Electrical Transmission and Distribution Reference Book

by
Central Station Engineers
of the
Westinghouse Electric Corporation
EAST PITTSBURGH, PENNSYLVANIA

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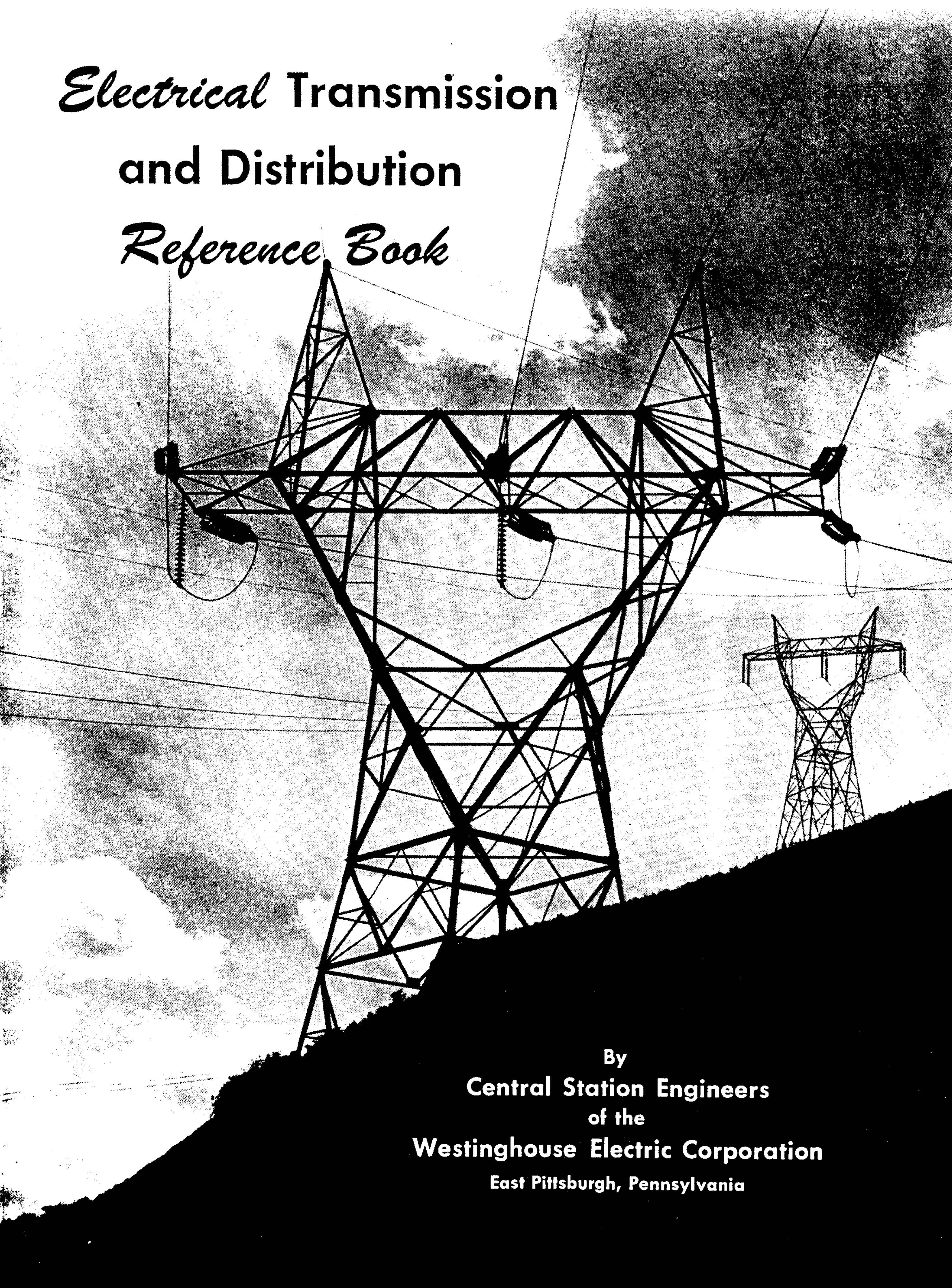

By

Central Station Engineers

of the

Westinghouse Electric Corporation

East Pittsburgh, Pennsylvania

This book is dedicated to the memory of

ROBERT D. EVANS

who contributed so greatly to the
transmission and distribution of electric power
and to the preparation of
the original edition of this book

Preface to fourth edition

Some thirty years ago a well-known electrical engineer was ordered by his physician to take a complete rest. During this period, as a diversion, he began to study transmission-line calculations. Out of that came, in 1922, a book that was quickly recognized as a classic on the subject because it was simple, practical, useful. The man was William Nesbit; the book, "Electrical Characteristics of Transmission Circuits."

In the two succeeding decades power-transmission systems grew tremendously in complexity. Voltages were doubled, longer lines were built, interconnections became more extensive, knowledge of how to protect against lightning was greatly increased, and new methods of calculating performances were adopted. With all this grew the need for a new book on transmission lines, one of broader scope that would meet the new conditions, but retain the entirely practical viewpoint of its predecessor. Fourteen men, all connected with the Central Station Engineering Group of the Westinghouse Electric Corporation, undertook to produce such a book. All of these men worked daily on actual problems such as are considered here. With this background of experience and with the reputation of the Nesbit book as inspiration, they presented in January, 1942 the first edition of a book which they hoped would be useful to all concerned with electric-power transmission as a practical reference book, helpful in solving everyday problems.

In 1943 a second edition was brought out in which two chapters that discussed the general features of the electrical distribution problem were added at the end of the book. The third edition differed from the second edition only in that the two chapters were introduced just before the appendix.

A fourth and completely rewritten edition is presented herewith. It contains essentially the material of the previous three editions, sometimes with new authors, and three new chapters—Excitation Systems, Application of Capacitors to Power Systems, and Power Line Carrier Application. As before, all of the authors are from the

Central Station Section or are closely associated with it. As was the case with previous editions, this one also bears the imprint of two outstanding engineers, who contributed so much to the transmission of power, Dr. Charles L. Fortescue and Mr. Robert D. Evans. The latter, before his recent death, was one of the active participants in the previous editions. The name or names of the original authors and the revising authors appear at the head of each chapter.

To conform to the original standards regarding the sign of reactive power, the authors in the first edition of this book found it necessary to change the curves and discussions from what they had used in their previous publications. With the recent change in the standards, the sign has again been changed so that the curves and discussions now use lagging kvar as positive.

The material presented here is naturally the results of research and investigations by many engineers. It is not feasible to list here the names of the companies and individuals whose work has been summarized. These acknowledgments are given in the individual chapters. Much of the material used has been the result of cooperative studies of mutual problems with engineers of electric-power companies, the conductor and cable manufacturers, and the communication companies. The authors gratefully acknowledge the hearty cooperation of those engineers whose work has assisted in the preparation of this book. The title page photograph is reproduced by permission of the Bureau of Reclamation, Grand Coulee, Washington. The acknowledgments would be incomplete without giving recognition to the fine cooperation of the editorial staff of the *Westinghouse* ENGINEER, in reviewing the material and making many helpful suggestions to the authors and to Mr. Raymond W. Ferguson, who assisted in editing the material.

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Vice President in Charge of Engineering

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September 1, 1950

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CHAPTER 1

GENERAL CONSIDERATIONS OF TRANSMISSION

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THROUGH discovery, invention, and engineering application, the engineer has made electricity of continually greater use to mankind. The invention of the dynamo first made engine power many times more effective in relieving the toil and increasing the opportunities and comforts not only of industry but also of the home. Its scope, however, was limited to relatively short distances from the power station because of the low voltage of the distribution circuits. This limitation, for economic reasons, kept the general use of electricity confined to city areas where a number of customers could be served from the same power station. The next step in the development of the present-day electric systems was the invention of the transformer. This invention was revolutionary in its effect on the electric industry because it made high voltage and long transmission distances possible, thus placing the engine power, through the medium of the alternating-current generator, at the doorstep of practically everyone.

The first alternating current system in America using transformers was put in operation at Great Barrington in Massachusetts in 1886. Mr. William Stanley, Westinghouse electrical expert who was responsible for the installation, gives an account of the plant, part of which reads:

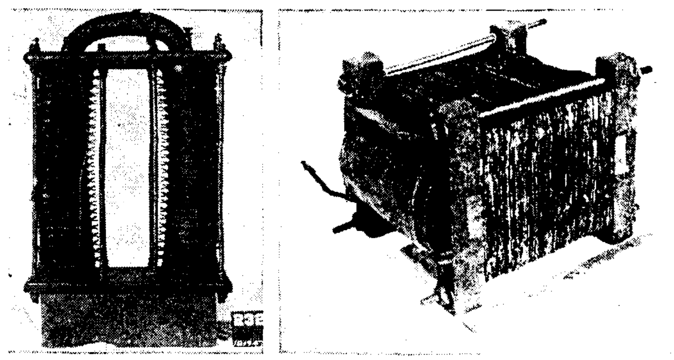
"Before leaving Pittsburgh I designed several induction coils, or transformers as we now call them, for parallel connection. The original was designed in the early summer of 1885 and wound for 500 volts primary and 100 volts secondary emf. Several other coils were constructed for experimental purposes.

"At the north end of the village of Great Barrington was an old deserted rubber mill which I leased for a trifling sum and erected in it a 25 hp boiler and engine that I purchased for the purpose. After what seemed an interminable delay I at last installed the Siemens alternator that Mr. Westinghouse had imported from London. It was wound to furnish 12 amperes of current with a maximum of 500 volts. In the meantime I had started the construction of a number of transformers in the laboratory and engaged a young man to canvass the town of Great Barrington for light customers. We built in all at Great Barrington 26 transformers, 10 of which were sent to Pittsburgh to be used in a demonstration plant between the Union Switch and Signal Company's factory* and East Liberty.

"We installed in the town plant at Great Barrington two 50-light and four 25-light transformers, the remainder being used in the laboratory for experimental work. The transformers in the village lit 13 stores, 2 hotels, 2 doctors' offices, one barber shop, and the telephone and post offices. The length of the line from the laboratory to the center of the town was about 4000 feet."

Our central-station industry today is, for all practical purposes, entirely alternating current. It can, therefore, be said to have grown from the small beginning at Great Barrington to its present size involving as it does a capitalization in the privately-owned power companies of some 17 billion dollars with an annual revenue of 4 billion dollars.

The growth since the beginning of this century in installed generating capacity of all electric power plants



(a) (b)

Fig. 1—(a) Gaulard and Gibbs transformer for which George Westinghouse had secured all rights in the United States. (b) First transformer designed by William Stanley. The prototype of all transformers since built, it definitely established the commercial feasibility of the alternating-current system, 1884-1886.

contributing to the public supply has been from about $1\frac{1}{2}$ million kilowatts to 55 million kilowatts in 1948. Of this 55 million kilowatts the privately-owned utilities accounted for 44 million kilowatts and government-owned utilities for 11 million kilowatts divided equally between the federal government and local governments. Thus, 80 percent of the generating capacity of the country is privately owned and 20 per cent government owned.

With this 55 million kilowatts of generating capacity, 282 billion kilowatt-hours, divided 228 billion kilowatt-hours by privately-owned generation and 54 billion public, were generated in 1948. The average use of the installed capacity for the country as a whole was, therefore, $\frac{282\,000}{55} = 5130$ hours, and the capacity factor for the country as a whole $\frac{5130}{8760} = 58.5$ percent.

This capacity factor of 58.5 percent is generally conceded as being too high. It does not allow sufficient margin to provide adequate spare capacity for maintenance and repairs. Fig. 2 illustrates how the spare and reserve capacity has shrunk in the past few years. A ratio of installed capacity to peak load of 1.15 to 1.20 is considered necessary to provide a safe margin for emergencies. Such

*About two miles.

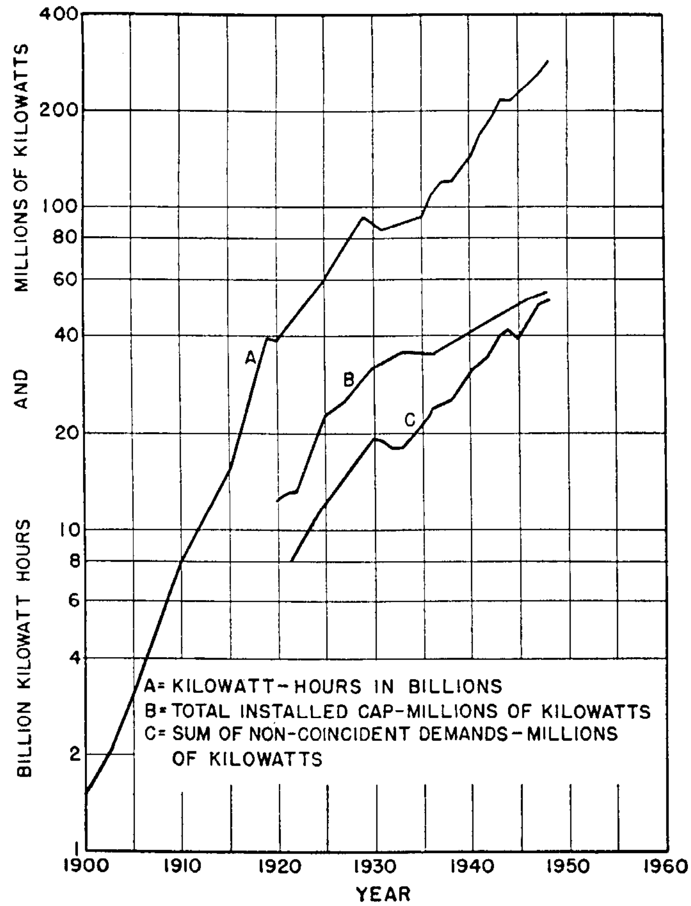


Fig. 2—Trend in production of electricity, installed capacity, and sum of peak demands.

a margin in 1948 would have given a capacity-factor of about 53 percent, instead of 58.5 percent.

The average cost of all electricity used for residential service has shown a steady downward trend since 1925 from 7 cents per kilowatt-hour to 3 cents in 1948. This is all the more remarkable as since 1939 all other items making up the cost-of-living index have shown increases ranging from 10 percent (for rents) to 121 percent (for food), the average increase of all items being 69 percent. The revenue from sales to residential customers accounts for about 36 percent of the total utility revenue; to large power customers about 29 percent; to small light and power customers 27 percent, and to miscellaneous customers (railroads, street lighting, etc.) 8 percent.

1. Sources of Energy

The sources of energy for large-scale generation of electricity are:

1. Steam, from (a) coal, (b) oil, or (c) natural gas
2. Water (hydro-electric)
3. Diesel power from oil

Other possible sources of energy are direct solar heat, windpower, tidal power, shale oil, and atomic energy, but none of these as yet has gone beyond the pilot-plant stage, for the reason that coal and petroleum are still abundantly available. But as fossil fuels become scarcer and more expensive, there is every reason to believe that all of these, as well as petroleum manufactured from vegetable matter, may become useful and economical supplementary sources of energy.

The estimated reserves of coal and lignite in the United States are about 3000 billion tons. This constitutes almost 99 percent of the mineral fuel energy reserves of the country; oil shale, petroleum and natural gas amounting to little more than 1 percent.[1]

By far the greater part of the electric energy generated in this country is obtained from fuel, the 55 million kilo-

Table 1—Preferred Standards for Large 3600-RPM 3-Phase 60-Cycle Condensing Steam Turbine-Generators

		Air-Cooled Generator	Hydrogen-Cooled Generators Rated for 0.5 Psig Hydrogen Pressure						
Turbine-generator rating, kw		11 500	15 000	20 000	30 000	40 000	60 000	90 000*	
Turbine capability, kw		12 650	16 500	22 000	33 000	44 000	66 000	99 000	
Generator rating, kva		13 529	17 647	23 529	35 294	47 058	70 588	105 882	
power factor		0.85	0.85	0.85	0.85	0.85	0.85	0.85	
short-circuit ratio		0.8	0.8	0.8	0.8	0.8	0.8	0.8	
Throttle pressure, psig		600	850	850	850	850 or 1250	850 or 1250	1450 or	1450**
Throttle temperature, F		825	900	900	900	900 or 950	900 or 950	1000	1000
Reheat temperature, F		. . .	. . .	. . .	. . .	. . .	. . .	. . .	1000
Number of extraction openings		4	4	4	5	5	5	5	5
Saturation temperatures at	1st	175	175	175	175	175	175	180	175
openings at "turbine-generator rating" with all ex-	2nd	235	235	235	235	235	235	245	240
	3rd	285	285	285	285	285	285	305	300
traction openings in serv-	4th	350	350	350	350	350	350	380	370
ice, F	5th	. . .	. . .	. . .	410	410	410	440	440
Exhaust pressure, inches Hg abs		1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
Generator capability at 0.85 power factor and 15 psig hydrogen pressure, kva		. . .	20 394	27 058	40 588	54 117	81 176	121 764	
Generator capability at 0.85 power factor and 30 psig hydrogen pressure, kva		. . .	. . .	. . .	. . .	. . .	. . .	132 353	

*A 10 percent pressure drop is assumed between the high pressure turbine exhaust and low pressure turbine inlet for the reheat machine.
**These are two different units; the first for regenerative cycle operation, and the second a machine for reheat cycle operation.

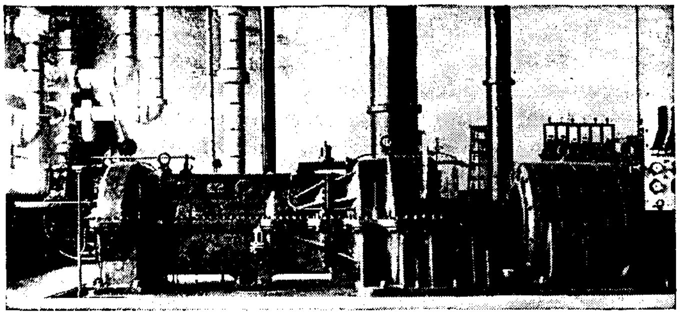


Fig. 3—The first central-station turbo-alternator installation in the United States—a 2000-kw turbine coupled to a 60-cycle generator, 2000 kw, 2400 volts, two-phase, 1200 rpm—at the Hartford Electric Light Company, Hartford, Connecticut, 1900. This turbine was about four times as large as any one built before that time and caused much comment the world over.

watts of installed capacity being made up of approximately 38 million kilowatts of steam turbines and one million kilowatts of diesel engines. Approximately 16 million kilowatts of the installed capacity are in hydro-electric stations. Of the 282 billion kilowatt-hours generated by all means in 1948, roughly 200 billion came from fuel; 76 percent from coal, 14 percent from natural gas, and 10 percent from oil.

2. Development of Steam Power

The modern steam-electric station can be dated from the installation by the Hartford Electric Company in 1900 of a 2000-kw unit (Fig. 3) which at that time was a large machine. Progress in design and efficiency from then on has been continuous and rapid. In 1925 the public utilities consumed in their fuel-burning plants an average of 2 pounds of coal (or coal equivalent) per kilowatt-hour, whereas today the corresponding figure is 1.3 pounds per kilowatt-hour. This average figure has not changed materially in the last 10 years. It would appear that the coal consumption curve is approaching an asymptote and that a much better overall performance is not to be expected, even though the best base-load stations generate power for less than one pound of coal per kilowatt-hour. The very high efficiency in the best base-load stations is obtained at a considerable increase in investment. It cannot be economically carried over to the system as a whole for the reason that there must be some idle or partly idle capacity on the system to allow for peaks (seasonal and daily), cleaning, adjustments, overhaul, and repairs. How much one can afford to spend for the improvement of station efficiency above "normal" depends on the shape of the system load curve, the role of the station in that curve, and the cost of fuel.

Most of the credit for the improvement in steam consumption goes to the boiler and turbine manufacturers who through continuous betterment of designs and materials have been able to raise steam pressures and temperatures. Between 1925 and 1942 the maximum throttle pressure was raised from 1000 psi to 2400 psi and the average from 350 to 1000 psi. In the same period the throttle temperature was raised from 725 to 1000 degrees F. and the

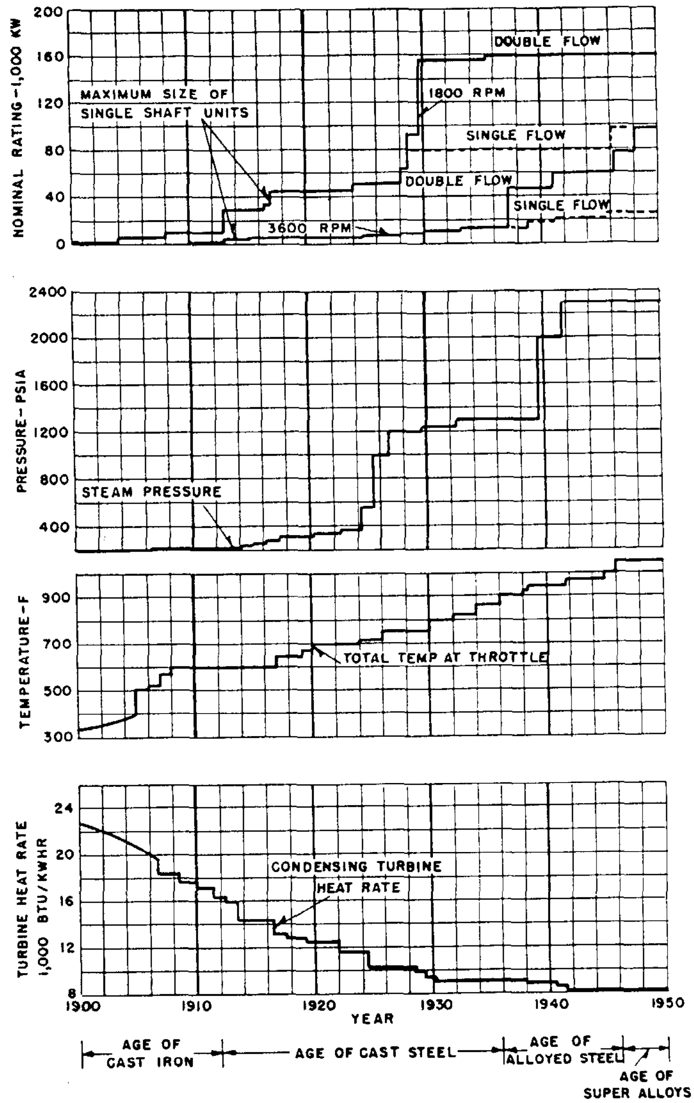


Fig. 4—Progress in turbine generator design.

average from 675 to 910 degrees. Generator losses in the meantime have been greatly reduced from about 6 percent in 1900 to 2 percent today, but these losses never did form a large part of the total, and their influence on the overall performance of the station has been minor.

The increase in maximum size of 60-cycle, two-and four-pole generating units over the years since 1900 is shown in Fig. 4. The remarkable increase has been due to improved materials and designs, particularly in large forgings, turbine blading, and generator ventilation.

In 1945 the American Society of Mechanical Engineers and the American Institute of Electrical Engineers adopted standard ratings for turbine-generator units. These were revised in November 1950 to include the 90 000 kw unit and are listed in Table 1. The machines are designed to meet their rating with 0.5 psi hydrogen pressure, but experience has shown that between 0.5 and 15 psi the output of the generator can be increased one percent for each pound increase in the gas pressure without exceeding the temperature rise guarantee at atmospheric pressure. In many locations operation at more than 15 psi gas pressure may be difficult because of codes regulating operation of "unfired pressure vessels" at greater pressures, but serious consideration is being given to operation at 30 lbs.

For a hydrogen-air mixture to be explosive, the percentage of hydrogen must lie between 5 and 75 percent. The control equipment is designed to operate an alarm if the purity of the hydrogen drops below 95 percent. The density meter and alarm system is in principle a small constant-speed fan circulating a sample of the mixture. If the density varies, the drop of pressure across the fan varies and registers on the meter.

3. Development of Water Power

The great transmission systems of this country received their impetus as a result of hydro-electric developments. Forty years ago conditions favored such developments, and in the early years of this century water-power plants costing $150 per kilowatt or less were common. Steam stations were relatively high in first cost and coal consumption per kilowatt hour was three times as much as today, and finally fuel oil was not readily available. As undeveloped water-power sites became economically less desirable, steam stations less costly and their efficiency higher, and as oil fuel and natural gas became more generally available through pipe lines, steam stations rapidly outgrew hydro-electric stations in number and capacity. Today very few water-power sites can be developed at such low cost as to be competitive with steam stations in economic energy production. For this reason hydro-electric developments of recent years have almost all been undertaken by Government agencies, which are in a position to include in the projects other considerations, such as, navigation, flood control, irrigation, conservation of resources, giving them great social value.

As the water-power developments within easy reach of the load centers were utilized and it became necessary to reach to greater distances for water power, only large developments could be considered, and stations of less than 100 000 kw became the exception rather than the rule, as witness Conowingo with 252 000 kw, Diablo with 135 000 kw, Fifteen Mile Falls with 140 000 kw, Osage with 200 000 kw, and many others. The developments of recent years undertaken by various government agencies have reached gigantic proportions, as for example Hoover Dam with 1 000 000 and Grand Coulee with 2 000 000 kw installed capacity.

A natural corollary to the increase in station capacity has been a gradual increase in the size of the individual generator units, the growth of which is shown in Fig. 5, culminating in the Grand Coulee generators of 120 000 kw at 120 rpm with an overall diameter of 45 feet.

Most of the multi-purpose hydraulic developments call for large, slow-speed machines. For such conditions vertical units are used to obtain maximum energy from the water passing through the turbine. The rotating parts are supported by a thrust bearing which is an integral part of the generator.

Two general types of generator design are used as distinguished by the arrangement of the guide and thrust bearings. Where the axial length of the generator is short in relation to its diameter, the "umbrella" design is preferred, in which a single combination guide and thrust bearing is located below the rotor (Fig. 1, Chapter 6). Where the axial length of the machines is too great an additional guide bearing must be provided. In this case the combination thrust and guide bearing is usually located above the rotor and the additional guide bearing below the rotor.

The advantages of the umbrella design are (a) reduction in overhead room to assemble and dismantle the unit during erection and overhaul, and (b) simplicity of the single bearing from the standpoint of cooling and minimum amount of piping. The design also lends itself readily to a totally-enclosed recirculating system of ventilation, which keeps dirt out of the machine and facilitates the use of fire-extinguishing equipment. It also reduces heat and noise in the power house.

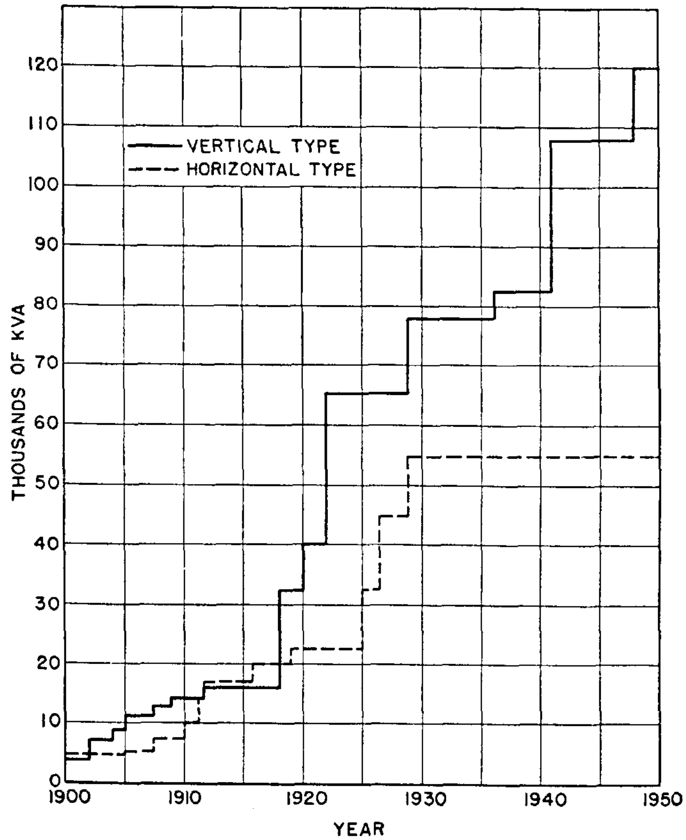


Fig. 5—Trend in maximum waterwheel generator ratings.

4. Combination of Water and Steam Power

There are very few locations today where an important market can be supplied entirely from water power because of seasonal variations in river flow, but in most cases a saving will be realized from combining water power and steam. The saving results from the combination of low operating cost of water-power plants with low investment cost of steam stations. Moreover, hydro-electric units in themselves have certain valuable advantages when used in combination with steam units. They start more quickly than steam-driven units, providing a high degree of standby readiness in emergency.

They are well adapted to maintenance of frequency, and also to providing wattless energy at times of low water flow. And finally, hydro-pondage can be drawn upon to relieve steam plants of short-time peaks to save banking extra boilers.

To what extent a water-power site can be developed economically involves a thorough investigation of individual cases. An economic balance must be struck between the steam and water power to give maximum economy. One might install enough generating capacity to take care of the maximum flow of the river during a short period. The cost per kilowatt installed would be low but the use made of the equipment (capacity factor) would also be low. Or one might put in only enough generating capacity to use the minimum river flow. In this case the cost of the development per kilowatt installed would be high, but the capacity factor would be high

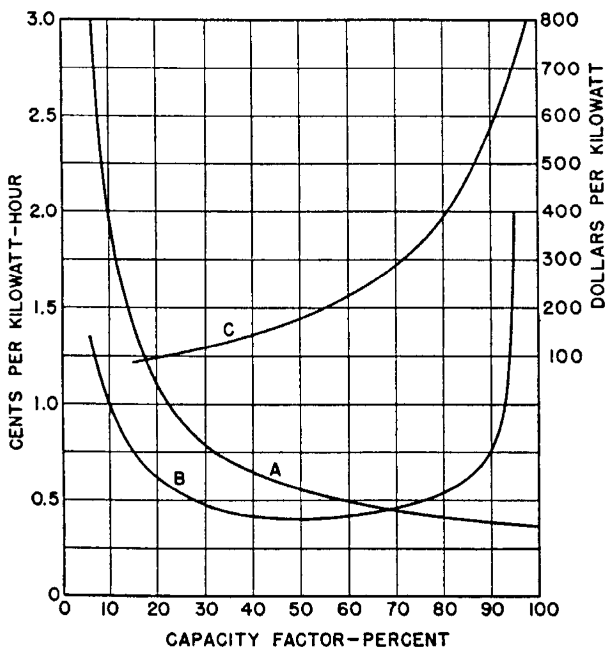


Fig. 6—Cost of energy at various capacity factors of steam and hydro-electric plants.

also. Obviously between these two extremes lies an optimum value. The ratio of installed water-power capacity to the peak load of the system that gives the minimum annual cost of power supply has been referred to as the "economic hydro ratio," and it can be determined without great difficulty for any particular set of conditions.

In a paper[2] presented before the American Society of Mechanical Engineers, Irwin and Justin discussed in an interesting and graphical manner the importance of incremental costs on the economics of any proposed development. Fig. 6, taken from their paper, shows in Curve C the capital cost per kilowatt of installation for various capacity factors. The costs were segregated in items that would be the same regardless of installation (land, water rights, dams) and those that vary with the amount of installation (power house, machinery, transmission). The latter group in this particular study was about $70 per kilowatt.

Curve A gives the total cost of energy per kilowatt hour for a modern steam plant costing $95 per kilowatt with fixed charges at 12 percent and coal at $4 a ton.

Curve B gives the total cost of energy from the water-power plant having the capital cost indicated in Curve C. To obtain such a curve it is necessary to determine the amount of energy available at the various capacity factors, the assumption being made that all hydro capacity installed is firm capacity†, that is, that the system load can absorb all of the energy generated.

Curve B shows the typically high cost of hydro-electric energy as compared with steam at high capacity factors and its low cost at low capacity factors.

5. Transmission Liability

In a hydro-electric development the transmission becomes a large factor of expense and in comparing such developments with equivalent steam plants, it is necessary to include the transmission as a charge against the hydro-electric plant. Figures of cost published on the Hoover Dam-Los Angeles 287-kv line indicate that this transmission costs over $90 a kilowatt, and other lines contemplated will probably show higher costs.

Under certain conditions it may be more costly to transmit electrical energy over wires than to transport the equivalent fuel to the steam station. It has been shown[3] that the cost of electric transmission for optimum load and voltages can be expressed as a linear function of power and distance, as follows:

$$\text{For 50\% load factor: mills/kw-hr} = 0.54 + \frac{0.61 \times \text{miles}}{100}$$

$$\text{For 90\% load factor: mills/kw-hr} = 0.30 + \frac{0.35 \times \text{miles}}{100}$$

It was also shown that fuel transportation can be expressed as a linear function of energy and distance, thus:

Railroad rates on coal
  $1.20+5½ mills per mile
Pipe-line rates on crude oil
  $5.00+4 cents per mile per 100 barrels
For pipe-line rates on natural gas two curves were given for estimated minimum and maximum interruptible contract rates
  $0 +12 cents per mile per million cubic feet
  $50+12 cents per mile per million cubic feet

The authors point out that a comparison between transmission costs alone for gas, oil, and coal are likely to be misleading because there is a wide difference in the costs of the fuels at their source. There is also a considerable variation in the transportation costs above and below the average.

†"Firm Capacity" or "Firm Power" in the case of an individual station is the capacity intended to be always available even under emergency conditions. "Hydro Firm Capacity" in the case of combined steam and hydro is the part of the installed capacity that is capable of doing the same work on that part of the load curve to which it is assigned as could be performend by an alternative steam plant.

The equivalence between the fuels is given as:

1 ton of coal .... 25 000 000 BTU
1 barrel of oil .... 6 250 000 BTU
1000 cubic feet of gas .... 1 000 000 BTU

6. Purpose of Transmission

Transmission lines are essential for three purposes.

a. To transmit power from a water-power site to a market. These may be very long and justified because of the subsidy aspect connected with the project.
b. For bulk supply of power to load centers from outlying steam stations. These are likely to be relatively short.
c. For interconnection purposes, that is, for transfer of energy from one system to another in case of emergency or in response to diversity in system peaks.

Frequent attempts have been made to set up definitions of "transmission lines," "distribution circuits" and "substations." None has proved entirely satisfactory or universally applicable, but for the purposes of accounting the Federal Power Commission and various state commissions have set up definitions that in essence read:

A transmission system includes all land, conversion structures and equipment at a primary source of supply; lines, switching and conversion stations between a generating or receiving point and the entrance to a distribution center or wholesale point, all lines and equipment whose primary purpose is to augment, integrate or tie together sources of power supply.

7. Choice of Frequency

The standard frequency in North America is 60 cycles per second. In most foreign countries it is 50 cycles. As a general-purpose distribution frequency 60 cycles has an economic advantage over 50 cycles in that it permits a maximum speed of 3600 rpm as against 3000 rpm. Where a large number of distribution transformers are used a considerable economic gain is obtained in that the saving in materials of 60-cycle transformers over 50-cycle transformers may amount to 10 to 15 percent. This is because in a transformer the induced voltage is proportional to the total flux-linkage and the frequency. The higher the frequency, therefore, the smaller the cross-sectional area of the core, and the smaller the core the shorter the length of the coils. There is a saving, therefore, in both iron and copper.

The only condition under which any frequency other than 50 to 60 cycles might be considered for a new project would be the case of a long transmission of, say, 500 or 600 miles. Such long transmission has been discussed in connection with remote hydro-electric developments at home and abroad, and for these a frequency less than 60 cycles might be interesting because as the frequency is decreased the inductive reactance of the line, $2\pi fL$, decreases and the capacitive reactance, $\frac{1}{2\pi fC}$, increases, resulting in higher load limits, transmission efficiency, and better regulation.

Full advantage of low frequency can be realized, however, only where the utilization is at low frequency. If the low transmission frequency must be converted to 60 cycles for utilization, most of the advantage is lost because of limitations of terminal conversion equipment.

Long-distance direct-current transmission has also been considered. It offers advantages that look attractive, but present limitations in conversion and inversion equipment make the prospect of any application in the near future unlikely.

In many industrial applications, particularly in the machine-tool industry, 60 cycles does not permit a high enough speed, and frequencies up to 2000 cycles may be necessary. Steps are being taken to standardize frequencies of more than 60 cycles.

8. Choice of Voltage

Transmission of alternating-current power over several miles dates from 1886 when a line was built at Cerchi, Italy, to transmit 150 hp 17 miles at 2000 volts. The voltage has progressively increased, as shown in Fig. 7, until in 1936 the Hoover Dam-Los Angeles line was put in service at 287 kv. This is still the highest operating voltage in use in the United States today, but consideration is being given to higher values. An investigation was begun in 1948 at the Tidd Station of the Ohio Power Company on an experimental line with voltages up to 500 kv.

The cost of transformers, switches, and circuit breakers increases rapidly with increasing voltage in the upper ranges of transmission voltages. In any investigation involving voltages above 230 000 volts, therefore, the unit cost of power transmitted is subject to the law of diminishing returns. Furthermore, the increase of the reactance of the terminal transformers also tends to counteract the gain obtained in the transmission line from the higher voltage. There is, therefore, some value of voltage in the range being investigated beyond which, under existing circumstances, it is uneconomical to go and it may be more profitable to give consideration to line compensation by means of capacitors to increase the economic limit of

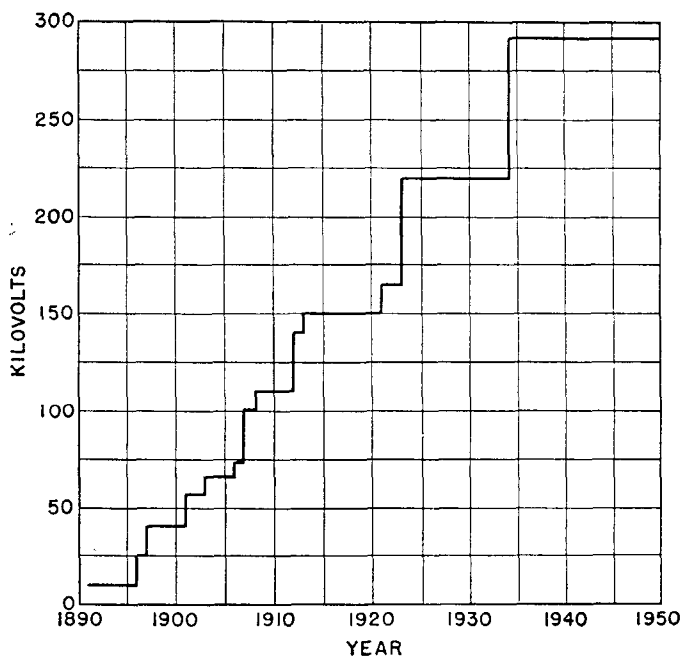


Fig. 7—Trend in transmission voltages in 60 years.

TABLE 2—FORM OF TABULATION FOR DETERMINING VOLTAGES AND CONDUCTOR SIZES

Based on the Transmission of 10 000 Kva for 10 Miles at 80 Percent Power Factor Lagging, 60-Cycle, 3-Phase

VOLTAGE IN KV			CONDUCTORS							VOLTAGE DROP AT FULL LOAD			FIRST COST							ANNUAL OPERATING COST		
						Total I^2R Loss: 10 000 Kva		2 500 Kva														
Between Conductors in KV	To Neutral in KV	Amperes for 10 000 Kva	B&S or Circular Mils	Total Weight in Pounds	Resistance in Ohms	Kw for 10 Hours	Loss in Percent	Kw for 14 Hours	Total Loss per Year in Kwhr.	Resistance IR in Percent	Reactance IX in Percent	Percent Regulation	Conductors at 30 Cents per Pound	Transformers 25 000 Kva	Circuit Breakers and Control	Lightning Arresters	Insulators	Total	Interest on First Cost at 6 Percent	Depreciation on First Cost 10 Percent	I^2R Losses at 1 Cent per Kwhr.	Total
22	12.7	262	300 000	147 000	1.97	405	5.1	25	1 610 000	4.1	12.6	11.0	$44 100	$69 600	$ 9 500	$1 600	$1 500	$126 300	$7 580	$12 630	$16 100	$36 310
			#000	82 100	3.50	720	9.0	45	2 860 000	7.2	13.4	14.0	24 600	69 600	9 500	1 600	1 500	106 800	6 410	10 680	28 600	45 690
			#0	51 600	5.55	1 140	14.3	71	4 520 000	11.5	14.0	17.5	15 500	69 600	9 500	1 600	1 500	97 700	5 860	9 770	45 200	60 830
33	19.1	175	#00	65 000	4.40	405	5.1	25	1 610 000	4.0	6.4	7.0	19 500	71 600	9 700	2 400	2 200	105 400	6 330	10 540	16 100	32 970
			#2	32 500	8.82	810	10.1	51	3 220 000	8.1	6.8	10.5	9 700	71 600	9 700	2 400	2 200	95 600	5 740	9 560	32 200	47 500
			#4	20 200	14.0	1 290	16.1	81	5 120 000	12.9	7.0	14.5	6 000	71 600	9 700	2 400	2 200	91 900	5 520	9 190	51 200	65 910
44	25.4	131	#2	32 500	8.82	453	5.7	28	1 800 000	4.5	3.8	5.9	9 700	74 400	10 900	3 400	3 000	101 400	6 090	10 140	18 000	34 230
			#5	16 000	17.7	910	11.4	57	3 610 000	9.1	4.0	9.7	4 800	74 400	10 900	3 400	3 000	96 500	5 790	9 650	36 100	51 540

TABLE 3—QUICK-ESTIMATING DATA ON THE LOAD CARRYING CAPACITY OF TRANSMISSION LINES†

Delivered Line Voltage	Kw Which Can Be Delivered Based on 5% Regulation and 90% Power Factor			
	Distance in Miles			
13.2 kv—3-foot spacing Stranded Copper	5	10	15	20
4	950	490	330	245
2	1 400	700	470	350
4/0	3 000	1 500	1 000	750
33 kv—4-foot spacing Stranded Copper	10	20	30	40
1	5 000	2 500	1 700	1 250
2/0	6 700	3 350	2 200	1 700
4/0	8 350	4 180	2 800	2 100
300 000	11 500	5 750	3 800	2 900
66 kv—8-foot spacing Stranded Copper	20	40	60	80
2/0	12 500	6 250	4 180	3 140
4/0	16 000	8 000	5 320	3 990
300 000	18 400	9 180	6 120	4 590
	Kw Which Can Be Delivered Based on 10% Loss and Equal Voltage at Sending and Receiving Ends			
	Distance in Miles			
132 kv—16-foot spacing Stranded Copper	40	80	120	160
4/0	116 000	58 000	39 500	30 100
300 000	172 000	86 000	58 800	44 800
500 000	297 000	150 000	101 000	77 100
220 kv—24-foot spacing Hollow Copper	80	160	240	320
500 000	425 000	219 000	151 000	119 000
ACSR—795 000	417 000	216 000	149 000	118 500

†Data obtained from Figs. 19 and 22 of Chap. 9.

power transmission than increase the voltage much above present practice.

The basic principles underlying system operation as regards voltages have been set forth in a report[4] which lists the voltages in common use, the recommended limits of voltage spread, and the equipment voltage ratings intended to fulfill the voltage requirements of the level for which the equipment is designed. The report should be carefully studied before any plans are made involving the adoption of or change in a system voltage.

In selecting the transmission voltage, consideration should be given to the present and probable future voltage of other lines in the vicinity. The advantages of being able to tie together adjoining power districts at a common voltage frequently outweighs a choice of voltage based on lowest immediate cost.

If the contemplated transmission is remote from any existing system, the choice of voltage should result from a complete study of all factors involved. Attempts have been made to determine by mathematical expression, based on the well-known Kelvin's Law, the most economical transmission voltage with all factors evaluated, but these are so numerous that such an expression becomes complicated, difficult, and unsatisfactory. The only satisfactory way to determine the voltage is to make a complete study of the initial and operating costs corresponding to various assumed transmission voltages and to various sizes of conductors.

For the purposes of the complete study, it is usually unnecessary to choose more than three voltages, because a fairly good guess as to the probable one is possible without knowing more than the length of the circuit. For this preliminary guess, the quick-estimating Table 3 is useful. This table assumes that the magnitude of power transmitted in the case of voltages 13.2, 33, and 66 kv is based on a regulation of 5 percent and a load power factor of 90 percent. In the case of 132 and 220 kv, the table is based on a loss of 10 percent and equal voltages at the sending and receiving ends of the line. The reason for this and the bases of the calculations are given in Chapter 9.

A representative study is given in Table 2. It is assumed that it is desired to transmit over a single-circuit ten miles long 8000 kw (10 000 kva) at 80 percent power-factor lagging for 10 hours a day followed by 2000 kw (2500 kva) at 80 percent power-factor for 14 hours. The preliminary guess indicates that 23, 34.5, or 46 kv are probably the economical nominal voltages. Equivalent conductor spacing and the number of insulators are as given in Table 4. Conductors of hard-drawn stranded copper are employed, the resistance being taken at 25 degrees C. The step-up and step-down transformers are assumed as $2.5 \times 10\,000$ kva, 12 500 kva at either end, and high-voltage circuit-breakers are used in anticipation of future additional circuits.

TABLE 4—CONSTRUCTION FEATURES OF TRANSMISSION LINES IN THE UNITED STATES*

Line Voltage in Kv	Length in Miles			Equivalent Spacing		Number of Insulators		
	Av.	Min.	Max.	Type**	Av.	Av.	Min.	Max.
13.8	...	..	...	SC-W	3	..	..	..
34.5	...	..	...	SC-W	4	..	..	..
69	35	25	100	SC-W	8	5	4	8
115	40	25	100	SC-W	17	7	6	11
138	40	25	140	SC-W	18	10	8	12
230	133	45	260	SC-ST	31	15	14	20

The costs of the pole line, right-of-way, building, and real estate are not included as they will be practically the same for the range of voltages studied.

Assuming that the cost figures in the table are correct, a 34 500-volt line with No. 00 copper conductor is the most economical. The transmission loss will be 5 percent and the regulation 7 percent at full load, which is deemed satisfactory. The voltage is sufficiently high for use as a subtransmission voltage if and when the territory develops and additional load is created. The likelihood of early growth of a load district is an important factor in selection of the higher voltage and larger conductor where the annual operating costs do not vary too widely.

9. Choice of Conductors

The preliminary choice of the conductor size can also be limited to two or three, although the method of selecting will differ with the length of transmission and the choice of voltage. In the lower voltages up to, say, 30 kv, for a given percentage energy loss in transmission, the cross section and consequently the weight of the conductors required to transmit a given block of power varies inversely as the square of the voltage. Thus, if the voltage is doubled, the weight of the conductors will be reduced to one-fourth with approximately a corresponding reduction in their cost. This saving in conducting material for a given energy loss in transmission becomes less as the higher voltages are reached, becoming increasingly less as voltages go higher. This is for the reason that for the higher voltages at least two other sources of loss, leakage over insulators and the escape of energy through the air between the conductors (known as "corona"—see Chap. 3) appear. In addition to these two losses, the charging current, which increases as the transmission voltage goes higher, may either increase or decrease the current in the circuit depending upon the power-factor of the load current and the relative amount of the leading and lagging components of the current in the circuit. Any change in the current of the circuit will consequently be accompanied by a corresponding change in the I^2R loss. In fact, these sources of additional losses may, in some cases of long circuits or extensive systems, materially contribute toward limiting the transmission voltage. The weight of copper conductors, from which their cost can be calculated, is given in Chap. 3. As an insurance against breakdown, important lines frequently are built with circuits in duplicate. In such cases the cost of conductors for two circuits should not be overlooked.

*This table is based on information published in *Electrical World* and in *Electrical Engineering*. While it does not include all lines, it is probably representative of general practice in the U.S.A.

**SC-W —Single-circuit wood.
SC-ST—Single-circuit steel.

10. Choice of Spacing

Conductor spacing depends upon the economic consideration given to performance against lightning surges. If maximum reliability is sought, the spacing loses its relation to the operating voltage and then a medium voltage line assumes most of the cost of a high-voltage transmission without the corresponding economy. (See Chap. 17) In general a compromise is adopted whereby the spacing is based on the dynamic voltage conditions with some allowance for reasonable performance against lightning surges.

Table 4 shows typical features of transmission lines in the United States including their "equivalent spacing" and the number of suspension insulators used. By equivalent spacing is understood the spacing that would give the same reactance and capacitance as if an equilateral triangular arrangement of conductors had been used. It is usually impractical to use an equilateral triangular arrangement for design reasons. The equivalent spacing is obtained from the formula $D = \sqrt[3]{ABC}$ where A, B, and C are the actual distances between conductors.

11. Choice of Supply Circuits

The choice of the electrical layout of the proposed power station is based on the conditions prevailing locally. It should take into consideration the character of the load and the necessity for maintaining continuity of service. It should be as simple in arrangement as practicable to secure the desired flexibility in operation and to provide the proper facilities for inspection of the apparatus.

A review of existing installations shows that the apparent combinations are innumerable, but an analysis indicates that in general they are combinations of a limited number of fundamental schemes. The arrangements vary from the simplest single-circuit layout to the involved duplicate systems installed for metropolitan service where the importance of maintaining continuity of service justifies a high capital expenditure.

The scheme selected for stations distributing power at bus voltage differs radically from the layout that would be desirable for a station designed for bulk transmission.

In some metropolitan developments supplying underground cable systems segregated-phase layouts have been and are still employed to secure the maximum of reliability in operation. However, their use seems to be on the decline, as the improvement in performance over the conventional adjacent phase grouping is not sufficiently better to justify the extra cost, particularly in view of the continuing improvement of protective equipment and the more reliable schemes of relaying available today for removing faulty equipment, buses, or circuits.

Several fundamental schemes for bus layouts supplying feeders at generator voltage are shown in Fig. 8. These vary from the simplest form of supply for a small industrial plant as shown in (a) to a reliable type of layout for central-station supply to important load areas shown in (e) and (f)†.

Sketch (a) shows several feeders connected to a common bus fed by only one generator. This type of construction should be used only where interruptions to service are relatively unimportant because outages must exist to all feeders simultaneously when the bus, generator breaker, generator or power source is out of service for any reason. Each feeder has a circuit breaker and a disconnect switch. The circuit breaker provides protection against short circuits on the feeder and enables the feeder to be removed from service while it is carrying load if necessary. The disconnect switch serves as additional backup protection for personnel, with the breaker open, during maintenance or repair work on the feeder. The disconnect also enables the breaker to be isolated from the bus for inspection and maintenance on the breaker. Quite frequently disconnect switches are arranged so that when opened the blade can be connected to a grounded clip for protection. If the bus is supplied by more than one generator, the reliability of supply to the feeders using this type of layout is considerably increased.

With more than one generator complete flexibility is obtained by using duplicate bus and switching equipment as shown in (b). It is often questionable whether the expense of such an arrangement is justified and it should be used only where the importance of the service warrants it. One breaker from each generator or feeder can be removed from service for maintenance with complete protection for maintenance personnel and without disrupting service to any feeder. Also, one complete bus section can be removed from service for cleaning and maintenance or for adding an additional feeder without interfering with the normal supply to other feeder circuits. There are many intermediate schemes that can be utilized that give a lesser degree of flexibility, an example of which is shown in (c). There are also several connections differing in degree of duplication that are intermediate to the three layouts indicated, as for instance in (d). An analysis of the connections in any station layout usually shows that they are built up from parts of the fundamental schemes depending upon the flexibility and reliability required.

The generating capacity connected to a bus may be so large that it is necessary to use current-limiting reactors in series with the generator leads or in series with each feeder. Sometimes both are required. Sketch (e) shows a double bus commonly used where reactors are in series with each generator and each feeder. Bus-tie reactors are also shown that, with all generators in service, keep the short-circuit currents within the interrupting ability of the breakers. These bus-tie reactors are important

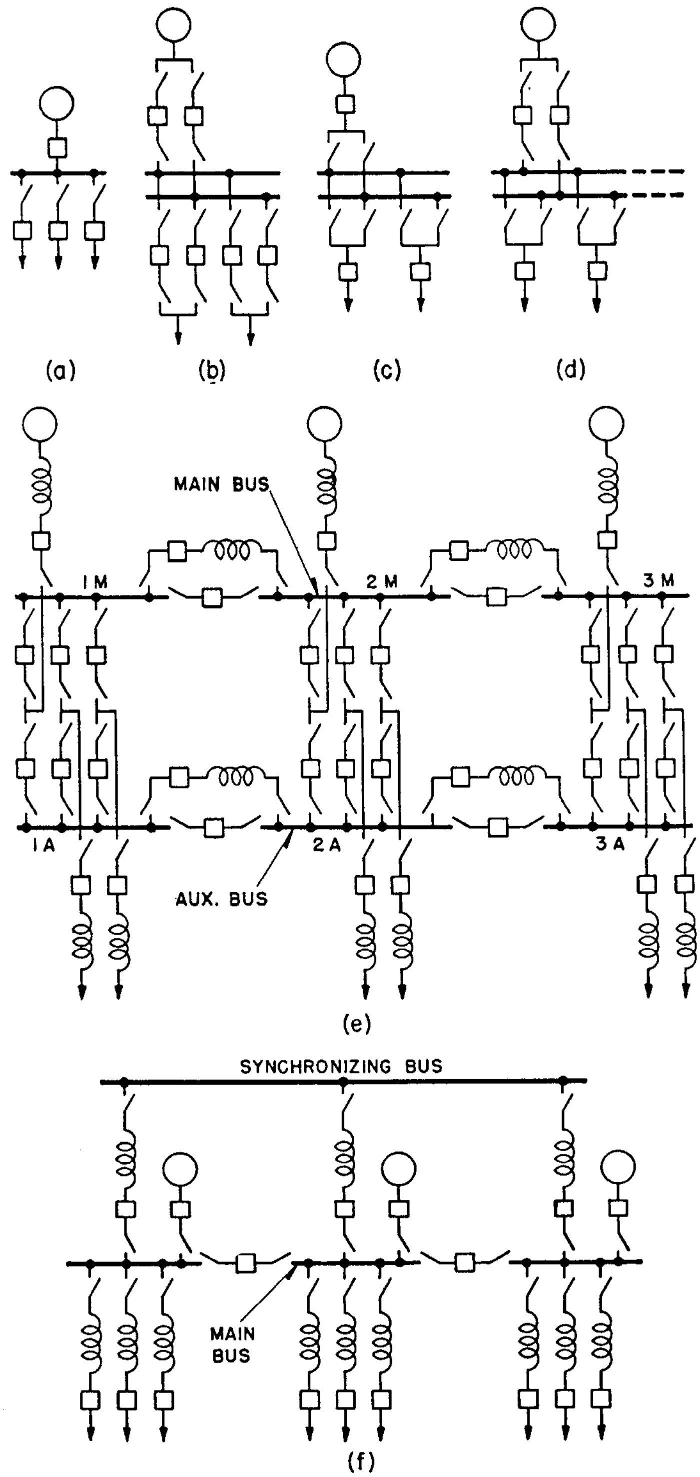


Fig. 8—Fundamental schemes of connections for supply at generator voltage.

†NELA Publications Nos. 164 and 278-20—Elec. App. Comm. give a number of station and substation layouts.

because they not only limit the current on short circuit but also serve as a source of supply to the feeders on a bus section if the generator on that bus section fails. Each feeder can be connected to either the main or auxiliary bus through what is called a selector breaker. A selector breaker is similar in every respect to the feeder breaker and serves as backup protection in case the feeder breaker does not function properly when it should open on a feeder fault. The bus-tie breakers can be used when one or more generators are out of service to prevent voltage and phase-angle differences between bus sections that would exist with the supply to a bus section through a reactor. The phase angle between bus sections becomes important when a station is supplying a network system and should be kept to a minimum to prevent circulating currents through the network. For a network supply at least four bus sections are generally used so that the network can still be supplied in case one bus section should trip out on a fault. Sketch (e) shows only three bus sections, the main and auxiliary buses serve as one bus for the feeders connected to that section.

Sketch (f) shows a more modern design for central stations with the feeder reactors next to the bus structure, in contrast with (e) where the reactors are on the feeder side of the breaker. This arrangement is possible because of the proven reliability of reactors, circuit breakers, and dust-tight metalclad bus structures. Continuous supply to all feeders is provided through reactor ties to a synchronizing bus should a generator fail. Bus-tie circuit breakers are provided to tie solidly adjacent bus sections for operation with one or more generators out of service. Stations of this type would be expected to have four to six or more bus sections especially if the station supplies network loads. The synchronizing bus also serves as a point where tie feeders from other stations can be connected and be available for symmetrical power supply to all feeder buses through the reactors. This is not the case for station design shown in (e) where a tie feeder must be brought in to a particular bus section.

For any type of generating-station design proper current and potential transformers must be provided to supply the various types of relays to protect all electrical parts of the station against any type of fault. Likewise, current and voltage conditions must be obtained from current and potential transformers through the proper metering equipment to enable the operating forces to put into service or remove any equipment without impairing the operation of the remainder of the station. A ground bus must be provided for grounding each feeder when it is out of service for safety to personnel. Also a high-potential test bus is necessary to test circuit breakers, bus work and feeders, following an outage for repairs or maintenance, before being reconnected to the station.

Fire walls are generally provided between bus sections or between each group of two bus sections to provide against the possibility of a fire in one section spreading to the adjacent sections. The separate compartments within the station should be locked and made as tight as possible for protection against accidental contact by operating personnel either physically or through the medium of a wire or any conducting material. Stray animals have caused considerable trouble by electrocuting themselves in accessible bus structures.

With stations supplying transmission systems the scheme of connections depends largely on the relative capacities of the individual generators, transformers and transmission circuits; and whether all the generated power is supplied in bulk over transmission lines or whether some must also be supplied at generator voltage. The simplest layout is obtained when each generator, transformer and transmission circuit is of the same capacity and can be treated as a single entity. Unfortunately, this is seldom the case because the number of generators do not equal the number of outgoing circuits. Even here, however, some simplification is possible if the transformers are selected of the same capacity as the generators, so that the combination becomes the equivalent of a high-voltage generator with all the switching on the high-voltage side of the transformer.

In Fig. 9, (a) shows the "unit scheme" of supply. The power system must be such that a whole unit comprising generator, transformer and transmission line can be dropped without loss of customer's load. The station auxiliaries that go with each unit are usually supplied

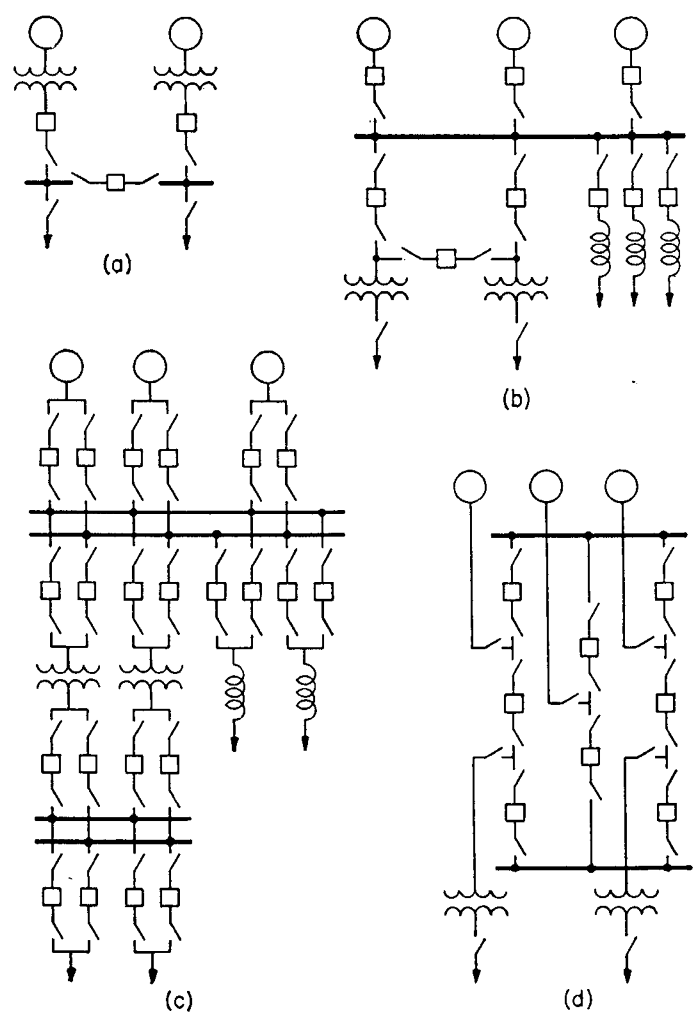


Fig. 9—Fundamental schemes of supply at higher than generated voltage.

through a station transformer connected directly to the generator terminals, an independent supply being provided for the initial start-up and for subsequent emergency restarts.

Sketch (b) shows the case where conditions do not permit of the transformers being associated directly with the generators because, perhaps, of outgoing feeders at generator voltage, but where the capacity of the transmission lines is such as to give an economical transformer size. Here it may be desirable to include the transformer bank as an integral part of the line and perform all switching operations on the low-voltage side. Sketch (b) shows the extreme of simplicity, which is permissible only where feeders and lines can be taken in and out of service at will, and (c) shows the other extreme where the feeders and lines are expected to be in service continuously. Sketch (d) shows an arrangement which is frequently applicable and which provides a considerable flexibility with the fewest breakers.

Figs. 8 and 9 include fundamental layouts from which almost any combination can be made to meet local conditions. The choice depends on the requirements of service continuity, the importance of which depends on two factors, the multiplicity of sources of supply, and the type of load. Some industrial loads are of such a nature that the relatively small risk of an outage does not justify duplication of buses and switching.

The same argument applies to the transmission line itself. Figure 10 shows an assumed transmission of 100 miles with two intermediate stations at 33 miles from either end. Sketch (a) is a fully-sectionalized scheme giving the ultimate in flexibility and reliability. Any section of either transmission circuit can be taken out for maintenance without the loss of generating capacity. Furthermore, except within that part of the transmission where one section is temporarily out of service, a fault on any section of circuit may also be cleared without loss of load. Sketch (b) shows the looped-in method of connection. Fewer breakers are required than for the fully sectionalized scheme, and as in (a) any section of the circuit can be removed from service without reducing power output. If, however, a second line trips out, part or all of the generating capacity may be lost. Relaying is somewhat more difficult than with (a), but not unduly so. Flexibility on the low-voltage side is retained as in (a). Sketch (c) is in effect an extension of the buses from station to station. The scheme is, of course, considerably cheaper than that in (a) and slightly less than that in (b) but can be justified only where a temporary outage of the transmission is unimportant. Relaying in (c) is complicated by the fact that ties between buses tend to equalize the currents so that several distinct relaying steps are required to clear a fault.

A proper balance must be kept between the reliability of the switching scheme used and the design of the line itself. Most line outages originate from lightning and a simplification and reduction in the cost of switching is permissible if the circuit is built lightning proof. (See Chap. 13.) On the other hand, if a line is of poor construction as regards insulation and spacing, it would not be good engineering to attempt to compensate for this by putting in an elaborate switching and relaying scheme.

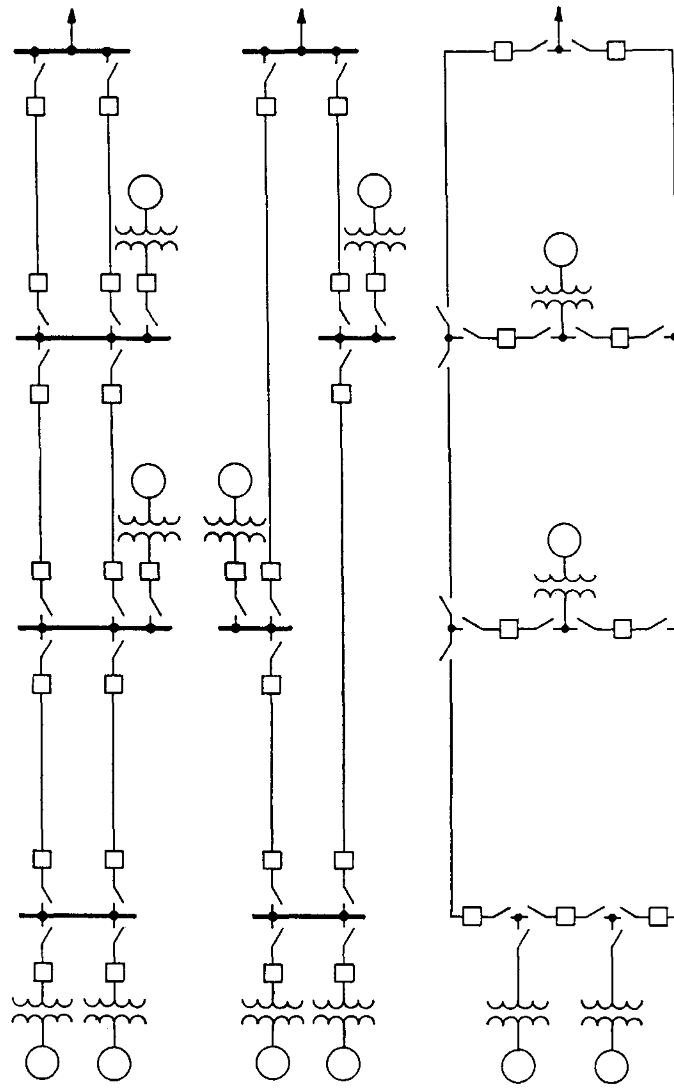


Fig. 10—Fundamental schemes of transmission. *(a) Fully-sectionalized supply. (b) Looped-in supply. (c) Bussed supply.*

Only a few fundamental ideas have been presented on the possible layout of station buses and the switching arrangements of transmission circuits. The possible combinations are almost infinite in number and will depend on local conditions and the expenditure considered permissible for the conditions prevailing.

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CHAPTER 2

SYMMETRICAL COMPONENTS

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THE analysis of a three-phase circuit in which phase voltages and currents are balanced (of equal magnitude in the three phases and displaced 120° from each other), and in which all circuit elements in each phase are balanced and symmetrical, is relatively simple since the treatment of a single-phase leads directly to the three-phase solution. The analysis by Kirchoff's laws is much more difficult, however, when the circuit is not symmetrical, as the result of unbalanced loads, unbalanced faults or short-circuits that are not symmetrical in the three phases. Symmetrical components is the method now generally adopted for calculating such circuits. It was presented to the engineering profession by Dr. Charles L. Fortescue in his 1918 paper, "Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks." This paper, one of the longest ever presented before the A.I.E.E., is now recognized as a classic in engineering literature. For several years symmetrical components remained the tool of the specialist; but the subsequent work of R. D. Evans, C. F. Wagner, J. F. Peters, and others in developing the sequence networks and extending the application to system fault calculations and stability calculations focused the attention of the industry on the simplification and clarification symmetrical components offered in the calculation of power system performance under unbalanced conditions.

The method was recognized immediately by a few engineers as being very useful for the analysis of unbalanced conditions on symmetrical machines. Its more general application to the calculation of power system faults and unbalances, and the simplification made possible by the use of symmetrical components in such calculations, was not appreciated until several years later when the papers by Evans, Wagner, and others were published. The use of symmetrical components in the calculation of unbalanced faults, unbalanced loads, and stability limits on three-phase power systems now overshadows the other applications.

The fundamental principle of symmetrical components, as applied to three-phase circuits, is that an unbalanced group of three related vectors (for example, three unsymmetrical and unbalanced vectors of voltage or current in a three-phase system) can be resolved into three sets of vectors. The three vectors of each set are of equal magnitude and spaced either zero or 120 degrees apart. Each set is a "symmetrical component" of the original unbalanced vectors. The same concept of resolution can be applied to rotating vectors, such as voltages or currents, or nonrotating vector operators such as impedances or admittances.

Stated in more general terms, an unbalanced group of n associated vectors, all of the same type, can be resolved into n sets of balanced vectors. The n vectors of each set are of equal length and symmetrically located with respect to each other. A set of vectors is considered to be symmetrically located if the angles between the vectors, taken in sequential order, are all equal. Thus three vectors of one set are symmetrically located if the angle between adjacent vectors is either zero or 120 degrees. Although the method of symmetrical components is applicable to the analysis of any multi-phase system, this discussion will be limited to a consideration of three-phase systems, since three phase systems are most frequently encountered.

This method of analysis makes possible the prediction, readily and accurately, of the behavior of a power system during unbalanced short-circuit or unbalanced load conditions. The engineer's knowledge of such phenomena has been greatly augmented and rapidly developed since its introduction. Modern concepts of protective relaying and fault protection grew from an understanding of the symmetrical component methods.

Out of the concept of symmetrical components have sprung, almost full-born, many electrical devices. The negative-sequence relay for the detection of system faults, the positive-sequence filter for causing generator voltage regulators to respond to voltage changes in all three phases rather than in one phase alone, and the connection of instrument transformers to segregate zero-sequence quantities for the prompt detection of ground faults are interesting examples. The HCB pilot wire relay, a recent addition to the list of devices originating in minds trained to think in terms of symmetrical components, uses a positive-sequence filter and a zero-sequence filter for the detection of faults within a protected line section and for initiating the high speed tripping of breakers to isolate the faulted section.

Symmetrical components as a tool in stability calculations was recognized in 1924–1926, and has been used extensively since that time in power system stability analyses. Its value for such calculations lies principally in the fact that it permits an unbalanced load or fault to be represented by an impedance in shunt with the single-phase representation of the balanced system.

The understanding of three-phase transformer performance, particularly the effect of connections and the phenomena associated with three-phase core-form units has been clarified by symmetrical components, as have been the physical concepts and the mathematical analysis of rotating machine performance under conditions of unbalanced faults or unbalanced loading.

The extensive use of the network calculator for the determination of short-circuit currents and voltages and for the application of circuit breakers, relays, grounding transformers, protector tubes, etc. has been furthered by the development of symmetrical components, since each sequence network may be set up independently as a single-phase system. A miniature network of an extensive power system, set up with three-phase voltages, separate impedances for each phase, and mutual impedances between phases would indeed be so large and cumbersome to handle as to be prohibitive. In this connnection it is of interest to note that the network calculator has become an indispensable tool in the analysis of power system performance and in power system design.

Not only has the method been an exceedingly valuable tool in system analyses, but also, by providing new and simpler concepts the understanding of power system behavior has been clarified. The method of symmetrical components is responsible for an entirely different manner of approach to predicting and analyzing power-system performance.

Symmetrical components early earned a reputation of being complex. This is unfortunate since the mathematical manipulations attendant with the method are quite simple, requiring only a knowledge of complex vector notation. It stands somewhat unique among mathematical tools in that it has been used not only to explain existing conditions, but also, as pointed out above, the physical concepts arising from a knowledge of the basic principles have led to the development of new equipment and new schemes for power system operation, protection, etc. Things men come to know lose their mystery, and so it is with this important tool.

Inasmuch as the theory and applications of symmetrical components are fully discussed elsewhere (see references) the intention here is only to summarize the important equations and to provide a convenient reference for those who are already somewhat familiar with the subject.

I. THE VECTOR OPERATOR "a"

For convenience in notation and manipulation a vector operator is introduced. Through usage it has come to be known as the vector a and is defined as

$$a=-\frac{1}{2}+j\frac{\sqrt{3}}{2}=\epsilon^{j120} \tag{1}$$

This indicates that the vector a has unit length and is oriented 120 degrees in a positive (counter-clockwise) direction from the reference axis. A vector operated upon by a is not changed in magnitude but is simply rotated in position 120 degrees in the forward direction. For example, $V'=aV$ is a vector having the same length as the vector V, but rotated 120 degrees forward from the vector V. This relationship is shown in Fig. 1. The square of the vector a is another unit vector oriented 120 degrees in a negative (clockwise) direction from the reference axis, or oriented 240 degrees forward in a positive direction.

$$a^2=(\epsilon^{j120})(\epsilon^{j120})=\epsilon^{j240}=-\frac{1}{2}-j\frac{\sqrt{3}}{2} \tag{2}$$

As shown in Fig. 1, the resultant of a^2 operating on a vector V is the vector V'' having the same length as V, but located 120 degrees in a clockwise direction from V. The three vectors $1+j0$, a^2, and a (taken in this order)

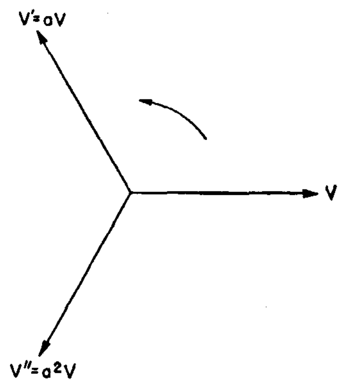


Fig. 1—Rotation of a vector by the operator a.

form a balanced, symmetrical, set of vectors of positive-phase-sequence rotation, since the vectors are of equal length, displaced equal angles from each other, and cross the reference line in the order 1, a^2, and a (following the usual convention of counter-clockwise rotation for the

TABLE 1—PROPERTIES OF THE VECTOR OPERATOR "a"

$$1=1+j0=\epsilon^{j0}$$
$$a=-\frac{1}{2}+j\frac{\sqrt{3}}{2}=\epsilon^{j120}$$
$$a^2=-\frac{1}{2}-j\frac{\sqrt{3}}{2}=\epsilon^{j240}$$
$$a^3=1+j0=\epsilon^{j0}$$
$$a^4=a=-\frac{1}{2}+j\frac{\sqrt{3}}{2}=\epsilon^{j120}$$
$$a^5=a^2=-\frac{1}{2}-j\frac{\sqrt{3}}{2}=\epsilon^{j240}$$
$$a+a^2+1=0$$
$$a+a^2=-1+j0=\epsilon^{j180}$$
$$a-a^2=0+j\sqrt{3}=\sqrt{3}\epsilon^{j90}$$
$$a^2-a=0-j\sqrt{3}=\sqrt{3}\epsilon^{j270}$$
$$1-a=\frac{3}{2}-j\frac{\sqrt{3}}{2}=ja^2\sqrt{3}=\sqrt{3}\epsilon^{j330}$$
$$1-a^2=\frac{3}{2}+j\frac{\sqrt{3}}{2}=-ja\sqrt{3}=\sqrt{3}\epsilon^{j30}$$
$$a-1=-\frac{3}{2}+j\frac{\sqrt{3}}{2}=-ja^2\sqrt{3}=\sqrt{3}\epsilon^{j150}$$
$$a^2-1=-\frac{3}{2}-j\frac{\sqrt{3}}{2}=ja\sqrt{3}=\sqrt{3}\epsilon^{j210}$$
$$1+a=-a^2=\frac{1}{2}+j\frac{\sqrt{3}}{2}=\epsilon^{j60}$$
$$1+a^2=-a=\frac{1}{2}-j\frac{\sqrt{3}}{2}=\epsilon^{j300}$$
$$(1+a)\ (1+a^2)=1+j0=\epsilon^{j0}$$
$$(1-a)\ (1-a^2)=3+j0=3\epsilon^{j0}$$
$$\frac{1+a}{1+a^2}=a=-\frac{1}{2}+j\frac{\sqrt{3}}{2}=\epsilon^{j120}$$
$$\frac{1-a}{1-a^2}=-a=\frac{1}{2}-j\frac{\sqrt{3}}{2}=\epsilon^{j300}$$
$$(1+a)^2=a=-\frac{1}{2}+j\frac{\sqrt{3}}{2}=\epsilon^{j120}$$
$$(1+a^2)^2=a^2=-\frac{1}{2}-j\frac{\sqrt{3}}{2}=\epsilon^{j240}$$

vector diagram). The vectors 1, a, and a^2 (taken in this order) form a balanced, symmetrical, set of vectors of negative-phase-sequence, since the vectors do not cross the reference line in the order named, keeping the same

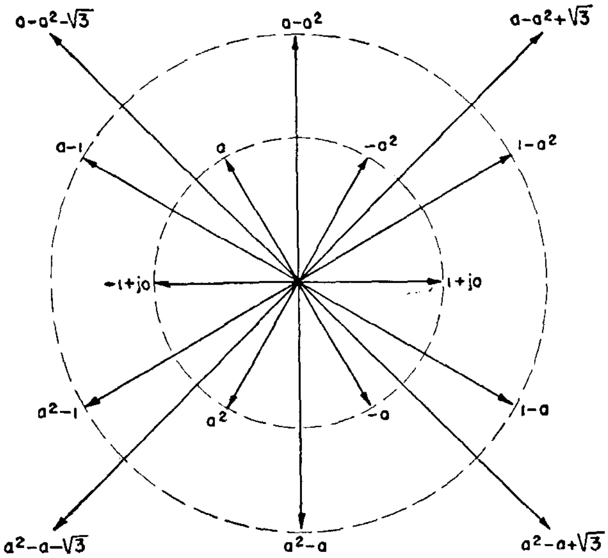


Fig. 2—Properties of the vector operator *a*.

convention of counterclockwise rotation, but the third named follows the first, etc.

Fundamental properties of the vector a are given in Table 1, and are shown on the vector diagram of Fig. 2.

II. RESOLUTION AND COMBINATION OF VECTOR COMPONENTS

1. Resolution of Unbalanced Three-Phase Voltages

A three-phase set of unbalanced voltage vectors is shown in Fig. 3. Any three unbalanced vectors such as those in Fig. 3 can be resolved into three balanced or symmetrical sets of vectors by the use of the following equations:

$$\begin{aligned} E_0 &= \tfrac{1}{3}(E_a+E_b+E_c) \\ E_1 &= \tfrac{1}{3}(E_a+aE_b+a^2E_c) \\ E_2 &= \tfrac{1}{3}(E_a+a^2E_b+aE_c) \end{aligned} \tag{3}$$

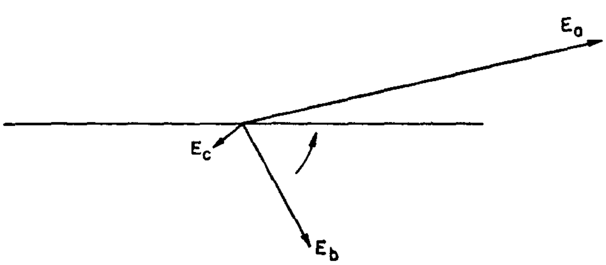


Fig. 3—Unbalanced vectors.

E_0 is the zero-sequence component of E_a, and is likewise the zero-sequence component of E_b and E_c, so that $E_0=E_{a0}=E_{b0}=E_{c0}$. This set of three-phase vectors is shown in Fig. 4.

Fig. 4—Zero-sequence components of the vectors in Fig. 3.

E_1 is the positive-sequence component of E_a, written as E_{a1}. The positive-sequence component of E_b, E_{b1}, is equal to a^2E_{a1}. The positive-sequence component of E_c, E_{c1}, is equal to aE_{a1}. E_{a1}, E_{b1}, E_{c1} form a balanced, symmetrical three-phase set of vectors of positive phase sequence since the vector E_{a1} is 120 degrees ahead of E_{b1} and 120 degrees behind E_{c1}, as shown in Fig. 5.

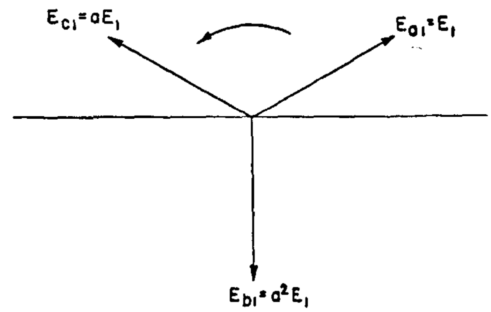


Fig. 5—Positive-sequence components of the vectors in Fig. 3.

E_2 is the negative-sequence component of E_a, written as E_{a2}. The negative-sequence components of E_b and E_c are, respectively, aE_{a2} and a^2E_{a2}, so that E_{a2}, E_{b2}, E_{c2} taken in order form a symmetrical set of negative-sequence vectors as in Fig. 6.

All three of the zero-sequence-component vectors are defined by E_0, since $E_{a0}=E_{b0}=E_{c0}$. Likewise, the three

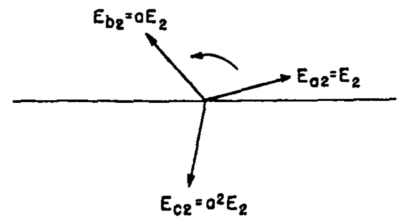


Fig. 6—Negative-sequence components of the vectors in Fig. 3.

positive-sequence vectors are defined by E_1, since $E_{a1}=E_1$, $E_{b1}=a^2E_1$, and $E_{c1}=aE_1$. Similarly the three negative-sequence vectors are defined by E_2. Thus all nine component vectors of the three original unbalanced vectors are completely defined by E_0, E_1, and E_2; and it is understood that E_0, E_1, and E_2, are the zero-, positive-, and negative-sequence components of E_a without writing E_{a0}, etc. The three original unbalanced vectors possess six degrees of freedom, since an angle and a magnitude are necessary to define each vector. The nine component vectors also possess six degrees of freedom, since each of the three sets of component vectors is described by one angle and one magnitude; for example, the three positive-sequence vectors E_{a1}, E_{b1}, and E_{c1}, are defined by the angular position and magnitude of E_1.

Note that all three sets of component vectors have the same counterclockwise direction of rotation as was assumed for the original unbalanced vectors. The negative-sequence set of vectors does not rotate in a direction opposite to the positive-sequence set; but the phase-sequence, that is, the order in which the maximum occur with time, of the negative-sequence set is a, c, b, a, and therefore opposite to the a, b, c, a, phase-sequence of the positive-sequence set.

The unbalanced vectors can be expressed as functions of the three components just defined:

$$\begin{aligned} E_a &= E_{a0}+E_{a1}+E_{a2}=E_0+E_1+E_2 \\ E_b &= E_{b0}+E_{b1}+E_{b2}=E_0+a^2E_1+aE_2 \\ E_c &= E_{c0}+E_{c1}+E_{c2}=E_0+aE_1+a^2E_2 \end{aligned} \tag{4}$$

The combination of the sequence component vectors to form the original unbalanced vectors is shown in Fig. 7.

In general a set of three unbalanced vectors such as those in Fig. 3 will have zero-, positive-, and negative-sequence components. However, if the vectors are balanced and symmetrical—of equal length and displaced 120 degrees from each other—there will be only a positive-sequence component, or only a negative-sequence component, depending upon the order of phase sequence for the original vectors.

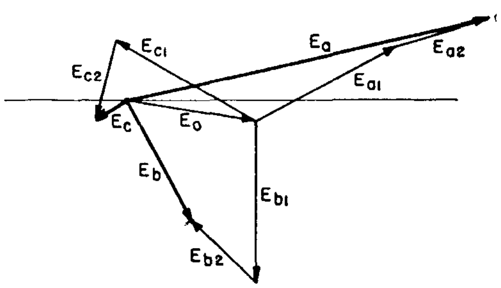


Fig. 7—Combination of the three symmetrical component sets of vectors to obtain the original unbalanced vectors in Fig. 3.

Equations (3) can be used to resolve either line-to-neutral voltages or line-to-line voltages into their components. Inherently, however, since three delta or line-to-line voltages must form a closed triangle, there will be no zero-sequence component for a set of three-phase line-to-line voltages, and $E_{0D}=\frac{1}{3}$ $(E_{ab}+E_{bc}+E_{ca})=0$. The subscript "D" is used to denote components of delta voltages or currents flowing in delta windings.

In many cases it is desirable to know the ratio of the negatives- to positive-sequence amplitudes and the phase angle between them. This ratio is commonly called the unbalance factor and can be conveniently obtained from the chart given in Fig. 8. The angle, θ, by which E_{a2} leads E_{a1} can be obtained also from the same chart. The chart is applicable only to three-phase, three-wire systems, since it presupposes no zero-sequence component. The only data needed to use the chart is the scalar magnitudes of the three line voltages. As an example the chart can be used to determine the unbalance in phase voltages permissible on induction motors without excessive heating. This limit has usually been expressed as a permissible

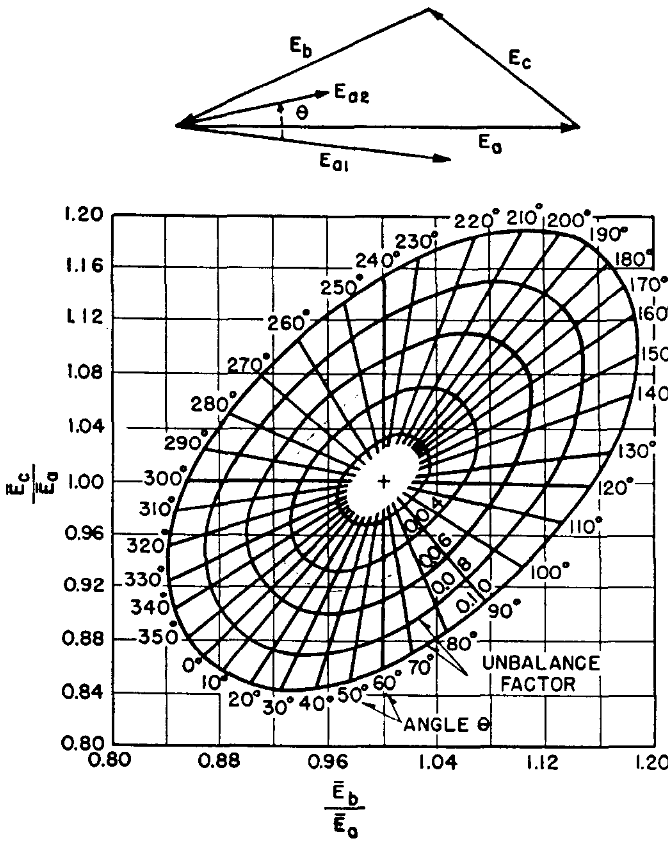


Fig. 8—Determination of unbalance factor.

negative sequence voltage whereas the phase voltages are of course more readily measured.

2. Resolution of Unbalanced Three-Phase Currents

Three line currents can be resolved into three sets of symmetrical component vectors in a manner analogous to that just given for the resolution of voltages.

Referring to Fig. 9:

$$\begin{aligned} I_0 &= I_{a0}=\tfrac{1}{3}(I_a+I_b+I_c) \\ I_1 &= I_{a1}=\tfrac{1}{3}(I_a+aI_b+a^2I_c) \\ I_2 &= I_{a2}=\tfrac{1}{3}(I_a+a^2I_b+aI_c) \end{aligned} \tag{5}$$

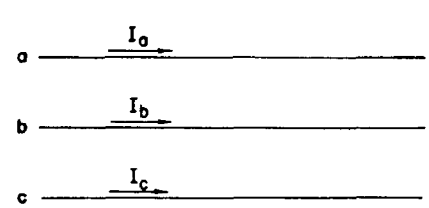


Fig. 9—Three-phase line currents.

The above are, respectively, the zero-, positive-, and negative-sequence components of I_a, the current in the reference phase.

$$\begin{aligned} I_a &= I_{a0}+I_{a1}+I_{a2}=I_0+I_1+I_2 \\ I_b &= I_{b0}+I_{b1}+I_{b2}=I_0+a^2I_1+aI_2 \\ I_c &= I_{c0}+I_{c1}+I_{c2}=I_0+aI_1+a^2I_2 \end{aligned} \tag{6}$$

Three delta currents, Fig. 10, can be resolved into components:

$$
\begin{aligned}
I_{0D}&=\tfrac{1}{3}(I_x+I_y+I_z)\\
I_{1D}&=\tfrac{1}{3}(I_x+aI_y+a^2I_z)\\
I_{2D}&=\tfrac{1}{3}(I_x+a^2I_y+aI_z)
\end{aligned}
\tag{7}
$$

Where I_x has been chosen as the reference phase current.

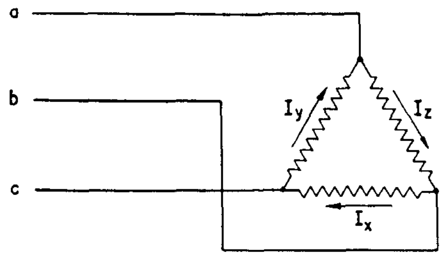


Fig. 10—Three-phase delta currents.

Three line currents flowing into a delta-connected load, or into a delta-connected transformer winding, cannot have a zero-sequence component since $I_a+I_b+I_c$ must obviously be equal to zero. Likewise the currents flowing into a star-connected load cannot have a zero-sequence component unless the neutral wire is returned or the neutral point is connected to ground. Another way of stating this fact is that zero-sequence current cannot flow into a delta-connected load or transformer bank; nor can zero-sequence current flow into a star-connected load or transformer bank unless the neutral is grounded or connected to a return neutral wire.

The choice of which phase to use as reference is entirely arbitrary, but once selected, this phase must be kept as the reference for voltages and currents throughout the system, and throughout the analysis. It is customary in symmetrical component notation to denote the reference phase as "phase a". The voltages and currents over an entire system are then expressed in terms of their components, all referred to the components of the reference phase. The components of voltage, current, impedance, or power found by analysis are directly the components of the reference phase, and the components of voltage, current, impedance, or power for the other phases are easily found by rotating the positive-or negative-sequence components of the reference-phase through the proper angle. The ambiguity possible where star-delta transformations of voltage and current are involved, or where the components of star voltages and currents are to be related to delta voltages and currents, is detailed in a following section.

3. Resolution of Unbalanced Impedances and Admittances

Self Impedances—Unbalanced impedances can be resolved into symmetrical components, although the impedances are vector operators, and not rotating vectors as are three-phase voltages and currents. Consider the three star-impedances of Fig. 11(a), which form an unbalanced load. Their sequence components are:

$$
\begin{aligned}
\mathbf{Z_0}&=\tfrac{1}{3}(Z_a+Z_b+Z_c)\\
\mathbf{Z_1}&=\tfrac{1}{3}(Z_a+aZ_b+a^2Z_c)\\
\mathbf{Z_2}&=\tfrac{1}{3}(Z_a+a^2Z_b+aZ_c)
\end{aligned}
\tag{8}
$$

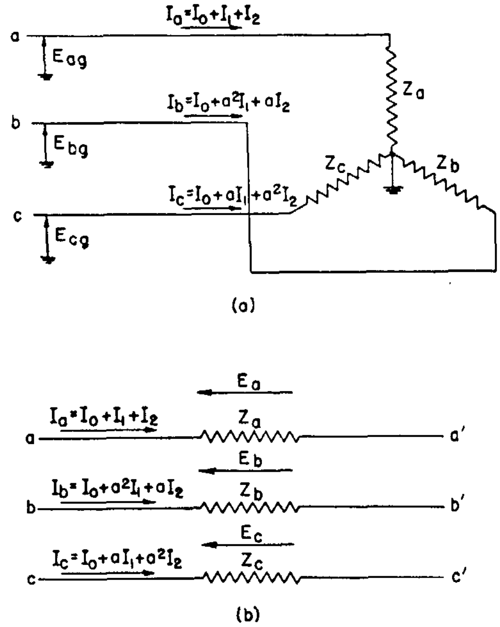


Fig. 11—Three unbalanced self impedances.

The sequence components of current through the impedances, and the sequence components of the line voltages impressed across them are interrelated by the following equations:

$$
\begin{aligned}
E_0&=\tfrac{1}{3}(E_{ag}+E_{bg}+E_{cg})=I_0\mathbf{Z_0}+I_1\mathbf{Z_2}+I_2\mathbf{Z_1}\\
E_1&=\tfrac{1}{3}(E_{ag}+aE_{bg}+a^2E_{cg})=I_0\mathbf{Z_1}+I_1\mathbf{Z_0}+I_2\mathbf{Z_2}\\
E_2&=\tfrac{1}{3}(E_{ag}+a^2E_{bg}+aE_{cg})=I_0\mathbf{Z_2}+I_1\mathbf{Z_1}+I_2\mathbf{Z_0}
\end{aligned}
\tag{9}
$$

The above equations illustrate the fundamental principle that there is mutual coupling between sequences when the circuit constants are not symmetrical. As the equations reveal, both positive- and negative-sequence current (as well as zero-sequence current) create a zero-sequence voltage drop. If $Z_a=Z_b=Z_c$, the impedances are symmetrical, $\mathbf{Z_1}=\mathbf{Z_2}=0$, and $\mathbf{Z_0}=Z_a$. For this condition,

$$
\begin{aligned}
E_0&=I_0\mathbf{Z_0}\\
E_1&=I_1\mathbf{Z_0}\\
E_2&=I_2\mathbf{Z_0}
\end{aligned}
\tag{10}
$$

and, as expected, the sequences are independent. If the neutral point is not grounded in Fig. 11(a), $I_0=0$ but $E_0=I_1\mathbf{Z_2}+I_2\mathbf{Z_1}$ so that there is a zero-sequence voltage, representing a neutral voltage shift, created by positive- and negative-sequence current flowing through the unbalanced load.

Equations (8) and (9) also hold for unsymmetrical series line impedances, as shown in Fig. 11(b), where E_0, E_1, and E_2 are components of E_a, E_b, and E_c, the voltage drops across the impedances in the three phases.

Mutual Impedances between phases can also be resolved into components. Consider Z_{mbc} of Fig. 12(a), as reference, then

$$
\begin{aligned}
\mathbf{Z_{m0}}&=\tfrac{1}{3}(Z_{mbc}+Z_{mca}+Z_{mab})\\
\mathbf{Z_{m1}}&=\tfrac{1}{3}(Z_{mbc}+aZ_{mca}+a^2Z_{mab})\\
\mathbf{Z_{m2}}&=\tfrac{1}{3}(Z_{mbc}+a^2Z_{mca}+aZ_{mab})
\end{aligned}
\tag{11}
$$

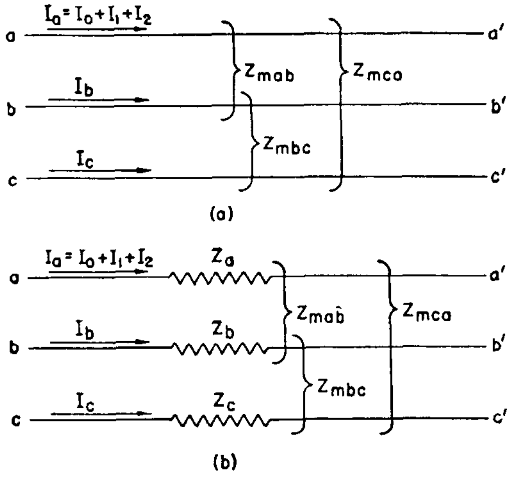


Fig. 12

(a) Three unbalanced mutual impedances.
(b) Unbalanced self and mutual impedances.

The components of the three-phase line currents and the components of the three-phase voltage drops created by the mutual impedances will be interrelated by the following equations:

$$\begin{aligned}E_0&=\tfrac{1}{3}(E_{aa'}+E_{bb'}+E_{cc'})=2I_0\mathbf{Z}_{m0}-I_1\mathbf{Z}_{m2}-I_2\mathbf{Z}_{m1}\\E_1&=\tfrac{1}{3}(E_{aa'}+aE_{bb'}+a^2E_{cc'})=-I_0\mathbf{Z}_{m1}-I_1\mathbf{Z}_{m0}+2I_2\mathbf{Z}_{m2}\\E_2&=\tfrac{1}{3}(E_{aa'}+a^2E_{bb'}+aE_{cc'})=-I_0\mathbf{Z}_{m2}+2I_1\mathbf{Z}_{m1}-I_2\mathbf{Z}_{m0}\end{aligned}\tag{12}$$

If, as in Fig. 12(b), both self and mutual impedances are present in a section of a three-phase circuit, the symmetrical components of the three voltage drops across the section are:

$$\begin{aligned}E_0&=\tfrac{1}{3}(E_{aa'}+E_{bb'}+E_{cc'})\\&=I_0(\mathbf{Z}_0+2\mathbf{Z}_{m0})+I_1(\mathbf{Z}_2-\mathbf{Z}_{m2})+I_2(\mathbf{Z}_1-\mathbf{Z}_{m1})\\E_1&=\tfrac{1}{3}(E_{aa'}+aE_{bb'}+a^2E_{cc'})\\&=I_0(\mathbf{Z}_1-\mathbf{Z}_{m1})+I_1(\mathbf{Z}_0-\mathbf{Z}_{m0})+I_2(\mathbf{Z}_2+2\mathbf{Z}_{m2})\\E_2&=\tfrac{1}{3}(E_{aa'}+a^2E_{bb'}+aE_{cc'})\\&=I_0(\mathbf{Z}_2-\mathbf{Z}_{m2})+I_1(\mathbf{Z}_1+2\mathbf{Z}_{m1})+I_2(\mathbf{Z}_0-\mathbf{Z}_{m0})\end{aligned}\tag{13}$$

Again, if both self and mutual impedances are symmetrical, in all three phases,

$$\begin{aligned}E_0&=I_0(\mathbf{Z}_0+2\mathbf{Z}_{m0})=I_0Z_0\\E_1&=I_1(\mathbf{Z}_0-\mathbf{Z}_{m0})=I_1Z_1\\E_2&=I_2(\mathbf{Z}_0-\mathbf{Z}_{m0})=I_2Z_2\end{aligned}\tag{14}$$

Where Z_0, Z_1, and Z_2 are, respectively, the impedance to zero-, positive-, and negative-sequence. For this condition positive-sequence currents produce only a positive-sequence voltage drop, etc. Z_0, Z_1, and Z_2 are commonly referred to as the zero-sequence, positive-sequence, and negative-sequence impedances. Note, however, that this is not strictly correct and that Z_1, the impedance to positive-sequence currents, should not be confused with $\mathbf{Z}_1$, the positive sequence component of self impedances. Since Z_0, Z_1, and Z_2 are used more frequently than $\mathbf{Z}_0$, $\mathbf{Z}_1$, and $\mathbf{Z}_2$ the shorter expression "zero-sequence impedance" is usually used to refer to Z_0 rather than $\mathbf{Z}$. For a circuit that has only symmetrical impedances, both self and mutual, the sequences are independent of each other, and positive-sequence currents produce only positive-sequence voltage drops, etc. Fortunately, except for unsymmetrical loads, unsymmetrical transformer connections, etc., the three-phase systems usually encountered are symmetrical (or balanced) and the sequences are independent.

Admittances can be resolved into symmetrical components, and the components used to find the sequence components of the currents through a three-phase set of line impedances, or star-connected loads, as functions of the symmetrical components of the voltage drops across the impedances. In Fig. 11(a), let $Y_a=\frac{1}{Z_a}$, $Y_b=\frac{1}{Z_b}$, $Y_c=\frac{1}{Z_c}$, then

$$\begin{aligned}\mathbf{Y}_0&=\tfrac{1}{3}(Y_a+Y_b+Y_c)\\\mathbf{Y}_1&=\tfrac{1}{3}(Y_a+aY_b+a^2Y_c)\\\mathbf{Y}_2&=\tfrac{1}{3}(Y_a+a^2Y_b+aY_c)\end{aligned}\tag{15}$$

and

$$\begin{aligned}I_0&=E_0\mathbf{Y}_0+E_1\mathbf{Y}_2+E_2\mathbf{Y}_1\\I_1&=E_0\mathbf{Y}_1+E_1\mathbf{Y}_0+E_2\mathbf{Y}_2\\I_2&=E_0\mathbf{Y}_2+E_1\mathbf{Y}_1+E_2\mathbf{Y}_0\end{aligned}\tag{16}$$

Note, however, that $\mathbf{Y}_0$ is not the reciprocal of $\mathbf{Z}_0$, as defined in Eq. 8, $\mathbf{Y}_1$ is not the reciprocal of $\mathbf{Z}_1$, and $\mathbf{Y}_2$ is not the reciprocal of $\mathbf{Z}_2$, unless $Z_a=Z_b=Z_c$; in other words, the components of admittance are not reciprocals of the corresponding components of impedance unless the three impedances (and admittances) under consideration are equal.

4. Star-Delta Conversion Equations

If a delta arrangement of impedances, as in Fig. 13(a), is to be converted to an equivalent star shown in Fig. 13(b), the following equations are applicable.

$$\begin{aligned}Z_a&=\frac{1}{Y_a}=\frac{Z_{ab}\times Z_{ca}}{Z_{ab}+Z_{bc}+Z_{ca}}\\Z_b&=\frac{1}{Y_b}=\frac{Z_{ab}\times Z_{bc}}{Z_{ab}+Z_{bc}+Z_{ca}}\\Z_c&=\frac{1}{Y_c}=\frac{Z_{bc}\times Z_{ca}}{Z_{ab}+Z_{bc}+Z_{ca}}\end{aligned}\tag{17}$$

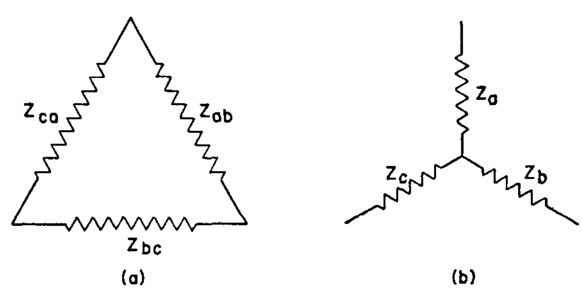


Fig. 13—Star-delta impedance conversions.

When the delta impedances form a three-phase load, no zero-sequence current can flow from the line to the load; hence, the equivalent star load must be left with neutral ungrounded.

The reverse transformation, from the star impedances of Fig. 13(b), to the equivalent delta Fig. 13(a), is given by

$$
\begin{aligned}
Z_{ab} &= Z_a + Z_b + \frac{Z_a Z_b}{Z_c} \\
Z_{bc} &= Z_b + Z_c + \frac{Z_b Z_c}{Z_a} \qquad (18) \\
Z_{ca} &= Z_c + Z_a + \frac{Z_c Z_a}{Z_b}
\end{aligned}
$$

An equivalent delta for a star-connected, three-phase load with neutral grounded cannot be found, since zero-sequence current can flow from the line to the star load and return in the ground, but cannot flow from the line to any delta arrangement.

III. RELATIONSHIP BETWEEN SEQUENCE COMPONENTS OF LINE-TO-LINE AND LINE-TO-NEUTRAL VOLTAGES

Assume that E_{ag}, E_{bg}, and E_{cg}, are a positive-sequence set of line-to-neutral vectors in Fig. 14(a). The line-to-line voltages will also form a positive-sequence set of vectors. The relationship between the two sets of three-phase vectors is shown in Fig. 14(b). Although E_{1D} (the positive-sequence component of the line-to-line voltages) will be numerically equal to $\sqrt{3}E_1$—E_1 is the positive-sequence component of the line-to-neutral voltages (which is equal in this case to E_{ag}); the angular relationship between E_1 and E_{1D} depends upon the line-to-line voltage taken as reference. The choice is arbitrary. Table 2 gives the relation between E_{1D} and E_1 for various line-to-line phases selected as reference.

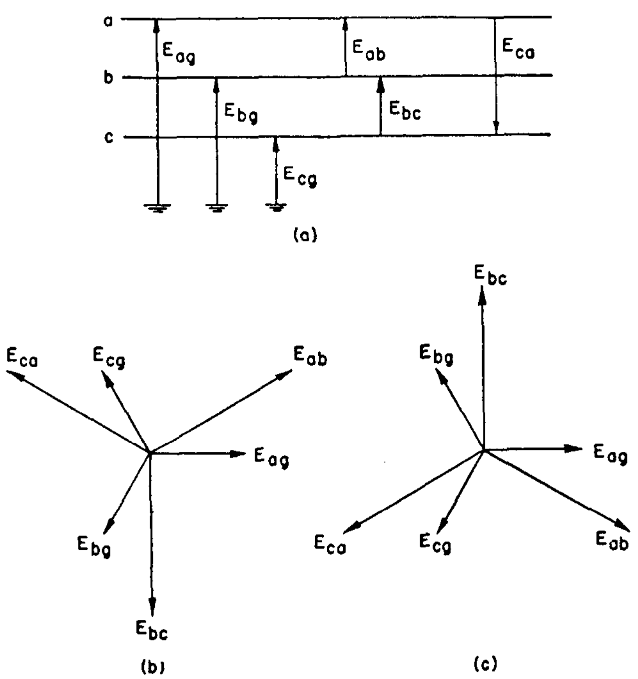


Fig. 14—Relationships between line-to-line and line-to-neutral components of voltage.

(b) Positive-sequence relationships.
(c) Negative-sequence relationships.

TABLE 2

Reference Phase Line-to-Line Voltages	Positive-Sequence Line-to-Line Voltage As a Function of Positive Sequence Line-to-Neutral Voltage
AB	$E_{1D} = E_{ab} = \sqrt{3}E_1\epsilon^{j30} = (1-a^2)E_1$
BC	$E_{1D} = E_{bc} = -j\sqrt{3}E_1 = (a^2-a)E_1$
CA	$E_{1D} = E_{ca} = \sqrt{3}E_1\epsilon^{j150} = (a-1)E_1$
BA	$E_{1D} = E_{ba} = \sqrt{3}E_1\epsilon^{-j150} = (a^2-1)E_1$
CB	$E_{1D} = E_{cb} = j\sqrt{3}E_1 = (a-a^2)E_1$
AC	$E_{1D} = E_{ac} = \sqrt{3}E_1\epsilon^{-j30} = (1-a)E_1$

If E_{ag}, E_{bg}, and E_{cg}, form a negative-sequence set of vectors, the vector diagram of Fig. 14(c) illustrates the relation between $E_2 = E_{ag}$, and E_{2D}, the negative-sequence component of the line-to-line voltages. Again, the algebraic relation expressing E_{2D} as a function of E_2 will depend upon the line-to-line phase selected for reference, as illustrated in Table 3.

TABLE 3

Reference Phase	Negative-Sequence Line-to-Line Voltage As a Function of Negative Sequence Line-to-Neutral Voltage
AB	$E_{2D} = E_{ab} = \sqrt{3}E_2\epsilon^{-j30} = (1-a)E_2$
BC	$E_{2D} = E_{bc} = j\sqrt{3}E_2 = (a-a^2)E_2$
CA	$E_{2D} = E_{ca} = \sqrt{3}E_2\epsilon^{-j150} = (a^2-1)E_2$
BA	$E_{2D} = E_{ba} = \sqrt{3}E_2\epsilon^{j150} = (a-1)E_2$
CB	$E_{2D} = E_{cb} = -j\sqrt{3}E_2 = (a^2-a)E_2$
AC	$E_{2D} = E_{ac} = \sqrt{3}E_2\epsilon^{j30} = (1-a^2)E_2$

Since the line-to-line voltages cannot have a zero-sequence component, $E_{0D}=0$ under all conditions, and E_0 is an indeterminate function of E_{0D}.

The equations expressing E_{1D} as a function of E_1, and E_{2D} as a function of E_2, can be solved to express E_1 and E_2 as functions of E_{1D} and E_{2D}, respectively. Refer to Table 4 for the relationships.

TABLE 4

Reference Phase		
AB	$E_1 = \frac{E_{1D}}{\sqrt{3}}\epsilon^{-j30} = \frac{1-a}{3}E_{1D}$	$E_2 = \frac{E_{2D}}{\sqrt{3}}\epsilon^{j30} = \frac{1-a^2}{3}E_{2D}$
BC	$E_1 = j\frac{E_{1D}}{\sqrt{3}} = \frac{a-a^2}{3}E_{1D}$	$E_2 = -j\frac{E_{2D}}{\sqrt{3}} = \frac{a^2-a}{3}E_{2D}$
CA	$E_1 = \frac{E_{1D}}{\sqrt{3}}\epsilon^{-j150} = \frac{a^2-1}{3}E_{1D}$	$E_2 = \frac{E_{2D}}{\sqrt{3}}\epsilon^{j150} = \frac{a-1}{3}E_{2D}$
BA	$E_1 = \frac{E_{1D}}{\sqrt{3}}\epsilon^{j150} = \frac{a-1}{3}E_{1D}$	$E_2 = \frac{E_{2D}}{\sqrt{3}}\epsilon^{-j150} = \frac{a^2-1}{3}E_{2D}$
CB	$E_1 = -j\frac{E_{1D}}{\sqrt{3}} = \frac{a^2-a}{3}E_{1D}$	$E_2 = j\frac{E_{2D}}{\sqrt{3}} = \frac{a-a^2}{3}E_{2D}$
AC	$E_1 = \frac{E_{1D}}{\sqrt{3}}\epsilon^{j30} = \frac{1-a^2}{3}E_{1D}$	$E_2 = \frac{E_{2D}}{\sqrt{3}}\epsilon^{-j30} = \frac{1-a}{3}E_{2D}$

Certain authors have arbitrarily adopted phase CB as reference, since the relations between the line-to-line and line-to-neutral components are easily remembered and the angular shift of 90 degrees is easy to carry in computations. Using this convention:

$$E_{1D}=j\sqrt{3}E_1 \qquad E_1=-j\frac{E_{1D}}{\sqrt{3}}$$

$$E_{2D}=-j\sqrt{3}E_2 \qquad E_2=j\frac{E_{2D}}{\sqrt{3}} \tag{19}$$

$$E_{0D}=0 \qquad E_0 \text{ is not a function of } E_{0D}$$

The equations and vector diagrams illustrate the interesting fact that the numerical relation between the line-to-line and line-to-neutral positive-sequence components is the same as for negative-sequence; but that the angular shift for negative-sequence is opposite to that for positive-sequence, regardless of the delta phase selected for reference. Also, a connection of power or regulating transformers giving a shift of θ degrees in the transformation for positive-sequence voltage and current will give a shift of $-\theta$ degrees in the transformation for negative-sequence voltage and current.

IV. SEQUENCE COMPONENTS OF LINE AND DELTA CURRENTS

The relation existing between the positive-sequence component of the delta currents and the positive-sequence component of the line currents flowing into a delta load or delta-connected transformer winding, and the relation existing for the negative-sequence components of the currents are given in Figs. 15(b) and 15(c). Although the components of line currents are $\sqrt{3}$ times the delta phase selected for reference, the angular relationship depends upon the phase selected for reference. I_a is taken as reference for the line currents. Refer to Table 5.

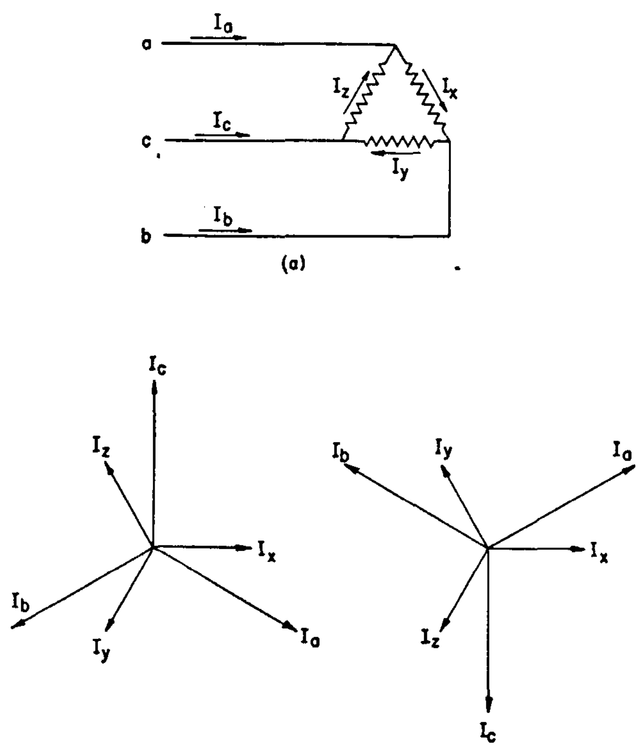


Fig. 15—Relationships between components of phase and delta currents.

(b) Positive-sequence relationships.
(c) Negative-sequence relationships.

TABLE 5

Delta Reference Current	Fig. 14(b)	Fig. 14(c)
I_x	$I_{1D}=I_x=\frac{1}{\sqrt{3}}I_1\epsilon^{j30}$	$I_{2D}=I_x=\frac{1}{\sqrt{3}}I_2\epsilon^{-j30}$
I_y	$I_{1D}=I_y=\frac{-j}{\sqrt{3}}I_1$	$I_{2D}=I_y=\frac{j}{\sqrt{3}}I_2$
I_z	$I_{1D}=I_z=\frac{1}{\sqrt{3}}I_1\epsilon^{j150}$	$I_{2D}=I_z=\frac{1}{\sqrt{3}}I_2\epsilon^{-j150}$
$-I_x$	$I_{1D}=-I_x=\frac{1}{\sqrt{3}}I_1\epsilon^{-j150}$	$I_{2D}=-I_x=\frac{1}{\sqrt{3}}I_2\epsilon^{j150}$
$-I_y$	$I_{1D}=-I_y=\frac{j}{\sqrt{3}}I_1$	$I_{2D}=-I_y=\frac{-j}{\sqrt{3}}I_2$
$-I_z$	$I_{1D}=-I_z=\frac{1}{\sqrt{3}}I_1\epsilon^{-j30}$	$I_{2D}=-I_z\frac{1}{\sqrt{3}}I_2\epsilon^{j30}$

If the current $(-I_y)$ is taken as reference, the relation are easily remembered; also, the j operator is convenien. to use in analysis.

$$I_{1D}=\frac{j}{\sqrt{3}}I_1 \qquad I_1=-j\sqrt{3}I_{1D}$$

$$I_{2D}=\frac{-j}{\sqrt{3}}I_2 \qquad I_2=j\sqrt{3}I_{2D} \tag{20}$$

V. STAR-DELTA TRANSFORMATIONS OF VOLTAGE AND CURRENT

Each sequence component of voltage and current must be followed separately through the transformer, and the angular shift of the sequence will depend upon the input and output phases arbitrarily selected for reference. In Fig. 16(a), the winding ratio is n and the overall transformation ratio is $N=\frac{n}{\sqrt{3}}$. Line-to-line or line-to-neutral voltages on the delta side will be N times the corresponding voltages on the star side of the transformer (neglecting impedance drop). If the transformer windings are symmetrical in the three phases, there will be no interaction between sequences, and each sequence component of voltage or current is transformed independently.

To illustrate the sequence transformations, phases a and a' have been selected as reference phases in the two circuits. Figs. 16(b), (c), (d), and (e) give the relationships for the three phases with each component of voltage and current considered separately.

From the vector diagrams

$$E_1'=NE_1\epsilon^{j30}$$
$$I_1'=\frac{1}{N}I_1\epsilon^{j30}$$
$$E_2'=NE_2\epsilon^{-j30} \tag{21}$$
$$I_2'=\frac{1}{N}I_2\epsilon^{-j30}$$

Regardless of the phases selected for reference, both positive-sequence current and voltage will be shifted *in the same direction by the same angle.* Negative-sequence current and voltage will also be shifted the *same angle* in

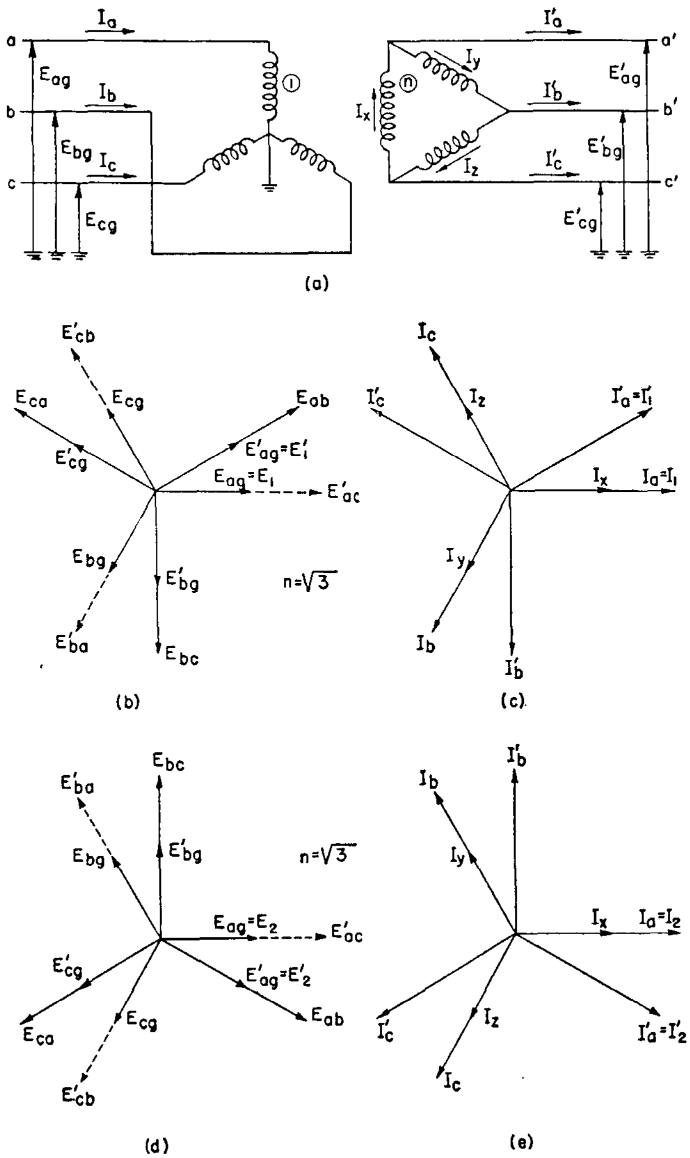


Fig. 16—Transformation of the sequence components of current and voltage in a star-delta transformer bank.

(b) Relationship of positive-sequence line-to-neutral and line-to-line voltages.
(c) Relationship of positive-sequence currents.
(d) Relationship of negative-sequence line-to-neutral and line-to-line voltages.
(e) Relationship of negative-sequence currents.

one direction, and the negative-sequence angular shift will be equal to the positive-sequence shift *but in the opposite direction*. As previously stated, this is a general rule for all connections of power and regulating transformers, wherever phase shift is involved in the transformation.

Since zero-sequence current cannot flow from the delta winding, there will be no zero-sequence component of I_a'. If the star winding is grounded, I_a may have a zero-sequence component. From the star side the transformer bank acts as a return path for zero-sequence current (if the neutral is grounded), and from the delta side the bank acts as an open circuit to zero-sequence. For zero-sequence current alone, $I_a=I_b=I_c=I_0$, and a current will circulate around the delta such that $I_x=I_y=I_z=I_{od}=\frac{1}{n}I_o$. The zero-sequence line-to-neutral voltages, E_o and E_o' are entirely independent; each being determined by conditions in its respective circuit. The transformation characteristics for the three sequence currents and voltages, and the sequence impedance characteristics, for common connections of power and regulating transformers are given in Chap. 5. The action of a transformer bank in the transformation of zero-sequence currents must be given particular attention, since certain connections do not permit zero-sequence current to flow, others permit it to pass through the bank without transformation, and still others transform zero-sequence quantities in the same manner as positive- or negative-sequence quantities are transformed.

VI. THREE-PHASE POWER

The total three-phase power of a circuit can be expressed in terms of the symmetrical components of the line currents and the symmetrical components of the line-to-neutral voltages.

$$P=3(E_0I_0\cos\theta_0+E_1I_1\cos\theta_1+E_2I_2\cos\theta_2) \qquad (22)$$

where θ_0 is the angle between E_0 and I_0, θ_1 the angle between E_1 and I_1, θ_2 the angle between E_2 and I_2. The equation shows that the total power is the sum of the three components of power; but the power in one phase of an unbalanced circuit is not one-third of the above expression, since each phase will contain components of power resulting from zero-sequence voltage and positive-sequence current, etc. This power "between sequences" is generated in one phase and absorbed by the others, and does not appear in the expression for total three-phase power.

Only positive-sequence power is developed by the generators. This power is converted to negative-sequence and zero-sequence power by circuit dissymmetry such as occurs from a single line-to-ground or a line-to-line fault. The unbalanced fault, unbalanced load, or other dissymmetry in the circuit thus acts as the "generator" for negative-sequence and zero-sequence power.

VII. CONJUGATE SETS OF VECTORS

Since power in an alternating-current circuit is defined as $E\hat{I}$ (the vector E times the conjugate of the vector I), some consideration should be given to conjugates of the symmetrical-component sets of vectors. A system of positive-sequence vectors are drawn in Fig. 17(a). In

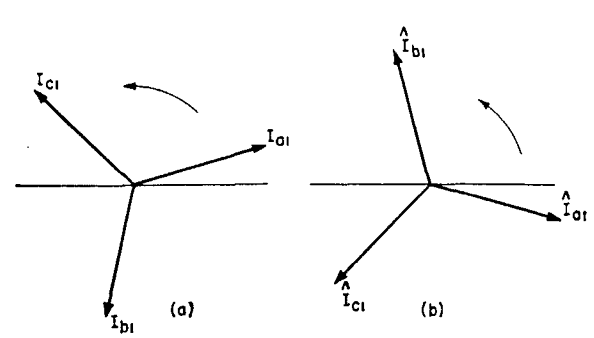


Fig. 17—Conjugates of a positive-sequence set of vectors.

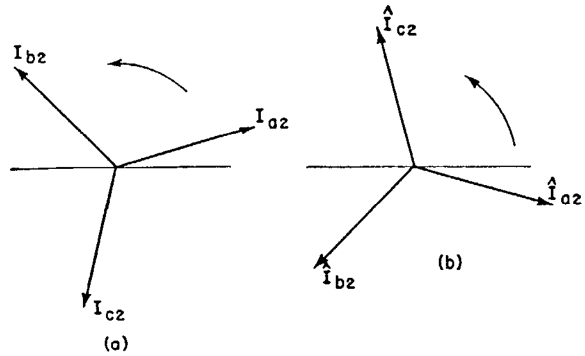


Fig. 18—Conjugates of a negative-sequence set of vectors.

accordance with the definition that the conjugate of a given vector is a vector of the same magnitude but displaced the same angle from the reference axis in the *opposite* direction to the given vector, the conjugates of the positive-sequence set of vectors are shown in Fig. 17(b). Note that the conjugates to a positive-sequence set of vectors form a negative-sequence set of vectors. Similarly, as in Fig. 18, the conjugates to a negative-sequence set of vectors form a positive-sequence set. The conjugate of a zero-sequence set of vectors is another zero-sequence set of vectors, see Fig. 19.

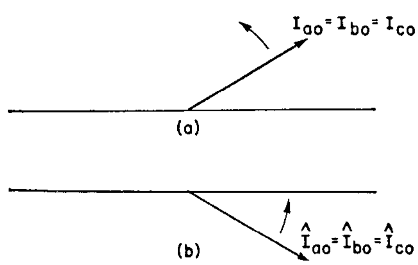


Fig. 19—Conjugates of a zero-sequence set of vectors.

VIII. SEQUENCE NETWORKS

5. General Considerations

One of the most useful concepts arising from symmetrical components is that of the sequence network, which is an equivalent network for the balanced power system under an imagined operating condition such that only one sequence component of voltages and currents is present in the system. As shown above for the case of balanced loads (and it can be readily shown in general) currents of one sequence will create voltage drops of that sequence only, if a power system is balanced (equal series impedances in all three phases, equal mutual impedances between phases, rotating machines symmetrical in all three phases, all banks of transformers symmetrical in all three phases, etc.). There will be no interaction between sequences and the sequences are independent. Nearly all power systems can be assumed to be balanced except for emergency conditions such as short-circuits, faults, unbalanced load, unbalanced open circuits, or unsymmetrical conditions arising in rotating machines. Even under such emergency unbalanced conditions, which usually occur at only one point in the system, the remainder of the power system remains balanced and an equivalent sequence network can be obtained for the balanced part of the system. The advantage of the sequence network is that, since currents and voltages of only one sequence are present, the three-phase system can be represented by an equivalent single-phase diagram. The entire sequence network can often be reduced by simple manipulation to a single voltage and a single impedance. The type of unbalance or dissymmetry in the circuit can be represented by an interconnection between the equivalent sequence networks.

The positive-sequence network is the only one of the three that will contain generated voltages, since alternators can be assumed to generate only positive-sequence voltages. The voltages appearing in the negative- and zero-sequence networks will be generated by the unbalance, and will appear as voltages impressed on the networks at the point of fault. Furthermore, the positive-sequence network represents the system operating under normal balanced conditions. For short-circuit studies the internal voltages are shorted and the positive sequence network is driven by the voltage appearing at the fault before the fault occurred according to the theory of Superposition and the Compensation Theorems (see Chapter 10, Section 11). This gives exactly the increments or changes in system quantities over the system. Since the fault current equals zero before the fault, the increment alone is the fault current total. However, the normal currents in any branch must be added to the calculated fault current in the same branch to get the total current in any branch after the fault occurs.

6. Setting Up the Sequence Networks

The equivalent circuits for each sequence are set up "as viewed from the fault," by imagining current of the particular sequence to be circulated through the network from the fault point, investigating the path of current flow and the impedance of each section of the network to currents of that sequence. Another approach is to imagine in each network a voltage impressed across the terminals of the network, and to follow the path of current flow through the network, dealing with each sequence separately. It is particularly necessary when setting up the zero-sequence network to start at the fault point, or point of unbalance, since zero-sequence currents might not flow over the entire system. Only parts of the system over which zero-sequence current will flow, as the result of a zero-sequence voltage impressed at the unbalanced point, are included in the zero-sequence network "as viewed from the fault." The two terminals for each network correspond to the two points in the three-phase system on either side of the unbalance. For the case of shunt faults between conductors and ground, one terminal of each network will be the fault point in the three-phase system, the other terminal will be ground or neutral at that point. For a series unbalance, such as an open conductor, the two terminals will correspond to the two points in the three-phase system immediately adjacent to the unbalance.

7. Sequence Impedances of Lines, Transformers, and Rotating Machinery

The impedance of any unit of the system—such as a generator, a transformer, or a section of line—to be inserted in a sequence network is obtained by imagining unit current of that sequence to be circulated through the apparatus or line in all three phases, and writing the equation for the voltage drop; or by actually measuring the voltage drop when current of the one sequence being investigated is circulated through the three phases of the apparatus. The impedance to negative-sequence currents for all static non-rotating apparatus will be equal to the impedance for positive-sequence currents. The impedance to negative-sequence currents for rotating apparatus will in general be different from the impedance to positive sequence. The impedance to zero-sequence currents for all apparatus will in general be different from either the impedance to positive-sequence or the impedance to negative-sequence. The sequence impedance characteristics of the component parts of a power system have been investigated in detail and are discussed in Chaps. 3, 4, 5, and 6.

An impedance in the neutral will not appear in either the positive-sequence network or the negative-sequence network, since the three-phase currents of either sequence add to zero at the neutral; an equivalent impedance equal to three times the ohmic neutral impedance will appear in the zero-sequence network, however, since the zero-sequence currents flowing in the three phases, I_0 add directly to give a neutral current of $3I_0$.

8. Assumed Direction of Current Flow

By convention, the positive direction of current flow in each sequence network is taken as being outward at the faulted or unbalanced point; thus the sequence currents are assumed to flow in the same direction in all three sequence networks. This convention of assumed current flow must be carefully followed to avoid ambiguity or error even though some of the currents are negative. After the currents flowing in each network have been determined, the sequence voltage at any point in the network can be found by *subtracting* the impedance drops of that sequence from the generated voltages, taking the neutral point of the network as the point of zero voltage. For example, if the impedances to positive-, negative-, and zero-sequence between neutral and the point in question are Z_1, Z_2, and Z_0, respectively, the sequence voltages at the point will be

$$\begin{aligned} E_1 &= E_{a1} - I_1 Z_1 \\ E_2 &= -I_2 Z_2 \\ E_0 &= -I_0 Z_0 \end{aligned} \tag{23}$$

where E_{a1} is the generated positive-sequence voltage, the positive-sequence network being the only one of the three having a generated voltage between neutral and the point for which voltages are to be found. In particular, if Z_1, Z_2 and Z_0 are the total equivalent impedances of the networks to the point of fault, then Eq. (23) gives the sequence voltages at the fault.

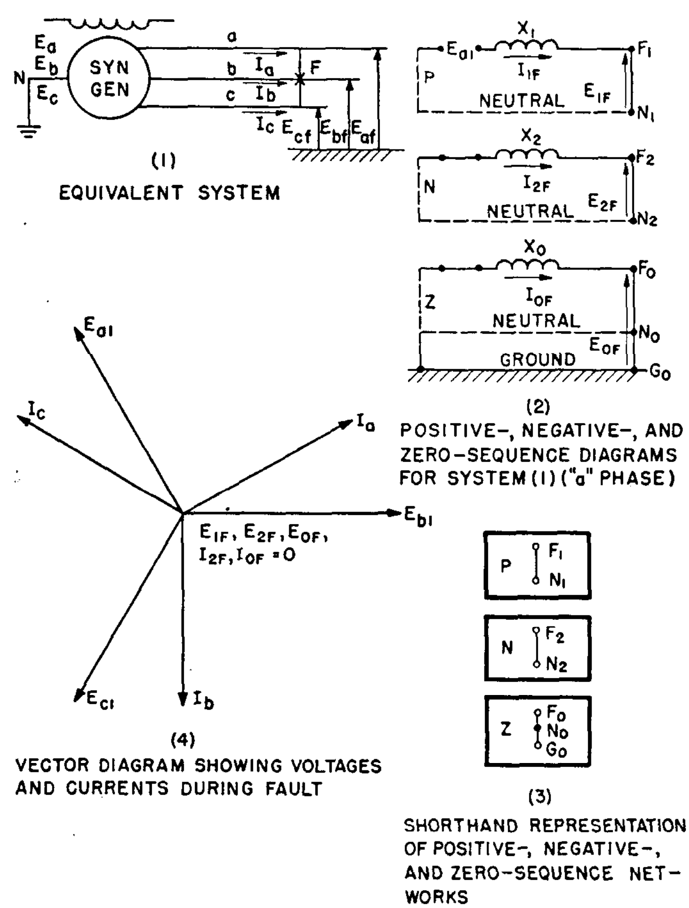
Distribution Factors—If several types of unbalance are to be investigated for one point in the system, it is convenient to find distribution factors for each sequence current by circulating unit sequence current in the terminals of each network, letting it flow through the network and finding how this current distributes in various branches. Regardless of the type of fault, and the magnitude of sequence current at the fault, the current will distribute through each network in accordance with the distribution factors found for unit current. This follows from the fact that within any one of the three networks the currents and voltages of that sequence are entirely independent of the other two sequences.

These points will be clarified by detailed consideration of a specific example at the end of this chapter.

IX. CONNECTIONS BETWEEN THE SEQUENCE NETWORKS

As discussed in Part II, Sec. 3 of this chapter, any unbalance or dissymmetry in the system will result in mutual action between the sequences, so that it is to be expected that the sequence networks will have mutual coupling, or possibly direct connections, between them at the point of unbalance. Equations can be written for the conditions existing at the point of unbalance that show the coupling or connections necessarily existing between the sequence networks at that point.

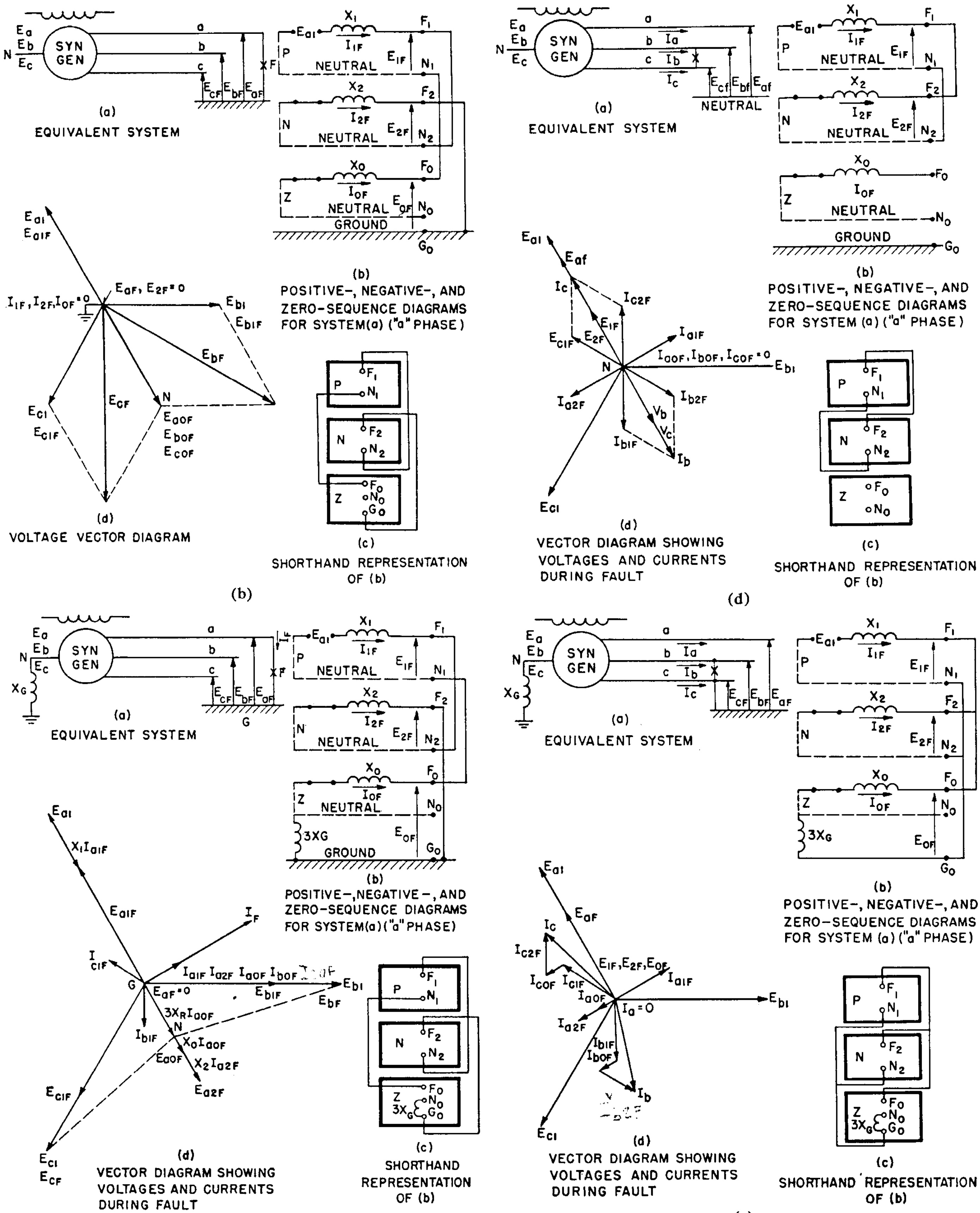
As pointed out in Sec. 5, it is usually sufficiently accurate to reduce a given system to an equivalent source and single reactance to the point of fault. This in effect means that the system is reduced to a single generator with a fault applied at its terminals. Figs. 20(a) through 20(e) show such an equivalent system with the more common types of faults applied. For example Fig. 20(a) is drawn for a three-



(a)

Fig. 20.

(a) Three-phase short circuit on generator.



(c) **Fig. 20** (e)

(b) Single-line-to-ground fault on ungrounded generator.
(c) Single-line-to-ground fault on generator grounded through a neutral reactor.
(d) Line-to-line fault on grounded or ungrounded generator.
(e) Double-line-to-ground fault on generator grounded through a neutral reactor.

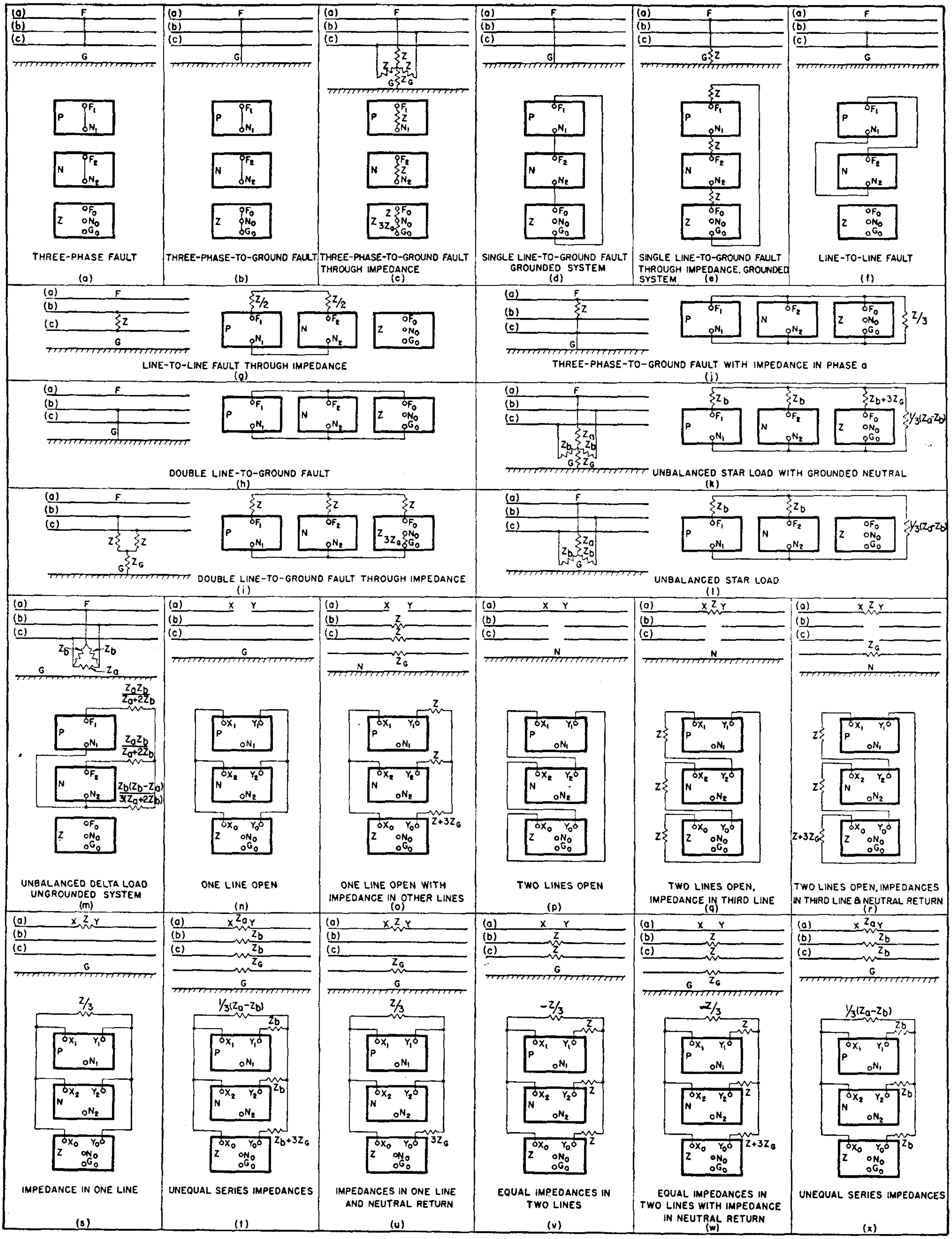


Fig. 21—Connection of the sequence networks to represent shunt and series unbalanced conditions. For shunt unbalances the faulted point in the system is represented by F and neutral by N. Corresponding points are represented in the sequence networks by the letter with a sequence subscript. P, N, and Z refer to the positive-, negative-, and zero-sequence networks, respectively. For series unbalances, points in the system adjacent to the unbalance are represented by X and Y. N is again the neutral.

phase fault on the system. Part (1) shows the equivalent system (2) the corresponding positive- negative- and zero-sequence diagrams, and (3) the shorthand representation of the sequence diagrams. Part (4) is a vector diagram showing graphically the relationship between the various voltages and currents. In the zero-sequence diagrams of (2) and (3) a distinction is made between "neutral", N, and "ground", G. In the positive- and negative-sequence networks no such distinction is necessary, since by their definition positive- and negative-sequence quantities are balanced with respect to neutral. For example, all positive- and negative-sequence currents add to zero at the system neutral so that the terms "neutral" and "ground" are synonymous. Zero-sequence quantities however, are not balanced with respect to neutral. Thus, by their nature zero-sequence currents require a neutral or ground return path. In many cases impedance exists between neutral and ground and when zero-sequence currents flow a voltage drop exists between neutral and ground. Therefore, it is necessary that one be specific when speaking of line-to-neutral and line-to-ground zero-sequence voltages. They are the same only when no impedance exists between the neutral and ground.

In parts (3) of Fig. 20(a) all portions of the network within the boxes are balanced and only the terminals at the point of unbalance are brought out. The networks as shown are for the "a" or reference phase only. In Eqs. (25) through (29) the zero-sequence impedance, Z_o, is infinite for the case of Fig. 20(b) and includes $3X_G$ in the case of Fig. 20(e). Fig. 21 gives a summary of the connections required to represent the more common types of faults encountered in power system work.

Equations for calculating the sequence quantities at the point of unbalance are given below for the unbalanced conditions that occur frequently. In these equations E_{1F}, E_{2F}, and E_{0F} are components of the line-to-neutral voltages at the point of unbalance; I_{1F}, I_{2F}, and I_{0F} are components of the fault current I_F; Z_1, Z_2, and Z_o are impedances of the system (as viewed from the unbalanced terminals) to the flow of the sequence currents; and E_a is the line-to-neutral positive-sequence generated voltage.

9. Three-Phase Fault—Fig. 20(a)

$$I_{1F}=I_F=\frac{E_{a1}}{Z_1} \tag{24}$$

10. Single Line-to-Ground Fault—Fig. 20(b)

$$I_{1F}=I_{2F}=I_{0F}=\frac{E_{a1}}{Z_1+Z_2+Z_0} \tag{25}$$

$$I_F=I_{1F}+I_{2F}+I_{0F}=3I_{0F} \tag{26}$$

$$E_{1F}=E_{a1}-I_{1F}Z_1=E_{a1}\frac{(Z_2+Z_0)}{Z_1+Z_2+Z_0} \tag{27}$$

$$E_{2F}=-I_{2F}Z_2=-\frac{E_{a1}Z_2}{Z_1+Z_2+Z_0} \tag{28}$$

$$E_{0F}=-I_{0F}Z_0=-\frac{E_{a1}Z_0}{Z_1+Z_2+Z_0} \tag{29}$$

11. Line-to-Line Fault—Fig. 20(d)

$$I_{1F}=-I_{2F}=\frac{E_{a1}}{Z_1+Z_2} \tag{30}$$

$$I_F=\sqrt{3}I_{1F} \tag{31}$$

$$E_{1F}=E_{a1}-I_{1F}Z_1=\frac{E_{a1}Z_2}{Z_1+Z_2} \tag{32}$$

$$E_{2F}=-I_{2F}Z_2=\frac{E_{a1}Z_2}{Z_1+Z_2} \tag{33}$$

12. Double Line-to-Ground Fault—Fig. 20(e)

$$I_{1F}=\frac{E_{a1}}{Z_1+\dfrac{Z_2Z_0}{Z_2+Z_0}}=\frac{E_{a1}(Z_2+Z_0)}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{34}$$

$$I_{2F}=-\frac{Z_0}{Z_2+Z_0}I_{1F}=\frac{-Z_0E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{35}$$

$$I_{0F}=-\frac{Z_2}{Z_2+Z_0}I_{1F}=\frac{-Z_2E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{36}$$

$$E_{1F}=E_{a1}-I_{1F}Z_1=\frac{Z_2Z_0E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{37}$$

$$E_{2F}=-I_{2F}Z_2=\frac{Z_2Z_0E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{38}$$

$$E_{0F}=-I_{0F}Z_0=\frac{Z_2Z_0E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{39}$$

13. One Line Open—Fig. 21(n)

$$I_{1F}=\frac{E_{a1}(Z_2+Z_0)}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{40}$$

$$I_{2F}=\frac{-Z_0E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{41}$$

$$I_{0F}=\frac{-Z_2E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{42}$$

$$E_{1x}-E_{1y}=E_{a1}-I_{1F}Z_1=\frac{Z_2Z_0E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{43}$$

$$E_{2x}-E_{2y}=-I_{2F}Z_2=\frac{Z_2Z_0E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{44}$$

$$E_{0x}-E_{0y}=-I_{0F}Z_0=\frac{Z_2Z_0E_{a1}}{Z_1Z_2+Z_1Z_0+Z_2Z_0} \tag{45}$$

14. Two Lines Open—Fig. 21(p)

$$I_{1F}=I_{2F}=I_{0F}=\frac{E_{a1}}{Z_1+Z_2+Z_0} \tag{46}$$

$$I_F=I_a=3I_{0F} \tag{47}$$

$$E_{1x}-E_{1y}=E_{a1}-I_{1F}Z_1=\frac{E_{a1}(Z_2+Z_0)}{Z_1+Z_2+Z_0} \tag{48}$$

$$E_{2x}-E_{2y}=-I_{2F}Z_2=-\frac{Z_2E_{a1}}{Z_1+Z_2+Z_0} \tag{49}$$

$$E_{0z}-E_{0y}=-I_{0F}Z_0=-\frac{Z_0E_{a1}}{Z_1+Z_2+Z_0} \tag{50}$$

15. Impedance in One Line—Fig. 21(s)

$$I_{1F}=\frac{E_{a1}(ZZ_0+ZZ_2+3Z_0Z_2)}{ZZ_1Z_0+ZZ_1Z_2+3Z_1Z_2Z_0+ZZ_2Z_0} \tag{51}$$

$$I_{2F}=-\frac{ZZ_0E_{a1}}{ZZ_1Z_0+ZZ_1Z_2+3Z_1Z_2Z_0+ZZ_2Z_0} \tag{52}$$

$$I_{0F}=-\frac{ZZ_2E_{a1}}{ZZ_1Z_0+ZZ_1Z_2+3Z_1Z_2Z_0+ZZ_2Z_0} \tag{53}$$

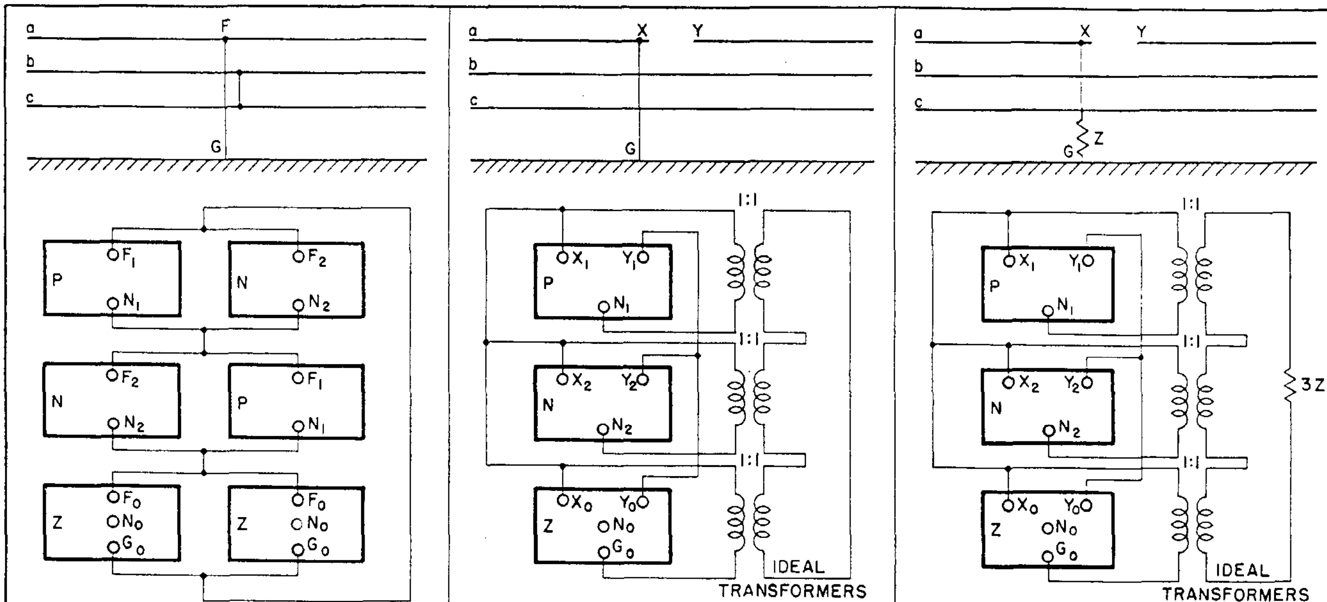
$$E_{1x}-E_{1y}=E_{a1}-I_{1F}Z_1=\frac{ZZ_2Z_0E_{a1}}{ZZ_1Z_0+ZZ_1Z_2+3Z_1Z_2Z_0+ZZ_2Z_0} \tag{54}$$

$$E_{2x}-E_{2y}=-\frac{ZZ_2Z_0E_{a1}}{I_{2f}Z_2+ZZ_1Z_0+ZZ_1Z_2+3Z_1Z_2Z_0+ZZ_2Z_0} \quad (55)$$

$$E_{0x}-E_{0y}=-\frac{ZZ_2Z_0E_{a1}}{I_{0f}Z_0=ZZ_1Z_0+ZZ_1Z_2+3Z_1Z_2Z_0+ZZ_2Z_0} \quad (56)$$

If two or more unbalances occur simultaneously, mutual coupling or connections will occur between the sequence networks at each point of unbalance, and if the unbalances are not symmetrical with respect to the same phase, the connections will have to be made through phase-shifting transformers. The analysis in the cases of simultaneous faults is considerably more complicated than for single unbalances.

No assumptions were made in the derivation of the representation of the shunt and series unbalances of Fig. 21 that would not permit the application of the same principles to simultaneous faults on multiple unbalances. In fact various cases of single unbalance can be combined to



SIMULTANEOUS SINGLE LINE-TO-GROUND ON PHASE A AND LINE-TO-LINE BETWEEN PHASES B AND C.
(a)

SIMULTANEOUS SINGLE LINE-TO-GROUND FAULT AND OPEN CONDUCTOR ON PHASE A.
(b)

SIMULTANEOUS SINGLE LINE-TO-GROUND THROUGH IMPEDANCE AND OPEN CONDUCTOR ON PHASE A.
(c)

SIMULTANEOUS SINGLE LINE-TO-GROUND FAULT ON PHASE B AND OPEN CONDUCTOR ON PHASE A.
(d)

SIMULTANEOUS SINGLE LINE-TO-GROUND FAULT ON PHASE C AND OPEN CONDUCTOR ON PHASE A.
(e)

SIMULTANEOUS SINGLE LINE-TO-GROUND FAULTS ON PHASES A AND B AT DIFFERENT LOCATIONS.
(f)

Fig. 22—Connections between the sequence networks for typical cases of multiple unbalances.

form the proper restraints or terminal connections to represent multiple unbalances. For example, the representation for a simultaneous single line-to-ground fault on phase "a" and a line-to-line fault on phases "b" and "c" can be derived by satisfying the terminal connections of Figs. 21(d) and 21(f). Fig. 21(d) dictates that the three networks be connected in series, while Fig. 21(f) shows the positive- and negative-sequence networks in parallel. Both of these requirements can be met simultaneously as shown in Fig. 22(a). Simultaneous faults that are not symmetrical to the reference phase can be represented by similar connections using ideal transformers or phase shifters to shift the sequence voltages and currents originating in all of the unbalances except the first or reference condition. The fault involving phase "a" is usually taken as the reference and all others are shifted by the proper amount before making the terminal connections required to satisfy that particular type of fault. The positive-, negative-, and zero-sequence shifts, respectively for an unbalance that is symmetrical to phase "a" are 1, 1, 1; "b" phase a^2, a, 1; to "c" phase a, a^2, 1. A few multiple unbalances that may occur at one point in a system simultaneously are given in Fig. 22, which also gives one illustration of simultaneous faults at different points in a system with one fault not symmetrical with respect to phase a.

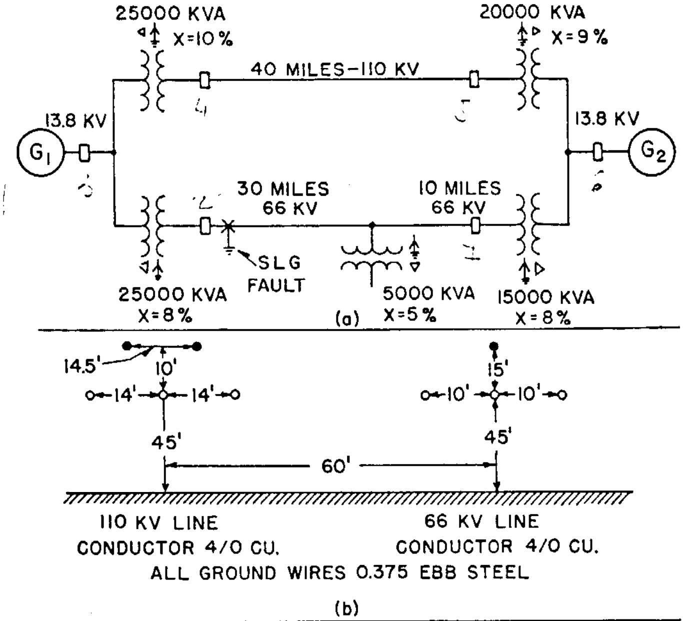
To summarize, the procedure in finding voltages and currents throughout a system during fault conditions is: (1) set up each sequence network as viewed from the fault, (2) find the distribution factors for each sequence current throughout its network, (3) reduce the network to as simple a circuit as possible, (4) make the proper connection between the networks at the fault point to represent the unbalanced condition, (5) solve the resulting single-phase circuit for the sequence currents at the fault, (6) find the sequence components of voltage and current at the desired locations in the system. The positive-sequence voltage to be used, and the machine impedances, in step (5) depend upon when the fault currents and voltages are desired; if immediately after the fault occurs, in general, use subtransient reactances and the voltage back of subtransient reactance immediately preceding the fault; if a few cycles after the fault occurs, use transient reactances and the voltage back of transient reactance immediately before the fault; and if steady-state conditions are desired, use synchronous reactances and the voltage back of synchronous reactance. If regulators are used, normal bus voltage can be used to find steady-state conditions and the machine reactance in the positive-sequence network taken as being zero.

X. EXAMPLE OF FAULT CALCULATION

16. Problem

Let us assume the typical transmission system shown in Fig. 23(a) to have a single line-to-ground fault on one end of the 66 kv line as shown. The line construction is given in Fig. 23(b) and the generator constants in Fig. 23(c). Calculate the following:

(a) Positive-sequence reactance to the point of fault.
(b) Negative-sequence reactance to the point of fault.
(c) Zero-sequence reactance to the point of fault.



GENERATOR G_1	GENERATOR G_2
50,000 KVA	37,500 KVA
NEUTRAL GROUNDED THROUGH 4% REACT.	NEUTRAL UNGROUNDED
X_d = 100%	X_d = 130%
X'_d = 21%	X'_d = 25%
X''_d = 12%	X''_d = 17%
X_2 = 12%	X_2 = 17%
X_0 = 6%	X_0 = 5%

(c)

Fig. 23—Typical system assumed for fault calculation.

(a) System single-line diagram.
(b) Line construction.
(c) Tabulation of generator constants.

(d) Fault current.
(e) Line currents, line-to-ground voltages, and line-to-line voltages at the breaker adjacent to the fault.
(f) Line currents, line-to-ground voltages, and line-to-line voltages at the terminals of G'.
(g) Line currents, line-to-ground voltages, and line-to-line voltages at the 110 kv breaker adjacent to the 25,000 kva transformer.

17. Assumptions

(1) That the fault currents are to be calculated using transient reactances.
(2) A base of 50,000 kva for the calculations.
(3) That all resistances can be neglected.
(4) That a voltage, positive-sequence, as viewed from the fault of j 100% will be used for reference. This is an assumed voltage of $j\dfrac{66{,}000}{\sqrt{3}}$ volts between line "a" and neutral.
(5) That the reference phases on either side of the star-delta transformers are chosen such that positive-sequence voltage on the high side is advanced 30° in phase position from the positive-sequence voltage on the low side of the transformer.

18. Line Reactances (Refer to Chap. 3)

Positive- and Negative-Sequence Reactances of the 110 kv Line.

For 4/0 copper conductors $x_a = 0.497$ ohms per mile.
$x_d = \frac{1}{3}(x_d \text{ for 14 feet} + x_d \text{ for 14 feet} + x_d \text{ for 28 feet})$.
$= \frac{1}{3}(0.320 + 0.320 + 0.404) = 0.348$ ohms per mile.
$x_1 = x_2 = x_a + x_d = 0.497 + 0.348 = 0.845$ ohms per mile.

Positive- and Negative-Sequence Reactances of the 66 kv Line.

$x_a = 0.497$ ohms per mile.
$x_d = \frac{1}{3}(x_d \text{ for 10 feet} + x_d \text{ for 10 feet} + x_d \text{ for 20 feet})$
$= \frac{1}{3}(0.279 + 0.279 + 0.364) = 0.307$ ohms per mile.
$x_1 = x_2 = x_a + x_d = 0.497 + 0.307 = 0.804$ ohms per mile.

Zero-Sequence Reactances—Since zero-sequence currents flowing in either the 110- or the 66-kv line will induce a zero-sequence voltage in the other line and in all three ground wires, the zero-sequence mutual reactances between lines, between each line and the two sets of ground wires, and between the two sets of ground wires, must be evaluated as well as the zero-sequence self reactances. Indeed, the zero-sequence self reactance of either the 110- or the 66-kv line will be affected by the mutual coupling existing with all of the ground wires. The three conductors of the 110-kv line, with ground return, are assumed to form one zero-sequence circuit, denoted by "a" in Fig. 24; the two ground conductors for this line, with ground return, form the zero-sequence circuit denoted "g"; the three conductors for the 66-kv line, with ground return, form the zero-sequence circuit denoted "a'"; and the single ground wire for the 66-kv line, with ground return, forms the zero-sequence circuit denoted "g'." Although not strictly correct, we assume the currents carried by the two ground wires of circuit "g" are equal. Then let:

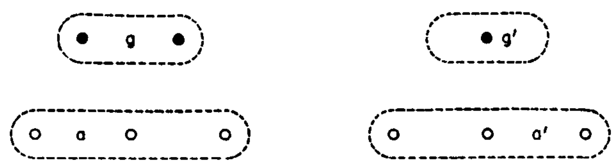


Fig. 24—Zero-sequence circuits formed by the 110 kv line (*a*), the 66 kv line (*a'*), the two ground wires (*g*), and the single ground wire (*g'*).

E_0 = zero-sequence voltage of circuit a
E_{g0} = zero-sequence voltage of circuit $g = 0$, since the ground wires are assumed to be continuously grounded.
E_0' = zero-sequence voltage of circuit a'
E_{g0}' = zero-sequence voltage of circuit $g' = 0$, since the ground wire is assumed to be continuously grounded.
I_0 = zero-sequence current of circuit a
I_g = zero-sequence current of circuit g
I_0' = zero-sequence current of circuit a'
I_g' = zero-sequence current of circuit g'

It should be remembered that unit I_0 is one ampere in each of the three line conductors with three amperes returning in ground; unit I_g is 3/2 amperes in each of the two ground wires with three amperes returning in the ground; unit I_0' is one ampere in each of the three line conductors with three amperes returning in the ground; and unit I_g' is three amperes in the ground wire with three amperes returning in the ground.

These quantities are inter-related as follows:

$$E_0 = I_0 z_{0(a)} + I_g z_{0(ag)} + I_0' z_{0(aa')} + I_g' z_{0(ag')}$$
$$E_{g0} = I_0 z_{0(ag)} + I_g z_{0(g)} + I_0' z_{0(a'g)} + I_g' z_{0(gg')} = 0$$
$$E_0' = I_0 z_{0(aa')} + I_g z_{0(a'g)} + I_0' z_{0(a')} + I_g' z_{0(a'g')}$$
$$E_{g0}' = I_0 z_{0(ag')} + I_g z_{0(gg')} + I_0' z_{0(a'g')} + I_g' z_{0(g')} = 0$$

where

$z_{0(a)}$ = zero-sequence self reactance of the a circuit
$= x_a + x_e - \frac{2}{3}(x_d \text{ for 14 feet} + x_d \text{ for 14 feet} + x_d \text{ for 28 feet})$
$= 0.497 + 2.89 - 2(0.348) = 2.69$ ohms per mile.

$z_{0(a')}$ = zero-sequence self reactance of the a' circuit
$= x_a + x_e - \frac{2}{3}(x_d \text{ for 10 feet} + x_d \text{ for 10 feet} + x_d \text{ for 20 feet})$
$= 0.497 + 2.89 - 2(0.307) = 2.77$ ohms per mile.

$z_{0(g)}$ = zero-sequence self reactance of the g circuit
$= \frac{3}{2}x_a + x_e - \frac{3}{2}(x_d \text{ for 14.5 feet})$
$= \frac{3}{2}(2.79) + 2.89 - \frac{3}{2}(0.324) = 6.59$ ohms per mile.

$z_{0(g')}$ = zero-sequence self reactance of the g' circuit
$= 3x_a + x_e$
$= 3(2.79) + 2.89 = 11.26$ ohms per mile.

$z_{0(ag)}$ = zero-sequence mutual reactance between the a and g circuits
$= x_e - \frac{3}{6}(x_d \text{ for 12.06 feet} + x_d \text{ for 12.06 feet} + x_d \text{ for 12.35 feet} + x_d \text{ for 12.35 feet} + x_d \text{ for 23.5 feet} + x_d \text{ for 23.5 feet})$
$= 2.89 - 3(0.3303) = 1.90$ ohms per mile.

$z_{0(aa')}$ = zero-sequence mutual reactance between the a and a' circuits
$= x_e - \frac{3}{9}(x_d \text{ for 60 feet} + x_d \text{ for 50 feet} + x_d \text{ for 70 feet} + x_d \text{ for 46 feet} + x_d \text{ for 36 feet} + x_d \text{ for 56 feet} + x_d \text{ for 74 feet} + x_d \text{ for 64 feet} + x_d \text{ for 84 feet})$
$= 2.89 - 3(0.493) = 1.411$ ohms per mile.

$z_{0(ag')}$ = zero-sequence mutual reactance between the a and g' circuits.
$= x_e - \frac{3}{3}(x_d \text{ for 75 feet} + x_d \text{ for 62 feet} + x_d \text{ for 48 feet})$
$= 2.89 - 3(0.498) = 1.40$ ohms per mile.

$z_{0(a'g')}$ = zero-sequence mutual reactance between the a' and g' circuits.
$= x_e - \frac{3}{3}(x_d \text{ for 15 feet} + x_d \text{ for 18.03 feet} + x_d \text{ for 18.03 feet})$
$= 2.89 - 3(0.344) = 1.86$ ohms per mile.

Similar definitions apply for $Z_{0(a'g)}$ and $Z_{0(gg')}$. In each case the zero-sequence mutual reactance between two circuits is equal to x_e minus three times the average of the x_d's for all possible distances between conductors of the two circuits.

The zero-sequence self reactance of the 110-kv line in the presence of all zero-sequence circuits is obtained by letting I_0' be zero in the above equations and solving for $\frac{E_0}{I_0}$. Carrying out this rather tedious process, it will be found that

$$\frac{E_0}{I_0}=2.05 \text{ ohms per mile.}$$

The zero-sequence self reactance of the 66-kv line in the presence of all zero-sequence circuits is obtained by letting I_0 be zero in the equations and solving for $\frac{E_0'}{I_0'}$. It will be found that

$$\frac{E_0'}{I_0'}=2.25 \text{ ohms per mile.}$$

The zero-sequence mutual reactance between the 66- and the 110-kv line in the presence of all zero-sequence

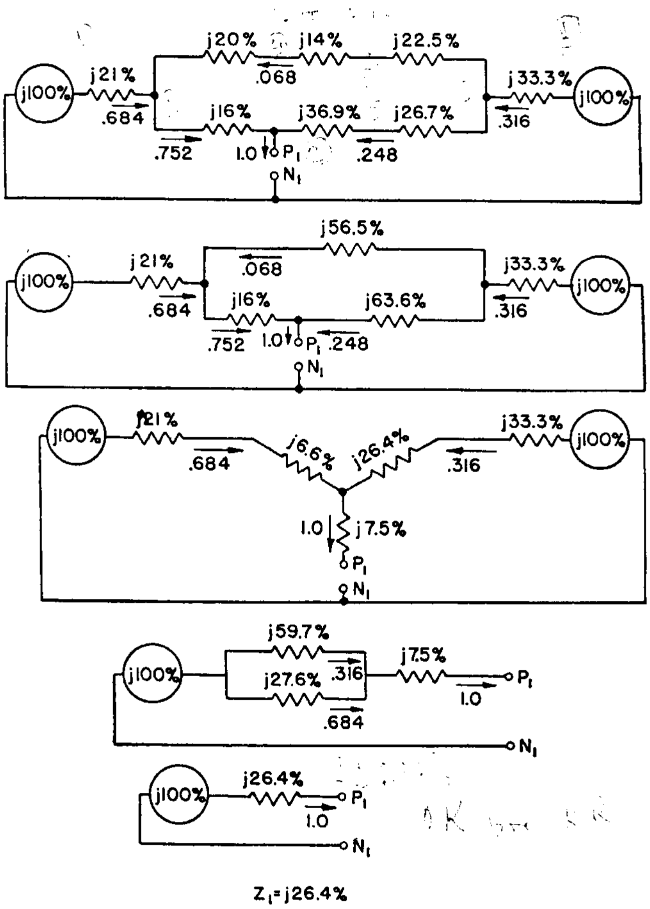


Fig. 25—Reduction of the positive-sequence network and the positive-sequence distribution factors.

circuits is obtained by letting I_0' be zero and solving for $\frac{E_0'}{I_0}$. When this is done, it will be found that

$$\frac{E_0'}{I_0}(\text{with } I_0'=0)=\frac{E_0}{I_0'}(\text{with } I_0=0)=0.87 \text{ ohms per mile.}$$

19. The Sequence Networks

The sequence networks are shown in Figs. 25, 26, and 27, with all reactances expressed in percent on a 50 000-kva base and the networks set up as viewed from the fault. Illustrative examples of expressing these reactances in percent on a 50 000-kva base follow:

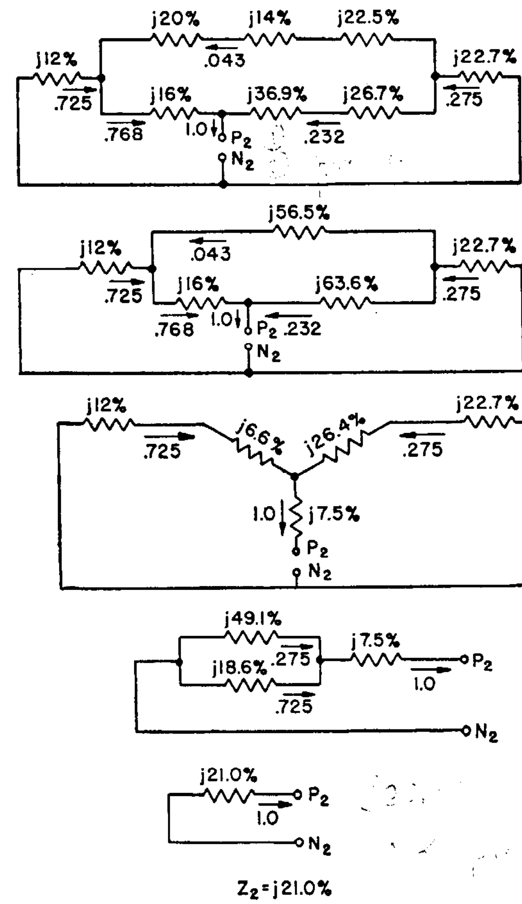


Fig. 26—Reduction of the negative-sequence network and the negative sequence distribution factors.

Positive-sequence reactance of $G_2=$

$$(25)\frac{(50\ 000)}{(37\ 500)}=33.3\%$$

Positive-sequence reactance of the 66-kv line =

$$\frac{(0.804)\ (40)\ (50\ 000)}{(66)\ (66)\ (10)}=36.9\%$$

Positive-sequence reactance of the 110-kv line =

$$\frac{(0.845)\ (40)\ (50\ 000)}{(110)\ (110)\ (10)}=14\%$$

Zero-sequence mutual reactance between the 66- and the 110-kv line for the 30 mile section =

$$\frac{(0.87)\ (30)\ (50\ 000)}{(110)\ (66)\ (10)}=18\%$$

The distribution factors are shown on each sequence network; obtained by finding the distribution of one ampere taken as flowing out at the fault.

Each network is finally reduced to one equivalent impedance as viewed from the fault.

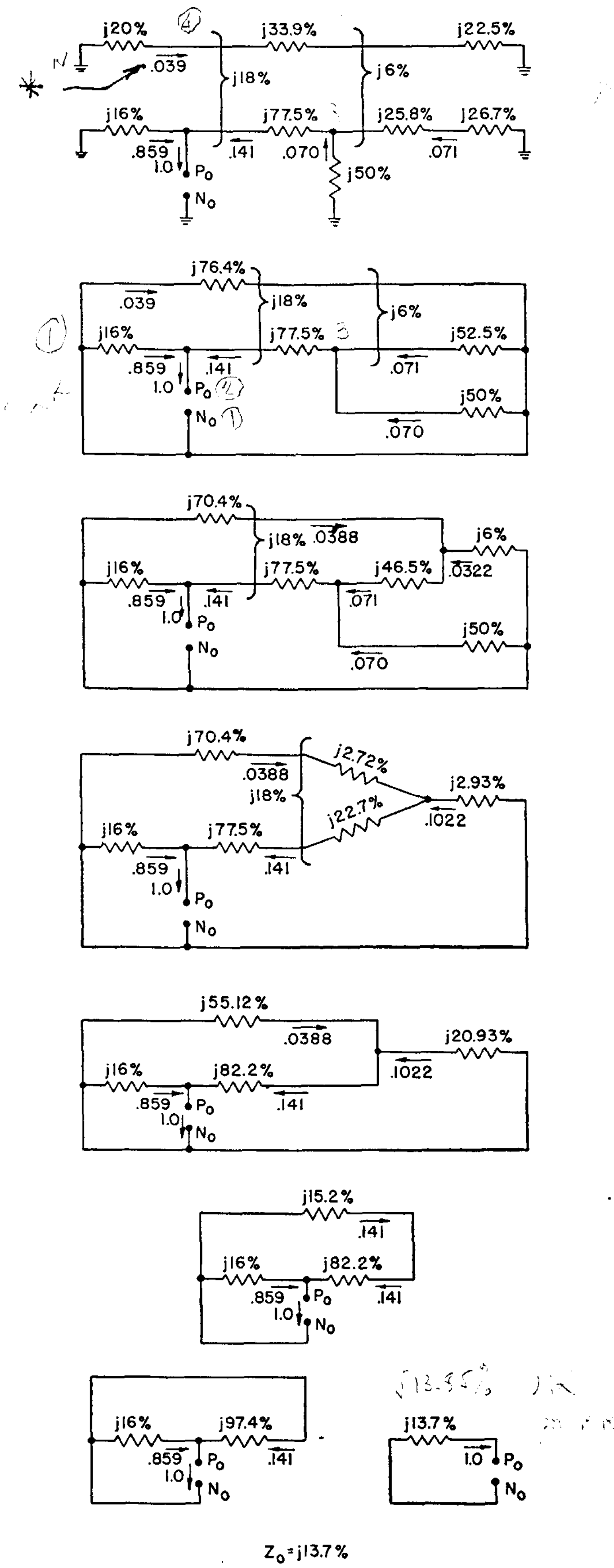


Fig. 27—Reduction of the zero-sequence network and the zero-sequence distribution factors.

20. Voltages and Currents at the Fault

The sequence networks are connected in series to represent a single line-to-ground fault. The total reactance of the resulting single-phase network is

$$Z_1\%+Z_2\%+Z_0\%=26.4\%+21.0\%+13.7\%=61.1\%.$$

Then: $$I_{0F}=I_{1F}=I_{2F}=\frac{j100\%}{j61.1\%}=1.637 \text{ p.u.}$$

Since normal current for the 66-kv circuit (for a base kva of 50 000)

$$=\frac{50\,000}{\sqrt{3}\times 66}=437.5 \text{ amperes.}$$

$$I_0=I_1=I_2=(1.637)(437.5)=715 \text{ amperes.}$$

The total fault current=

$$I_0+I_1+I_2=4.911 \text{ p.u.}=2145 \text{ amperes.}$$

The sequence voltages at the fault:

$E_1=E_{a1}-I_1Z_1=j100\%-j(1.637)(26.4)\%=j56.9\%$
$=j21\,700$ volts.

$E_2=-I_2Z_2=-j(1.637)(21)\%=-j34.4\%=-j13\,100$ volts.

$E_0=-I_0Z_0=-j(1.637)(13.7)\%=-j22.5\%=-j8\,600$ volts.

$E_{ag}=E_0+E_1+E_2=0$

$E_{bg}=E_0+a^2E_1+aE_2=30\,200-j12\,900$
$=32\,800$ volts.

$E_{cg}=E_0+aE_1+a^2E_2=-30\,200-j12\,900$
$=32\,800$ volts.

$E_{ab}=E_{ag}-E_{bg}=-30\,200+j12\,900=32\,800$ volts.

$E_{bc}=E_{bg}-E_{cg}=60\,400$ volts.

$E_{ca}=E_{cg}-E_{ag}=-30\,200-j12\,900=32\,800$ volts.

21. Voltages and Currents at the Breaker Adjacent to the Fault

Using the distribution factors in the sequence networks at this point:

$I_1=(0.752)(1.637)=1.231$ p.u. $=540$ amperes.
$I_2=(0.768)(1.637)=1.258$ p.u. $=550$ amperes.
$I_0=(0.859)(1.637)=1.407$ p.u. $=615$ amperes.
$I_a=I_0+I_1+I_2=1705$ amperes.
$I_b=I_0+a^2I_1+aI_2=70+j8.6=70.5$ amperes.
$I_c=I_0+aI_1+a^2I_2=70-j8.6=70.5$ amperes.

The line-to-ground and line-to-line voltages at this point are equal to those calculated for the fault.

22. Voltages and Currents at the Breaker Adjacent to Generator G_1

The base, or normal, voltage at this point is 13 800 volts line-to-line, or 7960 volts line-to-neutral.

The base, or normal, current at this point is $\frac{50\,000}{\sqrt{3}\times 13.8}$ $=2090$ amperes. Since a star-delta transformation is involved, there will be a phase shift in positive- and negative-sequence quantities.

$I_1=(0.684)(1.637)(2090)\epsilon^{-j30}=2340$ amperes
$=2030-j1170.$

$I_2=(0.725)(1.637)(2090)\epsilon^{+j30}=2480$ amperes
$=2150+j1240.$

$I_0 = 0.$
$I_a = I_0 + I_1 + I_2 = 4180 + j70 = 4180$ amperes.
$I_b = I_0 + a^2 I_1 + a I_2 = -4180 + j70 = 4180$ amperes.
$I_c = I_0 + a I_1 + a^2 I_2 = -j140 = 140$ amperes.

The sequence voltages at this point are:

$E_1 = (j100\% - j0.684 \times 21 \times 1.637\%)\epsilon^{-j30} = -a^2 76.5\%$
$= 3045 + j5270$ volts.
$E_2 = (-j0.725 \times 12 \times 1.637\%)\epsilon^{j30} = -a14.2\%$
$= 565 - j980$ volts.
$E_0 = 0.$
$E_{ag} = E_1 + E_2 = 3610 + j4290 = 5600$ volts.
$E_{bg} = a^2 E_1 + a E_2 = 3610 - j4290 = 5600$ volts.
$E_{cg} = a E_1 + a^2 E_2 = -7220 = 7220$ volts.
$E_{ab} = +j8580 = 8580$ volts.
$E_{bc} = 10\,830 - j4290 = 11\,650$ volts.
$E_{ca} = -10\,830 - j4290 = 11\,650$ volts.

23. Voltages and Currents at the 110-kv Breaker Adjacent to the 25 000 kva Transformer

The base, or normal, voltage at this point is 110 000 volts line-to-line; or 63 500 volts line-to-neutral.

The base, or normal, current at this point is $\frac{50\,000}{\sqrt{3} \times 110}$ $= 262$ amperes.

The sequence currents at this point are:

$I_1 = (-0.068)(1.637)(262) = -29.2$ amperes.
$I_2 = (-0.043)(1.637)(262) = -18.4$ amperes.
$I_0 = (0.039)(1.637)(262) = 16.7$ amperes.
$I_a = I_0 + I_1 + I_2 = -30.9 = 30.9$ amperes.
$I_b = I_0 + a^2 I_1 + a I_2 = 40.5 + j9.35 = 41.6$ amperes.
$I_c = I_0 + a I_1 + a^2 I_2 = 40.5 - j9.35 = 41.6$ amperes.

The sequence voltages at this point are:

$E_1 = j100\% - j(0.684)(1.637)(21)\%$
$-j(-0.068)(1.637)(20)\% = j78.7\%$
$= j50\,000$ volts.
$E_2 = -j(0.725)(1.637)(12)\%$
$-j(-0.043)(1.637)(20)\% = -j12.8\%$
$= -j8130$ volts.
$E_0 = -j(0.039)(1.637)(20)\% = -j1.3\% = -j825$ volts.
$E_{ag} = E_0 + E_1 + E_2 = j41\,000 = 41\,000$ volts.
$E_{bg} = E_0 + a^2 E_1 + a E_2 = 50\,300 - j21\,750 = 54\,800$ volts.
$E_{cg} = E_0 + a E_1 + a^2 E_2 = -50\,300 - j21\,750 = 54\,800$ volts.
$E_{ab} = -50\,300 + j62\,750 = 80\,400$ volts.
$E_{bc} = 100\,600 = 100\,600$ volts.
$E_{ca} = -50\,300 - j62\,750 = 80\,400$ volts.

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* CORRIENTE INDUCIDA

$$\frac{0.141 \times j18\% + 0.071 \times j6}{j76\%} = 0.03879$$

CHAPTER 3

CHARACTERISTICS OF AERIAL LINES

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IN the design, operation, and expansion of electrical power systems it is necessary to know electrical and physical characteristics of conductors used in the construction of aerial distribution and transmission lines.

This chapter presents a description of the common types of conductors along with tabulations of their important electrical and physical characteristics. General formulas are presented with their derivation to show the basis of the tabulated values and as a guide in calculating data for other conductors of similar shapes, dimensions, composition and operating conditions.

Also included are the more commonly used symmetrical-component-sequence impedance equations that are applicable to the solution of power system problems involving voltage regulation, load flow, stability, system currents, and voltages under fault conditions, or other system problems where the electrical characteristics of aerial lines are involved.

Additional formulas are given to permit calculation of approximate current-carrying capacity of conductors taking into account such factors as convection and radiation losses as influenced by ambient temperature, wind velocity, and permissible temperature rise.

I. TYPES OF CONDUCTORS

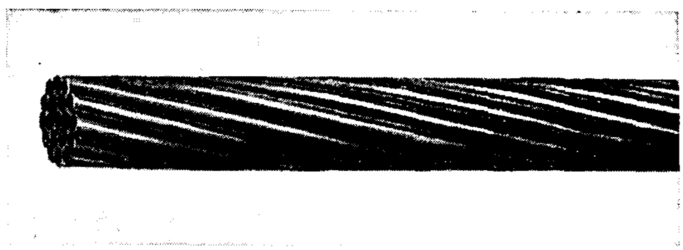
In the electric-power field the following types of conductors are generally used for high-voltage power transmission lines: stranded copper conductors, hollow copper conductors, and ACSR (aluminum cable, steel reinforced).

Other types of conductors such as Copperweld and Copperweld-Copper conductors are also used for transmission and distribution lines. Use is made of Copperweld, bronze, copper bronze, and steel for current-carrying conductors on rural lines, as overhead ground wires for transmission lines, as buried counterpoises at the base of transmission towers, and also for long river crossings.

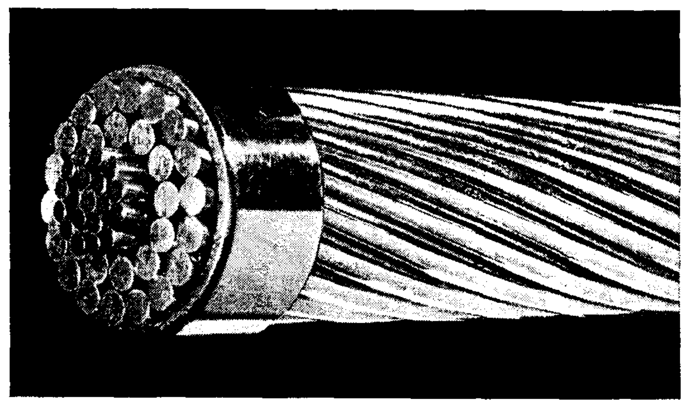
A stranded conductor, typical of both copper and steel conductors in the larger sizes, is shown in Fig. 1. A stranded conductor is easier to handle and is more flexible than a solid conductor, particularly in the larger sizes.

A typical ACSR conductor is illustrated in Fig. 2. In this type of conductor, aluminum strands are wound about a core of stranded steel. Varying relationships between tensile strength and current-carrying capacity as well as overall size of conductor can be obtained by varying the proportions of steel and aluminum. By the use of a filler, such as paper, between the outer aluminum strands and the inner steel strands, a conductor of large diameter can be obtained for use in high voltage lines. This type of conductor is known as "expanded" ACSR and is shown in Fig. 3.

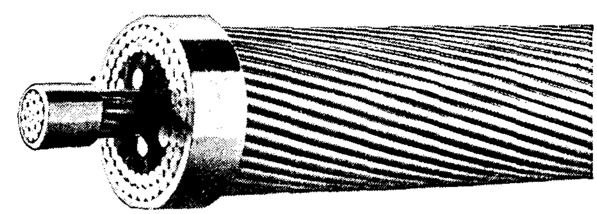
In Fig. 4 is shown a representative Anaconda Hollow Copper Conductor. It consists of a twisted copper "I"



Courtesy of General Cable Corporation
Fig. 1—A typical stranded conductor, (bare copper).

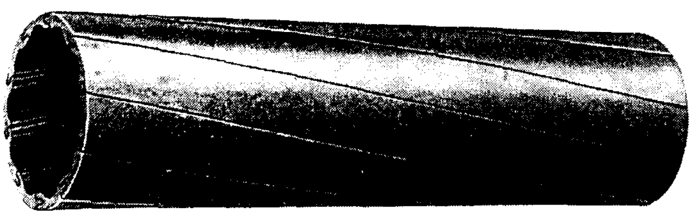


Courtesy of Aluminum Company of America
Fig. 2—A typical ACSR conductor.



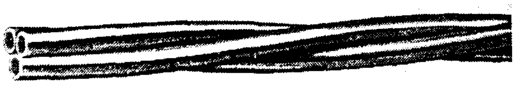
Courtesy of Aluminum Company of America
Fig. 3—A typical "expanded" ACSR conductor.

Courtesy of Anaconda Wire and Cable Company
Fig. 4—A typical Anaconda Hollow Copper Conductor.



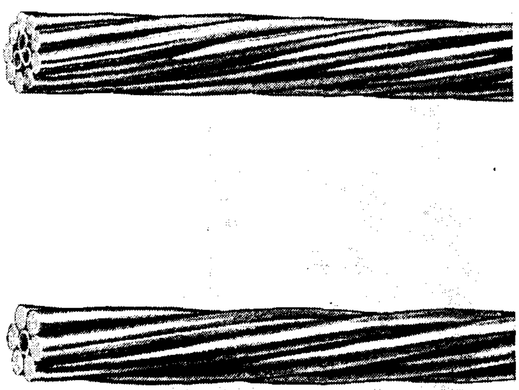
Courtesy of General Cable Corporation

Fig. 5—A typical General Cable Type HH.



Courtesy of Copperweld Steel Company

Fig. 6—A typical Copperweld conductor.



Courtesy of Copperweld Steel Company

Fig. 7—Typical Copperweld-Copper conductors

(a) Upper photograph–Type V
(b) Lower photograph–Type F

beam as a core about which strands of copper wire are wound. The "I" beam is twisted in a direction opposite to that of the inner layer of strands.

Another form of hollow copper conductor is shown in Fig. 5. Known as the General Cable Type HH hollow copper conductor, it is made up of segmental sections of copper mortised into each other to form a self-supporting hollow cylinder. Hollow copper conductors result in conductors of large diameter for a given cross section of copper. Corona losses are therefore smaller. This construction also produces a reduction in skin effect as well as inductance as compared with stranded conductors. A discussion of large diameter conductors and their characteristics is given in reference 1.

Copperweld conductors consist of different numbers of copper-coated steel strands, a typical conductor being illustrated in Fig. 6. Strength is provided by the core of steel and protection by the outer coating of copper.

When high current-carrying capacities are desired as well as high tensile strength, copper strands are used with Copperweld strands to form Copperweld-Copper conductors as shown in Fig. 7. Different relationships between current-carrying capacity, outside diameter, and tensile strength can be obtained by varying the number and size of the Copperweld and copper strands.

II. ELECTRICAL CHARACTERISTICS OF AERIAL CONDUCTORS

The following discussion is primarily concerned with the development of electrical characteristics and constants of aerial conductors, particularly those required for analysis of power-system problems. The constants developed are particularly useful in the application of the principles of symmetrical components to the solution of power-system problems involving positive-, negative-, and zero-sequence impedances of transmission and distribution lines. The basic quantities needed are the positive-, negative-, and zero-sequence resistances, inductive reactances and shunt capacitive reactances of the various types of conductors and some general equations showing how these quantities are used.

1. Positive- and Negative-Sequence Resistance

The resistance of an aerial conductor is affected by the three factors: temperature, frequency, current density. Practical formulas and methods will now be given to take into account these factors.

Temperature Effect on Resistance—The resistance of copper and aluminum conductors varies almost directly with temperature. While this variation is not strictly linear for an extremely wide range of temperatures, for practical purposes it can be considered linear over the range of temperatures normally encountered.

When the d-c resistance of a conductor at a given temperature is known and it is desired to find the d-c resistance at some other temperature, the following general formula may be used.

$$\frac{R_{t2}}{R_{t1}}=\frac{M+t_2}{M+t_1} \tag{1}$$

where

R_{t2} = d-c resistance at any temperature t_2 degree C.
R_{t1} = d-c resistance at any other temperature t_1 degree C.
M = a constant for any one type of conductor material.
= inferred absolute zero temperature.
= 234.5 for annealed 100 percent conductivity copper.
= 241.5 for hard drawn 97.3 percent conductivity copper.
= 228.1 for aluminum.

The above formula is useful for evaluating changes in d-c resistance only, and cannot be used to give a-c resistance variations unless skin effect can be neglected. For small conductor sizes the frequency has a negligible effect on resistance in the d-c to 60-cycle range. This is generally true for conductor sizes up to 2/0.

The variations of resistance with temperature are usually unimportant because the actual ambient temperature is indefinite as well as variable along a transmission line. An illustration of percentage change in resistance is when temperature varies from winter to summer over a range of 0 degree C to 40 degrees C (32 degrees F to 104 degrees F) in which case copper resistance increases 17 percent.

Skin Effect in Straight Round Wires—The resistance of non-magnetic conductors varies not only with temperature but also with frequency. This is due to skin effect. Skin effect is due to the current flowing nearer the outer surface of the conductor as a result of non-uniform flux distribution in the conductor. This increases the resistance of the conductor by reducing the effective cross section of the conductor through which the current flows.

The conductor tables give the resistance at commercial frequencies of 25, 50, and 60 cycles. For other frequencies the following formula should be used.

$$r_f = K r_{dc} \text{ ohms per mile} \tag{2}$$

where

r_f = the a-c resistance at the desired frequency (cycles per second).

r_{dc} = d-c resistance at any known temperature.

K = value given in Table 5.

In Table 5, K is given as a function of X, where

$$X = .063598\sqrt{\frac{\mu f}{r_{\text{mile}}}} \tag{3}$$

f = frequency in cycles per second.

μ = permeability = 1.0 for non-magnetic materials.

r_{mile} = d-c resistance of the conductor in ohms per mile.

Table 5 (skin effect table) is carried in the Bureau of Standards Bulletin No. 169 on pages 226–8, to values of $X = 100$. To facilitate interpolation over a small range of the table, it is accurate as well as convenient to plot a curve of the values of K vs. values of X.

Combined Skin Effect and Temperature Effect on Resistance of Straight Round Wires—When both temperature and skin effect are considered in determining conductor resistance, the following procedure is followed.

First calculate the d-c resistance at the new temperature using Eq. (1). Then substitute this new value of d-c resistance and the desired frequency in the equation defining X. Having calculated X, determine K from Table 5. Then using Eq. (2), calculate the new a-c resistance r_f, using the new d-c resistance for r_{dc} and the value of K obtained from Table 5.

Effect of Current on Resistance—The resistance of magnetic conductors varies with current magnitude as well as with the factors that affect non-magnetic conductors (temperature and frequency).

Current magnitude determines the flux and therefore the iron or magnetic losses inside magnetic conductors. The presence of this additional factor complicates the determination of resistance of magnetic conductors as well as any tabulation of such data. For these reasons the effect of current magnitude will not be analyzed in detail. However, Fig. 8 gives the resistance of steel conductors as a function of current, and the tables on magnetic conductors such as Copperweld-copper, Copperweld, and ACSR conductors include resistance tabulations at two current carrying levels to show this effect. These tabulated resistances are generally values obtained by tests.

Zero-Sequence Resistance—The zero-sequence resistance of aerial conductors is discussed in detail in the section on zero-sequence resistance and inductive reactance given later in the chapter since the resistance and inductive reactance presented to zero-sequence currents is influenced by the distribution of the zero-sequence current in the earth return path.

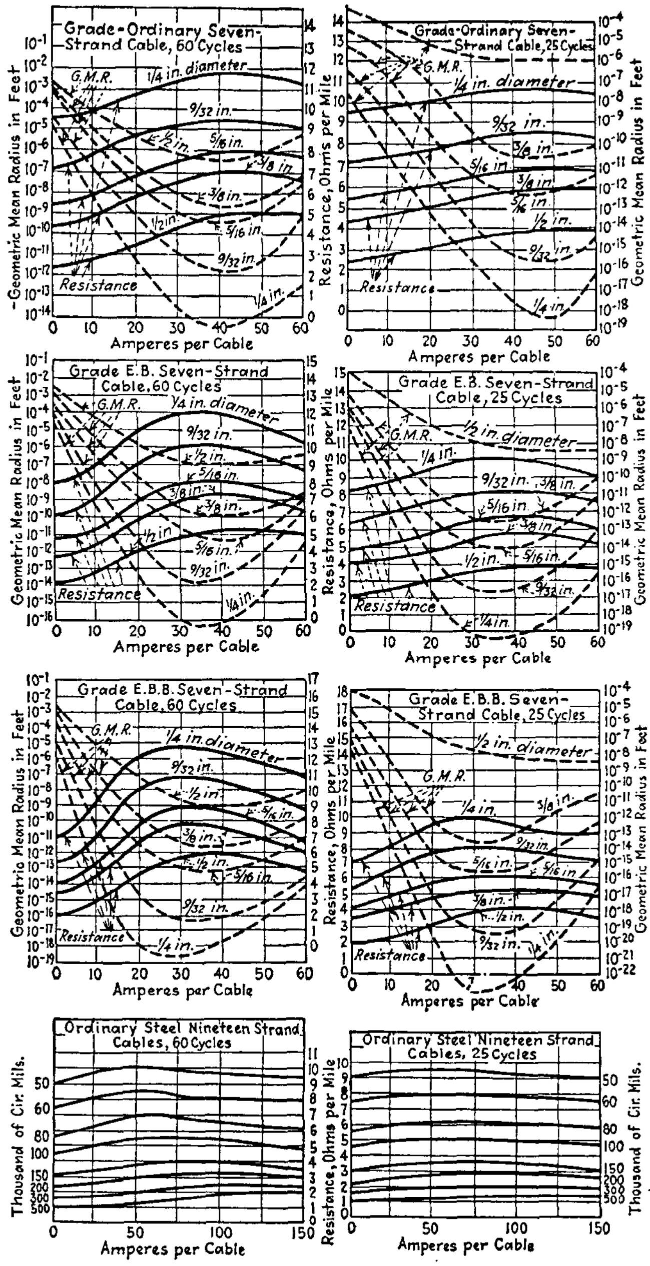


Fig. 8—Electrical Characteristics of Steel Ground Wires*

2. Positive- and Negative-Sequence Inductive Reactance

To develop the positive- and negative-sequence inductive reactance of three-phase aerial lines it is first necessary to develop a few concepts that greatly simplify the problem.

First, the total inductive reactance of a conductor carrying current will be considered as the sum of two components:

*This figure has been taken from *Symmetrical Components* (a book) by C. F. Wagner and R. D. Evans, McGraw-Hill Book Company, 1933.

(1) The inductive reactance due to the flux within a radius of one foot from the conductor center, including the flux inside the conductor.
(2) The inductive reactance due to the flux external to a radius of one foot and out to some finite distance.

This concept was first given in Wagner and Evans book on Symmetrical Components[2] and was suggested by W. A. Lewis.[48]

It can be shown most easily by considering a two-conductor single-phase circuit with the current flowing out in one conductor and returning in the other. In Fig. 9 such a circuit is shown with only the flux produced by conductor 1 for simplicity. Conductor 2 also produces similar lines of flux.

The classic inductance formula for a single round straight wire in the two-conductor single-phase circuit is:

$$L=\frac{\mu}{2}+2\ln\frac{D_{12}}{r} \text{ abhenries per cm. per conductor.} \quad (4)$$

where

μ = permeability of conductor material.
r = radius of conductor.
D_{12} = distance between conductor 1 and conductor 2.

D_{12} and r must be expressed in the same units for the above equation to be valid. For practical purposes one foot is used as the unit of length since most distances between aerial conductors are in feet. In cable circuits, however, the distance between conductors is less than one foot and the inch is a more common unit (see Chap. 4).

From derivation formulas a general term such as $2\ln\frac{b}{a}$ represents the flux and associated inductance between circles of radius a and radius b surrounding a conductor carrying current. (See Fig. 10).

Rewriting Eq. (4) keeping in mind the significance of the general term $2\ln\frac{b}{a}$,

$$L=\frac{\mu}{2}+2\ln\frac{1}{r}+2\ln\frac{D_{12}}{1} \text{ abhenries per cm. per conductor} \quad (5)$$

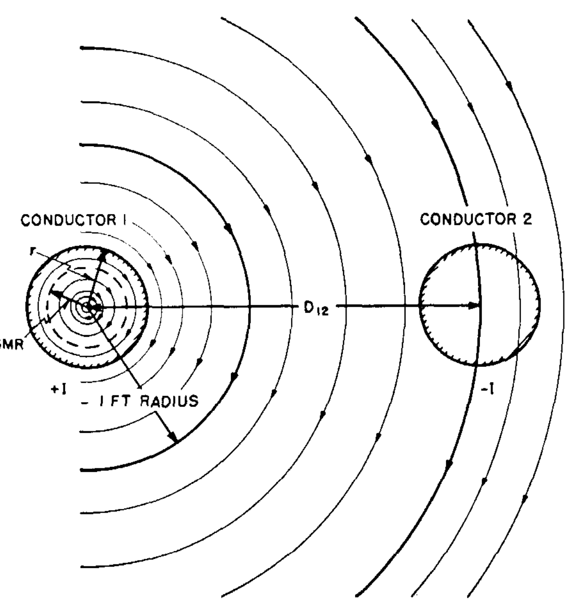


Fig. 9—A two conductor single phase circuit (inductance)

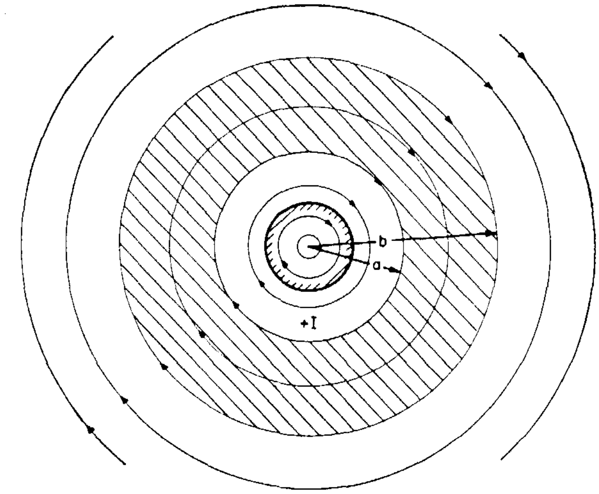


Fig. 10—Inductance due to flux between radius a and radius b ($2\ln\frac{b}{a}$ abhenries per cm.)

where $\frac{\mu}{2}$ = inductance due to the flux inside the conductor.

$2\ln\frac{1}{r}$ = inductance due to the flux outside the conductor to a radius of one foot.

$2\ln\frac{D_{12}}{1}$ = inductance due to the flux external to a one foot radius out to D_{12} feet where D_{12} is the distance between conductor 1 and conductor 2.

From Fig. 9 it can be seen that it is unnecessary to include the flux beyond the return conductor 2 because this flux does not link any net current and therefore does not affect the inductance of conductor 1.

Grouping the terms in Eq. (5) we have:

$$L=\underbrace{\frac{\mu}{2}+2\ln\frac{1}{r}}_{\substack{L \text{ due to flux out}\\ \text{to a one ft. radius}}}+\underbrace{2\ln\frac{D_{12}}{1}}_{\substack{L \text{ due to flux external}\\ \text{to a 1 ft. radius out to } D_{12} \text{ ft.}}} \text{ abhenries per cm. per conductor.} \quad (6)$$

Examining the terms in the first bracket, it is evident that this expression is the sum of the flux both inside the conductor $\left(\frac{\mu}{2}\right)$ and that external to the conductor out to a radius of one foot $\left(2\ln\frac{1}{r}\right)$. Furthermore this expression contains terms that are strictly a function of the conductor characteristics of permeability and radius.

The term in the second bracket of Eq. (6) is an expression for inductance due to flux external to a radius of one foot and out to a distance of D_{12}, which, in the two-conductor case, is the distance between conductor 1 and conductor 2. This term is not dependent upon the conductor characteristics and is dependent only upon conductor spacing.

Equation (6) can be written again as follows:

$$L = 2\ln\frac{1}{\text{GMR}} + 2\ln\frac{D_{12}}{1} \text{ abhenries per cm. per conductor.} \tag{7}$$

GMR in the first term is the conductor "geometric mean radius". It can be defined as the radius of a tubular conductor with an infinitesimally thin wall that has the same external flux out to a radius of one foot as the internal and external flux of solid conductor 1, out to a radius of one foot. In other words, GMR is a mathematical radius assigned to a solid conductor (or other configuration such as stranded conductors), which describes in one term the inductance of the conductor due to both its internal flux $\left(\frac{\mu}{2}\right)$ and the external flux out to a one foot radius $\left(2\ln\frac{1}{r}\right)$. GMR therefore makes it possible to replace the two terms $\left(\frac{\mu}{2}+2\ln\frac{1}{r}\right)$ with one term $\left(2\ln\frac{1}{\text{GMR}}\right)$ which is entirely dependent upon the conductor characteristics. GMR is expressed in feet.

Converting Eq. (7) to practical units of inductive reactance,

$$x = 0.2794\frac{f}{60}\log_{10}\frac{1}{\text{GMR}} + 0.2794\frac{f}{60}\log_{10}\frac{D_{12}}{1} \text{ ohms per conductor per mile} \tag{8}$$

where f = frequency in cps.
GMR = conductor geometric mean radius in feet.
D_{12} = distance between conductors 1 and 2 in feet.

If we let the first term be called x_a and the second term x_d, then

$$x = x_a + x_d \text{ ohms per conductor per mile} \tag{9}$$

where

x_a = inductive reactance due to both the internal flux and that external to conductor 1 to a radius of one foot.

x_d = inductive reactance due to the flux surrounding conductor 1 from a radius of one foot out to a radius of D_{12} feet.

For the two-conductor, single-phase circuit, then, the total inductive reactance is

$$x = 2(x_a + x_d) \text{ ohms per mile of circuit} \tag{10}$$

since the circuit has two conductors, or both a "go" and "return" conductor.

Sometimes a tabulated or experimental reactance with 1 foot spacing is known, and from this it is desired to calculate the conductor GMR. By derivation from Eq. (8)

$$\text{GMR} = \frac{1}{\text{Antilog}_{10}\dfrac{\text{Reactance with 1 ft spacing (60 cycles)}}{0.2794}} \text{ feet.} \tag{11}$$

When reactance is known not to a one-foot radius but out to the conductor surface, it is called the "internal reactance." The formula for calculating the GMR from the "internal reactance" is:

$$\text{GMR} = \frac{\text{physical radius}}{\text{Antilog}_{10}\dfrac{\text{"Internal Reactance" (60 cycles)}}{0.2794}} \text{ feet} \tag{12}$$

The values of GMR at 60 cycles and x_a at 25, 50, and 60 cycles for each type of conductor are given in the tables of electrical characteristics of conductors. They are given

Solid round conductor	0.779a
Full stranding	
7	0.726a
19	0.758a
38	0.768a
61	0.772a
91	0.774a
127	0.776a
Hollow stranded conductors and A.C.S.R. (neglecting steel strands)	
30-two layer	0.826a
26-two layer	0.809a
54-three layer	0.810a
Single layer A.C.S.R.	0.35a-070a
Point within circle to circle	a
Point outside circle to circle	distance to center of circle
Rectangular section of sides α and β	$0.2235(\alpha+\beta)$

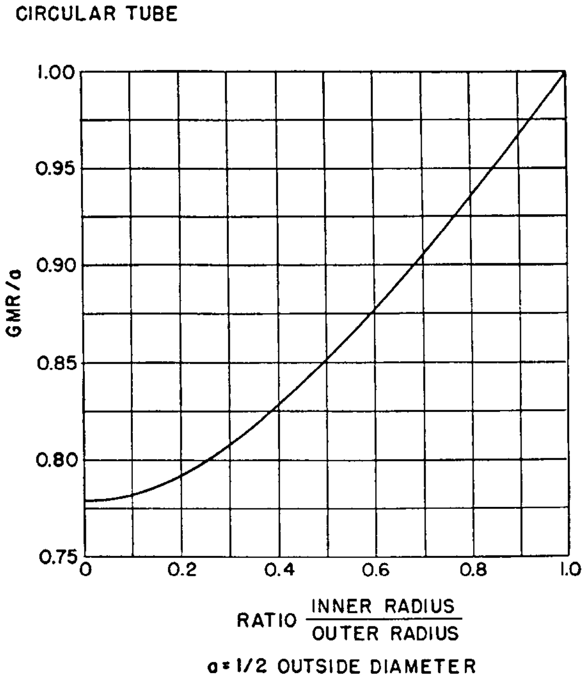


Fig. 11—Geometric Mean Radii and Distances.

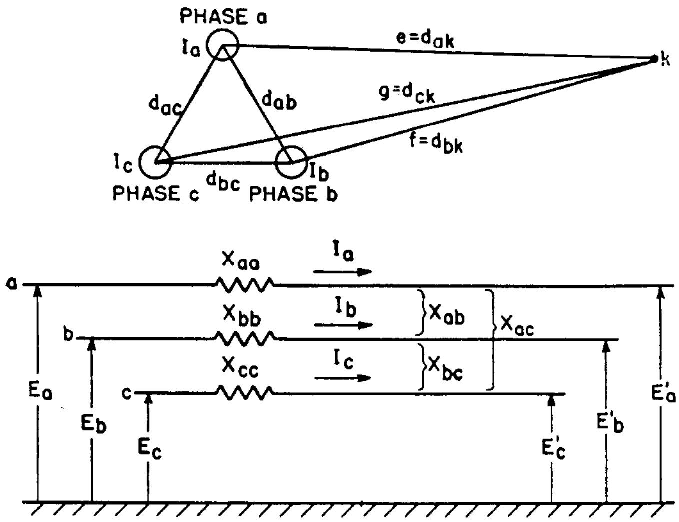


Fig. 12—A Three-conductor three-phase circuit (symmetrical spacing).

in these tables because they are a function of conductor characteristics of radius and permeability. Values of x_d for various spacings are given in separate tables in this Chapter for 25, 50, and 60 cycles. This factor is dependent on distance between conductors only, and is not associated with the conductor characteristics in any way.

In addition to the GMR given in the conductor characteristics tables, it is sometimes necessary to determine this quantity for other conductor configurations. Figure 11 is given for convenience in determining such values of GMR. This table is taken from the Wagner and Evans book *Symmetrical Components*, page 138.

Having developed x_a and x_d in terms of a two-conductor, single-phase circuit, these quantities can be used to determine the positive- and negative-sequence inductive reactance of a three-conductor, three-phase circuit.

Figure 12 shows a three-conductor, three-phase circuit carrying phase currents I_a, I_b, I_c produced by line to ground voltages E_a, E_b, and E_c. First, consider the case where the three conductors are symmetrically spaced in a triangular configuration so that no transpositions are required to maintain equal voltage drops in each phase along the line. Assume that the three-phase voltages E_a, E_b, E_c are balanced (equal in magnitude and 120° apart) so that they may be either positive- or negative-sequence voltages. Also assume the currents I_a, I_b, I_c are also balanced so that $I_a+I_b+I_c=0$. Therefore no return current flows in the earth, which practically eliminates mutual effects between the conductors and earth, and the currents I_a, I_b, I_c can be considered as positive- or negative-sequence currents. In the following solution, positive- or negative-sequence voltages E_a, E_b, E_c, are applied to the conductors and corresponding positive- or negative-sequence currents are assumed to flow producing voltage drops in each conductor. The voltage drop per phase, divided by the current per phase results in the positive- or negative-sequence inductive reactance per phase for the three-phase circuit. To simplify the problem further, consider only one current flowing at a time. With all three currents flowing simultaneously, the resultant effect is the sum of the effects produced by each current flowing alone.

Taking phase a, the voltage drop is:

$$E_a-E_a'=I_a x_{aa}+I_b x_{ab}+I_c x_{ac} \tag{13}$$

where

x_{aa} = self inductive reactance of conductor a.

x_{ab} = mutual inductive reactance between conductor a, and conductor b.

x_{ac} = mutual inductive reactance between conductor a and conductor c.

In terms of x_a and x_d, inductive reactance spacing factor,

$$x_{aa}=x_a+x_{d(ak)} \tag{14}$$

where only I_a is flowing and returning by a remote path e feet away, assumed to be the point k.

Considering only I_b flowing in conductor b and returning by the same remote path f feet away,

$$x_{ab}=x_{d(bk)}-x_{d(ba)} \tag{15}$$

where x_{ab} is the inductive reactance associated with the flux produced by I_b that links conductor a out to the return path f feet away.

Finally, considering only I_c flowing in conductor c and returning by the same remote path g feet away.

$$x_{ac}=x_{d(ck)}-x_{d(ca)} \tag{16}$$

where x_{ac} is the inductive reactance associated with the flux produced by I_c that links conductor a out to the return path g feet away.

With all three currents I_a, I_b, I_c flowing simultaneously, we have in terms of x_a and x_d factors:

$$E_a-E_a'=I_a(x_a+x_{d(ak)})+I_b(x_{d(bk)}-x_{d(ba)}) +I_c(x_{d(ck)}-x_{d(ca)}). \tag{17}$$

Expanding and regrouping the terms we have:

$$E_a-E_a'=I_a x_a-I_b x_{d(ba)}-I_c x_{d(ca)} +[I_a x_{d(ak)}+I_b x_{d(bk)}+I_c x_{d(ck)}]. \tag{18}$$

Since $I_c=-I_a-I_b$, the terms in the bracket may be written

$$I_a(x_{d(ak)}-x_{d(ck)})+I_b(x_{d(bk)}-x_{d(ck)}).$$

Using the definition of x_d, $0.2794\frac{f}{60}\log\frac{D_{12}}{1}$, this expression can be written

$$I_a\left(0.2794\frac{f}{60}\log\frac{d_{(ak)}}{d_{(ck)}}\right)+I_b\left(0.2794\frac{f}{60}\log\frac{d_{(bk)}}{d_{(ck)}}\right).$$

Assuming the distances $d_{(ak)}$, $d_{(ck)}$, and $d_{(bk)}$ to the remote path approach infinity, then the ratios $\frac{d_{(ak)}}{d_{(ck)}}$ and $\frac{d_{(bk)}}{d_{(ck)}}$ approach unity. Since the log of unity is zero, the two terms in the bracket are zero, and Eq. (18) reduces to

$$E_a-E_a'=I_a x_a-I_b x_{d(ba)}-I_c x_{d(ca)} \tag{19}$$

since

$$x_{d(ba)}=x_{d(ca)}=x_{d(bc)}=x_d, \text{ and } I_a=-I_b-I_c,$$
$$E_a-E_a'=I_a(x_a+x_d). \tag{20}$$

Dividing the equation by I_a,

$$x_1=x_2=\frac{E_a-E_a'}{I_a}=x_a+x_d \text{ ohms per phase per mile} \tag{21}$$

where

x_a = inductive reactance for conductor a due to the flux out to one foot.

x_d = inductive reactance corresponding to the flux external to a one-foot radius from conductor a out to the center of conductor b or conductor c since the spacing between conductors is symmetrical.

Therefore, the positive- or negative-sequence inductive reactance per phase for a three-phase circuit with equilateral spacing is the same as for one conductor of a single-phase circuit as previously derived. Values of x_a for various conductors are given in the tables of electrical characteristics of conductors later in the chapter, and the values of x_d are given in the tables of inductive reactance spacing factors for various conductor spacings.

When the conductors are unsymmetrically spaced, the voltage drop for each conductor is different, assuming the currents to be equal and balanced. Also, due to the unsymmetrical conductor spacing, the magnetic field external to the conductors is not zero, thereby causing induced voltages in adjacent electrical circuits, particularly telephone circuits, that may result in telephone interference.

To reduce this effect to a minimum, the conductors are transposed so that each conductor occupies successively the

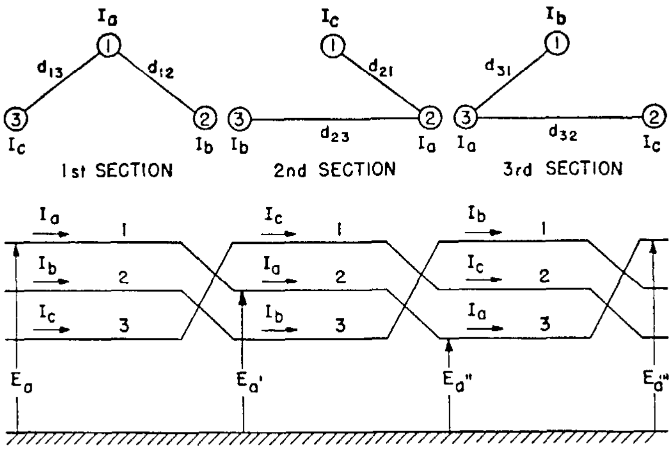


Fig. 13—A Three-conductor three-phase circuit (unsymmetrical spacing).

same positions as the other two conductors in two successive line sections. For three such transposed line sections, called a "barrel of transposition", the total voltage drop for each conductor is the same, and any electrical circuit parallel to the three transposed sections has a net voltage of very low magnitude induced in it due to normal line currents.

In the following derivation use is made of the general equations developed for the case of symmetrically spaced conductors. First, the inductive reactance voltage drop of phase a in each of the three line sections is obtained. Adding these together and dividing by three gives the average inductive reactance voltage drop for a line section. Referring to Fig. 13 and using Eq. (19) for the first line section where I_a is flowing in conductor 1,

$$E_a - E_a' = I_a x_a - I_b x_{d(12)} - I_c x_{d(13)}.$$

In the second line section where I_a is flowing in conductor 2,

$$E_a' - E_a'' = I_a x_a - I_b x_{d(23)} - I_c x_{d(21)}.$$

In the third line section where I_a is flowing in conductor 3,

$$E_a'' - E_a''' = I_a x_a - I_b x_{d(31)} - I_c x_{d(32)}.$$

Taking the average voltage drop per line section, we have

$$E_{avg} = \frac{(E_a - E_a') + (E_a - E_a'') + (E_a'' - E_a''')}{3}$$

$$= \frac{3I_a x_a}{3} - I_b \frac{(x_{d(12)} + x_{d(23)} + x_{d(31)})}{3} - \frac{I_c(x_{d(12)} + x_{d(23)} + x_{d(31)})}{3}$$

$$E_{avg} = I_a x_a - (I_b + I_c)\frac{(x_{d(12)} + x_{d(23)} + x_{d(31)})}{3}$$

Since

$$I_a + I_b + I_c = 0, \qquad I_a = -(I_b + I_c)$$

$$E_{avg} = I_a\left(x_a + \frac{x_{d(12)} + x_{d(23)} + x_{d(31)}}{3}\right).$$

Dividing by I_a, we have the positive- or negative-sequence inductive reactance per phase

$$x_1 = x_2 = (x_a + x_d) \text{ ohms per phase per mile}$$

where

$$x_d = \tfrac{1}{3}(x_{d(12)} + x_{d(23)} + x_{d(31)}) \text{ ohms per phase per mile.} \tag{22}$$

Expressed in general terms,

$$x_d = \tfrac{1}{3}\left(0.2794\frac{f}{60}\right)(\log d_{(12)} + \log d_{(23)} + \log d_{(31)})$$

$$x_d = \tfrac{1}{3}\, 0.2794\frac{f}{60} \log d_{12}d_{23}d_{31}$$

$$x_d = 0.2794\frac{f}{60} \log \sqrt[3]{d_{12}d_{23}d_{31}}$$

$$x_d = 0.2794\frac{f}{60} \log \text{GMD}$$

where GMD (geometrical mean distance) $= \sqrt[3]{d_{12}d_{23}d_{31}}$, and is mathematically defined as the nth root of an n-fold product.

For a three-phase circuit where the conductors are not symmetrically spaced, we therefore have an expression for the positive- or negative-sequence inductive reactance, which is similar to the symmetrically spaced case except x_d is the inductive-reactance spacing factor for the GMD (geometric mean distance) of the three conductor separations. For x_d, then, in the case of unsymmetrical conductor spacing, we can take the average of the three inductive-reactance spacing factors

$$x_d = \tfrac{1}{3}(x_{d(12)} + x_{d(23)} + x_{d(31)}) \text{ ohms per phase per mile}$$

or we can calculate the GMD of the three spacings

$$\text{GMD} = \sqrt[3]{d_{12}d_{23}d_{31}} \text{ feet} \tag{23}$$

and use the inductive-reactance spacing factor for this distance. This latter procedure is perhaps the easier of the two methods.

x_a is taken from the tables of electrical characteristics of conductors presented later in the chapter, and x_d is taken

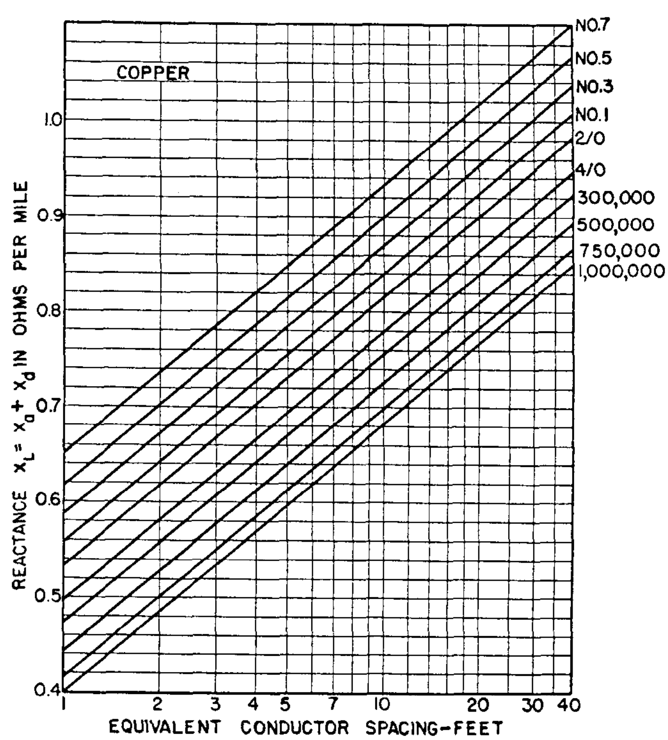


Fig. 14—Quick reference curves for 60-cycle inductive reactance of three-phase lines (per phase) using hard drawn copper conductors. For total reactance of single-phase lines multiply these values by two. See Eqs. (10) and (21).

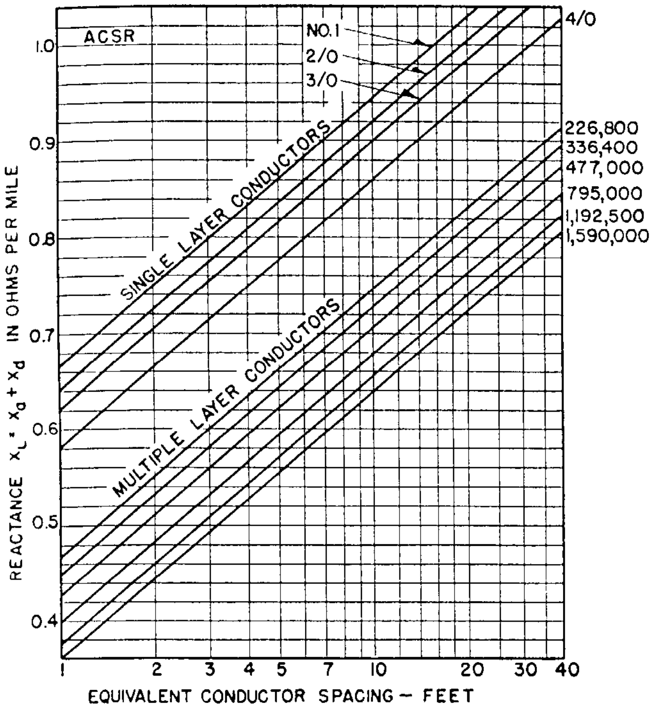


Fig. 15—Quick reference curves for 60-cycle inductive reactance of three-phase lines (per phase) using ACSR conductors. For total reactance of single-phase lines, multiply these values by two. See Eqs. (10) and (21).

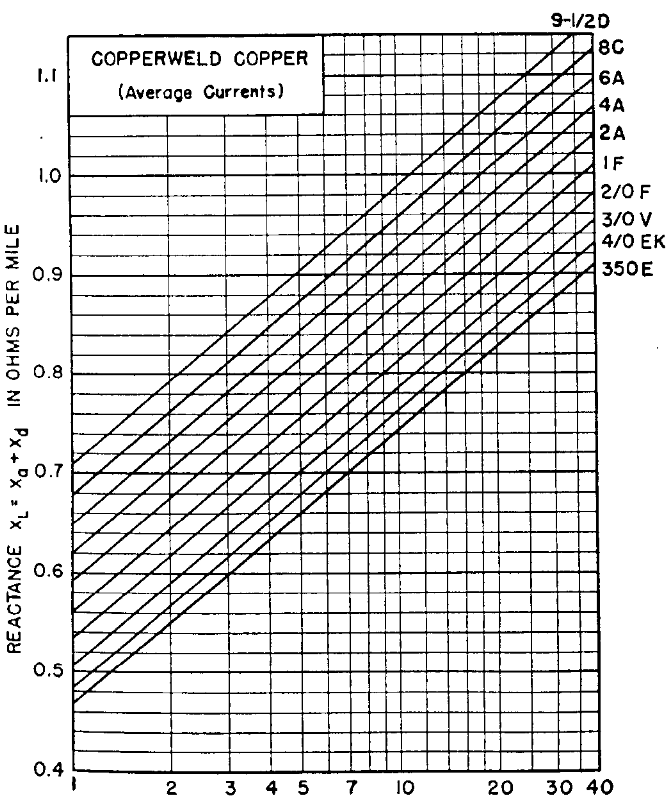


Fig. 16—Quick reference curves for 60-cycle inductive reactance of three-phase lines (per phase) using Copperweld-Copper conductors. For total reactance of single-phase lines multiply these values by two. See Eqs. (10) and (21).

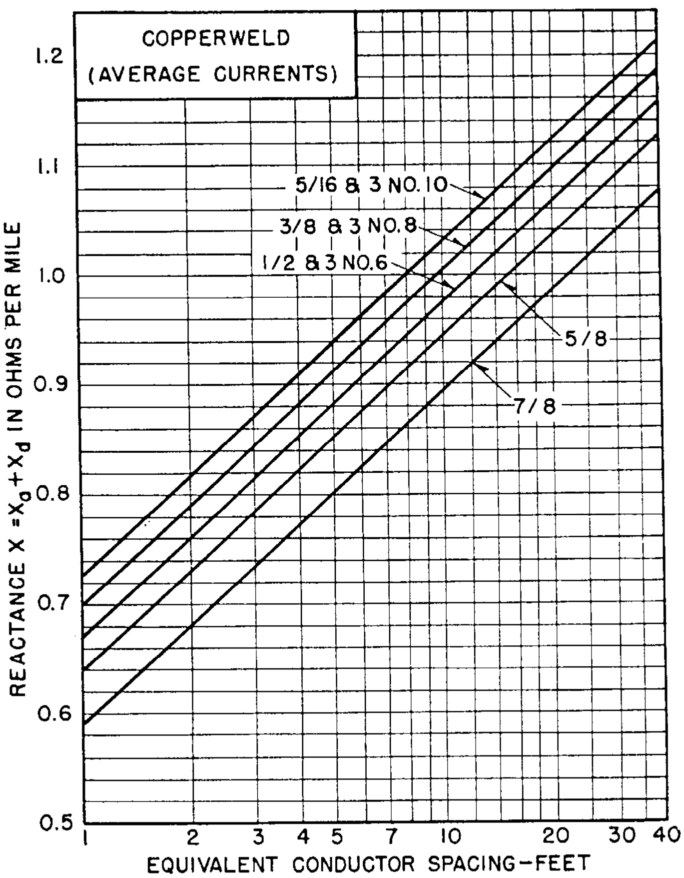


Fig. 17—Quick reference curves for 60-cycle inductive reactance of three-phase lines (per phase) using Copperweld conductors. For total reactance of single-phase lines multiply these values by two. See Eqs. (10) and (21).

from the tables of inductive-reactance spacing factors. Geometric mean distance (GMD) is sometimes referred to as "equivalent conductor spacing." For quick reference the curves of Figs. (14), (15), (16), and (17) have been plotted giving the reactance (x_a+x_d) for different conductor sizes and "equivalent conductor spacings."

Since most three-phase lines or circuits do not have conductors symmetrically spaced, the above formula for positive- or negative-sequence inductive reactance is generally used. This formula, however, assumes that the circuit is transposed.

When a single-circuit line or double-circuit line is not transposed, either the dissymmetry is to be ignored in the calculations, in which case the general symmetrical components methods can be used, or dissymmetry is to be considered, thus preventing the use of general symmetrical-components methods. In considering this dissymmetry, unequal currents and voltages are calculated for the three phases even when terminal conditions are balanced. In most cases of dissymmetry it is most practical to treat the circuit as transposed and use the equations for x_1 and x_2 derived for an unsymmetrically-spaced transposed circuit Some error results from this method but in general it is small as compared with the laborious calculations that must be made when the method of symmetrical components cannot be used.

Positive- and Negative-Sequence Reactance of Parallel Circuits—When two parallel three-phase circuits are close together, particularly on the same tower, the effect of mutual inductance between the two circuits is not entirely eliminated by transpositions. By referring to Fig. 18 showing two transposed circuits on a single tower, the positive- or negative-sequence reactance of the paralleled circuit is:

$$x_1 = x_2 = 0.2794\frac{f}{60}\left[\tfrac{1}{2}\log_{10}\frac{\sqrt[3]{d_{ab}d_{bc}d_{ca}}}{GMR_{\text{conductor}}} - \tfrac{1}{12}\log_{10}\frac{(d_{aa'})^4(d_{bb'})^2}{(d_{ab'})^2(d_{ca'})(d_{ac'})(d_{ba'})^2}\right]$$

ohms per phase per mile. (24)

in which the distances are those between conductors in the first section of transposition.

The first term in the above equation is the positive- or negative-sequence reactance for the combined circuits. The second term represents the correction factor due to the mutual reactance between the two circuits and may reduce the reactance three to five percent. The formula assumes transposition of the conductor as shown in Fig. 18.

The formula also assumes symmetry about the vertical axis but not necessarily about the horizontal axis.

As contrasted with the usual conductor arrangement as shown in Fig. 18, the arrangement of conductors shown in Fig. 19 might be used. However, this arrangement of conductors results in five to seven percent greater inductive reactance than the usual arrangement of conductors. This has been demonstrated in several references.[3]

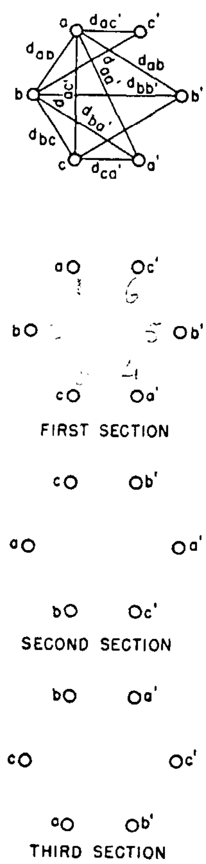


Fig. 18—Two parallel three-phase circuits on a single tower showing transpositions.

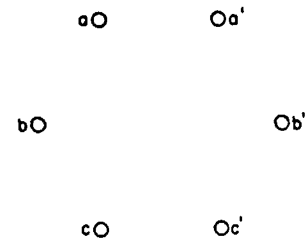


Fig. 19—Arrangement of conductors on a single tower which materially increases the inductance per phase.

3. Zero-Sequence Resistance and Inductive Reactance

The development of zero-sequence resistance and inductive reactance of aerial lines will be considered simultaneously as they are related quantities. Since zero-sequence currents for three-phase systems are in phase and equal in magnitude, they flow out through the phase conductors and return by a neutral path consisting of the earth alone, neutral conductor alone, overhead ground wires, or any combination of these. Since the return path often consists of the earth alone, or the earth in parallel with some other path such as overhead ground wires, it is necessary to use a method that takes into account the resistivity of the earth as well as the current distribution in the earth. Since both the zero-sequence resistance and inductive-reactance of three-phase circuits are affected by these two factors, their development is considered jointly.

As with the positive- and negative-sequence inductive reactance, first consider a single-phase circuit consisting of a single conductor grounded at its far end with the earth acting as a return conductor to complete the circuit. This permits the development of some useful concepts for calculating the zero-sequence resistance and inductive reactance of three-phase circuits.

Figure 20 shows a single-phase circuit consisting of a single outgoing conductor a, grounded at its far end with the return path for the current consisting of the earth. A second conductor, b, is shown to illustrate the mutual effects produced by current flowing in the single-phase circuit. The zero-sequence resistance and inductive reactance of this circuit are dependent upon the resistivity of the earth and the distribution of the current returning in the earth.

This problem has been analyzed by Rudenberg, Mayr,

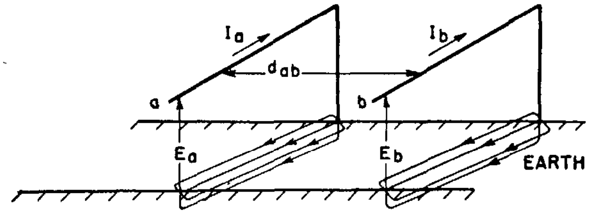


Fig. 20—A single conductor single phase circuit with earth return.

and Pollaczek in Europe, and Carson and Campbell in this country. The more commonly used method is that of Carson, who, like Pollaczek, considered the return current to return through the earth, which was assumed to have uniform resistivity and to be of infinite extent.

The solution of the problem is in two parts: (1) the determination of the self impedance z_g of conductor a with earth return (the voltage between a and earth for unit current in conductor a), and (2), the mutual impedance z_{gm} between conductors a and b with common earth return (the voltage between b and earth for unit current in a and earth return).

As a result of Carson's formulas, and using average heights of conductors above ground, the following fundamental simplified equations may be written:

$$z_g = r_c + 0.00159f + j0.004657f \log_{10} \frac{2160\sqrt{\frac{\rho}{f}}}{\text{GMR}} \quad \text{ohms per mile} \tag{25}$$

$$z_{gm} = 0.00159f + j0.004657f \log_{10} \frac{2160\sqrt{\frac{\rho}{f}}}{d_{ab}} \quad \text{ohms per mile} \tag{26}$$

where

r_c = resistance of conductor a per mile.
f = frequency in cps.
ρ = earth resistivity in ohms per meter cube.
GMR = geometric mean radius of conductor a in feet.
d_{ab} = distance between conductors a and b in feet.

A useful physical concept for analyzing earth-return circuits is that of concentrating the current returning through the earth in a fictitious conductor at some considerable depth below the outgoing conductor a. This equivalent depth of the fictitious return conductor is represented as D_e.

For the single-conductor, single-phase circuit with earth return now considered as a single-phase, two-wire circuit, the self-inductive reactance is given by the previously derived $j0.2794\frac{f}{60}\log_{10}\frac{D_{12}}{\text{GMR}}$ (See Eq. (8)) for a single-phase, two-wire circuit, or $j0.004657f \log_{10}\frac{D_e}{\text{GMR}}$ where D_e is substituted for D_{12}, the distance between conductor a and the fictitious return conductor in the earth. This expression is similar to the inductive-reactance as given in Carson's simplified equation for self impedance. Equating the logarithmic expressions of the two equations,

$$j0.004657f\log_{10}\frac{D_e}{\text{GMR}} = j0.004657f\log_{10}\frac{2160\sqrt{\frac{\rho}{f}}}{\text{GMR}}$$

$$\text{or } D_e = 2160\sqrt{\frac{\rho}{f}} \text{ feet.} \tag{27}$$

This defines D_e, equivalent depth of return, and shows that it is a function of earth resistivity, ρ, and frequency, f.

Also an inspection of Carson's simplified equations show that the self and mutual impedances contain a resistance component $0.00159f$, which is a function of frequency.

Rewriting Carson's equations in terms of equivalent depth of return, D_e,

$$z_g = r_c + 0.00159f + j0.004657f \log_{10}\frac{D_e}{\text{GMR}} \quad \text{ohms per mile.} \tag{28}$$

$$z_{gm} = 0.00159f + j0.004657f\log_{10}\frac{D_e}{d_{ab}} \text{ ohms per mile.} \tag{29}$$

These equations can be applied to multiple-conductor circuits if r_c, the GMR and d_{ab} refer to the conductors as a group. Subsequently the GMR of a group of conductors are derived for use in the above equations.

To convert the above equations to zero-sequence quantities the following considerations must be made. Considering three conductors for a three-phase system, unit zero-sequence current consists of one ampere in each phase conductor and three amperes in the earth return circuit. To use Eqs. (28) and (29), replace the three conductors by a single equivalent conductor in which three amperes flow for every ampere of zero-sequence current. Therefore the corresponding zero-sequence self and mutual impedances per phase are three times the values given in Carson's simplified equations. Calling the zero sequence impedances z_0 and z_{0m}, we have:

$$z_0 = 3r_c + 0.00477f + j0.01397f \log_{10}\frac{D_e}{\text{GMR}} \quad \text{ohms per phase per mile.} \tag{30}$$

$$z_{0(m)} = 0.00477f + j0.01397f \log_{10}\frac{D_e}{d_{ab}} \quad \text{ohms per phase per mile} \tag{31}$$

where f = frequency in cps.
r_c = resistance of a conductor equivalent to the three conductors in parallel. $3r_c$ therefore equals the resistance of one conductor for a three-phase circuit.
GMR = geometric mean radius for the group of phase conductors. This is different than the GMR for a single conductor and is derived subsequently as $\text{GMR}_{\text{circuit}}$.
d_{ab} = distance from the equivalent conductor to a parallel conductor, or some other equivalent conductor if the mutual impedance between two parallel three-phase circuits is being considered.

For the case of a single overhead ground wire, Eq. (30) gives the zero-sequence self impedance. Equation (31) gives the zero-sequence mutual impedance between two overhead ground wires.

Zero-sequence self impedance of two ground wires with earth return

Using Eq. (30) the zero-sequence self impedance of two ground wires with earth return can be derived.

$$z_0 = 3r_c + 0.00477f + j0.01397f \log_{10}\frac{D_e}{\text{GMR}} \quad \text{ohms per phase per mile} \tag{30}$$

where r_c = resistance of a single conductor equivalent to the two ground wires in parallel. (r_c therefore becomes $\frac{r_a}{2}$ where r_a is the resistance of one of the two ground wires).

GMR = geometric mean radius for the two ground wires. (GMR therefore becomes $\sqrt[4]{(\mathrm{GMR})^2_{\text{conductor}}\, d_{xy}^2}$ or $\sqrt[2]{(\mathrm{GMR})(d_{xy})}$

where d_{xy} is the distance between the two conductors x and y.)

Substituting $\frac{r_a}{2}$ for r_c and $\sqrt[2]{(\mathrm{GMR})(d_{xy})}$ for GMR in Eq. (30), the zero-sequence self impedance of two ground wires with earth return becomes

$$z_0 = \frac{3r_a}{2} + 0.00477f + j0.01397f \log_{10} \frac{D_e}{\sqrt[2]{(\mathrm{GMR})(d_{xy})}}$$

ohms per mile per phase. (32)

Zero-sequence self impedance of n ground wires with earth return

Again using Eq. (30), the zero-sequence self impedance of n ground wires with earth return can be developed.

$$z_0 = 3r_c + 0.00477f + j0.01397f \log_{10} \frac{D_e}{\mathrm{GMR}}$$

ohms per mile per phase. (30)

Since r_c is the resistance of a single conductor equivalent to n ground wires in parallel, then $r_c = \frac{r_a}{n}$ where r_a is the resistance of one of the n ground wires, in ohms per phase per mile.

GMR is the geometric mean radius of the n ground wires as a group, which may be written as follows in terms of all possible distances,

$$\mathrm{GMR} = \sqrt[n^2]{(\mathrm{GMR})^n_{\text{conductor}} (d_{(g_1g_2)} d_{(g_1g_3)} \text{—} d_{(g_1g_n)}) (d_{(g_2g_1)} d_{(g_2g_3)} \text{—} d_{(g_2g_n)}) (d_{(g_3g_1)} d_{(g_3g_2)} \text{—} d_{(g_3g_n)}) (d_{(g_ng_1)} d_{(g_ng_2)} \text{—} d_{(g_ng_{n-1})})} \text{ feet.}$$

This expression can also be written in terms of all possible pairs of distances as follows.

$$\mathrm{GMR} = \sqrt[n^2]{(\mathrm{GMR})^n_{\text{conductor}} (d_{(g_1g_2)} d_{(g_1g_3)} - d_{(g_1g_n)})^2 (d_{(g_2g_3)} - d_{(g_2g_n)})^2 \ (d_{(g_3g_n)})^2} \text{ feet.} \quad (33)$$

The equation for zero-sequence self impedance of n ground wires with earth return can therefore be obtained by substituting $\frac{r_a}{n}$ for r_c and Eq. (33) for GMR in Eq. (30).

Self impedance of parallel conductors with earth return

In the preceeding discussion the self and mutual impedances between single cylindrical conductors with earth return were derived from which the zero-sequence self and mutual reactances were obtained. These expressions were expanded to include the case of multiple overhead ground wires, which are not transposed. The more common case is that of three-phase conductors in a three-phase circuit which can be considered to be in parallel when zero-sequence currents are considered. Also the three conductors in a three-phase circuit are generally transposed. This factor was not considered in the preceeding cases for multiple overhead ground wires.

In order to derive the zero-sequence self impedance of three-phase circuits it is first necessary to derive the self impedance of three-phase circuits taking into account transpositions. The expression for self impedance is then converted to zero-sequence self impedance in a manner analogous to the case of single conductors with earth return.

Consider three phase conductors a, b, and c as shown in Fig. 21. With the conductors transposed the current

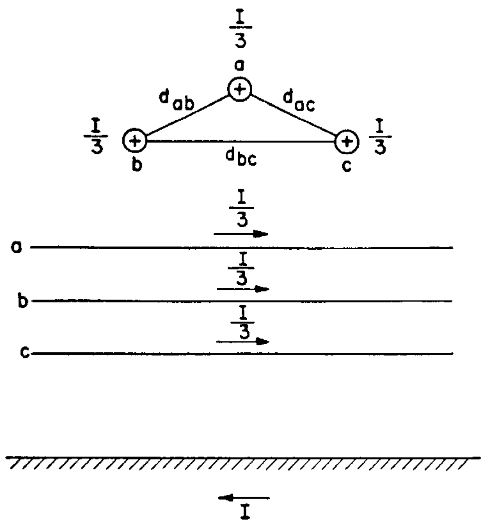


Fig. 21—Self impedance of parallel conductors with earth return.

divides equally between the conductors so that for a total current of unity, the current in each conductor is one third.

The voltage drop in conductor a for the position indicated in Fig. 21 is

$$\frac{z_{aa}}{3} + \frac{z_{ab}}{3} + \frac{z_{ac}}{3}$$

For conductor b:

$$\frac{z_{ab}}{3} + \frac{z_{bb}}{3} + \frac{z_{bc}}{3}$$

and for conductor c:

$$\frac{z_{ac}}{3} + \frac{z_{bc}}{3} + \frac{z_{cc}}{3}$$

in which z_{aa}, z_{bb}, and z_{cc} are the self impedances of the three conductors with ground return and z_{ab}, z_{bc}, and z_{ac} are the mutual impedances between the conductors.

Since conductor a takes each of the three conductor positions successively for a transposed line, the average drop per conductor is

$$\frac{1}{9}(z_{aa} + z_{bb} + z_{cc} + 2z_{ab} + 2z_{bc} + 2z_{ac}).$$

Substituting the values of self and mutual impedances given by Eqs. (28) and (29) in this expression,

$$z_g = \frac{1}{9}\left[3r_c + 9(0.00159f) + j0.004657f\left(3 \log_{10} \frac{D_e}{\mathrm{GMR}} + 2 \log_{10} \frac{D_e}{d_{ab}} + 2 \log_{10} \frac{D_e}{d_{bc}} + 2 \log_{10} \frac{D_e}{d_{ac}}\right)\right]$$

ohms per mile.

$$z_g = \frac{r_c}{3} + 0.00159f + j0.004657f \log_{10} \frac{D_e}{\sqrt[9]{(\mathrm{GMR})^3 d_{ab}^2 d_{bc}^2 d_{ca}^2}}$$

ohms per mile. (34)

The ninth root in the denominator of the logarithmic term is the GMR of the circuit and is equal to an infinitely thin tube which would have the same inductance as the three-conductor system with earth return shown in Fig. 21.

$$\mathrm{GMR}_{\mathrm{circuit}} = \sqrt[9]{(\mathrm{GMR})^3_{\mathrm{conductor}}\, d_{ab}^2 d_{bc}^2 d_{ca}^2} \quad \text{feet.}$$

$$\mathrm{GMR}_{\mathrm{circuit}} = \sqrt[9]{(\mathrm{GMR})^3_{\mathrm{conductor}}\, (d_{ab} d_{bc} d_{ca})^2} \text{ feet.}$$

$$\mathrm{GMR}_{\mathrm{circuit}} = \sqrt[3]{\mathrm{GMR}_{\mathrm{conductor}} (\sqrt[3]{d_{ab} d_{bc} d_{ca}})^2} \quad \text{feet.}$$

By previous derivation (See Eq. (23)), $\mathrm{GMD}_{\mathrm{separation}}$

$$= \sqrt[3]{d_{ab} d_{bc} d_{ca}} \quad \text{feet.}$$

Therefore $\mathrm{GMR}_{\mathrm{circuit}} = \sqrt[3]{(\mathrm{GMR})_{\mathrm{conductor}} (\mathrm{GMD})^2_{\mathrm{separation}}}$ feet. (35)

Substituting $\mathrm{GMR}_{\mathrm{circuit}}$ from equation (35) in equation (34),

$$z_g = \frac{r_c}{3} + 0.00159f + j0.004657f \log_{10} \frac{D_e}{\sqrt[3]{(\mathrm{GMR})_{\mathrm{conductor}} (\mathrm{GMD})^2_{\mathrm{separation}}}}$$

ohms per mile. (36)

In equations (34) and (36), r_c is the resistance per mile of one phase conductor.

Zero-sequence self impedance of three parallel conductors with earth return

Equation (36) gives the self impedance of three parallel conductors with earth return and was derived for a total current of unity divided equally among the three conductors. Since zero-sequence current consists of unit current in each conductor or a total of three times unit current for the group of three conductors, the voltage drop for zero-sequence currents is three times as great. Therefore Eq. (36) must be multiplied by three to obtain the zero-sequence self impedance of three parallel conductors with earth return. Therefore,

$$z_0 = r_c + 0.00477f + j0.01397f \log_{10} \frac{D_e}{\sqrt[3]{\mathrm{GMR}_{\mathrm{conductor}} (\mathrm{GMD})^2_{\mathrm{separation}}}}$$

ohms per phase per mile (37)

where $\sqrt[3]{\mathrm{GMR}_{\mathrm{conductor}} (\mathrm{GMD})^2_{\mathrm{separation}}}$ is the $\mathrm{GMR}_{\mathrm{circuit}}$ derived in equation (35) or $\sqrt[9]{(\mathrm{GMR})^3_{\mathrm{conductor}}\, d_{ab}^2 d_{bc}^2 d_{ca}^2}$

Zero-sequence mutual impedance between two circuits with earth return

Using a similar method of derivation the zero-sequence mutual impedance between 2 three-phase circuits with common earth return is found to be

$$z_{0(m)} = 0.00477f + j0.01397f \log_{10} \frac{D_e}{\mathrm{GMD}}$$

ohms per phase per mile (38)

where GMD is the geometric mean distance between the 2 three-phase circuits or the ninth root of the product of the nine possible distances between conductors in one group and conductors in the other group. Note the similarity between Eq. (38) and Eq. (31)

Zero-sequence self impedance of two identical parallel circuits with earth return

For the special case where the two parallel three-phase circuits are identical, following the same method of derivation

$$z_0 = \frac{r_c}{2} + 0.00477f + j0.01397f \log_{10} \frac{D_e}{\sqrt{(\mathrm{GMR})(\mathrm{GMD})}}$$

ohms per phase per mile (39)

in which GMR is the geometric mean radius of one set of conductors, $(\sqrt[3]{(\mathrm{GMR})_{\mathrm{conductor}} (\mathrm{GMD})^2_{\mathrm{separation}}})$, and GMD is the geometric mean distance between the two sets of conductors or the ninth root of the product of the nine possible distances between conductors in one circuit and conductors in the other circuit.

This equation is the same as $\frac{1}{2}(z_0 + z_{0(m)})$ where z_0 is the zero-sequence self impedance of one circuit by equation (37) and $z_{0(m)}$ is the zero-sequence mutual impedance between two circuits as given by Eq. (38). For non-identical circuits it is better to compute the mutual and self impedance for the individual circuits, and using $\frac{1}{2}(z_0 + z_{0(m)})$ compute the zero-sequence self impedance.

Zero-sequence mutual impedance between one circuit (with earth return) and n ground wires (with earth return)

Figure 22 shows a three-phase circuit with n ground

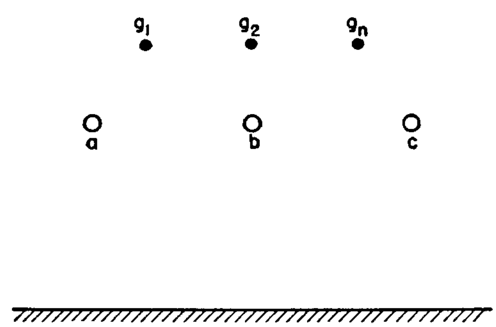


Fig. 22—A three-conductor three-phase circuit (with earth return) and *n* ground wires (with earth return)

wires. Equation (31) gives the zero sequence mutual impedance between two conductors:

$$z_{0(m)} = 0.00477f + j0.01397f \log_{10} \frac{D_e}{d_{ab}}$$

ohms per phase per mile (31)

where d_{ab} is the distance between the two conductors. This equation can be applied to two groups of conductors if d_{ab} is replaced by the GMD or geometric mean distance between the two groups. In Fig. 22, if the ground wires are considered as one group of conductors, and the phase conductors a, b, c, are considered as the second group of conductors, then the GMD between the two groups is

$$\mathrm{GMD} = \sqrt[3n]{d_{ag1} d_{bg1} d_{cg1} \text{---} d_{agn} d_{bgn} d_{cgn}} \quad \text{feet}$$

Substituting this quantity for d_{ab} in Eq. (31) results in an equation for the zero-sequence mutual impedance between one circuit and n ground wires. This $z_{0(m)}$ is $z_{0(ag)}$.

$$z_{0(m)} = 0.00477f + j0.01397f \log_{10} \frac{D_e}{\sqrt[3n]{d_{ag1}d_{bg1}d_{cg1} \text{---} d_{agn}d_{bgn}d_{cgn}}}$$
ohms per phase per mile. (40)

Zero-sequence impedance of one circuit with n ground wires (and earth) return.

Referring to Fig. 20 the zero-sequence self impedance of a single conductor, and the zero-sequence mutual impedance between a single conductor and another single conductor with the same earth return path was derived. These values are given in Eqs. (30) and (31). As stated before, these equations can be applied to multi-conductor circuits by substituting the circuit GMR for the conductor GMR in Eq. (30) and the GMD between the two circuits for d_{ab} in Eq. (31).

First, consider the single-conductor, single-phase circuit with earth return and one ground wire with earth return. Referring to Fig. 20 conductor a is considered as the single conductor of the single-phase circuit and conductor b will be used as the ground wire.

Writing the equations for E_a and E_b, we have:

$$E_a = I_a z_{aa} + I_b z_m \tag{41}$$

$$E_b = I_a z_m + I_b z_{bb}. \tag{42}$$

If we assume conductor b as a ground wire, then $E_b = 0$ since both ends of this conductor are connected to ground. Therefore solving Eq. (42) for I_b and substituting this value of I_b in Eq. (41),

$$E_a = I_a\left(z_{aa} - \frac{z_m^2}{z_{bb}}\right).$$

To obtain z_a, divide E_a by I_a, and the result is

$$z_a = z_{aa} - \frac{z_m^2}{z_{bb}} \tag{43}$$

The zero-sequence impedance of a single-conductor, single-phase circuit with one ground wire (and earth) return is therefore defined by Eq. (43) when zero-sequence self impedances of single-conductor, single-phase circuits are substituted for z_{aa} and z_{bb} and the zero-sequence mutual impedance between the two conductors is substituted for z_m. Equation (43) can be expanded to give the zero-sequence impedance of a three-phase circuit with n ground wires (and earth) return.

$$z_0 = z_{0(a)} - \frac{z_0^2{}_{(ag)}}{z_{0(g)}} \tag{44}$$

Where z_0 = zero-sequence impedance of one circuit with n ground wires (and earth) return.

$z_{0(a)}$ = zero-sequence self impedance of the three-phase circuit.

$z_{0(g)}$ = zero-sequence self impedance of n ground wires.

$z_{0(ag)}$ = zero-sequence mutual impedance between the phase conductors as one group of conductors and the ground wire(s) as the other conductor group.

Equation (44) results in the equivalent circuit of Fig. 23 for determining the zero-sequence impedance of one circuit with n ground wires (and earth) return.

General Method for Zero-Sequence Calculations—The preceding sections have derived the zero-sequence self and mutual impedances for the more common circuit arrangements both with and without ground wires. For more complex circuit and ground wire arrangements a general method must be used to obtain the zero-sequence impedance of a particular circuit in such arrangements.

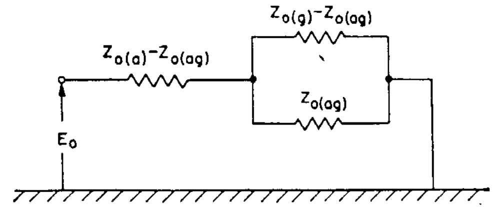


Fig. 23—Equivalent circuit for zero-sequence impedance of one circuit (with earth return) and *n* ground wires (with earth return).

The general method consists of writing the voltage drop for each conductor or each group of conductors in terms of zero-sequence self and mutual impedances with all conductors or groups of conductors present. Ground wire conductors or groups of conductors have their voltage drops equal to zero. Solving these simultaneous equations for $\frac{E_0}{I_0}$ of the desired circuit gives the zero-sequence impedance of that circuit in the presence of all the other zero-sequence circuits.

This general method is shown in detail in Chap. 2, Part X, Zero-Sequence Reactances. Two circuits, one with two overhead ground wires and one with a single overhead ground wire are used to show the details of this more general method.

Practical Calculation of Zero-Sequence Impedance of Aerial Lines—In the preceding discussion a number of equations have been derived for zero-sequence self and mutual impedances of transmission lines taking into account overhead ground wires. These equations can be further simplified to make use of the already familiar quantities r_a, x_a, and x_d. To do this two additional quantities, r_e and x_e are necessary that result from the use of the earth as a return path for zero-sequence currents. They are derived from Carson's formulas and can be defined as follows:

$r_e = 0.00477f$ ohms per phase per mile.

$x_e = 0.006985f \log_{10} 4.6655 \times 10^6 \frac{\rho}{f}$ ohms per phase per mile. (46)

It is now possible to write the previously derived equations for zero-sequence self and mutual impedances in terms of r_a, x_a, x_d, r_e, and x_e. The quantities r_a, x_a, x_d are given in the tables of Electrical Characteristics of Conductors and Inductive Reactance Spacing Factors. The quantities r_e and x_e are given in Table 7 as functions of earth resistivity, ρ, in meter ohms for 25, 50, and 60 cycles per second. The following derived equations are those most commonly used in the analysis of power system problems.

Zero-sequence impedance—one circuit (with earth return) but without ground wires

$$z_0 = r_e + 0.00477f + j0.01397f \log_{10} \frac{D_e}{\sqrt[3]{(\text{GMR})_{\text{conductor}}(\text{GMD})^2_{\text{separation}}}}$$ ohms per phase per mile. (37)

$$z_0 = r_a + r_e + j0.00698f \log_{10} 4.6656 \times 10^6 \frac{\rho}{f} + j0.2794\frac{f}{60} \log_{10} \frac{1}{\text{GMR}_{\text{conductor}}} - j2(0.2794\frac{f}{60} \log_{10} \text{GMD}_{\text{separation}})$$

$$z_0 = r_a + r_e + j(x_e + x_a - 2x_d) \text{ ohms per phase per mile} \quad (47)$$

where $x_d = \frac{1}{3}(x_{d(ab)} + x_{d(bc)} + x_{d(ca)})$

and $x_{d(ab)} = x_d$ from Table 6 for spacing a to b, etc.

Mutual zero-sequence impedance between two circuits (with earth return) but without ground wires

$$z_{0(m)} = 0.00477f + j0.01397f \log_{10} \frac{D_e}{\text{GMD}}$$ ohms per phase per mile. (38)

$$z_{0(m)} = r_e + j0.006985f \log_{10} 4.665 \times 10^6 \frac{\rho}{f} - j0.006985f \log_{10} \text{GMD}^2$$

$$z_{0(m)} = r_e + j(x_e - 3x_d) \text{ ohms per phase per mile} \quad (48)$$

where x_d is $\frac{1}{9}(x_{d(aa')} + x_{d(ab')} + x_{d(ac')} + x_{d(ba')} + x_{d(bb')} + x_{d(bc')} + x_{d(ca')} + x_{d(cb')} + x_{d(cc')})$

Zero-sequence self impedance—one ground wire (with earth return)

$$z_{0(g)} = 3r_e + 0.00477f + j0.01397f \log_{10} \frac{D_e}{\text{GMR}_{\text{conductor}}}$$ ohms per phase per mile. (30)

$$z_{0(g)} = 3r_a + r_e + j0.006985f \log_{10} 4.6656 \times 10^6 \frac{\rho}{f} + 0.006985f \log_{10} \frac{1}{(\text{GMR})^2_{\text{conductor}}}$$

$$z_{0(g)} = 3r_a + r_e + j(x_e + 3x_a) \text{ ohms per phase per mile.} \quad (49)$$

Zero sequence self impedance—two ground wires (with earth return)

$$z_{0(g)} = \frac{3r_a}{2} + 0.00477f + j0.01397f \log_{10} \frac{D_e}{\sqrt[2]{(\text{GMR})_{\text{conductor}} d_{xy}}}$$ ohms per phase per mile. (32)

$$z_{0(g)} = \frac{3r_a}{2} + r_e + j0.006985f \log_{10} 4.6656 \times 10^6 \frac{\rho}{f} + \frac{0.8382}{2} \log \frac{1}{\text{GMR}} - \frac{0.8382}{2} \log \frac{d_{xy}}{1}$$

$$z_{0(g)} = \frac{3r_a}{2} + r_e + j(x_e + \frac{3}{2}x_a - \frac{3}{2}x_d)$$ ohms per phase per mile (50)

where

$x_d = x_d$ from Table 6 for spacing between ground wires, d_{xy}.

Zero-sequence self impedance—n ground wires (with earth return)

$$z_{0(g)} = 3r_c + 0.00477f + j0.01397f \log_{10} \frac{D_e}{\text{GMR}}$$ ohms per phase per mile (30)

where $r_c = \frac{r_a}{n}$ ohms per phase per mile.

$$\text{GMR} = \sqrt[n^2]{(\text{GMR})^n_{\text{conductor}}(d_{g1g2}d_{g1g3}\text{---}d_{g1gn})(d_{g2g1}d_{g2g3}\text{---}d_{g2gn})(d_{g3g1}d_{g3g2}\text{---}d_{g3gn})(d_{gng1}d_{gng2}\text{---}d_{gng(n-1)})}$$

$$z_{0(g)} = \frac{3r_a}{n} + r_e + jx_e + j\frac{3}{2}(0.2794)\frac{f}{60} \log_{10} 4.665 \times 10^6 \frac{\rho}{f} + j\frac{3}{2}(0.2794)\frac{f}{60} \log_{10} \frac{1}{(\text{GMR})^{\frac{2}{n}}_{\text{conductor}}} - j\frac{3}{2}(0.2794)\frac{f}{60} \log_{10} \left[(d_{g1g2}d_{g1g3}\text{---}d_{g1gn})(d_{g2g1}d_{g2g3}\text{---}d_{g2gn})(d_{g3g1}d_{g3g2}\text{---}d_{g3gn})(d_{gng1}d_{gng2}\text{---}d_{gng(n-1)})\right]^{\frac{2}{n^2}}$$

$$z_{0(g)} = \frac{3}{n}r_a + r_e + j(x_e + \frac{3x_a}{n} - \frac{3(n-1)}{n}x_d)$$ ohms per mile per phase (51)

where $x_d = \frac{1}{n(n-1)}$(sum of $x_{d's}$ for all possible distances between all ground wires.)

or $x_d = \frac{2}{n(n-1)}$(sum of $x_{d's}$ for all possible distances between all possible pairs of ground wires).

Zero-sequence mutual impedance between one circuit (with earth return) and n ground wires (with earth return)

$$z_{0(ag)} = 0.00477f + j0.01397f \log_{10} \frac{D_e}{\sqrt[3n]{d_{ag1}d_{bg1}d_{cg1}\text{---}d_{agn}d_{bgn}d_{cgn}}}$$ ohms per phase per mile. (40)

$$z_{0(ag)} = r_e + j0.006985f \log_{10} 4.6656 \times 10^6 \frac{\rho}{f} - j0.006985f \log_{10} (\sqrt[3n]{d_{ag1}d_{bg1}d_{cg1}\text{---}d_{agn}d_{bgn}d_{cgn}})^2$$

$$z_{0(ag)} = r_e + j(x_e - 3x_d) \text{ ohms per phase per mile} \quad (52)$$

where $x_d = \frac{1}{3n}(x_{d(ag1)} + x_{d(bg1)} + x_{d(cg1)} \text{---} + x_{d(agn)} + x_{d(bgn)} + x_{d(cgn)})$.

Zero-sequence impedance—One circuit with n ground wires (and earth return)

$$z_0 = z_{0(a)} - \frac{z_0^2{}_{(ag)}}{z_{0(g)}} \quad (44)$$

where $z_{0(a)}$ = zero-sequence self impedance of the three-phase circuit.

$z_{0(g)}$ = zero-sequence self impedance of n ground wires.

$z_{0(ag)}$ = zero-sequence mutual impedance between the three-phase circuit as one group of conductors and the ground wire(s) as the other conductor group.

4. Positive-, Negative-, and Zero-sequence Shunt Capacitive Reactance

The capacitance of transmission lines is generally a negligible factor at the lower voltages under normal operating conditions. However, it becomes an appreciable effect for higher voltage lines and must be taken into consideration when determining efficiency, power factor, regulation, and voltage distribution under normal operating conditions. Use of capacitance in determining the performance of long high voltage lines is covered in detail in Chap. 9, "Regulation and Losses of Transmission Lines."

Capacitance effects of transmission lines are also useful in studying such problems as inductive interference, lightning performance of lines, corona, and transients on power systems such as those that occur during faults.

For these reasons formulas are given for the positive-, negative-, and zero-sequence shunt capacitive reactance for the more common transmission line configurations. The case of a two-conductor, single-phase circuit is considered to show some of the fundamentals used to obtain these formulas. For a more detailed analysis of the capacitance problem a number of references are available. [2,4,5].

In deriving capacitance formulas the distribution of a charge, q, on the conductor surface is assumed to be uniform. This is true because the spacing between conductors in the usual transmission circuit is large and therefore the charges on surrounding conductors produce negligible distortion in the charge distribution on a particular conductor. Also, in the case of a single isolated charged conductor, the voltage between any two points of distances x and y meters radially from the conductor can be defined as the work done in moving a unit charge of one coulomb from point P_2 to point P_1 through the electric field produced by the charge on the conductor. (See Fig. 24.) This is given

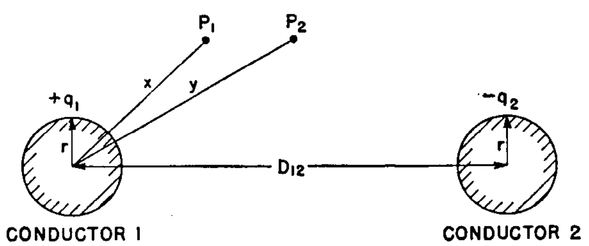


Fig. 24—A two conductor single phase circuit (capacitance).

by

$$V_{xy}=18\times10^9\, q \ln \frac{y}{x} \text{ volts} \tag{53}$$

where q is the conductor charge in coulombs per meter.

By use of this equation and the principle of superposition, the capacitances of systems of parallel conductors can be determined.

Applying Eq. (53) and the principle of superposition to the two-conductor, single-phase circuit of Fig. 24 assuming conductor 1 alone to have a charge q_1, the voltage between conductors 1 and 2 is

$$V_{12}=18\times10^9\, q_1 \ln \frac{D_{12}}{r} \text{ volts.} \tag{54}$$

This equation shows the work done in moving a unit charge from conductor 2 a distance D_{12} meters to the surface of conductor 1 through the electric field produced by q_1. Now assuming only conductor 2, having a charge q_2, the voltage between conductors 1 and 2 is

$$V_{12}=18\times10^9\, q_2 \ln \frac{r}{D_{12}} \text{ volts.} \tag{55}$$

This equation shows the work done in moving a unit charge from the outer radius of conductor 2 to conductor 1 a distance D_{12} meters away through the electric field produced by q_2.

With both charges q_1 and q_2 present, by the principle of superposition the voltage V_{12} is the sum of the voltages resulting from q_1 and q_2 existing one at a time. Therefore V_{12} is the sum of Eqs. (54) and (55) when both charges q_1 and q_2 are present.

$$V_{12}=18\times10^9\left(q_1 \ln \frac{D_{12}}{r}+q_2 \ln \frac{r}{D_{12}}\right) \text{ volts.} \tag{56}$$

Also if the charges on the two conductors are equal and their sum is zero,

$$q_1+q_2=0 \text{ or } q_2=-q_1$$

Substituting $-q_1$ for q_2 in equation (56)

$$V_{12}=36\times10^9 q_1 \ln \frac{D_{12}}{r} \text{ volts.} \tag{57}$$

The capacitance between conductors 1 and 2 is the ratio of the charge to the voltage or

$$\frac{q_1}{V_{12}}=C_{12}=\frac{1}{36\times10^9 \ln \frac{D_{12}}{r}} \text{ farads per meter.} \tag{58}$$

The capacitance to neutral is twice that given in Eq. (58) because the voltage to neutral is half of V_{12}.

$$C_n=\frac{1}{18\times10^9 \ln \frac{D_{12}}{r}} \text{ farads per meter.} \tag{59}$$

The shunt-capacitive reactance to neutral (or per conductor) is $x_{cn}=\frac{1}{2\pi fC}$ or in more practical units

$$x_{cn}=0.0683\frac{60}{f} \log_{10} \frac{D_{12}}{r} \text{ megohms per conductor per mile.} \tag{60}$$

This can be written as

$$x_{cn}=0.0683\frac{60}{f} \log_{10} \frac{1}{r}+0.0683\frac{60}{f} \log_{10}\frac{D_{12}}{1} \text{ megohms per conductor per mile} \tag{61}$$

where D_{12} and r are in feet and f is cycles per second. Eq. (61) may be written

$$x_{cn}=x_a'+x_d' \text{ megohms per conductor per mile.} \tag{62}$$

The derivation of shunt-capacitive reactance formulas brings about terms quite analogous to those derived for inductive reactance, and as in the case of inductive reactance, these terms can be resolved into components as shown in Eq. (62). The term x_a' accounts for the electrostatic flux within a one foot radius and is the term

$0.0683 \frac{60}{f} \log_{10} \frac{1}{r}$ in Eq. (61). It is a function of the conductor outside radius only. The term x_d' accounts for the electric flux between a one foot radius and the distance D_{12} to the other conductor and is the term $0.0683 \frac{60}{f} \log_{10} \frac{D_{12}}{1}$ in Eq. (61). Note that unlike inductive-reactance where the conductor geometric mean radius (GMR) is used, in capacitance calculations the only conductor radius used is the actual physical radius of the conductor in feet.

Zero-sequence capacitive reactance is, like inductive-reactance, divided into components x_a' taking into account the electrostatic flux within a one-foot radius, x_d' taking into account the electrostatic flux external to a radius of one foot out to a radius D feet, and x_e' taking into account the flux external to a radius of one foot and is a function of the spacing to the image conductor.

$$x'_e = \frac{12.30}{f} \log_{10} 2h \text{ megohms per mile per conductor} \tag{63}$$

where h = conductor height above ground.
f = frequency in cps.

x_a' is given in the tables of Electrical Characteristics of conductors, x_d' is given in Table 8, Shunt-Capacitive Reactance Spacing Factor, and x_e' is given in Table 9, Zero-Sequence Shunt-Capacitive Reactance Factor.

The following equations have been derived in a manner similar to those for the two-conductor, single-phase case, making use of the terms x_a', x_d' and x_e'. They are summarized in the following tabulation.

Shunt-Capacitive Reactance, x_c, of Three-Phase Circuits (Conductors a, b, c)

(a) Positive (and negative) sequence x_c.

$$x_1' = x_2' = x_a' + x_d' \text{ megohms per conductor per mile.} \tag{64}$$

$$x_d' = \frac{1}{3}(\text{sum of all three } x_d\text{'s for distances between all possible pairs}).$$

$$= \frac{1}{3}(x_{d\,ab}' + x_{d\,ac}' + x_{d\,bc}'). \text{ See Table (8)} \tag{65}$$

(b) Zero-Sequence x_c of one circuit (and earth).

$$x_{0(a)}' = x_a' + x_e' - 2x_d' \text{ megohms per conductor per mile.} \tag{66}$$

x_d' = value given in Eq. (65). Table (9) gives x_e'.

(c) Zero-Sequence x_c of one ground wire (and earth).

$$x_{0(g)}' = 3x_{a(g)}' + x_{e(g)}' \text{ megohms per conductor per mile.} \tag{67}$$

(d) Zero-Sequence x_c of two ground wires (and earth).

$$x_{0(g)}' = \frac{3}{2}x_{a(g)}' + x_{e(g)}' - \frac{3}{2}x_d' \text{ megohms per conductor per mile.} \tag{68}$$

$x_d' = x_{d(g1g2)}' = x_d'$ for distance between ground wires.

(e) Zero Sequence x_c of n ground wires (and earth).

$$x_{0(g)}' = x_e' + \frac{3}{n}x_a' - \frac{3(n-1)}{n}x_d' \text{ megohms per conductor per mile} \tag{69}$$

where

$$x_d' = \frac{2}{n(n-1)}(\text{sum of all } x_d\text{'s for all possible distances between all possible pairs of ground wires})$$

$$\text{or } x_d' = \frac{1}{n(n-1)}(\text{sum of all } x_d\text{'s for all possible distances between all ground wires}).$$

(f) Zero-Sequence x_c between one circuit (and earth) and n ground wires (and earth)

$$x_{0(ag)}' = x_e' - 3x_d' \text{ megohms per conductor per mile.} \tag{70}$$

$$x_d' = \frac{1}{3n}(x_{d(ag1)}' + x_{d(bg1)}' + x_{d(cg1)}' \cdots + x_{d(agn)}' + x_{d(bgn)}' + x_{d(cgn)}').$$

(g) Zero-Sequence x_c of one circuit with n ground wires

$$x_0' = x_{0(a)}' - \frac{x_{0(ag)}'^2}{x_{0(g)}'} \text{ megohms per conductor per mile.} \tag{71}$$

Shunt Capacitive Reactance, x_c, of Single-Phase Circuits (Conductors a and b)

(h) x_c of single-phase circuit of two identical conductors

$$x' = 2(x_a' + x_d') \text{ megohms per mile of circuit.} \tag{72}$$

$x_d' = x_d'$ for spacing between conductors.

(i) x_c of single-phase circuit of two non-identical conductors a and b.

$$x' = x_{a(a)}' + x_{a(b)}' + 2x_d' \text{ megohms per mile of circuit.} \tag{73}$$

(j) x_c of one conductor and earth.

$$x' = x_a' + \tfrac{1}{3}x_e' \text{ megohms per mile.} \tag{74}$$

In using the equations it should be remembered that the shunt capacitive reactance in megohms for more than one mile decreases because the capacitance increases. For more than one mile of line, therefore, the shunt-capacitive reactance as given by the above equations should be divided by the number of miles of line.

5. Conductor Temperature Rise and Current-Carrying Capacity

In distribution- and transmission-line design the temperature rise of conductors above ambient while carrying current is important. While power loss, voltage regulation, stability and other factors may determine the choice of a conductor for a given line, it is sometimes necessary to consider the maximum continuous current carrying capacity of a conductor. The maximum continuous current rating is necessary because it is determined by the maximum operating temperature of the conductor. This temperature affects the sag between towers or poles and determines the loss of conductor tensile strength due to annealing. For short tie lines or lines that must carry excessive loads under emergency conditions, the maximum continuous current-carrying capacity may be important in selecting the proper conductor.

The following discussion presents the Schurig and Frick[6] formulas for calculating the approximate current-carrying capacity of conductors under known conditions of ambient temperature, wind velocity, and limiting temperature rise.

The basis of this method is that the heat developed in the conductor by I^2R loss is dissipated (1) by convection in the surrounding air, and (2) radiation to surrounding objects. This can be expressed as follows:

$$I^2R=(W_c+W_r)A \text{ watts.} \tag{75}$$

where I = conductor current in amperes.
R = conductor resistance per foot.
W_c = watts per square inch dissipated by convection.
W_r = watts per square inch dissipated by radiation.
A = conductor surface area in square inches per foot of length.

The watts per square inch dissipated by convection, W_c, can be determined from the following equation:

$$W_c=\frac{0.0128\sqrt{pv}}{T_a^{0.123}\sqrt{d}}\Delta t \text{ watts per square inch} \tag{76}$$

where p = pressure in atmospheres ($p=1.0$ for atmospheric pressure).
v = velocity in feet per second.
T_a = (degrees Kelvin) average of absolute temperatures of conductor and air.
d = outside diameter of conductor in inches.
Δt = (degrees C) temperature rise.

This formula is an approximation applicable to conductor diameters ranging from 0.3 inch to 5 inches or more when the velocity of air is higher than free convection air currents (0.2—0.5 ft/sec).

The watts per square inch dissipated by radiation, W_r, can be determined from the following equation:

$$W_r=36.8E\left[\left(\frac{T}{1000}\right)^4-\left(\frac{T_0}{1000}\right)^4\right] \text{ watts per square inch} \tag{77}$$

where E = relative emissivity of conductor surface ($E=1.0$ for "black body," or 0.5 for average oxidized copper).
T = (degrees Kelvin) absolute temperature of conductor.
T_0 = (degrees Kelvin) absolute temperature of surroundings.

By calculating (W_c+W_r), A, and R, it is then possible to determine I from Eq. (75). The value of R to use is the a-c resistance at the conductor temperature (ambient temperature plus temperature rise) taking into account skin effect as discussed previously in the section on positive- and negative-sequence resistances.

This method is, in general, applicable to both copper and aluminum conductors. Tests have shown that aluminum conductors dissipate heat at about the same rate as copper conductors of the same outside diameter when the temperature rise is the same. Where test data is available on conductors, it should be used. The above general method can be used when test data is not available, or to check test results.

The effect of the sun upon conductor temperature rise is generally neglected, being some 3° to 8°C. This small effect is less important under conditions of high temperature rise above ambient.[6]

The tables of Electrical Characteristics of Conductors include tabulations of the approximate maximum current-

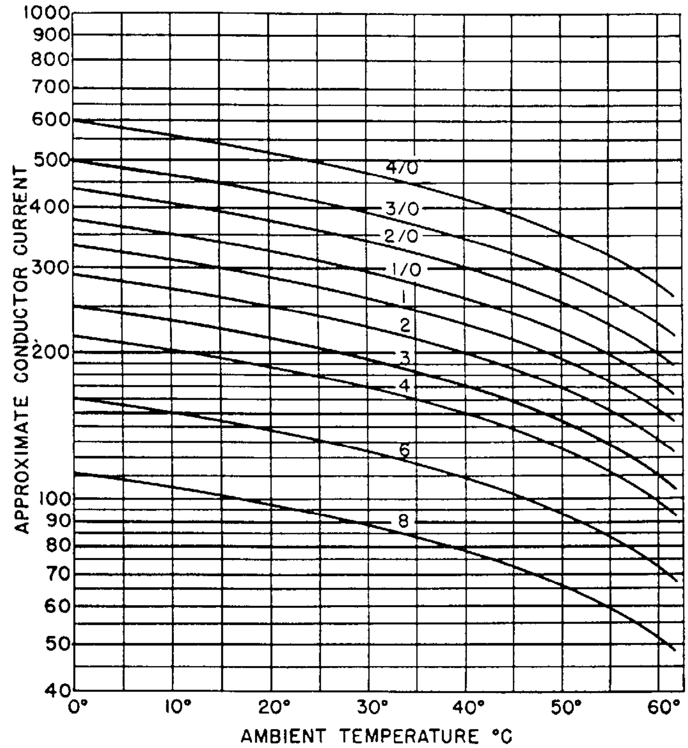


Fig. 25—Copper conductor current carrying capacity in Amperes VS. Ambient Temperature in °C. (Copper Conductors at 75 °C, wind velocity at 2 fps.).

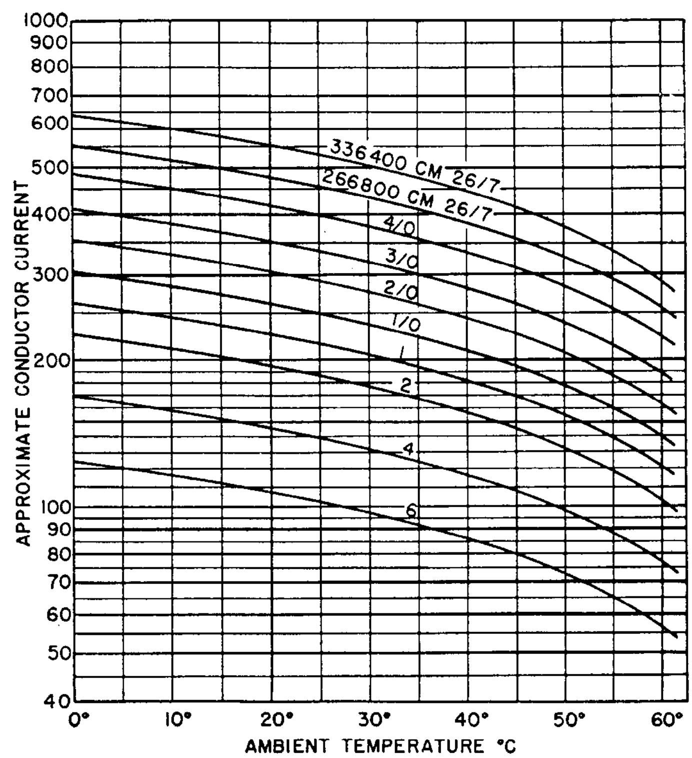


Fig. 26—Aluminum conductor current carrying capacity in Amperes VS. Ambient Temperature in °C. (Aluminum Conductors at 75°C, wind velocity at 2 fps).

TABLE 1—CHARACTERISTICS OF **COPPER** CONDUCTORS, HARD DRAWN, 97.3 PERCENT CONDUCTIVITY

Size of Conductor		Number of Strands	Diameter of Individual Strands Inches	Outside Diameter Inches	Breaking Strength Pounds	Weight Pounds per Mile	Approx. Current Carrying Capacity* Amps	Geometric Mean Radius at 60 Cycles Feet	r_a Resistance Ohms per Conductor per Mile								x_a Inductive Reactance Ohms per Conductor Per Mile At 1 Ft. Spacing			x_a' Shunt Capacitive Reactance Megohms per Conductor Per Mile At 1 Ft Spacing		
Circular Mils	A.W.G. or B. & S.								25°C. (77°F.)				50°C. (122°F.)									
									d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
1 000 000	...	37	0.1644	1.151	43 830	16 300	1 300	0.0368	0.0585	0.0594	0.0620	0.0634	0.0640	0.0648	0.0672	0.0685	0.1666	0.333	0.400	0.216	0.1081	0.0901
900 000	...	37	0.1560	1.092	39 510	14 670	1 220	0.0349	0.0650	0.0658	0.0682	0.0695	0.0711	0.0718	0.0740	0.0752	0.1693	0.339	0.406	0.220	0.1100	0.0916
800 000	...	37	0.1470	1.029	35 120	13 040	1 130	0.0329	0.0731	0.0739	0.0760	0.0772	0.0800	0.0806	0.0826	0.0837	0.1722	0.344	0.413	0.224	0.1121	0.0934
750 000	...	37	0.1424	0.997	33 400	12 230	1 090	0.0319	0.0780	0.0787	0.0807	0.0818	0.0853	0.0859	0.0878	0.0888	0.1739	0.348	0.417	0.226	0.1132	0.0943
700 000	...	37	0.1375	0.963	31 170	11 410	1 040	0.0308	0.0836	0.0842	0.0861	0.0871	0.0914	0.0920	0.0937	0.0947	0.1759	0.352	0.422	0.229	0.1145	0.0954
600 000	...	37	0.1273	0.891	27 020	9 781	940	0.0285	0.0975	0.0981	0.0997	0.1006	0.1066	0.1071	0.1086	0.1095	0.1799	0.360	0.432	0.235	0.1173	0.0977
500 000	...	37	0.1162	0.814	22 510	8 151	840	0.0260	0.1170	0.1175	0.1188	0.1196	0.1280	0.1283	0.1296	0.1303	0.1845	0.369	0.443	0.241	0.1205	0.1004
500 000	...	19	0.1622	0.811	21 590	8 151	840	0.0256	0.1170	0.1175	0.1188	0.1196	0.1280	0.1283	0.1296	0.1303	0.1853	0.371	0.445	0.241	0.1206	0.1005
450 000	...	19	0.1539	0.770	19 750	7 336	780	0.0243	0.1300	0.1304	0.1316	0.1323	0.1422	0.1426	0.1437	0.1443	0.1879	0.376	0.451	0.245	0.1224	0.1020
400 000	...	19	0.1451	0.726	17 560	6 521	730	0.0229	0.1462	0.1466	0.1477	0.1484	0.1600	0.1603	0.1613	0.1619	0.1909	0.382	0.458	0.249	0.1245	0.1038
350 000	...	19	0.1357	0.679	15 590	5 706	670	0.0214	0.1671	0.1675	0.1684	0.1690	0.1828	0.1831	0.1840	0.1845	0.1943	0.389	0.466	0.254	0.1269	0.1058
350 000	...	12	0.1708	0.710	15 140	5 706	670	0.0225	0.1671	0.1675	0.1684	0.1690	0.1828	0.1831	0.1840	0.1845	0.1918	0.384	0.460	0.251	0.1253	0.1044
300 000	...	19	0.1257	0.629	13 510	4 891	610	0.01987	0.1950	0.1953	0.1961	0.1966	0.213	0.214	0.214	0.215	0.1982	0.396	0.476	0.259	0.1296	0.1080
300 000	...	12	0.1581	0.657	13 170	4 891	610	0.0208	0.1950	0.1953	0.1961	0.1966	0.213	0.214	0.214	0.215	0.1957	0.392	0.470	0.256	0.1281	0.1068
250 000	...	19	0.1147	0.574	11 360	4 076	540	0.01813	0.234	0.234	0.235	0.235	0.256	0.256	0.257	0.257	0.203	0.406	0.487	0.266	0.1329	0.1108
250 000	...	12	0.1443	0.600	11 130	4 076	540	0.01902	0.234	0.234	0.235	0.235	0.256	0.256	0.257	0.257	0.200	0.401	0.481	0.263	0.1313	0.1094
211 600	4/0	19	0.1055	0.528	9 617	3 450	480	0.01668	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.207	0.414	0.497	0.272	0.1359	0.1132
211 600	4/0	12	0.1328	0.552	9 483	3 450	490	0.01750	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.205	0.409	0.491	0.269	0.1343	0.1119
211 600	4/0	7	0.1739	0.522	9 154	3 450	480	0.01579	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.210	0.420	0.503	0.273	0.1363	0.1136
167 800	3/0	12	0.1183	0.492	7 556	2 736	420	0.01559	0.349	0.349	0.349	0.350	0.381	0.381	0.382	0.382	0.210	0.421	0.505	0.277	0.1384	0.1153
167 800	3/0	7	0.1548	0.464	7 366	2 736	420	0.01404	0.349	0.349	0.349	0.350	0.381	0.381	0.382	0.382	0.216	0.431	0.518	0.281	0.1405	0.1171
133 100	2/0	7	0.1379	0.414	5 926	2 170	360	0.01252	0.440	0.440	0.440	0.440	0.481	0.481	0.481	0.481	0.222	0.443	0.532	0.289	0.1445	0.1205
105 500	1/0	7	0.1228	0.368	4 752	1 720	310	0.01113	0.555	0.555	0.555	0.555	0.606	0.607	0.607	0.607	0.227	0.455	0.546	0.298	0.1488	0.1240
83 690	1	7	0.1093	0.328	3 804	1 364	270	0.00992	0.699	0.699	0.699	0.699	0.765				0.233	0.467	0.560	0.306	0.1528	0.1274
83 690	1	3	0.1670	0.360	3 620	1 351	270	0.01016	0.692	0.692	0.692	0.692	0.757				0.232	0.464	0.557	0.299	0.1495	0.1246
66 370	2	7	0.0974	0.292	3 045	1 082	230	0.00883	0.881	0.882	0.882	0.882	0.964				0.239	0.478	0.574	0.314	0.1570	0.1308
66 370	2	3	0.1487	0.320	2 913	1 071	240	0.00903	0.873				0.955				0.238	0.476	0.571	0.307	0.1537	0.1281
66 370	2	1	......	0.258	3 003	1 061	220	0.00836	0.864				0.945				0.242	0.484	0.581	0.323	0.1614	0.1345
52 630	3	7	0.0867	0.260	2 433	858	200	0.00787	1.112				1.216				0.245	0.490	0.588	0.322	0.1611	0.1343
52 630	3	3	0.1325	0.285	2 359	850	200	0.00805	1.101				1.204				0.244	0.488	0.585	0.316	0.1578	0.1315
52 630	3	1	......	0.229	2 439	841	190	0.00745	1.090		Same as d-c		1.192		Same as d-c		0.248	0.496	0.595	0.331	0.1656	0.1380
41 740	4	3	0.1180	0.254	1 879	674	180	0.00717	1.388				1.518				0.250	0.499	0.599	0.324	0.1619	0.1349
41 740	4	1	......	0.204	1 970	667	170	0.00663	1.374				1.503				0.254	0.507	0.609	0.339	0.1697	0.1415
33 100	5	3	0.1050	0.226	1 505	534	150	0.00638	1.750				1.914				0.256	0.511	0.613	0.332	0.1661	0.1384
33 100	5	1	......	0.1819	1 591	529	140	0.00590	1.733				1.895				0.260	0.519	0.623	0.348	0.1738	0.1449
26 250	6	3	0.0935	0.201	1 205	424	130	0.00568	2.21				2.41				0.262	0.523	0.628	0.341	0.1703	0.1419
26 250	6	1	......	0.1620	1 280	420	120	0.00526	2.18				2.39				0.265	0.531	0.637	0.356	0.1779	0.1483
20 820	7	1	......	0.1443	1 030	333	110	0.00468	2.75				3.01				0.271	0.542	0.651	0.364	0.1821	0.1517
16 510	8	1	......	0.1285	826	264	90	0.00417	3.47				3.80				0.277	0.554	0.665	0.372	0.1862	0.1552

* For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency=60 cycles.

carrying capacity based on 50°C rise above an ambient of 25°C, (75°C total conductor temperature), tarnished surface ($E=0.5$), and an air velocity of 2 feet per second. These conditions were used after discussion and agreement with the conductor manufacturers. These thermal limitations are based on continuous loading of the conductors.

The technical literature shows little variation from these conditions as line design limits.[7] The ambient air temperature is generally assumed to be 25°C to 40°C whereas the temperature rise is assumed to be 10°C to 60°C. This gives a conductor total temperature range of 35°C to 100°C. For design purposes copper or ACSR conductor total temperature is usually assumed to be 75°C as use of this value has given good conductor performance from an annealing standpoint, the limit being about 100°C where annealing of copper and aluminum begins.

Using Schurig and Frick's formulas, Fig. 25 and Fig. 26 have been calculated to show how current-carrying capacity of copper and aluminum conductors varies with ambient temperature assuming a conductor temperature of 75.C and wind velocity of 2 feet per second. These values are conservative and can be used as a guide in normal line design. For those lines where a higher conductor temperature may be obtained that approaches 100°C, the conductor manufacturer should be consulted for test data or other more accurate information as to conductor temperature limitations. Such data on copper conductors has been presented rather thoroughly in the technical literature.[7]

III TABLES OF CONDUCTOR CHARACTERISTICS

The following tables contain data on copper, ACSR, hollow copper, Copperweld-copper, and Copperweld conductors, which along with the previously derived equations, permit the determination of positive-, negative-, and zero-sequence impedances of conductors for use in the solution of power-system problems. Also tabulated are such conductor characteristics as size, weight, and current-carrying capacity as limited by heating.

The conductor data (r_a, x_a, x_a') along with inductive and shunt-capacitive reactance spacing factors (x_d, x_d') and zero-sequence resistance, inductive and shunt-capacitive reactance factors (r_e, x_e, x_e') permit easy substitution in the previously derived equations for determining the symmetrical component sequence impedances of aerial circuits.

The cross-sectional inserts in the tables are for ease in

TABLE 2-A—CHARACTERISTICS OF ALUMINUM CABLE STEEL REINFORCED

(Aluminum Company of America)

Circular Mils or A.W.G. Aluminum	Aluminum Strands	Aluminum Layers	Aluminum Strand Dia. Inches	Steel Strands	Steel Strand Dia. Inches	Outside Diameter Inches	Copper Equivalent* Circular Mils or A.W.G.	Ultimate Strength Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity† Amps	r_a 25°C. (77°F.) Small Currents d-c	r_a 25°C. 25 cycles	r_a 25°C. 50 cycles	r_a 25°C. 60 cycles	r_a 50°C. (122°F.) Current Approx. 75% Capacity‡ d-c	r_a 50°C. 25 cycles	r_a 50°C. 50 cycles	r_a 50°C. 60 cycles	x_a 25 cycles	x_a 50 cycles	x_a 60 cycles	x_a' 25 cycles	x_a' 50 cycles	x_a' 60 cycles
1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	0.1495	0.299	0.359	0.1953	0.0977	0.0814
1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	0.1508	0.302	0.362	0.1971	0.0986	0.0821
1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	0.1522	0.304	0.365	0.1991	0.0996	0.0830
1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	0.1536	0.307	0.369	0.201	0.1006	0.0838
1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	0.1551	0.310	0.372	0.203	0.1016	0.0847
1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	0.1568	0.314	0.376	0.206	0.1028	0.0857
1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 200	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	0.1585	0.317	0.380	0.208	0.1040	0.0867
1 033 500	54	3	0.1384	7	0.1384	1.246	650 000	37 100	7 019	0.0420	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	0.1603	0.321	0.385	0.211	0.1053	0.0878
954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	0.1624	0.325	0.390	0.214	0.1068	0.0890
900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	0.1639	0.328	0.393	0.216	0.1078	0.0898
874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	0.1646	0.329	0.395	0.217	0.1083	0.0903
795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378	0.1670	0.334	0.401	0.220	0.1100	0.0917
795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.1660	0.332	0.399	0.219	0.1095	0.0912
795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.1637	0.327	0.393	0.217	0.1085	0.0904
715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	0.1697	0.339	0.407	0.224	0.1119	0.0932
715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.1687	0.337	0.405	0.223	0.1114	0.0928
715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.1664	0.333	0.399	0.221	0.1104	0.0920
666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601	0.1715	0.343	0.412	0.226	0.1132	0.0943
636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688	0.1726	0.345	0.414	0.228	0.1140	0.0950
636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.1718	0.344	0.412	0.227	0.1135	0.0946
636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.1693	0.339	0.406	0.225	0.1125	0.0937
605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	0.1739	0.348	0.417	0.230	0.1149	0.0957
605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720	0.1730	0.346	0.415	0.229	0.1144	0.0953
556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.1751	0.350	0.420	0.232	0.1159	0.0965
556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.1728	0.346	0.415	0.230	0.1149	0.0957
500 000	30	2	0.1291	7	0.1291	0.904	314 500	24 400	4 122	0.0311	690	0.187	0.187	0.187	0.187	0.206				0.1754	0.351	0.421	0.234	0.1167	0.0973
477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216				0.1790	0.358	0.430	0.237	0.1186	0.0988
477 000	30	2	0.1261	7	0.1261	0.883	300 000	23 300	3 933	0.0304	670	0.196	0.196	0.196	0.196	0.216				0.1766	0.353	0.424	0.235	0.1176	0.0980
397 500	26	2	0.1236	7	0.0961	0.783	250 000	16 190	2 885	0.0265	590	0.235				0.259				0.1836	0.367	0.441	0.244	0.1219	0.1015
397 500	30	2	0.1151	7	0.1151	0.806	250 000	19 980	3 277	0.0278	600	0.235	Same as d-c			0.259	Same as d-c			0.1812	0.362	0.435	0.242	0.1208	0.1006
336 400	26	2	0.1138	7	0.0885	0.721	4/0	14 050	2 442	0.0244	530	0.278				0.306				0.1872	0.376	0.451	0.250	0.1248	0.1039
336 400	30	2	0.1059	7	0.1059	0.741	4/0	17 040	2 774	0.0255	530	0.278				0.306				0.1855	0.371	0.445	0.248	0.1238	0.1032
300 000	26	2	0.1074	7	0.0835	0.680	188 700	12 650	2 178	0.0230	490	0.311				0.342				0.1908	0.382	0.458	0.254	0.1269	0.1057
300 000	30	2	0.1000	7	0.1000	0.700	188 700	15 430	2 473	0.0241	500	0.311				0.342				0.1883	0.377	0.452	0.252	0.1258	0.1049
266 800	26	2	0.1013	7	0.0788	0.642	3/0	11 250	1 936	0.0217	460	0.350				0.385				0.1936	0.387	0.465	0.258	0.1289	0.1074

Circular Mils or A.W.G. Aluminum	Aluminum Strands	Aluminum Layers	Aluminum Strand Dia. Inches	Steel Strands	Steel Strand Dia. Inches	Outside Diameter Inches	Copper Equivalent* Circular Mils or A.W.G.	Ultimate Strength Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet For Current Approx. 75% Capacity ‡	Approx. Current Carrying Capacity† Amps	r_a 25°C. d-c	r_a 25°C. 25 cycles	r_a 25°C. 50 cycles	r_a 25°C. 60 cycles	r_a 50°C. d-c	r_a 50°C. 25 cycles	r_a 50°C. 50 cycles	r_a 50°C. 60 cycles	x_a Single Layer Conductors, Small Currents 25 cycles	Small Currents 50 cycles	Small Currents 60 cycles	Current Approx. 75% Capacity‡ 25 cycles	Current Approx. 75% Capacity‡ 50 cycles	Current Approx. 75% Capacity‡ 60 cycles	x_a' 25 cycles	x_a' 50 cycles	x_a' 60 cycles
266 800	6	1	0.2109	7	0.0703	0.633	3/0	9 645	1 802	0.00684	460	0.351	0.351	0.351	0.352	0.386	0.430	0.510	0.552	0.194	0.388	0.466	0.252	0.504	0.605	0.259	0.1294	0.1079
4/0	6	1	0.1878	1	0.1878	0.563	2/0	8 420	1 542	0.00814	340	0.441	0.442	0.444	0.445	0.485	0.514	0.567	0.592	0.218	0.437	0.524	0.242	0.484	0.581	0.267	0.1336	0.1113
3/0	6	1	0.1672	1	0.1672	0.502	1/0	6 675	1 223	0.00600	300	0.556	0.557	0.559	0.560	0.612	0.642	0.697	0.723	0.225	0.450	0.540	0.259	0.517	0.621	0.275	0.1377	0.1147
2/0	6	1	0.1490	1	0.1490	0.447	1	5 345	970	0.00510	270	0.702	0.702	0.704	0.706	0.773	0.806	0.866	0.895	0.231	0.462	0.554	0.267	0.534	0.641	0.284	0.1418	0.1182
1/0	6	1	0.1327	1	0.1327	0.398	2	4 280	769	0.00446	230	0.885	0.885	0.887	0.888	0.974	1.01	1.08	1.12	0.237	0.473	0.568	0.273	0.547	0.656	0.292	0.1460	0.1216
1	6	1	0.1182	1	0.1182	0.355	3	3 480	610	0.00418	200	1.12	1.12	1.12	1.12	1.23	1.27	1.34	1.38	0.242	0.483	0.580	0.277	0.554	0.665	0.300	0.1500	0.1250
2	6	1	0.1052	1	0.1052	0.316	4	2 790	484	0.00418	180	1.41	1.41	1.41	1.41	1.55	1.59	1.66	1.69	0.247	0.493	0.592	0.277	0.554	0.665	0.308	0.1542	0.1285
2	7	1	0.0974	1	0.1299	0.325	4	3 525	566	0.00504	180	1.41	1.41	1.41	1.41	1.55	1.59	1.62	1.65	0.247	0.493	0.592	0.267	0.535	0.642	0.306	0.1532	0.1276
3	6	1	0.0937	1	0.0937	0.281	5	2 250	384	0.00430	160	1.78	1.78	1.78	1.78	1.95	1.98	2.04	2.07	0.252	0.503	0.604	0.275	0.551	0.661	0.317	0.1583	0.1320
4	6	1	0.0834	1	0.0834	0.250	6	1 830	304	0.00437	140	2.24	2.24	2.24	2.24	2.47	2.50	2.54	2.57	0.257	0.514	0.611	0.274	0.549	0.659	0.325	0.1627	0.1355
4	7	1	0.0772	1	0.1029	0.257	6	2 288	356	0.00452	140	2.24	2.24	2.24	2.24	2.47	2.50	2.53	2.55	0.257	0.515	0.618	0.273	0.545	0.655	0.323	0.1615	0.1346
5	6	1	0.0743	1	0.0743	0.223	7	1 460	241	0.00416	120	2.82	2.82	2.82	2.82	3.10	3.12	3.16	3.18	0.262	0.525	0.630	0.279	0.557	0.665	0.333	0.1666	0.1388
6	6	1	0.0661	1	0.0661	0.198	8	1 170	191	0.00394	100	3.56	3.56	3.56	3.56	3.92	3.94	3.97	3.98	0.268	0.536	0.643	0.281	0.561	0.673	0.342	0.1708	0.1423

* Based on copper 97 percent, aluminum 61 percent conductivity.
† For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency = 60 cycles.
‡ "Current Approx. 75% Capacity" is 75% of the "Approx. Current Carrying Capacity in Amps." and is approximately the current which will produce 50°C. conductor temp. (25°C. rise) with 25°C. air temp., wind 1.4 miles per hour.

TABLE 2-B—CHARACTERISTICS OF "EXPANDED" ALUMINUM CABLE STEEL REINFORCED

(Aluminum Company of America)

Circular Mils or A.W.G. Aluminum	Aluminum Strands	Aluminum Layers	Aluminum Strand Dia. Inches	Steel Strands	Steel Strand Dia. Inches	Filler Section Aluminum Strands	Filler Section Aluminum Strand Dia. Inches	Filler Section Paper Strands	Filler Section Paper Layers	Outside Diameter Inches	Copper Equivalent Circular Mils or A.W.G.	Ultimate Strength Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity Amps	r_a 25°C. (77°F.) Small Currents d-c	25 cycles	50 cycles	60 cycles	r_a 50°C. (122°F.) Current Approx. 75% Capacity d-c	25 cycles	50 cycles	60 cycles	x_a 25 cycles	x_a 50 cycles	x_a 60 cycles	x_a' 25 cycles	x_a' 50 cycles	x_a' 60 cycles
850 000	54	2	0.1255	19	0.0834	4	0.1182	23	2	1.38	534 000	35 371	7 200																
1 150 000	54	2	0.1409	19	0.0921	4	0.1353	24	2	1.55	724 000	41 900	9 070	(1)	(1)	(1)				(1)				(1)	(1)	(1)	(1)	(1)	(1)
1 338 000	66	2	0.1350	19	0.100	4	0.184	18	2	1.75	840 000	49 278	11 340																

(1) Electrical Characteristics not available until laboratory measurements are completed.

TABLE 3-A—CHARACTERISTICS OF ANACONDA HOLLOW COPPER CONDUCTORS

(Anaconda Wire & Cable Company)

Design Number	Size of Conductor Circular Mils or A.W.G.	Wires: Number	Wires: Diameter Inches	Outside Diameter Inches	Breaking Strain Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity Amps‡	r_a Resistance Ohms per Conductor per Mile: 25°C. (77°F.) d-c 25 cycles	25°C. (77°F.) 50 cycles 60 cycles	50°C. (122°F.) d-c 25 cycles	50°C. (122°F.) 50 cycles 60 cycles	x_a Inductive Reactance Ohms per Conductor per Mile at 1 Ft. Spacing: 25 cycles	50 cycles	60 cycles	x_a' Shunt Capacitive Reactance Megohms per Conductor per Mile at 1 Ft. Spacing: 25 cycles	50 cycles	60 cycles
966	890 500	28	0.1610	1.650	36 000	15 085	0.0612	1395	0.0671	0.0676	0.0734	0.0739	0.1412	0.282	0.339	0.1907	0.0953	0.0794
96R1	750 000	42	0.1296	1.155	34 200	12 345	0.0408	1160	0.0786	0.0791	0.0860	0.0865	0.1617	0.323	0.388	0.216	0.1080	0.0960
939	650 000	50	0.1097	1.126	29 500	10 761	0.0406	1060	0.0909	0.0915	0.0994	0.1001	0.1621	0.324	0.389	0.218	0.1089	0.0908
360R1	600 000	50	0.1053	1.007	27 500	9 905	0.0387	1020	0.0984	0.0991	0.1077	0.1084	0.1644	0.329	0.395	0.221	0.1105	0.0921
938	550 000	50	0.1009	1.036	25 200	9 103	0.0373	960	0.1076	0.1081	0.1177	0.1183	0.1663	0.333	0.399	0.224	0.1119	0.0932
4R5	510 000	50	0.0970	1.000	22 700	8 485	0.0360	910	0.1173	0.1178	0.1283	0.1289	0.1681	0.336	0.404	0.226	0.1131	0.0943
892R3	500 000	18	0.1558	1.080	21 400	8 263	0.0394.	900	0.1178	0.1184	0.1289	0.1296	0.1630	0.326	0.391	0.221	0.1104	0.0920
933	450 000	21	0.1353	1.074	19 300	7 476	0.0398	850	0.1319	0.1324	0.1443	0.1448	0.1630	0.326	0.391	0.221	0.1106	0.0922
924	400 000	21	0.1227	1.014	17 200	6 642	0.0376	810	0.1485	0.1491	0.1624	0.1631	0.1658	0.332	0.398	0.225	0.1126	0.0939
925R1	380 500	22	0.1211	1.003	16 300	6 331	0.0373	780	0.1565	0.1572	0.1712	0.1719	0.1663	0.333	0.399	0.226	0.1130	0.0942
565R1	350 000	21	0.1196	0.950	15 100	5 813	0.0353	750	0.1695	0.1700	0.1854	0.1860	0.1691	0.338	0.406	0.230	0.1150	0.0958
936	350 000	15	0.1444	0.860	15 400	5 776	0.0311	740	0.1690	0.1695	0.1849	0.1854	0.1754	0.351	0.421	0.237	0.1185	0.0988
378R1	350 000	30	0.1059	0.736	16 100	5 739	0.0253	700	0.1685	0.1690	0.1843	0.1849	0.1860	0.372	0.446	0.248	0.1241	0.1034
954	321 000	22	0.1113	0.920	13 850	5 343	0.0340	700	0.1851	0.1856	0.202	0.203	0.1710	0.342	0.410	0.232	0.1161	0.0968
935	300 000	18	0.1205	0.839	13 100	4 984	0.0307	670	0.1980	0.1985	0.216	0.217	0.1761	0.352	0.423	0.239	0.1194	0.0995
903R1	300 000	15	0.1338	0.797	13 200	4 953	0.0289	660	0.1969	0.1975	0.215	0.216	0.1793	0.359	0.430	0.242	0.1212	0.1010
178R2	300 000	12	0.1507	0.750	13 050	4 937	0.0266	650	0.1964	0.1969	0.215	0.216	0.1833	0.367	0.440	0.247	0.1234	0.1028
926	250 000	18	0.1100	0.766	10 950	4 155	0.0279	600	0.238	0.239	0.260	0.261	0.1810	0.362	0.434	0.245	0.1226	0.1022
915R1	250 000	15	0.1214	0.725	11 000	4 148	0.0266	590	0.237	0.238	0.259	0.260	0.1834	0.367	0.440	0.249	0.1246	0.1038
24R1	250 000	12	0.1368	0.683	11 000	4 133	0.0245	580	0.237	0.238	0.259	0.260	0.1876	0.375	0.450	0.253	0.1267	0.1066
923	4/0	18	0.1005	0.700	9 300	3 521	0.0255	530	0.281	0.282	0.307	0.308	0.1855	0.371	0.445	0.252	0.1258	0.1049
922	4/0	15	0.1109	0.663	9 300	3 510	0.0238	520	0.281	0.282	0.307	0.308	0.1889	0.378	0.453	0.256	0.1278	0.1065
50R2	4/0	14	0.1152	0.650	9 300	3 510	0.0234	520	0.280	0.281	0.306	0.307	0.1898	0.380	0.455	0.257	0.1285	0.1071
158R1	3/0	16	0.0961	0.606	7 500	2 785	0.0221	460	0.354	0.355	0.387	0.388	0.1928	0.386	0.463	0.262	0.1310	0.1091
495R1	3/0	15	0.0996	0.595	7 600	2 785	0.0214	460	0.353	0.354	0.386	0.387	0.1943	0.389	0.466	0.263	0.1316	0.1097
570R2	3/0	12	0.1123	0.560	7 600	2 772	0.0201	450	0.352	0.353	0.385	0.386	0.1976	0.395	0.474	0.268	0.1338	0.1115
909R2	2/0	15	0.0880	0.530	5 950	2 213	0.0191	370	0.446	0.446	0.487	0.487	0.200	0.400	0.481	0.271	0.1357	0.1131
412R2	2/0	14	0.0913	0.515	6 000	2 207	0.0184	370	0.446	0.446	0.487	0.487	0.202	0.404	0.485	0.274	0.1368	0.1140
937	2/0	13	0.0950	0.505	6 000	2 203	0.0181	370	0.446	0.446	0.487	0.487	0.203	0.406	0.487	0.275	0.1375	0.1146
930	125 600	14	0.0885	0.500	5 650	2 083	0.0180	360	0.473	0.473	0.517	0.517	0.203	0.466	0.487	0.276	0.1378	0.1149
934	121 300	15	0.0836	0.500	5 400	2 015	0.0179	350	0.491	0.491	0.537	0.537	0.203	0.407	0.488	0.276	0.1378	0.1149
901	119 400	12	0.0936	0.470	5 300	1 979	0.0165	340	0.507	0.507	0.555	0.555	0.207	0.415	0.498	0.280	0.1400	0.1167

‡For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency=60 cycles, average tarnished surface.

TABLE 3-B—CHARACTERISTICS OF GENERAL CABLE TYPE HH HOLLOW COPPER CONDUCTORS

(General Cable Corporation)

Conductor Size Circular Mils or A.W.G.	Outside(1) Diameter Inches	Wall Thickness Inches	Weight Pounds per Mile	Breaking Strength Pounds	Geometric Mean Radius Feet	Approx. Current Carrying Capacity(2) Amps	r_a Resistance Ohms per Conductor per Mile: 25°C. (77°F.) d-c	25°C. 25 cycles	25°C. 50 cycles	25°C. 60 cycles	50°C. (122°F.) d-c	50°C. 25 cycles	50°C. 50 cycles	50°C. 60 cycles	x_a Inductive Reactance Ohms per Conductor per Mile at 1 Foot Spacing: 25 cycles	50 cycles	60 cycles	x_a' Shunt Capacitive Reactance Megohms per Conductor per Mile at 1 Foot Spacing: 25 cycles	50 cycles	60 cycles
1 000 000	2.103	0.150*	16 160	43 190	0.0833	1620	0.0576	0.0576	0.0577	0.0577	0.0630	0.0630	0.0631	0.0631	0.1257	0.251	0.302	0.1734	0.0867	0.0722
950 000	2.035	0.147*	15 350	41 030	0.0805	1565	0.0606	0.0606	0.0607	0.0607	0.0663	0.0664	0.0664	0.0664	0.1274	0.255	0.306	0.1757	0.0879	0.0732
900 000	1.966	0.144*	14 540	38 870	0.0778	1505	0.0640	0.0640	0.0641	0.0641	0.0700	0.0701	0.0701	0.0701	0.1291	0.258	0.310	0.1782	0.0891	0.0742
850 000	1.901	0.140*	13 730	36 710	0.0751	1450	0.0677	0.0678	0.0678	0.0678	0.0741	0.0742	0.0742	0.0742	0.1309	0.262	0.314	0.1805	0.0903	0.0752
800 000	1.820	0.137*	12 920	34 550	0.0722	1390	0.0720	0.0720	0.0720	0.0721	0.0788	0.0788	0.0788	0.0788	0.1329	0.266	0.319	0.1833	0.0917	0.0764
790 000	1.650	0.131†	12 760	34 120	0.0646	1335	0.0729	0.0729	0.0730	0.0730	0.0797	0.0798	0.0799	0.0799	0.1385	0.277	0.332	0.1906	0.0953	0.0794
750 000	1.750	0.133*	12 120	32 390	0.0691	1325	0.0768	0.0768	0.0768	0.0769	0.0840	0.0840	0.0841	0.0841	0.1351	0.270	0.324	0.1864	0.0932	0.0777
700 000	1.686	0.130*	11 310	30 230	0.0665	1265	0.0822	0.0823	0.0823	0.0823	0.0900	0.0900	0.0901	0.0901	0.1370	0.274	0.329	0.1891	0.0945	0.0788
650 000	1.610	0.126*	10 500	28 070	0.0635	1200	0.0886	0.0886	0.0886	0.0887	0.0969	0.0970	0.0970	0.0970	0.1394	0.279	0.335	0.1924	0.0962	0.0802
600 000	1.558	0.123*	9 692	25 910	0.0615	1140	0.0959	0.0960	0.0960	0.0960	0.1050	0.1051	0.1051	0.1051	0.1410	0.282	0.338	0.1947	0.0974	0.0811
550 000	1.478	0.119*	8 884	23 750	0.0583	1075	0.1047	0.1048	0.1048	0.1048	0.1146	0.1146	0.1147	0.1147	0.1437	0.287	0.345	0.1985	0.0992	0.0827
512 000	1.400	0.115*	8 270	22 110	0.0551	1020	0.1124	0.1125	0.1125	0.1125	0.1230	0.1230	0.1231	0.1231	0.1466	0.293	0.352	0.202	0.1012	0.0843
500 000	1.390	0.115*	8 076	21 590	0.0547	1005	0.1151	0.1151	0.1152	0.1152	0.1259	0.1260	0.1260	0.1260	0.1469	0.294	0.353	0.203	0.1014	0.0845
500 000	1.268	0.109†	8 074	21 590	0.0494	978	0.1151	0.1152	0.1152	0.1152	0.1259	0.1260	0.1260	0.1261	0.1521	0.304	0.365	0.209	0.1047	0.0872
500 000	1.100	0.130†	8 068	21 590	0.0420	937	0.1150	0.1151	0.1152	0.1153	0.1258	0.1259	0.1260	0.1260	0.1603	0.321	0.385	0.219	0.1098	0.0915
500 000	1.020	0.144†	8 063	21 590	0.0384	915	0.1150	0.1150	0.1152	0.1152	0.1258	0.1259	0.1260	0.1261	0.1648	0.330	0.396	0.225	0.1124	0.0937
450 000	1.317	0.111*	7 268	19 430	0.0518	939	0.1279	0.1280	0.1280	0.1280	0.1400	0.1401	0.1401	0.1401	0.1496	0.299	0.359	0.207	0.1033	0.0861
450 000	1.188	0.105†	7 266	19 430	0.0462	910	0.1278	0.1279	0.1279	0.1280	0.1399	0.1400	0.1400	0.1401	0.1554	0.311	0.373	0.214	0.1070	0.0892
400 000	1.218	0.106*	6 460	17 270	0.0478	864	0.1439	0.1440	0.1440	0.1440	0.1575	0.1576	0.1576	0.1576	0.1537	0.307	0.369	0.212	0.1061	0.0884
400 000	1.103	0.100†	6 458	17 270	0.0428	838	0.1438	0.1439	0.1439	0.1440	0.1574	0.1575	0.1575	0.1576	0.1593	0.319	0.382	0.219	0.1097	0.0914
350 000	1.128	0.102*	5 653	15 110	0.0443	790	0.1644	0.1645	0.1645	0.1645	0.1799	0.1800	0.1800	0.1800	0.1576	0.315	0.378	0.218	0.1089	0.0907
350 000	1.014	0.096†	5 650	15 110	0.0393	764	0.1644	0.1645	0.1645	0.1646	0.1799	0.1800	0.1800	0.1801	0.1637	0.328	0.393	0.225	0.1127	0.0939
300 000	1.020	0.096*	4 845	12 950	0.0399	709	0.1918	0.1919	0.1919	0.1919	0.210	0.210	0.210	0.210	0.1628	0.326	0.391	0.225	0.1124	0.0937
300 000	0.919	0.091†	4 843	12 950	0.0355	687	0.1917	0.1918	0.1918	0.1919	0.210	0.210	0.210	0.210	0.1688	0.338	0.405	0.232	0.1162	0.0968
250 000	0.914	0.091*	4 037	10 790	0.0357	626	0.230	0.230	0.230	0.230	0 252	0.252	0.252	0.252	0.1685	0.337	0.404	0.233	0.1163	0.0970
250 000	0.818	0.086†	4 036	10 790	0.0315	606	0.230	0.230	0.230	0.230	0.252	0.252	0.252	0.252	0.1748	0.350	0.420	0.241	0.1203	0.1002
250 000	0.766	0.094†	4 034	10 790	0.0292	594	0.230	0.230	0.230	0.230	0.252	0.252	0.252	0.252	0.1787	0.357	0.429	0.245	0.1226	0.1022
214 500	0.650	0.098†	3 459	9 265	0.0243	524	0.268	0.268	0.268	0.268	0.293	0.293	0.293	0.294	0.1879	0.376	0.451	0.257	0.1285	0.1071
4/0	0.733	0.082†	3 415	9 140	0.0281	539	0.272	0.272	0.272	0.272	0.297	0.297	0.298	0.298	0.1806	0.361	0.433	0.248	0.1242	0.1035
3/0	0.608	0.080†	2 707	7 240	0.0230	454	0.343	0.343	0.343	0.343	0.375	0.375	0.375	0.375	0.1907	0.381	0.458	0.262	0.1309	0.1091
2/0	0.500	0.080†	2 146	5 750	0.0186	382	0.432	0.432	0.432	0.432	0.472	0.473	0.473	0.473	0.201	0.403	0.483	0.276	0.1378	0.1149

Notes: *Thickness at edges of interlocked segments. †Thickness uniform throughout.
(1) Conductors of smaller diameter for given cross-sectional area also available; in the naught sizes, some additional diameter expansion is possible.
(2) For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency=60 cycles.

TABLE 4-A—CHARACTERISTICS OF COPPERWELD-COPPER CONDUCTORS

(Copperweld Steel Company)

Size of Conductor									r_a Resistance Ohms per Conductor per Mile at 25°C.(77°F.) Small Currents				r_a Resistance Ohms per Conductor per Mile at 50°C. (122°F.) Current Approx. 75% of Capacity**				x_a Inductive Reactance Ohms per Conductor per Mile One ft. Spacing Average Currents			x_a' Capacitive Reactance Megohms per Conductor per Mile One ft. Spacing		
Nominal Designation	Number and Diameter of Wires: Copperweld	Number and Diameter of Wires: Copper	Outside Diameter Inches	Copper Equivalent Circular Mils or A.W.G.	Rated Breaking Load Lbs.	Weight Lbs. per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity at 60 Cycles Amps*	d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
350 E	7x.1576"	12x.1576"	0.788	350 000	32 420	7 409	0.0220	660	0.1658	0.1728	0.1789	0.1812	0.1812	0.1915	0.201	0.204	0.1929	0.386	0.463	0.243	0.1216	0.1014
350 EK	4x.1470"	15x.1470"	0.735	350 000	23 850	6 536	0.0245	680	0.1658	0.1682	0.1700	0.1705	0.1812	0.1845	0.1873	0.1882	0.1875	0.375	0.450	0.248	0.1241	0.1034
350 V	3x.1751"	9x.1893"	0.754	350 000	23 480	6 578	0.0226	650	0.1655	0.1725	0.1800	0.1828	0.1809	0.1910	0.202	0.206	0.1915	0.383	0.460	0.246	0.1232	0.1027
300 E	7x.1459"	12x.1459"	0.729	300 000	27 770	6 351	0.0204	600	0.1934	0.200	0.207	0.209	0.211	0.222	0.232	0.235	0.1969	0.394	0.473	0.249	0.1244	0.1037
300 EK	4x.1361"	15x.1361"	0.680	300 000	20 960	5 602	0.0227	610	0.1934	0.1958	0.1976	0.1981	0.211	0.215	0.218	0.219	0.1914	0.383	0.460	0.254	0.1269	0.1057
300 V	3x.1621"	9x.1752"	0.698	300 000	20 730	5 639	0.0209	590	0.1930	0.200	0.208	0.210	0.211	0.222	0.233	0.237	0.1954	0.391	0.469	0.252	0.1259	0.1050
250 E	7x.1332"	12x.1332"	0.666	250 000	23 920	5 292	0.01859	540	0.232	0.239	0.245	0.248	0.254	0.265	0.275	0.279	0.202	0.403	0.484	0.255	0.1276	0.1064
250 EK	4x.1242"	15x.1242"	0.621	250 000	17 840	4 669	0.0207	540	0.232	0.235	0.236	0.237	0.254	0.258	0.261	0.261	0.1960	0.392	0.471	0.260	0.1301	0.1084
250 V	3x.1480"	9x.1600"	0.637	250 000	17 420	4 699	0.01911	530	0.232	0.239	0.246	0.249	0.253	0.264	0.276	0.281	0.200	0.400	0.480	0.258	0.1292	0.1077
4/0 E	7x.1225"	12x.1225"	0.613	4/0	20 730	4 479	0.01711	480	0.274	0.281	0.287	0.290	0.300	0.312	0.323	0.326	0.206	0.411	0.493	0.261	0.1306	0.1088
4/0 G	2x.1944"	5x.1944"	0.583	4/0	15 640	4 168	0.01409	460	0.273	0.284	0.294	0.298	0.299	0.318	0.336	0.342	0.215	0.431	0.517	0.265	0.1324	0.1103
4/0 EK	4x.1143"	15x.1143"	0.571	4/0	15 370	3 951	0.01903	490	0.274	0.277	0.278	0.279	0.300	0.304	0.307	0.308	0.200	0.401	0.481	0.266	0.1331	0.1109
4/0 V	3x.1361"	9x.1472"	0.586	4/0	15 000	3 977	0.01758	470	0.274	0.281	0.288	0.291	0.299	0.311	0.323	0.328	0.204	0.409	0.490	0.264	0.1322	0.1101
4/0 F	1x.1833"	6x.1833"	0.550	4/0	12 290	3 750	0.01558	470	0.273	0.280	0.285	0.287	0.299	0.309	0.318	0.322	0.210	0.421	0.505	0.269	0.1344	0.1120
3/0 E	7x.1091"	12x.1091"	0.545	3/0	16 800	3 552	0.01521	420	0.346	0.353	0.359	0.361	0.378	0.391	0.402	0.407	0.212	0.423	0.508	0.270	0.1348	0.1123
3/0 J	3x.1851"	4x.1851"	0.555	3/0	16 170	3 732	0.01156	410	0.344	0.356	0.367	0.372	0.377	0.398	0.419	0.428	0.225	0.451	0.541	0.268	0.1341	0.1118
3/0 G	2x.1731"	2x.1731"	0.519	3/0	12 860	3 305	0.01254	400	0.344	0.355	0.365	0.369	0.377	0.397	0.416	0.423	0.221	0.443	0.531	0.273	0.1365	0.1137
3/0 EK	4x.1018"	4x.1018"	0.509	3/0	12 370	3 134	0.01697	420	0.346	0.348	0.350	0.351	0.378	0.382	0.386	0.386	0.206	0.412	0.495	0.274	0.1372	0.1143
3/0 V	3x.1311"	9x.1311"	0.522	3/0	12 220	3 154	0.01566	410	0.345	0.352	0.360	0.362	0.377	0.390	0.403	0.408	0.210	0.420	0.504	0.273	0.1363	0.1136
3/0 F	1x.1632"	6x.1632"	0.490	3/0	9 980	2 974	0.01388	410	0.344	0.351	0.356	0.358	0.377	0.388	0.397	0.401	0.216	0.432	0.519	0.277	0.1385	0.1155
2/0 K	4x.1780"	3x.1780"	0.534	2/0	17 600	3 411	0.00912	360	0.434	0.447	0.459	0.466	0.475	0.499	0.524	0.535	0.237	0.475	0.570	0.271	0.1355	0.1129
2/0 J	3x.1648"	4x.1648"	0.494	2/0	13 430	2 960	0.01029	350	0.434	0.446	0.457	0.462	0.475	0.498	0.520	0.530	0.231	0.463	0.555	0.277	0.1383	0.1152
2/0 G	2x.1542"	5x.1542"	0.463	2/0	10 510	2 622	0.01119	350	0.434	0.445	0.456	0.459	0.475	0.497	0.518	0.525	0.227	0.454	0.545	0.281	0.1406	0.1171
2/0 V	3x.1080"	9x.1167"	0.465	2/0	9 846	2 502	0.01395	360	0.435	0.442	0.450	0.452	0.476	0.489	0.504	0.509	0.216	0.432	0.518	0.281	0.1404	0.1170
2/0 F	1x.1454"	6x.1454"	0.436	2/0	8 094	2 359	0.01235	350	0.434	0.441	0.446	0.448	0.475	0.487	0.497	0.501	0.222	0.444	0.533	0.285	0.1427	0.1189
1/0 K	4x.1585"	3x.1585"	0.475	1/0	14 490	2 703	0.00812	310	0.548	0.560	0.573	0.579	0.599	0.625	0.652	0.664	0.243	0.487	0.584	0.279	0.1397	0.1164
1/0 J	3x.1467"	4x.1467"	0.440	1/0	10 970	2 346	0.00917	310	0.548	0.559	0.570	0.576	0.599	0.624	0.648	0.659	0.237	0.474	0.569	0.285	0.1423	0.1186
1/0 G	2x.1373"	5x.1373"	0.412	1/0	8 563	2 078	0.00996	310	0.548	0.559	0.568	0.573	0.599	0.623	0.645	0.654	0.233	0.466	0.559	0.289	0.1447	0.1206
1/0 F	1x.1294"	6x.1294"	0.388	1/0	6 536	1 870	0.01099	310	0.548	0.554	0.559	0.562	0.599	0.612	0.622	0.627	0.228	0.456	0.547	0.294	0.1469	0.1224
1 N	5x.1546"	2x.1546"	0.464	1	15 410	2 541	0.00638	280	0.691	0.705	0.719	0.726	0.755	0.787	0.818	0.832	0.256	0.512	0.614	0.281	0.1405	0.1171
1 K	4x.1412"	3x.1412"	0.423	1	11 900	2 144	0.00723	270	0.691	0.704	0.716	0.722	0.755	0.784	0.813	0.825	0.249	0.498	0.598	0.288	0.1438	0.1198
1 J	3x.1307"	4x.1307"	0.392	1	9 000	1 861	0.00817	270	0.691	0.703	0.714	0.719	0.755	0.783	0.808	0.820	0.243	0.486	0.583	0.293	0.1465	0.1221
1 G	2x.1222"	5x.1222"	0.367	1	6 956	1 649	0.00887	260	0.691	0.702	0.712	0.716	0.755	0.781	0.805	0.815	0.239	0.478	0.573	0.298	0.1488	0.1240
1 F	1x.1153"	6x.1153"	0.346	1	5 266	1 483	0.00980	270	0.691	0.698	0.704	0.705	0.755	0.769	0.781	0.786	0.234	0.468	0.561	0.302	0.1509	0.1258
2 P	6x.1540"	1x.1540"	0.462	2	16 870	2 487	0.00501	250	0.871	0.886	0.901	0.909	0.952	0.988	1.024	1.040	0.268	0.536	0.643	0.281	0.1406	0.1172
2 N	5x.1377"	2x.1377"	0.413	2	12 680	2 015	0.00568	240	0.871	0.885	0.899	0.906	0.952	0.986	1.020	1.035	0.261	0.523	0.627	0.289	0.1446	0.1205
2 K	4x.1257"	3x.1257"	0.377	2	9 730	1 701	0.00644	240	0.871	0.884	0.896	0.902	0.952	0.983	1.014	1.028	0.255	0.510	0.612	0.296	0.1479	0.1232
2 J	3x.1164"	4x.1164"	0.349	2	7 322	1 476	0.00727	230	0.871	0.883	0.894	0.899	0.952	0.982	1.010	1.022	0.249	0.498	0.598	0.301	0.1506	0.1255
2 A	1x.1699"	2x.1699"	0.366	2	5 876	1 356	0.00763	240	0.869	0.875	0.880	0.882	0.950	0.962	0.973	0.979	0.247	0.493	0.592	0.298	0.1489	0.1241
2 G	2x.1089"	5x.1089"	0.327	2	5 626	1 307	0.00790	230	0.871	0.882	0.892	0.896	0.952	0.980	1.006	1.016	0.245	0.489	0.587	0.306	0.1529	0.1275
2 F	1x.1026"	6x.1026"	0.308	2	4 233	1 176	0.00873	230	0.871	0.878	0.884	0.885	0.952	0.967	0.979	0.985	0.230	0.479	0.575	0.310	0.1551	0.1292
3 P	6x.1371"	1x.1371"	0.411	3	13 910	1 973	0.00445	220	1.098	1.113	1.127	1.136	1.200	1.239	1.273	1.296	0.274	0.547	0.657	0.290	0.1448	0.1207
3 N	5x.1226"	2x.1226"	0.368	3	10 390	1 598	0.00506	210	1.098	1.112	1.126	1.133	1.200	1.237	1.273	1.289	0.267	0.534	0.641	0.298	0.1487	1.1239
3 K	4x.1120"	3x.1120"	0.336	3	7 910	1 349	0.00574	210	1.098	1.111	1.123	1.129	1.200	1.233	1.267	1.281	0.261	0.522	0.626	0.304	0.1520	0.1266
3 J	3x.1036"	4x.1036"	0.311	3	5 955	1 171	0.00648	200	1.098	1.110	1.121	1.126	1.200	1.232	1.262	1.275	0.255	0.509	0.611	0.309	0.1547	0.1289
3 A	1x.1513"	2x.1513"	0.326	3	4 810	1 075	0.00679	210	1.096	1.102	1.107	1.109	1.198	1.211	1.225	1.229	0.252	0.505	0.606	0.306	0.1531	0.1275
4 P	6x.1221"	1x.1221"	0.366	4	11 420	1 564	0.00397	190	1.385	1.400	1.414	1.423	1.514	1.555	1.598	1.616	0.280	0.559	0.671	0.298	0.1489	0.1241
4 N	5x.1092"	2x.1092"	0.328	4	8 460	1 267	0.00451	180	1.385	1.399	1.413	1.420	1.514	1.554	1.593	1.610	0.273	0.546	0.655	0.306	0.1528	0.1274
4 D	2x.1615"	1x.1615"	0.348	4	7 340	1 191	0.00566	190	1.382	1.389	1.396	1.399	1.511	1.529	1.544	1.542	0.262	0.523	0.628	0.301	0.1507	0.1256
4 A	1x.1347"	2x.1347"	0.290	4	3 938	853	0.00604	180	1.382	1.388	1.393	1.395	1.511	1.525	1.540	1.545	0.258	0.517	0.620	0.314	0.1572	0.1310
5 P	6x.1087"	1x.1087"	0.326	5	9 311	1 240	0.00353	160	1.747	1.762	1.776	1.785	1.909	1.954	2.00	2.02	0.285	0.571	0.685	0.306	0.1531	0.1275
5 D	2x.1438"	1x.1438"	0.310	5	6 035	944	0.00504	160	1.742	1.749	1.756	1.759	1.905	1.924	1.941	1.939	0.268	0.535	0.642	0.310	0.1548	0.1290
5 A	1x.1200"	2x.1200"	0.258	5	3 193	676	0.00538	160	1.742	1.748	1.753	1.755	1.905	1.920	1.936	1.941	0.264	0.528	0.634	0.323	0.1614	0.1345
6 D	2x.1281"	1x.1281"	0.276	6	4 942	749	0.00449	140	2.20	2.21	2.21	2.22	2.40	2.42	2.44	2.44	0.273	0.547	0.656	0.318	0.1590	0.1325
6 A	1x.1068"	2x.1068"	0.230	6	2 585	536	0.00479	140	2.20	2.20	2.21	2.21	2.40	2.42	2.44	2.44	0.270	0.540	0.648	0.331	0.1655	0.1379
6 C	1x.1046"	2x.1046"	0.225	6	2 143	514	0.00469	130	2.20	2.20	2.21	2.21	2.40	2.42	2.44	2.44	0.271	0.542	0.651	0.333	0.1663	0.1386
7 D	2x.1141"	1x.1141"	0.246	7	4 022	594	0.00400	120	2.77	2.78	2.79	2.79	3.03	3.05	3.07	3.07	0.279	0.558	0.670	0.326	0.1631	0.1359
7 A	1x.1266"	2x.0895"	0.223	7	2 754	495	0.00441	120	2.77	2.78	2.78	2.78	3.03	3.05	3.07	3.07	0.274	0.548	0.658	0.333	0.1666	0.1388
8 D	2x.1016"	1x.1016"	0.219	8	3 256	471	0.00356	110	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.86	0.285	0.570	0.684	0.334	0.1672	0.1393
8 A	1x.1127"	2x.0797"	0.199	8	2 233	392	0.00394	100	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.87	0.280	0.560	0.672	0.341	0.1706	0.1422
8 C	1x.0808"	2x.0834"	0.179	8	1 362	320	0.00373	100	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.86	0.283	0.565	0.679	0.349	0.1744	0.1453
9½ D	2x.0808"	1x.0808"	0.174	9½	1 743	298	0.00283	85	4.91	4.92	4.92	4.93	5.37	5.39	5.42	5.42	0.297	0.593	0.712	0.351	0.1754	0.1462

*Based on a conductor temperature of 75°C. and an ambient of 25°C., wind 1.4 miles per hour (2 ft/sec.), frequency=60 cycles, average tarnished surface.
**Resistances at 50°C. total temperature, based on an ambient of 25°C. plus 25°C. rise due to heating effect of current. The approximate magnitude of current necessary to produce the 25° C. rise is 75% of the "Approximate Current Carrying Capacity at 60 cycles."

finding the appropriate table for a particular conductor. For these figures open circles, solid circles, and cross-hatched circles represent copper, steel, and aluminum conductors respectively. The double cross hatched area in the insert for Table 2-B, Characteristics of "EXPANDED" Aluminum Cable Steel Reinforced, represents stranded paper.

The authors wish to acknowledge the cooperation of the conductor manufacturers in supplying the information for compiling these tables.

TABLE 4-B—CHARACTERISTICS OF COPPERWELD CONDUCTORS

(Copperweld Steel Company)

Nominal Conductor Size	Number and Size of Wires	Outside Diameter Inches	Area of Conductor Circular Mils	Rated Breaking Load Pounds — Strength		Weight Pounds per Mile	Geometric Mean Radius at 60 cycles and Average Currents Feet	Approx. Current Carrying Capacity* Amps at 60 Cycles	r_a Resistance Ohms per Conductor per Mile at 25°C. (77°F.) Small Currents				r_a Resistance Ohms per Conductor per Mile at 75°C. (167°F.) Current Approx. 75% of Capacity**				x_a Inductive Reactance Ohms per Conductor per Mile One Ft. Spacing Average Currents			x_a' Capacitive Reactance Megohms per Conductor per Mile One Ft. Spacing		
				High	Extra High				d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
30% Conductivity																						
7/8″	19 No. 5	0.910	628 900	55 570	66 910	9 344	0.00758	620	0.306	0.316	0.326	0.331	0.363	0.419	0.476	0.499	0.261	0.493	0.592	0.233	0.1165	0.0971
13/16″	19 No. 6	0.810	498 800	45 830	55 530	7 410	0.00675	540	0.386	0.396	0.406	0.411	0.458	0.518	0.580	0.605	0.267	0.505	0.606	0.241	0.1206	0.1005
23/32″	19 No. 7	0.721	395 500	37 740	45 850	5 877	0.00601	470	0.486	0.496	0.506	0.511	0.577	0.643	0.710	0.737	0.273	0.517	0.621	0.250	0.1248	0.1040
21/32″	19 No. 8	0.642	313 700	31 040	37 690	4 660	0.00535	410	0.613	0.623	0.633	0.638	0.728	0.799	0.872	0.902	0.279	0.529	0.635	0.258	0.1289	0.1074
9/16″	19 No. 9	0.572	248 800	25 500	30 610	3 696	0.00477	360	0.773	0.783	0.793	0.798	0.917	0.995	1.075	1.106	0.285	0.541	0.649	0.266	0.1330	0.1109
5/8″	7 No. 4	0.613	292 200	24 780	29 430	4 324	0.00511	410	0.656	0.664	0.672	0.676	0.778	0.824	0.870	0.887	0.281	0.533	0.640	0.261	0.1306	0.1088
9/16″	7 No. 5	0.546	231 700	20 470	24 650	3 429	0.00455	360	0.827	0.835	0.843	0.847	0.981	1.030	1.080	1.099	0.287	0.545	0.654	0.269	0.1347	0.1122
1/2″	7 No. 6	0.486	183 800	16 890	20 460	2 719	0.00405	310	1.042	1.050	1.058	1.062	1.237	1.290	1.343	1.364	0.293	0.557	0.668	0.278	0.1388	0.1157
7/16″	7 No. 7	0.433	145 700	13 910	16 890	2 157	0.00361	270	1.315	1.323	1.331	1.335	1.560	1.617	1.675	1.697	0.299	0.569	0.683	0.286	0.1429	0.1191
3/8″	7 No. 8	0.385	115 600	11 440	13 890	1 710	0.00321	230	1.658	1.666	1.674	1.678	1.967	2.03	2.09	2.12	0.305	0.581	0.697	0.294	0.1471	0.1226
11/32″	7 No. 9	0.343	91 650	9 393	11 280	1 356	0.00286	200	2.09	2.10	2.11	2.11	2.48	2.55	2.61	2.64	0.311	0.592	0.711	0.303	0.1512	0.1260
5/16″	7 No. 10	0.306	72 680	7 758	9 196	1 076	0.00255	170	2.64	2.64	2.65	2.66	3.13	3.20	3.27	3.30	0.316	0.604	0.725	0.311	0.1553	0.1294
3 No. 5	3 No. 5	0.392	99 310	9 262	11 860	1 467	0.00457	220	1.926	1.931	1.936	1.938	2.29	2.31	2.34	2.35	0.289	0.545	0.654	0.293	0.1465	0.1221
3 No. 6	3 No. 6	0.349	78 750	7 639	9 754	1 163	0.00407	190	2.43	2.43	2.44	2.44	2.88	2.91	2.94	2.95	0.295	0.556	0.668	0.301	0.1506	0.1255
3 No. 7	3 No. 7	0.311	62 450	6 291	7 922	922.4	0.00363	160	3.06	3.07	3.07	3.07	3.63	3.66	3.70	3.71	0.301	0.568	0.682	0.310	0.1547	0.1289
3 No. 8	3 No. 8	0.277	49 530	5 174	6 282	731.5	0.00323	140	3.86	3.87	3.87	3.87	4.58	4.61	4.65	4.66	0.307	0.580	0.696	0.318	0.1589	0.1324
3 No. 9	3 No. 9	0.247	39 280	4 250	5 129	580.1	0.00288	120	4.87	4.87	4.88	4.88	5.78	5.81	5.85	5.86	0.313	0.591	0.710	0.326	0.1629	0.1358
3 No. 10	3 No. 10	0.220	31 150	3 509	4 160	460.0	0.00257	110	6.14	6.14	6.15	6.15	7.28	7.32	7.36	7.38	0.319	0.603	0.724	0.334	0.1671	0.1392
40% Conductivity																						
7/8″	19 No. 5	0.910	628 900	50 240	......	9 344	0.01175	690	0.229	0.239	0.249	0.254	0.272	0.321	0.371	0.391	0.236	0.449	0.539	0.233	0.1165	0.0971
13/16″	19 No. 6	0.810	498 800	41 600	......	7 410	0.01046	610	0.289	0.299	0.309	0.314	0.343	0.396	0.450	0.472	0.241	0.461	0.553	0.241	0.1206	0.1005
23/32″	19 No. 7	0.721	395 500	34 390	......	5 877	0.00931	530	0.365	0.375	0.385	0.390	0.433	0.490	0.549	0.573	0.247	0.473	0.567	0.250	0.1248	0.1040
21/32″	19 No. 8	0.642	313 700	28 380	......	4 660	0.00829	470	0.460	0.470	0.480	0.485	0.546	0.608	0.672	0.698	0.253	0.485	0.582	0.258	0.1289	0.1074
9/16″	19 No. 9	0.572	248 800	23 390	......	3 696	0.00739	410	0.580	0.590	0.600	0.605	0.688	0.756	0.826	0.853	0.259	0.496	0.595	0.266	0.1330	0.1109
5/8″	7 No. 4	0.613	292 200	22 310	......	4 324	0.00792	470	0.492	0.500	0.508	0.512	0.584	0.624	0.664	0.680	0.255	0.489	0.587	0.261	0.1306	0.1088
9/16″	7 No. 5	0.546	231 700	18 510	......	3 429	0.00705	410	0.620	0.628	0.636	0.640	0.736	0.780	0.843	0.840	0.261	0.501	0.601	0.269	0.1347	0.1122
1/2″	7 No. 6	0.486	183 800	15 330	......	2 719	0.00628	350	0.782	0.790	0.798	0.802	0.928	0.975	1.021	1.040	0.267	0.513	0.615	0.278	0.1388	0.1157
7/16″	7 No. 7	0.433	145 700	12 670	......	2 157	0.00559	310	0.986	0.994	1.002	1.006	1.170	1.220	1.271	1.291	0.273	0.524	0.629	0.286	0.1429	0.1191
3/8″	7 No. 8	0.385	115 600	10 460	......	1 710	0.00497	270	1.244	1.252	1.260	1.264	1.476	1.530	1.584	1.606	0.279	0.536	0.644	0.294	0.1471	0.1226
11/32″	7 No. 9	0.343	91 650	8 616	......	1 356	0.00443	230	1.568	1.576	1.584	1.588	1.861	1.919	1.978	2.00	0.285	0.548	0.658	0.303	0.1512	0.1260
5/16″	7 No. 10	0.306	72 680	7 121	......	1 076	0.00395	200	1.978	1.986	1.994	1.998	2.35	2.41	2.47	2.50	0.291	0.559	0.671	0.311	0.1553	0.1294
3 No. 5	3 No. 5	0.392	99 310	8 373	......	1 467	0.00621	250	1.445	1.450	1.455	1.457	1.714	1.738	1.762	1.772	0.269	0.514	0.617	0.293	0.1465	0.1221
3 No. 6	3 No. 6	0.349	78 750	6 934	......	1 163	0.00553	220	1.821	1.826	1.831	1.833	2.16	2.19	2.21	2.22	0.275	0.526	0.631	0.301	0.1506	0.1255
3 No. 7	3 No. 7	0.311	62 450	5 732	......	922.4	0.00492	190	2.30	2.30	2.31	2.31	2.73	2.75	2.78	2.79	0.281	0.537	0.645	0.310	0.1547	0.1289
3 No. 8	3 No. 8	0.277	49 530	4 730	......	731.5	0.00439	160	2.90	2.90	2.91	2.91	3.44	3.47	3.50	3.51	0.286	0.549	0.659	0.318	0.1589	0.1324
3 No. 9	3 No. 9	0.247	39 280	3 898	......	580.1	0.00391	140	3.65	3.66	3.66	3.66	4.33	4.37	4.40	4.41	0.292	0.561	0.673	0.326	0.1629	0.1358
3 No. 10	3 No. 10	0.220	31 150	3 221	......	460.0	0.00348	120	4.61	4.61	4.62	4.62	5.46	5.50	5.53	5.55	0.297	0.572	0.687	0.334	0.1671	0.1392
3 No. 12	3 No. 12	0.174	19 590	2 236	......	289.3	0.00276	90	7.32	7.33	7.33	7.34	8.69	8.73	8.77	8.78	0.310	0.596	0.715	0.351	0.1754	0.1462

*Based on conductor temperature of 125°C. and an ambient of 25°C.
**Resistance at 75°C. total temperature, based on an ambient of 25°C. plus 50°C. rise due to heating effect of current.
The approximate magnitude of current necessary to produce the 50°C. rise is 75% of the "Approximate Current Carrying Capacity at 60 Cycles."

TABLE 5—SKIN EFFECT TABLE

X	K	X	K	X	K	X	K
0.0	1.00000	1.0	1.00519	2.0	1.07816	3.0	1.31809
0.1	1.00000	1.1	1.00758	2.1	1.09375	3.1	1.35102
0.2	1.00001	1.2	1.01071	2.2	1.11126	3.2	1.38504
0.3	1.00004	1.3	1.01470	2.3	1.13069	3.3	1.41999
0.4	1.00013	1.4	1.01969	2.4	1.15207	3.4	1.45570
0.5	1.00032	1.5	1.02582	2.5	1.17538	3.5	1.49202
0.6	1.00067	1.6	1.03323	2.6	1.20056	3.6	1.52879
0.7	1.00124	1.7	1.04205	2.7	1.22753	3.7	1.56587
0.8	1.00212	1.8	1.05240	2.8	1.25620	3.8	1.60314
0.9	1.00340	1.9	1.06440	2.9	1.28644	3.9	1.64051

TABLE 6—INDUCTIVE REACTANCE SPACING FACTOR (x_d) OHMS PER CONDUCTOR PER MILE

25 CYCLES

SEPARATION

Feet	Inches 0	1	2	3	4	5	6	7	8	9	10	11
0	–	-0.1256	-0.0906	-0.0701	-0.0555	-0.0443	-0.0350	-0.0273	-0.0205	-0.0145	-0.0092	-0.0044
1	0	0.0040	0.0078	0.0113	0.0145	0.0176	0.0205	0.0232	0.0258	0.0283	0.0306	0.0329
2	0.0350	0.0371	0.0391	0.0410	0.0428	0.0446	0.0463	0.0480	0.0496	0.0511	0.0527	0.0541
3	0.0555	0.0569	0.0583	0.0596	0.0609	0.0621	0.0633	0.0645	0.0657	0.0668	0.0679	0.0690
4	0.0701	0.0711	0.0722	0.0732	0.0741	0.0751	0.0760	0.0770	0.0779	0.0788	0.0797	0.0805
5	0.0814	0.0822	0.0830	0.0838	0.0846	0.0854	0.0862	0.0869	0.0877	0.0884	0.0892	0.0899
6	0.0906	0.0913	0.0920	0.0927	0.0933	0.0940	0.0946	0.0953	0.0959	0.0965	0.0972	0.0978
7	0.0984	0.0990	0.0996	0.1002	0.1007	0.1013	0.1019	0.1024	0.1030	0.1035	0.1041	0.1046
8	0.1051											
9	0.1111											
10	0.1164											
11	0.1212											
12	0.1256											
13	0.1297											
14	0.1334											
15	0.1369											
16	0.1402											
17	0.1432											
18	0.1461											
19	0.1489											
20	0.1515											
21	0.1539											
22	0.1563											
23	0.1585											
24	0.1607											
25	0.1627											
26	0.1647											
27	0.1666											
28	0.1685											
29	0.1702											
30	0.1720											
31	0.1736											
32	0.1752											
33	0.1768											
34	0.1783											
35	0.1798											
36	0.1812											
37	0.1826											
38	0.1839											
39	0.1852											
40	0.1865											
41	0.1878											
42	0.1890											
43	0.1902											
44	0.1913											
45	0.1925											
46	0.1936											
47	0.1947											
48	0.1957											
49	0.1968											

x_d at 25 cycles

$x_d = 0.1164 \log_{10} d$
d = separation, feet.

FUNDAMENTAL EQUATIONS

$$z_1 = z_2 = r_a + j(x_a + x_d)$$
$$z_0 = r_a + r_e + j(x_a + x_e - 2x_d)$$

50 CYCLES

SEPARATION

Feet	Inches 0	1	2	3	4	5	6	7	8	9	10	11
0	–	-0.2513	-0.1812	-0.1402	-0.1111	-0.0885	-0.0701	-0.0545	-0.0410	-0.0291	-0.0184	-0.0088
1	0	0.0081	0.0156	0.0226	0.0291	0.0352	0.0410	0.0465	0.0517	0.0566	0.0613	0.0658
2	0.0701	0.0742	0.0782	0.0820	0.0857	0.0892	0.0927	0.0960	0.0992	0.1023	0.1053	0.1082
3	0.1111	0.1139	0.1166	0.1192	0.1217	0.1242	0.1267	0.1291	0.1314	0.1337	0.1359	0.1380
4	0.1402	0.1423	0.1443	0.1463	0.1483	0.1502	0.1521	0.1539	0.1558	0.1576	0.1593	0.1610
5	0.1627	0.1644	0.1661	0.1677	0.1693	0.1708	0.1724	0.1739	0.1754	0.1769	0.1783	0.1798
6	0.1812	0.1826	0.1839	0.1853	0.1866	0.1880	0.1893	0.1906	0.1918	0.1931	0.1943	0.1956
7	0.1968	0.1980	0.1991	0.2003	0.2015	0.2026	0.2037	0.2049	0.2060	0.2071	0.2081	0.2092
8	0.2103											
9	0.2222											
10	0.2328											
11	0.2425											
12	0.2513											
13	0.2594											
14	0.2669											
15	0.2738											
16	0.2804											
17	0.2865											
18	0.2923											
19	0.2977											
20	0.3029											
21	0.3079											
22	0.3126											
23	0.3170											
24	0.3214											
25	0.3255											
26	0.3294											
27	0.3333											
28	0.3369											
29	0.3405											
30	0.3439											
31	0.3472											
32	0.3504											
33	0.3536											
34	0.3566											
35	0.3595											
36	0.3624											
37	0.3651											
38	0.3678											
39	0.3704											
40	0.3730											
41	0.3755											
42	0.3779											
43	0.3803											
44	0.3826											
45	0.3849											
46	0.3871											
47	0.3893											
48	0.3914											
49	0.3935											

x_d at 50 cycles

$x_d = 0.2328 \log_{10} d$
d = separation, feet.

60 CYCLES

SEPARATION

Feet	**Inches 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
0	–	-0.3015	-0.2174	-0.1682	-0.1333	-0.1062	-0.0841	-0.0654	-0.0492	-0.0349	-0.0221	-0.0106
1	0	0.0097	0.0187	0.0271	0.0349	0.0423	0.0492	0.0558	0.0620	0.0679	0.0735	0.0789
2	0.0841	0.0891	0.0938	0.0984	0.1028	0.1071	0.1112	0.1152	0.1190	0.1227	0.1264	0.1299
3	0.1333	0.1366	0.1399	0.1430	0.1461	0.1491	0.1520	0.1549	0.1577	0.1604	0.1631	0.1657
4	0.1682	0.1707	0.1732	0.1756	0.1779	0.1802	0.1825	0.1847	0.1869	0.1891	0.1912	0.1933
5	0.1953	0.1973	0.1993	0.2012	0.2031	0.2050	0.2069	0.2087	0.2105	0.2123	0.2140	0.2157
6	0.2174	0.2191	0.2207	0.2224	0.2240	0.2256	0.2271	0.2287	0.2302	0.2317	0.2332	0.2347
7	0.2361	0.2376	0.2390	0.2404	0.2418	0.2431	0.2445	0.2458	0.2472	0.2485	0.2498	0.2511
8	0.2523											
9	0.2666											
10	0.2794											
11	0.2910											
12	0.3015											
13	0.3112											
14	0.3202											
15	0.3286											
16	0.3364											
17	0.3438											
18	0.3507											
19	0.3573											
20	0.3635											
21	0.3694											
22	0.3751											
23	0.3805											
24	0.3856											
25	0.3906											
26	0.3953											
27	0.3999											
28	0.4043											
29	0.4086											
30	0.4127											
31	0.4167											
32	0.4205											
33	0.4243											
34	0.4279											
35	0.4314											
36	0.4348											
37	0.4382											
38	0.4414											
39	0.4445											
40	0.4476											
41	0.4506											
42	0.4535											
43	0.4564											
44	0.4592											
45	0.4619											
46	0.4646											
47	0.4672											
48	0.4697											
49	0.4722											

x_d at 60 cycles

$x_d = 0.2794 \log_{10} d$
d = separation, feet.

TABLE 7—ZERO-SEQUENCE RESISTANCE AND INDUCTIVE REACTANCE FACTORS (r_e, x_e)*

Ohms per Conductor per Mile

	ρ Meter Ohm	FREQUENCY 25 Cycles	50 Cycles	60 Cycles
r_e	All	0.1192	0.2383	0.2860
x_e	1	0.921	1.736	2.050
	5	1.043	1.980	2.343
	10	1.095	2.085	2.469
	50	1.217	2.329	2.762
	100†	**1.270**	**2.434**	**2.888**
	500	1.392	2.679	3.181
	1000	1.444	2.784	3.307
	5000	1.566	3.028	3.600
	10 000	1.619	3.133	3.726

*From Formulas:
$r_e = 0.004764f$

$x_e = 0.006985f \log_{10} 4\,665\,600 \frac{\rho}{f}$

where f = frequency
ρ = Resistivity (meter, ohm)

†This is an average value which may be used in the absence of definite information.

TABLE 8—SHUNT **CAPACITIVE** REACTANCE SPACING FACTOR (x_d') MEGOHMS PER CONDUCTOR PER MILE

25 CYCLES

SEPARATION

Feet	INCHES 0	1	2	3	4	5	6	7	8	9	10	11
0	—	-0.1769	-0.1276	-0.0987	-0.0782	-0.0623	-0.0494	-0.0384	-0.0289	-0.0205	-0.0130	-0.0062
1	0	0.0057	0.0110	0.0159	0.0205	0.0248	0.0289	0.0327	0.0364	0.0398	0.0432	0.0463
2	0.0494	0.0523	0.0551	0.0577	0.0603	0.0628	0.0652	0.0676	0.0698	0.0720	0.0742	0.0762
3	0.0782	0.0802	0.0821	0.0839	0.0857	0.0875	0.0892	0.0909	0.0925	0.0941	0.0957	0.0972
4	0.0987	0.1002	0.1016	0.1030	0.1044	0.1058	0.1071	0.1084	0.1097	0.1109	0.1122	0.1134
5	0.1146	0.1158	0.1169	0.1181	0.1192	0.1203	0.1214	0.1225	0.1235	0.1246	0.1256	0.1266
6	0.1276	0.1286	0.1295	0.1305	0.1314	0.1324	0.1333	0.1342	0.1351	0.1360	0.1368	0.1377
7	0.1386	0.1394	0.1402	0.1411	0.1419	0.1427	0.1435	0.1443	0.1450	0.1458	0.1466	0.1473
8	0.1481											
9	0.1565											
10	0.1640											
11	0.1707											
12	0.1769											
13	0.1826											
14	0.1879											
15	0.1928											
16	0.1974											
17	0.2017											
18	0.2058											
19	0.2097											
20	0.2133											
21	0.2168											
22	0.2201											
23	0.2233											
24	0.2263											
25	0.2292											
26	0.2320											
27	0.2347											
28	0.2373											
29	0.2398											
30	0.2422											
31	0.2445											
32	0.2468											
33	0.2490											
34	0.2511											
35	0.2532											
36	0.2552											
37	0.2571											
38	0.2590											
39	0.2609											
40	0.2627											
41	0.2644											
42	0.2661											
43	0.2678											
44	0.2695											
45	0.2711											
46	0.2726											
47	0.2742											
48	0.2756											
49	0.2771											

x_d' at 25 cycles
$x_d' = .1640 \log_{10} d$
d=separation, feet.

FUNDAMENTAL EQUATIONS

$$x_1' = x_2' = x_a' + x_d'$$
$$x_0' = x_a' + x_e' - 2x_d'$$

50 CYCLES

SEPARATION

Feet	Inches 0	1	2	3	4	5	6	7	8	9	10	11
0	—	-0.0885	-0.0638	-0.0494	-0.0391	-0.0312	-0.0247	-0.0192	-0.0144	-0.0102	-0.0065	-0.0031
1	0	0.0028	0.0055	0.0079	0.0102	0.0124	0.0144	0.0164	0.0182	0.0199	0.0216	0.0232
2	0.0247	0.0261	0.0275	0.0289	0.0302	0.0314	0.0326	0.0338	0.0349	0.0360	0.0371	0.0381
3	0.0391	0.0401	0.0410	0.0420	0.0429	0.0437	0.0446	0.0454	0.0463	0.0471	0.0478	0.0486
4	0.0494	0.0501	0.0508	0.0515	0.0522	0.0529	0.0535	0.0542	0.0548	0.0555	0.0561	0.0567
5	0.0573	0.0579	0.0585	0.0590	0.0596	0.0601	0.0607	0.0612	0.0618	0.0623	0.0628	0.0633
6	0.0638	0.0643	0.0648	0.0652	0.0657	0.0662	0.0666	0.0671	0.0675	0.0680	0.0684	0.0689
7	0.0693	0.0697	0.0701	0.0705	0.0709	0.0713	0.0717	0.0721	0.0725	0.0729	0.0733	0.0737
8	0.0740											
9	0.0782											
10	0.0820											
11	0.0854											
12	0.0885											
13	0.0913											
14	0.0940											
15	0.0964											
16	0.0987											
17	0.1009											
18	0.1029											
19	0.1048											
20	0.1067											
21	0.1084											
22	0.1100											
23	0.1116											
24	0.1131											
25	0.1146											
26	0.1160											
27	0.1173											
28	0.1186											
29	0.1199											
30	0.1211											
31	0.1223											
32	0.1234											
33	0.1245											
34	0.1255											
35	0.1266											
36	0.1276											
37	0.1286											
38	0.1295											
39	0.1304											
40	0.1313											
41	0.1322											
42	0.1331											
43	0.1339											
44	0.1347											
45	0.1355											
46	0.1363											
47	0.1371											
48	0.1378											
49	0.1386											

x_d' at 50 cycles
$x_d' = 0.08198 \log_{10} d$
d=separation, feet.

60 CYCLES

SEPARATION

Feet	**Inches 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
0	—	-0.0737	-0.0532	-0.0411	-0.0326	-0.0260	-0.0206	-0.0160	-0.0120	-0.0085	-0.0054	-0.0026
1	0	0.0024	0.0046	0.0066	0.0085	0.0103	0.0120	0.0136	0.0152	0.0166	0.0180	0.0193
2	0.0206	0.0218	0.0229	0.0241	0.0251	0.0262	0.0272	0.0282	0.0291	0.0300	0.0309	0.0318
3	0.0326	0.0334	0.0342	0.0350	0.0357	0.0365	0.0372	0.0379	0.0385	0.0392	0.0399	0.0405
4	0.0411	0.0417	0.0423	0.0429	0.0435	0.0441	0.0446	0.0452	0.0457	0.0462	0.0467	0.0473
5	0.0478	0.0482	0.0487	0.0492	0.0497	0.0501	0.0506	0.0510	0.0515	0.0519	0.0523	0.0527
6	0.0532	0.0536	0.0540	0.0544	0.0548	0.0552	0.0555	0.0559	0.0563	0.0567	0.0570	0.0574
7	0.0577	0.0581	0.0584	0.0588	0.0591	0.0594	0.0598	0.0601	0.0604	0.0608	0.0611	0.0614
8	0.0617											
9	0.0652											
10	0.0683											
11	0.0711											
12	0.0737											
13	0.0761											
14	0.0783											
15	0.0803											
16	0.0823											
17	0.0841											
18	0.0858											
19	0.0874											
20	0.0889											
21	0.0903											
22	0.0917											
23	0.0930											
24	0.0943											
25	0.0955											
26	0.0967											
27	0.0978											
28	0.0989											
29	0.0999											
30	0.1009											
31	0.1019											
32	0.1028											
33	0.1037											
34	0.1046											
35	0.1055											
36	0.1063											
37	0.1071											
38	0.1079											
39	0.1087											
40	0.1094											
41	0.1102											
42	0.1109											
43	0.1116											
44	0.1123											
45	0.1129											
46	0.1136											
47	0.1142											
48	0.1149											
49	0.1155											

x_d' at 60 cycles
$x_d' = 0.06831 \log_{10} d$
d=separation, feet.

TABLE 9—ZERO-SEQUENCE SHUNT CAPACITIVE REACTANCE FACTOR
x_e'
Megohms per Conductor per Mile

Conductor Height Above Ground Feet	FREQUENCY 25 Cycles	50 Cycles	60 Cycles
10	0.640	0.320	0.267
15	0.727	0.363	0.303
20	0.788	0.394	0.328
25	0.836	0.418	0.348
30	0.875	0.437	0.364
40	0.936	0.468	0.390
50	0.984	0.492	0.410
60	1.023	0.511	0.426
70	1.056	0.528	0.440
80	1.084	0.542	0.452
90	1.109	0.555	0.462
100	1.132	0.566	0.472

$$x_e' = \frac{12.30}{f} \log_{10} 2h$$

where h=height above ground.
f=frequency.

IV CORONA

With the increased use of high-voltage transmission lines and the probability of going to still higher operating voltages, the common aspects of corona (radio influence and corona loss) have become more important in the design of transmission lines.

In the early days of high-voltage transmission, corona was something which had to be avoided, largely because of the energy loss associated with it. In recent years the RI (radio influence) aspect of corona has become more important. In areas where RI must be considered, this factor might establish the limit of acceptable corona performance.

Under conditions where abnormally high voltages are present, corona can affect system behavior. It can reduce the overvoltage on long open-circuited lines. It will attenuate lightning voltage surges (see Sec. 29 Chap. 15) and switching surges.[17] By increasing the electrostatic coupling between the shield wire and phase conductors, corona at times of lightning strokes to towers or shield wires reduces the voltage across the supporting string of insulators and thus, in turn, reduces the probability of flashover and improves system performance. On high-voltage lines grounded through a ground-fault neutralizer, the in-phase current due to corona loss can prevent extinction of the arc during a line to ground fault.[28]

6. Factors Affecting Corona

At a given voltage, corona is determined by conductor diameter, line configuration, type of conductor, condition of its surface, and weather. Rain is by far the most important aspect of weather in increasing corona. Hoarfrost and fog have resulted in high values of corona loss on experimental test lines. However, it is believed that these high losses were caused by sublimation or condensation of water vapor, which are conditions not likely to occur on an operating line because the conductor temperature would normally be above ambient. For this reason, measurements of loss made under conditions of fog and hoarfrost might be unreliable unless the conductors were at operating temperatures. Falling snow generally causes only a moderate increase in corona. Also, relative humidity, temperature, atmospheric pressure, and the earth's electric field can affect corona, but their effect is minor compared to that of rain. There are apparently other unknown factors found under desert conditions which can increase corona.[19]

The effect of atmospheric pressure and temperature is generally considered to modify the critical disruptive voltage of a conductor directly, or as the ⅔ power of the air density factor, δ, which is given by:

$$\delta=\frac{17.9b}{459+{}^\circ F} \tag{78}$$

where
b = barometric pressure in inches of mercury
$^\circ F$ = temperature in degrees Fahrenheit.

The temperature to be used in the above equation is generally considered to be the conductor temperature. Under standard conditions (29.92 in. of Hg. and 77°F) the air density factor equals 1.00. The air density factor should be considered in the design of transmission lines to be built in areas of high altitude or extreme temperatures. Table 10 gives barometric pressures as a function of altitude.

TABLE 10—STANDARD BAROMETRIC PRESSURE AS A FUNCTION OF ALTITUDE

Altitude, feet	Pressure, in. Hg.	Altitude, feet	Pressure, in. Hg.
−1000	31.02	4 000	25.84
− 500	30.47	5 000	24.89
		6 000	23.98
0	29.92	8 000	22.22
1000	28.86	10 000	20.58
2000	27.82	15 000	16.88
3000	26.81	20 000	13.75

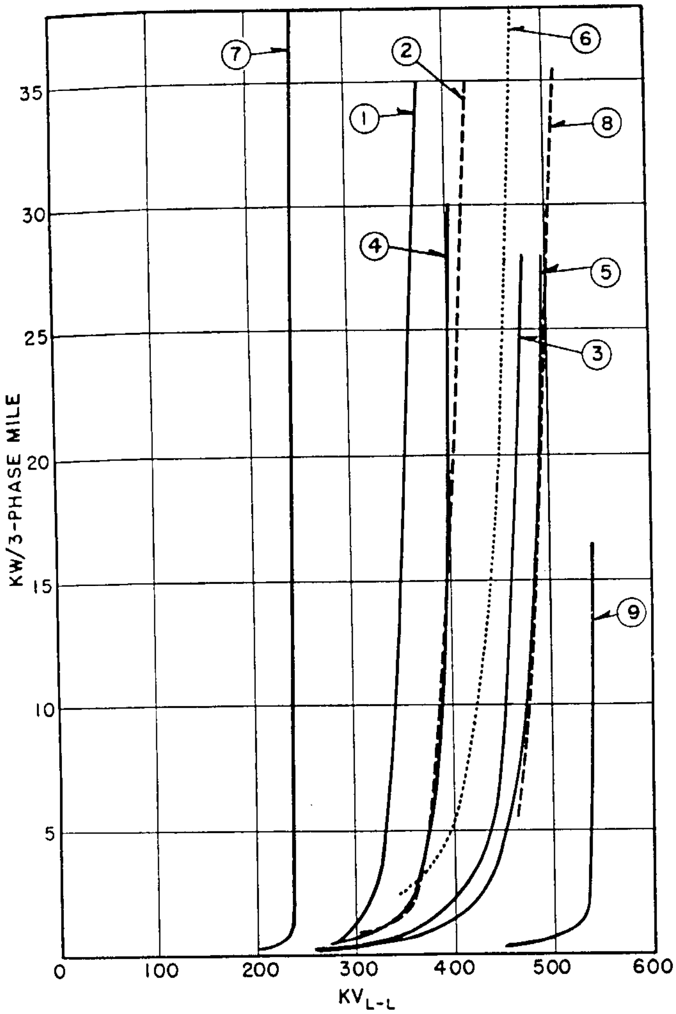
Corona in fair weather is negligible or moderate up to a voltage near the disruptive voltage for a particular conductor. Above this voltage corona effects increase very rapidly. The calculated disruptive voltage is an indicator of corona performance. A high value of critical disruptive voltage is not the only criterion of satisfactory corona performance. Consideration should also be given to the sensitivity of the conductor to foul weather. Corona increases somewhat more rapidly on smooth conductors than it does on stranded conductors. Thus the relative corona characteristics of these two types of conductors might interchange between fair and foul weather. The equation for critical disruptive voltage is:

$$E_o=g_o\,\delta^{2/3}\,r\,m\log_e D/r \tag{79a}$$

where:
E_o = critical disruptive voltage in kv to neutral
g_o = critical gradient in kv per centimeter. (Ref. 10 and 16 use $g_o=21.1$ Kv/cm rms. Recent work indicates value given in Sec. 10 is more accurate.)
r = radius of conductor in centimeters
D = the distance in centimeters between conductors, for single-phase, or the equivalent phase spacing, for three-phase voltages.
m = surface factor (common values, 0.84 for stranded, 0.92 for segmental conductors)
δ = air density factor

The more closely the surface of a conductor approaches a smooth cylinder, the higher the critical disruptive voltage assuming constant diameter. For equal diameters, a stranded conductor is usually satisfactory for 80 to 85 percent of the voltage of a smooth conductor. Any distortion of the surface of a conductor such as raised strands, die burrs, and scratches will increase corona. Care in handling conductors should be exercised, and imperfections in the surface should be corrected, if it is desired to obtain the best corona performance from a conductor. Die burrs and die grease on a new conductor, particularly the segmental type, can appreciably increase corona effects when it is first placed in service. This condition improves with time, taking some six months to become stable.

Strigel[44] concluded that the material from which a conductor is made has no effect on its corona performance. In



Curve 1—1.4 in. HH copper. $\delta = 0.88$. Ref. 19. Corona loss test made in desert at a location where abnormally high corona loss is observed on the Hoover–Los Angeles 287.5-kv line, which is strung with this conductor. Measurement made in three-phase test line. This particular curve is plotted for $\delta = 0.88$ to show operating condition in desert. All other curves are for $\delta = 1.00$.
Curve 2—Same as curve 1, except converted to $\delta = 1.00$.
Curve 3—1.4 in. HH copper. Ref. 12. Corona loss test made in California. Comparison with curve 2 shows effect of desert conditions. Measurements made on three-phase test line, 30-foot flat spacing, 16-foot sag, 30-foot ground clearance, 700 feet long.
Curve 4—1.1 in. HH. Ref. 13. Measurements made on three-phase test line, 22-foot flat spacing, 16-foot sag, 30-foot clearance to ground, 700 feet long.
Curve 5—1.65 in. smooth. Ref. 12. This conductor had a poor surface. Measurements made on three-phase test line, 30-foot spacing, 16-foot sag, 30-foot ground clearance, 700 feet long.
Curve 6—1.65 in. smooth aluminum. Ref. 27. Reference curve obtained by converting per-phase measurement to loss on three-phase line. Dimensions of line not given.
Curve 7—1.04 in. smooth cylinder. Ref. 23. In reference this conductor is referred to as having an infinite number of strands. Plotted curve obtained by conversion of per-phase measurements to three-phase values, using an estimated value for charging kva, to give loss on a line having 45-foot flat configuration.
Curve 8—1.96 in. smooth aluminum. Ref. 28. Reference curve gives three-phase loss, but line dimensions are not given.
Curve 9—1.57 in. smooth. Ref. 23. This conductor was smooth and clean. Reference curve gives per-phase values. Plotted curve is for 45-foot flat spacing.

Fig. 27—Fair-Weather Corona-Loss Curves for Smooth Conductors; Air Density Factor, $\delta = 1$.

industrial areas, foreign material deposited on the conductor can, in some cases, seriously reduce the corona performance. (Reference 28 gives some measurements made in an industrial area.)

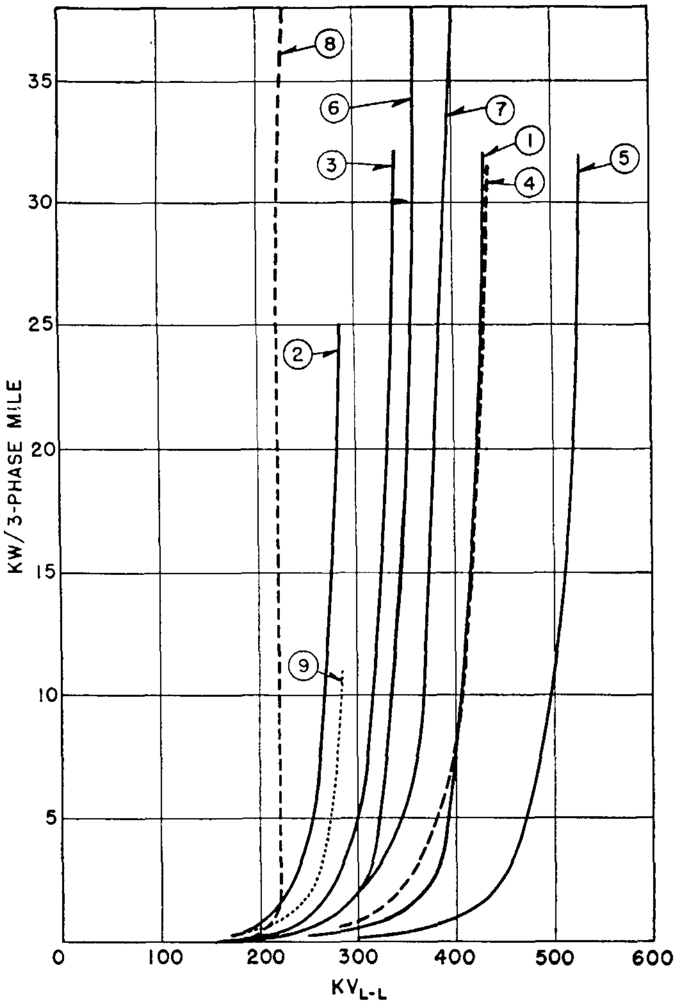
Corona is an extremely variable phenomenon. On a conductor energized at a voltage slightly above its fair weather corona-starting voltage, variations up to 10 to 1 in corona loss and radio-influence factor have been recorded during fair weather. The presence of rain produces corona loss on a conductor at voltages as low as 65 percent of the voltage at which the same loss is observed during fair-weather. Thus it is not practical to design a high-voltage line such that it will never be in corona. This also precludes expressing a ratio between fair- and foul-weather corona, since the former might be negligibly small.

If a conductor is de-energized for more than about a day, corona is temporarily increased. This effect is moderate compared to that of rain. It can be mitigated by re-energizing a line during fair weather where such a choice is possible.

7. Corona Loss

Extensive work by a large number of investigators has been done in determining corona loss on conductors operated at various voltages. This work has lead to the development of three formulas[10,14,16] generally used in this country (Reference 18 gives a large number of formulas). The Carroll-Rockwell and the Peterson formulas are considered the most accurate especially in the important low loss region (below 5 kw per three-phase mile). The Peterson formula, when judiciously used, has proved to be a reliable indicator of corona performance (see Sec. 9) for transmission voltages in use up to this time. Recent work on corona loss has been directed toward the extra-high-voltage range and indicates that more recent information should be used for these voltages.

Fair-weather corona-loss measurements made by a number of different investigators are shown in Figs. 27, 28, and 29. All curves are plotted in terms of kilowatts per three-phase mile. The data presented in these curves has been corrected for air density factor, δ, by multiplying the test voltage by $1/\delta^{2/3}$. Some error might have been introduced in these curves because in most cases it was necessary to convert the original data from per-phase measurements. The conversions were made on the basis of voltage gradient at the surface of each conductor. The curves should be used as an indicator of expected performance during fair weather. For a particular design, reference should be made to the original publications, and a conversion made for the design under consideration. The relation between fair-



Curve 1—1.4 in. ACSR. Ref. 12. Conductor was washed with gasoline then soap and water. Test configuration: three-phase line, 30-foot flat spacing, 16-foot sag, 30-foot ground clearance, 700 feet long.
Curve 2—1.0 in. ACSR. Ref. 11. Conductor weathered by exposure to air without continuous energization. Test configuration: three-phase line, 20-foot flat spacing, 700 feet long.
Curve 3—1.125 in. hollow copper. Ref. 14. Washed in same manner as for curve 1. Test configuration: three-phase line, 22-foot flat spacing.
Curve 4—1.49 in. hollow copper. Ref. 14. Washed in same manner as for curve 1. Test configuration: three-phase line, 30-foot flat spacing, 16-foot sag, 30-foot ground clearance, 700 feet long.
Curve 5—2.00 in. hollow aluminum. Ref. 14. Washed in same manner as for curve 1. Test configuration: three-phase line, 30-foot flat spacing, 16-foot sag, 30-foot ground clearance, 700 feet long.
Curve 6—1.09 in. steel-aluminum. Ref. 22. Reference curve is average fair-weather corona loss obtained by converting per-phase measurements to three-phase values, for a line 22.9 foot flat spacing, 32.8 feet high. This conductor used on 220-kv lines in Sweden which have above dimensions.
Curve 7—1.25 in. steel-aluminum. Ref. 22 App. A. Plotted curve obtained by estimating average of a number of fair-weather per-phase curves given in reference and converting to three-phase loss for line having 32-foot flat spacing, 50-foot average height.
Curve 8—1.04 in. steel-aluminum, 24-strand. Ref. 23. Plotted curve obtained by conversion of per-phase measurements to three-phase values, using an estimated value for charging kva, to give loss on a line having 45-foot flat configuration.
Curve 9—0.91 in. Hollow Copper. Ref. 11. Conductor washed. Test configuration: three-phase line, 20-foot flat spacing, 700 feet long.

Fig. 28—Fair-Weather Corona-Loss Curves for Stranded Conductors; Air Density Factor, $\delta = 1$.

and foul-weather corona loss and the variation which can be expected during fair weather is shown in Fig. 30 for one conductor.

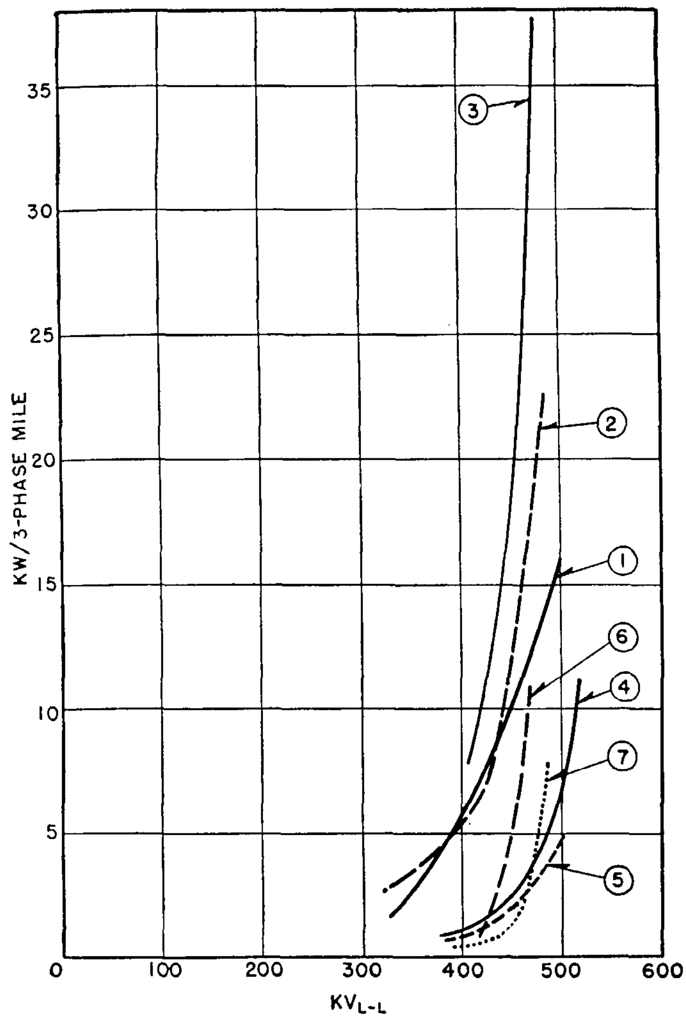
Corona loss on a satisfactory line is primarily caused by rain. This is shown by the fairly high degree of correlation between total rainfall and integrated corona loss which has been noted.[21,26,41] The corona loss at certain points on a transmission line can reach high values during bad storm conditions. However, such conditions are not likely to occur simultaneously all along a line. Borgquist and Vrethem expect only a variation from 1.6 to 16 kw per mile, with an average value of 6.5 kw per mile, on their 380-kv lines now under construction in Sweden. The measured loss on their experimental line varied from 1.6 to 81 kw per mile. The calculated fair-weather corona loss common in the U.S.A. is generally less than one kw per mile, based on calculations using Reference 16. Where radio-influence must be considered, the annual corona loss will not be of much economic importance[20], and the maximum loss will not constitute a serious load.

Corona loss is characterized on linear coordinates by a rather gradual increase in loss with increased voltage up to the so-called "knee" and above this voltage, a very rapid increase in loss. The knee of the fair-weather loss curve is generally near the critical disruptive voltage. A transmission line should be operated at a voltage well below the voltage at which the loss begins to increase rapidly under fair-weather eonditions. Operation at or above this point can result in uneconomical corona loss. A very careful analysis, weighing the annual energy cost and possibly the maximum demand against reduced capitalized line cost, must be made if operation at a voltage near or above the knee of the fair-weather loss curve is contemplated.

Corona loss on a conductor is a function of the voltage gradient at its surface. Thus the effect of reduced conductor spacing and lowered height is to increase the corona loss as a function of the increased gradient. On transmission lines using a flat conductor configuration, the gradient at the surface of the middle phase conductor is higher than on the outer conductor. This results in corona being more prevalent on the middle conductor.

8. Radio Influence (RI)

Radio influence is probably the factor limiting the choice of a satisfactory conductor for a given voltage. The RI performance of transmission lines has not been as thoroughly investigated as corona loss. Recent publications (see references) present most of the information available. RI plotted against voltage on linear graph paper is characterized by a gradual increase in RI up to a vol-



Curve 1—4/0.985/15.7* (Smooth) Ref. 25. δ not given, but assumed 1.10, which is average value for Germany. Reference curve obtained by converting single-phase measurements to three-phase values on the basis of surface gradient. Dimensions of line used in making conversion are not given.

Curve 2—4/0.827/15.7* (stranded aluminum-steel). Ref. 25. $\delta=1.092$. See discussion of Curve 1.

Curve 3—3/0.985/11.8* (Smooth). Ref. 26. $\delta=1.092$. Reference curve gives single-phase measurements versus line-to-ground voltage, but it is not clear whether actual test voltage or equivalent voltage at line height is given. Latter was used in making the conversion to three-phase. If this is wrong, curve is approximately 15 percent low in voltage. Converted to flat configuration of 45 feet.

Curve 4—2/1.09/17.7* (Stranded aluminum-steel). $\delta=1.01$. Ref. 12, App. A. Reference curve gives per-phase measurements versus gradient. Converted to three-phase corona loss on line of 42.5-foot average height, 39.4-foot flat configuration.

Curve 5—2/1.25/17.7* (Stranded aluminum-steel) δ not given, probably close to unity. Ref. 12. Reference curve, which gives three-phase corona loss, was converted from per-phase measurements. Dimensions 42.5 feet average height, 39.4 feet flat configuration. This conductor was selected for use on the Swedish 380-kv system. Original author probably selected a worse fair-weather condition than the writer did in plotting curve 4, which could account for their closeness.

Curve 6—2/1.04/23.7* (Stranded aluminum-steel). δ not given. Ref. 13. Plotted curve is average of two single-phase fair-weather curves, converted to three-phase loss for 45-foot flat configuration. See Curve 7.

Curve 7—2/1.04/15.7* (Stranded aluminum-steel). δ not given. Ref. 13. Plotted curve is average of two single-phase fair-weather curves, converted to three-phase loss for 45-foot flat configuration. Data for curves 6 and 7 were taken at same time in order to show effect of sub-conductor separation.

*Bundle-conductor designation—number of sub-conductors/outside diameter of each sub-conductor in inches/separation between adjacent sub-conductors in inches.

Fig. 29—Fair-Weather Corona-Loss Curves for Two-, Three-, and Four-conductor Bundles; Air Density Factor, $\delta=1.00$.

tage slightly below the minimum voltage at which measurable corona loss is detected. Above this voltage, the increase in the RI is very rapid. The rate of increase in RI is influenced by conductor surface and diameter, being higher for smooth conductors and large-diameter conductors. Above a certain voltage, the magnitude of the RI field begins to level off. For practical conductors, the leveling off value is *much* too high to be acceptable, and where RI is a factor, lines must be designed to operate below the voltage at which the rapid increase starts during fair weather. Figures 32 and 33 are characteristic RI curves. The relation between fair- and foul-weather corona performance is shown in Fig. 32.

An evaluation of RI in the design of a high-voltage line must consider not only its magnitude, but its effect on the various communication services which require protection. Amplitude-modulated broadcasting and power-line carrier are the most common services encountered but other services such as aviation, marine, ship-to-shore SOS calls, police and a number of government services might also have to be considered.

In determining the RI performance of a proposed line, the magnitude of the RI factors for the entire frequency range of communication services likely to be encountered, should be known. An evaluation of these factors in terms of their effect on various communication services must take into consideration many things. These are available signal intensities along the line, satisfactory signal-to-noise ratios, effect of weather on the RI factors and on the importance of particular communication services, number and type of receivers in vicinity of the line, proximity of particular receivers, transfer of RI to lower-voltage circuits, the general importance of particular communication services, and means for improvement of reception at individual receiver locations.[21] For extra-high-voltage and double-circuit high-voltage lines the tolerable limits of RI might be higher because the number of receivers affected, the coupling to lower voltage circuits, and the coupling to receiver antennas is reduced. Also fewer lines are required for the same power handling ability, and wider right-of-ways are used which tend to reduce the RI problem.

Although RI increases very rapidly with increased gradient at the surface of a conductor, theoretical considerations of the radiation characteristics of a transmission line as spacing is reduced, indicate that the RI from a transmission line will not be seriously affected by reduced spacing.[42]

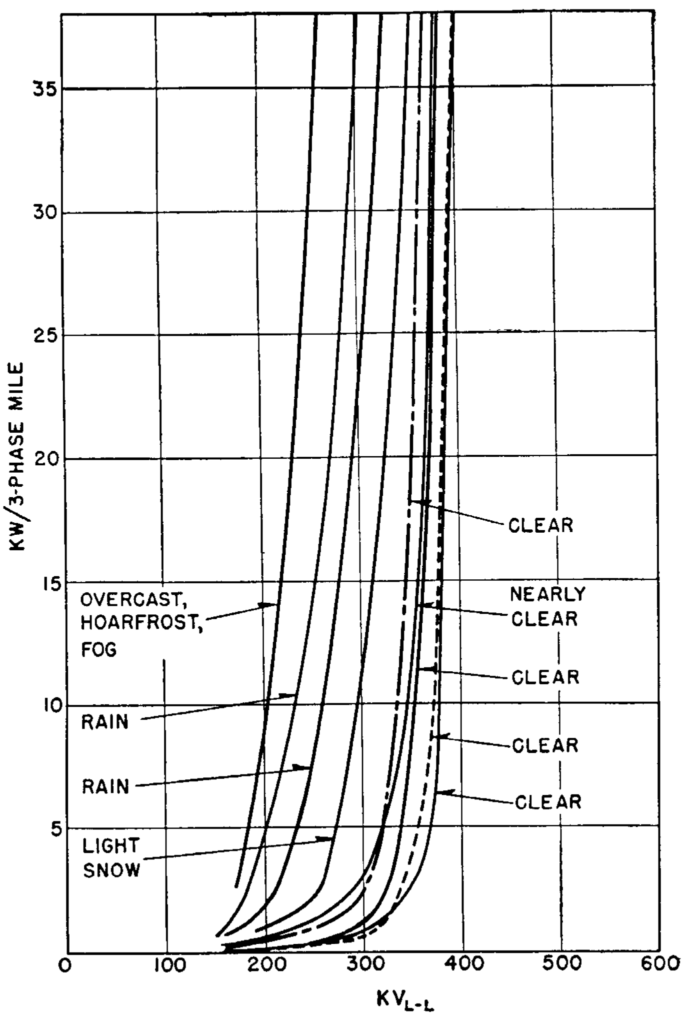


Fig. 30—Corona Loss on 1.09 Inch Stranded Aluminum-Steel Conductor under Different Weather Conditions. This conductor is in use on the Swedish 220-kv system. Note variation in fair-weather corona loss and the relation between fair- and foul-weather corona loss. Plotted curves obtained by converting per-phase measurements to three-phase values for a line having 32-foot flat spacing, 50-foot average height. No correction made for air density factor. Ref. 22, App. A.

The conductor configuration, the number of circuits, and the presence of ground wires affect the radiation from the line with a given RI voltage on the conductors. Very little is known about the radiation characteristics of transmission lines and caution should be exercised in applying data not taken on a line configuration closely approximating the design under consideration.

The RI field from a transmission line varies somewhat as the inverse of the radio frequency measured. Thus services in the higher-frequency bands, (television[37], frequency-modulated broadcasting, microwave relay, radar, etc.) are less apt to be affected. Directional antennas which are generally used at these frequencies, on the average, increase the signal-to-noise ratio. The lower signal strengths, and wider band-widths generally found in the high-frequency bands can alter this picture somewhat. Frequency-modulated broadcast is inherently less sensitive to RI because of its type of modulation.

Standard radio-noise meters[35,36] can measure the average, quasi-peak, and peak values of the RI field. The average value is the amplitude of the RI field averaged continuously over ½ second. For quasi-peak measurements, a circuit having a short time constant (0.001–0.01 sec.) for charging and a long time constant (0.3 to 0.6 sec.) for discharging is used, with the result that the meter indication is near the peak value of the RI field. Aural tests of radio reception indicate that quasi-peak readings interpreted in terms of broadcast-station field strengths represent more accurately the "nuisance" value of the RI field. The peak value is the maximum instantaneous value during a given period. The type of measurements made must be known before evaluating published RI information or misleading conclusions can be drawn.

The lateral attenuation of RI from a transmission line depends on the line dimensions and is independent of voltage. At distances between 40 and 150 feet from the outer conductor, the attenuation at 1000 kc varies from 0.1 to 0.3 db per foot, with the lower values applying generally to high-voltage lines. Typical lateral attenuation curves are shown in Fig. 34. Lateral attenuation is affected by local conditions. Because of the rapid attenuation of RI laterally from a line, a change of a few hundred feet in the location of a right-of-way can materially aid in protecting a communication service.

9. Selection of Conductor

In the selection of a satisfactory conductor from the standpoint of its corona performance for voltages up to 230 kv, operating experience and current practice are the best guide. Experience in this country indicates that the corona performance of a transmission line will be satisfactory when a line is designed so that the fair-weather corona loss according to Peterson's formula,[16] is less than one kw per three-phase mile. Unsatisfactory corona performance in areas where RI must be considered has been reported for lines on which the calculated corona loss is in excess of this value, or even less in the case of medium high-voltage lines. Figure 31 is based on Peterson's formula and indicates satisfactory conductors which can be used on high-voltage lines. For medium high-voltage lines (138 kv) considerably more margin below the one kw curve is necessary because of the increased probability of exposure of receivers to RI from the line, and a design approaching 0.1 kw should be used.

10. Bundle Conductors

A "bundle conductor" is a conductor made up of two or more "sub-conductors", and is used as one phase conductor. Bundle conductors are also called duplex, triplex, etc., conductors, referring to the number of sub-conductors and are sometimes referred to as grouped or multiple conductors. Considerable work on bundle conductors has been done by the engineers of Siemens-Schuckertwerke[27] who concluded that bundle conductors were not economical at 220 kv, but for rated voltages of 400 kv or more, are the best solution for overhead transmission. Rusck and Rathsman[46] state that the increase in transmitting capacity justifies economically the use of two-conductor bundles on 220-kv lines.

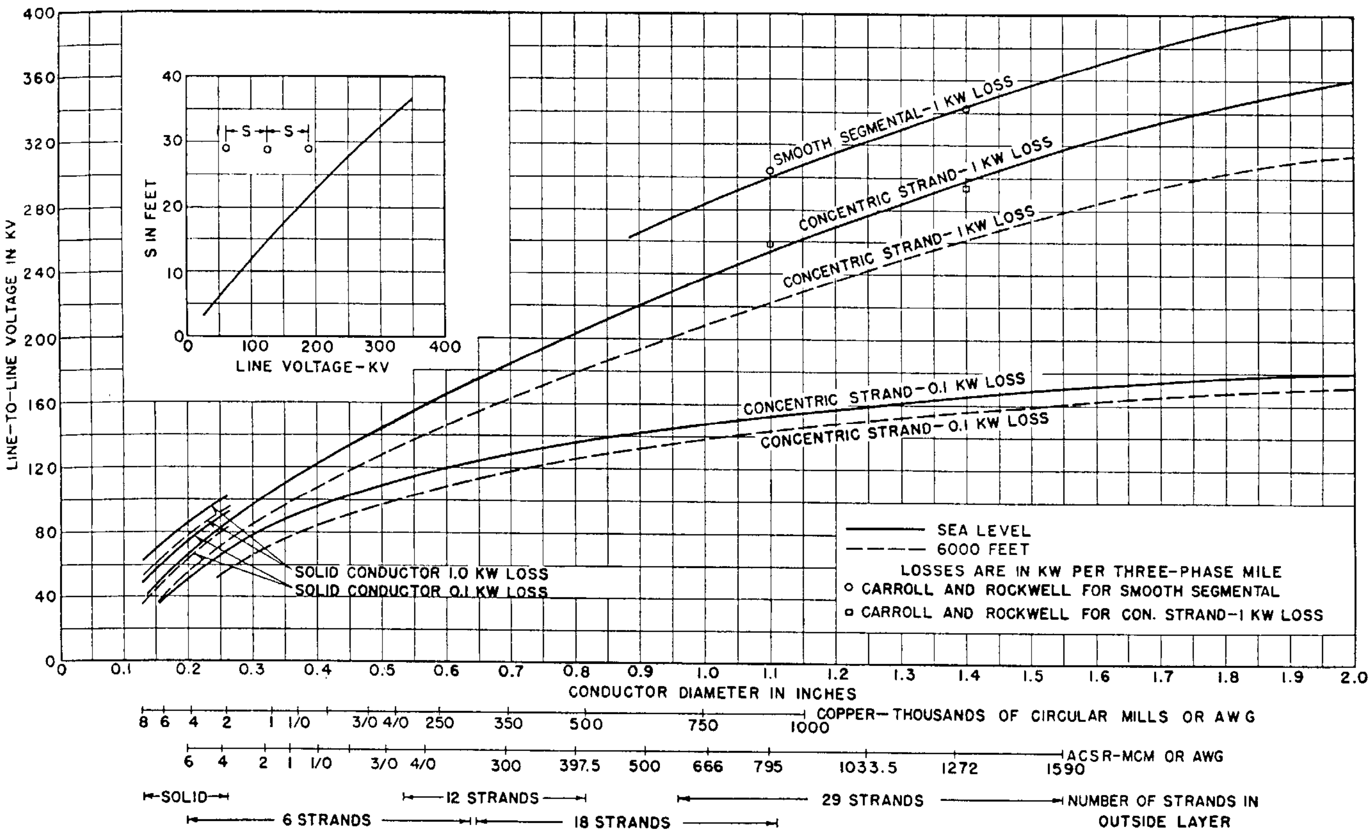


Fig. 31—Quick-Estimating Corona-Loss Curves. Curves based on Peterson's formula with a few check points from the Carrol and Rockwell paper for comparison.

The advantages of bundle conductors are higher disruptive voltage with conductors of reasonable dimensions, reduced surge impedance and consequent higher power capabilities, and less rapid increase of corona loss and RI with increased voltage.[22,27,28] These advantages must be weighed against increased circuit cost, increased charging kva if it cannot be utilized, and such other considerations as the large amount of power which would be carried by one circuit. It is possible with a two-conductor bundle composed of conductors of practical size to obtain electrical characteristics, excepting corona, equivalent to a single conductor up to eight inches in diameter.

Theoretically there is an optimum sub-conductor separation for bundle conductors that will give minimum crest gradient on the surface of a sub-conductor and hence highest disruptive voltage. For a two-conductor bundle, the separation is not very critical, and it is advantageous to use a larger separation than the optimum which balances the reduced corona performance and slightly increased circuit cost against the advantage of reduced reactance.

Assuming isolated conductors which are far apart compared to their diameter and have a voltage applied between them, the gradient at the surface of one conductor is given by:

$$g=\frac{e}{r\log_e D/r} \tag{79b}$$

where the symbols have the same meaning as used in Eq. (79a). This equation is the same as equation (79a), except that surface factor, m, and air density factor, δ, have been omitted. These factors should be added to Eqs. 80 and 81 for practical calculations. For a two-conductor bundle, the equation for maximum gradient at the surface of a sub-conductor[33] is:

$$g=\frac{e(1+2r/S)}{2r\log_e\dfrac{D}{\sqrt{rS}}} \tag{80}$$

where:
S = separation between sub-conductors in centimeters.

Because of the effect of the sub-conductors on each other, the gradient at the surface of a sub-conductor is not uniform. It varies in a cosinusoidal manner from a maximum at a point on the outside surface on the line-of-centers, to a minimum at the corresponding point on the inside surface. This effect modifies the corona performance of a bundle conductor such that its corona starting point corresponds to the voltage that would be expected from calculations, but the rate of increase of corona with increased voltage is less than for a single conductor. This effect can be seen by comparing curve 6 of Fig. 28 with curve 2 of Fig. 29. Cahen and Pelissier[21,24] concluded that the corona performance of a two-conductor bundle is more accurately indicated by the mean between the average

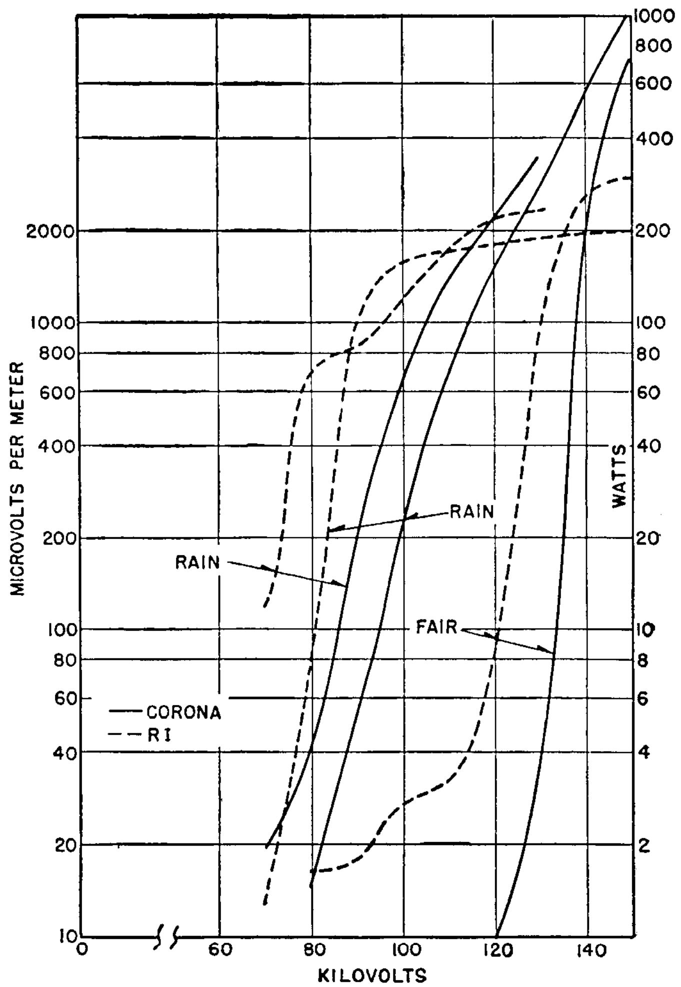


Fig. 32—Radio influence and corona loss measurements made on an experimental test line. Ref. 26.

and maximum gradient at the surface of a sub-conductor, which is given by:

$$g=\frac{e(1+r/S)}{2r\log_e\dfrac{D}{\sqrt{rS}}} \tag{81}$$

If it is desired to determine the approximate disruptive voltage of a conductor, $g_o=21.1\left(1+\dfrac{0.301}{\sqrt{r}}\right)$ kv per centimeter rms can be substituted for g and the equations solved for e_o in kv rms. This value neglects air density Factor and surface factor, which can be as low as 0.80 (consult references 10 and 16 for more accurate calculations).

380 kv Systems using bundle conductors are being built or under consideration in Sweden, France, and Germany.

Curve 1—Average lateral attenuation for a number of transmission lines from 138- to 450-kv. ○ × △ □ are plotted values which apply to this curve only. Test frequency 1000 kc. Ref. 21.

Curve 2—Lateral Attenuation from the 220-kv Eguzon-Chaingy line in France. Line has equilateral spacing, but dimensions not given. Distance measured from middle phase. Test frequency—868 kc. Ref. 24.

Curve 3—Lateral Attenuation from 230-kv Midway–Columbia Line of the Bonneville Power Administration. Conductor height 47.5 feet, test frequency 830 kc. Ref. 42.

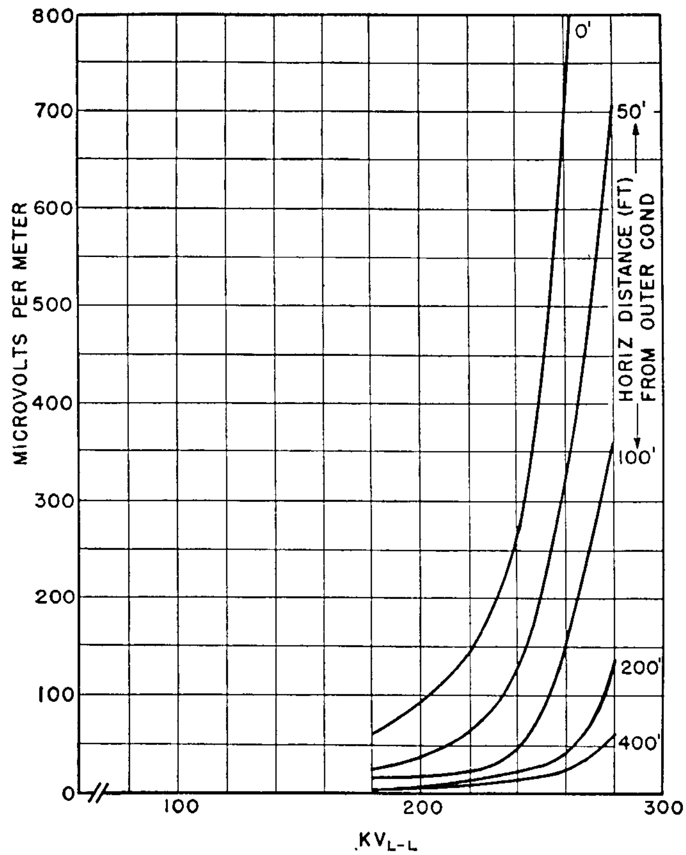


Fig. 33—Fair-Weather Radio-Influence Field from a Transmission Line as a Function of Voltage. Measurements made opposite mid-span on the 230-kv Covington-Grand Coulee Line No. 1 of the Bonneville Power Administration. RI values are quasi-peak. 1.108 inch ACSR conductor, 27-foot flat spacing, 41-foot height, test frequency—800 kc.

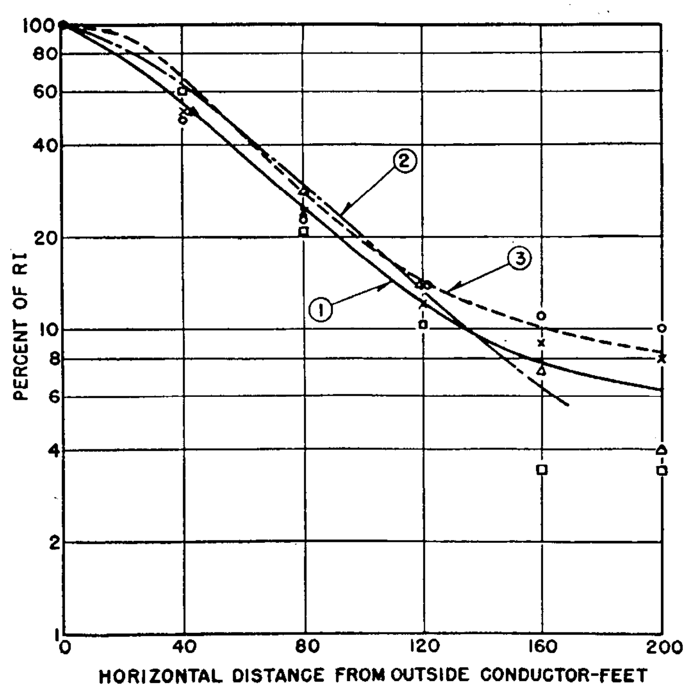


Fig. 34—Lateral Attenuation of Radio Influence in Vicinity of High-Voltage Transmission Lines.

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CHAPTER 4

ELECTRICAL CHARACTERISTICS OF CABLES

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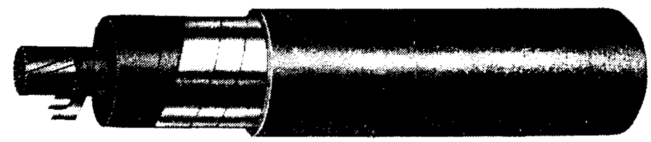
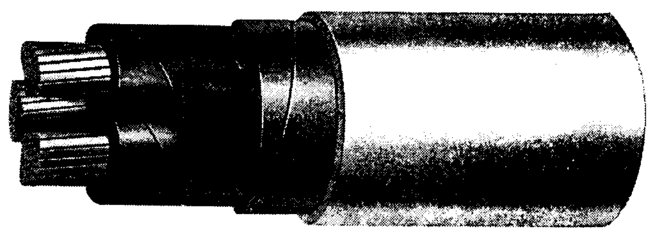
CABLES are classified according to their insulation as paper, varnished-cambric, rubber, or asbestos, each of these materials having unique characteristics which render it suitable for particular applications. Because cables for power transmission and distribution are composed of so many different types of insulation, conductors, and sheathing materials, the discussion here must be limited to those cable designs most commonly used. Reasonable estimates of electrical characteristics for cables not listed can be obtained in most cases by reading from the table for a cable having similar physical dimensions.

Paper can be wound onto a conductor in successive layers to achieve a required dielectric strength, and this is the insulation generally used for cables operating at 10 000 volts and higher. Paper insulation is impregnated in different ways, and accordingly cables so insulated can be sub-divided into solid, oil-filled, or gas-filled types.

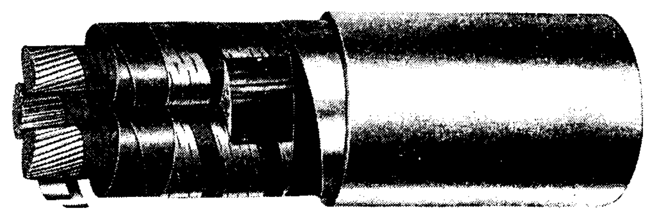
Solid paper-insulated cables are built up of layers of paper tape wound onto the conductor and impregnated with a viscous oil, over which is applied a tight-fitting, extruded lead sheath. Multi-conductor solid cables are also available, but the material shown here covers only single- and three-conductor types. Three-conductor cables are of either belted or shielded construction. The belted assembly consists of the three separately insulated conductors cabled together and wrapped with another layer of impregnated paper, or belt, before the sheath is applied. In the shielded construction each conductor is individually insulated and covered with a thin metallic non-magnetic shielding tape; the three conductors are then cabled together, wrapped with a metallic binder tape, and sheathed with lead. The purpose of the metallic shielding tape around each insulated conductor is to control the electrostatic stress, reduce corona formation, and decrease the thermal resistance. To minimize circulating current under normal operating conditions and thus limit the power loss, shielding tape only three mils in thickness is used. Solid single-conductor cables are standard for all voltages from 1 to 69 kv; solid three-conductor cables are standard from 1 to 46 kv. Sample sections of paper-insulated single-conductor, three-conductor belted, and three-conductor shielded cables are shown in Fig. 1(a), (b), and (c) respectively.



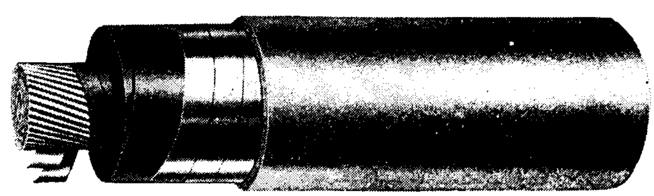
(a) Single-conductor solid, compact-round conductor.



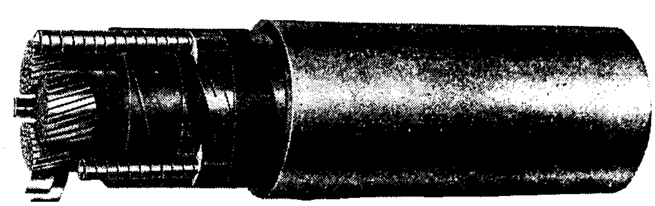
(b) Three-conductor belted, compact-sector conductors.



(c) Three-conductor shielded, compact-sector conductors.



(d) Single-conductor oil-filled, hollow-stranded conductor.

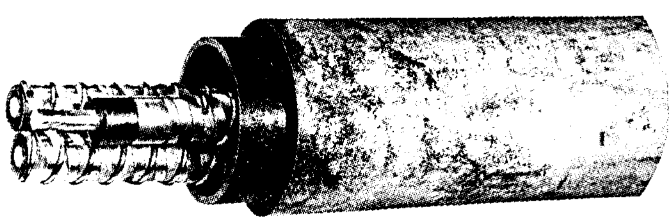


(e) Three-conductor oil-filled, compact-sector conductors.

Fig. 1—Paper-insulated cables.
Courtesy of General Cable Corporation

Oil-filled paper-insulated cables are available in single- or three-conductor designs. Single-conductor oil-filled cable consists of a concentric stranded conductor built around an open helical spring core, which serves as a channel for the flow of low-viscosity oil. This cable is insulated and sheathed in the same manner as solid cables, as a comparison of Figs. 1(a) and 1(d) indicates. Three-conductor oil-filled cables are all of the shielded design, and have three oil channels composed of helical springs that extend through the cable in spaces normally occupied by filler material. This construction is shown in Fig. 1(e). Oil-filled cables are relatively new and their application has become widespread in a comparatively short time. The oil used is only slightly more viscous than transformer oil, and remains fluid at all operating temperatures. The oil in the cable and its connected reservoirs is maintained under moderate pressure so that during load cycles oil may flow between the cable and the reservoirs to prevent the development of voids or excessive pressure in the cable. The prevention of void formation in paper insulation permits the use of greatly reduced insulation thickness for a given operating voltage. Another advantage of oil-filled cables is that oil will seep out through any crack or opening which develops in the sheath, thereby preventing the entrance of water at the defective point. This action prevents the occurrence of a fault caused by moisture in the insulation, and since operating records show that this cause accounts for a significant percentage of all high-voltage cable faults, it is indeed a real advantage. Single-conductor oil-filled cables are used for voltages ranging from 69 to 230 kv; the usual range for three-conductor oil-filled cables is from 23 to 69 kv.

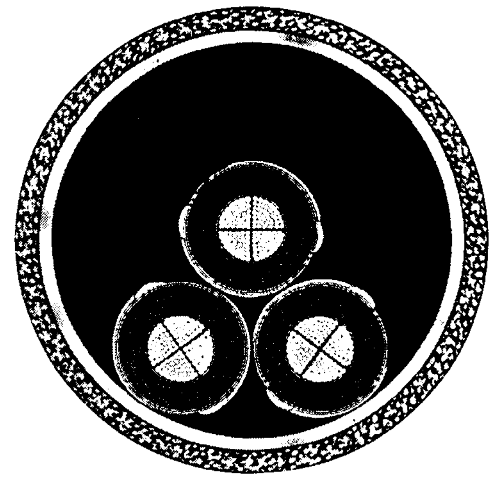
Gas-filled cables of the low-pressure type have recently become standard up to 46 kv. The single-conductor type employs construction generally similar to that of solid cables, except that longitudinal flutes or other channels are provided at the inner surface of the sheath to conduct nitrogen along the cable. The three-conductor design employs channels in the filler spaces among the conductors, much like those provided in oil-filled three-conductor cables. The gas is normally maintained between 10 and 15 pounds per square inch gauge pressure, and serves to fill all cable voids and exclude moisture at faulty points in the sheath or joints.



Courtesy of the Okonite-Callender Cable Company

Fig. 2—High-pressure pipe-type oil-filled cable.

High-pressure cables, of either the oil- or gas-filled variety, are being used widely for the higher range of voltages. The physical and electrical characteristics are fairly well known, but their specifications are not yet standardized. The usual application calls for pressure of about 200 pounds per square inch, contained by a steel pipe into which three single-conductor cables are pulled. The immediate presence of the iron pipe makes difficult the calculations of circuit impedance, particularly the zero-sequence quantities. Most high-pressure cables are designed so that the oil or gas filler comes into direct contact with the conductor insulation; in oil-filled pipe-type cables a temporary lead sheath can be stripped from the cable as it is pulled into the steel pipe; in gas-filled pipe-type cables the lead sheath surrounding each conductor remains in place, with nitrogen introduced both inside and outside the sheath so that no differential pressure develops across the sheath. Examples of oil- and gas-filled pipe-type cables are shown in Figs. 2 and 3.



Courtesy of General Cable Corporation

Fig. 3—Cross-section of high-pressure pipe-type gas-filled cable. Oil-filled pipe-type cable may have a similar cross-section.

Compression cable is another high-pressure pipe-type cable in which oil or nitrogen gas at high pressure is introduced within a steel pipe containing lead-sheathed solid-type single-conductor cables; no high-pressure oil or gas is introduced directly inside the lead sheaths, but voids within the solid-type insulation are prevented by pressure exerted externally on the sheaths. This construction is sketched in Fig. 4.

During recent years there has been a trend toward the modification of cable conductors to reduce cost and improve operating characteristics, particularly in multi-conductor cables. Referring to Fig. 5, the first departure from concentric round conductors was the adoption of sector-shaped conductors in three-conductor cables. More recently a crushed stranding that results in a compacted sector has been developed and has found widespread use for conductor sizes of 1/0 A.W.G. and larger. Its use in smaller conductors is not practical. The principal advantages of such a conductor are: reduced overall diameter for a given copper cross-section; elimination of space between the conductor and the insulation, which results in higher

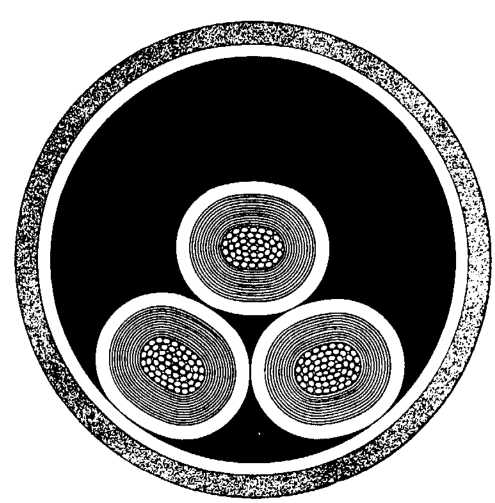
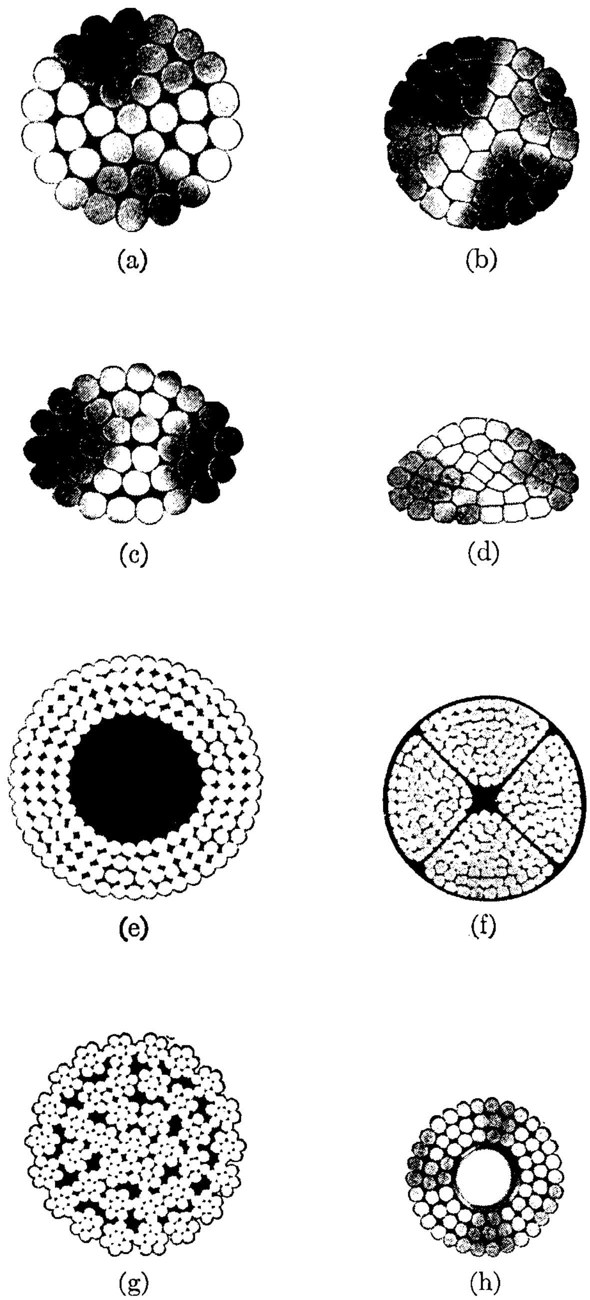


Fig. 4—Cross-sectional sketch of compression cable.



Photographs in this figure furnished by the Okonite-Callender Cable Company

Fig. 5—Cable conductors.

(a) Standard concentric stranded.
(b) Compact round.
(c) Non-compact sector.
(d) Compact sector.
(e) Annular stranded (rope core).
(f) Segmental.
(g) Rope stranded.
(h) Hollow core.

electrical breakdown; low a-c resistance due to minimizing of proximity effect; retention of the close stranding during bending; and for solid cables, elimination of many longitudinal channels along which impregnating compound can migrate. While most single-conductor cables are of the concentric-strand type, they may also be compact-round, annular-stranded, segmental, or hollow-core.

I. ELECTRICAL CHARACTERISTICS

The electrical characteristics of cables have been discussed comprehensively in a series of articles[1] upon which much of the material presented here has been based. This chapter is primarily concerned with the determination of the electrical constants most commonly needed for power-system calculations, particular emphasis being placed on quantities necessary for the application of symmetrical components.[2] A general rule is that regardless of the complexity of mutual inductive relations between component parts of individual phases, the method of symmetrical components can be applied rigorously whenever there is symmetry among phases. All the three-conductor cables inherently satisfy this condition by the nature of their construction; single-conductor cables may or may not, although usually the error is small in calculating short-circuit currents. Unsymmetrical spacing and change in permeability resulting from different phase currents when certain methods of eliminating sheath currents are used, may produce dissymmetry.

Those physical characteristics that are of general interest in electrical application problems have been included along with electrical characteristics in the tables of this section.

All linear dimensions of radius, diameter, separation, or distance to equivalent earth return are expressed in inches in the equations in this chapter. This is unlike overhead transmission line theory where dimensions are in feet; the use of inches when dealing with cable construction seems appropriate. Many equations contain a factor for frequency, f, which is the circuit operating frequency in cycles per second.

1. Geometry of Cables

The space relationship among sheaths and conductors in a cable circuit is a major factor in determining reactance, capacitance, charging current, insulation resistance, dielectric loss, and thermal resistance. The symbols used in this chapter for various cable dimensions, both for single-conductor and three-conductor types, are given in Figs. 6 and 7. Several factors have come into universal use for defining the cross-section geometry of a cable circuit, and some of these are covered in the following paragraphs.[1,2]

Geometric Mean Radius (GMR)—This factor is a property usually applied to the conductor alone, and depends on the material and stranding used in its construction. One component of conductor reactance[3] is normally calculated by evaluating the integrated flux-linkages both inside and outside the conductor within an overall twelve-inch radius. Considering a solid conductor, some of the flux lines lie within the conductor and contribute to total flux-linkages even though they link only a portion of the total conductor current; if a tubular conductor having an infinitely thin wall were substituted for the solid conductor, its flux would necessarily all be external to the tube. A theoretical tubular conductor, in order to be inductively equivalent to a solid conductor, must have a smaller radius so

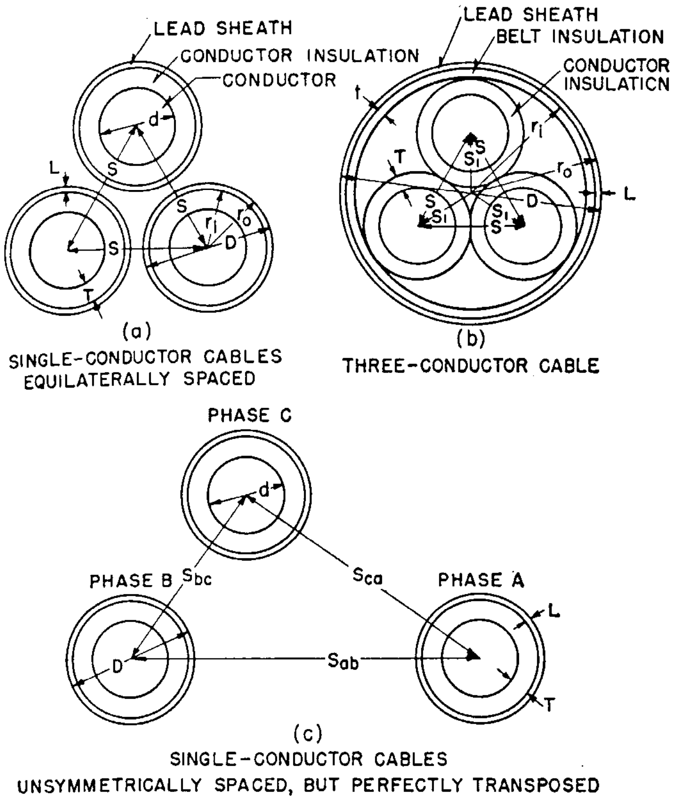


Fig. 6—Geometry of cables.

that the flux-linkages present inside the solid conductor but absent within the tube will be replaced by additional linkages between the tube surface and the limiting cylinder of twelve-inch radius. A solid copper conductor of radius $d/2$ can be replaced by a theoretical tubular conductor whose radius is $0.779\ d/2$. This equivalent radius is called the geometric mean radius of the actual conductor, denoted herein by GMR_{1c} where the subscript denotes reference to only a single actual conductor. This quantity can be used in reactance calculations without further reference to the shape or make-up of the conductor. The factor by which actual radius must be multiplied to obtain GMR_{1c} varies with

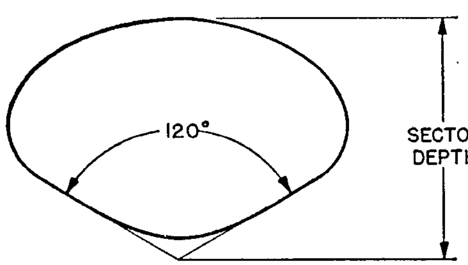


Fig. 7—Typical sector shape of conductor used in three-conductor cables.

stranding or hollow-core construction as shown in Chap. 3, Fig. 11. Sometimes in calculations involving zero-sequence reactances, simplification may result if the three conductors comprising a three-phase circuit are considered as a group and converted to a single equivalent conductor. This requires the use of a new GMR, denoted here as GMR_{3c}, which applies to the group as though it were one complex conductor. This procedure is illustrated later in Eq. (18).

Geometric Mean Distance (GMD)—Spacings among conductors, or between conductors and sheaths, are important in determining total circuit reactance. The total flux-linkages surrounding a conductor can be divided into two components, one extending inward from a cylinder of 12-inch radius as discussed in the preceding paragraph, and the other extending outward from this cylinder to the current return path beyond which there are no net flux-linkages.[3] The flux-linkages per unit conductor current between the 12-inch cylinder and the return path are a function of the separation between the conductor and its return. The return path can in many cases be a parallel group of wires, so that a geometric mean of all the separations between the conductor and each of its returns must be used in calculations. Geometric mean distance, therefore, is a term that can be used in the expression for external flux-linkages, not only in the simple case of two adjacent conductors where it is equal to the distance between conductor centers, but also in the more complex case where two circuits each composed of several conductors are separated by an equivalent GMD.

The positive- or negative-sequence reactance of a three-phase circuit depends on separation among phase conductors. If the conductors are equilaterally spaced the distance from one conductor center to another is equal to the GMD among conductors for that circuit. Using the terminology in Fig. 6,

$$GMD_{3c} = S \text{ for an equilateral circuit.}$$

The subscript here denotes that this GMD applies to separations among three conductors. If the conductors are arranged other than equilaterally, but transposed along their length to produce a balanced circuit, the equivalent separation may be calculated by deriving a geometric mean distance from the cube root of three distance products[3] (see Chap. 3):

$$GMD_{3c} = \sqrt[3]{S_{ab} \cdot S_{bc} \cdot S_{ca}} \tag{1}$$

The component of circuit reactance caused by flux outside a twelve inch radius is widely identified as "reactance spacing factor" (x_d) and can be calculated directly from the GMD:

$$x_d = 0.2794 \frac{f}{60} \log_{10} \frac{GMD_{3c}}{12} \text{ ohms per phase per mile.} \tag{2}$$

When the equivalent separation is less than twelve inches, as can occur in cable circuits, the reactance spacing factor is negative so as to subtract from the component of conductor reactance due to flux out to a twelve-inch radius.

The zero-sequence reactance of a three-phase circuit may depend on spacing among conductors and sheath as well as among conductors. A distance that represents the equivalent spacing between a conductor or a group of conductors and the enclosing sheath can be expressed as a GMD. Also, the equivalent separation between cable conductors and the sheath of a nearby cable, or the equivalent separation between two nearby sheaths, can be expressed as a GMD. Because these and other versions[2] of geometric mean distance may be used successively in a single problem, care must be taken to identify and distinguish among them during calculations.

Geometric Factor—The relation in space between the cylinders formed by sheath internal surface and conductor external surface in a single-conductor lead-sheathed cable can be expressed as a "geometric factor." This factor is applicable to the calculation of such cable characteristics as capacitance, charging current, dielectric loss, leakage current, and heat transfer, because these characteristics depend on a field or flow pattern between conductor and sheath. The mathematical expression for geometric factor G in a single conductor cable is

$$G = 2.303 \log_{10} \frac{2r_i}{d} \tag{3}$$

where:

r_i = inside radius of sheath.
d = outside diameter of conductor.

Geometric factors for single-conductor cables can be read from Fig. 8. Geometric factors for three-phase shielded cables having round conductors are identical, except for heat flow calculations, to those for single-conductor cables. The shielding layer establishes an equipotential surface surrounding each conductor just as a lead sheath does for single-conductor cables. The heat conductivity of the three-mil shielding tape is not high enough to prevent a temperature differential from developing around the shield circumference during operation: this poses a more complex problem than can be solved by the simple geometric factors given here.

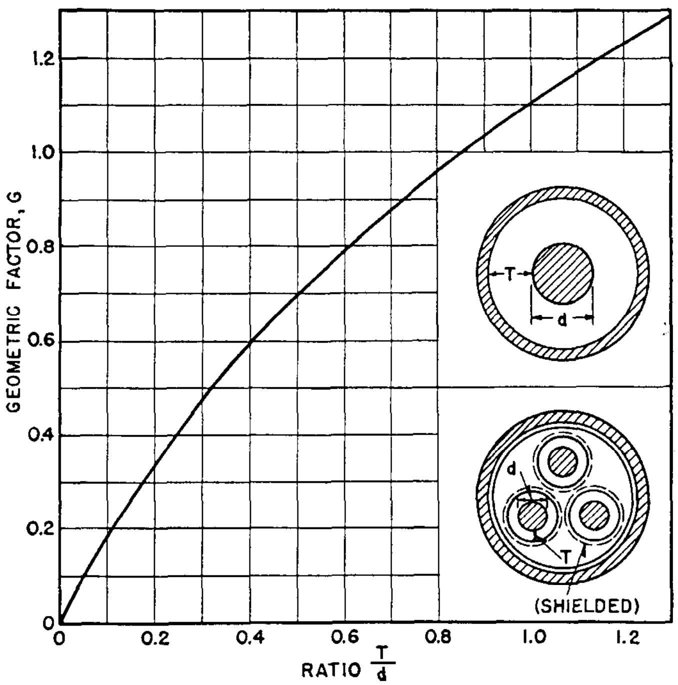


Fig. 8—Geometric factor for single-conductor cables, or three-conductor shielded cables having round conductors.

NOTE: This is approximately correct for shielded sector-conductor cables if curve is entered with the dimensions of a round-conductor cable having identical conductor area and insulation thickness. This geometric factor is not applicable for heat-flow calculations in shielded cables. See Secs. 5 and 6.

Because of the various possible combinations of conductors and sheaths that can be taken in a three-conductor belted cable, several geometric factors are required for complete definition. Two of these factors, the ones applicable to positive- and to zero-sequence electrical calculations, are shown in Fig. 9.

2. Positive- and Negative-Sequence Resistance

Skin Effect—It is well known that the resistance of a conductor to alternating current is larger than its resistance to direct current. The direct-current resistance in cables can be taken as the resistance of solid rod of the same length and cross-section, but increased two percent to take into account the effect of spiraling of the strands that compose the conductor. When alternating current flows in the conductor there is an unequal distribution of current, with the outer filaments of the conductor carrying more current than the filaments closer to the center. This results in a higher resistance to alternating current than to direct current, and is commonly called skin effect. The ratio of the two resistances is known as the skin-effect ratio. In small conductors this ratio is entirely negligible, but for larger conductors it becomes quite appreciable, and must be considered when figuring the 60-cycle resistances of large conductors. Some skin-effect ratios are tabulated in Table 1 for stranded and representative hollow conductors.[1]

TABLE 1—DIMENSIONS AND 60-CYCLE SKIN-EFFECT RATIO OF STRANDED COPPER CONDUCTORS AT 65°C.

Conductor Size (Circular Mils)	Round Concentric-Stranded		Inner Diameter of Annular Stranded Conductor, inches			
			0.50		0.75	
	Diameter inches	Ratio	Outer Diam.	Ratio	Outer Diam.	Ratio
211 600	0.528	1.00	.....	.....	.....	.....
250 000	0.575	1.005	.....	.....	.....	.....
300 000	0.630	1.006	.....	.....	.....	.....
400 000	0.728	1.012	.....	.....	.....	.....
500 000	0.814	1.018	0.97	1.01	.....	.....
600 000	0.893	1.026	1.04	1.01	.....	.....
800 000	1.031	1.046	1.16	1.02	1.28	1.01
1 000 000	1.152	1.068	1.25	1.03	1.39	1.02
1 500 000	1.412	1.145	1.52	1.09	1.63	1.06
2 000 000	1.631	1.239	1.72	1.17	1.80	1.12
2 500 000	1.825	1.336	1.91	1.24	2.00	1.20
3 000 000	1.998	1.439	2.08	1.36	2.15	1.29

Proximity Effect—The alternating magnetic flux in a conductor caused by the current flowing in a neighboring conductor gives rise to circulating currents, which cause an apparent increase in the resistance of a conductor. This phenomenon is called proximity effect. The increase in resistance is negligible except in very large conductors.

Proximity effect can, however, become important under certain conditions of cable installation. When cables are laid parallel to metal beams, walls, etc., as is frequently the case in buildings or ships, proximity effect increases the apparent impedance of these cables appreciably. Booth, Hutchings and Whitehead[4] have made extensive tests on

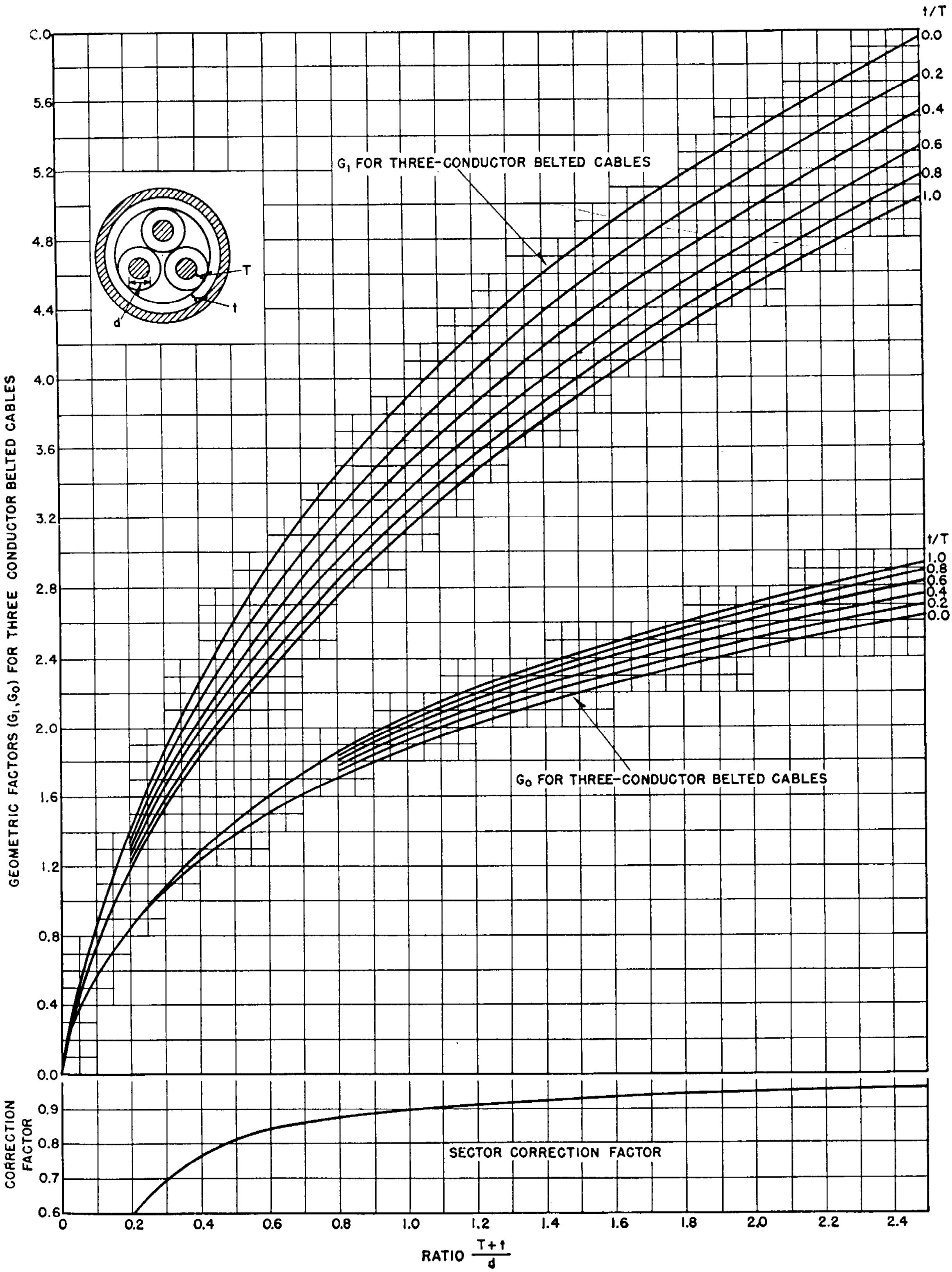


Fig. 9—Geometric factor for three-conductor belted cables having round or sector conductors.

NOTE: For cables having sector conductors, enter the curve with the dimensions of a round-conductor cable having identical conductor area and insulation thicknesses. Multiply the resultant geometric factor by the sector correction factor given above.

(G_1 is calculated for three-phase operation; G_0 is calculated for single-phase operation, with three conductors paralleled and return in sheath. See Secs. 5 and 6.)

the impedance and current-carrying capacity of cables, as they are affected by proximity to flat plates of conducting and magnetic material. Figures 11 and 12, taken from this work, illustrate forcefully that proximity effect can be significantly large. Although these tests were performed at 50 cycles it is believed that the results serve to indicate effects that would be experienced at 60 cycles. The results in an actual installation of cables close to metal surfaces are influenced so greatly by the material involved, and by the structural shape of the surface, that calculation and prediction is difficult.

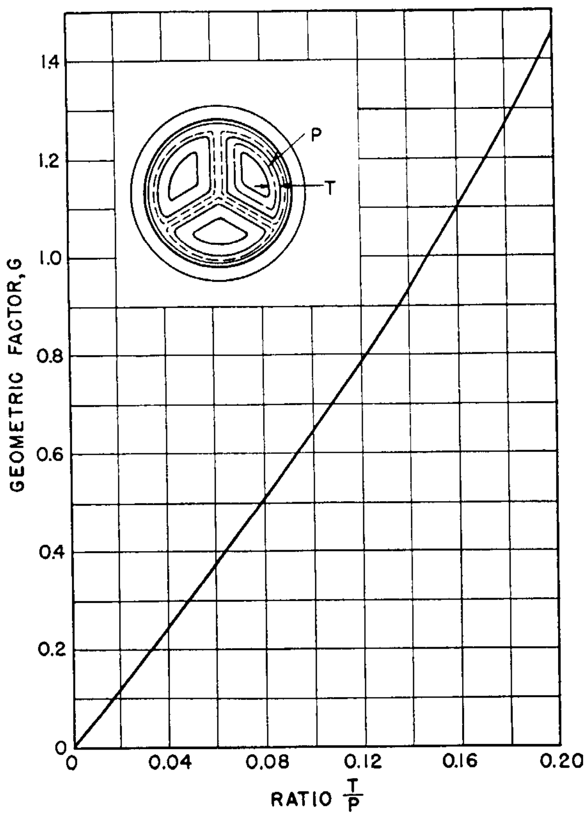


Fig. 10—Geometric factor for three-conductor shielded cables having sector conductors, in terms of insulation thickness T and mean periphery P.

The additional losses caused by placing a metal plate or other structural shape close to a cable circuit arise from both hysteresis and eddy-current effects within the plate. Hysteresis losses are large if the flux density within the plate is high throughout a large proportion of the plate volume. A material having high permeability and very high resistivity would promote hysteresis loss, because flux developed by cable currents could concentrate within the low-reluctance plate, and because the action of eddy-currents to counteract the incident flux would be comparatively small in a high-resistance material. Eddy-current losses depend on the magnetic field strength at the plate, and also upon the resistance of the paths available for flow within the plate.

Because the factors that affect hysteresis loss and those that affect eddy-current loss are interdependent, it is seldom easy to theorize on which material or combination of materials will contribute lowest losses. Some practical possibilities, drawn from experience in the design of switchgear, transformers, and generators, are listed here:

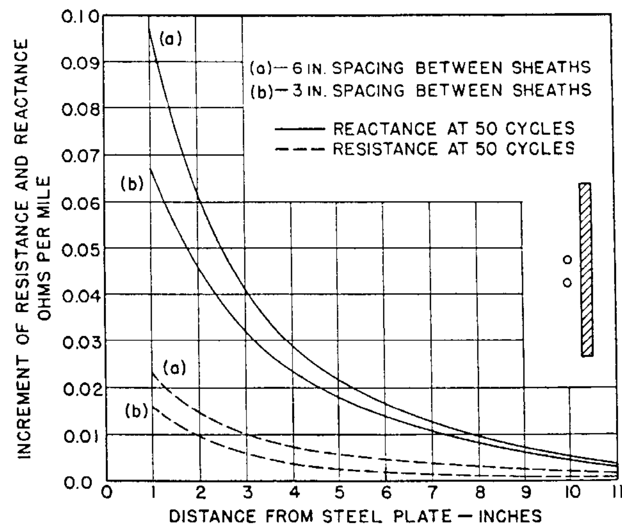


Fig. 11—Increase in cable resistance and reactance caused by proximity to steel plate for single phase systems (cable sheaths are insulated).

a. The magnetic plate can be shielded by an assembly of laminated punchings, placed between the cables and the plate, so that flux is diverted from the plate and into the laminations. The laminations normally have low eddy-current losses and they must be designed so that flux density is not excessive.
b. The magnetic plate can be shielded with a sheet of conducting material, such as copper or aluminum, placed so that the magnetic field acts to build up

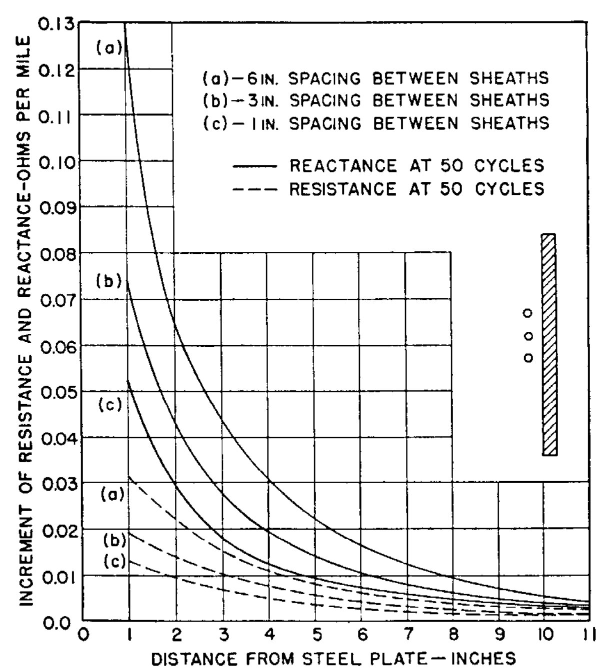


Fig. 12—Increase in cable resistance and reactance caused by proximity to steel plate for three-phase systems (cable sheaths are insulated).

counteracting circulating currents within the conducting sheet: these currents considerably reduce the magnetic field strength at the plate. The conducting sheet must have sufficient cross-sectional area to accommodate the currents developed.

c. The magnetic material can be interleaved with conducting bars that are bonded at the ends so that circulating currents develop to counteract the incident magnetic field as in (b).

d. The magnetic plate can be replaced, either entirely or partially, by a non-magnetic steel. Non-magnetic steel has low permeability and high resistivity when compared with conventional steel plate: these characteristics do not act in all respects to reduce losses, but the net effect is often a loss reduction. Non-magnetic steel is of particular benefit when the structure near the cable circuit partially or entirely surrounds individual phase conductors.

The effect of parallel metal on reactance is much larger than on resistance as Figs. 11 and 12 indicate. These figures also show that the magnitude of the increase in impedance is independent of conductor size. Actually, when large cables approach very close to steel, the resistance increments become higher and the reactance increments become somewhat lower. The curves of Figs. 11 and 12 are based on tests performed at approximately two-thirds of maximum current density for each cable used. The increments in resistance and reactance do not, however, change greatly with current density; the variation is only about 1 percent per 100 amperes. In three-phase systems the middle cable of the three is influenced less than the outer ones by the presence of the parallel steel. This variation again is less than variations in materials and has not been accounted for in Figs. 11 and 12. These curves cover only a few specific cases, and give merely an indication of the importance and magnitude of proximity effect. More detailed information can be found in the reference listed.[4]

Proximity effect also has an important bearing on the current-carrying capacity of cables when installed near steel plates or structures. This subject is discussed in the section on current-carrying capacity.

Sheath Currents in Cables—Alternating current in the conductors of single-conductor cables induces alternating voltages in the sheaths. When the sheaths are continuous and bonded together at their ends so that sheath currents may flow longitudinally, additional I^2R losses develop in the sheath. The common way to represent these losses is by increasing the resistance of the conductor involved. For single-conductor cables operating in three-phase systems, this increment in resistance can be calculated by the following equation, the derivation of which is given in references:[1,2]

$$r=\frac{x_m^2 r_s}{x_m^2+r_s^2}\text{ ohms per phase per mile.} \tag{4}$$

Here x_m is the mutual reactance between conductors and sheath in ohms per phase per mile, and r_s is the resistance of the sheath in ohms per phase per mile. These two quantities can be determined from the following equations:

$$x_m=0.2794\,\frac{f}{60}\log_{10}\frac{2S}{r_o+r_i}\text{ ohms per phase per mile.} \tag{5}$$

and

$$r_s=\frac{0.200}{(r_o+r_i)(r_o-r_i)}\text{ ohms per phase per mile, for lead sheath.} \tag{6}$$

in which

S = spacing between conductor centers in inches.
r_o = outer radius of lead sheath in inches.
r_i = inner radius of lead sheath in inches.

Thus the total resistance (r_a) to positive- or negative-sequence current flow in single-conductor cables, including the effect of sheath currents, is

$$r_a=r_c+\frac{x_m^2 r_s}{x_m^2+r_s^2}\text{ ohms per phase per mile.} \tag{7}$$

where r_c is the alternating-current resistance of the conductor alone including skin effect at the operating frequency. Eq. (7) applies rigorously only when the cables are in an equilateral triangular configuration. For other arrangements the geometric mean distance among three conductors, GMD_{3c}, can be used instead of S with results sufficiently accurate for most practical purposes.

The sheath loss in a three-conductor cable is usually negligible except for very large cables and then it is important only when making quite accurate calculations. In these largest cables the sheath losses are about 3 to 5 percent of the conductor loss, and are of relatively little importance in most practical calculations. When desired the sheath loss in three-conductor cables can be calculated from the equivalent resistance,

$$r=\frac{44160(S_1)^2}{r_s(r_o+r_i)^2}\times 10^{-6}\text{ ohms per phase per mile.} \tag{8}$$

where

r_s is sheath resistance from Eq. (6).
r_o and r_i are sheath radii defined for Eq. (5).

$$S_1=\frac{1}{\sqrt{3}}(d+2T),\text{ and is the distance between conductor center and sheath center for three-conductor cables made up of round conductors.} \tag{9}$$

d = conductor diameter.
T = conductor insulation thickness.

For sector-shaped conductors an approximate figure can be had by using Eq. (8), except that d should be 82 to 86 percent of the diameter of a round conductor having the same cross-sectional area.

Example 1—Find the resistance at 60 cycles of a 750 000 circular-mil, three-conductor belted cable having 156 mil conductor insulation and 133 mil lead sheath. The overall diameter of the cable is 2.833 inches and the conductors are sector shaped.

From conductor tables (see Table 10) the diameter of an equivalent round conductor is 0.998 inches. From Eq. (9),

$$S_1=\frac{1}{\sqrt{3}}[0.998(0.84)+2(0.156)]$$
$$=0.664\text{ inches.}$$

Since the overall diameter is 2.833 inches,

$$r_o=1.417\text{ inches}$$

and

$$r_i = 1.284 \text{ inches.}$$

From Eq. (6),

$$r_s = \frac{0.200}{(2.701)(0.133)}$$
$$= 0.557 \text{ ohms per phase per mile.}$$

Substituting in Eq. (8),

$$r = \frac{44160(0.664)^2}{0.557(2.701)^2} \times 10^{-6}$$
$$= 0.00479 \text{ ohms per phase per mile.}$$

From Table 6 it is found that r_c, the conductor resistance, including skin effect is 0.091 ohms per phase per mile. The total positive- and negative-sequence resistance is then,

$r_a = 0.091 + .005 = 0.096$ ohms per phase per mile.

Sheath currents obviously have little effect on the total alternating-current resistance of this cable.

Theoretically some allowance should be made for the losses that occur in the metallic tape on the individual conductors of shielded cable, but actual measurements indicate that for all practical purposes these losses are negligible with present designs and can be ignored in most cases. The resistance to positive- and negative-sequence in shielded cable can be calculated as though the shields were not present.

Three Conductors in Steel Pipe—Typical values for positive- and negative-sequence resistance of large pipe-type cables have been established by test[5], and an empirical calculating method has been proposed by Wiseman[6] that checks the tests quite closely. Because the calculations are complex, only an estimating curve is presented here. The ratio of actual resistance as installed to the d-c resistance of the conductor itself based on data obtained in laboratory tests is shown in Fig. 13. The increased resistance is due to conductor skin effect, conductor proximity effect in the presence of steel pipe, and to I^2R loss in the pipe itself. In preparing Fig. 13, the pipe size assumed for each cable size was such that 60 percent of the internal pipe cross-sectional area would have been unoccupied by cable material: choosing a nearest standard pipe size as a practical expedient does not affect the result appreciably. The conductor configuration for these tests was a triangular grouping, with the group lying at the bottom of the pipe. If, instead, the conductors were to be laid in an approximately flat cradled arrangement, some change in resistance would be expected. Actual tests on the flat arrangement produced variable results as conductor size was changed, some tests giving higher losses and some lower than the triangular. If a maximum value is desired, an estimated increase of 15 percent above the resistance for triangular configuration can be used. Field tests have been made on low-voltage circuits by Brieger[14], and these results are shown in Table 2.

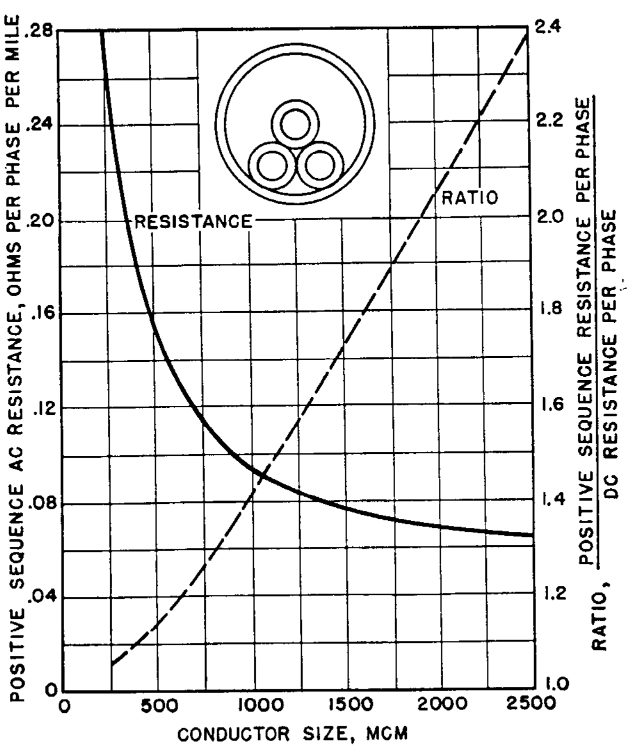


Fig. 13—Positive-sequence resistance of high-voltage cables in steel pipe (estimating curve).

3. Positive- and Negative-Sequence Reactances

Single-Conductor Cables—The reactance of single-conductor lead-sheathed cables to positive- and negative-sequence currents can be calculated from the following equation, which takes into account the effect of sheath currents.

$$x_1 = x_2 = 0.2794 \frac{f}{60} \log_{10} \frac{\text{GMD}_{3c}}{\text{GMR}_{1c}} - \frac{x_m^3}{x_m^2 + r_s^2}$$

ohms per phase per mile. (10)

or

$$x_1 = x_2 = x_a + x_d - \frac{x_m^3}{x_m^2 + r_s^2} \text{ ohms per phase per mile.} \quad (11)$$

The conductor component of reactance is

$$x_a = 0.2794 \frac{f}{60} \log_{10} \frac{12}{\text{GMR}_{1c}} \quad (12)$$

where

GMR_{1c} = geometric mean radius of one conductor.

The separation component of reactance is

$$x_d = 0.2794 \frac{f}{60} \log_{10} \frac{\text{GMD}_{3c}}{12} \quad (13)$$

where

GMD_{3c} = geometric mean distance among three conductors (see Eq. 1).

The component to be subtracted[1] because of the effect of sheath currents is composed of terms defined by Eqs. (5) and (6).

Three-Conductor Cables—Because negligible sheath current effects are present in three-conductor non-shielded cables, the reactance to positive- and negative-sequence currents can be calculated quite simply as:

$$x_1 = x_2 = 0.2794 \frac{f}{60} \log_{10} \frac{\text{GMD}_{3c}}{\text{GMR}_{1c}}$$

ohms per phase per mile (14)

or

$$x_1 = x_2 = x_a + x_d \text{ ohms per phase per mile} \quad (15)$$

TABLE 2—IMPEDANCE OF THREE-PHASE 120/208 VOLT CABLE CIRCUITS IN FIBRE AND IN IRON CONDUITS.[1]

Positive- and Negative-Sequence Impedance, Ohms per Phase per Mile at 60 Cycles.

Phase Conductor Size	Conductor Assembly	Duct Material (4 inch)	Cable Sheath (Phase Conductors)	Resistance (Ohms at 25°C.)	Reactance (Ohms)
500 MCM (1 per phase)	Uncabled[2]	Fibre	Non-leaded	0.120	0.189
			Lead	0.127	0.188
		Iron	Non-leaded	0.135	0.229
			Lead	0.156	0.236
	Cabled[3]	Fibre	Non-leaded	0.125	0.169
		Iron	Non-leaded	0.135	0.187
	Cabled[4]	Fibre	Non-leaded	0.136	0.144
		Iron	Non-leaded	0.144	0.159
0000 AWG (2 per phase)	Uncabled[5]	Fibre	Non-leaded	0.135	0.101
		Iron	Non-leaded	0.144	0.152
			Lead	0.143	0.113
	Cabled[6]	Fibre	Non-leaded	0.137	0.079
		Iron	Non-leaded	0.137	0.085

Zero-Sequence Impedance, Ohms Per Phase Per Mile at 60 Cycles.

Phase Conductor Size	Neutral Conductor Size	Conductor Assembly	Duct Material (4 inch)	Cable Sheath (Phase Conductors)	Resistance (Ohms at 25°C.)	Reactance (Ohms)
500 MCM (1 per phase)	0000 AWG (1 conductor, bare)	Uncabled[2]	Fibre	Non-leaded	0.972	0.814
				Lead	0.777	0.380
			Iron	Lead	0.729	0.349
	500 MCM (1 conductor, bare)	Uncabled[2]	Iron	Non-leaded	0.539	0.772
		Cabled[3]	Fibre	Non-leaded	0.539	0.566
			Iron	Non-leaded	0.534	0.603
	000 AWG (3 conductors, bare)	Cabled[4]	Fibre	Non-leaded	0.471	0.211
			Iron	Non-leaded	0.433	0.264
0000 AWG (2 per phase)	0000 AWG (1 conductor, bare)	Uncabled[5]	Fibre	Non-leaded	1.015	0.793
			Iron	Non-leaded	0.707	0.676
				Lead	0.693	0.328
	0000 AWG (2 conductors, bare)	Uncabled[5]	Fibre	Non-leaded	0.583	0.475
			Iron	Non-leaded	0.629	0.538
	500 MCM (1 conductor, bare)	Cabled[6]	Iron	Non-leaded	0.497	0.359

[1] Material taken from "Impedance of Three-Phase Secondary Mains in Non-Metallic and Iron Conduits," by L. Brieger, EEI Bulletin, Vol. 6, No. 2, pg. 61, February 1938.

[2] Assembly of four conductors arranged rectangularly, in the sequence (clockwise) A-B-C-neutral, while being pulled into the duct; conductors may assume a random configuration after entering the duct.

[3] Assembly as in note 2, except that conductors are cabled in position.

[4] Assembly of three phase conductors arranged triangularly with three neutral conductors interposed in the spaces between phase conductors. All conductors are cabled in position.

[5] Assembly of six phase conductors arranged hexagonally, in the sequence A-B-C-A-B-C, with either one or two neutral conductors inside the phase conductor group. This arrangement is maintained only at the duct entrance; a random configuration may develop within the duct.

[6] Assembly as in note 5, except that conductors are cabled in position.

where:

$GMD_{3c} = S =$ geometric mean distance among three conductors, and the remaining values are as defined in Eqs. (12) and (13).

For sector-shaped conductors no accurate data on change in reactance because of conductor shape is available, but Dr. Simmons can be quoted as authority for the statement that the reactance is from five to ten percent less than for round conductors of the same area and insulation thickness.

For shielded three-conductor cables the reactance to positive- and negative-sequence currents can be calculated as though the shields were not present, making it similar to belted three-conductor cable. This is true because the effect on reactance of the circulating currents in the shielding tapes has been calculated by the method used for determining sheath effects in single-conductor cables and proves to be negligible.

Three Conductors in Steel Pipe—Conductor skin effect and proximity effects influence the apparent reactance of high-voltage cables in steel pipe. Because the detailed calculation of these factors is complex, a curve is supplied in Fig. 14 that serves for estimating reactance within about five percent accuracy. The curve is drawn for triangular conductor grouping, with the group lying at the bottom of the pipe. If the grouping is instead a flat cradled arrangement, with the conductors lying side by-side at the bottom of the pipe, the curve results should be increased by 15 percent. A calculating method that accounts in detail for the variable factors in this problem has been presented by Del Mar[7]. Table 2 contains information[14] useful in estimating the impedance of low-voltage (120/208 volt) cables in iron conduit.

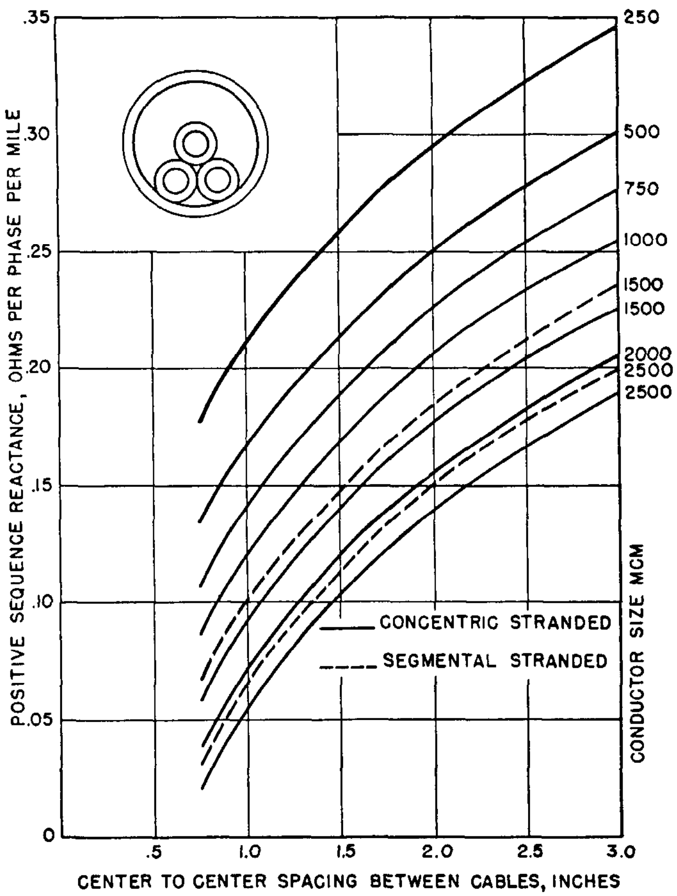


Fig. 14—Positive-sequence reactance of high-voltage cables in steel pipe (estimating curve).

4. Zero-Sequence Resistance and Reactance

When zero-sequence current flows along the phase conductors of a three-phase cable circuit, it must return in either the ground, or the sheaths, or in the parallel combination of both ground and sheaths.[2] As zero-sequence current flows through each conductor it encounters the a-c resistance of that conductor, and as it returns in the ground or sheaths it encounters the resistance of those paths. The zero-sequence current flowing in any one phase encounters also the reactance arising from conductor self-inductance, from mutual inductance to the other two phase conductors, from mutual inductance to the ground and sheath return paths, and from self-inductance of the return paths. Each of these inductive effects cannot always be identified individually from the equations to be used for reactance calculations because the theory of earth return circuits[3], and the use of one GMR to represent a paralleled conductor group, present in combined form some of the fundamental effects contributing to total zero-sequence reactance. The resistance and reactance effects are interrelated so closely that they are best dealt with simultaneously.

Cable sheaths are frequently bonded and grounded at several points, which allows much of the zero-sequence return current to flow in the sheath. On the other hand, when any of the various devices used to limit sheath current are employed, much or all of the return current flows in the earth. The method of bonding and grounding, therefore, has an effect upon the zero-sequence impedance of cables. An actual cable installation should approach one of these three theoretical conditions:

1 Return current in sheath and ground in parallel.
2 All return current in sheath, none in ground.
3 All return current in ground, none in sheath.

Three-Conductor Cables—Actual and equivalent circuits for a single-circuit three-conductor cable having a solidly bonded and grounded sheath are shown in Fig. 15 (a) and (c). The impedance of the group of three paralleled conductors, considering the presence of the earth return but ignoring for the moment the presence of the sheath, is given in Eqs. (16) or (17) in terms of impedance to zero-sequence currents.

$$z_c = r_c + r_e + j0.8382\,\frac{f}{60}\log_{10}\frac{D_e}{\mathrm{GMR}_{3c}}$$
ohms per phase per mile (16)

or

$$z_c = r_c + r_e + j(x_a + x_e - 2x_d)$$
ohms per phase per mile. (17)

TABLE 3—EQUIVALENT DEPTH OF EARTH RETURN (D_e), AND EARTH IMPEDANCE (r_e AND x_e), AT 60 CYCLES

Earth Resistivity (meter-ohm)	Equivalent Depth of Earth Return, D_e inches	Equivalent Depth of Earth Return, D_e feet	Equivalent Earth Resistance r_e (ohms per mile)	Equivalent Earth Reactance x_e (ohms per mile)
1	3.36x10³	280	0.286	2.05
5	7.44x10³	620	0.286	2.34
10	1.06x10⁴	880	0.286	2.47
50	2.40x10⁴	2 000	0.286	2.76
100	3.36x10⁴	2 800	0.286	2.89
500	7.44x10⁴	6 200	0.286	3.18
1 000	1.06x10⁵	8 800	0.286	3.31
5 000	2.40x10⁵	20 000	0.286	3.60
10 000	3.36x10⁵	28 000	0.286	3.73

where:

r_c = a-c resistance of one conductor, ohms per mile.

r_e = a-c resistance of earth return (See Table 3), ohms per mile.

D_e = distance to equivalent earth return path, (See Table 3), inches.

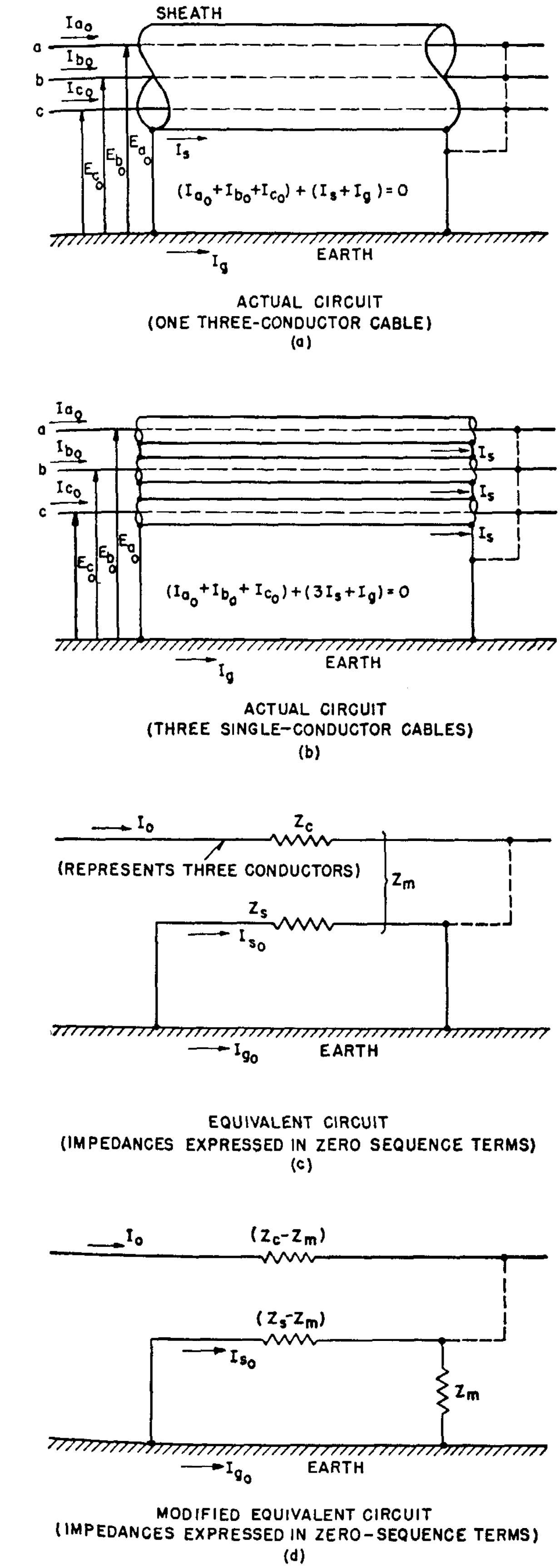


Fig. 15—Actual and equivalent zero-sequence circuits for three-conductor and single-conductor lead-sheathed cables.

GMR_{3c} = geometric mean radius of the conducting path made up of the three actual conductors taken as a group, inches.

$= \sqrt[3]{(GMR_{1c})(S)^2}$ for round conductors. (18)

GMR_{1c} = geometric mean radius of an individual conductor, inches.

x_a = reactance of an individual phase conductor at twelve inch spacing, ohms per mile.

x_e = reactance of earth return.

$=0.8382\,\frac{f}{60}\log_{10}\frac{D_e}{12}$ ohms per mile. (Refer to Table 3). (19)

$$x_d=0.2794\,\frac{f}{60}\log_{10}\left(\frac{GMD_{3c}}{12}\right),\ \text{ohms per mile.}$$

GMD_{3c} = geometric mean distance among conductor centers, inches.

$=S=(d+2T)$ for round conductors in three conductor cables.

The impedance of the sheath, considering the presence of the earth return path but ignoring for the moment the presence of the conductor group, is given in terms of impedance to zero-sequence currents:

$$z_s=3r_s+r_e+j0.8382\,\frac{f}{60}\log_{10}\frac{2D_e}{r_o+r_i}\ \text{ohms per phase per mile.} \quad (20)$$

or

$$z_s=3r_s+r_e+j(3x_s+x_e)\ \text{ohms per phase per mile.} \quad (21)$$

where:

r_s = sheath resistance, ohms per mile.

$=\frac{0.200}{(r_o+r_i)(r_o-r_i)}$ for lead sheaths.

r_i = inside radius of sheath, inches.

r_o = outside radius of sheath, inches.

x_s = reactance of sheath, ohms per mile.

$$=0.2794\,\frac{f}{60}\log_{10}\frac{24}{r_o+r_i}\ \text{ohms per mile.} \quad (22)$$

The mutual impedance between conductors and sheath, considering the presence of the earth return path which is common to both sheath and conductors, in zero-sequence terms is

$$z_m=r_e+j0.8382\,\frac{f}{60}\log_{10}\frac{2D_e}{r_o+r_i}\ \text{ohms per phase per mile.} \quad (23)$$

or

$$z_m=r_e+j(3x_s+x_e)\ \text{ohms per phase per mile.} \quad (24)$$

The equivalent circuit in Fig. 15(d) is a conversion from the one just above it, and combines the mutual impedance into a common series element. From this circuit, when both ground and sheath return paths exist, total zero-sequence impedance is:

$$z_0=(z_c-z_m)+\frac{(z_s-z_m)z_m}{z_s}$$

$$=z_c-\frac{z_m^2}{z_s}\ \text{ohms per phase per mile.} \quad (25)$$

If current returns in the sheath only, with none in the ground:

$$z_0=(z_c-z_m)+(z_s-z_m)$$
$$=z_c+z_s-2z_m \quad (26)$$
$$=r_c+3r_s+j0.8382\frac{f}{60}\log_{10}\frac{r_o+r_i}{2(\text{GMR}_{3c})}\text{ ohms per phase per mile.} \quad (27)$$
$$=r_c+3r_s+j(x_a-2x_d-3x_s)\text{ ohms per phase per mile.} \quad (28)$$

If current returns in ground only with none in the sheath, as would be the case with non-sheathed cables or with insulating sleeves at closely spaced intervals, the zero-sequence impedance becomes:

$$z_0=(z_c-z_m)+z_m$$
$$=z_c\text{ ohms per phase per mile.} \quad (29)$$

The zero-sequence impedance of shielded cables can be calculated as though the shielding tapes were not present because the impedance is affected only slightly by circulating currents in the shields.

The equivalent geometric mean radius (GMR_{3c}) for three-conductor cables having sector conductors is difficult to calculate accurately. The method used to calculate values of GMR_{3c} for the tables of characteristics is of practical accuracy, but is not considered to be appropriate for explanation here. As an alternate basis for estimations, it appears that the GMR_{3c} for three sector-conductors is roughly 90 percent of the GMR_{3c} for three round conductors having the same copper area and the same insulation thickness.

Example 2—Find the zero-sequence impedance of a three-conductor belted cable, No. 2 A.W.G. conductor (7 strands) with conductor diameter of 0.292 inches. Conductor insulation thickness is 156 mils, belt insulation is 78 mils, lead sheath thickness is 109 mils, and overall cable diameter is 1.732 inches. Assume $D_e=2800$ feet and resistance of one conductor $=0.987$ ohms per mile at 60 cycles. Distance between conductor centers is:

$$S=0.292+2\times0.156=0.604\text{ inches.}$$

GMR of one conductor is (see Chap. 3, Fig. 11):

$$\text{GMR}_{1c}=0.726\times0.146=0.106\text{ inches.}$$

GMR of three conductors is:

$$\text{GMR}_{3c}=\sqrt[3]{(0.106)(0.604)^2}=0.338\text{ inches.}$$

The conductor component of impedance is ($r_c=0.987$, $r_e=0.286$):

$$z_c=0.987+0.286+j0.8382\log_{10}\frac{2800\times12}{0.338}$$
$$=1.27+j4.18=4.37\text{ ohms per mile.}$$

This would represent total zero-sequence circuit impedance if all current returned in the ground, and none in the sheath.

For the sheath component of impedance:

$$r_s=\frac{0.200}{(1.623)(0.109)}=1.13\text{ ohms per mile}$$

$$z_s=3\times1.13+0.286+j0.8382\log_{10}\frac{2\times2800\times12}{1.623}$$
$$=3.68+j3.87\text{ ohms per mile}$$

The mutual component of impedance is:

$$z_m=0.286+j3.87$$

If all current returned the sheath, and none in the ground,

$$z_0=1.27+j4.18+3.68+j3.87-0.57-j7.74$$
$$=4.38+j0.31=4.39\text{ ohms per mile.}$$

If return current may divide between the ground and sheath paths,

$$z_0=1.27+j4.18-\frac{(0.286+j3.87)^2}{3.68+j3.87}$$
$$=1.27+j4.18+1.623-j2.31$$
$$=2.89+j1.87=3.44\text{ ohms per mile.}$$

The positive-sequence impedance of this cable is:

$$z_1=0.987+j0.203\text{ ohms per mile.}$$

Therefore the ratio of zero- to positive-sequence resistance is 2.9, and the ratio of zero- to positive-sequence reactance is 9.2.

Zero-sequence impedance is often calculated for all return current in the sheath and none in the ground, because the magnitude of the answer is usually close to that calculated considering a paralleled return. The actual nature of a ground-return circuit is usually indefinite, since it may be mixed up with water pipes and other conducting materials, and also because low-resistance connections between sheath and earth are sometimes difficult to establish.

Single-Conductor Cables—Fig. 15 also shows the actual and equivalent circuits for three single-conductor cables in a perfectly transposed three-phase circuit, where the sheaths are solidly bonded and grounded. The impedance expressions applying to single-conductor cables differ in some respects from those for three-phase cables:

$$z_c=r_c+r_e+j0.8382\frac{f}{60}\log_{10}\frac{D_e}{\text{GMR}_{3c}}\text{ ohms per phase per mile.} \quad (30)$$

or

$$z_c=r_c+r_e+j(x_a+x_e-2x_d)\text{ ohms per phase per mile.} \quad (31)$$

where:

r_c = a.c. resistance of one conductor, ohms per mile.

r_e = a.c. resistance of earth (see Table 3), ohms per mile.

D_e = distance to equivalent earth return path (see Table 3), inches.

GMR_{3c} = geometric mean radius of the conducting path made up of the three actual conductors taken as a group, inches.
$=\sqrt[3]{(\text{GMR}_{1c})(\text{GMD}_{3c})^2}$

x_a = reactance of an individual phase conductor at twelve-inch spacing, ohms per mile.

x_e = reactance of earth return.
$=0.8382\frac{f}{60}\log_{10}\frac{D_e}{12}$ ohms per mile.
(See Table 3.)

$x_d=0.2794\frac{f}{60}\log_{10}\left(\frac{\text{GMD}_{3c}}{12}\right)$, ohms per mile.

GMD_{3c} = geometric mean distance among conductor centers, inches.

$= \sqrt[3]{S_{ab} \cdot S_{bc} \cdot S_{ca}}$.

$$z_s = r_s + r_e + j0.8382 \frac{f}{60} \log_{10} \frac{D_e}{GMR_{3s}} \text{ ohms per phase per mile} \quad (32)$$

or

$$z_s = r_s + r_e + j(x_s + x_e - 2x_d) \text{ ohms per phase per mile} \quad (33)$$

where:

GMR_{3s} = geometric mean radius of the conducting path made up of the three sheaths in parallel

$= \sqrt[3]{\left(\frac{r_o + r_i}{2}\right)\left(GMD_{3c}\right)^2}$.

r_s = resistance of one sheath, ohms per mile

$= \frac{0.200}{(r_o + r_i)(r_o - r_i)}$ for lead sheaths.

r_i = inside radius of sheath, inches.

r_o = outside radius of sheath, inches.

x_s = reactance of one sheath, ohms per mile

$= 0.2794 \frac{f}{60} \log_{10} \frac{24}{r_o + r_i}$.

$$z_m = r_e + j0.8382 \frac{f}{60} \log_{10} \frac{D_e}{GMD_{3c-3s}} \text{ ohms per phase per mile.} \quad (34)$$

or

$$z_m = r_e + j(x_e + x_s - 2x_d) \text{ ohms per phase per mile.} \quad (35)$$

where:

GMD_{3c-3s} = geometric mean of all separations between sheaths and conductors.

$= \sqrt[9]{\left(\frac{r_o + r_i}{2}\right)^3 (GMD_{3c})^6} = \sqrt[3]{\left(\frac{r_o + r_i}{2}\right)(GMD_{3c})^2}$.

From the equivalent circuit of Fig. 15, total zero-sequence impedance when both ground and sheath paths exist is:

$$z_0 = z_c - \frac{z_m^2}{z_s} \text{ ohms per phase per mile.} \quad (25)$$

If current returns in the sheath only, with none in the ground:

$$z_0 = z_c + z_s - 2z_m \text{ ohms per phase per mile} \quad (26)$$

$$= r_c + r_s + 0.8382 \log_{10} \frac{GMR_{3s}}{GMR_{3c}} \text{ ohms per phase per mile.} \quad (36)$$

$$= r_c + r_s + j(x_a - x_s) \text{ ohms per phase per mile.} \quad (37)$$

If current returns in the ground only:

$$z_0 = (z_c - z_m) + z_m$$
$$= z_c \text{ ohms per phase per mile.} \quad (29)$$

Cables in Steel Pipes or Conduits—When cables are installed in iron conduits or steel pipes, the zero-sequence resistance and reactance are affected by the magnetic material because it closely surrounds the phase conductors and forms a likely return path for zero-sequence current. No method of calculating this zero-sequence impedance is available, but some rather complete results are available from field tests on installed low-voltage cables, as shown in Table 2. Some special tests of the zero-sequence impedance of high-voltage pipe-type cable have been made but the results are not yet of a sufficiently wide scope to be generally usable.

5. Shunt Capacitive Reactance

Shunt capacitive reactances of several types of cables are given in the Tables of Electrical Characteristics, directly in ohms per mile. In addition, shunt capacitance and charging current can be derived from the curves of geometric factors shown in Figs. 8 and 9, for any cable whose dimensions are known. The geometric factors given in these curves are identified by symmetrical-component terminology.

The positive-, negative-, and zero-sequence shunt capacitances for single-conductor metallic-sheathed cables are all equal, and can be derived from the curves of Fig. 8. Three-conductor shielded cables having round conductors are similar to single-conductor cable in that each phase conductor is surrounded by a grounded metallic covering; therefore the positive-, negative-, and zero-sequence values are equal and are dependent upon the geometric factor relating a conductor to its own shielding layer. The geometric factor for three-conductor shielded cables having sector-shaped conductors is approximately equal to the geometric factor, G, applying to round conductors. However, if the sector shape of a shielded cable is known, then the curve in Fig. 10, based on insulation thickness and mean periphery of insulation, is recommended as giving more accurate values of geometric factor.

For single-conductor and three-conductor shielded cables (see Fig. 8),

$$C_1 = C_2 = C_0 = \frac{0.0892k}{G} \text{ microfarads per phase per mile.} \quad (38)$$

$$x_{1'} = x_{2'} = x_{0'} = \frac{1.79G}{f \cdot k} \text{ megohms per phase per mile.} \quad (39)$$

$$I_{1'} = I_{2'} = I_{0'} = \frac{0.323f \cdot k \cdot kv}{1000G} \text{ amperes per phase per mile.} \quad (40)$$

Three-conductor belted cables having no conductor shielding have zero-sequence values which differ from the positive- and negative-sequence; the appropriate geometric factors are given in Fig. 9;

$$C_1 = C_2 = \frac{0.267k}{G_1} \text{ microfarads per phase per mile.} \quad (41)$$

$$C_0 = \frac{0.0892k}{G_0} \text{ microfarads per phase per mile.} \quad (42)$$

$$x_{1'} = x_{2'} = \frac{0.597G_1}{f \cdot k}, \text{ megohms per phase per mile.} \quad (43)$$

$$x_{0'} = \frac{1.79G_0}{f \cdot k} \text{ megohms per phase per mile.} \quad (44)$$

$$I_{1'} = I_{2'} = \frac{0.97f \cdot k \cdot kv}{1000G_1} \text{ amperes per phase per mile.} \quad (45)$$

$$I_{0'} = \frac{0.323f \cdot k \cdot kv}{1000G_0} \text{ amperes per phase per mile.} \quad (46)$$

When three-conductor belted cables have sector-shaped conductors, the geometric factor must be corrected from the value which applies to round conductors. This correction factor is plotted in Fig. 9, and its use is explained below the curve.

In the foregoing equations,

C_1, C_2, and C_0 are positive-, negative-, and zero-sequence capacitances.

$x_{1'}$, $x_{2'}$ and $x_{0'}$ are positive-, negative-, and zero-sequence capacitive reactances.

$I_{1'}$, $I_{2'}$ and $I_{0'}$ are positive-, negative-, and zero-sequence charging currents.

kv = line-to-line system voltage, kilovolts.

k = dielectric constant, according to the values in Table 4.

It is important to note that in converting shunt capacitive reactance from an "ohms per phase per mile" basis to a total "ohms per phase" basis, it is necessary to divide by the circuit length:

$$X_{c'} = \frac{x_{c'}}{l, \text{ length in miles}}, \text{ ohms per phase.} \qquad (47)$$

6. Insulation Resistance.

The calculation of cable insulation resistance is difficult because the properties of the insulation are generally predictable only within a wide range. The equations presented below are therefore quite dependent upon an accurate knowledge of insulation power factor.

For single-conductor and three-conductor shielded cables,

$$r_{1'} = r_{2'} = r_{0'} = \frac{1.79G}{f \cdot k \cdot \cos\phi} \cdot 10^6 \text{ ohms per phase per mile.} \qquad (48)$$

For three-conductor belted cables,

$$r_{1'} = r_{2'} = \frac{0.597G_1}{f \cdot k \cdot \cos\phi} \cdot 10^6 \text{ ohms per phase per mile.} \qquad (49)$$

$$r_{0'} = \frac{1.79G_0}{f \cdot k \cdot \cos\phi} \cdot 10^6 \text{ ohms per phase per mile.} \qquad (50)$$

In these equations,

$r_{1'}$, $r_{2'}$, and $r_{0'}$ are positive-, negative-, and zero-sequence shunt resistances.

k = dielectric constant (see Table 4).

$\cos\phi$ = power factor of insulation, in per unit.

In Table 5 are listed maximum values of insulation power factor, taken from specifications of the Association of Edison Illuminating Companies[15]. These standard values will very probably be several times larger than actual measured power factors on new cables.

TABLE 4—DIELECTRIC CONSTANTS OF CABLE INSULATION

Insulation	Range of k	Typical k
Solid Paper	3.0–4.0	3.7
Oil-Filled	3.0–4.0	3.5
Gas-Filled	3.0–4.0	3.7
Varnished Cambric	4.0–6.0	5.0
Rubber	4.0–9.0	6.0

TABLE 5—MAXIMUM POWER FACTORS* OF CABLE INSULATION

Temperature of Cable (Deg. C.)	Solid Paper	Oil-Filled (low-pressure)	Gas-Filled (low-pressure)
25 to 60	0.009	0.0060	0.009
70	0.015	0.0075	0.013
80	0.021	0.0090	0.018
85	0.025	0.0097	0.022
90	0.030	0.0105	0.027

*The power factor of new cable is usually below these values by a wide margin.[15]

II. TABLES OF ELECTRICAL CHARACTERISTICS

The 60-cycle electrical characteristics of the most usual sizes and voltage classes of paper insulated cable are contained in Tables 6 through 11. In each case the positive-, negative-, and zero-sequence resistances and reactances are tabulated, or else constants are given from which these quantities can be calculated. Also, included in these tables are other characteristics useful in cable work, such as typical weights per 1000 feet, sheath thicknesses and resistances, conductor diameters and GMR's, and the type of conductors normally used in any particular cable.

In each of these tables the electrical characteristics have been calculated by the equations and curves presented in the foregoing pages. Where sector-shaped conductors are used, some approximations are necessary as pointed out previously. In Table 6 the positive- and negative-sequence reactance for sectored cables has arbitrarily been taken 7.5 percent less than that of an equivalent round-conductor cable, in accordance with Dr. Simmons' recommendations. The equivalent GMR of three conductors in sectored cables is necessarily an approximation because the GMR of one sector cannot be determined accurately. This condition arises since the shape of sectors varies and a rigorous calculation is not justified. The variation in sector shapes probably is greater than any error present in the approximation given in the tables. The reactances calculated from these approximate GMR's are sufficiently accurate for all practical calculations.

Table 7 for shielded cables is similar in form to Table 6 and where sectored cables are listed the same approximations in GMR and reactance apply. Table 8 for three-conductor oil-filled cables is similar to both Tables 6 and 7 and the same considerations apply.

In these tables for three-conductor cables, the zero-sequence characteristics are calculated for the case of all return current in the sheath and none in the ground. As pointed out in the discussion of zero-sequence impedance, this is usually sufficiently accurate because of the indefinite nature of the ground return circuit. Where ground must be considered or where there are paralleled three-phase circuits, the impedance must be calculated as illustrated in the examples given.

From the quantities given in these tables of three-conductor cables, the overall diameter of any particular cable can be calculated.

$$D = 2.155(d+2T)+2(t+L) \qquad (51)$$

TABLE 6—60-CYCLE CHARACTERISTICS OF **THREE-CONDUCTOR BELTED** PAPER-INSULATED CABLES

Grounded Neutral Service

Voltage Class	Insulation Thickness Mils: Conductor	Insulation Thickness Mils: Belt	Circular Mils or AWG (B. & S.)	Type of (6) Conductor	Weight per 1000 Feet	Diameter or Sector Depth (5) inches	Resistance, Ohms Per Mile (1)	GMR of One Conductor—inches (2)	Positive & Negative—Seq.: Series Reactance Ohms per Mile	Positive & Negative—Seq.: Shunt Capacitive Reactance Ohms per Mile (3)	GMR—Three Conductors	Zero—Sequence: Series Resistance Ohms per Mile (4)	Zero—Sequence: Series Reactance Ohms per Mile (4)	Zero—Sequence: Shunt Capacitive Reactance Ohms per Mile (3)	Sheath: Thickness Mils	Sheath: Resistance, Ohms per Mile at 50°C
1 Kv	60	35	6	SR	1 500	0.184	2.50	0.067	0.185	6300	0.184	10.66	0.315	11 600	85	2.69
	60	35	4	SR	1 910	0.232	1.58	0.084	0.175	5400	0.218	8.39	0.293	10 200	90	2.27
	60	35	2	SR	2 390	0.292	0.987	0.106	0.165	4700	0.262	6.99	0.273	9 000	90	2.00
	60	35	1	SR	2 820	0.332	0.786	0.126	0.155	4300	0.295	6.07	0.256	8 400	95	1.76
	60	35	0	SR	3 210	0.373	0.622	0.142	0.152	4000	0.326	5.54	0.246	7 900	95	1.64
	60	35	00	CS	3 160	0.323	0.495	0.151	0.138	2800	0.290	5.96	0.250	5 400	95	1.82
	60	35	000	CS	3 650	0.364	0.392	0.171	0.134	2300	0.320	5.46	0.241	4 500	95	1.69
	60	35	0000	CS	4 390	0.417	0.310	0.191	0.131	2000	0.355	4.72	0.237	4 000	100	1.47
	60	35	250 000	CS	4 900	0.455	0.263	0.210	0.129	1800	0.387	4.46	0.224	3 600	100	1.40
	60	35	300 000	CS	5 660	0.497	0.220	0.230	0.128	1700	0.415	3.97	0.221	3 400	105	1.25
	60	35	350 000	CS	6 310	0.539	0.190	0.249	0.126	1500	0.446	3.73	0.216	3 100	105	1.18
	60	35	400 000	CS	7 080	0.572	0.166	0.265	0.124	1500	0.467	3.41	0.214	2 900	110	1.08
	60	35	500 000	CS	8 310	0.642	0.134	0.297	0.123	1300	0.517	3.11	0.208	2 600	110	0.993
	65	40	600 000	CS	9 800	0.700	0.113	0.327	0.122	1200	0.567	2.74	0.197	2 400	115	0.877
	65	40	750 000	CS	11 800	0.780	0.091	0.366	0.121	1100	0.623	2.40	0.194	2 100	120	0.771
3 Kv	70	40	6	SR	1 680	0.184	2.50	0.067	0.192	6700	0.192	9.67	0.322	12 500	90	2.39
	70	40	4	SR	2 030	0.232	1.58	0.084	0.181	5800	0.227	8.06	0.298	11 200	90	2.16
	70	40	2	SR	2 600	0.292	0.987	0.106	0.171	5100	0.271	6.39	0.278	9 800	95	1.80
	70	40	1	SR	2 930	0.332	0.786	0.126	0.161	4700	0.304	5.83	0.263	9 200	95	1.68
	70	40	0	SR	3 440	0.373	0.622	0.142	0.156	4400	0.335	5.06	0.256	8 600	100	1.48
	70	40	00	CS	3 300	0.323	0.495	0.151	0.142	3500	0.297	5.69	0.259	6 700	95	1.73
	70	40	000	CS	3 890	0.364	0.392	0.171	0.138	2700	0.329	5.28	0.246	5 100	95	1.63
	70	40	0000	CS	4 530	0.417	0.310	0.191	0.135	2400	0.367	4.57	0.237	4 600	100	1.42
	70	40	250 000	CS	5 160	0.455	0.263	0.210	0.132	2100	0.396	4.07	0.231	4 200	105	1.27
	70	40	300 000	CS	5 810	0.497	0.220	0.230	0.130	1900	0.424	3.82	0.228	3 800	105	1.20
	70	40	350 000	CS	6 470	0.539	0.190	0.249	0.129	1800	0.455	3.61	0.219	3 700	105	1.14
	70	40	400 000	CS	7 240	0.572	0.166	0.265	0.128	1700	0.478	3.32	0.218	3 400	110	1.05
	70	40	500 000	CS	8 660	0.642	0.134	0.297	0.126	1500	0.527	2.89	0.214	3 000	115	0.918
	75	40	600 000	CS	9 910	0.700	0.113	0.327	0.125	1400	0.577	2.68	0.210	2 800	115	0.855
	75	40	750 000	CS	11 920	0.780	0.091	0.366	0.123	1300	0.633	2.37	0.204	2 500	120	0.758
5 Kv	105	55	6	SR	2 150	0.184	2.50	0.067	0.215	8500	0.218	8.14	0.342	15 000	95	1.88
	100	55	4	SR	2 470	0.232	1.58	0.084	0.199	7600	0.250	6.86	0.317	13 600	95	1.76
	95	50	2	SR	2 900	0.292	0.987	0.106	0.184	6100	0.291	5.88	0.290	11 300	95	1.63
	90	45	1	SR	3 280	0.332	0.786	0.126	0.171	5400	0.321	5.23	0.270	10 200	100	1.48
	90	45	0	SR	3 660	0.373	0.622	0.142	0.165	5000	0.352	4.79	0.259	9 600	100	1.39
	85	45	00	CS	3 480	0.323	0.495	0.151	0.148	3600	0.312	5.42	0.263	9 300	95	1.64
	85	45	000	CS	4 080	0.364	0.392	0.171	0.143	3200	0.343	4.74	0.254	6 700	100	1.45
	85	45	0000	CS	4 720	0.417	0.310	0.191	0.141	2800	0.380	4.33	0.245	8 300	100	1.34
	85	45	250 000	CS	5 370	0.455	0.263	0.210	0.138	2600	0.410	3.89	0.237	7 800	105	1.21
	85	45	300 000	CS	6 050	0.497	0.220	0.230	0.135	2400	0.438	3.67	0.231	7 400	105	1.15
	85	45	350 000	CS	6 830	0.539	0.190	0.249	0.133	2200	0.470	3.31	0.225	7 000	110	1.04
	85	45	400 000	CS	7 480	0.572	0.166	0.265	0.131	2000	0.493	3.17	0.221	6 700	110	1.00
	85	45	500 000	CS	8 890	0.642	0.134	0.297	0.129	1800	0.542	2.79	0.216	6 200	115	0.885
	85	45	600 000	CS	10 300	0.700	0.113	0.327	0.128	1600	0.587	2.51	0.210	5 800	120	0.798
	85	45	750 000	CS	12 340	0.780	0.091	0.366	0.125	1500	0.643	2.21	0.206	5 400	125	0.707
8 Kv	130	65	6	SR	2 450	0.184	2.50	0.067	0.230	9600	0.236	7.57	0.353	16 300	95	1.69
	125	65	4	SR	2 900	0.232	1.58	0.084	0.212	8300	0.269	6.08	0.329	14 500	100	1.50
	115	60	2	SR	3 280	0.292	0.987	0.106	0.193	6800	0.307	5.25	0.302	12 500	100	1.42
	110	55	1	SR	3 560	0.332	0.786	0.126	0.179	6100	0.338	4.90	0.280	11 400	100	1.37
	110	55	0	SR	4 090	0.373	0.622	0.142	0.174	5700	0.368	4.31	0.272	10 700	105	1.23
	105	55	00	CS	3 870	0.323	0.495	0.151	0.156	4300	0.330	4.79	0.273	8 300	100	1.43
	105	55	000	CS	4 390	0.364	0.392	0.171	0.151	3800	0.362	4.41	0.263	7 400	100	1.34
	105	55	0000	CS	5 150	0.417	0.310	0.191	0.147	3500	0.399	3.88	0.254	6 600	105	1.19
	105	55	250 000	CS	5 830	0.455	0.263	0.210	0.144	3200	0.428	3.50	0.246	6 200	110	1.08
	105	55	300 000	CS	6 500	0.497	0.220	0.230	0.141	2900	0.458	3.31	0.239	5 600	110	1.03
	105	55	350 000	CS	7 160	0.539	0.190	0.249	0.139	2700	0.489	3.12	0.233	5 200	110	0.978
	105	55	400 000	CS	7 980	0.572	0.166	0.265	0.137	2500	0.513	2.86	0.230	4 900	115	0.899
	105	55	500 000	CS	9 430	0.642	0.134	0.297	0.135	2200	0.563	2.53	0.224	4 300	120	0.800
	105	55	600 000	CS	10 680	0.700	0.113	0.327	0.132	2000	0.606	2.39	0.218	3 900	120	0.758
	105	55	750 000	CS	12 740	0.780	0.091	0.366	0.129	1800	0.663	2.11	0.211	3 500	125	0.673
15 Kv	170	85	2	SR	4 350	0.292	0.987	0.106	0.217	8600	0.349	4.20	0.323	15 000	110	1.07
	165	80	1	SR	4 640	0.332	0.786	0.126	0.202	7800	0.381	3.88	0.305	13 800	110	1.03
	160	75	0	SR	4 990	0.373	0.622	0.142	0.193	7100	0.409	3.62	0.288	12 800	110	1.00
	155	75	00	SR	5 600	0.419	0.495	0.159	0.185	6500	0.439	3.25	0.280	12 000	115	0.918
	155	75	000	SR	6 230	0.470	0.392	0.178	0.180	6000	0.476	2.99	0.272	11 300	115	0.867
	155	75	0000	SR	7 180	0.528	0.310	0.200	0.174	5600	0.520	2.64	0.263	10 600	120	0.778
	155	75	250 000	SR	7 840	0.575	0.263	0.218	0.168	5300	0.555	2.50	0.256	10 200	120	0.744
	155	75	300 000	CS	7 480	0.497	0.220	0.230	0.155	5400	0.507	2.79	0.254	7 900	115	0.855
	155	75	350 000	CS	8 340	0.539	0.190	0.249	0.152	5100	0.536	2.54	0.250	7 200	120	0.784
	155	75	400 000	CS	9 030	0.572	0.166	0.265	0.149	4900	0.561	2.44	0.245	6 900	120	0.758
	155	75	500 000	CS	10 550	0.642	0.134	0.297	0.145	4600	0.611	2.26	0.239	6 200	125	0.680
	155	75	600 000	CS	12 030	0.700	0.113	0.327	0.142	4300	0.656	1.97	0.231	5 700	130	0.620
	155	75	750 000	CS	14 190	0.780	0.091	0.366	0.139	4000	0.712	1.77	0.226	5 100	135	0.558

[1]A-c resistance based upon 100% conductivity at 65°C. including 2% allowance for stranding.
[2]GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.
[3]For dielectric constant=3.7.
[4]Based upon all return current in the sheath; none in ground.
[5]See Fig. 7.
[6]The following symbols are used to designate the cable types; SR—Stranded Round; CS—Compact Sector.

TABLE 7—60-CYCLE CHARACTERISTICS OF **THREE-CONDUCTOR SHIELDED** PAPER-INSULATED CABLES

Grounded Neutral Service

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B. & S.)	Type of Conductor (6)	Weight per 1000 Feet	Diameter or Sector Depth (5) inches	Resistance—Ohms per Mile (1)	GMR of one Conductor (2) inches	POSITIVE & NEGATIVE SEQUENCE: Series Reactance Ohms per Mile	POSITIVE & NEGATIVE SEQUENCE: Shunt-Capacitive Reactance—Ohms per Mile (3)	GMR—Three Conductors	ZERO—SEQUENCE: Series Resistance Ohms per Mile (4)	ZERO—SEQUENCE: Series Reactance Ohms per Mile (4)	ZERO—SEQUENCE: Shunt-Capacitive Reactance—Ohms per Mile (3)	SHEATH: Thickness Mils	SHEATH: Resistance Ohms per Mile at 50°C
15 Kv	205	4	SR	3 860	0.232	1.58	0.084	0.248	8200	0.328	5.15	0.325	8200	105	1.19
	190	2	SR	4 260	0.292	0.987	0.106	0.226	6700	0.365	4.44	0.298	6700	105	1.15
	185	1	SR	4 740	0.332	0.786	0.126	0.210	6000	0.398	3.91	0.285	6000	110	1.04
	180	0	SR	5 090	0.373	0.622	0.141	0.201	5400	0.425	3.65	0.275	5400	110	1.01
	175	00	CS	4 790	0.323	0.495	0.151	0.178	5200	0.397	3.95	0.268	5200	105	1.15
	175	000	CS	5 510	0.364	0.392	0.171	0.170	4800	0.432	3.48	0.256	4800	110	1.03
	175	0000	CS	6 180	0.417	0.310	0.191	0.166	4400	0.468	3.24	0.249	4400	110	0.975
	175	250 000	CS	6 910	0.455	0.263	0.210	0.158	4100	0.498	2.95	0.243	4100	115	0.897
	175	300 000	CS	7 610	0.497	0.220	0.230	0.156	3800	0.530	2.80	0.237	3800	115	0.860
	175	350 000	CS	8 480	0.539	0.190	0.249	0.153	3600	0.561	2.53	0.233	3600	120	0.783
	175	400 000	CS	9 170	0.572	0.166	0.265	0.151	3400	0.585	2.45	0.228	3400	120	0.761
	175	500 000	CS	10 710	0.642	0.134	0.297	0.146	3100	0.636	2.19	0.222	3100	125	0.684
	175	600 000	CS	12 230	0.700	0.113	0.327	0.143	2900	0.681	1.98	0.215	2900	130	0.623
	175	750 000	CS	14 380	0.780	0.091	0.366	0.139	2600	0.737	1.78	0.211	2600	135	0.562
23 Kv	265	2	SR	5 590	0.292	0.987	0.106	0.250	8300	0.418	3.60	0.317	8300	115	0.870
	250	1	SR	5 860	0.332	0.786	0.126	0.232	7500	0.450	3.26	0.298	7500	115	0.851
	250	0	SR	6 440	0.373	0.622	0.141	0.222	6800	0.477	2.99	0.290	6800	120	0.788
	240	00	CS	6 060	0.323	0.495	0.151	0.196	6600	0.446	3.16	0.285	6600	115	0.890
	240	000	CS	6 620	0.364	0.392	0.171	0.188	6000	0.480	2.95	0.285	6000	115	0.851
	240	0000	CS	7 480	0.410	0.310	0.191	0.181	5600	0.515	2.64	0.268	5600	120	0.775
	240	250 000	CS	8 070	0.447	0.263	0.210	0.177	5200	0.545	2.50	0.261	5200	120	0.747
	240	300 000	CS	8 990	0.490	0.220	0.230	0.171	4900	0.579	2.29	0.252	4900	125	0.690
	240	350 000	CS	9 720	0.532	0.190	0.249	0.167	4600	0.610	2.10	0.249	4600	125	0.665
	240	400 000	CS	10 650	0.566	0.166	0.265	0.165	4400	0.633	2.03	0.246	4400	130	0.620
	240	500 000	CS	12 280	0.635	0.134	0.297	0.159	3900	0.687	1.82	0.237	3900	135	0.562
	240	600 000	CS	13 610	0.690	0.113	0.327	0.154	3700	0.730	1.73	0.230	3700	135	0.540
	240	750 000	CS	15 830	0.767	0.091	0.366	0.151	3400	0.787	1.56	0.225	3400	140	0.488
35 Kv	355	0	SR	8 520	0.288	0.622	0.141	0.239	9900	0.523	2.40	0.330	9900	130	0.594
	345	00	SR	9 180	0.323	0.495	0.159	0.226	9100	0.548	2.17	0.322	9100	135	0.559
	345	000	SR	9 900	0.364	0.392	0.178	0.217	8500	0.585	2.01	0.312	8500	135	0.538
	345	0000	CS	9 830	0.410	0.310	0.191	0.204	7200	0.594	2.00	0.290	7200	135	0.563
	345	250 000	CS	10 470	0.447	0.263	0.210	0.197	6800	0.628	1.90	0.280	6800	135	0.545
	345	300 000	CS	11 290	0.490	0.220	0.230	0.191	6400	0.663	1.80	0.273	6400	135	0.527
	345	350 000	CS	12 280	0.532	0.190	0.249	0.187	6000	0.693	1.66	0.270	6000	140	0.491
	345	400 000	CS	13 030	0.566	0.166	0.265	0.183	5700	0.721	1.61	0.265	5700	140	0.480
	345	500 000	CS	14 760	0.635	0.134	0.297	0.177	5200	0.773	1.46	0.257	5200	145	0.441
	345	600 000	CS	16 420	0.690	0.113	0.327	0.171	4900	0.819	1.35	0.248	4900	150	0.412
	345	750 000	CS	18 860	0.767	0.091	0.366	0.165	4500	0.879	1.22	0.243	4500	155	0.377

[1]A-c resistance based on 100% conductivity at 65°C. including 2% allowance for stranding.
[2]GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.
[3]For dielectric constant=3.7.
[4]Based on all return current in the sheath; none in ground.
[5]See Fig. 7.
[6]The following symbols are used to designate conductor types: SR—Stranded Round; CS—Compact Sector.

in which, according to Fig. 6,

D = outside diameter in inches.
d = diameter of individual conductor in inches.
T = conductor insulation thickness in inches.
t = belt insulation thickness in inches (when present).
L = lead sheath thickness in inches.

This equation refers to cables with round conductors. For sectored cables there is no exact rule, but a close approximation can be obtained by using an equivalent cable with round conductors and calculating the diameter D by Eq. (11), and then subtracting 0.3 to 0.4 times the round conductor diameter d, depending upon the shape of the sector.

A set of calculated constants is given in Table 10 for single-conductor cables, from which the positive-, negative- and zero-sequence characteristics can be quickly determined by using the equations given at the foot of the tabulation. These equations are derived directly from those given for the calculation of sequence impedances in the sections under Electrical Characteristics. Since

$$x_a = 0.2794 \frac{f}{60} \log_{10} \frac{12}{GMR_{1c}} \text{ ohms per phase per mile} \quad (12)$$

$$x_s = 0.2794 \frac{f}{60} \log_{10} \frac{24}{r_o + r_i} \text{ ohms per phase per mile} \quad (22)$$

$$x_d = 0.2794 \frac{f}{60} \log_{10} \frac{S}{12} \text{ ohms per phase per mile} \quad (13)$$

and r_c and r_s are conductor and sheath resistances respectively, the derivation of the equations given with Table 10 becomes evident. Table 12 gives the one other quantity, x_d, necessary for the use of Table 10. These reactance spacing factors are tabulated for equivalent cable spacings

TABLE 8—60-CYCLE CHARACTERISTICS OF **THREE-CONDUCTOR OIL-FILLED** PAPER-INSULATED CABLES

Grounded Neutral Service

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B. & S.)	Type of Conductor (6)	Weight per 1000 Feet	Diameter or Sector Depth (5)—inches	Resistance-Ohms Per Mile (1)	GMR of One Conductor (2)—inches	POSITIVE & NEGATIVE SEQ. Series Reactance Ohms Per Mile	POSITIVE & NEGATIVE SEQ. Shunt Capacitive Reactance-Ohms Per Mile (3)	GMR—Three Conductors	ZERO—SEQUENCE Series Resistance Ohms Per Mile (4)	ZERO—SEQUENCE Series Reactance Ohms Per Mile (4)	ZERO—SEQUENCE Shunt Capacitive Reactance-Ohms Per Mile (3)	SHEATH Thickness Mils	SHEATH Resistance-Ohms Per Mile at 50°C
35 Kv	190	00	CS	5 590	0.323	0.495	0.151	0.185	6030	0.406	3.56	0.265	6030	115	1.02
		000	CS	6 150	0.364	0.392	0.171	0.178	5480	0.439	3.30	0.256	5480	115	0.970
		0000	CS	6 860	0.417	0.310	0.191	0.172	4840	0.478	3.06	0.243	4840	115	0.918
		250 000	CS	7 680	0.455	0.263	0.210	0.168	4570	0.508	2.72	0.238	4570	125	0.820
		300 000	CS	9 090	0.497	0.220	0.230	0.164	4200	0.539	2.58	0.232	4200	125	0.788
		350 000	CS	9 180	0.539	0.190	0.249	0.160	3900	0.570	2.44	0.227	3900	125	0.752
		400 000	CS	9 900	0.572	0.166	0.265	0.157	3690	0.595	2.35	0.223	3690	125	0.729
		500 000	CS	11 550	0.642	0.134	0.297	0.153	3400	0.646	2.04	0.217	3400	135	0.636
		600 000	CS	12 900	0.700	0.113	0.327	0.150	3200	0.691	1.94	0.210	3200	135	0.608
		750 000	CS	15 660	0.780	0.091	0.366	0.148	3070	0.763	1.73	0.202	3070	140	0.548
46 Kv	225	00	CS	6 360	0.323	0.495	0.151	0.195	6700	0.436	3.28	0.272	6700	115	0.928
		000	CS	6 940	0.364	0.392	0.171	0.188	6100	0.468	2.87	0.265	6100	125	0.826
		0000	CS	7 660	0.410	0.310	0.191	0.180	5520	0.503	2.67	0.256	5520	125	0.788
		250 000	CS	8 280	0.447	0.263	0.210	0.177	5180	0.533	2.55	0.247	5180	125	0.761
		300 000	CS	9 690	0.490	0.220	0.230	0.172	4820	0.566	2.41	0.241	4820	125	0.729
		350 000	CS	10 100	0.532	0.190	0.249	0.168	4490	0.596	2.16	0.237	4490	135	0.658
		400 000	CS	10 820	0.566	0.166	0.265	0.165	4220	0.623	2.08	0.232	4220	135	0.639
		500 000	CS	12 220	0.635	0.134	0.297	0.160	3870	0.672	1.94	0.226	3870	135	0.603
		600 000	CS	13 930	0.690	0.113	0.327	0.156	3670	0.718	1.74	0.219	3670	140	0.542
		750 000	CS	16 040	0.767	0.091	0.366	0.151	3350	0.773	1.62	0.213	3350	140	0.510
		1 000 000	CS												
69 Kv	315	00	CR	8 240	0.376	0.495	0.147	0.234	8330	0.532	2.41	0.290	8330	135	0.639
		000	CS	8 830	0.364	0.392	0.171	0.208	7560	0.538	2.32	0.284	7560	135	0.642
		0000	CS	9 660	0.410	0.310	0.191	0.200	6840	0.575	2.16	0.274	6840	135	0.618
		250 000	CS	10 330	0.447	0.263	0.210	0.195	6500	0.607	2.06	0.266	6500	135	0.597
		300 000	CS	11 540	0.490	0.220	0.230	0.190	6030	0.640	1.85	0.260	6030	140	0.543
		350 000	CS	12 230	0.532	0.190	0.249	0.185	5700	0.672	1.77	0.254	5700	140	0.527
		400 000	CS	13 040	0.566	0.166	0.265	0.181	5430	0.700	1.55	0.248	5430	140	0.513
		500 000	CS	14 880	0.635	0.134	0.297	0.176	5050	0.750	1.51	0.242	5050	150	0.460
		600 000	CS	16 320	0.690	0.113	0.327	0.171	4740	0.797	1.44	0.235	4740	150	0.442
		750 000	CS	18 980	0.767	0.091	0.366	0.165	4360	0.854	1.29	0.230	4360	155	0.399
		1 000 000													

[1] A-c resistance based on 100% conductivity at 65°C. including 2% allowance for stranding.
[2] GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.
[3] For dielectric constant=3.5.
[4] Based on all return current in sheath; none in ground.
[5] See Fig. 7.
[6] The following symbols are used to designate the cable types: CR—Compact Round; CS—Compact Sector.

from 0.5 to 36.0 inches, which should cover the range met in practice. For all spacings less than 12 inches, x_d is negative.

The constants calculated in this manner apply to one three-phase circuit of single-conductor lead-sheath cables, assuming all zero-sequence return current to be in the sheaths, none in the ground.

The 60-cycle characteristics of single-conductor oil-filled cables are given in Table 11. This table is similar in form to Table 10 and the impedance characteristics are determined in precisely the same way. Here again the sequence constants apply to one three-phase circuit of three cables with zero-sequence return current assumed to be all in the cable sheaths. Single-conductor oil-filled cables have hollow conductors (the oil channel forms the core), consequently Table 11 includes cables of the two most common inside diameters, 0.5 and 0.69 inches.

In each of the tabulations, the voltage class listed in the first column refers specifically to grounded-neutral operation. Frequently cable systems are operated with other than a solidly grounded neutral. In low-voltage cables the same insulation thickness is used for both grounded and ungrounded operation, but in cables rated 7000 volts and above, a greater thickness of insulation is recommended for a given voltage class when cable is operated with an ungrounded neutral. A good approximation of the electrical characteristics of these higher voltage cables when operated with other than a solidly grounded neutral, can be had by referring in each specific case to the next higher voltage class listed in the tables.

The constants of several typical cables calculated by the methods outlined are listed in Table 13. These typical cases are included to be used as a check on the general magnitude of cable constants when making calculations for a specific case. Representative sizes and types of cable have been chosen to cover as many types of calculation as possible.

III. TABLES OF CURRENT CARRYING CAPACITY

One of the most common problems in cable calculations is that of determining the maximum permissible amperes

TABLE 9—60-CYCLE CHARACTERISTICS OF **THREE-CONDUCTOR GAS-FILLED** PAPER-INSULATED CABLES (SHIELDED TYPE)

Grounded Neutral Service

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B. & S.)	Type of Conductor (6)	Weight Per 1000 Feet	Diameter or Sector Depth (5)—inches	Resistance-Ohms Per Mile (1)	GMR of One Conductor (2)—inches	POSITIVE & NEGATIVE SEQ. Series Reactance Ohms Per Mile	POSITIVE & NEGATIVE SEQ. Shunt Capacitive Reactance-Ohms Per Mile (3)	GMR—Three Conductors	ZERO-SEQUENCE Series Resistance Ohms Per Mile (4)	ZERO-SEQUENCE Series Reactance Ohms Per Mile (4)	ZERO-SEQUENCE Shunt Capacitive Reactance-Ohms Per Mile (3)	SHEATH Thickness Mils	SHEATH Resistance-Ohms Per Mile at 50°C
15 Kv	130	2	SR	3 800	0.292	0.987	0.106	0.197	5100	0.321	4.86	0.289	5100	110	1.29
	130	1	SR	4 320	0.332	0.786	0.126	0.189	4600	0.354	4.42	0.274	4600	110	1.21
	130	0	CS	4 010	0.288	0.622	0.135	0.172	4500	0.326	4.52	0.279	4500	110	1.30
	130	00	CS	4 440	0.323	0.495	0.151	0.165	4200	0.355	4.34	0.267	4200	110	1.28
	130	000	CS	4 970	0.364	0.392	0.171	0.158	3800	0.392	3.90	0.245	3800	110	1.17
	130	0000	CS	5 620	0.417	0.310	0.191	0.156	3500	0.437	3.58	0.234	3500	110	1.09
	130	250 000	CS	6 180	0.455	0.263	0.210	0.153	3200	0.462	3.41	0.230	3200	110	1.05
	130	350 000	CS	7 530	0.539	0.190	0.249	0.146	2800	0.521	3.05	0.222	2800	110	0.953
	130	500 000	CS	9 540	0.642	0.134	0.297	0.141	2400	0.600	2.70	0.210	2400	110	0.854
	140	750 000	CS	12 900	0.780	0.091	0.366	0.137	2200	0.715	2.21	0.198	2200	115	0.707
	150	1 000 000	CS	16 450	0.900	0.070	0.430	0.134	2000	0.810	1.80	0.193	2000	125	0.578
23 Kv	200	2	SR	4 670	0.292	0.987	0.106	0.224	6900	0.376	4.17	0.302	6900	110	1.06
		1	SR	5 120	0.332	0.786	0.126	0.215	6300	0.410	3.82	0.286	6300	110	1.01
		0	CR	5 300	0.288	0.622	0.131	0.211	6200	0.398	3.62	0.302	6200	110	1.00
		00	CS	5 360	0.323	0.495	0.151	0.188	5800	0.412	3.56	0.281	5800	110	1.02
		000	CS	5 910	0.364	0.392	0.171	0.178	5300	0.445	3.31	0.271	5300	110	0.971
		0000	CS	6 570	0.417	0.310	0.191	0.175	4800	0.488	3.08	0.258	4800	110	0.922
		250 000	CS	7 160	0.455	0.263	0.210	0.171	4500	0.520	2.92	0.249	4500	110	0.885
		350 000	CS	8 540	0.539	0.190	0.249	0.163	4000	0.575	2.64	0.240	4000	110	0.816
		500 000	CS	10 750	0.642	0.134	0.297	0.155	3500	0.655	2.36	0.230	3500	110	0.741
		750 000	CS	14 650	0.780	0.091	0.466	0.147	2900	0.760	1.84	0.218	2900	125	0.582
		1 000 000	CS	18 560	0.900	0.070	0.430	0.144	2600	0.850	1.49	0.210	2600	140	0.473
35 Kv	310	0	CR	6 900	0.288	0.622	0.131	0.242	8400	0.477	3.00	0.320	8400	110	0.794
		00	CR	7 300	0.323	0.495	0.147	0.233	7900	0.509	2.69	0.310	7900	110	0.763
		000	CR	8 200	0.364	0.392	0.165	0.222	7300	0.545	2.58	0.284	7300	115	0.730
		0000	CS	8 660	0.410	0.310	0.191	0.201	6700	0.570	2.43	0.281	6700	115	0.707
		250 000	CS	9 380	0.447	0.263	0.210	0.195	6300	0.604	2.32	0.270	6300	115	0.685
		350 000	CS	11 200	0.532	0.190	0.249	0.185	5600	0.665	1.95	0.264	5600	125	0.587
		500 000	CS	12 790	0.635	0.134	0.297	0.175	4800	0.745	1.63	0.251	4800	135	0.500
		750 000	CS	18 190	0.767	0.091	0.366	0.165	4200	0.847	1.32	0.238	4200	150	0.409
		1 000 000	CS	22 100	0.898	0.070	0.430	0.158	3700	0.930	1.13	0.234	3700	160	0.353

[1] A-c resistance based on 100% conductivity at 65°C. including 2% allowance for stranding.
[2] GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.
[3] For dielectric constant=3.7.
[4] Based on all return current in sheath; none in ground.
[5] See Fig. 7.
[6] The following symbols are used to designate conductor types: SR—Stranded Round; CR—Compact Round; CS—Compact Sector.

per conductor for any given cable. The limiting factor in cable applications is not always the maximum permissible insulation temperature. Sometimes regulation, efficiency, economy, etc., may dictate the maximum permissible amperes. However because temperature rise is most often the controlling factor, the calculations of current-carrying capacity are usually based upon this limitation.

In Tables 14 through 19 earth temperature is assumed to be uniform at 20 degrees Centigrade. These tables were taken from a publication[16] of the Insulated Power Cable Engineers Association and give maximum allowable amperes per conductor for representative cable types. Corrections for earth temperatures other than 20 degrees Centigrade are given within the tables.

Special conditions may make it advisable to calculate a cable temperature problem in detail,[10,11] taking into account variable loading, "hot spots" along the cable route, and other factors not contemplated in making up the tabulated information.

Approximations can also be obtained for the current-carrying capacities of other types of insulation by applying multipliers to the tables presented for paper-insulated cables. The value for varnished cambric-insulated cables can be obtained by multiplying the value given in the tables for paper insulation by 0.91, the resulting figure being accurate to within five percent of the calculated value. Similarly, carrying capacities for rubber insulation can be determined with the same degree of accuracy by applying a factor of 0.85 to the figure given for an equivalent paper-insulated cable. For special heat-resisting rubber this factor becomes 0.95.

Circuits are frequently installed with each duct containing three cables. The current capacity of these circuits will be less than that tabulated here for one cable per duct, but will be somewhat higher than the capacity of an equivalent shielded three-conductor cable of the same conductor size and voltage rating.

The number of overhead power cables is a small percentage of the number in ducts, and for this reason space does not permit inclusion of loading tables for cables in air. Unfortunately there is no simple correction factor or curve that can be used to translate the figure for cables in ducts

TABLE 10—60-CYCLE CHARACTERISTICS OF **SINGLE-CONDUCTOR** CONCENTRIC-STRAND PAPER-INSULATED CABLES

Grounded Neutral Service

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Weight Per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[1]—inches	x_a Reactance at 12 Inches—Ohms Per Phase Per Mile	x_s Reactance of Sheath—Ohms Per Phase Per Mile	r_c Resistance of One Conductor—Ohms Per Phase Per Mile[2]	r_s Resistance of Sheath Ohms Per Phase Per Mile at 50°C	Shunt Capacitive Reactance—Ohms Per Phase Per Mile[3]	Lead Sheath Thickness—Mils
1 Kv	60	6	560	0.184	0.067	0.628	0.489	2.50	6.20	4040	75
	60	4	670	0.232	0.084	0.602	0.475	1.58	5.56	3360	75
	60	2	880	0.292	0.106	0.573	0.458	0.987	4.55	2760	80
	60	1	990	0.332	0.126	0.552	0.450	0.786	4.25	2490	80
	60	0	1 110	0.373	0.141	0.539	0.442	0.622	3.61	2250	80
	60	00	1 270	0.418	0.159	0.524	0.434	0.495	3.34	2040	80
	60	000	1 510	0.470	0.178	0.512	0.425	0.392	3.23	1840	85
	60	0000	1 740	0.528	0.200	0.496	0.414	0.310	2.98	1650	85
	60	250 000	1 930	0.575	0.221	0.484	0.408	0.263	2.81	1530	85
	60	350 000	2 490	0.681	0.262	0.464	0.392	0.190	2.31	1300	90
	60	500 000	3 180	0.814	0.313	0.442	0.378	0.134	2.06	1090	90
	60	750 000	4 380	0.998	0.385	0.417	0.358	0.091	1.65	885	95
	60	1 000 000	5 560	1.152	0.445	0.400	0.344	0.070	1.40	800	100
	60	1 500 000	8 000	1.412	0.543	0.374	0.319	0.050	1.05	645	110
	60	2 000 000	10 190	1.632	0.633	0.356	0.305	0.041	0.894	555	115
3 Kv	75	6	600	0.184	0.067	0.628	0.481	2.50	5.80	4810	75
	75	4	720	0.232	0.084	0.602	0.467	1.58	5.23	4020	75
	75	2	930	0.292	0.106	0.573	0.453	0.987	4.31	3300	80
	75	1	1 040	0.332	0.126	0.552	0.445	0.786	4.03	2990	80
	75	0	1 170	0.373	0.141	0.539	0.436	0.622	3.79	2670	80
	75	00	1 320	0.418	0.159	0.524	0.428	0.495	3.52	2450	80
	75	000	1 570	0.470	0.178	0.512	0.420	0.392	3.10	2210	85
	75	0000	1 800	0.528	0.200	0.496	0.412	0.310	2.87	2010	85
	75	250 000	1 990	0.575	0.221	0.484	0.403	0.263	2.70	1860	85
	75	350 000	2 550	0.681	0.262	0.464	0.389	0.190	2.27	1610	90
	75	500 000	3 340	0.814	0.313	0.442	0.375	0.134	1.89	1340	95
	75	750 000	4 570	0.998	0.385	0.417	0.352	0.091	1.53	1060	100
	75	1 000 000	5 640	1.152	0.445	0.400	0.341	0.070	1.37	980	100
	75	1 500 000	8 090	1.412	0.543	0.374	0.316	0.050	1.02	805	110
	75	2 000 000	10 300	1.632	0.633	0.356	0.302	0.041	0.877	685	115
5 Kv	120	6	740	0.184	0.067	0.628	0.456	2.50	4.47	6700	80
	115	4	890	0.232	0.084	0.573	0.447	1.58	4.17	5540	80
	110	2	1 040	0.292	0.106	0.573	0.439	0.987	3.85	4520	80
	110	1	1 160	0.332	0.126	0.552	0.431	0.786	3.62	4100	80
	105	0	1 270	0.373	0.141	0.539	0.425	0.622	3.47	3600	80
	100	00	1 520	0.418	0.159	0.524	0.420	0.495	3.09	3140	85
	100	000	1 710	0.470	0.178	0.512	0.412	0.392	2.91	2860	85
	95	0000	1 870	0.525	0.200	0.496	0.406	0.310	2.74	2480	85
	90	250 000	2 080	0.575	0.221	0.484	0.400	0.263	2.62	2180	85
	90	350 000	2 620	0.681	0.262	0.464	0.386	0.190	2.20	1890	90
	90	500 000	3 410	0.814	0.313	0.442	0.369	0.134	1.85	1610	95
	90	750 000	4 650	0.998	0.385	0.417	0.350	0.091	1.49	1350	100
	90	1 000 000	5 850	1.152	0.445	0.400	0.339	0.070	1.27	1140	105
	90	1 500 000	8 160	1.412	0.543	0.374	0.316	0.050	1.02	950	110
	90	2 000 000	10370	1.632	0.663	0.356	0.302	0.041	0.870	820	115
8 Kv	150	6	890	0.184	0.067	0.628	0.431	2.50	3.62	7780	80
	150	4	1 010	0.232	0.084	0.602	0.425	1.58	3.22	6660	85
	140	2	1 150	0.292	0.106	0.573	0.417	0.987	3.06	5400	85
	140	1	1 330	0.332	0.126	0.552	0.411	0.786	2.91	4920	85
	135	0	1 450	0.373	0.141	0.539	0.408	0.622	2.83	4390	85
	130	00	1 590	0.418	0.159	0.524	0.403	0.495	2.70	3890	85
	125	000	1 760	0.470	0.178	0.512	0.397	0.392	2.59	3440	85
	120	0000	1 980	0.528	0.200	0.496	0.389	0.310	2.29	3020	90
	120	250 000	2 250	0.575	0.221	0.484	0.383	0.263	2.18	2790	90
	115	350 000	2 730	0.681	0.262	0.464	0.375	0.190	1.90	2350	95
	115	500 000	3 530	0.814	0.313	0.442	0.361	0.134	1.69	2010	95
	115	750 000	4 790	0.998	0.385	0.417	0.341	0.091	1.39	1670	100
	115	1 000 000	6 000	1.152	0.415	0.400	0.330	0.070	1.25	1470	105
	115	1 500 000	8 250	1.412	0.543	0.374	0.310	0.050	0.975	1210	110
	115	2 000 000	10 480	1.632	0.663	0.356	0.297	0.041	0.797	1055	120

Voltage Class	Insulation thickness Mils	Circular Mils or AWG (B & S)	Weight Per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[1] inches	x_a Reactance at 12 inches—Ohms per Phase per Mile	x_s Reactance of Sheath Ohms per Phase Per Mile	r_c Resistance of One Conductor—Ohms per Phase per Mile[2]	r_s Resistance of Sheath Ohms per phase per Mile at 50°C	Shunt-Capacitive Reactance Ohms per phase per Mile[3]	Lead Sheath Thickness—Mils
15 Kv	220	4	1 340	0.232	0.084	0.602	0.412	1.58	2.91	8560	85
	215	2	1 500	0.292	0.106	0.573	0.406	0.987	2.74	7270	85
	210	1	1 610	0.332	0.126	0.552	0.400	0.786	2.64	6580	85
	200	0	1 710	0.373	0.141	0.539	0.397	0.622	2.59	5880	85
	195	00	1 940	0.418	0.159	0.524	0.391	0.495	2.32	5290	90
	185	000	2 100	0.470	0.178	0.512	0.386	0.392	2.24	4680	90
	180	0000	2 300	0.528	0.200	0.496	0.380	0.310	2.14	4200	90
	175	250 000	2 500	0.575	0.221	0.484	0.377	0.263	2.06	3820	90
	175	350 000	3 110	0.681	0.262	0.464	0.366	0.190	1.98	3340	95
	175	500 000	3 940	0.814	0.313	0.442	0.352	0.134	1.51	2870	100
	175	750 000	5 240	0.998	0.385	0.417	0.336	0.091	1.26	2420	105
	175	1 000 000	6 350	1.152	0.445	0.400	0.325	0.070	1.15	2130	105
	175	1 500 000	8 810	1.412	0.546	0.374	0.305	0.050	0.90	1790	115
	175	2 000 000	11 080	1.632	0.633	0.356	0.294	0.041	0.772	1570	120
23 Kv	295	2	1 920	0.292	0.106	0.573	0.383	0.987	2.16	8890	90
	285	1	2 010	0.332	0.126	0.552	0.380	0.786	2.12	8050	90
	275	0	2 120	0.373	0.141	0.539	0.377	0.622	2.08	7300	90
	265	00	2 250	0.418	0.159	0.524	0.375	0.495	2.02	6580	90
	260	000	2 530	0.470	0.178	0.512	0.370	0.392	1.85	6000	95
	250	0000	2 740	0.528	0.200	0.496	0.366	0.310	1.78	5350	95
	245	250 000	2 930	0.575	0.221	0.484	0.361	0.263	1.72	4950	95
	240	350 000	3 550	0.681	0.262	0.464	0.352	0.190	1.51	4310	100
	240	500 000	4 300	0.814	0.313	0.442	0.341	0.134	1.38	3720	100
	240	750 000	5 630	0.998	0.385	0.417	0.325	0.091	1.15	3170	105
	240	1 000 000	6 910	1.152	0.445	0.400	0.313	0.070	1.01	2800	110
	240	1 500 000	9 460	1.412	0.546	0.374	0.296	0.050	0.806	2350	120
	240	2 000 000	11 790	1.632	0.633	0.356	0.285	0.041	0.697	2070	125
35 Kv	395	0	2 900	0.373	0.141	0.539	0.352	0.622	1.51	9150	100
	385	00	3 040	0.418	0.159	0.524	0.350	0.495	1.48	8420	100
	370	000	3 190	0.470	0.178	0.512	0.347	0.392	1.46	7620	100
	355	0000	3 380	0.528	0.200	0.496	0.344	0.310	1.43	6870	100
	350	250 000	3 590	0.575	0.221	0.484	0.342	0.263	1.39	6410	100
	345	350 000	4 230	0.681	0.262	0.464	0.366	0.190	1.24	5640	105
	345	500 000	5 040	0.814	0.313	0.442	0.325	0.134	1.15	4940	105
	345	750 000	6 430	0.998	0.385	0.417	0.311	0.091	0.975	4250	110
	345	1 000 000	7 780	1.152	0.445	0.400	0.302	0.070	0.866	3780	115
	345	1 500 000	10 420	1.412	0.546	0.374	0.285	0.050	0.700	3210	125
	345	2 000 000	12 830	1.632	0.633	0.356	0.274	0.041	0.611	2830	130
46 Kv	475	000	3 910	0.470	0.178	0.512	0.331	0.392	1.20	8890	105
	460	0000	4 080	0.528	0.200	0.496	0.329	0.310	1.19	8100	105
	450	250 000	4 290	0.575	0.221	0.484	0.326	0.263	1 16	7570	105
	445	350 000	4 990	0.681	0.262	0.464	0.319	0.190	1.05	6720	110
	445	500 000	5 820	0.814	0.313	0.442	0.310	0.134	0.930	5950	115
	445	750 000	7 450	0.998	0.385	0.417	0.298	0.091	0.807	5130	120
	445	1 000 000	8 680	1.152	0.445	0.400	0.290	0.070	0.752	4610	120
	445	1 500 000	11 420	1.412	0.546	0.374	0.275	0.050	0.615	3930	130
	445	2 000 000	13 910	1.632	0.633	0.356	0.264	0.041	0.543	3520	135
69 Kv	650	350 000	6 720	0.681	0.262	0.464	0.292	0.190	0.773	8590	120
	650	500 000	7 810	0.814	0.313	0.442	0.284	0.134	0.695	7680	125
	650	750 000	9 420	0.998	0.385	0.417	0.275	0.091	0.615	6700	130
	650	1 000 000	10 940	1.152	0.445	0.400	0.267	0.070	0.557	6060	135
	650	1 500 000	13 680	1.412	0.546	0.374	0.256	0.050	0.488	5250	140
	650	2 000 000	16 320	1.632	0.633	0.356	0.246	0.041	0.437	4710	145

[1] Conductors are standard concentric-stranded, not compact round.
[2] A-c Resistance based on 100% conductivity at 65°C. including 2% allowance for stranding.
[3] For dielectric constant=3.7.

Positive- and Negative-Sequence Impedances:

(a) Neglecting Sheath Currents;

$$z_1 = z_2 = r_c + j(x_a + x_d).$$

(b) Including Sheath Currents;

$$z_1 = z_2 = r_c + \frac{x_m^2\, r_s}{x_m^2 + r_s^2} + j\left(x_a + x_d - \frac{x_m^3}{x_m^2 + r_s^2}\right)$$

Where $x_m = (x_s + x_d)$.

Note: x_d is obtained from Table 12.

Zero-Sequence Impedance:

(Based on all return current in sheath; none in ground)

$$z_0 = r_c + r_s + j(x_a - x_s)$$

TABLE 11—60-CYCLE CHARACTERISTICS OF **SINGLE-CONDUCTOR OIL-FILLED** (HOLLOW CORE) PAPER-INSULATED CABLES

Grounded Neutral Service

INSIDE DIAMETER OF SPRING CORE = 0.5 INCHES

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Weight per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[3]—inches	x_a Reactance at 12 Inches—Ohms Per Phase Per Mile	x_s Reactance of Sheath—Ohms Per Phase Per Mile	r_c Resistance of One Conductor—Ohms Per Phase Per Mile[1]	r_s Resistance of Sheath Ohms Per Phase Per Mile at 50°C.	Shunt Capacitive Reactance—Ohms Per Phase Per Mile[2]	Lead Sheath Thickness—Mils
69 Kv	315	00	3 980	0.736	0.345	0.431	0.333	0.495	1.182	5240	110
		000	4 090	0.768	0.356	0.427	0.331	0.392	1.157	5070	110
		0000	4 320	0.807	0.373	0.421	0.328	0.310	1.130	4900	110
		250 000	4 650	0.837	0.381	0.418	0.325	0.263	1.057	4790	115
		350 000	5 180	0.918	0.408	0.410	0.320	0.188	1.009	4470	115
		500 000	6 100	1.028	0.448	0.399	0.312	0.133	0.905	4070	120
		750 000	7 310	1.180	0.505	0.384	0.302	0.089	0.838	3620	120
		1 000 000	8 630	1.310	0.550	0.374	0.294	0.068	0.752	3380	125
		1 500 000	11 090	1.547	0.639	0.356	0.281	0.048	0.649	2920	130
		2 000 000	13 750	1.760	0.716	0.342	0.270	0.039	0.550	2570	140
115 Kv	480	0000	5 720	0.807	0.373	0.421	0.305	0.310	0.805	6650	120
		250 000	5 930	0.837	0.381	0.418	0.303	0.263	0.793	6500	120
		350 000	6 390	0.918	0.408	0.410	0.298	0.188	0.730	6090	125
		500 000	7 480	1.028	0.448	0.399	0.291	0.133	0.692	5600	125
		750 000	8 950	1.180	0.505	0.384	0.283	0.089	0.625	5040	130
		1 000 000	10 350	1.310	0.550	0.374	0.276	0.068	0.568	4700	135
		1 500 000	12 960	1.547	0.639	0.356	0.265	0.048	0.500	4110	140
		2 000 000	15 530	1.760	0.716	0.342	0.255	0.039	0.447	3710	145
138 Kv	560	0000	6 480	0.807	0.373	0.421	0.295	0.310	0.758	7410	125
		250 000	6 700	0.837	0.381	0.418	0.293	0.263	0.746	7240	125
		350 000	7 460	0.918	0.408	0.410	0.288	0.188	0.690	6820	130
		500 000	8 310	1.028	0.448	0.399	0.282	0.133	0.658	6260	130
		750 000	9 800	1.180	0.505	0.384	0.274	0.089	0.592	5680	135
		1 000 000	11 270	1.310	0.550	0.374	0.268	0.068	0.541	5240	140
		1 500 000	13 720	1.547	0.639	0.356	0.257	0.048	0.477	4670	145
		2 000 000	16 080	1.760	0.716	0.342	0.248	0.039	0.427	4170	150
161 Kv	650	250 000	7 600	0.837	0.381	0.418	0.283	0.263	0.660	7980	130
		350 000	8 390	0.918	0.408	0.410	0.279	0.188	0.611	7520	135
		500 000	9 270	1.028	0.448	0.399	0.273	0.133	0.585	6980	135
		750 000	10 840	1.180	0.505	0.384	0.266	0.089	0.532	6320	140
		1 000 000	12 340	1.310	0.550	0.374	0.259	0.068	0.483	5880	145
		1 500 000	15 090	1.547	0.639	0.356	0.246	0.048	0.433	5190	150
		2 000 000	18 000	1.760	0.716	0.342	0.241	0.039	0.391	4710	155

INSIDE DIAMETER OF SPRING CORE = 0.69 INCHES

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Weight per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[3]—inches	x_a Reactance at 12 Inches—Ohms Per Phase Per Mile	x_s Reactance of Sheath—Ohms Per Phase Per Mile	r_c Resistance of One Conductor—Ohms Per Phase Per Mile[1]	r_s Resistance of Sheath Ohms Per Phase Per Mile at 50°C.	Shunt Capacitive Reactance—Ohms Per Phase Per Mile[2]	Lead Sheath Thickness—Mils
69 Kv	315	000	4 860	0.924	0.439	0.399	0.320	0.392	1.007	4450	115
		0000	5 090	0.956	0.450	0.398	0.317	0.310	0.985	4350	115
		250 000	5 290	0.983	0.460	0.396	0.315	0.263	0.975	4230	115
		350 000	5 950	1.050	0.483	0.390	0.310	0.188	0.897	4000	120
		500 000	6 700	1.145	0.516	0.382	0.304	0.132	0.850	3700	120
		750 000	8 080	1.286	0.550	0.374	0.295	0.089	0.759	3410	125
		1 000 000	9 440	1.416	0.612	0.360	0.288	0.067	0.688	3140	130
		1 500 000	11 970	1.635	0.692	0.346	0.276	0.047	0.601	2750	135
		2 000 000	14 450	1.835	0.763	0.334	0.266	0.038	0.533	2510	140
115 Kv	480	0000	6 590	0.956	0.450	0.398	0.295	0.310	0.760	5950	125
		250 000	6 800	0.983	0.460	0.396	0.294	0.263	0.752	5790	125
		350 000	7 340	1.050	0.483	0.390	0.290	0.188	0.729	5540	125
		500 000	8 320	1.145	0 516	0.382	0.284	0.132	0.669	5150	130
		750 000	9 790	1.286	0.550	0.374	0.277	0.089	0.606	4770	135
		1 000 000	11 060	1.416	0.612	0.360	0.270	0.067	0.573	4430	135
		1 500 000	13 900	1.635	0.692	0.346	0.260	0.047	0.490	3920	145
		2 000 000	16 610	1.835	0.763	0.334	0.251	0.038	0.440	3580	150
138 Kv	560	0000	7 390	0.956	0.450	0.398	0.786	0.310	0.678	6590	130
		250 000	7 610	0.983	0.460	0.396	0.285	0.263	0.669	6480	130
		350 000	8 170	1.050	0.483	0.390	0.281	0.188	0.649	6180	130
		500 000	9 180	1.145	0.516	0.382	0.276	0.132	0.601	5790	135
		750 000	10 660	1.286	0.550	0.374	0.269	0.089	0.545	5320	140
		1 000 000	12 010	1.416	0.612	0.360	0.263	0.067	0.519	4940	140
		1 500 000	14 450	1.635	0.692	0.346	0.253	0.047	0.462	4460	145
		2 000 000	16 820	1.835	0.763	0.334	0.245	0.038	0.404	4060	155
161 Kv	650	250 000	8 560	0.983	0.460	0.396	0.275	0.263	0.596	7210	135
		350 000	9 140	1.050	0.483	0.390	0.272	0.188	0.580	6860	135
		500 000	10 280	1.145	0.516	0.382	0.267	0.132	0.537	6430	140
		750 000	11 770	1.286	0.550	0.374	0.261	0.089	0.492	5980	145
		1 000 000	13 110	1.416	0.612	0.360	0.255	0.067	0.469	5540	145
		1 500 000	15 840	1.635	0.692	0.346	0.246	0.047	0.421	4980	150
		2 000 000	18 840	1.835	0.763	0.334	0.238	0.038	0.369	4600	160
230 Kv	925	750 000	15 360	1.286	0.550	0.374	0.238	0.089	0.369	7610	160
		1 000 000	16 790	1.416	0.612	0.360	0.233	0.067	0.355	7140	160
		2 000 000	22 990	1.835	0.763	0.334	0.219	0.038	0.315	5960	170

[1]A-c Resistance based on 100% conductivity at 65°C. including 2% allowance for stranding. Above values calculated from "A Set of Curves for Skin Effect in Isolated Tubular Conductors" by A. W. Ewan, *G. E. Review*, Vol. 33, April 1930.
[2]For dielectric constant = 3.5.
[3]Calculated for circular tube as given in *Symmetrical Components* by Wagner & Evans, Ch. VII, page 138.

Positive- and Negative-Sequence Impedances:

(a) Neglecting Sheath Currents;

$$z_1 = z_2 = r_c + j(x_a + x_d).$$

(b) Including Sheath Currents;

$$z_1 = z_2 = r_c + \frac{x_m^2\, r_s}{x_m^2 + r_s^2} + j\left(x_a + x_d - \frac{x_m^3}{x_m^2 + r_s^2}\right)$$

Where $x_m = (x_s + x_d)$.

Note: x_d is obtained from Table 12.

Zero-Sequence Impedance:

(Based on all return current in sheath; none in ground)

$$z_0 = r_c + r_s + j(x_a - x_s)$$

to a reasonable figure for cables in air. The current-carrying capacities of cables in air have recently been revised by the IPCEA and are now available in the cable manufacturers' catalogs.

In the discussion on proximity effect it was mentioned that where cables are installed parallel to steel plates, the extra losses arising from proximity to the plate may affect the current-carrying capacity. This reduction in carrying capacity is given by the curves of Fig. 19 which are taken from the test values presented by Booth.

IV. CABLES IN PARALLEL

The problem of current division among paralleled cables is frequently encountered, because in many circuits more than one cable per phase is installed in order to carry the total current. Also, mutual effects may develop between cable circuits which are adjacent throughout their length but which terminate on separate busses. Depending upon the type of circuit, the cable type and configuration, and the system conditions being investigated, the problem may take any of several forms.

TABLE 12—REACTANCE SPACING FACTORS (x_d)*, OHMS PER MILE AT 60 CYCLES

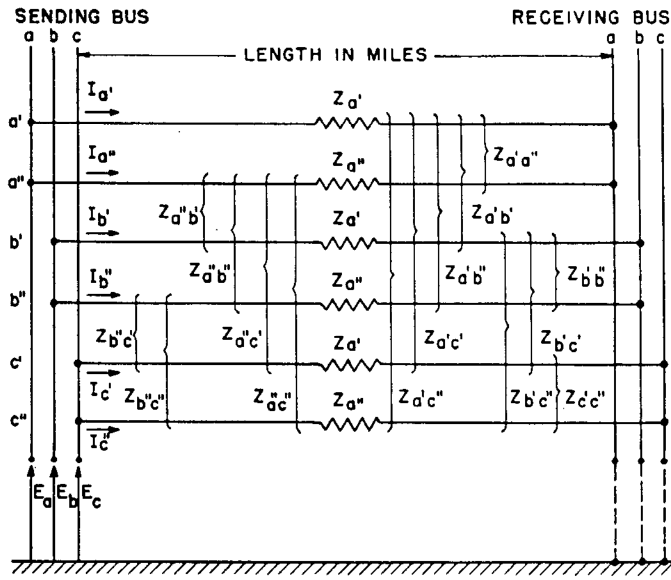
In.	x_d	In.	x_d	In.	x_d	In.	x_d	In.	x_d	In.	x_d	In.	x_d	In.	x_d
....	....	2.75	−0.179	5.25	−0.100	7.75	−0.053	10.5	−0.016	15.5	0.031	20.5	0.065	27.0	0.098
0.50	−0.385	3.00	−0.169	5.50	−0.095	8.00	−0.049	11.0	−0.011	16.0	0.035	21.0	0.068	28.0	0.103
0.75	−0.336	3.25	−0.159	5.75	−0.089	8.25	−0.045	11.5	−0.005	16.5	0.039	21.5	0.071	29.0	0.107
1.00	−0.302	3.50	−0.149	6.00	−0.084	8.50	−0.042	12.0	0.0	17.0	0.042	22.0	0.074	30.0	0.111
1.25	−0.274	3.75	−0.141	6.25	−0.079	8.75	−0.038	12.5	0.005	17.5	0.046	22.5	0.076	31.0	0.115
1.50	−0.252	4.00	−0.133	6.50	−0.074	9.00	−0.035	13.0	0.010	18.0	0.049	23.0	0.079	32.0	0.119
1.75	−0.234	4.25	−0.126	6.75	−0.070	9.25	−0.032	13.5	0.014	18.5	0.053	23.5	0.082	33.0	0.123
2.00	−0.217	4.50	−0.119	7.00	−0.065	9.50	−0.028	14.0	0.019	19.0	0.056	24.0	0.084	34.0	0.126
2.25	−0.203	4.75	−0.112	7.25	−0.061	9.75	−0.025	14.5	0.023	19.5	0.059	25.0	0.090	35.0	0.130
2.50	−0.190	5.00	−0.106	7.50	−0.057	10.00	−0.022	15.0	0.027	20.0	0.062	26.0	0.094	36.0	0.133

*$x_d = 0.2794 \frac{f}{60} \log_{10} \frac{S}{12}$, where S is spacing in inches.

It is difficult to anticipate in detail the problems met in practice, but the examples outlined here indicate methods of solution that can be modified to fit actual circumstances.

Almost any problem involving paralleled cables can be represented by simultaneous equations of voltage drops caused by self and mutual impedances but such equations often become numerous and cumbersome. Therefore in approaching most problems it becomes desirable to search about for one or more simplifying assumptions so that the problem can be reduced to simpler terms, still without introducing errors large enough to invalidate the solution. For example, when paralleled cable circuits connect a generating source to a balanced load, it is usually permissible to assume that the total current in each phase is composed only of the respective positive-sequence component: this assumption is based on the unsymmetrical cable-circuit impedances being much smaller than the symmetrical load impedances.

Three outlined examples of calculations on paralleled cables are included here, but they assist only by illustrating general methods, since there are so many different, and more complex, cases to be found in practice.



(ZERO CURRENT IN EARTH RETURN PATH)

Fig. 16—Equivalent circuit for parallel cables, with open-circuited sheaths and no net ground-return current (see Example 3).

TABLE 13—60-CYCLE CONSTANTS OF TYPICAL CABLES IN OHMS PER PHASE PER MILE

DESCRIPTION OF CABLE	Assumed Operating Kilovolts	POSITIVE- AND NEGATIVE-SEQUENCE: RESISTANCE*: No Sheath Currents	RESISTANCE*: Including Sheath Currents	REACTANCE: No Sheath Currents	REACTANCE: Including Sheath Currents	Shunt Capacitive Reactance	ZERO-SEQUENCE (ALL RETURN IN SHEATH): Resistance	Reactance	Shunt Capacitive Reactance
Single-Conductor, 1 000 MCM. 30/64 in. Insulation; ⅛ in. Sheath. Three Cables spaced 4 in. horizontally..........................	44	0.070	0.114	0.295	0.284	4 780	0.783	0.113	4 780
Single-Conductor, 500 MCM. 9/64 in. Insulation; 6/64 in. Sheath. Three Cables spaced 3, 3, 6 in..	6.9	0.134	0.162	0.302	0.299	2 440	1.87	0.081	2 440
Single-Conductor Oil-Filled, 750 MCM., inside diam. 0.50 in. 650 mils Insulation; 9/64 in. Sheath. Three Cables spaced 13 in. horizontally	161	0.089	0.221	0.422	0.347	6 300	0.631	0.115	6 300
Single-Conductor, 250 MCM. 6/64 in. insulation; 7/64 in. Sheath. Three Sheaths in contact and 4/0 Copper Neutral Wire..........	0.21	0.263	0.239	0.181	0.180	2 270	0.960	0.381	2 270
Three-Conductor belted; Sectored, 500 MCM. 7/64 in. Conductor Insulation, 4/64 in. Belt. 7.5/64 in. Sheath......................	6.9	0.134	0.135	0.135	....	2 410	2.53	0.231	4 670
Three-Conductor Type H; Sectored, 500 MCM. 13/64 in. Insulation, 8/64 in. Sheath...	15	0.134	0.135	0.156	....	3 400	2.10	0.226	3 400
Three-Conductor Oil-Filled Type H; Sectored, 500 MCM. 225 Mils Insulation, 8.5/64 in. Sheath....................................	44	0.134	0.135	0.160	....	3 870	1.94	0.226	3 870

*Conductor temperature 65°C.; Sheath temperature 50°C.

TABLE 14—CURRENT CARRYING CAPACITY OF **THREE-CONDUCTOR BELTED** PAPER-INSULATED CABLES

Conductor Size AWG or 1000 CM	Conductor type[1]	Number of Equally Loaded Cables in Duct Bank																			
		ONE				THREE				SIX				NINE				TWELVE			
		Per Cent Load Factor																			
		30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
		AMPERES PER CONDUCTOR[2]																			
4500 Volts																Copper Temperature 85°C					
6	S	82	80	78	75	81	78	73	68	79	74	68	63	78	72	65	58	76	69	61	54
4	SR	109	106	103	98	108	102	96	89	104	97	89	81	102	94	84	74	100	90	79	69
2	SR	143	139	134	128	139	133	124	115	136	127	115	104	133	121	108	95	130	117	101	89
1	SR	164	161	153	146	159	152	141	130	156	145	130	118	152	138	122	108	148	133	115	100
0	CS	189	184	177	168	184	175	162	149	180	166	149	134	175	159	140	122	170	152	130	114
00	CS	218	211	203	192	211	201	185	170	208	190	170	152	201	181	158	138	195	173	148	128
000	CS	250	242	232	219	242	229	211	193	237	217	193	172	229	206	179	156	223	197	167	145
0000	CS	286	276	264	249	276	260	240	218	270	246	218	194	261	234	202	176	254	223	189	163
250	CS	316	305	291	273	305	288	263	239	297	271	239	212	288	258	221	192	279	244	206	177
300	CS	354	340	324	304	340	321	292	264	332	301	264	234	321	285	245	211	310	271	227	195
350	CS	392	376	357	334	375	353	320	288	366	330	288	255	351	311	266	229	341	296	248	211
400	CS	424	406	385	359	406	380	344	309	395	355	309	272	380	334	285	244	367	317	264	224
500	CS	487	465	439	408	465	433	390	348	451	403	348	305	433	378	320	273	417	357	296	251
600	CS	544	517	487	450	517	480	430	383	501	444	383	334	480	416	350	298	462	393	323	273
750	CS	618	581	550	505	585	541	482	427	566	500	427	371	541	466	390	331	519	439	359	302
		(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]			
7500 Volts																Copper Temperature 83°C					
6	S	81	80	77	74	79	76	72	67	78	74	67	62	77	71	64	57	75	69	60	53
4	SR	107	105	101	97	104	100	94	87	103	96	87	79	100	92	82	73	98	89	77	68
2	SR	140	137	132	126	136	131	122	113	134	125	113	102	130	119	105	93	127	114	99	87
1	SR	161	156	150	143	156	149	138	128	153	142	128	115	149	136	120	105	145	130	112	98
0	CS	186	180	174	165	180	172	156	146	177	163	148	131	172	155	136	120	167	149	128	111
00	CS	214	206	198	188	206	196	181	166	202	186	166	148	196	177	155	135	191	169	145	125
000	CS	243	236	226	214	236	224	206	188	230	211	188	168	223	200	174	152	217	192	163	141
0000	CS	280	270	258	243	270	255	235	214	264	241	213	190	255	229	198	172	247	218	184	159
250	CS	311	300	287	269	300	283	259	235	293	266	235	208	282	252	217	188	273	240	202	174
300	CS	349	336	320	300	335	316	288	260	326	296	259	230	315	279	240	207	304	265	223	190
350	CS	385	369	351	328	369	346	315	283	359	323	282	249	345	305	261	224	333	289	242	206
400	CS	417	399	378	353	398	373	338	303	388	348	303	267	371	317	279	239	360	309	257	220
500	CS	476	454	429	399	454	423	381	341	440	392	340	298	422	369	312	267	406	348	288	245
600	CS	534	508	479	443	507	471	422	376	491	436	375	327	469	408	343	291	451	384	315	267
750	CS	607	576	540	497	575	532	473	418	555	489	418	363	529	455	381	323	507	428	350	295
		(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]			
15 000 Volts																Copper Temperature 75°C					
6	S	78	77	74	71	76	74	69	64	75	70	64	59	73	68	61	54	72	65	57	50
4	SR	102	99	96	92	98	95	89	83	97	91	83	75	95	87	78	69	93	85	73	64
2	SR	132	129	125	119	129	123	115	106	126	117	106	96	123	112	99	88	120	108	93	82
1	SR	151	147	142	135	146	140	131	120	144	133	120	109	140	128	112	99	136	122	107	92
0	CS	175	170	163	155	169	161	150	138	166	153	137	123	161	146	128	112	156	139	120	104
00	CS	200	194	187	177	194	184	170	156	189	175	156	139	183	166	145	127	178	158	135	117
000	CS	230	223	214	202	222	211	195	178	217	199	177	158	210	189	165	143	203	180	153	132
0000	CS	266	257	245	232	253	242	222	202	249	228	201	179	240	215	187	158	233	205	173	149
250	CS	295	284	271	255	281	268	245	221	276	251	220	196	266	239	204	177	257	225	189	163
300	CS	330	317	301	283	316	297	271	245	307	278	244	215	295	264	225	194	285	248	208	178
350	CS	365	349	332	310	348	327	297	267	339	305	266	235	324	289	245	211	313	271	227	193
400	CS	394	377	357	333	375	352	319	286	365	327	285	251	349	307	262	224	336	290	241	206
500	CS	449	429	406	377	428	399	359	321	414	396	319	280	396	346	293	250	379	326	269	229
600	CS	502	479	450	417	476	443	396	352	459	409	351	306	438	380	319	273	420	358	294	249
750	CS	572	543	510	468	540	499	444	393	520	458	391	341	494	425	356	302	471	399	326	275
		(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]				(1.09 at 10°C, 0.90° at 30°C 0.79 at 40°C, 0.66 at 50°C)[3]			

[1] The following symbols are used here to designate conductor types:
S—Solid copper, SR—standard round concentric-stranded, CS—compact-sector stranded.

[2] Current ratings are based on the following conditions:
a. Ambient earth temperature = 20°C.
b. 60 cycle alternating current.
c. Ratings include dielectric loss, and all induced a-c losses.
d. One cable per duct, all cables equally loaded and in outside ducts only.

[3] Multiply tabulated currents by these factors when earth temperature is other than 20°C.

Example 3—Type of Circuit: A three-phase 60-cycle cable circuit connected between a sending and a receiving bus, using single-conductor unsheathed cables, and having two paralleled cables per phase.

Conditions: The current flowing into the sending bus and out of the receiving bus is nearly balanced three-phase load current (positive-sequence only), and its magnitude is known. The cable conductors can be of different sizes, and their spacings can be entirely unsymmetrical.

Problem: To find the division of load current among all conductors.

Circuit: Refer to Figure 16.

TABLE 15—CURRENT CARRYING CAPACITY OF **THREE-CONDUCTOR SHIELDED** PAPER-INSULATED CABLES

Conductor Size AWG or 1000CM	Conductor Type[1]	Number of Equally Loaded Cables in Duct Bank																			
		ONE				THREE				SIX				NINE				TWELVE			
		Per Cent Load Factor																			
		30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
		AMPERES PER CONDUCTOR[2]																			
15 000 Volts																Copper Temperature 81°C					
6	S	94	91	88	83	91	87	81	75	89	83	74	66	87	78	69	60	84	75	64	56
4	SR	123	120	115	107	119	114	104	95	116	108	95	85	113	102	89	77	109	96	83	72
2	SR	159	154	146	137	153	144	139	121	149	136	120	107	144	129	112	97	139	123	104	90
1	SR	179	174	166	156	172	163	149	136	168	153	136	121	162	145	125	109	158	138	117	100
0	CS	203	195	182	176	196	185	169	154	190	173	154	137	183	164	141	122	178	156	131	112
00	CS	234	224	215	202	225	212	193	175	218	198	174	156	211	187	162	139	203	177	148	127
000	CS	270	258	245	230	258	242	220	198	249	225	198	174	241	212	182	157	232	202	168	144
0000	CS	308	295	281	261	295	276	250	223	285	257	224	196	275	241	205	176	265	227	189	162
250	CS	341	327	310	290	325	305	276	246	315	283	245	215	303	265	224	193	291	250	207	177
300	CS	383	365	344	320	364	339	305	272	351	313	271	236	337	293	246	211	322	276	227	194
350	CS	417	397	375	346	397	369	330	293	383	340	293	255	366	318	267	227	350	301	245	208
400	CS	453	428	403	373	429	396	354	314	413	366	313	273	394	340	285	242	376	320	262	222
500	CS	513	487	450	418	483	446	399	350	467	410	350	303	444	381	318	269	419	358	292	247
600	CS	567	537	501	460	534	491	437	385	513	450	384	330	488	416	346	293	465	390	317	269
750	CS	643	606	562	514	602	551	485	426	576	502	423	365	545	464	383	323	519	432	348	293
		(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.81 at 40°C, 0.70 at 50°C)[3]			
23 000 Volts																Copper Temperature 77°C					
2	SR	156	150	143	134	149	141	130	117	145	132	117	105	140	125	107	84	134	119	100	86
1	SR	177	170	162	152	170	160	145	133	164	149	132	117	159	140	121	105	154	133	112	97
0	CS	200	192	183	172	192	182	166	149	186	169	147	132	178	158	136	118	173	149	126	109
00	CS	227	220	210	197	221	208	189	170	212	193	168	149	202	181	156	134	196	172	144	123
000	CS	262	251	238	223	254	238	216	193	242	220	191	169	230	206	175	150	222	195	162	139
0000	CS	301	289	271	251	291	273	246	219	278	250	215	190	264	233	197	169	255	221	182	157
250	CS	334	315	298	277	321	299	270	239	308	275	236	207	290	258	216	184	279	242	199	170
300	CS	373	349	328	306	354	329	297	263	341	302	259	227	320	283	232	202	309	266	217	186
350	CS	405	379	358	331	384	356	318	283	369	327	280	243	347	305	255	217	335	285	233	199
400	CS	434	409	386	356	412	379	340	302	396	348	298	260	374	325	273	232	359	303	247	211
500	CS	492	465	436	401	461	427	379	335	443	391	333	288	424	363	302	257	400	336	275	230
600	CS	543	516	484	440	512	470	414	366	489	428	365	313	464	396	329	279	441	367	299	248
750	CS	616	583	541	495	577	528	465	407	550	479	402	347	520	439	364	306	490	408	329	276
		(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.65 at 50°C)[3]			
34 500 Volts																Copper Temperature 70°C					
0	CS	193	185	176	165	184	174	158	141	178	161	140	124	171	149	129	111	164	142	119	103
00	CS	219	209	199	187	208	197	178	160	202	182	158	140	194	170	145	126	185	161	134	115
000	CS	250	238	225	211	238	222	202	182	229	206	179	158	220	193	165	141	209	182	152	128
0000	CS	288	275	260	241	273	256	229	205	263	234	203	179	251	219	186	160	238	205	170	144
250	CS	316	302	285	266	301	280	253	224	289	258	222	196	276	240	202	174	262	222	187	157
300	CS	352	335	315	293	334	310	278	246	320	284	244	213	304	264	221	190	288	244	203	171
350	CS	384	364	342	318	363	336	301	267	346	308	264	229	329	285	238	204	311	263	217	184
400	CS	413	392	367	341	384	360	321	284	372	329	281	244	352	303	254	216	334	282	232	195
500	CS	468	442	414	381	436	402	358	317	418	367	312	271	393	337	281	238	372	313	256	215
600	CS	514	487	455	416	481	440	391	344	459	401	340	294	430	367	304	259	406	340	277	232
750	CS	584	548	510	466	541	496	435	383	515	447	378	324	481	409	337	284	452	377	304	255
		(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.61 at 50°C)[3]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[3]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[3]				(1.10 at 10°C, 0.88 at 30°C, 0.75 at 40°C, 0.58 at 50°C)[3]				(1.10 at 10°C, 0.88 at 30°C, 0.74 at 40°C, 0.56 at 50°C)[3]			

[1] The following symbols are used here to designate conductor types:
S—solid copper, SR—standard round concentric-stranded, CS—compact-sector stranded.

[2] Current ratings are based on the following conditions:
a. Ambient earth temperature = 20°C.
b. 60 cycle alternating current.
c. Ratings include dielectric loss, and all induced a-c losses.
d. One cable per duct, all cables equally loaded and in outside ducts only.

[3] Multiply tabulated currents by these factors when earth temperature is other than 20°C.

Complete voltage drop equations:

$$E_{a'} = I_{a'}Z_{a'} + I_{a''}Z_{a'a''} + I_{b'}Z_{a'b'} + I_{b''}Z_{a'b''} + I_{c'}Z_{a'c'} + I_{c''}Z_{a'c''}$$
$$E_{a''} = I_{a'}Z_{a'a''} + I_{a''}Z_{a''} + I_{b'}Z_{a''b'} + I_{b''}Z_{a''b''} + I_{c'}Z_{a''c'} + I_{c''}Z_{a''c''}$$
$$E_{b'} = I_{a'}Z_{a'b'} + I_{a''}Z_{a''b'} + I_{b'}Z_{b'} + I_{b''}Z_{b'b''} + I_{c'}Z_{b'c'} + I_{c''}Z_{b'c''}$$
$$E_{b''} = I_{a'}Z_{a'b''} + I_{a''}Z_{a''b''} + I_{b'}Z_{b'b''} + I_{b''}Z_{b''} + I_{c'}Z_{b''c'} + I_{c''}Z_{b''c''}$$
$$E_{c'} = I_{a'}Z_{a'c'} + I_{a''}Z_{a''c'} + I_{b'}Z_{b'c'} + I_{b''}Z_{b''c'} + I_{c'}Z_{c'} + I_{c''}Z_{c'c''}$$
$$E_{c''} = I_{a'}Z_{a'c''} + I_{a''}Z_{a''c''} + I_{b'}Z_{b'c''} + I_{b''}Z_{b''c''} + I_{c'}Z_{c'c''} + I_{c''}Z_{c'}$$

Simplifying assumptions: It is apparent that $E_{a'} = E_{a''}$, $E_{b'} = E_{b''}$, and $E_{c'} = E_{c''}$; therefore these voltages can be eliminated by subtraction. Also if one ampere positive sequence current is assumed to flow through the overall circuit, then $I_{a''} = 1.0 - I_{a'}$, $I_{b''} = a^2 - I_{b'}$, and $I_{c''} = a - I_{c'}$.

Applying these assumptions leads to a set of three simultaneous equations relating three currents:

Modified equations:

$$I_{a'}(Z_{a'} + Z_{a''} - 2Z_{a'a''}) + I_{b'}(Z_{a'b'} + Z_{a''b''} - Z_{a''b'} - Z_{a'b''}) + I_{c'}(Z_{a'c'} + Z_{a''c''} - Z_{a''c'} - Z_{a'c''}) = (Z_{a''} - Z_{a'a''}) + a^2(Z_{a''b''} - Z_{a'b''}) + a(Z_{a''c''} - Z_{a'c''}).$$

$$I_{a'}(Z_{a'b'} + Z_{a''b''} - Z_{a'b''} - Z_{a''b'}) + I_{b'}(Z_{b'} + Z_{b''} - 2Z_{b'b''}) + I_{c'}(Z_{b'c'} + Z_{b''c''} - Z_{b''c'} - Z_{b'c''}) = (Z_{a''b''} - Z_{a''b'}) + a^2(Z_{b''} - Z_{b'b''}) + a(Z_{b''c''} - Z_{b'c''}).$$

$$I_{a'}(Z_{a'c'} + Z_{a''c''} - Z_{a'c''} - Z_{a''c'}) + I_{b'}(Z_{b'c'} + Z_{b''c''} - Z_{b'c''} - Z_{b''c'}) + I_{c'}(Z_{c'} + Z_{c''} - 2Z_{c'c''}) = (Z_{a''c''} - Z_{a''c'}) + a^2(Z_{b''c''} - Z_{b''c'}) + a(Z_{c''} - Z_{c'c''}).$$

TABLE 16—CURRENT CARRYING CAPACITY FOR THREE-CONDUCTOR OIL-FILLED PAPER INSULATED CABLES (amperes per conductor)*

Circular Mils. or A.W.G. (B.& S.)	Rated Line Voltage—Grounded Neutral		
	34 500	46 000	69 000
	Maximum Copper Temperature—Deg. C.		
	75	75	75
0	168	...	...
00	190	190	...
000	210	210	210
0000	240	240	240
250 000	265	265	265
300 000	295	295	295
350 000	320	320	320
400 000	342	342	342
500 000	382	382	380
600 000	417	417	412
700 000	445	445	440
750 000	460	460	455
Deg. C.	Correction Factor for Various Earth Temps.		
10	1.08	1.08	1.08
20	1.00	1.00	1.00
30	0.90	0.90	0.90
40	.79	.79	.79

75% load factor assumed.
Ratings include dielectric loss and extra a-c. losses such as sheath and proximity loss.
Above values apply specifically to sector shaped conductors. For round conductors multiply by 0.99.
*Applies to three similar loaded cables in a duct bank; for six loaded cables in a duct bank, multiply above values by 0.88.

After substituting the proper self and mutual impedance values as defined later, these equations can be solved by the method of determinants for current distribution, based on a total of 1.0 ampere positive-sequence current in the circuit. To obtain actual currents, the distribution factors must be multiplied by the actual load current in amperes.

Apparent conductor impedances: Using the current-distribution factors for each conductor to solve the complete voltage drop equations, an "apparent" impedance for each phase of the circuit can be calculated. This apparent impedance is valid only for the particular current division calculated:

Apparent impedance of phase a

$$=\frac{E_{a'}}{I_{a'}+I_{a''}}=\frac{E_{a'}}{1.0}=E_{a'}=E_{a''}, \text{ ohms.}$$

Apparent impedance of phase b

$$=\frac{E_{b'}}{I_{b'}+I_{b''}}=\frac{E_{b'}}{a^2}=aE_{b'}=aE_{b''}, \text{ ohms.}$$

Apparent impedance of phase c

$$=\frac{E_{c'}}{I_{c'}+I_{c''}}=\frac{E_{c'}}{a}=a^2E_{c'}=a^2E_{c''}, \text{ ohms.}$$

Supplementary equations: The original assumption of positive-sequence current flow through the circuit precludes the existence of any net ground return current. This assumption simplifies the determination of the various self and mutual impedances, because the effects of a ground return path may be ignored with very small error:

$$Z_{a'}=l(r_c+jx_a)$$

where

l = circuit length in miles.

r_c = a-c. resistance of conductor a', ohms per mile.

x_a = reactance of conductor a', to a twelve inch radius, ohms per mile.

$$=j0.2794\log_{10}\frac{12}{\text{GMR}_1 \text{ of conductor } a', \text{ inches}}$$

$Z_{a''}$, $Z_{b'}$, $Z_{b''}$, $Z_{c'}$, and $Z_{c''}$ are determined similarly, based on the respective conductor characteristics.

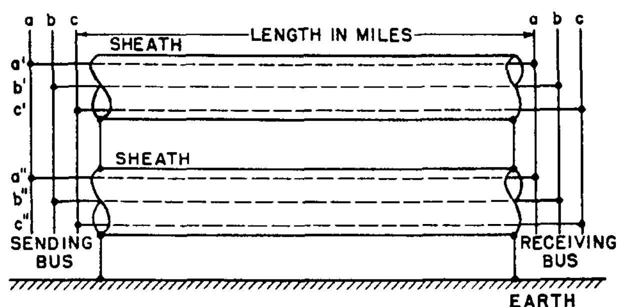
$Z_{a'a''}=l\cdot j0.2794\log_{10}\frac{12}{S_{a'a''}}=l(-x_d)$ where $S_{a'a''}$ is the axial spacing in inches between conductors a' and a''. The remaining mutual impedance are calculated similarly, using the appropriate spacing for each.

A series of more complex examples of the above type of problem is described by Wagner and Muller.[8]

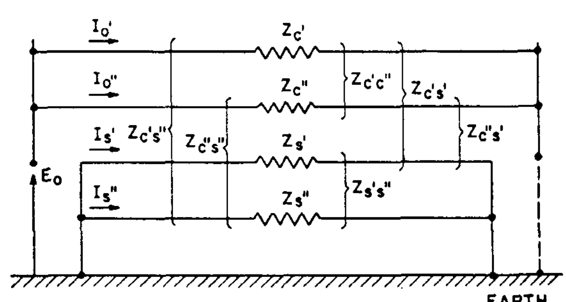
Example 4—Type of circuit: A three-phase 60-cycle cable circuit connected between a sending and a receiving bus, using two dissimilar three-conductor lead sheathed cables in parallel.

Conditions: Each cable contains three conductors that, by the nature of the cable construction, are symmetrically transposed so that the flow of positive- or negative-sequence currents will cause no zero-sequence voltage drops. Therefore, the sequence networks are not interdependent and an impedance value of each sequence may be calculated and used independently.

Problem: To find the zero-sequence impedance of the entire cable circuit, and to determine how zero-sequence current divides between cables.



(a) ACTUAL CIRCUIT



(b) EQUIVALENT CIRCUIT, WITH ALL QUANTITIES EXPRESSED IN ZERO-SEQUENCE TERMS

Fig. 17—Actual and equivalent zero-sequence circuit for two parallel three-conductor lead-sheathed cables (see Example 4).

TABLE 17—CURRENT CARRYING CAPACITY OF **THREE-CONDUCTOR GAS-FILLED** PAPER-INSULATED CABLES

Conductor Size AWG or MCM	Conductor Type (1)	Number of Equally Loaded Cables																			
		ONE				THREE				SIX				NINE				TWELVE			
		Per Cent Load Factor																			
		30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
		APPROXIMATE AMPERES PER CONDUCTOR[2]																			
15 000 Volts														Copper Temperature 81°C							
2	SR	159	154	146	137	153	144	133	121	149	136	120	107	144	129	112	97	139	123	104	90
1	SR	179	174	166	156	172	163	149	136	168	153	136	121	162	145	125	109	158	138	117	100
0	CS	203	195	182	176	196	185	169	154	190	173	154	137	183	164	141	122	178	156	131	112
00	CS	234	224	215	202	225	212	193	175	218	198	174	156	211	187	162	139	203	177	148	127
000	CS	270	258	245	230	258	242	220	198	249	225	198	174	241	212	182	157	232	202	168	144
0000	CS	308	295	281	261	295	276	250	223	285	257	224	196	275	241	205	176	265	227	189	162
250	CS	341	327	310	290	325	305	276	246	315	283	245	215	303	265	224	193	291	250	207	177
350	CS	417	397	375	346	397	369	330	293	383	340	293	255	366	318	267	227	350	301	245	208
500	CS	513	487	450	418	483	446	399	350	467	410	350	303	444	381	318	269	419	358	292	247
750	CS	643	606	562	514	602	551	485	426	576	502	423	365	545	464	383	323	519	432	348	293
		(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.70 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.81 at 40°C, 0.70 at 50°C)[3]			
23 000 Volts														Copper Temperature 77°C							
2	SR	156	150	143	134	149	141	130	117	145	132	117	105	140	125	107	84	134	119	100	86
1	SR	177	170	162	152	170	160	145	133	164	149	132	117	159	140	121	105	154	133	112	97
0	CS	200	192	183	172	192	182	166	149	186	169	147	132	178	158	136	118	173	149	126	109
00	CS	227	220	210	197	221	208	189	170	212	193	168	149	202	181	156	134	196	172	144	123
000	CS	262	251	238	223	254	238	216	193	242	220	191	169	230	206	175	150	222	195	162	139
0000	CS	301	289	271	251	291	273	246	219	278	250	215	190	264	233	197	169	255	221	182	157
250	CS	334	315	298	277	321	299	270	239	308	275	236	207	290	258	216	184	279	242	199	170
350	CS	405	379	358	331	384	356	318	283	369	327	280	243	347	305	255	217	335	285	233	199
500	CS	492	465	436	401	461	427	379	335	443	391	333	288	424	363	302	257	400	336	275	230
750	CS	616	583	541	495	577	528	465	407	550	479	402	347	520	439	364	306	490	408	329	276
		(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.65 at 50°C)[3]			
34 500 Volts														Copper Temperature 70°C							
0	CS	203	194	185	173	193	183	166	148	187	169	147	130	180	156	135	116	172	149	125	108
00	CS	230	220	209	196	218	207	187	168	212	191	166	147	203	178	152	132	194	169	141	121
000	CS	262	250	236	222	250	233	212	191	240	216	188	166	231	203	173	148	220	191	160	134
0000	CS	302	289	273	253	287	269	240	215	276	246	213	188	264	230	195	168	250	215	179	151
250	CS	332	317	300	279	316	294	266	235	304	271	233	206	290	252	212	183	275	233	196	165
350	CS	403	382	359	334	382	353	316	280	363	323	277	240	346	300	250	214	327	276	228	193
500	CS	492	464	435	400	458	422	376	333	439	386	328	285	413	354	295	250	390	329	269	226
750	CS	613	575	536	490	570	521	457	402	540	469	397	340	505	430	354	298	475	396	320	268
		(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.61 at 50°C)[3]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[3]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[3]				(1.10 at 10°C, 0.88 at 30°C, 0.75 at 40°C, 0.58 at 50°C)[3]				(1.10 at 10°C, 0.88 at 30°C, 0.74 at 40°C, 0.56 at 50°C)[3]			

[1] The following symbols are used here to designate conductor types:
SR—standard round concentric-stranded, CS—compact-sector stranded.

[2] Current ratings are based on the following conditions:
a. Ambient earth temperature = 20°C.
b. 60 cycle alternating current.
c. Ratings include dielectric loss, and all induced a-c losses.
d. One cable per duct, all cables equally loaded and in outside ducts only.

[3] Multiply tabulated currents by these factors when earth temperature is other than 20°C.

Circuit: Refer to Fig. 17. The three actual conductors in each cable have been reduced to one equivalent conductor in this figure, and all impedances are to be expressed in zero-sequence terms considering the earth as a return path for each circuit.

Complete voltage drop equations:

$$E_0 = I_{0'}Z_{c'} + I_{0''}Z_{c'c''} + I_{s'}Z_{c's'} + I_{s''}Z_{c's''}$$
$$E_0 = I_{0'}Z_{c'c''} + I_{0''}Z_{c''} + I_{s'}Z_{c''s'} + I_{s''}Z_{c''s''}$$
$$0 = I_{0'}Z_{c's'} + I_{0'}Z_{c''s'} + I_{s'}Z_{s'} + I_{s''}Z_{s's''}$$
$$0 = I_{0'}Z_{c's''} + I_{0''}Z_{c''s''} + I_{s'}Z_{s's''} + I_{s''}Z_{s''}$$

The voltage drop E_0 can be eliminated by subtraction, and the sheath currents can be solved in terms of conductor currents by using the last two equations only. Also, it is convenient to assume that the total zero-sequence current flowing into the sending bus is one ampere, which makes

$$I_{0''} = 1.0 - I_{0'}.$$

After making these changes, the following single equation results:

Modified equation:

$$\begin{aligned} I_{0'}[&(Z_{c'} - Z_{c'c''})(Z_{s'}Z_{s''} - Z^2_{s's''}) + (Z_{c''} - Z_{c'c''})(Z_{s'}Z_{s''} - Z^2_{s's''}) \\ &+ (Z_{c's'} - Z_{c''s'})(Z_{c's''}Z_{s's''} - Z_{c's'}Z_{s''}) + (Z_{c''s'} - Z_{c's'}) \\ &(Z_{c''s''}Z_{s's''} - Z_{c''s'}Z_{s''}) + (Z_{c's''} - Z_{c''s''})(Z_{c's'}Z_{s's''} - Z_{c's''}Z_{s'}) \\ &+ (Z_{c''s''} - Z_{c's''})(Z_{c''s'}Z_{s's''} - Z_{c''s''}Z_{s'})] = (Z_{c''} - Z_{c'c''}) \\ &(Z_{s'}Z_{s''} - Z^2_{s's''}) + (Z_{c''s'} - Z_{c's'})(Z_{c''s''}Z_{s's''} - Z_{c''s'}Z_{s''}) \\ &+ (Z_{c''s''} - Z_{c's''})(Z_{c''s'}Z_{s's''} - Z_{c''s''}Z_{s'}) \end{aligned}$$

This equation furnishes a solution for $I_{0'}$, from which $I_{0''}$ follows directly. To find the zero-sequence impedance of the entire circuit requires that one of the complete voltage drop equations be solved for E_0. Then

$$Z_0 = \frac{E_0}{I_{0'} + I_{0''}} = \frac{E_0}{1.0} = E_0, \text{ ohms.}$$

Supplementary equations: The equations necessary to determine each impedance value are shown here: every impedance must be expressed in zero-sequence terms, with the effect of earth as a return path included.

Table 18—Current Carrying Capacity of SINGLE-CONDUCTOR SOLID Paper-Insulated Cables

Number of Equally Loaded Cables in Duct Bank — Per Cent Load Factor — AMPERES PER CONDUCTOR[1]

7500 Volts — Copper Temperature, 85°C

Conductor Size AWG or MCM	THREE 30	THREE 50	THREE 75	THREE 100	SIX 30	SIX 50	SIX 75	SIX 100	NINE 30	NINE 50	NINE 75	NINE 100	TWELVE 30	TWELVE 50	TWELVE 75	TWELVE 100
6	116	113	109	103	115	110	103	96	113	107	98	90	111	104	94	85
4	154	149	142	135	152	144	134	125	149	140	128	116	147	136	122	110
2	202	196	186	175	199	189	175	162	196	183	167	151	192	178	159	142
1	234	226	214	201	230	218	201	185	226	210	190	172	222	204	181	162
0	270	262	245	232	266	251	231	212	261	242	219	196	256	234	208	184
00	311	300	283	262	309	290	270	241	303	278	250	224	295	268	236	208
000	356	344	324	300	356	333	303	275	348	319	285	255	340	308	270	236
0000	412	395	371	345	408	380	347	314	398	364	325	290	390	352	307	269
250	456	438	409	379	449	418	379	344	437	400	356	316	427	386	336	294
300	512	491	459	423	499	464	420	380	486	442	394	349	474	428	371	325
350	561	537	500	460	546	507	457	403	532	483	429	379	518	466	403	352
400	607	580	540	496	593	548	493	445	576	522	461	407	560	502	434	378
500	692	660	611	561	679	626	560	504	659	597	524	459	641	571	490	427
600	772	735	679	621	757	696	621	557	733	663	579	506	714	632	542	470
700	846	804	741	677	827	758	674	604	802	721	629	548	779	688	587	508
750	881	837	771	702	860	789	700	627	835	750	651	568	810	714	609	526
800	914	866	797	725	892	817	725	648	865	776	674	588	840	740	630	544
1000	1037	980	898	816	1012	922	815	725	980	874	756	657	950	832	705	606
1250	1176	1108	1012	914	1145	1039	914	809	1104	981	845	730	1068	941	784	673
1500	1300	1224	1110	1000	1268	1146	1000	884	1220	1078	922	794	1178	1032	855	731
1750	1420	1332	1204	1080	1382	1240	1078	949	1342	1166	992	851	1280	1103	919	783
2000	1546	1442	1300	1162	1500	1343	1162	1019	1442	1260	1068	914	1385	1190	986	839
	(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[2]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[2]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[2]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[2]			

15 000 Volts — Copper Temperature, 81°C

Conductor Size AWG or MCM	THREE 30	THREE 50	THREE 75	THREE 100	SIX 30	SIX 50	SIX 75	SIX 100	NINE 30	NINE 50	NINE 75	NINE 100	TWELVE 30	TWELVE 50	TWELVE 75	TWELVE 100
6	113	110	105	100	112	107	100	93	110	104	96	87	108	101	92	83
4	149	145	138	131	147	140	131	117	144	136	125	114	142	132	119	107
2	195	190	180	170	193	183	170	157	189	177	161	146	186	172	154	137
1	226	218	208	195	222	211	195	179	218	204	185	167	214	197	175	157
0	256	248	234	220	252	239	220	203	247	230	209	188	242	223	198	177
00	297	287	271	254	295	278	253	232	287	265	239	214	283	257	226	202
000	344	330	312	290	341	320	293	267	333	306	274	245	327	296	260	230
0000	399	384	361	335	392	367	335	305	383	352	315	280	374	340	298	263
250	440	423	396	367	432	404	367	334	422	387	345	306	412	372	325	286
300	490	470	439	406	481	449	406	369	470	429	382	338	457	413	359	316
350	539	516	481	444	527	491	443	401	514	468	416	367	501	450	391	342
400	586	561	522	480	572	530	478	432	556	506	447	395	542	485	419	366
500	669	639	592	543	655	605	542	488	636	577	507	445	618	551	474	412
600	746	710	656	601	727	668	598	537	705	637	557	488	685	608	521	452
700	810	772	712	652	790	726	647	581	766	691	604	528	744	659	564	488
750	840	797	736	674	821	753	672	602	795	716	625	547	772	684	584	505
800	869	825	762	696	850	780	695	622	823	741	646	565	800	707	604	522
1000	991	939	864	785	968	882	782	697	933	832	724	631	903	794	675	581
1250	1130	1067	975	864	1102	1000	883	784	1063	941	816	706	1026	898	759	650
1500	1250	1176	1072	966	1220	1105	972	856	1175	1037	892	772	1133	987	828	707
1750	1368	1282	1162	1044	1330	1198	1042	919	1278	1124	958	824	1230	1063	886	755
2000	1464	1368	1233	1106	1422	1274	1105	970	1360	1192	1013	869	1308	1125	935	795
	(1.08 at 10°C, 0.92 at 30°C, 0.82 at 40°C 0.71 at 50°C)[2]				(1.08 at 10°C, 0.92 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[2]				(1.08 at 10°C, 0.92 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[2]				(1.08 at 10°C, 0.92 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[2]			

23 000 Volts — Copper Temperature, 77°C

Conductor Size AWG or MCM	THREE 30	THREE 50	THREE 75	THREE 100	SIX 30	SIX 50	SIX 75	SIX 100	NINE 30	NINE 50	NINE 75	NINE 100	TWELVE 30	TWELVE 50	TWELVE 75	TWELVE 100
2	186	181	172	162	184	175	162	150	180	169	154	140	178	164	147	132
1	214	207	197	186	211	200	185	171	206	193	176	159	203	187	167	150
0	247	239	227	213	244	230	213	196	239	222	197	182	234	216	192	171
00	283	273	258	242	278	263	243	221	275	253	225	205	267	245	217	193
000	326	314	296	277	320	302	276	252	315	290	259	233	307	280	247	220
0000	376	362	340	317	367	345	315	288	360	332	297	265	351	320	281	250
250	412	396	373	346	405	380	346	316	396	365	326	290	386	351	307	272
300	463	444	416	386	450	422	382	349	438	404	360	319	428	389	340	301
350	508	488	466	422	493	461	418	380	481	442	393	347	468	424	369	326
400	548	525	491	454	536	498	451	409	521	478	423	373	507	458	398	349
500	627	600	559	514	615	570	514	464	597	546	480	423	580	521	450	392
600	695	663	616	566	684	632	568	511	663	603	529	466	645	577	496	431
700	765	729	675	620	744	689	617	554	725	656	574	503	703	627	538	467
750	797	759	702	643	779	717	641	574	754	681	596	527	732	650	558	483
800	826	786	726	665	808	743	663	595	782	706	617	540	759	674	576	500
1000	946	898	827	752	921	842	747	667	889	797	692	603	860	759	646	560
1250	1080	1020	935	848	1052	957	845	751	1014	904	781	676	980	858	725	630
1500	1192	1122	1025	925	1162	1053	926	818	1118	993	855	736	1081	940	791	682
1750	1296	1215	1106	994	1256	1130	991	875	1206	1067	911	785	1162	1007	843	720
2000	1390	1302	1180	1058	1352	1213	1053	928	1293	1137	967	831	1240	1073	893	760
	(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.68 at 50°C)[2]				(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.68 at 50°C)[2]				(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.68 at 50°C)[2]				(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.62 at 50°C)[2]			

Continued

TABLE 18—CURRENT CARRYING CAPACITY OF **SINGLE-CONDUCTOR SOLID** PAPER-INSULATED CABLES

(Continued)

Conductor Size AWG or MCM	Number of Equally Loaded Cables in Duct Bank: THREE				SIX				NINE				TWELVE			
	Per Cent Load Factor: 30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
	AMPERES PER CONDUCTOR[1]															
34 500 Volts														**Copper Temperature, 70°C**		
0	227	221	209	197	225	213	197	182	220	205	187	169	215	199	177	158
00	260	251	239	224	255	242	224	205	249	234	211	190	245	226	200	179
000	299	290	273	256	295	278	256	235	288	268	242	217	282	259	230	204
0000	341	330	312	291	336	317	291	267	328	304	274	246	321	293	259	230
250	380	367	345	322	374	352	321	294	364	337	303	270	356	324	286	253
300	422	408	382	355	416	390	356	324	405	374	334	298	395	359	315	278
350	464	446	419	389	455	426	388	353	443	408	364	324	432	392	343	302
400	502	484	451	419	491	460	417	379	478	440	390	347	466	421	368	323
500	575	551	514	476	562	524	474	429	547	500	442	392	532	479	416	364
600	644	616	573	528	629	584	526	475	610	556	491	433	593	532	459	401
700	710	675	626	577	690	639	574	517	669	608	535	470	649	580	500	435
750	736	702	651	598	718	664	595	535	696	631	554	486	675	602	518	450
800	765	730	676	620	747	690	617	555	723	654	574	503	700	624	535	465
1000	875	832	766	701	852	783	698	624	823	741	646	564	796	706	601	520
1250	994	941	864	786	967	882	782	696	930	833	722	628	898	790	670	577
1500	1098	1036	949	859	1068	972	856	760	1025	914	788	682	988	865	730	626
1750	1192	1123	1023	925	1156	1048	919	814	1109	984	845	730	1066	929	780	668
2000	1275	1197	1088	981	1234	1115	975	860	1182	1045	893	770	1135	985	824	704
2500	1418	1324	1196	1072	1367	1225	1064	936	1305	1144	973	834	1248	1075	893	760
	(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.61 at 50°C)[2]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.61 at 50°C)[2]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[2]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[2]			
46 000 Volts														**Copper Temperature, 65°C**		
000	279	270	256	240	274	259	239	221	268	249	226	204	262	241	214	191
0000	322	312	294	276	317	299	274	251	309	287	259	232	302	276	244	217
250	352	340	321	300	346	326	299	274	336	313	282	252	329	301	266	236
300	394	380	358	334	385	364	332	304	377	349	313	280	367	335	295	260
350	433	417	392	365	425	398	364	331	413	382	341	304	403	366	321	283
400	469	451	423	393	459	430	391	356	446	411	367	326	433	394	344	307
500	534	512	482	444	522	487	441	400	506	464	412	365	492	444	386	339
600	602	577	538	496	589	546	494	447	570	520	460	406	553	497	430	377
700	663	633	589	542	645	598	538	486	626	569	502	441	605	542	468	408
750	689	658	611	561	672	622	559	504	650	590	520	457	629	562	485	422
800	717	683	638	583	698	645	578	522	674	612	538	472	652	582	501	436
1000	816	776	718	657	794	731	653	585	766	691	604	528	740	657	562	487
1250	927	879	810	738	900	825	732	654	865	777	675	589	834	736	626	541
1500	1020	968	887	805	992	904	799	703	951	850	735	638	914	802	679	585
1750	1110	1047	959	867	1074	976	859	762	1028	915	788	682	987	862	726	623
2000	1184	1115	1016	918	1144	1035	909	805	1094	970	833	718	1048	913	766	656
2500	1314	1232	1115	1002	1265	1138	994	875	1205	1062	905	778	1151	996	830	708
	(1.11 at 10°C, 0.87 at 30°C, 0.73 at 40°C, 0.54 at 50°C)[2]				(1.11 at 10°C, 0.87 at 30°C, 0.72 at 40°C, 0.53 at 50°C)[2]				(1.11 at 10°C, 0.87 at 30°C, 0.72 at 40°C, 0.52 at 50°C)[2]				(1.12 at 10°C, 0.87 at 30°C, 0.70 at 40°C, 0.51 at 50°C)[2]			
69 000 Volts														**Copper Temperature, 60°C**		
350	395	382	360	336	387	364	333	305	375	348	312	279	365	332	293	259
400	428	413	389	362	418	393	358	328	405	375	335	300	394	358	315	278
500	489	470	441	409	477	446	406	370	461	425	379	337	447	405	354	312
600	545	524	490	454	532	496	450	409	513	471	419	371	497	448	391	343
700	599	573	536	495	582	543	490	444	561	514	455	403	542	489	425	372
750	623	597	556	514	605	562	508	460	583	533	472	417	563	506	439	384
800	644	617	575	531	626	582	525	475	603	554	487	430	582	523	453	396
1000	736	702	652	599	713	660	592	533	685	622	547	481	660	589	508	442
1250	832	792	734	672	806	742	664	595	772	698	610	535	741	659	564	489
1500	918	872	804	733	886	814	724	647	848	763	664	580	812	718	612	529
1750	994	942	865	788	957	876	776	692	913	818	711	618	873	770	653	563
2000	1066	1008	924	840	1020	931	822	732	972	868	750	651	927	814	688	592
2500	1163	1096	1001	903	1115	1013	892	791	1060	942	811	700	1007	880	741	635
	(1.13 at 10°C, 0.85 at 30°C, 0.67 at 40°C, 0.42 at 50°C)[2]				(1.13 at 10°C, 0.85 at 30°C, 0.66 at 40°C, 0.40 at 50°C)[2]				(1.13 at 10°C, 0.84 at 30°C, 0.65 at 40°C, 0.36 at 50°C)[2]				(1.14 at 10°C, 0.84 at 30°C, 0.64 at 40°C, 0.32 at 50°C)[2]			

[1]Current ratings are based on the following conditions:
a. Ambient earth temperature=20°C.
b. 60 cycle alternating current.
c. Sheaths bonded and grounded at one point only (open circuited sheaths).
d. Standard concentric stranded conductors.
e. Ratings include dielectric loss and skin effect.
f. One cable per duct, all cables equally loaded and in outside ducts only.

[2]Multiply tabulated values by these factors when earth temperature is other than 20°C.

$Z_{c'}=l[r_c+r_e+j(x_a+x_e-2x_d)]$ ohms, where l = circuit length in miles, and the other terms are defined as for Eq. (19).

$Z_{c''}$ is defined similarly.

$Z_{s'}=l[3r_s+r_e+j(3x_s+x_e)]$ ohms, where the terms are defined as for Eq. (23).

$Z_{s''}$ is determined similarly.

$Z_{c's'}=l[r_e+j(3x_s+x_e)]$ ohms, where the terms are defined as for Eq. (26).

$Z_{c''s''}$ is determined similarly.

$Z_{c'c''}=Z_{c's''}=Z_{s's''}=l[r_e+j(x_e-3x_d)]$ ohms, where

$x_d=0.2794\log_{10}\frac{S}{12}$, using for S the center-to-center spacing between cables,[3] in inches.

A more general version of the above type of problem, covering those cases where the cables are not necessarily bussed together, is described by Cheek.[9]

Example 5—The use of complex GMR's and GMD's will very often reduce a complicated problem to workable terms. The use and significance[2] of these factors should be studied thoroughly before attempting a solution by this method (see Chap. 3).

Table 19—Current Carrying Capacity for **SINGLE-CONDUCTOR OIL-FILLED** Paper-Insulated Cables (amperes per conductor)*

Circular Mils. or A.W.G. (B.& S.)	Rated Line Voltage—Grounded Neutral				
	34 500	46 000	69 000	115 000	138 000
	Maximum Copper Temperature—Deg. C.				
	75	75	75	70	70
0	256	.....	.....	.....	.....
00	287	286	282	.....	.....
000	320	310	300	.....	.....
0000	378	367	367	347	335
250 000	405	395	390	365	352
300 000	450	440	430	402	392
350 000	492	482	470	438	427
400 000	528	512	502	470	460
500 000	592	592	568	530	522
600 000	655	650	628	585	578
700 000	712	710	688	635	630
750 000	742	740	715	667	658
800 000	767	765	740	685	680
1 000 000	872	870	845	775	762
1 250 000	990	982	955	875	852
1 500 000	1 082	1 075	1 043	957	935
1 750 000	1 165	1 162	1 125	1 030	1 002
2 000 000	1 240	1 240	1 200	1 100	1 070
Deg. C.	Correction Factor for Various Earth Temps.				
10	1.08	1.08	1.08	1.08	1.09
20	1.00	1.00	1.00	1.00	1.00
30	0.90	0.90	0.90	0.89	0.89
40	.79	.79	.79	.77	.77

75% load factor assumed.
Ratings include dielectric loss and skin effect.
Ratings based on open-circuited sheath operation; i.e.—no sheath loss considered.
*Applies to three similar loaded cables in a duct bank; for six loaded cables in a duct bank, multiply above values by 0.91.

Table 20—Suggested Withstand Impulse Voltages for Cables with Metallic Covering*

Insulation Class kv	Basic Impulse Insulation Level for Equipment	Solid-Paper Insulation		Oil-Filled Paper Insulation	
		Insulation Thickness mils	Withstand Voltage kv	Insulation Thickness mils	Withstand Voltage kv
1.2	30	78	94	...	...
2.5	45	78	94	...	...
5.0	60	94	113	...	...
8.7	75	141	169	...	...
15	110	203	244	110	132
23	150	266	319	145	174
34.5	200	375	450	190	228
46	250	469	563	225	270
69	350	688	825	315	378
115	550	...	...	480	575
138	650	...	...	560	672
161	750	...	...	648	780
230	1050	...	...	925	1110

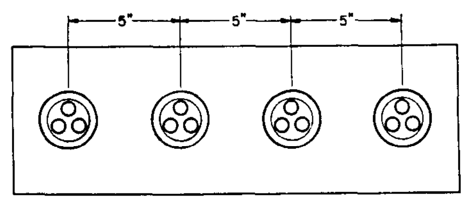
*Based on recommendations by Halperin and Shanklin.[29]

Circuit: Four paralleled cables similar to the three-conductor belted cable described in Example 1, and arranged in a duct bank as illustrated in Fig. 18.

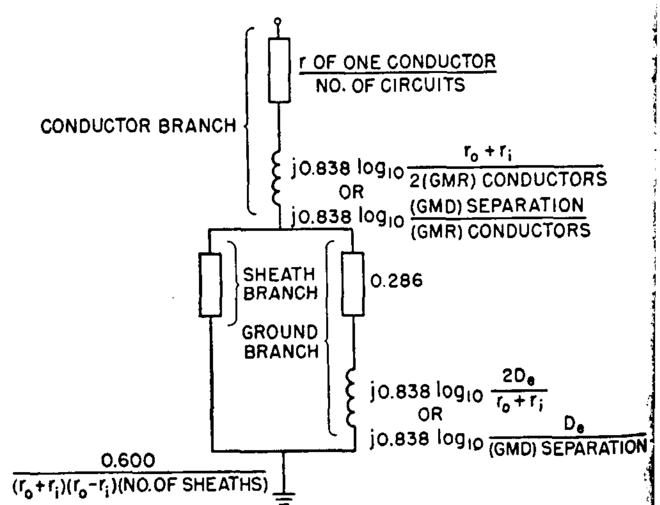
Problem: To find the overall zero-sequence impedance of the circuit, with sheaths and ground in parallel, or with return current only in the sheaths.

GMR of three conductors,

$GMR_{3c}=0.338$ inches (from example 1).



(a) Cable configuration.



(b) General equivalent circuit.

Fig. 18—Four three-conductor cables in a duct bank (see Example 5).

GMR of the four conductor groups,

$$GMR_{4g} = \sqrt[16]{(0.338)^4(5)^6(10)^4(15)^2} = 3.479 \text{ inches.}$$

Equivalent spacing of three conductors to their sheath,

$$S_{eq} = \frac{r_i + r_o}{2} = 0.812 \text{ inches.}$$

GMD among the conductors and the sheaths,

$$GMD_{(4g-4s)} = \sqrt[16]{(0.812)^4(5)^6(10)^4(15)^2} = 4.330 \text{ inches.}$$

From Fig. 18(b), resistance of the sheath branch,

$$\frac{0.600}{(1.623)(0.109)(4)} = 0.848 \text{ ohms per mile.}$$

Also from Fig. 18(b), impedance of the ground branch

$$= 0.286 + j0.838 \log_{10} \frac{2800 \times 12}{4.330}$$

$$= 0.286 + j3.260 \text{ ohms per mile.}$$

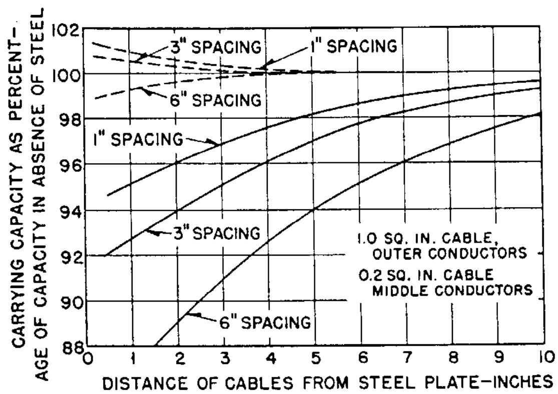


Fig. 19—Effect of steel plates on current-carrying capacity of single-conductor cables. Three phase system; flat configuration.

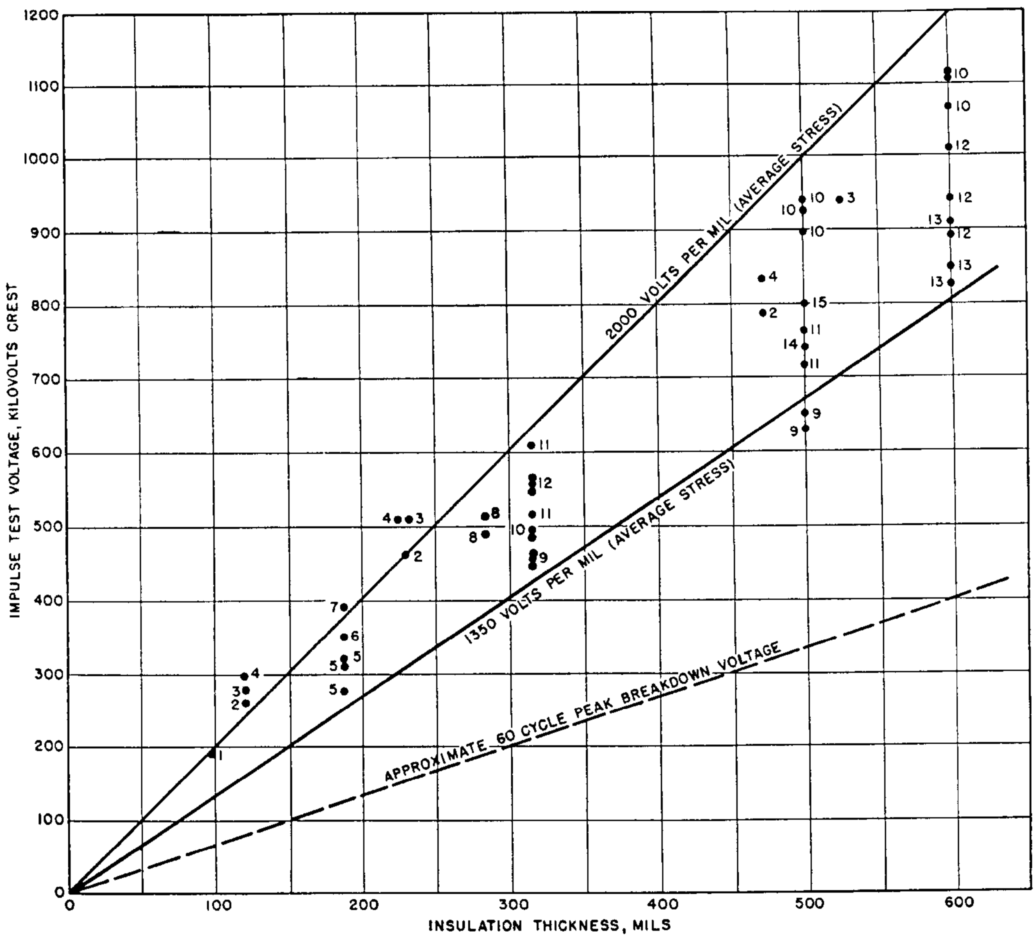


Fig. 20—Summary of some impulse tests on paper-insulated cables (based on information presented by Foust and Scott[13]).

Key:

1 Davis and Eddy,[12] 1 x 10 negative wave, high density paper, solid insulation (Simplex Wire and Cable Co.).
2 Held and Leichsenring,[17] negative wave, solid insulation.
3 Held and Leichsenring, positive and negative waves, oil-filled insulation.
4 Held and Leichsenring, positive wave, solid insulation.
5 An unpublished test, regular density paper, oil-filled insulation (General Cable Corporation).
6 Foust and Scott, average of five tests, 1 x 10 positive wave, regular density paper, solid insulation (General Electric Co.).
7 An unpublished test, high density paper, oil-filled insulation (General Cable Corporation).
8 An unpublished test, solid insulation (The Okonite Company).
9 Foust and Scott, 1.5 x 40 positive wave, regular density paper, solid insulation.
10 Foust and Scott, combination regular and medium density paper, solid insulation.
11 Foust and Scott, high density paper, solid insulation.
12 Foust and Scott, medium density paper, solid insulation.
13 Foust and Scott, 1.5 x 40 positive wave, combination regular and medium density paper, solid insulation.
14 Foust and Scott, 0.5 x 40 positive wave, regular density paper, solid insulation.
15 Foust and Scott, 0.5 x 5 positive wave, regular density paper, solid insulation.

The zero-sequence impedance with sheath and ground in parallel,

$$Z_0=\frac{0.848(0.286+j3.260)}{0.848+(0.286+j3.260)}+0.247+j0.0797$$
$$=1.022+j0.275 \text{ ohms per phase per mile.}$$

The absolute value of this impedance is 1.06 ohms per phase per mile.

The zero-sequence impedance considering all return current in the sheath and none in the ground,

$$Z_0=(0.247+j0.0797)+0.848$$
$$=1.095+j0.0797 \text{ ohms per phase per mile.}$$

The absolute value of this impedance is 1.1 ohms per phase per mile, or substantially the same as with the sheath and ground in parallel. In this case the effect of high sheath resistance is minimized by the fact that four sheaths are paralleled.

V. IMPULSE STRENGTH OF CABLES

Power-transmission circuits are often made up of cables and overhead-line sections connected in series, and this construction may impose lightning-surge voltages on the cable insulation. Even when circuits are totally underground, it is possible that cable insulation will be stressed by transient overvoltages caused by switching operations. For these reasons the impulse strength of cable insulation is information of some value for predicting cable performance in an actual installation.

No industry-wide standards have been established for cable impulse strength. Test data from various sources is available,[12,13] and some of these results for paper-insulated cables are shown in Fig. 20. Several variables are inherent in the curves, so that the spread of the test points is wider than might be obtained with uniformly controlled test conditions. The factors not yet completely investigated include the effect of normal insulation aging, the relation between actual voltage gradient within the insulation and the average gradient, wave shape and polarity of the test impulse voltage, and grade or compounding of insulation.

Using 1200 volts per mil average stress as a safe withstand impulse strength for paper-insulated cables, as suggested by Halperin and Shanklin,[18] the withstand voltages for representative cables may be listed as in Table 20.

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CHAPTER 5

POWER TRANSFORMERS AND REACTORS

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In this chapter are included the fundamental theory, operating practices, pertinent application data, and some of the physical characteristics of power transformers and reactors. No attempt is made to give a complete exposition of the material. It is expected that the listed references will be consulted for a more detailed consideration of each section. Although the fundamental theory presented here holds also for distribution transformers, the standards of operation and present practices regarding distribution transformer application are not included in this chapter. Grounding transformers are included since they are ordinarily associated with power systems.

I. THEORY

1. Fundamental Considerations

Before going into the various problems involved in the application of transformers and the methods used in analyzing their effect on system operation, it is well to review briefly the fundamental theory of transformer action.

Two windings on a common magnetic core are pictured in Fig. 1. Let the number of turns in the P winding be n_1,

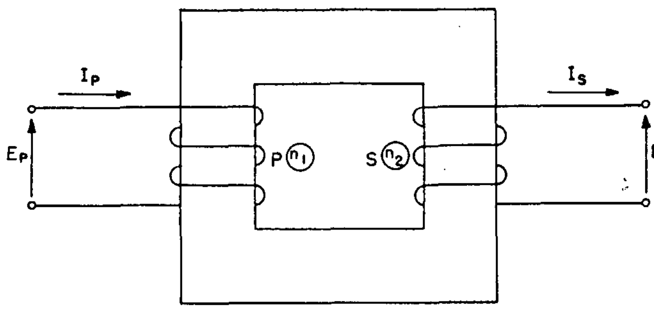


Fig. 1—Two-winding transformer.

and the number of turns in the S winding be n_2. Assume that there is a flux in the core which links both windings and is a sinusoidal function of time.

$$\phi=\phi_{max}\sin\omega t \tag{1}$$

Then the voltage induced in the P winding at any instant by the flux is

$$e_p=-n_1\frac{d\phi}{dt}\times10^{-8}\text{ volts} \tag{2}$$

$$=-n_1\omega\phi_{max}\cos\omega t\times10^{-8}\text{ volts}$$

where $\omega=2\pi f$

hence, $e_p=-2\pi f n_1\phi_{max}\cos\omega t\times10^{-8}$ volts

and the rms value of this voltage is

$$E_p=\frac{2\pi f}{\sqrt{2}}n_1\phi_{max}\times10^{-8}\text{ volts}$$

$$=4.44fn_1AB_{max}\times10^{-8}\text{ volts} \tag{3}$$

where, f = frequency in cycles per second.
A = cross sectional area of magnetic circuit in square centimeters (assumed uniform).
B_{max} = maximum flux density in the core in lines per square centimeter.

Similarly, the rms voltage induced in the S winding by the flux is given by

$$E_s=4.44fn_2AB_{max}\times10^{-8}\text{ volts.} \tag{4}$$

Thus it is evident that a sinusoidal flux linking a coil induces in it a voltage which is also sinusoidal and which lags the flux by 90 electrical degrees.

To apply the above principle to the operation of a transformer, refer again to Fig. 1 and consider the S winding as open and let a sinusoidal voltage be impressed on the P winding. The current, I_e, that flows in the P winding under this condition ($I_s=0$) is called the exciting current and sets up an alternating flux about that winding, which consists of two parts: a mutual flux whose path is wholly in the core and which, therefore, links both windings, and a leakage flux whose path is partly in air and which links only the P winding. The ratio of the leakage flux to the mutual flux depends on the relative reluctance of their respective paths, which in turn is a function of the saturation of the core and the magnitude of the current. It is convenient to consider the voltage induced in the P winding, by the flux linking it, as made up of two components, one produced by the linkages resulting from the mutual flux and the other produced by leakage flux. In the ordinary commercial transformer the leakage flux is small and can be neglected for the present. Then, if the small iR drop in the winding is also ignored, the voltage induced in the P winding by the mutual flux can, with close approximation, be set equal and opposite to the impressed voltage. If, as assumed, the latter is sinusoidal, then the mutual flux must also be sinusoidal and the induced voltage is given by Eq. (3),

$$E_p=4.44fn_1AB_{max}\times10^{-8}\text{ volts.}$$

By hypothesis, all of the mutual flux which has just been considered in connection with the P winding must also link the S winding. Hence, a voltage is induced in the S winding, which is expressed by Eq. (4),

$$E_s=4.44fn_2AB_{max}\times10^{-8}\text{ volts.}$$

If the circuit connected to the S winding is closed, a current, I_s, flows and, in the manner already described in connection with the P winding, sets up a mutual and leakage flux about the winding. The direction of this current is such that the mutual flux produced by it opposes that produced by the P winding and it, therefore, tends to nullify the flux in the core. Consideration of the energies involved shows that an additional component, I_p', must be added to the current in the P winding before the S winding is closed, such that the magnetomotive force acting on the magnetic circuit remains unchanged after S is closed. In other words, the resultant flux in the core produced by the combined action of the currents flowing in the P and S windings must equal the mutual flux present when the S winding is open. Therefore,

$$n_1 I_e = n_1 I_p - n_2 I_s, \tag{5}$$

remembering that the flux caused by I_s is opposite that caused by I_p which accounts for the negative sign. In a well-designed transformer, the exciting current is small in comparison to the normal load current I'_p, hence we can assume the total current, I_p, in the P winding to be equal to I'_p and obtain

$$I_s = +\frac{n_1}{n_2} I_p. \tag{6}$$

The leakage flux produced by I_s induces a voltage in the S winding opposing that produced by the mutual flux. However, it is small as in the case of the P winding, and, if neglected along with the resistance drop, permits writing the relation between the P and S voltages as

$$E_s = +\frac{n_2}{n_1} E_p. \tag{7}$$

The seven equations developed above summarize the general relationships between the flux, the induced voltages, and the primary and secondary voltages and currents involved in transformer action. However, they are based on a number of assumptions that, in analyzing the operation of the transformer or of the system to which it is connected, cannot always be made. A more rigorous development that takes into consideration the effects of exciting current, losses, and leakage fluxes is therefore required.

Referring again to Fig. 1, and considering instantaneous currents and voltages, the classical equations for the coupled circuits are

$$e_p = R_P i_p + L_P \frac{di_p}{dt} - M \frac{di_s}{dt} \tag{8}$$

$$e_s = M \frac{di_p}{dt} - R_S i_s - L_S \frac{di_s}{dt}$$

where R_P and R_S are, respectively, the effective resistances of the primary and secondary windings: L_P and L_S are the self-inductances of the primary and secondary windings: and M is the mutual inductance between the two windings. The positive direction of current flow in the two windings is taken such that the fluxes set up by the two currents will be in opposition.

The coefficients L_P, L_S, and M are not constant but vary with the saturation of the magnetic circuit[1]. As previously stated, the total flux linking either winding can be divided into two components, a leakage flux whose path is wholly or partly in air and a mutual flux most of which lies in the iron core. Furthermore, the mutual coupling between circuits must have an energy component to furnish the iron loss in the magnetic circuit. With the above considerations in mind the equivalent circuit representing the two coupled windings in Fig. 1 can be derived[2].

The equivalent circuit is shown in Fig. 2(a), where the mathematical artifice of an ideal transformer[2] is introduced to preserve actual voltage and current relationships at the terminals, and to insulate the two windings. The ideal transformer is defined as having no losses, no impedance drop, and requiring no exciting current. The ratio of transformation for the ideal transformer is N, where

$$N = \frac{n_2}{n_1}. \tag{9}$$

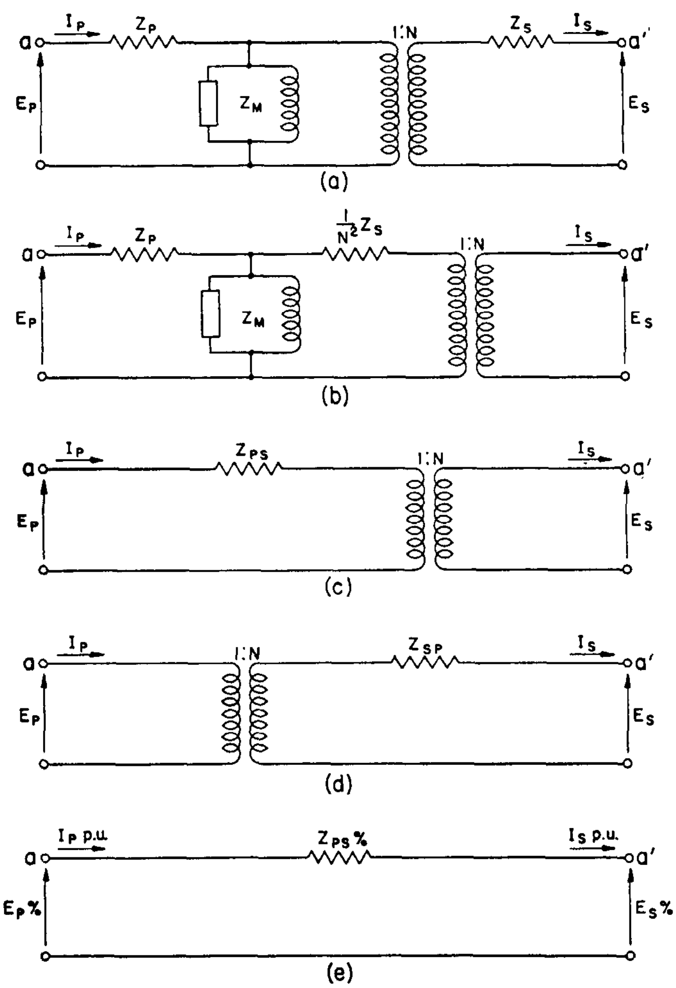


Fig. 2—Equivalent circuits for two-winding transformer.

(a) Equivalent circuit in ohms, with magnetizing current considered.
(b) Equivalent circuit in ohms, with all impedances on the primary voltage base.
(c) Equivalent circuit in ohms, with the magnetizing branch neglected.
(d) Equivalent circuit in ohms, with the leakage impedance referred to the secondary voltage base.
(e) Equivalent circuit in percent.

The shunt resistance branch in Z_M represents the iron losses and the shunt reactive branch $\left(j\omega\frac{n_1}{n_2}M\right)$ provides a path for the no load, or exciting current of the transformer. The variation in M during the cycle of instantaneous current and voltage variation is ignored, and a mean value is used. The branches, $Z_P=R_P+j\omega\left(L_P-\frac{n_1}{n_2}M\right)$ and, $Z_S=R_S+j\omega\left(L_S-\frac{n_2}{n_1}M\right)$ are essentially constant, regardless of instantaneous current variations, since their corresponding leakage fluxes lie mostly in air. Z_P and Z_S are components of the leakage impedance between the P and S windings such that

$$Z_{PS}=Z_P+\frac{1}{N^2}Z_S \quad (10)$$

Z_{PS} is defined as the leakage impedance between the P and S windings, as measured in ohms on the P winding with the S winding short-circuited. Actually it is not possible[3] to segregate Z_{PS} into two parts, Z_P associated with the P winding and Z_S associated with the S winding by any method of test; for example, Z_P, the portion of Z_{PS} associated with the primary winding, varies with excitation and load conditions. It is customary, in many calculations involving the equivalent circuit, to make

$$Z_P=\frac{1}{N^2}Z_S=\tfrac{1}{2}Z_{PS} \quad (11)$$

The ideal transformer can be shifted to the right, as in Fig. 2(b), to get all branches of the circuit on the same voltage base. Since the impedance of the shunt branch is large compared to Z_{PS}, it can be omitted for most calculations involving transformer regulation, and the equivalent circuit becomes that of Fig. 2(c). A notable exception to those cases where the shunt branch can be disregarded is the case of the three-phase core-form transformer excited with zero-sequence voltages. This will be discussed in detail later.

The form of the equivalent circuit given in Fig. 2(c) can be changed to show the leakage impedance referred to the secondary voltage, by shifting the ideal transformer to the left, as in Fig. 2(d). For this condition Z_{SP}, the leakage impedance between the P and S windings as measured in ohms on the S winding with the P winding short-circuited, is related to Z_{PS} as follows:

$$Z_{SP}=N^2Z_{PS}=\left(\frac{n_2}{n_1}\right)^2 Z_{PS} \quad (12)$$

The equivalent circuit using percentage impedances, percentage voltages, and currents in per unit is given in Fig. 2(e). An ideal transformer to maintain transformation ratios is not required.

2. Transformer Vector Diagram

The vector diagram illustrating the relationship between the terminal voltages, the internal induced voltages and the currents in the transformer of Fig. 1 can be drawn directly from the equivalent circuit for the transformer. This circuit is repeated in Fig. 3(a) and the various voltages and currents are identified there. The primary and secondary leakage impedances Z_P and Z_S are shown separately, and the primary and secondary resistances R_P and R_S are also indicated. I_h and I_m represent the core-loss component and the magnetizing component respectively of the exciting current I_e. The vector diagram in Fig. 3(b) is drawn for a 1:1 ratio of transformation and for a load of lagging power factor. The power-factor angles at the P winding terminals and the S winding terminals are designated in the diagram as θ_P and θ_S respectively.

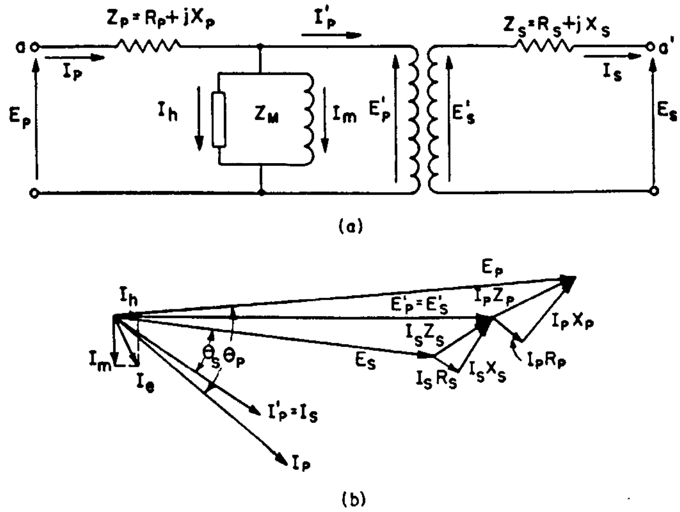


Fig. 3—Equivalent circuit and corresponding vector diagram for two-winding transformer.

II. ELECTRICAL CHARACTERISTICS

3. Transformer Impedances

The turns ratio of a two-winding transformer determines the ratio between primary and secondary terminal voltages, when the transformer load current is zero. However, when load is applied to the transformer, the load current encounters an apparent impedance within the transformer which causes the ratio of terminal voltages to depart from the actual turns ratio. This internal impedance consists of two components: (1) a reactance derived from the effect of leakage flux in the windings, and (2) an equivalent resistance which represents all losses traceable to the flow of load current, such as conductor I^2R loss and stray eddy-current loss.

Impedance drop is conveniently expressed in percent, and is the impedance-drop voltage expressed as a percentage of rated terminal voltage, when both voltages are referred to the same circuit; in three-phase transformer banks, it is usually appropriate to refer both impedance-drop voltage and rated voltage to a line-to-neutral basis. Percent impedance is also equal to measured ohmic impedance, expressed as a percentage of "normal" ohms. Normal ohms for a transformer circuit are defined as the rated current (per phase) divided into rated voltage (line-to-neutral).

Representative impedance values for distribution and power transformers are given in Table 1; for most purposes the impedances of power transformers may be considered

TABLE 1—TRANSFORMER IMPEDANCES

(a) Standard Reactances and Impedances for Ratings 500 kva and below (for 60-cycle transformers)

Single-Phase Kva Rating*	Rated-Voltage Class in kv							
	2.5		15		25		69	
	Average Reactance %	Average Impedance %	Average Reactance %	Average Impedance %	Average Reactance %	Average Impedance %	Average Reactance %	Average Impedance %
3	1.1	2.2	0.8	2.8				
10	1.5	2.2	1.3	2.4	4.4	5.2		
25	2.0	2.5	1.7	2.3	4.8	5.2		
50	2.1	2.4	2.1	2.5	4.9	5.2	6.3	6.5
100	3.1	3.3	2.9	3.2	5.0	5.2	6.3	6.5
500	4.7	4.8	4.9	5.0	5.1	5.2	6.4	6.5

*For three-phase transformers use ⅓ of the three-phase kva rating, and enter table with rated line-to-line voltages.

as equal to their reactances, because the resistance component is so small. The standard tolerances by which the impedances may vary are $\pm 7\frac{1}{2}$ percent of specified values for two-winding transformers and ± 10 percent for three-winding, auto, and other non-standard transformers.

The percent resistance of transformers is less consistent among various designs than is the impedance, and though the curves in Fig. 4 show definite values for transformer resistance, considerable deviation from these figures is possible.

Transformers can be designed to have impedances within closer tolerances than mentioned above, or impedances outside the normal range, but usually at extra cost.

A guide to the impedances of three-winding transformers is given below (this guide does not apply to auto-transformers).

(1) Select a kva base equal to the kva rating of the largest capacity winding, regardless of voltage rating. All impedances will be referred to this base.

(2) Select a percent impedance between the medium-voltage and the high-voltage circuits ($Z_{MH}\%$), lying between the limits shown for two-winding transformers in Table 1.

(3) The percent impedance between the medium-voltage and low-voltage circuits ($Z_{ML}\%$) may lie between the limits of 0.35 ($Z_{MH}\%$) and 0.80 ($Z_{MH}\%$). Select a value of $Z_{ML}\%$ lying within this range.

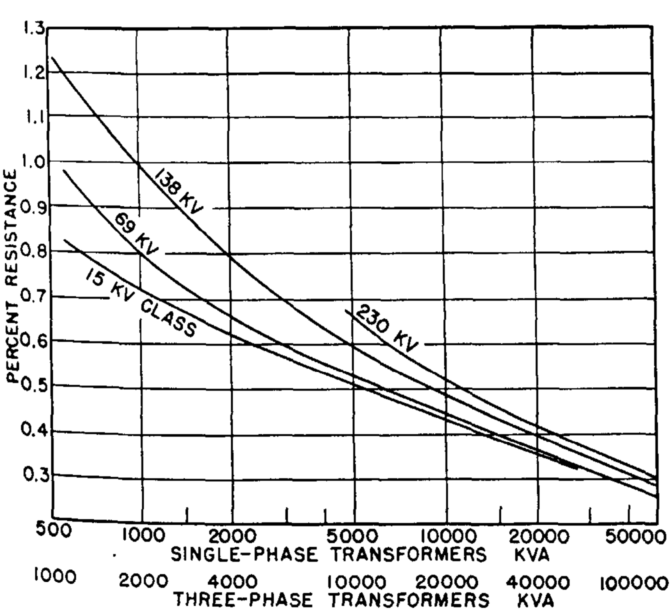


Fig. 4—Percent resistance of transformers, based on OA kva ratings.

TABLE 1—TRANSFORMER IMPEDANCES (Continued)

(b) Standard Range in Impedances for Two-Winding Power Transformers Rated at 55 C Rise

(Both 25- and 60-cycle transformers)

High-Voltage Winding Insulation Class kv	Low-Voltage Winding Insulation Class kv	Impedance Limit in Percent			
		Class OA OW OA/FA* OA/FA/FOA*		Class FOA FOW	
		Min.	Max.	Min.	Max.
15	15	4.5	7.0	6.75	10.5
25	15	5.5	8.0	8.25	12.0
34.5	15	6.0	8.0	9.0	12.0
	25	6.5	9.0	9.75	13.5
46	25	6.5	9.0	9.75	13.5
	34.5	7.0	10.0	10.5	15.0
69	34.5	7.0	10.0	10.5	15.0
	46	8.0	11.0	12.0	16.5
92	34.5	7.5	10.5	11.25	15.75
	69	8.5	12.5	12.75	18.75
115	34.5	8.0	12.0	12.0	18.0
	69	9.0	14.0	13.5	21.0
	92	10.0	15.0	15.0	23.25
138	34.5	8.5	13.0	12.75	19.5
	69	9.5	15.0	14.25	22.5
	115	10.5	17.0	15.75	25.5
161	46	9.5	15.0	13.5	21.0
	92	10.5	16.0	15.75	24.0
	138	11.5	18.0	17.25	27.0
196	46	10	15.0	15.0	22.5
	92	11.5	17.0	17.25	25.5
	161	12.5	19.0	18.75	28.5
230	46	11.0	16.0	16.5	24.0
	92	12.5	18.0	18.75	27.0
	161	14.0	20.0	21.0	30.0

*The impedances are expressed in percent on the self-cooled rating of OA/FA and OA/FA/FOA

Definition of transformer classes:

OA—Oil-immersed, self cooled OW—Oil-immersed, water-cooled.
OA/FA—Oil-immersed, self-cooled/forced-air-cooled.
OA/FA/FOA—Oil-immersed, self-cooled/forced-air-cooled/forced oil cooled.
FOA—Oil-immersed, forced-oil-cooled with forced air cooler.
FOW—Oil-immersed, forced-oil-cooled with water cooler.

Note: The through impedance of a two-winding autotransformer can be estimated knowing rated circuit voltages, by multiplying impedance obtained from this table by the factor $\left(\frac{HV-LV}{HV}\right)$.

(4) Having established $Z_{MH}\%$ and $Z_{ML}\%$, the percent impedance between the high-voltage and low-voltage circuits ($Z_{HL}\%$) is determined as follows:

$$Z_{HL}\% = 1.10(Z_{MH}\% + Z_{ML}\%) \tag{13}$$

When impedances outside the above ranges are required, a suitable transformer can usually be supplied but probably at increased cost.

4. Regulation

The full load regulation of a power transformer is the change in secondary voltage, expressed in percent of rated secondary voltage, which occurs when the rated kva output at a specified power factor is reduced to zero, with the primary impressed terminal voltage maintained constant. Percent regulation can be calculated at any load and any power factor by an approximate formula:

$$\text{Regulation} = \left[pr + qx + \frac{(px - qr)^2}{200} \right] \times \frac{\text{operating current}}{\text{rated current}} \tag{14}$$

where:

"Regulation" is a percent quantity;

r = percent resistance

$$= \frac{\text{load losses in kw, at rated kva}}{\text{rated kva}} \times 100$$

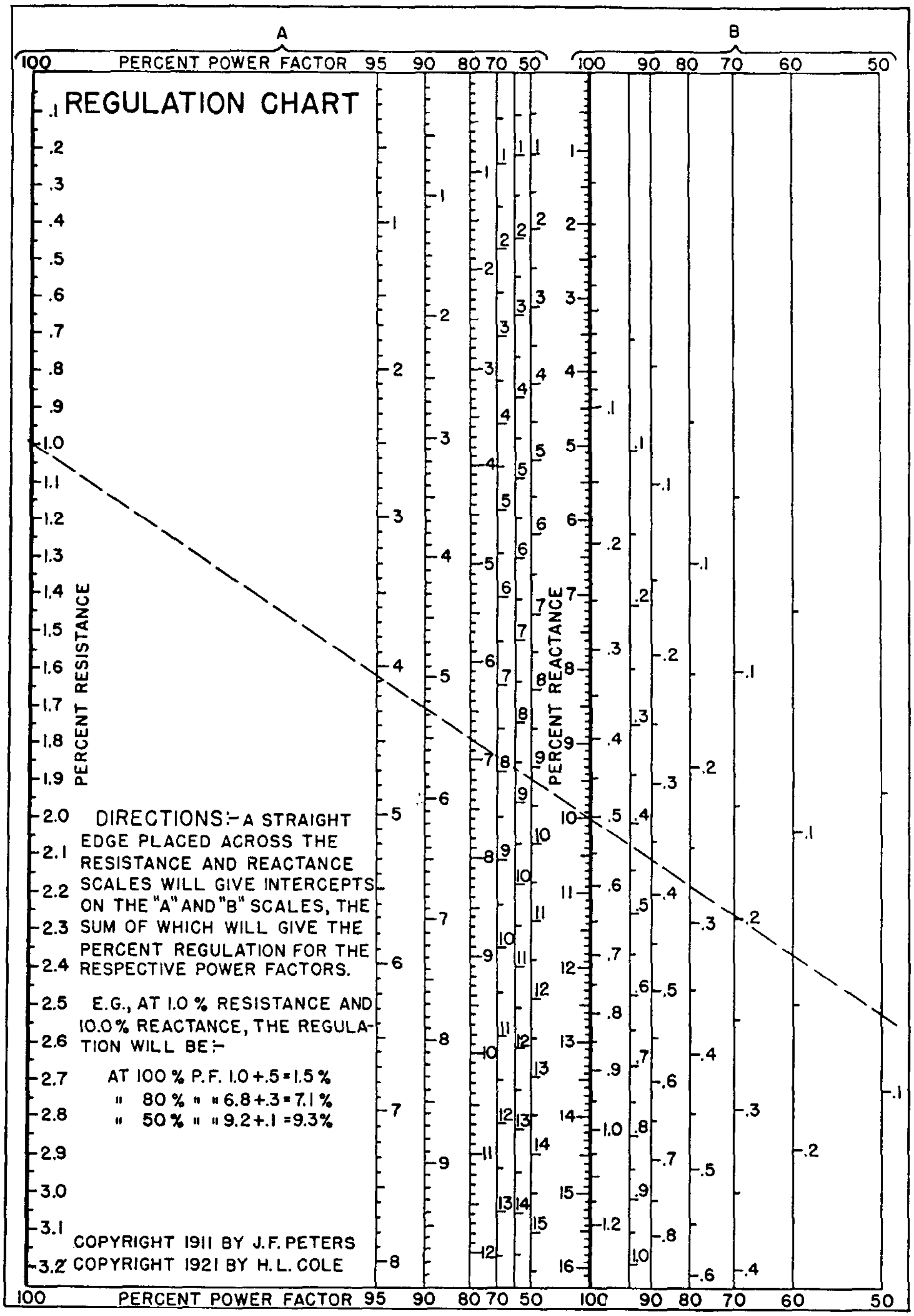


Fig. 5—Chart for calculating regulation of transformers.

z = percent impedance $= \dfrac{\text{impedance kva}}{\text{rated kva}} \times 100$

x = percent reactance $= \sqrt{z^2 - r^2}$

$p = \cos\theta$

$q = \sin\theta$

θ = power factor angle of load (taken as positive when current lags voltage).

The full-load regulation of a transformer can be determined for any power factor from the chart in Fig. 5; this chart is based on Eq. (14).

Typical regulation for three-phase transformers at full load and various power factors is shown in Table 2. These

TABLE 2—APPROXIMATE REGULATION FOR 60-CYCLE THREE-PHASE TYPE OA TRANSFORMERS AT FULL LOAD

Insulation Class kv	Lagging Power Factor Percent	Percent Regulation		
		1000 kva	10 000 kva	100 000 kva
15	80	4.2	3.9	
	90	3.3	3.1	
	100	1.1	0.7	
34.5	80	5.0	4.8	
	90	4.0	3.7	
	100	1.2	0.8	
69	80	6.1	5.7	5.5
	90	4.9	4.4	4.2
	100	1.4	0.9	0.6
138	80	7.7	7.2	7.0
	90	6.2	5.6	5.4
	100	1.8	1.2	0.9
230	80		9.7	9.4
	90		7.6	7.3
	100		1.7	1.3

Note: These figures apply also to OA/FA and OA/FA/FOA transformers, at loads corresponding to their OA ratings.

figures also apply, but less accurately, to transformer banks made up of three single-phase transformers; in this case the table should be entered with the three-phase bank kva rating.

The regulation of three-winding transformers can be calculated directly from transformer equivalent circuits, if the impedance branches and loading for each circuit are known. The regulation of four-winding transformers may also be calculated using formulas developed by R. D. Evans.[4]

5. Definition of Efficiency

The efficiency of a transformer, expressed in per unit, is the ratio of real power output to power input;

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = 1 - \frac{\text{Losses}}{\text{Input}}. \tag{15}$$

Total losses are the sum of the no-load losses and load losses. No-load losses are eddy-current loss, hysteresis loss, I^2R loss caused by exciting current, and dielectric loss; that is, all losses incident to magnetization at full voltage with the secondary circuit open. Load losses are I^2R loss caused by load current, eddy-current loss induced by stray fluxes within the transformer structure, and similar losses varying with load current.

No-load losses are measured at rated frequency and rated secondary voltage, and can be considered as independent of load. Load losses are measured at rated frequency and rated secondary current, but with the secondary short-circuited and with reduced voltage applied to the primary. Load losses can be assumed to vary as the square of the load current.

6. Methods of Calculating Efficiency

Conventional Method—This method is illustrated below for a transformer having 0.50 percent no-load loss and 1.0 percent load loss at full load. Percent no-load loss is determined by dividing the no-load loss in watts by 10 times the kva rating of the transformer, and the percent load loss (total minus no-load) is determined by dividing the load loss in watts by 10 times the kva rating of the transformer. Note that the no-load loss remains constant regardless of the load whereas the load loss varies directly as the square of the load.

Percent load........	100.00	75.00	50.00	25.00	(1)
Percent load loss....	1.00	.562	.25	.062	(2)
Percent no-load loss..	.50	.50	.50	.50	(3)
Sum of (2) and (3)..	1.50	1.062	.75	.562	(4)
Sum of (1) and (4)..	101.50	76.062	50.75	25.562	(5)
Dividing 100 times (4) by (5)........	1.48	1.40	1.48	2.20	(6)
Subtract (6) from 100	98.52	98.60	98.52	97.80	(efficiency)

Slide-Rule Method—This method is illustrated for the same transformer.

Percent load..............	100.00	75.00	50.00	25.00	(1)
Percent no load loss........	.50	.50	.50	.50	(2)
Percent load loss...........	1.00	.562	.25	.062	(3)
Sum of (2) and (3).........	1.50	1.062	.75	.562	(4)
Sum of (1) and (4).........	101.50	76.062	50.75	25.562	(5)

At this point the operations are continued on the slide rule, and are described here for the full load point only:

1. Set 1.5 (sum of no-load and load losses) on *D* scale.
2. Set 101.5 over this on the *C* scale.
3. Now starting at the right end of scale *D*, read the first figure (i.e., 1) as 90, the next (i.e., 9) as 91, the next (i.e., 8) as 92, etc., until 98.52 is read under the left end (i.e., 1) of scale *C*. This 98.52 is the percent efficiency at full load.

This procedure is repeated in a similar manner for other loads.

NOTE—If the sum of the percent no-load and load loss at full load is 1 percent or less, the first figure at the right end of *D* scale (i.e., 1) is read as 99 percent and the second figure (i.e., 9) is read as 99.1, the third figure (i.e., 8) is read as 99.2, etc.

If the sum of the percent no-load and load loss is greater than 1 percent as in the case illustrated above, the right end is read as 90 percent. In calculating the values for the other points, judgment will indicate whether 90 or 99 is to be used as the first figure on the right end of scale *D*.

Chart Method—The chart in Fig. 6 may be used to calculate transformer efficiency at various loads. The procedure is described in the caption below the chart.

7. Loss Ratio and Product

Maximum operating efficiency for a transformer results when the no-load (constant) losses equal the load (variable) losses. This condition will likely occur at some load less than rated kva:

$$Cu \times L^2 = Fe \tag{16}$$

$$L = \sqrt{\frac{Fe}{Cu}} = \frac{1}{\sqrt{R}} \tag{17}$$

where:

L = per unit kva load at which transformer operates most efficiently.

Cu = load losses at rated load, kw.

Fe = no-load losses, kw.

$R = \text{loss ratio} = \dfrac{\text{load loss at rated load}}{\text{no-load loss}}$.

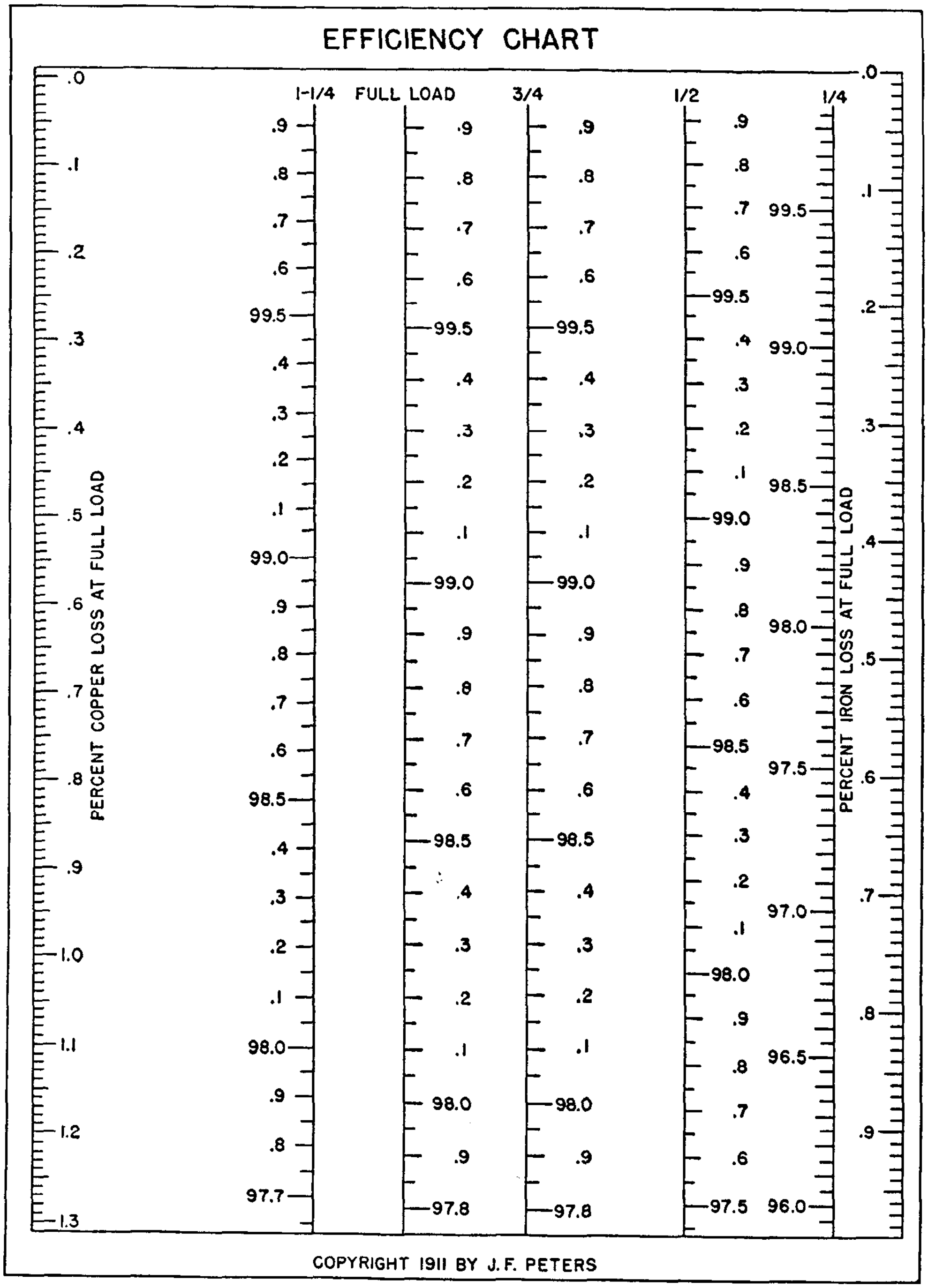


Fig. 6—Chart for calculation of efficiency. Directions: A straight-edge placed between the known full load copper loss and iron loss points will give intercepts on the efficiency scales for various loads.

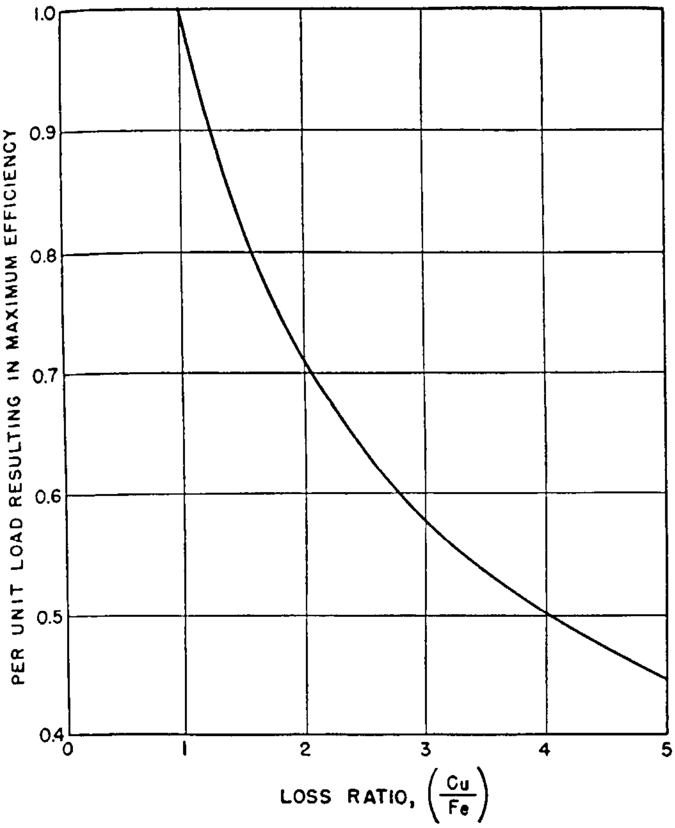


Fig. 7—Relation between transformer loss ratio and the most efficient loading.

The relation between loss ratio and most efficient transformer loading is shown in Fig. 7. The range through which loss ratio may vary in normal transformer designs is shown by Table 3.

The product of percent no-load and load losses is a quantity that has become standardized to the extent that it is predictable with fair accuracy for large power transformers.

TABLE 3

Normal Limits of Loss Ratio, R

Voltage Class kv	Loss Ratio, R = (Cu/Fe)	
	OA, OW OA/FA* OA/FA/FOA*	FOA** FOW**
46 and below	1.75 to 3.25	1.4 to 2.4
69 to 138, incl.	1.50 to 2.75	1.2 to 2.0
Above 138	1.25 to 2.00	1.0 to 1.8

*Based on losses at OA rating.
**Based on losses at 60 percent of FOA or FOW rating.

Fig. 8 shows typical values of the product of percent losses, as a function of transformer size and voltage rating. To estimate values of no-load and load losses for a particular transformer rating it is first necessary to select values of loss ratio R and loss product P from Table 3 and Fig. 8. Then the respective loss values, in kilowatts, are given below:

$$Fe = \frac{kva}{100}\sqrt{\frac{P}{R}},\ kw. \tag{18}$$

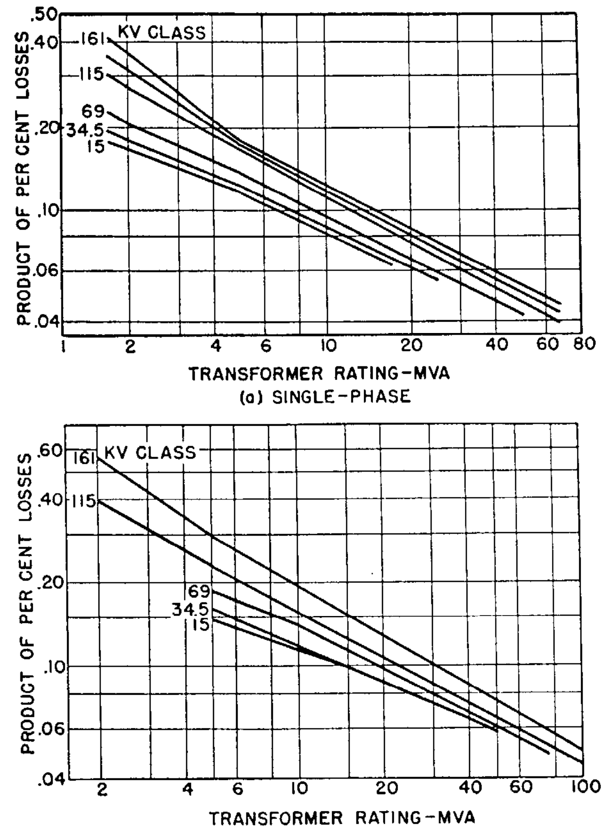


Fig. 8—Typical values of product of percent losses (percent full-load copper-loss times percent iron loss). For OA/FA or OA/FA/FOA units use OA rating to evaluate product. For FOA and FOW units use 60 percent of rated kva to evaluate product.

$$Cu = \frac{kva}{100} \times \sqrt{PR},\ kw. \tag{19}$$

where:

R = loss ratio, $\left(\frac{Cu}{Fe}\right)$.

P = product of the percent values of no-load and load losses, $\left(\frac{100\ Fe}{kva}\right) \times \left(\frac{100\ Cu}{kva}\right)$.

kva = transformer rating.

8. Typical Efficiency Values

Conventional transformer efficiency is given on the basis of losses calculated at (or corrected to) 75 degrees C and

TABLE 4—APPROXIMATE VALUES OF EFFICIENCY FOR 60-CYCLE, TWO-WINDING, OA, THREE-PHASE POWER TRANSFORMERS

(Full load, unity power factor, at 75°C)

kva	Voltage Class				
	15 kv	34.5 kv	69 kv	138 kv	161 kv
2000	98.97	98.89	98.83	98.56	98.47
10 000	99.23	99.22	99.17	99.12	99.11
50 000		99.47	99.45	99.44	99.44

Note: These figures apply also to OA/FA and OA/FA/FOA transformers, at loads corresponding to their OA ratings.

unity power-factor load unless otherwise specified. Table 4 gives approximate values for 60-cycle power transformers at full load, unity power-factor, and 75 degrees C.

III. TRANSFORMER CLASSIFICATIONS

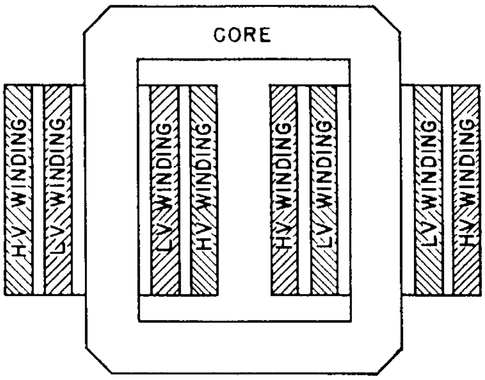
9. Forms of Construction.

Core-form construction for single-phase transformers consists of magnetic steel punchings arranged to provide a single-path magnetic circuit. High- and low-voltage coils are grouped together on each main or vertical leg of the core, as shown in Fig. 9. In general, the mean length of turn for the winding is comparatively short in the core-form design, while the magnetic path is long.

Shell-form construction for single-phase transformers consists of all windings formed into a single ring, with magnetic punchings assembled so as to encircle each side of the winding ring, as in Fig. 10. The mean length of turn is usually longer than for a comparable core-form design, while the iron path is shorter.

In the design of a particular transformer many factors such as insulation stress, mechanical stress, heat distribution, weight and cost must be balanced and compromised[5]. It appears that, for well-balanced design, both core-form and shell-form units have their respective fields of applicability determined by kva and kv rating.

In the larger sizes, shell-form construction is quite appropriate; the windings and magnetic iron can be assembled



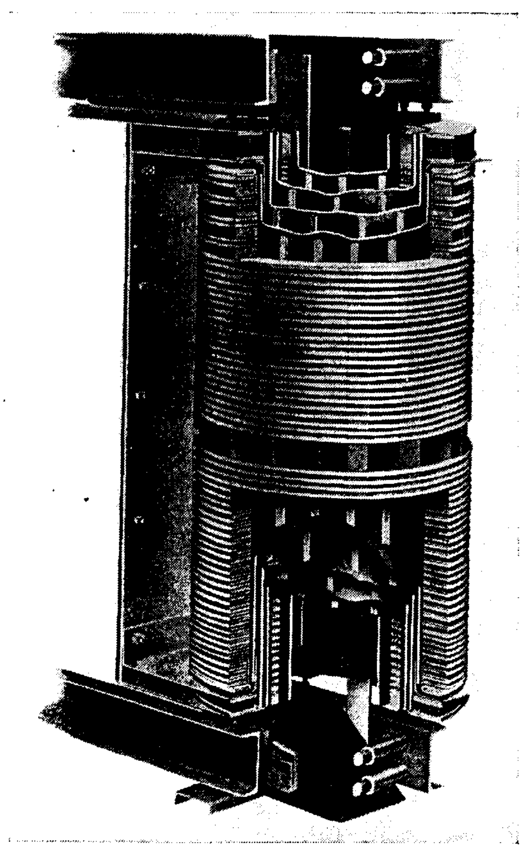
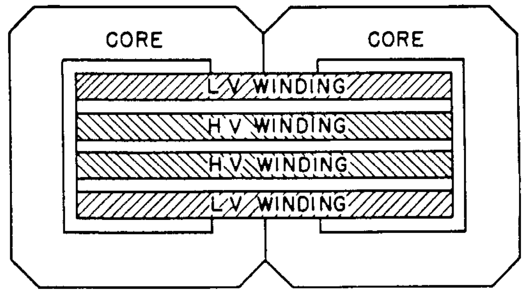


Fig. 9—Core-form construction.



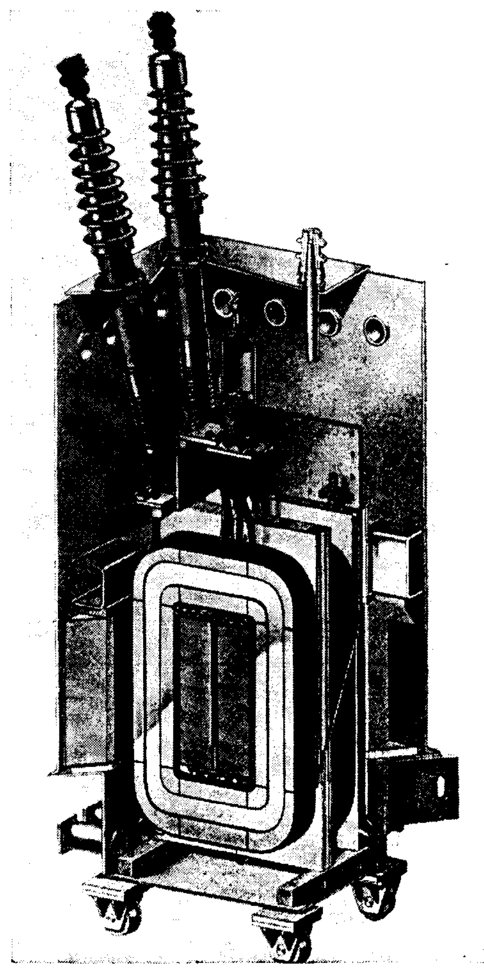


Fig. 10—Shell-form construction.

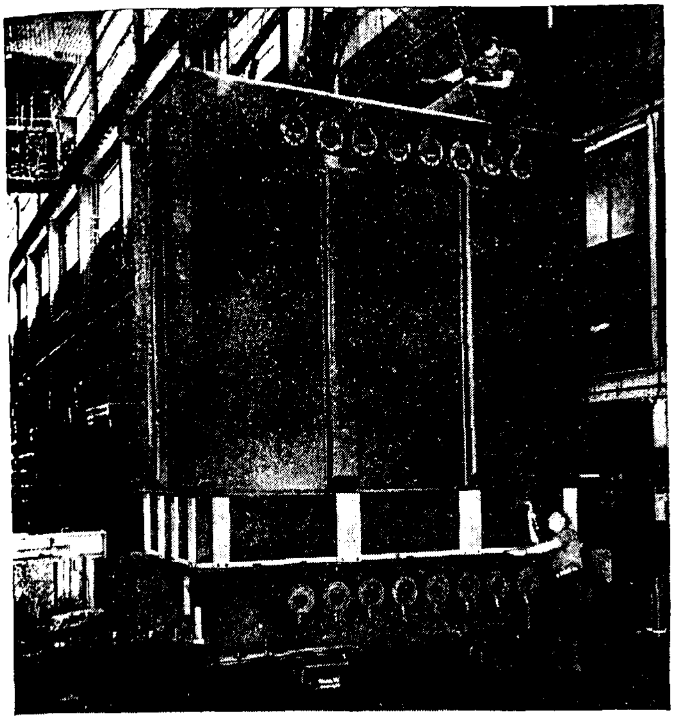


Fig. 11—Assembly of 15 000 kva three-phase transformer, showing "form-fit" tank being lowered into position.

on a steel base structure, with laminations laid in horizontally to link and surround the windings. A close-fitting tank member is then dropped over the core and coil assembly and welded to the steel base, completing the tank assembly and also securing the core to the base member. This "form-fit" construction is shown in Fig. 11; it is more compact than can be achieved by assembling a core form unit within a tank, and the flow of cooling oil can be directed more uniformly throughout the interior of the coil assembly.

10. Comparison of Single-Phase and Three-Phase Units for Three-Phase Banks

A three-phase power transformation can be accomplished either by using a three-phase transformer unit, or by interconnecting three single-phase units to form a three-phase bank. The three-phase unit has advantages of greater efficiency, smaller size, and less cost when compared with a bank having equal kva capacity made up of three single-phase units.

When three single-phase units are used in a bank, it is possible to purchase and install a fourth unit at the same location as an emergency spare. This requires only 33 percent additional investment to provide replacement capacity, whereas 100 percent additional cost would be required to provide complete spare capacity for a three-phase unit. However, transformers have a proven reliability higher than most other elements of a power system, and for this reason the provision of immediately available spare capacity is now considered less important than it once was. Three-phase units are quite generally used in the highest of circuit ratings, with no on-the-spot spare transformer capacity provided. In these cases parallel or interconnected circuits of the system may provide emergency capacity, or, for small and medium size transformers, portable substations can provide spare capacity on short notice.

If transportation or rigging facilities should not be adequate to handle the required transformer capacity as a single unit, a definite reason of course develops for using three single-phase units.

11. Types of Cooling

Basic types of cooling are referred to by the following designations.[6]

OA—Oil-Immersed Self-Cooled—In this type of transformer the insulating oil circulates by natural convection within a tank having either smooth sides, corrugated sides, integral tubular sides, or detachable radiators. Smooth tanks are used for small distribution transformers but because the losses increase more rapidly than the tank surface area as kva capacity goes up, a smooth tank transformer larger than 50 kva would have to be abnormally large to provide sufficient radiating surface. Integral tubular-type construction is used up to about 3000 kva and in some cases to larger capacities, though shipping restrictions usually limit this type of construction at the larger ratings. Above 3000 kva detachable radiators are usually supplied. Transformers rated 46 kv and below may also be filled with Inerteen fire-proof insulating liquid, instead of with oil.

The OA transformer is a basic type, and serves as a standard for rating and pricing other types.

OA/FA—Oil-Immersed Self-Cooled/Forced-Air Cooled—This type of transformer is basically an OA unit with the addition of fans to increase the rate of heat transfer from the cooling surfaces, thereby increasing the permissible transformer output. The OA/FA transformer is applicable in situations that require short-time peak loads to be carried recurrently, without affecting normal expected transformer life. This transformer may be purchased with fans already installed, or it may be purchased with the option of adding fans later.

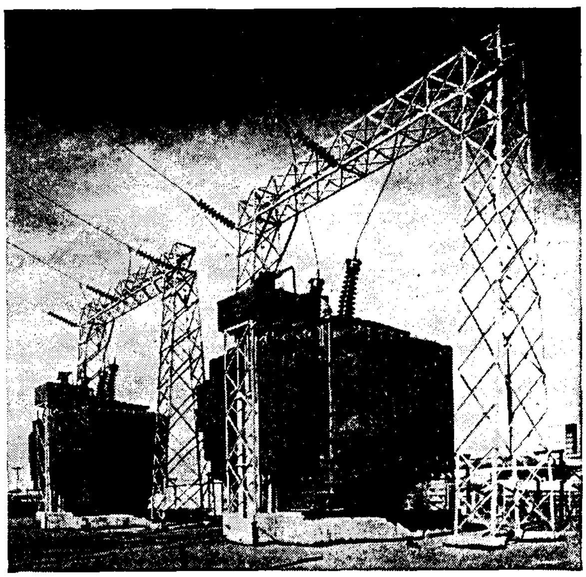


Fig. 12—Installation view of a 25 000 kva, 115-12 kv, three-phase, 60 cycle, OA/FA transformer.

The higher kva capacity attained by the use of fans is dependent upon the self-cooled rating of the transformer and may be calculated as follows:

For 2500 kva (OA) and below:

$$\text{kva (FA)} = 1.15 \times \text{kva(OA)}. \tag{20}$$

For 2501 to 9999 kva (OA) single-phase or 11 999 kva (OA) three-phase:

$$\text{kva (FA)} = 1.25 \times \text{kva (OA)}. \tag{21}$$

For 10 000 kva (OA) single-phase and 12 000 kva (OA) three-phase, and above:

$$\text{kva (FA)} = 1.333 \times \text{kva (OA)}. \tag{22}$$

These ratings are standardized, and are based on a hot-test-spot copper temperature of 65 degrees C above 30 degrees C average ambient.

OA/FOA/FOA—Oil-Immersed Self-Cooled/Forced-Oil Forced-Air Cooled/Forced-Oil Forced-Air Cooled—The rating of an oil-immersed transformer may be increased from its OA rating by the addition of some combination of fans and oil pumps. Such transformers are normally built in the range 10 000 kva (OA) single-phase or 12 000 kva (OA) three-phase, and above. Increased ratings are defined as two steps, 1.333 and 1.667 times the OA rating respectively. Recognized variations of these triple-rated transformers are the OA/FA/FA and the OA/FA/FOA types. Automatic controls responsive to oil temperature are normally used to start the fans and pumps in a selected sequence as transformer loading increases.

FOA—Oil-Immersed Forced-Oil-Cooled With Forced-Air Cooler—This type of transformer is intended for use only when both oil pumps and fans are operating, under which condition any load up to full rated kva may be carried. Some designs are capable of carrying excitation current with no fans or pumps in operation, but this is not universally true. Heat transfer from oil to air is accomplished in external oil-to-air heat exchangers.

OW—Oil-Immersed Water-Cooled—In this type of water-cooled transformer, the cooling water runs through coils of pipe which are in contact with the insulating oil of the transformer. The oil flows around the outside of these pipe coils by natural convection, thereby effecting the desired heat transfer to the cooling water. This type has no self-cooled rating.

FOW—Oil-Immersed Forced-Oil-Cooled With Forced-Water Cooler—External oil-to-water heat exchangers are used in this type of unit to transfer heat from oil to cooling water; otherwise the transformer is similar to the FOA type.

AA—Dry-Type Self-Cooled—Dry-type transformers, available at voltage ratings of 15 kv and below, contain no oil or other liquid to perform insulating and cooling functions.

Air is the medium which surrounds the core and coils, and cooling must be accomplished primarily by air flow inside the transformer. The self-cooled type is arranged to permit circulation of air by natural convection.

AFA—Dry-Type Forced-Air Cooled—This type of transformer has a single rating, based on forced circulation of air by fans or blowers.

AA/FA—Dry-Type Self-Cooled/Forced-Air Cooled—This design has one rating based on natural convection and a second rating based on forced circulation of air by fans or blowers.

IV. POLARITY AND TERMINAL MARKINGS

12. Single-Phase Transformers

Primary and secondary terminals of a single-phase transformer have the same polarity when, at a given instant of time, the current enters the primary terminal in question and leaves the secondary terminal. In Fig. 13 are illustrated

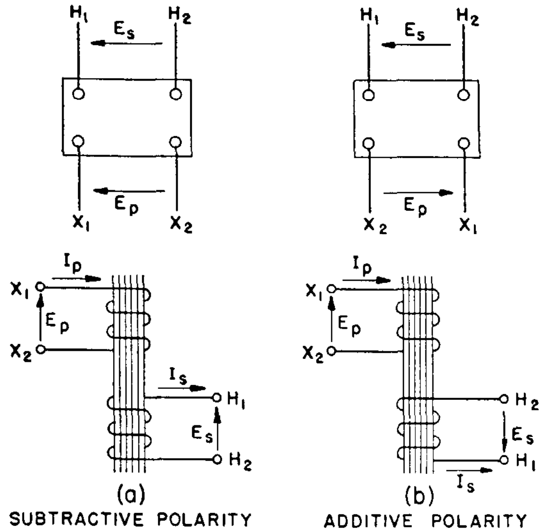


Fig. 13—Standard polarity markings for two-winding transformers.

single-phase transformers of additive and subtractive polarity. If voltage is applied to the primary of both transformers, and *adjacent* leads connected together, H_1 to X_1 in Fig. 13(a) and H_1 to X_2 in Fig. 13(b), a voltmeter across the other pair of terminals [H_2 and X_2 in Fig. 13(a) and H_2 and X_1 in Fig. 13(b)] indicates a voltage greater than E_s if the transformer is additive as Fig. 13(b), and less than E_s if the transformer is subtractive as Fig. 13(a).

Additive polarity is standard for all single-phase transformers 200 kva and smaller having high-voltage ratings 8660 volts (winding voltage) and below. Subtractive polarity is standard for all other single-phase transformers.[6]

13. Three-Phase Transformers

The polarity of a three-phase transformer is fixed by the connections between phases as well as by the relative locations of leads, and can be designated by a sketch showing lead marking and a vector diagram showing the electrical angular shift between terminals.

The standard angular displacement between reference phases of a delta-delta bank, or a star-star bank is zero degrees. The standard angular displacement between reference phases of a star-delta bank, or a delta-star bank, is 30 degrees. The present American standard for such three-phase banks is that the high-voltage reference phase is 30 degrees *ahead* of the reference phase on the low voltage, regardless of whether the bank connections are star-delta or delta-star.[6] The standard terminal markings

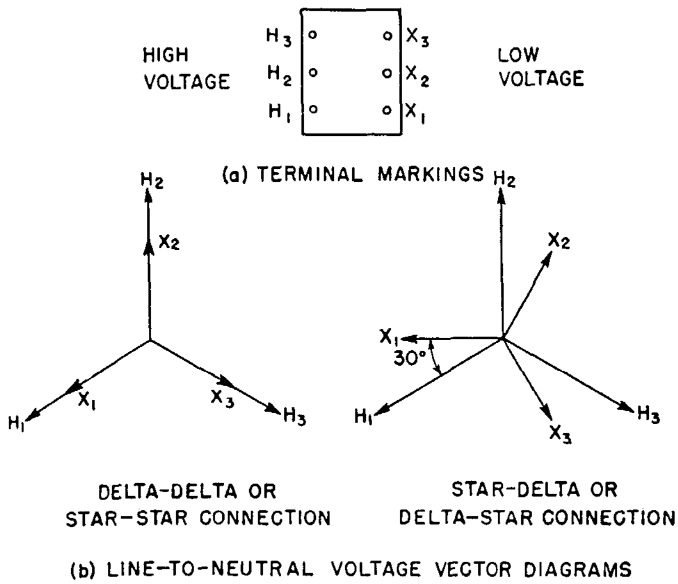


Fig. 14—Standard polarity markings and vector diagrams for three-phase transformers.

for a three-phase, two-winding transformer are illustrated in Fig. 14. Also included are the vector diagrams for delta-delta, star-star, star-delta and delta-star connected transformers. The phase rotations are assumed to be H_1–H_2–H_3 and X_1–X_2–X_3.

Fig. 15 summarizes the phase angles that can be obtained between high- and low-voltage sides of star-delta and delta-

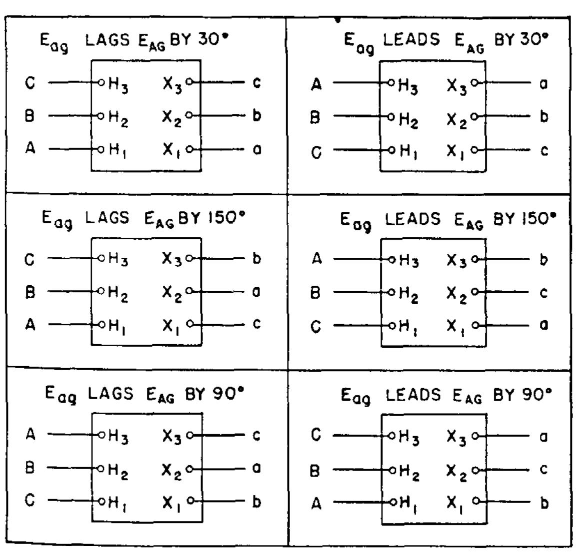


Fig. 15—Angular phase displacements obtainable with three-phase star-delta transformer units.

star, three-phase transformers built with standard connections and terminal markings. In this Figure A, B, and C represent the three phases of the high-voltage system, whereas a, b, and c represent the three phases of the low-voltage system. Phase rotations A–B–C and a–b–c are assumed.

V. STANDARD INSULATION CLASSES

14. Choice of Insulation Class

The standard insulation classes and dielectric tests for power transformers are given in Table 5. The insulation class of a transformer is determined by the dielectric tests which the unit can withstand, rather than by rated operating voltage.

On a particular system, the insulation class of the connected power transformers may be determined by the ratings and characteristics of the protective devices installed to limit surge voltages across the transformer windings. Ratings of the protective devices will in turn depend upon the type of system, its grounding connections, and some related factors. For example, when the system neutral is solidly grounded so that a grounded neutral (80 percent) arrester can be used, an insulation level corresponding to the arrester rating may be chosen rather than an insulation level corresponding to the system operating voltage. Many transformer banks having a star-connected three-phase winding, with the neutral permanently and solidly grounded, have an impulse strength corresponding to a lower line-to-line classification than indicated in Table 5 (See Chap. 18 for a more detailed discussion of this subject).

15. Dielectric Tests

The purpose of dielectric testing is to show that the design, workmanship, and insulation qualities of a transformer are such that the unit will actually meet standard or specified voltage test limits. Below is a description of the various dielectric tests which may be applied to power transformers:

(1) The *standard impulse test* consists of applying in succession, one *reduced full wave*, two *chopped waves*, and one *full wave*.

(a) A *full wave* is a 1.5×40 microsecond wave, usually of negative polarity for oil-immersed transformers, or positive polarity for dry type, and of the magnitude given in Table 5.

(b) *A reduced full wave* is a 1.5×40 microsecond wave, having a crest value between 50 and 70 percent of the full wave crest.

(c) A *chopped wave* is formed by connecting an air gap to cause voltage breakdown on the tail of the applied wave. The crest voltage and minimum time to flashover are specified in Table 5.

(2) The standard *applied-potential test* consists of applying a low-frequency voltage between ground and the winding under test, with all other windings grounded. The standard test voltage magnitude is listed in Table 5, and its specified duration is one minute.

(3) The standard *induced-potential test* in general consists of applying between the terminals of one winding a voltage equal to twice the normal operating voltage of that winding. A frequency of twice rated or more is used for this test, so that the transformer core will not be overexcited by the application of double voltage. The duration of the test is 7200 cycles of the test frequency, but not longer than one minute. Commonly used test frequencies

TABLE 5—STANDARD INSULATION CLASSES AND DIELECTRIC TESTS FOR DISTRIBUTION AND POWER TRANSFORMERS

(Taken from Table 11.030 ASA Standard C57.11-1948 for Transformers, Regulators and Reactors.)

Insulation Class	Rated Voltage Between Terminals of Power-Transformers (a)			Low Frequency Tests		Impulse Tests					
						Oil-Immersed Transformers 500 kva or Less			Oil-Immersed Transformers Above 500 kva		
	Single-Phase		3-Phase	Oil-Immersed Type	Dry Type (b)	Chopped Wave		Full Wave (e)	Chopped Wave		Full Wave (e)
kv	For Y-Connection on 3-Phase System (c) kv rms	For Delta-Connection on 3-Phase System kv rms	Delta or Y-Connected kv rms (c)	kv rms	kv rms	Kv Crest	Min Time to Flash-over Microsec.	Kv Crest	Kv Crest	Min Time to Flash-over Microsec.	Kv Crest
1.2	0.69	0.69 (d)	1.2	10	4	36	1.0	30	54	1.5	45
2.5	.....	......	2.5	15	10	54	1.25	45	....	...	....
5.0	2.89	2.89 (d)	5.0	19	12	69	1.5	60	88	1.6	75
8.66	5.0	5.00 (d)	8.66	26	19	88	1.6	75	110	1.8	95
15	8.66	15.0	15.0	34	31	110	1.8	95	130	2.0	110
25.0	14.4	25.0	25.0	50	..	175	3.0	150	175	3.0	150
34.5	19.9	34.5	34.5	70	..	230	3.0	200	230	3.0	200
46.0	26.6	46.0	46.0	95	..	290	3.0	250	290	3.0	250
69.0	39.8	69.0	69.0	140	..	400	3.0	350	400	3.0	350
92	53.1	92	92	185	..	520	3.0	450	520	3.0	450
115	66.4	115	115	230	..	630	3.0	550	630	3.0	550
138	79.7	138	138	275	..	750	3.0	650	750	3.0	650
161	93.0	161	161	325	..	865	3.0	750	865	3.0	750
196	113	196	196	395	..	1035	3.0	900	1035	3.0	900
230	133	230	230	460	..	1210	3.0	1050	1210	3.0	1050
287	166	287	287	575	..	1500	3.0	1300	1500	3.0	1300
345	199	345	690	690	..	1785	3.0	1550	1785	3.0	1550

Notes: (a) Intermediate voltage ratings are placed in the next higher insulation class unless otherwise specified.
(b) Standard impulse tests have not been established for dry-type distribution and power transformers. Present-day values for impulse tests of such apparatus are as follows:
1.2 kv class, 10 kv; 2.5 class, 20 kv; 5.0 class, 25 kv; 8.66 kv class, 35 kv; 15 kv class, 50 kv. These values apply to both chopped-wave and full-wave tests.
(c) Y-connected transformers for operation with neutral solidly grounded or grounded through an impedance may have reduced insulation at the neutral. When this reduced insulation is below the level required for delta operation, transformers cannot be operated delta-connected.
(d) These apparatus are insulated for the test voltages corresponding to the Y connection, so that a single line of apparatus serves for the Y and delta applications. The test voltages for such delta-connected single-phase apparatus are therefore one step higher than needed for their voltage rating.
(e) 1.5×40 microsecond wave.

are 120 cycles for 60-cycle transformers, and 60 cycles for 25-cycle transformers.

Combinations and modifications of the tests described above are contained in transformer standard publications, for example ASA C57.22-1948, and these publications should be consulted for detailed information.

16. Insulation Class of Transformer Neutrals

Transformers designed for wye connection only with the neutral brought out may have a lower insulation level at the neutral than at the line end. The following rules are included as a guide in selecting the permissible neutral insulation level:

(a) A solidly grounded transformer may have a minimum neutral insulation class in accordance with column 2 of Table 6.

(b) A transformer grounded through a neutral impedance must have a neutral insulation class at least as high as the maximum dynamic voltage at the transformer neutral during system short-circuit conditions. In no case should the neutral class be lower than that given in Column 2, Table 6.

(c) If the neutral of a transformer is connected to ground through the series winding of a regulating transformer, the neutral insulation class must be at least as high as the maximum raise or lower voltage (phase to neutral) of the regulating transformer. In no case should the neutral class be less than that given in Column 3 of Table 6.

(d) A transformer grounded through the series winding of a regulating transformer and a separate neutral impedance shall have a neutral insulation class at least as high as the sum of the maximum raise or lower voltage (line to neutral) of the regulating transformer and the maximum dynamic voltage across the neutral impedance during system short-circuit conditions. In no case should the neutral insulation class be less than that given in Column 3 of Table 6.

(e) If the neutral of a transformer is connected to ground through a ground fault neutralizer, or operated ungrounded but impulse protected, the minimum neutral

TABLE 6—MINIMUM INSULATION CLASS AT TRANSFORMER NEUTRAL

(1) Winding Insulation Class at Line End	(2) Grounded Solidly or Through Current Transformer	(3) Grounded Through Regulating Transformer	(4) Grounded Through Ground Fault Neutralizer or Isolated but Impulse Protected
1.2			
2.5			
5.0		Same as Line	End
8.66			
15	8.66	8.66	8.66
25	8.66	8.66	15
34.5	8.66	8.66	25
46	15	15	34.5
69	15	15	46
92	15	25	69
115	15	25	69
138	15	34.5	92
161	15	34.5	92
196	15	46	115
230	15	46	138
287	15	69	161
345	15	69	196

insulation class shall be in accordance with Column 4 of Table 6.

VI. TEMPERATURE AND SHORT-CIRCUIT STANDARDS.

17. Temperature Standards

The rating of electrical apparatus is inherently determined by the allowable operating temperatures of insulation, or the temperature rise of the insulation above ambient temperature. For transformers and voltage regulators with Class A insulation, either air or oil cooled, the rating is based on an observable temperature rise (by resistance or thermometer) of 55 C above an ambient temperature at no time in excess of 40 C, and the average during any 24-hour period not exceeding 30 C. Transformers and other induction apparatus are designed to limit the hottest-spot temperatures of the windings to not more than 10 C above their average temperatures under continuous rated conditions. The limits of observable temperature rise for air-cooled transformers with Class B insulation is 80 C by resistance measurement.

18. Short-Circuit Conditions

A proposed revision to American Standard C57.12-1948 (section 12.050) reads in part:

"1. Transformers shall be capable of withstanding without injury short circuits on any external terminals, with rated line voltages maintained on all terminals intended for connection to sources of power, provided:

(a) The magnitude of the symmetrical current in any winding of the transformer, resulting from the external short circuit, does not exceed 25 times the base current of the winding. The initial current is assumed to be completely displaced from zero.

(b) The duration of the short circuit is limited to the following time periods. Intermediate values may be determined by interpolation.

Symmetrical Current in Any Winding	Time Period in Seconds
25 times base current	2
20 times base current	3
16.6 times base current	4
14.3, or less, times base current	5

"2. Where kva is mentioned in paragraph 3 the following is intended:
When the windings have a self-cooled rating, the kva of the self-cooled rating shall be used. When the windings have no self-cooled ratings, the largest kva obtained from the ratings assigned for other means of cooling by the use of the following factors shall be used:

Type of Transformer	Multiplying Factor
Water-cooled (OW)	1.0
Dry-Type Forced-Air-Cooled (AFA)	0.75
Forced-oil-cooled (FOA or FOW)	0.60

"3. For multi-winding transformers:
The base current of any winding provided with external terminals, or of any delta-connected stabilizing winding without terminals, shall be determined from the rated kva of the winding or from not less than 35 percent of the rated kva of the largest winding of the transformer, whichever is larger.

"In some cases, the short-circuit current, as limited by transformer impedance alone, will exceed 25 times base current. It must be recognized that such cases can occur with transformers manufactured according to these standards and that the transformers built under these standards are not designed to withstand such short-circuit current."

Under short-circuit conditions the calculated copper temperatures for power and distribution transformers shall not exceed 250 C where Class A insulation is used assuming an initial copper temperature of 90 C, or 350 C where Class B insulation is used assuming an initial copper temperature of 125 C.

VII. TRANSFORMER TEMPERATURE-TIME CURVES

19. Constant Load

A "heat run" of a transformer on test is made to determine the temperature rise of the various parts at rated load. If the test were made by applying only rated load, with the transformer at room temperature, thirty hours or more would be required before stationary temperatures were reached. Such a process would be quite inefficient of time, energy, and in the use of testing facilities. Accelerated heat runs are made by closing radiator valves, etc., and applying loads in excess of rated load until the expected temperatures are reached. Radiation restrictions are then removed, the load reduced to normal, and the test continued until stable temperatures are reached.

It is evident that the temperature-time characteristics of a transformer cannot be obtained from the accelerated heat-run data. Information is secured from the heat run, however, which permits the temperatures to be calculated under assumed load conditions. Exact calculations are quite involved, but sufficiently accurate results can be obtained by the use of an approximate method due to S. B. Griscom for estimating the temperatures reached under variable load conditions, changing ambient temperatures, etc. Certain simplifying assumptions can be made that permit a quick estimate of the expected temperatures.

Let L = transformer load in kva.
- W = total losses (in kw) at load L.
- T_F = final temperature rise at load L in degrees C above the temperature at $t=0$.
- M = thermal capacity in kw hours per degree C.
- k = radiation constant in kw per degree C.
- T = oil temperature rise in degrees C at time t above the temperature at $t=0$.
- H = thermal time constant in hours.
- t = time in hours.

If the heat radiated is directly proportional to the temperature rise of the transformer above the ambient, the radiation constant can be obtained from the heat run data for W and T_F:

$$k=\frac{W}{T_F} \tag{23}$$

where the temperature at $t=0$ is taken as ambient.

Since the total heat generated is equal to the heat radiated plus the heat stored (heat consumed in raising the temperatures of the various parts)

$$W=kT+M\frac{dT}{dt} \tag{24}$$

This equation can be solved for T, giving

$$T=\frac{W}{k}\left(1-\epsilon^{\frac{-t}{M/k}}\right) \tag{25}$$

or

$$T=T_F\left(1-\epsilon^{-\frac{t}{H}}\right) \tag{26}$$

where

$$H=\frac{M}{k}=\text{the transformer time constant in hours.} \tag{27}$$

This derivation may be broadened to show that Eq. (26) is equally correct for the case where the oil temperature rises T and T_F are those above the temperature at $t=0$, whether the value then is the ambient temperature or otherwise.

The foregoing discussion has been based on the assumption that the temperature throughout all parts of the transformer is the same. This, of course, is not the case. When the transformer load is increased, the copper temperature is above that of the surrounding parts, and when the load is decreased, the copper tends to be more nearly the same temperature as the surrounding parts. Also, the top and bottom oil are at different temperatures. Eq. (26) is therefore commonly taken as referring to the top-oil temperature rise, that is, T and T_F are defined as before but refer to the top-oil specifically. Further, the final top-oil temperature rise T_F is not directly proportional to the losses for all types of transformers as Eq. (23) would indicate, but is more correctly represented by the relation

$$T_F=T_{F(fl)}\left(\frac{W}{\text{Total loss at full load}}\right)^{m} \tag{28}$$

where:
- m = 0.8 for type OA transformers.
- = 0.9 for type OA/FA transformers.
- = 1.0 for type FOA transformers.
- $T_{F(fl)}$ = final top-oil temperature rise at full load in degrees C.

The use of this relation when substituted in Eq. (23) indicates that for other than the type FOA transformer the radiation constant k and the time constant H are not completely independent of load but vary according to a small fractional power of the total loss. However for convenience in calculations this variation in k and H is normally overlooked and the values obtained from Equations (23) and (27) for the full load condition are taken as constant. The error introduced by the procedure is not large compared to that normally expected in transient thermal calculations.

To determine the temperature rise curve for any load L therefore, the radiation constant k under full load conditions is first determined from the heat run data using Eq. (23). The thermal capacity M is dependent on the thermal capacities of the various parts of the transformer. For convenience it can be assumed that the transformer parts can be separated into three elements: the core and coils, the case and fittings, and the oil. Although the core and coils are of copper, iron, and insulation the specific heats of those elements do not vary widely. Since, also, there is a reasonably constant proportion of these elements in different transformers, a single weighted coefficient of thermal capacity for the coils and core is warranted. The following relation is accordingly suggested:

$$\begin{aligned}M=\frac{1}{1000}[&0.06\ (\text{wt. of core and coils})\\&+0.04\ (\text{wt. of case and fittings})\\&+0.17\ (\text{wt. of oil})]\end{aligned} \tag{29}$$

Here the coefficients of the last two terms are also weighted to make further allowance for the fact that all parts of the case and fittings and the oil are not at a uniform temperature. The values of k and M found as above may be substituted in Eq. (27) to obtain H. The value of T_F for the desired load L is determined next by substitution of heat run data in Eq. (28). The quantity W for the load L may be evaluated by the relation

$$W=\left[\left(\frac{L}{\text{Full load kva}}\right)^2\times(\text{full load copper loss})+(\text{no-load loss})\right] \tag{30}$$

The quantities H and T_F may now be substituted in Eq. (26) from which the top-oil temperature-rise curve may be plotted directly.

For example, a 6000-kva, three-phase, self-cooled, 24 000-5040 volt transformer has the following full load performance data as supplied by the manufacturer:

Iron loss = 10 920 watts.
Copper loss = 43 540 watts.
Total = 54 460 watts.
Top-oil rise = 40 C (from heat-run test data).
LV copper rise = 46.3 C.
HV copper rise = 43.3 C.
Wt. of core and coils = 25 000 pounds.
Wt. of case and fittings = 18 000 pounds.
Wt. of oil = 17 400 pounds.

From this information the time constant H may be evaluated and the expression for T obtained for the load L equal to the rated load.

$$k=\frac{W}{T_F}=\frac{54.46}{40}=1.36 \text{ kw per degree C.}$$

$$M=\frac{1}{1000}[0.06\times 25\,000+0.04\times 18\,000+0.17\times 17\,400]$$

$$=5.18 \text{ kw hours per degree C.}$$

$$H=\frac{M}{k}=\frac{5.18}{1.36}=3.81 \text{ hours.}$$

$$T=T_F(1-\epsilon^{-t/H})=40(1-\epsilon^{-t/3.81}).$$

The full load top-oil temperature rise curve shown in Fig. 16 was calculated from this relation.

To plot the top-oil temperature-rise curve for half-load conditions for this transformer the same time constant H is used as found above.

From Eq. (28):

$$T_F=40\left(\frac{(0.5)^2\times 43.54+10.92}{54.46}\right)^{0.8}=19.2 \text{ C.}$$

$$T=19.2\left(1-\epsilon^{\frac{-t}{3.81}}\right).$$

The curve represented by this equation also appears in Fig. 16.

The rise of the hottest-spot copper temperature above the top-oil temperature is known as the hottest-spot copper gradient and at full load may be estimated from the relation

$$G_{H(fl)}=G_{C(fl)}+A. \tag{31}$$

where:

$G_{H(fl)}$ = hottest spot copper gradient at full load in degrees C.

$G_{C(fl)}$ = apparent copper gradient at full load in degrees C.

A = 10 C for type OA and OW transformers.
= 10 C for type OA/FA transformers.
= 5 C for type FOA and FOW transformers with directed flow over coils.

The apparent copper gradient at full load ($G_{C(fl)}$) is the difference between the average copper temperature rise and the top-oil temperature rise, both of which are de-

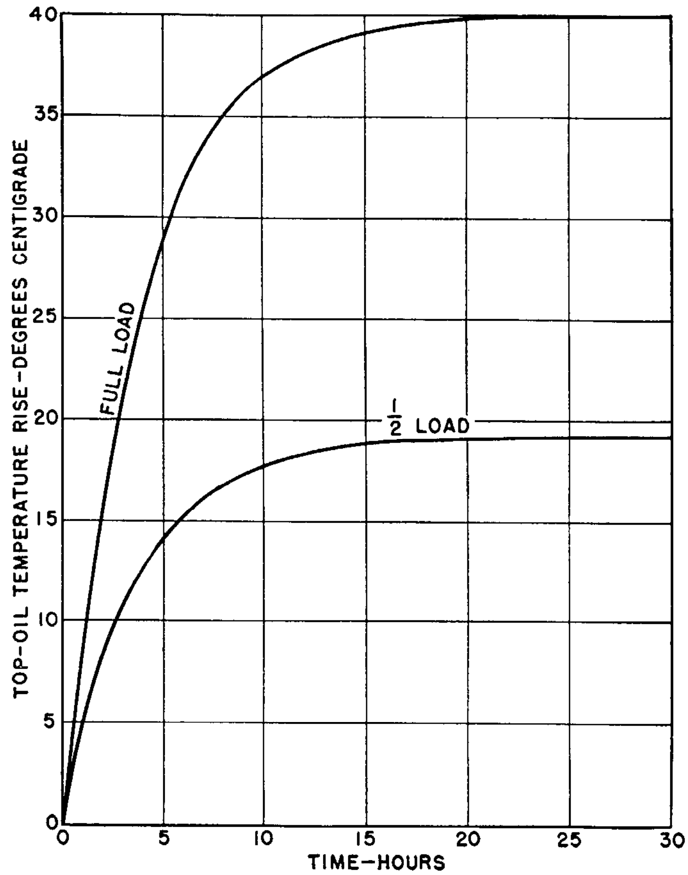


Fig. 16—Top-oil temperature rise versus time, for a typical transformer.

termined during the heat-run. The average copper temperature rise above ambient at full load is required by standards not to exceed 55 C for class A insulation. The use of that value to obtain the apparent copper gradient will generally lead to overly pessimistic results since the actual value of the average copper temperature rise is normally below the limit. Therefore it is advisable to use the value measured on the heat run and obtained from the manufacturer.

For any load L, the hottest-spot copper gradient may be calculated from the relation

$$G_{H(L)}=G_{H(fl)}\times\left(\frac{L}{\text{full load kva}}\right)^{1.6} \tag{32}$$

From the performance data of the transformer previously cited:

$G_{C(fl)}=46.3-40=6.3$ C for the LV winding.
$G_{H(fl)}=6.3+10=16.3$ C.

The hottest-spot copper temperature for full-load is thus 16.3 C above the top-oil temperature. For, say, half-load, Eq. (32) must be used to obtain

$$G_{H(L)}=16.3\times(0.5)^{1.6}=5.4 \text{ C.}$$

It is not feasible in a study of this kind to keep track of short time variations of copper or hottest-spot temperature, and it is suggested if it is desirable to show roughly how this varies, a time constant of 15 minutes be used.

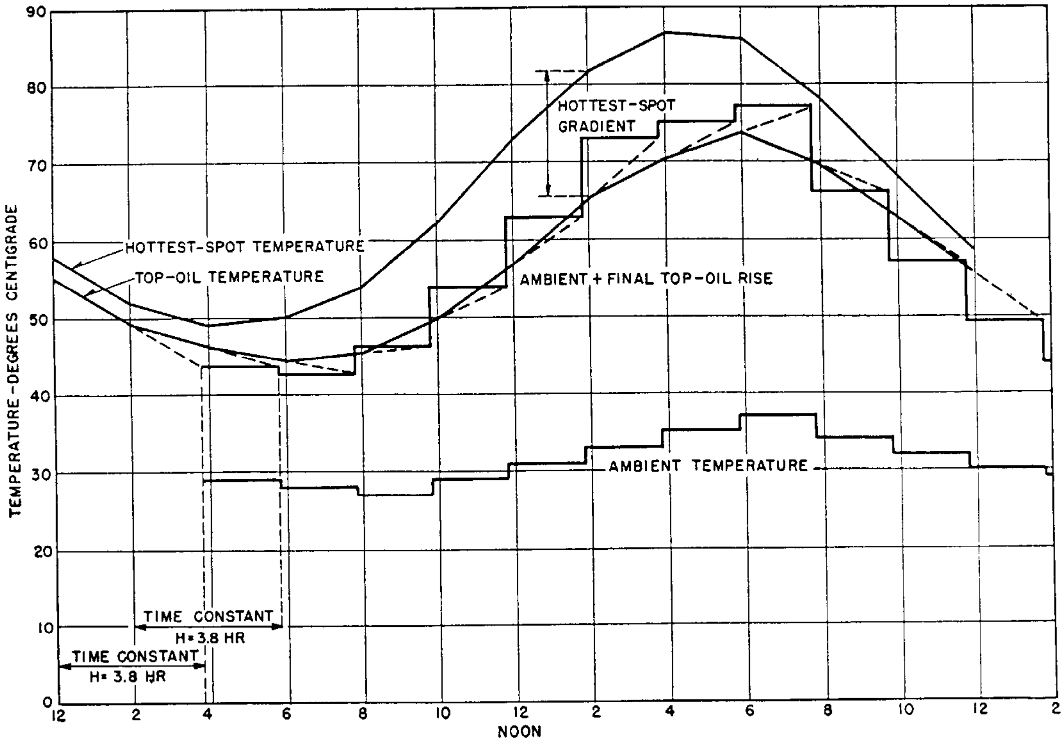


Fig. 17—Step-by-step graphical calculation of temperatures under changing load conditions.

20. Variable Load

A step-by-step analysis using Eqs. (28) to (32) can be made to consider conditions of variable load, changing ambient temperatures, etc. The method of approach is based on the fact that the initial rate of change of temperature is the slope of a line joining the initial and final temperatures, the two temperatures being separated by a time interval equal to the thermal time constant of the transformer. As before T_F is calculated from heat run data and the total loss W for each load condition through the use of Eq. (28). The loss W is obtained from Eq. (30). The final top-oil temperature is then found by adding T_F to the ambient temperature. Since the load is varying, the final temperature cannot be reached for each load condition and the step-by-step analysis must be employed to obtain the top-oil temperature curve. Points on the hottest-spot temperature time curve may then be obtained by adding the hottest-spot copper gradient G_H for each load to the top-oil temperature at the time corresponding to the load for which the gradient was calculated. G_H is obtained in the same manner as previously outlined.

To illustrate the step-by-step method, the oil temperature-time curve for the 6000-kva transformer previously described will be calculated, starting with an oil temperature of 55 C for an assumed load cycle as tabulated in the adjacent column.

Figure 17 illustrates the use of the calculated data in the graphical step-by-step process to plot the curve of top-oil temperature with time and the manner in which the hottest-spot gradients are added to obtain the hottest-spot temperature-time curve. The accuracy can be increased by using shorter time intervals.

Time	Ambient	Load (mva)	Loss Eq. (30)	Final Oil Rise Eq. (28)	Final Oil Temp. ambient plus final rise	Hottest-Spot Gradient Eq. (32)
12	29C	2	15.7	14.7C	43.7C	2.8C
2 AM	29	2	15.7	14.7	43.7	2.8
4	28	2	15.7	14.7	42.7	2.8
6	27	3	21.8	19.2	46.2	5.4
8	29	4	30.2	24.9	53.9	8.5
10	31	5	41.1	31.9	62.9	12.2
12	33	6	54.5	40.0	73.0	16.3
2 PM	35	6	54.5	40.0	75.0	16.3
4	37	6	54.5	40.0	77.0	16.3
6	34	5	41.1	31.9	65.9	12.2
8	32	4	30.2	24.9	56.9	8.5
10	30	3	21.8	19.2	49.2	5.4
12	29	2	15.7	14.7	43.7	2.8

VIII. GUIDES FOR LOADING OIL-IMMERSED POWER TRANSFORMERS

21. General

The rated kva output of a transformer is that load which it can deliver continuously at rated secondary voltage without exceeding a given temperature rise measured under prescribed test conditions. The actual test temperature rise may, in a practical case, be somewhat below the established limit because of design and manufacturing tolerances.

The output which a transformer can deliver in service without undue deterioration of the insulation may be more or less than its rated output, depending upon the following design characteristics and operating conditions as they exist at a particular time[6]:

(1) Ambient temperature.
(2) Top-oil rise over ambient temperature.
(3) Hottest-spot rise over top-oil temperature (hottest-spot copper gradient).
(4) Transformer thermal time constant.
(5) Ratio of load loss to no-load loss.

22. Loading Based on Ambient Temperature

Air-cooled oil-immersed transformers built to meet established standards will operate continuously with normal life expectancy at rated kva and secondary voltage, providing the ambient air temperature averages no more than 30 C throughout a 24-hour period with maximum air temperature never exceeding 40 C. Water-cooled transformers are built to operate continuously at rated output with ambient water temperatures averaging 25 C and never exceeding 30 C.

When the average temperature of the cooling medium is different from the values above, a modification of the transformer loading may be made according to Table 7. In

TABLE 7—PERCENT CHANGE IN KVA LOAD FOR EACH DEGREE CENTIGRADE CHANGE IN AVERAGE AMBIENT TEMPERATURE

Type of Cooling	Air above 30 C avg. or Water above 25 C avg.	Air below 30 C avg. or Water below 25 C avg.
Self-cooled	−1.5% per deg. C	+1.0% per deg. C
Water-cooled	−1.5	+1.0
Forced-Air-Cooled	−1.0*	+0.75*
Forced-Oil-Cooled	−1.0*	+0.75*

*Based on forced-cooled rating.

cases where the difference between maximum air temperature and average air temperature exceeds 10 C, a new temperature that is 10 C below the maximum should be used in place of the true average. The allowable difference between maximum and average temperature for water-cooled transformers is 5 C.

23. Loading Based on Measured Oil Temperatures

The temperature of the hottest-spot within a power transformer winding influences to a large degree the deterioration rate of insulation. For oil-immersed transformers the hottest-spot temperature limits have been set at 105 C maximum and 95 C average through a 24 hour period; normal life expectancy is based on these limits. The top-oil temperature, together with a suitable temperature increment called either *hottest-spot copper rise over top-oil temperature* or *hottest-spot copper gradient,* is often used as an indication of hottest-spot temperature. Allowable top-oil temperature for a particular constant load may be determined by subtracting the hottest-spot copper gradient for that load from 95 C. The hottest-spot copper gradient must be known from design information for accurate results, though typical values may be assumed for estimating purposes. If the hottest-spot copper gradient is known for one load condition, it may be estimated for other load conditions by reference to Fig. 18.

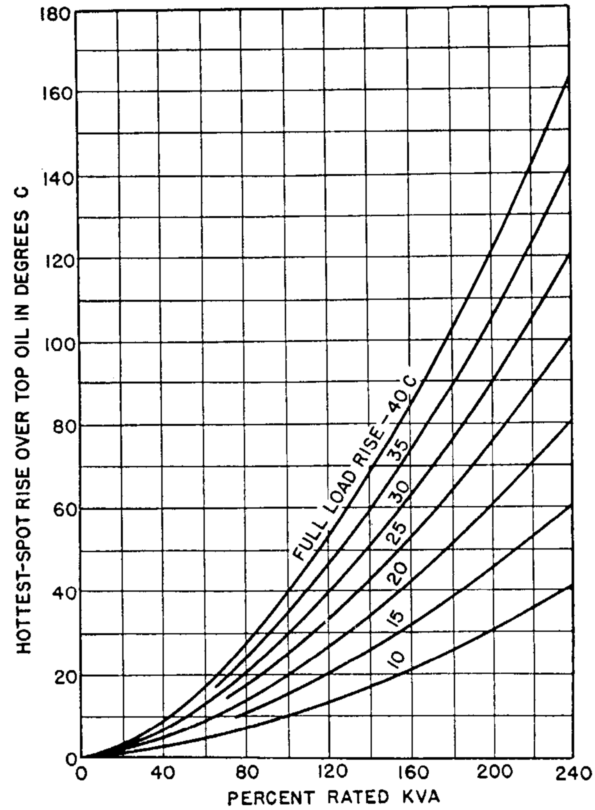


Fig. 18—Hottest-spot copper rise above top-oil temperature as a function of load, for various values of full load copper rise.

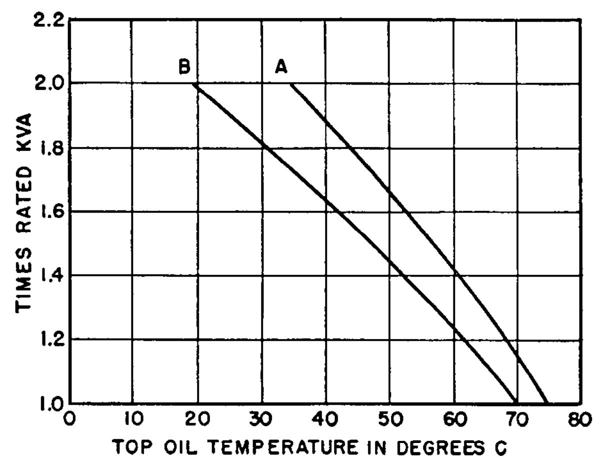


Fig. 19—Loading guide based on top-oil temperature.
(A) OA, OW, OA/FA types.
(B) OA/FA/FOA, FOA, FOW types.

A conservative loading guide, based on top-oil temperatures, is given in Fig. 19.

24. Loading Based on Capacity Factor

Transformer capacity factor (operating kva divided by rated kva) averaged throughout a 24-hour period may be well below 100 percent, and when this is true some compensating increase in maximum transformer loading may be made. The percentage increase in maximum loading

TABLE 8—PERMISSIBLE TRANSFORMER LOADING BASED ON AVERAGE PERCENT CAPACITY FACTORS*

Type of Cooling	Percent Increase Above Rated kva For each Percent By Which Capacity Factor Is Below 100	Maximum Percent Increase, Regardless of Capacity Factor
Self-Cooled	0.5	25
Water-cooled	0.5	25
Forced-Air-Cooled	0.4	20
Forced-Oil-Cooled	0.4	20

*Here, percent capacity factor is equal to $\frac{\text{operating kva}}{\text{rated kva}} \times 100$, averaged throughout a 24-hour period.

as a function of capacity factor, based on a normal transformer life expectancy, is given in Table 8.

25. Loading Based on Short-Time Overloads

Short-time loads which occur not more than once during any 24-hour period may be in excess of the transformer rating without causing any predictable reduction in transformer life. The permissible load is a function of the average load previous to the period of above-rated loading, according to Table 9. The load increase based on capacity factor and the increase based on short-time overloads cannot be applied concurrently; it is necessary to chose one method or the other.

Short time loads larger than those shown in Table 9 will cause a decrease in probable transformer life, but the amount of the decrease is difficult to predict in general terms. Some estimate of the sacrifice in transformer life can be obtained from Table 10(a) which is based on the theoretical conditions and limitations described in Table 10(b). These conditions were chosen to give results containing some probable margin, when compared with most conventional transformer designs. For special designs, or for a more detailed check on some particular unit, the hottest-spot copper temperature can be calculated by the method shown in section 19, and the probable sacrifice in transformer life can then be estimated from Table 11.

TABLE 9—PERMISSIBLE DAILY SHORT-TIME TRANSFORMER LOADING BASED ON NORMAL LIFE EXPECTANCY

Period of Increased Loading, Hours	Maximum Load In Per Unit of Transformer Rating[a]								
	OA, OW			OA/FA[b]			OA/FA/FOA,[c] FOA		
	Average[d] Initial Load, In Per Unit of Transformer Rating								
	0.90	0.70	0.50	0.90	0.70	0.50	0.90	0.70	0.50
0.5	1.59	1.77	1.89	1.45	1.58	1.68	1.36	1.47	1.50
1	1.40	1.54	1.60	1.31	1.38	1.50	1.24	1.31	1.34
2	1.24	1.33	1.37	1.19	1.23	1.26	1.14	1.18	1.21
4	1.12	1.17	1.19	1.11	1.13	1.15	1.09	1.10	1.10
8	1.06	1.08	1.08	1.06	1.07	1.07	1.05	1.06	1.06

(a) Ambient temperatures of 30C for air and 25C for water are assumed throughout this table.
(b) Based on FA rating.
(c) Based on FOA rating.
(d) Use either average load for two hours previous to overload period, or average load for 24 hours (less the overload period), whichever is greater.

26. Limiting of Load by Automatic Control

The loading of a transformer can be supervised by control devices to insure that hottest-spot copper temperatures

TABLE 10(a)—PERMISSIBLE SHORT-TIME TRANSFORMER LOADING, BASED ON REDUCED LIFE EXPECTANCY

Type of Cooling	Period of Increased Loading Hours	Following 50 percent or less of rated kva[a]				Following 100 percent of rated kva[b]			
		Probable Sacrifice In Percent of Normal Life Caused By Each Overload							
		0.10	0.25	0.50	1.00	0.10	0.25	0.50	1.00
		Maximum Load In Per Unit Of Transformer Rating							
OA or OW	0.5	2.00	2.00	2.00	2.00	1.75	1.92	2.00	2.00
	1.0	1.76	1.91	2.00	2.00	1.54	1.69	1.81	1.92
	2.0	1.50	1.62	1.72	1.82	1.35	1.48	1.58	1.68
	4.0	1.27	1.38	1.46	1.53	1.20	1.32	1.40	1.48
	8.0	1.13	1.21	1.30	1.37	1.11	1.20	1.28	1.35
	24.0	1.05	1.10	1.15	1.23	1.05	1.09	1.15	1.23
OA/FA[c]	0.5	1.97	2.00	2.00	2.00	1.67	1.82	1.94	2.00
	1.0	1.66	1.79	1.90	2.00	1.47	1.60	1.71	1.81
	2.0	1.39	1.51	1.59	1.68	1.29	1.41	1.50	1 58
	4.0	1.21	1.31	1.38	1.45	1.18	1 28	1.35	1.43
	8.0	1.11	1.19	1.26	1.33	1.10	1.18	1.26	1.33
	24.0	1.05	1.09	1.15	1.22	1.05	1.09	1.15	1.21
OA/FA/FOA[d] or FOA	0.5	1.78	1.92	2.00	2.00	1.56	1.70	1.80	1.90
	1.0	1.53	1.64	1.73	1.82	1.39	1.50	1.59	1.69
	2.0	1.32	1.42	1.49	1.57	1.26	1.36	1.43	1.51
	4.0	1.18	1.26	1.33	1.40	1.16	1.25	1.32	1.39
	8.0	1.10	1.17	1.24	1.31	1.10	1.18	1.24	1.31
	24.0	1.05	1.08	1.14	1.20	1.05	1.09	1.14	1.20

(a) More basically, following a top-oil rise of 25 C for OA and OA/FA transformers, or a 22 C rise for OA/FA/FOA and FOA units.
(b) More basically, following a top-oil rise of 45 C for OA and OA/FA transformers, or a 40 C rise for OA/FA/FOA and FOA units.
(c) Based on the FA kva rating.
(d) Based on the FOA kva rating.

TABLE 10(b)—CONDITIONS AND TRANSFORMER CHARACTERISTICS ASSUMED IN THE PREPARATION OF TABLE 10(a)

	OA OW	OA/FA	OA/FA/FOA FOA FOW
Hottest-spot rise (C)	65	65	65
Top-oil rise (C)	45	45	40
Time constant at full load (hours)	3.0	2.0	1.5
Ratio of full load copper to iron loss	2.5	3.5	5.0
Ambient temperature = 30 C.			
Maximum oil temperature = 100 C.[a]			
Maximum hottest-spot copper temperature = 150 C.			
Maximum short-time loading = 200 percent.[b]			

(a) Based on provision for oil expansion, and inert gas above the oil.
(b) Short-time loading for one-half hour or more. Terminals or tap-changers might in some cases impose a limit lower than 200 percent.

TABLE 11—PROBABLE SACRIFICE IN TRANSFORMER LIFE CAUSED BY PROLONGED HOTTEST-SPOT COPPER TEMPERATURE

Period of High Temperature, hours	0.10	0.25	0.50	1.00
	Temperature In Degrees Centigrade To Sacrifice Not More Than The Above Percent of Normal Life			
0.5	132	142	150	
1.0	124	134	142	150
2.0	117	126	134	142
4.0	111	119	126	134
8.0	105	112	119	126
24.0	99	104	109	115

are always within a permissible range and duration. This control may be accomplished with a thermal relay responsive to both top-oil temperature and to the direct heating effect of load current. The thermostatic element of this relay is immersed in the hot transformer oil, and it also carries a current proportional to load current: in this way the temperature of the element is geared to the total temperature that the transformer winding attains during operation. The relay can be arranged to close several sets of contacts in succession as the copper temperature climbs with increasing load: the first contacts to close can start fans or pumps for auxiliary cooling, the next contacts can warn of temperatures approaching the maximum safe limit, and the final contacts can trip a circuit breaker to remove load from the transformer.

Loading by copper temperature makes available the short-time overload capacity of a power transformer, so that emergency loads can be carried without interruption of power service, and so that peak loads can be carried without the use of over-size transformers.[7] The thermal relay can be coordinated with each transformer design to which it is applied, and it can inherently follow unpredictable factors that affect permissible safe loading for a particular installation.

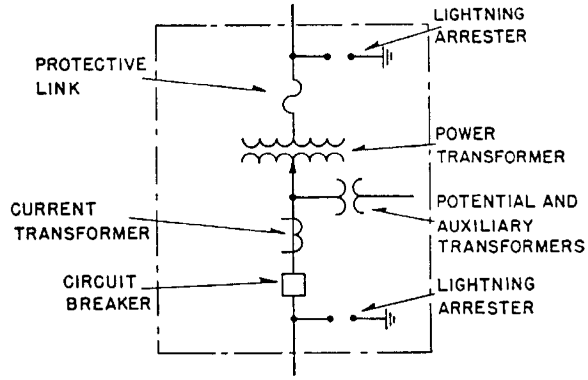


Fig. 20—Single-line diagram of CSP power transformer.

IX. THE COMPLETELY SELF-PROTECTED TRANSFORMER

A power transformer design may include protective devices capable of preventing damage to the unit when it is subjected to electrical conditions that would probably damage conventional transformers. Also, standard switching, metering, and voltage regulating functions may be included within a power transformer assembly. When these protective, switching, and metering features are all combined at the factory within a single unit, as indicated in Fig. 20, it may be designated a CSP power transformer.

Lightning Protection—Coordinated arresters are installed to protect both high- and low-voltage circuits from lightning or other voltage surges.

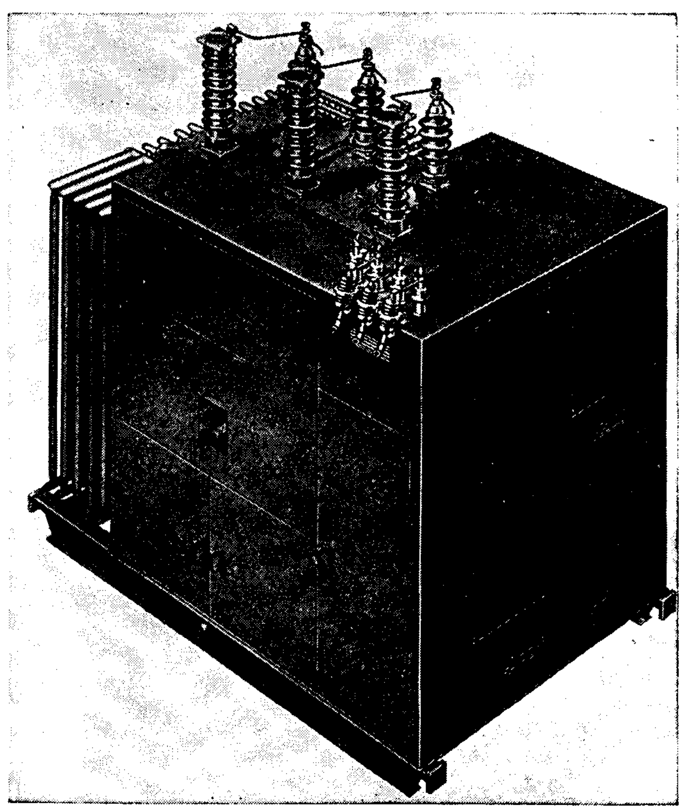


Fig. 21—Fully assembled 3000 kva, 33-4.16 kv CSP power transformer.

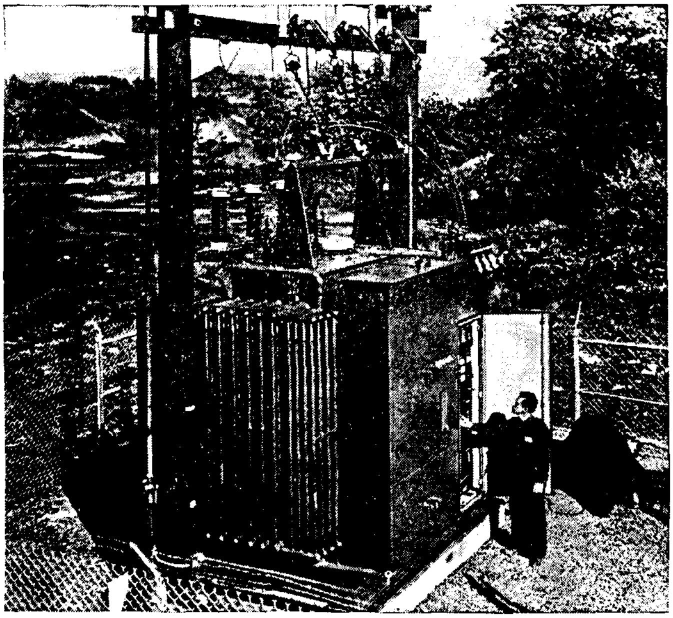


Fig. 22—Installation view of 1500 kva, 13.2-4.33 kv CSP power transformer.

Internal Fault Protection—Fusible protective links of high interrupting capacity are connected between the high-voltage bushings and the winding, so that the supply circuit can be cleared from internal transformer faults.

Overload Protection—A thermal relay, responsive to copper temperature (see section 26), operates to trip the secondary circuit breaker before damaging temperatures develop in the winding.

Relaying—Overcurrent relays normally are provided in the low-voltage circuit to protect for secondary faults.

Circuit Breaker—Load switching is accomplished by a circuit breaker in the low-voltage circuit of the transformer.

Voltage Regulation—Standard no-load taps are provided in the high-voltage winding. Tap-changing-under-load equipment for the secondary circuit may be built into the transformer housing.

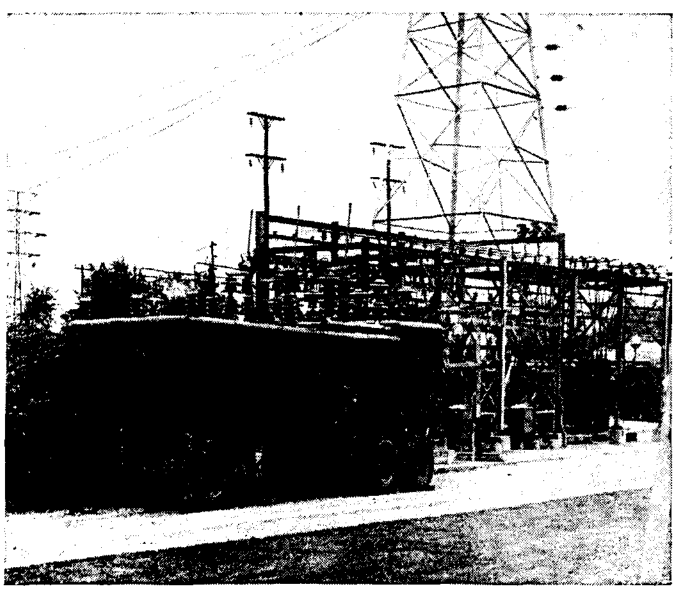


Fig. 23—Portable substation rated 2000 kva, 72 000/24 000—2.5/4.33/5.0/7.5 kv, shown in operation at a substation site.

Metering—Watthour meters and ammeters are usually supplied for circuit metering.

CSP transformers are available in kva ratings up to 3000, primary voltages up to 69 kv, and secondary voltages up to 15 kv. The units may be used to supply distribution circuits from high-voltage lines in either industrial or electric utility applications; if one unit is used individually on a radial circuit, a by-passing switch can be supplied across the low-voltage circuit breaker to permit withdrawal and maintenance of the breaker without a service interruption.

X. AUTOTRANSFORMERS

27. Two-Winding Autotransformer Theory

The single-phase two-winding autotransformer contains a primary winding and a secondary winding on a common core, just as a conventional two-winding transformer does. However, in the autotransformer the two windings are interconnected so that the kva to be transformed by actual magnetic coupling is only a portion of the total kva transmitted through the circuit to which the transformer is connected. Autotransformers are normally rated in terms of circuit kva, without reference to the internal winding kva.

The autotransformer circuit shown in Fig. 24 contains

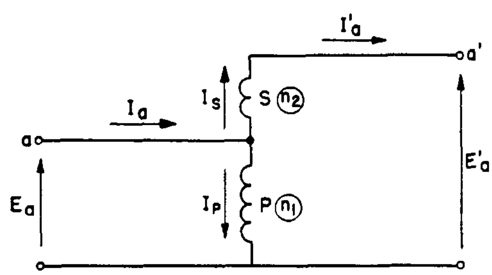


Fig. 24—Circuit for a two-winding autotransformer.

a primary winding P which is common to both low- and high-voltage circuits, and a secondary winding S which is connected directly in series with the high-voltage circuit. Under no-load conditions, high-side circuit voltage E'_a will be the sum of the primary and secondary winding voltages; low-side circuit voltage E_a will be equal to the primary winding voltage. The relation between primary and secondary winding voltages will depend upon the turns ratio $\frac{n_2}{n_1}$ between these windings.

$$\frac{E_a'}{E_a}=\frac{E_a+\frac{n_2}{n_1}E_a}{E_a}=1+\frac{n_2}{n_1}=N. \tag{33}$$

$$\frac{n_2}{n_1}=N-1. \tag{34}$$

Here N is the overall voltage ratio between high- and low-voltage circuits.

When the transformer is carrying load current, the primary ampere-turns should essentially balance the secondary ampere-turns (noting that $I'_a = I_S$):

$$n_1 I_P = n_2 I_S = n_2 I_a' \tag{35}$$

$$I_a' = \frac{n_1}{n_2} I_P \tag{36}$$

$$I_a = I_S + I_P = I_a' + \frac{n_2}{n_1} I_a' = N I_a' = N \frac{n_1}{n_2} I_P = \frac{N}{N-1} I_P. \tag{37}$$

The total circuit kva is given by $E_a \times I_a$ or $E_a' \times I_a'$ (expressing voltages in kv), but the winding kva is given by $E_a \times I_P$ or $\left(\frac{n_2}{n_1}\right) E_a \times I_S$. The ratio between winding kva (U_P or U_S) and circuit kva (U_C) is, referring to equation (37)

$$\frac{U_P}{U_C} = \frac{E_a \times I_P}{E_a \times I_a} = \frac{I_P}{\left(\frac{N}{N-1}\right) I_P} = \frac{N-1}{N}. \tag{38}$$

For example, an autotransformer rated 1000 kva, with a circuit voltage ratio of 22 kv to 33 kv $\left(N = \frac{33}{22} = 1.5\right)$ has an equivalent two-winding kva of

$$U_P = U_S = \frac{N-1}{N} U_C = \frac{1.5 - 1.0}{1.5} \times 1000 = 333 \text{ kva}$$

The reduced rating of transformer parts required in an autotransformer make it physically smaller, less costly, and of higher efficiency than a conventional two-winding unit for the same circuit kva rating. In the example just cited, the autotransformer would theoretically be only as large as a 333-kva conventional transformer, and this reduced kva would in practice furnish a fairly accurate basis for estimating the cost of the 1000-kva autotransformer. Total losses in the autotransformer would be comparable to those in a 333-kva conventional transformer, so that efficiency based on circuit transmitted power would be quite high.

An autotransformer will introduce series impedance, as well as current and voltage transformation, in the circuit where it is connected. The series impedance may be evaluated by referring to Fig. 25(a); here the low-voltage circuit terminals are short-circuited, so that the impedance measured at the high-voltage terminals will be equal to the series circuit impedance attributable to the autotransformer. Note that the circuit in Fig. 25(a) is exactly the same as the circuit that would be used to measure the leakage impedance if Z_{SP} were defined as the ohmic impedance measured across the secondary winding with the primary winding short-circuited. A circuit providing correct circuit voltage and current ratios, and also correct through impedance, is shown in Fig. 25(b). Two conversions may now be made, the first to move the series impedance to the low-voltage side, and the second to express impedance in terms of Z_{PS}.

$$Z_a = \frac{1}{N^2} Z_a' = \frac{1}{N^2} Z_{SP}. \tag{39}$$

$$Z_{SP} = \left(\frac{n_2}{n_1}\right)^2 Z_{PS} = (N-1)^2 Z_{PS}. \tag{40}$$

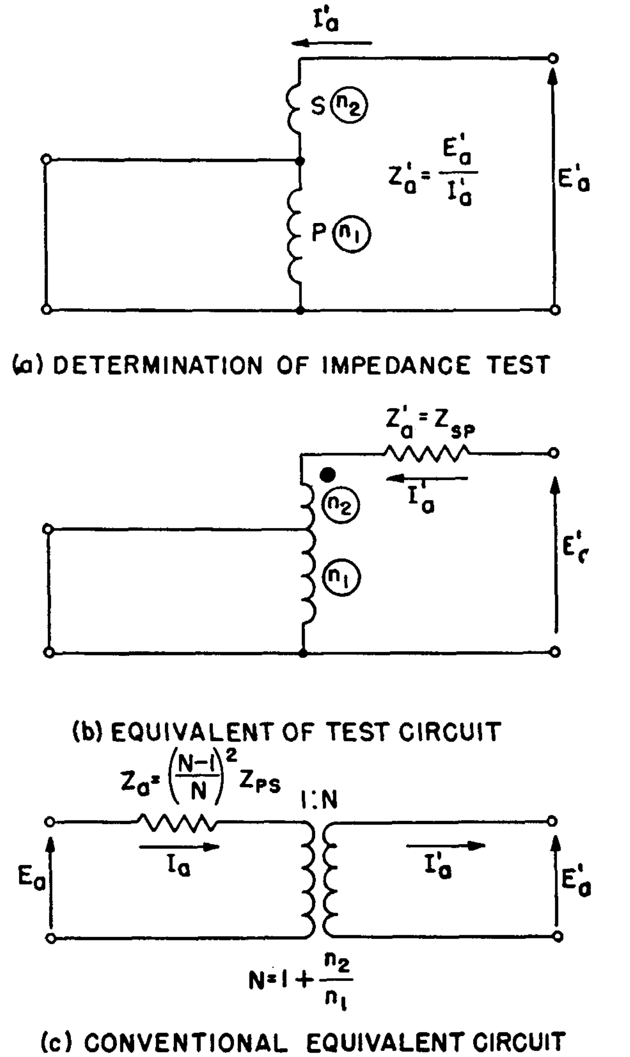


Fig. 25—Equivalent circuits for a two-winding autotransformer.

From this, the conventional form of equivalent circuit is shown in Fig. 25(c), where

$$Z_a = \left(\frac{N-1}{N}\right)^2 \times Z_{PS}. \tag{41}$$

Sequence equivalent circuits for the three-phase two-winding autotransformer are presented in the Appendix.

The circuit impedance of an autotransformer is smaller than that of a conventional two-winding unit of the same rating, as is evident from Eq. (41). This low series impedance, though advantageous in its effect on transformer regulation, may allow excessive short-circuit currents during system fault conditions. Often the through impedance will be less than four percent based on the autotransformer nameplate kva rating, which means that three-phase short circuit current could exceed the maximum of twenty-five times normal rated current for two seconds as permitted by standards. For this reason autotransformers, like voltage regulators, cannot always protect themselves against excessive fault current; reactors or other connected circuit elements may have to be relied on for this protection.

28. The Three-Winding Autotransformer

Three-phase autotransformers for power service are usually star-star connected with the neutral grounded, and in most of these cases it is desirable to have a third winding on the core delta-connected so as to carry the third harmonic component of exciting current. This winding could be very small in capacity if it were required to carry only harmonic currents, but its size is increased by the requirement that it carry high currents during system ground faults. A widely used rule sets the delta-winding rating at 35 percent of the autotransformer equivalent two-winding kva rating (not circuit kva rating).

Since it is necessary in most cases to have a delta-connected tertiary winding, it is often advantageous to design this winding so that load can be taken from it. This results in a three-winding autotransformer with terminals to accommodate three external circuits. The equivalent circuit for this type of transformer is given in section 59 of this chapter.

29. Autotransformer Taps

It is frequently necessary to place taps in the windings of an autotransformer to regulate either or both circuit voltages. It is not advisable to place taps adjacent to the line connections for voltages above 22 000 volts, because extra insulation is necessary on turns adjacent to the line terminals. If taps were placed at the ends of the winding, additional padding would be required throughout the tapped section. Furthermore, taps placed adjacent to the line, where the most severe voltage stresses occur, constitute a weakness that can be avoided by placing the taps in the middle of the winding as shown in Fig. 26. Taps

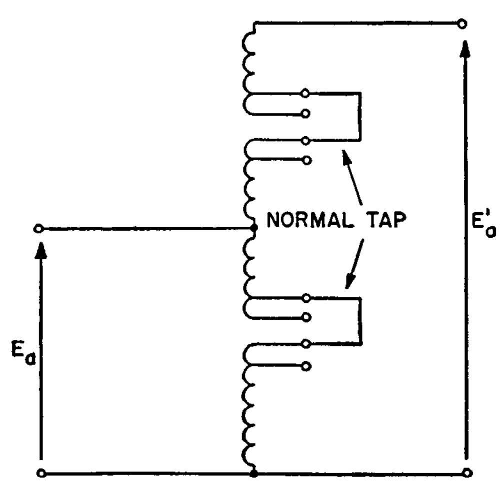


Fig. 26—Autotransformer taps.

may be placed in either the primary (common) winding, or in the secondary (series) winding, or in both windings: however, some tap combinations are more desirable than others, if the transformer materials are to be used most effectively.

The low-side and high-side circuit voltages may be related, under no-load conditions, by an equation which takes account of both primary and secondary taps:

$$E_a' = E_a + \frac{n_2 + t_2 n_2}{n_1 + t_1 n_1} E_a = \frac{n_1(1+t_1) + n_2(1+t_2)}{n_1(1+t_1)} E_a. \tag{42}$$

n_1 = turns on primary winding, not considering taps.
n_2 = turns on secondary winding, not considering taps.
t_1 = fractional part of n_1 included in primary winding tap ($+t_1 n_1$ indicates additional turns)
t_2 = fractional part of n_2 included in secondary winding tap. ($+t_2 n_2$ indicates additional turns).

If E_a is assumed constant at 1.0 per unit based on normal rated low-side circuit voltage, three cases are possible:

(1) *Taps in secondary winding only:*

$$E_a' = 1 + \frac{n_2}{n_1} + t_2 \frac{n_2}{n_1} \tag{43}$$

In this case the transformer volts per turn remain normal. The percent change in E_a' is:

$$\Delta E_a' = t_2 \frac{n_2}{n_1 + n_2} 100. \tag{44}$$

(2) *Taps in primary winding only:*

$$E_a' = 1 + \frac{n_2}{n_1} - \frac{t_1 n_2}{n_1 + n_1 t_1}. \tag{45}$$

The transformer volts per turn are $\left(\frac{1}{1+t_1}\right)$ times their normal value. The percent change in E_a' is:

$$\Delta E_a' = -\frac{t_1}{1+t_1} \times \frac{n_2}{n_1+n_2} 100. \tag{46}$$

(3) *Taps in both primary and secondary windings:*

$$E_a' = 1 + \frac{n_2}{n_1} + \frac{t_2 - t_1}{1+t_1} \times \frac{n_2}{n_1} \tag{47}$$

As in case (2), the transformer volts per turn are $\left(\frac{1}{1+t_1}\right)$ times their normal value. The percent change in E_a' is:

$$\Delta E_a' = \frac{t_2 - t_1}{1+t_1} \times \frac{n_2}{n_1+n_2} 100. \tag{48}$$

If E_a' is assumed constant at 1.0 per unit based on normal rated high-side circuit voltage, and E_a is allowed to vary, three more cases are possible:

(4) *Taps in secondary winding only:*

$$E_a = \frac{n_1}{n_1+n_2} - \frac{n_1}{n_1+n_2} \times \frac{n_2 t_2}{n_1 + n_2(1+t_2)} \tag{49}$$

The transformer volts per turn are $\left(\frac{n_1+n_2}{n_1+n_2(1+t_2)}\right)$ times their normal value.

The percent change in E_a will be:

$$\Delta E_a = -t_2 \frac{n_2}{n_1 + n_2(1+t_2)} 100. \tag{50}$$

(5) *Taps in primary winding only:*

$$E_a = \frac{n_1}{n_1+n_2} + \frac{n_1}{n_1+n_2} \times \frac{n_2 t_1}{n_1(1+t_1) + n_2} \tag{51}$$

Transformer volts per turn are $\left(\frac{n_1+n_2}{n_1(1+t_1)+n_2}\right)$ times their normal value.

The percent change in E_a' is:

$$\Delta E_a = t_1 \frac{n_2}{n_1(1+t_1)+n_2} 100. \tag{52}$$

(6) *Taps in both primary and secondary:*

$$E_a = \frac{n_1}{n_1+n_2} + \frac{n_1}{n_1+n_2} \times \frac{n_2(t_1-t_2)}{n_1(1+t_1)+n_2(1+t_2)}. \tag{53}$$

Transformer volts per turn are $\left(\frac{n_1+n_2}{n_1(1+t_1)+n_2(1+t_2)}\right)$ times their normal value. The percent change in E_a' is:

$$\Delta E_a = \frac{n_2(t_1-t_2)}{n_1(1+t_1)+n_2(1+t_2)} 100. \tag{54}$$

If the transformer were designed for constant volts per turn $\left(t_2 = -t_1 \frac{n_1}{n_2}\right)$, then the percent change in E_a would be:

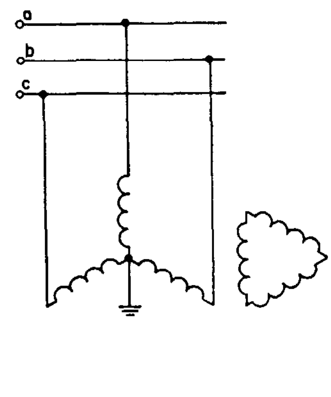
$$\Delta E_a = t_1 \times 100. \tag{55}$$

It is often advisable to specify a tap combination which will allow the autotransformer to operate at practically constant volts-per-turn, regardless of tap position. As indicated in some of the cases above, a tap change in only one winding may be less effective than would normally be anticipated, because of the nullifying effect of the accompanying change in volts-per-turn. Also, a significant increase in volts-per-turn at some tap setting would be reflected in a magnetic core of larger size than otherwise necessary.

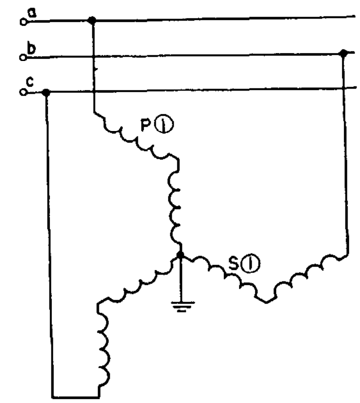
30. Autotransformer Operating Characteristics

An autotransformer inherently provides a metallic connection between its low- and high-voltage circuits; this is unlike the conventional two-winding transformer which isolates the two circuits. Unless the potential to ground of each autotransformer circuit is fixed by some means, the low-voltage circuit will be subject to overvoltages originating in the high-voltage circuit. These undesirable effects can be minimized by connecting the neutral of the autotransformer solidly to ground. If the neutral of an autotransformer is always to be grounded in service, an induced potential shop test is more appropriate than an applied potential test, because it represents more closely the field operating conditions; building a grounded autotransformer to withstand a full-voltage applied potential test would not be economical because of the excess insulation near the neutral.

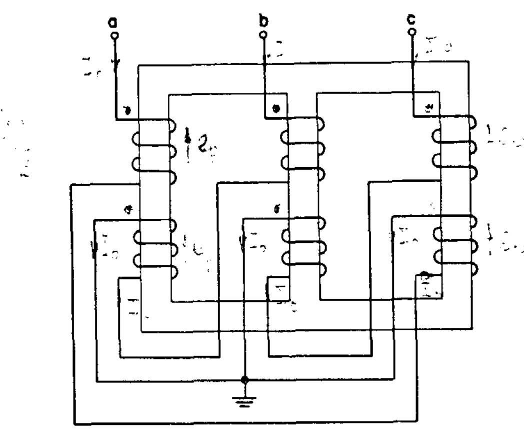
To summarize the preceding discussion, the autotransformer has advantages of lower cost, higher efficiency, and better regulation as compared with the two-winding transformer; it has disadvantages including low reactance which may make it subject to excessive short-circuit currents, the arrangement of taps is more complicated, the delta tertiary may have to carry fault currents exceeding its standard rating, the low- and high-voltage circuits cannot be isolated, and the two circuits must operate with no angular phase displacement unless a zig-zag connection is introduced. The advantages of lower cost and improved effi-



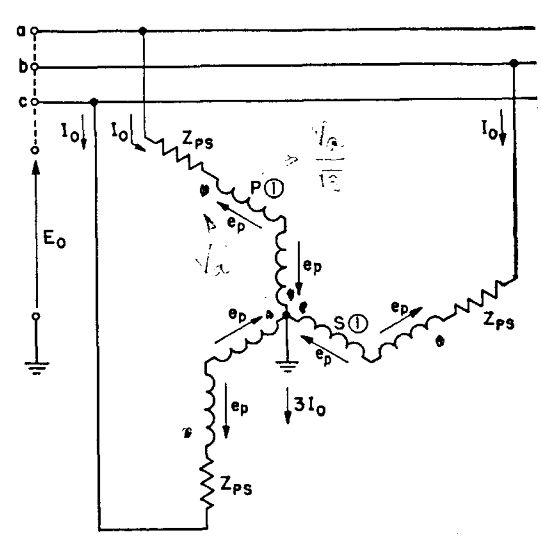
(a) STAR-DELTA GROUNDING-TRANSFORMER



(b) INTERCONNECTED-STAR GROUNDING TRANSFORMER WINDINGS DRAWN PARALLEL ARE ON THE SAME CORE



(c) SCHEMATIC WINDING ARRANGEMENT OF AN INTERCONNECTED-STAR GROUNDING TRANSFORMER OF THE THREE-PHASE CORE-FORM CONSTRUCTION



(d) EQUIVALENT CIRCUIT OF AN INTERCONNECTED-STAR GROUNDING TRANSFORMER

Fig. 27—Star-delta and zig-zag grounding transformers.

ciency become less apparent as the transformation ratio increases, so that autotransformers for power purposes are usually used for low transformation ratios, rarely exceeding 2 to 1.

XI. GROUNDING TRANSFORMERS

A grounding transformer is a transformer intended solely for establishing a neutral ground connection on a three-phase system. The transformer is usually of the star-delta or interconnected-star (zig-zag) arrangement as shown in Fig. 27.

The kva rating of a three-phase grounding transformer, or of a grounding bank, is the product of normal *line-to-neutral* voltage (kv) and the *neutral* or *ground* amperes that the transformer is designed to carry under fault conditions for a specified time. A one-minute time rating is often used for grounding transformers, though other ratings such as those suggested in AIEE Standard for "Neutral Grounding Devices" (No. 32, May 1947) can be specified depending upon the probable duty to be imposed on the unit in service.

Rated voltage of a grounding transformer is the line-to-line voltage for which the unit is designed.

When operated at rated three-phase balanced voltage, only exciting current circulates in the windings of a grounding transformer. Current of appreciable magnitude begins to flow in the grounding circuit only when a fault involving ground develops on the connected system.

Grounding transformers, particularly the zig-zag type, normally are designed so that rated neutral current flows when a solid single-line-to-ground fault is applied at the transformer terminals, assuming supply voltage to be fully maintained. This is equivalent to 100-percent zero-sequence voltage impressed at the transformer terminals resulting in the circulation of rated neutral current. Transformers so designed are said to have 100-percent impedance based on rated kva and rated voltage.

Sometimes a resistor or other impedance is connected in the transformer neutral, and in these cases it may be desirable to specify that the grounding transformer shall have less than the conventional 100 percent impedance. Equivalent circuits for star-delta and zig-zag grounding transformers with external neutral impedance are included in the Appendix.

Because a grounding transformer is a short-time device, its size and cost are less than for a continuous duty transformer of equal kva rating. The reduced size can be established in terms of an "equivalent two-winding 55 C kva" U_x by applying a reduction factor K to the short-time rated kva of the grounding transformer, and this reduced kva can be used for a price estimate.

$U_x = U_G \times K_3$ for a three-phase grounding unit. (56)

$U_x = 3U_G \times K_1$ for a bank of single-phase grounding units (57)

where

U_x = equivalent two-winding 55 C kva, three-phase.

U_G = (line-to-neutral kv) $\times$ (rated neutral amperes).

Values for K are listed in Table 12 for various types and classes of grounding transformers; the table includes values for both three-phase and single-phase units, though the single-phase type is uncommon.

Table 12—"K" Factors for Determining Equivalent Two-Winding 55 C kva of Grounding Transformers*

Time Rating	Star-Delta Connection	Zig-Zag Connection 2.4 to 13.8 kv	23 to 34.5 kv	46 kv	69 kv	92 kv
K_3, For A Three Phase Unit						
10 seconds	......	0.064	0.076	0.080	0.085	0.092
1 minute	0.170	0.104	0.110	0.113	0.118	0.122
2 minutes	0.240	0.139	0.153	0.160	0.167	0.174
3 minutes	0.295	0.170	0.187	0.196	0.204	0.212
4 minutes	0.340	0.196	0.216	0.225	0.235	0.245
5 minutes	0.380	0.220	0.242	0.253	0.264	0.275
K_1, For A Single Phase Unit (One of three in a bank)						
1 minute	0.057	0.033	0.037	0.040	0.043	0.046
2 minutes	0.080	0.046	0.051	0.055	0.060	0.064
3 minutes	0.098	0.057	0.064	0.068	0.074	0.080
4 minutes	0.113	0.065	0.073	0.078	0.084	0.091
5 minutes	0.127	0.073	0.082	0.088	0.095	0.102

*These values are calculated on the basis that the initial average winding temperature is not over 75C, that the heat from load losses is all stored in the transformer, and that the final temperature will not exceed values permitted by standards. The values are applicable only for grounding transformers designed to have 100 percent impedance.

Conventional power transformers may be connected to serve solely as grounding transformers, but the current and time ratings for grounding service are open to question depending upon the form and details of construction. When these modified ratings are desired, they should be obtained from the transformer manufacturer.

Star-Delta Impedances—The impedance to zero-sequence currents in each phase of a solidly-grounded star-delta grounding bank made up of single-phase units is equal to Z_{PS}, the ohmic leakage impedance between one primary (star) winding and the corresponding secondary (delta) winding:

$$Z_0 = Z_{PS} \tag{58}$$

Percent zero-sequence impedance is normally expressed in terms of short-time kva and line-to-line voltage:

$$Z_0\% = \frac{Z_{PS} \times U_G}{10 \times \text{kv}^2} \tag{59}$$

In a three-phase star-delta grounding transformer Z_0 may be smaller than Z_{PS} by an amount depending on the form of core construction: a typical ratio of Z_0 to Z_{PS} is 0.85, though variation from this value for different designs is likely.

Zig-zag Impedances—The impedance to zero-sequence currents in each phase of a solidly grounded zig-zag bank can be derived on a theoretical basis by reference to Fig. 27(d).

$$E_0 = I_0 \times Z_{PS} - e_p + e_p. \tag{60}$$

$$\frac{E_0}{I_0} = Z_0 = Z_{PS}. \tag{61}$$

Percent zero-sequence impedance for the zig-zag connection is normally expressed in terms of short-time kva and line-to-line voltage:

$$Z_0\% = \frac{Z_{PS} \times U_G}{10 \times \text{kv}^2} \tag{62}$$

XII. TAP CHANGING UNDER LOAD

The modern load tap changer had its beginning in 1925. Since that time the development of more complicated transmission networks has made tap changing under load more and more essential to control the in-phase voltage of power transformers, and in other cases to control the phase angle relation. Tap-changing-under-load equipment is applied to power transformers to maintain a constant secondary voltage with a variable primary voltage; to control the secondary voltage with a fixed primary voltage; to control the flow of reactive kva between two generating systems, or adjust the reactive flow between branches of loop circuits; and to control the division of power between branches of loop circuits by shifting the phase-angle position of transformer output voltages.

Various types of tap-changing equipment and circuits are used depending upon the voltage and kva and also upon whether voltage or phase angle control is required. Under-load-tap-changers are built for 8, 16, and 32 steps, with the trend in recent years being toward the larger number of steps so as to give a finer degree of regulation. The usual range of regulation is plus 10 percent and minus 10 percent of the rated line voltage, with plus and minus $7\frac{1}{2}$ percent and plus and minus 5 percent being second and third, respectively, in popularity. The 32 step, plus and minus 10 percent, tap-changing-under-load equipment has such wide acceptance as to be considered standard for many types of transformers.

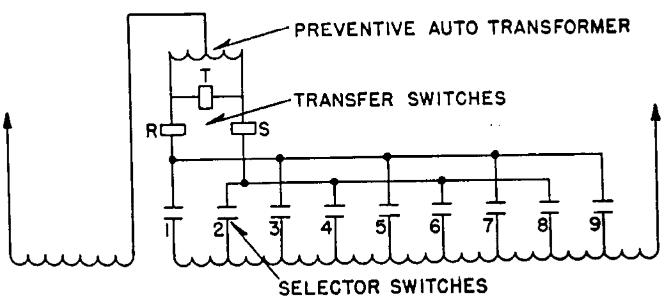
31. The UT Mechanism

Figure 28 illustrates schematically the operation of the type UT mechanism for changing taps under load. Taps from the transformer winding connect to selector switches 1 through 9. The selector switches are connected to load transfer switches *R*, *S*, and *T*. The connections for the tap changer positions are shown on the sequence chart of Fig. 28. The sequence of switching is so coordinated by the tap changing mechanism that the transfer switches perform all the switching operations, opening before and closing after the selector switches. All arcing is thus restricted to switches *R*, *S*, and *T*, while switches 1 to 9 merely select the transformer tap to which the load is to be transferred.

When the tap changer is on odd-numbered positions, the preventive auto-transformer is short-circuited. On all even-numbered positions, the preventive auto-transformer bridges two transformer taps. In this position, the relatively high reactance of the preventive auto-transformer to circulating currents between adjacent taps prevents damage to the transformer winding, while its relatively low impedance to the load current permits operation on this position to obtain voltages midway between the transformer taps.

32. The UNR Mechanism

Fig. 29 shows schematically the diagram of connections and sequence of operations of the type UNR tap changer. The operation of the selector and transfer switches is exactly as described for the type UT tap changer. But the type UNR tap changer also has a reversing switch which reverses the connections to the tapped section of the winding so that the same range and number of positions

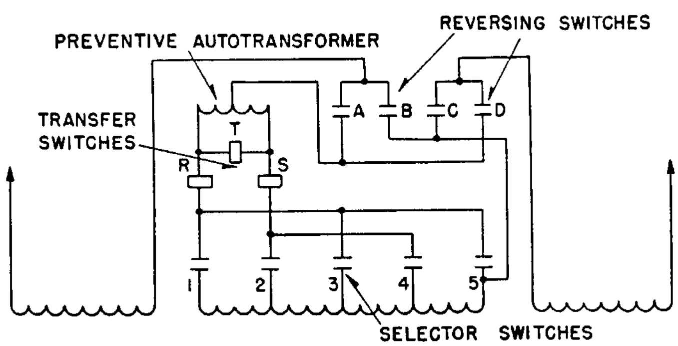


SEQUENCE OF OPERATION

POSITION	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
SWITCH-1	O	O															
" -2		O	O	O													
" -3				O	O	O											
" -4						O	O	O									
" -5								O	O	O							
" -6										O	O	O					
" -7												O	O	O			
" -8														O	O	O	
" -9																O	O
" -R	O	O		O	O	O		O	O	O		O	O	O		O	O
" -S		O	O	O		O	O	O		O	O	O		O	O	O	
" -T	O		O		O		O		O		O		O		O		O

O=SWITCH CLOSED

Fig. 28—Seventeen position, single-phase, Type UT tap changer.



SEQUENCE OF OPERATION

POSITION	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
SWITCH-1	O	O														O	O
" -2		O	O	O										O	O	O	
" -3				O	O	O						O	O	O			
" -4						O	O	O		O	O	O					
" -5								O	O	O							
" -A									O	O	O	O	O	O	O	O	O
" -B	O	O	O	O	O	O	O	O	O								
" -C									O	O	O	O	O	O	O	O	O
" -D	O	O	O	O	O	O	O	O	O								
" -R	O	O		O	O	O		O	O	O		O	O	O		O	O
" -S		O	O	O		O	O	O		O	O	O		O	O	O	
" -T	O		O		O		O		O		O		O		O		O

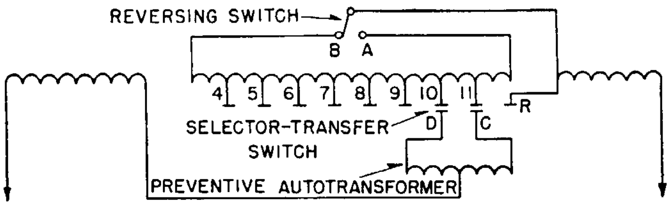
O=SWITCH CLOSED

Fig. 29—Seventeen position, single phase, Type UNR tap changer.

can be obtained with one-half the number of tap sections, or twice the range can be obtained with the same number of taps. The reversing switch is a close-before-open switch which operates at the time there is no voltage across its contacts.

33. The URS Mechanism

The type URS load tap changer is applied to small power transformers and large distribution transformers. The transfer switches are eliminated, and each selector switch serves as a transfer switch for the tap to which it is connected. The schematic circuit diagram and operations sequence chart is shown in Fig. 30.



SEQUENCE OF OPERATION																																	
POSITION	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
SW D-11	O																	O	O														
" C-11	O	O																	O	O													
" D-10		O	O																	O	O												
" C-10			O	O																	O	O											
" D-9				O	O																	O	O										
" C-9					O	O																	O	O									
" D-8						O	O																	O	O								
" C-8							O	O																	O	O							
" D-7								O	O																	O	O						
" C-7									O	O																	O	O					
" D-6										O	O																	O	O				
" C-6											O	O																	O	O			
" D-5												O	O																	O	O		
" C-5													O	O																	O	O	
" D-4														O	O																	O	O
" C-4															O	O																	O
" D-R																O	O																
" C-R																	O	O															
" A																	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
" B	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O																	

O = SWITCH CLOSED

Fig. 30—Thirty-three position, single-phase, Type URS tap changer.

Physically, the stationary selector switch contacts are arranged in circles, one for each phase. The moving selector switch contacts, as they rotate about a center shaft, both select the taps and make contact with them. The reversing switch operates when the selector switches are on position 17, at which time there is no current through the reversing switches and therefore no arcing on them.

The URS tap changer, like the other load tap changers, can be equipped for hand operation, remote manual operation, or for full automatic operation under the control of relays.

XIII. REGULATING TRANSFORMERS FOR VOLTAGE AND PHASE ANGLE CONTROL

Consider two systems A and B in Fig. 31 connected by a single transmission circuit. A and B may both be generating units, or one of them may be a generating unit and the other a load. Should A generate 10 000 kilowatts in excess of its own load, there can be but one result, the 10 000

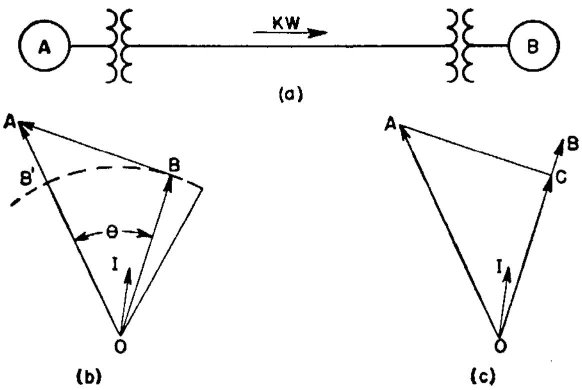


Fig. 31—Power interchange between systems:

(a) Two systems with tie.
(b) Vector diagram of voltages during interchange of power.
(c) Introduction of an in phase voltage, BC, to correct for excessive voltage drop.

kilowatts must go over the tie line to B. An increase in generator output by A must be accompanied by a corresponding decrease in output (increase in input) by B if there is to be no change in system frequency. The transmission of power from A to B results in a difference in magnitude between terminal voltages and also a shift in phase angle, as illustrated in Fig. 31 (b). AO is the terminal voltage at A, BO is the terminal voltage at B, AB is the vectorial voltage drop from A to B, created by the flow of load current I, and θ is the phase-angle difference between terminal voltages. In actual practice the phase angle is not always apparent, but the drop in voltage, AB', is often objectionable. An attempt to maintain satisfactory terminal voltages at A and B will often result in undesirable circulation of reactive kva between the systems. The flow of power from A to B, or vice versa, is determined by the governor settings. The flow of reactive power over the interconnecting line is determined by the terminal voltages held by the machine excitations at A and B. Excessive voltage drop between the systems can be readily corrected by transformer taps of a fixed nature or by tap-changing equipment, introduc-

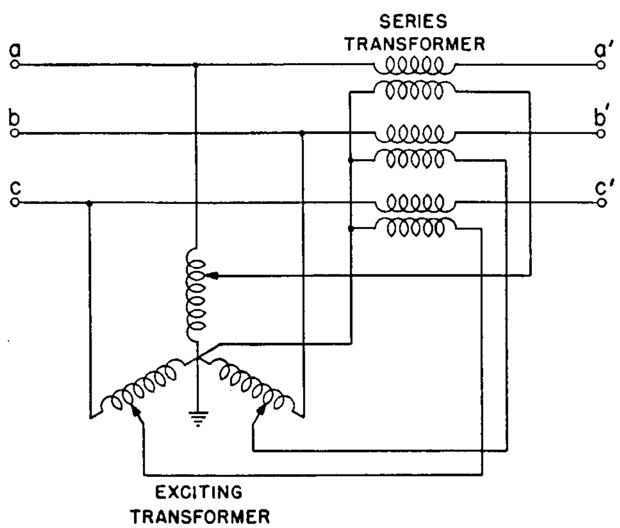


Fig. 32—Regulating transformer for voltage control.

ing an in-phase voltage, BC, to compensate for the voltage drop and bring the terminal voltage at B to a desired value. Figure 32 is a simplified sketch of a regulating transformer for voltage control, using an exciting autotransformer with automatic tap changing equipment indicated by the arrows.

Consider three systems interconnected with each other so that the interconnections from A to B, from B to C, and from C to A form a closed loop, as in Fig. 33 (a). An

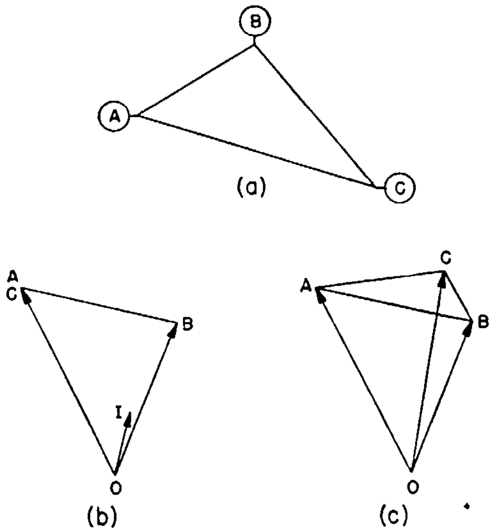


Fig. 33—Power interchange with three interconnected systems.

entirely new element enters, and adjustment of governors will not entirely control the flow of power over any one of the interconnecting lines. An attempt to adjust load on the tie between two systems results in a change of load on the other two tie lines. With the tie line from B to C open, and with power transmitted from A to B, the terminal voltages of A and C will be equal and in phase, with no power being transmitted from A to C, or vice versa (see Fig. 33 (b)). There now exists between B and C a difference in voltage and a difference in phase angle. If the tie line between B and C is closed under these conditions there is a redistribution of power flow between A and B, a part going over the line from A to B, and a part of the power going from A to B over the lines A-C and C-B (see Fig. 33 (c)). The distribution of power, both kw and reactive kva between the various lines is determined solely by the relative impedances of the interconnecting lines.

If at the time of closing B-C an adjustment of transformer taps were made, or a regulating transformer for voltage control were inserted in the loop, it would be possible to make the voltage at C equal in magnitude to that at B but it would not have the same phase relationship. There would still be a flow of power from A to C and from C to B.

Conditions similar to that just described occur on interconnected systems involving loop circuits. To control the circulation of kw and prevent overloading certain lines it is often necessary to introduce a quadrature voltage, any place in the loop, by the use of a regulating transformer for phase-angle control. This differs from the usual star-delta power transformation in that the angle of phase shift of current and voltage is not fixed but depends on the tap position. Figure 34 is a schematic diagram of a typical regulating transformer for phase angle control.

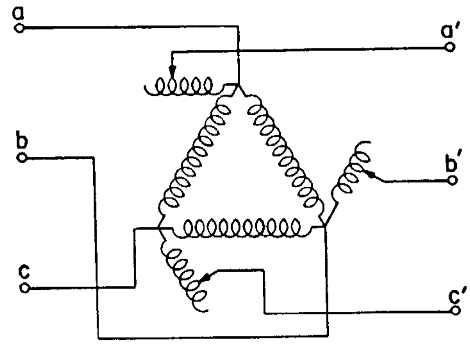


Fig. 34—Regulating transformer for phase-angle control.

In general the distribution of real power flow over the various interconnections found in loop circuits can be controlled by regulators for phase-angle control. The flow of reactive kva can be controlled by regulators for voltage control. The preceding statements follow from the fact that transmission-circuit impedances are predominantly reactive. The voltage regulator introduces a series in-phase voltage into the loop, and quadrature current (reactive kva) is circulated around the loop since the impedances are reactive. The regulator for phase-angle control introduces a quadrature series voltage in the loop resulting in the flow of currents lagging the impressed voltage by nearly 90 degrees, or the circulation of in-phase currents (kw).

For the case of correcting the voltage for line drop, a simple voltage control equipment can be used. This simply adds or subtracts a voltage in phase with the system voltage. For the case of phase-angle control, the equipment can be identical except the voltage selected to add or subtract is in quadrature. As the earlier discussion showed, there are cases where both voltage and phase angle control

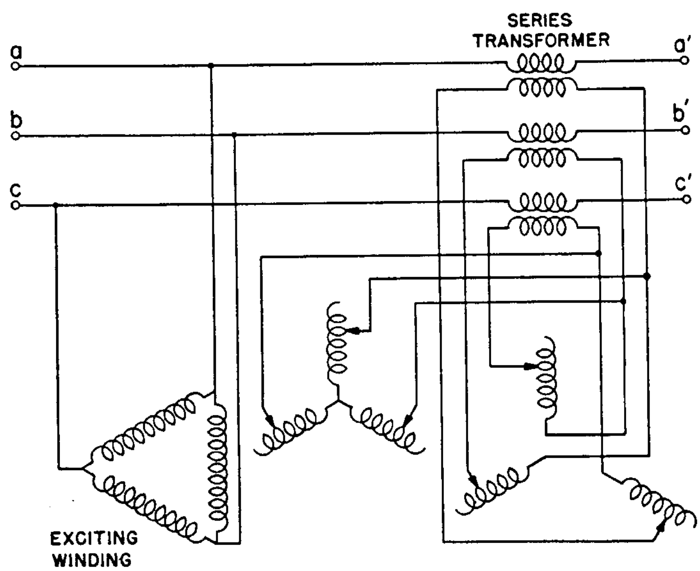


Fig. 35—Regulating transformer for independent phase-angle and voltage control.

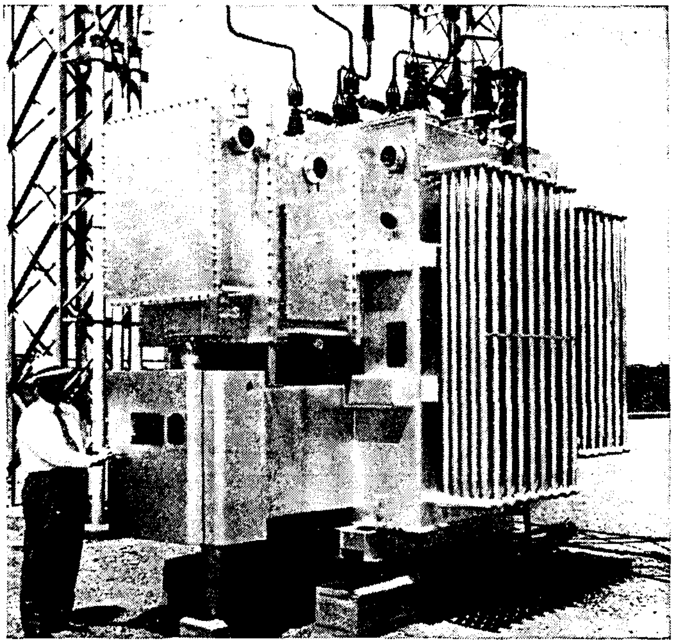


Fig. 36—Regulating transformer for voltage control, rated 20 000 kva, 12.47 kv, plus or minus 10 percent.

are required. There are a number of combinations of connections to accomplish this, one of them being shown in Fig. 35. Where the voltage and phase angle bear a close relation, one mechanism may suffice. However, where completely independent control is desired, two mechanisms with two regulating windings and one series winding, or with one regulating winding and two series windings are necessary. If it is desired to close the loop, and the flow of both real and reactive power over the various lines forming the loop must be controlled, the economical location for the control equipment is at the point of lowest load to be transferred. This may dictate the location in a loop, unless when in tying several companies together the boundary between systems determines the location. The voltage to be added or the phase-angle shift that must be obtained can be determined by calculation, considering the impedances of the tie line and the load conditions in the loop. When such calculations become involved, the use of the network calculator provides a quick and accurate tool for obtaining the solution.

Several common connections used for regulating transformers providing voltage control, phase angle control, or combined voltage and phase angle control, are tabulated in the Appendix under Equivalent Circuits of Power and Regulating Transformers. The equivalent circuits of the regulating transformers to positive-, negative-, and zero-sequence are given. It should be noted that the equivalent circuits for phase-angle control regulators involve an ideal transformer providing a phase shift of voltage and current. Positive-sequence voltage and current are always shifted by the *same* angle in the *same* direction. The angular shift for negative-sequence voltage and current is always equal to the angular shift for positive-sequence, but is in the *opposite* direction. Zero-sequence currents and voltages do not undergo an angular shift in being transformed. For example, refer to F-7 in Table 7 of the Appendix, which is the regulating transformer for phase-angle control shown in Fig. 34.

For positive-sequence, neglecting regulator impedance:

$$E_1' = N\epsilon^{j\alpha}E_1 = \sqrt{1+3n^2}\,\epsilon^{j\alpha}E_1 \tag{63}$$

$$I_1' = \frac{1}{N}\epsilon^{j\alpha}I_1 = \frac{1}{\sqrt{1+3n^2}}\epsilon^{j\alpha}I_1 \tag{64}$$

where $\alpha = \tan^{-1}\sqrt{3}n$

For negative-sequence, neglecting regulator impedance:

$$E_2' = N\epsilon^{-j\alpha}E_2 = \sqrt{1+3n^2}\,\epsilon^{-j\alpha}E_2 \tag{65}$$

$$I_2' = \frac{1}{N}\epsilon^{-j\alpha}I_2 = \frac{1}{\sqrt{1+3n^2}}\epsilon^{-j\alpha}I_2 \tag{66}$$

For zero-sequence, neglecting regulator impedance:

$$E_0' = E_0 \tag{67}$$

$$I_0' = I_0 \tag{68}$$

For this regulator zero-sequence voltage and current are not transformed; I_0 flows through the regulator as though it were a reactor.

It happens with several connections of regulating transformers that zero-sequence voltages and currents are not transformed at all, as in F-7; or are transformed with a different transformation ratio than for positive- or negative-sequence quantities as in G-1. This phenomenon, and the use of the sequence equivalent circuits for regulating transformers has been discussed in papers by Hobson and Lewis[2], and by J. E. Clem.[8]

XIV. EXCITING AND INRUSH CURRENTS

If normal voltage is impressed across the primary terminals of a transformer with its secondary open-circuited, a small exciting current flows. This exciting current consists of two components, the loss component and the magnetizing component. The loss component is in phase with the impressed voltage, and its magnitude depends upon the no-load losses of the transformer. The magnetizing component lags the impressed voltage by 90 electrical degrees, and its magnitude depends upon the number of turns in the primary winding, the shape of the transformer saturation curve and the maximum flux density for which the transformer was designed. A brief discussion of each of these components follows:

34. Magnetizing Component of Exciting Current

If the secondary of the transformer is open, the transformer can be treated as an iron-core reactor. The differential equation for the circuit consisting of the supply and the transformer can be written as follows:

$$e = Ri + n_1\frac{d\phi}{dt} \tag{69}$$

where, e = instantaneous value of supply voltage
i = instantaneous value of current
R = effective resistance of the winding
ϕ = instantaneous flux threading primary winding
n_1 = primary turns

Normally the resistance, R, and the exciting current, i, are small. Consequently the Ri term in the above equation has little effect on the flux in the transformer and can, for the purpose of discussion, be neglected. Under these conditions Eq. (69) can be rewritten:

$$e = n_1 \frac{d\phi}{dt} \tag{70}$$

If the supply voltage is a sine wave voltage,

$$e = \sqrt{2}E \sin(\omega t + \lambda), \tag{71}$$

where, E = rms value of supply voltage

$$\omega = 2\pi f$$

Substituting in Eq. (70)

$$\sqrt{2}E \sin(\omega t + \lambda) = n_1 \frac{d\phi}{dt}$$

Solving the above differential equation,

$$\phi = -\frac{\sqrt{2}E}{\omega n_1} \cos(\omega t + \lambda) + \phi_t \tag{72}$$

In this solution, $-\frac{\sqrt{2}E}{\omega n_1} \cos(\omega t + \lambda)$ is the normal steady-state flux in the transformer core. The second term, ϕ_t, represents a transient component of flux the magnitude of which depends upon the instant at which the transformer is energized, the normal maximum flux and the residual flux in the core at the time the transformer is energized. Under steady-state conditions this component is equal to zero; the magnitude of ϕ_t is discussed in Sec. 38.

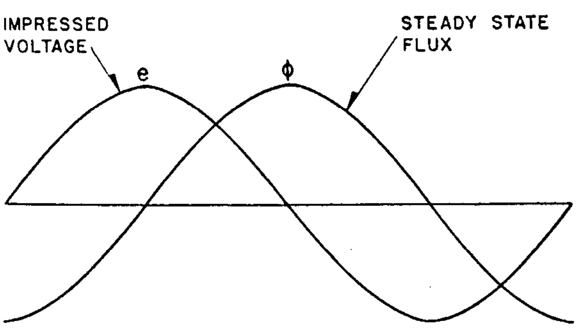
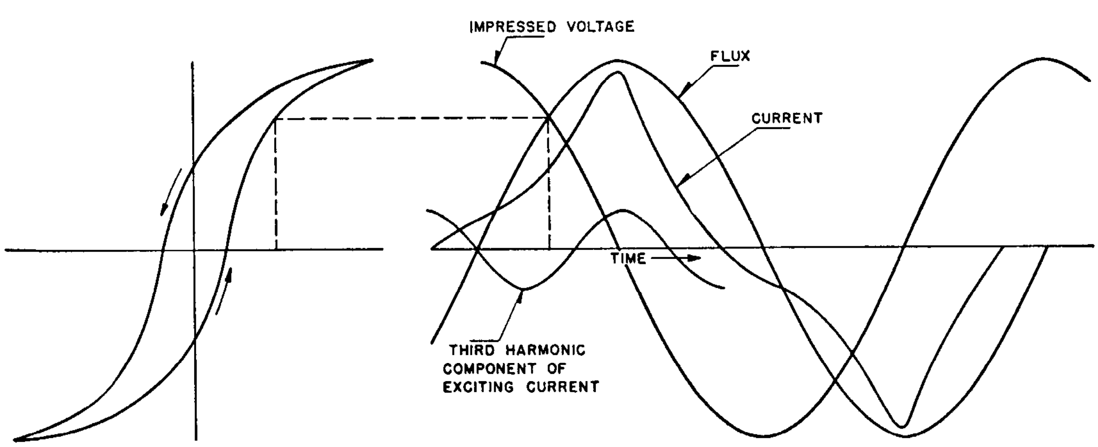


Fig. 37—Impressed voltage and steady-state flux.

From Eq. (72) it can be seen that the normal steady-state flux is a sine wave and lags the sine wave supply voltage by 90 degrees. The supply voltage and the normal flux are plotted in Fig. 37 as a function of time.

If there were no appreciable saturation in the magnetic circuit in a transformer, the magnetizing current and the flux would vary in direct proportion, resulting in a sinusoidal magnetizing current wave in phase with the flux. However, the economic design of a power transformer requires that the transformer iron be worked at the curved part of the saturation curve, resulting in appreciable saturation. Under this condition the magnetizing current is not a sine wave, and its shape depends upon the saturation characteristics (the B-H curve) of the transformer magnetic circuit. The shape of the current wave can be determined graphically as shown in Fig. 38. In Fig. 38(b) are shown the impressed voltage and the flux wave lagging the voltage by 90 degrees. For any flux the corresponding value of current can be found from the B-H curve. Following this procedure the entire current wave can be plotted. The current found in this manner does not consist of magnetizing current alone but includes a loss component required to furnish the hysteresis loss of the core. However, this component is quite small in comparison to the magnetizing component and has little effect on the maximum value of the total current.

A study of Fig. 38 shows that although the flux is a sine wave the current is a distorted wave. An analysis of this current wave shows that it contains odd-harmonic components of appreciable magnitude; the third harmonic component is included in Fig. 38. In a typical case the harmonics may be as follows: 45 percent third, 15 percent fifth, three percent seventh, and smaller percentages of higher frequency. The above components are expressed in percent of the equivalent sine wave value of the total exciting current. These percentages of harmonic currents will not change much with changes in transformer terminal voltage over the usual ranges in terminal voltage. In Fig. 39 are shown the variations in the harmonic content of the exciting current for a particular grade of silicon steel.



(a) TRANSFORMER B-H CURVE (b) VOLTAGE, FLUX AND CURRENT WAVES

Fig. 38—Graphical method of determining magnetizing current.

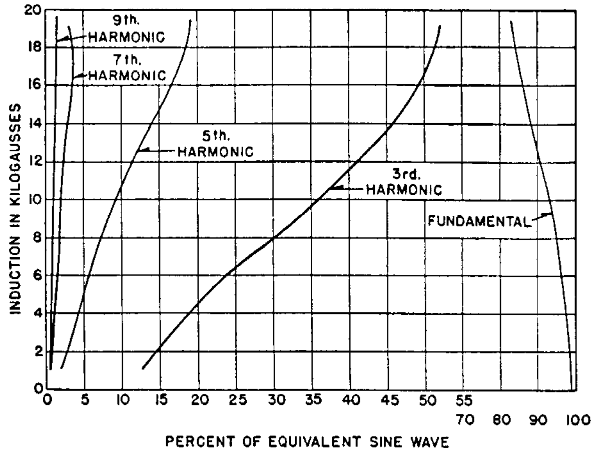


Fig. 39—Harmonic content of exciting current for a particular grade of silicon steel.

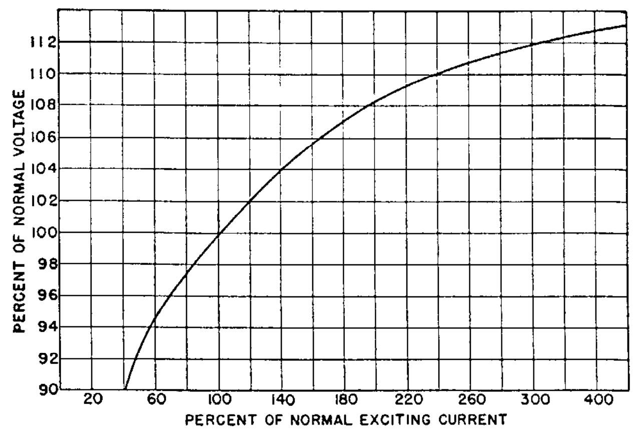


Fig. 40—Exciting current vs. terminal voltage. The above curve applies for one particular design of transformer; the shape of the curve may vary considerably depending upon the grade of steel and the transformer design.

35. Loss Component of Exciting Current

The no-load losses of a transformer are the iron losses, a small dielectric loss, and the copper loss caused by the exciting current. Usually only the iron losses, i.e., hysteresis and eddy current losses, are important. These losses depend upon frequency, maximum flux density, and the characteristics of the magnetic circuit.

In practice the iron losses are determined from laboratory tests on samples of transformer steel. However, the formulas given below are useful in showing the qualitative effect of the various factors on loss.

$$\text{Iron loss} = W_h + W_e \tag{73}$$
$$W_h = K_h f B^x_{max} \text{ watts per lb}$$
$$W_e = K_e f^2 t^2 B^2_{max} \text{ watts per lb}$$
$$W_h = \text{hysteresis loss}$$
$$W_e = \text{eddy current loss}$$
$$f = \text{frequency}$$
$$t = \text{thickness of laminations}$$
$$B_{max} = \text{maximum flux density}$$

K_h, K_e, and x are factors that depend upon the quality of the steel used in the core. In the original derivation of the hysteresis loss formula by Dr. Steinmetz, x was 1.6. For modern steels x may have a value as high as 3.0. The iron loss in a 60-cycle power transformer of modern design is approximately one watt per pound. The ratio of hysteresis loss to eddy current loss will be on the order of 3.0 with silicon steel and ⅔ with oriented steel. These figures should be used as a rough guide only, as they vary considerably with transformer design.

36. Total Exciting Current

As discussed above, the total exciting current of a transformer includes a magnetizing and a loss component. The economic design of a transformer dictates working the iron at the curved part of the saturation curve at normal voltage; hence any increase in terminal voltage above normal will greatly increase the exciting current. In Fig. 40 the exciting current of a typical transformer is given as a function of the voltage applied to its terminals. The exciting current increases far more rapidly than the terminal voltage. For example, 108-percent terminal voltage results in 200-percent exciting current.

37. Typical Magnitudes of Exciting Current

The actual magnitudes of exciting currents vary over fairly wide ranges depending upon transformer size, voltage class, etc. In Table 13 are given typical exciting currents for power transformers. The exciting currents vary directly with the voltage rating and inversely with the kva rating.

TABLE 13
TYPICAL EXCITING CURRENT VALUES FOR SINGLE-PHASE POWER TRANSFORMERS
(In percent of full load current)

The following values should be considered as very approximate for average standard designs and are predicated on prevailing performance characteristics. Test values will as a rule come below these values but a plus or minus variation must be expected depending upon purchaser's requirements. Should closer estimating data be required, the matter should be referred to the proper manufacturer's design engineers.

Three-phase KVA	Voltage Class (Full Insulation)						
	2.5 Kv	15 Kv	25 Kv	69 Kv	138 Kv	161 Kv	230 Kv
500	3.7%	3.7%	3.8%	4.9%			
1 000	3.3	3.3	3.6	4.3			
2 500	....	3.1	3.2	3.8			
5 000	....	....	2.8	3.1	2.5%	4.1%	
10 000	....	....	3.0	3.1	2.4	3.6	4.0%*
25 000	....	....	2.2	2.4	3.1	3.9	3.5 *
50 000	....	....	....	....	3.1	3.9	2.8 *

*Reduced Insulation.

38. Inrush Current

When a transformer is first energized, a transient exciting current flows to bridge the gap between the conditions existing before the transformer is energized and the conditions dictated by steady-state requirements. For any given transformer this transient current depends upon the magnitude of the supply voltage at the instant the transformer is energized, the residual flux in the core, and the impedance of the supply circuit. Often the magnitude of this transient current exceeds full-load current and may reach 8 to 10 times full-load current. These high inrush currents are important principally because of their effect on the operation of relays used for differential protection of transformers.

In studying the phenomena that occur when a transformer is energized it is more satisfactory to determine the flux in the magnetic circuit first and then derive the current from the flux. This procedure is preferable because the flux does not depart much from a sine wave even though the current wave is usually distorted.

The total flux in a transformer core is equal to the normal steady-state flux plus a transient component of flux, as shown in Eq. 72. This relation can be used to determine the transient flux in the core of a transformer immediately after the transformer is energized. As $\frac{\sqrt{2}E}{\omega n_1}$ represents the crest of the normal steady-state flux, Eq. (72) can be rewritten,

$$\phi = -\phi_m \cos(\omega t + \lambda) + \phi_t \tag{74}$$

where $\phi_m = \dfrac{\sqrt{2}E}{\omega n_1}$.

At $t = 0$,

$$\phi_0 = -\phi_m \cos\lambda + \phi_{t0} \tag{75}$$

where
ϕ_0 = transformer residual flux
$-\phi_m \cos\lambda$ = steady-state flux at $t = 0$
ϕ_{t0} = initial transient flux.

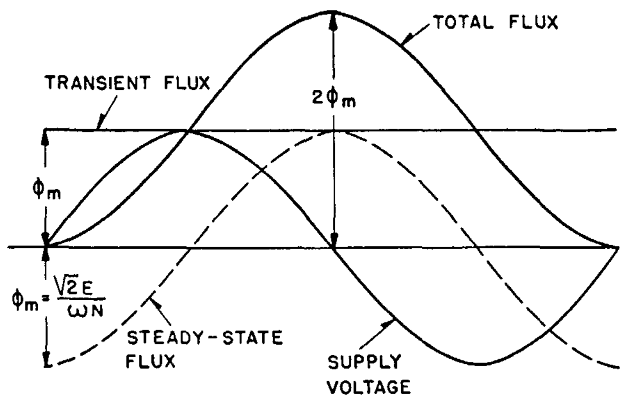
In the above equation the angle λ depends upon the instantaneous value of the supply voltage at the instant the transformer is energized. If the transformer is energized at zero voltage, λ is equal to 0, whereas if the transformer is energized where the supply voltage is at a positive maximum value, λ is equal to 90 degrees. Assume that a transformer having zero residual flux is energized when the supply voltage is at a positive maximum. For these conditions ϕ_0 and $\cos\lambda$ are both equal to zero so ϕ_{t0} is also equal to zero. The transformer flux therefore starts out under normal conditions and there would be no transient. However, if a transformer having zero residual is energized at zero supply voltage the following conditions exist:

$$\lambda = 0$$
$$-\phi_m \cos\lambda = -\phi_m$$
$$\phi_0 = 0$$
$$\phi_{t0} = \phi_m$$

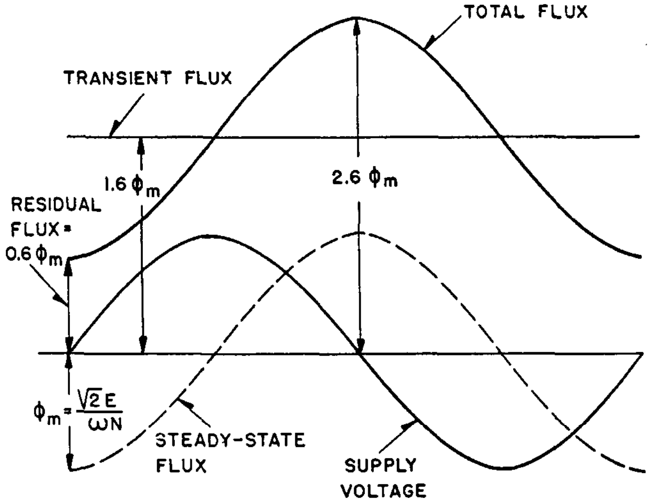
Substituting in Eq. (74)

$$\phi = -\phi_m \cos(\omega t) + \phi_m \tag{76}$$

The flux wave represented by Eq. (76) is plotted in Fig. 41a. The total flux wave consists of a sinusoidal flux wave plus a d-c flux wave and reaches a crest equal to twice the normal maximum flux. In this figure the transient flux has been assumed to have no decrement; if loss is considered the transient flux decreases with time and the crest value of the total flux is less than shown. In Fig. 41 (b) similar waves have been plotted for a transformer having 60 percent positive residual flux and energized at zero supply voltage. Sixty percent residual flux has been



(a) PRIMARY CLOSED AT ZERO VOLTAGE—ZERO RESIDUAL FLUX.



(b) PRIMARY CLOSED AT ZERO VOLTAGE—60% POSITIVE RESIDUAL FLUX.

Fig. 41—Transformer flux during transient conditions.

assumed for illustration only. Flux waves for any other initial conditions can be calculated in a similar manner using Eq. (74).

39. Determination of Current Inrush

After the flux variation has been determined by the method described, the current wave can be obtained graphically as shown in Fig. 42. In the case illustrated it was assumed that a transformer having zero residual flux was energized at zero supply voltage; the flux therefore is equal to twice normal crest flux. For any flux the corresponding current can be obtained from the transformer *B-H* curve. Although the maximum flux is only twice its normal value, the current reaches a value equal to many times the maximum value of the normal transformer exciting current. This high value of current is reached because of the high degree of saturation of the transformer magnetic circuit.

In the above discussion loss has been neglected in order to simplify the problem. Loss is important in an actual transformer because it decreases the maximum inrush current and reduces the exciting current to normal after a

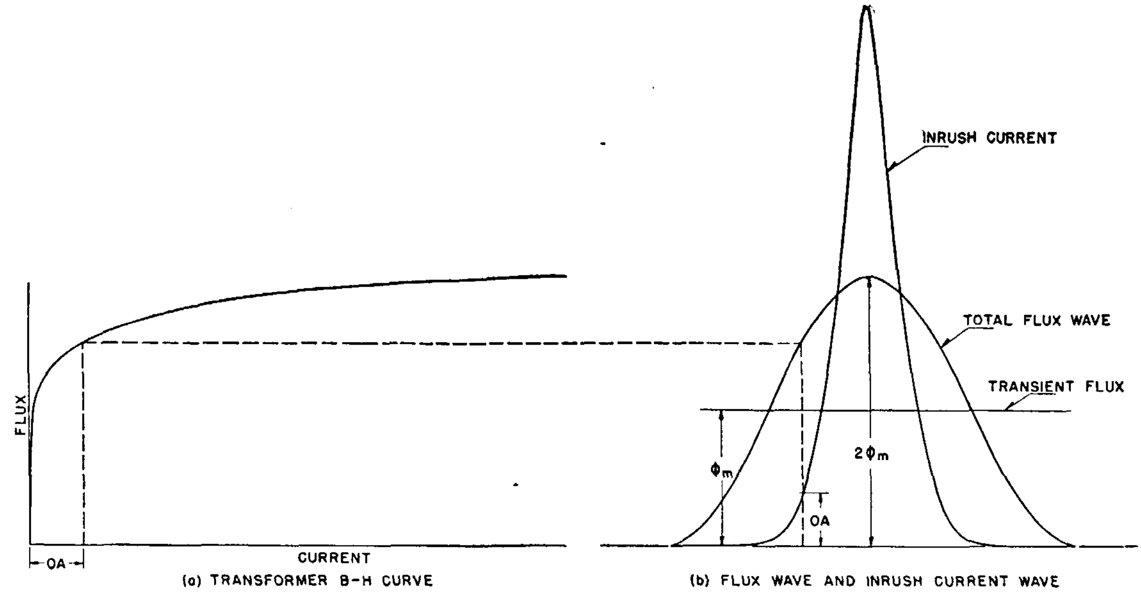


Fig. 42—Graphical method of determining inrush current.

period of time. The losses that are effective are the resistance loss of the supply circuit and the resistance and stray losses in the transformer. Figure 43 is an oscillogram of a typical exciting-current inrush for a single-phase transformer energized at the zero point on the supply voltage wave.[9] The transient has a rapid decrement during the first few cycles and decays more slowly thereafter. The damping coefficient, R/L, for this circuit is not constant because of the variation of the transformer inductance with saturation. During the first few current peaks, the degree of saturation of the iron is high, making L low. The inductance of the transformer increases as the saturation

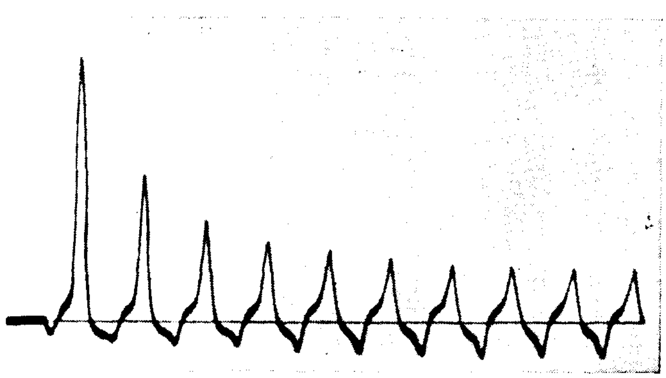


Fig. 43—Current inrush for a particular transformer energized at zero voltage.

decreases, and hence the damping factor becomes smaller as the current decays.

40. Estimating Inrush Currents

The calculation of the inrush current to a power transformer requires considerable detailed transformer design information not readily available to the application engineer. For this reason reference should be made to the manufacturer in those few cases where a reasonably accurate estimate is required. An order of magnitude of

TABLE 14—APPROXIMATE INRUSH CURRENTS TO 60-CYCLE POWER TRANSFORMERS ENERGIZED FROM THE HIGH-VOLTAGE SIDE

Transformer Rating Kva	Core Form	Shell Form
2000	5–8	
10 000	4–7	2.5–5
20 000		2.0–4

Note: The crest inrush currents are expressed in per unit of crest full-load current.

inrush currents to single-phase, 60-cycle transformers can be obtained from the data in Table 14. The values given are based on the transformer being energized from the high-voltage side at the instant the supply voltage passes through zero. Energizing a core-form transformer from the low-voltage side may result in inrush currrents approaching twice the values in the table. The per unit inrush current to a shell-form transformer is approximately the same on the high- and low-voltage sides.

The inrush currents in Table 14 are based on energizing a transformer from a zero-reactance source. When it is desired to give some weight to source reactance, the inrush current may be estimated from the relation

$$I=\frac{I_0}{1+I_0X} \tag{77}$$

where

I_0 = Inrush current neglecting supply reactance in per unit of rated transformer current.

X = Effective supply reactance in per unit on the transformer kva base.

XV. THIRD-HARMONIC COMPONENT OF EXCITING CURRENT

41. Suppression of the Third-Harmonic Component

As discussed in connection with Fig. 39, the exciting current of a transformer contains appreciable harmonic current. The third harmonic is by far the largest harmonic component, being as high as 40 to 50 percent of the equivalent sine-wave exciting current.

If the flux in a transformer magnetic circuit is sinusoidal, the exciting current must contain a third-harmonic component. If this component cannot flow, because of transformer or system connections, the flux will contain a third-harmonic component. The third-harmonic flux will, in turn, induce a third-harmonic voltage in the transformer windings. The magnitude of the third-harmonic voltage induced in a transformer winding, when the third-harmonic current is suppressed, will vary between 5 and 50 percent depending upon the type of transformers used. With single-phase transformers or with three-phase shell-form transformers the third-harmonic voltages may be as high as 50 percent of the fundamental-frequency voltage. In a three-phase core-form transformer the reluctance of the third-harmonic flux path is high (see Sec. 56); consequently the third-harmonic flux in the transformer magnetic circuit is small even if the third-harmonic component of the exciting current is suppressed. The third-harmonic voltage induced is therefore small, usually not more than five percent.

In a three-phase system, the third-harmonic currents of each phase are in phase with each other and hence constitute a zero-sequence set of currents of triple frequency. Likewise, the third-harmonic voltages will constitute a zero-sequence set of voltages of triple frequency. Thus, although a third-harmonic voltage may be present in the line-to-neutral voltages, there can be no third-harmonic component in the line-to-line voltage. The paths permitting the flow of third-harmonic currents are determined by the system and transformer zero-sequence circuits.

It has been shown that third harmonics must occur in either the exciting current or the voltage of a transformer. The exciting current will take the shape imposed by the particular connections used. It is always preferable to have at least one delta-connected winding in a three-phase transformer bank. The delta connection will furnish a path for the flow of third-harmonic currents and will minimize the third-harmonic current in the external circuits. This is very desirable because third-harmonic currents in the external circuits may, under some conditions, cause telephone interference. A discussion of telephone interference, as affected by transformer connections, is given in Chapter 23, Sec. 11.

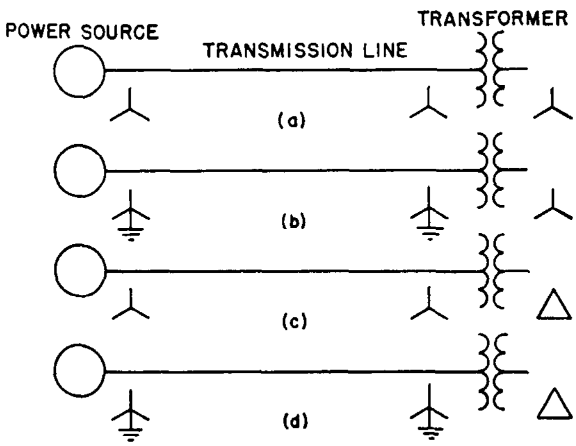


Fig. 44—Connections which influence the flow of third-harmonic exciting current.

42. Effect of Transformer Connections

The application of the above principles will be illustrated by consideration of a number of typical connections. In Fig. 44 is shown a three-phase transformer bank connected

TABLE 15—INFLUENCE OF TRANSFORMER CONNECTIONS ON THIRD-HARMONIC VOLTAGES AND CURRENTS

	SOURCE	TRANSFORMER CONNECTION		COMMENTS
		PRIM.	SEC.	
1	UNGROUNDED (SMALL CAPACITANCE TO GROUND, NO GROUNDED GENERATORS OR GROUNDED TRANSFORMER BANKS)	Y	Y	SEE NOTE 1
2		Y⏚	Y	" " 1
3		Y	Y⏚	" " 1,5
4		Y⏚	Y⏚	" " 1,5
5		Y	Δ	" " 3
6		Y⏚	Δ	" " 3
7		Δ	Y	" " 3
8		Δ	Y⏚	" " 3,6
9	GROUNDED (GROUNDED GENERATORS OR GROUNDED TRANSFORMER BANKS OR LARGE CAPACITANCE TO GROUND)	Y	Y	SEE NOTE 1
10		Y⏚	Y	" " 2
11		Y	Y⏚	" " 1,5
12		Y⏚	Y⏚	" " 2,5
13		Y	Δ	" " 3
14		Y⏚	Δ	" " 4
15		Δ	Y	" " 3
16		Δ	Y⏚	" " 3,6

Note:
1. The third-harmonic component of the exciting current is suppressed and so a third-harmonic component will be present in the transformer line-to-ground voltages.
2. The third-harmonic component of the exciting current flows over the line and may cause interference due to possible coupling with parallel telephone circuits.
3. The delta-connected winding furnishes a path for the third-harmonic exciting currents required to eliminate the third-harmonic voltages. No third-harmonic current will flow in the line between the source and the transformer and very little third-harmonic will be present in the system voltage.
4. The delta-connected winding furnishes a path for the third-harmonic exciting currents required to eliminate the third-harmonic voltages. Very little third-harmonic current will flow in the line and very little third-harmonic will be present in the system voltage.
5. If the capacitance-to-ground of the circuit connected to the transformer secondary is large, appreciable third-harmonic current can flow in the secondary windings. This factor will help decrease the magnitude of the third-harmonic voltages but may cause interference in telephone lines paralleling the secondary power circuits. The same comments would apply if other ground sources are connected to the secondary circuit. Resonance with the secondary capacitance may produce high harmonic voltages.
6. Some third-harmonic current can flow in the secondary windings if other ground sources are present on the secondary side of the transformer bank. The magnitude of this current will depend upon the impedance of the ground sources relative to the delta circuit impedance and is usually too small to cause trouble from telephone interference.

to a transmission line, the line in turn being connected to a power source. If the star-star connection in Fig. 44(a) is used the third-harmonic component of the exciting current is suppressed and a third-harmonic component will therefore be present in the line-to-neutral voltages. With the primary neutral and the generator neutral grounded, as in Fig. 44(b), a path is furnished for the third-harmonic exciting currents. If the impedance of this path is low, little third-harmonic voltage will be present on the system. However, if the line is long and is closely coupled with telephone circuits, telephone interference may result. If the transformer bank is close to the power source no telephone interference should result from the use of this connection.

When a delta-connected winding is present in the transformer such as in Fig. 44(c) and (d), the delta connection furnishes a path for the third-harmonic currents required to eliminate the third-harmonic voltages. If the primary is ungrounded or the generator is ungrounded, no third-harmonic current will flow in the line. If the primary is grounded and the generator is also grounded, a little third-harmonic current can flow over the line. With this connection the magnitude of the third-harmonic current in the line depends upon the relative impedances of the supply circuit and the delta circuit. This current is usually too small to cause any troublesome interference.

The same general comments apply when three-winding transformers are used. If one winding is delta connected, little or no third-harmonic current will flow in the supply circuit and little or no third-harmonic voltage will be present on the system.

In Table 15 is given a summary of a number of typical transformer connections with a brief description of the effect of the connections on the third-harmonic currents and voltages.

XVI. TRANSFORMER NOISE

Transformer noise is a problem because of its disturbing effect upon people. Noise may arise from several sources of force induced vibrations, including

(1) Magnetostriction, the small change in dimensions of ferromagnetic materials caused by induction.
(2) Magnetic forces tending to pull jointed core members together.
(3) Magnetic forces acting between two conductors, or between a conductor and a magnetic member.
(4) Fans, pumps, or other transformer auxiliaries.

The most persistent of these sources of noise is magnetostriction, which depends upon flux density and cannot be eliminated by tight core construction. The only means of reducing magnetostrictive force now at hand is to reduce flux density in the core.

Noise arising from any of the sources listed above may be amplified by mechanical resonance in the tank or fittings, and careful design is necessary to avoid such reinforcement of the original sound.

Standards[10] have been established for permissible sound pressure levels for various types of transformers, in terms of decibels referred to 0.002 dynes per square centimeter:

$$\text{db} = 20 \log_{10} \frac{P}{0.0002} \tag{78}$$

where P, the sound pressure, is expressed in dynes per square centimeter. Transformers designed to have sound levels below standard levels are available, but at extra cost because the magnetic material is worked at an induction below normal.

It is quite difficult to predetermine a sound level which will prove satisfactory in the surroundings where a new transformer is to be installed. Local conditions affect sound transmission, reflection, and resonance to a great degree, and these factors are hard to evaluate prior to transformer installation.

XVII. PARALLEL OPERATION OF TRANSFORMERS

43. Single-Phase Transformers

Transformers having different kva ratings may operate in parallel, with load division such that each transformer carries its proportionate share of the total load. To achieve accurate load division, it is necessary that the transformers be wound with the same turns ratio, and that the percent impedance of all transformers be equal, when each percentage is expressed on the kva base of its respective transformer. It is also necessary that the ratio of resistance to reactance in all transformers be equal, though most power transformers will likely be similar enough in this respect to permit calculations based on only the impedance magnitude.

The division of current between transformers having unequal turns ratios and unequal percent impedances may be calculated from an equivalent circuit similar to the one shown in Fig. 45. Either percent impedances or ohmic

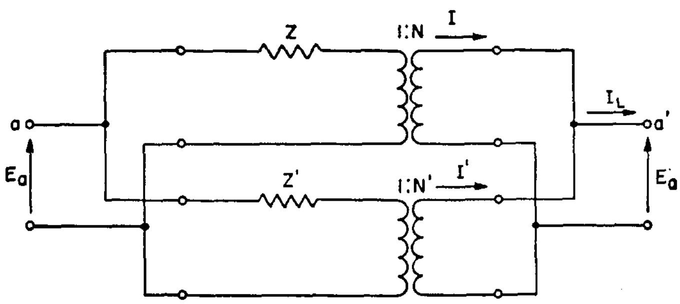


Fig. 45—Equivalent circuit for parallel connection of single-phase two-winding transformers.

impedances may be used in an equivalent circuit for paralleled transformers. The circuit in Fig. 45 contains ohmic impedances and actual turns ratios; this method is perhaps more appropriate when the circuit involves unequal turn ratios, because the use of percent values in this type of circuit involves extra complications. Solution of this circuit, with a load current I_L assumed, will indicate the division of current between transformers. Also, solution of this circuit with total load current set equal to zero will indicate the circulating current caused by unequal transformer ratios. For satisfactory operation the circulating