

# Image Processing : a DSP application

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TCD 2003 - 3C1: Digital Signal Processing

## Edge detection



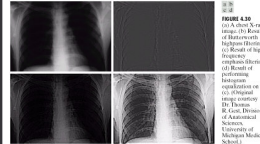
Image understanding



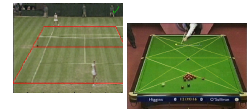
Segmentation

Object detection/recognition

Restoration/Image Enhancement



Video Indexing



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## Contents

- Image as a 2D continuous signal
- Image as a 2D digital signal
  - standard methods for derivation
  - smoothing before derivating
- Applications

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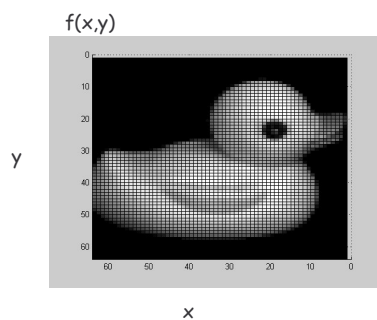
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## Image as a 2D Continuous Signal

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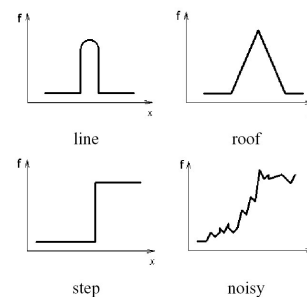
## Image: a surface



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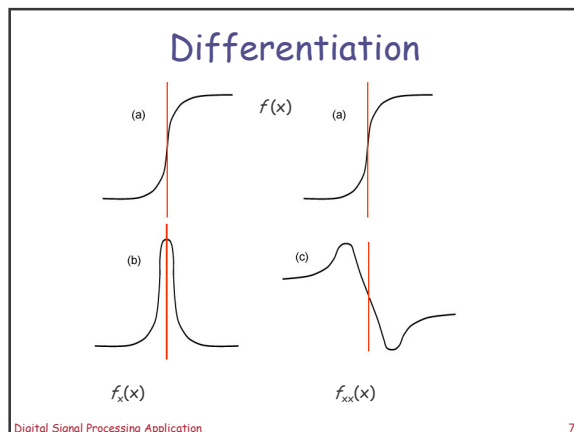
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## Type of Edges



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### The Gradient

First derivatives

$$\nabla f(x,y) = \begin{cases} f_x(x,y) = \frac{\partial f(x,y)}{\partial x} \\ f_y(x,y) = \frac{\partial f(x,y)}{\partial y} \end{cases}$$

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### The Laplacian

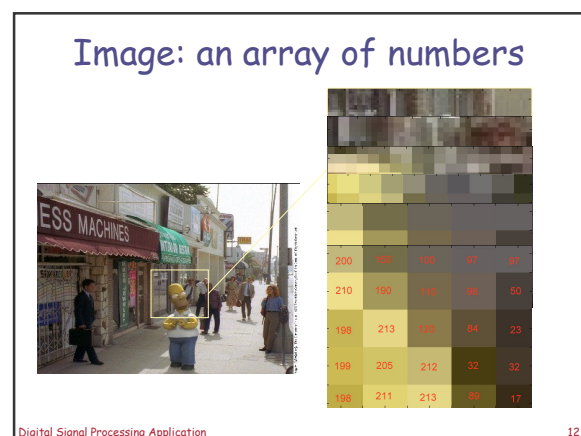
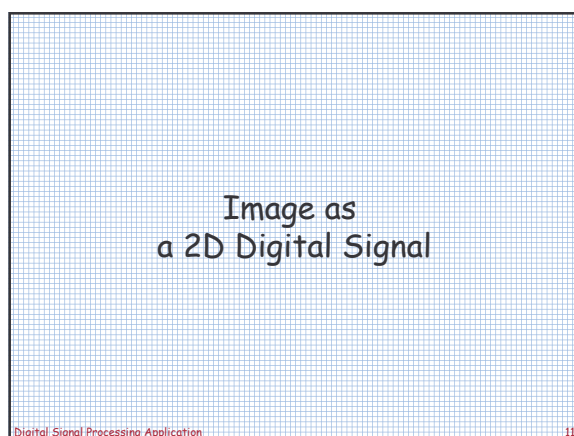
2<sup>nd</sup> derivatives

$$\Delta f(x,y) = f_{xx}(x,y) + f_{yy}(x,y)$$

Laplacian

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- ### Edge detection
1. Compute  $|\nabla f(x,y)|$
  2. Select  $|\nabla f(x,y)| > s$  threshold
  3. Thinning:
    1. Local maxima of  $|\nabla f(x,y)|$
    2. Zero-crossing of  $\Delta f(x,y)$
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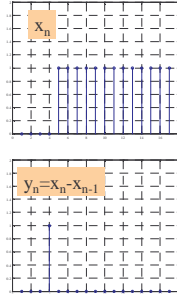
## The Gradient

- First derivatives

$$f_x(x, y) \approx f(x, y) - f(x-1, y)$$

$$f_y(x, y) \approx f(x, y) - f(x, y-1)$$

$(x, y)$  are now discrete !



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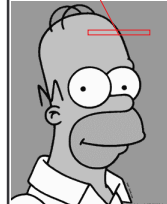
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## Computed by Convolution

212 212 212 212 212 212 10 10 150 150 150 150

1 Convolution

$$\otimes H_x = [1 \quad -1]$$



0 0 0 0 0 202 0 -140 0 0 0 0

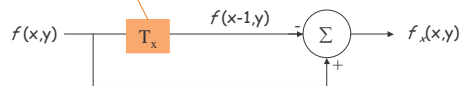


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## Block diagram

Shift in x operator



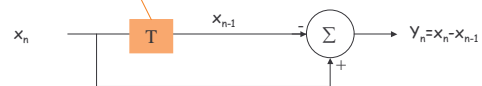
Impulsive response ( $f(0,0)=1$ ,  $f(x,y)=0 \forall (x,y) \neq (0,0)$ )  
cf. handout 8, p6:  
 $H_x(0,0)=1$   
 $H_x(1,0)=-1$   
 $H_x(x,y)=0$  otherwise

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## Block diagram

Shift in x operator



Impulsive response ( $x_0=1$ ,  $x_n=0 \forall n \neq 0$ )  
cf. handout 8, p6:  
 $H_0=1$   
 $H_1=-1$   
 $H_n=0$  otherwise

A one dimensional  
filter applied to  
rows and columns  
of the image

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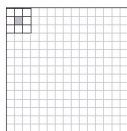
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## Computed by Convolution

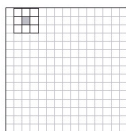
Other filters

averaging in the orthogonal direction :  $H_x = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$

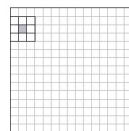
- Local processing within a moving window  
moving window calculations



output at: pixel 2, row 2



pixel 3, row 2



pixel 2, row 3

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## Magnitude of the Gradient



$f(x, y)$

$$H_x = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$H_y = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$



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## The Laplacian

- Second derivatives

$$\Delta f(x, y) \approx f(x+1, y) - f(x-1, y) + f(x, y+1) - f(x, y-1) - 4f(x, y)$$

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

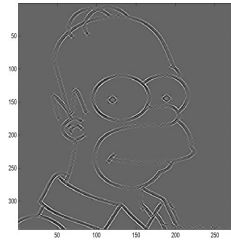
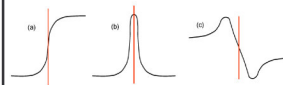
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## The Laplacian

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



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## Finding the edges



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## Remarks

- Enhancing the edge  $\nabla$
- Enhancing the noise  $\nabla^2$

Remember handout 2, p4 :

$$f'(x) \approx j\omega F(j\omega) - f(0)$$

$$f''(x) \approx (j\omega)^2 F(j\omega) - (j\omega) f(0) - f'(0)$$

High Frequencies  
are enhanced by  
differentiation

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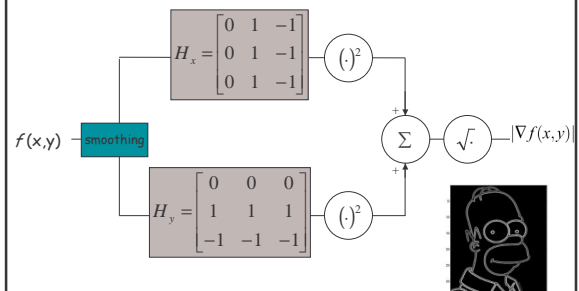
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## Smoothing before gradient Calculation using Gaussian filters

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## Smoothing before gradient calculation for noise removal



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## Gaussian filter

$$G(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) \cdot \exp\left(-\frac{y^2}{2\sigma^2}\right) \\ = G(x) \times G(y)$$

- Smoothing (low-pass) filter
- Separable filter (faster processing)

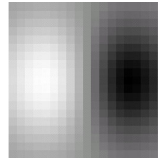


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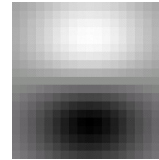
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## Derivative of Gaussian (DroG)

$$\nabla(\tilde{f} = G \otimes f) = \begin{cases} \tilde{f}_x = G_x(x) \otimes G(y) \otimes f \\ \tilde{f}_y = G(x) \otimes G_y(y) \otimes f \end{cases}$$



$$G_x(x) \otimes G(y)$$



$$G(x) \otimes G_y(y)$$

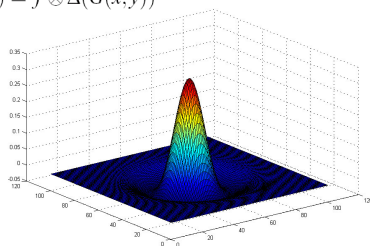
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## Laplacian of Gaussian (LoG)

$$\Delta(f \otimes G(x,y)) = f \otimes \Delta(G(x,y))$$

Mexican hat



$$\Delta(G(x,y)) = \frac{1}{\pi\sigma^4} \left[ 1 - \frac{x^2+y^2}{2\sigma^2} \right] \exp\left(-\frac{x^2}{2\sigma^2}\right) \cdot \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

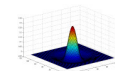
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## Laplacian of Gaussian (LoG)



Equivalent to :



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## Edge detection



Using Pixel difference



Using DroG

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## Applications

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## Unsharp Masking

Original image

Enhanced edge image

Laplacian of the image

Amplifying term

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## Unsharp Masking

HPF: High pass filter

K amplifies high frequencies

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## For image enhancement

Application to astronomical images, medical images (Xray photo), etc.

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## For Image Restoration

original

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## For Image Restoration

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## Sport Video Indexing

Inter-Shot Analysis in Sport Videos using the "sum of edges" in images

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## Sport Video Indexing

Intra-Shot Analysis in Sport Videos using the "Hough transform" (looking at the straight lines)



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## Sport Video Indexing



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Any questions ?

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