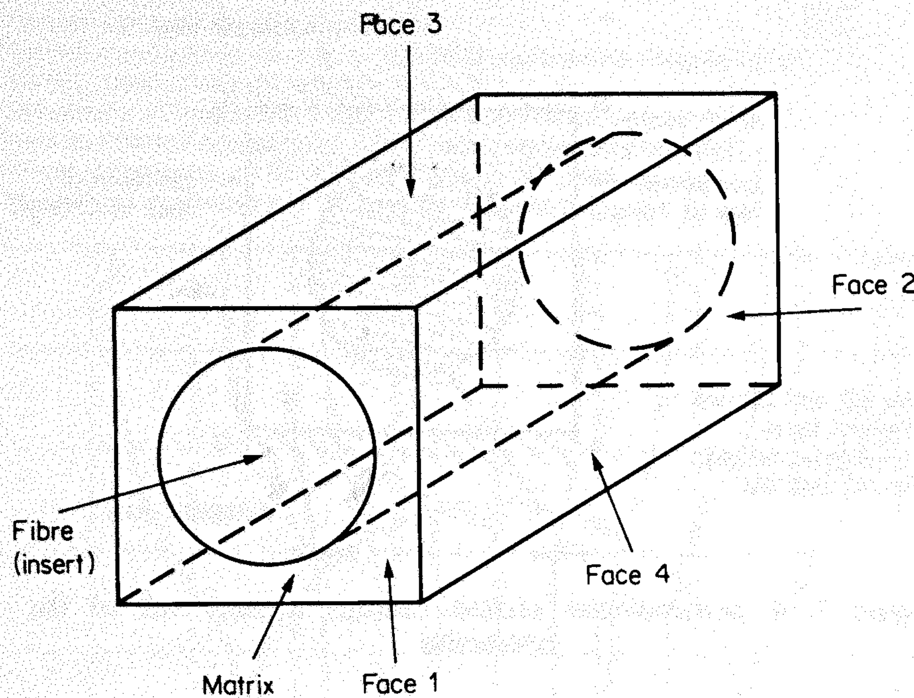

Figure 1. The basic rectangular prism or cell of a composite material containing a regular array of fibres

composite has an axis of three-, four-, or sixfold symmetry about the z-axis then $K_x = K_y$ and there is only one value for the effective transverse thermal conductivity.

2.2 Longitudinal heat flow

In Figure 1 suppose that a temperature difference is established between faces 1 and 2. By the reasoning developed above, the heat flow must be parallel to the axis of the fibre. The effective longitudinal thermal conductivity K_e of the whole cell is related to K_f the thermal conductivity of the fibre and K_m the thermal conductivity of the matrix in the following way:

$$K_e = \left(\frac{A_f}{A_f + A_m}\right)K_f + \left(\frac{A_m}{A_f + A_m}\right)K_m, \tag{2}$$

where A_f is the area of the fibre normal to its axis and A_m is the area of the matrix not occupied by the fibre. Actually the longitudinal heat flow problem is a straightforward application of the method of mixtures, since the two materials form parallel heat flow paths.

2.3 Transverse heat flow

If the cell in Figure 1 has a temperature difference between faces 3 and 4 the heat flow is transverse. Also if there are many basic cells, and hence lots

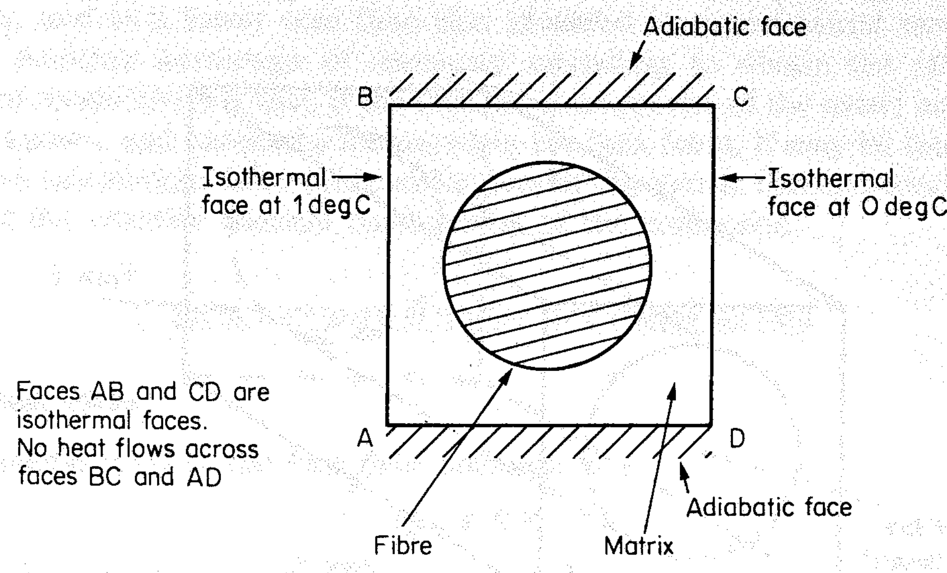

Figure 2. A perpendicular section through a basic cell of the composite

of fibres, no heat can flow across the other faces. Therefore in any section, perpendicular to the fibre, as shown in Figure 2, the temperature difference exists entirely between the faces AB and CD. One of the most informative computational pictures to construct in problems like this is the distribution of isotherms. A trivial case arises when the thermal conductivities of the fibre and matrix are equal. In this case the isotherms are parallel and, if they are drawn at constant temperature intervals, are equally spaced. The calculation of the effective thermal conductivity of the composite material is also a trivial matter.

On the other hand, if the two materials have different conductivities then the isotherms are no longer straight or parallel or evenly spaced. For a fibre with a lower conductivity than the matrix, a larger temperature gradient will exist in the fibre than in the matrix. Consequently, the majority of the heat will flow through the matrix around the fibre. This is shown in Figure 3 that shows isotherms, for a temperature difference across the cell of 1 degC, at intervals of 0.1 degC. If the fibre has a higher conductivity than the matrix, the reverse will occur, with the majority of the heat flowing through the fibre. This is illustrated in Figure 4.

The non-laminar redistribution of the flow of heat that occurs when $K_f \neq K_m$, is not a trivial matter to calculate and the remainder of this chapter is devoted to an electrical analogue technique that leads to the production of results, such as those shown in Figures 3 and 4. The electrical analogue of heat flow was considered a long time ago by Kayan.[3] It has been used both for calculations and for experimental investigations of heat flow problems. It is only necessary, with this technique, to specify the external boundary

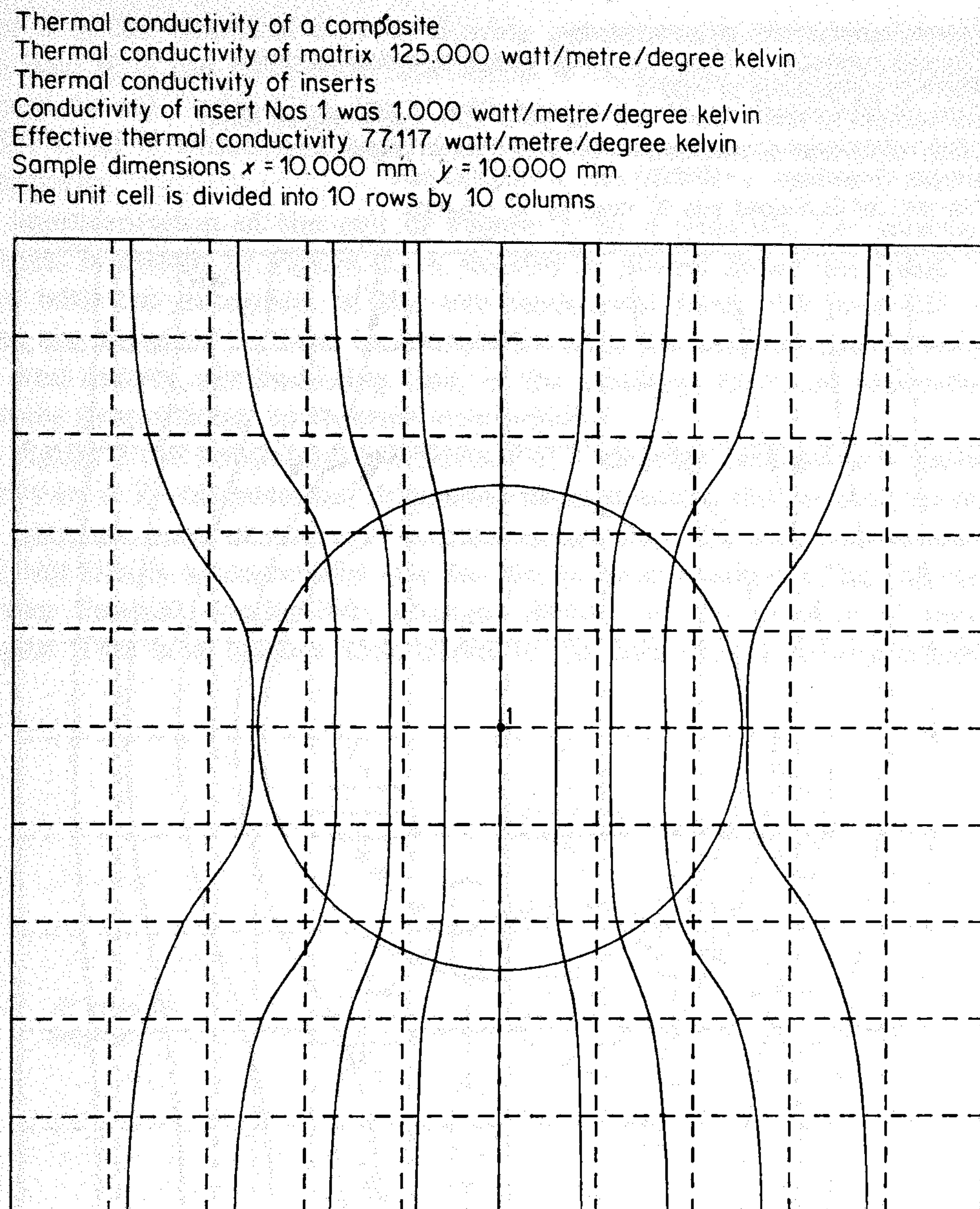

Figure 3. Section through a basic cell of a composite showing 0.1 deg C isotherms for high conductivity matrix and low conductivity fibre; n, the number of fibre inserts, is designated in the computer output as Nos n

conditions and the arrangement of the different materials of the structure to enable the temperature distribution and heat flow to be determined. An alternative approach, to the electrical analogy, is to use a relaxation method (Emmons[4]) that requires, in addition to the above information, an initial guess of the temperature distribution. It is not felt to be appropriate to the present problem, although Collier[5] has used it for problems involving complex arrangements of materials and boundary conditions.

Thermal conductivity of a composite
Thermal conductivity of matrix 1.000 watt/metre/degree kelvin
Thermal conductivity of inserts
Conductivity of insert Nos 1 was 125.000 watt/metre/degree kelvin
Effective thermal conductivity 1.471 watt/metre/degree kelvin
Sample dimensions x=10.000 mm y=10.000 mm
The unit cell is divided into 10 rows by 10 columns

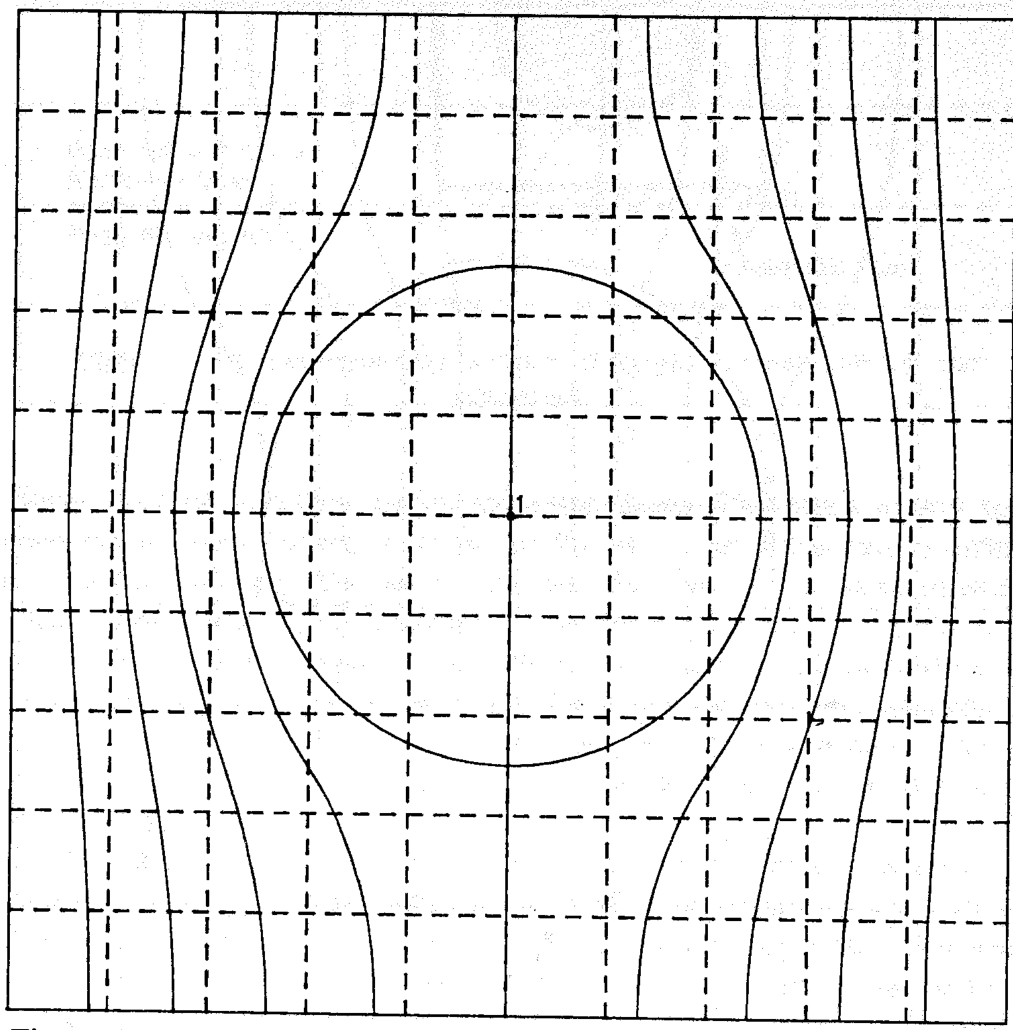

Figure 4. Section through a basic cell of a composite showing 0.1 degC isotherms for low-conductivity matrix and high-conductivity fibre; n, the number of fibre inserts is designated in the computer output as Nos n

3. THE ELECTRICAL ANALOGUE

The heat flow equation is analogous to Ohm's law of electrical resistivity. To put the matter very simply, Ohm's law for an isotropic solid, or along the longitudinal or transverse axes of a unidirectional solid, is

$$V = IR, \tag{3}$$

where I is the electrical current flowing per unit area, V is the potential drop, and R is the resistance. The terms in equation (3) are identified, by analogy, as: (a) I with Q/A, (b) V with temperature gradient $\partial T/\partial Z$, $\partial T/\partial X$, or $\partial T/\partial Y$, (c) R with $1/K$, where K is the longitudinal or transverse thermal conductivity.

A representation of the cell of Figure 2, by a network, i.e. meshes, of resistors, is shown in Figure 5. A resistor is placed along each side of a mesh, with the exception of the two isothermal faces AB and CD, and values are assigned to them in accordance with the thermal conductivity of fibre and matrix, remembering that, in the analogy, electrical resistance is inversely proportional to thermal conductivity.

In Figure 5 the temperature difference of T degrees centigrade is replaced directly by a T volt potential difference applied across the resistor network. This network may, of course, be much larger than the 3×3 system shown here, but this is a convenient size for the present example. The calculated effective thermal conductivity changes slightly as the number of meshes changes. This is a feature that needs to be considered, in any complete

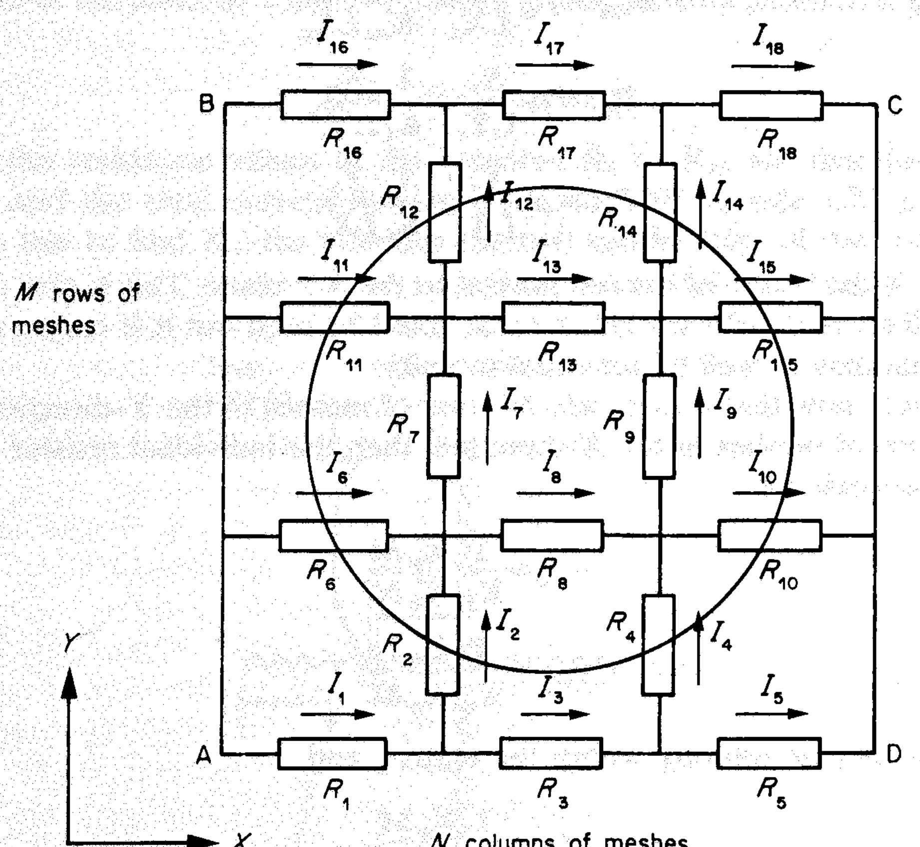

Figure 5. The resistor network to represent the composite shown in Figures 1 and 2. All points along AB are at 1.0 V with respect to points along CD, In this example $M=3$ and $N=3$

calculation, and is discussed later in the chapter. The method of calculating the values of the resistors R_1 to R_{18} is given below.

The thermal resistivities ρ_m and ρ_f, of the matrix and fibre, are $\rho_m = 1/K_m$ and $\rho_f = 1/K_f$ and a one-to-one correspondence between them and the electrical resistivities is taken, i.e. ρ_m and ρ_f are made numerically equal to the corresponding electrical resistivities. The individual network resistors have values that depend upon ρ_m and ρ_f, the geometry of the basic cell, and the number of rows and columns in the network.

The first stage in calculating the network resistor values is to find the apparent resistances in the X- and Y-directions of Figure 5 for a cell consisting entirely of matrix or entirely of fibre material. Suppose R_{mx} or R_{my} is the apparent matrix resistance in the X- or Y-direction and R_{fx} or R_{fy} is the apparent fibre resistance in the X- or Y-direction. If the cell dimensions are X_s and Y_s then the apparent resistances are

$$\begin{aligned}
R_{mx} &= \rho_m \frac{X_s}{LY_s} = \frac{1}{K_m}\frac{X_s}{LY_s}, \\
R_{my} &= \rho_m \frac{Y_s}{LX_s} = \frac{1}{K_m}\frac{Y_s}{LX_s}, \\
R_{fx} &= \rho_f \frac{X_s}{LY_s} = \frac{1}{K_f}\frac{X_s}{LY_s}, \\
R_{fy} &= \rho_f \frac{Y_s}{LX_s} = \frac{1}{K_f}\frac{Y_s}{LX_s},
\end{aligned} \tag{4}$$

where L is the length of the cell normal to the XY plane. This is now chosen to be 1.0 m; obviously any other value could be used but it is only a scaling factor that may as well be set equal to unity.

It is clear now that if there are M rows of meshes in the Y-direction, and N columns of meshes in the X-direction, then the individual resistor values in the network are

$$\begin{aligned}
r_{mx} &= \frac{1}{K_m}\frac{X_s}{Y_s}\frac{M}{N}, \\
r_{my} &= \frac{1}{K_m}\frac{Y_s}{X_s}\frac{N}{M},
\end{aligned} \tag{5}$$

provided they lie entirely within the matrix, and

$$\begin{aligned}
r_{fx} &= \frac{1}{K_f}\frac{X_s}{Y_s}\frac{M}{N}, \\
r_{fy} &= \frac{1}{K_f}\frac{Y_s}{X_s}\frac{N}{M},
\end{aligned} \tag{6}$$

provided they lie entirely within the fibre insert. These values, excluding those that lie partly in the matrix and partly in the fibre, are assigned to resistors $R_1 \ldots R_{18}$ in Figure 5.

Equations (5) and (6) assume that there are M lines of resistors in the X-direction between AB and CD, whereas Figure 5 shows that there are $M+1$. There are, however, effectively only M lines since the top and bottom lines, BC and AD, have a weighting factor of only $\frac{1}{2}$ since they are shared with adjacent cells. This means resistors in these network positions must have their resistance doubled. This difficulty does not arise in the Y-direction since the faces AB and CD are isothermal faces.

In Figure 5 the resistors are drawn symbolically as blocks, but this representation is possibly confusing when a resistor extends across a boundary between the fibre and the matrix. Each side of the mesh is, in fact, a resistor, as drawn in Figure 6. If a resistor happens to cross a fibre/matrix boundary then it cannot have the full $r_{mx,y}$ or $r_{fx,y}$ value. It is necessary to determine what proportion of it lies within the fibre or the matrix. For example, suppose that, as indicated in Figure 6, the boundary cuts the resistor at 0.45, measured from the junction point inside the fibre and taking the side of the mesh as 1 unit. A resistor in this network position is given the value

$$R = 0.45 r_{fx} + 0.55 r_{mx}. \tag{7}$$

All the resistance values of the resistors R_1 to R_{18} are therefore easily found and the total current flowing from side AB to side CD is used in Ohm's law to find K_e, the effective thermal conductivity of the composite.

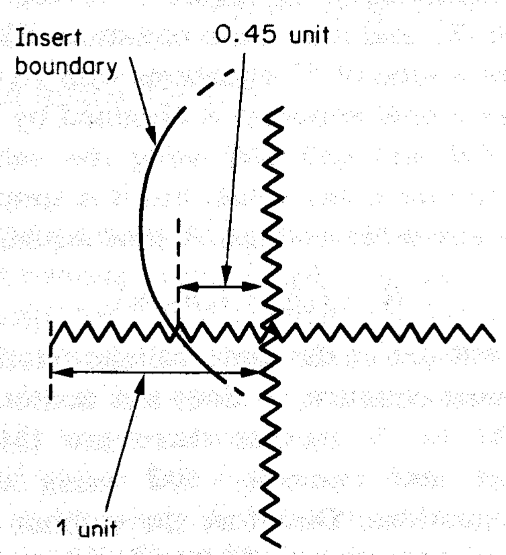

Figure 6. A resistor at the boundary between matrix and insert (fibre)

The total current is obviously the sum of the currents flowing from plane AB towards CD and to calculate these currents it is necessary to use Kirchhoff's laws.

4. KIRCHHOFF'S LAWS

The two Kirchhoff laws extend the applications of Ohm's law to a system comprising any number of branches, branch points, and electromotive force (emf) sources. They are:

(1) The junction law, that states that the sum of currents flowing in branches which form a junction is zero.
(2) The mesh law, that states that, in any closed mesh, the sum of the potential differences across the branches of which the circuit is composed, is equal to the sum of the electromotive forces in that mesh.

The application of the first law to a typical junction of four resistors in the network of Figure 5 gives

$$I_6+I_2-I_7-I_8=0. \tag{8}$$

Obviously, similar equations for all the junctions in the network exist.

For a typical mesh consisting of three resistors in Figure 5 the application of the second law gives

$$I_6R_6-I_2R_2-I_1R_1=0. \tag{9}$$

Similar equations for all the meshes of the network also exist.

In more detail an examination of Figure 5 reveals that eight junction equations like equation (8), and nine mesh equations like equation (9) exist. The fact that this makes a total of 17 equations with 18 unknown currents is a possible difficulty, but a final equation is obtained by considering any line of resistors between AB and CD and using the value of an externally applied emf. The emf can have any value, but it is simpler to use unity and scale the answers to fit any other voltage. A final equation is

$$I_1R_1+I_3R_3+I_5R_5=1. \tag{10}$$

Since all points along AB are at the same voltage relative to CD, there are other choices for the final equation. It does not matter which one is used.

In a network of M by N meshes there are $(M+1)N+(N-1)M=2MN+N-M$ resistors and currents, NM mesh equations and $MN+N-M-1$ junction equations. Therefore the number of equations that do not involve the external potential is $(2MN+N-M)-1$, i.e. one less than the total number of resistors.

The equations relevant to Figure 5 are, from the application of Kirchhoff's laws,

$$
\begin{aligned}
I_1-I_2-I_3&=0,\\
I_3-I_4-I_5&=0,\\
I_2+I_6-I_7-I_8&=0,\\
I_4+I_8-I_9-I_{10}&=0,\\
I_7+I_{11}-I_{12}-I_{13}&=0,\\
I_9+I_{13}-I_{14}-I_{15}&=0,\\
I_{12}+I_{16}-I_{17}&=0,\\
I_{14}+I_{17}-I_{18}&=0,
\end{aligned} \tag{11a}
$$

and

$$
\begin{aligned}
-I_1R_1-I_2R_2+I_6R_6&=0,\\
I_2R_2-I_3R_3-I_4R_4+I_8R_8&=0,\\
I_4R_4-I_5R_5+I_{10}R_{10}&=0,\\
-I_6R_6-I_7R_7+I_{11}R_{11}&=0,\\
I_7R_7-I_8R_8-I_9R_9+I_{13}R_{13}&=0,\\
I_9R_9-I_{10}R_{10}+I_{15}R_{15}&=0,\\
-I_{11}R_{11}-I_{12}R_{12}+I_{16}R_{16}&=0,\\
I_{12}R_{12}-I_{13}R_{13}-I_{14}R_{14}+I_{17}R_{17}&=0,\\
I_{14}R_{14}-I_{15}R_{15}+I_{18}R_{18}&=0,\\
I_1R_1+I_3R_3+I_5R_5&=1.0.
\end{aligned} \tag{11b}
$$

Equation (11), in matrix form, is

$$\mathbf{R}\cdot\mathbf{I}=\mathbf{V}, \tag{12}$$

where $\mathbf{R}$ is an 18×18 square matrix, $\mathbf{I}$ and $\mathbf{V}$ are current and voltage column vectors and it is the column vector $\mathbf{I}$ that is calculated. The matrix $\mathbf{R}$ consists mainly of zeros except for 3 or 4 elements in each row. The full $\mathbf{R}$ matrix is given in Figure 7, mainly to show that it presents some mathematical problems. Technically it is known as a non-diagonal sparse matrix and it becomes more so for larger networks.

After the column vector $\mathbf{I}$ is evaluated from equation (12) the effective transverse thermal conductivity K_e is obtained from the current flow through any plane parallel to AB. The effective electrical resistance R_e of the cell, found from I_T, the sum of the currents entering through plane AB, and the applied potential (1 Volt in this case), is $R_e=1.0/I_T$ and the effective

1	-1	-1															
		1	-1	-1													
	1				1	-1	-1										
			1				1	-1	-1								
						1				1	-1	-1					
								1				1	-1	-1			
											1				1	-1	
													1			1	-1
$-R_1$	$-R_2$				R_6												
	R_2	$-R_3$	$-R_4$				R_8										
			R_4	$-R_5$					R_{10}								
					$-R_6$	$-R_7$				R_{11}							
						R_7	$-R_8$	$-R_9$				R_{13}					
								R_9	$-R_{10}$					R_{15}			
										$-R_{11}$	$-R_{12}$				R_{16}		
											R_{12}	$-R_{13}$	$-R_{14}$			R_{17}	
													R_{14}	$-R_{15}$			R_{18}
R_1		R_3		R_5													

Figure 7. The full **R** matrix array with zeros omitted

resistivity is

$$\rho_e = R_e \frac{Y_s}{X_s} = \frac{1.0}{I_T} \frac{Y_s}{X_s}. \tag{13}$$

The one-to-one correspondence between the electrical and thermal resistivity enables the thermal conductivity to be written directly as

$$K_e = \frac{1}{\rho_e} = I_T \frac{X_s}{Y_s}. \tag{14}$$

The temperature distribution in the composite is found by calculating the potential at each network intersection. The locations of the isotherms (i.e. isopotential lines) are found by linear interpolation between the network intersections. The results are used to draw an isotherm map of the composite. Figures 3 and 4 show isotherms, plotted at intervals of 0.1 degC, for a composite, with only one fibre, in the case of a temperature difference of 1 degC.

5. THE COMPUTER PROGRAM

A modular approach is used since the program seems to split naturally into several sections, which are:

(1) Input of initial parameters that consists of the physical size of the sample, the number of meshes into which the sample is to be divided, the shape of the sample, the thermal conductivities of the constituents, and the number and sizes of the inserts.
(2) Calculation of the values of the resistances to represent inserts in the matrix, using equations (5) and (6).
(3) Setting up of the matrix and vectors of equation (12) from the above resistor values.
(4) Use of mathematical routines for calculating the current flowing in each resistor.
(5) Interpolation for isothermal values and calculation of the effective thermal conductivity of the composite from the results obtained in (4) above.
(6) Output of the temperature at each network intersection, the positions of the isothermals, and the effective thermal conductivity.

Some of these sections seem to subdivide further into many very simple modules. For example, in (3) five basic types of equations are needed to generate sufficient equations to solve for all of the currents, namely:

(a) currents flowing into and out of a four-resistor junction in the network;
(b) currents flowing into and out of a three-resistor junction in the network;
(c) currents flowing in a closed four-resistor mesh;

(d) currents flowing in a closed three-resistor mesh;
(e) currents flowing along one horizontal row of the mesh.

The main segment of the computer program calls a set of subroutines that perform nearly all these functions.

The subroutines are listed in Table 1 and their functions are outlined. The NAG library subroutines FO3AJF and FO4APF are used to evaluate equation (12). These NAG subroutines are specifically intended for use with sparse matrices. It is not absolutely necessary to use sparse matrix methods, but if they are not used the program needs a lot more storage space. Even now the program published here requires a storage of 62K to handle a 10×10 meshes network.

The format for the data required to run the program is shown below and annotated so that the numbers can be identified:

10 10	N and M
10.0 10.0	X_s and Y_s, sample cell size
1	the number of inserts in the sample
0.2041	K_m thermal conductivity of the matrix
1.998	K_f thermal conductivity of the fibre/insert
2	Insert type (1 = rectangular, 2 = circular)
10.0 0.0	the centre coordinates of the inserts
7.652	the radius of a circular insert or the length of sides of a rectangular insert in X, Y order

If the number of inserts is $n > 1$, the last four lines of the input data file occur n times.

A listing of the program, a typical data file, and the resulting output are given at the end of the chapter. There are three output channels which write to files or to the one line printer. The input data, the calculated effective thermal conductivity, and the positions of the isovoltage points, at intervals of 0.1 V, form the output on channel 2. The input data, the effective thermal conductivity, and the values of the voltage (synonymous with temperature) at the corners of the meshes form the output on channel 3. This output could be used together with a general contour plotting graphics program to produce an isotherm map of the composite. The output on channel 6 is in a form suitable for input to the accompanying graphics program and consists of the input data, the effective thermal conductivity, and the positions of the 0.1 deg C isotherms.

The graph plotting program that produces a picture of the isotherms makes use of the GINO-F software (widely available in the U.K. and elsewhere) and has been used to produce Figures 3, 4, and 8. A listing of this program is also given at the end of the chapter. Apart from drawing the isotherms the program also draws the resistor network and the position of the inserts.

Table 1. The subroutines of the main program

Subroutine name	Function of subroutine
ARRAYSIZE	Reads M and N the number of rows and columns of meshes in the network. Also reads the sample section dimensions X_s and Y_s.
CALCRESNUM	Calculates the number of resistors from given M and N.
ARRAYZERO	Sets all values of the resistance array to zero.
NUMOFINC	Reads the number of inserts or inclusions and repeats calls of ARRAYRES and INCLUSION an appropriate number of times.
ARRAYRES	Reads the conductivity of the matrix followed by the conductivity of the inserts.
INCLUSION	Reads the type of insert, coordinates of the centre of the insert, and the dimension(s) of the insert. Circular inserts are indicated by 1 and rectangular inserts by 2.
INCSC1	Called by INCLUSION for a rectangular insert to calculate the values of the resistors.
INCSC2	Called by INCLUSION for a cylindrical insert to calculate values of the resistors.
EDGERESCOMP	Doubles the values of the resistors which are at the edges of the network representing the cell of the composite.
SETVOLTARRAY	Sets $\mathbf{V}$ vector to zeros for all but one element which is set to 1.0.
MESH4I	Used for four-resistor meshes to set appropriate members of $\mathbf{R}$ equal to the appropriate values of resistance
MESH3I	Used for three-resistor meshes, otherwise the same as MESH4I
POINT4I	Used for four current junctions to set appropriate members of $\mathbf{R}$ equal to $+1$ or -1.
POINT3I	Used for three current junctions, otherwise the same as POINT4I.
OVERALLI	Sets up equation (9) for the network specified.
FO3AJF	Decomposes the sparse matrix $\mathbf{R}$ into triangular factors and evaluates the determinants.
FO4APF	Calculates the (approximate) solution $\mathbf{I}$ of the set of real sparse linear equations $\mathbf{R} \cdot \mathbf{I} = \mathbf{V}$.
EXITPARAM	If an error is detected in FO3AJF this provides the error exit.
EFFCOND	Calculates the effective thermal conductivity K_e from the values of $\mathbf{I}$.
WRITE	Calculates and outputs the voltage map for the network intersections.
WRITEGRAPH	Interpolates the above voltage map to find the positions of the 0.1 V (0.1 degC) steps and outputs results for use by a graphics program.

Thermal conductivity of a composite
Thermal conductivity of matrix 0.100 watt/metre/degree kelvin
Thermal conductivity of inserts
Conductivity ot insert Nos 1 was 1.000 watt/metre/degree kelvin
Effective thermal conductivity 0.261 watt/metre/degree kelvin
Sample dimensions x = 3.000 mm y = 3.000 mm
The unit cell is divided into 3 rows by 3 columns

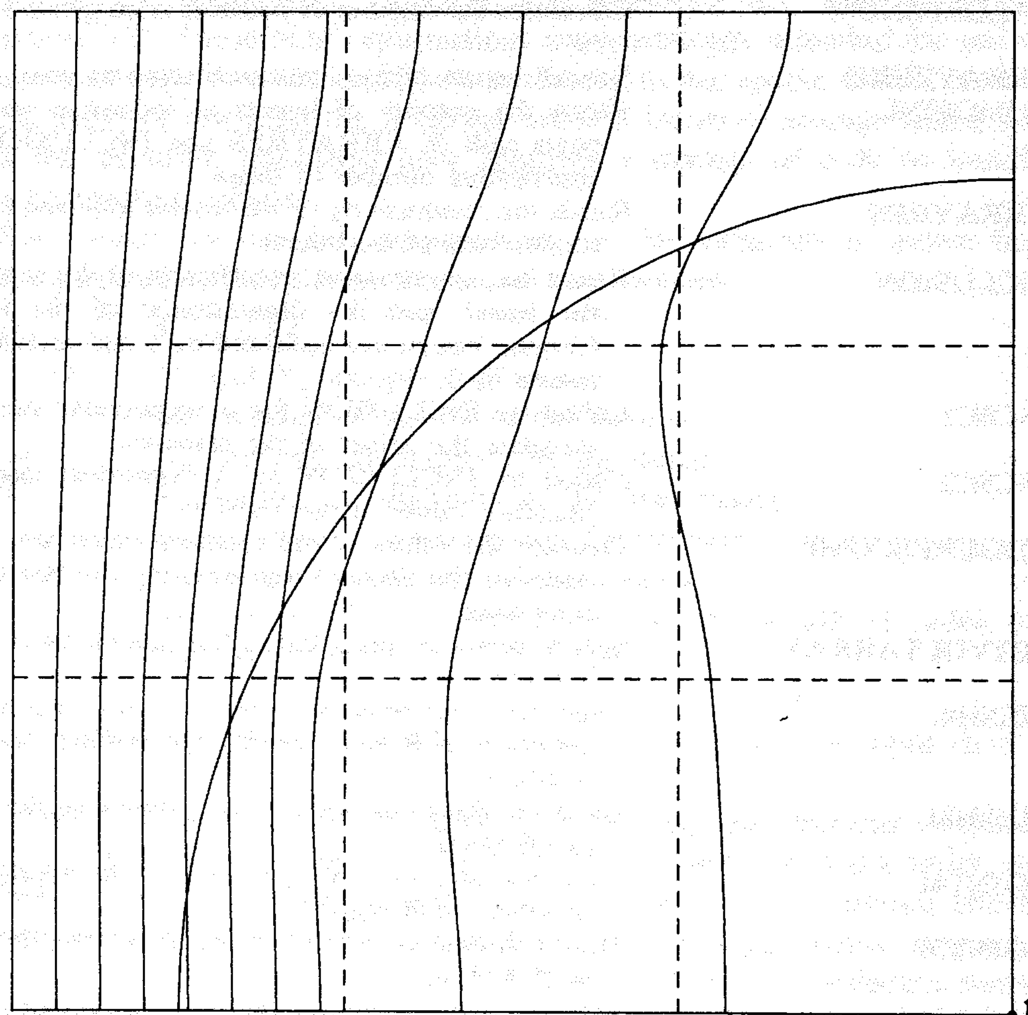

Figure 8. The output from the graphical program for the sample data given. Note that the centre of inserts is numbered as 1. These voltage contours, with 0.1 V steps, correspond to isotherms with 0.1 deg C steps

The data for the graphics program used for Figures 3, 4, and 8 is supplied from a file. Typical contents of this file are listed here under the title GRAPHICS PROGRAM INPUT FILE (p. 454). They consist of control and specification data, together with output from the subroutine WRITEGRAPH. The input file is typically.

1	Control integer
3 3 1	M N and number of inserts

3.0000 3.0000 0.1000	X_s Y_s K_m sample cell size and matrix thermal conductivity
1.000	K_f thermal conductivity of the fibre/matrix
2	Insert type (1 = rectangular, 2 = circular)
3.0000 0.0000 2.5000	The centre coordinates of the inserts and its radius
0.2614	K_e effective thermal conductivity
0.1325 .... 2.1421 .... 0.1887......... 2.3280	$\left.\right\}$ Positions of isotherms.
−1	Control integer to end data batch.

The same control and specification data can be used for a grid temperature plot together with the output on channel 3. An example of such a file is listed under GRID TEMPERATURES (p. 453).

6. RUNNING THE PROGRAM

The program was written for use at an interactive terminal. It is therefore provided with appropriate prompts. If interactive facilities are not available the data can be read from cards or a file of the format given in section 5. The prompts will then form part of the output and can be ignored.

The value of the effective thermal conductivity of a given composite varies slightly with the number of meshes in the resistor network and with the size of the representative cell, for a given number of meshes. The calculated value is nearest the true value for the largest number of meshes, and the approach to it as a function of the number of meshes should be investigated as it will illustrate the approximation contained in the model. As there is, inevitably, a limit to the storage available on a computer there is a consequential limit to the number of meshes that can be used. The program, listed here, can handle a maximum of 100 meshes, and requires about 62 K of storage. It is only limited to 100 meshes by the declared storage. A simple change will permit larger/smaller networks to be handled so long as they are compatible with the storage available on the computer. This feature may be useful in any development of the program beyond that given in this chapter. The greatest accuracy in the calculated value of the effective thermal conductivity is obtained by considering the smallest representative cell with the largest number of meshes possible.

In the example, given at the end of this chapter, only one-quarter of the basic cell of a square array of circular fibres is used (cf. Figure 8) since this is the minimum usable symmetry element compatible with the applied temperature gradient. (The minimum symmetry element of the cell is a sector of

45° about the centre because there are four planes of symmetry through the axis of the cell.) Apart from the square array of fibres considered in this chapter other possibilities are hexagonal and rectangular arrays of fibres. The minimum usable symmetry unit for a hexagonal array of fibres is a rectangle with side lengths in the ratio $\sqrt{3}:1$ and with a quarter fibre at opposite corners. In the case of a rectangular array of fibres one-quarter of a unit cell can be used with a quarter fibre in one corner. In this case the two orthogonal values of the effective thermal conductivity K_x and K_y have to be calculated separately. Composite systems with rectangular inserts can be considered by setting the appropriate control integer to 1 in the input data (see section 5). Other shapes of insert could also be considered by developing the appropriate subroutines.

Finally, some thermal conductivity data is given in Table 2 which will enable composites of some common materials to be investigated.

Table 2. Thermal conductivity of some common materials

Material	Thermal conductivity ($\mathrm{W\,m^{-1}\,(degK)^{-1}}$)
Silver	405
Copper	385
Aluminium	238
Brass	110
Steel	50
Concrete	1.5
Epoxy resin	0.23
Rock wool	0.042
Polystyrene (cellular)	0.035
Still air	0.024

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A PROGRAM TO CALCULATE THE THERMAL/CONDUCTIVITY OF COMPOSITES

```
      MASTER MAINPROGRAM
C
C     A PROGRAM TO CALCULATE THE THERMAL CONDUCTIVITY
C     OF COMPOSITE MATERIALS.
C
C
C     THE PROGRAM READS IN THE INSERT INFORMATION,
C     CALCULATES THE CURRENT THROUGH EACH RESISTOR,
C     AND INTERPOLATES THE 0.1 TO 0.9 VOLTAGE CONTOURS.
C
C     MAX. NUMBER OF RESISTOR VALUES IS 200 FOR STORAGE PROVIDED.
C     THIS IS OK FOR A SQUARE MESH OF SIDE UP TO & INCL. 10*10
C     OR A RECTANGULAR ONE WITH LESS THAN 200 RESISTORS.
C
      DIMENSION RNET(11000),R(210),IND(11000),IND2(11000),VOLTCURR(210)
      DIMENSION IW(210,13),IWW(22),INDW(22),W(220),VOLT(11)
      DIMENSION CONDINC(20),VOLTGRAPH(9)
C
      COMMON /NETWORK/RNET,IND,IL,IH,NUM
      COMMON /RESDATA/R,RXI,RYI,RXM,RYM
      COMMON /CURRVOLT/VOLTCURR
      COMMON /GENERAL/K
      COMMON /ANSCHECK/G,D1,ID2,IW
      COMMON /CONDDATA/CONDMAT,CONDINC,CONDEFF,XSIZE,YSIZE
C
      EQUIVALENCE (IW(1,6),W(1))
C
C     RECORD TIME OF RUN.
C
      CALL DATE(ADATE)
      CALL TIME(ATIME)
      WRITE(2,200) ADATE,ATIME
  200 FORMAT(1H1,' THIS RUN WAS DONE ON ',A8,' AT ',A8//)
C
C     READ IN DATA.
C
      CALL ARRAYSIZE(N,M)
C
C     GO TO ERROR LABEL IF THIS CONDITION IS TRUE.
C
      IF(N.EQ.1) GO TO 9
      CALL CALCRESNUM(N,M,IRNUM)
      CALL ARRAYZERO(IRNUM)
      CALL NUMOFINC(INCNUM)
C
C     CALCULATE ALL RESISTOR VALUES AND STORE THEM IN ARRAY R.
C
      DO 250 JJJ=1,INCNUM
      CALL ARRAYRES(IRNUM,JJJ,N,M)
      CALL INCLUSION(N,M,JJJ,INCNUM)
  250 CONTINUE
C
C     COMPENSATE FOR EDGE EFFECT.
C
      CALL EDGERESCOMP(N,M,IRNUM)
C
```

A PROGRAM TO CALCULATE THE THERMAL CONDUCTIVITY OF COMPOSITES

```
C     SET UP RIGHT HAND SIDES OF EQUATION.
C
      CALL SETVOLTARRAY(IRNUM)
C
      NUM=1
      IL=0
      IH=2
C
C     SET UP ALL THE SIMULTANEOUS EQUATIONS, FILLING ARRAYS
C     RNET AND IND, CONTROLLED BY THE SUBROUTINES BELOW.
C     THE RESISTOR VALUES ARE PUT INTO RNET IN COLUMN ORDER,
C     AND THE ROW INDEX IS STORED IN IND FOR THE NAG SUBROUTINES.
C     THE MESHES ARE WORKED THROUGH IN ROW ORDER,
C     LOOKING FOR THE PRESENCE OF THE PARTICULAR RESISTOR.
C
      DO 8 IC=1,IRNUM
C
      IW(IC,1)=NUM
C
      ICOUNT=0
C
C     CONCERNED WITH MESHES.
C
      DO 1 I=1,M
      DO 2 J=1,N
      ICOUNT=ICOUNT+1
      K=(I-1)*(2*N-1)
C
C     TEST FOR 3 OR 4 RESISTOR MESHES.
C
      IF(J.EQ.1.OR.J.EQ.N) GO TO 3
      CALL MESH4I(I,J,N,M,ICOUNT)
      GO TO 2
    3 CALL MESH3I(I,J,N,M,ICOUNT)
    2 CONTINUE
    1 CONTINUE
C
C     CONCERNED WITH MESH POINTS.
C
      MPLUS1=M+1
      NMINUS1=N-1
      DO 4 I=1,MPLUS1
      DO 5 J=1,NMINUS1
      ICOUNT=ICOUNT+1
      K=(I-1)*(2*N-1)
      IF(I.EQ.1.OR.I.EQ.MPLUS1) GO TO 6
C
C     TEST FOR 3 OR 4 RESISTOR POINT JUNCTIONS.
C
      CALL POINT4I(J,N,ICOUNT)
      GO TO 5
    6 CALL POINT3I(I,J,N,ICOUNT)
    5 CONTINUE
    4 CONTINUE
C
C     CONCERNED WITH ONE HORIZONTAL ROW.
```

```
A PROGRAM TO CALCULATE THE THERMAL CONDUCTIVITY OF COMPOSITES

C
      ICOUNT=ICOUNT+1
      CALL OVERALLI(N,ICOUNT)
C
      IL=IL+1
      IH=IH+1
C
    8 CONTINUE
C
      CALL TIME(ATIME)
      WRITE(2,202) ATIME
  202 FORMAT(' THE MATRIX IS NOW SET UP,TIME WAS ',A8//)
C
C     NOW SET UP THE FIRST UNOCCUPIED POSITION IN RNET.
C
      IW(IRNUM+1,1)=NUM
C
C     ERROR EXIT.
C
      IF(ICOUNT.NE.IRNUM) GO TO 7
C
C     DECREMENT NUM AS IT WAS AUTOINCREMENTED AFTER THE
C     LAST CORRECT POSITION FOUND.
C
      NUM=NUM-1
C
C     NUM NOW CONTAINS THE NUMBER OF THE LAST OCCUPIED POSITION
C     IN RNET.
C
C     SET UP PARAMETERS FOR NAG.
C
      IIW=IRNUM+10
      NW=(IRNUM/10)+1
      U=0.25
      JSCALE=0
      IFAIL=1
      IA=11000
C
C     SET IFAIL TO 1 FOR BOTH ROUTINES(SOFT FAILURE OPTION).
C
C     CALL NAG LIBRARY ROUTINES.
C
      CALL F03AJF(RNET,IND,IW,IIW,IND2,IRNUM,IWW,INDW,NW,6,U,
     CIA,D1,ID2,JSCALE,IFAIL)
      WRITE(2,100) IFAIL
  100 FORMAT(' FOR CALL TO F03AJF,IFAIL WA` ',I2//)
      IF(IFAIL.EQ.0) GO TO 10
      WRITE(2,104) JSCALE
  104 FORMAT(' PROGRAM FAILURE,JSCALE WAS ',I4//)
C
C     PRINT ERROR INFORMATION IF FAILURE.
C
      CALL EXITPARAM
      STOP
   10 MTYPE=1
      IFAIL=1
```