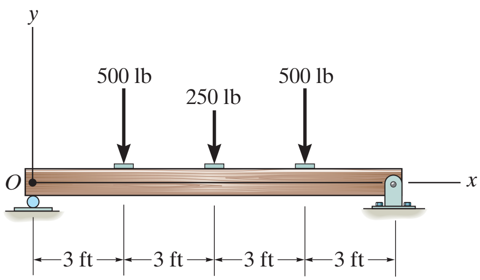
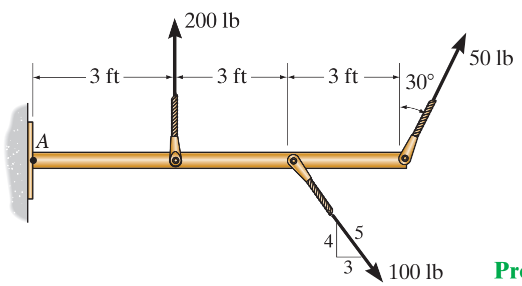
FUNDAMENTAL PROBLEMS

F4–31. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the beam measured from O.



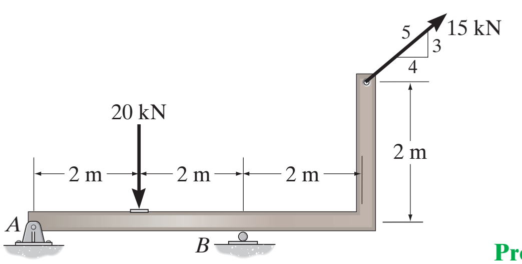
Prob. F4–31

F4–32. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member measured from A.



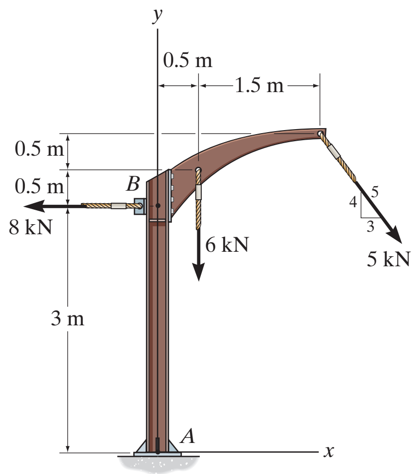
Prob. F4–32

F4–33. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the horizontal segment of the member measured from A.



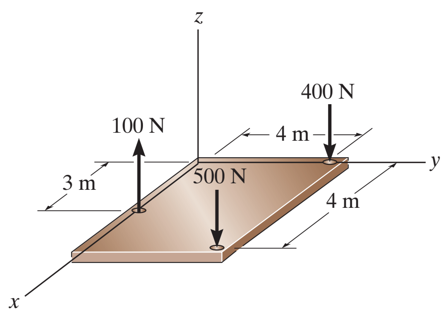
Prob. F4–33

F4–34. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member AB measured from A.



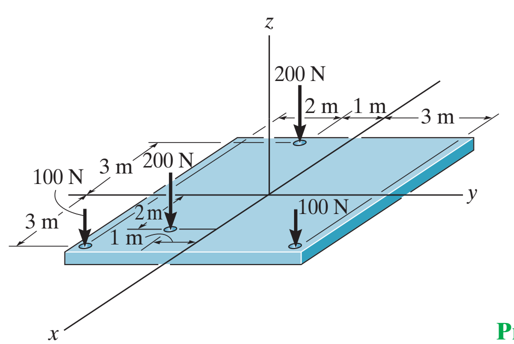
Prob. F4–34

F4–35. Replace the loading shown by an equivalent single resultant force and specify the x and y coordinates of its line of action.



Prob. F4–35

F4–36. Replace the loading shown by an equivalent single resultant force and specify the x and y coordinates of its line of action.

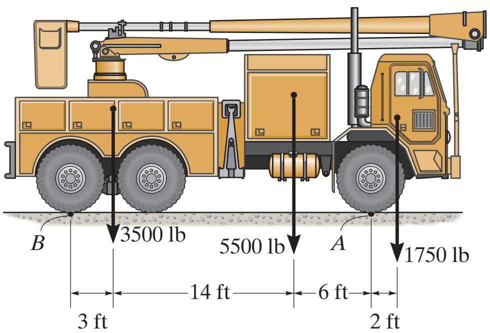


Prob. F4–36

PROBLEMS

4–113. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from B.

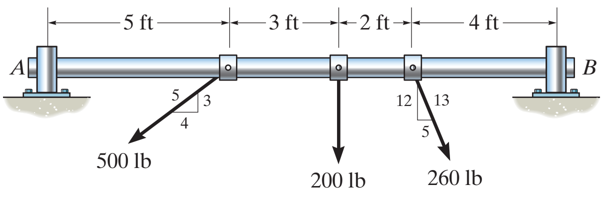
4–114. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point A.



Probs. 4–113/114

4–115. Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end A.

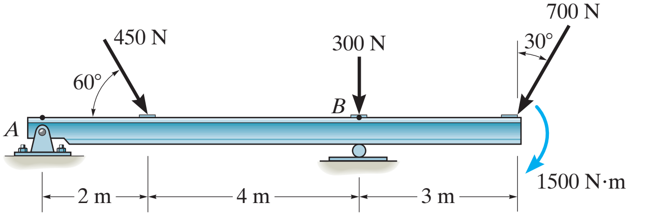
***4–116.** Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end B.



Probs. 4–115/116

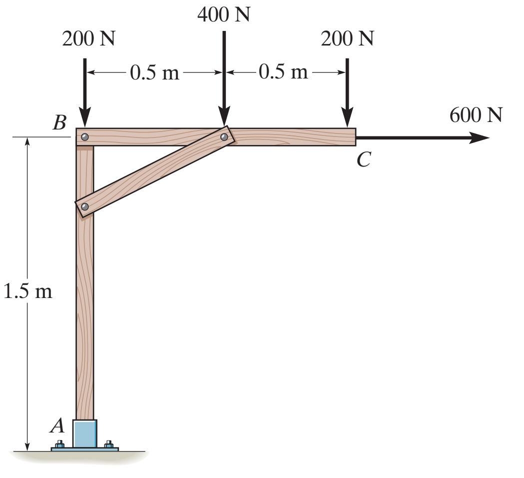
4–117. Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end A.

4–118. Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from B.



Probs. 4–117/118

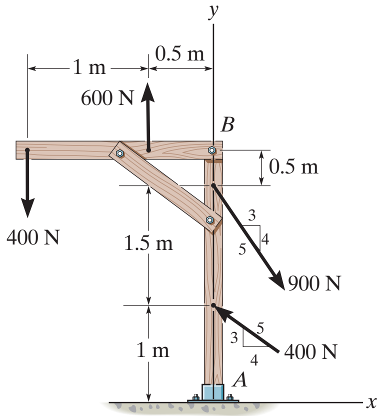
4–119. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.



Prob. 4–119

***4–120.** Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.

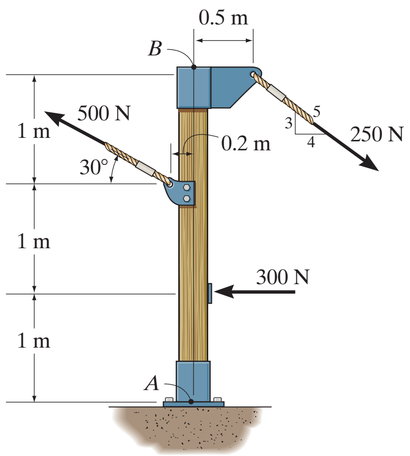
4–121. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a horizontal line along member CB, measured from end C.



Probs. 4–120/121

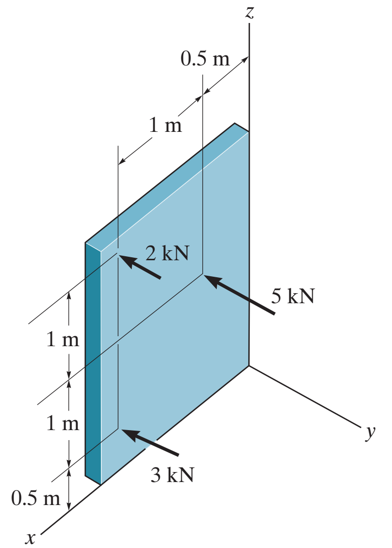
4–122. Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point A.

4–123. Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point B.



Probs. 4–122/123

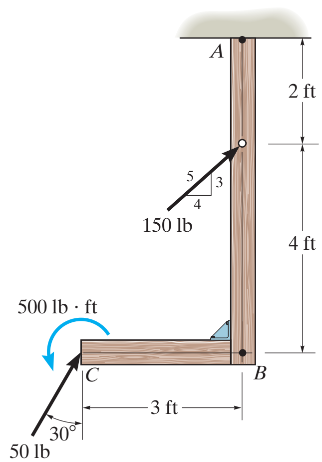
***4–124.** Replace the parallel force system acting on the plate by a resultant force and specify its location on the x-z plane.



Prob. 4–124

4–125. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member AB, measured from A.

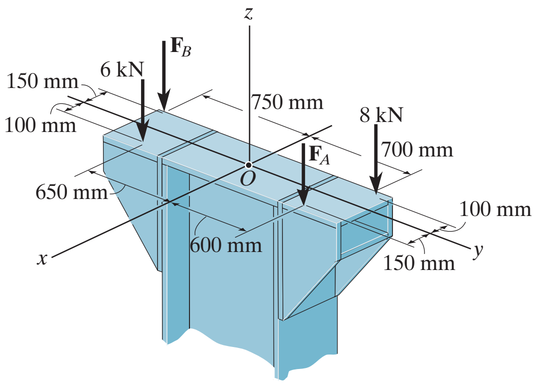
4–126. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member BC, measured from B.



Probs. 4–125/126

4–127. If $F_A = 7$ kN and $F_B = 5$ kN, represent the force system acting on the corbels by a resultant force, and specify its location on the x–y plane.

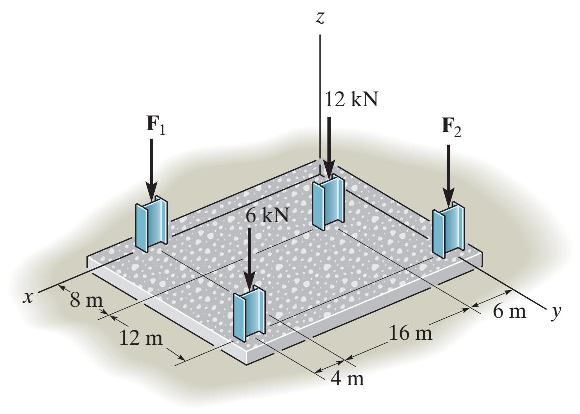
***4–128.** Determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$ so that the resultant force passes through point O of the column.



Probs. 4–127/128

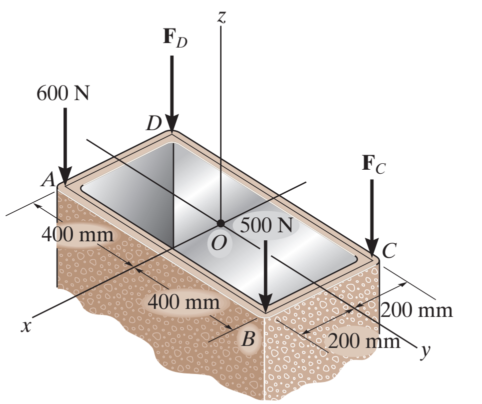
4–130. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 8$ kN and $F_2 = 9$ kN.

4–131. The building slab is subjected to four parallel column loadings. Determine $\mathbf{F}_1$ and $\mathbf{F}_2$ if the resultant force acts through point (12 m, 10 m).



Probs. 4–130/131

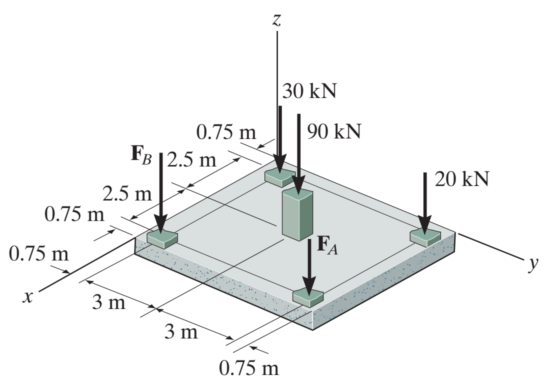
4–129. The tube supports the four parallel forces. Determine the magnitudes of forces $\mathbf{F}_C$ and $\mathbf{F}_D$ acting at C and D so that the equivalent resultant force of the force system acts through the midpoint O of the tube.



Prob. 4–129

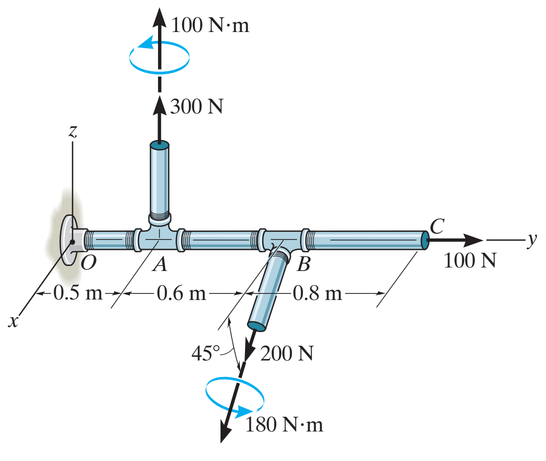
***4–132.** If $F_A = 40$ kN and $F_B = 35$ kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.

4–133. If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $\mathbf{F}_A$ and $\mathbf{F}_B$ and the magnitude of the resultant force.



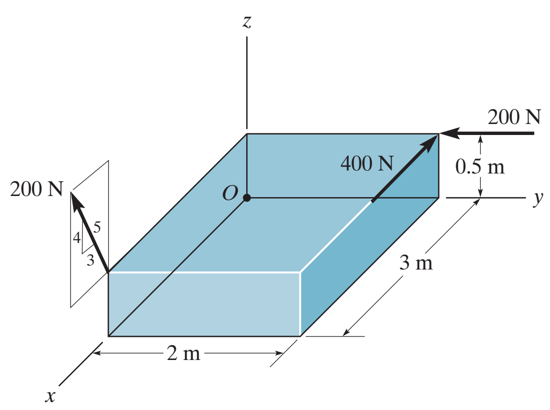
Probs. 4–132/133

4–134. Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point O.



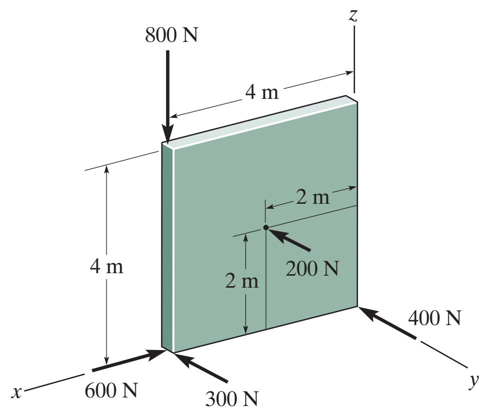
Prob. 4–134

4–135. Replace the force system by a wrench and specify the magnitude of the force and couple moment of the wrench and the point where the wrench intersects the x–z plane.



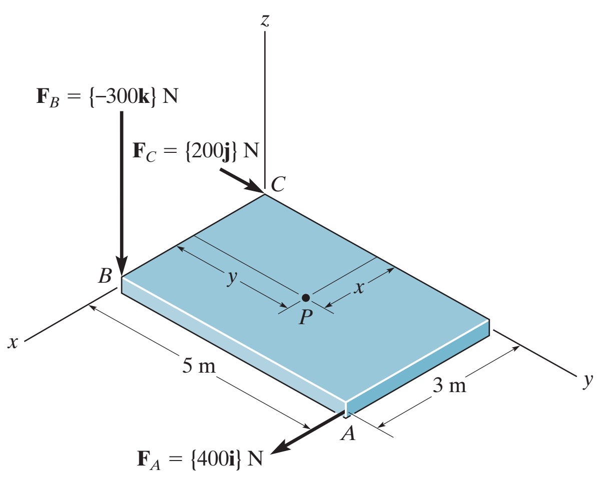
Prob. 4–135

***4–136.** Replace the five forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, z)$ where the wrench intersects the x–z plane.

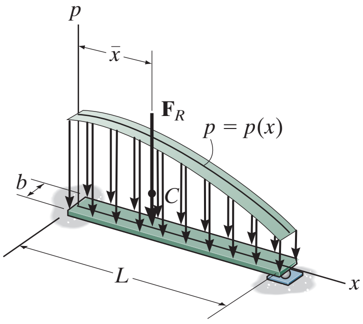


Prob. 4–136

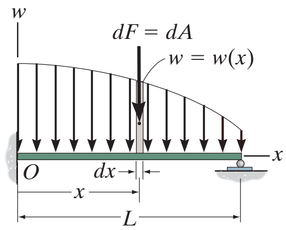
4–137. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where the wrench intersects the plate.



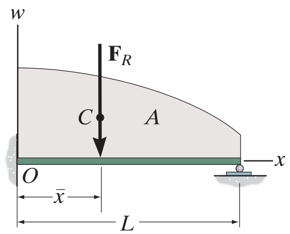
Prob. 4–137



(a)



(b)



(c)

Fig. 4–48

4.9 Reduction of a Simple Distributed Loading

Sometimes, a body may be subjected to a loading that is distributed over its surface. For example, the pressure of the wind on the face of a sign, the pressure of water within a tank, or the weight of sand on the floor of a storage container, are all ***distributed loadings***. The pressure exerted at each point on the surface indicates the intensity of the loading. It is measured using pascals Pa (or N/m^2) in SI units or lb/ft^2 in the U.S. Customary system.

Loading Along a Single Axis. The most common type of distributed loading encountered in engineering practice can be represented along a single axis.* For example, consider the beam (or plate) in Fig. 4–48*a* that has a constant width and is subjected to a pressure loading that varies only along the x axis. This loading can be described by the function $p = p(x)$ N/m^2. It contains only one variable x, and for this reason, we can also represent it as a *coplanar distributed load*. To do so, we multiply the loading function by the width b m of the beam, so that $w(x) = p(x)b$ N/m, Fig. 4–48*b*. Using the methods of Sec. 4.8, we can replace this coplanar parallel force system with a single equivalent resultant force $\mathbf{F}_R$ acting at a specific location on the beam, Fig. 4–48*c*.

Magnitude of Resultant Force. From Eq. 4–17 ($F_R = \Sigma F$), the magnitude of $\mathbf{F}_R$ is equivalent to the sum of all the forces in the system. In this case integration must be used since there is an infinite number of parallel forces $d\mathbf{F}$ acting on the beam, Fig. 4–48*b*. Since $d\mathbf{F}$ is acting on an element of length dx, and $w(x)$ is a force per unit length, then $dF = w(x)\,dx = dA$. In other words, the magnitude of $d\mathbf{F}$ is determined from the colored differential *area* dA under the loading curve. For the entire length L,

$$+\downarrow F_R = \Sigma F; \qquad \boxed{F_R = \int_L w(x)\,dx = \int_A dA = A} \qquad (4\text{–}19)$$

Therefore, the magnitude of the resultant force is equal to the area A under the loading diagram, Fig. 4–48*c*.

*The more general case of a surface loading acting on a body is considered in Sec. 9.5.

Location of Resultant Force. Applying Eq. 4–17 ($M_{R_O} = \Sigma M_O$), the location $\bar{x}$ of the line of action of $\mathbf{F}_R$ can be determined by equating the moments of the force resultant and the parallel force distribution about point O (the y axis). Since $d\mathbf{F}$ produces a moment of $x\,dF = xw(x)\,dx$ about O, Fig. 4–48b, then for the entire length, Fig. 4–48c,

$$\zeta+ (M_R)_O = \Sigma M_O; \qquad -\bar{x}F_R = -\int_L xw(x)\,dx$$

Solving for $\bar{x}$, using Eq. 4–19, we have

$$\boxed{\bar{x} = \frac{\displaystyle\int_L xw(x)\,dx}{\displaystyle\int_L w(x)\,dx} = \frac{\displaystyle\int_A x\,dA}{\displaystyle\int_A dA}} \qquad (4\text{–}20)$$

This coordinate $\bar{x}$, locates the geometric center or ***centroid*** of the *area* under the distributed loading. *In other words, the resultant force has a line of action which passes through the centroid C (geometric center) of the area under the loading diagram*, Fig. 4–48c. Detailed treatment of the integration techniques for finding the location of the centroid for areas is given in Chapter 9. In many cases, however, the distributed-loading diagram is in the shape of a rectangle, triangle, or some other simple geometric form. The centroid location for such common shapes does not have to be determined from the above equation but can be obtained directly from the tabulation given on the inside back cover.

Once $\bar{x}$ is determined, $\mathbf{F}_R$ by symmetry passes through point ($\bar{x}$, 0) on the surface of the beam, Fig. 4–48a. *Therefore, in this case the resultant force has a magnitude equal to the volume under the loading curve $p = p(x)$ and a line of action which passes through the centroid (geometric center) of this volume.*

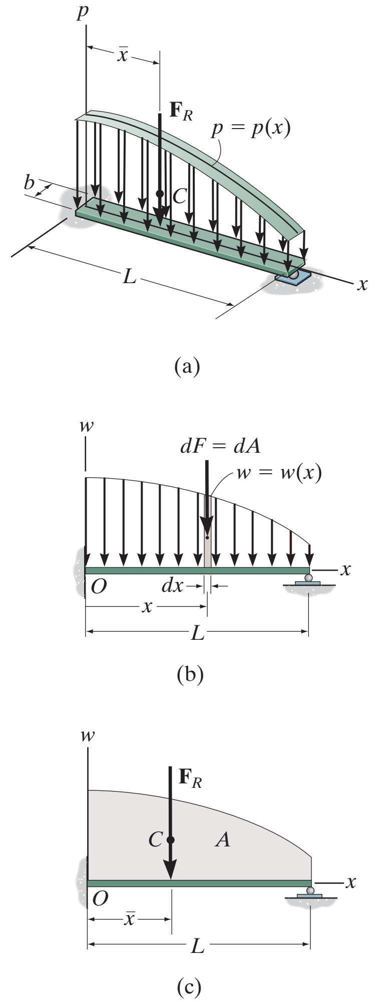
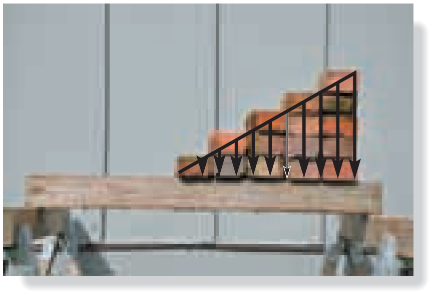


Fig. 4–48 (Repeated)

Important Points

- Coplanar distributed loadings are defined by using a loading function $w = w(x)$ that indicates the intensity of the loading along the length of a member. This intensity is measured in N/m or lb/ft.
- The external effects caused by a coplanar distributed load acting on a body can be represented by a single resultant force.
- This resultant force is equivalent to the *area* under the loading diagram, and has a line of action that passes through the *centroid* or geometric center of this area.



The pile of brick creates an approximate triangular distributed loading on the board. (© Russell C. Hibbeler)

EXAMPLE 4.21

Determine the magnitude and location of the equivalent resultant force acting on the shaft in Fig. 4–49*a*.

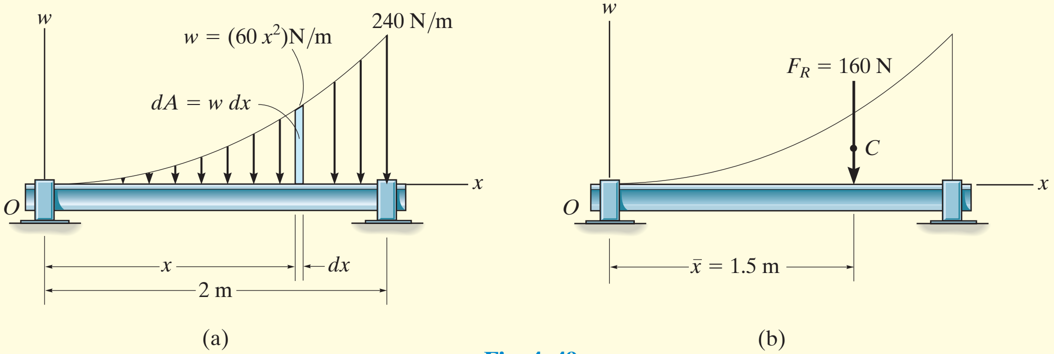


Fig. 4–49

SOLUTION

Since $w = w(x)$ is given, this problem will be solved by integration.

The differential element has an area $dA = w\,dx = 60x^2\,dx$. Applying Eq. 4–19,

$+\downarrow F_R = \Sigma F;$

$$F_R = \int_A dA = \int_0^{2\text{ m}} 60x^2\,dx = 60\left(\frac{x^3}{3}\right)\Big|_0^{2\text{ m}} = 60\left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

$$= 160\text{ N}$$ *Ans.*

The location $\bar{x}$ of $\mathbf{F}_R$ measured from O, Fig. 4–49*b*, is determined from Eq. 4–20.

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^{2\text{ m}} x(60x^2)\,dx}{160\text{ N}} = \frac{60\left(\frac{x^4}{4}\right)\Big|_0^{2\text{ m}}}{160\text{ N}} = \frac{60\left(\frac{2^4}{4} - \frac{0^4}{4}\right)}{160\text{ N}}$$

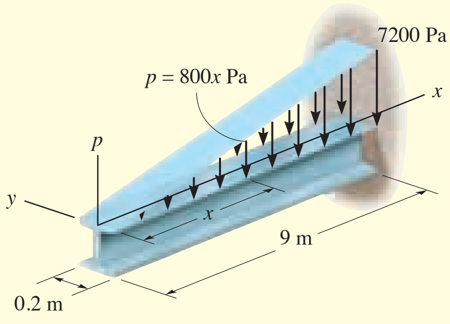
$$= 1.5\text{ m}$$ *Ans.*

NOTE: These results can be checked by using the table on the inside back cover, where it is shown that the formula for an exparabolic area of length a, height b, and shape shown in Fig. 4–49*a*, is

$$A = \frac{ab}{3} = \frac{2\text{ m}(240\text{ N/m})}{3} = 160\text{ N and } \bar{x} = \frac{3}{4}a = \frac{3}{4}(2\text{ m}) = 1.5\text{ m}$$

EXAMPLE 4.22

A distributed loading of $p = (800x)$ Pa acts over the top surface of the beam shown in Fig. 4–50*a*. Determine the magnitude and location of the equivalent resultant force.

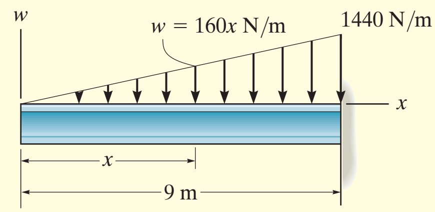


(a)

SOLUTION

Since the loading intensity is uniform along the width of the beam (the y axis), the loading can be viewed in two dimensions as shown in Fig. 4–50*b*. Here

$$w = (800x\ \mathrm{N/m^2})(0.2\ \mathrm{m})$$
$$= (160x)\ \mathrm{N/m}$$



(b)

At $x = 9$ m, note that $w = 1440$ N/m. Although we may again apply Eqs. 4–19 and 4–20 as in the previous example, it is simpler to use the table on the inside back cover.

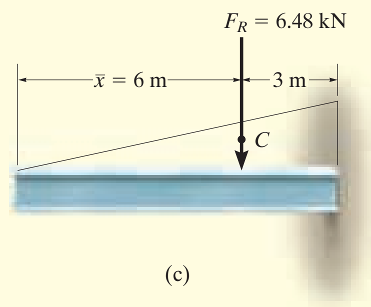
The magnitude of the resultant force is equivalent to the area of the triangle.

$$F_R = \tfrac{1}{2}(9\ \mathrm{m})(1440\ \mathrm{N/m}) = 6480\ \mathrm{N} = 6.48\ \mathrm{kN} \qquad \textit{Ans.}$$

The line of action of $\mathbf{F}_R$ passes through the *centroid* C of this triangle. Hence,

$$\bar{x} = 9\ \mathrm{m} - \tfrac{1}{3}(9\ \mathrm{m}) = 6\ \mathrm{m} \qquad \textit{Ans.}$$

The results are shown in Fig. 4–50*c*.



(c)

NOTE: We may also view the resultant $\mathbf{F}_R$ as *acting* through the *centroid* of the *volume* of the loading diagram $p = p(x)$ in Fig. 4–50*a*. Hence $\mathbf{F}_R$ intersects the x–y plane at the point (6 m, 0). Furthermore, the magnitude of $\mathbf{F}_R$ is equal to the volume under the loading diagram; i.e.,

$$F_R = V = \tfrac{1}{2}(7200\ \mathrm{N/m^2})(9\ \mathrm{m})(0.2\ \mathrm{m}) = 6.48\ \mathrm{kN} \qquad \textit{Ans.}$$

Fig. 4–50

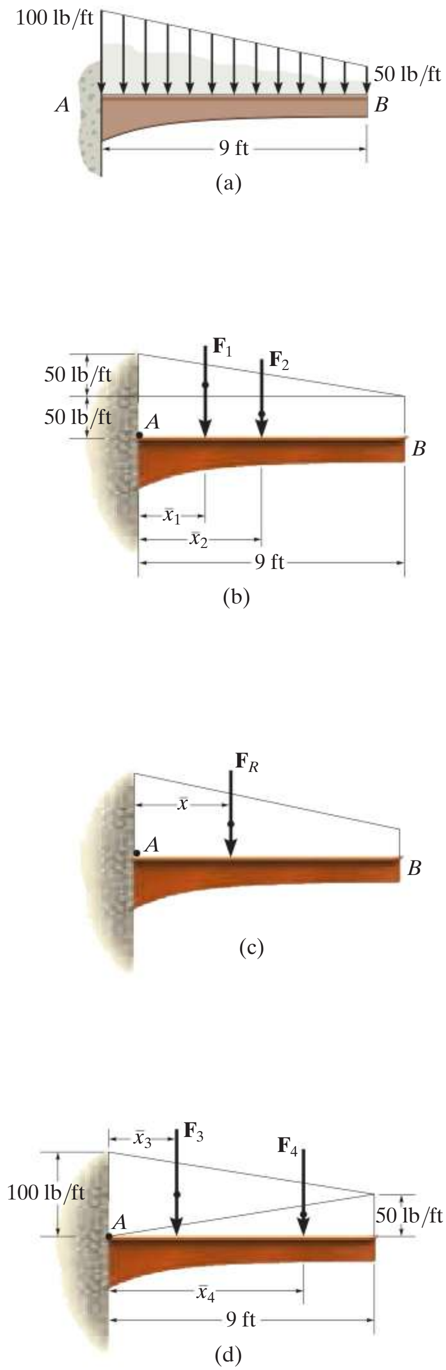


Fig. 4–51

The granular material exerts the distributed loading on the beam as shown in Fig. 4–51*a*. Determine the magnitude and location of the equivalent resultant of this load.

SOLUTION

The area of the loading diagram is a *trapezoid*, and therefore the solution can be obtained directly from the area and centroid formulas for a trapezoid listed on the inside back cover. Since these formulas are not easily remembered, instead we will solve this problem by using "composite" areas. Here we will divide the trapezoidal loading into a rectangular and triangular loading as shown in Fig. 4–51*b*. The magnitude of the force represented by each of these loadings is equal to its associated *area*,

$$F_1 = \tfrac{1}{2}(9 \text{ ft})(50 \text{ lb/ft}) = 225 \text{ lb}$$

$$F_2 = (9 \text{ ft})(50 \text{ lb/ft}) = 450 \text{ lb}$$

The lines of action of these parallel forces act through the respective *centroids* of their associated areas and therefore intersect the beam at

$$\bar{x}_1 = \tfrac{1}{3}(9 \text{ ft}) = 3 \text{ ft}$$

$$\bar{x}_2 = \tfrac{1}{2}(9 \text{ ft}) = 4.5 \text{ ft}$$

The two parallel forces $\mathbf{F}_1$ and $\mathbf{F}_2$ can be reduced to a single resultant $\mathbf{F}_R$. The magnitude of $\mathbf{F}_R$ is

$$+\downarrow F_R = \Sigma F; \qquad F_R = 225 + 450 = 675 \text{ lb} \qquad \textit{Ans.}$$

We can find the location of $\mathbf{F}_R$ with reference to point A, Figs. 4–51*b* and 4–51*c*. We require

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad \bar{x}(675) = 3(225) + 4.5(450)$$

$$\bar{x} = 4 \text{ ft} \qquad \textit{Ans.}$$

NOTE: The trapezoidal area in Fig. 4–51*a* can also be divided into two triangular areas as shown in Fig. 4–51*d*. In this case

$$F_3 = \tfrac{1}{2}(9 \text{ ft})(100 \text{ lb/ft}) = 450 \text{ lb}$$

$$F_4 = \tfrac{1}{2}(9 \text{ ft})(50 \text{ lb/ft}) = 225 \text{ lb}$$

and

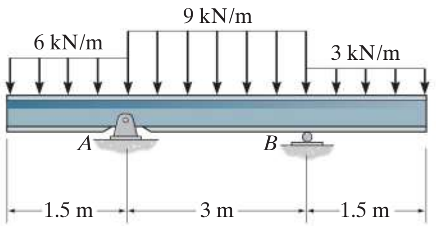
$$\bar{x}_3 = \tfrac{1}{3}(9 \text{ ft}) = 3 \text{ ft}$$

$$\bar{x}_4 = 9 \text{ ft} - \tfrac{1}{3}(9 \text{ ft}) = 6 \text{ ft}$$

Using these results, show that again $F_R = 675$ lb and $\bar{x} = 4$ ft.

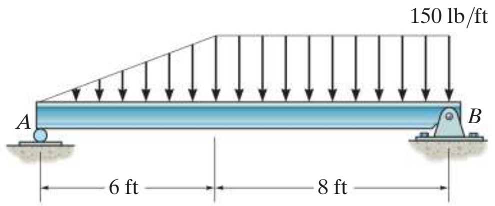
FUNDAMENTAL PROBLEMS

F4–37. Determine the resultant force and specify where it acts on the beam measured from A.



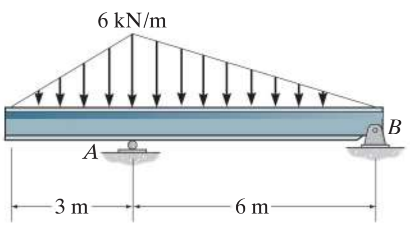
Prob. F4–37

F4–38. Determine the resultant force and specify where it acts on the beam measured from A.



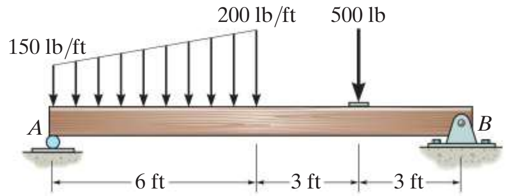
Prob. F4–38

F4–39. Determine the resultant force and specify where it acts on the beam measured from A.



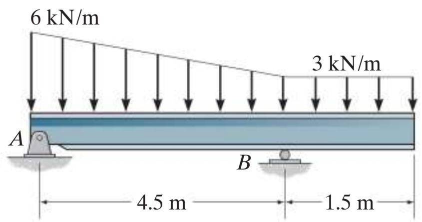
Prob. F4–39

F4–40. Determine the resultant force and specify where it acts on the beam measured from A.



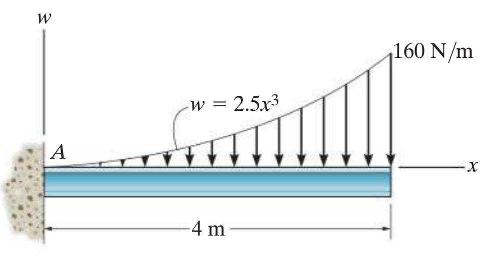
Prob. F4–40

F4–41. Determine the resultant force and specify where it acts on the beam measured from A.



Prob. F4–41

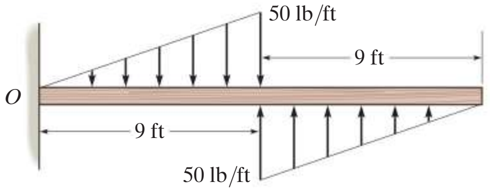
F4–42. Determine the resultant force and specify where it acts on the beam measured from A.



Prob. F4–42

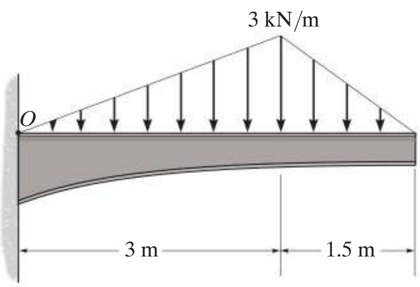
PROBLEMS

4–138. Replace the loading by an equivalent resultant force and couple moment acting at point O.



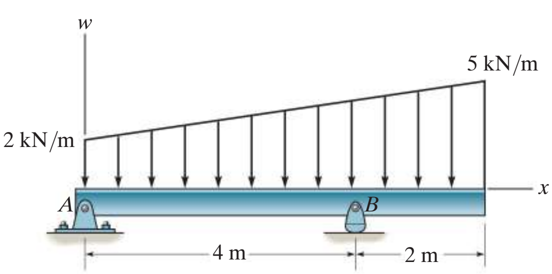
Prob. 4–138

4–139. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point O.



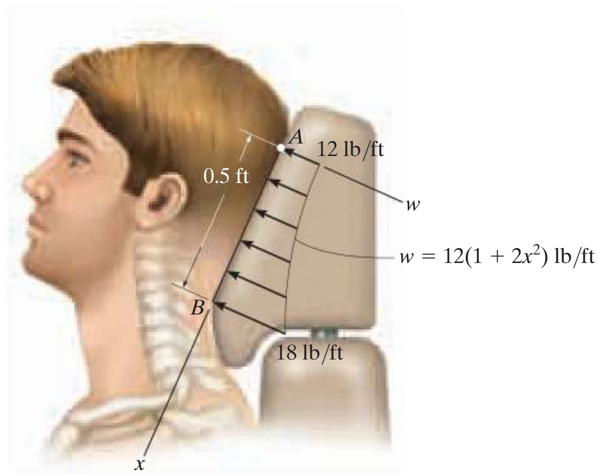
Prob. 4–139

***4–140.** Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point A.



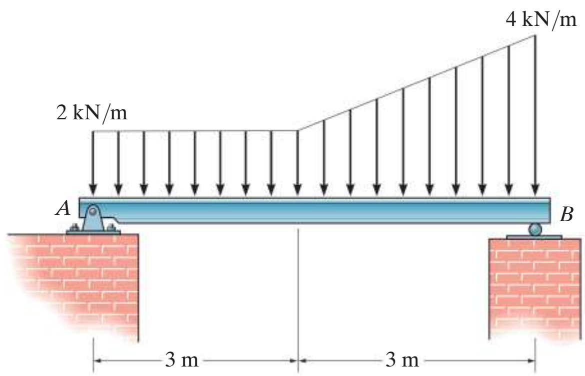
Prob. 4–140

4–141. Currently eighty-five percent of all neck injuries are caused by rear-end car collisions. To alleviate this problem, an automobile seat restraint has been developed that provides additional pressure contact with the cranium. During dynamic tests the distribution of load on the cranium has been plotted and shown to be parabolic. Determine the equivalent resultant force and its location, measured from point A.



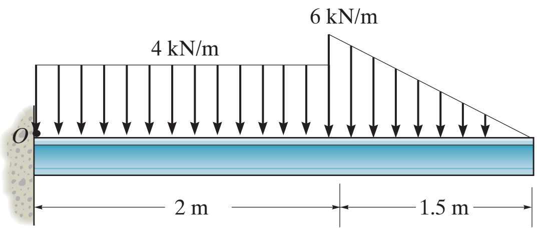
Prob. 4–141

4–142. Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at A.



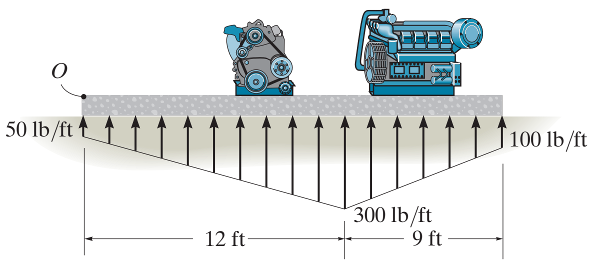
Prob. 4–142

4–143. Replace this loading by an equivalent resultant force and specify its location, measured from point O.



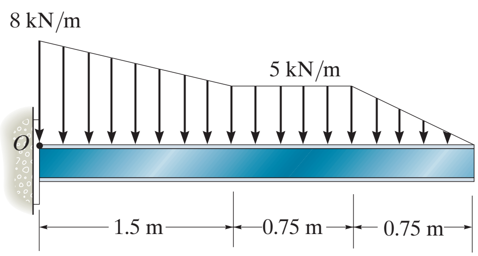
Prob. 4–143

***4–144.** The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.



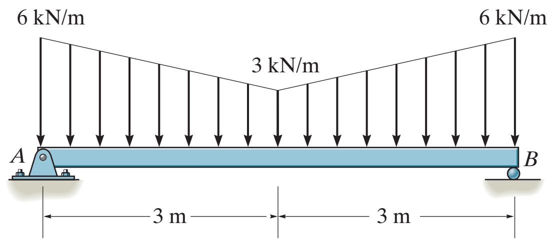
Prob. 4–144

4–145. Replace the loading by an equivalent resultant force and couple moment acting at point O.



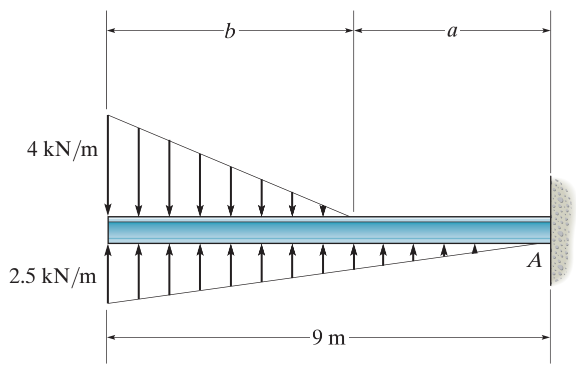
Prob. 4–145

4–146. Replace the distributed loading by an equivalent resultant force and couple moment acting at point A.



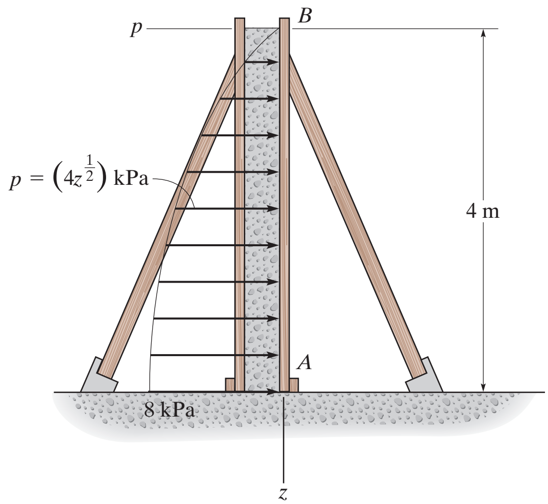
Prob. 4–146

4–147. Determine the length b of the triangular load and its position a on the beam such that the equivalent resultant force is zero and the resultant couple moment is 8 kN · m clockwise.



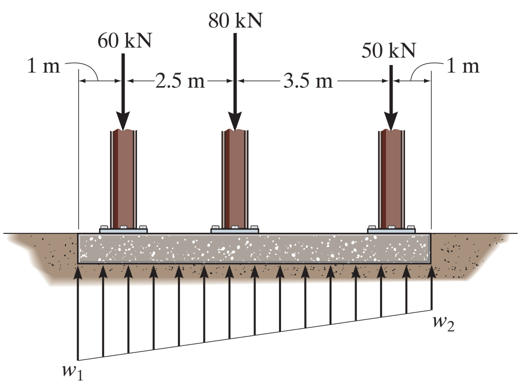
Prob. 4–147

*4–148. The form is used to cast a concrete wall having a width of 5 m. Determine the equivalent resultant force the wet concrete exerts on the form AB if the pressure distribution due to the concrete can be approximated as shown. Specify the location of the resultant force, measured from point B.



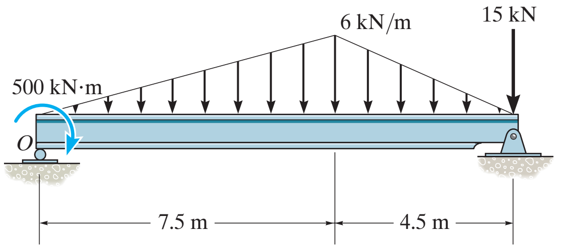
Prob. 4–148

4–149. If the soil exerts a trapezoidal distribution of load on the bottom of the footing, determine the intensities w_1 and w_2 of this distribution needed to support the column loadings.



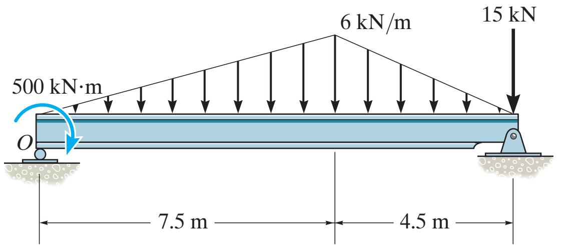
Prob. 4–149

4–150. Replace the loading by an equivalent force and couple moment acting at point O.



Prob. 4–150

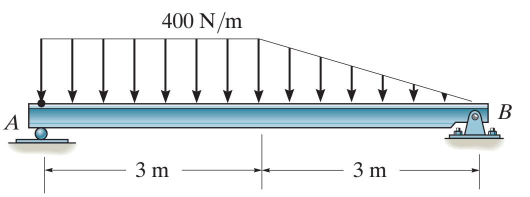
4–151. Replace the loading by a single resultant force, and specify the location of the force measured from point O.



Prob. 4–151

*4–152. Replace the loading by an equivalent resultant force and couple moment acting at point A.

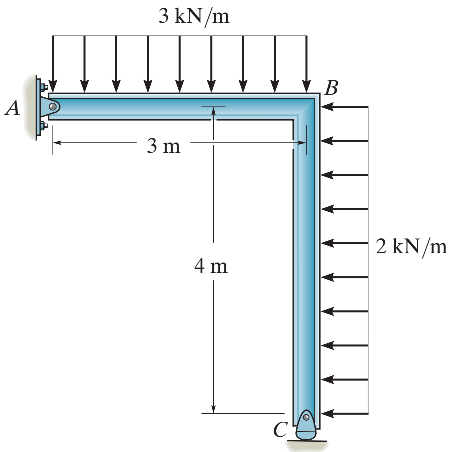
4–153. Replace the loading by a single resultant force, and specify its location on the beam measured from point A.



Probs. 4–152/153

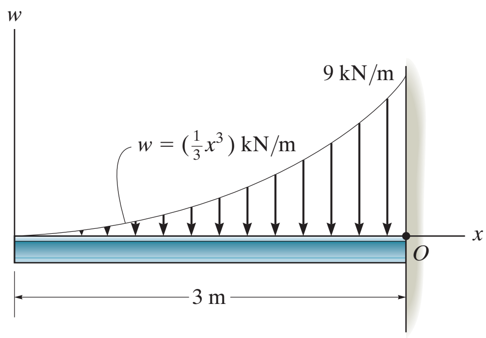
4–154. Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a horizontal line along member AB, measured from A.

4–155. Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a vertical line along member BC, measured from C.



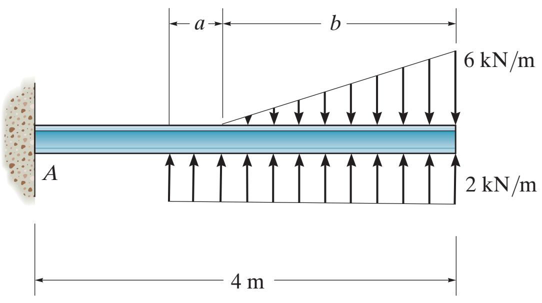
Probs. 4–154/155

4–157. Determine the equivalent resultant force and couple moment at point O.



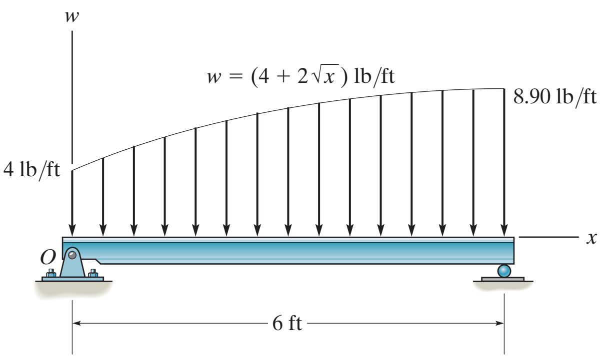
Prob. 4–157

***4–156.** Determine the length b of the triangular load and its position a on the beam such that the equivalent resultant force is zero and the resultant couple moment is 8 kN · m clockwise.



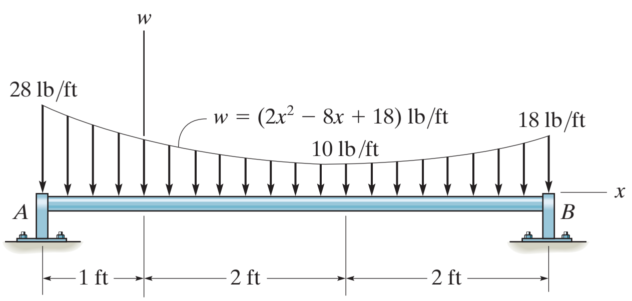
Prob. 4–156

4–158. Determine the magnitude of the equivalent resultant force and its location, measured from point O.



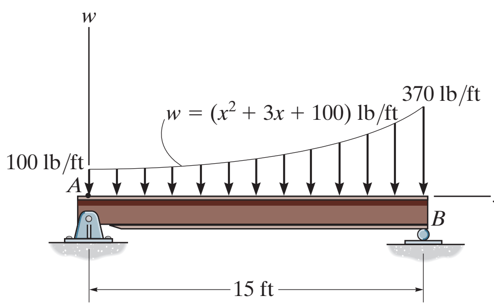
Prob. 4–158

4–159. The distributed load acts on the shaft as shown. Determine the magnitude of the equivalent resultant force and specify its location, measured from the support, A.



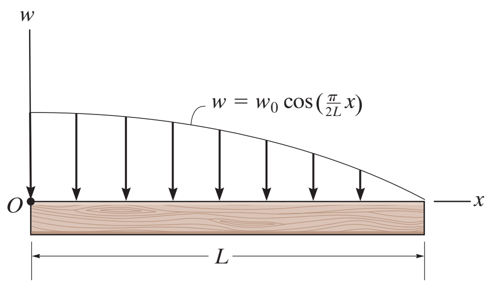
Prob. 4–159

***4–160.** Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



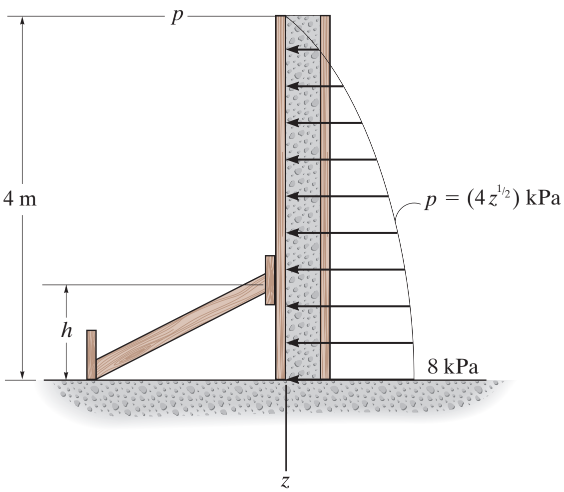
Prob. 4–160

4–161. Replace the loading by an equivalent resultant force and couple moment acting at point O.



Prob. 4–161

4–162. Wet concrete exerts a pressure distribution along the wall of the form. Determine the resultant force of this distribution and specify the height h where the bracing strut should be placed so that it lies through the line of action of the resultant force. The wall has a width of 5 m.



Prob. 4–162

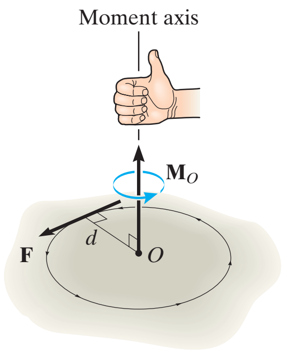
CHAPTER REVIEW

Moment of Force—Scalar Definition

A force produces a turning effect or moment about a point O that does not lie on its line of action. In scalar form, the moment *magnitude* is the product of the force and the moment arm or perpendicular distance from point O to the line of action of the force.

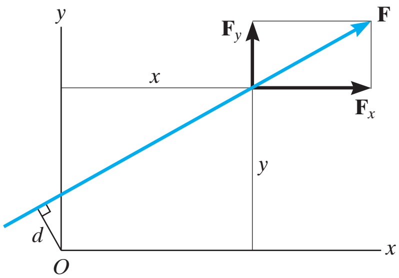
The *direction* of the moment is defined using the right-hand rule. $\mathbf{M}_O$ always acts along an axis perpendicular to the plane containing $\mathbf{F}$ and d, and passes through the point O.

$$M_O = Fd$$



Rather than finding d, it is normally easier to resolve the force into its x and y components, determine the moment of each component about the point, and then sum the results. This is called the principle of moments.

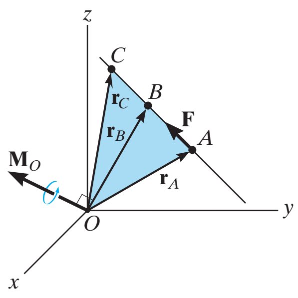
$$M_O = Fd = F_x y - F_y x$$



Moment of a Force—Vector Definition

Since three-dimensional geometry is generally more difficult to visualize, the vector cross product should be used to determine the moment. Here $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is a position vector that extends from point O to any point A, B, or C on the line of action of $\mathbf{F}$.

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \mathbf{r}_B \times \mathbf{F} = \mathbf{r}_C \times \mathbf{F}$$



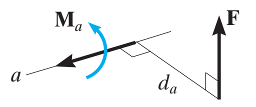
If the position vector $\mathbf{r}$ and force $\mathbf{F}$ are expressed as Cartesian vectors, then the cross product results from the expansion of a determinant.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Moment about an Axis

If the moment of a force **F** is to be determined about an arbitrary axis a, then for a scalar solution the moment arm, or shortest distance d_a from the line of action of the force to the axis must be used. This distance is perpendicular to both the axis and the force line of action.

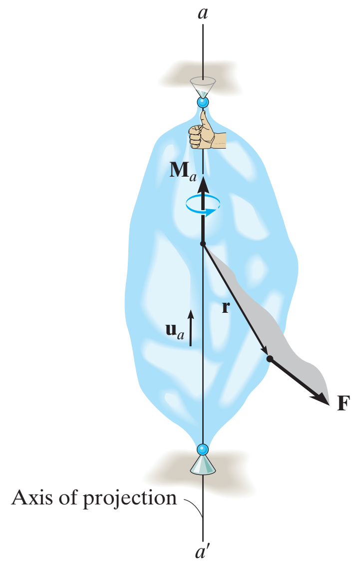
$$M_a = Fd_a$$



Note that when the line of action of **F** intersects the axis, then the moment of **F** about the axis is zero. Also, when the line of action of **F** is parallel to the axis, the moment of **F** about the axis is zero.

In three dimensions, the scalar triple product should be used. Here $\mathbf{u}_a$ is the unit vector that specifies the direction of the axis, and **r** is a position vector that is directed from any point on the axis to any point on the line of action of the force. If M_a is calculated as a negative scalar, then the sense of direction of $\mathbf{M}_a$ is opposite to $\mathbf{u}_a$.

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

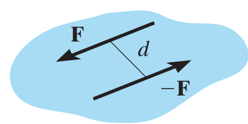


Couple Moment

A couple consists of two equal but opposite forces that act a perpendicular distance d apart. Couples tend to produce a rotation without translation.

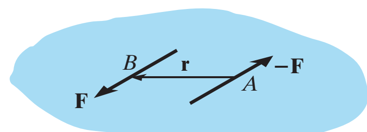
The magnitude of the couple moment is $M = Fd$, and its direction is established using the right-hand rule.

$$M = Fd$$



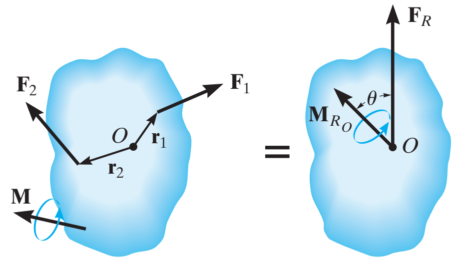
If the vector cross product is used to determine the moment of a couple, then **r** extends from any point on the line of action of one of the forces to any point on the line of action of the other force **F** that is used in the cross product.

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

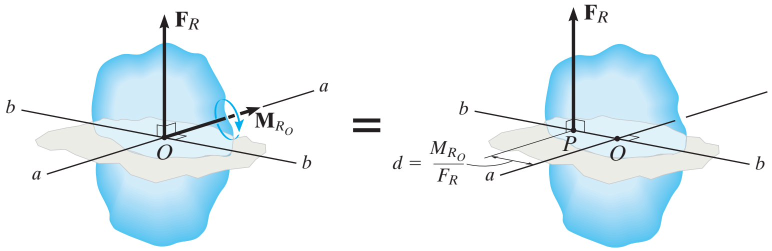


Simplification of a Force and Couple System

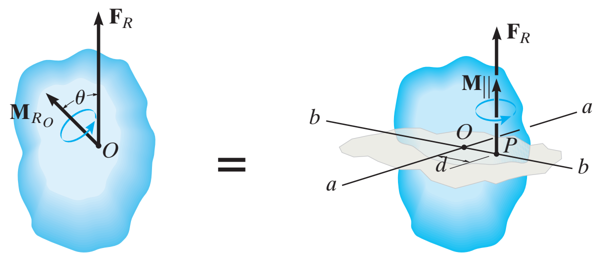
Any system of forces and couples can be reduced to a single resultant force and resultant couple moment acting at a point. The resultant force is the sum of all the forces in the system, $\mathbf{F}_R = \Sigma\mathbf{F}$, and the resultant couple moment is equal to the sum of all the moments of the forces about the point and couple moments. $\mathbf{M}_{R_O} = \Sigma\mathbf{M}_O + \Sigma\mathbf{M}$.



Further simplification to a single resultant force is possible provided the force system is concurrent, coplanar, or parallel. To find the location of the resultant force from a point, it is necessary to equate the moment of the resultant force about the point to the moment of the forces and couples in the system about the same point.

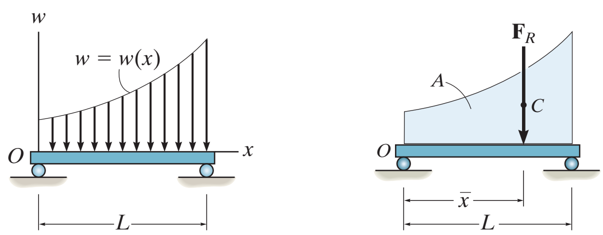


If the resultant force and couple moment at a point are not perpendicular to one another, then this system can be reduced to a wrench, which consists of the resultant force and collinear couple moment.



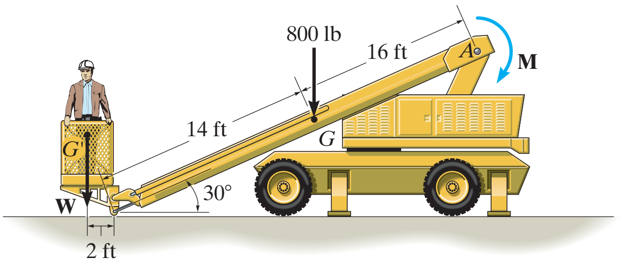
Coplanar Distributed Loading

A simple distributed loading can be represented by its resultant force, which is equivalent to the *area* under the loading curve. This resultant has a line of action that passes through the *centroid* or geometric center of the area or volume under the loading diagram.



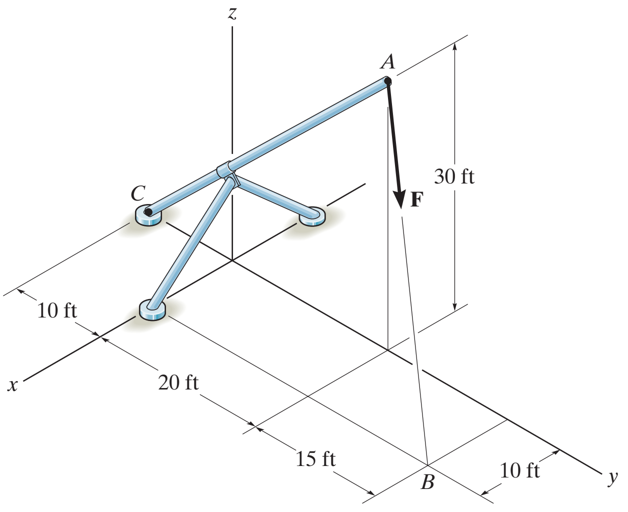
REVIEW PROBLEMS

R4–1. The boom has a length of 30 ft, a weight of 800 lb, and mass center at G. If the maximum moment that can be developed by a motor at A is $M = 20(10^3)$ lb · ft, determine the maximum load W, having a mass center at G', that can be lifted.



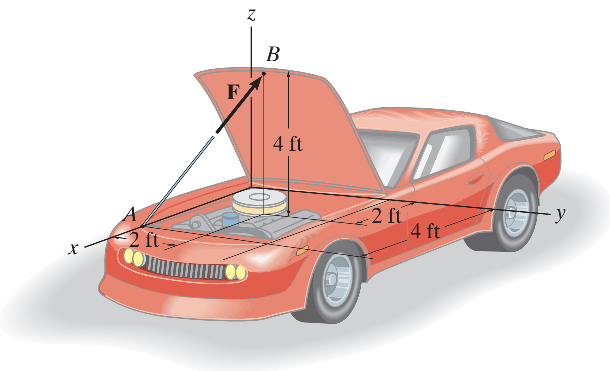
Prob. R4–1

R4–2. Replace the force **F** having a magnitude of $F = 50$ lb and acting at point A by an equivalent force and couple moment at point C.



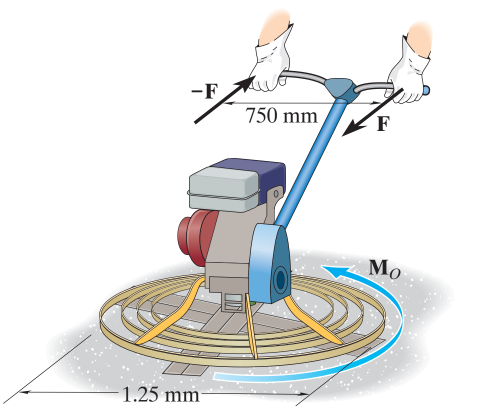
Prob. R4–2

R4–3. The hood of the automobile is supported by the strut AB, which exerts a force of $F = 24$ lb on the hood. Determine the moment of this force about the hinged axis y.



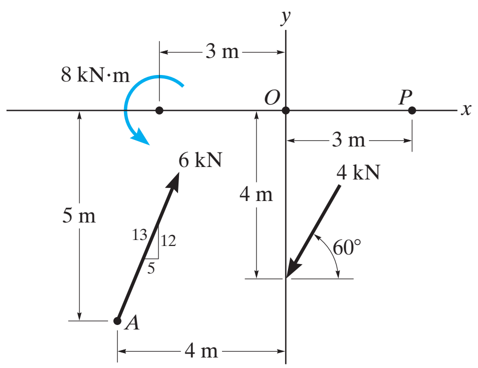
Prob. R4–3

R4–4. Friction on the concrete surface creates a couple moment of $M_O = 100$ N · m on the blades of the trowel. Determine the magnitude of the couple forces so that the resultant couple moment on the trowel is zero. The forces lie in the horizontal plane and act perpendicular to the handle of the trowel.



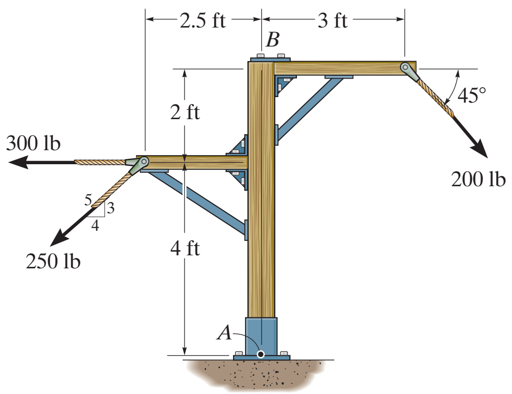
Prob. R4–4

R4–5. Replace the force and couple system by an equivalent force and couple moment at point P.



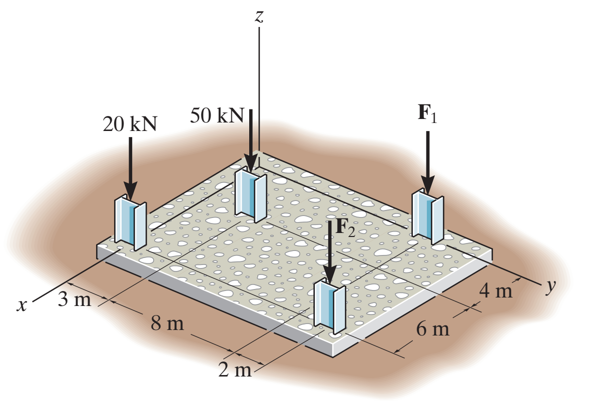
Prob. R4–5

R4–6. Replace the force system acting on the frame by a resultant force, and specify where its line of action intersects member AB, measured from point A.



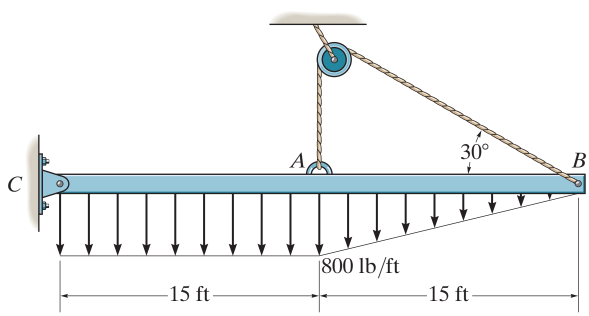
Prob. R4–6

R4–7. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 30$ kN, $F_2 = 40$ kN.



Prob. R4–7

R4–8. Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at C.



Prob. R4–8

Chapter 5

It is important to be able to determine the forces in the cables used to support this boom to ensure that it does not fail. In this chapter we will study how to apply equilibrium methods to determine the forces acting on the supports of a rigid body such as this.

Equilibrium of a Rigid Body

CHAPTER OBJECTIVES

- To develop the equations of equilibrium for a rigid body.
- To introduce the concept of the free-body diagram for a rigid body.
- To show how to solve rigid-body equilibrium problems using the equations of equilibrium.

5.1 Conditions for Rigid-Body Equilibrium

In this section, we will develop both the necessary and sufficient conditions for the equilibrium of the rigid body in Fig. 5–1*a*. As shown, this body is subjected to an external force and couple moment system that is the result of the effects of gravitational, electrical, magnetic, or contact forces caused by adjacent bodies. The internal forces caused by interactions between particles within the body are not shown in this figure because these forces occur in equal but opposite collinear pairs and hence will cancel out, a consequence of Newton's third law.

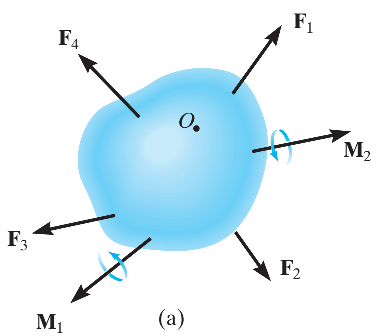
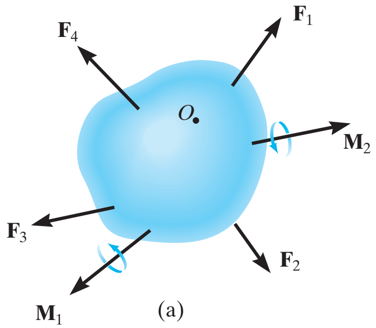
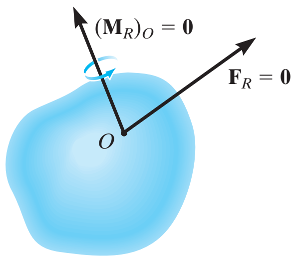


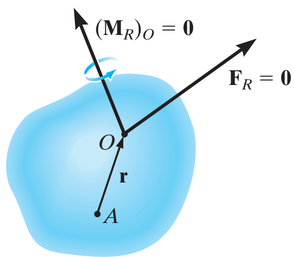
Fig. 5–1



(a)



(b)



(c)

Fig. 5–1 (cont.)

Using the methods of the previous chapter, the force and couple moment system acting on a body can be reduced to an equivalent resultant force and resultant couple moment at any arbitrary point O on or off the body, Fig. 5–1*b*. If this resultant force and couple moment are both equal to zero, then the body is said to be in ***equilibrium***. Mathematically, the equilibrium of a body is expressed as

$$\mathbf{F}_R = \Sigma \mathbf{F} = \mathbf{0}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O = \mathbf{0} \tag{5–1}$$

The first of these equations states that the sum of the forces acting on the body is equal to *zero*. The second equation states that the sum of the moments of all the forces in the system about point O, added to all the couple moments, is equal to *zero*. These two equations are not only necessary for equilibrium, they are also sufficient. To show this, consider summing moments about some other point, such as point A in Fig. 5–1*c*. We require

$$\Sigma \mathbf{M}_A = \mathbf{r} \times \mathbf{F}_R + (\mathbf{M}_R)_O = \mathbf{0}$$

Since $\mathbf{r} \neq \mathbf{0}$, this equation is satisfied if Eqs. 5–1 are satisfied, namely $\mathbf{F}_R = \mathbf{0}$ and $(\mathbf{M}_R)_O = \mathbf{0}$.

When applying the equations of equilibrium, we will assume that the body remains rigid. In reality, however, all bodies deform when subjected to loads. Although this is the case, most engineering materials such as steel and concrete are very rigid and so their deformation is usually very small. Therefore, when applying the equations of equilibrium, we can generally assume that the body will remain *rigid* and *not deform* under the applied load without introducing any significant error. This way the direction of the applied forces and their moment arms with respect to a fixed reference remain the same both before and after the body is loaded.

EQUILIBRIUM IN TWO DIMENSIONS

In the first part of the chapter, we will consider the case where the force system acting on a rigid body lies in or may be projected onto a *single* plane and, furthermore, any couple moments acting on the body are directed perpendicular to this plane. This type of force and couple system is often referred to as a two-dimensional or *coplanar* force system. For example, the airplane in Fig. 5–2 has a plane of symmetry through its center axis, and so the loads acting on the airplane are symmetrical with respect to this plane. Thus, each of the two wing tires will support the same load **T**, which is represented on the side (two-dimensional) view of the plane as 2**T**.

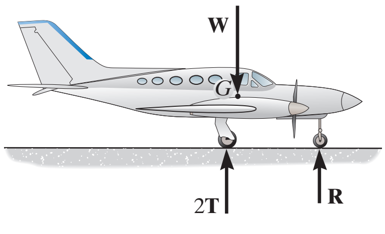


Fig. 5–2

5.2 Free-Body Diagrams

Successful application of the equations of equilibrium requires a complete specification of *all* the known and unknown external forces that act *on* the body. The best way to account for these forces is to draw a ***free-body diagram***. This diagram is a sketch of the outlined shape of the body, which represents it as being *isolated* or "free" from its surroundings, i.e., a "free body." On this sketch it is necessary to show *all* the forces and couple moments that the surroundings exert *on the body* so that these effects can be accounted for when the equations of equilibrium are applied. *A thorough understanding of how to draw a free-body diagram is of primary importance for solving problems in mechanics.*

Support Reactions. Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various types of reactions that occur at supports and points of contact between bodies subjected to coplanar force systems. As a general rule,

- A support prevents the translation of a body in a given direction by exerting a force on the body in the opposite direction.
- A support prevents the rotation of a body in a given direction by exerting a couple moment on the body in the opposite direction.

For example, let us consider three ways in which a horizontal member, such as a beam, is supported at its end. One method consists of a *roller* or cylinder, Fig. 5–3*a*. Since this support only prevents the beam from *translating* in the vertical direction, the roller will only exert a *force* on the beam in this direction, Fig. 5–3*b*.

The beam can be supported in a more restrictive manner by using a *pin*, Fig. 5–3*c*. The pin passes through a hole in the beam and two leaves which are fixed to the ground. Here the pin can prevent *translation* of the beam in *any direction* ϕ, Fig. 5–3*d*, and so the pin must exert a *force* **F** on the beam in the opposite direction. For purposes of analysis, it is generally easier to represent this resultant force **F** by its two rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$, Fig. 5–3*e*. If F_x and F_y are known, then F and ϕ can be calculated.

The most restrictive way to support the beam would be to use a *fixed support* as shown in Fig. 5–3*f*. This support will prevent both *translation and rotation* of the beam. To do this a *force and couple moment* must be developed on the beam at its point of connection, Fig. 5–3*g*. As in the case of the pin, the force is usually represented by its rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$.

Table 5–1 lists other common types of supports for bodies subjected to coplanar force systems. (In all cases the angle θ is assumed to be known.) Carefully study each of the symbols used to represent these supports and the types of reactions they exert on their contacting members.

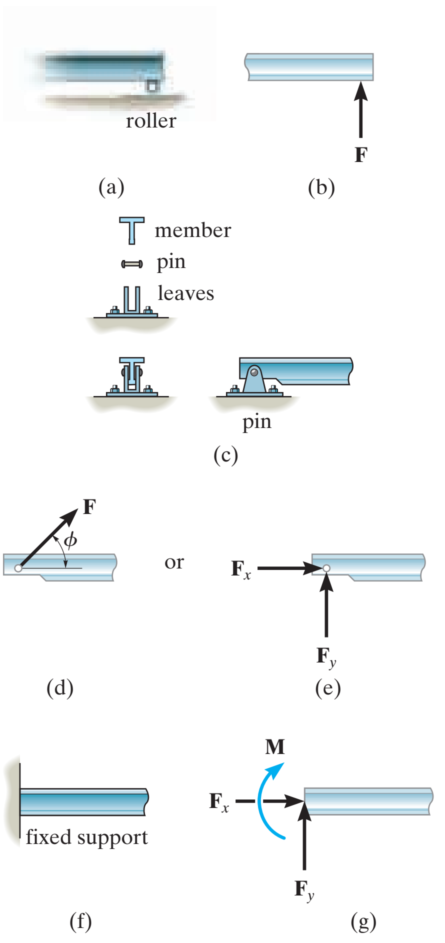
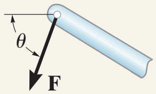
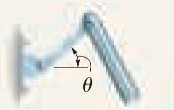
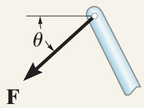
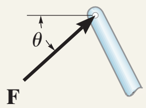
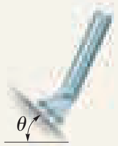
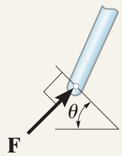
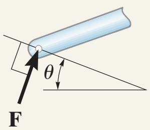
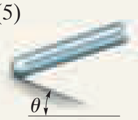
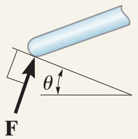
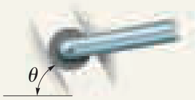
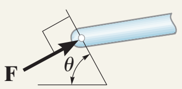
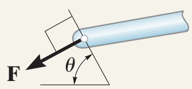
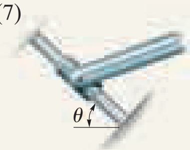
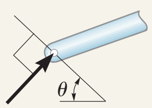
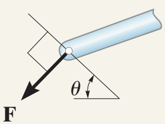


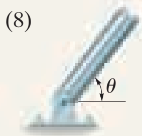
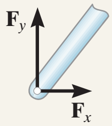
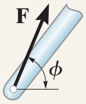
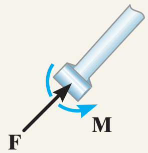
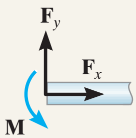
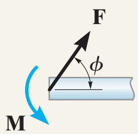
Fig. 5–3

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

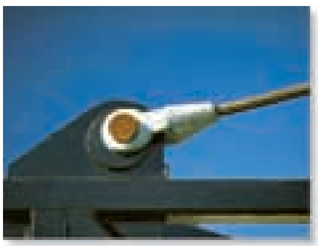
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

continued

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge	 or 	Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support	 or 	Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Typical examples of actual supports are shown in the following sequence of photos. The numbers refer to the connection types in Table 5–1.


(© Russell C. Hibbeler)

The cable exerts a force on the bracket in the direction of the cable. (1)

The rocker support for this bridge girder allows horizontal movement so the bridge is free to expand and contract due to a change in temperature. (4) (© Russell C. Hibbeler)

This concrete girder rests on the ledge that is assumed to act as a smooth contacting surface. (5) (© Russell C. Hibbeler)

Typical pin support for a beam. (8) (© Russell C. Hibbeler)

The floor beams of this building are welded together and thus form fixed connections. (10) (© Russell C. Hibbeler)

Internal Forces. As stated in Sec. 5.1, the internal forces that act between adjacent particles in a body always occur in collinear pairs such that they have the same magnitude and act in opposite directions (Newton's third law). Since these forces cancel each other, they will not create an *external effect* on the body. It is for this reason that the internal forces should not be included on the free-body diagram if the entire body is to be considered. For example, the engine shown in Fig. 5–4*a* has a free-body diagram shown in Fig. 5–4*b*. The internal forces between all its connected parts, such as the screws and bolts, will cancel out because they form equal and opposite collinear pairs. Only the external forces $\mathbf{T}_1$ and $\mathbf{T}_2$, exerted by the chains and the engine weight **W**, are shown on the free-body diagram.

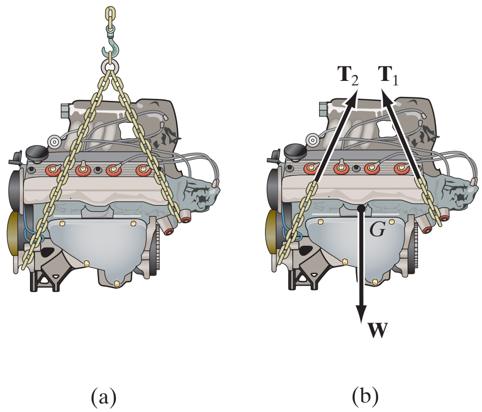


Fig. 5–4

Weight and the Center of Gravity. When a body is within a gravitational field, then each of its particles has a specified weight. It was shown in Sec. 4.8 that such a system of forces can be reduced to a single resultant force acting through a specified point. We refer to this force resultant as the *weight* **W** of the body and to the location of its point of application as the ***center of gravity***. The methods used for its determination will be developed in Chapter 9.

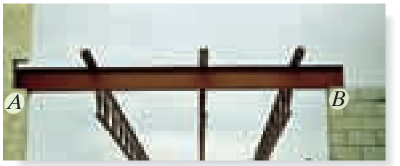
In the examples and problems that follow, if the weight of the body is important for the analysis, this force will be reported in the problem statement. Also, when the body is *uniform* or made from the same material, the center of gravity will be located at the body's *geometric center* or *centroid*; however, if the body consists of a nonuniform distribution of material, or has an unusual shape, then the location of its center of gravity G will be given.

Idealized Models. When an engineer performs a force analysis of any object, he or she considers a corresponding analytical or idealized model that gives results that approximate as closely as possible the actual situation. To do this, careful choices have to be made so that selection of the type of supports, the material behavior, and the object's dimensions can be justified. This way one can feel confident that any design or analysis will yield results which can be trusted. In complex cases this process may require developing several different models of the object that must be analyzed. In any case, this selection process requires both skill and experience.

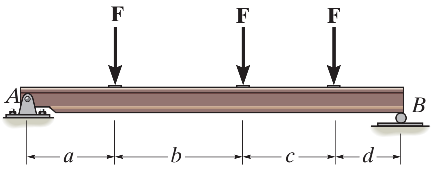
The following two cases illustrate what is required to develop a proper model. In Fig. 5–5*a*, the steel beam is to be used to support the three roof joists of a building. For a force analysis it is reasonable to assume the material (steel) is rigid since only very small deflections will occur when the beam is loaded. A bolted connection at A will allow for any slight rotation that occurs here when the load is applied, and so a *pin* can be considered for this support. At B a *roller* can be considered since this support offers no resistance to horizontal movement. Building code is used to specify the roof loading A so that the joist loads $\mathbf{F}$ can be calculated. These forces will be larger than any actual loading on the beam since they account for extreme loading cases and for dynamic or vibrational effects. Finally, the weight of the beam is generally neglected when it is small compared to the load the beam supports. The idealized model of the beam is therefore shown with average dimensions a, b, c, and d in Fig. 5–5b.

As a second case, consider the lift boom in Fig. 5–6*a*. By inspection, it is supported by a pin at A and by the hydraulic cylinder BC, which can be approximated as a weightless link. The material can be assumed rigid, and with its density known, the weight of the boom and the location of its center of gravity G are determined. When a design loading $\mathbf{P}$ is specified, the idealized model shown in Fig. 5–6b can be used for a force analysis. Average dimensions (not shown) are used to specify the location of the loads and the supports.

Idealized models of specific objects will be given in some of the examples throughout the text. It should be realized, however, that each case represents the reduction of a practical situation using simplifying assumptions like the ones illustrated here.

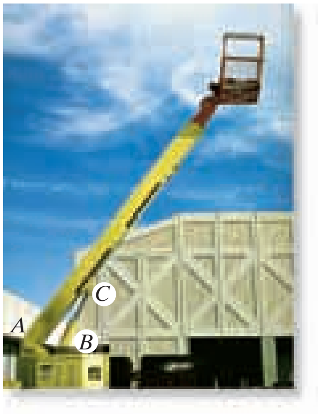


(a)

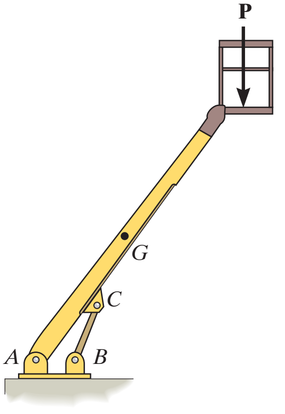


(b)

Fig. 5–5 (© Russell C. Hibbeler)



(a)



(b)

Fig. 5–6 (© Russell C. Hibbeler)

Important Points

- No equilibrium problem should be solved without *first drawing the free-body diagram*, so as to account for all the forces and couple moments that act on the body.
- If a support *prevents translation* of a body in a particular direction, then the support, when it is removed, exerts a *force* on the body in that direction.
- If *rotation is prevented*, then the support, when it is removed, exerts a *couple moment* on the body.
- Study Table 5–1.
- Internal forces are *never shown* on the free-body diagram since they occur in equal but opposite collinear pairs and therefore cancel out.
- The weight of a body is an external force, and its effect is represented by a single resultant force acting through the body's center of gravity G.
- *Couple moments* can be placed anywhere on the free-body diagram since they are *free vectors. Forces* can act at any point along their lines of action since they are *sliding vectors*.

Procedure for Analysis

To construct a free-body diagram for a rigid body or any group of bodies considered as a single system, the following steps should be performed:

Draw Outlined Shape.

Imagine the body to be *isolated* or cut "free" from its constraints and connections and draw (sketch) its outlined shape. Be sure to *remove all the supports* from the body.

Show All Forces and Couple Moments.

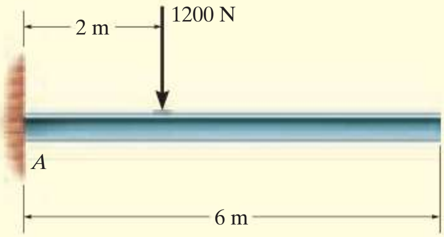
Identify all the known and unknown *external forces* and couple moments that *act on the body*. Those generally encountered are due to (1) applied loadings, (2) reactions occurring at the supports or at points of contact with other bodies (see Table 5–1), and (3) the weight of the body. To account for all these effects, it may help to trace over the boundary, carefully noting each force or couple moment acting on it.

Identify Each Loading and Give Dimensions.

The forces and couple moments that are known should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and direction angles of forces and couple moments that are unknown. Establish an x, y coordinate system so that these unknowns, A_x, A_y, etc., can be identified. Finally, indicate the dimensions of the body necessary for calculating the moments of forces.

EXAMPLE 5.1

Draw the free-body diagram of the uniform beam shown in Fig. 5–7*a*. The beam has a mass of 100 kg.

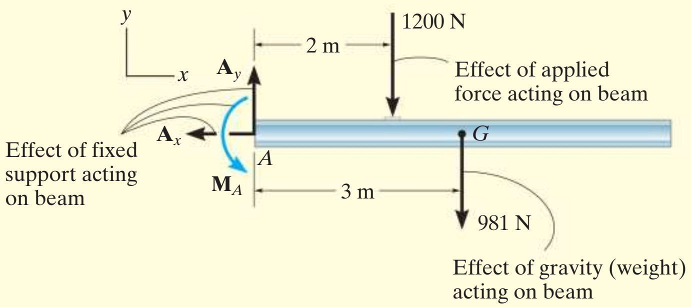


(a)

5

SOLUTION

The free-body diagram of the beam is shown in Fig. 5–7*b*. Since the support at A is fixed, the wall exerts three reactions *on the beam*, denoted as $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{M}_A$. The magnitudes of these reactions are *unknown*, and their sense has been *assumed*. The weight of the beam, $W = 100(9.81)\text{ N} = 981\text{ N}$, acts through the beam's center of gravity G, which is 3 m from A since the beam is uniform.

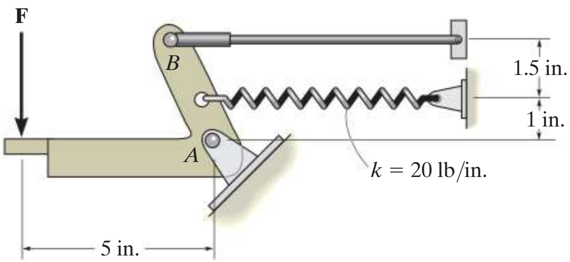


(b)

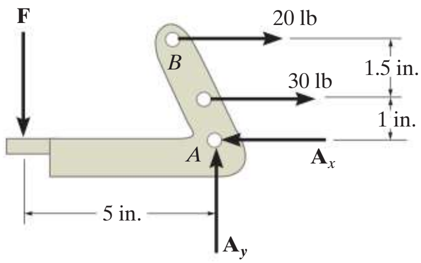
Fig. 5–7

Draw the free-body diagram of the foot lever shown in Fig. 5–8*a*. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force on the link at *B* is 20 lb.

(a)



(b)



(c)

Fig. 5–8 (© Russell C. Hibbeler)

5

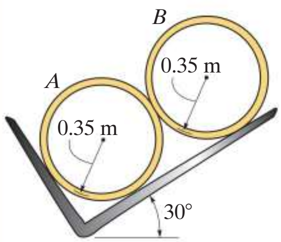
SOLUTION

By inspection of the photo the lever is loosely bolted to the frame at *A* and so this bolt acts as a pin. (See (8) in Table 5–1.) Although not shown here the link at *B* is pinned at both ends and so it is like (2) in Table 5–1. After making the proper measurements, the idealized model of the lever is shown in Fig. 5–8*b*. From this, the free-body diagram is shown in Fig. 5–8*c*. Since the pin at *A* is *removed*, it exerts force components $\mathbf{A}_x$ and $\mathbf{A}_y$ on the lever. The link exerts a force of 20 lb, acting in the direction of the link. In addition the spring also exerts a horizontal force on the lever. If the stiffness is measured and found to be $k = 20\text{ lb/in.}$, then since the stretch $s = 1.5\text{ in.}$, using Eq. 3–2, $F_s = ks = 20\text{ lb/in.}\,(1.5\text{ in.}) = 30\text{ lb}$. Finally, the operator's shoe applies a vertical force of **F** on the pedal. The dimensions of the lever are also shown on the free-body diagram, since this information will be useful when calculating the moments of the forces. As usual, the senses of the unknown forces at *A* have been assumed. The correct senses will become apparent after solving the equilibrium equations.

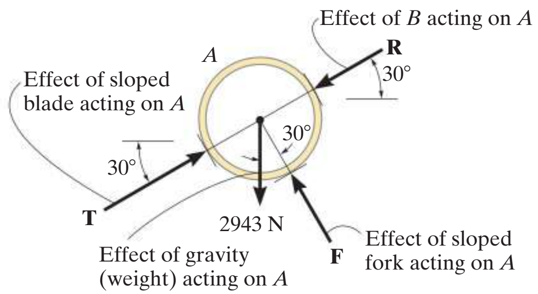
EXAMPLE 5.3

Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Fig. 5–9*a*. Draw the free-body diagrams for each pipe and both pipes together.

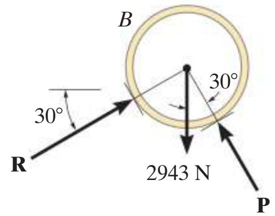
(a)



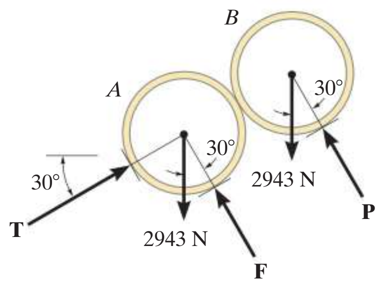
(b)



(c)



(d)



(e)

Fig. 5–9

SOLUTION

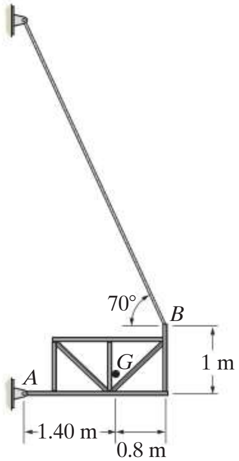
The idealized model from which we must draw the free-body diagrams is shown in Fig. 5–9*b*. Here the pipes are identified, the dimensions have been added, and the physical situation reduced to its simplest form.

Removing the surfaces of contact, the free-body diagram for pipe *A* is shown in Fig. 5–9*c*. Its weight is $W = 300(9.81)\text{ N} = 2943\text{ N}$. Assuming all contacting surfaces are *smooth*, the reactive forces **T**, **F**, **R** act in a direction *normal* to the tangent at their surfaces of contact.

The free-body diagram of the isolated pipe *B* is shown in Fig. 5–9*d*. Can you identify each of the three forces acting *on this pipe*? In particular, note that **R**, representing the force of *A* on *B*, Fig. 5–9*d*, is equal and opposite to **R** representing the force of *B* on *A*, Fig. 5–9*c*. This is a consequence of Newton's third law of motion.

The free-body diagram of both pipes combined ("system") is shown in Fig. 5–9*e*. Here the contact force **R**, which acts between *A* and *B*, is considered as an *internal* force and hence is not shown on the free-body diagram. That is, it represents a pair of equal but opposite collinear forces which cancel each other.

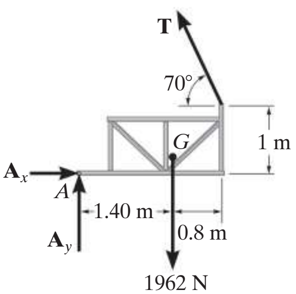
Draw the free-body diagram of the unloaded platform that is suspended off the edge of the oil rig shown in Fig. 5–10*a*. The platform has a mass of 200 kg.



(b)

(a)

Fig. 5–10 (© Russell C. Hibbeler)



(c)

SOLUTION

The idealized model of the platform will be considered in two dimensions because by observation the loading and the dimensions are all symmetrical about a vertical plane passing through its center, Fig. 5–10*b*. The connection at *A* is considered to be a pin, and the cable supports the platform at *B*. The direction of the cable and average dimensions of the platform are listed, and the center of gravity *G* has been determined. It is from this model that we have drawn the free-body diagram shown in Fig. 5–10*c*. The platform's weight is $200(9.81) = 1962$ N. The supports have been *removed*, and the force components $\mathbf{A}_x$ and $\mathbf{A}_y$ along with the cable force $\mathbf{T}$ represent the reactions that *both* pins and *both* cables exert on the platform, Fig. 5–10*a*. As a result, half their magnitudes are developed on each side of the platform.

5–1. Draw the free-body diagram for the following problems.

a) The cantilevered beam in Prob. 5–10.

b) The beam in Prob. 5–11.

c) The beam in Prob. 5–12.

d) The beam in Prob. 5–14.

5–2. Draw the free-body diagram for the following problems.

a) The truss in Prob. 5–15.

b) The beam in Prob. 5–16.

c) The man and load in Prob. 5–17.

d) The beam in Prob. 5–18.

5–3. Draw the free-body diagram for the following problems.

a) The man and beam in Prob. 5–19.

b) The rod in Prob. 5–20.

c) The rod in Prob. 5–21.

d) The beam in Prob. 5–22.

***5–4.** Draw the free-body diagram for the following problems.

a) The beam in Prob. 5–25.

b) The crane and boom in Prob. 5–26.

c) The bar in Prob. 5–27.

d) The rod in Prob. 5–28.

5–5. Draw the free-body diagram for the following problems.

a) The boom in Prob. 5–32.

b) The jib crane in Prob. 5–33.

c) The smooth pipe in Prob. 5–35.

d) The beam in Prob. 5–36.

5–6. Draw the free-body diagram for the following problems.

a) The jib crane in Prob. 5–37.

b) The bar in Prob. 5–39.

c) The bulkhead in Prob. 5–41.

d) The boom in Prob. 5–42.

5–7. Draw the free-body diagram for the following problems.

a) The rod in Prob. 5–44.

b) The hand truck and load when it is lifted in Prob. 5–45.

c) The beam in Prob. 5–47.

d) The cantilever footing in Prob. 5–51.

***5–8.** Draw the free-body diagram for the following problems.

a) The beam in Prob. 5–52.

b) The boy and diving board in Prob. 5–53.

c) The rod in Prob. 5–54.

d) The rod in Prob. 5–56.

5–9. Draw the free-body diagram for the following problems.

a) The beam in Prob. 5–57.

b) The rod in Prob. 5–59.

c) The bar in Prob. 5–60.

5.3 Equations of Equilibrium

In Sec. 5.1 we developed the two equations which are both necessary and sufficient for the equilibrium of a rigid body, namely, $\Sigma \mathbf{F} = \mathbf{0}$ and $\Sigma \mathbf{M}_O = \mathbf{0}$. When the body is subjected to a system of forces, which all lie in the x–y plane, then the forces can be resolved into their x and y components. Consequently, the conditions for equilibrium in two dimensions are

$$\boxed{\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_O &= 0 \end{aligned}} \qquad (5\text{–}2)$$

Here ΣF_x and ΣF_y represent, respectively, the algebraic sums of the x and y components of all the forces acting on the body, and ΣM_O represents the algebraic sum of the couple moments and the moments of all the force components about the z axis, which is perpendicular to the x–y plane and passes through the arbitrary point O.

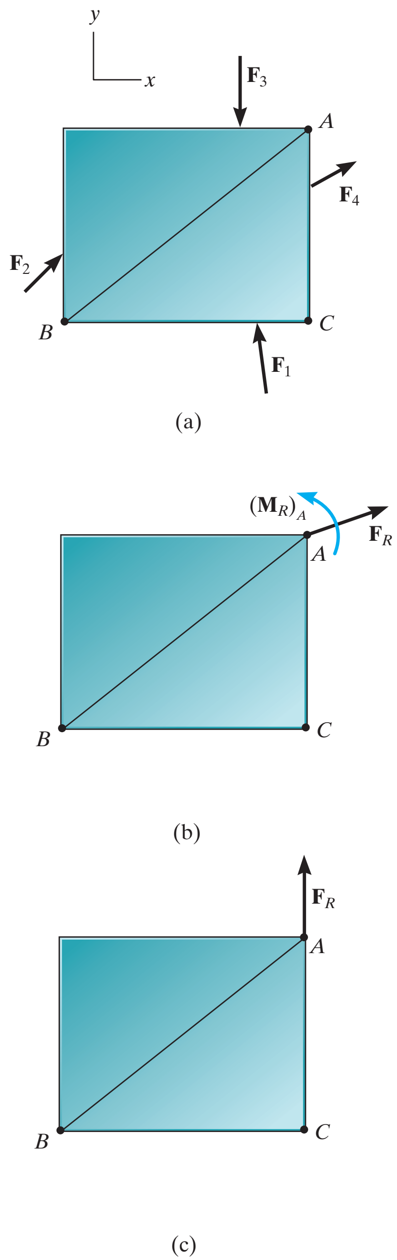


Fig. 5–11

Alternative Sets of Equilibrium Equations. Although Eqs. 5–2 are *most often* used for solving coplanar equilibrium problems, two *alternative* sets of three independent equilibrium equations may also be used. One such set is

$$\begin{aligned} \Sigma F_x &= 0 \\ \Sigma M_A &= 0 \\ \Sigma M_B &= 0 \end{aligned} \qquad (5\text{–}3)$$

When using these equations it is required that a line passing through points A and B is *not parallel* to the y axis. To prove that Eqs. 5–3 provide the *conditions* for equilibrium, consider the free-body diagram of the plate shown in Fig. 5–11a. Using the methods of Sec. 4.7, all the forces on the free-body diagram may be replaced by an equivalent resultant force $\mathbf{F}_R = \Sigma \mathbf{F}$, acting at point A, and a resultant couple moment $(\mathbf{M}_R)_A = \Sigma \mathbf{M}_A$, Fig. 5–11$b$. If $\Sigma M_A = 0$ is satisfied, it is necessary that $(\mathbf{M}_R)_A = \mathbf{0}$. Furthermore, in order that $\mathbf{F}_R$ satisfy $\Sigma F_x = 0$, it must have *no component* along the x axis, and therefore $\mathbf{F}_R$ must be parallel to the y axis, Fig. 5–11c. Finally, if it is required that $\Sigma M_B = 0$, where B does not lie on the line of action of $\mathbf{F}_R$, then $\mathbf{F}_R = \mathbf{0}$. Since Eqs. 5–3 show that both of these resultants are zero, indeed the body in Fig. 5–11a must be in equilibrium.

A second alternative set of equilibrium equations is

$$\begin{aligned} \Sigma M_A &= 0 \\ \Sigma M_B &= 0 \\ \Sigma M_C &= 0 \end{aligned} \qquad (5\text{–}4)$$

Here it is necessary that points A, B, and C do not lie on the same line. To prove that these equations, when satisfied, ensure equilibrium, consider again the free-body diagram in Fig. 5–11b. If $\Sigma M_A = 0$ is to be satisfied, then $(\mathbf{M}_R)_A = \mathbf{0}$. $\Sigma M_C = 0$ is satisfied if the line of action of $\mathbf{F}_R$ passes through point C as shown in Fig. 5–11c. Finally, if we require $\Sigma M_B = 0$, it is necessary that $\mathbf{F}_R = \mathbf{0}$, and so the plate in Fig. 5–11a must then be in equilibrium.

Procedure for Analysis

Coplanar force equilibrium problems for a rigid body can be solved using the following procedure.

Free-Body Diagram.

- Establish the x, y coordinate axes in any suitable orientation.
- Remove all supports and draw an outlined shape of the body.
- Show all the forces and couple moments acting on the body.
- Label all the loadings and specify their directions relative to the x or y axis. The sense of a force or couple moment having an *unknown* magnitude but known line of action can be *assumed*.
- Indicate the dimensions of the body necessary for computing the moments of forces.

Equations of Equilibrium.

- Apply the moment equation of equilibrium, $\Sigma M_O = 0$, about a point (O) that lies at the intersection of the lines of action of two unknown forces. In this way, the moments of these unknowns are zero about O, and a *direct solution* for the third unknown can be determined.
- When applying the force equilibrium equations, $\Sigma F_x = 0$ and $\Sigma F_y = 0$, orient the x and y axes along lines that will provide the simplest resolution of the forces into their x and y components.
- If the solution of the equilibrium equations yields a negative scalar for a force or couple moment magnitude, this indicates that the sense is opposite to that which was assumed on the free-body diagram.

Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown in Fig. 5–12*a*. Neglect the weight of the beam.

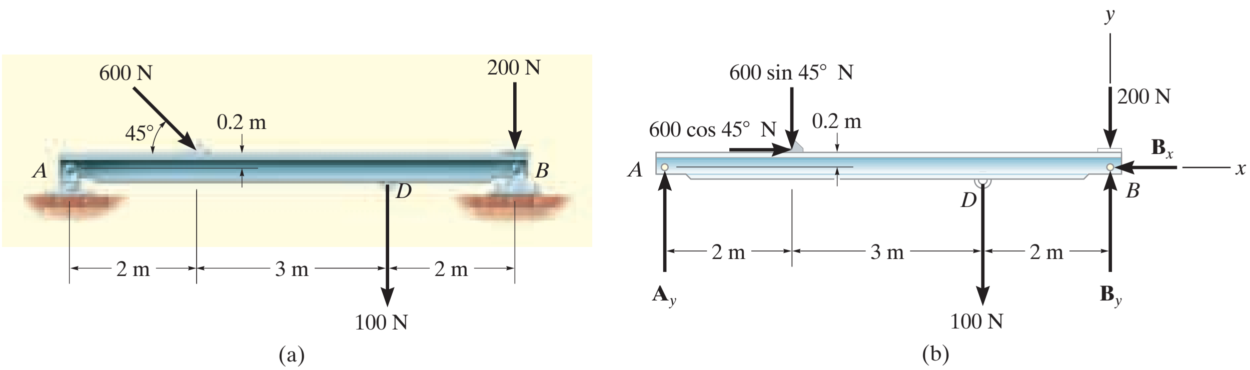
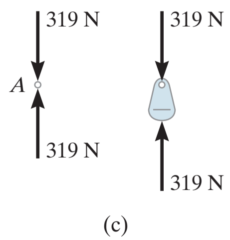


Fig. 5–12



SOLUTION

Free-Body Diagram. The supports are *removed*, and the free-body diagram of the beam is shown in Fig. 5–12*b*. (See Example 5.1.) For simplicity, the 600-N force is represented by its x and y components as shown in Fig. 5–12*b*.

Equations of Equilibrium. Summing forces in the x direction yields

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 600 \cos 45^\circ \text{ N} - B_x = 0$$

$$B_x = 424 \text{ N} \qquad \textit{Ans.}$$

A direct solution for $\mathbf{A}_y$ can be obtained by applying the moment equation $\Sigma M_B = 0$ about point B.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 100 \text{ N}(2 \text{ m}) + (600 \sin 45^\circ \text{ N})(5 \text{ m})$$

$$- (600 \cos 45^\circ \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$$

$$A_y = 319 \text{ N} \qquad \textit{Ans.}$$

Summing forces in the y direction, using this result, gives

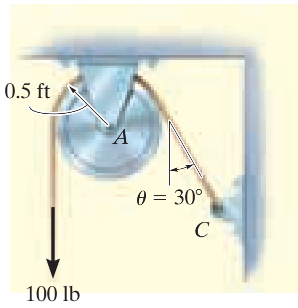
$$+\uparrow \Sigma F_y = 0; \qquad 319 \text{ N} - 600 \sin 45^\circ \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$$

$$B_y = 405 \text{ N} \qquad \textit{Ans.}$$

NOTE: Remember, the support forces in Fig. 5–12*b* are the result of pins that *act on the beam*. The opposite forces act on the pins. For example, Fig. 5–12*c* shows the equilibrium of the pin at A and the rocker.

EXAMPLE 5.6

The cord shown in Fig. 5–13*a* supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.

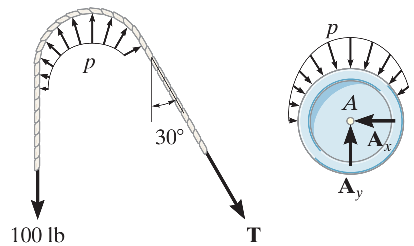


(a)

Fig. 5–13

SOLUTION

Free-Body Diagrams. The free-body diagrams of the cord and pulley are shown in Fig. 5–13*b*. Note that the principle of action, equal but opposite reaction must be carefully observed when drawing each of these diagrams: the cord exerts an unknown load distribution p on the pulley at the contact surface, whereas the pulley exerts an equal but opposite effect on the cord. For the solution, however, it is simpler to *combine* the free-body diagrams of the pulley and this portion of the cord, so that the distributed load becomes *internal* to this "system" and is therefore eliminated from the analysis, Fig. 5–13*c*.

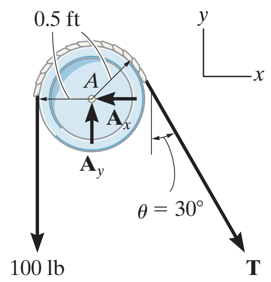


(b)

Equations of Equilibrium. Summing moments about point A to eliminate $\mathbf{A}_x$ and $\mathbf{A}_y$, Fig. 5–13*c*, we have

$$\circlearrowleft +\Sigma M_A = 0; \qquad 100 \text{ lb } (0.5 \text{ ft}) - T(0.5 \text{ ft}) = 0$$

$$T = 100 \text{ lb} \qquad \textit{Ans.}$$



(c)

Using this result,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -A_x + 100 \sin 30° \text{ lb} = 0$$

$$A_x = 50.0 \text{ lb} \qquad \textit{Ans.}$$

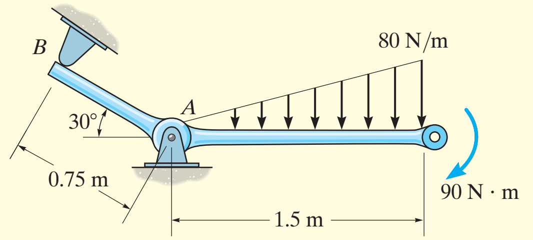
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 100 \text{ lb} - 100 \cos 30° \text{ lb} = 0$$

$$A_y = 187 \text{ lb} \qquad \textit{Ans.}$$

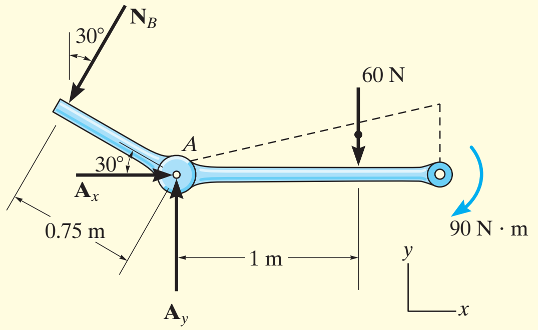
NOTE: From the moment equation, it is seen that the tension remains *constant* as the cord passes over the pulley. (This of course is true for *any angle* θ at which the cord is directed and for *any radius r* of the pulley.)

EXAMPLE 5.7

The member shown in Fig. 5–14*a* is pin connected at A and rests against a smooth support at B. Determine the horizontal and vertical components of reaction at the pin A.



(a)



(b)

Fig. 5–14

SOLUTION

Free-Body Diagram. As shown in Fig. 5–14*b*, the supports are removed and the reaction $\mathbf{N}_B$ is perpendicular to the member at B. Also, horizontal and vertical components of reaction are represented at A. The resultant of the distributed loading is $\frac{1}{2}(1.5\text{ m})(80\text{ N/m}) = 60\text{ N}$. It acts through the centroid of the triangle, 1 m from A as shown.

Equations of Equilibrium. Summing moments about A, we obtain a direct solution for N_B,

$$\circlearrowleft +\Sigma M_A = 0; \quad -90\text{ N}\cdot\text{m} - 60\text{ N}(1\text{ m}) + N_B(0.75\text{ m}) = 0$$

$$N_B = 200\text{ N}$$

Using this result,

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 200\sin 30^\circ\text{ N} = 0$$

$$A_x = 100\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 200\cos 30^\circ\text{ N} - 60\text{ N} = 0$$

$$A_y = 233\text{ N} \qquad \textit{Ans.}$$

EXAMPLE 5.8

The box wrench in Fig. 5–15*a* is used to tighten the bolt at *A*. If the wrench does not turn when the load is applied to the handle, determine the torque or moment applied to the bolt and the force of the wrench on the bolt.

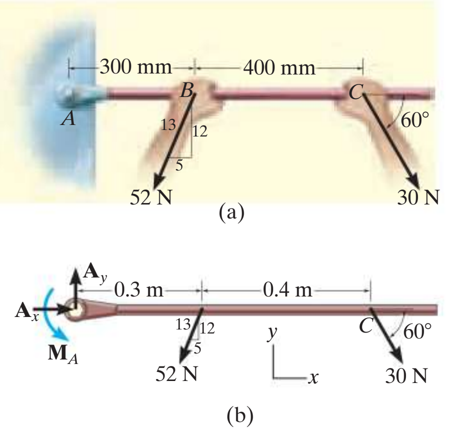


Fig. 5–15

SOLUTION

Free-Body Diagram. The free-body diagram for the wrench is shown in Fig. 5–15*b*. Since the bolt acts as a "fixed support," when it is removed, it exerts force components $\mathbf{A}_x$ and $\mathbf{A}_y$ and a moment $\mathbf{M}_A$ on the wrench at *A*.

Equations of Equilibrium.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x - 52\left(\tfrac{5}{13}\right)\text{ N} + 30\cos 60^\circ\text{ N} = 0$$

$$A_x = 5.00\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y - 52\left(\tfrac{12}{13}\right)\text{ N} - 30\sin 60^\circ\text{ N} = 0$$

$$A_y = 74.0\text{ N} \qquad \textit{Ans.}$$

$$\circlearrowleft+\Sigma M_A = 0; \quad M_A - \left[52\left(\tfrac{12}{13}\right)\text{N}\right](0.3\text{ m}) - (30\sin 60^\circ\text{ N})(0.7\text{ m}) = 0$$

$$M_A = 32.6\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

Note that $\mathbf{M}_A$ must be *included* in this moment summation. This couple moment is a free vector and represents the twisting resistance of the bolt on the wrench. By Newton's third law, the wrench exerts an equal but opposite moment or torque on the bolt. Furthermore, the resultant force on the wrench is

$$F_A = \sqrt{(5.00)^2 + (74.0)^2} = 74.1\text{ N} \qquad \textit{Ans.}$$

NOTE: Although only *three* independent equilibrium equations can be written for a rigid body, it is a good practice to *check* the calculations using a fourth equilibrium equation. For example, the above computations may be verified in part by summing moments about point *C*:

$$\circlearrowleft+\Sigma M_C = 0; \quad \left[52\left(\tfrac{12}{13}\right)\text{N}\right](0.4\text{ m}) + 32.6\text{ N}\cdot\text{m} - 74.0\text{ N}(0.7\text{ m}) = 0$$

$$19.2\text{ N}\cdot\text{m} + 32.6\text{ N}\cdot\text{m} - 51.8\text{ N}\cdot\text{m} = 0$$

Determine the horizontal and vertical components of reaction on the member at the pin A, and the normal reaction at the roller B in Fig. 5–16*a*.

SOLUTION

Free-Body Diagram. All the supports are removed and so the free-body diagram is shown in Fig. 5–16*b*. The pin at A exerts two components of reaction on the member, $\mathbf{A}_x$ and $\mathbf{A}_y$.

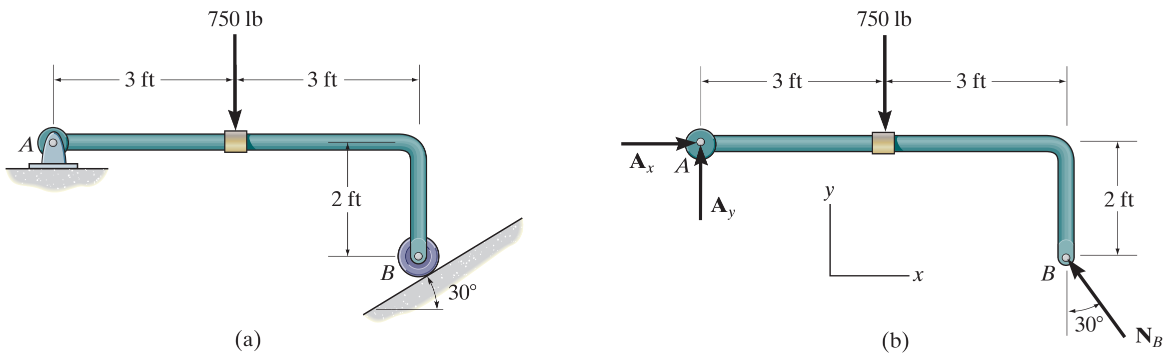
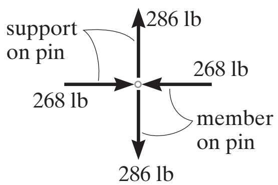


Fig. 5–16



(c)

Equations of Equilibrium. The reaction N_B can be obtained *directly* by summing moments about point A, since $\mathbf{A}_x$ and $\mathbf{A}_y$ produce no moment about A.

$$\circlearrowleft +\Sigma M_A = 0;$$

$$[N_B \cos 30°](6 \text{ ft}) - [N_B \sin 30°](2 \text{ ft}) - 750 \text{ lb}(3 \text{ ft}) = 0$$

$$N_B = 536.2 \text{ lb} = 536 \text{ lb} \qquad \textit{Ans.}$$

Using this result,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - (536.2 \text{ lb}) \sin 30° = 0$$

$$A_x = 268 \text{ lb} \qquad \textit{Ans.}$$

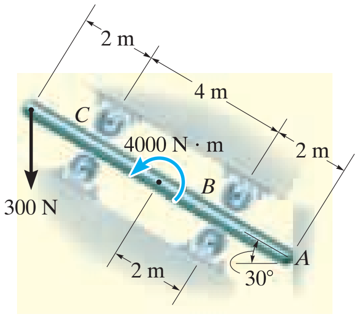
$$+\uparrow \Sigma F_y = 0; \qquad A_y + (536.2 \text{ lb}) \cos 30° - 750 \text{ lb} = 0$$

$$A_y = 286 \text{ lb} \qquad \textit{Ans.}$$

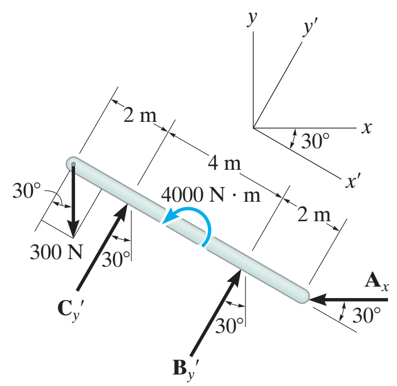
Details of the equilibrium of the pin at A are shown in Fig. 5–16*c*.

EXAMPLE 5.10

The uniform smooth rod shown in Fig. 5–17*a* is subjected to a force and couple moment. If the rod is supported at A by a smooth wall and at B and C either at the top or bottom by rollers, determine the reactions at these supports. Neglect the weight of the rod.



(a)



(b)

Fig. 5–17

SOLUTION

Free-Body Diagram. Removing the supports as shown in Fig. 5–17*b*, all the reactions act normal to the surfaces of contact since these surfaces are smooth. The reactions at B and C are shown acting in the positive y' direction. This assumes that only the rollers located on the bottom of the rod are used for support.

Equations of Equilibrium. Using the x, y coordinate system in Fig. 5–17*b*, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad C_{y'} \sin 30^\circ + B_{y'} \sin 30^\circ - A_x = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad -300 \text{ N} + C_{y'} \cos 30^\circ + B_{y'} \cos 30^\circ = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -B_{y'}(2 \text{ m}) + 4000 \text{ N}\cdot\text{m} - C_{y'}(6 \text{ m})$$
$$+ (300 \cos 30^\circ \text{ N})(8 \text{ m}) = 0 \qquad (3)$$

When writing the moment equation, it should be noted that the line of action of the force component 300 sin 30° N passes through point A, and therefore this force is not included in the moment equation.

Solving Eqs. 2 and 3 simultaneously, we obtain

$$B_{y'} = -1000.0 \text{ N} = -1 \text{ kN} \qquad \textit{Ans.}$$
$$C_{y'} = 1346.4 \text{ N} = 1.35 \text{ kN} \qquad \textit{Ans.}$$

Since $B_{y'}$ is a negative scalar, the sense of $\mathbf{B}_{y'}$ is opposite to that shown on the free-body diagram in Fig. 5–17*b*. Therefore, the top roller at B serves as the support rather than the bottom one. Retaining the negative sign for $B_{y'}$ (Why?) and substituting the results into Eq. 1, we obtain

$$1346.4 \sin 30^\circ \text{ N} + (-1000.0 \sin 30^\circ \text{ N}) - A_x = 0$$
$$A_x = 173 \text{ N} \qquad \textit{Ans.}$$

EXAMPLE 5.11

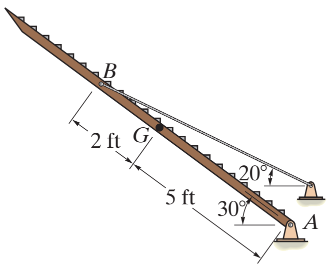
The uniform truck ramp shown in Fig. 5–18*a* has a weight of 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the tension in the cables.

(a)

(© Russell C. Hibbeler)

SOLUTION

The idealized model of the ramp, which indicates all necessary dimensions and supports, is shown in Fig. 5–18*b*. Here the center of gravity is located at the midpoint since the ramp is considered to be uniform.



(b)

Free-Body Diagram. Removing the supports from the idealized model, the ramp's free-body diagram is shown in Fig. 5–18*c*.

Equations of Equilibrium. Summing moments about point A will yield a direct solution for the cable tension. Using the principle of moments, there are several ways of determining the moment of $\mathbf{T}$ about A. If we use x and y components, with $\mathbf{T}$ applied at B, we have

$$\circlearrowleft +\Sigma M_A = 0; \qquad -T\cos 20^\circ(7\sin 30^\circ\text{ ft}) + T\sin 20^\circ(7\cos 30^\circ\text{ ft})$$

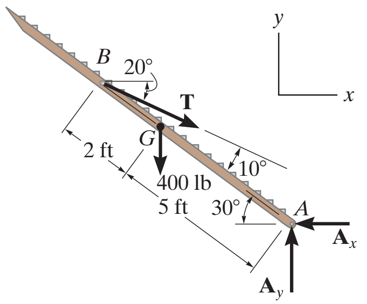
$$+ 400\text{ lb}\,(5\cos 30^\circ\text{ ft}) = 0$$

$$T = 1425\text{ lb}$$

We can also determine the moment of $\mathbf{T}$ about A by resolving it into components along and perpendicular to the ramp at B. Then the moment of the component along the ramp will be zero about A, so that

$$\circlearrowleft +\Sigma M_A = 0; \qquad -T\sin 10^\circ(7\text{ ft}) + 400\text{ lb}\,(5\cos 30^\circ\text{ ft}) = 0$$

$$T = 1425\text{ lb}$$



(c)

Fig. 5–18

Since there are two cables supporting the ramp,

$$T' = \frac{T}{2} = 712\text{ lb} \qquad \textit{Ans.}$$

NOTE: As an exercise, show that $A_x = 1339$ lb and $A_y = 887$ lb.

EXAMPLE 5.12

Determine the support reactions on the member in Fig. 5–19*a*. The collar at A is fixed to the member and can slide vertically along the vertical shaft.

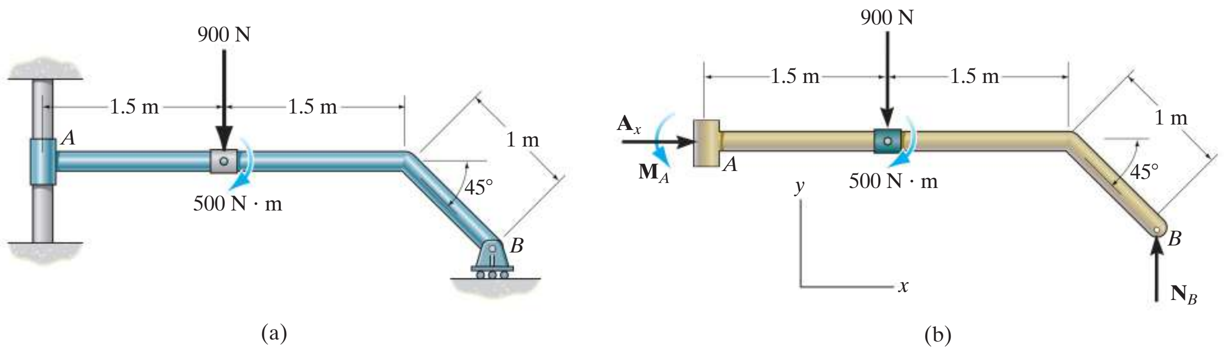


Fig. 5–19

SOLUTION

Free-Body Diagram. Removing the supports, the free-body diagram of the member is shown in Fig. 5–19*b*. The collar exerts a horizontal force $\mathbf{A}_x$ and moment $\mathbf{M}_A$ on the member. The reaction $\mathbf{N}_B$ of the roller on the member is vertical.

Equations of Equilibrium. The forces A_x and N_B can be determined directly from the force equations of equilibrium.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - 900\text{ N} = 0$$

$$N_B = 900\text{ N} \qquad \textit{Ans.}$$

The moment M_A can be determined by summing moments either about point A or point B.

$$\circlearrowleft + \Sigma M_A = 0;$$

$$M_A - 900\text{ N}(1.5\text{ m}) - 500\text{ N}\cdot\text{m} + 900\text{ N}\,[3\text{ m} + (1\text{ m})\cos 45^\circ] = 0$$

$$M_A = -1486\text{ N}\cdot\text{m} = 1.49\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

or

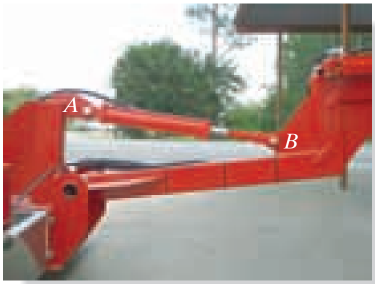
$$\circlearrowleft + \Sigma M_B = 0; \quad M_A + 900\text{ N}\,[1.5\text{ m} + (1\text{ m})\cos 45^\circ] - 500\text{ N}\cdot\text{m} = 0$$

$$M_A = -1486\text{ N}\cdot\text{m} = 1.49\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

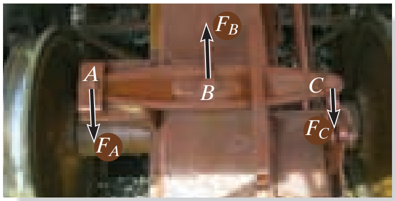
The negative sign indicates that $\mathbf{M}_A$ has the opposite sense of rotation to that shown on the free-body diagram.

5.4 Two- and Three-Force Members

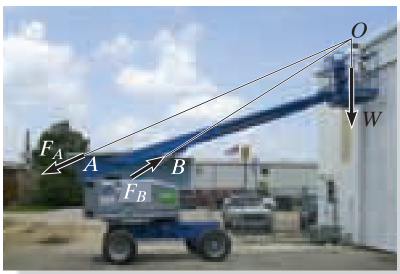
The solutions to some equilibrium problems can be simplified by recognizing members that are subjected to only two or three forces.



The hydraulic cylinder AB is a typical example of a two-force member since it is pin connected at its ends and, provided its weight is neglected, only the pin forces act on this member. (© Russell C. Hibbeler)

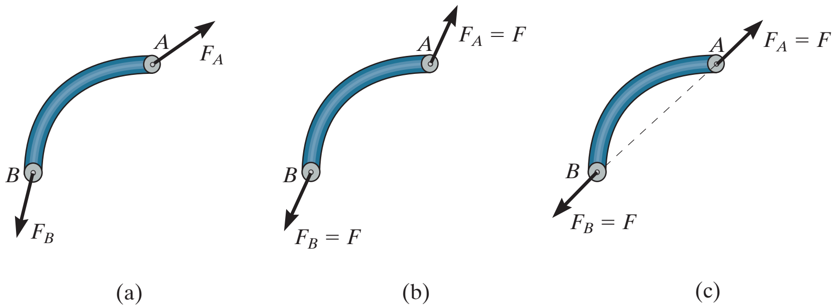


The link used for this railroad car brake is a three-force member. Since the force $\mathbf{F}_B$ in the tie rod at B and $\mathbf{F}_C$ from the link at C are parallel, then for equilibrium the resultant force $\mathbf{F}_A$ at the pin A must also be parallel with these two forces. (© Russell C. Hibbeler)



The boom and bucket on this lift is a three-force member, provided its weight is neglected. Here the lines of action of the weight of the worker, $\mathbf{W}$, and the force of the two-force member (hydraulic cylinder) at B, $\mathbf{F}_B$, intersect at O. For moment equilibrium, the resultant force at the pin A, $\mathbf{F}_A$, must also be directed towards O. (© Russell C. Hibbeler)

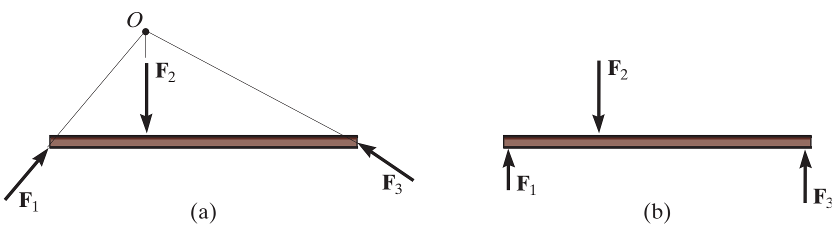
Two-Force Members. As the name implies, a ***two-force member*** has forces applied at only two points on the member. An example of a two-force member is shown in Fig. 5–20*a*. To satisfy force equilibrium, $\mathbf{F}_A$ and $\mathbf{F}_B$ must be equal in magnitude, $F_A = F_B = F$, but opposite in direction ($\Sigma \mathbf{F} = \mathbf{0}$), Fig. 5–20*b*. Furthermore, moment equilibrium requires that $\mathbf{F}_A$ and $\mathbf{F}_B$ share the same line of action, which can only happen if they are directed along the line joining points A and B ($\Sigma \mathbf{M}_A = \mathbf{0}$ or $\Sigma \mathbf{M}_B = \mathbf{0}$), Fig. 5–20*c*. Therefore, for any two-force member to be in equilibrium, the two forces acting on the member *must have the same magnitude, act in opposite directions, and have the same line of action, directed along the line joining the two points where these forces act.*



Two-force member

Fig. 5–20

Three-Force Members. If a member is subjected to only *three forces*, it is called a ***three-force member***. Moment equilibrium can be satisfied only if the three forces form a *concurrent* or *parallel* force system. To illustrate, consider the member subjected to the three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, shown in Fig. 5–21*a*. If the lines of action of $\mathbf{F}_1$ and $\mathbf{F}_2$ intersect at point O, then the line of action of $\mathbf{F}_3$ must *also* pass through point O so that the forces satisfy $\Sigma \mathbf{M}_O = \mathbf{0}$. As a special case, if the three forces are all parallel, Fig. 5–21*b*, the location of the point of intersection, O, will approach infinity.



Three-force member

Fig. 5–21

EXAMPLE 5.13

The lever ABC is pin supported at A and connected to a short link BD as shown in Fig. 5–22*a*. If the weight of the members is negligible, determine the force of the pin on the lever at A.

SOLUTION

Free-Body Diagrams. As shown in Fig. 5–22*b*, the short link BD is a *two-force member*, so the *resultant forces* from the pins D and B must be equal, opposite, and collinear. Although the magnitude of the force is unknown, the line of action is known since it passes through B and D.

Lever ABC is a *three-force member*, and therefore, in order to satisfy moment equilibrium, the three nonparallel forces acting on it must be concurrent at O, Fig. 5–22*c*. In particular, note that the force $\mathbf{F}$ on the lever at B is equal but opposite to the force $\mathbf{F}$ acting at B on the link. Why? The distance CO must be 0.5 m since the lines of action of $\mathbf{F}$ and the 400-N force are known.

Equations of Equilibrium. By requiring the force system to be concurrent at O, since $\Sigma M_O = 0$, the angle θ which defines the line of action of $\mathbf{F}_A$ can be determined from trigonometry,

$$\theta = \tan^{-1}\left(\frac{0.7}{0.4}\right) = 60.3^\circ$$

Using the x, y axes and applying the force equilibrium equations,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_A \cos 60.3^\circ - F\cos 45^\circ + 400\text{ N} = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad F_A \sin 60.3^\circ - F\sin 45^\circ = 0$$

Solving, we get

$$F_A = 1.07\text{ kN} \qquad \textit{Ans.}$$

$$F = 1.32\text{ kN}$$

NOTE: We can also solve this problem by representing the force at A by its two components $\mathbf{A}_x$ and $\mathbf{A}_y$ and applying $\Sigma M_A = 0$, $\Sigma F_x = 0$, $\Sigma F_y = 0$ to the lever. Once A_x and A_y are determined, we can get F_A and θ.

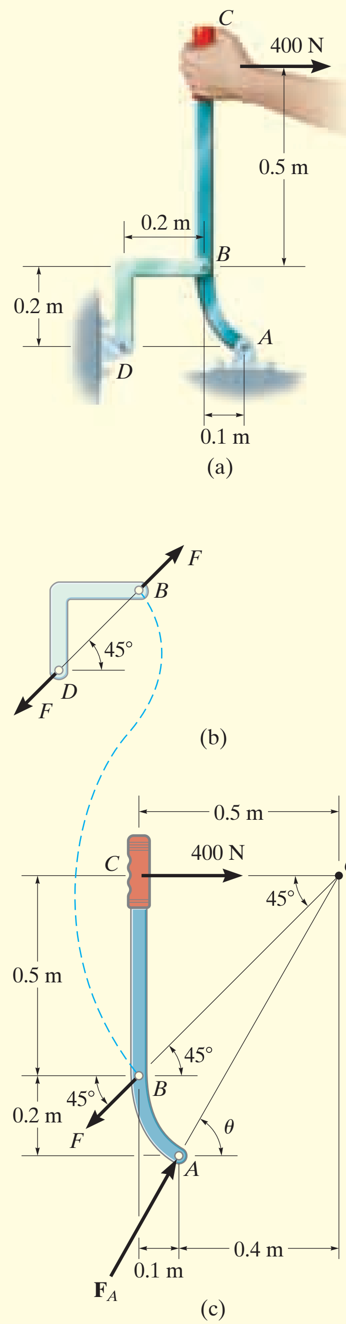
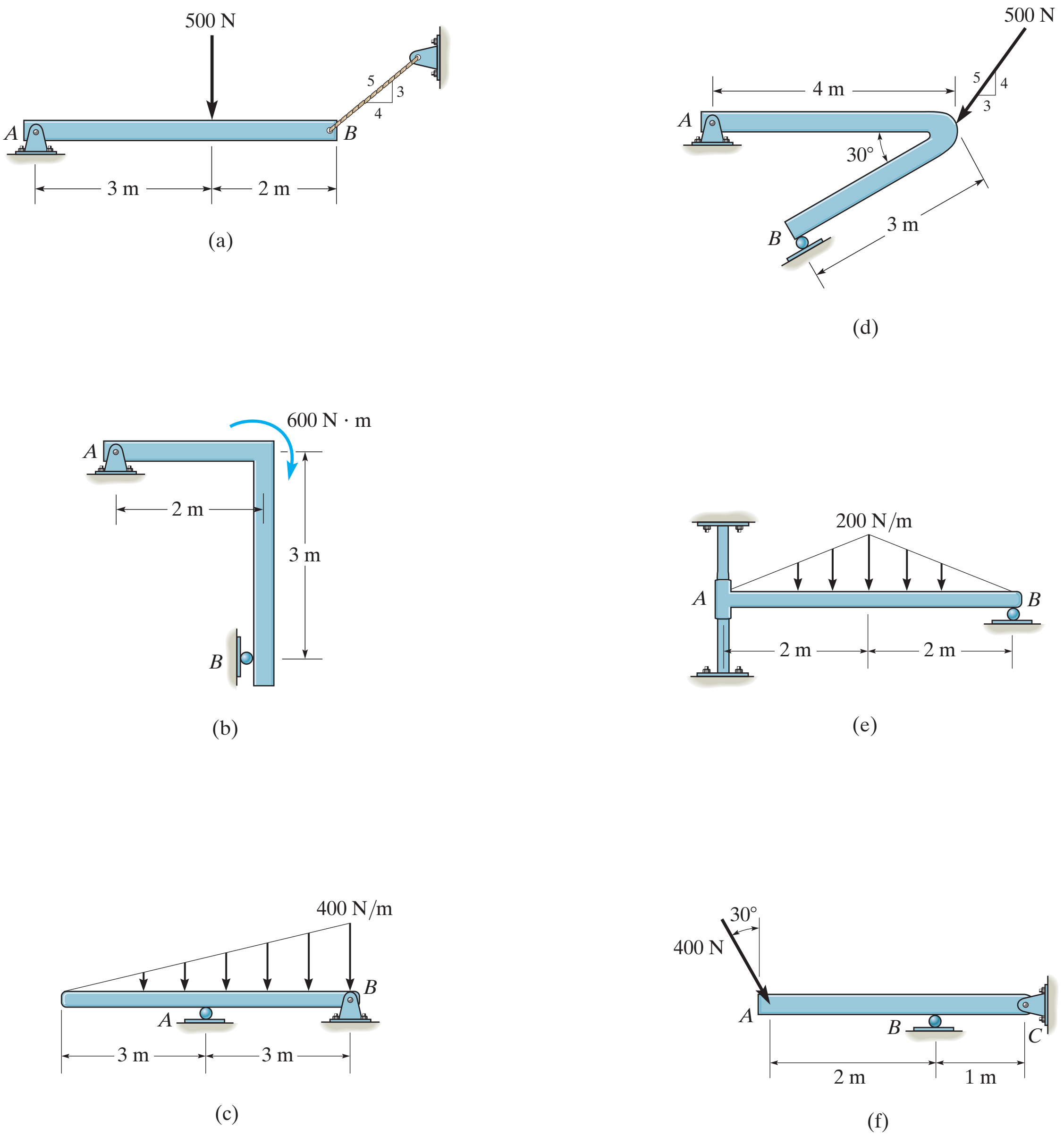


Fig. 5–22

PRELIMINARY PROBLEMS

P5–1. Draw the free-body diagram of each object.

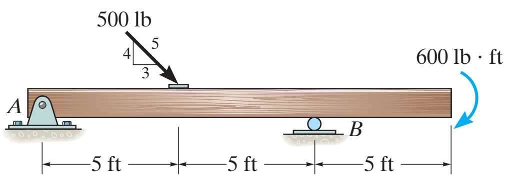


Prob. P5–1

FUNDAMENTAL PROBLEMS

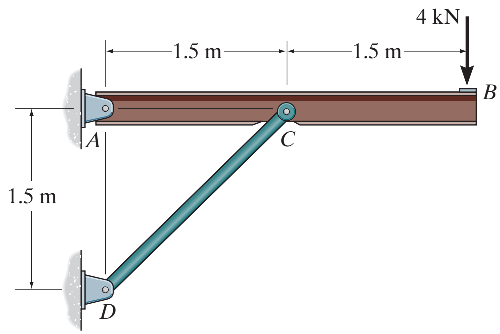
All problem solutions must include an FBD.

F5–1. Determine the horizontal and vertical components of reaction at the supports. Neglect the thickness of the beam.



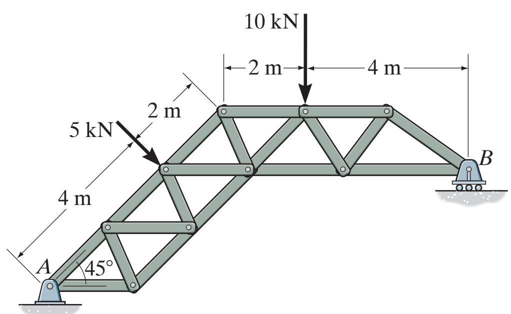
Prob. F5–1

F5–2. Determine the horizontal and vertical components of reaction at the pin A and the reaction on the beam at C.



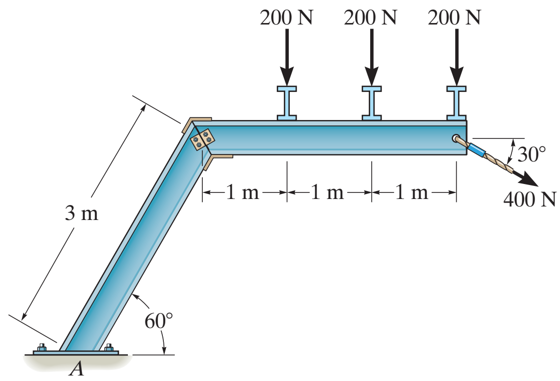
Prob. F5–2

F5–3. The truss is supported by a pin at A and a roller at B. Determine the support reactions.



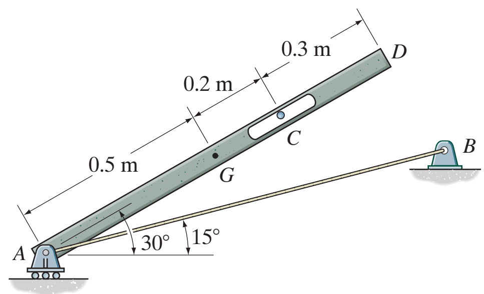
Prob. F5–3

F5–4. Determine the components of reaction at the fixed support A. Neglect the thickness of the beam.



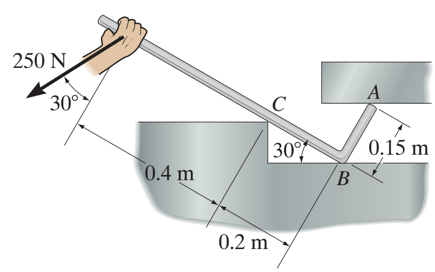
Prob. F5–4

F5–5. The 25-kg bar has a center of mass at G. If it is supported by a smooth peg at C, a roller at A, and cord AB, determine the reactions at these supports.



Prob. F5–5

F5–6. Determine the reactions at the smooth contact points A, B, and C on the bar.

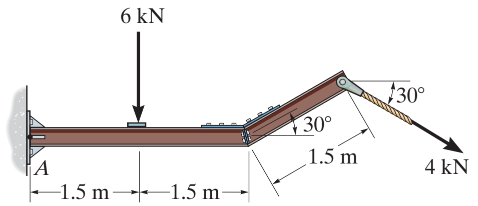


Prob. F5–6

PROBLEMS

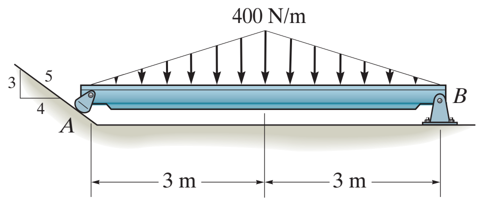
All problem solutions must include an FBD.

5–10. Determine the components of the support reactions at the fixed support A on the cantilevered beam.



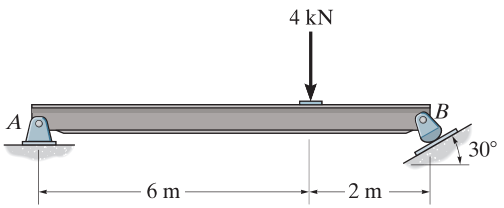
Prob. 5–10

5–11. Determine the reactions at the supports.



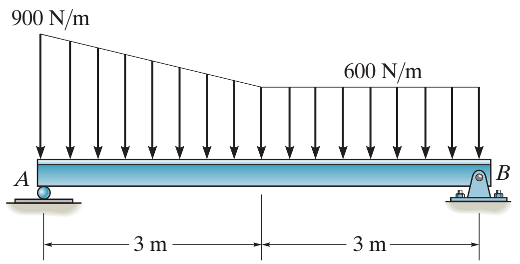
Prob. 5–11

***5–12.** Determine the horizontal and vertical components of reaction at the pin A and the reaction of the rocker B on the beam.



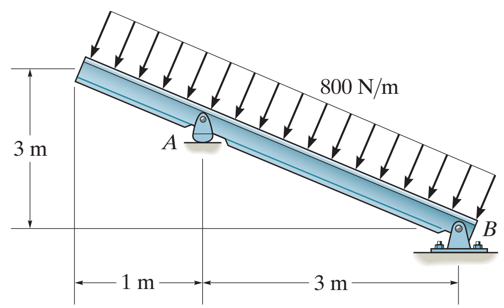
Prob. 5–12

5–13. Determine the reactions at the supports.



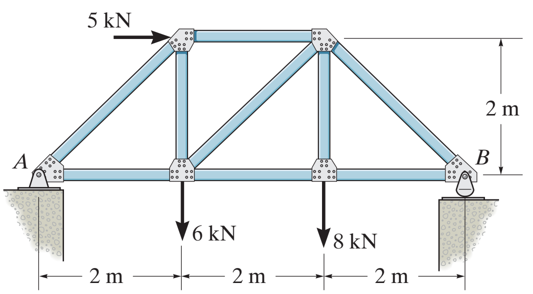
Prob. 5–13

5–14. Determine the reactions at the supports.



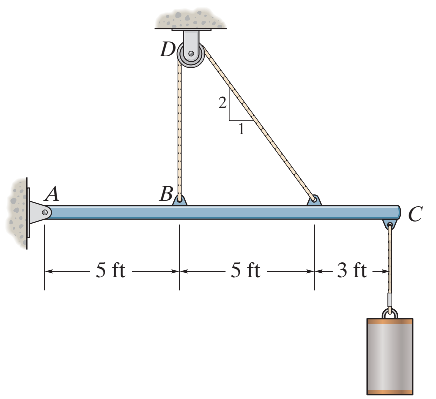
Prob. 5–14

5–15. Determine the reactions at the supports.



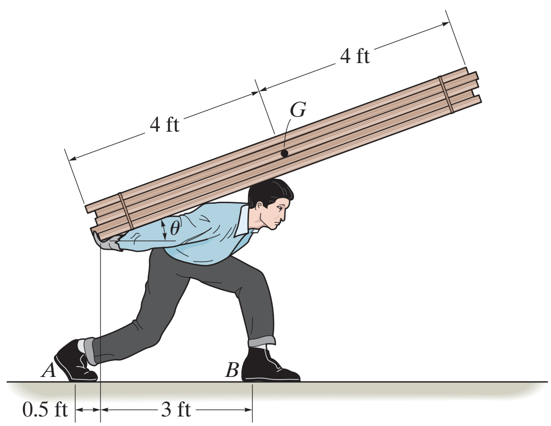
Prob. 5–15

***5–16.** Determine the tension in the cable and the horizontal and vertical components of reaction of the pin A. The pulley at D is frictionless and the cylinder weighs 80 lb.



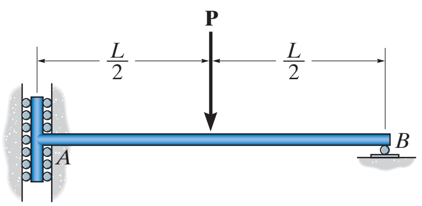
Prob. 5–16

5–17. The man attempts to support the load of boards having a weight W and a center of gravity at G. If he is standing on a smooth floor, determine the smallest angle θ at which he can hold them up in the position shown. Neglect his weight.



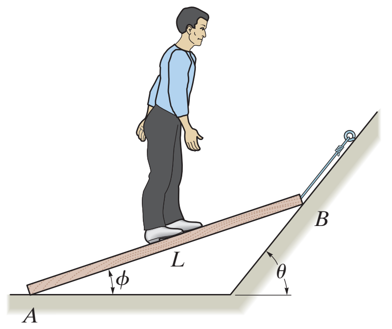
Prob. 5–17

5–18. Determine the components of reaction at the supports A and B on the rod.



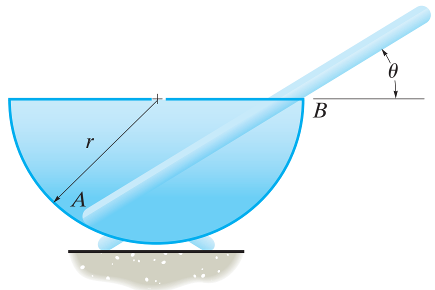
Prob. 5–18

5–19. The man has a weight W and stands at the center of the plank. If the planes at A and B are smooth, determine the tension in the cord in terms of W and θ.



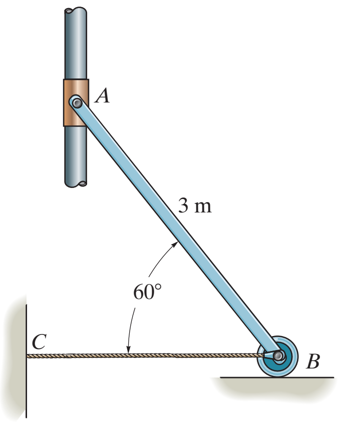
Prob. 5–19

***5–20.** A uniform glass rod having a length L is placed in the smooth hemispherical bowl having a radius r. Determine the angle of inclination θ for equilibrium.



Prob. 5–20

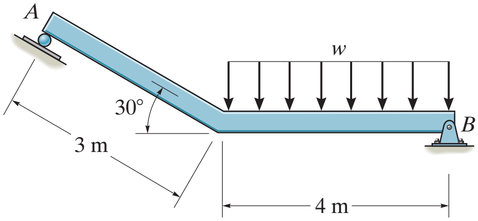
5–21. The uniform rod AB has a mass of 40 kg. Determine the force in the cable when the rod is in the position shown. There is a smooth collar at A.



Prob. 5–21

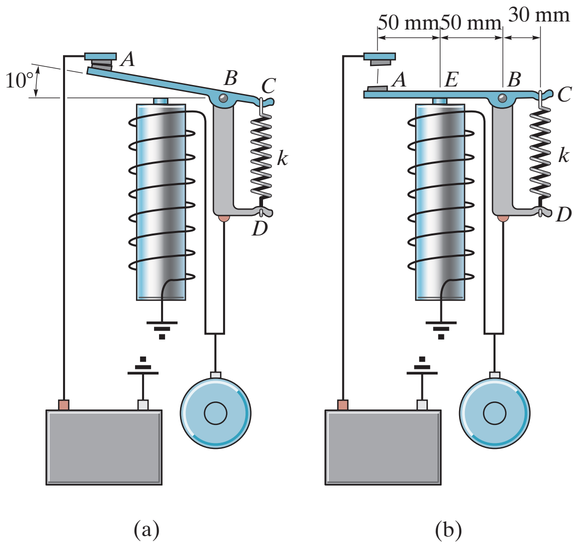
5–22. If the intensity of the distributed load acting on the beam is $w = 3\ \text{kN/m}$, determine the reactions at the roller A and pin B.

5–23. If the roller at A and the pin at B can support a load up to 4 kN and 8 kN, respectively, determine the maximum intensity of the distributed load w, measured in kN/m, so that failure of the supports does not occur.



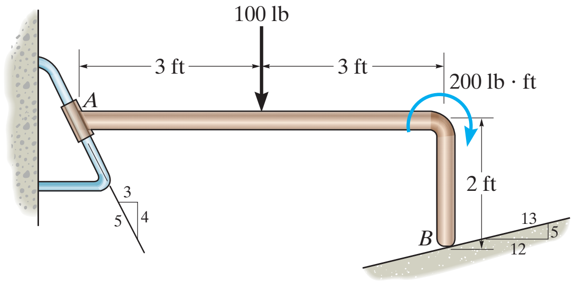
Probs. 5–22/23

***5–24.** The relay regulates voltage and current. Determine the force in the spring CD, which has a stiffness of $k = 120\ \text{N/m}$, so that it will allow the armature to make contact at A in figure (a) with a vertical force of 0.4 N. Also, determine the force in the spring when the coil is energized and attracts the armature to E, figure (b), thereby breaking contact at A.



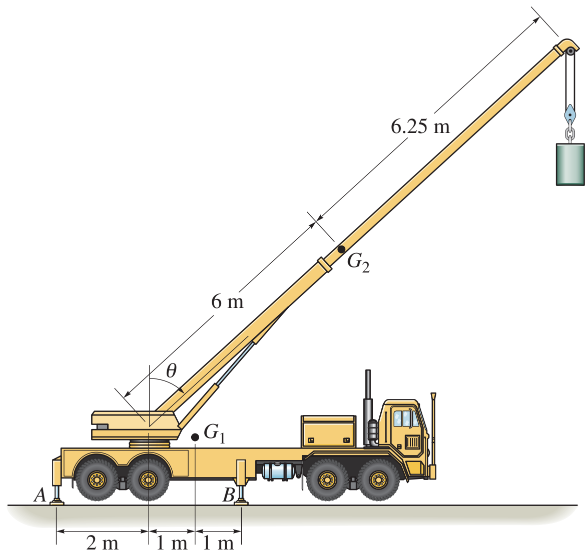
Prob. 5–24

5–25. Determine the reactions on the bent rod which is supported by a smooth surface at B and by a collar at A, which is fixed to the rod and is free to slide over the fixed inclined rod.



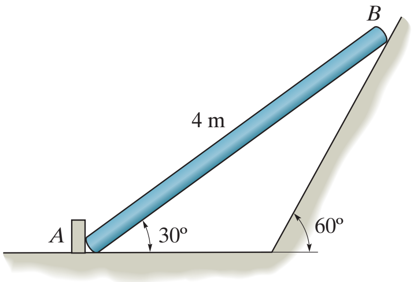
Prob. 5–25

5–26. The mobile crane is symmetrically supported by two outriggers at A and two at B in order to relieve the suspension of the truck upon which it rests and to provide greater stability. If the crane boom and truck have a mass of 18 Mg and center of mass at G_1, and the boom has a mass of 1.8 Mg and a center of mass at G_2, determine the vertical reactions at each of the four outriggers as a function of the boom angle θ when the boom is supporting a load having a mass of 1.2 Mg. Plot the results measured from $\theta = 0°$ to the critical angle where tipping starts to occur.



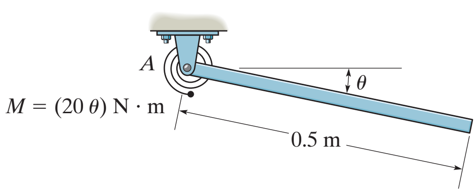
Prob. 5–26

5–27. Determine the reactions acting on the smooth uniform bar, which has a mass of 20 kg.



Prob. 5–27

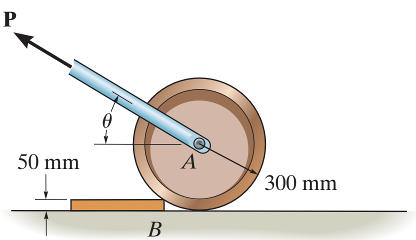
***5–28.** A linear *torsional spring* deforms such that an applied couple moment M is related to the spring's rotation θ in radians by the equation $M = (20\,\theta)\ \text{N}\cdot\text{m}$. If such a spring is attached to the end of a pin-connected uniform 10-kg rod, determine the angle θ for equilibrium. The spring is undeformed when $\theta = 0°$.



Prob. 5–28

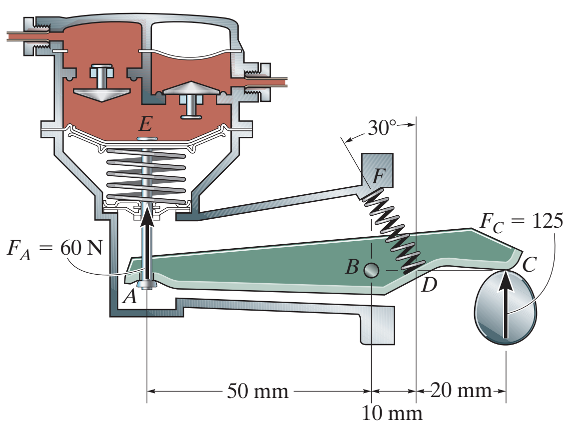
5–29. Determine the force P needed to pull the 50-kg roller over the smooth step. Take $\theta = 30°$.

5–30. Determine the magnitude and direction θ of the minimum force P needed to pull the 50-kg roller over the smooth step.



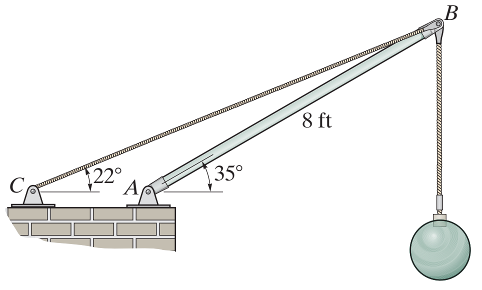
Probs. 5–29/30

5–31. The operation of the fuel pump for an automobile depends on the reciprocating action of the rocker arm ABC, which is pinned at B and is spring loaded at A and D. When the smooth cam C is in the position shown, determine the horizontal and vertical components of force at the pin and the force along the spring DF for equilibrium. The vertical force acting on the rocker arm at A is $F_A = 60$ N, and at C it is $F_C = 125$ N.



Prob. 5–31

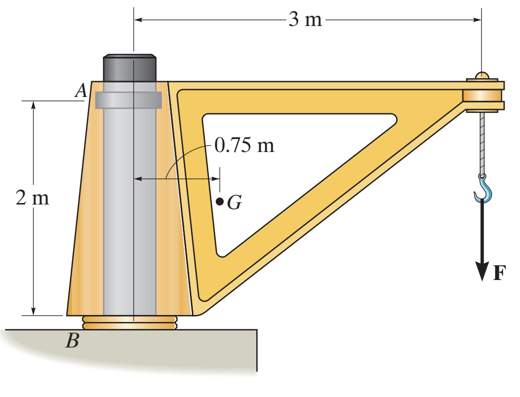
***5–32.** Determine the magnitude of force at the pin A and in the cable BC needed to support the 500-lb load. Neglect the weight of the boom AB.



Prob. 5–32

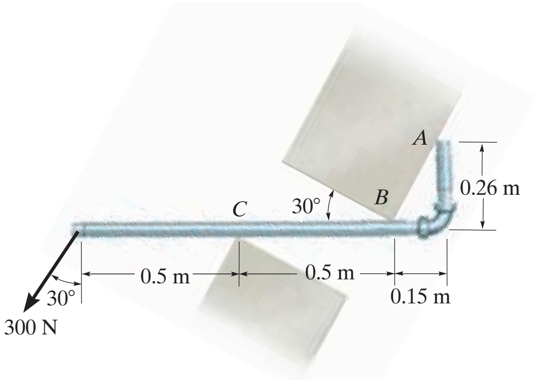
5–33. The dimensions of a jib crane, which is manufactured by the Basick Co., are given in the figure. If the crane has a mass of 800 kg and a center of mass at G, and the maximum rated force at its end is $F = 15$ kN, determine the reactions at its bearings. The bearing at A is a journal bearing and supports only a horizontal force, whereas the bearing at B is a thrust bearing that supports both horizontal and vertical components.

5–34. The dimensions of a jib crane, which is manufactured by the Basick Co., are given in the figure. The crane has a mass of 800 kg and a center of mass at G. The bearing at A is a journal bearing and can support a horizontal force, whereas the bearing at B is a thrust bearing that supports both horizontal and vertical components. Determine the maximum load F that can be suspended from its end if the selected bearings at A and B can sustain a maximum resultant load of 24 kN and 34 kN, respectively.



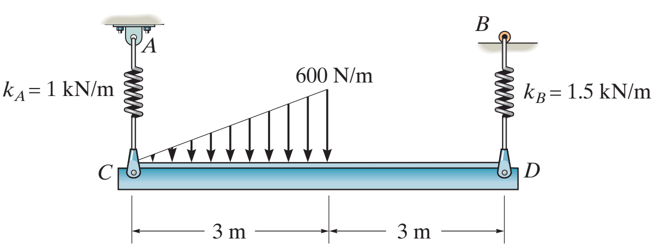
Probs. 5–33/34

5–35. The smooth pipe rests against the opening at the points of contact A, B, and C. Determine the reactions at these points needed to support the force of 300 N. Neglect the pipe's thickness in the calculation.



Prob. 5–35

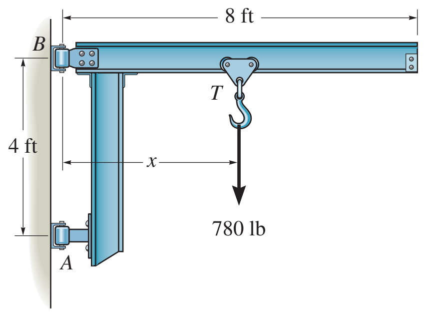
***5–36.** The beam of negligible weight is supported horizontally by two springs. If the beam is horizontal and the springs are unstretched when the load is removed, determine the angle of tilt of the beam when the load is applied.



Prob. 5–36

5–37. The cantilevered jib crane is used to support the load of 780 lb. If $x = 5$ ft, determine the reactions at the supports. Note that the supports are collars that allow the crane to rotate freely about the vertical axis. The collar at B supports a force in the vertical direction, whereas the one at A does not.

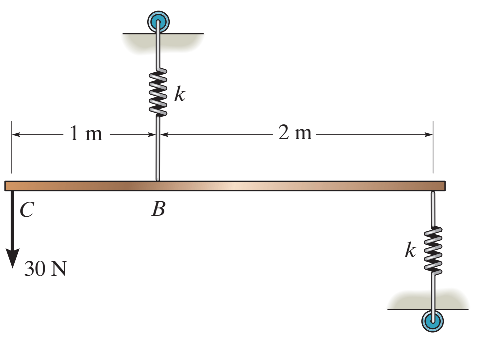
5–38. The cantilevered jib crane is used to support the load of 780 lb. If the trolley T can be placed anywhere between $1.5 \text{ ft} \leq x \leq 7.5 \text{ ft}$, determine the maximum magnitude of reaction at the supports A and B. Note that the supports are collars that allow the crane to rotate freely about the vertical axis. The collar at B supports a force in the vertical direction, whereas the one at A does not.



Probs. 5–37/38

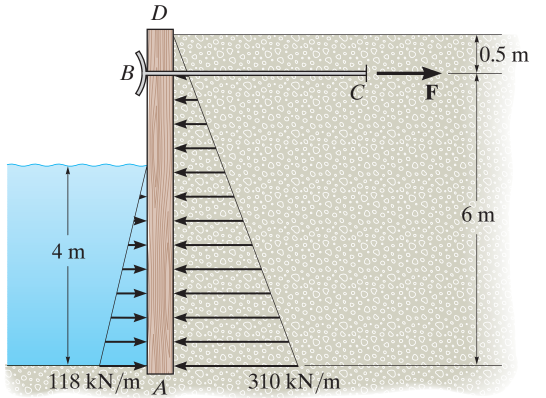
5–39. The bar of negligible weight is supported by two springs, each having a stiffness $k = 100$ N/m. If the springs are originally unstretched, and the force is vertical as shown, determine the angle θ the bar makes with the horizontal, when the 30-N force is applied to the bar.

***5–40.** Determine the stiffness k of each spring so that the 30-N force causes the bar to tip $\theta = 15°$ when the force is applied. Originally the bar is horizontal and the springs are unstretched. Neglect the weight of the bar.



Probs. 5–39/40

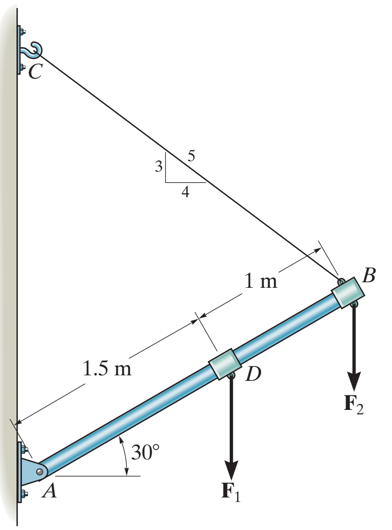
5–41. The bulk head AD is subjected to both water and soil-backfill pressures. Assuming AD is "pinned" to the ground at A, determine the horizontal and vertical reactions there and also the required tension in the ground anchor BC necessary for equilibrium. The bulk head has a mass of 800 kg.



Prob. 5–41

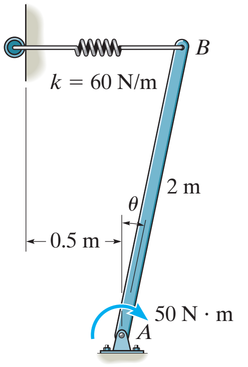
5–42. The boom supports the two vertical loads. Neglect the size of the collars at D and B and the thickness of the boom, and compute the horizontal and vertical components of force at the pin A and the force in cable CB. Set $F_1 = 800$ N and $F_2 = 350$ N.

5–43. The boom is intended to support two vertical loads, $\mathbf{F}_1$ and $\mathbf{F}_2$. If the cable CB can sustain a maximum load of 1500 N before it fails, determine the critical loads if $F_1 = 2F_2$. Also, what is the magnitude of the maximum reaction at pin A?



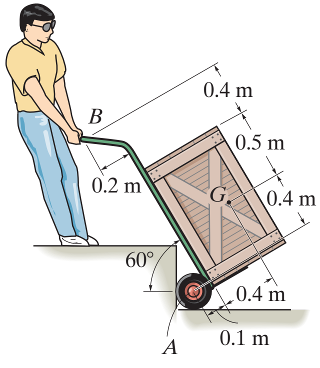
Probs. 5–42/43

***5–44.** The 10-kg uniform rod is pinned at end A. If it is also subjected to a couple moment of 50 N·m, determine the smallest angle θ for equilibrium. The spring is unstretched when $\theta = 0$, and has a stiffness of $k = 60$ N/m.



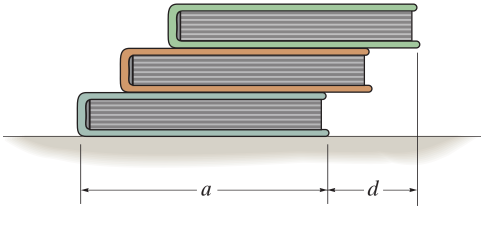
Prob. 5–44

5–45. The man uses the hand truck to move material up the step. If the truck and its contents have a mass of 50 kg with center of gravity at G, determine the normal reaction on both wheels and the magnitude and direction of the minimum force required at the grip B needed to lift the load.



Prob. 5–45

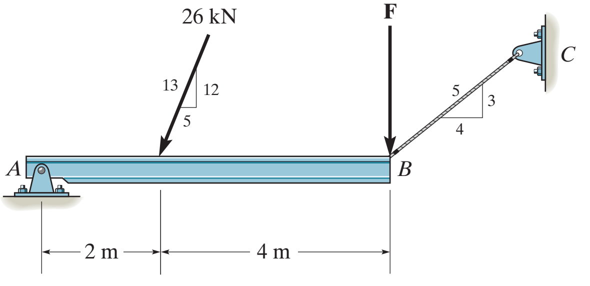
5–46. Three uniform books, each having a weight W and length a, are stacked as shown. Determine the maximum distance d that the top book can extend out from the bottom one so the stack does not topple over.



Prob. 5–46

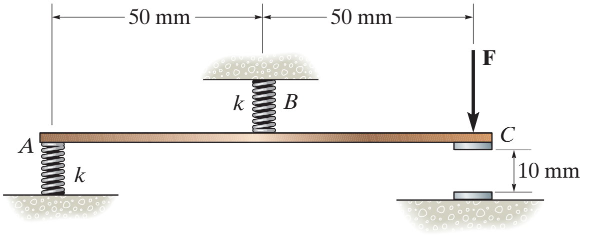
5–47. Determine the reactions at the pin A and the tension in cord BC. Set $F = 40$ kN. Neglect the thickness of the beam.

***5–48.** If rope BC will fail when the tension becomes 50 kN, determine the greatest vertical load F that can be applied to the beam at B. What is the magnitude of the reaction at A for this loading? Neglect the thickness of the beam.



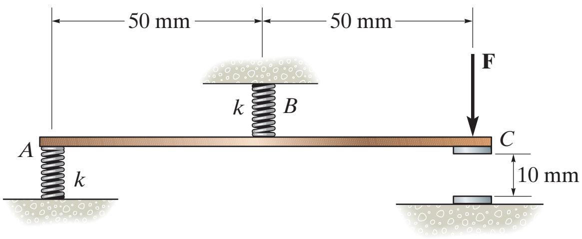
Probs. 5–47/48

5–49. The rigid metal strip of negligible weight is used as part of an electromagnetic switch. If the stiffness of the springs at A and B is $k = 5$ N/m and the strip is originally horizontal when the springs are unstretched, determine the smallest force F needed to close the contact gap at C.



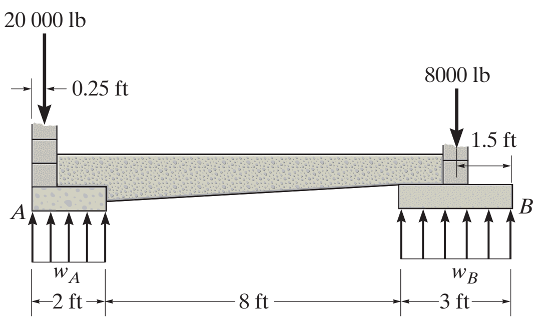
Prob. 5–49

5–50. The rigid metal strip of negligible weight is used as part of an electromagnetic switch. Determine the maximum stiffness k of the springs at A and B so that the contact at C closes when the vertical force developed there is $F = 0.5$ N. Originally the strip is horizontal as shown.



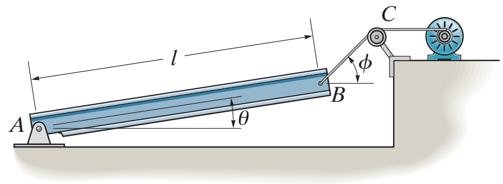
Prob. 5–50

5–51. The cantilever footing is used to support a wall near its edge A so that it causes a uniform soil pressure under the footing. Determine the uniform distribution loads, w_A and w_B, measured in lb/ft at pads A and B, necessary to support the wall forces of 8000 lb and 20 000 lb.



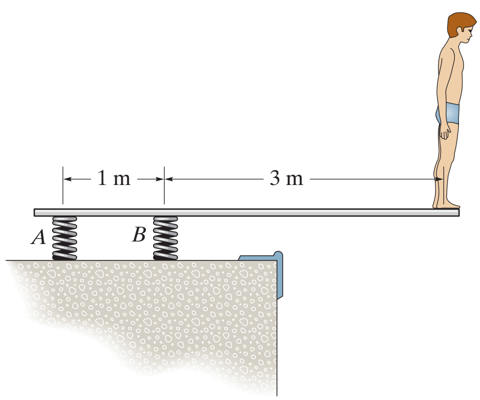
Prob. 5–51

***5–52.** The uniform beam has a weight W and length l and is supported by a pin at A and a cable BC. Determine the horizontal and vertical components of reaction at A and the tension in the cable necessary to hold the beam in the position shown.



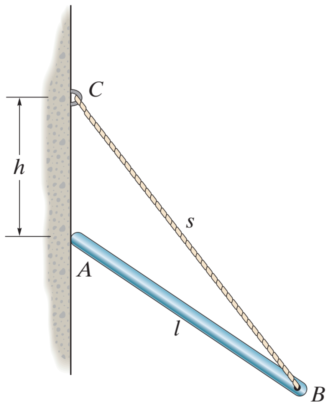
Prob. 5–52

5–53. A boy stands out at the end of the diving board, which is supported by two springs A and B, each having a stiffness of $k = 15$ kN/m. In the position shown the board is horizontal. If the boy has a mass of 40 kg, determine the angle of tilt which the board makes with the horizontal after he jumps off. Neglect the weight of the board and assume it is rigid.



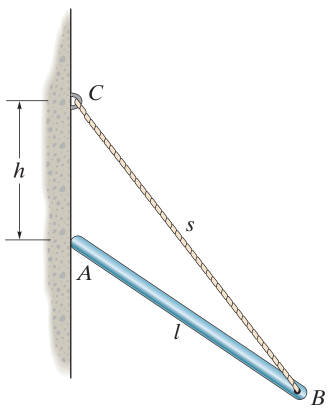
Prob. 5–53

5–54. The 30-N uniform rod has a length of $l = 1$ m. If $s = 1.5$ m, determine the distance h of placement at the end A along the smooth wall for equilibrium.



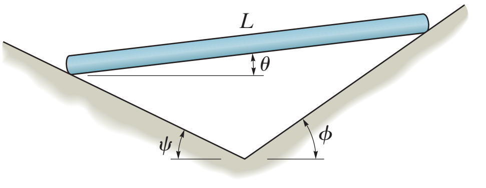
Prob. 5–54

5–55. The uniform rod has a length l and weight W. It is supported at one end A by a smooth wall and the other end by a cord of length s which is attached to the wall as shown. Determine the placement h for equilibrium.



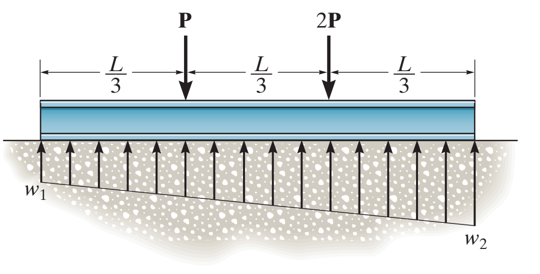
Prob. 5–55

***5–56.** The uniform rod of length L and weight W is supported on the smooth planes. Determine its position θ for equilibrium. Neglect the thickness of the rod.



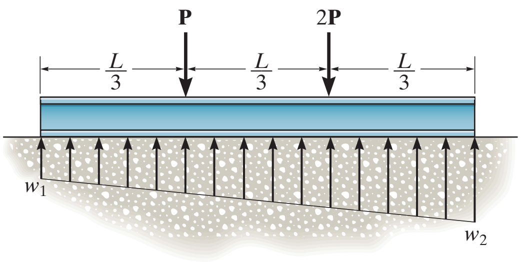
Prob. 5–56

5–57. The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium if $P = 500$ lb and $L = 12$ ft.



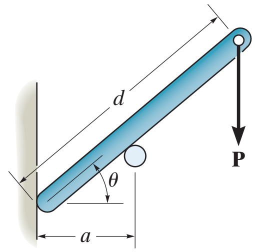
Prob. 5–57

5–58. The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium in terms of the parameters shown.



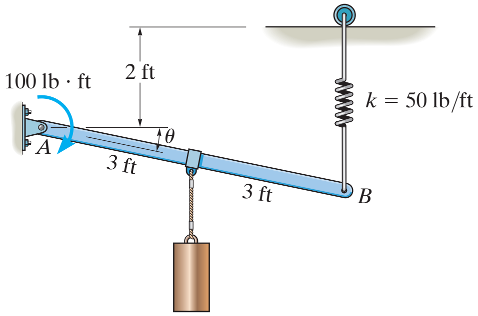
Prob. 5–58

***5–60.** Determine the distance d for placement of the load **P** for equilibrium of the smooth bar in the position θ as shown. Neglect the weight of the bar.



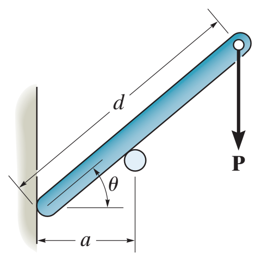
Prob. 5–60

5–59. The rod supports a weight of 200 lb and is pinned at its end A. If it is also subjected to a couple moment of 100 lb · ft, determine the angle θ for equilibrium. The spring has an unstretched length of 2 ft and a stiffness of $k = 50$ lb/ft.



Prob. 5–59

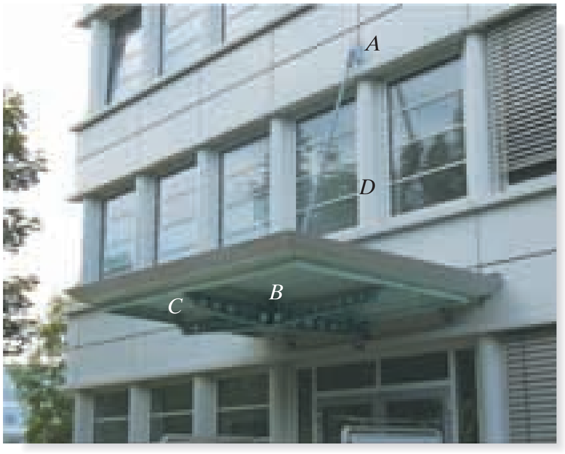
5–61. If $d = 1$ m, and $\theta = 30°$, determine the normal reaction at the smooth supports and the required distance a for the placement of the roller if $P = 600$ N. Neglect the weight of the bar.



Prob. 5–61

CONCEPTUAL PROBLEMS

C5–1. The tie rod is used to support this overhang at the entrance of a building. If it is pin connected to the building wall at A and to the center of the overhang B, determine if the force in the rod will increase, decrease, or remain the same if (a) the support at A is moved to a lower position D, and (b) the support at B is moved to the outer position C. Explain your answer with an equilibrium analysis, using dimensions and loads. Assume the overhang is pin supported from the building wall.



Prob. C5–1 (© Russell C. Hibbeler)

C5–2. The man attempts to pull the four wheeler up the incline and onto the trailer. From the position shown, is it more effective to pull on the rope at A, or would it be better to pull on the rope at B? Draw a free-body diagram for each case, and do an equilibrium analysis to explain your answer. Use appropriate numerical values to do your calculations.

Prob. C5–2 (© Russell C. Hibbeler)

C5–3. Like all aircraft, this jet plane rests on three wheels. Why not use an additional wheel at the tail for better support? (Can you think of any other reason for not including this wheel?) If there was a fourth tail wheel, draw a free-body diagram of the plane from a side (2 D) view, and show why one would not be able to determine all the wheel reactions using the equations of equilibrium.

Prob. C5–3 (© Russell C. Hibbeler)

C5–4. Where is the best place to arrange most of the logs in the wheelbarrow so that it minimizes the amount of force on the backbone of the person transporting the load? Do an equilibrium analysis to explain your answer.

Prob. C5–4 (© Russell C. Hibbeler)

EQUILIBRIUM IN THREE DIMENSIONS

5.5 Free-Body Diagrams

The first step in solving three-dimensional equilibrium problems, as in the case of two dimensions, is to draw a free-body diagram. Before we can do this, however, it is first necessary to discuss the types of reactions that can occur at the supports.

Support Reactions. The reactive forces and couple moments acting at various types of supports and connections, when the members are viewed in three dimensions, are listed in Table 5–2. It is important to recognize the symbols used to represent each of these supports and to understand clearly how the forces and couple moments are developed. As in the two-dimensional case:

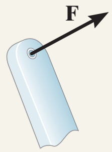
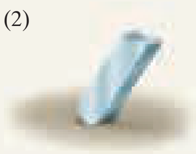
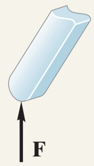
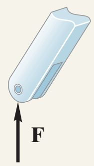
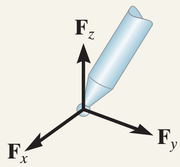
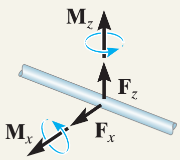
- A force is developed by a support that restricts the translation of its attached member.
- A couple moment is developed when rotation of the attached member is prevented.

For example, in Table 5–2, item (4), the ball-and-socket joint prevents any translation of the connecting member; therefore, a force must act on the member at the point of connection. This force has three components having unknown magnitudes, F_x, F_y, F_z. Provided these components are known, one can obtain the magnitude of force, $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$, and the force's orientation defined by its coordinate direction angles α, β, γ, Eqs. 2–5.* Since the connecting member is allowed to rotate freely about *any* axis, no couple moment is resisted by a ball-and-socket joint.

It should be noted that the *single* bearing supports in items (5) and (7), the *single* pin (8), and the *single* hinge (9) are shown to resist both force and couple-moment components. If, however, these supports are used in conjunction with *other* bearings, pins, or hinges to hold a rigid body in equilibrium and the supports are *properly aligned* when connected to the body, then the *force reactions* at these supports *alone* are adequate for supporting the body. In other words, the couple moments become redundant and are not shown on the free-body diagram. The reason for this should become clear after studying the examples which follow.

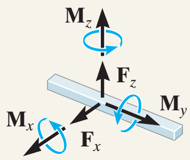
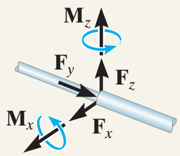
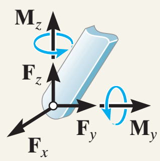
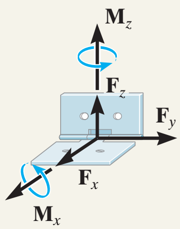
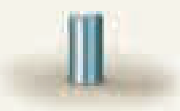
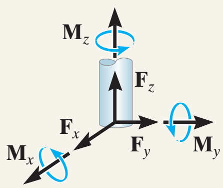
* The three unknowns may also be represented as an unknown force magnitude F and two unknown coordinate direction angles. The third direction angle is obtained using the identity $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, Eq. 2–8

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

continued

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

Typical examples of actual supports that are referenced to Table 5–2 are shown in the following sequence of photos.

This ball-and-socket joint provides a connection for the housing of an earth grader to its frame. (4) (© Russell C. Hibbeler)

The journal bearings support the ends of the shaft. (5) (© Russell C. Hibbeler)

This thrust bearing is used to support the drive shaft on a machine. (7) (© Russell C. Hibbeler)

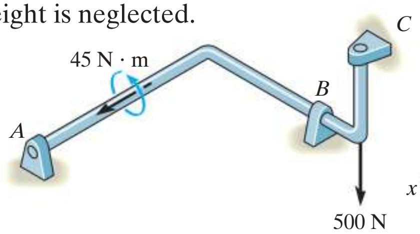
This pin is used to support the end of the strut used on a tractor. (8) (© Russell C. Hibbeler)

Free-Body Diagrams. The general procedure for establishing the free-body diagram of a rigid body has been outlined in Sec. 5.2. Essentially it requires first "isolating" the body by drawing its outlined shape. This is followed by a careful *labeling* of *all* the forces and couple moments with reference to an established x, y, z coordinate system. As a general rule, it is suggested to show the unknown components of reaction as acting on the free-body diagram in the *positive sense*. In this way, if any negative values are obtained, they will indicate that the components act in the negative coordinate directions.

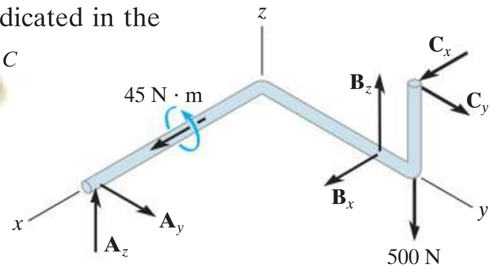
EXAMPLE 5.14

Consider the two rods and plate, along with their associated free-body diagrams, shown in Fig. 5–23. The *x, y, z* axes are established on the diagram and the unknown reaction components are indicated in the *positive sense*. The weight is neglected.

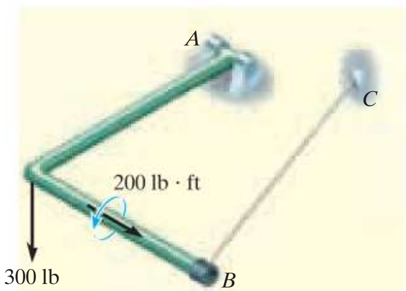
SOLUTION



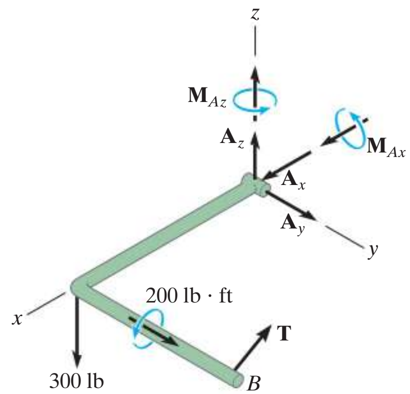
Properly aligned journal bearings at *A*, *B*, *C*.



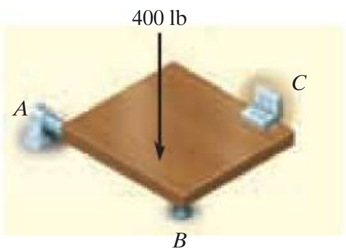
The force reactions developed by the bearings are *sufficient* for equilibrium since they prevent the shaft from rotating about each of the coordinate axes. No couple moments at each bearing are developed.



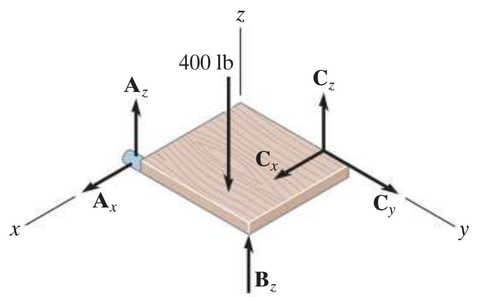
Pin at *A* and cable *BC*.



Moment components are developed by the pin on the rod to prevent rotation about the *x* and *z* axes.



Properly aligned journal bearing at *A* and hinge at *C*. Roller at *B*.



Only force reactions are developed by the bearing and hinge on the plate to prevent rotation about each coordinate axis. No moments are developed at the hinge.

Fig. 5–23

5.6 Equations of Equilibrium

As stated in Sec. 5.1, the conditions for equilibrium of a rigid body subjected to a three-dimensional force system require that both the *resultant* force and *resultant* couple moment acting on the body be equal to *zero*.

Vector Equations of Equilibrium. The two conditions for equilibrium of a rigid body may be expressed mathematically in vector form as

$$\boxed{\begin{aligned}\Sigma \mathbf{F} &= \mathbf{0}\\ \Sigma \mathbf{M}_O &= \mathbf{0}\end{aligned}} \tag{5–5}$$

where $\Sigma \mathbf{F}$ is the vector sum of all the external forces acting on the body and $\Sigma \mathbf{M}_O$ is the sum of the couple moments and the moments of all the forces about any point O located either on or off the body.

Scalar Equations of Equilibrium. If all the external forces and couple moments are expressed in Cartesian vector form and substituted into Eqs. 5–5, we have

$$\Sigma \mathbf{F} = \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = \mathbf{0}$$

$$\Sigma \mathbf{M}_O = \Sigma M_x \mathbf{i} + \Sigma M_y \mathbf{j} + \Sigma M_z \mathbf{k} = \mathbf{0}$$

Since the **i**, **j**, and **k** components are independent from one another, the above equations are satisfied provided

$$\boxed{\begin{aligned}\Sigma F_x &= 0\\ \Sigma F_y &= 0\\ \Sigma F_z &= 0\end{aligned}} \tag{5–6a}$$

and

$$\boxed{\begin{aligned}\Sigma M_x &= 0\\ \Sigma M_y &= 0\\ \Sigma M_z &= 0\end{aligned}} \tag{5–6b}$$

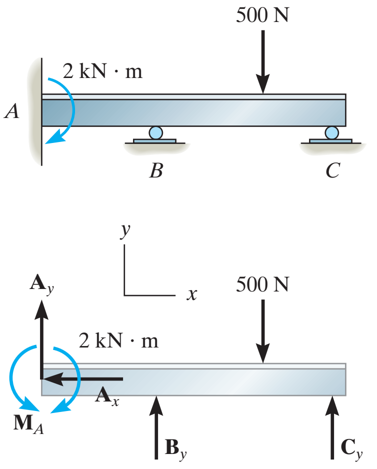
These *six scalar equilibrium equations* may be used to solve for at most six unknowns shown on the free-body diagram. Equations 5–6*a* require the sum of the external force components acting in the x, y, and z directions to be zero, and Eqs. 5–6*b* require the sum of the moment components about the x, y, and z axes to be zero.

5.7 Constraints and Statical Determinacy

To ensure the equilibrium of a rigid body, it is not only necessary to satisfy the equations of equilibrium, but the body must also be properly held or constrained by its supports. Some bodies may have more supports than are necessary for equilibrium, whereas others may not have enough or the supports may be arranged in a particular manner that could cause the body to move. Each of these cases will now be discussed.

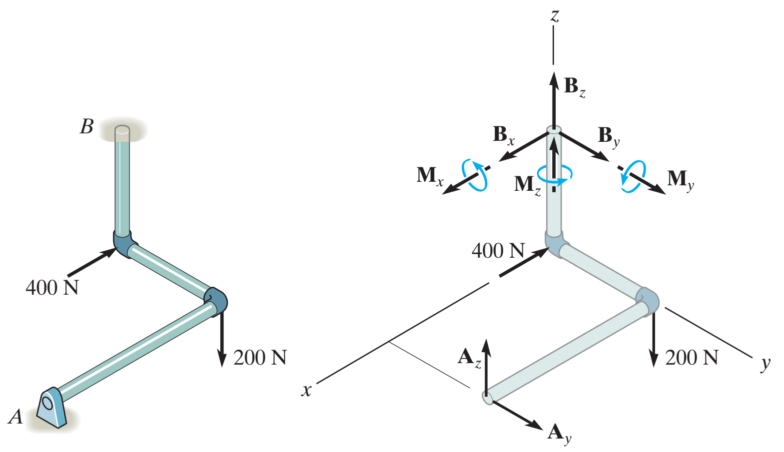
Redundant Constraints. When a body has redundant supports, that is, more supports than are necessary to hold it in equilibrium, it becomes statically indeterminate. ***Statically indeterminate*** means that there will be more unknown loadings on the body than equations of equilibrium available for their solution. For example, the beam in Fig. 5–24*a* and the pipe assembly in Fig. 5–24*b*, shown together with their free-body diagrams, are both statically indeterminate because of additional (or redundant) support reactions. For the beam there are five unknowns, M_A, A_x, A_y, B_y, and C_y, for which only three equilibrium equations can be written ($\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M_O = 0$, Eq. 5–2). The pipe assembly has eight unknowns, for which only six equilibrium equations can be written, Eqs. 5–6.

The additional equations needed to solve statically indeterminate problems of the type shown in Fig. 5–24 are generally obtained from the deformation conditions at the points of support. These equations involve the physical properties of the body which are studied in subjects dealing with the mechanics of deformation, such as “mechanics of materials.”*



(a)

Fig. 5–24

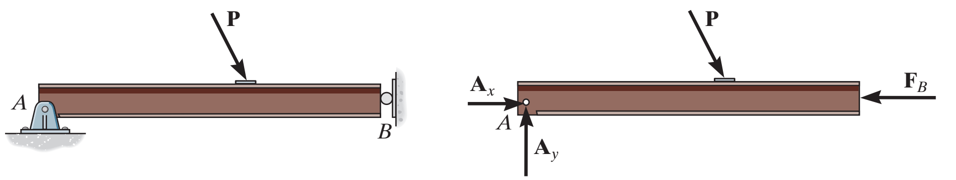


(b)

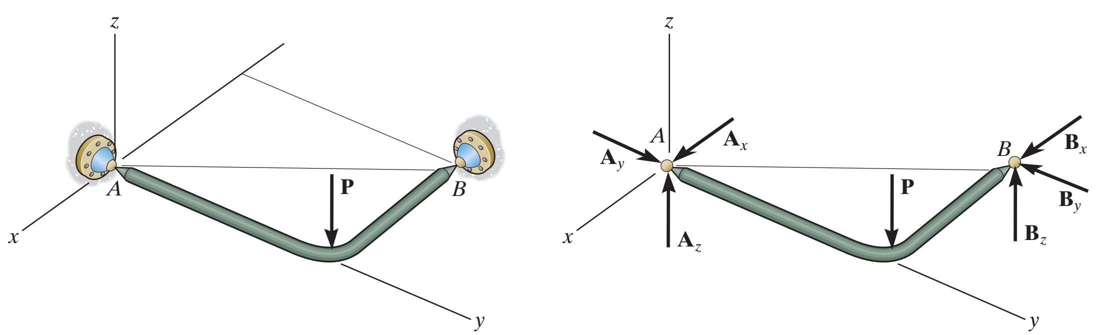
* See R. C. Hibbeler, *Mechanics of Materials*, 8th edition, Pearson Education/Prentice Hall, Inc.

Improper Constraints. Having the same number of unknown reactive forces as available equations of equilibrium does not always guarantee that a body will be stable when subjected to a particular loading. For example, the pin support at A and the roller support at B for the beam in Fig. 5–25*a* are placed in such a way that the lines of action of the reactive forces are *concurrent* at point A. Consequently, the applied loading $\mathbf{P}$ will cause the beam to rotate slightly about A, and so the beam is improperly constrained, $\Sigma M_A \neq 0$.

In three dimensions, a body will be improperly constrained if the lines of action of all the reactive forces intersect a common axis. For example, the reactive forces at the ball-and-socket supports at A and B in Fig. 5–25*b* all intersect the axis passing through A and B. Since the moments of these forces about A and B are all zero, then the loading $\mathbf{P}$ will rotate the member about the AB axis, $\Sigma M_{AB} \neq 0$.



(a)



(b)

Fig. 5–25

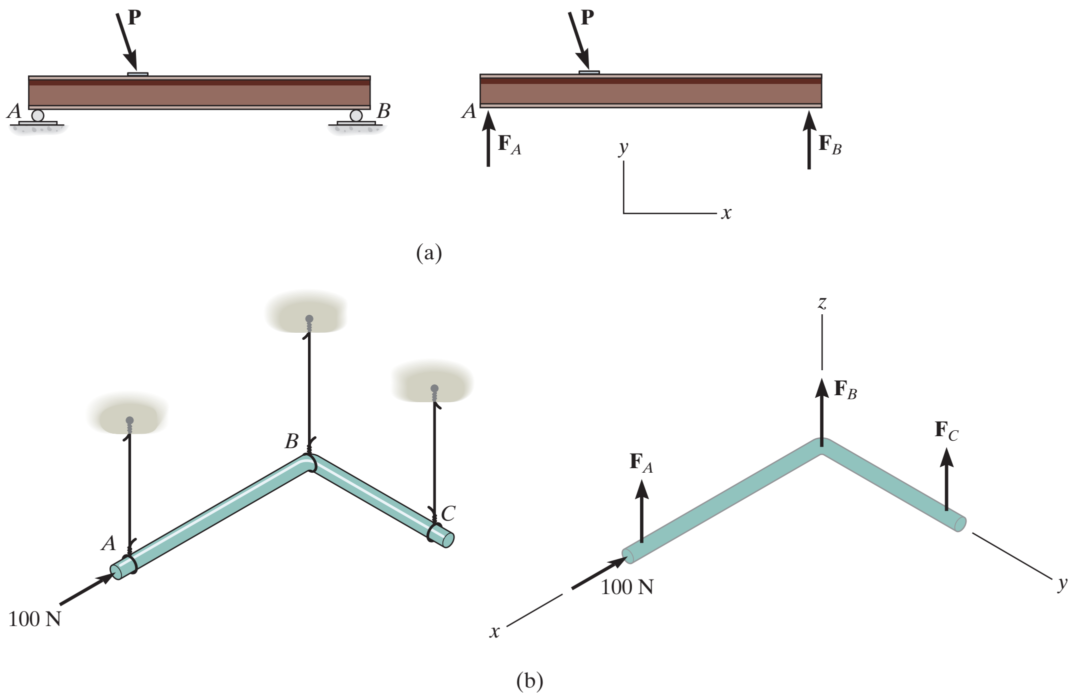


Fig. 5–26

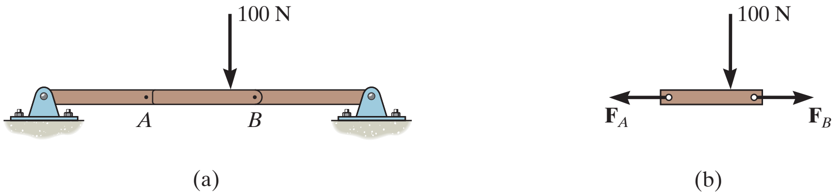


Fig. 5–27

Another way in which improper constraining leads to instability occurs when the *reactive forces* are all *parallel*. Two- and three-dimensional examples of this are shown in Fig. 5–26. In both cases, the summation of forces along the x axis will not equal zero.

In some cases, a body may have *fewer* reactive forces than equations of equilibrium that must be satisfied. The body then becomes only *partially constrained*. For example, consider member AB in Fig. 5–27*a* with its corresponding free-body diagram in Fig. 5–27*b*. Here $\Sigma F_y = 0$ will not be satisfied for the loading conditions and therefore equilibrium will not be maintained.

To summarize these points, a body is considered ***improperly constrained*** if all the reactive forces intersect at a common point or pass through a common axis, or if all the reactive forces are parallel. In engineering practice, these situations should be avoided at all times since they will cause an unstable condition.

Stability is always an important concern when operating a crane, not only when lifting a load, but also when moving it about. (© Russell C. Hibbeler)

Important Points

- Always draw the free-body diagram first when solving any equilibrium problem.
- If a support *prevents translation* of a body in a specific direction, then the support exerts a *force* on the body in that direction.
- If a support *prevents rotation about an axis*, then the support exerts a *couple moment* on the body about the axis.
- If a body is subjected to more unknown reactions than available equations of equilibrium, then the problem is *statically indeterminate*.
- A stable body requires that the lines of action of the reactive forces do not intersect a common axis and are not parallel to one another.

Procedure for Analysis

Three-dimensional equilibrium problems for a rigid body can be solved using the following procedure.

Free-Body Diagram.

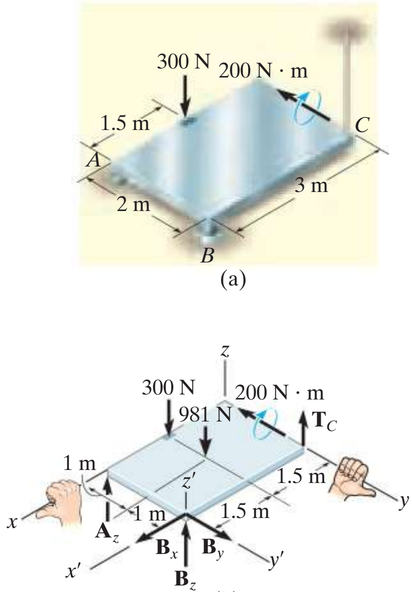
- Draw an outlined shape of the body.
- Show all the forces and couple moments acting on the body.
- Establish the origin of the x, y, z axes at a convenient point and orient the axes so that they are parallel to as many of the external forces and moments as possible.
- Label all the loadings and specify their directions. In general, show all the *unknown* components having a *positive sense* along the x, y, z axes.
- Indicate the dimensions of the body necessary for computing the moments of forces.

Equations of Equilibrium.

- If the x, y, z force and moment components seem easy to determine, then apply the six scalar equations of equilibrium; otherwise use the vector equations.
- It is not necessary that the set of axes chosen for force summation coincide with the set of axes chosen for moment summation. Actually, an axis in any arbitrary direction may be chosen for summing forces and moments.
- Choose the direction of an axis for moment summation such that it intersects the lines of action of as many unknown forces as possible. Realize that the moments of forces passing through points on this axis and the moments of forces which are parallel to the axis will then be zero.
- If the solution of the equilibrium equations yields a negative scalar for a force or couple moment magnitude, it indicates that the sense is opposite to that assumed on the free-body diagram.

EXAMPLE 5.15

The homogeneous plate shown in Fig. 5–28*a* has a mass of 100 kg and is subjected to a force and couple moment along its edges. If it is supported in the horizontal plane by a roller at A, a ball-and-socket joint at B, and a cord at C, determine the components of reaction at these supports.



(a)

(b)

Fig. 5–28

SOLUTION (*SCALAR ANALYSIS*)

Free-Body Diagram. There are five unknown reactions acting on the plate, as shown in Fig. 5–28*b*. Each of these reactions is assumed to act in a positive coordinate direction.

Equations of Equilibrium. Since the three-dimensional geometry is rather simple, a *scalar analysis* provides a *direct solution* to this problem. A force summation along each axis yields

$\Sigma F_x = 0; \qquad B_x = 0$ *Ans.*

$\Sigma F_y = 0; \qquad B_y = 0$ *Ans.*

$$\Sigma F_z = 0; \qquad A_z + B_z + T_C - 300\text{ N} - 981\text{ N} = 0 \tag{1}$$

Recall that the moment of a force about an axis is equal to the product of the force magnitude and the perpendicular distance (moment arm) from the line of action of the force to the axis. Also, forces that are parallel to an axis or pass through it create no moment about the axis. Hence, summing moments about the positive x and y axes, we have

$$\Sigma M_x = 0; \qquad T_C(2\text{ m}) - 981\text{ N}(1\text{ m}) + B_z(2\text{ m}) = 0 \tag{2}$$

$$\Sigma M_y = 0; \qquad 300\text{ N}(1.5\text{ m}) + 981\text{ N}(1.5\text{ m}) - B_z(3\text{ m}) - A_z(3\text{ m}) - 200\text{ N}\cdot\text{m} = 0 \tag{3}$$

The components of the force at B can be eliminated if moments are summed about the x' and y' axes. We obtain

$$\Sigma M_{x'} = 0; \qquad 981\text{ N}(1\text{ m}) + 300\text{ N}(2\text{ m}) - A_z(2\text{ m}) = 0 \tag{4}$$

$$\Sigma M_{y'} = 0; \qquad -300\text{ N}(1.5\text{ m}) - 981\text{ N}(1.5\text{ m}) - 200\text{ N}\cdot\text{m} + T_C(3\text{ m}) = 0 \tag{5}$$

Solving Eqs. 1 through 3 or the more convenient Eqs. 1, 4, and 5 yields

$A_z = 790\text{ N} \quad B_z = -217\text{ N} \quad T_C = 707\text{ N}$ *Ans.*

The negative sign indicates that $\mathbf{B}_z$ acts downward.

NOTE: The solution of this problem does not require a summation of moments about the z axis. The plate is partially constrained since the supports cannot prevent it from turning about the z axis if a force is applied to it in the x–y plane.

EXAMPLE 5.16

Determine the components of reaction that the ball-and-socket joint at A, the smooth journal bearing at B, and the roller support at C exert on the rod assembly in Fig. 5–29*a*.

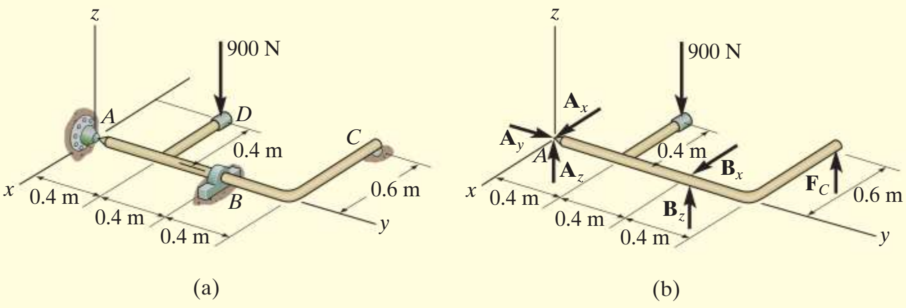


Fig. 5–29

SOLUTION (*SCALAR ANALYSIS*)

Free-Body Diagram. As shown on the free-body diagram, Fig. 5–29*b*, the reactive forces of the supports will prevent the assembly from rotating about each coordinate axis, and so the journal bearing at B only exerts reactive forces on the member. No couple moments are required.

Equations of Equilibrium. Because all the forces are either horizontal or vertical, it is convenient to use a scalar analysis. A direct solution for A_y can be obtained by summing forces along the y axis.

$\Sigma F_y = 0;\qquad A_y = 0$ *Ans.*

The force F_C can be determined directly by summing moments about the y axis.

$\Sigma M_y = 0;\qquad F_C(0.6\text{ m}) - 900\text{ N}(0.4\text{ m}) = 0$

$F_C = 600\text{ N}$ *Ans.*

Using this result, B_z can be determined by summing moments about the x axis.

$\Sigma M_x = 0;\qquad B_z(0.8\text{ m}) + 600\text{ N}(1.2\text{ m}) - 900\text{ N}(0.4\text{ m}) = 0$

$B_z = -450\text{ N}$ *Ans.*

The negative sign indicates that $\mathbf{B}_z$ acts downward. The force B_x can be found by summing moments about the z axis.

$\Sigma M_z = 0;\qquad -B_x(0.8\text{ m}) = 0\quad B_x = 0$ *Ans.*

Thus,

$\Sigma F_x = 0;\qquad A_x + 0 = 0\qquad A_x = 0$ *Ans.*

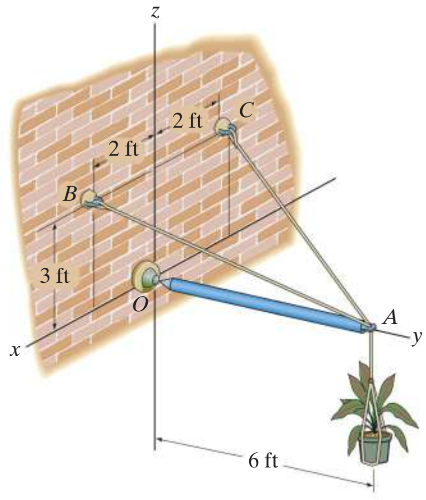
Finally, using the results of B_z and F_C.

$\Sigma F_z = 0;\qquad A_z + (-450\text{ N}) + 600\text{ N} - 900\text{ N} = 0$

$A_z = 750\text{ N}$ *Ans.*

EXAMPLE 5.17

The boom is used to support the 75-lb flowerpot in Fig. 5–30*a*. Determine the tension developed in wires *AB* and *AC*.



(a)

Fig. 5–30

SOLUTION (*VECTOR ANALYSIS*)

Free-Body Diagram. The free-body diagram of the boom is shown in Fig. 5–30*b*.

Equations of Equilibrium. Here the cable forces are directed at angles with the coordinate axes, so we will use a vector analysis.

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = F_{AB}\left(\frac{\{2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\text{ ft}}{\sqrt{(2\text{ ft})^2 + (-6\text{ ft})^2 + (3\text{ ft})^2}}\right)$$

$$= \tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = F_{AC}\left(\frac{\{-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\text{ ft}}{\sqrt{(-2\text{ ft})^2 + (-6\text{ ft})^2 + (3\text{ ft})^2}}\right)$$

$$= -\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}$$

We can eliminate the force reaction at *O* by writing the moment equation of equilibrium about point *O*.

$$\Sigma\mathbf{M}_O = \mathbf{0}; \qquad \mathbf{r}_A \times (\mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{W}) = \mathbf{0}$$

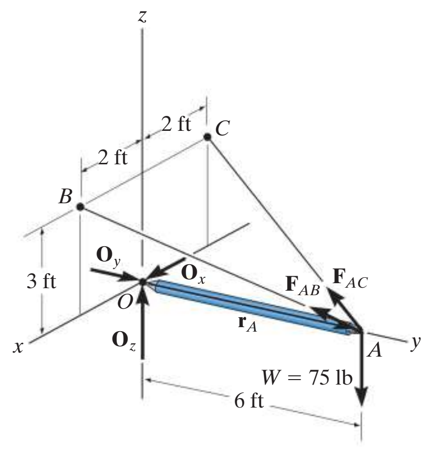
$$(6\mathbf{j}) \times \left[\left(\tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}\right) + \left(-\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}\right) + (-75\mathbf{k})\right] = \mathbf{0}$$

$$\left(\tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450\right)\mathbf{i} + \left(-\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC}\right)\mathbf{k} = \mathbf{0}$$

$$\Sigma M_x = 0; \qquad \tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450 = 0 \qquad (1)$$

$$\Sigma M_y = 0; \qquad 0 = 0$$

$$\Sigma M_z = 0; \qquad -\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC} = 0 \qquad (2)$$

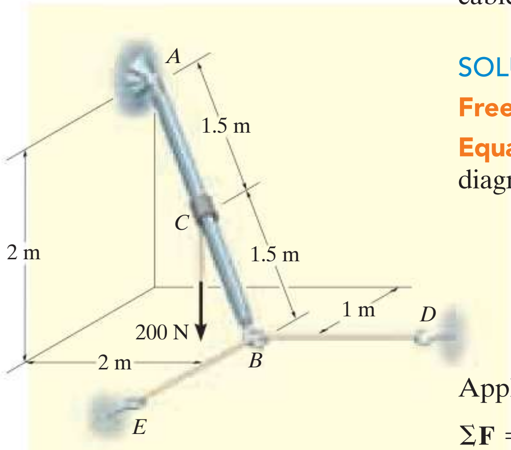


(b)

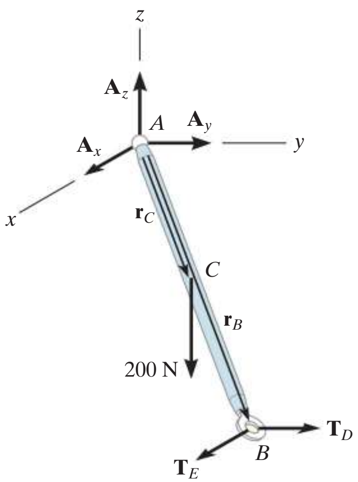
Solving Eqs. (1) and (2) simultaneously,

$$F_{AB} = F_{AC} = 87.5\text{ lb} \qquad \textit{Ans.}$$

Rod AB shown in Fig. 5–31a is subjected to the 200-N force. Determine the reactions at the ball-and-socket joint A and the tension in the cables BD and BE. The collar at C is fixed to the rod.



(a)



(b)

Fig. 5–31

SOLUTION (*VECTOR ANALYSIS*)

Free-Body Diagram. Fig. 5–31b.

Equations of Equilibrium. Representing each force on the free-body diagram in Cartesian vector form, we have

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$
$$\mathbf{T}_E = T_E\mathbf{i}$$
$$\mathbf{T}_D = T_D\mathbf{j}$$
$$\mathbf{F} = \{-200\mathbf{k}\}\text{ N}$$

Applying the force equation of equilibrium.

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_A + \mathbf{T}_E + \mathbf{T}_D + \mathbf{F} = \mathbf{0}$$
$$(A_x + T_E)\mathbf{i} + (A_y + T_D)\mathbf{j} + (A_z - 200)\mathbf{k} = \mathbf{0}$$

$$\Sigma F_x = 0; \qquad A_x + T_E = 0 \qquad (1)$$
$$\Sigma F_y = 0; \qquad A_y + T_D = 0 \qquad (2)$$
$$\Sigma F_z = 0; \qquad A_z - 200 = 0 \qquad (3)$$

Summing moments about point A yields

$$\Sigma\mathbf{M}_A = \mathbf{0}; \qquad \mathbf{r}_C \times \mathbf{F} + \mathbf{r}_B \times (\mathbf{T}_E + \mathbf{T}_D) = \mathbf{0}$$

Since $\mathbf{r}_C = \frac{1}{2}\mathbf{r}_B$, then

$$(0.5\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}) \times (-200\mathbf{k}) + (1\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) \times (T_E\mathbf{i} + T_D\mathbf{j}) = \mathbf{0}$$

Expanding and rearranging terms gives

$$(2T_D - 200)\mathbf{i} + (-2T_E + 100)\mathbf{j} + (T_D - 2T_E)\mathbf{k} = \mathbf{0}$$

$$\Sigma M_x = 0; \qquad 2T_D - 200 = 0 \qquad (4)$$
$$\Sigma M_y = 0; \qquad -2T_E + 100 = 0 \qquad (5)$$
$$\Sigma M_z = 0; \qquad T_D - 2T_E = 0 \qquad (6)$$

Solving Eqs. 1 through 5, we get

$$T_D = 100\text{ N} \qquad \textit{Ans.}$$
$$T_E = 50\text{ N} \qquad \textit{Ans.}$$
$$A_x = -50\text{ N} \qquad \textit{Ans.}$$
$$A_y = -100\text{ N} \qquad \textit{Ans.}$$
$$A_z = 200\text{ N} \qquad \textit{Ans.}$$

NOTE: The negative sign indicates that $\mathbf{A}_x$ and $\mathbf{A}_y$ have a sense which is opposite to that shown on the free-body diagram, Fig. 5–31b. Also, notice that Eqs. 1–6 can be set up *directly* using a scalar analysis.

EXAMPLE 5.19

The bent rod in Fig. 5–32*a* is supported at A by a journal bearing, at D by a ball-and-socket joint, and at B by means of cable BC. Using only *one equilibrium equation*, obtain a direct solution for the tension in cable BC. The bearing at A is capable of exerting force components only in the z and y directions since it is properly aligned on the shaft. In other words, no couple moments are required at this support.

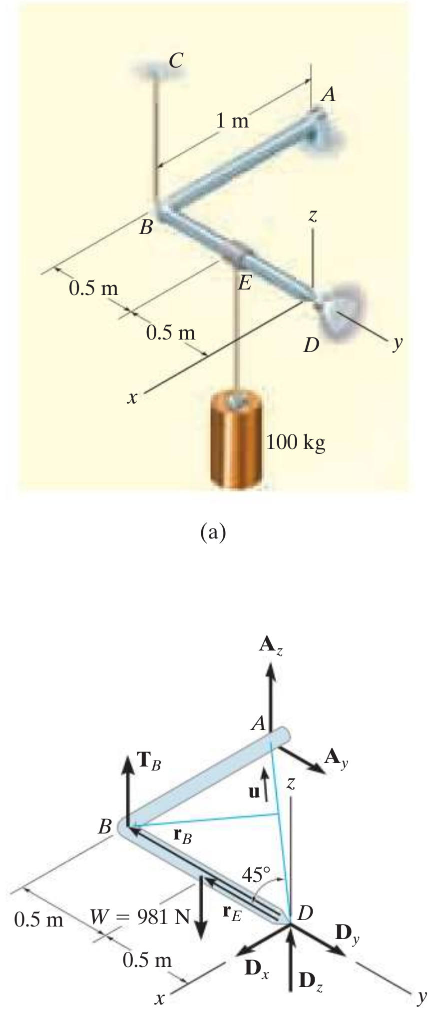


Fig. 5–32

SOLUTION (*VECTOR ANALYSIS*)

Free-Body Diagram. As shown in Fig. 5–32*b*, there are six unknowns.

Equations of Equilibrium. The cable tension $\mathbf{T}_B$ may be obtained *directly* by summing moments about an axis that passes through points D and A. Why? The direction of this axis is defined by the unit vector $\mathbf{u}$, where

$$\mathbf{u} = \frac{\mathbf{r}_{DA}}{r_{DA}} = -\frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$$
$$= -0.7071\mathbf{i} - 0.7071\mathbf{j}$$

Hence, the sum of the moments about this axis is zero provided

$$\Sigma M_{DA} = \mathbf{u} \cdot \Sigma(\mathbf{r} \times \mathbf{F}) = 0$$

Here $\mathbf{r}$ represents a position vector drawn from *any point* on the axis DA to any point on the line of action of force $\mathbf{F}$ (see Eq. 4–11). With reference to Fig. 5–32*b*, we can therefore write

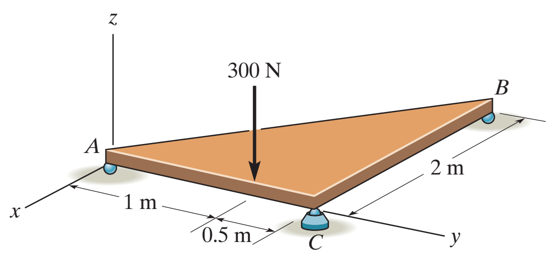
$$\mathbf{u} \cdot (\mathbf{r}_B \times \mathbf{T}_B + \mathbf{r}_E \times \mathbf{W}) = \mathbf{0}$$
$$(-0.7071\mathbf{i} - 0.7071\mathbf{j}) \cdot \big[(-1\mathbf{j}) \times (T_B\mathbf{k}) + (-0.5\mathbf{j}) \times (-981\mathbf{k})\big] = \mathbf{0}$$
$$(-0.7071\mathbf{i} - 0.7071\mathbf{j}) \cdot [(-T_B + 490.5)\mathbf{i}] = \mathbf{0}$$
$$-0.7071(-T_B + 490.5) + 0 + 0 = 0$$
$$T_B = 490.5 \text{ N}$$ *Ans.*

NOTE: Since the moment arms from the axis to $\mathbf{T}_B$ and $\mathbf{W}$ are easy to obtain, we can also determine this result using a scalar analysis. As shown in Fig. 5–32*b*,

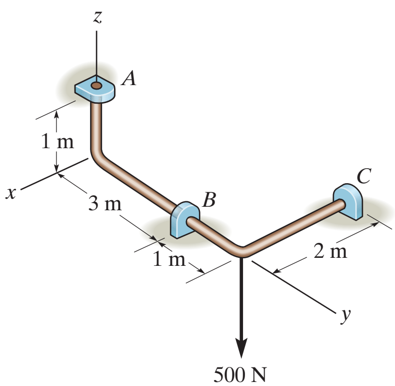
$$\Sigma M_{DA} = 0; \quad T_B(1 \text{ m} \sin 45^\circ) - 981 \text{ N}(0.5 \text{ m} \sin 45^\circ) = 0$$
$$T_B = 490.5 \text{ N}$$ *Ans.*

PRELIMINARY PROBLEMS

P5–2. Draw the free-body diagram of each object.



(a)



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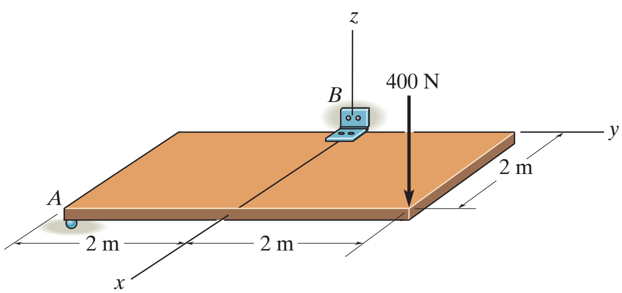
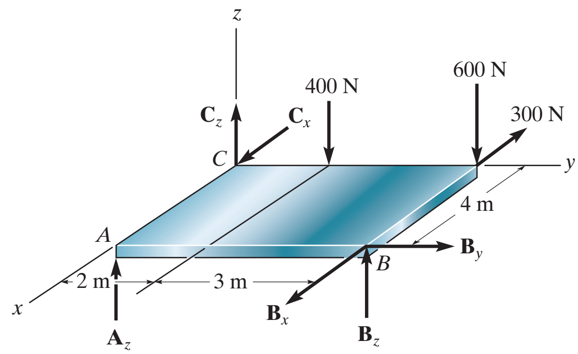


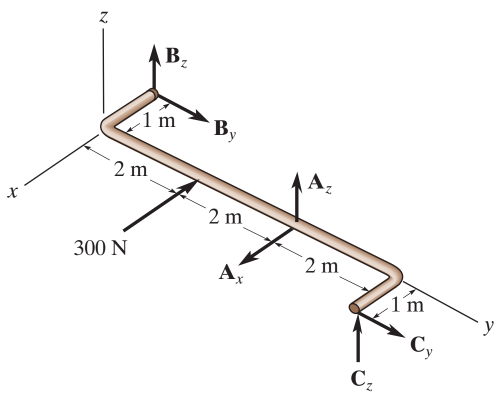
(c)

Prob. P5–2

P5–3. In each case, write the moment equations about the x, y, and z axes.



(a)



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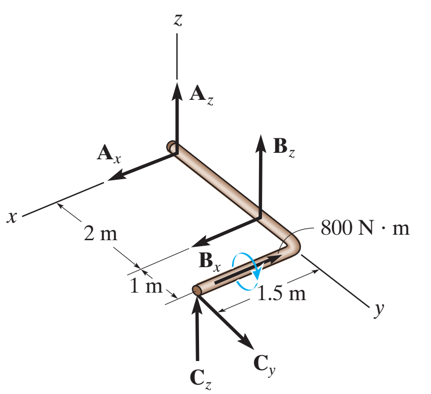


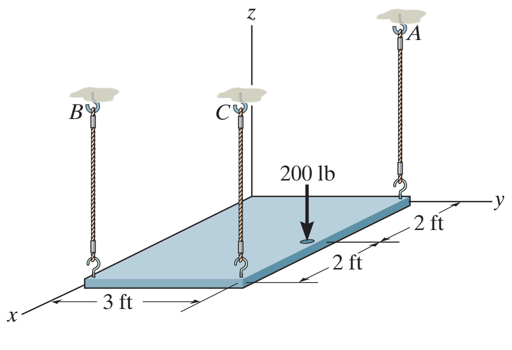
(c)

Prob. P5–3

FUNDAMENTAL PROBLEMS

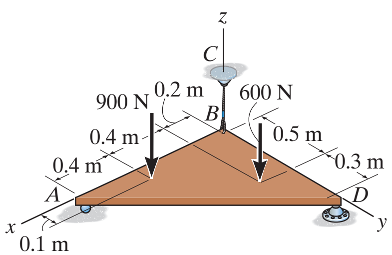
All problem solutions must include an FBD.

F5–7. The uniform plate has a weight of 500 lb. Determine the tension in each of the supporting cables.



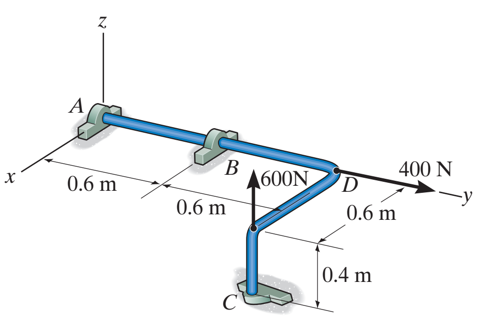
Prob. F5–7

F5–8. Determine the reactions at the roller support A, the ball-and-socket joint D, and the tension in cable BC for the plate.



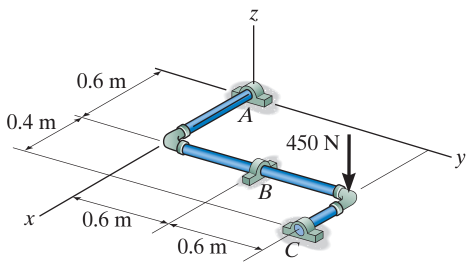
Prob. F5–8

F5–9. The rod is supported by smooth journal bearings at A, B, and C and is subjected to the two forces. Determine the reactions at these supports.



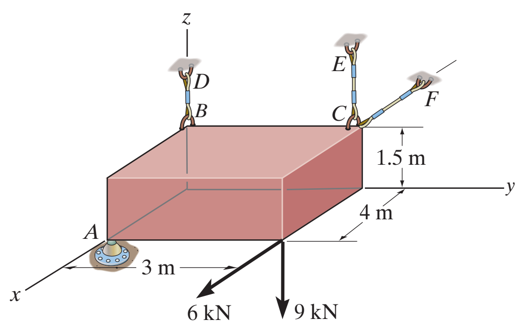
Prob. F5–9

F5–10. Determine the support reactions at the smooth journal bearings A, B, and C of the pipe assembly.



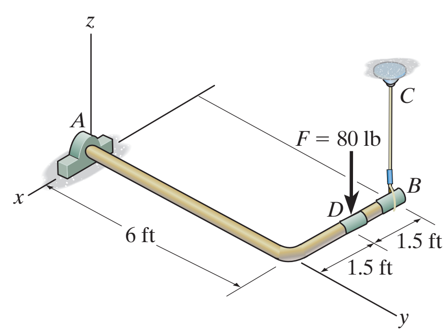
Prob. F5–10

F5–11. Determine the force developed in the short link BD, and the tension in the cords CE and CF, and the reactions of the ball-and-socket joint A on the block.



Prob. F5–11

F5–12. Determine the components of reaction that the thrust bearing A and cable BC exert on the bar.

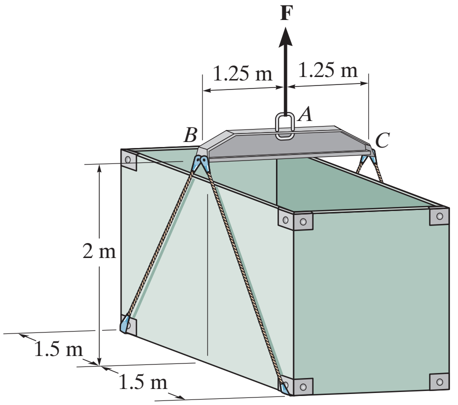


Prob. F5–12

PROBLEMS

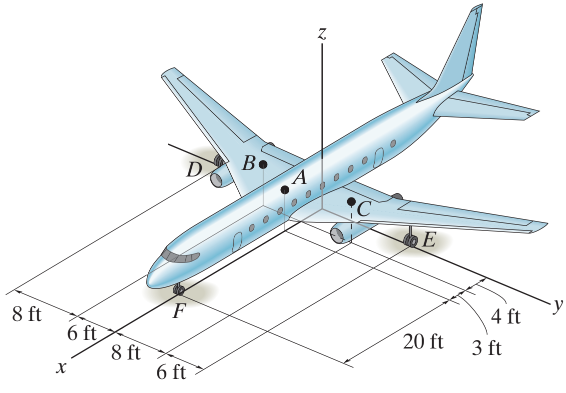
All problem solutions must include an FBD.

5–62. The uniform load has a mass of 600 kg and is lifted using a uniform 30-kg strongback beam BAC and four ropes as shown. Determine the tension in each rope and the force that must be applied at A.



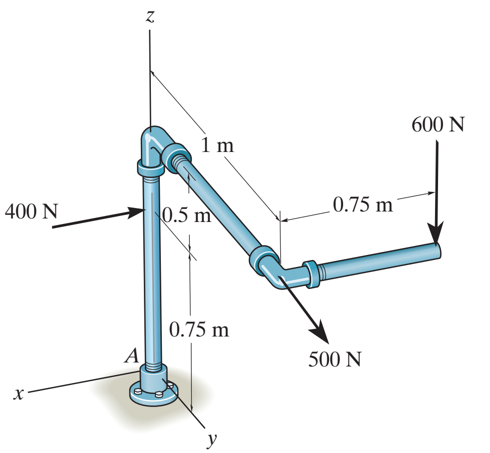
Prob. 5–62

5–63. Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage A and wings B and C are located as shown. If these components have weights $W_A = 45\,000$ lb, $W_B = 8000$ lb, and $W_C = 6000$ lb, determine the normal reactions of the wheels D, E, and F on the ground.



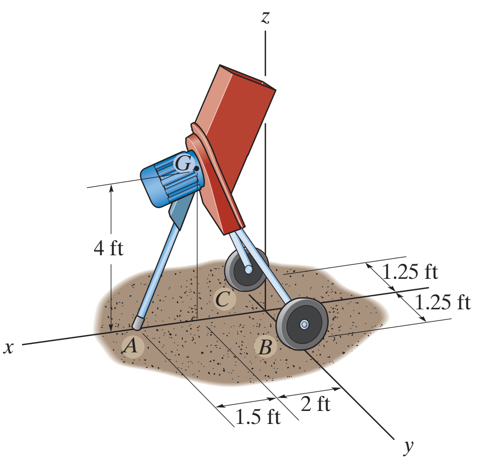
Prob. 5–63

***5–64.** Determine the components of reaction at the fixed support A. The 400 N, 500 N, and 600 N forces are parallel to the x, y, and z axes, respectively.



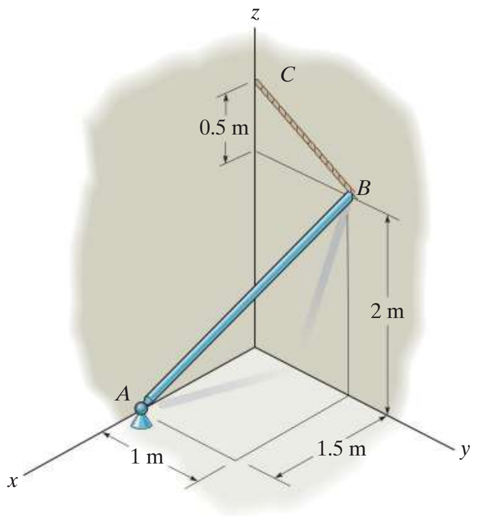
Prob. 5–64

5–65. The 50-lb mulching machine has a center of gravity at G. Determine the vertical reactions at the wheels C and B and the smooth contact point A.



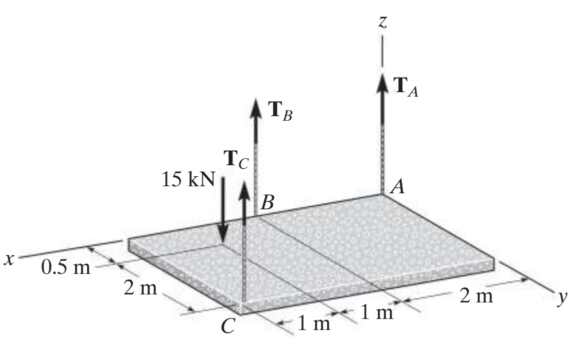
Prob. 5–65

5–66. The smooth uniform rod AB is supported by a ball-and-socket joint at A, the wall at B, and cable BC. Determine the components of reaction at A, the tension in the cable, and the normal reaction at B if the rod has a mass of 20 kg.



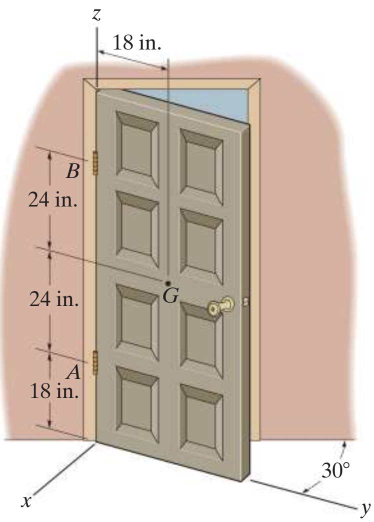
Prob. 5–66

5–67. The uniform concrete slab has a mass of 2400 kg. Determine the tension in each of the three parallel supporting cables when the slab is held in the horizontal plane as shown.



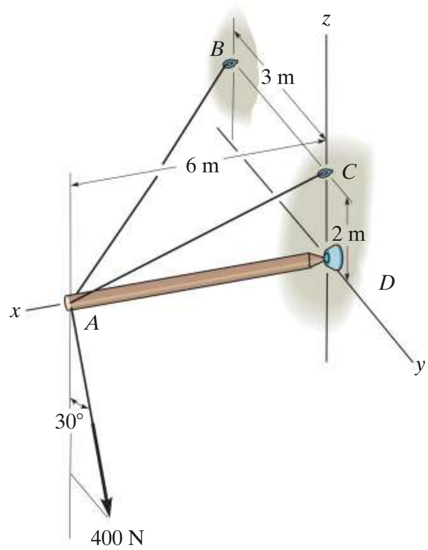
Prob. 5–67

***5–68.** The 100-lb door has its center of gravity at G. Determine the components of reaction at hinges A and B if hinge B resists only forces in the x and y directions and A resists forces in the x, y, z directions.



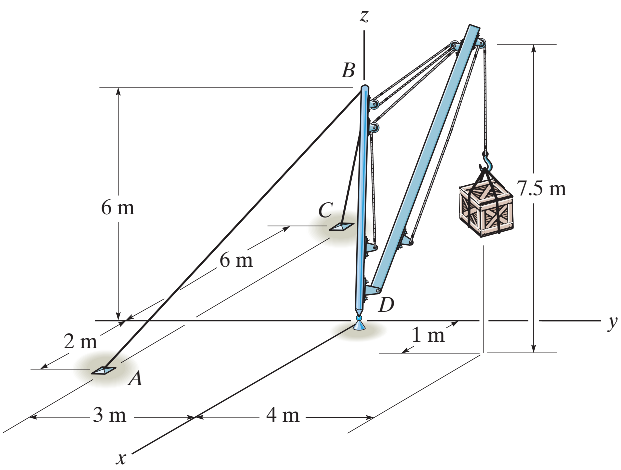
Prob. 5–68

5–69. Determine the tension in each cable and the components of reaction at D needed to support the load.



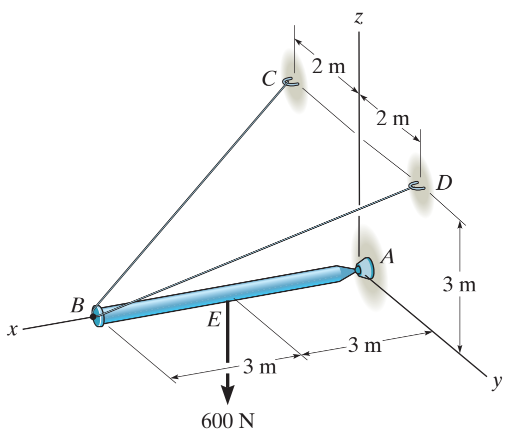
Prob. 5–69

5–70. The stiff-leg derrick used on ships is supported by a ball-and-socket joint at D and two cables BA and BC. The cables are attached to a smooth collar ring at B, which allows rotation of the derrick about z axis. If the derrick supports a crate having a mass of 200 kg, determine the tension in the cables and the x, y, z components of reaction at D.



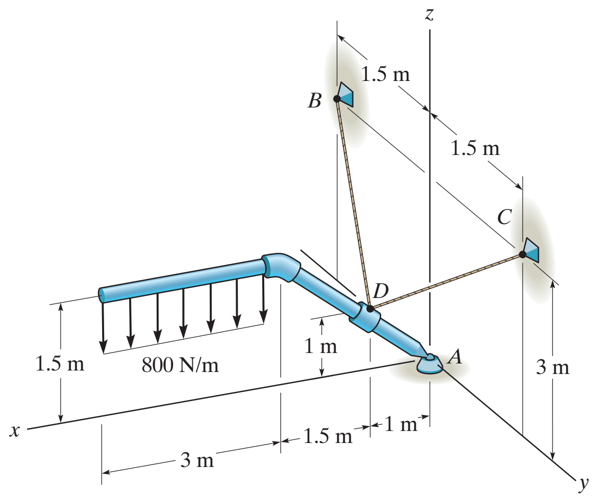
Prob. 5–70

5–71. Determine the components of reaction at the ball-and-socket joint A and the tension in each cable necessary for equilibrium of the rod.



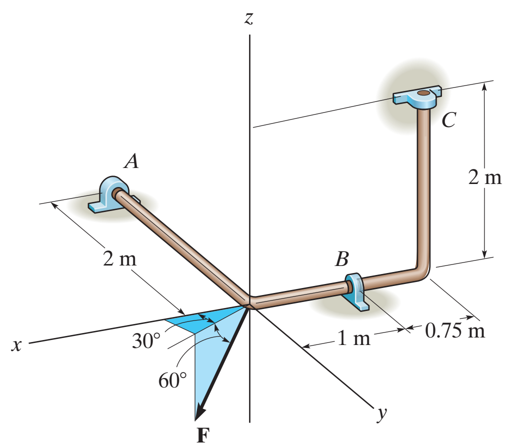
Prob. 5–71

***5–72.** Determine the components of reaction at the ball-and-socket joint A and the tension in the supporting cables DB and DC.



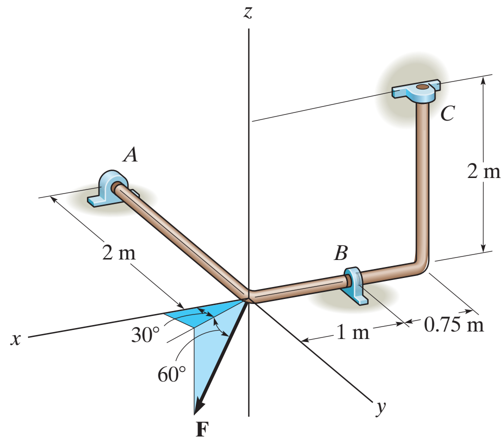
Prob. 5–72

5–73. The bent rod is supported at A, B, and C by smooth journal bearings. Determine the components of reaction at the bearings if the rod is subjected to the force $F = 800$ N. The bearings are in proper alignment and exert only force reactions on the rod.



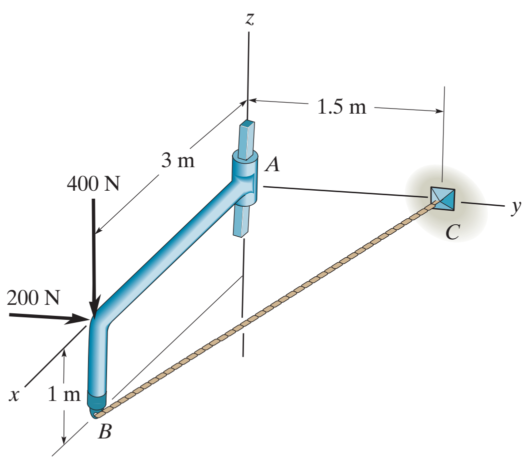
Prob. 5–73

5–74. The bent rod is supported at A, B, and C by smooth journal bearings. Determine the magnitude of **F** which will cause the positive x component of reaction at the bearing C to be $C_x = 50$ N. The bearings are in proper alignment and exert only force reactions on the rod.



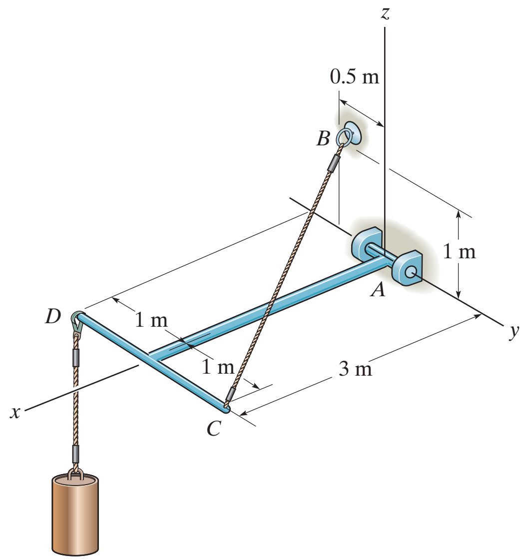
Prob. 5–74

5–75. Member AB is supported by a cable BC and at A by a *square* rod which fits loosely through the square hole in the collar fixed to the member as shown. Determine the components of reaction at A and the tension in the cable needed to hold the rod in equilibrium.



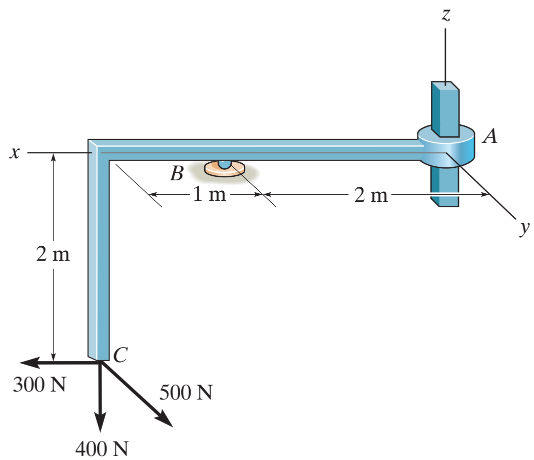
Prob. 5–75

***5–76.** The member is supported by a pin at A and cable BC. Determine the components of reaction at these supports if the cylinder has a mass of 40 kg.



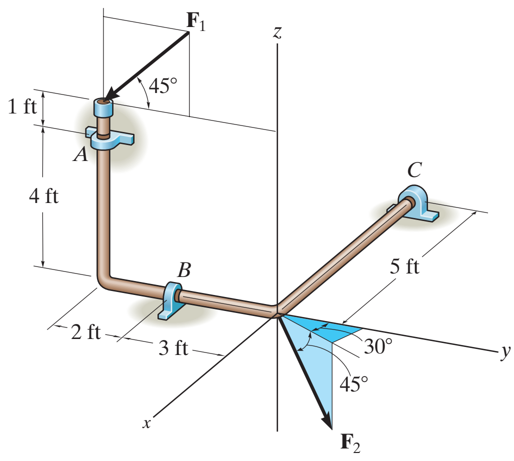
Prob. 5–76

5–77. The member is supported by a square rod which fits loosely through the smooth square hole of the attached collar at A and by a roller at B. Determine the components of reaction at these supports when the member is subjected to the loading shown.



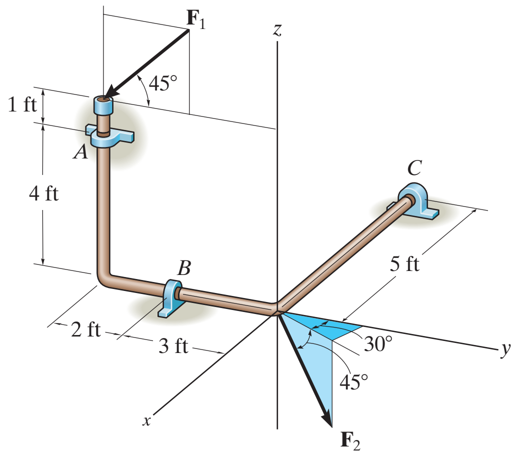
Prob. 5–77

5–78. The bent rod is supported at A, B, and C by smooth journal bearings. Compute the x, y, z components of reaction at the bearings if the rod is subjected to forces $F_1 = 300$ lb and $F_2 = 250$ lb. $\mathbf{F}_1$ lies in the y–z plane. The bearings are in proper alignment and exert only force reactions on the rod.



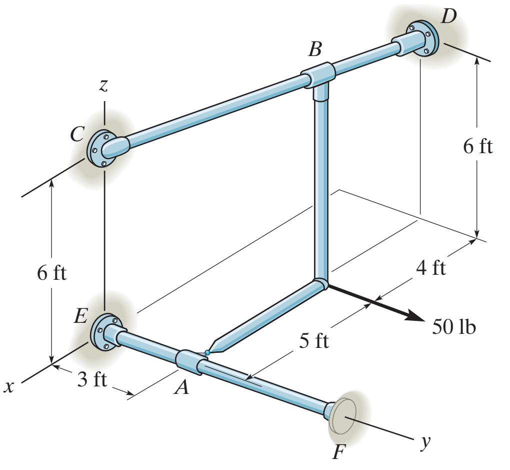
Prob. 5–78

5–79. The bent rod is supported at A, B, and C by smooth journal bearings. Determine the magnitude of $\mathbf{F}_2$ which will cause the reaction $\mathbf{C}_y$ at the bearing C to be equal to zero. The bearings are in proper alignment and exert only force reactions on the rod. Set $F_1 = 300$ lb.



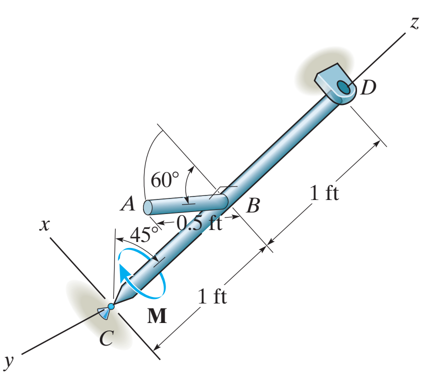
Prob. 5–79

***5–80.** The bar AB is supported by two smooth collars. At A the connection is with a ball-and-socket joint and at B it is a rigid attachment. If a 50-lb load is applied to the bar, determine the x, y, z components of reaction at A and B.



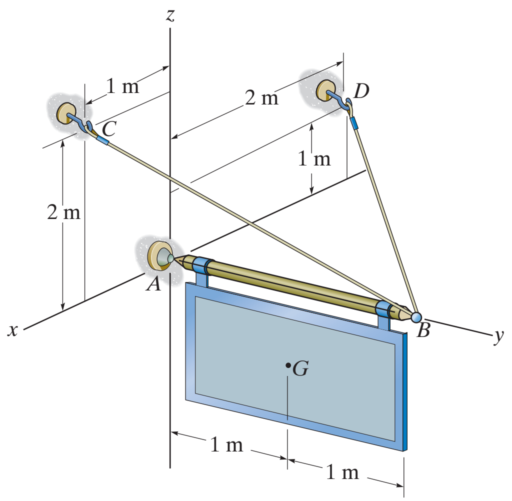
Prob. 5–80

5–81. The rod has a weight of 6 lb/ft. If it is supported by a ball-and-socket joint at C and a journal bearing at D, determine the x, y, z components of reaction at these supports and the moment M that must be applied along the axis of the rod to hold it in the position shown.



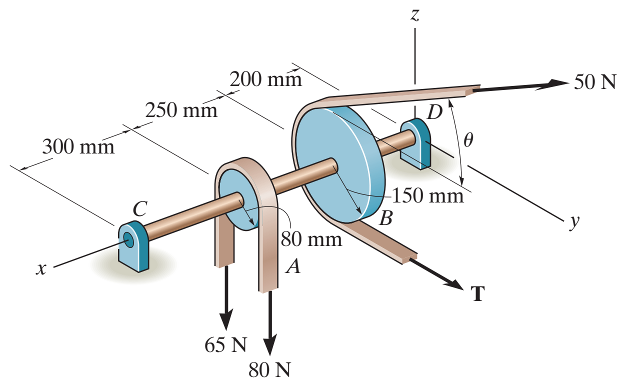
Prob. 5–81

5–82. The sign has a mass of 100 kg with center of mass at G. Determine the x, y, z components of reaction at the ball-and-socket joint A and the tension in wires BC and BD.



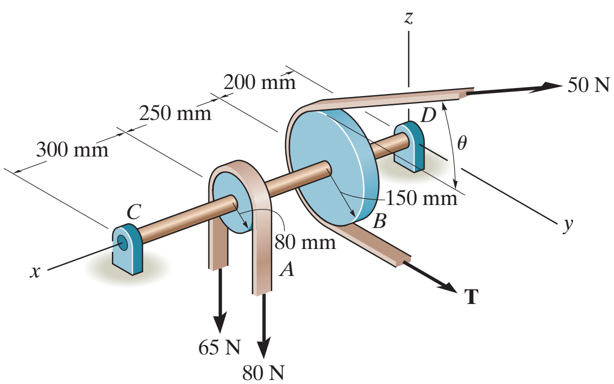
Prob. 5–82

***5–84.** Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension $\mathbf{T}$ in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 45°$. The bearings are in proper alignment and exert only force reactions on the shaft.



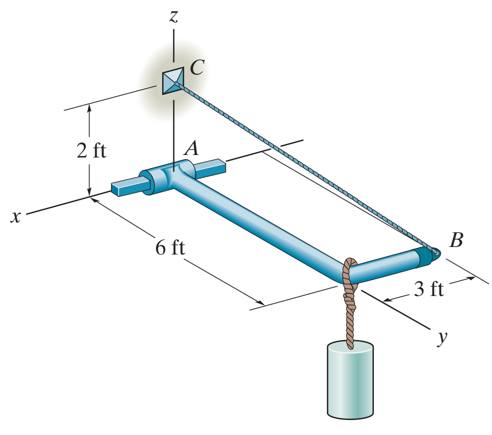
Prob. 5–84

5–83. Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension $\mathbf{T}$ in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 0°$. The bearings are in proper alignment and exert only force reactions on the shaft.



Prob. 5–83

5–85. Member AB is supported by a cable BC and at A by a *square* rod which fits loosely through the square hole at the end joint of the member as shown. Determine the components of reaction at A and the tension in the cable needed to hold the 800-lb cylinder in equilibrium.



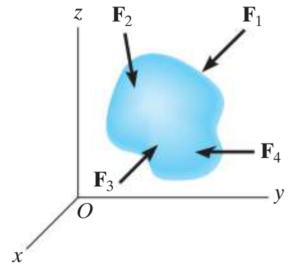
Prob. 5–85

CHAPTER REVIEW

Equilibrium

A body in equilibrium is at rest or can translate with constant velocity.

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma \mathbf{M} = \mathbf{0}$$



Two Dimensions

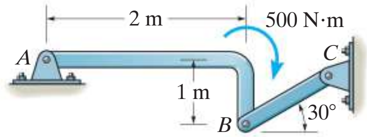
Before analyzing the equilibrium of a body, it is first necessary to draw its free-body diagram. This is an outlined shape of the body, which shows all the forces and couple moments that act on it.

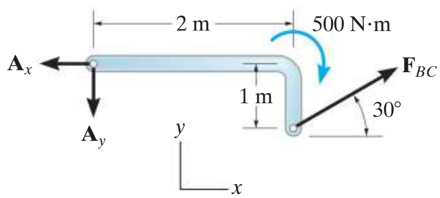
Couple moments can be placed anywhere on a free-body diagram since they are free vectors. Forces can act at any point along their line of action since they are sliding vectors.

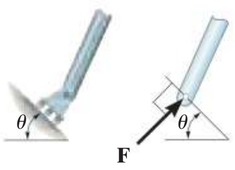
Angles used to resolve forces, and dimensions used to take moments of the forces, should also be shown on the free-body diagram.

Some common types of supports and their reactions are shown below in two dimensions.

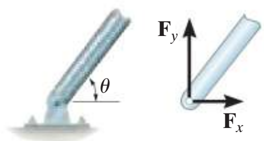
Remember that a support will exert a force on the body in a particular direction if it prevents translation of the body in that direction, and it will exert a couple moment on the body if it prevents rotation.



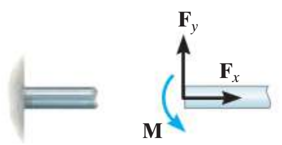




roller



smooth pin or hinge



fixed support

The three scalar equations of equilibrium can be applied when solving problems in two dimensions, since the geometry is easy to visualize.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

For the most direct solution, try to sum forces along an axis that will eliminate as many unknown forces as possible. Sum moments about a point A that passes through the line of action of as many unknown forces as possible.

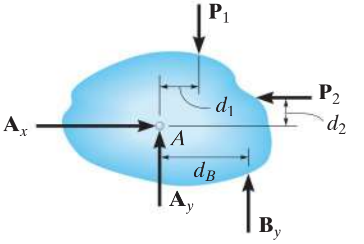
$\Sigma F_x = 0;$

$$A_x - P_2 = 0 \quad A_x = P_2$$

$\Sigma M_A = 0;$

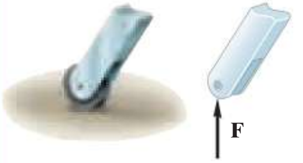
$$P_2 d_2 + B_y d_B - P_1 d_1 = 0$$

$$B_y = \frac{P_1 d_1 - P_2 d_2}{d_B}$$

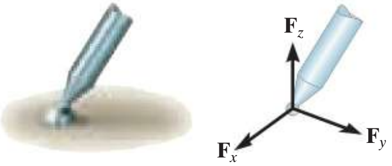


Three Dimensions

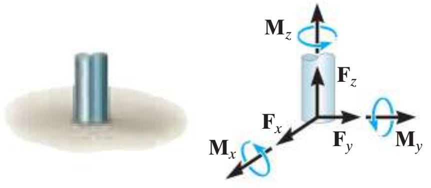
Some common types of supports and their reactions are shown here in three dimensions.



roller



ball and socket



fixed support

In three dimensions, it is often advantageous to use a Cartesian vector analysis when applying the equations of equilibrium. To do this, first express each known and unknown force and couple moment shown on the free-body diagram as a Cartesian vector. Then set the force summation equal to zero. Take moments about a point O that lies on the line of action of as many unknown force components as possible. From point O direct position vectors to each force, and then use the cross product to determine the moment of each force.

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma \mathbf{M}_O = \mathbf{0}$$

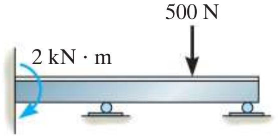
The six scalar equations of equilibrium are established by setting the respective **i**, **j**, and **k** components of these force and moment summations equal to zero.

$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$

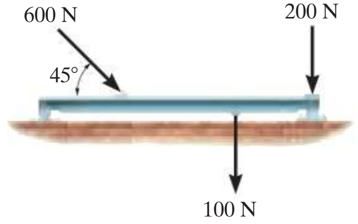
Determinacy and Stability

If a body is supported by a minimum number of constraints to ensure equilibrium, then it is statically determinate. If it has more constraints than required, then it is statically indeterminate.

To properly constrain the body, the reactions must not all be parallel to one another or concurrent.



Statically indeterminate, five reactions, three equilibrium equations

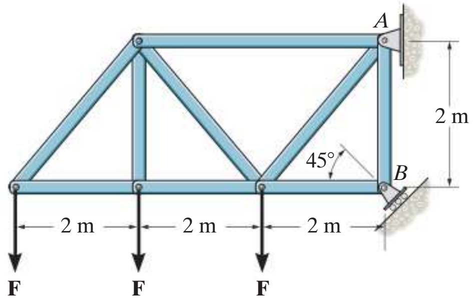


Proper constraint, statically determinate

REVIEW PROBLEMS

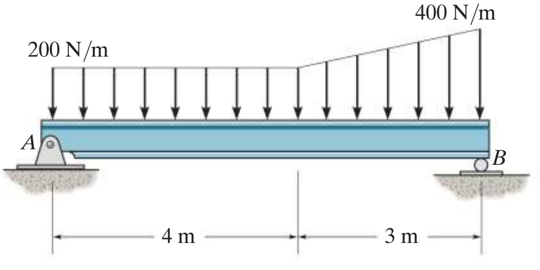
All problem solutions must include an FBD.

R5–1. If the roller at B can sustain a maximum load of 3 kN, determine the largest magnitude of each of the three forces **F** that can be supported by the truss.



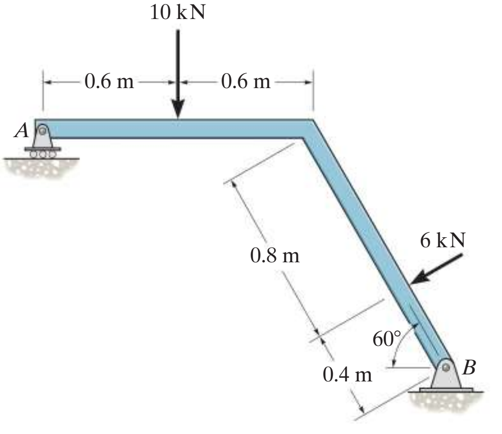
Prob. R5–1

R5–2. Determine the reactions at the supports A and B for equilibrium of the beam.



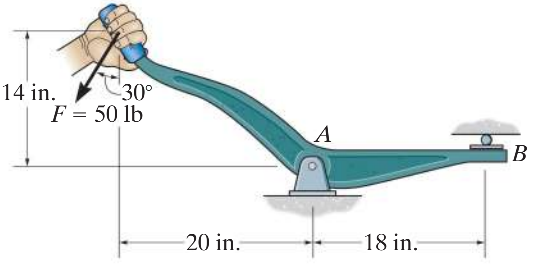
Prob. R5–2

R5–3. Determine the normal reaction at the roller A and horizontal and vertical components at pin B for equilibrium of the member.



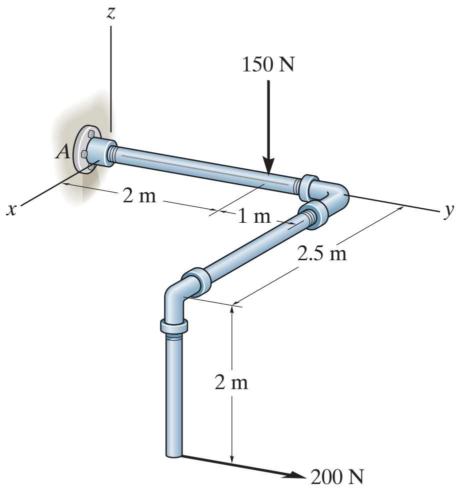
Prob. R5–3

R5–4. Determine the horizontal and vertical components of reaction at the pin at A and the reaction of the roller at B on the lever.



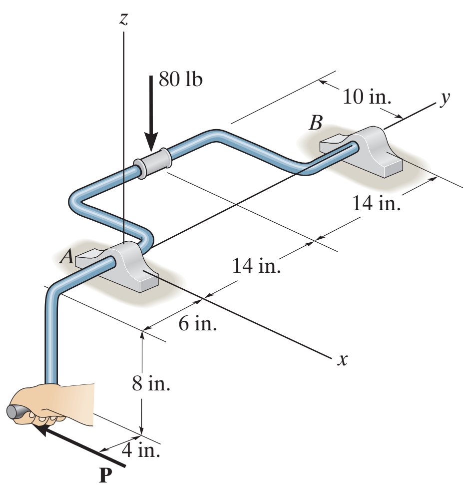
Prob. R5–4

R5–5. Determine the x, y, z components of reaction at the fixed wall A. The 150-N force is parallel to the z axis and the 200-N force is parallel to the y axis.



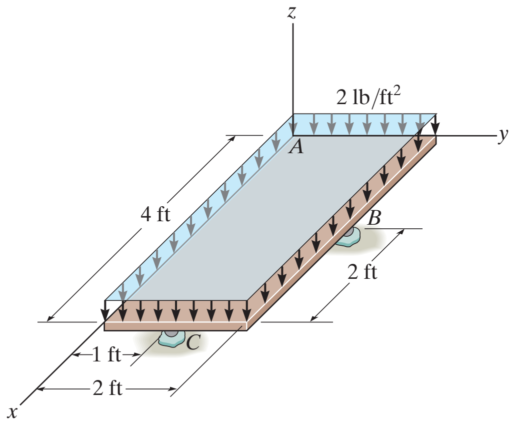
Prob. R5–5

R5–6. A vertical force of 80 lb acts on the crankshaft. Determine the horizontal equilibrium force **P** that must be applied to the handle and the x, y, z components of reaction at the journal bearing A and thrust bearing B. The bearings are properly aligned and exert only force reactions on the shaft.



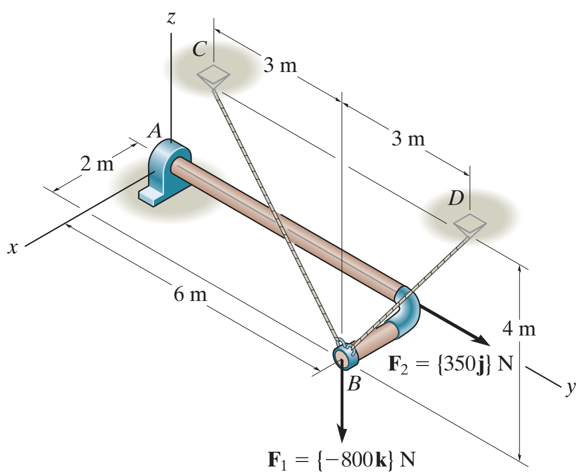
Prob. R5–6

R5–7. Determine the x, y, z components of reaction at the ball supports B and C and the ball-and-socket A (not shown) for the uniformly loaded plate.



Prob. R5–7

R5–8. Determine the x and z components of reaction at the journal bearing A and the tension in cords BC and BD necessary for equilibrium of the rod.



Prob. R5–8

Chapter 6

(© Tim Scrivener/Alamy)

In order to design the many parts of this boom assembly it is required that we know the forces that they must support. In this chapter we will show how to analyze such structures using the equations of equilibrium.

Structural Analysis

CHAPTER OBJECTIVES

- To show how to determine the forces in the members of a truss using the method of joints and the method of sections.
- To analyze the forces acting on the members of frames and machines composed of pin-connected members.

6.1 Simple Trusses

A ***truss*** is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts or metal bars. In particular, *planar* trusses lie in a single plane and are often used to support roofs and bridges. The truss shown in Fig. 6–1*a* is an example of a typical roof-supporting truss. In this figure, the roof load is transmitted to the truss *at the joints* by means of a series of *purlins*. Since this loading acts in the same plane as the truss, Fig. 6–1*b*, the analysis of the forces developed in the truss members will be two-dimensional.

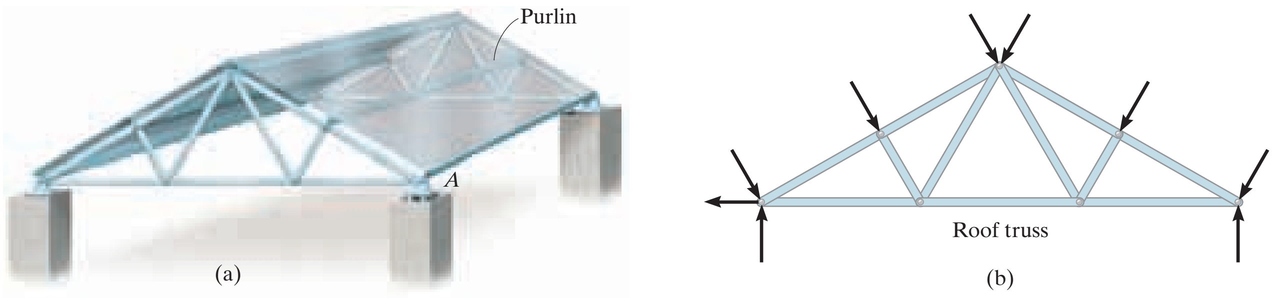
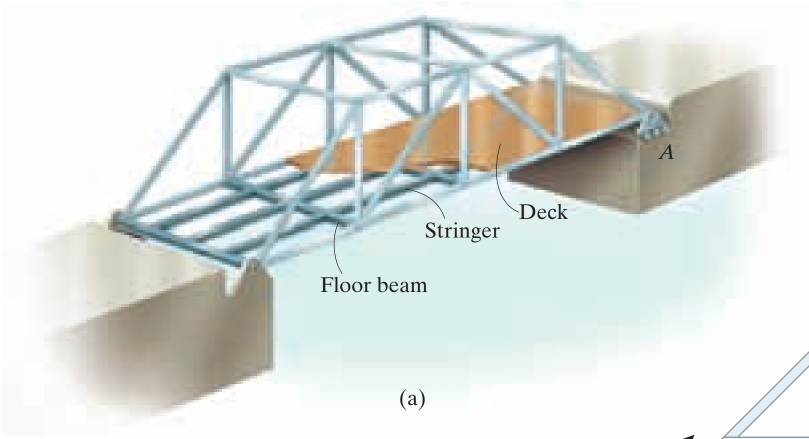
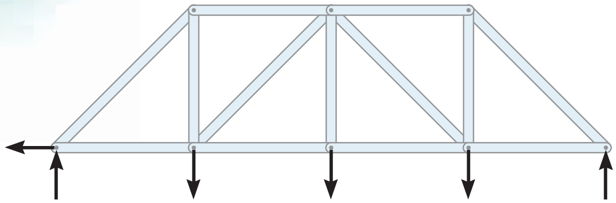


Fig. 6–1

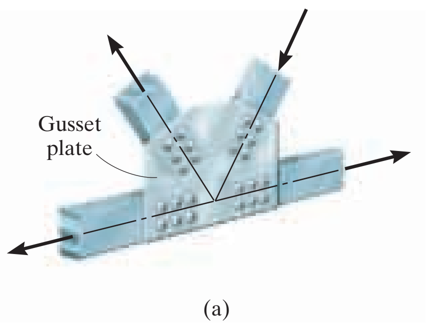


(a)

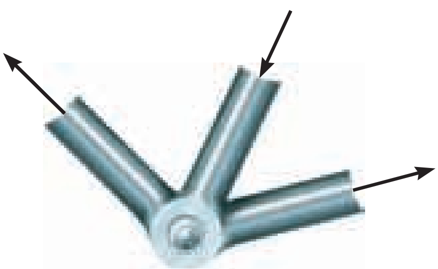


(b)

Fig. 6–2



(a)



(b)

Fig. 6–3

In the case of a bridge, such as shown in Fig. 6–2*a*, the load on the deck is first transmitted to *stringers*, then to *floor beams*, and finally to the *joints* of the two supporting side trusses. Like the roof truss, the bridge truss loading is also coplanar, Fig. 6–2*b*.

When bridge or roof trusses extend over large distances, a rocker or roller is commonly used for supporting one end, for example, joint *A* in Figs. 6–1*a* and 6–2*a*. This type of support allows freedom for expansion or contraction of the members due to a change in temperature or application of loads.

Assumptions for Design. To design both the members and the connections of a truss, it is necessary first to determine the *force* developed in each member when the truss is subjected to a given loading. To do this we will make two important assumptions:

- ***All loadings are applied at the joints.*** In most situations, such as for bridge and roof trusses, this assumption is true. Frequently the weight of the members is neglected because the force supported by each member is usually much larger than its weight. However, if the weight is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, with half of its magnitude applied at each end of the member.
- ***The members are joined together by smooth pins.*** The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a *gusset plate*, as shown in Fig. 6–3*a*, or by simply passing a large bolt or pin through each of the members, Fig. 6–3*b*. We can assume these connections act as pins provided the center lines of the joining members are *concurrent*, as in Fig. 6–3.

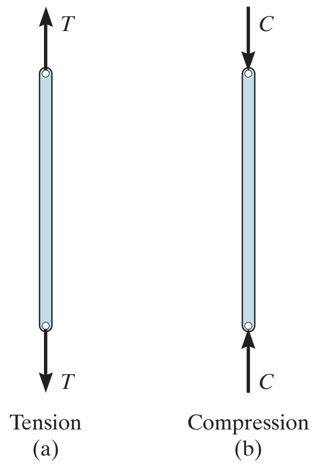


Fig. 6–4

Because of these two assumptions, *each truss member will act as a two-force member*, and therefore the force acting at each end of the member will be directed along the axis of the member. If the force tends to *elongate* the member, it is a ***tensile force*** (T), Fig. 6–4*a*; whereas if it tends to *shorten* the member, it is a ***compressive force*** (C), Fig. 6–4*b*. In the actual design of a truss it is important to state whether the nature of the force is tensile or compressive. Often, compression members must be made *thicker* than tension members because of the buckling or column effect that occurs when a member is in compression.

The use of metal gusset plates in the construction of these Warren trusses is clearly evident. (© Russell C. Hibbeler)

Simple Truss. If three members are pin connected at their ends, they form a *triangular truss* that will be *rigid*, Fig. 6–5. Attaching two more members and connecting these members to a new joint D forms a larger truss, Fig. 6–6. This procedure can be repeated as many times as desired to form an even larger truss. If a truss can be constructed by expanding the basic triangular truss in this way, it is called a ***simple truss***.

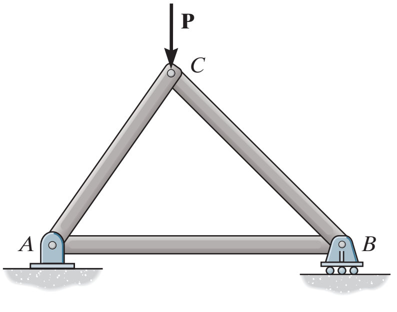


Fig. 6–5

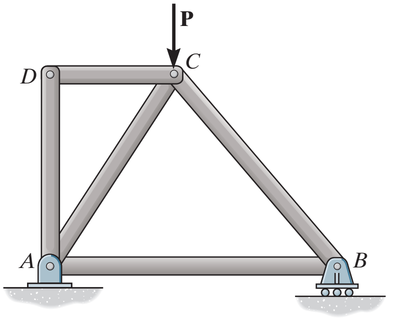


Fig. 6–6

6.2 The Method of Joints

In order to analyze or design a truss, it is necessary to determine the force in each of its members. One way to do this is to use the ***method of joints***. This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium. Therefore, if the free-body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint. Since the members of a *plane truss* are straight two-force members lying in a single plane, each joint is subjected to a force system that is *coplanar and concurrent*. As a result, only $\Sigma F_x = 0$ and $\Sigma F_y = 0$ need to be satisfied for equilibrium.

For example, consider the pin at joint *B* of the truss in Fig. 6–7*a*. Three forces act on the pin, namely, the 500-N force and the forces exerted by members *BA* and *BC*. The free-body diagram of the pin is shown in Fig. 6–7*b*. Here, $\mathbf{F}_{BA}$ is "pulling" on the pin, which means that member *BA* is in *tension;* whereas $\mathbf{F}_{BC}$ is "pushing" on the pin, and consequently member *BC* is in *compression.* These effects are clearly demonstrated by isolating the joint with small segments of the member connected to the pin, Fig. 6–7*c*. The pushing or pulling on these small segments indicates the effect of the member being either in compression or tension.

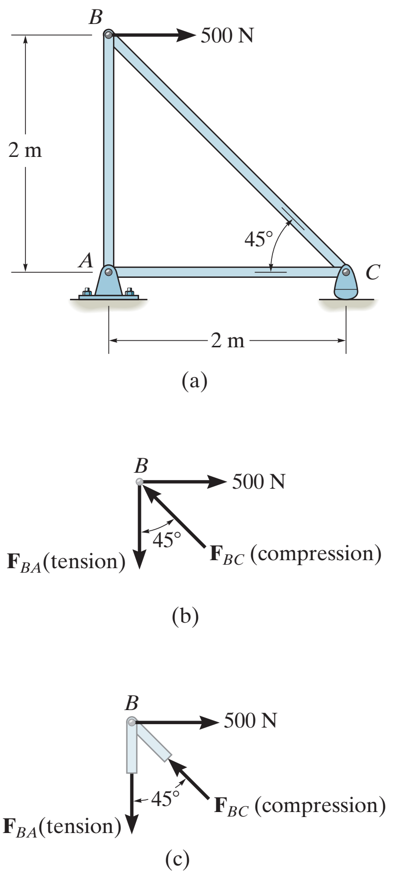


Fig. 6–7

The forces in the members of this simple roof truss can be determined using the method of joints. (© Russell C. Hibbeler)

When using the method of joints, always start at a joint having at least one known force and at most two unknown forces, as in Fig. 6–7*b*. In this way, application of $\Sigma F_x = 0$ and $\Sigma F_y = 0$ yields two algebraic equations which can be solved for the two unknowns. When applying these equations, the correct sense of an unknown member force can be determined using one of two possible methods.

- The *correct* sense of direction of an unknown member force can, in many cases, be determined "by inspection." For example, $\mathbf{F}_{BC}$ in Fig. 6–7*b* must push on the pin (compression) since its horizontal component, $F_{BC} \sin 45°$, must balance the 500-N force ($\Sigma F_x = 0$). Likewise, $\mathbf{F}_{BA}$ is a tensile force since it balances the vertical component, $F_{BC} \cos 45°$ ($\Sigma F_y = 0$). In more complicated cases, the sense of an unknown member force can be *assumed*; then, after applying the equilibrium equations, the assumed sense can be verified from the numerical results. A *positive* answer indicates that the sense is *correct*, whereas a *negative* answer indicates that the sense shown on the free-body diagram must be *reversed.*
- *Always assume* the *unknown member forces* acting on the joint's free-body diagram to be in *tension*; i.e., the forces "pull" on the pin. If this is done, then numerical solution of the equilibrium equations will yield *positive scalars for members in tension and negative scalars for members in compression.* Once an unknown member force is found, use its *correct* magnitude and sense (T or C) on subsequent joint free-body diagrams.

Important Points

- Simple trusses are composed of triangular elements. The members are assumed to be pin connected at their ends and loads applied at the joints.
- If a truss is in equilibrium, then each of its joints is in equilibrium. The internal forces in the members become external forces when the free-body diagram of each joint of the truss is drawn. A force pulling on a joint is caused by tension in a member, and a force pushing on a joint is caused by compression.

Procedure for Analysis

The following procedure provides a means for analyzing a truss using the method of joints.

- Draw the free-body diagram of a joint having at least one known force and at most two unknown forces. (If this joint is at one of the supports, then it may be necessary first to calculate the external reactions at the support.)
- Use one of the two methods described above for establishing the sense of an unknown force.
- Orient the x and y axes such that the forces on the free-body diagram can be easily resolved into their x and y components and then apply the two force equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Solve for the two unknown member forces and verify their correct sense.
- Using the calculated results, continue to analyze each of the other joints. Remember that a member in *compression* "pushes" on the joint and a member in *tension* "pulls" on the joint. Also, be sure to choose a joint having at most two unknowns and at least one known force.

EXAMPLE 6.1

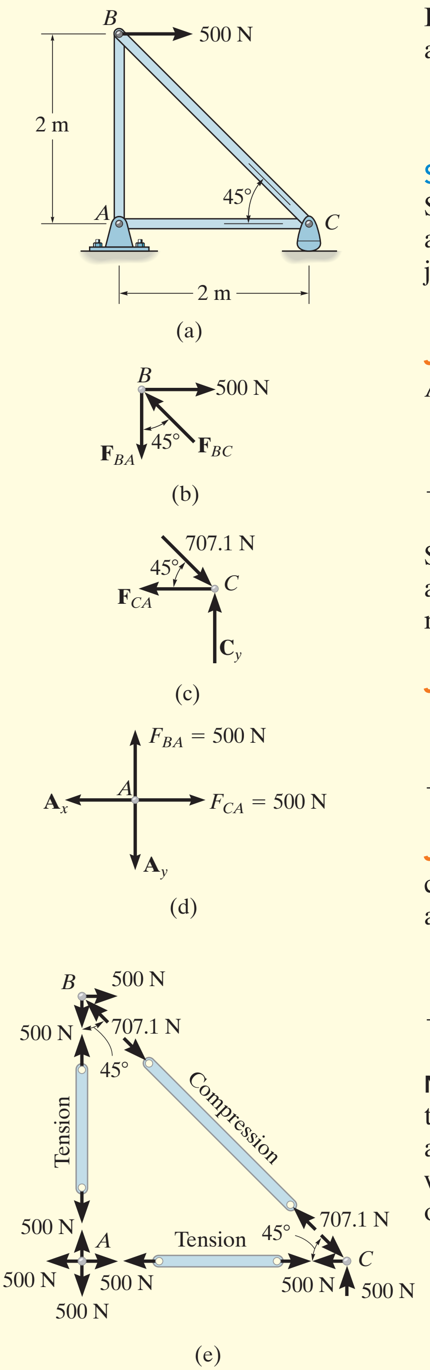


Fig. 6–8

Determine the force in each member of the truss shown in Fig. 6–8*a* and indicate whether the members are in tension or compression.

SOLUTION

Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint *B*.

Joint *B*. The free-body diagram of the joint at *B* is shown in Fig. 6–8*b*. Applying the equations of equilibrium, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 500\text{ N} - F_{BC}\sin 45^\circ = 0 \quad F_{BC} = 707.1\text{ N (C)} \quad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad F_{BC}\cos 45^\circ - F_{BA} = 0 \quad F_{BA} = 500\text{ N (T)} \quad \textit{Ans.}$$

Since the force in member *BC* has been calculated, we can proceed to analyze joint *C* to determine the force in member *CA* and the support reaction at the rocker.

Joint *C*. From the free-body diagram of joint *C*, Fig. 6–8*c*, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad -F_{CA} + 707.1\cos 45^\circ\text{ N} = 0 \quad F_{CA} = 500\text{ N (T)} \quad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad C_y - 707.1\sin 45^\circ\text{ N} = 0 \quad C_y = 500\text{ N} \quad \textit{Ans.}$$

Joint *A*. Although it is not necessary, we can determine the components of the support reactions at joint *A* using the results of F_{CA} and F_{BA}. From the free-body diagram, Fig. 6–8*d*, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 500\text{ N} - A_x = 0 \quad A_x = 500\text{ N}$$

$$+\uparrow\Sigma F_y = 0; \quad 500\text{ N} - A_y = 0 \quad A_y = 500\text{ N}$$

NOTE: The results of the analysis are summarized in Fig. 6–8*e*. Note that the free-body diagram of each joint (or pin) shows the effects of all the connected members and external forces applied to the joint, whereas the free-body diagram of each member shows only the effects of the end joints on the member.

EXAMPLE 6.2

Determine the forces acting in all the members of the truss shown in Fig. 6–9*a*.

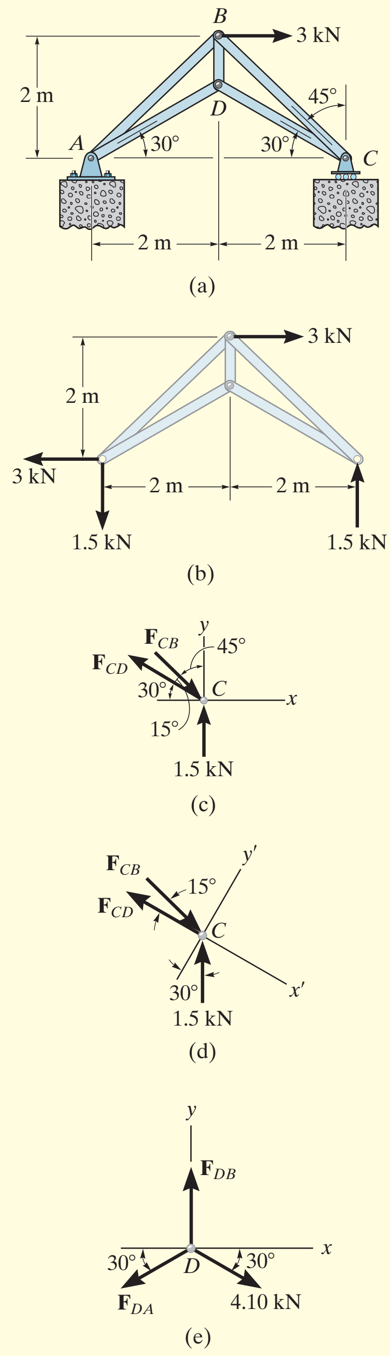


Fig. 6–9

SOLUTION

By inspection, there are more than two unknowns at each joint. Consequently, the support reactions on the truss must first be determined. Show that they have been correctly calculated on the free-body diagram in Fig. 6–9*b*. We can now begin the analysis at joint *C*. Why?

Joint *C*. From the free-body diagram, Fig. 6–9*c*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad -F_{CD}\cos 30^\circ + F_{CB}\sin 45^\circ = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad 1.5\text{ kN} + F_{CD}\sin 30^\circ - F_{CB}\cos 45^\circ = 0$$

These two equations must be solved *simultaneously* for each of the two unknowns. Note, however, that a *direct solution* for one of the unknown forces may be obtained by applying a force summation along an axis that is *perpendicular* to the direction of the other unknown force. For example, summing forces along the y' axis, which is perpendicular to the direction of $\mathbf{F}_{CD}$, Fig. 6–9*d*, yields a *direct solution* for F_{CB}.

$$+\nearrow\Sigma F_{y'} = 0; \qquad 1.5\cos 30^\circ\text{ kN} - F_{CB}\sin 15^\circ = 0$$

$$F_{CB} = 5.019\text{ kN} = 5.02\text{ kN (C)} \qquad \textit{Ans.}$$

Then,

$$+\searrow\Sigma F_{x'} = 0;$$

$$-F_{CD} + 5.019\cos 15^\circ - 1.5\sin 30^\circ = 0; \qquad F_{CD} = 4.10\text{ kN (T)} \qquad \textit{Ans.}$$

Joint *D*. We can now proceed to analyze joint *D*. The free-body diagram is shown in Fig. 6–9*e*.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad -F_{DA}\cos 30^\circ + 4.10\cos 30^\circ\text{ kN} = 0$$

$$F_{DA} = 4.10\text{ kN (T)} \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{DB} - 2(4.10\sin 30^\circ\text{ kN}) = 0$$

$$F_{DB} = 4.10\text{ kN (T)} \qquad \textit{Ans.}$$

NOTE: The force in the last member, *BA*, can be obtained from joint *B* or joint *A*. As an exercise, draw the free-body diagram of joint *B*, sum the forces in the horizontal direction, and show that $F_{BA} = 0.776$ kN (C).

Determine the force in each member of the truss shown in Fig. 6–10*a*. Indicate whether the members are in tension or compression.

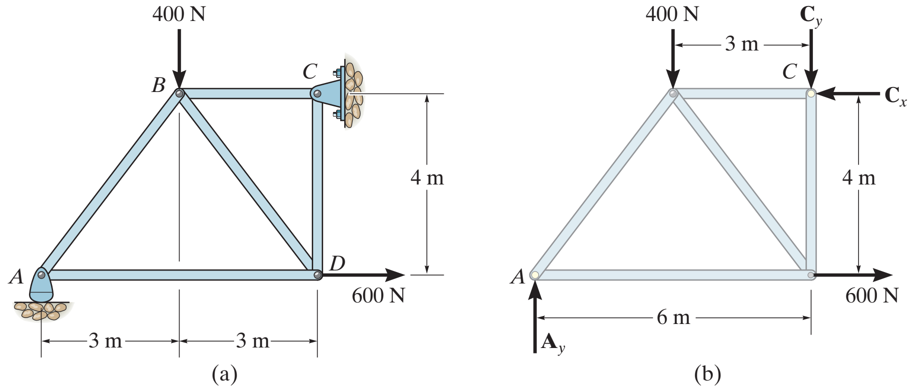


Fig. 6–10

SOLUTION

Support Reactions. No joint can be analyzed until the support reactions are determined, because each joint has at least three unknown forces acting on it. A free-body diagram of the entire truss is given in Fig. 6–10*b*. Applying the equations of equilibrium, we have

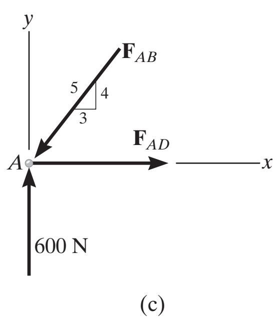
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 600\text{ N} - C_x = 0 \qquad C_x = 600\text{ N}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -A_y(6\text{ m}) + 400\text{ N}(3\text{ m}) + 600\text{ N}(4\text{ m}) = 0$$

$$A_y = 600\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad 600\text{ N} - 400\text{ N} - C_y = 0 \qquad C_y = 200\text{ N}$$

The analysis can now start at either joint *A* or *C*. The choice is arbitrary since there are one known and two unknown member forces acting on the pin at each of these joints.



Joint *A*. (Fig. 6–10*c*). As shown on the free-body diagram, $\mathbf{F}_{AB}$ is assumed to be compressive and $\mathbf{F}_{AD}$ is tensile. Applying the equations of equilibrium, we have

$$+\uparrow \Sigma F_y = 0; \qquad 600\text{ N} - \tfrac{4}{5}F_{AB} = 0 \qquad F_{AB} = 750\text{ N} \quad (\text{C}) \qquad \textit{Ans.}$$

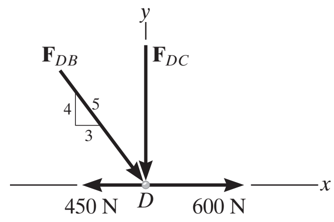
$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AD} - \tfrac{3}{5}(750\text{ N}) = 0 \qquad F_{AD} = 450\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$

Joint D. (Fig. 6–10*d*). Using the result for F_{AD} and summing forces in the horizontal direction, Fig. 6–10*d*, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -450\text{ N} + \tfrac{3}{5}F_{DB} + 600\text{ N} = 0 \qquad F_{DB} = -250\text{ N}$$

The negative sign indicates that $\mathbf{F}_{DB}$ acts in the *opposite sense* to that shown in Fig. 6–10*d*.* Hence,

$$F_{DB} = 250\text{ N (T)} \qquad \textit{Ans.}$$



(d)

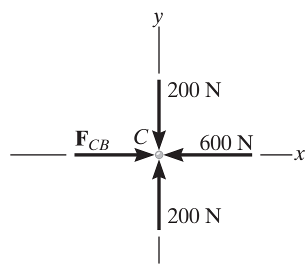
To determine $\mathbf{F}_{DC}$, we can either correct the sense of $\mathbf{F}_{DB}$ on the free-body diagram, and then apply $\Sigma F_y = 0$, or apply this equation and retain the negative sign for F_{DB}, i.e.,

$$+\uparrow \Sigma F_y = 0; \qquad -F_{DC} - \tfrac{4}{5}(-250\text{ N}) = 0 \qquad F_{DC} = 200\text{ N} \quad \text{(C)} \qquad \textit{Ans.}$$

Joint C. (Fig. 6–10*e*).

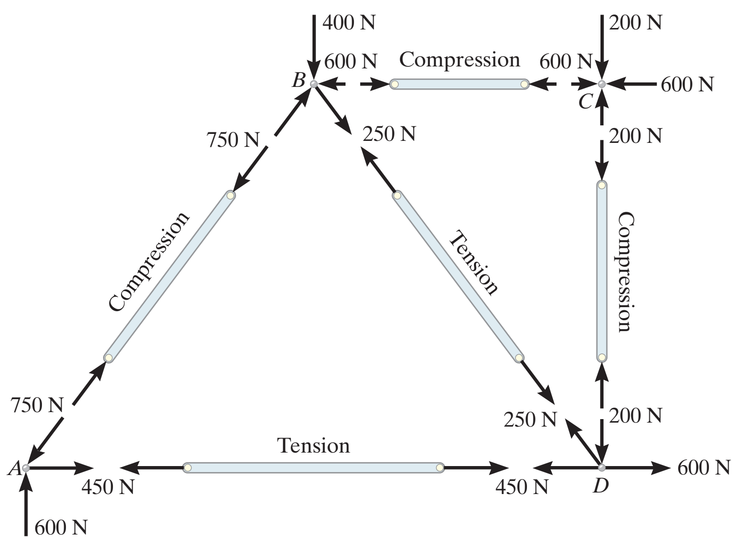
$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CB} - 600\text{ N} = 0 \qquad F_{CB} = 600\text{ N} \quad \text{(C)} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 200\text{ N} - 200\text{ N} \equiv 0 \quad \text{(check)}$$



(e)

NOTE: The analysis is summarized in Fig. 6–10*f*, which shows the free-body diagram for each joint and member.



(f)

Fig. 6–10 (cont.)

*The proper sense could have been determined by inspection, prior to applying $\Sigma F_x = 0$.

6.3 Zero-Force Members

Truss analysis using the method of joints is greatly simplified if we can first identify those members which support *no loading*. These *zero-force members* are used to increase the stability of the truss during construction and to provide added support if the loading is changed.

The zero-force members of a truss can generally be found *by inspection* of each of the joints. For example, consider the truss shown in Fig. 6–11*a*. If a free-body diagram of the pin at joint A is drawn, Fig. 6–11*b*, it is seen that members AB and AF are zero-force members. (We could not have come to this conclusion if we had considered the free-body diagrams of joints F or B simply because there are five unknowns at each of these joints.) In a similar manner, consider the free-body diagram of joint D, Fig. 6–11*c*. Here again it is seen that DC and DE are zero-force members. From these observations, we can conclude that *if only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members.* The load on the truss in Fig. 6–11*a* is therefore supported by only five members as shown in Fig. 6–11*d*.

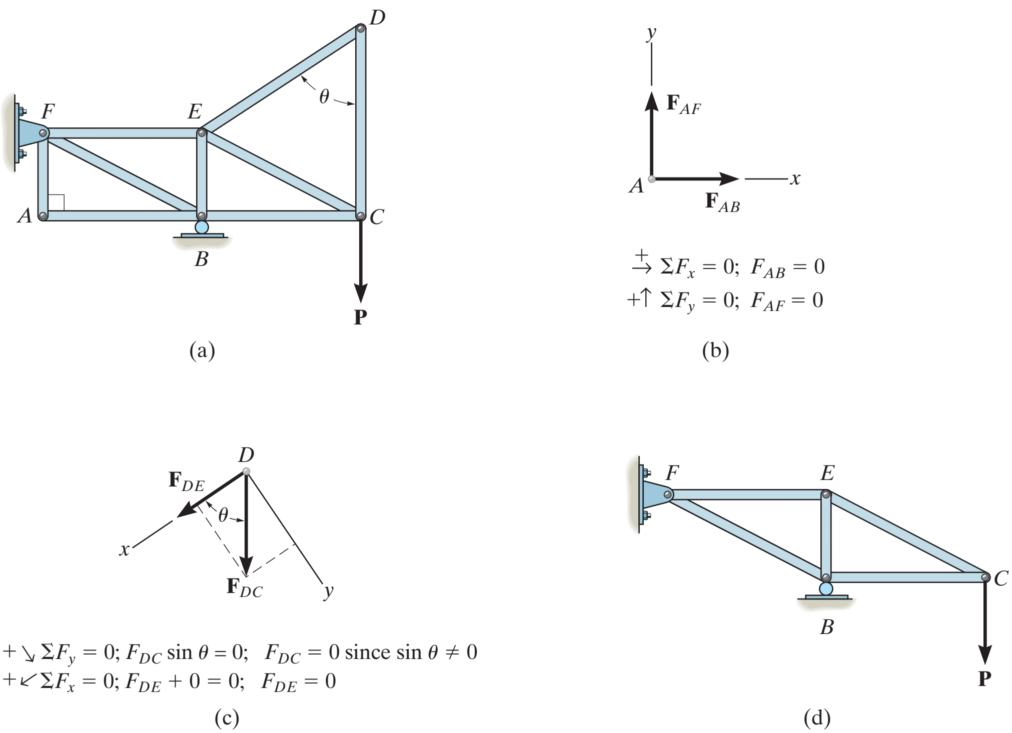
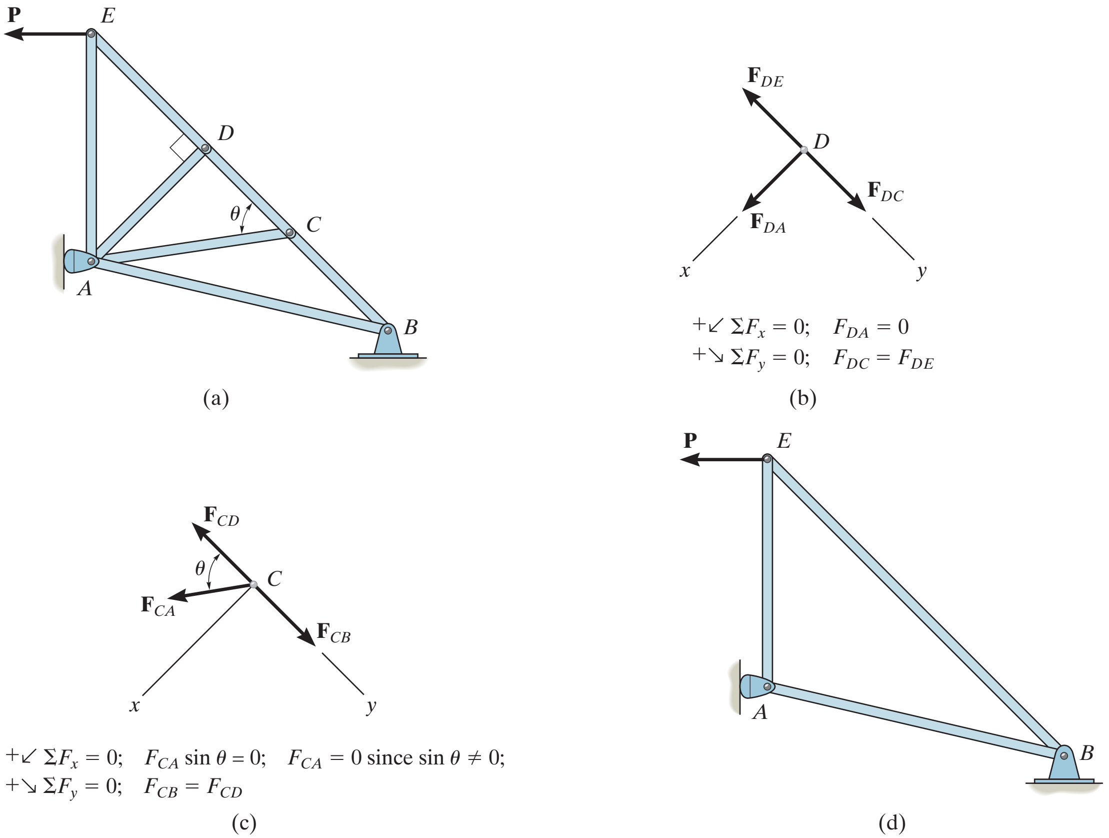


Fig. 6–11

Now consider the truss shown in Fig. 6–12*a*. The free-body diagram of the pin at joint D is shown in Fig. 6–12*b*. By orienting the y axis along members DC and DE and the x axis along member DA, it is seen that DA is a zero-force member. Note that this is also the case for member CA, Fig. 6–12*c*. In general then, *if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction has a component that acts along this member.* The truss shown in Fig. 6–12*d* is therefore suitable for supporting the load **P**.



$+\swarrow \Sigma F_x = 0; \quad F_{DA} = 0$

$+\searrow \Sigma F_y = 0; \quad F_{DC} = F_{DE}$

$+\swarrow \Sigma F_x = 0; \quad F_{CA} \sin\theta = 0; \quad F_{CA} = 0 \text{ since } \sin\theta \neq 0;$

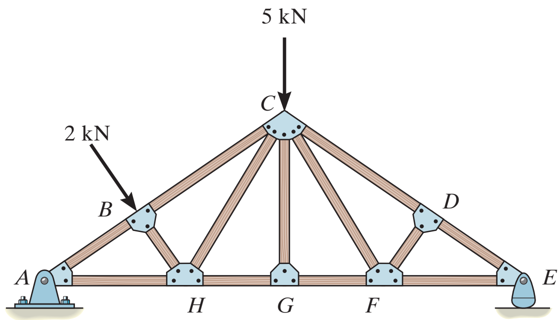
$+\searrow \Sigma F_y = 0; \quad F_{CB} = F_{CD}$

Fig. 6–12

Important Point

- Zero-force members support no load; however, they are necessary for stability, and are available when additional loadings are applied to the joints of the truss. These members can usually be identified by inspection. They occur at joints where only two members are connected and no external load acts along either member. Also, at joints having two collinear members, a third member will be a zero-force member if no external force components act along this member.

Using the method of joints, determine all the zero-force members of the *Fink roof truss* shown in Fig. 6–13*a*. Assume all joints are pin connected.

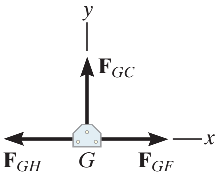


(a)

Fig. 6–13

SOLUTION

Look for joint geometries that have three members for which two are collinear. We have

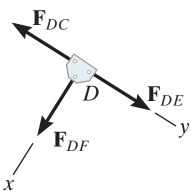


(b)

Joint *G*. (Fig. 6–13*b*).

$$+\uparrow\Sigma F_y = 0; \qquad F_{GC} = 0 \qquad \textit{Ans.}$$

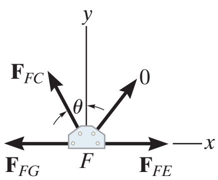
Realize that we could not conclude that GC is a zero-force member by considering joint C, where there are five unknowns. The fact that GC is a zero-force member means that the 5-kN load at C must be supported by members CB, CH, CF, and CD.



(c)

Joint *D*. (Fig. 6–13*c*).

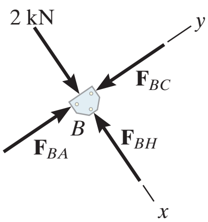
$$+\swarrow\Sigma F_x = 0; \qquad F_{DF} = 0 \qquad \textit{Ans.}$$



(d)

Joint *F*. (Fig. 6–13*d*).

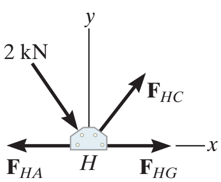
$$+\uparrow\Sigma F_y = 0; \qquad F_{FC}\cos\theta = 0 \quad \text{Since } \theta \neq 90^\circ, \quad F_{FC} = 0 \qquad \textit{Ans.}$$



(e)

NOTE: If joint B is analyzed, Fig. 6–13*e*,

$$+\searrow\Sigma F_x = 0; \qquad 2\text{ kN} - F_{BH} = 0 \quad F_{BH} = 2\text{ kN} \quad \text{(C)}$$



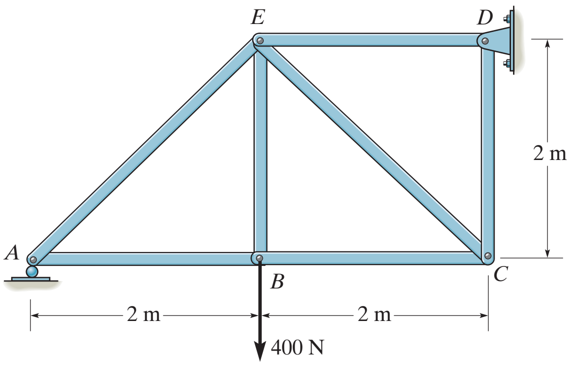
(f)

Also, F_{HC} must satisfy $\Sigma F_y = 0$, Fig. 6–13*f*, and therefore HC is *not* a zero-force member.

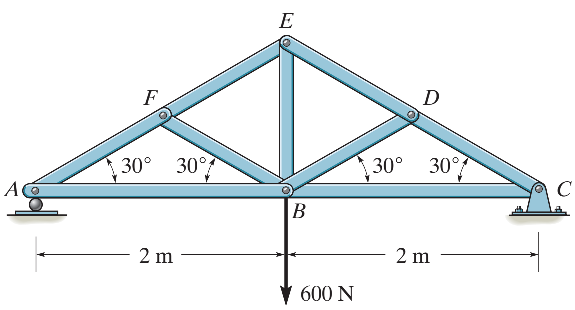
PRELIMINARY PROBLEMS

P6–1. In each case, calculate the support reactions and then draw the free-body diagrams of joints *A*, *B*, and *C* of the truss.

P6–2. Identify the zero-force members in each truss.

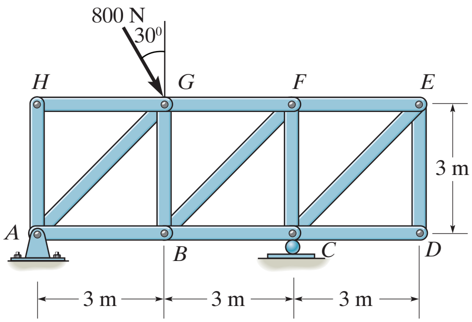


(a)

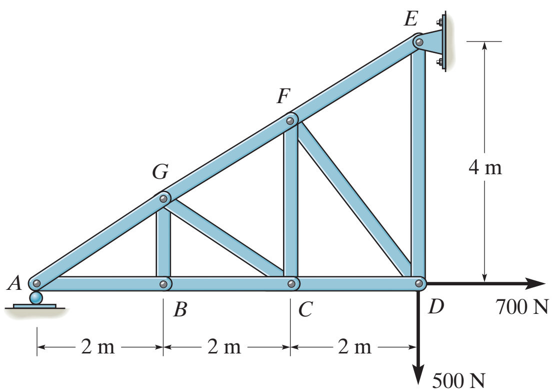


(b)

Prob. P6–1



(a)

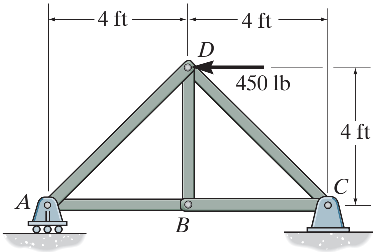


(b)

Prob. P6–2

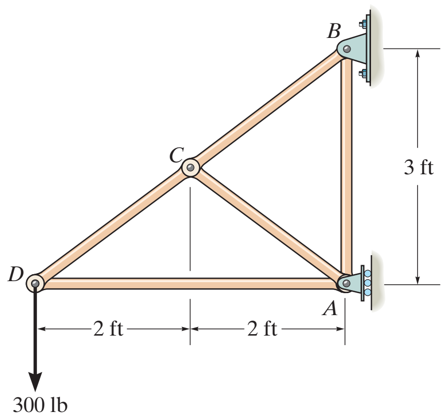
All problem solutions must include FBDs.

F6–1. Determine the force in each member of the truss. State if the members are in tension or compression.



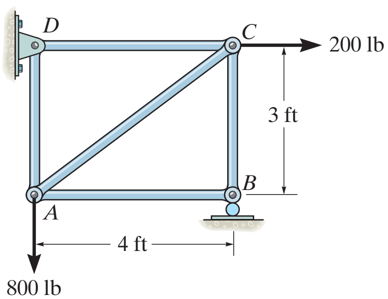
Prob. F6–1

F6–2. Determine the force in each member of the truss. State if the members are in tension or compression.



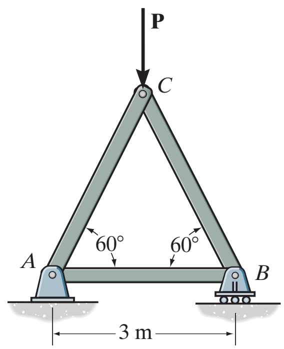
Prob. F6–2

F6–3. Determine the force in each member of the truss. State if the members are in tension or compression.



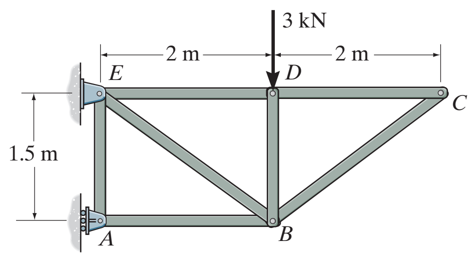
Prob. F6–3

F6–4. Determine the greatest load P that can be applied to the truss so that none of the members are subjected to a force exceeding either 2 kN in tension or 1.5 kN in compression.



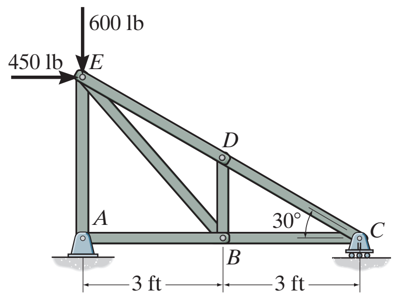
Prob. F6–4

F6–5. Identify the zero-force members in the truss.



Prob. F6–5

F6–6. Determine the force in each member of the truss. State if the members are in tension or compression.

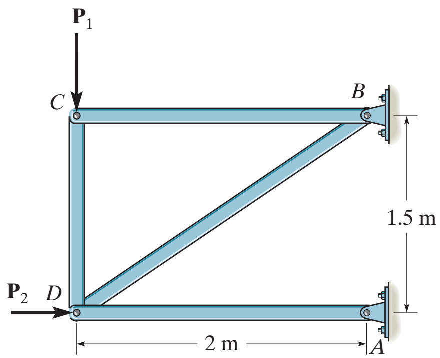


Prob. F6–6

All problem solutions must include FBDs.

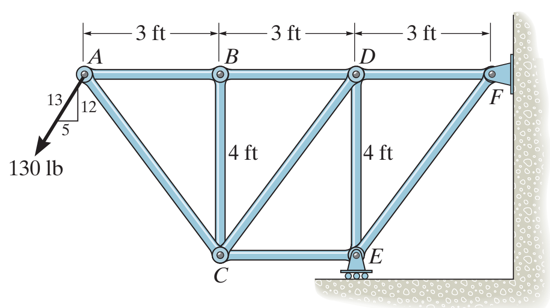
6–1. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 20$ kN, $P_2 = 10$ kN.

6–2. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 45$ kN, $P_2 = 30$ kN.



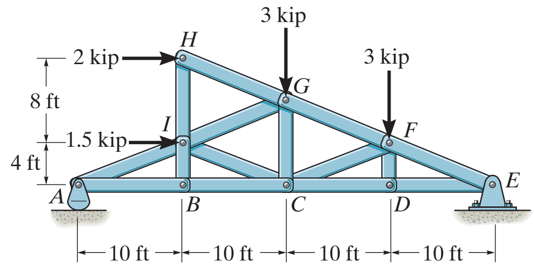
Probs. 6–1/2

6–3. Determine the force in each member of the truss. State if the members are in tension or compression.



Prob. 6–3

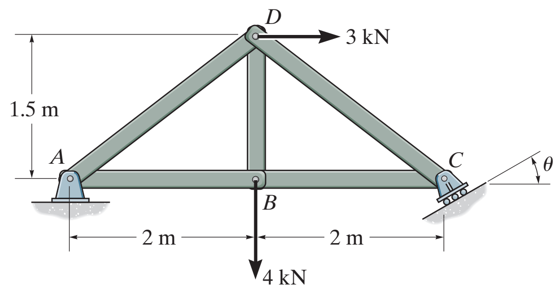
***6–4.** Determine the force in each member of the truss and state if the members are in tension or compression.



Prob. 6–4

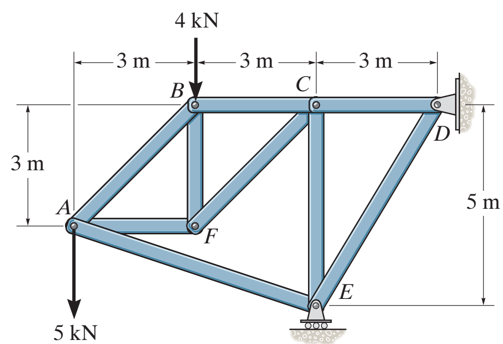
6–5. Determine the force in each member of the truss, and state if the members are in tension or compression. Set $\theta = 0°$.

6–6. Determine the force in each member of the truss, and state if the members are in tension or compression. Set $\theta = 30°$.



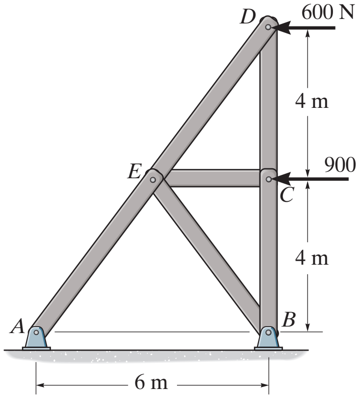
Probs. 6–5/6

6–7. Determine the force in each member of the truss and state if the members are in tension or compression.



Prob. 6–7

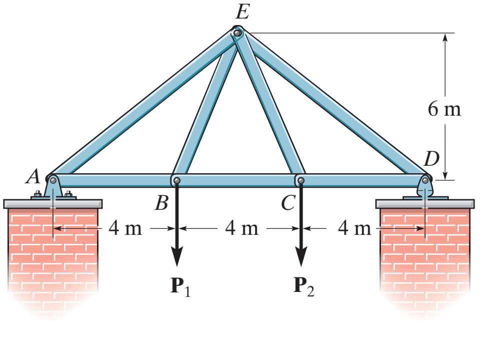
***6–8.** Determine the force in each member of the truss and state if the members are in tension or compression.



Prob. 6–8

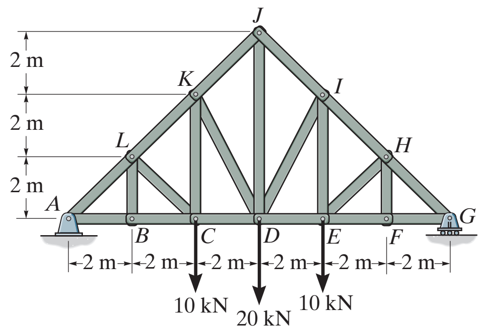
6–9. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 3$ kN, $P_2 = 6$ kN.

6–10. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 6$ kN, $P_2 = 9$ kN.



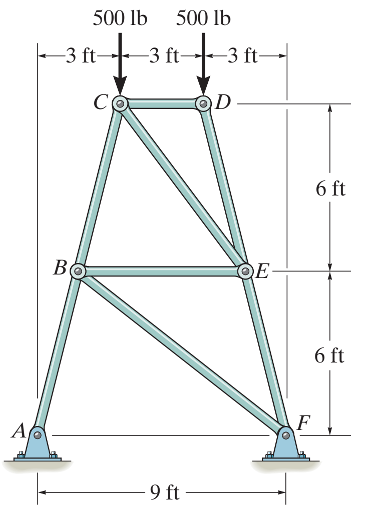
Probs. 6–9/10

6–11. Determine the force in each member of the *Pratt truss*, and state if the members are in tension or compression.



Prob. 6–11

***6–12.** Determine the force in each member of the truss and state if the members are in tension or compression.

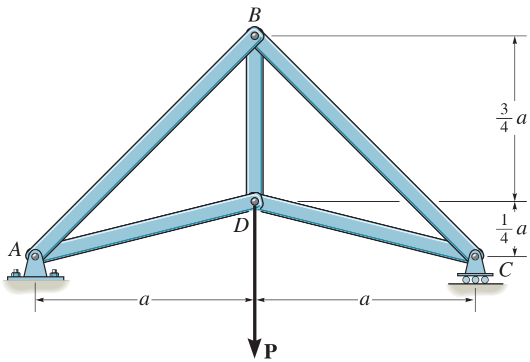


Prob. 6–12

6–13. Determine the force in each member of the truss in terms of the load P and state if the members are in tension or compression.

6–14. Members AB and BC can each support a maximum compressive force of 800 lb, and members AD, DC, and BD can support a maximum tensile force of 1500 lb. If $a = 10$ ft, determine the greatest load P the truss can support.

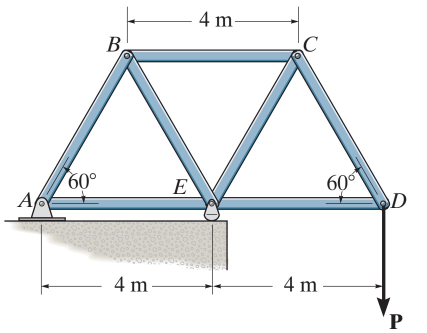
6–15. Members AB and BC can each support a maximum compressive force of 800 lb, and members AD, DC, and BD can support a maximum tensile force of 2000 lb. If $a = 6$ ft, determine the greatest load P the truss can support.



Probs. 6–13/14/15

***6–16.** Determine the force in each member of the truss. State whether the members are in tension or compression. Set $P = 8$ kN.

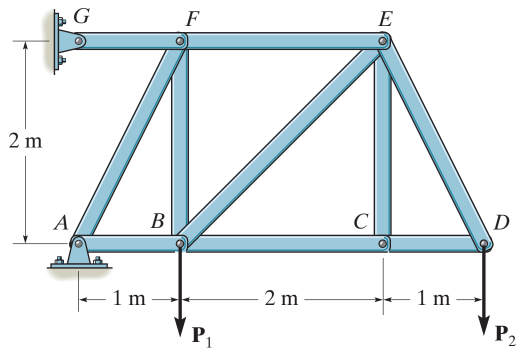
6–17. If the maximum force that any member can support is 8 kN in tension and 6 kN in compression, determine the maximum force P that can be supported at joint D.



Probs. 6–16/17

6–18. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 10$ kN, $P_2 = 8$ kN.

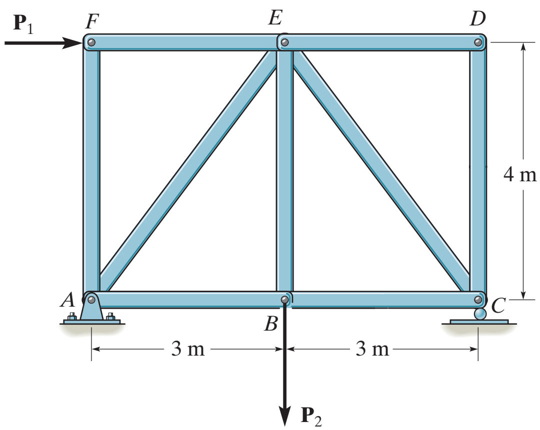
6–19. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 8$ kN, $P_2 = 12$ kN.



Probs. 6–18/19

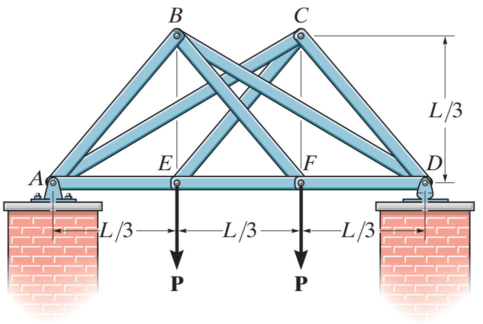
***6–20.** Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 9$ kN, $P_2 = 15$ kN.

6–21. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 30$ kN, $P_2 = 15$ kN.



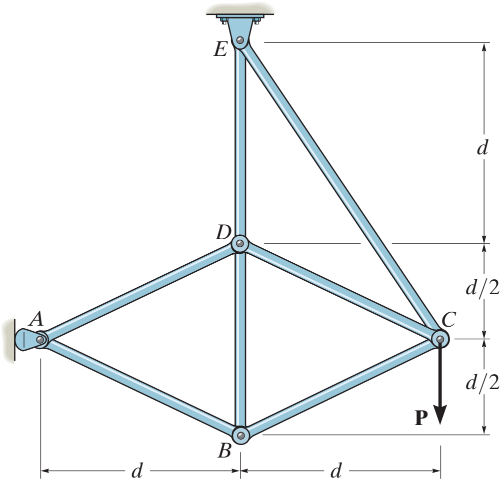
Probs. 6–20/21

6–22. Determine the force in each member of the double scissors truss in terms of the load P and state if the members are in tension or compression.



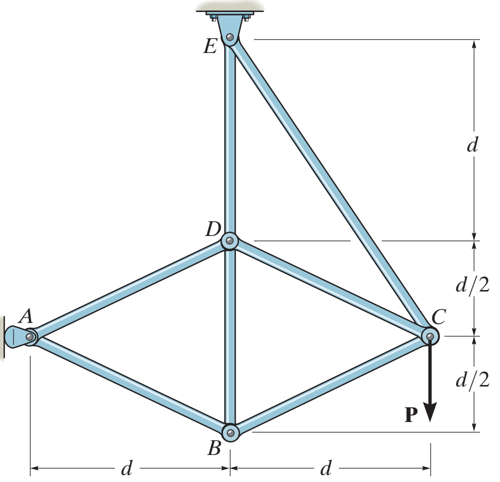
Prob. 6–22

6–23. Determine the force in each member of the truss in terms of the load P and state if the members are in tension or compression.



Prob. 6–23

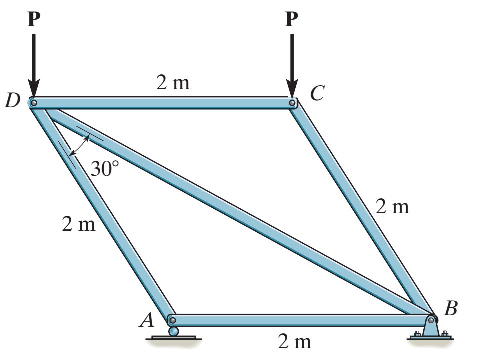
***6–24.** The maximum allowable tensile force in the members of the truss is $(F_t)_{max} = 5$ kN, and the maximum allowable compressive force is $(F_c)_{max} = 3$ kN. Determine the maximum magnitude of load $\mathbf{P}$ that can be applied to the truss. Take $d = 2$ m.



Prob. 6–24

6–25. Determine the force in each member of the truss in terms of the external loading and state if the members are in tension or compression. Take $P = 2$ kN.

6–26. The maximum allowable tensile force in the members of the truss is $(F_t)_{max} = 5$ kN, and the maximum allowable compressive force is $(F_c)_{max} = 3$ kN. Determine the maximum magnitude P of the two loads that can be applied to the truss.



Probs. 6–25/26

6.4 The Method of Sections

When we need to find the force in only a few members of a truss, we can analyze the truss using the ***method of sections***. It is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium. For example, consider the two truss members shown on the left in Fig. 6–14. If the forces within the members are to be determined, then an imaginary section, indicated by the blue line, can be used to cut each member into two parts and thereby "expose" each internal force as "external" to the free-body diagrams shown on the right. Clearly, it can be seen that equilibrium requires that the member in tension (T) be subjected to a "pull," whereas the member in compression (C) is subjected to a "push."

The method of sections can also be used to "cut" or section the members of an entire truss. If the section passes through the truss and the free-body diagram of either of its two parts is drawn, we can then apply the equations of equilibrium to that part to determine the member forces at the "cut section." Since only *three* independent equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$) can be applied to the free-body diagram of any segment, then we should try to select a section that, in general, passes through not more than *three* members in which the forces are unknown. For example, consider the truss in Fig. 6–15*a*. If the forces in members BC, GC, and GF are to be determined, then section aa would be appropriate. The free-body diagrams of the two segments are shown in Figs. 6–15*b* and 6–15*c*. Note that the line of action of each member force is specified from the *geometry* of the truss, since the force in a member is along its axis. Also, the member forces acting on one part of the truss are equal but opposite to those acting on the other part—Newton's third law. Members BC and GC are assumed to be in *tension* since they are subjected to a "pull," whereas GF in *compression* since it is subjected to a "push."

The three unknown member forces $\mathbf{F}_{BC}$, $\mathbf{F}_{GC}$, and $\mathbf{F}_{GF}$ can be obtained by applying the three equilibrium equations to the free-body diagram in Fig. 6–15*b*. If, however, the free-body diagram in Fig. 6–15*c* is considered, the three support reactions $\mathbf{D}_x$, $\mathbf{D}_y$ and $\mathbf{E}_x$ will have to be known, because only three equations of equilibrium are available. (This, of course, is done in the usual manner by considering a free-body diagram of the *entire truss*.)

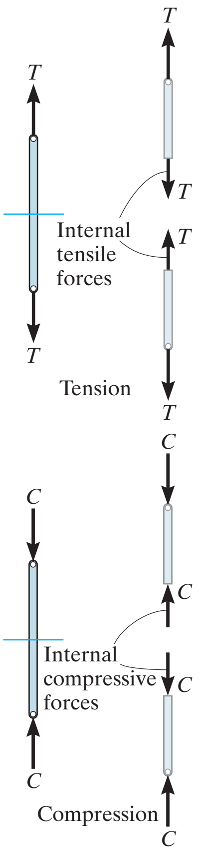


Fig. 6–14

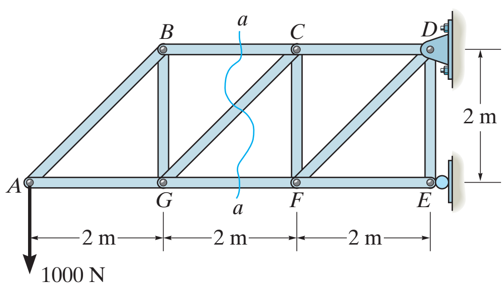


Fig. 6–15

The forces in selected members of this Pratt truss can readily be determined using the method of sections. (© Russell C. Hibbeler)

When applying the equilibrium equations, we should carefully consider ways of writing the equations so as to yield a *direct solution* for each of the unknowns, rather than having to solve simultaneous equations. For example, using the truss segment in Fig. 6–15*b* and summing moments about C would yield a direct solution for $\mathbf{F}_{GF}$ since $\mathbf{F}_{BC}$ and $\mathbf{F}_{GC}$ create zero moment about C. Likewise, $\mathbf{F}_{BC}$ can be directly obtained by summing moments about G. Finally, $\mathbf{F}_{GC}$ can be found directly from a force summation in the vertical direction since $\mathbf{F}_{GF}$ and $\mathbf{F}_{BC}$ have no vertical components. This ability to *determine directly* the force in a particular truss member is one of the main advantages of using the method of sections.*

As in the method of joints, there are two ways in which we can determine the correct sense of an unknown member force:

- The correct sense of an unknown member force can in many cases be determined "by inspection." For example, $\mathbf{F}_{BC}$ is a tensile force as represented in Fig. 6–15*b* since moment equilibrium about G requires that $\mathbf{F}_{BC}$ create a moment opposite to that of the 1000-N force. Also, $\mathbf{F}_{GC}$ is tensile since its vertical component must balance the 1000-N force which acts downward. In more complicated cases, the sense of an unknown member force may be *assumed*. If the solution yields a *negative* scalar, it indicates that the force's sense is *opposite* to that shown on the free-body diagram.
- *Always assume* that the unknown member forces at the cut section are *tensile* forces, i.e., "pulling" on the member. By doing this, the numerical solution of the equilibrium equations will yield *positive scalars for members in tension and negative scalars for members in compression.*

*Notice that if the method of joints were used to determine, say, the force in member GC, it would be necessary to analyze joints A, B, and G in sequence.

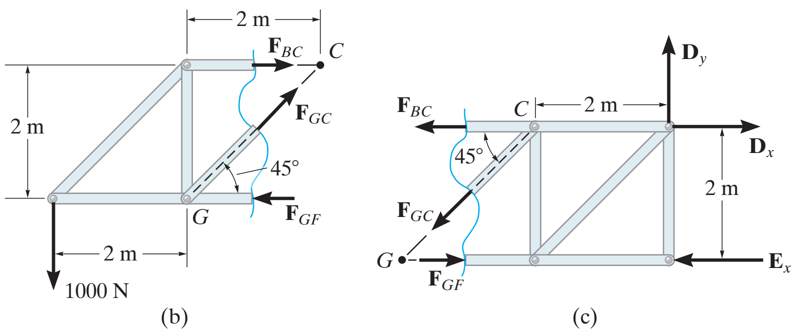


Fig. 6–15 (cont.)

Important Point

- If a truss is in equilibrium, then each of its segments is in equilibrium. The internal forces in the members become external forces when the free-body diagram of a segment of the truss is drawn. A force pulling on a member causes tension in the member, and a force pushing on a member causes compression.

Simple trusses are often used in the construction of large cranes in order to reduce the weight of the boom and tower. (© Russell C. Hibbeler)

Procedure for Analysis

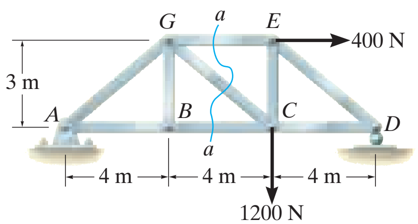
The forces in the members of a truss may be determined by the method of sections using the following procedure.

Free-Body Diagram.

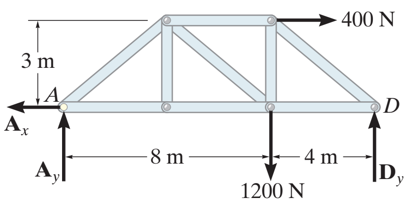
- Make a decision on how to "cut" or section the truss through the members where forces are to be determined.
- Before isolating the appropriate section, it may first be necessary to determine the truss's support reactions. If this is done then the three equilibrium equations will be available to solve for member forces at the section.
- Draw the free-body diagram of that segment of the sectioned truss which has the least number of forces acting on it.
- Use one of the two methods described above for establishing the sense of the unknown member forces.

Equations of Equilibrium.

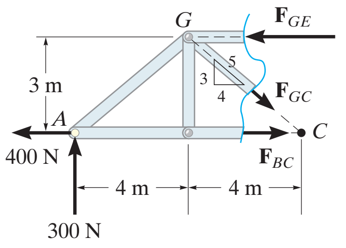
- Moments should be summed about a point that lies at the intersection of the lines of action of two unknown forces, so that the third unknown force can be determined directly from the moment equation.
- If two of the unknown forces are *parallel*, forces may be summed *perpendicular* to the direction of these unknowns to determine *directly* the third unknown force.



(a)



(b)



(c)

Fig. 6–16

Determine the force in members GE, GC, and BC of the truss shown in Fig. 6–16*a*. Indicate whether the members are in tension or compression.

SOLUTION

Section *aa* in Fig. 6–16*a* has been chosen since it cuts through the *three* members whose forces are to be determined. In order to use the method of sections, however, it is *first* necessary to determine the external reactions at A or D. Why? A free-body diagram of the entire truss is shown in Fig. 6–16*b*. Applying the equations of equilibrium, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 400\text{ N} - A_x = 0 \qquad A_x = 400\text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -1200\text{ N}(8\text{ m}) - 400\text{ N}(3\text{ m}) + D_y(12\text{ m}) = 0$$

$$D_y = 900\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 1200\text{ N} + 900\text{ N} = 0 \qquad A_y = 300\text{ N}$$

Free-Body Diagram. For the analysis the free-body diagram of the left portion of the sectioned truss will be used, since it involves the least number of forces, Fig. 6–16*c*.

Equations of Equilibrium. Summing moments about point G eliminates $\mathbf{F}_{GE}$ and $\mathbf{F}_{GC}$ and yields a direct solution for F_{BC}.

$$\circlearrowleft + \Sigma M_G = 0; \quad -300\text{ N}(4\text{ m}) - 400\text{ N}(3\text{ m}) + F_{BC}(3\text{ m}) = 0$$

$$F_{BC} = 800\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$

In the same manner, by summing moments about point C we obtain a direct solution for F_{GE}.

$$\circlearrowleft + \Sigma M_C = 0; \quad -300\text{ N}(8\text{ m}) + F_{GE}(3\text{ m}) = 0$$

$$F_{GE} = 800\text{ N} \quad (\text{C}) \qquad \textit{Ans.}$$

Since $\mathbf{F}_{BC}$ and $\mathbf{F}_{GE}$ have no vertical components, summing forces in the y direction directly yields F_{GC}, i.e.,

$$+\uparrow \Sigma F_y = 0; \qquad 300\text{ N} - \tfrac{3}{5}F_{GC} = 0$$

$$F_{GC} = 500\text{ N} \quad (\text{T}) \qquad \textit{Ans.}$$

NOTE: Here it is possible to tell, by inspection, the proper direction for each unknown member force. For example, $\Sigma M_C = 0$ requires $\mathbf{F}_{GE}$ to be *compressive* because it must balance the moment of the 300-N force about C.

EXAMPLE 6.6

Determine the force in member *CF* of the truss shown in Fig. 6–17*a*. Indicate whether the member is in tension or compression. Assume each member is pin connected.

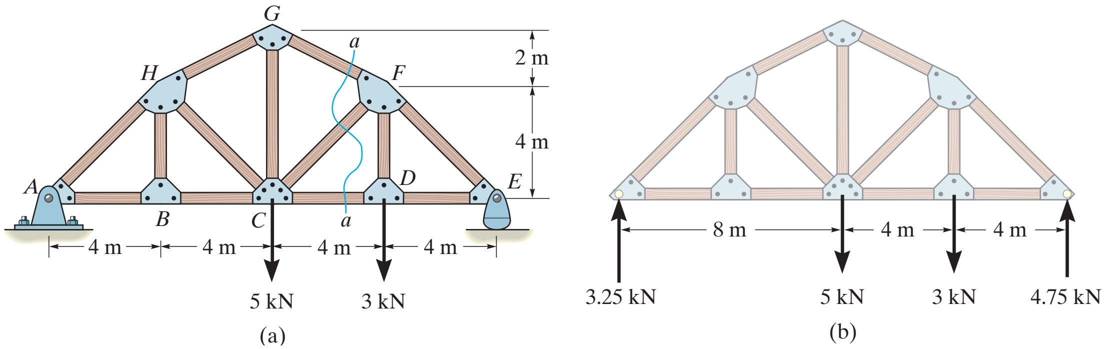
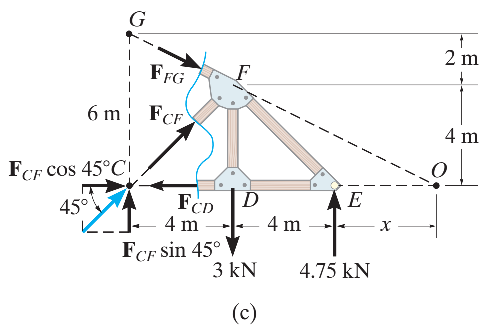


Fig. 6–17

SOLUTION

Free-Body Diagram. Section *aa* in Fig. 6–17*a* will be used since this section will "expose" the internal force in member *CF* as "external" on the free-body diagram of either the right or left portion of the truss. It is first necessary, however, to determine the support reactions on either the left or right side. Verify the results shown on the free-body diagram in Fig. 6–17*b*.

The free-body diagram of the right portion of the truss, which is the easiest to analyze, is shown in Fig. 6–17*c*. There are three unknowns, F_{FG}, F_{CF}, and F_{CD}.



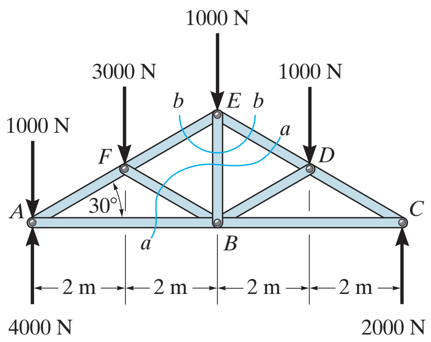
Equations of Equilibrium. We will apply the moment equation about point O in order to eliminate the two unknowns F_{FG} and F_{CD}. The location of point O measured from E can be determined from proportional triangles, i.e., $4/(4+x) = 6/(8+x)$, $x = 4$ m. Or, stated in another manner, the slope of member *GF* has a drop of 2 m to a horizontal distance of 4 m. Since *FD* is 4 m, Fig. 6–17*c*, then from *D* to *O* the distance must be 8 m.

An easy way to determine the moment of $\mathbf{F}_{CF}$ about point O is to use the principle of transmissibility and slide $\mathbf{F}_{CF}$ to point C, and then resolve $\mathbf{F}_{CF}$ into its two rectangular components. We have

$$\circlearrowleft + \Sigma M_O = 0;$$

$$-F_{CF} \sin 45^\circ (12 \text{ m}) + (3 \text{ kN})(8 \text{ m}) - (4.75 \text{ kN})(4 \text{ m}) = 0$$

$$F_{CF} = 0.589 \text{ kN} \quad \text{(C)} \qquad \textit{Ans.}$$



(a)

Determine the force in member EB of the roof truss shown in Fig. 6–18a. Indicate whether the member is in tension or compression.

SOLUTION

Free-Body Diagrams. By the method of sections, any imaginary section that cuts through EB, Fig. 6–18a, will also have to cut through three other members for which the forces are unknown. For example, section aa cuts through ED, EB, FB, and AB. If a free-body diagram of the left side of this section is considered, Fig. 6–18b, it is possible to obtain $\mathbf{F}_{ED}$ by summing moments about B to eliminate the other three unknowns; however, $\mathbf{F}_{EB}$ cannot be determined from the remaining two equilibrium equations. One possible way of obtaining $\mathbf{F}_{EB}$ is first to determine $\mathbf{F}_{ED}$ from section aa, then use this result on section bb, Fig. 6–18a, which is shown in Fig. 6–18c. Here the force system is concurrent and our sectioned free-body diagram is the same as the free-body diagram for the joint at E.

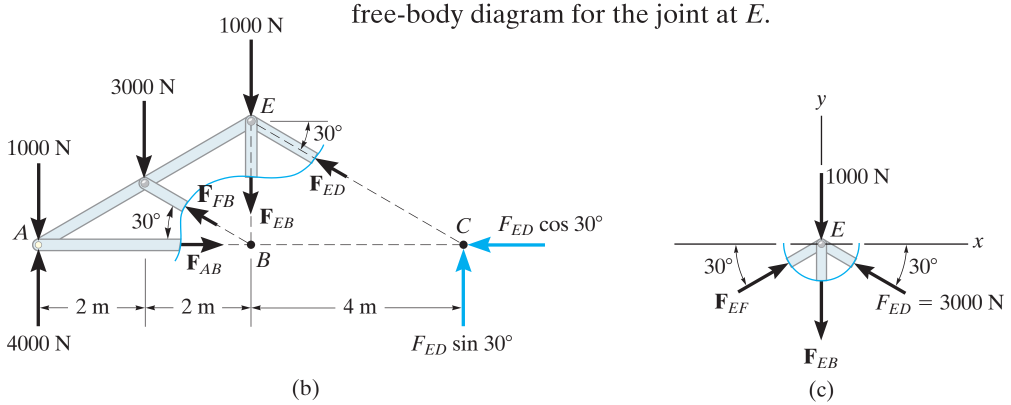


Fig. 6–18

Equations of Equilibrium. In order to determine the moment of $\mathbf{F}_{ED}$ about point B, Fig. 6–18b, we will use the principle of transmissibility and slide the force to point C and then resolve it into its rectangular components as shown. Therefore,

$$\circlearrowleft + \Sigma M_B = 0; \qquad 1000\text{ N}(4\text{ m}) + 3000\text{ N}(2\text{ m}) - 4000\text{ N}(4\text{ m})$$

$$+ F_{ED}\sin 30^\circ(4\text{ m}) = 0$$

$$F_{ED} = 3000\text{ N} \quad \text{(C)}$$

Considering now the free-body diagram of section bb, Fig. 6–18c, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{EF}\cos 30^\circ - 3000\cos 30^\circ\text{ N} = 0$$

$$F_{EF} = 3000\text{ N} \quad \text{(C)}$$

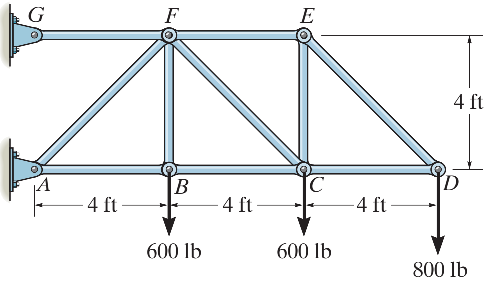
$$+\uparrow \Sigma F_y = 0; \qquad 2(3000\sin 30^\circ\text{ N}) - 1000\text{ N} - F_{EB} = 0$$

$$F_{EB} = 2000\text{ N} \quad \text{(T)} \qquad \textit{Ans.}$$

FUNDAMENTAL PROBLEMS

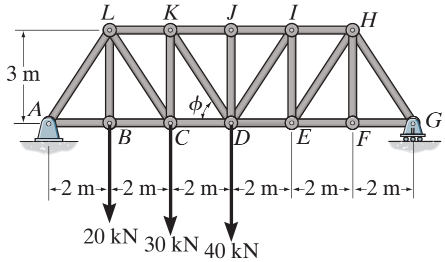
All problem solutions must include FBDs.

F6–7. Determine the force in members *BC*, *CF*, and *FE*. State if the members are in tension or compression.



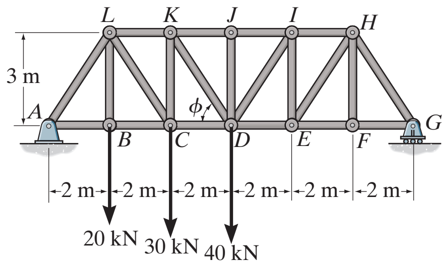
Prob. F6–7

F6–8. Determine the force in members *LK*, *KC*, and *CD* of the Pratt truss. State if the members are in tension or compression.



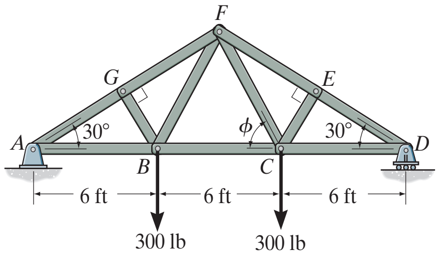
Prob. F6–8

F6–9. Determine the force in members *KJ*, *KD*, and *CD* of the Pratt truss. State if the members are in tension or compression.



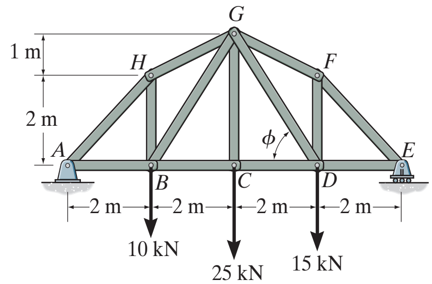
Prob. F6–9

F6–10. Determine the force in members *EF*, *CF*, and *BC* of the truss. State if the members are in tension or compression.



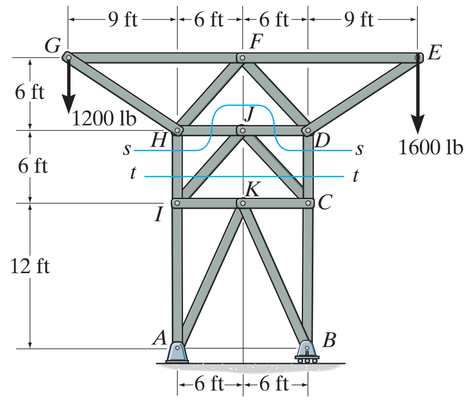
Prob. F6–10

F6–11. Determine the force in members *GF*, *GD*, and *CD* of the truss. State if the members are in tension or compression.



Prob. F6–11

F6–12. Determine the force in members *DC*, *HI*, and *JI* of the truss. State if the members are in tension or compression. *Suggestion:* Use the sections shown.



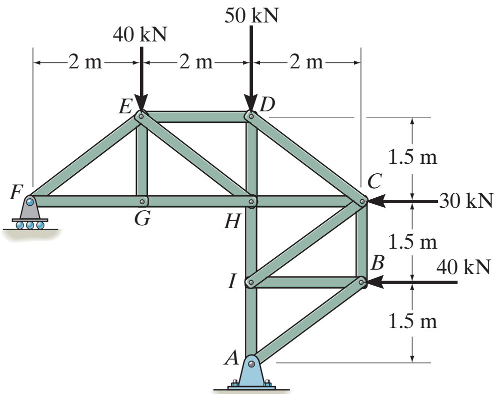
Prob. F6–12

PROBLEMS

All problem solutions must include FBDs.

6–27. Determine the force in members DC, HC, and HI of the truss, and state if the members are in tension or compression.

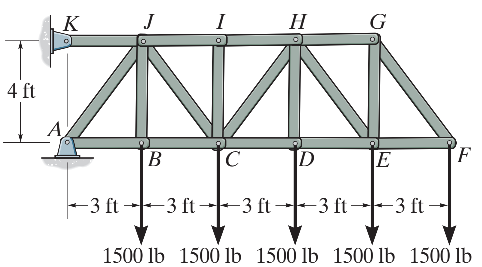
***6–28.** Determine the force in members ED, EH, and GH of the truss, and state if the members are in tension or compression.



Probs. 6–27/28

6–29. Determine the force in members HG, HE and DE of the truss, and state if the members are in tension or compression.

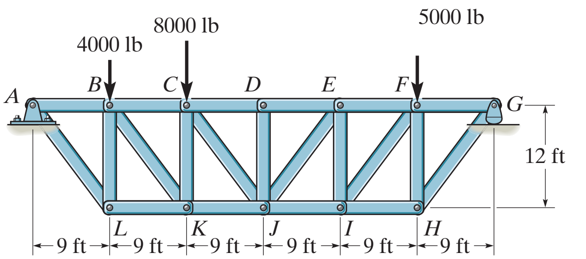
6–30. Determine the force in members CD, HI, and CH of the truss, and state if the members are in tension or compression.



Probs. 6–29/30

6–31. Determine the force in members CD, CJ, KJ, and DJ of the truss which serves to support the deck of a bridge. State if these members are in tension or compression.

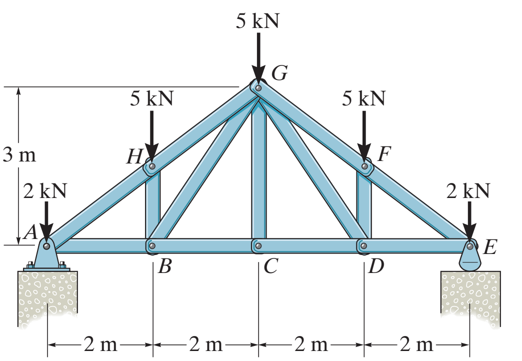
***6–32.** Determine the force in members EI and JI of the truss which serves to support the deck of a bridge. State if these members are in tension or compression.



Probs. 6–31/32

6–33. The *Howe truss* is subjected to the loading shown. Determine the force in members GF, CD, and GC, and state if the members are in tension or compression.

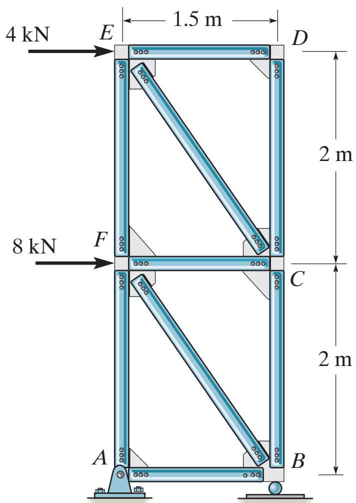
6–34. The *Howe truss* is subjected to the loading shown. Determine the force in members GH, BC, and BG of the truss and state if the members are in tension or compression.



Probs. 6–33/34

6–35. Determine the force in members *EF*, *CF*, and *BC*, and state if the members are in tension or compression.

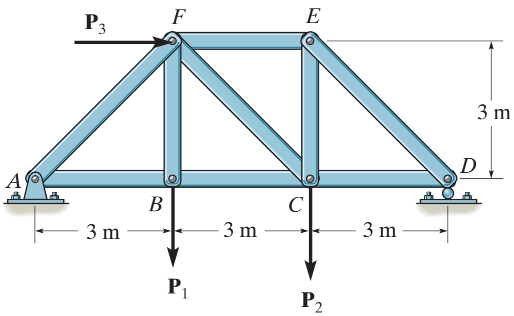
***6–36.** Determine the force in members *AF*, *BF*, and *BC*, and state if the members are in tension or compression.



Probs. 6–35/36

6–37. Determine the force in members *EF*, *BE*, *BC* and *BF* of the truss and state if these members are in tension or compression. Set $P_1 = 9$ kN, $P_2 = 12$ kN, and $P_3 = 6$ kN.

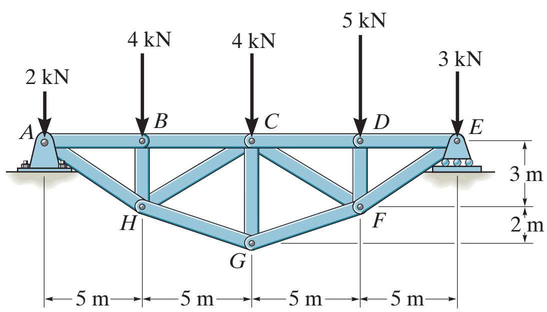
6–38. Determine the force in members *BC*, *BE*, and *EF* of the truss and state if these members are in tension or compression. Set $P_1 = 6$ kN, $P_2 = 9$ kN, and $P_3 = 12$ kN.



Probs. 6–37/38

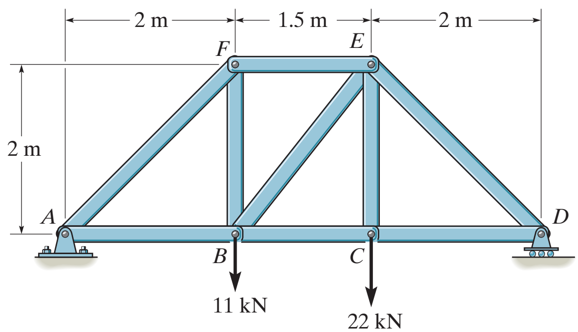
6–39. Determine the force in members *BC*, *HC*, and *HG*. After the truss is sectioned use a single equation of equilibrium for the calculation of each force. State if these members are in tension or compression.

***6–40.** Determine the force in members *CD*, *CF*, and *CG* and state if these members are in tension or compression.



Probs. 6–39/40

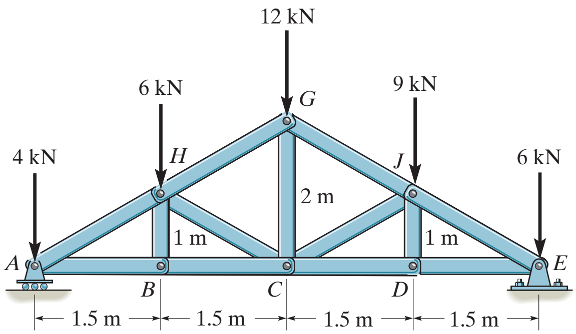
6–41. Determine the force developed in members *FE*, *EB*, and *BC* of the truss and state if these members are in tension or compression.



Prob. 6–41

6–42. Determine the force in members *BC*, *HC*, and *HG*. State if these members are in tension or compression.

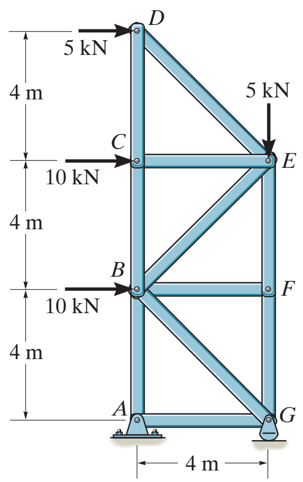
6–43. Determine the force in members *CD*, *CJ*, *GJ*, and *CG* and state if these members are in tension or compression.



Probs. 6–42/43

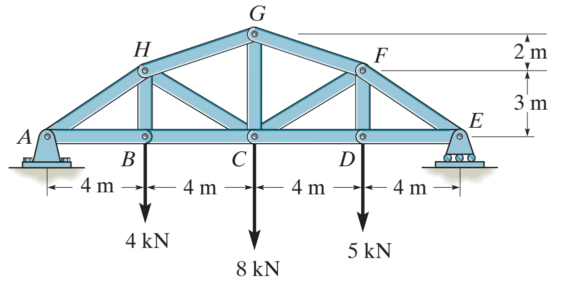
***6–44.** Determine the force in members *BE*, *EF*, and *CB*, and state if the members are in tension or compression.

6–45. Determine the force in members *BF*, *BG*, and *AB*, and state if the members are in tension or compression.



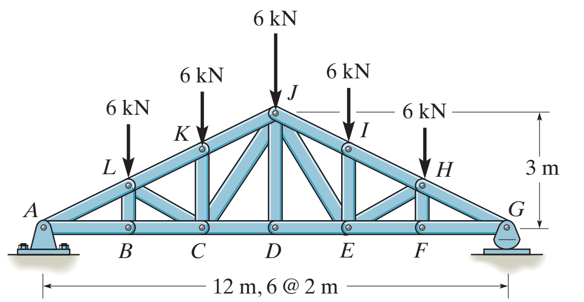
Probs. 6–44/45

6–46. Determine the force in members *BC*, *CH*, *GH*, and *CG* of the truss and state if the members are in tension or compression.



Prob. 6–46

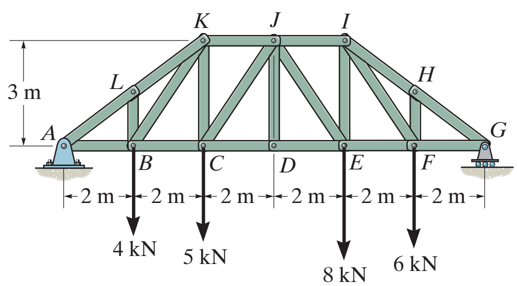
6–47. Determine the force in members *CD*, *CJ*, and *KJ* and state if these members are in tension or compression.



Prob. 6–47

***6–48.** Determine the force in members *JK*, *CJ*, and *CD* of the truss, and state if the members are in tension or compression.

6–49. Determine the force in members *HI*, *FI*, and *EF* of the truss, and state if the members are in tension or compression.



Probs. 6–48/49

*6.5 Space Trusses

A ***space truss*** consists of members joined together at their ends to form a stable three-dimensional structure. The simplest form of a space truss is a ***tetrahedron***, formed by connecting six members together, as shown in Fig. 6–19. Any additional members added to this basic element would be redundant in supporting the force **P**. A *simple space truss* can be built from this basic tetrahedral element by adding three additional members and a joint, and continuing in this manner to form a system of multiconnected tetrahedrons.

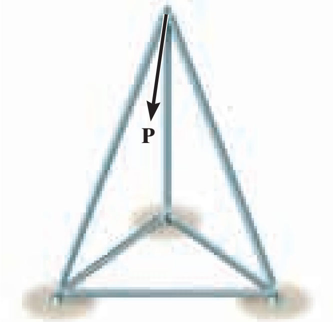
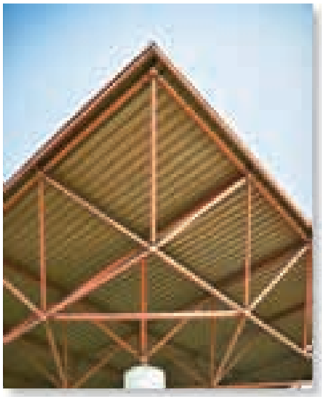


Fig. 6–19

Assumptions for Design. The members of a space truss may be treated as two-force members provided the external loading is applied at the joints and the joints consist of ball-and-socket connections. These assumptions are justified if the welded or bolted connections of the joined members intersect at a common point and the weight of the members can be neglected. In cases where the weight of a member is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, half of its magnitude applied at each end of the member.



Typical roof-supporting space truss. Notice the use of ball-and-socket joints for the connections. (© Russell C. Hibbeler)

Procedure for Analysis

Either the method of joints or the method of sections can be used to determine the forces developed in the members of a simple space truss.

Method of Joints.

If the forces in *all* the members of the truss are to be determined, then the method of joints is most suitable for the analysis. Here it is necessary to apply the three equilibrium equations $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$ to the forces acting at each joint. Remember that the solution of many simultaneous equations can be avoided if the force analysis begins at a joint having at least one known force and at most three unknown forces. Also, if the three-dimensional geometry of the force system at the joint is hard to visualize, it is recommended that a Cartesian vector analysis be used for the solution.

Method of Sections.

If only a *few* member forces are to be determined, the method of sections can be used. When an imaginary section is passed through a truss and the truss is separated into two parts, the force system acting on one of the segments must satisfy the *six* equilibrium equations: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$ (Eqs. 5–6). By proper choice of the section and axes for summing forces and moments, many of the unknown member forces in a space truss can be computed *directly*, using a single equilibrium equation.

For economic reasons, large electrical transmission towers are often constructed using space trusses. (© Russell C. Hibbeler)

Determine the forces acting in the members of the space truss shown in Fig. 6–20*a*. Indicate whether the members are in tension or compression.

SOLUTION

Since there are one known force and three unknown forces acting at joint *A*, the force analysis of the truss will begin at this joint.

Joint *A*. (Fig. 6–20*b*). Expressing each force acting on the free-body diagram of joint *A* as a Cartesian vector, we have

$$\mathbf{P} = \{-4\mathbf{j}\}\ \text{kN}, \qquad \mathbf{F}_{AB} = F_{AB}\,\mathbf{j}, \quad \mathbf{F}_{AC} = -F_{AC}\,\mathbf{k},$$

$$\mathbf{F}_{AE} = F_{AE}\left(\frac{\mathbf{r}_{AE}}{r_{AE}}\right) = F_{AE}(0.577\mathbf{i} + 0.577\mathbf{j} - 0.577\mathbf{k})$$

For equilibrium,

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{P} + \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AE} = \mathbf{0}$$

$$-4\mathbf{j} + F_{AB}\,\mathbf{j} - F_{AC}\,\mathbf{k} + 0.577F_{AE}\,\mathbf{i} + 0.577F_{AE}\,\mathbf{j} - 0.577F_{AE}\,\mathbf{k} = \mathbf{0}$$

$$\Sigma F_x = 0; \qquad 0.577F_{AE} = 0$$

$$\Sigma F_y = 0; \qquad -4 + F_{AB} + 0.577F_{AE} = 0$$

$$\Sigma F_z = 0; \qquad -F_{AC} - 0.577F_{AE} = 0$$

$$F_{AC} = F_{AE} = 0 \qquad \textit{Ans.}$$

$$F_{AB} = 4\ \text{kN} \quad \text{(T)} \qquad \textit{Ans.}$$

Since F_{AB} is known, joint *B* can be analyzed next.

Joint *B*. (Fig. 6–20*c*).

$$\Sigma F_x = 0; \qquad F_{BE}\frac{1}{\sqrt{2}} = 0$$

$$\Sigma F_y = 0; \qquad -4 + F_{CB}\frac{1}{\sqrt{2}} = 0$$

$$\Sigma F_z = 0; \qquad -2 + F_{BD} - F_{BE}\frac{1}{\sqrt{2}} + F_{CB}\frac{1}{\sqrt{2}} = 0$$

$$F_{BE} = 0, \qquad F_{CB} = 5.65\ \text{kN (C)} \qquad F_{BD} = 2\ \text{kN (T)} \qquad \textit{Ans.}$$

The *scalar* equations of equilibrium can now be applied to the forces acting on the free-body diagrams of joints *D* and *C*. Show that

$$F_{DE} = F_{DC} = F_{CE} = 0 \qquad \textit{Ans.}$$

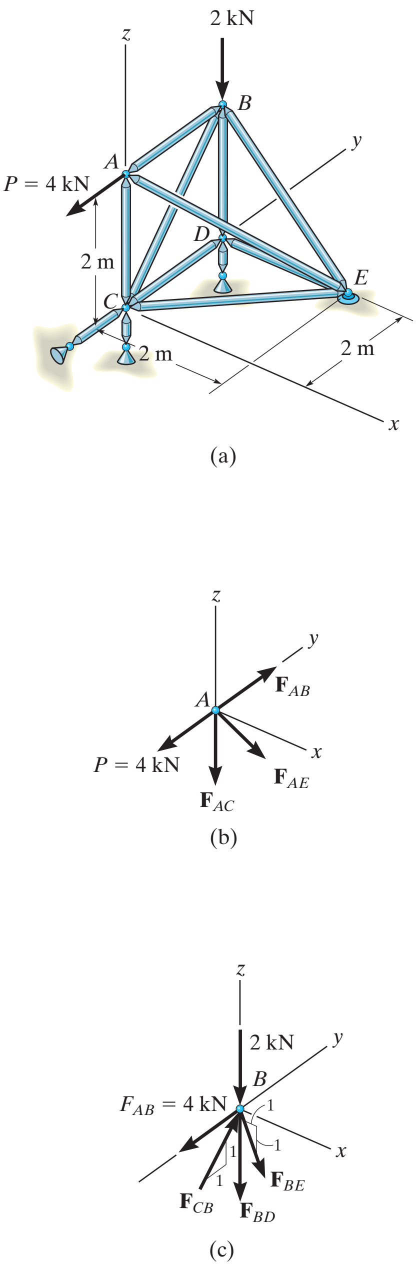
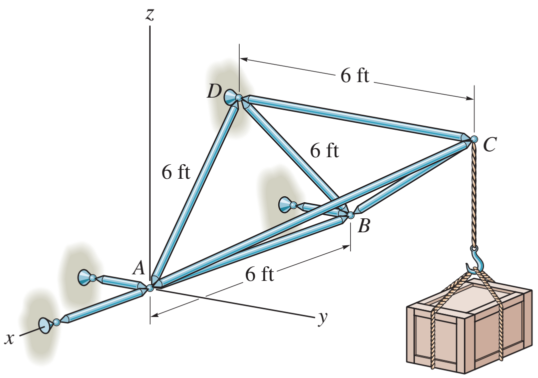


Fig. 6–20

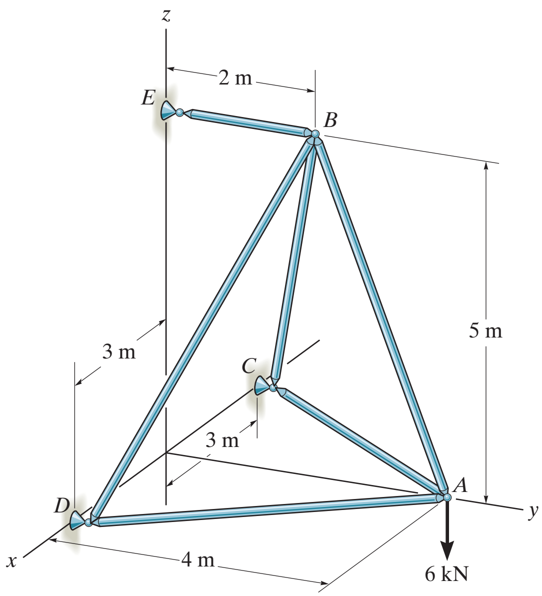
All problem solutions must include FBDs.

6–50. Determine the force developed in each member of the space truss and state if the members are in tension or compression. The crate has a weight of 150 lb.



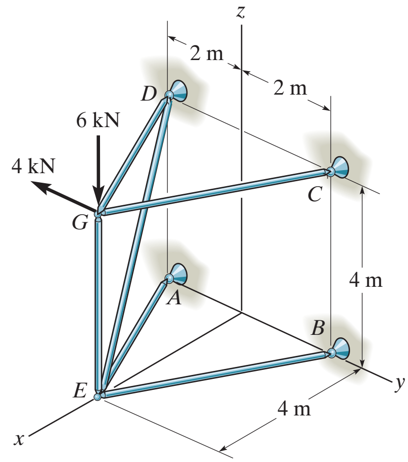
Prob. 6–50

6–51. Determine the force in each member of the space truss and state if the members are in tension or compression. *Hint*: The support reaction at E acts along member EB. Why?



Prob. 6–51

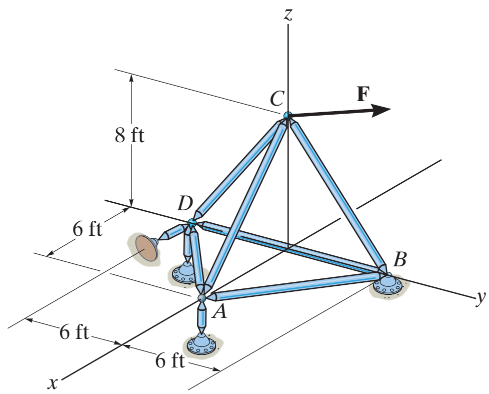
***6–52.** Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at $A, B, C,$ and D.



Prob. 6–52

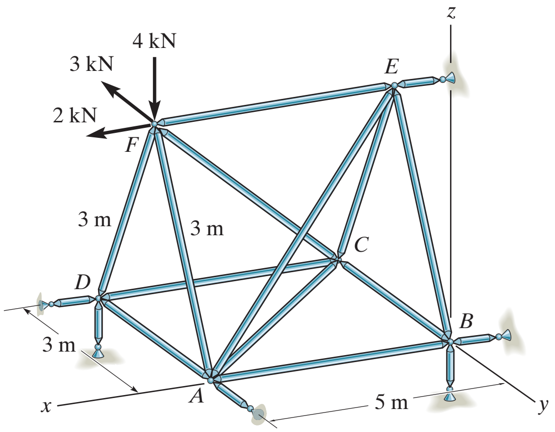
6–53. The space truss supports a force $\mathbf{F} = \{-500\mathbf{i} + 600\mathbf{j} + 400\mathbf{k}\}$ lb. Determine the force in each member, and state if the members are in tension or compression.

6–54. The space truss supports a force $\mathbf{F} = \{600\mathbf{i} + 450\mathbf{j} - 750\mathbf{k}\}$ lb. Determine the force in each member, and state if the members are in tension or compression.



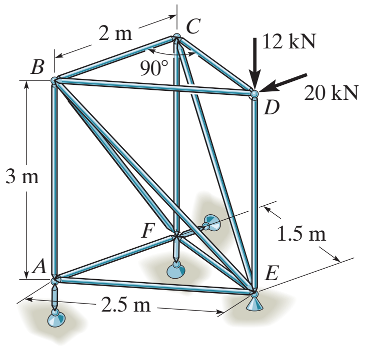
Probs. 6–53/54

6–55. Determine the force in members *EF*, *AF*, and *DF* of the space truss and state if the members are in tension or compression. The truss is supported by short links at *A*, *B*, *D*, and *E*.



Prob. 6–55

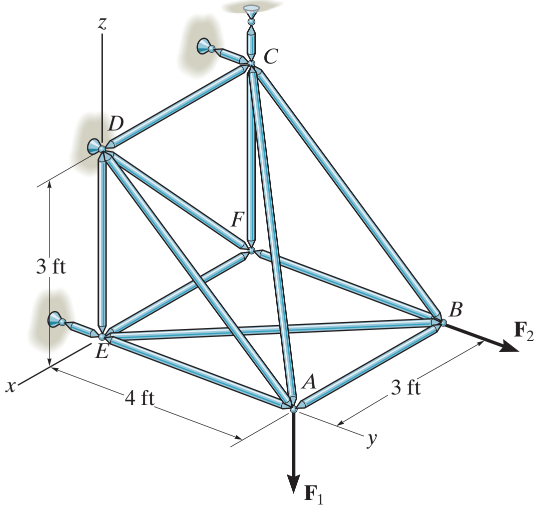
***6–56.** The space truss is used to support the forces at joints *B* and *D*. Determine the force in each member and state if the members are in tension or compression.



Prob. 6–56

6–57. The space truss is supported by a ball-and-socket joint at *D* and short links at *C* and *E*. Determine the force in each member and state if the members are in tension or compression. Take $\mathbf{F}_1 = \{-500\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{400\mathbf{j}\}$ lb.

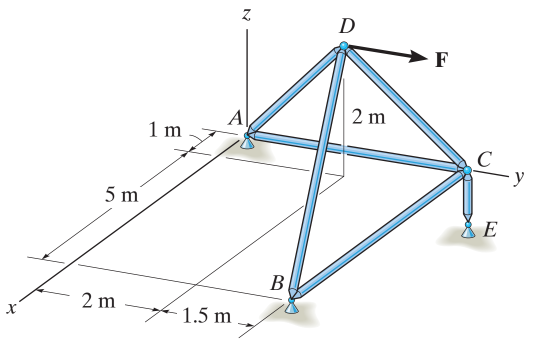
6–58. The space truss is supported by a ball-and-socket joint at *D* and short links at *C* and *E*. Determine the force in each member and state if the members are in tension or compression. Take $\mathbf{F}_1 = \{200\mathbf{i} + 300\mathbf{j} - 500\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{400\mathbf{j}\}$ lb.



Probs. 6–57/58

6–59. Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at *A*, *B*, and *E*. Set $\mathbf{F} = \{800\mathbf{j}\}$ N. *Hint*: The support reaction at *E* acts along member *EC*. Why?

***6–60.** Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at *A*, *B*, and *E*. Set $\mathbf{F} = \{-200\mathbf{i} + 400\mathbf{j}\}$ N. *Hint*: The support reaction at *E* acts along member *EC*. Why?



Probs. 6–59/60

6.6 Frames and Machines

Frames and machines are two types of structures which are often composed of pin-connected ***multiforce members***, i.e., members that are subjected to more than two forces. ***Frames*** are used to support loads, whereas ***machines*** contain moving parts and are designed to transmit and alter the effect of forces. Provided a frame or machine contains no more supports or members than are necessary to prevent its collapse, the forces acting at the joints and supports can be determined by applying the equations of equilibrium to each of its members. Once these forces are obtained, it is then possible to *design* the size of the members, connections, and supports using the theory of mechanics of materials and an appropriate engineering design code.

This crane is a typical example of a framework. (© Russell C. Hibbeler)

Free-Body Diagrams. In order to determine the forces acting at the joints and supports of a frame or machine, the structure must be disassembled and the free-body diagrams of its parts must be drawn. The following important points *must* be observed:

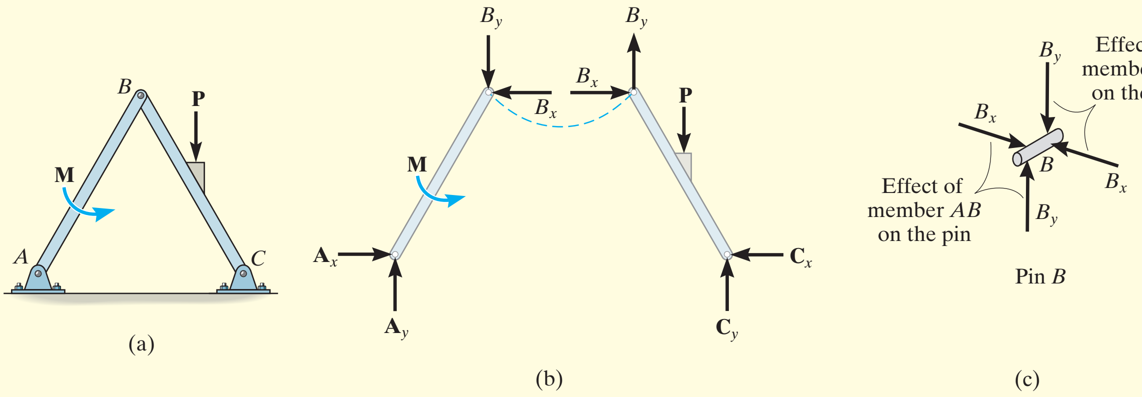
- Isolate each part by drawing its *outlined shape*. Then show all the forces and/or couple moments that act on the part. Make sure to *label* or *identify* each known and unknown force and couple moment with reference to an established x, y coordinate system. Also, indicate any dimensions used for taking moments. Most often the equations of equilibrium are easier to apply if the forces are represented by their rectangular components. As usual, the sense of an unknown force or couple moment can be assumed.
- Identify all the two-force members in the structure and represent their free-body diagrams as having two equal but opposite collinear forces acting at their points of application. (See Sec. 5.4.) By recognizing the two-force members, we can avoid solving an unnecessary number of equilibrium equations.
- Forces common to *any* two *contacting* members act with equal magnitudes but opposite sense on the respective members. If the two members are treated as a *"system" of connected members*, then these forces are *"internal"* and are *not shown* on the *free-body diagram of the system*; however, if the free-body diagram of *each member* is drawn, the forces are *"external"* and *must* be shown as equal in magnitude and opposite in direction on each of the two free-body diagrams.

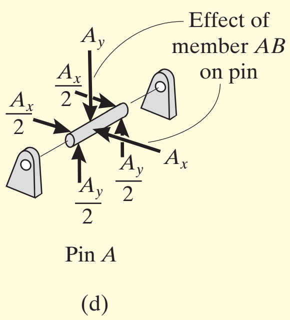
Common tools such as these pliers act as simple machines. Here the applied force on the handles creates a much larger force at the jaws. (© Russell C. Hibbeler)

The following examples graphically illustrate how to draw the free-body diagrams of a dismembered frame or machine. In all cases, the weight of the members is neglected.

EXAMPLE 6.9

For the frame shown in Fig. 6–21*a*, draw the free-body diagram of (a) each member, (b) the pins at *B* and *A*, and (c) the two members connected together.





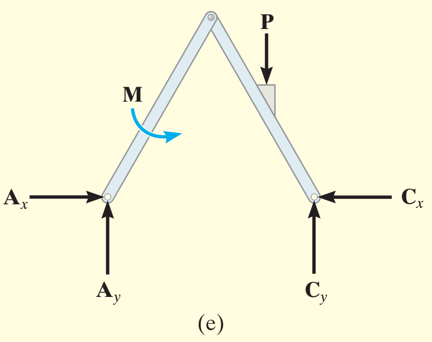


Fig. 6–21

SOLUTION

Part (a). By inspection, members *BA* and *BC* are *not* two-force members. Instead, as shown on the free-body diagrams, Fig. 6–21*b*, *BC* is subjected to a force from each of the pins at *B* and *C* and the external force **P**. Likewise, *AB* is subjected to a force from each of the pins at *A* and *B* and the external couple moment **M**. The pin forces are represented by their *x* and *y* components.

Part (b). The pin at *B* is subjected to only *two forces*, i.e., the force of member *BC* and the force of member *AB*. For *equilibrium* these forces (or their respective components) must be equal but opposite, Fig. 6–21*c*. Realize that Newton's third law is applied between the pin and its connected members, i.e., the effect of the pin on the two members, Fig. 6–21*b*, and the equal but opposite effect of the two members on the pin, Fig. 6–21*c*. In the same manner, there are three forces on pin *A*, Fig. 6–21*d*, caused by the force components of member *AB* and each of the two pin leafs.

Part (c). The free-body diagram of both members connected together, yet removed from the supporting pins at *A* and *C*, is shown in Fig. 6–21*e*. The force components $\mathbf{B}_x$ and $\mathbf{B}_y$ are *not shown* on this diagram since they are *internal* forces (Fig. 6–21*b*) and therefore cancel out. Also, to be consistent when later applying the equilibrium equations, the unknown force components at *A* and *C* must act in the *same sense* as those shown in Fig. 6–21*b*.

EXAMPLE 6.10

A constant tension in the conveyor belt is maintained by using the device shown in Fig. 6–22*a*. Draw the free-body diagrams of the frame and the cylinder (or pulley) that the belt surrounds. The suspended block has a weight of W.

(a)

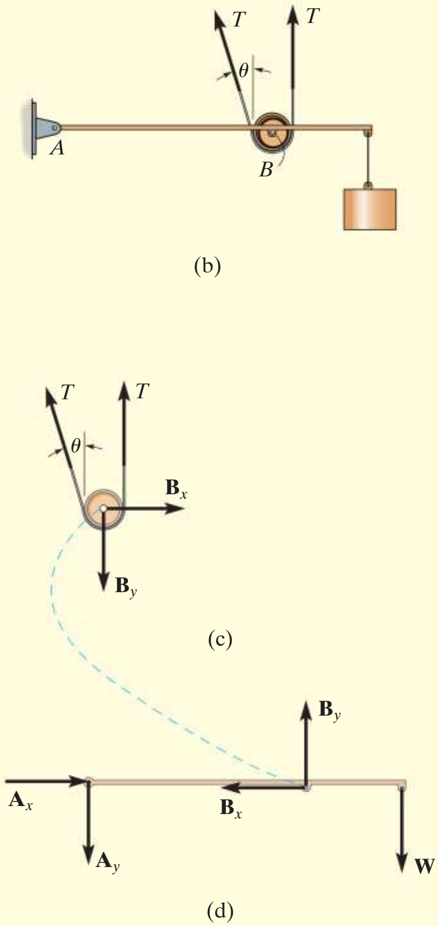


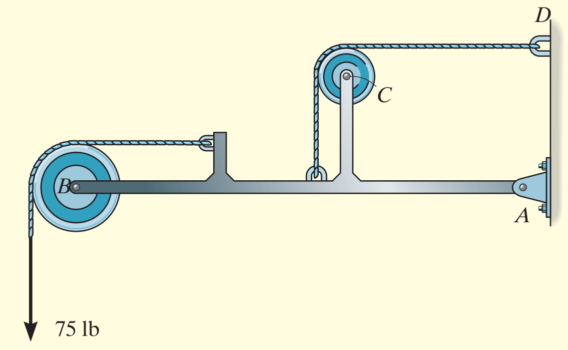
Fig. 6–22 (© Russell C. Hibbeler)

SOLUTION

The idealized model of the device is shown in Fig. 6–22*b*. Here the angle θ is assumed to be known. From this model, the free-body diagrams of the pulley and frame are shown in Figs. 6–22*c* and 6–22*d*, respectively. Note that the force components $\mathbf{B}_x$ and $\mathbf{B}_y$ that the pin at B exerts on the pulley must be equal but opposite to the ones acting on the frame. See Fig. 6–21*c* of Example 6.9.

EXAMPLE 6.11

For the frame shown in Fig. 6–23*a*, draw the free-body diagrams of (a) the entire frame including the pulleys and cords, (b) the frame without the pulleys and cords, and (c) each of the pulleys.



(a)

SOLUTION

Part (a). When the entire frame including the pulleys and cords is considered, the interactions at the points where the pulleys and cords are connected to the frame become pairs of *internal* forces which cancel each other and therefore are not shown on the free-body diagram, Fig. 6–23*b*.

Part (b). When the cords and pulleys are removed, their effect *on the frame* must be shown, Fig. 6–23*c*.

Part (c). The force components $\mathbf{B}_x$, $\mathbf{B}_y$, $\mathbf{C}_x$, $\mathbf{C}_y$ of the pins on the pulleys, Fig. 6–23*d*, are equal but opposite to the force components exerted by the pins on the frame, Fig. 6–23*c*. See Example 6.9.

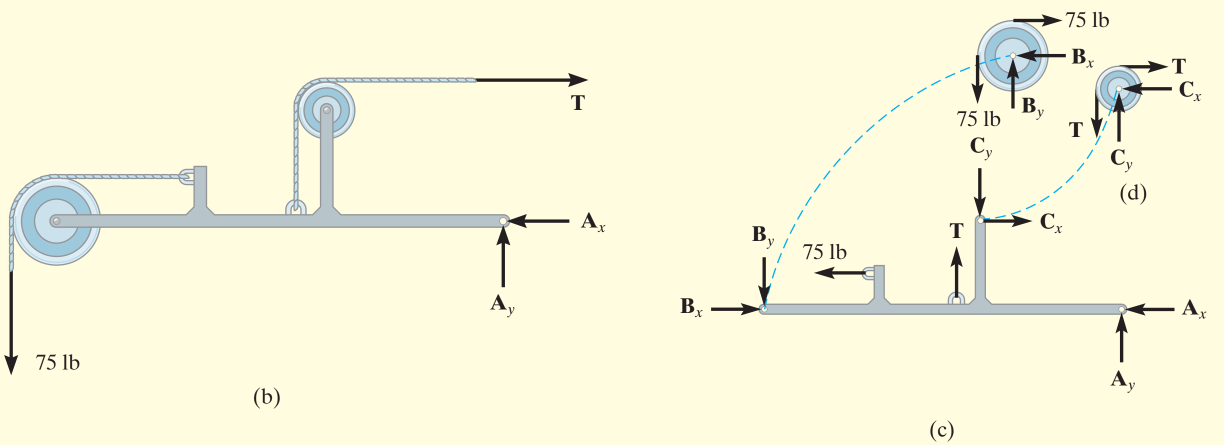
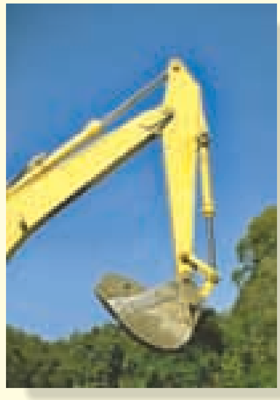


Fig. 6–23

EXAMPLE 6.12

Draw the free-body diagrams of the members of the backhoe, shown in the photo, Fig. 6–24*a*. The bucket and its contents have a weight *W*.

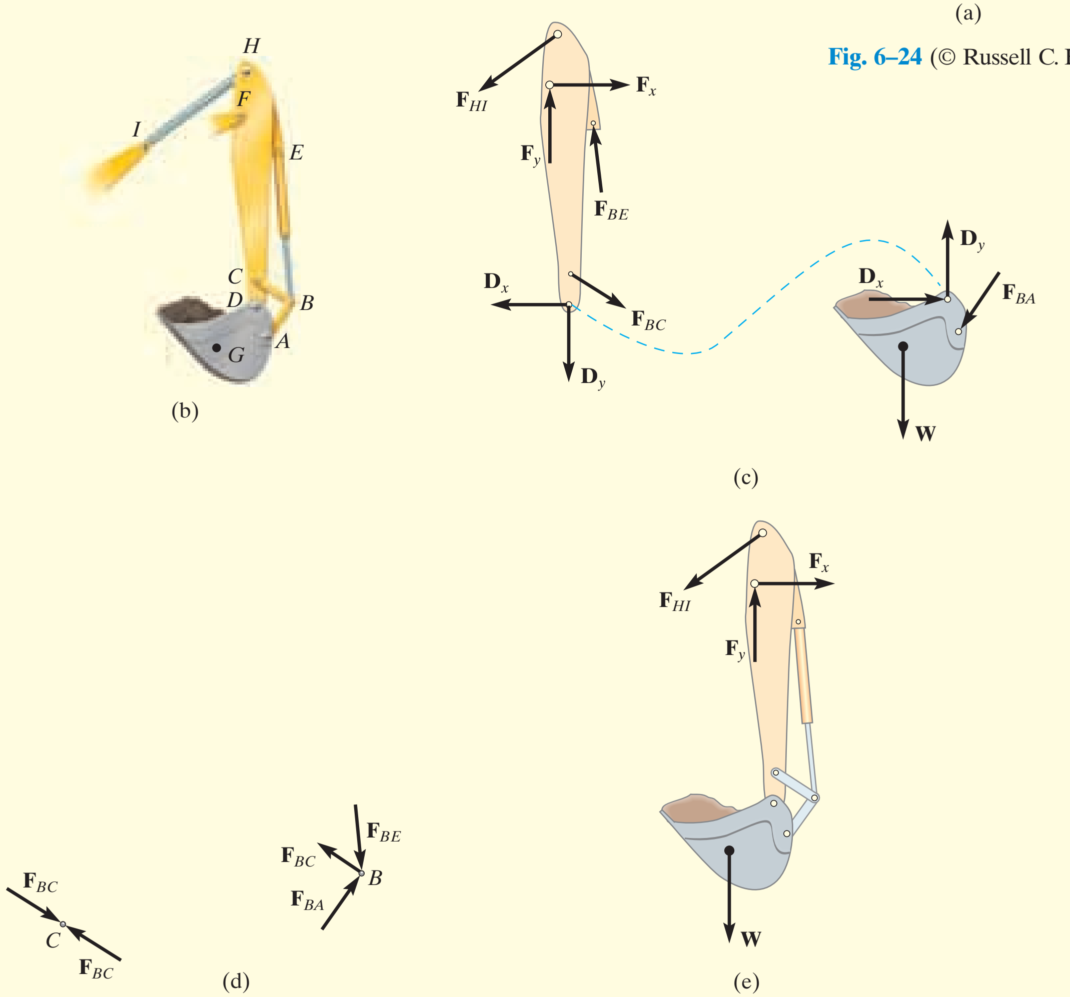


(a)

Fig. 6–24 (© Russell C. Hibbeler)

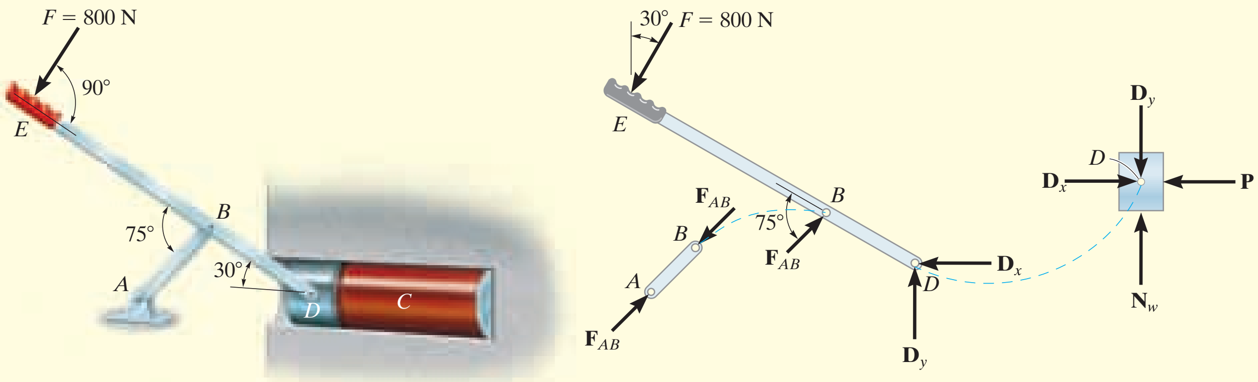
SOLUTION

The idealized model of the assembly is shown in Fig. 6–24*b*. By inspection, members *AB*, *BC*, *BE*, and *HI* are all two-force members since they are pin connected at their end points and no other forces act on them. The free-body diagrams of the bucket and the stick are shown in Fig. 6–24*c*. Note that pin *C* is subjected to only two forces, whereas the pin at *B* is subjected to three forces, Fig. 6–24*d*. The free-body diagram of the entire assembly is shown in Fig. 6–24*e*.



EXAMPLE 6.13

Draw the free-body diagram of each part of the smooth piston and link mechanism used to crush recycled cans, Fig. 6–25*a*.

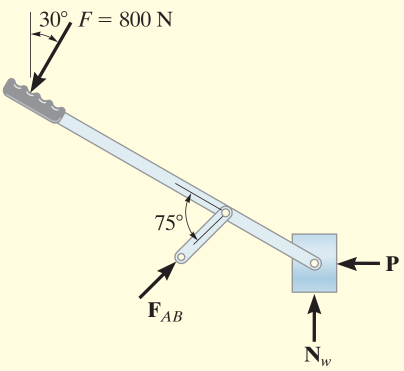


(a) (b)

Fig. 6–25

SOLUTION

By inspection, member AB is a two-force member. The free-body diagrams of the three parts are shown in Fig. 6–25*b*. Since the pins at B and D *connect only two parts together*, the forces there are shown as equal but opposite on the separate free-body diagrams of their connected members. In particular, four components of force act on the piston: $\mathbf{D}_x$ and $\mathbf{D}_y$ represent the effect of the pin (or lever EBD), $\mathbf{N}_w$ is the *resultant force* of the wall support, and $\mathbf{P}$ is the resultant compressive force caused by the can C. The directional sense of each of the unknown forces is assumed, and the correct sense will be established after the equations of equilibrium are applied.



(c)

NOTE: A free-body diagram of the entire assembly is shown in Fig. 6–25*c*. Here the forces between the components are internal and are not shown on the free-body diagram.

Before proceeding, it is highly recommended that you cover the solutions of these examples and attempt to draw the requested free-body diagrams. When doing so, make sure the work is neat and that all the forces and couple moments are properly labeled.

Procedure for Analysis

The joint reactions on frames or machines (structures) composed of multiforce members can be determined using the following procedure.

Free-Body Diagram.

- Draw the free-body diagram of the entire frame or machine, a portion of it, or each of its members. The choice should be made so that it leads to the most direct solution of the problem.
- Identify the two-force members. Remember that regardless of their shape, they have equal but opposite collinear forces acting at their ends.
- When the free-body diagram of a group of members of a frame or machine is drawn, the forces between the connected parts of this group are internal forces and are not shown on the free-body diagram of the group.
- Forces common to two members which are in contact act with equal magnitude but opposite sense on the respective free-body diagrams of the members.
- In many cases it is possible to tell by inspection the proper sense of the unknown forces acting on a member; however, if this seems difficult, the sense can be assumed.
- Remember that once the free-body diagram is drawn, a couple moment is a free vector and can act at any point on the diagram. Also, a force is a sliding vector and can act at any point along its line of action.

Equations of Equilibrium.

- Count the number of unknowns and compare it to the total number of equilibrium equations that are available. In two dimensions, there are three equilibrium equations that can be written for each member.
- Sum moments about a point that lies at the intersection of the lines of action of as many of the unknown forces as possible.
- If the solution of a force or couple moment magnitude is found to be negative, it means the sense of the force is the reverse of that shown on the free-body diagram.

EXAMPLE 6.14

Determine the tension in the cables and also the force **P** required to support the 600-N force using the frictionless pulley system shown in Fig. 6–26*a*.

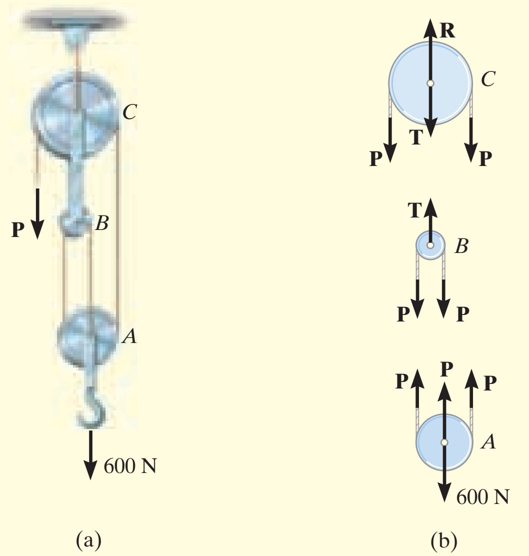


Fig. 6–26

SOLUTION

Free-Body Diagram. A free-body diagram of each pulley *including* its pin and a portion of the contacting cable is shown in Fig. 6–26*b*. Since the cable is *continuous*, it has a *constant tension* P acting throughout its length. The link connection between pulleys B and C is a two-force member, and therefore it has an unknown tension T acting on it. Notice that the *principle of action, equal but opposite reaction* must be carefully observed for forces **P** and **T** when the *separate* free-body diagrams are drawn.

Equations of Equilibrium. The three unknowns are obtained as follows:

Pulley A

$$+\uparrow \Sigma F_y = 0; \qquad 3P - 600\text{ N} = 0 \qquad P = 200\text{ N} \qquad \textit{Ans.}$$

Pulley B

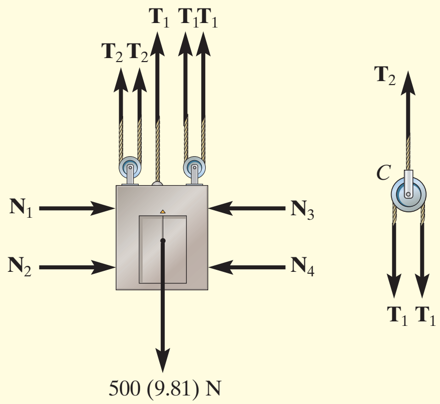
$$+\uparrow \Sigma F_y = 0; \qquad T - 2P = 0 \qquad T = 400\text{ N} \qquad \textit{Ans.}$$

Pulley C

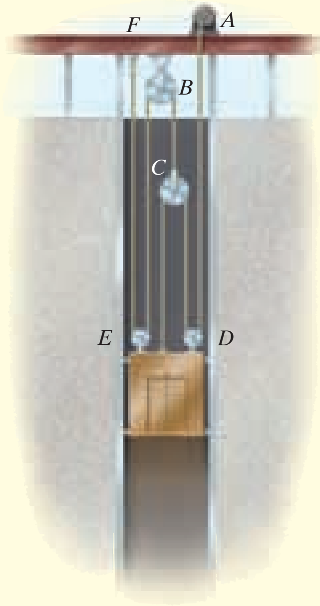
$$+\uparrow \Sigma F_y = 0; \qquad R - 2P - T = 0 \qquad R = 800\text{ N} \qquad \textit{Ans.}$$

EXAMPLE 6.15

A 500-kg elevator car in Fig. 6–27*a* is being hoisted by motor *A* using the pulley system shown. If the car is traveling with a constant speed, determine the force developed in the two cables. Neglect the mass of the cable and pulleys.



(b)



(a)

Fig. 6–27

SOLUTION

Free-Body Diagram. We can solve this problem using the free-body diagrams of the elevator car and pulley *C*, Fig. 6–27*b*. The tensile forces developed in the cables are denoted as T_1 and T_2.

Equations of Equilibrium. For pulley *C*,

$$+\uparrow \Sigma F_y = 0; \qquad T_2 - 2T_1 = 0 \qquad \text{or} \qquad T_2 = 2T_1 \tag{1}$$

For the elevator car,

$$+\uparrow \Sigma F_y = 0; \qquad 3T_1 + 2T_2 - 500(9.81)\text{ N} = 0 \tag{2}$$

Substituting Eq. (1) into Eq. (2) yields

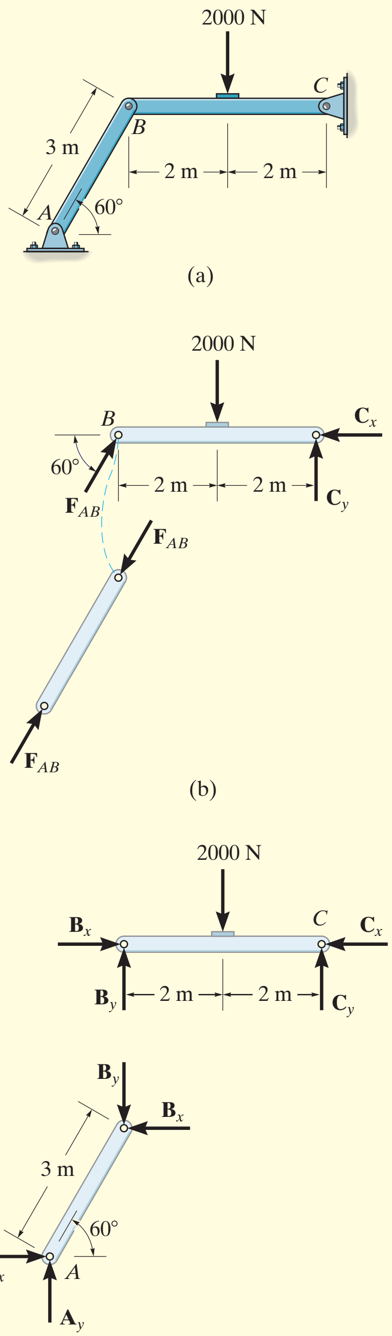
$$3T_1 + 2(2T_1) - 500(9.81)\text{ N} = 0$$

$$T_1 = 700.71\text{ N} = 701\text{ N} \qquad \textit{Ans.}$$

Substituting this result into Eq. (1),

$$T_2 = 2(700.71)\text{ N} = 1401\text{ N} = 1.40\text{ kN} \qquad \textit{Ans.}$$

EXAMPLE 6.16



(a) (b) (c)

Fig. 6–28

Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the frame in Fig. 6–28*a*.

SOLUTION I

Free-Body Diagrams. By inspection it can be seen that AB is a two-force member. The free-body diagrams are shown in Fig. 6–28*b*.

Equations of Equilibrium. The *three unknowns* can be determined by applying the three equations of equilibrium to member BC.

$$\circlearrowleft +\Sigma M_C = 0;\quad 2000\text{ N}(2\text{ m}) - (F_{AB}\sin 60^\circ)(4\text{ m}) = 0\quad F_{AB} = 1154.7\text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 1154.7\cos 60^\circ\text{ N} - C_x = 0\quad C_x = 577\text{ N}\qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0;\quad 1154.7\sin 60^\circ\text{ N} - 2000\text{ N} + C_y = 0$$

$$C_y = 1000\text{ N}\qquad \textit{Ans.}$$

SOLUTION II

Free-Body Diagrams. If one does not recognize that AB is a two-force member, then more work is involved in solving this problem. The free-body diagrams are shown in Fig. 6–28*c*.

Equations of Equilibrium. The *six unknowns* are determined by applying the three equations of equilibrium to each member.

Member AB

$$\circlearrowleft +\Sigma M_A = 0;\quad B_x(3\sin 60^\circ\text{ m}) - B_y(3\cos 60^\circ\text{ m}) = 0 \qquad (1)$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad A_x - B_x = 0 \qquad (2)$$

$$+\uparrow\Sigma F_y = 0;\quad A_y - B_y = 0 \qquad (3)$$

Member BC

$$\circlearrowleft +\Sigma M_C = 0;\quad 2000\text{ N}(2\text{ m}) - B_y(4\text{ m}) = 0 \qquad (4)$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad B_x - C_x = 0 \qquad (5)$$

$$+\uparrow\Sigma F_y = 0;\quad B_y - 2000\text{ N} + C_y = 0 \qquad (6)$$

The results for C_x and C_y can be determined by solving these equations in the following sequence: 4, 1, 5, then 6. The results are

$$B_y = 1000\text{ N}$$

$$B_x = 577\text{ N}$$

$$C_x = 577\text{ N}\qquad \textit{Ans.}$$

$$C_y = 1000\text{ N}\qquad \textit{Ans.}$$

By comparison, Solution I is simpler since the requirement that F_{AB} in Fig. 6–28*b* be equal, opposite, and collinear at the ends of member AB automatically satisfies Eqs. 1, 2, and 3 above and therefore eliminates the need to write these equations. ***As a result, save yourself some time and effort by always identifying the two-force members before starting the analysis!***

EXAMPLE 6.17

The compound beam shown in Fig. 6–29*a* is pin connected at *B*. Determine the components of reaction at its supports. Neglect its weight and thickness.

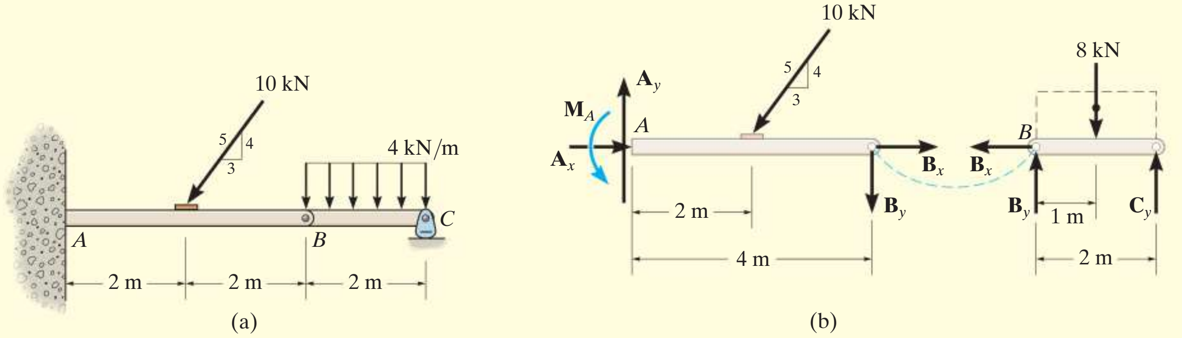


Fig. 6–29

SOLUTION

Free-Body Diagrams. By inspection, if we consider a free-body diagram of the *entire beam ABC*, there will be three unknown reactions at *A* and one at *C*. These four unknowns cannot all be obtained from the three available equations of equilibrium, and so for the solution it will become necessary to dismember the beam into its two segments, as shown in Fig. 6–29*b*.

Equations of Equilibrium. The six unknowns are determined as follows:

Segment BC

$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad B_x = 0$

$\circlearrowleft + \Sigma M_B = 0; \qquad -8\text{ kN}(1\text{ m}) + C_y(2\text{ m}) = 0$

$+\uparrow \Sigma F_y = 0; \qquad B_y - 8\text{ kN} + C_y = 0$

Segment AB

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - (10\text{ kN})\left(\frac{3}{5}\right) + B_x = 0$

$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - (10\text{ kN})\left(\frac{4}{5}\right)(2\text{ m}) - B_y(4\text{ m}) = 0$

$+\uparrow \Sigma F_y = 0; \qquad A_y - (10\text{ kN})\left(\frac{4}{5}\right) - B_y = 0$

Solving each of these equations successively, using previously calculated results, we obtain

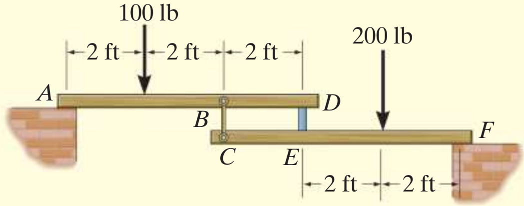
$A_x = 6\text{ kN} \qquad A_y = 12\text{ kN} \qquad M_A = 32\text{ kN}\cdot\text{m}$ *Ans.*

$B_x = 0 \qquad B_y = 4\text{ kN}$

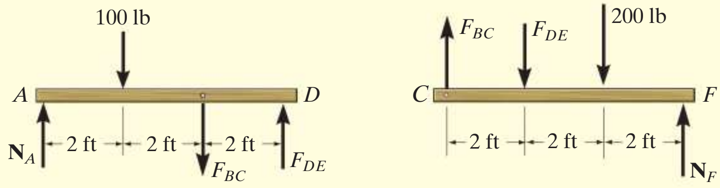
$C_y = 4\text{ kN}$ *Ans.*

EXAMPLE 6.18

The two planks in Fig. 6–30*a* are connected together by cable *BC* and a smooth spacer *DE*. Determine the reactions at the smooth supports *A* and *F*, and also find the force developed in the cable and spacer.



(a)



(b)

Fig. 6–30

SOLUTION

Free-Body Diagrams. The free-body diagram of each plank is shown in Fig. 6–30*b*. It is important to apply Newton's third law to the interaction forces F_{BC} and F_{DE} as shown.

Equations of Equilibrium. For plank *AD*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_{DE}(6 \text{ ft}) - F_{BC}(4 \text{ ft}) - 100 \text{ lb } (2 \text{ ft}) = 0$

For plank *CF*,

$\circlearrowleft + \Sigma M_F = 0;$ $\quad F_{DE}(4 \text{ ft}) - F_{BC}(6 \text{ ft}) + 200 \text{ lb } (2 \text{ ft}) = 0$

Solving simultaneously,

$$F_{DE} = 140 \text{ lb} \quad F_{BC} = 160 \text{ lb}$$ *Ans.*

Using these results, for plank *AD*,

$+\uparrow \Sigma F_y = 0;$ $\quad N_A + 140 \text{ lb} - 160 \text{ lb} - 100 \text{ lb} = 0$

$$N_A = 120 \text{ lb}$$ *Ans.*

And for plank *CF*,

$+\uparrow \Sigma F_y = 0;$ $\quad N_F + 160 \text{ lb} - 140 \text{ lb} - 200 \text{ lb} = 0$

$$N_F = 180 \text{ lb}$$ *Ans.*

NOTE: Draw the free-body diagram of the system of *both* planks and apply $\Sigma M_A = 0$ to determine N_F. Then use the free-body diagram of *CEF* to determine F_{DE} and F_{BC}.

EXAMPLE 6.19

The 75-kg man in Fig. 6–31*a* attempts to lift the 40-kg uniform beam off the roller support at B. Determine the tension developed in the cable attached to B and the normal reaction of the man on the beam when this is about to occur.

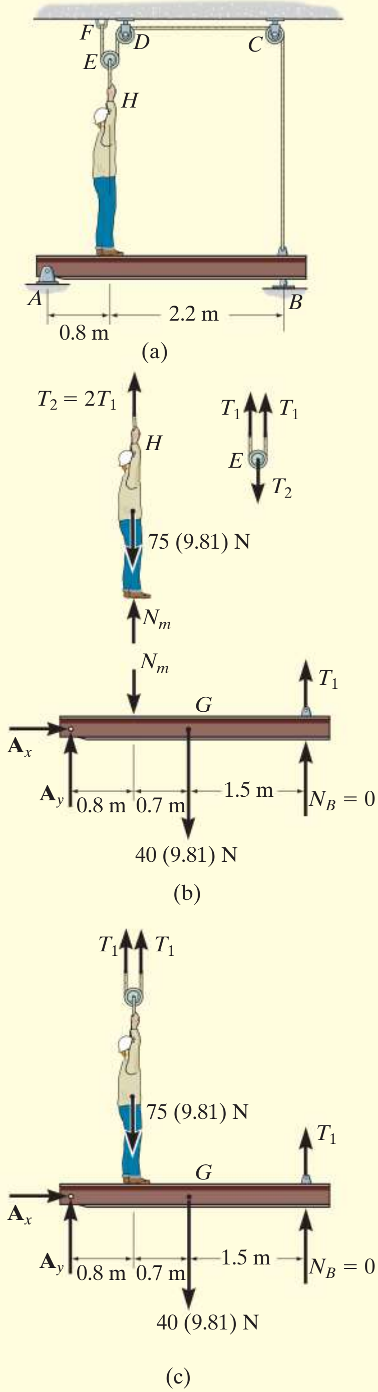


Fig. 6–31

SOLUTION

Free-Body Diagrams. The tensile force in the cable will be denoted as T_1. The free-body diagrams of the pulley E, the man, and the beam are shown in Fig. 6–31*b*. Since the man must lift the beam off the roller B then $N_B = 0$. When drawing each of these diagrams, it is very important to apply Newton's third law.

Equations of Equilibrium. Using the free-body diagram of pulley E,

$$+\uparrow\Sigma F_y = 0; \qquad 2T_1 - T_2 = 0 \quad \text{or} \quad T_2 = 2T_1 \tag{1}$$

Referring to the free-body diagram of the man using this result,

$$+\uparrow\Sigma F_y = 0 \qquad N_m + 2T_1 - 75(9.81)\text{ N} = 0 \tag{2}$$

Summing moments about point A on the beam,

$$\circlearrowleft+\Sigma M_A = 0; \; T_1(3\text{ m}) - N_m(0.8\text{ m}) - [40(9.81)\text{ N}]\,(1.5\text{ m}) = 0 \tag{3}$$

Solving Eqs. 2 and 3 simultaneously for T_1 and N_m, then using Eq. (1) for T_2, we obtain

$$T_1 = 256\text{ N} \qquad N_m = 224\text{ N} \qquad T_2 = 512\text{ N}$$ *Ans.*

SOLUTION II

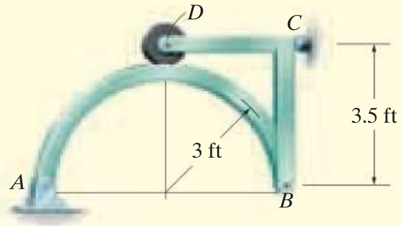
A direct solution for T_1 can be obtained by considering the beam, the man, and pulley E as a *single system*. The free-body diagram is shown in Fig. 6–31*c*. Thus,

$$\circlearrowleft+\Sigma M_A = 0; \qquad 2T_1(0.8\text{ m}) - [75(9.81)\text{ N}](0.8\text{ m})$$
$$- [40(9.81)\text{ N}](1.5\text{ m}) + T_1(3\text{ m}) = 0$$
$$T_1 = 256\text{ N}$$ *Ans.*

With this result Eqs. 1 and 2 can then be used to find N_m and T_2.

EXAMPLE 6.20

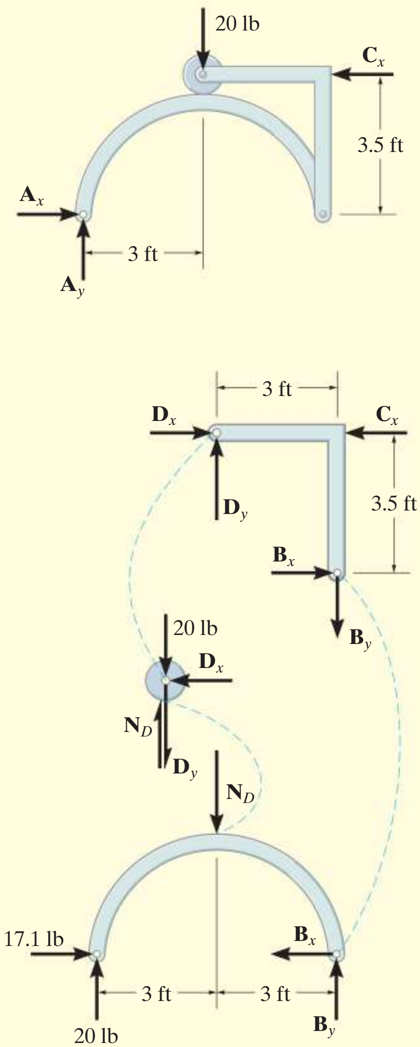
The smooth disk shown in Fig. 6–32*a* is pinned at *D* and has a weight of 20 lb. Neglecting the weights of the other members, determine the horizontal and vertical components of reaction at pins *B* and *D*.



(a)

SOLUTION

Free-Body Diagrams. The free-body diagrams of the entire frame and each of its members are shown in Fig. 6–32*b*.



(b)

Fig. 6–32

Equations of Equilibrium. The eight unknowns can of course be obtained by applying the eight equilibrium equations to each member—three to member AB, three to member BCD, and two to the disk. (Moment equilibrium is automatically satisfied for the disk.) If this is done, however, all the results can be obtained only from a simultaneous solution of some of the equations. (Try it and find out.) To avoid this situation, it is best first to determine the three support reactions on the *entire* frame; then, using these results, the remaining five equilibrium equations can be applied to two other parts in order to solve successively for the other unknowns.

Entire Frame

$$\circlearrowleft + \Sigma M_A = 0; \quad -20\text{ lb }(3\text{ ft}) + C_x(3.5\text{ ft}) = 0 \qquad C_x = 17.1\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 17.1\text{ lb} = 0 \qquad A_x = 17.1\text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 20\text{ lb} = 0 \qquad A_y = 20\text{ lb}$$

Member AB

$$\xrightarrow{+} \Sigma F_x = 0; \quad 17.1\text{ lb} - B_x = 0 \qquad B_x = 17.1\text{ lb} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad -20\text{ lb }(6\text{ ft}) + N_D(3\text{ ft}) = 0 \qquad N_D = 40\text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad 20\text{ lb} - 40\text{ lb} + B_y = 0 \qquad B_y = 20\text{ lb} \qquad \textit{Ans.}$$

Disk

$$\xrightarrow{+} \Sigma F_x = 0; \quad D_x = 0 \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 40\text{ lb} - 20\text{ lb} - D_y = 0 \qquad D_y = 20\text{ lb} \qquad \textit{Ans.}$$

EXAMPLE 6.21

The frame in Fig. 6–33*a* supports the 50-kg cylinder. Determine the horizontal and vertical components of reaction at A and the force at C.

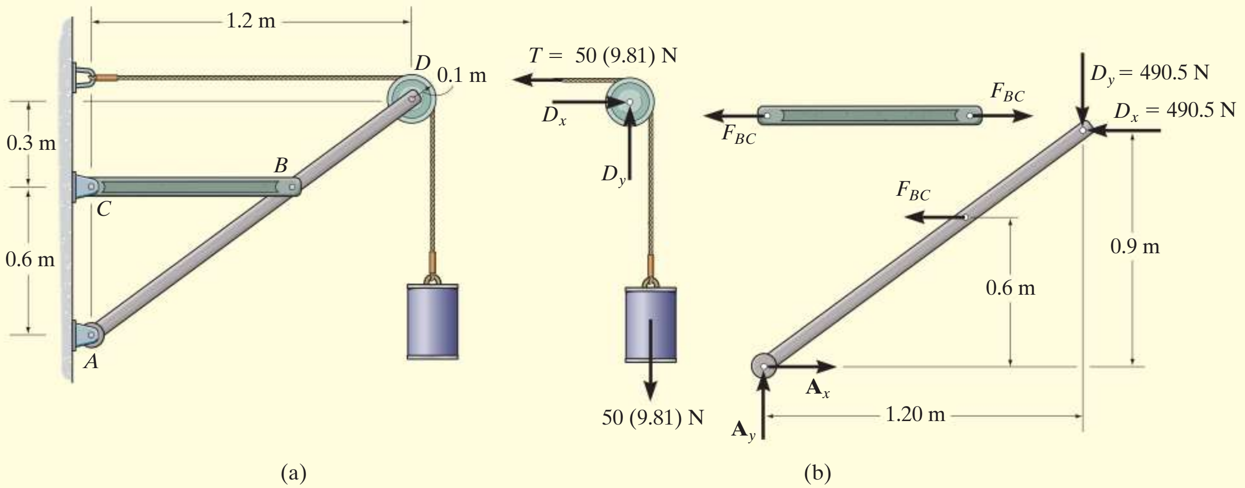


Fig. 6–33

SOLUTION

Free-Body Diagrams. The free-body diagram of pulley D, along with the cylinder and a portion of the cord (a system), is shown in Fig. 6–33*b*. Member BC is a two-force member as indicated by its free-body diagram. The free-body diagram of member ABD is also shown.

Equations of Equilibrium. We will begin by analyzing the equilibrium of the pulley. The moment equation of equilibrium is automatically satisfied with $T = 50(9.81)$ N, and so

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad D_x - 50(9.81)\text{ N} = 0 \quad D_x = 490.5\text{ N}$$

$$+\uparrow\Sigma F_y = 0; \qquad D_y - 50(9.81)\text{ N} = 0 \quad D_y = 490.5\text{ N} \qquad \textit{Ans.}$$

Using these results, F_{BC} can be determined by summing moments about point A on member ABD.

$$\circlearrowleft +\Sigma M_A = 0; \; F_{BC}\,(0.6\text{ m}) + 490.5\text{ N}(0.9\text{ m}) - 490.5\text{ N}(1.20\text{ m}) = 0$$

$$F_{BC} = 245.25\text{ N} \qquad \textit{Ans.}$$

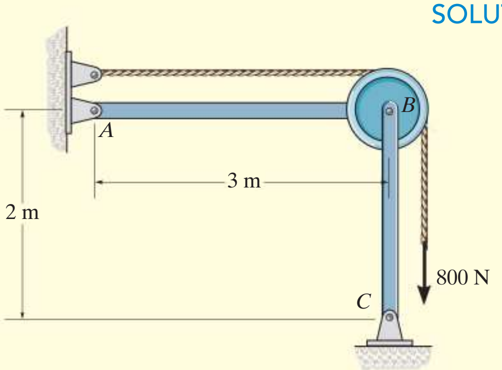
Now A_x and A_y can be determined by summing forces.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad A_x - 245.25\text{ N} - 490.5\text{ N} = 0 \quad A_x = 736\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y - 490.5\text{ N} = 0 \quad A_y = 490.5\text{ N} \qquad \textit{Ans.}$$

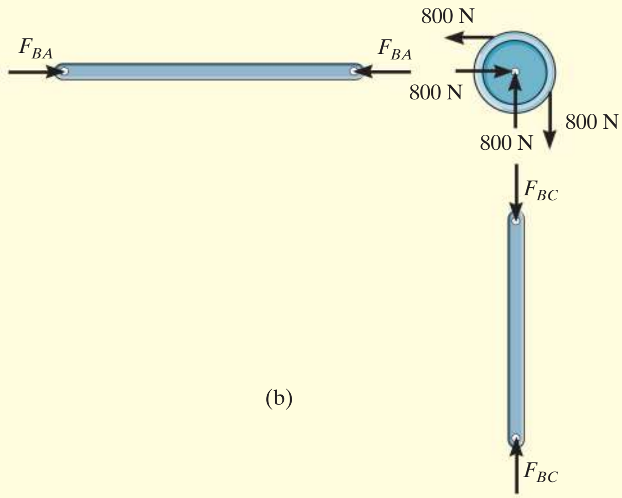
EXAMPLE 6.22

Determine the force the pins at A and B exert on the two-member frame shown in Fig. 6–34*a*.



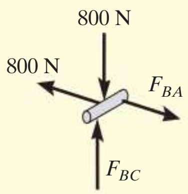
(a)

SOLUTION I



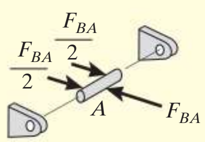
(b)

Free-Body Diagrams. By inspection AB and BC are two-force members. Their free-body diagrams, along with that of the pulley, are shown in Fig. 6–34*b*. In order to solve this problem we must also include the free-body diagram of the pin at B because this pin connects all *three members* together, Fig. 6–34*c*.



Pin B

(c)



Pin A

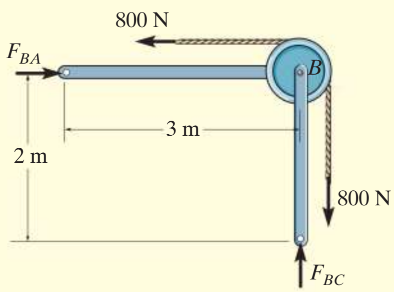
(d)

Equations of Equilibrium: Apply the equations of force equilibrium to pin B.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BA} - 800\text{ N} = 0; \qquad F_{BA} = 800\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{BC} - 800\text{ N} = 0; \qquad F_{BC} = 800\text{ N} \qquad \textit{Ans.}$$

NOTE: The free-body diagram of the pin at A, Fig. 6–34*d*, indicates how the force F_{AB} is balanced by the force $(F_{AB}/2)$ exerted on the pin by each of the two pin leaves.



(e)

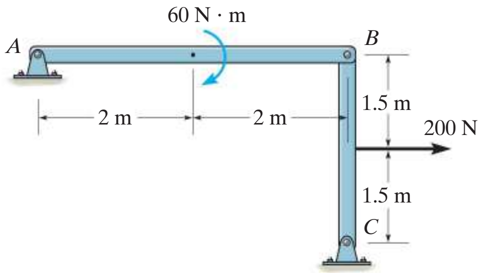
Fig. 6–34

SOLUTION II

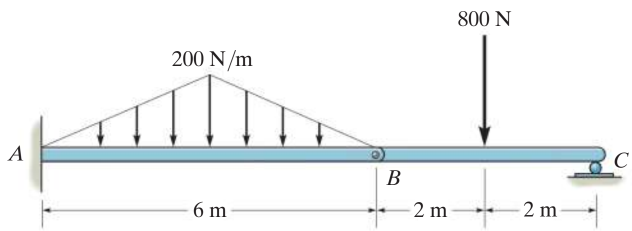
Free-Body Diagram. If we realize that AB and BC are two-force members, then the free-body diagram of the *entire frame* produces an easier solution, Fig. 6–34*e*. The force equations of equilibrium are the same as those above. Note that moment equilibrium will be satisfied, regardless of the radius of the pulley.

PRELIMINARY PROBLEMS

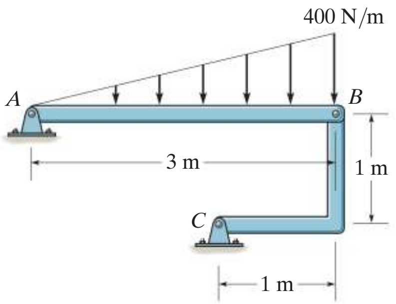
P6–3. In each case, identify any two-force members, and then draw the free-body diagrams of each member of the frame.



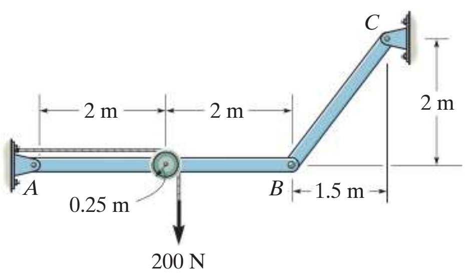
(a)



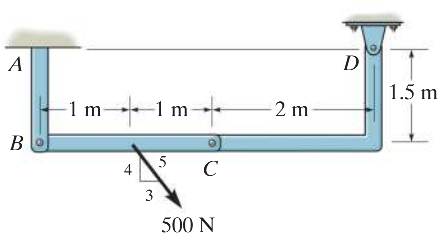
(d)



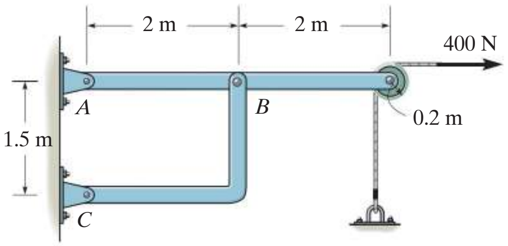
(b)



(e)



(c)



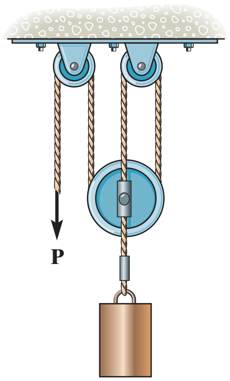
(f)

Prob. P6–3

FUNDAMENTAL PROBLEMS

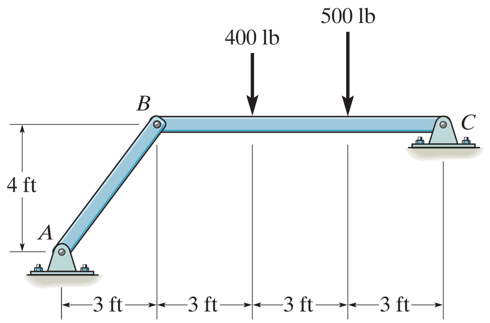
All problem solutions must include FBDs.

F6–13. Determine the force P needed to hold the 60-lb weight in equilibrium.



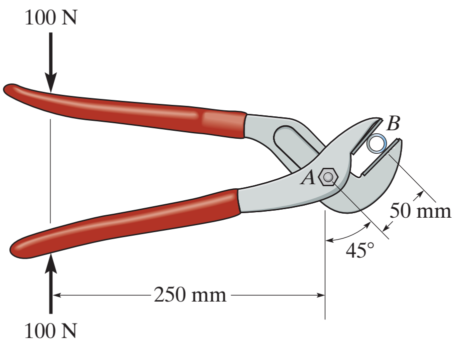
Prob. F6–13

F6–14. Determine the horizontal and vertical components of reaction at pin C.



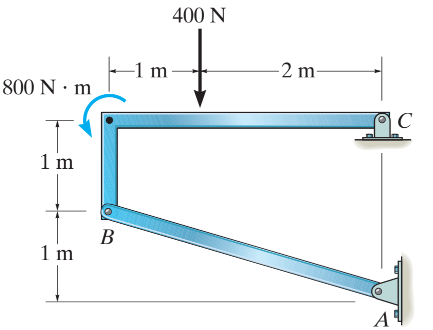
Prob. F6–14

F6–15. If a 100-N force is applied to the handles of the pliers, determine the clamping force exerted on the smooth pipe B and the magnitude of the resultant force that one of the members exerts on pin A.



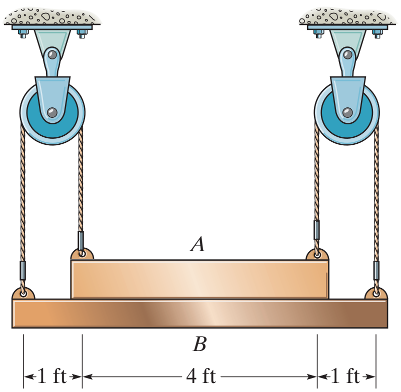
Prob. F6–15

F6–16. Determine the horizontal and vertical components of reaction at pin C.



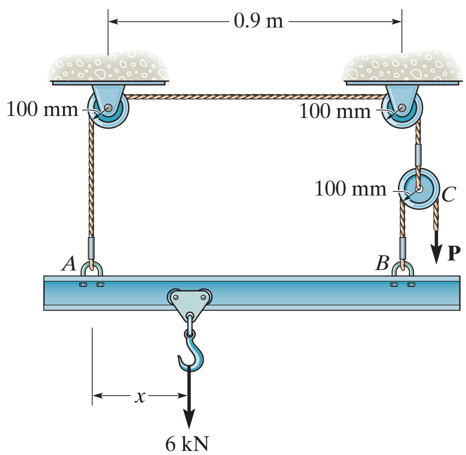
Prob. F6–16

F6–17. Determine the normal force that the 100-lb plate A exerts on the 30-lb plate B.



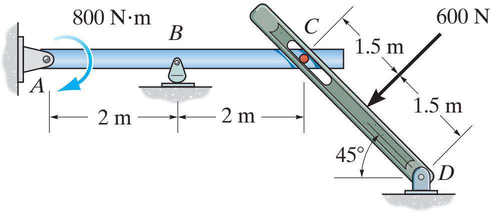
Prob. F6–17

F6–18. Determine the force P needed to lift the load. Also, determine the proper placement x of the hook for equilibrium. Neglect the weight of the beam.



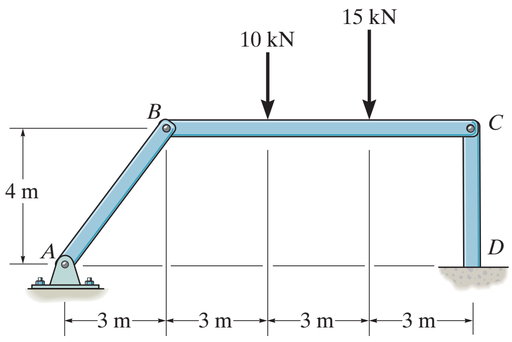
Prob. F6–18

F6–19. Determine the components of reaction at A and B.



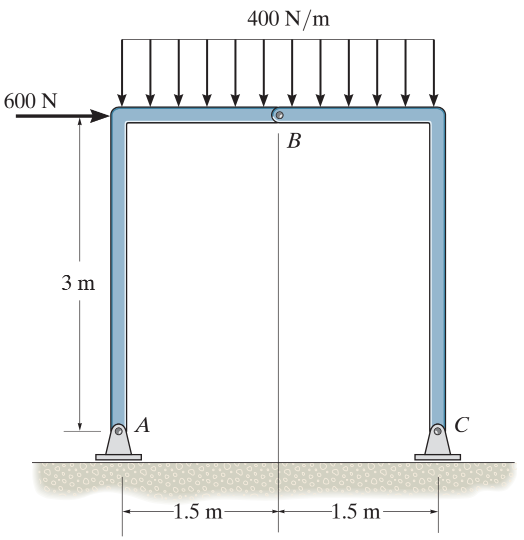
Prob. F6–19

F6–20. Determine the reactions at D.



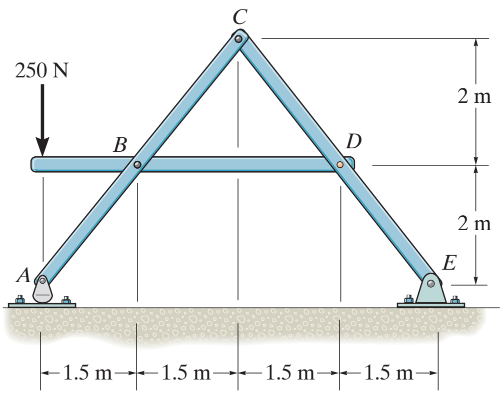
Prob. F6–20

F6–21. Determine the components of reaction at A and C.



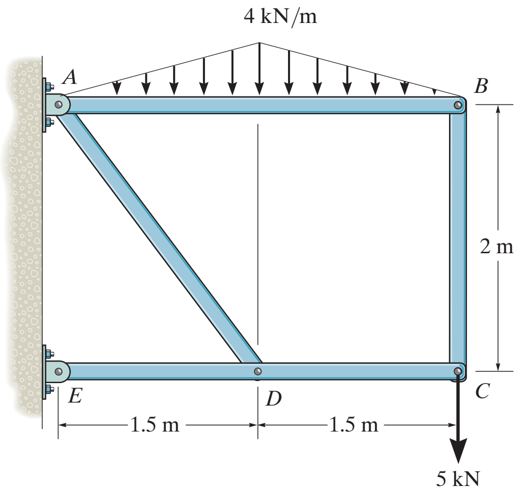
Prob. F6–21

F6–22. Determine the components of reaction at C.



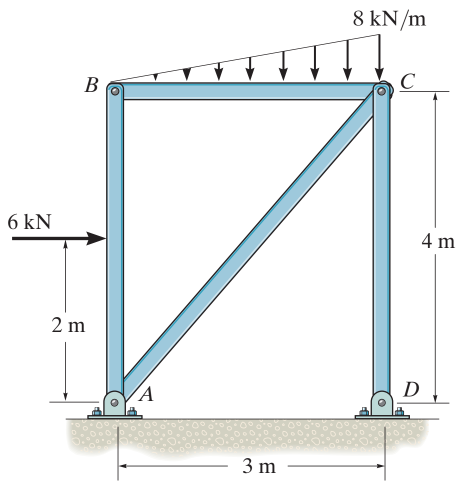
Prob. F6–22

F6–23. Determine the components of reaction at E.



Prob. F6–23

F6–24. Determine the components of reaction at D and the components of reaction the pin at A exerts on member BA.

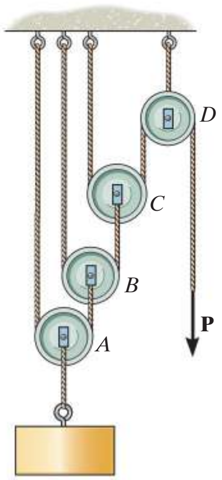


Prob. F6–24

PROBLEMS

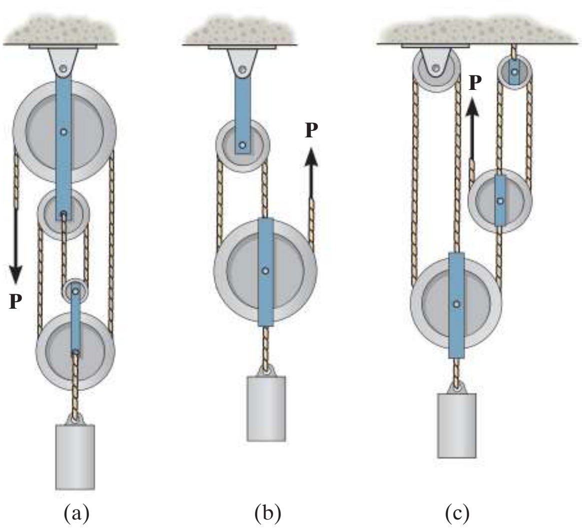
All problem solutions must include FBDs.

6–61. Determine the force **P** required to hold the 100-lb weight in equilibrium.



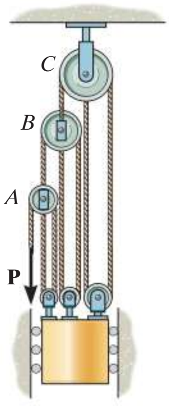
Prob. 6–61

6–62. In each case, determine the force **P** required to maintain equilibrium. The block weighs 100 lb.



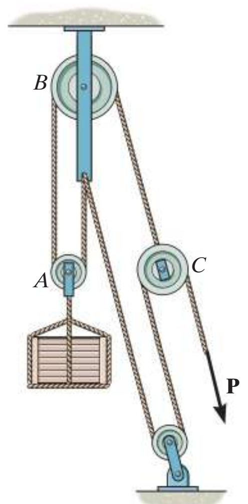
Prob. 6–62

6–63. Determine the force **P** required to hold the 50-kg mass in equilibrium.



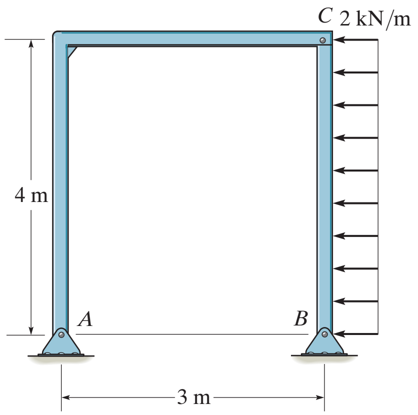
Prob. 6–63

***6–64.** Determine the force **P** required to hold the 150-kg crate in equilibrium.



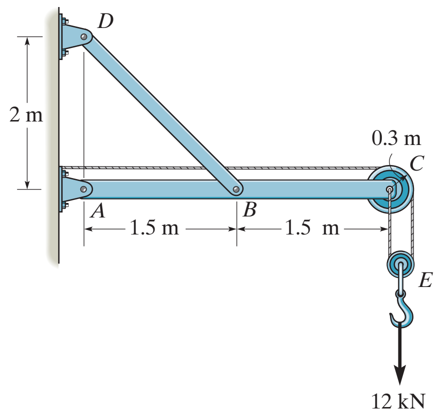
Prob. 6–64

6–65. Determine the horizontal and vertical components of force that pins A and B exert on the frame.



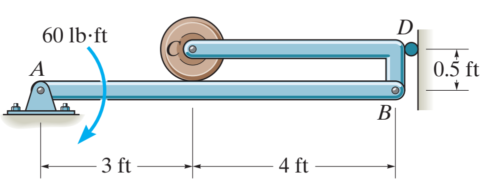
Prob. 6–65

6–66. Determine the horizontal and vertical components of force at pins A and D.



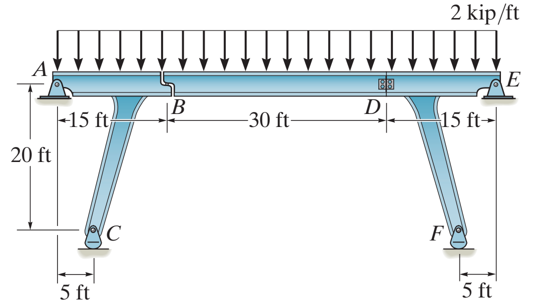
Prob. 6–66

6–67. Determine the force that the smooth roller C exerts on member AB. Also, what are the horizontal and vertical components of reaction at pin A? Neglect the weight of the frame and roller.



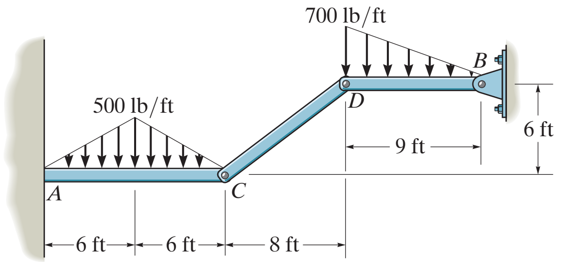
Prob. 6–67

***6–68.** The bridge frame consists of three segments which can be considered pinned at A, D, and E, rocker supported at C and F, and roller supported at B. Determine the horizontal and vertical components of reaction at all these supports due to the loading shown.



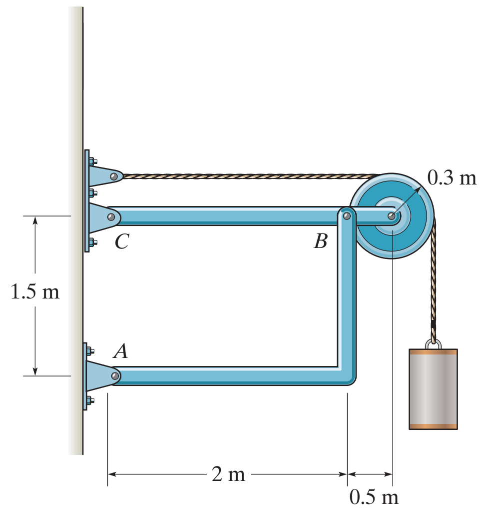
Prob. 6–68

6–69. Determine the reactions at supports A and B.



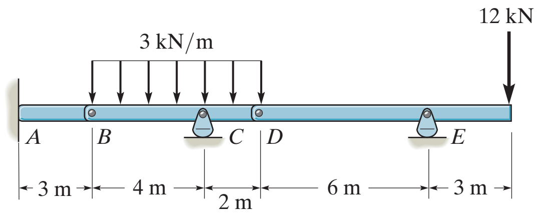
Prob. 6–69

6–70. Determine the horizontal and vertical components of force at pins B and C. The suspended cylinder has a mass of 75 kg.



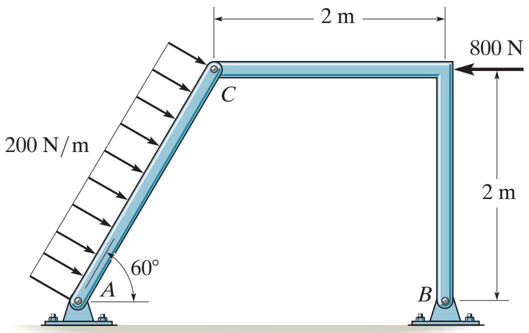
Prob. 6–70

6–71. Determine the reactions at the supports A, C, and E of the compound beam.



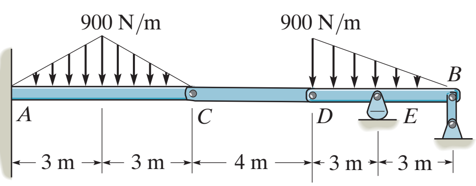
Prob. 6–71

***6–72.** Determine the resultant force at pins A, B, and C on the three-member frame.



Prob. 6–72

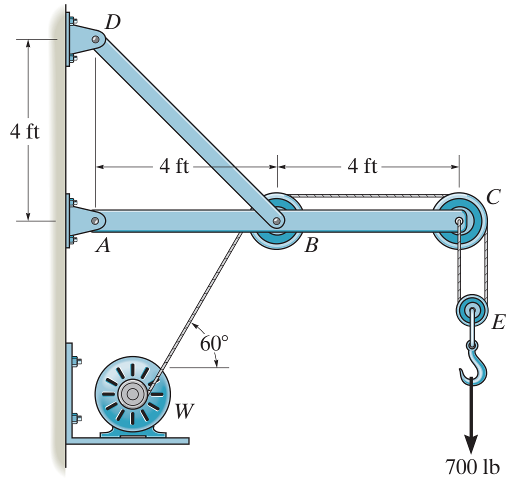
6–73. Determine the reactions at the supports at A, E, and B of the compound beam.



Prob. 6–73

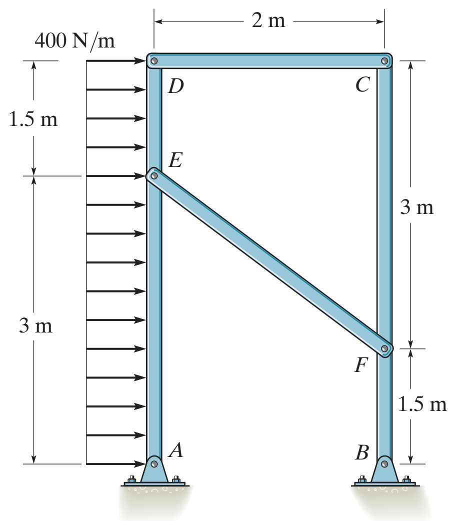
6–74. The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W?

6–75. The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W? The jib ABC has a weight of 100 lb and member BD has a weight of 40 lb. Each member is uniform and has a center of gravity at its center.



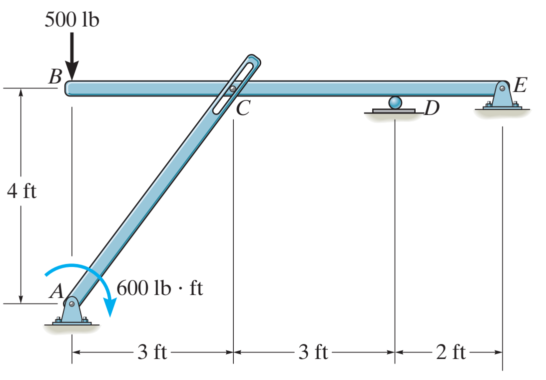
Probs. 6–74/75

***6–76.** Determine the horizontal and vertical components of force which the pins at A and B exert on the frame.



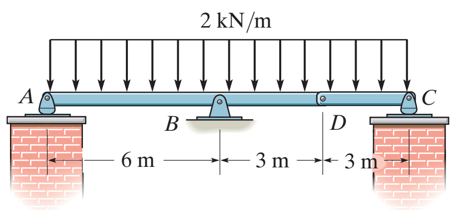
Prob. 6–76

6–77. The two-member structure is connected at C by a pin, which is fixed to BDE and passes through the smooth slot in member AC. Determine the horizontal and vertical components of reaction at the supports.



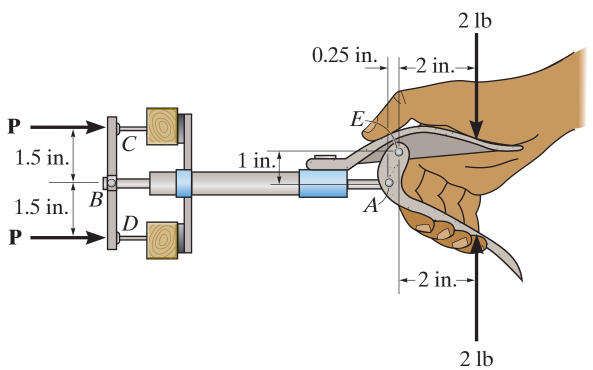
Prob. 6–77

6–78. The compound beam is pin supported at B and supported by rockers at A and C. There is a hinge (pin) at D. Determine the reactions at the supports.



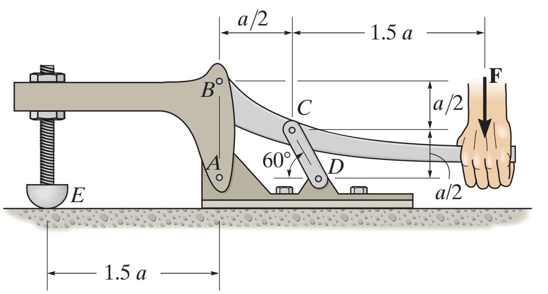
Prob. 6–78

6–79. When a force of 2 lb is applied to the handles of the brad squeezer, it pulls in the smooth rod AB. Determine the force **P** exerted on each of the smooth brads at C and D.



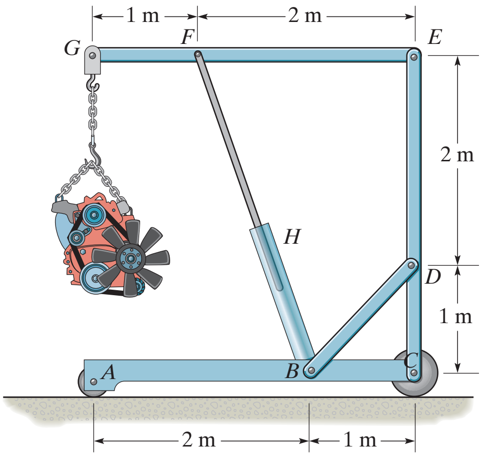
Prob. 6–79

***6–80.** The toggle clamp is subjected to a force **F** at the handle. Determine the vertical clamping force acting at E.



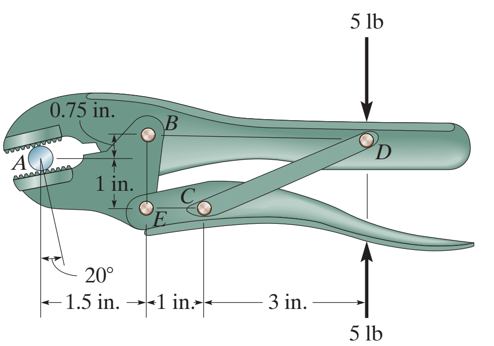
Prob. 6–80

6–81. The hoist supports the 125-kg engine. Determine the force the load creates in member DB and in member FB, which contains the hydraulic cylinder H.



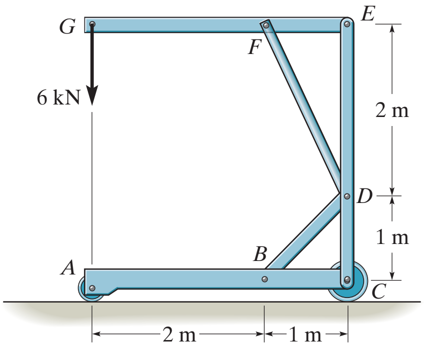
Prob. 6–81

6–82. A 5-lb force is applied to the handles of the vise grip. Determine the compressive force developed on the smooth bolt shank A at the jaws.



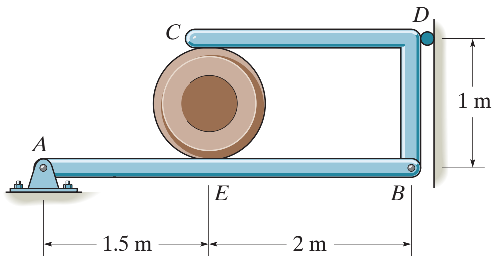
Prob. 6–82

6–83. Determine the force in members FD and DB of the frame. Also, find the horizontal and vertical components of reaction the pin at C exerts on member ABC and member EDC.



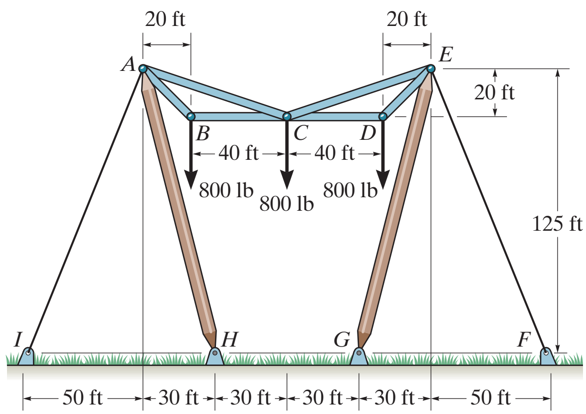
Prob. 6–83

***6–84.** Determine the force that the smooth 20-kg cylinder exerts on members AB and CDB. Also, what are the horizontal and vertical components of reaction at pin A?



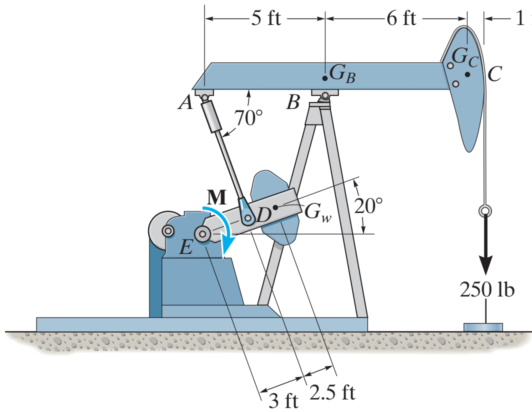
Prob. 6–84

6–85. The three power lines exert the forces shown on the pin-connected members at joints B, C, and D, which in turn are pin connected to the poles AH and EG. Determine the force in the guy cable AI and the pin reaction at the support H.



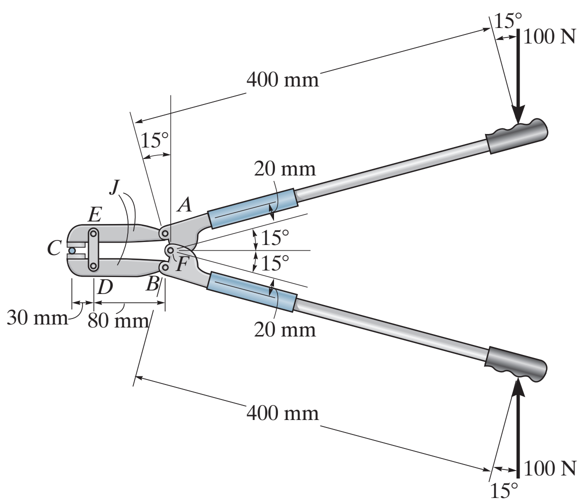
Prob. 6–85

6–86. The pumping unit is used to recover oil. When the walking beam ABC is horizontal, the force acting in the wireline at the well head is 250 lb. Determine the torque **M** which must be exerted by the motor in order to overcome this load. The horse-head C weighs 60 lb and has a center of gravity at G_C. The walking beam ABC has a weight of 130 lb and a center of gravity at G_B, and the counterweight has a weight of 200 lb and a center of gravity at G_W. The pitman, AD, is pin connected at its ends and has negligible weight.



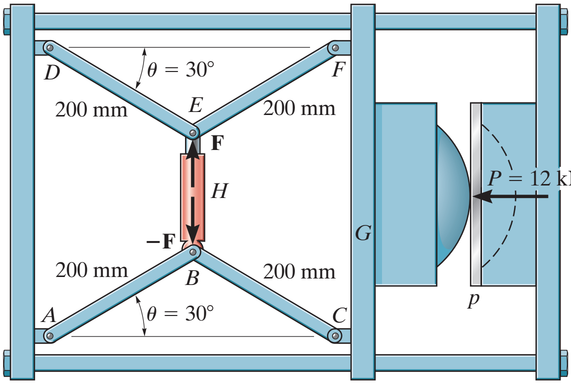
Prob. 6–86

6–87. Determine the force that the jaws J of the metal cutters exert on the smooth cable C if 100-N forces are applied to the handles. The jaws are pinned at E and A, and D and B. There is also a pin at F.



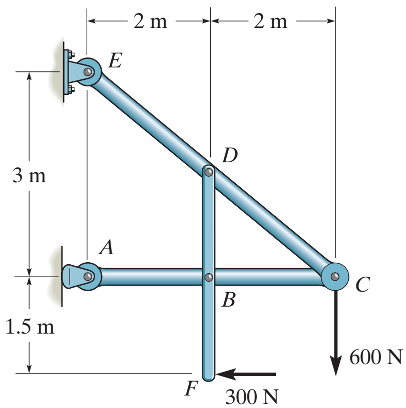
Prob. 6–87

***6–88.** The machine shown is used for forming metal plates. It consists of two toggles *ABC* and *DEF*, which are operated by the hydraulic cylinder *H*. The toggles push the movable bar *G* forward, pressing the plate *p* into the cavity. If the force which the plate exerts on the head is $P = 12$ kN, determine the force *F* in the hydraulic cylinder when $\theta = 30°$.



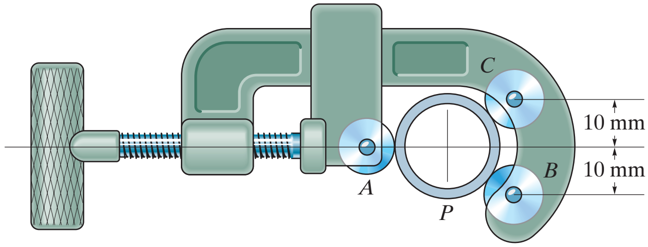
Prob. 6–88

6–89. Determine the horizontal and vertical components of force which pin *C* exerts on member *ABC*. The 600-N load is applied to the pin.



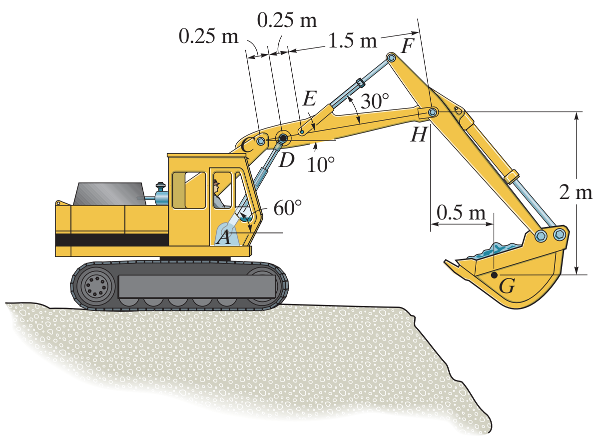
Prob. 6–89

6–90. The pipe cutter is clamped around the pipe *P*. If the wheel at *A* exerts a normal force of $F_A = 80$ N on the pipe, determine the normal forces of wheels *B* and *C* on the pipe. Also compute the pin reaction on the wheel at *C*. The three wheels each have a radius of 7 mm and the pipe has an outer radius of 10 mm.



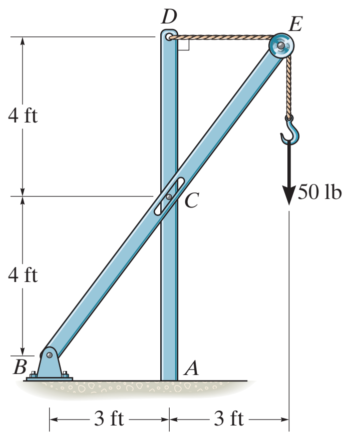
Prob. 6–90

6–91. Determine the force created in the hydraulic cylinders *EF* and *AD* in order to hold the shovel in equilibrium. The shovel load has a mass of 1.25 Mg and a center of gravity at *G*. All joints are pin connected.



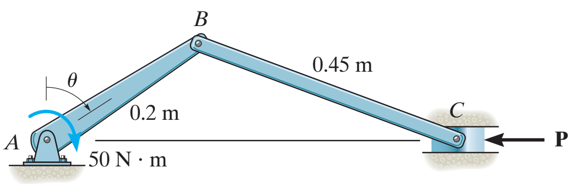
Prob. 6–91

***6–92.** Determine the horizontal and vertical components of force at pin B and the normal force the pin at C exerts on the smooth slot. Also, determine the moment and horizontal and vertical reactions of force at A. There is a pulley at E.



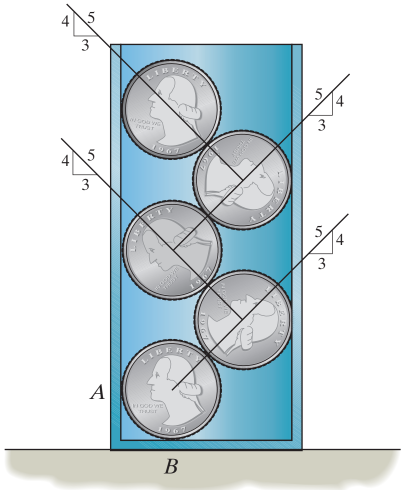
Prob. 6–92

6–93. The constant moment of 50 N·m is applied to the crank shaft. Determine the compressive force P that is exerted on the piston for equilibrium as a function of θ. Plot the results of P (vertical axis) versus θ (horizontal axis) for $0° \leq \theta \leq 90°$.



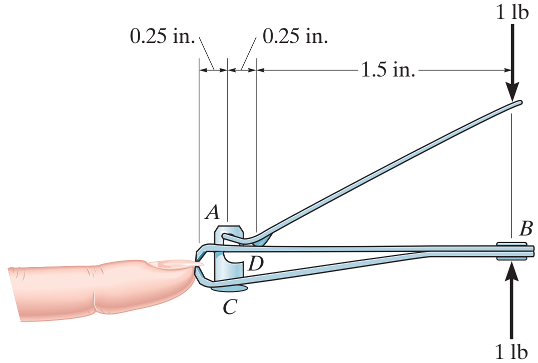
Prob. 6–93

6–94. Five coins are stacked in the smooth plastic container shown. If each coin weighs 0.0235 lb, determine the normal reactions of the bottom coin on the container at points A and B.



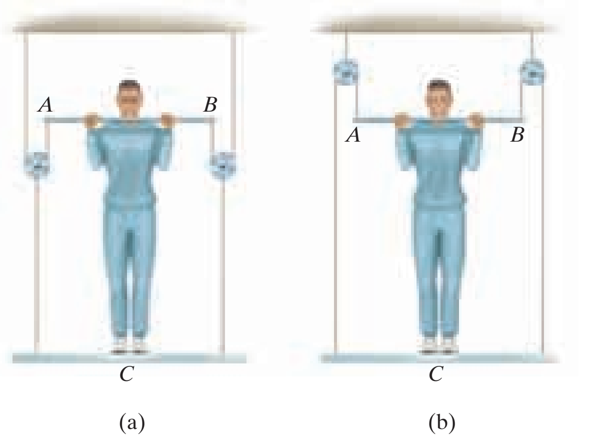
Prob. 6–94

6–95. The nail cutter consists of the handle and the two cutting blades. Assuming the blades are pin connected at B and the surface at D is smooth, determine the normal force on the fingernail when a force of 1 lb is applied to the handles as shown. The pin AC slides through a smooth hole at A and is attached to the bottom member at C.



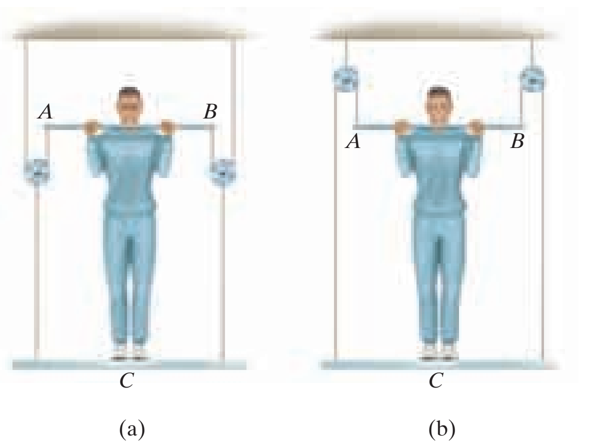
Prob. 6–95

*6–96. A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. Neglect the weight of the platform.



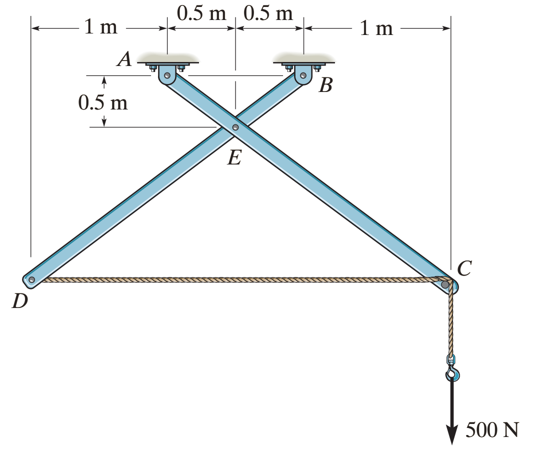
Prob. 6–96

6–97. A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. The platform has a weight of 30 lb.



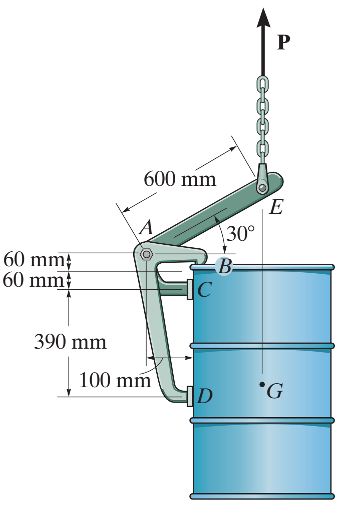
Prob. 6–97

6–98. The two-member frame is pin connected at E. The cable is attached to D, passes over the smooth peg at C, and supports the 500-N load. Determine the horizontal and vertical reactions at each pin.



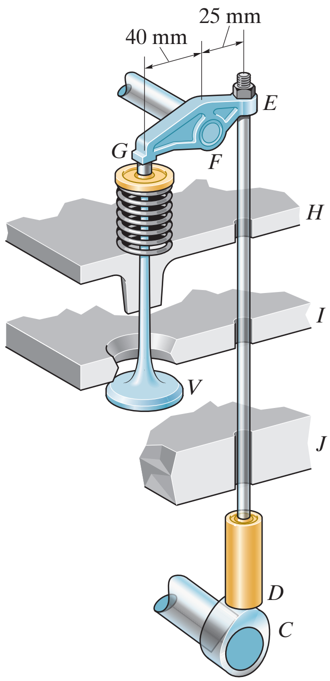
Prob. 6–98

6–99. If the 300-kg drum has a center of mass at point G, determine the horizontal and vertical components of force acting at pin A and the reactions on the smooth pads C and D. The grip at B on member DAB resists both horizontal and vertical components of force at the rim of the drum.



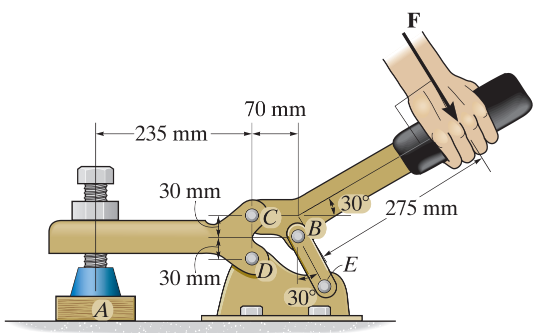
Prob. 6–99

***6–100.** Operation of exhaust and intake valves in an automobile engine consists of the cam C, push rod DE, rocker arm EFG which is pinned at F, and a spring and valve, V. If the compression in the spring is 20 mm when the valve is open as shown, determine the normal force acting on the cam lobe at C. Assume the cam and bearings at H, I, and J are smooth. The spring has a stiffness of 300 N/m.



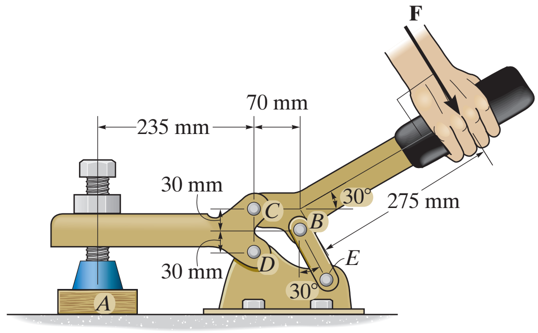
Prob. 6–100

6–101. If a clamping force of 300 N is required at A, determine the amount of force **F** that must be applied to the handle of the toggle clamp.



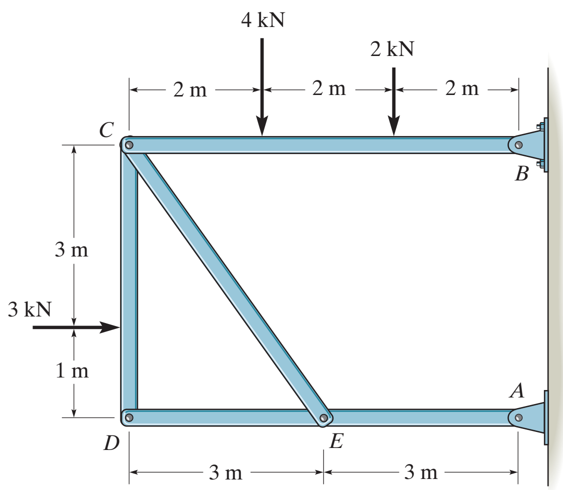
Prob. 6–101

6–102. If a force of $F = 350$ N is applied to the handle of the toggle clamp, determine the resulting clamping force at A.



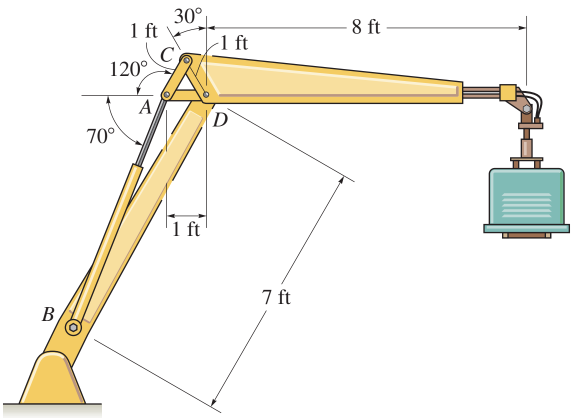
Prob. 6–102

6–103. Determine the horizontal and vertical components of force that the pins at A and B exert on the frame.



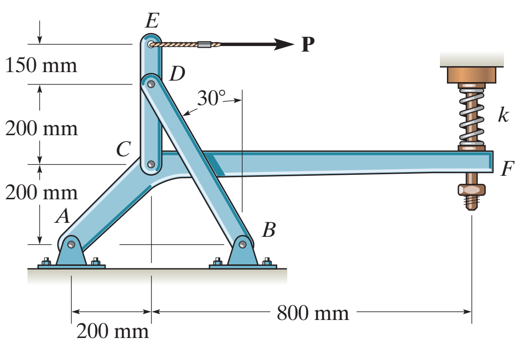
Prob. 6–103

***6–104.** The hydraulic crane is used to lift the 1400-lb load. Determine the force in the hydraulic cylinder AB and the force in links AC and AD when the load is held in the position shown.



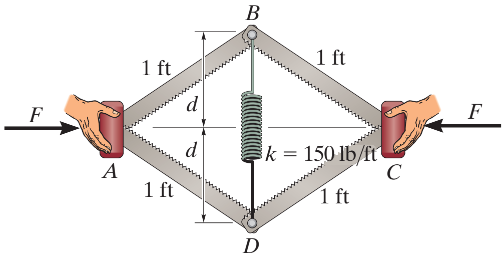
Prob. 6–104

6–105. Determine force **P** on the cable if the spring is compressed 0.025 m when the mechanism is in the position shown. The spring has a stiffness of $k = 6$ kN/m.



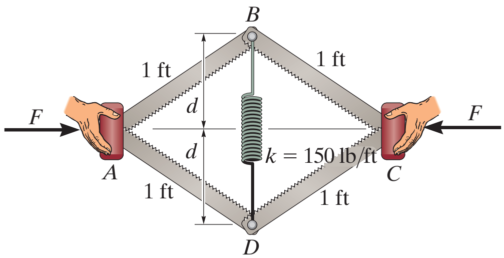
Prob. 6–105

6–106. If $d = 0.75$ ft and the spring has an unstretched length of 1 ft, determine the force F required for equilibrium.



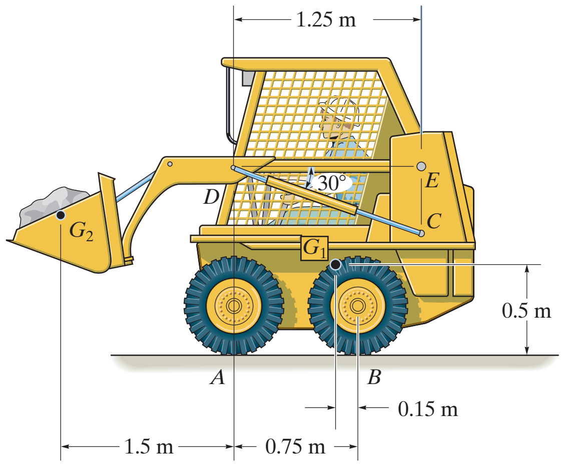
Prob. 6–106

6–107. If a force of $F = 50$ lb is applied to the pads at A and C, determine the smallest dimension d required for equilibrium if the spring has an unstretched length of 1 ft.



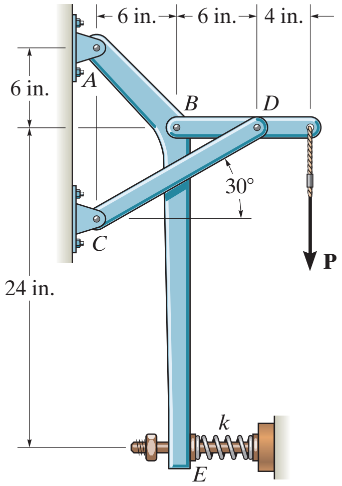
Prob. 6–107

***6–108.** The skid-steer loader has a mass of 1.18 Mg, and in the position shown the center of mass is at G_1. If there is a 300-kg stone in the bucket, with center of mass at G_2, determine the reactions of each pair of wheels A and B on the ground and the force in the hydraulic cylinder CD and at the pin E. There is a similar linkage on each side of the loader.



Prob. 6–108

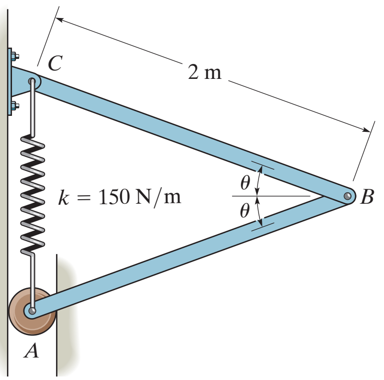
6–109. Determine the force **P** on the cable if the spring is compressed 0.5 in. when the mechanism is in the position shown. The spring has a stiffness of $k = 800$ lb/ft.



Prob. 6–109

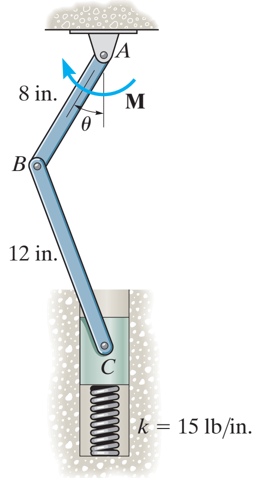
6–110. The spring has an unstretched length of 0.3 m. Determine the angle θ for equilibrium if the uniform bars each have a mass of 20 kg.

6–111. The spring has an unstretched length of 0.3 m. Determine the mass m of each uniform bar if each angle $\theta = 30°$ for equilibrium.



Probs. 6–110/111

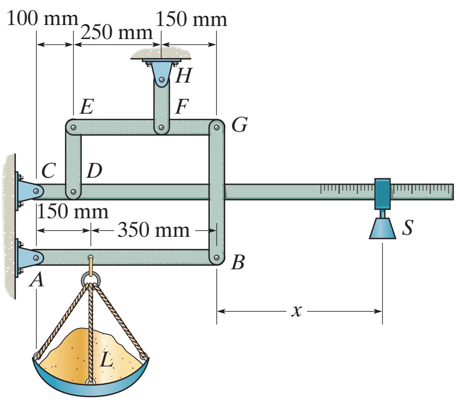
***6–112.** The piston C moves vertically between the two smooth walls. If the spring has a stiffness of $k = 15$ lb/in., and is unstretched when $\theta = 0°$, determine the couple **M** that must be applied to AB to hold the mechanism in equilibrium when $\theta = 30°$.



Prob. 6–112

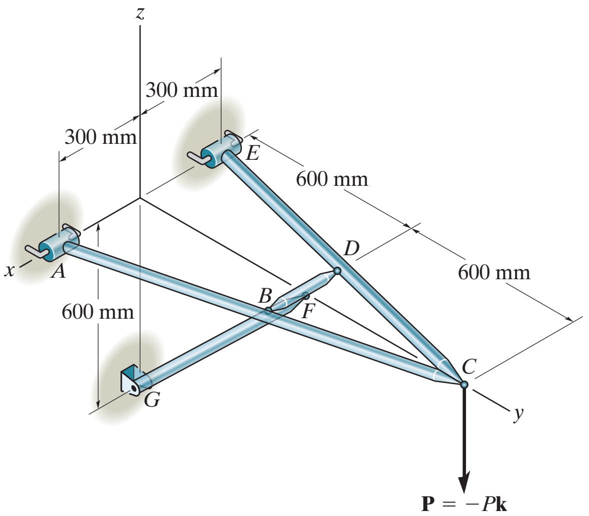
6–113. The platform scale consists of a combination of third and first class levers so that the load on one lever becomes the effort that moves the next lever. Through this arrangement, a small weight can balance a massive object. If $x = 450$ mm, determine the required mass of the counterweight S required to balance a 90-kg load, L.

6–114. The platform scale consists of a combination of third and first class levers so that the load on one lever becomes the effort that moves the next lever. Through this arrangement, a small weight can balance a massive object. If $x = 450$ mm, and the mass of the counterweight S is 2 kg, determine the mass of the load L required to maintain the balance.



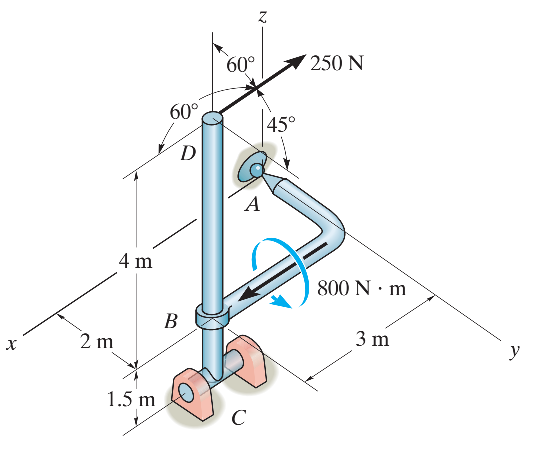
Probs. 6–113/114

6–115. The four-member "A" frame is supported at A and E by smooth collars and at G by a pin. All the other joints are ball-and-sockets. If the pin at G will fail when the resultant force there is 800 N, determine the largest vertical force P that can be supported by the frame. Also, what are the x, y, z force components which member BD exerts on members EDC and ABC? The collars at A and E and the pin at G only exert force components on the frame.



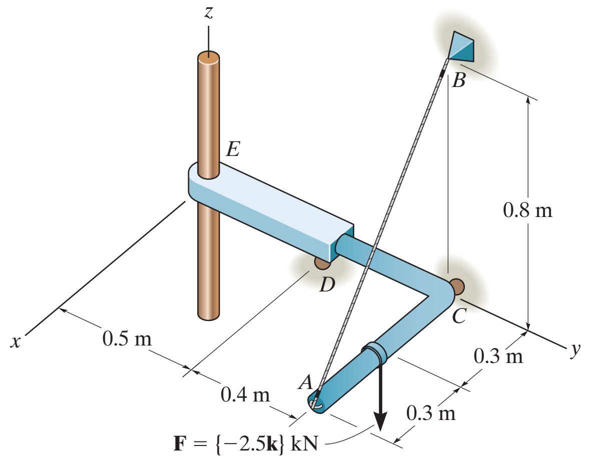
Prob. 6–115

***6–116.** The structure is subjected to the loadings shown. Member AB is supported by a ball-and-socket at A and smooth collar at B. Member CD is supported by a pin at C. Determine the x, y, z components of reaction at A and C.



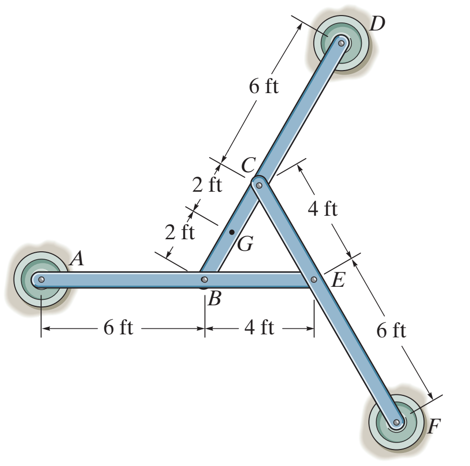
Prob. 6–116

6–117. The structure is subjected to the loading shown. Member AD is supported by a cable AB and roller at C and fits through a smooth circular hole at D. Member ED is supported by a roller at D and a pole that fits in a smooth snug circular hole at E. Determine the x, y, z components of reaction at E and the tension in cable AB.



Prob. 6–117

6–118. The three pin-connected members shown in the *top view* support a downward force of 60 lb at G. If only vertical forces are supported at the connections B, C, E and pad supports A, D, F, determine the reactions at each pad.

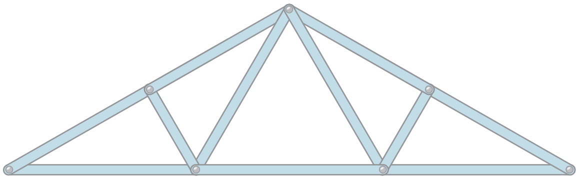


Prob. 6–118

CHAPTER REVIEW

Simple Truss

A simple truss consists of triangular elements connected together by pinned joints. The forces within its members can be determined by assuming the members are all two-force members, connected concurrently at each joint. The members are either in tension or compression, or carry no force.



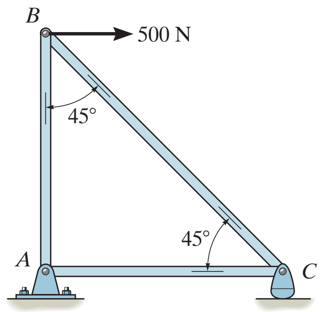
Roof truss

Method of Joints

The method of joints states that if a truss is in equilibrium, then each of its joints is also in equilibrium. For a plane truss, the concurrent force system at each joint must satisfy force equilibrium.

$$\Sigma F_x = 0$$

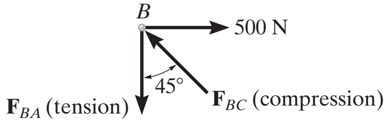
$$\Sigma F_y = 0$$



To obtain a numerical solution for the forces in the members, select a joint that has a free-body diagram with at most two unknown forces and one known force. (This may require first finding the reactions at the supports.)

Once a member force is determined, use its value and apply it to an adjacent joint.

Remember that forces that are found to *pull* on the joint are *tensile forces*, and those that *push* on the joint are *compressive forces*.

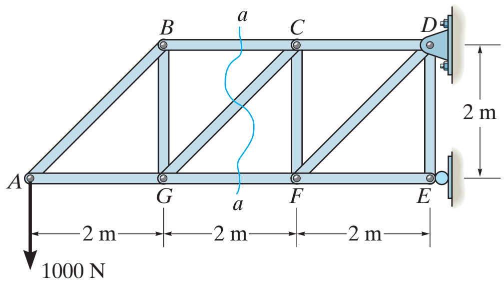


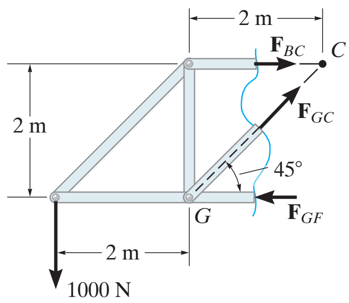
To avoid a simultaneous solution of two equations, set one of the coordinate axes along the line of action of one of the unknown forces and sum forces perpendicular to this axis. This will allow a direct solution for the other unknown.

The analysis can also be simplified by first identifying all the zero-force members.

Method of Sections

The method of sections states that if a truss is in equilibrium, then each segment of the truss is also in equilibrium. Pass a section through the truss and the member whose force is to be determined. Then draw the free-body diagram of the sectioned part having the least number of forces on it.





Sectioned members subjected to *pulling* are in *tension*, and those that are subjected to *pushing* are in *compression*.

Three equations of equilibrium are available to determine the unknowns.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

If possible, sum forces in a direction that is perpendicular to two of the three unknown forces. This will yield a direct solution for the third force.

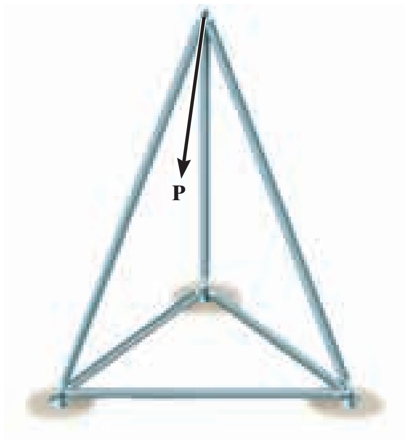
$$+\uparrow \Sigma F_y = 0$$
$$-1000 \text{ N} + F_{GC} \sin 45^\circ = 0$$
$$F_{GC} = 1.41 \text{ kN (T)}$$

Sum moments about the point where the lines of action of two of the three unknown forces intersect, so that the third unknown force can be determined directly.

$$\circlearrowleft + \Sigma M_C = 0$$
$$1000 \text{ N}(4 \text{ m}) - F_{GF}(2 \text{ m}) = 0$$
$$F_{GF} = 2 \text{ kN (C)}$$

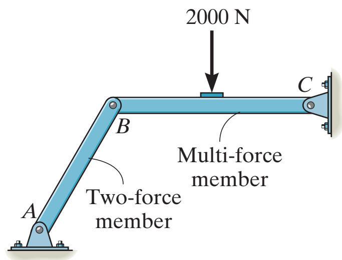
Space Truss

A space truss is a three-dimensional truss built from tetrahedral elements, and is analyzed using the same methods as for plane trusses. The joints are assumed to be ball-and-socket connections.



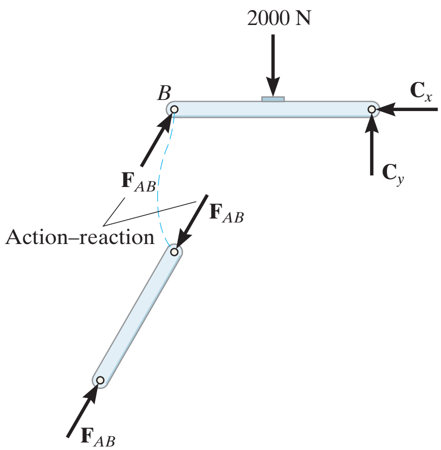
Frames and Machines

Frames and machines are structures that contain one or more multiforce members, that is, members with three or more forces or couples acting on them. Frames are designed to support loads, and machines transmit and alter the effect of forces.



The forces acting at the joints of a frame or machine can be determined by drawing the free-body diagrams of each of its members or parts. The principle of action–reaction should be carefully observed when indicating these forces on the free-body diagram of each adjacent member or pin. For a coplanar force system, there are three equilibrium equations available for each member.

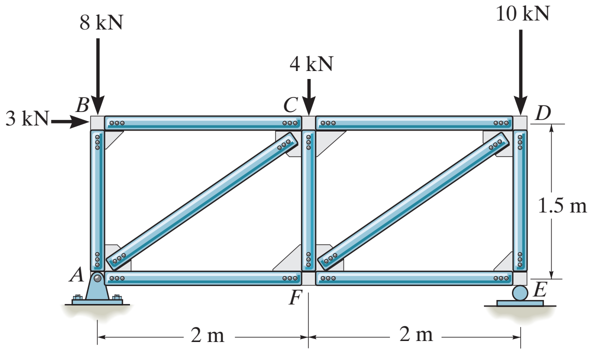
To simplify the analysis, be sure to recognize all two-force members. They have equal but opposite collinear forces at their ends.



REVIEW PROBLEMS

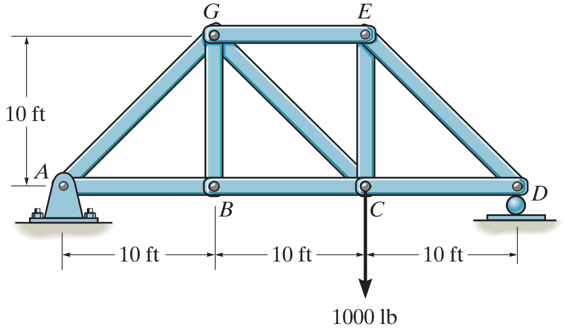
All problem solutions must include FBDs.

R6–1. Determine the force in each member of the truss and state if the members are in tension or compression.



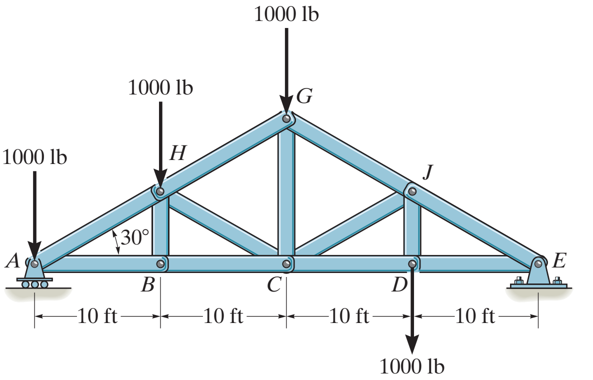
Prob. R6–1

R6–2. Determine the force in each member of the truss and state if the members are in tension or compression.



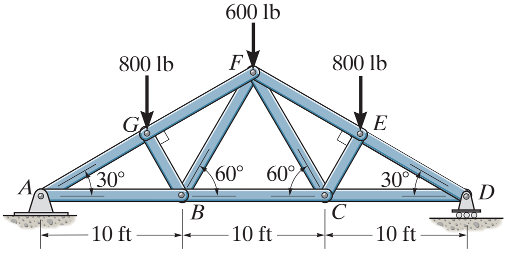
Prob. R6–2

R6–3. Determine the force in member *GJ* and *GC* of the truss and state if the members are in tension or compression.



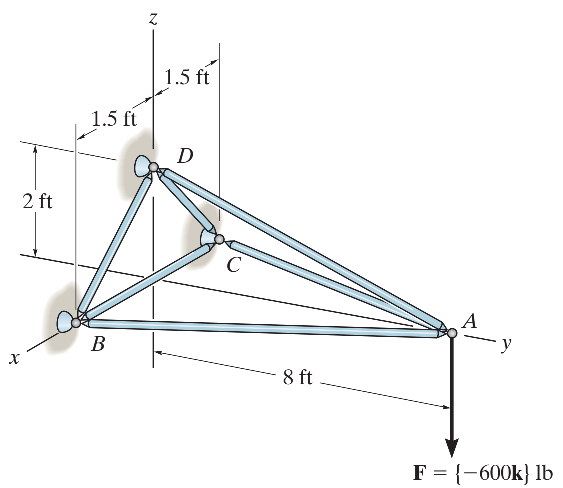
Prob. R6–3

R6–4. Determine the force in members *GF*, *FB*, and *BC* of the *Fink truss* and state if the members are in tension or compression.



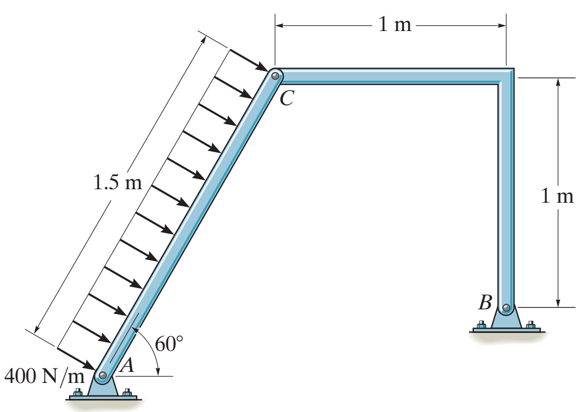
Prob. R6–4

R6–5. Determine the force in members AB, AD, and AC of the space truss and state if the members are in tension or compression.



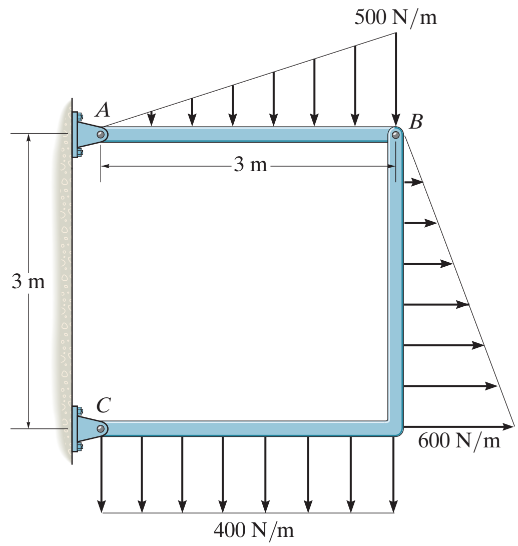
Prob. R6–5

R6–6. Determine the horizontal and vertical components of force that the pins A and B exert on the two-member frame.



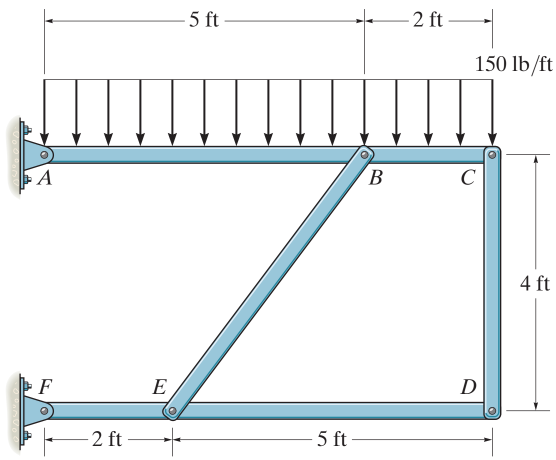
Prob. R6–6

R6–7. Determine the horizontal and vertical components of force at pins A and C of the two-member frame.



Prob. R6–7

R6–8. Determine the resultant forces at pins B and C on member ABC of the four-member frame.



Prob. R6–8

Chapter 7

(© Tony Freeman/Science Source)

When external loads are placed upon these beams and columns, the loads within them must be determined if they are to be properly designed. In this chapter we will study how to determine these internal loadings.

Internal Forces

CHAPTER OBJECTIVES

- To use the method of sections to determine the internal loadings in a member at a specific point.
- To show how to obtain the internal shear and moment throughout a member and express the result graphically in the form of shear and moment diagrams.
- To analyze the forces and the shape of cables supporting various types of loadings.

7.1 Internal Loadings Developed in Structural Members

To design a structural or mechanical member it is necessary to know the loading acting within the member in order to be sure the material can resist this loading. Internal loadings can be determined by using the ***method of sections***. To illustrate this method, consider the cantilever beam in Fig. 7–1*a*. If the internal loadings acting on the cross section at point *B* are to be determined, we must pass an imaginary section *a–a* perpendicular to the axis of the beam through point *B* and then separate the beam into two segments. The internal loadings acting at *B* will then be exposed and become *external* on the free-body diagram of each segment, Fig. 7–1*b*.

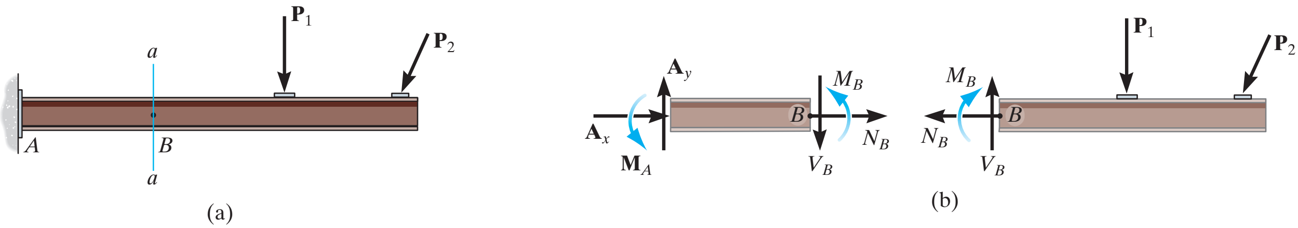


Fig. 7–1

In each case, the link on the backhoe is a two-force member. In the top photo it is subjected to both bending and an axial load at its center. It is more efficient to make the member straight, as in the bottom photo; then only an axial force acts within the member. (© Russell C. Hibbeler)

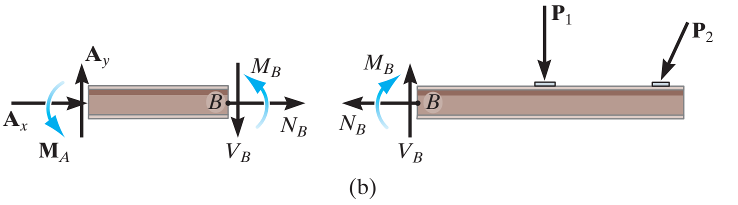
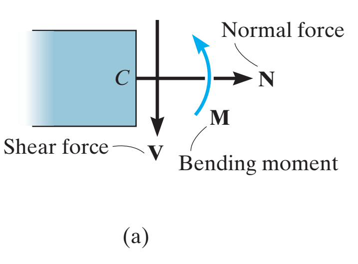


Fig. 7–1 (Repeated)

The force component $\mathbf{N}_B$ that acts *perpendicular* to the cross section is termed the ***normal force***. The force component $\mathbf{V}_B$ that is tangent to the cross section is called the ***shear force***, and the couple moment $\mathbf{M}_B$ is referred to as the ***bending moment***. The force components prevent the relative translation between the two segments, and the couple moment prevents the relative rotation. According to Newton's third law, these loadings must act in opposite directions on each segment, as shown in Fig. 7–1*b*. They can be determined by applying the equations of equilibrium to the free-body diagram of either segment. In this case, however, the right segment is the better choice since it does not involve the unknown support reactions at *A*. A direct solution for $\mathbf{N}_B$ is obtained by applying $\Sigma F_x = 0$, $\mathbf{V}_B$ is obtained from $\Sigma F_y = 0$, and $\mathbf{M}_B$ can be obtained by applying $\Sigma M_B = 0$, since the moments of $\mathbf{N}_B$ and $\mathbf{V}_B$ about *B* are zero.

In two dimensions, we have shown that three internal loading resultants exist, Fig. 7–2*a*; however in three dimensions, a general resultant internal force and couple moment resultant will act at the section. The *x, y, z* components of these loadings are shown in Fig. 7–2*b*. Here $\mathbf{N}_y$ is the *normal force*, and $\mathbf{V}_x$ and $\mathbf{V}_z$ are *shear force components*. $\mathbf{M}_y$ is a *torsional or twisting moment*, and $\mathbf{M}_x$ and $\mathbf{M}_z$ are *bending moment components*. For most applications, these *resultant loadings* will act at the geometric center or centroid (*C*) of the section's cross-sectional area. Although the magnitude for each loading generally will be different at various points along the axis of the member, the method of sections can always be used to determine their values.



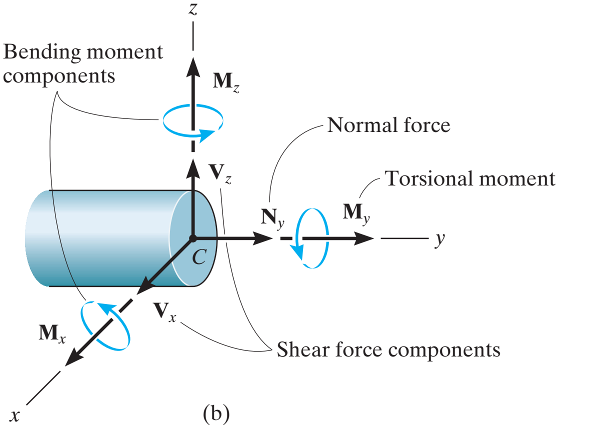
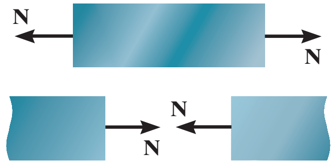
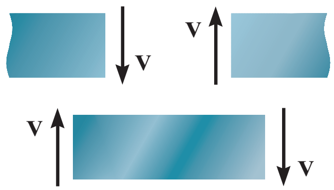


Fig. 7–2

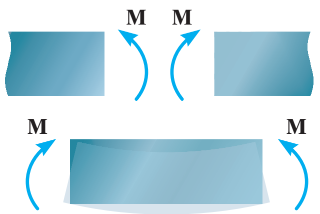
Sign Convention. For problems in two dimensions engineers generally use a sign convention to report the three internal loadings **N, V,** and **M**. Although this sign convention can be arbitrarily assigned, the one that is widely accepted will be used here, Fig. 7–3. The normal force is said to be positive if it creates *tension*, a positive shear force will cause the beam segment on which it acts to rotate clockwise, and a positive bending moment will tend to bend the segment on which it acts in a concave upward manner. Loadings that are opposite to these are considered negative.



Positive normal force



Positive shear



Positive moment

Fig. 7–3

Important Point

- There can be four types of resultant internal loads in a member. They are the normal and shear forces and the bending and torsional moments. These loadings generally vary from point to point. They can be determined using the method of sections.

Procedure for Analysis

The method of sections can be used to determine the internal loadings on the cross section of a member using the following procedure.

Support Reactions.

- Before the member is sectioned, it may first be necessary to determine its support reactions.

Free-Body Diagram.

- It is important to *keep* all distributed loadings, couple moments, and forces acting on the member in their *exact locations*, *then* pass an imaginary section through the member, perpendicular to its axis at the point where the internal loadings are to be determined.

- After the section is made, draw a free-body diagram of the segment that has the least number of loads on it, and indicate the components of the internal force and couple moment resultants at the cross section acting in their positive directions in accordance with the established sign convention.

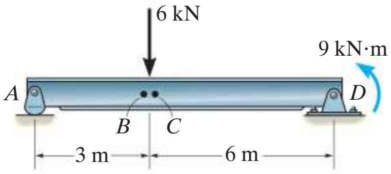
Equations of Equilibrium.

- Moments should be summed at the section. This way the normal and shear forces at the section are eliminated, and we can obtain a direct solution for the moment.

- If the solution of the equilibrium equations yields a negative scalar, the sense of the quantity is opposite to that shown on the free-body diagram.

The designer of this shop crane realized the need for additional reinforcement around the joint at *A* in order to prevent severe internal bending of the joint when a large load is suspended from the chain hoist. (© Russell C. Hibbeler)

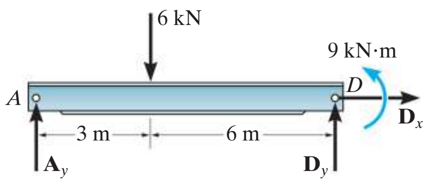
Determine the normal force, shear force, and bending moment acting just to the left, point *B*, and just to the right, point *C*, of the 6-kN force on the beam in Fig. 7–4*a*.



(a)

SOLUTION

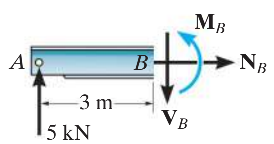
Support Reactions. The free-body diagram of the beam is shown in Fig. 7–4*b*. When determining the *external reactions*, realize that the 9-kN · m couple moment is a free vector and therefore it can be placed *anywhere* on the free-body diagram of the entire beam. Here we will only determine $\mathbf{A}_y$, since the left segments will be used for the analysis.



(b)

$$\circlearrowleft +\Sigma M_D = 0; \quad 9 \text{ kN}\cdot\text{m} + (6 \text{ kN})(6 \text{ m}) - A_y(9 \text{ m}) = 0$$
$$A_y = 5 \text{ kN}$$

Free-Body Diagrams. The free-body diagrams of the left segments *AB* and *AC* of the beam are shown in Figs. 7–4*c* and 7–4*d*. In this case the 9-kN · m couple moment is *not included* on these diagrams since it must be kept in its *original position* until *after* the section is made and the appropriate segment is isolated.

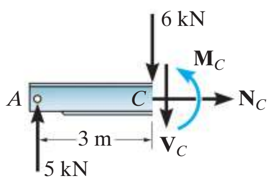


(c)

Equations of Equilibrium.

Segment AB

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_B = 0 \qquad \textit{Ans.}$$
$$+\uparrow \Sigma F_y = 0; \quad 5 \text{ kN} - V_B = 0 \quad V_B = 5 \text{ kN} \qquad \textit{Ans.}$$
$$\circlearrowleft +\Sigma M_B = 0; \quad -(5 \text{ kN})(3 \text{ m}) + M_B = 0 \quad M_B = 15 \text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$



(d)

Fig. 7–4

Segment AC

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0 \qquad \textit{Ans.}$$
$$+\uparrow \Sigma F_y = 0; \quad 5 \text{ kN} - 6 \text{ kN} - V_C = 0 \quad V_C = -1 \text{ kN} \qquad \textit{Ans.}$$
$$\circlearrowleft +\Sigma M_C = 0; \quad -(5 \text{ kN})(3 \text{ m}) + M_C = 0 \quad M_C = 15 \text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

NOTE: The negative sign indicates that $\mathbf{V}_C$ acts in the opposite sense to that shown on the free-body diagram. Also, the moment arm for the 5-kN force in both cases is approximately 3 m since *B* and *C* are "almost" coincident.

7

Determine the normal force, shear force, and bending moment at C of the beam in Fig. 7–5a.

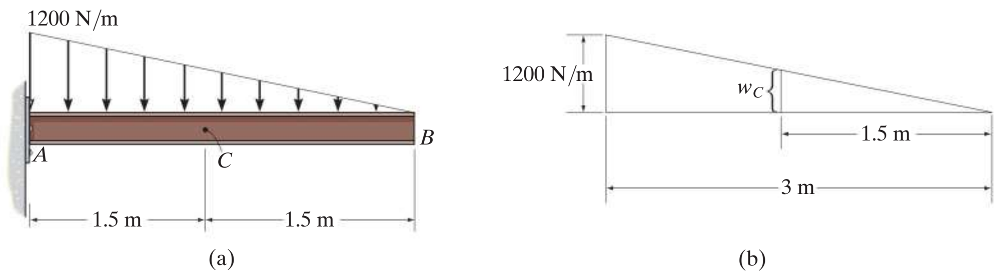
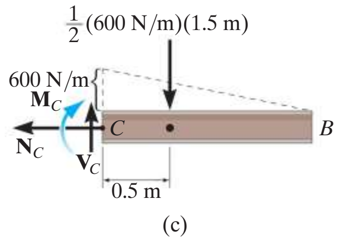


Fig. 7–5

SOLUTION

Free-Body Diagram. It is not necessary to find the support reactions at A since segment BC of the beam can be used to determine the internal loadings at C. The intensity of the triangular distributed load at C is determined using similar triangles from the geometry shown in Fig. 7–5b, i.e.,



$$w_C = (1200 \text{ N/m})\left(\frac{1.5 \text{ m}}{3 \text{ m}}\right) = 600 \text{ N/m}$$

The distributed load acting on segment BC can now be replaced by its resultant force, and its location is indicated on the free-body diagram, Fig. 7–5c.

Equations of Equilibrium.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad N_C = 0 \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad V_C - \tfrac{1}{2}(600 \text{ N/m})(1.5 \text{ m}) = 0$$

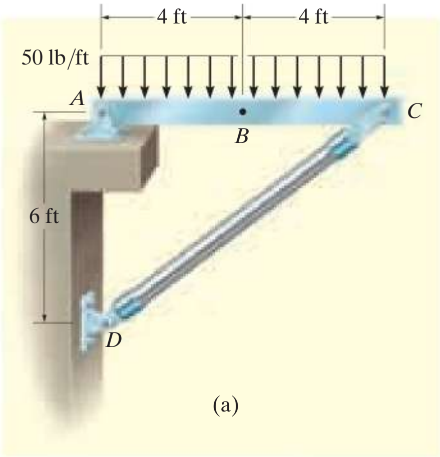
$$V_C = 450 \text{ N} \qquad \textit{Ans.}$$

$$\circlearrowleft +\Sigma M_C = 0; \quad -M_C - \tfrac{1}{2}(600 \text{ N/m})(1.5 \text{ m})(0.5 \text{ m}) = 0$$

$$M_C = -225 \text{ N} \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_C$ acts in the opposite sense to that shown on the free-body diagram.

Determine the normal force, shear force, and bending moment acting at point B of the two-member frame shown in Fig. 7–6*a*.



(a)

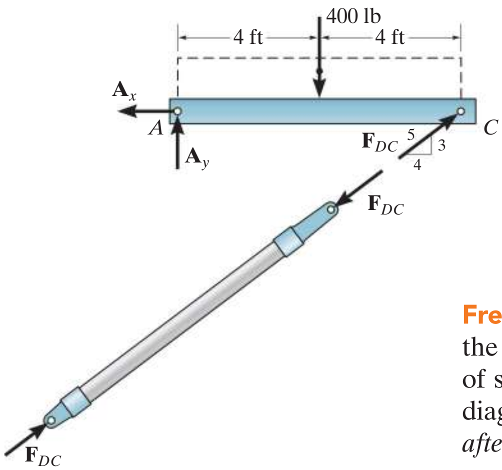
SOLUTION

Support Reactions. A free-body diagram of each member is shown in Fig. 7–6*b*. Since CD is a two-force member, the equations of equilibrium need to be applied only to member AC.

$$\circlearrowleft + \Sigma M_A = 0; \quad -400 \text{ lb } (4 \text{ ft}) + \left(\tfrac{3}{5}\right) F_{DC}(8 \text{ ft}) = 0 \quad F_{DC} = 333.3 \text{ lb}$$

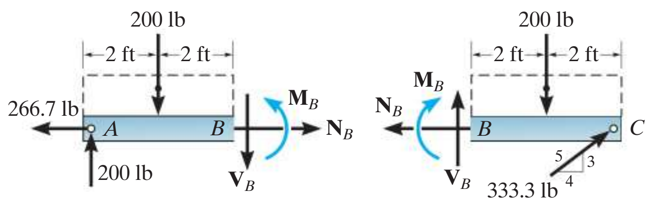
$$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + \left(\tfrac{4}{5}\right)(333.3 \text{ lb}) = 0 \quad A_x = 266.7 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 400 \text{ lb} + \left(\tfrac{3}{5}\right)(333.3 \text{ lb}) = 0 \quad A_y = 200 \text{ lb}$$



(b)

Fig. 7–6



(c)

Free-Body Diagrams. Passing an imaginary section perpendicular to the axis of member AC through point B yields the free-body diagrams of segments AB and BC shown in Fig. 7–6*c*. When constructing these diagrams it is important to keep the distributed loading where it is until *after the section is made*. Only then can it be replaced by a single resultant force.

Equations of Equilibrium. Applying the equations of equilibrium to segment AB, we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_B - 266.7 \text{ lb} = 0 \quad N_B = 267 \text{ lb} \quad \textit{Ans.}$$

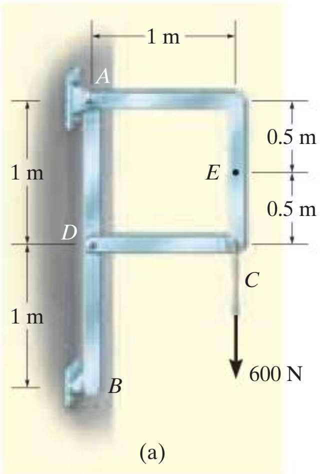
$$+\uparrow \Sigma F_y = 0; \quad 200 \text{ lb} - 200 \text{ lb} - V_B = 0 \quad V_B = 0 \quad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad M_B - 200 \text{ lb } (4 \text{ ft}) + 200 \text{ lb } (2 \text{ ft}) = 0$$

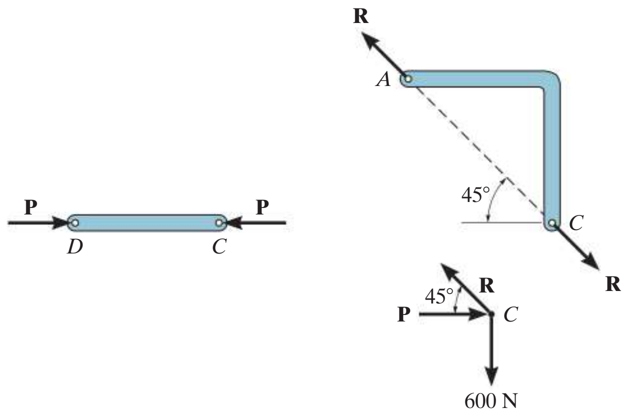
$$M_B = 400 \text{ lb} \cdot \text{ft} \quad \textit{Ans.}$$

NOTE: As an exercise, try to obtain these same results using segment BC.

Determine the normal force, shear force, and bending moment acting at point E of the frame loaded as shown in Fig. 7–7a.



(a)



(b)

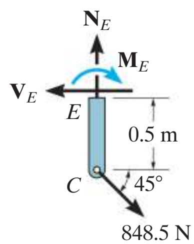
SOLUTION

Support Reactions. By inspection, members AC and CD are two-force members, Fig. 7–7b. In order to determine the internal loadings at E, we must first determine the force $\mathbf{R}$ acting at the end of member AC. To obtain it, we will analyze the equilibrium of the pin at C.

Summing forces in the vertical direction on the pin, Fig. 7–7b, we have

$$+\uparrow \Sigma F_y = 0; \qquad R \sin 45^\circ - 600 \text{ N} = 0 \qquad R = 848.5 \text{ N}$$

Free-Body Diagram. The free-body diagram of segment CE is shown in Fig. 7–7c.



(c)

Fig. 7–7

Equations of Equilibrium.

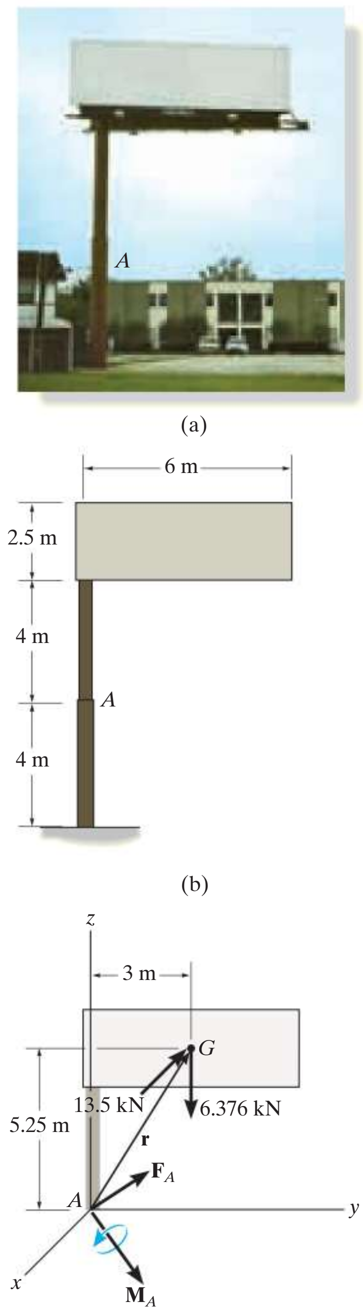
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 848.5 \cos 45^\circ \text{ N} - V_E = 0 \qquad V_E = 600 \text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -848.5 \sin 45^\circ \text{ N} + N_E = 0 \qquad N_E = 600 \text{ N} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_E = 0; \qquad 848.5 \cos 45^\circ \text{ N}(0.5 \text{ m}) - M_E = 0$$

$$M_E = 300 \text{ N} \cdot \text{m} \qquad \textit{Ans.}$$

NOTE: These results indicate a poor design. Member AC should be *straight* (from A to C) so that bending within the member is *eliminated*. If AC were straight then the internal force would only create tension in the member.



(a)

(b)

(c)

Fig. 7–8

(© Russell C. Hibbeler)

The uniform sign shown in Fig. 7–8*a* has a mass of 650 kg and is supported on the fixed column. Design codes indicate that the expected maximum uniform wind loading that will occur in the area where it is located is 900 Pa. Determine the internal loadings at *A*.

SOLUTION

The idealized model for the sign is shown in Fig. 7–8*b*. Here the necessary dimensions are indicated. We can consider the free-body diagram of a section above point *A* since it does not involve the support reactions.

Free-Body Diagram. The sign has a weight of $W = 650(9.81)\text{ N} = 6.376\text{ kN}$, and the wind creates a resultant force of $F_w = 900\text{ N/m}^2(6\text{ m})(2.5\text{ m}) = 13.5\text{ kN}$, which acts perpendicular to the face of the sign. These loadings are shown on the free-body diagram, Fig. 7–8*c*.

Equations of Equilibrium. Since the problem is three dimensional, a vector analysis will be used.

$$\Sigma \mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_A - 13.5\mathbf{i} - 6.376\mathbf{k} = \mathbf{0}$$

$$\mathbf{F}_A = \{13.5\mathbf{i} + 6.38\mathbf{k}\}\text{ kN} \qquad \textit{Ans.}$$

$$\Sigma \mathbf{M}_A = \mathbf{0}; \qquad \mathbf{M}_A + \mathbf{r} \times (\mathbf{F}_w + \mathbf{W}) = \mathbf{0}$$

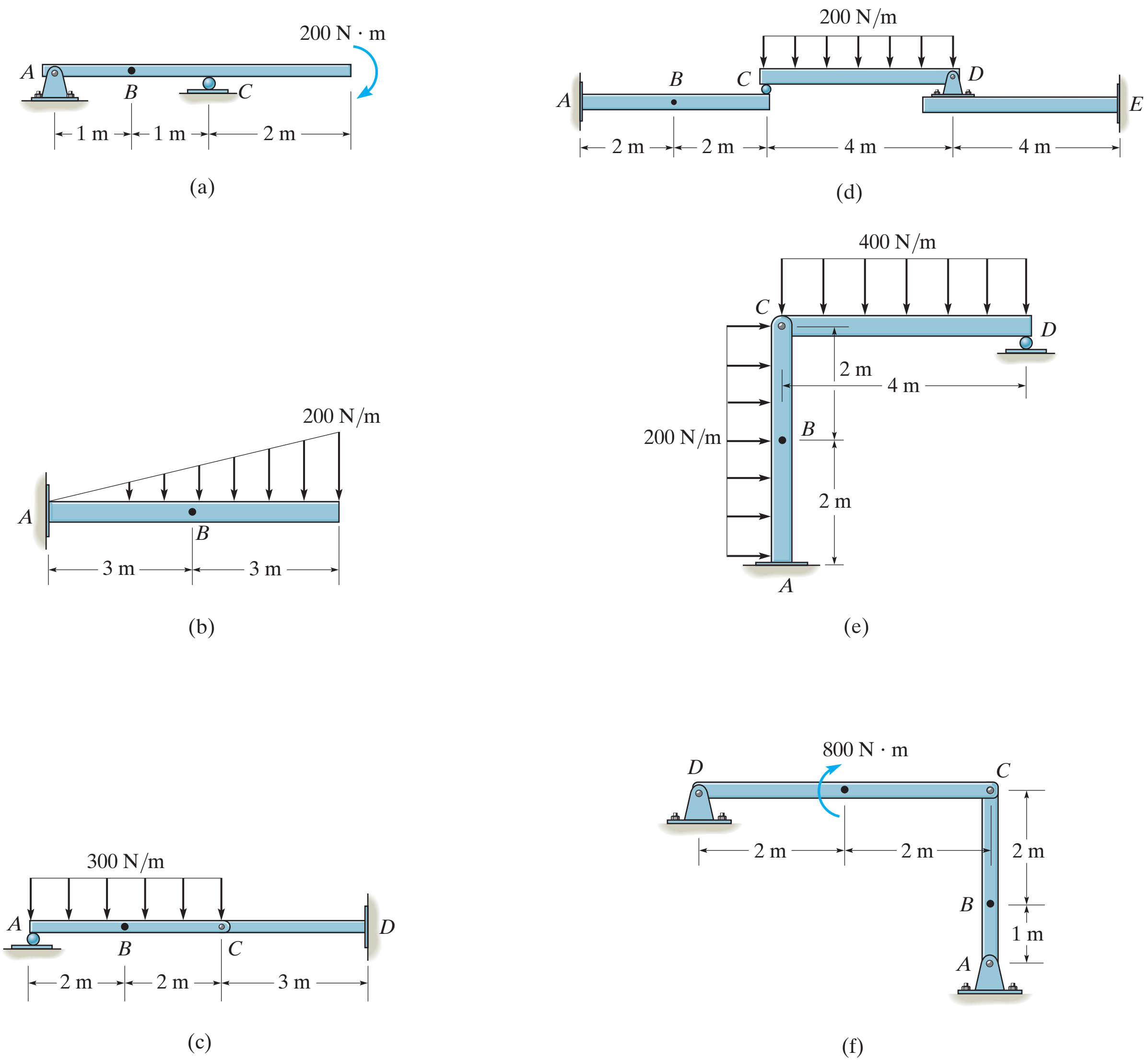
$$\mathbf{M}_A + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 5.25 \\ -13.5 & 0 & -6.376 \end{vmatrix} = \mathbf{0}$$

$$\mathbf{M}_A = \{19.1\mathbf{i} + 70.9\mathbf{j} - 40.5\mathbf{k}\}\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

NOTE: Here $\mathbf{F}_{A_z} = \{6.38\mathbf{k}\}$ kN represents the normal force, whereas $\mathbf{F}_{A_x} = \{13.5\mathbf{i}\}$ kN is the shear force. Also, the torsional moment is $\mathbf{M}_{A_z} = \{-40.5\mathbf{k}\}\text{ kN}\cdot\text{m}$, and the bending moment is determined from its components $\mathbf{M}_{A_x} = \{19.1\mathbf{i}\}\text{ kN}\cdot\text{m}$ and $\mathbf{M}_{A_y} = \{70.9\mathbf{j}\}\text{ kN}\cdot\text{m}$; i.e., $(M_b)_A = \sqrt{(M_A)_x^2 + (M_A)_y^2} = 73.4\text{ kN}\cdot\text{m}$.

PRELIMINARY PROBLEMS

P7–1. In each case, calculate the reaction at A and then draw the free-body diagram of segment AB of the beam in order to determine the internal loading at B.

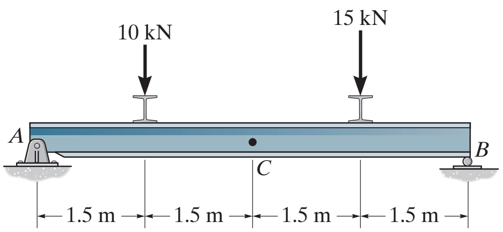


Prob. P7–1

FUNDAMENTAL PROBLEMS

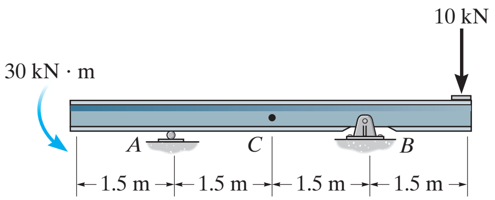
All problem solutions must include FBDs.

F7–1. Determine the normal force, shear force, and moment at point C.



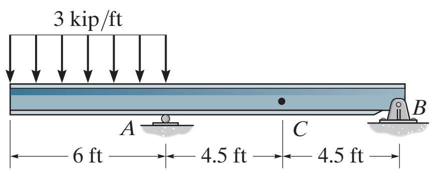
Prob. F7–1

F7–2. Determine the normal force, shear force, and moment at point C.



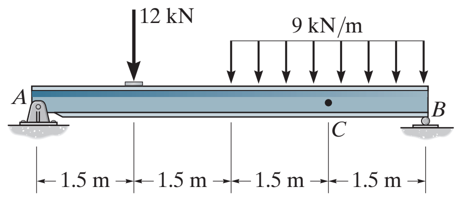
Prob. F7–2

F7–3. Determine the normal force, shear force, and moment at point C.



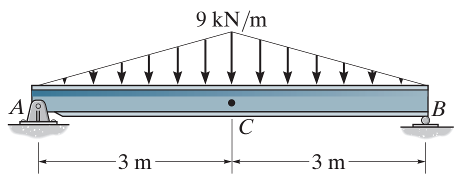
Prob. F7–3

F7–4. Determine the normal force, shear force, and moment at point C.



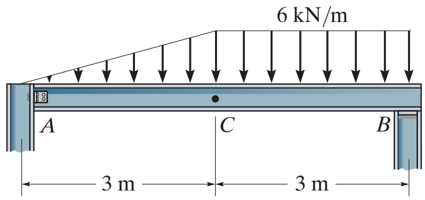
Prob. F7–4

F7–5. Determine the normal force, shear force, and moment at point C.



Prob. F7–5

F7–6. Determine the normal force, shear force, and moment at point C. Assume A is pinned and B is a roller.

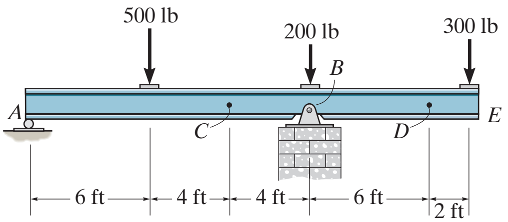


Prob. F7–6

PROBLEMS

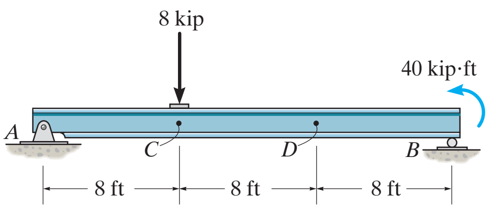
All problem solutions must include FBDs.

7–1. Determine the shear force and moment at points C and D.



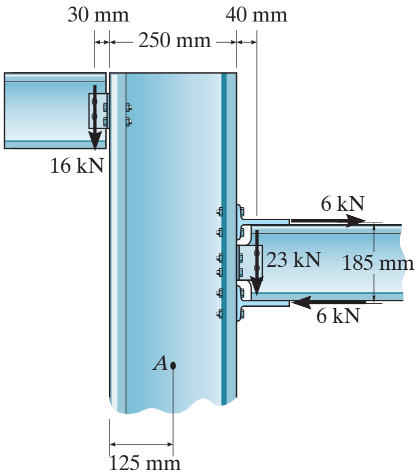
Prob. 7–1

7–2. Determine the internal normal force and shear force, and the bending moment in the beam at points C and D. Assume the support at B is a roller. Point C is located just to the right of the 8-kip load.



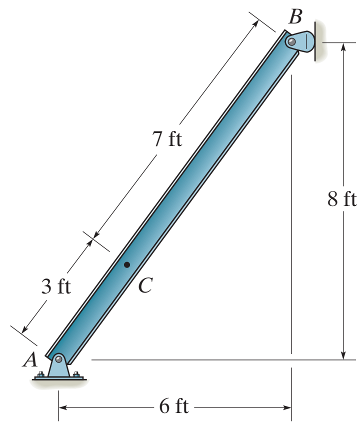
Prob. 7–2

7–3. Two beams are attached to the column such that structural connections transmit the loads shown. Determine the internal normal force, shear force, and moment acting in the column at a section passing horizontally through point A.



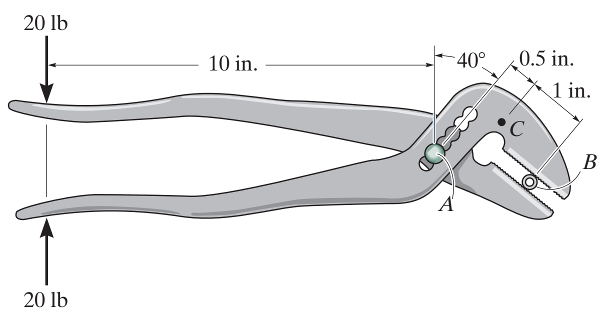
Prob. 7–3

***7–4.** The beam weighs 280 lb/ft. Determine the internal normal force, shear force, and moment at point C.



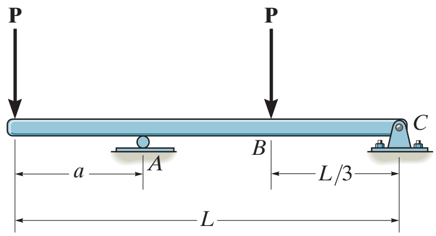
Prob. 7–4

7–5. The pliers are used to grip the tube at B. If a force of 20 lb is applied to the handles, determine the internal shear force and moment a point C. Assume the jaws of the pliers exert only normal forces on the tube.



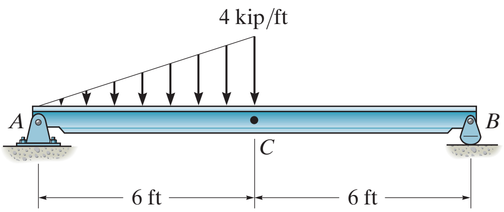
Prob. 7–5

7–6. Determine the distance a as a fraction of the beam's length L for locating the roller support so that the moment in the beam at B is zero.



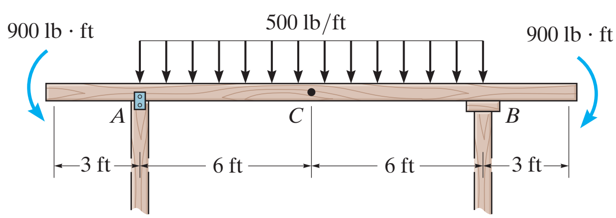
Prob. 7–6

7–7. Determine the internal shear force and moment acting at point C in the beam.



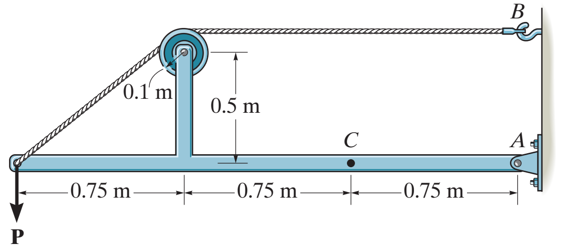
Prob. 7–7

***7–8.** Determine the internal shear force and moment acting at point C in the beam.



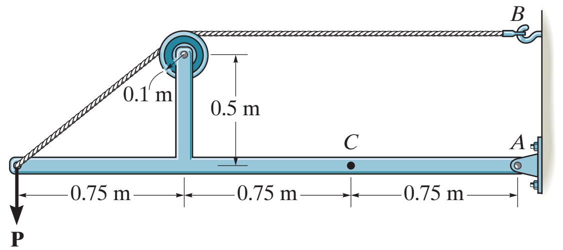
Prob. 7–8

7–9. Determine the normal force, shear force, and moment at a section passing through point C. Take $P = 8$ kN.



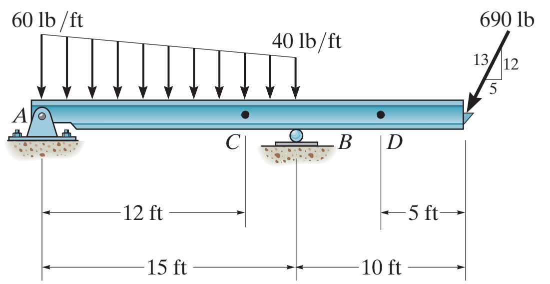
Prob. 7–9

7–10. The cable will fail when subjected to a tension of 2 kN. Determine the largest vertical load P the frame will support and calculate the internal normal force, shear force, and moment at a section passing through point C for this loading.



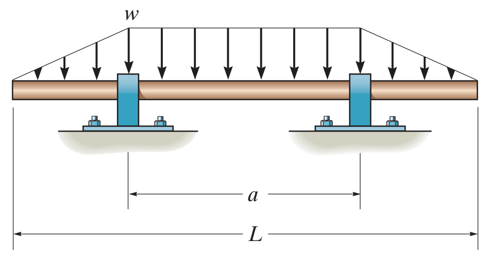
Prob. 7–10

7–11. Determine the internal normal force, shear force, and moment at points C and D of the beam.



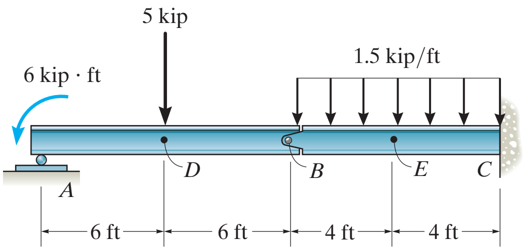
Prob. 7–11

*7–12. Determine the distance a between the bearings in terms of the shaft's length L so that the moment in the *symmetric* shaft is zero at its center.



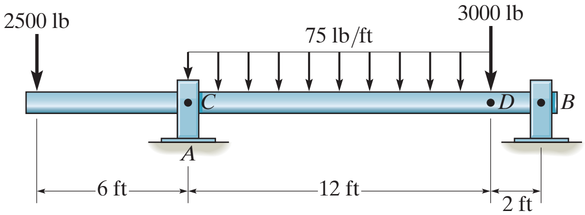
Prob. 7–12

7–13. Determine the internal normal force, shear force, and moment in the beam at sections passing through points D and E. Point D is located just to the left of the 5-kip load.



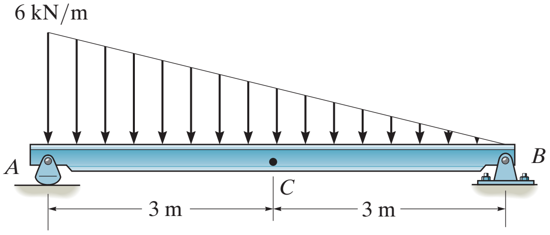
Prob. 7–13

7–14. The shaft is supported by a journal bearing at A and a thrust bearing at B. Determine the normal force, shear force, and moment at a section passing through (a) point C, which is just to the right of the bearing at A, and (b) point D, which is just to the left of the 3000-lb force.



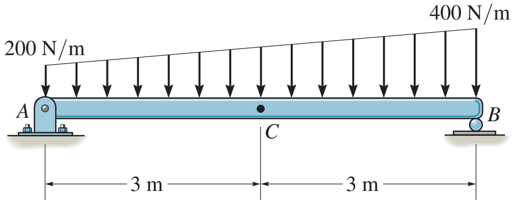
Prob. 7–14

7–15. Determine the internal normal force, shear force, and moment at point C.



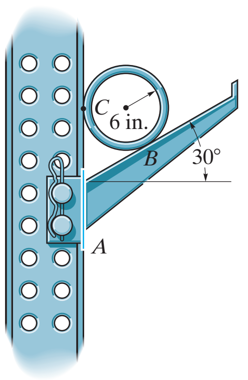
Prob. 7–15

*7–16. Determine the internal normal force, shear force, and moment at point C of the beam.



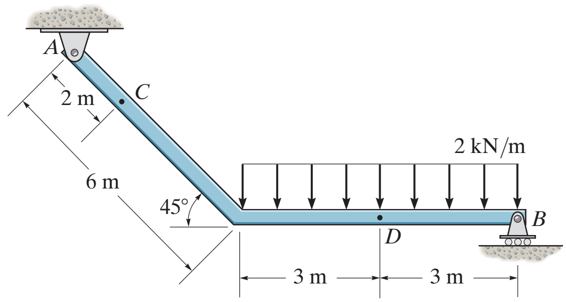
Prob. 7–16

7–17. The cantilevered rack is used to support each end of a smooth pipe that has a total weight of 300 lb. Determine the normal force, shear force, and moment that act in the arm at its fixed support A along a vertical section.



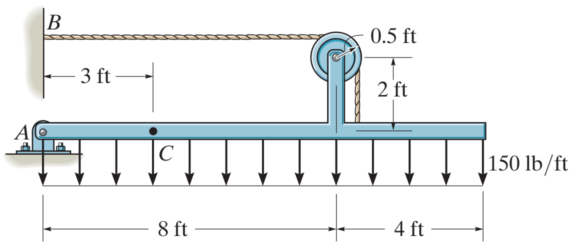
Prob. 7–17

7–18. Determine the internal normal force, shear force, and the moment at points C and D.



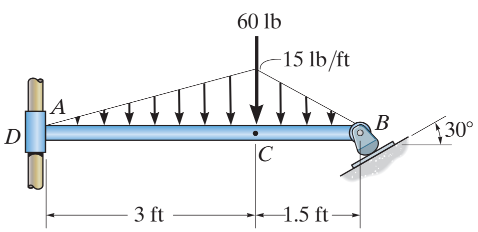
Prob. 7–18

7–19. Determine the internal normal force, shear force, and moment at point C.



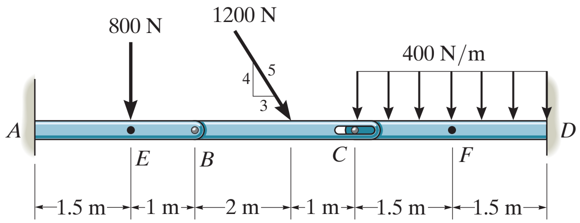
Prob. 7–19

***7–20.** Rod AB is fixed to a smooth collar D, which slides freely along the vertical guide. Determine the internal normal force, shear force, and moment at point C, which is located just to the left of the 60-lb concentrated load.



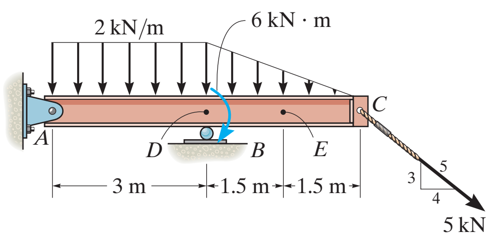
Prob. 7–20

7–21. Determine the internal normal force, shear force, and moment at points E and F of the compound beam. Point E is located just to the left of 800 N force.



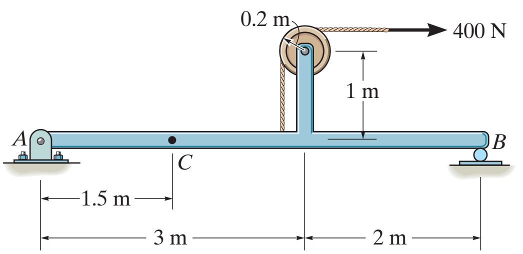
Prob. 7–21

7–22. Determine the internal normal force, shear force, and moment at points D and E in the overhang beam. Point D is located just to the left of the roller support at B, where the couple moment acts.



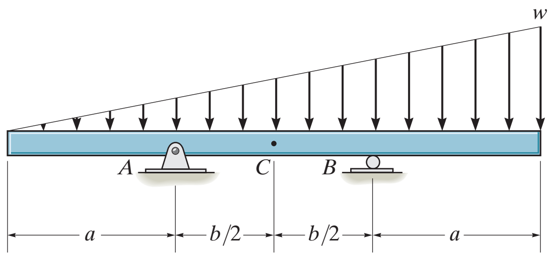
Prob. 7–22

7–23. Determine the internal normal force, shear force, and moment at point C.



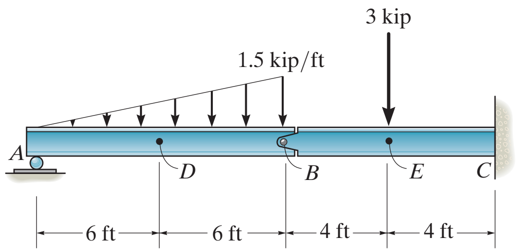
Prob. 7–23

***7–24.** Determine the ratio of a/b for which the shear force will be zero at the midpoint C of the beam.



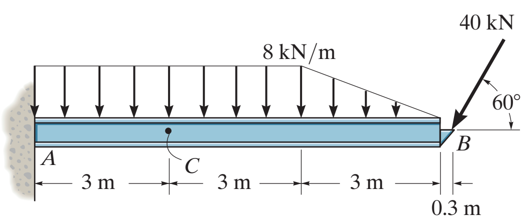
Prob. 7–24

7–25. Determine the normal force, shear force, and moment in the beam at sections passing through points D and E. Point E is just to the right of the 3-kip load.



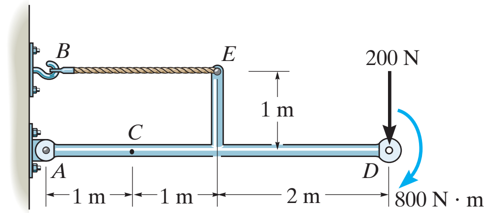
Prob. 7–25

7–26. Determine the internal normal force, shear force, and bending moment at point C.



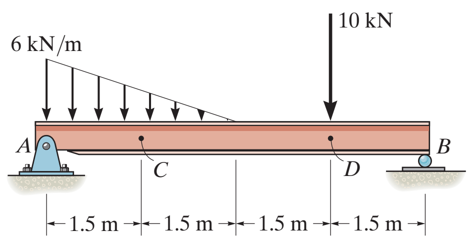
Prob. 7–26

7–27. Determine the internal normal force, shear force, and moment at point C.



Prob. 7–27

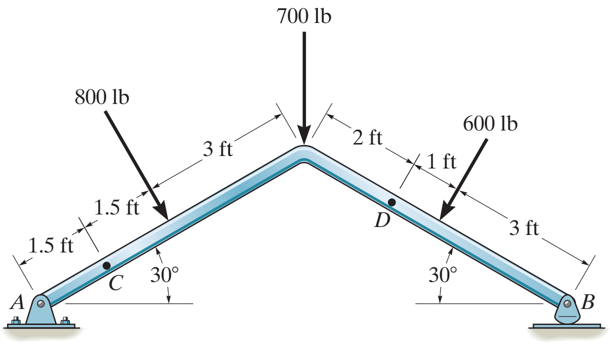
***7–28.** Determine the internal normal force, shear force, and moment at points C and D in the simply supported beam. Point D is located just to the left of the 10-kN concentrated load.



Prob. 7–28

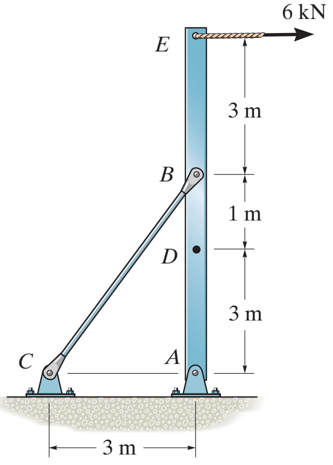
7–29. Determine the normal force, shear force, and moment acting at a section passing through point C.

7–30. Determine the normal force, shear force, and moment acting at a section passing through point D.



Probs. 7–29/30

***7–32.** Determine the internal normal force, shear force, and moment at point D.

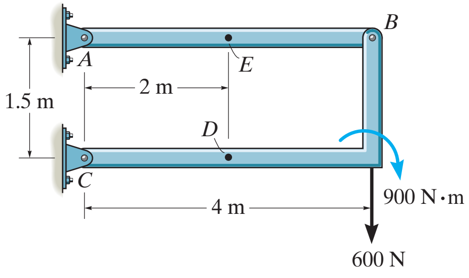


Prob. 7–32

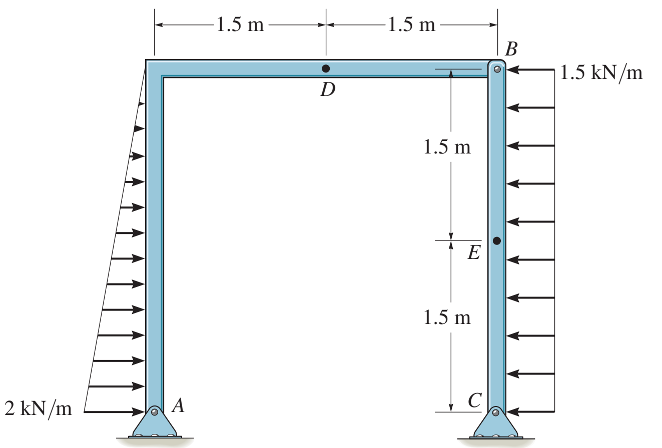
7–33. Determine the internal normal force, shear force, and moment at point D of the two-member frame.

7–34. Determine the internal normal force, shear force, and moment at point E.

7–31. Determine the internal normal force, shear force, and moment acting at points D and E of the frame.

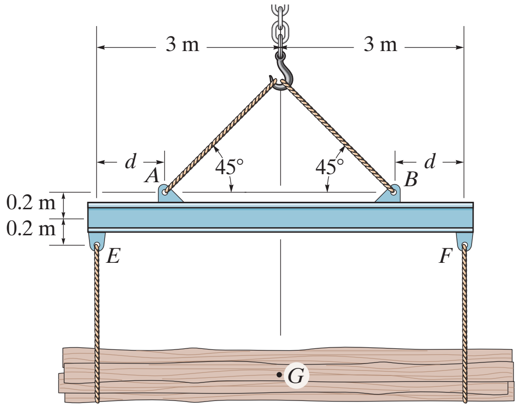


Prob. 7–31



Probs. 7–33/34

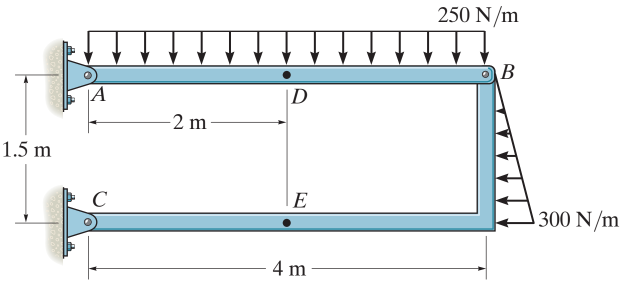
7–35. The strongback or lifting beam is used for materials handling. If the suspended load has a weight of 2 kN and a center of gravity of G, determine the placement d of the padeyes on the top of the beam so that there is no moment developed within the length AB of the beam. The lifting bridle has two legs that are positioned at 45°, as shown.



Prob. 7–35

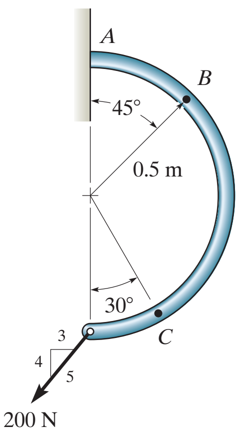
7–37. Determine the internal normal force, shear force, and moment at point D of the two-member frame.

7–38. Determine the internal normal force, shear force, and moment at point E of the two-member frame.



Probs. 7–37/38

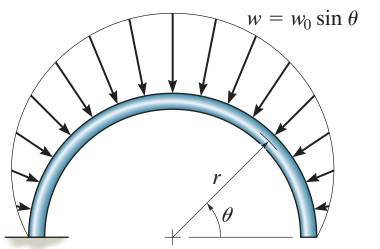
***7–36.** Determine the internal normal force, shear force, and moment acting at points B and C on the curved rod.



Prob. 7–36

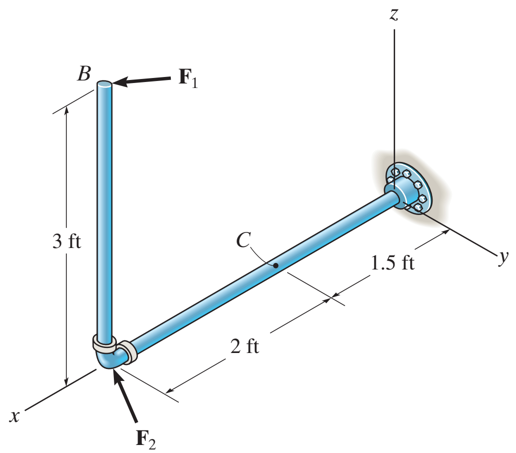
7–39. The distributed loading $w = w_0 \sin \theta$, measured per unit length, acts on the curved rod. Determine the internal normal force, shear force, and moment in the rod at $\theta = 45°$.

***7–40.** Solve Prob. 7–39 for $\theta = 120°$.



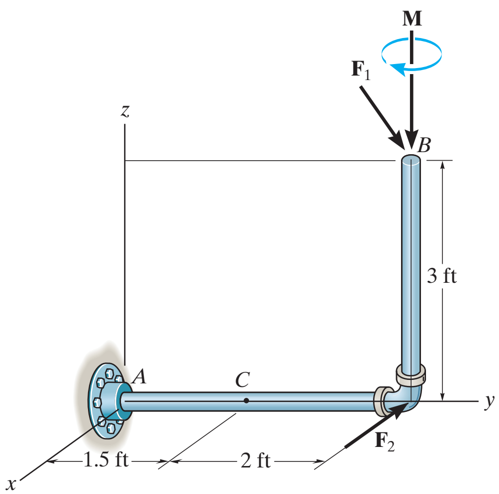
Probs. 7–39/40

7–41. Determine the *x, y, z* components of force and moment at point *C* in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{350\mathbf{i} - 400\mathbf{j}\}$ lb and $\mathbf{F}_2 = \{-300\mathbf{j} + 150\mathbf{k}\}$ lb.



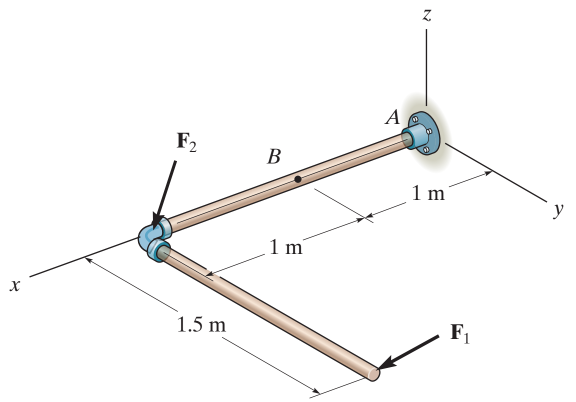
Prob. 7–41

7–42. Determine the *x, y, z* components of force and moment at point *C* in the pipe assembly. Neglect the weight of the pipe. The load acting at (0, 3.5 ft, 3 ft) is $\mathbf{F}_1 = \{-24\mathbf{i} - 10\mathbf{k}\}$ lb and $\mathbf{M} = \{-30\mathbf{k}\}$ lb · ft and at point (0, 3.5 ft, 0) $\mathbf{F}_2 = \{-80\mathbf{i}\}$ lb.



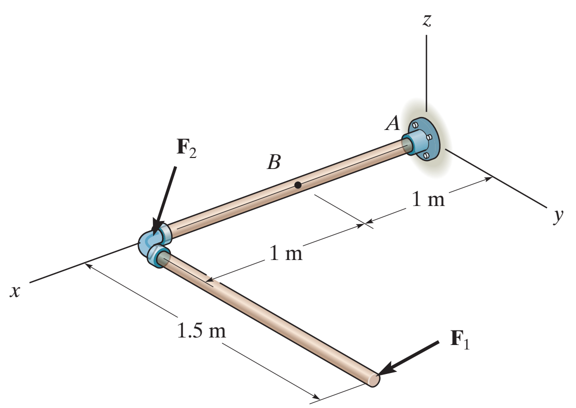
Prob. 7–42

7–43. Determine the *x, y, z* components of internal loading at a section passing through point *B* in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{200\mathbf{i} - 100\mathbf{j} - 400\mathbf{k}\}$ N and $\mathbf{F}_2 = \{300\mathbf{i} - 500\mathbf{k}\}$ N.



Prob. 7–43

***7–44.** Determine the *x, y, z* components of internal loading at a section passing through point *B* in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{100\mathbf{i} - 200\mathbf{j} - 300\mathbf{k}\}$ N and $\mathbf{F}_2 = \{100\mathbf{i} + 500\mathbf{j}\}$ N.



Prob. 7–44

*7.2 Shear and Moment Equations and Diagrams

Beams are structural members designed to support loadings applied perpendicular to their axes. In general, they are long and straight and have a constant cross-sectional area. They are often classified as to how they are supported. For example, a ***simply supported beam*** is pinned at one end and roller supported at the other, as in Fig. 7–9*a*, whereas a ***cantilevered beam*** is fixed at one end and free at the other. The actual design of a beam requires a detailed knowledge of the *variation* of the internal shear force V and bending moment M acting at *each point* along the axis of the beam.*

These *variations* of V and M along the beam's axis can be obtained by using the method of sections discussed in Sec. 7.1. In this case, however, it is necessary to section the beam at an arbitrary distance x from one end and then apply the equations of equilibrium to the segment having the length x. Doing this we can then obtain V and M as functions of x.

In general, the internal shear and bending-moment functions will be discontinuous, or their slopes will be discontinuous, at points where a distributed load changes or where concentrated forces or couple moments are applied. Because of this, these functions must be determined for *each segment* of the beam located between any two discontinuities of loading. For example, segments having lengths x_1, x_2, and x_3 will have to be used to describe the variation of V and M along the length of the beam in Fig. 7–9*a*. These functions will be valid *only* within regions from 0 to a for x_1, from a to b for x_2, and from b to L for x_3. If the resulting functions of x are plotted, the graphs are termed the ***shear diagram*** and ***bending-moment diagram***, Fig. 7–9*b* and Fig. 7–9*c*, respectively.

To save on material and thereby produce an efficient design, these beams, also called girders, have been tapered, since the internal moment in the beam will be larger at the supports, or piers, than at the center of the span. (© Russell C. Hibbeler)

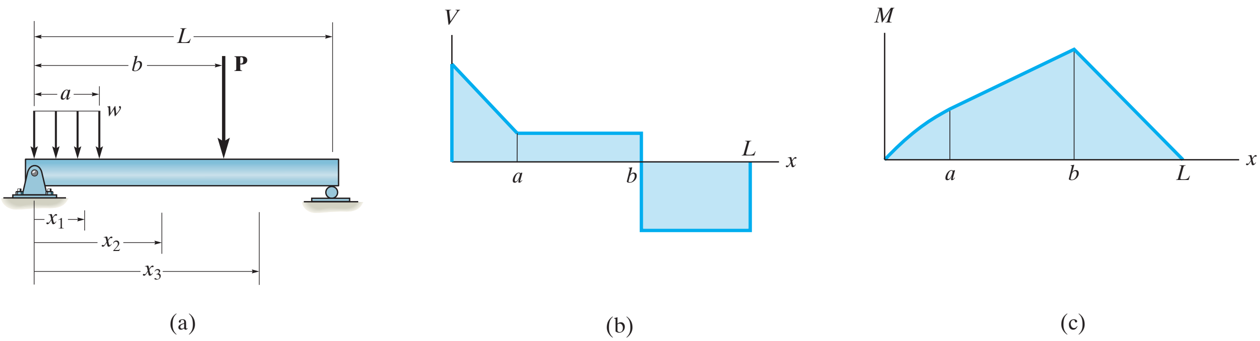


Fig. 7–9

*The internal normal force is not considered for two reasons. In most cases, the loads applied to a beam act perpendicular to the beam's axis and hence produce only an internal shear force and bending moment. And for design purposes, the beam's resistance to shear, and particularly to bending, is more important than its ability to resist a normal force.

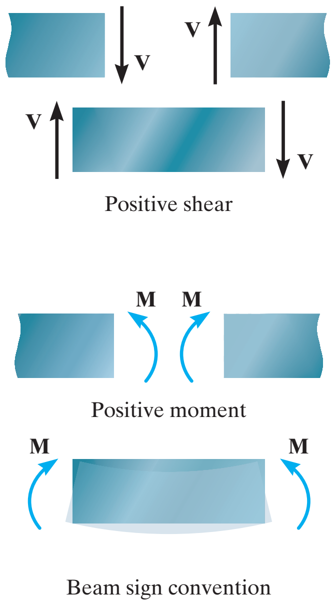


Fig. 7–10

Important Points

- Shear and moment diagrams for a beam provide graphical descriptions of how the internal shear and moment vary throughout the beam's length.
- To obtain these diagrams, the method of sections is used to determine V and M as functions of x. These results are then plotted. If the load on the beam suddenly changes, then regions between each load must be selected to obtain each function of x.

Procedure for Analysis

The shear and bending-moment diagrams for a beam can be constructed using the following procedure.

Support Reactions.

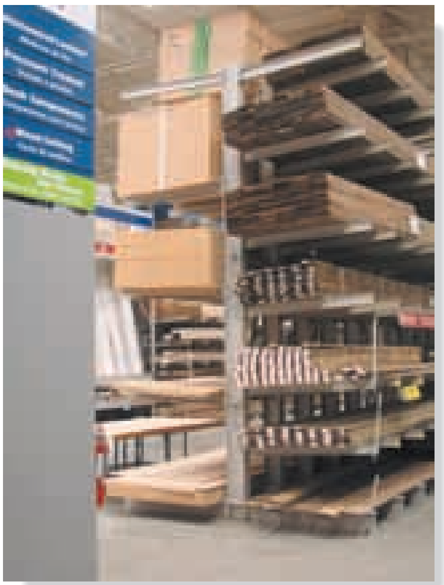
- Determine all the reactive forces and couple moments acting on the beam and resolve all the forces into components acting perpendicular and parallel to the beam's axis.

Shear and Moment Functions.

- Specify separate coordinates x having an origin at the beam's left end and extending to regions of the beam *between* concentrated forces and/or couple moments, or where the distributed loading is continuous.
- Section the beam at each distance x and draw the free-body diagram of one of the segments. Be sure **V** and **M** are shown acting in their *positive sense*, in accordance with the sign convention given in Fig. 7–10.
- The shear V is obtained by summing forces perpendicular to the beam's axis, and the moment M is obtained by summing moments about the sectioned end of the segment.

Shear and Moment Diagrams.

- Plot the shear diagram (V versus x) and the moment diagram (M versus x). If computed values of the functions describing V and M are *positive*, the values are plotted above the x axis, whereas *negative* values are plotted below the x axis.



The shelving arms must be designed to resist the internal loading in the arms caused by the lumber. (© Russell C. Hibbeler)

Draw the shear and moment diagrams for the shaft shown in Fig. 7–11*a*. The support at A is a thrust bearing and the support at C is a journal bearing.

SOLUTION

Support Reactions. The support reactions are shown on the shaft's free-body diagram, Fig. 7–11*d*.

Shear and Moment Functions. The shaft is sectioned at an arbitrary distance x from point A, extending within the region AB, and the free-body diagram of the left segment is shown in Fig. 7–11*b*. The unknowns **V** and **M** are assumed to act in the *positive sense* on the right-hand face of the segment according to the established sign convention. Applying the equilibrium equations yields

$$+\uparrow \Sigma F_y = 0; \qquad V = 2.5 \text{ kN} \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad M = 2.5x \text{ kN}\cdot\text{m} \qquad (2)$$

A free-body diagram for a left segment of the shaft extending from A a distance x, within the region BC is shown in Fig. 7–11*c*. As always, **V** and **M** are shown acting in the positive sense. Hence,

$$+\uparrow \Sigma F_y = 0; \qquad 2.5 \text{ kN} - 5 \text{ kN} - V = 0$$

$$V = -2.5 \text{ kN} \qquad (3)$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + 5 \text{ kN}(x - 2 \text{ m}) - 2.5 \text{ kN}(x) = 0$$

$$M = (10 - 2.5x) \text{ kN}\cdot\text{m} \qquad (4)$$

Shear and Moment Diagrams. When Eqs. 1 through 4 are plotted within the regions in which they are valid, the shear and moment diagrams shown in Fig. 7–11*d* are obtained. The shear diagram indicates that the internal shear force is always 2.5 kN (positive) within segment AB. Just to the right of point B, the shear force changes sign and remains at a constant value of -2.5 kN for segment BC. The moment diagram starts at zero, increases linearly to point B at $x = 2$ m, where $M_{\text{max}} = 2.5 \text{ kN}(2 \text{ m}) = 5 \text{ kN}\cdot\text{m}$, and thereafter decreases back to zero.

NOTE: It is seen in Fig. 7–11*d* that the graphs of the shear and moment diagrams "jump" or changes abruptly where the concentrated force acts, i.e., at points A, B, and C. For this reason, as stated earlier, it is necessary to express both the shear and moment functions separately for regions between concentrated loads. It should be realized, however, that all loading discontinuities are mathematical, arising from the *idealization of a concentrated force and couple moment*. Physically, loads are always applied over a finite area, and if the actual load variation could be accounted for, the shear and moment diagrams would then be continuous over the shaft's entire length.

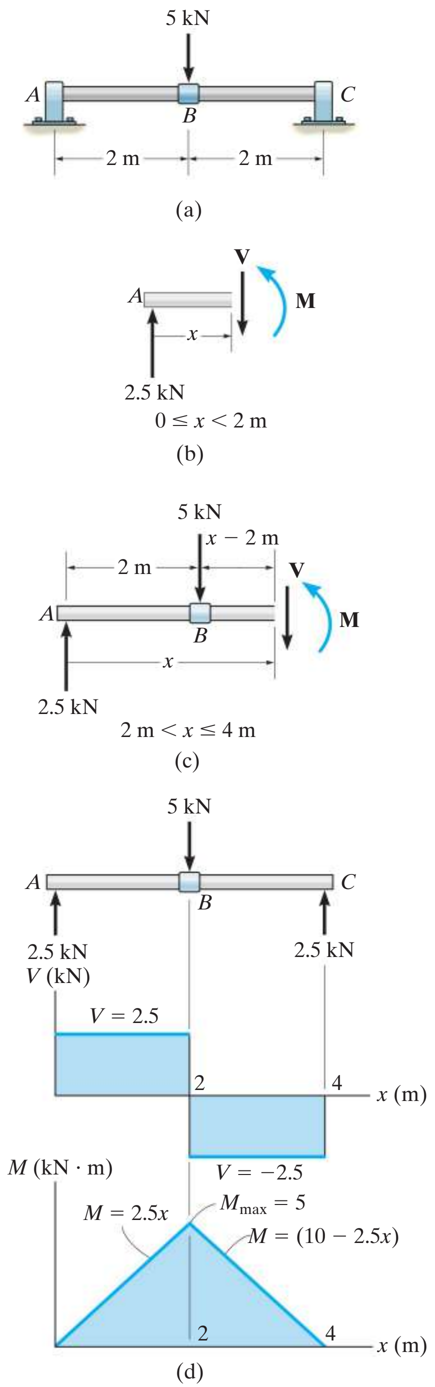


Fig. 7–11

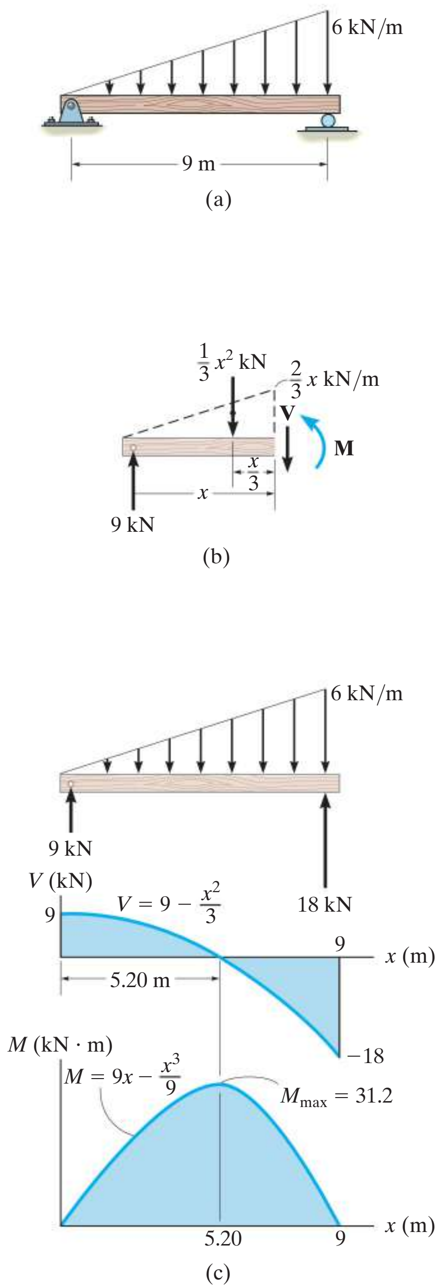


Fig. 7–12

Draw the shear and moment diagrams for the beam shown in Fig. 7–12*a*.

SOLUTION

Support Reactions. The support reactions are shown on the beam's free-body diagram, Fig. 7–12*c*.

Shear and Moment Functions. A free-body diagram for a left segment of the beam having a length x is shown in Fig. 7–12*b*. Due to proportional triangles, the distributed loading acting at the end of this segment has an intensity of $w/x = 6/9$ or $w = (2/3)x$. It is replaced by a resultant force *after* the segment is isolated as a free-body diagram. The *magnitude* of the resultant force is equal to $\frac{1}{2}(x)\left(\frac{2}{3}x\right) = \frac{1}{3}x^2$. This force *acts through the centroid* of the distributed loading area, a distance $\frac{1}{3}x$ from the right end. Applying the two equations of equilibrium yields

$$+\uparrow \Sigma F_y = 0; \qquad 9 - \frac{1}{3}x^2 - V = 0$$

$$V = \left(9 - \frac{x^2}{3}\right) \text{kN} \tag{1}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{3}x^2\left(\frac{x}{3}\right) - 9x = 0$$

$$M = \left(9x - \frac{x^3}{9}\right) \text{kN} \cdot \text{m} \tag{2}$$

Shear and Moment Diagrams. The shear and moment diagrams shown in Fig. 7–12*c* are obtained by plotting Eqs. 1 and 2.

The point of *zero shear* can be found using Eq. 1:

$$V = 9 - \frac{x^2}{3} = 0$$

$$x = 5.20 \text{ m}$$

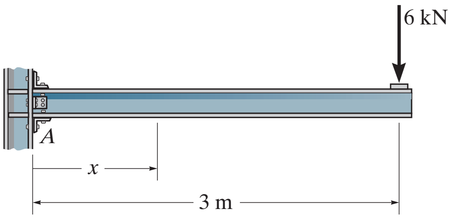
NOTE: It will be shown in Sec. 7.3 that this value of x happens to represent the point on the beam where the *maximum moment* occurs. Using Eq. 2, we have

$$M_{max} = \left(9(5.20) - \frac{(5.20)^3}{9}\right) \text{kN} \cdot \text{m}$$

$$= 31.2 \text{ kN} \cdot \text{m}$$

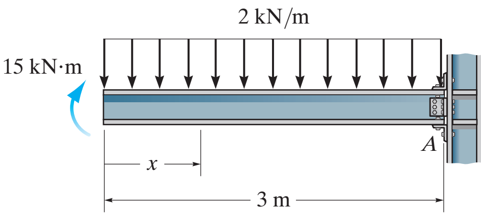
FUNDAMENTAL PROBLEMS

F7–7. Determine the shear and moment as a function of x, and then draw the shear and moment diagrams.



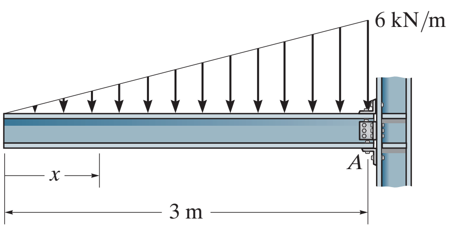
Prob. F7–7

F7–8. Determine the shear and moment as a function of x, and then draw the shear and moment diagrams.



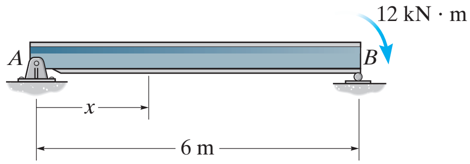
Prob. F7–8

F7–9. Determine the shear and moment as a function of x, and then draw the shear and moment diagrams.



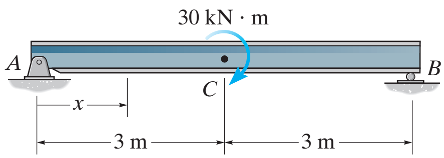
Prob. F7–9

F7–10. Determine the shear and moment as a function of x, and then draw the shear and moment diagrams.



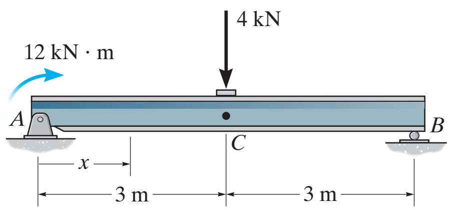
Prob. F7–10

F7–11. Determine the shear and moment as a function of x, where $0 \le x < 3$ m and $3 \text{ m} < x \le 6$ m, and then draw the shear and moment diagrams.



Prob. F7–11

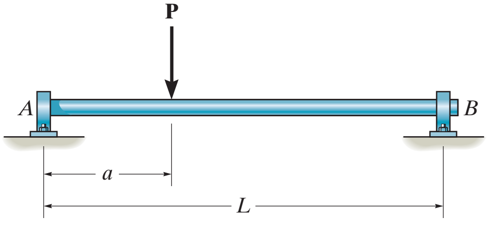
F7–12. Determine the shear and moment as a function of x, where $0 \le x < 3$ m and $3 \text{ m} < x \le 6$ m, and then draw the shear and moment diagrams.



Prob. F7–12

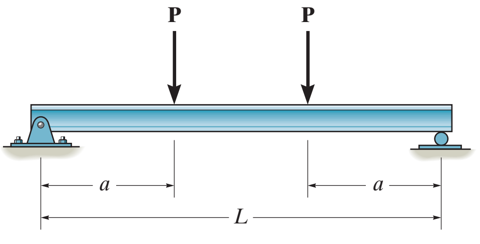
PROBLEMS

7–45. Draw the shear and moment diagrams for the shaft (a) in terms of the parameters shown; (b) set $P = 9$ kN, $a = 2$ m, $L = 6$ m. There is a thrust bearing at A and a journal bearing at B.



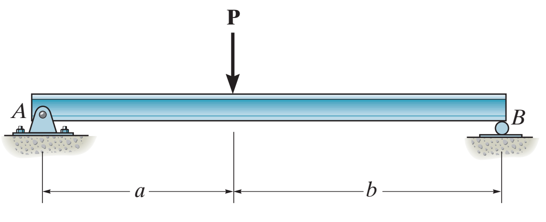
Prob. 7–45

7–46. Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 800$ lb, $a = 5$ ft, $L = 12$ ft.



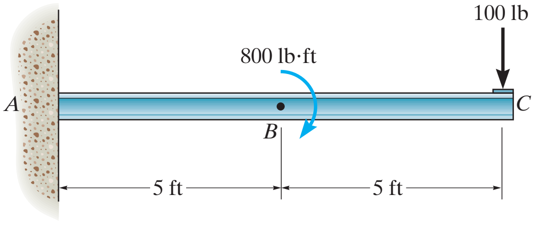
Prob. 7–46

7–47. Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 600$ lb, $a = 5$ ft, $b = 7$ ft.



Prob. 7–47

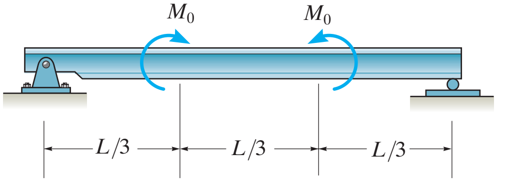
***7–48.** Draw the shear and moment diagrams for the cantilevered beam.



Prob. 7–48

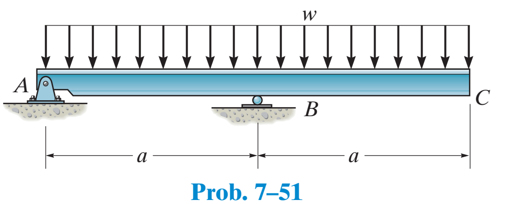
7–49. Draw the shear and moment diagrams of the beam (a) in terms of the parameters shown; (b) set $M_0 = 500$ N · m, $L = 8$ m.

7–50. If $L = 9$ m, the beam will fail when the maximum shear force is $V_{max} = 5$ kN or the maximum bending moment is $M_{max} = 2$ kN · m. Determine the magnitude M_0 of the largest couple moments it will support.



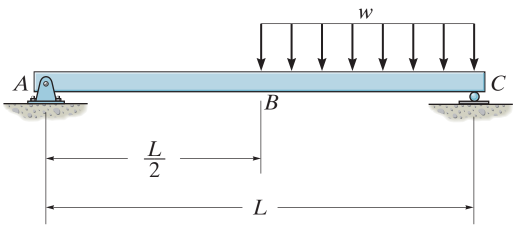
Probs. 7–49/50

7–51. Draw the shear and moment diagrams for the beam.



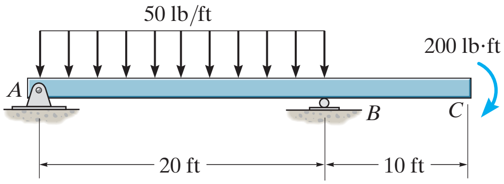
Prob. 7–51

***7–52.** Draw the shear and moment diagrams for the beam.



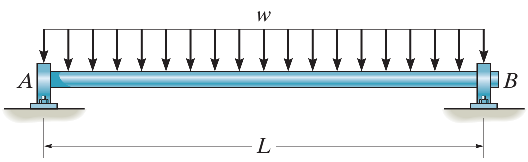
Prob. 7–52

7–53. Draw the shear and bending-moment diagrams for the beam.



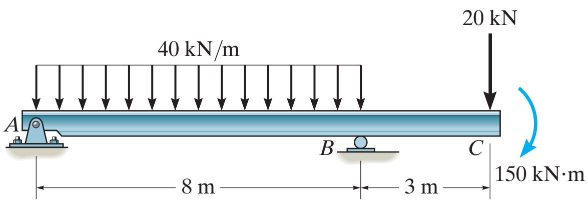
Prob. 7–53

7–54. The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft (a) in terms of the parameters shown; (b) set $w = 500$ lb/ft, $L = 10$ ft.



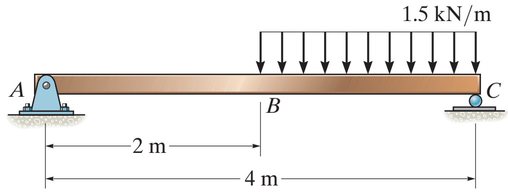
Prob. 7–54

7–55. Draw the shear and moment diagrams for the beam.



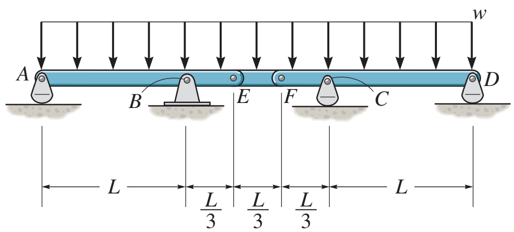
Prob. 7–55

***7–56.** Draw the shear and moment diagrams for the beam.



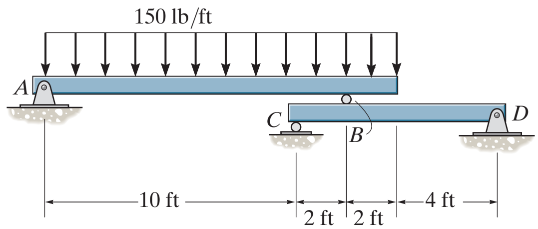
Prob. 7–56

7–57. Draw the shear and moment diagrams for the compound beam. The beam is pin connected at E and F.



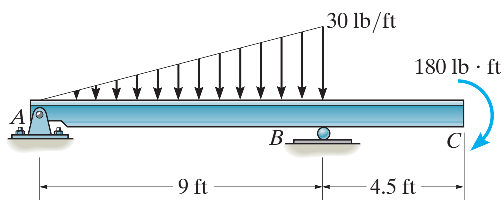
Prob. 7–57

7–58. Draw the shear and bending-moment diagrams for each of the two segments of the compound beam.



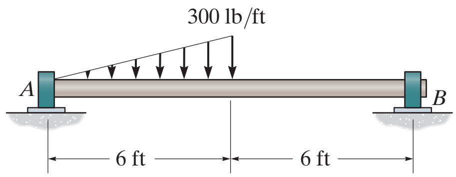
Prob. 7–58

7–59. Draw the shear and moment diagrams for the beam.



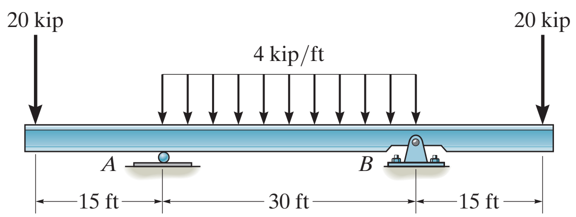
Prob. 7–59

***7–60.** The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.



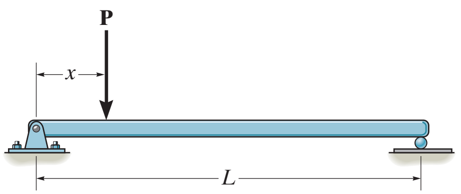
Prob. 7–60

7–61. Draw the shear and moment diagrams for the beam.



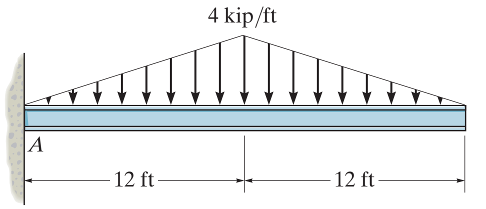
Prob. 7–61

7–62. The beam will fail when the maximum internal moment is M_{max}. Determine the position x of the concentrated force **P** and its smallest magnitude that will cause failure.



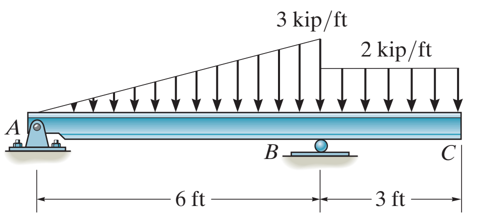
Prob. 7–62

7–63. Draw the shear and moment diagrams for the beam.



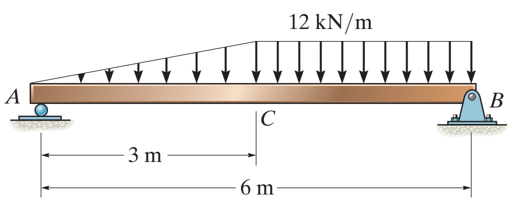
Prob. 7–63

***7–64.** Draw the shear and moment diagrams for the beam.



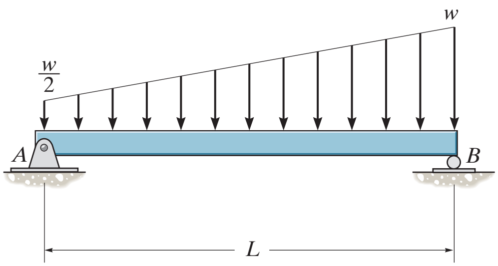
Prob. 7–64

7–65. Draw the shear and moment diagrams for the beam.



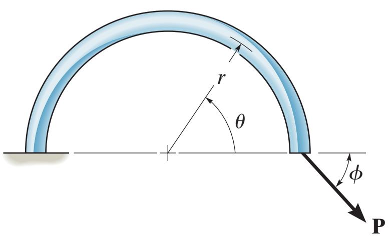
Prob. 7–65

7–66. Draw the shear and moment diagrams for the beam.



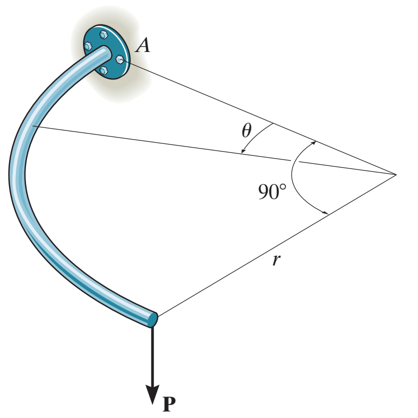
Prob. 7–66

7–67. Determine the internal normal force, shear force, and moment in the curved rod as a function of θ. The force **P** acts at the constant angle ϕ.



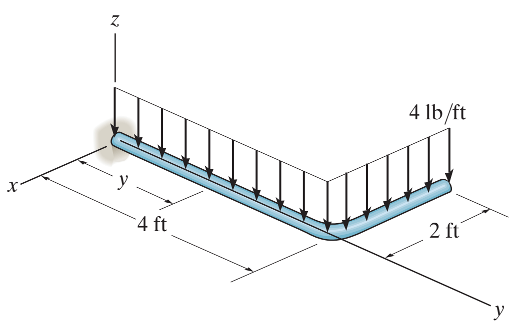
Prob. 7–67

***7–68.** The quarter circular rod lies in the horizontal plane and supports a vertical force **P** at its end. Determine the magnitudes of the components of the internal shear force, moment, and torque acting in the rod as a function of the angle θ.



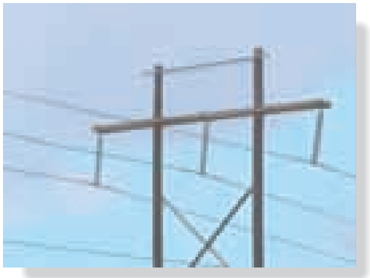
Prob. 7–68

7–69. Express the internal shear and moment components acting in the rod as a function of y, where $0 \le y \le 4$ ft.



Prob. 7–69

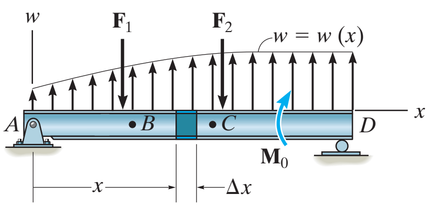
*7.3 Relations between Distributed Load, Shear, and Moment



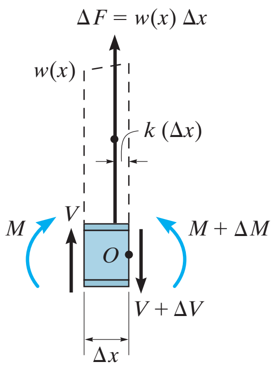
In order to design the beam used to support these power lines, it is important to first draw the shear and moment diagrams for the beam. (© Russell C. Hibbeler)

If a beam is subjected to several concentrated forces, couple moments, and distributed loads, the method of constructing the shear and bending-moment diagrams discussed in Sec. 7.2 may become quite tedious. In this section a simpler method for constructing these diagrams is discussed—a method based on differential relations that exist between the load, shear, and bending moment.

Distributed Load. Consider the beam AD shown in Fig. 7–13a, which is subjected to an arbitrary load $w = w(x)$ and a series of concentrated forces and couple moments. In the following discussion, the *distributed load* will be considered *positive* when the *loading acts upward* as shown. A free-body diagram for a small segment of the beam having a length Δx is chosen at a point x along the beam which is *not* subjected to a concentrated force or couple moment, Fig. 7–13b. Hence any results obtained will not apply at these points of concentrated loading. The internal shear force and bending moment shown on the free-body diagram are assumed to act in the *positive sense* according to the established sign convention. Note that both the shear force and moment acting on the right-hand face must be increased by a small, finite amount in order to keep the segment in equilibrium. The distributed loading has been replaced by a resultant force $\Delta F = w(x)\,\Delta x$ that acts at a fractional distance $k(\Delta x)$ from the right end, where $0 < k < 1$ [for example, if $w(x)$ is *uniform*, $k = \frac{1}{2}$].



(a)



(b)

Fig. 7–13

Relation between the Distributed Load and Shear. If we apply the force equation of equilibrium to the segment, then

$$+\uparrow \Sigma F_y = 0; \qquad V + w(x)\Delta x - (V + \Delta V) = 0$$
$$\Delta V = w(x)\Delta x$$

Dividing by Δx, and letting $\Delta x \rightarrow 0$, we get

$$\boxed{\begin{array}{ccc} & \dfrac{dV}{dx} = w(x) & \\ \text{Slope of shear diagram} & = & \text{Distributed load intensity} \end{array}} \qquad (7\text{–}1)$$

If we rewrite the above equation in the form $dV = w(x)dx$ and perform an integration between any two points B and C on the beam, we see that

$$\Delta V = \int w(x)\, dx \qquad (7\text{–}2)$$

Change in shear = Area under loading curve

Relation between the Shear and Moment.

If we apply the moment equation of equilibrium about point O on the free-body diagram in Fig. 7–13b, we get

$$\circlearrowleft +\Sigma M_O = 0; \quad (M + \Delta M) - [w(x)\Delta x]\, k\Delta x - V\Delta x - M = 0$$
$$\Delta M = V\Delta x + k\, w(x)\Delta x^2$$

Dividing both sides of this equation by Δx, and letting $\Delta x \to 0$, yields

$$\frac{dM}{dx} = V \qquad (7\text{–}3)$$

Slope of moment diagram = Shear

In particular, notice that a maximum bending moment $|M|_{\max}$ will occur at the point where the slope $dM/dx = 0$, since this is where the shear is equal to zero.

If Eq. 7–3 is rewritten in the form $dM = \int V\, dx$ and integrated between any two points B and C on the beam, we have

$$\Delta M = \int V\, dx \qquad (7\text{–}4)$$

Change in moment = Area under shear diagram

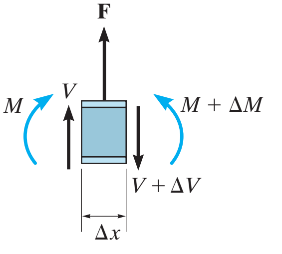
As stated previously, the above equations do not apply at points where a *concentrated* force or couple moment acts. These two special cases create *discontinuities* in the shear and moment diagrams, and as a result, each deserves separate treatment.

Force.

A free-body diagram of a small segment of the beam in Fig. 7–13a, taken from under one of the forces, is shown in Fig. 7–14a. Here force equilibrium requires

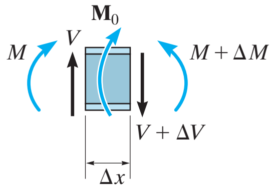
$$+\uparrow \Sigma F_y = 0; \qquad \Delta V = F \qquad (7\text{–}5)$$

Since the *change in shear is positive*, the shear diagram will "jump" *upward when* **F** *acts upward* on the beam. Likewise, the jump in shear (ΔV) is downward when **F** acts downward.



(a)

Fig. 7–14



(b)

Fig. 7–14 (cont.)

Couple Moment. If we remove a segment of the beam in Fig. 7–13*a* that is located at the couple moment $\mathbf{M}_0$, the free-body diagram shown in Fig. 7–14*b* results. In this case letting $\Delta x \rightarrow 0$, moment equilibrium requires

$$\circlearrowright +\Sigma M = 0; \qquad \Delta M = M_0 \qquad (7\text{–}6)$$

Thus, the *change in moment is positive*, or the moment diagram will "jump" *upward if* $\mathbf{M}_0$ *is clockwise*. Likewise, the jump ΔM is downward when $\mathbf{M}_0$ is counterclockwise.

The examples which follow illustrate application of the above equations when used to construct the shear and moment diagrams. After working through these examples, it is recommended that you also go back and solve Examples 7.6 and 7.7 using this method.

This concrete beam is used to support the deck. Its size and the placement of steel reinforcement within it can be determined once the shear and moment diagrams have been established. (© Russell C. Hibbeler)

Important Points

- The slope of the shear diagram at a point is equal to the intensity of the distributed loading, where positive distributed loading is upward, i.e., $dV/dx = w(x)$.
- The change in the shear ΔV between two points is equal to *the area* under the distributed-loading curve between the points.
- If a concentrated force acts upward on the beam, the shear will jump upward by the same amount.
- The slope of the moment diagram at a point is equal to the shear, i.e., $dM/dx = V$.
- The change in the moment ΔM between two points is equal to the *area* under the shear diagram between the two points.
- If a *clock*wise couple moment acts on the beam, the shear will not be affected; however, the moment diagram will jump *upward* by the amount of the moment.
- Points of *zero shear* represent points of *maximum or minimum moment* since $dM/dx = 0$.
- Because two integrations of $w = w(x)$ are involved to first determine the change in shear, $\Delta V = \int w(x)\,dx$, then to determine the change in moment, $\Delta M = \int V\,dx$, then if the loading curve $w = w(x)$ is a polynomial of degree n, $V = V(x)$ will be a curve of degree $n + 1$, and $M = M(x)$ will be a curve of degree $n + 2$.

EXAMPLE 7.8

Draw the shear and moment diagrams for the cantilever beam in Fig. 7–15*a*.

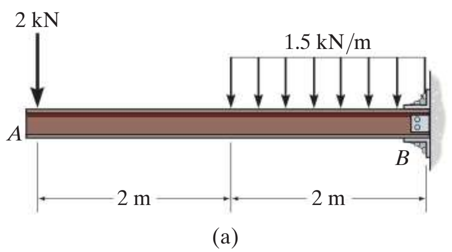
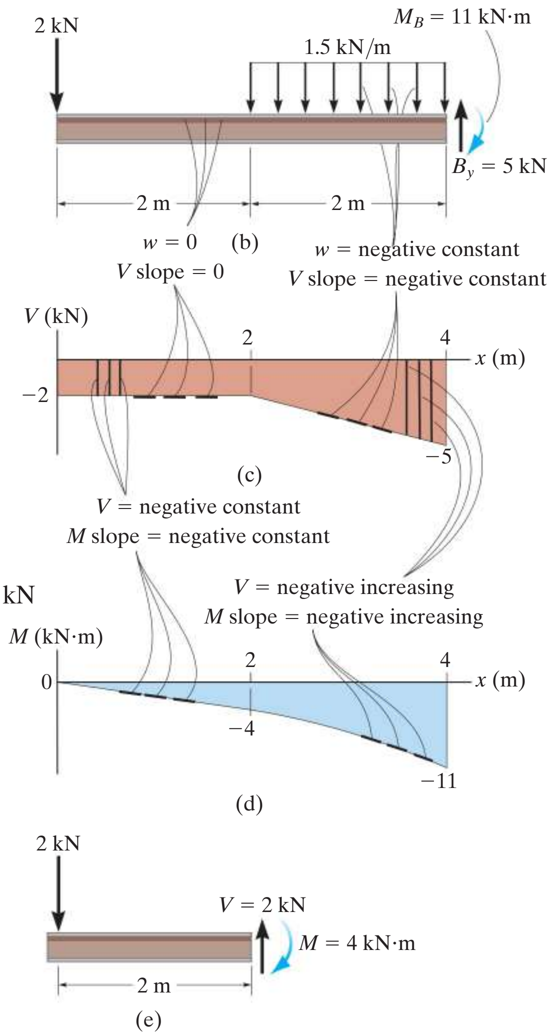


Fig. 7–15



SOLUTION

The support reactions at the fixed support B are shown in Fig. 7–15*b*.

Shear Diagram. The shear at end A is -2 kN. This value is plotted at $x = 0$, Fig. 7–15*c*. Notice how the shear diagram is constructed by following the slopes defined by the loading w. The shear at $x = 4$ m is -5 kN, the reaction on the beam. This value can be verified by finding the area under the distributed loading; i.e.,

$$V\big|_{x=4\text{ m}} = V\big|_{x=2\text{ m}} + \Delta V = -2\text{ kN} - (1.5\text{ kN/m})(2\text{ m}) = -5\text{ kN}$$

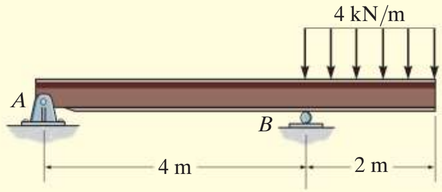
Moment Diagram. The moment of zero at $x = 0$ is plotted in Fig. 7–15*d*. Construction of the moment diagram is based on knowing that its slope is equal to the shear at each point. The change of moment from $x = 0$ to $x = 2$ m is determined from the area under the shear diagram. Hence, the moment at $x = 2$ m is

$$M\big|_{x=2\text{ m}} = M\big|_{x=0} + \Delta M = 0 + [-2\text{ kN}(2\text{ m})] = -4\text{ kN}\cdot\text{m}$$

This same value can be determined from the method of sections, Fig. 7–15*e*.

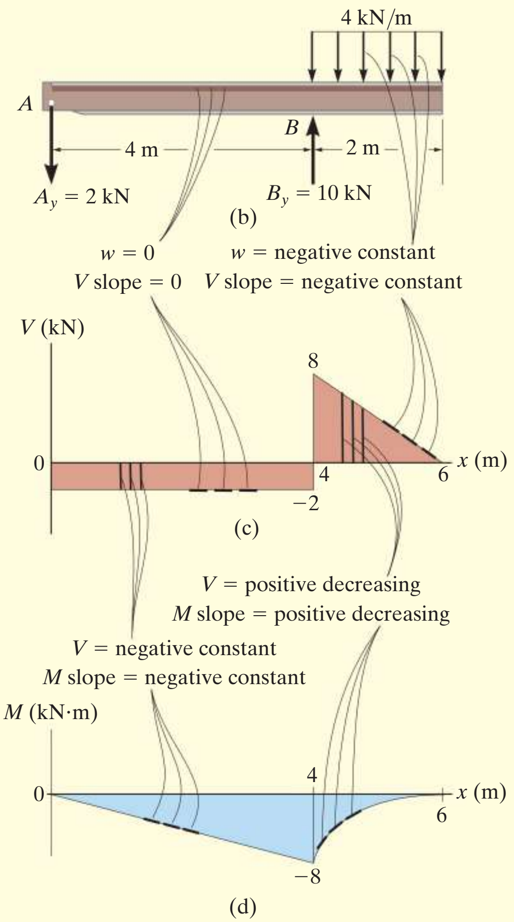
EXAMPLE 7.9

Draw the shear and moment diagrams for the overhang beam in Fig. 7–16a.



(a)

Fig. 7–16



SOLUTION

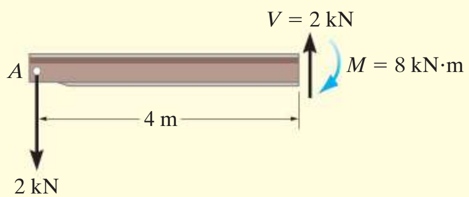
The support reactions are shown in Fig. 7–16b.

Shear Diagram. The shear of -2 kN at end A of the beam is plotted at $x = 0$, Fig. 7–16c. The slopes are determined from the loading and from this the shear diagram is constructed, as indicated in the figure. In particular, notice the positive jump of 10 kN at $x = 4$ m due to the force B_y, as indicated in the figure.

Moment Diagram. The moment of zero at $x = 0$ is plotted, Fig. 7–16d, then following the behavior of the slope found from the shear diagram, the moment diagram is constructed. The moment at $x = 4$ m is found from the area under the shear diagram.

$$M\big|_{x=4\text{ m}} = M\big|_{x=0} + \Delta M = 0 + [-2\text{ kN}(4\text{ m})] = -8\text{ kN}\cdot\text{m}$$

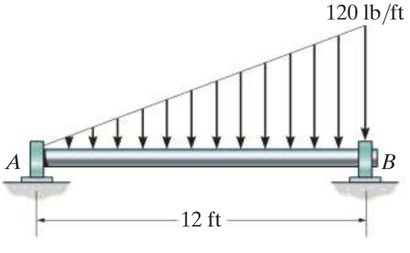
We can also obtain this value by using the method of sections, as shown in Fig. 7–16e.



(e)

EXAMPLE 7.10

The shaft in Fig. 7–17*a* is supported by a thrust bearing at *A* and a journal bearing at *B*. Draw the shear and moment diagrams.



(a)

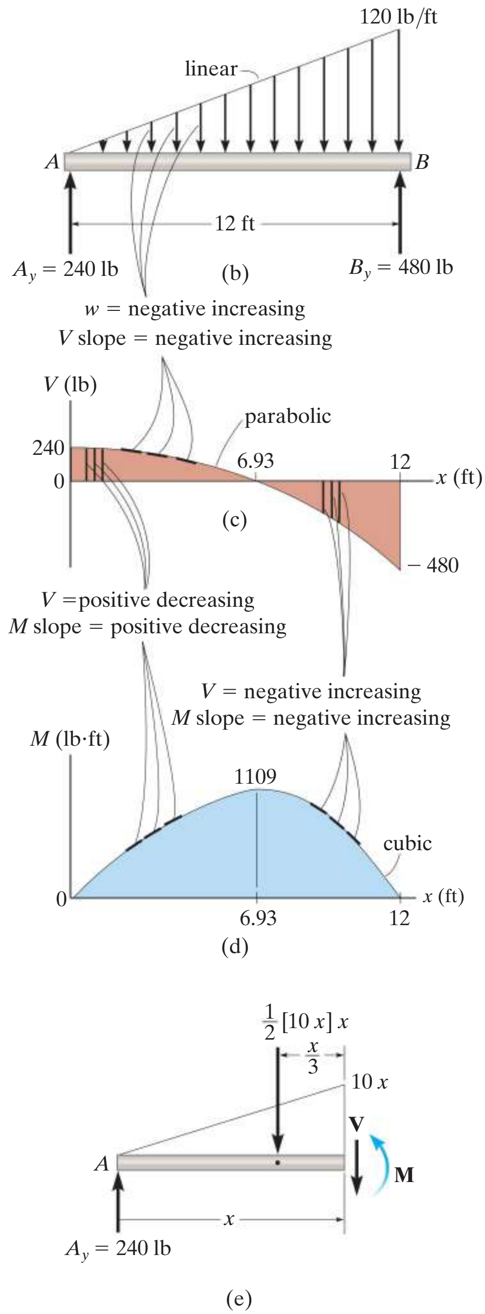


Fig. 7–17

SOLUTION

The support reactions are shown in Fig. 7–17*b*.

Shear Diagram. As shown in Fig. 7–17*c*, the shear at $x = 0$ is +240. Following the slope defined by the loading, the shear diagram is constructed, where at *B* its value is −480 lb. Since the shear changes sign, the point where $V = 0$ must be located. To do this we will use the method of sections. The free-body diagram of the left segment of the shaft, sectioned at an arbitrary position *x* within the region $0 \le x < 12$ ft, is shown in Fig. 7–17*e*. Notice that the intensity of the distributed load at *x* is $w = 10x$, which has been found by proportional triangles, i.e., $120/12 = w/x$.

Thus, for $V = 0$,

$$+\uparrow \Sigma F_y = 0; \qquad 240 \text{ lb} - \tfrac{1}{2}(10x)x = 0$$

$$x = 6.93 \text{ ft}$$

Moment Diagram. The moment diagram starts at 0 since there is no moment at *A*, then it is constructed based on the slope as determined from the shear diagram. The maximum moment occurs at $x = 6.93$ ft, where the shear is equal to zero, since $dM/dx = V = 0$, Fig. 7–17*e*,

$$\circlearrowleft + \Sigma M = 0;$$

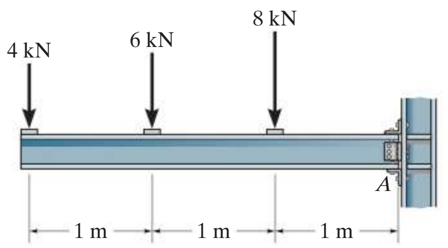
$$M_{\max} + \tfrac{1}{2}[(10)(6.93)]\,6.93\left(\tfrac{1}{3}(6.93)\right) - 240(6.93) = 0$$

$$M_{\max} = 1109 \text{ lb}\cdot\text{ft}$$

Finally, notice how integration, first of the loading *w* which is linear, produces a shear diagram which is parabolic, and then a moment diagram which is cubic.

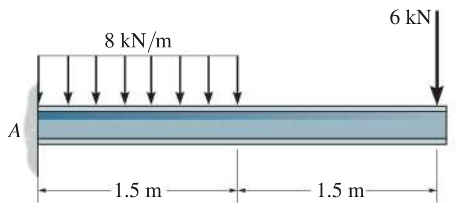
FUNDAMENTAL PROBLEMS

F7–13. Draw the shear and moment diagrams for the beam.



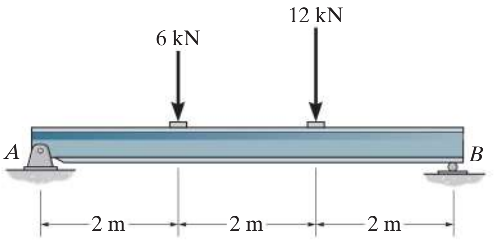
Prob. F7–13

F7–14. Draw the shear and moment diagrams for the beam.



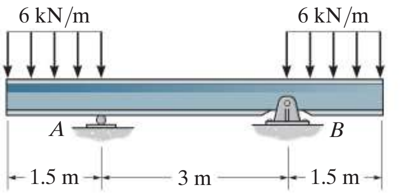
Prob. F7–14

F7–15. Draw the shear and moment diagrams for the beam.



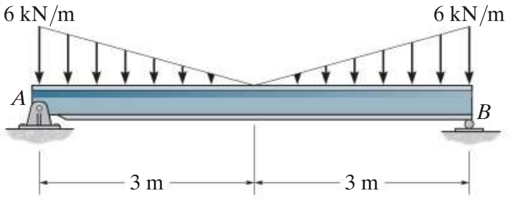
Prob. F7–15

F7–16. Draw the shear and moment diagrams for the beam.



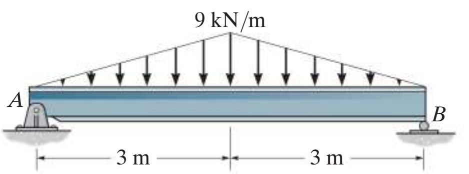
Prob. F7–16

F7–17. Draw the shear and moment diagrams for the beam.



Prob. F7–17

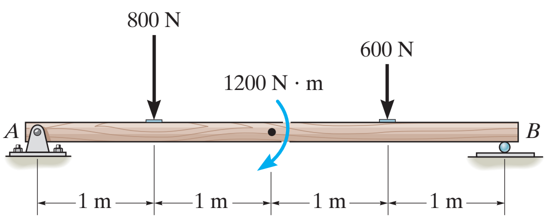
F7–18. Draw the shear and moment diagrams for the beam.



Prob. F7–18

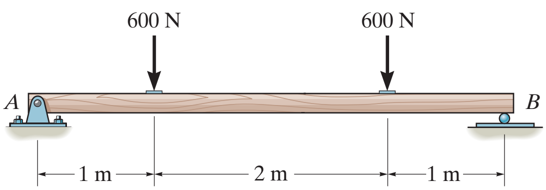
PROBLEMS

7–70. Draw the shear and moment diagrams for the beam.



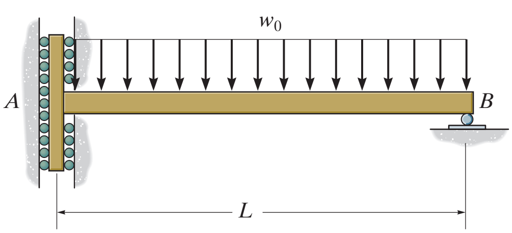
Prob. 7–70

7–71. Draw the shear and moment diagrams for the beam.



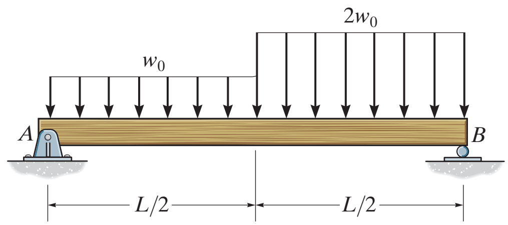
Prob. 7–71

***7–72.** Draw the shear and moment diagrams for the beam. The support at A offers no resistance to vertical load.



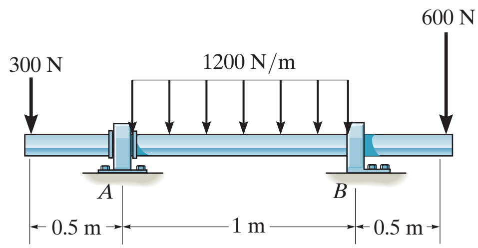
Prob. 7–72

7–73. Draw the shear and moment diagrams for the simply-supported beam.



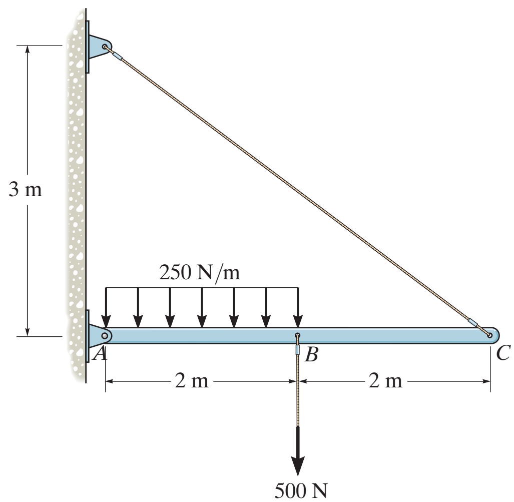
Prob. 7–73

7–74. Draw the shear and moment diagrams for the beam. The supports at A and B are a thrust bearing and journal bearing, respectively.



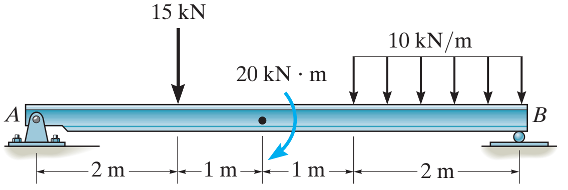
Prob. 7–74

7–75. Draw the shear and moment diagrams for the beam.



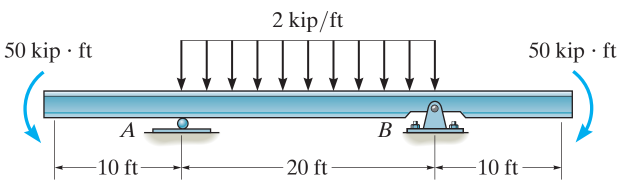
Prob. 7–75

***7–76.** Draw the shear and moment diagrams for the beam.



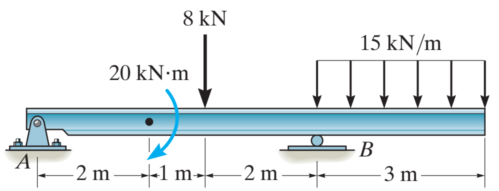
Prob. 7–76

7–77. Draw the shear and moment diagrams for the beam.



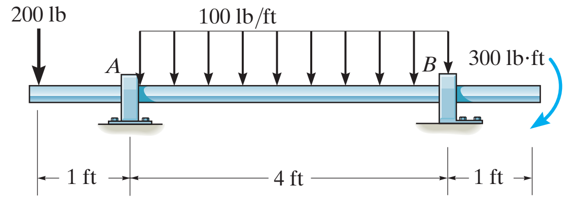
Prob. 7–77

7–78. Draw the shear and moment diagrams for the beam.



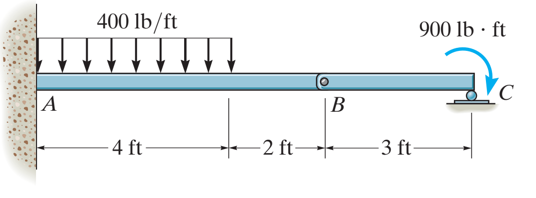
Prob. 7–78

7–79. Draw the shear and moment diagrams for the shaft. The support at A is a journal bearing and at B it is a thrust bearing.



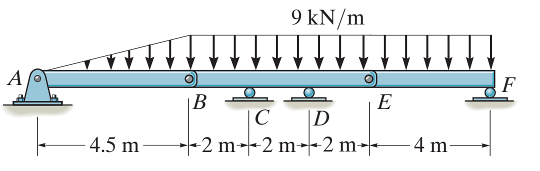
Prob. 7–79

***7–80.** Draw the shear and moment diagrams for the beam.



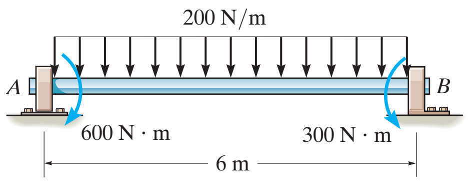
Prob. 7–80

7–81. The beam consists of three segments pin connected at B and E. Draw the shear and moment diagrams for the beam.



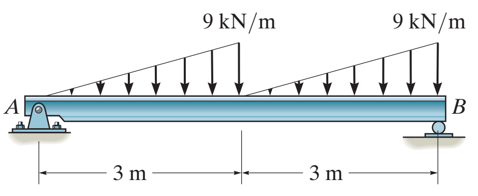
Prob. 7–81

7–82. Draw the shear and moment diagrams for the beam. The supports at *A* and *B* are a thrust and journal bearing, respectively.



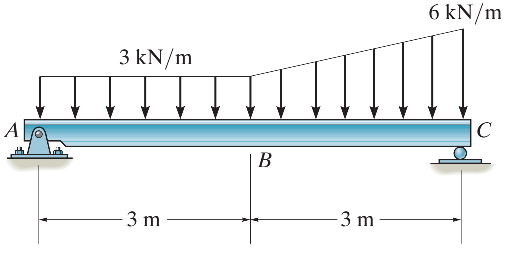
Prob. 7–82

7–83. Draw the shear and moment diagrams for the beam.



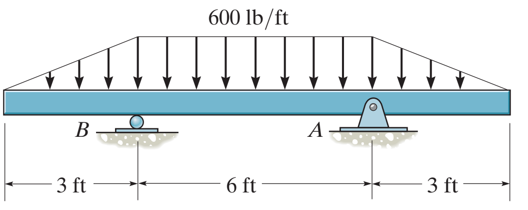
Prob. 7–83

***7–84.** Draw the shear and moment diagrams for the beam.



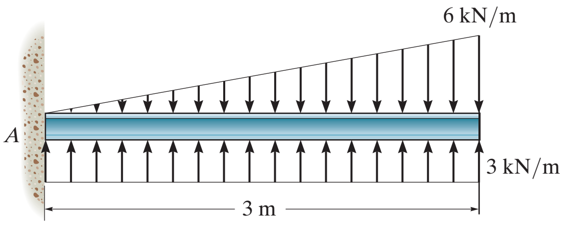
Prob. 7–84

7–85. Draw the shear and moment diagrams for the beam.



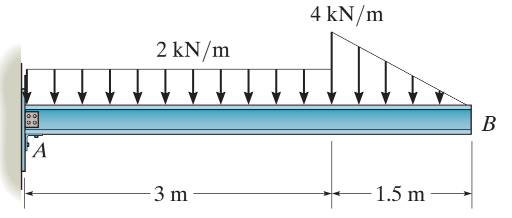
Prob. 7–85

7–86. Draw the shear and moment diagrams for the beam.



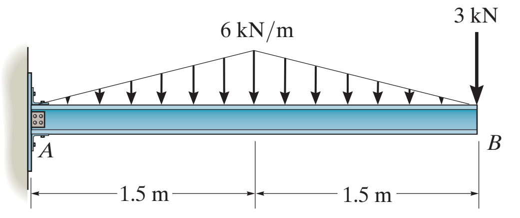
Prob. 7–86

7–87. Draw the shear and moment diagrams for the beam.



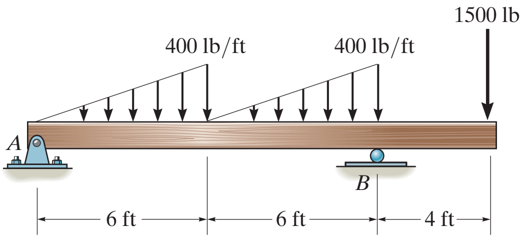
Prob. 7–87

***7–88.** Draw the shear and moment diagrams for the beam.



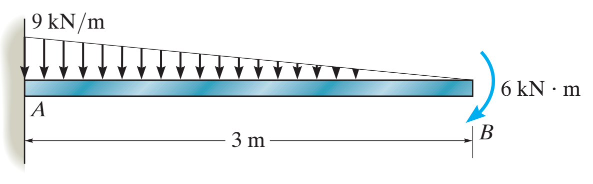
Prob. 7–88

7–89. Draw the shear and moment diagrams for the beam.



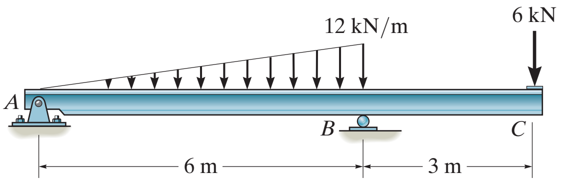
Prob. 7–89

7–90. Draw the shear and moment diagrams for the beam.



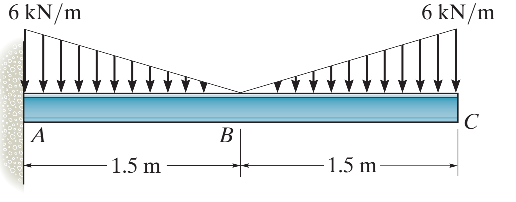
Prob. 7–90

7–91. Draw the shear and moment diagrams for the beam.



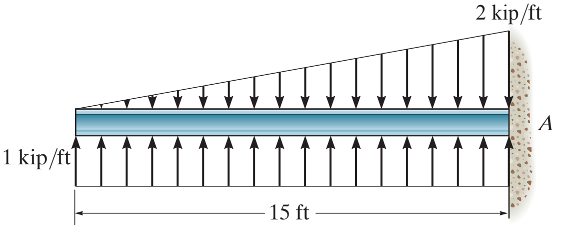
Prob. 7–91

***7–92.** Draw the shear and moment diagrams for the beam.



Prob. 7–92

7–93. Draw the shear and moment diagrams for the beam.

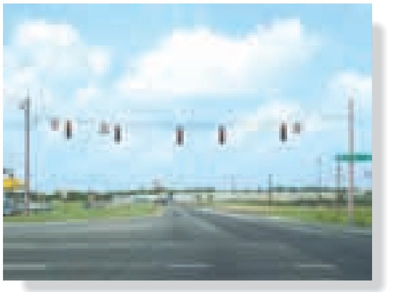


Prob. 7–93

*7.4 Cables

Flexible cables and chains combine strength with lightness and often are used in structures for support and to transmit loads from one member to another. When used to support suspension bridges and trolley wheels, cables form the main load-carrying element of the structure. In the force analysis of such systems, the weight of the cable itself may be neglected because it is often small compared to the load it carries. On the other hand, when cables are used as transmission lines and guys for radio antennas and derricks, the cable weight may become important and must be included in the structural analysis.

Three cases will be considered in the analysis that follows. In each case we will make the assumption that the cable is *perfectly flexible* and *inextensible*. Due to its flexibility, the cable offers no resistance to bending, and therefore, the tensile force acting in the cable is always tangent to the cable at points along its length. Being inextensible, the cable has a constant length both before and after the load is applied. As a result, once the load is applied, the geometry of the cable remains unchanged, and the cable or a segment of it can be treated as a rigid body.



Each of the cable segments remains approximately straight as they support the weight of these traffic lights. (© Russell C. Hibbeler)

Cable Subjected to Concentrated Loads. When a cable of negligible weight supports several concentrated loads, the cable takes the form of several straight-line segments, each of which is subjected to a constant tensile force. Consider, for example, the cable shown in Fig. 7–18, where the distances h, L_1, L_2, and L_3 and the loads $\mathbf{P}_1$ and $\mathbf{P}_2$ are known. The problem here is to determine the *nine unknowns* consisting of the tension in each of the *three* segments, the *four* components of reaction at A and B, and the *two* sags y_C and y_D at points C and D. For the solution we can write *two* equations of force equilibrium at each of points A, B, C, and D. This results in a total of *eight equations*.* To complete the solution, we need to know something about the geometry of the cable in order to obtain the necessary ninth equation. For example, if the cable's total *length* L is specified, then the Pythagorean theorem can be used to relate each of the three segmental lengths, written in terms of h, y_C, y_D, L_1, L_2, and L_3, to the total length L. Unfortunately, this type of problem cannot be solved easily by hand. Another possibility, however, is to specify one of the sags, either y_C or y_D, instead of the cable length. By doing this, the equilibrium equations are then sufficient for obtaining the unknown forces and the remaining sag. Once the sag at each point of loading is obtained, the length of the cable can then be determined by trigonometry. The following example illustrates a procedure for performing the equilibrium analysis for a problem of this type.

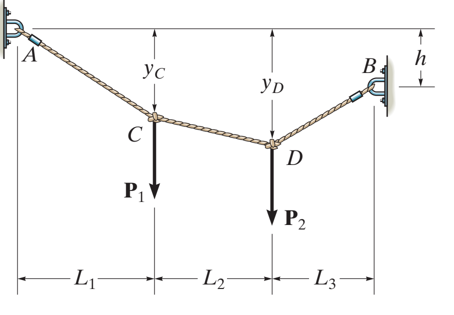
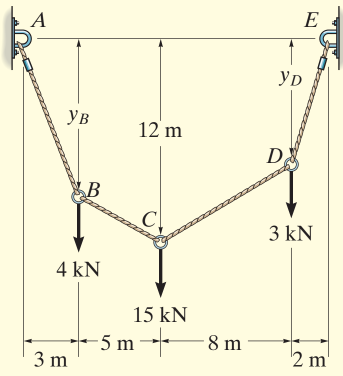


Fig. 7–18

*As will be shown in the following example, the eight equilibrium equations *also* can be written for the entire cable, or any part thereof. But *no more* than *eight* independent equations are available.

EXAMPLE 7.11

Determine the tension in each segment of the cable shown in Fig. 7–19*a*.



(a)

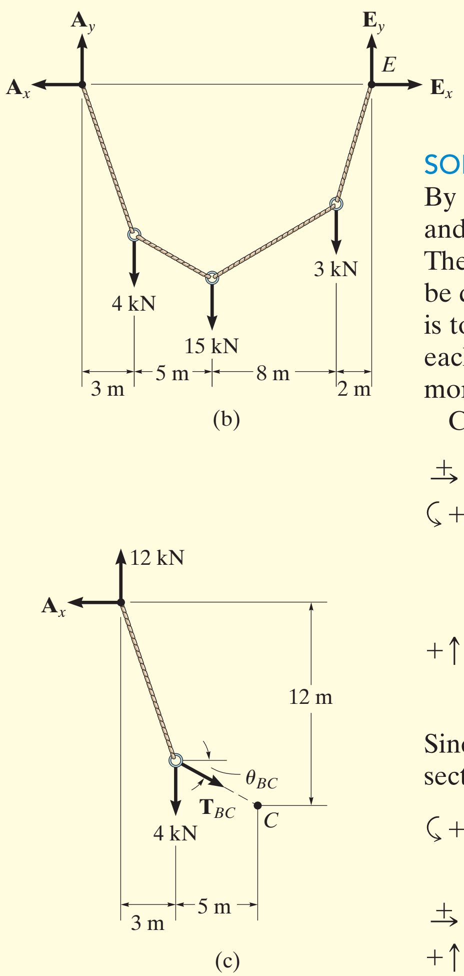


Fig. 7–19

SOLUTION

By inspection, there are four unknown external reactions (A_x, A_y, E_x, and E_y) and four unknown cable tensions, one in each cable segment. These eight unknowns along with the two unknown sags y_B and y_D can be determined from *ten* available equilibrium equations. One method is to apply the force equations of equilibrium ($\Sigma F_x = 0$, $\Sigma F_y = 0$) to each of the five points A through E. Here, however, we will take a more direct approach.

Consider the free-body diagram for the entire cable, Fig. 7–19*b*. Thus,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -A_x + E_x = 0$$

$$\circlearrowleft + \Sigma M_E = 0;$$

$$-A_y(18\text{ m}) + 4\text{ kN}\,(15\text{ m}) + 15\text{ kN}\,(10\text{ m}) + 3\text{ kN}\,(2\text{ m}) = 0$$

$$A_y = 12\text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad 12\text{ kN} - 4\text{ kN} - 15\text{ kN} - 3\text{ kN} + E_y = 0$$

$$E_y = 10\text{ kN}$$

Since the sag $y_C = 12$ m is known, we will now consider the leftmost section, which cuts cable BC, Fig. 7–19*c*.

$$\circlearrowleft + \Sigma M_C = 0; \; A_x(12\text{ m}) - 12\text{ kN}\,(8\text{ m}) + 4\text{ kN}\,(5\text{ m}) = 0$$

$$A_x = E_x = 6.33\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{BC}\cos\theta_{BC} - 6.33\text{ kN} = 0$$

$$+\uparrow \Sigma F_y = 0; \quad 12\text{ kN} - 4\text{ kN} - T_{BC}\sin\theta_{BC} = 0$$

Thus,

$$\theta_{BC} = 51.6^\circ$$

$$T_{BC} = 10.2\text{ kN} \qquad \textit{Ans.}$$

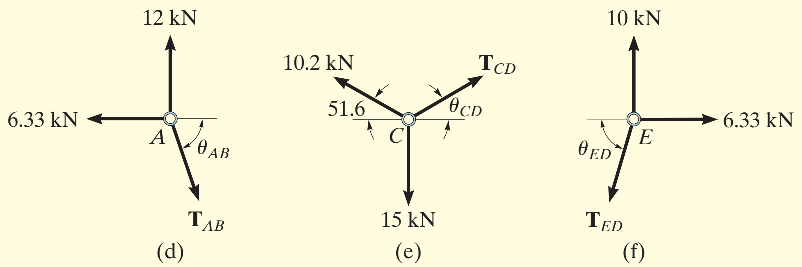


Fig. 7–19 (cont.)

Proceeding now to analyze the equilibrium of points A, C, and E in sequence, we have

Point A. (Fig. 7–19*d*).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{AB} \cos \theta_{AB} - 6.33 \text{ kN} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -T_{AB} \sin \theta_{AB} + 12 \text{ kN} = 0$$

$$\theta_{AB} = 62.2^\circ$$

$$T_{AB} = 13.6 \text{ kN} \qquad \textit{Ans.}$$

Point C. (Fig. 7–19*e*).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{CD} \cos \theta_{CD} - 10.2 \cos 51.6^\circ \text{ kN} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad T_{CD} \sin \theta_{CD} + 10.2 \sin 51.6^\circ \text{ kN} - 15 \text{ kN} = 0$$

$$\theta_{CD} = 47.9^\circ$$

$$T_{CD} = 9.44 \text{ kN} \qquad \textit{Ans.}$$

Point E. (Fig. 7–19*f*).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 6.33 \text{ kN} - T_{ED} \cos \theta_{ED} = 0$$

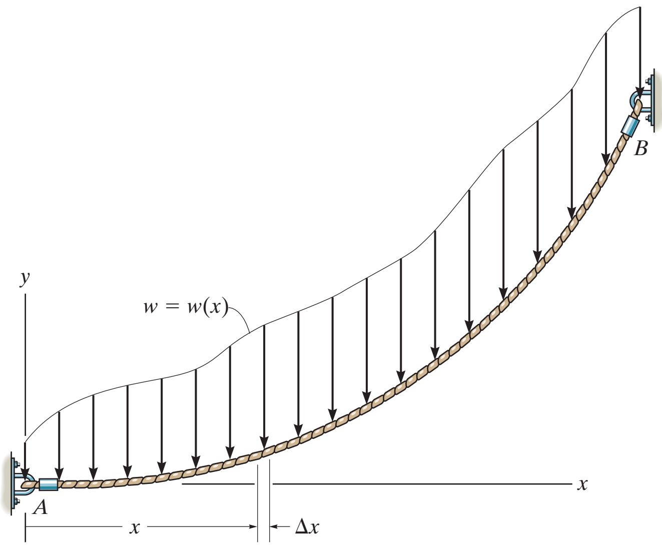
$$+\uparrow \Sigma F_y = 0; \qquad 10 \text{ kN} - T_{ED} \sin \theta_{ED} = 0$$

$$\theta_{ED} = 57.7^\circ$$

$$T_{ED} = 11.8 \text{ kN} \qquad \textit{Ans.}$$

NOTE: By comparison, the maximum cable tension is in segment AB since this segment has the greatest slope (θ) and it is required that for any cable segment the horizontal component $T \cos \theta = A_x = E_x$ (a constant). Also, since the slope angles that the cable segments make with the horizontal have now been determined, it is possible to determine the sags y_B and y_D, Fig. 7–19*a*, using trigonometry.

The cable and suspenders are used to support the uniform load of a gas pipe which crosses the river. (© Russell C. Hibbeler)



(a)

Fig. 7–20

Cable Subjected to a Distributed Load. Let us now consider the weightless cable shown in Fig. 7–20*a*, which is subjected to a distributed loading $w = w(x)$ that is *measured in the x direction*. The free-body diagram of a small segment of the cable having a length Δs is shown in Fig. 7–20*b*. Since the tensile force changes in both magnitude and direction along the cable's length, we will denote this change on the free-body diagram by ΔT. Finally, the distributed load is represented by its resultant force $w(x)(\Delta x)$, which acts at a fractional distance $k(\Delta x)$ from point O, where $0 < k < 1$. Applying the equations of equilibrium, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -T\cos\theta + (T + \Delta T)\cos(\theta + \Delta\theta) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -T\sin\theta - w(x)(\Delta x) + (T + \Delta T)\sin(\theta + \Delta\theta) = 0$$

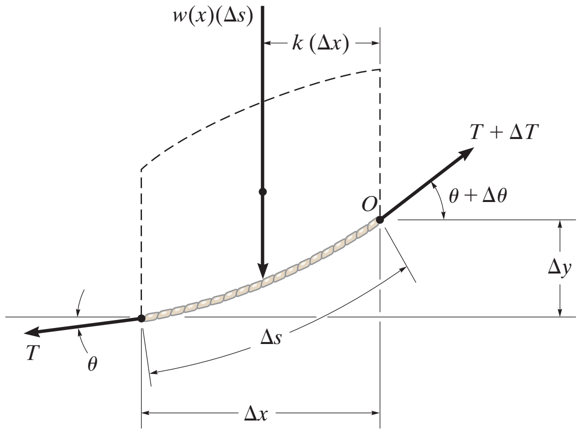
$$\circlearrowleft + \Sigma M_O = 0; \qquad w(x)(\Delta x)k(\Delta x) - T\cos\theta\,\Delta y + T\sin\theta\,\Delta x = 0$$

Dividing each of these equations by Δx and taking the limit as $\Delta x \rightarrow 0$, and therefore $\Delta y \rightarrow 0$, $\Delta\theta \rightarrow 0$, and $\Delta T \rightarrow 0$, we obtain

$$\frac{d(T\cos\theta)}{dx} = 0 \qquad (7\text{–}7)$$

$$\frac{d(T\sin\theta)}{dx} - w(x) = 0 \qquad (7\text{–}8)$$

$$\frac{dy}{dx} = \tan\theta \qquad (7\text{–}9)$$



(b)

Fig. 7–20 (cont.)

Integrating Eq. 7–7, we have

$$T\cos\theta = \text{constant} = F_H \tag{7–10}$$

where F_H represents the horizontal component of tensile force at *any point* along the cable.

Integrating Eq. 7–8 gives

$$T\sin\theta = \int w(x)\,dx \tag{7–11}$$

Dividing Eq. 7–11 by Eq. 7–10 eliminates T. Then, using Eq. 7–9, we can obtain the slope of the cable.

$$\tan\theta = \frac{dy}{dx} = \frac{1}{F_H}\int w(x)\,dx$$

Performing a second integration yields

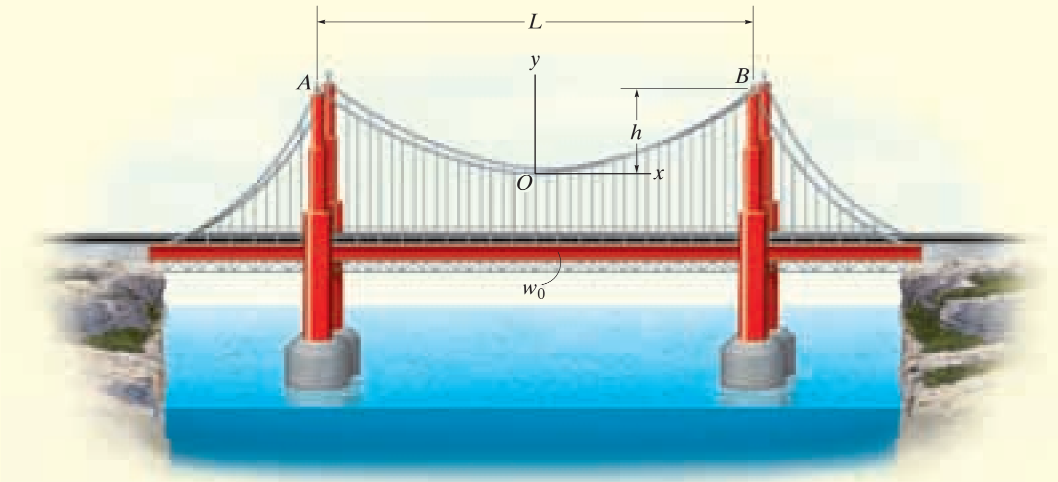
$$\boxed{y = \frac{1}{F_H}\int\left(\int w(x)\,dx\right)dx} \tag{7–12}$$

This equation is used to determine the curve for the cable, $y = f(x)$. The horizontal force component F_H and the additional two constants, say C_1 and C_2, resulting from the integration are determined by applying the boundary conditions for the curve.

The cables of the suspension bridge exert very large forces on the tower and the foundation block which have to be accounted for in their design. (© Russell C. Hibbeler)

EXAMPLE 7.12

The cable of a suspension bridge supports half of the uniform road surface between the two towers at A and B, Fig. 7–21a. If this distributed loading is w_0, determine the maximum force developed in the cable and the cable's required length. The span length L and sag h are known.



(a)

Fig. 7–21

SOLUTION

We can determine the unknowns in the problem by first finding the equation of the curve that defines the shape of the cable using Eq. 7–12. For reasons of symmetry, the origin of coordinates has been placed at the cable's center. Noting that $w(x) = w_0$, we have

$$y = \frac{1}{F_H}\int\left(\int w_0\,dx\right)dx$$

Performing the two integrations gives

$$y = \frac{1}{F_H}\left(\frac{w_0 x^2}{2} + C_1 x + C_2\right) \tag{1}$$

The constants of integration may be determined using the boundary conditions $y = 0$ at $x = 0$ and $dy/dx = 0$ at $x = 0$. Substituting into Eq. 1 and its derivative yields $C_1 = C_2 = 0$. The equation of the curve then becomes

$$y = \frac{w_0}{2F_H}x^2 \tag{2}$$

This is the equation of a *parabola*. The constant F_H may be obtained using the boundary condition $y = h$ at $x = L/2$. Thus,

$$F_H = \frac{w_0 L^2}{8h} \tag{3}$$

Therefore, Eq. 2 becomes

$$y = \frac{4h}{L^2} x^2 \tag{4}$$

Since F_H is known, the tension in the cable may now be determined using Eq. 7–10, written as $T = F_H/\cos\theta$. For $0 \le \theta < \pi/2$, the maximum tension will occur when θ is *maximum*, i.e., at point B, Fig. 7–21a. From Eq. 2, the slope at this point is

$$\left.\frac{dy}{dx}\right|_{x=L/2} = \tan\theta_{\max} = \left.\frac{w_0}{F_H} x\right|_{x=L/2}$$

or

$$\theta_{\max} = \tan^{-1}\left(\frac{w_0 L}{2F_H}\right) \tag{5}$$

Therefore,

$$T_{\max} = \frac{F_H}{\cos(\theta_{\max})} \tag{6}$$

Using the triangular relationship shown in Fig. 7–21b, which is based on Eq. 5, Eq. 6 may be written as

$$T_{\max} = \frac{\sqrt{4F_H^2 + w_0^2 L^2}}{2}$$

Substituting Eq. 3 into the above equation yields

$$T_{\max} = \frac{w_0 L}{2}\sqrt{1 + \left(\frac{L}{4h}\right)^2} \qquad \textit{Ans.}$$

For a differential segment of cable length ds, we can write

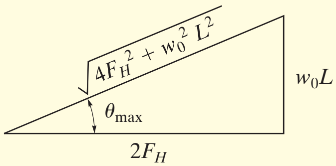
$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Hence, the total length of the cable can be determined by integration. Using Eq. 4, we have

$$\mathcal{L} = \int ds = 2\int_0^{L/2} \sqrt{1 + \left(\frac{8h}{L^2}x\right)^2}\, dx \tag{7}$$

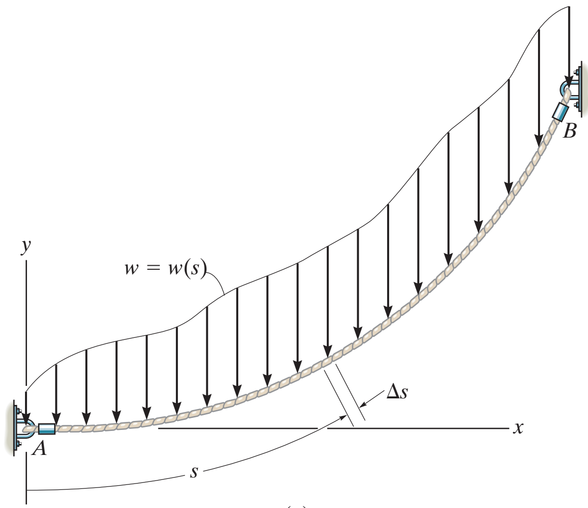
Integrating yields

$$\mathcal{L} = \frac{L}{2}\left[\sqrt{1 + \left(\frac{4h}{L}\right)^2} + \frac{L}{4h}\sinh^{-1}\left(\frac{4h}{L}\right)\right] \qquad \textit{Ans.}$$



(b)

Fig. 7–21 (cont.)



(a)

Fig. 7–22

Cable Subjected to Its Own Weight. When the weight of a cable becomes important in the force analysis, the loading function along the cable will be a function of the arc length s rather than the projected length x. To analyze this problem, we will consider a generalized loading function $w = w(s)$ acting along the cable, as shown in Fig. 7–22*a*. The free-body diagram for a small segment Δs of the cable is shown in Fig. 7–22*b*. Applying the equilibrium equations to the force system on this diagram, one obtains relationships identical to those given by Eqs. 7–7 through 7–9, but with s replacing x in Eqs. 7–7 and 7–8. Therefore, we can show that

$$T\cos\theta = F_H$$

$$T\sin\theta = \int w(s)\,ds \tag{7–13}$$

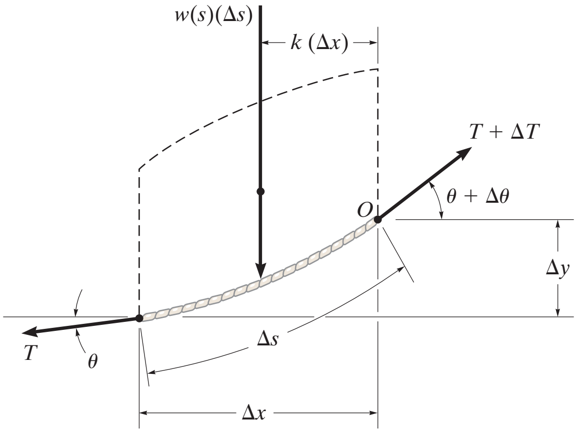
$$\frac{dy}{dx} = \frac{1}{F_H}\int w(s)\,ds \tag{7–14}$$

To perform a direct integration of Eq. 7–14, it is necessary to replace dy/dx by ds/dx. Since

$$ds = \sqrt{dx^2 + dy^2}$$

then

$$\frac{dy}{dx} = \sqrt{\left(\frac{ds}{dx}\right)^2 - 1}$$



(b)

Fig. 7–22 (cont.)

Therefore,

$$\frac{ds}{dx} = \left[1 + \frac{1}{F_H^2}\left(\int w(s)\,ds\right)^2\right]^{1/2}$$

Separating the variables and integrating we obtain

$$\boxed{x = \int \frac{ds}{\left[1 + \frac{1}{F_H^2}\left(\int w(s)\,ds\right)^2\right]^{1/2}}} \qquad (7\text{–}15)$$

The two constants of integration, say C_1 and C_2, are found using the boundary conditions for the curve.

Electrical transmission towers must be designed to support the weights of the suspended power lines. The weight and length of the cables can be determined since they each form a catenary curve. (© Russell C. Hibbeler)

EXAMPLE 7.13

Determine the deflection curve, the length, and the maximum tension in the uniform cable shown in Fig. 7–23. The cable has a weight per unit length of $w_0 = 5$ N/m.

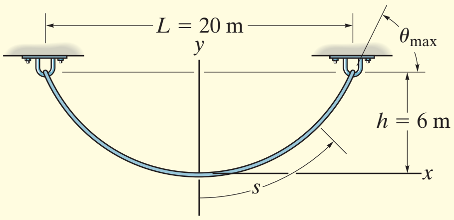


Fig. 7–23

SOLUTION

For reasons of symmetry, the origin of coordinates is located at the center of the cable. The deflection curve is expressed as $y = f(x)$. We can determine it by first applying Eq. 7–15, where $w(s) = w_0$.

$$x = \int \frac{ds}{\left[1 + (1/F_H^2)\left(\int w_0\, ds\right)^2\right]^{1/2}}$$

Integrating the term under the integral sign in the denominator, we have

$$x = \int \frac{ds}{[1 + (1/F_H^2)(w_0 s + C_1)^2]^{1/2}}$$

Substituting $u = (1/F_H)(w_0 s + C_1)$ so that $du = (w_0/F_H)\, ds$, a second integration yields

$$x = \frac{F_H}{w_0}(\sinh^{-1} u + C_2)$$

or

$$x = \frac{F_H}{w_0}\left\{\sinh^{-1}\left[\frac{1}{F_H}(w_0 s + C_1)\right] + C_2\right\} \tag{1}$$

To evaluate the constants note that, from Eq. 7–14,

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds \quad \text{or} \quad \frac{dy}{dx} = \frac{1}{F_H}(w_0 s + C_1)$$

Since $dy/dx = 0$ at $s = 0$, then $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{w_0 s}{F_H} \tag{2}$$

The constant C_2 may be evaluated by using the condition $s = 0$ at $x = 0$ in Eq. 1, in which case $C_2 = 0$. To obtain the deflection curve, solve for s in Eq. 1, which yields

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right) \tag{3}$$

Now substitute into Eq. 2, in which case

$$\frac{dy}{dx} = \sinh\left(\frac{w_0}{F_H}x\right)$$

Hence,

$$y = \frac{F_H}{w_0}\cosh\left(\frac{w_0}{F_H}x\right) + C_3$$

If the boundary condition $y = 0$ at $x = 0$ is applied, the constant $C_3 = -F_H/w_0$, and therefore the deflection curve becomes

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right] \qquad (4)$$

This equation defines the shape of a ***catenary curve***. The constant F_H is obtained by using the boundary condition that $y = h$ at $x = L/2$, in which case

$$h = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0 L}{2F_H}\right) - 1\right] \qquad (5)$$

Since $w_0 = 5\text{ N/m}$, $h = 6\text{ m}$, and $L = 20\text{ m}$, Eqs. 4 and 5 become

$$y = \frac{F_H}{5\text{ N/m}}\left[\cosh\left(\frac{5\text{ N/m}}{F_H}x\right) - 1\right] \qquad (6)$$

$$6\text{ m} = \frac{F_H}{5\text{ N/m}}\left[\cosh\left(\frac{50\text{ N}}{F_H}\right) - 1\right] \qquad (7)$$

Equation 7 can be solved for F_H by using a trial-and-error procedure. The result is

$$F_H = 45.9\text{ N}$$

and therefore the deflection curve, Eq. 6, becomes

$$y = 9.19[\cosh(0.109x) - 1]\text{ m} \qquad \textit{Ans.}$$

Using Eq. 3, with $x = 10\text{ m}$, the half-length of the cable is

$$\frac{\mathscr{L}}{2} = \frac{45.9\text{ N}}{5\text{ N/m}}\sinh\left[\frac{5\text{ N/m}}{45.9\text{ N}}(10\text{ m})\right] = 12.1\text{ m}$$

Hence,

$$\mathscr{L} = 24.2\text{ m} \qquad \textit{Ans.}$$

Since $T = F_H/\cos\theta$, the maximum tension occurs when θ is maximum, i.e., at $s = \mathscr{L}/2 = 12.1\text{ m}$. Using Eq. 2 yields

$$\left.\frac{dy}{dx}\right|_{s=12.1\text{ m}} = \tan\theta_{\max} = \frac{5\text{ N/m}(12.1\text{ m})}{45.9\text{ N}} = 1.32$$

$$\theta_{\max} = 52.8^\circ$$

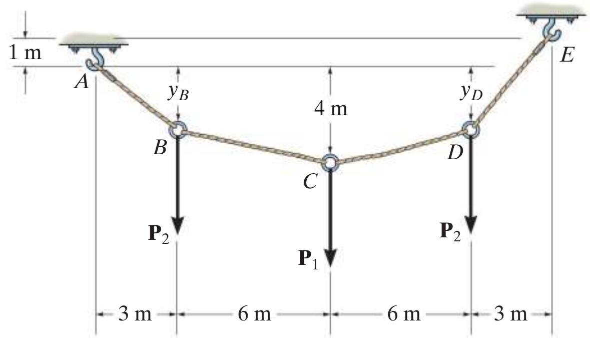
And so,

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{45.9\text{ N}}{\cos 52.8^\circ} = 75.9\text{ N} \qquad \textit{Ans.}$$

PROBLEMS

7–94. The cable supports the three loads shown. Determine the sags y_B and y_D of B and D. Take $P_1 = 800$ N, $P_2 = 500$ N.

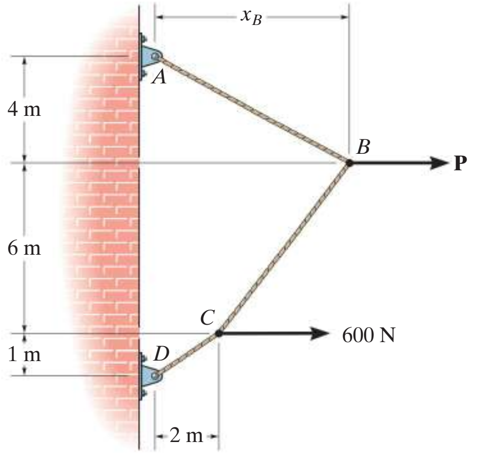
7–95. The cable supports the three loads shown. Determine the magnitude of $\mathbf{P}_1$ if $P_2 = 600$ N and $y_B = 3$ m. Also find sag y_D.



Probs. 7–94/95

7–97. The cable supports the loading shown. Determine the distance x_B the force at B acts from A. Set $P = 800$ N.

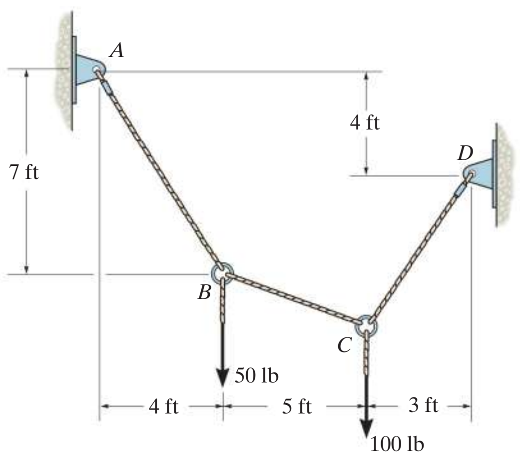
7–98. The cable supports the loading shown. Determine the magnitude of the horizontal force $\mathbf{P}$ so that $x_B = 5$ m.



Probs. 7–97/98

7

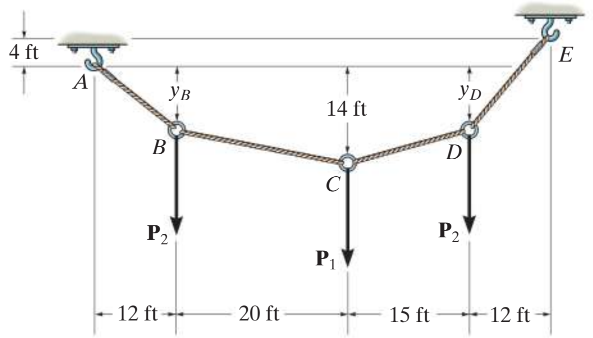
***7–96.** Determine the tension in each segment of the cable and the cable's total length.



Prob. 7–96

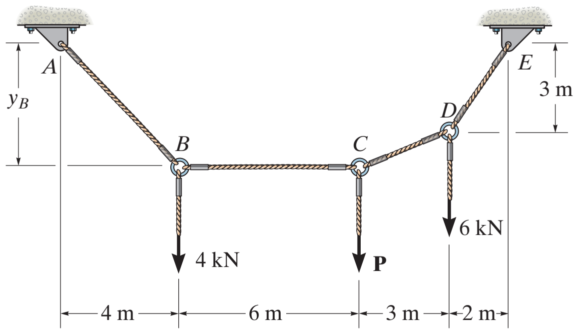
7–99. The cable supports the three loads shown. Determine the sags y_B and y_D of points B and D. Take $P_1 = 400$ lb, $P_2 = 250$ lb.

***7–100.** The cable supports the three loads shown. Determine the magnitude of $\mathbf{P}_1$ if $P_2 = 300$ lb and $y_B = 8$ ft. Also find the sag y_D.



Probs. 7–99/100

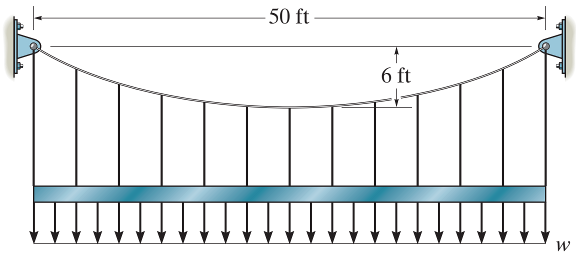
7–101. Determine the force P needed to hold the cable in the position shown, i.e., so segment BC remains horizontal. Also, compute the sag y_B and the maximum tension in the cable.



Prob. 7–101

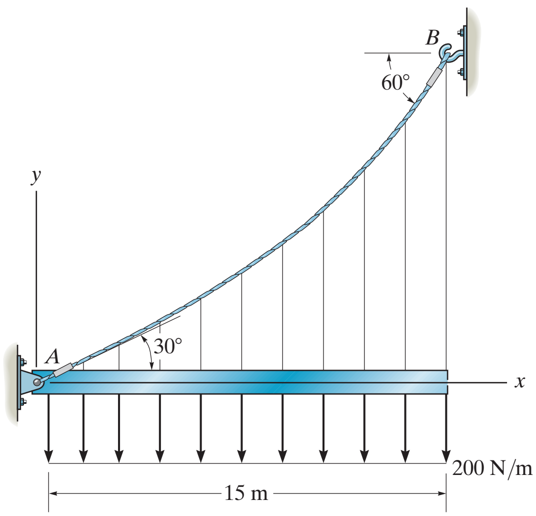
7–102. Determine the maximum uniform loading w, measured in lb/ft, that the cable can support if it is capable of sustaining a maximum tension of 3000 lb before it will break.

7–103. The cable is subjected to a uniform loading of $w = 250$ lb/ft. Determine the maximum and minimum tension in the cable.



Probs. 7–102/103

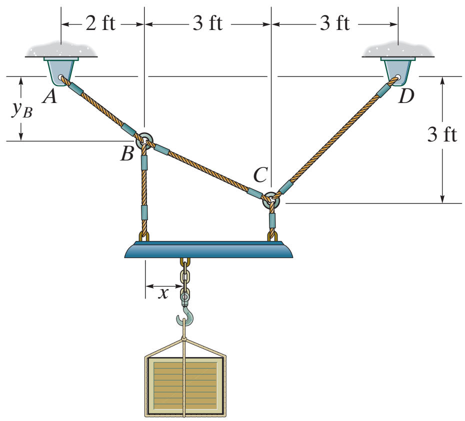
***7–104.** The cable AB is subjected to a uniform loading of 200 N/m. If the weight of the cable is neglected and the slope angles at points A and B are 30° and 60°, respectively, determine the curve that defines the cable shape and the maximum tension developed in the cable.



Prob. 7–104

7–105. If $x = 2$ ft and the crate weighs 300 lb, which cable segment AB, BC, or CD has the greatest tension? What is this force and what is the sag y_B?

7–106. If $y_B = 1.5$ ft, determine the largest weight of the crate and its placement x so that neither cable segment AB, BC, or CD is subjected to a tension that exceeds 200 lb.



Probs. 7–105/106