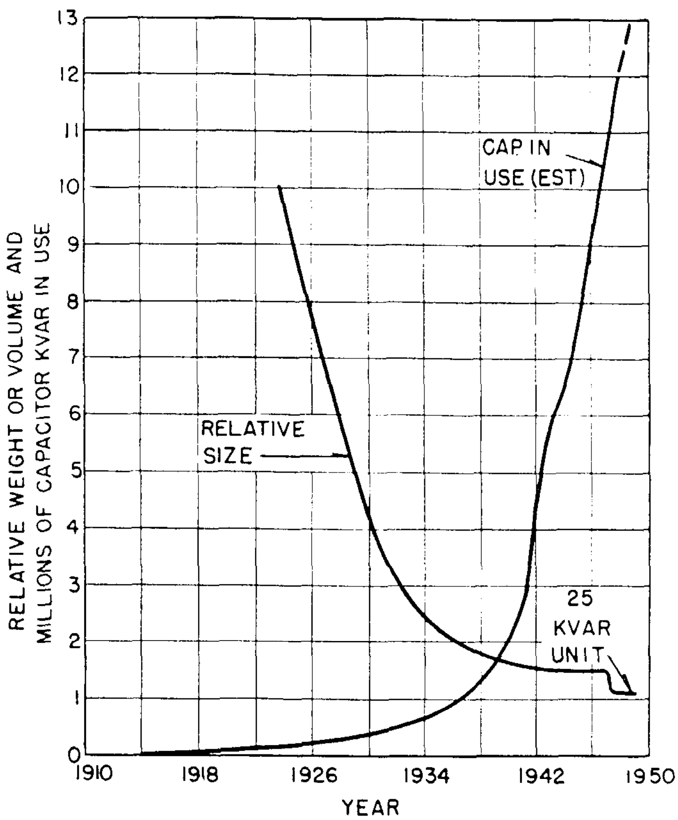

Fig. 2—Evaluation of the size and use of Shunt Capacitors.

1. Reduction in selling price.
2. Improved design and manufacturing methods resulting in small size and weight.
3. Development of outdoor, pole-type units and standardized mounting brackets.
4. Reduction in failures.
5. Better understanding of system benefits that accrue from their use.
6. By force of circumstances, during the war emergency of 1939 to 1945, manufacturing facilities for capacitors were more available than other means of supplying kilovars. Also less critical material was required for capacitors than for other kvar generators.

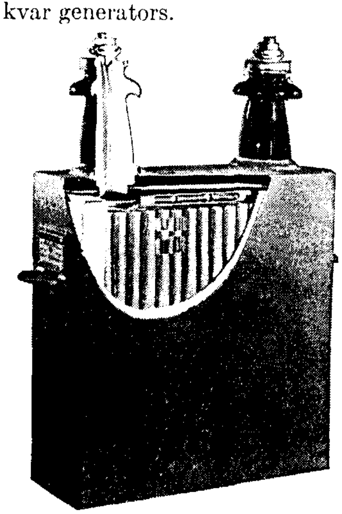

Fig. 3—Cut-away view of 25 kvar 2400 volt outdoor capacitor unit.

7. Due to the large volume of production during the war and since, the economics of using capacitors is favorable.

2. Capacitor Failure Rates

To evaluate the operation and economics of shunt capacitors, it is helpful to predict the number of unit failures that may occur. Not only do unit failures mean the loss of the units but also, under certain conditions a unit failure may damage other good units. Prediction of failures can be based on past experience, such as given in Curve A, Fig. 4. This curve gives cumulative unit failures per 1000 units in service regardless of how or where they are installed or how they are protected. Curve B represents unit failures of small groups of capacitors distributed over a system without lightning protection and subject to other hazards. In view of the benefits a performance as given by Curve B has been considered economical and satisfactory. Curve C represents performance of large banks of capacitors where careful attention has been given to operating conditions and protective devices. For such performance each unit should be inspected and tested at the installation to weed out units damaged in transportation. Individual capacitor fuses are also essential for best performance as discussed later under Capacitor fusing.

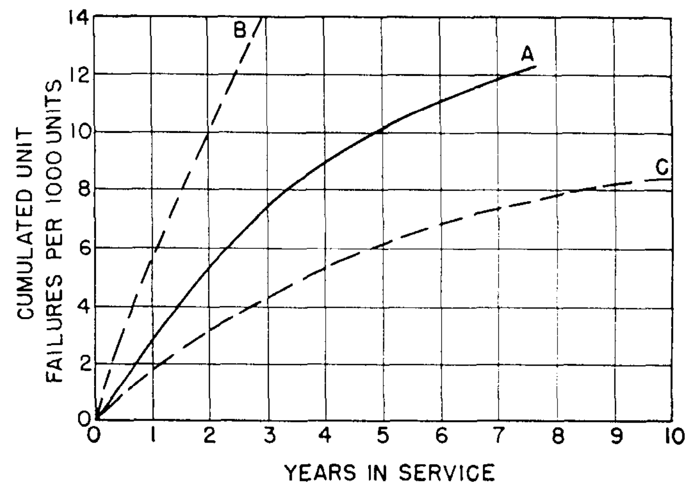

Fig. 4—Failure rate of shunt capacitors.

A—Average of all types of installations.
B—Average of unprotected, exposed installations.
C—Average of well protected larger installations.

3. Fundamental Effects

To illustrate the effects of shunt capacitors, assume that a 100-kva circuit or piece of apparatus has to carry 100 kva at various power factors. By adding shunt capacitors at the load, the kva from the source is reduced materially. The lower the load power factor, the more effective the capacitors are. This situation is illustrated in Fig. 5.

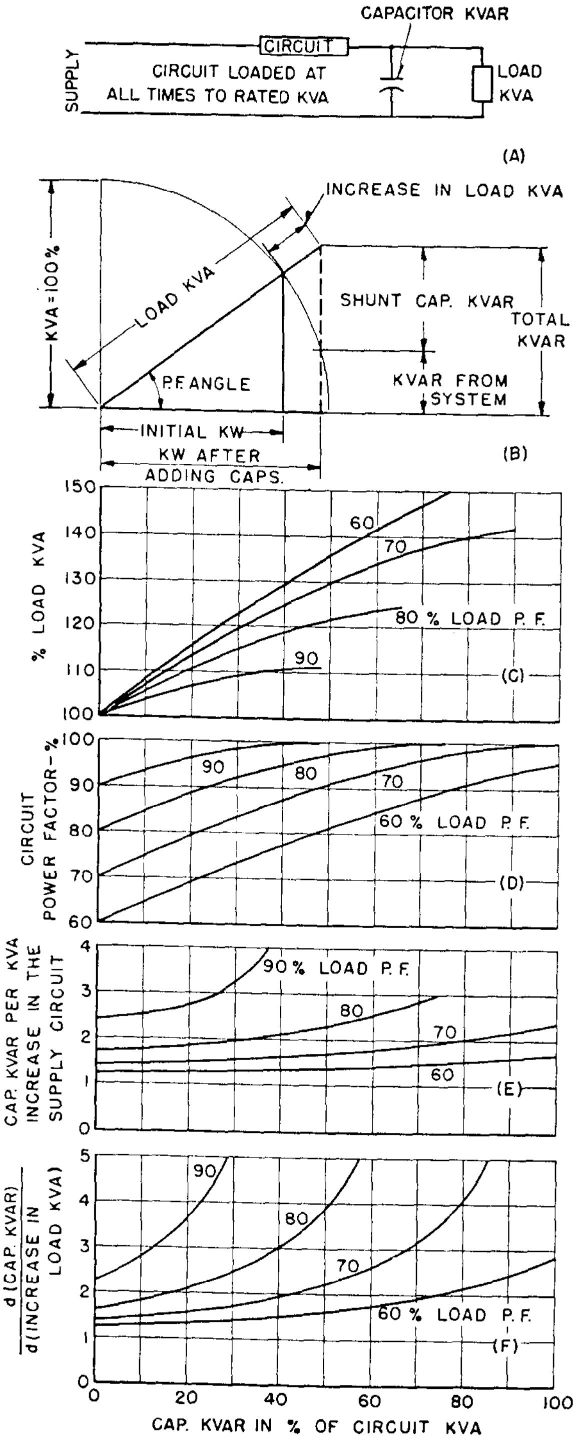

Fig. 5—Fundamental effects of shunt capacitors on power circuits.

Increasing the capacitors lessens the current carried by the supply circuit from the source (Fig. 5(D)), up to the ultimate point at which capacitors supply all of the kilovars required by the load and the circuit supplies only the kilowatt component. For a constant load in the circuit, adding various amounts of capacitors allows the useful load to be increased. By adding 40 kva of capacitors to a 100-kva load of 70 percent power factor, the load can be increased from 100 kva to about 124 kva, as Fig. 5(C) suggests. (If the load should be 10 000 kva at 70 percent power factor, then adding 4000 kvar of capacitors permits the kw to be increased from 7000 to 8700 without increasing the circuit loading above 10 000 kva. The load kva can thus be increased to 12 400 kva at 70 percent power factor.)

Shunt capacitors can be viewed in two lights. Adding capacitors releases circuit capacity for more load, and adding capacitors relieves overloaded circuits.

The capacitor kvar per kva of load increase, Fig. 5(E), is of particular interest, because multiplying this quantity by the cost per capacitor kvar, the product is the average cost of supplying each additional kva of load. This cost, neglecting other advantages of the capacitor, can be compared with the cost per kva of increasing the transformer or supply circuit rating. Thus if the load power factor is 70 percent and a capacitor kvar of 40 percent is added, the capacitor kvar per increase in kva of the load is 1.65. If capacitor cost is \$7.00 per kvar, then the increase in ability to supply load is obtained at a cost of 1.65 times \$7.00 or \$11.55 per kva. The incremental cost of adding transformer capacity may be much greater per kva of increased capacity.

The same data apply equally well to any equipment other than transformers in which current might constitute a limiting factor such as generators, cables, regulators, as well as transmission and distribution lines.

In the example taken (Fig. 5) as the load through the transformer approaches unity power factor, smaller and smaller incremental gains in load are obtained for incremental increases in capacitor kvar. The incremental capacitor kvar required for an increment in kva of the load is Fig. 5(F). Expressed mathematically, the ordinate in this

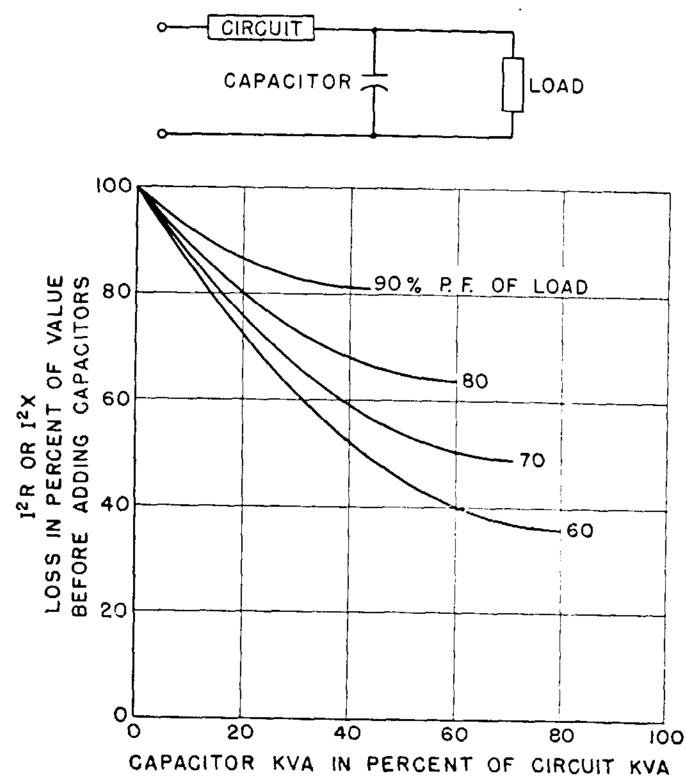

Fig. 6—Reduction in losses in the source circuit to shunt capacitors.

curve is equal to $\frac{d\,(\text{Cap. kvar})}{d\,(\text{Increase in load kva})}$. These curves show that the final increment is attained at much greater expense than the initial increment.

Capacitors applied to a given load reduce the I^2R and I^2X loss in the supply circuit in accordance with Fig. 6. For a 70 percent power factor load with 40 kvar of capacitors added for each 100 kva of circuit capacity, the I^2R and I^2X loss will be 59 percent of its former value. This loss in the particular circuit supplying the load can be calculated directly and may be a big factor, particularly if the circuit impedance is high. The resistance and reactance losses are also reduced in all circuits and transformers back to and including the source generators.

To illustrate the effect of shunt capacitors applied to a large load, the curves in Fig. 7 are shown where it is assumed that the load bus voltage is maintained constant at 4160 volts and the generator voltage varies with load.

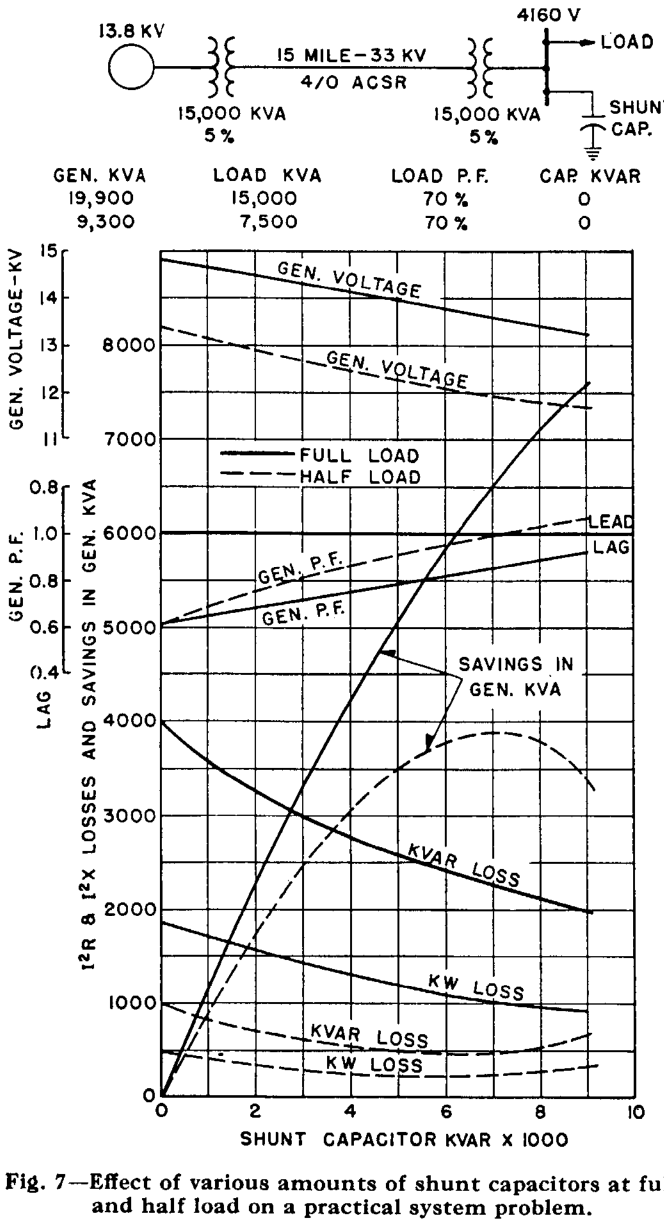

Fig. 7—Effect of various amounts of shunt capacitors at full and half load on a practical system problem.

A 15 000-kva, 70 percent power factor load is supplied over 15 miles of 33-kv circuit. Without shunt capacitors the generators must supply 19 900 kva at a power factor of 62 percent, whereas with the use of 6000 kvar of capacitors the generator power factor is raised to 82 percent. The 6000 capacitor kvar reduces the loading on the generator by 5850 kva, which is almost equal to the capacitor kvar. The I^2R loss in the circuit is reduced by about 800 kw (1900–1100) and the I^2X losses are reduced by about 1600 kvar (4000–2400). Curves are also shown for half load or 7500 kva at 70 percent power factor.

In the case cited, it is desirable to switch part or all of the capacitors off during light-load periods. The voltage and power factor at the generating end determine whether switching in steps should be applied. As Fig. 7 indicates the voltage at the generator would have to vary from 13.8 kv at full load with 6000 kvar of capacitors to 12.1 kv at half load with 6000 kvar of capacitors, assuming a constant voltage of 4160 at the load. By providing 3 steps of capacitors and removing 4000 kvar from the system at ½ load, the remaining 2000 kvar gives a voltage of 12.9 kv at the generator; removing all capacitors from service, a generator voltage of 13.4 kv is required for 4160 volts at the load.

4. Voltage Drop

The voltage drop in feeders or short lines can be expressed approximately by the relation

$$\text{Voltage drop} = RI_r + XI_x \tag{1}$$

where R is the resistance, X the reactance, I_r the power component of the current, and I_x the reactive component

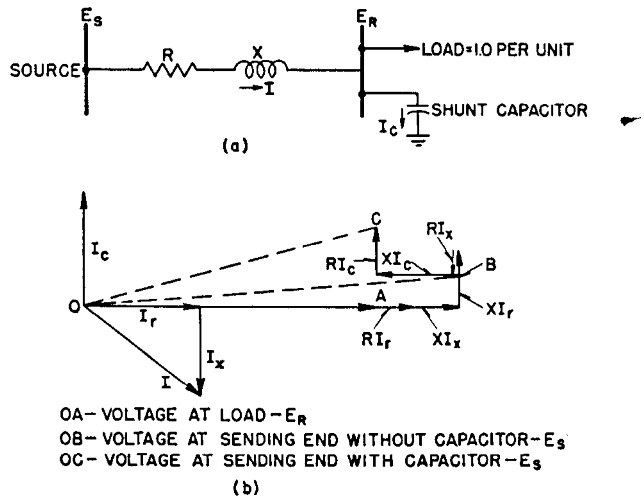

Fig. 8—Effect of shunt capacitors on voltage drop in source circuit.

as shown in Fig. 8. If a capacitor is placed in shunt across the end of the line, the drop immediately decreases or the voltage rises. The new voltage drop becomes approximately:

$$\text{Voltage drop} = RI_r + XI_x - XI_c \tag{2}$$

where I_c is the current drawn by the capacitor. Thus if I_c be made sufficiently large, both the RI_r and the XI_x drops can be neutralized. This expression also shows that if the

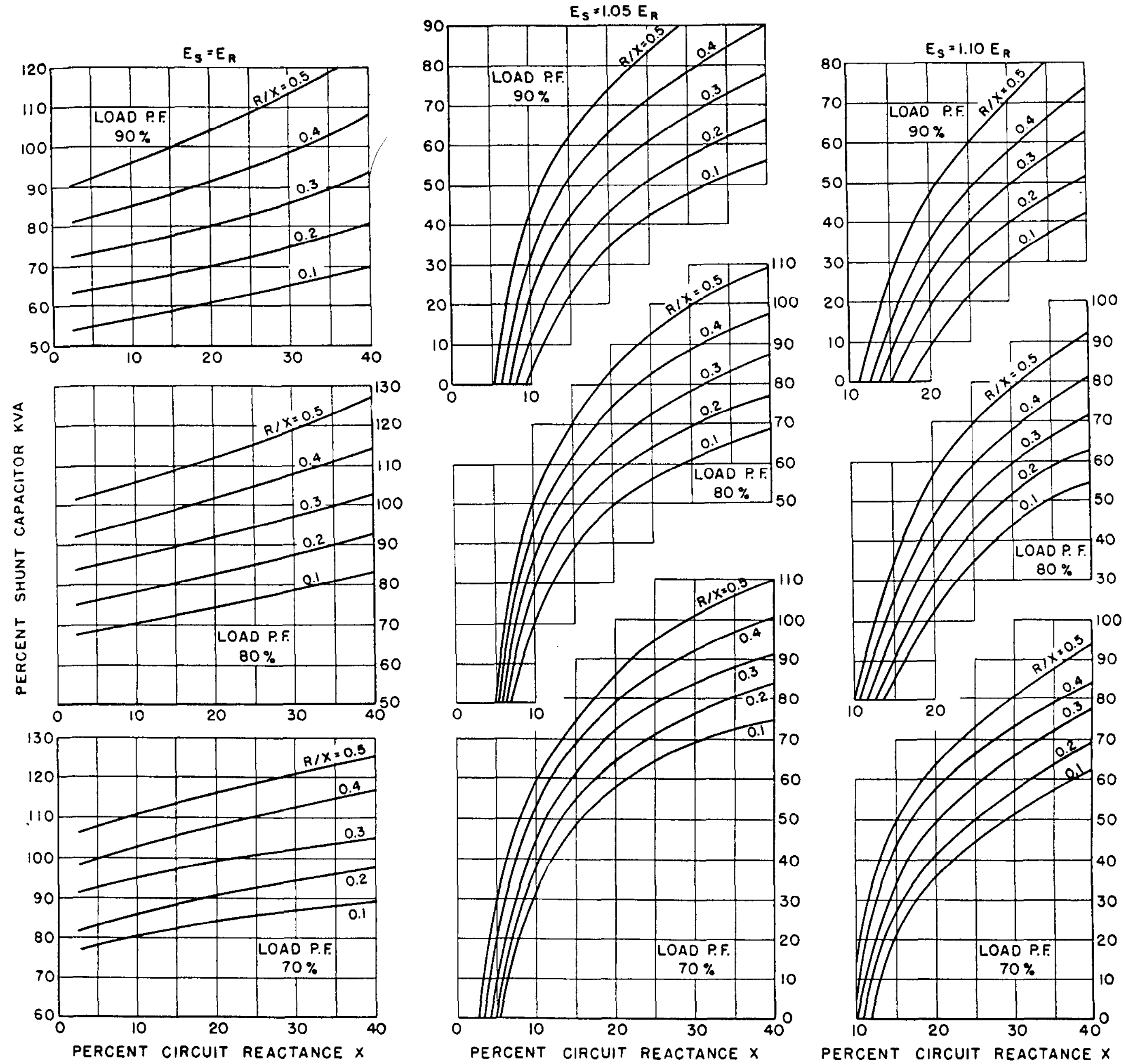

Fig. 9—Shunt Capacitors required for various power factor loads to give 0, 5 or 10 percent voltage drop in the source circuit. All percent values are referred to full load kva as 100 percent base.

voltage drop is compensated at full load with permanently connected capacitors, then at light loads I_r and I_x become smaller and the line is over-compensated because I_c is dependent only upon voltage and not upon load. Regulation of the line is practically unchanged by the capacitor because the capacitor effects an increase in voltage both at light load and at full load. At light loads the voltage rise might be so much in excess of normal as to represent an undesirable or even intolerable condition; a solution is to provide manual or automatic switching to add or remove groups of capacitors as desired.

The curves of Fig. 9 show the amount of shunt capacitor kvar required for loads of three power factors and for 0, 5 and 10 percent voltage drop over the supply circuit. To illustrate their use, assume a 20-mile, 33-kv line of 2/0 copper conductors which steps down through a 10 000-kva, 7-percent reactance transformer to 13.8 kv. Assume the full load is 10 000 kva at 80 percent power factor. Also assume: line impedance $9.62+j15.36$ ohms or $0.0886+j0.142$ per unit on 10 000-kva base; transformer impedance $0.008+j0.07$ per unit; total impedance $0.096+j0.212$ per unit. Therefore, ratio $R/X=\frac{0.096}{0.212}=0.45$. Referring to Fig. 9 for R/X ratio of 0.45 and a circuit reactance of 0.212 per unit, the shunt capacitor kvar required for a 10 percent voltage drop on the line is 0.54 per unit. In this case 1.0 per unit is 10 000 so 5400 kvar of shunt capacitors are necessary. These data and the capacitor kvar required for 0 and 5 percent voltage drop are given in the following table. In addition, calculated losses in the circuit are given, as well as the power factors at the sending (E_S) and receiving (E_R) ends of the circuit with the selected capacitor kvar in use. To give a more complete view of the use of Fig. 9 curves the shunt capacitor kvar required for 5000-kva 80-percent power factor load is included in Table 1.

TABLE 1—DATA FOR 20 MILE 33 KV LINE WITH TRANSFORMATION TO 13.8 KV LOAD BUS

Voltage Conditions	Capacitors At the Load		Circuit Loss Kw	Percent Power Factor At	
	Per Unit	Kva		E_S	E_R
10 000 Kva, 80% P.F. Load					
$E_S=E_R$	1.07	10,700	826	95.1 lead	86.2 lead
$E_S=1.05\ E_R$	0.81	8,100	657	99.7 lead	96.7 lead
$E_S=1.10\ E_R$	0.54	5,400	617	97.5 lag	99.7 lag
5000 Kva, 80% P.F. Load					
$E_S=E_R$	1.01	5,050	194	94.0 lead	88.9 lead
$E_S=1.05\ E_R$	0.50	2,500	156	97.9 lag	99.2 lag
$E=1.10\ E$	0.02	100	234	77.5 lag	81.0 lag

For 5000 kva the circuit reactance is 0.106. The ratio R/X remains constant for all loads. Thus the capacitor kvar can be determined, for a given voltage drop in the circuit, for any part of full load by using the per unit reactance based on the partial load.

5. Overvoltage on Capacitors

Capacitors are designed for operation on circuits whose average voltage over a 24-hour period does not exceed the rated voltage by more than 5 percent. The variations above the average may go to 115 percent in the case of 230, 460, and 575 volt capacitors, or 110 percent in the case of higher voltage units. For short periods of time, shunt capacitors can safely withstand higher voltages. For example, during the starting of large induction motors the voltage rating of capacitors applied in shunt with the motor may be as low as 67 percent of the voltage applied to the motor, which means that the voltage applied to the capacitor is 150 percent of its rating. The maximum momentary voltage, such as in welding applications, should not exceed 165 percent of the rated voltage.

TABLE 2—STANDARD CAPACITOR RATINGS

Indoor Type			Outdoor Type		
Volts	KVAR	Phase	Volts	KVAR	Phase
230	5-7½	1 & 3	230	1, 2½, 5, 7½	1 & 3
460	10 & 15	1 & 3	460	5, 10 & 15	1 & 3
575	10 & 15	1 & 3	575	5, 10 & 15	1 & 3
2 400	15 & 25	1 & 3	2400	10, 15 & 25	1 & 3
4 160	15 & 25	1 & 3	4160	10, 15 & 25	1 & 3
4 800	15 & 25	1 & 3	4800	15 & 25	1 & 3
7 200	15 & 25	1	7200	15 & 25	1
7 960	15 & 25	1	7960	15 & 25	1
12 470	15	1	12470	15	1
13 800	15	1	13800	15	1

Note: 25 KVAR Units are only single phase

6. Standard Ratings and Tests on Capacitors

Table 2 gives the standard ratings of capacitor units for indoor and outdoor types. Table 3 gives the standard ratings of indoor and outdoor housed capacitors. Table 4 gives the factory test voltage which are applied to capacitors.

The average operating loss for capacitors, in kw, is one-third of one percent of the kvar rating. Each capacitor has a built-in high resistance device which automatically discharges the capacitor for safety. The ambient temperature limit covering all capacitors is 40°C; for outdoor open mounted units it is 50°C and for housed units between 40°C and 50°C depending on rack type.

TABLE 3—STANDARD RATINGS FOR INDOOR AND OUTDOOR HOUSED CAPACITORS BANKS VOLTAGE AND KVAR RATINGS

230 V	460-575 V	2400-4160 V		4800-7200-7960 12,470-13,800 V	
15	30	30	600*	90	600*
30	60	45	900*	180	900*
45	90	60	1500*	360	1500*
60	120	90	2100*	540	2100*
90	180	135	2700*	720	2700*
135	270	180	3300*	1080	3300*
180	360	270	4200*	1260	4200*
270	540	360		1440	5100*
360	720	540		1800	
540	1080	720		2160	
630	1260	1080		2520	
		1260			

*Using 25 KVAR Units

TABLE 4—FACTORY TEST VOLTAGES ON CAPACITORS

Voltage Rating of Capacitors	Terminal-to-Terminal Test Voltage	Terminal-to-Ground Test Voltage	
		Indoor	Outdoor
230	500	3000	10 000
460	1000	5000	10 000
575	1200	5000	10 000
2 400	5000	19 000	19 000
4 160	9000	19 000	19 000
4 800	10 000	26 000	26 000
7 200	15 000	26 000	26 000
7 960	16 600	26 000	26 000
12 470	25 000	34 000	34 000
13 800	28 800	34 000	34 000

Application period: 3600 cycles—25 or 60 cycles.

II. CAPACITOR ON INDUSTRIAL PLANT CIRCUITS

A capacitor can be installed in shunt with any load of low power factor to supply the magnetizing current required by the load. The load may be a single motor, or it may be a large industrial plant. The capacitor can be chosen to supply the magnetizing current under peak load conditions, or it can be chosen only large enough to supply the reactive kva hours accumulated over the month. It can be located at the service entrance, thus removing magnetizing current from the utility system only; or units can be applied to the individual loads, thus removing magnetizing current from the plant circuit also, reducing their loss, and increasing their load capacity, and better maintaining voltage at the loads.

The selection of the capacitor size, and its location is dependent on what is to be accomplished. This varies with

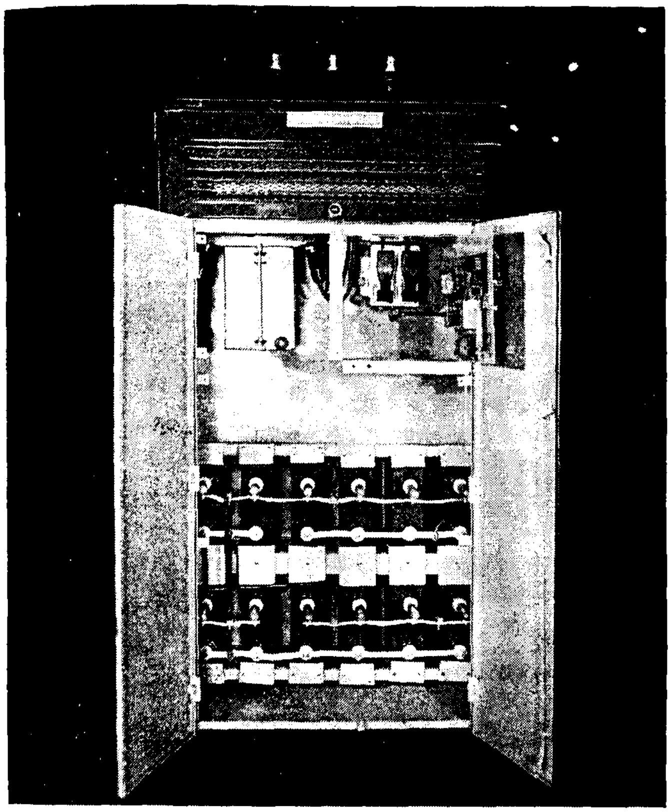

Fig. 10—Enclosed indoor bank of 2400/4160 volt shunt capacitor units with protective screen removed. This is one step voltage control with a RCOC oil contactor.

the power rates, and local conditions. An outdoor bank of capacitor units is shown in Fig. 10.

7. Location of Capacitors

Many factors influence the location of the capacitor such as the circuits in the plant, the length of the circuits, the variation in load, the load factor, types of motors, distribution of loads, constancy of load distribution.

The capacitors can be located in many ways as follows:

(a) Group correction—at primary of transformer.
(b) Group correction—at secondary of transformer.
(c) Group correction—out in a plant, as for example for one building.
(d) Localized correction on small feeders.
(e) Localized correction on branch motor circuits.
(f) Localized correction direct on motors, or groups of motors and switched with the motor.

8. Group Correction

The two principal conditions under which group correction is better are:

1. Where loads shift radically as to feeders.
2. Where motor voltages are low such as 230 volts.

If the power flows from the service entrance to various widely-separated parts of the plant and if the loads shift about a great deal from one feeder to another, the correction may be needed first in one part of the plant and later in another. A centrally-located group capacitor in this case would be an advantage since it would tend to be the same distance from the loads at all times.

If a group capacitor remains connected during light loads the voltage rise is less if this capacitor is installed at or near the transformer bank since the reactance of the plant circuits does not contribute to voltage rise. In this case, application of capacitors to individual motor would represent a larger investment because of the diversity factor. It, therefore, would be better for the operator to switch off portions of the central capacitor to meet the varying load conditions. Exceptions will arise where feeders are long and where the gain from individual load application warrants the greater initial investment in capacitors. Because of the higher cost of low-voltage capacitors their application to 230-volt motor circuits may more than double their cost. This gives considerable advantage to group installation if this can be on the primary side, 2400 to 7200 volts. Capacitors placed ahead of the main bank of transformers do not benefit the transformers; no transformer kva is released. Thus, use of the 230-volt capacitors on the feeders or near the motors is frequently warranted.

9. Localized Correction

Capacitors should be placed as near the load as possible or near the ends of feeders for three main reasons:

1. Losses are reduced in the circuits between the loads and the metering point.
2. Voltage is raised near the loads, giving better motor performance.
3. Capacitor kvar can be reduced automatically as the load drops off by installing some of the capacitors direct on loads so they are switched off with the loads.

The first point can be evaluated easily by investigating the length of the circuits, and the transformations, if any. Whatever gains are found in released transformer capacity and reduction in losses in transformers and circuits are added gains.

The effect of the capacitor is to raise the voltage permanently at any given point where it is connected. This voltage boost, superimposed on the normal voltage, is practically constant from no load to full load on the feeder.

10. Rates and Capacitor

For the purpose of analyzing the different types of rates a typical application can be considered, such as an industrial plant with a day load averaging 960 kw and 67 percent power factor, with peak loads running up to 1200 kw and 75 percent power factor. It is obvious that a large magnetizing current is drawn from the line, and considerable savings can be made by supplying this magnetizing current with capacitors. The size of the capacitor or the merits of their use can only be determined by systematic analysis.

One of the following conditions may exist.

(a) Power factor is not considered in the rates.
(b) Power factor is taken into account in demand charge.
(c) Power factor is checked by test and used to determine energy charge thereafter.
(d) Power factor is determined by the ratio of kw hours and rkva hours and is used in different ways to calculate the demand charge or energy charge or both.

(a) If power factor is not taken into account in the rate structure, the capacitor can be used only to secure savings in the plant, such as to reduce current in circuits, reduce loads on transformers, and to reduce loads on customer-operated generators. The capacitor should usually be located near the loads of low power factor. The size can be determined by calculating the reactive kva. By using a capacitor large enough to supply all or part of this reactive kva, the current in the circuit is reduced to the desired figure.

(b) If the rates include a kva demand charge, the kva can be reduced by raising the power factor during the demand peak. With a demand of 1200 kw at 75 percent power factor the kva demand is $\frac{1200}{0.75} = 1600$ kva.

If the power factor is raised to 95 percent the demand kva is $\frac{1200}{0.95} = 1260$ kva. The size of the capacitor required to accomplish this is determined from the reactive kva at the two values of power factor as follows.

Reactive kva at 75 percent power factor

$$= \sqrt{1600^2 - 1200^2} = 1060$$

Reactive kva at 95 percent power factor

$$= \sqrt{1260^2 - 1200^2} = 387$$

Kvar rating of capacitor is 1060 minus 387 which equals 673 kva.

The reduction in the kva demand from 1600 to 1260 may result in either a reduced kva demand charge, or it may reduce the energy charge depending on the rate structure. Some rates involve several energy charges for successive blocks of power, the size of the blocks depending on the kva demand. For example:

Size of block = (70) × (kva demand).
1st block—5c per kw hour
2nd block—1¼c per kw hour
3rd block—1c per kw hour
Additional ¾c per kw hour

In this case the energy cost is reduced by a decrease in kva demand, because if the blocks are smaller, the lower rate applies to a larger proportion of the energy consumed.

(c) Sometimes a check is made on the average power factor under day load conditions, and the billing thereafter based on this check until some future check is made. The energy charge, or the net billing is adjusted up or down according to this power factor. In such cases it is necessary to determine how this check is to be made, and under what conditions, in order to install capacitors to raise the power factor as high as warranted by the expected savings. Such a capacitor usually is made proportional to day load requirements. In the case above, the day load averaged 960 kw at 67 percent power factor. Assuming this is to be brought up to 95 percent power factor, 720 kva of capacitors are required as follows:

$$\frac{960 \text{ kw}}{67 \text{ percent}} = 1430 \text{ kva}$$

Reactive kva at 67 percent power factor

$$= \sqrt{1430^2 - 960^2} = 1035 \text{ kvar}$$

kva at 95 percent power factor $= \frac{960}{0.95} = 1010$ kva.

Reactive kva at 95 percent power factor

$$= \sqrt{1010^2 - 960^2} = 315.$$

Capacitor required is 1035 minus 315 which equals 720 kvar.

(d) A method commonly encountered in industrial plants takes into account monthly power factor obtained by integrating kw hours and rkva hours. Assuming the plant mentioned above is billed for 322 250 kw hours, and that the reactive kva hours equals 346 000. This ratio amounts to a power factor of 68 percent.

Assuming that rates indicate that it will be worthwhile to reduce this rkva hours to a point corresponding to 95 percent power factor.

kva hours at 95 percent power factor $= \frac{322\ 250}{95} = 339\ 000$

Reactive kva hours at 95 percent power factor

$$= \sqrt{339\ 000^2 - 322\ 250^2} = 106\ 000$$

Using 730 hours per month the capacitor kvar required equals $\frac{339\ 000 - 106\ 000}{730}$ or 319 where the kvar meter has no ratchet so that full credit results even if the power

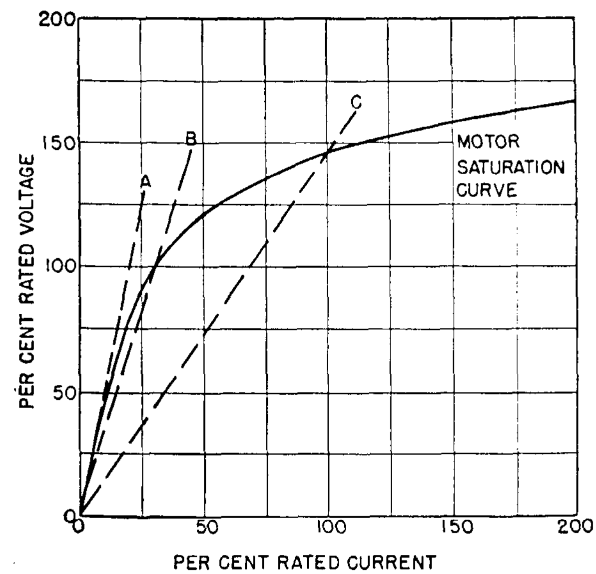

Fig. 11—Self excitation of induction motor with various amounts of shunt capacitors when supply breaker is opened.

A—Capacitor current less than motor current at no load rated voltage.
B—Capacitor current equal to motor current at no load and rated voltage.
C—Capacitor current equal to 100 percent.

factor is leading at times. When the meter has a ratchet the capacitor must be large enough to build up accumulated kvar-hours while the power factor is not leading.

Detail analysis of the load and its variations at each plant, taking into consideration the type of rates, should be made to obtain the greatest benefit from using capacitors. In some cases part of the capacitors may have to be switched off during light load periods to prevent excessive voltage on plant circuits.

11. Capacitors on Induction Motor Terminals

Capacitors frequently are installed across the terminals of induction motors and switched with the motor. The amount of kvar so connected should be limited to values that do not cause excessive voltage at the motor due to self-

TABLE 5—MAXIMUM CAPACITOR KVAR FOR USE WITH OPEN TYPE THREE PHASE 60 CYCLE INDUCTION MOTOR

Motor Rating HP	3600* RPM		1800* RPM		1200* RPM		900* RPM		720* RPM		600* PM	
	Kvar	**	Kvar	**	Kvar	**	Kvar	**	Kvar	**	Kvar	**
10	2.5	9	4	11	4	12	5	17	5	23	7.5	28
15	2.5	9	5	11	5	11	7.5	16	7.5	21	10	26
20	5	9	5	10	5	11	7.5	15	10	20	12.5	24
25	5	9	7.5	9	7.5	10	10	14	10	19	15	22
30	7.5	9	10	9	10	10	10	13	12.5	18	15	21
40	10	9	10	9	10	10	12.5	12	15	16	17.5	19
50	12.5	9	12.5	8	12.5	9	15	12	20	15	22.5	17
60	15	9	15	8	15	9	17.5	11	22.5	14	25	16
75	17.5	9	17.5	8	17.5	8	20	11	27.5	13	30	15
100	22.5	9	22.5	8	22.5	8	25	10	35	12	37.5	14
125	25	9	27.5	8	27.5	8	30	9	40	11	47.5	13
150	32.5	9	35	8	35	8	37.5	9	47.5	11	55	13
200	42.5	9	42.5	8	42.5	8	45	9	60	10	67.5	12

*Synchronous speed
**Percent reduction in line current using capacitor KVAR shown

excitation when the breaker is opened, as Fig. 11 shows. Table 5 gives the maximum recommended capacitor kvar for direct connection to the terminals of induction motors taken from the 1947 National Electrical Code.

III. CAPACITORS ON DISTRIBUTION CIRCUITS

Shunt capacitors offer a convenient and practical means of relieving lines and source equipment of wattless current. They can be installed in relatively small banks and placed near the load points. They usually are arranged in three-phase banks of 45 kvar or more and are distributed over the system at distribution voltage, usually 2400 volts and up, in accordance with local requirements. A 180 kvar installation is shown in Fig. 12. At present it is not economical to apply capacitors on the secondary side of distribution transformers because of the much greater cost. Where the transformers are expensive, such as network units, secondary capacitors may be justified.

The capacity of a distribution feeder can be limited by current or by voltage drop. Where current is the limiting factor, the effect of capacitors in reducing the current is dependent upon load power factor. If the power factor is

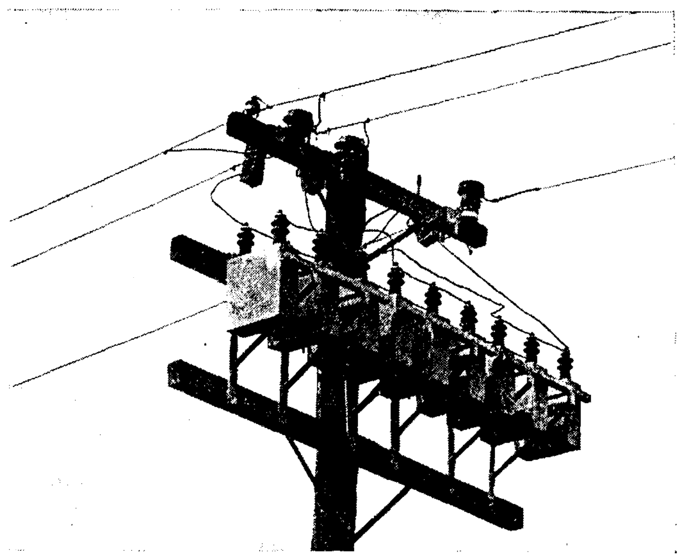

Fig. 12—180 kvar, group-fused, pole-mounted capacitor installation.

low, a large reduction in feeder current or kva can be obtained as indicated by the curves in Fig. 5. If the load power factor is high, shunt capacitors cannot materially change feeder loading. Where voltage is the limiting factor, the capacitor kvar to decrease voltage drop is dependent not only on load power factor but also on the ratio of resistance to reactance of the distribution feeder.

12. Application Factors

In applying shunt capacitors to distribution circuits, certain system data are required.

1. Determine variation, preferably by graphic instruments, of kw and kva on each feeder for a typical 24-hour period at both minimum and maximum daily loads. Usually the minimum reactive kva determines the amount of fixed capacitors to apply without automatic control. This gives about unity power factor at minimum load. In certain cases more fixed capacitor kvar can be applied where voltage conditions at light load permit and where leading power factor is not objectionable.
2. Obtain actual voltage measurements on the feeder during full load and light load at a sufficient number of points to determine the optimum location for capacitors. Fixed shunt capacitors raise the voltage level at the point where they are applied on a given circuit by practically a constant value as given by XI_c in Eq. 2.

To calculate the voltage at various points on the feeder the circuit characteristics and the load distribution must be known. Where the individual loads are not known, it is reasonable to assume they are proportional to the installed transformer capacity for minimum and maximum feeder load. To simplify calculations single-phase loads can be grouped together to form balanced three-phase loads and adjacent three-phase loads can be grouped to simplify the calculations.

3. It is desirable to supply the kvar required by the load as close to the load as possible to reduce feeder losses. Therefore, capacitor units should be located at load centers or near the ends of feeders. Ideally each load point would have the exact amount of capacitor kvar to supply the necessary load kvar. This, however, is not possible because standard size units must be used. Also it is more economical to use the large size units, namely, 15 or 25 kvar. Overcompensation of feeder branch circuits with capacitors to obtain a higher voltage results in increased copper losses because at lower and lower leading power factors, the current increases.
4. Calculate the released feeder capacity in kw and kva for the capacitor kvar installed. This may involve capacitors installed at several locations on a given feeder. Released substation, transmission, and generator capacity is also immediately available.
5. Calculate the reduction in kw losses and the reduction in kvar losses in the feeder. The effect on all equipment back to and including the source generator should also be evaluated when the total capacitor kvar become appreciable relative to the total source circuit or system reactive kva.
6. Summarize the tangible effects namely, the released feeder capacity, the released capacity back to and including the source generator, the reduction in losses, the effect on voltage, etc. and evaluate the economics to determine whether or not capacitors are justified. Also compare the cost of capacitors with other ways of doing an acceptable job, such as construction of a new feeder, installation of voltage regulators, raising the distribution voltage, etc.

From the above brief summary on applying shunt capacitors to distribution systems, it is evident that no fixed rules can be stated regarding the location of capacitors nor can the degree of importance of each of their effects be stated. Each case is different and requires a complete study in more detail than has been given in this general discussion.

IV. LARGE CAPACITOR BANKS

Shunt capacitors have been applied at substations and at the ends of primary feeders in banks ranging in size up to about 20 000 kvar. The usual large sizes are between 5 000 and 10 000 kvar. A capacitor bank can be switched all in one step, but general practice is to provide switching so that a large bank is connected to the system as needed in several equal steps. Three equal steps are quite common although more or less steps are used, depending on the voltage change per step and the variation in load.

Several typical layouts for switching large capacitor banks are shown in Fig. 13. Fig. 13(a) is for one group of capacitors switched by one automatic circuit breaker. Fig. 13(b) shows four automatic breakers controlling four equal steps in a large capacitor bank. The circuit breakers must be capable of handling short circuit currents. Figure 13(c) shows three equal steps where one automatic breaker supplies the entire bank and trips for short circuits in any one of the three groups of capacitors. Two non-automatic

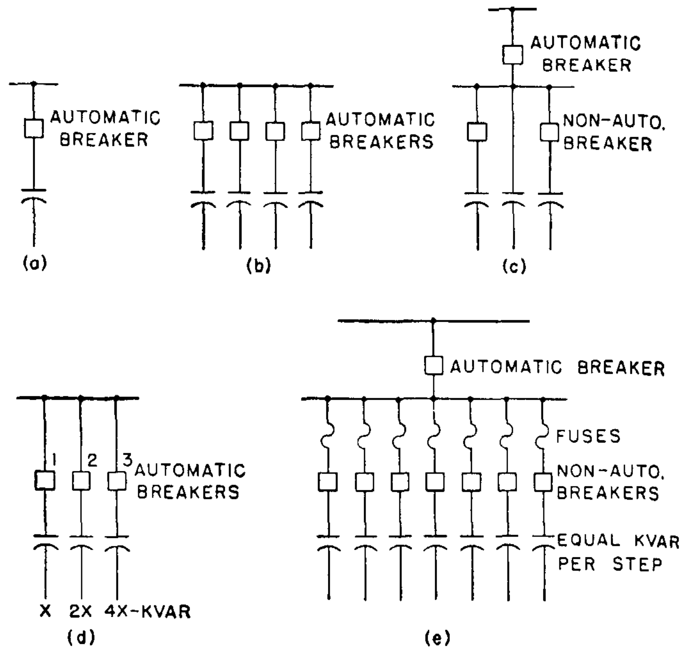

Fig. 13—Schematic arrangements for switching large capacitor banks.

breakers are provided for controlling two steps, the third step being controlled by the main breaker. Figure 13(d) is a scheme in which three groups of capacitors properly proportioned provide seven equal steps. Switch 1 gives $^1/_7$ of the total; switch 2 gives $^2/_7$; switches 1 and 2 give $^3/_7$, and so on for all three switches giving the full capacity of the bank. The disadvantage to this scheme is that during the switching process, large changes of capacitor kvar are made to get from one kvar to another. The worst condition is changing from $^3/_7$ to $^4/_7$ of the total kvar where switches 1 and 2 must be opened, thus, disconnecting all capacitors before closing switch 3, or switch 3 must be closed putting all of the capacitors in service before switches 1 and 2 are opened. If the voltage change during these changes can be tolerated, then seven steps in capacitor kvar can be obtained with three circuit breakers. Figure 13(e) is another scheme where one automatic circuit breaker supplies a number of non-automatic breakers which control equal amounts of capacitor kvar. Each non-automatic breaker has a high-capacity fuse that will clear a faulted capacitor group ahead of tripping the main supply breaker. There are many combinations of the use of automatic breakers, non-automatic breakers and high-capacity fuses for capacitor banks that can be applied, depending upon the operating requirements and economics.

13. High Voltage Banks

Supplying kilovars direct to high-voltage circuits is often desirable to meet certain system requirements even though a greater portion of the system is benefited by placing the capacitor nearer the load and on lower voltages. For many years, transformers were used to step down the voltage to the range of the capacitor unit ratings. A few years ago the practice of connecting low-voltage capacitors in series parallel groups and directly to the high-voltage line was established because they are more economical than the use of high-voltage capacitors or transformers and low-voltage units. One of the first such installation consisted of six groups of 2400–volt outdoor capacitor units operating in series on the phase to neutral voltage of a 24-kv circuit. Each group of 2400-volt units consisted of 10–15-kvar

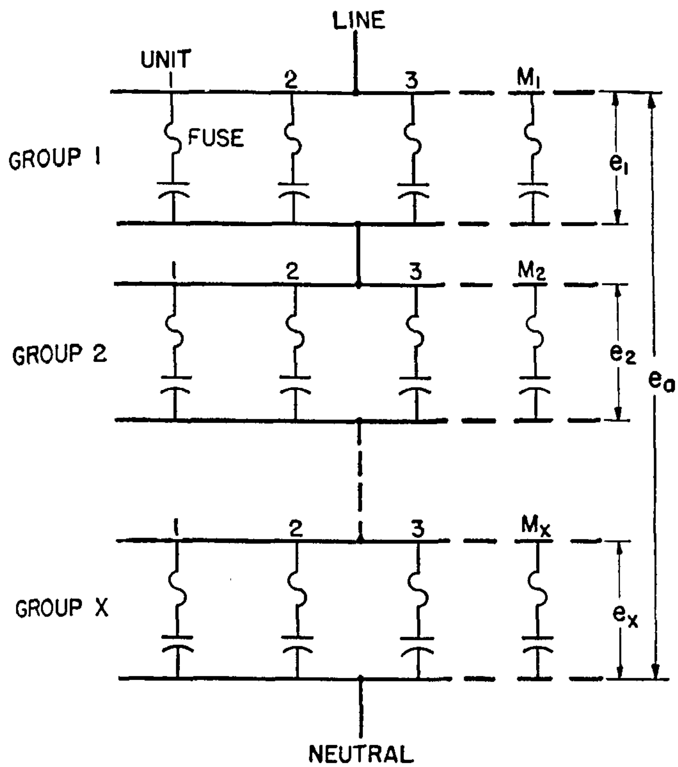

Fig. 14—Connection for fused capacitor units for one phase of a three phase bank. Symbols apply to Eqs. (3) to (11).

X—Number of capacitor groups in series.
M—Normal number of capacitor units per group
N—Number of units out of one group.
e_1 —Actual voltage across group 1.
e_{01}—Rated voltage across group 1.
e_a —Normal system voltage to neutral.

units in parallel, and these 150 kvar groups were supported on insulators to take care of the line to ground voltage. Figure 14 shows how capacitor units are assembled for one phase of a bank.

Initially, operation of capacitor units in series was looked upon as risky due to the ever-present possibility of subjecting capacitors to overvoltage as a result of changes in voltage distribution either due to a change in impedance of portions of the phase leg or due to grounds at some point on the assembly. Most of these risks are minimized or entirely eliminated, however, when proper thought is given to such factors as fusing, number of units in parallel, connection of one bushing of capacitor to the insulated platform on which it rests and means of detecting unbalance conditions before the unbalance becomes excessive. Each capacitor unit in a high-voltage bank should be provided with a fuse of the indicating type. These fuses need not be of high interrupting capacity because there are always two or more capacitor groups in series, and, when a unit becomes short circuited for any reason, the current through the fuse is limited. With individual fuses a faulty unit can be located without resorting to the risky procedure of searching for the source of noise or arcing, or making inconvenient tests. It is also easy to make a check and determine if all units in the bank are operating properly. The fuses can be omitted but at a sacrifice in the protection to the capacitor bank.

The number of units in parallel in a single group is important. Several things affect this. First the number should be sufficiently large to insure that the fuse on a single unit blows when the unit becomes short circuited and the fuse is called upon to carry the total phase current. Second, the voltage on the remaining units in a group should not become excessive with the operation of one fuse in a group. If the number of parallel units is too small, the current through the fuse may be so low that it will not blow, or take too long in doing so. An arc of 50 amperes inside a capacitor unit may rupture its case if allowed to continue for a long time and such a rupture may endanger other units in the bank. After considering the size of fuses that must be used to avoid operation on switching transients, and taking into account the arc energy required to rupture the capacitor case, it has been established that the current through the fuse when a unit becomes shorted should

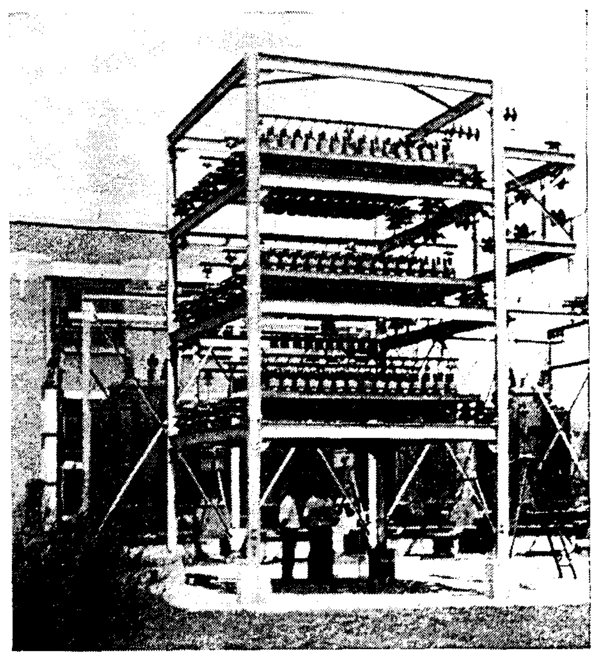

Fig. 15—6000 kvar 34.5 kv outdoor capacitor bank with bus-mounted fuses.

never be less than 10 times the normal capacitor current through the fuse.

It is also desirable to avoid voltages in excess of 110 percent on the remaining units in a group following the operation of one fuse. This assumes that in the case of the minimum size bank not more than one fuse operation is permitted. To accomplish this, periodic checks are necessary.

The amount of current that flows through a fuse when a unit is shorted is also affected by the number of series groups and whether or not the neutral of the capacitor bank is grounded.

Tables 6 and 7 show the recommended minimum number of fused capacitor units that should be used in parallel for a given number of groups in series in each phase leg, for ungrounded or grounded-wye connections respectively

TABLE 6—UNGROUNDED WYE CAPACITOR CURRENT AND VOLTAGE RELATIONSHIPS WITH SHORTING AND REMOVAL OF ONE UNIT IN ONE PHASE LEG

Number Groups Series	Minimum Units per Group	Current During Fault Through Fuse Times Normal	Voltage on Remaining Units in Group Percent
1	4	12.0	109
2	8	12.0	109
3	9	11.6	109.5
4	9	10.8	110
5	10	11.5	110
6	10	11.2	110
7	10	11.0	110
8	10	10.9	110
9	11	11.9	Less than 110
10	11	11.8	Less than 110
11	11	11.7	Less than 110
12	11	11.6	Less than 110
13	11	11.6	Less than 110
14	11	11.5	Less than 110
15	11	11.5	Less than 110
16	11	11.5	Less than 110

TABLE 7—GROUNDED WYE CURRENT AND VOLTAGE RELATIONSHIPS WITH SHORTING AND REMOVAL OF ONE UNIT IN ONE PHASE LEG

Number Groups Series	Minimum Units per Group	Current During Fault Through Fuse Times Normal	Voltage on Remaining Units in Group Percent
1	1	Line Fault	
2	6	12	109
3	8	12	109
4	9	12	109
5	9	11.2	109.8
6	9	10.8	110.0
7	10	11.7	109.4
8	10	11.4	119.5
9	10	11.2	Less than 110
10	10	11.1	Less than 110
11	10	11.0	Less than 110
12	10	10.9	Less than 110
13	10	10.8	Less than 110
14	11	11.8	Less than 110
15	11	11.8	Less than 110
16	11	11.7	Less than 110

based on meeting the previously discussed requirements. All capacitor units are assumed to be the same voltage and kvar rating.

Very often large banks contain many more than the minimum number of units in parallel. When this is the case, more than one fuse can operate and still not seriously raise the voltage across remaining units. In such cases

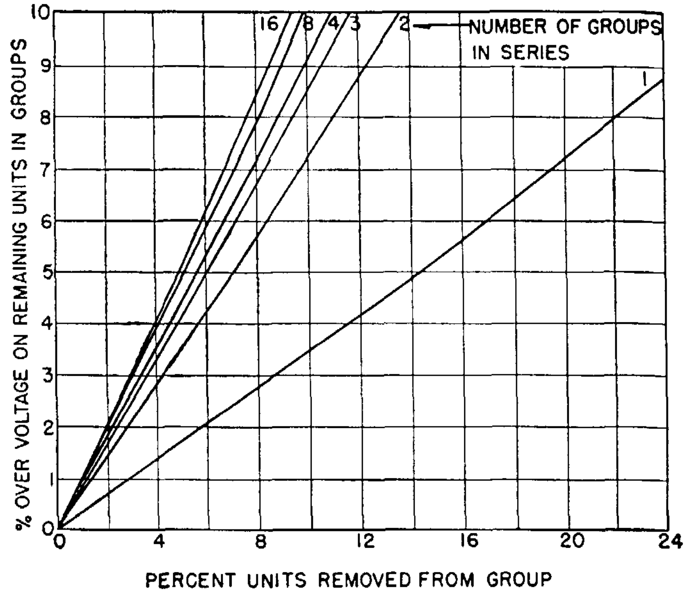

Fig. 16—Ungrounded wye connected shunt capacitor bank. Curves give the percent overvoltage across the remaining units in a group.

periodic checks of fuses are necessary to avoid abuse of good capacitors as result of a faulty one. The voltage across the remaining capacitors can be determined from Tables 6 and 7, the curves of Figs. 16 and 17 or calculated from the equations given below. For all equations the system impedance up to the capacitor bank was neglected.

Refer to Fig. 14 for identification of symbols in the following equations. The equations simplify quickly; all units have the same voltage rating.

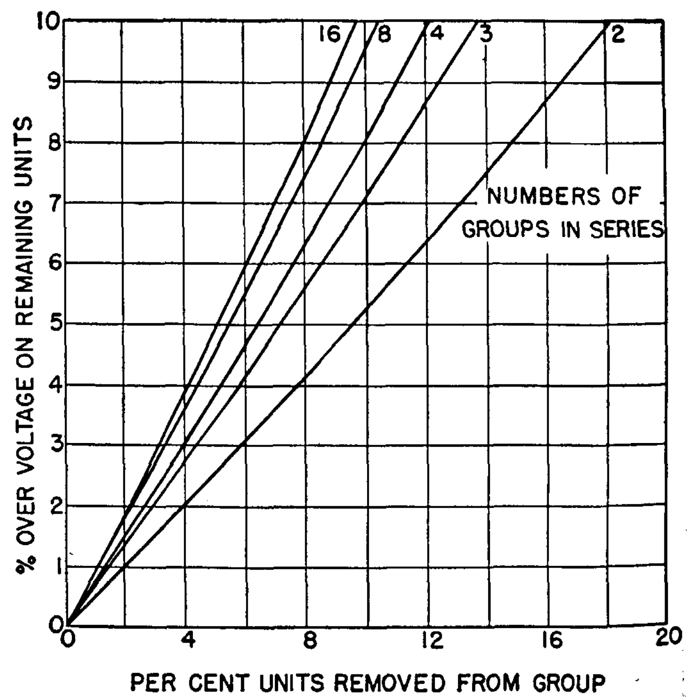

Fig. 17—Grounded wye-connected shunt capacitor bank. Curves give the percent overvoltage across the remaining units in a group.

14. Ungrounded Neutral Capacitor Bank

Normal voltage across group 1 is

$$e_{1N}=\frac{\left(\frac{e_{c1}^2}{M_1}\right)(e_a)}{\frac{e_{c1}^2}{M_1}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}} \tag{3}$$

With N_1 units removed from group 1, the voltage e_1 across the remaining units is

$$e_1=\frac{\left(\frac{e_{c1}^2}{(M_1-N_1)}\right)(e_a)}{\frac{(3M_1-N_1)(e_{c1}^2)}{3M_1(M_1-N_1)}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}} \tag{4}$$

With N_1 units removed from group 1 the voltage shift of the neutral of the capacitor bank e_{No} is

$$e_{No}=\frac{\frac{N_1}{M_1}\left(\frac{e_{c1}^2}{M_1-N_1}\right)(e_a)}{3\left[\frac{(3M_1-N_1)e_{c1}^2}{3M_1(M_1-N_1)}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}\right]} \tag{5}$$

The current through the fuse for a completely short-circuited capacitor unit in group 1 in times normal operating current is

$$I_f=(M_1)\left[\frac{\frac{e_{c1}^2}{M_1}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}}{\frac{e_{c1}^2}{3M_1}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}}\right] \tag{6}$$

15. Grounded—Neutral Capacitor Bank

Normal voltage e_1 across group 1 is same as for ungrounded neutral bank as given in Eq. (3).

With N_1 units removed from group 1 the voltage e_1 across the remaining units is

$$e_1=\frac{\left(\frac{e_{c1}^2}{(M_1-N_1)}\right)(e_a)}{\frac{e_{c1}^2}{M_1-N_1}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}} \tag{7}$$

The current through the fuse of a completely short-circuited capacitor unit in group 1 in times normal operating current for a grounded-neutral capacitor is

$$I_f=(M_1)\left[\frac{\frac{e_{c1}^2}{M_1}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}}{\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}}\right] \tag{8}$$

16. Two Identical Capacitor Banks with Neutrals Solidly Tied Together and Ungrounded

The normal voltage across any group of capacitors in an installation consisting of two similar groups with the neutrals tied solidly together and ungrounded is e_1 as given by Eq. (3) for any bank. With N_1 units out of group 1 in one bank the voltage across the remaining units in group 1 is

$$e_{1N}=\frac{\left(\frac{e_{c1}^2}{M_1-N_1}\right)(e_a)}{\frac{\left(6-\frac{N_1}{M_1}\right)(e_{c1}^2)}{6(M_1-N_1)}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}} \tag{9}$$

The current in the fuse of a completely short-circuited capacitor unit in group 1 of one bank of two similar banks with the neutrals solidly connected and ungrounded in terms of normal current in one capacitor unit is

$$I_f=(M_1)\left[\frac{\frac{e_{c1}^2}{M_1}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}}{\frac{e_{c1}^2}{6M_1}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}}\right] \tag{10}$$

The current in the neutral connection between two similar banks of capacitors, with N units out of group 1 in one bank, in terms of the normal current through one capacitor is

$$I_{No}=\tfrac{1}{2}\left[\frac{e_{c1}^2}{\frac{\left(6-\frac{N_1}{M_1}\right)(e_{c1}^2)}{6(M_1-N_1)}+\frac{e_{c2}^2}{M_2}+\cdots+\frac{e_{cx}^2}{M_x}}\right]\left[\frac{N_1}{M_1-N_1}\right] \tag{11}$$

17. Protection of Large Banks of Shunt Capacitors

The usual types of protection for large capacitor banks are:

1. Individual capacitor fuses.
2. Capacitor group (or bank) fuses.
3. Overcurrent relays or trip coils to trip a bank circuit breaker.
4. Potential transformers connected across each phase or each series group per phase of ungrounded wye banks to trip the bank circuit breaker on phase or group voltage unbalance. This scheme can be used for delta or wye grounded-neutral banks that have two or more groups in series.
5. Potential or current transformers connected between the neutrals of two or more wye ungrounded banks to detect unbalance in one bank and operate a relay to trip a single breaker through which all banks, in the protective scheme, are supplied.
6. Potential transformer placed between the neutral and ground of a wye ungrounded bank connected to a grounded system to operate a relay and trip the bank breaker on a shift in the neutral voltage.

Large capacitor banks can be connected in wye ungrounded, wye grounded or delta. However, the wye ungrounded connection is preferable from a protection standpoint. Individual single-phase 15- and 25-kvar capacitor units are protected usually by a fuse whether installed in an outdoor or indoor bank for any type of capacitor connection. For the wye ungrounded system of connecting single capacitor units in parallel across phase-to-neutral voltage the fault current through any fuse is limited by the capacitors in the two sound phases. In addition the ground path for harmonic currents is not present for the ungrounded bank. For wye grounded or delta-connected banks, however, the fault current can reach the full short-circuit value from the system because the sound phases cannot limit the current. Thus, with the wye ungrounded connection smaller fuses and less material are needed for protecting the capacitors. With two or more groups of capacitors in series per phase, the short-circuit current is limited by the capacitors in the unfaulted group. The capacitor bank should have a protective device to disconnect the bank from the system if individual units become defective thereby causing a bad unbalance of capacitor kvar among the three phases.

Two protective schemes for wye connected ungrounded banks for all voltage classes are shown in Fig. 18. The scheme shown in Fig. 18(a) is preferred because the potential transformers serve the dual purpose of protecting against unbalanced capacitor kvar per phase leg as well as

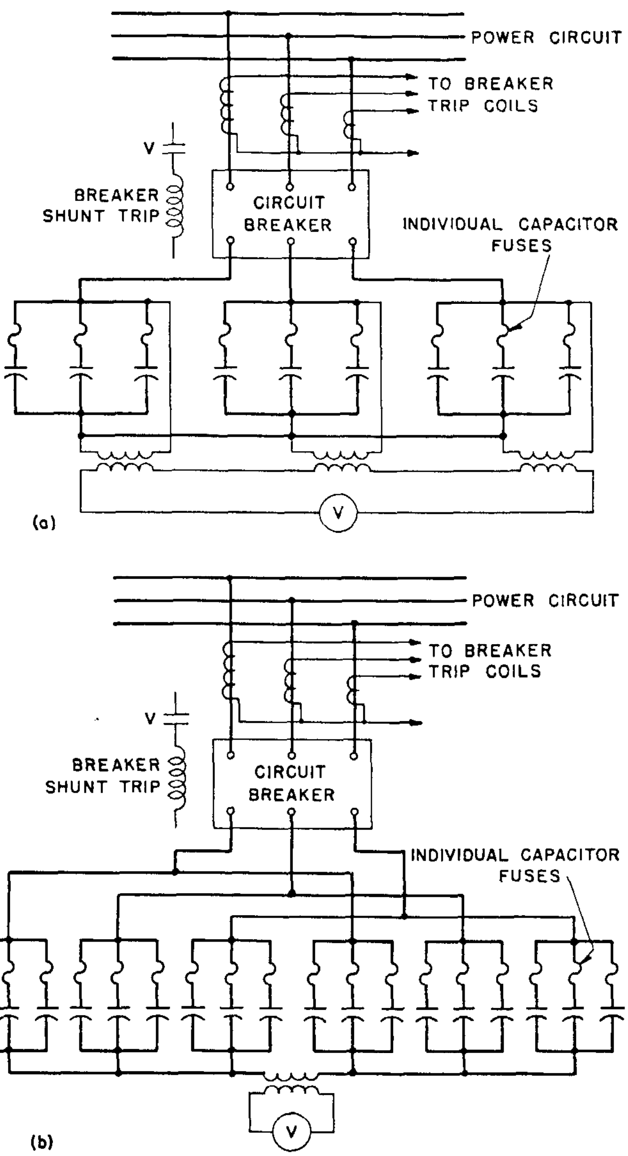

Fig. 18—Two protective schemes for large banks of ungrounded wye-connected capacitors.

(a) Residual voltage trip in event of unbalance among the three phases due to failure of capacitor units.

(b) Residual voltage trip in event of unbalance between the two 3-phase groups of capacitors. Current flow between the two groups can also be used for protection.

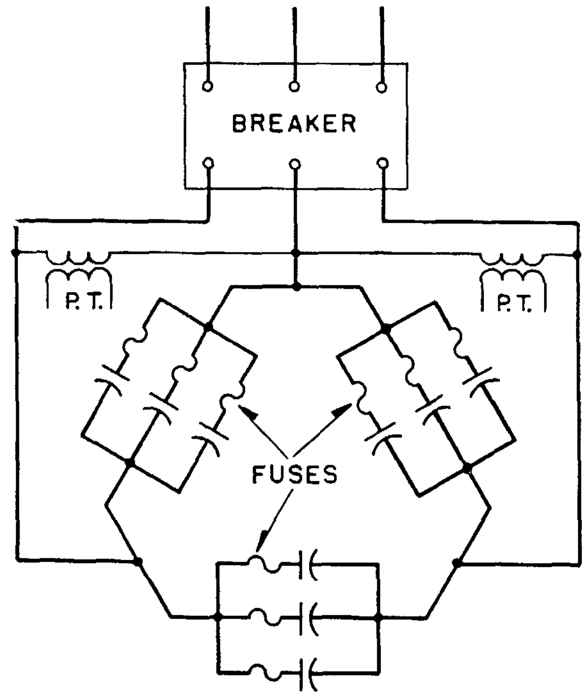

Fig. 19—Delta-connected, fused capacitor units usually used at 2400 volts or less.

providing a discharge path to dissipate quickly charges left on capacitor units when the supply is disconnected. A current or potential transformer connected between the neutral points of two equal parts of a group of capacitors provides protection for unbalanced kvar per phase as shown in Fig. 18(b). In addition, however, two potential transformers connected in open delta should be used on automatically controlled banks across the supply leads to the group to provide a fast discharge path when the capacitors are de-energized. One of the potential transformers can also be used for an indicating lamp to show when the group is energized.

A delta-connected bank of capacitors, Fig. 19, usually applies to voltage classes of 2400 volts or less. Individual capacitor fuses are provided for each unit. If the bank is controlled automatically, potential transformers should be applied across each phase leg to provide fast discharge when the group is de-energized. The individual capacitor units have a very high resistance provided across the terminals inside the case to discharge the capacitors in five minutes after being disconnected from the source. This time of five minutes is considered to be too long for banks that are controlled automatically because when the group is switched on again before the charge is dissipated high transient switching currents result. In special cases such as for indoor capacitor banks, it can be compulsory that potential transformers be applied for rapid dissipation of charges remaining on capacitor units.

18. Capacitor Fusing

General—Each capacitor unit contains a large area of insulation and the probability of unit failures must be recognized even though the record is good, as shown in Fig. 4. When the number of units in a single installation

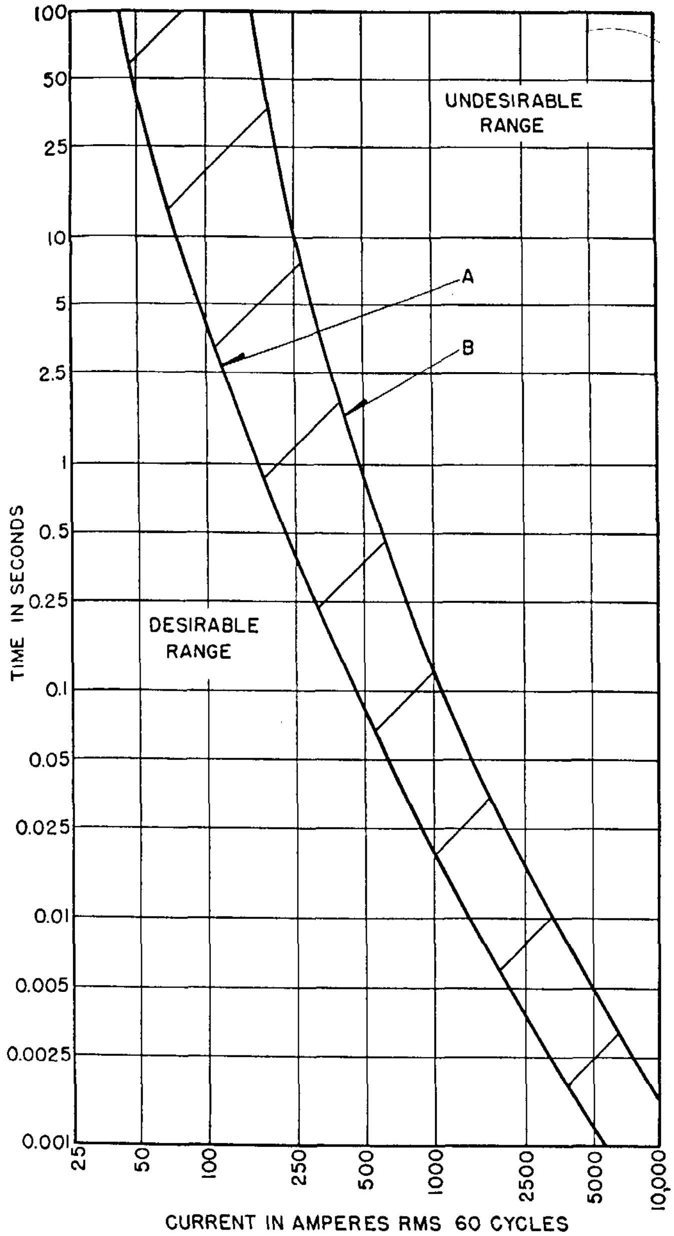

Fig. 20—Capacitor fault current and its relation to case rupture.

Curve A—Where fault currents are cleared in a time to the left of this curve the case is not likely to rupture.

Curve B—Where fault currents are on for a time to the right of this curve the case is likely to rupture with sufficient force to damage other units.

Area AB—Fault currents in this area may open case seams. This area may be used for fuse selectivity with reasonable safety.

is large the probability of a unit failure of insulation is greater. The removal of faulted units is important for the protection of the remaining good units.

About sixty-five percent of existing capacitor kvar on utility systems are "pole type" and usually total about 180 kvar per installation. These are usually on circuits where the fault currents are moderate and group fusing has been satisfactory. When a capacitor unit becomes shorted, it usually does not result in case rupture or damage to other units.

Large capacitor banks are generally on circuits capable of producing high fault current, and additional problems are created due to the close association of large numbers of capacitor units.

The ability of a short-circuited capacitor to pass current is limited by the current-carrying capacity of the thin aluminum foil that forms the electrode surfaces. If these foils are allowed to carry heavy fault current, the foil may act as its own fuse. This has considerable bearing on the fusing problem because a fault within a capacitor can melt the foil rather easily and the fault tends to clear and sometimes restrike. The presence of other capacitors in parallel with and discharging into the shorted capacitor increases the tendency to melt the fault clear. Under certain conditions the arc restrikes each half cycle, thus allowing the adjacent capacitors to be repetitively charged and discharged. This may damage the current-carrying connections of some adjacent units and cause simultaneous or later failure. The current a capacitor unit can pass before case rupture is likely to occur is shown in Fig. 20. If the fault current in a capacitor is limited to a few hundred amperes, the pressure builds up slowly and many cycles of current flow may be endured before case rupture takes place. When the current exceeds about 3000 amperes a rupture results in mechanical damage to adjacent units and often in short-circuited bus connections; the greater the short-circuit current the more violent the case rupture.

If the arc in a capacitor unit is allowed to persist until the case is ruptured, other units and parts in the bank may be damaged either mechanically or by consequent arcs. It is, therefore, desirable to provide adequate protection against short-circuited capacitor units. The function of this protection is:

(a) To protect the circuit and capacitor bank so as to minimize the chance of an outage.
(b) To protect other capacitors in the bank against electrical damage due to current transients.
(c) To protect the other units in the bank from mechanical damage due to a unit case rupture.
(d) To minimize the hazard to the operators and maintenance personnel.

Protection Inherent in Breakers—Breakers with overload protection, and adequate interrupting rating protect the circuit, but usually do not protect the capacitors against damage in case of a short-circuited unit, unless supplemented by individual capacitor fuses, or relay means to trip the breaker as a result of current or voltage unbalance. Use of breakers alone, however, does not remove the hazard associated with a bank where unit fault currents are high.

A breaker should be considered primarily as a switching device and circuit protective device, and not as protection against high fault current within an individual capacitor unit. It may, however, be considered as back-up protection in case the individual unit protection or other protection fails.

Group Fusing—A short-circuited capacitor is in reality a conducting path having time-melting characteristics, which has a bearing on the maximum size of the group fuse. The size of the group fuse is also determined by the normal current of the bank and harmonic currents.

In general, the following rules are recommended for group fusing:

(a) It is preferable not to apply group fuses greater than 85 amperes in rating (on a 100 per cent rating basis.)
(b) The circuit is protected adequately by group fuses if they have sufficient interrupting capacity.
(c) To minimize the danger of mechanical damage, group fuses should be supplemented with individual fuses when the unit fault current is expected to exceed 3000 amperes, even though the group fuse interrupting rating is adequate for the expected fault current.

Large banks of capacitors have been installed with dependence placed solely on group fuses or breakers. Where fault currents are high, the failure of one unit is likely to damage other units in the bank, thereby multiplying the damage considerably. Other units may also fail at a later date when the reasons are not immediately apparent.

Some of these large capacitor banks without individual fuses are wye connected with the neutral ungrounded, or are made up of series groups, so that the problem of high fault currents does not exist. Unbalance in these cases is detected by voltage transformer and relay schemes so as to trip the breaker under abnormal conditions such as might occur if a unit becomes short-circuited. The objection to this arrangement is that it is difficult to identify a defective unit and there is the possibility of electrical damage to parallel units before the breaker de-energizes the bank. Individual capacitor fuses give indication of a blown fuse and give electrical as well as mechanical protection to parallel units.

Individual Fuse—The individual fuse rating is dependent upon the normal current rating of the capacitor unit, harmonic currents and the number of times in rapid succession a fuse must carry discharge current from a good capacitor unit to a defective unit. To provide for the later requirement, the current rating of the fuse is usually at least twice the current rating of the capacitor.

Individual fuses are used primarily to remove units following failure of the dielectric. Since only one fuse is used with each unit, this fuse is not expected to clear for ground faults within the unit. Relaying should be provided where possible to detect ground faults even though their occurrence is very rare.

Individual capacitor fuses should be used, particularly in large banks, so that a faulted unit is disconnected promptly from the circuit for a number of reasons:

(a) Their current rating is small and coordinated with the time-current characteristics of the capacitor.
(b) They indicate the defective unit.
(c) They reduce to a minimum the chance of unit case rupture and subsequent mechanical damage.
(d) They remove a short-circuited unit before the inside foil material is fused to the point where repetitive clearing creates high transient current in adjacent units.
(e) They protect units against transient currents set up by parallel arcs in the bank such as bus flashovers, roof bushing flashovers, or failures in potheads or accessories, or arcs in short-circuited units in the bank.
(f) They permit uninterrupted use of the capacitor bank since a faulty unit need not take the bank out of service.

Table 7 shows there is a minimum number of capacitor units required in parallel per group to give sufficient current for positive operation of an individual fuse on a failed unit. Likewise there is a maximum safe number of individually fused capacitors that can be placed in parallel per group because if a unit fails all other parallel units discharge their stored energy, at high current, through one fuse to the fault. If too many units are in parallel per group, the current is high enough to cause mechanical rupture of the fuse with the possibility of damage to other

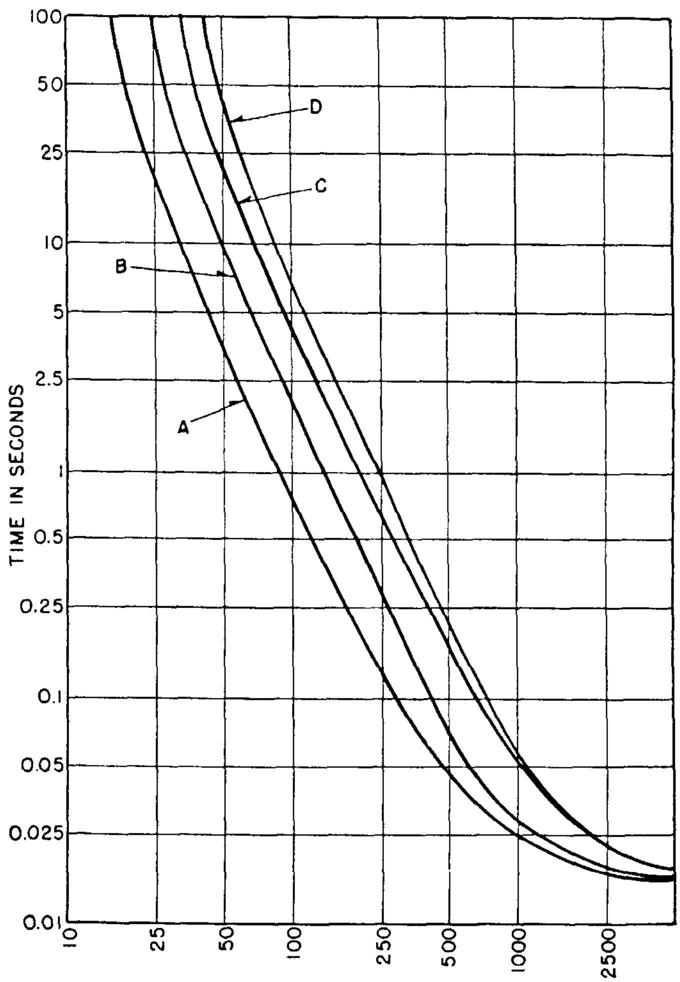

Fig. 21—Typical type BAC capacitor fuse characteristics for use with housed units where the fault current is less than 15 000 amperes from the system.

Fuse A—4160 volt delta connected 15 kvar units.
4160 volt ungrounded wye 15 kvar units.
7200 and 7960 volt ungrounded wye 15 kvar units.
Fuse B—2400 volt delta connected 15 kvar units.
2400 volt grounded wye 15 kvar units.
4160 volt delta connected 25 kvar units.
2775 volt ungrounded wye 15 kvar units.
4160 volt ungrounded wye 25 kvar units.
7200 and 7960 v. ungrounded wye 25 kvar units.
Fuse C—2775 volt ungrounded wye connected 25 kvar units.
Fuse D—2400 volt delta connected 25 kvar units.
2400 volt grounded wye-connected 25 kvar units.

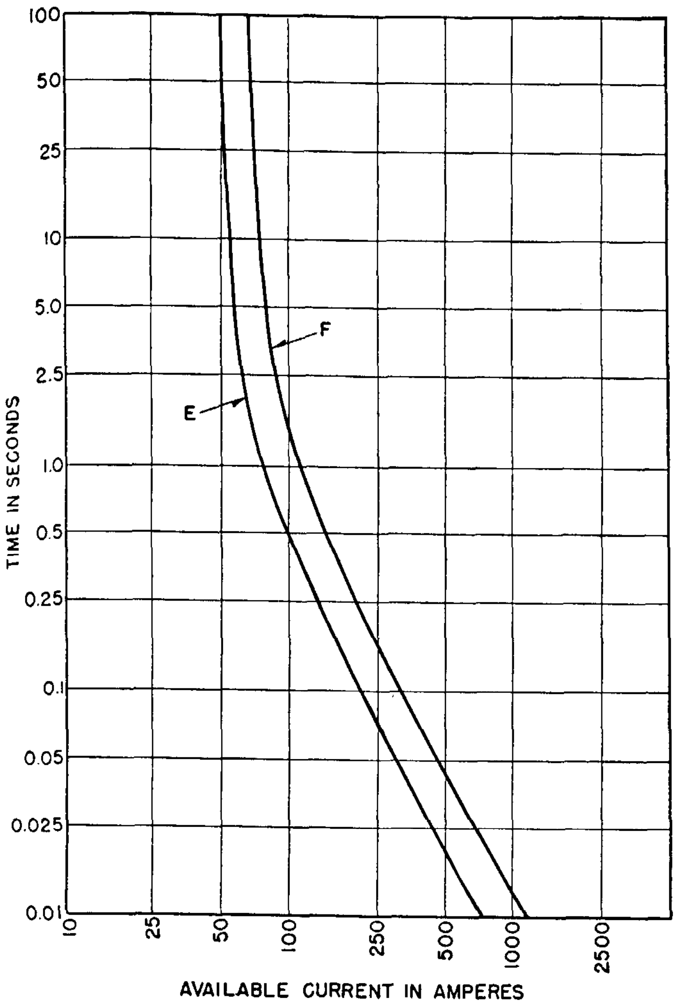

Fig. 22—Typical type CLC current limiting fuse characteristics for use where the fault current is high or in excess of 15 000 amperes from the system.

Fuse E—2400 volt delta connected 15 kvar units.
2400 volt grounded wye 15 kvar units.
Fuse F—2400 volt delta connected 25 kvar units.
2400 volt grounded wye 25 kvar units.

units. Therefore, on large banks of capacitors, when the number of units in parallel per group exceeds two or three times the minimum required number, special consideration should be given to the application particularly with regard to arrangement. Where such limitations are involved, the bank can be divided into two or more parts where there are two or more groups in series. Lower voltage units with a fewer number in parallel per group with more groups in series may be a solution also.

Individual Fuse Characteristics

(a) Housed Banks—2400- and 4160-volt delta-connected and 2400-volt wye-connected grounded-neutral.

Housed banks usually contain indoor-type individual unit fuses. Where the fault current is less than 15 000 amperes type BAC fuses are used, the characteristics of which are shown in Fig. 21. Actually the discharge current from the good capacitor units operating in parallel with the faulted unit supplies a considerable portion of the energy to blow the fuse on the faulted unit. If it were not for the current from the parallel units the system short-circuit current would have to be limited to about 3000 amperes to prevent rupture of the capacitor case. The discharge current from the parallel capacitors is high in magnitude as shown in Fig. 28 and reaches half value in about 0.02 second or less.

Where the fault current exceeds 15 000 amperes from the system, individual capacitor current limiting fuses (CLC) are used, the characteristics of which are shown on Fig. 22.

(b) Housed Banks (Ungrounded Wye)

Housed banks for circuit voltages of 4800 volts and above are usually wye connected with the capacitor neutral ungrounded, whether or not the source neutral is grounded.

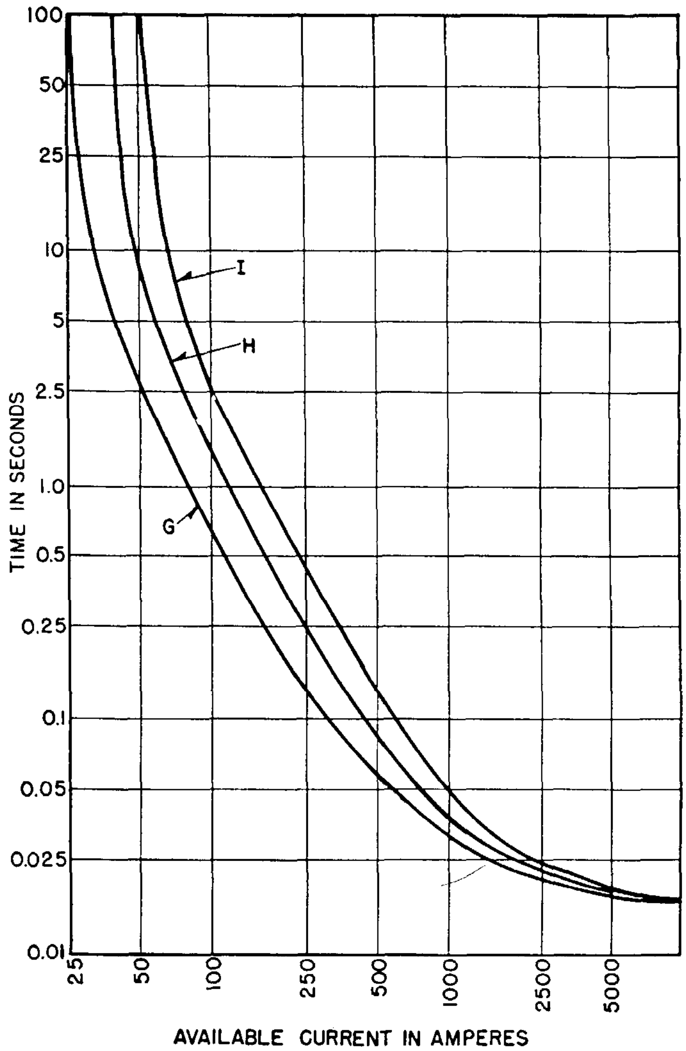

Fig. 23—Typical UT fuse characteristics used on ungrounded wye-connected outdoor capacitor banks.

Fuse G—4160 volt 15 kvar units
7200 volt 15 kvar units.
Fuse H—2775 volt 15 kvar units.
4160 volt 25 kvar units.
7200 volt 25 kvar units.
Fuse I—2775 volt 25 kvar units.

This arrangement limits fault current and the type BAC fuses are used, the characteristics of which are shown on Fig. 21.

(c) Outdoor Structural Type Banks (Delta or Grounded Wye)

Where the fault current is likely to be high as for a delta connected or grounded wye, outdoor bank current limiting individual fuses (CLC) are desirable. This applies to delta connected 2400 volt banks, wye connected 2400 volt and delta connected 4160 volt banks of capacitors. The characteristics of the fuses are the same as for similar indoor banks as shown on Fig. 22.

(d) Outdoor Structural Type Banks (Ungrounded Wye)

Outdoor structural type banks for voltages of 4800 volts and above are usually wye connected with the neutral of the capacitor ungrounded, whether or not the source neutral is grounded. This arrangement limits fault current and permits fuses of lower interrupting rating. The characteristics for these fuses are given on Fig. 23.

19. Automatic Control for Capacitor Banks

The intelligence required to switch banks of shunt capacitors automatically depends upon the reason for their use. If they are used primarily to control voltage, then the capacitors can be switched on when the voltage is low or off when the voltage is high, and a voltage relay supplies the control. If the system voltage is regulated by other means and the capacitors are used for power-factor correction, then the load kvar or total current must be used as the means for control.

It is always desirable to use the simplest type of control that will accomplish the desired result. Current control is commonly used where the voltage is regulated by other means and the power factor is practically constant through wide variations in load. Kvar control is used where the load power factor varies over a wide range as the load changes.

Whether the control is accomplished by voltage, current, or kvar, the control systems are similar. In addition to the master control relay, other devices are required in the control scheme such as time-delay relays, control switches, etc. For one-step automatic control the master relay energizes the "closing" element of a time-delay relay, and if the master-relay contacts stay closed for the time required for the time-delay relay contacts to make, then the operating circuit is energized and the capacitor breaker closes.

A similar process in reverse trips the capacitor breaker. For a two-step control the sequence is the same as for one-step control except that auxiliary contacts on the No. 1 breaker set up the circuits for the control of the second step. If the No. 1 breaker is closed, the circuit is set up to either trip No. 1 or to close No. 2. The sequence of operation is the same in all cases, that is, No. 1 breaker always closes first and trips last.

For more than two-step control, each additional breaker, by means of auxiliary contacts, sets up the control circuits for the next operation whether it be to add or remove capacitor kvar. The control circuits become numerous and involved, but their operation is accurate, reliable, and thoroughly proved by many applications.

Where the need for capacitor kvar follows a fixed schedule, the capacitors can be switched by a time relay that initiates *on* or *off* at predetermined times.

20. Inrush Current

When the first step of a capacitor bank is energized, it is possible for a large instantaneous current from the system to flow. Curves in Fig. 24 show for several line-to-line

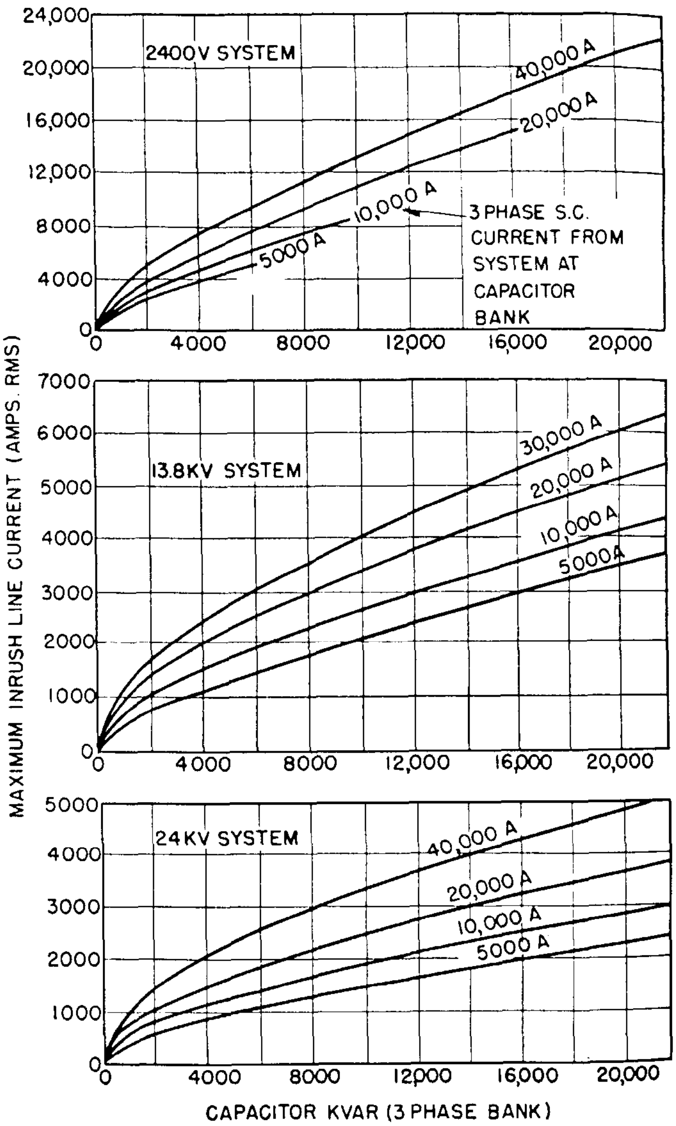

Fig. 24—Inrush current from system when energizing capacitor bank.

voltages the maximum rms inrush current for different system short-circuit currents available at the capacitor terminals. This current can be calculated using the following formula:

$$I=\frac{E_{LG}}{X_c-X_L}\left[1+\sqrt{\frac{X_c}{X_L}}\right] \qquad (12)$$

Where E_{LG} is line-to-ground operating voltage on the capacitor bank.

X_c is the capacitive reactance in ohms of one phase to neutral of the capacitor bank.

X_L is the inductive reactance in ohms per phase of the source.

The above formula applies to delta-connected capacitor banks if X_c is determined as the reactance of the equivalent line-to-neutral capacitor kvar. The current values are for the first step of a bank. If one or more steps in the capacitor bank are already energized, then the maximum peak current that flows into the next capacitor group to be energized is determined largely by the momentary discharge from those capacitor units already in service.

The breaker controlling the last step in a bank of capacitors is the one that is subjected to maximum peak current when this step is energized. The peak currents if no charge is on the step being energized, can be determined approximately by using the following equation:

$$I\text{peak} = (1.2)\ (\sqrt{2})\ \left(E_{LN}\sqrt{\frac{C}{L}}\right) \qquad (13)$$

If the step being energized is fully charged, the peak inrush current can be about twice this value. E_{LN} is rms line-to-neutral voltage applied to the capacitors. C is the total capacitance per phase of the capacitors already energized combined with the capacitance of the step being energized. For a three step bank with two steps energized and with the third step being energized then

$$C = \frac{1}{\dfrac{1}{C_1+C_2}+\dfrac{1}{C_3}} \qquad (14)$$

For delta-connected banks the equivalent single-phase-to-ground capacitor kvar must be used as though the bank was wye connected. L is the inductance between the step being energized and that portion of the bank already energized. This value of L is difficult to determine accurately, but, due to inductance in the capacitor leads and bus structure, the estimated L is usually a low value rather than a high one, thus giving a current that is too high and, therefore, on the safe side. The 1.2 factor is applied to account for some feed in from the system and also possible current unbalance due to unequal pole operation of the breaker.

The inrush current and frequency when a bank of capacitors is energized in parallel with one or more existing banks is given in Fig. 25. To illustrate its use assume a 13.8-kv, three-step capacitor installation consisting of three 2520-kvar banks, two being energized and the third step to be energized. The percent capacitive reactance for each step on 2520 kvar is 100. The two capacitor steps already energized in parallel are 50 percent on 2520 kva. These two steps in series with the one step to be energized are 150 percent. So the X_c for use with Fig. 25 is 150 percent. Now assume that each capacitor step has a series inductive reactance of 0.0076 ohm in all of its leads between the capacitor units and a common point on the bus which is 0.01 percent expressed on 2520 kvar. Two such units in parallel plus one in series gives 0.015 percent X_L for use with Fig. 25. Using this data the X_L $(X_c/100)$ equals 0.0225 which for switching in the third 2520 kvar step of capacitors allows a maximum peak inrush current of about 69 times normal rms rated current of each step or 69×105, or 7250 amperes. The frequency of this current is about 6000 cycles. If the inductive reactance of the leads is less than 0.0076 ohm, the maximum inrush current is greater than 7250 amperes.

Where the inrush current when switching banks of capacitors is excessive, it can be limited by the insertion

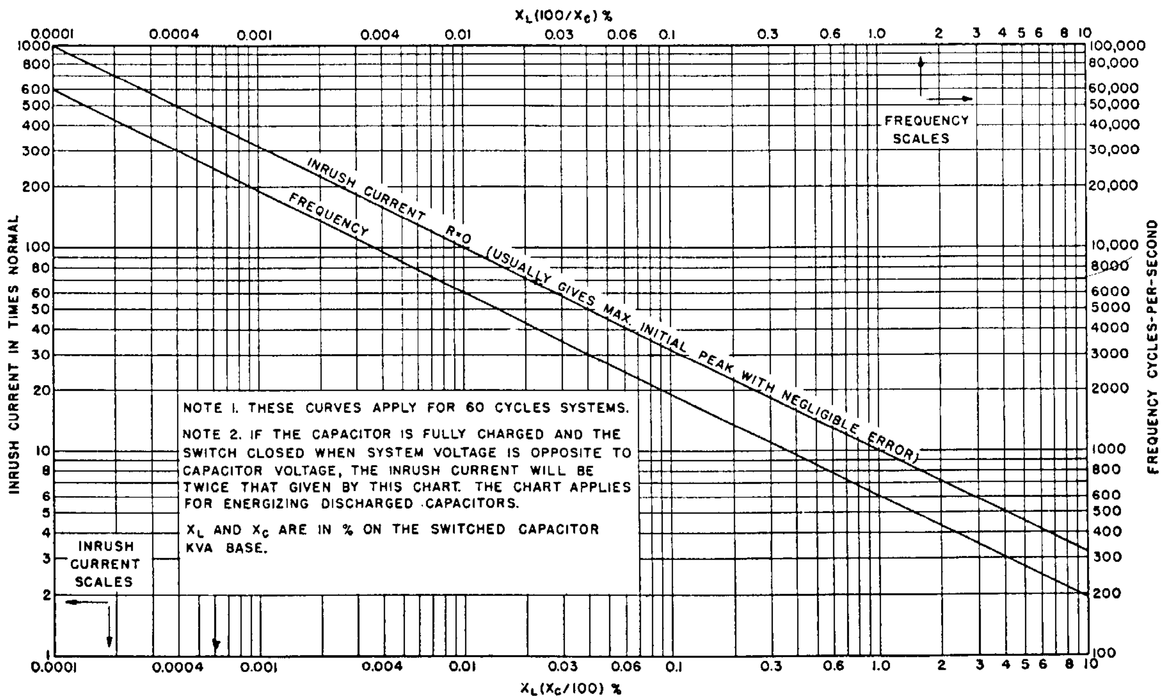

Fig. 25—Magnitude and Frequency of transient inrush current when energizing a bank in parallel with one existing bank.

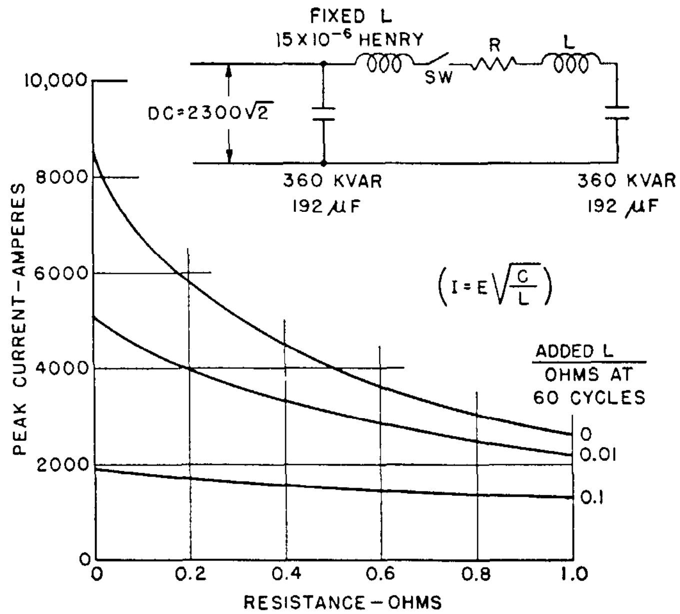

Fig. 26—Test results indicating the effect of reactance and resistance on limiting energizing inrush current.

of reactance or resistance into the circuit. Reactance is much more effective than resistance. The curves in Fig. 26 give the results of tests showing the effect of adding resistance or reactance in reducing the peak inrush current. D-c voltage was used to charge one group of capacitors; the voltage was then removed and when the switch was closed between the two groups, the peak current was measured.

21. Voltages When Switching Off Capacitors

Since the current goes out at normal current zero when de-energizing a bank of shunt capacitors, the rms voltages resulting can be calculated. The voltages to ground, recovery voltage across circuit breakers, and the line-to-line and line-to-neutral voltages across the capacitors are important. The voltages of Table 8 expressed in percent of normal peak line to neutral voltage are obtained when the supply system is grounded solidly and does not suffer neutral displacement while switching a wye-connected ungrounded capacitor bank. For a normal breaker opening, one phase is interrupted first even for a well adjusted breaker, at current zero, and 90 degrees later the other two phase currents are interrupted simultaneously at current zero by the clearing of either B or C breaker contact.

TABLE 8

Phase of Wye Connected Bank	Percent of Peak Voltage		
	A	B	C
Sequence of Opening	1st	2nd	2nd
Maximum E to Ground after Opening	150	87	87
Maximum E across Corresponding Breaker Pole	250	187	187
Maximum Voltage across Capacitor Leg Following Interruption	100	37	137

The voltages of Table 8 are brought about by the fact that 100 percent voltage is left on A phase, the first phase to open. The very instant A phase opens, a charge of 50-percent voltage is left on phases B and C because the instantaneous voltage across these two phases is 50 percent. The neutral point of the capacitor bank remains at a potential of 50 percent above ground, which appears across the capacitance to ground. The subsequent voltage applied across B and C when B or C clears is 173 percent, half of which is across B capacitor and half across C capacitor. But the 50-percent charge left on these two phases, when A opened, is still present and adds or subtracts from half of 173 percent giving a net of 37 percent or 137 percent. Similar analyses can be made for delta-connected capacitors.

The voltage across the contacts of the circuit breaker is important because if the recovery rate or the magnitude is too great, restriking occurs across the contacts. Such restriking cause switching surges that may produce peak voltages of several times the normal peak voltage to ground. Special consideration should be given to this problem in each case. The problem is more acute at voltages above 15 kv. Careful adjustment of the breaker can make an otherwise unsatisfactory condition one which is acceptable. Special treatment with respect to the oil flow in the breaker grid during interruption usually solves the problem. In extreme cases it may be necessary to limit restriking on de-energizing by inserting in series or parallel with each phase of the capacitor circuit a suitable resistor just prior to the operation of the circuit breaker to de-energize the bank. A careful analysis of the problem should be made for each application; laboratory and field tests may be necessary.

22. System Harmonic Voltages

Since the reactance of a capacitor varies inversely as the applied frequency relatively small harmonic voltages cause relatively large current-wave distortion. Capacitors are therefore built to permit combined harmonic and 60-cycle kvar to equal not more than 135 percent (AIEE Standard) of the capacitor nameplate rating. The kvar loading of a capacitor expressed as a fraction of its rating with harmonic voltages applied can be obtained as follows: $KVA = E_1^2 + 3E_3^2 + 5E_5^2 + \ldots\ldots$ where all voltages are expressed as a fraction of the rated voltage. If only one harmonic is present, it can have a value of

$$E_N = \sqrt{\frac{1.35 - E_1^2}{n}} \qquad (15)$$

where n is the order of the harmonic.

The standard margins in capacitors are usually more than sufficient for the amounts of harmonic voltages present in most systems and, therefore, very little trouble is experienced. The principal cause of harmonic currents in capacitors is the magnetizing requirements of system transformers. If the transformers are operated near their rated voltages, the harmonic voltages are limited to minimum values. Capacitors do not generate harmonic voltages.

Harmonic frequencies usually encountered are the third and fifth. The capacitor has lower reactances to higher frequencies and therefore allows proportionately larger currents. Figure 27 shows the amount of total rms current, fundamental and one harmonic, which standard capacitors can carry, depending on how much total rms voltage, fundamental and harmonic, exists at the same time. For example, suppose the fifth harmonic and the fundamental are present and the total rms voltage is 105 percent. Then

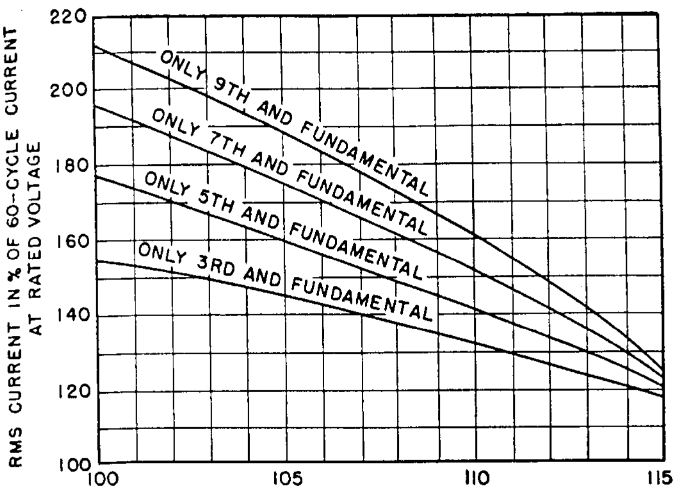

Fig. 27—Permissible harmonic currents. For 135 percent kvar for different fundamental voltages without exceeding thermal limits.

the total current the capacitor can carry is 161 percent. This is made up of about 102-percent rated amperes at fundamental frequency and about 125-percent rated amperes at fifth harmonic. The corresponding voltages are 102-percent fundamental and 25-percent fifth harmonic.

Breakers applied with shunt capacitors must have sufficient continuous current-carrying capacity to handle expected harmonic currents along with the rated-frequency current.

23. Discharge Current

When a capacitor is short circuited, either at its terminals or through a length of feeder, it discharges its stored energy determined by

$$\text{Stored energy} = \frac{1}{2} CE^2 \qquad (16)$$

If the short circuit occurs at the instant the voltage on the capacitor is a maximum, then the stored energy is a maximum. The stored energy is dissipated in the resistance of the circuit which includes the capacitor and the feeder up to the short circuit. The peak current, the frequency of the current and the time constant of the circuit can be calculated for a given situation. Figure 28 shows the peak value of current calculated for various lengths of bus consisting of single-conductor cables with an equivalent delta spacing of four feet. The peak current is high in magnitude but since the frequency is high and the time constant of the circuit low, the current decreases rapidly. For all practical sizes of capacitor banks, the discharge current reaches half value in about 0.02 second, or less. Breakers normally applied with capacitor banks are capable of handling these currents.

24. Harmonics and Coordination with Telephone Circuits

The principal cause of harmonic voltages and currents in capacitors is the magnetizing requirements of transformers. Because of the lower impedance of capacitors at higher frequencies, the harmonic currents may become so high as to endanger the life of the capacitor, or cause excessive fuse blowing, or overheating of breakers and switches. The standard margins built into capacitors, which were mentioned previously, are usually sufficient so that for the amount of harmonic voltage present in most systems no undue amount of trouble is experienced. For the transformer magnetizing current the third harmonic components and their multiples are supplied usually by circulation around the delta connected windings. The higher harmonics are usually so small that they give no appreciable trouble as long as the transformers are operated near their rated voltage.

An unbalanced fault on a system supplied by waterwheel generators without damper windings may produce harmonic voltages. By resonance or partial resonance with capacitors these voltages can be magnified. While the duration of the fault might not be sufficiently long to injure the capacitor, it may result in blowing of capacitor fuses all over the system. This hazard is reduced by properly designed damper windings and system arrangement.

Considerable study has been given the effects of shunt capacitors on the inductive coordination of power systems

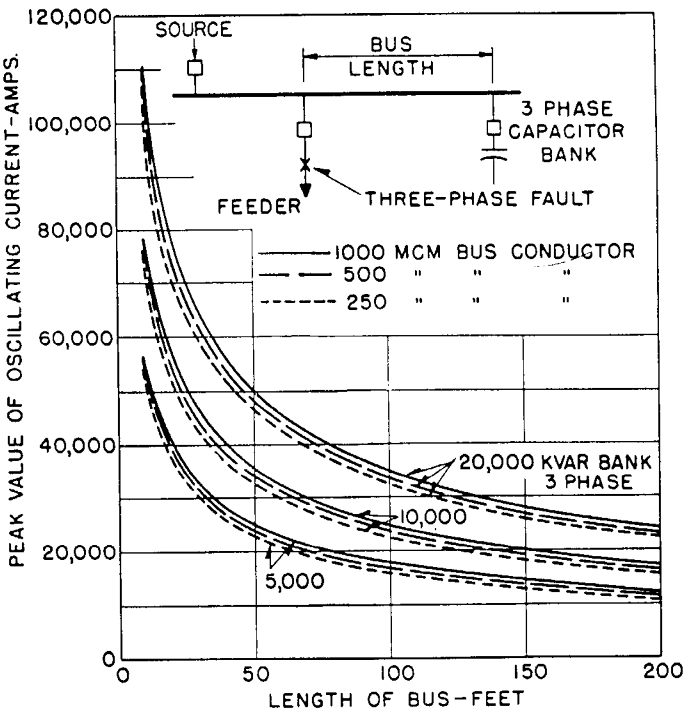

Fig. 28—Peak current supplied to a three phase fault through various lengths of bus from shunt capacitor banks.

and exposed telephone circuits at noise frequencies. These studies have been carried on by the Joint Subcommittee on Development and Research of the Edison Electric Institute and Bell Telephone System. The results of their preliminary study of the problem were included in an article published in the August, 1938 issue of the Edison Electric Institute Bulletin. It has been found that the use of capacitors may be either detrimental or beneficial from the inductive coordination standpoint, depending on the particular conditions in each case. Advance planning by the power and communication industries has reduced the number of troublesome situations to a small percentage of the capacitor installations. Where capacitors have resulted in increased noise, it has generally been practicable to improve conditions by relatively simple measures applied to either the power or communication systems or both. A summary of the available measures is included in the article mentioned above and in Chap. 23 of this book.

25. Portable Capacitors

Portable capacitor units such as shown in Fig. 29, are effective in relieving overloaded facilities until more permanent changes in the system can be made. Two single-phase mobile capacitor units can be used to reduce the overload on open-delta banks of transformers occasioned by the failure of one transformer of a three-phase delta-connected bank. In the open-delta application the most effective use of the capacitors is to plan twice as much capacitive kvar across the phase lagging the open side of the delta as is placed across the open side.

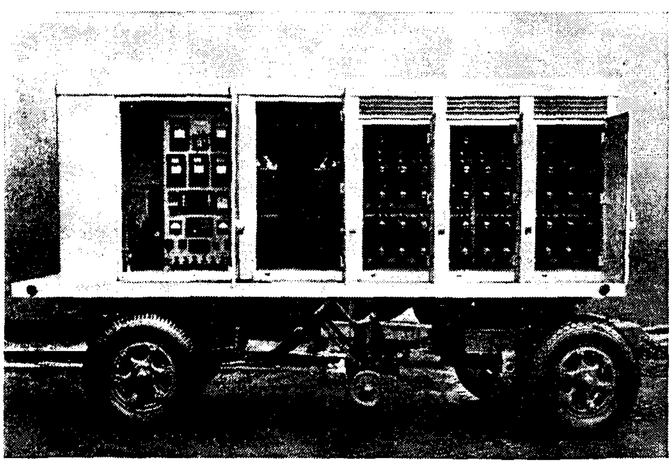

Fig. 29—Portable capacitor bank.

26. Capacitors and System Stability

Shunt capacitors reduce the static stability limits of generators (and systems) because they reduce the field currents used for a given kw load and terminal voltage. The effect is noticed by an increase in generator power factor as more and more shunt capacitors are added. Actually many factors are involved in determining the static stability limits of generators, some of which are difficult to evaluate. However, the effect of shunt capacitors can be determined rather directly.

The static stability limit of a generator for a given set of conditions is directly proportional to the voltage on the air-gap line of the generator corresponding to the excitation current. Therefore, as more shunt capacitors are added to a system, the power factor of the generators increase and consequently the exciting current decreases. As the exciting current is decreased, the voltage on the generator air-gap line decreases. The static stability limit is therefore proportional to generator exciting current. Generally on turbo-generators, if the operating power factor at full load is no greater than 95 percent lagging, experience has shown that the operation is safe. In some cases generators are operated between 95 percent lagging and 100 percent power factor with satisfactory performance. Few, if any, generators are operated consistently at power factors in the lead unless the generators are designed specifically for such service. Hydro-generators may also be affected by shunt capacitors, but usually these generators are so far removed electrically from capacitors that the generators are affected more by other factors such as the characteristics of transmission lines and the sending of power over relatively long distances.

Any generator, regardless of its prime-mover, may be affected by system shunt capacitors and therefore the problem should always be taken into consideration. This is particularly important where large amounts of shunt capacitors are planned for systems where generators are already operating at high power factors. A few power systems have this problem now and more will probably have the problem as future plans are made to get better overall system economy by taking advantage of the characteristics of shunt capacitors. This problem also has a direct bearing on how much capacitor kvar can be permanently connected through minimum-load periods with few generators in service and how much capacitor kvar can be installed with switching to provide needed kvar during maximum load periods and maximum generation.

27. Surge Protection of Shunt Capacitors

On circuits exposed to lightning it is recommended that lightning arresters be provided on all delta-connected capacitors either housed or hanger type large or small banks. Likewise arresters are recommended for all wye-connected capacitor banks where the neutral is ungrounded. Where the capacitor bank is switched, it is best practice to provide arresters on the capacitor side of the circuit breaker.

A capacitor bank connected in wye with the neutral grounded has the ability of sloping off the front and reducing the crest of traveling waves, so that it affords added lightning protection to the capacitor bank itself and to transformers and other adjacent equipment. Thus there is some question as to whether or not arresters are needed. In addition, for those surges where arresters are required there is also some hazard to the arrester because the capacitor discharges through the arrester when the arrester operates. When the capacitors are connected to a bus with transformers and other circuits, arresters are required to protect this other equipment whenever the capacitor bank is disconnected. The arresters are therefore available and in service at all times. Where the capacitor bank is the only load on a transformer winding the arresters can be omitted if the transformer is removed from service when all capacitors are disconnected. Where the capacitors are supplied from a third winding of the transformer, arresters may be required on this winding if all of the capacitors are to be out of service at times.

From a surge-protection point of view for greatest safety to the arresters, wye-connected capacitor banks should be operated ungrounded. For best surge protection of the capacitors, the neutral should be grounded and arresters provided. There are other problems with capacitor banks, however, which make the wye-grounded bank undesirable. The grounded-neutral bank provides a path for the third or residual harmonics, thereby increasing the probability of communication interference; if a capacitor unit becomes shorted, where there is a single unit between line and neutral, the fault current can exceed the ability of the fuse to clear before the capacitor unit is ruptured.

Lightning arresters protecting high-voltage capacitor banks above 15 kv are subjected to switching surges, when the capacitors are switched, whether or not the capacitor bank neutral is grounded. With restriking across breaker contacts, which may occur, the arresters may be damaged. Therefore it is necessary to provide means of limiting the restriking in the breaker to protect the arresters. The solution in a given case may require special field tests to determine the proper adjustment of the breaker or to determine what changes are necessary.

28. Capacitors Versus Synchronous Condensers

In large units synchronous condensers constitute a competitor of shunt capacitors. The following points should be considered in comparing these two types of equipment.

1. A standard synchronous condenser is capable of supplying kvars equal to its rating to the system as well as absorbing them to an extent equal to 50 percent of its rating. For those applications requiring these characteristics, the comparison should be on a basis of the synchronous condenser against the capacitor at full kvar plus a shunt reactor of 50 percent kvar.

2. The fineness of control of the synchronous condenser cannot be duplicated by the capacitor unless a large number of switching steps are used.

3. An instantaneous drop in terminal voltage, within practical limits, increases the kvar supplied to the system in the case of a synchronous condenser whereas a similar change in the case of capacitors decreases the kvar supplied to the system. In this regard the synchronous condenser has greater stabilizing effect upon system voltages and likewise tends to maintain synchronism between machines. Its mechanical inertia, in general, has a further stabilizing effect upon the other synchronous machines comprising the system. By reason of these same characteristics, a synchronous condenser reduces the effects of sudden load changes or rapidly varying loads, such as drop in system voltage occasioned by starting of a large motor or operation of large welders.

4. For short periods the synchronous condenser can supply kvar in excess of its rating at normal voltage, whereas this is not the case for capacitors.

5. The losses of synchronous condensers are much greater than those of capacitors. For synchronous condensers the full load losses vary from about 3 percent of the kva rating for 3000 kva units to about $1\frac{1}{2}$ percent for very large units of 50 000 to 100 000 kva. For capacitors the losses are about one-third of one percent of the kva rating. The no-load losses of air-cooled synchronous condensers are about 60 percent of the full-load losses and for hydrogen-cooled synchronous condensers about 40 percent; therefore, at fractional loads the losses of the synchronous condenser are not in proportion to the output in kva. For a capacitor, however, the losses are proportional to the kvar connected to the system.

6. A comparison of the cost of synchronous condensers and capacitors involves an evaluation of the losses. Figure 30 gives an idea of the relative cost of air-cooled outdoor

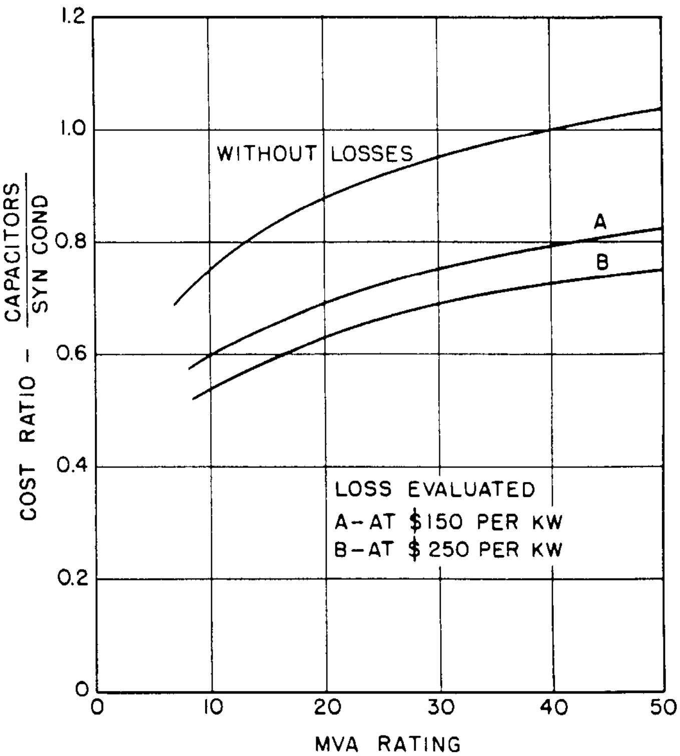

Fig. 30—Approximate relative cost of shunt capacitors and synchronous condensers. (Capacitors connected in wye and switched in five steps. Costs do not include main circuit breaker, land space, foundations, or space parts, but do include freight, automatic control, erection, capacitor fuses, coolers on synchronous condensers, and so forth.)

synchronous condensers and capacitors. Three evaluations for losses were assumed 0, \$150, and \$250 per kw. The low losses of the capacitors should not be evaluated as highly as those for the synchronous condenser because, as just mentioned, at fractional loads the losses decrease more rapidly than for the synchronous condenser.

7. Capacitors lend themselves to distribution at several locations throughout the system, which is difficult to do economically with small synchronous condensers. Thus, capacitors can be located at points closer to the load and be more effective.

8. The kvar rating of a capacitor installation can be increased or decreased as the loads and system requirements dictate, which is impractical with synchronous condensers. Capacitors can be installed easily. By moving

capacitors from point to point as required, the installation of other equipment such as transformers, may be deferred. Foundations are less important than for synchronous condensers, and auxiliaries are fewer and simple.

9. A failure of a single fused unit in a bank of capacitors affects only that unit and does not jeopardize operation of the entire bank. A failure in a condenser removes the entire ability to produce kva. On the other hand, failure of a synchronous condenser is less likely to occur than failure of a single unit in a bank of capacitors.

10. Synchronous condensers add to the short-circuit current of a system and may increase the size of breakers required. This is rarely, if ever, the case with shunt capacitors. On the other hand, breakers used in the switching large banks of capacitors may involve large currents of short duration. In general, however, these currents fall within circuit-breaker ratings dictated by the power system.

29. Capacitors and Synchronous Condensers

Banks of shunt capacitors have been used in conjunction with synchronous condenser where fluctuating loads of low power factor are prevalent or where the steps in the capacitor bank were too coarse to give the desired fineness of voltage control. In this way the economy of using shunt capacitors for part of the kvar correction can be had by using one or several steps of capacitors with breakers. Where the voltage of the bus is controlled by the combination of capacitors and condenser, the master control would be from the bus voltage. It is more likely though that the bus voltage will be controlled by other means such as a tap-changing-under-load supply transformer, and that the object of using the kvar corrective equipment is for power-factor regulation. In such cases the control of the kvar must be accomplished by a power-factor regulator.

V. SERIES CAPACITORS FUNDAMENTALS

Like the shunt capacitor, the series unit has application on transmission and distribution lines. Behavior of the shunt capacitor is generally well understood and can be accurately predicted. The same is not always true of the series type. Many questions are still unanswered and many problems are still unsolved. However, developments and experience of recent years are bringing new knowledge and maturity to the science of applying series capacitors to improve conditions on distribution and transmission lines.

Constructionwise, shunt and series capacitors are identical. In fact, should the need for a series capacitor disappear, the capacitor units can be removed and reinstalled as shunt units. The two types differ in their method of connection. The shunt unit is connected in parallel across full line voltage. The series unit is connected in series in the circuit and hence conducts full line current. While the voltage on a shunt installation remains substantially constant, the drop across the series bank changes instantaneously with load, as with any series device. It is this characteristic, which produces an effect dependent on load, that makes the series capacitor extremely valuable in certain applications by compensating for line series inductive reactance.

A series capacitor in an a-c circuit introduces negative or leading reactance. Current through this negative reactance causes a voltage drop that leads the current by 90 degrees. This drop is opposite from that across an inductive reactance. Thus a series capacitor at rated frequency compensates for the drop, or part of the drop, through the inductive reactance of a feeder. The effects of this compensation are valuable in two classes of applications: one, on radial feeders to reduce voltage drop and light flicker; and, two, on tie feeders to increase the ability of the feeder to transfer power and help the stability of the system.

30. Effects on Radial Feeders

The action of a series capacitor to reduce voltage drop is illustrated in Fig. 31. The voltage drop through a feeder is approximately

$$IR \cos \theta + IX_L \sin \theta \qquad (17)$$

where R is feeder resistance, X_L feeder reactance, and θ the power-factor angle. If the second term is equal to or greater than the voltage improvement desired, a series capacitor may be applicable. The magnitude of the second term is a relatively larger part of the total voltage drop where power factor is low and where the ratio of feeder resistance to reactance is small. With a series capacitor inserted, Fig. 31(b), the voltage drop becomes

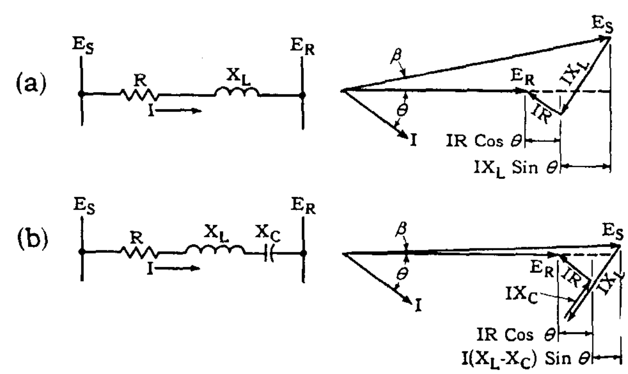

Fig. 31—Voltage vector diagrams for a circuit of lagging power factor (a) without and (b) with series capacitors. The series capacitor increases the receiving-end voltage, thus reducing voltage drop.

$$IR \cos \theta + I(X_L - X_C) \sin \theta \qquad (18)$$

or simply $IR \cos \theta$ when X_C equals X_L. In most applications the capacitive reactance is made smaller than feeder reactance. Should the reverse be true, a condition of overcompensation exists. Overcompensation has been employed where feeder resistance is relatively high to make $I(X_L - X_C) \cos \theta$ negative. However, overcompensation may not be a satisfactory condition if the amount of capacitance is selected for normal load, because during the starting of a large motor the lagging current may cause an excessive voltage rise, as shown by Fig. 32. This is harmful to lights and introduces light flicker.

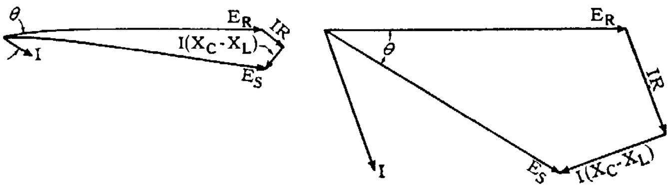

Fig. 32—The high lagging current due to motor starting rapidly raises the receving-end voltage of a circuit which is over-compensated with series capacitors.

The power factor of the load current through a circuit must be lagging for a series capacitor to decrease the voltage drop appreciably between the sending and receiving ends. If power factor is leading, the receiving-end voltage is decreased by the addition of a series capacitor, as indicated by Fig. 33. If the power factor is near unity, sin θ and consequently the second term of Eq. (18) are near zero. In such cases, series capacitors have comparatively little value.

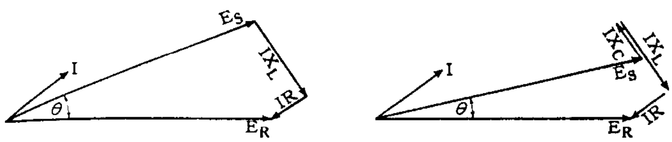

Fig. 33—When the load power factor is leading, a series capacitor is undesirable because it decreases the receiving end voltage.

When properly applied, a series capacitor reduces the impedance of a line and thereby raises the delivered voltage. This increases the kva capacity of a radial feeder and, for the same delivered load kva, slightly reduces line current. A series capacitor, however, is not a substitute for line copper.

31. Light Flicker

Series capacitors are suited particularly to radial circuits where light flicker is encountered due to rapid and repetitive load fluctuations, such as frequent motor starting, varying motor loads, electric welders, and electric furnaces. A transient voltage drop, which causes light flicker, is reduced almost instantaneously in the same manner as voltage drop due to a slowly increased load. To predict accurately the reduction in voltage flicker by series capacitors, the current and power factor of the sudden load increment must be known. It is obvious that to improve voltage conditions or reduce light flicker at a given load point the series capacitors must be on the source side of that point. The series capacitors must compensate for line inductance between the source and the point where it is desired to reduce light flicker. This sometimes makes the application of capacitors difficult because one feeder from a bus with several feeders may have a fluctuating load that produces sufficient voltage change on the bus to cause light flicker on all feeders. To use series capacitors to reduce the flicker, they must be installed in the supply circuit or circuits to the bus.

Shunt capacitors cannot be switched fast enough to prevent light flicker. In fact, an attempt to use shunt capacitors for this purpose might aggravate the situation. Step voltage or induction voltage regulators, also, are not sufficiently rapid to follow sudden voltage fluctuations. The voltage dip cannot be prevented by shunt capacitors or regulators as the dip itself is used to initiate the correction.

32. Effects on Tie Feeders

Series capacitors can be applied to tie feeders to increase power-transfer ability and improve system stability rather than to improve voltage regulation as on radial feeders. The vector diagrams and Eq. (17) and Eq. (18) still apply but the emphasis is now on power transfer and stability. For simplicity, assume the feeder impedance consists only of inductive reactance. Since the effect of resistance is small in most tie-feeder circuits, it can be neglected without materially affecting the results. Referring to Fig. 34,

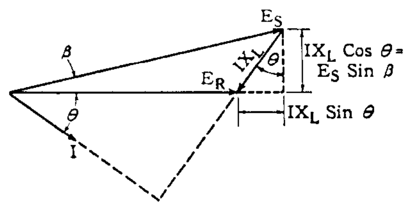

Fig. 34—Vector diagram for a tie feeder in which resistance effects are neglected.

the simplified equation for the amount of power transferred through a tie feeder is:

$$P=E_R\frac{E_S \sin\beta}{X_L\cos\theta}(\cos\theta)=\frac{E_RE_S}{X_L}\sin\beta \qquad (19)$$

where β is the angle between the sending (E_S) and receiving (E_R) voltages. With a series capacitor, the expression for power transfer is

$$P=\frac{E_SE_R}{X_L-X_C}\sin\beta \qquad (20)$$

Therefore, for a given phase-angle difference between the voltages, the power transfer is greater with a series capacitor. Thus by making possible a greater interchange of power, the normal load transfer and the synchronizing power flowing during transient conditions are increased, thereby helping stability. This is illustrated in Fig. 35,

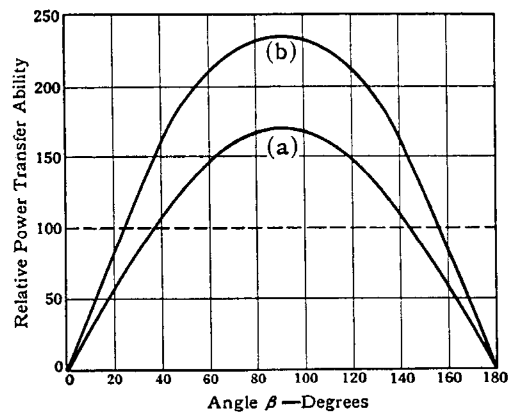

Fig. 35—The power-transfer ability of a tie feeder may be increased from curve (a) without series capacitors, to curve (b) with series capacitors.

which shows that for the same angle, a series capacitor effects a 40-percent increase in power-transfer ability—and also the maximum power that can be transferred. Furthermore, to transfer the same amount of power through the tie feeder, angle β is smaller, which aids stability of the system.

A series capacitor on a radial feeder is ineffectual unless the load power factor is lagging. This is not as important in most tie feeders as can be seen from Eqs. (19) and (20). Power transfer is affected primarily by the angle between the sending and receiving voltages and not as much by power factor.

33. System Power Factor Improved

The lagging kilovars supplied by a series capacitor improve system power factor, just as a shunt capacitor or an overexcited synchronous machine, but to a much smaller extent. In effect, the capacitor compensates for the I^2X_L "lost" in the feeder reactance. The amount of compensation varies, of course, as the square of the current since the kilovars supplied equal I^2X_C. At half load, for example, only one-quarter rated kilovars is provided.

34. Relative Effect of Power-Factor Correction

A shunt capacitor improves load voltage by neutralizing part of the lagging current in a circuit, thereby reducing the line current and voltage drop. A series capacitor improves load voltage more effectively by compensating directly for part of the feeder reactance, which causes the voltage drop. Consequently, the same voltage correction is obtained with a smaller rating of series capacitors than shunt, usually in the ratio of one half to one fourth. However, because the amount of power-factor correction increases with capacitor kvar rating, the shunt capacitor corrects power factor to a greater extent.

For example, on a 10 000-kva circuit having a load power factor of 80 percent and an R/X ratio of 0.3, 1100 kilovars of series capacitors are required to limit the voltage drop to 10 percent. This capacitor raises the source power factor from about 74 to about 78 percent. If a shunt capacitor is used in this circuit to obtain the same voltage correction, 3800 kilovars are required, but the source power factor is raised from 74 percent to 91 percent lagging.

To increase materially the source power factor as well as improve voltage, shunt capacitors at or near the load offer the best solution. Usually shunt capacitors must be switched in one or more groups to keep within desired voltage limits as load varies. Shunt capacitors do not reduce light flicker because they cannot be switched on and off fast enough to counteract rapid fluctuations in voltage.

VI. APPLICATION OF SERIES CAPACITORS

In general, series capacitors are applicable to radial circuits supplying loads of about 70 to 95 percent lagging power factor. Below 70 percent, shunt capacitors are more advantageous (unless the power factor changes over such a wide range, making it impossible to switch shunt capacitors fast enough to supply the kvar required by the load). Above 95 percent, the small value of sin θ limits the beneficial effect of series capacitors. Applications to radial circuits supplying loads of 70 to 90 percent power factor are most likely to be successful.

The application of series capacitors differs materially from that of shunt capacitors. Where voltage correction is the primary function of shunt capacitors the correction is obtained by raising the power factor of the load. To determine the shunt capacitor kvar required, the most important data needed are the magnitude of the load, its power factor and the impedance of the source circuit. While similar data are required for voltage correction with series capacitors, the effect of series capacitors is to reduce the reactance of the source circuit. Series capacitors affect power factor to a limited extent as compared with shunt capacitors because usually the kvar in a series capacitor is much smaller, being one-fourth to one-half of the shunt capacitor kvar for the same change in load voltage. In addition, the series capacitor contributes its kvar to the system as the square of the load current.

35. Determination of Capacitor Rating

A three-phase circuit containing a series capacitor consists of line resistance, line inductive reactance, and capacitive reactance. The kva ratings of these components are $3I^2R$, $3I^2X_L$, and $3I^2X_C$. These values as a percent of the total circuit rating are useful in considering the usefulness of series capacitors. The percent rating is obtained by dividing the kva rating of each element times 100 by the total circuit kva rating ($\sqrt{3}E_RI$) which must be known. The percent rating of the capacitor equals 300 $IX_C/\sqrt{3}E_R$ (or 173 IX_C/E_R) where I is full-load rating of the circuit and E_R is the load line-to-line voltage.

Calculation of kva ratings as a percent of circuit rating can be extended to voltage. The voltage drops, IR, IX_L, and IX_C times 100, are divided by the circuit voltage rating $E_R/\sqrt{3}$. The percent of the capacitor again equals 173 IX_C/E_R. Consequently, the percent ratings of each component on a kva base and on a voltage base are identical. Therefore, a series capacitor rated 20 percent on the base of circuit kva is also rated 20 percent on the base of circuit voltage. These ratings mean that at full load, the capacitor "consumes" 20 percent of rated circuit kva and the voltage drop across its terminals is 20 percent of rated circuit voltage.

The rating of a series capacitor (kilovars, voltage, and current) for a radial feeder depends on the desired voltage regulation, the load power factor, and the amount of resistance and reactance in the feeder relative to each other and to the circuit rating. The capacitor kilovar rating can be determined for 80 or 90 percent load power factor and 5 or 10 percent circuit voltage drop from data given in the curves of Fig. 36. To use these data, the feeder rating is taken as 100 percent kva and all other figures are calculated in percent on this base. For example, assume a 10 000-kva feeder having an inductive reactance of 20 percent and a ratio of resistance to reactance of 0.3 supplying a load whose power factor is 80 percent. From Fig. 36, to limit the voltage drop to 5 percent at full load, the series capacitor must be rated 20 percent of the circuit rating. This is 20 percent of 10 000 kva or 2000 kilovars. The capacitor voltage rating is also 20 percent of the rated circuit voltage.

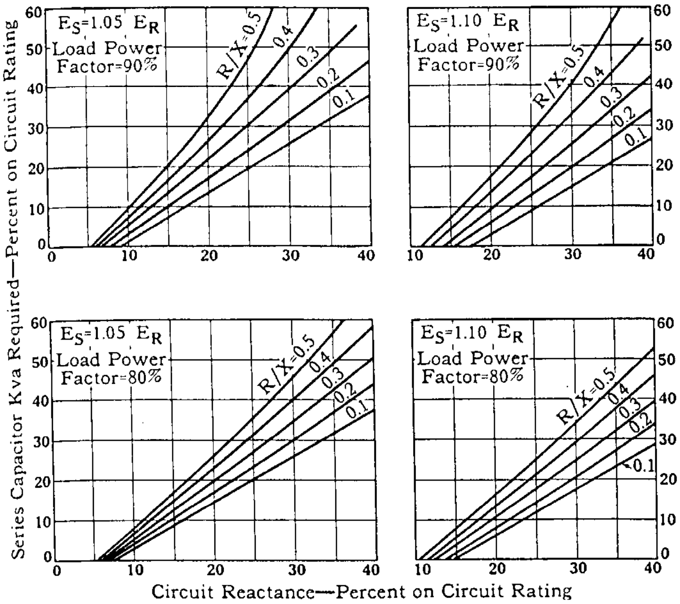

Fig. 36—Kilovar and voltage ratings of a series capacitor for a radial feeder may be determined for many cases from these curves.

Thus if the circuit is rated 20 000 volts, phase to neutral, the capacitor is rated 4000 volts. If a voltage regulation of 10 percent is permissible, only 1100 kilovars (at 2200 volts) are required. Had load power factor been 90 percent, 2200 kilovars (at 4400 volts) would be necessary for 5-percent regulation and 900 kilovars (at 1800 volts) for 10-percent regulation.

Other factors being equal, the ratio of R/X_L has a large effect on capacitor rating, as Fig. 36 indicates. Higher ratios require more capacitors; this is seen vectorially in Fig. 37.

The current rating of the capacitor equals that of the circuit because the bank must be able to carry rated circuit

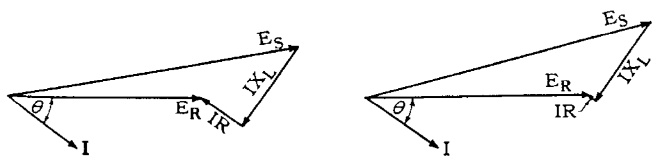

Fig. 37—Radial feeders having a higher ratio of resistance to reactance (for the same percent reactance) necessitate more capacitors.

current continuously. In addition, when circuits supply relatively large motors, the capacitor must be able to carry temporarily the starting current of the largest motor plus the current of other loads already in service. The total of the transient and steady-state currents through the capacitors should not exceed 1.5 times rated.

The rating of a series capacitor applied to a tie feeder is determined from a study of the power-transfer and stability requirements. No definite rules can be stated, but in general, the capacitive reactance of a series capacitor is less than (probably not more than 70 percent) the inductive reactance of the tie line. If the maximum transient current during a system disturbance is greater than about 1.5 times rated current, stability requirements rather than load transfer may dictate the capacitor rating for a tie-feeder application.

36. Arrangement of Capacitor Units

When the kilovar, voltage, and current ratings of the bank are known, capacitor units are arranged in series-parallel connections to obtain the desired values. Series connection builds up the voltage rating and parallel connections the current rating. Each bank must be "tailored" to fulfill the requirements of that specific application.

Capacitor banks can be assembled for any current rating and for almost any voltage rating, standard or non-standard. If the voltage across the bank is less than 230 volts, it may be economical to install a series step-up transformer to permit using standard capacitor units of higher voltage and lower cost.

37. Location of Capacitors

In general, a series capacitor can be located at any convenient place on a feeder provided that certain requirements are met. First, the voltage level at the output terminals of the bank must not be too high for the line insulation and lightning arresters; and second, a capacitor on a radial feeder must be located between the source and the load whose voltage is to be improved. Where a radial circuit has a number of tapped loads distributed throughout its length, the best location of the series capacitor is at about one third of the electrical impedance of the feeder from the source bus. If a feeder is long, two banks of capacitors may be preferable as more uniform voltage is obtained throughout the circuit. Where short-circuit current is high, it may be advisable to locate the capacitor so that fault current through the protective gaps and switches is a minimum.

A series capacitor located in a substation can be connected in each phase on the neutral ends of wye-connected transformer windings to permit use of a lower voltage class in the capacitor insulation. However, this practice raises the voltage-to-ground level of the transformer windings. This must be checked carefully. The effectiveness of a series capacitor is independent of whether it is connected on the neutral ends or on the line ends of the transformer windings.

VII. PROTECTION OF SERIES CAPACITORS

38. Protection During Line Fault

For most circuits in which series capacitors are applied, the currents and corresponding capacitor voltages during fault conditions are several times the maximum working value. As standard capacitor units can withstand about 200 percent of their rated working voltage for brief periods without damage to the dielectric, it is necessary to use capacitors with continuous current ratings equal to 50 percent of the maximum current that may flow during a fault, or to use a voltage-limiting device. For a given reactance, the cost of capacitors increases approximately as the square of the rated current so that it is more economical to use capacitors whose ratings are based on the working current and to limit the voltage that can appear across their terminals by means of auxiliary apparatus.

Care must be used that the voltage rating of the series capacitor and its associated protective equipment is made high enough so the capacitor is not by-passed during working loads. On radial circuits to insure availability of the series capacitor during motor starting currents, when its effects are most useful, the capacitor rating must be at least 67 percent of the greatest motor inrush current that may be imposed on the line plus other operating load. With protective devices set to by-pass the capacitor at 200-percent rating, the capacitor remains in service during such transient loads.

The device used to protect a series capacitor during a fault should limit the voltage rise to about twice the rated value even for a short time. The capacitor must therefore be by-passed during the first half cycle of fault current. A properly designed gap (shown in Fig. 38) fulfills this

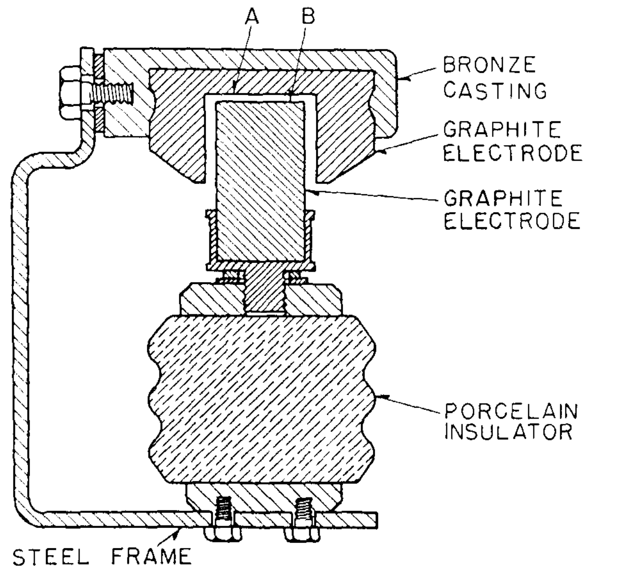

Fig. 38—Special gap for parallel protection of series capacitors.

requirement and materials can be selected to give a stable arc and a low arc drop without repetitive restriking. Under most conditions, some means must be provided for shunting this gap and transferring the arc current to another path. After the circuit current again falls to normal, the by-pass equipment must open to transfer current back through the capacitor. This commonly is done with a thermal or magnetic contactor or by an automatic circuit breaker that closes to by-pass the gap and capacitor and opens some time after the fault has cleared to restore the capacitor to service. If the installation consists of two or more groups of capacitor units in series within a bank, each can be protected by its own parallel gap.

Where the insulation class of the series capacitor is low, for example, where 230-volt capacitor units are used and the gap must break down at 460 volts, it is not possible to set the gap for sufficiently low breakdown voltage. In such instances, a trigger circuit is used to initiate the break-down of the gap.

On large series capacitors in transmission tie lines special gaps or special high-speed circuit breakers or both may be required to protect the capacitors and re-insert them into the circuit within a half cycle or a cycle after the fault is

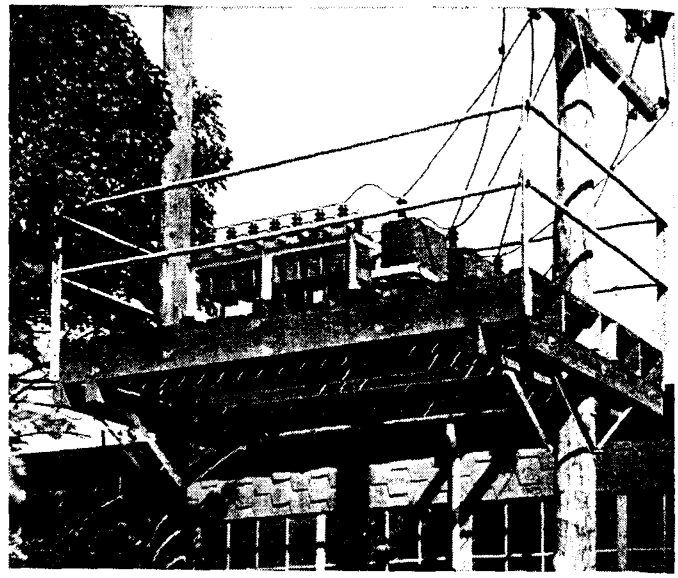

Fig. 39—A typical series capacitor on a distribution circuit.

cleared. This is necessary to enable the series capacitors to provide system stability. If the capacitors are not re-inserted within a cycle or less, their full benefit cannot be realized and their usefulness on tie lines would be reduced materially both electrically and economically.

39. Protection Against Continuous Overload

Standard series capacitors should not be used for continuous operation at an average more than 105 percent of their rated voltage. Consequently, average working current through a series capacitor should not exceed the rated working current by more than five percent. The short-circuit protective device is not designed to function at less than 200 percent of the rated current; therefore, it is sometimes desirable to provide overload as well as short-circuit protection. The overload protective device should have an inverse time-current characteristic that can be coordinated with the capacitor to allow momentary overloads but not continuous ones. Series capacitors have a 30-minute rating of 1.35 times rated current and a 5-minute rating of 1.5 times rated current. A thermally operated switch can also be used for this purpose.

This special type of protection usually is not warranted except on large series capacitor banks. The absence of overload protection on small distribution installations further emphasizes the need for care in choosing the continuous current rating.

40. Dielectric Failure Protection

Dielectric failure protection rarely is used except on large banks. This also is a feature that is sacrificed on small distribution series capacitors in the interest of simplicity and low first cost.

Dielectric protection is a means of detecting a faulted capacitor unit in a series-capacitor assembly. In an unfused capacitor bank a short-circuited capacitor may sustain an internal arc, which causes gas to be generated in the unit. Continued operation causes the internal pressure to reach a value that rupture the case and possibly damage other units and equipment. If the units are equipped with individual fuses—and they should be—a fuse operation to remove a faulted unit increases the reactance of the bank and operation at the rated current of the original bank subjects the remaining units to overvoltage. Protection is afforded by detecting with proper relaying when the currents become unequal in two equal branches of the capacitor. When the unbalance in current exceeds the selected value, the capacitor is by-passed until the defective unit is replaced.

41. Circuit Relaying

On radial circuits, fault-protection relaying usually is not affected by the addition of series capacitors. Fault currents practically always considerably exceed twice rated current. Consequently the parallel gap breaks down on the first half cycle of fault current. This happens faster than most types of relays operate and thus relay and circuit-breaker operations are the same as without capacitors. Relaying of line-to-ground faults is accomplished usually by residual or neutral current, which is not affected greatly by a series capacitor. Fault-protective relaying on a tie feeder, however, may be affected considerably by the installation of a series capacitor. Detailed studies must be made for each case prior to installation of the capacitor.

VIII. OPERATING PROBLEMS

Along with the desirable characteristics of series capacitors, there is the possibility of undesirable phenomena, usually involving some kind of resonance, which until recently has deterred the installation of large banks of series capacitors even where they otherwise could solve difficult system problems. In many cases the difficulties can be anticipated and suitable precautions taken to make an installation practical.

Three major phenomena may be encountered in a circuit employing a series capacitor: sub-synchronous resonance of a motor during starting, ferro-resonance of a transformer, and hunting of a motor during steady-state operation. One, two, or all of these may occur.

42. Sub-synchronous Resonance During Motor Starting

When an induction or a synchronous motor is started, (the latter as an induction motor) through a series capacitor, the rotor may lock in and continue to rotate at a speed below normal or synchronous. This condition is known as sub-synchronous resonance. It is caused by the capacitor, whose capacitive reactance in conjunction with the inductive reactance of the circuit and motor establishes a circuit resonant at a frequency below that of the power supply. The rotor, in effect, acts as a stable asynchronous generator. It receives power at rated frequency from the stator windings and transposes it to the sub-synchronous frequency, which it returns to the circuit containing the capacitor. This circuit, being resonant, imposes a minimum of impedance to the sub-synchronous voltage and consequently conducts a large current. A motor operating under these conditions may be damaged by excessive vibration or heating.

The sub-synchronous frequency is dependent on the relative sizes of the motor and the capacitor. The capacitor rating is determined by the circuit rating (other conditions remaining the same, the ratings are proportional). Consequently, the resonant frequency is related, indirectly, to the rating of the motor in proportion to that of the feeder. This frequency is usually 20 to 30 cycles for a 60-cycle motor whose rating equals half the circuit rating.

As the motor size decreases with respect to the capacitor and circuit ratings, its reactance increases. During resonance, capacitive and inductive reactance are equal. Because capacitive reactance increases with decreasing frequency, the sub-synchronous resonant frequency is lower when the motor requires a smaller proportion of the circuit rating. A motor requiring less than five percent of the circuit rating can be resonant at a sub-synchronous frequency of five cycles or less if it starts under load.

The most common method of preventing sub-synchronous resonance is to damp out this frequency by placing a resistor in parallel with the capacitor. While the resistance to use can be calculated, the results thus obtained are usually one half to one tenth the values that experience proves necessary. Calculations are inaccurate because of the difficulty of giving precise consideration to such variables as inertia of the motor and load, starting load, speed of acceleration, the type of starter, and other load on the circuit. For example, load elsewhere on the circuit, when a motor is started, reduces the possibility of sub-synchronous resonance by providing a damping effect similar to that of parallel resistance.

The resistance should be as high as possible in order to hold to a minimum its continuous losses, which are equal to the square of the voltage across the capacitor bank divided by the resistance. It is common practice, then, to apply resistors that are adjustable over a predetermined range, particularly in the larger installations.

When low ohmic resistance is used, the resistor can be disconnected after the motor reaches full speed and the risk of resonance has passed. Switching could be accomplished manually or by remote control over a pilot wire or power-line carrier channel with electrically-operated switching equipment.

Sub-synchronous resonance can also be avoided by use of parallel resistors across only two phases of a three-phase series capacitor. Such a solution is permissible where the omission of resistors from one phase does not unbalance the voltage appreciably. The amount of unbalance is determined by the resistance. The higher the resistance, the less the unbalance. But the resistance necessary, not the degree of unbalance, determines the ohmic value. At least one such installation is in service and is operating satisfactorily.

Sub-synchronous resonance can exist only during motor starting. Hence, resonance can be prevented by inserting resistance in series in the supply leads to the motor instead of in parallel with the capacitor. A contactor is required to short circuit the series resistance after the motor reaches full speed. If the circuit contains only a few motors such a scheme may be more economical than a single large resistance in parallel across the capacitors. To be effective, the series resistance must be in the stator circuit of the motor.

Resistance in the rotor circuit of a slip-ring motor does not give the desired damping but affects primarily the amount of slip between the sub-synchronous frequency and the frequency of the current through the rotor circuit.

If motors are started infrequently, sub-synchronous resonance can be avoided without using resistance by short circuiting the capacitor during starting. If a temporary unbalance is tolerable, the same result can be achieved in some cases by short circuiting only one phase of the bank, which simplifies the switching equipment.

The reactance of a capacitor is inversely proportional to frequency, while that of an inductor is directly proportional. Hence, in a series circuit consisting of capacitance and inductance the voltage drop across the former increases as frequency is reduced. Therefore, a condition of sub-synchronous resonance in a power circuit causes an increase in the voltage drop across the capacitor. This voltage may be large enough to cause the protective gap in parallel with the capacitor bank to flash over, thus short circuiting the capacitor. This halts the resonant condition and permits the motor to accelerate normally to full speed. After a time delay the capacitor is automatically restored to the circuit. This sequence of operations may make it possible in some installations (particularly where motors are started rarely) to use the gap alone to prevent sub-synchronous resonance and perhaps eliminate the need for parallel resistors. However, heavy-duty gaps in series with resistors to dissipate the energy stored in the capacitors may be required.

The gap is set to break down at twice rated current (twice rated voltage) at rated frequency. Consequently, during sub-synchronous resonance at half rated frequency the gap flashes over at rated current since the capacitive reactance is doubled. The lower the frequency the smaller the current required to break down the gap.

In general, the possibility of sub-synchronous resonance should be checked for all circuits in which the largest motor requires more than five percent of the circuit rating. Experience indicates that standard motors rated less than ten percent of circuit rating encounter no difficulty if started at no load. In fact, motors rated up to 20 percent usually accelerate satisfactorily if started at no load and across the line. However, when high-inertia loads are involved, the circuit must be checked for sub-synchronous resonance even if the power requirement of the largest motor is as low as five percent of the circuit rating.

43. Ferro-Resonance in Transformers

A transformer bank when energized draws a high transient exciting current. If a series capacitor is in the circuit, it may create a resonant condition that causes the high current to continue. This is known as ferro-resonance.

Ferro-resonance is cured automatically in most cases by the parallel gap. The magnetizing inrush current is probably of sufficient magnitude and low enough in frequency to cause a voltage drop to appear across the capacitor (and across the gap) high enough to break down the gap. As the transient period approaches its end, the current in the gap decreases. The steady-state current through the gap for a short period is usually too small to maintain the arc and therefore the gap clears, restoring the capacitor to the circuit. The possibility that the gap alone can prevent ferro-resonance is checked by oscillographic tests after the capacitor is installed.

If tests indicate that the gap is inadequate, ferro-resonance can be eliminated by shunting the capacitor with a resistor or by having a certain minimum load on the transformer side of the capacitor when the bank is energized. Of course, a parallel resistor applied to prevent sub-synchronous resonance of motors also prevents ferro-resonance of transformers.

In some cases, such as 2400- or 4160-volt circuits, the voltage rating of a series bank would be very low (and its cost high) if installed directly in the line. To permit application of a capacitor having a higher voltage rating, a transformer in series with the line is sometimes employed to step the voltage up from the required drop in the line to the capacitor rating. Such transformers must be designed carefully to prevent ferro-resonance.

A series capacitor, when installed in a long circuit supplying a transformer of abnormally high steady-state exciting current, may resonate during normal operation at a frequency corresponding to a harmonic component of the exciting current. Fluctuating loads may cause such resonance even though it does not appear when the transformer is energized. Resonance in this case is eliminated by a parallel resistor, by changing the transformer winding, or by replacing the transformer with another having a normal exciting current.

44. Hunting of Motors During Normal Operation

Hunting of a lightly-loaded synchronous motor can be caused by disturbances such as switching of power circuits and changes in load or excitation of the motor itself. Such hunting cannot be directly attributed to resonance. The principal factor in predicting hunting is the ratio of feeder resistance to total feeder reactance (including the series capacitor) between the power source and the motor terminals. If the ratio is less than one and is not negative, hunting is unlikely. Violent hunting of a synchronous motor was encountered upon application of a series capacitor in one instance because the ratio of feeder resistance to reactance was approximately four.

A synchronous motor, when fed through a long line excessively compensated by a series capacitor, may hunt if started during periods of light load. Such hunting is avoided if the power-factor angle of the load (after the motor is started) is equal to or greater than the impedance angle of the circuit (including the capacitor). The tangent of this impedance angle is the ratio of total circuit reactance (feeder reactance minus capacitor reactance) to feeder resistance.

Hunting is not limited to synchronous motors. Series capacitors should not be applied to circuits supplying either synchronous or induction motors driving reciprocating loads such as pumps or compressors. In addition to problems of sub-synchronous resonance, the motors once started may hunt, causing objectionable light flicker. The frequency of hunting is sometimes equal to, or a direct multiple of, the frequency of power pulsation, which further aggravates the situation. A cure for hunting may be the installation of a heavy flywheel to increase the

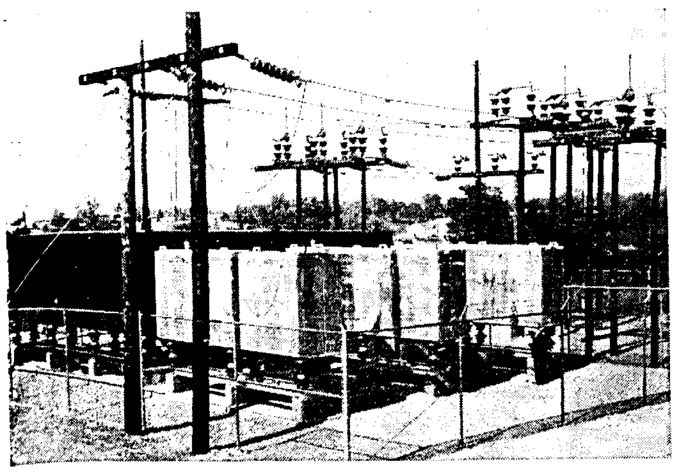

Fig. 40—10 000 Kvar series capacitor in 66-kv line showing large capacitor housings with somewhat smaller housings for parallel resistor units.

rotating mass. However, this solution may enhance the possibility of sub-synchronous resonance, which is equally undesirable.

IX. 10 000-KVAR SERIES CAPACITOR

A 10 000-kvar series capacitor is in service on a 66-kv radial circuit having a rated load of 500 amperes. Each phase of this capacitor bank consists of 240 standard 15-kvar, 2400-volt units, divided into 3 groups connected in series. Each group, which contains 80 units in parallel, is protected by its own gap and accompanying by-pass thermal switch.

This series capacitor, Fig. 40, was installed because the desired voltage improvement is obtained more efficiently and at less cost than by any other method. The principal function of the bank is to improve the voltage level and decrease flicker voltage at a steel plant where the bulk of the load consists of four 10 000-kva electric-arc furnaces. The heaviest load normally encountered is approximately 37 000-kva at about 78 percent power factor. The change in voltage conditions effected by the series capacitor is shown in Fig. 41, which indicates that the fluctuations are reduced and the average voltage level during periods of peak load is increased about 10 percent. Before installation of the capacitors, the voltage at the bus dropped from 12 000 volts at no load to 10 000 at full load. The full-load voltage is now about 11 300 volts. Furthermore, voltage conditions at the tapped point (Fig. 42), which was previously used only as an emergency supply to a nearby town, are so improved that this source now provides everyday power service.

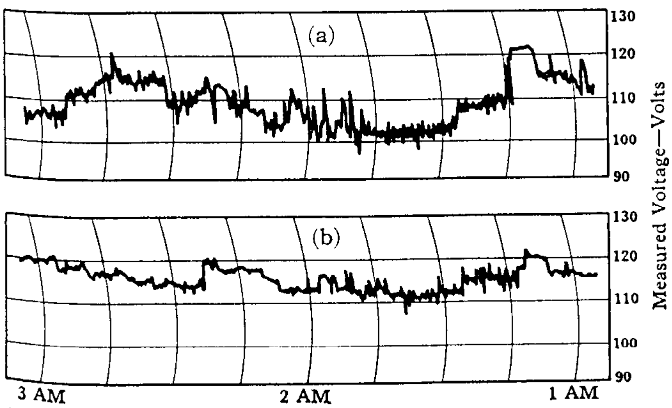

Fig. 41—Voltage conditions at the steel-mill bus (a) before and (b) after the series capacitor was installed.

The series capacitor compensates for 57 percent of the total circuit reactance up to the 11-kv steel-plant bus. This decreases by over 50 percent the magnitude of the change in voltage level during periods of heavy load and also reduces flicker voltage. However, the capacitor compensates for 100 percent of the total reactance up to the tapped load point. As a consequence the change in voltage level is reduced even more (about 80 percent) than at the steel-mill bus. Furthermore, the sudden fluctuations at the tapped point are almost entirely eliminated.

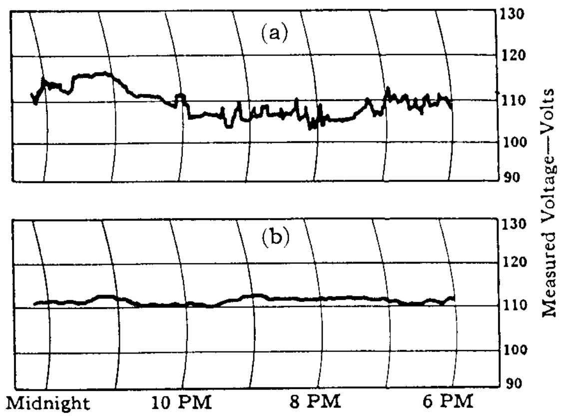

Fig. 42—Voltage conditions at tapped load point (a) before and (b) after the 10 000-kvar, 66-kv series capacitor was installed.

In addition to the furnace load the steel plant has several motors, the largest being a 4000-hp wound-rotor induction motor. A 400-ohm resistor across the capacitor gives sufficient damping for successful motor starting and prevents self-excitation or sub-synchronous resonance. Such phenomena sometimes occur when large motors (relative to the circuit rating) are started through a feeder containing a series capacitor. The resistor, because of its continuous losses, is undesirable but experience has indicated that it is essential for successful motor starting.

This large series capacitor has been successful. Had a synchronous condenser been installed at the load instead of a series capacitor, the initial cost would have been at least doubled and the continuous losses would have been much greater. The installed cost of such a capacitor is estimated to be about sixteen dollars per kilovar.

X. PROGRESS OF SERIES CAPACITORS

About 100 installations of series capacitors are in service on power circuits throughout the United States. The best results are obtained where there are no relatively large motors and where the capacitive reactance provided by the series capacitor is less than the inductive reactance of the circuit up to the principal load point.

Good results have been obtained with capacitors in circuits supplying electric-arc furnaces, one of the worst types of industrial loads. Series capacitors are ideal for resistance-welding devices where they can reduce the kva demand by 50 to 75 percent. Welders can be provided with built-in capacitors. If a series capacitor is applied to an existing welder, modifications to the welding transformer must be made to prevent excessive current flow.

While most of the improper operations of series capacitors are due to the fact that circuits with series capacitance resonate at some frequency, some trouble with protective devices has been encountered. But with new developments and information and experience gained on recent applications, more reliable performance is now expected. Some types of equipment should not be supplied through series capacitors because of difficulties that at present cannot be overcome. Overcompensation except in very special cases should be avoided as it produces undesirable results.

Twenty years ago shunt capacitors were used to a very limited extent. Today thay have been universally accepted as practical, reliable, and economical solutions to many problems involving voltage level, power-factor correction, equipment loading, etc. Many shunt capacitors rated over 5000 kva and a few over 10 000 kva are in operation. Undoubtedly the same evolution is now in process with series capacitors. Several series capacitors rated over 1000 kva and one installation of 10 000 kva have been installed. Perhaps the "ice" has been broken and other large installations will follow. Experience gained on the 10 000-kva installation certainly indicates that large series capacitors applied carefully are economical and successful in operation. Still further progress is likely to result from studies now being made on the application of large series capacitors to extra-high-voltage transmission lines. A large series capacitor is now being installed and tested in a 230-kv line in the Pacific Northwest.

REFERENCES

Shunt Capacitors

1. Capacitors Reduce Losses and Raise Voltage, W. H. Cuttino *Southern Power and Industry*, October 1941.
2. Use the Right Capacitor with Induction Motors, J. B. Owens, *Factory Management and Maintenance*, May 1945.
3. Safe Capacitor Selection for Power Factor Improvement, J. E. Barkle, *Power*, April 1943.
4. Uses of Capacitors, R. E. Marbury, *Electric Journal*, Vol. 33, July 1936, pp. 303–306.
5. Capacitors—Design, Application, Performance, M. C. Miller, *Electric Light and Power*, Vol. 16, October 1938, pp. 46–50.
6. Shunt Capacitors Reduce KVA Loads, C. M. Lytle and S. H. Pollock, *Electric Light and Power*, Vol. 15, November 1937, pp. 52–54.
7. Capacitors Defer $135,000 Investment in Synchronous Unit, J. F. Roberts, *Electrical West*, Vol. 83, October 1939, pp. 42–43.
8. Shunt Capacitor Application Problems, J. W. Butler, *General Electric Review*, Vol. 43, May 1940, pp. 206–212.
9. Current Control Broadens Capacitor Application, A. D. Caskey, *Electric Light and Power*, Vol. 18, February 1940, pp. 49–51.
10. Seventeen Systems Report Smooth Capacitor Performance, M. C. Miller, *Electrical World*, Vol. 113, January 27, 1940 pp. 289, 339–340.
11. Facilities for the Supply of Kilowatts and Kilovars, Hollis K. Sels and Theodore Seely, *A.I.E.E. Transactions*, Vol. 61, May 1942, p. 249.
12. Kilowatts, Kilovars, and System Investment, J. W. Butler, *A.I.E.E. Transactions*, Vol. 62, March 1943, pp. 133–137.
13. Mobile Capacitor Units for Emergency Loading of Transformers in Open Delta, H. B. Wolf and G. G. Mattison, *A.I.E.E. Transactions*, Vol. 62, February 1943, pp. 83–86.
14. The 13,500 KVAR Capacitor Installation at Newport News, E. L. Harder and V. R. Parrack, Southeastern Meeting of AIEE, Roanoke, Va., November 16, 1943.
15. Analysis of Factors Which Influence the Application, Operation, and Design of Shunt Capacitor Equipments Switched in Large Banks, J. W. Butler, AIEE Great Lakes District Meeting, Minneapolis, Minn., September 27–29, 1939.
16. Extending the Use of Shunt Capacitors by Means of Automatic Switching, W. H. Cuttino, AIEE Summer Meeting, St. Louis, Missouri, June 26–30, 1944.
17. Tests and Analysis of Circuit Breaker Performance When Switching Large Capacitor Banks, T. W. Schroeder, E. W. Boehne, and J. W. Butler, *A.I.E.E. Transactions*, Vol. 62, 1943, pp. 821, 831.
18. Automatic Switching Schemes for Capacitors, W. H. Cuttino, *A.I.E.E. Transactions*, Vol. 66, 1947.
19. Power Capacitors (Book), R. E. Marbury, McGraw-Hill Book Co., Inc., New York, N. Y., 1949.
20. The Why of a 25-KVAR Capacitor, M. E. Scoville, *General Electric Review*, Vol. 52, No. 5, May 1949.

Series Capacitors

21. Series Capacitors, R. E. Marbury and W. H. Cuttino, *Electric Journal*, March 1936.
22. Analysis of Series Capacitor Application Problems, J. W. Butler and C. Concordia, *A.I.E.E. Transactions*, Vol. 56, 1937, p. 975.
23. Series Capacitors for Transmission Circuits, E. C. Starr and R. D. Evans, *A.I.E.E. Transactions*, Vol. 61, 1942, p. 963.
24. Characteristics of 400-Mile 230-KV Series Capacitors, B. V. Hoard, *A.I.E.E. Transactions*, Vol. 65, 1946, p. 1102.
25. Design and Protection of 10,000-KVA Series Capacitor for 66-KV Transmission Line, A. A. Johnson, R. E. Marbury, J. M. Arthur, *A.I.E.E. Transactions*, Vol. 67, 1948.
26. 10 000 KVA Series Capacitor Improves Voltage in 66-KV Line Supplying Large Electric Furnace Load, B. M. Jones, J. M. Arthur, C. M. Stearns, A. A. Johnson, *A.I.E.E. Transactions*, Vol. 67, 1948.
27. Design and Layout of 66-KV 10 000-KVA Series Capacitor Substation, G. B. Miller, *A.I.E.E. Transactions*, Vol. 67, 1948.
28. New Series Capacitor Protective Device, R. E. Marbury and J. B. Owens, *A.I.E.E. Transactions*, Vol. 65, 1946, p. 142.
29. Self-Excitation of Induction Motors with Series Capacitors, C. F. Wagner, *A.I.E.E. Transactions*, Vol. 60, 1941 p. 1241.
30. Steady-State and Transient Stability Analysis of Series Capacitors in Long Transmission Lines, Butler, Paul, Schroeder, *A.I.E.E. Transactions*, Vol. 62, 1943, p. 58.
31. Series Capacitors Approach Maturity, A. A. Johnson, *Westinghouse ENGINEER*, Vol. 8, July 1948, pp. 106–111.
32. Application Considerations of Series Capacitors, A. A. Johnson, *Westinghouse ENGINEER*, Vol. 8, September 1948, pp. 155–156.

CHAPTER 9

REGULATION AND LOSSES OF TRANSMISSION LINES

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THIS chapter deals with problems relating to the performance of transmission lines under normal operating conditions. The analytical expressions for currents and voltages and the equivalent circuits for transmission lines are first developed for "short" lines and for "long" lines (where the effects of distributed line capacitance must be taken into account). A simplification is presented in the treatment of long lines that greatly clarifies their analysis and reduces the amount of work necessary for calculations. Problems relating to the regulation and losses of lines and their operation under conditions of fixed terminal voltages are then considered. The circle diagrams are developed for short lines, long lines, the general equivalent π circuit, and for the general circuit using *ABCD* constants. The circle diagrams are revised from the previous editions of the book to conform with the convention for reactive power which is now accepted by the American Institute of Electrical Engineers, so that lagging reactive power is positive and leading reactive power is negative.

When determining the relations between voltages and currents on a three-phase system it is customary to treat them on a "per phase" basis. The voltages are given from line to neutral, the currents for one phase, the impedances for one conductor, and the equations written for one phase. The three-phase system is thus reduced to an equivalent single-phase system. However, vector relationships between voltages and currents developed on this basis are applicable to line-to-line voltages and line currents if the impedance drops are multiplied by $\sqrt{3}$ for three-phase systems and by 2 for single-phase two-wire systems.

Most equations developed will relate the terminal conditions at the two ends of the line since they are of primary importance. These terminals will be called the sending end and receiving end with reference to the direction of normal flow of power, and the corresponding quantities designated by the subscripts *S* and *R*.

I. EQUIVALENT CIRCUITS FOR TRANSMISSION LINES

1. Short Transmission Lines

For all types of problems it is usually safe to apply the short transmission line analysis to lines up to 30 miles in length or all lines of voltages less than about 40 kv. The importance of distributed capacitance and its charging current varies not only with the characteristics of the line but also with the different types of problems. For this reason no definite length can be stipulated as the dividing point between long and short lines.

Neglecting the capacitance a transmission line can be treated as a simple, lumped, constant impedance,

$$Z = R + jX = zs = rs + jxs$$

Where

z = series impedance of one conductor in ohms per mile
r^* = resistance of one conductor in ohms per mile
x^* = inductive reactance of one conductor in ohms per mile
s = length of line in miles

The corresponding "per phase" or equivalent single-phase circuit is shown in Fig. 1 together with the vector diagram

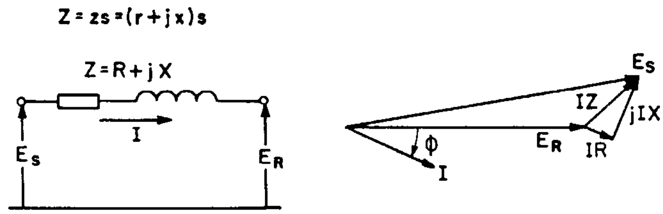

Fig. 1—Equivalent circuit and vector diagram for short transmission lines.

relating the line current and the line-to-neutral voltages at the two ends of the line.

The analytical expression for this relationship is given by the equation:

$$E_S = E_R + ZI \tag{1}$$

Throughout this chapter, the following symbols are used:

E—is a vector quantity
$\bar{E}$—is the absolute magnitude of the quantity
$\hat{E}$—is the conjugate of the vector quantity

2. Long Transmission Lines

The relative importance of the charging current of the line for all types of problems varies directly with the voltage of the line and inversely with the load current. To appreciate this fully it is necessary to consider the analysis of "long" lines.

A "long" transmission line can be considered as an infinite number of series impedances and shunt capacitances connected as shown in Fig. 2. The current I_R is unequal to I_S in both magnitude and phase position because some current is shunted through the capacitance between phase

*These quantities can be obtained from the tables of conductor characteristics of Chap. 3.

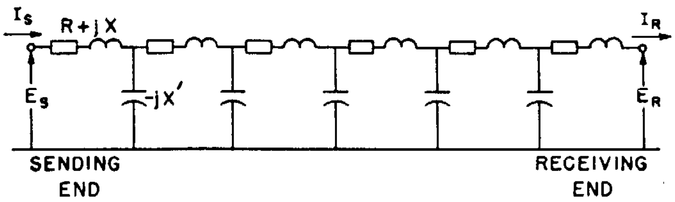

Fig. 2—Diagram representing long transmission lines.

and neutral. The relationship between E_S and E_R for a "long" line is different from the case of the short line because of the progressive change in the line current due to the shunt capacitance. If E_S and E_R are considered as phase-to-neutral voltages and I_S and I_R are the phase currents, the classical equations relating the sending-end voltages and currents to the receiving-end quantities are:

$$E_S = E_R \cosh (s\sqrt{zy}) + I_R\sqrt{\frac{z}{y}} \sinh (s\sqrt{zy}) \tag{2}$$

$$I_S = \frac{E_R}{\sqrt{\frac{z}{y}}} \sinh (s\sqrt{zy}) + I_R \cosh (s\sqrt{zy}) \tag{3}$$

The susceptance, y, heretofore has been used most frequently in these expressions. However, with the advent of the new form of tables giving characteristics of conductors, the shunt-capacitive reactance is obtained as a function of the conductor size and equivalent spacing. The reciprocal of y, which is x' is therefore a more convenient quantity to use. For this reason the concept of shunt-capacitive reactance is used throughout this chapter. Eqs. (2) and (3) then become:

$$E_S = E_R \cosh\left(s\sqrt{\frac{z}{z'}}\right) + I_R\sqrt{zz'} \sinh\left(s\sqrt{\frac{z}{z'}}\right) \tag{4}$$

$$I_S = \frac{E_R}{\sqrt{zz'}} \sinh\left(s\sqrt{\frac{z}{z'}}\right) + I_R \cosh\left(s\sqrt{\frac{z}{z'}}\right) \tag{5}$$

where z is the series impedance of one conductor in ohms per mile, z' is the shunt impedance of the line in ohms per mile, s is the distance in miles.

$z' = -jx'(10)^6$

x'* = capacitive reactance in megohms per mile.

Equations (4) and (5) can be written conveniently in terms of the conventional $ABCD$ constants.[4] For the case of a transmission line the circuit is symmetrical and D is equal to A. (Refer to Chapter 10, Section 21 for definition of $ABCD$ constants.)

$$E_S = AE_R + BI_R \tag{6}$$

$$I_S = CE_R + DI_R = CE_R + AI_R \tag{7}$$

$$E_R = AE_S - BI_S \tag{8}$$

$$I_R = -CE_S + DI_S = -CE_S + AI_S \tag{9}$$

where

$$A = \cosh\left(s\sqrt{\frac{z}{z'}}\right) = \cosh\sqrt{\frac{Z}{Z'}} \tag{10}$$

*This quantity can be obtained from the tables of conductor characteristics in Chap. 3. It is given in megohms in tables as it is then of the same order of magnitude as the inductive reactance.

in which

$$Z = zs \text{ and } Z' = \frac{z'}{s}.$$

$$B = \sqrt{zz'} \sinh\left(s\sqrt{\frac{z}{z'}}\right) = \sqrt{ZZ'} \sinh\sqrt{\frac{Z}{Z'}} \tag{11}$$

$$C = \frac{1}{\sqrt{zz'}} \sinh\left(s\sqrt{\frac{z}{z'}}\right) = \frac{1}{\sqrt{ZZ'}} \sinh\sqrt{\frac{Z}{Z'}} \tag{12}$$

The values of the hyperbolic functions can be obtained from tables[2] or charts[3] or from evaluation of their equivalent series expressions

$$\cosh\left(s\sqrt{\frac{z}{z'}}\right) = \cosh\theta = \left(1 + \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \frac{\theta^6}{6!} + \cdots\right) \tag{13}$$

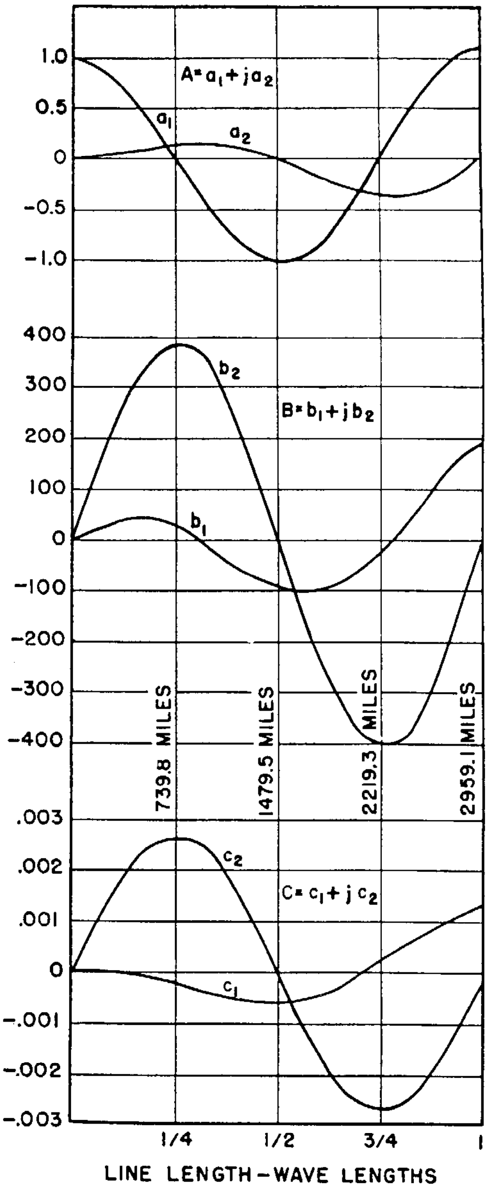

Fig. 3—Variation of the real and imaginary components of A, B, and C for a 795 000 circular mils ACSR, 25-foot equivalent spacing, transmission line.

$r = 0.117$ ohm per mile.
$x = 0.7836$ ohm per mile.
$x' = 0.1859$ megohm per mile.

$$\sinh\left(s\sqrt{\frac{z}{z'}}\right)=\sinh\theta=\left(\theta+\frac{\theta^3}{3!}+\frac{\theta^5}{5!}+\frac{\theta^7}{7!}+\cdots\right) \tag{14}$$

Expressed in terms of their equivalent series expansions, the ABC constants become

$$A=\left[1+\frac{Z}{2Z'}+\frac{Z^2}{24Z'^2}+\frac{Z^3}{720Z'^3}+\frac{Z^4}{40\,320Z'^4}+\cdots\right] \tag{15}$$

$$B=Z\left[1+\frac{Z}{6Z'}+\frac{Z^2}{120Z'^2}+\frac{Z^3}{5040Z'^3}+\frac{Z^4}{362\,880Z'^4}+\cdots\right] \tag{16}$$

$$C=\frac{1}{Z'}\left[1+\frac{Z}{6Z'}+\frac{Z^2}{120Z'^2}+\frac{Z^3}{5040Z'^3}+\frac{Z^4}{362\,880Z'^4}+\cdots\right] \tag{17}$$

The series are carried out far enough so that the ABC constants can be determined to a high degree of accuracy. However, for lines approaching one quarter wave length, the series do not converge rapidly enough. In such a case it is better to determine the ABC constants for the line in two sections and combine them as described in Chapter 10, Table 9.

The ABC constants can be determined easily for any length of line by an evaluation of the cosh and sinh functions using the hyperbolic and trigonometric functions. The procedure is outlined briefly here.

$$\theta=s\sqrt{\frac{z}{z'}}=\alpha+j\beta$$

where α and β are in radians.

$$\cosh\theta=\cosh\alpha\cos\beta+j\sinh\alpha\sin\beta$$
$$\sinh\theta=\sinh\alpha\cos\beta+j\cosh\alpha\sin\beta$$

where:

$$\cosh\alpha=\frac{\epsilon^{\alpha}+\epsilon^{-\alpha}}{2}$$

$$\sinh\alpha=\frac{\epsilon^{\alpha}-\epsilon^{-\alpha}}{2}$$

Figure 3 shows the variation of the ABC constants as a function of line length for the line of Fig. 18. The real and imaginary parts of A, B, and C are shown for a complete wave length.

3. The Equivalent π of a Transmission Line

There are several equivalent circuits that represent the above transmission line equations and thus can be used for the representation of transmission lines. One such circuit is the equivalent π shown in Fig. 4.

Referring to this figure the equations relating the terminal conditions for this circuit are

$$I_R'=\frac{E_R}{Z'_{eq}}$$

$$E_S=E_R+Z_{eq}\left(I_R+\frac{E_R}{Z'_{eq}}\right)$$

$$E_S=E_R\left(1+\frac{Z_{eq}}{Z'_{eq}}\right)+Z_{eq}I_R \tag{18}$$

A B

$$I_S'=\frac{E_S}{Z'_{eq}}$$

$$I_S=I_R+I_R'+I_S'=\frac{E_R}{Z'_{eq}}+\frac{E_S}{Z'_{eq}}+I_R$$

$$I_S=\left(\frac{2}{Z'_{eq}}+\frac{Z_{eq}}{Z'^2_{eq}}\right)E_R+\left(1+\frac{Z_{eq}}{Z'_{eq}}\right)I_R \tag{19}$$

C D = A

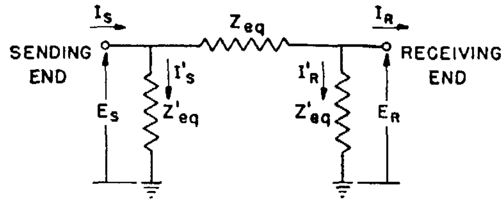

Fig. 4—Equivalent π circuit for representing long transmission lines.

By equating like coefficients of the equivalent Eqs. (18) and (6)

$$Z_{eq}=B \tag{20}$$

$$1+\frac{Z_{eq}}{Z'_{eq}}=A \tag{21}$$

Giving for the equivalent impedance Z'_{eq}

$$Z'_{eq}=\frac{B}{A-1} \tag{22}$$

Expressed in terms of the corresponding hyperbolic functions and their equivalent series the equations for the impedances are

$$Z_{eq}=\sqrt{ZZ'}\sinh\sqrt{\frac{Z}{Z'}}=Z\left(1+\frac{Z}{6Z'}+\frac{Z^2}{120Z'^2}+\frac{Z^3}{5040Z'^3}+\frac{Z^4}{362\,880Z'^4}\right) \tag{23}$$

$$Z'_{eq}=\frac{\sqrt{ZZ'}\sinh\sqrt{\frac{Z}{Z'}}}{\left(\cosh\sqrt{\frac{Z}{Z'}}\right)-1}=2Z'\left(1+\frac{Z}{12Z'}-\frac{Z^2}{720Z'^2}+\frac{Z^3}{30\,240Z'^3}-\frac{Z^4}{1\,207\,600Z'^4}+\cdots\right) \tag{24}$$

4. Equivalent T of a Transmission Line

Another equivalent circuit for a transmission line is shown in Fig. 5. The equations for the impedances of this circuit are

$$Z_T=\frac{A-1}{C}=\frac{Z}{2}\left(1-\frac{Z}{12Z'}+\frac{Z^2}{120Z'^2}-\frac{17Z^3}{20\,160Z'^3}+\frac{31Z^4}{3\,628\,800Z'^4}-\cdots\right) \tag{25}$$

$$Z'_T=\frac{1}{C}=Z'\left(1-\frac{Z}{6Z'}+\frac{7Z^2}{360Z'^2}-\frac{31Z^3}{15120Z'^3}+\frac{127Z^4}{604\,800Z'^4}-\cdots\right) \tag{26}$$

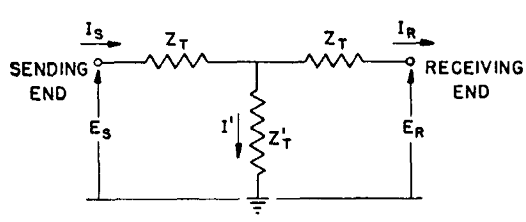

Fig. 5—Equivalent T circuit for representing long transmission lines.

5. Comparison of the Equivalent π vs. ABCD Constants

The choice of the use of the equivalent π vs. $ABCD$ constants in calculating transmission-line constants is largely a matter of personal preference. However, each offers certain advantages over the other. When the network calculator is to be used, it is necessary to set up an actual circuit in the form of the equivalent π. The equivalent π affords a better physical picture of transmission-line performance and makes the comparison between long and short lines and the effect of charging current easier to visualize.

On the other hand, when a problem is to be solved analytically, the use of $ABCD$ constants has a definite advantage over the equivalent π because of the availability of the independent check: $AD-BC=1$. This is particularly desirable when other circuits are to be combined with the transmission line circuit.

The equivalent π or $ABCD$ constants can be used to represent any line, section of line, or combination of lines and connected equipment. Either one represents accurately all conditions at the two terminals of the system. The equivalent circuit or $ABCD$ constants being considered here pertains only to a single line or line section. The general equivalent circuit and general $ABCD$ constants, if so desired, can be determined by the combination of the equivalent circuits for the rest of the system as discussed in Chapter 10.

6. Expressions for Transmission Line Constants by First Two Terms of Their Series

When considering the accuracy with which transmission line circuit constants need be determined, it should be realized that the resistance, inductance, and capacitance of a line can rarely be known to within 3 or 4 percent and probably never within one per cent. This is due to conductor sag, its variation with different spans, and the variation that exists in conductor spacing together with the effects of temperature upon conductor resistivity and sag. For this reason equations for the above circuit constants that are accurate to within 0.5 percent should be satisfactory.

The effect of neglecting all but the first two terms of the series in the above expressions can best be shown by considering an actual line. For a 300-mile line with 250 000 circular mil stranded copper and a 35-foot spacing the third term in all of the above series expressions is larger than normal.

For this line, from the conductor tables of Chap. 3

$$r=0.257 \text{ ohms per mile}$$

$$x=x_a+x_d=0.487+0.431=0.918 \text{ ohms per mile}$$

$$x'=x_a'+x_d'=0.111+0.106=0.217 \text{ megohms per mile}$$

$$Z=rs+jxs=(77.1+j275.4) \text{ ohms}$$

$$Z'=-j\frac{x'10^6}{s}=-j723.3 \text{ ohms}$$

$$\frac{Z}{Z'}=\frac{77.1+j275.4}{-j723.3}=-0.3807+j0.1066$$

$$\frac{Z^2}{Z'^2}=0.1335-j0.08117$$

For the third term in the series expression for A

$$\frac{Z^2}{24Z'^2}=0.0056-j0.0034$$

This term is thus about 0.6 percent of one (the first term).

For the third term in the expression for Z_T.

$$\frac{Z^2}{120Z'^2}=0.0011-j0.00067$$

which is about 0.1 percent of one (the first term).

For all the rest of the constants the term is less than 0.1 percent.

Since these terms vary with the fourth power of the length of the line, they decrease rapidly for lines less than 300 miles in length and can be neglected. For instance for a 150-mile line the terms are one-sixteenth as large as for a 300-mile line.

Thus the above transmission line constants can be expressed sufficiently accurately by the following equations which were derived from Eqs. (15), (16), (17), (23), (24), (25), and (26) by neglecting all but the first two terms of each series expression.

$$A=\left(1-\frac{xS^2}{200x'}\right)+j\frac{rS^2}{200x'} \tag{27}$$

$$B=Z_{eq}=100rS\left(1-\frac{xS^2}{300x'}\right)+j100xS\left(1-\frac{xS^2}{600x'}+\frac{r^2S^2}{600xx'}\right) \tag{28}$$

$$C=\frac{jS}{x'}\left[\left(1-\frac{xS^2}{600x'}\right)+j\frac{rS^2}{600x'}\right]10^{-4} \tag{29}$$

$$Z'_{eq}=-j\frac{2x'}{S}\left[\left(1-\frac{xS^2}{1200x'}\right)+j\frac{rS^2}{1200x'}\right]10^4 \tag{30}$$

$$Z_T=50rS\left(1-\frac{xS^2}{1200x'}\right)+j50xS\left(1-\frac{xS^2}{1200x'}+\frac{r^2S^2}{1200xx'}\right) \tag{31}$$

$$Z_T'=-j\frac{x'}{S}\left[\left(1+\frac{xS^2}{600x'}\right)-j\frac{rS^2}{600x'}\right]10^4 \tag{32}$$

In these equations:

S = length of line in hundreds of miles.

x and r are in ohms per mile, and x' in megohms per mile.

7. Simplified Method of Determining the Impedances of the Equivalent π Circuit for Transmission Lines

The following method greatly simplifies the determination of the impedances of the equivalent π circuit and still enables them to be determined to within 0.5 percent for all practical power transmission lines.

Equations (28) and (30) can be expressed in the following form:

$$Z_{eq}=100rSK_r+j100xSK_x \tag{33}$$

$$Z'_{eq}=-j\frac{2x'}{S}(k_r+jk_x)10^4 \tag{34}$$

where

$$K_r=1-\frac{xS^2}{300x'} \tag{35}$$