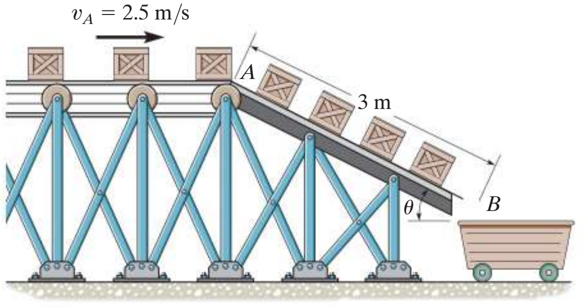
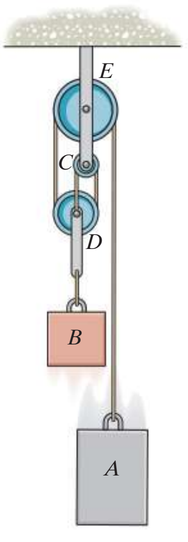
***13–20.** The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5\ \text{m/s}$, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.

13–21. The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5\ \text{m/s}$, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the smallest incline θ of the ramp so that the crates will slide off and fall into the cart.



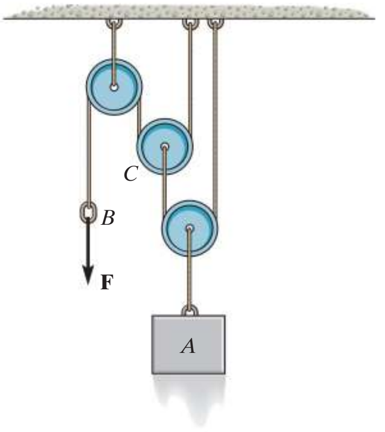
Probs. 13–20/21

13–22. The 50-kg block A is released from rest. Determine the velocity of the 15-kg block B in 2 s.



Prob. 13–22

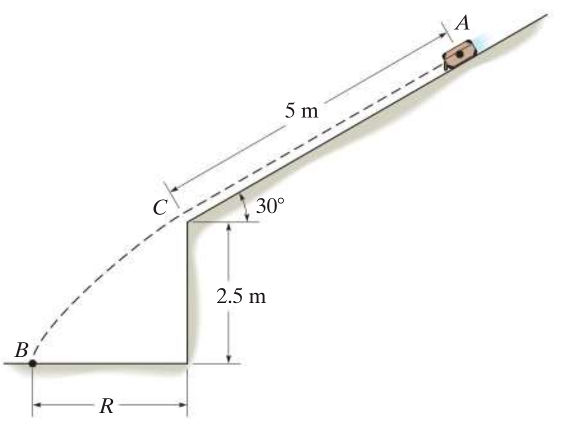
13–23. If the supplied force $F = 150$ N, determine the velocity of the 50-kg block A when it has risen 3 m, starting from rest.



Prob. 13–23

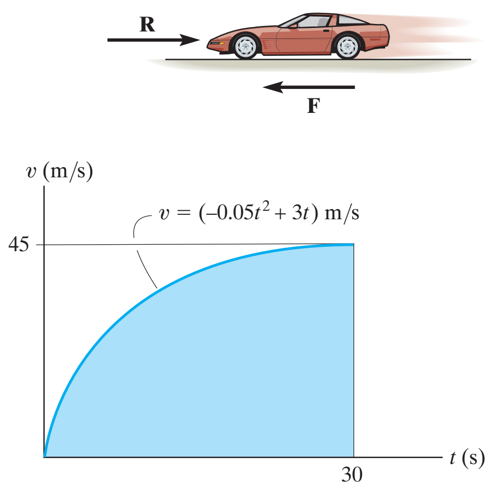
***13–24.** A 60-kg suitcase slides from rest 5 m down the smooth ramp. Determine the distance R where it strikes the ground at B. How long does it take to go from A to B?

13–25. Solve Prob. 13–24 if the suitcase has an initial velocity down the ramp of $v_A = 2\ \text{m/s}$, and the coefficient of kinetic friction along AC is $\mu_k = 0.2$.



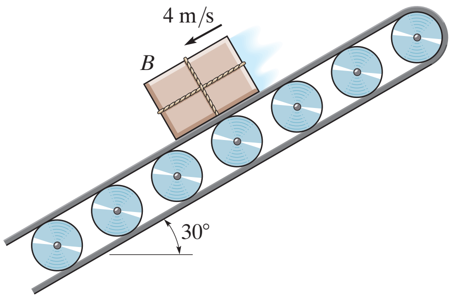
Probs. 13–24/25

13–26. The 1.5 Mg sports car has a tractive force of $F = 4.5$ kN. If it produces the velocity described by v-t graph shown, plot the air resistance R versus t for this time period.



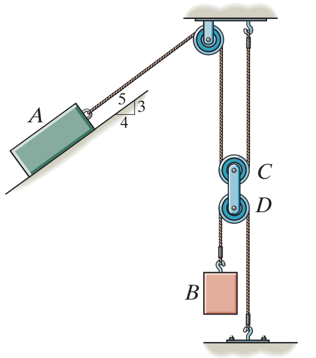
Prob. 13–26

13–27. The conveyor belt is moving downward at 4 m/s. If the coefficient of static friction between the conveyor and the 15-kg package B is $\mu_s = 0.8$, determine the shortest time the belt can stop so that the package does not slide on the belt.



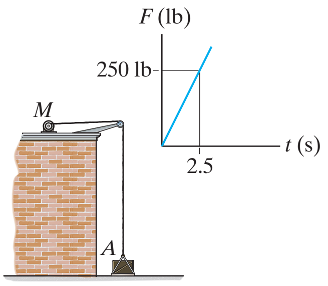
Prob. 13–27

***13–28.** At the instant shown the 100-lb block A is moving down the plane at 5 ft/s while being attached to the 50-lb block B. If the coefficient of kinetic friction between the block and the incline is $\mu_k = 0.2$, determine the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.



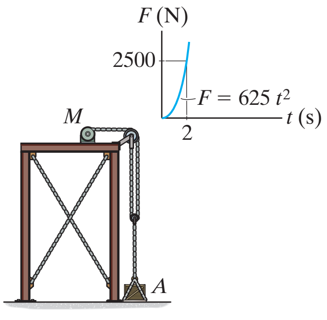
Prob. 13–28

13–29. The force exerted by the motor on the cable is shown in the graph. Determine the velocity of the 200-lb crate when $t = 2.5$ s.



Prob. 13–29

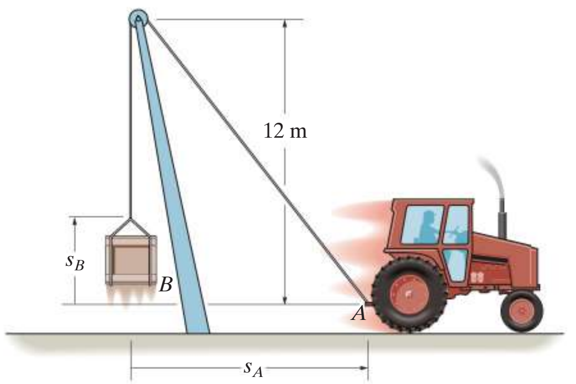
13–30. The force of the motor M on the cable is shown in the graph. Determine the velocity of the 400-kg crate A when $t = 2$ s.



Prob. 13–30

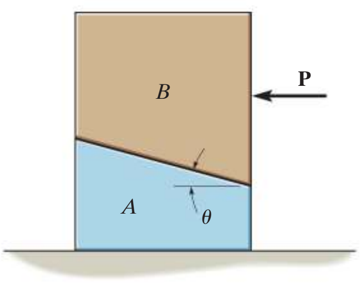
13–31. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.

***13–32.** The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



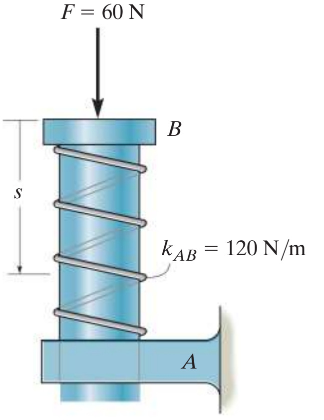
Probs. 13–31/32

13–33. Block A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that it will not slide on A. Also, what is the corresponding acceleration? The coefficient of static friction between A and B is μ_s. Neglect any friction between A and the horizontal surface.



Prob. 13–33

13–34. The 4-kg smooth cylinder is supported by the spring having a stiffness of $k_{AB} = 120$ N/m. Determine the velocity of the cylinder when it moves downward $s = 0.2$ m from its equilibrium position, which is caused by the application of the force $F = 60$ N.

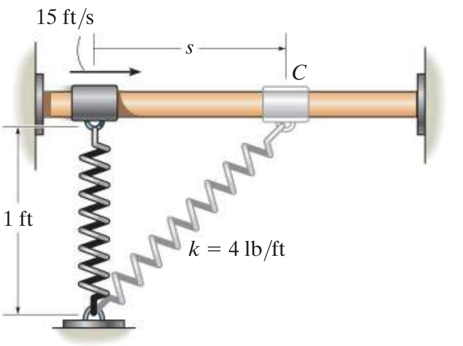


Prob. 13–34

13–35. The coefficient of static friction between the 200-kg crate and the flat bed of the truck is $\mu_s = 0.3$. Determine the shortest time for the truck to reach a speed of 60 km/h, starting from rest with constant acceleration, so that the crate does not slip.

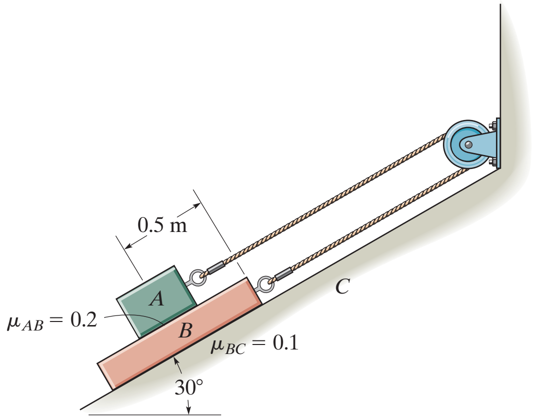
Prob. 13–35

***13–36.** The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



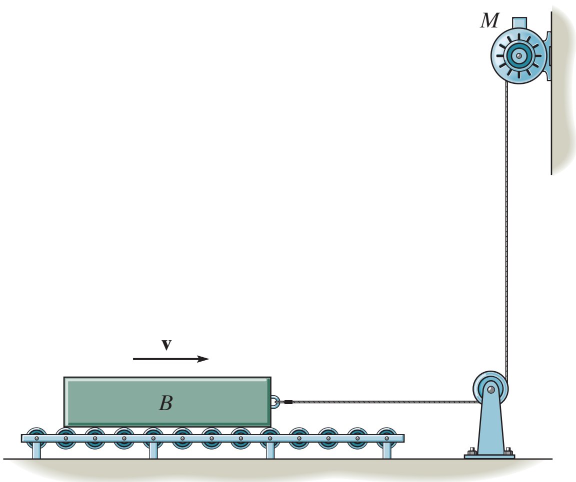
Prob. 13–36

13–37. The 10-kg block A rests on the 50-kg plate B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide 0.5 m *on the plate* when the system is released from rest.



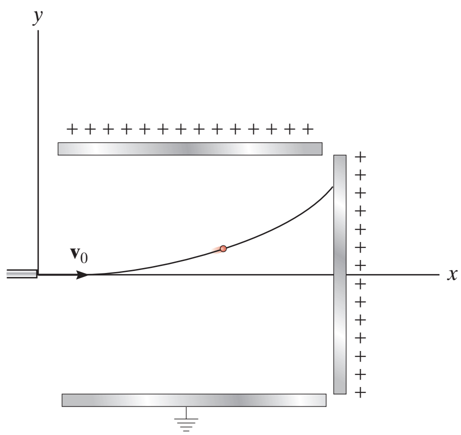
Prob. 13–37

13–38. The 300-kg bar B, originally at rest, is being towed over a series of small rollers. Determine the force in the cable when $t = 5$ s, if the motor M is drawing in the cable for a short time at a rate of $v = (0.4t^2)$ m/s, where t is in seconds $(0 \leq t \leq 6\text{ s})$. How far does the bar move in 5 s? Neglect the mass of the cable, pulley, and the rollers.



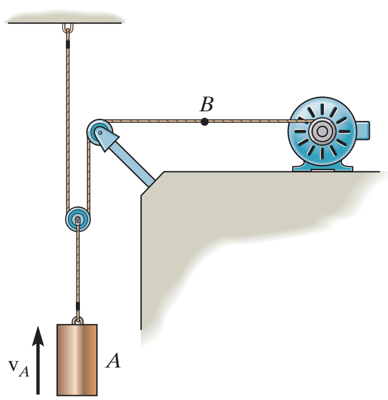
Prob. 13–38

13–39. An electron of mass m is discharged with an initial horizontal velocity of $\mathbf{v}_0$. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$, where F_0 is constant, determine the equation of the path, and the speed of the electron at any time t.



Prob. 13–39

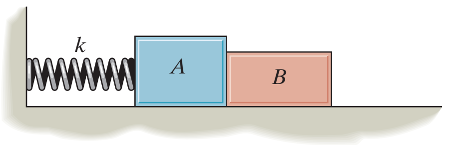
***13–40.** The 400-lb cylinder at A is hoisted using the motor and the pulley system shown. If the speed of point B on the cable is increased at a constant rate from zero to $v_B = 10$ ft/s in $t = 5$ s, determine the tension in the cable at B to cause the motion.



Prob. 13–40

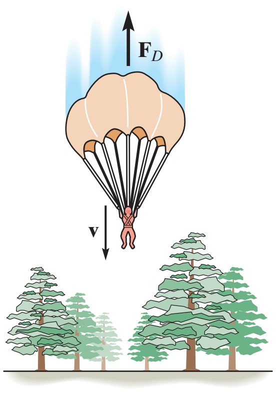
13–41. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?

13–42. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



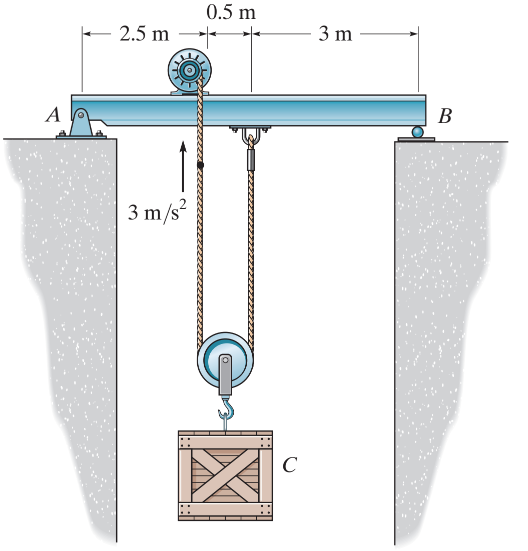
Probs. 13–41/42

13–43. A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.



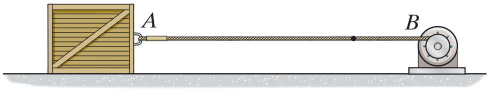
Prob. 13–43

*__13–44.__ If the motor draws in the cable with an acceleration of 3 m/s^2, determine the reactions at the supports A and B. The beam has a uniform mass of 30 kg/m, and the crate has a mass of 200 kg. Neglect the mass of the motor and pulleys.



Prob. 13–44

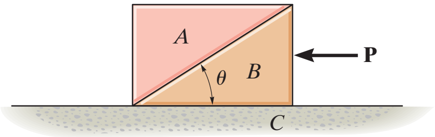
13–45. If the force exerted on cable AB by the motor is $F = (100t^{3/2})$ N, where t is in seconds, determine the 50-kg crate's velocity when $t = 5$ s. The coefficients of static and kinetic friction between the crate and the ground are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. Initially the crate is at rest.



Prob. 13–45

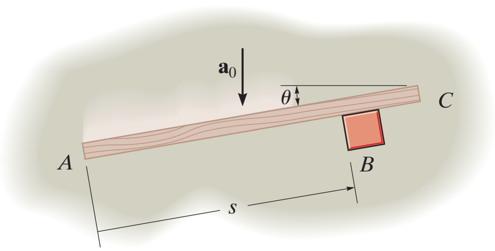
13–46. Blocks A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not move relative to B. All surfaces are smooth.

13–47. Blocks A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.



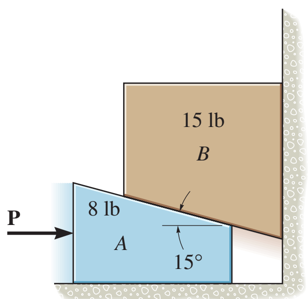
Probs. 13–46/47

***13–48.** The smooth block B of negligible size has a mass m and rests on the horizontal plane. If the board AC pushes on the block at an angle θ with a constant acceleration $\mathbf{a}_0$, determine the velocity of the block along the board and the distance s the block moves along the board as a function of time t. The block starts from rest when $s = 0$, $t = 0$.



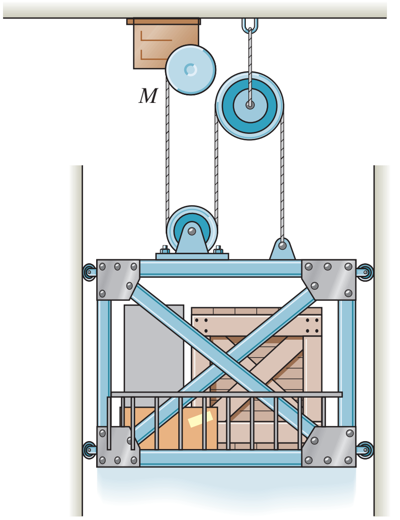
Prob. 13–48

13–49. If a horizontal force $P = 12$ lb is applied to block A determine the acceleration of the block B. Neglect friction.



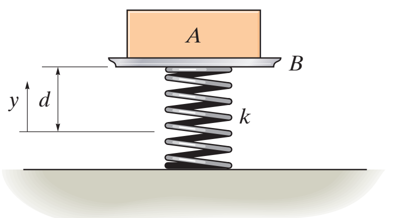
Prob. 13–49

13–50. A freight elevator, including its load, has a mass of 1 Mg. It is prevented from rotating due to the track and wheels mounted along its sides. If the motor M develops a constant tension $T = 4$ kN in its attached cable, determine the velocity of the elevator when it has moved upward 6 m starting from rest. Neglect the mass of the pulleys and cables.



Prob. 13–50

13–51. The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.



Prob. 13–51

13.5 Equations of Motion: Normal and Tangential Coordinates

When a particle moves along a curved path which is known, the equation of motion for the particle may be written in the tangential, normal, and binormal directions, Fig. 13–11. Note that there is no motion of the particle in the binormal direction, since the particle is constrained to move along the path. We have

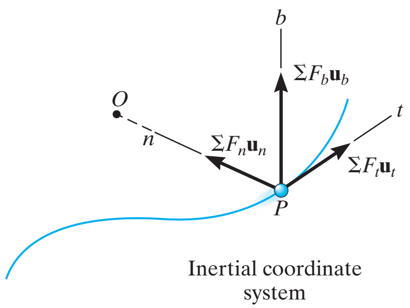


Fig. 13–11

$$\Sigma \mathbf{F} = m\mathbf{a}$$

$$\Sigma F_t \mathbf{u}_t + \Sigma F_n \mathbf{u}_n + \Sigma F_b \mathbf{u}_b = m\mathbf{a}_t + m\mathbf{a}_n$$

This equation is satisfied provided

$$\boxed{\begin{aligned} \Sigma F_t &= ma_t \\ \Sigma F_n &= ma_n \\ \Sigma F_b &= 0 \end{aligned}} \qquad (13\text{–}8)$$

Recall that $a_t (= dv/dt)$ represents the time rate of change in the magnitude of velocity. So if $\Sigma \mathbf{F}_t$ acts in the direction of motion, the particle's speed will increase, whereas if it acts in the opposite direction, the particle will slow down. Likewise, $a_n (= v^2/\rho)$ represents the time rate of change in the velocity's direction. It is caused by $\Sigma \mathbf{F}_n$, which *always* acts in the positive n direction, i.e., toward the path's center of curvature. For this reason it is often referred to as the *centripetal force*.

As a roller coaster falls downward along the track, the cars have both a normal and a tangential component of acceleration. (© R.C. Hibbeler)

Procedure for Analysis

When a problem involves the motion of a particle along a *known curved path*, normal and tangential coordinates should be considered for the analysis since the acceleration components can be readily formulated. The method for applying the equations of motion, which relate the forces to the acceleration, has been outlined in the procedure given in Sec. 13.4. Specifically, for t, n, b coordinates it may be stated as follows:

Free-Body Diagram.

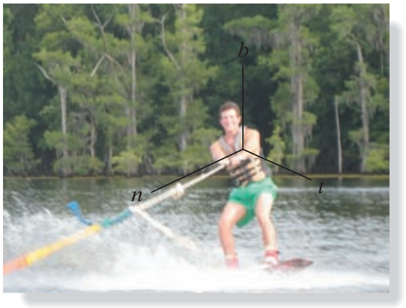
- Establish the inertial t, n, b coordinate system at the particle and draw the particle's free-body diagram.
- The particle's normal acceleration $\mathbf{a}_n$ *always* acts in the positive n direction.
- If the tangential acceleration $\mathbf{a}_t$ is unknown, assume it acts in the positive t direction.
- There is no acceleration in the b direction.
- Identify the unknowns in the problem.

Equations of Motion.

- Apply the equations of motion, Eq. 13–8.

Kinematics.

- Formulate the tangential and normal components of acceleration; i.e., $a_t = dv/dt$ or $a_t = v\,dv/ds$ and $a_n = v^2/\rho$.
- If the path is defined as $y = f(x)$, the radius of curvature at the point where the particle is located can be obtained from $\rho = [1 + (dy/dx)^2]^{3/2}/|d^2y/dx^2|$.



The unbalanced force of the rope on the skier gives him a normal component of acceleration. (© R.C. Hibbeler)

EXAMPLE 13.6

Determine the banking angle θ for the race track so that the wheels of the racing cars shown in Fig. 13–12*a* will not have to depend upon friction to prevent any car from sliding up or down the track. Assume the cars have negligible size, a mass m, and travel around the curve of radius ρ with a constant speed v.

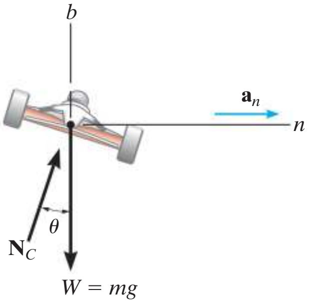
(© R.C. Hibbeler)

(a)

SOLUTION

Before looking at the following solution, give some thought as to why it should be solved using *t, n, b* coordinates.

Free-Body Diagram. As shown in Fig. 13–12*b*, and as stated in the problem, no frictional force acts on the car. Here $\mathbf{N}_C$ represents the *resultant* of the ground on all four wheels. Since a_n can be calculated, the unknowns are N_C and θ.



(b)

Fig. 13–12

Equations of Motion. Using the *n, b* axes shown,

$$\overset{+}{\rightarrow} \Sigma F_n = ma_n; \qquad N_C \sin\theta = m\frac{v^2}{\rho} \qquad (1)$$

$$+\uparrow \Sigma F_b = 0; \qquad N_C \cos\theta - mg = 0 \qquad (2)$$

Eliminating N_C and m from these equations by dividing Eq. 1 by Eq. 2, we obtain

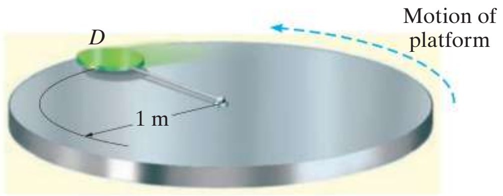
$$\tan\theta = \frac{v^2}{g\rho}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{g\rho}\right) \qquad \textit{Ans.}$$

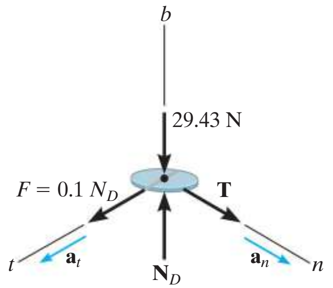
NOTE: The result is independent of the mass of the car. Also, a force summation in the tangential direction is of no consequence to the solution. If it were considered, then $a_t = dv/dt = 0$, since the car moves with *constant speed*. A further analysis of this problem is discussed in Prob. 21–53.

EXAMPLE 13.7

The 3-kg disk D is attached to the end of a cord as shown in Fig. 13–13*a*. The other end of the cord is attached to a ball-and-socket joint located at the center of a platform. If the platform rotates rapidly, and the disk is placed on it and released from rest as shown, determine the time it takes for the disk to reach a speed great enough to break the cord. The maximum tension the cord can sustain is 100 N, and the coefficient of kinetic friction between the disk and the platform is $\mu_k = 0.1$.



(a)



(b)

Fig. 13–13

SOLUTION

Free-Body Diagram. The frictional force has a magnitude $F = \mu_k N_D = 0.1N_D$ and a sense of direction that opposes the *relative motion* of the disk with respect to the platform. It is this force that gives the disk a tangential component of acceleration causing v to increase, thereby causing T to increase until it reaches 100 N. The weight of the disk is $W = 3(9.81) = 29.43$ N. Since a_n can be related to v, the unknowns are N_D, a_t, and v.

Equations of Motion.

$$\Sigma F_n = ma_n; \qquad T = 3\left(\frac{v^2}{1}\right) \qquad (1)$$

$$\Sigma F_t = ma_t; \qquad 0.1N_D = 3a_t \qquad (2)$$

$$\Sigma F_b = 0; \qquad N_D - 29.43 = 0 \qquad (3)$$

Setting $T = 100$ N, Eq. 1 can be solved for the critical speed v_{cr} of the disk needed to break the cord. Solving all the equations, we obtain

$$N_D = 29.43 \text{ N}$$

$$a_t = 0.981 \text{ m/s}^2$$

$$v_{cr} = 5.77 \text{ m/s}$$

Kinematics. Since a_t is *constant*, the time needed to break the cord is

$$v_{cr} = v_0 + a_t t$$

$$5.77 = 0 + (0.981)t$$

$$t = 5.89 \text{ s} \qquad \textit{Ans.}$$

EXAMPLE 13.8

Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximated by the parabola shown in Fig. 13–14*a*, determine the normal force on the 150-lb skier the instant she arrives at the end of the jump, point *A*, where her velocity is 65 ft/s. Also, what is her acceleration at this point?

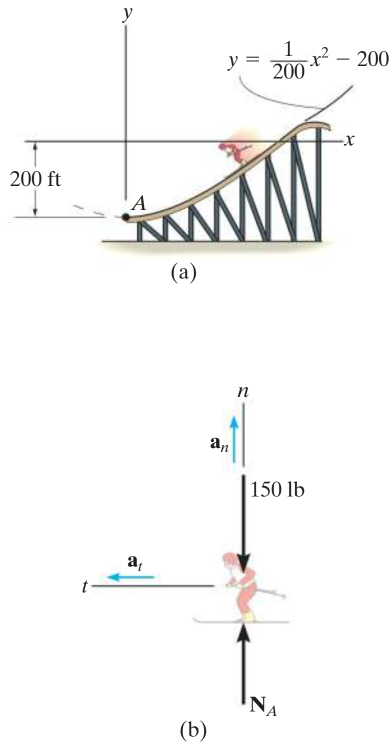


Fig. 13–14

SOLUTION

Why consider using *n, t* coordinates to solve this problem?

Free-Body Diagram. Since $dy/dx = x/100|_{x=0} = 0$, the slope at A is horizontal. The free-body diagram of the skier when she is at A is shown in Fig. 13–14*b*. Since the path is *curved*, there are two components of acceleration, $\mathbf{a}_n$ and $\mathbf{a}_t$. Since a_n can be calculated, the unknowns are a_t and N_A.

Equations of Motion.

$$+\uparrow \Sigma F_n = ma_n; \qquad N_A - 150 = \frac{150}{32.2}\left(\frac{(65)^2}{\rho}\right) \qquad (1)$$

$$\overset{+}{\leftarrow} \Sigma F_t = ma_t; \qquad 0 = \frac{150}{32.2}a_t \qquad (2)$$

The radius of curvature ρ for the path must be determined at point $A(0, -200 \text{ ft})$. Here $y = \frac{1}{200}x^2 - 200$, $dy/dx = \frac{1}{100}x$, $d^2y/dx^2 = \frac{1}{100}$, so that at $x = 0$,

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}\bigg|_{x=0} = \frac{[1 + (0)^2]^{3/2}}{|\frac{1}{100}|} = 100 \text{ ft}$$

Substituting this into Eq. 1 and solving for N_A, we obtain

$$N_A = 347 \text{ lb} \qquad \textit{Ans.}$$

Kinematics. From Eq. 2,

$$a_t = 0$$

Thus,

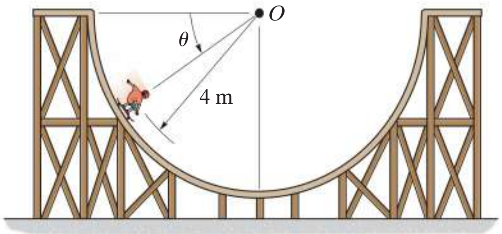
$$a_n = \frac{v^2}{\rho} = \frac{(65)^2}{100} = 42.2 \text{ ft/s}^2$$

$$a_A = a_n = 42.2 \text{ ft/s}^2 \uparrow \qquad \textit{Ans.}$$

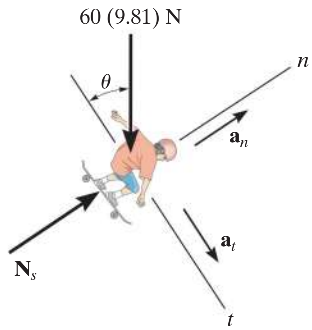
NOTE: Apply the equation of motion in the y direction and show that when the skier is in midair her downward acceleration is 32.2 ft/s^2.

EXAMPLE 13.9

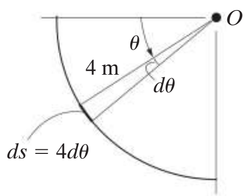
The 60-kg skateboarder in Fig. 13–15*a* coasts down the circular track. If he starts from rest when $\theta = 0^\circ$, determine the magnitude of the normal reaction the track exerts on him when $\theta = 60^\circ$. Neglect his size for the calculation.



(a)



(b)



(c)

Fig. 13–15

SOLUTION

Free-Body Diagram. The free-body diagram of the skateboarder when he is at an *arbitrary position* θ is shown in Fig. 13–15*b*. At $\theta = 60^\circ$ there are three unknowns, N_s, a_t, and a_n (or v).

Equations of Motion.

$$+\nearrow \Sigma F_n = ma_n; \quad N_s - [60(9.81)\text{N}]\sin\theta = (60\text{ kg})\left(\frac{v^2}{4\text{ m}}\right) \qquad (1)$$

$$+\searrow \Sigma F_t = ma_t; \quad [60(9.81)\text{N}]\cos\theta = (60\text{ kg})\,a_t$$

$$a_t = 9.81\cos\theta$$

Kinematics. Since a_t is expressed in terms of θ, the equation $v\,dv = a_t\,ds$ must be used to determine the speed of the skateboarder when $\theta = 60^\circ$. Using the geometric relation $s = \theta r$, where $ds = r\,d\theta = (4\text{ m})\,d\theta$, Fig. 13–15*c*, and the initial condition $v = 0$ at $\theta = 0^\circ$, we have,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^{60^\circ} 9.81\cos\theta(4\,d\theta)$$

$$\frac{v^2}{2}\Big|_0^v = 39.24\sin\theta\Big|_0^{60^\circ}$$

$$\frac{v^2}{2} - 0 = 39.24(\sin 60^\circ - 0)$$

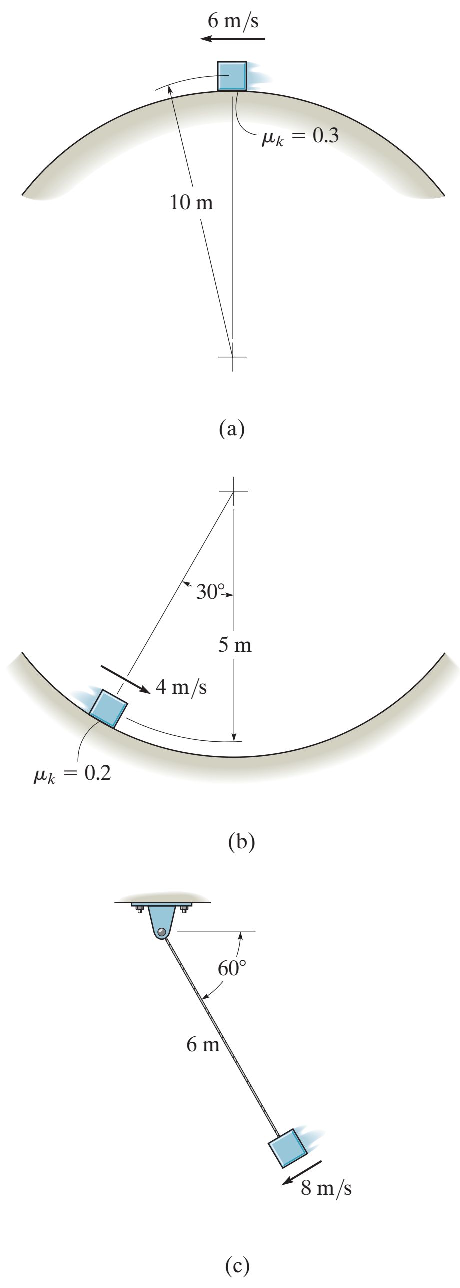
$$v^2 = 67.97\text{ m}^2/\text{s}^2$$

Substituting this result and $\theta = 60^\circ$ into Eq. (1), yields

$$N_s = 1529.23\text{ N} = 1.53\text{ kN}$$ *Ans.*

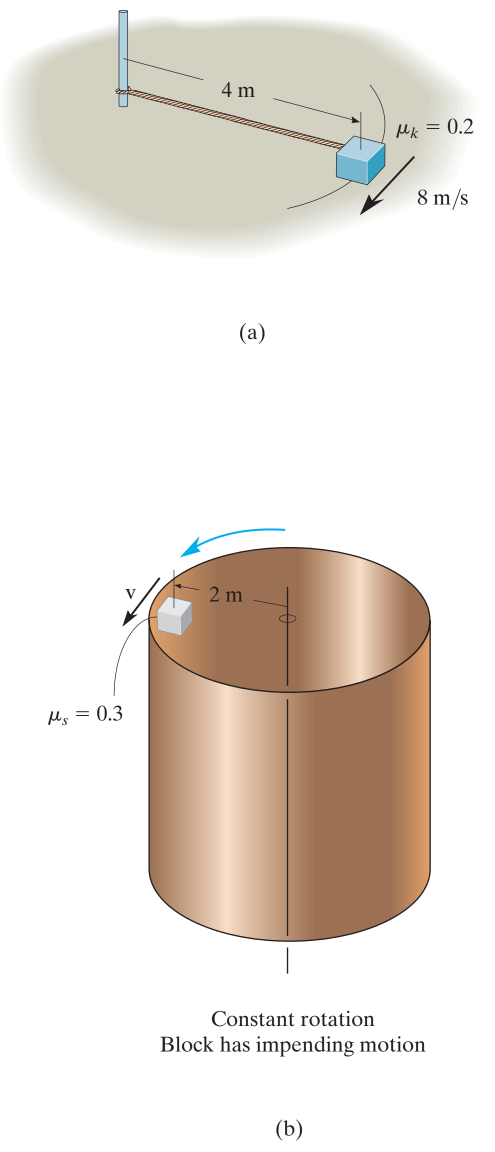
PRELIMINARY PROBLEMS

P13–5. Set up the n, t axes and write the equations of motion for the 10-kg block along each of these axes.



Prob. P13–5

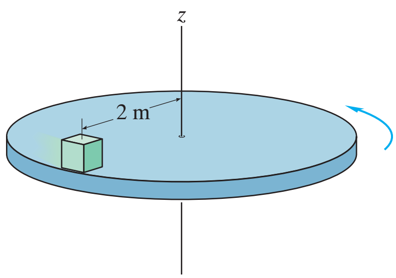
P13–6. Set up the n, b, t axes and write the equations of motion for the 10-kg block along each of these axes.



Prob. P13–6

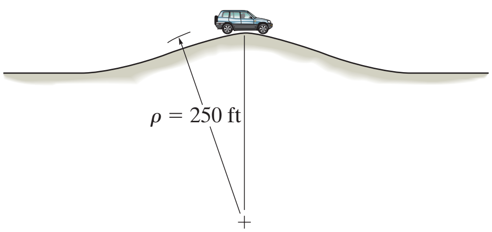
FUNDAMENTAL PROBLEMS

F13–7. The block rests at a distance of 2 m from the center of the platform. If the coefficient of static friction between the block and the platform is $\mu_s = 0.3$, determine the maximum speed which the block can attain before it begins to slip. Assume the angular motion of the disk is slowly increasing.



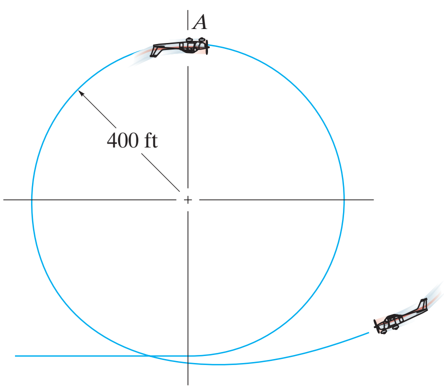
Prob. F13–7

F13–8. Determine the maximum speed that the jeep can travel over the crest of the hill and not lose contact with the road.



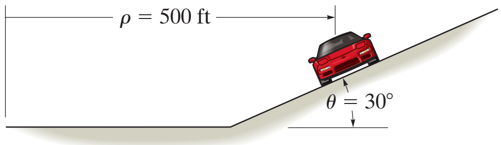
Prob. F13–8

F13–9. A pilot weighs 150 lb and is traveling at a constant speed of 120 ft/s. Determine the normal force he exerts on the seat of the plane when he is upside down at A. The loop has a radius of curvature of 400 ft.



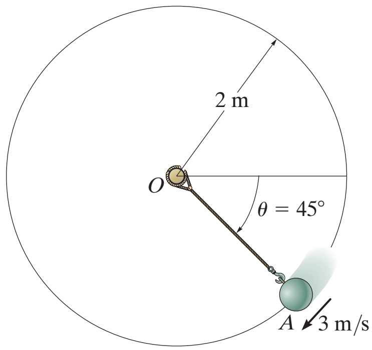
Prob. F13–9

F13–10. The sports car is traveling along a 30° banked road having a radius of curvature of $\rho = 500$ ft. If the coefficient of static friction between the tires and the road is $\mu_s = 0.2$, determine the maximum safe speed so no slipping occurs. Neglect the size of the car.



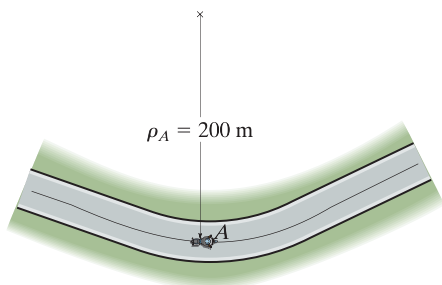
Prob. F13–10

F13–11. If the 10-kg ball has a velocity of 3 m/s when it is at the position A, along the vertical path, determine the tension in the cord and the increase in the speed of the ball at this position.



Prob. F13–11

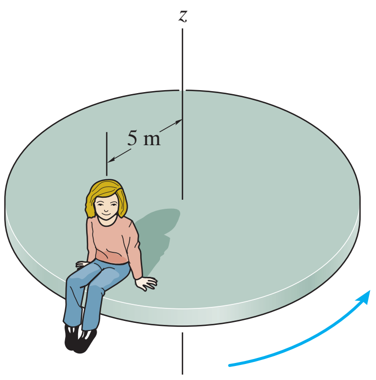
F13–12. The motorcycle has a mass of 0.5 Mg and a negligible size. It passes point A traveling with a speed of 15 m/s, which is increasing at a constant rate of 1.5 m/s². Determine the resultant frictional force exerted by the road on the tires at this instant.



Prob. F13–12

PROBLEMS

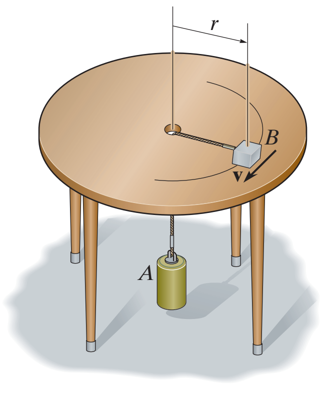
***13–52.** A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.



Prob. 13–52

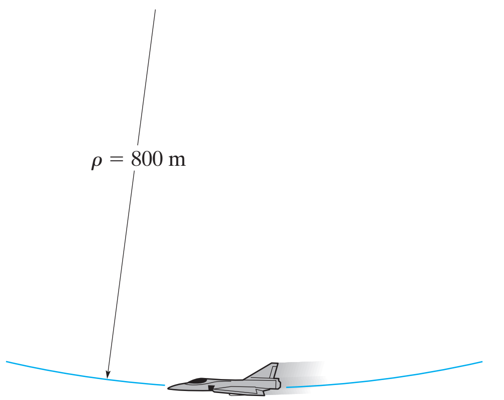
13–53. The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10$ m/s, determine the radius r of the circular path along which it travels.

13–54. The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5$ m, determine the speed of the block.



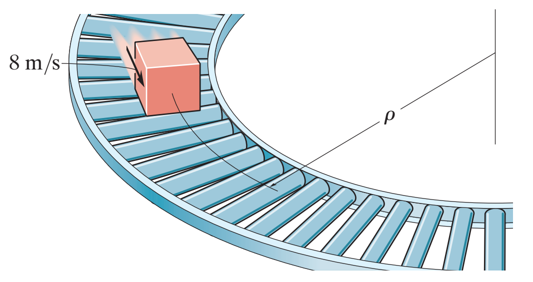
Probs. 13–53/54

13–55. Determine the maximum constant speed at which the pilot can travel around the vertical curve having a radius of curvature $\rho = 800$ m, so that he experiences a maximum acceleration $a_n = 8g = 78.5$ m/s^2. If he has a mass of 70 kg, determine the normal force he exerts on the seat of the airplane when the plane is traveling at this speed and is at its lowest point.



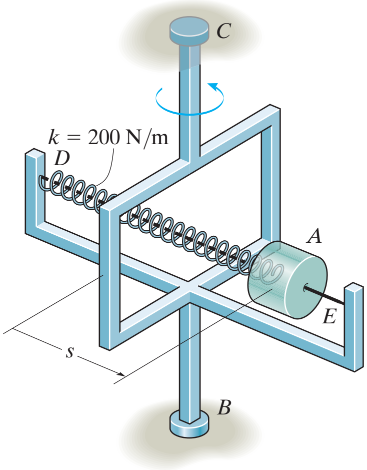
Prob. 13–55

***13–56.** Cartons having a mass of 5 kg are required to move along the assembly line at a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



Prob. 13–56

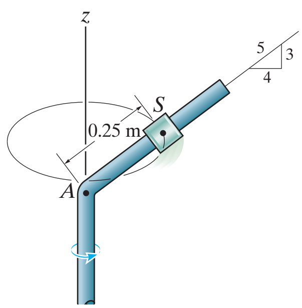
13–57. The collar A, having a mass of 0.75 kg, is attached to a spring having a stiffness of $k = 200$ N/m. When rod BC rotates about the vertical axis, the collar slides outward along the smooth rod DE. If the spring is unstretched when $s = 0$, determine the constant speed of the collar in order that $s = 100$ mm. Also, what is the normal force of the rod on the collar? Neglect the size of the collar.



Prob. 13–57

13–58. The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.

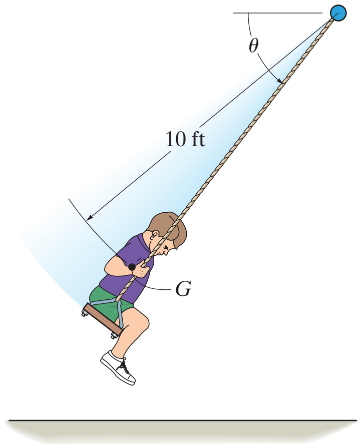
13–59. The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.



Probs. 13–58/59

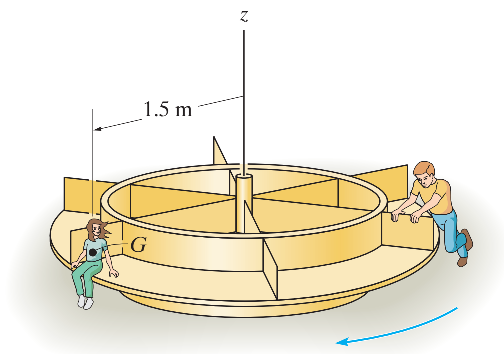
***13–60.** At the instant $\theta = 60^\circ$, the boy's center of mass G has a downward speed $v_G = 15$ ft/s. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.

13–61. At the instant $\theta = 60^\circ$, the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90^\circ$. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



Probs. 13–60/61

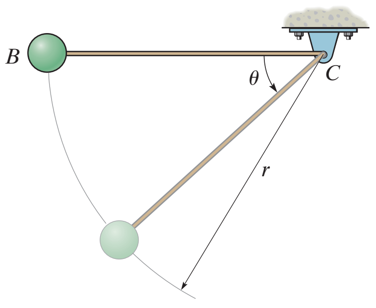
13–62. A girl having a mass of 25 kg sits at the edge of the merry-go-round so her center of mass G is at a distance of 1.5 m from the axis of rotation. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which she can have before she begins to slip off the merry-go-round. The coefficient of static friction between the girl and the merry-go-round is $\mu_s = 0.3$.



Prob. 13–62

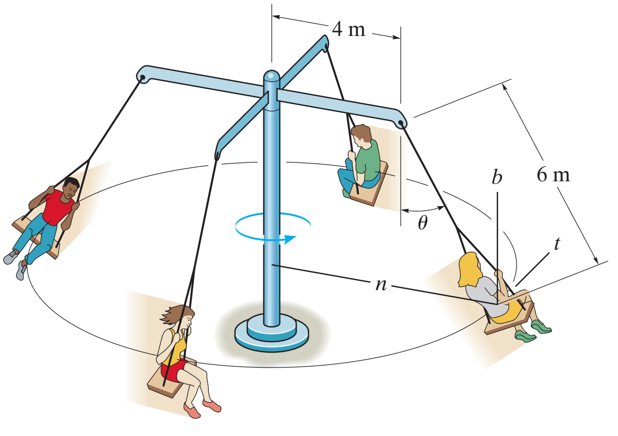
13–63. The pendulum bob B has a weight of 5 lb and is released from rest in the position shown, $\theta = 0°$. Determine the tension in string BC just after the bob is released, $\theta = 0°$, and also at the instant the bob reaches $\theta = 45°$. Take $r = 3$ ft.

***13–64.** The pendulum bob B has a mass m and is released from rest when $\theta = 0°$. Determine the tension in string BC immediately afterwards, and also at the instant the bob reaches the arbitrary position θ.



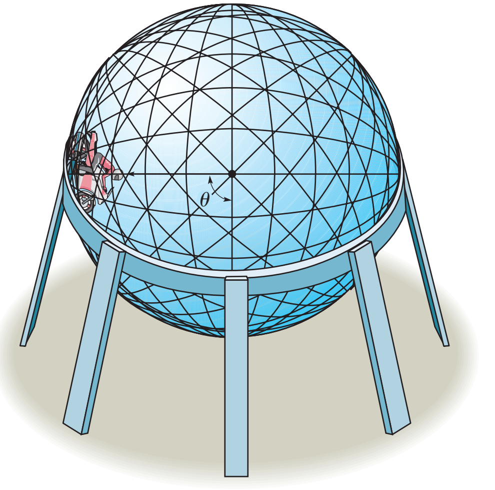
Probs. 13–63/64

13–65. Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the n, t, and b directions which the chair exerts on a 50-kg passenger during the motion?



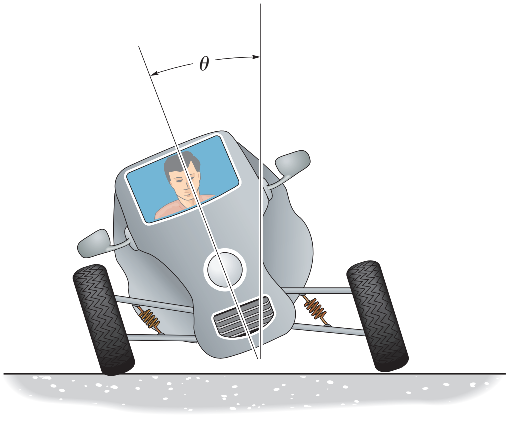
Prob. 13–65

13–66. A motorcyclist in a circus rides his motorcycle within the confines of the hollow sphere. If the coefficient of static friction between the wheels of the motorcycle and the sphere is $\mu_s = 0.4$, determine the minimum speed at which he must travel if he is to ride along the wall when $\theta = 90°$. The mass of the motorcycle and rider is 250 kg, and the radius of curvature to the center of gravity is $\rho = 20$ ft. Neglect the size of the motorcycle for the calculation.



Prob. 13–66

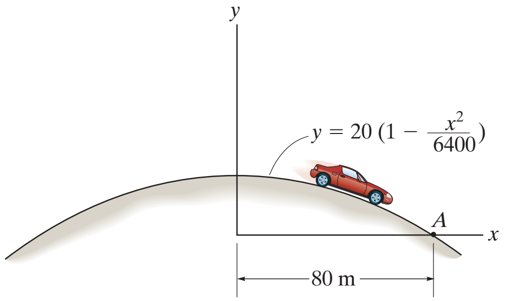
13–67. The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.



Prob. 13–67

***13–68.** The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.

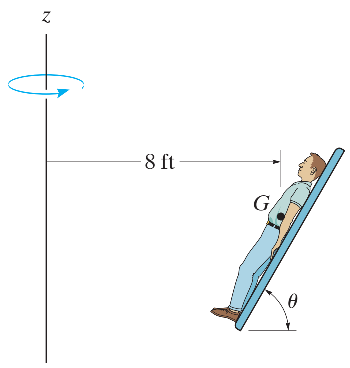
13–69. The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.



Probs. 13–68/69

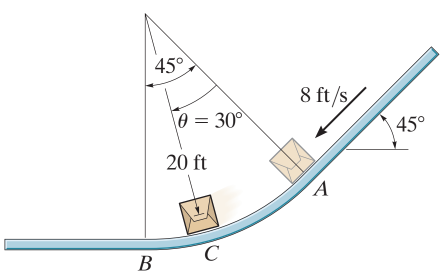
13–71. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, it due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60°$.

***13–72.** The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip off.



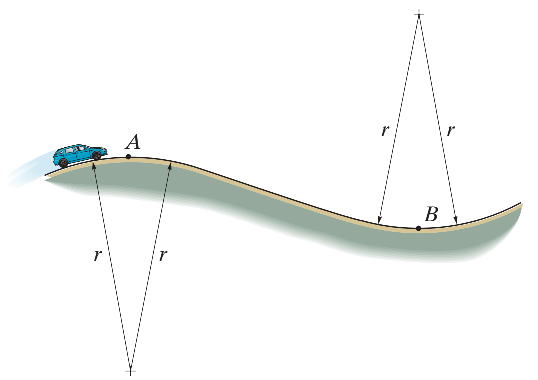
Probs. 13–71/72

13–70. The package has a weight of 5 lb and slides down the chute. When it reaches the curved portion AB, it is traveling at 8 ft/s ($\theta = 0°$). If the chute is smooth, determine the speed of the package when it reaches the intermediate point C ($\theta = 30°$) and when it reaches the horizontal plane ($\theta = 45°$). Also, find the normal force on the package at C.



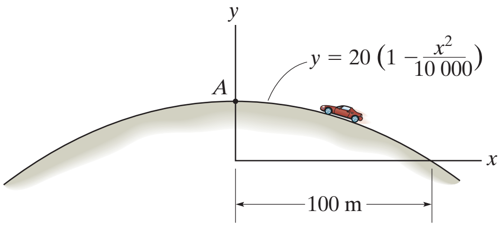
Prob. 13–70

13–73. Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?



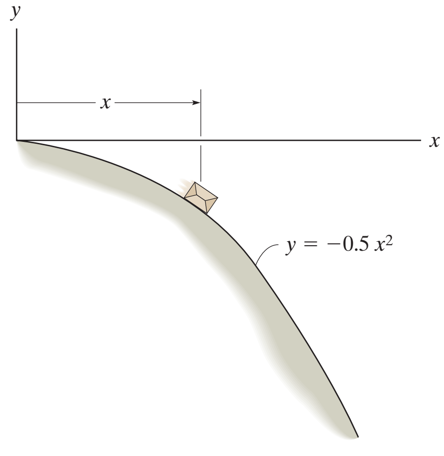
Prob. 13–73

13–74. Determine the maximum constant speed at which the 2-Mg car can travel over the crest of the hill at A without leaving the surface of the road. Neglect the size of the car in the calculation.



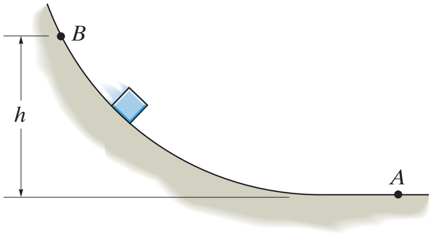
Prob. 13–74

13–75. The box has a mass m and slides down the smooth chute having the shape of a parabola. If it has an initial velocity of v_0 at the origin determine its velocity as a function of x. Also, what is the normal force on the box, and the tangential acceleration as a function of x?



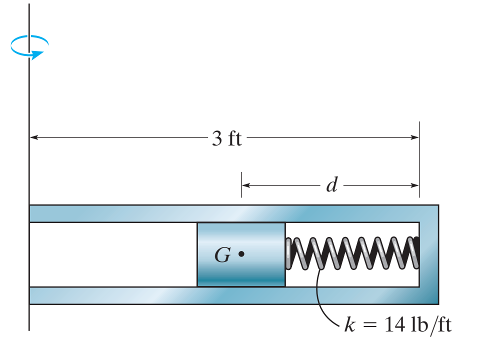
Prob. 13–75

***13–76.** Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



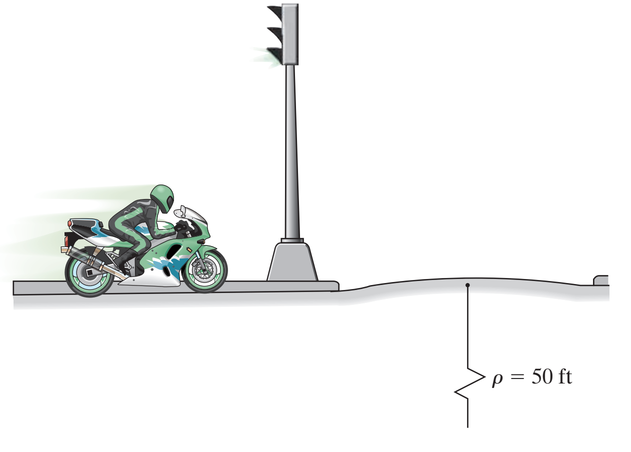
Prob. 13–76

13–77. The cylindrical plug has a weight of 2 lb and it is free to move within the confines of the smooth pipe. The spring has a stiffness $k = 14$ lb/ft and when no motion occurs the distance $d = 0.5$ ft. Determine the force of the spring on the plug when the plug is at rest with respect to the pipe. The plug is traveling with a constant speed of 15 ft/s, which is caused by the rotation of the pipe about the vertical axis.



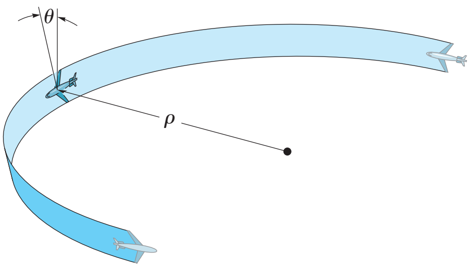
Prob. 13–77

13–78. When crossing an intersection, a motorcyclist encounters the slight bump or crown caused by the intersecting road. If the crest of the bump has a radius of curvature $\rho = 50$ ft, determine the maximum constant speed at which he can travel without leaving the surface of the road. Neglect the size of the motorcycle and rider in the calculation. The rider and his motorcycle have a total weight of 450 lb.



Prob. 13–78

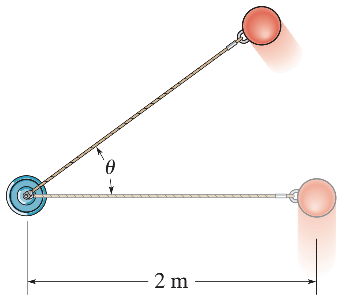
13–79. The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15^\circ$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg.



Prob. 13–79

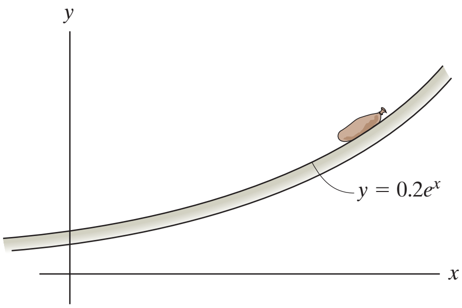
***13–80.** The 2-kg pendulum bob moves in the vertical plane with a velocity of 8 m/s when $\theta = 0^\circ$. Determine the initial tension in the cord and also at the instant the bob reaches $\theta = 30^\circ$. Neglect the size of the bob.

13–81. The 2-kg pendulum bob moves in the vertical plane with a velocity of 6 m/s when $\theta = 0^\circ$. Determine the angle θ where the tension in the cord becomes zero.



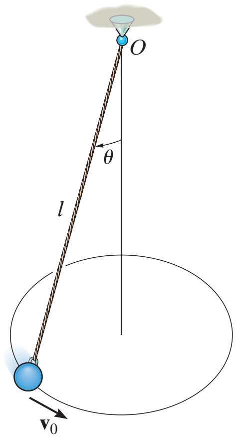
Probs. 13–80/81

13–82. The 8-kg sack slides down the smooth ramp. If it has a speed of 1.5 m/s when $y = 0.2$ m, determine the normal reaction the ramp exerts on the sack and the rate of increase in the speed of sack at this instant.



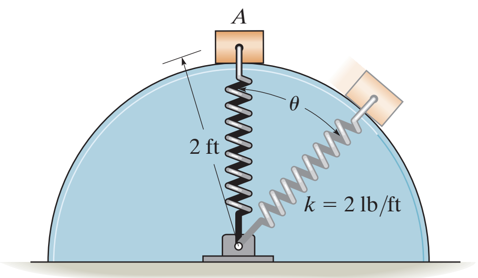
Prob. 13–82

13–83. The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta \sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.

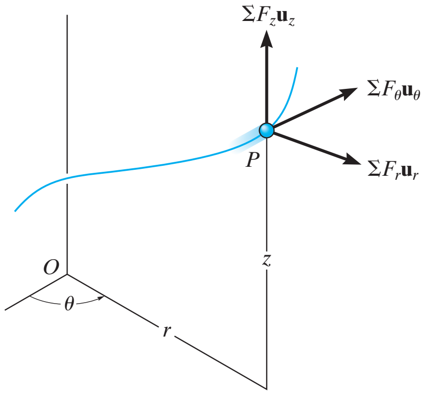


Prob. 13–83

***13–84.** The 2-lb block is released from rest at A and slides down along the smooth cylindrical surface. If the attached spring has a stiffness $k = 2$ lb/ft, determine its unstretched length so that it does not allow the block to leave the surface until $\theta = 60^\circ$.



Prob. 13–84



Inertial coordinate system

Fig. 13–16

13.6 Equations of Motion: Cylindrical Coordinates

When all the forces acting on a particle are resolved into cylindrical components, i.e., along the unit-vector directions $\mathbf{u}_r$, $\mathbf{u}_\theta$, $\mathbf{u}_z$, Fig. 13–16, the equation of motion can be expressed as

$$\Sigma \mathbf{F} = m\mathbf{a}$$

$$\Sigma F_r \mathbf{u}_r + \Sigma F_\theta \mathbf{u}_\theta + \Sigma F_z \mathbf{u}_z = ma_r \mathbf{u}_r + ma_\theta \mathbf{u}_\theta + ma_z \mathbf{u}_z$$

To satisfy this equation, we require

$$\boxed{\begin{aligned} \Sigma F_r &= ma_r \\ \Sigma F_\theta &= ma_\theta \\ \Sigma F_z &= ma_z \end{aligned}} \qquad (13\text{–}9)$$

If the particle is constrained to move only in the r–θ plane, then only the first two of Eq. 13–9 are used to specify the motion.

Tangential and Normal Forces. The most straightforward type of problem involving cylindrical coordinates requires the determination of the resultant force components ΣF_r, ΣF_θ, ΣF_z which cause a particle to move with a *known* acceleration. If, however, the particle's accelerated motion is not completely specified at the given instant, then some information regarding the directions or magnitudes of the forces acting on the particle must be known or calculated in order to solve Eqs. 13–9. For example, the force $\mathbf{P}$ causes the particle in Fig. 13–17*a* to move along a path $r = f(\theta)$. The *normal force* $\mathbf{N}$ which the path exerts on the particle is always *perpendicular to the tangent of the path*, whereas the frictional force $\mathbf{F}$ always acts along the tangent in the opposite direction of motion. The *directions* of $\mathbf{N}$ and $\mathbf{F}$ can be specified relative to the radial coordinate by using the angle ψ (psi), Fig. 13–17*b*, which is defined between the *extended* radial line and the tangent to the curve.

Motion of the roller coaster along this spiral can be studied using cylindrical coordinates. (© R.C. Hibbeler)

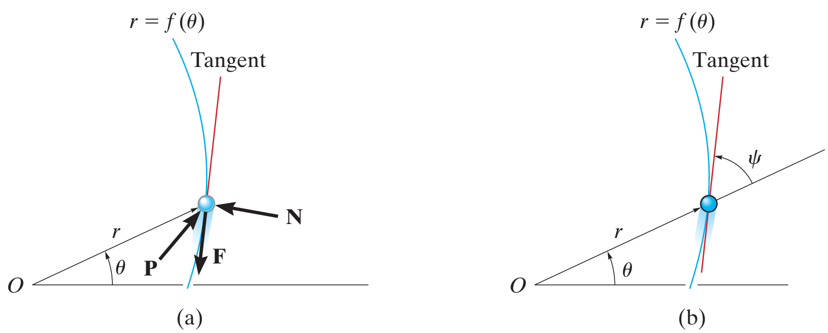
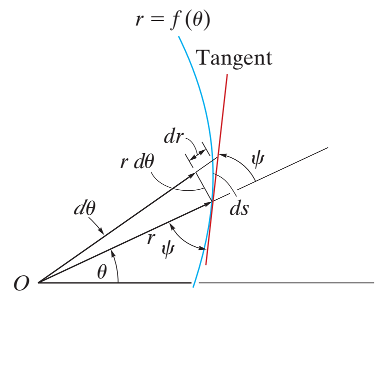


Fig. 13–17

This angle can be obtained by noting that when the particle is displaced a distance ds along the path, Fig. 13–17c, the component of displacement in the radial direction is dr and the component of displacement in the transverse direction is $r\,d\theta$. Since these two components are mutually perpendicular, the angle ψ can be determined from $\tan\psi = r\,d\theta/dr$, or

$$\tan\psi = \frac{r}{dr/d\theta} \tag{13–10}$$

If ψ is calculated as a positive quantity, it is measured from the *extended radial line* to the tangent in a counterclockwise sense or in the positive direction of θ. If it is negative, it is measured in the opposite direction to positive θ. For example, consider the cardioid $r = a(1 + \cos\theta)$, shown in Fig. 13–18. Because $dr/d\theta = -a\sin\theta$, then when $\theta = 30°$, $\tan\psi = a(1 + \cos 30°)/(-a\sin 30°) = -3.732$, or $\psi = -75°$, measured clockwise, opposite to $+\theta$ as shown in the figure.



(c)

Fig. 13–17

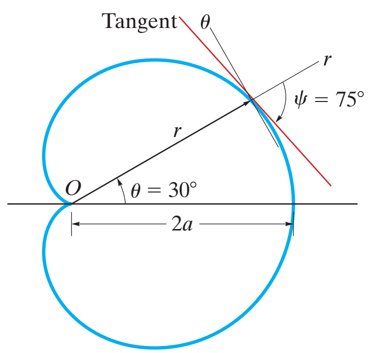


Fig. 13–18

Procedure for Analysis

Cylindrical or polar coordinates are a suitable choice for the analysis of a problem for which data regarding the angular motion of the radial line r are given, or in cases where the path can be conveniently expressed in terms of these coordinates. Once these coordinates have been established, the equations of motion can then be applied in order to relate the forces acting on the particle to its acceleration components. The method for doing this has been outlined in the procedure for analysis given in Sec. 13.4. The following is a summary of this procedure.

Free-Body Diagram.

- Establish the r, θ, z inertial coordinate system and draw the particle's free-body diagram.
- Assume that $\mathbf{a}_r$, $\mathbf{a}_\theta$, $\mathbf{a}_z$ act in the positive directions of r, θ, z if they are unknown.
- Identify all the unknowns in the problem.

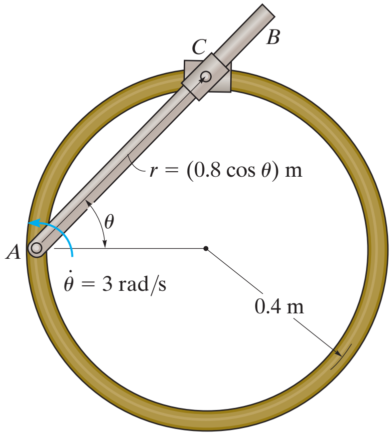
Equations of Motion.

- Apply the equations of motion, Eq. 13–9.

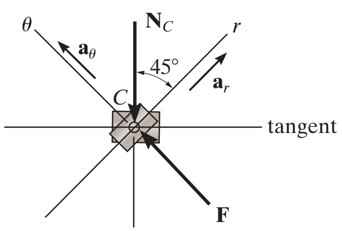
Kinematics.

- Use the methods of Sec. 12.8 to determine r and the time derivatives $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, $\ddot{\theta}$, $\ddot{z}$, and then evaluate the acceleration components $a_r = \ddot{r} - r\dot{\theta}^2$, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, $a_z = \ddot{z}$.
- If any of the acceleration components is computed as a negative quantity, it indicates that it acts in its negative coordinate direction.
- When taking the time derivatives of $r = f(\theta)$, it is very important to use the chain rule of calculus, which is discussed in Appendix C.

EXAMPLE 13.10



(a)



(b)

Fig. 13–19

The smooth 0.5-kg double-collar in Fig. 13–19*a* can freely slide on arm AB and the circular guide rod. If the arm rotates with a constant angular velocity of $\dot{\theta} = 3$ rad/s, determine the force the arm exerts on the collar at the instant $\theta = 45°$. Motion is in the horizontal plane.

SOLUTION

Free-Body Diagram. The normal reaction $\mathbf{N}_C$ of the circular guide rod and the force $\mathbf{F}$ of arm AB act on the collar in the plane of motion, Fig. 13–19*b*. Note that $\mathbf{F}$ acts perpendicular to the axis of arm AB, that is, in the direction of the θ axis, while $\mathbf{N}_C$ acts perpendicular to the tangent of the circular path at $\theta = 45°$. The four unknowns are N_C, F, a_r, a_θ.

Equations of Motion.

$$+\nearrow\Sigma F_r = ma_r: \qquad -N_C \cos 45° = (0.5\text{ kg})\, a_r \tag{1}$$

$$+\nwarrow\Sigma F_\theta = ma_\theta: \qquad F - N_C \sin 45° = (0.5\text{ kg})\, a_\theta \tag{2}$$

Kinematics. Using the chain rule (see Appendix C), the first and second time derivatives of r when $\theta = 45°$, $\dot{\theta} = 3$ rad/s, $\ddot{\theta} = 0$, are

$$\begin{aligned}
r &= 0.8 \cos \theta = 0.8 \cos 45° = 0.5657\text{ m}\\
\dot{r} &= -0.8 \sin \theta\, \dot{\theta} = -0.8 \sin 45°(3) = -1.6971\text{ m/s}\\
\ddot{r} &= -0.8\left[\sin \theta\, \ddot{\theta} + \cos \theta\, \dot{\theta}^2\right]\\
&= -0.8[\sin 45°(0) + \cos 45°(3^2)] = -5.091\text{ m/s}^2
\end{aligned}$$

We have

$$\begin{aligned}
a_r &= \ddot{r} - r\dot{\theta}^2 = -5.091\text{ m/s}^2 - (0.5657\text{ m})(3\text{ rad/s})^2 = -10.18\text{ m/s}^2\\
a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.5657\text{ m})(0) + 2(-1.6971\text{ m/s})(3\text{ rad/s})\\
&= -10.18\text{ m/s}^2
\end{aligned}$$

Substituting these results into Eqs. (1) and (2) and solving, we get

$$N_C = 7.20\text{ N}$$

$$F = 0 \qquad \textit{Ans.}$$

EXAMPLE 13.11

The smooth 2-kg cylinder C in Fig. 13–20a has a pin P through its center which passes through the slot in arm OA. If the arm is forced to rotate in the *vertical plane* at a constant rate $\dot{\theta} = 0.5$ rad/s, determine the force that the arm exerts on the peg at the instant $\theta = 60°$.

SOLUTION

Why is it a good idea to use polar coordinates to solve this problem?

Free-Body Diagram. The free-body diagram for the cylinder is shown in Fig. 13–20b. The force on the peg, $\mathbf{F}_P$, acts perpendicular to the slot in the arm. As usual, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are assumed to act in the directions of *positive* r and θ, respectively. Identify the four unknowns.

Equations of Motion. Using the data in Fig. 13–20b, we have

$$+\swarrow \Sigma F_r = ma_r; \qquad 19.62 \sin\theta - N_C \sin\theta = 2a_r \qquad (1)$$

$$+\searrow \Sigma F_\theta = ma_\theta; \qquad 19.62 \cos\theta + F_P - N_C \cos\theta = 2a_\theta \qquad (2)$$

Kinematics. From Fig. 13–20a, r can be related to θ by the equation

$$r = \frac{0.4}{\sin\theta} = 0.4 \csc\theta$$

Since $d(\csc\theta) = -(\csc\theta \cot\theta)\, d\theta$ and $d(\cot\theta) = -(\csc^2\theta)\, d\theta$, then r and the necessary time derivatives become

$$\dot{\theta} = 0.5 \qquad r = 0.4 \csc\theta$$

$$\ddot{\theta} = 0 \qquad \dot{r} = -0.4(\csc\theta \cot\theta)\dot{\theta}$$

$$= -0.2 \csc\theta \cot\theta$$

$$\ddot{r} = -0.2(-\csc\theta \cot\theta)(\dot{\theta}) \cot\theta - 0.2 \csc\theta(-\csc^2\theta)\dot{\theta}$$

$$= 0.1 \csc\theta(\cot^2\theta + \csc^2\theta)$$

Evaluating these formulas at $\theta = 60°$, we get

$$\dot{\theta} = 0.5 \qquad r = 0.462$$

$$\ddot{\theta} = 0 \qquad \dot{r} = -0.133$$

$$\ddot{r} = 0.192$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0.192 - 0.462(0.5)^2 = 0.0770$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.133)(0.5) = -0.133$$

Substituting these results into Eqs. 1 and 2 with $\theta = 60°$ and solving yields

$$N_C = 19.4 \text{ N} \qquad F_P = -0.356 \text{ N} \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{F}_P$ acts opposite to the direction shown in Fig. 13–20b.

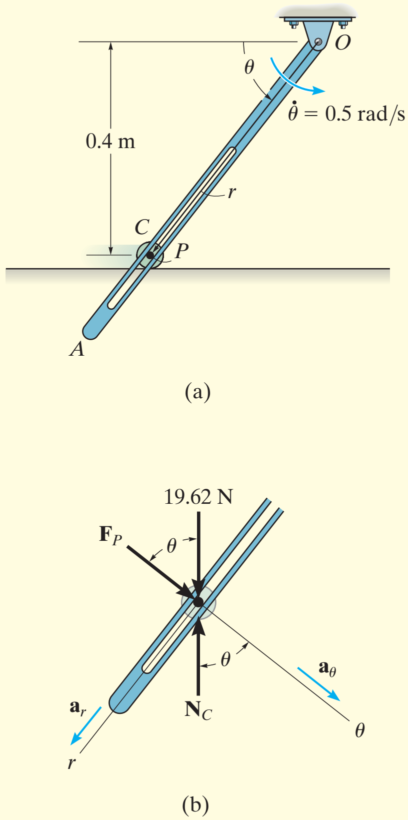
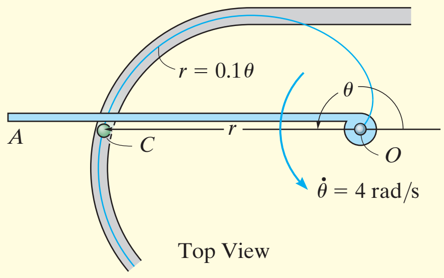


Fig. 13–20

EXAMPLE 13.12

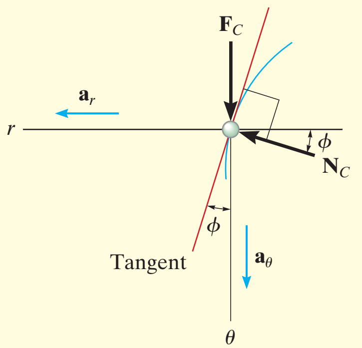


(a)

A can C, having a mass of 0.5 kg, moves along a grooved horizontal slot shown in Fig. 13–21*a*. The slot is in the form of a spiral, which is defined by the equation $r = (0.1\theta)$ m, where θ is in radians. If the arm OA rotates with a constant rate $\dot{\theta} = 4$ rad/s in the horizontal plane, determine the force it exerts on the can at the instant $\theta = \pi$ rad. Neglect friction and the size of the can.

SOLUTION

Free-Body Diagram. The driving force $\mathbf{F}_C$ acts perpendicular to the arm OA, whereas the normal force of the wall of the slot on the can, $\mathbf{N}_C$, acts perpendicular to the tangent to the curve at $\theta = \pi$ rad, Fig. 13–21*b*. As usual, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are assumed to act in the *positive directions* of r and θ, respectively. Since the path is specified, the angle ψ which the extended radial line r makes with the tangent, Fig. 13–21*c*, can be determined from Eq. 13–10. We have $r = 0.1\theta$, so that $dr/d\theta = 0.1$, and therefore



(b)

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.1\theta}{0.1} = \theta$$

When $\theta = \pi$, $\psi = \tan^{-1}\pi = 72.3°$, so that $\phi = 90° - \psi = 17.7°$, as shown in Fig. 13–21*c*. Identify the four unknowns in Fig. 13–21*b*.

Equations of Motion. Using $\phi = 17.7°$ and the data shown in Fig. 13–21*b*, we have

$$\xrightarrow{+} \Sigma F_r = ma_r; \qquad N_C \cos 17.7° = 0.5a_r \qquad (1)$$

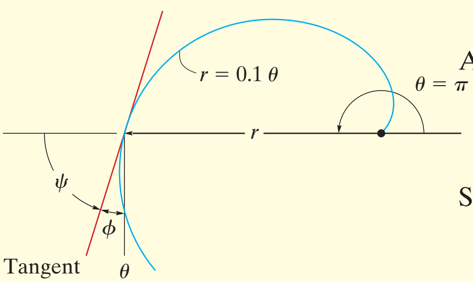
$$+\downarrow \Sigma F_\theta = ma_\theta; \qquad F_C - N_C \sin 17.7° = 0.5a_\theta \qquad (2)$$

Kinematics. The time derivatives of r and θ are

$$\dot{\theta} = 4 \text{ rad/s} \qquad r = 0.1\theta$$

$$\ddot{\theta} = 0 \qquad \dot{r} = 0.1\dot{\theta} = 0.1(4) = 0.4 \text{ m/s}$$

$$\ddot{r} = 0.1\ddot{\theta} = 0$$



(c)

Fig. 13–21

At the instant $\theta = \pi$ rad,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.1(\pi)(4)^2 = -5.03 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.4)(4) = 3.20 \text{ m/s}^2$$

Substituting these results into Eqs. 1 and 2 and solving yields

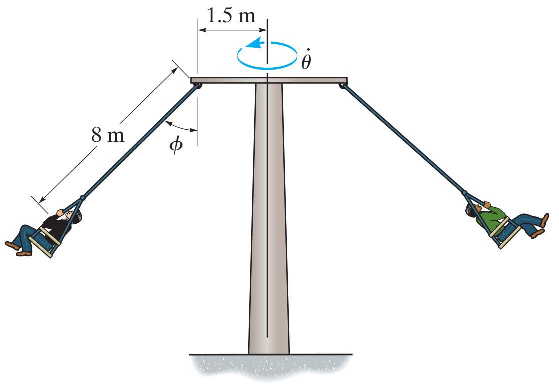
$$N_C = -2.64 \text{ N}$$

$$F_C = 0.800 \text{ N} \qquad \textit{Ans.}$$

What does the negative sign for N_C indicate?

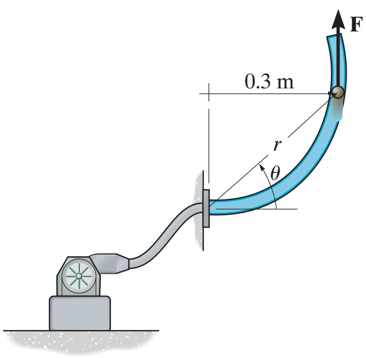
FUNDAMENTAL PROBLEMS

F13–13. Determine the constant angular velocity $\dot{\theta}$ of the vertical shaft of the amusement ride if $\phi = 45°$. Neglect the mass of the cables and the size of the passengers.



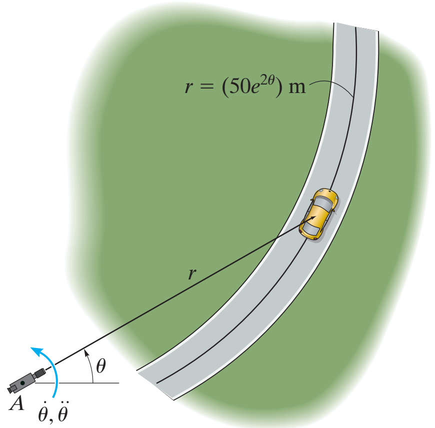
Prob. F13–13

F13–14. The 0.2-kg ball is blown through the smooth vertical circular tube whose shape is defined by $r = (0.6 \sin \theta)$ m, where θ is in radians. If $\theta = (\pi t^2)$ rad, where t is in seconds, determine the magnitude of force **F** exerted by the blower on the ball when $t = 0.5$ s.



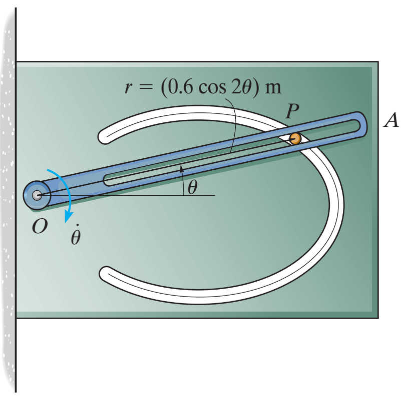
Prob. F13–14

F13–15. The 2-Mg car is traveling along the curved road described by $r = (50e^{2\theta})$ m, where θ is in radians. If a camera is located at A and it rotates with an angular velocity of $\dot{\theta} = 0.05$ rad/s and an angular acceleration of $\ddot{\theta} = 0.01$ rad/s^2 at the instant $\theta = \frac{\pi}{6}$ rad, determine the resultant friction force developed between the tires and the road at this instant.



Prob. F13–15

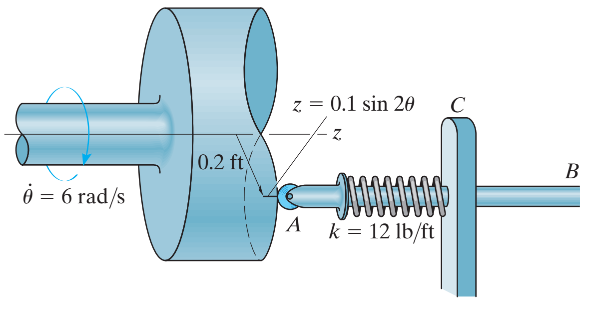
F13–16. The 0.2-kg pin P is constrained to move in the smooth curved slot, which is defined by the lemniscate $r = (0.6 \cos 2\theta)$ m. Its motion is controlled by the rotation of the slotted arm OA, which has a constant clockwise angular velocity of $\dot{\theta} = -3$ rad/s. Determine the force arm OA exerts on the pin P when $\theta = 0°$. Motion is in the vertical plane.



Prob. F13–16

PROBLEMS

13–85. The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin 2\theta)$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the force at the end A of the follower when $\theta = 45°$. In this position the spring is compressed 0.4 ft. Neglect friction at the bearing C.

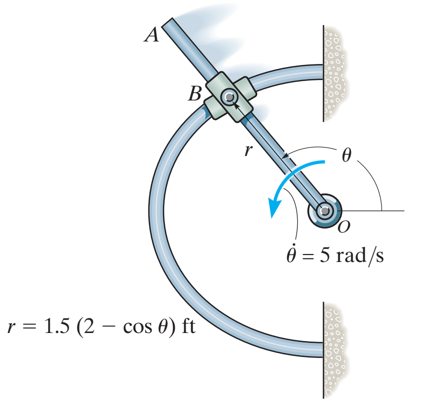


Prob. 13–85

13–86. Determine the magnitude of the resultant force acting on a 5-kg particle at the instant $t = 2$ s, if the particle is moving along a horizontal path defined by the equations $r = (2t + 10)$ m and $\theta = (1.5t^2 - 6t)$ rad, where t is in seconds.

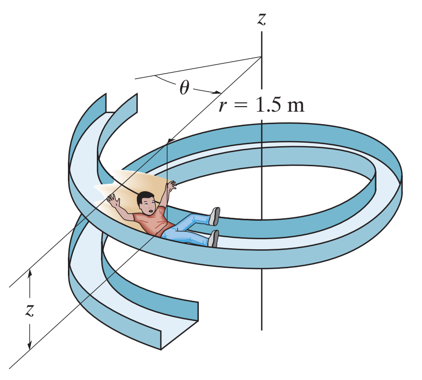
13–87. The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the unbalanced force acting on the particle when $t = 2$ s.

***13–88.** Rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other slides over the *horizontal* curved rod, of which the shape is described by the equation $r = 1.5(2 - \cos \theta)$ ft. If both collars weigh 0.75 lb, determine the normal force which the curved rod exerts on one collar at the instant $\theta = 120°$. Neglect friction.



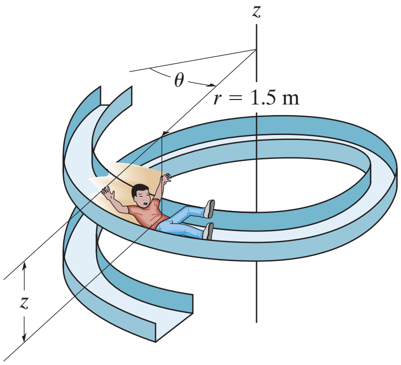
Prob. 13–88

13–89. The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.



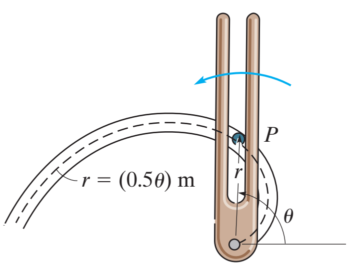
Prob. 13–89

13–90. The 40-kg boy is sliding down the smooth spiral slide such that $z = -2$ m/s and his speed is 2 m/s. Determine the r, θ, z components of force the slide exerts on him at this instant. Neglect the size of the boy.



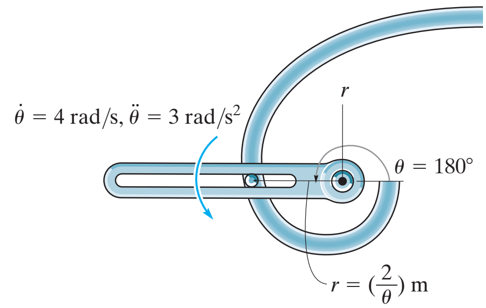
Prob. 13–90

13–91. Using a forked rod, a 0.5-kg smooth peg P is forced to move along the *vertical slotted* path $r = (0.5\,\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (\frac{\pi}{8}t^2)$ rad, where t is in seconds, determine the force of the rod on the peg and the normal force of the slot on the peg at the instant $t = 2$ s. The peg is in contact with only *one edge* of the rod and slot at any instant.



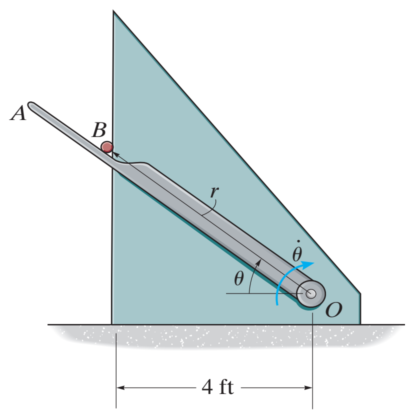
Prob. 13–91

***13–92.** The arm is rotating at a rate of $\dot{\theta} = 4$ rad/s when $\ddot{\theta} = 3$ rad/s^2 and $\theta = 180°$. Determine the force it must exert on the 0.5-kg smooth cylinder if it is confined to move along the slotted path. Motion occurs in the horizontal plane.



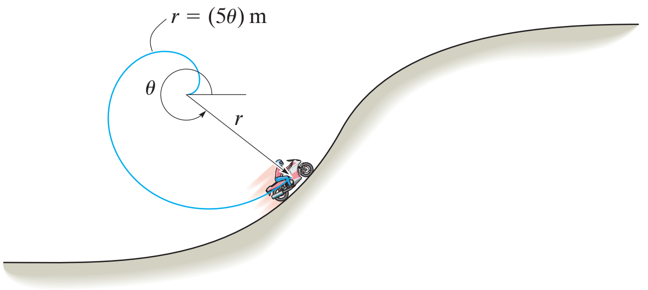
Prob. 13–92

13–93. If arm OA rotates with a constant clockwise angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the force arm OA exerts on the smooth 4-lb cylinder B when $\theta = 45°$.



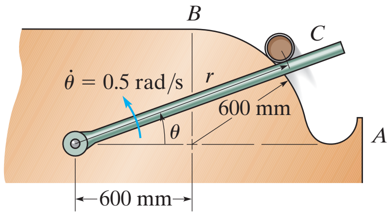
Prob. 13–93

13–94. Determine the normal and frictional driving forces that the partial spiral track exerts on the 200-kg motorcycle at the instant $\theta = \frac{5}{3}\pi$ rad, $\dot{\theta} = 0.4$ rad/s, $\ddot{\theta} = 0.8$ rad/s^2. Neglect the size of the motorcycle.



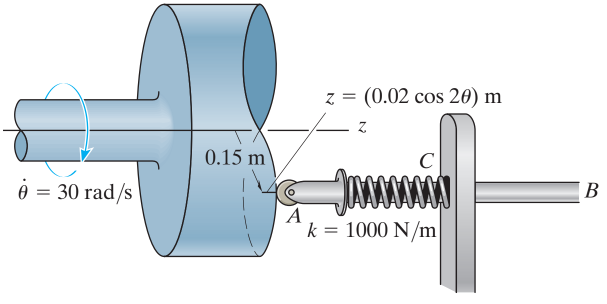
Prob. 13–94

13–95. A smooth can C, having a mass of 3 kg, is lifted from a feed at A to a ramp at B by a rotating rod. If the rod maintains a constant angular velocity of $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the can at the instant $\theta = 30°$. Neglect the effects of friction in the calculation and the size of the can so that $r = (1.2 \cos \theta)$ m. The ramp from A to B is circular, having a radius of 600 mm.



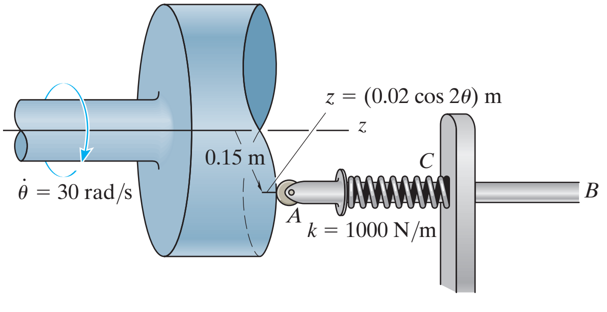
Prob. 13–95

***13–96.** The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the force component F_z at the end A of the follower when $\theta = 30°$. The spring is uncompressed when $\theta = 90°$. Neglect friction at the bearing C.



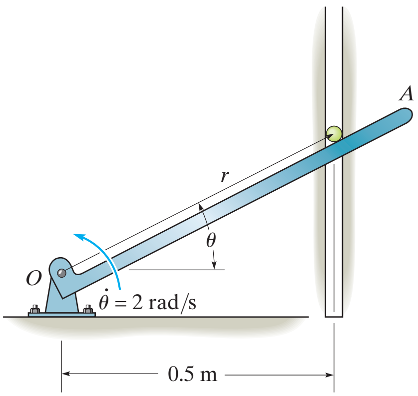
Prob. 13–96

13–97. The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the maximum and minimum force components F_z the follower exerts on the cam if the spring is uncompressed when $\theta = 90°$.



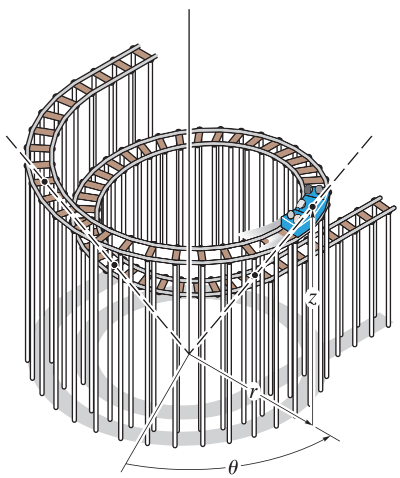
Prob. 13–97

13–98. The particle has a mass of 0.5 kg and is confined to move along the smooth vertical slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



Prob. 13–98

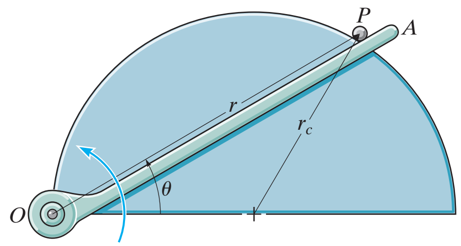
13–99. A car of a roller coaster travels along a track which for a short distance is defined by a conical spiral, $r = \frac{3}{4}z$, $\theta = -1.5z$, where r and z are in meters and θ in radians. If the angular motion $\dot{\theta} = 1$ rad/s is always maintained, determine the r, θ, z components of reaction exerted on the car by the track at the instant $z = 6$ m. The car and passengers have a total mass of 200 kg.



Prob. 13–99

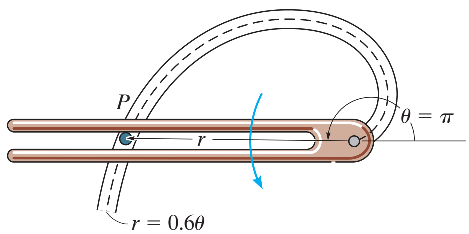
*__13–100.__ The 0.5-lb ball is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ ft.

13–101. The ball of mass m is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \leq 45°$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.



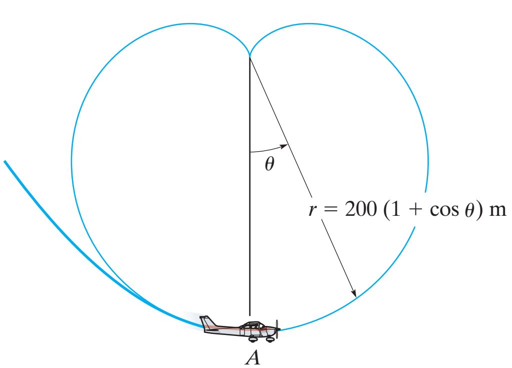
Probs. 13–100/101

13–102. Using a forked rod, a smooth cylinder P, having a mass of 0.4 kg, is forced to move along the *vertical slotted* path $r = (0.6\theta)$ m, where θ is in radians. If the cylinder has a constant speed of $v_C = 2$ m/s, determine the force of the rod and the normal force of the slot on the cylinder at the instant $\theta = \pi$ rad. Assume the cylinder is in contact with only *one edge* of the rod and slot at any instant. *Hint:* To obtain the time derivatives necessary to compute the cylinder's acceleration components a_r and a_θ, take the first and second time derivatives of $r = 0.6\theta$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v} = 0$ to determine $\ddot{\theta}$.



Prob. 13–102

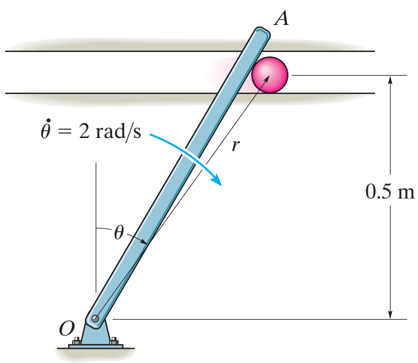
13–103. The pilot of the airplane executes a vertical loop which in part follows the path of a cardioid, $r = 200(1 + \cos\theta)$ m, where θ is in radians. If his speed at A is a constant $v_p = 85$ m/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He has a mass of 80 kg. *Hint:* To determine the time derivatives necessary to calculate the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 200(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$.



Prob. 13–103

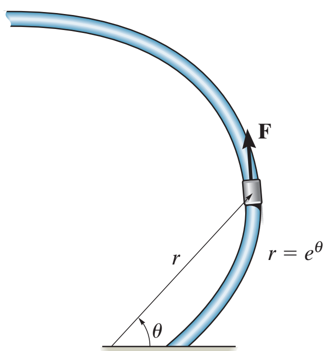
13–105. The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.

13–106. Solve Prob. 13–105 if the arm has an angular acceleration of $\ddot{\theta} = 3$ rad/s^2 when $\dot{\theta} = 2$ rad/s at $\theta = 30°$.



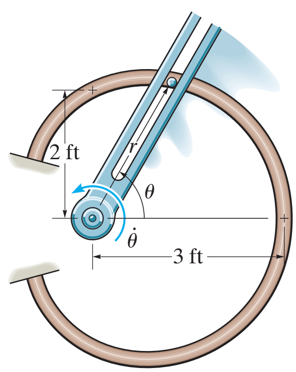
Probs. 13–105/106

***13–104.** The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^\theta)$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 45°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



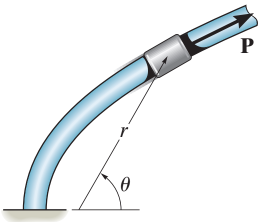
Prob. 13–104

13–107. The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.



Prob. 13–107

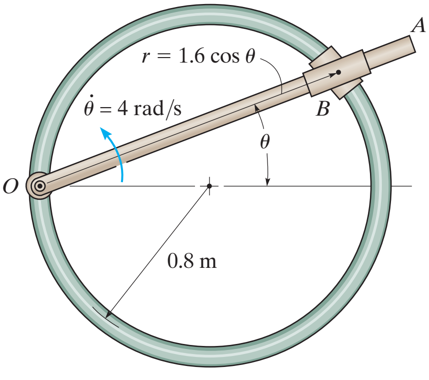
***13–108.** The collar, which has a weight of 3 lb, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = 4/(1 - \cos\theta)$, where θ is in radians and r is in feet. If the collar's angular rate is constant and equals $\dot{\theta} = 4$ rad/s, determine the tangential retarding force $\mathbf{P}$ needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = 90°$.



Prob. 13–108

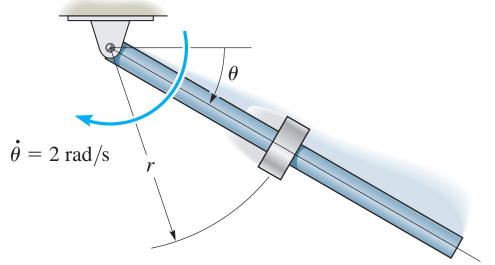
13–109. Rod OA rotates counterclockwise at a constant angular rate $\dot{\theta} = 4$ rad/s. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other collar slides over the circular rod described by the equation $r = (1.6\cos\theta)$ m. If *both* collars have a mass of 0.5 kg, determine the force which the circular rod exerts on one of the collars and the force that OA exerts on the other collar at the instant $\theta = 45°$. Motion is in the horizontal plane.

13–110. Solve Prob. 13–109 if motion is in the vertical plane.



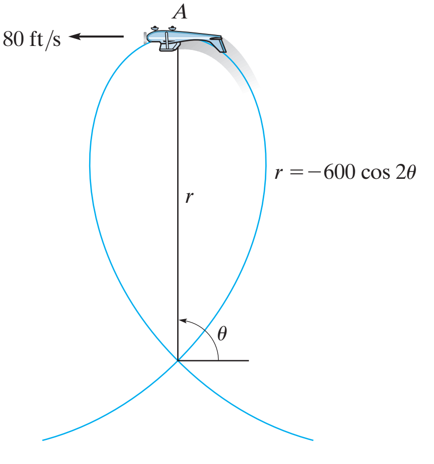
Probs. 13–109/110

13–111. A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81\sin\theta = 0$ and $0.8\dot{r} + N_s - 1.962\cos\theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1e^{-2t} + C_2e^{2t} - (9.81/8)\sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 determine r at the instant $\theta = \pi/4$ rad.

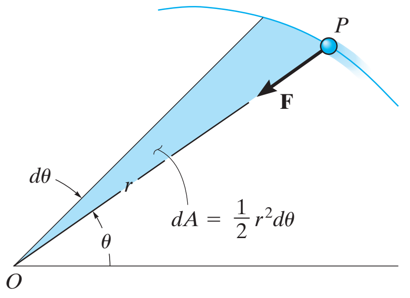


Prob. 13–111

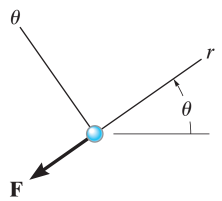
***13–112.** The pilot of an airplane executes a vertical loop which in part follows the path of a "four-leaved rose," $r = (-600\cos 2\theta)$ ft, where θ is in radians. If his speed is a constant $v_P = 80$ ft/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He weights 130 lb. *Hint*: To determine the time derivatives necessary to compute the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 400(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_P = 0$ to determine $\ddot{\theta}$.



Prob. 13–112



(a)



(b)

Fig. 13–22

*13.7 Central-Force Motion and Space Mechanics

If a particle is moving only under the influence of a force having a line of action which is always directed toward a fixed point, the motion is called *central-force motion*. This type of motion is commonly caused by electrostatic and gravitational forces.

In order to analyze the motion, we will consider the particle P shown in Fig. 13–22*a*, which has a mass m and is acted upon only by the central force $\mathbf{F}$. The free-body diagram for the particle is shown in Fig. 13–22*b*. Using polar coordinates (r, θ), the equations of motion, Eq. 13–9, become

$$\Sigma F_r = ma_r; \qquad -F = m\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right] \tag{13–11}$$

$$\Sigma F_\theta = ma_\theta; \qquad 0 = m\left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)$$

The second of these equations may be written in the form

$$\frac{1}{r}\left[\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right] = 0$$

so that integrating yields

$$r^2\frac{d\theta}{dt} = h \tag{13–12}$$

Here h is the constant of integration.

From Fig. 13–22*a* notice that the shaded area described by the radius r, as r moves through an angle $d\theta$, is $dA = \frac{1}{2}r^2\, d\theta$. If the *areal velocity* is defined as

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{h}{2} \tag{13–13}$$

then it is seen that the areal velocity for a particle subjected to central-force motion is *constant*. In other words, the particle will sweep out equal segments of area per unit of time as it travels along the path. To obtain the *path of motion*, $r = f(\theta)$, the independent variable t must be eliminated from Eqs. 13–11. Using the chain rule of calculus and Eq. 13–12, the time derivatives of Eqs. 13–11 may be replaced by

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{h}{r^2}\frac{dr}{d\theta}$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{h}{r^2}\frac{dr}{d\theta}\right) = \frac{d}{d\theta}\left(\frac{h}{r^2}\frac{dr}{d\theta}\right)\frac{d\theta}{dt} = \left[\frac{d}{d\theta}\left(\frac{h}{r^2}\frac{dr}{d\theta}\right)\right]\frac{h}{r^2}$$

Substituting a new dependent variable (xi) $\xi = 1/r$ into the second equation, we have

$$\frac{d^2r}{dt^2} = -h^2\xi^2\frac{d^2\xi}{d\theta^2}$$

Also, the square of Eq. 13–12 becomes

$$\left(\frac{d\theta}{dt}\right)^2 = h^2\xi^4$$

Substituting these two equations into the first of Eqs. 13–11 yields

$$-h^2\xi^2\frac{d^2\xi}{d\theta^2} - h^2\xi^3 = -\frac{F}{m}$$

or

$$\frac{d^2\xi}{d\theta^2} + \xi = \frac{F}{mh^2\xi^2} \qquad (13\text{–}14)$$

This differential equation defines the path over which the particle travels when it is subjected to the central force **F**.*

For application, the force of gravitational attraction will be considered. Some common examples of central-force systems which depend on gravitation include the motion of the moon and artificial satellites about the earth, and the motion of the planets about the sun. As a typical problem in space mechanics, consider the trajectory of a space satellite or space vehicle launched into free-flight orbit with an initial velocity $\mathbf{v}_0$, Fig. 13–23. It will be assumed that this velocity is initially *parallel* to the tangent at the surface of the earth, as shown in the figure.† Just after the satellite is released into free flight, the only force acting on it is the gravitational force of the earth. (Gravitational attractions involving other bodies such as the moon or sun will be neglected, since for orbits close to the earth their effect is small in comparison with the earth's gravitation.) According to Newton's law of gravitation, force **F** will always act between the mass centers of the earth and the satellite, Fig. 13–23. From Eq. 13–1, this force of attraction has a magnitude of

$$F = G\frac{M_e m}{r^2}$$

where M_e and m represent the mass of the earth and the satellite, respectively, G is the gravitational constant, and r is the distance between

This satellite is subjected to a central force and its orbital motion can be closely predicted using the equations developed in this section. (UniversalImagesGroup/Getty Images)

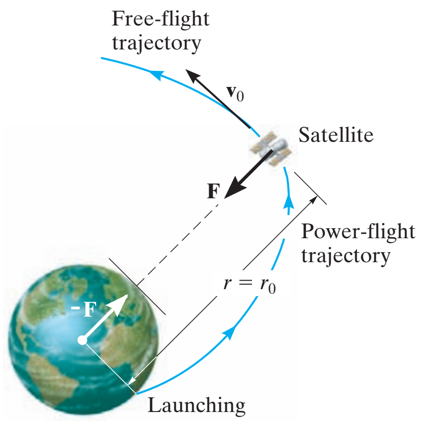


Fig. 13–23

*In the derivation, **F** is considered positive when it is directed toward point O. If **F** is oppositely directed, the right side of Eq. 13–14 should be negative.

†The case where $\mathbf{v}_0$ acts at some initial angle θ to the tangent is best described using the conservation of angular momentum.

the mass centers. To obtain the orbital path, we set $\xi = 1/r$ in the foregoing equation and substitute the result into Eq. 13–14. We obtain

$$\frac{d^2\xi}{d\theta^2} + \xi = \frac{GM_e}{h^2} \tag{13–15}$$

This second-order differential equation has constant coefficients and is nonhomogeneous. The solution is the sum of the complementary and particular solutions given by

$$\xi = \frac{1}{r} = C\cos(\theta - \phi) + \frac{GM_e}{h^2} \tag{13–16}$$

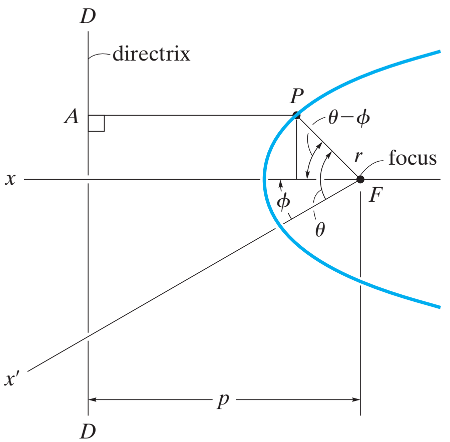


Fig. 13–24

This equation represents the *free-flight trajectory* of the satellite. It is the equation of a conic section expressed in terms of polar coordinates.

A geometric interpretation of Eq. 13–16 requires knowledge of the equation for a conic section. As shown in Fig. 13–24, a conic section is defined as the locus of a point P that moves in such a way that the ratio of its distance to a *focus*, or fixed point F, to its perpendicular distance to a fixed line DD called the *directrix*, is constant. This constant ratio will be denoted as e and is called the *eccentricity*. By definition

$$e = \frac{FP}{PA}$$

From Fig. 13–24,

$$FP = r = e(PA) = e[p - r\cos(\theta - \phi)]$$

or

$$\frac{1}{r} = \frac{1}{p}\cos(\theta - \phi) + \frac{1}{ep}$$

Comparing this equation with Eq. 13–16, it is seen that the fixed distance from the focus to the directrix is

$$p = \frac{1}{C} \tag{13–17}$$

And the eccentricity of the conic section for the trajectory is

$$\boxed{e = \frac{Ch^2}{GM_e}} \tag{13–18}$$

Provided the polar angle θ is measured from the x axis (an axis of symmetry since it is perpendicular to the directrix), the angle ϕ is zero, Fig. 13–24, and therefore Eq. 13–16 reduces to

$$\frac{1}{r} = C\cos\theta + \frac{GM_e}{h^2} \tag{13–19}$$

The constants h and C are determined from the data obtained for the position and velocity of the satellite at the end of the *power-flight trajectory.* For example, if the initial height or distance to the space vehicle is r_0, measured from the center of the earth, and its initial speed is v_0 at the beginning of its free flight, Fig. 13–25, then the constant h may be obtained from Eq. 13–12. When $\theta = \phi = 0°$, the velocity $\mathbf{v}_0$ has no radial component; therefore, from Eq. 12–25, $v_0 = r_0(d\theta/dt)$, so that

$$h = r_0^2\frac{d\theta}{dt}$$

or

$$\boxed{h = r_0 v_0} \tag{13–20}$$

To determine C, use Eq. 13–19 with $\theta = 0°$, $r = r_0$, and substitute Eq. 13–20 for h:

$$\boxed{C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)} \tag{13–21}$$

The equation for the free-flight trajectory therefore becomes

$$\boxed{\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\cos\theta + \frac{GM_e}{r_0^2 v_0^2}} \tag{13–22}$$

The type of path traveled by the satellite is determined from the value of the eccentricity of the conic section as given by Eq. 13–18. If

$$\boxed{\begin{array}{ll} e = 0 & \text{free-flight trajectory is a circle} \\ e = 1 & \text{free-flight trajectory is a parabola} \\ e < 1 & \text{free-flight trajectory is an ellipse} \\ e > 1 & \text{free-flight trajectory is a hyperbola} \end{array}} \tag{13–23}$$

Parabolic Path. Each of these trajectories is shown in Fig. 13–25. From the curves it is seen that when the satellite follows a parabolic path, it is "on the border" of never returning to its initial starting point. The initial launch velocity, $\mathbf{v}_0$, required for the satellite to follow a parabolic path is called the *escape velocity.* The speed, v_e, can be determined by using the second of Eqs. 13–23, $e = 1$, with Eqs. 13–18, 13–20, and 13–21. It is left as an exercise to show that

$$v_e = \sqrt{\frac{2GM_e}{r_0}} \tag{13–24}$$

Circular Orbit. The speed v_c required to launch a satellite into a *circular orbit* can be found using the first of Eqs. 13–23, $e = 0$. Since e is related to h and C, Eq. 13–18, C must be zero to satisfy this equation (from Eq. 13–20, h cannot be zero); and therefore, using Eq. 13–21, we have

$$v_c = \sqrt{\frac{GM_e}{r_0}} \tag{13–25}$$

Provided r_0 represents a minimum height for launching, in which frictional resistance from the atmosphere is neglected, speeds at launch which are less than v_c will cause the satellite to reenter the earth's atmosphere and either burn up or crash, Fig. 13–25.

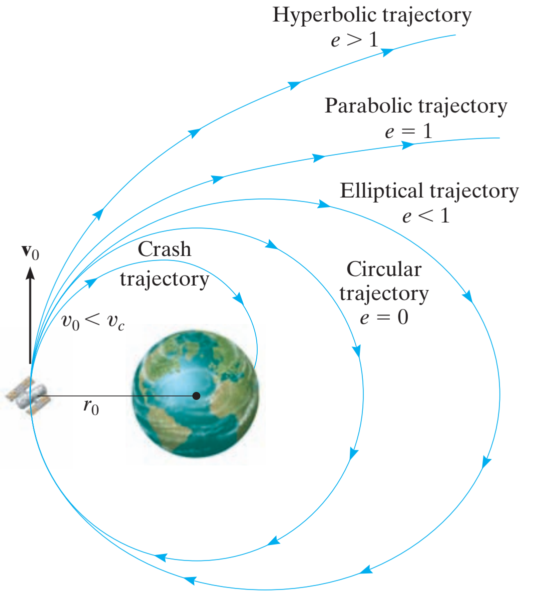


Fig. 13–25

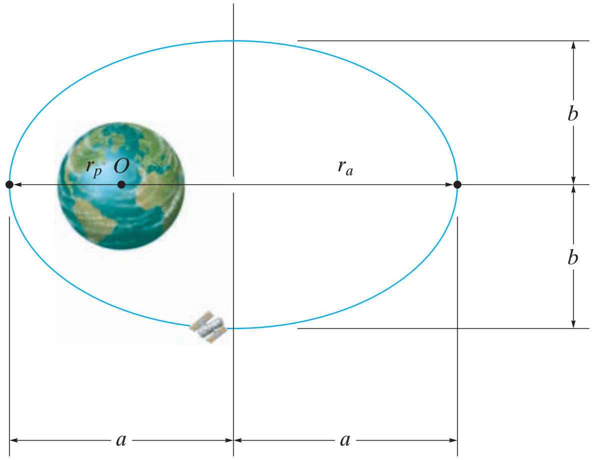


Fig. 13–26

Elliptical Orbit. All the trajectories attained by planets and most satellites are elliptical, Fig. 13–26. For a satellite's orbit about the earth, the *minimum distance* from the orbit to the center of the earth O (which is located at one of the foci of the ellipse) is r_p and can be found using Eq. 13–22 with $\theta = 0°$. Therefore;

$$\boxed{r_p = r_0} \tag{13–26}$$

This minimum distance is called the *perigee* of the orbit. The *apogee* or maximum distance r_a can be found using Eq. 13–22 with $\theta = 180°$.* Thus,

$$\boxed{r_a = \frac{r_0}{(2GM_e/r_0 v_0^2) - 1}} \tag{13–27}$$

With reference to Fig. 13–26, the half-length of the major axis of the ellipse is

$$a = \frac{r_p + r_a}{2} \tag{13–28}$$

Using analytical geometry, it can be shown that the half-length of the minor axis is determined from the equation

$$b = \sqrt{r_p r_a} \tag{13–29}$$

*Actually, the terminology perigee and apogee pertains only to orbits about the *earth*. If any other heavenly body is located at the focus of an elliptical orbit, the minimum and maximum distances are referred to respectively as the *periapsis* and *apoapsis* of the orbit.

Furthermore, by direct integration, the area of an ellipse is

$$A = \pi ab = \frac{\pi}{2}(r_p + r_a)\sqrt{r_p r_a} \tag{13–30}$$

The areal velocity has been defined by Eq. 13–13, $dA/dt = h/2$. Integrating yields $A = hT/2$, where T is the *period* of time required to make one orbital revolution. From Eq. 13–30, the period is

$$\boxed{T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}} \tag{13–31}$$

In addition to predicting the orbital trajectory of earth satellites, the theory developed in this section is valid, to a surprisingly close approximation, at predicting the actual motion of the planets traveling around the sun. In this case the mass of the sun, M_s, should be substituted for M_e when the appropriate formulas are used.

The fact that the planets do indeed follow elliptic orbits about the sun was discovered by the German astronomer Johannes Kepler in the early seventeenth century. His discovery was made *before* Newton had developed the laws of motion and the law of gravitation, and so at the time it provided important proof as to the validity of these laws. Kepler's laws, developed after 20 years of planetary observation, are summarized as follows:

1. Every planet travels in its orbit such that the line joining it to the center of the sun sweeps over equal areas in equal intervals of time, whatever the line's length.

2. The orbit of every planet is an ellipse with the sun placed at one of its foci.

3. The square of the period of any planet is directly proportional to the cube of the major axis of its orbit.

A mathematical statement of the first and second laws is given by Eqs. 13–13 and 13–22, respectively. The third law can be shown from Eq. 13–31 using Eqs. 13–19, 13–28, and 13–29. (See Prob. 13–117.)

PROBLEMS

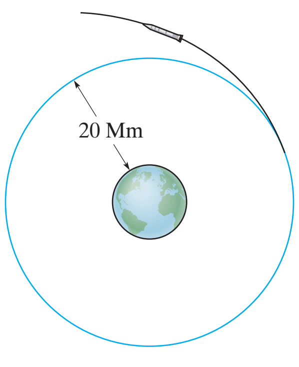
In the following problems, except where otherwise indicated, assume that the radius of the earth is 6378 km, the earth's mass is $5.976(10^{24})$ kg, the mass of the sun is $1.99(10^{30})$ kg, and the gravitational constant is $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg} \cdot \text{s}^2)$.

13–113. The earth has an orbit with eccentricity 0.0167 around the sun. Knowing that the earth's minimum distance from the sun is $146(10^6)$ km, find the speed at which the earth travels when it is at this distance. Determine the equation in polar coordinates which describes the earth's orbit about the sun.

13–114. A communications satellite is in a circular orbit above the earth such that it always remains directly over a point on the earth's surface. As a result, the period of the satellite must equal the rotation of the earth, which is approximately 24 hours. Determine the satellite's altitude h above the earth's surface and its orbital speed.

13–115. The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

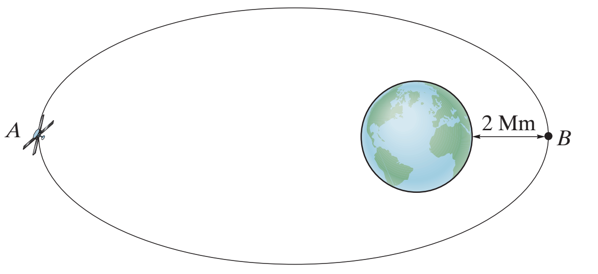
***13–116.** The rocket is in circular orbit about the earth at an altitude of 20 Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



Prob. 13–116

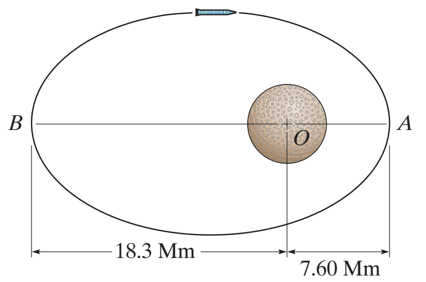
13–117. Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

13–118. The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



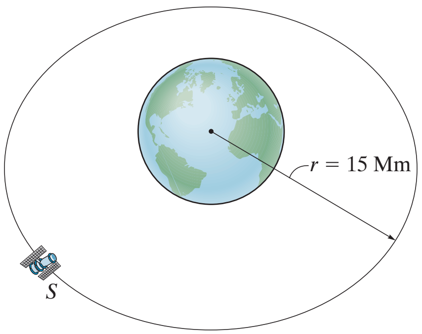
Prob. 13–118

13–119. The rocket is traveling in free flight along the elliptical orbit. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's speed when it is at A and at B.



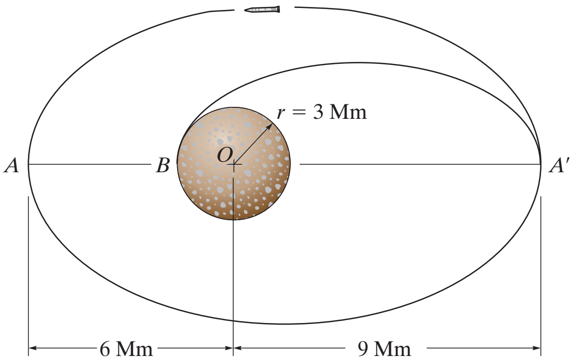
Prob. 13–119

***13–120.** Determine the constant speed of satellite S so that it circles the earth with an orbit of radius $r = 15$ Mm. *Hint:* Use Eq. 13–1.



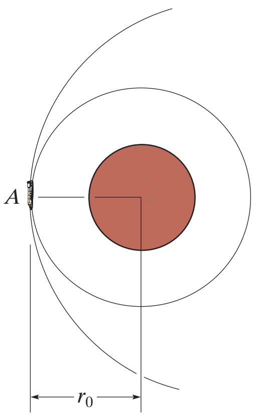
Prob. 13–120

13–121. The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.



Prob. 13–121

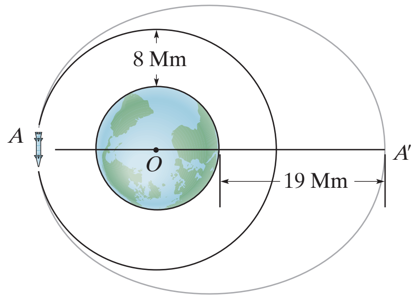
13–122. The Viking Explorer approaches the planet Mars on a parabolic trajectory as shown. When it reaches point A its velocity is 10 Mm/h. Determine r_0 and the required change in velocity at A so that it can then maintain a circular orbit as shown. The mass of Mars is 0.1074 times the mass of the earth.



Prob. 13–122

13–123. The rocket is initially in free-flight circular orbit around the earth. Determine the speed of the rocket at A. What change in the speed at A is required so that it can move in an elliptical orbit to reach point A'?

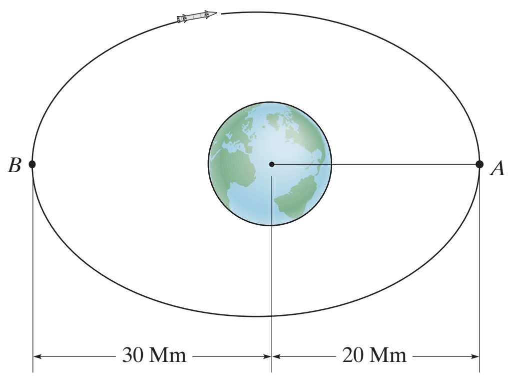
***13–124.** The rocket is in free-flight circular orbit around the earth. Determine the time needed for the rocket to travel from the innner orbit at A to the outer orbit at A'.



Probs. 13–123/124

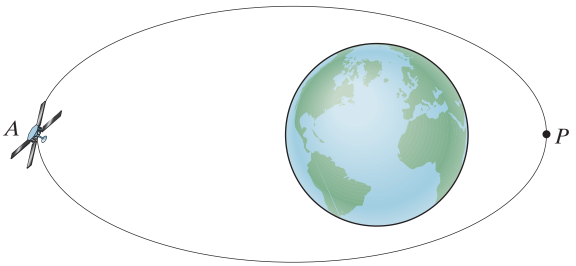
13–125. A satellite is launched at its apogee with an initial velocity $v_0 = 2500$ mi/h parallel to the surface of the earth. Determine the required altitude (or range of altitudes) above the earth's surface for launching if the free-flight trajectory is to be (a) circular, (b) parabolic, (c) elliptical, with launch at apogee, and (d) hyperbolic. Take $G = 34.4(10^{-9})(\text{lb}\cdot\text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, the earth's radius $r_e = 3960$ mi, and 1 mi = 5280 ft.

13–126. The rocket is traveling around the earth in free flight along the elliptical orbit. If the rocket has the orbit shown, determine the speed of the rocket when it is at A and at B.



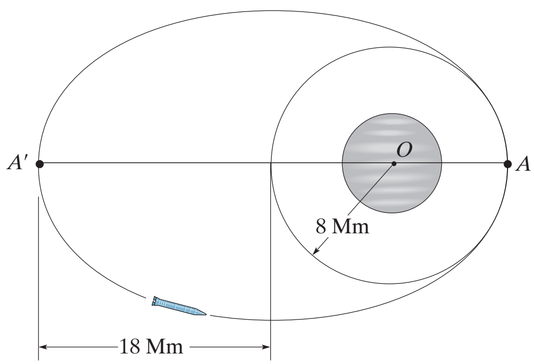
Prob. 13–126

13–127. An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?



Prob. 13–127

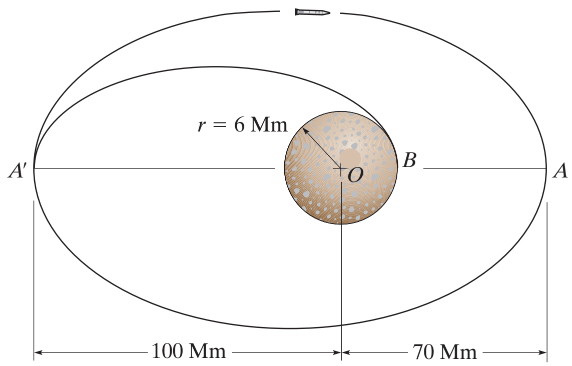
***13–128.** A rocket is in free-flight elliptical orbit around the planet Venus. Knowing that the periapsis and apoapsis of the orbit are 8 Mm and 26 Mm, respectively, determine (a) the speed of the rocket at point A', (b) the required speed it must attain at A just after braking so that it undergoes an 8-Mm free-flight circular orbit around Venus, and (c) the periods of both the circular and elliptical orbits. The mass of Venus is 0.816 times the mass of the earth.



Prob. 13–128

13–129. The rocket is traveling in a free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.

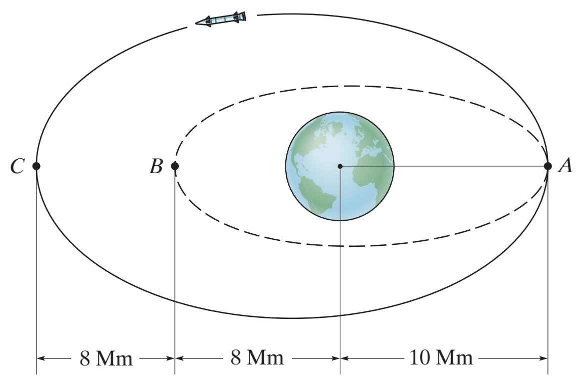
13–130. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth.



Probs. 13–129/130

13–131. The rocket is traveling around the earth in free flight along an elliptical orbit AC. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.

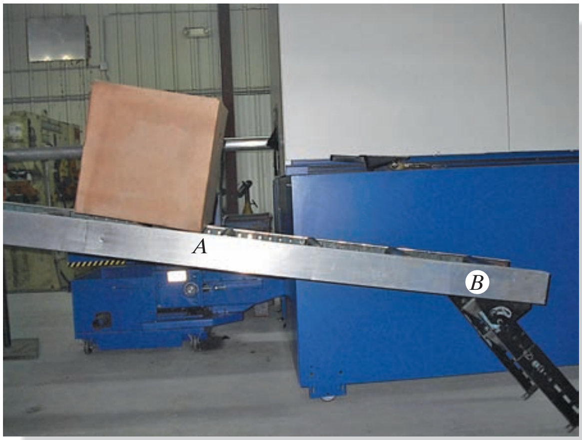
***13–132.** The rocket is traveling around the earth in free flight along the elliptical orbit AC. Determine its change in speed when it reaches A so that it travels along the elliptical orbit AB.



Probs. 13–131/132

CONCEPTUAL PROBLEMS

C13–1. If the box is released from rest at A, use numerical values to show how you would estimate the time for it to arrive at B. Also, list the assumptions for your analysis.



Prob. C13–1 (© R.C. Hibbeler)

C13–2. The tugboat has a known mass and its propeller provides a known maximum thrust. When the tug is fully powered you observe the time it takes for the tug to reach a speed of known value starting from rest. Show how you could determine the mass of the barge. Neglect the drag force of the water on the tug. Use numerical values to explain your answer.

Prob. C13–2 (© R.C. Hibbeler)

C13–3. Determine the smallest speed of each car A and B so that the passengers do not lose contact with the seat while the arms turn at a constant rate. What is the largest normal force of the seat on each passenger? Use numerical values to explain your answer.

Prob. C13–3 (© R.C. Hibbeler)

C13–4. Each car is pin connected at its ends to the rim of the wheel which turns at a constant speed. Using numerical values, show how to determine the resultant force the seat exerts on the passenger located in the top car A. The passengers are seated toward the center of the wheel. Also, list the assumptions for your analysis.

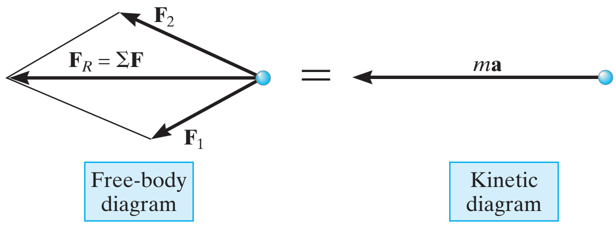
Prob. C13–4 (© R.C. Hibbeler)

CHAPTER REVIEW

Kinetics

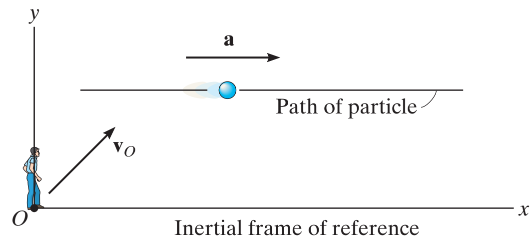
Kinetics is the study of the relation between forces and the acceleration they cause. This relation is based on Newton's second law of motion, expressed mathematically as $\Sigma \mathbf{F} = m\mathbf{a}$.

Before applying the equation of motion, it is important to first draw the particle's *free-body diagram* in order to account for all of the forces that act on the particle. Graphically, this diagram is equal to the *kinetic diagram*, which shows the result of the forces, that is, the $m\mathbf{a}$ vector.



Inertial Coordinate Systems

When applying the equation of motion, it is important to measure the acceleration from an inertial coordinate system. This system has axes that do not rotate but are either fixed or translate with a constant velocity. Various types of inertial coordinate systems can be used to apply $\Sigma \mathbf{F} = m\mathbf{a}$ in component form.



Rectangular x, y, z axes are used to describe the motion along each of the axes.

$$\Sigma F_x = ma_x,\ \Sigma F_y = ma_y,\ \Sigma F_z = ma_z$$

Normal, tangential, and binormal axes n, t, b, are often used when the path is known. Recall that $\mathbf{a}_n$ is always directed in the $+n$ direction. It indicates the change in the velocity direction. Also recall that $\mathbf{a}_t$ is tangent to the path. It indicates the change in the velocity magnitude.

$$\Sigma F_t = ma_t,\ \Sigma F_n = ma_n,\ \Sigma F_b = 0$$

$$a_t = dv/dt \quad \text{or} \quad a_t = v\,dv/ds$$

$$a_n = v^2/\rho \qquad \text{where} \quad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$

Cylindrical coordinates are useful when angular motion of the radial line r is specified or when the path can conveniently be described with these coordinates.

$$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

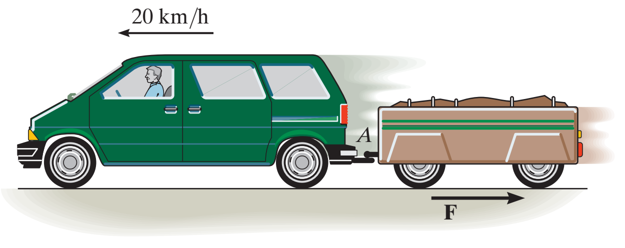
$$\Sigma F_z = m\ddot{z}$$

Central-Force Motion

When a single force acts upon a particle, such as during the free-flight trajectory of a satellite in a gravitational field, then the motion is referred to as central-force motion. The orbit depends upon the eccentricity e; and as a result, the trajectory can either be circular, parabolic, elliptical, or hyperbolic.

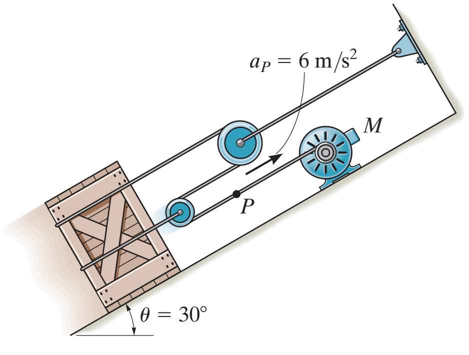
REVIEW PROBLEMS

R13–1. The van is traveling at 20 km/h when the coupling of the trailer at A fails. If the trailer has a mass of 250 kg and coasts 45 m before coming to rest, determine the constant horizontal force F created by rolling friction which causes the trailer to stop.



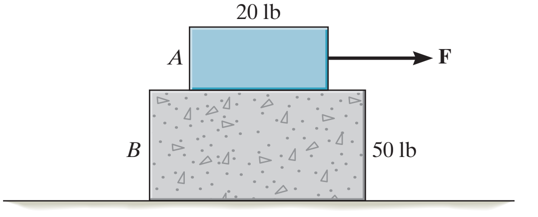
Prob. R13–1

R13–2. The motor M pulls in its attached rope with an acceleration $a_p = 6\ \text{m/s}^2$. Determine the towing force exerted by M on the rope in order to move the 50-kg crate up the inclined plane. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$. Neglect the mass of the pulleys and rope.



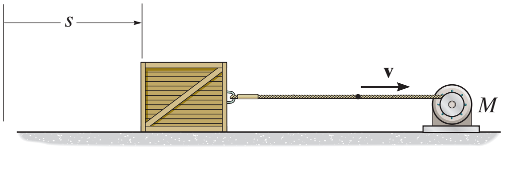
Prob. R13–2

R13–3. Block B rests on a smooth surface. If the coefficients of friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$, determine the acceleration of each block if $F = 50$ lb.



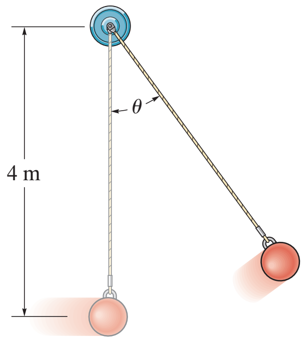
Prob. R13–3

R13–4. If the motor draws in the cable at a rate of $v = (0.05s^{3/2})$ m/s, where s is in meters, determine the tension developed in the cable when $s = 10$ m. The crate has a mass of 20 kg, and the coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



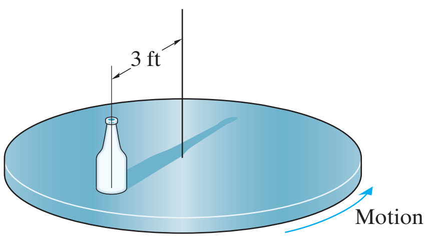
Prob. R13–4

R13–5. The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



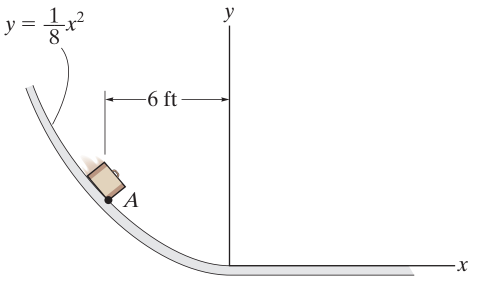
Prob. R13–5

R13–6. The bottle rests at a distance of 3 ft from the center of the horizontal platform. If the coefficient of static friction between the bottle and the platform is $\mu_s = 0.3$, determine the maximum speed that the bottle can attain before slipping. Assume the angular motion of the platform is slowly increasing.



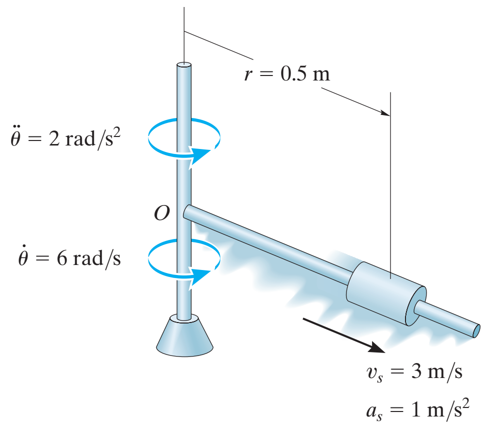
Prob. R13–6

R13–7. The 10-lb suitcase slides down the curved ramp for which the coefficient of kinetic friction is $\mu_k = 0.2$. If at the instant it reaches point A it has a speed of 5 ft/s, determine the normal force on the suitcase and the rate of increase of its speed.



Prob. R13–7

R13–8. The spool, which has a mass of 4 kg, slides along the rotating rod. At the instant shown, the angular rate of rotation of the rod is $\dot{\theta} = 6$ rad/s and this rotation is increasing at $\ddot{\theta} = 2$ rad/s^2. At this same instant, the spool has a velocity of 3 m/s and an acceleration of 1 m/s^2, both measured relative to the rod and directed away from the center O when $r = 0.5$ m. Determine the radial frictional force and the normal force, both exerted by the rod on the spool at this instant.



Prob. R13–8

Chapter 14

(© Oliver Furrer/Ocean/Corbis)

As the woman falls, her energy will have to be absorbed by the bungee cord. The principles of work and energy can be used to predict the motion.

Kinetics of a Particle: Work and Energy

CHAPTER OBJECTIVES

- To develop the principle of work and energy and apply it to solve problems that involve force, velocity, and displacement.
- To study problems that involve power and efficiency.
- To introduce the concept of a conservative force and apply the theorem of conservation of energy to solve kinetic problems.

14.1 The Work of a Force

In this chapter, we will analyze motion of a particle using the concepts of work and energy. The resulting equation will be useful for solving problems that involve force, velocity, and displacement. Before we do this, however, we must first define the work of a force. Specifically, a force **F** will do *work* on a particle only when the particle undergoes a *displacement in the direction of the force*. For example, if the force **F** in Fig. 14–1 causes the particle to move along the path s from position **r** to a new position **r**′, the displacement is then $d\mathbf{r} = \mathbf{r}' - \mathbf{r}$. The magnitude of $d\mathbf{r}$ is ds, the length of the differential segment along the path. If the angle between the tails of $d\mathbf{r}$ and **F** is θ, Fig. 14–1, then the work done by **F** is a *scalar quantity*, defined by

$$dU = F\,ds\cos\theta$$

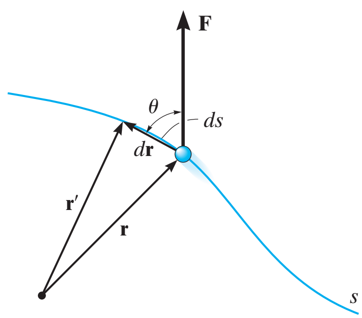


Fig. 14–1

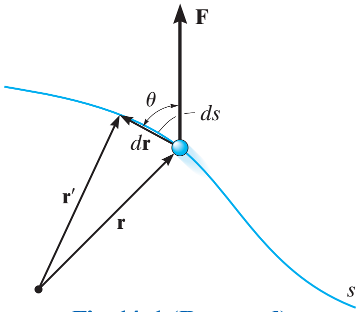


Fig. 14–1 (Repeated)

By definition of the dot product (see Eq. B–14) this equation can also be written as

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

This result may be interpreted in one of two ways: either as the product of F and the component of displacement $ds \cos \theta$ in the direction of the force, or as the product of ds and the component of force, $F \cos \theta$, in the direction of displacement. Note that if $0° \leq \theta < 90°$, then the force component and the displacement have the *same sense* so that the work is *positive;* whereas if $90° < \theta \leq 180°$, these vectors will have *opposite sense*, and therefore the work is *negative*. Also, $dU = 0$ if the force is *perpendicular* to displacement, since $\cos 90° = 0$, or if the force is applied at a *fixed point*, in which case the displacement is zero.

The unit of work in SI units is the joule (J), which is the amount of work done by a one-newton force when it moves through a distance of one meter in the direction of the force (1 J = 1 N · m). In the FPS system, work is measured in units of foot-pounds (ft · lb), which is the work done by a one-pound force acting through a distance of one foot in the direction of the force.*

Work of a Variable Force. If the particle acted upon by the force $\mathbf{F}$ undergoes a finite displacement along its path from $\mathbf{r}_1$ to $\mathbf{r}_2$ or s_1 to s_2, Fig. 14–2*a*, the work of force $\mathbf{F}$ is determined by integration. Provided $\mathbf{F}$ and θ can be expressed as a function of position, then

$$\boxed{U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} F \cos \theta \, ds} \tag{14–1}$$

Sometimes, this relation may be obtained by using experimental data to plot a graph of $F \cos \theta$ vs. s. Then the area under this graph bounded by s_1 and s_2 represents the total work, Fig. 14–2*b*.

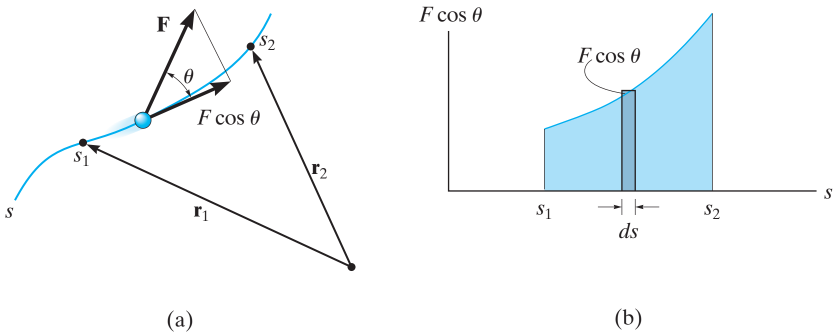


Fig. 14–2

*By convention, the units for the moment of a force or torque are written as lb · ft, to distinguish them from those used to signify work, ft · lb.

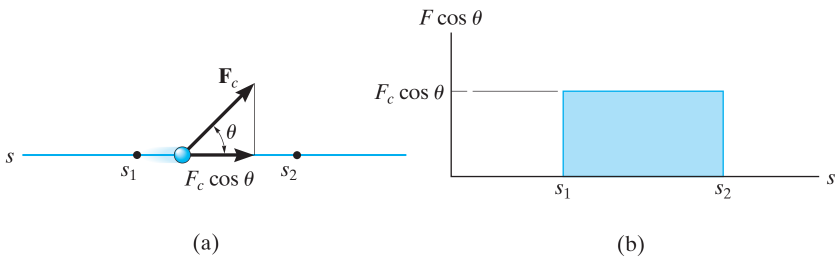


Fig. 14–3

Work of a Constant Force Moving Along a Straight Line.

If the force $\mathbf{F}_c$ has a constant magnitude and acts at a constant angle θ from its straight-line path, Fig. 14–3*a*, then the component of $\mathbf{F}_c$ in the direction of displacement is always $F_c \cos\theta$. The work done by $\mathbf{F}_c$ when the particle is displaced from s_1 to s_2 is determined from Eq. 14–1, in which case

$$U_{1-2} = F_c \cos\theta \int_{s_1}^{s_2} ds$$

or

$$\boxed{U_{1-2} = F_c \cos\theta(s_2 - s_1)} \qquad (14\text{–}2)$$

Here the work of $\mathbf{F}_c$ represents the *area of the rectangle* in Fig. 14–3*b*.

The crane must do work in order to hoist the weight of the pipe. (© R.C. Hibbeler)

Work of a Weight.

Consider a particle of weight $\mathbf{W}$, which moves up along the path s shown in Fig. 14–4 from position s_1 to position s_2. At an intermediate point, the displacement $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$. Since $\mathbf{W} = -W\mathbf{j}$, applying Eq. 14–1 we have

$$U_{1-2} = \int \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} (-W\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$= \int_{y_1}^{y_2} -W\,dy = -W(y_2 - y_1)$$

or

$$\boxed{U_{1-2} = -W\,\Delta y} \qquad (14\text{–}3)$$

Thus, the work is independent of the path and is equal to the magnitude of the particle's weight times its vertical displacement. In the case shown in Fig. 14–4 the work is *negative*, since W is downward and Δy is upward. Note, however, that if the particle is displaced *downward* $(-\Delta y)$, the work of the weight is *positive*. Why?

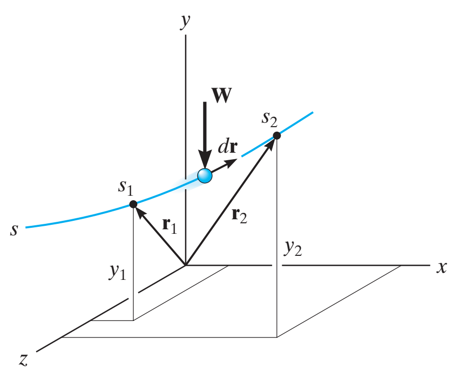


Fig. 14–4

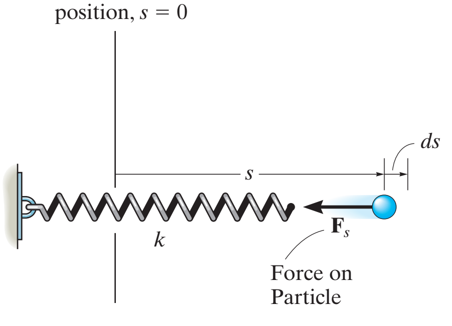
Work of a Spring Force. If an elastic spring is elongated a distance ds, Fig. 14–5a, then the work done by the force that acts on the attached particle is $dU = -F_s ds = -ks\,ds$. The work is *negative* since $\mathbf{F}_s$ acts in the opposite sense to ds. If the particle displaces from s_1 to s_2, the work of $\mathbf{F}_s$ is then

$$U_{1-2} = \int_{s_1}^{s_2} F_s\,ds = \int_{s_1}^{s_2} -ks\,ds$$

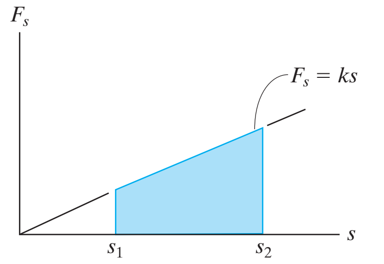
$$\boxed{U_{1-2} = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)} \qquad (14\text{–}4)$$

This work represents the trapezoidal area under the line $F_s = ks$, Fig. 14–5b.

A mistake in sign can be avoided when applying this equation if one simply notes the direction of the spring force acting on the particle and compares it with the sense of direction of displacement of the particle—if both are in the *same sense, positive work* results; if they are *opposite* to one another, the *work is negative.*

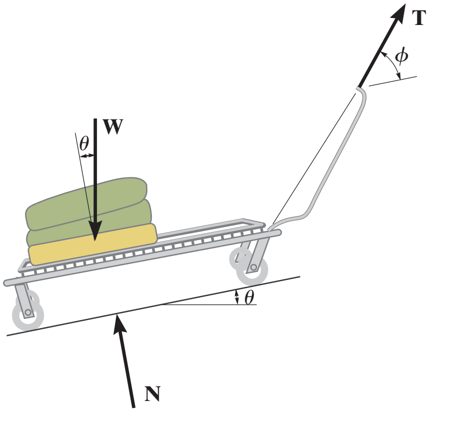


(a)



(b)

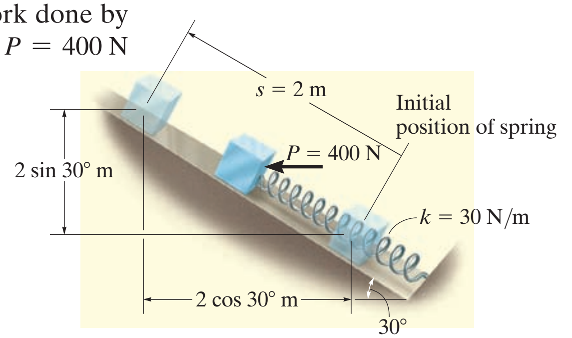
Fig. 14–5



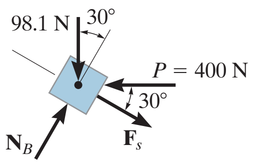
The forces acting on the cart, as it is pulled a distance s up the incline, are shown on its free-body diagram. The constant towing force $\mathbf{T}$ does positive work of $U_T = (T\cos\phi)s$, the weight does negative work of $U_W = -(W\sin\theta)s$, and the normal force $\mathbf{N}$ does no work since there is no displacement of this force along its line of action. (© R.C. Hibbeler)

EXAMPLE 14.1

The 10-kg block shown in Fig. 14–6*a* rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force $P = 400$ N pushes the block up the plane $s = 2$ m.



(a)



(b)

Fig. 14–6

SOLUTION

First the free-body diagram of the block is drawn in order to account for all the forces that act on the block, Fig. 14–6*b*.

Horizontal Force P. Since this force is *constant*, the work is determined using Eq. 14–2. The result can be calculated as the force times the component of displacement in the direction of the force; i.e.,

$$U_P = 400 \text{ N } (2 \text{ m} \cos 30^\circ) = 692.8 \text{ J}$$

or the displacement times the component of force in the direction of displacement, i.e.,

$$U_P = 400 \text{ N} \cos 30^\circ (2 \text{ m}) = 692.8 \text{ J}$$

Spring Force F_s. In the initial position the spring is stretched $s_1 = 0.5$ m and in the final position it is stretched $s_2 = 0.5 \text{ m} + 2 \text{ m} = 2.5$ m. We require the work to be negative since the force and displacement are opposite to each other. The work of $\mathbf{F}_s$ is thus

$$U_s = -\left[\tfrac{1}{2}(30 \text{ N/m})(2.5 \text{ m})^2 - \tfrac{1}{2}(30 \text{ N/m})(0.5 \text{ m})^2\right] = -90 \text{ J}$$

Weight W. Since the weight acts in the opposite sense to its vertical displacement, the work is negative; i.e.,

$$U_W = -(98.1 \text{ N})\,(2 \text{ m} \sin 30^\circ) = -98.1 \text{ J}$$

Note that it is also possible to consider the component of weight in the direction of displacement; i.e.,

$$U_W = -(98.1 \sin 30^\circ \text{ N})\,(2 \text{ m}) = -98.1 \text{ J}$$

Normal Force N_B. This force does *no work* since it is *always* perpendicular to the displacement.

Total Work. The work of all the forces when the block is displaced 2 m is therefore

$$U_T = 692.8 \text{ J} - 90 \text{ J} - 98.1 \text{ J} = 505 \text{ J} \qquad \textit{Ans.}$$

14.2 Principle of Work and Energy

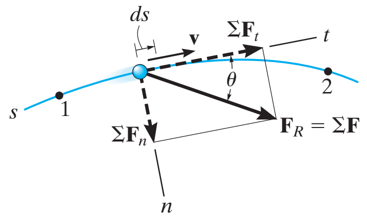


Fig. 14–7

Consider the particle in Fig. 14–7, which is located on the path defined relative to an inertial coordinate system. If the particle has a mass m and is subjected to a system of external forces represented by the resultant $\mathbf{F}_R = \Sigma\mathbf{F}$, then the equation of motion for the particle in the tangential direction is $\Sigma F_t = ma_t$. Applying the kinematic equation $a_t = v\,dv/ds$ and integrating both sides, assuming initially that the particle has a position $s = s_1$ and a speed $v = v_1$, and later at $s = s_2$, $v = v_2$, we have

$$\Sigma\int_{s_1}^{s_2} F_t\,ds = \int_{v_1}^{v_2} mv\,dv$$

$$\Sigma\int_{s_1}^{s_2} F_t\,ds = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \qquad (14\text{–}5)$$

From Fig. 14–7, note that $\Sigma F_t = \Sigma F\cos\theta$, and since work is defined from Eq. 14–1, the final result can be written as

$$\Sigma U_{1-2} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \qquad (14\text{–}6)$$

This equation represents the *principle of work and energy* for the particle. The term on the left is the sum of the work done by *all* the forces acting on the particle as the particle moves from point 1 to point 2. The two terms on the right side, which are of the form $T = \frac{1}{2}mv^2$, define the particle's final and initial *kinetic energy*, respectively. Like work, kinetic energy is a *scalar* and has units of joules (J) and ft · lb. However, unlike work, which can be either positive or negative, the kinetic energy is *always positive*, regardless of the direction of motion of the particle.

When Eq. 14–6 is applied, it is often expressed in the form

$$\boxed{T_1 + \Sigma U_{1-2} = T_2} \qquad (14\text{–}7)$$

which states that the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to its final position is equal to the particle's final kinetic energy.

As noted from the derivation, the principle of work and energy represents an integrated form of $\Sigma F_t = ma_t$, obtained by using the kinematic equation $a_t = v\,dv/ds$. As a result, this principle will provide a convenient *substitution* for $\Sigma F_t = ma_t$ when solving those types of kinetic problems which involve *force*, *velocity*, and *displacement* since these quantities are involved in Eq. 14–7. For application, it is suggested that the following procedure be used.

Procedure for Analysis

Work (Free-Body Diagram).

- Establish the inertial coordinate system and draw a free-body diagram of the particle in order to account for all the forces that do work on the particle as it moves along its path.

Principle of Work and Energy.

- Apply the principle of work and energy, $T_1 + \Sigma U_{1-2} = T_2$.
- The kinetic energy at the initial and final points is *always positive*, since it involves the speed squared $\left(T = \frac{1}{2}mv^2\right)$.
- A force does work when it moves through a displacement in the direction of the force.
- Work is *positive* when the force component is in the *same sense of direction* as its displacement, otherwise it is negative.
- Forces that are functions of displacement must be integrated to obtain the work. Graphically, the work is equal to the area under the force-displacement curve.
- The work of a weight is the product of the weight magnitude and the vertical displacement, $U_W = \pm Wy$. It is positive when the weight moves downwards.
- The work of a spring is of the form $U_s = \frac{1}{2}ks^2$, where k is the spring stiffness and s is the stretch or compression of the spring.

Numerical application of this procedure is illustrated in the examples following Sec. 14.3.

If an oncoming car strikes these crash barrels, the car's kinetic energy will be transformed into work, which causes the barrels, and to some extent the car, to be deformed. By knowing the amount of energy that can be absorbed by each barrel it is possible to design a crash cushion such as this. (© R.C. Hibbeler)

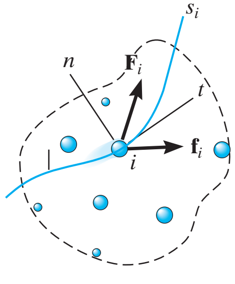
14.3 Principle of Work and Energy for a System of Particles

The principle of work and energy can be extended to include a system of particles isolated within an enclosed region of space as shown in Fig. 14–8. Here the arbitrary ith particle, having a mass m_i, is subjected to a resultant external force $\mathbf{F}_i$ and a resultant internal force $\mathbf{f}_i$ which all the other particles exert on the ith particle. If we apply the principle of work and energy to this and each of the other particles in the system, then since work and energy are scalar quantities, the equations can be summed algebraically, which gives

$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2 \tag{14–8}$$

In this case, the initial kinetic energy of the system plus the work done by all the external and internal forces acting on the system is equal to the final kinetic energy of the system.

If the system represents a *translating rigid body*, or a series of connected translating bodies, then all the particles in each body will undergo the *same displacement*. Therefore, the work of all the internal forces will occur in equal but opposite collinear pairs and so it will cancel out. On the other hand, if the body is assumed to be *nonrigid*, the particles of the body may be displaced along *different paths*, and some of the energy due to force interactions would be given off and lost as heat or stored in the body if permanent deformations occur. We will discuss these effects briefly at the end of this section and in Sec. 15.4. Throughout this text, however, the principle of work and energy will be applied to problems where direct accountability of such energy losses does not have to be considered.



Inertial coordinate system

Fig. 14–8

Work of Friction Caused by Sliding. A special class of problems will now be investigated which requires a careful application of Eq. 14–8. These problems involve cases where a body slides over the surface of another body in the presence of friction. Consider, for example, a block which is translating a distance s over a rough surface as shown in Fig. 14–9*a*. If the applied force **P** just balances the *resultant* frictional force $\mu_k N$, Fig. 14–9*b*, then due to equilibrium a constant velocity **v** is maintained, and one would expect Eq. 14–8 to be applied as follows:

$$\tfrac{1}{2}mv^2 + Ps - \mu_k Ns = \tfrac{1}{2}mv^2$$

Indeed this equation is satisfied if $P = \mu_k N$; however, as one realizes from experience, the sliding motion will *generate heat*, a form of energy which seems not to be accounted for in the work-energy equation. In order to explain this paradox and thereby more closely represent the nature of friction, we should actually model the block so that the surfaces of contact are *deformable* (nonrigid).* Recall that the rough portions at the bottom of the block act as "teeth," and when the block slides these teeth *deform slightly* and either break off or vibrate as they pull away from "teeth" at the contacting surface, Fig. 14–9*c*. As a result, frictional forces that act on the block at these points are displaced slightly, due to the localized deformations, and later they are replaced by other frictional forces as other points of contact are made. At any instant, the *resultant* **F** of all these frictional forces remains essentially constant, i.e., $\mu_k N$; however, due to the many *localized deformations*, the actual displacement s' of $\mu_k N$ is *not* the same as the displacement s of the applied force **P**. Instead, s' will be *less* than s $(s' < s)$, and therefore the *external work* done by the resultant frictional force will be $\mu_k Ns'$ and not $\mu_k Ns$. The remaining amount of work, $\mu_k N(s - s')$, manifests itself as an increase in *internal energy*, which in fact causes the block's temperature to rise.

In summary then, Eq. 14–8 can be applied to problems involving sliding friction; however, it should be fully realized that the work of the resultant frictional force is not represented by $\mu_k Ns$; instead, this term represents *both* the external work of friction $(\mu_k Ns')$ *and* internal work $[\mu_k N(s - s')]$ which is converted into various forms of internal energy, such as heat.†

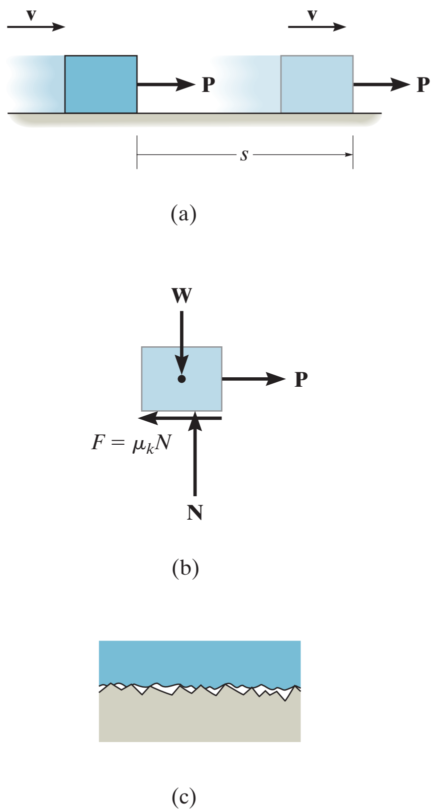
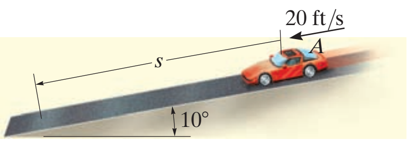


Fig. 14–9

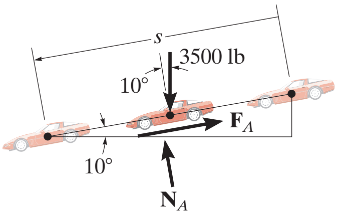
*See Chapter 8 of *Engineering Mechanics: Statics.*

†See B. A. Sherwood and W. H. Bernard, "Work and Heat Transfer in the Presence of Sliding Friction," *Am. J. Phys.* 52, 1001 (1984).

EXAMPLE 14.2



(a)



(b)

Fig. 14–10

The 3500-lb automobile shown in Fig. 14–10*a* travels down the 10° inclined road at a speed of 20 ft/s. If the driver jams on the brakes, causing his wheels to lock, determine how far s the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.

SOLUTION

This problem can be solved using the principle of work and energy, since it involves force, velocity, and displacement.

Work (Free-Body Diagram). As shown in Fig. 14–10*b*, the normal force $\mathbf{N}_A$ does no work since it never undergoes displacement along its line of action. The weight, 3500 lb, is displaced $s \sin 10°$ and does positive work. Why? The frictional force $\mathbf{F}_A$ does both external and internal work when it undergoes a displacement s. This work is negative since it is in the opposite sense of direction to the displacement. Applying the equation of equilibrium normal to the road, we have

$$+\nwarrow \Sigma F_n = 0; \qquad N_A - 3500 \cos 10° \text{ lb} = 0 \qquad N_A = 3446.8 \text{ lb}$$

Thus,

$$F_A = \mu_k N_A = 0.5\,(3446.8 \text{ lb}) = 1723.4 \text{ lb}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(20 \text{ ft/s})^2 + 3500 \text{ lb}(s \sin 10°) - (1723.4 \text{ lb})s = 0$$

Solving for s yields

$$s = 19.5 \text{ ft} \qquad \textit{Ans.}$$

NOTE: If this problem is solved by using the equation of motion, *two steps* are involved. First, from the free-body diagram, Fig. 14–10*b*, the equation of motion is applied along the incline. This yields

$$+\swarrow \Sigma F_s = ma_s; \qquad 3500 \sin 10° \text{ lb} - 1723.4 \text{ lb} = \frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2}a$$

$$a = -10.3 \text{ ft/s}^2$$

Then, since a is constant, we have

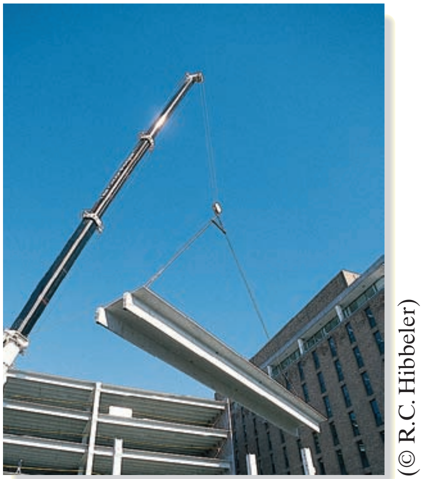
$$(+\swarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0);$$

$$(0)^2 = (20 \text{ ft/s})^2 + 2(-10.3 \text{ ft/s}^2)(s - 0)$$

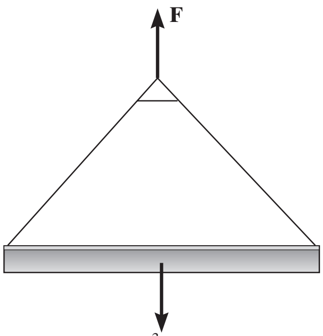
$$s = 19.5 \text{ ft} \qquad \textit{Ans.}$$

EXAMPLE 14.3

For a short time the crane in Fig. 14–11*a* lifts the 2.50-Mg beam with a force of $F = (28 + 3s^2)$ kN. Determine the speed of the beam when it has risen $s = 3$ m. Also, how much time does it take to attain this height starting from rest?



(a)



(b)

Fig. 14–11

SOLUTION

We can solve part of this problem using the principle of work and energy since it involves force, velocity, and displacement. Kinematics must be used to determine the time. Note that at $s = 0$, $F = 28(10^3)\text{N} > W = 2.50(10^3)(9.81)\text{N}$, so motion will occur.

Work (Free-Body Diagram). As shown on the free-body diagram, Fig. 14–11*b*, the lifting force **F** does positive work, which must be determined by integration since this force is a variable. Also, the weight is constant and will do negative work since the displacement is upward.

Principles of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^s (28 + 3s^2)(10^3)\,ds - (2.50)(10^3)(9.81)s = \tfrac{1}{2}(2.50)(10^3)v^2$$

$$28(10^3)s + (10^3)s^3 - 24.525(10^3)s = 1.25(10^3)v^2$$

$$v = (2.78s + 0.8s^3)^{\frac{1}{2}} \qquad (1)$$

When $s = 3$ m,

$$v = 5.47 \text{ m/s} \qquad \textit{Ans.}$$

Kinematics. Since we were able to express the velocity as a function of displacement, the time can be determined using $v = ds/dt$. In this case,

$$(2.78s + 0.8s^3)^{\frac{1}{2}} = \frac{ds}{dt}$$

$$t = \int_0^3 \frac{ds}{(2.78s + 0.8s^3)^{\frac{1}{2}}}$$

The integration can be performed numerically using a pocket calculator. The result is

$$t = 1.79 \text{ s} \qquad \textit{Ans.}$$

NOTE: The acceleration of the beam can be determined by integrating Eq. (1) using $v\,dv = a\,ds$, or more directly, by applying the equation of motion, $\Sigma F = ma$.

EXAMPLE 14.4

The platform P, shown in Fig. 14–12a, has negligible mass and is tied down so that the 0.4-m-long cords keep a 1-m-long spring compressed 0.6 m when *nothing* is on the platform. If a 2-kg block is placed on the platform and released from rest after the platform is pushed down 0.1 m, Fig. 14–12b, determine the maximum height h the block rises in the air, measured from the ground.

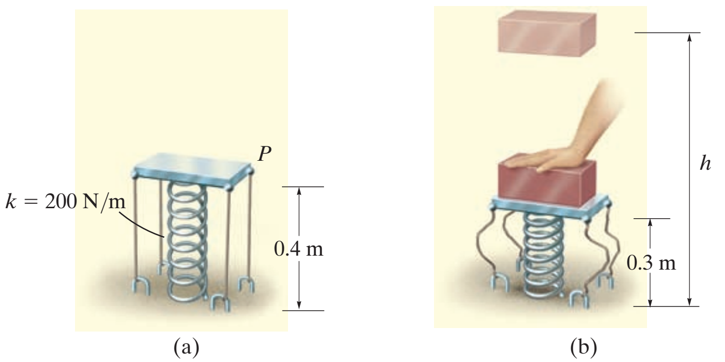
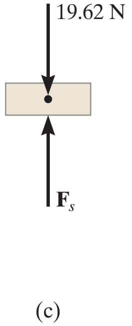


Fig. 14–12

SOLUTION

Work (Free-Body Diagram). Since the block is released from rest and later reaches its maximum height, the initial and final velocities are zero. The free-body diagram of the block when it is still in contact with the platform is shown in Fig. 14–12c. Note that the weight does negative work and the spring force does positive work. Why? In particular, the *initial compression* in the spring is $s_1 = 0.6\text{ m} + 0.1\text{ m} = 0.7\text{ m}$. Due to the cords, the spring's *final compression* is $s_2 = 0.6\text{ m}$ (after the block leaves the platform). The bottom of the block rises from a height of $(0.4\text{ m} - 0.1\text{ m}) = 0.3\text{ m}$ to a final height h.



Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\tfrac{1}{2}mv_1^2 + \left\{-\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right) - W\,\Delta y\right\} = \tfrac{1}{2}mv_2^2$$

Note that here $s_1 = 0.7\text{ m} > s_2 = 0.6\text{ m}$ and so the work of the spring as determined from Eq. 14–4 will indeed be positive once the calculation is made. Thus,

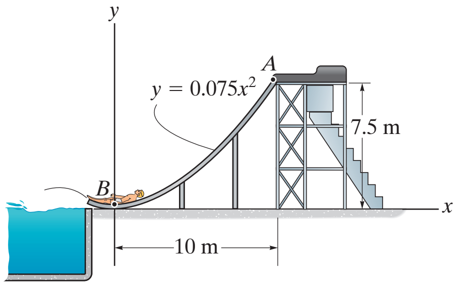
$$0 + \left\{-\left[\tfrac{1}{2}(200\text{ N/m})(0.6\text{ m})^2 - \tfrac{1}{2}(200\text{ N/m})(0.7\text{ m})^2\right] - (19.62\text{ N})[h - (0.3\text{ m})]\right\} = 0$$

Solving yields

$$h = 0.963\text{ m} \qquad \textit{Ans.}$$

EXAMPLE 14.5

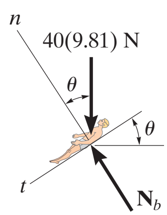
The 40-kg boy in Fig. 14–13*a* slides down the smooth water slide. If he starts from rest at A, determine his speed when he reaches B and the normal reaction the slide exerts on the boy at this position.



(a)

SOLUTION

Work (Free-Body Diagram). As shown on the free-body diagram, Fig. 14–13*b*, there are two forces acting on the boy as he goes down the slide. Note that the normal force does no work.



(b)

Principle of Work and Energy.

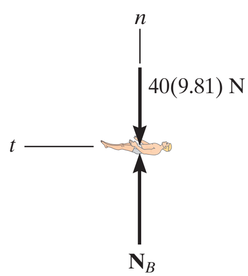
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + (40(9.81)\text{N})\,(7.5\text{ m}) = \tfrac{1}{2}(40\text{ kg})v_B^2$$

$$v_B = 12.13\text{ m/s} = 12.1\text{ m/s} \qquad \textit{Ans.}$$

Equation of Motion. Referring to the free-body diagram of the boy when he is at B, Fig. 14–13*c*, the normal reaction $\mathbf{N}_B$ can now be obtained by applying the equation of motion along the n axis. Here the radius of curvature of the path is

$$\rho_B = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (0.15x)^2\right]^{3/2}}{|0.15|}\right|_{x=0} = 6.667\text{ m}$$



(c)

Fig. 14–13

Thus,

$$+\!\uparrow\Sigma F_n = ma_n; \qquad N_B - 40(9.81)\text{ N} = 40\text{ kg}\left(\frac{(12.13\text{ m/s})^2}{6.667\text{ m}}\right)$$

$$N_B = 1275.3\text{ N} = 1.28\text{ kN} \qquad \textit{Ans.}$$

EXAMPLE 14.6

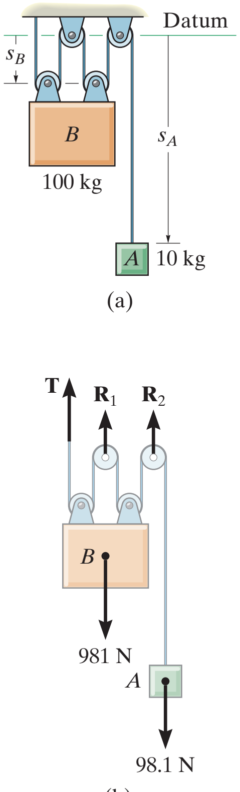


Fig. 14–14

Blocks A and B shown in Fig. 14–14a have a mass of 10 kg and 100 kg, respectively. Determine the distance B travels when it is released from rest to the point where its speed becomes 2 m/s.

SOLUTION

This problem may be solved by considering the blocks separately and applying the principle of work and energy to each block. However, the work of the (unknown) cable tension can be eliminated from the analysis by considering blocks A and B together as a *single system*.

Work (Free-Body Diagram). As shown on the free-body diagram of the system, Fig. 14–14b, the cable force $\mathbf{T}$ and reactions $\mathbf{R}_1$ and $\mathbf{R}_2$ do *no work*, since these forces represent the reactions at the supports and consequently they do not move while the blocks are displaced. The weights both do positive work if we *assume* both move downward, in the positive sense of direction of s_A and s_B.

Principle of Work and Energy. Realizing the blocks are released from rest, we have

$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

$$\left\{\tfrac{1}{2}m_A(v_A)_1^2 + \tfrac{1}{2}m_B(v_B)_1^2\right\} + \{W_A\,\Delta s_A + W_B\,\Delta s_B\} = \left\{\tfrac{1}{2}m_A(v_A)_2^2 + \tfrac{1}{2}m_B(v_B)_2^2\right\}$$

$$\{0 + 0\} + \{98.1\text{ N}\,(\Delta s_A) + 981\text{ N}\,(\Delta s_B)\} = \left\{\tfrac{1}{2}(10\text{ kg})(v_A)_2^2 + \tfrac{1}{2}(100\text{ kg})(2\text{ m/s})^2\right\} \quad (1)$$

Kinematics. Using methods of kinematics, as discussed in Sec. 12.9, it may be seen from Fig. 14–14a that the total length l of all the vertical segments of cable may be expressed in terms of the position coordinates s_A and s_B as

$$s_A + 4s_B = l$$

Hence, a change in position yields the displacement equation

$$\Delta s_A + 4\,\Delta s_B = 0$$
$$\Delta s_A = -4\,\Delta s_B$$

Here we see that a downward displacement of one block produces an upward displacement of the other block. Note that Δs_A and Δs_B must have the *same* sign convention in both Eqs. 1 and 2. Taking the time derivative yields

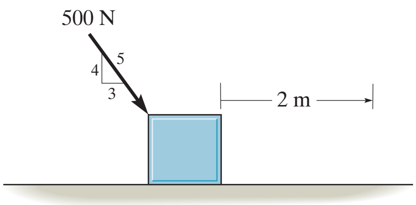
$$v_A = -4v_B = -4(2\text{ m/s}) = -8\text{ m/s} \quad (2)$$

Retaining the negative sign in Eq. 2 and substituting into Eq. 1 yields

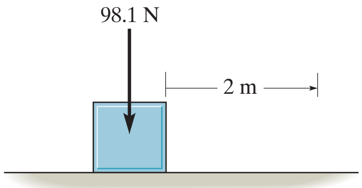
$$\Delta s_B = 0.883\text{ m}\downarrow \qquad \textit{Ans.}$$

PRELIMINARY PROBLEMS

P14–1. Determine the work of the force when it displaces 2 m.



(a)



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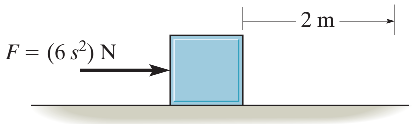

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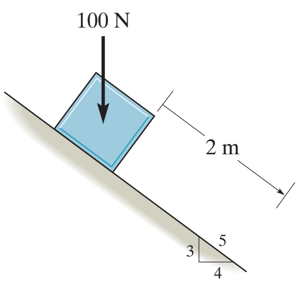

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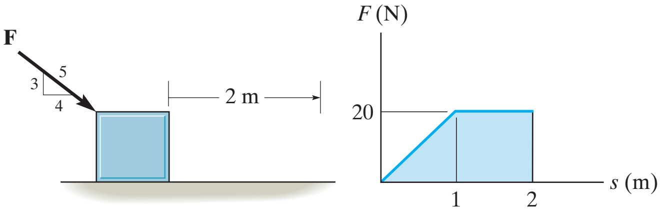

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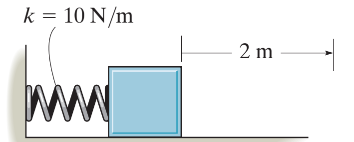
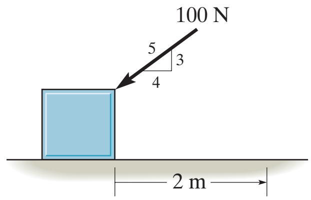


Spring is originally compressed 3 m.

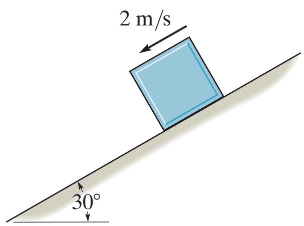
(f)



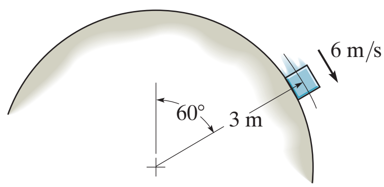
(g)

Prob. P14–1

P14–2. Determine the kinetic energy of the 10-kg block.



(a)

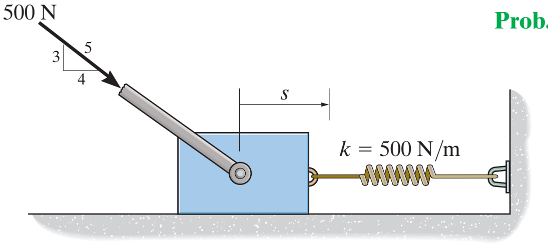


(b)

Prob. P14–2

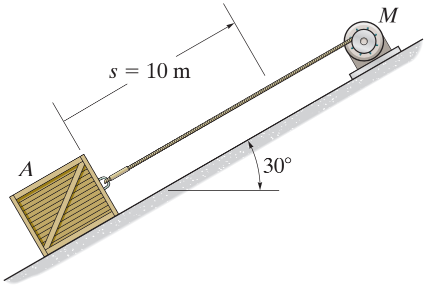
FUNDAMENTAL PROBLEMS

F14–1. The spring is placed between the wall and the 10-kg block. If the block is subjected to a force of $F = 500$ N, determine its velocity when $s = 0.5$ m. When $s = 0$, the block is at rest and the spring is uncompressed. The contact surface is smooth.



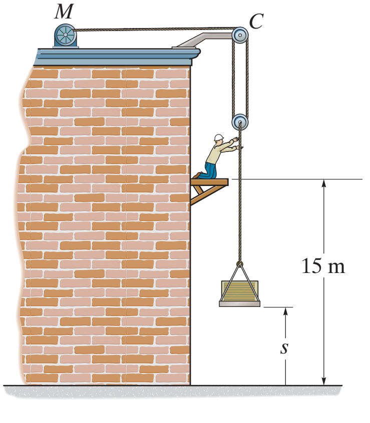
Prob. F14–1

F14–2. If the motor exerts a constant force of 300 N on the cable, determine the speed of the 20-kg crate when it travels $s = 10$ m up the plane, starting from rest. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.



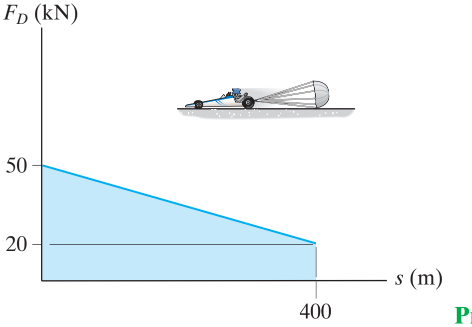
Prob. F14–2

F14–3. If the motor exerts a force of $F = (600 + 2s^2)$ N on the cable, determine the speed of the 100-kg crate when it rises to $s = 15$ m. The crate is initially at rest on the ground.



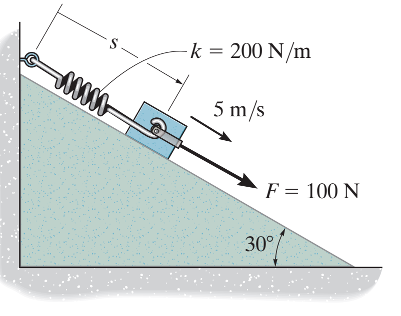
Prob. F14–3

F14–4. The 1.8-Mg dragster is traveling at 125 m/s when the engine is shut off and the parachute is released. If the drag force of the parachute can be approximated by the graph, determine the speed of the dragster when it has traveled 400 m.



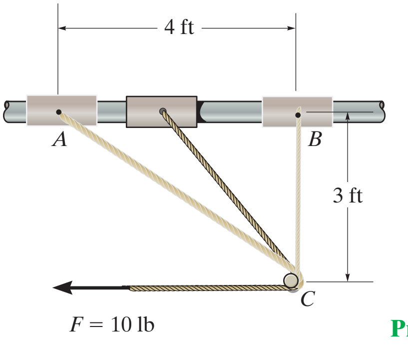
Prob. F14–4

F14–5. When $s = 0.6$ m, the spring is unstretched and the 10-kg block has a speed of 5 m/s down the smooth plane. Determine the distance s when the block stops.



Prob. F14–5

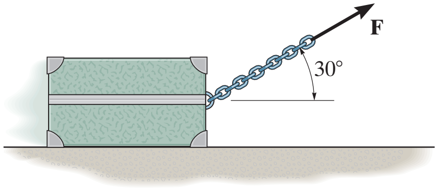
F14–6. The 5-lb collar is pulled by a cord that passes around a small peg at C. If the cord is subjected to a constant force of $F = 10$ lb, and the collar is at rest when it is at A, determine its speed when it reaches B. Neglect friction.



Prob. F14–6

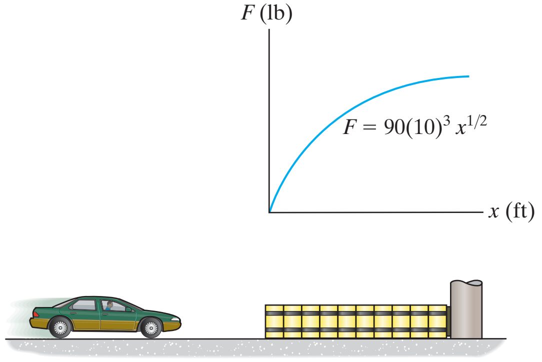
PROBLEMS

14–1. The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



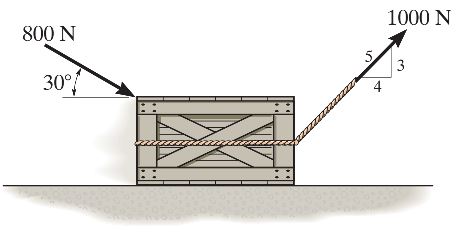
Prob. 14–1

14–2. For protection, the barrel barrier is placed in front of the bridge pier. If the relation between the force and deflection of the barrier is $F = (90(10^3)x^{1/2})$ lb, where x is in ft, determine the car's maximum penetration in the barrier. The car has a weight of 4000 lb and it is traveling with a speed of 75 ft/s just before it hits the barrier.



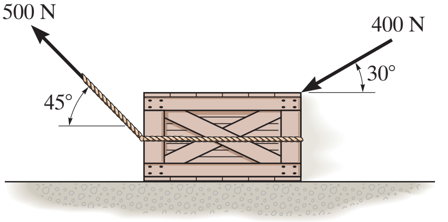
Prob. 14–2

14–3. The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



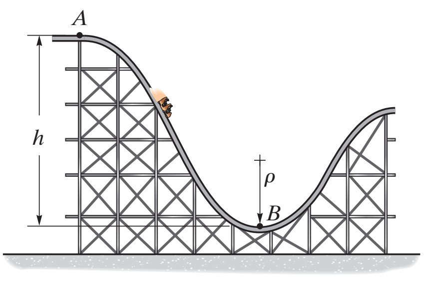
Prob. 14–3

***14–4.** The 100-kg crate is subjected to the forces shown. If it is originally at rest, determine the distance it slides in order to attain a speed of $v = 8$ m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



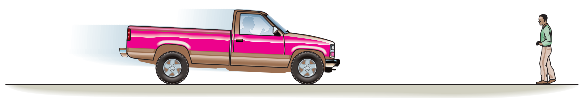
Prob. 14–4

14–5. Determine the required height h of the roller coaster so that when it is essentially at rest at the crest of the hill A it will reach a speed of 100 km/h when it comes to the bottom B. Also, what should be the minimum radius of curvature ρ for the track at B so that the passengers do not experience a normal force greater than $4mg = (39.24m)$ N? Neglect the size of the car and passenger.



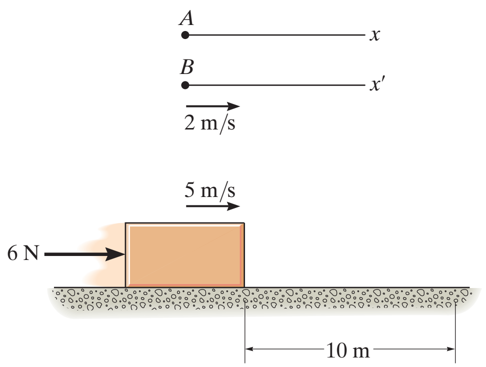
Prob. 14–5

14–6. When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



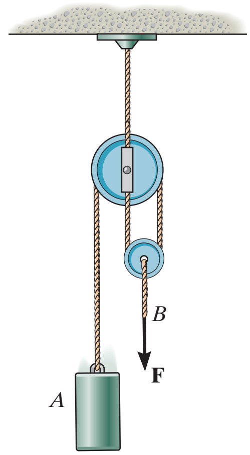
Prob. 14–6

14–7. As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint*: The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



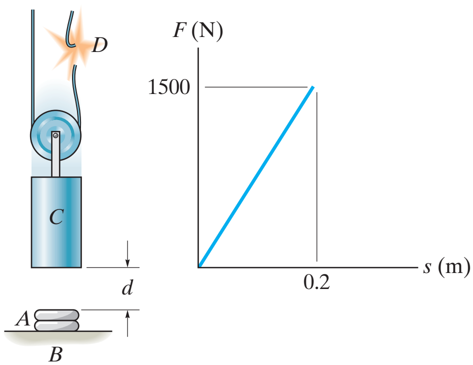
Prob. 14–7

***14–8.** A force of $F = 250$ N is applied to the end at B. Determine the speed of the 10-kg block when it has moved 1.5 m, starting from rest.



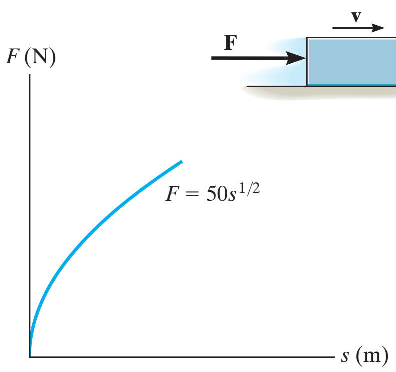
Prob. 14–8

14–9. The "air spring" A is used to protect the support B and prevent damage to the conveyor-belt tensioning weight C in the event of a belt failure D. The force developed by the air spring as a function of its deflection is shown by the graph. If the block has a mass of 20 kg and is suspended a height $d = 0.4$ m above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.



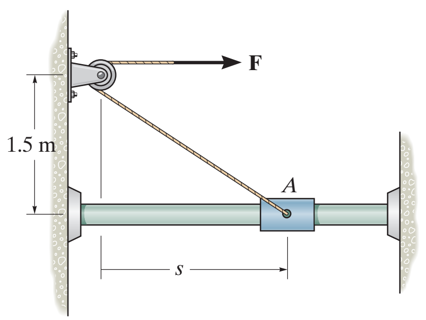
Prob. 14–9

14–10. The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block must slide before its velocity becomes 15 m/s. When $s = 0$ the block is moving to the right at $v = 6$ m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



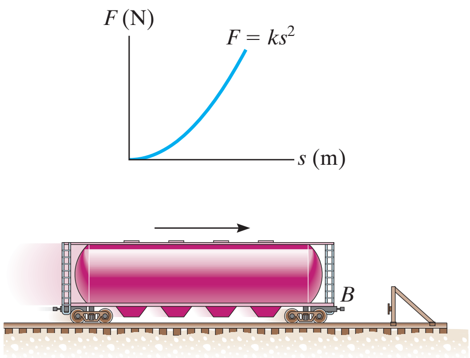
Prob. 14–10

14–11. The force of $F = 50$ N is applied to the cord when $s = 2$ m. If the 6-kg collar is orginally at rest, determine its velocity at $s = 0$. Neglect friction.



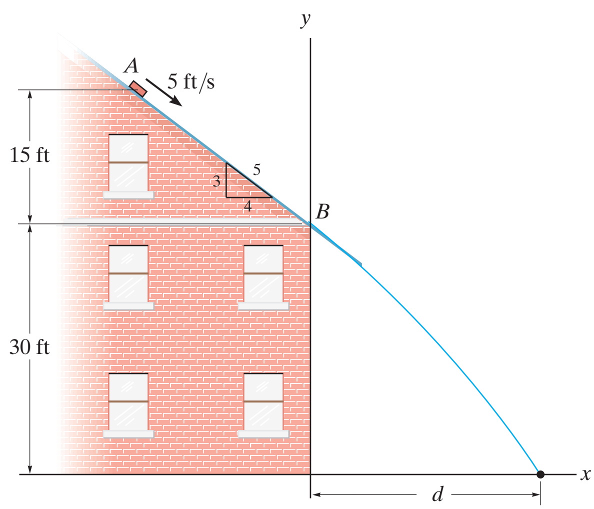
Prob. 14–11

*__14–12.__ Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.



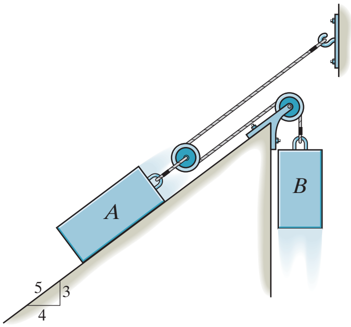
Prob. 14–12

14–13. The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the brick just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.



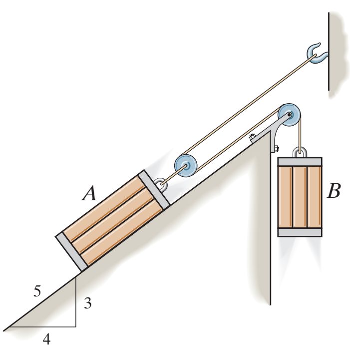
Prob. 14–13

14–14. Block A has a weight of 60 lb and block B has a weight of 10 lb. Determine the speed of block A after it moves 5 ft down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.



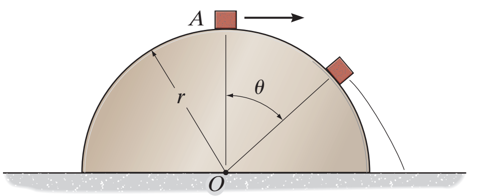
Prob. 14–14

14–15. The two blocks A and B have weights $W_A = 60$ lb and $W_B = 10$ lb. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0.2$, determine the speed of A after it moves 3 ft down the plane starting from rest. Neglect the mass of the cord and pulleys.



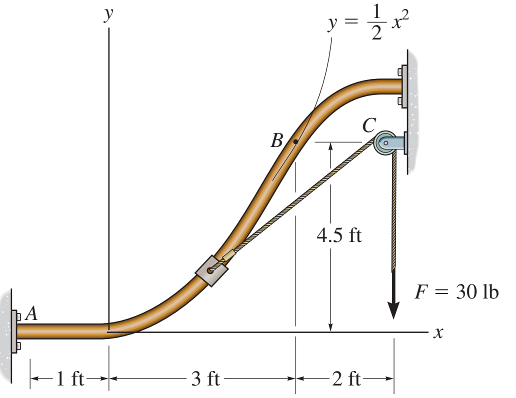
Prob. 14–15

***14–16.** A small box of mass m is given a speed of $v = \sqrt{\frac{1}{4}gr}$ at the top of the smooth half cylinder. Determine the angle θ at which the box leaves the cylinder.



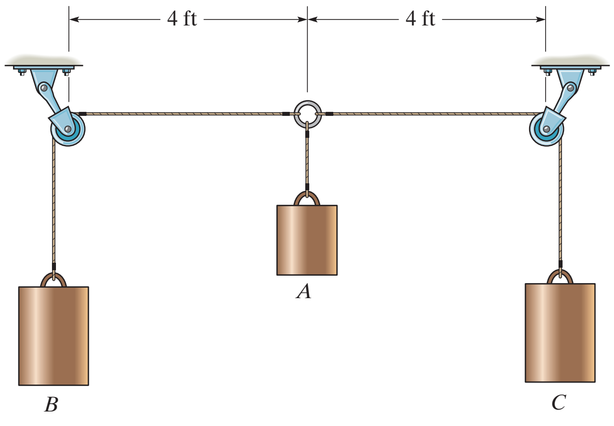
Prob. 14–16

14–17. If the cord is subjected to a constant force of $F = 30$ lb and the smooth 10-lb collar starts from rest at A, determine its speed when it passes point B. Neglect the size of pulley C.



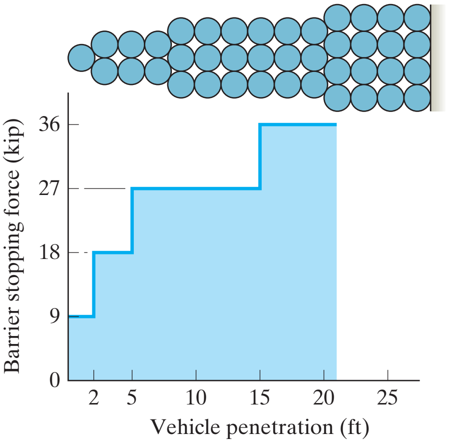
Prob. 14–17

14–18. When the 12-lb block A is released from rest it lifts the two 15-lb weights B and C. Determine the maximum distance A will fall before its motion is momentarily stopped. Neglect the weight of the cord and the size of the pulleys.



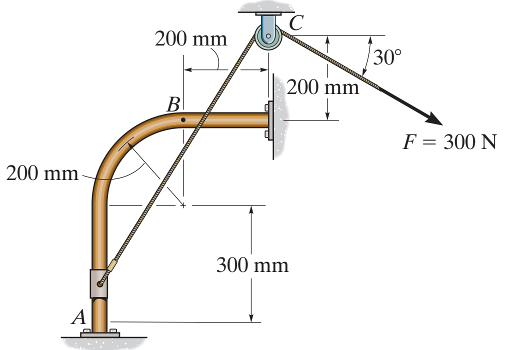
Prob. 14–18

***14–20.** The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having a weight of 4000 lb will penetrate the barrier if it is originally traveling at 55 ft/s when it strikes the first barrel.



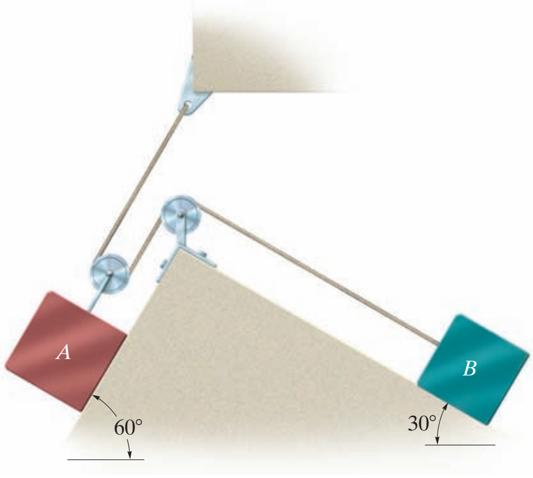
Prob. 14–20

14–19. If the cord is subjected to a constant force of $F = 300$ N and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



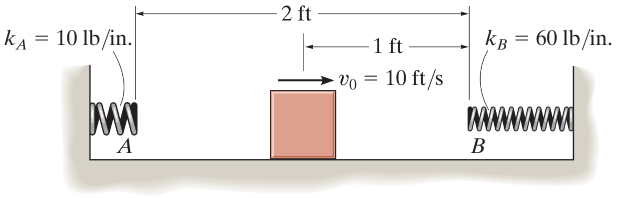
Prob. 14–19

14–21. Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



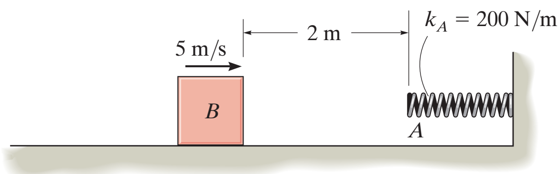
Prob. 14–21

14–22. The 25-lb block has an initial speed of $v_0 = 10 \text{ ft/s}$ when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



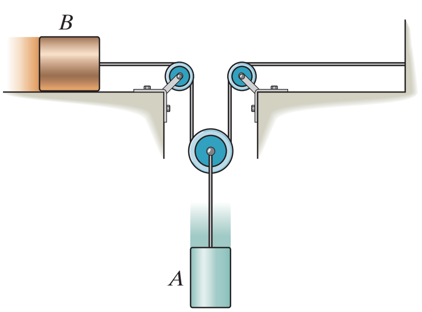
Prob. 14–22

14–23. The 8-kg block is moving with an initial speed of 5 m/s. If the coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$, determine the compression in the spring when the block momentarily stops.



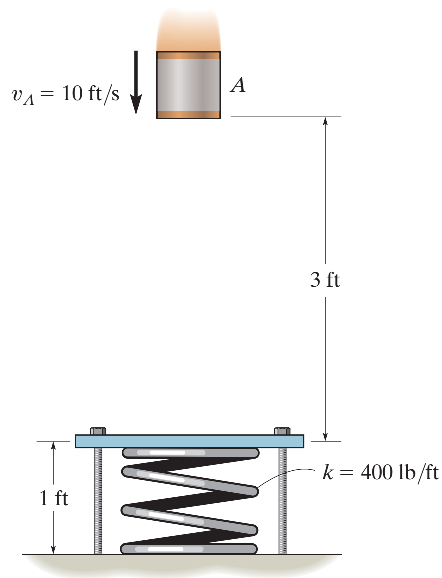
Prob. 14–23

***14–24.** At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the cord and pulleys.



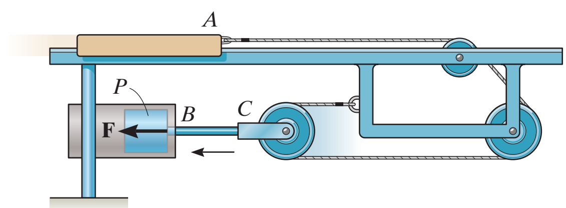
Prob. 14–24

14–25. The 5-lb cylinder is falling from A with a speed $v_A = 10 \text{ ft/s}$ onto the platform. Determine the maximum displacement of the platform, caused by the collision. The spring has an unstretched length of 1.75 ft and is originally kept in compression by the 1-ft long cables attached to the platform. Neglect the mass of the platform and spring and any energy lost during the collision.



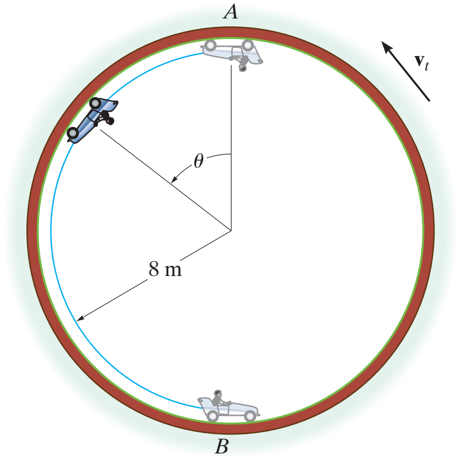
Prob. 14–25

14–26. The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20 \text{ kN}$ to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.



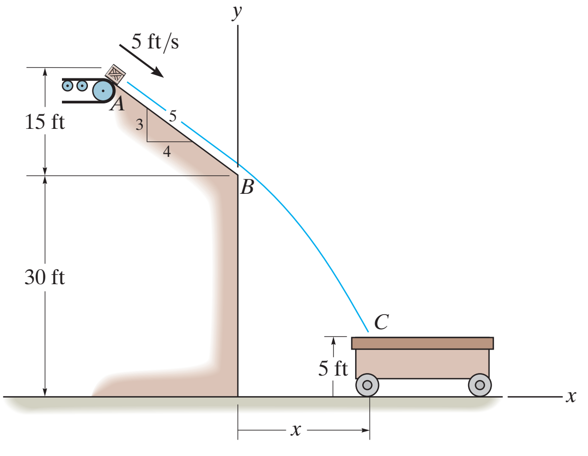
Prob. 14–26

14–27. The "flying car" is a ride at an amusement park which consists of a car having wheels that roll along a track mounted inside a rotating drum. By design the car cannot fall off the track, however motion of the car is developed by applying the car's brake, thereby gripping the car to the track and allowing it to move with a constant speed of the track, $v_t = 3\text{ m/s}$. If the rider applies the brake when going from B to A and then releases it at the top of the drum, A, so that the car coasts freely down along the track to B ($\theta = \pi$ rad), determine the speed of the car at B and the normal reaction which the drum exerts on the car at B. Neglect friction during the motion from A to B. The rider and car have a total mass of 250 kg and the center of mass of the car and rider moves along a circular path having a radius of 8 m.



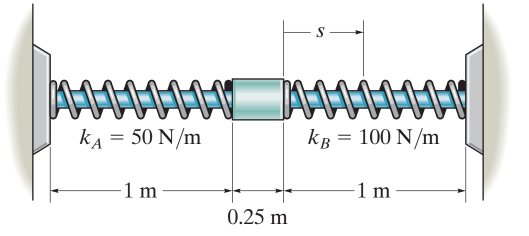
Prob. 14–27

***14–28.** The 10-lb box falls off the conveyor belt at 5-ft/s. If the coefficient of kinetic friction along AB is $\mu_k = 0.2$, determine the distance x when the box falls into the cart.



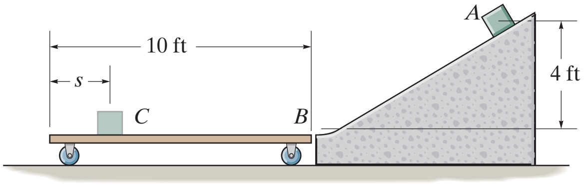
Prob. 14–28

14–29. The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



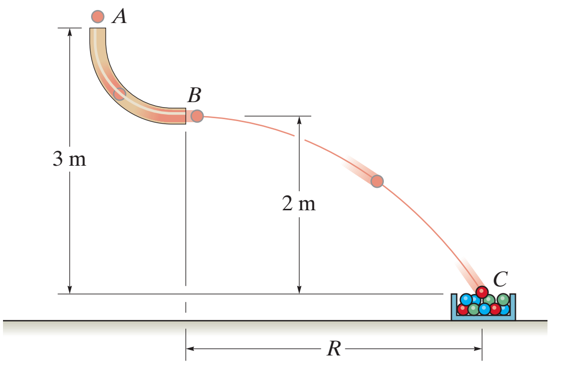
Prob. 14–29

14–30. The 30-lb box A is released from rest and slides down along the smooth ramp and onto the surface of a cart. If the cart is *prevented from moving*, determine the distance s from the end of the cart to where the box stops. The coefficient of kinetic friction between the cart and the box is $\mu_k = 0.6$.



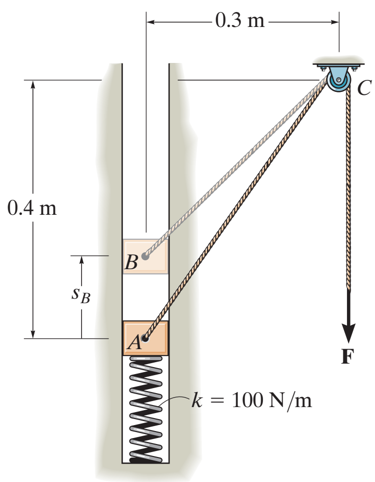
Prob. 14–30

14–31. Marbles having a mass of 5 g are dropped from rest at A through the smooth glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.



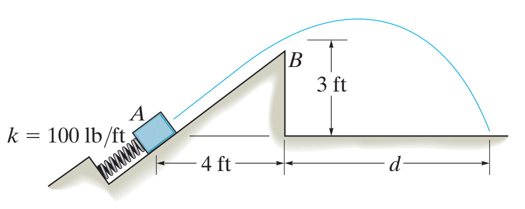
Prob. 14–31

***14–32.** The block has a mass of 0.8 kg and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the size and mass of the pulley. *Hint*: The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\,\Delta l$.



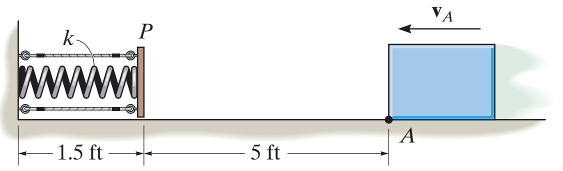
Prob. 14–32

14–33. The 10-lb block is pressed against the spring so as to compress it 2 ft when it is at A. If the plane is smooth, determine the distance d, measured from the wall, to where the block strikes the ground. Neglect the size of the block.



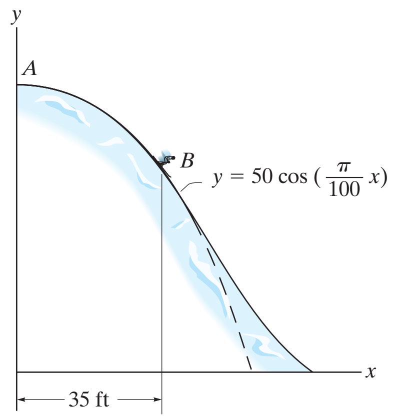
Prob. 14–33

14–34. The spring bumper is used to arrest the motion of the 4-lb block, which is sliding toward it at $v = 9$ ft/s. As shown, the spring is confined by the plate P and wall using cables so that its length is 1.5 ft. If the stiffness of the spring is $k = 50$ lb/ft, determine the required unstretched length of the spring so that the plate is not displaced more than 0.2 ft after the block collides into it. Neglect friction, the mass of the plate and spring, and the energy loss between the plate and block during the collision.



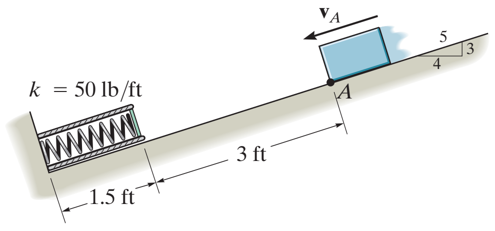
Prob. 14–34

14–35. When the 150-lb skier is at point A he has a speed of 5 ft/s. Determine his speed when he reaches point B on the smooth slope. For this distance the slope follows the cosine curve shown. Also, what is the normal force on his skis at B and his rate of increase in speed? Neglect friction and air resistance.



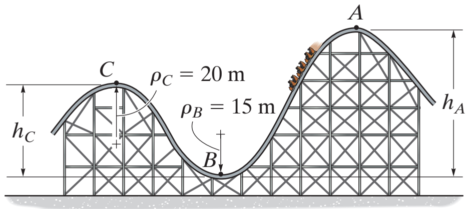
Prob. 14–35

***14–36.** The spring has a stiffness $k = 50$ lb/ft and an *unstretched length* of 2 ft. As shown, it is confined by the plate and wall using cables so that its length is 1.5 ft. A 4-lb block is given a speed v_A when it is at A, and it slides down the incline having a coefficient of kinetic friction $\mu_k = 0.2$. If it strikes the plate and pushes it forward 0.25 ft before stopping, determine its speed at A. Neglect the mass of the plate and spring.



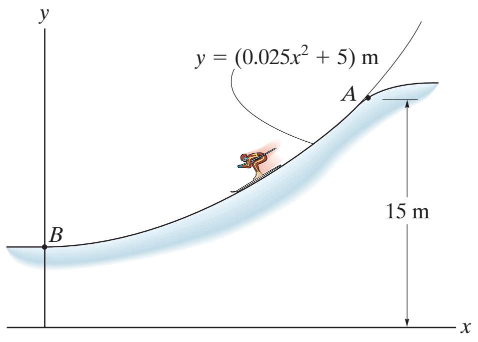
Prob. 14–36

14–37. If the track is to be designed so that the passengers of the roller coaster do not experience a normal force equal to zero or more than 4 times their weight, determine the limiting heights h_A and h_C so that this does not occur. The roller coaster starts from rest at position A. Neglect friction.



Prob. 14–37

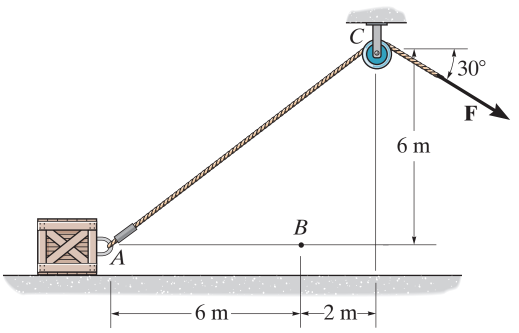
14–38. If the 60-kg skier passes point A with a speed of 5 m/s, determine his speed when he reaches point B. Also find the normal force exerted on him by the slope at this point. Neglect friction.



Prob. 14–38

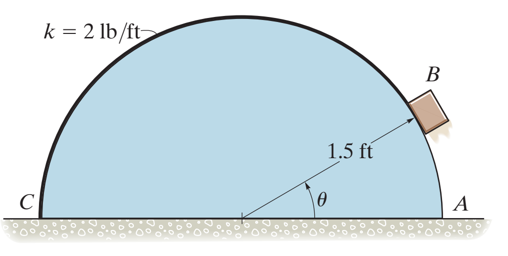
14–39. If the 75-kg crate starts from rest at A, determine its speed when it reaches point B. The cable is subjected to a constant force of $F = 300$ N. Neglect friction and the size of the pulley.

***14–40.** If the 75-kg crate starts from rest at A, and its speed is 6 m/s when it passes point B, determine the constant force **F** exerted on the cable. Neglect friction and the size of the pulley.



Probs. 14–39/40

14–41. A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at A ($\theta = 0°$), determine the unstretched length of the cord so the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.



Prob. 14–41

14.4 Power and Efficiency

Power. The term "power" provides a useful basis for choosing the type of motor or machine which is required to do a certain amount of work in a given time. For example, two pumps may each be able to empty a reservoir if given enough time; however, the pump having the larger power will complete the job sooner.

The *power* generated by a machine or engine that performs an amount of work dU within the time interval dt is therefore

$$\boxed{P = \frac{dU}{dt}} \tag{14–9}$$

If the work dU is expressed as $dU = \mathbf{F} \cdot d\mathbf{r}$, then

$$P = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$$

or

$$\boxed{P = \mathbf{F} \cdot \mathbf{v}} \tag{14–10}$$

The power output of this locomotive comes from the driving frictional force developed at its wheels. It is this force that overcomes the frictional resistance of the cars in tow and is able to lift the weight of the train up the grade. (© R.C. Hibbeler)

Hence, power is a *scalar*, where in this formulation $\mathbf{v}$ represents the velocity of the particle which is acted upon by the force $\mathbf{F}$.

The basic units of power used in the SI and FPS systems are the watt (W) and horsepower (hp), respectively. These units are defined as

$$1 \text{ W} = 1 \text{ J/s} = 1 \text{ N} \cdot \text{m/s}$$

$$1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s}$$

For conversion between the two systems of units, $1 \text{ hp} = 746 \text{ W}$.

Efficiency. The *mechanical efficiency* of a machine is defined as the ratio of the output of useful power produced by the machine to the input of power supplied to the machine. Hence,

$$\boxed{\varepsilon = \frac{\text{power output}}{\text{power input}}} \tag{14–11}$$

If energy supplied to the machine occurs during the *same time interval* at which it is drawn, then the efficiency may also be expressed in terms of the ratio

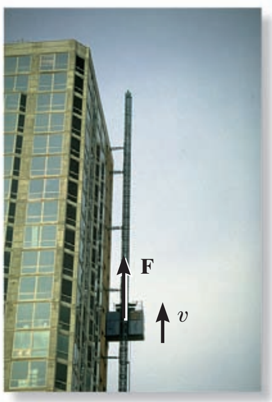
$$\boxed{\varepsilon = \frac{\text{energy output}}{\text{energy input}}} \qquad (14\text{–}12)$$

Since machines consist of a series of moving parts, frictional forces will always be developed within the machine, and as a result, extra energy or power is needed to overcome these forces. Consequently, power output will be less than power input and so *the efficiency of a machine is always less than 1*.

The power supplied to a body can be determined using the following procedure.

Procedure for Analysis

- First determine the external force **F** acting on the body which causes the motion. This force is usually developed by a machine or engine placed either within or external to the body.
- If the body is accelerating, it may be necessary to draw its free-body diagram and apply the equation of motion ($\Sigma\mathbf{F} = m\mathbf{a}$) to determine **F**.
- Once **F** and the velocity **v** of the particle where **F** is applied have been found, the power is determined by multiplying the force magnitude with the component of velocity acting in the direction of **F**, (i.e., $P = \mathbf{F} \cdot \mathbf{v} = Fv \cos \theta$).
- In some problems the power may be found by calculating the work done by **F** per unit of time ($P_{avg} = \Delta U/\Delta t$,).



The power requirement of this hoist depends upon the vertical force **F** that acts on the elevator and causes it to move upward. If the velocity of the elevator is **v**, then the power output is $P = \mathbf{F} \cdot \mathbf{v}$. (© R.C. Hibbeler)

EXAMPLE 14.7

The man in Fig. 14–15*a* pushes on the 50-kg crate with a force of $F = 150$ N. Determine the power supplied by the man when $t = 4$ s. The coefficient of kinetic friction between the floor and the crate is $\mu_k = 0.2$. Initially the create is at rest.

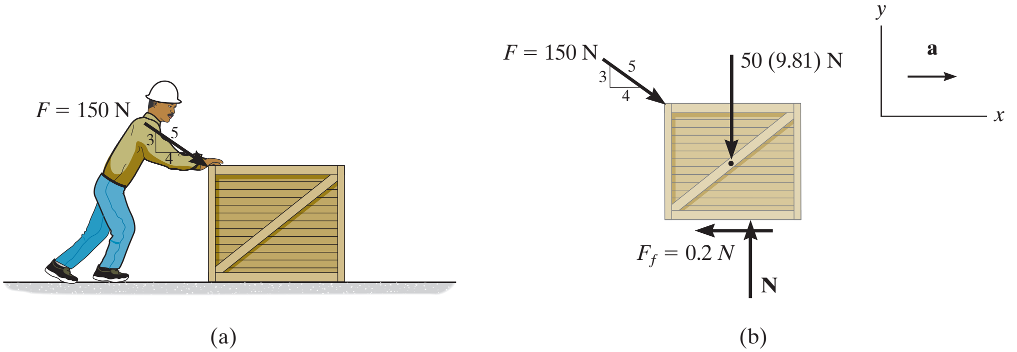


Fig. 14–15

SOLUTION

To determine the power developed by the man, the velocity of the 150-N force must be obtained first. The free-body diagram of the crate is shown in Fig. 14–15*b*. Applying the equation of motion,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - \left(\tfrac{3}{5}\right)150\text{ N} - 50(9.81)\text{ N} = 0$$

$$N = 580.5\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad \left(\tfrac{4}{5}\right)150\text{ N} - 0.2(580.5\text{ N}) = (50\text{ kg})a$$

$$a = 0.078\text{ m/s}^2$$

The velocity of the crate when $t = 4$ s is therefore

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v = 0 + (0.078\text{ m/s}^2)(4\text{ s}) = 0.312\text{ m/s}$$

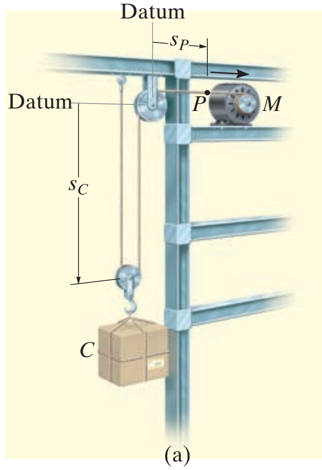
The power supplied to the crate by the man when $t = 4$ s is therefore

$$P = \mathbf{F}\cdot\mathbf{v} = F_x v = \left(\tfrac{4}{5}\right)(150\text{ N})(0.312\text{ m/s})$$

$$= 37.4\text{ W} \qquad \textit{Ans.}$$

EXAMPLE 14.8

The motor M of the hoist shown in Fig. 14–16a lifts the 75-lb crate C so that the acceleration of point P is 4 ft/s^2. Determine the power that must be supplied to the motor at the instant P has a velocity of 2 ft/s. Neglect the mass of the pulley and cable and take $\varepsilon = 0.85$.



(a)

SOLUTION

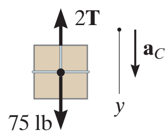
In order to find the power output of the motor, it is first necessary to determine the tension in the cable since this force is developed by the motor.

From the free-body diagram, Fig. 14–16b, we have

$$+\downarrow \Sigma F_y = ma_y; \qquad -2T + 75\text{ lb} = \frac{75\text{ lb}}{32.2\text{ ft/s}^2}a_c \qquad (1)$$

The acceleration of the crate can be obtained by using kinematics to relate it to the known acceleration of point P, Fig. 14–16a. Using the methods of absolute dependent motion, the coordinates s_C and s_P can be related to a constant portion of cable length l which is changing in the vertical and horizontal directions. We have $2s_C + s_P = l$. Taking the second time derivative of this equation yields

$$2a_C = -a_P \qquad (2)$$



(b)

Fig. 14–16

Since $a_P = +4\text{ ft/s}^2$, then $a_C = -(4\text{ ft/s}^2)/2 = -2\text{ ft/s}^2$. What does the negative sign indicate? Substituting this result into Eq. 1 and *retaining* the negative sign since the acceleration in *both* Eq. 1 and Eq. 2 was considered positive downward, we have

$$-2T + 75\text{ lb} = \left(\frac{75\text{ lb}}{32.2\text{ ft/s}^2}\right)(-2\text{ ft/s}^2)$$

$$T = 39.83\text{ lb}$$

The power output, measured in units of horsepower, required to draw the cable in at a rate of 2 ft/s is therefore

$$P = \mathbf{T}\cdot\mathbf{v} = (39.83\text{ lb})(2\text{ ft/s})[1\text{ hp}/(550\text{ ft}\cdot\text{lb/s})]$$
$$= 0.1448\text{ hp}$$

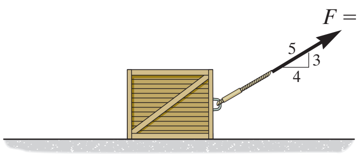
This *power output* requires that the motor provide a *power input* of

$$\text{power input} = \frac{1}{\varepsilon}(\text{power output})$$
$$= \frac{1}{0.85}(0.1448\text{ hp}) = 0.170\text{ hp} \qquad \textit{Ans.}$$

NOTE: Since the velocity of the crate is constantly changing, the power requirement is *instantaneous*.

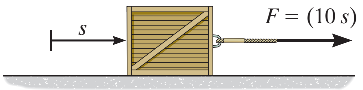
FUNDAMENTAL PROBLEMS

F14–7. If the contact surface between the 20-kg block and the ground is smooth, determine the power of force **F** when $t = 4$ s. Initially, the block is at rest.



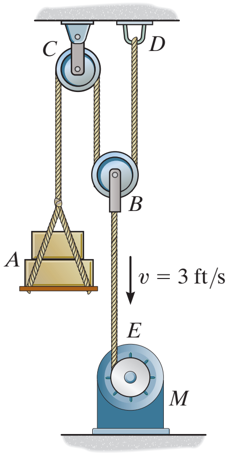
Prob. F14–7

F14–8. If $F = (10\,s)$ N, where s is in meters, and the contact surface between the block and the ground is smooth, determine the power of force **F** when $s = 5$ m. When $s = 0$, the 20-kg block is moving at $v = 1$ m/s.



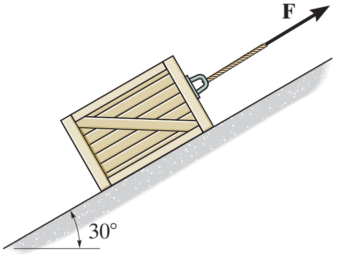
Prob. F14–8

F14–9. If the motor winds in the cable with a constant speed of $v = 3$ ft/s, determine the power supplied to the motor. The load weighs 100 lb and the efficiency of the motor is $\varepsilon = 0.8$. Neglect the mass of the pulleys.



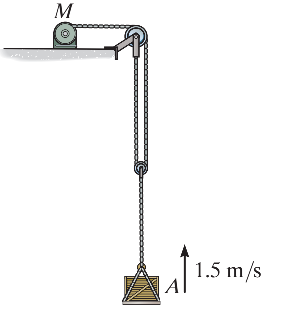
Prob. F14–9

F14–10. The coefficient of kinetic friction between the 20-kg block and the inclined plane is $\mu_k = 0.2$. If the block is traveling up the inclined plane with a constant velocity $v = 5$ m/s, determine the power of force **F**.



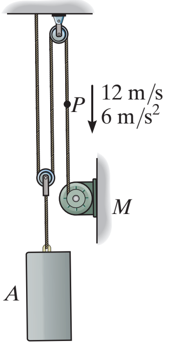
Prob. F14–10

F14–11. If the 50-kg load A is hoisted by motor M so that the load has a constant velocity of 1.5 m/s, determine the power input to the motor, which operates at an efficiency $\varepsilon = 0.8$.



Prob. F14–11

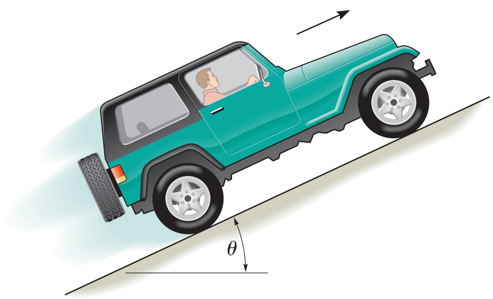
F14–12. At the instant shown, point P on the cable has a velocity $v_P = 12$ m/s, which is increasing at a rate of $a_P = 6$ m/s^2. Determine the power input to motor M at this instant if it operates with an efficiency $\varepsilon = 0.8$. The mass of block A is 50 kg.



Prob. F14–12

PROBLEMS

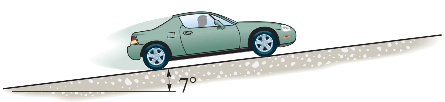
14–42. The jeep has a weight of 2500 lb and an engine which transmits a power of 100 hp to *all* the wheels. Assuming the wheels do not slip on the ground, determine the angle θ of the largest incline the jeep can climb at a constant speed $v = 30$ ft/s.



Prob. 14–42

14–43. Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\varepsilon = 0.65$.

***14–44.** An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\varepsilon = 0.65$.



Prob. 14–44

14–45. The Milkin Aircraft Co. manufactures a turbojet engine that is placed in a plane having a weight of 13000 lb. If the engine develops a constant thrust of 5200 lb, determine the power output of the plane when it is just ready to take off with a speed of 600 mi/h.

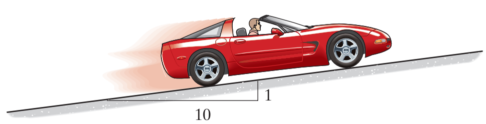
14–46. To dramatize the loss of energy in an automobile, consider a car having a weight of 5000 lb that is traveling at 35 mi/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy. (1 mi = 5280 ft.)

14–47. Escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.

***14–48.** The man having the weight of 150 lb is able to run up a 15-ft-high flight of stairs in 4 s. Determine the power generated. How long would a 100-W light bulb have to burn to expend the same amount of energy? *Conclusion*: Please turn off the lights when they are not in use!

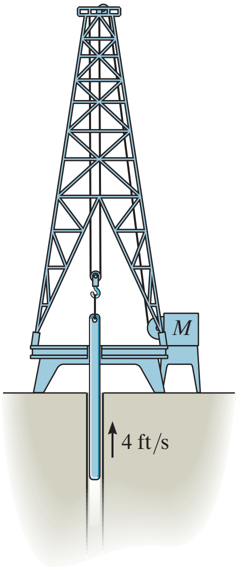
Prob. 14–48

14–49. The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\varepsilon = 0.8$. Also, find the average power supplied by the engine.



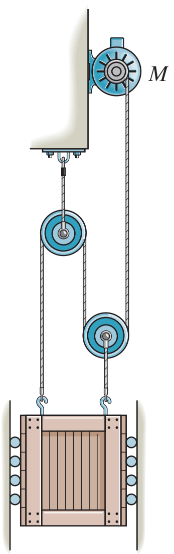
Prob. 14–49

14–50. Determine the power output of the draw-works motor M necessary to lift the 600-lb drill pipe upward with a constant speed of 4 ft/s. The cable is tied to the top of the oil rig, wraps around the lower pulley, then around the top pulley, and then to the motor.



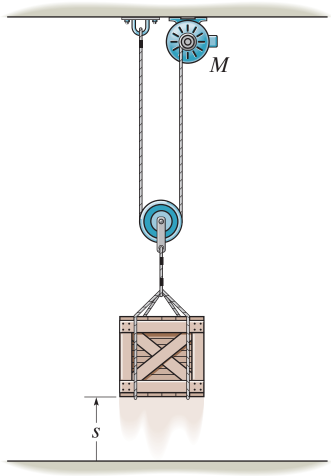
Prob. 14–50

14–51. The 1000-lb elevator is hoisted by the pulley system and motor M. If the motor exerts a constant force of 500 lb on the cable, determine the power that must be supplied to the motor at the instant the load has been hoisted $s = 15$ ft starting from rest. The motor has an efficiency of $\varepsilon = 0.65$.



Prob. 14–51

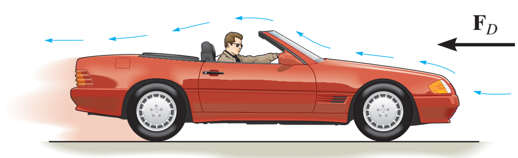
***14–52.** The 50-lb crate is given a speed of 10 ft/s in $t = 4$ s starting from rest. If the acceleration is constant, determine the power that must be supplied to the motor when $t = 2$ s. The motor has an efficiency $\varepsilon = 0.65$. Neglect the mass of the pulley and cable.



Prob. 14–52

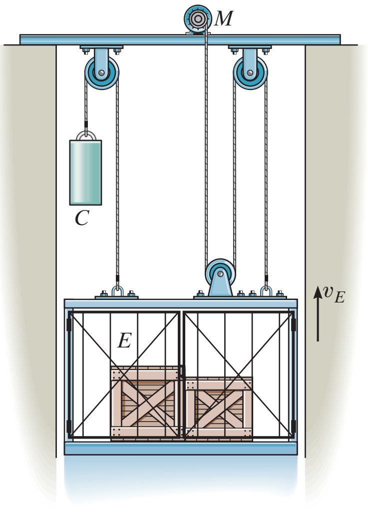
14–53. The sports car has a mass of 2.3 Mg, and while it is traveling at 28 m/s the driver causes it to accelerate at 5 m/s^2. If the drag resistance on the car due to the wind is $F_D = (0.3v^2)$ N, where v is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of $\varepsilon = 0.68$.

14–54. The sports car has a mass of 2.3 Mg and accelerates at 6 m/s^2, starting from rest. If the drag resistance on the car due to the wind is $F_D = (10v)$ N, where v is the velocity in m/s, determine the power supplied to the engine when $t = 5$ s. The engine has a running efficiency of $\varepsilon = 0.68$.



Probs. 14–53/54

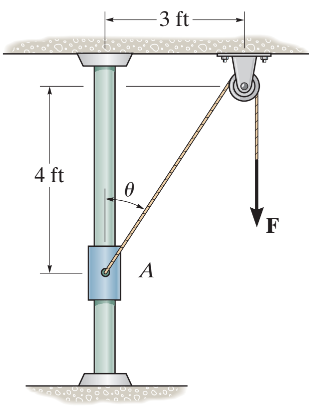
14–55. The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\varepsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



Prob. 14–55

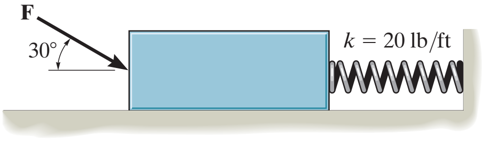
***14–56.** The 10-lb collar starts from rest at A and is lifted by applying a constant vertical force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.

14–57. The 10-lb collar starts from rest at A and is lifted with a constant speed of 2 ft/s along the smooth rod. Determine the power developed by the force **F** at the instant shown.



Prob. 14–57

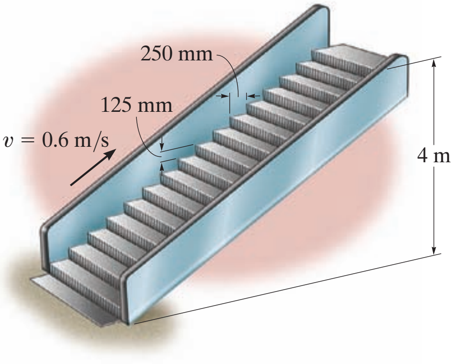
14–58. The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (40 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched ($s = 0$) and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.5$ ft.



Prob. 14–58

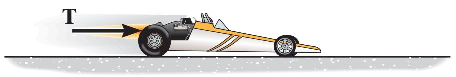
14–59. The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.

***14–60.** If the escalator in Prob. 14–46 is not moving, determine the constant speed at which a man having a mass of 80 kg must walk up the steps to generate 100 W of power—the same amount that is needed to power a standard light bulb.



Probs. 14–59/60

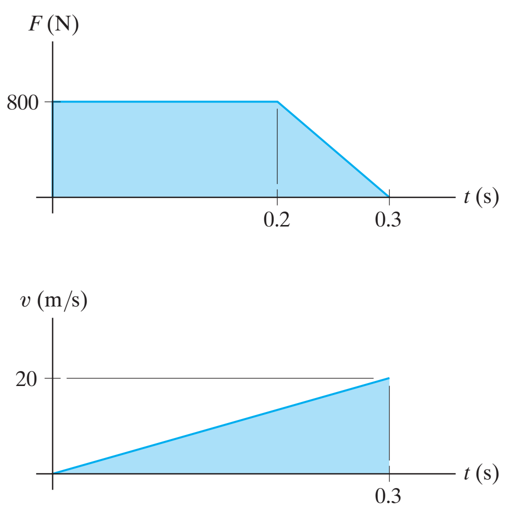
14–61. If the jet on the dragster supplies a constant thrust of $T = 20$ kN, determine the power generated by the jet as a function of time. Neglect drag and rolling resistance, and the loss of fuel. The dragster has a mass of 1 Mg and starts from rest.



Prob. 14–61

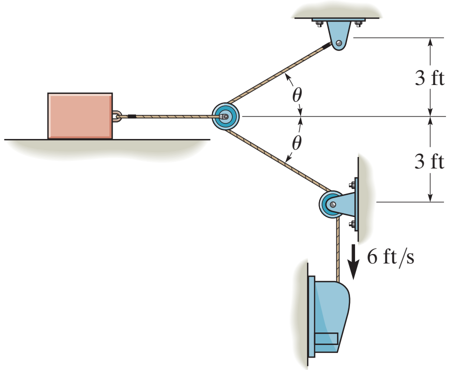
14–62. An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.

14–63. An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.



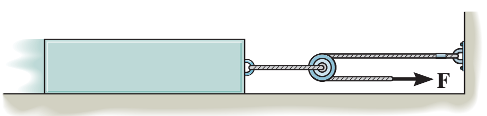
Probs. 14–62/63

***14–64.** The block has a weight of 80 lb and rests on the floor for which $\mu_k = 0.4$. If the motor draws in the cable at a constant rate of 6 ft/s, determine the output of the motor at the instant $\theta = 30°$. Neglect the mass of the cable and pulleys.



Prob. 14–64

14–65. The block has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. If a force $F = (60t^2)$ N, where t is in seconds, is applied to the cable, determine the power developed by the force when $t = 5$ s. *Hint*: First determine the time needed for the force to cause motion.

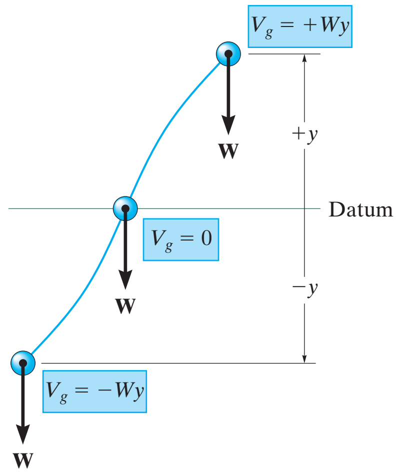


Prob. 14–65

14.5 Conservative Forces and Potential Energy

Conservative Force. If the work of a force is *independent of the path* and depends only on the force's initial and final positions on the path, then we can classify this force as a *conservative force*. Examples of conservative forces are the weight of a particle and the force developed by a spring. The work done by the weight depends *only* on the *vertical displacement* of the weight, and the work done by a spring force depends *only* on the spring's *elongation* or *compression*.

In contrast to a conservative force, consider the force of friction exerted *on a sliding object* by a fixed surface. The work done by the frictional force *depends on the path*—the longer the path, the greater the work. Consequently, *frictional forces are nonconservative*. The work is dissipated from the body in the form of heat.



Gravitational potential energy

Fig. 14–17

Energy. Energy is defined as the capacity for doing work. For example, if a particle is originally at rest, then the principle of work and energy states that $\Sigma U_{1\rightarrow 2} = T_2$. In other words, the kinetic energy is equal to the work that must be done on the particle to bring it from a state of rest to a speed v. Thus, the *kinetic energy* is a measure of the particle's *capacity to do work*, which is associated with the *motion* of the particle. When energy comes from the *position* of the particle, measured from a fixed datum or reference plane, it is called potential energy. Thus, *potential energy* is a measure of the amount of work a conservative force will do when it moves from a given position to the datum. In mechanics, the potential energy created by gravity (weight) and an elastic spring is important.

Gravitational Potential Energy. If a particle is located a distance y *above* an arbitrarily selected datum, as shown in Fig. 14–17, the particle's weight **W** has positive *gravitational potential energy*, V_g, since **W** has the capacity of doing positive work when the particle is moved back down to the datum. Likewise, if the particle is located a distance y *below* the datum, V_g is negative since the weight does negative work when the particle is moved back up to the datum. At the datum $V_g = 0$.

In general, if y is *positive upward*, the gravitational potential energy of the particle of weight W is*

$$\boxed{V_g = Wy} \qquad (14\text{–}13)$$

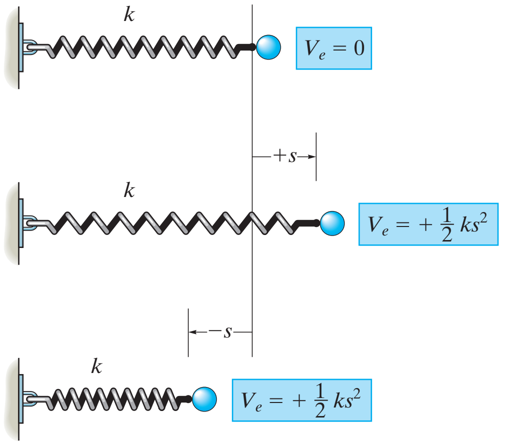
Gravitational potential energy of this weight is increased as it is hoisted upward. (© R.C. Hibbeler)

*Here the weight is assumed to be *constant*. This assumption is suitable for small differences in elevation Δy. If the elevation change is significant, however, a variation of weight with elevation must be taken into account (see Prob. 14–82).

Elastic Potential Energy. When an elastic spring is elongated or compressed a distance s from its unstretched position, elastic potential energy V_e can be stored in the spring. This energy is

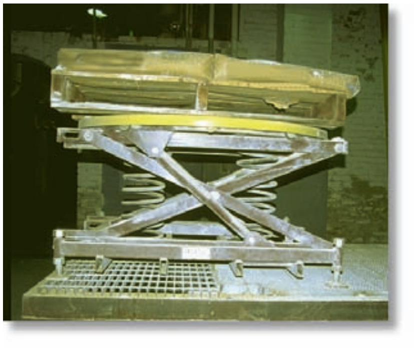
$$\boxed{V_e = +\tfrac{1}{2}ks^2} \tag{14–14}$$

Here V_e is *always positive* since, in the deformed position, the force of the spring has the *capacity* or "potential" for always doing positive work on the particle when the spring is returned to its unstretched position, Fig. 14–18.



Elastic potential energy

Fig. 14–18



The weight of the sacks resting on this platform causes potential energy to be stored in the supporting springs. As each sack is removed, the platform will *rise* slightly since some of the potential energy within the springs will be transformed into an increase in gravitational potential energy of the remaining sacks. Such a device is useful for removing the sacks without having to bend over to pick them up as they are unloaded. (© R.C. Hibbeler)

Potential Function. In the general case, if a particle is subjected to both gravitational and elastic forces, the particle's potential energy can be expressed as a *potential function*, which is the algebraic sum

$$\boxed{V = V_g + V_e} \tag{14–15}$$

Measurement of V depends on the location of the particle with respect to a selected datum in accordance with Eqs. 14–13 and 14–14.

The work done by a conservative force in moving the particle from one point to another point is measured by the *difference* of this function, i.e.,

$$U_{1-2} = V_1 - V_2 \tag{14–16}$$

For example, the potential function for a particle of weight W suspended from a spring can be expressed in terms of its position, s, measured from a datum located at the unstretched length of the spring, Fig. 14–19. We have

$$\begin{aligned} V &= V_g + V_e \\ &= -Ws + \tfrac{1}{2}ks^2 \end{aligned}$$

If the particle moves from s_1 to a lower position s_2, then applying Eq. 14–16 it can be seen that the work of $\mathbf{W}$ and $\mathbf{F}_s$ is

$$\begin{aligned} U_{1-2} = V_1 - V_2 &= \left(-Ws_1 + \tfrac{1}{2}ks_1^2\right) - \left(-Ws_2 + \tfrac{1}{2}ks_2^2\right) \\ &= W(s_2 - s_1) - \left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right) \end{aligned}$$

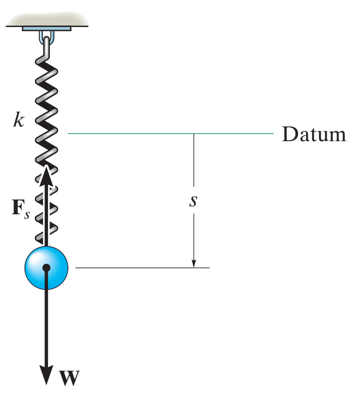


Fig. 14–19

When the displacement along the path is infinitesimal, i.e., from point (x, y, z) to $(x + dx, y + dy, z + dz)$, Eq. 14–16 becomes

$$\begin{aligned} dU &= V(x, y, z) - V(x + dx, y + dy, z + dz) \\ &= -dV(x, y, z) \end{aligned} \tag{14–17}$$

If we represent both the force and its displacement as Cartesian vectors, then the work can also be expressed as

$$\begin{aligned} dU &= \mathbf{F} \cdot d\mathbf{r} = (F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}) \\ &= F_x\,dx + F_y\,dy + F_z\,dz \end{aligned}$$

Substituting this result into Eq. 14–17 and expressing the differential $dV(x, y, z)$ in terms of its partial derivatives yields

$$F_x\,dx + F_y\,dy + F_z\,dz = -\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right)$$

Since changes in x, y, and z are all independent of one another, this equation is satisfied provided

$$F_x = -\frac{\partial V}{\partial x}, \qquad F_y = -\frac{\partial V}{\partial y}, \qquad F_z = -\frac{\partial V}{\partial z} \tag{14–18}$$

Thus,

$$\begin{aligned} \mathbf{F} &= -\frac{\partial V}{\partial x}\mathbf{i} - \frac{\partial V}{\partial y}\mathbf{j} - \frac{\partial V}{\partial z}\mathbf{k} \\ &= -\left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right)V \end{aligned}$$

or

$$\mathbf{F} = -\nabla V \tag{14–19}$$

where ∇ (del) represents the vector operator $\nabla = (\partial/\partial x)\mathbf{i} + (\partial/\partial y)\mathbf{j} + (\partial/\partial z)\mathbf{k}$.

Equation 14–19 relates a force **F** to its potential function V and thereby provides a mathematical criterion for proving that **F** is conservative. For example, the gravitational potential function for a weight located a distance y above a datum is $V_g = Wy$. To prove that **W** is conservative, it is necessary to show that it satisfies Eq. 14–18 (or Eq. 14–19), in which case

$$F_y = -\frac{\partial V}{\partial y}; \qquad\qquad F_y = -\frac{\partial}{\partial y}(Wy) = -W$$

The negative sign indicates that **W** acts downward, opposite to positive y, which is upward.

14.6 Conservation of Energy

When a particle is acted upon by a system of *both* conservative and nonconservative forces, the portion of the work done by the *conservative forces* can be written in terms of the difference in their potential energies using Eq. 14–16, i.e., $(\Sigma U_{1-2})_{\text{cons.}} = V_1 - V_2$. As a result, the principle of work and energy can be written as

$$T_1 + V_1 + (\Sigma U_{1-2})_{\text{noncons.}} = T_2 + V_2 \tag{14–20}$$

Here $(\Sigma U_{1-2})_{\text{noncons.}}$ represents the work of the nonconservative forces acting on the particle. If *only conservative forces* do work then we have

$$\boxed{T_1 + V_1 = T_2 + V_2} \tag{14–21}$$

This equation is referred to as the *conservation of mechanical energy* or simply the *conservation of energy*. It states that during the motion the sum of the particle's kinetic and potential energies remains *constant*. For this to occur, kinetic energy must be transformed into potential energy, and vice versa. For example, if a ball of weight $\mathbf{W}$ is dropped from a height h above the ground (datum), Fig. 14–20, the potential energy of the ball is maximum before it is dropped, at which time its kinetic energy is zero. The total mechanical energy of the ball in its initial position is thus

$$E = T_1 + V_1 = 0 + Wh = Wh$$

When the ball has fallen a distance $h/2$, its speed can be determined by using $v^2 = v_0^2 + 2a_c(y - y_0)$, which yields $v = \sqrt{2g(h/2)} = \sqrt{gh}$. The energy of the ball at the mid-height position is therefore

$$E = T_2 + V_2 = \frac{1}{2}\frac{W}{g}\left(\sqrt{gh}\right)^2 + W\left(\frac{h}{2}\right) = Wh$$

Just before the ball strikes the ground, its potential energy is zero and its speed is $v = \sqrt{2gh}$. Here, again, the total energy of the ball is

$$E = T_3 + V_3 = \frac{1}{2}\frac{W}{g}\left(\sqrt{2gh}\right)^2 + 0 = Wh$$

Note that when the ball comes in contact with the ground, it deforms somewhat, and provided the ground is hard enough, the ball will rebound off the surface, reaching a new height h', which will be *less* than the height h from which it was first released. Neglecting air friction, the difference in height accounts for an energy loss, $E_l = W(h - h')$, which occurs during the collision. Portions of this loss produce noise, localized deformation of the ball and ground, and heat.

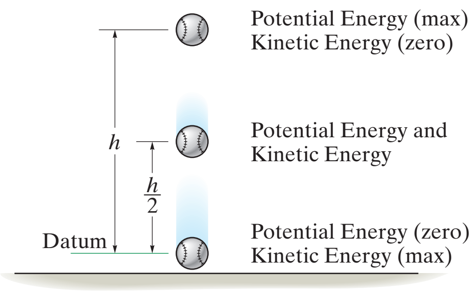


Fig. 14–20

System of Particles. If a system of particles is *subjected only to conservative forces*, then an equation similar to Eq. 14–21 can be written for the particles. Applying the ideas of the preceding discussion, Eq. 14–8 ($\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$) becomes

$$\boxed{\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2} \tag{14–22}$$

Here, the sum of the system's initial kinetic and potential energies is equal to the sum of the system's final kinetic and potential energies. In other words, $\Sigma T + \Sigma V = \text{const.}$

Procedure for Analysis

The conservation of energy equation can be used to solve problems involving *velocity, displacement*, and *conservative force systems*. It is generally *easier to apply* than the principle of work and energy because this equation requires specifying the particle's kinetic and potential energies at only *two points* along the path, rather than determining the work when the particle moves through a *displacement*. For application it is suggested that the following procedure be used.

Potential Energy.

- Draw two diagrams showing the particle located at its initial and final points along the path.
- If the particle is subjected to a vertical displacement, establish the fixed horizontal datum from which to measure the particle's gravitational potential energy V_g.
- Data pertaining to the elevation y of the particle from the datum and the stretch or compression s of any connecting springs can be determined from the geometry associated with the two diagrams.
- Recall $V_g = Wy$, where y is positive upward from the datum and negative downward from the datum; also for a spring, $V_e = \frac{1}{2}ks^2$, which is *always positive*.

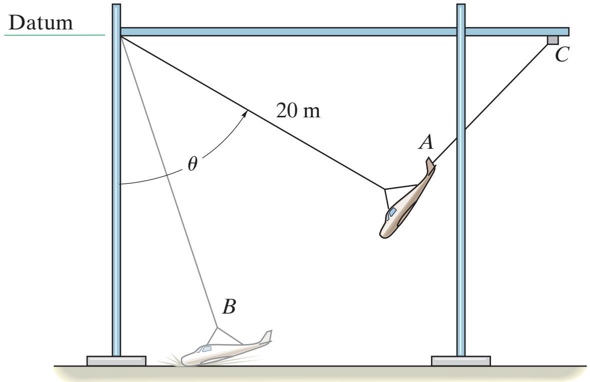
Conservation of Energy.

- Apply the equation $T_1 + V_1 = T_2 + V_2$.
- When determining the kinetic energy, $T = \frac{1}{2}mv^2$, remember that the particle's speed v must be measured from an inertial reference frame.

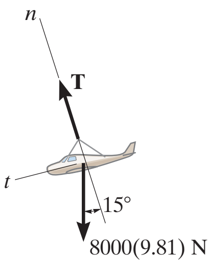
EXAMPLE 14.9

The gantry structure in the photo is used to test the response of an airplane during a crash. As shown in Fig. 14–21*a*, the plane, having a mass of 8 Mg, is hoisted back until $\theta = 60^\circ$, and then the pull-back cable *AC* is released when the plane is at rest. Determine the speed of the plane just before it crashes into the ground, $\theta = 15^\circ$. Also, what is the maximum tension developed in the supporting cable during the motion? Neglect the size of the airplane and the effect of lift caused by the wings during the motion.

(© R.C. Hibbeler)



(a)



(b)

Fig. 14–21

SOLUTION

Since the force of the cable does *no work* on the plane, it must be obtained using the equation of motion. First, however, we must determine the plane's speed at *B*.

Potential Energy. For convenience, the datum has been established at the top of the gantry, Fig. 14–21*a*.

Conservation of Energy.

$$T_A + V_A = T_B + V_B$$

$$0 - 8000 \text{ kg } (9.81 \text{ m/s}^2)(20 \cos 60^\circ \text{ m}) =$$

$$\tfrac{1}{2}(8000 \text{ kg})v_B^2 - 8000 \text{ kg } (9.81 \text{ m/s}^2)(20 \cos 15^\circ \text{ m})$$

$$v_B = 13.52 \text{ m/s} = 13.5 \text{ m/s} \qquad \textit{Ans.}$$

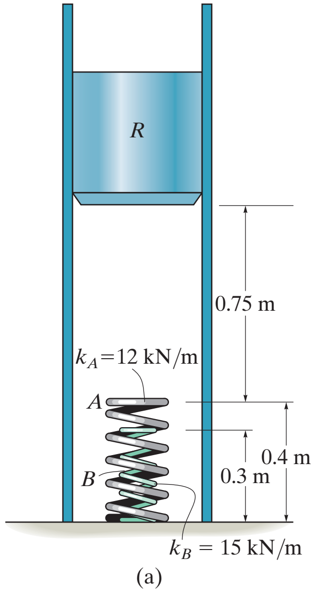
Equation of Motion. From the free-body diagram when the plane is at *B*, Fig. 14–21*b*, we have

$+\nwarrow \quad \Sigma F_n = ma_n;$

$$T - (8000(9.81) \text{ N}) \cos 15^\circ = (8000 \text{ kg})\frac{(13.52 \text{ m/s})^2}{20 \text{ m}}$$

$$T = 149 \text{ kN} \qquad \textit{Ans.}$$

EXAMPLE 14.10



The ram R shown in Fig. 14–22*a* has a mass of 100 kg and is released from rest 0.75 m from the top of a spring, A, that has a stiffness $k_A = 12\text{ kN/m}$. If a second spring B, having a stiffness $k_B = 15\text{ kN/m}$, is "nested" in A, determine the maximum displacement of A needed to stop the downward motion of the ram. The unstretched length of each spring is indicated in the figure. Neglect the mass of the springs.

SOLUTION

Potential Energy. We will *assume* that the ram compresses *both* springs at the instant it comes to rest. The datum is located through the center of gravity of the ram at its initial position, Fig. 14–22*b*. When the kinetic energy is reduced to zero ($v_2 = 0$), A is compressed a distance s_A and B compresses $s_B = s_A - 0.1$ m.

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \left\{\tfrac{1}{2}k_A s_A^2 + \tfrac{1}{2}k_B(s_A - 0.1)^2 - Wh\right\}$$

$$0 + 0 = 0 + \left\{\tfrac{1}{2}(12\,000\text{ N/m})s_A^2 + \tfrac{1}{2}(15\,000\text{ N/m})(s_A - 0.1\text{ m})^2 - 981\text{ N}\,(0.75\text{ m} + s_A)\right\}$$

Rearranging the terms,

$$13\,500 s_A^2 - 2481 s_A - 660.75 = 0$$

Using the quadratic formula and solving for the positive root, we have

$$s_A = 0.331\text{ m} \qquad \textit{Ans.}$$

Since $s_B = 0.331\text{ m} - 0.1\text{ m} = 0.231\text{ m}$, which is positive, the assumption that *both* springs are compressed by the ram is correct.

NOTE: The second root, $s_A = -0.148$ m, does not represent the physical situation. Since positive s is measured downward, the negative sign indicates that spring A would have to be "extended" by an amount of 0.148 m to stop the ram.

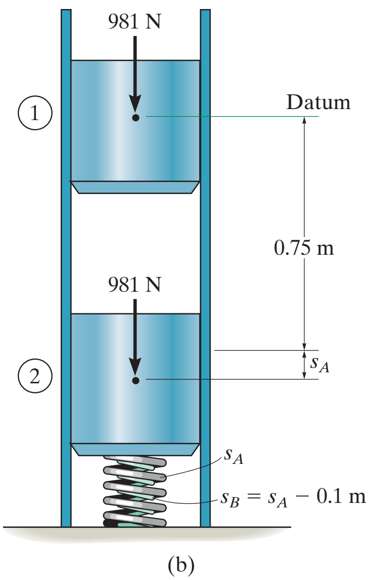
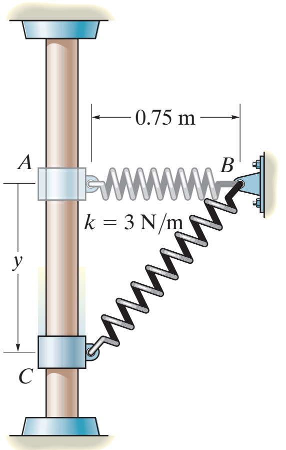


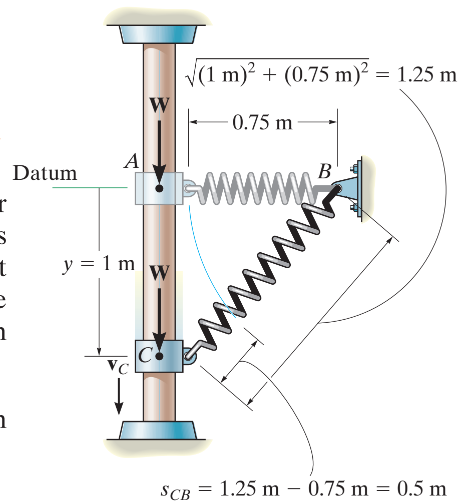
Fig. 14–22

EXAMPLE 14.11

A smooth 2-kg collar, shown in Fig. 14–23*a*, fits loosely on the vertical shaft. If the spring is unstretched when the collar is in the position A, determine the speed at which the collar is moving when $y = 1$ m, if (a) it is released from rest at A, and (b) it is released at A with an *upward* velocity $v_A = 2$ m/s.



(a)



(b)

Fig. 14–23

SOLUTION

Part (a) Potential Energy. For convenience, the datum is established through AB, Fig. 14–23*b*. When the collar is at C, the gravitational potential energy is $-(mg)y$, since the collar is *below* the datum, and the elastic potential energy is $\frac{1}{2}ks_{CB}^2$. Here $s_{CB} = 0.5$ m, which represents the *stretch* in the spring as shown in the figure.

Conservation of Energy.

$$T_A + V_A = T_C + V_C$$

$$0 + 0 = \tfrac{1}{2}mv_C^2 + \left\{\tfrac{1}{2}ks_{CB}^2 - mgy\right\}$$

$$0 + 0 = \left\{\tfrac{1}{2}(2\text{ kg})v_C^2\right\} + \left\{\tfrac{1}{2}(3\text{ N/m})(0.5\text{ m})^2 - 2(9.81)\text{ N }(1\text{ m})\right\}$$

$$v_C = 4.39\text{ m/s}\downarrow \qquad \textit{Ans.}$$

This problem can also be solved by using the equation of motion or the principle of work and energy. Note that for *both* of these methods the variation of the magnitude and direction of the spring force must be taken into account (see Example 13.4). Here, however, the above solution is clearly advantageous since the calculations depend *only* on data calculated at the initial and final points of the path.

Part (b) Conservation of Energy. If $v_A = 2$ m/s, using the data in Fig. 14–23*b*, we have

$$T_A + V_A = T_C + V_C$$

$$\tfrac{1}{2}mv_A^2 + 0 = \tfrac{1}{2}mv_C^2 + \left\{\tfrac{1}{2}ks_{CB}^2 - mgy\right\}$$

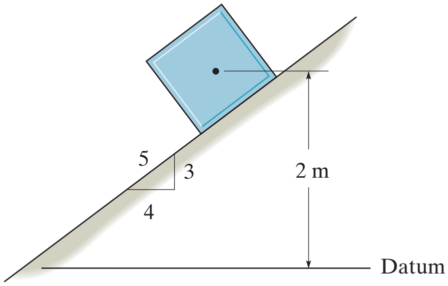
$$\tfrac{1}{2}(2\text{ kg})(2\text{ m/s})^2 + 0 = \tfrac{1}{2}(2\text{ kg})v_C^2 + \left\{\tfrac{1}{2}(3\text{ N/m})(0.5\text{ m})^2 - 2(9.81)\text{ N }(1\text{ m})\right\}$$

$$v_C = 4.82\text{ m/s}\downarrow \qquad \textit{Ans.}$$

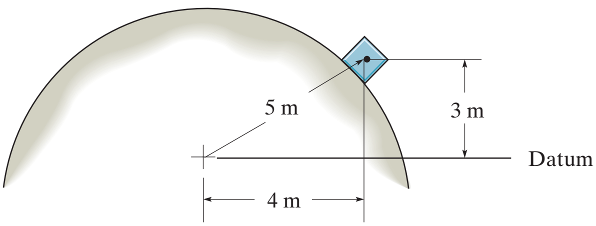
NOTE: The kinetic energy of the collar depends only on the *magnitude* of velocity, and therefore it is immaterial if the collar is moving up or down at 2 m/s when released at A.

PRELIMINARY PROBLEMS

P14–3. Determine the potential energy of the block that has a weight of 100 N.



(a)



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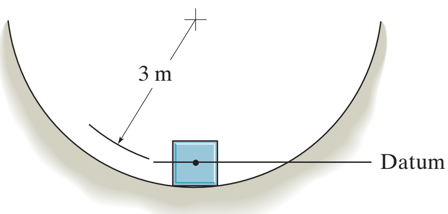
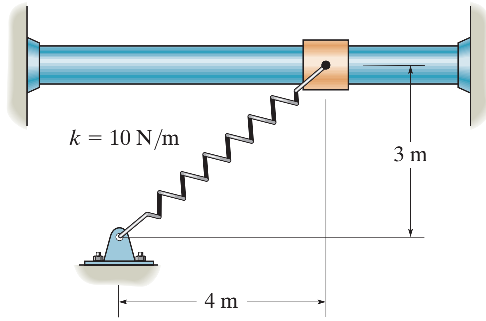


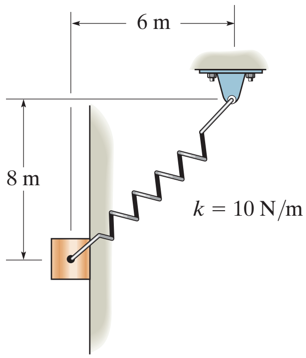
(c)

Prob. P14–3

P14–4. Determine the potential energy in the spring that has an unstretched length of 4 m.



(a)



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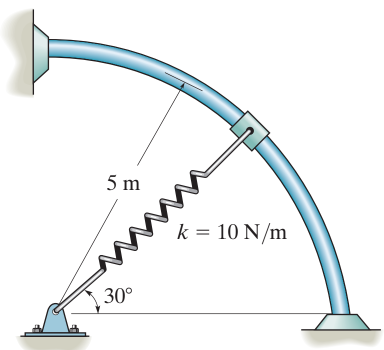
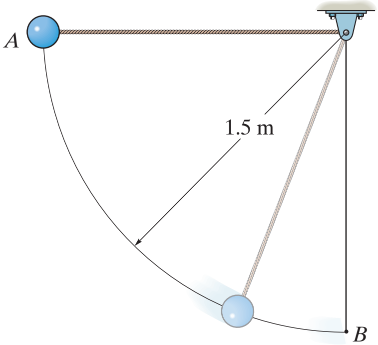


(c)

Prob. P14–4

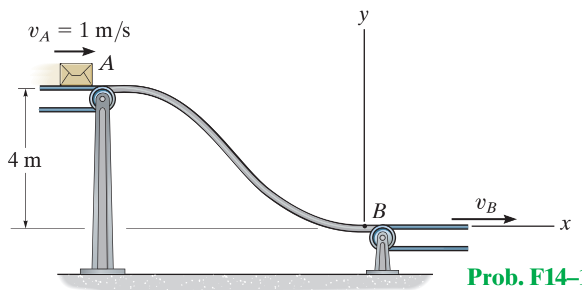
FUNDAMENTAL PROBLEMS

F14–13. The 2-kg pendulum bob is released from rest when it is at A. Determine the speed of the bob and the tension in the cord when the bob passes through its lowest position, B.



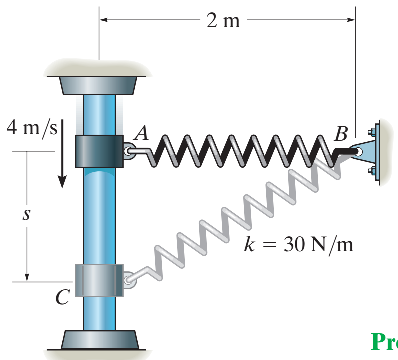
Prob. F14–13

F14–14. The 2-kg package leaves the conveyor belt at A with a speed of $v_A = 1$ m/s and slides down the smooth ramp. Determine the required speed of the conveyor belt at B so that the package can be delivered without slipping on the belt. Also, find the normal reaction the curved portion of the ramp exerts on the package at B if $\rho_B = 2$ m.



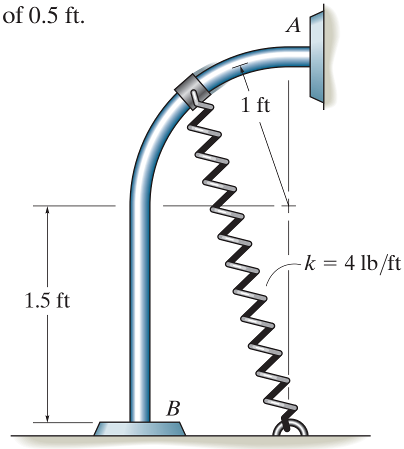
Prob. F14–14

F14–15. The 2-kg collar is given a downward velocity of 4 m/s when it is at A. If the spring has an unstretched length of 1 m and a stiffness of $k = 30$ N/m, determine the velocity of the collar at $s = 1$ m.



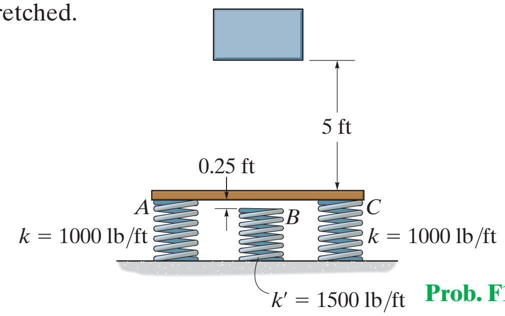
Prob. F14–15

F14–16. The 5-lb collar is released from rest at A and travels along the frictionless guide. Determine the speed of the collar when it strikes the stop B. The spring has an unstretched length of 0.5 ft.



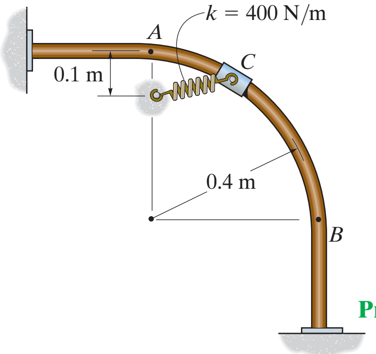
Prob. F14–16

F14–17. The 75-lb block is released from rest 5 ft above the plate. Determine the compression of each spring when the block momentarily comes to rest after striking the plate. Neglect the mass of the plate. The springs are initially unstretched.



Prob. F14–17

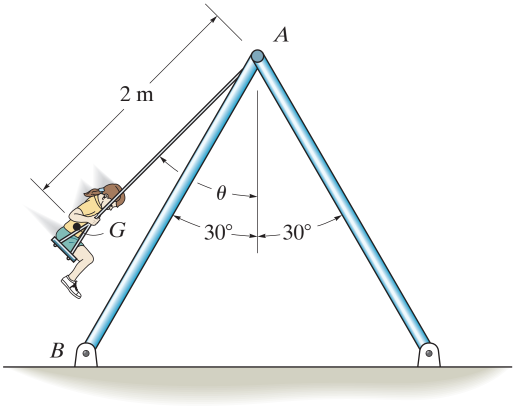
F14–18. The 4-kg collar C has a velocity of $v_A = 2$ m/s when it is at A. If the guide rod is smooth, determine the speed of the collar when it is at B. The spring has an unstretched length of $l_0 = 0.2$ m.



Prob. F14–18

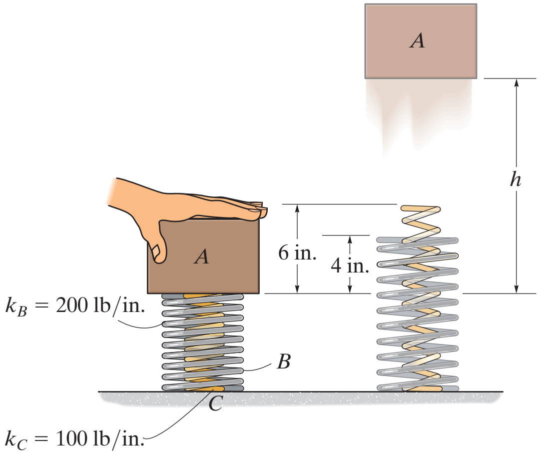
PROBLEMS

14–66. The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60°$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.



Prob. 14–66

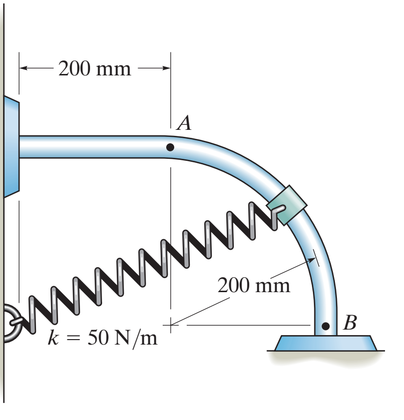
14–67. The 30-lb block A is placed on top of two nested springs B and C and then pushed down to the position shown. If it is then released, determine the maximum height h to which it will rise.



Prob. 14–67

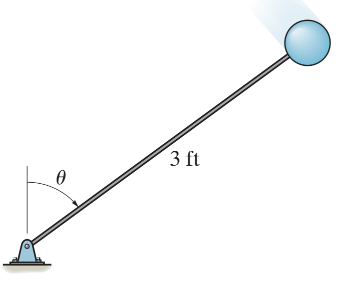
***14–68.** The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels down along the smooth guide. Determine the speed of the collar when it reaches point B, which is located just before the end of the curved portion of the rod. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.

14–69. The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels along the smooth guide. Determine its speed when its center reaches point B and the normal force it exerts on the rod at this point. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.



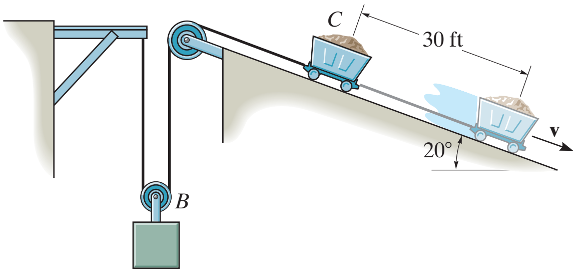
Probs. 14–68/69

14–70. The ball has a weight of 15 lb and is fixed to a rod having a negligible mass. If it is released from rest when $\theta = 0°$, determine the angle θ at which the compressive force in the rod becomes zero.



Prob. 14–70

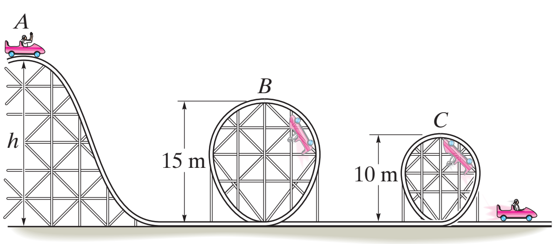
14–71. The car C and its contents have a weight of 600 lb, whereas block B has a weight of 200 lb. If the car is released from rest, determine its speed when it travels 30 ft down the 20° incline. *Suggestion*: To measure the gravitational potential energy, establish separate datums at the initial elevations of B and C.



Prob. 14–71

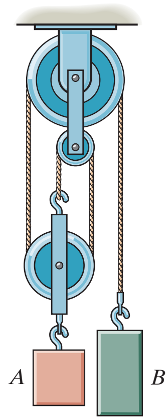
***14–72.** The roller coaster car has a mass of 700 kg, including its passenger. If it starts from the top of the hill A with a speed $v_A = 3\text{ m/s}$, determine the minimum height h of the hill crest so that the car travels around the inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5$ m and $\rho_C = 5$ m.

14–73. The roller coaster car has a mass of 700 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill crest so that the car travels around both inside the loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5$ m and $\rho_C = 5$ m.



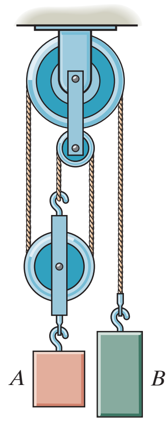
Probs. 14–72/73

14–74. The assembly consists of two blocks A and B which have a mass of 20 kg and 30 kg, respectively. Determine the speed of each block when B descends 1.5 m. The blocks are released from rest. Neglect the mass of the pulleys and cords.



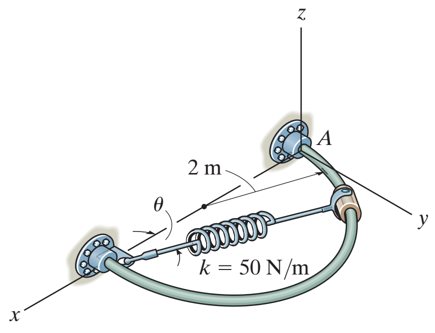
Prob. 14–74

14–75. The assembly consists of two blocks A and B, which have a mass of 20 kg and 30 kg, respectively. Determine the distance B must descend in order for A to achieve a speed of 3 m/s starting from rest.



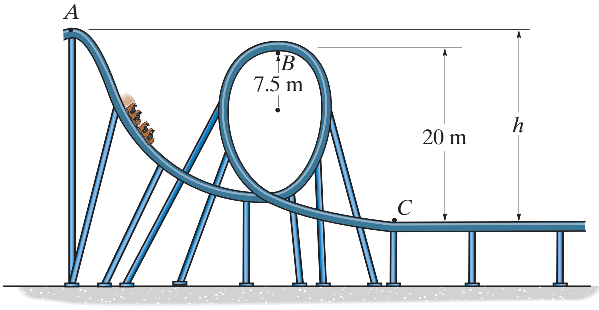
Prob. 14–75

***14–76.** The spring has a stiffness $k = 50$ N/m and an unstretched length of 0.3 m. If it is attached to the 2-kg smooth collar and the collar is released from rest at A ($\theta = 0°$), determine the speed of the collar when $\theta = 60°$. The motion occurs in the horizontal plane. Neglect the size of the collar.



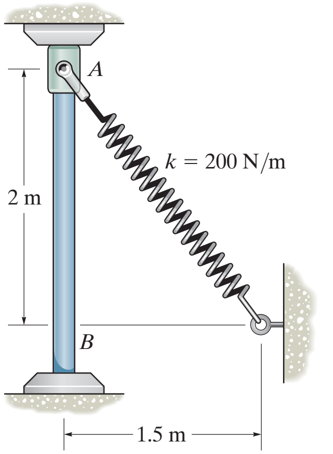
Prob. 14–76

14–77. The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



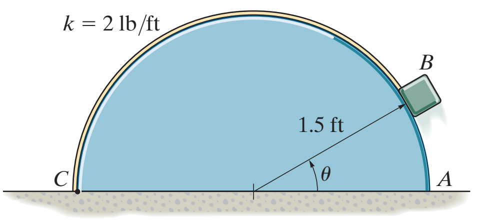
Prob. 14–77

14–78. The spring has a stiffness $k = 200$ N/m and an unstretched length of 0.5 m. If it is attached to the 3-kg smooth collar and the collar is released from rest at A, determine the speed of the collar when it reaches B. Neglect the size of the collar.



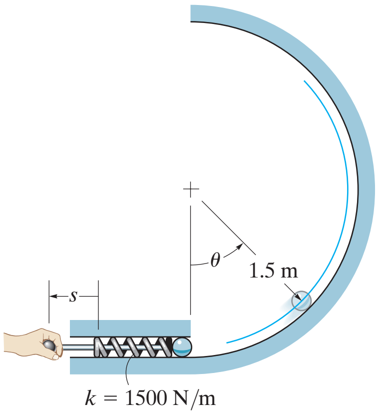
Prob. 14–78

14–79. A 2-lb block rests on the smooth semicylindrical surface at A. An elastic cord having a stiffness of $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at C. If the block is released from rest at $\theta = 0$, A, determine the longest unstretched length of the cord so the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.



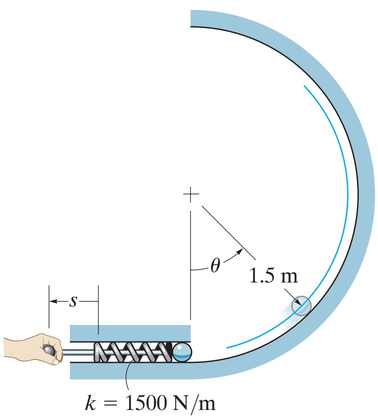
Prob. 14–79

***14–80.** When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the speed of the 0.3-kg ball and the normal reaction of the circular track on the ball when $\theta = 60°$. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



Prob. 14–80

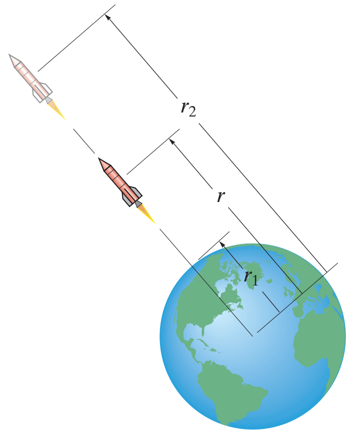
14–81. When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the maximum angle θ the ball will travel without leaving the circular track. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



Prob. 14–81

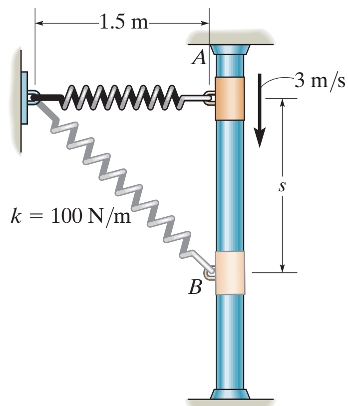
14–82. If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \rightarrow \infty$. Also, prove that F is a conservative force.

14–83. A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming that no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



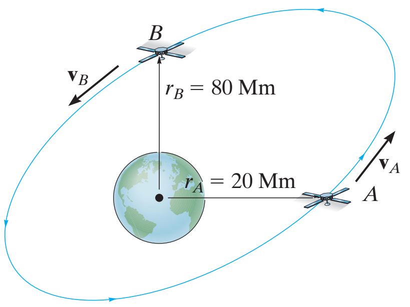
Probs. 14–82/83

***14–84.** The 4-kg smooth collar has a speed of 3 m/s when it is at $s = 0$. Determine the maximum distance s it travels before it stops momentarily. The spring has an unstretched length of 1 m.



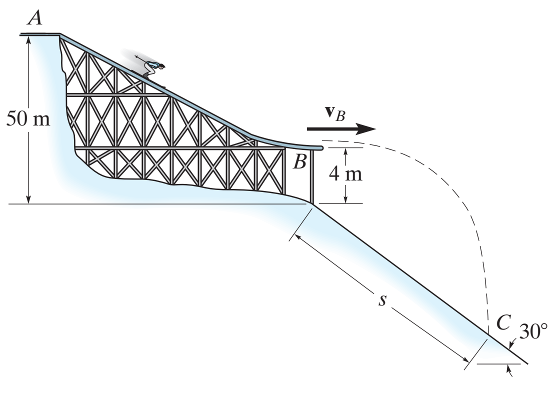
Prob. 14–84

14–85. A 60-kg satellite travels in free flight along an elliptical orbit such that at A, where $r_A = 20$ Mm, it has a speed $v_A = 40$ Mm/h. What is the speed of the satellite when it reaches point B, where $r_B = 80$ Mm? *Hint*: See Prob. 14–82, where $M_e = 5.976(10^{24})$ kg and $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$.



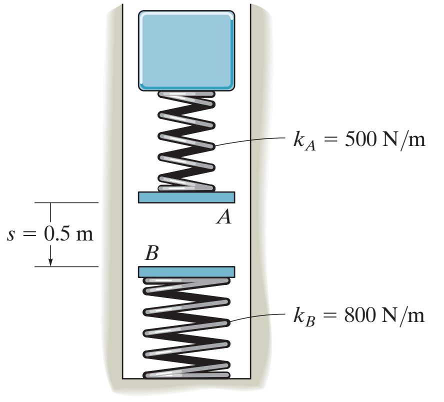
Prob. 14–85

14–86. The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, compute the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



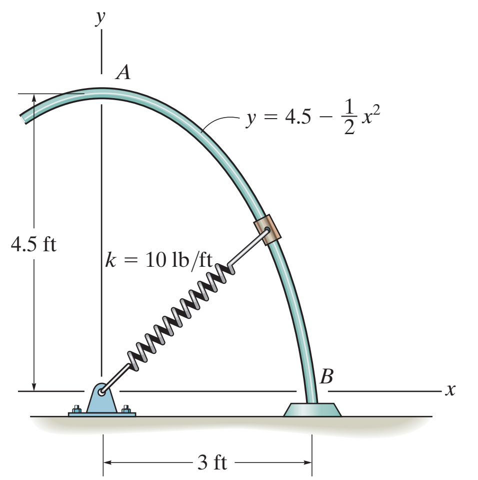
Prob. 14–86

14–87. The block has a mass of 20 kg and is released from rest when $s = 0.5$ m. If the mass of the bumpers A and B can be neglected, determine the maximum deformation of each spring due to the collision.



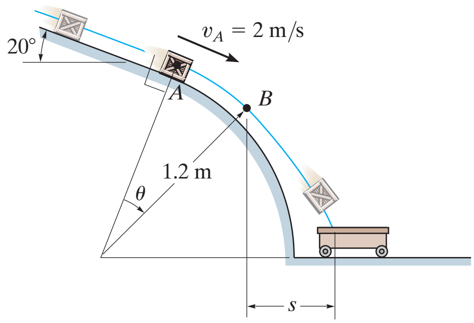
Prob. 14–87

***14–88.** The 2-lb collar has a speed of 5 ft/s at A. The attached spring has an unstretched length of 2 ft and a stiffness of $k = 10$ lb/ft. If the collar moves over the smooth rod, determine its speed when it reaches point B, the normal force of the rod on the collar, and the rate of decrease in its speed.



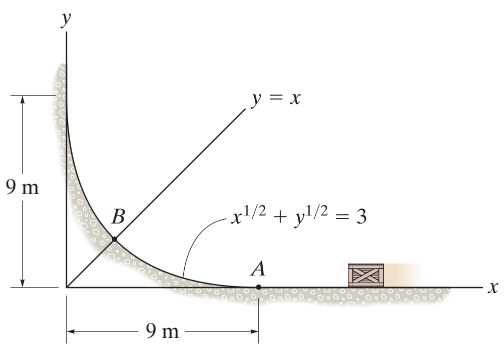
Prob. 14–88

14–89. When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.



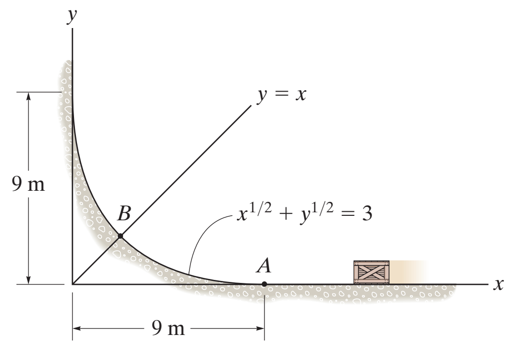
Prob. 14–89

14–90. When the 5-kg box reaches point A it has a speed $v_A = 10$ m/s. Determine the normal force the box exerts on the surface when it reaches point B. Neglect friction and the size of the box.



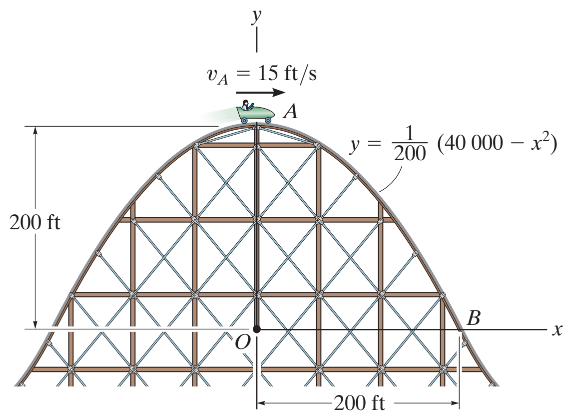
Prob. 14–90

14–91. When the 5-kg box reaches point A it has a speed $v_A = 10$ m/s. Determine how high the box reaches up the surface before it comes to a stop. Also, what is the resultant normal force on the surface at this point and the acceleration? Neglect friction and the size of the box.



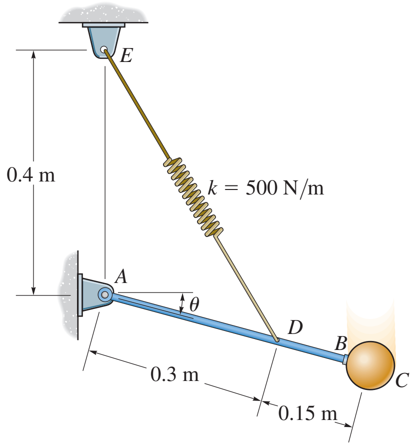
Prob. 14–91

***14–92.** The roller coaster car has a speed of 15 ft/s when it is at the crest of a vertical parabolic track. Determine the car's velocity and the normal force it exerts on the track when it reaches point B. Neglect friction and the mass of the wheels. The total weight of the car and the passengers is 350 lb.



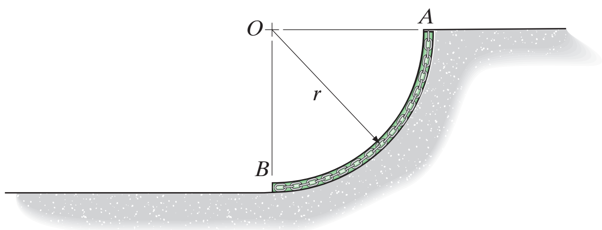
Prob. 14–92

14–93. The 10-kg sphere C is released from rest when $\theta = 0°$ and the tension in the spring is 100 N. Determine the speed of the sphere at the instant $\theta = 90°$. Neglect the mass of rod AB and the size of the sphere.



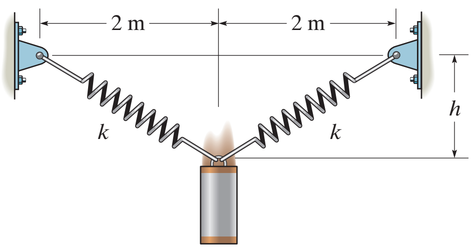
Prob. 14–93

14–94. A quarter-circular tube AB of mean radius r contains a smooth chain that has a mass per unit length of m_0. If the chain is released from rest from the position shown, determine its speed when it emerges completely from the tube.



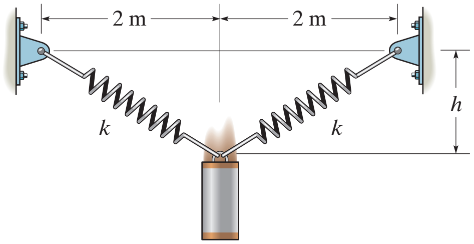
Prob. 14–94

14–95. The cylinder has a mass of 20 kg and is released from rest when $h = 0$. Determine its speed when $h = 3$ m. Each spring has a stiffness $k = 40$ N/m and an unstretched length of 2 m.



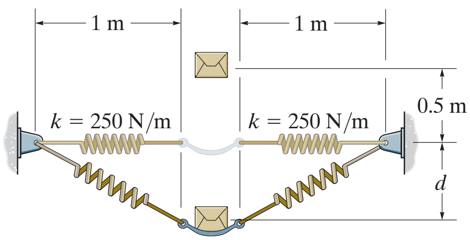
Prob. 14–95

***14–96.** If the 20-kg cylinder is released from rest at $h = 0$, determine the required stiffness k of each spring so that its motion is arrested or stops when $h = 0.5$ m. Each spring has an unstretched length of 1 m.



Prob. 14–96

14–97. A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



Prob. 14–97

CONCEPTUAL PROBLEMS

C14–1. The roller coaster is momentarily at rest at A. Determine the approximate normal force it exerts on the track at B. Also determine its approximate acceleration at this point. Use numerical data, and take scaled measurements from the photo with a known height at A.

Prob. C14–1 (© R.C. Hibbeler)

C14–2. As the large ring rotates, the operator can apply a breaking mechanism that binds the cars to the ring, which then allows the cars to rotate with the ring. Assuming the passengers are not belted into the cars, determine the smallest speed of the ring (cars) so that no passenger will fall out. When should the operator release the brake so that the cars can achieve their greatest speed as they slide freely on the ring? Estimate the greatest normal force of the seat on a passenger when this speed is reached. Use numerical values to explain your answer.

Prob. C14–2 (© R.C. Hibbeler)

C14–3. The woman pulls the water balloon launcher back, stretching each of the four elastic cords. Estimate the maximum height and the maximum range of a ball placed within the container if it is released from the position shown. Use numerical values and any necessary measurements from the photo. Assume the unstretched length and stiffness of each cord is known.

Prob. C14–3 (© R.C. Hibbeler)

C14–4. The girl is momentarily at rest in the position shown. If the unstretched length and stiffness of each of the two elastic cords is known, determine approximately how far the girl descends before she again becomes momentarily at rest. Use numerical values and take any necessary measurements from the photo.

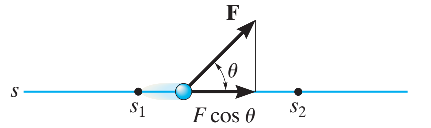
Prob. C14–4 (© R.C. Hibbeler)

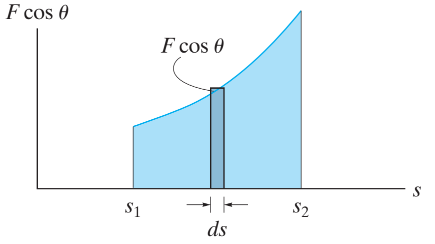
CHAPTER REVIEW

Work of a Force

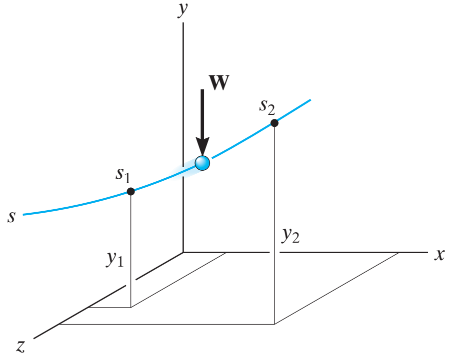
A force does work when it undergoes a displacement along its line of action. If the force varies with the displacement, then the work is $U = \int F \cos\theta \, ds$.

Graphically, this represents the area under the F–s diagram.



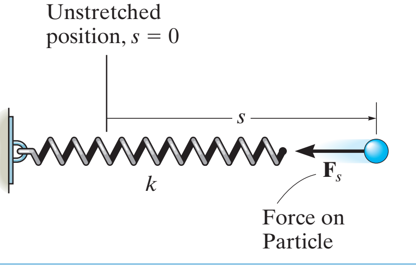


If the force is constant, then for a displacement Δs in the direction of the force, $U = F_c \, \Delta s$. A typical example of this case is the work of a weight, $U = -W\,\Delta y$. Here, Δy is the vertical displacement.



The work done by a spring force, $F = ks$, depends upon the stretch or compression s of the spring.

$$U = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)$$



The Principle of Work and Energy

If the equation of motion in the tangential direction, $\Sigma F_t = ma_t$, is combined with the kinematic equation, $a_t\,ds = v\,dv$, we obtain the principle of work and energy. This equation states that the initial kinetic energy T, plus the work done ΣU_{1-2} is equal to the final kinetic energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

The principle of work and energy is useful for solving problems that involve force, velocity, and displacement. For application, the free-body diagram of the particle should be drawn in order to identify the forces that do work.

Power and Efficiency

Power is the time rate of doing work. For application, the force **F** creating the power and its velocity **v** must be specified.

$$P = \frac{dU}{dt}$$

$$P = \mathbf{F} \cdot \mathbf{v}$$

Efficiency represents the ratio of power output to power input. Due to frictional losses, it is always less than one.

$$\varepsilon = \frac{\text{power output}}{\text{power input}}$$

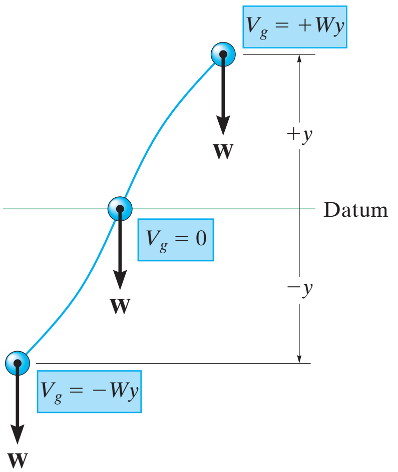
Conservation of Energy

A conservative force does work that is independent of its path. Two examples are the weight of a particle and the spring force.

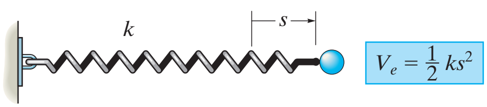
Friction is a nonconservative force since the work depends upon the length of the path. The longer the path, the more work done.

The work done by a conservative force depends upon its position relative to a datum. When this work is referenced from a datum, it is called potential energy. For a weight, it is $V_g = \pm Wy$, and for a spring it is $V_e = +\frac{1}{2}ks^2$.

Mechanical energy consists of kinetic energy T and gravitational and elastic potential energies V. According to the conservation of energy, this sum is constant and has the same value at any position on the path. If only gravitational and spring forces cause motion of the particle, then the conservation-of-energy equation can be used to solve problems involving these conservative forces, displacement, and velocity.



Gravitational potential energy

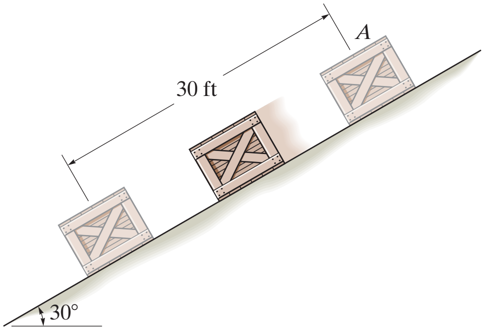


Elastic potential energy

$$T_1 + V_1 = T_2 + V_2$$

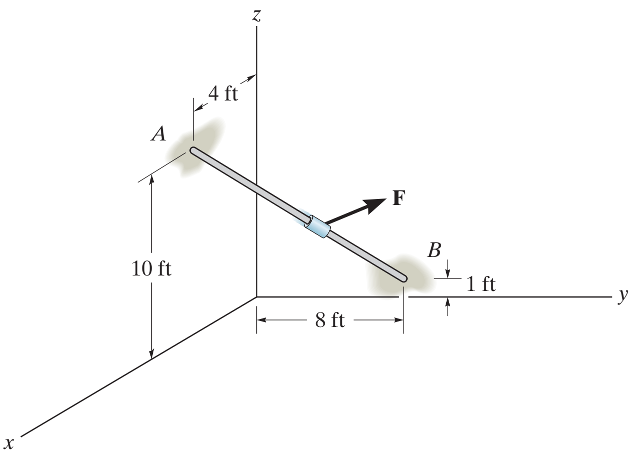
REVIEW PROBLEMS

R14–1. If a 150-lb crate is released from rest at A, determine its speed after it slides 30 ft down the plane. The coefficient of kinetic friction between the crate and plane is $\mu_k = 0.3$.



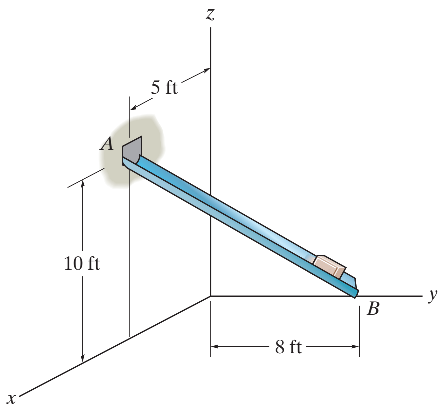
Prob. R14–1

R14–2. The small 2-lb collar starting from rest at A slides down along the smooth rod. During the motion, the collar is acted upon by a force $\mathbf{F} = \{10\mathbf{i} + 6y\mathbf{j} + 2z\mathbf{k}\}$ lb, where x, y, z are in feet. Determine the collar's speed when it strikes the wall at B.



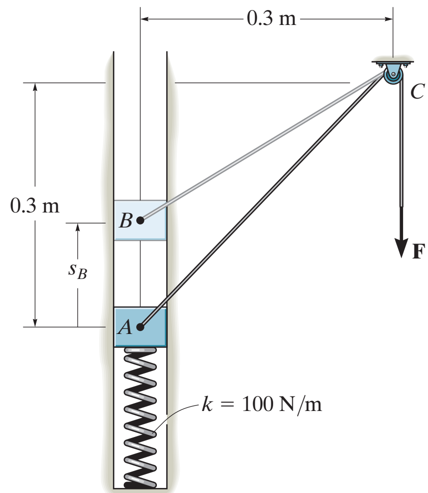
Prob. R14–2

R14–3. The block has a weight of 1.5 lb and slides along the smooth chute AB. It is released from rest at A, which has coordinates of $A(5\text{ ft}, 0, 10\text{ ft})$. Determine the speed at which it slides off at B, which has coordinates of $B(0, 8\text{ ft}, 0)$.



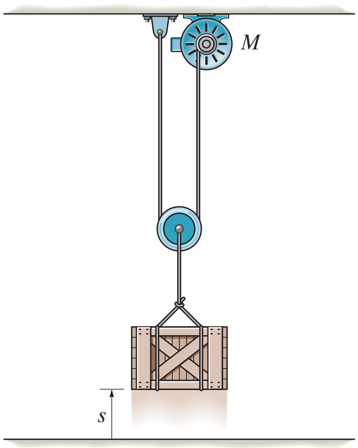
Prob. R14–3

R14–4. The block has a mass of 0.5 kg and moves within the smooth vertical slot. If the block starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the mass of the cord and pulley.



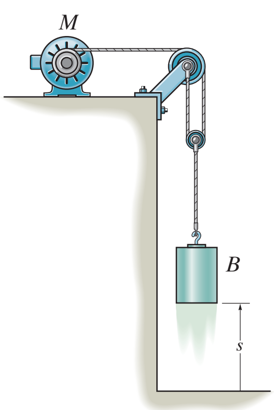
Prob. R14–4

R14–5. The crate, having a weight of 50 lb, is hoisted by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 12 ft/s after rising 10 ft, determine the power that must be supplied to the motor at the instant $s = 10$ ft. The motor has an efficiency $\varepsilon = 0.74$.



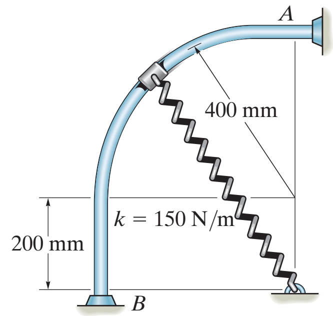
Prob. R14–5

R14–6. The 50-lb load is hoisted by the pulley system and motor M. If the motor exerts a constant force of 30 lb on the cable, determine the power that must be supplied to the motor if the load has been hoisted $s = 10$ ft starting from rest. The motor has an efficiency of $\varepsilon = 0.76$.



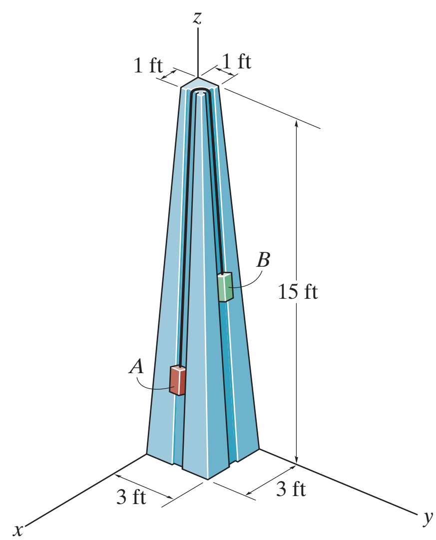
Prob. R14–6

R14–7. The collar of negligible size has a mass of 0.25 kg and is attached to a spring having an unstretched length of 100 mm. If the collar is released from rest at A and travels along the smooth guide, determine its speed just before it strikes B.



Prob. R14–7

R14–8. The blocks A and B weigh 10 and 30 lb, respectively. They are connected together by a light cord and ride in the frictionless grooves. Determine the speed of each block after block A moves 6 ft up along the plane. The blocks are released from rest.



Prob. R14–8

Chapter 15

(© David J. Green/Alamy)

The design of the bumper cars used for this amusement park ride requires knowledge of the principles of impulse and momentum.

Kinetics of a Particle: Impulse and Momentum

CHAPTER OBJECTIVES

- To develop the principle of linear impulse and momentum for a particle and apply it to solve problems that involve force, velocity, and time.
- To study the conservation of linear momentum for particles.
- To analyze the mechanics of impact.
- To introduce the concept of angular impulse and momentum.
- To solve problems involving steady fluid streams and propulsion with variable mass.

15.1 Principle of Linear Impulse and Momentum

In this section we will integrate the equation of motion with respect to time and thereby obtain the principle of impulse and momentum. The resulting equation will be useful for solving problems involving force, velocity, and time.

Using kinematics, the equation of motion for a particle of mass m can be written as

$$\Sigma \mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} \qquad (15\text{–}1)$$

where $\mathbf{a}$ and $\mathbf{v}$ are both measured from an inertial frame of reference. Rearranging the terms and integrating between the limits $\mathbf{v} = \mathbf{v}_1$ at $t = t_1$ and $\mathbf{v} = \mathbf{v}_2$ at $t = t_2$, we have

$$\Sigma \int_{t_1}^{t_2} \mathbf{F}\,dt = m\int_{\mathbf{v}_1}^{\mathbf{v}_2} d\mathbf{v}$$

The impulse tool is used to remove the dent in the trailer fender. To do so its end is first screwed into a hole drilled in the fender, then the weight is gripped and jerked upwards, striking the stop ring. The impulse developed is transferred along the shaft of the tool and pulls suddenly on the dent. (© R.C. Hibbeler)

or

$$\Sigma \int_{t_1}^{t_2} \mathbf{F}dt = m\mathbf{v}_2 - m\mathbf{v}_1 \tag{15–2}$$

This equation is referred to as the *principle of linear impulse and momentum*. From the derivation it can be seen that it is simply a time integration of the equation of motion. It provides a *direct means* of obtaining the particle's final velocity $\mathbf{v}_2$ after a specified time period when the particle's initial velocity is known and the forces acting on the particle are either constant or can be expressed as functions of time. By comparison, if $\mathbf{v}_2$ was determined using the equation of motion, a two-step process would be necessary; i.e., apply $\Sigma\mathbf{F} = m\mathbf{a}$ to obtain $\mathbf{a}$, then integrate $\mathbf{a} = d\mathbf{v}/dt$ to obtain $\mathbf{v}_2$.

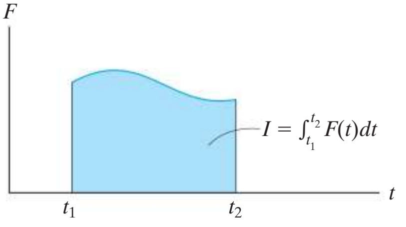
Linear Momentum. Each of the two vectors of the form $\mathbf{L} = m\mathbf{v}$ in Eq. 15–2 is referred to as the particle's linear momentum. Since m is a positive scalar, the linear-momentum vector has the same direction as $\mathbf{v}$, and its magnitude mv has units of mass times velocity, e.g., $\text{kg}\cdot\text{m/s}$, or $\text{slug}\cdot\text{ft/s}$.

Linear Impulse. The integral $\mathbf{I} = \int \mathbf{F}\,dt$ in Eq. 15–2 is referred to as the *linear impulse*. This term is a vector quantity which measures the effect of a force during the time the force acts. Since time is a positive scalar, the impulse acts in the same direction as the force, and its magnitude has units of force times time, e.g., $\text{N}\cdot\text{s}$ or $\text{lb}\cdot\text{s}$.*

If the force is expressed as a function of time, the impulse can be determined by direct evaluation of the integral. In particular, if the force is constant in both magnitude and direction, the resulting impulse becomes

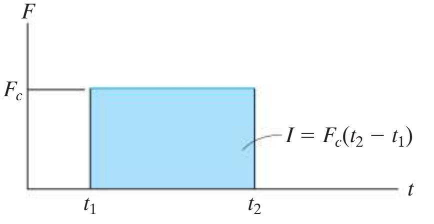
$$\mathbf{I} = \int_{t_1}^{t_2}\mathbf{F}_c\,dt = \mathbf{F}_c(t_2 - t_1).$$

Graphically the magnitude of the impulse can be represented by the shaded area under the curve of force versus time, Fig. 15–1. A constant force creates the shaded rectangular area shown in Fig. 15–2.



Variable Force

Fig. 15–1



Constant Force

Fig. 15–2

*Although the units for impulse and momentum are defined differently, it can be shown that Eq. 15–2 is dimensionally homogeneous.

Principle of Linear Impulse and Momentum. For problem solving, Eq. 15–2 will be rewritten in the form

$$\boxed{m\mathbf{v}_1 + \Sigma\int_{t_1}^{t_2}\mathbf{F}\,dt = m\mathbf{v}_2} \tag{15–3}$$

which states that the initial momentum of the particle at time t_1 plus the sum of all the impulses applied to the particle from t_1 to t_2 is equivalent to the final momentum of the particle at time t_2. These three terms are illustrated graphically on the *impulse and momentum diagrams* shown in Fig. 15–3. The two *momentum diagrams* are simply outlined shapes of the particle which indicate the direction and magnitude of the particle's initial and final momenta, $m\mathbf{v}_1$ and $m\mathbf{v}_2$. Similar to the free-body diagram, the *impulse diagram* is an outlined shape of the particle showing all the impulses that act on the particle when it is located at some intermediate point along its path.

The study of many types of sports, such as golf, requires application of the principle of linear impulse and momentum. (© R.C. Hibbeler)

If each of the vectors in Eq. 15–3 is resolved into its x, y, z components, we can write the following three scalar equations of linear impulse and momentum.

$$\boxed{\begin{aligned} m(v_x)_1 + \Sigma\int_{t_1}^{t_2}F_x\,dt &= m(v_x)_2 \\ m(v_y)_1 + \Sigma\int_{t_1}^{t_2}F_y\,dt &= m(v_y)_2 \\ m(v_z)_1 + \Sigma\int_{t_1}^{t_2}F_z\,dt &= m(v_z)_2 \end{aligned}} \tag{15–4}$$

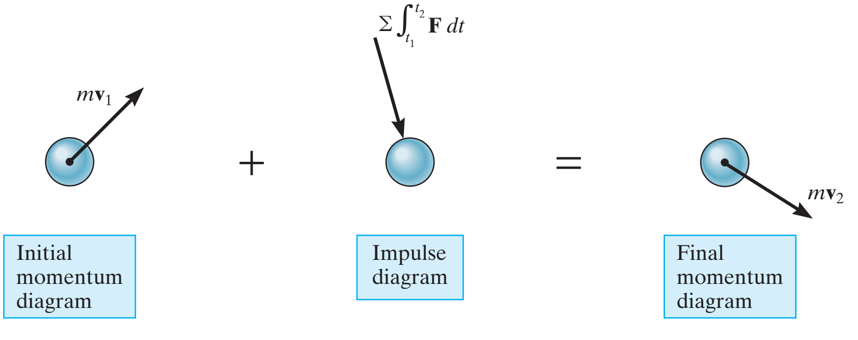


Fig. 15–3

15.2 Principle of Linear Impulse and Momentum for a System of Particles

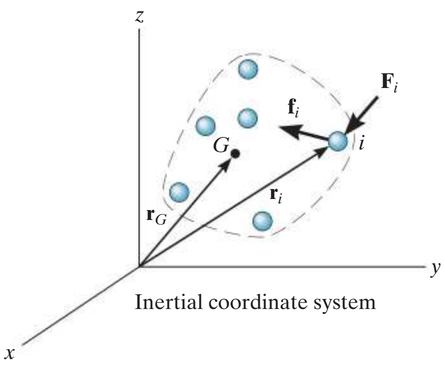


Fig. 15–4

The principle of linear impulse and momentum for a system of particles moving relative to an inertial reference, Fig. 15–4, is obtained from the equation of motion applied to all the particles in the system, i.e.,

$$\Sigma \mathbf{F}_i = \Sigma m_i \frac{d\mathbf{v}_i}{dt} \tag{15–5}$$

The term on the left side represents only the sum of the *external forces* acting on the particles. Recall that the internal forces $\mathbf{f}_i$ acting between particles do not appear with this summation, since by Newton's third law they occur in equal but opposite collinear pairs and therefore cancel out. Multiplying both sides of Eq. 15–5 by dt and integrating between the limits $t = t_1$, $\mathbf{v}_i = (\mathbf{v}_i)_1$ and $t = t_2$, $\mathbf{v}_i = (\mathbf{v}_i)_2$ yields

$$\boxed{\Sigma m_i(\mathbf{v}_i)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}_i\, dt = \Sigma m_i(\mathbf{v}_i)_2} \tag{15–6}$$

This equation states that the initial linear momentum of the system plus the impulses of all the *external forces* acting on the system from t_1 to t_2 is equal to the system's final linear momentum.

Since the location of the mass center G of the system is determined from $m\mathbf{r}_G = \Sigma m_i \mathbf{r}_i$, where $m = \Sigma m_i$ is the total mass of all the particles, Fig. 15–4, then taking the time derivative, we have

$$m\mathbf{v}_G = \Sigma m_i \mathbf{v}_i$$

which states that the total linear momentum of the system of particles is equivalent to the linear momentum of a "fictitious" aggregate particle of mass $m = \Sigma m_i$ moving with the velocity of the mass center of the system. Substituting into Eq. 15–6 yields

$$m(\mathbf{v}_G)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}_i\, dt = m(\mathbf{v}_G)_2 \tag{15–7}$$

Here the initial linear momentum of the aggregate particle plus the external impulses acting on the system of particles from t_1 to t_2 is equal to the aggregate particle's final linear momentum. As a result, the above equation justifies application of the principle of linear impulse and momentum to a system of particles that compose a rigid body.

Procedure for Analysis

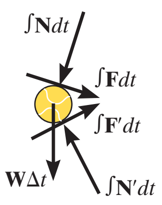
The principle of linear impulse and momentum is used to solve problems involving *force, time*, and *velocity*, since these terms are involved in the formulation. For application it is suggested that the following procedure be used.*

Free-Body Diagram.

- Establish the x, y, z inertial frame of reference and draw the particle's free-body diagram in order to account for all the forces that produce impulses on the particle.
- The direction and sense of the particle's initial and final velocities should be established.
- If a vector is unknown, assume that the sense of its components is in the direction of the positive inertial coordinate(s).
- As an alternative procedure, draw the impulse and momentum diagrams for the particle as discussed in reference to Fig. 15–3.

Principle of Impulse and Momentum.

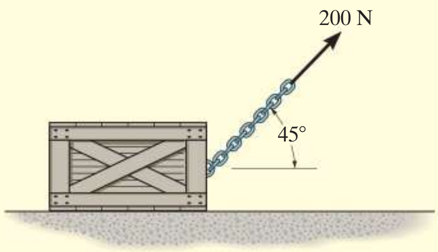
- In accordance with the established coordinate system, apply the principle of linear impulse and momentum, $m\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2$. If motion occurs in the x–y plane, the two scalar component equations can be formulated by either resolving the vector components of $\mathbf{F}$ from the free-body diagram, or by using the data on the impulse and momentum diagrams.
- Realize that every force acting on the particle's free-body diagram will create an impulse, even though some of these forces will do no work.
- Forces that are functions of time must be integrated to obtain the impulse. Graphically, the impulse is equal to the area under the force–time curve.



As the wheels of the pitching machine rotate, they apply frictional impulses to the ball, thereby giving it a linear momentum. These impulses are shown on the impulse diagram. Here both the frictional and normal impulses vary with time. By comparison, the weight impulse is constant and is very small since the time Δt the ball is in contact with the wheels is very small. (© R.C. Hibbeler)

*This procedure will be followed when developing the proofs and theory in the text.

EXAMPLE 15.1



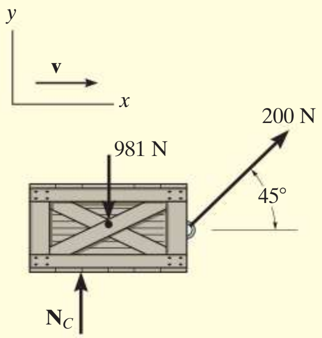
(a)

The 100-kg crate shown in Fig. 15–5*a* is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied for 10 s, determine the final velocity and the normal force which the surface exerts on the crate during this time interval.

SOLUTION

This problem can be solved using the principle of impulse and momentum since it involves force, velocity, and time.

Free-Body Diagram. See Fig. 15–5*b*. Since all the forces acting are *constant*, the impulses are simply the product of the force magnitude and 10 s $[\mathbf{I} = \mathbf{F}_c(t_2 - t_1)]$. Note the alternative procedure of drawing the crate's impulse and momentum diagrams, Fig. 15–5*c*.



(b)

Principle of Impulse and Momentum. Applying Eqs. 15–4 yields

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + 200\text{ N}\cos 45°(10\text{ s}) = (100\text{ kg})v_2$$

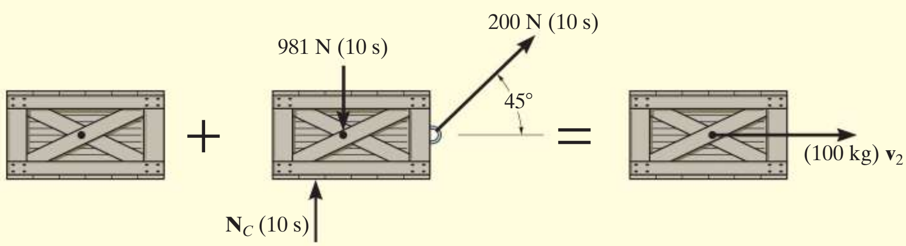
$$v_2 = 14.1\text{ m/s} \qquad \textit{Ans.}$$

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$0 + N_C(10\text{ s}) - 981\text{ N}(10\text{ s}) + 200\text{ N}\sin 45°(10\text{ s}) = 0$$

$$N_C = 840\text{ N} \qquad \textit{Ans.}$$

NOTE: Since no motion occurs in the y direction, direct application of the equilibrium equation $\Sigma F_y = 0$ gives the same result for N_C. Try to solve the problem by first applying $\Sigma F_x = ma_x$, then $v = v_0 + a_c t$.

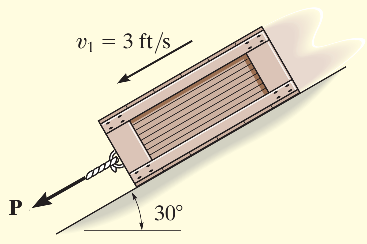


(c)

Fig. 15–5

EXAMPLE 15.2

The 50-lb crate shown in Fig. 15–6*a* is acted upon by a force having a variable magnitude $P = (20t)$ lb, where t is in seconds. Determine the crate's velocity 2 s after **P** has been applied. The initial velocity is $v_1 = 3$ ft/s down the plane, and the coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.



(a)

SOLUTION

Free-Body Diagram. See Fig. 15–6*b*. Since the magnitude of force $P = 20t$ varies with time, the impulse it creates must be determined by integrating over the 2-s time interval.

Principle of Impulse and Momentum. Applying Eqs. 15–4 in the x direction, we have

$$(+\swarrow) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$\frac{50\text{ lb}}{32.2\text{ ft/s}^2}(3\text{ ft/s}) + \int_0^{2\text{ s}} 20t\,dt - 0.3N_C(2\text{ s}) + (50\text{ lb})\sin 30°(2\text{ s}) = \frac{50\text{ lb}}{32.2\text{ ft/s}^2}v_2$$

$$4.658 + 40 - 0.6N_C + 50 = 1.553v_2$$

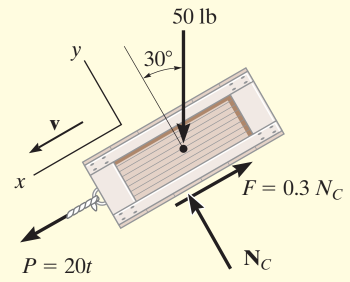
The equation of equilibrium can be applied in the y direction. Why?

$$+\nwarrow\Sigma F_y = 0; \qquad N_C - 50\cos 30°\text{ lb} = 0$$

Solving,

$$N_C = 43.30\text{ lb}$$

$$v_2 = 44.2\text{ ft/s} \swarrow \qquad \textit{Ans.}$$



(b)

Fig. 15–6

NOTE: We can also solve this problem using the equation of motion. From Fig. 15–6*b*,

$$+\swarrow\Sigma F_x = ma_x; \quad 20t - 0.3(43.30) + 50\sin 30° = \frac{50}{32.2}a$$

$$a = 12.88t + 7.734$$

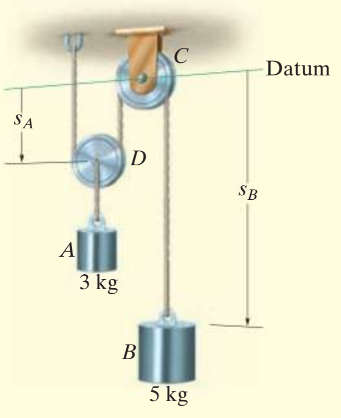
Using kinematics

$$+\swarrow dv = a\,dt; \qquad \int_{3\text{ ft/s}}^{v} dv = \int_0^{2\,s}(12.88t + 7.734)dt$$

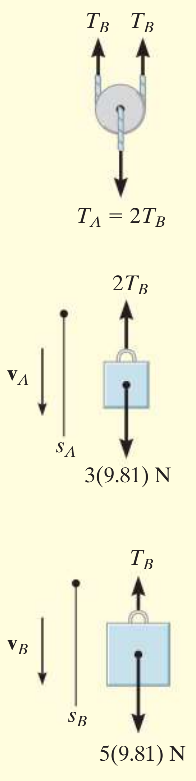
$$v = 44.2\text{ ft/s} \qquad \textit{Ans.}$$

By comparison, application of the principle of impulse and momentum eliminates the need for using kinematics ($a = dv/dt$) and thereby yields an easier method for solution.

EXAMPLE 15.3



(a)



(b)

Fig. 15–7

Blocks A and B shown in Fig. 15–7a have a mass of 3 kg and 5 kg, respectively. If the system is released from rest, determine the velocity of block B in 6 s. Neglect the mass of the pulleys and cord.

SOLUTION

Free-Body Diagram. See Fig. 15–7b. Since the weight of each block is constant, the cord tensions will also be constant. Furthermore, since the mass of pulley D is neglected, the cord tension $T_A = 2T_B$. Note that the blocks are both assumed to be moving downward in the positive coordinate directions, s_A and s_B.

Principle of Impulse and Momentum.

Block A:

$$(+\downarrow) \qquad m(v_A)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_A)_2$$

$$0 - 2T_B(6\text{ s}) + 3(9.81)\text{ N}(6\text{ s}) = (3\text{ kg})(v_A)_2 \tag{1}$$

Block B:

$$(+\downarrow) \qquad m(v_B)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_B)_2$$

$$0 + 5(9.81)\text{ N}(6\text{ s}) - T_B(6\text{ s}) = (5\text{ kg})(v_B)_2 \tag{2}$$

Kinematics. Since the blocks are subjected to dependent motion, the velocity of A can be related to that of B by using the kinematic analysis discussed in Sec. 12.9. A horizontal datum is established through the fixed point at C, Fig. 15–7a, and the position coordinates, s_A and s_B, are related to the constant total length l of the vertical segments of the cord by the equation

$$2s_A + s_B = l$$

Taking the time derivative yields

$$2v_A = -v_B \tag{3}$$

As indicated by the negative sign, when B moves downward A moves upward. Substituting this result into Eq. 1 and solving Eqs. 1 and 2 yields

$$(v_B)_2 = 35.8\text{ m/s}\downarrow \qquad \textit{Ans.}$$

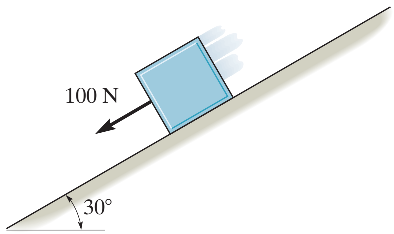
$$T_B = 19.2\text{ N}$$

NOTE: Realize that the *positive* (downward) direction for $\mathbf{v}_A$ and $\mathbf{v}_B$ is *consistent* in Figs. 15–7a and 15–7b and in Eqs. 1 to 3. This is important since we are seeking a simultaneous solution of equations.

PRELIMINARY PROBLEMS

15–1. Determine the impulse of the force for $t = 2$ s.

a)



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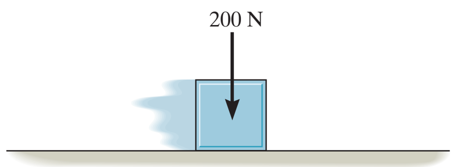

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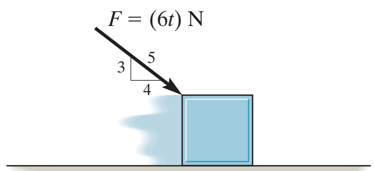
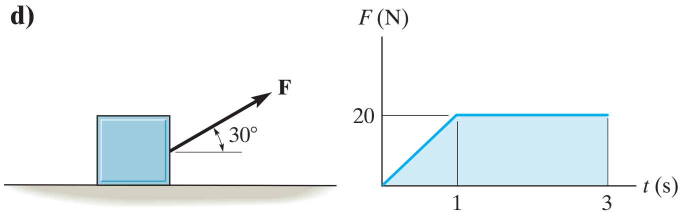
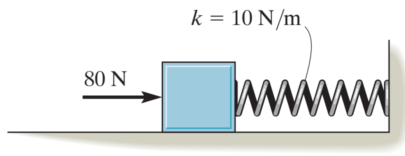


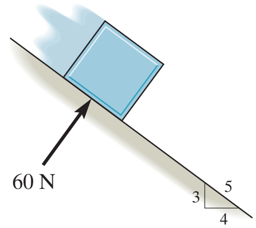
d)



e)



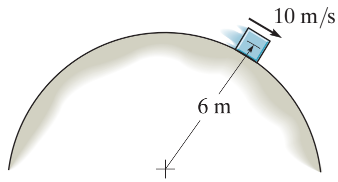
f)



Prob. P15–1

15–2. Determine the linear momentum of the 10-kg block.

a)



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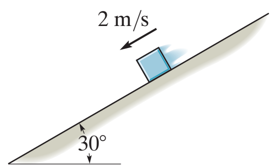

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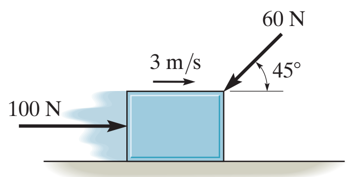
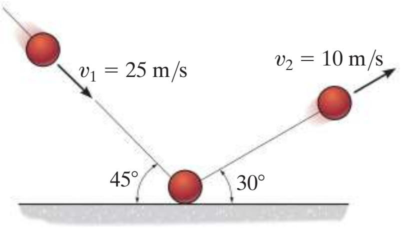


Prob. P15–2

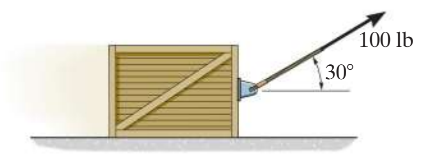
FUNDAMENTAL PROBLEMS

F15–1. The 0.5-kg ball strikes the rough ground and rebounds with the velocities shown. Determine the magnitude of the impulse the ground exerts on the ball. Assume that the ball does not slip when it strikes the ground, and neglect the size of the ball and the impulse produced by the weight of the ball.



Prob. F15–1

F15–2. If the coefficient of kinetic friction between the 150-lb crate and the ground is $\mu_k = 0.2$, determine the speed of the crate when $t = 4$ s. The crate starts from rest and is towed by the 100-lb force.

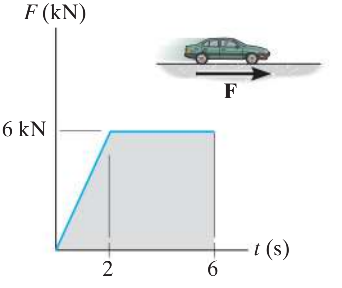


Prob. F15–2

F15–3. The motor exerts a force of $F = (20t^2)$ N on the cable, where t is in seconds. Determine the speed of the 25-kg crate when $t = 4$ s. The coefficients of static and kinetic friction between the crate and the plane are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively.

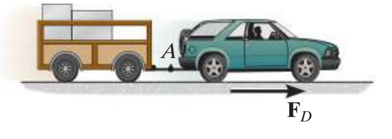
Prob. F15–3

F15–4. The wheels of the 1.5-Mg car generate the traction force **F** described by the graph. If the car starts from rest, determine its speed when $t = 6$ s.



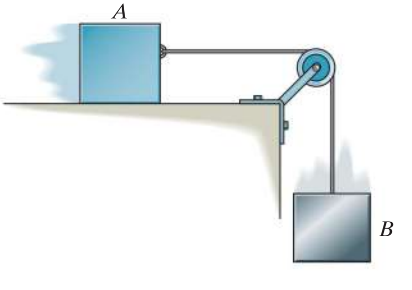
Prob. F15–4

F15–5. The 2.5-Mg four-wheel-drive SUV tows the 1.5-Mg trailer. The traction force developed at the wheels is $F_D = 9$ kN. Determine the speed of the truck in 20 s, starting from rest. Also, determine the tension developed in the coupling, A, between the SUV and the trailer. Neglect the mass of the wheels.



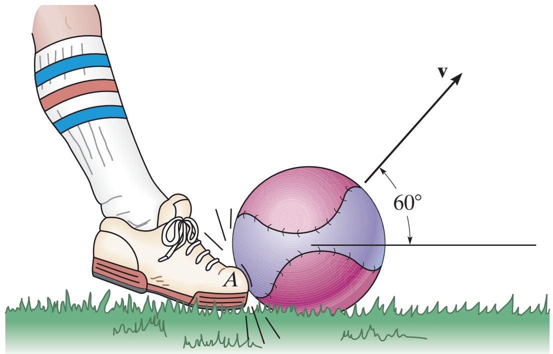
Prob. F15–5

F15–6. The 10-lb block A attains a velocity of 1 ft/s in 5 seconds, starting from rest. Determine the tension in the cord and the coefficient of kinetic friction between block A and the horizontal plane. Neglect the weight of the pulley. Block B has a weight of 8 lb.



Prob. F15–6

15–1. A man kicks the 150-g ball such that it leaves the ground at an angle of 60° and strikes the ground at the same elevation a distance of 12 m away. Determine the impulse of his foot on the ball at A. Neglect the impulse caused by the ball's weight while it's being kicked.

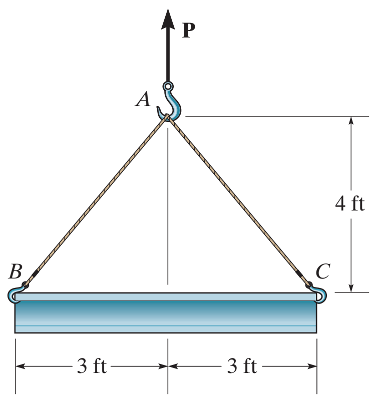


Prob. 15–1

15–2. A 20-lb block slides down a 30° inclined plane with an initial velocity of 2 ft/s. Determine the velocity of the block in 3 s if the coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$.

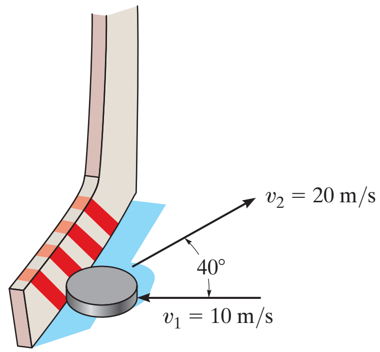
15–3. The uniform beam has a weight of 5000 lb. Determine the average tension in each of the two cables AB and AC if the beam is given an upward speed of 8 ft/s in 1.5 s starting from rest. Neglect the mass of the cables.

***15–4.** Each of the cables can sustain a maximum tension of 5000 lb. If the uniform beam has a weight of 5000 lb, determine the shortest time possible to lift the beam with a speed of 10 ft/s starting from rest.



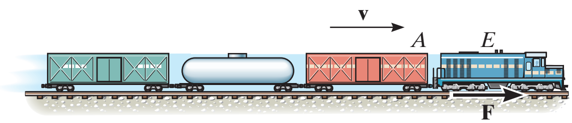
Probs. 15–3/4

15–5. A hockey puck is traveling to the left with a velocity of $v_1 = 10$ m/s when it is struck by a hockey stick and given a velocity of $v_2 = 20$ m/s as shown. Determine the magnitude of the net impulse exerted by the hockey stick on the puck. The puck has a mass of 0.2 kg.



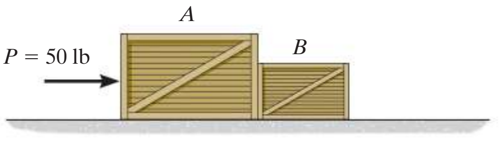
Prob. 15–5

15–6. A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force $\mathbf{F}$ which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.



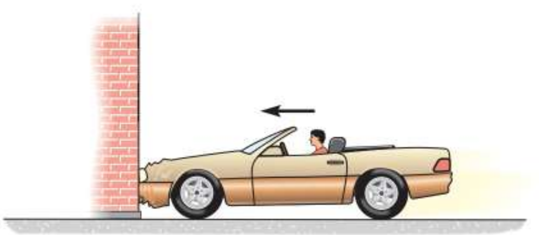
Prob. 15–6

15–7. Crates A and B weigh 100 lb and 50 lb, respectively. If they start from rest, determine their speed when $t = 5$ s. Also, find the force exerted by crate A on crate B during the motion. The coefficient of kinetic friction between the crates and the ground is $\mu_k = 0.25$.



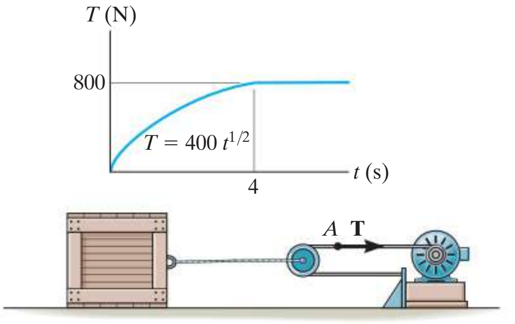
Prob. 15–7

***15–8.** The automobile has a weight of 2700 lb and is traveling forward at 4 ft/s when it crashes into the wall. If the impact occurs in 0.06 s, determine the average impulsive force acting on the car. Assume the brakes are *not applied*. If the coefficient of kinetic friction between the wheels and the pavement is $\mu_k = 0.3$, calculate the impulsive force on the wall if the brakes *were applied* during the crash. The brakes are applied to all four wheels so that all the wheels slip.



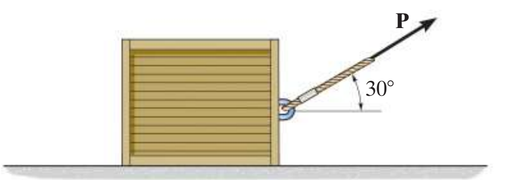
Prob. 15–8

15–9. The 200-kg crate rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force $\boldsymbol{T}$ to its cable at A which varies as shown in the graph. Determine the speed of the crate when $t = 4$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the crate.



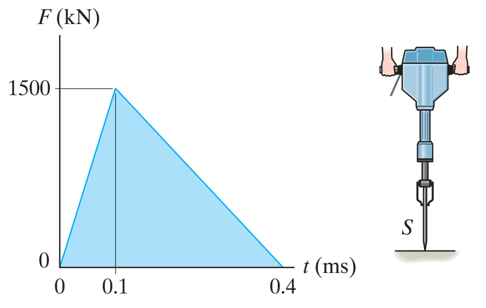
Prob. 15–9

15–10. The 50-kg crate is pulled by the constant force **P**. If the crate starts from rest and achieves a speed of 10 m/s in 5 s, determine the magnitude of **P**. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



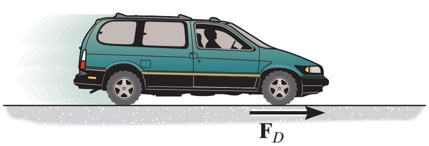
Prob. 15–10

15–11. During operation the jack hammer strikes the concrete surface with a force which is indicated in the graph. To achieve this the 2-kg spike S is fired into the surface at 90 m/s. Determine the speed of the spike just after rebounding.



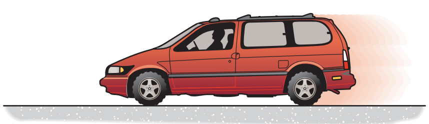
Prob. 15–11

***15–12.** For a short period of time, the frictional driving force acting on the wheels of the 2.5-Mg van is $F_D = (600t^2)$ N, where t is in seconds. If the van has a speed of 20 km/h when $t = 0$, determine its speed when $t = 5$ s.



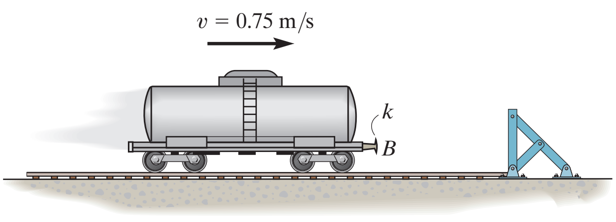
Prob. 15–12

15–13. The 2.5-Mg van is traveling with a speed of 100 km/h when the brakes are applied and all four wheels lock. If the speed decreases to 40 km/h in 5 s, determine the coefficient of kinetic friction between the tires and the road.



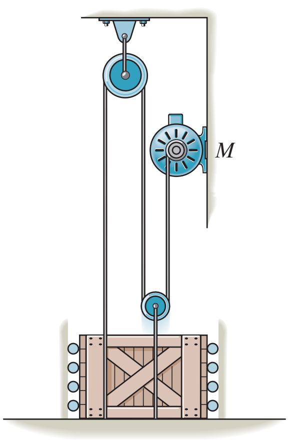
Prob. 15–13

15–14. A tankcar has a mass of 20 Mg and is freely rolling to the right with a speed of 0.75 m/s. If it strikes the barrier, determine the horizontal impulse needed to stop the car if the spring in the bumper B has a stiffness (a) $k \rightarrow \infty$ (bumper is rigid), and (b) $k = 15$ kN/m.



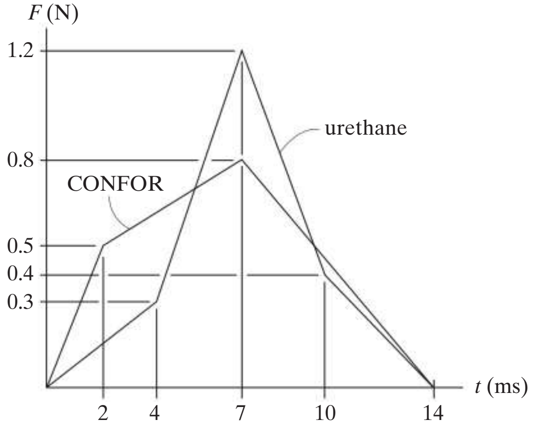
Prob. 15–14

15–15. The motor, M, pulls on the cable with a force $F = (10t^2 + 300)$ N, where t is in seconds. If the 100 kg crate is originally at rest at $t = 0$, determine its speed when $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



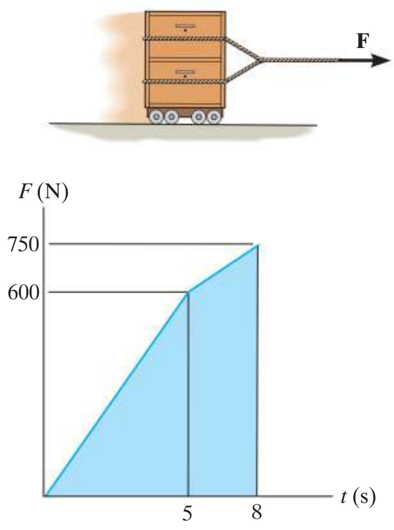
Prob. 15–15

***15–16.** The choice of a seating material for moving vehicles depends upon its ability to resist shock and vibration. From the data shown in the graphs, determine the impulses created by a falling weight onto a sample of urethane foam and CONFOR foam.



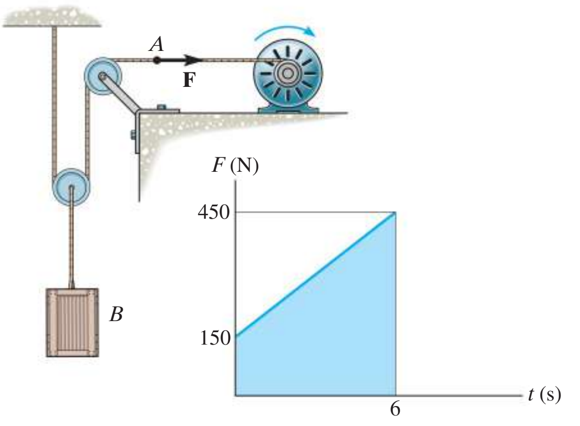
Prob. 15–16

15–17. The towing force acting on the 400-kg safe varies as shown on the graph. Determine its speed, starting from rest, when $t = 8$ s. How far has it traveled during this time?



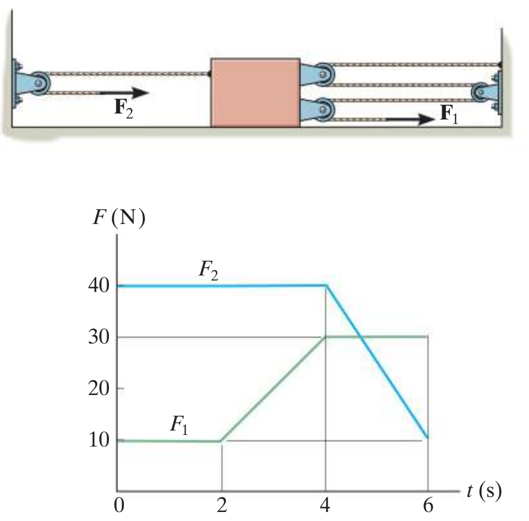
Prob. 15–17

15–18. The motor exerts a force $\boldsymbol{F}$ on the 40-kg crate as shown in the graph. Determine the speed of the crate when $t = 3$ s and when $t = 6$ s. When $t = 0$, the crate is moving downward at 10 m/s.



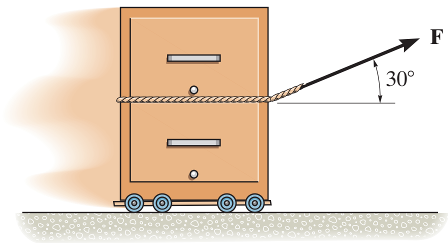
Prob. 15–18

15–19. The 30-kg slider block is moving to the left with a speed of 5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.



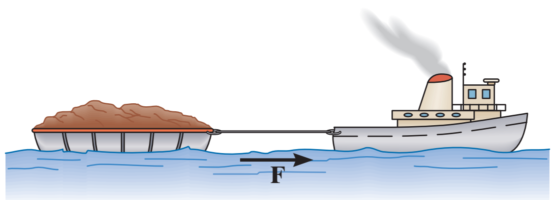
Prob. 15–19

***15–20.** The 200-lb cabinet is subjected to the force $F = 20(t + 1)$ lb where t is in seconds. If the cabinet is initially moving to the left with a velocity of 20 ft/s, determine its speed when $t = 5$ s. Neglect the size of the rollers.



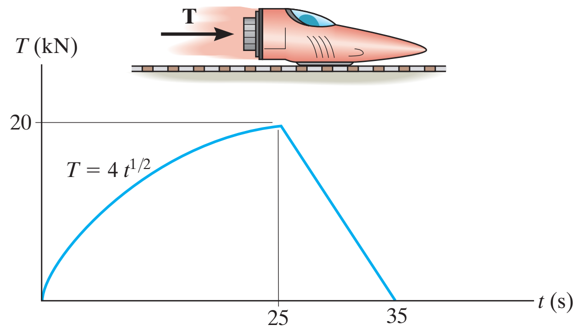
Prob. 15–20

15–21. If it takes 35 s for the 50-Mg tugboat to increase its speed uniformly to 25 km/h, starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine F acting on the tugboat. The barge has a mass of 75 Mg.



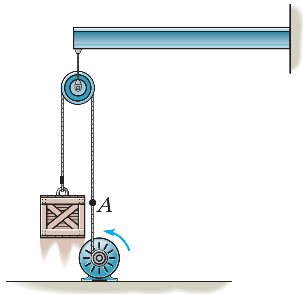
Prob. 15–21

15–22. The thrust on the 4-Mg rocket sled is shown in the graph. Determine the sleds maximum velocity and the distance the sled travels when $t = 35$ s. Neglect friction.



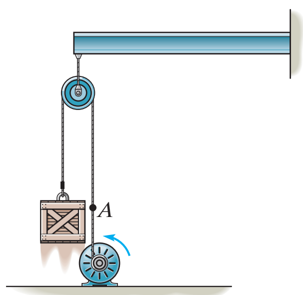
Prob. 15–22

15–23. The motor pulls on the cable at A with a force $F = (30 + t^2)$ lb, where t is in seconds. If the 34-lb crate is originally on the ground at $t = 0$, determine its speed in $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



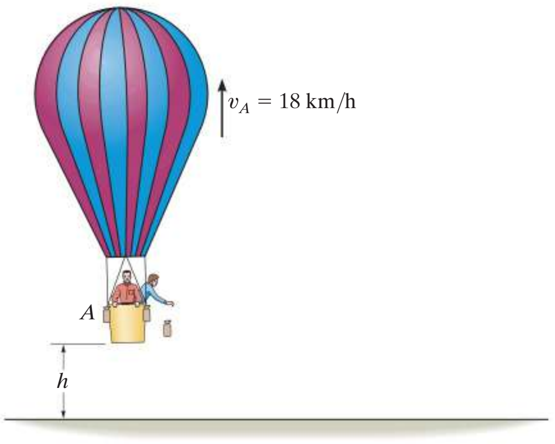
Prob. 15–23

***15–24.** The motor pulls on the cable at A with a force $F = (e^{2t})$ lb, where t is in seconds. If the 34-lb crate is originally at rest on the ground at $t = 0$, determine the crate's velocity when $t = 2$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



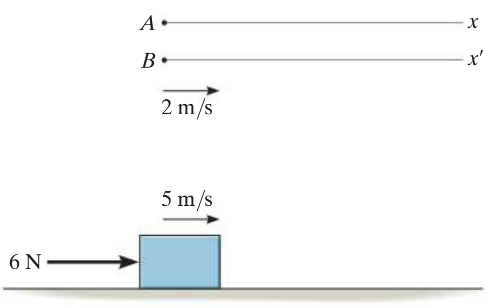
Prob. 15–24

15–25. The balloon has a total mass of 400 kg including the passengers and ballast. The balloon is rising at a constant velocity of 18 km/h when $h = 10$ m. If the man drops the 40-kg sand bag, determine the velocity of the balloon when the bag strikes the ground. Neglect air resistance.



Prob. 15–25

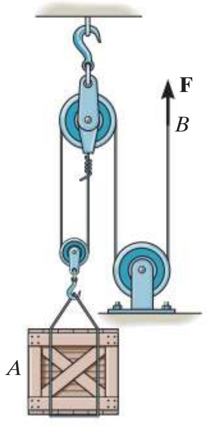
15–26. As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which slides along the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block in 4 s if it has an initial speed of 5 m/s measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis that moves at a constant velocity of 2 m/s relative to A.



Prob. 15–26

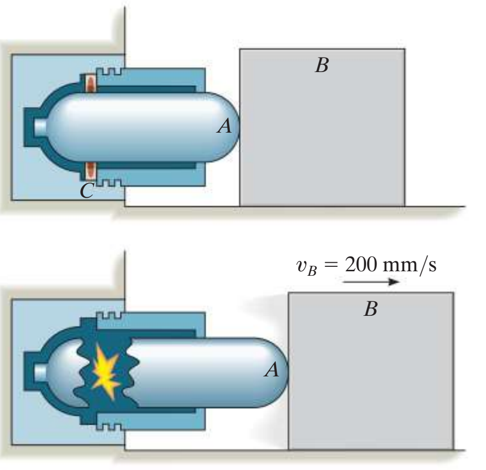
15–27. The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where t is in seconds. Determine the speed of the crate when $t = 3$ s, starting from rest.

***15–28.** The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where t is in seconds. Determine how high the crate has moved upward when $t = 3$ s, starting from rest.



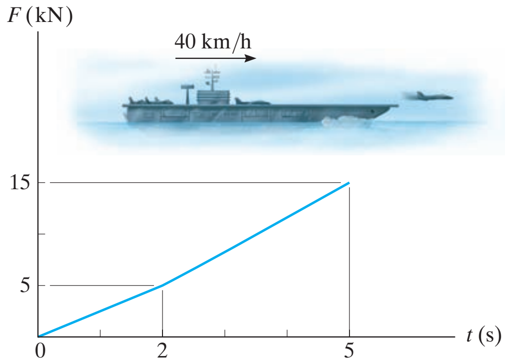
Probs. 15–27/28

15–29. In case of emergency, the gas actuator is used to move a 75-kg block B by exploding a charge C near a pressurized cylinder of negligible mass. As a result of the explosion, the cylinder fractures and the released gas forces the front part of the cylinder, A, to move B forward, giving it a speed of 200 mm/s in 0.4 s. If the coefficient of kinetic friction between B and the floor is $\mu_k = 0.5$, determine the impulse that the actuator imparts to B.



Prob. 15–29

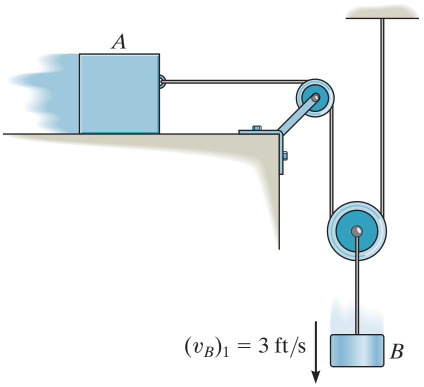
15–30. A jet plane having a mass of 7 Mg takes off from an aircraft carrier such that the engine thrust varies as shown by the graph. If the carrier is traveling forward with a speed of 40 km/h, determine the plane's airspeed after 5 s.



Prob. 15–30

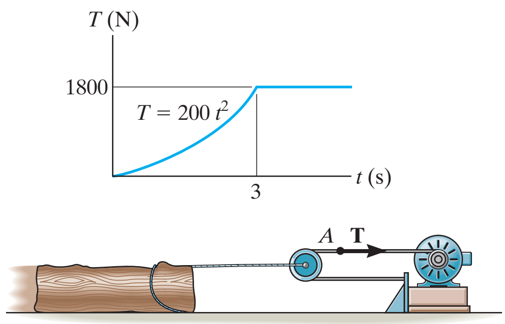
15–31. Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(v_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of A when $t = 1$ s. Assume that the horizontal plane is smooth. Neglect the mass of the pulleys and cords.

***15–32.** Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(v_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of A when $t = 1$ s. The coefficient of kinetic friction between the horizontal plane and block A is $\mu_A = 0.15$.



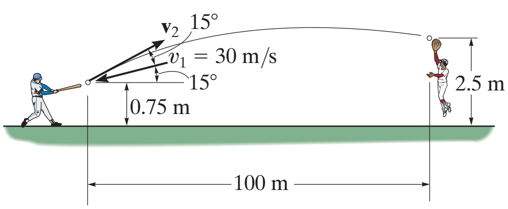
Probs. 15–31/32

15–33. The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.



Prob. 15–33

15–34. The 0.15-kg baseball has a speed of $v = 30$ m/s just before it is struck by the bat. It then travels along the trajectory shown before the outfielder catches it. Determine the magnitude of the average impulsive force imparted to the ball if it is in contact with the bat for 0.75 ms.



Prob. 15–34

(© R.C. Hibbeler)

The hammer in the top photo applies an impulsive force to the stake. During this extremely short time of contact the weight of the stake can be considered nonimpulsive, and provided the stake is driven into soft ground, the impulse of the ground acting on the stake can also be considered nonimpulsive. By contrast, if the stake is used in a concrete chipper to break concrete, then two impulsive forces act on the stake: one at its top due to the chipper and the other on its bottom due to the rigidity of the concrete. (© R.C. Hibbeler)

15.3 Conservation of Linear Momentum for a System of Particles

When the sum of the *external impulses* acting on a system of particles is *zero*, Eq. 15–6 reduces to a simplified form, namely,

$$\boxed{\Sigma m_i(\mathbf{v}_i)_1 = \Sigma m_i(\mathbf{v}_i)_2} \tag{15–8}$$

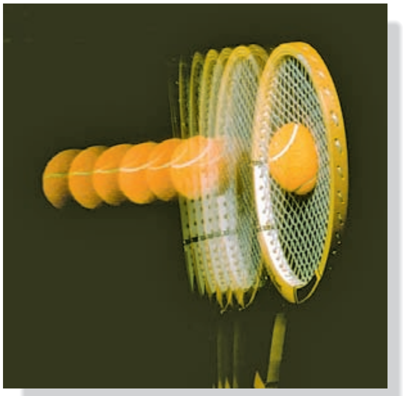
This equation is referred to as the *conservation of linear momentum*. It states that the total linear momentum for a system of particles remains constant during the time period t_1 to t_2. Substituting $m\mathbf{v}_G = \Sigma m_i\mathbf{v}_i$ into Eq. 15–8, we can also write

$$(\mathbf{v}_G)_1 = (\mathbf{v}_G)_2 \tag{15–9}$$

which indicates that the velocity $\mathbf{v}_G$ of the mass center for the system of particles does not change if no external impulses are applied to the system.

The conservation of linear momentum is often applied when particles collide or interact. For application, a careful study of the free-body diagram for the *entire* system of particles should be made in order to identify the forces which create either external or internal impulses and thereby determine in what direction(s) linear momentum is conserved. As stated earlier, the *internal impulses* for the system will always cancel out, since they occur in equal but opposite collinear pairs. If the time period over which the motion is studied is *very short*, some of the external impulses may also be neglected or considered approximately equal to zero. The forces causing these negligible impulses are called *nonimpulsive forces*. By comparison, forces which are very large and act for a very short period of time produce a significant change in momentum and are called *impulsive forces*. They, of course, cannot be neglected in the impulse–momentum analysis.

Impulsive forces normally occur due to an explosion or the striking of one body against another, whereas nonimpulsive forces may include the weight of a body, the force imparted by a slightly deformed spring having a relatively small stiffness, or for that matter, any force that is very small compared to other larger (impulsive) forces. When making this distinction between impulsive and nonimpulsive forces, it is important to realize that this only applies during the time t_1 to t_2. To illustrate, consider the effect of striking a tennis ball with a racket as shown in the photo. During the *very short* time of interaction, the force of the racket on the ball is impulsive since it changes the ball's momentum drastically. By comparison, the ball's weight will have a negligible effect on the change in momentum, and therefore it is nonimpulsive. Consequently, it can be neglected from an impulse–momentum analysis during this time. If an impulse–momentum analysis is considered during the much longer time of flight after the racket–ball interaction, then the impulse of the ball's weight is important since it, along with air resistance, causes the change in the momentum of the ball.



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Procedure for Analysis

Generally, the principle of linear impulse and momentum or the conservation of linear momentum is applied to a *system of particles* in order to determine the final velocities of the particles *just after* the time period considered. By applying this principle to the entire system, the internal impulses acting within the system, which may be unknown, are *eliminated* from the analysis. For application it is suggested that the following procedure be used.

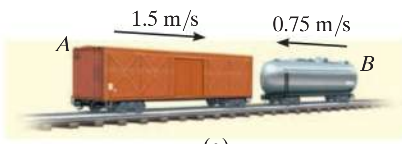
Free-Body Diagram.

- Establish the x, y, z inertial frame of reference and draw the free-body diagram for each particle of the system in order to identify the internal and external forces.
- The conservation of linear momentum applies to the system in a direction which either has no external forces or the forces can be considered nonimpulsive.
- Establish the direction and sense of the particles' initial and final velocities. If the sense is unknown, assume it is along a positive inertial coordinate axis.
- As an alternative procedure, draw the impulse and momentum diagrams for each particle of the system.

Momentum Equations.

- Apply the principle of linear impulse and momentum or the conservation of linear momentum in the appropriate directions.
- If it is necessary to determine the *internal impulse* $\int F\,dt$ acting on only one particle of a system, then the particle must be *isolated* (free-body diagram), and the principle of linear impulse and momentum must be applied *to this particle*.
- After the impulse is calculated, and provided the time Δt for which the impulse acts is known, then the *average impulsive force* F_{avg} can be determined from $F_{avg} = \int F\,dt/\Delta t$.

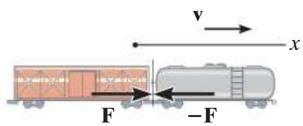
The 15-Mg boxcar A is coasting at 1.5 m/s on the horizontal track when it encounters a 12-Mg tank car B coasting at 0.75 m/s toward it as shown in Fig. 15–8*a*. If the cars collide and couple together, determine (a) the speed of both cars just after the coupling, and (b) the average force between them if the coupling takes place in 0.8 s.



(a)

SOLUTION

Part (a) Free-Body Diagram.* Here we have considered *both* cars as a single system, Fig. 15–8*b*. By inspection, momentum is conserved in the x direction since the coupling force $\mathbf{F}$ is *internal* to the system and will therefore cancel out. It is assumed both cars, when coupled, move at $\mathbf{v}_2$ in the positive x direction.



(b)

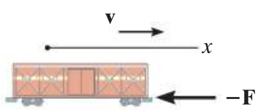
Conservation of Linear Momentum.

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$(15\,000 \text{ kg})(1.5 \text{ m/s}) - 12\,000 \text{ kg}(0.75 \text{ m/s}) = (27\,000 \text{ kg})v_2$$

$$v_2 = 0.5 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$

Part (b). The average (impulsive) coupling force, $\mathbf{F}_{avg}$, can be determined by applying the principle of linear momentum to *either one* of the cars.



(c)

Fig. 15–8

Free-Body Diagram. As shown in Fig. 15–8*c*, by isolating the boxcar the coupling force is *external* to the car.

Principle of Impulse and Momentum. Since $\int F\,dt = F_{avg}\,\Delta t = F_{avg}(0.8 \text{ s})$, we have

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + \Sigma\int F\,dt = m_A v_2$$

$$(15\,000 \text{ kg})(1.5 \text{ m/s}) - F_{avg}(0.8 \text{ s}) = (15\,000 \text{ kg})(0.5 \text{ m/s})$$

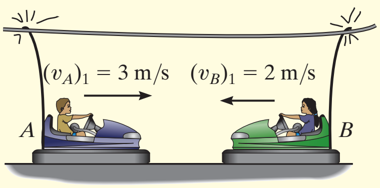
$$F_{avg} = 18.8 \text{ kN} \qquad \textit{Ans.}$$

NOTE: Solution was possible here since the boxcar's final velocity was obtained in Part (a). Try solving for F_{avg} by applying the principle of impulse and momentum to the tank car.

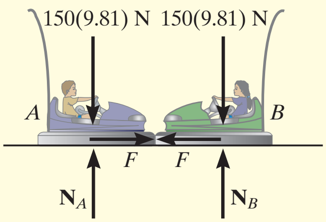
*Only horizontal forces are shown on the free-body diagram.

EXAMPLE 15.5

The bumper cars A and B in Fig. 15–9a each have a mass of 150 kg and are coasting with the velocities shown before they freely collide head on. If no energy is lost during the collision, determine their velocities after collision.



(a)



(b)

Fig. 15–9

SOLUTION

Free-Body Diagram. The cars will be considered as a single system. The free-body diagram is shown in Fig. 15–9b.

Conservation of Momentum.

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(150\text{ kg})(3\text{ m/s}) + (150\text{ kg})(-2\text{ m/s}) = (150\text{ kg})(v_A)_2 + (150\text{ kg})(v_B)_2$$

$$(v_A)_2 = 1 - (v_B)_2 \qquad (1)$$

Conservation of Energy. Since no energy is lost, the conservation of energy theorem gives

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}m_B(v_B)_1^2 + 0 = \frac{1}{2}m_A(v_A)_2^2 + \frac{1}{2}m_B(v_B)_2^2 + 0$$

$$\frac{1}{2}(150\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(150\text{ kg})(2\text{ m/s})^2 + 0 = \frac{1}{2}(150\text{ kg})(v_A)_2^2 + \frac{1}{2}(150\text{ kg})(v_B)_2^2 + 0$$

$$(v_A)_2^2 + (v_B)_2^2 = 13 \qquad (2)$$

Substituting Eq. (1) into (2) and simplifying, we get

$$(v_B)_2^2 - (v_B)_2 - 6 = 0$$

Solving for the two roots,

$$(v_B)_2 = 3\text{ m/s} \qquad \text{and} \qquad (v_B)_2 = -2\text{ m/s}$$

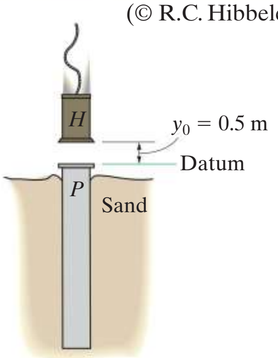
Since $(v_B)_2 = -2$ m/s refers to the velocity of B just *before* collision, then the velocity of B just after the collision must be

$$(v_B)_2 = 3\text{ m/s} \rightarrow \qquad \textit{Ans.}$$

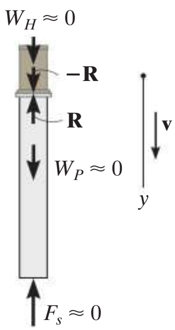
Substituting this result into Eq. (1), we obtain

$$(v_A)_2 = 1 - 3\text{ m/s} = -2\text{ m/s} = 2\text{ m/s} \leftarrow \qquad \textit{Ans.}$$

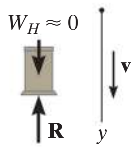
(© R.C. Hibbeler)



(a)



(b)



(c)

Fig. 15–10

An 800-kg rigid pile shown in Fig. 15–10*a* is driven into the ground using a 300-kg hammer. The hammer falls from rest at a height $y_0 = 0.5$ m and strikes the top of the pile. Determine the impulse which the pile exerts on the hammer if the pile is surrounded entirely by loose sand so that after striking, the hammer does *not* rebound off the pile.

SOLUTION

Conservation of Energy. The velocity at which the hammer strikes the pile can be determined using the conservation of energy equation applied to the hammer. With the datum at the top of the pile, Fig. 15–10*a*, we have

$$T_0 + V_0 = T_1 + V_1$$

$$\frac{1}{2}m_H(v_H)_0^2 + W_H y_0 = \frac{1}{2}m_H(v_H)_1^2 + W_H y_1$$

$$0 + 300(9.81)\text{ N}(0.5\text{ m}) = \frac{1}{2}(300\text{ kg})(v_H)_1^2 + 0$$

$$(v_H)_1 = 3.132\text{ m/s}$$

Free-Body Diagram. From the physical aspects of the problem, the free-body diagram of the hammer and pile, Fig. 15–10*b*, indicates that during the *short time* from *just before* to *just after* the *collision*, the weights of the hammer and pile and the resistance force $\mathbf{F}_s$ of the sand are all *nonimpulsive*. The impulsive force $\mathbf{R}$ is internal to the system and therefore cancels. Consequently, momentum is conserved in the vertical direction during this short time.

Conservation of Momentum. Since the hammer does not rebound off the pile just after collision, then $(v_H)_2 = (v_P)_2 = v_2$.

$$(+\downarrow) \qquad m_H(v_H)_1 + m_P(v_P)_1 = m_H v_2 + m_P v_2$$

$$(300\text{ kg})(3.132\text{ m/s}) + 0 = (300\text{ kg})v_2 + (800\text{ kg})v_2$$

$$v_2 = 0.8542\text{ m/s}$$

Principle of Impulse and Momentum. The impulse which the pile imparts to the hammer can now be determined since v_2 is known. From the free-body diagram for the hammer, Fig. 15–10*c*, we have

$$(+\downarrow) \qquad m_H(v_H)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m_H v_2$$

$$(300\text{ kg})(3.132\text{ m/s}) - \int R\,dt = (300\text{ kg})(0.8542\text{ m/s})$$

$$\int R\,dt = 683\text{ N}\cdot\text{s} \qquad \textit{Ans.}$$

NOTE: The equal but opposite impulse acts on the pile. Try finding this impulse by applying the principle of impulse and momentum to the pile.

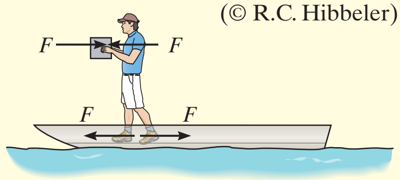
EXAMPLE 15.7

The 80-kg man can throw the 20-kg box horizontally at 4 m/s when standing on the ground. If instead he firmly stands in the 120-kg boat and throws the box, as shown in the photo, determine how far the boat will move in three seconds. Neglect water resistance.

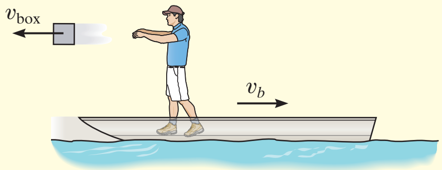
(© R.C. Hibbeler)

SOLUTION

Free-Body Diagram. If the man, boat, and box are considered as a single system, the horizontal forces between the man and the boat and the man and the box become internal to the system, Fig. 15–11*a*, and so linear momentum will be conserved along the *x* axis.



(a)



(b)

Fig. 15–11

Conservation of Momentum. When writing the conservation of momentum equation, it is *important* that the velocities be measured from the same inertial coordinate system, assumed here to be fixed. From this coordinate system, we will assume that the boat and man go to the right while the box goes to the left, as shown in Fig. 15–11*b*.

Applying the conservation of linear momentum to the man, boat, box system,

$$(\overset{+}{\rightarrow}) \qquad 0 + 0 + 0 = (m_m + m_b)\, v_b - m_{box}\, v_{box}$$

$$0 = (80 \text{ kg} + 120 \text{ kg})\, v_b - (20 \text{ kg})\, v_{box}$$

$$v_{box} = 10\, v_b \qquad (1)$$

Kinematics. Since the velocity of the box *relative* to the man (and boat), $v_{box/b}$, is known, then v_b can also be related to v_{box} using the relative velocity equation.

$$(\overset{+}{\rightarrow}) \qquad v_{box} = v_b + v_{box/b}$$

$$-v_{box} = v_b - 4 \text{ m/s} \qquad (2)$$

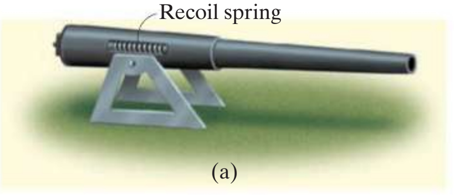
Solving Eqs. (1) and (2),

$$v_{box} = 3.64 \text{ m/s} \leftarrow$$

$$v_b = 0.3636 \text{ m/s} \rightarrow$$

The displacement of the boat in three seconds is therefore

$$s_b = v_b t = (0.3636 \text{ m/s})(3 \text{ s}) = 1.09 \text{ m} \qquad \textit{Ans.}$$



(a)

The 1200-lb cannon shown in Fig. 15–12*a* fires an 8-lb projectile with a muzzle velocity of 1500 ft/s measured relative to the cannon. If firing takes place in 0.03 s, determine the recoil velocity of the cannon just after firing. The cannon support is fixed to the ground, and the horizontal recoil of the cannon is absorbed by two springs.

SOLUTION

Part (a) Free-Body Diagram.* As shown in Fig. 15–12*b*, we will consider the projectile and cannon as a single system, since the impulsive forces, **F** and −**F**, between the cannon and projectile are *internal* to the system and will therefore cancel from the analysis. Furthermore, during the time $\Delta t = 0.03$ s, the two recoil springs which are attached to the support each exert a *nonimpulsive force* $\mathbf{F}_s$ on the cannon. This is because Δt is very short, so that during this time the cannon only moves through a *very small* distance s. Consequently, $F_s = ks \approx 0$, where k is the spring's stiffness, which is also considered to be relatively small. Hence it can be concluded that momentum for the system is conserved in the *horizontal direction*.

Conservation of Linear Momentum.

$$(\overset{+}{\rightarrow}) \qquad m_c(v_c)_1 + m_p(v_p)_1 = -m_c(v_c)_2 + m_p(v_p)_2$$

$$0 + 0 = -\left(\frac{1200\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_c)_2 + \left(\frac{8\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_p)_2$$

$$(v_p)_2 = 150\,(v_c)_2 \qquad (1)$$

These unknown velocities are measured by a *fixed* observer. As in Example 15–7, they can also be related using the relative velocity equation.

$$\overset{+}{\rightarrow} \qquad (v_p)_2 = (v_c)_2 + v_{p/c}$$

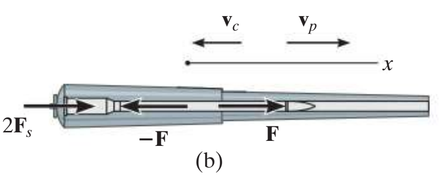
$$(v_p)_2 = -(v_c)_2 + 1500\text{ ft/s} \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$(v_c)_2 = 9.93\text{ ft/s} \qquad \textit{Ans.}$$

$$(v_p)_2 = 1490\text{ ft/s}$$

Apply the principle of impulse and momentum to the projectile (or the cannon) and show that the average impulsive force on the projectile is 12.3 kip.



(b)

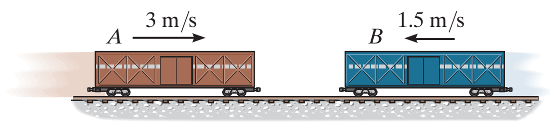
Fig. 15–12

NOTE: If the cannon is firmly fixed to its support (no springs), the reactive force of the support on the cannon must be considered as an external impulse to the system, since the support would allow no movement of the cannon. In this case momentum is *not* conserved.

*Only horizontal forces are shown on the free-body diagram.

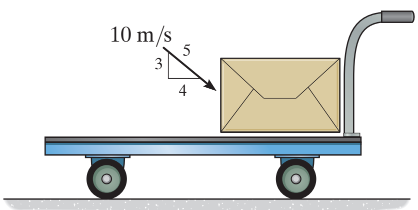
FUNDAMENTAL PROBLEMS

F15–7. The freight cars A and B have a mass of 20 Mg and 15 Mg, respectively. Determine the velocity of A after collision if the cars collide and rebound, such that B moves to the right with a speed of 2 m/s. If A and B are in contact for 0.5 s, find the average impulsive force which acts between them.



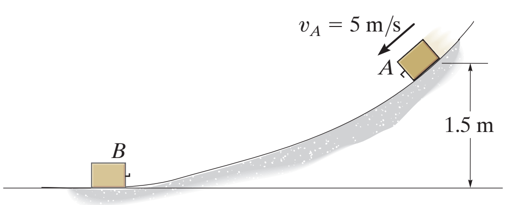
Prob. F15–7

F15–8. The cart and package have a mass of 20 kg and 5 kg, respectively. If the cart has a smooth surface and it is initially at rest, while the velocity of the package is as shown, determine the final common velocity of the cart and package after the impact.



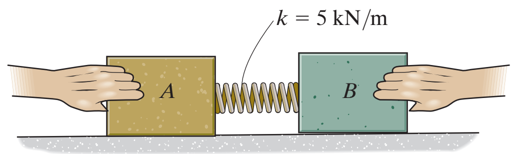
Prob. F15–8

F15–9. The 5-kg block A has an initial speed of 5 m/s as it slides down the smooth ramp, after which it collides with the stationary block B of mass 8 kg. If the two blocks couple together after collision, determine their common velocity immediately after collision.



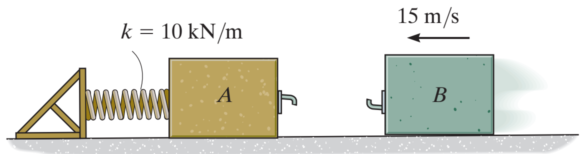
Prob. F15–9

F15–10. The spring is fixed to block A and block B is pressed against the spring. If the spring is compressed $s = 200$ mm and then the blocks are released, determine their velocity at the instant block B loses contact with the spring. The masses of blocks A and B are 10 kg and 15 kg, respectively.



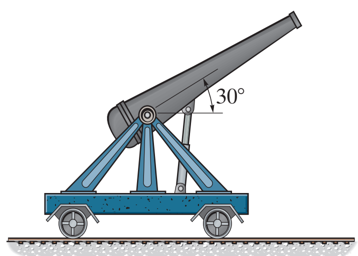
Prob. F15–10

F15–11. Blocks A and B have a mass of 15 kg and 10 kg, respectively. If A is stationary and B has a velocity of 15 m/s just before collision, and the blocks couple together after impact, determine the maximum compression of the spring.



Prob. F15–11

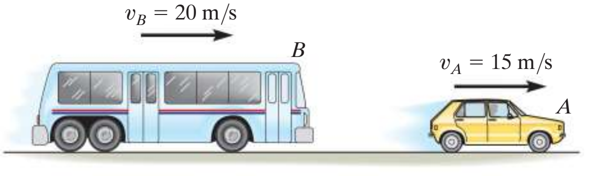
F15–12. The cannon and support without a projectile have a mass of 250 kg. If a 20-kg projectile is fired from the cannon with a velocity of 400 m/s, measured *relative* to the cannon, determine the speed of the projectile as it leaves the barrel of the cannon. Neglect rolling resistance.



Prob. F15–12

PROBLEMS

15–35. The 5-Mg bus B is traveling to the right at 20 m/s. Meanwhile a 2-Mg car A is traveling at 15 m/s to the right. If the vehicles crash and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.

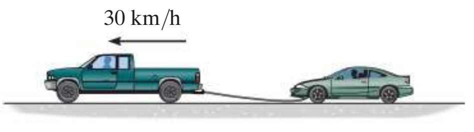


Prob. 15–35

***15–36.** The 50-kg boy jumps on the 5-kg skateboard with a horizontal velocity of 5 m/s. Determine the distance s the boy reaches up the inclined plane before momentarily coming to rest. Neglect the skateboard's rolling resistance.

Prob. 15–36

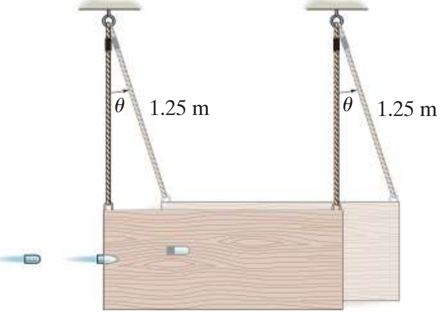
15–37. The 2.5-Mg pickup truck is towing the 1.5-Mg car using a cable as shown. If the car is initially at rest and the truck is coasting with a velocity of 30 km/h when the cable is slack, determine the common velocity of the truck and the car just after the cable becomes taut. Also, find the loss of energy.



Prob. 15–37

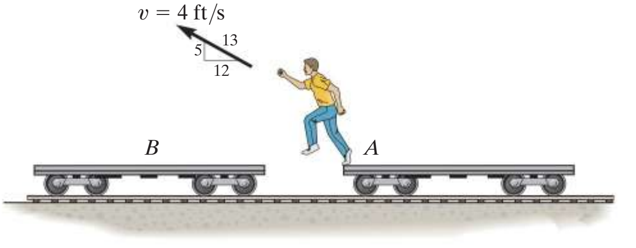
15–38. A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

15–39. A ballistic pendulum consists of a 4-kg wooden block originally at rest, $\theta = 0°$. When a 2-g bullet strikes and becomes embedded in it, it is observed that the block swings upward to a maximum angle of $\theta = 6°$. Estimate the initial speed of the bullet.



Prob. 15–39

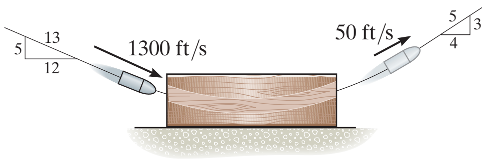
***15–40.** The boy jumps off the flat car at A with a velocity of $v = 4$ ft/s relative to the car as shown. If he lands on the second flat car B, determine the final speed of both cars after the motion. Each car has a weight of 80 lb. The boy's weight is 60 lb. Both cars are originally at rest. Neglect the mass of the car's wheels.



Prob. 15–40

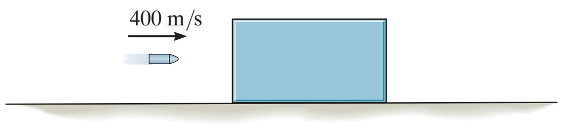
15–41. A 0.03-lb bullet traveling at 1300 ft/s strikes the 10-lb wooden block and exits the other side at 50 ft/s as shown. Determine the speed of the block just after the bullet exits the block, and also determine how far the block slides before it stops. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$.

15–42. A 0.03-lb bullet traveling at 1300 ft/s strikes the 10-lb wooden block and exits the other side at 50 ft/s as shown. Determine the speed of the block just after the bullet exits the block. Also, determine the average normal force on the block if the bullet passes through it in 1 ms, and the time the block slides before it stops. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$.



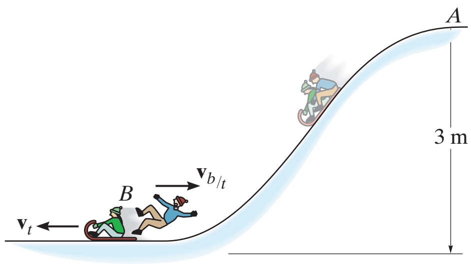
Probs. 15–41/42

15–43. The 20-g bullet is traveling at 400 m/s when it becomes embedded in the 2-kg stationary block. Determine the distance the block will slide before it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.



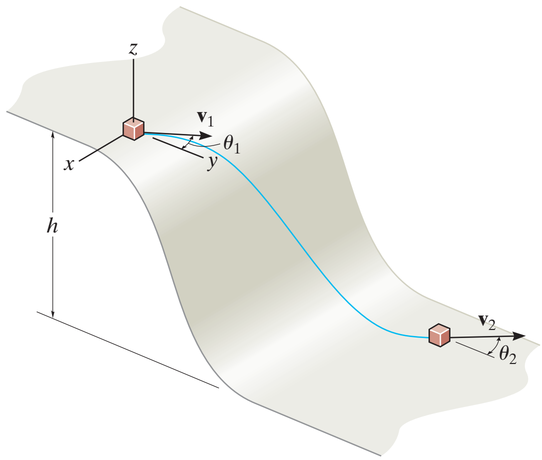
Prob. 15–43

***15–44.** A toboggan having a mass of 10 kg starts from rest at A and carries a girl and boy having a mass of 40 kg and 45 kg, respectively. When the toboggan reaches the bottom of the slope at B, the boy is pushed off from the back with a horizontal velocity of $v_{b/t} = 2$ m/s, measured relative to the toboggan. Determine the velocity of the toboggan afterwards. Neglect friction in the calculation.



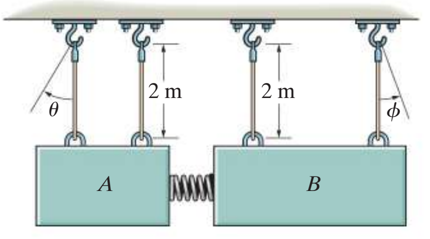
Prob. 15–44

15–45. The block of mass m travels at v_1 in the direction θ_1 shown at the top of the smooth slope. Determine its speed v_2 and its direction θ_2 when it reaches the bottom.



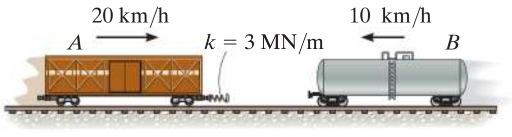
Prob. 15–45

15–46. The two blocks A and B each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of $k = 60$ N/m, is attached to B and is compressed 0.3 m against A and B as shown. Determine the maximum angles θ and ϕ of the cords when the blocks are released from rest and the spring becomes unstretched.



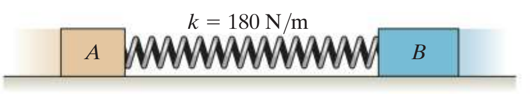
Prob. 15–46

15–47. The 30-Mg freight car A and 15-Mg freight car B are moving towards each other with the velocities shown. Determine the maximum compression of the spring mounted on car A. Neglect rolling resistance.



Prob. 15–47

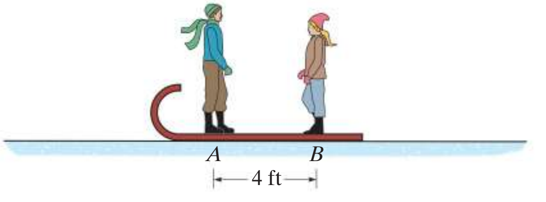
***15–48.** Blocks A and B have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.



Prob. 15–48

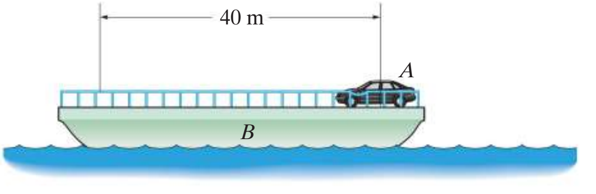
15–49. A boy A having a weight of 80 lb and a girl B having a weight of 65 lb stand motionless at the ends of the toboggan, which has a weight of 20 lb. If they exchange positions, A going to B and then B going to A's original position, determine the final position of the toboggan just after the motion. Neglect friction between the toboggan and the snow.

15–50. A boy A having a weight of 80 lb and a girl B having a weight of 65 lb stand motionless at the ends of the toboggan, which has a weight of 20 lb. If A walks to B and stops, and both walk back together to the original position of A, determine the final position of the toboggan just after the motion stops. Neglect friction between the toboggan and the snow.



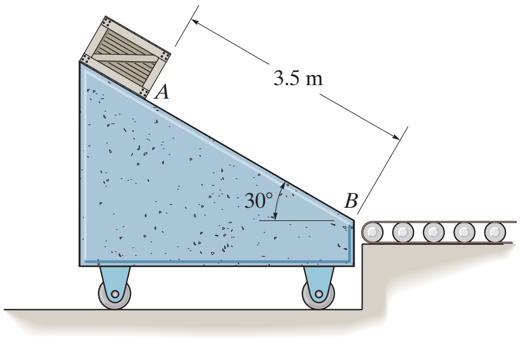
Probs. 15–49/50

15–51. The 10-Mg barge B supports a 2-Mg automobile A. If someone drives the automobile to the other side of the barge, determine how far the barge moves. Neglect the resistance of the water.



Prob. 15–51

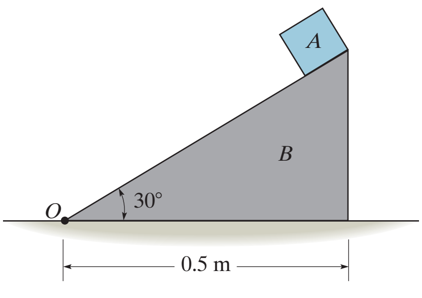
*15–52. The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?



Prob. 15–52

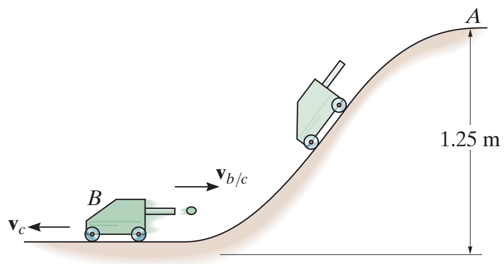
15–53. Block A has a mass of 5 kg and is placed on the smooth triangular block B having a mass of 30 kg. If the system is released from rest, determine the distance B moves from point O when A reaches the bottom. Neglect the size of block A.

15–54. Solve Prob. 15–53 if the coefficient of kinetic friction between A and B is $\mu_k = 0.3$. Neglect friction between block B and the horizontal plane.



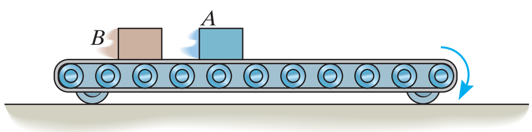
Probs. 15–53/54

15–55. The cart has a mass of 3 kg and rolls freely from A down the slope. When it reaches the bottom, a spring loaded gun fires a 0.5-kg ball out the back with a horizontal velocity of $v_{b/c} = 0.6$ m/s, measured relative to the cart. Determine the final velocity of the cart.



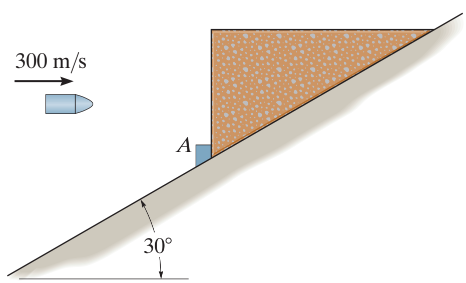
Prob. 15–55

*15–56. Two boxes A and B, each having a weight of 160 lb, sit on the 500-lb conveyor which is free to roll on the ground. If the belt starts from rest and begins to run with a speed of 3 ft/s, determine the final speed of the conveyor if (a) the boxes are not stacked and A falls off then B falls off, and (b) A is stacked on top of B and both fall off together.



Prob. 15–56

15–57. The 10-kg block is held at rest on the smooth inclined plane by the stop block at A. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.



Prob. 15–57

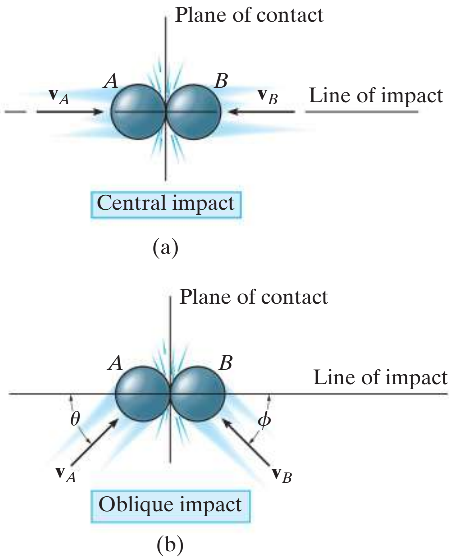


Fig. 15–13

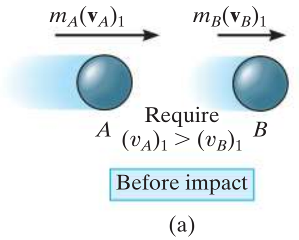
15.4 Impact

Impact occurs when two bodies collide with each other during a very *short* period of time, causing relatively large (impulsive) forces to be exerted between the bodies. The striking of a hammer on a nail, or a golf club on a ball, are common examples of impact loadings.

In general, there are two types of impact. *Central impact* occurs when the direction of motion of the mass centers of the two colliding particles is along a line passing through the mass centers of the particles. This line is called the *line of impact*, which is perpendicular to the plane of contact, Fig. 15–13*a*. When the motion of one or both of the particles make an angle with the line of impact, Fig. 15–13*b*, the impact is said to be *oblique impact*.

Central Impact. To illustrate the method for analyzing the mechanics of impact, consider the case involving the central impact of the two particles *A* and *B* shown in Fig. 15–14.

- The particles have the initial momenta shown in Fig. 15–14*a*. Provided $(v_A)_1 > (v_B)_1$, collision will eventually occur.
- During the collision the particles must be thought of as *deformable* or nonrigid. The particles will undergo a *period of deformation* such that they exert an equal but opposite deformation impulse $\int \mathbf{P}\,dt$ on each other, Fig. 15–14*b*.
- Only at the instant of *maximum deformation* will both particles move with a common velocity $\mathbf{v}$, since their relative motion is zero, Fig. 15–14*c*.
- Afterward a *period of restitution* occurs, in which case the particles will either return to their original shape or remain permanently deformed. The equal but opposite *restitution impulse* $\int \mathbf{R}\,dt$ pushes the particles apart from one another, Fig. 15–14*d*. In reality, the physical properties of any two bodies are such that the deformation impulse will *always be greater* than that of restitution, i.e., $\int P\,dt > \int R\,dt$.
- Just after separation the particles will have the final momenta shown in Fig. 15–14*e*, where $(v_B)_2 > (v_A)_2$.



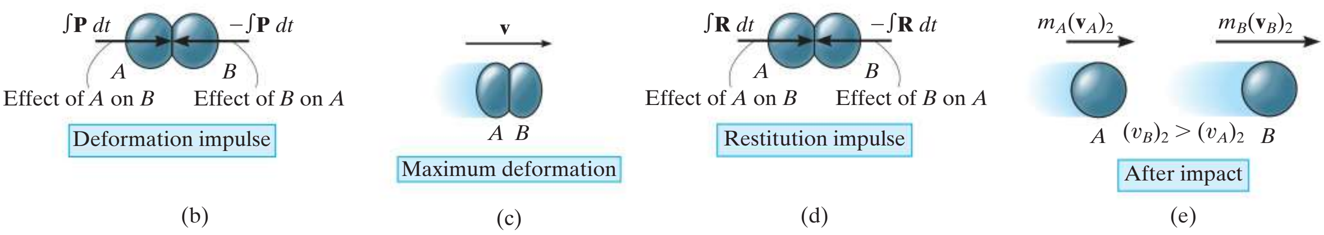


Fig. 15–14

In most problems the initial velocities of the particles will be *known*, and it will be necessary to determine their final velocities $(v_A)_2$ and $(v_B)_2$. In this regard, *momentum* for the *system of particles* is *conserved* since during collision the internal impulses of deformation and restitution *cancel*. Hence, referring to Fig. 15–14*a* and Fig. 15–14*e* we require

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2 \qquad (15\text{–}10)$$

In order to obtain a second equation necessary to solve for $(v_A)_2$ and $(v_B)_2$, we must apply the principle of impulse and momentum to *each particle*. For example, during the deformation phase for particle A, Figs. 15–14*a*, 15–14*b*, and 15–14*c*, we have

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 - \int P\,dt = m_A v$$

For the restitution phase, Figs. 15–14*c*, 15–14*d*, and 15–14*e*,

$$(\overset{+}{\rightarrow}) \qquad m_A v - \int R\,dt = m_A(v_A)_2$$

The ratio of the restitution impulse to the deformation impulse is called the *coefficient of restitution*, e. From the above equations, this value for particle A is

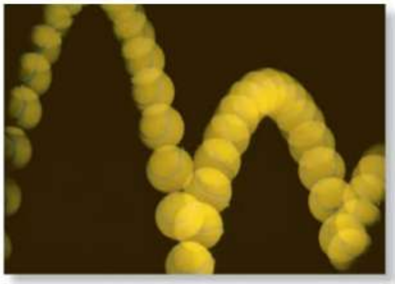
$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{v - (v_A)_2}{(v_A)_1 - v}$$

In a similar manner, we can establish e by considering particle B, Fig. 15–14. This yields

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{(v_B)_2 - v}{v - (v_B)_1}$$

If the unknown v is eliminated from the above two equations, the coefficient of restitution can be expressed in terms of the particles' initial and final velocities as

$$(\overset{+}{\rightarrow}) \qquad \boxed{e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}} \qquad (15\text{–}11)$$



The quality of a manufactured tennis ball is measured by the height of its bounce, which can be related to its coefficient of restitution. Using the mechanics of oblique impact, engineers can design a separation device to remove substandard tennis balls from a production line. (© Gary S. Settles/Science Source)

Provided a value for e is specified, Eqs. 15–10 and 15–11 can be solved simultaneously to obtain $(v_A)_2$ and $(v_B)_2$. In doing so, however, it is important to carefully establish a sign convention for defining the positive direction for both $\mathbf{v}_A$ and $\mathbf{v}_B$ and then use it *consistently* when writing *both* equations. As noted from the application shown, and indicated symbolically by the arrow in parentheses, we have defined the positive direction to the right when referring to the motions of both A and B. Consequently, if a negative value results from the solution of either $(v_A)_2$ or $(v_B)_2$, it indicates motion is to the left.

Coefficient of Restitution. From Figs. 15–14*a* and 15–14*e*, it is seen that Eq. 15–11 states that e is equal to the ratio of the relative velocity of the particles' separation *just after impact*, $(v_B)_2 - (v_A)_2$, to the relative velocity of the particles' approach *just before impact*, $(v_A)_1 - (v_B)_1$. By measuring these relative velocities experimentally, it has been found that e varies appreciably with impact velocity as well as with the size and shape of the colliding bodies. For these reasons the coefficient of restitution is reliable only when used with data which closely approximate the conditions which were known to exist when measurements of it were made. In general e has a value between zero and one, and one should be aware of the physical meaning of these two limits.

Elastic Impact ($e = 1$). If the collision between the two particles is *perfectly elastic*, the deformation impulse $\left(\int \mathbf{P}\,dt\right)$ is equal and opposite to the restitution impulse $\left(\int \mathbf{R}\,dt\right)$. Although in reality this can never be achieved, $e = 1$ for an elastic collision.

The mechanics of pool depends upon application of the conservation of momentum and the coefficient of restitution. (© R.C. Hibbeler)

Plastic Impact ($e = 0$). The impact is said to be *inelastic or plastic* when $e = 0$. In this case there is no restitution impulse $\left(\int \mathbf{R}\,dt = \mathbf{0}\right)$, so that after collision both particles couple or stick *together* and move with a common velocity.

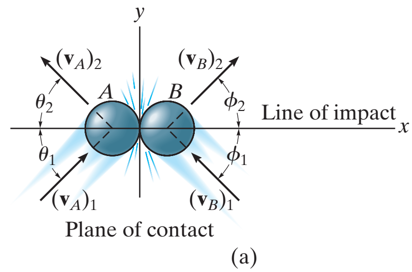
From the above derivation it should be evident that the principle of work and energy cannot be used for the analysis of impact problems since it is not possible to know how the *internal forces* of deformation and restitution vary or displace during the collision. By knowing the particle's velocities before and after collision, however, the energy loss during collision can be calculated on the basis of the difference in the particle's kinetic energy. This energy loss, $\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$, occurs because some of the initial kinetic energy of the particle is transformed into thermal energy as well as creating sound and localized deformation of the material when the collision occurs. In particular, if the impact is *perfectly elastic*, no energy is lost in the collision; whereas if the collision is *plastic*, the energy lost during collision is a maximum.

Procedure for Analysis (Central Impact)

In most cases the *final velocities* of two smooth particles are to be determined *just after* they are subjected to direct central impact. Provided the coefficient of restitution, the mass of each particle, and each particle's initial velocity *just before* impact are known, the solution to this problem can be obtained using the following two equations:

- The conservation of momentum applies to the system of particles, $\Sigma mv_1 = \Sigma mv_2$.
- The coefficient of restitution, $e = [(v_B)_2 - (v_A)_2]/[(v_A)_1 - (v_B)_1]$, relates the relative velocities of the particles along the line of impact, just before and just after collision.

When applying these two equations, the sense of an unknown velocity can be assumed. If the solution yields a negative magnitude, the velocity acts in the opposite sense.



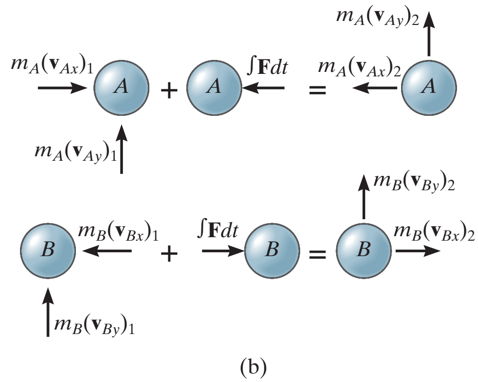


Fig. 15–15

Oblique Impact. When oblique impact occurs between two smooth particles, the particles move away from each other with velocities having unknown directions as well as unknown magnitudes. Provided the initial velocities are known, then four unknowns are present in the problem. As shown in Fig. 15–15*a*, these unknowns may be represented either as $(v_A)_2, (v_B)_2, \theta_2$, and ϕ_2, or as the x and y components of the final velocities.

Procedure for Analysis (Oblique Impact)

If the y axis is established within the plane of contact and the x axis along the line of impact, the impulsive forces of deformation and restitution act *only in the x direction*, Fig. 15–15*b*. By resolving the velocity or momentum vectors into components along the x and y axes, Fig. 15–15*b*, it is then possible to write four independent scalar equations in order to determine $(v_{Ax})_2, (v_{Ay})_2, (v_{Bx})_2$, and $(v_{By})_2$.

- Momentum of the system is conserved *along the line of impact*, x axis, so that $\Sigma m(v_x)_1 = \Sigma m(v_x)_2$.
- The coefficient of restitution, $e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]$, relates the relative-velocity *components* of the particles *along the line of impact* (x axis).
- If these two equations are solved simultaneously, we obtain $(v_{Ax})_2$ and $(v_{Bx})_2$.
- Momentum of particle A is conserved along the y axis, perpendicular to the line of impact, since no impulse acts on particle A in this direction. As a result $m_A(v_{Ay})_1 = m_A(v_{Ay})_2$ or $(v_{Ay})_1 = (v_{Ay})_2$
- Momentum of particle B is conserved along the y axis, perpendicular to the line of impact, since no impulse acts on particle B in this direction. Consequently $(v_{By})_1 = (v_{By})_2$.

Application of these four equations is illustrated in Example 15.11.

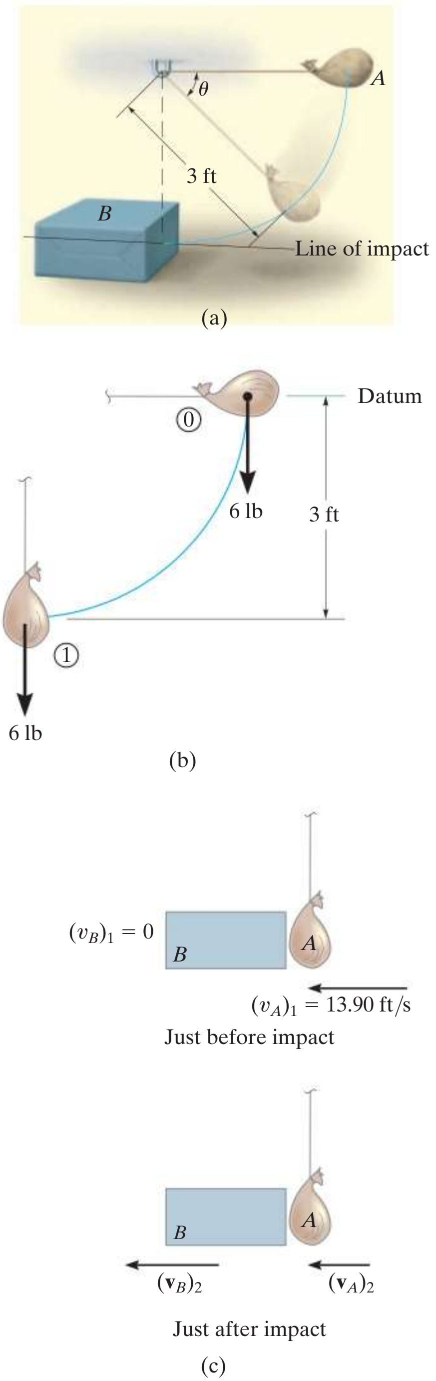


Fig. 15–16

The bag A, having a weight of 6 lb, is released from rest at the position $\theta = 0°$, as shown in Fig. 15–16*a*. After falling to $\theta = 90°$, it strikes an 18-lb box B. If the coefficient of restitution between the bag and box is $e = 0.5$, determine the velocities of the bag and box just after impact. What is the loss of energy during collision?

SOLUTION

This problem involves central impact. Why? Before analyzing the mechanics of the impact, however, it is first necessary to obtain the velocity of the bag *just before* it strikes the box.

Conservation of Energy. With the datum at $\theta = 0°$, Fig. 15–16*b*, we have

$$T_0 + V_0 = T_1 + V_1$$

$$0 + 0 = \frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_A)_1^2 - 6\text{ lb}(3\text{ ft}); (v_A)_1 = 13.90\text{ ft/s}$$

Conservation of Momentum. After impact we will assume A and B travel to the left. Applying the conservation of momentum to the system, Fig. 15–16*c*, we have

$$(\overset{+}{\leftarrow}) \qquad m_B(v_B)_1 + m_A(v_A)_1 = m_B(v_B)_2 + m_A(v_A)_2$$

$$0 + \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(13.90\text{ ft/s}) = \left(\frac{18\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_B)_2 + \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_A)_2$$

$$(v_A)_2 = 13.90 - 3(v_B)_2 \qquad (1)$$

Coefficient of Restitution. Realizing that for separation to occur after collision $(v_B)_2 > (v_A)_2$, Fig. 15–16*c*, we have

$$(\overset{+}{\leftarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.5 = \frac{(v_B)_2 - (v_A)_2}{13.90\text{ ft/s} - 0}$$

$$(v_A)_2 = (v_B)_2 - 6.950 \qquad (2)$$

Solving Eqs. 1 and 2 simultaneously yields

$$(v_A)_2 = -1.74\text{ ft/s} = 1.74\text{ ft/s} \rightarrow \quad \text{and} \quad (v_B)_2 = 5.21\text{ ft/s} \leftarrow \qquad \textit{Ans.}$$

Loss of Energy. Applying the principle of work and energy to the bag and box just before and just after collision, we have

$$\Sigma U_{1-2} = T_2 - T_1;$$

$$\Sigma U_{1-2} = \left[\frac{1}{2}\left(\frac{18\text{ lb}}{32.2\text{ ft/s}^2}\right)(5.21\text{ ft/s})^2 + \frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(1.74\text{ ft/s})^2\right] - \left[\frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(13.9\text{ ft/s})^2\right]$$

$$\Sigma U_{1-2} = -10.1\text{ ft}\cdot\text{lb} \qquad \textit{Ans.}$$

NOTE: The energy loss occurs due to inelastic deformation during the collision.

EXAMPLE 15.10

Ball B shown in Fig. 15–17*a* has a mass of 1.5 kg and is suspended from the ceiling by a 1-m-long elastic cord. If the cord is *stretched* downward 0.25 m and the ball is released from rest, determine how far the cord stretches after the ball rebounds from the ceiling. The stiffness of the cord is $k = 800\ \text{N/m}$, and the coefficient of restitution between the ball and ceiling is $e = 0.8$. The ball makes a central impact with the ceiling.

SOLUTION

First we must obtain the velocity of the ball *just before* it strikes the ceiling using energy methods, then consider the impulse and momentum between the ball and ceiling, and finally again use energy methods to determine the stretch in the cord.

Conservation of Energy. With the datum located as shown in Fig. 15–17*a*, realizing that initially $y = y_0 = (1 + 0.25)\ \text{m} = 1.25\ \text{m}$, we have

$$T_0 + V_0 = T_1 + V_1$$

$$\tfrac{1}{2}m(v_B)_0^2 - W_B y_0 + \tfrac{1}{2}ks^2 = \tfrac{1}{2}m(v_B)_1^2 + 0$$

$$0 - 1.5(9.81)\text{N}(1.25\ \text{m}) + \tfrac{1}{2}(800\ \text{N/m})(0.25\ \text{m})^2 = \tfrac{1}{2}(1.5\ \text{kg})(v_B)_1^2$$

$$(v_B)_1 = 2.968\ \text{m/s}\uparrow$$

The interaction of the ball with the ceiling will now be considered using the principles of impact.* Since an unknown portion of the mass of the ceiling is involved in the impact, the conservation of momentum for the ball–ceiling system will not be written. The "velocity" of this portion of ceiling is zero since it (or the earth) are assumed to remain at rest *both* before and after impact.

Coefficient of Restitution. Fig. 15–17*b*.

$$(+\uparrow)\quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1};\qquad 0.8 = \frac{(v_B)_2 - 0}{0 - 2.968\ \text{m/s}}$$

$$(v_B)_2 = -2.374\ \text{m/s} = 2.374\ \text{m/s}\downarrow$$

Conservation of Energy. The maximum stretch s_3 in the cord can be determined by again applying the conservation of energy equation to the ball just after collision. Assuming that $y = y_3 = (1 + s_3)\ \text{m}$, Fig. 15–17*c*, then

$$T_2 + V_2 = T_3 + V_3$$

$$\tfrac{1}{2}m(v_B)_2^2 + 0 = \tfrac{1}{2}m(v_B)_3^2 - W_B y_3 + \tfrac{1}{2}ks_3^2$$

$$\tfrac{1}{2}(1.5\ \text{kg})(2.37\ \text{m/s})^2 = 0 - 9.81(1.5)\ \text{N}(1\ \text{m} + s_3) + \tfrac{1}{2}(800\ \text{N/m})s_3^2$$

$$400s_3^2 - 14.715s_3 - 18.94 = 0$$

Solving this quadratic equation for the positive root yields

$$s_3 = 0.237\ \text{m} = 237\ \text{mm} \qquad \textit{Ans.}$$

*The weight of the ball is considered a nonimpulsive force.

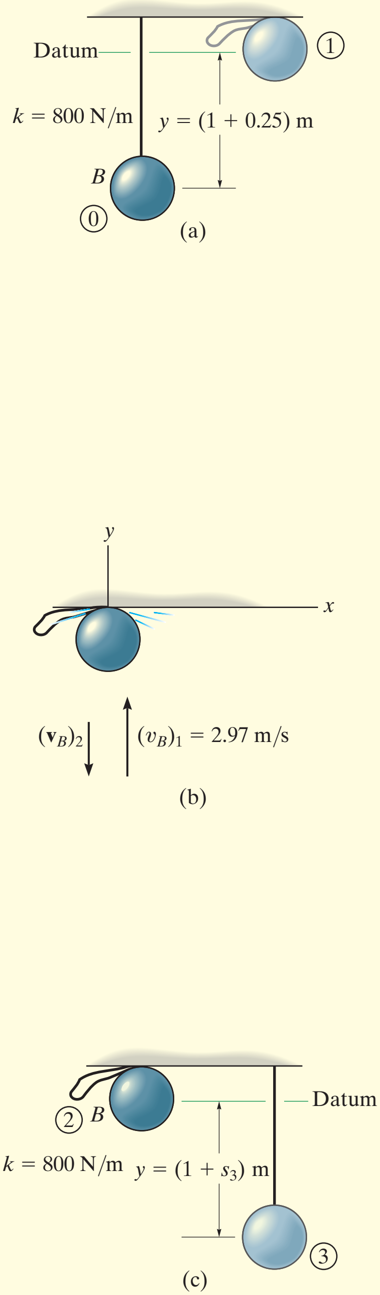
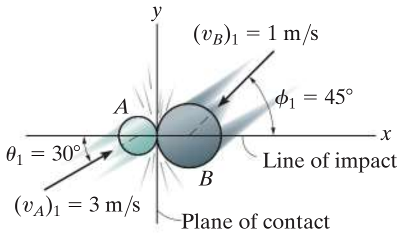


Fig. 15–17



(a)

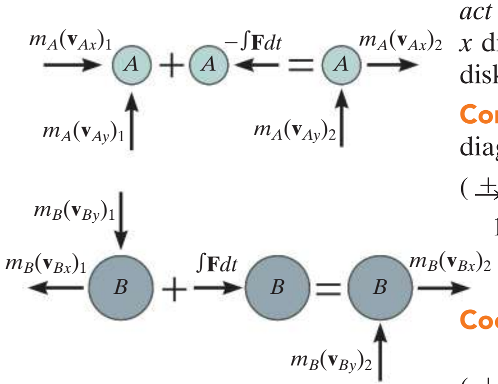
Two smooth disks A and B, having a mass of 1 kg and 2 kg, respectively, collide with the velocities shown in Fig. 15–18*a*. If the coefficient of restitution for the disks is $e = 0.75$, determine the x and y components of the final velocity of each disk just after collision.

SOLUTION

This problem involves *oblique impact*. Why? In order to solve it, we have established the x and y axes along the line of impact and the plane of contact, respectively, Fig. 15–18*a*.

Resolving each of the initial velocities into x and y components, we have

$$(v_{Ax})_1 = 3 \cos 30° = 2.598 \text{ m/s} \qquad (v_{Ay})_1 = 3 \sin 30° = 1.50 \text{ m/s}$$
$$(v_{Bx})_1 = -1 \cos 45° = -0.7071 \text{ m/s} \qquad (v_{By})_1 = -1 \sin 45° = -0.7071 \text{ m/s}$$



(b)

The four unknown velocity components after collision are *assumed to act in the positive directions*, Fig. 15–18*b*. Since the impact occurs in the x direction (line of impact), the conservation of momentum for *both* disks can be applied in this direction. Why?

Conservation of "x" Momentum. In reference to the momentum diagrams, we have

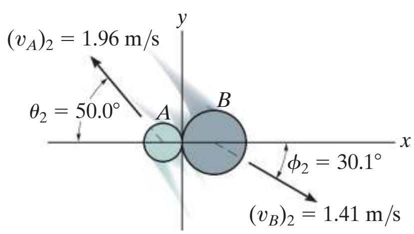
$$(\overset{+}{\rightarrow}) \qquad m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$
$$1 \text{ kg}(2.598 \text{ m/s}) + 2 \text{ kg}(-0.707 \text{ m/s}) = 1 \text{ kg}(v_{Ax})_2 + 2 \text{ kg}(v_{Bx})_2$$
$$(v_{Ax})_2 + 2(v_{Bx})_2 = 1.184 \tag{1}$$

Coefficient of Restitution (*x*).

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}; \quad 0.75 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{2.598 \text{ m/s} - (-0.7071 \text{ m/s})}$$
$$(v_{Bx})_2 - (v_{Ax})_2 = 2.482 \tag{2}$$

Solving Eqs. 1 and 2 for $(v_{Ax})_2$ and $(v_{Bx})_2$ yields

$$(v_{Ax})_2 = -1.26 \text{ m/s} = 1.26 \text{ m/s} \leftarrow \qquad (v_{Bx})_2 = 1.22 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$



(c)

Fig. 15–18

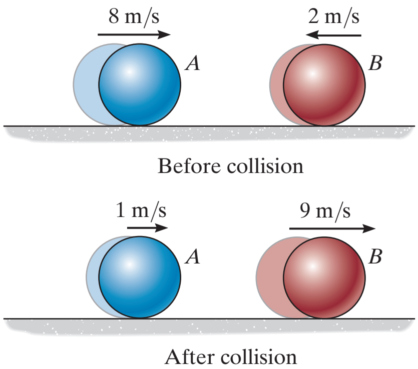
Conservation of "y" Momentum. The momentum of *each disk* is *conserved* in the y direction (plane of contact), since the disks are smooth and therefore *no* external impulse acts in this direction. From Fig. 15–18*b*,

$$(+\uparrow) \; m_A(v_{Ay})_1 = m_A(v_{Ay})_2; \quad (v_{Ay})_2 = 1.50 \text{ m/s} \uparrow \qquad \textit{Ans.}$$
$$(+\uparrow) \; m_B(v_{By})_1 = m_B(v_{By})_2; \quad (v_{By})_2 = -0.707 \text{ m/s} = 0.707 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

NOTE: Show that when the velocity components are summed vectorially, one obtains the results shown in Fig. 15–18*c*.

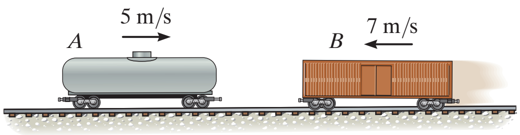
FUNDAMENTAL PROBLEMS

F15–13. Determine the coefficient of restitution e between ball A and ball B. The velocities of A and B before and after the collision are shown.



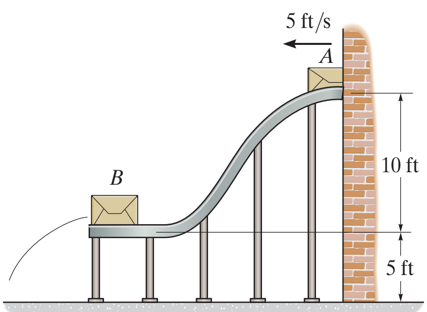
Prob. F15–13

F15–14. The 15-Mg tank car A and 25-Mg freight car B travel toward each other with the velocities shown. If the coefficient of restitution between the bumpers is $e = 0.6$, determine the velocity of each car just after the collision.



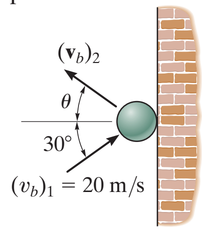
Prob. F15–14

F15–15. The 30-lb package A has a speed of 5 ft/s when it enters the smooth ramp. As it slides down the ramp, it strikes the 80-lb package B which is initially at rest. If the coefficient of restitution between A and B is $e = 0.6$, determine the velocity of B just after the impact.



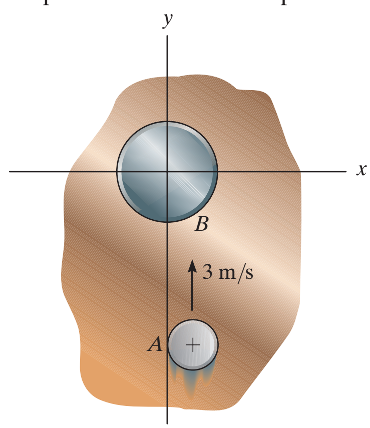
Prob. F15–15

F15–16. The ball strikes the smooth wall with a velocity of $(v_b)_1 = 20$ m/s. If the coefficient of restitution between the ball and the wall is $e = 0.75$, determine the velocity of the ball just after the impact.



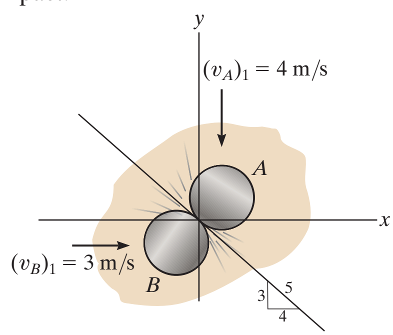
Prob. F15–16

F15–17. Disk A has a mass of 2 kg and slides on the smooth horizontal plane with a velocity of 3 m/s. Disk B has a mass of 11 kg and is initially at rest. If after impact A has a velocity of 1 m/s, parallel to the positive x axis, determine the speed of disk B after impact.



Prob. F15–17

F15–18. Two disks A and B each have a mass of 1 kg and the initial velocities shown just before they collide. If the coefficient of restitution is $e = 0.5$, determine their speeds just after impact.



Prob. F15–18

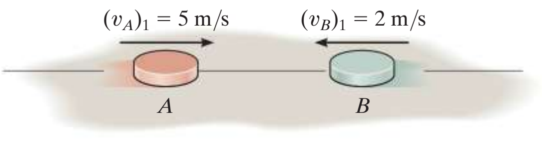
PROBLEMS

15–58. Disk A has a mass of 250 g and is sliding on a *smooth* horizontal surface with an initial velocity $(v_A)_1 = 2$ m/s. It makes a direct collision with disk B, which has a mass of 175 g and is originally at rest. If both disks are of the same size and the collision is perfectly elastic ($e = 1$), determine the velocity of each disk just after collision. Show that the kinetic energy of the disks before and after collision is the same.

15–59. The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right *relative* to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.

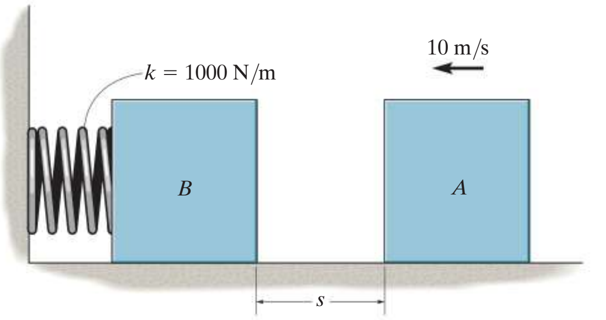
Prob. 15–59

***15–60.** Disk A has a mass of 2 kg and is sliding forward on the *smooth* surface with a velocity $(v_A)_1 = 5$ m/s when it strikes the 4-kg disk B, which is sliding towards A at $(v_B)_1 = 2$ m/s, with direct central impact. If the coefficient of restitution between the disks is $e = 0.4$, compute the velocities of A and B just after collision.



Prob. 15–60

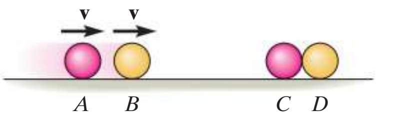
15–61. The 15-kg block A slides on the surface for which $\mu_k = 0.3$. The block has a velocity $v = 10$ m/s when it is $s = 4$ m from the 10-kg block B. If the unstretched spring has a stiffness $k = 1000$ N/m, determine the maximum compression of the spring due to the collision. Take $e = 0.6$.



Prob. 15–61

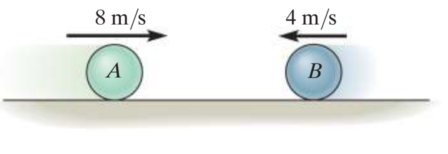
15–62. The four smooth balls each have the same mass m. If A and B are rolling forward with velocity **v** and strike C, explain why after collision C and D each move off with velocity **v**. Why doesn't D move off with velocity 2**v**? The collision is elastic, $e = 1$. Neglect the size of each ball.

15–63. The four balls each have the same mass m. If A and B are rolling forward with velocity **v** and strike C, determine the velocity of each ball after the first three collisions. Take $e = 0.5$ between each ball.



Probs. 15–62/63

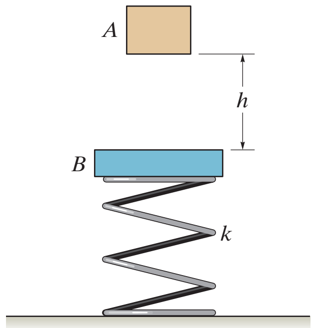
***15–64.** Ball A has a mass of 3 kg and is moving with a velocity of 8 m/s when it makes a direct collision with ball B, which has a mass of 2 kg and is moving with a velocity of 4 m/s. If $e = 0.7$, determine the velocity of each ball just after the collision. Neglect the size of the balls.



Prob. 15–64

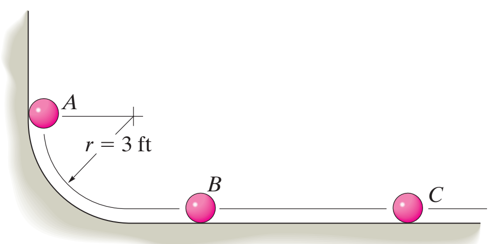
15–65. A 1-lb ball A is traveling horizontally at 20 ft/s when it strikes a 10-lb block B that is at rest. If the coefficient of restitution between A and B is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the time for the block B to stop sliding.

15–66. Block A, having a mass m, is released from rest, falls a distance h and strikes the plate B having a mass $2m$. If the coefficient of restitution between A and B is e, determine the velocity of the plate just after collision. The spring has a stiffness k.



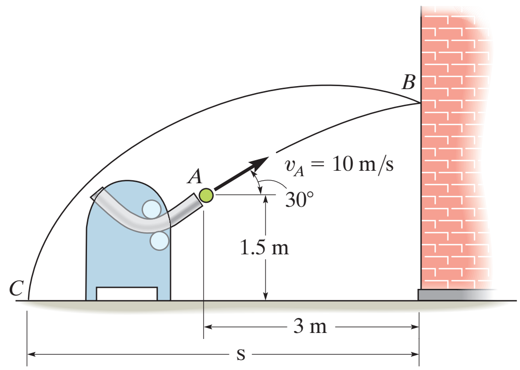
Prob. 15–66

15–67. The three balls each weigh 0.5 lb and have a coefficient of restitution of $e = 0.85$. If ball A is released from rest and strikes ball B and then ball B strikes ball C, determine the velocity of each ball after the second collision has occurred. The balls slide without friction.



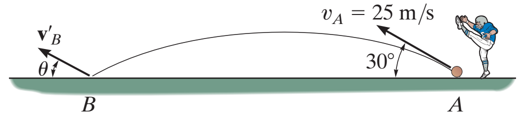
Prob. 15–67

***15–68.** A pitching machine throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10$ m/s as shown. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



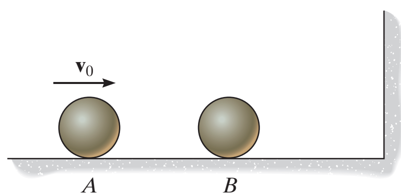
Prob. 15–68

15–69. A 300-g ball is kicked with a velocity of $v_A = 25$ m/s at point A as shown. If the coefficient of restitution between the ball and the field is $e = 0.4$, determine the magnitude and direction θ of the velocity of the rebounding ball at B.



Prob. 15–69

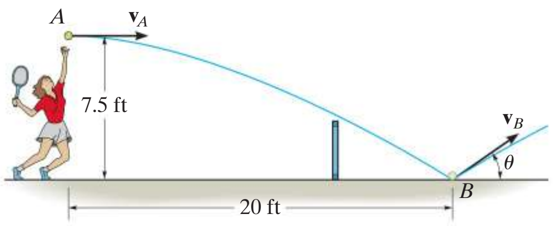
15–70. Two smooth spheres A and B each have a mass m. If A is given a velocity of v_0, while sphere B is at rest, determine the velocity of B just after it strikes the wall. The coefficient of restitution for any collision is e.



Prob. 15–70

15–71. It was observed that a tennis ball when served horizontally 7.5 ft above the ground strikes the smooth ground at B 20 ft away. Determine the initial velocity $\mathbf{v}_A$ of the ball and the velocity $\mathbf{v}_B$ (and θ) of the ball just after it strikes the court at B. Take $e = 0.7$.

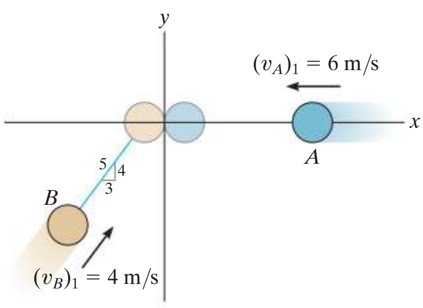
***15–72.** The tennis ball is struck with a horizontal velocity $\mathbf{v}_A$, strikes the smooth ground at B, and bounces upward at $\theta = 30°$. Determine the initial velocity $\mathbf{v}_A$, the final velocity $\mathbf{v}_B$, and the coefficient of restitution between the ball and the ground.



Probs. 15–71/72

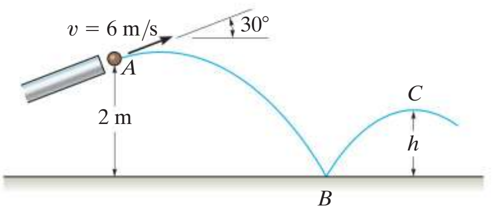
15–73. Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.

15–74. Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line, 30° counterclockwise from the y axis.



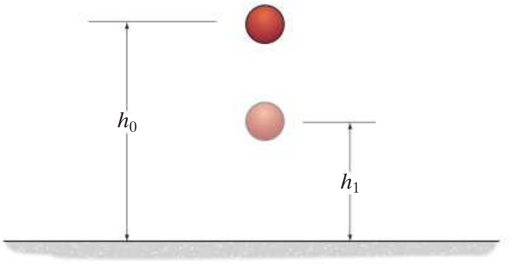
Probs. 15–73/74

15–75. The 0.5-kg ball is fired from the tube at A with a velocity of $v = 6$ m/s. If the coefficient of restitution between the ball and the surface is $e = 0.8$, determine the height h after it bounces off the surface.



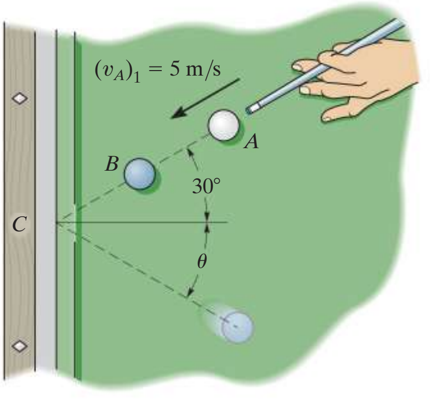
Prob. 15–75

***15–76.** A ball of mass m is dropped vertically from a height h_0 above the ground. If it rebounds to a height of h_1, determine the coefficient of restitution between the ball and the ground.



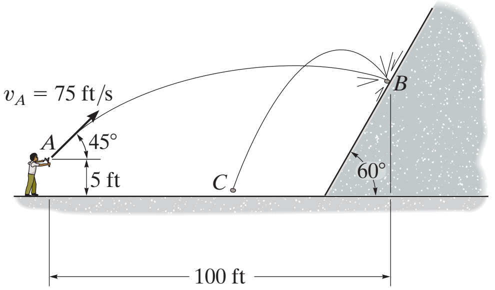
Prob. 15–76

15–77. The cue ball A is given an initial velocity $(v_A)_1 = 5$ m/s. If it makes a direct collision with ball B ($e = 0.8$), determine the velocity of B and the angle θ just after it rebounds from the cushion at C ($e' = 0.6$). Each ball has a mass of 0.4 kg. Neglect their size.



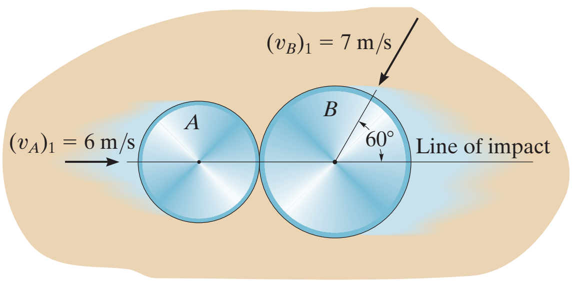
Prob. 15–77

15–78. Using a slingshot, the boy fires the 0.2-lb marble at the concrete wall, striking it at B. If the coefficient of restitution between the marble and the wall is $e = 0.5$, determine the speed of the marble after it rebounds from the wall.



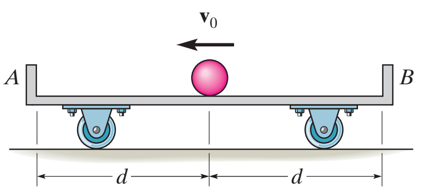
Prob. 15–78

15–79. The two disks A and B have a mass of 3 kg and 5 kg, respectively. If they collide with the initial velocities shown, determine their velocities just after impact. The coefficient of restitution is $e = 0.65$.



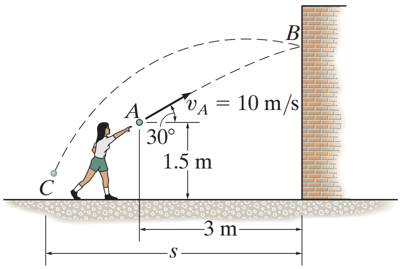
Prob. 15–79

***15–80.** A ball of negligible size and mass m is given a velocity of $\mathbf{v}_0$ on the center of the cart which has a mass M and is originally at rest. If the coefficient of restitution between the ball and walls A and B is e, determine the velocity of the ball and the cart just after the ball strikes A. Also, determine the total time needed for the ball to strike A, rebound, then strike B, and rebound and then return to the center of the cart. Neglect friction.



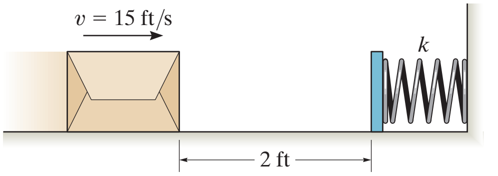
Prob. 15–80

15–81. The girl throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10$ m/s. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if the coefficient of restitution $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



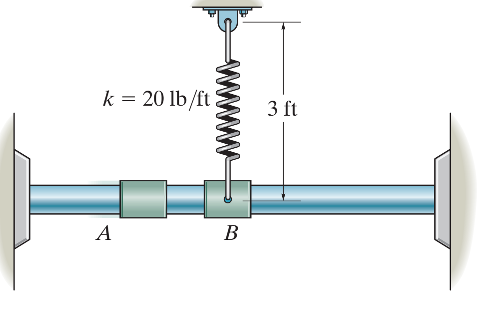
Prob. 15–81

15–82. The 20-lb box slides on the surface for which $\mu_k = 0.3$. The box has a velocity $v = 15$ ft/s when it is 2 ft from the plate. If it strikes the smooth plate, which has a weight of 10 lb and is held in position by an unstretched spring of stiffness $k = 400$ lb/ft, determine the maximum compression imparted to the spring. Take $e = 0.8$ between the box and the plate. Assume that the plate slides smoothly.



Prob. 15–82

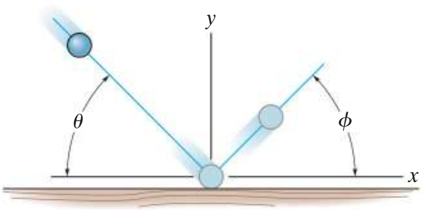
15–83. The 10-lb collar B is at rest, and when it is in the position shown the spring is unstretched. If another 1-lb collar A strikes it so that B slides 4 ft on the smooth rod before momentarily stopping, determine the velocity of A just after impact, and the average force exerted between A and B during the impact if the impact occurs in 0.002 s. The coefficient of restitution between A and B is $e = 0.5$.



Prob. 15–83

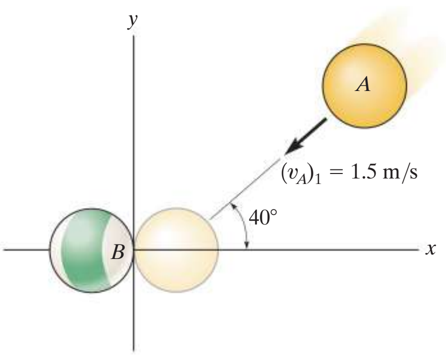
***15–84.** A ball is thrown onto a rough floor at an angle θ. If it rebounds at an angle ϕ and the coefficient of kinetic friction is μ, determine the coefficient of restitution e. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x\,\Delta t = \mu F_y\,\Delta t$ or $F_x = \mu F_y$.

15–85. A ball is thrown onto a rough floor at an angle of $\theta = 45°$. If it rebounds at the same angle $\phi = 45°$, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is $e = 0.6$. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x\,\Delta t = \mu F_y\,\Delta t$ or $F_x = \mu F_y$.



Probs. 15–84/85

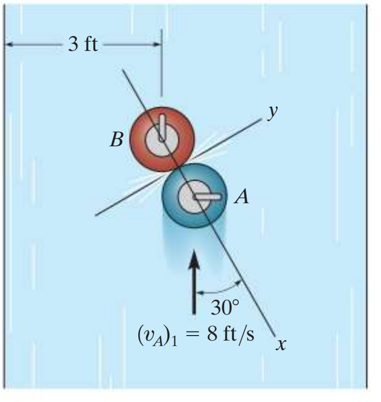
15–86. Two smooth billiard balls A and B each have a mass of 200 g. If A strikes B with a velocity $(v_A)_1 = 1.5$ m/s as shown, determine their final velocities just after collision. Ball B is originally at rest and the coefficient of restitution is $e = 0.85$. Neglect the size of each ball.



Prob. 15–86

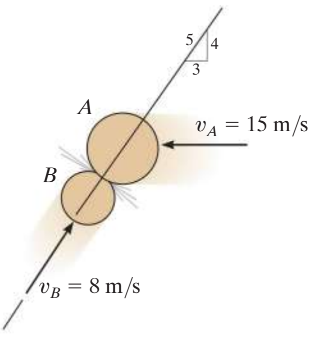
15–87. The "stone" A used in the sport of curling slides over the ice track and strikes another "stone" B as shown. If each "stone" is smooth and has a weight of 47 lb, and the coefficient of restitution between the "stones" is $e = 0.8$, determine their speeds just after collision. Initially A has a velocity of 8 ft/s and B is at rest. Neglect friction.

***15–88.** The "stone" A used in the sport of curling slides over the ice track and strikes another "stone" B as shown. If each "stone" is smooth and has a weight of 47 lb, and the coefficient of restitution between the "stone" is $e = 0.8$, determine the time required just after collision for B to slide off the runway. This requires the horizontal component of displacement to be 3 ft.



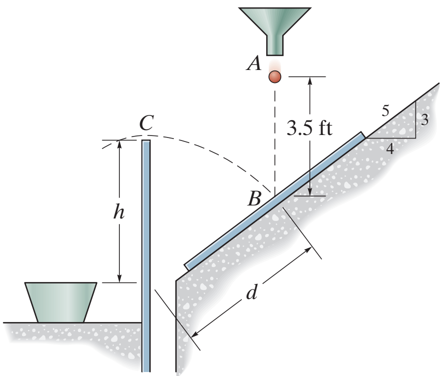
Probs. 15–87/88

15–89. Two smooth disks A and B have the initial velocities shown just before they collide. If they have masses $m_A = 4$ kg and $m_B = 2$ kg, determine their speeds just after impact. The coefficient of restitution is $e = 0.8$.



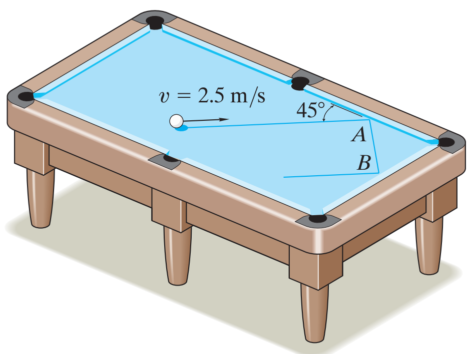
Prob. 15–89

15–90. Before a cranberry can make it to your dinner plate, it must pass a bouncing test which rates its quality. If cranberries having an $e \geq 0.8$ are to be accepted, determine the dimensions d and h for the barrier so that when a cranberry falls from rest at A it strikes the incline at B and bounces over the barrier at C.



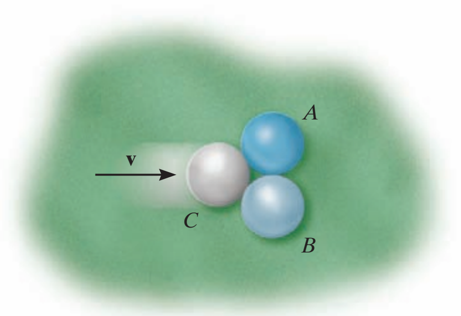
Prob. 15–90

15–91. The 200-g billiard ball is moving with a speed of 2.5 m/s when it strikes the side of the pool table at A. If the coefficient of restitution between the ball and the side of the table is $e = 0.6$, determine the speed of the ball just after striking the table twice, i.e., at A, then at B. Neglect the size of the ball.



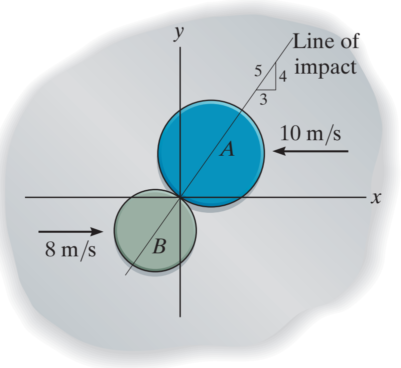
Prob. 15–91

***15–92.** The two billiard balls A and B are originally in contact with one another when a third ball C strikes each of them at the same time as shown. If ball C remains at rest after the collision, determine the coefficient of restitution. All the balls have the same mass. Neglect the size of each ball.



Prob. 15–92

15–93. Disks A and B have a mass of 15 kg and 10 kg, respectively. If they are sliding on a smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.8$.



Prob. 15–93

15.5 Angular Momentum

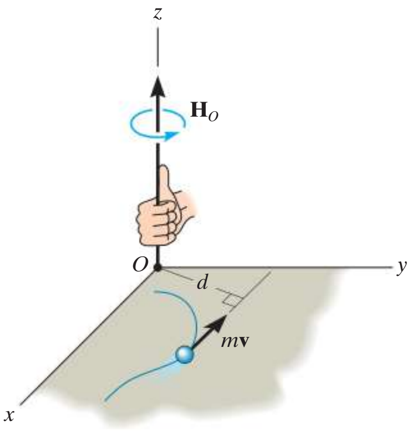


Fig. 15–19

The *angular momentum* of a particle about point O is defined as the "moment" of the particle's linear momentum about O. Since this concept is analogous to finding the moment of a force about a point, the angular momentum, $\mathbf{H}_O$, is sometimes referred to as the *moment of momentum*.

Scalar Formulation. If a particle moves along a curve lying in the x–y plane, Fig. 15–19, the angular momentum at any instant can be determined about point O (actually the z axis) by using a scalar formulation. The *magnitude* of $\mathbf{H}_O$ is

$$\boxed{(H_O)_z = (d)(mv)} \qquad (15\text{–}12)$$

Here d is the moment arm or perpendicular distance from O to the line of action of $m\mathbf{v}$. Common units for $(H_O)_z$ are $\text{kg}\cdot\text{m}^2/\text{s}$ or $\text{slug}\cdot\text{ft}^2/\text{s}$. The *direction* of $\mathbf{H}_O$ is defined by the right-hand rule. As shown, the curl of the fingers of the right hand indicates the sense of rotation of $m\mathbf{v}$ about O, so that in this case the thumb (or $\mathbf{H}_O$) is directed perpendicular to the x–y plane along the $+z$ axis.

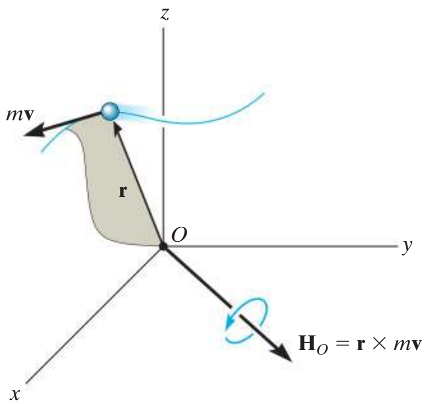


Fig. 15–20

Vector Formulation. If the particle moves along a space curve, Fig. 15–20, the vector cross product can be used to determine the *angular momentum* about O. In this case

$$\boxed{\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}} \qquad (15\text{–}13)$$

Here $\mathbf{r}$ denotes a position vector drawn from point O to the particle. As shown in the figure, $\mathbf{H}_O$ is *perpendicular* to the shaded plane containing $\mathbf{r}$ and $m\mathbf{v}$.

In order to evaluate the cross product, $\mathbf{r}$ and $m\mathbf{v}$ should be expressed in terms of their Cartesian components, so that the angular momentum can be determined by evaluating the determinant:

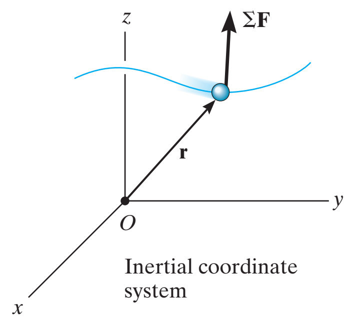
$$\boxed{\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}} \qquad (15\text{–}14)$$

15.6 Relation Between Moment of a Force and Angular Momentum

The moments about point O of all the forces acting on the particle in Fig. 15–21*a* can be related to the particle's angular momentum by applying the equation of motion. If the mass of the particle is constant, we may write

$$\Sigma \mathbf{F} = m\dot{\mathbf{v}}$$

The moments of the forces about point O can be obtained by performing a cross-product multiplication of each side of this equation by the position vector $\mathbf{r}$, which is measured from the x, y, z inertial frame of reference. We have



(a)

Fig. 15–21

$$\Sigma \mathbf{M}_O = \mathbf{r} \times \Sigma \mathbf{F} = \mathbf{r} \times m\dot{\mathbf{v}}$$

From Appendix B, the derivative of $\mathbf{r} \times m\mathbf{v}$ can be written as

$$\dot{\mathbf{H}}_O = \frac{d}{dt}(\mathbf{r} \times m\mathbf{v}) = \dot{\mathbf{r}} \times m\mathbf{v} + \mathbf{r} \times m\dot{\mathbf{v}}$$

The first term on the right side, $\dot{\mathbf{r}} \times m\mathbf{v} = m(\dot{\mathbf{r}} \times \dot{\mathbf{r}}) = \mathbf{0}$, since the cross product of a vector with itself is zero. Hence, the above equation becomes

$$\boxed{\Sigma \mathbf{M}_O = \dot{\mathbf{H}}_O} \qquad (15\text{–}15)$$

which states that *the resultant moment about point O of all the forces acting on the particle is equal to the time rate of change of the particle's angular momentum about point O*. This result is similar to Eq. 15–1, i.e.,

$$\boxed{\Sigma \mathbf{F} = \dot{\mathbf{L}}} \qquad (15\text{–}16)$$

Here $\mathbf{L} = m\mathbf{v}$, so that *the resultant force acting on the particle is equal to the time rate of change of the particle's linear momentum*.

From the derivations, it is seen that Eqs. 15–15 and 15–16 are actually another way of stating Newton's second law of motion. In other sections of this book it will be shown that these equations have many practical applications when extended and applied to problems involving either a system of particles or a rigid body.

System of Particles. An equation having the same form as Eq. 15–15 may be derived for the system of particles shown in Fig. 15–21*b*. The forces acting on the arbitrary *i*th particle of the system consist of a resultant *external force* $\mathbf{F}_i$ and a resultant *internal force* $\mathbf{f}_i$. Expressing the moments of these forces about point O, using the form of Eq. 15–15, we have

$$(\mathbf{r}_i \times \mathbf{F}_i) + (\mathbf{r}_i \times \mathbf{f}_i) = (\dot{\mathbf{H}}_i)_O$$

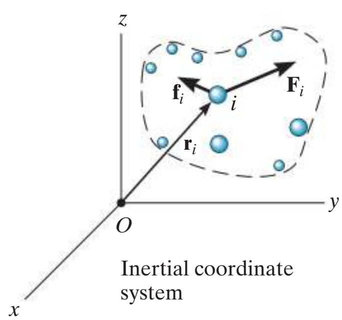
Here $(\dot{\mathbf{H}}_i)_O$ is the time rate of change in the angular momentum of the *i*th particle about O. Similar equations can be written for each of the other particles of the system. When the results are summed vectorially, the result is

$$\Sigma(\mathbf{r}_i \times \mathbf{F}_i) + \Sigma(\mathbf{r}_i \times \mathbf{f}_i) = \Sigma(\dot{\mathbf{H}}_i)_O$$

The second term is zero since the internal forces occur in equal but opposite collinear pairs, and hence the moment of each pair about point O is zero. Dropping the index notation, the above equation can be written in a simplified form as

$$\Sigma \mathbf{M}_O = \dot{\mathbf{H}}_O \tag{15–17}$$

which states that *the sum of the moments about point O of all the external forces acting on a system of particles is equal to the time rate of change of the total angular momentum of the system about point O*. Although O has been chosen here as the origin of coordinates, it actually can represent any *fixed point* in the inertial frame of reference.

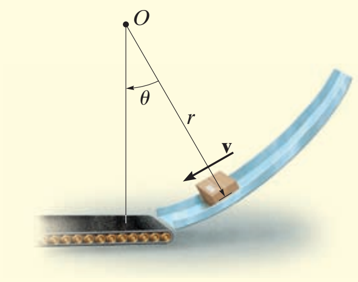


(b)

Fig. 15–21 (cont.)

EXAMPLE 15.12

The box shown in Fig. 15–22*a* has a mass m and travels down the smooth circular ramp such that when it is at the angle θ it has a speed v. Determine its angular momentum about point O at this instant and the rate of increase in its speed, i.e., a_t.



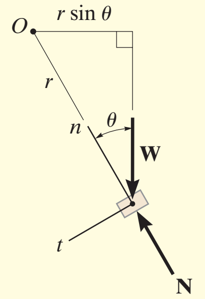


Fig. 15–22

SOLUTION

Since $\mathbf{v}$ is tangent to the path, applying Eq. 15–12 the angular momentum is

$$H_O = r\,m\,v \circlearrowright \qquad \textit{Ans.}$$

The rate of increase in its speed (dv/dt) can be found by applying Eq. 15–15. From the free-body diagram of the box, Fig. 15–22*b*, it can be seen that only the weight $W = mg$ contributes a moment about point O. We have

$$\circlearrowleft +\Sigma M_O = \dot{H}_O; \qquad mg(r\sin\theta) = \frac{d}{dt}(r\,m\,v)$$

Since r and m are constant,

$$m\,g\,r\sin\theta = r\,m\frac{dv}{dt}$$

$$\frac{dv}{dt} = g\sin\theta \qquad \textit{Ans.}$$

NOTE: This same result can, of course, be obtained from the equation of motion applied in the tangential direction, Fig. 15–22*b*, i.e.,

$$+\swarrow \Sigma F_t = ma_t; \qquad mg\sin\theta = m\left(\frac{dv}{dt}\right)$$

$$\frac{dv}{dt} = g\sin\theta \qquad \textit{Ans.}$$

15.7 Principle of Angular Impulse and Momentum

Principle of Angular Impulse and Momentum. If Eq. 15–15 is rewritten in the form $\Sigma \mathbf{M}_O\,dt = d\mathbf{H}_O$ and integrated, assuming that at time $t = t_1$, $\mathbf{H}_O = (\mathbf{H}_O)_1$ and at time $t = t_2$, $\mathbf{H}_O = (\mathbf{H}_O)_2$, we have

$$\Sigma \int_{t_1}^{t_2} \mathbf{M}_O\,dt = (\mathbf{H}_O)_2 - (\mathbf{H}_O)_1$$

or

$$(\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O\,dt = (\mathbf{H}_O)_2 \qquad (15\text{–}18)$$

This equation is referred to as the *principle of angular impulse and momentum*. The initial and final angular momenta $(\mathbf{H}_O)_1$ and $(\mathbf{H}_O)_2$ are defined as the moment of the linear momentum of the particle $(\mathbf{H}_O = \mathbf{r} \times m\mathbf{v})$ at the instants t_1 and t_2, respectively. The second term on the left side, $\Sigma \int \mathbf{M}_O\,dt$, is called the *angular impulse*. It is determined by integrating, with respect to time, the moments of all the forces acting on the particle over the time period t_1 to t_2. Since the moment of a force about point O is $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$, the angular impulse may be expressed in vector form as

$$\boxed{\text{angular impulse} = \int_{t_1}^{t_2} \mathbf{M}_O\,dt = \int_{t_1}^{t_2} (\mathbf{r} \times \mathbf{F})\,dt} \qquad (15\text{–}19)$$

Here $\mathbf{r}$ is a position vector which extends from point O to any point on the line of action of $\mathbf{F}$.

In a similar manner, using Eq. 15–18, the principle of angular impulse and momentum for a system of particles may be written as

$$\Sigma(\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O\,dt = \Sigma(\mathbf{H}_O)_2 \qquad (15\text{–}20)$$

Here the first and third terms represent the angular momenta of all the particles $[\Sigma \mathbf{H}_O = \Sigma(\mathbf{r}_i \times m\mathbf{v}_i)]$ at the instants t_1 and t_2. The second term is the sum of the angular impulses given to all the particles from t_1 to t_2. Recall that these impulses are created only by the moments of the external forces acting on the system where, for the ith particle, $\mathbf{M}_O = \mathbf{r}_i \times \mathbf{F}_i$.

Vector Formulation. Using impulse and momentum principles, it is therefore possible to write two equations which define the particle's motion, namely, Eqs. 15–3 and Eqs. 15–18, restated as

$$\boxed{\begin{aligned} m\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}\, dt &= m\mathbf{v}_2 \\ (\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O\, dt &= (\mathbf{H}_O)_2 \end{aligned}} \tag{15–21}$$

Scalar Formulation. In general, the above equations can be expressed in x, y, z component form. If the particle is confined to move in the x–y plane, then three scalar equations can be written to express the motion, namely,

$$\boxed{\begin{aligned} m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt &= m(v_x)_2 \\ m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt &= m(v_y)_2 \\ (H_O)_1 + \Sigma \int_{t_1}^{t_2} M_O\, dt &= (H_O)_2 \end{aligned}} \tag{15–22}$$

The first two of these equations represent the principle of linear impulse and momentum in the x and y directions, which has been discussed in Sec. 15–1, and the third equation represents the principle of angular impulse and momentum about the z axis.

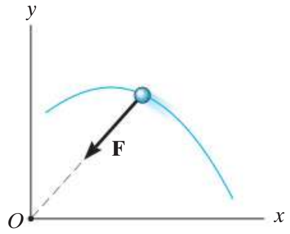


Fig. 15–23

Conservation of Angular Momentum. When the angular impulses acting on a particle are all zero during the time t_1 to t_2, Eq. 15–18 reduces to the following simplified form:

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2 \tag{15–23}$$

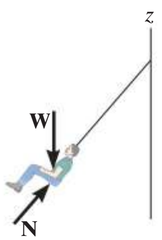
This equation is known as the *conservation of angular momentum*. It states that from t_1 to t_2 the particle's angular momentum remains constant. Obviously, if no external impulse is applied to the particle, both linear and angular momentum will be conserved. In some cases, however, the particle's angular momentum will be conserved and linear momentum may not. An example of this occurs when the particle is subjected *only* to a *central force* (see Sec. 13.7). As shown in Fig. 15–23, the impulsive central force **F** is always directed toward point O as the particle moves along the path. Hence, the angular impulse (moment) created by **F** about the z axis is always zero, and therefore angular momentum of the particle is conserved about this axis.

From Eq. 15–20, we can also write the conservation of angular momentum for a system of particles as

$$\Sigma(\mathbf{H}_O)_1 = \Sigma(\mathbf{H}_O)_2 \tag{15–24}$$

In this case the summation must include the angular momenta of all particles in the system.

(© Petra Hilke/Fotolia)



Provided air resistance is neglected, the passengers on this amusement-park ride are subjected to a conservation of angular momentum about the z axis of rotation. As shown on the free-body diagram, the line of action of the normal force **N** of the seat on the passenger passes through this axis, and the passenger's weight **W** is parallel to it. Thus, no angular impulse acts around the z axis.

Procedure for Analysis

When applying the principles of angular impulse and momentum, or the conservation of angular momentum, it is suggested that the following procedure be used.

Free-Body Diagram.

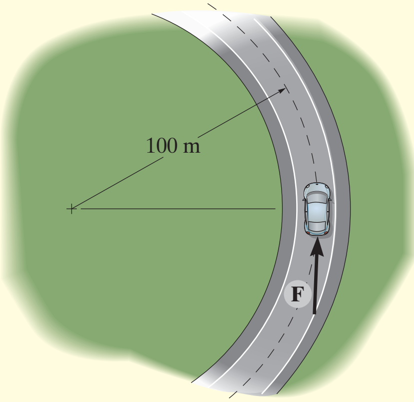
- Draw the particle's free-body diagram in order to determine any axis about which angular momentum may be conserved. For this to occur, the moments of all the forces (or impulses) must either be parallel or pass through the axis so as to create zero moment throughout the time period t_1 to t_2.
- The direction and sense of the particle's initial and final velocities should also be established.
- An alternative procedure would be to draw the impulse and momentum diagrams for the particle.

Momentum Equations.

- Apply the principle of angular impulse and momentum, $(\mathbf{H}_O)_1 + \Sigma\int_{t_1}^{t_2}\mathbf{M}_O dt = (\mathbf{H}_O)_2$, or if appropriate, the conservation of angular momentum, $(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$.

EXAMPLE 15.13

The 1.5-Mg car travels along the circular road as shown in Fig. 15–24*a*. If the traction force of the wheels on the road is $F = (150t^2)$ N, where t is in seconds, determine the speed of the car when $t = 5$ s. The car initially travels with a speed of 5 m/s. Neglect the size of the car.



(a)

Free-Body Diagram. The free-body diagram of the car is shown in Fig. 15–24*b*. If we apply the principle of angular impulse and momentum about the z axis, then the angular impulse created by the weight, normal force, and radial frictional force will be eliminated since they act parallel to the axis or pass through it.

Principle of Angular Impulse and Momentum.

$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z\,dt = (H_z)_2$$

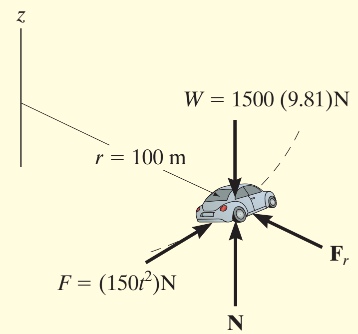
$$r\,m_c(v_c)_1 + \int_{t_1}^{t_2} r\,F\,dt = r\,m_c(v_c)_2$$

$$(100\text{ m})(1500\text{ kg})(5\text{ m/s}) + \int_0^{5\text{ s}} (100\text{ m})[(150t^2)\text{ N}]\,dt$$

$$= (100\text{ m})(1500\text{ kg})(v_c)_2$$

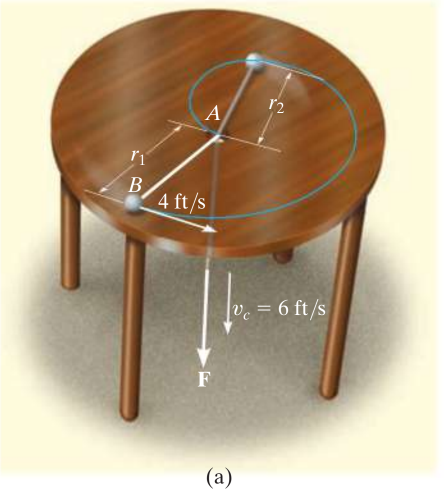
$$750(10^3) + 5000t^3\Big|_0^{5\text{ s}} = 150(10^3)(v_c)_2$$

$$(v_c)_2 = 9.17\text{ m/s} \qquad \textit{Ans.}$$

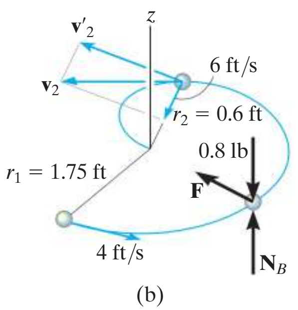


(b)

Fig. 15–24



(a)



(b)

Fig. 15–25

The 0.8-lb ball B, shown in Fig. 15–25a, is attached to a cord which passes through a hole at A in a smooth table. When the ball is $r_1 = 1.75$ ft from the hole, it is rotating around in a circle such that its speed is $v_1 = 4$ ft/s. By applying the force $\mathbf{F}$ the cord is pulled downward through the hole with a constant speed $v_c = 6$ ft/s. Determine (a) the speed of the ball at the instant it is $r_2 = 0.6$ ft from the hole, and (b) the amount of work done by $\mathbf{F}$ in shortening the radial distance from r_1 to r_2. Neglect the size of the ball.

SOLUTION

Part (a) Free-Body Diagram. As the ball moves from r_1 to r_2, Fig. 15–25b, the cord force $\mathbf{F}$ on the ball always passes through the z axis, and the weight and $\mathbf{N}_B$ are parallel to it. Hence the moments, or angular impulses created by these forces, are all *zero* about this axis. Therefore, angular momentum is conserved about the z axis.

Conservation of Angular Momentum. The ball's velocity $\mathbf{v}_2$ is resolved into two components. The radial component, 6 ft/s, is known; however, it produces zero angular momentum about the z axis. Thus,

$$\mathbf{H}_1 = \mathbf{H}_2$$

$$r_1 m_B v_1 = r_2 m_B v_2'$$

$$1.75\text{ ft}\left(\frac{0.8\text{ lb}}{32.2\text{ ft/s}^2}\right)4\text{ ft/s} = 0.6\text{ ft}\left(\frac{0.8\text{ lb}}{32.2\text{ ft/s}^2}\right)v_2'$$

$$v_2' = 11.67\text{ ft/s}$$

The speed of the ball is thus

$$v_2 = \sqrt{(11.67\text{ ft/s})^2 + (6\text{ ft/s})^2}$$

$$= 13.1\text{ ft/s}$$

Part (b). The only force that does work on the ball is $\mathbf{F}$. (The normal force and weight do not move vertically.) The initial and final kinetic energies of the ball can be determined so that from the principle of work and energy we have

$$T_1 + \Sigma U_{1-2} = T_2$$

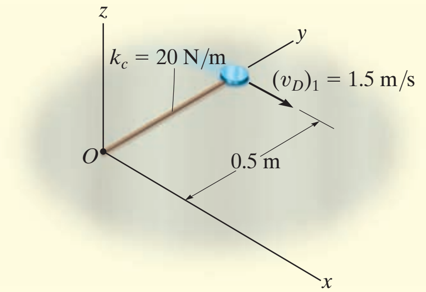
$$\frac{1}{2}\left(\frac{0.8\text{ lb}}{32.2\text{ ft/s}^2}\right)(4\text{ ft/s})^2 + U_F = \frac{1}{2}\left(\frac{0.8\text{ lb}}{32.2\text{ ft/s}^2}\right)(13.1\text{ ft/s})^2$$

$$U_F = 1.94\text{ ft}\cdot\text{lb} \qquad \textit{Ans.}$$

NOTE: The force F is not constant because the normal component of acceleration, $a_n = v^2/r$, changes as r changes.

EXAMPLE 15.15

The 2-kg disk shown in Fig. 15–26*a* rests on a smooth horizontal surface and is attached to an elastic cord that has a stiffness $k_c = 20$ N/m and is initially unstretched. If the disk is given a velocity $(v_D)_1 = 1.5$ m/s, perpendicular to the cord, determine the rate at which the cord is being stretched and the speed of the disk at the instant the cord is stretched 0.2 m.



(a)

SOLUTION

Free-Body Diagram. After the disk has been launched, it slides along the path shown in Fig. 15–26*b*. By inspection, angular momentum about point O (or the z axis) is *conserved*, since none of the forces produce an angular impulse about this axis. Also, when the distance is 0.7 m, only the transverse component $(\mathbf{v}'_D)_2$ produces angular momentum of the disk about O.

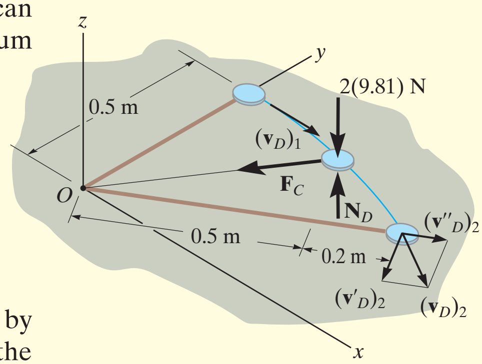
Conservation of Angular Momentum. The component $(\mathbf{v}'_D)_2$ can be obtained by applying the conservation of angular momentum about O (the z axis).

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

$$r_1 m_D (v_D)_1 = r_2 m_D (v'_D)_2$$

$$0.5 \text{ m } (2 \text{ kg})(1.5 \text{ m/s}) = 0.7 \text{ m}(2 \text{ kg})(v'_D)_2$$

$$(v'_D)_2 = 1.071 \text{ m/s}$$



(b)

Fig. 15–26

Conservation of Energy. The speed of the disk can be obtained by applying the conservation of energy equation at the point where the disk was launched and at the point where the cord is stretched 0.2 m.

$$T_1 + V_1 = T_2 + V_2$$

$$\tfrac{1}{2}m_D(v_D)_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}m_D(v_D)_2^2 + \tfrac{1}{2}kx_2^2$$

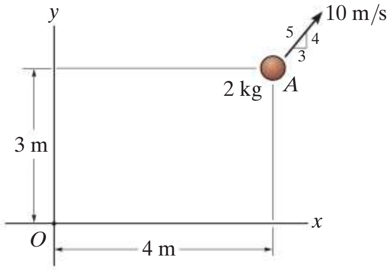
$$\tfrac{1}{2}(2 \text{ kg})(1.5 \text{ m/s})^2 + 0 = \tfrac{1}{2}(2 \text{ kg})(v_D)_2^2 + \tfrac{1}{2}(20 \text{ N/m})(0.2 \text{ m})^2$$

$$(v_D)_2 = 1.360 \text{ m/s} = 1.36 \text{ m/s} \qquad \textit{Ans.}$$

Having determined $(v_D)_2$ and its component $(v'_D)_2$, the rate of stretch of the cord, or radial component, $(v''_D)_2$ is determined from the Pythagorean theorem,

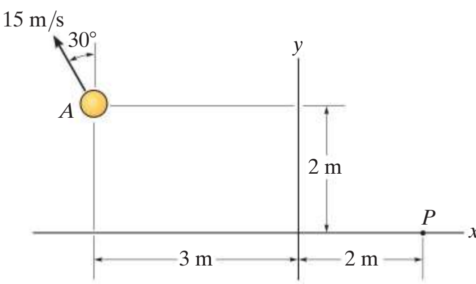
$$\begin{aligned}(v''_D)_2 &= \sqrt{(v_D)_2^2 - (v'_D)_2^2} \\ &= \sqrt{(1.360 \text{ m/s})^2 - (1.071 \text{ m/s})^2} \\ &= 0.838 \text{ m/s} \qquad \textit{Ans.}\end{aligned}$$

F15–19. The 2-kg particle A has the velocity shown. Determine its angular momentum $\mathbf{H}_O$ about point O.



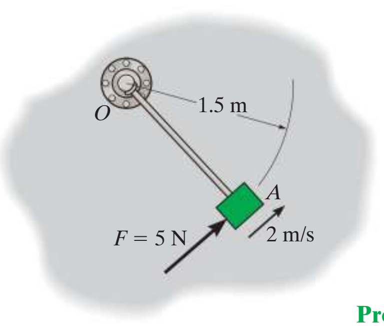
Prob. F15–19

F15–20. The 2-kg particle A has the velocity shown. Determine its angular momentum $\mathbf{H}_P$ about point P.



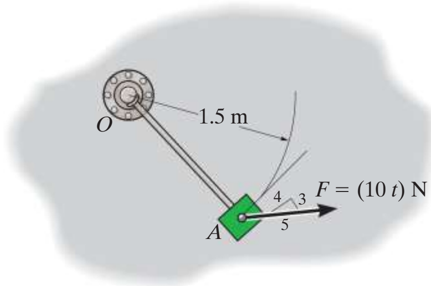
Prob. F15–20

F15–21. Initially the 5-kg block is moving with a constant speed of 2 m/s around the circular path centered at O on the smooth horizontal plane. If a constant tangential force $F = 5$ N is applied to the block, determine its speed when $t = 3$ s. Neglect the size of the block.



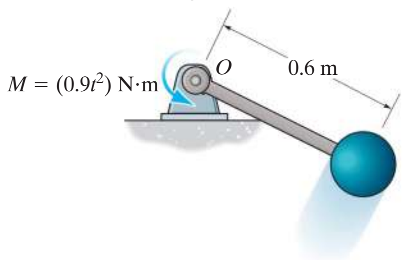
Prob. F15–21

F15–22. The 5-kg block is moving around the circular path centered at O on the smooth horizontal plane when it is subjected to the force $F = (10t)$ N, where t is in seconds. If the block starts from rest, determine its speed when $t = 4$ s. Neglect the size of the block. The force maintains the same constant angle tangent to the path.



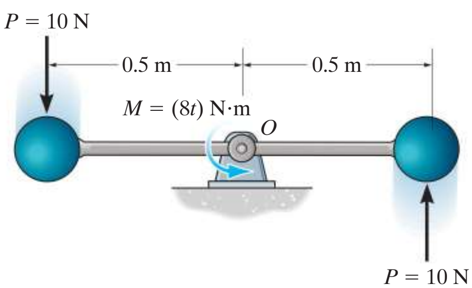
Prob. F15–22

F15–23. The 2-kg sphere is attached to the light rigid rod, which rotates in the *horizontal plane* centered at O. If the system is subjected to a couple moment $M = (0.9t^2)$ N·m, where t is in seconds, determine the speed of the sphere at the instant $t = 5$ s starting from rest.



Prob. F15–23

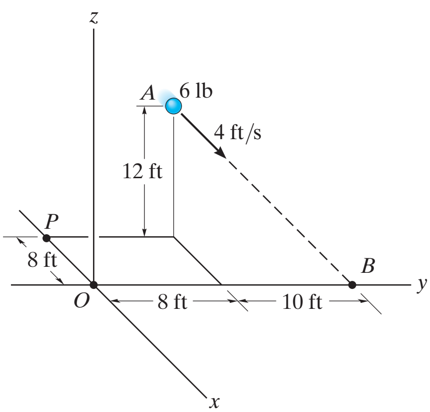
F15–24. Two identical 10-kg spheres are attached to the light rigid rod, which rotates in the horizontal plane centered at pin O. If the spheres are subjected to tangential forces of $P = 10$ N, and the rod is subjected to a couple moment $M = (8t)$ N·m, where t is in seconds, determine the speed of the spheres at the instant $t = 4$ s. The system starts from rest. Neglect the size of the spheres.



Prob. F15–24

15–94. Determine the angular momentum $\mathbf{H}_O$ of the 6-lb particle about point O.

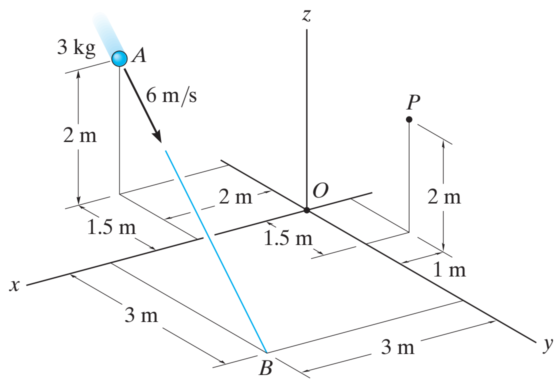
15–95. Determine the angular momentum $\mathbf{H}_P$ of the 6-lb particle about point P.



Probs. 15–94/95

15–98. Determine the angular momentum $\mathbf{H}_O$ of the 3-kg particle about point O.

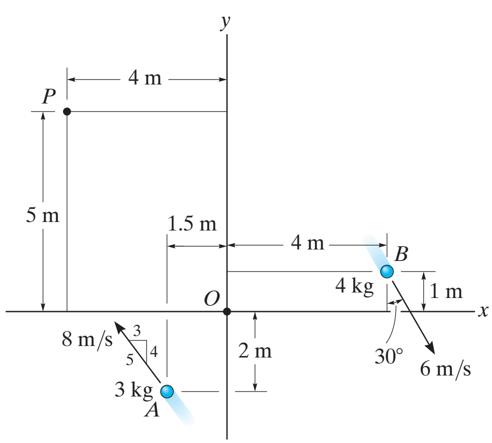
15–99. Determine the angular momentum $\mathbf{H}_P$ of the 3-kg particle about point P.



Probs. 15–98/99

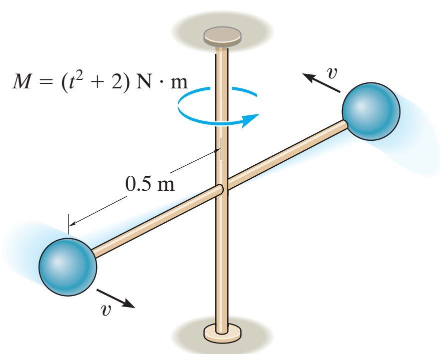
***15–96.** Determine the angular momentum $\mathbf{H}_o$ of each of the two particles about point O.

15–97. Determine the angular momentum $\mathbf{H}_p$ of each of the two particles about point P.



Probs. 15–96/97

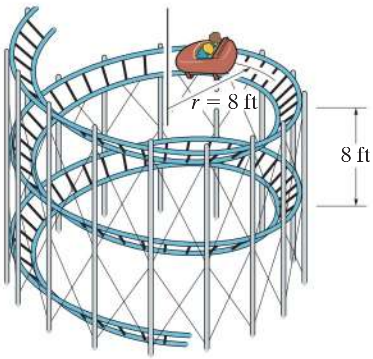
***15–100.** Each ball has a negligible size and a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (t^2 + 2)\ \text{N}\cdot\text{m}$, where t is in seconds, determine the speed of each ball when $t = 3$ s. Each ball has a speed $v = 2$ m/s when $t = 0$.



Prob. 15–100

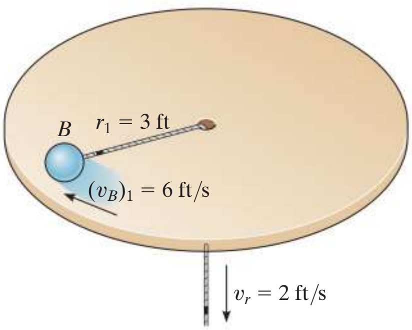
15–101. The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the speed of the car when $t = 4$ s. Also, how far has the car descended in this time? Neglect friction and the size of the car.

15–102. The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the time required for the car to attain a speed of 60 ft/s. Neglect friction and the size of the car.



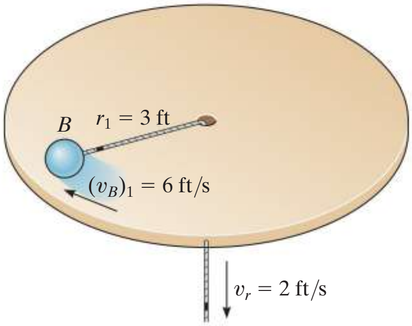
Probs. 15–101/102

15–103. A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine the ball's speed at the instant $r_2 = 2$ ft. How much work has to be done to pull down the cord? Neglect friction and the size of the ball.



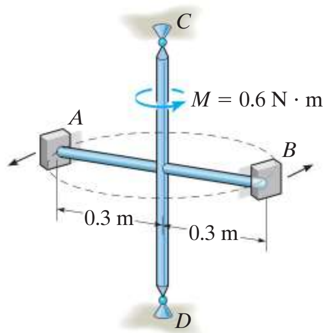
Prob. 15–103

***15–104.** A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine how much time is required for the ball to reach a speed of 12 ft/s. How far r_2 is the ball from the hole when this occurs? Neglect friction and the size of the ball.



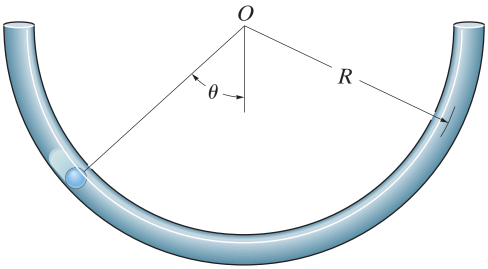
Prob. 15–104

15–105. The two blocks A and B each have a mass of 400 g. The blocks are fixed to the horizontal rods, and their initial velocity along the circular path is 2 m/s. If a couple moment of $M = (0.6)$ N·m is applied about CD of the frame, determine the speed of the blocks when $t = 3$ s. The mass of the frame is negligible, and it is free to rotate about CD. Neglect the size of the blocks.



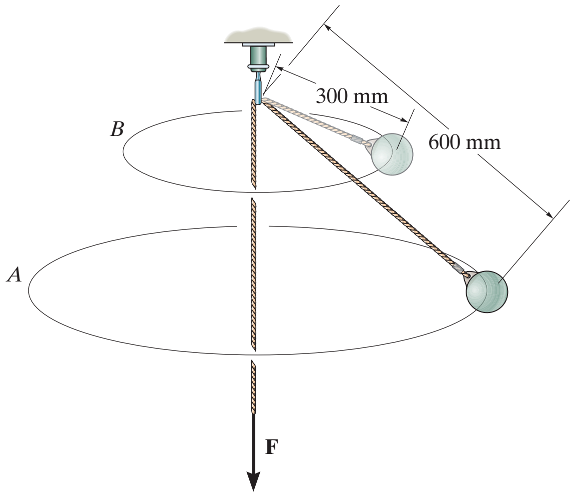
Prob. 15–105

15–106. A small particle having a mass m is placed inside the semicircular tube. The particle is placed at the position shown and released. Apply the principle of angular momentum about point O ($\Sigma M_O = \dot{H}_O$), and show that the motion of the particle is governed by the differential equation $\ddot{\theta} + (g/R)\sin\theta = 0$.



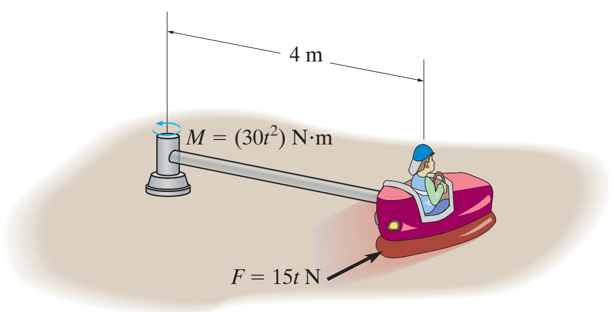
Prob. 15–106

***15–108.** When the 2-kg bob is given a horizontal speed of 1.5 m/s, it begins to move around the horizontal circular path A. If the force **F** on the cord is increased, the bob rises and then moves around the horizontal circular path B. Determine the speed of the bob around path B. Also, find the work done by force **F**.



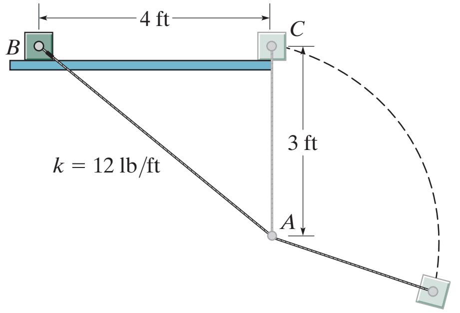
Prob. 15–108

15–107. If the rod of negligible mass is subjected to a couple moment of $M = (30t^2)$ N·m, and the engine of the car supplies a traction force of $F = (15t)$ N to the wheels, where t is in seconds, determine the speed of the car at the instant $t = 5$ s. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.



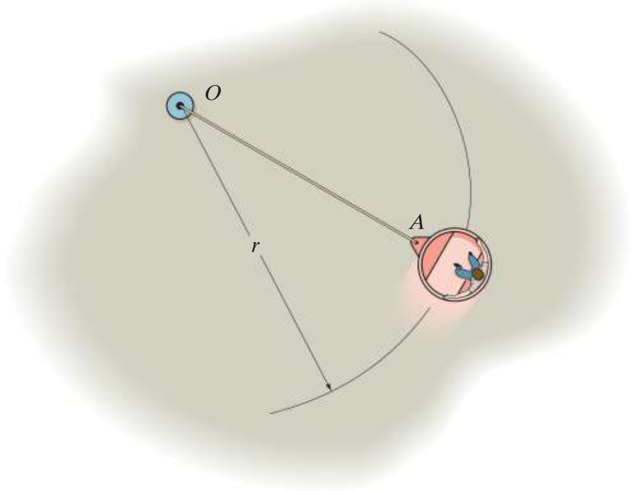
Prob. 15–107

15–109. The elastic cord has an unstretched length $l_0 = 1.5$ ft and a stiffness $k = 12$ lb/ft. It is attached to a fixed point at A and a block at B, which has a weight of 2 lb. If the block is released from rest from the position shown, determine its speed when it reaches point C after it slides along the smooth guide. After leaving the guide, it is launched onto the smooth *horizontal* plane. Determine if the cord becomes unstretched. Also, calculate the angular momentum of the block about point A, at any instant after it passes point C.



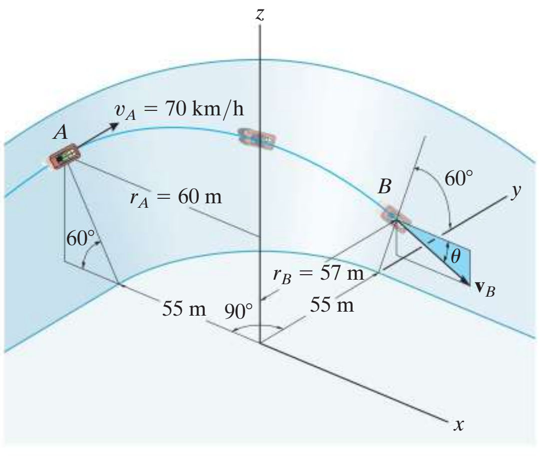
Prob. 15–109

15–110. The amusement park ride consists of a 200-kg car and passenger that are traveling at 3 m/s along a circular path having a radius of 8 m. If at $t = 0$, the cable OA is pulled in toward O at 0.5 m/s, determine the speed of the car when $t = 4$ s. Also, determine the work done to pull in the cable.



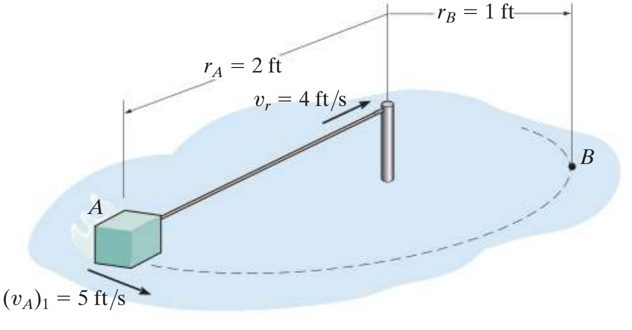
Prob. 15–110

***15–112.** A toboggan and rider, having a total mass of 150 kg, enter horizontally tangent to a 90° circular curve with a velocity of $v_A = 70$ km/h. If the track is flat and banked at an angle of 60°, determine the speed v_B and the angle θ of "descent," measured from the horizontal in a vertical x–z plane, at which the toboggan exists at B. Neglect friction in the calculation.



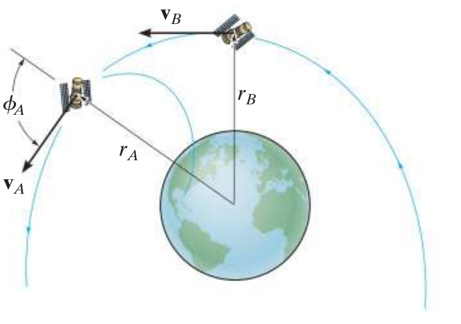
Prob. 15–112

15–111. A box having a weight of 8 lb is moving around in a circle of radius $r_A = 2$ ft with a speed of $(v_A)_1 = 5$ ft/s while connected to the end of a rope. If the rope is pulled inward with a constant speed of $v_r = 4$ ft/s, determine the speed of the box at the instant $r_B = 1$ ft. How much work is done after pulling in the rope from A to B? Neglect friction and the size of the box.



Prob. 15–111

15–113. An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10$ km/s when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70°$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy.

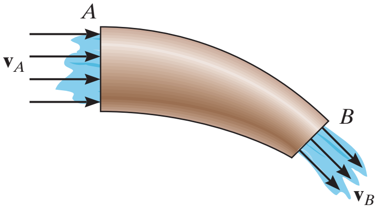


Prob. 15–113

15.8 Steady Flow of a Fluid Stream

Up to this point we have restricted our study of impulse and momentum principles to a system of particles contained within a *closed volume*. In this section, however, we will apply the principle of impulse and momentum to the steady mass flow of fluid particles entering into and then out of a *control volume*. This volume is defined as a region in space where fluid particles can flow into or out of the region. The size and shape of the control volume is frequently made to coincide with the solid boundaries and openings of a pipe, turbine, or pump. Provided the flow of the fluid into the control volume is equal to the flow out, then the flow can be classified as *steady flow*.

Principle of Impulse and Momentum. Consider the steady flow of a fluid stream in Fig. 15–27*a* that passes through a pipe. The region within the pipe and its openings will be taken as the control volume. As shown, the fluid flows into and out of the control volume with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$, respectively. The change in the direction of the fluid flow within the control volume is caused by an impulse produced by the resultant external force exerted on the control surface by the wall of the pipe. This resultant force can be determined by applying the principle of impulse and momentum to the control volume.

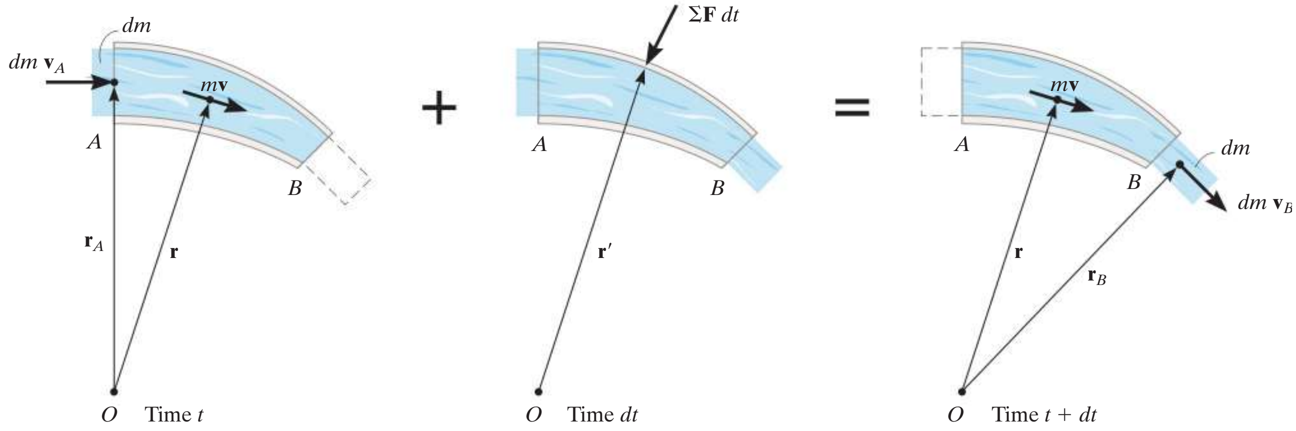


(a)

Fig. 15–27

The conveyor belt must supply frictional forces to the gravel that falls upon it in order to change the momentum of the gravel stream, so that it begins to travel along the belt. (© R.C. Hibbeler)

The air on one side of this fan is essentially at rest, and as it passes through the blades its momentum is increased. To change the momentum of the air flow in this manner, the blades must exert a horizontal thrust on the air stream. As the blades turn faster, the equal but opposite thrust of the air on the blades could overcome the rolling resistance of the wheels on the ground and begin to move the frame of the fan. (© R.C. Hibbeler)



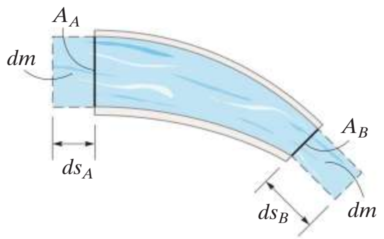
(b)

As indicated in Fig. 15–27*b*, a small amount of fluid having a mass dm is about to enter the control volume through opening A with a velocity of $\mathbf{v}_A$ at time t. Since the flow is considered steady, at time $t + dt$, the same amount of fluid will leave the control volume through opening B with a velocity $\mathbf{v}_B$. The momenta of the fluid entering and leaving the control volume are therefore $dm\,\mathbf{v}_A$ and $dm\,\mathbf{v}_B$, respectively. Also, during the time dt, the momentum of the fluid mass within the control volume remains constant and is denoted as $m\mathbf{v}$. As shown on the center diagram, the resultant external force exerted on the control volume produces the impulse $\Sigma\mathbf{F}\,dt$. If we apply the principle of linear impulse and momentum, we have

$$dm\,\mathbf{v}_A + m\mathbf{v} + \Sigma\mathbf{F}\,dt = dm\,\mathbf{v}_B + m\mathbf{v}$$

If $\mathbf{r}$, $\mathbf{r}_A$, $\mathbf{r}_B$ are position vectors measured from point O to the geometric centers of the control volume and the openings at A and B, Fig. 15–27*b*, then the principle of angular impulse and momentum about O becomes

$$\mathbf{r}_A \times dm\,\mathbf{v}_A + \mathbf{r} \times m\mathbf{v} + \mathbf{r}' \times \Sigma\mathbf{F}\,dt = \mathbf{r} \times m\mathbf{v} + \mathbf{r}_B \times dm\,\mathbf{v}_B$$



(c)

Fig. 15–27 (cont.)

Dividing both sides of the above two equations by dt and simplifying, we get

$$\boxed{\Sigma\mathbf{F} = \frac{dm}{dt}(\mathbf{v}_B - \mathbf{v}_A)} \qquad (15\text{–}25)$$

$$\boxed{\Sigma\mathbf{M}_O = \frac{dm}{dt}(\mathbf{r}_B \times \mathbf{v}_B - \mathbf{r}_A \times \mathbf{v}_A)} \qquad (15\text{–}26)$$

The term dm/dt is called the *mass flow*. It indicates the constant amount of fluid which flows either into or out of the control volume per unit of time. If the cross-sectional areas and densities of the fluid at the entrance A are A_A, ρ_A and at exit B, A_B, ρ_B, Fig. 15–27c, then for an incompressible fluid, the *continuity of mass* requires $dm = \rho dV = \rho_A(ds_A A_A) = \rho_B(ds_B A_B)$. Hence, during the time dt, since $v_A = ds_A/dt$ and $v_B = ds_B/dt$, we have $dm/dt = \rho_A v_A A_A = \rho_B v_B A_B$ or in general,

$$\boxed{\frac{dm}{dt} = \rho v A = \rho Q} \qquad (15\text{–}27)$$

The term $Q = vA$ measures the volume of fluid flow per unit of time and is referred to as the *discharge* or the *volumetric flow*.

Procedure for Analysis

Problems involving steady flow can be solved using the following procedure.

Kinematic Diagram.

- Identify the control volume. If it is *moving*, a *kinematic diagram* may be helpful for determining the entrance and exit velocities of the fluid flowing into and out of its openings since a *relative-motion analysis* of velocity will be involved.
- The measurement of velocities v_A and v_B must be made by an observer fixed in an inertial frame of reference.
- Once the velocity of the fluid flowing into the control volume is determined, the mass flow is calculated using Eq. 15–27.

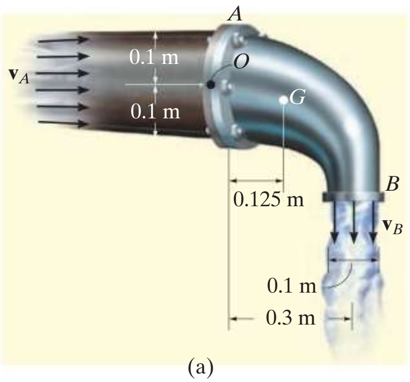
Free-Body Diagram.

- Draw the free-body diagram of the control volume in order to establish the forces $\Sigma \mathbf{F}$ that act on it. These forces will include the support reactions, the weight of all solid parts and the fluid contained within the control volume, and the static gauge pressure forces of the fluid on the entrance and exit sections.* The gauge pressure is the pressure measured above atmospheric pressure, and so if an opening is exposed to the atmosphere, the gauge pressure there will be zero.

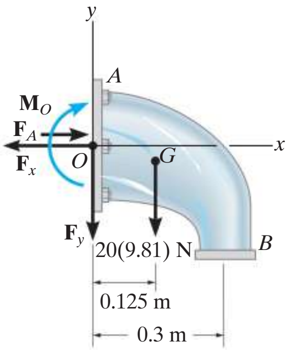
Equations of Steady Flow.

- Apply the equations of steady flow, Eq. 15–25 and 15–26, using the appropriate components of velocity and force shown on the kinematic and free-body diagrams.

* In the SI system, pressure is measured using the pascal (Pa), where $1\text{Pa} = 1\ \text{N/m}^2$.



(a)



(b)

Fig. 15–28

Determine the components of reaction which the fixed pipe joint at A exerts on the elbow in Fig. 15–28a, if water flowing through the pipe is subjected to a static gauge pressure of 100 kPa at A. The discharge at B is $Q_B = 0.2 \text{ m}^3/\text{s}$. Water has a density $\rho_w = 1000 \text{ kg/m}^3$, and the water-filled elbow has a mass of 20 kg and center of mass at G.

SOLUTION

We will consider the control volume to be the outer surface of the elbow. Using a fixed inertial coordinate system, the velocity of flow at A and B and the mass flow rate can be obtained from Eq. 15–27. Since the density of water is constant, $Q_B = Q_A = Q$. Hence,

$$\frac{dm}{dt} = \rho_w Q = (1000 \text{ kg/m}^3)(0.2 \text{ m}^3/\text{s}) = 200 \text{ kg/s}$$

$$v_B = \frac{Q}{A_B} = \frac{0.2 \text{ m}^3/\text{s}}{\pi(0.05 \text{ m})^2} = 25.46 \text{ m/s} \downarrow$$

$$v_A = \frac{Q}{A_A} = \frac{0.2 \text{ m}^3/\text{s}}{\pi(0.1 \text{ m})^2} = 6.37 \text{ m/s} \rightarrow$$

Free-Body Diagram. As shown on the free-body diagram of the control volume (elbow) Fig. 15–28b, the *fixed* connection at A exerts a resultant couple moment $\mathbf{M}_O$ and force components $\mathbf{F}_x$ and $\mathbf{F}_y$ on the elbow. Due to the static pressure of water in the pipe, the pressure force acting on the open control surface at A is $F_A = p_A A_A$. Since $1 \text{ kPa} = 1000 \text{ N/m}^2$,

$$F_A = p_A A_A = [100(10^3) \text{ N/m}^2][\pi(0.1 \text{ m})^2] = 3141.6 \text{ N}$$

There is no static pressure acting at B, since the water is discharged at atmospheric pressure; i.e., the pressure measured by a gauge at B is equal to zero, $p_B = 0$.

Equations of Steady Flow.

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax}); \quad -F_x + 3141.6 \text{ N} = 200 \text{ kg/s}(0 - 6.37 \text{ m/s})$$

$$F_x = 4.41 \text{ kN} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay}); \quad -F_y - 20(9.81) \text{ N} = 200 \text{ kg/s}(-25.46 \text{ m/s} - 0)$$

$$F_y = 4.90 \text{ kN} \qquad \textit{Ans.}$$

If moments are summed about point O, Fig. 15–28b, then $\mathbf{F}_x$, $\mathbf{F}_y$, and the static pressure $\mathbf{F}_A$ are eliminated, as well as the moment of momentum of the water entering at A, Fig. 15–28a. Hence,

$$\circlearrowleft + \Sigma M_O = \frac{dm}{dt}(d_{OB} v_B - d_{OA} v_A)$$

$$M_O + 20(9.81) \text{ N}(0.125 \text{ m}) = 200 \text{ kg/s}[(0.3 \text{ m})(25.46 \text{ m/s}) - 0]$$

$$M_O = 1.50 \text{ kN} \cdot \text{m} \qquad \textit{Ans.}$$

EXAMPLE 15.17

A 2-in.-diameter water jet having a velocity of 25 ft/s impinges upon a single moving blade, Fig. 15–29*a*. If the blade moves with a constant velocity of 5 ft/s away from the jet, determine the horizontal and vertical components of force which the blade is exerting on the water. What power does the water generate on the blade? Water has a specific weight of $\gamma_w = 62.4 \text{ lb/ft}^3$.

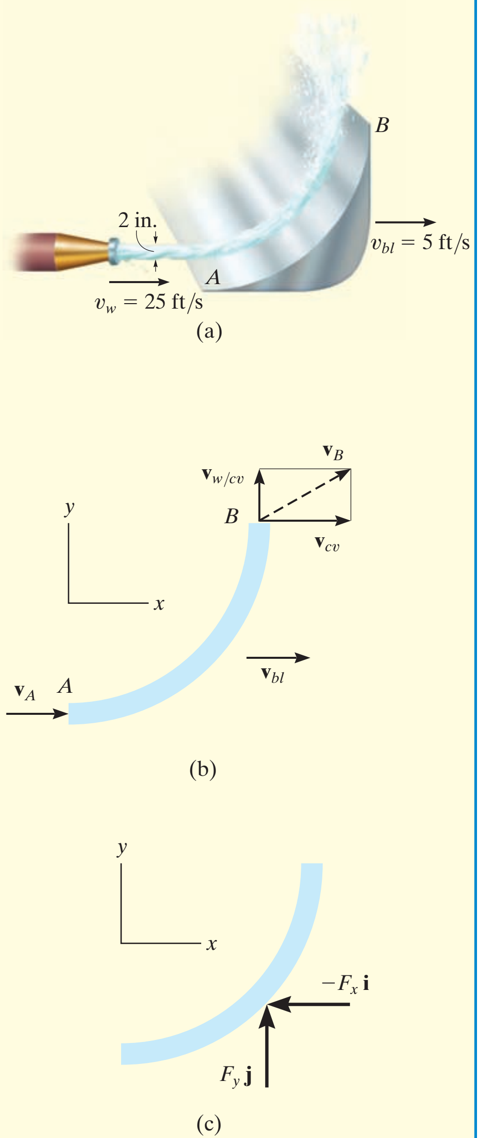


Fig. 15–29

SOLUTION

Kinematic Diagram. Here the control volume will be the stream of water on the blade. From a fixed inertial coordinate system, Fig. 15–29*b*, the rate at which water enters the control volume at *A* is

$$\mathbf{v}_A = \{25\mathbf{i}\} \text{ ft/s}$$

The *relative-flow velocity* within the control volume is $\mathbf{v}_{w/cv} = \mathbf{v}_w - \mathbf{v}_{cv} = 25\mathbf{i} - 5\mathbf{i} = \{20\mathbf{i}\}$ ft/s. Since the control volume is moving with a velocity of $\mathbf{v}_{cv} = \{5\mathbf{i}\}$ ft/s, the velocity of flow at *B* measured from the fixed *x, y* axes is the vector sum, shown in Fig. 15–29*b*. Here,

$$\begin{aligned}\mathbf{v}_B &= \mathbf{v}_{cv} + \mathbf{v}_{w/cv} \\ &= \{5\mathbf{i} + 20\mathbf{j}\} \text{ ft/s}\end{aligned}$$

Thus, the mass flow of water *onto* the control volume that undergoes a momentum change is

$$\frac{dm}{dt} = \rho_w (v_{w/cv}) A_A = \left(\frac{62.4}{32.2}\right)(20)\left[\pi\left(\frac{1}{12}\right)^2\right] = 0.8456 \text{ slug/s}$$

Free-Body Diagram. The free-body diagram of the control volume is shown in Fig. 15–29*c*. The weight of the water will be neglected in the calculation, since this force will be small compared to the reactive components $\mathbf{F}_x$ and $\mathbf{F}_y$.

Equations of Steady Flow.

$$\Sigma \mathbf{F} = \frac{dm}{dt}(\mathbf{v}_B - \mathbf{v}_A)$$

$$-F_x\mathbf{i} + F_y\mathbf{j} = 0.8456(5\mathbf{i} + 20\mathbf{j} - 25\mathbf{i})$$

Equating the respective **i** and **j** components gives

$$F_x = 0.8456(20) = 16.9 \text{ lb} \leftarrow \qquad \textit{Ans.}$$

$$F_y = 0.8456(20) = 16.9 \text{ lb} \uparrow \qquad \textit{Ans.}$$

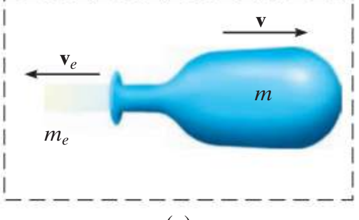
The water exerts equal but opposite forces on the blade.

Since the water force which causes the blade to move forward horizontally with a velocity of 5 ft/s is $F_x = 16.9$ lb, then from Eq. 14–10 the power is

$$P = \mathbf{F} \cdot \mathbf{v}; \qquad P = \frac{16.9 \text{ lb}(5 \text{ ft/s})}{550 \text{ hp/(ft} \cdot \text{lb/s)}} = 0.154 \text{ hp}$$

*15.9 Propulsion with Variable Mass

A Control Volume That Loses Mass. Consider a device such as a rocket which at an instant of time has a mass m and is moving forward with a velocity $\mathbf{v}$, Fig. 15–30*a*. At this same instant the amount of mass m_e is expelled from the device with a mass flow velocity $\mathbf{v}_e$. For the analysis, the control volume will include *both the mass m of the device and the expelled mass* m_e. The impulse and momentum diagrams for the control volume are shown in Fig. 15–30*b*. During the time dt, its velocity is increased from $\mathbf{v}$ to $\mathbf{v} + d\mathbf{v}$ since an amount of mass dm_e has been ejected and thereby gained in the exhaust. This increase in forward velocity, however, does not change the velocity $\mathbf{v}_e$ of the expelled mass, as seen by a fixed observer, since this mass moves with a constant velocity once it has been ejected. The impulses are created by $\Sigma\mathbf{F}_{cv}$, which represents the resultant of all the external forces, such as drag or weight, that *act on the control volume* in the direction of motion. This force resultant *does not include* the force which causes the control volume to move forward, since this force (called a *thrust*) is *internal to the control volume*; that is, the thrust acts with equal magnitude but opposite direction on the mass m of the device and the expelled exhaust mass m_e.* Applying the principle of impulse and momentum to the control volume, Fig. 15–30*b*, we have

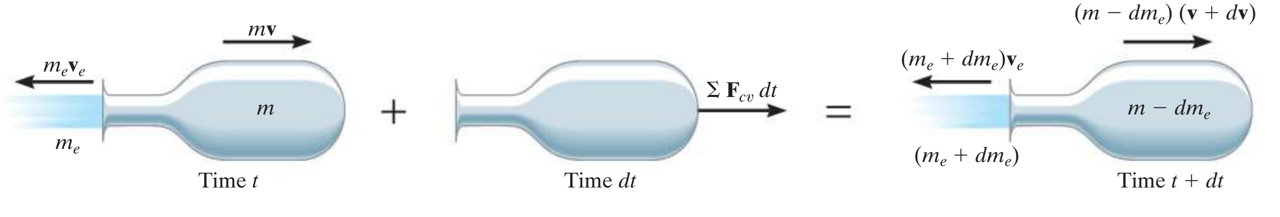


(a)

$$(\overset{+}{\rightarrow}) \qquad mv - m_e v_e + \Sigma F_{cv}\, dt = (m - dm_e)(v + dv) - (m_e + dm_e)v_e$$

or

$$\Sigma F_{cv}\, dt = -v\, dm_e + m\, dv - dm_e\, dv - v_e\, dm_e$$



(b)

Fig. 15–30

*$\Sigma\mathbf{F}$ represents the external resultant force *acting on the control volume*, which is different from $\mathbf{F}$, the resultant force acting only on the device.

Without loss of accuracy, the third term on the right side may be neglected since it is a "second-order" differential. Dividing by dt gives

$$\Sigma F_{cv} = m\frac{dv}{dt} - (v + v_e)\frac{dm_e}{dt}$$

The velocity of the device as seen by an observer moving with the particles of the ejected mass is $v_{D/e} = (v + v_e)$, and so the final result can be written as

$$\boxed{\Sigma F_{cv} = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}} \qquad (15\text{–}28)$$

Here the term dm_e/dt represents the rate at which mass is being ejected.

To illustrate an application of Eq. 15–28, consider the rocket shown in Fig. 15–31, which has a weight **W** and is moving upward against an atmospheric drag force $\mathbf{F}_D$. The control volume to be considered consists of the mass of the rocket and the mass of ejected gas m_e. Applying Eq. 15–28 gives

$$(+\uparrow) \qquad -F_D - W = \frac{W}{g}\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

The last term of this equation represents the *thrust* **T** which the engine exhaust exerts on the rocket, Fig. 15–31. Recognizing that $dv/dt = a$, we can therefore write

$$(+\uparrow) \qquad T - F_D - W = \frac{W}{g}a$$

If a free-body diagram of the rocket is drawn, it becomes obvious that this equation represents an application of $\Sigma\mathbf{F} = m\mathbf{a}$ for the rocket.

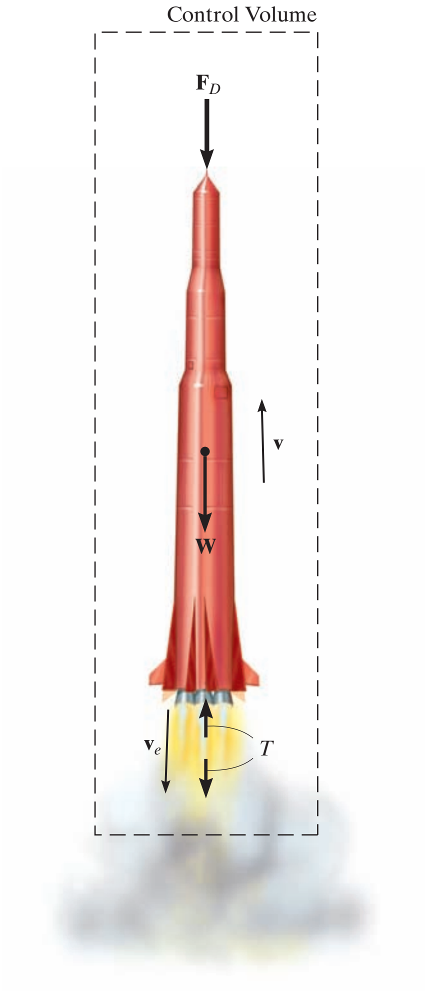
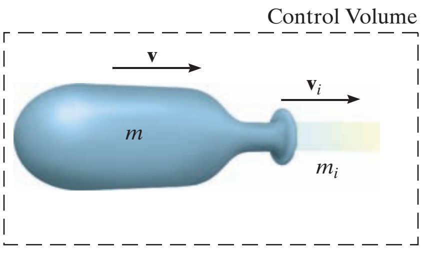


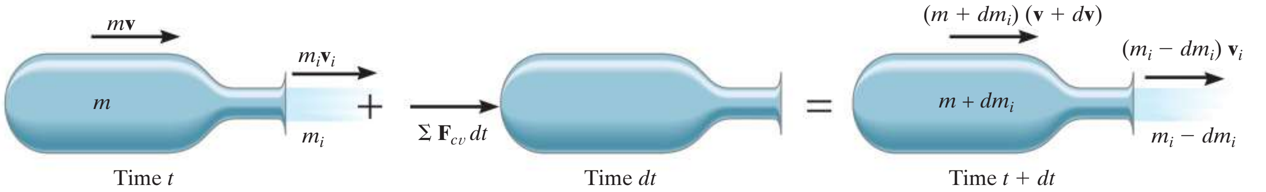
Fig. 15–31

A Control Volume That Gains Mass. A device such as a scoop or a shovel may gain mass as it moves forward. For example, the device shown in Fig. 15–32*a* has a mass m and moves forward with a velocity **v**. At this instant, the device is collecting a particle stream of mass m_i. The flow velocity $\mathbf{v}_i$ of this injected mass is constant and independent of the velocity **v** such that $v > v_i$. The control volume to be considered here includes both the mass of the device and the mass of the injected particles.



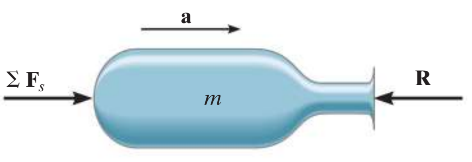
(a)

Fig. 15–32



(b)

Fig. 15–32 (cont.)



(c)

The scraper box behind this tractor represents a device that gains mass. If the tractor maintains a constant velocity v, then $dv/dt = 0$ and, because the soil is originally at rest, $v_{D/i} = v$. Applying Eq. 15–29, the horizontal towing force on the scraper box is then $T = 0 + v(dm/dt)$, where dm/dt is the rate of soil accumulated in the box. (© R.C. Hibbeler)

The impulse and momentum diagrams are shown in Fig. 15–32*b*. Along with an increase in mass dm_i gained by the device, there is an assumed increase in velocity $d\mathbf{v}$ during the time interval dt. This increase is caused by the impulse created by $\Sigma\mathbf{F}_{cv}$, the resultant of all the external forces *acting on the control volume* in the direction of motion. The force summation does not include the retarding force of the injected mass acting on the device. Why? Applying the principle of impulse and momentum to the control volume, we have

$$(\overset{+}{\rightarrow}) \qquad mv + m_i v_i + \Sigma F_{cv}\,dt = (m + dm_i)(v + dv) + (m_i - dm_i)v_i$$

Using the same procedure as in the previous case, we may write this equation as

$$\Sigma F_{cv} = m\frac{dv}{dt} + (v - v_i)\frac{dm_i}{dt}$$

Since the velocity of the device as seen by an observer moving with the particles of the injected mass is $v_{D/i} = (v - v_i)$, the final result can be written as

$$\boxed{\Sigma F_{cv} = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}} \qquad (15\text{–}29)$$

where dm_i/dt is the rate of mass injected into the device. The last term in this equation represents the magnitude of force $\mathbf{R}$, which the injected mass *exerts on the device*, Fig. 15–32*c*. Since $dv/dt = a$, Eq. 15–29 becomes

$$\Sigma F_{cv} - R = ma$$

This is the application of $\Sigma\mathbf{F} = m\mathbf{a}$.

As in the case of steady flow, problems which are solved using Eqs. 15–28 and 15–29 should be accompanied by an identified control volume and the necessary free-body diagram. With this diagram one can then determine ΣF_{cv} and isolate the force exerted on the device by the particle stream.

EXAMPLE 15.18

The initial combined mass of a rocket and its fuel is m_0. A total mass m_f of fuel is consumed at a constant rate of $dm_e/dt = c$ and expelled at a constant speed of u relative to the rocket. Determine the maximum velocity of the rocket, i.e., at the instant the fuel runs out. Neglect the change in the rocket's weight with altitude and the drag resistance of the air. The rocket is fired vertically from rest.

(© NASA)

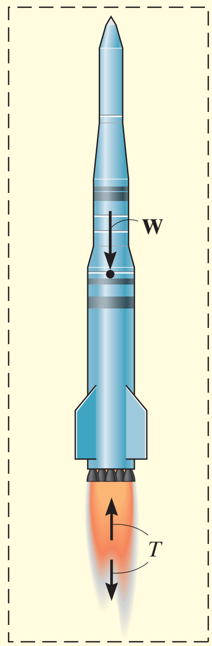


Fig. 15–33

SOLUTION

Since the rocket loses mass as it moves upward, Eq. 15–28 can be used for the solution. The only *external force* acting on the *control volume* consisting of the rocket and a portion of the expelled mass is the weight **W**, Fig. 15–33. Hence,

$$+\uparrow \Sigma F_{cv} = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad -W = m\frac{dv}{dt} - uc \tag{1}$$

The rocket's velocity is obtained by integrating this equation.

At any given instant t during the flight, the mass of the rocket can be expressed as $m = m_0 - (dm_e/dt)t = m_0 - ct$. Since $W = mg$, Eq. 1 becomes

$$-(m_0 - ct)g = (m_0 - ct)\frac{dv}{dt} - uc$$

Separating the variables and integrating, realizing that $v = 0$ at $t = 0$, we have

$$\int_0^v dv = \int_0^t \left(\frac{uc}{m_0 - ct} - g\right) dt$$

$$v = -u\ln(m_0 - ct) - gt\Big|_0^t = u\ln\left(\frac{m_0}{m_0 - ct}\right) - gt \tag{2}$$

Note that liftoff requires the first term on the right to be greater than the second during the initial phase of motion. The time t' needed to consume all the fuel is

$$m_f = \left(\frac{dm_e}{dt}\right)t' = ct'$$

Hence,

$$t' = m_f/c$$

Substituting into Eq. 2 yields

$$v_{max} = u\ln\left(\frac{m_0}{m_0 - m_f}\right) - \frac{gm_f}{c} \qquad \textit{Ans.}$$

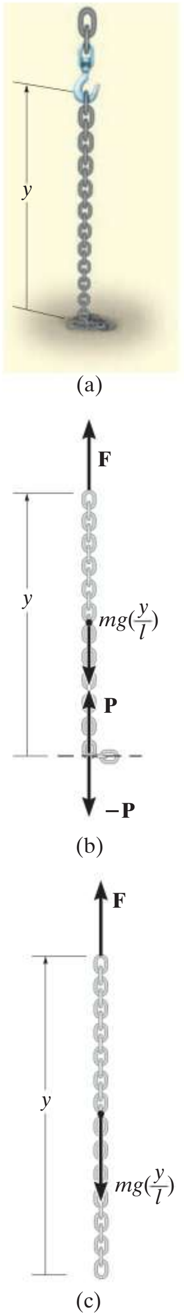


Fig. 15–34

A chain of length l, Fig. 15–34a, has a mass m. Determine the magnitude of force $\mathbf{F}$ required to (a) raise the chain with a constant speed v_c, starting from rest when $y = 0$; and (b) lower the chain with a constant speed v_c, starting from rest when $y = l$.

SOLUTION

Part (a). As the chain is raised, all the suspended links are given a sudden downward impulse by each added link which is lifted off the ground. Thus, the *suspended portion* of the chain may be considered as a device which is *gaining mass*. The control volume to be considered is the length of chain y which is suspended by $\mathbf{F}$ at any instant, including the next link which is about to be added but is still at rest, Fig. 15–34b. The forces acting on the control volume *exclude* the internal forces $\mathbf{P}$ and $-\mathbf{P}$, which act between the added link and the suspended portion of the chain. Hence, $\Sigma F_{cv} = F - mg(y/l)$.

To apply Eq. 15–29, it is also necessary to find the rate at which mass is being added to the system. The velocity $\mathbf{v}_c$ of the chain is equivalent to $\mathbf{v}_{D/i}$. Why? Since v_c is constant, $dv_c/dt = 0$ and $dy/dt = v_c$. Integrating, using the initial condition that $y = 0$ when $t = 0$, gives $y = v_c t$. Thus, the mass of the control volume at any instant is $m_{cv} = m(y/l) = m(v_c t/l)$, and therefore the *rate* at which mass is *added* to the suspended chain is

$$\frac{dm_i}{dt} = m\left(\frac{v_c}{l}\right)$$

Applying Eq. 15–29 using this data, we have

$$+\uparrow \Sigma F_{cv} = m\frac{dv_c}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$F - mg\left(\frac{y}{l}\right) = 0 + v_c m\left(\frac{v_c}{l}\right)$$

Hence,

$$F = (m/l)(gy + v_c^2) \qquad \textit{Ans.}$$

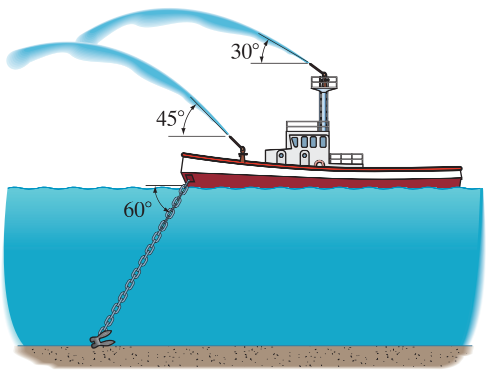
Part (b). When the chain is being lowered, the links which are expelled (given zero velocity) *do not* impart an impulse to the *remaining* suspended links. Why? Thus, the control volume in Part (a) will not be considered. Instead, the equation of motion will be used to obtain the solution. At time t the portion of chain still off the floor is y. The free-body diagram for a suspended portion of the chain is shown in Fig. 15–34c. Thus,

$$+\uparrow \Sigma F = ma; \qquad F - mg\left(\frac{y}{l}\right) = 0$$

$$F = mg\left(\frac{y}{l}\right) \qquad \textit{Ans.}$$

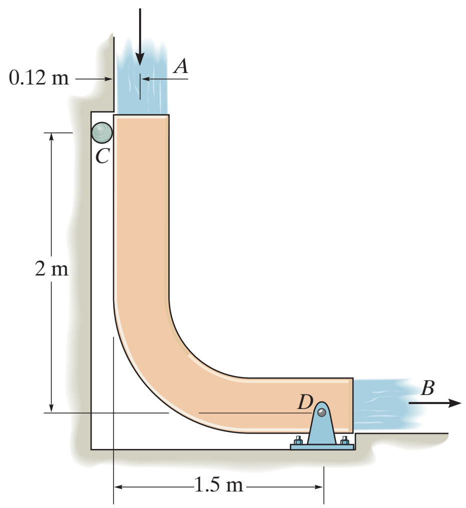
PROBLEMS

15–114. The fire boat discharges two streams of seawater, each at a flow of 0.25 m^3/s and with a nozzle velocity of 50 m/s. Determine the tension developed in the anchor chain, needed to secure the boat. The density of seawater is $\rho_{sw} = 1020\ kg/m^3$.



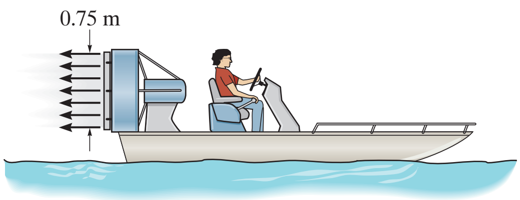
Prob. 15–114

15–115. The chute is used to divert the flow of water, $Q = 0.6\ m^3/s$. If the water has a cross-sectional area of 0.05 m^2, determine the force components at the pin D and roller C necessary for equilibrium. Neglect the weight of the chute and weight of the water on the chute. $\rho_w = 1\ Mg/m^3$.



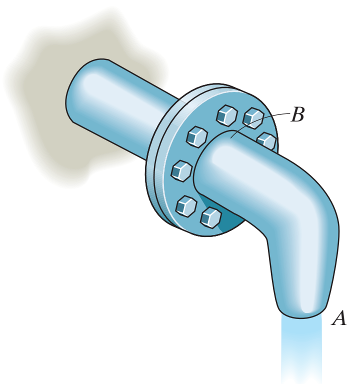
Prob. 15–115

***15–116.** The 200-kg boat is powered by the fan which develops a slipstream having a diameter of 0.75 m. If the fan ejects air with a speed of 14 m/s, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density of $\rho_w = 1.22\ kg/m^3$ and that the entering air is essentially at rest. Neglect the drag resistance of the water.



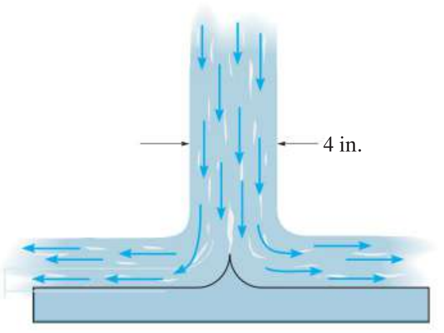
Prob. 15–116

15–117. The nozzle discharges water at a constant rate of 2 ft^3/s. The cross-sectional area of the nozzle at A is 4 in^2, and at B the cross-sectional area is 12 in^2. If the static gauge pressure due to the water at B is 2 lb/in^2, determine the magnitude of force which must be applied by the coupling at B to hold the nozzle in place. Neglect the weight of the nozzle and the water within it. $\gamma_w = 62.4\ lb/ft^3$.



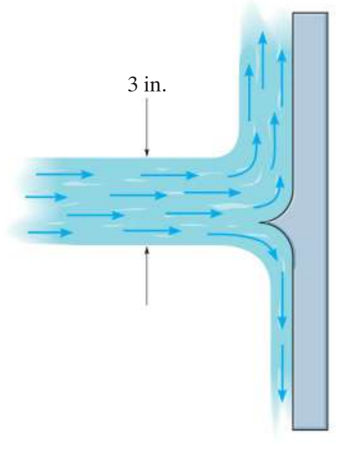
Prob. 15–117

15–118. The blade divides the jet of water having a diameter of 4 in. If one-half of the water flows to the right while the other half flows to the left, and the total flow is $Q = 1.5\ \text{ft}^3/\text{s}$, determine the vertical force exerted on the blade by the jet, $\gamma_\omega = 62.4\ \text{lb}/\text{ft}^3$.



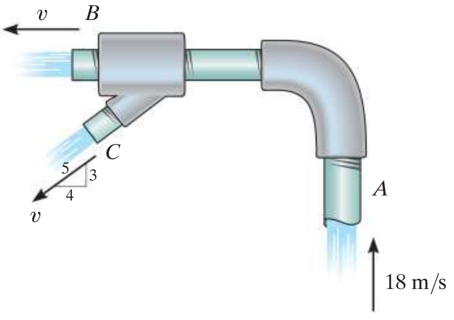
Prob. 15–118

15–119. The blade divides the jet of water having a diameter of 3 in. If one-fourth of the water flows downward while the other three-fourths flows upward, and the total flow is $Q = 0.5\ \text{ft}^3/\text{s}$, determine the horizontal and vertical components of force exerted on the blade by the jet, $\gamma_w = 62.4\ \text{lb}/\text{ft}^3$.



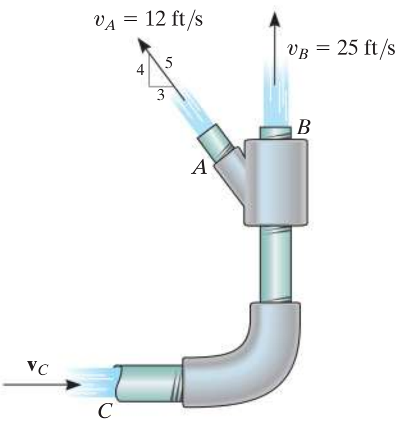
Prob. 15–119

***15–120.** The gauge pressure of water at A is 150.5 kPa. Water flows through the pipe at A with a velocity of 18 m/s, and out the pipe at B and C with the same velocity v. Determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 50 mm at A, and at B and C the diameter is 30 mm. $\rho_w = 1000\ \text{kg}/\text{m}^3$.



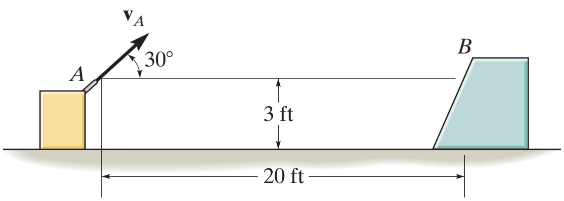
Prob. 15–120

15–121. The gauge pressure of water at C is $40\ \text{lb}/\text{in}^2$. If water flows out of the pipe at A and B with velocities $v_A = 12\ \text{ft}/\text{s}$ and $v_B = 25\ \text{ft}/\text{s}$, determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 0.75 in. at C, and at A and B the diameter is 0.5 in. $\gamma_w = 62.4\ \text{lb}/\text{ft}^3$.



Prob. 15–121

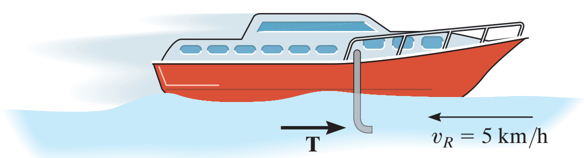
15–122. The fountain shoots water in the direction shown. If the water is discharged at 30° from the horizontal, and the cross-sectional area of the water stream is approximately 2 in^2, determine the force it exerts on the concrete wall at *B*. $\gamma_w = 62.4 \text{ lb/ft}^3$.



Prob. 15–122

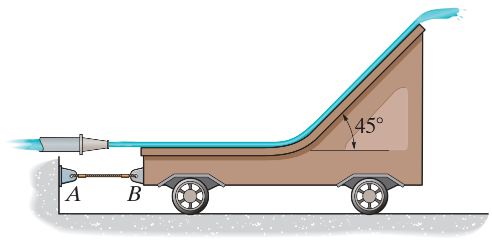
15–123. A plow located on the front of a locomotive scoops up snow at the rate of 10 ft^3/s and stores it in the train. If the locomotive is traveling at a constant speed of 12 ft/s, determine the resistance to motion caused by the shoveling. The specific weight of snow is $\gamma_s = 6 \text{ lb/ft}^3$.

***15–124.** The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust *T* on the tube that is required to overcome the resistance due to the water collection and yet maintain the constant speed of the boat. $\rho_w = 1 \text{ Mg/m}^3$.



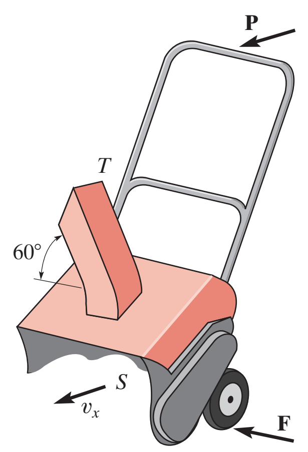
Prob. 15–124

15–125. Water is discharged from a nozzle with a velocity of 12 m/s and strikes the blade mounted on the 20-kg cart. Determine the tension developed in the cord, needed to hold the cart stationary, and the normal reaction of the wheels on the cart. The nozzle has a diameter of 50 mm and the density of water is $\rho_w = 1000 \text{ kg/m}^3$.



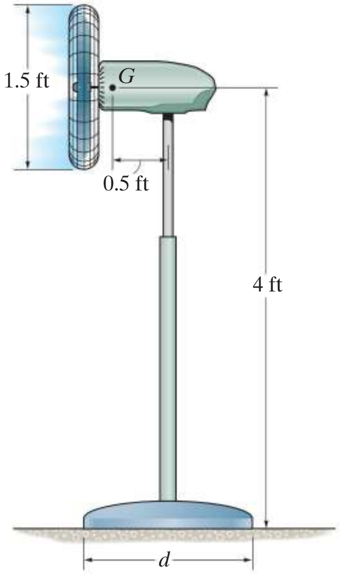
Prob. 15–125

15–126. A snowblower having a scoop *S* with a cross-sectional area of $A_s = 0.12 \text{ m}^3$ is pushed into snow with a speed of $v_s = 0.5$ m/s. The machine discharges the snow through a tube *T* that has a cross-sectional area of $A_T = 0.03 \text{ m}^2$ and is directed 60° from the horizontal. If the density of snow is $\rho_s = 104 \text{ kg/m}^3$, determine the horizontal force *P* required to push the blower forward, and the resultant frictional force *F* of the wheels on the ground, necessary to prevent the blower from moving sideways. The wheels roll freely.



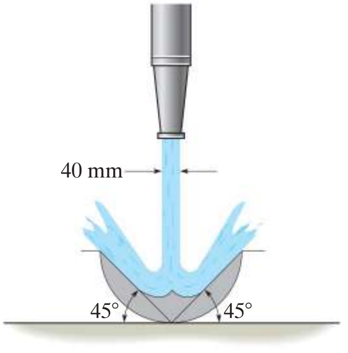
Prob. 15–126

15–127. The fan blows air at 6000 ft^3/min. If the fan has a weight of 30 lb and a center of gravity at G, determine the smallest diameter d of its base so that it will not tip over. The specific weight of air is $\gamma = 0.076 \text{ lb/ft}^3$.



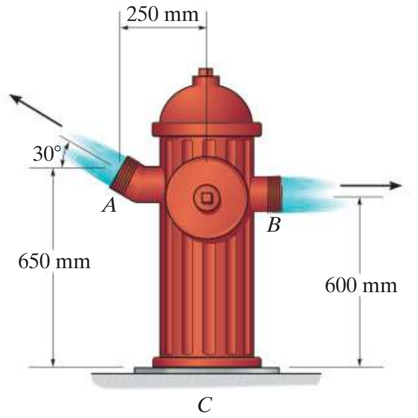
Prob. 15–127

***15–128.** The nozzle has a diameter of 40 mm. If it discharges water uniformly with a downward velocity of 20 m/s against the fixed blade, determine the vertical force exerted by the water on the blade. $\rho_w = 1 \text{ Mg/m}^3$.



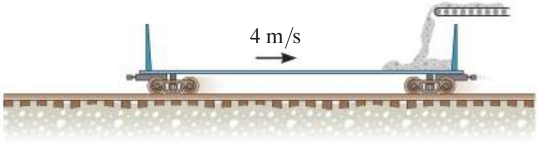
Prob. 15–128

15–129. The water flow enters below the hydrant at C at the rate of $0.75 \text{ m}^3/\text{s}$. It is then divided equally between the two outlets at A and B. If the gauge pressure at C is 300 kPa, determine the horizontal and vertical force reactions and the moment reaction on the fixed support at C. The diameter of the two outlets at A and B is 75 mm, and the diameter of the inlet pipe at C is 150 mm. The density of water is $\rho_w = 1000 \text{ kg/m}^3$. Neglect the mass of the contained water and the hydrant.



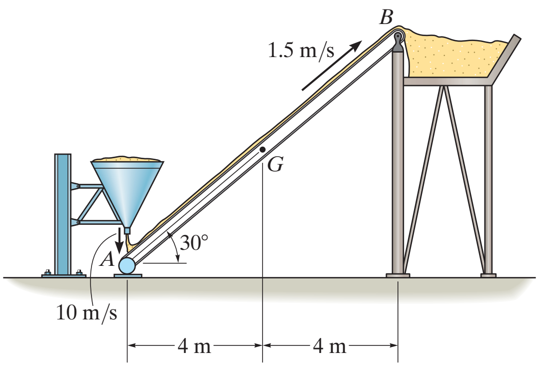
Prob. 15–129

15–130. Sand drops onto the 2-Mg empty rail car at 50 kg/s from a conveyor belt. If the car is initially coasting at 4 m/s, determine the speed of the car as a function of time.



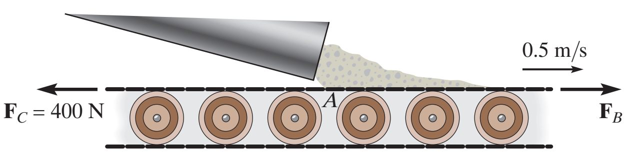
Prob. 15–130

15–131. Sand is discharged from the silo at A at a rate of 50 kg/s with a vertical velocity of 10 m/s onto the conveyor belt, which is moving with a constant velocity of 1.5 m/s. If the conveyor system and the sand on it have a total mass of 750 kg and center of mass at point G, determine the horizontal and vertical components of reaction at the pin support B and roller support A. Neglect the thickness of the conveyor.



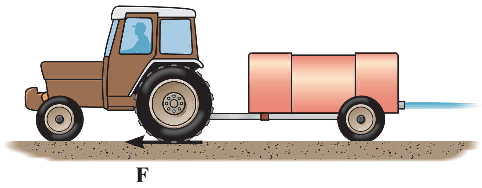
Prob. 15–131

*15–132. Sand is deposited from a chute onto a conveyor belt which is moving at 0.5 m/s. If the sand is assumed to fall vertically onto the belt at A at the rate of 4 kg/s, determine the belt tension F_B to the right of A. The belt is free to move over the conveyor rollers and its tension to the left of A is $F_C = 400$ N.



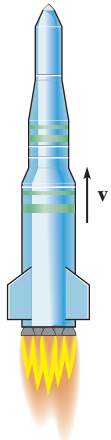
Prob. 15–132

15–133. The tractor together with the empty tank has a total mass of 4 Mg. The tank is filled with 2 Mg of water. The water is discharged at a constant rate of 50 kg/s with a constant velocity of 5 m/s, measured relative to the tractor. If the tractor starts from rest, and the rear wheels provide a resultant traction force of 250 N, determine the velocity and acceleration of the tractor at the instant the tank becomes empty.



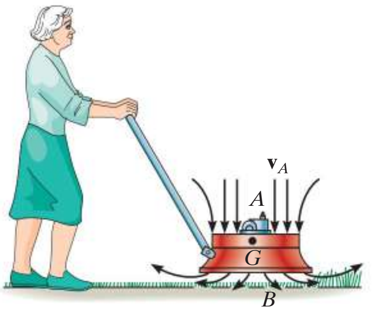
Prob. 15–133

15–134. A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 15 lb/s and ejected with a relative velocity of 4400 ft/s, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.



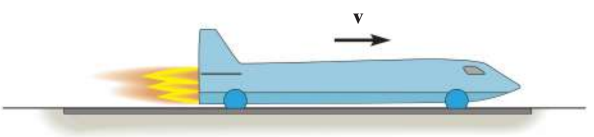
Prob. 15–134

15–135. A power lawn mower hovers very close over the ground. This is done by drawing air in at a speed of 6 m/s through an intake unit A, which has a cross-sectional area of $A_A = 0.25\ \text{m}^2$, and then discharging it at the ground, B, where the cross-sectional area is $A_B = 0.35\ \text{m}^2$. If air at A is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has a mass of 15 kg with center of mass at G. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$.



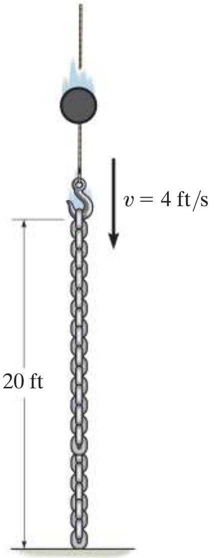
Prob. 15–135

*__15–136.__ The rocket car has a mass of 2 Mg (empty) and carries 120 kg of fuel. If the fuel is consumed at a constant rate of 6 kg/s and ejected from the car with a relative velocity of 800 m/s, determine the maximum speed attained by the car starting from rest. The drag resistance due to the atmosphere is $F_D = (6.8v^2)$ N, where v is the speed in m/s.



Prob. 15–136

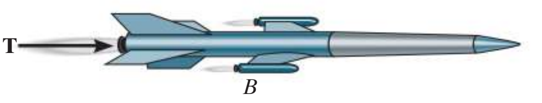
15–137. If the chain is lowered at a constant speed $v = 4$ ft/s, determine the normal reaction exerted on the floor as a function of time. The chain has a weight of 5 lb/ft and a total length of 20 ft.



Prob. 15–137

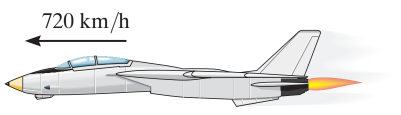
15–138. The second stage of a two-stage rocket weighs 2000 lb (empty) and is launched from the first stage with a velocity of 3000 mi/h. The fuel in the second stage weighs 1000 lb. If it is consumed at the rate of 50 lb/s and ejected with a relative velocity of 8000 ft/s, determine the acceleration of the second stage just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravitation.

15–139. The missile weighs 40 000 lb. The constant thrust provided by the turbojet engine is $T = 15\,000$ lb. Additional thrust is provided by *two* rocket boosters B. The propellant in each booster is burned at a constant rate of 150 lb/s, with a relative exhaust velocity of 3000 ft/s. If the mass of the propellant lost by the turbojet engine can be neglected, determine the velocity of the missile after the 4-s burn time of the boosters. The initial velocity of the missile is 300 mi/h.



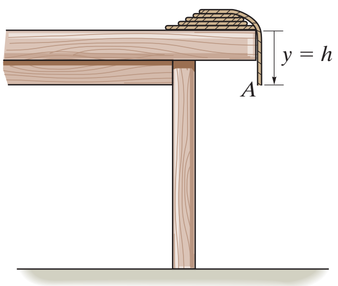
Prob. 15–139

***15–140.** The jet is traveling at a speed of 720 km/h. If the fuel is being spent at 0.8 kg/s, and the engine takes in air at 200 kg/s, whereas the exhaust gas (air and fuel) has a relative speed of 12 000 m/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (55\,v^2)$, where the speed is measured in m/s. The jet has a mass of 7 Mg.



Prob. 15–140

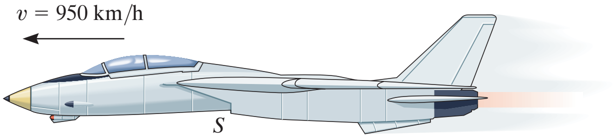
15–141. The rope has a mass m' per unit length. If the end length $y = h$ is draped off the edge of the table, and released, determine the velocity of its end A for any position y, as the rope uncoils and begins to fall.



Prob. 15–141

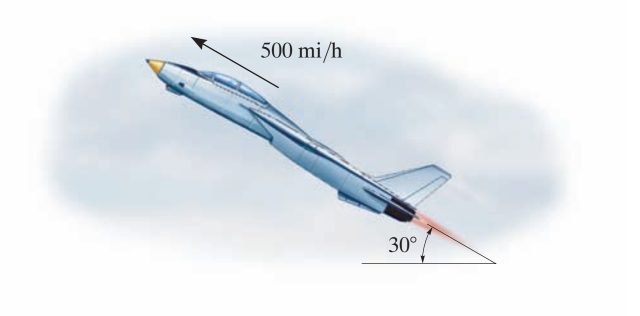
15–142. The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3. *Hint:* Since mass both enters and exits the plane, Eqs. 15–28 and 15–29 must be combined to yield

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}.$$



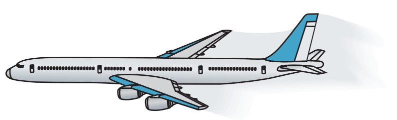
Prob. 15–142

15–143. The jet is traveling at a speed of 500 mi/h, 30° with the horizontal. If the fuel is being spent at 3 lb/s, and the engine takes in air at 400 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 32 800 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.7v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. *Hint:* See Prob. 15–142.



Prob. 15–143

***15–144.** A four-engine commercial jumbo jet is cruising at a constant speed of 800 km/h in level flight when all four engines are in operation. Each of the engines is capable of discharging combustion gases with a velocity of 775 m/s relative to the plane. If during a test two of the engines, one on each side of the plane, are shut off, determine the new cruising speed of the jet. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. Neglect the loss of mass due to fuel consumption.



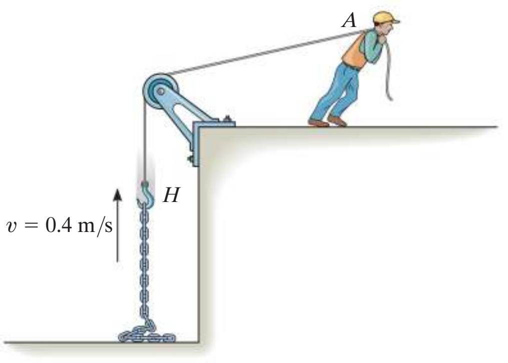
Prob. 15–144

15–145. The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to the helicopter, determine the initial upward acceleration the helicopter experiences as the water is being released.

Prob. 15–145

15–146. A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 1.5 lb/s and ejected with a velocity of 4400 ft/s relative to the rocket, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

15–147. Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed $v = 0.4\ \text{m/s}$. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.

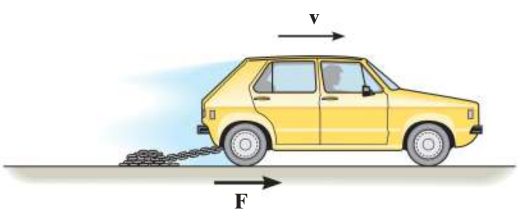


Prob. 15–147

***15–148.** The truck has a mass of 50 Mg when empty. When it is unloading 5 m^3 of sand at a constant rate of 0.8 m^3/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the load begins to empty. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is $\rho_s = 1520\ \text{kg/m}^3$.

Prob. 15–148

15–149. The car has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force F that must be supplied by the rear wheels of the car, necessary to maintain a constant speed v while the chain is being drawn out.



Prob. 15–149

CONCEPTUAL PROBLEMS

C15–1. The ball travels to the left when it is struck by the bat. If the ball then moves horizontally to the right, determine which measurements you could make in order to determine the net impulse given to the ball. Use numerical values to give an example of how this can be done.

Prob. C15–1 (© R.C. Hibbeler)

C15–2. The steel wrecking "ball" is suspended from the boom using an old rubber tire A. The crane operator lifts the ball then allows it to drop freely to break up the concrete. Explain, using appropriate numerical data, why it is a good idea to use the rubber tire for this work.

Prob. C15–2 (© R.C. Hibbeler)

C15–3. The train engine on the left, A, is at rest, and the one on the right, B, is coasting to the left. If the engines are identical, use numerical values to show how to determine the maximum compression in each of the spring bumpers that are mounted in the front of the engines. Each engine is free to roll.

Prob. C15–3 (© R.C. Hibbeler)

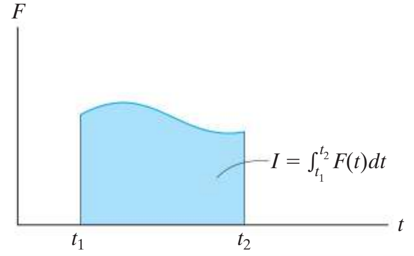
C15–4. Three train cars each have the same mass and are rolling freely when they strike the fixed bumper. Legs AB and BC on the bumper are pin connected at their ends and the angle BAC is 30° and BCA is 60°. Compare the average impulse in each leg needed to stop the motion if the cars have no bumper and if the cars have a spring bumper. Use appropriate numerical values to explain your answer.

Prob. C15–4 (© R.C. Hibbeler)

CHAPTER REVIEW

Impulse

An impulse is defined as the product of force and time. Graphically it represents the area under the F–t diagram. If the force is constant, then the impulse becomes $I = F_c(t_2 - t_1)$.



Principle of Impulse and Momentum

When the equation of motion, $\Sigma \mathbf{F} = m\mathbf{a}$, and the kinematic equation, $a = dv/dt$, are combined, we obtain the principle of impulse and momentum. This is a vector equation that can be resolved into rectangular components and used to solve problems that involve force, velocity, and time. For application, the free-body diagram should be drawn in order to account for all the impulses that act on the particle.

$$m\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2$$

Conservation of Linear Momentum

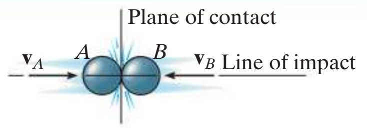
If the principle of impulse and momentum is applied to a *system of particles*, then the collisions between the particles produce internal impulses that are equal, opposite, and collinear, and therefore cancel from the equation. Furthermore, if an external impulse is small, that is, the force is small and the time is short, then the impulse can be classified as nonimpulsive and can be neglected. Consequently, momentum for the system of particles is conserved.

The conservation-of-momentum equation is useful for finding the final velocity of a particle when internal impulses are exerted between two particles and the initial velocities of the particles is known. If the internal impulse is to be determined, then one of the particles is isolated and the principle of impulse and momentum is applied to this particle.

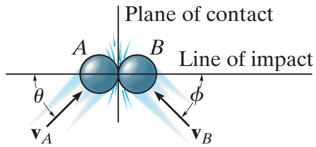
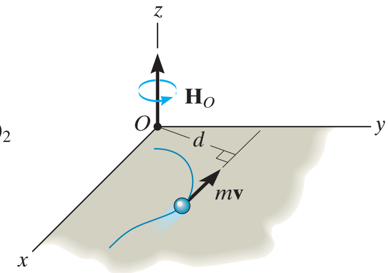
$$\Sigma m_i(\mathbf{v}_i)_1 = \Sigma m_i(\mathbf{v}_i)_2$$

Impact

When two particles A and B have a direct impact, the internal impulse between them is equal, opposite, and collinear. Consequently the conservation of momentum for this system applies along the line of impact.

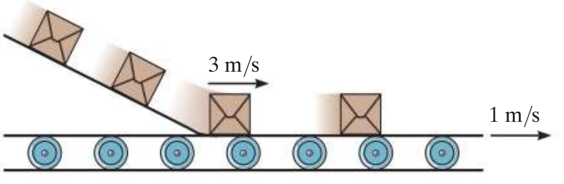


$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

If the final velocities are unknown, a second equation is needed for solution. We must use the coefficient of restitution, e. This experimentally determined coefficient depends upon the physical properties of the colliding particles. It can be expressed as the ratio of their relative velocity after collision to their relative velocity before collision. If the collision is elastic, no energy is lost and $e = 1$. For a plastic collision $e = 0$.	$e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$
If the impact is oblique, then the conservation of momentum for the system and the coefficient-of-restitution equation apply along the line of impact. Also, conservation of momentum for each particle applies perpendicular to this line (plane of contact) because no impulse acts on the particles in this direction.	
Principle of Angular Impulse and Momentum The moment of the linear momentum about an axis (z) is called the angular momentum. The principle of angular impulse and momentum is often used to eliminate unknown impulses by summing the moments about an axis through which the lines of action of these impulses produce no moment. For this reason, a free-body diagram should accompany the solution.	$(H_O)_z = (d)(mv)$ $(\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O \, dt = (\mathbf{H}_O)_2$ 
Steady Fluid Streams Impulse-and-momentum methods are often used to determine the forces that a device exerts on the mass flow of a fluid—liquid or gas. To do so, a free-body diagram of the fluid mass in contact with the device is drawn in order to identify these forces. Also, the velocity of the fluid as it flows into and out of a control volume for the device is calculated. The equations of steady flow involve summing the forces and the moments to determine these reactions.	$\Sigma \mathbf{F} = \dfrac{dm}{dt}(\mathbf{v}_B - \mathbf{v}_A)$ $\Sigma \mathbf{M}_O = \dfrac{dm}{dt}(\mathbf{r}_B \times \mathbf{v}_B - \mathbf{r}_A \times \mathbf{v}_A)$
Propulsion with Variable Mass Some devices, such as a rocket, lose mass as they are propelled forward. Others gain mass, such as a shovel. We can account for this mass loss or gain by applying the principle of impulse and momentum to a control volume for the device. From this equation, the force exerted on the device by the mass flow can then be determined.	$\Sigma F_{cv} = m\dfrac{dv}{dt} - v_{D/e}\dfrac{dm_e}{dt}$ Loses Mass $\Sigma F_{cv} = m\dfrac{dv}{dt} + v_{D/i}\dfrac{dm_i}{dt}$ Gains Mass

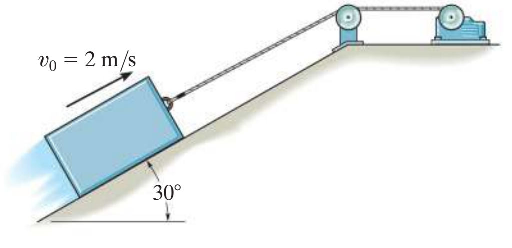
FUNDAMENTAL REVIEW PROBLEMS

R15–1. Packages having a mass of 6 kg slide down a smooth chute and land horizontally with a speed of 3 m/s on the surface of a conveyor belt. If the coefficient of kinetic friction between the belt and a package is $\mu_k = 0.2$, determine the time needed to bring the package to rest on the belt if the belt is moving in the same direction as the package with a speed $v = 1$ m/s.



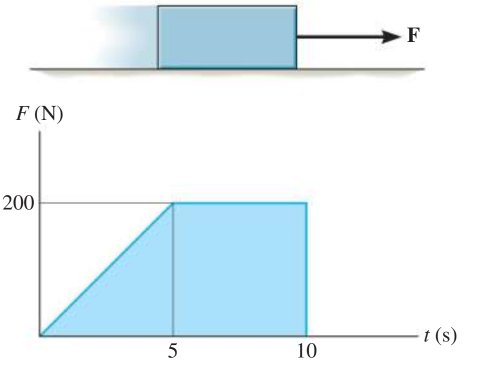
Prob. R15–1

R15–2. The 50-kg block is hoisted up the incline using the cable and motor arrangement shown. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.4$. If the block is initially moving up the plane at $v_0 = 2$ m/s, and at this instant ($t = 0$) the motor develops a tension in the cord of $T = (300 + 120\sqrt{t})$ N, where t is in seconds, determine the velocity of the block when $t = 2$ s.



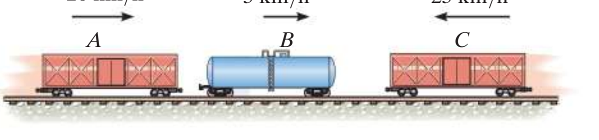
Prob. R15–2

R15–3. A 20-kg block is originally at rest on a horizontal surface for which the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.5$. If a horizontal force F is applied such that it varies with time as shown, determine the speed of the block in 10 s. *Hint:* First determine the time needed to overcome friction and start the block moving.



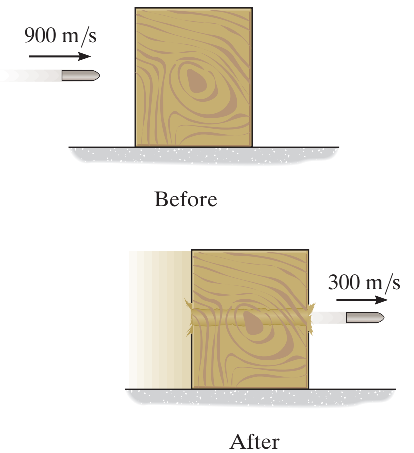
Prob. R15–3

R15–4. The three freight cars A, B, and C have masses of 10 Mg, 5 Mg, and 20 Mg, respectively. They are traveling along the track with the velocities shown. Car A collides with car B first, followed by car C. If the three cars couple together after collision, determine the common velocity of the cars after the two collisions have taken place.



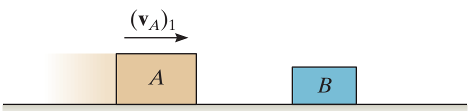
Prob. R15–4

R15–5. The 200-g projectile is fired with a velocity of 900 m/s towards the center of the 15-kg wooden block, which rests on a rough surface. If the projectile penetrates and emerges from the block with a velocity of 300 m/s, determine the velocity of the block just after the projectile emerges. How long does the block slide on the rough surface, after the projectile emerges, before it comes to rest again? The coefficient of kinetic friction between the surface and the block is $\mu_k = 0.2$.



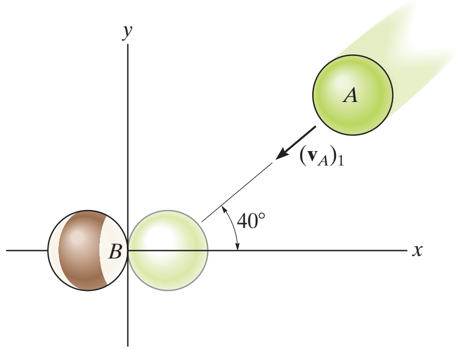
Prob. R15–5

R15–6. Block A has a mass of 3 kg and is sliding on a rough horizontal surface with a velocity $(v_A)_1 = 2$ m/s when it makes a direct collision with block B, which has a mass of 2 kg and is originally at rest. If the collision is perfectly elastic ($e = 1$), determine the velocity of each block just after collision and the distance between the blocks when they stop sliding. The coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.3$.



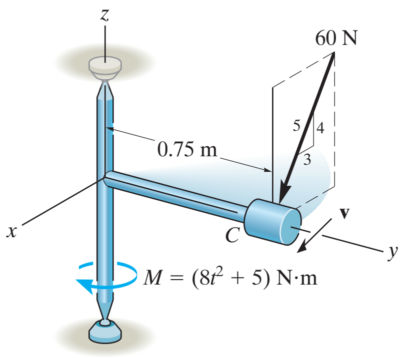
Prob. R15–6

R15–7. Two smooth billiard balls A and B have an equal mass of $m = 200$ g. If A strikes B with a velocity of $(v_A)_1 = 2$ m/s as shown, determine their final velocities just after collision. Ball B is originally at rest and the coefficient of restitution is $e = 0.75$.



Prob. R15–7

R15–8. The small cylinder C has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the frame is subjected to a couple $M = (8t^2 + 5)$ N·m, where t is in seconds, and the cylinder is subjected to a force of 60 N, which is always directed as shown, determine the speed of the cylinder when $t = 2$ s. The cylinder has a speed $v_0 = 2$ m/s when $t = 0$.



Prob. R15–8

Chapter 16

(© TFoxFoto/Shutterstock)

Kinematics is important for the design of the mechanism used on this dump truck.

Planar Kinematics of a Rigid Body

CHAPTER OBJECTIVES

- To classify the various types of rigid-body planar motion.
- To investigate rigid-body translation and angular motion about a fixed axis.
- To study planar motion using an absolute motion analysis.
- To provide a relative motion analysis of velocity and acceleration using a translating frame of reference.
- To show how to find the instantaneous center of zero velocity and determine the velocity of a point on a body using this method.
- To provide a relative-motion analysis of velocity and acceleration using a rotating frame of reference.

16.1 Planar Rigid-Body Motion

In this chapter, the planar kinematics of a rigid body will be discussed. This study is important for the design of gears, cams, and mechanisms used for many mechanical operations. Once the kinematics is thoroughly understood, then we can apply the equations of motion, which relate the forces on the body to the body's motion.

The *planar motion* of a body occurs when all the particles of a rigid body move along paths which are equidistant from a fixed plane. There are three types of rigid-body planar motion. In order of increasing complexity, they are

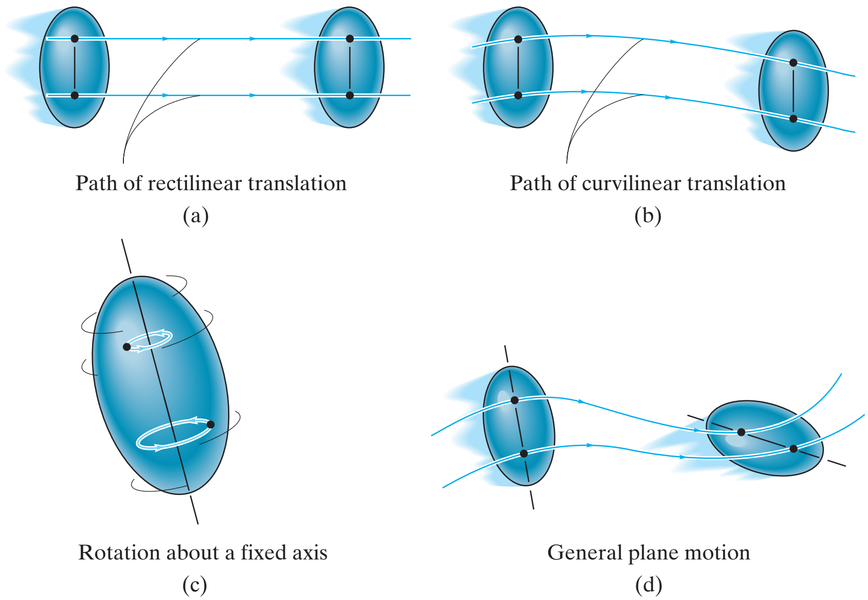


Fig. 16–1

- *Translation.* This type of motion occurs when a line in the body remains parallel to its original orientation throughout the motion. When the paths of motion for any two points on the body are parallel lines, the motion is called *rectilinear translation*, Fig. 16–1*a*. If the paths of motion are along curved lines, the motion is called *curvilinear translation*, Fig. 16–1*b*.
- *Rotation about a fixed axis.* When a rigid body rotates about a fixed axis, all the particles of the body, except those which lie on the axis of rotation, move along circular paths, Fig. 16–1*c*.
- *General plane motion.* When a body is subjected to general plane motion, it undergoes a combination of translation *and* rotation, Fig. 16–1*d*. The translation occurs within a reference plane, and the rotation occurs about an axis perpendicular to the reference plane.

In the following sections we will consider each of these motions in detail. Examples of bodies undergoing these motions are shown in Fig. 16–2.

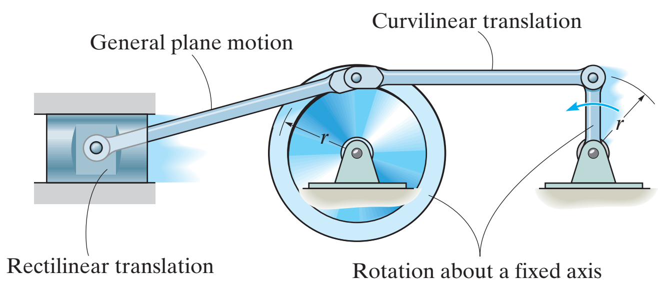


Fig. 16–2

16.2 Translation

Consider a rigid body which is subjected to either rectilinear or curvilinear translation in the x–y plane, Fig. 16–3.

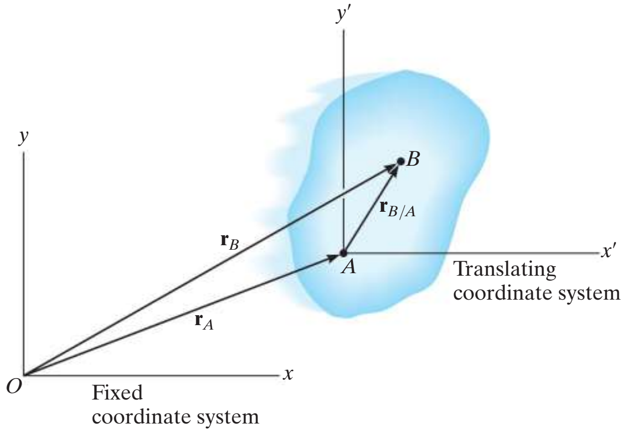


Fig. 16–3

Position. The locations of points A and B on the body are defined with respect to fixed x, y reference frame using *position vectors* $\mathbf{r}_A$ and $\mathbf{r}_B$. The translating x', y' coordinate system is *fixed in the body* and has its origin at A, hereafter referred to as the *base point*. The position of B with respect to A is denoted by the *relative-position vector* $\mathbf{r}_{B/A}$ ("$\mathbf{r}$ of B with respect to A"). By vector addition,

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

Velocity. A relation between the instantaneous velocities of A and B is obtained by taking the time derivative of this equation, which yields $\mathbf{v}_B = \mathbf{v}_A + d\mathbf{r}_{B/A}/dt$. Here $\mathbf{v}_A$ and $\mathbf{v}_B$ denote *absolute velocities* since these vectors are measured with respect to the x, y axes. The term $d\mathbf{r}_{B/A}/dt = \mathbf{0}$, since the *magnitude* of $\mathbf{r}_{B/A}$ is *constant* by definition of a rigid body, and because the body is translating the *direction* of $\mathbf{r}_{B/A}$ is also *constant*. Therefore,

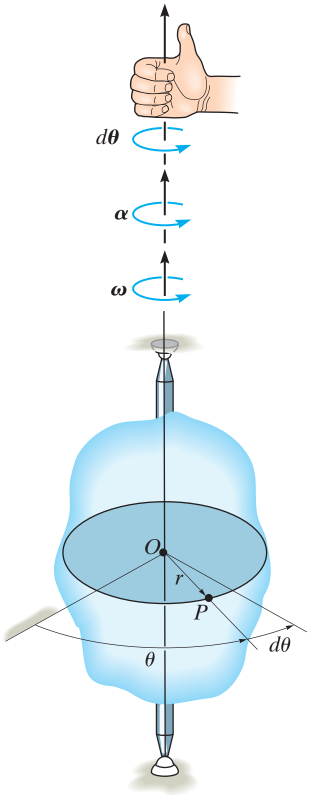
$$\mathbf{v}_B = \mathbf{v}_A$$

Acceleration. Taking the time derivative of the velocity equation yields a similar relationship between the instantaneous accelerations of A and B:

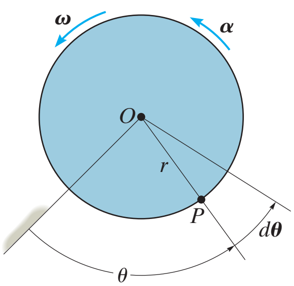
$$\mathbf{a}_B = \mathbf{a}_A$$

The above two equations indicate that *all points in a rigid body subjected to either rectilinear or curvilinear translation move with the same velocity and acceleration*. As a result, the kinematics of particle motion, discussed in Chapter 12, can also be used to specify the kinematics of points located in a translating rigid body.

Riders on this Ferris wheel are subjected to curvilinear translation, since the gondolas move in a circular path, yet it always remains in the upright position. (© R.C. Hibbeler)



(a)



(b)

Fig. 16–4

16.3 Rotation about a Fixed Axis

When a body rotates about a fixed axis, any point P located in the body travels along a *circular path*. To study this motion it is first necessary to discuss the angular motion of the body about the axis.

Angular Motion. Since a point is without dimension, it cannot have angular motion. *Only lines or bodies undergo angular motion*. For example, consider the body shown in Fig. 16–4*a* and the angular motion of a radial line r located within the shaded plane.

Angular Position. At the instant shown, the *angular position* of r is defined by the angle θ, measured from a *fixed* reference line to r.

Angular Displacement. The change in the angular position, which can be measured as a differential $d\boldsymbol{\theta}$, is called the *angular displacement*.* This vector has a *magnitude* of $d\theta$, measured in degrees, radians, or revolutions, where 1 rev $= 2\pi$ rad. Since motion is about a *fixed axis*, the direction of $d\boldsymbol{\theta}$ is *always* along this axis. Specifically, the *direction* is determined by the right-hand rule; that is, the fingers of the right hand are curled with the sense of rotation, so that in this case the thumb, or $d\boldsymbol{\theta}$, points upward, Fig. 16–4*a*. In two dimensions, as shown by the top view of the shaded plane, Fig. 16–4*b*, both θ and $d\theta$ are counterclockwise, and so the thumb points outward from the page.

Angular Velocity. The time rate of change in the angular position is called the *angular velocity* $\boldsymbol{\omega}$ (omega). Since $d\boldsymbol{\theta}$ occurs during an instant of time dt, then,

$$(\circlearrowleft +) \qquad \boxed{\omega = \frac{d\theta}{dt}} \qquad (16\text{–}1)$$

This vector has a *magnitude* which is often measured in rad/s. It is expressed here in scalar form since its *direction* is also along the axis of rotation, Fig. 16–4*a*. When indicating the angular motion in the shaded plane, Fig. 16–4*b*, we can refer to the sense of rotation as clockwise or counterclockwise. Here we have *arbitrarily* chosen counterclockwise rotations as *positive* and indicated this by the curl shown in parentheses next to Eq. 16–1. Realize, however, that the directional sense of $\boldsymbol{\omega}$ is actually outward from the page.

*It is shown in Sec. 20.1 that finite rotations or finite angular displacements are *not* vector quantities, although differential rotations $d\boldsymbol{\theta}$ are vectors.

Angular Acceleration. The *angular acceleration* $\boldsymbol{\alpha}$ (alpha) measures the time rate of change of the angular velocity. The *magnitude* of this vector is

$$(\circlearrowleft +) \qquad \boxed{\alpha = \frac{d\omega}{dt}} \qquad (16\text{–}2)$$

Using Eq. 16–1, it is also possible to express α as

$$(\circlearrowleft +) \qquad \alpha = \frac{d^2\theta}{dt^2} \qquad (16\text{–}3)$$

The line of action of $\boldsymbol{\alpha}$ is the same as that for $\boldsymbol{\omega}$, Fig. 16–4*a*; however, its sense of *direction* depends on whether $\boldsymbol{\omega}$ is increasing or decreasing. If $\boldsymbol{\omega}$ is decreasing, then $\boldsymbol{\alpha}$ is called an *angular deceleration* and therefore has a sense of direction which is opposite to $\boldsymbol{\omega}$.

By eliminating dt from Eqs. 16–1 and 16–2, we obtain a differential relation between the angular acceleration, angular velocity, and angular displacement, namely,

$$(\circlearrowleft +) \qquad \boxed{\alpha\, d\theta = \omega\, d\omega} \qquad (16\text{–}4)$$

The similarity between the differential relations for angular motion and those developed for rectilinear motion of a particle ($v = ds/dt$, $a = dv/dt$, and $a\,ds = v\,dv$) should be apparent.

Constant Angular Acceleration. If the angular acceleration of the body is *constant*, $\boldsymbol{\alpha} = \boldsymbol{\alpha}_c$, then Eqs. 16–1, 16–2, and 16–4, when integrated, yield a set of formulas which relate the body's angular velocity, angular position, and time. These equations are similar to Eqs. 12–4 to 12–6 used for rectilinear motion. The results are

$$(\circlearrowleft +) \qquad \omega = \omega_0 + \alpha_c t \qquad (16\text{–}5)$$

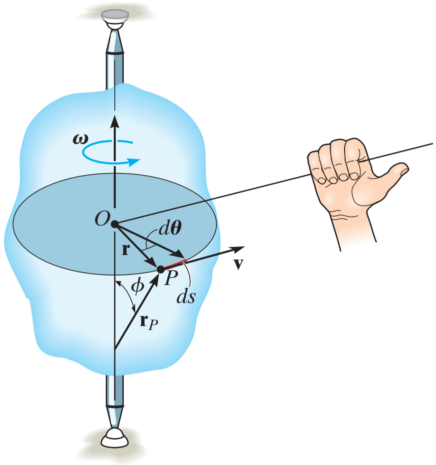
$$(\circlearrowleft +) \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha_c t^2 \qquad (16\text{–}6)$$

$$(\circlearrowleft +) \qquad \omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0) \qquad (16\text{–}7)$$

Constant Angular Acceleration

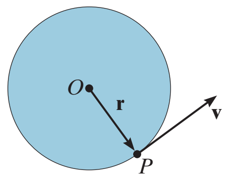
Here θ_0 and ω_0 are the initial values of the body's angular position and angular velocity, respectively.

The gears used in the operation of a crane all rotate about fixed axes. Engineers must be able to relate their angular motions in order to properly design this gear system. (© R.C. Hibbeler)



(c)

Fig. 16–4 (cont.)



(d)

Motion of Point *P*. As the rigid body in Fig. 16–4*c* rotates, point *P* travels along a *circular path* of radius *r* with center at point *O*. This path is contained within the shaded plane shown in top view, Fig. 16–4*d*.

Position and Displacement. The position of *P* is defined by the position vector **r**, which extends from *O* to *P*. If the body rotates $d\theta$ then *P* will displace $ds = r d\theta$.

Velocity. The velocity of *P* has a magnitude which can be found by dividing $ds = r d\theta$ by dt so that

$$\boxed{v = \omega r} \tag{16–8}$$

As shown in Figs. 16–4*c* and 16–4*d*, the *direction* of **v** is *tangent* to the circular path.

Both the magnitude and direction of **v** can also be accounted for by using the cross product of $\boldsymbol{\omega}$ and $\mathbf{r}_P$ (see Appendix B). Here, $\mathbf{r}_P$ is directed from *any point* on the axis of rotation to point *P*, Fig. 16–4*c*. We have

$$\boxed{\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}_P} \tag{16–9}$$

The order of the vectors in this formulation is important, since the cross product is not commutative, i.e., $\boldsymbol{\omega} \times \mathbf{r}_P \neq \mathbf{r}_P \times \boldsymbol{\omega}$. Notice in Fig. 16–4*c* how the correct direction of **v** is established by the right-hand rule. The fingers of the right hand are curled from $\boldsymbol{\omega}$ toward $\mathbf{r}_P$ ($\boldsymbol{\omega}$ "cross" $\mathbf{r}_P$). The thumb indicates the correct direction of **v**, which is tangent to the path in the direction of motion. From Eq. B–8, the magnitude of **v** in Eq. 16–9 is $v = \omega r_P \sin \phi$, and since $r = r_P \sin \phi$, Fig. 16–4*c*, then $v = \omega r$, which agrees with Eq. 16–8. As a special case, the position vector **r** can be chosen for $\mathbf{r}_P$. Here **r** lies in the plane of motion and again the velocity of point *P* is

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} \tag{16–10}$$

Acceleration. The acceleration of P can be expressed in terms of its normal and tangential components. Applying Eq. 12–19 and Eq. 12–20, $a_t = dv/dt$ and $a_n = v^2/\rho$, where $\rho = r$, $v = \omega r$, and $\alpha = d\omega/dt$, we get

$$\boxed{a_t = \alpha r} \tag{16–11}$$

$$\boxed{a_n = \omega^2 r} \tag{16–12}$$

The *tangential component of acceleration*, Figs. 16–4*e* and 16–4*f*, represents the time rate of change in the velocity's magnitude. If the speed of P is increasing, then $\mathbf{a}_t$ acts in the same direction as $\mathbf{v}$; if the speed is decreasing, $\mathbf{a}_t$ acts in the opposite direction of $\mathbf{v}$; and finally, if the speed is constant, $\mathbf{a}_t$ is zero.

The *normal component of acceleration* represents the time rate of change in the velocity's direction. The *direction* of $\mathbf{a}_n$ is always toward O, the center of the circular path, Figs. 16–4*e* and 16–4*f*.

Like the velocity, the acceleration of point P can be expressed in terms of the vector cross product. Taking the time derivative of Eq. 16–9 we have

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r}_P + \boldsymbol{\omega} \times \frac{d\mathbf{r}_P}{dt}$$

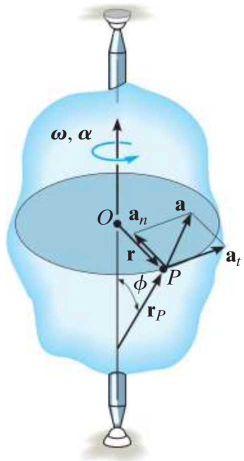
Recalling that $\boldsymbol{\alpha} = d\boldsymbol{\omega}/dt$, and using Eq. 16–9 ($d\mathbf{r}_P/dt = \mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}_P$), yields

$$\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r}_P + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_P) \tag{16–13}$$

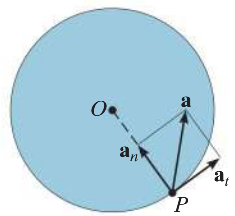
From the definition of the cross product, the first term on the right has a magnitude $a_t = \alpha r_P \sin\phi = \alpha r$, and by the right-hand rule, $\boldsymbol{\alpha} \times \mathbf{r}_P$ is in the direction of $\mathbf{a}_t$, Fig. 16–4*e*. Likewise, the second term has a magnitude $a_n = \omega^2 r_P \sin\phi = \omega^2 r$, and applying the right-hand rule twice, first to determine the result $\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}_P$ then $\boldsymbol{\omega} \times \mathbf{v}_P$, it can be seen that this result is in the same direction as $\mathbf{a}_n$, shown in Fig. 16–4*e*. Noting that this is also the *same* direction as $-\mathbf{r}$, which lies in the plane of motion, we can express $\mathbf{a}_n$ in a much simpler form as $\mathbf{a}_n = -\omega^2\mathbf{r}$. Hence, Eq. 16–13 can be identified by its two components as

$$\boxed{\begin{aligned} \mathbf{a} &= \mathbf{a}_t + \mathbf{a}_n \\ &= \boldsymbol{\alpha} \times \mathbf{r} - \omega^2\mathbf{r} \end{aligned}} \tag{16–14}$$

Since $\mathbf{a}_t$ and $\mathbf{a}_n$ are perpendicular to one another, if needed the magnitude of acceleration can be determined from the Pythagorean theorem; namely, $a = \sqrt{a_n^2 + a_t^2}$, Fig. 16–4*f*.

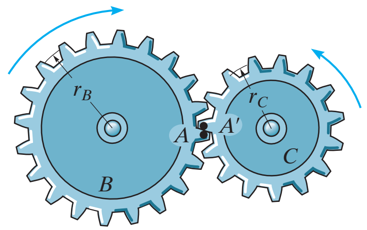


(e)



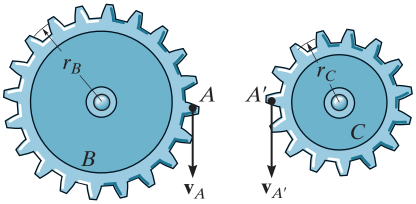
(f)

Fig. 16–4 (cont.)

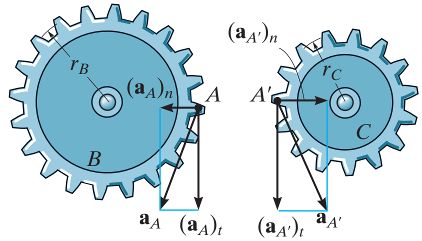


(a)

If two rotating bodies contact one another, then the *points in contact* move along *different circular paths*, and the velocity and *tangential components* of acceleration of the points will be the *same*: however, the *normal components* of acceleration will *not* be the same. For example, consider the two meshed gears in Fig. 16–5*a*. Point A is located on gear B and a coincident point A' is located on gear C. Due to the rotational motion, $\mathbf{v}_A = \mathbf{v}_{A'}$, Fig. 16–5*b*, and as a result, $\omega_B r_B = \omega_C r_C$ or $\omega_B = \omega_C(r_C/r_B)$. Also, from Fig. 16–5*c*, $(\mathbf{a}_A)_t = (\mathbf{a}_{A'})_t$, so that $\alpha_B = \alpha_C(r_C/r_B)$; however, since both points follow different circular paths, $(\mathbf{a}_A)_n \neq (\mathbf{a}_{A'})_n$ and therefore, as shown, $\mathbf{a}_A \neq \mathbf{a}_{A'}$.



(b)



(c)

Fig. 16–5

Important Points

- A body can undergo two types of translation. During rectilinear translation all points follow parallel straight-line paths, and during curvilinear translation the points follow curved paths that are the same shape.
- All the points on a translating body move with the same velocity and acceleration.
- Points located on a body that rotates about a fixed axis follow circular paths.
- The relation $\alpha\, d\theta = \omega\, d\omega$ is derived from $\alpha = d\omega/dt$ and $\omega = d\theta/dt$ by eliminating dt.
- Once angular motions ω and α are known, the velocity and acceleration of any point on the body can be determined.
- The velocity always acts tangent to the path of motion.
- The acceleration has two components. The tangential acceleration measures the rate of change in the magnitude of the velocity and can be determined from $a_t = \alpha r$. The normal acceleration measures the rate of change in the direction of the velocity and can be determined from $a_n = \omega^2 r$.

Procedure for Analysis

The velocity and acceleration of a point located on a rigid body that is rotating about a fixed axis can be determined using the following procedure.

Angular Motion.

- Establish the positive sense of rotation about the axis of rotation and show it alongside each kinematic equation as it is applied.
- If a relation is known between any *two* of the four variables α, ω, θ, and t, then a third variable can be obtained by using one of the following kinematic equations which relates all three variables.

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} \qquad \alpha\, d\theta = \omega\, d\omega$$

- If the body's angular acceleration is *constant*, then the following equations can be used:

$$\omega = \omega_0 + \alpha_c t$$
$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha_c t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

- Once the solution is obtained, the sense of θ, ω, and α is determined from the algebraic signs of their numerical quantities.

Motion of Point *P*.

- In most cases the velocity of P and its two components of acceleration can be determined from the scalar equations

$$v = \omega r$$
$$a_t = \alpha r$$
$$a_n = \omega^2 r$$

- If the geometry of the problem is difficult to visualize, the following vector equations should be used:

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}_P = \boldsymbol{\omega} \times \mathbf{r}$$
$$\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}_P = \boldsymbol{\alpha} \times \mathbf{r}$$
$$\mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_P) = -\omega^2\mathbf{r}$$

- Here $\mathbf{r}_P$ is directed from any point on the axis of rotation to point P, whereas $\mathbf{r}$ lies in the plane of motion of P. Either of these vectors, along with $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$, should be expressed in terms of its $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components, and, if necessary, the cross products determined using a determinant expansion (see Eq. B–12).

EXAMPLE 16.1

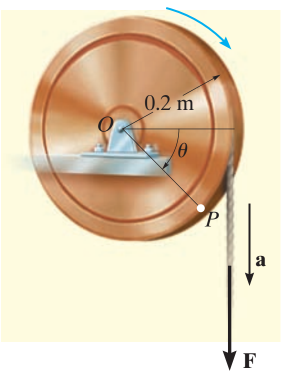


Fig. 16–6

A cord is wrapped around a wheel in Fig. 16–6, which is initially at rest when $\theta = 0$. If a force is applied to the cord and gives it an acceleration $a = (4t)\ \text{m/s}^2$, where t is in seconds, determine, as a function of time, (a) the angular velocity of the wheel, and (b) the angular position of line OP in radians.

SOLUTION

Part (a). The wheel is subjected to rotation about a fixed axis passing through point O. Thus, point P on the wheel has motion about a circular path, and the acceleration of this point has *both* tangential and normal components. The tangential component is $(a_P)_t = (4t)\ \text{m/s}^2$, since the cord is wrapped around the wheel and moves *tangent* to it. Hence the angular acceleration of the wheel is

$$(\circlearrowright +) \qquad (a_P)_t = \alpha r$$

$$(4t)\ \text{m/s}^2 = \alpha(0.2\ \text{m})$$

$$\alpha = (20t)\ \text{rad/s}^2 \circlearrowright$$

Using this result, the wheel's angular velocity ω can now be determined from $\alpha = d\omega/dt$, since this equation relates α, t, and ω. Integrating, with the initial condition that $\omega = 0$ when $t = 0$, yields

$$(\circlearrowright +) \qquad \alpha = \frac{d\omega}{dt} = (20t)\ \text{rad/s}^2$$

$$\int_0^\omega d\omega = \int_0^t 20t\ dt$$

$$\omega = 10t^2\ \text{rad/s} \circlearrowright \qquad \textit{Ans.}$$

Part (b). Using this result, the angular position θ of OP can be found from $\omega = d\theta/dt$, since this equation relates θ, ω, and t. Integrating, with the initial condition $\theta = 0$ when $t = 0$, we have

$$(\circlearrowright +) \qquad \frac{d\theta}{dt} = \omega = (10t^2)\ \text{rad/s}$$

$$\int_0^\theta d\theta = \int_0^t 10t^2\ dt$$

$$\theta = 3.33t^3\ \text{rad} \qquad \textit{Ans.}$$

NOTE: We cannot use the equation of constant angular acceleration, since α is a function of time.

The motor shown in the photo is used to turn a wheel and attached blower contained within the housing. The details are shown in Fig. 16–7*a*. If the pulley A connected to the motor begins to rotate from rest with a constant angular acceleration of $\alpha_A = 2\ \text{rad/s}^2$, determine the magnitudes of the velocity and acceleration of point P on the wheel, after the pulley has turned two revolutions. Assume the transmission belt does not slip on the pulley and wheel.

(© R.C. Hibbeler)

SOLUTION

Angular Motion. First we will convert the two revolutions to radians. Since there are 2π rad in one revolution, then

$$\theta_A = 2\ \text{rev}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 12.57\ \text{rad}$$

Since α_A is constant, the angular velocity of pulley A is therefore

$$(\circlearrowright +) \qquad \omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega_A^2 = 0 + 2(2\ \text{rad/s}^2)(12.57\ \text{rad} - 0)$$

$$\omega_A = 7.090\ \text{rad/s}$$

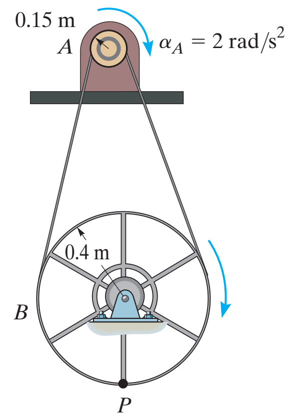
The belt has the same speed and tangential component of acceleration as it passes over the pulley and wheel. Thus,

$$v = \omega_A r_A = \omega_B r_B; \quad 7.090\ \text{rad/s}\ (0.15\ \text{m}) = \omega_B(0.4\ \text{m})$$

$$\omega_B = 2.659\ \text{rad/s}$$

$$a_t = \alpha_A r_A = \alpha_B r_B; \quad 2\ \text{rad/s}^2\ (0.15\ \text{m}) = \alpha_B(0.4\ \text{m})$$

$$\alpha_B = 0.750\ \text{rad/s}^2$$



(a)

Motion of *P*. As shown on the kinematic diagram in Fig. 16–7*b*, we have

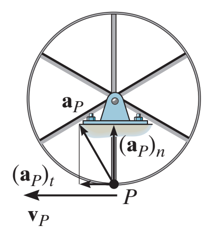
$$v_P = \omega_B r_B = 2.659\ \text{rad/s}\ (0.4\ \text{m}) = 1.06\ \text{m/s} \qquad \textit{Ans.}$$

$$(a_P)_t = \alpha_B r_B = 0.750\ \text{rad/s}^2\ (0.4\ \text{m}) = 0.3\ \text{m/s}^2$$

$$(a_P)_n = \omega_B^2 r_B = (2.659\ \text{rad/s})^2(0.4\ \text{m}) = 2.827\ \text{m/s}^2$$

Thus

$$a_P = \sqrt{(0.3\ \text{m/s}^2)^2 + (2.827\ \text{m/s}^2)^2} = 2.84\ \text{m/s}^2 \qquad \textit{Ans.}$$

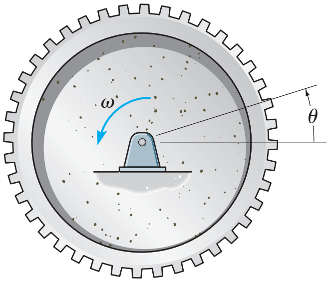


(b)

Fig. 16–7

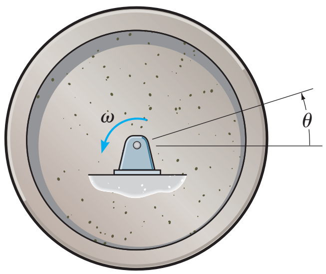
FUNDAMENTAL PROBLEMS

F16–1. When the gear rotates 20 revolutions, it achieves an angular velocity of $\omega = 30$ rad/s, starting from rest. Determine its constant angular acceleration and the time required.



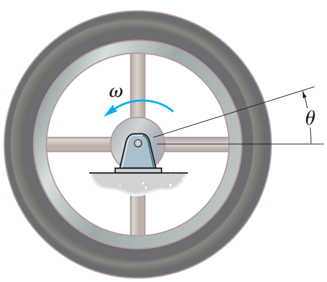
Prob. F16–1

F16–2. The flywheel rotates with an angular velocity of $\omega = (0.005\theta^2)$ rad/s, where θ is in radians. Determine the angular acceleration when it has rotated 20 revolutions.



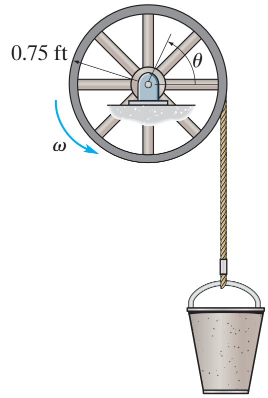
Prob. F16–2

F16–3. The flywheel rotates with an angular velocity of $\omega = (4\,\theta^{1/2})$ rad/s, where θ is in radians. Determine the time it takes to achieve an angular velocity of $\omega = 150$ rad/s. When $t = 0$, $\theta = 1$ rad.



Prob. F16–3

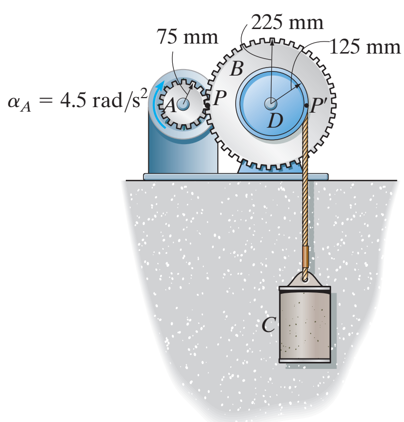
F16–4. The bucket is hoisted by the rope that wraps around a drum wheel. If the angular displacement of the wheel is $\theta = (0.5t^3 + 15t)$ rad, where t is in seconds, determine the velocity and acceleration of the bucket when $t = 3$ s.



Prob. F16–4

F16–5. A wheel has an angular acceleration of $\alpha = (0.5\,\theta)$ rad/s^2, where θ is in radians. Determine the magnitude of the velocity and acceleration of a point P located on its rim after the wheel has rotated 2 revolutions. The wheel has a radius of 0.2 m and starts at $\omega_0 = 2$ rad/s.

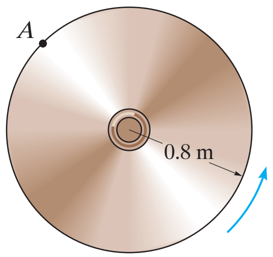
F16–6. For a short period of time, the motor turns gear A with a constant angular acceleration of $\alpha_A = 4.5$ rad/s^2, starting from rest. Determine the velocity of the cylinder and the distance it travels in three seconds. The cord is wrapped around pulley D which is rigidly attached to gear B.



Prob. F16–6

PROBLEMS

16–1. The angular velocity of the disk is defined by $\omega = (5t^2 + 2)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and acceleration of point A on the disk when $t = 0.5$ s.

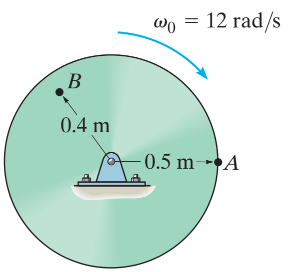


Prob. 16–1

16–2. The angular acceleration of the disk is defined by $\alpha = 3t^2 + 12$ rad/s, where t is in seconds. If the disk is originally rotating at $\omega_0 = 12$ rad/s, determine the magnitude of the velocity and the n and t components of acceleration of point A on the disk when $t = 2$ s.

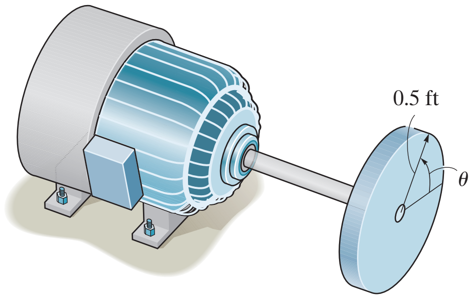
16–3. The disk is originally rotating at $\omega_0 = 12$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 20$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant $t = 2$ s.

*__16–4.__ The disk is originally rotating at $\omega_0 = 12$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 20$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point B when the disk undergoes 2 revolutions.



Probs. 16–2/3/4

16–5. The disk is driven by a motor such that the angular position of the disk is defined by $\theta = (20t + 4t^2)$ rad, where t is in seconds. Determine the number of revolutions, the angular velocity, and angular acceleration of the disk when $t = 90$ s.

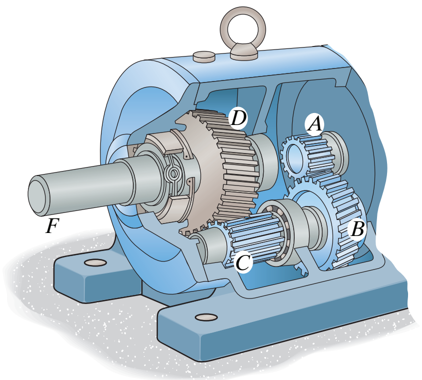


Prob. 16–5

16–6. A wheel has an initial clockwise angular velocity of 10 rad/s and a constant angular acceleration of 3 rad/s^2. Determine the number of revolutions it must undergo to acquire a clockwise angular velocity of 15 rad/s. What time is required?

16–7. If gear A rotates with a constant angular acceleration of $\alpha_A = 90$ rad/s^2, starting from rest, determine the time required for gear D to attain an angular velocity of 600 rpm. Also, find the number of revolutions of gear D to attain this angular velocity. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.

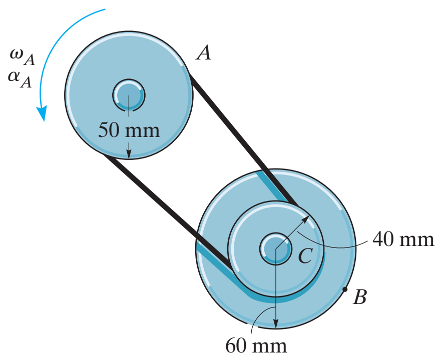
*__16–8.__ If gear A rotates with an angular velocity of $\omega_A = (\theta_A + 1)$ rad/s, where θ_A is the angular displacement of gear A, measured in radians, determine the angular acceleration of gear D when $\theta_A = 3$ rad, starting from rest. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



Probs. 16–7/8

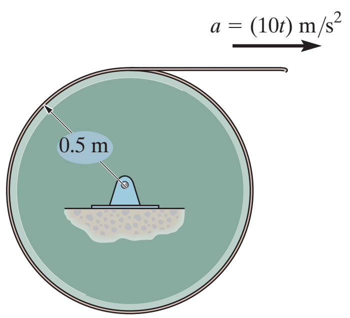
16–9. At the instant $\omega_A = 5$ rad/s, pulley A is given an angular acceleration $\alpha = (0.8\theta)$ rad/s^2, where θ is in radians. Determine the magnitude of acceleration of point B on pulley C when A rotates 3 revolutions. Pulley C has an inner hub which is fixed to its outer one and turns with it.

16–10. At the instant $\omega_A = 5$ rad/s, pulley A is given a constant angular acceleration $\alpha_A = 6$ rad/s^2. Determine the magnitude of acceleration of point B on pulley C when A rotates 2 revolutions. Pulley C has an inner hub which is fixed to its outer one and turns with it.



Probs. 16–9/10

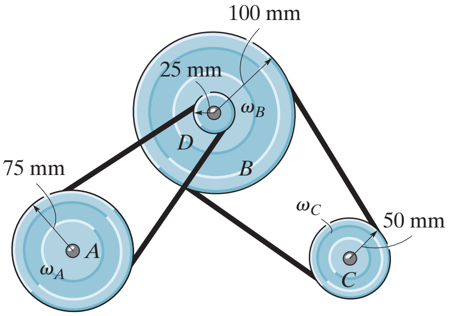
16–11. The cord, which is wrapped around the disk, is given an acceleration of $a = (10t)$ m/s^2, where t is in seconds. Starting from rest, determine the angular displacement, angular velocity, and angular acceleration of the disk when $t = 3$ s.



Prob. 16–11

***16–12.** The power of a bus engine is transmitted using the belt-and-pulley arrangement shown. If the engine turns pulley A at $\omega_A = (20t + 40)$ rad/s, where t is in seconds, determine the angular velocities of the generator pulley B and the air-conditioning pulley C when $t = 3$ s.

16–13. The power of a bus engine is transmitted using the belt-and-pulley arrangement shown. If the engine turns pulley A at $\omega_A = 60$ rad/s, determine the angular velocities of the generator pulley B and the air-conditioning pulley C. The hub at D is rigidly *connected* to B and turns with it.

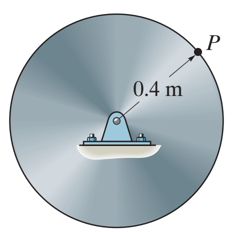


Probs. 16–12/13

16–14. The disk starts from rest and is given an angular acceleration $\alpha = (2t^2)$ rad/s^2, where t is in seconds. Determine the angular velocity of the disk and its angular displacement when $t = 4$ s.

16–15. The disk starts from rest and is given an angular acceleration $\alpha = (5t^{1/2})$ rad/s^2, where t is in seconds. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $t = 2$ s.

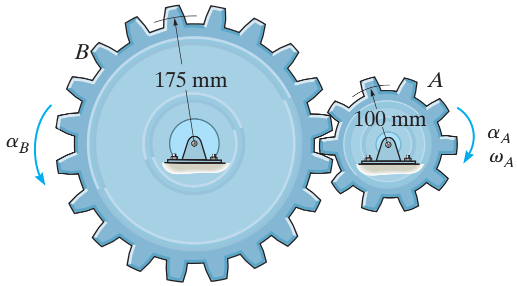
***16–16.** The disk starts at $\omega_0 = 1$ rad/s when $\theta = 0$, and is given an angular acceleration $\alpha = (0.3\theta)$ rad/s^2, where θ is in radians. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $\theta = 1$ rev.



Probs. 16–14/15/16

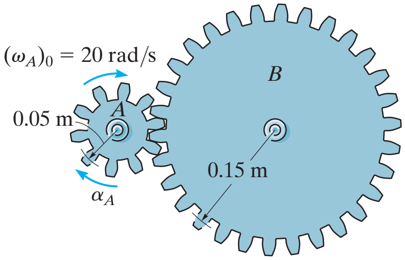
16–17. A motor gives gear A an angular acceleration of $\alpha_A = (2 + 0.006\ \theta^2)$ rad/s^2, where θ is in radians. If this gear is initially turning at $\omega_A = 15$ rad/s, determine the angular velocity of gear B after A undergoes an angular displacement of 10 rev.

16–18. A motor gives gear A an angular acceleration of $\alpha_A = (2t^3)$ rad/s^2, where t is in seconds. If this gear is initially turning at $\omega_A = 15$ rad/s, determine the angular velocity of gear B when $t = 3$ s.



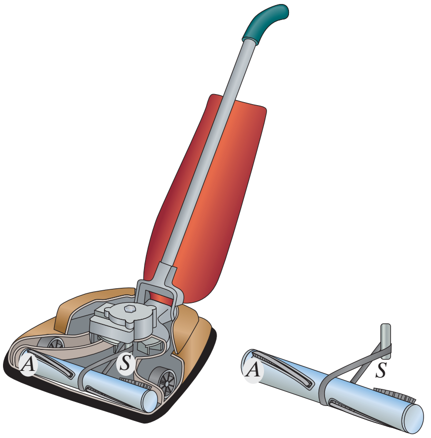
Probs. 16–17/18

*16–20. A motor gives gear A an angular acceleration of $\alpha_A = (4t^3)$ rad/s^2, where t is in seconds. If this gear is initially turning at $(\omega_A)_0 = 20$ rad/s, determine the angular velocity of gear B when $t = 2$ s.



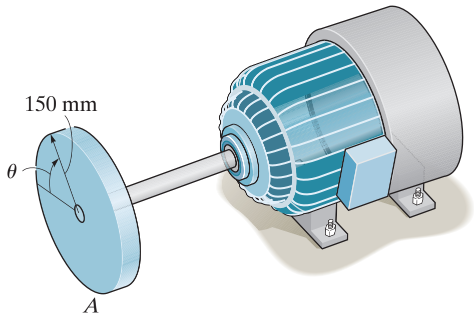
Prob. 16–20

16–19. The vacuum cleaner's armature shaft S rotates with an angular acceleration of $\alpha = 4\omega^{3/4}$ rad/s^2, where ω is in rad/s. Determine the brush's angular velocity when $t = 4$ s, starting from $\omega_0 = 1$ rad/s, at $\theta = 0$. The radii of the shaft and the brush are 0.25 in. and 1 in., respectively. Neglect the thickness of the drive belt.



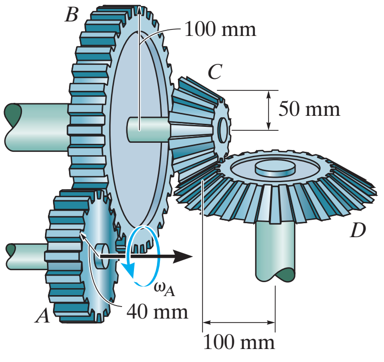
Prob. 16–19

16–21. The motor turns the disk with an angular velocity of $\omega = (5t^2 + 3t)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and the n and t components of acceleration of the point A on the disk when $t = 3$ s.



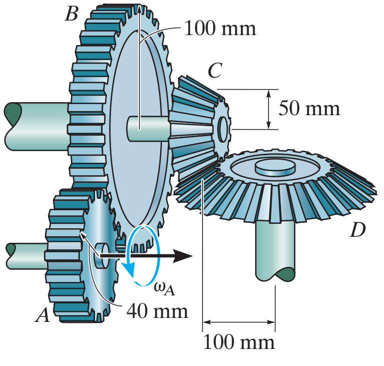
Prob. 16–21

16–22. If the motor turns gear A with an angular acceleration of $\alpha_A = 2 \text{ rad/s}^2$ when the angular velocity is $\omega_A = 20 \text{ rad/s}$, determine the angular acceleration and angular velocity of gear D.



Prob. 16–22

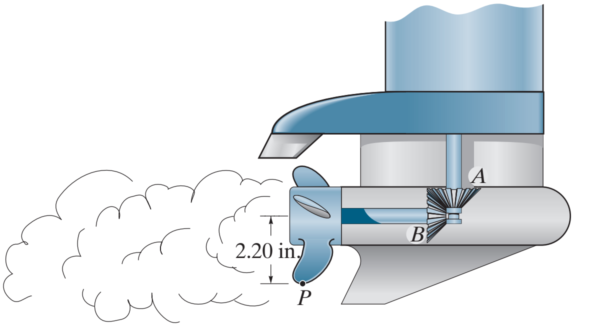
16–23. If the motor turns gear A with an angular acceleration of $\alpha_A = 3 \text{ rad/s}^2$ when the angular velocity is $\omega_A = 60 \text{ rad/s}$, determine the angular acceleration and angular velocity of gear D.



Prob. 16–23

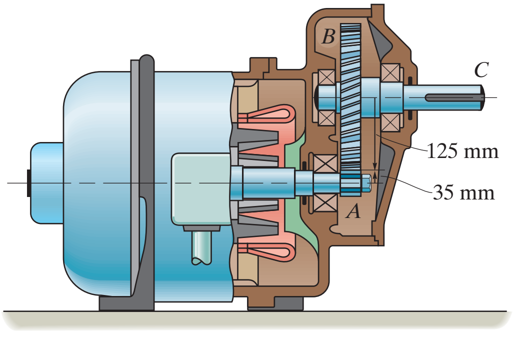
***16–24.** The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.5$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.2$ in. Determine the angular velocity of the propeller in $t = 1.5$ s, if the drive shaft rotates with an angular acceleration $\alpha = (400t^3) \text{ rad/s}^2$, where t is in seconds. The propeller is originally at rest and the motor frame does not move.

16–25. For the outboard motor in Prob. 16–24, determine the magnitude of the velocity and acceleration of point P located on the tip of the propeller at the instant $t = 0.75$ s.



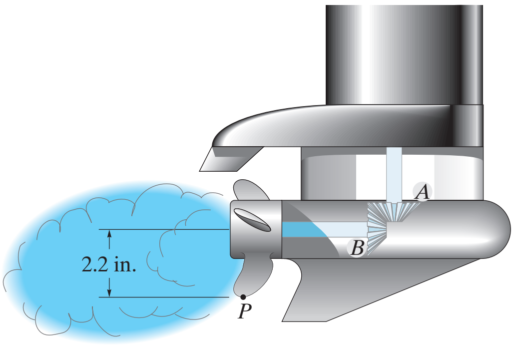
Probs. 16–24/25

16–26. The pinion gear A on the motor shaft is given a constant angular acceleration $\alpha = 3 \text{ rad/s}^2$. If the gears A and B have the dimensions shown, determine the angular velocity and angular displacement of the output shaft C, when $t = 2$ s starting from rest. The shaft is fixed to B and turns with it.



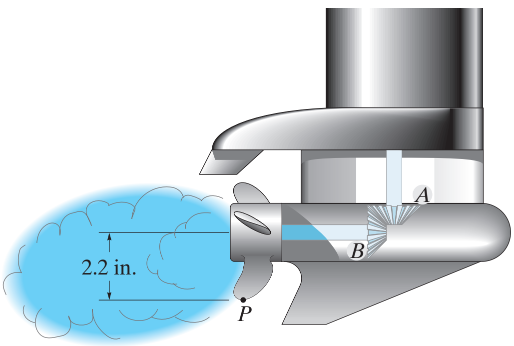
Prob. 16–26

16–27. The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.7$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.4$ in. Determine the angular velocity of the propeller in $t = 1.3$ s if the drive shaft rotates with an angular acceleration $\alpha = (300\sqrt{t})$ rad/s^2, where t is in seconds. The propeller is originally at rest and the motor frame does not move.



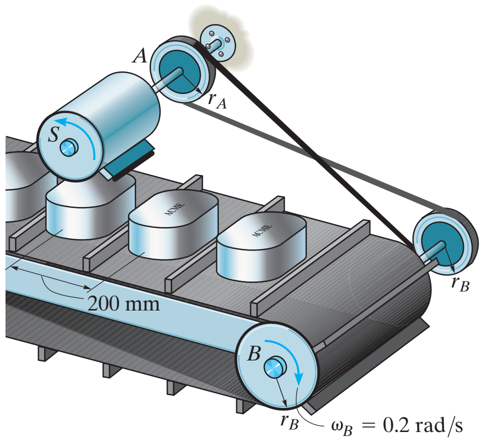
Prob. 16–27

***16–28.** The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.7$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.4$ in. Determine the magnitudes of the velocity and acceleration of a point P located on the tip of the propeller at the instant $t = 0.75$ s. The drive shaft rotates with an angular acceleration $\alpha = (300\sqrt{t})$ rad/s^2, where t is in seconds. The propeller is originally at rest and the motor frame does not move.



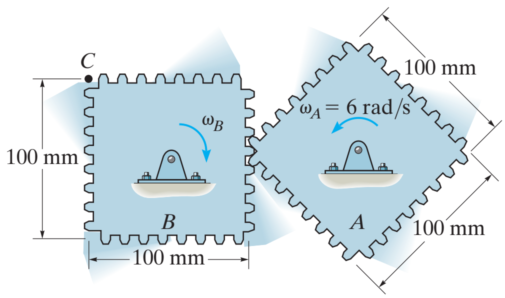
Prob. 16–28

16–29. A stamp S, located on the revolving drum, is used to label canisters. If the canisters are centered 200 mm apart on the conveyor, determine the radius r_A of the driving wheel A and the radius r_B of the conveyor belt drum so that for each revolution of the stamp it marks the top of a canister. How many canisters are marked per minute if the drum at B is rotating at $\omega_B = 0.2$ rad/s? Note that the driving belt is twisted as it passes between the wheels.



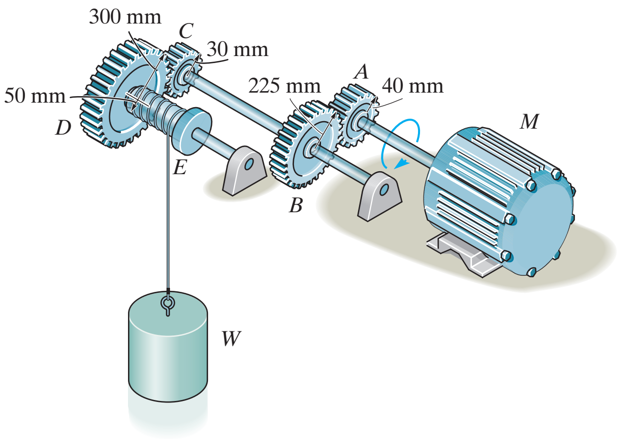
Prob. 16–29

16–30. At the instant shown, gear A is rotating with a constant angular velocity of $\omega_A = 6$ rad/s. Determine the largest angular velocity of gear B and the maximum speed of point C.



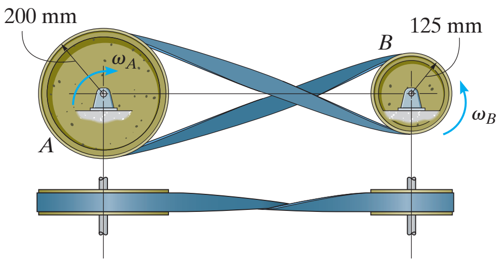
Prob. 16–30

16–31. Determine the distance the load W is lifted in $t = 5$ s using the hoist. The shaft of the motor M turns with an angular velocity $\omega = 100(4 + t)$ rad/s, where t is in seconds.



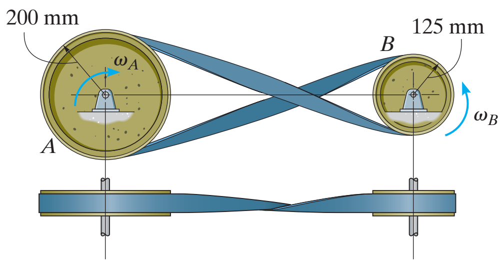
Prob. 16–31

***16–32.** The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If A has a constant angular acceleration of $\alpha_A = 30$ rad/s^2, determine the tangential and normal components of acceleration of a point located at the rim of B when $t = 3$ s, starting from rest.



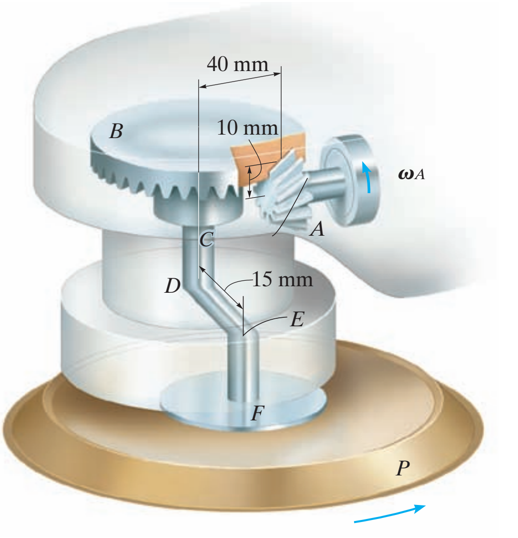
Prob. 16–32

16–33. The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If the angular displacement of A is $\theta_A = (5t^3 + 10t^2)$ rad, where t is in seconds, determine the angular velocity and angular acceleration of B when $t = 3$ s.



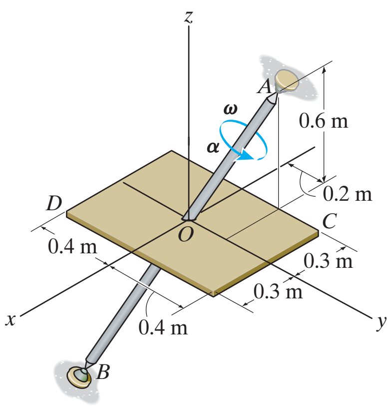
Prob. 16–33

16–34. For a short time a motor of the random-orbit sander drives the gear A with an angular velocity of $\omega_A = 40(t^3 + 6t)$ rad/s, where t is in seconds. This gear is connected to gear B, which is fixed connected to the shaft CD. The end of this shaft is connected to the eccentric spindle EF and pad P, which causes the pad to orbit around shaft CD at a radius of 15 mm. Determine the magnitudes of the velocity and the tangential and normal components of acceleration of the spindle EF when $t = 2$ s after starting from rest.



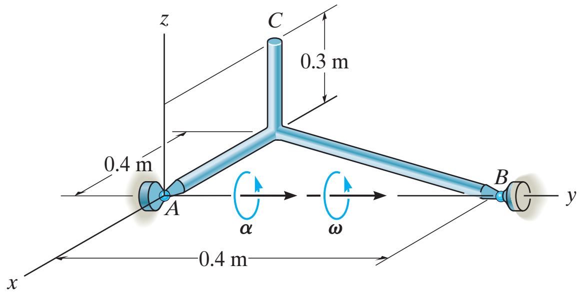
Prob. 16–34

16–35. If the shaft and plate rotates with a constant angular velocity of $\omega = 14\ \text{rad/s}$, determine the velocity and acceleration of point C located on the corner of the plate at the instant shown. Express the result in Cartesian vector form.



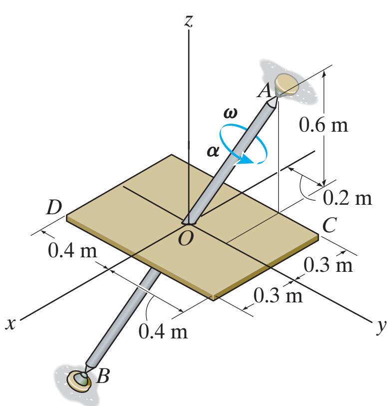
Prob. 16–35

16–37. The rod assembly is supported by ball-and-socket joints at A and B. At the instant shown it is rotating about the y axis with an angular velocity $\omega = 5\ \text{rad/s}$ and has an angular acceleration $\alpha = 8\ \text{rad/s}^2$. Determine the magnitudes of the velocity and acceleration of point C at this instant. Solve the problem using Cartesian vectors and Eqs. 16–9 and 16–13.



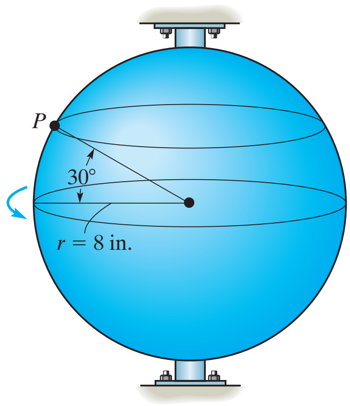
Prob. 16–37

***16–36.** At the instant shown, the shaft and plate rotates with an angular velocity of $\omega = 14\ \text{rad/s}$ and angular acceleration of $\alpha = 7\ \text{rad/s}^2$. Determine the velocity and acceleration of point D located on the corner of the plate at this instant. Express the result in Cartesian vector form.



Prob. 16–36

16–38. The sphere starts from rest at $\theta = 0°$ and rotates with an angular acceleration of $\alpha = (4\theta + 1)\ \text{rad/s}^2$, where θ is in radians. Determine the magnitudes of the velocity and acceleration of point P on the sphere at the instant $\theta = 6\ \text{rad}$.



Prob. 16–38

16.4 Absolute Motion Analysis

The dumping bin on the truck rotates about a fixed axis passing through the pin at A. It is operated by the extension of the hydraulic cylinder BC. The angular position of the bin can be specified using the angular position coordinate θ, and the position of point C on the bin is specified using the rectilinear position coordinate s. Since a and b are fixed lengths, then the two coordinates can be related by the cosine law, $s = \sqrt{a^2 + b^2 - 2ab\cos\theta}$. The time derivative of this equation relates the speed at which the hydraulic cylinder extends to the angular velocity of the bin. (© R.C. Hibbeler)

A body subjected to *general plane motion* undergoes a *simultaneous* translation and rotation. If the body is represented by a thin slab, the slab translates in the plane of the slab and rotates about an axis perpendicular to this plane. The motion can be completely specified by knowing *both* the angular rotation of a line fixed in the body and the motion of a point on the body. One way to relate these motions is to use a rectilinear position coordinate s to locate the point along its path and an angular position coordinate θ to specify the orientation of the line. The two coordinates are then related using the geometry of the problem. By *direct application* of the time-differential equations $v = ds/dt$, $a = dv/dt$, $\omega = d\theta/dt$, and $\alpha = d\omega/dt$, the *motion* of the point and the *angular motion* of the line can then be related. This procedure is similar to that used to solve dependent motion problems involving pulleys, Sec. 12.9. In some cases, this same procedure may be used to relate the motion of one body, undergoing either rotation about a fixed axis or translation, to that of a connected body undergoing general plane motion.

Procedure for Analysis

The velocity and acceleration of a point P undergoing rectilinear motion can be related to the angular velocity and angular acceleration of a line contained within a body using the following procedure.

Position Coordinate Equation.

- Locate point P on the body using a position coordinate s, which is measured from a *fixed origin* and is *directed along the straight-line path of motion* of point P.
- Measure from a fixed reference line the angular position θ of a line lying in the body.
- From the dimensions of the body, relate s to θ, $s = f(\theta)$, using geometry and/or trigonometry.

Time Derivatives.

- Take the first derivative of $s = f(\theta)$ with respect to time to get a relation between v and ω.
- Take the second time derivative to get a relation between a and α.
- In each case the chain rule of calculus must be used when taking the time derivatives of the position coordinate equation. See Appendix C.

EXAMPLE 16.3

The end of rod R shown in Fig. 16–8 maintains contact with the cam by means of a spring. If the cam rotates about an axis passing through point O with an angular acceleration $\boldsymbol{\alpha}$ and angular velocity $\boldsymbol{\omega}$, determine the velocity and acceleration of the rod when the cam is in the arbitrary position θ.

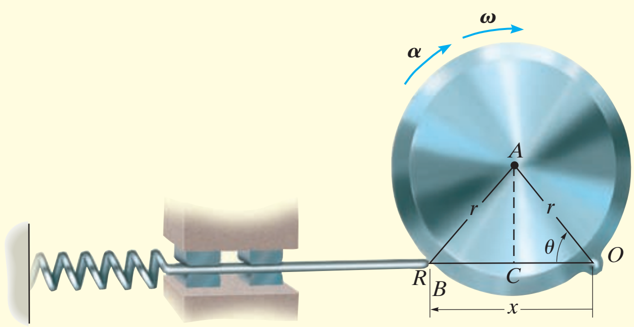


Fig. 16–8

SOLUTION

Position Coordinate Equation. Coordinates θ and x are chosen in order to relate the *rotational motion* of the line segment OA on the cam to the *rectilinear translation* of the rod. These coordinates are measured from the *fixed point O* and can be related to each other using trigonometry. Since $OC = CB = r\cos\theta$, Fig. 16–8, then

$$x = 2r\cos\theta$$

Time Derivatives. Using the chain rule of calculus, we have

$$\frac{dx}{dt} = -2r(\sin\theta)\frac{d\theta}{dt}$$

$$v = -2r\omega\sin\theta \qquad \textit{Ans.}$$

$$\frac{dv}{dt} = -2r\left(\frac{d\omega}{dt}\right)\sin\theta - 2r\omega(\cos\theta)\frac{d\theta}{dt}$$

$$a = -2r(\alpha\sin\theta + \omega^2\cos\theta) \qquad \textit{Ans.}$$

NOTE: The negative signs indicate that v and a are opposite to the direction of positive x. This seems reasonable when you visualize the motion.

EXAMPLE 16.4

At a given instant, the cylinder of radius r, shown in Fig. 16–9, has an angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$. Determine the velocity and acceleration of its center G if the cylinder rolls without slipping.

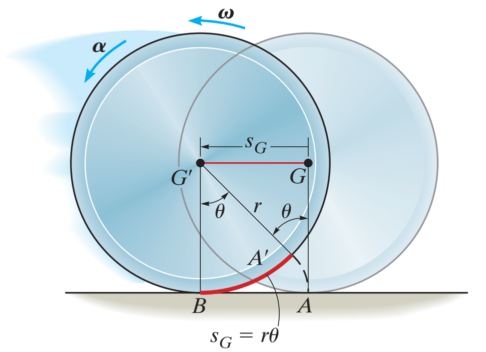


Fig. 16–9

SOLUTION

Position Coordinate Equation. The cylinder undergoes general plane motion since it simultaneously translates and rotates. By inspection, point G moves in a *straight line* to the left, from G to G', as the cylinder rolls, Fig. 16–9. Consequently its new position G' will be specified by the *horizontal* position coordinate s_G, which is measured from G to G'. Also, as the cylinder rolls (without slipping), the arc length $A'B$ on the rim which was in contact with the ground from A to B, is equivalent to s_G. Consequently, the motion requires the radial line GA to rotate θ to the position $G'A'$. Since the arc $A'B = r\theta$, then G travels a distance

$$s_G = r\theta$$

Time Derivatives. Taking successive time derivatives of this equation, realizing that r is constant, $\omega = d\theta/dt$, and $\alpha = d\omega/dt$, gives the necessary relationships:

$$s_G = r\theta$$

$$v_G = r\omega \qquad \textit{Ans.}$$

$$a_G = r\alpha \qquad \textit{Ans.}$$

NOTE: Remember that these relationships are valid only if the cylinder (disk, wheel, ball, etc.) rolls *without* slipping.