THE ELECTROSTATIC FIELD

Law of Force between two Charges

The magnitude of the force between two electrically charged bodies was studied by Coulomb in 1875. He showed that, if the bodies were small compared with the distance between them, then the force F was inversely proportional to the square of the distance r, i.e.

$$F \propto \frac{1}{r^2}. \qquad . \quad . \quad . \quad . \quad (1)$$

This result is known as the *inverse square law*, or Coulomb's law.

It is not possible to verify the law accurately by direct measurement of the force between two charged bodies. In 1936 Plimton and Lawton showed, by an indirect method, that the power in the law cannot differ from 2 by more than $\pm 2 \times 10^{-9}$. We have no reason to suppose, therefore, that the inverse square law is other than exactly true.

Quantity of Charge

The SI unit of charge is the *coulomb* (C). The *ampere* (A), the unit of current, is defined later (p. 939). The coulomb is defined as that quantity of charge which passes a section of a conductor in one second when the current flowing is one ampère.

By measuring the force F between two charges when their respective magnitudes Q and Q' are varied, it is found that F is proportional to the product QQ'. Thus

$$F \propto QQ' \qquad . \quad . \quad . \quad . \quad (2)$$

Law of Force

Combining (1) and (2), we have

$$F \propto \frac{QQ'}{r^2}$$

$$\therefore F = k\frac{QQ'}{r^2}, \qquad . \quad . \quad . \quad . \quad (3)$$

where k is a constant. For reasons explained later, k is written as $1/4\pi\varepsilon_0$, where ε_0 is a constant called the *permittivity* of free space if we suppose the charges are situated in a vacuum. Thus

$$F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2} \qquad . \quad . \quad . \quad (4)$$

In this expression, F is measured in newtons (N), Q in coulombs (C) and r in metres (m). Now, from (4),

$$\varepsilon_0 = \frac{QQ'}{4\pi F r^2}.$$

Hence the units of ε_0 are coulomb^2 newton^{-1} metre^{-2} ($C^2\,N^{-1}m^{-2}$). Another unit of ε_0, more widely used, is *farad metre*$^{-1}$ ($F\,m^{-1}$). See p. 774.

We shall see later that ε_0 has the numerical value of $8{\cdot}854 \times 10^{-12}$, and $1/4\pi\varepsilon_0$ then has the value 9×10^9 approximately.

Permittivity

So far we have considered charges in a vacuum. If charges are situated in other media such as water, then the force between the charges is reduced. Equation (4) is true only in a vacuum. In general, we write

$$F = \frac{1}{4\pi\varepsilon}\frac{QQ'}{r^2} \quad . \quad . \quad . \quad . \quad (1)$$

where ε is the *permittivity* of the medium. The permittivity of air at normal pressure is only about 1·005 times that, ε_0, of a vacuum. For most purposes, therefore, we may assume the value of ε_0 for the permittivity of air. The permittivity of water is about eighty times that of a vacuum. Thus the force between charges situated in water is eighty times less than if they were situated the same distance apart in a vacuum.

EXAMPLE

(a) Calculate the value of two equal charges if they repel one another with a force of 0·1 N when situated 50 cm apart in a vacuum.

(b) What would be the size of the charges if they were situated in an insulating liquid whose permittivity was ten times that of a vacuum?

(a) From (4),

$$F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2}.$$

Since $Q = Q'$ here,

$$0{\cdot}1 = \frac{9 \times 10^9\, Q^2}{(0{\cdot}5)^2}$$

or

$$Q^2 = \frac{0{\cdot}1 \times (0{\cdot}5)^2}{9 \times 10^9}$$

$$Q = 1{\cdot}7 \times 10^{-6}\ \text{C (coulomb), approx.}$$

$$= 1{\cdot}7\ \mu\text{C (microcoulomb).}$$

(b) The permittivity of the liquid $\varepsilon = 10\,\varepsilon_0$.

$$F = \frac{1}{4\pi\varepsilon}\frac{QQ'}{r^2}$$

$$= \frac{1}{10(4\pi\varepsilon_0)}\frac{Q^2}{r^2}$$

$$Q^2 = \frac{(0{\cdot}1) \times (0{\cdot}5)^2 \times 10}{9 \times 10^9}$$

$$Q = 5{\cdot}3 \times 10^{-6}\ \text{C} = 5{\cdot}3\ \mu\text{C}.$$

Electric Intensity or Field-strength. Lines of Force

An 'electric field' can be defined as a region where an electric force is experienced. As in magnetism, electric fields can be mapped out by electrostatic lines of force, which may be defined as a line such that the tangent to it is in the direction of the force on a small positive charge at that point. Arrows on the lines of force show the direction of the force on a positive charge; the force on a negative charge is in the opposite direction. Fig 30.18 shows the lines of force, also called *electric flux*, in some electrostatic fields.

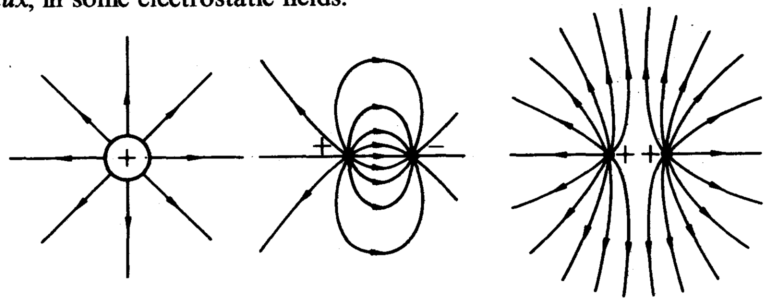


FIG. 30.18. Lines of electrostatic force.

The force exerted on a charged body in an electric field depends on the charge of the body and on the *intensity* or *strength* of the field. If we wish to explore the variation in intensity of an electric field, then we must place a test charge Q' at the point concerned which is small enough not to upset the field by its introduction. The intensity E of an electrostatic field at any point is defined as *the force per unit charge* which it exerts at that point. Its direction is that of the force exerted on a positive charge.

From this definition,

$$E = \frac{F}{Q'}$$

$$F = EQ' \qquad . \quad . \quad . \quad . \quad (1)$$

Since F is measured in newtons and Q' in coulombs, it follows that *intensity E has units of newton per coulomb* (N C^{-1}). We shall see later that a more practical unit of E is *volt metre*$^{-1}$ (V m^{-1}) (see p. 755).

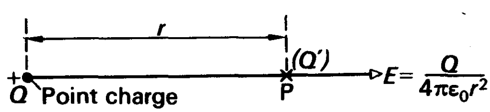


FIG. 30.19. Electric field intensity due to point charge.

We can easily find an expression for the strength E of the electric field due to a point charge Q situated in a vacuum (Fig. 30.19). We start from equation (4), p. 743, for the force between two such charges:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2}$$

If the test charge Q' is situated at the point P in Fig. 30.19, the electric field strength at that point is given by (1).

$$\therefore E = \frac{F}{Q'} = \frac{Q}{4\pi\varepsilon_0 r^2} \quad . \quad . \quad . \quad (2)$$

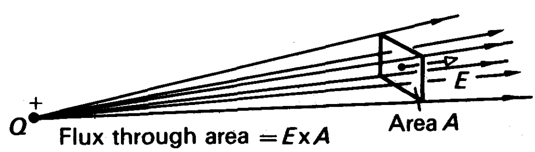
The direction of the field is radially outward if the charge Q is positive (Fig. 30.18(i)); it is radially inward if the charge Q is negative. If the charge were surrounded by a material of permittivity ε then,

$$E = \frac{Q}{4\pi\varepsilon r^2} \quad . \quad . \quad . \quad . \quad (3)$$

Flux from a Point Charge

We have already shown how electric fields can be described by lines of force. From Fig. 30.18(i) it can be seen that the density of the lines increases near the charge where the field intensity is high. The intensity E at a point can thus be represented by *the number of lines per unit area* through a surface perpendicular to the lines of force at the point considered. The *flux* through an area perpendicular to the lines of force is the name given to the product of $E \times$ *area*, where E is the intensity at that place. This is illustrated in Fig. 30.20(i).

(i)



(ii)

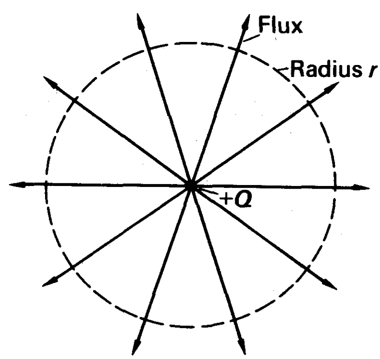


FIG. 30.20. Flux from a point charge.

Consider a sphere of radius r drawn in space concentric with a point charge (Fig. 30.20(ii)). The value of E at this place is given by (3),

p. 746. The total flux through the sphere is,

$$E \times \text{area} = E \times 4\pi r^2$$

$$= \frac{Q}{4\pi\varepsilon r^2} \times 4\pi r^2$$

$$= \frac{Q}{\varepsilon}$$

$$= \frac{\text{charge inside sphere}}{\text{permittivity}} \quad . \quad . \quad (1)$$

This demonstrates the important fact that the total flux crossing any sphere drawn outside and concentrically around a point charge is a constant. It does not depend on the distance from the charged sphere. It should be noted that this result is only true if the inverse square law is true.

To see this, suppose some other force law were valid, i.e. $E = Q/4\pi\varepsilon r^n$. Then the total flux through the area

$$= \frac{Q}{4\pi\varepsilon r^n} \times 4\pi r^2$$

$$= \frac{Q}{\varepsilon} r^{(2-n)}$$

This is only independent of r if $n = 2$.

Field due to Charged Sphere and Plane Conductor

Equation (1) can be shown to be generally true. Thus the flux passing through any *closed* surface whatever its shape, is always equal to Q/ε, where Q is the total charge enclosed by the surface. This relation, called *Gauss's Theorem*, can be used to find the value of E in other common cases.

(1) *Outside a charged sphere*

The flux across a spherical surface of radius r, concentric with a small sphere carrying a charge Q (Fig. 30.21), is given by,

$$\text{Flux} = \frac{Q}{\varepsilon}$$

$$\therefore E \times 4\pi r^2 = \frac{Q}{\varepsilon}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon r^2}$$

This is the same answer as that for a point charge. This means that *outside* a charged sphere, the field behaves as if all the charge on the sphere were concentrated at the centre.

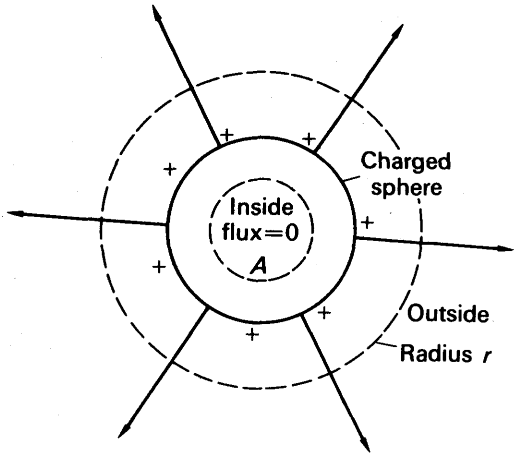


FIG. 30.21. Electric field of a charged sphere.

(2) *Inside a charged empty sphere*

Suppose a spherical surface A is drawn *inside* a charge sphere, as shown in Fig. 30.21. Inside this sphere there are no charges and so Q in equation (1), p. 747, is zero. This result is independent of the radius drawn, provided that it is less than that of the charged sphere. Hence from (1), p. 747, *E must be zero everywhere inside a charged sphere.*

(3) *Outside a Charged Plane Conductor*

Now consider a charged *plane* conductor S, with a surface charge density of σ coulomb metre^{-2}. Fig. 30.22 shows a plane surface P, drawn outside S, which is parallel to S and has an area A metre^2. Applying equation (1),

$$\therefore\ E \times \text{area} = \frac{\text{Charge inside surface}}{\varepsilon}$$

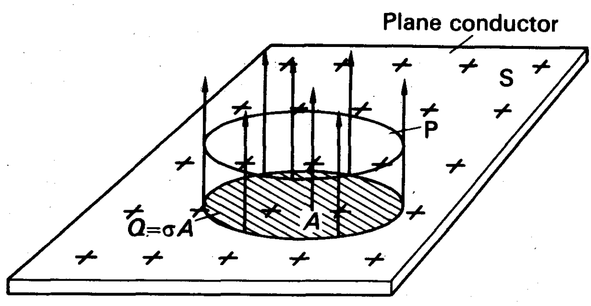


FIG. 30.22. Field of a charged plane conductor.

Now by symmetry, the intensity in the field must be perpendicular to the surface. Further, the charges which produce this field are those in the projection of the area P on the surface S, i.e. those within the shaded area A in Fig. 30.22. The total charge here is thus σA coulomb.

$$\therefore E.A = \frac{\sigma A}{\varepsilon}$$

$$\therefore E = \frac{\sigma}{\varepsilon}$$

Field Round Points

On p. 742 we saw that the surface-density of charge (charge per unit area) round a point of a conductor is very great. Consequently, the strength of the electric field near the point is very great. The intense electric field breaks down the insulation of the air, and sends a stream of charged molecules away from the point. The mechanism of the breakdown, which is called a 'corona discharge', is complicated, and we shall not discuss it here; some of the processes in it are similar to those in conduction through a gas at low pressure, which we shall describe in Chapter 40. Corona breakdown starts when the electric field strength is about 3 million volt metre^{-1}. The corresponding surface-density is about $2{\cdot}7 \times 10^{-5}$ coulomb metre^{-2}.

EXAMPLE

An electron of charge $1{\cdot}6 \times 10^{-19}$ C is situated in a uniform electric field of intensity 1200 volt cm^{-1}. Find the force on it, its acceleration, and the time it takes to travel 2 cm from rest (electronic mass, m, $= 9{\cdot}10 \times 10^{-31}$ kg).

Force on electron $F = eE$.
Now $E = 1200$ volt cm^{-1} $= 120\,000$ volt m^{-1}.

$$\therefore \quad F = 1{\cdot}6 \times 10^{-19} \times 1{\cdot}2 \times 10^{5}$$

$$= 1{\cdot}92 \times 10^{-14} \text{ N (newton).}$$

Acceleration, $$a = \frac{F}{m} = \frac{1{\cdot}92 \times 10^{-14}}{9{\cdot}1 \times 10^{-31}}$$

$$= 2{\cdot}12 \times 10^{16} \text{ m s}^{-2} \text{ (metre second}^{-2}\text{)}$$

Time for 2 cm travel is given by

$$s = \tfrac{1}{2}at^2$$

$$\therefore \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 0{\cdot}02}{2{\cdot}12 \times 10^{16}}}$$

$$= 1{\cdot}37 \times 10^{-9} \text{ seconds.}$$

The extreme shortness of this time is due to the fact that the ratio of charge-to-mass for an electron is very great:

$$\frac{e}{m} = \frac{1{\cdot}6 \times 10^{-19}}{9{\cdot}1 \times 10^{-31}} = 1{\cdot}8 \times 10^{11} \text{ C kg}^{-1}.$$

In an electric field, the charge e determines the force on an electron, while the mass m determines its inertia. Because of the large ratio e/m, the electron moves almost instantaneously, and requires very little energy to displace it. Also it can respond to changes in an electric field which take place even millions of times per second. Thus it is the large value of e/m for electrons which makes electronic tubes, for example, useful in electrical communication and remote control.

ELECTRIC POTENTIAL

Potential in Fields

When an object is held at a height above the earth it is said to have potential energy. A heavy body tends to move under the force of attraction of the earth from a point of great height to one of less, and we say that points in the earth's gravitational field have potential values depending on their height.

Electric potential is analogous to gravitational potential, but this time we think of points in an electric field. Thus in the field round a positive charge, for example, a positive charge moves from points near the charge to points further away. Points round the charge are said to have an 'electric potential'.

Potential Difference

In mechanics we are always concerned with differences of height; if a point A on a hill is h metre higher than a point B, and our weight is w newton, then we do wh joule of work in climbing from B to A (Fig. 30.23 (i)). Similarly in electricity we are often concerned with differences of potential; and we define these also in terms of work.

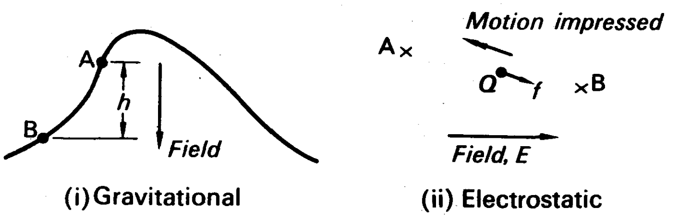


FIG. 30.23. Work done, in gravitational and electrostatic fields.

Let us consider two points A and B in an electrostatic field, and let us suppose that the force on a positive charge Q has a component f in the direction AB (Fig. 30.23 (ii)). Then if we move a positively charged body from B to A, we do work against this component of the field E. *We define the potential difference between A and B as the work done in moving a unit positive charge from B to A.* We denote it by the symbol V_{AB}.

The work done will be measured in joules (J). The unit of potential difference is called the volt and may be defined as follows: *The potential difference between two points A and B is one volt if the work done in taking one coulomb of positive charge from B to A is one joule.*

From this definition, if a charge of Q coulombs is moved through a p.d. of V volt, then the work done W in joules is given by

$$W = QV \qquad . \quad . \quad . \quad . \quad . \qquad (1)$$

Potential and Energy

Let us consider two points A and B in an electrostatic field, A being at a higher potential than B. The potential difference between A and

B we denote as usual by V_{AB}. If we take a positive charge Q from B to A, we do work on it of amount QV_{AB}: the charge gains this amount of potential energy. If we now let the charge go back from A to B, it loses that potential energy: work is done on it by the electrostatic force, in the same way as work is done on a falling stone by gravity. This work may become kinetic energy, if the charge moves freely, or external work if the charge is attached to some machine, or a mixture of the two.

The work which we must do in first taking the charge from B to A does not depend on the path along which we carry it, just as the work done in climbing a hill does not depend on the route we take. If this were not true, we could devise a perpetual motion machine, in which we did less work in carrying a charge from B to A via X than it did for us in returning from A to B via Y (Fig. 30.24).

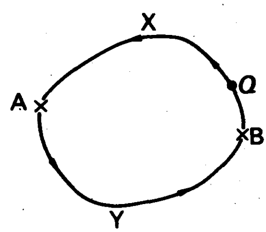


FIG. 30.24. A closed path in an electrostatic field.

The fact that the potential differences between two points is a constant, independent of the path chosen between the points, is the most important property of potential in general; we shall see why later on. This property can be conveniently expressed by saying that the work done in carrying a charge round a closed path in an electrostatic field, such as BXAYB in Fig. 30.24 is zero.

Potential Difference Formula

To obtain a formula for potential difference, let us calculate the potential difference between two points in the field of a single point positive charge, Q in Fig. 30.25. For simplicity we will assume that the points, A and B, lie on a line of force at distances a and b respectively

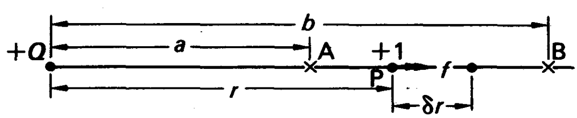


FIG. 30.25. Calculation of potential.

from the charge. When a unit positive charge is at a distance r from the charge Q in free space the force on it is

$$f = \frac{Q \times 1}{4\pi\varepsilon_0 r^2}$$

The work done in taking the charge from B to A, against the force f, is equal to the work which the force f would do if the charge were allowed to go from A to B. Over the short distance δr, the work done by the force f is

$$\delta W = f\delta r.$$

Over the whole distance AB, therefore, the work done by the force on the unit charge is

$$\int_{A}^{B}\delta W = \int_{r=a}^{r=b} f dr = \int_{a}^{b}\frac{Q}{4\pi\varepsilon_0 r^2}dr$$

$$= -\left[\frac{Q}{4\pi\varepsilon_0 r}\right]_a^b = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$

This, then, is the value of the work which an external agent must do to carry a unit positive charge from B to A. The work per coulomb is the potential difference V_{AB} between A and B.

$$\therefore\ V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \quad . \quad . \quad . \quad (1)$$

V_{AB} will be in volts if Q is in coulombs, a and b are in metres and ε_0 is taken as $8{\cdot}85 \times 10^{-12}$ or $1/4\pi\varepsilon_0$ as 9×10^9 approximately (see p. 744).

EXAMPLE

Two positive point charges, of 12 and 8 microcoulomb respectively, are 10 cm apart. Find the work done in bringing them 4 cm closer. (Assume $1/4\pi\varepsilon_0 = 9 \times 10^9$.)

Suppose the 12 μC (microcoulomb) charge is fixed in position. Since 6 cm = 0·06 m and 10 cm = 0·1 m, then the potential difference between points 6 and 10 cm from it is given by (1).

$$\therefore \quad V = \frac{12 \times 10^{-6}}{4\pi\varepsilon_0}\left(\frac{1}{0{\cdot}06} - \frac{1}{0{\cdot}1}\right)$$

$$= 12 \times 10^{-6} \times 9 \times 10^9\ (16\tfrac{2}{3} - 10)$$

$$= 720\,000\ \text{V}.$$

(Note the very high potential difference due to quite small charges.)

The work done in moving the 8 μC charge from 10 cm to 6 cm away from the other is given by, using $W = QV$,

$$W = 8 \times 10^6 \times V$$

$$= 8 \times 10^{-6} \times 720\,000$$

$$= 5{\cdot}8\ \text{J}.$$

Zero Potential

Instead of speaking continually of potential differences between pairs of points, we may speak of the potential at a single point—provided we always refer it to some other, agreed, reference point. This procedure is analogous to referring the heights of mountains to sea-level.

For practical purposes we generally choose as our reference point the electric potential of the surface of the earth. Although the earth is large it is all at the same potential, because it is a good conductor of electricity; if one point on it were at a higher potential than another, electrons would flow from the lower to the higher potential. As a result, the higher potential would fall, and the lower would rise; the flow of electricity would cease only when the potentials became equalized.

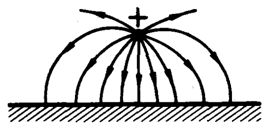


FIG. 30.26. Electric field of positive charge near earth.

In general it is difficult to calculate the potential of a point relative to the earth. This is because the electric field due to a charged body near a conducting surface is complicated, as shown by the lines of force diagram in Fig. 30.26. In theoretical calculations, therefore, we often find it convenient to consider charges so far from the earth that the effect of the earth on their field is negligible; we call these 'isolated' charges.

Thus we define the potential at a point A as V volts if V joules of work is done in bringing one coulomb of positive charge from infinity to A.

Potential Formula

Equation (1), p. 752, gives the potential difference between two points in the field of an isolated point charge Q. If we let the point B retreat to infinity, then $b \gg a$, and the equation gives for the potential at A:

$$V_A = \frac{Q}{4\pi\varepsilon_0 a} \quad . \quad . \quad . \quad . \quad (1)$$

The derivation of this equation shows us what we mean by the word 'infinity': the distance b is infinite if $1/b$ is negligible compared with $1/a$. If a is 1 cm, and b is 1 m, we make an error of only 1 per cent. in ignoring it; if b is 100 m, then for all practical purposes the point B is at infinity. In atomic physics, where the distances concerned have the order of 10^{-8} cm, a fraction of a millimetre is infinite.

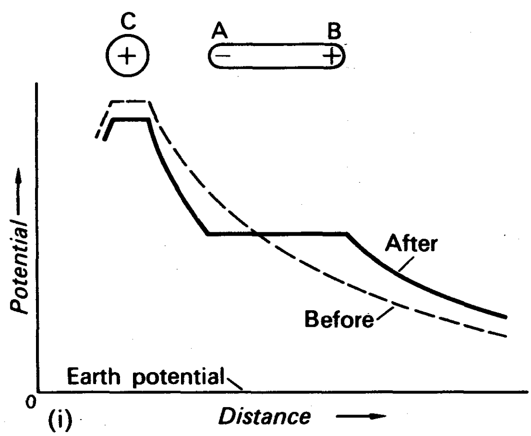


FIG. 30.27 (i). Potential distribution near a positive charge before and after bringing up an uncharged conductor.

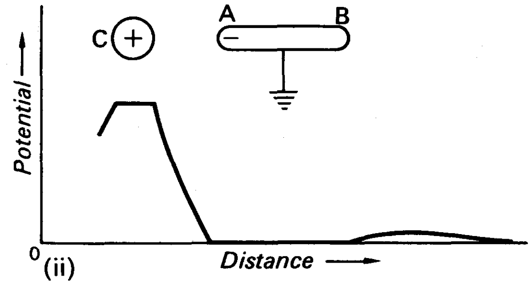


FIG. 30.27 (ii). Potential distribution near a positive charge in the presence of an earthed conductor.

In the neighbourhood of an isolated negative charge, the potential is negative, because Q in equation (1) is negative. The potential is also negative in the neighbourhood of a negative charge near the earth: the earth is at zero potential, and a positive charge will tend to move from it towards the negative charge. A negative potential is analogous to the depth of a mine below sea-level. Fig. 30.27 (i) shows the potential variation near a positive charge C before and after a conductor AB is brought near. Fig. 30.27 (ii) shows the potential variation when AB is earthed.

Potential Difference and Intensity

We shall now see how potential difference is related to intensity or field-strength. Suppose A, B are two neighbouring points on a line of force, so close together that the electric field-intensity between them is constant and equal to E (Fig. 30.28). If V is the potential at A, $V+\delta V$ is that at B, and the respective distances of A, B from the origin are x and $x+\delta x$, then

$$V_{AB} = \text{potential difference between A, B}$$
$$= V_A - V_B = V-(V+\delta V) = -\delta V.$$

The work done in taking a unit charge from B to A

$$= \text{force} \times \text{distance} = E \times \delta x = V_{AB} = -\delta V.$$

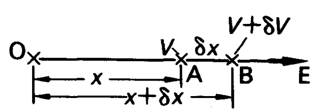


FIG. 30.28. Field strength and potential gradient.

Hence $$E = -\frac{\delta V}{\delta x},$$

or, in the limit,

$$E = -\frac{dV}{dx}. \quad . \quad . \quad . \quad . \quad (1)$$

The quantity dV/dx is the rate at which the potential rises with distance, and is called the potential gradient. Equation (1) shows that the strength of the electric field is equal to the negative of the potential gradient, and strong and weak fields in relation to potential are illustrated in Fig. 30.29.

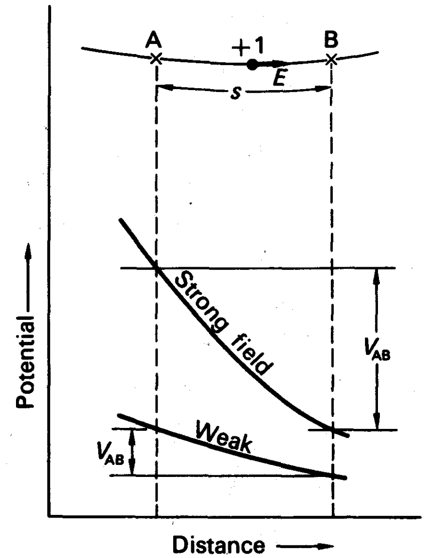


FIG. 30.29. Relationship between potential and field strength.

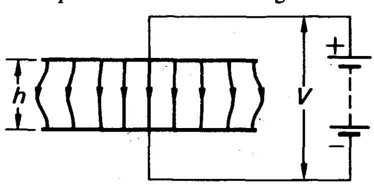


FIG. 30.30. Electric field between parallel plates.

In Fig. 30.30 the electric intensity $= V/h$, the potential gradient, and this is uniform in magnitude in the middle of the plates. At the edge of the plates the field becomes non-uniform.

We can now see why E is usually given in units of 'volt per metre' ($\mathrm{V\,m^{-1}}$).

From (1), $E = -(dV/dx)$. Since V is measured in volts and x in metres, then E will be in volt per metre ($\mathrm{V\,m^{-1}}$). From the original definition of E, summarized by equation (1) on p. 745, the units of E were newton coulomb^{-1}. To show that these are equivalent, from (1),

$$1 \text{ volt} = 1 \text{ joule coulomb}^{-1}$$
$$= 1 \text{ newton metre coulomb}^{-1}$$

Since $$1 \text{ joule} = 1 \text{ newton} \times 1 \text{ metre}$$

$$\therefore 1 \text{ volt metre}^{-1} = 1 \text{ newton coulomb}^{-1}$$

EXAMPLES

1 An electron is liberated from the lower of two large parallel metal plates separated by a distance $h = 2$ cm. The upper plate has a potential of 2400 volts relative to the lower. How long does the electron take to reach it?

Between large parallel plates, close together, the electric field is uniform except near the edges of the plates, as shown in Fig. 30.30. Except near the edges, therefore, the potential gradient between the plates is uniform; its magnitude is V/h,

$$\begin{aligned}\text{electric intensity } E &= \text{potential gradient}\\ &= 2400/0{\cdot}02\ \mathrm{V\,m^{-1}}\\ &= 1{\cdot}2 \times 10^5\ \mathrm{V\,m^{-1}}.\end{aligned}$$

The rest of this problem may now be worked out exactly as the example on p. 749.

2 An electron is liberated from a hot filament, and attracted by an anode, of potential 1200 volts positive with respect to the filament. What is the speed of the electron when it strikes the anode?

$$e = \text{electronic charge} = 1{\cdot}6 \times 10^{-19}\ \mathrm{C}.$$

$$V = 1200\ \mathrm{V},\ m = \text{mass of electron} = 9{\cdot}1 \times 10^{-31}\ \mathrm{kg}.$$

The energy which the electron gains from the field $= QV = eV$.

Kinetic energy gained

$$= \tfrac{1}{2}mv^2 = eV,$$

where v is the speed gained from rest.

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1{\cdot}6 \times 10^{-19} \times 1200}{9{\cdot}1 \times 10^{-31}}}$$

$$= 2{\cdot}1 \times 10^7\ \mathrm{m\,s^{-1}}.$$

The Electron-Volt

The kinetic energy gained by an electron which has been accelerated through a potential difference of 1 volt is called an *electron-volt* (eV). Since the energy gained in moving a charge Q through a p.d. $V = QV$,

$$\therefore\ 1\ \mathrm{eV} = \text{electronic charge} \times 1 = 1{\cdot}6 \times 10^{-19} \times 1\ \text{joule} = 1{\cdot}6 \times 10^{-19}\ \mathrm{J}.$$

The electron-volt is a useful unit of energy in atomic physics. For example, the work necessary to extract a conduction electron from tungsten is 4·52 electron-volt. This quantity determines the magnitude of the thermionic emission from the metal at a given temperature (p. 1026); it is analogous to the latent heat of evaporation of a liquid.

EQUIPOTENTIALS

Equipotentials

We have already said that the earth must have the same potential all over, because it is a conductor. In a conductor there can be no differences of potential, because these would set up a potential gradient or electric field; electrons would then redistribute themselves throughout the conductor, under the influence of the field, until they had destroyed the field. This is true whether the conductor has a net charge, positive or negative, or whether it is uncharged; it is true whatever the actual potential of the conductor, relative to any other body.

Any surface or volume over which the potential is constant is called an *equipotential.* The volume or surface may be that of a material body, or simply a surface or volume in space. For example, as we shall see later, the space inside a hollow charged conductor is an equipotential volume. Equipotential surfaces can be drawn throughout any space in which there is an electric field, as we shall now explain.

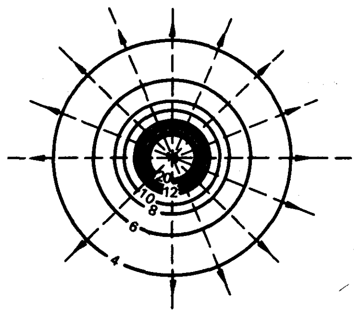


FIG. 30.31. Equipotentials and lines of force around a point charge.

Let us consider the field of an isolated point-charge Q. At a distance a from the charge, the potential is $Q/4\pi\varepsilon_0 a$; a sphere of radius a and centre at Q is therefore an equipotential surface, of potential $Q/4\pi\varepsilon_0 a$. In fact, all spheres centred on the charge are equipotential surfaces, whose potentials are inversely proportional to their radii (Fig. 30.31). An equipotential surface has the property that, along any direction lying in the surface, there is no electric field; for there is no potential gradient. *Equipotential surfaces are therefore always at right angles to lines of force*, as shown in Fig. 30.31. This also shows numerical values proportional to their potentials. Since conductors are always equipotentials, if any conductors appear in an electric-field diagram the lines of force must always be drawn to meet them at right angles.

Potential due to a System of Charges

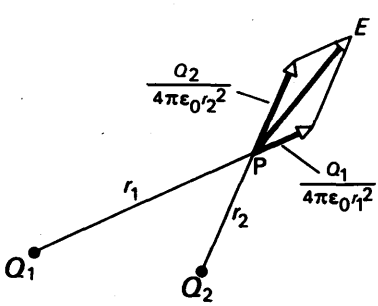
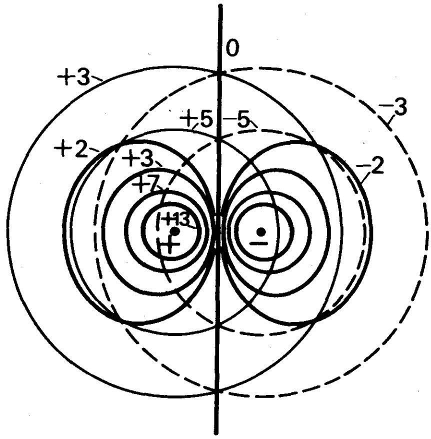


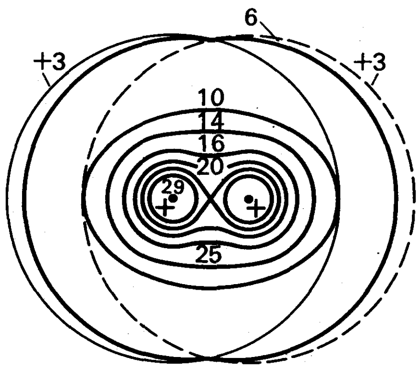
FIG. 30.32. Finding resultant field of two point-charges.

When we set out to consider the electric field due to more charges than one, then we see the advantages of the idea of potential over the idea of field-strength. If we wish to find the field-strength E at the point P in Fig. 30.32, due to the two charges Q_1 and Q_2, we have first to find the force exerted by each on a unit charge at P, and then to compound these forces by the parallelogram method. See Fig. 30.32. On the other hand, if we wish to find the potential at P, we merely calculate the potential due to each charge, and *add the potentials algebraically.*

Quantities which can be added algebraically are called 'scalars'; they may have signs—positive or negative, like a bank balance— but



(i) Opposite charges



(ii) Like charges

FIG. 30.33. Equipotentials in the field of two point charges.

they have no direction: they do not point north, east, south, or west. Quantities which have direction, like forces, are called 'vectors'; they have to be added either by resolution into components, or by the parallelogram method. Either way is slow and clumsy, compared with the addition of scalars. For example, we can draw the equipotentials round a point-charge with compasses; if we draw two sets of them, as in Fig. 30.33 (i) or (ii), then by simple addition we can rapidly sketch the equipotentials around the two charges together.

And when we have plotted the equipotentials, they turn out to be more useful than lines of force. A line of force diagram appeals to the imagination, and helps us to see what would happen to a charge in the field. But it tells us little about the strength of the field—at the best, if it is more carefully drawn than most, we can only say that the field is strongest where the lines are closest. But equipotentials can be labelled with the values of potential they represent; and from their spacing we can find the actual value of the potential gradient, and hence the field-strength. The only difficulty in interpreting equipotential diagrams lies in visualizing the direction of the force on a charge; this is always at right angles to the curves.

Field inside Hollow Conductor. Potential Difference and Gold-leaf Electroscope

If a hollow conductor contains no charged bodies, then, whatever charge there may be on its outside, there is none on its inside. Inside it, therefore, there is no electric field; the space within the conductor is an equipotential volume. If the conductor has an open end, like a can, then most of the space inside it is equipotential, but near its mouth there is a weak field (Fig. 30.34).

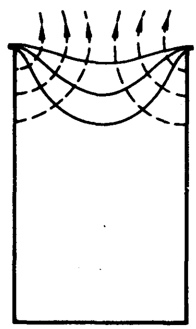


FIG. 30.34. Equipotentials and lines of force near mouth of an open charged can.

The behaviour of the *gold-leaf electroscope* illustrates this point. If we stand the case on an insulator, and connect the cap to it with a wire, then, no matter what charge we give to the cap, the leaves do not diverge (Fig. 30.35). Any charge we give to the cap spreads over the case

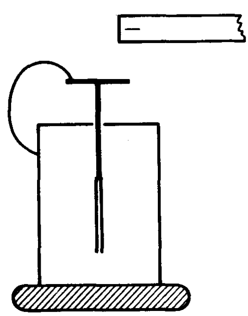


FIG. 30.35. Electroscope cap joined to case.

of the electroscope, but none appears on the leaves, and there is no force acting to diverge them. When, as usual, the cap is insulated and the case earthed, charging the cap sets up a potential difference between it and the case. Charges appear on the leaves, and the field between them and the case makes them diverge (p. 733). If the case is insulated from earth, as well as from the cap, the leaves diverge less; the charge on them and the cap raises the potential of the case and reduces the potential difference between it and the leaves. The field acting on the leaves is thus made weaker, and the force on the leaves less. We can sum up these observations by saying that *the electroscope indicates the potential difference between its leaves and its case.*

Potential of Pear-shaped Conductor

On p. 742 we saw that the surface-density of the charge on a pear-shaped conductor was greatest where the curvature was greatest. The potential of the conductor at various points can be examined by means of the gold-leaf electroscope, the case being earthed. One end of a wire is connected to the cap; some of the wire is then wrapped round an insulating rod, and the free end of the wire is placed on the conductor. As the free end is moved over the conductor, it is observed that the divergence of the leaf remains constant. This result was explained on pp. 756, 757.

Electrostatic Shielding

The fact that there is no electric field inside a close conductor, when it contains no charged bodies, was demonstrated by Faraday in a spectacular manner. He made for himself a large wire cage, supported it on insulators, and sat inside it with his electroscopes. He then had the cage charged by an induction machine—a forerunner of the type we described on p. 737—until painful sparks could be drawn from its outside. Inside the cage Faraday sat in safety and comfort, and there was no deflection to be seen on even his most sensitive electroscope.

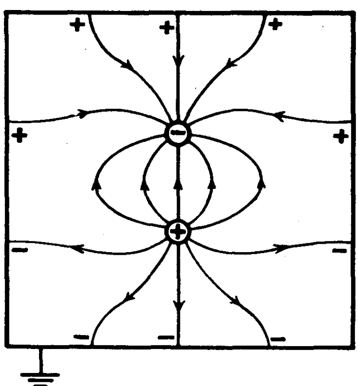


FIG. 30.36. Lines of force round charges.

If we wish to protect any persons or instruments from intense electric fields, therefore, we enclose them in hollow conductors; these are called 'Faraday cages', and are widely used in high-voltage measurements in industry.

We may also wish to prevent charges in one place from setting up an electric field beyond their immediate neighbourhood. To do this we surround the charges with a Faraday cage, and connect the cage to earth (Fig. 30.36). The charge induced on the outside of the cage then runs to earth, and there is no external field. (When a cage is used to shield something *inside* it, it does not have to be earthed.)

Comparison of Static and Current Phenomena

Broadly speaking, we may say that in electrostatic phenomena we meet small quantities of charge, but great differences of potential. On the other hand in the phenomena of current electricity discussed later, the potential differences are small but the amounts of charge transported by the currents are great. Sparks and shocks are common in electrostatics, because they require great potential differences; but they are rarely dangerous, because the total amount of energy available is usually small. On the other hand, shocks and sparks in current electricity are rare, but, when the potential difference is great enough to cause them, they are likely to be dangerous.

These quantitative differences make problems of insulation much more difficult in electrostatic apparatus than in apparatus for use with currents. The high potentials met in electrostatics make leakage currents relatively great, and the small charges therefore tend to disappear rapidly. Any wood, for example, ranks as an insulator for current electricity, but a conductor in electrostatics. In electrostatic experiments we sometimes wish to connect a charged body to earth; all we have then to do is to touch it.

EXAMPLE

Three charges $-1\ \mu$C, $2\ \mu$C and $3\ \mu$C are placed respectively at the corners A, B, C of an equilateral triangle of side 2 metres. Calculate (*a*) the potential, (*b*) the electric field, at a point X which is half-way along BC. Fig. 30.37.

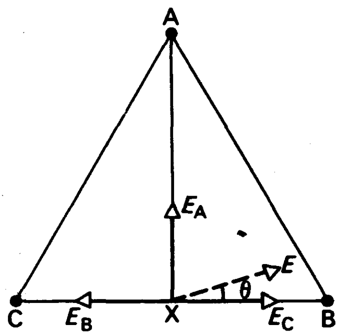


FIG. 30.37. Example.

(*a*) Potential at X due to charge at B is

$$\frac{Q}{4\pi\varepsilon_0 r} = \frac{2\times10^{-6}}{4\pi\varepsilon_0\times1}$$
$$= 18\times10^3\ \text{V}.$$

Similarly potential at X due to the charge at $C = 27\times10^3$ V, and the potential due to A is

$$V_A = \frac{Q}{4\pi\varepsilon_0 r} = \frac{-10^{-6}}{4\pi\varepsilon_0\times\sqrt{3}}$$
$$= -5\times10^3\ \text{V (approx.)}.$$

Since potential is a scalar quantity and can be added algebraically, the net potential at X

$$= (18+27-5)\times10^3\ \text{V}$$
$$= 40\times10^3\ \text{V}.$$

(*b*) The resultant field at X is due to the three electric fields from the three charges.

The field due to B, E_B has magnitude given by

$$E_B = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{2\times10^{-6}}{4\pi\varepsilon_0\times1^2}$$
$$= 18\times10^3\ \text{V m}^{-1}.$$

Similarly $\qquad E_C = 27\times10^3\ \text{V m}^{-1}.$

Since these act along the same straight line the resultant of E_B and $E_C = 9\times10^3$ V m^{-1} directed from C to B.

Also,
$$E_A = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{10^{-6}}{4\pi\varepsilon_0\times(\sqrt{3})^2}$$
$$= 3\times10^3 \text{ V m}^{-1}.$$

The resultant field has magnitude, E, given by
$$E^2 = E_A^2+(E_C-E_B)^2$$
$$= (9+81)\times10^6$$
$$= 90\times10^6$$
$$E = 3\sqrt{10}\times10^3 \text{ V m}^{-1} = 9{\cdot}5\times10^3 \text{ V m}^{-1}.$$

This makes an angle θ with CB, in the direction shown by the dotted line, where
$$\tan\theta = \frac{E_A}{E_C-E_B} = \frac{3\times10^3}{9\times10^3} = \frac{1}{3}$$
$$\therefore \quad \theta = 18^\circ\, 25'.$$

EXERCISES 30

1. Describe experiments with a gold leaf electroscope:
(*a*) to demonstrate that this instrument indicates the potential difference between its leaves and its case and not necessarily the total charge on its leaves and cap;
(*b*) to compare the quantities of electricity on two conductors of unequal size;
(*c*) to investigate the distribution of electricity on a charged conductor.
In case (*c*) state the results you would expect if the conductor were spherical with a pointed rod attached to it, and describe and explain two practical applications of pointed conductors. (*L.*)

2. Describe, with the aid of a labelled diagram, a Van de Graaff generator, explaining the physical principles of its action.

The high voltage terminal of such a generator consists of a spherical conducting shell of radius 50 cm. Estimate the maximum potential to which it can be raised in air for which electrical breakdown occurs when the electric intensity exceeds 30000 volt cm^{-1}.

State two ways in which this maximum potential could be increased. (*N.*)

3. Define *potential at a point* in an electric field.

Sketch a graph illustrating the variation of potential along a radius from the centre of a charged isolated conducting sphere to infinity.

Assuming the expression for the potential of a charged isolated conducting sphere in air, determine the change in the potential of such a sphere caused by surrounding it with an earthed concentric thin conducting sphere having three times its radius. (*N.*)

4. What is meant by the terms (*a*) *potential* and (*b*) *field strength* in electrostatics? State whether each quantity is a scalar or a vector.

Write down the law which gives the force between two point charges q_1 and q_2 at a distance r apart and use it to derive the electric field strength and the potential due to a point charge q at a distance x from it.

The points A, B and C form an equilateral triangle of side z. Point charges of equal magnitude q are placed at A and B. Find the electric field strength and the potential at C due to these charges when (i) both charges are positive and (ii) the charge at A is positive and the charge at B is negative.

O is the midpoint of AB and POQ is the perpendicular bisector of AB; PO and OQ are very large distances compared with AB. Draw rough graphs to show how the magnitude of the electric field strength and the potential vary along the line POQ in cases (i) and (ii) above. (*O. & C.*)

5. Describe simple electrostatic experiments to illustrate *two* of the following: (*a*) the production of equal and opposite charges by induction; (*b*) the action of points; (*c*) the effect on a charged conductor of the approach of an earthed conductor. (*L.*)

6. Discuss the following, giving examples: *Electrostatic induction, electric discharge from points* and *dielectric strength.*

Describe a modern form of apparatus for obtaining a small current at a very high voltage. (*L.*)

7. An isolated conducting spherical shell of radius 10 cm, in vacuo, carries a positive charge of $1{\cdot}0 \times 10^{-7}$ coulomb. Calculate (*a*) the electric field intensity, (*b*) the potential, at a point on the surface of the conductor. Sketch a graph to show how one of these quantities varies with distance along a radius from the centre to a point well outside the spherical shell. Point out the main features of the graph. (*N.*)

8. Define (*a*) electric intensity, (*b*) difference of potential. How are these quantities related?

A charged oil-drop of radius 0·00013 cm is prevented from falling under gravity by the vertical field between two horizontal plates charged to a difference of potential of 8340 volts. The distance between the plates is 1·6 cm, and the density of oil is 920 kg m^{-3}. Calculate the magnitude of the charge on the drop ($g = 9{\cdot}81$ m s^{-2}). (*O. & C.*)

9. Two plane parallel conducting plates 1·50 cm apart are held horizontal, one above the other, in air. The upper plate is maintained at a positive potential of 1500 volts while the lower plate is earthed. Calculate the number of electrons which must be attached to a small oil drop of mass $4{\cdot}90 \times 10^{-12}$ g, if it remains stationary in the air between the plates. (Assume that the density of air is negligible in comparison with that of oil.)

If the potential of the upper plate is suddenly changed to -1500 volts what is the initial acceleration of the charged drop? Indicate, giving reasons, how the acceleration will change.

10. Show how (i) the surface density, (ii) the intensity of electric field, (iii) the potential, varies over the surface of an elongated conductor charged with electricity. Describe experiments you would perform to support your answer in cases (i) and (iii).

Describe and explain the action of points on a charged conductor; and give two practical applications of the effect. (*L.*)

11. Describe carefully Faraday's ice-pail experiments and discuss the deductions to be drawn from them. How would you investigate experimentally the charge distribution over the surface of a conductor? (*C.*)

12. What is an *electric field*? With reference to such a field define *electric potential.*

Two plane parallel conducting plates are held horizontal, one above the other, in a vacuum. Electrons having a speed of $6{\cdot}0 \times 10^{8}$ cm s^{-1} and moving normally to the plates enter the region between them through a hole in the lower plate which is earthed. What potential must be applied to the other plate so that the electrons just fail to reach it? What is the subsequent motion of these electrons? Assume that the electrons do not interact with one another.

(Ratio of charge to mass of electron is $1{\cdot}8 \times 10^{11}$ coulomb kg^{-1}.) (*N.*)

13. Define the *electric potential V* and the *electric field strength E* at a point in an electrostatic field. How are they related? Write down an expression for the electric field strength at a point close to a charged conducting surface, in terms of the surface density of charge.

Corona discharge into the air from a charged conductor takes place when the potential gradient at its surface exceeds 3×10^6 volt metre^{-1}; a potential gradient of this magnitude also breaks down the insulation afforded by a solid dielectric. Calculate the greatest charge that can be placed on a conducting sphere of radius 20 cm supported in the atmosphere on a long insulating pillar; also calculate the corresponding potential of the sphere. Discuss whether this potential could be achieved if the pillar of insulating dielectric was only 50 cm long. (Take ε_0 to be $8{\cdot}85 \times 10^{-12}$ farad metre^{-1}.) (*O.*)

14. (i) A needle is mounted vertically, point upwards, on the plate (cap) of a gold leaf electroscope, the blunt end being in metallic contact with the plate. When a negatively charged body is brought close to the needle point, without touching it, and is then withdrawn the gold leaf is left with a permanent deflection. What is the sign of the charge causing this deflection, and how was this charge produced?

(ii) A gold leaf electroscope is so constructed that for a few degrees deflection the gold leaf touches the case and is thereby earthed. Describe and explain the behaviour of the leaf when:

(*a*) a positively charged, insulated body is brought towards the plate (cap) of the electroscope until the leaf touches the case;
(*b*) the positively charged body is then moved slowly closer to the plate;
(*c*) the positively charged body is fixed close to the plate and the air between them is feebly ionized. (*O. & C.*)

chapter thirty-one

Capacitors

A capacitor (or 'condenser'), is a device for storing electricity. The earliest capacitor was invented—almost accidentally—by van Musschenbroek of Leyden, in about 1746, and became known as a Leyden jar. A present-day form of it is shown in Fig. 31.1 (i), J is a glass jar, FF are tin-foil coatings over the lower parts of its walls, and T is a knob connected to the inner coating. Modern forms of capacitor are shown at (ii) and (iv) in the figure. Essentially, all capacitors consist of two metal plates separated by an insulator. The insulator is called the dielectric; in some capacitors it is oil or air. Fig. 31.1 (iii) shows the conventional symbol for a capacitor.

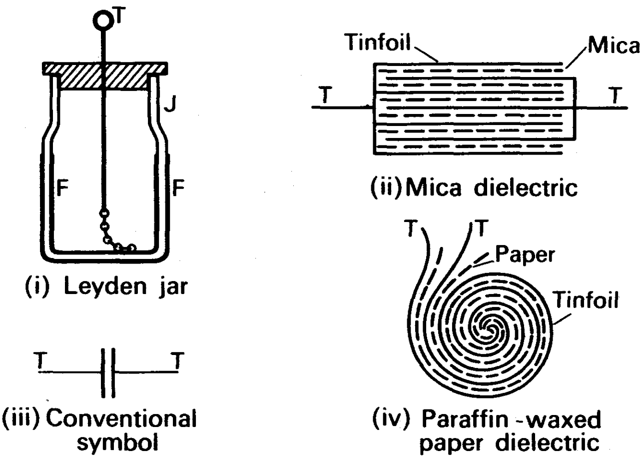


FIG. 31.1. Types of capacitor.

Charging a Capacitor

To study the action of a capacitor we need a paper one of about 4 microfarad capacitance (see later), a couple of high-tension batteries D and a high impedance voltmeter V such as a valve voltmeter reading to about 300 volts. We also need a two-way key (K in Fig. 31.2) and a poor conductor (R). The latter is a short stick of powdered and compressed carbon; it is called a radio resistor, and should have a resistance of about 5 megohms (p. 790). We connect the batteries in series, and measure their total voltage, V_0, with the voltmeter (Fig. 31.2 (i)). We then connect up all the apparatus as shown in Fig. 31.2 (ii). If we close the key at A, the capacitor is connected via the resistor to the battery, and the potential difference across the capacitor, V, which is measured

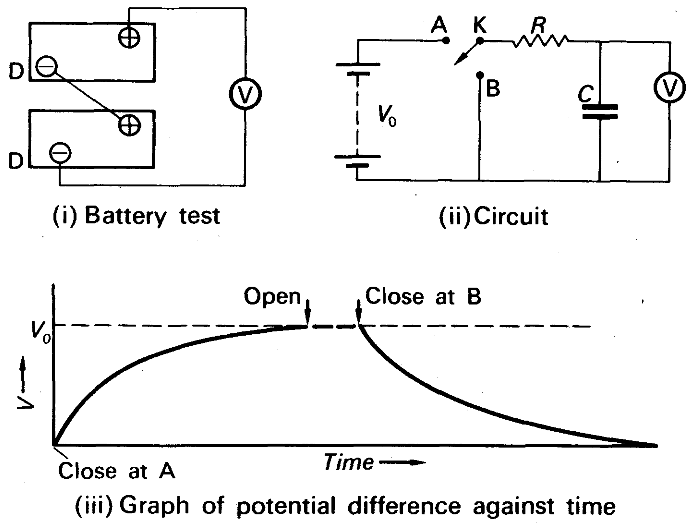


FIG. 31.2. Charging and discharging a capacitor through a resistor.

by the voltmeter, begins to rise (Fig. 31.2 (iii)). The potential difference becomes steady when it is equal to the battery voltage V_0. If we now open the key, the voltmeter reading stays unchanged (unless the capacitor is leaky). The capacitor is said to be charged, to the battery voltage; its condition does not depend at all on the resistor, whose only purpose was to slow down the charging process, so that we could follow it on the voltmeter.

Discharging a Capacitor

We can show that the charged capacitor is storing electricity by discharging it: if we put a piece of wire across its terminals, a fat spark passes just as the wire makes contact, and the voltmeter reading falls to zero.

If we now recharge the capacitor and then close the key at B, in Fig 31.2 (ii), we allow the capacitor to discharge through the resistor R. The potential difference across it now falls to zero as slowly as it rose during charging.

Charging and Discharging Processes

When we connect a capacitor to a battery, electrons flow from the negative terminal of the battery on to the plate A of the capacitor connected to it (Fig. 31.3); and, at the same rate, electrons flow from the other plate B of the capacitor towards the positive terminal of the battery. Positive and negative charges thus appear on the plates, and oppose the flow of electrons which causes them. As the charges accumulate, the potential difference between the plates increases, and the charging current falls to zero when the potential difference becomes equal to the battery voltage V_0.

When the battery is disconnected and the plates are joined together by a wire, electrons flow back from plate A to plate B until the positive charge on B is completely neutralized. A current thus flows for a time in the wire, and at the end of the time the charges on the plates become zero.

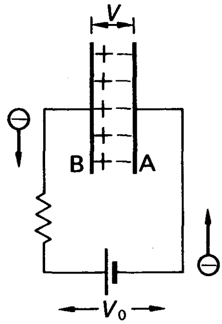


FIG. 31.3. A capacitor charging. (Resistance is shown because some is always present, even if only that of the connecting wires.)

Capacitors in A.C. Circuits

Capacitors are widely used in alternating current and radio circuits, because they can transmit alternating currents. To see how they do so, let us consider the circuit of Fig. 31.4, in which the capacitor may be connected across either of the batteries X, Y. When the key is closed at A, current flows from the battery X, and charges the plate D of the capacitor positively. If the key is now closed at B instead, current flows from the battery Y; the plate D loses its positive charge and becomes negatively charged. Thus if the key is rocked rapidly between A and B, current surges backwards and forwards along the wires connected to the capacitor. An alternating voltage, as we shall see in Chapter 39, is one which reverses many times a second; when such a voltage is applied to a capacitor, therefore, an alternating current flows in the connecting wires.

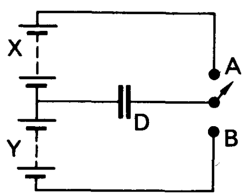


FIG. 31.4. Reversals of voltage applied to capacitor.

Capacitance Definition, and Units

Experiments with a ballistic galvanometer, which measures quantity of electricity, show that, when a capacitor is charged to a potential difference V, the charges stored on its plates, $\pm Q$, are proportional to V. The ratio of the charge on either plate to the potential difference between the plates is called the *capacitance*, C, of the capacitor:

$$C = \frac{Q}{V}. \qquad (1)$$

Thus

$$Q = CV, \qquad (2)$$

and

$$V = \frac{Q}{C}. \qquad (3)$$

When Q is in coulombs (C) and V is in volts (V), then capacitance C is in farads (F). One farad (1F) is the capacitance of an extremely large capacitor. In practical circuits, such as in radio receivers, the capacitance of capacitors used are therefore expressed in *microfarads* (μF). One microfarad is one millionth part of a farad, that is, $1\mu F = 10^{-6} F$. It is also quite usual to express small capacitors, such as those used on radiograms for altering tone, in *picofarads* (pF). A picofarad is one millionth part of a microfarad, that is, $1 pF = 10^{-6}\mu F = 10^{-12} F$.

Comparison of Capacitances

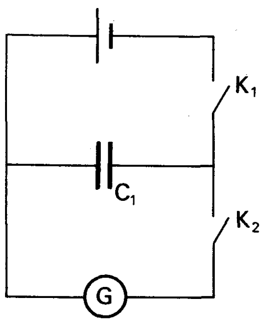


FIG. 31.5. Comparison of capacitances.

Large capacitances, of the order of microfarads, can be compared with the aid of a ballistic galvanometer. In this instrument, as explained on p. 920, the first 'throw' or deflection is proportional to the quantity of electricity discharged through it.

The circuit required is shown in Fig. 31.5. The capacitor of capacitance C_1 is charged by a battery of e.m.f. V, and then discharged through the ballistic galvanometer G. The corresponding first deflection θ_1 is observed. The capacitor is now replaced by another of capacitance C_2, charged again by the battery, and the new deflection θ_2 is observed when the capacitor is discharged.

Now

$$\frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2}.$$

$$\therefore \frac{C_1 V}{C_2 V} = \frac{\theta_1}{\theta_2}.$$

$$\therefore \frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}. \qquad (4)$$

If C_2 is a standard capacitor, whose value is known, then the capacitance of C_1 can be found.

Factors determining Capacitance. Variable Capacitor

We shall now find out by experiment what factors influence capacitance. To interpret our observations we shall require the formula for potential difference:

$$V = \frac{Q}{C}.$$

This shows that, when a capacitor is given a fixed charge, the potential difference between its plates is inversely proportional to its capacitance.

Distance between plates. In Fig. 31.6 (i), A and B are two metal plates, B being earthed, while A is insulated and connected to an electroscope.

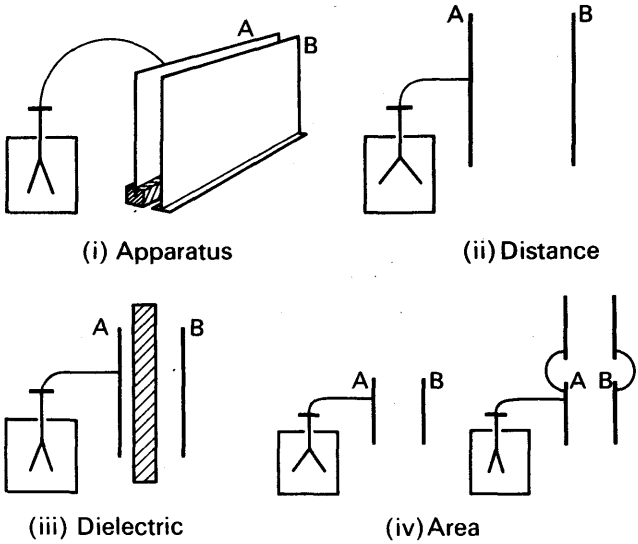


FIG. 31.6. Factors determining capacitance.

We set the plates close together, but not touching, and charge A from an electrophorus. The leaves of the electroscope diverge by an amount which measures the potential difference between the plates. If we move the plates further apart the leaves diverge further, showing that the potential difference has increased (Fig. 31.6 (ii)). Since we have done nothing to increase the charge on the plates, the increase in potential difference must be due to a decrease in capacitance (see above equation). The capacitance of a capacitor therefore decreases when the separation of its plates is increased; we shall see in the next chapter that the capacitance is inversely proportional to the separation.

Dielectric. Let us now put a dielectric—a sheet of glass or ebonite—between the plates. The leaves diverge less, showing that the potential difference has decreased (Fig. 31.6 (iii)). The capacitance has therefore increased, and the increase is due to the dielectric. By using several sheets of it we can show that the effect of the dielectric increases with its thickness. In practical capacitors the dielectric completely fills the space between the plates.

Area of plates. To see how the capacitance depends on the area of the plates, we set them at a known distance apart. We then take another pair of plates, at the same distance apart, and connect them to the first pair by wires held on insulating handles (Fig. 31.6 (iv)). The leaves diverge less, showing that the capacitance has increased; it is, in fact, directly proportional to the area of the plates.

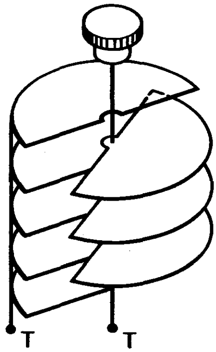


FIG. 31.7. Variable air capacitor.

A capacitor in which the effective area of the plates can be adjusted is called a *variable capacitor*. In the type shown in Fig. 31.7, the plates are semicircular and one set can be swung into or out of the other. The capacitance is proportional to the area of overlap of the plates. The plates are made of brass or aluminium, and the dielectric may be air, oil, or mica.

We shall see shortly that a capacitor with parallel plates, having a vacuum (or air, if we assume the permittivity of air is the same as a vacuum) between them, has a capacitance given by

$$C = \frac{\varepsilon_0 A}{d}$$

where C = capacitance in farads (F), A = area of overlap of plates in m^2, d = distance between plates in m and $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}$ farad metre^{-1}.

If a material of permittivity ε completely fills the space between the plates, then the capacitance becomes:

$$C = \frac{\varepsilon A}{d}$$

Capacitance Values. Isolated Sphere

Suppose a sphere of radius r metre situated in air is given a charge of Q coulombs. We assume, as on p. 747, that the charge on a sphere gives rise to potentials *on and outside* the sphere as if all the charge were concentrated at the *centre*. From p. 757, the surface of the sphere has a potential given by:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\therefore \frac{Q}{V} = 4\pi\varepsilon_0 r$$

$$\therefore \text{Capacitance, } C = 4\pi\varepsilon_0 r \quad . \quad . \quad . \quad . \quad (1)$$

The other 'plate' of the capacitor is the earth.

Suppose $r = 10$ cm $= 0{\cdot}1$ m. Then,

$$C = 4\pi\varepsilon_0 r = 4\pi \times 8{\cdot}85 \times 10^{-12} \times 0{\cdot}1 \text{ F}$$

$$= 11 \times 10^{-12} \text{ F (approx.)} = 11 \text{ pF}.$$

Concentric Spheres

Faraday used two concentric spheres to investigate the dielectric constant of liquids. Suppose a, b are the respective radii of the inner and outer spheres (Fig. 31.8). Let $+Q$ be the charge given to the inner sphere and let the outer sphere be earthed, with air between them.

The induced charge on the outer sphere is $-Q$ (see p. 740). The potential V_a of the inner sphere = potential due to $+Q$ and potential due to $-Q$ = $+\frac{Q}{4\pi\varepsilon_0 a}-\frac{Q}{4\pi\varepsilon_0 b}$, since the potential due to the charge $-Q$ is $-Q/4\pi\varepsilon_0 b$ everywhere inside the larger sphere (see p. 757).

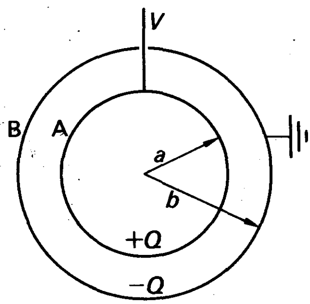


FIG. 31.8. Concentric spherical capacitor.

But $V_b = 0$, as the outer sphere is earthed.

$$\therefore \text{ potential difference, } V, = V_a - V_b = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{a}-\frac{Q}{b}\right)$$

$$\therefore V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right).$$

$$\therefore \frac{Q}{V} = \frac{4\pi\varepsilon_0 ab}{b-a},$$

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As an example, suppose $b = 10$ cm and $a = 9$ cm

$$\therefore C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

$$= \frac{4\pi \times 8{\cdot}85 \times 10^{-12} \times 0{\cdot}1 \times 0{\cdot}09}{(0{\cdot}1-0{\cdot}09)}$$

$$= 100 \text{ pF (approx).}$$

Note that the inclusion of a nearby second plate to the capacitor increases the capacitance. For an *isolated* sphere of radius 10 cm, the capacitance was 11 pF (p. 770).

Parallel Plate Capacitor

Suppose two parallel plates of a capacitor each have a charge numerically equal to Q, Fig. 31.9. The surface density σ is then Q/A

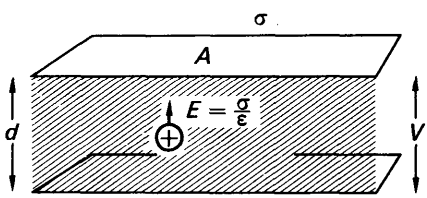


FIG. 31.9. Parallel-plate capacitor.

where A is the area of either plate, and the intensity between the plates, E, is given, from p. 749, by

$$E = \frac{\sigma}{\varepsilon} = \frac{Q}{\varepsilon A}$$

Now the work done in taking a unit charge from one plate to the other = force × distance = $E \times d$ where d is the distance between the plates. But the work done per unit charge = V, the p.d. between the plates.

$$\therefore V = \frac{\sigma}{\varepsilon}d = \frac{Q}{\varepsilon A}d$$

$$\therefore \frac{Q}{V} = \frac{\varepsilon A}{d}$$

$$\therefore C = \frac{\varepsilon A}{d} \qquad . \quad . \quad . \quad . \quad . \qquad (1)$$

It should be noted that this formula for C is approximate, as the field becomes non-uniform at the edges. See Fig. 30.30, p. 755.

Dielectric Constant (Relative Permittivity) and Strength

To study the effect of the dielectric in a capacitor, the capacitance of a given capacitor must be measured: first without a dielectric, and then with one. We shall see later how this can be done (p. 776).

The ratio of the capacitance with and without the dielectric between the plates is called the *dielectric constant* (or *relative permittivity*) of the material used. The expression 'without a dielectric' strictly means 'with the plates in a vacuum'; but the effect of air on the capacitance of a capacitor is so small that for most purposes it may be neglected. The dielectric constant of a substance is denoted by the letter ε: thus

$$\varepsilon = \frac{\text{capacitance of given capacitor, with space between plates filled with dielectric}}{\text{capacitance of same capacitor with plates } \textit{in vacuo}}$$

If we take the case of a parallel plate capacitor as an example, then

$$\varepsilon_r = \frac{\varepsilon A/d}{\varepsilon_0 A/d} = \frac{\varepsilon}{\varepsilon_0}$$

Thus the dielectric constant is the ratio of the permittivity of the substance to that of free space. It is for this reason that dielectric constant is also known as 'relative permittivity'. Note that the dielectric constant is a pure number and has no dimensions, unlike ε or ε_0.

The following table gives the value of dielectric constant, and also of *dielectric strength*, for various substances. The strength of a dielectric is the potential gradient at which its insulation breaks down, and a spark passes through it. A solid dielectric is ruined by such a breakdown, but a liquid or gaseous one heals up as soon as the applied potential difference is reduced.

Water is not suitable for a dielectric in practice, because it is a good insulator only when it is very pure, and to remove all matter dissolved in it is almost impossible.

PROPERTIES OF DIELECTRICS

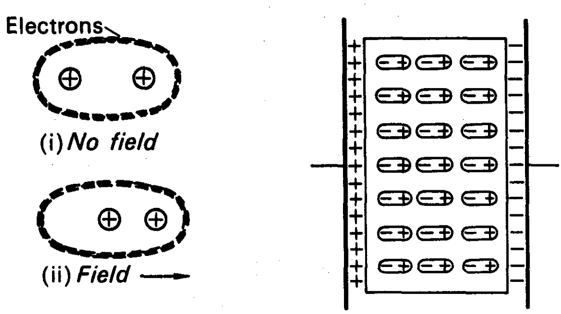
Substance	Dielectric constant	Dielectric strength, kilovolts per mm
Glass	5–10	30–150
Mica	6	80–200
Ebonite	2·8	30–110
Ice*	94	—
Paraffin wax	2	15–50
Paraffined paper	2	40–60
Paraffin oil	4·7	—
Methyl alcohol*	32	—
Water*	81	—
Air	1·0005	—
Sulphur dioxide*	1·01	—

* Polar molecules (see p. 774).

Action of Dielectric

The explanation of dielectric action which we shall now give is similar in principle to Faraday's, but expressed in modern terms—there was no knowledge of electrons in his day.

We regard a molecule as a collection of atomic nuclei, positively charged, and surrounded by a cloud of negative electrons. When a dielectric is in a charged capacitor, its molecules are in an electric field; the nuclei are urged in the direction of the field, and the electrons in the opposite direction (Fig. 31.10 (i)). Thus each molecule is distorted, or polarized: one end has an excess of positive charge, the other an excess of negative. At the surfaces of the dielectric, therefore, charges appear, as shown in Fig. 31.10 (ii). These charges are of opposite sign to the charges on the plates, and so reduce the potential difference between the plates.



(i) Single molecule (ii) Between capacitor plates

FIG. 31.10. Polarization of a dielectric.

If the capacitor is connected to a battery, then its potential difference is constant; but the surface charges on the dielectric still increase its capacitance. They do so because they offset the mutual repulsions of the charges on the plates, and so enable greater charges to accumulate there before the potential difference rises to the battery voltage.

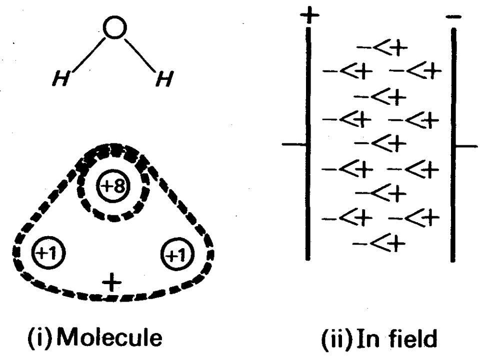


FIG. 31.11. Water as a dielectric.

Some molecules, we believe, are permanently polarized: they are called *polar molecules*. For example, the water molecule consists of an oxygen atom, O, with two hydrogen atoms H, making roughly a right-angled structure (Fig. 31.11 (i)). Oxygen has a nuclear charge of $+8e$, where e is the electronic charge, and has eight electrons. Hydrogen has a nuclear charge of $+e$, and one electron. In the water molecule, the two electrons from the hydrogen atom move in paths which surround the oxygen nucleus. Thus they are partly added to the oxygen atom, and partly withdrawn from the hydrogen atoms. On the average, therefore, the apex of the triangle is negatively charged, and its base is positively charged. In an electric field, water molecules tend to orient themselves as shown in Fig. 31.11 (ii). The effect of this, in a capacitor, is to increase the capacitance in the way already described. The increase is, in fact, much greater than that obtained with a dielectric which is polarized merely by the action of the field.

ε_0 and its measurement

We can now see how the units of ε_0 may be stated in a more convenient manner and how its magnitude may be measured.

Units. From $C = \dfrac{\varepsilon_0 A}{d}$, we have $\varepsilon_0 = \dfrac{Cd}{A}$

$$\text{Thus the unit of } \varepsilon_0 = \frac{\text{farad} \times \text{metre}}{\text{metre}^2}$$

$$= \text{farad metre}^{-1} \text{ (see also p. 743).}$$

Measurement. In order to find the magnitude of ε_0, the circuit in Fig. 31.12 is used.

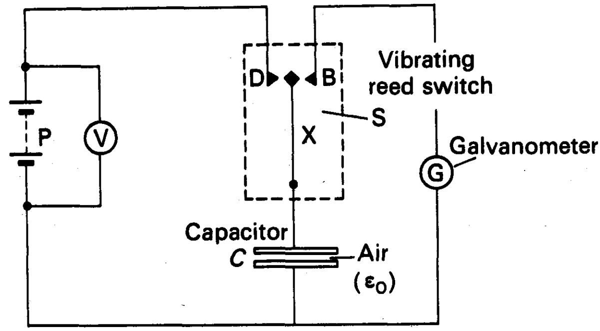


FIG. 31.12. Measurement of ε_0.

C is a parallel plate capacitor, which may be made of sheets of glass or perspex coated with aluminium foil or aquadag. The two conducting surfaces are placed facing inwards, so that only air is present between these plates. The area A of the plates in metre^2, and the separation d in metres, are measured. P is a high tension supply capable of delivering about 200 V, and G is a calibrated sensitive galvanometer, such as a 'Scalamp' type. S is a *vibrating switch* unit, energized by a low a.c. voltage from the mains. When operating, the vibrating bar X touches and then B, and the motion is repeated at the mains frequency, fifty times a second. When the switch is in contact with D, the capacitor is charged from the supply P to a potential difference of V volt, measured on the voltmeter. When the contact moves over to B, the capacitor discharges through the galvanometer. The galvanometer thus receives fifty pulses of charge per second. This gives an average steady reading on the galvanometer, corresponding to a mean current I.

Now from previous, $C = \varepsilon_0 A/d$. Thus on charging, the charge stored, Q, is given by

$$Q = CV = \frac{\varepsilon_0 VA}{d}.$$

The capacitor is discharged fifty times per second. Since the current is the charge flowing per second,

$$\therefore \quad I = \frac{\varepsilon_0 VA\,.\,50}{d}\text{ ampere}$$

$$\therefore \quad \varepsilon_0 = \frac{Id}{50VA}\text{ farad metre}^{-1}$$

The following results were obtained in one experiment:

$A = 0{\cdot}0317\text{ m}^2, d = 1{\cdot}0\text{ cm} = 0{\cdot}010\text{ m}, V = 150\text{ V}, I = 0{\cdot}21 \times 10^{-6}\text{A}$

$$\therefore \varepsilon_0 = \frac{Id}{50\,VA}$$

$$= \frac{0{\cdot}21 \times 10^{-6} \times 0{\cdot}01}{50 \times 150 \times 0{\cdot}0317}$$

$$= 8{\cdot}8 \times 10^{-12}\text{ farad metre}^{-1}$$

As very small currents are concerned, care must be taken to make the apparatus of high quality insulating material, otherwise leakage currents will lead to serious error.

This method can also be used to find the permittivity of various materials. Thus if the experiment is repeated with a material of permittivity ε completely filling the space between the plates, then $\varepsilon = I'd/50\ VA$, where I' is the new current.

If only the relative permittivity or dielectric constant, ε_r, is required, there is no need to know the p.d. supplied or the dimensions of the capacitor. In this case,

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{I'd/50\ VA}{Id/50\ VA} = \frac{I'}{I}$$

Thus ε_r is the ratio of the respective currents in G with and without the dielectric between the plates.

Measurement of Capacitance

If a standard capacitor is available, an unknown capacitance can be measured as described on p. 768.

If no standard capacitor is available, the method of the last section can be employed. The unknown capacitor replaces C in the circuit of Fig. 31.12. The current I in the galvanometer, and the p.d. supplied, V, are then measured. Now the charge $Q = CV$. This is discharged fifty times per second. Since the current is the charge flowing per second,

$$\therefore\ I = 50\ CV$$

$$\therefore\ C = \frac{I}{50\ V}\ \text{farad.}$$

Arrangements of Capacitors

In radio circuits, capacitors often appear in arrangements whose resultant capacitances must be known. To derive expressions for these, we need the equation defining capacitance in its three possible forms:

$$C = \frac{Q}{V}, \qquad V = \frac{Q}{C}, \qquad Q = CV.$$

In Parallel. Fig. 31.13 shows three capacitors, having all their left-hand plates connected together, and all their right-hand plates likewise.

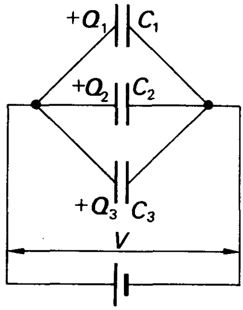


FIG. 31.13. Capacitors in parallel.

They are said to be connected in parallel. If a cell is now connected across them, they all have the same potential difference V. (For, if they had not, current would flow from one to another until they had.) The charges on the individual capacitors are respectively

$$\begin{aligned} Q_1 &= C_1 V \\ Q_2 &= C_2 V \\ Q_3 &= C_3 V \end{aligned} \qquad . \quad . \quad . \quad . \quad (1)$$

The total charge on the system of capacitors is

$$Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)V.$$

And the system is therefore equivalent to a single capacitor, of capacitance

$$C = \frac{Q}{V} = C_1 + C_2 + C_3.$$

Thus when capacitors are connected in parallel, their resultant capacitance is the sum of their individual capacitances. It is greater than the greatest individual one.

In Series. Fig. 31.14 shows three capacitors having the right-hand plate of one connected to the left-hand of the next, and so on— connected in series. When a cell is connected across the ends of the system, a charge Q is transferred from the plate H to the plate A, a charge $-Q$ being left on H. This charge induces a charge $+Q$ on plate G; similarly, charges

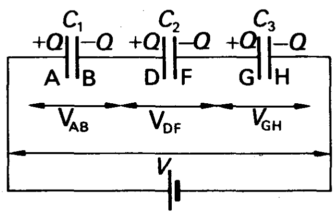


FIG. 31.14. Capacitors in series.

appear on all the other capacitor plates, as shown in the figure. (The induced and inducing charges are equal because the capacitor plates are very large and very close together, in effect, either may be said to enclose the other.) The potential differences across the individual capacitors are, therefore, given by

$$\left.\begin{aligned} V_{AB} &= \frac{Q}{C_1} \\ V_{DF} &= \frac{Q}{C_2} \\ V_{GH} &= \frac{Q}{C_3} \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad (2)$$

The sum of these is equal to the applied potential difference V because the work done in taking a unit charge from H to A is the sum of the work done in taking it from H to G, from F to D, and from B to A. Therefore

$$V = V_{AB} + V_{DF} + V_{GH}$$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right). \quad . \quad . \quad . \quad (3)$$

The resultant capacitance of the system is the ratio of the charge stored to the applied potential difference, V. The charge stored is equal to Q, because, if the battery is removed, and the plates HA joined by a wire, a charge Q will pass through that wire, and the whole system will be discharged. The resultant capacitance is therefore given by

$$C = \frac{Q}{V}, \quad \text{or} \quad \frac{1}{C} = \frac{V}{Q},$$

whence, by equation (3),

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \quad . \quad . \quad . \quad (4)$$

Thus, to find the resultant capacitance of capacitors in series, we must add their reciprocals of the individual capacitances. The resultant is less than the smallest individual.

Comparison of Series and Parallel Arrangements. Let us compare Figs. 31.13 and 13.14. In Fig. 31.14, where the capacitors are in series, all the capacitors carry the same charge, which is equal to the charge carried by the system as a whole, Q. The potential difference applied to the system, however, is divided amongst the capacitors, in inverse proportion to their capacitances (equations (2)). In Fig. 31.13, where the capacitors are in parallel, they all have the same potential difference; but the charge stored is divided amongst them, in direct proportion to the capacitances (equations (1)).

EXAMPLE

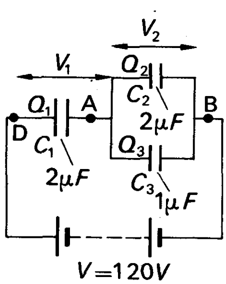


FIG. 31.15. Example.

Find the charges on the capacitors in Fig. 31.15, and the potential differences across them.

Capacitance between A and B,

$$C' = C_2 + C_3 = 3\ \mu\text{F}.$$

Overall capacitance A to D,

$$C = \frac{C_1 C'}{C_1 + C'} = \frac{2 \times 3}{2+3} = 1{\cdot}2\ \mu\text{F}.$$

Charge stored in this

$$= Q_1 = Q_2 + Q_3 = CV = 1{\cdot}2 \times 10^{-6} \times 120$$
$$= 144 \times 10^{-6} \text{ coulomb},$$

$$\therefore \quad V_1 = \frac{Q_1}{C_1} = \frac{144\times 10^{-6}}{2\times 10^{-6}} = 72 \text{ volt},$$

$$V_2 = V - V_1 = 120 - 72 = 48 \text{ volt},$$

$$Q_2 = C_2V_2 = 2\times 10^{-6}\times 48 = 96\times 10^{-6} \text{ coulomb},$$

$$Q_3 = C_3V_2 = 10^{-6}\times 48 = 48\times 10^{-6} \text{ coulomb}.$$

Energy of a Charged Capacitor

A charged capacitor is a store of electrical energy, as we may see from the vigorous spark it can give on discharge. To find the energy stored, let us suppose that the capacitor, of capacitance C, is already charged to a potential difference V. And let us suppose that we wish to increase the charge on its plates from Q to $Q+\delta Q$, where δQ is very small. Then we must transfer a charge δQ from the negative plate to the positive. In doing so, we shall increase the potential difference by the amount

$$\delta V = \frac{\delta Q}{C}.$$

But if δQ is very small compared with Q, δV will be very small compared with V, and the potential difference will be almost constant at the value V. Then the work done in displacing the charge δQ will be

$$\delta W = V\delta Q.$$

from the definition of potential difference. But

$$V = \frac{Q}{C},$$

and therefore

$$\delta W = \frac{Q}{C}\delta Q.$$

If we now suppose that the capacitor is at first completely discharged, and the charged until the final charge on the plates has some definite value Q_1, then the work done in charging it is

$$\int_{Q=0}^{Q=Q_1} dW = \int_0^{Q_1} \frac{QdQ}{C} = \left[\frac{1}{2}\frac{Q^2}{C}\right]_0^{Q_1} = \frac{1}{2}\frac{Q_1^2}{C}.$$

In general, therefore, the energy stored by a capacitor of capacitance C, carrying a charge Q, at a potential difference V, is

$$\begin{aligned} W &= \tfrac{1}{2}(Q^2/C) \\ &= \tfrac{1}{2}QV \qquad \qquad (1) \\ &= \tfrac{1}{2}CV^2 \end{aligned}$$

If C is measured in farad, Q in coulomb and V in volt, then the expressions derived in (1) will give the energy W in joules.

EXAMPLES

1. Define a *capacitance* of a capacitor. Explain how, using a capacitor in conjunction with a gold-leaf electroscope, the voltage sensitivity of the electroscope may be increased.

Two capacitors, of capacitance 4 μF and 2 μF respectively, are joined in series with a battery of e.m.f. 100 volts. The connexions are broken and the like terminals of the capacitors are then joined. Find the final charge on each capacitor. (*L.*)

The combined capacitance, C, of the capacitors is given by

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2},$$

$$\therefore \quad \frac{1}{C}=\frac{1}{4}+\frac{1}{2}=\frac{3}{4}, \quad \text{or} \quad C=\frac{4}{3}\mu\text{F}.$$

$\therefore$ charge on each capacitor = charge on 'equivalent' capacitor

$$= CV=\frac{4}{3}\times 100=\frac{400}{3}\text{ microcoulomb.}$$

When like terminals are joined together, the p.d. across each capacitor, which is different at first, becomes equalized. Suppose it reaches a p.d. V. Then, as the total charge remains constant,

$$\text{initial total charge}=\frac{400}{3}+\frac{400}{3}=\text{final total charge}=4V+2V.$$

$$\therefore \quad 6V=\frac{800}{3},$$

$$\therefore \quad V=\frac{400}{9}\text{ volt,}$$

$$\therefore \text{ charge on larger capacitor}=4\times\frac{400}{9}=\frac{1600}{9}\text{ microcoulomb,}$$

$$\text{and charge on smaller capacitor}=2\times\frac{400}{9}=\frac{800}{9}\text{ microcoulomb.}$$

2. Define the *electrostatic potential* of an isolated conductor. Obtain an expression relating the energy of a charged conductor to the charge on it and its capacity.

Two insulated spherical conductors of radii 5·00 cm and 10·00 cm are charged to potentials of 600 volts and 300 volts respectively. Calculate the total energy of the system. Also calculate the energy after the spheres have been connected by a fine wire. Comment on the difference between the two results. (*N.*)

Capacitance C of a sphere of radius $r = 4\pi\varepsilon_0 r$.

For 5 cm radius, or 5×10^{-2} m, $C_1=4\pi\varepsilon_0\times5\times10^{-2}=\dfrac{5\times10^{-2}}{9\times10^{9}}=\dfrac{5}{9}\times10^{-11}$ F,

using $4\pi\varepsilon_0 = 1/(9\times10^9)$ approx.

For 10 cm radius, $C_2=\dfrac{10}{9}\times10^{-11}$ F.

Since energy, $W=\frac{1}{2}CV^2$,

$$\therefore \text{ total energy}=\frac{1}{2}\times\frac{5}{9}\times10^{-11}\times600^2+\frac{1}{2}\times\frac{15}{9}\times10^{-11}\times300^2$$

$$=15\times10^{-7}\text{ J.} \qquad \text{(i)}$$

When the spheres are connected by a fine wire, the potentials become equalized. Suppose this is V. Then, since $Q = CV$ *and the total charge is constant,*

$$\text{original total charge} = \text{total final charge}.$$

$$\therefore\ 4\pi\varepsilon_0(5\times10^{-2}\times600+10\times10^{-2}\times300) = 4\pi\varepsilon_0(5\times10^{-2}\ V+10\times10^{-2}\ V).$$

Solving, $V = 400$ volt

$$\therefore\ \text{total final energy} = \tfrac{1}{2}C_1V^2+\tfrac{1}{2}C_2V^2 = \tfrac{1}{2}(C_1+C_2)V^2$$

$$= \tfrac{1}{2}\times\frac{1}{9\times10^9}(5\times10^{-2}+10\times10^{-2})\times400^2$$

$$= 13\tfrac{1}{3}\times10^{-7}\ \text{J}. \qquad \text{(ii)}$$

From (i) and (ii), $1\frac{2}{3}\times10^{-7}$ J of electrical energy have been converted into heat. This occurred because a current flows in the connecting wire when the two spheres are connected.

3. Define *electrical capacitance.* Describe experiments to demonstrate the factors which determine its value for a parallel plate capacitance.

The plates of a parallel plate air capacitor consisting of two circular plates, each of 10 cm radius, placed 2 mm apart, are connected to the terminals of an electrostatic voltmeter. The system is charged to give a reading of 100 on the voltmeter scale. The space between the plates is then filled with oil of dielectric constant 4·7 and the voltmeter reading falls to 25. Calculate the capacitance of the voltmeter. You may assume that the voltage recorded by the voltmeter is proportional to the scale reading. (*N.*)

Suppose V is the initial p.d. across the air capacitor and voltmeter, and let C_1 be the voltmeter capacitance.

Then $$\text{total charge} = CV+C_1V = (C+C_1)V. \qquad \text{(i)}$$

When the plates are filled with oil the capacitance increases to $4{\cdot}7C$, and the p.d. falls to V_1. But the total charge remains constant.

$$\therefore\ 4{\cdot}7CV_1+C_1V_1 = (C+C_1)V, \quad \text{from (i).}$$

$$\therefore\ (4{\cdot}7C+C_1)V_1 = (C+C_1)V,$$

$$\therefore\ \frac{4{\cdot}7C+C_1}{C+C_1} = \frac{V}{V_1} = \frac{100}{25} = 4,$$

$$\therefore\ 0{\cdot}7C = 3C_1,$$

$$\therefore\ C_1 = \frac{0{\cdot}7C}{3} = \frac{7}{30}C.$$

Now $C = \varepsilon_0A/d$, where A is in metre^2 and d is in metres.

$$\therefore\ C = \frac{8{\cdot}85\times10^{-12}\times\pi\times(10\times10^{-2})^2}{2\times10^{-3}}\ \text{F}$$

$$= 1{\cdot}4\times10^{-10}\ \text{F (approx.).}$$

$$\therefore\ C_1 = \frac{7}{30}\times1{\cdot}4\times10^{-10}\ \text{F} = 3{\cdot}3\times10^{-11}\ \text{F}.$$

EXERCISES 31

1. Derive an expression for the capacitance of a parallel plate capacitor. Describe an experiment to demonstrate the variation of capacitance with *one* of the physical quantities involved.

A 2·5 μF capacitor is charged to a potential difference of 100 volts and is disconnected from the supply. Its terminals are then connected to those of an uncharged 10 μF capacitor. Find (*a*) the resulting potential difference across the two capacitors and (*b*) the total energy stored in them. Compare the result in (*b*) with the energy originally stored in the 2·5 μF capacitor and comment on the difference. (*L.*)

2. Explain the meaning of the term *capacitance* as used in electrostatics.

A potential difference of 90 V is applied across uncharged capacitors of 2 μF, 3 μF and 1·5 μF connected in series. Across which capacitor is the potential difference least? Explain this and find the numerical value of this potential difference. (*N.*)

3. Define *capacitance* of a conductor. State the factors on which its value depends and describe simple experiments to justify your answers.

The circular plates, *A* and *B*, of a parallel plate air capacitor have each an effective diameter of 10·0 cm and are 2·0 mm apart. The plates, *C* and *D*, of a similar capacitor have each an effective diameter of 12·0 cm and are 3·0 mm apart. *A* is earthed, *B* and *C* are connected together and *D* is connected to the positive pole of a 120-volt battery whose negative pole is earthed. Calculate (*a*) the combined capacitance of the arrangement, (*b*) the energy stored in it, (*c*) the energy stored in each capacitor. (*L.*)

4. Explain what is meant by *dielectric constant* (*relative permittivity*). State two physical properties desirable in a material to be used as the dielectric in a capacitor.

A sheet of paper 4·0 cm wide and $1{\cdot}5 \times 10^{-3}$ cm thick between metal foil of the same width is used to make a 2·0 μF capacitor. If the dielectric constant (relative permittivity) of the paper is 2·5, what length of paper is required? ($\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$ farad m^{-1}.) (*N.*)

5. Define *capacitance.* Obtain from first principles a formula for the capacitance of a parallel-plate capacitor.

Outline, without full experimental details, a method of determining the relative permittivity (dielectric constant) of a dielectric material.

A sensitive moving-coil meter with a suspended system that can swing freely is calibrated both as a ballistic galvanometer and as a galvanometer to record steady currents. Its sensitivity is 8×10^{-8} coulombs per division ballistically, and $2{\cdot}5 \times 10^{-7}$ amperes per division for steady currents. When a capacitor is charged from a 24 volt battery and discharged through the meter, the ballistic throw is 30 divisions. When a 2 volt battery is used, and the capacitor is charged and discharged through the meter many times per second by means of a vibrating-reed switch, the steady deflection is 80 divisions. Find (*a*) the value of the capacitance, and (*b*) the number of times the capacitor is discharged per second. (*O.*)

6. Explain what is meant by the capacitance of a capacitor and define a unit in which it is measured.

Explain the principle of the guard ring capacitor. How could it be used to pass a charge of known value through a galvanometer?

A voltmeter of high internal resistance is connected across the terminals of a 10 μF capacitor which is initially charged to 410 V. When the charging source is removed the voltmeter reading falls and reaches 390 V after 10·0 seconds.

Calculate the internal resistance of the voltmeter. (You may use an approximate method of calculation, but must explain why it is approximate.) (*O. & C.*)

7. Derive an expression for the energy stored in a capacitor C when there is a potential difference V between the plates. If C is in microfarads and V is in volts, express the result in joules.

Show that when a battery is used to charge a capacitor through a resistor, the heat dissipated in the circuit is equal to the energy stored in the capacitor.

Describe the structure of a 1 microfarad capacitor and describe an experiment to compare the capacitances of two capacitors of this type. (*N.*)

8. Define *electrostatic potential* and *capacitance.* Assuming that the electric intensity between the plates of a parallel-plate air capacitor is $1/\varepsilon_0$ times the charge density on one of the plates, obtain an expression for its capacitance. If you were provided with a capacitor of unknown capacitance and a standard capacitor of comparable capacitance, how would you find the value of the unknown capacitance? A parallel-plate air capacitor is charged to a potential difference of 300 volts and is then connected in parallel with another capacitor of equal dimensions with ebonite as dielectric. The potential of the combination is found to be 75 volts. Calculate the dielectric constant of the ebonite. (*N.*)

9. Deduce expressions for the combined capacitance of two capacitors (*a*) connected in series, (*b*) connected in parallel. Describe how *one* of these expressions may be verified by experiment.

A fixed capacitor of capacitance 10^{-4} microfarad is connected in series with a variable capacitor the capacitance of which may be varied from zero to 10^{-4} microfarad in steps of 10^{-6} microfarad. These two in series are connected in parallel with a third capacitor of capacitance 5×10^{-4} microfarad. Calculate (i) the maximum capacitance of the whole combination, (ii) the smallest change in this capacitance which can be produced by the arrangement. (*N.*)

10. Explain how the strength of the electric field at any point is related to the electric potential at and near the point.

A parallel plate capacitor consists of two large plates 2 cm apart, the dielectric on one side of the middle plane between the plates consisting of air, and on the other side of an insulating material of dielectric constant 4·2. Calculate the strength of the electric field in the half occupied by air, if the difference of potential between the plates is 500 volts. (*L.*)

11. Define *capacitance* (or capacity), *relative permittivity* (or dielectric constant), microfarad (μF).

Derive an approximate expression for the capacitance of an air capacitor consisting of two parallel opposite circular plates of radius r cm at a distance t cm apart. Explain on what the degree of approximation depends.

Two capacitors of capacitances respectively 2 μF and 3 μF are joined in series between points A and B. What capacitance must be placed in parallel with the 2 μF capacitor in order to increase the capacitance from A to B by 0·8 μF? (*L.*)

12. State the law of force between electric charges. Write down expressions for (*a*) the electric field strength, and (*b*) the electric potential, at a point in air situated at a distance r from a point charge Q. State the units in which each of the quantities is measured.

Describe the Faraday ice-pail experiment. Illustrate the successive steps in the experiment by a series of simple diagrams, and state the conclusions that can be drawn from the experiment.

The sensitivity of a ballistic galvanometer is found by charging a 0·001 microfarad capacitor from a 12-volt battery, and discharging it through the meter, which gives a corrected swing of 16·8 divisions. A large hollow metal can is then connected to one terminal of the galvanometer, while the other is earthed; an insulated conducting sphere, of 5 cm radius, is charged to an unknown potential V, thrust right inside the can, and allowed to touch its inner surface; as a result, the galvanometer gives a corrected swing of 21·0 divisions. Find, in coulombs, the charge on the sphere, and also the value in V in volts. (*O.*)

13. Define *potential, capacitance.*

Obtain from first principles a formula for the capacitance of a parallel-plate capacitor.

The plates of such a capacitor are each 0·4 m square, and separated by 10^{-3} m, the space between being filled with a medium of relative permittivity 5. A vibrating contact, with frequency 50 seconds^{-1}, repeatedly connects the capacitor across a 120-volt battery and then discharges it through a galvanometer whose resistance is of the order of 50 ohm. Calculate the current recorded, and explain why this is independent of the actual value of the galvanometer resistance. (Take the permittivity of space to be $8{\cdot}85 \times 10^{-12}$ F m^{-1}.) (*O.*)

14. Define 'the capacitance of a capacitor' and show how it leads to a practical unit of capacitance. Why is it necessary to specify 'capacitor' in this definition?

State the physical factors which affect the magnitude of a capacitance, and describe an experiment to demonstrate the effect of one of them.

A charged 20 microfarad capacitor A is connected to an electrostatic voltmeter of infinite resistance and negligible capacitance which reads 500 volts. A 0·25 microfarad capacitor B is now connected in parallel with A. The capacitor B is then disconnected from A, discharged, and the process of connection and discharge repeated ten times in all. Calculate (*a*) what fraction of the charge on A remains after the first connexion, (*b*) what fraction remains eventually, and (*c*) the final reading of the voltmeter (*C.*)

chapter thirty-two

Current Electricity

OHM'S AND JOULE'S LAWS: RESISTANCE AND POWER

Discovery and Electric Current

BY the middle of the eighteenth century, electrostatics was a well-established branch of physics. Machines had been invented which could produce by friction great amounts of charge, giving sparks and electric shocks. The momentary current (as we would now call it) carried by the spark or the body was called a 'discharge'.

In 1786 Galvani, while dissecting a frog, noticed that its leg-muscle twitched when one of his assistants produced an electric spark in another part of the room. He also found that, when a frog's leg-muscle was hung by a copper hook from an iron stand, the muscle twitched whenever it swung so as to touch the stand. Galvani supposed that the electricity which caused the twitching was generated within the muscle, but his fellow-Italian Volta believed that it arose from the contact of the two different metals. Volta turned out to be right, and

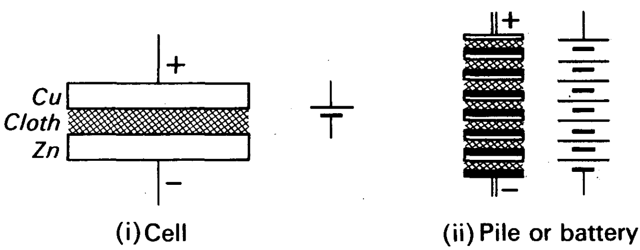


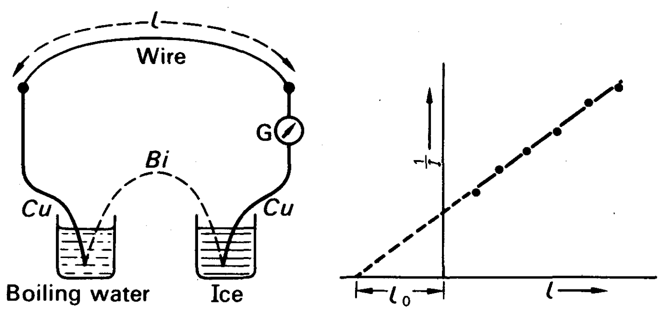
FIG. 32.1. Voltaic cell and pile, with conventional symbols.

in 1799 he discovered how to obtain from two metals a continuous supply of electricity: he placed a piece of cloth soaked in brine between copper and zinc plates (Fig. 32.1 (i)). The arrangement is called a *voltaic cell*, and the metal plates its 'poles'; the copper is known as the positive pole, the zinc as the negative cell. Volta increased the power by building a pile of cells, with the zinc of one cell resting on the copper of the other (Fig. 32.1 (ii)). From this pile he obtained sparks and shocks similar to those given by electrostatic machines.

Shortly after, it was found that water was decomposed into hydrogen and oxygen when connected to a voltaic pile. This was the earliest discovery of the chemical effect of an electric current. The heating effect was also soon found, but the magnetic effect, the most important effect, was discovered some twenty years later.

Ohm's Law

The properties of an electric circuit, as distinct from the effects of a current, were first studied by Ohm in 1826. He set out to find how the length of wire in a circuit affected the current through it—in modern language, he investigated electrical resistance. In his first experiments he used voltaic piles as sources of current, but he found that the current which they gave fluctuated considerably, and he later replaced them by thermocouples (p. 803). He passed the currents through various lengths of brass wire, 0·037 cm in diameter, and observed them on a torsion balance galvanometer; this is represented by G in Fig. 32.2 (i). No unit of current had been defined at the time of these experiments, but physicists had agreed to take the strength of a current as proportional to its magnetic field. Ohm found that the



(i) Apparatus (ii) Results

FIG. 32.2. Ohm's experiment.

current I in his experiments was almost inversely proportional to the length of wire, l, in the circuit. He plotted the reciprocal of the current (in arbitrary units) against the length l, and got a straight line, as shown in Fig. 32.2 (ii). Thus

$$I \propto \frac{1}{l_0 + l},$$

where l_0 is the intercept of the line on the axis of length. Ohm explained this result by supposing, naturally, that the thermocouples and galvanometer, as well as the wire, offered resistance to the current. He interpreted the constant l_0 as the length of wire equal in resistance to the galvanometer and thermocouples. The *ohm*, symbol Ω, is the unit of electrical resistance (see p. 790).

Mechanism of Metallic Conduction

The conduction of electricity in metals is due to free electrons. Free electrons have thermal energy, and wander randomly through the metal from atom to atom. When a battery is connected across the ends of the metal, an electric field is set up. The electrons are now accelerated by the field, so they gain velocity and energy. When they 'collide' with an atom vibrating about its fixed mean position (called a 'lattice site'), they give up some of their energy to it. The amplitude of the vibrations is then increased and the temperature of the metal rises. The electrons are then again accelerated by the field and again give up some energy. Although their movement is erratic, on the average the electrons drift in the direction of the field with a mean speed we calculate shortly. This drift constitutes an 'electric current'. It will be noted that heat is generated by the collision of electrons whichever way they flow. Thus the heating effect of a current—called *Joule heating* (p. 790)—is irreversible, that is, it still occurs when the current in a wire is reversed.

A simple calculation enables the average drift speed to be estimated. Fig. 32.3 shows a portion of a copper wire of cross-sectional area A

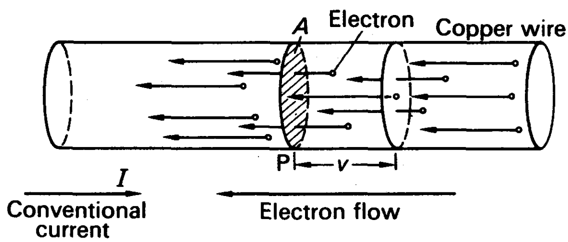


FIG. 32.3. Theory of metallic conduction.

through which a current I is flowing. We suppose that there are n electrons per unit volume, and that each electron carries a charge e. Now in one second all those electrons within a distance v to the right of the plane at P, that is, in a volume Av, will flow through this plane, as shown. This volume contains nAv electrons and hence a charge $nAve$. Thus a charge of $nAve$ per second passes P, and so the current I is given by

$$I = nAve \qquad . \quad . \quad . \quad . \quad (1)$$

To find the order of magnitude of v, suppose $I = 10\text{A}$, $A = 1\ \text{mm}^2 = 10^{-6}\text{m}^2$, $e = 1{\cdot}6 \times 10^{-19}\text{C}$, and $n = 10^{28}$ electrons m^{-3}. Then, from (1),

$$v = \frac{I}{nAe} = \frac{10}{10^{28} \times 10^{-6} \times 1{\cdot}6 \times 10^{-19}}$$

$$= \frac{1}{160}\ \text{m s}^{-1}\ \text{(approx.)}$$

This is a surprisingly slow drift compared with the average thermal speeds, which are of the order of several hundred metres per second (p. 234).

Resistivity

Ohm showed, by using wires of different length and diameter, that the resistance of a wire, R, is proportional to its length, l, and inversely proportional to its cross-sectional area A. The truth of this can easily be demonstrated today by experiments with a Wheatstone bridge (see p. 829) and suitable lengths of wire. We have, then, for a given wire,

$$R \propto \frac{l}{A};$$

we may therefore write

$$R = \rho\frac{l}{A}, \qquad (1)$$

where ρ is a constant for the material of the wire. It is called the *resistivity* of that material.

To define it in words, we imagine a rectangular prism of the material, of unit length and unit cross-section. Then $l = 1$, $A = 1$, and $R = \rho$. Thus the resistivity of a substance is the resistance between the faces of a rectangular prism of the substance, which is 1 cm long and whose cross-sectional area is 1 cm^2. One unit of resistivity is 1 ohm cm, because, from equation (1),

$$\rho = R\frac{A}{l}, \qquad (2)$$

which has the units

$$\text{ohms} \times \frac{\text{cm}^2}{\text{cm}} = \text{ohms} \times \text{cm}.$$

Resistivities are thus often expressed in microhm centimetres; 1 microhm $= 10^{-6}$ ohm. The SI unit is the *ohm metre* (Ω m). Using it, R is in ohms when l is in metres and A is in metre^2 in equation (2).

RESISTIVITIES

Substance	Resistivity ρ, ohm m (at 20°C)	Temperature coefficient α, K^{-1}
Aluminium	$2{\cdot}82 \times 10^{-8}$	0·0039
Brass	*c.* 8×10^{-8}	*c.* 0·0015
Constantan[1]	*c.* 49×10^{-8}	0·00001
Copper	$1{\cdot}72 \times 10^{-8}$	0·0043
Iron	*c.* $9{\cdot}8 \times 10^{-8}$	0·0056
Manganin[2]	*c.* 44×10^{-8}	*c.* 0·00001
Mercury	$95{\cdot}77 \times 10^{-8}$	0·00091
Nichrome[3]	*c.* 100×10^{-8}	0·0004
Silver	$1{\cdot}62 \times 10^{-8}$	0·0039
Tungsten[4]	$5{\cdot}5 \times 10^{-8}$	0·0058
Carbon (graphite)	33 to 185×10^{-8}	$-0{\cdot}0006$ to $-0{\cdot}0012$

[1] Also called Eureka; 60 per cent Cu, 40 per cent Ni.
[2] 84 per cent Cu, 12 per cent Mn, 4 per cent Ni; used for resistance boxes and shunts.
[3] Ni–Cu–Cr; used for electric fires—does not oxidize at 1000°C.
[4] Used for lamp filaments—melts at 3380°C.

The resistivity of a metal is increased by even small amounts of impurity; and alloys, such as Constantan, may have resistivities far greater than any of their constituents.

Ohm's Theory of the Circuit

We have seen that Ohm abandoned the use of voltaic piles in his experiments, because the currents which they gave were not steady. He attributed this to fluctuations in their 'exciting force'—electromotive force, as we now call it. Similarly, when he used thermocouples, he found that the current through a given circuit increased when the difference in temperature between the couples was increased. He was thus led to propose a 'mathematical law of the galvanic circuit':

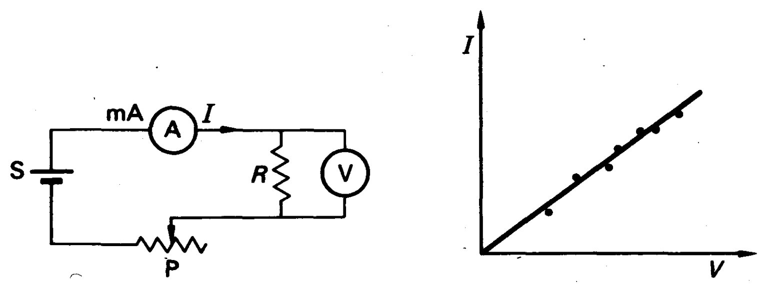
$$I = \frac{E}{R}. \quad \quad (3)$$

Here I stands for the strength of the current, E for the exciting force (e.m.f.), and R for the total resistance of the circuit.

Demonstration of Ohm's Law

Ohm showed that his law applied not only to a complete circuit but to any part of it. To understand this, let us consider an experiment which can easily be done with modern apparatus. As shown in Fig. 32.4 (*a*), we connect in series the following apparatus:

(i) one or more accumulators, S;
(ii) a milliammeter reading to 15 milliamperes;
(iii) a wire-wound resistor R of the order of 50 ohms;
(iv) a suitable variable resistance or *rheostat* P of the same order of resistance.



(a) Circuit (b) Results

FIG. 32.4. Demonstration of Ohm's law.

Across the resistor R we connect a voltmeter to measure the potential difference V across R. This must be a voltmeter such as a potentiometer (p. 817) whose calibration does not depend on Ohm's law, otherwise the experiment would not be valid. The milliammeter calibration likewise must not depend on Ohm's law. By adjusting the resistor P we vary the current I through the circuit, and at each value of I we measure V. On plotting V against I we get a straight line through the origin, as in Fig. 32.4 (*b*); this shows that the potential difference across the resistor R is proportional to the current through it:

$$V \propto I. \quad . \quad . \quad . \quad . \quad (4)$$

Thus, taking into account that the resistance of a conductor depends on its temperature and on other physical conditions such as mechanical strain, Ohm's law can be stated as follows:

Under constant physical conditions, the potential difference across a conductor is proportional to the current through it.

The law is obeyed by the most important class of conductors—metals—and by some others, such as carbon. It is not obeyed by some crystals, such as silicon carbide, nor by some conducting solutions, nor by diode valves, nor—as in a neon lamp— by gases.

Resistance

From Ohm's law, it follows that

$$\frac{V}{I} = R, \text{ a constant.} \quad . \quad . \quad . \quad (5)$$

R is *defined* as the 'resistance' of the conductor.

The unit of potential difference, V, is the *volt*, symbol V; the unit of current, I, is the *ampere*, symbol A; the unit of resistance, R, is the *ohm*, symbol Ω. The ohm is thus the resistance of a conductor through which a current of one ampere flows when a potential difference (p.d.) of one volt is across it.

From the above equation, it also follows that

$$V = IR, \quad \text{and} \quad I = \frac{V}{R}. \quad . \quad . \quad . \quad (6)$$

Smaller units of current are the milliampere (one-thousandth of an ampere), symbol mA and the micro-ampere (one-millionth of an ampere), symbol μA. Smaller units of p.d. are the millivolt (1/1000 V) and the microvolt ($1/10^6$ V). A small unit of resistance is the microhm ($1/10^6$ ohm); larger units are the kilohm (1,000 ohms) and the megohm (10^6 ohms).

HEAT AND POWER

Electrical Heating. Joule's Laws

In 1841 Joule studied the heating effect of an electric current by passing it through a coil of wire in a jar of water (Fig. 32.5). He used various currents, measured by an early form of galvanometer G, and various lengths of wire, but always the same mass of water. The rise in temperature of the water, in a given time, was then proportional to the heat developed by the current in that time. Joule found that the heat produced in a given time, with a given wire, was proportional to

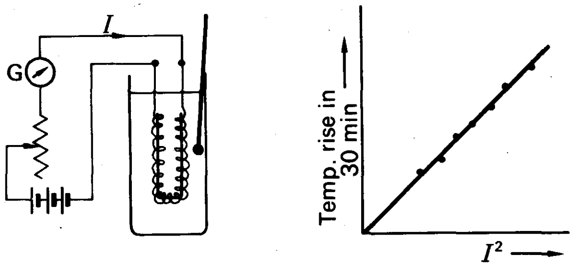


FIG. 32.5. Joule's experiment on heating effect of current.

I^2, where I is the current flowing. If H is the heat produced per second, then

$$H \propto I^2. \quad \quad (7)$$

Joule also made experiments on the heat produced by a given current in different wires. He used wires of different lengths, but of the same diameter, and of the same material; he found that the rate at which heat was produced, by a given current, was proportional to the length of the wire. That is to say, he found that the rate of heat production was proportional to what Ohm had already called the resistance of the wire:

$$H \propto R. \quad \quad (8)$$

Relationships (7) and (8) together give

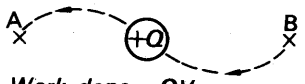
$$H \propto I^2R. \quad \quad (9)$$

Mechanism of the Heating Effect

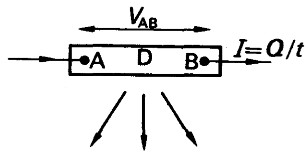
Heat is a form of energy. The heat produced per second by a current in a wire is therefore a measure of the energy which it liberates in one second, as it flows through the wire. The heat is produced, we suppose, by the free electrons as they move through the metal. On their way they collide frequently with atoms; at each collision they lose some of their kinetic energy, and give it to the atoms which they strike. Thus, as the current flows through the wire, it increases the kinetic energy of vibration of the metal atoms: it generates heat in the wire. The electrical resistance of the metal is due, we say, to its atoms obstructing the drift of the electrons past them: it is analogous to mechanical friction. As the current flows through the wire, the energy lost per second by the electrons is the electrical power supplied by the battery which maintains the current. That power comes, as we shall see later, from the chemical energy liberated by these actions within the battery.

Potential Difference and Energy

On p. 750 we defined the potential difference V_{AB} between two points, A and B, as the work done by an external agent in taking a unit positive charge from B to A (Fig. 32.6 (i)). This definition applies equally well to points in an electrostatic field and to points on a conductor carrying a current.



(i) Definition



(ii) Application to a current

FIG. 32.6. Potential difference and energy.

In Fig. 32.6 (ii), D represents any electrical device or circuit element: a lamp, motor, or battery on charge, for example. A current of I amperes flows through it from the terminal A to the terminal B; if it flows for t seconds, the charge Q which it carries from A to B is, since a current is the quantity of electricity per second flowing,

$$Q = It \text{ coulombs.} \quad . \quad (10)$$

Let us suppose that the device D liberates a total amount of energy W joules in the time t; this total may be made up of heat, light, sound mechanical work, chemical transformation, and any other forms of energy. Then W is the amount of electrical energy given up by the charge Q in passing through the device D from A to B.

$$\therefore W = QV_{AB}, \quad . \quad . \quad . \quad . \quad (11)$$

where V_{AB} is the potential difference between A and B in volts.

The work, in all its forms, which the current I does in t seconds as it flows through the device, is therefore

$$W = IV_{AB}t, \quad . \quad . \quad . \quad . \quad (12)$$

by equations (10) and (11).

Electrical Power

The energy liberated per second in the device is defined as its electrical *power*. The electrical power, P, supplied is given, from above, by

$$P = \frac{W}{t} = \frac{IV_{AB}t}{t}$$

or

$$P = IV_{AB}. \quad . \quad . \quad . \quad . \quad (13)$$

When an electric current flows through a wire or 'passive' resistor, all the power which it conveys to the wire appears as heat. If I is the current, R is the resistance, then $V_{AB} = IR$, Fig. 32.7.

$$\therefore P = I^2R. \quad . \quad . \quad . \quad . \quad (14)$$

Also,

$$P = \frac{V_{AB}{}^2}{R}. \quad . \quad . \quad . \quad . \quad (15)$$

The power, P, is in *watts* (W) when I is in amp, R is in ohms, and V_{AB} is in volts. 1 kilowatt (kW) = 1000 watts.

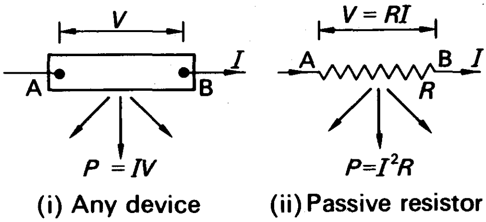


FIG. 32.7. Power equations.

The formulae for power, $P = I^2R$ or V^2/R, is true only when all the electrical power supplied is dissipated as heat. As we shall see, the formulae do not hold when part of the electrical energy supplied is converted into mechanical work, as in a motor, or into chemical energy, as in an accumulator being charged. A device which converts all the electrical energy supplied to it into heat is called a 'passive' resistor; it may be a wire, or a strip of carbon, or a liquid which conducts electricity but is not decomposed by it. Since the joule (J) is the unit of heat, it follows that, for a resistor, the heat H in it in joules is given by

$$H = IVt$$

or by

$$H = I^2Rt \quad . \quad . \quad . \quad . \quad (16)$$

or by

$$H = \frac{V^2t}{R}$$

The units of I, V, R are amperes (A), volts (V), ohms(Ω) respectively.

High-tension Transmission

When electricity has to be transmitted from a source, such as a power station, to a distant load, such as a factory, the two must be connected by cables. These cables have resistance, which is in effect added to the internal resistance of the generator; power is wasted in them as heat. If r is the total resistance of the cables, and I the supply current, the power wasted is I^2r. The power delivered to the factory is IV, where V is the potential difference at the factory. Economy requires the waste power, I^2r, to be small; but it also requires the cables to be thin, and therefore cheap to buy and erect. The thinner the cables, however, the higher their resistance r. Thus the most economical way to transmit the power is to make the current, I, as small as possible; this means making the potential difference V as high as possible. When large amounts of power are to be transmitted, therefore, very high voltages are used: 132000 volts on the main lines of the British grid, 6000 volts on subsidiary lines. These voltages are much too high to be brought into a house, or even a factory. They are stepped down by transformers, in a way which we shall describe later; stepping-down in that way is possible only with alternating current, which is one of the main reasons why alternating current is so widely used.

EXAMPLE

An electric heating element to dissipate 480 watts on 240 V mains is to be made from Nichrome ribbon 1 mm wide and thickness 0·05 mm. Calculate the length of ribbon required if the resistivity of Nichrome is $1{\cdot}1\times10^{-6}$ ohm metre.

Power, $$P=\frac{V^2}{R}$$

$$\therefore R=\frac{V^2}{P}=\frac{240^2}{480}=120\Omega$$

The area A of cross-section of the ribbon $=1\times0{\cdot}05\ \text{mm}^2=0{\cdot}05\times10^{-6}\ \text{m}^2$.

From $$R=\frac{\rho l}{A}$$

$$\therefore l=\frac{R.A}{\rho}=\frac{120\times0{\cdot}05\times10^{-6}}{1{\cdot}1\times10^{-6}}=5{\cdot}45\ \text{metre}$$

Summary of Formulae Related to Power and Ohm's Law

In any device whatever (Fig. 32.7 (i)):

Electrical power consumed = power developed in other forms,

$$P=IV,$$

watts = amperes × volts.

In a passive resistor (Fig. 32.7 (ii)):

(i) $$V=IR;\qquad I=\frac{V}{R};\qquad R=\frac{V}{I},$$

volts = ohms × amperes.

(ii) Power consumed = heat developed per second, in watts.

$$P=I^2R=IV=\frac{V^2}{R}.$$

(iii) Heat developed in time t:

Electrical energy consumed = heat developed in joules

$$I^2Rt=IVt=\frac{V^2}{R}t.$$

Board of Trade (commercial) unit = kilowatt hour (kWh) = kilowatt × hour

$= 3{\cdot}6\times10^6$ joule.

ELECTROMOTIVE FORCE

E.M.F. Internal Resistance

If we take a high-tension battery, and connect a high resistance voltmeter across it, the meter reads about 120 volts. Across two batteries

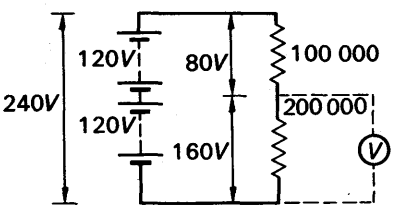


FIG. 32.8. P.D. with moving-coil voltmeter.

in series it reads 240 volts. Let us now connect the two batteries in series with two resistors, of resistance 200000 and 100000 ohms, as in Fig. 32.8. By using the voltmeter, we find that the potential difference across the 200000-ohm resistor is 160 volts, and across the 100000-ohm resistor 80 volts. These add up to 240 volts; if we inserted a third resistor, we should find that the potential difference across all three added up to the same value. It appears that the batteries always maintain a total potential difference of 240 volts across any circuit to which they are joined. This constant potential difference represents what Ohm called the 'exciting force' of the batteries. Since it is the property which enables the batteries to maintain a flow of electricity in a circuit, we may call it their electromotive force.

Now let us connect a lower resistance across the batteries of Fig. 32.8—say 10000 ohms. We find the potential difference across their terminals falls slightly. And if we use a still lower resistance—say 1000 ohms—then the potential difference falls greatly, perhaps to about $\frac{2}{3}$ of its open-circuit value.

We suppose, therefore, that the batteries have some internal resistance—the resistance r of the chemical solutions between the plates (Fig. 32.9 (i)). This is analogous to the resistance of the wires of Ohm's thermocouples (p. 780). When an appreciable current I flows from the battery, it sets up a potential difference rI across the internal resistance, and by that amount makes the external potential difference less than the electromotive force, E (Fig. 32.9 (ii)). In a rough-and-ready way we may represent the battery as a source of constant potential difference E, in series with a passive resistor r (Fig. 32.9 (iii)).

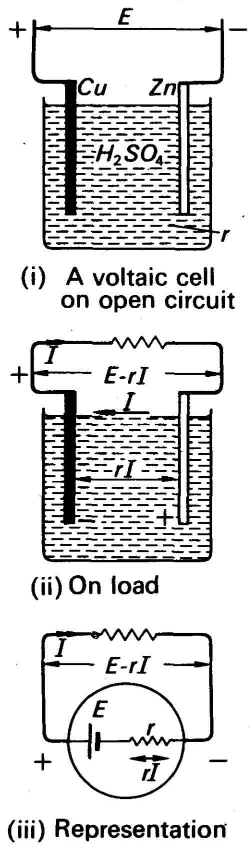
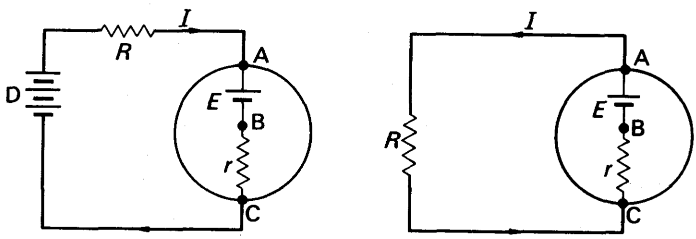


FIG. 32.9. E.m.f. and internal resistance.

Electromotive Force and Energy

To get a rigorous definition of electromotive force, let us first imagine that we pass a current through the device of Fig. 32.10 in opposition to its e.m.f. We can do this by connecting its terminals AC, via a resistance

R, to a battery D which has a greater e.m.f. (Fig. 32.10 (i)). If a charge Q passes round the circuit in a given time, then the work done in carrying it from A to B, against the potential difference E, is QE joules. This work appears as chemical changes in the source of E. Now suppose that we remove the battery D, and connect a resistor across the terminals AC (Fig. 32.10 (ii)). The potential difference will now send a current round the circuit in the opposite direction to the previous current. And when a



(i) Current in opposition to e.m.f.

(ii) Current in direction of e.m.f.

FIG. 32.10. Illustrating definition of e.m.f.

charge Q has passed, the energy delivered by the source of e.m.f. E will be QE joules. The chemical changes in the source will have been reversed, and will have given up this amount of energy, as electrical energy. The current, in passing round the circuit, will have converted this energy into heat. Some of the heat will have been dissipated in the external resistance R, some in the internal resistance r.

Our picture of a source of current as a constant potential difference in series with a resistance is over-simplified, but it has brought us to the point where we can make a definition of e.m.f. which is both rigorous and intelligible. We shall make it first in terms of charge, later in terms of current.

In terms of charge: *if a device has an electromotive force E, then, in passing a charge Q round a circuit joined to it, it liberates an amount of electrical energy equal to QE.* If a charge Q is passed through the source against its e.m.f., then the work done against the e.m.f. is QE. The above definition of e.m.f. does not depend on any assumptions about the nature of its source.

If a device of e.m.f. E passes a steady current I for a time t, then the charge that it circulates is

$$Q = It.$$

Thus:

$$\text{electrical energy liberated, } W, = QE = IEt, \qquad (17)$$

and

$$\text{electrical power generated, } P, = \frac{W}{t} = EI. \qquad (18)$$

We can now define e.m.f. in terms of power and current, and therefore in a way suitable for dealing with circuit problems. From equation (18)

$$P = EI,$$

or

$$E = \frac{P}{I}.$$

Thus *the e.m.f. of a device is the ratio of the electrical power which it generates, to the current which it delivers.* If current is forced through a device in opposition to its e.m.f., then equation (18) gives the power consumed in overcoming the e.m.f.

Electromotive force resembles potential difference in that both can be defined as the ratio of power to current. The unit of e.m.f. is therefore 1 watt per ampere, or 1 volt; and *the e.m.f. of a source, in volts, is numerically equal to the power which it generates when it delivers a current of* 1 *ampere.*

Representation of an E.M.F. and Internal Resistance

Sources of e.m.f. differ widely in their nature. In a thermocouple, the e.m.f. arises at the junction of the two metals—this point is sometimes called the seat of the e.m.f. In a voltaic cell, we believe, the e.m.f. arises at the interfaces of the plates and solutions—part at one interface, the rest at the other. In each of these sources we can distinguish between the seat, or seats, of the e.m.f., and that of the internal resistance, which is in the bulk of the solution or the wires of the thermocouple. The e.m.f. of a dynamo, however, does not arise at a point: it acts along the wires of the armature coil as they move in the magnetic field (p. 915). Here we cannot distinguish between the seats of the e.m.f. and the internal resistance. In solving circuit problems, however, it is helpful to show the e.m.f. and internal resistance separately, although as a rule they are physically inextricable.

Ohm's Law for Complete Circuit

Fig. 32.11 shows a source of current connected to a passive resistor—called the load—of resistance R. To find the current I, we equate the power generated by the source to the heat developed per second in the resistances:

$$EI = I^2r + I^2R.$$

Thus

$$E = Ir + IR, \qquad (19)$$

whence

$$I = \frac{E}{R+r}. \qquad (20)$$

Equation (20) asserts that the current is equal to the e.m.f. of the source divided by the total resistance of the circuit; it is Ohm's original statement of his law (p. 786). Equation (19) asserts that the sum of the potential differences across the resistances is equal to the e.m.f. The potential difference Ir appears across the internal resistance, and is often called the voltage drop; because of it, the

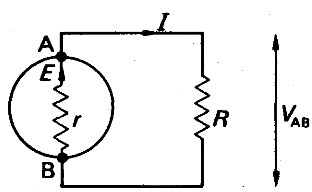


FIG. 32.11. A complete circuit.

potential difference between the terminals of the cell, V_{AB}, falls when the current taken, I, is increased:

$$V_{AB} = E - Ir. \quad . \quad . \quad . \quad . \quad (21)$$

Terminal Potential Difference

The quantity V_{AB} is often called the terminal potential difference of the cell; it is also the potential difference across the load. Equations (19) and (21) give

$$V_{AB} = IR, \quad . \quad . \quad . \quad . \quad (22)$$

which is Ohm's law for the load alone; we could have written it down directly. Equations (20) and (22) together give the terminal potential difference in terms of the e.m.f. and the resistances:

$$V_{AB} = IR = \frac{E}{R+r}R. \quad . \quad . \quad . \quad (23)$$

Output and Efficiency

The power delivered to the load in Fig. 32.10 is called the output power, P_{out}; its value is

$$P_{out} = IV_{AB}$$
$$= I^2R. \quad . \quad . \quad . \quad . \quad (24)$$

The power generated by the source of current is

$$P_{gen} = IE. \quad . \quad . \quad . \quad . \quad (25)$$

The difference between the power generated and the output is the power wasted as heat in the source: I^2r. The ratio of the power output to the power generated is the efficiency, η, of the circuit as a whole:

$$\eta = \frac{P_{out}}{P_{gen}}. \quad . \quad . \quad . \quad . \quad (26)$$

By equations (24) and (25), therefore,

$$\eta = \frac{P_{out}}{P_{gen}} = \frac{IV_{AB}}{IE}.$$
$$= \frac{V_{AB}}{E}.$$

Equation (23) now gives

$$\eta = \frac{R}{R+r}. \quad . \quad . \quad . \quad . \quad (27)$$

This shows that the efficiency tends to unity (or 100 per cent) as the load resistance R tends to infinity. For high efficiency the load resistance must be several times the internal resistance of the source. When the load resistance is equal to the internal resistance, the efficiency is 50 per cent. (See Fig. 32.12 (i).)

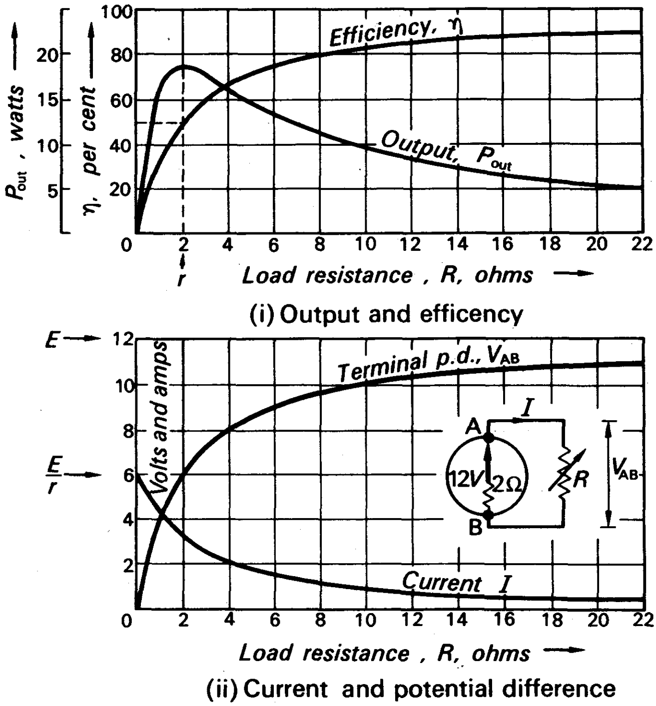


FIG. 32.12. Effect of varying load resistance.

Power Variation

Now let us consider how the power output varies with the load resistance. Equations (24) and (20) give

$$P_{out} = I^2R,$$

and
$$I = \frac{E}{R+r},$$

whence
$$P_{out} = \frac{E^2R}{(R+r)^2}.$$

If we take fixed values of E and r, and plot P_{out} as a function of R, we find that it passes through a maximum when $R = r$ (Fig. 32.12 (i)). We can get the same result in a more general way by differentiating P_{out} with respect to R, and equating the differential coefficient to zero. Physically, this result means that the power output is very small when R is either very large or very small, compared with r. When R is very large, the terminal potential difference, V_{AB}, approaches a constant value equal to the e.m.f. E (Fig. 32.12 (ii)); as R is increased the current I falls, and the power IV_{AB} falls with it. When R is very small, the current approaches the constant value E/r, but the potential difference (which is equal to IR) falls steadily with R; the power output therefore falls likewise. Consequently the power output is greatest for a moderate value of R; the mathematics show that this value is actually $R = r$.

To prove $R = r$, differentiate the expression for P_{out}, given on p. 799, with respect to R. Then

$$E^2\frac{(R+r)^2-R\,.\,2(R+r)}{(R+r)^4} = 0, \text{ for a maximum.}$$

From the numerator, $r^2-R^2 = 0$, or $R = r$.

Examples of Loads in Electrical Circuits

The loading on a dynamo or battery is generally adjusted for high efficiency, because that means greatest economy. Also, if a large dynamo were used with a load not much greater than its internal resistance, the current would be so large that the heat generated in the internal resistance would ruin the machine. With batteries and dynamos, therefore, the load resistance is made many times greater than the internal resistance.

Loading for greatest power output is common in communication engineering. For example, the last transistor in a receiver delivers electrical power to the loudspeaker, which the speaker converts into mechanical power as sound-waves (p. 596). Because it converts electrical energy into mechanical energy, and not heat, the loudspeaker is not a passive resistor, and the simple equations above do not apply to it. Nevertheless, circuit conditions can be specified which enable the transistor to deliver the greatest power to the speaker; these are similar to the condition of equal load and internal resistances, and are usually satisfied in practice.

Load not a Passive Resistor

As an example of a load which is not a passive resistor, we shall take an accumulator being charged. The charging is done by connecting the accumulator X in opposition to a source of greater e.m.f., Y in Fig. 32.13,

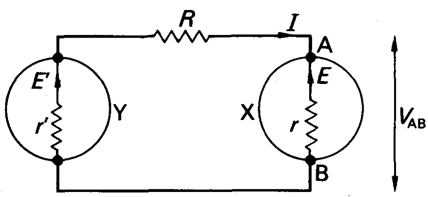


FIG. 32.13. Accumulator charging.

via a controlling resistor R. If E, E' and r, r' are the e.m.f. and internal resistances of X and Y respectively, then the current I is given by the equation:

$$\left.\begin{array}{c}\text{power generated}\\ \text{in Y}\end{array}\right\} = \left\{\begin{array}{c}\text{power converted to}\\ \text{chemical energy}\\ \text{in X}\end{array}\right\} + \left\{\begin{array}{c}\text{power dissipated}\\ \text{as heat in all}\\ \text{resistances}\end{array}\right.$$

$$E'I \quad = \quad EI \quad + \quad I^2R+I^2r'+I^2r. \qquad (28)$$

Thus $$(E'-E)I = I^2(R+r'+r),$$

whence $$I = \frac{E'-E}{R+r'+r}. \qquad (29)$$

The potential difference across the accumulator, V_{AB}, is given by

$$\left.\begin{array}{c}\text{power delivered}\\ \text{to X}\end{array}\right\} = \left\{\begin{array}{c}\text{power converted to}\\ \text{chemical energy}\end{array}\right\} + \left\{\begin{array}{c}\text{power dissipated}\\ \text{as heat}\end{array}\right.$$

$$IV_{AB} \quad = \quad IE \quad + \quad I^2r.$$

Hence $$V_{AB} = E+Ir. \qquad (30)$$

Equation (30) shows that, when current is driven through a generator in opposition to its e.m.f., then the potential difference across the generator is equal to the *sum* of its e.m.f. and the voltage drop across its internal resistance. This result follows at once from energy considerations, as we have just seen.

Cells in Series and Parallel

When cells or batteries are in series and assist each other, then the total e.m.f.

$$E = E_1+E_2+E_3+\ldots, \qquad (31)$$

and the total internal resistance

$$r = r_1+r_2+r_3+\ldots, \qquad (32)$$

where E_1, E_2 are the individual e.m.f.s and r_1, r_2 are the corresponding internal resistances. If one cell, e.m.f. E_2 say, is turned round 'in opposition' to the others, then $E = E_1-E_2+E_3+\ldots$; but the total internal resistance remains unaltered.

When *similar* cells are in parallel, the total e.m.f. = E, the e.m.f. of any one of them. The internal resistance r is here given by

$$\frac{1}{r} = \frac{1}{r_1}+\frac{1}{r_1}+\ldots, \qquad (33)$$

where r_1 is the internal resistance of each cell. If different cells are in parallel, there is no simple formula for the total e.m.f. and the total internal resistance, and any calculations involving circuits with such cells are dealt with by applying Kirchhoff's laws (see p. 827).

Summary of Formulae Involving E.M.F.

(i) *Any load*:

(*a*) $$\left.\text{power generated}\right\} = \left\{\begin{array}{c}\text{power supplied}\\ \text{to load}\end{array}\right\} + \left\{\begin{array}{c}\text{power dissipated}\\ \text{in internal}\\ \text{resistance}\end{array}\right.$$

$$EI \quad = \quad IV_{AB} \quad + \quad I^2r,$$

(*b*) terminal p.d. = e.m.f. − voltage drop in internal resistance

$$V_{AB} = E-rI.$$

(ii) *Passive resistance load*:

power equation: $EI = I^2R+I^2r$,

current: $I = \dfrac{E}{R+r}$,

terminal p.d.: $V_{AB} = E-rI = RI$

$$= E\frac{R}{R+r}.$$

EXAMPLES

1. What is meant by the *electromotive force* of a cell?

A voltmeter is connected in parallel with a variable resistance, R, which is in series with an ammeter and a cell. For one value of R the meters read 0·3 amp and 0·9 volt. For another value of R the readings are 0·25 amp and 1·0 volt. Find the values of R, the e.m.f. of the cell, and the internal resistance of the cell. What assumptions are made about the resistance of the meters in the calculation?

If in this experiment the ammeter had a resistance of 10 ohms and the voltmeter a resistance of 100 ohms and R was 2 ohms, what would the meters read? (*L.*)

First part (see p. 796).

Second part. The voltmeter reads the p.d. across the cell if the resistances of the meters are neglected. Thus, with the usual notation,

$$E-Ir = 0{\cdot}9, \quad \text{or} \quad E-0{\cdot}3r = 0{\cdot}9 \qquad \text{(i)}$$

and

$$E-0{\cdot}25r = 1{\cdot}0. \qquad \text{(ii)}$$

Subtracting (i) from (ii),

$$0{\cdot}05r = 0{\cdot}1, \quad \text{i.e.} \quad r = 2 \text{ ohms.}$$

Also, from (i),

$$E = 0{\cdot}3r+0{\cdot}9 = 0{\cdot}6+0{\cdot}9 = 1{\cdot}5 \text{ volts.}$$

Further,

$$R_1 = \frac{V}{I} = \frac{0{\cdot}9}{0{\cdot}3} = 3 \text{ ohms.}$$

and

$$R_2 = \frac{1{\cdot}0}{0{\cdot}25} = 4 \text{ ohms.}$$

If the voltmeter has 100 ohms resistance and is in parallel with the 2 ohms resistance, the combined resistance R is given by

$$\frac{1}{R} = \frac{1}{2}+\frac{1}{100} = \frac{51}{100}, \quad \text{or} \quad R = \frac{100}{51} \text{ ohms.}$$

$$\therefore \text{ current, } I = \frac{E}{\text{Total resistance}}$$

$$= \frac{1{\cdot}5}{\frac{100}{51}+10+2} = 0{\cdot}11 \text{ A.}$$

Also, voltmeter reading $= IR = 0{\cdot}11 \times \dfrac{100}{51} = 0{\cdot}21$ volt.

2. Define *internal resistance* of a voltaic cell. Describe one method of finding by experiment the internal resistance of a primary cell.

Two Daniell cells A and B are connected in series with a coil of resistance 9·8 ohms. A voltmeter of very high resistance connected to the terminals of A reads 0·96 volt and when connected to the terminals of B it reads 1·00 volt. Find the internal resistance of each cell. (Take the e.m.f. of a Daniell cell as 1·08 volts.) (*L.*)

First part. The 'internal resistance' is the resistance of the chemicals inside the cell between the poles, and is given by (drop in terminal p.d.)/current, when the cell is used. The potentiometer may be used to measure the internal resistance. See p. 820.

Second part. The p.d. across both cells $= 0{\cdot}96+1{\cdot}00 = 1{\cdot}96$ volts
$= $ p.d. across 9·8 ohms.

$$\therefore \text{ current flowing, } I, = \frac{V}{R} = \frac{1{\cdot}96}{9{\cdot}8} = 0{\cdot}2 \text{ A.}$$

Now terminal p.d. across each cell $= E - Ir$.

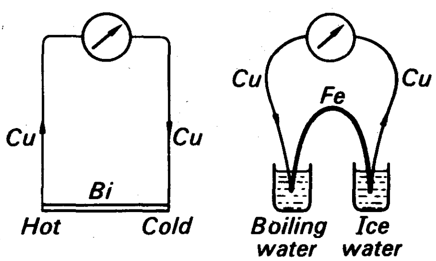
$$\therefore \text{ for cell A,} \quad 0{\cdot}96 = 1{\cdot}08 - 0{\cdot}2r, \quad \text{or} \quad r = 0{\cdot}6 \text{ ohm.}$$

$$\text{for cell B,} \quad 1{\cdot}00 = 1{\cdot}08 - 0{\cdot}2r, \quad \text{or} \quad r = 0{\cdot}4 \text{ ohm.}$$

THE THERMOELECTRIC EFFECT

Seebeck Effect

The heating effect of the current converts electrical energy into heat, but we have not so far described any mechanism which converts heat into electrical energy. This was discovered by Seebeck in 1822. In his experiments he connected a plate of bismuth between copper wires leading to a galvanometer, as shown in Fig. 32.14 (i). He found that if



(i) Seebeck's experiment (ii) Demonstration

FIG. 32.14. The thermo-electric effect.

one of the bismuth-copper junctions was heated, while the other was kept cool, then a current flowed through the galvanometer. The direction of the current was from the copper to the bismuth at the cold junction. We can easily repeat Seebeck's experiment, using copper and iron wires and a galvanometer capable of indicating a few micro-amperes (p. 882) (Fig. 32.14 (ii)).

Thermocouples

Seebeck went on to show that a current flowed, without a battery, in any circuit containing two different metals, with their two junctions at different temperatures. Currents obtained in this way are called thermo-electric currents, and a pair of metals, with their junctions at different temperatures, are said to form a thermocouple. The following is a list of metals, such that if any two of them form a thermocouple, then the current will flow from the higher to the lower in the list, across the cold junction:

Antimony, Iron, Zinc, Lead, Copper, Platinum, Bismuth.

Thermo-electric currents often appear when they are not wanted; they may arise from small differences in purity of two samples of the same metal, and from small differences of temperature—due, perhaps, to the warmth of the hand. They can cause a great deal of trouble in circuits used for precise measurements, or for detecting other small currents, not of thermal origin. As sources of electrical energy, thermo-electric currents are neither convenient nor economical, but they have been used—in gas-driven radio sets. Their only wide application is in the measurement of temperature, and of other quantities, such as radiant energy, which can be measured by a temperature rise.

Variation of Thermoelectric E.M.F. with Temperature

On p. 825 we shall see how thermo-electric e.m.f.s are measured. When the cold junction of a given thermocouple is kept constant at 0°C, and the hot junction temperature t°C is varied, the e.m.f. E is found to vary as $E = at+bt^2$, where a, b are constants. This is a parabola-shaped curve (Fig. 32.15). The temperature A corresponding to the maximum e.m.f. is known as the *neutral temperature*; it is about 250°C for a copper-iron thermocouple. Beyond the temperature B, known as the *inversion temperature*, the e.m.f. reverses. Thermo-electric thermometers, which utilize thermocouples, are used only as far as the neutral temperature, as the same e.m.f. is obtained at two different temperatures, from Fig. 32.15.

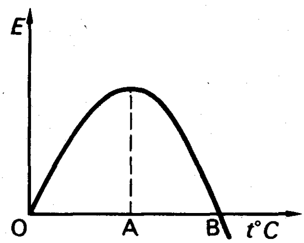


FIG. 32.15. Thermo-electric e.m.f. variation with temperature.

Peltier and Thomson Effects

When a current flows along the junction A of two metals in series, heat is evolved or absorbed at A depending on the current direction. This is known as the *Peltier effect*. It has no connexion with the usual heating or Joule effect of a current, discussed on p. 791. The Joule effect is irreversible, that is, heat is obtained in both directions of the current. In the Peltier effect, however, the effect is reversed when the current is reversed; that is, a cooling is produced at the junction of two metals in one direction, and an evolution of heat in the other direction.

Sir William Thomson, later Lord Kelvin, also found that heat was evolved or absorbed when a current flows along a metal whose ends are kept at different temperatures. The *Thomson effect*, like the Peltier effect, is also reversible.

EXERCISES 32

1. State the laws of the development of heat when an electric current flows (*a*) through a wire of uniform material, (*b*) across the junction between two metals.

An electric heating coil is connected in series with a resistance of X ohms across the 240-volt mains, the coil being immersed in a kilogramme of water at 20°C. The temperature of the water rises to boiling-point in 10 minutes. When a second heating experiment is made with the resistance X short-circuited, the time required to develop the same quantity of heat is reduced to 6 minutes. Calculate the value of X. (Heat losses may be neglected.) (*L.*)

2. Define *electromotive force* and explain with the help of an example the difference between electromotive force and potential difference.

A thermocouple whose junctions are maintained at constant temperatures has a resistance of 5 ohms and its e.m.f. as measured using a potentiometer is 3·9 mV. What will be the reading on a millivoltmeter of resistance 60 ohms connected directly to the thermocouple? (*N.*)

3. Deduce an expression for the heat developed in a wire by the passage of an electric current.

The temperature of 300 g of paraffin oil in a vacuum flask rises 1·0°C per minute with an immersion heater of 12·3 watts input. On repeating with 400 g of oil the temperature rises by 1·20°C per minute for an input of 19·2 watts. Find the specific heat of the oil and the thermal capacity (assumed constant) of the flask. (*L.*)

4. Describe an experiment to determine the resistance of a wire by a calorimetric method.

It is desired to construct a 5-amp fuse from tin wire which has a melting-point of 230°C and resistivity 22×10^{-8} ohm m at that temperature. Estimate the diameter of the wire required if the emissivity of its surface is 88×10^{-5} J per sq. cm per second per °C excess temperature above the surroundings whose temperature is 20°C. Neglect the heat loss by conduction along the wire. (*N.*)

5. Describe the chief thermo-electric effects which occur in a circuit which includes two metals such as copper and iron.

Make a labelled diagram showing clearly the arrangement of a potentiometer circuit suitable for measuring a thermo-electric e.m.f. of about 2 mV. (*L.*)

6. Indicate, by means of graphs, the relation between the current and voltage (*a*) for a uniform manganin wire; (*b*) for a water voltameter; (*c*) for a diode valve. How do you account for the differences between the three curves?

An electric hot plate has two coils of manganin wire, each 20 metres in length and 0·23 mm² cross-sectional area. Show that it will be possible to arrange for three different rates of heating, and calculate the wattage in each case when the heater is supplied from 200-volt mains. The resistivity of manganin is $4{\cdot}6 \times 10^{-7}$ ohm m. (*O. & C.*)

7. Describe an experiment for determining the variation of the resistance of a coil of wire with temperature.

An electric fire dissipates 1 kW when connected to a 250-volt supply. Calculate to the nearest whole number the percentage change that must be made in the resistance of the heating element in order that it may dissipate 1 kW on a 200-volt supply. What percentage change in the length of the heating element will produce this change of resistance if the consequent increase in the temperature of the wire causes its resistivity to increase by a factor 1·05? The cross-sectional area may be assumed constant. (*N.*)

8. What is a thermocouple? Explain the use of a potentiometer to measure the small electromotive forces developed by a thermocouple.

What are the relative advantages and disadvantages of a thermocouple used as a thermometer as compared with the resistance thermometer? (*L.*)

9. Give a general account of the thermo-electric effect. Describe how you would calibrate a thermocouple for use over the range of 0°–100°C on the mercury-in-glass scale of temperature. (*C.*)

10. Derive, from first principles, an expression for the rate at which heat is generated in a resistance R by the passage of a current I.

An electric lamp takes 60 watts on a 240-volt circuit. How many dry cells, each of e.m.f. 1·45 volts and internal resistance 1·0 ohm, would be required to light the lamp? How much zinc would be consumed by the battery in 1 hour? (1 faraday = 96,500 coulombs; equivalent weight of zinc = 32·5.) (*O. & C.*)

11. Describe an experimental method of producing a thermo-electric e.m.f. How may a thermojunction be used to measure temperatures?

Why is a copper-iron junction not used to measure temperatures above 250°C, although a copper-constantan junction is often so employed? (*L.*)

12. Describe an instrument which measures the strength of an electric current by making use of its heating effect. State the advantages and disadvantages of this method.

A surge suppressor is made of a material whose conducting properties are such that the current passing through is directly proportional to the fourth power of the applied voltage. If the suppressor dissipates energy at a rate of 6·0 watts when the potential difference across it is 240 volts, estimate the power dissipated when the potential difference rises to 1200 volts. (*C.*)

13. Give a short account of the thermoelectric effect first discovered by Seebeck.

If you were given ice, boiling water, a thermocouple, a variable resistor and a sensitive galvanometer (with a linear scale) but no thermometer, describe how you would determine the temperature inside a domestic refrigerator. How would you test the assumption you are making? No other apparatus is available, but you may assume body temperature is 36·9°C.

Describe how you would measure, in the laboratory, the resistance of a pair of headphones, which is damaged if the current through it exceeds 100 milli-amp. (*C.*)

chapter thirty-three

Applications of Ohm's Law

MEASUREMENTS. NETWORKS

IN this chapter we shall apply Ohm's law to circuits more complicated than those of Chapter 32. We shall see that some special types of circuits can be used to make electrical measurements more accurately than with pointer instruments.

RESISTORS AND THEIR ARRANGEMENTS

Series Resistors

The resistors of an electric circuit may be arranged in series, so that the charges carrying the current flow through each in turn (Fig. 33.1); or they may be arranged in parallel, so that the flow of charge divides between them (Fig. 33.2), p. 808.

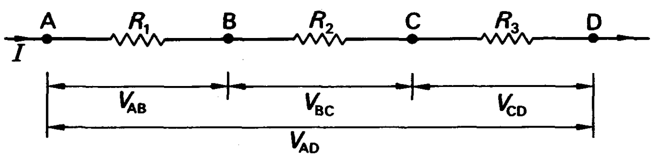


FIG. 33.1. Resistances in series.

Fig. 33.1 shows three passive resistors in series, carrying a current I. If V_{AD} is the potential difference across the whole system, the electrical energy supplied to the system per second is IV_{AD}. This is equal to the electrical energy dissipated per second in all the resistors; therefore

$$IV_{AD} = IV_{AB}+IV_{BC}+IV_{CD},$$

whence
$$V_{AD} = V_{AB}+V_{BC}+V_{CD}, \quad . \quad . \quad . \quad . \quad (1)$$

The individual potential differences are given by Ohm's law:

$$\left.\begin{aligned} V_{AB} &= IR_1 \\ V_{BC} &= IR_2 \\ V_{CD} &= IR_3 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

and
Hence, by equation (1),

$$\begin{aligned} V_{AD} &= IR_1+IR_2+IR_3 \\ &= I(R_1+R_2+R_3). \quad . \quad . \quad . \quad (3) \end{aligned}$$

And the effective resistance of the system is

$$R = \frac{V_{AD}}{I} = R_1 + R_2 + R_3. \quad . \quad . \quad (4)$$

The physical facts are:

(i) *Current same through all resistors.*
(ii) *Total potential difference = sum of individual potential difference* (equation (1)).
(iii) *Individual potential differences directly proportional to individual resistances* (equation (2)).
(iv) *Total resistance greater than greatest individual resistance* (equation (4)).
(v) *Total resistance = sum of individual resistances.*

Resistors in Parallel

Fig. 33.2 shows three passive resistors connected in parallel, between the points A, B. A current I enters the system at A and leaves at B,

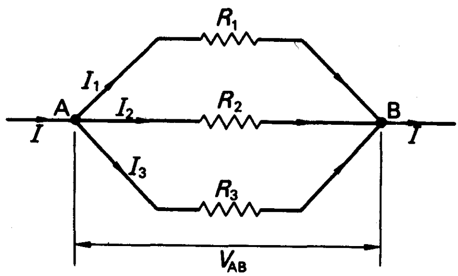


FIG. 33.2. Resistances in parallel.

setting up a potential difference V_{AB} between those points. The current branches into I_1, I_2, I_3, through the three elements, and

$$I = I_1 + I_2 + I_3. \quad . \quad . \quad . \quad (5)$$

Now $$I_1 = \frac{V_{AB}}{R_1}, \quad I_2 = \frac{V_{AB}}{R_2}, \quad I_3 = \frac{V_{AB}}{R_3}.$$

$$\therefore I = V_{AB}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

$$\therefore \frac{I}{V_{AB}} = \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \quad . \quad . \quad (6)$$

where R is the effective resistance (V_{AB}/I) of the system.

The physical facts about resistors in parallel may be summarized as follows:

(i) *Potential difference same across each resistor.*
(ii) *Total current = sum of individual currents* (equation (5)).

(iii) *Individual currents inversely proportional to individual resistances.*
(iv) *Effective resistance less than least individual resistance* (equation (6)).

Resistance Boxes

In many electrical measurements variable known resistances are required; they are called resistance boxes. As shown in Fig. 33.3 (i) ten coils, each of resistance 1 ohm, for example, are connected in series. A rotary switch with eleven contacts enables any number of these coils to be connected between the terminals AA′. A resistance box contains several sets of coils and switches, the first giving resistances 0–10 ohms in steps of 1 ohm, the next 0–100 ohms in steps of 10 ohms, and so on. These are called decade boxes.

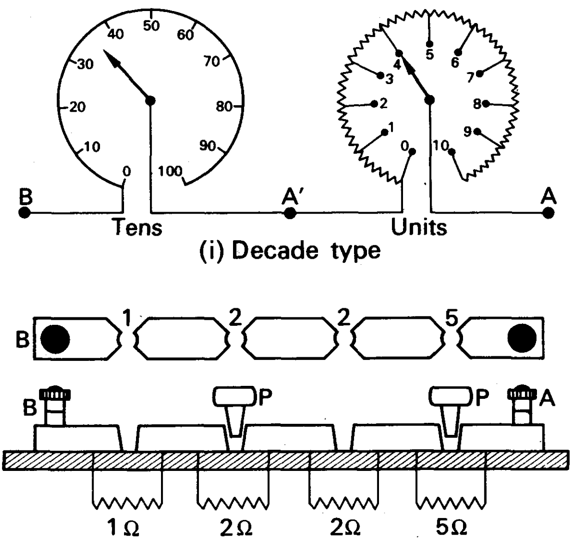


FIG. 33.3. Resistance boxes.

The switches used in a decade box are of very high quality; their contact resistances are negligible compared with the resistances of the coils which they select. Switches of this kind have been developed only in the last twenty years or so: in older boxes no switches are used. Instead, the resistances are varied by means of plugs. As shown in Fig. 33.3 (ii), the resistance coils are joined across gaps in a thick brass bar, and the gaps are formed into tapered sockets to receive short-circuiting plugs P. The resistance between the terminals A and B in Fig. 33.3 (ii) is the sum of the unplugged resistances between them—3 ohms in this example.

The coils of a resistance box are wound in a particular way, which we shall describe and explain later (p. 924). They are not intended to carry large currents, and must not be allowed to dissipate more than one watt. Therefore, since $P = I^2R$, the greatest safe current for a 1-ohm coil is 1 amp, and for a 10-ohm coil about 0·3 amp. If the one-watt limit is exceeded, the insulation will be damaged, or the wire burnt out.

The Potential Divider

Two resistance boxes in series are often used in the laboratory to provide a known fraction of a given potential difference—for example, of one which is too large to measure easily. Fig. 33.4 (i) shows the

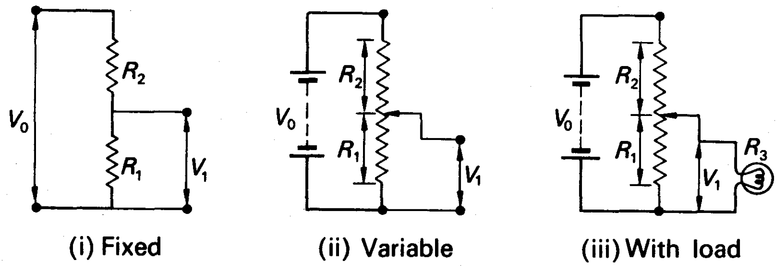


FIG. 33.4. Potential divider.

arrangement, which is called a resistance 'potential divider'. The current flowing, I, is given by

$$I = \frac{V_0}{R_1 + R_2},$$

$$\therefore\ V_1 = IR_1 = \frac{R_1}{R_1 + R_2}V_0. \qquad (7)$$

A resistor with a sliding contact can similarly be used, as shown in Fig. 33.4 (ii), to provide a continuously variable potential difference, from zero to the full supply value V_0. This is a convenient way of controlling the voltage applied to a load, such as a lamp (Fig. 33.4 (iii)). The resistance of the load, R_3, however, acts in parallel with the resistance R_1; equation (7) is therefore no longer true, and the voltage V_1 must be measured with a voltmeter. It can be calculated, as in the following example, if R_3 is known; but if the load is a lamp its resistance varies greatly with the current through it, because its temperature varies.

EXAMPLE

A load of 2000 ohms is connected, via a potential divider of resistance 4000 ohms, to a 10-volt supply (Fig. 33.5). What is the potential difference across the load when the slider is (*a*) one-quarter, (*b*) half-way up the divider?

Since $\frac{1}{R} = \frac{1}{2000} + \frac{1}{1000}$,

(a) $R_{BC} = \frac{2000 \times 1000}{2000 + 1000} = \frac{2000}{3}$ ohms.

$\therefore R_{AC} = R_{AB} + R_{BC} = 3000 + \frac{2000}{3}$ ohms,

$\therefore V_{BC} = \frac{R_{BC}}{R_{AC}} V_C$

$= \frac{2000/3}{11000/3} \times 10 = \frac{2}{11} \times 10$

$= 1{\cdot}8$ volts.

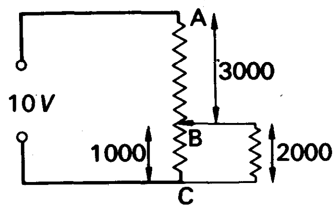


FIG. 33.5. A loaded potential divider.

If the load were removed, V_{BC} would be 2·5 volts.

(b) It is left for the reader to show similarly that $V_{BC} = 3{\cdot}3$ volts. Without the load it would be 5 volts.

MEASURING INSTRUMENTS

Conversion of a Milliammeter into a Voltmeter

Ohm's law enables us to use a milliammeter as a voltmeter. Let us suppose that we have a moving-coil instrument which requires 5 milliamperes for full-scale deflection (f.s.d.). And let us suppose that the resistance of its coil, r, is 20 ohms (Fig. 33.6). Then, when it is fully deflected, the potential difference across it is

$V = rI$

$= 20 \times 5 \times 10^{-3} = 100 \times 10^{-3}$ volt

$= 0{\cdot}1$ volt.

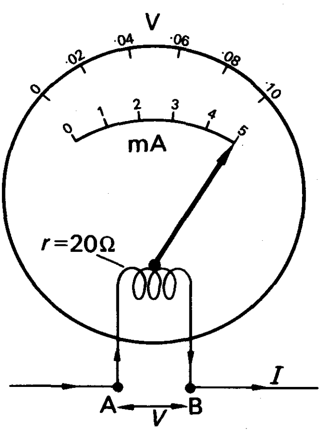


FIG. 33.6. P.D. across moving-coil meter.

Since the coil obeys Ohm's law, the current through it is proportional to the potential difference across it; and since the deflection of the pointer is proportional to the current it is therefore also proportional to the potential difference. Thus the instrument can be used as a voltmeter, giving full-scale deflection for a potential difference of 0·1 volt, or 100 millivolts. Its scale could be engraved as shown at the top of Fig. 33.6.

The potential differences to be measured in the laboratory are usually greater than 100 millivolts, however. To measure such a potential difference, we insert a resistor R in series with the coil, as shown in Fig. 33.7. If we wish to measure up to 10 volts we must choose the resistance R so that, when 10 volts are applied between the

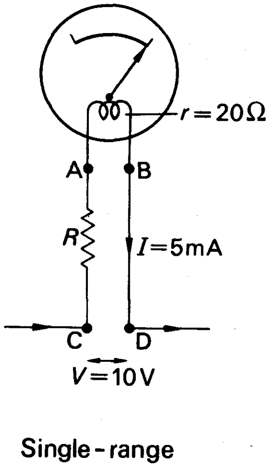


FIG. 33.7. Single-range voltmeter.

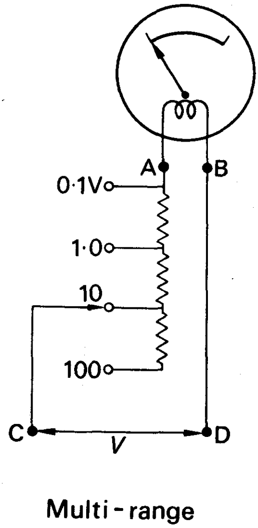


FIG. 33.8. Multi-range voltmeter.

terminals CD, then a current of 5 milliamperes flows through the moving coil. By Ohm's law

$$V = (R+r)I,$$

$$\therefore\ 10 = (R+20)\times 5\times 10^{-3}$$

or
$$R+20 = \frac{10}{5\times 10^{-3}} = 2\times 10^3 = 2000 \text{ ohms.}$$

$$\therefore\ R = 2000-20$$

$$= 1980 \text{ ohms.} \qquad (8)$$

The resistance R is called a *multiplier.* Many voltmeters contain a series of multipliers of different resistances, which can be chosen by a switch or plug-and-socket arrangement (Fig. 33.8).

Conversion of a Milliammeter into an Ammeter

Moving-coil instruments give full-scale deflection for currents smaller than those generally encountered in the laboratory. If we wish to measure a current of the order of an ampere or more we connect a low resistance S, called a shunt, across the terminals of a moving-coil meter (Fig. 33.9). The shunt diverts most of the current to be measured, I, away from the coil—hence its name. Let us suppose that, as before, the coil of the meter has a resistance r of 20 ohms and is fully deflected by a current, I_C, of 5 milliamperes. And let us suppose that we wish to shunt it so that it gives f.s.d. for 5 amperes to be measured. Then the current through the shunt is

$$I_S = I - I_C$$
$$= 5 - 0{\cdot}005$$
$$= 4{\cdot}995 \text{ amp.}$$

The potential difference across the shunt is the same as that across the coil, which is

$$V = rI_C = 20 \times 0{\cdot}005 = 0{\cdot}1 \text{ volt.}$$

The resistance of the shunt must therefore be

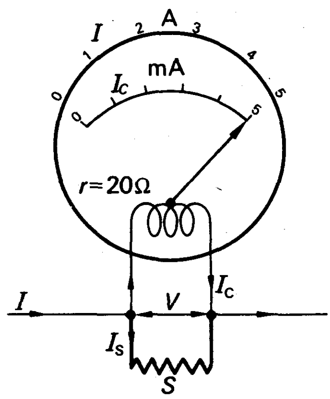


FIG. 33.9. Conversion of milliammeter to ammeter.

$$S = \frac{V}{I_S} = \frac{0{\cdot}1}{4{\cdot}995} = 0{\cdot}02002 \text{ ohms.} \qquad (9)$$

The ratio of the current measured to the current through the coil is

$$\frac{I}{I_C} = \frac{5}{5 \times 10^{-3}} = 1000.$$

This ratio is the same whatever the current I, because it depends only on the resistances S and r; the reader may easily show that its value is $(S+r)/S$. The deflection of the coil is therefore proportional to the measured current, as indicated in the figure, and the shunt is said to have a 'power' of 1000 when used with this instrument.

The resistance of shunts and multipliers are always given with four-figure accuracy. The moving-coil instrument itself has an error of the order of 1 per cent.; a similar error in the shunt or multiplier would therefore double the error in the instrument as a whole. On the other hand, there is nothing to be gained by making the error in the shunt less than about 0·1 per cent., because at that value it is swamped by the error of the moving system.

Multimeters

A *multimeter* instrument is one which is adapted for measuring both current and voltage. It has a shunt R as shown, and a series of voltage multipliers R' (Fig. 33.10). The shunt is connected permanently across the coil, and the resistances in R' are adjusted to give the desired full-scale voltages with the shunt in position. A switch or plug enables the various full-scale values of current or voltage to be chosen, but the user does the mental arithmetic. The instrument shown in the figure is reading 1·7 volts; if it were on the 10-volt range, it would be reading 6·4.

The terminals of a meter, multimeter or otherwise, are usually marked + and −; the pointer is deflected to the right when current passes through the meter from + to −.

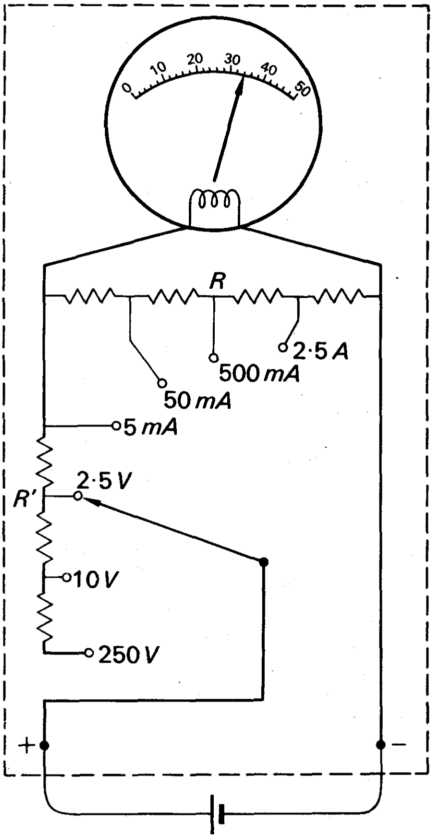


FIG. 33.10. A multimeter.

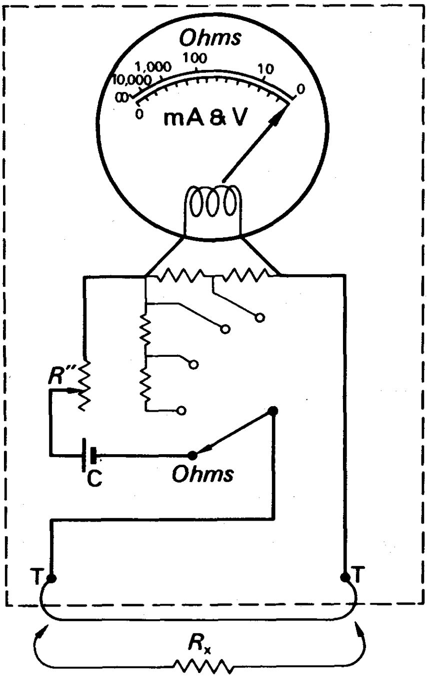


FIG. 33.11. Measurement of resistance with multimeter.

The multimeters are generally arranged to measure resistance as well as current and voltage. An extra position on the switch, marked 'R' or 'ohms', puts a dry cell C and a variable resistor R'' in series with the moving coil (Fig. 33.11). Before the instrument is used to measure a resistance, its terminals TT are short-circuited, and R'' is adjusted until the pointer is fully deflected. As shown in the figure, it is then opposite the zero on the ohms scale. The short-circuit is next removed, and the unknown resistance R_x is connected across the terminals. The current falls, and the pointer moves to the left, indicating on the ohms scale the value of R_x. The ohms scale is calibrated by the makers with known resistances.

Use of Voltmeter and Ammeter

A moving-coil voltmeter is a current-operated instrument. It can be used to measure potential differences only because the current which it draws is proportional to the potential difference applied to it, from Ohm's law. Since its action depends on Ohm's law, a moving-coil voltmeter cannot be used in any experiment to demonstrate that law; that is why, when describing such an experiment on p. 789, we specified measuring instruments whose readings are not dependent on Ohm's law.

Having once established Ohm's law, however, we can use moving-coil voltmeters freely; they are both more sensitive and more accurate than other forms of voltmeters. The current which they take does, however, sometimes complicate their use. To see how it may do so, let us suppose that we wish to measure a resistance R of about 100 ohms. As shown in Fig. 33.12, we connect it in series with a cell, a milliammeter, and a

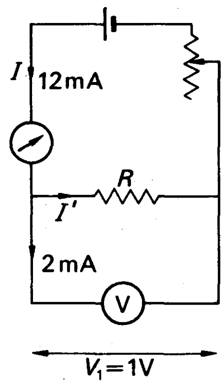


FIG. 33.12

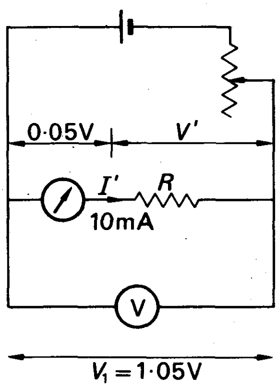


FIG. 33.13.

Use of ammeter and voltmeter.

variable resistance; across it we place the voltmeter. We adjust the current until the voltmeter reads, say, $V_1 = 1$ volt; let us suppose that the milliammeter then reads $I = 12$ mA. The value of the resistance then appears to be

$$R = \frac{V_1}{I} = \frac{1}{12 \times 10^{-3}} = \frac{10^3}{12}$$

$$= 83 \text{ ohms (approx.).}$$

But the milliammeter reading includes the current drawn by the voltmeter. If that is 2 mA, then the current through the resistor, I', is only 10 mA and its resistance is actually

$$R = \frac{V_1}{I'} = \frac{1}{10 \times 10^{-3}} = \frac{1}{10^{-2}}$$

$$= 100 \text{ ohms.}$$

The current drawn by the voltmeter has made the resistance appear 17 per cent. lower than its true value.

In an attempt to avoid this error, we might connect the voltmeter as shown in Fig. 33.13: across both the resistor and the milliammeter. But its reading would then include the potential difference across the milliammeter. Let us suppose that this is 0·05 volt when the current through the milliammeter is 10 mA. Then the potential difference V' across the resistor would be 1 volt, and the voltmeter would read 1·05 volt. The resistance would appear to be

$$R = \frac{1{\cdot}05}{10 \times 10^{-3}} = \frac{1{\cdot}05}{10^{-2}}$$

$$= 105 \text{ ohms.}$$

Thus the voltage drop across the milliammeter would make the resistance appear 5 per cent. higher than its true value.

Errors of this kind are negligible only when the voltmeter current is much less than the current through the resistor, or when the voltage drop across the ammeter is much less than the potential difference across the resistor. If we were measuring a resistance of about 1 ohm, for example, the current I' in Fig. 33.12 would be 1 amp, and I would be 1·002 amp. The error in measuring R would then be only 0·2 per cent—less than the intrinsic error of the meter. But the circuit of Fig. 33.13 would give the same error as before. It could do so because, as we saw when considering shunts, the shunt across the milliammeter would have been chosen to make the voltage drop still 0·05 volt. Thus V_1 would still be 1·05 volt when V' was 1 volt, and the error would be 5 per cent as before.

In low-resistance circuits, therefore, the voltmeter should be connected as in Fig. 33.12, so that its reading does not include the voltage drop across the ammeter. But in high-resistance circuits the voltmeter should be connected as in Fig. 33.13, so that the ammeter does not carry its current.

If a moving-coil voltmeter is connected across a cell, it will not read its true e.m.f., because the current which it draws will set up a voltage drop across the internal resistance of the cell. The drop will be negligible only if the resistance of the voltmeter is very high compared with the internal resistance. E.m.f.s are thus compared by a potentiometer method, discussed shortly.

Figure of Merit of a Voltmeter

If a milliammeter of 1 mA f.s.d. (full scale deflection) is converted into a voltmeter, then if it is to have 1 volt f.s.d. its total resistance—coil plus multiplier—must be 1000 ohms. (One volt across its terminals will send through it a current of 1/1000 amp = 1 mA.) If it is to have 10 volts f.s.d., then its total resistance must be 10000 ohms; for 20 volts f.s.d., 20000 ohms, and so on. It will have a resistance of 1000 ohms for every volt of its full-scale deflection. Such a meter is said to have a *figure of merit* of 1000 ohms per volt. Similarly, a voltmeter which takes 10 mA, or 1/100 amp, for full-scale deflection has a figure of merit of 100 ohms per volt. The greater the figure of merit of a voltmeter, expressed in this way, the less will it disturb any circuit to which it is connected, and the less error will its current cause in any measurements made with it. On the other hand, the greater the figure of merit, the more delicate the moving system of the meter and the greater its intrinsic error. First-grade, and particularly 'sub-standard', meters therefore have medium or low figures of merit: from 500 to 66·7 ohms per volt.

When a voltmeter of low figure of merit is being used, it may be necessary to allow for the current which it draws. The allowance is made in the way indicated on p. 815, where the use of a voltmeter and ammeter together was discussed.

THE POTENTIOMETER

Pointer instruments are useless for very accurate measurements: the best of them have an intrinsic error of abour 1 per cent of full scale. Where greater accuracy than this is required, elaborate measuring circuits are used.

One of the most versatile of these, due to Poggendorf, is the potentiometer. It consists of a uniform wire, AB in Fig. 33.13(i), about a metre long; through it an accumulator X maintains a steady current I. Since the wire is uniform, its resistance per centimetre, R, is constant; the voltage drop across 1 cm of the wire, RI, is therefore also constant.

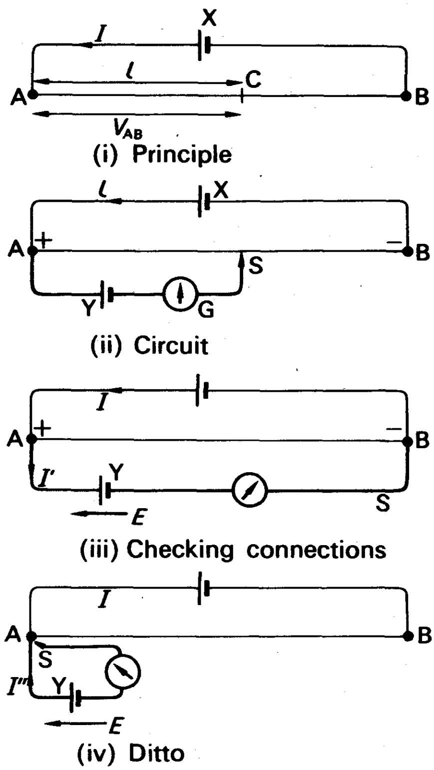


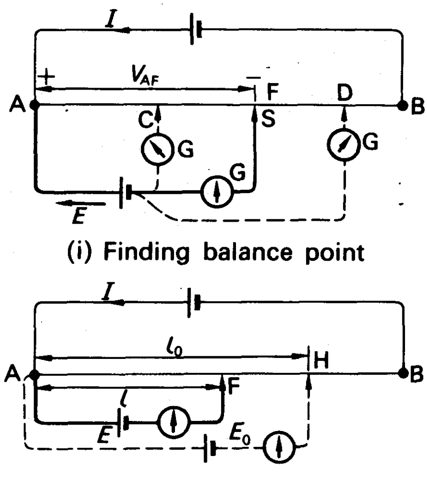
FIG. 33.14. The potentiometer.

The potential difference between the end A of the wire, and any point C upon it, is thus proportional to the length of wire l between A and C:

$$V_{AC} \propto l. \qquad (10)$$

Comparison of E.M.F.s

To illustrate the use of the potentiometer, let us suppose that we take a cell, Y in Fig. 33.14(ii), and join its positive terminal to the point A (to which the positive terminal of X is also joined). We connect the negative terminal of Y, via a sensitive galvanometer, to a slider S, which we can press on to any point in the wire. Let us suppose that the cell Y has an e.m.f. E, which is less than the potential difference V_{AB} across the whole of the wire. Then if we press the slider on B, a current I' will flow through Y in opposition to its e.m.f. (Fig. 33.14(iii)). This current will deflect the galvanometer G—let us say to the right. If we now press the slider on A, the cell Y will be connected straight across the galvanometer, and will deliver a current I'' in the direction of its e.m.f. (Fig. 33.14(iv)). The galvanometer will therefore show a deflection to the left. If the deflections at A and B are not opposite, then either the e.m.f. of Y is greater than the potential difference across the whole wire, or we have connected the circuit wrongly. The commonest mistake in connecting up is not joining both positives to A.



(i) Finding balance point

(ii) Comparison of e.m.f.

FIG. 33.15. Use of potentiometer.

Now let us suppose that we place the slider on to the wire at a point a few centimetres from A, then at a point a few centimetres farther on, and so forth. (We do not run the slider continuously along the wire, because the scraping would destroy the uniformity.) When the slider is at a point C near A (Fig. 33.15(i)) the potential difference V_{AC} is less than the e.m.f. E of Y; current therefore flows through G in the direction of E, and G may deflect to the left. When the slider is at D near B, V_{AD} is greater than E, current flows through G in opposition to E, and G deflects to the right. By trial and error (but no scraping of the slider) we can find a point F such that, when the slider is pressed upon it, the galvanometer shows no deflection. The potential difference V_{AF} is then equal to the e.m.f. E; no current flows through the galvanometer because E and V_{AF} act in opposite directions in the galvanometer circuit (Fig. 33.15(i)). Because no current flows, the resistance of the galvanometer, and the internal resistance of the cell, cause no voltage drop; the full e.m.f. E therefore appears, between the points, A and S, and is balanced by V_{AF}:

$$E = V_{AF}.$$

If we now take another cell of e.m.f. E_0, and balance it in the same way, at a point H (Fig. 33.15(ii)), then

$$E_0 = V_{AH}.$$

Therefore

$$\frac{E}{E_0} = \frac{V_{AF}}{V_{AH}}.$$

The potential differences V_{AF}, V_{AH} are proportional to the lengths l, l_0 from A to F, and from A to H, respectively. Therefore

$$\frac{E}{E_0} = \frac{l}{l_0}. \qquad (11)$$

Accuracy

When the potentiometer is used to compare the e.m.f.s of cells, no errors are introduced by the internal resistances, because no current flows at the balance-points.

The potentiometer is more accurate than an electrometer instrument, which, like a moving-coil voltmeter, has an intrinsic error of about 1 per cent of full-scale. The accuracy of a potentiometer is limited by the non-uniformity of the slide-wire, the uncertainty of the balance-point, and the error in measuring the length l of wire from the balance-point to the end A. With even crude apparatus, the balance-point can be located to within about 0·5 mm; if the length l is 50 cm, or 500 mm, then the error in locating the balance-point is 1:1000. If the wire has been carefully treated, its non-uniformity may introduce an error of about the same magnitude. The overall error is then about ten times less than that of a pointer instrument. A refined potentiometer has a still smaller error.

The precision with which the balance-point of a potentiometer can be found depends on the sensitivity of the galvanometer—the smallness of the current which will give a just-discernible deflection. A moving-coil galvanometer must be protected by a series resistance R of several thousand ohms, which is shorted out when the balance is nearly reached (Fig. 33.16). A series resistance is preferable to a shunt, because it re-

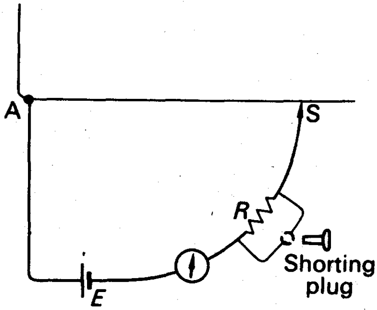


FIG. 33.16. Use of protective resistance with galvanometer.

duces the current drawn from the cell under test, when the potentiometer is unbalanced. The process of seeking the balance-point then causes less change in the chemical condition of the cell, and therefore in its e.m.f.

It is important to realize that the accuracy of a potentiometer does not depend on the accuracy of the galvanometer, but only on its sensitivity. The galvanometer is used not to measure a current but merely to show one when the potentiometer is off balance. It is said to be used as a null-indicator, and the potentiometer method of measurement, like the bridge methods which we shall describe shortly, is called a null method.

The current through the potentiometer wire must be steady—it must not change appreciably between the finding of one balance-point and the next. The accumulator which provides it should therefore be neither freshly charged nor nearly run-down; when an accumulator is in either of those conditions its e.m.f. falls with time. Errors in potentiometer measurements may be caused by non-uniformity of the wire, and by the resistance of its connexion to the terminal at A. This resistance is added to the resistance of the length l of the wire between A and the balance-point, and if it is appreciable it makes equation (11) invalid. Both these sources of error are eliminated in the Rayleigh potentiometer, which we shall describe later (p. 825).

Uses of the Potentiometer. E.M.F. and Internal Resistance

All the uses of the potentiometer depend on the fact that it can measure potential difference accurately, and without drawing current from the circuit under test.

If one of the cells in Fig. 33.15 (ii) has a known e.m.f., say E_0 then the e.m.f. of the other, E, is given by equation:

$$\frac{E}{E_0} = \frac{l}{l_0}. \qquad (12)$$

A cell of known e.m.f. is called a standard cell. The e.m.f.s of standard cells are determined absolutely—that is to say, without reference to the e.m.f.s of any other cell—by methods which depend, in principle, on the definition of e.m.f. (power/current, p. 797). Standard cells are described on p. 865, along with the precautions which must be taken in their use. For simple experiments a Daniell cell (p. 860), whose e.m.f. is about 1·1 volt, may be used as a standard.

Equation (12) is true only if the current I through the potentiometer wire has remained constant. The easiest way to check that it has done so is to balance the standard cell against the wire before and after balancing the unknown cell. If the lengths to the balance-point are equal—within the limits of experimental error— then the current I may be taken as constant. A check of this kind should be made in each of the experiments to be described.

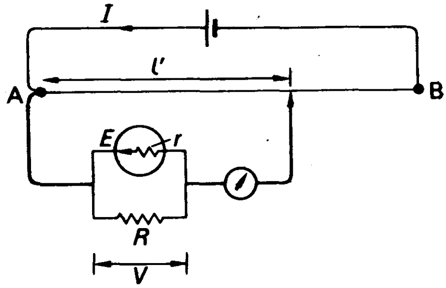


FIG. 33.17. Measurement of internal resistance.

The internal resistance of a cell, r, can be found with a potentiometer by balancing first its e.m.f., E, and the its terminal potential difference, V, when a known resistance R is connected across it (Fig. 33.17). Ohm's law for the complete circuit gives

$$\frac{V}{E}=\frac{R}{R+r}. \qquad (13)$$

But

$$\frac{V}{E}=\frac{l'}{l}, \qquad (14)$$

where l and l' are the lengths of potentiometer wire required to balance E and V. From equations (13) and (14), r can be found from

$$r=\left(\frac{l}{l'}-1\right)R.$$

Calibration of Voltmeter

Fig. 33.18 shows how a potentiometer can be used to calibrate a voltmeter. A standard cell is first used to find the p.d. per cm or volts

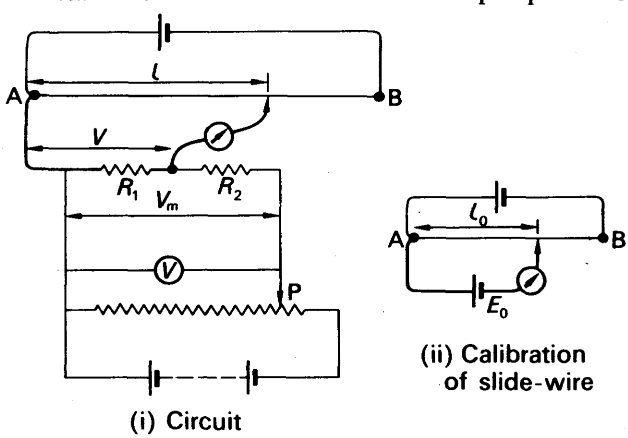


FIG. 33.18. Calibration of voltmeter with potentiometer.

per cm of the wire (Fig. 33.18(ii)): if its e.m.f. E_0 is balanced by a length l_0, then

$$\text{volts per cm} = \frac{E_0}{l_0}. \qquad (15)$$

Different voltages V_m are now applied to the voltmeter by the adjustable potential divider P (Fig. 33.18(i)). The fixed potential divider, comprising R_1R_2, gives a known fraction V of each value of V_m, which is then balanced on the potentiometer:

$$\frac{V}{V_m} = \frac{R_1}{R_1+R_2}. \qquad (16)$$

(The resistances R_1 and R_2 are high—of the order of 1000 to 10000 ohms, so that the voltage adjustment by P is fairly uniform. Their ratio is chosen so that the greatest value of V is measurable on the potentiometer—about 1·5 volts.) If l is the lengths of potentiometer wire which balances a given value of V, then

$$V = l \times (\text{volt/cm of wire})$$
$$= l\frac{E_0}{l_0}.$$

From each value of V, the value of V_m is calculated by equation (16). If the voltmeter reading is V_{obs}, then the correction to be added to it is $V_m - V_{obs}$. This is plotted against V_{obs}, as in Fig. 33.19.

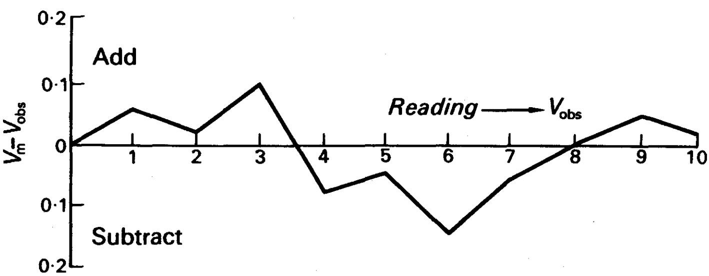
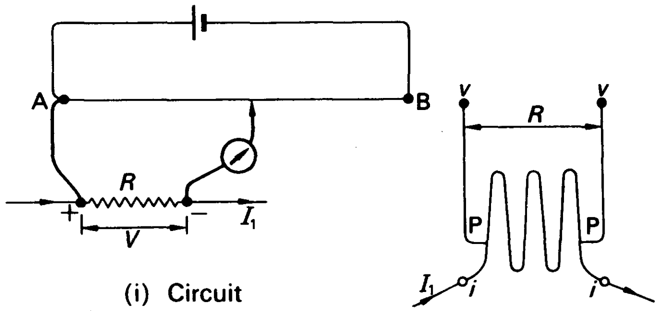


FIG. 33.19. Correction curve of voltmeter.

Measurement of Current

A current can be measured on a potentiometer by means of the potential difference which it sets up across a known resistance, R in Fig. 33.20(i). The resistance is low, being chosen so that the potential difference across it is of the order of 0·1 volt. (A higher value is not chosen, because the voltage drop across the resistor disturbs the circuit in which it is inserted.) Fig. 33.20(ii) shows in detail the kind of resistor used, which is often called a standard shunt. It consists of a broad strip of alloy, such as manganin, whose resistance varies very little with temperature (p. 837). The current is led in and out as the terminals i, i. The terminals v, v are connected to fine wires soldered to points PP on the strip; they are called the potential terminals. The marked value R of the resistance is the value between the points PP; it is adjusted by making hack-saw cuts into the edges of the strip.



(i) Circuit

(ii) Resistor

FIG. 33.20. Measurement of current with potentiometer.

As shown in Fig. 33.20(i), the current to be measured, I_1, is passed through the shunt, and the potential difference between its potential terminals, V, is balanced on the potentiometer wire. If l is the length of wire to the balance-point, then

$$\frac{V}{E_0} = \frac{l}{l_0}, \qquad (17)$$

where E_0 and l_0 refer to a standard cell as before. Equation (17) enables the current to be found in terms of E_0, l_0, l, and R, since

$$V = I_1 R.$$

The resistance of the wires connecting the potential terminals to the points PP, and to the potentiometer circuit, do not affect the result, because at the balance-point the current through them is zero.

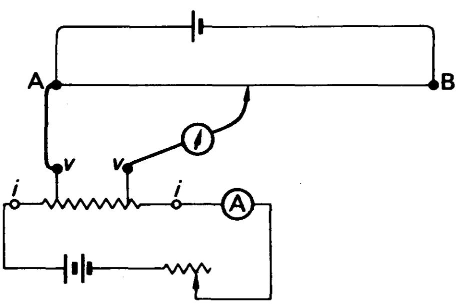


FIG. 33.21. Calibration of ammeter with potentiometer.

This method of measuring a current can be used to calibrate an ammeter, A. The circuit is shown in Fig. 33.21; its principle and use should explain themselves. The results are treated in the same way as in the calibration of a voltmeter.

Comparison of Resistances

A potentiometer can be used to compare resistances, by comparing the potential differences across them when they are carrying the same current I_1 (Fig. 33.22). This method is particularly useful for very low resistances, because, as we have just seen, the resistances of the connecting wires do not affect the result of the experiment. It can, however, be used for higher resistances if desired. With low resistances the ammeter A′ and rheostat P are necessary to adjust the current to a value which will neither exhaust the accumulator Y, nor overheat the resistors. No standard cell is required. The potential difference across the first resistor, $V_1 = R_1 I_1$, is balanced against a length l_1 of the potentiometer wire, as shown by the full lines in the figure. Both potential terminals

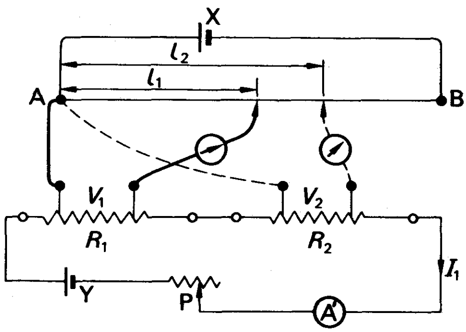


FIG. 33.22.
Comparison of resistances with potentiometer.

of R_1 are then disconnected from the potentiometer, and those of R_2 are connected in their place. If l_2 is the length to the new balance-point, then

$$\frac{l_1}{l_2} = \frac{V_1}{V_2} = \frac{R_1 I_1}{R_2 I_1} = \frac{R_1}{R_2}.$$

This result is true only if the current I_1 is constant; as well as the potentiometer current. The accumulator Y, as well as X, must therefore be in good condition. To check the constancy of the current I_1, the ammeter A′ is not accurate enough. The reliability of the experiment as a whole can be checked by balancing the potential V_1 a second time, after V_2. If the new value of l_1 differs from the original, then at least one of the accumulators is running down and must be replaced.

The Rayleigh Potentiometer

Fig. 33.23 shows a potentiometer devised by Lord Rayleigh (1842–1919), which is free from errors due to non-uniformity of the wire and to contact resistance at the end A (p. 820). It consists of two plug-type resistance boxes, R_1, R_2, joined in series. (These boxes may well be the R-sections of two similar Post Office boxes (p. 834). At the start of a measurement all the plugs of R_1 are inserted, and all of R_2 taken out. Then R_1 is zero, and the main current I sets up no potential difference across it; but when the key K is pressed, the unknown e.m.f. E deflects the galvanometer. R_1 is now increased by, say, 100 ohms, and R_2 is

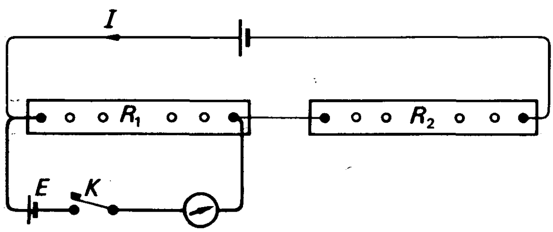


FIG. 33.23. Rayleigh potentiometer.

decreased by the same amount. In this way R_1+R_2 is kept constant, and the current I does not change. But there is now a potential difference across R_1, which opposes E. Plugs are taken out of R_1 and put into R_2, so as to keep R_1+R_2 constant, until the galvanometer shows no deflection when K is pressed. If R_1' is the value of R_1 at this point, then

$$E = R_1'I.$$

The procedure is now repeated with a standard cell of e.m.f. E_0, in place of E. Since R_1+R_2 has been kept constant, the current I is the same as before; hence, if R_1'' is the new value of R_1 at balance,

$$E_0 = R_1''I.$$

Consequently,
$$\frac{E}{E_0} = \frac{R_1'I}{R_1''I} = \frac{R_1'}{R_1''}.$$

Measurement of Thermal E.M.F.

The e.m.f.s of thermojunctions (p. 803) are small—of the order of a millivolt. If we attempted to measure such an e.m.f. on a simple potentiometer we should find the balance-point very near one end of the wire, so that the end-error would be serious. The Rayleigh potentiometer, although it is free from end-errors, is not suitable for measuring small e.m.f.s; if, in Fig. 33.23, $R_1+R_2 = 10000\ \Omega$, and the e.m.f.

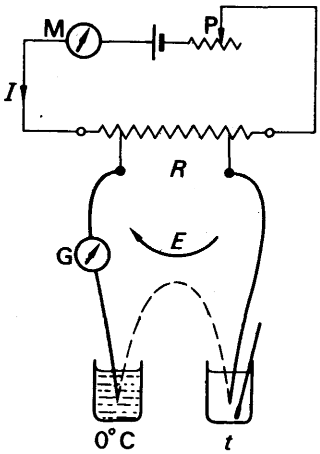


FIG. 33.24. Measurement of thermal e.m.f.

of the accumulator = 2 volts, then $I = 2/10000 = 2\times10^{-4}$ amp = 0·2 mA. To balance a thermal e.m.f. of 2 mV, R_1 would therefore have to be 10 Ω; and since R_1 cannot be adjusted in steps smaller than 1 Ω, the e.m.f. cannot be measured to a greater accuracy than 10 per cent.

For accurate measurement of thermal e.m.f.s special potentiometers have been devised, but the simple circuit of Fig. 33.24 will do for a laboratory experiment. The e.m.f. E is applied via a sensitive galvanometer G across a standard shunt R of about 1 ohm. A current I, of a few milliamperes, is passed through the shunt, and measured on the milliammeter M. Its value is adjusted by the rheostat P until G shows no deflection. The potential difference RI is then equal and opposite to the thermal e.m.f.

$$E = RI.$$

If a balance cannot be found, the connexions of the junction to R should be reversed.

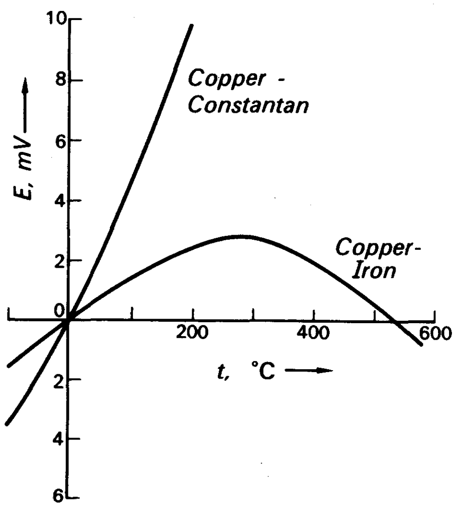


FIG. 33.25. E.m.f.s of thermocouples.
(Reckoned positive when into copper at the cold junction.)

Fig. 33.25 shows the results of measuring the e.m.f. E when the cold junction is at 0°C and the hot is at various temperatures t. The curves approximate to parabolas:

$$E = at + bt^2. \qquad (18)$$

THERMO-ELECTRIC E.M.F.s

(E in micro-volts when t is in °C
and cold junction at 0°C)

Junction	a	b	Range for a and b, °C	Limits of use, °C
Cu/Fe . . .	14	−0·02	0–100	*See* 1
Cu/Constantan[2] .	41	0·04	−50 to +300	−200 to +300
Pt/Pt−Rh[3] . .	6·4	0·006	0–200	0–1700
Chromel[4]/Alumel[5] .	41	0·00	0–900	0–1300

[1] Simple demonstrations.
[2] See p. 788.
[3] 10 per cent Rh; used only for accurate work or very high temperatures.
[4] 90 per cent Ni, 10 per cent Cr.
[5] 94 per cent Ni, 3 per cent Mn, 2 per cent Al, 1 per cent Si.

NETWORKS

Kirchhoff's Laws

A 'network' is usually a complicated system of electrical conductors. Kirchhoff (1824–87) extended Ohm's law to networks, and gave two laws, which together enabled the current in any part of the network to be calculated.

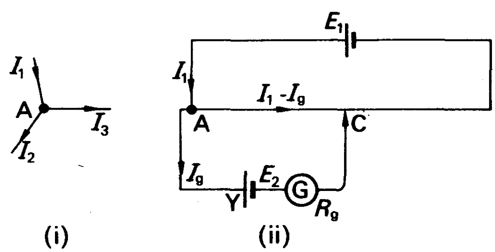


FIG. 33.26. Kirchhoff's laws.

The *first law* refers to any point in the network, such as A in Fig. 33.26 (*a*); it states that the total current flowing into the point is equal to the total current flowing out of it:

$$I_1 = I_2+I_3.$$

The law follows from the fact that electric charges do not accumulate at the points of a network. It is often put in the form that *the algebraic sum of the currents at a junction of a circuit is zero*, or

$$\Sigma I = 0,$$

where a current, I, is reckoned positive if it flows towards the point, and negative if it flows away from it. Thus at A in Fig. 33.26 (i),

$$I_1-I_2-I_3 = 0.$$

Kirchhoff's first law gives a set of equations which contribute towards the solving of the network; in practice, however, we can shorten the work by putting the first law straight into the diagram, as shown in Fig. 33.26 (ii) for example, since

$$\text{current along AC} = I_1 - I_g.$$

Kirchhoff's *second law* is a generalization of Ohm's law for the complete circuit. It refers to any closed loop, such as AYCA in Fig. 33.26 (ii); and it states that, round such a loop, the algebraic sum of the voltage drops is equal to the algebraic sum of the e.m.f.s:

$$\Sigma RI = \Sigma E.$$

Thus, clockwise round the loop,

$$R_{AC}(I_1 - I_g) - R_g I_g = E_2.$$

We have used the potentiometer to illustrate Kirchhoff's laws merely because it is already familiar to us; we shall not go on and solve it as a network, because we have already dealt with as much of the theory of it as we need.

EXAMPLE

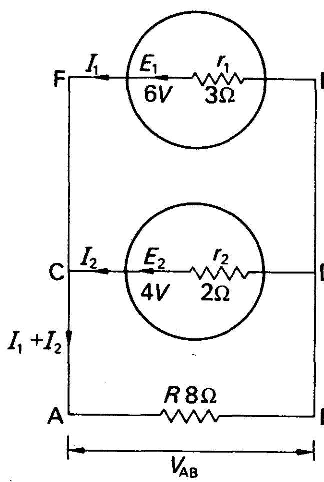


FIG. 33.27. Load across cells in parallel.

Fig. 33.27 shows a network which can be solved by Kirchhoff's laws. From the first law, the current in the 8-ohm wire is (I_1+I_2), assuming I_1, I_2 are the currents through the cells. Taking closed circuits formed by each cell with the 8-ohm wire, we have, from the second law,

$$E_1 = 6 = 3I_1+8(I_1+I_2) = 11I_1+8I_2$$

and

$$E_2 = 4 = 2I_2+8(I_1+I_2) = 8I_1+10I_2.$$

Solving the two equations, we find $I_1 = \frac{14}{23} = 0{\cdot}61$ amp, $I_2 = -\frac{2}{23} = -0{\cdot}09$ amp.

The minus sign indicates that the current I_2 flows in the sense opposite to that shown in the diagram; i.e. it flows against the e.m.f. of the generator E_2. It does so because the potential difference V_{AB} is greater than E_2:

$$V_{AB} = \left(R\ I_1+I_2\right) = 8\left(\frac{14}{23}-\frac{2}{23}\right)$$

$$= 8\times\frac{12}{23} = \frac{96}{23} = 4{\cdot}2 \text{ volts.}$$

This is equal to the e.m.f. E_2 *plus* the drop across the internal resistance r_2 (p. 828):

$$V_{CD} = 4+2\times\frac{2}{23} = 4+\frac{4}{23}$$

$$= \frac{96}{23}\text{ volts} = V_{AB}.$$

It is also equal to the e.m.f. E_1 *minus* the drop across r_1, because the current flows through the upper generator in the direction of its e.m.f.:

$$V_{FH} = 6-3\times\frac{14}{23} = 6-\frac{42}{23} = \frac{138-42}{23}$$

$$= \frac{96}{23}\text{ volts} = V_{AB}.$$

WHEATSTONE BRIDGE
MEASUREMENT OF RESISTANCE

Wheatstone Bridge Circuit

About 1843 Wheatstone designed a circuit called a 'bridge circuit' which gave an accurate method for measuring resistance. We shall deal later with the practical aspects. In Fig. 33.28, X is the unknown resistance, and P, Q, R are resistance boxes. One of these—usually R— is adjusted until the galvanometer between A, C, represented by its resistance R_g, shows no deflection: that is to say,

$$I_g = 0.$$

Then, as we shall show,

$$\frac{P}{Q} = \frac{R}{X},$$

whence

$$X = \frac{Q}{P}R.$$

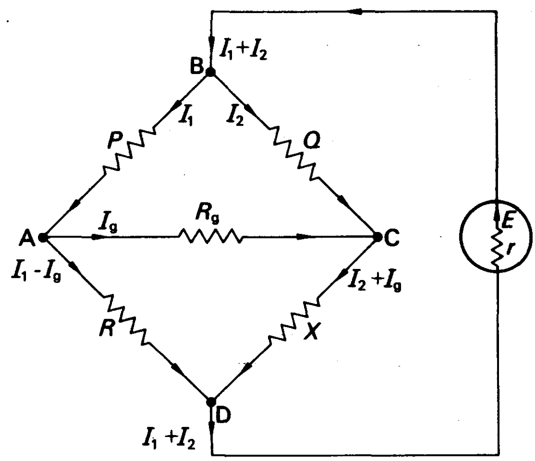


FIG. 33.28. Analysis of Wheatstone bridge.

Fig 33.28 shows Kirchhoff's first law applied to the circuit. From the second law, we have:

$$\text{loop ACBA:} \quad R_gI_g - QI_2 + PI_1 = 0, \qquad \text{(i)}$$

$$\text{loop ACDA:} \quad R_gI_g + X(I_2+I_g) - R(I_1-I_g) = 0,$$

or

$$I_g(R_g+X+R)+XI_2+RI_1 = 0. \qquad \text{(ii)}$$

If we wished to find I_g, we would have to set up a third equation, by going round one of the loops, including the battery (p. 831). But if we wish only to find the condition for no deflection of the galvanometer, we have merely to put $I_g = 0$ in equations (i) and (ii). Then

$$-QI_2+PI_1 = 0, \quad \text{or} \quad PI_1 = QI_2,$$

whence

$$\frac{P}{Q} = \frac{I_2}{I_1};$$

and

$$IX_2 - RI_1 = 0, \quad \text{or} \quad XI_2 = RI_1,$$

whence

$$\frac{R}{X} = \frac{I_2}{I_1}$$

Therefore, as already stated,

$$\frac{P}{Q} = \frac{R}{X}. \qquad (19)$$

This is the condition for balance of the bridge. It is the same, as the reader may easily show, if the battery and galvanometer are interchanged in the citcuit.

Alternative Wheatstone Bridge Proof

Equation (19) for the balance condition can be got without the use of Kirchhoff's laws. At balance, since no current flows through the galvanometer, the points A and C must be at the same potential (Fig. 33.29). Therefore

$$V_{AB} = V_{CB}$$

and

$$V_{AD} = V_{CD},$$

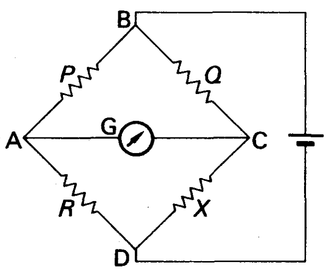


FIG. 33.29. Wheatstone bridge.

whence

$$\frac{V_{AB}}{V_{AD}} = \frac{V_{CB}}{V_{CD}}. \qquad \text{(i)}$$

Also, since $I_g = 0$, P and R carry the same current, I_1, and X and Q carry the same current, I_2. Therefore

$$\frac{V_{AB}}{V_{AD}} = \frac{I_1P}{I_1R} = \frac{P}{R}$$

and

$$\frac{V_{CB}}{V_{CD}} = \frac{I_2Q_2}{I_2X} = \frac{Q}{X} \qquad \text{(ii)}$$

Hence by equations (i) and (ii),

$$\frac{P}{R} = \frac{Q}{X}.$$

or

$$\frac{P}{Q} = \frac{R}{X}.$$

Galvanometer Position

We shall now show, by taking a numerical example, how the galvanometer in a bridge circuit can best be positioned.

Fig. 33.30 shows an unbalanced Wheatstone bridge, fed from a cell

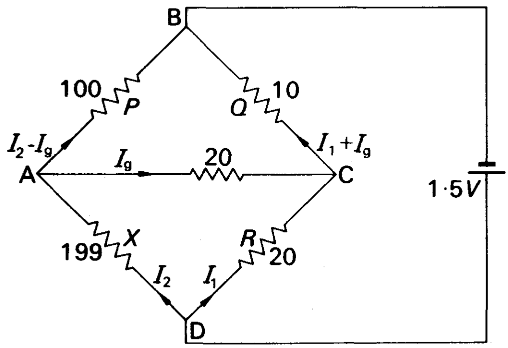


FIG. 33.30. An unbalanced Wheatstone bridge.

of negligible internal resistance. The figures give the resistance in ohms, and I_g is to be found. Applying Kirchhoff's laws:

$$\text{Loop ACBA:}\quad 20I_g + 10(I_1 + I_g) - 100(I_2 - I_g) = 0$$

or

$$130I_g + 10I_1 - 100I_2 = 0,$$

whence

$$I_1 = 10I_2 - 13I_g. \qquad \text{(i)}$$

$$\text{Loop ADCA:}\quad -139I_2 + 20I_1 - 20I_g = 0.$$

$$\text{Substituting for } I_1\text{:}\quad -199I_2 + 200I_2 - 260I_g - 20^{9}{}_{g} = 0,$$

or

$$I_2 - 280I_g = 0,$$

whence

$$I_2 = 280I_g.$$

$$\therefore \text{ by (i),}\quad I_1 = 2800I_g - 13I_g = 2787I_g. \qquad \text{(ii)}$$

$$\text{Loop DCBXD:}\quad 20I_1 + 10(I_1 + I_g) = 1{\cdot}5,$$

or

$$30I_1 + 10I_g = 1{\cdot}5.$$

$$\text{Substituting from (ii) for } I_1\text{:}\quad 30 \times 2787I_g + 10I_g = 1{\cdot}5$$

or

$$83620I_g = 1{\cdot}5,$$

whence

$$I_g = \frac{1{\cdot}5}{83620} = 1{\cdot}79 \times 10^{-5}\ \text{A}$$

$$= 17{\cdot}9 \text{ microamperes.}$$

The reader should now show that, if the battery and galvanometer were interchanged, the current I_g would be 13·2 microamperes. This result illustrates an important point in the use of the Wheatstone bridge: with a given unbalance, the galvanometer current is greatest when the galvanometer is connected from the junction of the highest resistances to the junction of the lowest. Therefore, unless $P = Q$, which is unusual, the galvanometer should be connected across PQ.

Practical Arrangement

A practical form of Wheatstone bridge is shown in Fig. 33.31. The resistances P and Q can be given values of 10, 100, or 1000 ohms by three-point switches. The resistance R has four decade dials by which it can be varied from 1 ohm to more than 10000 ohms. Pairs of terminals are provided for connecting the unknown resistance, the battery, and the galvanometer, X, B, G; and keys K_1 and K_2 are fitted in the battery and galvanometer circuits.

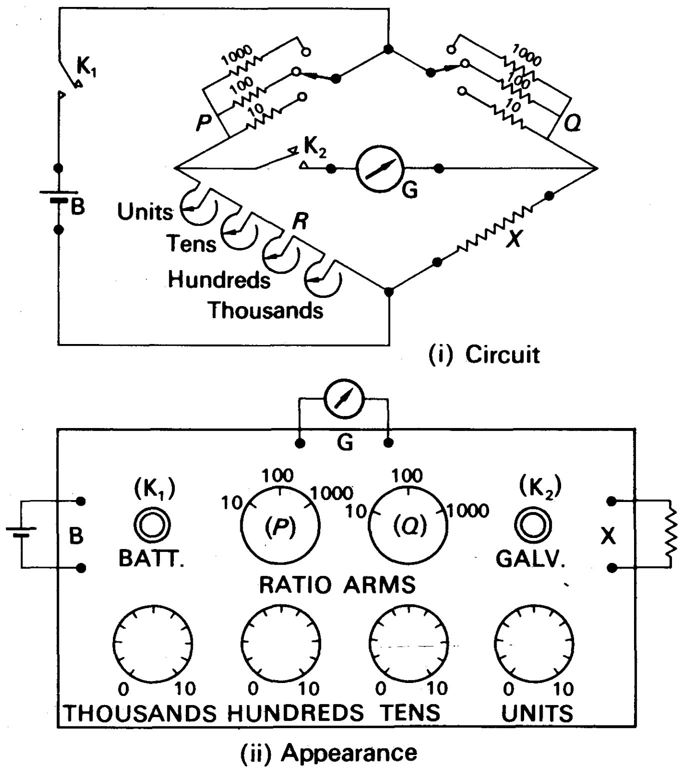


FIG. 33.31. Practical form of Wheatstone bridge.

To measure a resistance, we first set $P = Q = 10\,\Omega$. We set $R = 0$ and press first K_1 then K_2; the small interval between pressing K_1 and K_2 gives time for the currents in the bridge to become steady (Chapter

31). Let us suppose that, when we press K_2, the galvanometer deflects to the right. We then set $R = 10000\ \Omega$ and again press K_1, K_2. If the galvanometer deflects to the left we can proceed; if it deflects again to the right, then either we have made a wrong connexion —which with this form of bridge is almost impossible— or X is greater than 10000 ohms. If the galvanometer deflects to the left, we try again with $R = 1000\ \Omega$; and so on with 100 Ω and 10 Ω, if necessary. Let us suppose that the galvanometer deflects to the left with $R = 100\ \Omega$, but to the right with 10 Ω. We then adjust the 10's dial until we get, say, a leftward deflection with 40 Ω and a rightward with 30 Ω. With the unit's dial we now narrow the limits to, let us say, 36 Ω (left) and 35 Ω (right). We have

$$\frac{X}{R} = \frac{Q}{P} = \frac{10}{10} = 1.$$

$$\therefore X = R.$$

It follows that X lies between 35 and 36 Ω.

We now set $P = 100\ \Omega$, so that

$$\frac{X}{R} = \frac{Q}{P} = \frac{10}{100} = \frac{1}{10}$$

or

$$X = \frac{R}{10}.$$

The balance-point now lies between $R = 350\ \Omega$ (right) and $R = 360\ \Omega$ (left); by using the unit's dial we can now locate it between, say, 353 and 354. Then, from the equation, X lies between 35·3 and 35·4 ohms. If we finally make $P = 1000\ \Omega$, we have

$$\frac{X}{R} = \frac{Q}{P} = \frac{10}{1000} = \frac{1}{100}$$

or

$$X = \frac{R}{100}.$$

Only a sensitive galvanometer will give considerable deflections near the balance-point in this condition; if it locates the balance-point between $R = 3536\ \Omega$ and $3537\ \Omega$ then X lies between 35·36 and 35·37 ohms. If a moving-coil galvanometer is used, it must be protected by a high series resistance while balance is being sought.

Range of Measurable Resistance

The resistors P and Q are often called the ratio arms of the bridge, because their resistances determine the ratio of R to X. If X is greater than the greatest value of R, it can be measured by making $Q = 100$, $P = 10$. Then

$$\frac{X}{R} = \frac{Q}{P} = \frac{100}{10} = 10$$

and

$$X = 10R.$$

A balance-point between $R = 14620$ and 14630, say, would mean that R lay between 146 200 and 146 300 ohms. Similarly, by making $Q = 1000$, $P = 10$, resistances can be measured up to 100 times the greatest value of R: that is to say, up to a little more than 1 000 000 ohms. With these high resistances, however, the near-balance currents are very small, and a sensitive galvanometer is necessary.

The lowest resistance which a bridge of this type can measure with reasonable accuracy is about 1 ohm; R can be adjusted in steps of 1 ohm, and P/Q can be made 1/100, so that measurements can be made to within 1/100 ohm. Resistances lower than about 1 ohm cannot be measured accurately on a Wheatstone bridge, whatever the ratios available, or the smallest steps in R. They cannot because of the resistances of the wires connecting them to the X terminals, and of the contacts between those wires and the terminals to which they are, at each end, attached. This is the reason why the potentiometer method is more satisfactory for low resistances.

The Post Office Box

An old-fashioned type of Wheatstone bridge, with plugs instead of switches, is called the Post Office box, and is illustrated in Fig. 33.32. It is connected up and used in the same way as the dial type of bridge,

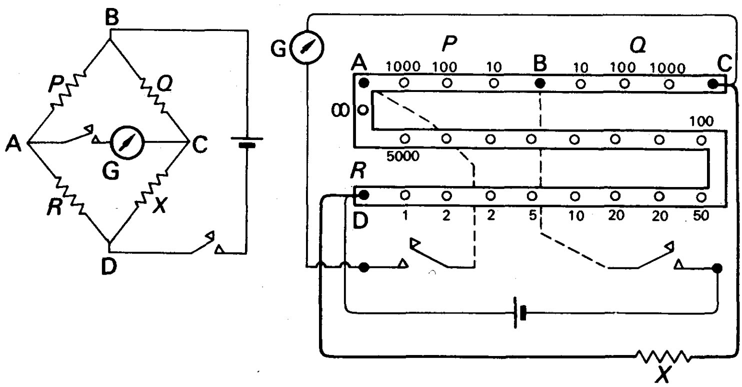


FIG. 33.32. Post Office box.

but requires far more skill by its operator. Anyone who has to use a Post Office box should observe the following rules:

(i) do not attempt to memorize the wiring-up; the circuit should be worked out from the Wheatstone bridge diagram (Fig. 33.29);

(ii) take the 10 Ω plugs out of each ratio arm P, Q before testing the circuit;

(iii) test for correct connexions by seeing whether the galvanometer gives opposite deflections with $R = 0$ and $R = \infty$ (for the latter an 'infinity' plug is provided, whose gap is not bridged by any resistor—Fig. 33.32);

(iv) press plugs home firmly, with a half-turn to the right;
(v) never mix the plugs from different boxes—always put loose plugs in the lid of their box, never on the bench.

The Slide-wire (Metre) Bridge

Fig. 33.33 shows a simple and cheap form of Wheatstone bridge; it is sometimes called a metre bridge, for no better reason than that the wire AB is often a metre long. The wire is uniform, as in a potentiometer, and can be explored by a slider S. The unknown resistance X and a known resistance R are connected as shown in the figure; heavy brass or copper strip is used for the connexions AD, FH, KB, whose resistances are generally negligible. When the slider is at a point C in

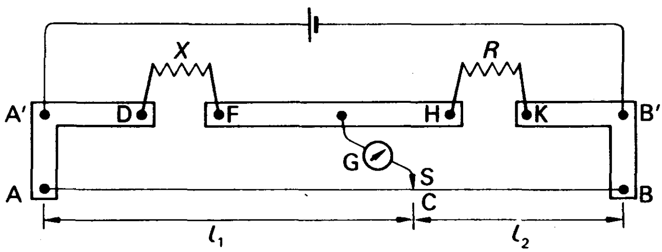


FIG. 33.33. Slide-wire (metre) bridge.

the wire it divides the wire into two parts, of resistances R_{AC} and R_{CB}; these, with X and R, form a Wheatstone bridge. (The galvanometer and battery are interchanged relative to the circuits we have given earlier; that enables the slider S to be used as the galvanometer key. We have already seen that the interchange does not affect the condition for balance (p. 830).) The connexions are checked by placing S first on A, then on B. The balance-point is found by trial and error—not by scraping S along AB. At balance,

$$\frac{X}{R}=\frac{R_{AC}}{R_{CB}}.$$

Since the wire is uniform, the resistances R_{AC} and R_{CB} are proportional to the lengths of wire, l_1 and l_2. Therefore

$$\frac{X}{R}=\frac{l_1}{l_2}. \qquad \qquad (20)$$

The resistance R should be chosen so that the balance-point C comes fairly near to the centre of the wire—within, say, its middle third. If either l_1 or l_2 is small, the resistance of its end connexion AA′ or BB′ in Fig. 33.33 is not negligible in comparison with its own resistance; equation (20) then does not hold. Some idea of the accuracy of a particular measurement can be got by interchanging R and X, and balancing again. If the new ratio agrees with the old within about 1 per cent, then their average may be taken as the value of X.

Resistance by Substitution

Fig. 33.34 illustrates a simple way of measuring a resistance X. It is connected in series with a rheostat S, an ammeter A, and a cell. S is adjusted until A gives a large deflection. X is then replaced by a box of known resistances, R, which can be selected by plugs or dials. R is varied until the ammeter gives the same reading as before. Then, if the e.m.f. of the cell has not fallen, $R = X$. The accuracy of this method is limited, by the inherent error of the ammeter, to about 1 per cent. It does not depend on the accuracy of *calibration* of the ammeter, but on the accuracy with which it reproduces a given deflection for a given current. The ammeter is used simple as a 'transfer instrument'—to indicate when the current in the second part of the experiment is the same as in the first. This principle is very useful in measurements more difficult than that of resistance by direct method. For example, the power output of a small radio transmitter can be measured by making it light a lamp, which is placed near to a lightmeter (p. 571). The lamp is then connected to a source of direct current, and the current through it is adjusted until the light-meter gives the same reading. Simple measurements of the current, and the voltage across the lamp, then give the power supplied to it—which is equal to the power output of the radio transmitter.

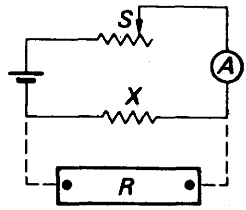
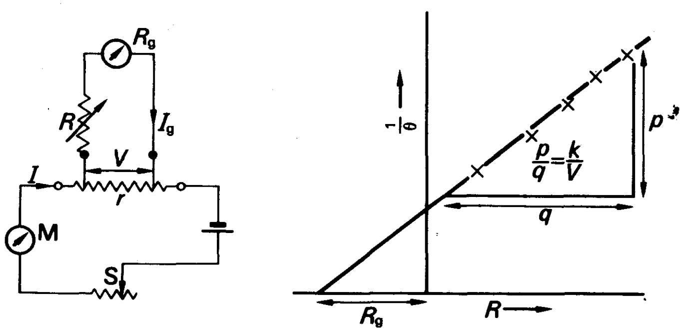


FIG. 33.34. Measurement of resistance by substitution.

Resistance and Sensitivity of a Galvanometer

It is often necessary to know the resistance, R_g, and sensitivity of a suspended-coil galvanometer. To find them, we may use the circuit of Fig. 33.35 (i). S is a rheostat of about 1000 ohms maximum, r is a standard shunt of about 0·01 ohm, M is a milliammeter, and R is a resistance box. The current in the main circuit, I, is adjusted to a value which can be accurately read on M—say 10 milliamperes. Since r is very



(i) Circuit (ii) Results

FIG. 33.35. Galvanometer calibration.

small compared with $R+R_g$, the galvanometer current I_g is negligible compared with I, and we may say that the potential difference across r is

$$V = rI.$$

In this example it would be $0{\cdot}01 \times 10 \times 10^{-3} = 10^{-4}$ volt $= 0{\cdot}1$ millivolt. The galvanometer current is therefore

$$I_g = \frac{V}{R+R_g}.$$

If $R+R_g$ is 1000 ohms, then $I_g = 10^{-4}/10^3 = 10^{-7}$ amp $= 0{\cdot}1$ microampere, which is a reasonable value for a galvanometer of moderate sensitivity. If θ is the deflection of the galvanometer, then

$$I_g = k\theta,$$

where k is a constant (the reduction factor) which we wish to find.
From the equation for I_g above.

$$\frac{V}{R+R_g} = k\theta,$$

whence $$\frac{1}{\theta} = \frac{k}{V}(R+R_g).$$

Therefore, if we vary R and plot $1/\theta$ against it, we get a straight line, as shown in Fig. 33.35 (ii). The line makes an intercept on the R axis, which gives the value of R_g. And its slope p/q is k/V. Since we know V from above, we can hence find k; and $1/k$, the deflection per unit current, is the sensitivity.

Temperature Coefficient of Resistance

We have seen that the resistance of a given wire increases with its temperature. If we put a coil of fine copper wire into a water bath, and use a Wheatstone bridge to measure its resistance at various moderate temperatures t, we find that the resistance, R, increases

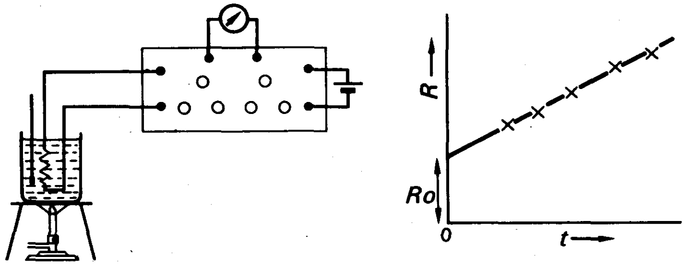


FIG. 33.36. Measurement of temperature coefficient.

uniformly with the temperature (Fig. 33.36). We may therefore define a temperature coefficient of resistance, α, such that

$$R = R_0(1+\alpha t), \qquad . \quad . \quad . \quad (21)$$

where R_0 is the resistance at 0°C. In words,

$$\alpha = \frac{\text{increase of resistance per deg. C rise of temperature}}{\text{resistance at 0°C}}$$

If R_1 and R_2 are the resistances at t_1°C and t_2°C, then

$$\frac{R_1}{R_2} = \frac{1+\alpha t_1}{1+\alpha t_2}. \qquad . \quad . \quad . \quad (22)$$

Values of α for pure metals are of the order of 0·004 per deg. C. They are much less for alloys than for pure metals, a fact which enhances the value of alloys as materials for resistance boxes and shunts.

Equation (21) represents the change of resistance with temperature fairly well, but not as accurately as it can be measured. More accurate equations are given on p. 370 in the Heat section of this book, where resistance thermometers are discussed.

EXAMPLES

1. How would you compare the resistances of two wires A and B, using (*a*) a Wheatstone bridge method and (*b*) a potentiometer? For each case draw a circuit diagram and indicate the method of calculating the result.

In an experiment carried out at 0°C, A was 120 cm of Nichrome wire of resistivity 100×10^{-6} ohm cm and diameter 1·20 mm, and B a German silver wire 0·80 mm diameter and resistivity 28×10^{-6} ohm cm. The ratio of the resistances A/B was 1·20. What was the length of the wire B?

If the temperature coefficient of resistance of Nichrome is 0·00040 per deg. C and of German silver is 0·00030 per deg. C, what would the ratio of the resistances become if the temperature were raised by 100 deg. C? (*L.*)

First part (see pp. 830, 824).

Second part. With usual notation,

for A, $$R_1 = \frac{\rho_1 l_1}{a_1},$$

and for B, $$R_2 = \frac{\rho_2 l_2}{a_2}.$$

$$\therefore \frac{R_1}{R_2} = \frac{\rho_1}{\rho_2} \cdot \frac{l_1}{l_2} \cdot \frac{a_2}{a_1} = \frac{\rho_1}{\rho_2} \cdot \frac{l_1}{l_2} \cdot \frac{d_2^2}{d_1^2},$$

where d_2, d_1 are the respective diameters of B and A.

$$\therefore 1{\cdot}20 = \frac{100}{28} \times \frac{120}{l_2} \times \frac{0{\cdot}8^2}{10{\cdot}2^2}.$$

$$\therefore l_2 = \frac{100 \times 120 \times 0{\cdot}64}{1{\cdot}20 \times 28 \times 1{\cdot}44} = 159 \text{ cm} \qquad . \quad . \quad (i)$$

When the temperature is raised by 100°C, the resistance increases according to the relation $R_t = R_0(1+\alpha t)$. Thus

$$\text{new Nichrome resistance, } R_A = R_1(1+\alpha\,.\,100) = R_1 \times 1{\cdot}04,$$

$$\text{and new German silver resistance, } R_B = R_2(1+\alpha'\,.\,100) = R_2 \times 1{\cdot}03.$$

$$\therefore \frac{R_A}{R_B} = \frac{R_1}{R_2} \times \frac{1{\cdot}04}{1{\cdot}03} = 1{\cdot}20 \times \frac{1{\cdot}04}{1{\cdot}03} = 1{\cdot}21 \quad . \quad . \quad . \quad \text{(ii)}$$

2. State Kirchhoff's laws relating to the currents in network of conductors. Two cells of e.m.f. 1·5 volts and 2 volts respectively and internal resistances of 1 ohm and 2 ohms respectively are connected in parallel to an external resistance of 5 ohms. Calculate the currents in each of the three branches of the network. (*N.*)

First part (see p. 828).

Second part. Suppose x, y amp are the respective currents through the cells (Fig. 33.37). Then, from Kirchhoff's first law, the current through the 5-ohm wire is $(x+y)$ amp.

Applying Kirchhoff's second law to the complete circuit with the cell of e.m.f. 1·5 volts and external resistance 5 ohms, we have

$$1{\cdot}5 = x+5(x+y) = 6x+5y \quad . \quad . \quad . \quad \text{(i)}$$

Applying the law to the complete circuit with the cell of e.m.f. 2 volts and external resistance 5 ohms, then

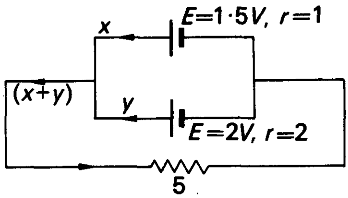


FIG. 33.37. Example.

$$2 = 2y+5(x+y) = 5x+7y. \quad . \quad . \quad . \quad \text{(ii)}$$

Solving (i) and (ii), we find $y = 9/34$ amp, $x = 1/34$ amp. Thus the current through the 5-ohm resistor $= x+y = 10/34 = 5/17$ amp.

EXERCISES 33

Circuit Calculations

1. State Ohm's law and describe an experiment to verify it.

A resistance of 1000 ohms and one of 2000 ohms are placed in series with a 100-volt mains supply. What will be the reading on a voltmeter of internal resistance 2000 ohms when placed across (*a*) the 1000 ohms resistance, (*b*) the 2000 ohms resistance? (*L.*)

2. Define the practical unit of potential difference and hence show that the rate of production of heat in a wire of constant resistance is proportional to the square of the current passing through it.

A cell A, e.m.f. 1·1 volts and internal resistance 3 ohms, is joined in parallel with another cell B, e.m.f. 1·4 volts and internal resistance 1 ohm, similar poles being connected together. The ends of a wire, of resistance 4 ohms, are joined to the terminals of A. Find (*a*) the current through the wire, (*b*) the rate of dissipation of energy in watts in each of the cells A and B. (*L.*)

3. Twelve cells each of e.m.f. 2 volts and of internal resistance $\frac{1}{2}$ ohm are arranged in a battery of n rows and an external resistance of $\frac{3}{8}$ ohm is connected to the poles of the battery. Determine the current flowing through the resistance in terms of n.

Obtain numerical values of the current for the possible values which n may take and draw a graph of current against n by drawing a smooth curve through the points. Give the value of the current corresponding to the maximum of the curve and find the internal resistance of the battery when the maximum current is produced. (*L.*)

4. Describe with full experimental details an experiment to test the validity of Ohm's law for a metallic conductor.

An accumulator of e.m.f. 2 volts and of negligible internal resistance is joined in series with a resistance of 500 ohms and an unknown resistance X ohms. The readings of a voltmeter successively across the 500-ohm resistance and X are 2/7 and 8/7 volts respectively. Comment on this and calculate the value of X and the resistance of the voltmeter. (*N.*)

5. State with reasons the essential requirement for the resistance of (*a*) an ammeter, (*b*) a voltmeter.

A voltmeter having a resistance of 1800 ohms is used to measure the potential difference across a 200 ohm resistance which is connected to the terminals of a d.c. power supply having an e.m.f. of 50 volts and an internal resistance of 20 ohms. Determine the percentage change in the potential difference across the 200 ohm resistor as a result of connecting the voltmeter across it. (*N.*)

6. State Ohm's law and describe how you would test its validity. Why would an experiment involving the use of a moving-coil ammeter and a moving-coil voltmeter be unsatisfactory?

In order to calibrate a galvanometer an accumulator of e.m.f. 200 volts and negligible resistance is connected in series with two resistances P and Q. A resistance R and the galvanometer, resistance G, are joined in series and then connected to the ends of P. The galvanometer is shunted by a resistance S. If $P = 200$ ohms, $Q = 1880$ ohms, $R = 291$ ohms, $S = 10$ ohms, $G = 90$ ohms, and the deflection is 20 divisions, calculate the current sensitivity in micro-amperes per division.

7. State Kirchhoff's laws for flow of electricity through a network containing sources of e.m.f.

Wheatstone bridge with slide wire of 0·5 ohm and length 50 cm is used to compare two resistances, each of 2 ohms. The cell has an e.m.f. of 2 volts and no internal resistance, and the galvanometer has resistance of 100 ohms. Find the current through the galvanometer when the bridge is 1 cm off balance. Compare the result with that of approximate calculation. (*L.*)

8. State Ohm's law, and describe the experiments you would make in order to verify it. The positive poles A and C of two cells are connected by a uniform wire of resistance 4 ohms and their negative poles B and D by a uniform wire of resistance 6 ohms. The middle point of BD is connected to earth. The e.m.f.s of the cells AB and CD are 2 volts and 1 volt respectively, their resistances 1 ohm and 2 ohms respectively. Find the potential at the middle point of AC. (*O. & C.*)

9. From the adoption of either the fundamental units cm, g, second, or the fundamental units m, kg, second, trace the steps necessary to define the *volt* and the *ohm* in terms of the ampere.

Discuss the suitability of (*a*) a moving-coil voltmeter, and (*b*) a slide-wire potentiometer for determining the potential differences in an experiment designed to verify Ohm's law.

Four resistors AB, BC, CD and DA of resistance 4 ohms, 8 ohms, 4 ohms and 8 ohms respectively are connected to form a closed loop, and a 6-volt battery of negligible resistance is connected between A and C. Calculate (i) the potential difference between B and D, and (ii) the value of the additional resistance which must be connected between A and D so that no current flows through a galvanometer connected between B and D. (*O. & C.*)

Measurements

10. Describe and explain how you would use a potentiometer (*a*) to compare the electromotive forces of two cells, (*b*) to test the accuracy of the 1-amp reading of an ammeter.

The e.m.f. of a cell is balanced by the fall in potential along 150 cm of a potentiometer wire. When the cell is shunted by a resistance of 14 ohms the length required is 140 cm. What is the internal resistance of the cell? (*N.*)

11. Describe a metre bridge and the method of using it to compare the resistances of two conductors, giving the theory of the method. Why is the method unsatisfactory if the two resistances (*a*) differ widely from one another, (*b*) are very small?

A metre bridge is balanced with a piece of aluminium wire of resistance 7·30 ohm in the left-hand gap, the slide contact being 42·6 cm from the left-hand end of the bridge wire and the temperature 17°C. If the temperature of the aluminium wire is raised to 57°C, how may the balance be restored (*a*) by adjusting the slide contact, (*b*) by keeping the contact at 42·6 cm and connecting a conductor in parallel with the aluminium wire? (The temperature coefficient of resistance for aluminium may be taken as $3{\cdot}8 \times 10^{-3}\ K^{-1}$) (*L.*)

12. Describe and explain how a potentiometer is used to test the accuracy of the 1 volt reading of a voltmeter.

A potentiometer consists of a fixed resistance of 2030 ohms in series with a slide wire of resistance 4 ohm metre^{-1}. When a constant current flows in the potentiometer circuit a balance is obtained when (*a*) a Weston cell of e.m.f. 1·018 volt is connected across the fixed resistance and 150 cm of the slide wire and also when (*b*) a thermocouple is connected across 125 cm of the slide wire only. Find the current in the potentiometer circuit and the e.m.f. of the thermocouple.

Find the value of the additional resistance which must be present in the above potentiometer circuit in order that the constant current shall flow through it, given that the driver cell is a lead accumulator of e.m.f. 2 volt and of negligible resistance and the length of the slide wire is 2 metres. (*L.*)

13. Describe the Wheatstone bridge circuit and deduce the condition for 'balance'. State clearly the fundamental electrical principles on which you base your argument. Upon what factors do (*a*) the sensitivity of the bridge, (*b*) the accuracy of the measurement made with it, depend?

Using such a circuit, a coil of wire was found to have a resistance of 5 ohms in melting ice. When the coil was heated to 100°C, a 100 ohm resistor had to be connected in parallel with the coil in order to keep the bridge balanced at the same point. Calculate the temperature coefficient of resistance of the coil. (*C.*)

14. Explain the use of the Weston cell and the precautions to be taken when using it.

A steady current is passed through a manganin potentiometer wire AB of length 10 metres and diameter 0·56 mm connected to a resistance box BC with a resistance of 1001 ohms. A Weston cell of e.m.f. 1·018 volts, with a sensitive galvanometer in series with it, is connected in parallel between the points A and C. It is seen that no deflection is produced in the galvanometer. The Weston cell is removed and a thermocouple is connected *via* the galvanometer to points on the potentiometer wire 524 cm apart. Again there is no deflection. Draw the two circuits and calculate the e.m.f. of the thermocouple. (Resistivity of manganin $= 41{\cdot}87 \times 10^{-6}$ ohm cm.) (*L.*)

15. Describe, giving full experimental details, how you would compare the values of two unknown resistances each of the order of 0·1 ohm using a potentiometer. Draw a circuit diagram and give the theory of the method.

Suppose that, having set up the circuit to carry out this experiment, you found that no balance point could be obtained along the potentiometer. Discuss three possible reasons for this and the procedure you would adopt in order to trace it. (*N.*)

16. A two-metre potentiometer wire is used in an experiment to determine the internal resistance of a voltaic cell. The e.m.f. of the cell is balanced by the fall of potential along 90·6 cm of wire. When a standard resistance of 10 ohms is connected across the cell the balance length is found to be 75·5 cm. Draw a labelled circuit diagram and calculate, from first principles, the internal reistance of the cell.

How may the accuracy of this determination be improved? Assume that other electrical components are available if required. (*N.*)

17. Define *resistivity* and *temperature coefficient of resistance.*

Explain, with the help of a clear circuit diagram, how you would use a Post Office box to determine the resistance of an electric lamp filament at room temperature.

A carbon lamp filament was found to have a resistance 375 ohms at the laboratory temperature of 20°C. The lamp was them connected in series with an ammeter and a d.c. supply, and a voltmeter of resistance 1050 ohms was connected in parallel with the lamp. The ammeter and voltmeter indicated 0·76 amp and 100 volt respectively. The temperature of the carbon filament was estimated to be 1200°C. Estimate the mean value of the temperature coefficient of resistance of carbon between 20°C and 1200°C, and comment on your result. (*L.*)

18. Explain the principle of the potentiometer.

If a slide wire potentiometer of total resistance 5 ohms and a Weston standard cell of e.m.f. 1·0187 volt were available, together with the necessary auxiliary apparatus, describe in detail how you would (*a*) determine the variation of the e.m.f. of a thermocouple with the temperature difference between its junctions (maximum e.m.f. 3 millivolt), (*b*) compare the values of two resistances nominally 0·010 ohm and 0·005 ohm. (*L.*)

19. State Ohm's law. Deduce a formula for the resistance of a number of resistors connected in parallel.

If you were given a cell of constant internal resistance, an ammeter of negligible resistance and various wires of different lengths but otherwise identical, describe how you would determine how many cm of wire had the same resistance as the internal resistance of the cell.

Describe how you would compare the resistances of two resistors each about 10^6 ohms using only an accumulator of e.m.f. of about 2 volts, a low-resistance uncalibrated galvanometer with a linear scale and full-scale deflection for about 10^{-7} amp and a three-terminal slider rheostat of about 20 ohms resistance. Justify the calculation of the resistance ratio from the readings you would take. (*C.*)

20. Describe how you would calibrate an ammeter using a standard resistor, a rheostat, accumulators, potentiometer slide wire with usual accessories, and a standard cell.

A standard low resistor is accidentally connected across a 100 volt d.c. main. It emits a momentary flash of light, vapourizes, and immediately breaks the circuit without further sparking. Estimate the duration of the flash if the wire becomes incandescent at 550°C, melts at 900°C, has specific heat 0·42 kJ kg^{-1} K^{-1}, density 5500 kg m^{-3}, is 10 cm long and has constant resistivity of 40 microhm cm. Assume that the rate of energy loss by the wire is small compared with the rate of heat development within it.

Give a physical explanation of the fact that this time is independent of the cross-sectional area of the wire. (*C.*)

chapter thirty-four

The Chemical Effect of the Current

In this chapter we shall deal both with the effects of an electric current when it is passed through a chemical solution, and with chemical generators of electric current, or cells.

ELECTROLYSIS

The chemical effect of the electric current was first studied quantitatively by Faraday, who introduced most of the technical terms which are now used in describing it. A conducting solution is called an *electrolyte* and the chemical changes which occur when a current passes through it are called *electrolysis* (*lysis* = decomposition). Solutions in water of acids, bases, and salts are electrolytes, and so are their solutions in some other solvents, such as alcohol. The plates or wires which dip into the electrolyte to connect it to the circuit are called electrodes; the one by which the current enters the solution is called the *anode*, and the one by which it leaves is called the *cathode* (Fig. 34.1 (i)).

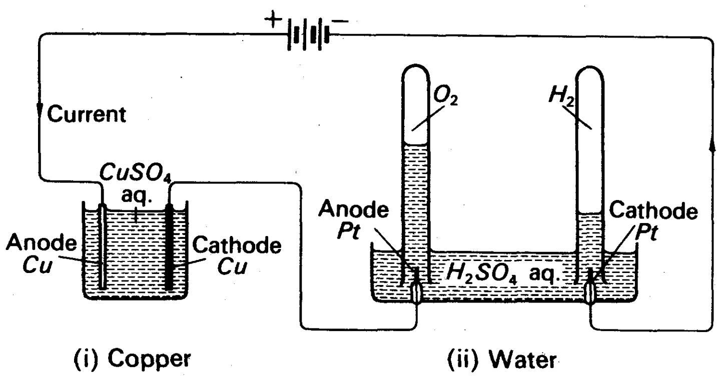
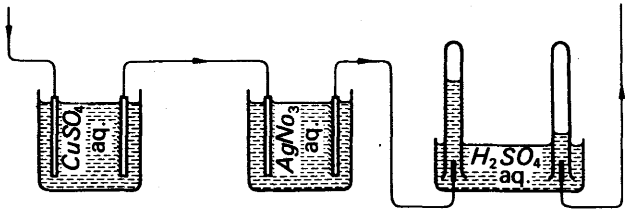


FIG. 34.1. Voltameters.

The whole arrangement is called a *voltameter*, presumably because it can be used to measure the current delivered by a voltaic cell; if the electrolyte is a solution of a copper or silver salt, the voltameter is called a copper or silver voltameter. If the electrolyte is acidulated water, then the voltameter is called a water voltameter, because when a current passes through it, the water, not the acid, is decomposed (Fig. 34.2 (ii)). We shall see why later.

Faraday's Laws of Electrolysis

When a current is passed through copper sulphate solution with copper electrodes, copper is deposited on the cathode and lost from the anode. Faraday showed that the mass dissolved off the anode by a given current in a given time is equal to the mass deposited on the cathode. He also showed that the mass is proportional to the product of the current, and the time for which it flows: that is to say, to the quantity of charge which passes through the voltameter. When he studied the electrolysis of water, he found that the masses of hydrogen and oxygen, though not equal, were each proportional to the quantity of charge that flowed. He therefore put forward his first law of electrolysis: *the mass of any substance liberated in electrolysis is proportional to the quantity of electric charge that liberated it.*



Element liberated	Cu	Ag	O_2	H_2
Relative mass	31·8	107·9	8·00	1·008
Chemical equivalent	31·8	107·9	8·00	1·008

FIG. 34.2. Voltameters in series (same quantity of charge passes through each).

Faraday's second law of electrolysis concerns the masses of different substances liberated by the same quantity of charge. An experiment to illustrate it is in Fig. 34.2. The experiment shows that *the masses of different substances, liberated in electrolysis by the same quantity of electric charge, are proportional to the ratio of the relative atomic mass to the valency* (also called the *chemical equivalent*). This is Faraday's second law; it implies that the same quantity of charge is required to liberate one mole divided by the valency (also called the 'gramme-equivalent') of any substance. Recent measurements give this quantity as 96500 coulombs; it is called Faraday's constant, symbol F. It is also known as the *faraday*.

Electrochemical Equivalent

The mass of a substance which is liberated by one coulomb is called its *electrochemical equivalent.* It is expressed in kilogrammes per coulomb (kg C^{-1}) in SI units. If z is the electrochemical equivalent of a substance, the mass of it in kilogrammes liberated by I amperes in t seconds is

$$m = zIt. \qquad . \quad . \quad . \quad . \quad . \qquad (1)$$

Since the chemical equivalent of hydrogen is 1·008 then 1·008 g of hydrogen is liberated by 96500 coulombs, and the electrochemical equivalent of hydrogen is

$$z_H = \frac{1{\cdot}008}{06{,}500} = 10{\cdot}5 \times 10^{-5}\ \text{g C}^{-1} = 1{\cdot}05 \times 10^{-8}\ \text{kg C}^{-1}$$

And similarly, since the chemical equivalent of copper is 31·8, its electrochemical equivalent is

$$z_{Cu} = \frac{31{\cdot}8}{96{,}500} = 3{\cdot}29 \times 10^{-4}\ \text{g C}^{-1} = 3{\cdot}29 \times 10^{-7}\ \text{kg C}^{-1}$$

Measurement of Current by Electrolysis

In the past, the chemical effect of the current was used to define the ampere, because measurements with the current balance (p. 940) could not be made as accurately as simple weighings. Nowadays they can, and the ampere is defined as on p. 939. In those days the ampere was defined as the current which, when flowing steadily, would deposit 0·001118 g of silver per second.

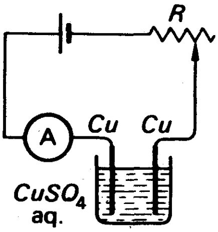


FIG. 34.3. Current measurement by chemical effect.

In the absence of a standard ammeter, the chemical effect can be used to find the error in a particular reading on an ammeter A (Fig. 34.3). A copper voltameter is connected in series with the ammeter, and a steady current passed for a known time t. The current is kept constant by adjusting the rheostat R to keep the deflection constant. The cathode is weighed before and after the experiment. Its increase in mass, m, gives the current I, in terms of the electrochemical equivalent of copper, z:

$$m = zIt,$$

$$I = \frac{m}{zt}. \qquad \qquad (2)$$

The error in the ammeter is then the difference in the reading on A and the current calculated from (2).

Great care must be taken in this experiment over the cleanliness of the electrodes. They must be cleaned with sandpaper at the start; and, at the finish, the cathode must be rinsed with water and dried with alcohol, or over a gentle spirit flame: strong heating will oxidize the copper deposit.

The Mechanism of Conduction; Ions

The theory of electrolytic conduction is generally attributed to Arrhenius (1859–1927), although Faraday had stated some of its essentials in 1834. Faraday suggested that the current through an electrolyte was carried by charged particles, which he called ions (Greek *ion* = go). A solution of silver nitrate, he supposed, contained silver ions and 'nitrate' ions. The silver ions were silver atoms with a positive charge; they were positive because silver was deposited at the cathode, or negative electrode (Fig. 34.4). The nitrate ions were groups of

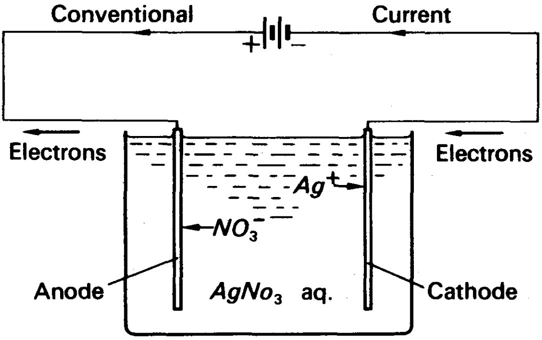


FIG. 34.4. Ions in electrolysis.

atoms—NO_3 groups—with a negative charge; they travelled towards the anode, or positive electrode, and, when silver electrodes are used, formed silver nitrate.

Nowadays, we consider that a silver ion is a silver atom which has lost an electron; this electron transfers itself to the NO_3 group when the silver nitrate molecule is formed, and gives the nitrate ion its negative charge. We denote nitrate and silver ions, respectively, by the symbols NO_3^- and Ag^+. When the ions appear at the electrodes of a voltameter they are discharged. The current in the external circuit brings electrons to the cathode, and takes them away from the anode (Fig. 34.4). At the anode silver atoms lose electrons and go into solution as positive ions. *In effect*, the negative charges carried across the cell by the NO_3^- ions flow away through the external circuit. At the cathode, each silver ions gains an electron, and becomes a silver atom, which is deposited upon the electrode.

Ionization

The splitting up of a compound into ions in solution is called ionization, or ionic dissociation. Faraday does not seem to have paid much attention to how it took place, and the theory of it was given by Arrhenius in 1887. For a reason which we will consider later, Arrhenius suggested that an electrolyte ionized as soon as it was dissolved: that its ions were not produced by the current through it, but were present as such in the solution, before ever the current was passed.

We now consider that salts of strong bases and acids, such as silver nitrate, copper sulphate, sodium chloride, ionize completely as soon as they are dissolved in water. That is to say, a solution contains no molecules of these salts, but only their ions. Such salts are called strong electrolytes; so are the acids and bases from which they are formed, for these also ionize completely when dissolved in water.

Other salts, such as sodium carbonate, do not appear to ionize completely on solution in water. They are the salts of weak acids, and are called weak electrolytes. The weak acids themselves are also incompletely ionized in water.

Formation of Ions; Mechanism of Ionization

In the Heat section of this book we described the structure of the solid state (p. 294). In solid crystalline salts such as sodium chloride the structure is made up of sodium and chlorine ions: not of atoms, nor of NaCl molecules, but of Na^+ and Cl^- ions. In other words, we think today that ions exist in solid crystalline salts, as well as in their solutions. We do so for the reason that the idea enables us to build up a consistent theory of chemical combination, of the solid state, and of electrolytic dissociation.

A sodium atom contains eleven electrons, ten of which move in orbits close to the nucleus, and one of which ranges much more widely; for our present purposes we may represent it as in Fig. 34.5 (i). A chlorine atom has ten inner electrons and seven outer ones; for our present purposes we may lump these into two groups, as in Fig. 34.5 (ii). The outer electron of the sodium atom is weakly attracted to its nucleus, but the outer electrons of the chlorine atom are strongly attracted (because the ten inner electrons are a more effective shield round the +11 nucleus of sodium than round the +17 nucleus of chlorine). Therefore, when a sodium and a chlorine atom approach one another, the outer electron of the sodium atom is attracted more strongly by the chlorine nucleus than by the sodium nucleus. It leaves the sodium atom, and joins the outer electrons of

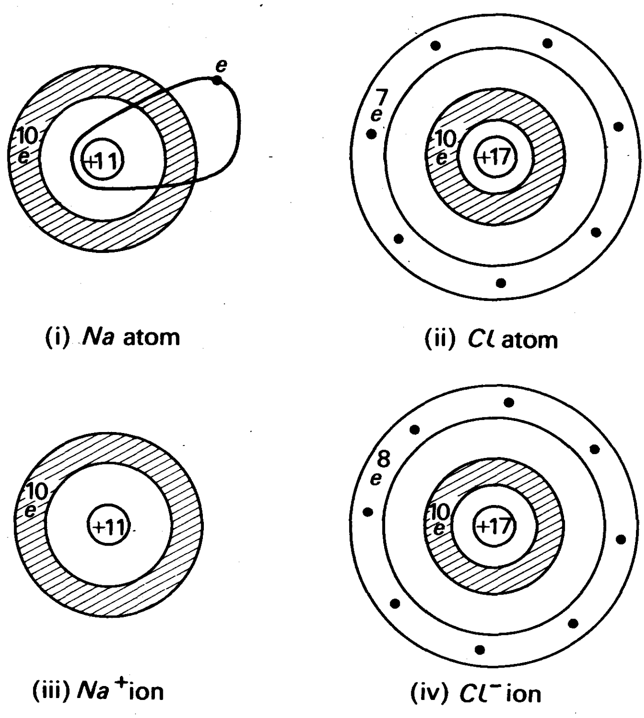


FIG. 34.5. Sodium and chlorine, atoms and ions.

the chlorine; the sodium atom becomes a positively charged sodium ion. Fig. 34.5 (iii), and the chlorine atom a negatively charged chlorine ion, Fig. 34.5 (iv). Between these two ions there now appears a strong electrostatic attraction, which holds them together as a molecule of NaCl. In the solid state, the ions are arranged alternately positive and negative; the forces between them bind the whole into a rigid crystal.

When such a crystal is dropped into water, it dissolves and ionizes. We can readily understand this when we remember that water has a very high dielectric constant: 81 (p. 774). It therefore reduces the forces between the ions 81 times, and the crystal falls apart into ions. In the same way we explain the ionization of other salts, and bases and acids. The idea that these dissociate because they are held together by electrostatic forces, which the solvent weakens, is supported by the fact that they ionize in some other solvents as well as water. These solvents also are liquids which have a high dielectric constant, such as methyl and ethyl alcohols (32 and 26 respectively). In these liquids, however, it seems that strong electrolytes behave as weak ones do in water: only a fraction of the dissolved molecules, not all of them, dissociate. In the electronic theory of atomic structure, the chemical behaviour of an element is determined by the number of its outer electrons. If it can readily lose one or two it is metallic, and forms positive ions; if it can readily gain one or two, it is acidic, and forms negative ions. Ions are not chemically active in the way that atoms are. Sodium atoms, in the form of a lump of the metal, react violently with water; but the hydroxide which they form ionizes ($NaOH \rightarrow Na^+ + OH^-$), and the sodium ions drift peaceably about in the solution—which is still mainly water.

Pure water is a feeble conductor of electricity, and we consider that it is but feebly ionized into H^+ and OH^- ions. These, we believe, are continually joining up to form water molecules, and then dissociating again in a dynamical equilibrium:

$$H_2O \rightleftharpoons H^+ + OH^-.$$

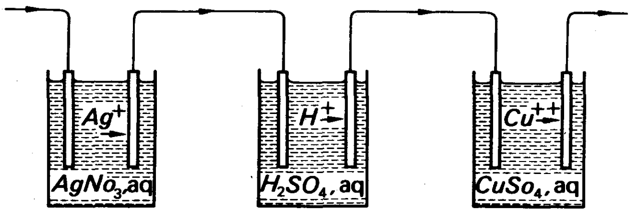
If, as we shall find in the electrolysis of water, H^+ and OH^- ions are removed from water, then more molecules dissociate, to restore the equilibrium.

The concentrations of H^+ and OH^- in water are so small that they do not contribute appreciably to the conduction of electricity when an electrolyte is dissolved in the water; but, as we shall see, they sometimes take part in reactions at the electrodes.

Explanation of Faraday's Laws

The theory of dissociation neatly explains Faraday's laws and some other phenomena of electrolysis. If an $AgNO_3$ molecule splits up into Ag^+ and NO_3^- ions, then each NO_3^- ion that reaches the anode dissolves one silver atom off it. At the same time, one silver atom is deposited on the cathode. Thus the gain in mass of the cathode is equal to the loss in mass of the anode. Also the total mass of silver nitrate in solution is unchanged; experiment shows that this is true. The mass of silver deposited is proportional to the number of ions reaching the cathode; if all the ions carry the same charge—a reasonable assumption—then the number deposited is proportional to the quantity of charge which deposits them. This is Faraday's first law.

To see how the ionic theory explains Faraday's second law, let us again consider a number of voltameters in series (Fig. 34.6). When a



Relative mass liberated	107·9	1·008	31·8
Atomic weight	107·9	1·008	63·6

FIG. 34.6. Illustrating Faraday's second law.

current flows through them all, the same quantity of charge passes through each in a given time. Experiment shows that

$$\frac{\text{mass of silver deposited}}{\text{mass of hydrogen liberated}} = 107{\cdot}0.$$

From experiments on chemical combination, we know that

$$\frac{\text{mass of silver atom}}{\text{mass of hydrogen atom}} = \frac{107{\cdot}9}{1{\cdot}008} = 107{\cdot}0.$$

Therefore we may say that, each time a silver ion is discharged and deposited as an atom, a hydrogen ion is also discharged and becomes an atom. The hydrogen atoms thus formed join up in pairs, and escape as molecules of hydrogen gas. The theory fits the facts, on the simple assumption that the hydrogen and silver atoms carry equal charges: we now say that each is an atom which has lost one electron.

But when we consider the copper voltameter in Fig. 34.6, we find a complication. For

$$\frac{\text{mass of copper deposited}}{\text{mass of hydrogen liberated}} = \frac{31{\cdot}8}{1{\cdot}008},$$

whereas

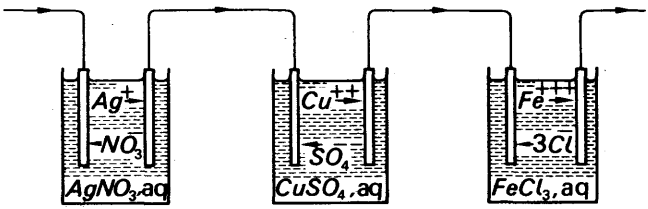
$$\frac{\text{mass of copper atom}}{\text{mass of hydrogen atom}} = \frac{63{\cdot}6}{1{\cdot}008}.$$

To explain this result we must suppose that only one copper atom is deposited for every two hydrogen atoms liberated. In terms of the ionic theory, therefore, only one copper ion is discharged for every two hydrogen ions. It follows that a copper ion must have twice as great a charge as a hydrogen ion: it must be an atom which has lost two electrons.

This conclusion fits in with our knowledge of the chemistry of copper. One atom of copper can replace two of hydrogen, as it does, for example, in the formation of copper sulphate, $CuSO_4$, from sulphuric acid, H_2SO_4. We therefore suppose that the sulphate ion also is doubly charged: SO_4^{2-}. When sulphuric acid is formed, two hydrogen atoms each lose an electron, and the SO_4 group gains two. When copper sulphate is formed, each copper atom gives up two electrons to an SO_4 group. And when copper sulphate ionizes, each molecule splits into two doubly charged ions:

$$CuSO_4 \rightarrow Cu^{2+} + SO_4^{2-}. \quad (3)$$

In general, if we express the charge on an ion in units of the electronic charge, we find that it is equal to the valency of the atom from which the ion was formed. That is to say, it is equal to the number of hydrogen atoms which the atom can combine with or replace.



Ion arriving	NO_3^-	Ag^+	SO_4^{--}	Cu^{++}	Cl^-	Fe^{+++}
Valency	1	1	2	2	1	3

FIG. 34.7. Movement of ions in electrolysis.

This assertion, which is illustrated in Fig. 34.7, explains Faraday's second law (mass deposited $\propto$ chemical equivalent). For if a current I passes through a voltameter for a time t, the total charge carried through it is It. And if q is the charge on an ion, the number of ions liberated is It/q. If M is the mass of an ion, the mass liberated is $M(It/q)$, and is therefore proportional to M/q. But M is virtually equal to the relative atomic mass, since the mass of an electron is negligible. And q, we have just seen, is equal, in electronic units, to the valency. Therefore the mass liberated is proportional to the ratio of relative atomic mass to valency. See also p. 845.

Electrolysis of Copper Sulphate Solution

If copper sulphate solution is electrolysed with platinum or carbon electrodes, copper is deposited on the cathode, but the anode is not dissolved away: instead, oxygen is evolved from it (Fig. 34.8). The SO_4^{2-} ions which approach the anode do not attack it; neither carbon nor platinum forms a sulphate, and each is said to be insoluble, in the electrolysis of copper sulphate. As the electrolysis proceeds, the solution becomes paler in colour; chemical tests show that it is gradually losing copper sulphate, but gaining sulphuric acid—Cu^{2+} ions are disappearing from the solution, but the SO_4^{2-} are remaining in it.

The oxygen which is evolved comes from the water of the solution. We have already seen that water is always slightly ionized, into H^+ and OH^- ions. When copper sulphate is electrolyzed with platinum or

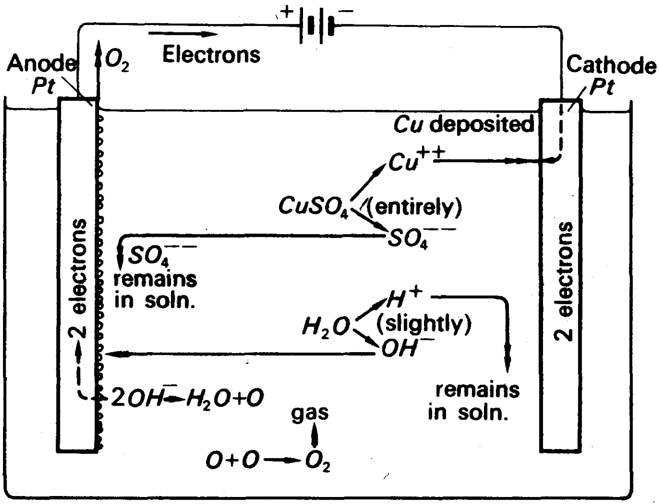


FIG. 34.8. Electrolysis of copper sulphate with insoluble electrodes.

carbon electrodes, the OH^- ions of the water are discharged at the anode. Each gives up an electron; they then combine in pairs to give a water molecule and an oxygen atom:

$$2OH^- - 2\text{ electrons} \rightarrow H_2O + O.$$

The oxygen atoms combine in pairs and come off as molecules:

$$2O \rightarrow O_2 \uparrow$$

As the OH^- ions disappear, an excess of H^+ ions appear in the solution. If the electrolysis were carried to the point where all the copper originally in the solution was deposited on the cathode, the solution would become simply one of sulphuric acid. This would be ionized into H^+ and SO_4^{2-} ions, in the proportion two H^+ to one SO_4^{2-}.

Electrolysis of Water

When a current is passed through water acidulated with dilute sulphuric acid, and platinum electrodes are used, oxygen and hydrogen are produced at the anode and cathode respectively. The amount of acid in solution remains unaltered, and the net effect is thus the electrolysis of water. The sulphuric acid ionizes into hydrogen and sulphate ions (Fig. 34.9):

$$H_2SO_4 \rightarrow 2H^+ + SO_4^{2-}.$$

The hydrogen ions from the acid greatly outnumber those from the water, but we cannot distinguish between them. All we can say is that, for every SO_4^{2-} ion that approaches the anode, two H^+ ions approach the cathode. At the cathode, the H^+ ions collect electrons, join up in pairs, and come off as molecules of hydrogen gas, H_2. At the anode,

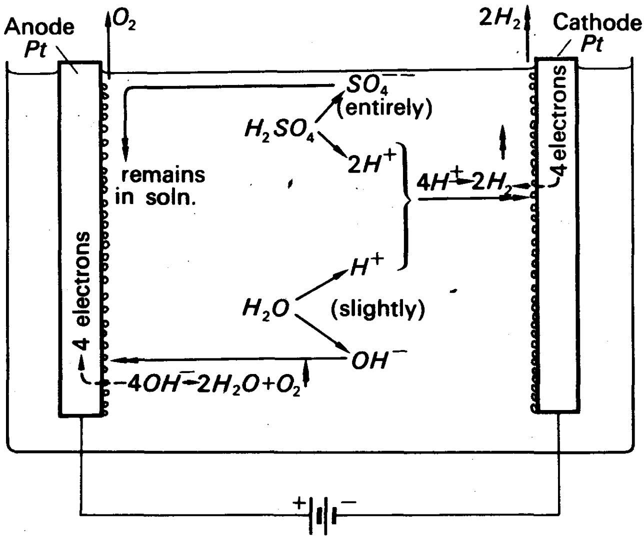


FIG. 34.9. Electrolysis of water.

however, the SO_4^{2-} ions remain in solution, and OH^- ions are discharged; as before, they form water and oxygen molecules, and the oxygen comes off as gas.

To produce one molecule of oxygen, four OH^- ions must be discharged:

$$4OH^- \rightarrow 2H_2O + O_2\uparrow + 4 \text{ electrons to circuit.} \qquad (4)$$

And every time four OH^- ions are discharged at the anode, four H^+ ions are discharged at the cathode. (If this were not so, a net positive or negative charge would accumulate in the solution, and we could draw sparks from it.) Thus the reaction is accompanied by

$$4H^+ + 4 \text{ electrons} \rightarrow 2H_2\uparrow \qquad (5)$$

Equations (4) and (5) agree with the experimental fact that hydrogen and oxygen come off in the proportions in which they are found in water: 2 to 1 by volume. This can be shown by collecting the gases in tubes filled with electrolyte and inverted over the electrodes (Fig. 34.1 (ii)). The electrolysis decomposes the water only, and leaves the acid unchanged. Equal numbers of H^+ and OH^- ions are discharged, and in the solution H^+ ions remain, in the proportion of two H^+ to one SO_4^{2-}.

The only function of the sulphuric acid is to increase the concentration of ions in the solution, and so to enable it to carry a greater current with a given potential difference than would pure water. The greater current discharges H^+ ions at a greater rate, and so causes the water to dissociate faster into H^+ and OH^-. Thus more ions are formed to carry the current.

The Electrolytic Capacitor

An electrolytic capacitor is one in which the dielectric is formed by electrolysis—by a secondary reaction at an insoluble electrode. It is made from two coaxial aluminium tubes, A and K in Fig. 34.10, with a solution or paste of ammonium borate between them. A current is passed through from A (anode) to K (cathode) and a secondary reaction at A liberates oxygen. The oxygen does not come off as a gas, however, but combines with the aluminium to form a layer of aluminium oxide over the electrode A. This layer is about 1/400 cm thick, and is an insulator. When the layer has been formed, the whole system can be used as a capacitor, one of whose electrodes is the cylinder A, and the other the surface of the liquid or paste adjacent to A. Because the dielectric layer is so thin, the capacitance is much greater than that of a paper capacitor of the same size.

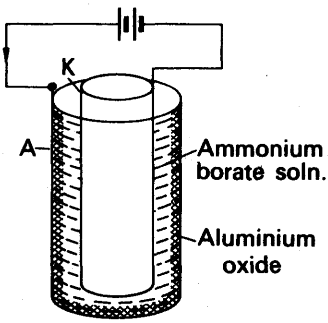


FIG. 34.10. Forming an electrolytic capacitor.

In the use of an electrolytic capacitor, some precautions must be taken. The voltage applied to it must not exceed a value determined by the thickness of the dielectric, and marked on the condenser; otherwise the layer of aluminium oxide will break down (see 'dielectric strength', p. 772). And the voltage must always be applied in the same sense as when the layer was being formed. If the plate A is made negative with respect to K, the oxide layer is rapidly dissolved away. Consequently an alternating voltage must never be applied to an electrolytic capacitor. This condition limits the usefulness of these capacitors.

Electrolytic capacitors are not very reliable, because the oxide layer is apt to break down with age. Domestic radio receivers abound in them, but in high-grade apparatus they are avoided.

Application of Ohm's Law to Electrolytes

Fig. 34.11 (i) shows how the current through an electrolyte, and the potential difference across it, may be measured. If the electrodes are soluble—copper in copper sulphate, for example—then the current is proportional to the potential difference (Fig. 34.11 (ii)). The best results in this experiment are obtained with very small currents. If the current is large, the solution becomes non-uniform: it becomes stronger near the anode, where copper is dissolved by the attack of the SO_4^{2-} ions, and weaker near the cathode, where copper is deposited, and SO_4^{2-} ions drift away. Near the cathode the solution becomes paler in colour; near the anode it becomes deeper. The total amount of copper sulphate in solution remains constant, but is gradually transferred to the neighbourhood of the anode. As the solution round the cathode becomes weaker, its resistance increases, and more than offsets the decreasing resistance of the solution round the anode; with a given potential difference, therefore, the current gradually falls. A small current makes

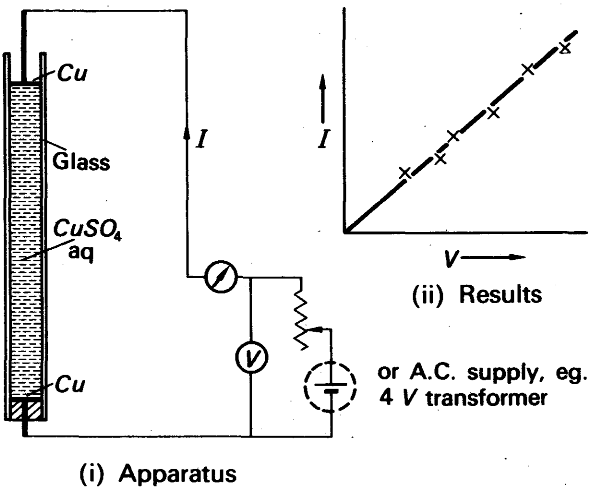


FIG. 34.11. Current/Voltage characteristic of electrolyte with soluble electrodes.

this effect unimportant, because it makes the electrolysis very slow. But the best way to do the experiment is to use an alternating current; the electrolysis reverses with the current fifty or more times per second, and no changes of concentration build up. Rectifier-type meters (p. 1011) are most suitable for measuring the current and potential difference in this case.

When the measurements are properly made they show, as we have said, that the current is proportional to the potential difference: for example, copper sulphate solution, with copper electrodes, obeys Ohm's law. The voltameter behaves as a passive resistor; all the electrical energy delivered to it by the current appears as heat (pp. 791–2); no electrical energy is converted into mechanical or chemical work. In particular, therefore, no electrical energy is used to break up the molecules of copper sulphate into ions. This is the argument which led Arrhenius to suggest that the electrolyte dissociates into ions as soon as it is dissolved; dissociation is a result of solution, not of electrolysis.

Measurement of Resistance of Electrolyte

The resistance of an electrolyte can be measured on a Wheatstone bridge, preferably with an alternating current supply (Fig. 34.12). A telephone earpiece T is used as the detector, in place of the galvanometer—it gives minimum sound at the balance-point. 50 Hz mains

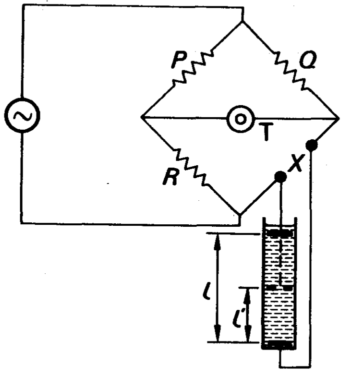


FIG. 34.12. Measurement of electrolyte resistance.

give an uncomfortably low-pitched note for listening, and a supply of frequency 400 to 1000 Hz is more satisfactory. This may be a small induction coil of a type sold for the purpose, which has fewer secondary turns than the common spark-coil, because a high voltage is not required. By sliding the upper electrode of the cell, the resistances of two lengths of electrolyte, l and l' are measured; their difference, r, is the resistance of a length l–l', and is free from end-errors. If A is the cross-section of the tube, then the specific resistance of the electrolyte, ρ, is given by

$$r = \frac{(l-l')\rho}{A}.$$

It is usual, however, not to give the resistivity of an electrolyte, but to give instead its conductivity, σ. This is defined as the reciprocal of its resistivity, and is expressed in mho per centimetre:

$$\sigma = \frac{1}{\rho} = \frac{l-l'}{Ar}.$$

Electrical Behaviour with Insoluble Electrodes

If we set out to find the current/voltage relationship for a water voltameter, using a d.c. supply, we find that the voltameter does not obey Ohm's law. If we apply to it a voltage E less than 1·7 volts, the current flows only for a short time and then stops, as though the voltameter were charged like a capacitor. To study the matter further, we may arrange a two-way key and a second galvanometer, G, as in Fig. 34.13. We first press the key at Y, and pass a current through the voltameter until it stops; then we press the key at X, and connect the galvanometer G straight across the voltameter. A brief current I flows through G, whose direction shows that the anode of the voltameter is acting as the positive pole of a current supply. It appears, therefore, that the voltameter is setting up a back-e.m.f. which prevents a steady current from flowing, unless the supply voltage is greater than 1·7 volts.

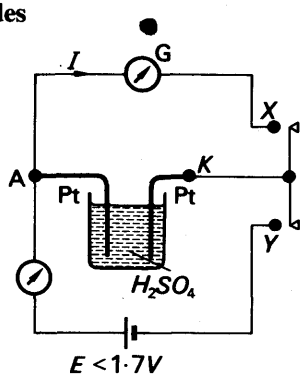
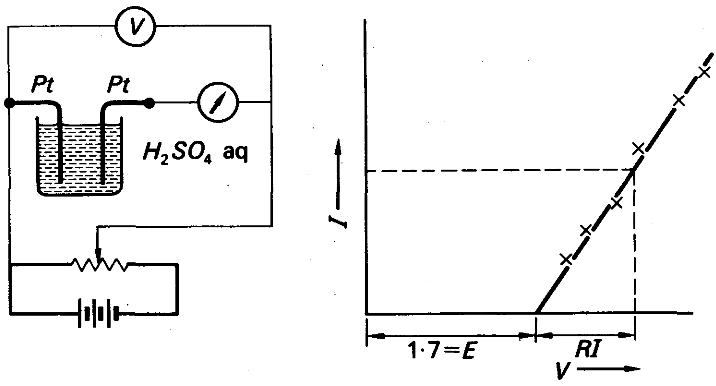


FIG. 34.13. Demonstrating back-e.m.f. of water voltameter.

To investigate this behaviour further, let us gradually increase the potential difference across the voltameter, as in Fig. 34.14 (i). We then



(i) Apparatus (ii) Results

FIG. 34.14. Current/Voltage characteristic of water voltameter.

find that, if we work with small currents, the current increases linearly with the potential difference V, when the latter is greater than 1·7 volts (Fig. 34.14 (ii)). It follows that the voltameter does indeed exert a back-e.m.f., E, equal to 1·7 volts. If V is the potential difference applied to it, then the current through it obeys the relationship

$$I \propto (V-E).$$

We may write this as

$$I = \frac{V-E}{R},$$

where R is the resistance of the electrolyte.

Electrical Energy Consumed in Decomposition

We have not yet explained the origin of the back-e.m.f. E, but we shall try to do so later. Meanwhile let us notice that the behaviour of the voltameter is somewhat like that of an electric motor. When the armature of a motor rotates, a back-e.m.f. is induced in it, and the current through it is given by an equation similar to that above. The back-e.m.f. in the motor, we shall see, represents the electrical power converted into mechanical work. So here the back-e.m.f. E represents this electrical power converted into chemical work—used in breaking up the water molecules. The potential difference across the voltameter is

$$V = IR + E$$

from the equation for I; the power equation is therefore

$$IV = I^2R + EI.$$

The left-hand term is the electrical power input; the first term on the right is the heat produced per second in the electrolyte; and the second term is the work done per second in decomposing the water.

Chemists tell us that when oxygen and hydrogen combine to form one mole of water (18 g) then 286 000 joules of heat are evolved. The energy set free is therefore 286 000 joules.

When one mole of water is decomposed, this much work must be done. In the process two moles of hydrogen are liberated (because the formula for water is H_2O). The quantity of electricity required to decompose one mole of water is therefore 2×96500 coulombs. If the back-e.m.f. is E, the corresponding amount of energy is $2 \times 96500 \times E$ joules. Therefore

$$2 \times 96500 \times E = 286\,000$$

whence

$$E = \frac{286\,000}{2 \times 96500}$$

$$= 1{\cdot}48 \text{ volts.}$$

The lowest value of E which anyone has ever got by experiment is 1·67 volts—from which it appears that we have something yet to learn about what happens in a water voltameter.

CELLS

If we put plates of copper and zinc into a beaker of dilute sulphuric acid, we have a voltaic cell (p. 785). It is often called a simple cell. If we join its plates via a galvanometer, current flows through the galvanometer from the copper to the zinc (Fig. 34.15); the cell sets up an e.m.f. which acts, in the external circuit, from copper to zinc. Its value is about one volt. The copper plate is at a higher potential than the zinc plate, and is the positive terminal of the cell; the zinc is the negative terminal. Within the cell there must be some agency which

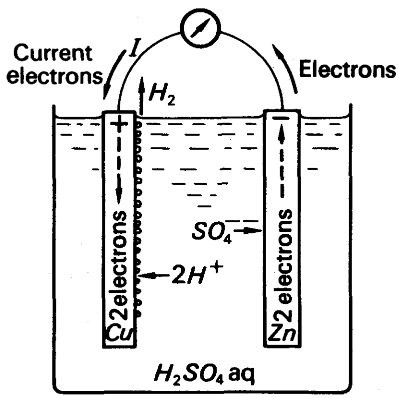


FIG. 34.15. A simple cell.

carries the current from the zinc to the copper. This is the agency which gives rise to the e.m.f. of the cell; it is analogous to the force exerted by the magnetic field on the moving electrons in the armature winding of a dynamo.

In a voltaic cell, the agency which gives rise to the e.m.f. is not so easy to track down as in a dynamo; there has been much argument about what is called the 'seat' of the e.m.f. We may start to seek it by placing a penknife blade into a strong solution of copper sulphate: a pink film of metallic copper is deposited on the blade. It appears, then, that copper ions have a tendency to go out of solution on to iron. In the same way we can show that they tend to go out on to zinc.

Do metal ions ever tend to go the other way—from solid metal into solution? They certainly do if the solution is sulphuric acid and the metal zinc or iron: the metal enters the solution in the form of ions, displaces the hydrogen ions, which are discharged and come off as gas, and turns the solution into one of iron or zinc sulphate. But this happens only if the zinc or iron is impure. Pure zinc in sulphuric acid gives no action at all—no zinc sulphate, no hydrogen.

Action in a Simple Cell

If we want to make pure zinc react with sulphuric acid, we must make it into part of a voltaic cell: we must connect it to a plate of a different metal, such as copper which also dips in the acid. Then the zinc is eaten away, and hydrogen bubbles off; but the hydrogen appears at the copper plate, and not at the zinc (Fig. 34.15). At the same time the solution becomes one of zinc sulphate—which simply means that it contains zinc ions in place of hydrogen ones. We can now form a picture of what happens when a stick of pure zinc is put alone into dilute sulphuric acid (Fig. 34.16). At first zinc ions leave the metal, and go into the liquid. But they leave negative charges on the zinc rod, which attract the zinc ions, and prevent any more from leaving. Nothing further happens. But if we introduce a plate of copper, and connect it to the zinc, electrons can flow from the zinc to the copper (Fig. 34.15). At the copper they can neutralize the charges on hydrogen ions, and to enable molecules to form and come off in bubbles. As the electrons flow away from the zinc, more zinc ions can go into solution, and so the zinc can continuously dissolve in the acid. In doing so, it maintains a continuous electric current in the wire connecting it to the copper. (When the zinc is impure each speck of impurity acts as the other plate of a minute cell, and enables the zinc around it to react with the acid. This 'local action', as it is called, makes impure zinc undesirable in voltaic cells, for it consumes the zinc without giving any useful current. It can be prevented by rubbing the zinc with mercury, which dissolves it and presents a surface of pure zinc to the acid. The process is called amalagamating the zinc.)

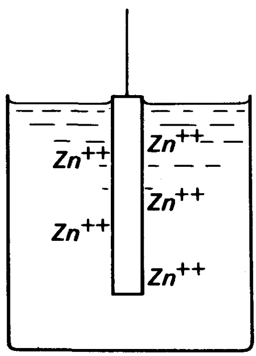


FIG. 34.16.
Pure zinc in dilute sulphuric acid.

To explain the voltaic cell, therefore, we must suppose that zinc ions tend to dissolve from zinc into sulphuric acid, but copper ions do not. This is consistent with the fact that copper does not react chemically with cold dilute sulphuric acid. The passage of ions from metals to solutions, and oppositely, was studied by Nernst about 1889; we shall give a slight account of his theory later.

Daniell's Cell

Fig 34.17 shows a cell, developed by Daniell about 1850, which has some advantages over Volta's. Daniell's cell consists of a zinc rod, Zn, in a porous pot, P, containing sulphuric acid; this in turn stands in a strong solution of copper sulphate in a copper vessel, Cu. (Sometimes the copper is just a thin sheet in a glass vessel.) When the copper and

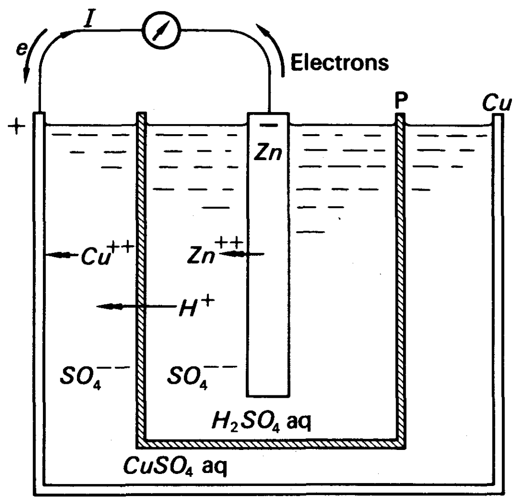


FIG. 34.17. A Daniell cell.

zinc are connected by a wire, current flows through the wire from copper to zinc. The copper is therefore the positive terminal of the cell, and the zinc the negative. As in Volta's cell, zinc ions go into solution at the zinc rod, leaving electrons on it. But at the copper plate, copper ions go out of solution—a metallic film of copper is deposited on the vessel. When the zinc and copper are joined by a wire, electrons from the zinc can go along it to the copper vessel, and discharge the copper ions as they reach it. To complete the action of the cell hydrogen ions from the sulphuric acid pass through the porous pot into the copper sulphate solution (Fig. 34.17). Thus zinc is dissolved, the acid gradually changes to zinc sulphate, the copper sulphate gradually changes to sulphuric acid, and copper is deposited on the copper vessel.

The e.m.f. of a Daniell cell is about 1·08 volts. Its internal resistance depends on its size and condition—the size is usually about that of a plant-pot, and the internal resistance is of the order of several ohms.

Polarization

The great disadvantage of the simple cell is that it does not give a steady current; from the moment of making the circuit, the current starts to fall, and after a minute or two it almost ceases to flow. The current decays because a layer of hydrogen gas forms over the copper plate; scraping the plate enables the current to start once more, but it soon decays again. The hydrogen layer increases the internal resistance of the cell, but we do not believe that this is the main reason for the decay of the current. If the copper plate is replaced by one of platinum black (platinum with a finely grained surface), bubbles form on it very easily, and escape readily. The hydrogen layer may then be no more than one molecule thick; but the current decays as before.

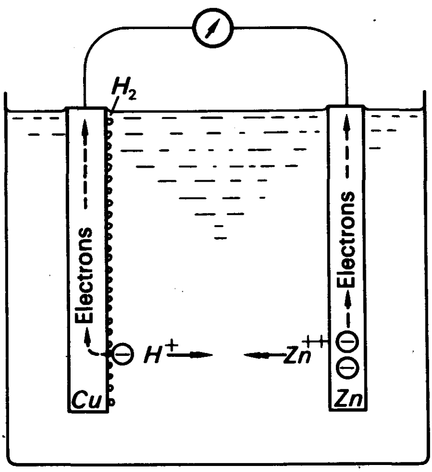


FIG. 34.18. Polarization in simple cell.

To explain this decay of the current we think that the hydrogen layer replaces the copper *as an electrode of the cell.* We suppose that the hydrogen tends to go back into solution as positive hydrogen ions (Fig. 34.18). In other words, it tends to behave in the same way as the zinc rod, on the other side of the cell, which also goes into solution as positive ions. Thus the hydrogen sets up an e.m.f. which opposes the original e.m.f. of the cell, and cuts down the current; the hydrogen thus sets up a 'back-e.m.f.' in the circuit. This behaviour is called polarization of the cell.

Depolarization

The advantage of Daniell's cell over Volta's is that it does not polarize. Hydrogen ions drift from the acid compartment into the copper sulphate compartment, but they are never discharged, no hydrogen molecules are formed, and no layer of hydrogen appears on the copper electrode. The copper sulphate solution is often called the depolarizer, because it prevents the formation of hydrogen gas.

Polarization in Water Voltameter

We can find support for the idea of polarization in the behaviour of the water voltameter (p. 856). When the potential difference across the voltameter is less than 1·7 volts, the current through it falls to zero in a minute or less. The voltameter itself can then deliver a current for a short time. Its positive terminal, as a source, is that which was its anode, and its negative terminal is that which was its cathode. While current was being sent through the voltameter, the cathode became covered with hydrogen, and the anode with oxygen. When the voltameter acts as a source of current it has, in effect, electrodes of oxygen and hydrogen and the current through the external circuit flows from the oxygen plate to the hydrogen (Fig. 34.19). In the simple cell, when it is polarized, there is no oxygen plate, but there is a hydrogen one, and this lies over the copper plate. We may therefore suppose

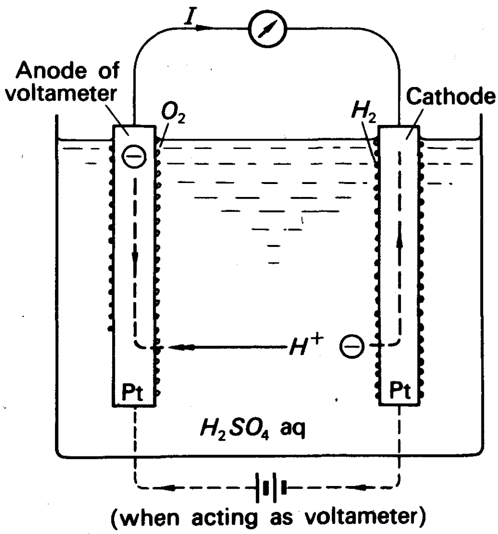


FIG. 34.19. Polarization of water voltameter.

that it tends to drive a current, through the external circuit, from zinc to copper; that is to say, it sets up an e.m.f. opposing that of the cell with the copper plate clean.

Nernst's Theory of the Voltaic cell; Electrode Potentials

If a metal is in contact with a solution of one of its own salts, it is surrounded by its own ions. Whether the ions deposit themselves on the metal, or the metal goes into solution, depends partly on the particular metal concerned, and partly on the strength of the solution: the stronger the solution the greater its tendency to deposit ions on the metal. If the solution deposits ions, the metal comes to a positive potential with respect to it; if the metal goes into solution as ions, it becomes negative with respect to the solution (Fig. 34.20 (i)). By methods beyond the scope of this book, the potential difference between a metal and a solution can be measured. These show that copper in normal copper sulphate solution (1 g-equivalent weight per litre) becomes 0·08 volt positive with respect to the solution.

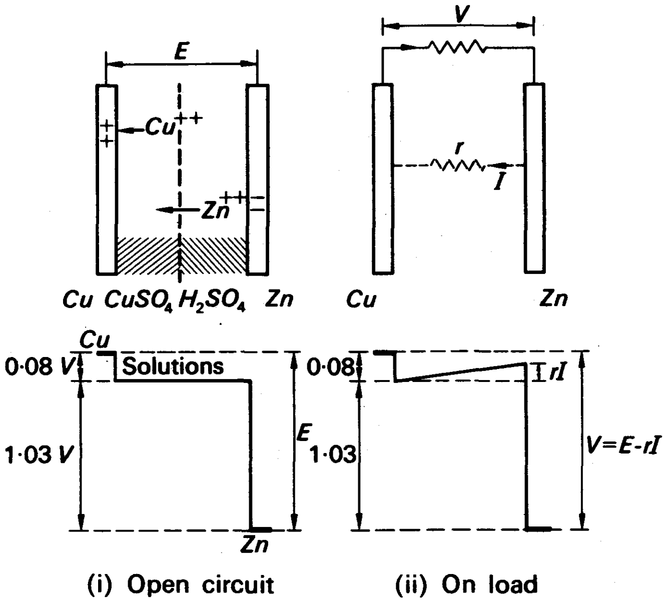


FIG. 34.20. Potential differences in voltaic cell.

Zinc in normal zinc sulphate solution becomes 1·03 volts negative. Now let us imagine a Daniell cell in which zinc sulphate replaces the sulphuric acid, and both solutions are normal. If we suppose that the solutions themselves set up no appreciable potential difference at their interface, then we get a potential distribution like that shown in Fig. 33.20 (i). The difference in potential between the copper and zinc is very nearly equal to the e.m.f. of a Daniell cell: 1·11 volts compared with 1·08. We may attribute the difference to the fact that a Daniell cell in practice has sulphuric acid, not zinc sulphate solution, in contact with the zinc; also the solutions are not normal: the acid is usually 1 to 4 of water, and the copper sulphate is saturated.

These considerations explain a striking experimental fact about all cells: the e.m.f. depends only on the nature and concentration of the constituent chemicals. The size of a cell affects only its internal resistance.

When a current I is drawn from a cell, there is a voltage drop across the internal resistance r, that is to say, across the solution or solutions. This modifies the potential diagram as shown in Fig. 34.20 (ii). The terminal voltage V, which is the observed potential difference between the copper and zinc, is now less than its open-circuit value, which is the e.m.f. of the cell.

The Leclanché Cell

Daniell's cell has the great practical disadvantage that it cannot be left set up; the solutions gradually mix by diffusion through the porous pot. It is now used only in teaching laboratories as a simple standard of e.m.f.; its e.m.f. is more nearly constant than that of any other

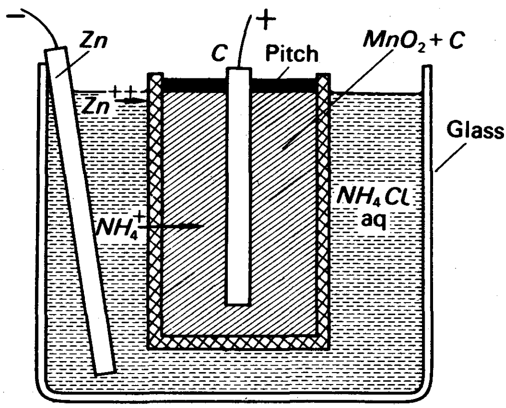


FIG. 34.21. Leclanché cell.

cheap and robust type of cell, and it is remarkably free from polarization.

A practically more useful cell is that devised by Leclanché. Its negative electrode is a zinc rod in a strong solution of ammonium chloride (Fig. 34.21). Its positive electrode is a carbon plate in a porous pot packed with manganese dioxide, which acts as the depolarizer. Manganese dioxide is a poor conductor of electricity, and powdered carbon is therefore packed in the pot with it.

The ammonium chloride ionizes into ammonium ions and chlorine ions:

$$NH_4Cl \rightarrow NH_4^+ \text{ and } Cl^-.$$

The zinc goes into solution, as zinc ions, and the ammonium ions drift through the porous pot towards the carbon plate. When a current is drawn from the cell, electrons flow from the zinc to the carbon, and discharge the NH_4^+ ions. The chemical action in this cell is complicated, but may be crudely represented as

$$2NH_4^+ + 2 \text{ electrons} \rightarrow 2NH_3 + H_2.$$

The hydrogen tends to polarize the cell, but is gradually oxidized by the manganese dioxide; again the action is complicated, but it reduces to

$$2MnO_2 + 2H_2O + H_2 \rightarrow 2Mn(OH)_3.$$

The depolarizing action is slow, and a Leclanché cell is therefore not suitable for giving a large current for a long time. A short rest, however, enables the manganese dioxide to remove the hydrogen and restore the e.m.f. of the cell. Thus Leclanché cells are suitable for giving intermittent currents: they are widely used, for example, with electric bells. They are also suitable for Wheatstone bridges, because they cannot give a current large enough to burn out the resistance coils when a wrong connexion is made. The e.m.f. of a Leclanché cell, before polarization sets in, is about 1·5 volts, and its internal resistance about 1 ohm.

Dry Cell

Fig. 34.22 shows a dry form of Leclanché cell, which has the obvious advantage that it is portable. The ammonium chloride is made into a paste with water, zinc chloride, flour, and gum; and the porous pot is replaced by a muslin bag. A cardboard spacer prevents the bag from touching the zinc and short-circuiting the cell. A dry Leclanché cell has the same e.m.f. as a wet one, but, for a given size, a lower internal resistance, because the thickness of solution between the zinc and carbon is less. It depolarizes better, because the volume of manganese dioxide is greater in relation to the overall size of the cell: the cycle-lamp size will give a useful light continuously for two hours or more.

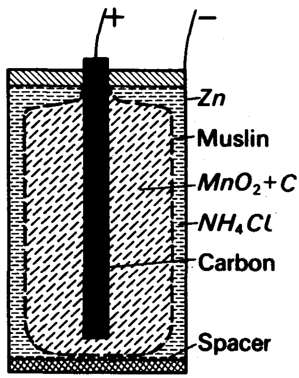


FIG. 34.22. A dry cell.

Standard Cells

A standard cell is one whose e.m.f. varies very little with time, and with temperature, so that it can be used as a standard of potential difference in potentiometer experiments. The commonest type is the Weston cadmium cell (Fig. 34.23). It is housed in an H-shaped glass

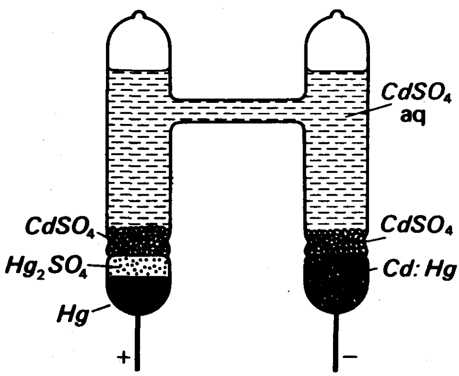


FIG. 34.23. A Weston cadmium cell.

tube because its electrodes are liquid or semi-liquid. The negative electrode is an amalgam of cadmium in mercury; the solution is of cadmium sulphate; the depolarizer is a paste of mercurous sulphate; and the positive electrode is mercury. In some cells, crystals of cadmium sulphate are placed on top of the electrodes to keep the solution saturated. The e.m.f. of one of these, in volts at a temperature t°C, is

$$E = 1{\cdot}01830 - 0{\cdot}0000406\,(t-20) \\ + 0{\cdot}00000095\,(t-20)^2 + 0{\cdot}00000001\,(t-20)^3.$$

The e.m.f. of the type without crystals is about 1·0186 volts between 0°C and 40°C.

A standard cell without crystals of cadmium sulphate is called an unsaturated cell; one with crystals is called a saturated cell, because the crystals keep the solution saturated. Saturated cells give an accurately reproducible e.m.f., because the concentration of the solution is sharply defined at any given temperature. Unsaturated cells do not agree among one another so well, because the solution may vary a little from one to the other. But the e.m.f. of a given unsaturated cell varies less with temperature than that of a saturated cell, because the concentration of the solution is constant.

The depolarizer of a standard cell is effective only for very small currents, and the e.m.f. of the cell will change appreciably if more than about 10 microamperes are drawn from it. *A standard cell must not, in any circumstances, be used as a source of current.* In the early stages of balancing a standard cell against a potentiometer wire, a protective resistance of about 100 000 ohms should be connected in series with the cell.

Primary and Secondary Cells

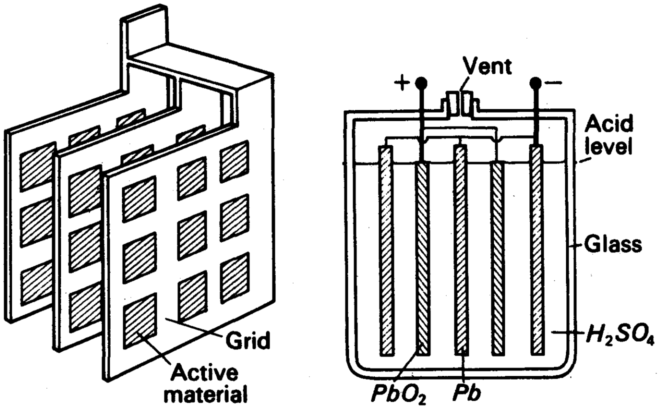
The cells which we have so far described are called primary cells. When they are run-down, their active materials must be renewed; the cells cannot be recharged by passing a current through them from another source. A secondary cell is one which can be recharged in this way.

SECONDARY CELLS

The Lead Accumulator

The commonest secondary cell is the lead-acid accumulator. Its active materials are spongy lead, Pb (for the negative plate), lead dioxide, PbO_2 (the for positive plate), and sulphuric acid. The active materials of the plates are supported in grids of hard lead-antimony alloy (Fig. 34.24 (i)). These are assembled in interchanging groups, closely spaced to give a low internal resistance, and often held apart by strips of wood or celluloid (Fig. 34.24 (ii)).

When the cell is discharging—giving a current—hydrogen ions drift to the positive plate, and SO_4^{2-} ions to the negative. As they give up their charges they attack the plates, and reduce the active materials of each to lead sulphate.



(i) Structure of plates (ii) Assembly of cell

FIG. 34.24. Lead-acid accumulator.

At the negative plate the reaction is

$$Pb+SO_4^{2-}-2\text{ electrons}\rightarrow PbSO_4. \qquad (6)$$

The chemical action at the positive plate is generally given as

(i) $PbO_2+2H^++2\text{ electrons}\rightarrow PbO+H_2O$;
(ii) $PbO+H_2SO_4\rightarrow PbSO_4+H_2O$;

whence, altogether

$$PbO_2+H_2SO_4+2H^++2\text{ electrons}\rightarrow PbSO_4+2H_2O. \qquad (7)$$

However, H_2SO_4 molecules do not exist in the solution—they are dissociated into $2H^+$ and $SO_4^{\,-}$ ions. We may therefore write equation (7) as

$$PbO_2+SO_4^{2-}+4H^++2\text{ electrons}\rightarrow PbSO_4+2H_2O. \qquad (8)$$

The lead sulphate produced in these reactions is a soft form, which is chemically more active than the hard, insoluble lead sulphate familiar in the general chemistry of lead. In the discharging reactions water is formed and sulphuric acid consumed: the concentration of the acid, and therefore its specific gravity, fall.

Charging the Accumulator

When the cell is to be charged it is connected, in opposition, to a supply of greater e.m.f., via a rheostat and ammeter (Fig. 34.25). The

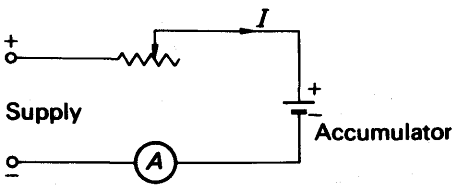


FIG. 34.25 Charging an accumulator.

supply forces a current I through the cell in the opposite direction to the discharging current, so that hydrogen ions are carried to the negative plate, and SO_4^{2-} ions to the positive. The chemical reactions are as follows.

At the negative plate:

$$PbSO_4 + 2H^+ + 2 \text{ electrons} \rightarrow Pb + H_2SO_4. \qquad (9)$$

At the positive plate:

(i) $PbSO_4 + SO_4^{2-} - 2 \text{ electrons} \rightarrow PbO_2 + 2SO_3$;
(ii) $2SO_3 + 2H_2O \rightarrow 2H_2SO_4$;

altogether:

$$PbSO_4 + 2H_2O + SO_4^{2-} - 2 \text{ electrons} \rightarrow PbO_2 + 2H_2SO_4. \qquad (10)$$

The active materials are converted back to lead and lead dioxide, water is consumed, and sulphuric acid is formed. The acid therefore becomes more concentrated during charge, and its specific gravity rises.

Properties and Care of the Lead Accumulator

The e.m.f. of a freshly charged lead accumulator is about 2·2 volts, and the specific gravity of the acid about 1·25. When the cell is being discharged its e.m.f. falls rapidly to about 2 volts, and then becomes steady (Fig. 34.26); but towards the end of the discharge the e.m.f. begins to fall again. When the terminal voltage load has dropped below about 1·9 volts, or the specific gravity of the acid below about 1·15, the cell should be recharged. If the cell is discharged too far, or left in a discharged condition, hard lead sulphate forms on its plates, and it becomes useless.

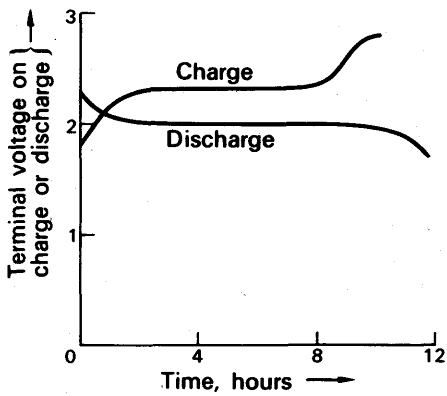


FIG. 34.26. Voltage-time curves of lead accumulator.

The internal resistance of a lead accumulator, like that of any other cell, depends on the area and spacing of its plates. It is much lower than that of any primary cell, however, being usually of the order of 1/10 to 1/100 ohm. The amount of electricity which an accumulator can store is called its *capacity*. It is a vague quantity, but a particular accumulator may give, for example, 4 amperes for 20 hours before needing a recharge. The capacity of this accumulator would be 80 ampere-hours. (One ampere-hour = 3600 coulombs.) If the accumulator were discharged faster—at 8 amperes, say—then it would probably need recharging after rather less than 10 hours; and if it were discharged more slowly—say at 2 amperes—it might hold out for more than 40 hours. The capacity of an accumulator therefore depends on its rate of discharge; it is usually specified at the '10-hour' or '20-hour' rate. Discharging an accumulator faster than at about the 10-hour rate causes the active material to fall out of the plates.

Accumulators are usually charged at about the '8-hour' rate—say 5 amperes for the cell discussed above. The charging is continued until gas is bubbling freely off the plates. When the plates are gassing, the chemical reactions (9) and (10) have been completed, and the current through the cell is simply decomposing the water in it. Before the charge is started the vent-plugs in the cell-case must be removed to let the gases out; the gases are hydrogen and oxygen and naked lights near are dangerous. The water lost at the end of each charge must be made up by pouring in distilled water until the acid rises to the level marked on the case. If the specific gravity of the acid is then less than 1·25, the charging must be continued. Near the end of the charging, the back-e.m.f. of the cell rises sharply to about 2·6 volts (Fig. 34.26). It never gives a forward e.m.f. as great as this: as soon as it is put on discharge, its e.m.f. falls to about 2·2 volts.

Efficiency of Accumulator

The number of ampere-hours put into an accumulator on charge is greater than the number which can be got out of it without discharging it too far. The ratio

$$\frac{\text{ampere-hours on discharge}}{\text{ampere-hours on charge}}$$

is called the ampere-hour efficiency of the cell; its value is commonly about 90 per cent. However, to judge an accumulator by its ampere-hour efficiency is to flatter it; not only does it take in more ampere-hours on charge than it gives out on discharge, but it takes them in at a higher voltage. The electrical energy put into an accumulator on charge is the integral of current, e.m.f., and time:

$$W = \int I\, E dt.$$

For simplicity we may say

$$\text{energy put in} = \text{quantity of electricity put in} \times \text{average e.m.f. on charge.}$$

If the quantity of electricity is measured in ampere-hours, the energy is in watt-hours instead of joules. Similarly, on discharge,

$$\text{energy given out} = \text{quantity of electricity given out} \times \text{average e.m.f. on discharge.}$$

The energy efficiency of the cell is

$$\frac{\text{energy given out on discharge}}{\text{energy taken in on charge}}$$

$$= \frac{\text{amp-hours} \times \text{average e.m.f. on discharge}}{\text{amp-hours} \times \text{average e.m.f. on charge}}$$

$$= \text{amp-hour efficiency} \times \frac{\text{average e.m.f. on discharge}}{\text{average e.m.f. on charge}}$$

$$\simeq \text{amp-hour efficiency} \times \frac{2{\cdot}0}{2{\cdot}2}.$$

The energy efficiency is more often called the watt-hour efficiency of the cell; it is about 80 per cent.

The Nickel-iron Accumulator

The nickel-iron (NIFE) accumulator has active materials of nickel hydroxide (positive), iron (negative) and caustic potash solution. Its e.m.f. varies from 1·3 volt to 1·0 on discharge; it has a higher internal resistance than a lead accumulator of similar size; and it is less efficient. Its advantages are that it is more rugged, both mechanically and electrically. Very rapid charging and discharging do not harm it, nor do overdischarging and overcharging. Vibration does not make the active materials fall out of the plates, as it does with a lead cell. Nickel-iron accumulators are therefore used in electric trucks and at sea.

EXAMPLES

1. A copper refining cell consists of two parallel copper plate electrodes, 6 cm apart and 1 metre square, immersed in a copper sulphate solution of resistivity $1{\cdot}2 \times 10^{-2}$ ohm metre. Calculate the potential difference which must be established between the plates to provide a constant current to deposit 480 g of copper on the cathode in one hour (e.c.e. of copper $= 3{\cdot}29 \times 10^{-7}$ kg C^{-1}).

From $m = zIt$, since 480 g = 0·48 kg,

$$I = \frac{m}{zt} = \frac{0{\cdot}48}{3{\cdot}29 \times 10^{-7} \times 3600}\ \text{A} \qquad \text{(i)}$$

The resistance of the cell, $R, = \dfrac{\rho l}{A}$

$$= \frac{1{\cdot}2 \times 10^{-2} \times 6 \times 10^{-2}}{1^2}$$

Hence, from (i), the p.d. $V = IR = \dfrac{0{\cdot}48 \times 1{\cdot}2 \times 10^{-2} \times 6 \times 10^{-2}}{3{\cdot}29 \times 10^{-7} \times 3600 \times 1^2}$

$$= 0{\cdot}3\ \text{V (approx.).}$$

2. State Faraday's laws of electrolysis and show that the ionic dissociation theory offers an explanation of them. Acidulated water is electrolyzed between platinum electrodes. Sketch a graph showing the relation between the strength of the current and the reading of a voltmeter connected to the electrodes. Comment on the nature of the graph.

Give a circuit diagram showing how you would charge a series battery of 12 lead accumulators, each of e.m.f. 2 volts and internal resistance 1/24 ohm, from 240-volt d.c. mains, if the charging current is not to exceed 3 amp. What percentage of the energy taken from the mains would be wasted? (*L.*)

First part (see text). When the water is electrolyzed, no current flows until the p.d. is greater than about 1·7 volts, when the back-e.m.f. of the liberated product is overcome. After this, a straight-line graph is obtained between V and I.

Second part. A series resistance R is required, given by

$$I = 3 = \frac{240-12\times 2}{R+\frac{12}{24}}.$$

$$\therefore\ 3R+1{\cdot}5 = 216.$$

$$\therefore\ R = \frac{214{\cdot}5}{3} = 71{\cdot}5 \text{ ohms.}$$

$$\begin{aligned} \text{Energy taken from mains} &= EIt = 240\times 3t = 720t, \text{ where } t \text{ is the time.}\\ \text{Energy wasted} &= (I^2R+I^2r)t = (3^2\times 71{\cdot}5+3^2\times 0{\cdot}5)t\\ &= 648t. \end{aligned}$$

$$\therefore\ \text{percentage wasted} = \frac{648t}{720t}\times 100\% = 90\%.$$

3. State Faraday's laws of electrolysis. How would you verify the laws experimentally? Discuss briefly the phenomenon of polarization in electrolysis and how it is overcome in the Daniell cell.

Calculate a value for the e.m.f. of a Daniell cell from energy considerations, using the following data: 1 g of zinc dissolved in copper sulphate solution liberates 3327 J (e.c.e. of zinc = 0·000340 g C^{-1}). (*L.*)

The e.m.f. E of a cell can be defined as the energy per coulomb delivered by the cell (p. 797).

When 1 g of zinc is dissolved, number of coulombs flowing, $Q = \frac{1}{0{\cdot}00034}$.

Energy liberated, $W = 3327$ J.

$$\begin{aligned} \therefore\ \text{energy liberated per coulomb} &= \frac{W}{Q}\\ &= 3327\times 0{\cdot}00034 \text{ joules/coulomb}\\ &= 1{\cdot}13 \text{ volts.} \end{aligned}$$

EXERCISES 34

1. State Faraday's laws of electrolysis and show how they can be interpreted in terms of the ionic theory.

A voltameter with large platinum plates contains dilute sulphuric acid in which 10^{-2} g of metallic copper has been dissolved. A constant current of 10^{-2} amp is then passed through the solution for 25 minutes. Calculate (*a*) the mass of copper deposited on the cathode, (*b*) the volume of oxygen liberated at the anode if the pressure is 750 mm Hg and the temperature is 20°C and (*c*) the additional time for which the same current must be passed before hydrogen is liberated at the cathode. (The chemical equivalents of copper and oxygen are 31·8 and 8 respectively; the Faraday is 95600 coulombs; 32 g of oxygen occupy 22·4 litres at S.T.P.) (*O. & C.*)

2. Describe the electrolytic processes which occur in a Daniell cell when its terminals are joined through a small resistance.

A steady current of 5 A is passed through a silver voltameter in series with a coil of wire of 10 ohms resistance immersed in 200 g of water. What will be the rise of temperature of the latter when 0·10 g of silver has been deposited? (Assume that the e.c.e. of silver = $1{\cdot}118 \times 10^{-6}$ kg C^{-1}; thermal capacity of the coil and vessel = 42 J K^{-1}.) (*L.*)

3. State Faraday's laws of electrolysis. Explain why it is necessary to have a potential greater than about 1·5 volts in order to maintain a large steady current through acidulated water.

In the electrolysis of water 83·7 cm^3 of hydrogen were collected at a pressure of 68 cm of mercury at 25°C when a current of 0·5 A had been passed for 20 minutes. What is the electrochemical equivalent of copper in copper sulphate ($CuSO_4$)? (Atomic weight of copper = 63·57, atomic weight of hydrogen = 1·008, density of hydrogen at S.T.P. = 0·08987 kg m^{-3}.) (*L.*)

4. What do you understand by (*a*) a *cathode*, and (*b*) an *ion*? Describe how the simple ionic theory accounts for the fact that the liberation of 1 g of hydrogen in the electrolysis of dilute sulphuric acid, using platinum electrodes, always requires the passage of a definite quantity of charge.

If the heat produced when 1 g of hydrogen burns to form water is $1{\cdot}44 \times 10^5$ joules, calculate the minimum e.m.f. which must be applied in such electrolysis before a continuous current will flow. Justify and explain your calculation by reference to the principle of conservation of energy. (1 Faraday = $9{\cdot}65 \times 10^4$ coulombs.)

Describe an experiment to determine the actual value of the back e.m.f. generated by a gas at the electrodes in such electrolysis. (*C.*)

5. State (*a*) Faraday's laws of electrolysis, (*b*) the main features of the theory of ionic dissociation, showing that the theory is in accordance with these laws.

A potential difference of 14 volts is applied to a water voltameter whose total thermal capacity is 2121 J K^{-1}. The temperature of the voltameter rises 1°C in the time taken to liberate 20 cm^3 of hydrogen at S.T.P. Calculate the back e.m.f. of the voltameter and state the source of this e.m.f. (Assume the electrochemical equivalent of hydrogen to be $1{\cdot}044 \times 10^{-8}$ kg C^{-1} and its density at S.T.P. to be 9×10^{-2} kg m^{-3}.) (*L.*)

6. To answer this question you are asked to assume that no galvanometer of any kind is available and that you cannot take for granted that the same current flows continuously round a series circuit.

A circuit consists only of a straight horizontal length of wire and a copper voltmeter connected in series to a suitable d.c. supply. How would you show that a current is flowing through the wire as well as through the voltmeter?

How would you show that the direction of the current through the wire is the same as that of the current through the voltameter?

Outline the measurements you would make to discover whether the strength of the current in the wire is the same as that of the current through the voltameter.

Give an account of the actual physical processes by which a current passes through a metal wire. Explain briefly why the resistance normally increases as the temperature rises. (*O.*)

7. Explain the general nature of the chemical changes that take place in a lead accumulator during charging and discharging.

A battery of accumulators, of e.m.f. 50 volts and internal resistance 2 ohms, is charged on a 100-volt direct-current mains. What series resistance will be required to give a charging current of 2 A? If the price of electrical energy is 1*d.* per kilowatt-hour, what will it cost to charge the battery for 8 hours, and what percentage of the energy supplied will be wasted in the form of heat? (*C.*)

8. State the laws of electrolysis and give a concise account of an elementary theory of electrolysis which is consistent with the laws.

If an electric current passes through a copper voltameter and a water voltameter in series, calculate the volume of hydrogen which will be liberated in the latter, at 25°C and 78 cm of mercury pressure, whilst 0·05 g of copper is deposited in the former. (Take e.c.e. of hydrogen as $1{\cdot}04 \times 10^{-8}$ kg C^{-1}, e.c.e. of copper as $3{\cdot}3 \times 10^{-7}$ kg C^{-1}, density of hydrogen as 9×10^{-2} kg m^{-3} at S.T.P.) (*L.*)

9. State Faraday's laws of electrolysis and describe experiments to verify them.

A difference of potential of 60 volts is maintained between two electrodes 12 cm apart in a solution of common salt. How long will it take a chlorine ion to travel 3 cm in the solution? (The mobility of chlorine ions may be taken as 0·00053 cm s^{-1} per V cm^{-1}.) (*L.*)

10. Explain what happens when an e.m.f. is applied to platinum electrodes immersed in dilute sulphuric acid. What is the relation between the e.m.f. and the current in such a cell?

If the electrochemical equivalent of hydrogen is $1{\cdot}04 \times 10^{-8}$ kg coulomb^{-1}, and if 1 g of hydrogen on burning to form water liberates 147000 joules, calculate the back-e.m.f. produced in a water voltameter when it is connected to a 2-volt accumulator. (*C.*)

chapter thirty-five

Magnetic Field and Force on Currents

NATURAL magnets were known some thousands of years ago, and in the eleventh century A.D. the Chinese invented the magnetic compass. This consisted of a magnet, floating on a buoyant support in a dish of water. The respective ends of the magnet, where iron filings are attracted most, are called the north and south poles.

In the thirteenth century the properties of magnets were studied by Peter Peregrinus. He showed that

like poles repel and *unlike poles attract*

His work was forgotten, however, and his results were rediscovered in the sixteenth century by Dr. Gilbert, who is famous for his researches in magnetism and electrostatics.

Ferromagnetism

About 1823 Sturgeon placed an iron core into a coil carrying a current, and found that the magnetic effect of the current was increased enormously. On switching off the current the iron lost nearly all its magnetism. Iron, which can be magnetized strongly, is called a *ferromagnetic* material. Steel, made by adding a small percentage of carbon to iron, is also ferromagnetic. It retains its magnetism, however, after removal from a current-carrying coil, and is more difficult to magnetize than iron. Nickel and cobalt are the only other ferromagnetic elements in addition to iron, and are widely used for modern magnetic apparatus. A modern alloy for permanent magnets, called *alnico*, has the composition 54 per cent iron, 18 per cent nickel, 12 per cent cobalt, 6 per cent copper, 10 per cent aluminium. It retains its magnetism extremely well, and, by analogy with steel, is therefore said to be magnetically very hard. Alloys which are very easily magnetized, but do not retain their magnetism, are said to be magnetically soft. An example is *mumetal*, which contains 76 per cent nickel, 17 per cent iron, 5 per cent copper, 2 per cent chromium.

Magnetic Fields

The region round a magnet, where a magnetic force is experienced, is called a *magnetic field*. The appearance of a magnetic field is quickly obtained by iron filings, and accurately plotted with a small compass, as the reader knows. The *direction* of a magnetic field is taken as the direction which a north pole would move if placed in the field.

Fig. 35.1 shows a few typical fields. The field round a bar-magnet is 'non-uniform', that is, its strength and direction vary from place to place (Fig. 35.1 (i)). The earth's field locally, however, is uniform (Fig. 35.1 (ii)). A bar of soft iron placed north–south becomes magnetized by

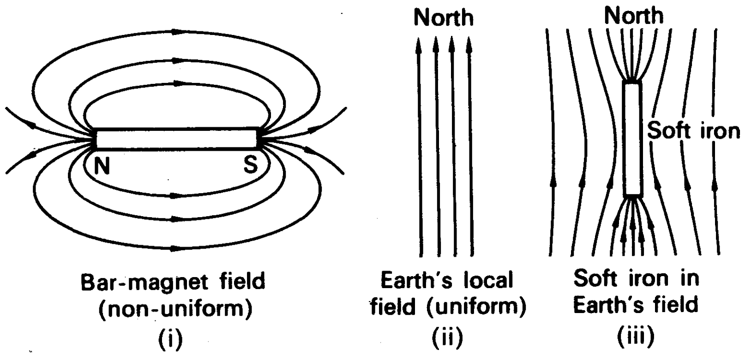


FIG. 35.1. Magnetic fields.

induction by the earth's field, and the lines of force become concentrated in the soft iron (Fig. 35.1 (iii)). The tangent to a line of force at a point gives the direction of the magnetic field at that point.

Oersted's Discovery

The magnetic effect of the electric current was discovered by Oersted in 1820. Like many others, Oersted suspected a relationship between electricity and magnetism, and was deliberately looking for it. In the course of his experiments, he happened to lead a wire carrying a current

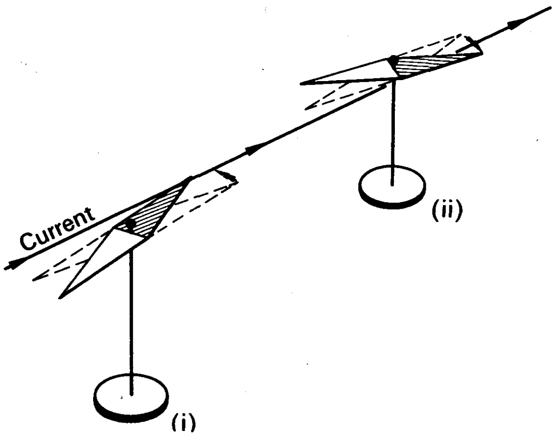


FIG. 35.2. Deflection of compass-needle by electric current.

over, but parallel to, a compass-needle, as shown in Fig. 35.2(i); the needle was deflected. Oersted then found that if the wire was led under

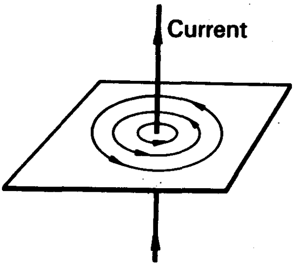


FIG. 35.3. Magnetic field of long straight conductor.

the needle, it was deflected in the opposite sense (Fig. 35.2(ii)). From these observations he concluded that 'the electric conflict performs gyrations'. What he meant by this we can see by plotting the lines of force of a long vertical wire, as shown in Fig. 35.3. To get a clear result a strong current is needed, and we must work close to the wire, so that the effect of the earth's field is negligible. It is then seen that the lines of force are circles, concentric with the wire.

Directions of Current and Field; Corkscrew Rule

The relationship between the direction of the lines of force and of the current is expressed in Maxwell's corkscrew rule: if we imagine ourselves driving a corkscrew in the direction of the current, then the direction of rotation of the corkscrew is the direction of the lines of force. Fig. 35.4 illustrates this rule, the small, heavy circle representing the wire, and the large light one a line of force. At (i) the current is flowing into the paper; its direction is indicated by a cross, which stands for the tail of a retreating arrow. At (ii) the current is flowing out of the paper; the dot in the centre of the wire stands for the point of an approaching arrow.

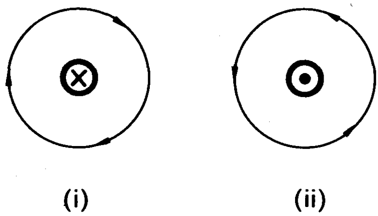


FIG. 35.4. Illustrating corkscrew rule.

If we plot the magnetic field of a circular coil carrying a current, we get the result shown in Fig. 35.5. Near the circumference of the coil, the lines of force are closed loops, which are not circular, but whose

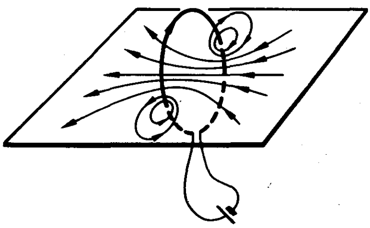


FIG. 35.5. Magnetic field of narrow coil.

directions are still given by the corkscrew rule, as in Fig. 35.5. Near the centre of the coil, the lines are almost straight and parallel. Their direction here is again given by the corkscrew rule, but the current and the lines of force are interchanged: if we turn the screw in the direction of the current, then it travels in the direction of the lines.

The Solenoid

The same is true of the magnetic field of a long cylindrical coil, shown in Fig. 35.6. Such a coil is called a solenoid; it has a field similar to that of a bar-magnet, whose poles are indicated in the figure. If an iron or steel core were put into the coil, it would become magnetized, with the polarity shown.

If the terminals of a battery are joined by a wire which is simply

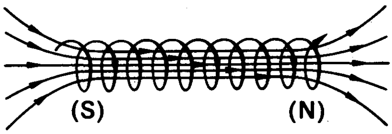


FIG. 35.6. Magnetic field of solenoid.

doubled back on itself, as in Fig. 35.7, there is no magnetic field at all: each element of the outward run, such as AB, in effect cancels the field of the corresponding element of the inward run, CD. But as soon as the wire is opened out into a loop, its magnetic field appears (Fig. 35.8).

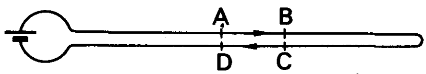


FIG. 35.7. A double-back current has no magnetic field.

Within the loop, the field is strong, because all the elements of the loop give magnetic fields in the same sense, as we can see by applying the corkscrew rule to each side of the square ABCD. Outside the loop, for example at the point P, corresponding elements of the loop give

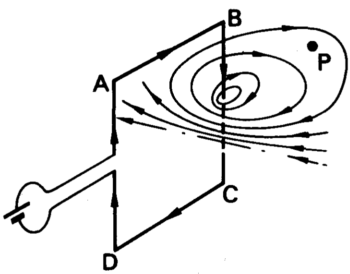


FIG. 35.8. An open loop of current has magnetic field.

opposing fields (for example, DA opposes BC); but these elements are at different distances from P (DA is farther away than BC). Thus there is a resultant field at P, but it is weak compared with the field inside the loop. A magnetic field can thus be set up either by wires carrying a current, or by the use of permanent magnets.

Force on Conductor. Fleming's Rule

When a conductor carrying a current is placed in a magnetic field due to some source other than itself, it experiences a mechanical force.

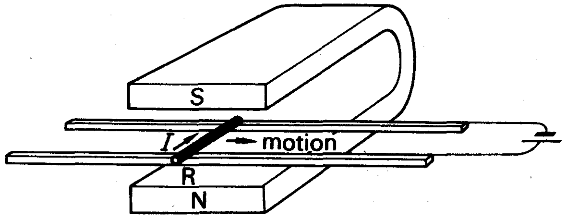


FIG. 35.9. Force on current in magnetic field.

To demonstrate this, a short brass rod R is connected across a pair of brass rails, as shown in Fig. 35.9. A horseshoe magnet is placed so that the rod lies in the field between its poles. When we pass a current through the rod, from an accumulator, the rod rolls along the rails. The relative directions of the current, the applied field, and the motion are shown in Fig. 35.10; they are the same as those of the middle finger, the fore-finger, and the thumb of the left hand when held all at right angles to one another. If we place the horseshoe magnet so that its field lies along the rod carrying the current, then the rod experiences no force.

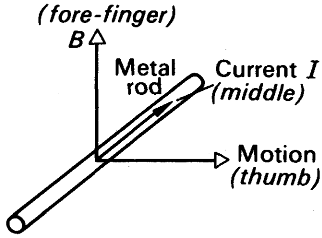


FIG. 35.10. Left-hand rule.

Experiments like this were first made by Ampère in 1820. As a result of them, he concluded that the force on a conductor is always *at right angles to the plane which contains both the conductor and the direction of the field in which it is placed.* He also showed that, if the conductor makes an angle α with the field, the force on it is proportional to sin α, so that the maximum force is exerted when the conductor is perpendicular to the field.

Dependence of Force on Physical Factors

Since the magnitude of the force on a current-carrying conductor is given by

$$F \propto \sin\alpha, \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

where α is the angle between the conductor and the field, it follows that F is zero when the conductor is parallel to the field direction. This defines the direction of the magnetic field. To find which way it points, we can apply Fleming's rule to the case when the conductor is placed at right angles to the field. The sense of the field then corresponds to the direction of the forefinger.

Variation of *F* with *I* and *l*

To investigate how the magnitude of the force F depends on the current I and the length l of the conductor, we may use the apparatus of Fig. 35.11.

Here the conductor AC is situated in the field of a solenoid S. The current flows into, and out of, the wire via the pivot points X and Y. The scale pan T is placed at the same distance from the pivot as the straight wire AC, which is perpendicular to the axis of the coil. The

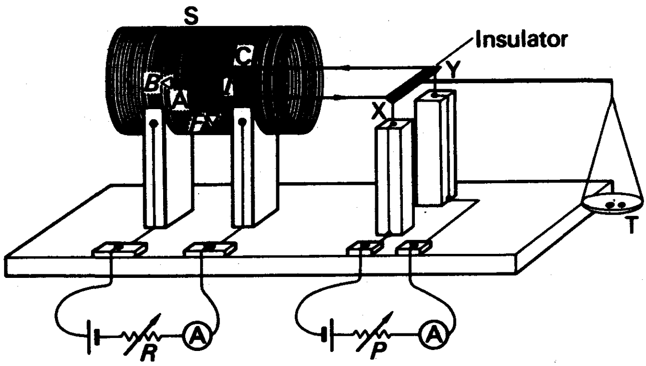


FIG. 35.11. Experiment to show how F varies with I and B.

frame is first balanced with no current flowing in AC. A current is then passed, and the extra weight needed to restore the frame to a horizontal position is equal to the force on the wire AC. By varying the current in AC with the rheostat P, it may be shown that:

$$F \propto I \qquad . \quad . \quad . \quad . \quad (2)$$

If different frames are used so that the length, l, of AC is changed, it can be shown that

$$F \propto l \qquad . \quad . \quad . \quad . \quad (3)$$

Effect of *B*

The magnetic field due to the solenoid will depend on the current flowing in it. If this current is varied by adjusting the rheostat R, it can be shown that the larger the current *in the solenoid*, S, the larger is the force F. It is reasonable to suppose that a larger current in S produces a stronger magnetic field. Thus the force F increases if the magnetic field strength is increased. The magnetic field is represented by a vector quantity which is given the symbol B. This is called the *magnetic induction* or *flux density* in the field. We assume that:

$$F \propto B \qquad . \quad . \quad . \quad . \quad (4)$$

Magnitude of F

From the results expressed in equations (1) to (4), we obtain

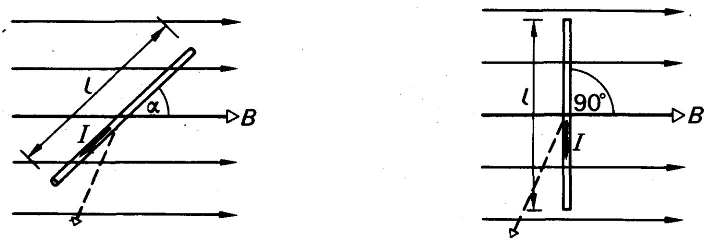
$$F \propto BIl \sin \alpha,$$

or

$$F = \kappa BIl \sin \alpha \quad . \quad . \quad . \quad . \quad (5)$$

where κ is a constant.

In the SI system of units, the unit of B is the tesla (T). 1 T = 1 weber metre^{-2} (Wb m^{-2}). One tesla may be defined as the flux density in a field when the force on a conductor 1 metre long, placed perpendicular



$F = BIl \sin \alpha$ (i) $\qquad$ $F = BIl$ (ii)

FIG. 35.12. Magnitude of F.

to the field and carrying a current of 1 ampere, is 1 newton. Substituting $F = 1$, $B = 1$, $l = 1$ and $\sin \alpha = \sin 90° = 1$ in (5), then $k = 1$. Thus in Fig. 35.12 (i), with the above units,

$$F = BIl \sin \alpha \quad . \quad . \quad . \quad . \quad (5)$$

When the whole length of the conductor is perpendicular to the field B, Fig. 35.12 (ii), then, since $\alpha = 90°$ in this case,

$$F = BIl \quad . \quad . \quad . \quad . \quad (6)$$

It may be noted that the apparatus of Fig. 35.11 can be used to determine the flux density B of the field in the solenoid. In this case, $\alpha = 90°$ and $\sin \alpha = 1$, so that measurement of F, I and l enables B to be found from (6).

EXAMPLE

A wire carrying a current of 10 A and 2 metres in length is placed in a field of flux density 0·15 T (Wb m^{-2}). What is the force on the wire if it is placed (*a*) at right angles to the field, (*b*) at 45° to the field, (*c*) along the field.

From (5) $\qquad F = BIl \sin \alpha$

(*a*) $\qquad \therefore F = 0{\cdot}15 \times 10 \times 2 \times \sin 90°$
$= 3$ newtons.

(*b*) $\qquad \therefore F = 0{\cdot}15 \times 10 \times 2 \times \sin 45°$
$= 2{\cdot}12$ newtons.

(*c*) $\qquad \therefore F = 0$, since $\sin 0° = 0$.

Force on Moving Charges

It was explained in Chapter 32 that an electric current in a wire can be regarded as a drift of electrons in the wire, superimposed on their random thermal motions. If the electrons in the wire drift with average velocity v, and the wire lies at right angles to the field, then the force on *each* electron is given by

$$F = Bev \quad . \quad . \quad . \quad . \quad (1)$$

If B is in tesla (T) or weber metre^{-2} (Wb m^{-2}), e is in coulomb (C) and v in metre second^{-1} (m s^{-1}), then F will be in newtons (N) (Fig. 35.13(i)).

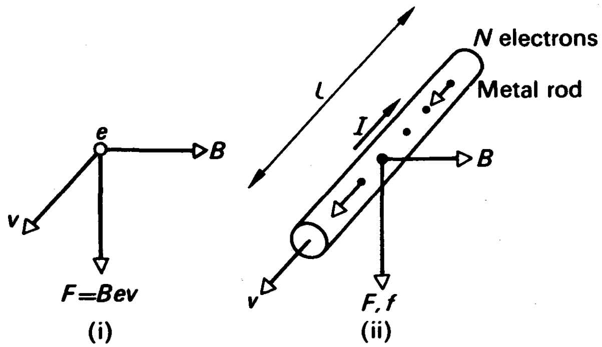


FIG. 35.13. Force on moving electron in magnetic field (v at right angles to page).

The proof of equation (1) is easily obtained. If there are N free electrons in a wire of length l, their total charge is Ne, Fig. 35.13 (ii). If they have a drift velocity v, the time which any one of them takes to travel the distance l is

$$t = \frac{l}{v}.$$

In this time, therefore, the N electrons are swept out of the wire by the current and are replaced by another N electrons. The rate at which charge flows along the wire, which is the current through it, is therefore

$$I = \frac{Ne}{t} = \frac{Nev}{l}.$$

If the wire is at right angles to a magnetic field B, the force on it is

$$f = BIl = BNev.$$

Therefore the force on a single electron is

$$F = \frac{f}{N} = Bev.$$

An electron moving across a magnetic field experiences a force whether it is in a wire or not—for example, it may be one of a beam of electrons in a vacuum tube. Because of this force, a magnetic field can be used to focus or deflect an electron beam, instead of an electrostatic field as on p. 1002. Magnetic deflection and focusing are common in cathode ray tubes used for television.

The Moving-coil Galvanometer

All except the most accurate of current measurements are made today with the moving-coil galvanometer. In this instrument a coil of fine insulated copper wire, ABDF in Fig. 35.14 (i), hangs in a strong magnetic field. The field is set up between soft iron pole-pieces, NS, attached to a powerful permanent magnet.

The pole-pieces are curved to form parts of a cylinder coaxial with the suspension of the coil. And between them lies a cylindrical core of soft iron, C; it is supported on a brass pin, T in Fig. 35.14(iii), which is placed so that it does not foul the coil. As this figure shows, the magnetic field B is radial to the core and pole-pieces, over the region in which the coil can swing.

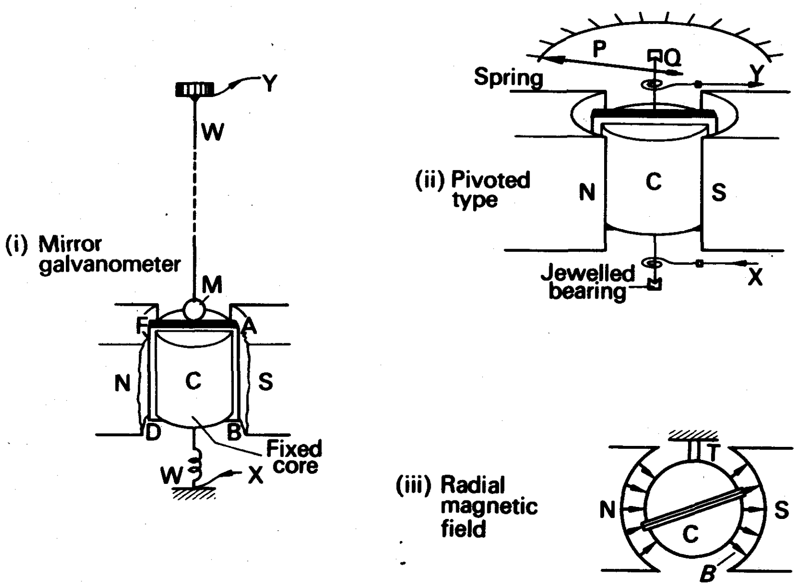


FIG. 35.14. Moving-coil galvanometers.

In the more sensitive instruments, the coil is suspended on a phosphor-bronze wire, WW, which is kept taut. The current is led into and out of the coil through the suspension, at X and Y, and the deflection of the coil is shown by a mirror, M. This is known as a *mirror galvanometer*. More robust but less sensitive forms of the galvanometer have hair-springs and jewelled bearings, instead of the phosphor-bronze suspension (Fig. 35.14 (ii)). The coil is wound on a rigid but light former, of bamboo or aluminium, which also carries the pivots. The pivots are insulated from the former if it is aluminium, and the current is led in and out through the springs. The framework, which carries the springs and jewels, is made from brass or aluminium—if it were steel it would affect the magnetic field. An aluminium pointer, P, shows the deflection of the coil; it is balanced by a counterweight, Q.

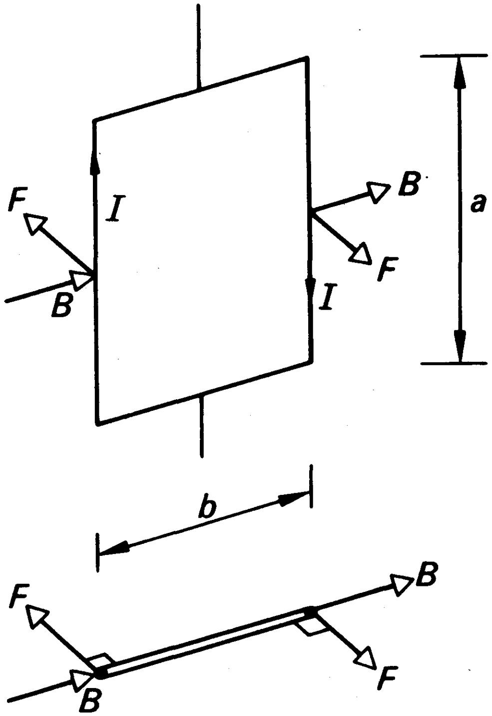


FIG. 35.15. Couple on coil in radial field.

Theory of Moving-coil Instrument

Fig. 35.15 shows a current I flowing through the coil of a moving-coil instrument. The magnetic field exerts forces F on the vertical limbs of each turn of the coil. These forces tend to rotate the coil about the suspension. Since the magnetic field B is at right angles to the vertical limbs, the forces are given by

$$F = BIl,$$

where l is the height of the coil; F acts at right angles to its plane because the magnetic field is radial. Therefore, if b is the width of the coil, the couple which the forces F exert on each turn is:

$$C = Fb = BIlb = BIA,$$

If the coil has N turns, then the total couple which the current exerts on it is

$$C' = NC = BIAN \quad . \quad . \quad . \quad (1)$$

The coil will turn until the restoring couple due to the twist in the suspension is equal to C'. This couple is proportional to the twist θ, which is also the deflection of the coil; thus

$$C' = k\theta,$$

where k is a constant of the suspension. Hence

$$BIAN = k\theta$$

or

$$I = \frac{k}{BAN}\theta. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Equation (2) shows that the deflection is proportional to the current. The pointer type of instrument (Fig. 35.15 (ii)) usually has a scale calibrated directly in milliamperes or microamperes. Full-scale reading on such an instrument corresponds to a deflection θ of 90° to 120°; it may represent a current of 50 microamperes to 15 milliamperes, according to the strength of the hair-springs, the geometry of the coil, and the strength of the magnetic field. The less sensitive models are more accurate, because their pivots and springs are more robust, and therefore are less affected by dust, vibration, and hard use. Models known as 'first grade' (FG) have an error not greater than 1 per cent of full-scale deflection. This error is constant over the scale, so that the inaccuracy may be 10 per cent when the reading is one-tenth of full scale. Readings less than about half full scale must therefore be avoided in accurate work. The best moving-coil instruments have an error of about 0·2 per cent of full-scale deflection.

When a galvanometer is of the suspended-coil type (Fig. 35.15 (i)), its sensitivity is generally expressed in terms of the displacement of the spot of light reflected from the mirror on to the scale. At a scale distance of 1 metre a moderately sensitive instrument of this type will give a deflection of 100 cm per microampere.

All forms of moving-coil galvanometer have one disadvantage: they are easily damaged by overload. A current much greater than that which the instrument is intended to measure will burn out its hair-springs or suspension.

Couple on a Coil in a Uniform Field

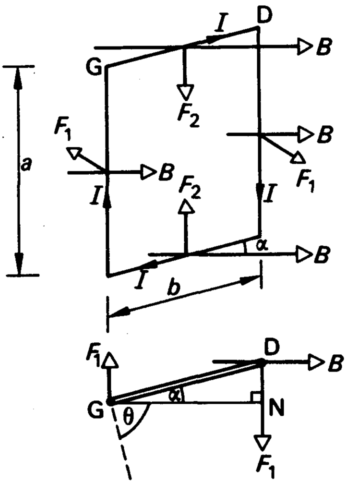


FIG. 35.16. Couple on coil in uniform field.

So far we have considered a coil in a radial magnetic field. Now let us consider one in a uniform field. Fig. 35.16 shows a rectangular coil of one turn, whose plane makes an angle α with a uniform magnetic field B. If it carries a current I amperes, the forces F_1 on its vertical limbs are given by

$$F_1 = BIl = BIa,$$

where a is the height of the coil. And the forces F_2 on its horizontal limbs are given by

$$F_2 = BIb \sin \alpha,$$

where b is the width of the coil. The forces F_2 merely compress the coil, and are resisted by its rigidity. The forces F_1 set up a couple, whose moment is, from the lower figure,

$$C = F_1 \times \mathrm{GN}$$

$$= F_1 b \cos \alpha.$$

Hence $$C = BIab \cos \alpha$$

$$= BIA \cos \alpha, \quad . \quad . \quad . \quad . \quad (1)$$

where A is the area of the coil. If θ is the angle between the field and the normal to the plane of the coil, then $\theta = 90° - \alpha$, and

$$C = BIA \sin \theta. \quad . \quad . \quad . \quad . \quad (2)$$

With a coil of N turns,

$$C = BNIA \quad . \quad . \quad . \quad . \quad (3)$$

Magnetic Moment

The couple on the coil thus depends on the magnitude of the flux density B, the current I in it and the Area A. We can write equation (3) as

$$C = mB \sin \theta \quad . \quad . \quad . \quad . \quad (1)$$

where $m = NIA$. m is a property of the coil and the current in it and is called the *magnetic moment* of the coil. (Fig. 35.17 (i)). In general, the magnetic moment of a coil is defined as the couple exerted on it when it is placed with its plane parallel to a field of unit induction. In this case $B = 1$, $\sin \theta = \sin 90° = 1$ and hence $C = m$ (Fig. 35.17 (ii)). It should be noted that the magnetic moment of a coil can be calculated from

$$m = NIA \quad . \quad . \quad . \quad . \quad (2)$$

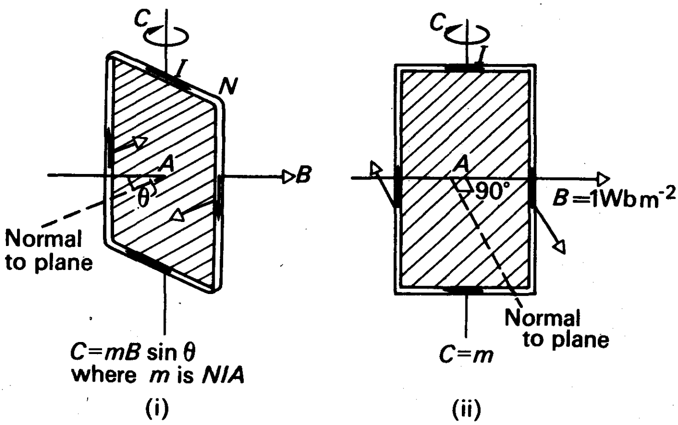


FIG. 35.17. Magnetic moment and couple.

whatever the shape of the coil. From this expression the unit of m is ampere metre^2 (A m^2).

EXAMPLE

What couple is needed to hold a small single-turn coil of area 5 cm^2 in equilibrium, when it carries a current of 10 A and is placed with its axis at right angles to a field of flux density 0·15 T? What couple would be required if the coil had 20 turns?

Area $A = 5\ \text{cm}^2 = 5\times 10^{-4}\ \text{m}^2$.

$\therefore$ Magnetic moment of single-turn coil $= m = IA$
$= 10\times(5\times 10^{-5})\ \text{A m}^2$.

$$\therefore C = mB\sin\theta$$
$$= (10\times 5\times 10^{-5})\times 0{\cdot}15\times \sin 90^\circ$$
$$= 0{\cdot}75\times 10^{-4}\ \text{newton metre (N m)}.$$

If the coil has 20 turns, each will have the same magnetic moment. Hence

$$\text{total couple} = 20\times 0{\cdot}75\times 10^{-4}$$
$$= 1{\cdot}5\times 10^{-3}\ \text{N m}.$$

Vibration Magnetometer

Magnetism is due to circulating and spinning electrons inside atoms (see p. 948). The moving charges are equivalent to electric currents. Consequently, like a current-carrying coil, permanent magnets also have a couple acting on them when they are placed with their axis at an angle to a magnetic field. Like the coil, they turn and settle in equilibrium with their axis along the field direction.

Consider a magnet suspended from a torsionless silk thread in a draught-shield. If it is gently disturbed, for example by momentarily bringing a piece of iron towards it, the magnet *vibrates* about the

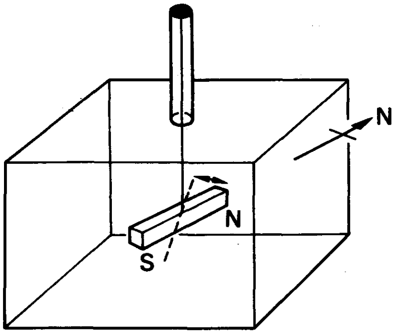


FIG. 35.18. Oscillation of bar-magnet.

magnetic meridian (Fig. 35.18).

The oscillation occurs because, when the magnet is deflected from the meridian, a couple acts on it, tending to bring it back into the meridian. On p. 885 it was shown that the couple had a moment $mB \sin \theta$, where m is the moment of the magnet, B is the field intensity, and θ is the angle between the magnet and the field. If the angular displacement, θ, is small—say not more than 10°—then, to a close approximation, $\sin \theta = \theta$ in radians. The restoring couple C is then given by

$$C = mB\theta. \quad . \quad . \quad . \quad . \quad (1)$$

Thus the restoring couple is proportional to the angular displacement θ, and the magnet therefore makes simple harmonic vibrations (see p. 89).

Period of Vibration

The period of vibration, T, depends on the restoring couple per unit deflection of the magnet; from (1),

$$\frac{C}{\theta} = mB.$$

The period also depends on the moment of inertia K of the magnet, which is determined by its mass, size, and shape. Now from equation (1), p. 89,

$$T = 2\pi\sqrt{\frac{K}{\text{restoring couple per unit deflection}}},$$

$$\therefore T = 2\pi\sqrt{\frac{K}{mB}}. \quad . \quad . \quad . \quad (1)$$

The number of vibrations made per second, n, or the *frequency* of the vibration, is given by

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{mB}{K}},$$

$$\therefore n^2 = \frac{mB}{4\pi^2 K},$$

or

$$B \propto n^2. \quad . \quad . \quad . \quad . \quad (2)$$

This relationship shows that a vibrating magnet may be used to compare magnetic flux densities, for example, to explore the earth's field over the laboratory. When used in this way it is called a *vibration magnetometer*.

It should be noted that the relations (1) and (2) are true only when the fibre suspending the magnet is torsionless, that is, it exerts a negligible restoring couple when it is twisted. The restoring couple on the magnet is then simply that due to the magnetic field ($mB \sin \theta$), as already assumed. A common material used for the suspension is unspun silk. The magnet must be small so that the whole of it is in the uniform field, and it must be protected from draughts, which would upset the oscillation.

Comparison of Flux Densities

By means of a vibration magnetometer, the values of B in two magnetic fields can easily be compared (see also p. 892). The magnetometer is set up in one field of magnitude B_1 say and the time period T_1 of small swings is measured. The new time period T_2 is then measured in the other field of magnitude B_2. Then, from previous,

$$T_1 = 2\pi\sqrt{\frac{K}{mB_1}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{K}{mB_2}}$$

Dividing the two equations,

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{B_2}{B_1}}, \quad \text{or} \quad \frac{B_1}{B_2} = \frac{T_2^2}{T_1^2}.$$

Hence the flux densities in the two fields may be compared.

Comparison of Moments

The method of vibration can also be used to compare the magnetic moments of two magnets. The magnets are rigidly fixed together, with similar poles adjacent, and allowed to vibrate in the earth's field (Fig. 35.19 (i)). If m_1, m_2, K_1, K_2, are their magnetic moments and moments

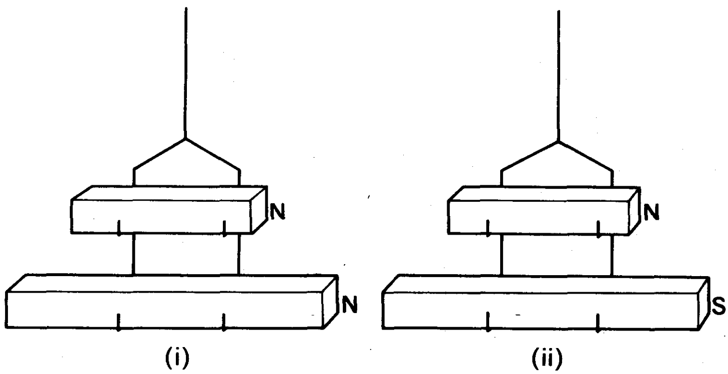


FIG. 35.19. Comparison of moments by vibration.

of inertia respectively, and if B_H is the earth's horizontal field, then the period of the magnets is

$$T_1 = 2\pi\sqrt{\frac{K_1+K_2}{(m_1+m_2)B_H}}.$$

The magnets are now fixed together with opposite poles adjacent, and again allowed to vibrate. Fig. 35.19 (ii). Their period is now

$$T_2 = 2\pi\sqrt{\frac{K_1+K_2}{(m_1-m_2)B_H}}.$$

Therefore

$$\frac{T_2}{T_1} = \sqrt{\frac{m_1+m_2}{m_1-m_2}},$$

whence

$$\frac{m_1}{m_2} = \frac{T_2^2+T_1^2}{T_2^2-T_1^2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The Wattmeter

The wattmeter is an instrument for measuring electrical power. In construction and appearance it resembles a moving-coil voltmeter or ammeter, but it has no permanent magnet. Instead it has two fixed coils, FF in Fig. 35.20; these set up the magnetic field in which the suspended coil, M, moves. When the instrument is in use, the coils FF are connected in series with the device X whose power consumption is

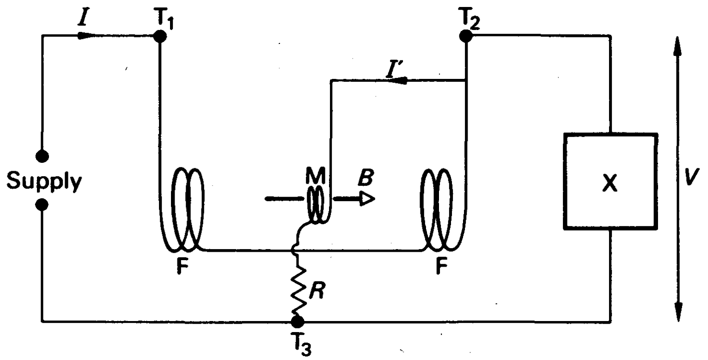


FIG. 35.20. Principle of wattmeter.

to be measured. The magnetic field B, set up by FF, is then proportional to the current I drawn by X:

$$B \propto I.$$

The moving coil M is connected across the device X. In series with M is a high resistance R, similar to the multiplier of a voltmeter; M is, indeed, often called the volt-coil. The current I' through the volt-coil is small compared with the main current I, and is proportional to the potential difference V across the device X:

$$I' \propto V.$$

The couple acting on the moving coil is proportional to the current through it, and to the magnetic field in which it is placed:

$$C \propto BI'.$$

Consequently $$C \propto IV.$$

That is to say, the couple on the coil is proportional to the product of the current through the device X, and the voltage across it. The couple is therefore proportional to the power consumed by X, and the power can be measured by the deflection of the coil.

The diagram shows that, because the volt-coil draws current, the current through the fixed coils is a little greater than the current through X. As a rule, the error arising from this is negligible; if not, it can be allowed for as when a voltmeter and ammeter are used separately.

Hall Effect

In 1879, Hall found that an e.m.f. is set up *transversely* or *across* a current-carrying conductor when a perpendicular magnetic field is applied. This is called the *Hall effect.*

To explain the Hall effect, consider a slab of metal carrying a current (Fig. 35.21). The flow of electrons is in the opposite direction to the conventional current. If the metal is placed in a magnetic field B at

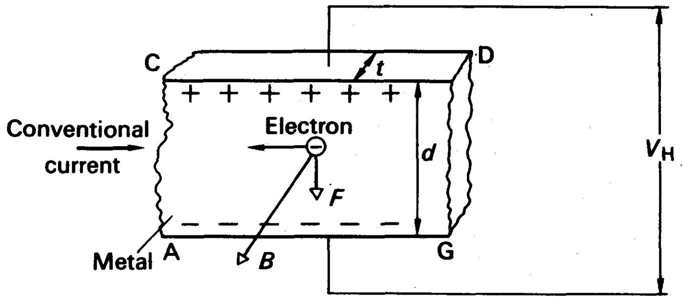


FIG. 35.21. Hall voltage.

right angles to the face AGDC of the slab and directed out of the plane of the paper, a force Bev then acts on each electron in the direction from CD to AG. Thus electrons accumulate along the side AG of the metal, which will make AG negatively charged and lower its potential with respect to CD. Thus a potential difference or e.m.f. opposes the electron flow. The flow ceases when the e.m.f. reaches a particular value V_H called the *Hall voltage*, which may be measured by using a high impedance voltmeter as shown in Fig. 35.22.

Magnitude of Hall Voltage

Suppose V_H is the magnitude of the Hall voltage and d is the width of the slab. Then the electric field intensity E set up across the slab is numerically equal to the potential gradient and hence $E = V_H/d$. Hence the force on each electron $= Ee = V_He/d$.

This force, which is directed upwards from AG to CD, is equal to the force produced by the magnetic field when the electrons are in equilibrium.

$$\therefore Ee = Bev$$

$$\therefore \frac{V_He}{d} = Bev$$

$$\therefore V_H = Bvd \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From p. 787, the drift velocity of the electrons is given by

$$I = NevA, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where N is the number of electrons per unit volume and A is the area of cross section of the conductor. In this case $A = td$ where t is the thickness. Hence, from (2),

$$v = \frac{I}{Netd}$$

Substituting in (1),

$$\therefore V_H = \frac{BI}{Net} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We now take some typical values for copper to see the order of magnitude of V_H. Suppose $B = 1$ T, a field obtained by using a large laboratory electromagnet. For copper, $N \simeq 10^{29}$ electrons per metre^3, and the charge on the electron is $1{\cdot}6 \times 10^{-19}$ coulomb. Suppose the specimen carries a current of 10 A and that its thickness is about 1 mm or 10^{-3} m. Then

$$V_H = \frac{1 \times 10}{10^{29} \times 1{\cdot}6 \times 10^{-19} \times 10^{-3}} = 0{\cdot}6\ \mu\text{V (approx.)}.$$

This e.m.f. is very small and would be difficult to measure. The importance of the Hall effect becomes apparent when semiconductors are used, as we now see.

Hall Effect in Semiconductors

In semiconductors, the charge carriers which produce a current when they move may be positively or negatively charged (see p. 1024). The Hall effect helps us to find the sign of the charge carried. In Fig. 35.21, p. 890, suppose that electrons were not responsible for carrying the current, and that the current was due to the movement of positive charges in the *same* direction as the conventional current. The magnetic force on these charges would also be downwards, in the same direction as if the current were carried by electrons. This is because the sign *and* the direction of movement of the charge carriers have both been reversed. Thus AB would now become positively charged, and the polarity of the Hall voltage would be reversed. Experimental investigation of the polarity of the Hall voltage hence tells us whether the current is predominantly due to the drift of positive charges or to the drift of negative charges. In this way it was shown that the current in a metal such as copper is due to movement of negative charges, but that in impure semiconductors such as germanium or silicon, the current may be predominantly due to movement of either negative or positive charges (p. 945).

The magnitude of the Hall voltage V_H in metals was shown on p. to be very small. In semiconductors it is much larger because the number N of charge carriers per metre^3 is much *less* than in a metal and $V_H = BI/Net$. Suppose that N is about 10^{25} per metre^3 in a semiconductor, and $B = 1$ T (Wb m^{-2}), $t = 10^{-3}$ m, $e = 1{\cdot}6 \times 10^{-19}$ C, as on p. 891. Then

$$V_H = \frac{1 \times 10}{10^{25} \times 1{\cdot}6 \times 10^{-19} \times 10^{-3}} = 6 \times 10^{-3} \text{ V (approx.)} = 6 \text{ mV}.$$

The Hall voltage is thus much more measurable in semiconductors than in metals.

Use of Hall Effect

An instrument called a *Hall probe* may now be used to measure the flux density B of a magnetic field. A simple Hall probe is shown in Fig. 35.22. Here a wafer of semiconductor has two contacts on opposite

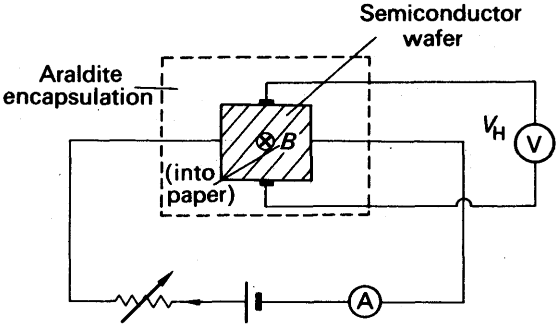


FIG. 35.22. *B* by Hall voltage.

sides which are connected to a high impedance voltmeter, V. A current, generally less than one ampere, is passed through the semiconductor and is measured on the ammeter, A. The 'araldite' encapsulation prevents the wires from being detached from the wafer. Now, from (3) on p. 891.

$$V_H = \frac{BI}{Net}$$

$$\therefore B = \frac{V_H Net}{I}$$

Now Net is a constant for the given semiconductor, which can be determined previously. Thus from the measurement of V_H and I, B can be found.

EXERCISES 35

1. With the aid of clear labelled diagrams describe the structure of a suspended moving-coil galvanometer and explain its mode of action. Derive an expression for its current sensitivity.

A certain galvanometer when shunted with a resistance 0·0500 ohm gives a full-scale deflection for 2 A, while if placed in series with a resistance 495·05 ohms it gives a full-scale deflection for 10 volts. Deduce the resistance of the galvanometer and the current required to produce a full-scale deflection when it is used alone. (*L.*)

2. A rectangular coil of wire of n turns and area A is suspended at the mid-point of one side by a fibre of torsional constant c so that its plane is parallel to a horizontal uniform field of magnetic induction (flux density) B. Derive an expression for the deflection of the coil when a steady current I flows through it.

Explain, with reasons, how a modern moving coil galvanometer of the suspended type has been developed from this simple arrangement.

Distinguish between the accuracy and sensitivity of such a galvanometer and explain the factors on which the sensitivity depends. (*N.*)

3. Describe a moving-coil type of galvanometer and deduce a relation between its deflection and the steady current passing through it.

A galvanometer, with a scale divided into 150 equal divisions, has a current sensitivity of 10 divisions per milliampere and a voltage sensitivity of 2 divisions per millivolt. How can the instrument be adapted to serve (*a*) as an ammeter reading to 6 A, (*b*) as a voltmeter in which each division represents 1 volt? (*L.*)

4. Describe an experiment to show that a force is exerted on a conductor carrying a current when it is placed in a magnetic field. Give a diagram showing the directions of the current, the field, and the force.

A rectangular coil of 50 turns hangs vertically in a uniform magnetic field of 10^{-2} T (Wb m^{-2}), so that the plane of the coil is parallel to the field. The mean height of the coil is 5 cm and its mean width 2 cm. Calculate the strength of the current that must pass through the coil in order to deflect it 30° if the torsional constant of the suspension is 10^{-9} newton metre per degree. Give a labelled diagram of a moving-coil galvanometer. (*L.*)

5. Explain the terms *magnetic flux*, *magnetic flux density* (*magnetic induction*), *magnetic flux linkage*.

Write down an expression for the force experienced by a long straight wire of length L carrying a current I when it is situated at right angles to a uniform field of flux-density B. State the units in which each of the quantities in the expression is measured, and show with the aid of a diagram the directions of the current, the magnetic field and the force.

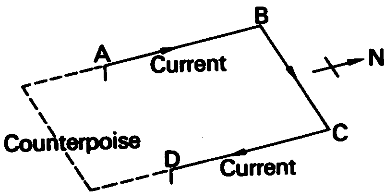


FIG. 35.23.

Fig. 35.23 represents a rigid rectangular wire frame with three members AB = 30 cm, BC = 60 cm, and CD = 30 cm. It is pivoted at the contacts A and D and suitably counterpoised. When BC is at right angles to the magnetic meridian and a current of 5 A passes along ABCD as shown, the frame is found to be in equilibrium when its plane is horizontal. Find the magnitude and direction of the force on each of the members AB, BC, and CD, and also the moment of the deflecting couple which results. (Take the flux density of the earth's magnetic field to be 6×10^{-5} T (Wb m^{-2}) and the angle of dip to be 70°.)

State, and explain briefly without attempting any numerical calculation, what would be obtained (*a*) if the current through the frame were reversed, (*b*) if the whole system were turned through 90° about a vertical axis, the counterpoising couple in each case remaining the same as before. (*O.*)

6. Define *magnetic moment.*

A magnetized uniform rod of length 0·15 m is pivoted about a horizontal axis passing through its centre of mass. The axis of rotation is perpendicular to a uniform magnetic field of 64 A m^{-1} inclined at 60° to the horizontal. When a small mass of 0·2 g is fixed to one end of the rod, the rod sets horizontally. Draw a diagram to show the couples acting on the rod in this position. Calculate the magnetic moment of the rod. (*N.*)

7. Draw a clear labelled diagram showing the essential features of a moving-coil ballistic galvanometer. What differences in structure would be necessary and for what reasons, to make the galvanometer dead beat?

Describe, with a circuit diagram, how a ballistic galvanometer may be used to compare the capacitances of two capacitors and state the precautions necessary to obtain a reliable result. (*L.*)

8. Describe the construction of a sensitive moving-coil galvanometer. How could the instrument be adapted for use a millivoltmeter?

A standard cell of e.m.f. 1·018 volts and internal resistance 1000 ohms is joined to two resistances in series of values 149000 and 2 ohms respectively. The ends of the 2-ohm resistance are also connected to the terminals of a galvanometer of resistance 8 ohms, when a scale deflection of 100 mm is recorded. What is the sensitivity of the instrument expressed in microamperes per scale division? (*L.*)

9. With the help of a labelled diagram or diagrams describe the construction and explain the action of a pivoted moving-coil galvanometer. Indicate on diagrams how the direction of deflection is related to the direction of current flow and the polarity of the magnet.

If such an instrument has a resistance of 10 ohms and gives a full-scale deflection when a current of 20 milliamps is passing, how would it be converted into a voltmeter with 3- and 150-volt ranges? What reading would this voltmeter give when connected to a battery of e.m.f. 120 volts and internal resistance 300 ohms? (*L.*)

chapter thirty-six

Electromagnetic Induction

Faraday's Discovery

AFTER Ampere and others had investigated the magnetic effect of a current Faraday attempted to find its converse: he tried to produce a current by means of a magnetic field. He began work on the problem in 1825 but did not succeed until 1831.

The apparatus with which he worked is represented in Fig. 36.1; it consists of two coils of insulated wire, A, B, wound on a wooden core.

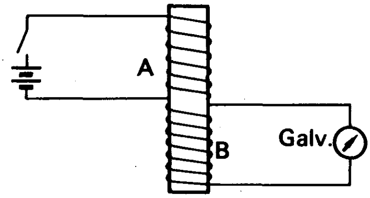


FIG. 36.1. Faraday's experiment on induction.

One coil was connected to a galvanometer, and the other to a battery. No current flowed through the galvanometer, as in all Faraday's previous attempts. But when he disconnected the battery Faraday happened to notice that the galvanometer needle gave a kick. And when he connected the battery back again, he noticed a kick in the opposite direction. However often he disconnected and reconnected the battery, he got the same results. The 'kicks' could hardly be all accidental—they must indicate momentary currents. Faraday had been looking for a steady current, but the effect he sought turned out to be a transient one—that was why it took him six years to find it.

Conditions for Generation of Induced Current

The results of Faraday's experiments showed that a current flowed in coil B of Fig. 36.1 only while the magnetic field due to coil A was changing—the field building up as the current in A was switched on, decaying as the current in A was switched off. And the current which flowed in B while the field was decaying was in the opposite direction to the current which flowed while the field was building up. Faraday called the current in B an induced current. He found that it could be made much greater by winding the two coils on an iron core, instead of a wooden one.

Once he had realized that an induced current was produced only by a change in the magnetic field inducing it, Faraday was able to find induced currents wherever he had previously sought them. In place of the coil A he used a magnet, and showed that as long as the coil and the magnet were at rest, there was no induced current (Fig. 36.2 (i)). But when he moved either the coil or the magnet an induced current flowed as long as the motion continued (Fig. 36.2 (ii)). If the current flowed one way when the north pole of the magnet was approaching

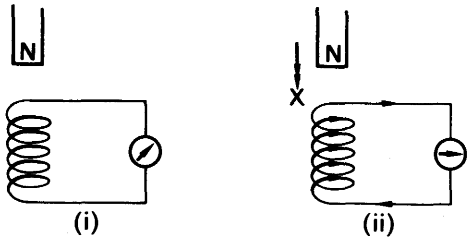


FIG. 36.2. Induction of current by moving magnet.

the end X of the coil, it flowed the other way when the north pole was retreating from X, or the south pole approached X.

Since a flow of current implies the presence of an e.m.f., Faraday's experiments showed that an e.m.f. could be induced in a coil by moving it relatively to a magnetic field. In discussing induction it is more fundamental to deal with the e.m.f. than the current, because the current depends on both the e.m.f. and the resistance.

Direction of E.M.F.; Lenz's Law

Before considering the magnitude of an induced e.m.f., let us investigate its direction. To do so we must first see which way the galvanometer deflects when a current passes through it in a known direction: we can find this out with a battery and a megohm resistor (Fig. 36.3 (i)). We then take a coil whose direction of winding we know, and connect this to the galvanometer. In turn we plunge each pole of a magnet into and out of the coil; and we get the results shown in Fig. 36.3 (ii), (iii), (iv). These results were generalized most elegantly into a rule by Lenz in 1835. He said that *the induced current flows always in such a direction as to oppose the change which is giving rise to it.* If the reader will sketch with a pencil on Fig. 36.3 the magnetic fields of the induced currents, then he will see what Lenz meant: when the magnet is approaching the coil, the coil repels it; when the magnet is retreating from the coil, the coil attracts it.

Lenz's law is a beautiful example of the conservation of energy: the induced current sets up a force on the magnet, which the mover of the magnet must overcome: the work done in overcoming this force provides the electrical energy of the current. (This energy is dissipated as heat in the coil.) If the induced current flowed in the opposite direction to

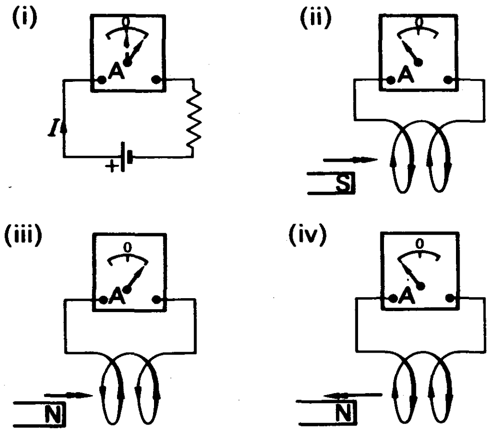


FIG. 36.3. Direction of induced currents.

that which it actually takes, then it would aid—it would speed up—the motion of the magnet. It would enhance its own cause, and grow indefinitely; at the same time, it would continuously increase the kinetic energy of the magnet. Thus both mechanical and electrical energy would be produced, without any agent having to do work. The system would be a perpetual motion machine.

The direction of the induced e.m.f., E, is specified by that of the current, as in Fig. 36.4. If we wished to reword Lenz's law, substituting e.m.f. for current, we would have to speak of the e.m.f.s *tending* to oppose the change... etc., because there can be no opposing force unless the circuit is closed and a current can flow.

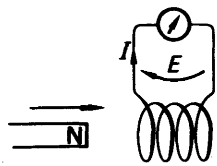


FIG. 36.4. Direction of induced e.m.f.

Magnitude of E.M.F.

Accurate experiments on induction are difficult to contrive with simple apparatus; but rough-and-ready experiments will show on what factors the magnitude depends. We require coils of the same diameter but different numbers of turns, coils of the same number of turns but different diameters, and two similar magnets, which we can use singly or together. If we use a high-resistance galvanometer, the current will not vary much with the resistance of the coil in which the e.m.f. is induced, and we can take the deflection as a measure of the e.m.f. There is no need to plunge the magnet into and out of the coil: we can

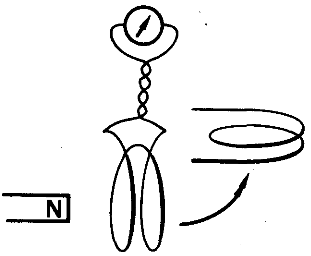


FIG. 36.5. E.m.f. induced by turning coil.

get just as great a deflection by simply turning the coil through a right angle, so that its plane changes from parallel to perpendicular to the magnet, or vice versa (Fig. 36.5). We find that the induced e.m.f. increases with:

(i) the speed with which we turn the coil;
(ii) the area of the coil;
(iii) the strength of the magnetic field (two magnets give a greater e.m.f. than one);
(iv) the number of turns in the coil.

To generalize these results and to build up useful formulae, we use the idea of *magnetic flux*, or field lines passing through a coil. Fig. 36.6 shows a coil, of area A, whose normal makes an angle θ with a uniform magnetic field of induction B. The component of the field at right angles to the plane of the coil is $B\cos\theta$, and we say that the magnetic flux Φ through the coil is

$$\Phi = AB\cos\theta \quad . \quad . \quad . \quad . \quad (1)$$

(We get the same result if we multiply the field-strength B by the area projected at right angles to the field, $A\cos\theta$.) If either the strength of the field is changed, or the coil is turned so as to change the angle θ, then the flux through the coil changes.

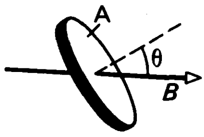


FIG. 36.6. Magnetic flux.

Results (i) to (iii) above, therefore, show that the e.m.f. induced in a coil increases with the *rate of change of the magnetic flux* through it. More accurate experiments show that the induced e.m.f. is actually proportional to the rate of change of flux through the coil; this result is sometimes called *Faraday's*, or *Neumann's*, *law*.

The unit of magnetic flux Φ is called the *weber* (Wb). Hence the unit of B is the *weber per metre*^{2} (Wb m^{-2}) or *tesla* (T).

Flux Linkage

If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. We call this the *flux linkage* through the whole coil. If the magnetic field is uniform, the flux through one turn is given, from (1), by $AB\cos\theta$. If the coil has N turns, the total flux linkage Φ is given by

$$\Phi = NAB\cos\theta \quad . \quad . \quad . \quad . \quad (2)$$

From Faraday's or Neumann's law, the e.m.f. induced in a coil is proportional to the rate of change of the flux linkage, Φ. Hence

$$E \propto \frac{d\Phi}{dt},$$

or

$$E = -k\frac{d\Phi}{dt}, \qquad (3)$$

where k is a positive constant. The minus sign expresses Lenz's law. It means that the induced e.m.f. is in such a direction that, if the circuit is closed, the induced current *opposes* the change of flux. Note that an induced e.m.f. exists across the terminals of a coil when the flux linkage changes, even though the coil is on 'open circuit'. A current, of course, does not flow in the latter case.

On p. 901, it is shown that $E = -kd\Phi/dt$ is consistent with the expression $F = BIl$ for the force on a conductor only if $k = 1$. We may therefore say that

$$E = -\frac{d\Phi}{dt}, \qquad (4)$$

where Φ is the flux linkage in webers, t is in seconds, and E is in volts.

From (4), it follows that one weber is the flux linking a circuit if the induced e.m.f. is one volt when the flux is reduced uniformly to zero in one second.

E.M.F. Induced in Moving Rod

Generators at power stations produce high induced voltages by rotating long *straight conductors*. Fig. 36.7 (i) shows a simple apparatus

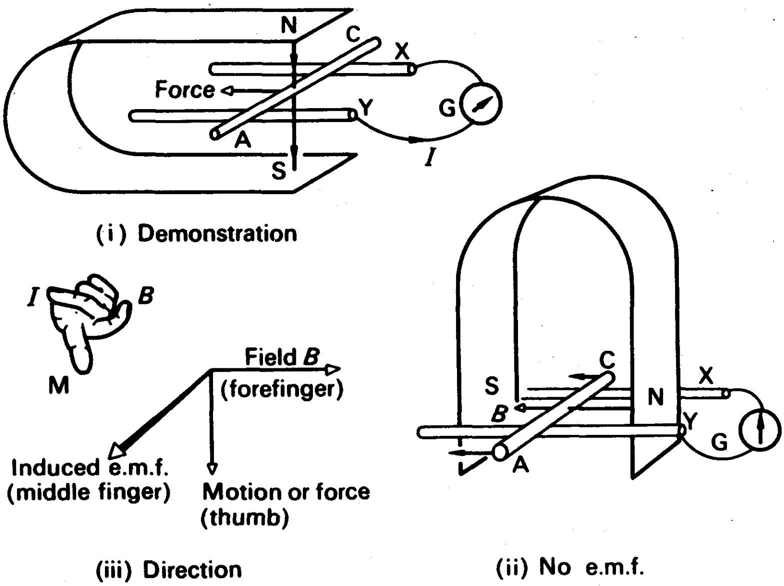


FIG. 36.7. E.m.f. induced in moving rod.

for demonstrating that an e.m.f. may be induced in a straight rod or wire, when it is moved across a magnetic field. The apparatus consists of a rod AC resting on rails XY, and lying between the poles NS of a permanent magnet. The rails are connected to a galvanometer G.

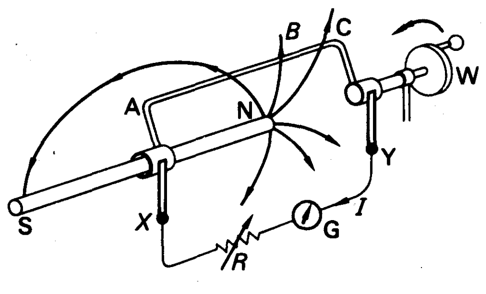
If we move the rod to the left, so that it cuts across the field B of the magnet, a current I flows as shown. If we move the rod to the right, the current reverses. We notice that the current flows only while the rod is moving, and we conclude that the motion of the rod AC induces an e.m.f. in it.

By turning the magnet into a vertical position (Fig. 36.7 (ii)) we can show that no e.m.f. is induced in the rod when it moves parallel to the field B. We conclude that an e.m.f. is induced in the rod only when it *cuts across* the field. And, whatever the direction of the field, no e.m.f. is induced when we slide the rod parallel to its own length. The induced e.m.f. is greatest when we move the rod at right angles, both to its own length and to the magnetic field. These results may be summarized in *Fleming's right-hand rule*:

If we extend the thumb and first two fingers of the right hand, so that they are all at right angles to one another, then the directions of field, motion, and induced e.m.f. are related as in Fig. 36.7 (iii).

To show E.M.F. $\propto$ Rate of Change

The variation of the magnitude of the e.m.f. in a rod with the speed of 'cutting' magnetic flux can be demonstrated with the apparatus in Fig. 36.8 (i).



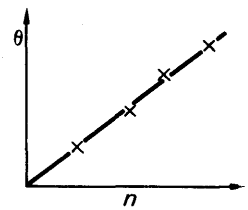


FIG. 36.8. Induced e.m.f.

Here AC is a copper rod, which can be rotated by a wheel W around one pole N of a long magnet. Brushing contacts at X and Y connect the rod to a galvanometer G and a series resistance R. When we turn the wheel, the rod AC cuts across the field B of the magnet, and an e.m.f. is induced in it. If we turn the wheel steadily, the galvanometer gives a steady deflection, showing that a steady current is flowing round the circuit.

To find how the current and e.m.f. depends on the speed of the rod, we keep the circuit resistance constant, and vary the rate at which we turn the wheel. We time the revolutions with a stop-watch, and find that the deflection θ is proportional to the number of revolutions per second, n (Fig. 36.8 (ii)). It follows that the induced e.m.f. is proportional to the speed of the rod.

Calculation of E.M.F. in Rod

Consider the circuit shown in Fig. 36.9. PQ is a straight wire touching the two connected parallel wires QR, PS and free to move over them. All the conductors are situated in a uniform magnetic field of induction B, perpendicular to the plane of PQRS.

Suppose the rod PQ is pulled with a uniform velocity v by an external force F. There will then be a change of flux linkage in the area PQRS and so an e.m.f. will be induced in the circuit. This produces a current I which flows round the circuit. A *force* will now act on the wire PQ

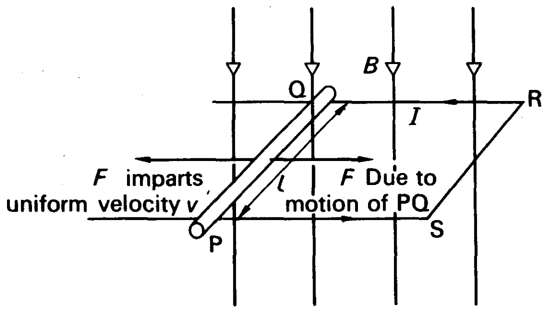


FIG. 36.9 Calculation of induced e.m.f.

due to the current flowing and to the presence of the magnetic field (p. 878). By Lenz's law, the direction of this force will oppose the movement of PQ. If the current flowing is I, and the length of PQ is l, the force on PQ is BIl. This is equal to the external force F, since PQ is not accelerating.

Because energy is conserved, the rate of working by the external force is equal to the rate at which energy is supplied to the electrical circuit. Now in one second, PQ moves a distance v. Hence

$$\text{work done per second} = \text{force} \times \text{distance moved per second.}$$
$$= BIlv$$

If the induced e.m.f. is E, the electrical energy used in one second, or power, $= EI$.

$$\therefore EI = BIlv$$
$$\therefore E = Blv. \qquad (6)$$

This result has been derived without using the relation $E = -d\Phi/dt$. To see if the same result as (6) can be obtained, consider the flux changes. In one second the area of PQRS changes by vl. Hence the change in flux linkage per second, $d\Phi/dt = B \times$ area change per second $= Blv$. Hence, numerically,

$$\therefore E = Blv.$$

This means that the relation $E = -d\Phi/dt$ may be used to find the induced e.m.f. in a straight wire.

Induced E.M.F. and Force on Moving Electrons

We have seen that an electron moving across a magnetic field experiences a mechanical force (p. 881). This explains neatly the e.m.f. induced in a wire: when we move the wire across the field, we move each free electron in it, likewise across the field. As Fig. 36.10 shows, the force on the electrons, F, is at right angles to the plane containing the velocity v of the wire, and the magnetic field B. Thus it tends to drive the electrons along the wire. The direction in which it does so agrees with the direction of the conventional e.m.f., which is the direction of the force on a positive charge.

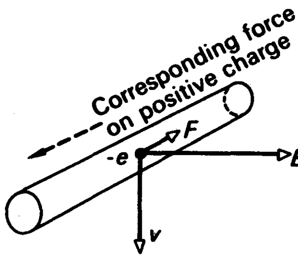


FIG. 36.10. Force on a moving electron.

When a wire AC is swept, as shown in Fig. 36.11, across a magnetic field B, the force on the electrons in it acts from A to C. Therefore, if the wire is not connected to a closed circuit, electrons will pile up at C: the end C will gain a negative

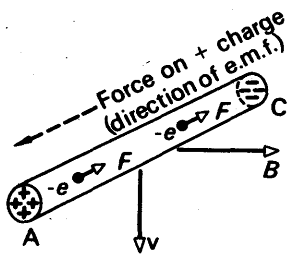


FIG. 36.11. Induced e.m.f. arising from force on moving electrons.

charge, and A will be left with a positive charge. The end A will therefore be at a higher potential than C.

If we now clear our minds of electrons, we see that the conventional e.m.f. acts from C to A. If positive electricity were free to move, it would accumulate at A;

in other words, the tendency of the e.m.f., acting from C to A, is to give A a higher potential than C. The *potential difference* between A and C tends to drive positive electricity the other way. Equilibrium is reached when the potential difference V_{AC} is equal to the e.m.f. acting from C to A.

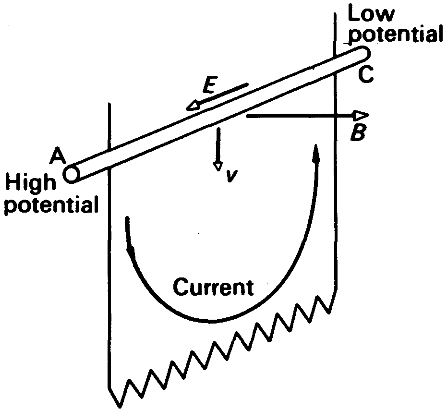


FIG. 36.12. E.m.f. and potential difference.

When the wire is connected to a closed circuit, current flows from A to C round the external circuit (Fig. 36.12). Within the source of current—the wire AC—the e.m.f. drives the current (of positive charge) from C to A: from low potential to high. *This is the essential function of an e.m.f.*; an e.m.f. is an agency which can drive an electric current *against* a potential difference. When the e.m.f. arises in a wire moving across a magnetic field, this agency is the force on the electrons moving with the wire.

The e.m.f. induced in a wire can easily be calculated from the force on a moving electron. If the wire moves with a velocity v at right angles to a field B, then so do the electrons in it. Each of them therefore experiences a force

$$F = Bev,$$

(equation (1), p. 881), where e is the electronic charge. The work which this force does in carrying the electron along the length l of the wire is Fl. But it is also, by definition, equal to the product of the e.m.f. E, and the charge e. Therefore

$$Ee = Fl = Bevl,$$

whence

$$E = Blv.$$

APPLICATIONS OF INDUCTION

The Induction Coil

The induction coil is a device for getting a high voltage from a low one. It was at one time used for X-ray tubes (p. 1067), and is nowadays used in car radios. It consists of a core of iron wires, around which is wrapped a coil of about a hundred turns of thick insulated wire, called the primary (Fig. 36.13 (i)). Around the primary is wound the secondary coil, which has many thousands of turns of fine insulated wire. The primary is connected to a battery of accumulators, via a make-and-break M, which works in the same way as the contact-breaker of an electric bell: it switches the current on and off many times a second, thus varying the magnetic flux.

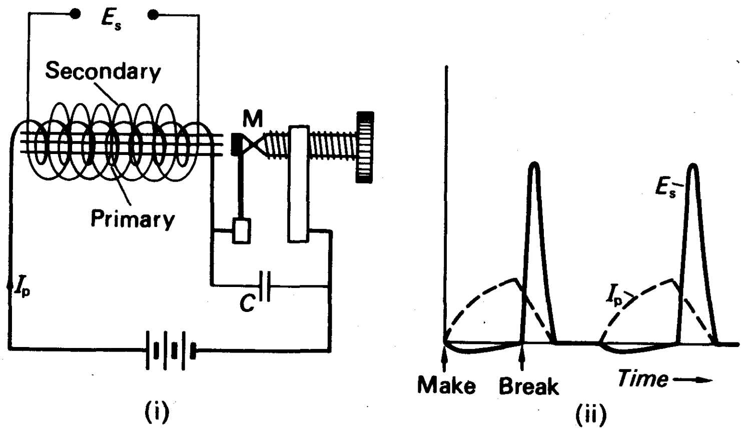


FIG. 36.13. Induction coil.

When the primary current I_p is switched on, the rise of its magnetic field induces an e.m.f. E_s in the secondary. A similar e.m.f., but in the opposite sense, is induced in the secondary when the primary current is switched off, by the collapse of the magnetic field. The secondary e.m.f.s are determined by the number of turns in the secondary coil, and by the rate of change of the magnetic flux through the iron core. Because of the great number of secondary turns, the secondary e.m.f.s may be high and of the order of thousands of volts (Fig. 36.13 (ii)).

In practice, an induction coil such as we have described—consisting simply of primary, secondary, and contact-breaker—would not give high secondary e.m.f.s. For, at the make of the primary current, the current would rise slowly, because of the self-inductance (see p. 924) of the primary winding. The rate of change of flux linked with the secondary would therefore be small, and the secondary e.m.f. low. And at the break of the primary current a spark would pass between the contacts of the make-and-break. The spark would allow primary current to continue to flow, and the primary current would fall slowly. At the instant of break, before the spark began, the primary current would be falling rapidly and the secondary e.m.f. would be high; but the e.m.f. would remain high for only a very short time: as soon as the spark passed the secondary e.m.f. would fall to a value about as low as at make.

Nothing can be done about the low secondary e.m.f. at make. But the secondary e.m.f. at break can be made high, by preventing sparking at the contact-breaker. To prevent sparking, a capacitor, C in Fig. 36.13, is connected across the contacts.

As we shall see on p. 926, the capacitor actually *slows down* the fall of the primary current at the instant of break; but in doing so it prevents the induced e.m.f. in the primary from rising high enough to start a spark. And the rate at which the primary current falls, in charging the capacitor, is greater than the rate at which it would fall if a spark were passing. Thus, with a capacitor, the secondary e.m.f. is less at the instant of break than without one, but it is greater throughout the rest of the fall of the primary current. Consequently the average secondary voltage at break is higher with a capacitor than without; in practice it is much higher. To get the greatest possible secondary voltage, the capacitance of the capacitor is chosen so that it just suppresses sparking at the contacts. The secondary voltage is then a series of almost unidirectional pulses, as shown in Fig. 36.13 (ii).

The iron core of an induction coil is made from a bundle of wires, to minimize eddy-currents (p. 913). If eddy-currents were to flow they would, by Lenz's law, set up a flux opposing the change of primary current. Thus they would reduce the secondary e.m.f.

The Dynamo and Generator

Faraday's discovery of electromagnetic induction was the beginning of electrical engineering. Nearly all the electric current used today is generated by induction, in machines which contain coils moving continuously in a magnetic field. Fig. 36.14 illustrates the principle of such a machine, which is called a dynamo, or generator. A coil DEFG, shown for simplicity as having only one turn, rotates on a shaft, which is not shown, between the poles NS of a horseshoe magnet. The ends of the coil are connected to flat brass rings R, which are supported on the shaft by discs of insulating material, also not shown. Contact with the rings is made by small blocks of carbon B, supported on springs, and shown connected to a lamp L. As the coil rotates, the flux linking it changes, and a current is induced in it which flows, via the carbon blocks, through the lamp.

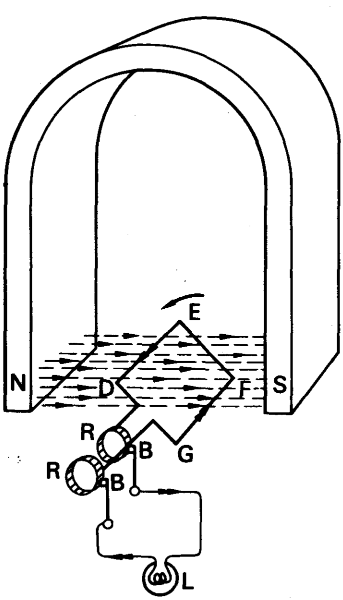


FIG. 36.14. A simple dynamo.

The magnitude (which we study shortly) and the direction of the current are not constant. Thus when the coil is in the position shown, the limb ED is moving downwards through the lines of force, and GF is moving upwards. Half a revolution later, ED and GF will have interchanged their positions, and ED will be moving upwards. Consequently, applying Fleming's right-hand rule, the current round the coil must reverse as ED changes from downward to upward motion. The actual direction of the current at the instant shown on the diagram is indicated by the double arrows, from Fleming's rule. By applying this rule, it can be seen that the current reverses every time the plane of the coil passes the vertical position.

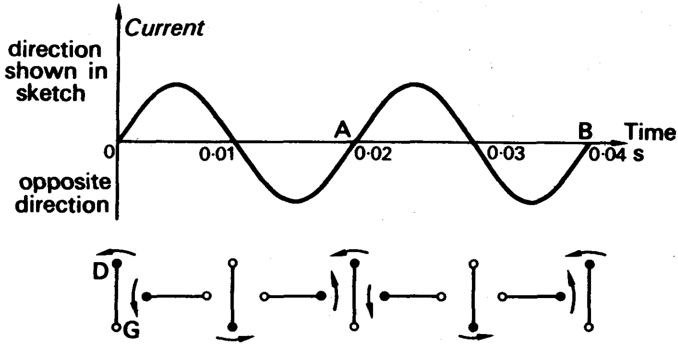


FIG. 36.15. Current generated by dynamo of Fig. 36.14, plotted against time and coil position.

We shall see shortly that the magnitude of the e.m.f. and current varies with time as shown in Fig. 36.15; this diagram also shows the corresponding position of DG. This type of current is called an *alternating current* (*A.C.*). A complete alternation, such as from A to B in the figure, is called a 'cycle'; and the number of cycles which the current goes through in one second is called its 'frequency'. The frequency of the current represented in the figure is that of most domestic supplies in Britain—50 Hz (cycles per second).

E.M.F. in Dynamo

We can now calculate the e.m.f. in the rotating coil. If the coil has an area A, and its normal makes an angle θ with the magnetic field B, as in Fig. 36.16, then the flux through the coil

$$= AB\cos\theta \text{ (see p. 898).}$$

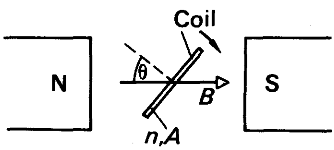


FIG. 36.16. Coil inclined to magnetic field.

The flux linkages with the coil, if it has N turns, are

$$\Phi = NAB\cos\theta.$$

If the coil turns with a steady angular velocity ω or $d\theta/dt$, then the e.m.f. induced in volts in the coil is

$$E = -\frac{d\Phi}{dt}$$

$$= -NAB\frac{d}{dt}(\cos\theta)$$

$$= NAB\sin\theta\frac{d\theta}{dt} \quad . \quad . \quad . \quad . \quad (1)$$

In terms of the number of revolutions per second, f, which the coil makes, we have

$$\frac{d\theta}{dt} = 2\pi f$$

and $$\theta = 2\pi f t,$$

$$\therefore E = 2\pi f NAB \sin 2\pi f t. \quad . \quad . \quad (2)$$

Thus the e.m.f. varies sinusoidally with time, like the pressure in a sound-wave, the frequency being f cycles per second.

The maximum (peak) value or amplitude of E occurs when $\sin 2\pi f t$ reaches the value 1. If the maximum value is denoted by E_0, it follows that

$$E_0 = 2\pi f NAB,$$

and $$E = E_0 \sin 2\pi f t. \quad . \quad . \quad . \quad (3)$$

The e.m.f. E sends an alternating current, of a similar sine equation, through a resistor connected across the coil.

Alternators

Generators of alternating current are often called *alternators.* In all but the smallest, the magnetic field of an alternator is provided by an electromagnet called a field-magnet or *field*, as shown in Fig. 36.17; it has a core of cast steel, and is fed with direct current from a separate d.c.

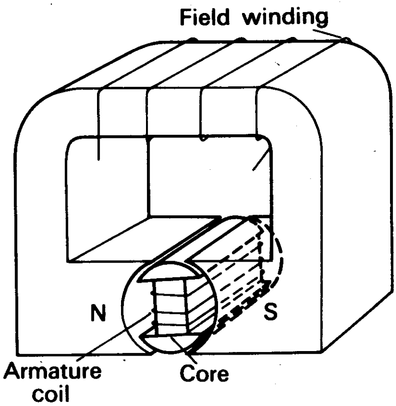


FIG. 36.17. Field magnet and armature.

generator. The rotating coil, called the armature, is wound on an iron core, which is shaped so that it can turn within the pole-pieces of the field-magnet. With the field-magnet, the armature core forms a system which is almost wholly iron, and can be strongly magnetized by a small current through the field winding. The field in which the armature turns is much stronger than if the coil had no iron core, and the e.m.f. is proportionately greater. In the small alternators used for bicycle lighting the armature is stationary, and the field is provided by permanent magnets, which rotate around it. In this way rubbing contacts, for leading the current into and out of the armature, are avoided.

When no current is being drawn from a generator, the horse-power required to turn its armature is merely that needed to overcome friction, since no electrical energy is produced. But when a current is drawn, the horse-power required increases, to provide the electrical power. The current, flowing through the armature winding, causes the magnetic field to set up a couple which opposes the rotation of the armature, and so demands the extra horse-power. The reader should check the truth of this statement by marking the direction of the e.m.f., current, and force on the limbs of the coil in Fig. 36.17.

The Transformer

A transformer is a device for stepping up—or down—an alternating voltage. It has primary and secondary windings, as in an induction coil, but no make-and-break (Fig. 36.18). It has an iron core, which is made from E-shaped laminations, interleaved so that the magnetic flux does not pass through air at all; in this way the greatest flux is obtained

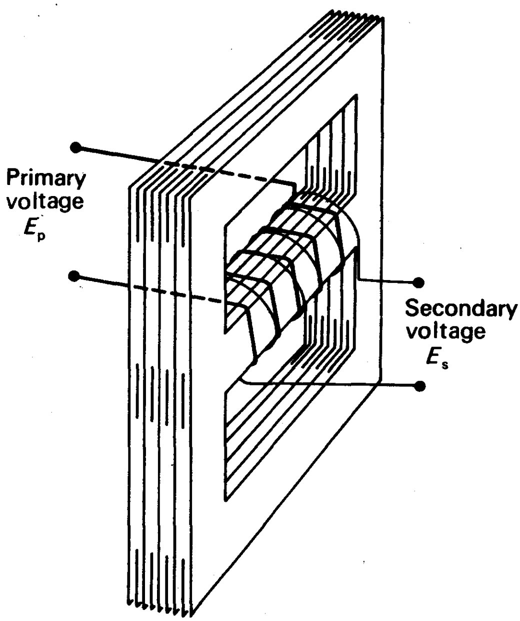


FIG. 36.18. Transformer.

with a given current. When an alternating e.m.f. E_p is impressed on the primary winding, it sends an alternating current through it, which sets up an alternating flux in the core of magnitude BA, where B is the induction and A is the cross-sectional area. This induces an alternating e.m.f. in the secondary E_s. If N_p, N_s are the number of turns in the primary and secondary coils, their linkages with the flux Φ are:

$$\Phi_p = N_p AB \qquad \Phi_s = N_s AB$$

The magnitude of the e.m.f. induced in the secondary is, from the formula on p. 899:

$$E_s = \frac{d\Phi_s}{dt} = N_s A \frac{dB}{dt}$$

The changing flux also induces a back-e.m.f. in the primary, whose magnitude is

$$E_p = \frac{d\Phi_p}{dt} = N_p A \frac{dB}{dt}$$

Because the primary winding has inevitably some resistance, the current flowing through it sets up a voltage drop across the resistance. But in practice this is negligible compared with the back-e.m.f. due to the changing flux. Consequently we may say that the voltage applied to the primary, from the source of current, is used simply in overcoming the back-e.m.f. E_p. Therefore it is equal in magnitude to E_p. (This is analogous to saying, in mechanics, that action and reaction are equal and opposite.) Consequently we have

$$\frac{\textit{e.m.f. induced in secondary}}{\textit{voltage applied to primary}} = \frac{E_s}{E_p} = \frac{N_s}{N_p} \quad . \quad . \quad (1)$$

Thus the transformer steps voltage up or down according to its '*turns-ratio*':

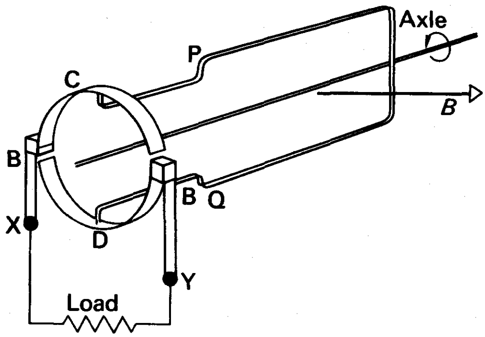
$$\frac{\text{secondary voltage}}{\text{primary voltage}} = \frac{\text{secondary turns}}{\text{primary turns}}.$$

When a load is connected to the secondary winding, a current flows in it. This current flows in such a direction as to reduce the flux in the core. At the instant that the load is connected, therefore, the back-e.m.f. in the primary falls. The primary current then increases. The increase in primary current increases the flux through the core, and continues until the flux is restored to its original value. The back-e.m.f. in the primary is then again equal to the applied voltage, and equilibrium is restored. But now a greater primary current is flowing than before the secondary was loaded. Thus the power drawn from the secondary is drawn, in turn, from the supply to which the primary is connected.

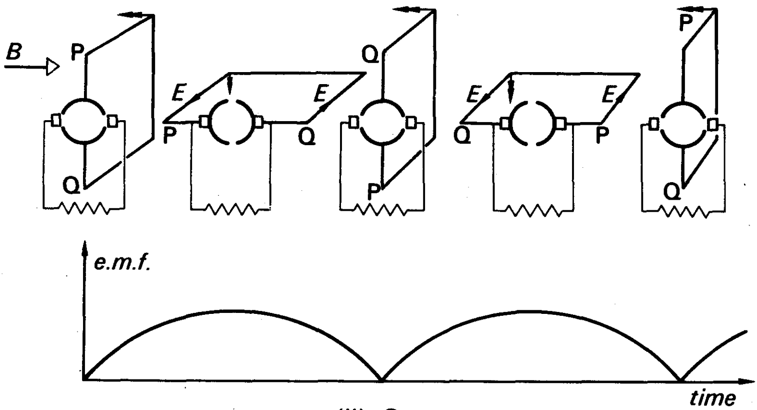
Transformers are used to step up the voltage generated at a power station, from 11000 to 132 000 volts for high-tension transmission (p. 793). After transmission they are used to step it down again to a value safer for distribution (240 volts in houses). Inside a house a transformer may be used to step the voltage down from 240 to 4, for ringing bells. Transformers with several secondaries are used in, for example, radio-receivers, where several different voltages are required.

D.C. Generators

Fig. 36.19 (i) is a diagram of a direct-current generator or dynamo. Its essential difference from an alternator is that the armature winding is connected to a *commutator* instead of slip-rings. The commutator consists of two half-rings of copper C, D, insulated from one another, and turning with the coil. Brushes BB, with carbon tips, press against the commutator and are connected to the external circuit. The commutator



(i) Principle



(ii) Output

FIG. 36.19. D.C. generator.

is oriented so that it reverses the connexions from the coil to the circuit at the instant when the e.m.f. reverses in the coil. Fig. 36.19 (ii) shows several positions of the coil and commutator, and the e.m.f. observed at the terminals XY. This e.m.f. pulsates in magnitude, but it acts always in the same sense round the circuit connected to XY. It is a pulsating direct e.m.f. The average value in this case can be shown to be $2/\pi$ of the maximum e.m.f. E_0, given in equation (3), p. 907.

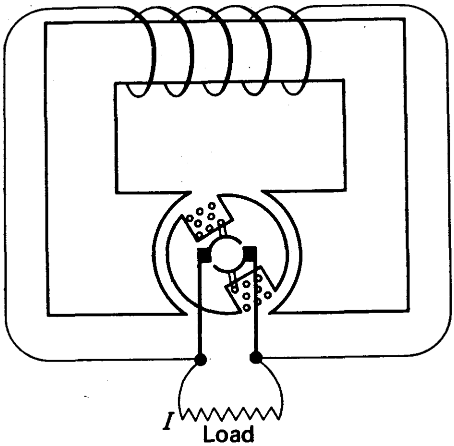


FIG. 36.20. D.C. generator with energized field.

In practice, as in an alternator, the armature coil is wound with insulated wire on a soft iron core, and the field-magnet is energized by a current (Fig. 36.20). This current is provided by the dynamo itself. The steel of the field-magnet has always a small residual magnetism, so that as soon as the armature is turned an e.m.f. is induced in it. This then sends a current through the field winding, which increases the field and the e.m.f.; the e.m.f. rapidly builds up to its working value.

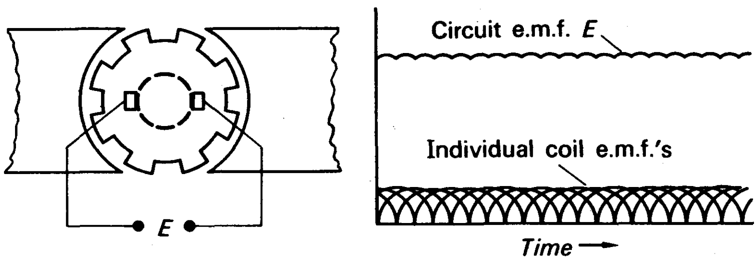


FIG. 36.21. D.C. generator with segment commutator.

Most consumers of direct current wish it to be steady, not pulsating as in Fig. 36.19. A reasonably steady e.m.f. is given by an armature with many coils, inclined to one another, and a commutator with a correspondingly large number of segments. The coils are connected to the commutator in such a way that their e.m.f.s add round the external circuit (Fig. 36.21).

Homopolar Generator

Another type of generator, which gives a very steady e.m.f., is illustrated in Fig. 36.22. It consists of a copper disc which rotates between the poles of a magnet; connexions are made to its axle and circumference. If we assume (as is not true) that the magnetic field B is uniform over the radius XY, then we can calculate the induced e.m.f. E. In one revolution the radius XY sweeps out an area $(\pi(r_1^2 - r_2^2)$, where r_1 and r_2 are the radii of the wheel and the axle. If T is the time for one revolution, then the rate at which XY sweeps out are is $\pi(r_1^2 - r_2^2)/T$.

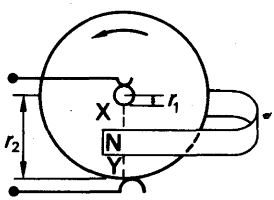


FIG. 36.22.
E.m.f. induced in a disc.

The rate at which it sweeps out flux is therefore

$$\frac{\pi(r_1^2 - r_2^2)}{T}B = \pi(r_1^2 - r_2^2)Bf,$$

where f denotes the revolutions of the wheel per second. Thus

$$E = \pi(r_1^2 - r_2^2)Bf$$
$$= \pi(r_1^2 - r_2^2)Bf$$

Generators of this kind are called *homopolar* because the e.m.f. induced in the moving conductor is always in the same sense. They are sometimes used for electroplating, where only a small voltage is required, but they are not useful for most purposes, because they give too small an e.m.f. The e.m.f. of a commutator dynamo can be made large by having many turns in the coil; but the e.m.f. of a homopolar dynamo is limited to that induced in one radius of the disc.

Applications of Alternating and Direct Currents

Direct currents are less easy to generate than alternating currents, and alternating e.m.f.s are more convenient to step up and to step down, and to distribute over a wide area. The national grid system, which supplies electricity to the whole country, is therefore fed with alternating current. Alternating current is just as suitable for heating as is direct current, because the heating effect of a current is independent of its direction. It is also equally suitable for lighting, because filament lamps depend on the heating effect, and gas-discharge lamps —neon, sodium, mercury—run as well on alternating current as on direct. Small motors, of the size used in vacuum-cleaners and common machine-tools, run satisfactorily on alternating current, but large ones, as a general rule, do not. Direct current is therefore used on most electric railway and tramway systems. These systems either have their own generating stations, or convert alternating current from the grid into direct current. One way of conveying alternating current into direct is to use a valve rectifier, whose principle we shall describe later.

For electro-chemical processes alternating current is useless. The chemical effect of a current reverses with its direction, and if, therefore, we tried to deposit a metal by alternating current, we would merely cause a small amount of the metal to be alternately deposited and dissolved. For electroplating, and for battery charging, alternating current must be rectified.

Eddy-currents

The core of the armature of a dynamo is built up from thin sheets of soft iron insulated from one another by an even thinner film of oxide, as shown in Fig. 36.23 (i). These are called laminations, and the armature is said to be laminated. If the armature were solid, then, since iron is a conductor, currents would be induced in it by its motion across the magnetic field (Fig. 36.23 (ii)). These currents would absorb power by opposing the rotation of the armature, and they would dissipate that power as heat, which would damage the insulation of the winding; but

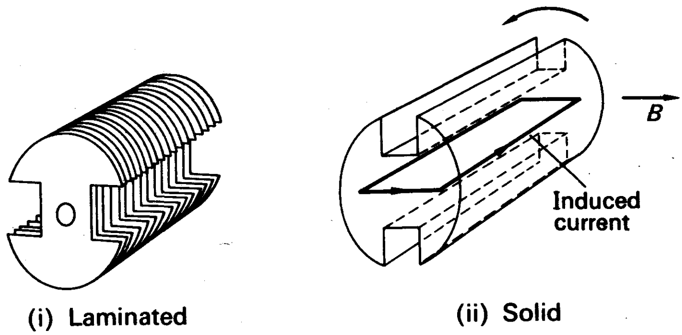


FIG. 36.23. Armature cores.

when the armature is laminated, these currents cannot flow, because the induced e.m.f. acts at right angles to the laminations, and therefore to the insulation between them. The magnetization of the core, however, is not affected, because it acts along the laminations. Thus the eddy-currents are suppressed, while the desired e.m.f.—in the armature coil—is not.

Eddy-currents, by Lenz's law, always tend to oppose the motion of a solid conductor in a magnetic field. The opposition can be shown in many ways. One of the most impressive is to make a chopper with a thick copper blade, and to try to slash it between the poles of a strong electromagnet; then to hold it delicately and allow it to drop between them. The resistance to the motion in the former case can be felt.

Sometimes eddy-currents can be made use of—for example, in damping a galvanometer. When a current is passed through the coil of a galvanometer, it applies a couple to the coil which sets it swinging. If the swings are opposed only by the viscosity of the air, they decay very slowly and are said to be naturally damped (Fig. 36.24). The pointer or light-spot takes a long time to come to its final steady deflection θ. To bring the spot or pointer more rapidly to rest, the damping must be increased. One way of increasing the damping is to wind the coil on a metal former. Then, as the coil swings, the field of the permanent magnet induces eddy-currents in it; and these, by Lenz's law, oppose its motion. They therefore slow down the turning of the coil towards its eventual position, but they also suppress its swings about that position; in the end the coil comes to rest sooner than if it

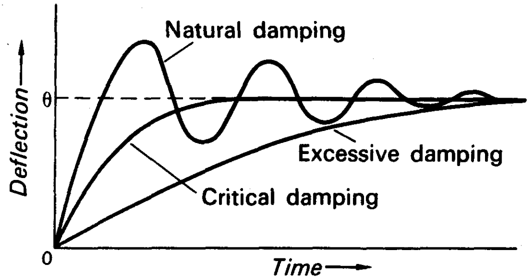


FIG. 36.24. Damping of galvanometer.

were not damped. Galvanometer coils which are wound on insulating formers can be damped by short-circuiting a few of their turns, or by connecting an external shunt across the whole coil. With a shunt the eddy-currents circulate round the coil and shunt, independently of the current to be measured. The smaller the shunt, the greater the eddy-currents and the damping; if the coil is overdamped, as shown in Fig. 36.24, it may take almost as long to come to rest as when it is undamped. The damping which is just sufficient to prevent overshoot is called 'critical' damping.

Electric Motors

If a simple direct-current dynamo, of the kind described on p. 910, is connected to a battery it will run as a motor (Fig. 36.25). Current flows round the armature coil, and the magnetic field exerts a couple on this, as in a moving-coil galvanometer. The commutator reverses the current just as the limbs of the coil are changing from upward to

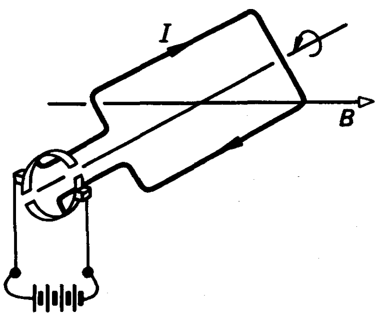


FIG. 36.25. Principle of D.C. motor.

downward movement and vice versa. Thus the couple on the armature is always in the same sense, and the shaft turns continuously. (The reader should verify these statements with the help of Fig. 36.25.)

The armature of a motor is laminated, in the same way and for the same reason, as the armature of a dynamo.

Back-e.m.f. in Motor

When the armature of a motor rotates, an e.m.f. is induced in its windings; by Lenz's law this e.m.f. opposes the current which is making the coil turn. It is therefore called a back-e.m.f. If its magnitude is E, and V is the potential difference applied to the armature by the supply, then the armature current is

$$I_a = \frac{V-E}{R_a}. \qquad (1)$$

Here R_a is the resistance of the armature, which is generally small—of the order of 1 ohm.

The back-e.m.f. E is proportional to the strength of the magnetic field, and the speed of rotation of the armature. When the motor is first switched on, the back-e.m.f. is zero: it rises as the motor speeds up. In a large motor the starting current would be ruinously great; to limit it, a variable resistance is inserted in series with the armature, which is gradually reduced to zero as the motor gains speed.

When a motor is running, the back-e.m.f. in its armature E is not much less than the supply voltage V. For example, a motor running off the mains ($V = 230$ volts) might develop a back-e.m.f. $E = 220$ volts. If the armature had a resistance of 1 ohm, the armature current would then be 10 amp (equation (1)). When the motor was switched on, the armature current would be 230 amp if no starting resistor were used.

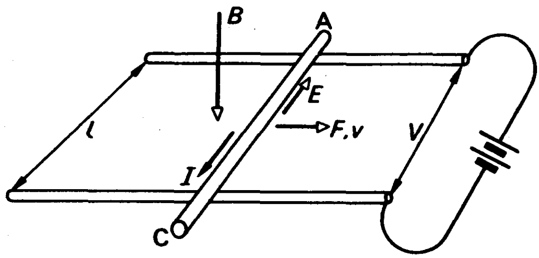


FIG. 36.26. Back-e.m.f. and mechanical power.

Back-e.m.f. and Power

The back-e.m.f. in the armature of a motor represents the mechanical power which it develops. To see that this is so, we use an argument similar to that which we used in finding an expression for the e.m.f. induced in a conductor. We consider a rod AC, able to slide along rails, in a plane at right angles to a magnetic field B (Fig. 36.26). But we now suppose that a current I is maintained in the rod by a battery, which sets up a potential difference V between the rails. The magnetic field then exerts a force F on the rod, given by

$$F = BIl.$$

The force F makes the rod move; if its velocity is v, the mechanical power developed by the force F is

$$P_m = Fv = BIlv \qquad (1)$$

As the rod moves, a back-e.m.f. is induced in it, whose magnitude E is given by the expression for the e.m.f. in a moving rod (p. 901):

$$E = Blv. \qquad (2)$$

Equations (1) and (2) together give

$$P_m = EI. \qquad (3)$$

Thus the mechanical power developed is equal to the product of the back-e.m.f. and the current.

Before returning to consider motors, we may complete the analysis of the action represented in Fig. 36.26. If R is the resistance of the rails and rod, the heat developed in them is I^2R. The power supplied by the battery is IV, and the battery is the only source of power in the whole system. Therefore

$$IV = I^2R + P_m; \qquad (4)$$

the power supplied by the battery goes partly into heat, and partly into useful mechanical power. Also, by equation (3),

$$IV = I^2R + EI, \qquad (5)$$

whence

$$V = IR + E$$

or

$$I = \frac{V - E}{R}.$$

This is equation (1), p. 915, which we previously obtained simply from Ohm's law.

Let us apply this theory to the example which we were considering. We had:

supply voltage, V, = 230 volts;

back-e.m.f., E, = 220 volts;

armature resistance, R_a, = 1 ohm;

armature current, I_a, = 10 amp.

The power dissipated as heat in the armature is $I_a^2R_a = 100 \times 1 = 100$ watts. The power supplied to the armature is $I_aV = 10 \times 230 = 2300$ watts, and the mechanical power is $I_aE = 10 \times 220 = 2200$ watts. Of the power supplied to the armature, the fraction which appears as mechanical power is $2200/2300 = 96$ per cent. This is not, however, the efficiency of the motor as a whole, because current is taken by the winding on the field magnet.

The Field Winding

The field winding of a motor may be connected in series or in parallel with the armature. If it is connected in series, it carries the armature current, which is large (Fig. 36.27). The field winding therefore has few turns of thick wire, to keep down its resistance, and so the power wasted in it as heat. The few turns are enough to magnetize the iron, because the current is large. If the field coil is connected in parallel with the armature, as in Fig. 36.28, the motor is said to be 'shunt-wound'. The field winding has many turns of fine wire to keep down

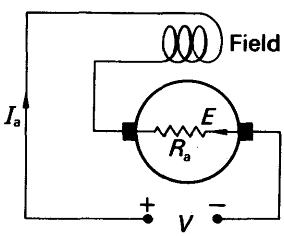


FIG. 36.27. Series-wound motor.

the current which it consumes. In the above example, if the motor is shunt-wound and the field current is 0·5 A, then the power dissipated as heat in the field is $0{\cdot}5 \times 230 = 115$ watts. The power consumption of the motor is therefore $2300 + 115 = 2415$ watts, and its efficiency is

$$\frac{\text{mechanical power developed}}{\text{electrical power consumed}} = \frac{2220}{2415}$$

$$= 92 \text{ per cent.}$$

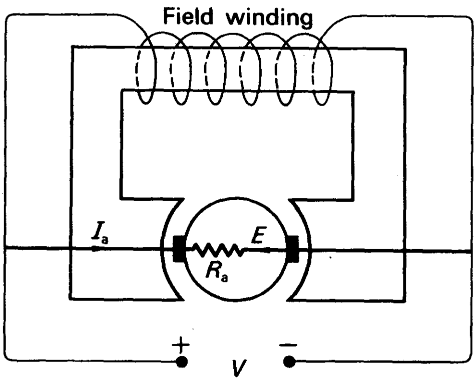


FIG. 36.28. Current and voltages in shunt-wound motor.

The working efficiency of the motor will be a little less than this, because some of the mechanical power will be used in overcoming friction in the bearings.

Shunt Field

Shunt-wound motors are used for driving machine-tools, and in other jobs where a steady speed is required. A shunt motor keeps a nearly steady speed for the following reason. If the load is increased, the speed falls a little; the back-e.m.f. then falls in proportion to the speed, and the current rises, enabling the motor to develop more power to overcome the increased load. In the example, p. 916, if the speed falls by 1 part in 220, the back-e.m.f. falls from 220 to 219 V. The current then rises from $\frac{230-220}{1} = 10$ A to $\frac{230-219}{1} = 11$ A. And the mechanical power increases from $220 \times 10 = 2200$ watts to $219 \times 11 = 2409$ watts ($\simeq 2400$). Thus an increase in load of $\frac{2400-2200}{2200} = 9$ per cent causes a fall in speed of 1 part in 220—less than $\frac{1}{2}$ per cent.

Series Field

Series motors are used where great torque is required in starting—for example, in cranes.

They develop a great starting torque because the armature current flows through the field coil. At the start the armature back-e.m.f. is small, and the current is great—as great as the starting resistance will allow. The field-magnet is therefore very strongly magnetized. The torque on the armature is proportional to the field and to the armature current; since both are great at the start, the torque is very great.

A series motor does not keep such a steady speed as a shunt motor. Just as in a shunt motor, when the load increases the speed falls; and the fall in speed decreases the back-e.m.f., and allows more current to flow. But, as we will see in a moment, the back-e.m.f., in a series motor, does not fall with the speed as sharply as it does in a shunt motor. To meet a given increase in load, the armature current must increase by a definite amount. And therefore the back-e.m.f. must fall by a definite amount. But it falls less with the speed than it does in a shunt motor. Consequently, to meet a given increase in load, the speed of a series motor must fall more than that of a shunt motor.

We now show that the back-e.m.f. in a series motor falls less with the speed than in a shunt one. The argument is best given in steps:

(i) when the speed falls, the back-e.m.f. falls;
(ii) the current through both armature and field winding increases;
(iii) the field becomes stronger;
(iv) the increase in the field tends to *increase* the back-e.m.f., i.e. to offset its initial fall;
(v) thus the very fall of the back-e.m.f., by permitting a greater current and strengthening the field, tends to offset itself;
(vi) therefore the back-e.m.f. falls slowly with the speed—more slowly than in a shunt motor, where the field is constant;
(vii) as we have already seen, this means that the speed must fall further, to meet a given increase in load.

CHARGE AND FLUX LINKAGE

Flux and Charge relation

We have already seen that an electromotive force is induced in a circuit when the magnetic flux linked with it changes. If the circuit is closed, a current flows, and electric charge is carried round the circuit. As we shall now show, there is a simple relationship between the charge and the change of flux.

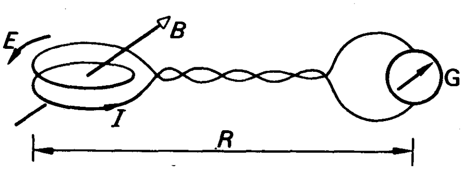


FIG. 36.29. Coil with changing flux.

Consider a closed circuit of total resistance R ohm, which has a total flux linkage Φ with a magnetic field (Fig. 36.29). If the flux linkages start to change,

$$\text{induced e.m.f., } E = -\frac{d\Phi}{dt}.$$

$$\therefore \text{ current, } I = \frac{E}{R} = -\frac{1}{R}\frac{d\Phi}{dt} \qquad . \quad . \quad (1)$$

In general, the flux linkage will not change at a steady rate, and the current will not be constant. But, throughout its change, charge is being carried round the circuit. If a time t seconds is taken to reach a new constant value, the charge carried round the circuit in that time is

$$Q = \int_0^t I dt.$$

From (1),

$$\therefore Q = -\frac{1}{R}\int_0^t \frac{d\Phi}{dt} . dt$$

$$= -\frac{1}{R}\int_{\Phi_0}^{\Phi_t} d\Phi,$$

where Φ_0 is the number of linkages at $t = 0$, and Φ_t is the number of linkages at time t. Thus

$$Q = -\frac{\Phi_t - \Phi_0}{R} = \frac{\Phi_0 - \Phi_t}{R}.$$

The quantity $\Phi_0 - \Phi_t$ is positive if the linkages Φ have decreased, and negative if they have increased. But as a rule we are interested only in the magnitude of the charge, and we may write

$$Q = \frac{\textit{change of flux linkage}}{R}. \qquad . \quad . \quad . \quad (2)$$

Equation (2) shows that the charge circulated is proportional to the change of flux-linkages, and independent of the time.

Ballistic Galvanometer

It can be seen from the last section that the charge which flows round a given circuit is directly proportional to the change of flux linkage. If the charge flowing is measured by a *ballistic galvanometer* G, as shown in Fig. 36.29, then we have a measure of the change in flux linkage, Φ.

Ballistics is the study of the motion of a body, such as a projectile, which is set off by a blow, and then allowed to move freely. By freely, we mean without friction. A ballistic galvanometer is one used to measure an electrical blow, or impulse: for example, the charge Q which circulates when a capacitor is discharged through it. A galvanometer which is intended to be used ballistically has a heavier coil than one which is not; and it has as little damping as possible—an insulating former, no short-circuited turns, no shunt. The mass of its coil makes it swing slowly; in the example above, for instance, the capacitor has discharged, and the charge has finished circulating, while the galvanometer coil is just beginning to turn. The galvanometer coil continues to turn, however; and as it does so it twists the suspension. The coil stops turning when its kinetic energy, which it gained from the forces set up by the current, has been converted into potential energy of the suspending fibre. The coil then swings back, as the suspension untwists itself, and it continues to swing back and forth for some time. Eventually it comes to rest, but only because of the damping due to the viscosity of the air, and to the internal friction of the fibre. Theory shows that, if the damping is negligible, *the first deflection of the galvanometer is proportional to the quantity of electricity, Q, that passed through its coil, as it began to move.* This first deflection, θ, is often called the 'throw' of the galvanometer; we have, then,

$$Q = k\theta, \qquad . \quad . \quad . \quad . \quad (1)$$

where k is a constant of the galvanometer.

Equation (1) is true only if all the energy given to the coil is spent in twisting the suspension. If an appreciable amount of energy is used to overcome damping—i.e. dissipated as heat by eddy currents—then the galvanometer is not ballistic, and θ is not proportional to Q.

To calibrate the ballistic galvanometer, a capacitor of known capacitance, e.g. 2 μF, is charged by a battery of known e.m.f., e.g. 50 volt, and then discharged through the instrument. See p. 768. Suppose the deflection is 200 divisions. The charge $Q = CV = 100$ microcoulomb, and thus the galvanometer sensitivity is 2 divisions per microcoulomb.

Measurement of Induction

Fig. 36.30 illustrates the principle of measuring the induction B in the field between the poles of a powerful magnet. A small coil, called a *search coil*, with a known area and number of turns, is connected to a ballistic galvanometer G. It is positioned at right angles to the field to be measured, as shown, so that the flux enters the coil face normally.

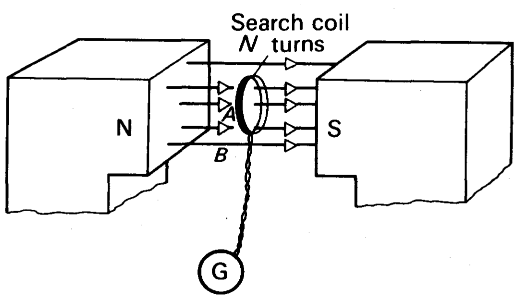


FIG. 36.20. Induction by ballistic galvanometer.

The coil is then pulled completely out of the field by moving it smartly downwards, for example, and the throw θ produced in the galvanometer is observed. The charge q which passes round the circuit is proportional to θ, from above.

Suppose B is the field-strength in Wb m^{-2} or tesla (T), A is the area of the coil in m^2 and N is the number of turns. Then

$$\text{change of flux-linkages} = NAB$$

$$\therefore \text{ quantity, } Q, \text{ through galvanometer} = \frac{NAB}{R},$$

where R is the *total* resistance of the galvanometer and search coil. But

$$Q = c\theta,$$

where c is the quantity per unit deflection of the ballistic galvanometer.

$$\therefore \frac{NAB}{R} = c\theta$$

$$\therefore B = \frac{Rc\theta}{NA}. \qquad . \quad . \quad . \quad . \quad . \qquad (1)$$

The constant c is found by discharging a capacitor through the galvanometer (see p. 768). If C is the capacitance in farads, V the p.d. in volts of the battery originally charging it, and α the deflection of the galvanometer, then $c = CV/\alpha$ coulomb per unit deflection.

The Earth Inductor

As another example of the use of a search coil and ballistic galvanometer, we describe a method that has been used for measuring the angle of dip of the earth's magnetic field (p. 944). The earth's field is so nearly uniform that the search coil may be large, usually about 30 cm square; but the field is also so weak that even a large coil must have many turns—of the order of 100. The coil, which is called an earth inductor, is pivoted in a wooden frame, and this is fitted with stops so that the coil can be turned rapidly through 180° (Fig. 36.31). To find

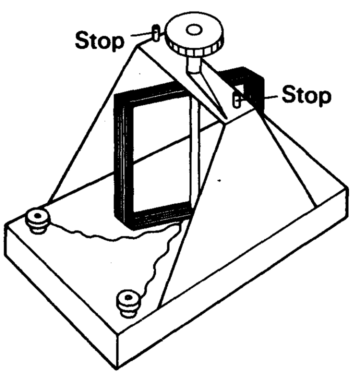


FIG. 36.31. Earth inductor.

the angle of dip, we connect the coil to a ballistic galvanometer, and set it with its plane horizontal, as shown at (i) in Fig. 36.32. The flux linking the coil at N turns is then

$$\Phi = NAB_V,$$

where A is the area of the coil, and B_V is the vertical component of the earth's field. If we were to turn the coil through 90°, the flux would fall to zero; and if we were to turn it through a further 90°, the flux linkage would become NAB_V once more, but it would thread the coil in the opposite direction. Therefore we turn the coil through 180°, and change the flux linkage by $2NAB_V$; at the same time we observe the throw, θ, of the galvanometer.

If R is the total resistance of galvanometer and search coil, the circulated charge is, by equation (2) on p. 919,

$$Q = \frac{\Phi}{R} = \frac{2NAB_V}{R}.$$

But

$$Q = c\theta,$$

where c is the constant of the galvanometer. Therefore

$$\frac{2NAB_V}{R} = c\theta. \qquad . \quad . \quad . \quad (1)$$

We now set the frame of the earth inductor so that the axis of the coil is vertical, and so that, when the coil is held by one of the stops, its plane lies East–West. See Fig. 36.32 (ii). The flux threading the coil is now NAB_H, where B_H is the horizontal component of the earth's field. Therefore, when we turn the coil through 180°, the throw θ' of the galvanometer is given by

$$\frac{2NAB_H}{R} = c\theta' \qquad . \quad (2)$$

Now the angle of dip, δ, is given by

$$\tan\delta = \frac{B_V}{B_H}.$$

Therefore, from equations (1) and (2),

$$\tan\delta = \frac{\theta}{\theta'}.$$

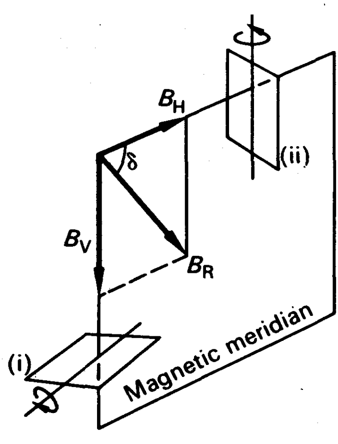


FIG. 36.32. Measurement of dip.

Self-induction

The phenomenon which we call self-induction was discovered by the American, Joseph Henry, in 1832. He was led to it by a theoretical argument, starting from the phenomena of induced e.m.f., which he had discovered at about the same time as Faraday.

When a current flows through a coil, it sets up a magnetic field. And that field threads the coil which produces it. Fig. 36.33 (i). If the current through the coil is changed—by means of a variable resistance, for example—the flux linked with the turn of the coil changes. An e.m.f. is therefore induced in the coil. By Lenz's law the direction of the induced e.m.f. will be such as to oppose the change of curent; the e.m.f. will be against the current if it is increasing, with it if it is decreasing (Fig. 36.33 (ii)).

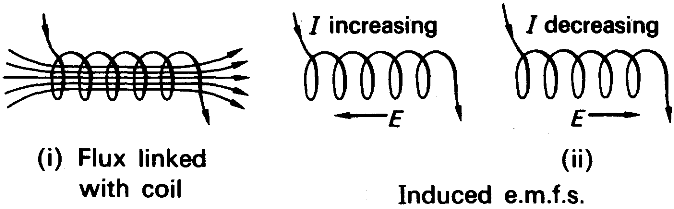


FIG. 36.33. Self-induction.

Back-E.M.F.

When an e.m.f. is induced in a circuit by a change in the current through that circuit, the process of induction is called self-induction. The e.m.f. induced is called a back-e.m.f. Self-induction opposes the growth of current in a coil, and so makes it gradual. This effect can be demonstrated by connecting an iron-cored coil of many turns in series with an ammeter and a few accumulators (Fig. 36.34 (i)). (The ammeter

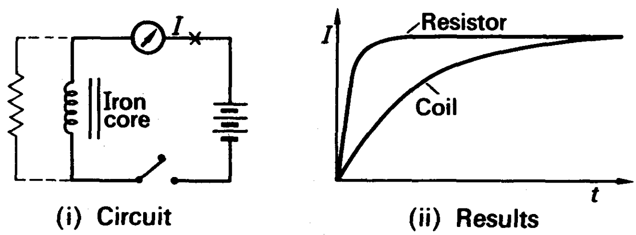


FIG. 36.34. Self-induction experiment.

should be of the 'short-period' type and critically damped.) When the current is switched on, the pointer of the ammeter moves slowly over to its final position. If the coil is now replaced by a rheostat of the same *resistance*, the pointer moves much more swiftly to the same reading (Fig. 36.34 (ii)).

Just as self-induction opposes the rise of an electric current when it is switched on, so also it opposes the decay of the current when it is switched off. When the circuit is broken, the current starts to fall very rapidly, and a correspondingly great e.m.f. is induced, which tends to maintain the current. This e.m.f. is often great enough to break down the insulation of the air between the switch contacts, and produce a spark. To do so, the e.m.f. must be about 350 volts or more, because air will not break down—not over any gaps, narrow or wide—when the voltage is less than that value. The e.m.f. at break may be much greater than the e.m.f. of the supply which maintained the current: a spark can easily be obtained, for example, by breaking a circuit consisting of an iron-cored coil and an accumulator.

Non-inductive Coils

In bridge circuits, such as are used for resistance measurements, self-induction is a nuisance. When the galvanometer key of a bridge is closed, the currents in the arms of the bridge are redistributed, unless the bridge happens to be balanced. While the currents are being redistributed they are changing, and self-induction delays the reaching of a new equilibrium. Thus the galvanometer deflection at the instant of closing the key, does not correspond to the steady state which the bridge will eventually reach. It may therefore be misleading. To minimize their self-inductance, the coils of bridges and resistance boxes are wound so as to set up extremely small magnetic fields: as shown in Fig. 36.35, the wire is doubled-back on itself before being coiled up. Every part of the coil is then traversed by the same current travelling in opposite directions, and its magnetic field is negligible. Such a coil is said to be non-inductive.

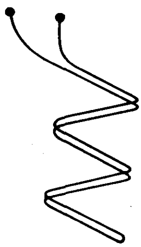


FIG. 36.35. Non-inductive winding.

When describing the use of a bridge, we said that the battery key should be pressed before the galvanometer key. Doing so gives time for the currents in the arms of the bridge to become steady before the galvanometer key is pressed. It therefore minimizes any possible effects of self-induction.

Self-inductance

To discuss the effects of self-induction we must define the property of a coil which gives rise to them. This property is called the self-inductance of the coil, and is defined as follows:

$$\textit{self-inductance} = \frac{\textit{back-e.m.f. induced in coil by a changing current}}{\textit{rate of change of current through coil}}$$

Self-inductance is denoted by the symbol L; we may therefore write its definition as

$$L = \frac{E_{back}}{dI/dt}.$$

or

$$E_{back} = L\frac{dI}{dt}. \quad \quad (1)$$

Equation (1) is the simplest form in which to remember the definition.

The unit of self-inductance is the henry (H). It is defined by making each term in equation (1) equal to unity; thus *a coil has a self-inductance of* 1 *henry if the back-e.m.f. in it is* 1 *volt, when the current through it is changing at the rate of* 1 *ampere per second.* Equation (1) then becomes:

$$E_{back}\,\text{(volts)} = L\,\text{(henrys)} \times \frac{dI}{dt}\,\text{(ampere/second)}.$$

The iron-cored coils used for smoothing the rectified supply current to a radio receiver (p. 1011) are usually very large and have an inductance of about 30 henrys.

L for Coil

Since the induced e.m.f. $E = d\Phi/dt = L = dI/dt$, numerically, it follows by integration from a limit of zero that

$$\Phi = LI.$$

Thus $L = \Phi/I$. Hence the self-inductance may be defined as the *flux linkage per unit current*. When Φ is in webers and I in amperes, then L is in henrys. Thus if a current of 2A produces a flux linkage of 4 Wb in a coil, the inductance $L = 4$ Wb/2A $= 2$H.

We shall see later that when a long coil of N turns and length l carries a current I, (i) the magnetizing field $H = NI/l$ and (ii) this produces a flux-density or induction B inside the coil given by $B = \mu H$, where μ is the permeability of the material inside the coil (p. 941). Hence

$$\text{flux linkage } \Phi = NAB = NA\mu H = \frac{\mu N^2 AI}{l}$$

$$\therefore L = \frac{\Phi}{I} = \frac{\mu N^2 A}{l} \quad . \quad . \quad . \quad (1)$$

This formula may be used to find the approximate value of the inductance of a coil. L is in henrys when A is in metre^2, l in metre and μ is in henry metre^{-1}.

Energy Stored; E.M.F. at Break

The spark which passes when the current in a coil is interrupted liberates energy in the form of heat and light. This energy has been stored in the magnetic field of the coil, just as the energy of a charged capacitor is stored in the electrostatic field between its plates (p. 779). When the current in the coil is first switched on, the back-e.m.f. opposes the rise of current; the current flows against the back-e.m.f. and therefore does work against it (p. 795). When the current becomes steady, there is no back-e.m.f. and no more work done against it. The total work done in bringing the current to its final value is stored in the magnetic field of the coil. It is liberated when the current collapses; for then the induced e.m.f. tends to maintain the current, and to do external work of some kind.

To calculate the energy stored in a coil, we suppose that the current through it is rising at a rate dI/dt ampere per second. Then, if L is its self-inductance in henrys, the back-e.m.f. across it is given by

$$E = L\frac{dI}{dt} \text{ volt.}$$

If the value of the current, at the instant concerned, is I amperes, then the rate at which work is being done against the back-e.m.f. is

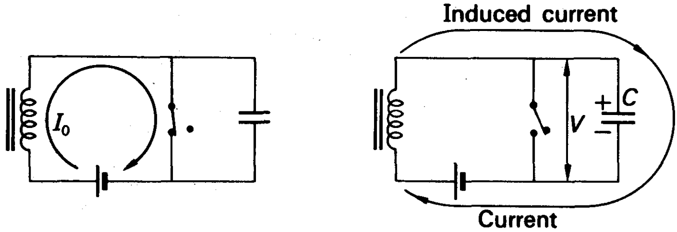
$$P = EI = LI\frac{dI}{dt} \text{ watt.}$$

The total work done in bringing the current from zero to a steady value I_0 is therefore

$$W = \int P dt = \int_0^{I_0} LI\frac{dI}{dt}dt = \int_0^{I_0} LIdt$$
$$= \tfrac{1}{2}LI_0{}^2 \text{ joule.}$$

This is the energy stored in the coil.

To calculate the e.m.f. induced at break is, in general, a complicated business. But we can easily do it for one important practical circuit. To prevent sparking



(i) Steady state, switch closed

(ii) Switch open

FIG. 36.36. Prevention of sparking by capacitor.

at the contacts of the switch in an inductive circuit, a capacitor is often connected across them (Fig. 36.36 (i)). When the circuit is broken, the collapsing flux through the coil tends to maintain the current; but now the current can continue to flow for a brief time; it can flow by charging the capacitor (Fig. 36.36 (ii)). Consequently the current does not decay as rapidly as it would without the capacitor, and the back-e.m.f. never rises as high. If the capacitance of the capacitor is great enough, the potential difference across it (and therefore across the switch) never rises high enough to cause a spark.

To find the value to which the potential difference does rise, we assume that all the energy originally stored in the magnetic field of the coil is now stored in the electrostatic field of the capacitor.

If C is the capacitance of the capacitor in farad, and V_0 the final value of potential difference across it in volt, then the energy stored in it is $\frac{1}{2}CV_0^2$ joule (p. 779). Equating this to the original value of the energy stored in the coil, we have

$$\tfrac{1}{2}CV_0^2 = \tfrac{1}{2}LI_0^2.$$

Let us suppse that a current of 1 ampere is to be broken, without sparking, in a circuit of self-inductance 1 henry. To prevent sparking, the potential difference across the capacitor must not rise above 350 volt. The least capacitance that must be connected across the switch is therefore given by

$$\tfrac{1}{2}C \times 350^2 = \tfrac{1}{2} \times 1 \times 1^2.$$

Hence
$$C = \frac{1}{350^2} = 8 \times 10^{-6} \text{ farad} = 8\ \mu\text{F}.$$

A paper capacitor of capacitance 8 μF, and able to withstand 350 volts, would therefore be required.

Mutual Induction

We have already seen than an e.m.f. may be induced in one circuit by a changing current in another (Fig. 36.1, p. 895). The phenomenon is often called mutual induction, and the pair of circuits which show it are said to have mutual inductance. The mutual inductance, M, between two circuits is defined by the equation:

$$\left.\begin{array}{l}\text{e.m.f. induced in B, by}\\ \text{changing current in A}\end{array}\right\} = M \times \left\{\begin{array}{l}\text{rate of change of}\\ \text{current in A.}\end{array}\right.$$

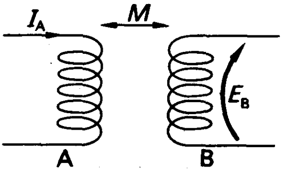


FIG. 36.37. Mutual induction.

See Fig. 36.37. In symbols,

$$E_B = M\frac{dI_A}{dt}.$$

Mutual inductance is truly mutual; it is the same from B to A as A to B. Its unit is the same as that of self-inductance, the henry.

EXERCISES 36

1. State Lenz's law of electromagnetic induction and describe, with explanation, an experiment which illustrates its truth.

Describe the structure of a transformer suitable for supplying 12 volts from 240-volt mains and explain its action. Indicate the energy losses which occur in the transformer and explain how they are reduced to a minimum.

When the primary of a transformer is connected to the a.c. mains the current in it (*a*) is very small if the secondary circuit is open, but (*b*) increases when the secondary circuit is closed. Explain these facts. (*L.*)

2. Describe, with the aid of a large labelled diagram, the structure of a simple form of a.c. generator.

Explain (*a*) how it could be modified to produce direct current;
(*b*) the features that enable is to produce a high e.m.f. (compared with a cell);
(*c*) the features that minimize the heat wasted. (*L.*)

3. Define *electromotive force* and state the *laws of electromagnetic induction.* Using the definition and the laws, derive an expression for the e.m.f. induced in a conductor moving in a magnetic field.

When a wheel with metal spokes 120 cm long is rotated in a magnetic field of flux density $0{\cdot}5 \times 10^{-4}$ Wb m^{-2} normal to the plane of the wheel, an e.m.f. of 10^{-2} volt is induced between the rim and the axle. Find the rate of rotation of the wheel. (*L.*)

4. Describe the differences in structure and action between a non-ballistic and a ballistic moving-coil galvanometer.

A corrected deflection of 24 scale divisions of a ballistic galvanometer is obtained *either* by charging a capacitor of 3 μF capacitance to a potential difference of 2 volt and discharging it through the galvanometer, or by connecting the ballistic galvanometer in series with a flat circular coil of 80 turns each of diameter 1 cm, the combined resistance of coil and galvanometer being 2000 ohms, and quickly thrusting the coil into a strong magnetic field so that the plane of the coil is perpendicular to the direction of the field. State the sensitivity of the galvanometer and calculate the strength of the magnetic field. (The strength of the earth's magnetic field may be neglected.) (*L.*)

5. Explain what is meant by *self inductance* and define the practical unit in which it is measured.

Describe and explain an experiment which demonstrates the phenomenon of self-induction. (*N.*)

6. State the laws relating to (*a*) the direction, (*b*) the magnitude of an electromagnetically induced electromotive force, and describe very briefly an experiment illustrating each.

Deduce the relation between the quantity of electricity flowing through a circuit and the flux change producing it.

A flat coil of 150 turns, each of area 300 cm^2 and of total resistance 50 ohms, is connected to a circuit whose resistance is 40 ohms. Starting with its plane horizontal, the coil is rotated quickly through a half-turn about a diametral axis pointing along the magnetic meridian. If the quantity of electricity which then flows round the circuit is 4 microcoulombs, find the intensity of the vertical component of the earth's magnetic field. (*N.*)

7. What are eddy currents?

Describe and explain an experiment in which eddy currents are produced.

Describe one useful application of eddy currents. (*N.*)

8. Define *self inductance* and *mutual inductance.*

Explain the differences in structure and action between a ballistic and an aperiodic galvanometer.

A ballistic galvanometer of resistance 15 ohms and sensitivity 5 divisions per microcoulomb is connected in series with a resistance of 100 ohms and a secondary coil of 500 turns and of resistance 50 ohms. This coil is wound round the middle of a long solenoid of radius 3 cm having 10 turns cm^{-1} and carrying a current of 0·6 A. Assuming no damping, calculate the deflection produced in the galvanometer when the current in the solenoid is switched off. (*L.*)

9. Describe experiments (one in each case) involving the use of a moving-coil ballistic galvanometer to (*a*) compare two capacitances of approximately the same magnitude, (*b*) compare the magnetic induction (flux density) between the poles of one electromagnet with that between the poles of another. In each case justify the method used to calculate the result.

Explain two special features of a galvanometer suitable for use in these experiments. (*N.*)

10. State Lenz's law and describe how you would demonstrate it using a solenoid with two separate superimposed windings with clearly visible turns, a cell with marked polarity, and a centre-zero galvanometer. Illustrate your answer with diagrams.

A metal aircraft with a wing span of 40 m flies with a ground speed of 1000 km h^{-1} in a direction due east at constant altitude in a region of the northern hemisphere where the horizontal component of the earth's magnetic field is $1{\cdot}6 \times 10^{-5}$ T (Wb m^{-2}) and the angle of dip is 71·6°. Find the potential difference in volts that exists between the wing tips and state, with reasons, which tip is at the higher potential. (*N.*)

11. State the laws of electromagnetic induction and describe briefly experiments to show their validity.

A coil *A* passes a current of 1·25 A when a steady potential difference of 5 V is maintained across it, and an r.m.s. current of 1 A when it has across it a sinusoidal potential difference of 5 V r.m.s. at a frequency of 60 Hz (cycles per second). Explain why the current is less in the second case, and calculate the resistance and the inductance of the coil.

The same coil A, which has 100 turns, has a second coil B with 500 turns wound on it so that all the magnetic flux produced by A is linked by B. Find the r.m.s. value of the e.m.f. that appears across the open-circuit ends of B when a sinusoidal alternating current of 1 A r.m.s. at a frequency of 50 Hz is passed through A. Why is the ratio of this e.m.f. to the r.m.s. potential difference across A not the same as the ratio of the number of turns in B and A, i.e. 5:1?

Explain why the insertion of an iron core into the coils would decrease the current in A and increase the e.m.f. across B, if the alternating potential difference across A were kept unchanged; the effects of hysteresis and eddy currents in the iron may be neglected. (*O. & C.*)

12. A choke of large self inductance and small resistance, a battery and a switch are connected in series. Sketch and explain a graph illustrating how the current varies with time after the switch is closed. If the self inductance and resistance of the coil are 10 henrys and 5 ohms respectively and the battery has an e.m.f. of 20 volts and negligible resistance, what are the greatest values after the switch is closed of (*a*) the current, (*b*) the rate of change of current? (*N.*)

13. State the laws of electromagnetic induction, and describe an experiment by which one of them can be verified.

A piece of wire 8 cm long, of resistance 0·020 ohm and mass 22 mg, is bent to form a closed square *ABCD*. It is mounted so as to turn without friction about a horizontal axis through *AB*; a uniform horizontal magnetic field of flux-density 0·50 Wb m^{-2} is applied at right angles to this axis. The side *CD* is raised until the plane of the square is horizontal and then released. Calculate approximately the time taken for the plane of the square to become vertical. You can assume that, during the falling, the couple due to gravity is equal and opposite to that due to electromagnetic forces. (*C.*)

14. State the laws of electromagnetic induction. Hence derive an expression for the time variation of the electromotive force induced in a single turn of wire rotating about an axis in its plane, the axis being perpendicular to a uniform magnetic field. Explain the action of a simple alternating current generator.

What modification of this generator is required to produce a direct current? Indicate by a sketch how the e.m.f. across the output terminals of a single coil would vary with time in the case of (*a*) an a.c. and (*b*) a d.c. generator. How may a more uniform output e.m.f. be obtained in the latter case?

A rectangular coil of wire having 100 turns, of dimensions 30 cm $\times$ 30 cm, is rotated at a constant speed of 600 r.p.m. in a magnetic field of 0·1 Wb m^{-2}, the axis of rotation being in the plane of the coil and perpendicular to the field. Calculate the induced e.m.f. (*L.*)

15. Describe with the aid of diagrams (*a*) a transformer and (*b*) an induction coil. Explain the action of each.

Draw diagrams to show, in a general way, how the voltage output from each of these appliances varies with the time. (*N.*)

16. What are eddy-currents? Give two examples of the practical use of such currents.

A metal disc of diameter 20 cm rotates at a constant speed of 600 r.p.m. about an axis through its centre and perpendicular to its plane in a uniform magnetic field of 5×10^{-3} Wb m^{-2} established parallel to the axis of rotation. Calculate the e.m.f. in volts between the centre and rim of the disc. Show clearly on a diagram the direction of rotation of the disc and the direction of the magnetic field and of the e.m.f. induced. (*L.*)

17. How would you show that a change in the number of lines of magnetic force, however produced, threading through a circuit produces an induced e.m.f.?

A magnet is suspended by a thin wire so that its axis is horizontal and its centre is above the centre of a circular copper disc, mounted horizontally. Explain what happens to the magnet when the disc is rotated.

What would be the effect of replacing the disc by one of identical dimensions but made of a substance of high resistivity? (*L.*)

18. State Lenz's law and describe fully a method by which you could verify this law experimentally.

A horizontal metal disc of radius 10 cm is rotated about a central vertical axis at a region where the value of the earth's magnetic flux density is $5{\cdot}3 \times 10^{-5}$ T (Wb m^{-2}) and the angle of dip is 70°. A sensitive galvanometer of resistance 150 ohms is connected between the centre of the disc and a brush pressing on the rim. Assuming the resistance of the disc to be negligible, what will be the current through the galvanometer when the disc is rotated at 1500 rev. min.$^{-1}$? If the system is frictionless, calculate the power required to maintain the motion. (*C.*)

19. State the laws of electromagnetic induction and describe experiments you would perform to illustrate the factors which determine the magnitude of the induced current set up in a closed circuit.

A simple electric motor has an armature of 0·1 ohm resistance. When the motor is running on a 50-volt supply the current is found to be 5 amp. Explain this and show what bearing it has on the method of starting large motors. (*L.*)

20. State the laws relating to the electromotive force induced in a conductor which is moving in a magnetic field.

Describe the mode of action of a simple dynamo.

Find in volts the e.m.f. induced in a straight conductor of length 20 cm, on the armature of a dynamo and 10 cm from the axis when the conductor is moving in a uniform radial field of 0·5 Wb m^{-2} and the armature is rotating at 1000 r.p.m. (*L.*)

chapter thirty-seven

Magnetic Fields of Current-Carrying Conductors

In the previous chapters, the induction or flux density B in a magnetic field was used to find the force on conductors and the e.m.f. induced in conductors. In this chapter we shall see how the magnitude of B is calculated. This depends on the geometry of the conductor, that is, whether it is a straight wire, or a coil, or a solenoid. The geometry also determines the pattern of the lines of force in the field.

Law of Biot and Savart

To calculate B for any shape of conductor, Biot and Savart gave a law which can now be stated as follows: The induction or flux density δB at a point P due to a small element δl of a conductor carrying a current is given by

$$\delta B \propto \frac{I\delta l \sin\alpha}{r^2}, \qquad \text{. . . . (1)}$$

where r is the distance from the point P to the element and α is the angle between the element and the line joining it to P (Fig. 37.1).

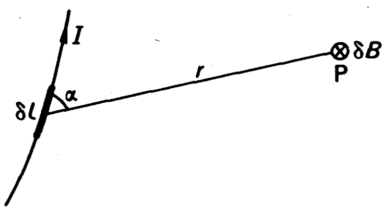


FIG. 37.1. Biot and Savart law.

The formula in (1) cannot be proved directly, as we cannot experiment with an infinitesimally small conductor. We believe in its truth because the deductions for large practical conductors turn out to be true.

The constant of proportionality in equation (1) depends on the medium in which the conductor is situated. In air (or, more exactly, in a vacuum), we write

$$\delta B = \frac{\mu_0}{4\pi}\frac{I\delta l \sin\alpha}{r^2} \qquad \text{. . . . (2)}$$

The value of μ_0 is defined to be,

$$\mu_0 = 4\pi \times 10^{-7},$$

and its unit is '*henry per metre*' ($\mathrm{H\ m^{-1}}$) as will be shown later.

Induction Formula for Narrow Coil

The formula for the induction B at the centre of a narrow circular coil can be immediately deduced from (2). Here the radius r is constant for all the elements δl, and the angle α is constant and equal to 90° (Fig. 37.2 (i)). If the coil has N turns, the length of wire in it is $2\pi rN$, and the field at its centre is therefore given, if the current is I, by

$$B = \int dB = \frac{\mu_0}{4\pi}\int_0^{2\pi rn} \frac{I\,dl \sin 90^\circ}{r^2}$$

$$= \frac{\mu_0 I}{4\pi r^2}\int_0^{2\pi rn} dl = \frac{\mu_0 I}{4\pi r^2}2\pi rN$$

$$= \frac{\mu_0 NI}{2r} \qquad . \quad . \quad . \quad . \quad . \qquad (1)$$

From (1), $B \propto I$ when r and N are constant, $B \propto 1/r$ when I and N are constant, and $B \propto N$ when I and r are constant. Any one of these relations may be verified with the apparatus shown in Fig. 37.2 (ii). This

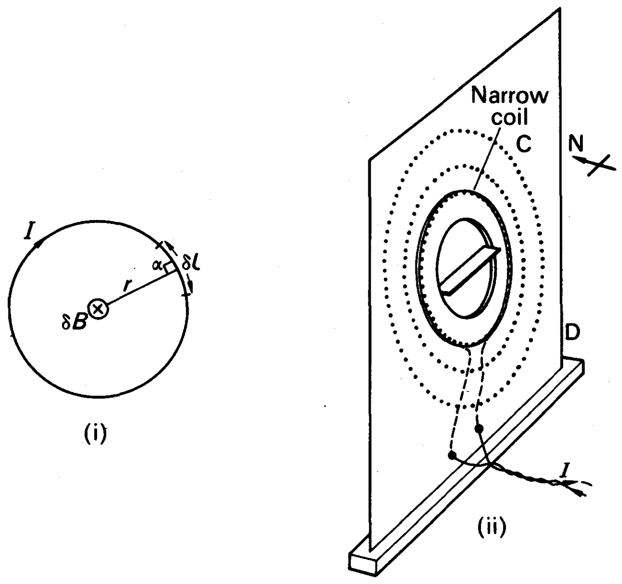


FIG. 37.2. Field of circular coil.

is a board D with sets of pegs, round which wire can be wound to form narrow coils of various radii. If a vibration magnetometer is used to measure B, the board is turned so that the axis of the coils C is in the direction of the magnetic meridian. The number of vibrations per minute is then found with the current in one direction and again when the current is reversed.

Suppose n_1 is the number when the field B of the current assists the field B_H due to the earth, so that $(B+B_H)$ is the resultant field. Suppose n_2 is the number when the field B opposes B_H and B is stronger than B_H, so that $(B-B_H)$ is the resultant field. Now on p. 887, it was shown that the flux density in a field was directly proportional to the square of the frequency of the vibration magnetometer. Hence

$$B+B_H = kn_1^2 \quad \text{and} \quad B-B_H = kn_2^2.$$

Adding,

$$\therefore 2B = k(n_1^2+n_2^2), \quad \text{or} \quad B \propto (n_1^2+n_2^2).$$

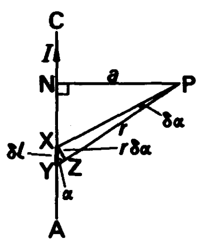
To verify $B \propto 1/r$ when I and N are constant, $(n_1^2+n_2^2)$ is evaluated when the wire is wound round other sets of pegs and I and N are kept constant each time. A graph of $(n_1^2+n_2^2)$ against $1/r$ is then plotted. This is found to be a straight line passing through the origin. Hence $B \propto 1/r$ when I and N are constant.

In a similar way, by varying the current I it can be shown that $B \propto I$ when N and r are constant. Similar experiments show that $B \propto N$ when I and r are constant. Thus $B \propto NI/r$ for a narrow circular coil.

The induction B in a magnetic field may also be measured by means of a *ballistic galvanometer* or by an *a.c. method*. See pp. 920, 934.

Field due to Long Straight Wire

We now deduce the induction at a point outside a long straight wire. In Fig. 37.3 (i), AC represents part of a long straight wire. P is taken as a point so near it that, from P, the wire looks infinitely long—it subtends very nearly 180°. An element XY of this wire, of length δl, makes an angle α with the radius vector, r, from P. It therefore contributes to the magnetic field at P an amount



(i)

FIG. 37.3 (i). Field of a long, straight wire.

$$\delta B = \frac{\mu_0 I \delta l \sin \alpha}{4\pi r^2} \quad . \quad \text{(i)}$$

when the wire carries a current I. If α is the perpendicular distance, PN, from P to the wire, then

$$\text{PN} = \text{PX} \sin \alpha$$

or

$$a = r \sin \alpha,$$

whence

$$r = \frac{a}{\sin \alpha}. \quad . \quad . \quad . \quad \text{(ii)}$$

Also, if we draw XZ perpendicular to PY, we have

$$\text{XZ} = \text{XY} \sin \alpha = \delta l \sin \alpha.$$

If δl subtends an angle $\delta\alpha$ at P, then

$$\text{XZ} = r\delta\alpha = \delta l \sin \alpha.$$

From (i),

$$\therefore \delta B = \frac{\mu_0 I \delta l \sin\alpha}{4\pi r^2} = \frac{\mu_0 I r \delta\alpha}{4\pi r^2} = \frac{\mu_0 I \delta\alpha}{4\pi r}$$

From (ii),

$$\therefore \delta B = \frac{\mu_0 I \sin\alpha\delta\alpha}{4\pi a}$$

When the point Y is at the bottom end A of the wire, $\alpha = 0$; and when Y is at the top C of the wire, $\alpha = \pi$. Therefore the total magnetic field at P is

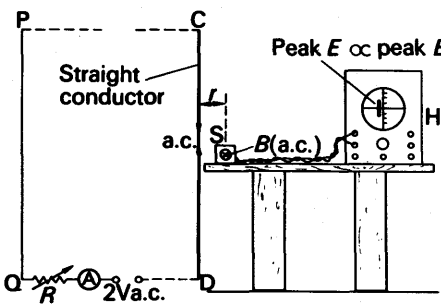
$$B = \frac{\mu_0}{4\pi}\int_0^\pi \frac{I \sin\alpha\delta\alpha}{a} = \frac{\mu_0 I}{4\pi a}\Big[-\cos\alpha\Big]_0^\pi$$

$$\therefore B = \frac{\mu_0 I}{2\pi a}. \qquad (1)$$

Equation (1) shows that the magnetic field of a long straight wire, at a point near it, is inversely proportional to the distance of the point from the wire. The result was discovered experimentally by Biot and Savart using a vibration magnetometer method, and led to their general formula in (i) which we used to derive (1).

Variation of *B* with Distance—A.C. Method

An apparatus suitable for finding the variation of B with distance from a long straight wire CD is shown in Fig. 37.3 (ii). Alternating current (a.c.) of the order of 10 A, from a low voltage mains transformer, is passed through CD by using another long wire PQ at least one metre away, a rheostat B and an a.c. ammeter A. A small search coil S, with thousands of turns of wire, such as the coil from an output transformer, is placed near CD. It is positioned with its axis at a small distance r from CD



(ii)

FIG. 37.3 (ii). Investigation of B due to long straight conductor.

and so that the flux from CD enters its face normally. S is joined by long twin flex to the Y-plates of an oscilloscope H and the greatest sensitivity, such as 5 mV/cm, is used.

When the a.c. supply is switched on, the varying flux through S produces an induced alternating e.m.f. E. The peak value of E can be determined by switching off the time-base and measuring the length of the line trace, Fig. 37.3 (ii). See p. 1014. Now the peak value of B, the magnetic induction, is proportional to the peak value of E, as shown in the case of the simple dynamo on p. 907. Thus the length of the trace gives a measure of the peak value of B.

The distance r of the coil from CD is then increased and the corresponding length of the trace is measured. The length of the trace plotted against $1/r$ gives a straight line graph passing through the origin. Hence $B \propto 1/r$. A similar method can be used for investigating the induction B for the case of a narrow circular coil or for a solenoid (p. 939).

EXAMPLE

Calculate the flux density at a distance of 1 cm or 0·01 m from a very long vertical straight wire carrying a current of 10 A. At what distance from the wire will the field induction neutralize that due to the earth's horizontal component flux-density, $0{\cdot}2 \times 10^{-4}$ T?

(i)
$$B = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 10^{-2}}$$
$$= 2 \times 10^{-4}\ \text{T}.$$

(ii) 1 cm from the wire, the induction due to the current is 2×10^{-4} T.

Now B is inversely-proportional to a, the distance from the wire. Thus B is $0{\cdot}2 \times 10^{-4}$ Wb m^{-2}, or ten times smaller than at 1 cm, at a distance 10 times as great. Thus the distance is 10 cm.

Note that the actual position of the point where the two fields neutralize must take account of the fact that B is a *vector*, that is, it has direction and magnitude. For a downward current of 10 A, the point concerned is due east of the wire. It is called a *neutral* point.

Field along Axis of a Narrow Circular Coil

We will now find the magnetic field at a point anywhere on the axis of a narrow circular coil (P in Fig. 37.4). We consider an element δl of the coil, at right angles to the plane of the paper. This sets up a field δB at P, in the plane of the paper, and at right angles to the radius vector r. If β is the angle between r and the axis of the coil, then the field δB has components $\delta B \sin \beta$ along the axis, and

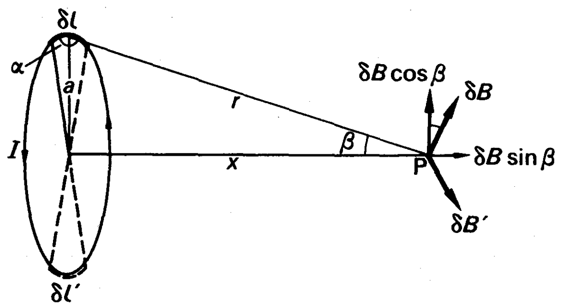


FIG. 37.4. Field on axis of flat coil.

$\delta B \cos \beta$ at right angles to the axis. If we now consider the element $\delta l'$ diametrically opposite to δl, we see that it sets up a field $\delta B'$ equal in magnitude to δB. This also has a component, $\delta B' \cos \beta$, at right angles to the axis; but this component acts in the opposite direction to $\delta B \cos \beta$ and therefore cancels it. By considering elements such as δl and $\delta l'$ all round the circumference of the coil, we see that the field at P can have no component at right angles to the axis. Its value along the axis is

$$B = \int dB \sin \beta.$$

From Fig. 37.4, we see that the length of the radius vector r is the same for all points on the circumference of the coil, and that the angle α is also constant, being 90°. This, if the coil has a single turn, and carries a current I,

$$\delta B = \frac{\mu_0 I \delta l \sin \alpha}{4\pi r^2} = \frac{\mu_0 I}{4\pi r^2}\delta l.$$

And, if the coil has a radius a, then

$$B = \int dB \sin \beta = \int_0^{2\pi a} \frac{\mu_0 I}{4\pi r^2} dl \sin \beta$$

$$= \frac{\mu_0 I a \sin \beta}{2r^2}. \qquad \text{(i)}$$

When the coil has more than one turn, the distance r varies slightly from one turn to the next. But if the width of the coil is small compared with all its other dimensions, we may neglect it, and write,

$$B = \frac{\mu_0 N I a \sin \beta}{2r^2}, \qquad \text{(ii)}$$

where N is the number of turns.

Equation (i) can be put into a variety of forms, by using the facts that

$$\sin \beta = \frac{a}{r},$$

and

$$r^2 = x^2 + a^2,$$

where x is the distance from P to the centre of the coil. Thus

$$B = \frac{\mu_0 N I a^2}{2r^3} = \frac{\mu_0 N A I}{2\pi (x^2 + a^2)^{3/2}}, \qquad (1)$$

When the distance x is large compared with a, the expression (1) reduced to:

$$B = \frac{\mu_0 N A I}{2\pi x^3} = \frac{\mu_0 m}{2\pi x^3}, \qquad (2)$$

where $m = NAI$ = the *magnetic moment* of the circular coil, p. 885.

Helmholtz Coils

The field along the axis of a single coil varies with the distance x from the coil. In order to obtain a *uniform* field, Helmholtz used two coaxial parallel coils of equal radius R, separated by a distance R. In this case, when the same current flows round each coil in the same direction, the resultant field B is uniform for some distance on either side of the point on their axis midway between the coils. This may be seen roughly by adding the fields due to each coil alone. Helmholtz coils were used in Thomson's determination of e/m (p. 1003).

The magnitude of the resultant field B at the midpoint can be found from our previous formula for a single coil. We now have $a = R$ and $x = R/2$. Thus, for the two coils,

$$B = 2\times\frac{\mu_0 NIR^2}{2(R^2/4+R^2)^{3/2}} = \left(\frac{4}{5}\right)^{3/2}\times\frac{\mu_0 NI}{R}$$

$$= 0{\cdot}72\frac{\mu_0 NI}{R}\text{ (approx.).}$$

Field on Axis of a Long Solenoid

We may regard a solenoid as a long succession of narrow coils; if it has n turns per metre, then in an element δx of it there are $n\delta x$ coils (Fig. 37.5). At a point P on the axis of the solenoid, the field due to these is, by equation (ii),

$$\delta B = \frac{\mu_0 Ia\sin\beta}{2r^2}n\delta x,$$

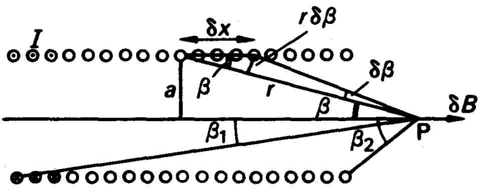


FIG. 37.5. Field on axis of solenoid.

in the notation which we have used for the flat coil. If the element δx subtends an angle $\delta\beta$ at P, then, from the figure,

$$r\delta\beta = \delta x\sin\beta;$$

whence

$$\delta x = \frac{r\delta\beta}{\sin\beta}.$$

Also,

$$a = r\sin\beta.$$

Thus

$$\delta B = \frac{\mu_0 Ir\sin^2\beta}{2r^2}n\frac{r\delta\beta}{\sin\beta}.$$

$$= \frac{\mu_0 nI}{2}\sin\beta\delta\beta.$$

If the radii of the coil, at its ends, subtend the angles β_1 and β_2 at P, then the field at P is

$$H = \int_{\beta^1}^{\beta_2}\frac{\mu_0 nI}{2}\sin\beta d\beta$$

$$= \frac{\mu_0 nI}{2}\Big[-\cos\beta\Big]_{\beta_2}^{\beta_2},$$

$$= \frac{\mu_0 nI}{2}(\cos\beta_1-\cos\beta_2). \quad . \quad . \quad . \quad . \quad (1)$$

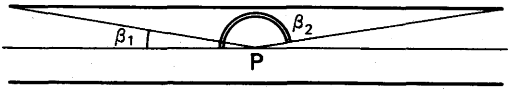


FIG. 37.6. A very long solenoid.

If the point P is inside a very long solenoid—so long that we may regard it as infinite—then $\beta_1 = 0$ and $\beta_2 = \pi$, as shown in Fig. 37.6. Then, by equation (1):

$$B = \frac{\mu_0 nI}{2}\Big[-\cos\beta\Big]_0^{\pi}.$$

whence

$$B = \mu_0 nI \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The quantity nI is often called the 'ampere-turns per metre'.

Very Long Solenoid or Toroid

Equation (1) shows that the field along the axis of an infinite solenoid is constant: it depends only on the number of turns per centimetre, and the current. By methods beyond the scope of this book, it can also be shown that the field is the same at points not on the axis. An infinite solenoid therefore gives us a means of producing a uniform magnetic field.

In practice, solenoids cannot be made infinitely long. But if the length of a solenoid is about ten times its diameter, the field near its middle is fairly uniform, and has the value given by equation (1).

A form of coil which gives a very nearly uniform field is shown in Fig. 37.7. It is a solenoid of N turns and length L metre wound on a

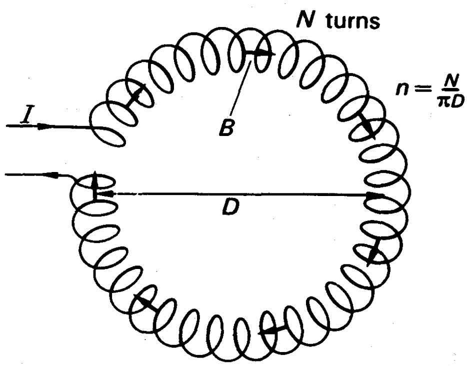


FIG. 37.7. A toroid.

circular support instead of a straight one, and is called a toroid. If its average diameter D is several times its core diameter d, then the turns of wire are almost equally spaced around its inside and outside circumferences; their number per metre is therefore

$$n = \frac{N}{L} = \frac{N}{\pi D} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The magnetic field within a toroid is very nearly uniform, because the coil has no ends. The coil is equivalent to an infinitely long solenoid, and the field-strength at all points within it is given by

$$B = \mu_0 nI \quad . \quad . \quad . \quad . \quad (2)$$

A 'Slinky' is a coil which can be stretched to provide simply a solenoid with a varying number of turns per metre, n. A small search coil with many thousands of turns, placed coaxially inside the solenoid, can be connected to an oscilloscope to provide a measure of B when alternating current is passed into the solenoid. See p. 934. Since $n \propto 1/L$, where L is the length of the coil, a graph of B against $1/L$ can be plotted for various values of L, the current being the same each time. A straight line through the origin is obtained, showing that $B \propto n$. A search coil connected to a ballistic galvanometer, and a direct current which is reversed in the coil, may also be used to provide a measure of B (p. 919).

Forces between Currents

Ampère carried out many experiments on the forces of attraction and repulsion between two current-carrying conductors. Each was acted on by the field of the other, as shown in Fig. 37.8. Currents flowing in the same direction ('like' currents) attracted each other,

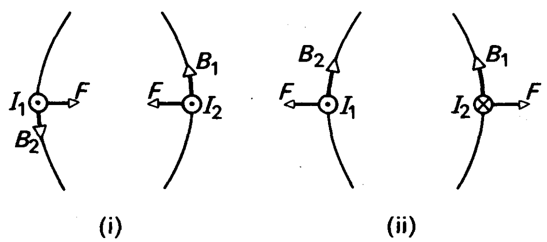


FIG. 37.8. Forces between currents.

Fig. 37.8 (i), while 'unlike' currents (in opposite directions) repelled each other, Fig. 37.8 (ii). This difference from the laws governing poles and charges greatly impressed Ampère.

If two long straight conductors lie parallel and close together at a distance r apart, and carry currents I, I' respectively, then the current I is in a magnetic field of flux density B equal to $\mu_0 I'/2\pi r$ due to the current I (p. 934). The force per metre length F is hence given by

$$F = BIl = BI \times 1 = \frac{\mu_0 I'}{2\pi r} \times I \times 1$$

$$\therefore F = \frac{\mu_0 II'}{2\pi r} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Nowadays the ampere is *defined* in terms of the force between conductors. It is *that current, which flowing in each of two infinitely-long* *parallel straight wires of negligible cross-sectional area separated by a distance of* 1 *metre in vacuo, produces a force between the wires of* 2×10^{-7} *newton metre*$^{-1}$.

Taking $I = I' = 1$ A, $r = 1$ metre, $F = 2 \times 10^{-7}$ newton metre^{-1}, then, from (1),

$$2 \times 10^{-7} = \frac{\mu_0 \times 1 \times 1}{2\pi \times 1}$$

$$\therefore \mu_0 = 4\pi \times 10^{-7} \text{ henry metre}^{-1},$$

which is the value quoted above.

Unit of μ_0

The permeability of free space μ_0 was defined previously from the relation

$$\delta B = \frac{\mu_0 I \delta s \sin\theta}{4\pi r^2}$$

From this relation, the unit of μ_0 is

$$\frac{\text{weber metre}^{-2} \times \text{metre}^2}{\text{ampere} \times \text{metre}} \text{ or Wb A}^{-1}\text{ m}^{-1} \quad . \quad (2)$$

Now the unit of inductance L is the henry (H), which can be defined from the relation $\Phi = LI$ (p. 925). Thus, since $L = \Phi/I$

$$1 \text{ H} = 1 \text{ Wb A}^{-1}.$$

From (2), it follows that the unit of μ_0 can be written as

$$\text{H m}^{-1} \text{ (henry per metre)},$$

and this is the SI unit of μ_0 and of permeability μ generally.

Absolute Determination of Current

A laboratory form of an *ampere balance*, which measures current by measuring the force between current-carrying conductors, is shown in Fig. 37.9.

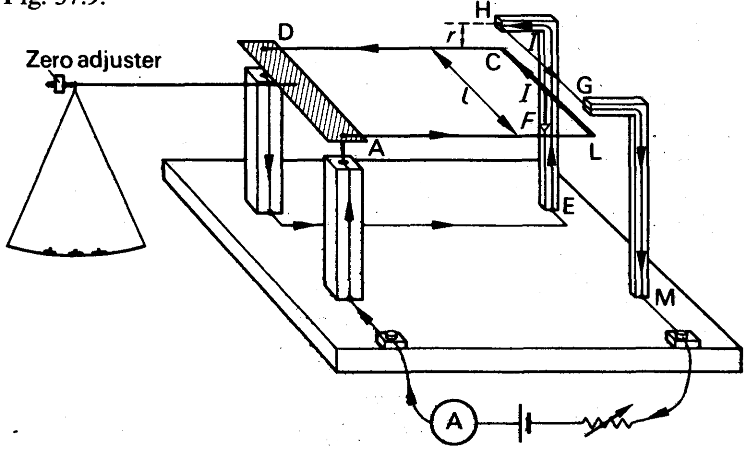


FIG. 37.9. Laboratory form of Ampere Balance.

With no current flowing, the zero screw is adjusted until the plane of ALCD is horizontal. The current I to be measured is then switched on so that it flows through ALCD and EHGM in series and HG repels CL. The mass m necessary to restore balance is then measured, and mg is the force between the conductors since the respective distances of CL and the scale pan from the pivot are equal. The equal lengths l of the straight wires CL and HG, and their separation r, are all measured.

From equation (1) on p. 939,

$$\text{force per metre} = \frac{4\pi \times 10^{-7} I^2}{2\pi r}$$

$$\therefore mg = \frac{4\pi \times 10^{-7} I^2 l}{2\pi r}$$

$$\therefore I = \sqrt{\frac{mgr}{2 \times 10^{-7} l}}$$

In this expression, I will be in amperes if m is in kilogrammes, $g = 9{\cdot}8 \text{ m s}^{-2}$ and l and r are measured in metres.

Magnetizing Force, or Intensity, *H*

Biot and Savart's law has been stated as

$$\delta B = \frac{\mu_0 I \delta l \sin \alpha}{4\pi r^2}$$

This is only true in air or a vacuum. In other materials the flux density may be altered, even though the currents remain the same. To take account of this we write

$$\delta B = \frac{\mu I \delta l \sin \alpha}{4\pi r^2}$$

where μ is the *permeability* of the medium. μ_0 is called the permeability of 'free space' or vacuum.

So far we have only used the flux density, B, in a field. Another field quantity, symbol H, is also used. It is called the *magnetizing force or intensity*, and is defined by the relation

$$H = \frac{B}{\mu}.$$

We may thus write δH arising from a current I in an element of length δl as

$$\delta H = \frac{I \delta l \sin \alpha}{4\pi r^2} \qquad . \quad . \quad . \quad . \qquad (1)$$

From this it can be seen that the unit of H is

$$\frac{\text{ampere metre}}{\text{metre}^2} \quad \text{or} \quad \text{ampere metre}^{-1}\ (\text{A m}^{-1});$$

whereas the unit of B in the tesla (T) or weber metre^{-2} (Wb m^{-2}).

In any medium δB has a value depending on the permeability of the medium. From (1), it can be seen that H does not depend on μ but only the currents and their geometry. H is *independent*, therefore, of the medium in which the conductors are situated. It is for this reason that H is regarded as being due directly to the currents. H is then a 'cause' which gives rise to a flux density B given by μH, and so B is dependent on the medium used. Because of this interpretation, H is often called the 'magnetizing force', or 'magnetizing intensity'.

From equation (1) on p. 934, the magnetizing force due to a straight wire is given by

$$H = \frac{I}{2\pi a}.$$

Ampère's Theorem

In the calculation of magnetic fields, we have used so far only the Biot and Savart law. Another law useful for calculating magnetic field strengths is *Ampère's theorem.*

Consider Fig. 37.10, in which a continuous closed line or loop L is drawn round the wires P, Q, R which carry currents of I_1, I_2, I_3 respectively. The total current enclosed by L is $(I_1+I_2+I_3)$. Now if H is the magnetizing force or *magnetic field intensity* at any element dl

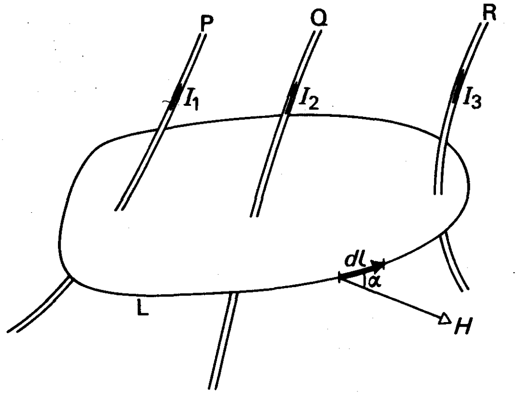


FIG. 37.10. Ampere's theorem.

in the loop L, obtained from the Biot and Savart law, it is possible to calculate the quantity $H.dl.\cos\alpha$, where α is the angle between the element and the field H. When the quantity $H.dl.\cos\alpha$ is summed for every element in the closed loop L, calculation beyond the scope of this book shows that the result is $(I_1+I_2+I_3)$. This is expressed by

$$\oint H.dl.\cos\alpha = I_1+I_2+I_3,$$

where the symbol $\oint$ represents the integral taken completely round the closed loop. Ampère's theorem is the general statement

$$\oint H.dl.\cos\alpha = I,$$

where I is the total current enclosed by the loop.

We now apply the theorem to two special cases of current-carrying conductors.

1. *Straight wire*

Fig. 37.11 shows a circular loop L of radius r, drawn concentrically round a straight wire carrying a current I. The lines of force are circles and hence, at *every* part of a closed line, H is directed along the line itself. Thus $\alpha = 0°$ all round the line. Further, by symmetry, H has the same value everywhere on the line.

$$\therefore \oint H\,.\,dl\,.\cos\alpha = H\oint dl = H\,.\,2\pi r,$$

since $\alpha = 0°$ and H is constant. Hence, from Ampère's theorem,

$$H\,.\,2\pi r = I$$

$$\therefore H = \frac{I}{2\pi r}, \quad \text{and} \quad B = \mu_0 H = \frac{\mu_0 I}{2\pi r}.$$

This agrees with the result for B on p. 934.

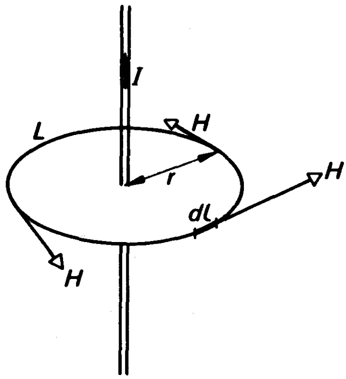


FIG. 37.11. Field intensity of straight wire.

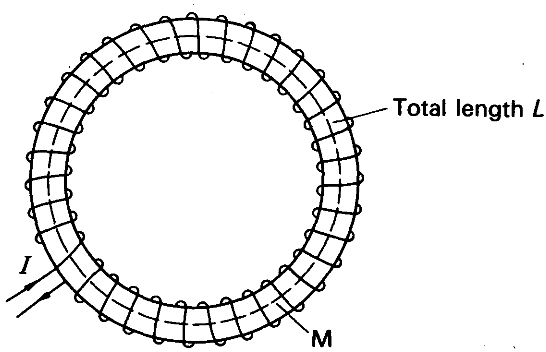


FIG. 37.12. Field intensity of toroid.

2. *Toroid*

Consider the closed loop M indicated by the broken line in Fig. 37.12. Again H is everywhere the same on M and is directed along the loop.

$$\therefore \oint H\,.\,dl\cos\alpha = H\oint dl = HL,$$

where L is the total length of the loop M. Hence, from Ampere's theorem,

$$HL = NI$$

$$\therefore H = \frac{NI}{L} = nI,$$

where N is the total number of turns, and n is the number of turns per metre. This agrees with p. 938.

Earth's Magnetism

It was Dr. Gilbert who first showed that a magnetized needle, when freely suspended about its centre of gravity, dipped downwards towards the north at about 70° to the horizontal in England. He also found that this *angle of dip* increased with latitude, as shown in Fig. 37.13, and concluded that the earth itself was, or contained, a magnet. The points where the angle of dip is 90° are called the earth's magnetic poles; they

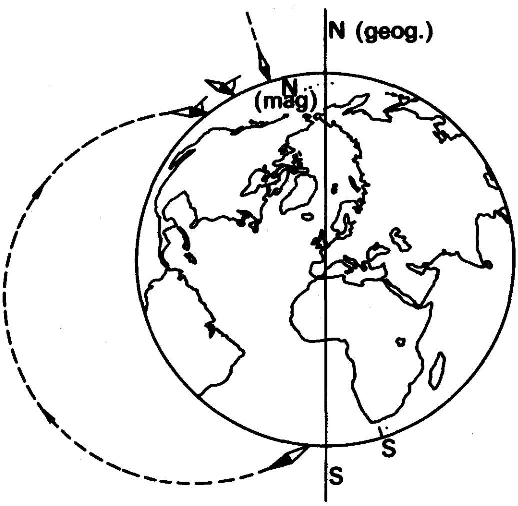


FIG. 37.13. Illustrating the angle of dip.

are fairly near to the geographic poles, but their positions are continuously, though slowly, changing. Gilbert's simple idea of the earth as a magnet has had to be rejected. The earth's crust does not contain enough magnetic material to make a magnet of the required strength; the earth's core is, we believe, molten—and molten iron is non-magnetic. The origin of the earth's magnetism is, in fact, one of the great theoretical problems of the present day.

Horizontal and Vertical Components. Variation and Dip

Since a freely suspended magnetic needle dips downward at some angle δ to the horizontal, the earth's *resultant magnetic field*, B_R acts at an angle δ to the horizontal. The 'angle of dip', or *inclination*, can thus be defined as the angle between the resultant earth's field and the horizontal. The earth's field has a *vertical component*, B_V, given by

$$B_V = B_R \sin \delta, \qquad (1)$$

and a *horizontal component*, B_H, given by

$$B_H = B_R \cos \delta. \qquad (2)$$

Also,
$$\frac{B_V}{B_H} = \tan \delta. \qquad (3)$$

To specify the earth's magnetic field at any point, we must state its

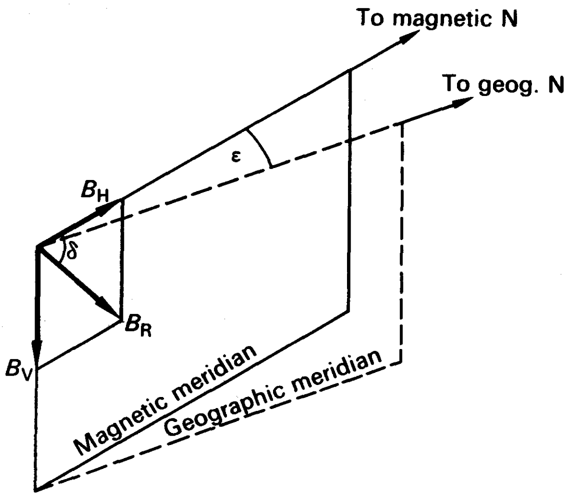


FIG. 37.14. Magnetic and geographic meridians. Dip.

strength and direction. To specify its direction we must give the direction of the magnetic meridian, and the angle of dip δ (Fig. 37.14). In most parts of the world the magnetic meridian does not lie along the geographic meridian (the vertical plane running geographically north–south). The angle between the magnetic and geographic meridians, ε, is called the magnetic variation, or sometimes the declination, at the place concerned; it is shown on the margins of maps. The horizontal and vertical components of the earth's field, and the angle of dip, can be measured by a large coil or 'earth inductor' (see p. 921).

EXERCISES 37

1. Define the *ampere.* Write down expressions for (i) the magnetic field strength (magnetizing force) at a distance of d from a very long straight conductor carrying a current I, and (ii) the mechanical force acting on a straight conductor of length l carrying a current I at right angles to a uniform magnetic field of flux density B.

Show how these two expressions may be used to deduce a formula for the force per unit length between two long straight parallel conductors in vacuo carrying currents I_1 and I_2 separated by a distance d.

A horizontal straight wire 5 cm long weighing $1{\cdot}2\ \text{g m}^{-1}$ is placed perpendicular to a uniform horizontal magnetic field of flux density $0{\cdot}6\ \text{Wb m}^{-2}$. If the resistance of the wire is $3{\cdot}8\ \text{ohm m}^{-1}$, calculate the p.d. that has to be applied between the ends of the wire to make it just self-supporting. Draw a diagram showing the direction of the field and the direction in which the current would have to flow in the wire. (*C.*)

2. State the law of force acting on a conductor carrying an electric current in a magnetic field. Indicate the direction of the force and show how its magnitude depends on the angle between the conductor and the direction of the field.

Sketch the magnetic field due solely to two long parallel conductors carrying respectively currents of 12 and 8 A in the same direction. If the wires are 10 cm apart, find where a third parallel wire also carrying a current must be placed so that the force experienced by it shall be zero. (*L.*)

3. Define the *ampere.*

Two long vertical wires, set in a plane at right angles to the magnetic meridian, carry equal currents flowing in opposite directions. Draw a diagram showing the pattern, in a horizontal plane, of the magnetic flux due to the currents alone—that is, for the moment ignoring the earth's magnetic field.

Next, taking into account the earth's magnetic field, discuss the various situations that can give rise to neutral points in the plane of the diagram.

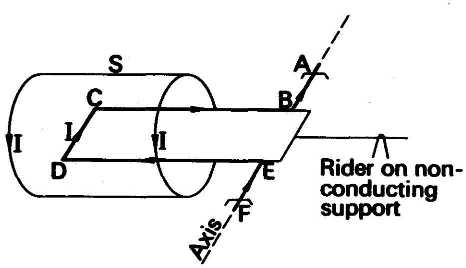


FIG. 37.15.

Fig. 37.15 shows a simple form of current balance. The 'long' solenoid S, which has 2000 turns per metre, is in series with the horizontal rectangular copper loop ABCDEF, where BC = 10 cm and CD = 3 cm. The loop, which is freely pivoted on the axis AF, goes well inside the solenoid, and CD is perpendicular to the axis of the solenoid. When the current is switched on, a rider of mass 0·2 g placed 5 cm from the axis is needed to restore equilibrium. Calculate the value of the current, I. (*O.*)

4. Define *magnetic moment.*

A small magnet, suspended with its axis horizontal so as to be able to rotate freely about a vertical axis, is situated at the centre of a long horizontal solenoid, the axis of which lies at right angles to the magnetic meridian. If the solenoid has 20 turns per cm, determine the value of the current passing through it which would cause the magnet to rotate through 50°. (Horizontal component of earth's magnetic field intensity = 14 A m^{-1}.) (*N.*)

5. Describe with experimental details how you would carry out any *two* of the following in the laboratory:
(*a*) Determine the current sensitivity of a moving-coil galvanometer.
(*b*) Determine the internal resistance of a dry cell using a potentiometer.
(*c*) Investigate, using a vibration magnetometer, how the magnetic field due to a long straight wire carrying a steady current varies with distance from the wire. (*L.*)

6. Define the *ampere.*

Draw a labelled diagram of an instrument suitable for measuring a current absolutely in terms of the ampere, and describe the principle of it.

A very long straight wire PQ of negligible diameter carries a steady current I_1. A square coil ABCD of side l with n turns of wire also of negligible diameter is set up with sides AB and DC parallel to and coplanar with PQ; the side AB is nearest to PQ and is at a distance d from it. Derive an expression for the resultant force on the coil when a steady current I_2 flows in it, and indicate on a diagram the direction of this force when the current flows in the same direction in PQ and AB.

Calculate the magnitude of the force when $I_1 = 5$ A, $I_2 = 3$ A, $d = 3$ cm, $n = 48$ and $l = 5$ cm. (*O. & C.*)

7. (i) Draw a diagram showing the pattern of the magnetic field lines in a horizontal plane passing through the centre of a plane vertical circular coil carrying a steady current. Indicate on your diagram possible positions for neutral points if the plane of the coil were to be set in the magnetic meridian.

(ii) How would you show experimentally that the value of B at the centre of such a coil is proportional to nI/r, where n is the number of turns and r the radius?

8. A long straight wire carries a steady current I. Write down a formula for the magnetic intensity (magnetizing force) H due to the current at a point distant y from the wire. Give a consistent set of units for the quantities in your formula.

Show on a diagram the lines of magnetic force due to the currents when two parallel wires separated by a distance $2a$ carry equal currents in the same direction. Use your formula to derive an expression for the magnetic intensity at a point P in a plane perpendicular to the wires at a distance x from the point midway between the wires and along the right bisector of the line joining them in this plane. Hence derive the condition that the intensity at P shall be a maximum. (*N.*)

chapter thirty-eight

Magnetic Properties of Materials

THE magnetic properties of materials require investigation to decide whether they are suitable for permanent magnets such as loudspeaker magnets, for temporary magnets such as electromagnets, or for cores of electromagnetic induction machines such as transformers.

Induction or Flux Density in Magnetic Material

Consider a toroid of length L, wound with N turns each carrying a current I round a ring of magnetic material, Fig. 38.1.

The *total flux density B* in the material is partly due to the currents

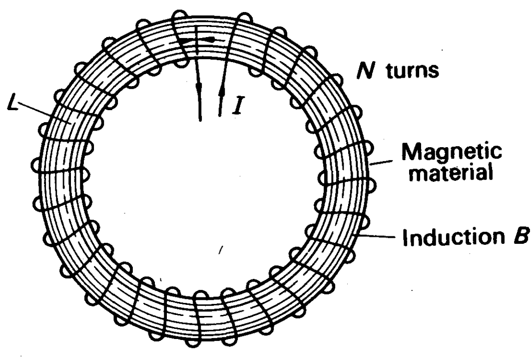


FIG. 38.1. Induction in magnetic material.

flowing in the wire and partly due to the magnetization of the material. We thus write

$$B = B_0 + B_M, \quad . \quad . \quad . \quad . \quad (1)$$

where B_0 is the flux density due directly to the current in the wire, and B_M is the flux density due to the magnetization of the material.

We now assume that the induction B_M is produced by many small circulating currents inside the magnetic material, due to the circulating and spinning electrons in the atoms. Fig. 38.2 shows that the effect of many small adjacent current loops may be thought of as one current loop. In the same way, the internal circulating and spinning currents can be replaced by a single current I_M flowing in the coil wound round the core. This theoretical surface or magnetization current is additional to the real or actual current I flowing in the coil.

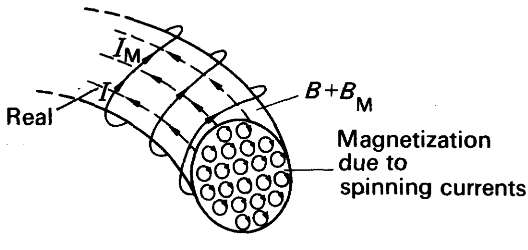


FIG. 38.2. Magnetizing surface current.

By itself, the real current I produces a flux-density B_0. If the toroid has n turns per unit length ($n = N/L$), then, from p. 938,

$$B_0 = \mu_0 nI.$$

The surface current I_M may be imagined to flow in n turns per metre of the solenoid, as the real current does. The flux density or induction B_M is then given by

$$B_M = \mu_0 n I_M.$$

This is the induction which would be produced by the current I_M if the material were not present in the toroid, that is, a current I_M in the coil would produce a flux density equal to that due to the magnetization in the material.

$$\therefore \text{ total induction } B = B_0 + B_M = \mu_0 n(I + I_M) \quad . \quad (2)$$

Intensity of Magnetization

It is possible to write B_M, the induction due to the magnetization of the material, in a different way. The magnetic moment of each turn due to this imaginary surface current $= A \times I_M$, where A is the area of each turn. See p. 885. The magnetic moment of the whole toroid is then $nLAI_M$, since nL is the total number of turns. Hence the *magnetic moment per unit volume*

$$= \frac{nLAI_M}{\text{volume}} = \frac{nLAI_M}{LA} = nI_M$$

The 'magnetic moment per unit volume' is called the *intensity of magnetization, M,* of the magnetic core in the toroid. Thus $B_M = \mu_0 nI_M = \mu_0 M$. Further, nI is the magnetic field intensity, H, in the toroid due to the current I.

Thus the *total induction or flux-density* B in the core when the coil carries a current is given by

$$B = B_0 + B_M = \mu_0 nI + \mu_0 nI_M$$

$$\therefore B = \mu_0(H + M) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) relates the total induction B to the magnetizing field intensity H due to the current and to the intensity of magnetization M of the material produced. Note that H and M have the same units from (3). *Thus the unit of M is ampere per metre* (A m^{-1}).

EXAMPLE

A solenoid 1 m long is wound with 10^4 turns of copper wire each carrying a current of 10 A. If the cross-sectional area of the coil is 10 cm^2, calculate (*a*) the magnetizing force or intensity in the solenoid, (*b*) the couple exerted on the solenoid when it is placed at right angles to an external field of induction 10^{-2} T.

If the solenoid is now filled with certain magnetic material, the induction in the material is 1·5 T. Calculate (*c*) the intensity of magnetization in the material, (*d*) the total couple exerted when the solenoid and material inside is placed at right angles to a field of induction 10^{-2} T.

(*a*) We have $H = nI$, where n is the number of turns per metre,

$$= 10^4 \times 10$$

$$= 10^5 \text{ ampere metre}^{-1} \text{ (A m}^{-1}\text{)}.$$

(*b*) Magnetic moment of each solenoid turn $= IA$.

$$\therefore \text{ magnetic moment of empty solenoid, } m, = NIA.$$

Since $N = 10^4$, $I = 10$ A, $A = 10$ cm^2 $= 10 \times 10^{-4}$ m^2,

$$\therefore m = 10^4 \times 10 \times 10 \times 10^{-4}$$

$$= 100 \text{ A m}^2.$$

Now couple $C = mB \sin \alpha$, where α is the angle between the solenoid axis and the external field B.

$$\therefore C = mB \sin 90^\circ$$

$$= 100 \times 10^{-2}$$

$$= 1 \text{ newton metre (N m)}$$

(*c*) We have $B = \mu_0(H+M)$

$$\therefore M = \frac{B-\mu_0 H}{\mu_0}$$

$$= \frac{1{\cdot}5 - 4\pi \times 10^{-7} \times 10^5}{4\pi \times 10^{-7}}$$

$$= 10{\cdot}9 \times 10^5 \text{ ampere metre}^{-1}$$

(*d*) Total magnetic moment, m, of the specimen is given by,

$$m = MV$$

$$= 10{\cdot}9 \times 10^5 \times (1 \times 10^{-3})$$

$$= 10{\cdot}9 \times 10^2 \text{ A m}^2$$

$\therefore$ Total magnetic moment of the solenoid and the material inside

$$= 100 + 10{\cdot}9 \times 10^2$$

$$= 11{\cdot}9 \times 10^2 \text{ A m}^2$$

$$\therefore \text{Couple} = mB \sin \alpha \text{ newton metre}$$
$$= 11{\cdot}9 \times 10^2 \times 10^{-2} \times \sin 90°$$
$$= 11{\cdot}9 \text{ N m.}$$

Relative Permeability

The permeability of a material is defined by the relation $\mu = B/H$. Now, in general, $B = \mu_0(H+M)$

$$\therefore \mu H = \mu_0(H+M)$$

or
$$\frac{\mu}{\mu_0} = 1+\frac{M}{H}$$

The ratio μ/μ_0 is called the *relative permeability*, μ_r, of the material. The ratio M/H is called the *susceptibility*, χ, of the material. Hence, from above,

$$\mu_r = 1+\chi. \qquad . \quad . \quad . \quad . \qquad (4)$$

It should be noted that μ_r and χ are dimensionless; each is the ratio of two quantities having the same dimensions. Permeability, μ, however, which is the ratio B/H, has dimensions; its unit is *henry per metre* (H m^{-1}). See p. 940.

We shall describe shortly how the variation of B with H may be measured. Here we shall anticipate the results. As a magnetic material, originally unmagnetized, is subjected to an increasing field, the intensity of magnetization M increases until it reaches a maximum value (Fig. 38.3 (*a*)). The material is then 'saturated'; that is, its magnetic

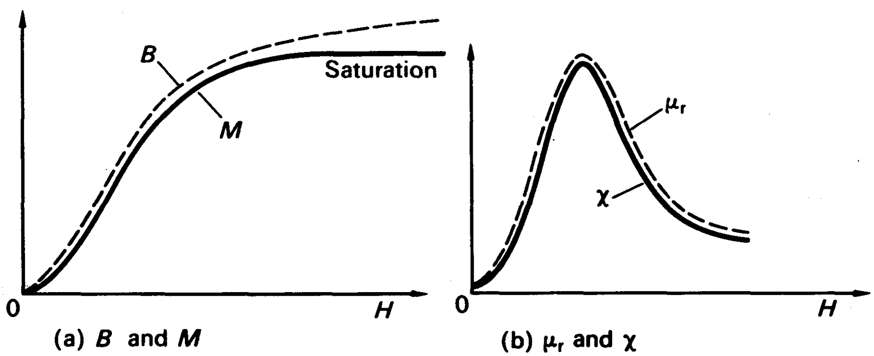


FIG. 38.3. Variation of B, M, μ_r, χ.

'domains' are completely aligned with the field H. B, however, continues to increase with H, since $B = \mu_0(H+M)$. Fig. 38.3 (*b*) also shows how the relative permeability μ_r and the susceptibility χ varies with H. It increases at first, and passes through a maximum value. As the material approaches saturation the domains cannot yield much further, and the susceptibility falls to a low value.

Variation of B with H

Ferromagnetic materials are those which have a high susceptibility. This is generally very much greater than 1, for example, 3000. Some materials are capable of retaining their magnetization and forming strong permanent magnets. Others form temporary magnets (p. 954).

The relationship between B, the flux density in a material, and the applied magnetizing field or force H, is best investigated experimentally by using a toroid shaped specimen. This eliminates the reduction in the field due to the effect of poles at the ends of a cylindrical rod. However, in the experiment described shortly, a specimen is placed inside a long current-carrying solenoid. This does not produce as large a value B as that obtained with a toroid, but the essential features of the variation of B with H are still observed.

One form of apparatus is shown in Fig. 38.4. The current I through the solenoid is measured on the ammeter A, and since $H = nI$, the current I is directly proportional to the magnetizing force H. C is a

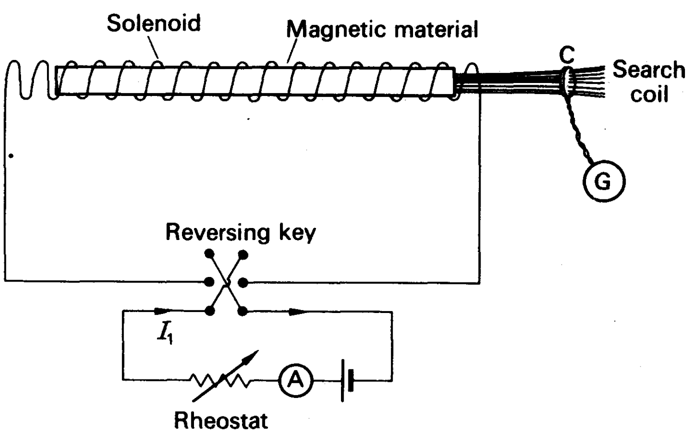


FIG. 38.4. Variation of B with H by experiment.

small search coil, which is placed in contact with the end of the specimen, and which is connected to a ballistic galvanometer G. When the coil is sharply removed from the vicinity of the specimen the throw on the galvanometer gives a measure of the flux change (see p. 920). Since the number of turns and area of the coil is constant, this throw will be proportional to the flux density B.

Ths specimen is first demagnetized (see p. 958) and placed in the solenoid. The current I is now increased in steps from zero, and the corresponding deflections θ on the ballistic galvanometer are observed. The specimen is taken through a *magnetic cycle* of magnetization. This is done by increasing I until θ is nearly constant, when the specimen has become saturated, then reducing I to zero and reversing it until saturation is reached in the opposite direction, and finally reducing I to zero and increasing it once more in the opposite direction.

H can be calculated from the relation $H = nI$. If the search coil has N turns of area A, and the ballistic galvanometer is calibrated, then B can be found from:

$$\text{Charge} = \frac{\text{Change in flux}}{\text{Resistance of circuit, } R}$$

or

$$Q = \frac{BAN}{R}$$

If R and Q are measured, then B can be found.

Sometimes it is required to plot M against H rather than B against H. In this case M is calculated, using the relation (3) on p. 949, from

$$M = \frac{B - \mu_0 H}{\mu_0}.$$

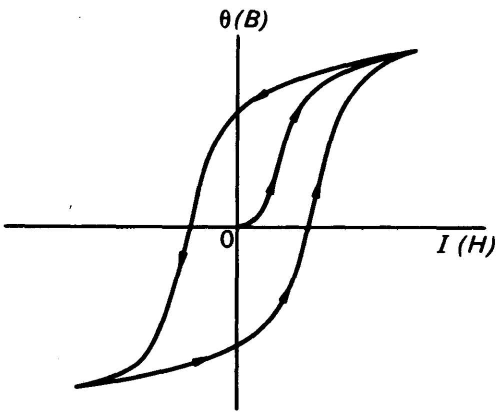


FIG. 38.5. *B-H* variation.

If only the general form of the $B-H$ curve is wanted, it is sufficient to plot θ against I. Fig. 38.5 illustrates a typical graph obtained; the arrows show the sequence in going round the magnetic cycle.

Hysteresis. Remanence. Coercive Force

Fig. 38.6 shows the variation of magnetic induction, B, with the applied field, H, when the specimen is taken through a complete cycle. After the specimen has become saturated, and the field is reduced to zero, the iron is still quite strongly magnetized, setting up a flux-density B_r. This flux-density is called the *remanence*; it is due to the tendency of groups of molecules, or domains, to stay put once they have been aligned.

When the field is reversed, the residual magnetism is opposed. Each increase of magnetizing field now causes a decrease of flux-density, as the domains are twisted farther out of alignment. Eventually, the flux-density is reduced to zero, when the opposing field H has the value

H_c. This value of H is called the *coercive force* of the iron; it is a measure of the difficulty of breaking up the alignment of the domains.

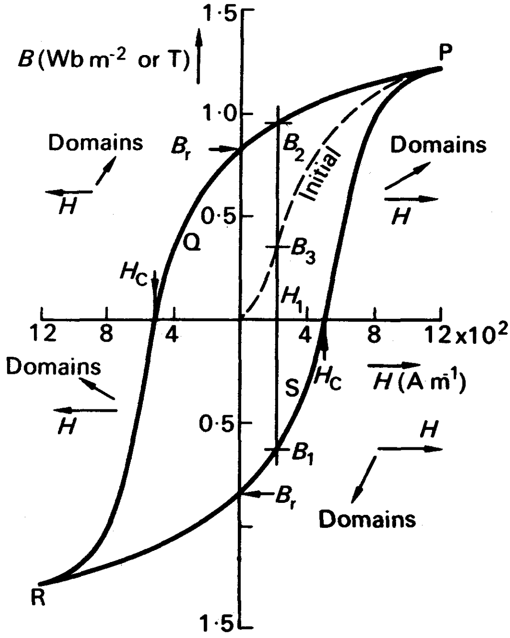


FIG. 38.6. Hysteresis loop.

We now see that, when once the iron has been magnetized, its magnetization curve never passes through the origin again. Instead, it forms the closed loop PQRS, which is called a *hysteresis loop.* Hysteresis, which comes from a Greek work meaning 'delayed', can be defined as *the lagging of the magnetic induction, B, behind the magnetizing field, H,* when the specimen is taken through a magnetic cycle.

Properties of Magnetic Materials

Fig 38.7 shows the hysteresis loops of iron and steel. Steel is more suitable for permanent magnets, because its high coercivity means that it is not easily demagnetized by shaking. The fact that the remanence of iron is a little greater than that of steel is completely outweighed by its much smaller coercivity, which makes it very easy to demagnetize. On the other hand, iron is much more suitable for electromagnets, which have to be switched on and off, as in relays. Iron is also more suitable for the cores of transformers and the armatures of machines. Both of these go through complete magnetizing cycles continuously: transformer cores because they are magnetized by alternating current, armatures because they are turning round and round in a constant field. In each cycle the iron passes through two parts of its hysteresis loop (near Q and S in Fig. 38.6), where the magnetizing field is having to demagnetize the iron. There the field is doing work against the internal

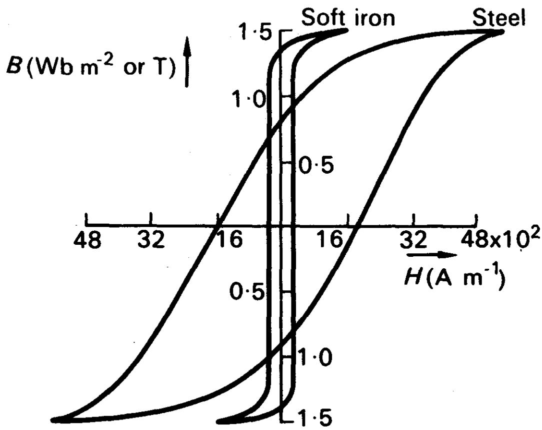


FIG. 38.7. Hysteresis curves for iron and steel.

friction of the domains. This work, like all work that is done against friction, is dissipated as heat. The energy dissipated in this way, per cycle, is less for iron than for steel, because iron is easier to demagnetize. It is called the *hysteresis loss*; we will show soon that it is proportional to the area of the B-H loop (p. 957).

In a large transformer the hysteresis loss, together with the heat developed by the current in the resistance of the windings, liberates so much heat that the transformer must be artificially cooled. The cooling is done by circulating oil, which itself is cooled by the atmosphere: it passes through pipes which can be seen outside the transformer, running from top to bottom.

The table on p. 956 gives the properties of some typical magnetic materials; mumetal and ticonal are inventions of the last twenty years, the results of deliberate attempts to develop materials with extreme properties.

Hysteresis Loss; Area of Loop

To calculate the work done in carrying a piece of iron round a hysteresis loop, we adopt the method which we used to calculate the energy stored in the magnetic field of a coil: we consider the back-e.m.f. induced during a change of flux.

Fig 38.8 shows a ring of iron, wound with a uniform magnetizing coil of N turns, and mean length l. If the current through the coil is I, the magnetizing field is

$$H = \frac{NI}{l} \quad . \quad . \quad . \quad \text{(i)}$$

And if B is the flux density in the iron, and A its cross-sectional area, the flux through it $= AB$. The flux linkages with the magnetizing coil are therefore

$$\Phi = NAB.$$

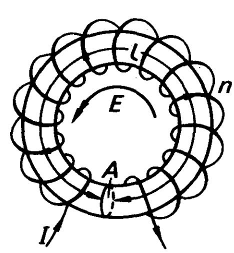


FIG. 38.8. Work done in magnetization of iron.

PROPERTIES OF MAGNETIC MATERIALS

Material	Relative Permeability μ_r	Saturation B_m 10^{-4} T	Remanence B_r 10^{-4} T	Coercivity H_c A m^{-1}	Hysteresis loss W J m^{-3} per cycle	Critical temp. t,* °C	Resistivity† ρ (approx.) 10^{-8} Ω-m	Applications
	SOFT MATERIALS (iron-like, used for electromagnets with varying currents)							
Iron (99·94%)	5500	21500	13000	81·0	500	770	10	
Nickel . .	600	6100	3600	272	30	360	7	
Cobalt . .	240	18000	5000	800	200	1120	10	
Silicon Iron (Stalloy) . (Fe 96, Si 4)	6700	20000	12000	40	350	690	55	Armatures, relays, large transformer cores, telephone diaphragms
Mumetal (Ni 74, Fe 20, Cu 5, Mn 1)	80000	8500	6000	4	20	—	—	Small transformer cores, magnetic shields
	HARD MATERIALS (steel-like, used for permanent magnets)							
Carbon Steel	—	10000	8000	4800	20000	—	—	
Cobalt Steel.	—	—	8000–10000	12000–19200	—	—	—	Moving-coil instruments, loudspeakers, microphones, telephone earpiece magnets
Ticonal (Fe 51, Co 24, Ni 14, Al 8, Cu 3)	—	—	12500	44000	—	—	—	

* See page 963.

† The higher the resistivity of a magnetic material, the less the eddy-currents in it when the flux through it is changed; and therefore the less the energy lost as heat.

Now let us suppose that, in a brief time δt, we increase the current by a small amount, and so increase the flux and flux linkages. During the change a back-e.m.f. will be induced in the coil, of magnitude

$$E = \frac{d\Phi}{dt}$$

Hence, from above,

$$E = NA\frac{dB}{dt}$$

To overcome this back-e.m.f., the source of the current I must supply energy to the coil at the rate

$$P = EI$$

Thus the total energy supplied to the coil, in increasing the flux through the iron, is

$$\begin{aligned} \delta W = P\delta t &= EI\delta t \\ &= INA\frac{dB}{dt}\delta t \\ &= INA\delta B \end{aligned}$$

where δB is the increase in flux-density.

Let us now substitute for the current I in terms of the magnetizing field H. From equation (i) we have

$$I = \frac{Hl}{N}$$

Hence

$$\begin{aligned} \delta W &= \frac{lHNA}{N}\delta B \\ &= VH\delta B \qquad \text{(ii)} \end{aligned}$$

where V, $= lA$, is the volume of the iron.

Equation (ii) shows that, in taking the flux from any value B_1 to any other B_2, the work done is

$$W = V\int_{B_1}^{B_2} HdB \qquad \text{(iii)}$$

On unit volume of the iron, the work is

$$\int_{B_1}^{B_2} HdB$$

The integral in this expression is the area between the B-H curve and the axis of B. Round the complete hysteresis loop, the work done per unit volume is

$$W = \oint HdB$$

Here the symbol $\oint$ denotes integration round the closed loop (PQRSP in Fig. 38.6); the integral is proportional to the area of the loop.

Subsidiary Hysteresis Loops

When a piece of iron is magnetized, first one way and then the other, it goes round a hysteresis loop even if it is not magnetized to saturation at any point (Fig. 38.9). The subsidiary loops, *ab* for example, may represent the magnetization of a transformer core by an alternating

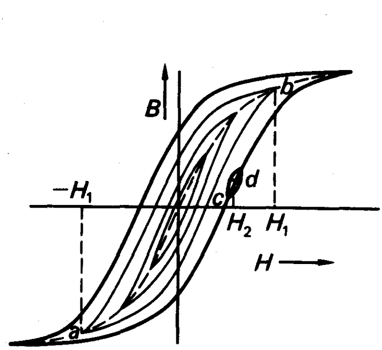


FIG. 38.9. Subsidiary hysteresis loops.

current in the primary winding: the amplitude H_1 of the magnetizing field is proportional to the amplitude of the current. A transformer core is designed so that it is never saturated under working conditions. For, if it were saturated, the flux through it would not follow the changes in primary current; and the e.m.f. induced in the secondary would be less than it should.

The energy dissipated as heat in going round a subsidiary hysteresis loop is proportional to the area of the loop, just as in going round the main one.

Another kind of subsidiary loop is shown at *cd* in the figure. The iron goes round such a loop when the field is varied above and below the value H_2. This happens in a transformer when the primary carries a fluctuating direct current.

Demagnetization

The only satisfactory way to demagnetize a piece of iron or steel is to carry it round a series of hysteresis loops, shrinking gradually to the origin. If the iron is the core of a toroid, we can do this by connecting the winding to an a.c. supply via a potential divider, as in Fig. 38.10, and reducing the current to zero. Since the iron goes through fifty loops per second, we do not have to reduce the current very slowly.

To demagnetize a loose piece of iron or steel, such as a watch, we merely put it into, and take it out of, a coil of many turns connected to an a.c. supply. As we draw the watch out, it moves into an ever-weakening field, and is demagnetized.

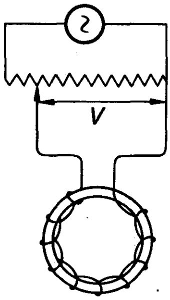


FIG. 38.10. Demagnetization.

Electromagnets and Magnetic Circuits

Consider the electromagnet shown in Fig. 38.11. The magnetic material, which is almost a complete toroid, is wound with N turns

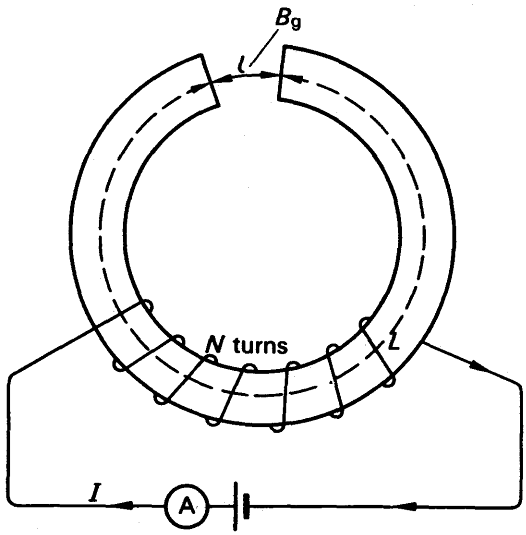


FIG. 38.11. Magnetic circuit.

each carrying a current I. The length of the magnetic material is L and of the small air gap l. We now calculate the induction B_g in the gap between the faces of magnetic material. From $B = \mu H$,

$$B_g = \mu_0 H_g \qquad (1)$$

and

$$B_m = \mu H_m, \qquad (2)$$

where H_g is the value of the magnetizing force in the gap, and H_m and B_m are respectively the magnetizing force and flux density inside the material. μ is the permeability of the material.

Taking the closed loop indicated in Fig. 38.11, by Ampere's theorem,

$$H_g l + H_m L = NI.$$

Thus from (1) and (2):

$$\frac{B_g l}{\mu_0} + \frac{B_m L}{\mu} = NI.$$

The total flux Φ round the magnetic circuit is constant. Since the area of cross-section of the air gap is the same as that of the material, it follows that

$$B_g = B_m.$$

$$\therefore B_g\left(\frac{l}{\mu_0}+\frac{L}{\mu}\right) = NI$$

or
$$B_g = \frac{NI}{(l/\mu_0 + L/\mu)} \quad . \quad . \quad . \quad (3)$$

$$\therefore \Phi = B_g A = \frac{NI}{(l/\mu_0 A + L/\mu A)}$$

In general, the value of μ depends on the current flowing in the electromagnet coil.

EXAMPLE

A toroid made from an iron bar of length 6 cm and of cross-sectional area 4 cm^2, has an air gap of length 1 cm. If it is wound with 500 turns of wire carrying a current of 20 A, find the flux density in the gap. The material may be assumed to have a permeability of 3000 times that of free space under these conditions (that is $\mu_r = 3000$). If there were no air gap, what would be the induction or flux density in the iron?

We have $N = 500$, $I = 20$ A, $l = 1$ cm $= 0{\cdot}01$ m, $\mu = \mu_r\mu_0 = 3000 \times 4\pi \times 10^{-7}$. From (3) above,

$$\therefore B_g = \frac{500 \times 20}{1/\mu_0(l + L/3000)} = \frac{500 \times 20 \times 4\pi \times 10^{-7}}{(0{\cdot}01 + 0{\cdot}0002)}$$
$$= 1{\cdot}3 \text{ Wb m}^{-2} \text{ (approx.).}$$

If $l = 0$, so that there is no air gap, and assuming the value of μ does not alter, then the flux density in the iron is now given by

$$B = \frac{NI}{L/\mu} = \frac{500 \times 20 \times 3000 \times 4\pi \times 10^{-7}}{0{\cdot}6}$$
$$= 53 \text{ Wb m}^{-2}.$$

Note that even a small air gap considerably reduces the value of the flux density obtained.

Magnetomotive Force. Reluctance.

In equation (3), NI is called the *magnetomotive force*, M.M.F., in the magnetic circuit comprising the iron and air gap. It is analogous to the e.m.f. in an electric circuit. The flux Φ through a cross-section of the circuit, which is given by BA, is analogous to the current.

The quantity $(l/A\mu_0 + L/A\mu)$ in the denominator in (3) is called the *reluctance* of the magnetic circuit. It is analogous to electrical resistance R, since $R = \rho l/A$ (p. 788). Each term in the expression for the reluctance is due to a separate part of the magnetic circuit. Thus $l/\mu_0 A$ is the reluctance of the air gap and $L/\mu A$ is that of the iron. In the above example, the reluctance of the air gap was $0{\cdot}01/(4\pi \times 10^{-7} \times A)$ whilst that of the iron was $0{\cdot}6/(3000 \times 4\pi \times 10^{-7} \times A)$. Thus the reluctance of the air gap is fifty times greater than that of the iron. This accounts for the considerable reduction in the value of B when the air gap is introduced. It should be noted that the reluctances are in series in Fig. 38.11. They are added together, as are series resistances in electric circuits.

Permanent Magnets

A permanent magnet is shown in Fig. 38.12 (i). We apply Ampère's theorem to the circuit shown by the dotted line. Here there are no external currents flowing so that, using the same notation as in the last section,

$$H_m L + H_g L = 0$$

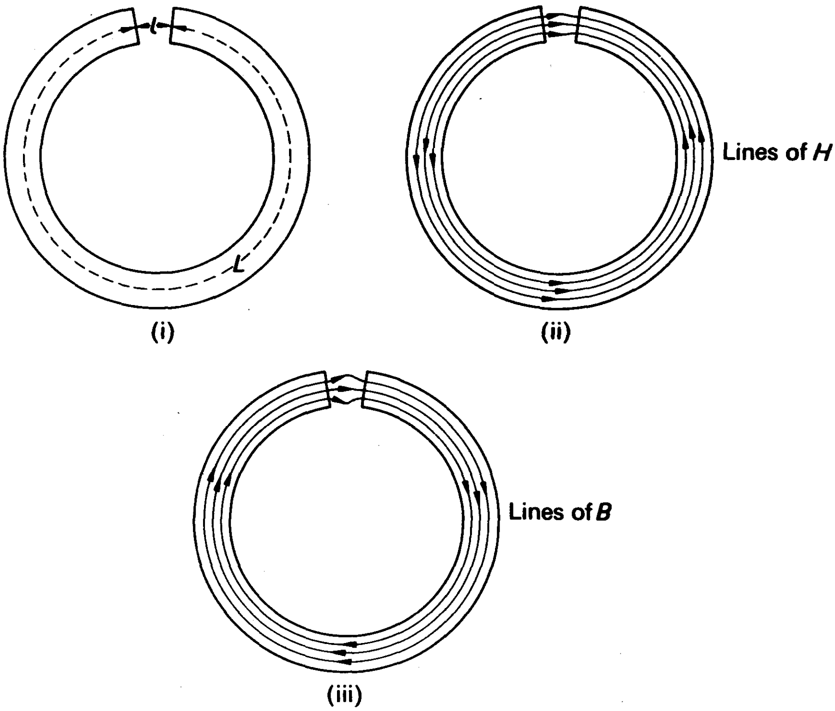


FIG. 38.12. Permanent magnet. B and H.

Also,

$$B_g = \mu H_g \qquad \text{and} \qquad B_g = B_m,$$

for the same reasons as in the case of electromagnets.
Combining these equations,

$$\therefore \frac{B_m}{H_m} = -\mu_0 \frac{L}{l} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This result expresses the fact that B *and* H *are oppositely directed inside the medium.* The lines of B and H are shown in Fig. 38.12 (ii) and (iii) respectively. Since $B = \mu H$, the value of μ in the case of this permanent magnet is $-\mu_0 L/l$. Fig. 38.13 shows the points, A or C, representing the magnetic state of the material of a permanent magnet.

The relationship between B and H is provided by the hysteresis loop. Also, B and H inside the material satisfy equation (1), which is represented by the line AOC. The only two points which satisfy both these conditions are A and C, the points of intersection of the line with the curve. At A or C, H is oppositely directed to B and hence μ is negative. For any given shape of toroidal magnet, one can predict, using (1), the greatest possible induction in the air gap.

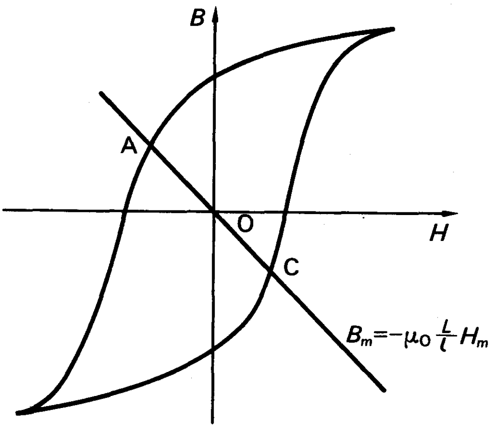


FIG. 38.13. Permanent magnet.

Diamagnetism, Paramagnetism, Ferromagnetism

We have already mentioned that the magnetic properties of materials are due to circulating and spinning electrons within the atoms.

Diamagnetism

If a magnetic field is produced in the neighbourhood of a magnetic material, a changing flux occurs in the current loops within the atoms. An e.m.f. or electric field will then be set up which causes the electrons to alter their motions, so that an extra or induced current is produced. By Lenz's law, this current gives rise to a magnetic field which *opposes* the applied magnetic field H. Thus the induced magnetization will be in the opposite direction to H, that is, M/H is negative. Hence the susceptibility χ is negative. This phenomenon is called *diamagnetism*. For a diamagnetic material, χ is generally very small, about $-0{\cdot}000015$ for bismuth, for example. The relative permeability, μ, which is given by $\mu_r = 1+\chi$, is thus generally slightly less than 1. All substances have a diamagnetic contribution to their susceptibility, since the induced currents always oppose the applied field. In many substances, the diamagnetism is completely masked by another magnetic phenomenon (p. 953).

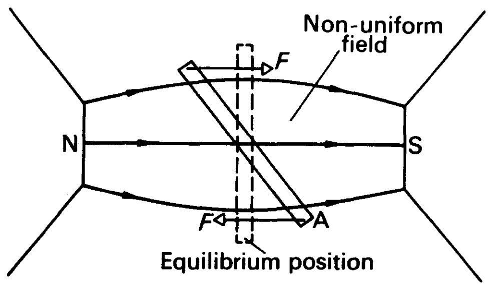


FIG. 38.14. Rod of diamagnetic material in strong field.

If a rod of diamagnetic material is placed in a non-uniform magnetic field, it will settle at right angles to the field. Fig. 38.14 shows the specimen slightly displaced from this position. The magnetization will oppose the applied field so that the end A will now effectively be a weak S pole. It will then experience a force as shown by the arrow, so that a restoring couple turns the specimen back to its position at right angles to the field.

It should be noted that diamagnetism is a natural 'reaction' to an applied magnetic field and that it is independent of temperature.

Paramagnetism

In contrast to bismuth, a rod of a material such as platinum will settle along the same direction as the applied magnetic field. Further, the induced magnetism will be in the same direction as the field. Fig. 38.15. Platinum is an example of a *paramagnetic* material. The susceptibility, χ, of a paramagnetic substance is very small and positive, $+0{\cdot}0001$ for example, so that its relative permeability μ_r is very slightly greater than 1 from $\mu_r = 1+\chi$.

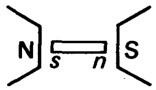


FIG. 38.15. Rod of paramagnetic material in strong field.

As we have already mentioned, atoms contain circulating and spinning electrons. Each electron possesses a resultant magnetic moment on account of its orbital motion and its spin motion. In a diamagnetic atom, all these contributions to the magnetic moment cancel. In a paramagnetic atom, however, there is a resultant magnetic moment. Generally, the thermal motions of the atoms will cause these magnetic moments to be oriented purely at random and there will be no resultant magnetization. If, however, a field is applied, each atomic moment will try to set in the direction of the field but the thermal motions will prevent complete alignment. In this case there will be overall weak magnetization in the direction of the applied field. This accounts for the phenomenon of paramagnetization.

It is clear that paramagnetism is temperature dependent. At low temperatures, the thermal motions will be less successful at preventing the alignment of the atomic moments and so the susceptibility will be larger. At higher temperatures thermal motion will make alignment difficult. At very high temperatures, the material may become diamagnetic, for the diamagnetic contribution to χ is not affected by temperature whilst the paramagnetic contribution falls.

Ferromagnetism. Magnetic Domain Theory

A ferromagnetic material has a very high value of susceptibility, χ, and hence of relative permeability, μ_r. The value of μ_r can be several thousands. Like a paramagnetic material, the magnetization is in the direction of the applied field and a rod of a ferromagnetic material will align itself along the field.

In a paramagnetic substance which is not subjected to a magnetic field, the magnetic moments are oriented purely at random due to the thermal vibrations. In a ferromagnetic material, however, strong 'interactions' are present between the moments, the nature of which requires quantum theory to understand it and is outside the scope of this work. These cause neighbouring moments to align, even in the absence of an applied field, with the result that tiny regions of very strong magnetism are obtained inside the unmagnetized material called *magnetic domains*. Above a critical temperature called the *Curie point*, ferromagnetics become paramagnetics (see Table, p. 956).

Domain Formation

A crystal of ferromagnetic material is shown in Fig. 38.16 (i). If all the domains were aligned completely, the material would behave like one enormous domain and the energy in the magnetic field outside is then considerable, as represented by the flux shown. Now all physical systems settle in equilibrium when their energy is a minimum. Fig. 38.16(ii) is therefore more stable than Fig. 38.16(i)

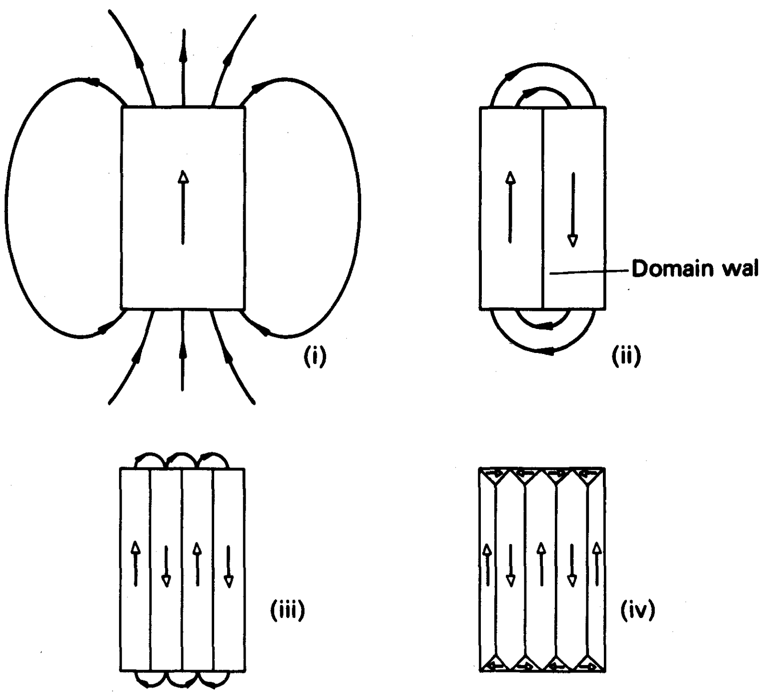


FIG. 38.16. Magnetic domains.

because the external magnetic field energy is less. Thus the domains grow in the material, as shown in Fig. 38.16 (iii) and (iv). The region between two domains, where the magnetization changes direction, is called a *domain wall* and also contains energy. When the formation of a new domain wall requires more energy than is gained by the reduction in the external magnetic field, no more domains are formed. Thus there is a limit to the number of domains formed. This occurs when the volume of the domains is of the order 10^{-4} cm^3 or less.

Domains and Magnetization

Some of the phenomena in magnetization of ferromagnetic material can now be explained. In an unmagnetized specimen, the domains are oriented in different directions. The net magnetization is then zero. If a small magnetic field H is applied, there is some small rotation of the magnetization within the domains, which produces an overall component of magnetization in the direction of H. This occurs in the region AB of the magnetization-field (M-H) curve shown in Fig. 38.17.

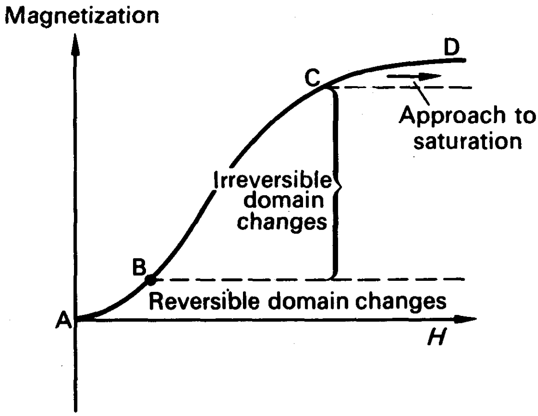


FIG. 38.17. Domain movement in magnetization.

If the field H is removed, the domain magnetization returns to its original direction. Thus the magnetization returns to zero. The changes in the part AB of the curve are hence reversible. If the field H is increased beyond B in the region BC, the magnetization becomes greater. On removal of the field the magnetization does not return to zero, and so remanence occurs. Along BC, then, irreversible changes take place; the domains grow in the direction of the field, by movement of domain walls, at the expense of those whose magnetization is in the opposite direction. At very high applied fields H there is complete alignment of the domains and so the magnetization M approaches 'saturation' along CD.

EXERCISES 38

1. Define *intensity of magnetization*, *susceptibility* and *permeability*. Two substances, A and B, have relative permeabilities slightly greater and slightly less than unity respectively. What does this signify about their magnetic properties? To what group of magnetic substances fo A and B each belong?

A soft iron ring of cross-sectional diameter 8 cm and mean circumference 200 cm has 400 turns of wire wound uniformly on it. Calculate the current necessary to produce magnetic flux to the value of 5×10^{-4} Wb if the relative permeability of the iron in the condition stated is 1800. Why is it not possible to say from this information what the flux would be if the current were reduced to 1/10 of its calculated value? (*L.*)

2. Define *permeability* (*relative permeability*) of a magnetic substance.

Indicate the orders of magnetide of the relative permeabilities of ferromagnetic, paramagnetic and diamagnetic substances respectively. Discuss in relation to its (relative) permeability one aspect of the behaviour of a specimen of each of these substances when placed in turn in the same magnetic field.

An iron ring of mean circumference 30 cm and of area of cross-section 1·5 cm^2 has 240 turns of wire uniformly wound on it, through which passes a current of 2 A. If the flux in the iron is found to be $7{\cdot}5 \times 10^{-4}$ Wb, find the relative permeability of the iron. (*L.*)

3. Give diagrams to illustrate the distribution of the lines of induction when a sphere of (*a*) soft iron, (*b*) bismuth (which is diamagnetic) is placed in a magnetic field, initially uniform. Compare and contrast the magnetic properties of these materials.

The magnetic induction (flux density) in a uniformly magnetized specimen of cast iron is 0·3 Wb m^{-2} when the strength of the magnetizing field is 1000 A m^{-1}. Find (i) the intensity of magnetization, (ii) the relative permeability, (iii) the magnetic susceptibility of the specimen. (*L.*)

4. What is meant by *intensity of magnetization* and *hysteresis*? Describe an experiment from the observations of which a graph of intensity of magnetization (M) against magnetizing field strength (H) over a complete hysteresis cycle may be plotted for a ferromagnetic material. What information may be obtained from the graph by measuring (*a*) the area enclosed by the loop, (*b*) the intercept on the M axis, (*c*) the intercept on the H axis, (*d*) the slope of a line joining a point on the graph to the origin? (*L.*)

5. Define *intensity of magnetization M* and *magnetic susceptibility* χ_m, and with the aid of a diagram explain what is meant by *hysteresis.*

Sketch a graph showing how M varies with the applied magnetizing field as the field applied to a specimen of soft iron, initially unmagnetized, is slowly increased from zero. On the same diagram sketch the corresponding graph for steel.

By reference to your earlier account state, with reasons, desirable magnetic properties of materials to be used as (*a*) the core of an electromagnet, (*b*) the core of a transformer, (*c*) a permanent magnet. (*L.*)

6. Define *magnetic moment* and explain why this concept is useful in the study of magnetism.

A bar of magnetic material is magnetized in the uniform field inside a solenoid. How would you study experimentally the changes in its magnetic moment when the magnetizing current is varied in magnitude and direction?

How would you represent the results of your measurements graphically? Give freehand sketches of the graphs you would expect to obtain for (i) material suitable for the core of a transformer, (ii) material suitable for a permanent magnet. Point out in each case the features of the graph which indicate that the material is suitable for its purpose. (*O. & C.*)

7. Define *susceptibility, permeability,* and state the relation between them.

Describe briefly how you would test if a small rod is diamagnetic, paramagnetic, or ferromagnetic. Draw intensity of magnetization graphs for each type of material and comment on their special features. (*N.*)

8. Explain what is meant by a *ferromagnetic material.* Give a general account of the wide range of magnetic properties exhibited by different ferromagnetic materials and indicate the practical applications of these properties. (*N.*)

chapter thirty-nine

A.C. Circuits. Electromagnetic Waves

Measurement of A.C.

IF an alternating current (a.c.) is passed through a moving-coil meter, the pointer does not move. The coil is urged clockwise and anticlockwise at the frequency of the current—50 times per second if it is drawn from the British grid—and does not move at all. In a delicate instrument the pointer may be seen to vibrate, with a small amplitude. Instruments for measuring alternating currents must be so made that the pointer deflects the same way when the current flows through the instrument in either direction.

Moving-iron Instrument

A fairly common type of such instrument, called the moving-iron type, is shown in Fig. 39.1. It consists of two iron rods XY, PQ, surrounded by a coil which carries the current. The coil is fixed to the framework of the meter, and so is one of the rods PQ. The other rod is attached to an axle, which also carries the pointer, its motion is controlled by hair-springs. When a current flows through the coil, it

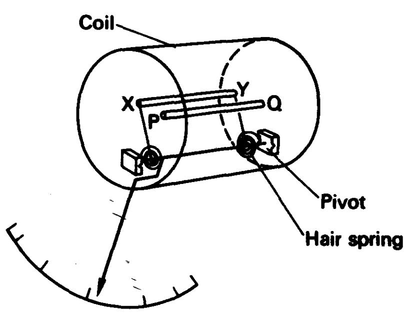


FIG. 39.1. Moving-iron meter, repulsion type.

magnetizes the rods, so that their adjacent ends have the same polarity. The polarity of each pair of ends reverses with the current, but whichever direction the current has, the iron rods repel each other. The force on the pivoted rod is therefore always in the same direction, and the pointer is deflected through an angle which is proportional to the average force. To a fair approximation, the magnetization of the rods at any instant is proportional to the current at that instant; the force between the rods is therefore roughly proportional to the square of the current. The deflection of the pointer is therefore roughly proportional to the average value of the square of the current.

Hot-wire Instrument

Another type of 'square law' instrument is the hot-wire ammeter (Fig. 39.2). In it the current flows through a fine resistance-wire XY, which it heats. The wire warms up to such a temperature that it loses heat—mainly by convection—at a rate equal to the average rate at which heat is developed in the wire. The rise in temperature of the wire makes it expand and sag; the sag is taken up by a second fine wire PQ, which is held taut by a spring. The wire PQ passes round a pulley attached to the pointer of the instrument, which rotates as the wire XY sags. The deflection of the pointer is roughly proportional to the average rate at which heat is developed in the wire XY; it is therefore roughly proportional to the average value of the square of the alternating current, and the scale is a square-law one.

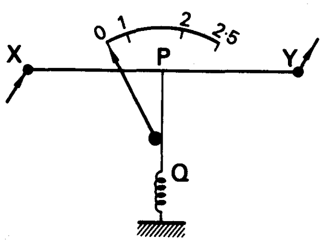


FIG. 39.2. Hot wire meter.

Root-mean-square value of A.C.

On p. 907 we saw that an alternating current I varied sinusoidally; that is, it could be represented by the equation $I = I_m \sin \omega t$, where I_m was the peak (maximum) value of the current. In commercial practice, alternating currents are always measured and expressed in terms of their *root-mean-square* (*r.m.s.*) value, which we shall now consider.

Consider two resistors of equal resistance R, one carrying an alternating current and the other a direct current. Suppose both are dissipating the same power, P, as heat. The root-mean-square (r.m.s.) value of the alternating current, I_r, is defined as equal to the direct current, I_d. Thus:

the root-mean-square value of an alternating current is defined as that value of steady current which would dissipate heat at the same rate in a given resistance.

Since the power dissipated by the direct current is

$$P = I_d^2 R,$$

our definition means that, in the a.c. circuit,

$$P = I_r^2 R. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Whatever the wave-form of the alternating current, if I is its value at any instant, the power which it delivers to the resistance R at that instant is I^2R. Consequently, the average power P is given by

$$P = \text{average value of } (I^2R)$$

$$= R \times \text{average value of } (I^2),$$

since R is a constant. Therefore, by equation (1),

$$I_r^2 R = R \times \text{average value of } (I^2)$$

or

$$I_r^2 = \text{average value of } (I^2). \qquad (2)$$

The average value of (I^2) is called the mean-square current; the meaning of the term is illustrated for a non-sinusoidal current in Fig. 39.3 (i).

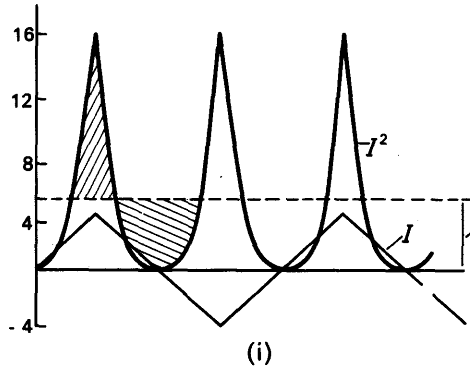


FIG. 39.3 (i). Mean-square values.

$$\therefore I_r = \sqrt{\text{average value of } (I^2)}. \qquad (3)$$

For a sinusoidal current, the average value of I^2 is $I_m^2/2$, where I_m is the maximum value of the current

$$\therefore I_r = \frac{I_m}{\sqrt{2}}. \qquad (4)$$

Equation (3) shows the origin of the term 'root-mean-square'. We therefore require a meter whose deflection measures not the current through it but the average value of the square of the current. As we have already seen, moving-iron and hot-wire meters have just this property (p. 968).

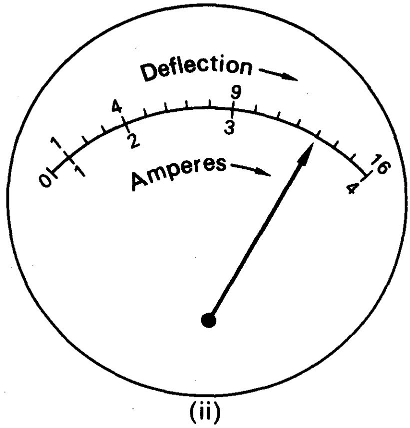


FIG. 39.3 (ii). Scale of A.C. ammeter.

For convenience, such meters are scaled to read amperes, not (amperes)^2, as in Fig. 39.3 (ii). The scale reading is then proportional to the square-root of the deflection, and indicates directly the root-mean-square value of the current I_r. An a.c. meter of the moving-iron or hot-wire type can be calibrated in direct current, as shown in Fig. 39.3 (ii). This follows at once from the definition of the r.m.s. value of current.

A.C. through a Capacitor

In many radio circuits, resistors, capacitors, and coils are present. An alternating current can flow through a resistor, but it is not obvious at first that it can flow through a capacitor. This can be demonstrated, however, by wiring a capacitor, of capacitance one or more microfarads, in series with a neon lamp, and connecting them to the a.c. mains via a plug. In place of the neon lamp we could use a mains filament lamp of low rating, such as 25 watts. The lamp lights up, showing that a current is flowing through it.

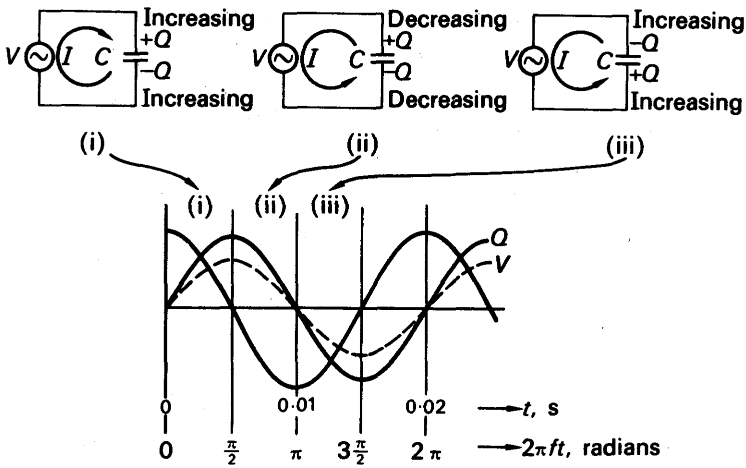


FIG. 39.4. Flow of A.C. through capacitor, frequency 50 Hz.

The current flows because the capacitor plates are being continually charged, discharged, and charged the other way round by the alternating voltage of the mains (Fig. 39.4). The current thus flows round the circuit, and can be measured by an a.c. milliammeter inserted in any of the connecting wires.

To find the amplitude of the current, let us denote the amplitude of the voltage by V_m, and its frequency by f. Then, as in equation (3), p.907, the instantaneous voltage at any time t is

$$V = V_m \sin 2\pi ft.$$

If C is the capacitance of the capacitor, then the charge Q on its plates is

$$Q = CV,$$

whence

$$Q = CV_m \sin 2\pi ft.$$

The current flowing at any instant, I, is equal to the rate at which charge is accumulating on the capacitor plates. Thus

$$I = \frac{dQ}{dt}$$

$$= \frac{d}{dt}(CV_m \sin 2\pi ft)$$

$$= 2\pi fCV_m \cos 2\pi ft. \quad . \quad . \quad . \quad (1)$$

Equation (1) shows that the amplitude of the current is $2\pi fCV_m$; proportional to the frequency, the capacitance, and the voltage amplitude. These results are easy to explain. The greater the voltage, or the capacitance, the greater the charge on the plates, and therefore the greater the current required to charge or discharge the capacitor. And the higher the frequency, the more rapidly is the capacitor charged and discharged, and therefore again the greater the current.

A more puzzling feature of equation (1) is the factor giving the time variation of the current, $\cos 2\pi ft$. It shows that the current varies a quarter-cycle out of step with the voltage—or, as is more often said, $\pi/2$ out of phase. Fig. 39.4 shows this variation, and also helps to explain it. When the voltage is a maximum, so is the charge on the capacitor. It is therefore not charging and the current is zero. When the voltage starts to fall, the capacitor starts to discharge; the rate of discharging, or current, reaches its maximum when the capacitor is completely discharged and the voltage across it is zero.

A.C. through an Inductor

Since a coil is made from conducting wire, we have no difficulty in seeing that an alternating current can flow through it. However, if the coil has appreciable self-inductance, the current is less than would flow through a non-inductive coil of the same resistance. We have already seen how self-inductance opposes changes of current; it must therefore oppose an alternating current, which is continuously changing.

Let us suppose that the resistance of the coil is negligible, a condition which can easily be satisfied in practice. We can simplify the theory by considering first the current, and working back to find the potential difference across the coil. Let us therefore denote the current by

$$I = I_m \sin 2\pi ft, \quad . \quad . \quad . \quad . \quad (1)$$

where I_m is its amplitude (Fig. 39.5). If L is the inductance of the coil, the changing current sets up a back-e.m.f. in the coil, of magnitude

$$E = L\frac{dI}{dt}.$$

Thus

$$E = L\frac{d}{dt}(I_m \sin 2\pi ft)$$

$$= 2\pi fLI_m \cos 2\pi ft.$$

To maintain the current, the applied supply voltage must be equal to the back-e.m.f. The voltage applied to the coil must therefore be given by

$$V = 2\pi f L I_m \cos 2\pi f t. \quad . \quad . \quad . \quad . \quad (2)$$

Equation (2) shows that the amplitude of the voltage across a pure inductance (without resistance) is proportional to the frequency, the magnitude of the self-inductance, and the amplitude of the current. The reader is left to work out the physical explanation of these three facts for himself.

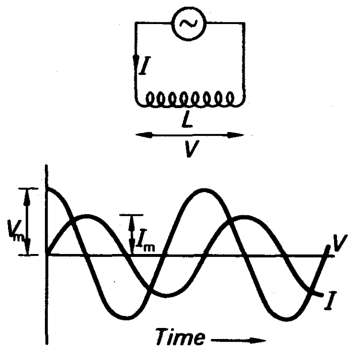


FIG. 39.5. Passage of a.c. through a coil.

The voltage across a pure inductance is a quarter-cycle, or $\pi/2$ radians, out of phase with the current. Fig. 39.5 shows this relationship, which follows from comparing equations (1) and (2). It can be explained by considering the relationship between the back-e.m.f., and the changing current, which the reader should do for himself.

The phase relationship between current and voltage for a pure inductance is, however, different from that for a capacitor. For, as Fig. 39.5 shows, the current through an inductance lags $\pi/2$ behind the voltage; that is to say, it passes its maxima a quarter-cycle later than the voltage. On the other hand, in a capacitor the current passes its maxima a quarter-cycle ahead of the voltage (Fig. 39.4): we say that it leads the voltage by $\pi/2$.

Reactance

The term reactance is used to denote the opposition which an inductance or capacitor offers to the passage of an alternating current. We do not use the term resistance for this, because we have already defined resistance as the property of a circuit which enables it to dissipate electrical power as heat; and we have just seen that a capacitor or a pure inductance dissipates no electrical power at all.

We define the reactance of an inductor L, or capacitor C, by the equation

$$\textit{reactance} = \frac{\textit{amplitude of voltage across L or C}}{\textit{amplitude of current through it}}$$

Denoting reactance by X, we have

$$X = \frac{V_m}{I_m}.$$

Since V_m is measured in volts, and I_m in amperes, the reactance X is expressed in ohms.

We have already seen (p. 971) that the amplitude of the current through a capacitor is given by

$$I_m = 2\pi f C V_m.$$

The reactance of the capacitor is therefore

$$X_C = \frac{V_m}{I_m} = \frac{1}{2\pi f C}.$$

X_C is in ohms when f is in Hz (cycles per second), and C in farads.

The amplitude of the voltage across a pure inductor is

$$V_m = 2\pi f L I_m.$$

Hence the reactance of the inductor is

$$X_L = \frac{V_m}{I_m} = 2\pi f L.$$

It is in ohms when f is in Hz, and L is in henrys (H).

For convenience we often write $\omega = 2\pi f$.

The quantity ω is called the angular frequency of the current and voltage. It is expressed in radians per second. Then an alternating voltage, for example, may be written as

$$V = V_m \sin \omega t.$$

And reactances become

$$X_L = \omega L \qquad X_C = \frac{1}{\omega C}.$$

EXAMPLE

1. A capacitor C of 1 μF is used in a radio circuit where the frequency is 1000 Hz and the current flowing is 2 mA. Calculate the voltage across C.

$$\text{Reactance, } X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 1/10^6} = 159 \text{ ohms (approx.)}.$$

$$\therefore V = IX_C = \frac{2}{1000} \times 159 = 0{\cdot}32 \text{ V (approx.)}.$$

2. An inductor of 2 H and negligible resistance is connected to a 12 V mains supply, $f = 50$ Hz. Find the current flowing.

$$\text{Reactance, } X_L = 2\pi f L = 2\pi \times 50 \times 2 = 628 \text{ ohms}.$$

$$\therefore I = \frac{V}{X_L} = \frac{12}{628} \text{ A} = 19 \text{ mA (approx.)}.$$

Vector Diagram

In the Mechanics section of this book, it is shown that a quantity which varies sinusoidally with time may be represented as the projection of a rotating vector (p. 44). Alternating currents and voltages may therefore be represented in this way. Fig. 39.6 shows, on the left, the vectors representing the current through a capacitor, and

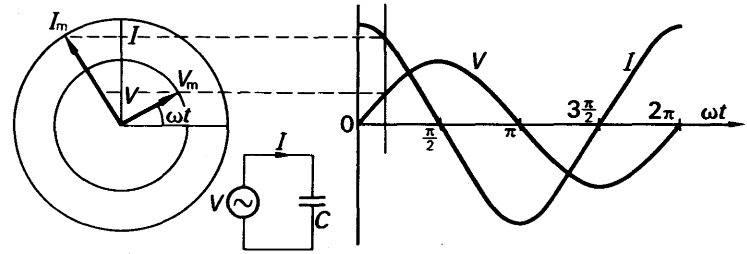


FIG. 39.6. Vector diagram for capacitor.

the voltage across it. Since the current leads the voltage by $\pi/2$, the current vector I is displaced by 90° ahead of the voltage vector V.

Fig. 39.7 shows the vector diagram for a pure inductor. In drawing it, the voltage has been taken as $V = V_m \sin \omega t$, and the current drawn

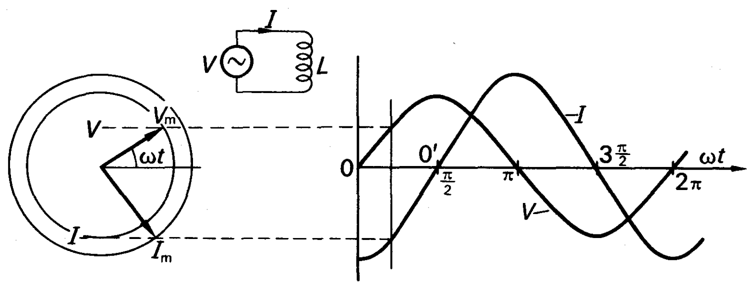


FIG. 39.7. Vector diagram for pure inductance.

lagging $\pi/2$ behind it. This enables the diagram to be readily compared with that for a capacitor. To show that it is essentially the same as Fig. 39.5, we have only to shift the origin by $\pi/2$ to the right, from 0 to 0′.

SERIES CIRCUITS

L and *R* in series

Consider an inductor L in series with resistance R, with an alternating voltage V (r.m.s.) of frequency f connected across both components (Fig. 39.8 (i)).

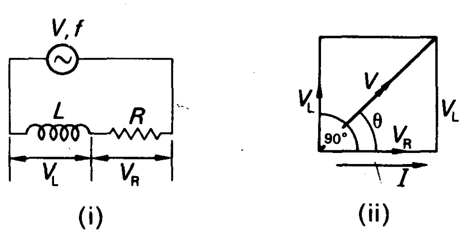


FIG. 39.8. Inductance and resistance in series.

The sum of the respective voltages V_L and V_R across L and R is equal to V. But the voltage V_L leads by 90° on the current I, and the voltage V_R is in phase with I (see page 972). Thus the two voltages can be drawn to scale as shown in Fig. 39.8 (ii), and hence, by Pythagoras' theorem, it follows that the vector sum V is given by

$$V^2 = V_L^2 + V_R^2.$$

But $V_L = IX_L$, $V_R = IR$.

$$\therefore V^2 = I^2 X_L{}^2 + I^2 R^2 = I^2(X_L{}^2 + R^2),$$

$$\therefore I = \frac{V}{\sqrt{X_L{}^2 + R^2}}. \qquad \text{(i)}$$

Also, from Fig. 39.8 (ii), the current I lags on the applied voltage V by an angle θ given by

$$\tan\theta = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}. \qquad \text{(ii)}$$

From (i), it follows that the 'opposition' Z to the flow of alternating current is given in ohms by

$$Z = \frac{V}{I} = \sqrt{X_L{}^2 + R^2}. \qquad \text{(iii)}$$

This 'opposition', Z, is known as the *impedance* of the circuit.

EXAMPLE

An iron-cored coil of 2 H and 50 ohms resistance is placed in series with a resistor of 450 ohms, and a 100 V, 50 Hz, a.c. supply is connected across the arrangement. Find the current flowing in the coil.

The reactance $X_L = 2\pi f L = 2\pi \times 50 \times 2 = 628$ ohms.

Total resistance $R = 50 + 450 = 500$ ohms.

$$\therefore \text{circuit impedance } Z = \sqrt{X_L^2 + R^2} = \sqrt{628^2 + 500^2} = 803 \text{ ohms}$$

$$\therefore I = \frac{V}{Z} = \frac{100}{803}\text{ A} = 12{\cdot}5 \text{ mA (approx.)}.$$

C and *R* in series

A similar analysis enables the impedance to be found of a capacitance C and resistance R in series. Fig. 39.9 (i). In this case the voltage V_C

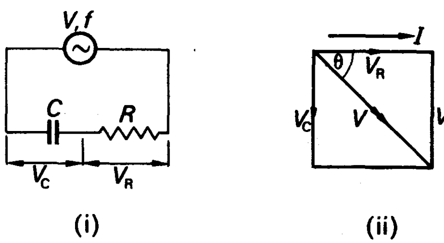


FIG. 39.9. Capacitance and resistance in series.

across the capacitor lags by 90° on the current I (see p. 971), and the voltage V_R across the resistance is in phase with the current I. As the vector sum is V, the applied voltage, it follows by Pythagoras' theorem that

$$V^2 = V_C^2 + V_R^2 = I^2 X_C{}^2 + I^2 R^2 = I^2(X_C{}^2 + R^2),$$

$$\therefore I = \frac{V}{\sqrt{X_C{}^2 + R^2}}. \qquad . \quad . \quad . \quad \text{(i)}$$

Also, from Fig. 39.9 (ii), the current I leads on V by an angle θ given by

$$\tan\theta = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}. \qquad . \quad . \quad \text{(ii)}$$

It follows from (i) that the impedance Z of the C–R series circuit is

$$Z = \frac{V}{I} = \sqrt{X_C{}^2 + R^2}.$$

It should be noted that although the impedance formula for a C–R series circuit is of the same mathematical form as that for a L–R series circuit, the current in the former case leads on the applied voltage but the current in the latter case lags on the applied voltage.

L, C, R in series

The most general series circuit is the case of L, C, R in series (Fig. 39.10 (i)). The vector diagram has V_L leading by 90° on V_R, V_C lagging by 90° on V_R, with the current I in phase with V_R (Fig. 39.10 (ii)). If V_L is greater than V_C, their resultant is $(V_L - V_C)$ in the direction of V_L,

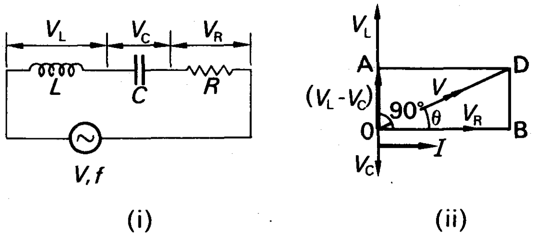


FIG. 39.10. *L, C, R* in series.

as shown. Thus, from Pythagoras' theorem for triangle ODB, the applied voltage V is given by

$$V^2 = (V_L - V_C)^2 + V_R^2.$$

But $V_L = IX_L$, $V_C = IX_C$, $V_R = IR$.

$$\therefore V^2 = (IX_L - IX_C)^2 + I^2R^2 = I^2[(X_L - X_C)^2 + R^2],$$

$$\therefore I = \frac{V}{\sqrt{(X_L - X_C)^2 + R^2}}. \qquad . \quad . \quad . \quad \text{(i)}$$

Also, I lags on V by an angle θ given by

$$\tan\theta = \frac{DB}{OB} = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}. \quad . \quad \text{(ii)}$$

Resonance in the *L*, *C*, *R* series circuit

From (i), it follows that the impedance Z of the circuit is given by

$$Z = \sqrt{(X_L - X_C)^2 + R^2}.$$

The impedance varies as the frequency, f, of the applied voltage varies, because X_L and X_C both vary with frequency. Since $X_L = 2\pi fL$, then $X_L \propto f$, and thus the variation of X_L with frequency is a straight line passing through the origin (Fig. 39.11 (i)). Also, since $X_C = 1/2\pi fC$, then $X_C \propto 1/f$, and thus the variation of X_C with frequency is a curve approaching the two axes (Fig. 39.11 (i)). The resistance R is independent of frequency, and is thus represented by a line

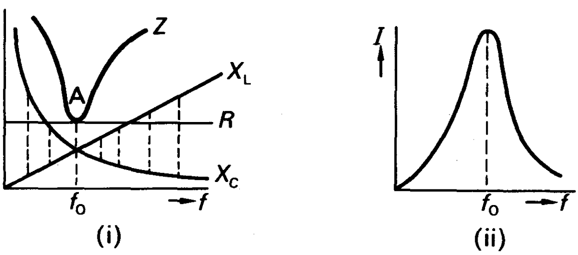


FIG. 39.11. Resonance curves.

parallel to the frequency axis. The difference $(X_L - X_C)$ is represented by the dotted lines shown in Fig. 39.11 (i), and it can be seen that $(X_L - X_C)$ decreases to zero for a particular frequency f_0, and thereafter increases again. Thus, from $Z = \sqrt{(X_L - X_C)^2 + R^2}$, the impedance diminishes and then increases as the frequency f is varied. The variation of Z with f is shown in Fig. 39.11 (i), and since the current $I = V/Z$, the current varies as shown in Fig. 39.11 (ii). Thus the current has a maximum value at the frequency f_0, and this is known as the *resonant frequency* of the circuit.

The magnitude of f_0 is given by $X_L - X_C = 0$, or $X_L = X_C$.

$$\therefore 2\pi f_0 L = \frac{1}{2\pi f_0 C}, \quad \text{or} \quad 4\pi^2 LCf_0 = 1.$$

$$\therefore f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

At frequencies above and below the resonant frequency, the current is less than the maximum current, see Fig. 39.11 (ii), and the phenomenon is thus basically the same as the forced and resonant vibrations obtained in Sound or Mechanics (p. 581).

The series resonance circuit is used for tuning a radio receiver. In this case the incoming waves of frequency f say from a distant transmitting station induces a varying voltage in the aerial, which in turn induces a voltage V of the same frequency in a coil and capacitor circuit in the receiver (Fig. 39.12). When the capacitance C is varied the resonant frequency is changed; and at one setting of C the resonant frequency becomes f, the frequency of the incoming waves. The maximum current is then obtained, and the station is now heard very loudly.

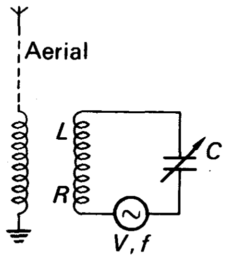


FIG. 39.12. Tuning a receiver.

Power in A.C. circuits

Resistance R. The power absorbed generally is $P = IV$. In the case of a resistance, $V = IR$, and $P = I^2R$. The variation of power is shown in Fig. 39.13 (i), from which it follows that the average power absorbed $P = I_m^2R/2$, where $I_0 = I_m$ = the peak (maximum) value of the current. Since the r.m.s. value of the current is $I_m/\sqrt{2}$, it follows that

$$P = I^2R,$$

where I is the r.m.s. value (see p. 968).

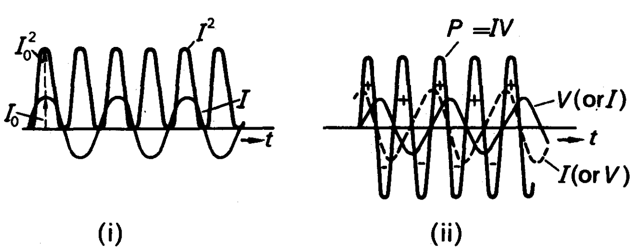


FIG. 39.13. Power in A.C. circuits.

Inductance L. In the case of a pure inductor, the voltage V across it leads by 90° on the current I. Thus if $I = I_m \sin \omega t$, then $V = V_m \sin(90° + \omega t) = V_m \cos \omega t$. Hence, at any instant,

$$\text{power absorbed} = IV = I_m V_m \sin \omega t . \cos \omega t = \tfrac{1}{2} I_m V_m \sin 2\omega t.$$

The variation of power, P, with time t is shown in Fig. 39.13 (ii); it is a sine curve with an average of zero. *Hence no power is absorbed in a pure inductance.* This is explained by the fact that on the first quarter of the current cycle, power is absorbed (+) in the magnetic field of the coil (see p. 925). On the next quarter-cycle the power is returned (−) to the generator, and so on.

Capacitance. With a pure capacitance, the voltage V across it lags by 90° in the current I (p. 971). Thus is $I = I_m \sin \omega t$,

$$V = V_m \sin(\omega t - 90°) = -V_m \cos \omega t.$$

Hence, numerically,

$$\text{power at an instant}, P, = IV = I_m V_m \sin \omega t \cos \omega t = \frac{I_m V_m}{2} \sin 2\omega t.$$

Thus, as in the case of the inductance, *the power absorbed in a cycle is zero* (Fig. 39.13 (ii)). This is explained by the fact that on the first quarter of the cycle, energy is stored in the electrostatic field of the capacitor. On the next quarter the capacitor discharges, and the energy is returned to the generator.

Formulae for A.C. Power

It can now be seen that, if I is the r.m.s. value of the current in amps in a circuit containing a resistance R ohms, the power absorbed is I^2R watts. Care should be taken to exclude the inductances and capacitances in the circuit, as no power is absorbed in them.

If the voltage V across a circuit leads by an angle θ on the current I, the voltage can be resolved into a component $V \cos \theta$ in phase with the current, and a voltage $V \sin \theta$ perpendicular to the current (Fig. 39.14). The former component, $V \cos \theta$, represents that part of the

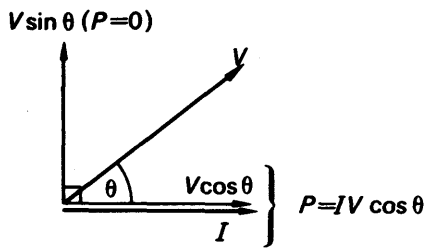


FIG. 39.14. Power absorbed.

voltage across the total *resistance* in the circuit, and hence the power absorbed is

$$P = IV \cos \theta.$$

The component $V \sin \theta$ is that part of the applied voltage across the total inductance and capacitance. Since the power absorbed here is zero, it is sometimes called the 'wattless component' of the voltage.

EXAMPLE

A circuit consists of a capacitor of 2 μF and a resistor of 1000 ohms. An alternating e.m.f. of 12 V (r.m.s.) and frequency 50 Hz is applied. Find (1) the current flowing, (2) the voltage across the capacitor, (3) the phase angle between the applied e.m.f. and current, (4) the average power supplied.

The reactance X_C of the capacitor is given by

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 2/10^6} = 1590 \text{ ohms (approx.)}$$

$\therefore$ total impedance $Z = \sqrt{R^2+X_C^2} = \sqrt{1000^2+1600^2} = 1880$ ohms (approx.).

(1) $\therefore$ current, $I = \dfrac{V}{Z} = \dfrac{12}{1880} = 6{\cdot}4 \times 10^{-3}$ A.

(2) voltage across, $V_C = IX_C = \dfrac{12}{1880} \times 1590 = 10{\cdot}2$ V (approx.).

(3) The phase angle ϕ is given by

$$\tan \phi = \frac{X_C}{R} = \frac{1590}{1000} = 1{\cdot}59$$

$$\therefore \phi = 58^\circ \text{ (approx.)}.$$

(4) Power supplied $= I^2R = \left(\dfrac{12}{1880}\right)^2 \times 1000 = 0{\cdot}04$ W (approx.).

ELECTROMAGNETIC WAVES

Alternating current circuits containing a coil and a capacitor, such as those we have just described, produce varying magnetic and electric fields confined to the neighbourhood of the two components (p. 977). We now discuss how varying magnetic and electric fields can travel through space in the form of *electromagnetic waves.*

Production of Electric Field from Moving Magnetic Field

In chapter 36, we saw that a changing magnetic flux linkage produces an induced e.m.f. It was shown there that the e.m.f. induced in a rod of length l moving in a field of induction B with velocity v, when B, v and l are mutually perpendicular, is given by

$$\text{induced e.m.f.} = Blv.$$

The same e.m.f., Blv, is produced if the rod remains stationary, whilst a finite field moves with velocity v in the opposite direction to the moving

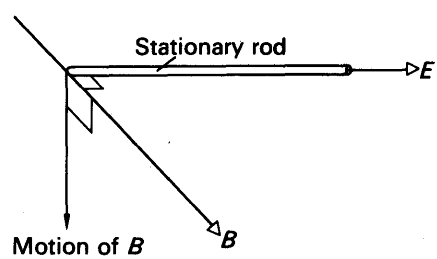


FIG. 39.15. Electric field E produced by moving B.

rod in Fig. 39.15. Hence the *electric field intensity* E, set up along the rod by the moving magnetic field is given by

$$E = \text{potential gradient}$$

$$= \frac{\text{e.m.f.}}{l} = \frac{Blv}{l} = Bv \quad . \quad . \quad . \quad (1)$$

Notice that the electric field E produced is at right angles to B and v, and is in the same direction as the induced e.m.f. The directions of E, B and v are shown in Fig. 39.15.

Production of Magnetic Field from Moving Electric Field

It has already been shown that moving charges in a wire produce a magnetic field, p. 931. We can, however, look at this in a different way. We can say that each charge produces an electric field moving with the charge, and that *this moving electric field sets up a magnetic field.*

Fig. 39.16 shows a wire carrying a current due to moving electrons. The positive ions in a wire are fixed and so the electric fields of the positive ions and the negative electrons cancel. The ions are not, however, moving. Thus the magnetic field due to the moving electric

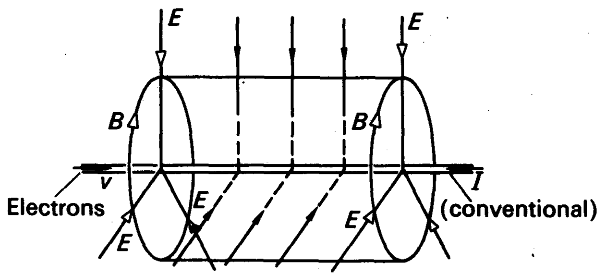


FIG. 39.16. Magnetic field B produced by moving E.

field of the electrons is *not* cancelled. Consider now a cylindrical surface of length l round the wire. If the charge per metre of the wire is ρ, then the total flux of the electric field entering the surface due to the negative charges inside $= E \times area = E.2\pi rl$, where E is the electric intensity. From Gauss's theorem (p. 747), this is equal to $charge/\varepsilon_0$ or $\rho l/\varepsilon_0$.

$$\therefore E = \frac{\rho}{2\pi\varepsilon_0 r}.$$

But the current I = charge per second crossing a section of wire $= \rho v$. Hence

$$E = \frac{I}{2\pi\varepsilon_0 rv}.$$

From p. 934, the magnetic field produced by the current has an induction B given by $B = \mu_0 I/2\pi r$. Substituting $I/2\pi r = B/\mu_0$,

$$\therefore E = \frac{B}{\varepsilon_0\mu_0 v}$$

$$\therefore B = \mu_0\varepsilon_0 vE \quad . \quad . \quad . \quad (2)$$

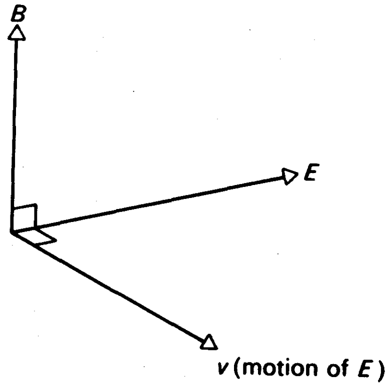


FIG. 39.17. Directions of E, B, v.

The direction of B, v and E are shown in Fig. 39.17. Note that the electric field intensity E, and the magnetic field induction B produced, are perpendicular to each other and to v.

It should also be noted that Fig. 39.17 has exactly the *same* relation between the directions of v, B and E as Fig. 39.15.

Electromagnetic Waves

Consider a pattern of electric fields E and magnetic fields B which are imagined to move into the paper with speed v as shown in Fig. 39.18.

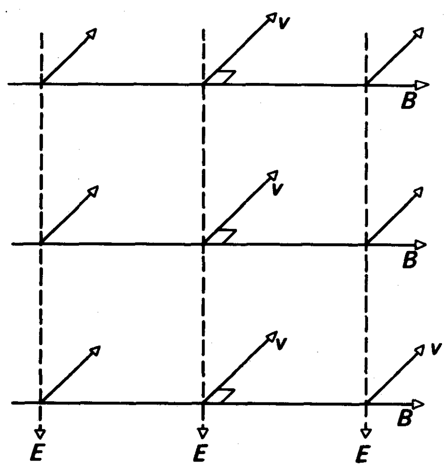


FIG. 39.18. Electromagnetic waves.

From equation (1), p. 980, the moving magnetic field creates an electric field of strength E' given by,

$$E' = vB \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This electric field is in the same direction as E, as can be seen by applying Fig. 39.15. From equation (2), p. 981, the moving electric field itself creates a magnetic field of induction B' given by

$$B' = \mu_0 \varepsilon_0 v E \quad . \quad . \quad . \quad . \quad (4)$$

This field is in the direction of the magnetic field B, as can be seen by applying Fig. 39.17. Now if $E = E'$ and $B = B'$, *the magnetic field creates the electric field* and vice versa. The system is then *self sustaining*. This occurs if

$$\begin{aligned} E' &= vB' \\ &= v(\mu_0 \varepsilon_0 v E') \\ &= \mu_0 \varepsilon_0 v^2 E', \end{aligned}$$

or if

$$v^2 = \frac{1}{\mu_0 \varepsilon_0} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence if the pattern moves with a speed $1/\sqrt{\mu_0 \varepsilon_0}$ the pattern will propagate through space. Now $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1} by definition, and $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$ F m^{-1} by experiment.

$$\therefore v = \frac{1}{\sqrt{(4\pi \times 10^{-7} \times 8{\cdot}85 \times 10^{-12})}} = 3 \times 10^8 \text{ m s}^{-1} \text{ (approx.)}.$$

This is just the speed of light and radio waves, as determined by experiment. It is reasonable to suppose, therefore, that waves such as light and radio waves which move with this speed are *electromagnetic* in nature.

Demonstration of Electromagnetic Waves. Transmission Line

Electromagnetic waves of high frequency, such as radio waves, can be radiated into space by *aerials*. The electrical circuit producing the waves is called an *oscillator* (p. 1019) and it is connected to the aerial by wires called a *transmission line*.

The effect of electromagnetic waves travelling in a transmission line can be demonstrated with the aid of a very high frequency (v.h.f.) oscillator producing electrical oscillations of about 150 MHz frequency. The output from this oscillator is fed to two parallel transmission lines a few centimetres apart. Fig. 39.19. The line is terminated by a short circuit (that is, a piece of wire clipped to the two lines),

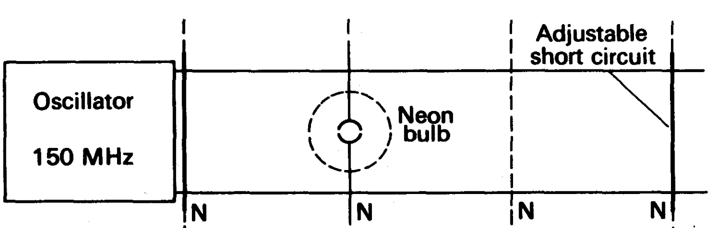


FIG. 39.19. Transmission line. Neon detector.

whose position along the line may be varied. As a neon bulb is moved along the line, variations in intensity are produced; the peaks of intensity are separated by a distance of about 100 cm. The greatest variations are obtained by adjusting the position of the short circuit. At the points marked by N in the diagram the neon may then be extinguished; mid-way between these positions the neon is brightest.

At the points marked N, therefore, there is no voltage and these points are *voltage nodes* (compare *sound waves*, p. 642). Mid-way between consecutive nodes, the high frequency p.d. has a maximum value. At these points on the wire, therefore, there are *voltage antinodes.* Thus *a voltage standing wave is produced on the line.* Since nodes are separated by half a wavelength (p. 590), the wavelength λ can be measured from this experiment; and if the oscillator frequency f is known, the speed c of the waves can be found from the general relation $c = f\lambda$. A value of about 3×10^8 m s^{-1} is obtained for electromagnetic waves.

If the neon bulb is replaced by a small filament lamp (which, unlike the neon, has a very low resistance) it is found that the lamp glows brightest, not at the voltage antinodes, but at the voltage nodes. This is because the low resistance lamp is sensitive to *current* in the line and not to voltage. Thus there are current antinodes at voltage nodes, and current nodes at voltage antinodes, as illustrated in Fig. 39.20.

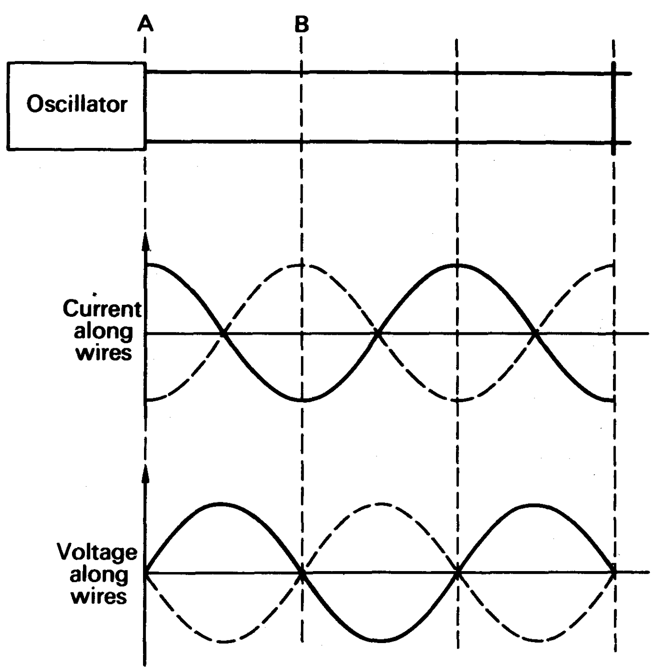


FIG. 39.20. Standing waves of voltage and current.

This situation is very like the standing waves set up in a resonating pipe open at both ends (p. 648). In this electrical case, resonance is obtained by adjusting the short circuit. In a pipe, resonance may be obtained by adjusting the length of the tube. As illustrated in Fig. 39.31,

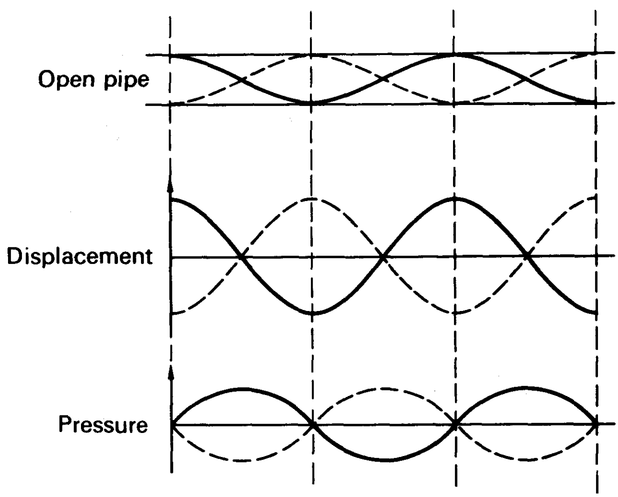


FIG. 39.21. Standing sound waves.

pressure nodes in a resonating pipe exist at displacement antinodes, and vice-versa.

To see what is happening on the transmission line during the course of one oscillation of the oscillator, consider the section of the line from A to B (Fig. 39.22). As the oscillator begins to force charge from the lower wire to the upper wire, a current will be flowing past A to the right on the upper wire, and to the left on the lower wire. Current will also be flowing past B but to the left on the upper wire and to the right on the lower wire, as shown in Fig. 39.22.

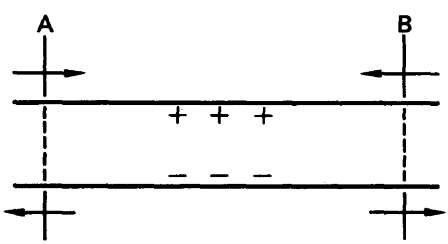


FIG. 39.22. Charge movement on transmission line.

This means that positive charge accumulates between A and B on the upper wire, so that the centre point will increase in potential. On the lower wire there will be a deficit of charge. Thus the mid-point of AB on the lower wire will be at a low potential. It follows that the p.d. across the wire at the mid-point increases. This will continue for one quarter of a cycle, when the process starts to reverse and the directions of current flow change. Later in the cycle, there will be an accumulation of positive charge on the lower wire. The process is then repeated and a standing wave pattern on the wire is therefore produced. The moving charges create electric and magnetic fields between the wires which constitute an electromagnetic wave. Here the wave is a standing wave; it is due to two electromagnetic waves travelling in opposite directions.

Radiation of Electromagnetic Waves into Space

Consider an oscillator with a connected transmission line, and suppose the transmission line is bent as shown in Fig. 39.23 (i).

We choose the points C and D to be at a current antinode, and A and B at a current node. It is clear that no current can flow at A and B as the

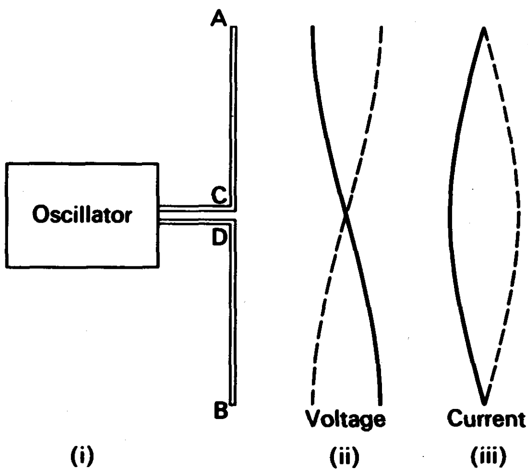


FIG. 39.23. Aerial. Half-wave dipole.

wires terminate at these points. A standing wave pattern will be set up on the wires, as we have seen, and if the lengths AC and DB are chosen correctly, there will be no additional nodes on the wires. In this case AB must be half a wavelength. Such an arrangement acts as an *aerial*, as discussed shortly. It is called a *half-wave dipole* aerial.

The charges moving along AB are forced up to A during one half cycle of oscillation and then down to B during the next half cycle. The charge, therefore, oscillates between A and B. *This accelerating charge radiates energy in the form of electromagnetic waves.* In contrast, charges moving in a line with a steady speed create a static magnetic field, and no electromagnetic wave is radiated.

If this aerial is used then, electromagnetic waves are radiated into space. They may be detected by a similar receiving aerial, of the same length as the transmitting aerial. Fig. 39.24. Detection of the signal is obtained using a wireless receiver tuned to the correct frequency.

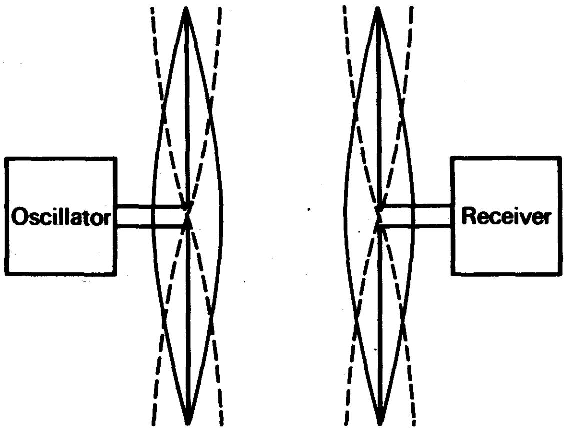


FIG. 39.24. Transmitting and receiving aerials.

Polarization of Electromagnetic Waves

By rotating the receiving aerial about the axis AB, it can be shown that no signal is obtained in the receiver when the aerial is directed along the line MN. Fig. 39.25. This corresponds to an angle of rotation α of 90°. Thus the radiation from the transmitting aerial is *plane polarized*, that is, the electric and magnetic vibrations in the electromagnetic wave each occur in one plane. The distribution of electric and

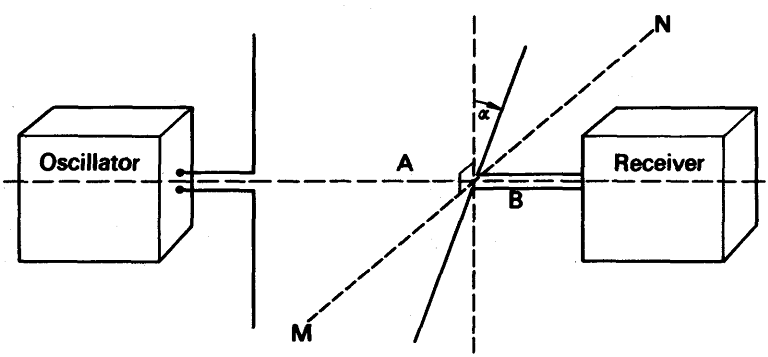


FIG. 39.25. Polarization of electromagnetic wave.

magnetic fields in the plane-polarized wave is shown in Fig. 39.26. The plane containing the magnetic field B is at right angles to the plane containing the electric field E, and the directions of E, B and v are mutually perpendicular (compare Fig. 39.15 and 39.17).

As we have seen, light is an example of an electromagnetic wave. The phenomenon of polarization of light is explained in an exactly

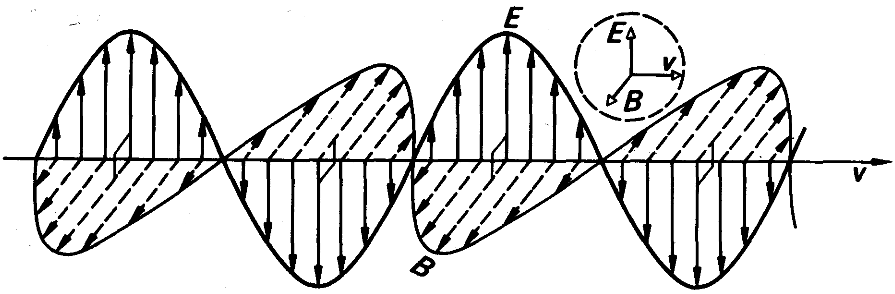


FIG. 39.26. Plane-polarized wave.

similar way (see p. 716). Plane polarized light has its electric field in one plane and its magnetic field in a perpendicular plane. Unpolarized light has electric and magnetic vibrations in *all* planes perpendicular to the direction of propagation.

Wave Properties of Electromagnetic Waves

We have already seen how standing (stationary) waves on transmission lines (guided waves), and the polarization of electromagnetic waves, can be demonstrated. It can also be shown that electromagnetic waves, like all waves, can undergo reflection, refraction, interference and diffraction. The apparatus described on p. 987 has a frequency of 150 MHz and hence the wavelength $\lambda = c/f = 3 \times 10^8/(150 \times 10^6)$ metres = 2 metres = 200 cm. This is rather long for demonstration experiments in a teaching laboratory. *Microwaves* of about 3 cm wavelength are therefore used. These are radiated from a horn waveguide T and are received by a similar waveguide R or smaller *probe*. The detected wave then produces a deflection in a connected meter. Some experiments which can be performed in a school laboratory are illustrated in Fig. 39.27 (i)–(vi).

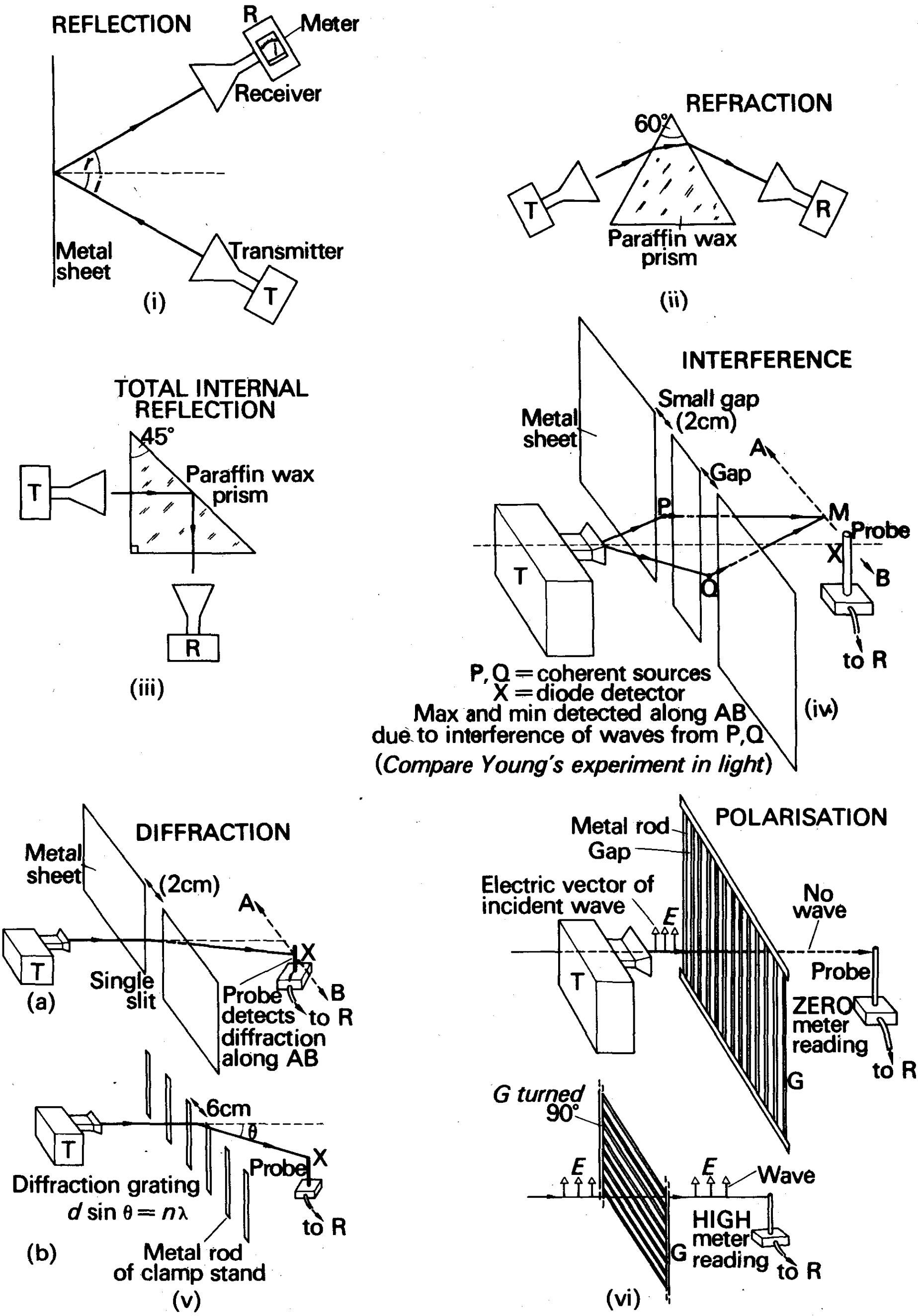


FIG. 39.27. Experiments with microwaves: (i) Reflection; (ii) Refraction; (iii) Total internal reflection; (iv) Interference; (v) Diffraction; (vi) Polarization.

EXERCISES 39

1. Explain what is meant by the *peak value* and *root mean square value* of an alternating current. Establish the relation between these quantities for a sinusoidal waveform.

What is the r.m.s. value of the alternating current which must pass through a resistor immersed in oil in a calorimeter so that the initial rate of rise of temperature of the oil is three times that produced when a direct current of 2 amps passes through the resistor under the same conditions? (*N.*)

2. Define the *impedance* of an a.c. circuit.

A 2·5 μF capacitor is connected in series with a non-inductive resistor of 300 ohms across a source of p.d. of r.m.s. value 50 volts alternating at $1000/2\pi$ Hz. Calculate (*a*) the r.m.s. values of the current in the circuit and the p.d. across the capacitor, (*b*) the mean rate at which energy is supplied by the source. (*N.*)

3. Describe and explain the mode of action of an ammeter suitable for the measurement of alternating current.

A constant a.c. supply is connected to a series circuit consisting of a resistance of 300 ohms in series with a capacitance of 6·67 microfarads, the frequency of the supply being $3000/2\pi$ Hz. It is desired to reduce the current in the circuit to half its value. Show how this could be done by placing either (*a*) an additional resistance, or (*b*) an inductance, in series. Calculate in each case the magnitude of the extra component. (*L.*)

4. Describe and explain an instrument for measuring alternating current. What do you understand by the r.m.s. value of an alternating current? How is it related to the peak value in the case of a sinusoidal current?

When the coils of an electromagnet are connected to a 240-volt d.c. supply, the current taken is 10 amps. When connected to a 240-volt a.c. supply the current taken is only 1 amp. Explain why there is a smaller current on a.c. and calculate the resistance of the coils. Using the same time axis, draw curves showing how the current through the coils and the applied voltage vary with time when the supply is a.c. (*C.*)

5. A rectangular coil with n turns each of area A is rotated with uniform angular velocity ω about an axis at right angles to a uniform magnetic field of flux density B. Show that a sinusoidal alternating e.m.f. is generated in the coil and write down an expression for its peak value.

Explain what is meant by the root-mean-square (r.m.s.) value of an alternating current or voltage, and show how for a sinusoidal alternation it can be calculated from the peak value.

A resistor and an inductor are connected in series across the output terminals of a variable frequency a.c. generator. The r.m.s. voltage between the terminals of the generator is 10 volts. When the frequency is 20 cycles per second, the r.m.s. current in the circuit is 0·20 amp, the p.d. across the resistor is 8 volts, and the p.d. across the inductor is 6 volts. With the help of a vector diagram, explain these readings, and find the values of the resistance and the inductance.

With the generator voltage unchanged at 10 volts r.m.s. its frequency is adjusted until at some value f the p.d. across the resistor and that across the inductor both have the same value V. Calculate f and V. (*O.*)

6. Define *self inductance.* Explain and justify a method of calculating the self inductance of an endless solenoid whose axis is a circle.

A coil of self inductance 0·987 millihenry and resistance 25 ohms is connected in series with a variable capacitance and an a.c. supply of constant e.m.f. and of frequency $15000/\pi$ Hz. Find the value of the capacitance for which the current in the coil would be a maximum. Represent on a vector diagram the relative potential differences across the coil and the capacitance in this condition and find the phase angle between them. What is the power factor of the coil? (*L.*)

7. A box X and a coil Y are connected in series with a variable frequency a.c. supply of constant e.m.f. 10 volts. X contains a capacitance 1 μF in series with a resistance 32 ohms, and Y has a self-inductance 5·1 millihenry and resistance 68 ohms. The frequency is adjusted until maximum current flows in X and Y. Determine the impedances of X and Y at this frequency and hence find the potential differences across X and Y separately. Show these potential differences on a vector diagram indicating their phase relations with the applied e.m.f. (*L.*)

8. Explain what is meant by *self-induction.* Describe one experiment in each case to illustrate the effect of a coil having self-inductance in (*a*) a direct current circuit, (*b*) an alternating current circuit.

Describe the essential features of an instrument for measuring alternating current. (*N.*)

9. A filament lamp of negligible inductance is found to glow when connected in series with a capacitor across an alternating voltage supply of negligible impedance. Explain what will happen to the brightness of the lamp if the frequency of the supply is doubled. Show how a second capacitor can be included in the circuit so as to increase the brightness of the lamp.

What is the advantage of using a capacitor rather than a resistor to control the brightness of a lamp operated on an alternating voltage power supply? (*N.*)

10. Explain what is meant by the terms *impedance* and *phase angle* as applied to a circuit carrying alternating current.

An a.c. generator of negligible internal impedance whose output voltage is 100 volts r.m.s. is connected in series with a resistance of 100 ohms and a pure inductance of 2 henrys. If an a.c. voltmeter gives the same reading whether connected across the resistance or the inductance what does it read? What is the frequency of the generator? Calculate the power dissipated in the resistor at the instant when the current is half its maximum value. Calculate also the power supplied to the inductance at the same instant. Show that the sum of these two quantities is equal to the power delivered by the generator at this instant. (*O. & C.*)

11. An alternating current is represented by the equation $I = I_0 \sin \omega t$. Write down (*a*) the r.m.s. value of the current, (*b*) the frequency of the current. Deduce the maximum value of the rate of change of the current.

A long solenoid wound with 10 turns per cm carries an alternating current of frequency 50 cycles per second^{-1} and of r.m.s. value 1 amp. In the centre of the solenoid is mounted coaxially a five-turn coil of diameter 1 cm. Calculate the r.m.s. voltage induced in this coil.

If the inner coil is surrounded by a copper tube, the voltage induced in it decreases. Explain. (*C.*)

12. The r.m.s. potential difference V volts alternating at f Hz across a coil is adjusted until the r.m.s. current through the coil is 10 mA. Corresponding values of V and f are given in the table, the current being adjusted to 10 mA in each instance.

f(Hz)	1000	1500	2000	2500	3000
V(volts)	3·88	5·78	7·58	9·46	11·30

(*a*) Why does V change when f changes? (*b*) Plot a graph of V^2 against f^2. (*c*) Use the graph to find the inductance of the coil. (*d*) Comment on the possible use of the graph in a determination of the resistance of the coil. (*e*) If a corresponding set of readings were obtained using a capacitor instead of a coil, state with reasons how V would vary with f. (*N.*)

13. State the laws of electromagnetic induction and describe briefly experiments to show their validity.

A coil A passes current of 1·25 A when a steady potential difference of 5 V is maintained across it, and an r.m.s. current of 1 A when it has across it a sinusoidal potential difference of 5 V r.m.s. at a frequency of 50 Hz (cycles per second). Explain why the current is less in the second case, and calculate the resistance and the inductance of the coil.

The same coil A, which has 100 turns, has a second coil B with 500 turns wound on it so that all the magnetic flux produced by A is linked by B. Find the r.m.s. value of the e.m.f. that appears across the open-circuit ends of B when a sinusoidal alternating current of 1 A r.m.s. at a frequency of 50 Hz is passed through A. Why is the ratio of this e.m.f. to the r.m.s. potential difference across A not the same as the ratio of the number of turns in B and A, i.e. 5:1?

Explain why the insertion of an iron core into the coils would decrease the current in A and increase the e.m.f. across B, if the alternating potential difference across A were kept unchanged; the effects of hysteresis and eddy currents in the iron may be neglected. (*O. & C.*)

Electromagnetic waves

14. A uniform magnetic field of 0·5 Wb m^{-2} is moving with a constant velocity of 4 m s^{-1} past a stationary straight conductor of length 20 cm. If the field, velocity and conductor are mutually perpendicular, calculate the electric field intensity in the conductor observed by an observer moving with the field and the force on an electron in the conductor ($e = 1{\cdot}6 \times 10^{-19}$ C).

15. A wire in the form of a circle radius 5 cm carries a charge of 10^{-8} C and rotates with a constant speed of 10 rev. second^{-1} about its centre. Considering the moving charge as a current, calculate the magnetic field at the centre of the field.

16. 'A moving electric field E produces a magnetic field B.' Explain this statement and derive a relation for B.

17. Show that a magnetic field B, moving with uniform velocity v, generates an electric field $E = Bv$.

18. Stationary electromagnetic waves can be set up in a transmission line. Explain, with diagrams, how this can be demonstrated in the laboratory and how you would verify the existence of the stationary waves.

19. The length of a half wave dipole is 15 m and the frequency of the oscillator feeding the aerial is increased slowly from 5 MHz to 15 MHz. State and explain what will be observed on a receiver connected to an exactly similar aerial, which is placed parallel to the first aerial and a short distance away.

How would you show that the electromagnetic waves from the dipole are plane polarized?

chapter forty
Electrons. Motion in Fields

Particle Nature of Matter

MATTER is made up of many millions of molecules, which are particles whose dimensions are about 3×10^{-10} m. Evidence of the existence of molecules is given by experiments demonstrating *Brownian motion*, with which we assume the reader is familiar. One example is the irregular motion of smoke particles in air, which can be observed by means of a microscope. This is due to continuous bombardment of a tiny smoke particle by numerous air molecules all round it. The air molecules move with different velocities in different directions. The resultant force on the smoke particle is therefore unbalanced, and irregular in magnitude and direction. Larger particles do not show Brownian motion when struck on all sides by air molecules. The resultant force is then relatively negligible.

More evidence of the existence of molecules is supplied by the successful predictions made by the *kinetic theory of gases*. This theory assumes that a gas consists of millions of separate particles or molecules moving about in all directions. *X-ray diffraction patterns* of crystals also provides evidence for the particle nature of matter. The symmetrical patterns of spots obtained are those which one would expect from a three-dimensional grating or lattice formed from particles. A smooth continuous medium would not give a diffraction pattern of spots.

The size of atoms and molecules can be estimated in several different ways. By allowing an oil drop to spread on water, for example, an upper limit of about 5×10^{-7} cm is obtained for the size of an oil molecule. X-ray diffraction experiments enable the interatomic spacing between atoms in a crystal to be accurately found. The results are of the order of a few angstrom units, such as 3 Å or 3×10^{-10} m or 0·3 nm (nanometre).

A simple calculation shows the order of magnitude of the enormous number of molecules present in a small volume. One gram of water occupies 1 cm^3. One mole has a mass of 18 g, and thus occupies a volume of 18 cm^3. Assuming the diameter of a molecule is 3×10^{-8} cm, its volume is roughly $(3 \times 10^{-8})^3$ or 27×10^{-24} cm^3. Hence the number of molecules in 18 cm^3 $= 18/(27 \times 10^{-24}) = 6 \times 10^{23}$ approximately.

Avogadro's constant, N_A, is the number of molecules in one mole of a substance. Accurate values show that $N_A = 6{\cdot}02 \times 10^{23}$ mol^{-1}, or $6{\cdot}02 \times 10^{26}$ kmol^{-1}, where 'kmol' represents a kilomole.

Particle Nature of Electricity

In electrolysis, we assume that the carriers of current through an acid or salt solution are ions, which may be positively and negatively charged (p. 846). From Faraday's laws of electrolysis, the charge carried by each ion is proportional to its valency (p. 845). We can find the charge on a monovalent ion using the following argument.

Avogadro's constant, about $6{\cdot}02 \times 10^{23}$, is the number of molecules in one mole. In electrolysis, 96500 coulombs (one faraday) is the quantity of electricity required to deposit one mole of a monovalent element (see p. 845). When the element is monatomic, the number of ions of one kind which carry this charge is equal to the number of molecules. Thus the charge on each ion is given by $96500/6{\cdot}02 \times 10^{23}$ or $1{\cdot}6 \times 10^{-19}$C. If $1{\cdot}6 \times 10^{-19}$C is denoted by the symbol e, the charge on any ion is then e, $2e$, or $3e$, etc., depending on its valency. Thus e is a basic unit of charge.

All charges, whether produced in electrostatics, current electricity or any other method, are multiples of the basic unit e. Evidence that this is the case was obtained by Millikan, who, in 1909, designed an experiment to measure the unit e.

Theory of Millikan's experiment

Millikan first measured the terminal velocity of an oil-drop falling through air. He then charged the oil-drop and applied an electric field to oppose gravity. The drop now moved with a different terminal velocity, which was again measured.

Suppose the radius of the oil-drop is a, the densities of oil and air are ρ and σ respectively, and the viscosity of air is η. When the drop, without a charge, falls steadily under gravity with a terminal velocity v_1,

$$\text{upthrust} + \text{viscous force} = \text{weight of drop.}$$

$$\therefore\ \tfrac{4}{3}\pi a^3 \sigma g + 6\pi\eta a v_1 \text{ (Stokes's law)} = \tfrac{4}{3}\pi a^3 \rho g,$$

$$\therefore\ 6\pi\eta a v_1 = \tfrac{4}{3}\pi a^3(\rho - \sigma)g, \qquad \text{(i)}$$

$$\therefore\ a = \left[\frac{9\eta v_1}{2(\rho-\sigma)g}\right]^{\frac{1}{2}} \qquad \text{(ii)}$$

Suppose the drop now acquires a negative charge e' and an electric field of intensity E is applied to oppose gravity, so that the drop now has a terminal velocity v_2. Then, since the force due to E on the drop is Ee', we have

$$\tfrac{4}{3}\pi a^3 \sigma g + 6\pi\eta a v_2 = \tfrac{4}{3}\pi a^3 \rho g - Ee'.$$

$$\therefore\ Ee' = \tfrac{4}{3}\pi a^3(\rho - \sigma)g - 6\pi\eta a v_2.$$

Hence, from (i), $\quad Ee' = 6\pi\eta a v_1 - 6\pi\eta a v_2 = 6\pi\eta a(v_1 - v_2).$

Thus, with (ii) $\quad e' = \dfrac{6\pi\eta}{E}\left[\dfrac{9\eta v_1}{2(\rho-\sigma)g}\right]^{\frac{1}{2}}(v_1 - v_2). \qquad \text{(iii)}$

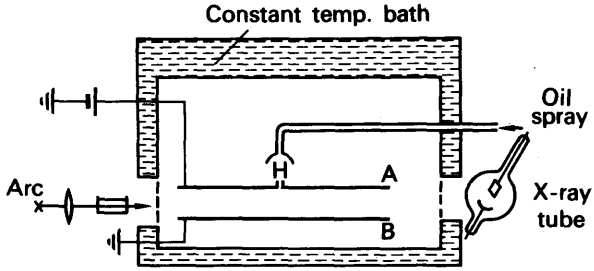


FIG. 40.1. Principle of Millikan's experiment.

Experiment

In his experiments Millikan used two horizontal plates A, B about 20 cm in diameter and 1·5 cm apart, with a small hole H in the centre of the upper plate (Fig. 40.1). He used a fine spray to 'atomize' the oil and create tiny drops above H, and occasionally one would find its way through H, and would be observed in a low-power microscope by reflected light when the chamber was brightly illuminated. The drop was seen as a pin-point of light, and its downward velocity was measured by timing its fall through a known distance by means of a scale in the eyepiece. The field was applied by connecting a battery of several thousand volts across the plates A, B, and its intensity E was known, since $E = V/d$, where V is the p.d. between the plates and d is their distance apart. Millikan found that the friction between the drops when they were formed by the spray created electric charge, but to give a drop an increased charge an X-ray tube was operated near the chamber to ionize the air.

From equation (iii), it follows that when v_1, v_2, E, ρ, σ and η are all known, the charge e' on the drop can be calculated. Millikan found, working with hundreds of drops, that the charge e' was always a simple multiple of a basic unit, $1{\cdot}6 \times 10^{-19}$ coulomb. He thus concluded that the charge e was numerically $1{\cdot}6 \times 10^{-19}$ coulomb.

EXAMPLE

Calculate the radius of a drop of oil, density 900 kg m^{-3}, which falls with a terminal velocity of $2{\cdot}9 \times 10^{-2}$ cm s^{-1} through air of viscosity $1{\cdot}8 \times 10^{-5}$ N s m^{-2}. Ignore the density of the air.

If the charge on the drop is $-3e$, what p.d. must be applied between two plates 5 cm apart for the drop to be held stationary between them? ($e = 1{\cdot}6 \times 10^{-19}$ C.)

When the drop falls with a terminal velocity, force due to viscous drag = weight of sphere. With the usual notation, if ρ is the oil density, we have

$$6\pi\eta a v = \text{volume} \times \text{density} \times g = \tfrac{4}{3}\pi a^3 \rho g$$

$$\therefore\ a = \sqrt{\frac{9\eta v}{2\rho g}} = \sqrt{\frac{9 \times 1{\cdot}8 \times 10^{-5} \times 2{\cdot}9 \times 10^{-4}}{2 \times 900 \times 9{\cdot}8}}$$

$$= 1{\cdot}6 \times 10^{-6}\ \text{m} = 1{\cdot}6 \times 10^{-4}\ \text{cm}, \quad . \quad . \quad . \quad (1)$$

since $v = 2{\cdot}9 \times 10^{-2}$ cm s^{-1} $= 2{\cdot}9 \times 10^{-4}$ m s^{-1} and $g = 9{\cdot}8$ m s^{-2}.

Suppose the upper plate is V volts higher than the lower plate when the drop is stationary, so that the electric field intensity E between the plates is V/d. Then upward force on drop $= E \times 3e =$ weight of drop.

$$\therefore E \times 3e = \tfrac{4}{3}\pi a^3 \rho$$

$$\therefore E = \frac{4\pi a^3 \rho}{9e} = \frac{V}{d}$$

$$\therefore V = \frac{4\pi a^3 \rho d}{9e}$$

$$= \frac{4\pi \times (1{\cdot}6 \times 10^{-6})^3 \times 900 \times 5 \times 10^{-2}}{9 \times 1{\cdot}6 \times 10^{-19}}$$

$$= 1600 \text{ V.}$$

CATHODE RAYS (ELECTRONS) AND PROPERTIES

Atomic physics can be said to have begun with the study of the conduction of electricity through gases. The passage of electricity through a gas, called a 'discharge', was familiar to Faraday, but the steady conduction—as distinct from sparks—takes place when the pressure of the gas is less than about 50 mm Hg; in a neon lamp it is about 10 mm Hg.

The Gaseous Discharge at Various Pressures

Fig. 40.2 (i) represents a glass tube, about 0·5 metre long, connected to a vacuum pump P and a pressure gauge G. It contains an anode A and a cathode K, connected respectively to the positive and negative terminals of the secondary of an induction coil. As the air is pumped out, nothing happens until the pressure has fallen to about 100 mm Hg (mercury). Then thin streamers of luminous gas appear between the electrodes (Fig. 40.2 (ii)).

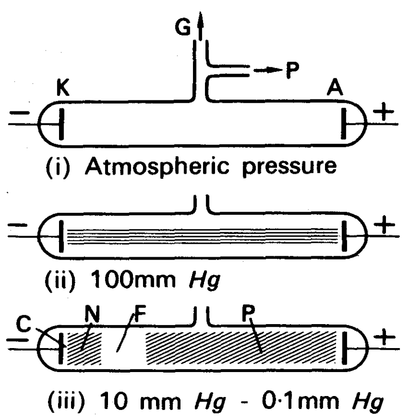


FIG. 40.2. Stages in development of gaseous discharge.

At about 10 mm Hg the discharge becomes a steady glow, spread throughout the tube (Fig. 40.2 (iii)). It is broken up by two darker regions, of which the one nearest the cathode, K in the figure, is narrow and hard to see. The dark region C is called the cathode dark space, or sometimes, after its discoverer, the Crookes' dark space. Beyond the cathode dark space is a bright region N called the negative glow, and beyond that the Faraday dark space F—also called after its discoverer. Beyond the Faraday dark space stretches a luminous column P, called the positive column, which fills the rest of the discharge tube. Sometimes the positive column breaks up into alternating bright and dark segments, called striations, shown in Fig. 40.3. In all the photographs

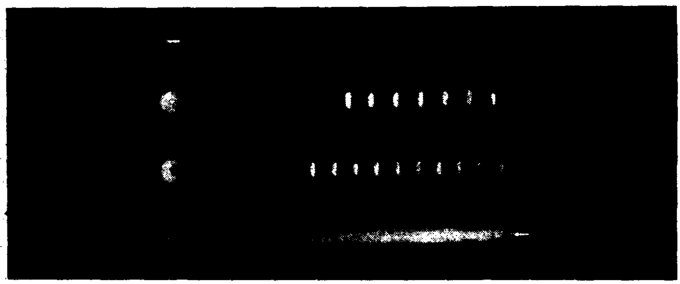


FIG. 40.3.
The lowest photograph shows the positive column. As the pressure decreases, the positive column breaks up into striations and shrinks towards the anode on the right. The top photograph shows the dark space completely filling the tube, when the pressure is about 0·01 mm Hg.

of Fig. 40.3 the cathode is on the left. The cathode dark space can hardly be seen—it lies just around the cathode—but the negative glow and Faraday dark space are clear.

The positive column is the most striking part of the discharge, but the cathode dark space is electrically the most important. In it the electrons from the cathode are being violently accelerated by the electric field, and gaining energy with which to ionize the gas atoms. In the positive column some atoms are being ionized by collisions with electrons; others are being excited, in a way which we cannot describe here, and made to emit their characteristic spectra.

When the pressure of the gas in the discharge tube is reduced still further, the dark spaces swell, and the positive column shrinks. At about 1 mm Hg the cathode dark space becomes distinct, and at 0·1 mm Hg it is several centimetres long. Eventually, as the pressure falls, the cathode dark space stretches from the anode to the cathode, and the negative glow and positive column vanish. This happens at about 0·01 mm Hg in a tube about a half-metre long.

When the cathode dark space occupies the whole discharge tube, the walls of the tube fluoresce, in the way we have already described. The electrons flying across the space are called *cathode rays.* Where they strike the anode they produce X-rays (p. 1067).

The Mechanism of Conduction

How are the ions and electrons in a gaseous discharge produced? A luminous discharge requires a voltage, V, of at least a hundred volts across the gas at pressures of about 1 mm Hg. At much higher or lower pressures, it may require thousands of volts. But with a voltage of about ten, although there is no glow, a very weak current, I, can be detected—of the order of 10^{-15} amp (Fig. 40.4 (i)). This we attribute

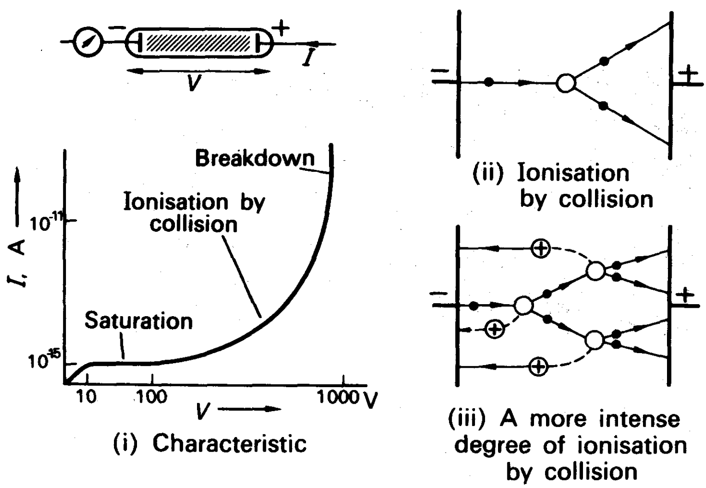


FIG. 40.4. Discharge through gas at low pressure. [In (i) currents and voltages are order of magnitude only: in (ii) and (iii), the numerous non-ionizing collisions are not shown.]

to electrons emitted from the cathode by the photo-electric effect (p. 1076); a trace of ultra-violet light in the laboratory would account for the emission. When the voltage is increased the electrons are accelerated by the electric field to a higher speed, and strike the gas atoms more violently on their way to the anode. When the voltage is high enough the electrons strike the atoms with sufficient kinetic energy to knock electrons out of them (Fig. 40.4 (ii)). This process is called ionization by collision; the atoms become ions, and move towards the cathode; the extra electrons join the original ones in their flight to the anode. At higher voltages the knocked-out electrons are accelerated enough to produce more ions and electrons on the way (Fig. 40.4 (iii)). Eventually a point is reached at which the current grows uncontrollably —the gas is said to break down. In practice, the current is limited by a resistor, in series with the discharge tube; in a commercial neon lamp this resistor, of resistance about 5000 ohms, is hidden in the base.

The current through a gas, like that through an electrolyte, is carried by carriers of both signs—positive and negative. At the anode, the negative electrons enter the wires of the outside circuit, and eventually come round to the cathode. There they meet positive ions, which they now enter, and so re-form neutral gas atoms. Positive ions arriving at the cathode knock off some of the atoms, which diffuse into the body of the discharge, and there, eventually, they are ionized again. Thus a limited amount of gas can carry a current indefinitely.

Once a gas has broken down, current can continue to pass through it even in the dark: that is to say, when there is no ultra-violet light to make the cathode emit electrons. The electrons from the cathode are now simply knocked out of it by the violent bombardment of the positive ions.

Ultra-violet light is not, as a rule, necessary even for starting a gaseous discharge. The somewhat mysterious cosmic rays, which reach the earth from outer space, are able to ionize a gas; they may therefore enable a discharge to start. Once it has started, the emission of electrons by bombardment of the cathode keeps it going.

Modern Production of Cathode Rays

The discharge tube method is not a convenient one for producing and studying cathode rays or electrons. Firstly, a gas is needed at the appropriate low pressure; secondly, a very high p.d. is needed across the tube; thirdly, X-rays are produced (p. 1068) which may be dangerous.

Nowadays a *hot cathode* is used to produce a supply of electrons. This may consist of a fine tungsten wire, which is heated to a high temperature when a low voltage source of 4–6 V is connected to it. Metals contain free electrons, moving about rather like the molecules in a gas. If the temperature of the metal is raised, the thermal velocities of the electrons will be increased. The chance of electrons escaping from the attraction of the positive ions, fixed in the lattice, will then also be raised. Thus by heating a metal such as tungsten to a high temperature, electrons can be 'boiled off'. This is called *thermionic emission* (see also p. 1009).

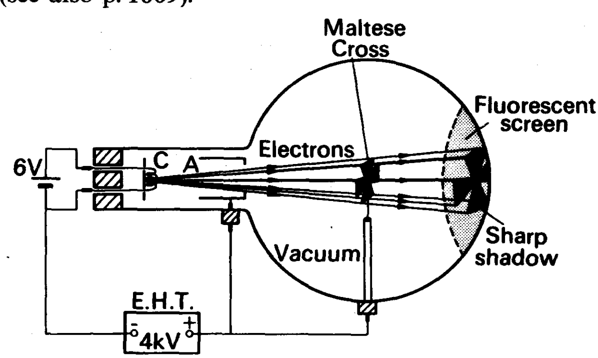


FIG. 40.5. Electrons travel in straight lines.

Fig. 40.5 shows a tungsten filament C inside an evacuated tube. When heated by a low voltage supply, electrons are produced, and they are accelerated by a positive voltage of several thousand volts applied between C and a metal cylinder A. The electrons travel unimpeded across the tube past A, and produce a glow when they collide with a fluorescent screen and give up their energy.

Properties of Cathode Rays

Fast-moving electrons emitted from C produce a sharp shadow of a Maltese cross on the fluorescent screen, as shown in Fig. 40.5. Thus the cathode rays travel in straight lines. They also produce heat when incident on a metal—a fine piece of platinum glows, for example.

When a magnet is brought near to the electron beam, the glow on the fluorescent screen moves, Fig. 40.6. If Fleming's left hand rule is

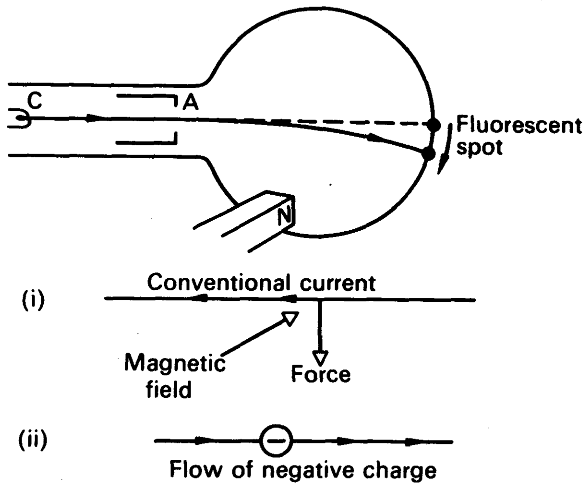


FIG. 40.6. Deflection shows electrons are negatively charged.

applied to the motion, the middle finger points in a direction *opposite* to the electron flow. Thus electrons appear to be particles which carry a *negative* charge.

This is confirmed by collecting electrons inside a *Perrin tube*, shown in Fig. 40.7. The electrons are deflected by the magnet S until they

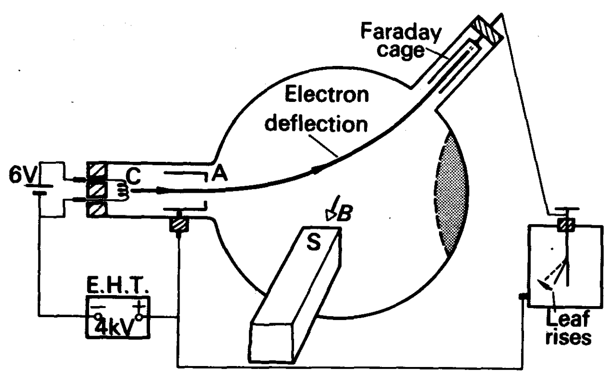


FIG. 40.7. Perrin tube. Direct method for testing electron charge.

pass into a metal cylinder called a 'Faraday cage' (see p. 760). The cylinder is connected to the plate of an electroscope, which has been negatively charged using an ebonite rod and fur. As soon as the electrons are deflected into the cage the leaf rises further, showing that an extra *negative* charge has been collected by the cage. This supports the idea that cathode rays are fast moving electrons.

ELECTRON MOTION IN ELECTRIC AND MAGNETIC FIELDS

Deflection in an Electric Field

Suppose a horizontal beam of electrons, moving with velocity v, passes between two parallel plates, Fig. 40.8. If the p.d. between the plates is V and their distance apart is d, the field intensity $E = V/d$. Hence the force on an electron of charge e moving between the plates $= Ee = eV/d$ and is directed towards the positive plate.

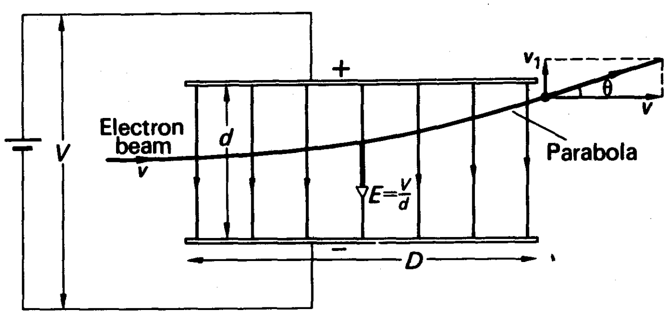


FIG. 40.8. Deflection in electric field.

Since the electric intensity E is vertical, no horizontal force acts on the electron entering the plates. Thus the horizontal velocity, v, of the beam is unaffected. This is similar to the motion of a projectile projected horizontally under gravity. The vertical acceleration due to gravity does not affect the horizontal motion.

In a vertical direction the displacement, y, $= \frac{1}{2}at^2$, where $a =$ acceleration $=$ force/mass $= Ee/m_e$ and t is the time.

$$\therefore y = \frac{1}{2}\frac{Ee}{m_e}t^2 \quad . \quad . \quad . \quad . \quad \text{(i)}$$

In a horizontal direction, the displacement, x, $= vt$ . . (ii)

Eliminating t between (i) and (ii), we obtain

$$y = \frac{1}{2}\left(\frac{Ee}{m_e}\right)\frac{x^2}{v^2} = \frac{Ee}{2m_e v^2}x^2$$

The path is therefore a *parabola*.

When the electron just passes the plates, $x = D$. The value of y is then $y = EeD^2/2m_e v^2$. The beam then moves in a straight line, as shown in Fig. 40.8. The time for which the electron is between the plates is D/v. Thus the component of the velocity v_1, gained in the direction of the field during this time, is given by

$$v_1 = \text{acceleration} \times \text{time} = \frac{Ee}{m_e} \times \frac{D}{v}.$$

Hence the angle θ at which the beam emerges from the field is given by:

$$\tan\theta = \frac{v_1}{v} = \frac{EeD}{m_e v} \cdot \frac{1}{v} = \frac{EeD}{m_e v^2}.$$

The energy of the electron is increased by an amount of $\frac{1}{2}mv_1^2$ as it passes through the plates, since the energy due to the horizontal motion is unaltered.

Deflection in a Magnetic Field

Consider an electron beam, moving with a speed v, which enters a uniform magnetic field of induction B acting perpendicular to the direction of motion. Fig. 40.9. The force F on an electron is then Bev. The direction of the force is perpendicular to both B and v.

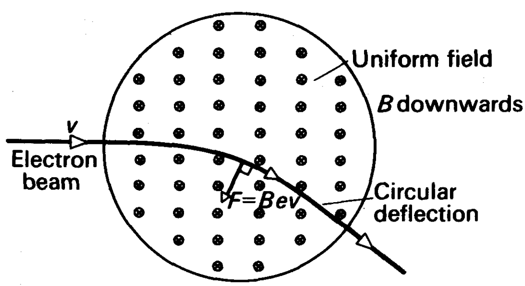


FIG. 40.9. Circular motion in uniform magnetic field.

Consequently, unlike the electric force, the magnetic force cannot change the *energy* of the electron.

The force Bev is always normal to the path of the beam. If the field is uniform, the force is constant in magnitude and the beam then travels in a *circle* of radius r. Since Bev is the centripetal force towards the centre,

$$Bev = \frac{m_e v^2}{r}$$

$$\therefore r = \frac{m_e v}{Be} = \frac{\text{momentum}}{Be}$$

Thomson's Experiment for e/m

In 1897, Sir J. J. Thomson devised an experiment for measuring the ratio *charge/mass* or e/m_e for an electron, sometimes called its *specific charge*.

Thomson's apparatus is shown simplified in Fig. 40.10. C and A are the cathode and anode respectively, and narrow slits are cut in opposite plates at A so that the cathode rays passing through are limited to a narrow beam. The rays then strike the glass at O, producing a glow there. The rays can be deflected electrostatically by means of connecting a large battery to the horizontal plates P, Q, or magnetically by means of a current passing through Helmholtz coils R, S, on either side of the tube near P and Q, as shown by the small circles in Fig. 40.10.

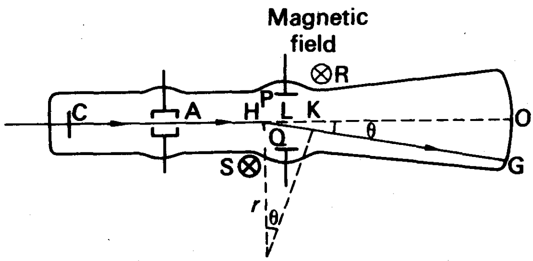


FIG. 40.10. Thomson's determination of e/m for electron (*not to scale*).

The magnetic field is perpendicular to the paper, and if it is uniform a constant force acts on the cathode rays (electrons) normal to its motion. The particles thus begin to move along the arc HK of a circle of radius r. When they leave the field, the particles move in a straight line and strike the glass at G.

With the usual notation, see p. 1000,

$$\text{force } F = Bev = \frac{m_e v^2}{r},$$

where e is the charge on an electron and m is its mass.

$$\therefore \frac{e}{m_e} = \frac{v}{rB} \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

To find the radius r, we note that, from Fig. 40.10, $\tan\theta = OG/OL = HK/r$.

$$\therefore r = \frac{HK \,.\, OL}{OG}.$$

L is about the middle of the solenoid surrounding the plates.

The velocity v was found by applying an electric field between P, Q of such an intensity E as to bring the beam back to O. Then

$$Ee = Bev$$

$$\therefore v = \frac{E}{B}.$$

Thomson found that v was considerably less than the velocity of light, 3×10^8 m s^{-1}, so that cathode rays were certainly not electromagnetic waves.

On substituting for v and r in (i), the ratio charge/mass (e/m_e) for an electron was obtained. Modern determinations show that

$$\frac{e}{m_e} = 1{\cdot}76 \times 10^{11} \text{coulomb per kg (C kg}^{-1})$$

or
$$\frac{m_e}{e} = \frac{1}{1{\cdot}76} \times 10^{-11} \text{ kg per coulomb (kg C}^{-1}) \quad . \quad \text{(ii)}$$

Now from electrolysis the electrochemical equivalent of hydrogen is 0·0000104 g per coulomb, or $1{\cdot}04 \times 10^{-5}$ g C^{-1}.

$$\therefore \frac{m_H}{e} = 1{\cdot}04 \times 10^{-8} \text{ kg C}^{-1},$$

assuming the hydrogen ion carries a charge e numerically equal to that on an electron, m_H being the mass of the hydrogen ion. Hence, with (ii),

$$\frac{m_e}{m_H} = \frac{1}{1{\cdot}76 \times 1{\cdot}04 \times 10^3} = \frac{1}{1830}.$$

Thus the electron is nearly two thousand times as light as the hydrogen atom.

Until Sir J. J. Thomson's experiment, it was believed that the hydrogen atom was the lightest particle in existence. Note also that in Thomson's experiment, the speed v of the electron beam is measured by means of perpendicular ('crossed') magnetic and electric fields.

Fine Beam Tube

The *fine beam tube* also enables the ratio e/m to be determined. One form of apparatus is shown in Fig. 40.11. It consists of an electron gun which produces an electron beam from a heated cathode. This is accelerated by a voltage V applied to the anode A. The heater and

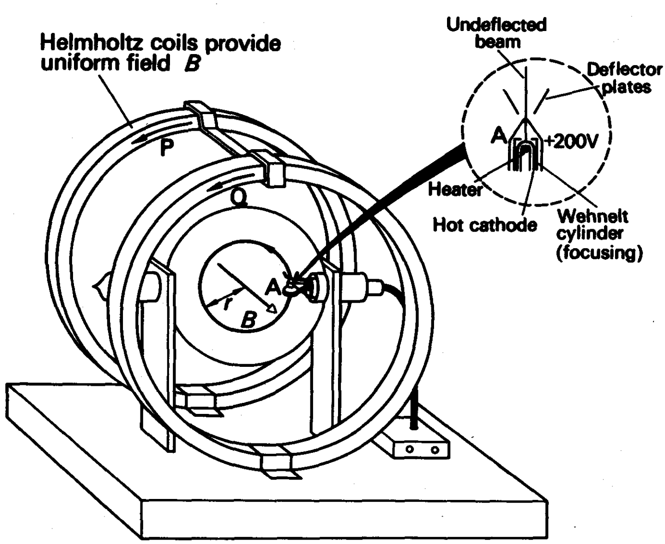


FIG. 40.11. Fine beam tube method for e/m_e.

other voltage supplies are shown in Fig. 40.12. The beam is made visible by having a small quantity of hydrogen gas inside the tube. Collisions of the electrons with the gas molecules cause the latter to emit light, and a straight beam of light is seen, thus showing the electron path.

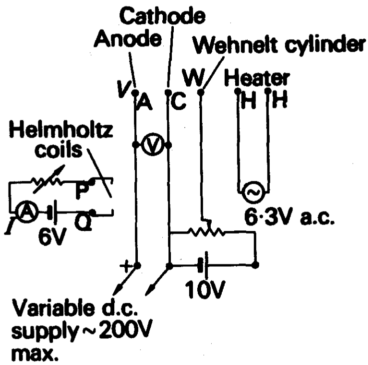


FIG. 40.12. Fine beam tube voltage supplies.

The two large Helmholtz coils P and Q are placed a distance apart equal to their radius to provide a *uniform magnetic field* over the path of the electrons (see p. 936). The value of B can be varied by changing the current I in the coils. Fig. 40.12. When the magnetic field is switched on, the electrons are deflected into a circle of radius r as shown in Fig. 40.11. r may be measured with the aid of a travelling microscope, or by using markers and a mirror. The anode voltage, V, should be measured using a voltmeter. To calculate B, the current I in the Helmholtz coils and their mean radius R are measured. If N is the number of turns in each coil, then from p. 936, B is given by

$$B = 0{\cdot}72\frac{\mu_0 NI}{R}$$

Theory

If we assume that the initial velocity of the electrons is zero, the kinetic energy of the electrons in moving through a p.d. V is given by

$$\tfrac{1}{2}m_e v^2 = eV,$$

where m_e is the mass and v the velocity of each electron.

$$\therefore v^2 = \frac{2eV}{m_e} \qquad . \quad . \quad . \quad . \quad . \qquad \text{(i)}$$

Since

$$Bev = \frac{m_e v^2}{r},$$

$$\therefore v = \frac{rBe}{m_e} \qquad . \quad . \quad . \quad . \quad . \qquad \text{(ii)}$$

From (i) and (ii)

$$\frac{2eV}{m_e} = \left(\frac{rBe}{m_e}\right)^2$$

$$\therefore \frac{e}{m_e} = \frac{2V}{r^2B^2},$$

from which e/m_e may be evaluated if V, r and B are known.

EXAMPLE

Describe and give the theory of a method to determine e the electronic charge. Why is it considered that all electric charges are multiples of e?

An electron having 450 electron-volts of energy moves at right angles to a uniform magnetic field of magnetic induction (flux density) $1{\cdot}50 \times 10^{-3}$ weber metre^{-2}. Show that the path of the electron is a circle and find its radius. Assume that the specific charge of the electron is $1{\cdot}76 \times 10^{11}$ coulomb kilogramme^{-1}. (*L.*)

With the usual notation, the velocity v of the electron is given by

$$\tfrac{1}{2}m_e v^2 = eV, \qquad \text{where } V \text{ is } 450 \text{ V}.$$

$$\therefore v = \sqrt{\frac{2eV}{m_e}} \qquad \qquad (1)$$

The path of the electron is a circle because the force Bev is constant and always normal to the electron path. Its radius r is given by

$$Bev = \frac{m_e v^2}{r}$$

$$\therefore r = \frac{m_e}{e}.\frac{v}{B} = \frac{m_e}{e}.\frac{1}{B}\sqrt{\frac{2eV}{m_e}}, \text{ from (1)}$$

$$\therefore r = \frac{1}{B}\sqrt{2\frac{m_e}{e}V}$$

Now $e/m_e = 1{\cdot}76 \times 10^{11}$ C kg^{-1}, $V = 450$ V, $B = 1{\cdot}5 \times 10^{-3}$ T (Wb m^{-2})

$$\therefore r = \frac{1}{1{\cdot}5 \times 10^{-3}}\sqrt{\frac{2 \times 450}{1{\cdot}76 \times 10^{11}}} \text{ metre}$$

$$= 4{\cdot}8 \times 10^{-2} \text{ metre} = 4{\cdot}8 \text{ cm}.$$

EXERCISES 40

1. Describe an experiment to determine the magnitude of the charge associated with an electron.

State the nature of the path traversed by a charged particle when it is projected at right angles to (*a*) a uniform magnetic field, (*b*) a uniform electric field. A uniform magnetic field and a uniform electric field are superimposed so that they allow a charged particle of velocity v to proceed in a straight line in a vacuum. Explain the relations between (i) the directions of the fields and of the particle velocity, (ii) the magnitudes of the fields. (*L.*)

2. Give an account of a method by which the charge associated with an electron has been measured.

Taking this electronic charge to be $-1{\cdot}60 \times 10^{-19}$ coulomb, calculate the potential difference in volts necessary to be maintained between two horizontal conducting plates, one 0·50 cm above the other, so that a small oil drop, of mass $1{\cdot}31 \times 10^{-11}$ g with two electrons attached to it, remains in equilibrium between them. Which plate would be at the positive potential? (*L.*)

3. Describe an experiment to determine the ratio of the charge to the mass of electrons. Draw labelled diagrams of (*a*) the apparatus, (*b*) any necessary electrical circuits, and show how the result is calculated from the observations.

Two plane metal plates 4·0 cm long are held horizontally 3·0 cm apart in a vacuum, one being vertically above the other. The upper plate is at a potential of 300 volts and the lower is earthed. Electrons having a velocity of $1{\cdot}0 \times 10^7$ m s^{-1} are injected horizontally midway between the plates and in a direction parallel to the 4·0 cm edge. Calculate the vertical deflection of the electron beam as it emerges from the plates. (e/m for electron $= 1{\cdot}8 \times 10^{11}$ C kg^{-1}.) (*N.*)

4. Show that if a free electron moves at right angles to a magnetic field the path is a circle. Show also that the electron suffers no force if it moves parallel to the field. Point out how the steps in your proof are related to fundamental definitions.

If the path of the electron is a circle, prove that the time for a complete revolution is independent of the speed of the electron.

In the ionosphere electrons execute $1{\cdot}4 \times 10^6$ revolutions in a second. Find the strength of the magnetic induction B in this region. (Mass of an electron $= 9{\cdot}1 \times 10^{-28}$ g; electronic charge $= 1{\cdot}6 \times 10^{-19}$ coulomb.) (*C.*)

5. The electron is stated to have a mass of approximately 10^{-27} g and a negative charge of approximately $1{\cdot}6 \times 10^{-19}$ C. Outline the experimental evidence for this statement. Formulae may be quoted without proof. You are not required to justify the actual numerical values quoted.

An oil drop of mass $3{\cdot}25 \times 10^{-12}$ g falls vertically, with uniform velocity, through the air between *vertical* parallel plates which are 2 cm apart. When a p.d. of 1000 V is applied to the plates the drop moves towards the negatively charged plate, its path being inclined at 45° to the vertical. Explain why the vertical component of its velocity remains unchanged and calculate the charge on the drop.

If the path of the drop suddenly changes to one at 26° 30′ to the vertical, and subsequently to one at 37° to the vertical, what conclusions can be drawn? (*O. & C.*)

6. Give an account of Millikan's experiment for determining the value of the electronic charge e.

In a Millikan-type apparatus the horizontal plates are 1·5 cm apart. With the electric field switched off an oil drop is observed to fall with the steady velocity $2{\cdot}5 \times 10^{-2}$ cm s^{-1}. When the field is switched on the upper plate being positive, the drop just remains stationary when the p.d. between the two plates is 1500 volts.

(*a*) Calculate the radius of the drop. (*b*) How many electronic charges does it carry? (*c*) If the p.d. between the two plates remains unchanged, with what velocity will the drop move when it has collected two more electrons as a result of exposure to ionizing radiation? (*O. & C.*)

7. Give a short account of the phenomena observed when an electric discharge passes through a gas at very low pressure (10^{-6} atmosphere). Describe very briefly experiments which reveal the nature of the discharge.

What is the direction of the force acting on a negatively charged particle moving through a magnetic field? Deduce the shape of the path of a charged particle projected at right angles to a uniform magnetic field. (*L.*)

8. Describe a method for measuring the charge per unit mass for the electron, showing how the value is calculated from the observations.

An ion, for which the charge per unit mass is $4{\cdot}40 \times 10^7$ C kg^{-1}, has a velocity of $3{\cdot}52 \times 10^7$ cm s^{-1} and moves in a circular orbit in a magnetic field of induction 0·4 Wb m^{-2}. What will be the radius of this orbit? (*L.*)

9. Describe an oil drop method of determining the electronic charge, *e*. How may Avogadro's constant be found when *e* is known?

An oil drop of radius $1{\cdot}000 \times 10^{-3}$ cm falls freely in air, midway between two vertical parallel metal plates of large extent, which are 0·5000 cm apart, and its terminal velocity is 1·066 cm s^{-1}. When a potential difference of 3000 volts is applied between the plates, the path of the drop becomes a straight line inclined at an angle of 31° 36′ to the vertical. Find the charge on the drop. (Assume the viscosity of air to be $1{\cdot}816 \times 10^{-5}$ kg m^{-1} s^{-1}.) (*L.*)

10. An electron with a velocity of 10^7 m s^{-1} enters a region of uniform magnetic flux density of 0·10 Wb m^{-2}, the angle between the direction of the field and the initial path of the electron being 25°. By resolving the velocity of the electron find the axial distance between two turns of the helical path. Assume that the motion occurs in a vacuum and illustrate the path with a diagram. ($e/m = 1{\cdot}8 \times 10^{11}$ coulomb kg^{-1}.) (*N.*)

11. Describe an apparatus for determining the ratio of the charge *e* to the mass *m* of the electron. Explain how measurements are carried out with the apparatus, and derive the relationship between e/m and the experimentally measured quantities.

Indicate briefly how you would attempt to test whether the particles emitted in the photoelectric effect and in thermionic emission are the same.

When low energy electrons are moving at right angles to a uniform magnetic field of flux density 10^{-3} Wb m^{-2}, they describe circular orbits $2{\cdot}82 \times 10^7$ times per second. Deduce a value for e/m. (*O. & C.*)

12. Describe and give the theory of the Millikan oil drop experiment for the determination of the electronic charge. What is the importance of the experiment?

In one such experiment a single charged drop was found to fall under gravity at a terminal velocity of 0·0040 cm per second and to rise at 0·0120 cm per second when a field of 2×10^5 volt per m was suitably applied. Calculate the electronic charge given that the radius, *a*, of the drop was $6{\cdot}0 \times 10^{-7}$ m and that the viscosity, η, of the gas under the conditions of the experiment was 180×10^{-5} N s m^{-2} (*N.*)

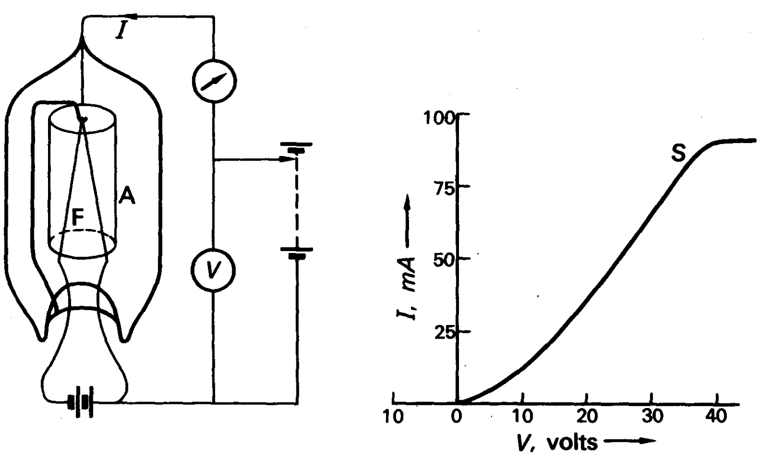
chapter forty-one

Radio Valves. C.R.O. Junction Diode. Transistor

RADIO VALVES. CATHODE RAY OSCILLOGRAPH

Rectification by A.C. Diode Valve

ALTERNATING current is easier to distribute that direct current, because alternating voltage can be transformed easily up or down. For electrolysis, battery-charging, and the operation of radio-receivers and transmitters, however, direct current is essential. It can be obtained from an alternating current supply by means of a *rectifier*, which is a device that will only pass current in one direction. A common type of rectifier is that called a *diode valve*. It contains a metal filament, F in Fig. 41.1 (*a*), surrounded by a metal anode A. The filament is heated by a current drawn from a low voltage supply, and emits electrons. A circuit for varying the anode potential is shown in Fig. 41.1 (*a*).



(i) Diode (ii) Characteristic

FIG. 41.1. Diode valve.

Since electrons are negative charges, such a device passes current when its anode is made positive with respect to its filament, but not when the anode is made negative. Fig. 41.1 (*b*) shows the curve of anode current against anode potential for a small diode; it is called the diode's characteristic curve, or simply its *characteristic*. The current increases with the positive anode potential as far as the point S. Beyond this point the current does not increase, because the anode is collecting all the electrons emitted by the filament; the current is said to be saturated.

At first sight we might expect that any positive anode potential, however small, would draw the full saturation current from the filament to the anode. But it does not, because the charges on the electrons make them repel one another. Thus the cloud of electrons between the anode and filament repels the electrons leaving the filament, and turns some of them back. The electrons round the filament are like the molecules in a cloud of vapour above a liquid; they are continually escaping from it and returning to it. The positive anode draws some away from the cloud, as a wind carries water vapour away from a pool. The wind, or the anode, thins out the cloud, so that more electrons or molecules escape than return. The higher the anode potential, the fewer electrons return to the filament; as the anode potential rises, the current increases, to its saturation limit.

Rectifier Circuit

When a diode is used as a rectifier it is connected in a circuit such as Fig. 41.2 (i). The low-voltage secondary of the transformer simply provides the heating current from the filament. The current to be rectified is drawn from the high-voltage secondary. One end of this

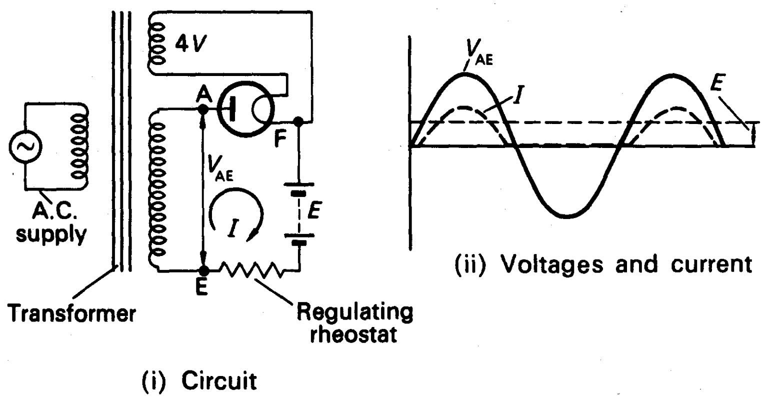


FIG. 41.2. Rectifier circuit with diode.

secondary is connected to the load, which could be, as shown, an accumulator on charge; the other end of the load is connected to the anode of the diode, and the other end of the secondary to one of the filament connexions. When the transformer secondary voltage V_{AE} is greater than the e.m.f. E of the accumulator, the anode is positive with respect to the filament; electrons from the hot filament are then drawn to the anode, and a current flows through the load (Fig. 41.2 (ii)). On half-cycles when the anode is negative, the electrons are repelled, and no current flows. Because it only allows current to flow through it in one direction, a thermionic diode is often called a valve.

Some rectifying valves contain a little mercury vapour. When electrons flow through them, they ionize the mercury atoms, as explained on p. 1016. The ions and electrons thus produced make the valve a very good conductor, and reduce the voltage drop across it; they therefore allow more of the voltage from the transformer to appear across the load.

The current from a rectifier flows in pulses, whenever the anode is positive with respect to the filament. Sometimes a smoother current is required, as, for example, in a radio-receiver, where the pulses would cause a humming sound in the loudspeaker. The current can be smoothed by connecting an inductance coil of about 30 henrys in series with the load. The inductance prevents rapid fluctuations in current. So also does a capacitor of about 16 microfarads connected across the load. Generally the two are used together to give a very smooth output.

Metal Rectifier

Yet other rectifiers are not thermionic at all. One such type consists of an oxidized copper disc, Cu_2O/Cu, pressed against a disc of lead, Pb (Fig. 41.3 (i)). These conduct well when the lead is made positive, but very badly when it is made negative; they are called metal rectifiers. A

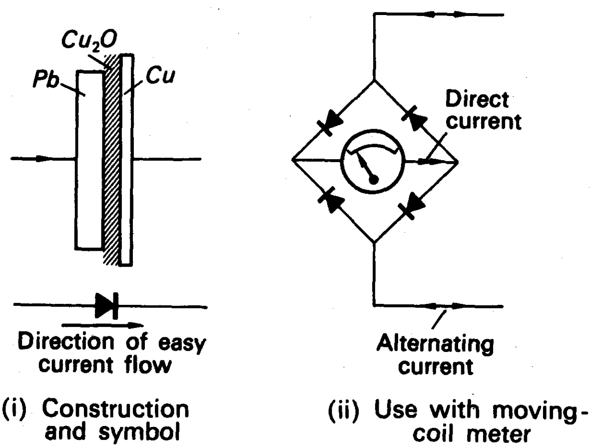


FIG. 41.3. Metal rectifier and use.

metal rectifier can be used to convert a moving-coil milliammeter into an alternating-current ammeter or voltmeter (Fig. 41.3 (ii)). Such a meter is more sensitive than a moving-iron or hot-wire instrument, and has a more open scale near zero: its deflection is roughly proportional to the average value of the current or voltage.

Cathode-Ray Oscillograph

An oscillograph is an instrument for plotting one varying physical quantity—potential difference, sound-pressure, heart-beat—against another—current, displacement, time. A cathode-ray oscillograph, of the kind we are about to describe, plots alternating potential difference against time. It is so called because it traces the desired wave-form with a beam of electrons, and beams of electrons were originally called cathode rays.

A cathode-ray oscillograph is essentially an electrostatic instrument. It consists of a highly evacuated glass tube, T in Fig. 41.4, one end of

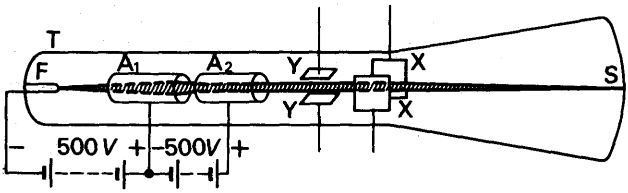
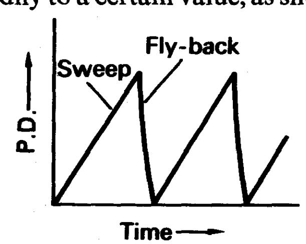


FIG. 41.4. A cathode-ray oscillograph tube.

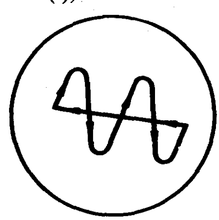
which opens out to form a screen S which is internally coated with zinc sulphide. A hot filament F, at the other end of the tube, emits electrons. These are then attracted by the cylinders A_1 and A_2, which have increasing positive potentials with respect to the filament. Many of the electrons, however, shoot through the cylinders and strike the screen; where they do so, the zinc sulphide fluoresces in a green spot. On their way to the screen, the electrons pass through two pairs of metal plates, XX and YY, called the deflecting plates.

Deflection; Time-base

If a battery were connected between the Y-plates, so as to make the upper one positive, the electrons in the beam would be attracted towards that plate, and the beam would be deflected upwards. In the same way, the beam can be deflected horizontally by a potential difference applied between the X-plates. When the oscillograph is in use, the alternating potential difference to be examined is applied between the Y-plates. If that were all, then the spot would be simply drawn out into a vertical line. To trace the wave-form of the alternating potential difference, the X-plates are used to provide a time-axis. A special valve circuit generates a potential difference which rises steadily to a certain value, as shown in Fig. 41.5 (i), and then falls rapidly



(i) P.D. applied to X - plates



(ii) Trace of spot on screen

FIG. 41.5. Action of a C.R.O.

to zero; it can be made to go through these changes tens, hundreds, or thousands of times per second. This potential difference is applied between the X-plates, so that the spot is swept steadily to the right, and then flies swiftly back and starts out again. This horizontal motion provides what is called the time-base of the oscillograph. On it is superimposed the vertical motion produced by the Y-plates; thus, as shown in Fig. 41.5 (ii), the wave-form of the potential difference to be examined is displayed on the screen.

Focusing

To give a clear trace on the screen, the electron beam must be focused to a sharp spot. This is the function of the cylinders A_1 and A_2, called the first and second anodes. Fig. 41.6 shows the equipotentials of the field between them, when their difference of potential is 500 volts.

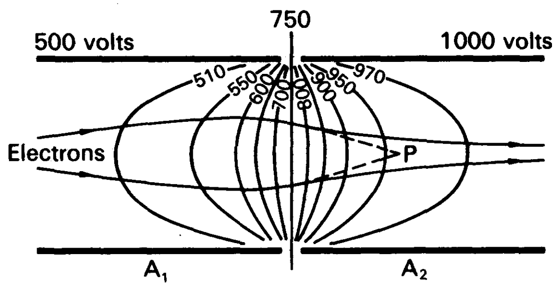


FIG. 41.6. Focusing in a C.R.O. tube.

Electrons entering the field from the filament experience forces from low potential to high at right angles to the equipotentials. They have, however, considerable momentum, because they have been accelerated by a potential difference of about 500 volts, and are travelling fast. Consequently the field merely deflects them, and, because of its cylindrical symmetry, it converges the beam towards the point P. Before they can reach this point, however, they enter the second cylinder. Here the potential rises from the axis, and the electrons are deflected outwards. However, they are now travelling faster than when they were in the first cylinder, because the potential is everywhere higher. Consequently their momentum is greater, and they are less deflected than before. The second cylinder, therefore, diverges the beam less than the first cylinder converged it, and the beam emerges from the second anode still somewhat convergent. By adjusting the potential of the first anode, the beam can be focused upon the screen, to give a spot a millimetre or less in diameter.

Electron-focusing devices are called electron-lenses, or electron-optical systems. For example, the action of the anodes A_1 and A_2 is roughly analogous to that of a pair of glass lenses on a beam of light, the first glass lens being converging, and the second diverging, but weaker.

Uses of Oscillograph

In addition to displaying waveforms, the oscillograph can be used for measurement of voltage, frequency and phase.

1. *A.C. voltage*

An unknown a.c. voltage, whose peak value is required, is connected to the Y-plates. With the time-base switched off, the vertical line on the screen is centred and its length then measured. Fig. 41.7 (i). This is proportional to twice the amplitude or peak voltage, V_0. By measuring the length corresponding to a known a.c. voltage V, then V_0 can be found by proportion.

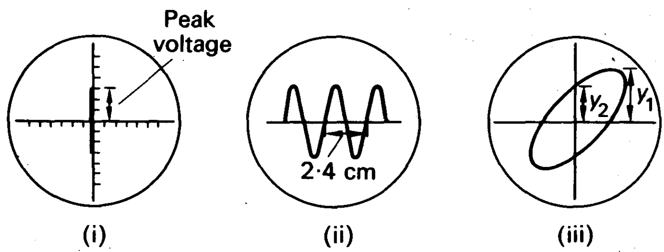


FIG. 41.7. Uses of oscillograph.

Alternatively, using the same gain, the waveforms of the unknown and known voltages, V_0 and V, can be displayed on the screen. The ratio V_0/V is then obtained from measurement of the respective peak-to-peak heights.

2. *Comparison of frequency*

If a calibrated time-base is available, frequency measurements can be made. In Fig. 41.7 (ii), for example, the trace shown is that of an alternating waveform with the time-base switched to the '5 millisec/cm' scale. This means that the time taken for the spot to move 1 cm horizontally across the screen is 5 milliseconds. The horizontal distance on the screen for one cycle is 2·4 cm. This corresponds to a time of $5 \times 2{\cdot}4$ ms or $12{\cdot}0$ ms $= 12 \times 10^{-3}$ seconds, which is the period T.

$$\therefore \text{ frequency} = \frac{1}{T} = \frac{1}{12 \times 10^{-3}} = 83 \text{ Hz.}$$

If a comparison of frequencies f_1, f_2 is required, then the corresponding horizontal distances on the screen are measured. Suppose these are d_1, d_2 respectively. Then, since $f \propto 1/T$,

$$\frac{f_1}{f_2} = \frac{T_2}{T_1} = \frac{d_2}{d_1}.$$

3. *Measurement of phase*

The use of a double beam oscillograph to measure phase difference is given on p. 586. If only a single beam tube is available, an elliptical trace can be obtained. With the time-base switched off, one input is joined to the X-plates and the other to the Y-plates. We consider only the case when the frequencies of the two signals are the same. An ellipse will then be seen generally on the screen, as shown in Fig. 41.7 (iii) (see p. 1014).

The trace is centred, and the peak vertical displacement y_2 at the middle O, and the peak vertical displacement y_1 of the ellipse, are then both measured. Suppose the x-displacement is given by $x = a \sin \omega t$, where a is the amplitude in the x-direction, and the y-displacement by $y = y_1 \sin(\omega t + \phi)$, where y_1 is the amplitude in the y-direction and ϕ is the phase angle. When $x = 0$, $\sin \omega t = 0$, so that $\omega t = 0$. In this case, $y = y_2 = y_1 \sin \phi$. Hence $\sin \phi = y_2/y_1$, from which ϕ can be found.

TRIODE VALVE—AMPLIFIER, OSCILLATOR, DETECTOR

Triode Valve

A few years after the invention of the diode valve Lee de Forest introduced the triode valve. This had three electrodes: a cathode C, the emitter of electrons; an anode A, the collector of electrons; and a *grid* G, a wire with open spaces, placed between the anode and cathode (Fig. 41.8 (i)). The function of the grid is to control the electron flow to the anode, and for this purpose the grid has a small negative potential relative to the cathode. The grid is nearer the cathode than the anode, and its potential thus affects the electric field round the cathode more, with the result that the grid potential has a more delicate control than the anode potential over the anode current. As we shall see later, this enables the triode to act as an amplifier of alternating voltages as well as a detector.

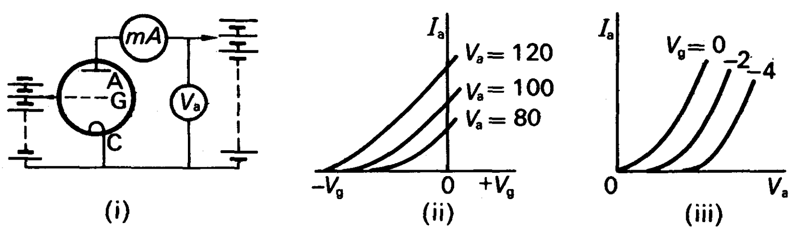


FIG. 41.8. Triode valve characteristics.

Triode Valve Characteristics

In order to predict the performance of a valve in a circuit, the 'characteristics' of the valve must be first determined. The chief characteristics are I_a-V_g (V_a constant), the variation of anode current with grid voltage when the anode voltage is constant; and I_a-V_a (V_g constant), the variation of anode current with anode voltage when grid voltage is constant. The I_a-V_g curves are known as the valve's *mutual characteristics*; the I_a-V_a curves as the *anode characteristics*.

The mutual characteristics obtained are shown in Fig. 41.8 (ii). When the anode voltage is 80 volts, a negative voltage on the grid such as -15 volts creates a resultant negative electric intensity at the cathode, and hence no electrons flow past the grid. As the negative voltage is reduced and reaches a certain value the attractive effect of the positive anode voltage overcomes the repulsive effect of the grid voltage, and electrons now reach the anode. As the negative voltage is reduced further, more electrons reach the anode, and the current increases as shown. The general shape of the I_a-V_g curves is an initial curvature, followed by a straight line.

The anode characteristics, I_a-V_a curves, are shown in Fig. 41.8 (iii), and are explained in a similar way. As the anode voltage, V_a, increases, the anode current increases. Generally, the anode current begins to flow at higher values of V_a when the grid voltage is increased more negatively. If the anode voltage is increased sufficiently, all the electrons emitted by the cathode are collected, and the current has then reached its saturation value (p. 1010).

Valve Constants

There are three main constants or properties of a radio valve. These are:

1. *Anode or A.C. Resistance*, R_a, which is defined by

$$R_a = \frac{\delta V_a}{\delta I_a} (V_g \text{ constant}),$$

the changes in V_a and I_a being taken on the *straight* part of the anode characteristics.

2. *Mutual conductance*, g_m, which is defined by

$$g_m = \frac{\delta I_a}{\delta V_g} (V_a \text{ constant}),$$

the changes in I_a and V_g being taken on the straight part of the mutual characteristics.

3. *Amplification factor*, μ, which is defined by

$$\mu = \frac{\delta V_a}{\delta V_g},$$

where δV_a produces the same change in anode current (V_g constant) as δV_g (V_a constant).

Thus, generally, R_a is the 'resistance' of the valve when the anode circuit variations are considered, g_m is the change in anode current produced by unit grid voltage variation, and μ is a measure of the 'step-up' in voltage produced in the anode circuit by a change in the grid voltage.

Triode as Voltage Amplifier

When a valve is used as a voltage amplifier in radio circuits, it is important to realize at the outset that it amplifies *alternating* voltages, and that these voltages are applied in the grid-cathode circuit, as represented by V in Fig. 41.9 (i). The action of the valve should not only result in an increased alternating voltage V_0 in the anode circuit, known as the 'output voltage', but the waveform of V_0 should be exactly the same as V, the applied voltage, so that there is no distortion. In order to obtain no distortion, a steady negative p.d. (grid-bias, G.B.) is also connected in the grid-cathode circuit, as shown in Fig. 41.9 (i).

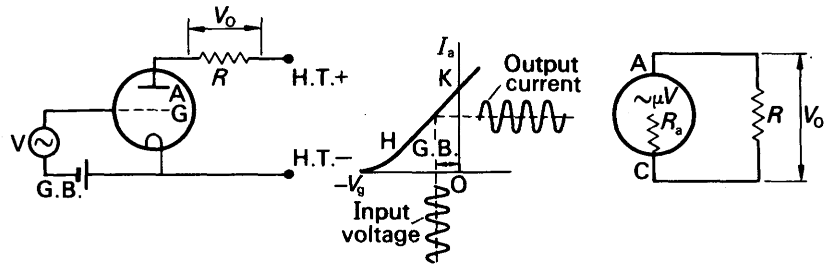


FIG. 41.9. Triode amplification.

The most suitable value of the grid-bias p.d. is OX volts, where X (not shown) corresponds to the middle of the straight part HK of the I_a-V_g characteristic (Fig. 41.9 (ii)). Then, if the applied alternating voltage V has a peak value less than OX, the actual grid potential values will produce anode current variations corresponding to the straight part of the characteristic. The anode or output current will then have a waveform exactly the same as the applied or input voltage V (Fig. 41.9 (ii)). As we shall now show, the triode acts as a 'voltage amplifier' in this case.

Voltage Gain or Amplification Factor

The magnitude of the voltage amplification can be found by replacing the valve circuit in Fig. 41.9 (i) by an 'equivalent A.C. circuit'. Since a change of p.d. δV_g in the grid-cathode circuit is equivalent to a change of $\mu\delta V_g$ in the anode circuit, the alternating voltage V is equivalent to an alternating voltage μV in the anode circuit. We therefore consider that, between the anode and cathode, the valve is an alternating voltage generator of e.m.f. μV, with a internal resistance R_a, the a.c. resistance discussed on p. 1016. See Fig. 41.9 (iii).

To convert the alternating current in the anode circuit to an alternating voltage, a large resistance R is needed, of the order of thousands of ohms. The internal resistance of the H.T. battery and that of the G.B. battery can be neglected by comparison, and since varying voltages are now considered, the magnitudes of the steady H.T. and G.B. voltages can also be ignored. The complete valve equivalent a.c. circuit is therefore as shown in Fig. 41.9 (iii).

The total resistance of the circuit is $R+R_a$. Thus the alternating current, I, flowing

$$=\frac{\mu V}{R+R_a}.$$

$$\therefore \text{ output alternating voltage, } V_0, = IR = \frac{\mu VR}{R+R_a}.$$

$$\therefore \text{ voltage gain or amplification factor} = \frac{V_0}{V} = \frac{\mu R}{R+R_a} \qquad \text{(i)}$$

Thus is a triode has an amplification factor μ of 10, an internal resistance R_a of 8000 ohms and a resistance R of 10000 ohms,

$$\text{voltage gain} = \frac{\mu R}{R+R_a} = \frac{10 \times 10000}{10000+8000} = 5{\cdot}6.$$

Hence if the applied alternating voltage is 0·2 volt (r.m.s.),

$$\text{amplified output voltage} = 5{\cdot}6 \times 0{\cdot}2 = 1{\cdot}1 \text{ volts (r.m.s.)}.$$

Couplings

Fig. 41.10 shows how the output voltage V_0 across R is passed from the valve V1 to the next valve V2. Fig. 41.10 (i) illustrates *resistance–capacitance* coupling. Here one end of the load R is joined to the coupling capacitor, C_g. The other end of R is joined to the lower end of the resistor R_g through the relatively low internal resistance of the h.t. battery. Thus, from an a.c. point of view, C_g and R_g are effectively in

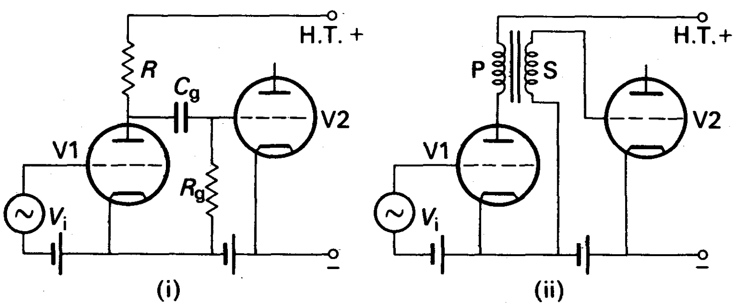


FIG. 41.10. Couplings: (i) Resistance-capacitance; (ii) Transformer.

parallel with R. R_g thus passes to V2 a fraction of the output (a.c.) voltage across R. Note that C_g is necessary to isolate the grid of $V2$ from the positive potential of the h.t. battery.

Fig. 4.10 (ii) illustrates *transformer coupling*. The load in the anode circuit is the primary coil P. The output (a.c.) voltage across this coil is amplified by the transformer and passed to the next value V2 by the secondary coil S.

Basic Oscillatory Circuit

About 1862 Lord Kelvin showed theoretically that when an electrical disturbance is made in a capacitor consisting of a capacitor and a coil and then left, *oscillations* of current occur (see also p. 578). Thus suppose a current I flows at an instant t in a circuit consisting of a coil of inductance L and negligible resistance, in series with a capacitor of capacitance C (Fig. 41.11). Then, from p. 924, if Q is the charge on the capacitor,

$$\text{p.d. across inductance} = -L\frac{dI}{dt}$$

$$= \text{p.d. across capacitor} = \frac{Q}{C}$$

$$\therefore -L\frac{dI}{dt} = \frac{Q}{C}$$

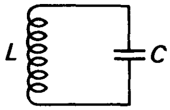


FIG. 41.11.
Basic oscillatory circuit.

$$\text{But } I = \frac{dQ}{dt}. \quad \therefore -L\frac{d^2Q}{dt^2} = \frac{Q}{C}$$

$$\therefore \frac{d^2Q}{dt^2} = -\frac{I}{LC}\cdot Q \quad \text{(i)}$$

This is a 'simple harmonic' equation. Thus Q, the charge circulating, varies with time t according to the relation

$$Q = Q_0 \sin \omega t, \quad . \quad . \quad . \quad \text{(ii)}$$

where Q is the maximum value of the varying charge and ω is a constant given by $\omega^2 = 1/LC$, or

$$\omega = \frac{1}{\sqrt{LC}}.$$

The *frequency, f,* of the oscillatory charge is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}. \quad . \quad . \quad . \quad \text{(iii)}$$

A coil-capacitor series circuit is thus a basic oscillatory circuit, and the frequency of the oscillations of charge (or current) depends on the magnitudes of the inductance L and capacitance C.

The physical reason for the oscillations is the constant interchange of energy between the capacitor and the coil. When the bob of a pendulum is oscillating, its energy at the end of a swing is wholly potential. This gradually changes into kinetic energy until it is wholly kinetic at the middle of the swing, and then becomes potential energy again at the end of the swing. In a similar way, the capacitor becomes fully charged at one instant, the energy being electrostatic energy, and as the capacitor discharges, the energy is stored in the magnetic field of the coil. When the capacitor is fully discharged, the energy is wholly in the magnetic field. After this, the capacitor charges up the other way round, storing electrical energy, and when it is fully charged, there is then no energy in the coil's magnetic field. See p. 578.

Damped and Undamped Waves

The oscillations of charge or current in a circuit containing only an inductor and a capacitor will theoretically last indefinitely with a constant amplitude (Fig. 41.12 (i)). In practice, however, some of the energy is dissipated as heat in the resistance of the coil. Since this energy is no longer available as oscillatory energy, the amplitude of the oscillations gradually diminish, and a *damped* oscillation is thus obtained (Fig. 41.12 (ii)). This is analogous to the case of a vibrating

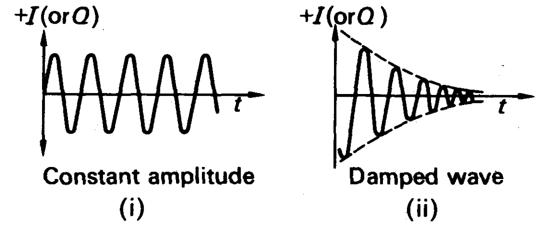


FIG. 41.12. Undamped (continuous) waves and damped waves.

tuning-fork. If there were no air friction the oscillations of the prongs would proceed indefinitely with constant amplitude, but in practice the amplitude of the oscillations gradually diminishes to zero.

Triode as an Oscillator

A triode valve can be used to maintain oscillations in a coil (L, R)-capacitor (C) electrical circuit. The principle of the method is shown in Fig. 41.13, which consists of the basic oscillatory circuit connected in the grid-cathode circuit of the valve; a coil L_1, close to the oscillator coil, in the anode circuit; a high-tension (h.t.) battery; and a capacitor C_g and a resistance R_g in the grid circuit, for a reason to be explained later.

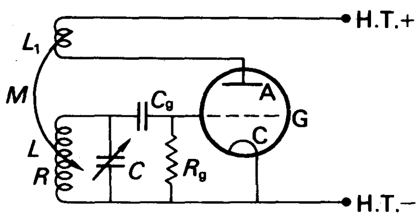


FIG. 41.13. Triode as oscillator.

When the circuit is made, oscillations of current occur in the coil (L, R)-capacitor (C) circuit, as already explained. The oscillatory coil alternating p.d., across C, is amplified by the valve, and oscillatory currents are then obtained in the anode circuit and hence in the coil L_1. By mutual induction M between the coils, *some energy is fed back to the oscillator circuit.* If the feed-back is correctly phased and is of the required amount, it will help to maintain the oscillations of current in the oscillatory current, which will then become undamped. The magnitude and phase of the feed-back can be varied by altering the position of the coil L_1 and, if necessary, reversing its connections in the anode circuit. With audio-frequency oscillations, a continuous whistle can usually be heard.

Efficiency of Oscillator

The source of the oscillatory energy is the h.t. battery in the circuit. An oscillator is thus said to be a device for converting d.c. energy into a.c. energy. The *efficiency* of the circuit is defined as:

$$\frac{\text{output (oscillatory) energy, a.c.}}{\text{input energy, d.c.}} \times 100\%$$

Once the balance-wheel of a watch is set in motion, energy is imparted to it regularly only at certain times of its oscillation. In this way the balance-wheel is supplied with the least amount of energy needed to maintain undamped oscillations. For the same reason, the triode oscillator circuit has a capacitor C_g and a high-resistance R_g in the grid-circuit. See Fig. 41.13. When the circuit is first made the p.d. across R_g is zero, and hence the grid is at zero potential. The alternating or oscillatory voltage across the capacitor C makes the grid positive in potential for some part of its cycle. Some electrons are therefore drawn into the grid circuit, charging the capacitor C_g. During the oscillatory voltage some charge (electrons) leaks away through R_g, and the grid thus becomes more negative in potential. Fig. 41.14 shows roughly how the grid potential decreases, and with suitably chosen values of C_g and R_g it soon settles down to some steady negative value E_g, which is the grid-bias while the valve is functioning. In this condition the oscillatory voltage across the capacitor C only produces a pulse of current in the anode circuit at brief intervals, as shown in Fig. 41.14, and by mutual induction, energy is fed back simultaneously by the coil L_1 into the oscillatory circuit (L, R and C) to make up for the energy lost as heat in the resistance R.

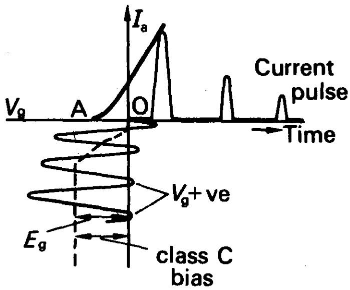


FIG. 41.14. Variation of grid p.d. and anode current (*not to scale*).

The valve is here said to be operating under 'class C' conditions, that is, the negative grid-bias E_g is at least twice the grid-bias value OA which cuts off the anode current in the I_a-V_g characteristic (Fig. 41.14). During part of the cycle the grid potential becomes positive, as shown. A fixed negative grid-bias equal to E_g is unsuitable in a valve oscillator circuit. The alternating (oscillatory) voltage across the capacitor C, once obtained, would not produce any current in the anode circuit and oscillations would then not continue owing to lack of feed-back of energy.

Radio Waves and Modulation

In 1887 Hertz found by experiment that when an oscillatory voltage of high frequency was connected to two capacitor plates far apart, some of the oscillatory energy travelled in space some distance from the plates and was detected. This was the first discovery of the existence of *radio waves.* A transmitting aerial is a form of capacitor in which one 'plate' is high above the other 'plate', the earth. Theory and experiment show that radio waves will not travel out far from a transmitting aerial unless their frequency is very high. Valve oscillator circuits (p. 1020) therefore usually produce alternating voltages of the order of a million (10^6) Hz, 1 MHz, or more, known as *radio-frequencies* (*R.F.*). The Radio 4 station in Britain, broadcasting on 330 metres wavelength, sends out radio waves of a frequency of 908000 Hz (908 kHz). Another station sends out radio waves of very high frequency (V.H.F.) 90·0 MHz.

At broadcasting stations, the oscillator alone would produce a radio wave of constant amplitude (Fig. 41.15 (i), (ii)). When *audio-frequency* (*A.F.*) currents, due to speech or music, are fed through a microphone into the oscillatory circuit, the radio waves are affected or 'modulated' accordingly. In *amplitude modulation* (*A.M.*), the amplitude of the radio-frequency wave varies exactly as the audio-frequency (Fig. 41.15 (iii)). In *frequency modulation* (*F.M.*) the amplitude of the radio-frequency wave is constant but the audio-frequency is superimposed on the frequency of the radio wave (Fig. 41.15 (iv)).

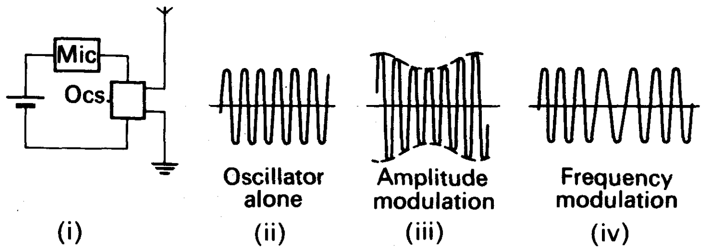


FIG. 41.15. Amplitude and frequency modulation.

Diode Valve Detection

The principle of the diode valve was discussed on p. 1009. There we showed that a diode valve, which consists of a nickel plate or anode placed in a vacuum opposite a cathode emitting electrons, allowed current to flow through it only when the anode was positive in potential relative to the cathode.

The diode can be used to convert alternating to direct voltage (see p. 1010). It can also be used to 'detect' the audio-frequency variation carried along with the modulated wave sent out by transmitters. If a modulated wave is applied between the anode and cathode of a diode, with a resistor R in the circuit (Fig. 41.16 (i)), the valve conducts on the

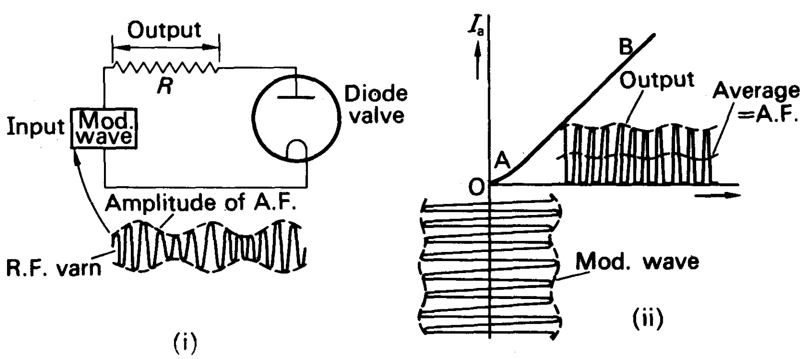


FIG. 41.16. Diode valve detection.

positive parts of the cycle. The variation of current I_a in the anode circuit, the output current, is then as shown in Fig. 41.16 (ii), where OAB is the I_a-V_a curve. The average value of the current, it will be noted, follows the variation of the amplitude of the modulated wave, and hence the voltage across R, called the *output voltage*, has the same audio-frequency variation. In this way the diode is said to act as a 'detector' of the audio-frequency. If the modulated wave were applied to the resistance R without using the diode, the average current obtained would be zero.

Triode as a Detector

We have just shown that the diode can act as a 'detector' of the audio-frequency carried with a modulated wave. The triode can also act as a detector, and in one method, known as *anode-bend detection*, the modulated wave is applied in the grid-cathode circuit, together with a steady grid-bias (G.B.) corresponding to a point on the bend of the I_a-V_g characteristic (Fig. 41.17 (i), (ii)). The swings of the modu-

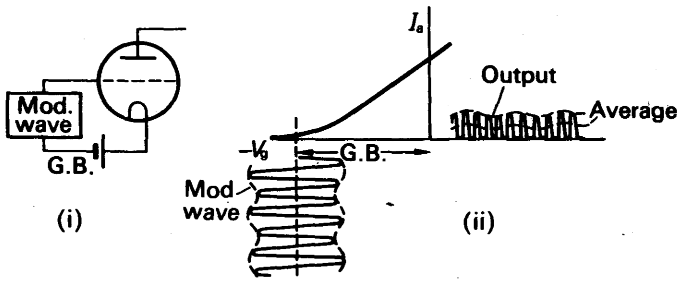


FIG. 41.17. Triode detection.

lated wave on one half of the cycles now produce an anode-current variation, but very little current flows on the other half-cycles. The output, or anode current, thus varies as shown in Fig. 41.17 (ii), and hence the *average* current variation follows the variation of the peaks of the current, which is the audio-frequency variation carried along by the modulated wave (see p. 1022). By means of high resistance earphones and a suitable capacitor across it, the audio-frequency variation can be heard.

JUNCTION DIODE. TRANSISTOR

Semiconductors

In receivers, the radio valve has been superseded by components made from *semiconductors*, which perform the same function as the valve. Semiconductors are a class of solids with electrical resistivity between that of a conductor and an insulator. For example, the resistivity of a conductor is of the order 10^{-8} Ω m, that of an insulator is 10^4 Ω m and higher, and that of a semiconductor is 10^{-1} Ω m. Silicon and germanium are examples of semiconductor elements widely used in industry.

Electrons and Holes

Silicon and germanium atoms are tetravalent. They have four electrons in their outermost shell, called *valence electrons*. One valence electron is shared with each of four surrounding atoms in a tetrahedral arrangement, forming 'covalent bonds' which maintain the crystalline solid structure. Fig. 41.18 (i) is a two-dimensional representation of the structure.

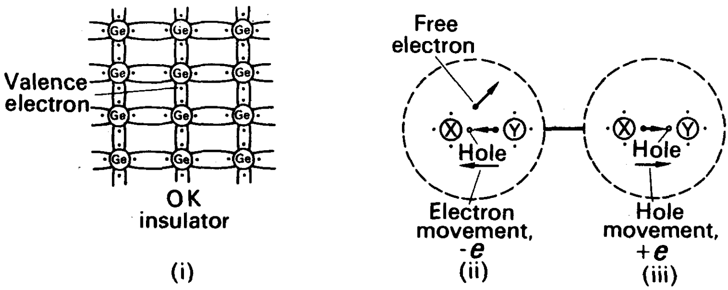


FIG. 41.18. Semiconductor. Electron (−) and hole (+) movement.

At 0 K, all the valence electrons are firmly bound to the nucleus of their particular atom. At room temperature, however, the thermal energy of a valence electron may become greater than the energy binding it to its nucleus. The covalent bond is then broken. The electron leaves the atom, X say, and becomes a free electron. This leaves X with a vacancy or *hole*. Fig. 41.18 (ii). Since X now has a net positive charge, an electron in a neighbouring atom may then be attracted. Thus the hole appears to move to Y.

The hole movement through a semiconductor is random. But if a battery is connected, the valence electrons are urged to move in one direction and to fill the holes. The holes then drift in the direction of the field. Thus the holes move as if they were carriers with a positive charge $+e$, where e is the numerical value of the charge on an electron. Fig. 41.18 (iii). The current in the semiconductor is also carried by the free electrons present. These are equal in number to the holes in a pure semiconductor and drift in the opposite direction since they are negative charges. The mobility of an electron, its average velocity per unit electric field intensity, is usually much greater than that of a hole.

In electrolytes (p. 844), the current is also carried by moving negative and positive charges but the carriers here are *ions*. It should be noted that, in a pure semiconductor, there are equal numbers of electrons and holes. *Electron-hole pairs* are said to be produced by the movement of an electron from bound state in an atom to a higher energy level, where it becomes a free electron.

Effect of Temperature Rise

In contrast to a semiconductor, the carriers of electricity in a metal such as copper are only free electrons. Further, as the temperature of the metal rises, the amplitude of vibration of the atoms increases and more 'collisions' with atoms are then made by drifting electrons. Thus, as stated on p. 837, the resistance of a pure metal increases with temperature rise.

In the case of a semiconductor, however, the increase in thermal energy of the valence electrons due to temperature rise enables more of them to break the covalent bonds and become free electrons. Thus more electron-hole pairs are produced which can act as carriers of current. Hence, in contrast to a pure metal, the electrical resistance of a semiconductor *decreases* with temperature rise. This is one way of distinguishing between a pure metal and a semiconductor.

P- and N-type Semiconductors

By 'doping' a semiconductor with a tiny amount of impurity such as one part in a million, a considerable increase can be made to the number of charge carriers.

Arsenic atoms, for example, have five electrons in their outermost or valence band. When an atom of arsenic is added to a germanium crystal, the atom settles in a lattice site with four of its electrons shared with neighbouring germanium atoms. Fig. 41.19 (i). The fifth electron

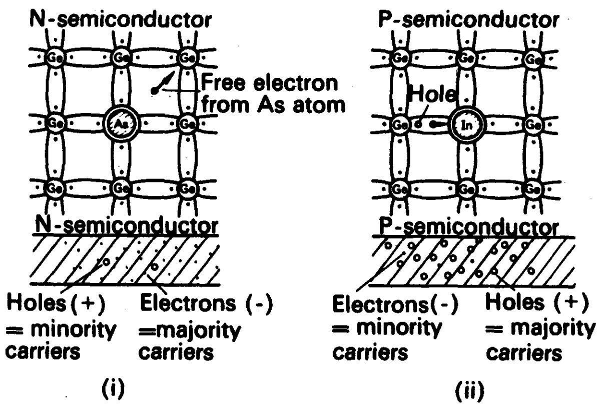


FIG. 41.19. N- and P- semiconductors.

may thus become free to wander through the crystal. Since an impurity atom may provide one free electron, an enormous increase occurs in the number of electron carriers. The impure semiconductor is called an 'n-type semiconductor' or *n-semiconductor*, where 'n' represents the negative charge on an electron. Thus the *majority carriers* in an n-semiconductor are electrons. Positive charges or holes are also present in the n-semiconductor. These are thermally generated, as previously explained, and since they are relatively few they are called the *minority carriers*. The impurity (arsenic) atoms are called *donors* because they donate electrons as carriers.

P-semiconductors are made by adding foreign atoms which are trivalent to pure germanium or silicon. Examples are boron or indium. In this case the reverse happens to that previously described. Each trivalent atom at a lattice site attracts an electron from a neighbouring atom, thereby completing the four valence bonds and forming a hole in the neighbouring atom. Fig. 41.19 (ii). In this way an enormous increase occurs in the number of holes. Thus in a p-semiconductor, the majority carriers are holes or positive charges. The minority carriers are electrons, negative charges, which are thermally generated. The impurity atoms are called *acceptors* in this case because each 'accepts' an electron when the atom is introduced into the crystal.

Summarizing: In a n-semiconductor, conduction is due mainly to negative charges or electrons, with positive charges (holes) as minority carriers. In a p-semiconductor, conduction is due mainly to positive charges or holes, with negative charges (electrons) as minority carriers.

P-N Junction

By a special manufacturing process, p- and n-semiconductors can be melted so that a boundary or *junction* is formed between them. This junction is extremely thin and of the order 10^{-4} cm. It is called a *p-n junction*. Fig. 41.20 (i). When a scent bottle is opened, the high

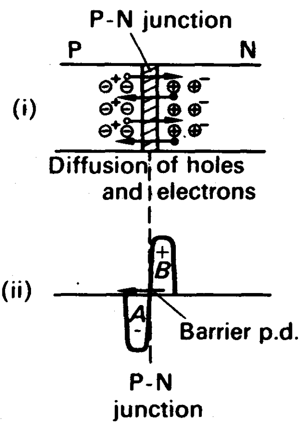


FIG. 41.20. P-n junction and barrier p.d.

concentration of scent molecules in the bottle causes the molecules to diffuse into the air. In the same way, the high concentration of holes (positive charges) on one side of a p-n junction, and the high concentration of electrons on the other side, causes the two carriers to diffuse respectively to the other side of the junction, as shown. The electrons which move to the p-semiconductor side recombine with holes there. These holes therefore disappear, and an excess negative charge A appears on this side. Fig. 41.20 (ii).

In a similar way, an excess positive charge B builds up in the n-semiconductor when holes diffuse across the junction. Together with the negative charge A on the p-side, an e.m.f. or p.d. is produced which opposes the diffusion of charges across the junction. This is called a *barrier p.d.* and when the flow ceases it has a magnitude of a few tenths of a volt.

Junction Diode

When a battery B, with an e.m.f. greater than the barrier p.d., is joined with its positive pole to the p-semiconductor, P, and its negative pole to the n-semiconductor, N, p-charges (holes) are urged across the p-n junction from P to N and n-charges (electrons) from N to P. Fig. 41.21 (i). Thus an appreciable current is obtained. The p-n junction is now said to be *forward-biased*, and when the applied p.d. is increased, the current increases.

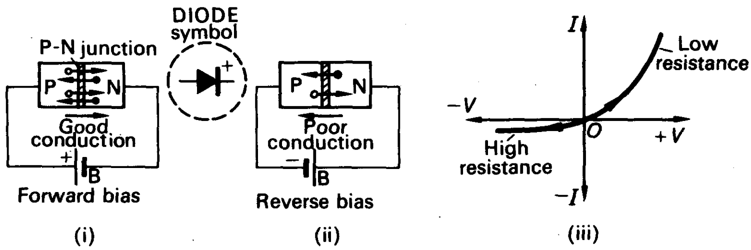


FIG. 41.21. Junction diode characteristic.

When the poles of the battery are reversed, only a very small current flows. Fig. 41.21 (ii). In this case the p-n junction is said to be *reverse-biased.* This time only the minority carriers, negative charges in the p-semiconductor and positive charges in the n-semiconductor, are urged across the p-n junction by the battery. Since the minority carriers are thermally-generated, the magnitude of the reverse current depends only on the temperature of the semiconductors.

It can now be seen that the p-n junction acts as a *rectifier.* It has a low resistance for one direction of p.d. and a high resistance for the opposite direction, as shown by the characteristic curve in Fig. 41.21 (iii). It is therefore called a *junction diode.* The junction diode has advantages over a diode valve; for example, it needs only a low voltage battery B to function; it does not need time to warm up; it is less bulky, and it is cheaper to manufacture in large numbers. On this account, the junction diode has replaced the diode valve in receivers.

Zener Diode

When the reverse bias or p.d. is increased across a p-n junction, a large increase in current is suddenly obtained at a voltage Z. Fig. 41.22 (i). This is called the *Zener effect*, after the discoverer. It is partly due to the high electric field which exists across the narrow p-n junction at the breakdown or Zener voltage Z, which drags more electrons from

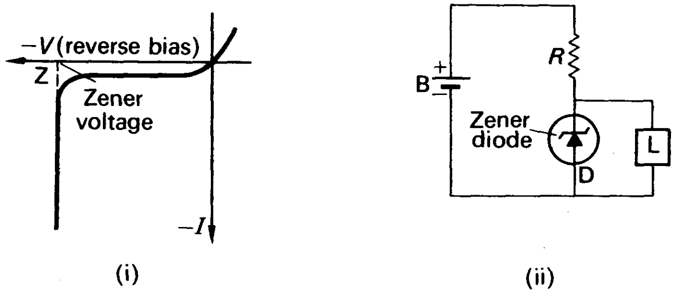


FIG. 41.22. Zener diode and voltage regulation.

their atoms and thus increases considerably the number of electron-hole pairs. Ionization by collision also contributes to the increase in carriers.

Zener diodes are used as voltage regulators or stabilizers in circuits. In Fig. 41.22 (ii), a suitable diode D is placed across a circuit L. Although the battery supply B may fluctuate, and produce changes of current in L and D, if R is suitably chosen, the voltage across D remains practically constant over a reverse current range of tens of milliamperes at the Zener voltage. See Fig. 41.22 (i). The voltage across L thus remains stable.

The Transistor

The junction diode is a component which can only rectify. The *transistor* is a more useful component; it is a *current amplifier*. A transistor is made from three layers of p- and n-semiconductors. They are called respectively the *emitter* (E), *base* (B) and collector (C). Fig. 41.23 (i) illustrates a *p-n-p transistor*, with electrodes connected to the

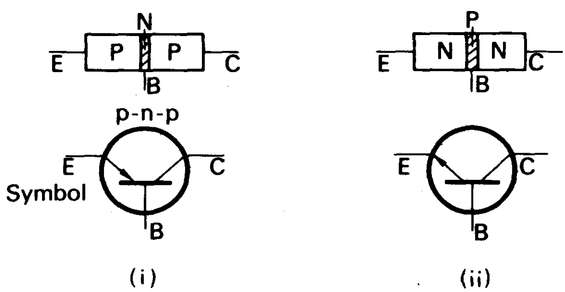


FIG. 41.23. Transistors and symbols.

respective three layers. In a *n-p-n transistor*, the emitter is n-type, the base is p-type and the collector is n-type. Fig. 41.23 (ii). The base is deliberately made very thin in manufacture. The transistor, like the triode valve, is thus a three-terminal device.

Fig. 41.23 shows the circuit symbols for p-n-p and n-p-n transistors. In an actual transistor, the collector terminal is displaced more than the others for recognition or has a dot near it.

Common-Base (C-B) Arrangement

The transistor may be regarded as two p-n junctions back-to-back. Fig. 41.24 (i) shows batteries correctly connected to a p-n-p transistor. The emitter-base is forward-biased; the collector-base is *reverse*-biased; and the base is common. This is called the *common-base* (C-B) mode of using a transistor. Note carefully the polarities of the two batteries. The positive pole of the supply voltage X is joined to the emitter E but the *negative* pole of the supply voltage Y is joined to the collector C.

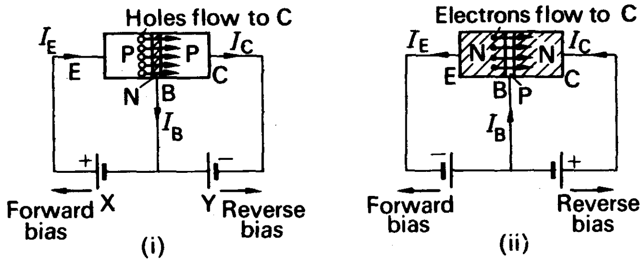


FIG. 41.24. Transistor action.

If batteries are connected the wrong way round to a transistor the latter may be seriously damaged. In the case of a n-p-n transistor, therefore, the negative pole of one battery is joined to the emitter and the positive pole of the other is joined to the collector. Fig. 41.24 (ii).

Consider Fig. 41.24 (i). Here the emitter-base is forward biased by X, so that positive charges or holes flow across the junction from E to the base B. The base is so thin, however, that the great majority of the holes are urged across the base to the collector by the battery Y. Thus a current I_C flows in the collector circuit. The remainder of the holes flow in the base circuit, so that a small current I_B is obtained here. From Kirchhoff's first law, it follows that, if I_E is the emitter current,

$$I_E = I_C + I_B.$$

Typical values for a.f. amplifier transistors are: $I_E = 1{\cdot}0$ mA, $I_C = 0{\cdot}98$ mA, $I_B = 0{\cdot}02$ mA.

Although the action of n-p-n transistors is similar in principle to p-n-p transistors, the carriers of the current in the former case are mainly electrons and in the latter case holes. Electrons are more speedy carriers than holes (p. 1024). Thus n-p-n transistors are used in high-frequency circuits, where the carriers are required to respond very quickly to signals.

Common-Base Characteristics

The behaviour of a particular transistor in the common-base arrangement can be obtained from its characteristic curves. Fig. 41.25 shows a circuit for determining the curves. V_{CC} represents the supply voltage, for example 9 V; A_1, A_2 are current measuring instruments; V are voltmeters; and the two potentiometers of 1 megohm and 50

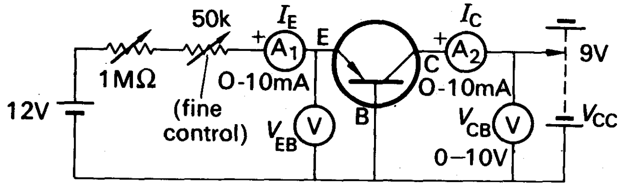


FIG. 41.25. Common-base characteristics investigation.

kilohm are used to vary the input or emitter current, I_E. The more important curves are:

(1) *Output characteristics* (I_C v. V_{CB}, with I_E constant), (2) *input characteristics* (I_E v. V_{EB}, with V_{CB} constant), (3) *transfer characteristics* (I_C v. I_E, with V_{CB} constant).

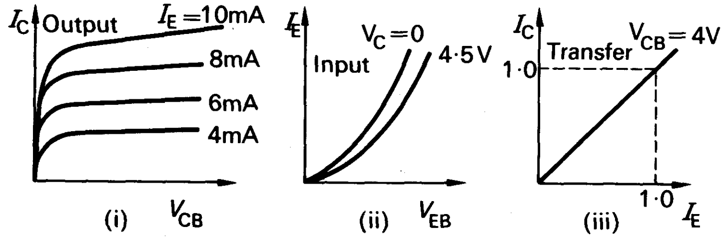


FIG. 41.26. C-B characteristics.

Typical results are shown in Fig. 41.26 (i), (ii), (iii). The flat output characteristics in Fig. 41.26 (i) show that the *output resistance*, $\Delta V_{CB}/\Delta I_C$, is high. The *input resistance*, $\Delta V_{EB}/\Delta I_E$, varies with the slope of the curve in Fig. 41.26 (ii) and is generally low. From the straight line graph of the transfer characteristic in Fig. 41.26 (iii), it follows that a linear relation exists between I_C and I_E.

Common-Emitter (C-E) Arrangement

The slope of the transfer characteristic in Fig. 41.26 (iii) provides the current gain of the transistor, $\Delta I_C/\Delta I_E$. It is practically 1, showing that the common-base arrangement is unsuitable for current amplification. Now in a typical transistor, as already seen, $I_C = 0{\cdot}98$ mA and $I_B = 0{\cdot}02$ mA. Thus I_C is 49 times as large as I_B and a similar order of magnitude for current gain occurs with changes in I_B. On this account the common-emitter (or grounded-emitter) arrangement is widely used in a.f. amplifiers.

Fig. 41.27 shows a circuit for determining the output characteristics,

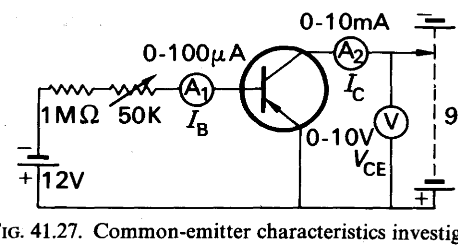


FIG. 41.27. Common-emitter characteristics investigation.

input characteristics and transfer characteristic in the common-emitter (C-E) arrangement. The results are shown in Fig. 41.28 (i), (ii), (iii).

Output characteristic. Since the knee of the curve occurs at a low voltage of the order of 1 V, only low battery supply voltages are needed to operate a transistor in the linear region beyond the knee. This is an advantage of the transistor compared with the valve. Further, the small slope of the straight line shows that the output resistance is high. Thus although the load in the collector circuit may vary, the collector current is constant for a given alternating input or base current. Hence the transistor can be considered as a constant *current generator* in circuitry, whereas the triode valve is treated as a constant *voltage generator* with a given input (p. 1017).

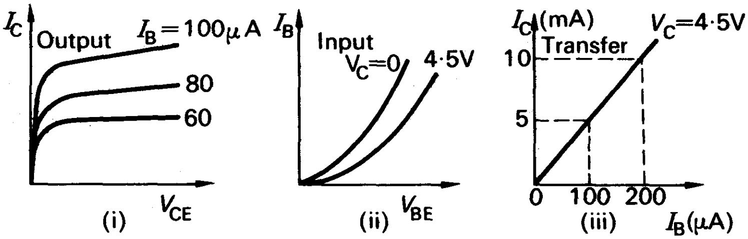


FIG. 41.28. C-E characteristics.

Transfer characteristic. The output current I_C varies fairly linearly with the input current I_B. The current gain, denoted by β, is the ratio $\Delta I_C/\Delta I_B$, V_C constant. From Fig. 41.28 (iii), $\beta = (10-5)$ mA/$(200-100)$ μA $= 50$. The reader should note that 'current amplification' usually refers to variations in current in amplifier circuit analysis. The ratio I_C/I_B provides the d.c. current amplification.

Input characteristic. The input resistance, r_i, is the ratio $\Delta V_{BE}/\Delta I_B$ with V_C constant. It varies at different points of the curve and has a medium value such as 1000Ω or 1 kΩ.

Relation between Current Gain in C-E and C-B Arrangements

In the C-B arrangement, the current gain is denoted by α and is the ratio $\Delta I_C/\Delta I_E$. In the C-E arrangement, the current gain is denoted by β and is the ratio $\Delta I_C/\Delta I_B$. Now from p. 1029, it is always true that $I_E = I_C+I_B$. Hence $\Delta I_E = \Delta I_C+\Delta I_B$. Using $\Delta I_C/\Delta I_B = \beta$, then $\Delta I_B = \Delta I_C/\beta$. Thus, by substitution for ΔI_B in $\Delta I_E = \Delta I_C+\Delta I_B$,

$$\Delta I_E = \Delta I_C + \frac{\Delta I_C}{\beta}$$

$$\therefore \frac{\Delta I_E}{\Delta I_C} = \frac{1}{\alpha} = 1+\frac{1}{\beta}$$

Simplifying,

$$\therefore \beta = \frac{\alpha}{1-\alpha} \qquad . \quad . \quad . \quad . \qquad (1)$$

If $\alpha = 0{\cdot}98$, then $\beta = 0{\cdot}98/0{\cdot}02 = 49$ from (1).

Leakage Current

When the base current I_B is zero, some current still flows in the collector circuit in the common-emitter arrangement. This is due to the minority carriers present in the collector-base part of the transistor, which is reverse-biased. The collector current when I_B is zero is denoted by I_{CEO} and is called the *leakage current.*

In the common-base arrangement, the leakage current obtained when I_E is zero is denoted by I_{CBO}. This is also due to minority carriers in the collector-base, which is reverse-biased. Thus the leakage current flows when a transistor is in the C-E or C-B arrangement.

Since the current gain β in the C-E arrangement is the ratio $\Delta I_C/\Delta I_B$, with the usual notation, it follows that, generally,

$$I_C = \beta I_B + I_{CEO} \qquad . \quad . \quad . \quad (1)$$

Similarly, in the C-B arrangement,

$$I_C = \alpha I_E + I_{CBO} \qquad . \quad . \quad . \quad (2)$$

Now any change in I_{CEO} or minority carriers is magnified β times in the C-E arrangement, since I_{CEO} also flows in the base-emitter circuit when the transistor is operating. A temperature change from 25°C to 45°C, which would increase the current I_{CEO} by 10 μA say, would thus be amplified to about 49×10 μA or 490 μA, if β is 49. This increase in current, nearly 0·5 mA, would have a considerable effect on the output in the collector circuit, and it would lead to a distorted output, for example.

On the other hand, $\alpha = 0{\cdot}98$ for the same transistor. Thus in the C-B arrangement, a similar temperature rise and current increase of 10 μA would produce a change in $0{\cdot}98 \times 10$ μA, or nearly 10 μA, in the output or collector circuit. This is only a very small change compared to the C-E case. On this account the C-E arrangement, which is very sensitive to temperature change, *must* be stabilized for excessive temperature rise. Silicon transistors are much less sensitive to temperature change than germanium transistors and are hence becoming used more widely.

Simple C-E Amplifier Circuit

Fig. 41.29 shows a p-n-p transistor in a simple or basic C-E arrangement. It uses one battery supply, V_{CC}. A load, R_L, is placed in the collector or output circuit. A resistor R provides the necessary bias, V_{BE},

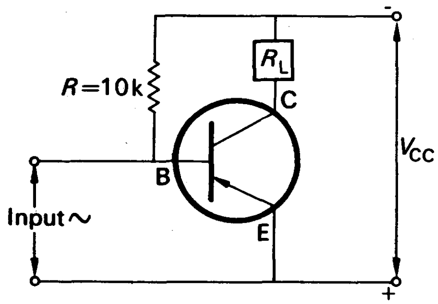


FIG. 41.29. Simple amplifier.

for the base-emitter circuit. The base-emitter is then forward-biased but the collector-base is reverse-biased, that is, the potential of B is negative relative to E but positive relative to C.

Suppose a small signal is applied, so that the base current changes by an amount ΔI_B. Then I_C changes by $\beta \Delta I_B$.

$$\therefore \text{ voltage gain } = \frac{\text{output voltage}}{\text{input voltage}} = \frac{\beta \Delta I_B \,.\, R_L}{V_i}$$

$$= \frac{\beta \Delta I_B \,.\, R_L}{\Delta I_B \,.\, r_i},$$

where r_i is the *input resistance* or resistance to a.c. between base-emitter.

$$\therefore \text{ voltage gain } = \frac{\beta \,.\, R_L}{r_i}.$$

If $\beta = 49$, $R_L = 4700\ \Omega$, $r_i = 1000\ \Omega$, the voltage gain $= 49 \times 4{\cdot}7$ $= 230$ (approx.).

C-E Amplifier Circuit

In practice, Fig. 41.29 is unsuitable as an amplifier circuit since there is no arrangement for temperature stabilization. A more reliable C-E a.f. amplifier circuit is shown in Fig. 41.30. Its principal features are:

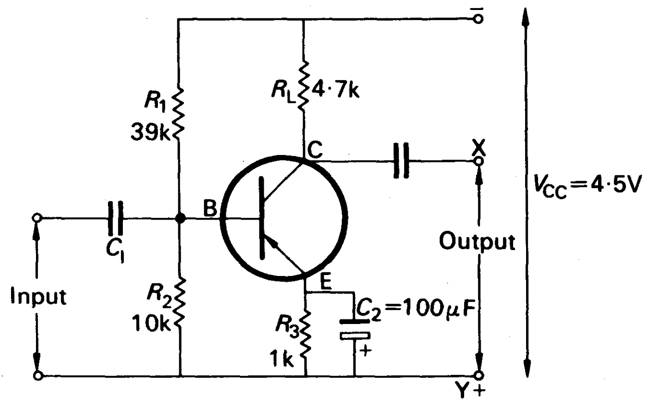


FIG. 41.30. Amplifier circuit.

(i) a potential divider arrangement, R_1, R_2, which provides the necessary base-bias;
(ii) a load R_L which produces the output across X, Y;
(iii) a capacitor C_1 which isolates the d.c. component in the input signal from the circuit;
(iv) a large capacitor C_2 across a resistor R_3, which prevents undesirable feedback of the amplified signal to the base-emitter circuit;

(v) an emitter resistance R_3, which stabilizes the circuit for excessive temperature rise. Thus if the collector current rises, the current through R_3 increases. This lowers the p.d. between E and B, so that the collector current is automatically lowered.

Transistor Oscillator Circuit

Like the triode valve, a transistor can be arranged to provide 'positive feedback' to an oscillatory (L-C) circuit. Oscillations in the L-C circuit can thus be maintained, as explained on p. 1020.

Fig. 41.31 shows one form of transistor oscillator circuit. Its main features are:

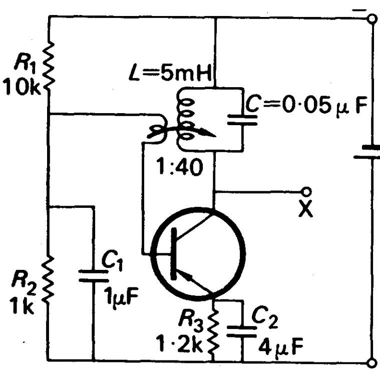


FIG. 41.31. Transistor oscillator.

(i) a coil-capacitor, L-C, load in the collector circuit;
(ii) positive feedback through the coil L_1 to maintain the oscillations in the L-C circuit;
(iii) a potential divider arrangement, R_1, R_2, to provide the necessary base bias;
(iv) an emitter resistor R_3 to stabilize the circuit for excessive temperature rise;
(v) large capacitors C_1 and C_2 across R_2 and R_3 respectively, which prevent undesirable feedback to the base circuit.

Approximately, the frequency of oscillation is given by $f = 1/2\pi\sqrt{LC}$, in this case an audio-frequency. Other frequencies may be obtained by changing the magnitude of C.

Thermistor

There are numerous semiconductor devices other than the transistor. A *thermistor* is a heat-sensitive resistor usually made from semiconductor materials which have a high negative temperature coefficient of resistance. Its resistance thus decreases appreciably with temperature rise.

One use of a thermistor is to safeguard against current surges in circuits where this could be harmful, for example, in a circuit where the heaters of radio valves are in series. A thermistor, T, is included in the circuit, as shown (Fig.

41.32). When the supply voltage is switched on, the thermistor has a high resistance at first because it is cold. It thus limits the current to a moderate value. As it warms up, the thermistor resistance drops appreciably and an increased current then flows through the heaters. Thermistors are also used in transistor receiver circuits to compensate for excessive rise in collector current.

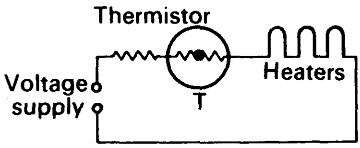


FIG. 41.32. Use of Thermistor.

Phototransistor

A *photodiode* is a junction diode sensitive to light. When the diode is reverse-biased, minority carriers flow in the circuit and constitute a so-called 'dark' current. If the junction of the diode is now illuminated, the light energy produces more electron-hole pairs, which are then swept across the junction. The increased current which flows is the 'light' current.

A *phototransistor* is a transistor sensitive to light in which the base is usually left disconnected. When light falls on the emitter side, more electron-hole pairs are produced in the base. This is amplified by transistor action, and a larger collector current is obtained. In principle the phototransistor is a photodiode plus amplifier.

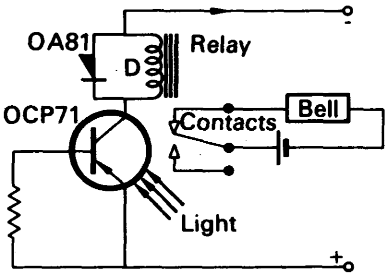


FIG. 41.33. Phototransistor operating relay.

Fig. 41.33 shows a circuit in which a Mullard phototransistor OCP71 is connected in series with a relay coil D and a d.c. supply voltage. When the phototransistor is illuminated, the increase in collector current closes the contacts of a magnetic relay. Current then flows in a circuit connected to the relay, and a bell, for example, may then ring. Fig. 41.33. When the light is switched off, the falling current in the relay coil produces an induced voltage in the same direction as the battery supply. This would raise the collector voltage and prevent the switch-off at the contacts. The diode OA81 across the coil acts as a safeguard. As soon as the rising induced voltage becomes equal to the battery voltage the diode conducts, and prevents any further rise in collector voltage.

EXERCISES 41

1. Sketch graphs using the same axes showing how the current through a thermionic diode varies with the d.c. potential difference applied between the anode and filament for two filament temperatures. Explain three special features of the graphs.

What is meant by (*a*) half-wave rectification, (*b*) full-wave rectification? Explain with the aid of labelled circuit diagrams how each of these may be achieved using thermionic diodes. (*N*.)

2. Figure 41.34 represents a simple rectifier circuit. A sinusoidal 50 cycle alternating voltage of peak value 100 V is applied between *D* and *E*, and a load circuit can be connected between the output terminals *X* and *Y*.

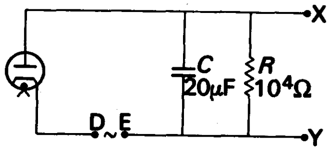


FIG. 41.34.

Draw a diagram showing the variation with time of the p.d. between *X* and *Y* on open circuit. Explain how this is modified when a load is connected across *XY*.

What changes, if any, in the behaviour of the circuit would result from an increase in (*a*) the temperature of the cathode of the diode, (*b*) the value of *C*, (*c*) the value of *R*?

Draw a diagram of a full-wave rectifier-smoother circuit, labelling each component and explaining briefly its purpose. (*O*.)

3. Give an account of thermionic emission. What analogy exists between thermionic emission and the evaporation of molecules from the surface of a liquid?

Show how the introduction of the grid in a triode enables it to be used as a voltage amplifier. Draw a circuit diagram to show (i) where the voltage to be amplified is applied to the triode, and (ii) where the amplified voltage is tapped off. (*O. & C.*)

4. Describe the structure of a diode and describe an experiment to justify the term 'valve'.

Explain how a triode (*a*) differs in structure and operation from a diode, (*b*) may be used to amplify small alternating potential differences. (*L*.)

5. Describe the structure of a triode valve and explain the functions of its component parts.

Draw clearly labelled diagrams to show the arrangements necessary (*a*) to determine the grid characteristics of a triode for various fixed anode potentials, (*b*) to maintain electrical oscillations by means of a triode. (*L*.)

6. For a triode, sketch curves to show (*a*) the form of the anode current/grid voltage static characteristics, (*b*) the form of the anode current/anode voltage static characteristics. How may the amplification factor of the valve be deduced from these curves?

Explain, with the aid of a circuit diagram and with reference to the static characteristics, how the triode may be used to amplify a small alternating voltage. (*N*.)

7. Describe the essential features of a triode valve and comment on any *one* feature of its construction which you consider of special importance.

Describe *briefly*, with the aid of a diagram, how you would investigate the variation of anode current with anode potential, the grid potential being constant and negative to the filament. Sketch the curve you would expect to obtain.

A triode valve is to be used to amplify a direct current of 10^{-7} amps flowing in a circuit incorporating a resistance of 10^5 ohms. The valve has a mutual conductance of 2 milliamp volt^{-1} and an anode slope resistance (impedance) of 10^6 ohms. Draw a diagram of a suitable circuit and calculate the current amplification. (*N.*)

8. What is meant by *thermionic emission*? Describe how this phenomenon is used in the action of a radio valve and give some other use to which it is put.

Show how a triode can be used (*a*) to detect, (*b*) to amplify, radio signals. (*L.*)

9. Give a brief description of the construction of a high-vacuum diode. Draw a graph which shows the variation of the current through such a diode with the potential difference across it, and account for the main features of the curve.

Describe how the introduction of a third electrode, the grid, makes possible (*a*) control of the current which reaches the anode, and (*b*) the amplification of a voltage. (*O. & C.*)

10. Describe with the aid of a circuit diagram how a triode valve can be used as an oscillator. What factors determine (*a*) the frequency, and (*b*) the amplitude of the oscillation?

A certain triode valve has an amplification factor of 100 and a mutual conductance of 2·5 mA V^{-1}. For reasons of stability, the makers recommend that a resistance of not more than 1·0 megohm shall be connected between grid and cathode. If the valve is used as an amplifier, calculate the maximum possible value of

$$\frac{\text{Power delivered into load resistance}}{\text{Power delivered into the input terminals of the amplifier}}. \qquad (C.)$$

11. Draw a clear labelled diagram showing the structure of a cathode ray tube.

The potential difference between cathode and anode of a cathode ray tube is 500 volts. The tube is set up with its axis along the direction of the earth's resultant magnetic field and the spot is focussed on the screen which is 15·0 cm from the anode. On rotating the tube about a horizontal axis to a position at right angles to the earth's resultant field the spot is deflected through 0·75 cm. Find a value for e/m for an electron. (Assume the magnetic induction in the earth's magnetic field to be $0{\cdot}50 \times 10^{-4}$ Wb m^{-2}.) (*L.*)

12. Describe an experiment to determine the deflection sensitivity of a cathode-ray tube in volts per cm.

Give an account of any experiment you have performed, or seen performed, in which a cathode ray oscilloscope is used to obtain information. Explain the purpose of the experiment, and the nature of the information obtained from the oscillograph. (*C.*)

13. Draw a sketch to show the essential parts of a cathode ray oscillograph having electrostatic deflection.

With the help of your sketch explain how in a cathode ray oscillograph: (*a*) the electrons are produced; (*b*) the electrons are focused; (*c*) the spot is made visible; (*d*) the brightness of the spot is controlled.

What is meant by stating that a cathode ray oscillograph is fitted with a linear time base of variable frequency. (*N.*)

14. Explain what is meant by (*a*) a linear time-base, (*b*) a sinusoidal time-base, in a cathode ray oscilloscope.

The X and Y deflection sensitivities of a cathode ray oscilloscope are each 5 volt cm^{-1}. A sinusoidal potential difference alternating at 50 Hz and of r.m.s. value 20 volts is applied to the Y plates of the instrument. A potential difference of the same form and frequency but of r.m.s. value 10 volts is simultaneously applied to the X plates. Sketch and explain the pattern seen on the oscilloscope when the potential differences are (*a*) in phase, (*b*) 90° out of phase. Indicate the appropriate dimensions on your sketches. (*N*.)

15. Give an account of the cathode ray tube, and explain how the 'brilliance', 'focus' and 'shift' controls operate.

How could you use the cathode ray tube to measure d.c. voltages?

It is proposed to use a cathode ray tube to study the amplification of a thermionic triode by applying a sinusoidal alternating voltage V_C between the grid and the cathode and comparing this with the alternating voltage V developed across a resistor in the anode circuit. Draw a suitable circuit diagram, showing how the voltages V_C and V may be applied in turn across the Y plates of the cathode ray tube.

How would you expect the ratio V/V_C to depend on the value of the anode resistor? (*O*.)

16. Explain, with special reference to a cathode ray tube how a stream of electrons may be produced and its direction and intensity controlled.

The focussing of a C.R.O. may be done by using two cylindrical anodes at different potentials. Sketch the lines of force between two such anodes, and use your sketch to explain how a diverging beam of electrons may be made converging.

100 volts a.c. applied to the Y-plates of a C.R.O. give a sinusoidal trace which measures 6 cm vertically (peak to peak). When 10 volts sinusoidal a.c. are applied to the input of a simple triode amplifier and the output from the amplifier is connected to the Y-plates of the C.R.O., the trace produced is as shown in Fig. 41B. Describe the output from the amplifier. How may it be explained in terms of triode characteristics? (*N*.)

Junction Diode. Transistor

17. Explain, with reference to the carriers, the effect of temperature rise on the resistance of a pure metal and on the resistance of a pure semiconductor.

18. Explain what is meant by (i) a *p*- and a *n-semiconductor*. (ii) a *p-n junction*.

19. Draw a sketch of the characteristic of a p-n junction diode. Explain, in terms of the movement of carriers, why the resistance of the diode is low in one direction and high in the reverse direction.

20. Draw a sketch of a p-n-p transistor used in (i) a common-base (CB) and (ii) a common-emitter (CE) arrangement, showing clearly the polarities of the batteries. Explain why the common-emitter arrangement is preferred in an a.f. amplifier circuit.

21. Draw a circuit showing how the collector current-collector voltage and the emitter current-emitter voltage characteristics of a transistor can be found for the common-emitter (CE) arrangement. Sketch the characteristics obtained.

Draw a sketch of a simple CE a.f. amplifier.

22. A transistor in the common-emitter arrangement provides the following results for I_C, collector current, and V_C, collector voltage, for various constant base currents I_B:

I_C(mA)

V_C (V)	$I_B = 20\,\mu A$	$= 40\,\mu A$	$= 60\,\mu A$	$= 80\,\mu A$
3	0·91	1·60	2·30	3·00
5	0·93	1·70	2·50	3·25
7	0·97	1·85	2·70	3·55
9	1·00	2·05	3·00	4·05

Plot the characteristics, and from them find (i) the current gain at 8 V, (ii) the output resistance for a base current of 40 μA.

23. Explain what is meant by p-type and n-type semiconductors. Describe a p-n junction diode. Draw a graph which shows the variation of the current through such a diode with the potential difference across it, and explain why the diode behaves differently when the potential difference across it is reversed.

Describe the junction transistor. Sketch curves to show the variation of the collector current with the collector-base voltage for various values of the emitter current and explain their form. (*O. & C.*)

chapter forty-two

Radioactivity. The Nucleus

IN 1896 Becquerel found that a uranium compound affected a photographic plate wrapped in light-proof paper. He called the phenomenon *radioactivity*, and we shall see later that natural radioactivity is due to one or more of three types of radiation emitted from heavy elements such as uranium. These were originally called α-, β- and γ-rays but α- and β-'rays' were soon shown to be actually particles.

α- and β-particles and γ-rays all produce ionization as they move through a gas. On average, α-particles produce about 1000 times as many ions per unit length of their path as β-particles, which in turn produce about 1000 times as many ions as γ-rays. There are numerous detectors of ionizing radiations such as α- and β-particles and γ-rays. We begin by describing two detectors used in laboratories.

Geiger-Müller Tube

A Geiger-Muller (GM) tube is widely used for detecting ionizing particles. In one form it consists of an insulated wire A mounted in a thin-walled glass tube B coated with aquadag (a colloidal suspension of graphite) and earthed (Fig. 42.1). A p.d. V of the order of 400 volts is maintained across A, B. When a single ionizing particle enters the

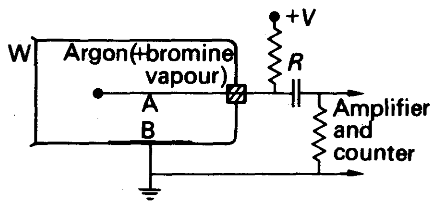


FIG. 42.1. Principle of Geiger-Muller tube.

chamber, a few electrons and ions are produced in the gas. If V is above the breakdown potential of the gas, the number of electrons and ions are multiplied enormously (see p. 998). The electrons are attracted by and move towards A, and the positive ions move towards B. Thus a 'discharge' is suddenly obtained between A and B. The current flowing in the high resistance R produces a p.d. which is amplified and passed to a counter, discussed on p. 1041. This registers the passage of an ionizing particle or radiation through the tube.

The discharge persists for a short time, as secondary electrons are emitted from the cathode by the positive ions which arrive there. This would upset the recording of other ionizing particles following fast on the first one recorded. The air in the tube is therefore replaced by argon mixed with bromine vapour, which has the property of quenching the discharge quickly. Electrical methods are also used for quenching.

Solid State Detector

A *solid state detector*, Fig. 42.2, is made from semiconductors. Basically, it has a p-n junction which is given a small bias in the non-conduction direction (p. 1026). When an energetic ionizing particle such

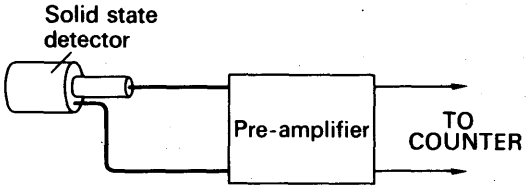


FIG. 42.2. Solid State Detector.

as an α-particle falls on the detector, more electron-hole pairs are created near the junction. These charge carriers move under the influence of the biasing potential and so a pulse of current is produced. The pulse is fed to an amplifier and the output passed to a counter.

The solid state detector is particularly useful for α-particle detection. If the amplifier is specially designed, β-particles and γ-rays of high energy may also be detected. This type of detector can thus be used for all three types of radiation.

Dekatron Counter. Ratemeter

As we have seen, each ionizing particle or radiation produces a pulse voltage in the external circuit of a Geiger-Muller or solid state detector. In order to measure the number of pulses from the detectors, some form of counter must be used.

A *dekatron counter* consists of two or more dekatron tubes, each containing a glow or discharge which can move round a circular scale graduated in numbers 0–9, together with a mechanical counter (Fig. 42.3). Each impulse causes the discharge in the first tube, which

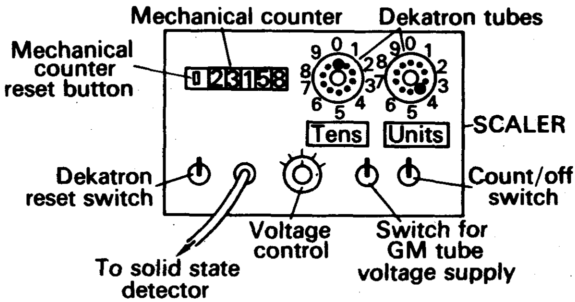


FIG. 42.3. Dekatron counter.

counts units, to advance one digit. The circuit is designed so that on the tenth pulse, which returns the first counter to zero, a pulse is sent to the second tube. The glow here then moves on one place. The second tube thus counts the number of tens of pulses. After ten pulses are sent to the second tube, corresponding to a count of 100, the output pulse from the second tube is fed to the mechanical counter. This, therefore, registers the hundreds, thousands and so on. Dekatron tubes are used in radioactive experiments because they can respond to a rate of about 1000 counts per second. This is greatly in excess of the count rate possible with a mechanical counter.

In contrast to a scaler, which counts the actual number of pulses, a *ratemeter* is a device which provides directly the average number of pulses per second or *count rate*. The principle is shown in Fig. 42.4. The

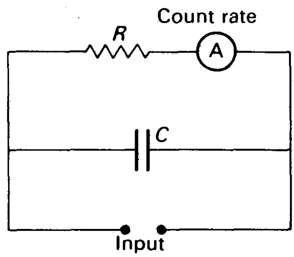


FIG. 42.4. Principle of a Ratemeter.

pulses received are passed to a capacitor C, which then stores the charge. C discharges slowly through a high resistor R and the average discharge current is recorded on a microammeter A. The greater the rate at which the pulses arrive, the greater will be the meter reading. The meter thus records a current which is proportional to the count rate.

If a large capacitor C is used, it will take a long time to charge and the pulses will be averaged over a long time. A switch marked 'time constant' on most ratemeters allows the magnitude of C to be chosen. If a large value of C is used, the capacitor will take a relatively long time to charge and correspondingly it will be a long time before a reading can be taken. The reading obtained, however, will be more accurate since the count rate is then averaged over a longer time (see below). For high accuracy, a small value of C may be used only if the count rate is very high.

Errors in Counting Experiments

Radioactive decay is random in nature (p. 1048). If the count rate is high, it is not necessary to wait so long before readings are obtained which vary relatively slightly from each other. If the count rate is low, successive counts will have larger percentage differences from each other, unless a much longer counting time is employed.

The accuracy of a count does not depend on the time involved but on *the total count obtained.* If N counts are received, the statistics of random processes show that this is subject to a statistical error of

$\pm\sqrt{N}$. The proof of this is beyond the scope of this book. The percentage error is thus

$$\frac{\sqrt{N}}{N} \times 100 = \frac{100}{\sqrt{N}}\%.$$

If 10 per cent accuracy is required, $\sqrt{N} = 10$ and hence $N = 100$. Thus 100 counts must be obtained. If the counts are arriving at about 10 every second, it will be necessary to wait for 10 seconds to obtain a count of 100 and so achieve 10 per cent accuracy. Thus a ratemeter circuit must be arranged with a time constant (CR) of 10 seconds, so that an average is obtained over this time. If, however, the counts are arriving at a rate of 1000 per second on average, it will be necessary to wait only 1/10th second to achieve 10 per cent accuracy. Thus the 1 second time constant scale on the ratemeter will be more than adequate.

Existence of α-, β-particles and γ-rays

The existence of different ionizing particles or radiations from radioactive substances can be shown by an absorption experiment, using a counter or ratemeter.

A radium source S, producing α- and β-particles and γ-rays, is placed at a fixed small distance from a solid state detector A and sensitive low-noise pre-amplifier, which is connected to a counter C (or ratemeter) (Fig. 42.5 (i)). Foils of increasing thickness are placed over the

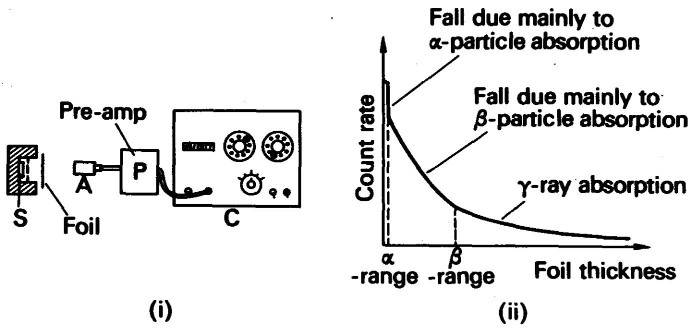


FIG. 42.5. Existence of α-, β-particles and γ-rays.

source, starting with *very* thin foils of paper and then aluminium foils. For each thickness the count rate is measured. A graph of count rate against thickness of foil is then plotted.

The resulting curve, Fig. 42.5 (ii), shows three distinct portions. To begin with, α, β and γ all pass through the very thin foils such as paper. After a particular thickness the α-particles are absorbed, and beyond this point the curve does not then fall off with distance as quickly. A similar change takes place when the β-particles are all absorbed at a particular thickness of aluminium plate, leaving another radiation, the γ-rays. This straightforward experiment shows the existence of three different types of radiation, α- and β-particles and γ-rays.

Alpha-particles

It is found that α-particles have a fairly definite range in air at atmospheric pressure. This can be shown by slowly increasing the distance between a pure α-source and a detector. The count rate is observed to fall rapidly to zero at a separation greater than a particular value, which is called the 'range' of the α-particles. The range depends on the source and on the air pressure.

Using the apparatus of Fig. 42.6, it can be shown the α-particles are *positively* charged. When there is no magnetic field, the solid state detector is placed so that

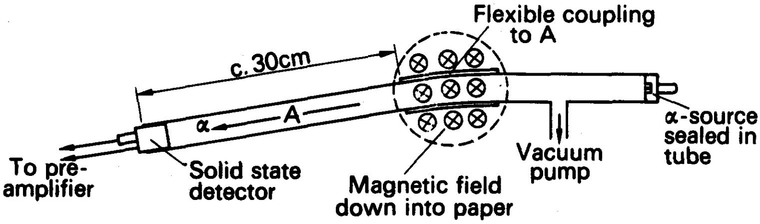


FIG. 42.6. Charge on an α-particle.

the tube A is horizontal in order to get the greatest count. When the magnetic field is applied, the detector has to be moved *downwards* in order to get the greatest count. This shows that the α-particles are deflected by a small amount downwards. By applying Fleming's left-hand rule, we find that particles are *positively* charged. The vacuum pump is needed in the experiment, as the range of α-particles in air at normal pressures is too small.

Nature of α-particle

Lord Rutherford and his collaborators found by deflection experiments than an α-particle had a mass about four times that of a hydrogen atom, and carried a charge $+2e$, where e was the numerical value of the charge on an electron. The atomic weight of helium is about four. It was thus fairly certain that an *α-particle was a helium nucleus,* that is, a helium atom which has lost two electrons.

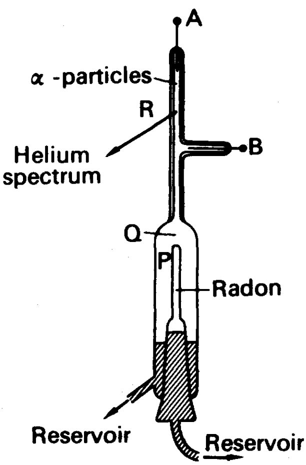


FIG. 42.7. Rutherford and Royds' experiment on α-particles.

In 1909 Rutherford and Royds showed conclusively that α-particles were helium nuclei. Radon, a gas given off by radium which emits α-particles, was collected above mercury in a thin-walled tube P (Fig. 42.7). After several days some of the α-particles passed through P into a surrounding vacuum Q, and in about a week, the space in Q was reduced in volume by raising mercury reservoirs. A gas was collected in a capillary tube R at the top of Q. A high voltage from an induction coil was then connected to electrodes at A and B, and the spectrum of the discharge was observed to be exactly the same as the characteristic spectrum of helium.

Beta-particles and Gamma-rays

By deflecting β-particles with perpendicular magnetic and electric fields, their charge-mass ratio could be estimated. This is similar to Thomson's experiment, p. 1003. These experiments showed that *β-particles are electrons moving at high speeds.* Generally, β-particles have a greater penetrating power of materials than α-particles. They also have a greater range in air than α-particles, since their ionization of air is relatively smaller, but their path is not so well defined.

Using a Ticonal bar magnet, it can be shown that β-particles are strongly deflected by a magnetic field. The direction of the deflection corresponds to a stream of *negatively*-charged particles, that is, opposite to the deflection of α-particles in the same field. This is consistent with the idea that β-particles are usually fast-moving electrons.

The nature of γ-rays was shown by experiments with crystals. Diffraction phenomena are obtained in this case, which suggest that *γ-rays are electromagnetic waves* (compare X-rays, p. 1067). Measurement of their wavelengths, by special techniques with crystals, show they are shorter than the wavelengths of X-rays and of the order 10^{-9} cm. γ-rays can penetrate large thicknesses of metals, but they have far less ionizing power in gases than β-particles.

If a beam of γ-rays are allowed to pass through a very strong magnetic field no deflection is observed. This is consistent with the fact that γ-rays are electromagnetic waves and carry no charge.

Inverse-square Law for γ-rays

If γ-rays are a form of electromagnetic radiation and undergo negligible absorption in air, then the intensity I should vary inversely as the square of the distance between the source and the detector. The apparatus shown in Fig. 42.8 can be used to investigate if this is the case. A pure γ-source is placed at a suitable distance from a GM tube connected to a scaler, and I will then be proportional to the count rate C.

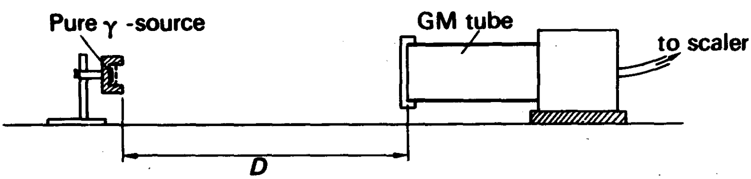


FIG. 42.8. Inverse square law for γ-rays.

Suppose D is the measured distance from a fixed point on the γ-source support to the front of the GM tube. To obtain the true distance from the source to the region of gas inside the tube where ionization occurs, we need to add an unknown but constant distance h to D. Then, assuming an inverse-square law, $I \propto 1/(D+h)^2$. Thus

$$D+h \propto \frac{1}{\sqrt{I}} \propto \frac{1}{\sqrt{C}}.$$

A graph of $1/\sqrt{C}$ is therefore plotted against D for varying values of D. If the inverse-square law is true, a straight line graph is obtained which has an intercept on the D-axis of $-h$. Note that if I is plotted against $1/D^2$ and h is not zero, a straight line graph is *not* obtained from the relation $I \propto 1/(D+h)^2$. Consequently we need to plot D against $1/\sqrt{I}$.

If a pure β-source is substituted for the γ-source and the experiment is repeated, a straight-line graph is not obtained. The absorption of β-particles in air is thus appreciable compared with γ-rays.

Half-life Period

Radioactivity, or the emission of α- or β-particles and γ-rays, is due to disintegrating nuclei of atoms (p. 1052). The disintegrations obey the statistical law of chance. Thus although we can not tell which particular atom is likely to disintegrate next, the number of atoms disintegrating per second, dN/dt, is directly proportional to the number of atoms, N, present at that instant. Hence:

$$\frac{dN}{dt} = -\lambda N;$$

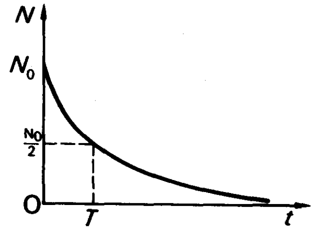


FIG. 42.9. Radioactive decay with time.

where λ is a constant characteristic of the atom concerned called the *radioactivity decay constant*. Thus, if N_0 is the number of radioactive atoms present at a time $t = 0$, and N is the number at the end of a time t, we have, by integration,

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt.$$

$$\therefore \left[\log_e N\right]_{N_0}^{N} = -\lambda t.$$

$$\therefore N = N_0 e^{-\lambda t} \quad . \quad . \quad . \quad \text{(i)}$$

Thus the number N of radioactive atoms left decreases exponentially with the time t, and this is illustrated in Fig. 42.9.

The *half-life period* $T_{1/2}$ of a radioactive element is defined as the time taken for half the atoms to disintegrate (see Fig. 42.9), that is, in a time $T_{1/2}$ the radioactivity of the element diminishes to half its value. Hence, from (i),

$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

$$\therefore T_{1/2} = \frac{1}{\lambda}\log_e 2 = \frac{0{\cdot}693}{\lambda}. \quad . \quad . \quad . \quad \text{(ii)}$$

The half-life period varies considerably in a particular radioactive series. In the uranium series shown in the Table on p. 1053, for example, uranium I has a half-life period of the order of 4500 million years, radium has one of about 1600 years, radium F about 138 days, radium B about 27 minutes, and radium C′ about 10^{-4} second.

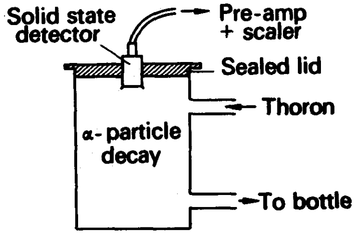


FIG. 42.10. Half life of thoron.

The half-life of thoron, a radioactive gas with a short half-life, can be measured by the apparatus shown in Fig. 42.10. A solid state detector is mounted inside a closed chamber and some thoron gas is passed in from a bottle containing thorium hydroxide which produces the gas. The total count is measured every 30 seconds for about 5 minutes, when counting has virtually stopped, and a graph of total count v. time is then plotted.

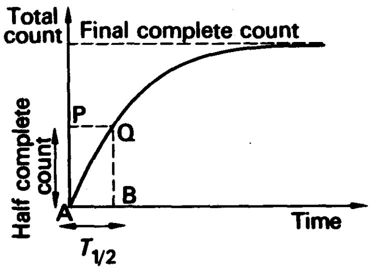


FIG. 42.11. Count in half life experiment.

Fig 42.11 shows the shape of the graph obtained. To find the half-life, a line PQ is drawn at half the final count. Half the atoms have disintegrated in the time AB and hence AB is the half-life, $T_{1/2}$, of thoron. This is read from the axis.

EXAMPLE

At a certain instant, a piece of radioactive material contains 10^{12} atoms. The half-life of the material is 30 days.

(1) Calculate the number of disintegrations in the first second. (2) How long will elapse before 10^4 atoms remain? (3) What is the count rate at this time?

(1) We have $$N = N_0 e^{-\lambda t}$$

$$\therefore \frac{dN}{dt} = -N_0 \lambda e^{-\lambda t} = -\lambda N.$$

Hence, when $N = 10^{12}, \dfrac{dN}{dt} = -\lambda 10^{12}$.

Now $$\lambda = \frac{0{\cdot}693}{T} = \frac{0{\cdot}693}{30 \times 24 \times 60 \times 60}\ \mathrm{s}^{-1}$$

$\therefore$ number of disintegrations per second

$$= \frac{10^{12} \times 0{\cdot}693}{30 \times 24 \times 60 \times 60} = 2{\cdot}7 \times 10^5$$

(2) When $N = 10^4$, we have

$$10^4 = 10^{12}e^{-\lambda t}$$

$$\therefore 10^{-8} = e^{-\lambda t}$$

Taking logs to base 10,

$$\therefore \log 10^{-8} = \log_{10}e^{-\lambda t}$$

$$\therefore -8 = -\lambda t \log_{10}e$$

$$\therefore t = \frac{8}{\lambda}\frac{1}{\log_{10}e} = \frac{8T}{0{\cdot}693 \log_{10}e}$$

$$= 64 \text{ days (approx.)}.$$

(3) Since $$\frac{dN}{dt} = -\lambda N$$

$$\therefore \text{number of disintegrations per hour} = \frac{0{\cdot}693}{30 \times 24} \times 10^4$$

$$= 9{\cdot}6.$$

Wilson's Cloud Chamber

C. T. R. Wilson's *cloud chamber*, invented in 1911, was one of the most useful early inventions for studying radioactivity. It enabled photographs to be obtained of ionizing particles or radiation.

Basically, Wilson's cloud chamber consists of a chamber Y into which saturated water-vapour is introduced (Fig. 42.12). When the pressure is suddenly reduced below a hollow glass piston X, the latter drops down and the air in Y undergoes an adiabatic expansion and cools. The dust nuclei are all carried away after a few expansions by drops forming on them, and then the dust-free air in Y is subjected to a controlled adiabatic expansion of about 1·31 to 1·38 times its original volume. The air is now supersaturated, that is, the vapour pressure is greater than the saturation vapour pressure at the reduced temperature reached but no water-vapour condenses. Simultaneously, the air is exposed to ionizing agents such as α-, β-particles or γ-rays, and water droplets immediately collect round the ions produced which act as centres of formation. The drops are photographed by light scattered from them, and in this way the tracks of ionizing particles or radiation are made visible. Wilson's cloud chamber has proved of immense value in the study of radioactivity and nuclear structure.

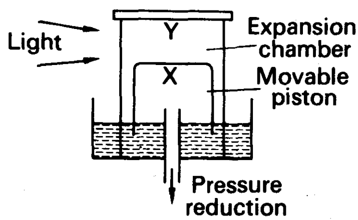


FIG. 42.12. Principle of cloud chamber—Wilson.

The random nature of radioactive decay can be seen by using a Wilson cloud chamber. Particles emitted by a radioactive substance do not appear at equal intervals of time but are sporadic or entirely random. The length of the track of an emitted particle is a measure of its initial energy. The tracks of α-particles are nearly all the same, showing that the α-particles were all emitted with the same energy. Sometimes two different lengths of tracks are obtained, showing that the α-particles may have one of two energies on emission.

Glaser's Bubble Chamber

In the same way as air can be super-saturated with water vapour, a liquid under pressure can be heated to a temperature higher than that at which boiling normally takes place and is then said to be *superheated*. If the pressure is suddenly released, bubbles may not form in the liquid for perhaps 30 seconds or more. During this quiet period, if ionizing particles or radiation are introduced into the liquid, nuclei are obtained for bubble formation. The liquid quickly evaporates into the bubble, which grows rapidly, and the bubble track when photographed shows the path of the ionizing particle.

Glaser invented the bubble chamber in 1951. It is now widely used in nuclear investigations all over the world, and it is superior to the cloud chamber. The density of the liquid ensures shorter tracks than in air, so that a nuclear collision of interest by a particle will be more likely to take place in a given length of liquid than in the same length of air. Photographs of the tracks are much clearer than those taken in the cloud chamber, and they can be taken more rapidly. In 1963 a 1·5 metre liquid hydrogen bubble chamber was constructed for use at the Rutherford High Energy Laboratory, Didcot, England. High energy protons, accelerated by millions of volts are used to bombard hydrogen nuclei in the chamber. The products of the reaction are bent into a curved track by a very powerful magnetic field, and the appearance and radius of the track then provides information about the nature, momentum or energy of the particles emitted.

Scintillations and Photomultiplier

In the early experiments on radioactivity, Rutherford observed the scintillations produced when an α-particle was incident on a material such as zinc sulphide. This is now utilized in the scintillation photomultiplier, whose principle is illustrated in Fig. 42.13. When an ionizing

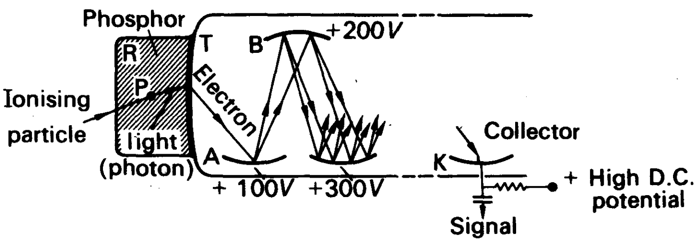


FIG. 42.13. Principle of photomultiplier.

particle strikes the scintillation material or *phosphor* S, the light falls on a photo-sensitive material A and ejects electrons. In one type of tube, these are now focused towards and accelerated to an electrode B, coated with a material which emits secondary electrons four or five times as numerous as those incident on it. The secondary electrons then strike an electrode C after further acceleration, thus multiplying the number of electrons further, and so on along the tube. A single ionizing particle can produce a million electrons in a photomultiplier tube, and the pulse of current is amplified further and recorded. By choosing a suitable phosphor, scintillation counters can detect electrons and gamma rays, as well as fast neutrons.

Emulsions

Special photographic emulsions have been designed for investigating nuclear reactions. The emulsions are much thicker than those used in ordinary photography, and the concentration of silver bromide in gelatine is many times greater than in ordinary photography. α-particles, protons and neutrons can be detected in specially-prepared emulsions by the track of silver granules produced, which has usually a very short range of the order of a millimetre or less. Consequently, after the plate is developed the track is observed under a high power microscope, or a photomicrograph is made. Nuclear emulsions were particularly useful in investigations of cosmic rays at various altitudes.

THE NUCLEUS

Discovery of Nucleus

In 1909 Geiger and Marsden, at Lord Rutherford's suggestion, investigated the scattering of α-particles by thin films of metal of high atomic weight, such as gold foil. They used a radon tube S in a metal block as a source of α-particles, and limited the particles to a narrow pencil (Fig. 42.14). The thin metal foil A was placed in the centre of an evacuated vessel, and the scattering of the particles after passing through A was observed on a fluorescent screen B, placed at the focal plane of a microscope M. Scintillations are seen on B whenever it is struck by α-particles.

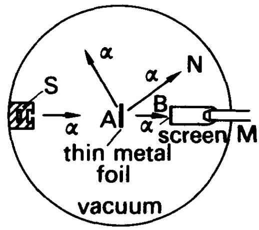


FIG. 42.14. Discovery of nucleus—Geiger and Marsden.

Geiger and Marsden found that α-particles struck B not only in the direction SA, but also when the microscope M was moved round to N and even to P. Thus though the majority of α-particles were scattered through small angles, some particles were scattered through very large angles. Rutherford found this very exciting news. It meant that some α-particles had come into the repulsive field of a highly concentrated positive charge at the heart or centre of the atom, and on the basis of an inverse-square law repulsion he calculated the number of α-particles scattered in a definite direction. The relationship was verified by Geiger and Marsden in subsequent experiments. An atom thus has a *nucleus*, in which all the positive charge and most of its mass is concentrated.

Atomic Mass and Atomic Number

In 1911 Rutherford proposed the basic structure of the atom which is accepted today, and which subsequent experiments by Moseley and others have confirmed. A neutral atom consists of a very tiny nucleus of diameter about 10^{-13} cm which contains practically the whole mass of the atom. The atom is largely empty. If a drop of water was magnified until it reached the size of the earth, the atoms inside would then be only a few metres in diameter and the atomic nucleus would have a diameter of only about 10^{-2} millimetre.

The nucleus of hydrogen is called a *proton*, and it carries a charge of $+e$, where e is the numerical value of the charge on an electron. The helium nucleus has a charge of $+2e$. The nucleus of copper has a charge of $+29e$, and the uranium nucleus carries a charge of $+92e$. Generally, the positive charge on a nucleus is $+Ze$, where Z is the *atomic number* of the element and is defined as the number of protons in the nucleus (see also p. 1072). Under the attractive influence of the positively-charged nucleus, a number of electrons equal to the atomic number move round the nucleus and surround it like a negatively-charged cloud.

Discovery of Protons in Nucleus

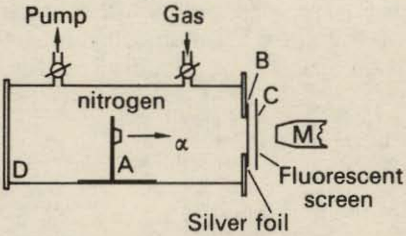


FIG. 42.15. Discovery of protons in the nucleus—Rutherford.

In 1919 Rutherford found that energetic α-particles could penetrate nitrogen atoms and that protons were ejected after the collision. The apparatus used is shown in Fig. 42.15. A source of α-particles, A, was placed in a container D from which all the air had been pumped out and replaced by nitrogen. Silver foil, B, sufficiently thick to stop α-particles, was then placed between A and a fluorescent screen C, and scintillations were observed by a microscope M. The particles which have passed through B were shown to have a similar range, and the same charge, as protons.

Protons were also obtained with the gas fluorine, and with other elements such as the metals sodium and aluminium. It thus became clear that the nuclei of all elements contain

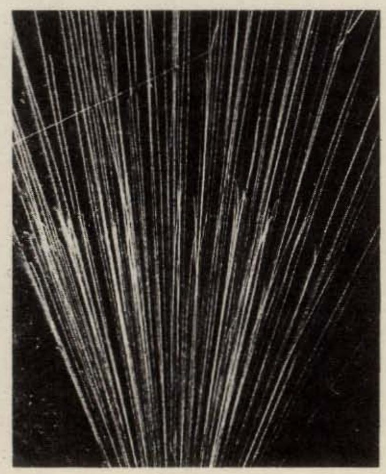


FIG. 42.16. Transmutation of Nitrogen by collision with α-particle. An oxygen nucleus, right-curved track, and a proton, left straight track, are produced.

protons. The number of protons must equal the number of electrons surrounding the nucleus, so that each is equal to the atomic number, Z, of the element. A proton is represented by the symbol, ${}^{1}_{1}\mathrm{H}$; the top number denotes the *mass number*, the whole number nearest to the relative atomic mass, and the bottom number the nuclear charge in units of $+e$. The helium nucleus such as an α-particle is represented by ${}^{4}_{2}\mathrm{He}$; its mass number is 4 and its nuclear charge is $+2e$, so that the nucleus contains two protons. One of the heaviest nuclei, uranium, can be represented by ${}^{238}_{92}\mathrm{U}$; it has a mass number of 238 and a nuclear charge of $+92e$, so that its nucleus contains 92 protons.

Discovery of Neutron in Nucleus

In 1930 Bothe and Becker found that a very penetrating radiation was produced when α-particles were incident on beryllium. Since the radiation had no charge it was thought to be γ-radiation of very great energy. In 1932 Curie-Joliot placed a block of paraffin-wax in front of the penetrating radiation, and showed that protons of considerable range were ejected from the paraffin-wax. The energy of the radiation could be calculated from the range of the ejected proton, and it was then found to be improbably high.

In 1932 Chadwick measured the velocity of protons and of nitrogen nuclei when they were ejected from materials containing hydrogen and nitrogen by the penetrating radiation. He used polonium, A, as a source of α-particles and the unknown radiation X, obtained by impact with beryllium, B, was then incident on a slab C of paraffin-wax (Fig.

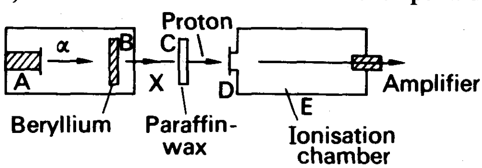


FIG. 42.17. Discovery of the neutron—Chadwick.

42.17). The velocity of the protons emitted from C could be found from their range in air, which was determined by placing various thicknesses of mica, D, in front of an ionization chamber, E, until no effect was produced here. By previous calibration of the thickness of mica in terms of air thickness, the range in air was found.

Chadwick repeated the experiment with a slab of material containing nitrogen in place of paraffin-wax. He then applied the laws of conservation of linear momentum and energy to the respective collisions with the hydrogen and nitrogen atoms, assuming that the unknown radiation was a *particle* carrying no charge and the collisions were elastic. From the equations obtained, he calculated the mass of the particle, and found it to be about the same mass as the proton. Chadwick called the new particle a *neutron*, and it is now considered that all nuclei contain protons and neutrons. The neutron is represented by the symbol ${}^{1}_{0}\mathrm{n}$ as it has an atomic mass of 1 and zero charge.

We can now see that a helium nucleus, ${}^{4}_{2}\mathrm{He}$, has 2 protons and 2 neutrons, a total mass number of 4 and a total charge of $+2e$. The sodium nucleus, $^{23}_{11}Na$, has 11 protons and 12 neutrons. The uranium nucleus, $^{238}_{92}U$, has 92 protons and 146 neutrons. Generally, a nucleus represented by $^{A}_{Z}X$ has Z protons and $(A-Z)$ neutrons.

Radioactive Disintegration

Naturally occurring radioactive elements such as uranium, actinium and thorium disintegrate to form new elements, and these in turn are unstable and form other elements. Between 1902 and 1909 Rutherford and Soddy made a study of the elements formed from a particular 'parent' element, and the *uranium series* is listed in the table below.

Element	Symbol	Atomic Number	Mass Number	Half-life Period (T)	Particle emitted
Uranium I (U)	UI	92	238	4,500 million years	α
Uranium X_1 (Th)	UX_1	90	234	24 days	β, γ
Uranium X_2 (Pa)	UX_2	91	234	1·2 minutes	β, γ
Uranium II (U)	UII	92	234	250,000 years	α
Ionium (Th)	Io	90	230	80,000 years	α, γ
Radium	Ra	88	226	1,600 years	α, γ
Radon	Rn	86	222	3·8 days	α
Radium A (Po)	RaA	84	218	3 minutes	α
Radium B (Pb)	RaB	82	214	27 minutes	β, γ
Radium C (Bi)	RaC	83	214	20 minutes	β, γ
Radium C′ (Po)	RaC′	84	214	$1{\cdot}6 \times 10^{-4}$ seconds	α
Radium C″ (Tl)	RaC″	81	210	1·3 minutes	β
Radium D (Pb)	RaD	82	210	19 years	β, γ
Radium E (Bi)	RaE	83	210	5 days	β
Radium F (Po)	RaF	84	210	138 days	α, γ
Lead	Pb	82	206	(stable)	

The new element formed after disintegration can be identified by considering the particles emitted from the nucleus of the parent atom. An α-particle, a helium nucleus, has a charge of $+2e$ and a mass number 4. Uranium I, of atomic number 92 and mass number 238, emits an α-particle from its nucleus of charge $+92e$, and hence the new nucleus formed has an atomic number 90 and a mass number 234. This was called uranium X_1, and since the element thorium (Th) has an atomic number 90, uranium X_1 is actually thorium.

A β-particle, an electron, and a γ-ray, an electromagnetic wave, have a negligible effect on the mass of a nucleus when they are emitted. A β-particle has a charge of $-e$. Now uranium X_1 has a nuclear charge of $+90e$ and a mass number 234, and emits β and γ rays. Consequently the mass number is unaltered, but the nuclear charge increases to $+91e$, and hence a new element is formed of atomic number 91. This is uranium X_2 in the series, and is actually the element protactinium. The symbols of the new elements formed are shown in brackets in the column of elements in the Table. The series contains isotopes of uranium $(U)_2$, lead (Pb), thorium (Th) and bismuth (Bi), that is, elements which have the same atomic numbers but different mass numbers (see p. 1056).

Summarizing, we can say that:

(i) when the nucleus of an element loses an α-particle, the element is displaced two places to the left in the periodic table of the elements, which follows in the order of its atomic number, and lowers its mass number by two units;

(ii) when the nucleus of an element loses a β-particle, the element is displaced one place to the right in the periodic table and its mass number is unaltered.

This law was stated in 1913 by Soddy, Russell and Fajans.

NUCLEAR MASS, NUCLEAR ENERGY

Positive Rays and Atomic Mass

As we saw in the discharge tube (p. 996), at low pressures electrons (negative charges) will flow from the cathode to the anode. If a hole is made in the cathode, some rays appear to pass through the metal, as shown in Fig. 42.18. These rays, which move in the opposite direction

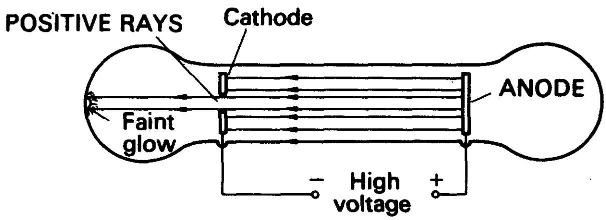


FIG. 42.18. Positive rays.

to the cathode rays (electrons), were called *positive rays*. They were first thought to come from the anode. They are now known to be formed when electrons from the cathode collide with the gas atoms and strip some electrons from the atom. Positive gas ions are then produced. These move slowly towards the cathode on account of the electric field between anode and cathode, and if the cathode is pierced they pass through the hole.

In 1911, Sir. J. J. Thomson measured the masses of individual atoms for the first time. The gas concerned was passed slowly through a bulb B a low pressure, and a high voltage was applied. Cathode rays or electrons then flow from the cathode to the anode (not shown), and positive rays due to ionization flow to the cathode, C, whose axis was pierced by a fine tube (Fig. 42.19). The positive rays are *ions*, that is, atoms which have lost one or more electrons. After flowing through C they are subjected to parallel

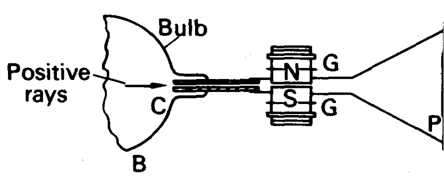


FIG. 42.19. Principle of Thomson's experiment on positive rays.

magnetic and electric fields set at right angles to the incident beam, which are applied between the poles N, S of an electromagnet. Pieces of mica, G, G are used to insulate N, S from the magnet core. The ions were deflected by the fields and were incident on a photographic plate P. After development, parabolic traces were found on P.

Theory

Suppose a positive ray or ion has a charge Q and mass M, and the electric and magnetic field intensities are E, B respectively, acting over a distance D of the ray's path (Fig. 42.20, 42.21).

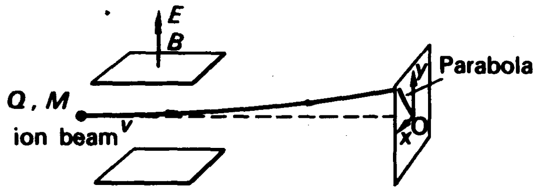


FIG. 42.20. Theory of Thomson's experiment.

(i) From p. 1002, the *electric field* causes the rays to emerge from the plates at an angle θ to the incident direction given by

$$\tan\theta = \frac{QED}{Mv^2},$$

since $e = Q$ here. Hence if y is the deflection in a vertical direction from O, and l is the horizontal distance from the middle of the plates to O,

$$y = l\tan\theta = \frac{QEDl}{Mv^2}. \qquad \text{(i)}$$

(ii) The *magnetic field* deflects the beam into a circular arc AC of radius R. From p. 1002, this is given by $R = Mv/BQ$.

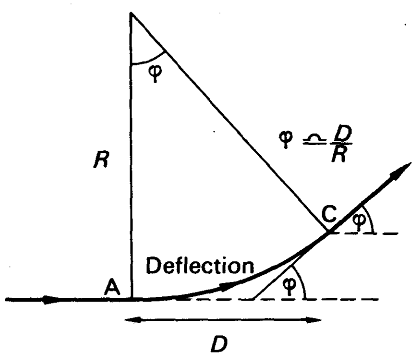


FIG. 42.21. Deflection in magnetic field.

If ϕ is the angle made by the emergent beam with the incident or x-direction, then, if ϕ is small, $\phi = D/R$ (Fig. 42.21). Hence the deflection x in a horizontal direction from O is given by

$$x = l\tan\phi = l\phi = \frac{lD}{R} = \frac{lDBQ}{Mv} \qquad \text{(ii)}$$

Eliminating v from (i) and (ii), we obtain

$$x^2 = \frac{Q}{M}\left(\frac{B^2Dl}{E}\right)y \qquad \text{(iii)}$$

From (iii), it follows that ions with the same charge-mass ratio Q/M, although moving with different velocities, all lie on a *parabola* of the form $x^2 = \text{constant} \times y$.

Determination of Masses. Isotopes

As the zero of the parabola was ill-defined, Thomson reversed the field to obtain a parabolic trace on the other side of the y-axis, as

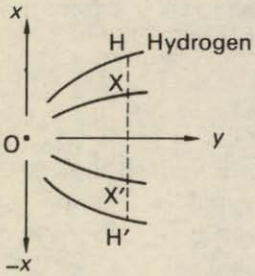


FIG. 42.22. Determination of mass of atom.

FIG. 42.23. Mass Spectrometer. Positive-ray parabolas due to mercury, carbon monoxide, oxygen and carbon ions.

shown in Fig. 42.22. Now from (iii), $x^2 \propto Q/M$ for a given value of y. Consequently any hydrogen ions present would produce the outermost parabola H, since they have the greatest value of Q/M. The masses of ions can thus be measured by comparing the squares of the x-values of the individual parabolas, such as the squares of X′X and H′H, for example. In this way Thomson obtained a *mass spectrometer*, one which gave the masses of individual atoms.

With chlorine gas, two parabolas were obtained which gave atomic masses of 35 and 37 respectively. Thus the atoms of chlorine have different masses but the same chemical properties, and these atoms are said to be *isotopes* of chlorine. In chlorine, there are three times as many atoms of mass 35 as there are of mass 37, so that the average atomic weight is $(3 \times 35 + 1 \times 37)/4$, or 35·5. The element xenon has as many as nine isotopes. One part in 5000 of hydrogen consists of an isotope of mass 2 called deuterium, or heavy hydrogen. An unstable isotope of hydrogen of mass 3 is called tritium.

Bainbridge Mass Spectrometer

Thomson's earliest form of mass spectrometer was followed by more sensitive forms. In 1933 Bainbridge devised a mass spectrometer in which the ions were photographed after being deflected by a magnetic field. The principle of the spectrometer is shown in Fig. 42.24. Positive ions were produced in a discharge tube (not shown) and admitted as a fine beam through slits S_1, S_2. The beam then passed between insulated plates P, Q, connected to a battery, which created an electric field of intensity E. A uniform magnetic field B_1, perpendicular to E, was also applied over the region of the plates, and all ions, charge $\bar{e}$, with the same velocity v given by $B_1\bar{e}v = E\bar{e}$ will then pass undeflected through the plates and through a slit S_3. The selected ions are now deflected in a circular path of radius r by a uniform perpendicular magnetic field B_2, and an image is produced on a photographic plate A, as shown. In this case,

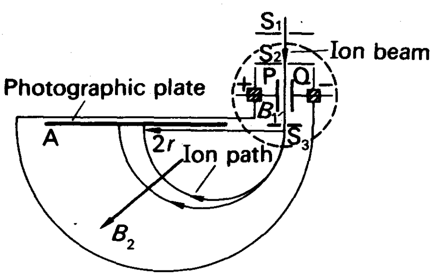


FIG. 42.24. Principle of Bainbridge's mass spectrometer.

$$\frac{mv^2}{r} = B_2\bar{e}v.$$

$$\therefore \frac{m}{\bar{e}} = \frac{rB_2}{v}.$$

But for the selected ions, $v = E/B_1$ from above.

$$\therefore \frac{m}{\bar{e}} = \frac{rB_2B_1}{E}$$

$$\therefore \frac{m}{\bar{e}} \propto r,$$

for given magnetic and electric fields.

Since the ions strike the photographic plate at a distance $2r$ from the middle of the slit S_3, it follows that the separation of ions carrying the same charge is directly proportional to their mass. Thus a 'linear' mass scale is achieved. A resolution of 1 in 30000 was obtained with a later type of spectrometer.

Einstein's Mass-Energy Relation

In 1905 Einstein showed from his Theory of Relativity that mass and energy can be changed from one form to the other. The energy E produced by a change of mass m is given by the relation:

$$E = mc^2,$$

where c is the numerical value of the velocity of light. E is in joules when m is in kg and c has the numerical value 3×10^8 (p. 560). Thus a change in mass of 1 g could theoretically produce 9×10^{13} joules of energy. Now 1 kilowatt-hour of energy is 1000×3600 or $3{\cdot}6 \times 10^6$ joules, and hence 9×10^{13} joules is $2{\cdot}5 \times 10^7$ or 25 million kilowatt-hours. Consequently a change in mass of 1 g could be sufficient to keep the electric lamps in a million houses burning for about a week in winter, on the basis of about seven hours' use per day.

In electronics and in nuclear energy, the unit of energy called an *electron-volt* (eV) is often used. This is defined as the energy gained by a charge equal to that on an electron moving through a p.d. of one volt.

$$1 \text{ eV} = 1{\cdot}6 \times 10^{-19} \text{ joule (p. 756).}$$

The *megelectron-volt* (MeV) is a larger energy unit, and is defined as 1 million eV.

Atomic Mass Unit

If another unit of energy is needed, then one may use a unit of mass, since mass and energy are interchangeable. The *atomic mass unit* (*a.m.u.*) is defined as one-twelfth of the mass of the carbon atom ${}^{12}_{6}C$. Now the number of molecules in 1 mole of carbon is $6{\cdot}02 \times 10^{23}$, Avogadro's constant, and since carbon is monoatomic, there are $6{\cdot}02 \times 10^{23}$ atoms of carbon. These have a mass 12 g.

$$\therefore \text{ mass of 1 atom of carbon} = \frac{12}{6{\cdot}02 \times 10^{23}} \text{ g} = \frac{12}{6{\cdot}02 \times 10^{26}} \text{ kg}$$

$$= 12 \text{ a.m.u.}$$

$$\therefore 1 \text{ a.m.u.} = \frac{12}{12 \times 6{\cdot}02 \times 10^{26}} \text{ kg}$$

$$= 1{\cdot}66 \times 10^{-27} \text{ kg.}$$

We have seen that 1 kg change in mass produces 9×10^{16} joules, and that 1 MeV $= 1{\cdot}6 \times 10^{-13}$ joule.

$$\therefore 1 \text{ a.m.u.} = \frac{1{\cdot}66 \times 10^{-27} \times 9 \times 10^{16}}{1{\cdot}6 \times 10^{-13}} \text{ MeV}$$

$$\therefore 1 \text{ a.m.u.} = 931 \text{ MeV} \qquad . \quad . \quad . \quad . \quad (1)$$

This relation is used to change mass units to MeV, and vice-versa, as we shall see shortly. An electron mass, $9{\cdot}1 \times 10^{-31}$ kg, corresponds to about $0{\cdot}5$ MeV.

Binding Energy

The protons and neutrons in the nucleus of an atom are called *nucleons*. The work or energy needed to take all the nucleons apart so that they are completely separated is called the *binding energy* of the nucleus. Hence, from Einstein's mass-energy relation, it follows that the total mass of all the individual nucleons is greater than that of the nucleus, in which they are together. *The difference in mass is a measure of the binding energy.*

As an example, consider a helium nucleus ^4_2He. This has 4 nucleons, 2 protons and 2 neutrons. The mass of a proton is 1·0076 and the mass of a neutron is 1·009 a.m.u.

$$\therefore \text{ total mass of 2 protons plus 2 neutrons} = 2 \times 1{\cdot}0076 + 2 \times 1{\cdot}009$$
$$= 4{\cdot}0332 \text{ a.m.u.}$$

But the helium nucleus has a mass of 4·0028 a.m.u.

$$\therefore \text{ binding energy} = \text{mass difference of nucleons and nucleus}$$
$$= 4{\cdot}0332 - 4{\cdot}0028 = 0{\cdot}0304 \text{ a.m.u.}$$
$$= 0{\cdot}0304 \times 931 \text{ MeV} = 28{\cdot}3 \text{ MeV.}$$

The *binding energy per nucleon* of a nucleus is binding energy divided by the total number of nucleons. In the case of the helium nucleus, since there are four nucleons (2 protons and 2 neutrons), the binding energy per nucleon is 28·3/4 or about 7·1 MeV. Fig. 42.25 shows roughly the variation of the binding energy per nucleon among the elements. The great majority have a value of about 8 MeV per nucleon. In spite of considerable binding energy, elements with high mass numbers may have a tendency to disintegrate. This is not surprising because a very

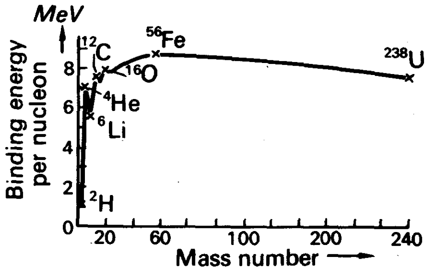


FIG. 42.25. Variation of binding energy per nucleon with mass number.

heavy nucleus contains many protons (and neutrons) packed into a very tiny volume, and strong forces of repulsion may then exist. An α-particle, perhaps formed by two neutron-proton pairs, may then be expelled from the nucleus. A β-particle is emitted when a neutron changes into a proton in the nucleus.

Stability of Nuclei

It is instructive to consider, from an energy point of view, whether a particular nucleus is likely to disintegrate with the mission of an α-particle. As an illustration, consider radium F or polonium, $^{210}_{84}Po$. If an α-particle could be emitted from the nucleus, the reaction products would be the α-particle or helium nucleus, ^4_2He, and a lead nucleus, $^{206}_{82}Pb$, a reaction which could be represented by:

$$^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^4_2He. \qquad \text{(i)}$$

It should be noted that the sum of the mass numbers, 210, and the sum of the nuclear charges, $+84e$, of the lead and helium nuclei is equal to the mass number and nuclear charge of the polonium nucleus.

If we require to find whether energy has been released or absorbed in the reaction, we should calculate the total mass of the lead and helium nuclei and compare this with the mass of the polonium nucleus. It is more convenient to use atomic masses rather than nuclear masses, and since the total number of electrons required on each side of (i) to convert the nuclei into atoms is the same, we may use atomic masses in the reaction. These are as follows:

$$\text{lead } {}^{206}_{82}\text{Pb}, = 206{\cdot}034 \text{ a.m.u.}$$

$$\alpha\text{-particle, } {}^{4}_{2}\text{He}, = 4{\cdot}004 \text{ a.m.u.}$$

$$\therefore \text{ total mass} = 210{\cdot}038 \text{ a.m.u.}$$

Now $$\text{polonium } {}^{210}_{84}\text{Po}, = 210{\cdot}049 \text{ a.m.u.}$$

Thus the atomic masses of the products of the reaction are together *less* than the original polonium nucleus, that is,

$${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He} + Q,$$

where Q is the energy released. It therefore follows that polonium can disintegrate with the emission of an α-particle and a release of energy (see *uranium series*, p. 1053), that is, the polonium is unstable.

Suppose we now consider the possibility of a lead nucleus, ${}^{206}_{82}$, disintegrating with the emission of an α-particle, ${}^{4}_{2}$He. If this were possible, a mercury nucleus, ${}^{202}_{80}$Hg, would be formed. The atomic masses are as follows:

$$\text{mercury, } {}^{202}_{80}\text{Hg}, = 202{\cdot}035 \text{ a.m.u.}$$

$$\alpha\text{-particle, } {}^{4}_{2}\text{He}, = 4{\cdot}004 \text{ a.m.u.}$$

$$\therefore \text{ total mass} = 206{\cdot}039 \text{ a.m.u.}$$

Now $$\text{lead } {}^{206}_{82}\text{Pb}, = 206{\cdot}034 \text{ a.m.u.}$$

Thus, unlike the case previously considered, the atomic masses of the mercury nucleus and α-particle are together *greater* than the lead nucleus, that is,

$${}^{206}_{82}\text{Pb} + Q \rightarrow {}^{202}_{80}\text{Hg} + {}^{4}_{2}\text{He},$$

where Q is the energy which must be *given* to the lead nucleus to obtain the reaction products. It follows that the lead nucleus by itself is stable.

Generally, then, a nucleus would tend to be unstable and emit an α-particle if the sum of the atomic masses of the products are together less than that of the nucleus, and it would be stable if the sum of the atomic masses of the possible reaction products are together greater than the atomic mass of the nucleus.

Artificial Disintegration

Uranium, thorium and actinium are elements which disintegrate naturally. The artificial disintegration of elements began in 1919, when Rutherford used α-particles to bombard nitrogen and found that protons were produced (p. 1051). Some nuclei of nitrogen had changed into nuclei of oxygen, that is, transmutation had occurred, a reaction which can be represented by:

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H.$$

In 1932 Cockcroft and Walton produced nuclear disintegrations by accelerating protons with a high-voltage machine producing about half a million volts, and then bombarding elements with the high-speed protons. When the light element lithium was used, photographs of the reaction taken in the cloud chamber showed that α-particles were produced. The latter shot out in opposite direction from the point of impact of the protons, and as their range in air was equal, the α-particles had initially equal energy. The nuclear reaction was:

$$^{7}_{3}Li + ^{1}_{1}H \rightarrow ^{4}_{2}He + ^{4}_{2}He + Q, \quad . \quad . \quad . \quad \text{(i)}$$

where Q is the energy released in the reaction.

To calculate Q, we should calculate the total mass of the lithium and hydrogen nuclei and subtract the total mass of the two helium nuclei. As already explained, however, the total number of electrons required to convert the nuclei to neutral atoms is the same on both sides of equation (i), and hence atomic masses can be used in the calculation in place of nuclear masses. The atomic masses of lithium and hydrogen are 7·018 and 1·008 a.m.u. respectively, a total of 8·026 a.m.u. The atomic mass of the two α-particles is $2 \times 4{\cdot}004$ a.m.u. or 8·008 a.m.u. Thus:

$$\begin{aligned} \text{energy released, } Q, &= 8{\cdot}026 - 8{\cdot}008 = 0{\cdot}018 \text{ a.m.u.} \\ &= 0{\cdot}018 \times 931 \text{ MeV} = 16{\cdot}8 \text{ MeV}. \end{aligned}$$

Each α-particle has therefore an initial energy of 8·4 MeV, and this theoretical value agreed closely with the energy of the α-particle measured from its range in air.

Cockcroft and Walton were the first scientists to use protons for disrupting atomic nuclei after accelerating them by high voltage. Today, giant high-voltage machines are being built at Atomic Energy centres for accelerating protons to enormously high speeds, and the products of the nuclear explosion with light atoms such as hydrogen will yield valuable information on the structure of the nucleus.

Energy released in Fission

In 1934 Fermi began using neutrons to produce nuclear disintegration. These particles are generally more effective than α-particles or protons for this purpose, because they have no charge and are therefore able to penetrate more deeply into the positively-charged nucleus. Usually the atomic nucleus charges only slightly after disintegration, but in 1939 Frisch and Meitner showed that a uranium nucleus had disintegrated into two relatively-heavy nuclei. This is called *nuclear fission*, and as we shall now show, a large amount of energy is released in this case.

Natural uranium consists of about 1 part by weight of uranium atoms $^{235}_{92}U$ and 140 parts by weight of uranium atoms $^{238}_{92}U$. In a nuclear reaction with natural uranium and slow neutrons, it is usually the nucleus $^{235}_{92}U$ which is fissioned. If the resulting nuclei are lanthanum $^{148}_{57}La$ and bromine $^{85}_{35}Br$, together with several neutrons, then:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{148}_{57}La + ^{85}_{35}Br + 3^{1}_{0}n \quad . \quad . \quad . \quad \text{(i)}$$

Now $^{235}_{02}U$ and $^{1}_{0}n$ together have a mass of $(235{\cdot}1 + 1{\cdot}009)$ or $236{\cdot}1$ a.m.u. The lanthanum, bromine and neutrons produced together have a mass

$$= 148{\cdot}0 + 84{\cdot}9 + 3 \times 1{\cdot}00 = 235{\cdot}9 \text{ a.m.u.}$$

.. energy released = mass difference

$$= 0{\cdot}2 \text{ a.m.u.} = 0{\cdot}2 \times 931 \text{ MeV} = 186 \text{ MeV}.$$
$$= 298 \times 10^{-13} \text{ J (approx.)}.$$

This is the energy released per atom of uranium fissioned. In 1 kg of uranium there are about

$$\frac{1000}{235} \times 6 \times 10^{23} \text{ or } 26 \times 10^{23} \text{ atoms,}$$

since Avogadro's number, the number of atoms in a mole of any element, is $6{\cdot}02 \times 10^{23}$. Thus if all the atoms in 1 kg of uranium were fissioned, total energy released

$$= 26 \times 10^{23} \times 298 \times 10^{-13} \text{ joules}$$
$$= 2 \times 10^{7} \text{ kilowatt-hours (approx.),}$$

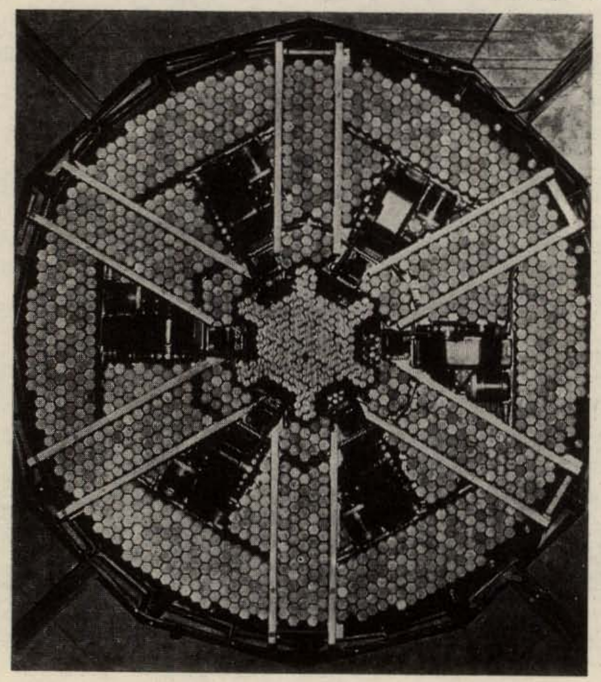


FIG. 42.26. Nuclear Research Reactor, ZEUS. This view shows the heart of the reactor, containing a highly enriched uranium central core surrounded by a natural uranium blanket for breeding studies.

which is the amount of energy given out by burning about 3 million tonnes of coal. The energy released per gramme of uranium fissioned = 8×10^{10} joules (approx.).

To make practical use of nuclear fission, the incident neutrons must be moderated in speed so that they are 'captured' by the nuclei in a mass of uranium. Carbon rods are used as moderators. The neutrons produced in the nuclear reaction in equation (i), p. 1062, in turn produce fission swiftly in other uranium nuclei, and so on, thus creating a multiplying rapid *chain reaction* throughout the mass of uranium. Details of nuclear reactors can be obtained from the United Kingdom Atomic Energy Authority, London.

Energy released in Fusion

In fission, energy is released when a heavy nucleus is split into two lighter nuclei. Energy is also released if light nuclei are *fused* together to form heavier nuclei, and a fusion reaction, as we shall see, is also a possible source of considerable energy. As an illustration, consider the fusion of the nuclei of deuterium, ${}^2_1H$. Deuterium is an isotope of hydrogen known as 'heavy hydrogen', and its nucleus is called a 'deuteron'. The fusion of two deuterons can result in a helium nucleus, ${}^3_2He$, as follows:

$$ {}^2_1H + {}^2_1H \rightarrow {}^3_2He + {}^1_0n. $$

Now mass of two deuterons

$$ = 2 \times 2{\cdot}015 = 4{\cdot}03 \text{ a.m.u.,} $$

and mass of helium plus neutron

$$ = 3{\cdot}017 + 1{\cdot}009 = 4{\cdot}026 \text{ a.m.u.} $$

$\therefore$ mass converted to energy by fusion

$$ \begin{aligned} &= 4{\cdot}03 - 4{\cdot}026 = 0{\cdot}004 \text{ a.m.u.} \\ &= 0{\cdot}004 \times 931 \text{ MeV} = 3{\cdot}7 \text{ MeV} \\ &= 3{\cdot}7 \times 1{\cdot}6 \times 10^{-13} \text{ J} = 6{\cdot}0 \times 10^{-13} \text{ J} \end{aligned} $$

$\therefore$ energy released per deuteron

$$ = 3{\cdot}0 \times 10^{-13} \text{ J.} $$

6×10^{23} is the number of atoms in a mole of deuterium, which is about 2 grammes. Thus if all the atoms could undergo fusion,

energy released per gramme

$$ \begin{aligned} &= 3{\cdot}0 \times 10^{-13} \times 3 \times 10^{23} \text{ J} \\ &= 9 \times 10^{10} \text{ J (approx.).} \end{aligned} $$

Other fusion reactions can release much more energy, for example, the fusion of the nuclei of deuterium, ${}^2_1H$, and tritium, ${}^3_1H$, isotopes of hydrogen, releases about 30×10^{10} joules of energy according to the reaction:

$$ {}^2_1H + {}^3_1H \rightarrow {}^4_2He + {}^1_0n. $$

In addition, the temperature required for this fusion reaction is less than that needed for the fusion reaction between two deuterons given above, which is an advantage. Hydrogen contains about 1/5000th by weight of deuterium or heavy hydrogen, needed in fusion reactions, and this can be obtained by electrolysis of sea-water, which is cheap and in plentiful supply.

Thermonuclear Reaction

The binding energy curve in Fig. 42.23 shows that elements with low atomic mass, up to about 56, can produce energy by fusion of their nucleons. For fusion to take place, the nuclei must at least overcome their nuclear repulsion when approaching each other. Consequently, for practical purposes, fusion reactions can best be achieved with the lightest elements such as hydrogen, whose nuclei carry the smallest charges and hence repel each other least.

In attempts to obtain fusion, isotopes of hydrogen such as deuterium, ${}^2_1H$, and tritium, ${}^3_1H$, are heated to tens of millions of degrees centigrade. The thermal energy of the nuclei at these high temperatures is sufficient for fusion to occur. One technique of promoting this *thermonuclear reaction* is to pass enormously high currents through the gas, which heat it. A very high percentage of the atoms are then ionized and the name *plasma* is given to the gas. Interstellar space or the *aurora borealis* contains a weak form of plasma, but the interior of stars contains a highly concentrated form of plasma. The gas discharge consists of parallel currents, carried by ions, and the powerful magnetic field round one current due to a neighbouring current (see p. 939) draws the discharge together. This is the so-called 'pinch effect'. The plasma, however, wriggles and touches the sides of the containing vessel, thereby losing heat. The main difficulty in thermonuclear experiments in the laboratory is to retain the heat in the gas for a sufficiently long time for a fusion reaction to occur, and the stability of plasma is now the subject of considerable research.

It is believed that the energy of the sun is produced by thermonuclear reactions in the heart of the sun, where the temperature is many millions of degrees centigrade. Bethe has proposed a cycle of nuclear reactions in which, basically, protons are converted to helium by fusion, with the liberation of a considerable amount of energy.

EXERCISES 42

1. ${}^{24}_{11}Na$ is a *radioactive isotope* of sodium which has a *half-life period* of 15 hours and disintegrates with the emission of *β-particles* and *γ-rays.* It emits β-particles that have energies of 4·2 *MeV.*)

Explain the meaning of the five terms that are italicized in the statement above. (*L.*)

2. Give concisely the important facts about mass, charge and velocity associated with α, β and γ radiations respectively. State the effect, if any, of the emission of each of these radiations on (*a*) the mass number and (*b*) the atomic number, of the element concerned.

Describe an experiment *either* to measure the range of α particles in air *or* to verify that the intensity of γ radiation varies inversely as the square of the distance from the source, being provided with a suitable radioactive source for the experiment chosen. (*L.*)

3. 'Gamma rays obey the inverse square law.' What does this mean? What conditions must be satisfied for the statement to be valid?

Describe an experiment designed to verify the statement for gamma rays in air.

The window area of a gamma ray detector is 5·0 cm^2. The window is placed horizontally and lies 80 cm vertically below a small source of gamma rays, 60 photons per minute from the source are incident on it. Estimate the rate of emission of photons from the source. (*N.*)

4. Describe the nature of α, β and γ radiations.

What is meant by the statement that the stable isotope of gold has an atomic number of 79 and a mass number of 197? A sample of pure gold is irradiated with neutrons to produce a small proportion of the radioactive isotope of gold of mass number 198. What experiments would you perform to examine the radiation emitted by the sample to establish whether it was α, β or γ radiation? If chemical analysis of the sample subsequently showed that it contained a trace of mercury (atomic number 80) what would you conclude from this about the nature of the radiation from the radioactive gold? What would you expect the mass number of the isotope of mercury present in the gold to be? (*O. & C.*)

5. Given a standard set of radioactive sources, which include nearly pure α-, β- and γ-emitters, and also a thorium hydroxide preparation, describe with full experimental details how you would demonstrate two of the following:

(i) that each source emits ionizing radiation;
(ii) the characteristic differences between α-, β- and γ-radiations;
(iii) that radioactivity involves a decay process, the half-life of which can be measured;
(iv) that radioactive decay is a random process. (*O.*)

6. A Geiger-Müller tube is placed close to a source of beta particles of constant activity. Sketch a graph showing how the count-rate, measured using a suitable scaler or ratemeter, varies with the potential difference applied to the G.M. tube. Discuss how the form of the graph determines the choice of operating conditions for the tube.

Describe how you would investigate the absorption of beta particles of aluminium using a G.M. tube. Sketch a graph showing the results you would expect to obtain. How would the form of the graph change if (*a*) the same source was used with lead substituted for aluminium, (*b*) a different source emitting beta particles of higher energy was used, aluminium being the absorbing material? (*N.*)

7. Compare and contrast the properties of α-particles, protons and neutrons. Discuss briefly the part played by the two last-named particles in atomic structure.

Compare the velocities attained by a proton and an α-particle each of which has been accelerated from rest through the same potential difference. (*L.*)

8. In nuclear fusion, deuterium nuclei ^2_1H might fuse together to form a single helium nucleus. If the atomic masses of deuterium and helium are 2·010 and 4·004 a.m.u. respectively, and 1 a.m.u. = 931 MeV, calculate the energy released in MeV.

9. What is gamma-radiation? Explain one way in which it originates.

An experiment was conducted to investigate the absorption by aluminium of the radiation from a radioactive source by inserting aluminium plates of different thicknesses between the source and a Geiger tube connected to a rate-meter (or scaler). The observations are summarized in the following table:

Thickness of aluminium (cm)	*Corrected mean* count rate ($min.^{-1}$)
2·3	1326
6·9	802
11·4	496
16·0	300

Use these data to plot a graph and hence determine for this radiation in aluminium the *linear absorption coefficient*, μ (defined by $\mu = -dI/I \times 1/dx$ where I is the intensity of the incident radiation and dI is the part of the incident radiation absorbed in thickness dx).

Draw a diagram to illustrate the arrangement of the apparatus used in the experiment and describe its preliminary adjustment.

What significance do you attach to the words 'corrected' and 'mean' printed in italics in the table? (*N.*)

10. Using the information on atomic masses given below, show that a nucleus of uranium 238 can disintegrate with the emission of an alpha particle according to the reaction:

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^4_2He.$$

Calculate (*a*) the total energy released in the disintegration, (*b*) the kinetic energy of the alpha particle, the nucleus being at rest before disintegration.

Mass of ^{238}U = 238·12492 a.m.u. Mass of ^{234}Th = 234·11650 a.m.u. Mass of ^4He = 4·00387 a.m.u. 1 a.m.u. (atomic mass unit) is equivalent to 930 MeV. (*N.*)

11. Compare and contrast the properties of the *proton*, *neutron* and *electron*. Explain the role played by each of these particles in the structure of the atom.

How is your account of the arrangement of the electrons in the atom supported by experimental evidence? (*L.*)

12. The decay of a certain radioactive source is found to be exponential. Explain this statement.

The disintegration of a radioactive atom is considered to be a matter of chance. How do you reconcile this idea with the observed regular law of decay?

To determine the half life of the radioactive gas thoron an experimenter uses an ionization chamber into which he introduces air loaded with thoron. The ionization current is 90×10^{-12} A initially and falls to half of this value in 60 seconds, which is taken to be the half life of thoron. Unknown to the experimenter, the ionization chamber is contaminated, from previous use, with a radioactive substance whose half life is 11 hours, which is responsible for 4×10^{-12} A of the ionization current at the outset of the experiment. Calculate a more accurate value for the half life of thoron.

What experimental procedure would have revealed this source of error? (*O. & C.*)

13. What is meant by the *half-life period* (*half-life*) of a radioactive material? Describe how the nature of α-particles has been established experimentally.

The half-life period of the body polonium-210 is about 140 days. During this period the average number of α-emissions per day from a mass of polonium initially equal to 1 microgram is about 12×10^{12}. Assuming that one emission takes place per atom and that the approximate density of polonium is 10 g cm^{-3}, estimate the number of atoms in 1 cm^3 of polonium. (*N.*)

14. Describe, with the aid of a labelled diagram, the apparatus used in J. J. Thomson's investigations into the nature and properties of positive rays. Describe, and in qualitative terms explain the pattern observed on the screen of the apparatus.

What conclusions were drawn from the observations made in these experiments? (*N.*)

15. Give an account of the structure of atoms including the significance of the terms *atomic mass* and *atomic number*. State how the fundamental particles of which atoms are composed differ from each other as regards mass and electric charge.

A potential difference of 600 V is maintained between two identical horizontal metal plates placed 4·0 cm apart one above the other in an evacuated vessel. Particles each with mass $9{\cdot}1 \times 10^{-31}$ kg and electric charge $1{\cdot}6 \times 10^{-19}$ C are emitted with negligible velocity from the plate at the lower potential. For one of the particles calculate (*a*) the ratio of the electric force to the gravitational force on it, (*b*) its acceleration, (*c*) the kinetic energy it acquires on reaching the other plate. (Assume $g = 10$ m s^{-2}.) (*N.*)

chapter forty-three

X-Rays. Photoelectricity. Energy Levels

X-RAYS

X-rays

In 1895, Röntgen found that some photographic plates, kept carefully wrapped in his laboratory, had become fogged. Instead of merely throwing them aside he set out to find the cause of the fogging. He traced it to a gas-discharge tube, which he was using with a low pressure and high voltage. This tube appeared to emit a radiation that could penetrate paper, wood, glass, rubber, and even aluminium a centimetre and a half thick. Röntgen could not find out whether the radiation was a stream of particles or a train of waves—Newton had the same difficulty with light—and he decided to call it X-rays.

Nature and Production of X-rays

We now regard X-rays as waves, similar to light waves, but of much shorter wavelength: about 10^{-8} cm, or 1 Ångström unit. They are produced when fast electrons, or cathode rays, strike a target, such as the walls or anode of a low-pressure discharge tube. In a modern X-ray tube there is no gas, or as little as high-vacuum technique can achieve: the pressure is about 10^{-5} mm Hg. The electrons are provided by thermionic emission from a white-hot tungsten filament (p. 999). In

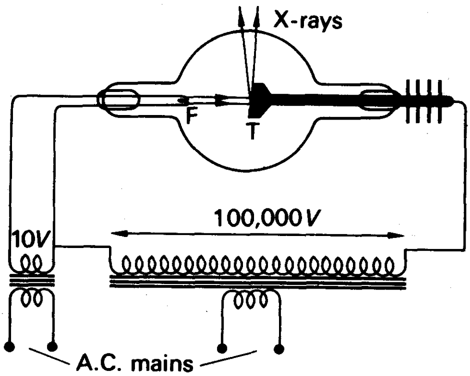


FIG. 43.1. An X-ray tube.

Fig. 43.1 F is the filament and T is the target, or anode. Because there is so little gas, the electrons on their way to the anode do not lose any perceptible amount of their energy in ionizing atoms. From the a.c.

mains, transformers provide about 10 volts for heating the filament, and about 100 000 volts for accelerating the electrons. On the half-cycles when the target is positive, the electrons bombard it, and generate X-rays. On the half-cycles when the target is negative, nothing happens at all—there is too little gas in the tube for it to break down. Thus the tube acts, in effect, as its own rectifier (p. 1010), providing pulses of direct current between target and filament. The heat generated at the target by the electronic bombardment is so great that the target must be cooled artificially. In the figure, fins for air-cooling are shown, but in large tubes the target is made hollow, and is cooled by circulating water or oil. The target in an X-ray tube is usually tungsten, which has a high melting-point.

Effects and Uses of X-rays

When X-rays strike many minerals, such as zinc sulphide, they make them fluoresce. (It was while studying this fluorescence that Becquerel discovered the radiations from uranium.) If a human—or other—body is placed between an X-ray tube and a fluorescent screen, the shadows of its bones can be seen on the screen, because they absorb X-rays more than flesh does. Unusual objects, such as swallowed safety-pins, if they are dense enough, can also be located. X-ray photographs can likewise be taken, with the plate in place of the screen. In this way cracks and flaws can be detected in metal castings.

When X-rays are passed through a crystal, they are scattered by its atoms and diffracted, as light is by a diffraction grating (p. 707). By recording the diffraction pattern on a photographic plate, and measuring it up, the structure of the crystal can be discovered. This was developed by Sir William Bragg and his son, Sir Lawrence Bragg.

X-ray Spectra

In an X-ray tube, very energetic electrons bombard atoms in a metal target such as tungsten, and an electron may be ejected from the innermost shell, the K shell. The atom is then in an excited state and is unstable. If an electron from the L shell now moves into the vacancy in the K shell, the energy of the atom is decreased and simultaneously there is emission of radiation. If E is the change in energy of the atom when the electron moves from the L to the K shell, then $E = h\nu$, where ν is the frequency of the radiation, from Bohr's theory. Thus $\nu = E/h$, and as E is very high for metals, the frequency ν is very high and the wavelength is correspondingly short. It is commonly of the order of 10^{-8} cm, the wavelengths of X-rays.

The X-ray spectra of different metals such as copper, iron and tungsten are similar in appearance. Each indicates energy changes of electrons in the interior of the atom close to the nucleus. By contrast, the optical spectra of metals are related to the energy changes of electrons in the outermost shells of the atoms, which are different for different metals. The optical spectra are therefore different.

Crystal Diffraction

The first proof of the wave-nature of X-rays was due to Laue in 1913, many years after X-rays were discovered. He suggested that the regular small spacing of atoms in crystals might provide a natural diffraction grating if the wavelengths of the rays were too short to be used with an optical line grating. Experiments by Friedrich and Knipping showed that X-rays were indeed diffracted by a thin crystal, and produced a pattern of intense spots round a central image on a photographic plate placed to receive them (Fig. 43.2). The rays had thus

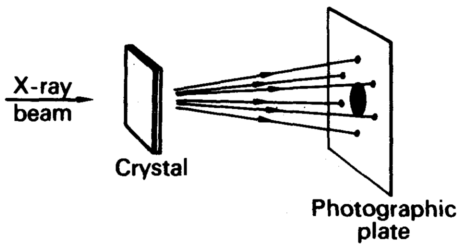


FIG. 43.2. Laue crystal diffraction.

been scattered by interaction with electrons in the atoms of the crystal, and the pattern obtained gave information on the geometrical spacing of the atoms.

Bragg's Law

The study of the atomic structure of crystals by X-ray analysis was initiated in 1914 by Sir William Bragg and his son Sir Lawrence Bragg, with notable achievements. They soon found that a monochromatic beam of X-rays was reflected from a plane in the crystal rich in atoms, a so-called atomic plane, as if the latter acted like a mirror.

This important effect can be explained by Huyghens's wave theory in the same way as the reflection of light by a plane surface. Suppose a monochromatic parallel X-ray beam is incident on a crystal and interacts with atoms such as A, B, C, D in an atomic plane P (Fig. 43.3 (i)). Each atom scatters the X-rays. Using Huyghens's construction, wavelets can be drawn with the atoms as centres, which all lie on a plane

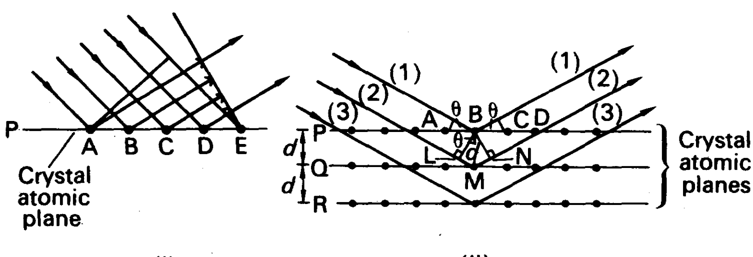


FIG. 43.3. Reflection (diffraction) at crystal atomic planes.

wavefront reflected at an equal angle to the atomic plane P. When the X-ray beam penetrates the crystal to other atomic planes such as Q, R parallel to P, reflection occurs in a similar way (Fig. 43.3 (ii)). Usually, the beam or ray reflected from one plane is weak in intensity. If, however, the reflected beams or rays from all planes are *in phase* with each other, an intense reflected beam is produced by the crystal.

Suppose, then, that the glancing angle on an atomic plane in the crystal is θ, and d is the distance apart of consecutive parallel atomic planes (Fig. 43.3 (ii)). The path difference between the rays marked (1) and (2) $=$ LM + MN $=$ 2LM $= 2d \sin \theta$. Thus an intense X-ray beam is reflected when

$$2d \sin \theta = n\lambda,$$

where λ is the wavelength and n has integral values. This is known as *Bragg's law.* Hence, as the crystal is rotated so that the glancing angle is increased from zero, and the beam reflected at an equal angle is observed each time, an intense beam is suddenly produced for a glancing angle θ_1 such that $2d \sin \theta_1 = \lambda$. When the crystal is rotated further, an intense reflected beam is next obtained for an angle θ_2 when $2d \sin \theta_2 = 2\lambda$. Thus several orders of diffraction images may be observed. Many orders are obtained if λ is small compared with $2d$. Conversely, no images are obtained if λ is greater than $2d$.

The intense diffraction (reflection) images from an X-ray tube are due to X-ray lines *characteristic of the metal used* as the target, or 'anti-cathode' as it was originally known. This is because the quantum of energy $h\nu$ in the emitted X-ray depends on the nuclear charge $+Ze$ of the atom, which affects electron energy changes near the nucleus (p. 1084). The frequency, or wavelength λ, thus depends on the atomic number, Z.

X-ray Analysis

A special form of spectrometer was designed by Sir William Bragg for his experiments. The crystal was fixed on the table, and an X-ray beam, limited by lead shields A, B, was incident on the crystal at various glancing angles, θ (Fig. 43.4 (i)). An ionization chamber, Q, was used to measure the intensity of the X-rays reflected by the crystal. Q contained a heavy gas such as methyl iodide, and the intensity of the

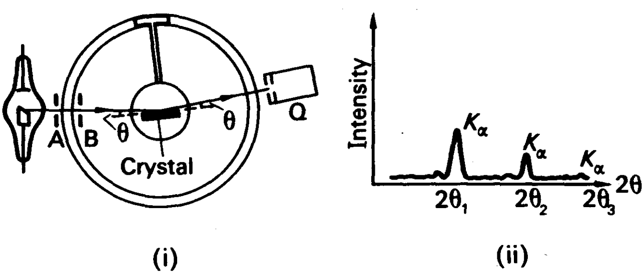


FIG. 43.4. X-ray spectrometer and results.

X-rays was proportional to the ionization current flowing, which was measured by means of an electrometer (not shown) connected to Q. The table and Q were geared so that Q turned through twice the angle of rotation of the crystal, and was always ready to measure the intensity of X-rays satisfying the law $2d \sin \theta = n\lambda$.

Typical results with particular parallel atomic planes in a crystal such as sylvine (KCl) or rocksalt (NaCl) are shown roughly in Fig. 43.4 (ii). A characteristic X-ray line such as K_α produces peaks of intensity at glancing angles θ_1, θ_2 and θ_3 for the first three orders. Measurement shows that $\sin \theta_1 : \sin \theta_2 : \sin \theta_3 = 1:2:3$, thus verifying Bragg's law, $2d \sin \theta = n\lambda$.

Crystal Atomic Spacing

Before the wavelength λ can be calculated, the distance d between consecutive parallel atomic planes is required. As an illustration of the calculation, consider the distance d between those atomic planes of a rock-salt crystal, NaCl, which are parallel to the face ABCD of a unit cell or cube of the crystal (Fig. 43.5). In this case $d = a$, the side of the cube. We thus require the distance a between consecutive atoms (ions) of sodium and chlorine in the crystal.

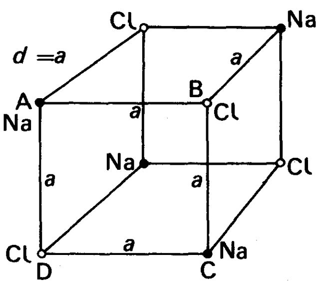


FIG. 43.5. Unit cell of rock salt.

The mass of one mole of sodium chloride is 58·5 g, the sum of the relative atomic masses of sodium and chlorine. This contains about 6×10^{23} molecules, Avogadro's constant. The mass of a molecule is thus $58{\cdot}5\ \mathrm{g}/6 \times 10^{23}$. Since the density of rocksalt is about $2{\cdot}2\ \mathrm{g\ cm^{-3}}$,

$$\text{volume occupied by 1 molecule (2 atoms)} = \frac{58{\cdot}5}{6 \times 10^{23} \times 2{\cdot}2}\ \mathrm{cm^3}.$$

$$\therefore \text{ volume associated with each atom} = \frac{58{\cdot}5}{2 \times 6 \times 10^{23} \times 2{\cdot}2}\ \mathrm{cm^3}.$$

$$\therefore \text{ separation of atoms} = \left[\frac{58{\cdot}5}{2 \times 6 \times 10^{23} \times 2{\cdot}2}\right]^{1/3}\ \mathrm{cm}.$$

$$= 2{\cdot}8 \times 10^{-8}\ \mathrm{cm}.$$

Thus if the first order diffraction image is obtained for a glancing angle θ of 5·4° for a particular X-ray wavelength λ, then

$$\lambda = 2d \sin \theta = 2 \times 2{\cdot}8 \times 10^{-8} \times \sin 5{\cdot}4^\circ$$

$$= 0{\cdot}5 \times 10^{-8}\ \mathrm{cm} = 0{\cdot}5\ \text{Å}.$$

Knowing λ, the atomic spacing d in other crystals can then be found, thus leading to analysis of crystal structure.

Moseley's Law

In 1914 Moseley measured the frequency ν of the characteristic X-rays from many metals, and found that, for a particular type of emitted X-ray such as K_α, the frequency ν varied in a regular way with the atomic number Z of the metal. When a graph of Z v. $\nu^{1/2}$ was plotted, an almost perfect straight line was obtained (Fig. 43.6). Moseley therefore gave an empirical relation, known as *Moseley's law*, between ν and Z as

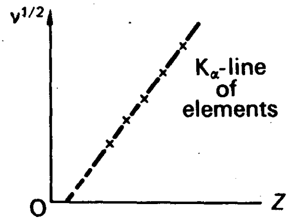


FIG. 43.6. Moseley's law.

$$\nu = a(Z-b)^2,$$

where a, b are constants.

Since the regularity of the graph was so marked, Moseley predicted the discovery of elements with atomic numbers 43, 61, 72 and 75, which were missing from the graph at that time. These were later discovered. He also found that though the atomic weights of iron, nickel and cobalt increased in this order, their positions from the graph were: iron ($Z = 26$), cobalt ($Z = 27$) and nickel ($Z = 28$). The chemical properties of the three elements agree with the order by atomic number and not by atomic weight. Rutherford's experiments on the scattering of α-particles (p. 1050) showed that the atom contained a central nucleus of charge $+Ze$ where Z is the atomic number, and Moseley's experiments confirm the importance of Z in atomic theory (see also p. 1090).

Continuous X-ray Background Radiation

The characteristic X-ray spectrum from a metal is usually superimposed on a background of continuous, or so-called 'white', radiation of small intensity. Fig. 43.7 illustrates the characteristic lines, K_α, K_β, of a metal and the continuous background of radiation for two values of p.d., 40000 and 32000 volts, across an X-ray tube. It should be

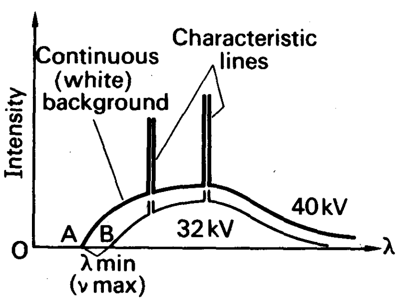


FIG. 43.7. X-ray characteristic lines and background.

noted that (i) the wavelengths of the characteristic lines are independent of the p.d.—they are characteristic of the metal, (ii) the background of continuous radiation has increasing wavelengths which slowly diminish in intensity, but as the wavelengths diminish they are cut off *abruptly*, as at A and B.

When the bombarding electrons collide with the metal atoms in the target, most of their energy is lost as heat. A little energy is also lost in the form of electromagnetic radiation. The existence of a sharp minimum wavelength at A or B can be explained only by the quantum theory. The energy of an electron before striking the metal atoms of the target is eV, where V is the p.d. across the tube. If a direct collision is made with an atom and *all* the energy is absorbed, then, on quantum theory, the X-ray quantum produced has maximum energy.

$$\therefore h\nu_{max} = eV$$

$$\therefore \nu_{max} = \frac{eV}{h} \quad . \quad . \quad . \quad . \quad (i)$$

$$\therefore \lambda_{min} = \frac{c}{\nu_{max}} = \frac{ch}{eV} \quad . \quad . \quad . \quad (ii)$$

Verification of Quantum Theory

These conclusions are borne out by experiment. Thus for a particular metal target, experiment shows that the minimum wavelength is obtained for p.d.s of 40 kV and 32 kV at glancing angles of about 3·0° and 3·8° respectively. The ratio of the minimum wavelengths is hence, from Bragg's law,

$$\frac{\lambda_1}{\lambda_2} = \frac{\sin 3{\cdot}0^\circ}{\sin 3{\cdot}8^\circ} = 0{\cdot}8 \text{ (approx.)}.$$

From (ii), $\lambda_{min} \propto 1/V$.

$$\therefore \frac{\lambda_1}{\lambda_2} = \frac{32}{40} = 0{\cdot}8.$$

With a tungsten target and a p.d. of 30 kV, experiment shows that a minimum wavelength of $0{\cdot}42 \times 10^{-8}$ cm is obtained, as calculated from values of d and θ. From (ii),

$$\therefore \lambda_{min} = \frac{ch}{eV} = \frac{3{\cdot}0 \times 10^8 \times 6{\cdot}6 \times 10^{-34}}{1{\cdot}6 \times 10^{-19} \times 30000} \text{ m}$$

$$= 0{\cdot}41 \times 10^{-10} \text{ m} = 0{\cdot}41 \times 10^{-8} \text{ cm},$$

using $c = 3{\cdot}0 \times 10^8$ m s^{-1}, $h = 6{\cdot}6 \times 10^{-34}$ J s, $e = 1{\cdot}6 \times 10^{-19}$ C, $V = 30000$ volts. This is in good agreement with the experimental result.

WAVE NATURE OF MATTER

Electron Diffraction

We have just seen how the wave nature of X-rays has been established by X-ray diffraction experiments. Similar experiments, first performed by Davisson and Germer, show that streams of *electrons* produce diffraction patterns and hence also exhibit wave properties. Electron diffraction is now as useful a research tool as X-ray diffraction.

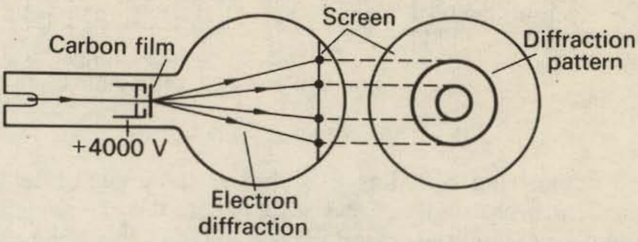


FIG. 43.8 (a). Electron diffraction tube.

A TELTRON tube available for demonstrating electron diffraction, is shown diagrammatically in Fig. 43.8 (a). A beam of electrons impinges on a layer of graphite which is extremely thin, and a diffraction pattern, consisting of rings, is seen on the tube face. Sir George Thomson first obtained such a diffraction pattern using a very thin gold film. If the voltage V on the anode is increased, the velocity, v, of the electrons is increased. The rings are then seen to become narrow, showing that the wavelength λ of the electron waves decreases with increasing v or increasing voltage V.

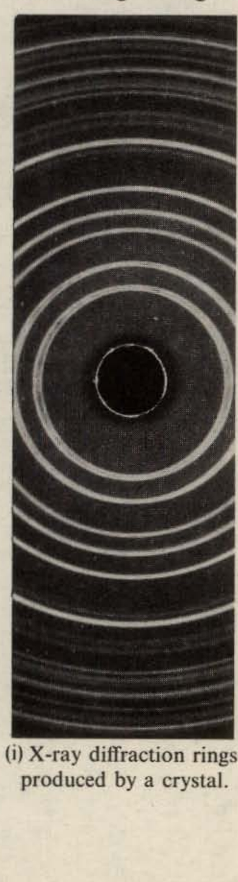
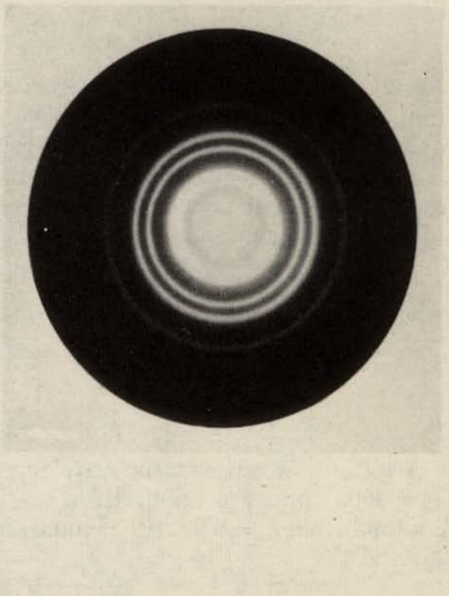


FIG. 43.8 (b). Similarity of Wave (i) and Particle (ii)



(i) X-ray diffraction rings produced by a crystal.

(ii) Electron diffraction rings produced by a thin gold film.

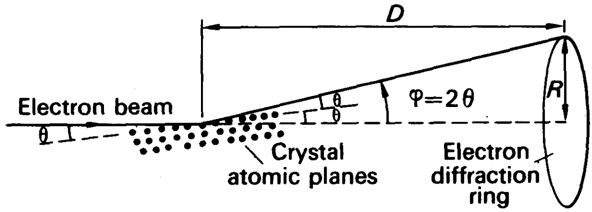


FIG. 43.9. Theory of diffraction experiment.

If a particular ring of radius R is chosen, the angle of deviation ϕ of the incident beam is given by $\phi = 2\theta$, where θ is the angle between the incident beam and the crystal planes. Fig. 43.9. Now $\tan\phi = R/D$, and if ϕ is small, $\phi = R/D$ to a good approximation. Hence $\theta = R/2D$. If the Bragg law is true for electron diffraction as well as X-ray diffraction, then, with the usual notation, $2d\sin\theta = n\lambda$.

$$\therefore \lambda \propto \sin\theta \propto \theta \propto R \quad . \quad . \quad . \quad . \quad \text{(i)}$$

On plotting a graph of R against $1/\sqrt{V}$ for different values of accelerating voltage V, a straight line graph passing through the origin is obtained. Now $\frac{1}{2}m_e v^2 = e\text{V}$, or $1/\sqrt{V} \propto 1/v$, where v is the velocity of the electrons accelerated from rest. Hence the electrons appear to act as waves whose wavelength is inversely-proportional to their velocity. This is in agreement with de Broglie's theory, now discussed.

De Broglie's Theory

In 1925, before the discovery of electron diffraction, de Broglie proposed that

$$\lambda = \frac{h}{p}, \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

where λ is the wavelength of waves associated with particles of momentum p, and h is *Planck's constant*, $6{\cdot}63 \times 10^{-34}$ joule second. The quantity h was first used by Planck in his theory of heat radiation and is a constant which enters into all branches of atomic physics. It is easy to see that de Broglie's relation is consistent with the experimental result (ii). The gain in kinetic energy is eV so that

$$\tfrac{1}{2}m_e v^2 = eV,$$

where v is the velocity of the electrons. Thus $v = \sqrt{2eV/m_e}$ and hence

$$p = m_e v = \sqrt{2eVm_e}$$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{h}{\sqrt{2eVm_e}} \propto V^{-1/2}.$$

We can now estimate the wavelength of an electron beam. Suppose $V = 3600$ volts. For an electron, $m = 9{\cdot}1 \times 10^{-31}$ kg, $e = 1{\cdot}6 \times 10^{-19}$ coulomb, and $h = 6{\cdot}6 \times 10^{-34}$ joule second.

$$\therefore \lambda = \frac{h}{\sqrt{2eVm_e}} = \frac{6{\cdot}6 \times 10^{-34}}{\sqrt{2 \times 1{\cdot}6 \times 10^{-19} \times 3600 \times 9{\cdot}1 \times 10^{-31}}}$$

$$= 2 \times 10^{-11} \text{ metres.}$$

This is about 30000 times smaller than the wavelength of visible light. On this account electron beams are used in *electron microscopes*. These instruments can produce resolving powers far greater than that of an optical microscope.

Wave Nature of Matter

Electrons are not the only particles which behave as waves. The effects are less noticeable with more massive particles because their momenta are generally much higher, and so the wavelength is correspondingly shorter. Since appreciable diffraction is observed only when the wavelength is of the same order as the grating spacing, the heavier particles, such as protons, are diffracted much less. Slow neutrons, however, are used in diffraction experiments, since the low velocity and high mass combine to give a momentum similar to that of electrons used in electron diffraction. The wave nature of α-particles is important in explaining α-decay.

PARTICLE NATURE OF WAVES

We have already seen that γ-rays behave as electromagnetic waves of very short wavelength. Now γ-rays can be detected by Geiger-Müller (GM) tubes and solid state detectors, where *individual* pulses are counted. Thus, on detection, γ-rays behave as *particles*. Other evidence for the particle nature of electromagnetic waves is given by the photoelectric effect, which we now discuss.

Photoelectricity

In 1888 Hallwachs discovered that an insulated zinc plate, negatively charged, lost its charge if exposed to ultra-violet light. Hertz had previously noticed that a spark passed more easily across the gap of an induction coil when the negative metal terminal was exposed to sunlight. Later investigators such as Lenard and others showed that electrons were ejected from a zinc plate when exposed to ultra-violet light. Light thus gives energy to the electrons in the surface atoms of the metal, and enables them to break through the surface. This is called the *photoelectric effect*.

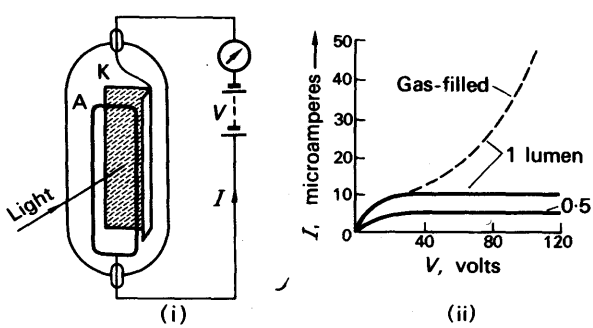


FIG. 43.10. (i) A photo-cell (photo-emissive). (ii) Characteristic.

Fig. 43.10 (i) is a diagram of a modern photoelectric cell. The cathode K is a V-shaped plate coated with caesium or some more complicated and very sensitive surface. In front of it is a wire ring, the anode A, which collects the photo-electrons. The heavy curve at (ii) in the figure shows how the current through the cell, I, varies with the potential difference across it, V. At first the current rises, but at a potential difference of about 30 volts is saturates. We suppose that the anode is then collecting all the electrons emitted by the cathode. This curve is drawn for a light-flux of 1 lumen upon the cathode (p. 562). If the flux is halved, the saturation current also falls by a half, showing that the number of electrons emitted per second is proportional to the light-flux falling upon the cathode.

Some photoelectric cells contain an inert gas—such as argon—at a pressure of a few millimetres of mercury. The current in such a cell does not saturate, because the electrons ionize the gas atoms by collision. Fig. 43.10 (ii). The greater the potential difference, the greater the kinetic energy of the electrons, and the more intense the ionization of the gas.

Photoelectric cells are used in photometry, in industrial control and counting operations, in television, and in many other ways. Their use in reproducing sound from film is explained in the Sound section of this book (p. 599).

Photo-voltaic Cells

Photoelectric cells of the kind we have just described are called photo-emissive cells, because in them light causes electrons to be emitted. Another type of cell is called photo-voltaic, because it generates an e.m.f. and can therefore provide a current without a battery. One form of such a cell consists of a copper disc, oxidized on one

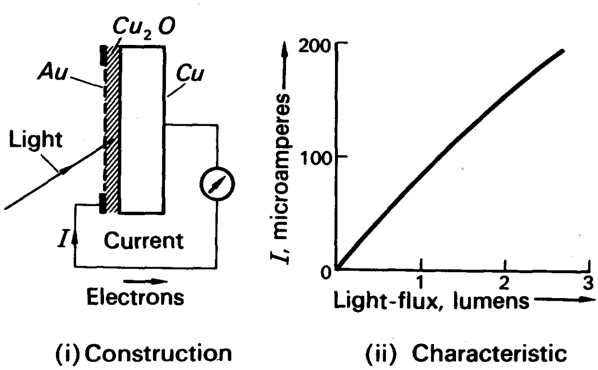


FIG. 43.11. A photo-voltaic cell.

face (Cu_2O/Cu), as shown in Fig. 43.11 (i). Over the exposed surface of the oxide a film of gold (Au) is deposited, by evaporation in a vacuum; the film is so thin that light can pass through it. When it does so it generates an e.m.f. in a way which we cannot describe here.

Photo-voltaic cells are sensitive to visible light. Fig. 43.11 (ii) shows how the current from such a cell, through a galvanometer of resistance about 100 ohms, varies with the light-flux falling upon it. The current is not quite proportional to the flux. Photo-voltaic cells are obviously convenient for photographic exposure meters, for measuring illumination in factories, and so on, but as measuring instruments they are less accurate than photo-emissive cells.

Photo-conductive Cells

A photo-conductive cell is one whose resistance changes when it is illuminated. A common form consists of a pair of interlocking comb-like electrodes made of gold (Au) deposited on glass (Fig. 43.12); over these a thin film of selenium (Se) is deposited. In effect, the selenium

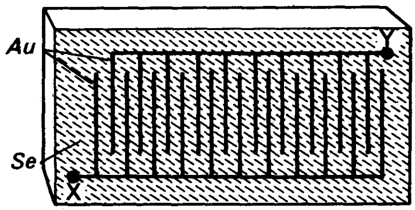


FIG. 43.12. A selenium cell.

forms a large number of strips, electrically in parallel; this construction is necessary because selenium has a very high resistivity (about 700 ohm m in the dark). The resistance between the terminals, XY, falls from about 10^7 ohms in the dark to about 10^6 ohms in bright light. In conjunction with valve amplifiers, photo-conductive cells were used as fire alarms during the last war. They were the first photo-cells to be discovered—in 1873—but they were the least useful. They are sluggish, taking about a second to respond fully to a change of illumination; and they show hysteresis—one change of illumination affects their response to the next.

Velocity of Photo-electrons

In 1902 Lenard found that the velocity of ejection of the electron from an illuminated metal was independent of the intensity of the particular incident monochromatic light. It appeared to vary only with the wavelength or frequency of the incident light, and above a particular wavelength no electrons were emitted. This was a very surprising result. It could not be explained on classical grounds, which predicts that the greater the light energy incident on the metal, the greater should be the energy of the liberated electrons, and that electrons should always be ejected, irrespective of the incident wavelength, if the incident energy is large enough.

Theories of Light. The Photon

About 1660 Newton had proposed a *corpuscular theory* of light, that is, light consists of particles or corpuscles, and he explained the phenomena of reflection and refraction by applying the laws of mechanics to the particles (p. 680). About the same period Huyghens proposed a *wave theory* of light, that is, light travels by the propagation of a wave or disturbance in the medium (p. 676), and this was applied with particular success to the phenomena of interference and diffraction. Newton's theory was abandoned soon after 1800 when Thomas Young revived interest in Huyghen's wave theory. Among other difficulties, Newton's theory led to the conclusion that the velocity of light in water was greater than in air, which was shown to be untrue experimentally.

In 1902 Planck had shown that the experimental observations in black-body radiation could be explained on the basis that the energy from the body was emitted in separate or discrete packets of energy, known as *quanta* of energy, of amounts $h\nu$, where ν is the frequency of the radiation and h is a constant known as *Planck's constant*. This is the *quantum theory of radiation.* With characteristic genius, Einstein asserted in 1905 that the unexpected experimental result of Lenard—that the energy of the ejected electron was independent of the intensity of the incident light and depended only on the frequency of the light—could be explained by applying a quantum theory of light. He assumed that light of frequency ν contains packets or quanta of energy $h\nu$. On this basis, light consists of *particles,* and these are called *photons.* The number of photons per unit area of cross-section of the beam of light per unit time is proportional to its intensity, but the energy of a photon is proportional to its frequency.

The minimum amount of work or energy to take a free electron out of the surface of a metal against the attractive forces of the positive ions is known as the *work function,* w_0, of the metal. When light of sufficiently high frequency is incident on the metal, an amount w_0 of the incident energy $h\nu$ is used to liberate the electron, leaving an excess energy $h\nu - w_0$, which is given to the ejected electron. The maximum kinetic energy, $\frac{1}{2}m_e v_{max}^2$, of the latter is thus, on Einstein's theory:

$$\tfrac{1}{2}m_e v_{max}^2 = h\nu - w_0. \qquad \text{(i)}$$

Millikan's Experiment

To test the linear relationship between the kinetic energy of the ejected electron and the frequency expressed in (i), Millikan carried out experiments in 1916 using the alkali metals lithium, sodium and potassium. These metals emit electrons when illuminated by ordinary (visible) light, and cylinders of them, A, B, C, were placed round a wheel W (Fig. 43.13). To avoid tarnishing and the formation of

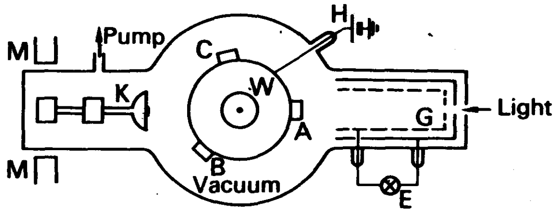


FIG. 43.13. Millikan's photoelectric experiment.

oxide films on the metal surface, which lead to considerable error, the metals were housed in a vacuum. Their surfaces were kept clean by a cutting knife K, which could be moved and turned by means of a magnet M outside.

The metal, A say, was kept at a variable positive potential by a battery H, and illuminated by a beam of monochromatic light of wavelength λ_1 from a spectrometer. Any photo-electrons emitted could reach a gauze cylinder G, which was connected to one side of an electrometer E whose other terminal was earthed, and a current I would then flow in E. When the potential of A is increased, G has an increasing negative potential relative to A, $-V$ say, and the current I then decreases. At some negative value, $-V_1$, the current becomes zero (Fig. 43.14 (i)). The potential of G is then the same as the other terminal of E, which is earthed, and the negative potential of G relative to A is thus now given numerically by the p.d. of the battery H. Millikan obtained variations of current, I, with monochromatic light of other wavelengths λ_2, λ_3, using light of constant intensity.

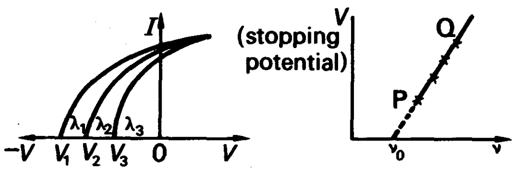


FIG. 43.14. Results of Millikan's experiment.

Deduction from Millikan's Results

The negative potential V of G relative to A when no electrons reach G is called the 'stopping potential' of G. In this case the maximum kinetic energy of the ejected electrons is just equal to the work eV done in moving against the opposing p.d. Thus

$$eV = h\nu - w_0. \qquad \text{(i)}$$

This is a linear relation between V and ν, and when the stopping potential was plotted against the frequency, a straight line PQ was obtained (Fig. 43.14 (ii)). Now from (i), the slope of the line is h/e, and knowing e, Millikan calculated h. The result was $6{\cdot}26 \times 10^{-34}$ joule second, which was very close to the value of h found from experiments on black-body radiation. This confirmed Einstein's photoelectric theory that light can be considered to consist of particles with energy $h\nu$.

In (i), we can write the work function energy w_0 as $h\nu_0$, where $\nu_0 = w_0/h$. Hence, for the electrons with maximum energy,

$$eV = \text{kinetic energy of electron} = h\nu - w_0 = h(\nu - \nu_0).$$

It then follows that no electrons are emitted from a metal when the incident light has a frequency less than ν_0. The magnitude of ν_0 is called the *threshold frequency* of the metal concerned, and is given by the intercept of PQ with the axis of ν (Fig. 43.14 (ii)).

EXAMPLE

Caesium has a work function of 1·9 electron-volts. Find (i) its threshold wavelength, (ii) the maximum energy of the liberated electrons when the metal is illuminated by light of wavelength $4{\cdot}5 \times 10^{-5}$ cm (1 electron-volt $= 1{\cdot}6 \times 10^{-19}$ J, $h = 6{\cdot}6 \times 10^{-34}$ J s, $c = 3{\cdot}0 \times 10^{8}$ m s^{-1}).

(i) The threshold frequency, ν_0, is given by $h\nu_0 = w_0 = 1{\cdot}9 \times 1{\cdot}6 \times 10^{-19}$ J. Now threshold wavelength, λ_0, $= c/\nu_0$

$$\therefore \lambda_0 = \frac{c}{w_0/h} = \frac{ch}{w_0}$$

$$= \frac{3 \times 10^{8} \times 6{\cdot}6 \times 10^{-34}}{1{\cdot}9 \times 1{\cdot}6 \times 10^{-19}}$$

$$= 6{\cdot}5 \times 10^{-7} \text{ m.}$$

(ii) Maximum energy of liberated electrons $= h\nu - w_0$, where ν is the frequency of the incident light. But $\nu = c/\lambda$.

$$\therefore \text{max. energy} = \frac{hc}{\lambda} - w_0$$

$$= \frac{6{\cdot}6 \times 10^{-34} \times 3 \times 10^{8}}{4{\cdot}5 \times 10^{-7}} - 1{\cdot}9 \times 1{\cdot}6 \times 10^{-19}$$

$$= 1{\cdot}4 \times 10^{-19} \text{ J.}$$

Duality

From what has been said, it is clear that particles can exhibit wave properties, and that waves can sometimes behave as particles. It would appear, therefore, that a paradox exists since wave and particle structure appear mutually exclusive.

Scientists gradually realized, however, that the dual aspect of wave-particle properties are completely general in nature. All physical entities can be described either as waves or particles; the description to choose is entirely a matter of convenience. The two aspects, wave and particle, are linked through the two relations

$$E = h\nu; \qquad p = h/\lambda.$$

On the left of each of these relations, E and p refer to a particle description. On the right, ν and λ refer to a wave description. Note that Planck's constant is the constant of proportionality in *both* these equations, a fact which can be predicted by Einstein's Special Theory of Relativity.

QUANTIZATION OF ENERGY

Energy of Atoms

The average energy of a monatomic molecule moving in a gas at room temperature is $\frac{3}{2}kT$ or $\frac{3}{2} \times 1{\cdot}4 \times 10^{-23} \times 300$ joule, which is $6{\cdot}3 \times 10^{-21}$ joule. Since 1 eV is $1{\cdot}6 \times 10^{-19}$ joule, this energy corresponds to about 0·04 eV. Thus when collisions between molecules take place, the energy exchange is of the order of 0·04 eV. In these conditions the collisions are *perfectly elastic*, that is, the internal energy of the atoms is not increased and all the energy remains in the form of translational kinetic energy.

In 1914, Franck and Hertz bombarded atoms by electrons of much higher energy, of the order of several electron volts. They used sodium vapour at a very low pressure of about 1 mm of mercury in a tube containing a heated tungsten filament F, a grid plate G, and a plate A (Fig. 43.15 (i)). Electrons were emitted from F, and the distance FG was arranged to be much greater than the mean free path of the electrons in the gas, in which case the electrons would make collisions with the atoms before reaching G. The p.d. V between F and G could be varied by the potentiometer S. The electrons emitted from F were accelerated to kinetic energies depending on the magnitude of V, measured by a voltmeter. A small p.d., less than 1 volt, was applied

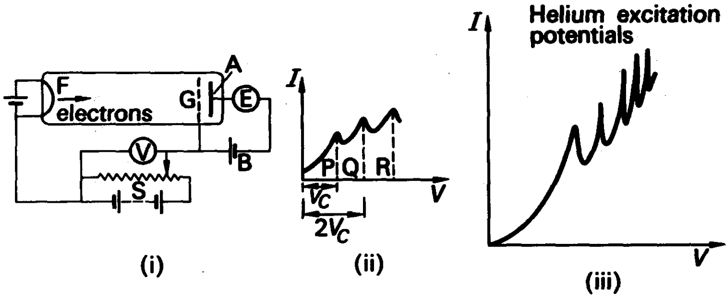


FIG. 43.15. Franck and Hertz experiment.

between A and G so that A was negative in potential relative to G. The plate A was close to G, and electrons reaching G and passing through to A were subjected to a retarding field. The number per second reaching A was measured by an electrometer E.

When the accelerating p.d. V between F and G was increased from zero, the current in E rose until the p.d. reached a value P (Fig. 43.15 (ii)). As V was increased further the current diminished to a minimum, rose again to a new peak at a higher p.d. Q, then diminished again and rose to another peak at a higher p.d. R. The p.d. V_c between successive peaks was found to be constant and equal to 2·10 volts for sodium vapour. Similar results were found for other gases.

Energy Levels

From the graph, it can be seen that the current begins to drop at the critical potential V_c. Here the electrons have an energy of V_c electron-volts or eV_c joules. This energy is just sufficient to raise the internal energy of the sodium atom by collision. Energies less than eV_c fail to increase the energy of the atom. After giving up this energy, the electrons then have insufficient energy to overcome the small retarding p.d. between G and A. Thus the current starts to fall. At a p.d. of $2V_c$ a dip again begins to form. This is due to the electrons giving up energy equal to $2eV_c$ to two atoms at separate collisions.

It thus appears that the energy of the atom cannot be increased unless the energy of the colliding particle is greater than V_c electron-volts. In this case an *inelastic* collision takes place. The atom now takes up an energy equal to eV_c joules, which the colliding particle loses. This process of increasing the energy of an atom is called *excitation*. The interval, V_c, between successive peaks of the graph is the *excitation potential* of the atom.

The results of the Franck-Hertz experiment show that the energy of an atom is constant unless the atom is given enough energy to raise this by a *definite* amount. *No intermediate energy change is allowed.* An atom, therefore, exists in one of a set of well defined *energy levels.* If helium, for example, is used in a Franck-Hertz tube, a graph shown in Fig. 43.15 (iii) is obtained. Here each peak corresponds to a different energy level of the atom. Thus a whole sequence of different energy levels can be found in the helium atom.

As we have seen, at ordinary temperatures, the thermal energy of molecules in a gas is insufficient to cause excitation. If the gas can be heated to an enormously high temperature, of the order of 100 000 K. the molecules can gain enough energy to cause excitation.

Energy Levels in Spectra

If a gas is excited by a high voltage to produce a discharge, and the light is examined in a spectrometer, an emission spectrum is seen. A number of gases such as neon produce a *line spectrum*, that is, the spectrum consists of a number of well defined lines, each having a particular wavelength or frequency. These lines are also experimental evidence for the existence of separate or 'quantized' energy levels in the atom, as we now explain.

As they move through the discharge in the gas, some electrons have sufficient energy to excite atoms to a higher energy level. We suppose that a given atom has a series of defined and discrete (separated) energy levels of the atom, E_0, E_1, E_2, and that no other or intermediate energy level is possible. The lowest energy level E_0 is called the *ground state energy.* All physical systems are in stable equilibrium in the lowest energy state. Thus once an atom has been excited to a higher energy level E_n, it will try to reduce its energy. The energy lost if the atom reverts directly to the ground state is $(E_n - E_0)$. *This energy is radiated in the form of a photon of electromagnetic radiation.* The energy of the photon is $h\nu$ where ν is the frequency of the radiation, and thus

$$h\nu = E_n - E_0.$$

It can now be seen that a number of frequencies ν of electromagnetic radiation may be produced from a hot gas in a discharge tube. Each frequency corresponds to a possible energy change in the atom. Sometimes it is possible for the energy to change back to the ground state *via* an intermediate energy level E_m. In this case two different frequencies ν_1, ν_2 are radiated which are given respectively by the equations:

$$h\nu_1 = E_n - E_m, \qquad h\nu_2 = E_m - E_0.$$

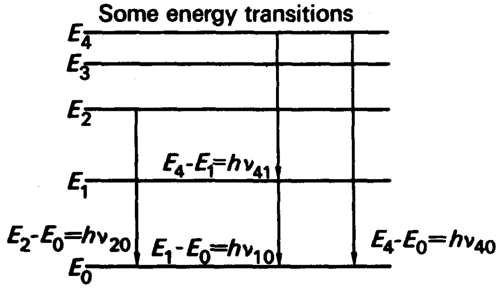


FIG. 43.16. Energy transitions.

It is customary to draw the energy levels on a vertical scale and to mark the *transitions* from one energy level to another with an arrow. Fig. 43.16.

Bohr's Theory of Hydrogen Atom

A *model* of the hydrogen atom was proposed by Bohr in 1911. This explained satisfactorily the existence of energy levels and the spectrum of the hydrogen atom. Later, however, it was shown that the model could not be applied to other atoms and a more satisfactory 'quantum theory' of the atom has since been developed.

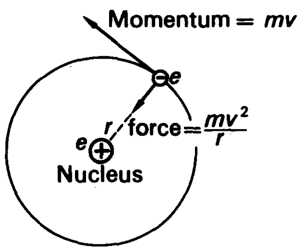


FIG. 43.17. Bohr's theory of hydrogen atom.

Bohr considered one electron of charge $-e$ and mass m, moving in a circular orbit round a central hydrogen nucleus of charge $+e$ (Fig. 43.17). The energy of the electron is partly kinetic and partly potential. If v is the velocity in the orbit, then

$$\text{kinetic energy} = \tfrac{1}{2}mv^2 \qquad . \quad . \quad . \quad \text{(i)}$$

If the electron is removed a very long way against the attraction of the nucleus, that is, to infinity, its potential energy is a maximum, but in calculations this energy is given the value of 'zero'. In practice, this means that the hydrogen atom then loses an electron completely and becomes an *ion.* Consider now an electron in the atom at a distance r from the nucleus. Since work is required to move the electron from this point to infinity against the attraction of the nucleus, it follows that the potential energy of the electron is *negative.* The potential due to the nuclear charge $+e$ at a distance r is given by $+e/4\pi\varepsilon_0 r$. Since this is the work done per unit charge, then

$$\text{potential energy of electron} = \frac{e}{4\pi\varepsilon_0 r} \times -e = \frac{-e^2}{4\pi\varepsilon_0 r} \quad . \quad \text{(ii)}$$

Hence, from (i) and (ii),

$$\text{total energy, } E = \tfrac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_0 r} \quad . \quad . \quad \text{(iii)}$$

For circular motion,

$$\frac{mv^2}{r} = \text{centripetal force} = \frac{e^2}{4\pi\varepsilon_0 r^2} \quad . \quad . \quad \text{(iv)}$$

$$\therefore \tfrac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r}$$

$$\therefore E = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 r} \quad . \quad . \quad \text{(v)}$$

If the electron can behave as a *wave*, it must be possible to fit a whole number of wavelengths around the orbit. In this case a standing wave pattern is set up and the energy in the wave is confined to the atom. A progressive wave would imply that the electron is moving from the atom and is not in a stationary orbit.

If there are n waves in the orbit and λ is the wavelength,

$$n\lambda = 2\pi r \quad . \quad . \quad . \quad . \quad \text{(vi)}$$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{mv} \quad . \quad . \quad . \quad . \quad \text{(vii)}$$

Hence, from (vi) and (vii),

$$\frac{nh}{2\pi} = mvr \quad . \quad . \quad . \quad . \quad \text{(viii)}$$

Now $mv \times r$ is the moment of momentum or *angular momentum* of the electron about the nucleus. Thus equation (viii) states that the *angular momentum is a multiple of* $h/2\pi$. This quantization of angular momentum, a key point in atomic theory, was first proposed by Bohr in 1913, twelve years before de Broglie proposed the wave-particle relation $\lambda = h/p$.

From (iv), $mv^2 r = e^2/4\pi\varepsilon_0$.

Hence, with (viii), $$v = \frac{2\pi e^2}{4\pi\varepsilon_0 nh} = \frac{e^2}{2\varepsilon_0 nh}$$

and thus $$r = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \quad . \quad . \quad . \quad . \quad . \quad \text{(ix)}$$

In classical physics, charges undergoing acceleration emit radiation and, therefore, lose energy (see p. 986). On this basis the electron would spiral towards the nucleus and the atom would collapse. Bohr, therefore, suggested (*a*) that in those orbits where the angular momentum is a multiple of $h/2\pi$ the energy is constant, (*b*) that the electron, or atom, can pass from one allowed energy level E_1 to another E_2 of smaller value, but not to a value between, and that the difference in energy is released in the form of radiation of energy $h\nu$, where ν is the frequency of the radiation emitted. Thus,

$$E_1 - E_2 = h\nu \quad . \quad . \quad . \quad . \quad \text{(x)}$$

From (v) and (ix),

$$\therefore E = -\frac{e^2}{8\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0} \times \frac{\pi m e^2}{\varepsilon_0 n^2 h^2}$$

$$= -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

$$\therefore E_1 - E_2 = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_2{}^2} - \frac{1}{n_1{}^2}\right) = h\nu$$

$$\therefore \nu = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_2{}^2} - \frac{1}{n_1{}^2}\right)$$

or

$$\frac{1}{\lambda} = \frac{\nu}{c} = \bar{\nu} = \frac{me^4}{8\varepsilon_0^2 c h^3}\left(\frac{1}{n_2{}^2} - \frac{1}{n_1{}^2}\right) \quad . \quad . \quad . \quad \text{(xi)}$$

where λ is the wavelength and $\bar{\nu}$, the ***wave number***, is the number of wavelengths per metre.

Spectral Series of Hydrogen

Before Bohr's theory of the hydrogen atom it had been found that the wave numbers of the hydrogen spectrum could be arranged in the form of a series, named after its discoverer. Among the wave numbers were:

1. *Lyman (ultra-violet) series*—$\bar{\nu} = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right)$.
2. *Balmer (visible) series*—$\bar{\nu} = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right)$.
3. *Paschen (infra-red) series*—$\bar{\nu} = R\left(\frac{1}{3^2} - \frac{1}{m^2}\right)$,

where R is a constant known as *Rydberg's constant* and m is an integer. From Bohr's formula in (xi), it follows that all the spectral series can

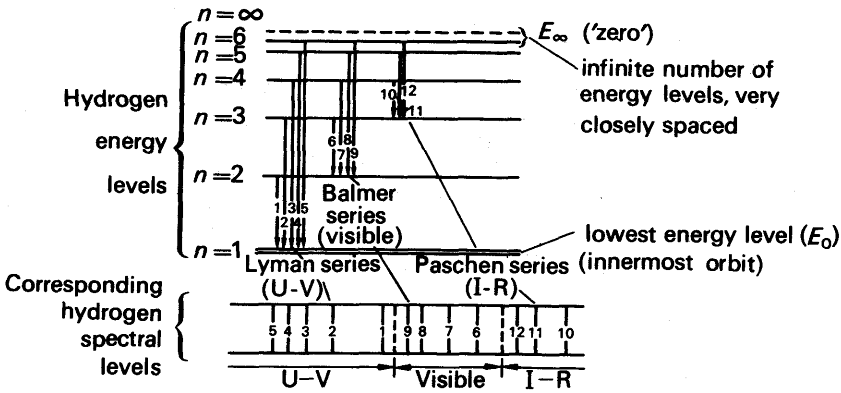


FIG. 43.18. Energy levels and spectra of hydrogen (*not to scale*).

be obtained simply by putting $n_2 = 1, 2, 3$ respectively and $n_1 = m$ (see Fig. 43.18). Moreover, (*a*) the agreement between the experimental and theoretical values of the wave numbers is excellent, (*b*) Rydberg's constant determined experimentally is $1{\cdot}09678 \times 10^7\ \mathrm{m}^{-1}$ and from $R = me^4/8\varepsilon_0^2ch^3$ it is $1{\cdot}09700 \times 10^7\ \mathrm{m}^{-1}$, (*c*) the radius of the first circular orbit calculated from $r = \varepsilon_0 h^2/\pi me^2$, equation (ix), is $5{\cdot}29 \times 10^{-11}$ m. This radius, which corresponds to $n = 1$ and is called the 'first Bohr radius', is the radius of the stable hydrogen atom, since the energy E is a minimum when $n = 1$, from $E = -me^4/8\varepsilon_0^2n^2h^2$. The value of r is in good agreement with the atomic radius calculated from the kinetic theory of gases.

Excitation and Ionization Potentials

Bohr's theory of the hydrogen atom was unable to predict the energy levels in complex atoms, which had many electrons. Quantum or wave mechanics, beyond the scope of this book, is used to explain the spectral frequencies of these atoms. The fundamental ideas of Bohr's theory, however, are still retained, for example, the angular momentum of the electron has quantum values and the energy levels of the atom have only allowed discrete values.

Generally, an atom is most stable when it has a minimum energy E_0, and it is then said to be in its *ground state* (p. 1084). If the atom absorbs energy, and the energy of the atom reaches one of its allowed values E_1, the atom is said to be in an *excited state.*

The energy required to raise an atom from its ground state to an excited state is called the *excitation energy* of the atom. If the energy is eV, where e is the electronic charge, V is known as the *excitation potential* of the atom.

If an atom is in its ground state with energy E_0, and absorbs an amount of energy eV which just removes an electron completely from the atom, then V is said to be the *ionization potential* of the atom. The potential energy of the atom is here denoted by E_∞, as the ejected electron is so far away from the attractive influence as to be, in effect, at infinity.

E_∞ is taken as the 'zero' energy of the atom, and its other values are thus negative. The ionization potential is given by $E_\infty - E_0 = e\overline{V}$, or by $-E_0 = eV$. Fig. 43.19 shows roughly the energy levels of an atom,

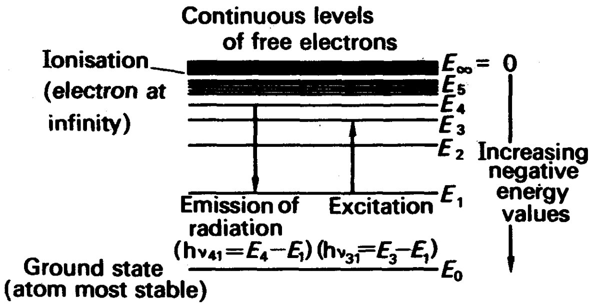


FIG. 43.19. Energy levels in the atom.

namely, its ground state E_0, its excited states, E_1, E_2, ..., and its ionization state, E_∞. It will be noted that the energy levels become more closely spaced at the higher excited states. Beyond the value E_∞, when the ejected electron is no longer under the attractive influence of the nucleus, the energy of the 'free' electron, $\frac{1}{2}mv^2$, can have one of a continuous range of energies, whereas inside the atom it could only have one of a number of separated allowed values.

Optical Spectra

As an illustration of Bohr's theory of energy levels, the ionization potential, E_∞, of helium is 24·6 eV (electron-volt). The ground state thus corresponds to an energy level of E_0 of $-24{\cdot}6$ eV. Suppose there is an excitation level, E_n, of helium of $-21{\cdot}4$ eV. Then if the helium atom is excited to this level and falls directly to the ground state, the frequency ν_n of the radiation emitted is given by

$$h\nu_n = E_n - E_0$$

$$\therefore \nu_n = \frac{[(-21{\cdot}4)-(-24{\cdot}6)] \times 1{\cdot}6 \times 10^{-19}}{6{\cdot}6 \times 10^{-34}} \text{ Hz},$$

since h = Planck's constant $= 6{\cdot}6 \times 10^{-34}$ J s and 1 eV $= 1{\cdot}6 \times 10^{-19}$ J. Thus the wavelength, λ_n, is given by

$$\lambda_n = \frac{c}{\nu_n} = \frac{3 \times 10^8 \times 6{\cdot}6 \times 10^{-34}}{3{\cdot}1 \times 1{\cdot}6 \times 10^{-19}}$$

$$= 3{\cdot}9 \times 10^{-7} \text{ m}.$$

This is a wavelength in the violet end of the spectrum. Fig. 43.19 illustrates the emission of radiation as the energy of the atom falls from one level to another.

Emission spectra are classified into *continuous*, *line* and *band* spectra. With few exceptions, incandescent solids and liquids produce a continuous spectrum, one in which all wavelengths are found over a wide range. Line spectra are obtained from *atoms* in gases such as hydrogen in a discharge tube, and the spectrum of a sodium salt vaporized in a Bunsen flame consists of two lines close together. Gases such as carbon dioxide in a discharge tube also produce a band spectrum, each band consisting of a series of lines very close together at the sharp edge or head of the band and farther apart at the other end or tail. Band spectra are essentially due to *molecules.* The different band heads in a band system are due to small allowed discrete energy changes in the vibrational states of the molecule. The fine lines in a given band are due to still smaller allowed discrete energy changes in the rotational states of the molecule.

Limitations of Bohr's Theory—Quantum Theory

The Bohr theory predicts successfully the energy levels of the hydrogen atom, as we have just seen. There are, however, some details of the spectrum which are not explained. When the radiating atoms are placed in a magnetic field each energy level splits into a number of energy levels near to the main energy level. This causes the spectral lines to split and is known as the *Zeeman effect.* The full quantum theory predicts this behaviour and utilizes four quantum numbers:

1. *Principal quantum number, n.* This specifies the main energy level and corresponds to the number n in the Bohr theory.
2. *Orbital quantum number, l.* This can take values $0, 1, \ldots, n-1$, and specifies the angular momentum of the electron. In the Bohr theory, one value of angular momentum corresponds to one value of energy. In quantum theory there are n values of l corresponding to the principal quantum number n. For $n = 1$, the ground state, quantum theory states that the angular momentum is zero, unlike the Bohr theory.
3. *Magnetic quantum number, m.* This can take values $-l, -(l-1) \ldots 0, 1, \ldots +l$. It specifies the orientation of the electron orbitals in a magnetic field and plays an important part in explaining the Zeeman effect.
4. *Spin quantum number,* m_s. It has already been mentioned that electrons are *spinning* and have a magnetic moment because of this spin.

The two possible values of m_s, $\pm\frac{1}{2}$, specify whether the spin is aligned with, or counter to, an applied magnetic field.

A full use of these quantum numbers gives a more accurate description of the fine details of the hydrogen spectrum.

Pauli Exclusion Principle

Pauli's exclusion principle is needed in describing the electron configuration of multi-electron atoms. It states that *no two electrons in an atom can have the same set of four quantum numbers.* In the lowest energy level, $n = 1$ and hence $l = 0$. This in turn restricts the value of m to zero. m_s, however, may take either of its two values $\pm\frac{1}{2}$ and so there are two electron states with $n = 1$. These two electrons form a *shell.* The two electrons in the shell have energy levels which are very near to one another. This first shell is called the K shell.

When $n = 2$, l may take the value 0 or 1 and m the value $-1, 0$, or 1. The following set of quantum numbers are, therefore, possible:

$$
\begin{array}{llll}
(2 & 0 & 0 & +\tfrac{1}{2}), \\
(2 & 1 & -1 & +\tfrac{1}{2}), \\
(2 & 1 & 0 & +\tfrac{1}{2}), \\
(2 & 1 & +1 & +\tfrac{1}{2}),
\end{array}
\qquad
\begin{array}{llll}
(2 & 0 & 0 & -\tfrac{1}{2}). \\
(2 & 1 & -1 & -\tfrac{1}{2}). \\
(2 & 1 & 0 & -\tfrac{1}{2}), \\
(2 & 1 & +1 & -\tfrac{1}{2}),
\end{array}
$$

There are thus 8 electron states with $n = 2$ and these form the L shell. It is left as an exercise to the reader to show that, for $n = 3$, there are 18 electrons. These form the M shell.

On this basis, hydrogen has 1 electron in the K shell; helium, $Z = 2$, has 2 electrons in the K shell. The maximum number of electrons in the K shell is 2, and hence lithium, $Z = 3$, has 2 electrons in the K shell and 1 electron in the L shell. As the atomic number increases, the electrons fill up the L shell. Fluorine, $Z = 9$, has 2 electrons in the K shell and 7 electrons in the L shell. Neon, $Z = 10$, has 2 electrons in the K shell and 8 electrons in the L shell, the maximum possible. Sodium, $Z = 11$, has 2 electrons in the K, 8 in the L, and 1 in the M shells. Chlorine, $Z = 17$, has 2 electrons in the K, 8 in the L, and 7 in the M shells. The sodium atom has thus 1 'available' electron in its outermost shell, that is, it is an electron donor; on the other hand, the chlorine atom can accommodate 1 more electron in its outermost shell, that is, it is an electron acceptor. When the very stable compound sodium chloride is formed, the outer electron in the sodium atom passes to the chlorine atom. The outermost shells of each atom are now complete, which is a very stable electron arrangement (see p. 848).

The chemical activity, or inactivity, of all elements is explained by their electron shell arrangement. Thus fluorine ($K = 2, L = 7$ electrons), chlorine ($K = 2, L = 8, M = 7$ electrons) and bromine ($K = 2, L = 8, M = 8$, N $= 7$ electrons) have each a vacancy for 1 electron in their respective outermost shells to make them complete, and have similar chemical properties. Lithium ($K = 2, L = 1$ electron), sodium ($K = 2, L = 8, M = 1$ electron) and potassium ($K = 2, L = 8, M = 8, N = 1$ electron) are all electron donors and have similar chemical activity. In contrast, helium ($K = 2$ electrons), and neon ($K = 2, L = 8$ electrons) have complete shells of electrons and are therefore chemically inactive.

The total number of electrons in all the shells is equal to Z, the atomic number, which is 1 for hydrogen and 92 for uranium.

EXERCISES 43

1. Describe the properties of X-rays and compare them with those of ultra-violet radiation. Outline the evidence for the wave nature of X-rays.

The energy of an X-ray photon is $h\nu$ joules where $h = 6{\cdot}63 \times 10^{-34}$ J s and ν is the frequency in hertz (cycles per second). X-rays are emitted from a target bombarded by electrons which have been accelerated from rest through 10^5 V. Calculate the minimum possible wavelength of the X-rays assuming that the corresponding energy is equal to the whole of the kinetic energy of one electron. (Charge of an electron $= 1{\cdot}60 \times 10^{-19}$ C; velocity of electromagnetic waves *in vacuo* $= 3{\cdot}00 \times 10^8$ m s^{-1}.) (*O. & C.*)

2. In an experiment on the photoelectric effect using radiation of wavelength $4{\cdot}00 \times 10^{-7}$ m the maximum electron energy was observed to be $1{\cdot}40 \times 10^{-19}$ joule. With radiation of wavelength $3{\cdot}00 \times 10^{-7}$ m the maximum energy was $3{\cdot}06 \times 10^{-19}$ joule. Derive a value for Planck's constant.

Mention one other physical phenomenon involving Planck's constant. (Velocity of light $= 3{\cdot}00 \times 10^8$ m s^{-1}.) (*N.*)

3. Describe briefly experiments to demonstrate *three* of the following:

(*a*) That gases absorb strongly some of the characteristic radiations of their emission spectra.

(*b*) That light waves are transverse.

(*c*) That X-radiation does not consist of electrically charged particles.

(*d*) That X-radiation is more strongly absorbed by a sheet of lead than by a sheet of aluminium of the same thickness. (*O. & C.*)

4. Describe a modern form of X-ray tube and explain its action.

Outline the evidence for believing (*a*) that X-rays are an electromagnetic radiation, (*b*) that wavelengths in the X-ray region are of the order of 10^{-3} times those of visible light. (*O.*)

5. Write down Einstein's equation for photoelectric emission. Explain the meaning of the terms in the equation and discuss its significance.

Describe briefly how Einstein's equation may be verified experimentally.

An effective point source emits monochromatic light of wavelength 4500 Å at a rate of 0·11 watt. How many photons leave the source per second? Light from the source is emitted uniformly in all directions and falls normally on the cathode of area π cm^2 of a photocell at a distance of 50 cm from the source. Calculate the photoelectric current, assuming 10 per cent of the photons incident on the cathode liberate electrons. (Planck's constant $= 6{\cdot}6 \times 10^{-34}$ J s; charge of electron $= 1{\cdot}6 \times 10^{-19}$ C.) (*O. & C.*)

6. Describe the atomic processes in the target of an X-ray tube whereby X-ray line spectra are produced. Determine the ratio of the energy of a photon of X-radiation of wavelength 1 Å to that of a photon of visible radiation of wave-length 5000 Å. Why is the potential difference applied across an X-ray tube very much greater than that applied across a sodium lamp producing visible radiation? (*N.*)

7. Draw a labelled diagram of some form of X-ray tube and of its electrical connexions when in actual use.

Electrons starting from rest and passed through a potential difference of 1000 volts are found to acquire a velocity of $1{\cdot}88 \times 10^7$ m s^{-1}. Calculate the ratio of the charge to the mass of the electron in coulombs per kg. (*N.*)

8. You are provided with a glass tube containing an electrode at each end, an exhaust pump, and a source of high potential. Under what conditions (*a*) does the gas within the tube become a relatively good conductor, (*b*) is a beam of electrons (cathode rays) produced within the tube?

What modifications must be made in the tube in order that a strong beam of X-rays may be produced? What would happen in each case if the potential applied was increased?

What experiment would you perform to show the effect of a magnetic field on the conducting particles in (*a*) and (*b*) and on a beam of X-rays? State the result you would expect. (*L.*)

9. When light is incident in a metal plate electrons are emitted only when the frequency of the light exceeds a certain value. How has this been explained?

The maximum kinetic energy of the electrons emitted from a metallic surface is $1{\cdot}6 \times 10^{-19}$ joule when the frequency of the incident radiation is $7{\cdot}5 \times 10^{14}$ Hz. Calculate the minimum frequency of radiation for which electrons will be emitted. Assume that Planck's constant $= 6{\cdot}6 \times 10^{-34}$ J s. (*N.*)

10. Describe a modern form of X-ray tube and explain briefly the energy changes that take place while it is operating.

Calculate the energy in electron-volts of a quantum of X-radiation of wavelength 1·5 Å.

An X-ray tube is operated at 8000 volts. Why is there no radiation of shorter wavelength than that calculated above (while there is a great deal of longer wavelength)? When the voltage is increased considerably above this value why does the spectrum of the radiation include a few (*but only a few*) sharp strong lines, the wavelengths of which depend on the material of the target? (Take $e = 1{\cdot}6 \times 10^{-19}$ C; $h = 6{\cdot}5 \times 10^{-34}$ J s; $c = 3 \times 10^8$ m s^{-1}.) (*O.*)

11. How are X-rays produced, and how are their wavelengths determined?

Discuss briefly the origin of the lines in the spectrum produced by an X-ray tube that are characteristic of the target metal.

Give a brief account of Moseley's work and the part it played in establishing the idea of atomic number. (*O.*)

12. Describe and explain one experiment in which light exhibits a wave-like character and one experiment which illustrates the existence of photons.

Light of frequency $5{\cdot}0 \times 10^{14}$ Hz liberates electrons with energy $2{\cdot}31 \times 10^{-19}$ joule from a certain metallic surface. What is the wavelength of ultra-violet light which liberates electrons of energy $8{\cdot}93 \times 10^{-19}$ joule from the same surface? (Take the velocity of light to be $3{\cdot}0 \times 10^8$ m s^{-1}, and Planck's constant (h) to be $6{\cdot}62 \times 10^{-34}$ J s.) (*L.*)

13. What are the chief characteristics of a line spectrum? Explain how line spectra are used in analysis for the identification of elements.

Fig. 43.20, which represents the lowest energy levels of the electron in the

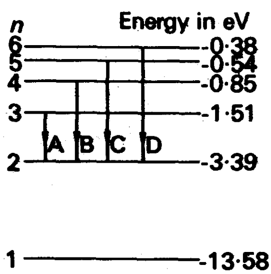


FIG. 43.20.

hydrogen atom, specifies the value of the principal quantum number n associated with each state and the corresponding value of the energy of the level, measured in electron volts. Work out the wavelengths of the lines associated with the transitions A, B, C, D marked in the figure. Show that the other transitions that can occur give rise to lines that are either in the ultra-violet or the infra-red regions of the spectrum. (Take 1 eV to be $1{\cdot}6 \times 10^{-19}$ J; Planck's constant h to be $6{\cdot}5 \times 10^{-34}$ J s; and c, the velocity of light *in vacuo*, to be 3×10^8 m s^{-1}.) (*O.*)

14. Einstein's equation for the photoelectric emission of electrons from a metal surface can be written $hv = \frac{1}{2}mv^2 + \phi$, where ϕ is the work function of the metal, and consistent energy units are used for each term in the equation. Explain briefly the physical process that this equation represents. Outline an experiment by which you could determine the values of h (or of h/e) and ϕ.

For caesium the value of ϕ is $1{\cdot}35$ electron-volts. (*a*) What is the longest wavelength that can cause photo-electric emission from a caesium surface? (*b*) What is the minimum velocity with which photoelectrons will be emitted from a caesium surface illuminated with light of wavelength 4000 Å? (*c*) What potential difference will just prevent a current passing through a caesium photocell illuminated with light of 4000 Å wavelength? (*O.*)

Summary of C.G.S. and SI units

QUANTITY AND SYMBOL	C.G.S UNIT	SI UNIT	RELATIONSHIP
mass (m)	gramme (g)	**kilogramme (kg)**	1000 g = 1 kg
length (l)	centimetre (cm)	**metre (m)**	100 cm = 1 m
time (t)	second(s)	**second (s)**	

MECHANICS, FLUIDS

Linear motion			
velocity (v)	$\mathrm{cm\,s^{-1}}$	$\mathbf{m\,s^{-1}}$	$1\ \mathrm{cm\,s^{-1}} = 10^{-2}\ \mathrm{m\,s^{-1}}$
acceleration (a)	$\mathrm{cm\,s^{-2}}$	$\mathbf{m\,s^{-2}}$	$1\ \mathrm{cm\,s^{-2}} = 10^{-2}\ \mathrm{m\,s^{-2}}$
force (F)	dyne (dyn)	**newton (N)**	$1\ \mathrm{dyn} = 10^{-5}\ \mathrm{N}$
momentum (p)	$\mathrm{g\,cm\,s^{-1}}$	$\mathbf{kg\,m\,s^{-1}}$	$1\ \mathrm{g\,cm\,s^{-1}} = 10^{-5}\ \mathrm{kg\,m\,s^{-1}}$
work, energy (W)	erg	**joule (J)**	$1\ \mathrm{erg} = 10^{-7}\ \mathrm{J}$
power (P)	$\mathrm{erg\,s^{-1}}$	**watt (W)**	$1\ \mathrm{erg\,s^{-1}} = 10^{-7}\ \mathrm{W}$
Rotational motion			
angular velocity (ω)	$\mathrm{rad\,s^{-1}}$	$\mathbf{rad\,s^{-1}}$	
angular acceleration ($d\omega/dt$)	$\mathrm{rad\,s^{-2}}$	$\mathbf{rad\,s^{-2}}$	
moment of inertia (I)	$\mathrm{g\,cm^2}$	$\mathbf{kg\,m^2}$	$1\ \mathrm{g\,cm^2} = 10^{-7}\ \mathrm{kg\,m^2}$
couple or torque (T)	dyn cm	**N m**	$1\ \mathrm{dyn\,cm} = 10^{-7}\ \mathrm{N\,m}$
angular momentum (L)	$\mathrm{g\,cm^2\,s^{-1}}$	$\mathbf{kg\,m^2\,s^{-1}}$	$1\ \mathrm{g\,cm^2\,s^{-1}} = 10^{-7}\ \mathrm{kg\,m^2\,s^{-1}}$
Fluids			
volume (V)	$\mathrm{cm^3}$	$\mathbf{m^3}$	$1\ \mathrm{cm^3} = 10^{-6}\ \mathrm{m^3}$
density (ρ)	$\mathrm{g\,cm^{-3}}$	$\mathbf{kg\,m^{-3}}$	$1\ \mathrm{g\,cm^{-3}} = 10^3\ \mathrm{kg\,m^{-3}}$
pressure (p)	$\mathrm{dyn\,cm^{-2}}$	$\mathbf{N\,m^{-2}}$	$1\ \mathrm{dyn\,cm^{-2}} = 10^{-1}\ \mathrm{N\,m^{-2}}$

length	1 micron (μm) = 10^{-6} m; 1 nanometre (nm) = 10^{-9} m; 1 Angstrom unit, Å = 10^{-10} m = 10^{-1} nm
time	1 millisecond (ms) = 10^{-3} s; 1 microsecond (μs) = 10^{-6} s; 1 nanosecond (ns) = 10^{-9} s
volume	1 c.c. = 1 cm^3 = 10^{-6} m^3; 1 litre = 10^{-3} m^3
density	water (ρ_W) = 1000 kg m^{-3}; mercury (ρ_{Hg}) = 13 600 kg m^{-3}; air at s.t.p. (ρ_{air}) = 1·29 kg m^{-3}

gravitational	$g = 981$ cm s^{-2} = 9·81 m s^{-2} = 10 m s^{-2} (*approx.*)
	$G = 6{\cdot}7\times10^{-8}$ dyn cm^2 g^{-2} = $6{\cdot}7\times10^{-11}$ N m^2 kg^{-2} (*approx.*)
force	1 kgf = 9·8 N (*approx.*) = 10 N (*approx.*)
power	1 h.p. = 746 W = $\frac{3}{4}$ kW (*approx.*)
pressure	1 bar = 10^6 dyn cm^{-2} = 10^5 N m^{-2} = 750 torr (mmHg) (*approx.*)
	standard atmospheric pressure = 760 torr = $1{\cdot}01325\times10^5$ N m^{-2} (*exact*)

PROPERTIES OF MATTER

QUANTITY AND SYMBOL	C.G.S. UNIT	SI UNIT	RELATIONSHIP
surface tension coefficient (γ)	dyn cm^{-1}	**N m^{-1}**	1 dyn cm^{-1} = 10^{-3} N m^{-1}
modulus of elasticity (E, K or G)	dyn cm^{-2}	**N m^{-2}**	1 dyn cm^{-2} = 10^{-1} N m^{-2}
viscosity coefficient (η)	poise	**N s m^{-2}**	1 dyn s cm^{-2} = 10^{-1} N s m^{-2}

γ (water–air at 15°C) = 74 dyn cm^{-1} = $7{\cdot}4\times10^{-2}$ N m^{-1}
E (steel) = $2{\cdot}1\times10^{12}$ dyn cm^{-2} = $2{\cdot}1\times10^{11}$ N m^{-2}
η (air at 15°C) = $1{\cdot}8\times10^{-4}$ poise = $1{\cdot}8\times10^{-5}$ N s m^{-2}

HEAT

quantity of heat (Q)	calorie (cal)	**joule (J)**	1 cal = 4·2 J (4·1868 J = 1 cal$_{IT}$
	kilocal	**kilojoule (kJ)**	1 kcal = 4·2 kJ
temperature (t)	°C	**K**	T(K) = 273·15 + t(°C)
temperature change (t)	deg C (°C)	**K**	1 deg C = 1 K
heat capacity (C)	cal deg C^{-1}	**J K^{-1}**	1 cal deg C^{-1} = 4·2 J K^{-1}
specific heat capacity (c)	cal g^{-1} deg C^{-1}	**J kg^{-1} K^{-1}**	1 cal g^{-1} deg C^{-1} = 4·2 kJ kg^{-1} K^{-1}
specific latent heat (l)	cal g^{-1}	**J kg^{-1}**	1 cal g^{-1} = 4·2 kJ kg^{-1}
linear, cubic expansivity (α, γ)	deg C^{-1}	**K^{-1}**	deg C^{-1} = K^{-1}
gas constant per unit mass	cal g^{-1} °K^{-1}	**J kg^{-1} K^{-1}**	1 cal g^{-1} °K^{-1} = 4·2 kJ kg^{-1} K^{-1}
molar gas constant	cal g-mol^{-1} °K^{-1}	**J mol^{-1} K^{-1}**	1 cal g-mol °K^{-1} = 4·2 J mol^{-1} K^{-1}
thermal conductivity (k)	cal s^{-1} cm^{-1} °C^{-1}	**W m^{-1} K^{-1}**	1 cal s^{-1} cm^{-1} °K^{-1} = 420 W m^{-1} K^{-1}
radiation (heat per second)	cal s^{-1}	**W**	1 cal s^{-1} = 4·2 W

	WATER (*approximate values*) C.G.S.	WATER SI	COPPER (*approximate values*) C.G.S.	COPPER SI
specific heat capacity	1 cal g^{-1} deg C^{-1}	**4·2 kJ kg^{-1} K^{-1}**	0·095 cal g^{-1} deg C^{-1}	**0·4 kJ kg^{-1} K^{-1}**
specific latent heat of fusion	80 cal g^{-1}	**336 kJ kg^{-1}**	49 cal g^{-1}	**206 kJ kg^{-1}**
specific latent heat of vaporisation	540 cal g^{-1}	**2268 kJ kg^{-1}**	1250 cal g^{-1}	**5250 kJ kg^{-1}**
thermal conductivity (20°C)	$1{\cdot}4\times10^{-3}$ cal s^{-1} cm^{-1} °C^{-1}	**0·6 W m^{-1} K^{-1}**	0·91 cal s^{-1} cm^{-1} °C^{-1}	**380 W m^{-1} K^{-1}**
expansivity (20°C)	(γ) 2×10^{-4} deg C^{-1}	**2×10^{-4} K^{-1}**	(α) 17×10^{-6} deg C^{-1}	**17×10^{-6} K^{-1}**

	C.G.S.	SI
gas constant per unit mass, hydrogen	1 cal g^{-1} K^{-1}	**4·15 kJ kg^{-1} K^{-1}**
molar gas constant	2 cal g-mol^{-1} deg C^{-1}	**8·3 J mol^{-1} K^{-1} = 8·3 kJ kmol^{-1} K^{-1}**
Avogadro constant (N_A or L)	$6{\cdot}02\times10^{23}$ g-mol^{-1}	**$6{\cdot}02\times10^{23}$ mol^{-1} = $6{\cdot}02\times10^{26}$ kmol^{-1}**
Stefan constant (σ)	$5{\cdot}7\times10^{-5}$ erg s^{-1} cm^{-2} K^{-4}	**$5{\cdot}7\times10^{-8}$ W m^{-2} K^{-4}**

ELECTRICITY

QUANTITY AND SYMBOL	C.G.S. UNIT	SI UNIT	RELATIONSHIP
electric charge (Q)	e.s.u./e.m.u.	**coulomb (C)**	1 e.s.u. = $1/(3\times10^9)$C
permittivity (ε)	e.s.u.	**F m^{-1}**	1 e.s.u. = $8{\cdot}85\times10^{-12}$ F m^{-1}
electric intensity (E)	e.s.u.	**V m^{-1} or N C^{-1}**	1 e.s.u. = 3×10^4 V m^{-1}
electric potential, p.d. (V)	e.s.u.	**V**	{1 e.s.u. = 300 V; 1 e.m.u. = 10^{-8} V}
capacitance (C)	e.s.u.	**F**	1 e.s.u. = $1/(9\times10^{11})$ F
surface density (σ)	e.s.u.	**C m^{-2}**	1 e.s.u. = $1/(3\times10^5)$ C m^{-2}
current (I)	e.m.u.	**A**	1 e.m.u. = 10 A
resistance (R)	e.m.u.	**Ω**	1 e.m.u. = 10^{-9}Ω
resistivity (ρ)	Ω cm	**Ω m**	1 Ω cm = 10^{-2} Ω m
electrochem. equivalent (z)	g C^{-1}	**kg C^{-1}**	1 g C^{-1} = 10^{-3} kg C^{-1}
magnetic flux density (B) or magnetic induction	gauss	**tesla, T (= Wb m^{-2})**	1 gauss = 10^{-4} T
magnetic flux (Φ)	maxwell	**Wb**	1 maxwell = 10^{-8} Wb
permeability (μ)	e.m.u.	**H m^{-1}**	1 e.m.u. = $4\pi\times10^{-7}$ H m^{-1}
self inductance (L)	e.m.u.	**H**	1 e.m.u. = 10^{-9} H
magnetising field strength (H)	oersted (Oe)	**A m^{-1}**	1 Oe = $(10^3/4\pi)$ A m^{-1}
magnetic moment (m)	e.m.u.	**A m^2**	1 e.m.u. = 10^{-3} A m^2

(*Approximate values*)

permittivity of vacuum, $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$ F m^{-1}

permeability of vacuum, $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1} (*exact*)

resistivity of copper = $1{\cdot}7 \times 10^{-6}\ \Omega$ cm = $1{\cdot}7 \times 10^{-8}\ \Omega$ m

e.c.e of hydrogen = $1{\cdot}1 \times 10^{-5}$ g C^{-1} = $1{\cdot}1 \times 10^{-8}$ kg C^{-1}

earth's horizontal component field = 0·2 gauss = 2×10^{-5} T

moving-coil meter field = 5000 gauss = 0·5 T

OPTICS

QUANTITY AND SYMBOL	C.G.S. UNIT	SI UNIT	RELATIONSHIP
luminous flux (Φ)	lumen (lm)	**lumen (lm)**	
luminous intensity (I)	c.p.	**candela (cd)**	1 c.p. = 1 cd (*approx.*)
illumination (E)	cm-candle	**lux**	1 cm-candle = 10^4 lux
luminance (L)	lm cm^{-2}	**lm m^{-2}**	1 lm cm^{-2} = 10^{-4} lm m^{-2}

SOUND

frequency (f)	Hz (c.p.s.)	**Hz**	
intensity (I)	W cm^{-2}	**W m^{-2}**	1 W cm^{-2} = 10^{-4} W m^{-2}

speed of light in vacuo = 3×10^{10} cm s^{-1} = 3×10^8 m s^{-1}

speed of sound in air at 0°C = 33 200 cm s^{-1} = 332 m s^{-1}

threshold of hearing = 10^{-16} W cm^{-2} = 10^{-12} W m^{-2}

wavelength of sodium (yellow) light = $5{\cdot}89 \times 10^{-5}$ cm = $5{\cdot}89 \times 10^{-7}$ m = 589 nm

SOME ATOMIC CONSTANTS (*approximate values*)

	C.G.S.	SI
electron charge (e)	$4{\cdot}8 \times 10^{-10}$ e.s.u.; $1{\cdot}6 \times 10^{-20}$ e.m.u.	**$1{\cdot}6 \times 10^{-19}$ C**
electron rest mass (m_e)	$9{\cdot}1 \times 10^{-28}$ g	**$9{\cdot}1 \times 10^{-31}$ kg**
electron specific charge (e/m_e)	$5{\cdot}3 \times 10^{17}$ e.s.u. g^{-1}; $1{\cdot}76 \times 10^{7}$ e.m.u. g^{-1}	**$1{\cdot}76 \times 10^{11}$ C kg^{-1}**
proton rest mass (m_p)	$1{\cdot}7 \times 10^{-24}$ g	**$1{\cdot}7 \times 10^{-27}$ kg**
proton specific charge (e/m_p)	9600 e.m.u. g^{-1}	**$9{\cdot}6 \times 10^{7}$ C kg^{-1}**
speed of e–m waves (c)	$3{\cdot}0 \times 10^{10}$ cm s^{-1}	**$3{\cdot}0 \times 10^{8}$ m s^{-1}**
Planck constant (h)	$6{\cdot}6 \times 10^{-27}$ erg s	**$6{\cdot}6 \times 10^{-34}$ J s**
electron-volt (eV)	$1{\cdot}6 \times 10^{-12}$ erg	**$1{\cdot}6 \times 10^{-19}$ J**
atomic mass unit (a.m.u.)	$1{\cdot}66 \times 10^{-24}$ g	**$1{\cdot}66 \times 10^{-27}$ kg**

References

Symbols, Signs and Abbreviations—The Royal Society.
Physico-Chemical Quantities and Units—Prof. M. L. McGlashan, Royal Institute of Chemistry.
The Use of SI Units—PD5686—British Standards Institution.
Changing to the Metric System—National Physical Laboratory—HMSO.
Signs, Symbols and Abbreviations—Association for Science Education.

Answers to Exercises

MECHANICS

EXERCISES 1 (p. 32)

1. LT^{-1}. **2.** MLT^{-2}. **3.** (i) scalar, (ii) vector, (iii) scalar, (iv) vector. **4.** mass × velocity. **5.** momentum, energy. **6.** total linear momentum, external. **7.** 1 joule. **8.** 10 N (approx.). **9.** vector. **10.** rate. **11.** *C*. **12.** *B*. **13.** *D*. **14.** *A*. **15.** (i) 5 s, (ii) 62·5 m, (iii) 18 m s^{-1}. **16.** (i) 4 s, (ii) 20 m, (iii) 10, 10 J. **17.** 19·8 km h^{-1}, N. 30·5° W. **18.** (i) 100, (ii) 500 J. **19.** (i) 5·2 m s^{-1}, loss = 58 J, (ii) 1·2 s^{-1}, loss = 314 J. **20.** 26·5 km h^{-1}, S. 41° W., 7·6 km. **21.** (i) $1\frac{3}{7}$ m s^{-1}, (ii) $1\frac{3}{7}$ m s^{-2}, (iii) 86 J. **22.** 10·5° from vertical. **23.** $2E/103$. **24.** (*a*) 10/3 N, (*b*) 5/9 W, (*c*) 5/18 W. **25.** 167 kgf; 83330 J. **26.** 21675 m, 3,125 m, 25 s; 465 m s^{-1}. **27.** 1/3. **28.** (*a*) ML^2T^{-2}, (*b*) a: ML^5T^{-2}, b: L^3. **29.** 14·4 minutes, 8 km, 37° S. of E. **30.** $v/2$ at 60° to initial velocity of first sphere. **31.** 2 m s^{-1}. **32.** 800 kgf, 22 kW. **33.** 70% 0·023. **34.** $22·5 \times 10^4$ N m^{-2}.

EXERCISES 2 (p. 69)

1. centripetal. **2.** middle. **3.** $g = GM/r^2$. **4.** end. **5.** 1/distance^2. **6.** 24 h. **7.** *B*. **8.** *D*. **9.** *C*. **10.** *D*. **11.** (i) 2 rad s^{-1}. (ii) 9·6 kgf. **12.** (i) 11·8 kgf, (ii) 32°. **13.** 42°, 1555 kgf. **14.** 22·4, 6·4 kgf. **15.** (*a*) mgl, (*b*) $\sqrt{2gl}$, (*c*) $2g$ up, (*d*) 3 mg. **16.** Break when stone vertically below point of suspension; 7·7 rad s^{-1}; 122 cm from point below point of suspension. **17.** (i) 1/20 s, (ii) 0, $3200\pi^2$, (iii) $40\sqrt{3}\pi$, 0. **18.** 101 cm, (i) 0, $2\pi^2$ cm s^{-2}, (ii) 2π cm s^{-1}, 0, (iii) $\sqrt{3}\pi$ cm s^{-1}, π^2 cm s^{-2}. **19.** 0·32 s. **20.** 1·6 s. **21.** 10 m s^{-2}, 4·5 m. **22.** (*a*) 2·5 m s^{-1}, (*b*) 790 m s^{-2}. **23.** 6·3 s. **26.** 1·6 Hz. **27.** (*a*) $4\pi^2$, (*b*) $2\sqrt{3}\pi^2$. (*c*) $4\pi^3$; $16\pi^4 mr$. **30.** $\frac{1}{2}m\omega^2(a^2-x^2)$. **32.** 5×10^{-2} m. **33.** 3×10^{-7} N. **34.** $6·0 \times 10^{24}$ kg. **35.** 9·9 m s^{-2}. **38.** $1·4 \times 10^{-5}$ rad s^{-1}. **39.** 9·77 m s^{-2}. **40.** 5500 kg m^{-3}. **42.** 24 hours.

EXERCISES 3 (p. 93)

1. $\frac{1}{2}I\omega^2$. **2.** $I\omega$. **3.** couple. **4.** $2\pi\sqrt{I/mgh}$. **5.** *D*. **6.** *B*. **7.** *A*. **8.** *E*. **9.** (i) 2×10^{-4}, (ii) 8×10^{-4} kg m^2. **10.** (i) 8, (ii) 24 kg m^2. **11.** (i) 4×10^{-3}. (ii) 8×10^{-3}, (iii) 2×10^{-3} kg m^2. **12.** 15 joule. **13.** 3·6 m s^{-2}, 6·0 m s^{-1}. **14.** (i) 1·9, (ii) 2·2 rad s^{-1}. **15.** 4·0 s. **16.** (*a*) 20 rad s^{-2}. (*b*) 0·32 N m. **17.** $8·1 \times 10^{-4}$ kg m^2. **18.** $4·02 \times 10^{-3}$ kg m^2. **19.** $25Mh^2/7$. **20.** 3·5 m s^{-1}. **21.** $6·2 \times 10^{-4}$ kg m^2. **23.** 1:12·5.

EXERCISES 4 (p. 121)

1. newton metre. **2.** rises. **3.** meet. **4.** $F \cos \theta$. **5.** centre of gravity. **6.** weight of object. **7.** velocity. **8.** *B*. **9.** *D*. **10.** *C*. **11.** 22·6 cm. **12.** 69,139 kgf. **13.** 50 kgf m; 16·7 kgf. **14.** (i) 0·65, (ii) 0·60 m. **15.** $4\frac{1}{6}$, $4\frac{1}{6}$ kgf. **16.** 30°. **18.** 171 kgf. **19.** 0·04 cm; 0·1°. **20.** $\pm\cos^{-1}[(M+m)r \sin \phi/Ml]$. **22.** 18, 15·9 kgf. **23.** stable if $r > t/2$. **25.** (i) 3:2, (ii) 40:9. **26.** 162 gf. **27.** 5·1 cm. **28.** 2:3. **29.** Wp/σ, (*a*) 0·95, (*b*) 1·19. **30.** (*a*) 75·0 cm, (*b*) 1003·3 cm. **33.** 1·25. **34.** 5·7 cm^3.

PROPERTIES OF MATTER

EXERCISES 5 (p. 148)

1. N m^{-1}. **2.** MT^{-2}. **3.** minimum. **4.** $4\gamma/r$. **5.** $2\gamma/r$. **6.** obtuse. **7.** per unit length on one side of a line in the surface. **8.** free surface energy. **9.** *C*. **10.** *B*. **11.** *D*. **12.** *A*. **13.** $1{\cdot}4\times10^{-2}$, $8{\cdot}68\times10^{-3}$ N. **14.** MT^{-2}, (i) 6·6, (ii) 5·5 cm. **15.** 2·7 cm. **16.** (i) $1{\cdot}00056\times10^{5}$ N m^{-2}. (ii) 0·14 cm. **19.** 0·75 cm. **20.** 2·6 cm. **21.** $8\pi T(b^2-a^2)$. **22.** 7·35 cm, angle of contact now 47°. **25.** $\pi r^2 h\rho - 2\pi r\gamma\cos\alpha$. **26.** 5·6 cm, angle of contact 44°. **27.** 0·18 cm. **28.** (*a*) 3·2 cm, (*b*) $7{\cdot}1\times10^{-2}$ N m^{-1}. **30.** 0·8. **31.** $7{\cdot}4\times10^{-3}$ N.

EXERCISES 6 (p. 167)

1. tensile. **2.** N m^{-2}. **3.** elastic limit. **4.** $\frac{1}{2}$ load. **5.** pressure. **6.** shear. **7.** *C*. **8.** *B*. **9.** *A*. **10.** *D*. **11.** $6{\cdot}2\times10^{6}$ N m^{-2}, 6×10^{-5}, $1{\cdot}0\times10^{11}$ N m^{-2}. **12.** 6·8 kg. **13.** 0·08 mm. **15.** 7·13 kgf. **16.** 1·2 N. **17.** 20 m s^{-1}. **18.** 117 gf. **19.** 2%. **20.** 1/120 J. **21.** $1{\cdot}1\times10^{4}$ J. **22.** (*a*) $2{\cdot}2\times10^{-6}$ m, (*b*) $5{\cdot}46\times10^{-5}$ J. **23.** 0·5. **24.** 144°C, 3360 kg. **25.** (*a*) 2×10^{11} N m^{-2}, (*b*) $4{\cdot}7\times10^{-2}$ J. **26.** $1{\cdot}1\times10^{4}$ N. **27.** $7{\cdot}8\times10^{10}$ N m^{-2}. **28.** 40, 74 N. **29.** 28·6 kgf. **30.** 12×10^{7} N m^{-2}, 3 kgf.

EXERCISES 7 (p. 183)

1. F/R. **2.** independent. **3.** $\pi p a^4/8\eta l$. **4.** ML^{-1}T^{-1}. **5.** $6\pi\eta a v$. **6.** volume. **7.** *C*. **8.** *B*. **9.** *D*. **10.** *A*. **11.** 4·97 cm. **12.** (*a*) 17°, (*b*) 36 gf. **13.** 0·6. **14.** 160 J. **15.** (*a*) 36, (*b*) 60 rev min^{-1}. **17.** 0·5, 13·5 cm. **18.** $x = 2$, $y = 1$, $z = 2$; $k = 2{\cdot}3$. **20.** 0·025 m s^{-1}. **23.** 17 s.

HEAT

EXERCISES 9 (p. 216)

1. 2·2 kJ kg^{-1} K^{-1} (2·2 J g^{-1} K^{-1}). **2.** 351 kJ kg^{-1} (351 J g^{-1}). **3.** 378 kJ kg^{-1} (378 J g^{-1}). **4.** 904 m. **5.** 8·8 g. **6.** 74 km h^{-1}. **7.** 0·76 kJ kg^{-1} K^{-1} (0·76 J g^{-1} K^{-1}), 30·8 min. **8.** 0·68 m. **9.** 417 kJ kg^{-1} (417 J g^{-1}), 6·7 W. **11.** 2230 kJ kg^{-1} (2230 J g^{-1}). **13.** 59·3 kg. **14.** 0·010 s. **15.** 50 g; 328 kJ kg^{-1} (328 J g^{-1}). **16.** 42 J K^{-1}. **17.** 2·22 kJ kg^{-1} K^{-1} (2·22 J g^{-1} K^{-1}). **18.** 93·5 g. **19.** −186°C. **20.** 4·2 kJ kg^{-1} K^{-1} (4·2 J g^{-1} K^{-1}).

EXERCISES 10 (p. 264)

1. 0·014 m^3. **2.** 3·75 at. **3.** 878 mmHg. **4.** (i) 2·08 kJ kg^{-1}, (ii) 842 mmHg. **5.** 100°C. **8.** 146·5 mmHg. **9.** 164 J. **11.** 0·53 kJ kg^{-1} K^{-1} (0·53 J g^{-1} K^{-1}); 1/3. **13.** (*a*) 586 K, (*b*) 101·2 J, (*c*) 355 J. **14.** 0·725 kJ kg^{-1} K^{-1} (0·725 J g^{-1} K^{-1}). **15.** 222 K, 38·4 cmHg. **16.** 1·67. **17.** 56·8 cmHg, 227 K. **18.** 597 m s^{-1}. **21.** 0·21 mmHg (27·5 N m^{-2}). **22.** 1305 m s^{-1}, 0°C. **26.** (*b*) 0·31 J. **27.** 164 J, 30 J.

EXERCISES 11 (p. 291)

1. 79°C. **2.** 434°C. **3.** 762·4 mmHg. **4.** $8{\cdot}5\times10^{-6}$ K^{-1}. **5.** $3{\cdot}8\times10^{-4}$ cm^2. **6.** 964 g. **7.** 62·4 g. **8.** 12×10^{-6} K^{-1}. **9.** 4 s. **10.** 300 N. **12.** 20·5 cm. **13.** 270°C. **14.** 0·436, 0·444 cm^3.

EXERCISES 12 (p. 327)

3. 91·7 torr. **5.** 26°C. **7.** 4·45 cmHg. **10.** 707 mm. **11.** air: $1{\cdot}18\times10^{3}$ kg, vapour: 8·96 kg. **13.** 78%.

EXERCISES 13 (p. 361)

1. 42 $W m^{-1} K^{-1}$, 0·12 $W m^{-1} K^{-1}$. **2.** 89°C = junction temp. **3.** (*a*) 90°C, (*b*) 2·86 W. **4.** 354 J. **5.** 1700 g. **6.** 41:1. **7.** $2{\cdot}9\times10^{-2}$ W, 0·36 $kJ\,kg^{-1}\,K^{-1}$. **8.** 2·8 W. **9.** 22 W, 10%. **12.** $5{\cdot}7\times10^{-8}$ $W m^{-2} K^{-4}$. **13.** 2140 K. **14.** Newton: (*a*) 16, (*b*) 136 degC min^{-1}; Stefan: (*a*) 19, (*b*) 1374 degC min^{-1}. **15.** $5{\cdot}6\times10^{7}$ J. **17.** 5450°C. **19.** 40 $J m^{-2}$, 2 degC min^{-1}. **20.** 87600 J. **21.** 5490°C.

EXERCISES 14 (p. 381)

1. 68°C, −272°C. **3.** 385°C. **4.** (*a*) −274°C, (*b*) 99·45 cm. **8.** 50·4°C. **10.** 815 mmHg. **14.** 0·89°C. **15.** 309°C. **16.** 77·5 cmHg.

OPTICS

EXERCISES 16 (p. 400)

4. $4a$; $2na$.

EXERCISES 17 (p. 417)

1. (i) 15 cm, 1·5, (ii) 12 cm, 3. **3.** 6 cm, 0·4. **6.** 4/21 m. **7.** Object distance = 10 cm, r = 40 cm; concave. **9.** $2R$. **10.** 2 radians, or 114°. **11.** Inverted. **12.** 4·5 cm behind mirror; 0·25 mm; 5/38. **13.** (*a*) 240 cm, (*b*) 1·3 cm.

EXERCISES 18 (p. 437)

1. 35·3°. **2.** 41·8°. **3.** (i) 26·3°. (ii) 56·4°. **4.** (*b*) 3 cm from bottom. **6.** (i) 41·8, (ii) 48·8°, (iii) 62·5°. **9.** 1·47. **11.** (*b*) 12 cm above mirror. **15.** 1·60. **17.** 1·41. **20.** Nearer by $(n-1)\,d/n$.

EXERCISES 19 (p. 451)

1. 42°. **2.** (i) 1·52, (ii) 52·2°. **3.** 60°, 55° 30′, 1·648. **4.** 4° 48′. **5.** angle i on second face = 60·7°, c = 41·8°. **7.** 43° 35′. **8.** 37° 45′; 10° 8′; 180°. **10.** 55°, 1·53. **12.** 27·9°.

EXERCISES 20 (p. 468)

1. Crown: (i) 3·07°, (ii) 3·14°, (iii) 3·11°; flint: (i) 2·58°, (ii) 2·66°, (iii) 2·62°. **2.** 0·023, 0·031. **3.** 3·92°, 0·021°. **4.** 0·54 mm; 0·54 mm. **5.** 1·75. **8.** 0·144°. **11.** (*a*) 49° 12′, (*b*) 50° 38′, (*c*) 1° 26′. **12.** 6·67°, 0·83°. **13.** 1170 $km\,s^{-1}$.

EXERCISES 21 (p. 504)

1. (i) 40 cm virtual, (ii) 80 cm real. **2.** 7·2, 18 cm from nearest point on sphere. **3.** (*a*) 1·51, (*b*) 7·5 cm. **4.** $v = 6r$, where r is radius. **5.** (i) 12 cm, m = 1, (ii) 12 cm, m = 3. **6.** $6\frac{2}{3}$ cm, $\frac{3}{5}$. **7.** $13\frac{1}{3}$, 40 cm. **8.** (i) $5\frac{1}{7}$ cm, (ii) $22\frac{1}{2}$ cm. **9.** (i) $9\frac{3}{5}$ cm, (ii) $16\frac{2}{3}$ cm. **10.** 10 cm. **11.** 1·4. **12.** 12·0, 18·7 cm. **13.** 80 cm. **14.** (*a*) 40 cm, (*b*) 160 cm. **15.** (*a*) $11\frac{1}{9}$, (*b*) $5\frac{5}{9}$ cm; $3\frac{19}{27}$ cm. **16.** $27\frac{1}{4}$ cm from lens. **18.** 1·44. **19.** radii = 9·9, 24·8 cm; n = 1·51. **20.** 4 cm. **21.** $12\frac{6}{7}$, $37\frac{6}{7}$ cm above water surface. **22.** 1·4. **23.** (*a*) 20·5 cm, (*b*) 12·85 cm, (*c*) 1·63. **24.** (*a*) 120 cm from converging lens, (*b*) 92·2 cm, (*c*) 2·2. **25.** (*a*) Beside object O, (*b*) 72·5 cm from O.

EXERCISES 22 (p. 522)

1. Diverging, f = 200 cm; $22\frac{2}{3}$ cm. **2.** Converging, f = $28\frac{4}{7}$ cm; $35\frac{5}{17}$ to 25 cm. **3.** near pt.: 50 cm, far pt.: 200 cm, r = $16\frac{2}{3}$, +100 cm. **5.** 220 cm. **6.** Diverging,

$f = 20$ cm. Infinity to 20 cm from eye. **8.** Diverging, $f = 30$ cm; 30 cm. **9.** 15·3 cm; 39·7 cm, diverging. **10.** 40 cm; converging. **12.** 64 cm; $106\frac{2}{3}$ cm. **14.** 0·98, 1·02 cm; 3·8. **15.** 10 cm.

EXERCISES 23 (p. 548)

1. 8:5:2. **2.** (*a*) infinity, (*b*) least distance of distinct vision. **3.** (*a*) 4, (*b*) 4·8. **4.** long sight, $f = +38\frac{8}{9}$ cm. **5.** 30 cm from scale; $f = 6$ cm. **8.** $f_e = 4$ cm, dia. = 0·5 cm; $r = 4{\cdot}8$ cm; 0·013 cm. **9.** 3·2, 8·8 cm. **10.** 4·0, 21·0 cm. $M = 6{\cdot}0$. **11.** (*a*) 250, (*b*) 0·12 cm. **12.** 89° or 91°. **13.** 8·7 cm, 46·7. **14.** 0·55 cm; 2:1. **15.** 2; 22·5 cm. **16.** (*a*) 22·4, (*b*) 4·9 cm diameter. **17.** $6{\cdot}25 \times 10^{-3}$ rad.

EXERCISES 24 (p. 573)

7. 22·5 m; 0·02 m. **8.** 25; 6×10^4. **9.** 25 rev s^{-1}, 3×10^4 cycles. **12.** 187·5 lux (m-candle), 67·5 cd. **13.** (i) $3\frac{1}{8}$, (ii) 1·6 lux. **14.** 125 cd. **15.** 2, 10 lux. **16.** 0·5. **17.** 1, 0·83 lux; 1·41, 1·2 lux. **18.** 2·63 metres. **19.** 64 lumen m^{-2}. **20.** 63·6° **22.** 90·3%.

WAVES AND SOUND

EXERCISES 25 (p. 604)

2. L = sound, T = remainder. **3.** (i) 133 cm, (ii) 400 Hz. **4.** (i) 100 Hz, (ii) 1·7 m, (iii) 170 m s^{-1}, (iv) π, (v) 0·2 sin $(400\pi t + 20\pi x/17)$. **6.** 380, 391 Hz. **7.** 625·9 Hz. **10.** (i) $5\pi/3$, (ii) $y = 0{\cdot}03 \sin 2\pi(250t - 25x/3)$, (iii) 6 cm, (iv) 0·01 sin $50\pi x/3$. cos $500\pi t$ (x in m).

EXERCISES 26 (p. 635)

1. (*a*) 333, (*b*) 360·4 m s^{-1}; 342 m s^{-1}. **2.** 349 m s^{-1}. **3.** 332 m s^{-1}. **4.** 53·1°. **6.** 680 Hz. **9.** 1366 m. **11.** 1115 Hz. **13.** 1·83 s. **14.** 6·7 m s^{-1}. **15.** $6{\cdot}44 \times 10^6$ m s^{-1}. **16.** 1174, 852 Hz. **17.** (*b*) (i) 66 cm, (ii) (1) 550, (2) 545, (3) 600 Hz. **21.** 10·8 db. **22.** 25 W. **23.** 5×10^{-17} W m^{-2}.

EXERCISES 27 (p. 671)

1. (i) $\lambda/2$, (ii) $\lambda/4$, (iii) $\lambda/2$; 567 Hz. **2.** 20 cm. **3.** (*a*) 0·322, (*b*) 0·645 m. **4.** (*b*) Amp.: (i) max, (ii) 0, (iii) max, (iv) half-max. **5.** 267 Hz. **8.** (i) 15·95 cm, (ii) 4·5 Hz. **9.** +5·2°C. **12.** −3·2, +6·7%. **13.** 239 Hz. **15.** (*a*) 2 m, (*b*) 100 Hz. **16.** touch 1/6th from end. **18.** 2·08 kgf. **19.** 100, 300, 500 Hz.

OPTICS

EXERCISES 28 (p. 685)

2. 39°. **3.** 18·6°. **4.** 47° 10′; 41° 48′ with vertical at oil surface. **8.** $4{\cdot}0 \times 10^{-7}$ m.

EXERCISES 29 (p. 723)

2. $6{\cdot}25 \times 10^{-7}$ m. **3.** $2{\cdot}27 \times 10^{-5}$ m. **4.** 0·34 mm. **5.** 1·5. **7.** 1·64 mm; oil—centre bright, fringes closer. **8.** $2{\cdot}52 \times 10^5$. **10.** $1{\cdot}42 \times 10^{-6}$ m, receded. **11.** $1\frac{1}{3}$. **13.** inwards, 0·133 cm. **15.** $2{\cdot}32 \times 10^{-7}$ m. **16.** 2° 35′. **18.** 3; 1·2 cm. **19.** 6,800. **20.** (*b*) 5 m s^{-1}. **21.** 13·3°. **22.** 3; $5{\cdot}895 \times 10^{-7}$ m. **23.** 2·35 mm, 0·0785 rad. **25.** 67·5°. **29.** 49·0°. **32.** 2·37 cm.

ELECTRICITY AND ATOMIC PHYSICS

EXERCISES 30 (p. 762)

2. $1{\cdot}5\times10^6$ V. **4.** (i) $E = \sqrt{3}q/4\pi\varepsilon_0 z^2$, $V = q/2\pi\varepsilon_0 z$, (ii) $E = q/4\pi\varepsilon_0 z^2$ parallel to AB, $V = 0$. **7.** (*a*) 9×10^4 N C^{-1} (V m^{-1}), (*b*) 9000 V. **8.** $1{\cdot}59\times10^{-19}$ C. **9.** 3 electrons, 19·6 m s^{-2} (2*g*). **12.** 100 V. **13.** $1{\cdot}33\times10^{-5}$ C, 6×10^5 V, yes.

EXERCISES 31 (p. 782)

1. (*a*) 20 V, (*b*) $2{\cdot}5\times10^{-3}$ J, originally $12{\cdot}5\times10^{-3}$ J. **2.** 3 μF; 20 V. **3.** (*a*) $1{\cdot}7\times10^{-5}$ μF, (*b*) $1{\cdot}22\times10^{-7}$ J, (*c*) A—$6{\cdot}2\times10^{-8}$ J; B—$6{\cdot}0\times10^{-8}$ J. **4.** 33·9 m. **5.** (*a*) 0·1 μF, (*b*) 100. **6.** 20 MΩ. **8.** 3. **9.** $5{\cdot}5\times10^{-4}$, $0{\cdot}0025\times10^{-4}$ μF. **10.** 40,400 V m^{-1}. **11.** 4 μF. **12.** $1{\cdot}5\times10^{-8}$ C, 2700 V. **13.** $4{\cdot}25\times10^{-5}$ A. **14.** (*a*) 0·988, (*b*) 0·88, (*c*) 440 V.

EXERCISES 32 (p. 805)

1. 18Ω. **2.** 3·6 mV. **3.** 2·2 kJ kg^{-1} K^{-1} (2·2 J g^{-1} K^{-1}). **4.** 1·1 mm. **6.** $\frac{1}{2}$, 1, 2 kW. **7.** 36, 39%. **10.** 200 cells, 0·30 g. **12.** 18·75 kW.

EXERCISES 33 (p. 839)

1. 25, 50 V. **2.** (*a*) 0·3 A, (*b*) 83×10^{-6}, 81×10^{-3} W. **3.** $64n/(n^2+16)$, 8 A, 3/8 Ω. **4.** X = 2000 Ω; 1000 Ω. **5.** −1%. **6.** 2μA div^{-1}. **7.** $3{\cdot}96\times10^{-4}$, 4×10^{-4} A. **8.** $1\frac{7}{13}$ V. **9.** V, 2·7 Ω. **10.** 1 Ω. **11.** (*a*) 45·9 cm from left hand, (*b*) 58·6 Ω parallel. **12.** 5×10^{-4} A, $2{\cdot}5\times10^{-3}$ V; 1962 Ω. **13.** $5{\cdot}3\times10^{-4}$ K^{-1} (degC^{-1}). **14.** 8·91 mV. **16.** 2 Ω. **17.** $-5{\cdot}1\times10^{-4}$ K^{-1} (degC^{-1}). **20.** $3{\cdot}23\times10^{-4}$ s.

EXERCISES 34 (p. 871)

1. (*a*) $4{\cdot}4\times10^{-3}$ g, (*b*) $1{\cdot}1\times10^{-3}$ g, (*c*) 1860 s (31 min). **2.** 5·1 degC. **3.** 33×10^{-8} kg C^{-1}. **4.** 1·49 V. **5.** 1·7 V. **7.** 23 Ω, 1·6 d, 50%. **8.** 1·8 cm^3. **9.** 18·9 min. **10.** 1·5 V.

EXERCISES 35 (p. 893)

1. 4·95 Ω, 20 mA. **2.** $nABi/c$. **3.** (*a*) shunt 0·0125 Ω, (*b*) series 9995 Ω. **4.** 69 μA. **5.** AB, CD: $8{\cdot}49\times10^{-5}$ N horizontal; BC: $1{\cdot}8\times10^{-4}$ N at 20° to horizontal; $1{\cdot}85\times10^{-5}$ N m, (*a*) BC goes down, (*b*) BC goes down. **6.** 2·1 A m^{-2}. **8.** 0·0136 μA/div. **9.** series 140, 7490 Ω. 115 V.

EXERCISES 36 (p. 927)

3. 44·2 s^{-1}. **4.** 4 div μC^{-1}, 1·9 T (Wb m^{-2}). **6.** 1·05 V. **8.** 32·3 div. **10.** 0·53 V. **11.** 4 Ω, 8×10^{-3} H; 12·5 V. **12.** (*a*) 4 A, (*b*) 2 A s^{-1}. **13.** 0·93 s. **14.** 75·4 sin ωt. **16.** 1·6 mV. **18.** $2{\cdot}61\times10^{-7}$ A, $1{\cdot}02\times10^{-11}$ W. **20.** 1·05 V.

EXERCISES 37 (p. 946)

1. $8{\cdot}7\times10^{-3}$ V. **2.** 6 cm from 12 A wire. **3.** 3·64 A. **4.** 8·4 mA. **6.** $\mu_0 n I_1 I_2 l^2/2\pi d(d+l)$, $1{\cdot}5\times10^{-4}$ N. **8.** $\mu_0 ix/\pi(x^2+a^2)$; $x = a$.

EXERCISES 38 (p. 965)

1. 0·22 A. **2.** 2500. **3.** (i) 0·298 T (Wb m^{-2}), (ii) 238, (iii) 237.

EXERCISES 39 (p. 990)

1. 3·46 A. **2.** (*a*) 0·1 A, (*b*) 3 J. **3.** (*a*) 754 Ω, (*b*) 5·42 H. **4.** 24 Ω. **5.** 40 Ω, 0·24 H; 26·7 Hz, 7·1 V. **6.** 1·126 μF; 140°; 0·645. **7.** X = 78 Ω, 7·8 V; Y = 99 Ω, 9·9 V. X: +46°, Y: −66°. **10.** 71 V, 25/π, 25 W, 25√3 W. **11.** $1{\cdot}57 \times 10^{-4}$ V. **12.** 0·06 H. **13.** 4 Ω, 9·55 mH. **14.** 0·4 V, 2 V m^{-1}, $3{\cdot}2 \times 10^{-19}$ N. **15.** $1{\cdot}3 \times 10^{-12}$ T.

EXERCISES 40 (p. 1007)

2. 2·0006 V. **3.** $14{\cdot}4 \times 10^{-3}$ m. **4.** $0{\cdot}5 \times 10^{-4}$ T (Wb m^{-2}). **5.** 4*e*; changes to 2*e* and 3*e*. **6.** (*a*) $1{\cdot}5 \times 10^{-6}$ m, (*b*) 1, (*c*) $5{\cdot}0 \times 10^{-4}$ m s^{-1}. **8.** 2 cm. **9.** $3{\cdot}72 \times 10^{-17}$ C. **10.** 0·32 cm. **11.** $1{\cdot}8 \times 10^{11}$ C kg^{-1}. **12.** $1{\cdot}6 \times 10^{-19}$ C.

EXERCISES 41 (p. 1036)

7. 200. **10.** 62000. **11.** $1{\cdot}8 \times 10^{11}$ C kg^{-1}. **22.** (i) 45, (ii) 13 kΩ.

EXERCISES 42 (p. 1065)

3. $9{\cdot}6 \times 10^{5}$ min^{-1}. **4.** β-particles emitted, 198. **8.** 14·9 MeV. **9.** 0·11. **10.** (*a*) 4·23, (*b*) 4·16 MeV. **12.** 56 s. **13.** $3{\cdot}36 \times 10^{22}$. **15.** (*a*) $2{\cdot}6 \times 10^{12}$:1, (*b*) $2{\cdot}6 \times 10^{13}$ m s^{-2}, (*c*) $9{\cdot}6 \times 10^{-17}$ J.

EXERCISES 43 (p. 1092)

1. $1{\cdot}24 \times 10^{-11}$ m. **2.** $6{\cdot}64 \times 10^{-34}$ J s. **5.** $2{\cdot}3 \times 10^{17}$, 4×10^{-7} A. **6.** 5000:1. **7.** $1{\cdot}8 \times 10^{11}$ C kg^{-1}. **9.** $3{\cdot}1 \times 10^{4}$ Hz. **10.** 8125 eV. **12.** $2{\cdot}0 \times 10^{-7}$ m. **13.** A, B, C, D = 6·5, 4·8, 4·3, $4{\cdot}0 \times 10^{-7}$ m. **14.** (*a*) 9×10^{-7} m, (*b*) $7{\cdot}8 \times 10^{5}$ m s^{-1}, (*c*) 1·7 V.

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