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The **Cornea** is a tough, transparent tissue that covers the anterior surface of the eye.

The **Sclera** is an opaque membrane that encloses the remainder of the optic globe

The **Choroid** contrarians the blood vessels, which are the major source of nutrition to the eye.

The **lens** is made up of concentric layer of fibers cells and is supported by fibers that attach to the ciliary body.

The **Retina** lines the inside of the wall's entire posterior portion. Its surface include two classes of light receptors: **cones** and **rods**.

The **cones** lies on the fovea and highly sensitive to color (6-7 million in each eye).

The rods (75-150 million), which are distributed over the retinal surface and serves to give general overall picture of the field of view and are sensitive to low level illumination.

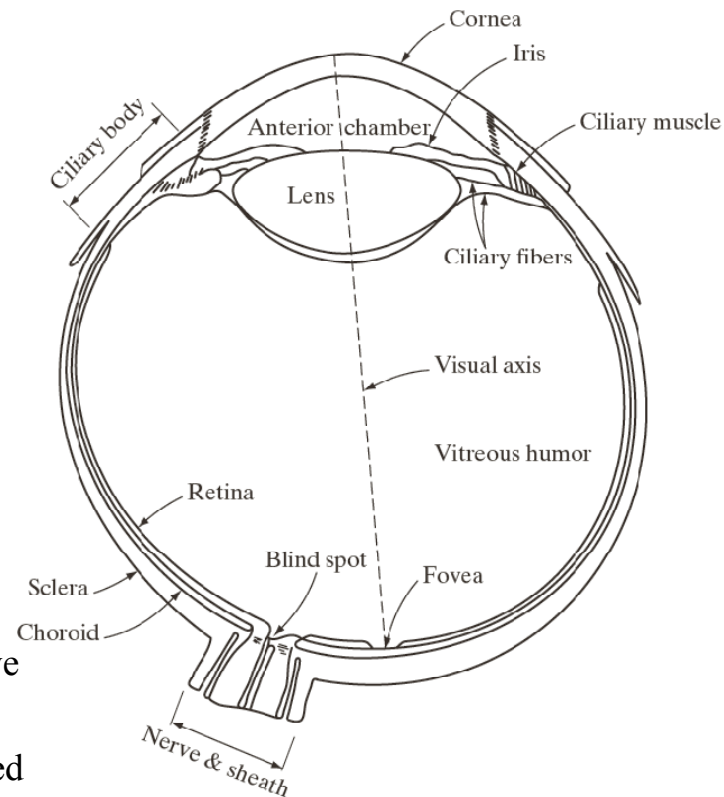
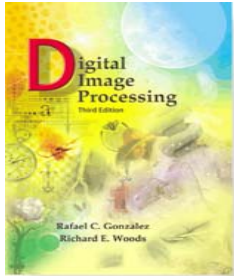


FIGURE 2.1
Simplified
diagram of a cross
section of the
human eye.



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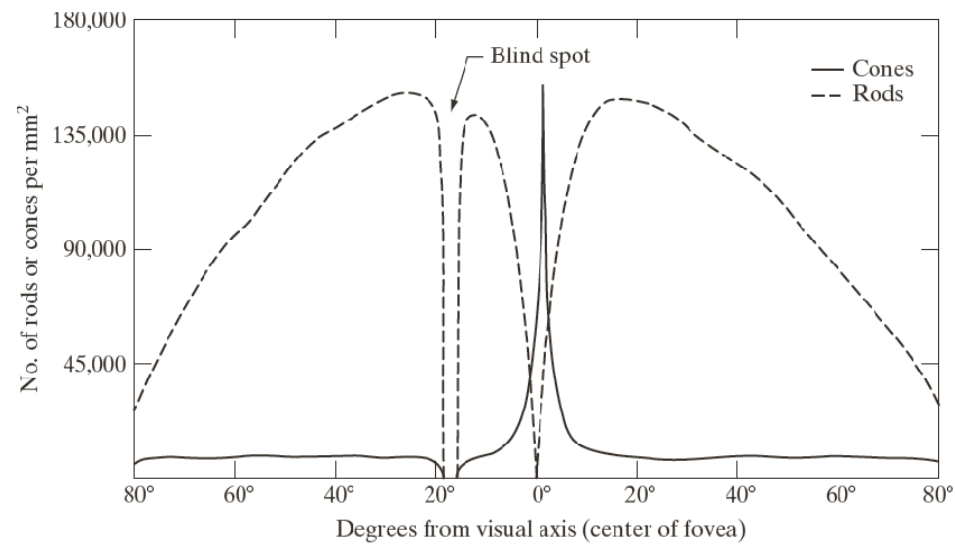
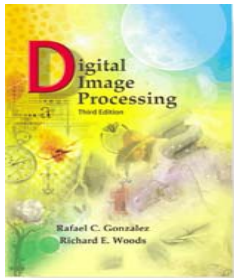


FIGURE 2.2
Distribution of
rods and cones in
the retina.



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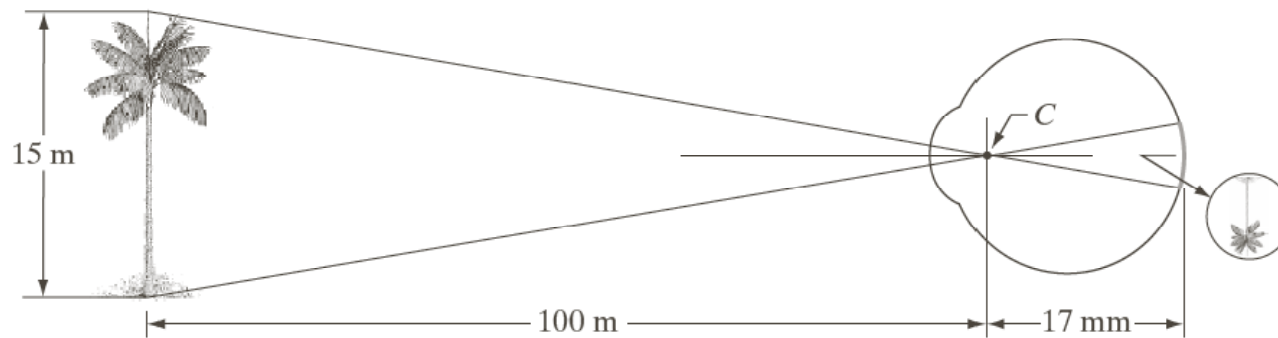
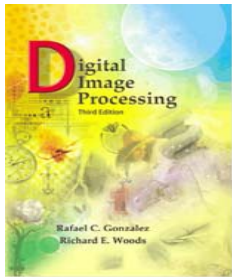


FIGURE 2.3

Graphical representation of the eye looking at a palm tree. Point *C* is the optical center of the lens.



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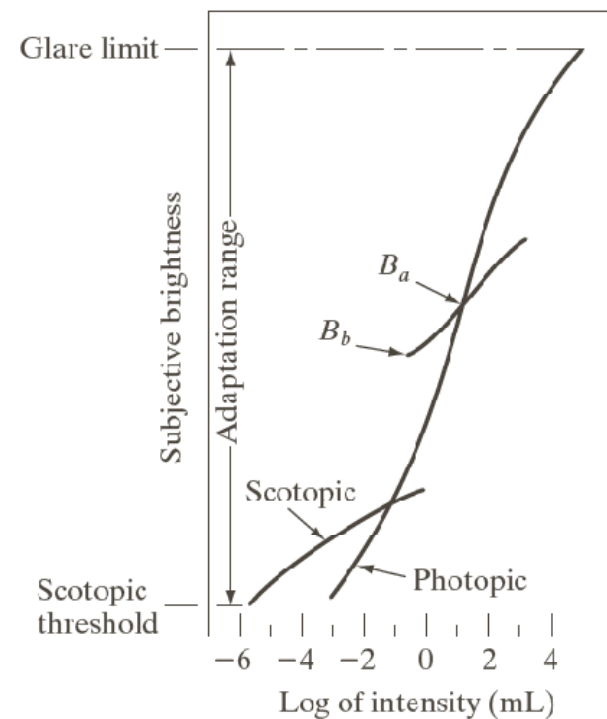
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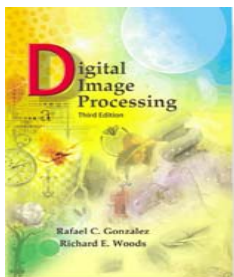
FIGURE 2.4
Range of
subjective
brightness
sensations
showing a
particular
adaptation level.

The human eye can adapt to enormous range of light intensity levels – on the order of 10^{10} . The subjective brightness (the intensity perceived by the eye) of the eye is a logarithmic function.

Photopic vision is the vision of the eye under well-lit conditions. In humans and many other animals, photopic vision allows color perception, mediated by cone cells.

Scotopic vision is the vision of the eye under low light conditions.





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FIGURE 2.5 Basic experimental setup used to characterize brightness discrimination.

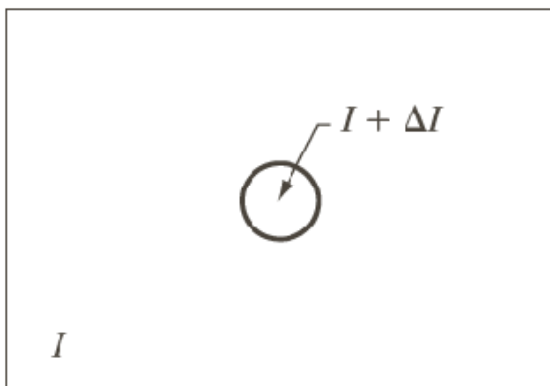
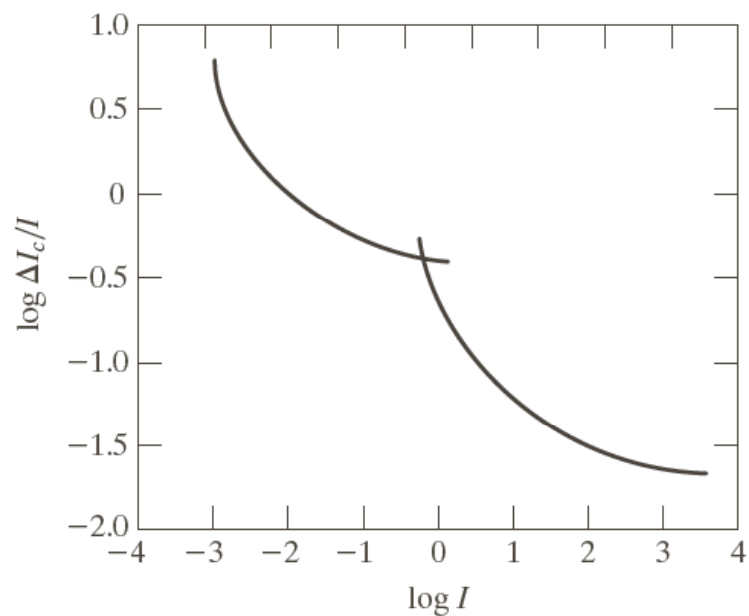
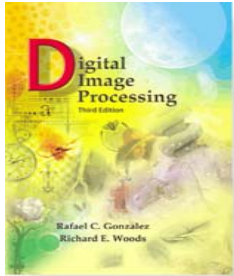


FIGURE 2.6 Typical Weber ratio as a function of intensity.



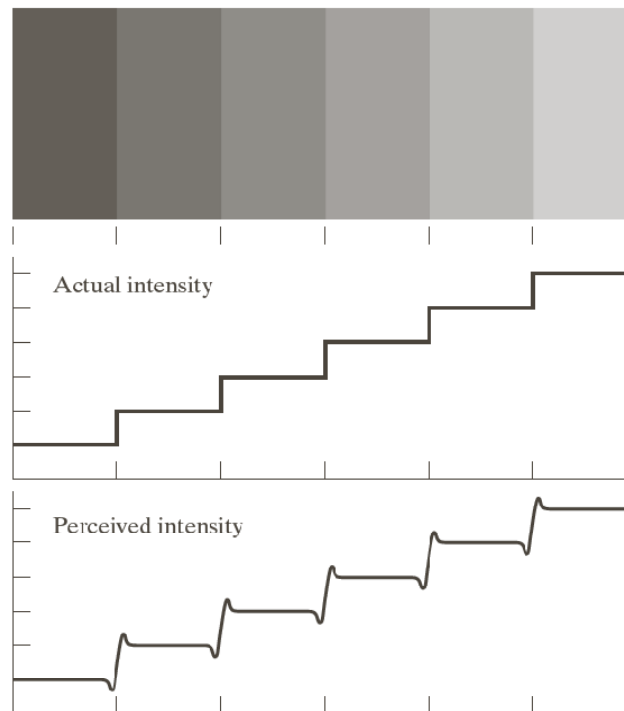


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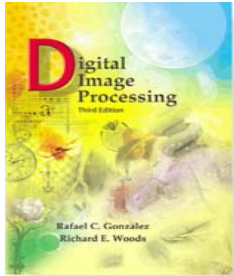
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a
b
c

FIGURE 2.7

Illustration of the Mach band effect. Perceived intensity is not a simple function of actual intensity.



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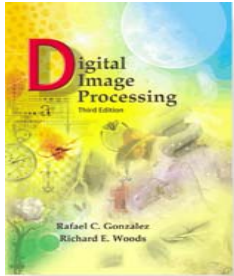
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FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.



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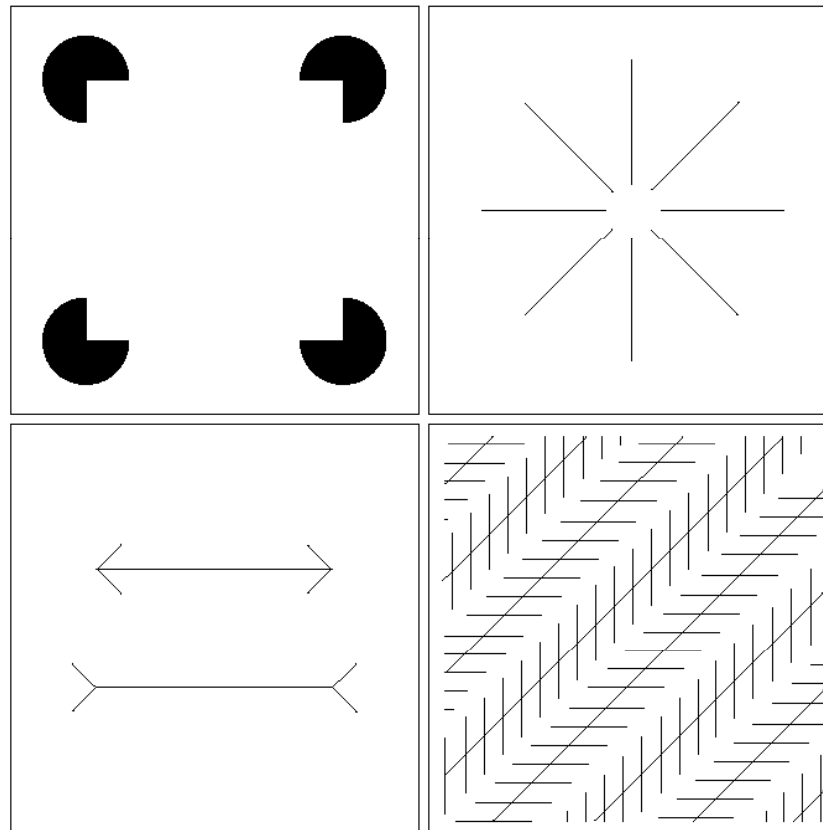
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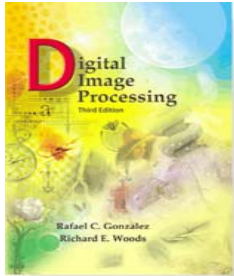
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a b
c d

FIGURE 2.9 Some well-known optical illusions.





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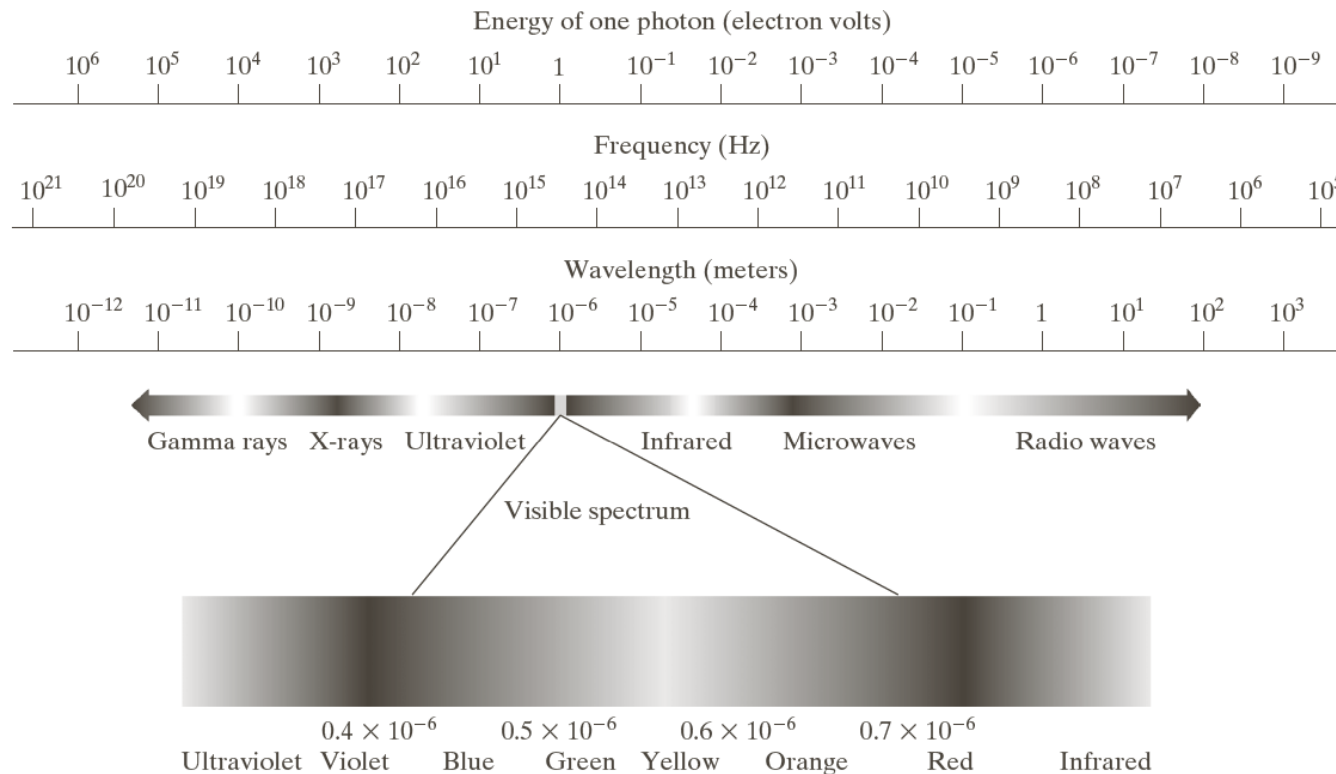
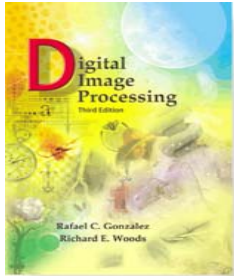


FIGURE 2.10 The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.



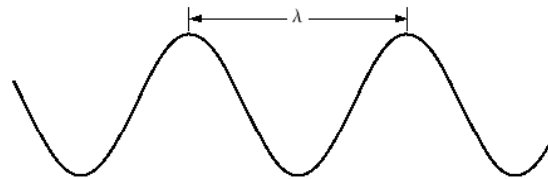
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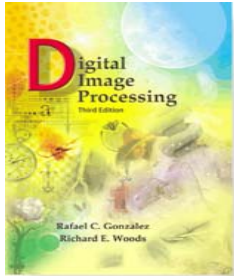
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FIGURE 2.11
Graphical
representation of
one wavelength.





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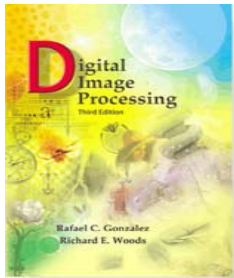
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a
b
c

FIGURE 2.12

- (a) Single imaging sensor.
- (b) Line sensor.
- (c) Array sensor.



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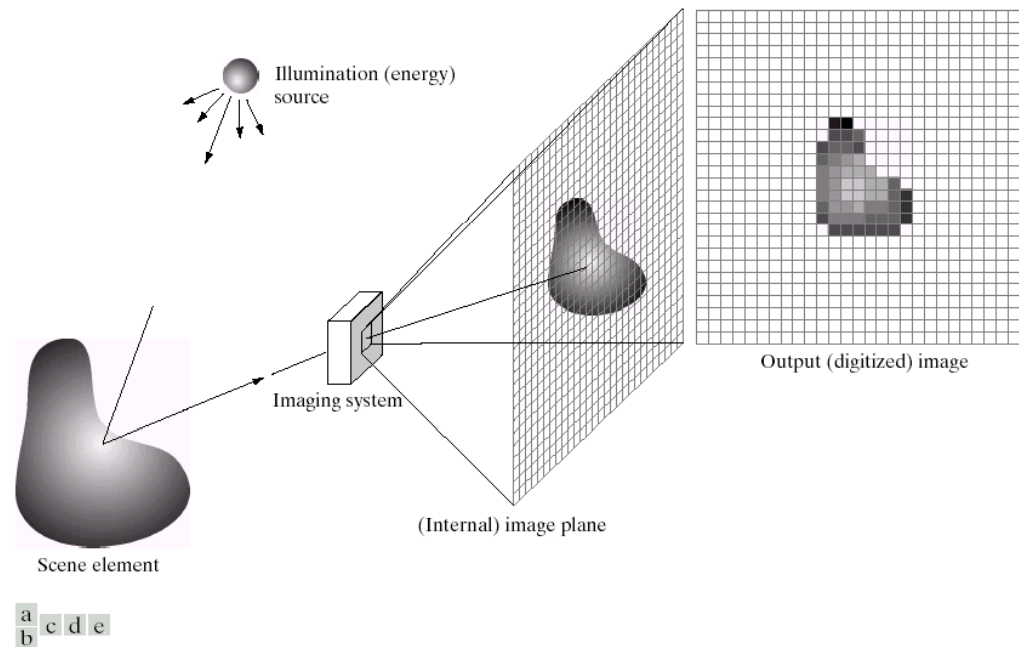
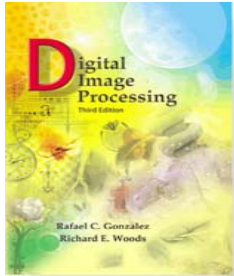


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

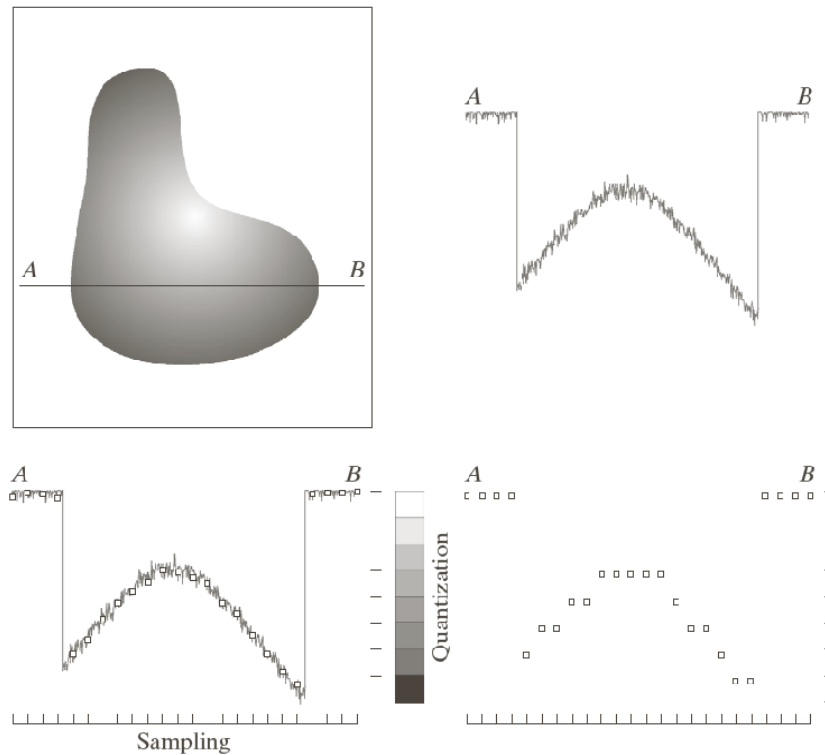


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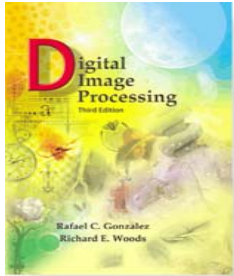
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a	b
c	d

FIGURE 2.16

Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

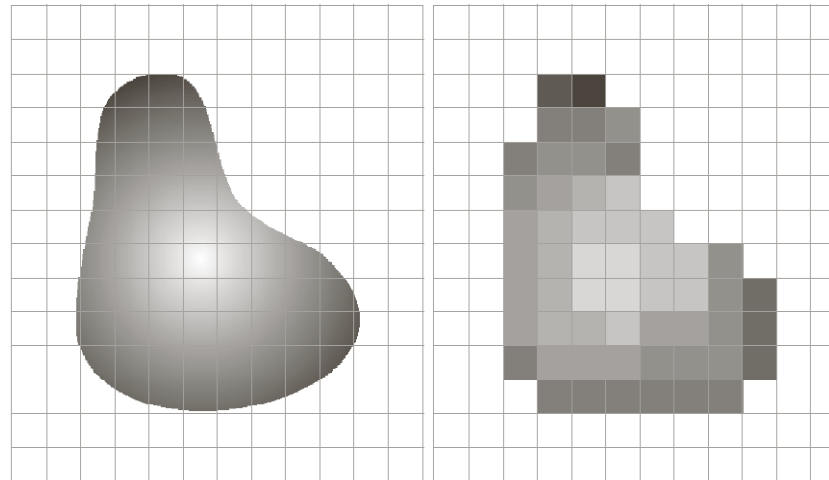


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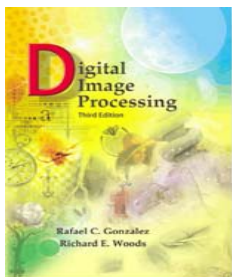
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a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

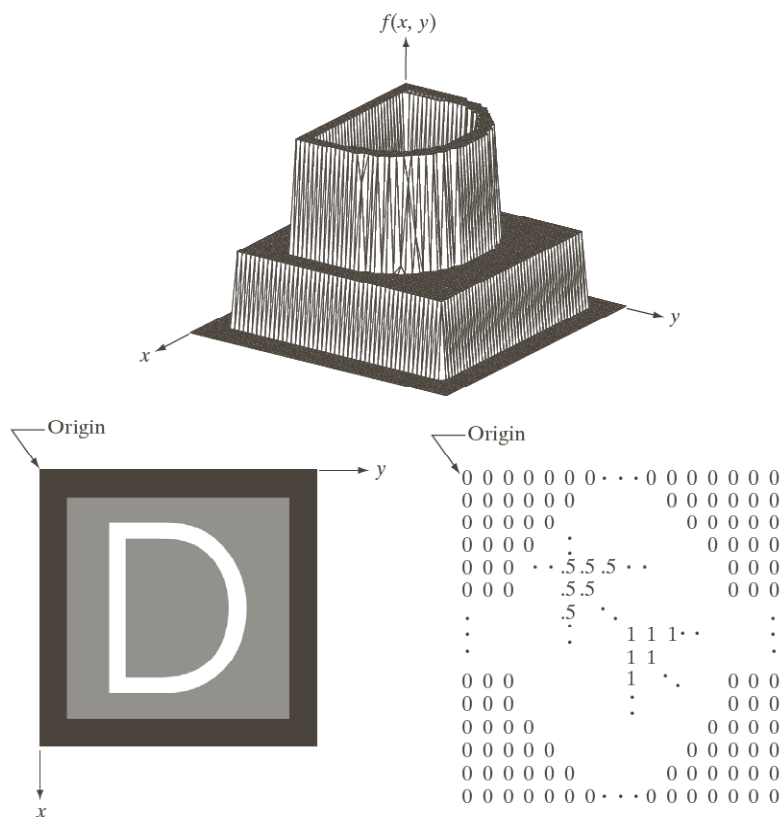


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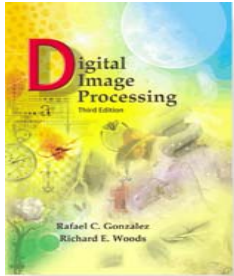
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a
b c

FIGURE 2.18

(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).



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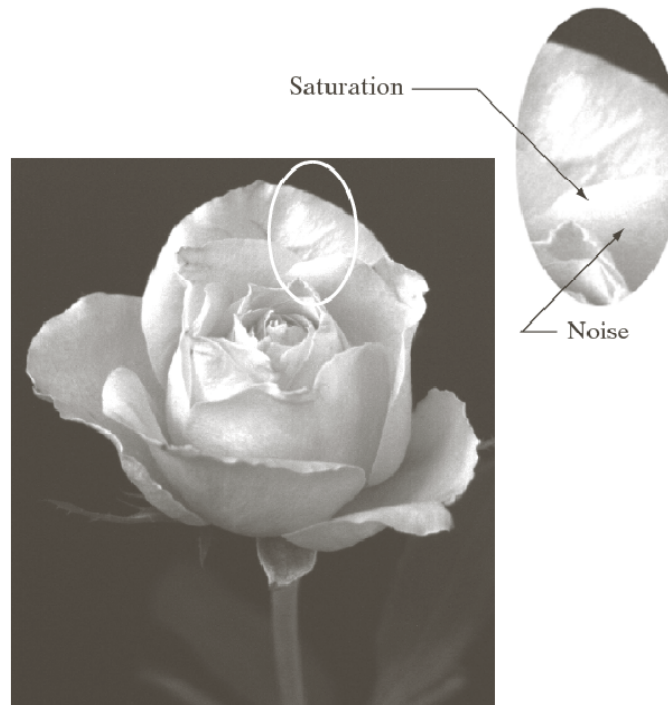
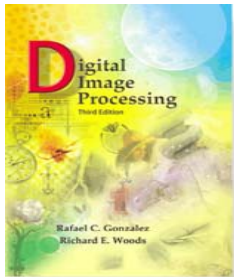


FIGURE 2.19 An image exhibiting saturation and noise. Saturation is the highest value beyond which all intensity levels are clipped (note how the entire saturated area has a high, *constant* intensity level). Noise in this case appears as a grainy texture pattern. Noise, especially in the darker regions of an image (e.g., the stem of the rose) masks the lowest detectable true intensity level.



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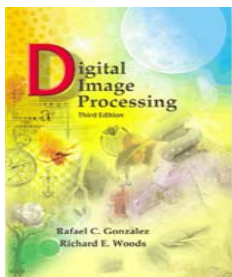
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a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.



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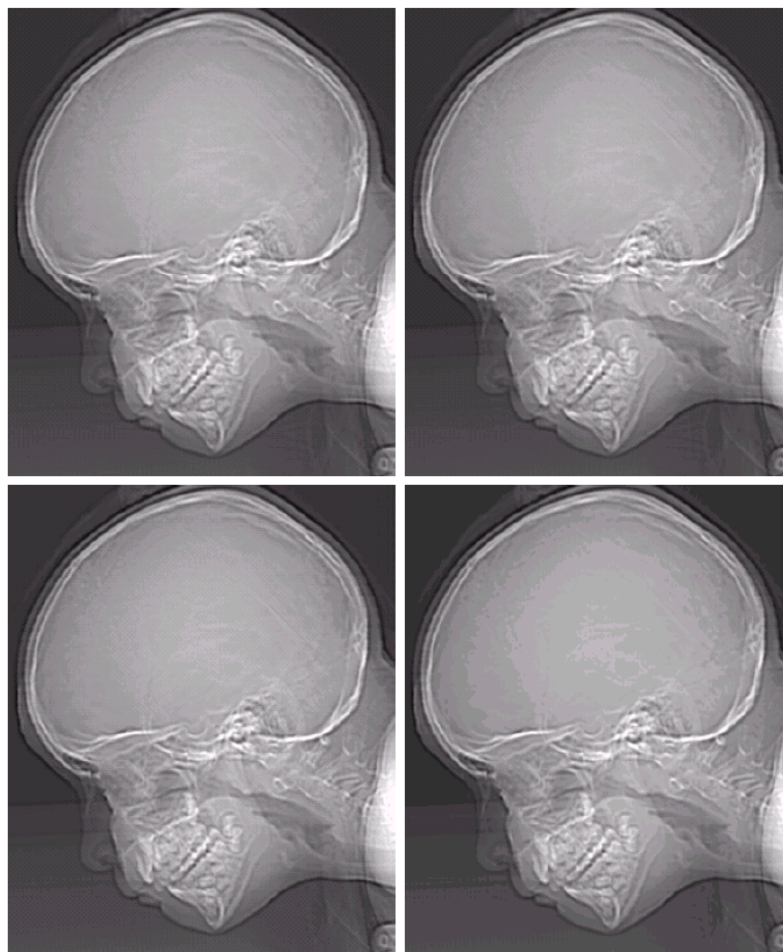
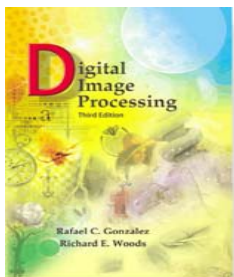


FIGURE 2.21
(a) 452×374 ,
256-level image.
(b)–(d) Image
displayed in 128,
64, and 32 gray
levels, while
keeping the
spatial resolution
constant.



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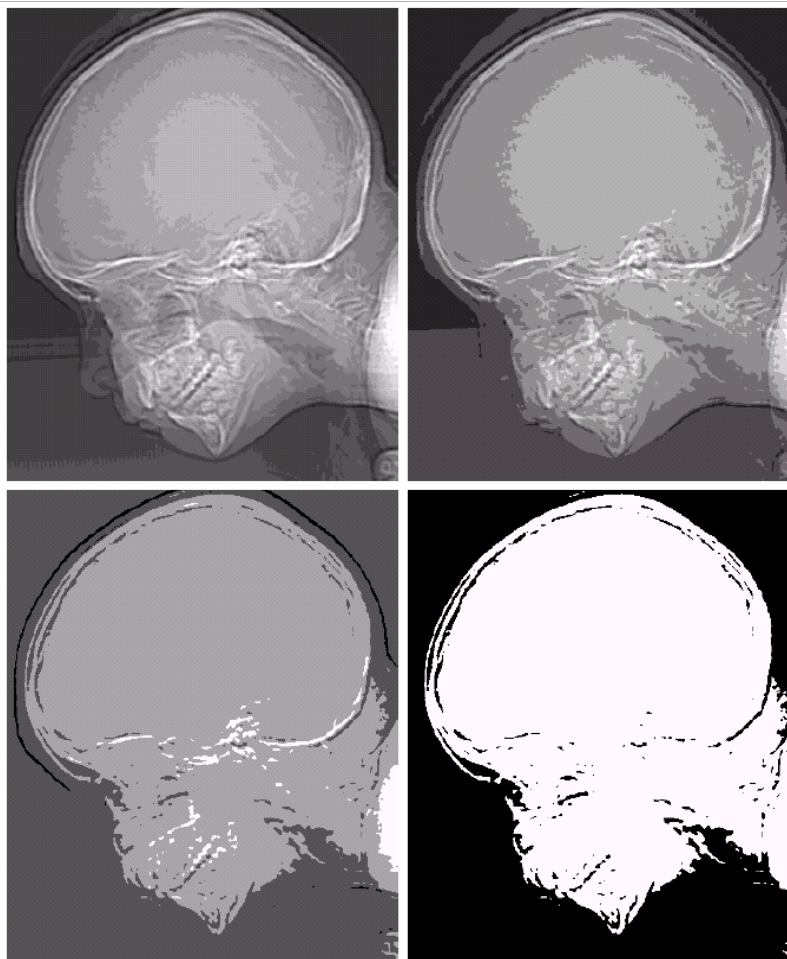
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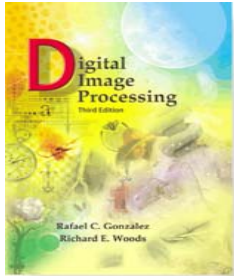
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e f
g h

FIGURE 2.21
(Continued)
(e)–(h) Image
displayed in 16, 8,
4, and 2 gray
levels. (Original
courtesy of
Dr. David
R. Pickens,
Department of
Radiology &
Radiological
Sciences,
Vanderbilt
University
Medical Center.)





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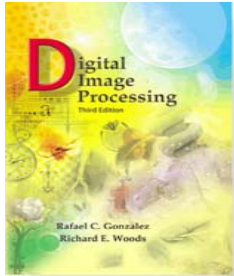
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a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)



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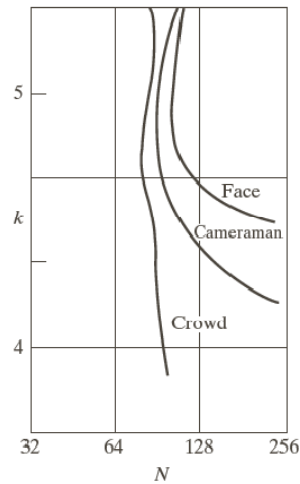
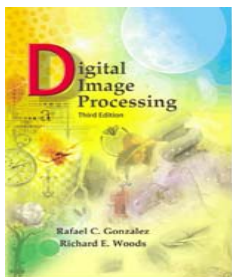


FIGURE 2.23
Typical
isopreference
curves for the
three types of
images in
Fig. 2.22.



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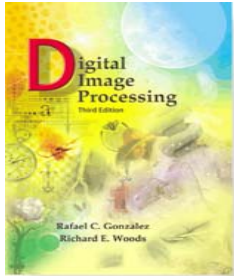
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FIGURE 2.24 (a) Image reduced to 72 dpi and zoomed back to its original size (3692×2812 pixels) using nearest neighbor interpolation. This figure is the same as Fig. 2.20(d). (b) Image shrunk and zoomed using bilinear interpolation. (c) Same as (b) but using bicubic interpolation. (d)–(f) Same sequence, but shrinking down to 150 dpi instead of 72 dpi [Fig. 2.24(d) is the same as Fig. 2.20(c)]. Compare Figs. 2.24(e) and (f), especially the latter, with the original image in Fig. 2.20(a).



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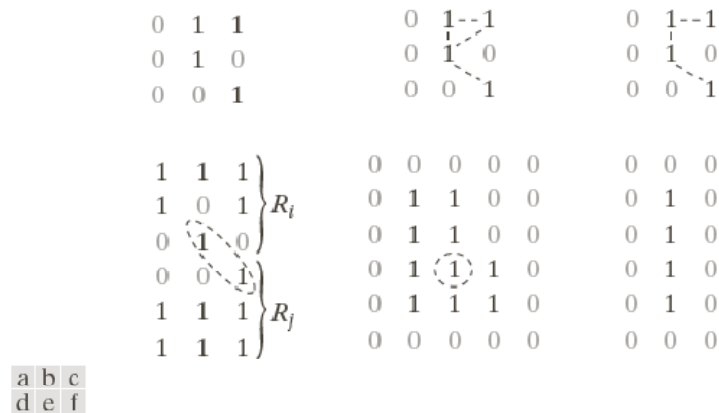
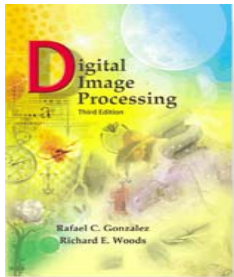


FIGURE 2.25 (a) An arrangement of pixels. (b) Pixels that are 8-adjacent (adjacency is shown by dashed lines; note the ambiguity). (c) m -adjacency. (d) Two regions that are adjacent if 8-adjacency is used. (e) The circled point is part of the boundary of the 1-valued pixels only if 8-adjacency between the region and background is used. (f) The inner boundary of the 1-valued region does not form a closed path, but its outer boundary does.

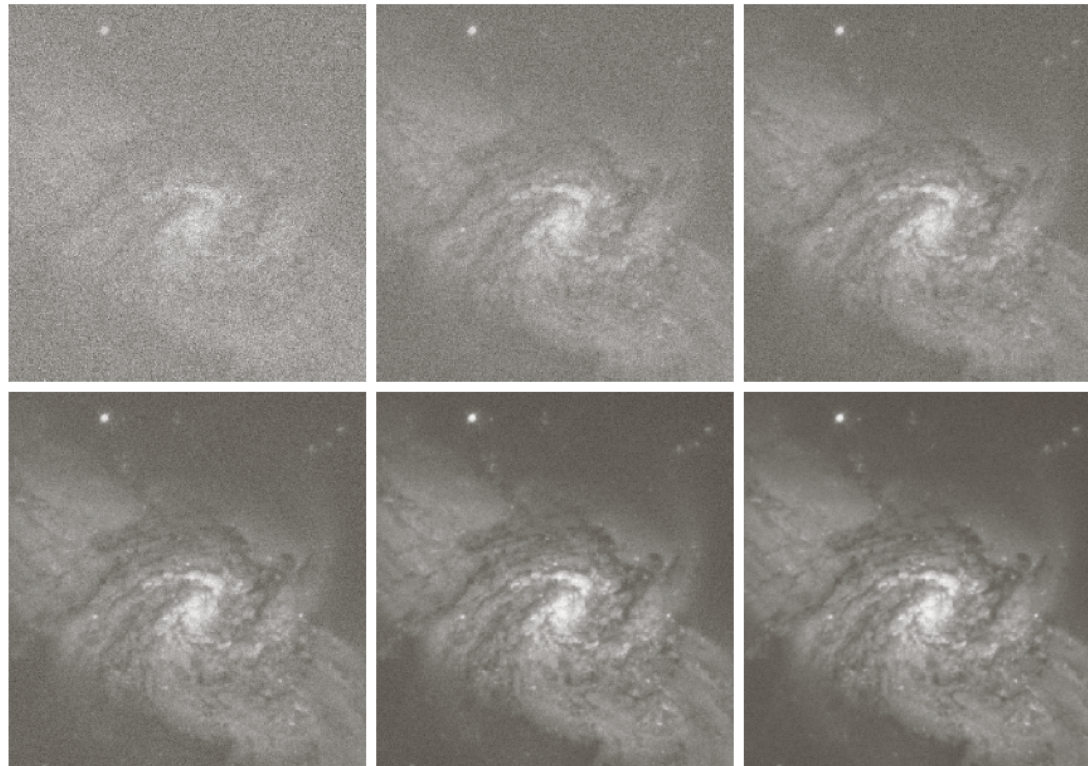


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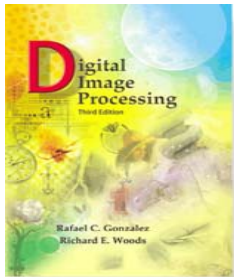
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a	b	c
d	e	f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

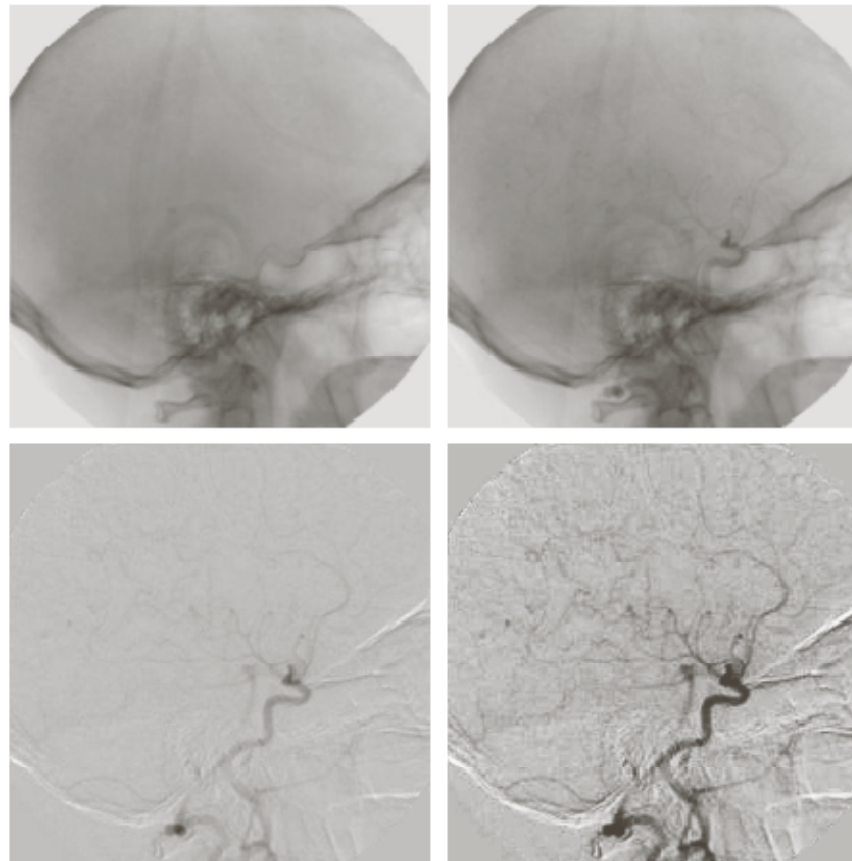


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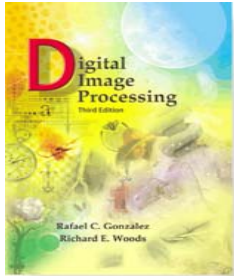
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a b
c d

FIGURE 2.28

Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

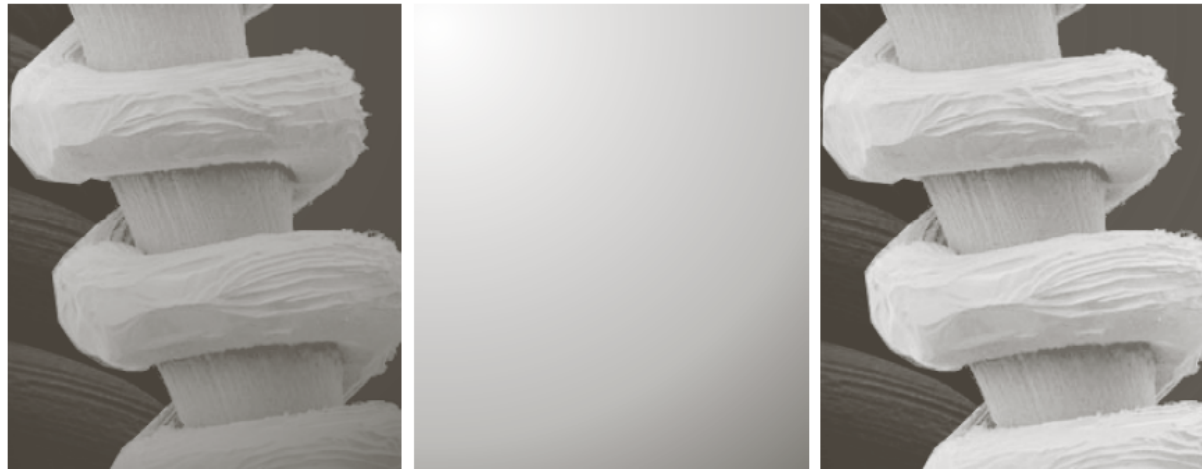


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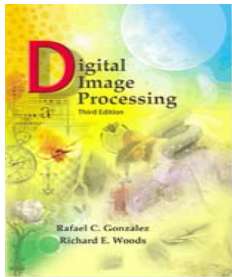
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a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)



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Simple Image Operation

We assume the image pixels as elements of a set and use the set operation to manipulate images

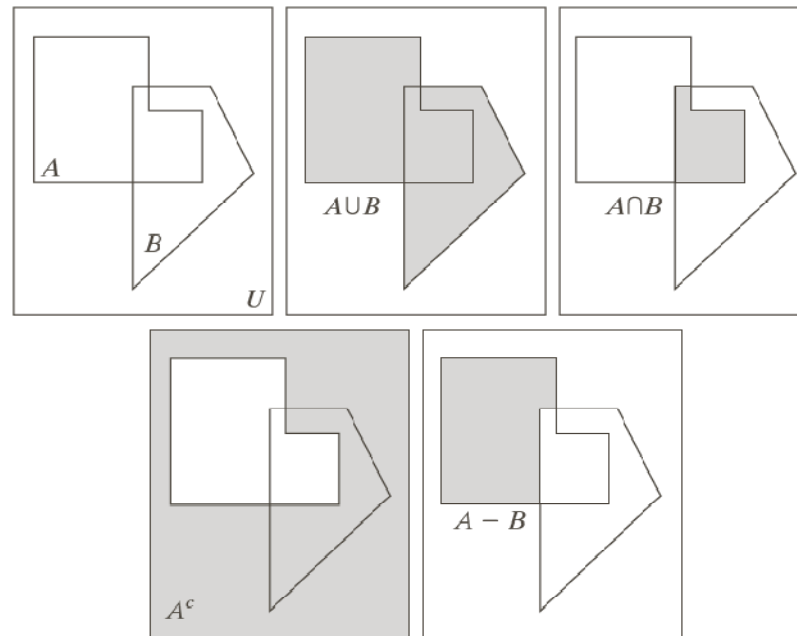
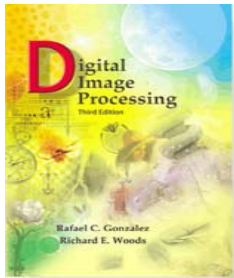


FIGURE 2.31

(a) Two sets of coordinates, A and B , in 2-D space. (b) The union of A and B . (c) The intersection of A and B . (d) The complement of A . (e) The difference between A and B . In (b)–(e) the shaded areas represent the member of the set operation indicated.



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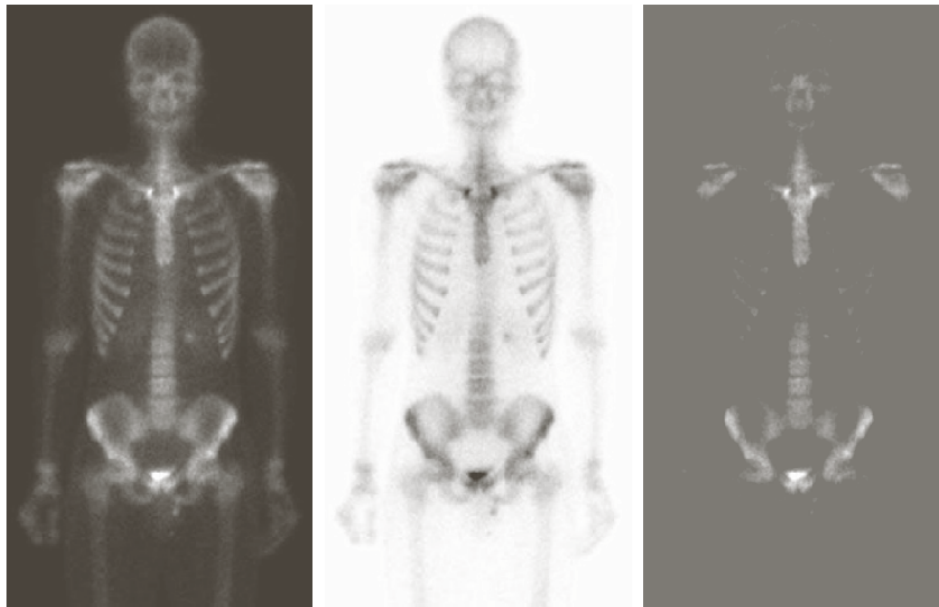
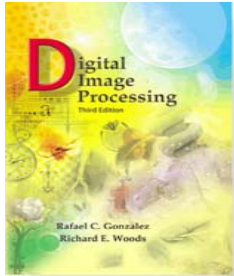


FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

Simple Image Operation

Negating a binary image is done simply by exchanging the white pixels “1” by black pixels “0” and vice versa. For gray scale images this is done by subtracting 255 from the intensity of each pixel; i.e., $C_i = 255 - C_i$



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Logical Operations on Images

We assume binary values for pixels of the image (B/W) and treat the image as a binary buffer.

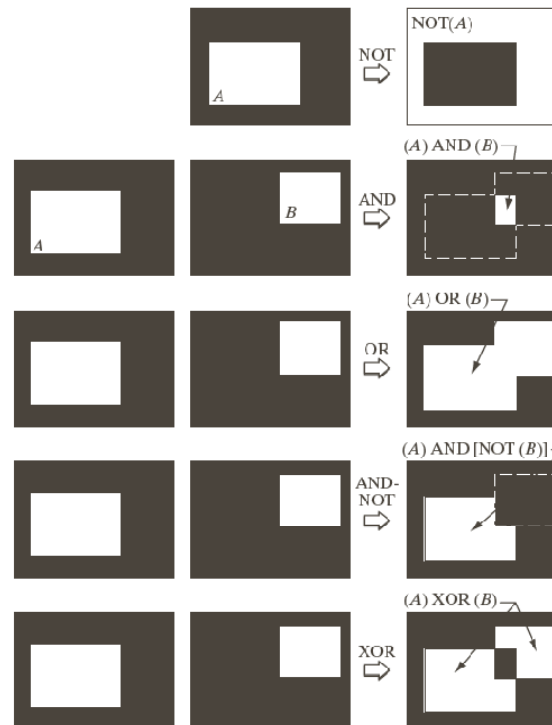
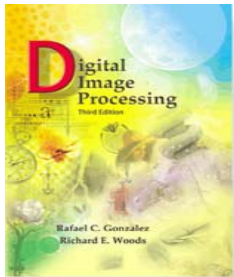


FIGURE 2.33
Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.



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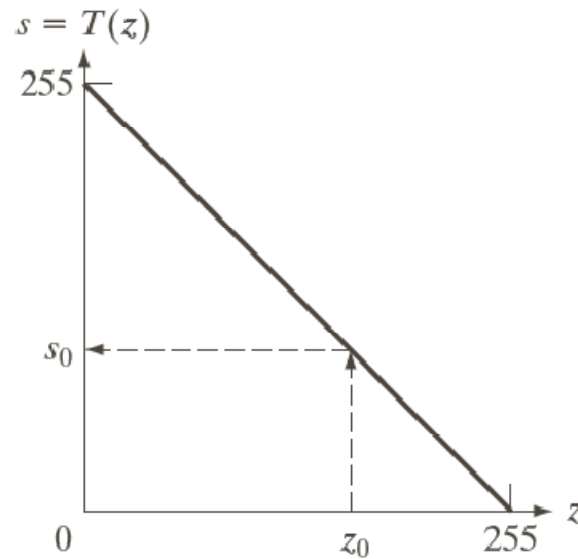
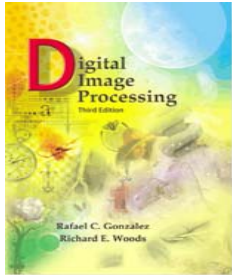


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0 .



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Image Operation

Most of image operation take into account the adjacent pixels when computing the resulting values (for pixels).

Example: The value of a pixel, in the resulting image, is taken as the average of of its adjacent neighbors.

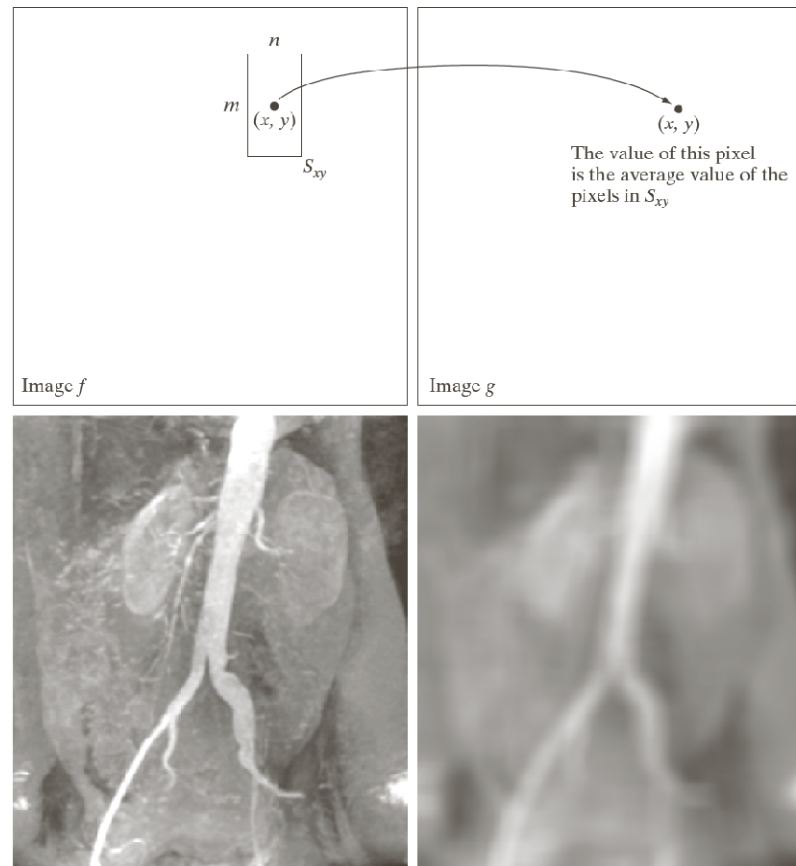
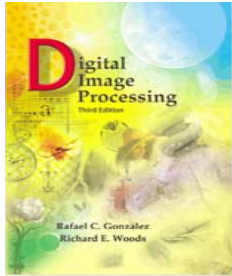


FIGURE 2.35

Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.



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TABLE 2.2

Affine transformations based on Eq. (2.6.–23).

Geometric Spatial Transformation

These operations modify the spatial relationship between pixels in an image.

In digital images Geometric Transformations involves

1. Spatial Transformation of coordinates.
2. Intensity interpolations.

The geometric coordinate could be expressed as

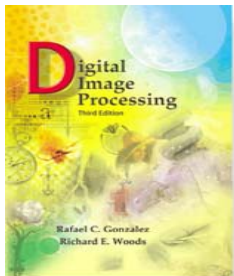
$$(x_p, y_p) = \mathbf{T}\{(u_p, v_p)\}, \text{ where}$$

(u_p, v_p) are the coordinate of the pixel \mathbf{p} before applying the transformation \mathbf{T} and (x_p, y_p) are the coordinate of the pixel \mathbf{p} before applying \mathbf{T}

It is often written as vector operations

$$(x_p, y_p) = [u_p, v_p]T$$

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v \cos \theta - w \sin \theta$ $y = v \sin \theta + w \cos \theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	



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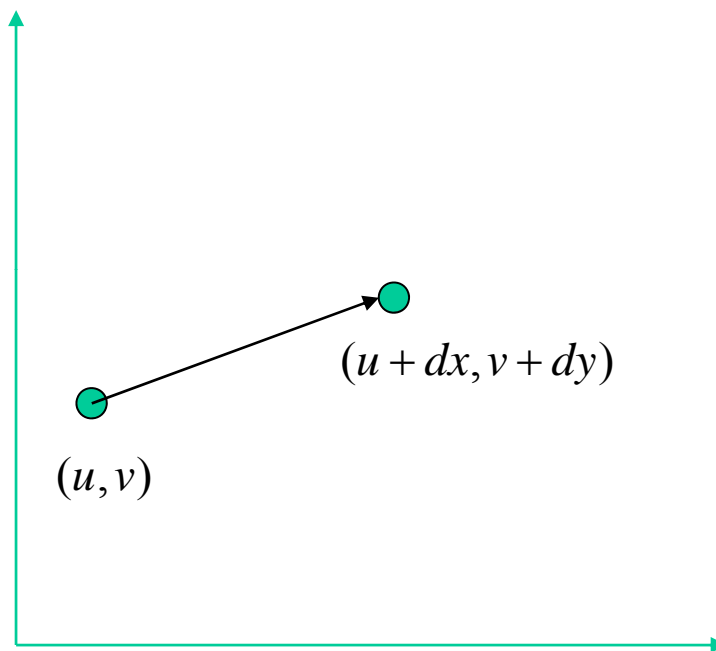
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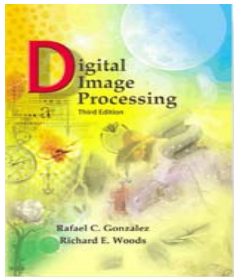
Translation

Alter the position of a point or an object in 2D space.

$$(x, y) = T_{dx, dy}(u, v) = (u + dx, v + dy)$$

$$(x, y) = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$





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Rotation

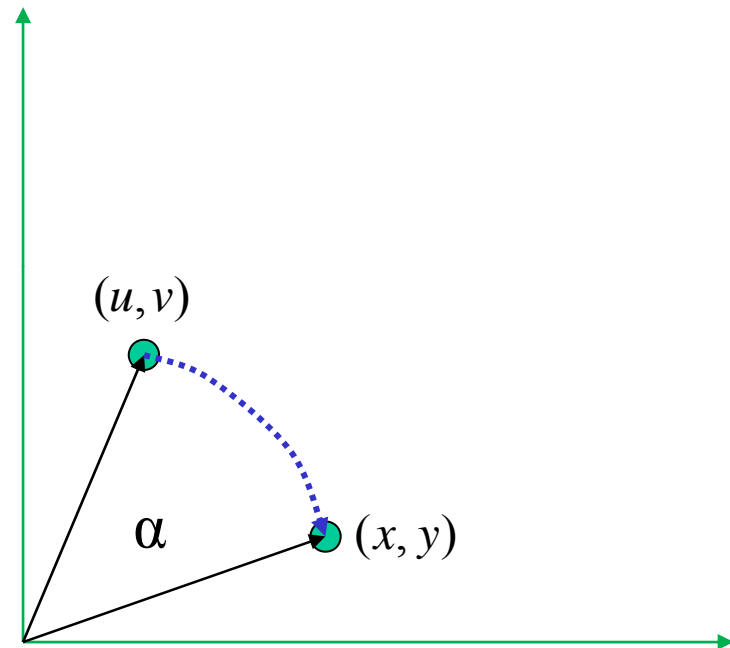
- Rotate a pixel $P(x, y)$ around the origin by an angle α .
- The results pixel position is $P'(x', y')$

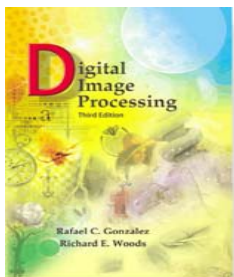
$$x = u \cos(\alpha) - v \sin(\alpha)$$

$$y = u \sin(\alpha) + v \cos(\alpha)$$

$$R_{\alpha}(u, v) = (u \cos(\alpha) - v \sin(\alpha), u \sin(\alpha) + v \cos(\alpha))$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$



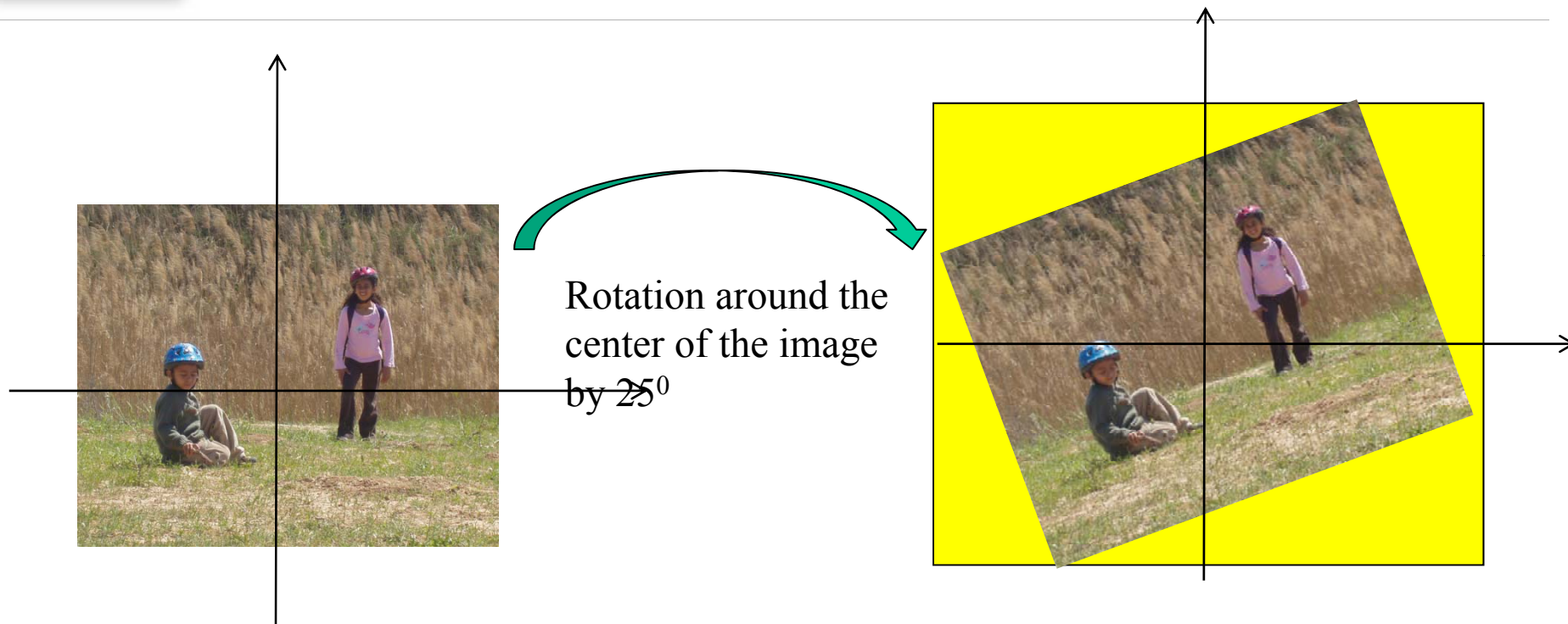


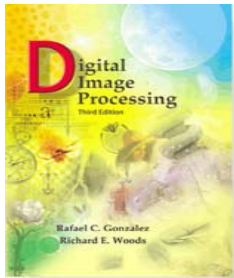
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Image Scale

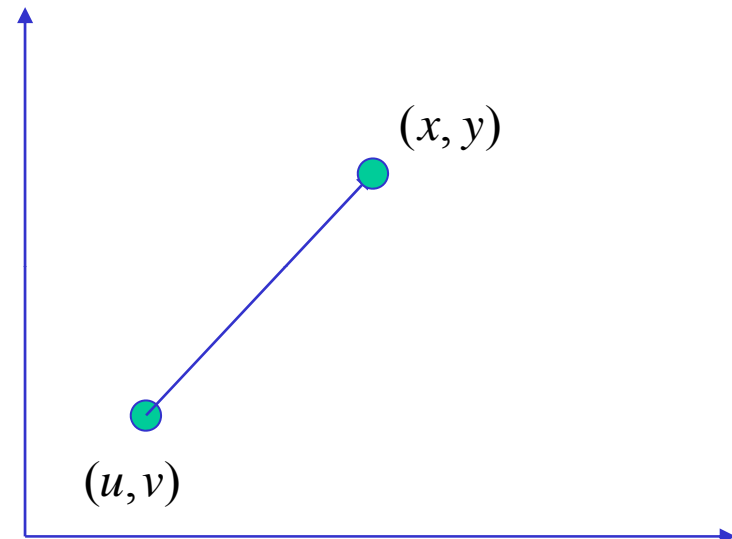
- Scale a region of pixels by a factor (s_x, s_y)
- Each pixel in this region $P(u, v)$ is “scaled” to a new position is $P'(x, y)$

$$x = u \cdot s_x$$

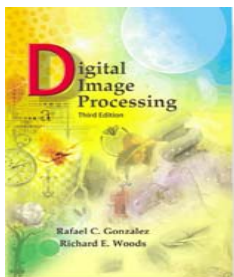
$$y = v \cdot s_y$$

$$(x, y) = (u \cdot s_x, v \cdot s_y)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$



The scale of include additional translation when no point of the scaled region is in the origin

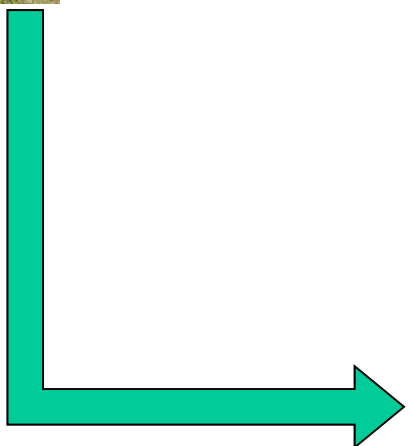
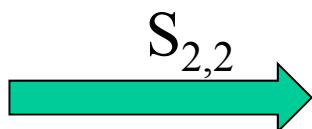


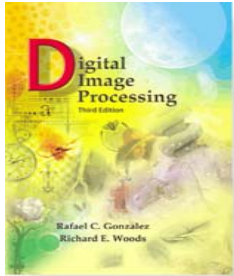
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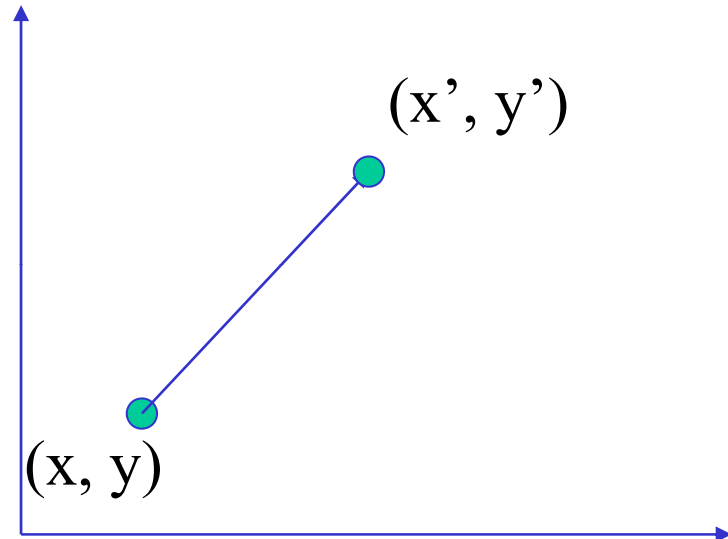
Image Sheer

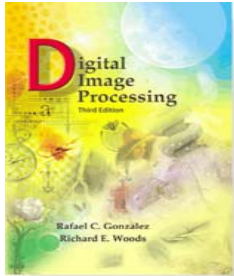
- Scale a region of pixels by a factor (S_{hx}, S_{hy}) .
- Each pixel in this region $P(u, v)$ is “Sheered” to a new position is $P'(x, y)$

$$x = u + v \cdot s_{hx}$$

$$y = v + u \cdot s_{hy}$$

$$(x, y) = (u + v \cdot s_{hx}, v + u \cdot s_{hy})$$





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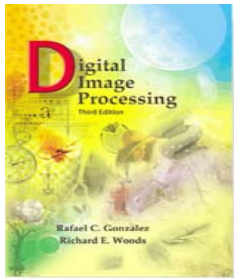
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Horizontal Shear





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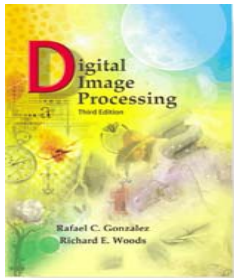
Homogenous Coordinates

Let us consider applying several transformation T_0, T_1, \dots, T_k on the same image.

$$T_k \{T_{k-1} \{ \dots \{T_1 \{T_0 \begin{bmatrix} u \\ v \end{bmatrix} \} \} \dots \} \} \quad \text{This requires } k \text{ matrix multiplication for each pixel}$$

If we can perform the following, where $T = T_0 T_1 \dots T_k$, we are required to perform one time k matrix multiplication and one matrix multiplication for each vertex

$$T_k T_{k-1} \dots T_1 T_0 \begin{bmatrix} u \\ v \end{bmatrix} = T \begin{bmatrix} u \\ v \end{bmatrix}$$



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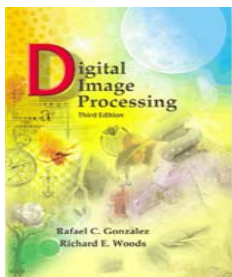
Example:

Let us consider the following transformation

$$S_{1,2}T_{5,3}R_{30}S_{7,5}T_{2,4}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} \cos(30) & -\sin(30) \\ \sin(30) & \cos(30) \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

It is impossible to multiply these matrices.

To overcome this limitation we use a homogeneous coordinate (x, y, w), where w=1. This also update all the spatial transformation to 3×3 matrices.



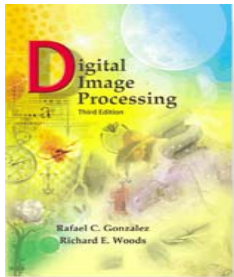
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Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v \cos \theta - w \sin \theta$ $y = v \sin \theta + w \cos \theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	



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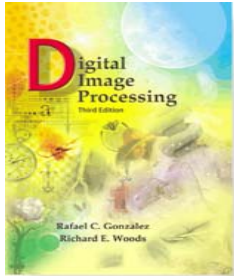
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FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.



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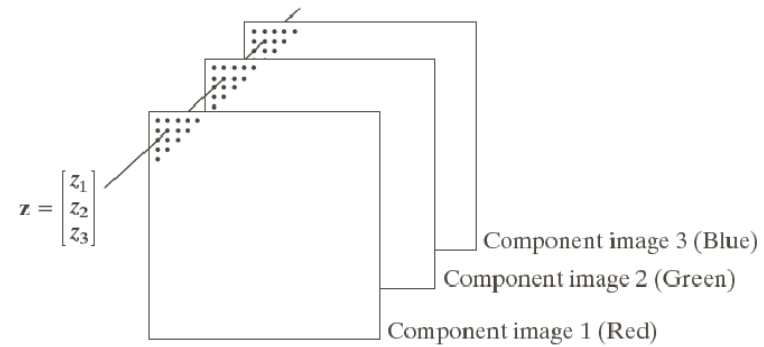
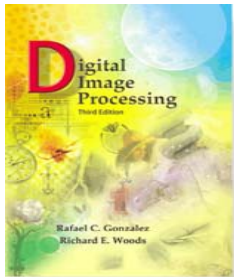


FIGURE 2.38

Formation of a vector from corresponding pixel values in three RGB component images.



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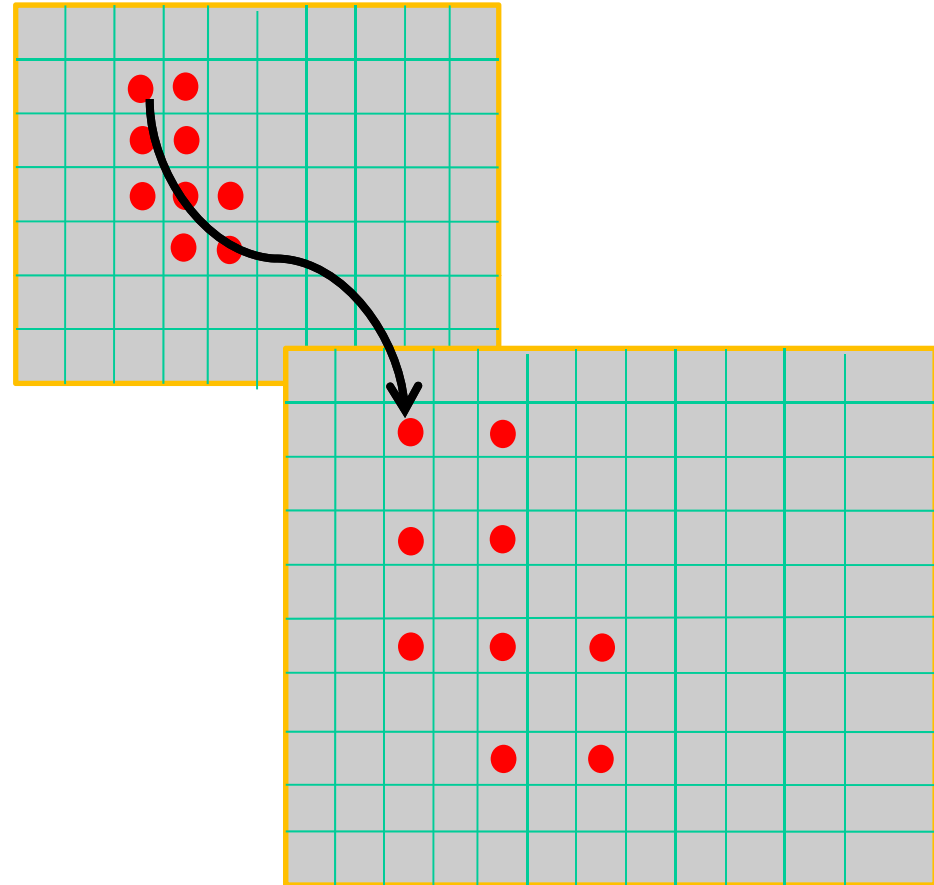
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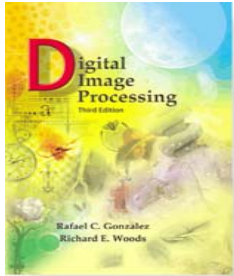
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Spatial Transformation

- Scaling an image by mapping each pixel to a target.
- In the resulting image
 - The number of pixels increases
 - Some pixels are “empty”
 - How to determine their values?





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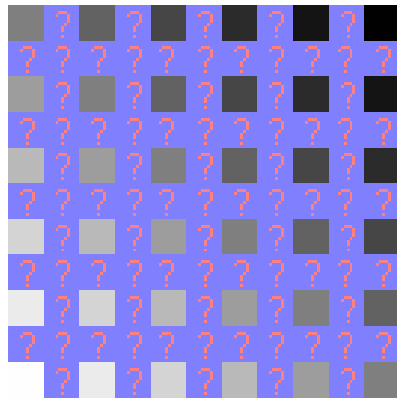
Gonzalez & Woods

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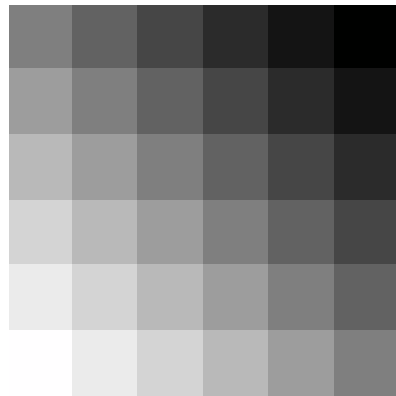
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Original
Image

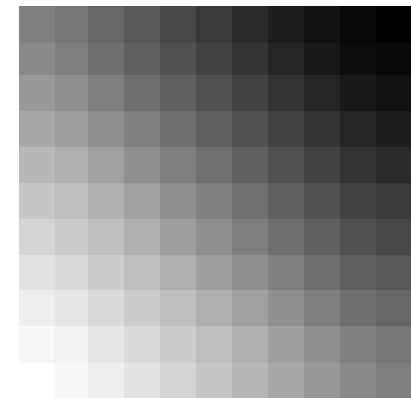


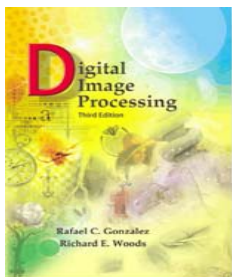
After Scaling



After Scaling with
Nearest Pixel

A better Result





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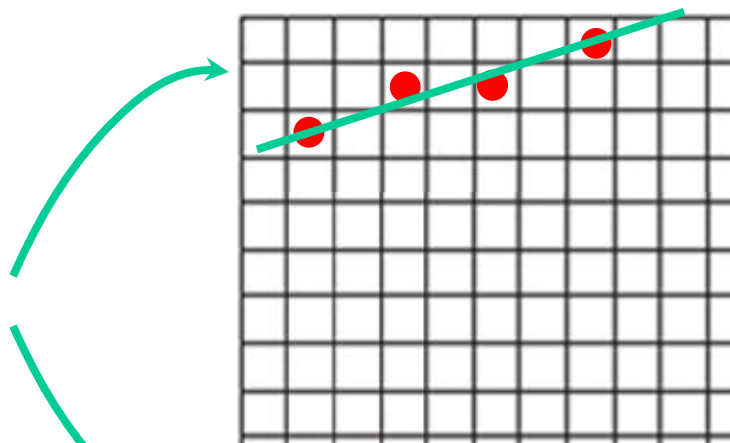
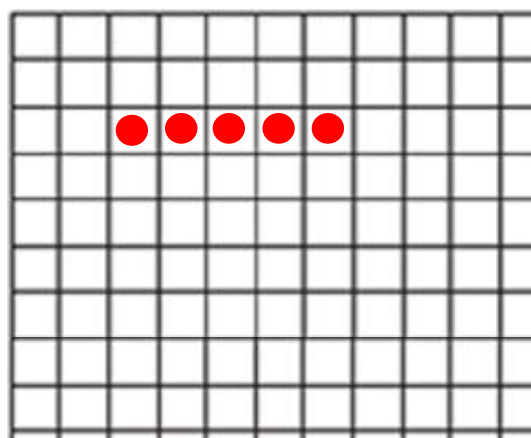
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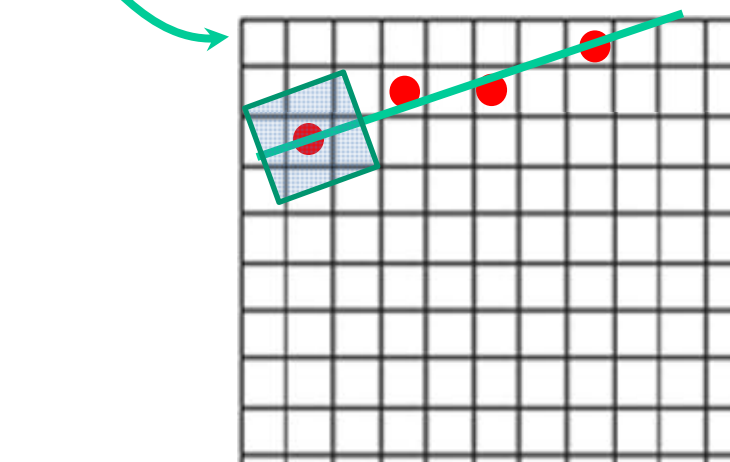
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Spatial Transformation - Forward Scanning

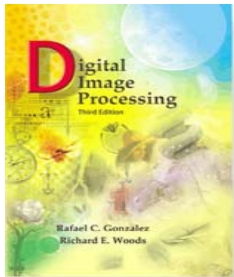
Scale
+
Rotate



In this approach missing pixel are clearly visible. To fill the missing pixel, one could replicate pixels. However, the quality of the resulting is image



Mapping the image of the square pixel covers all the pixels in the target image. The value of a pixel is determined by the mapped squares that cover it.



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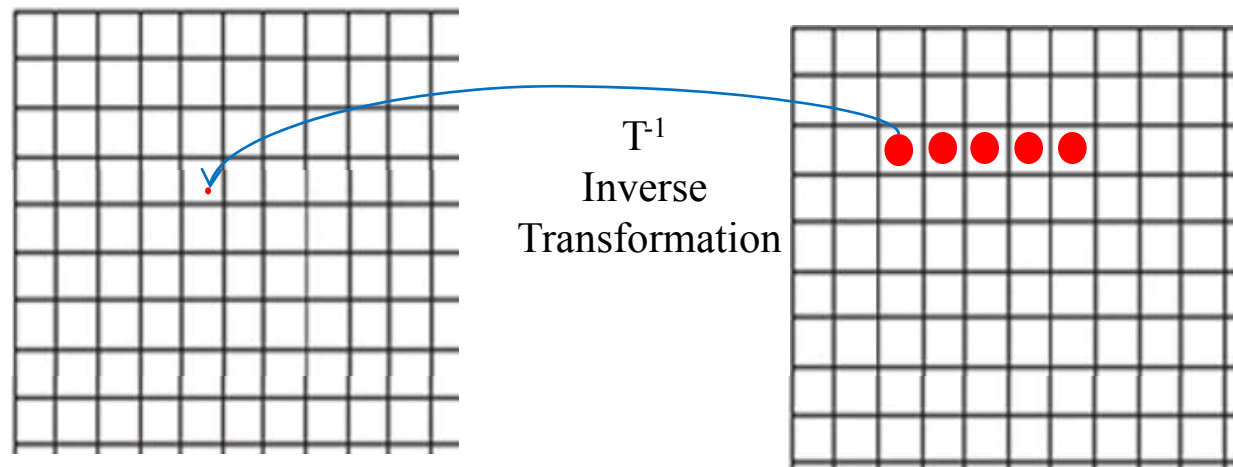
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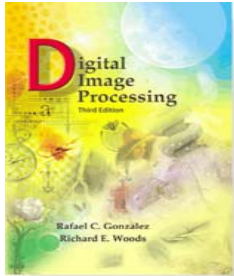
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Spatial Transformation - Backward Scanning

In this approach we fill the target image pixel by pixel. For each pixel we compute its source position. The value of the target pixel is computed by interpolating the pixels around the source position.



This approach does not need to take into account the type of transformation to determine the adjacent pixels of the source position



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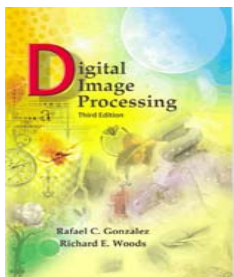
Interpolation

- According to Wikipedia

In the mathematical subfield of numerical analysis, interpolation is a method of constructing new data points within the range of a discrete set of known data points.

- In Image Processing

It is a techniques to assign values to pixels in an images based on their adjacent pixels, or adjacent samples that my not be correlate with pixel location in the image.



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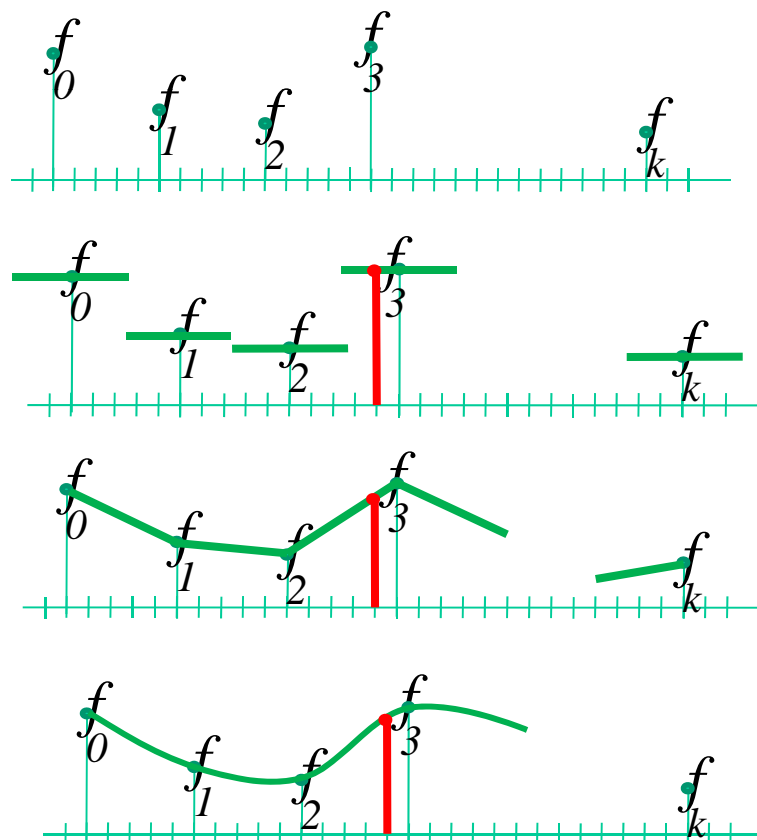
Gonzalez & Woods

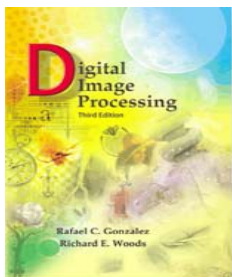
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Value Interpolation

- Let us assume we have several samples of a function f
 - $f^0, f^1, f^2, \dots, f^n$
- Nearest (Zero Order)
-
- Bilinear (First Order)
- Bicubic (Third Order)





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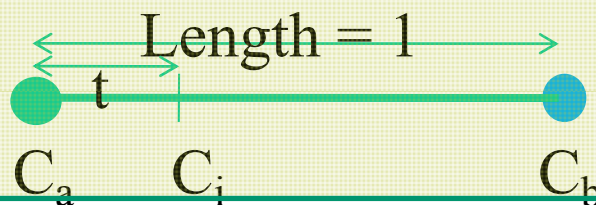
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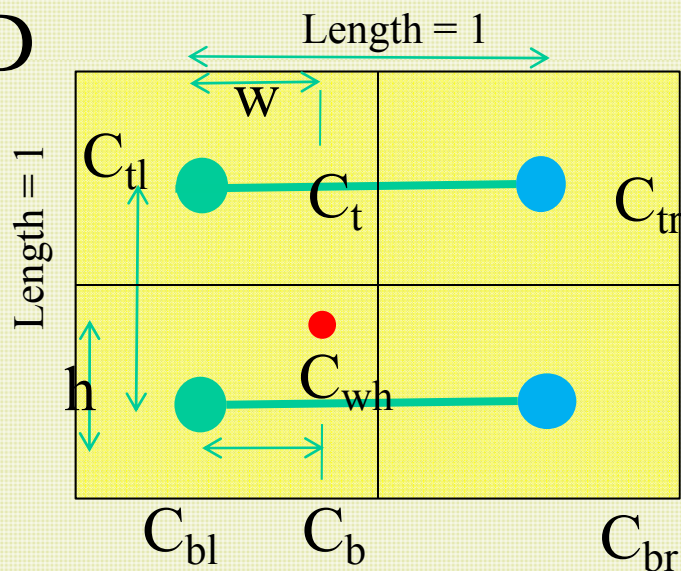
Bilinear Interpolation

1D



$$C_i = (1 - t)C_a + tC_b$$

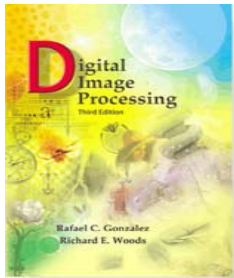
2D



$$C_t = (1 - w)C_{tl} + wC_{tr}$$

$$C_b = (1 - w)C_{bl} + wC_{br}$$

$$C_{wh} = (1 - h)C_b + hC_t$$



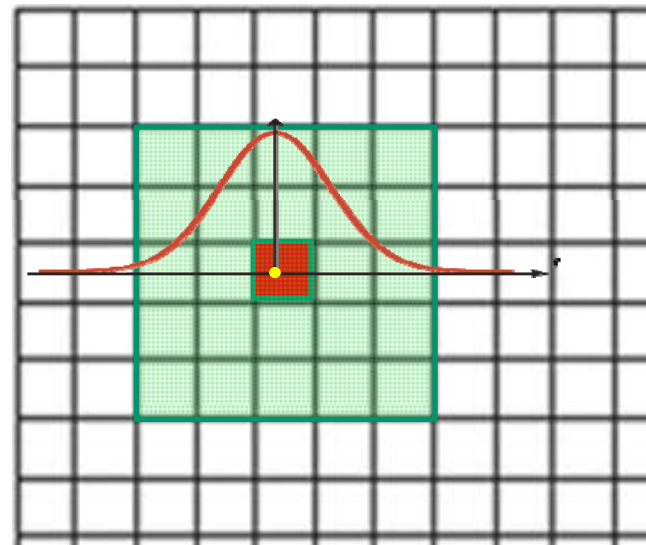
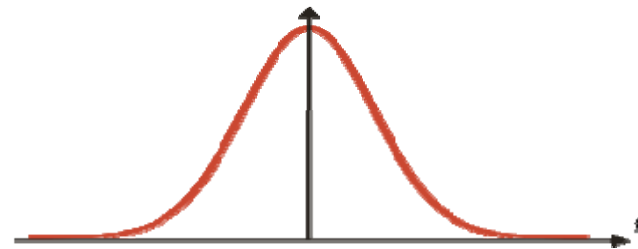
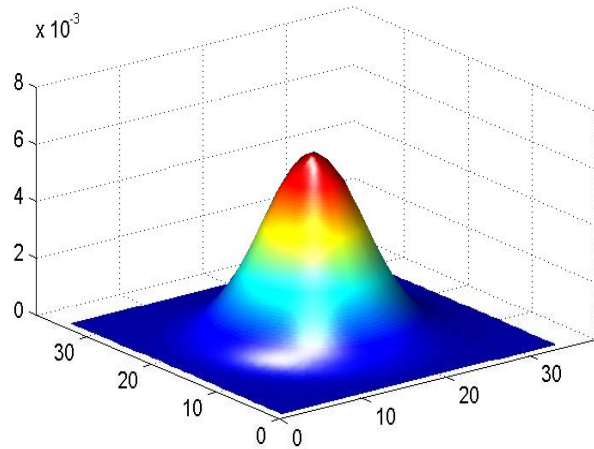
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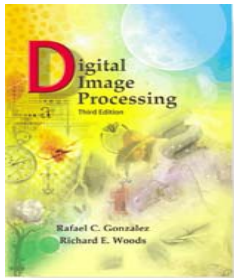
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Quadric and Cubic Interpolation





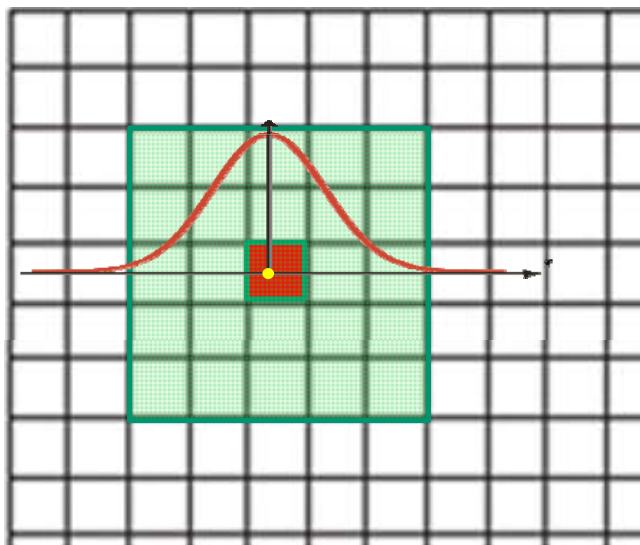
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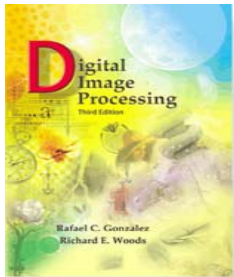
Quadric and Cubic Interpolation



The value of the sample position $f(x+dx, y+dy)$ is determined based on the following equation.

$$f(x+dx, y+dy) = \sum_{i=-1}^2 \sum_{j=-1}^2 f(x+i, y+j) C_a(dx+i) C_a(dy+j)$$

$$C_a(x) = \begin{cases} (a+2)|x|^3 + (a+3)|x|^2 + 1 & , \quad 0 \leq |x| < 1 \\ a|x|^3 + 5a|x|^2 + 8a|x| + 4a & , \quad 1 \leq |x| < 2 \\ 0 & , \quad 2 \leq |x| \end{cases}$$



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1D Quadric Interpolation

General Quadric Equation

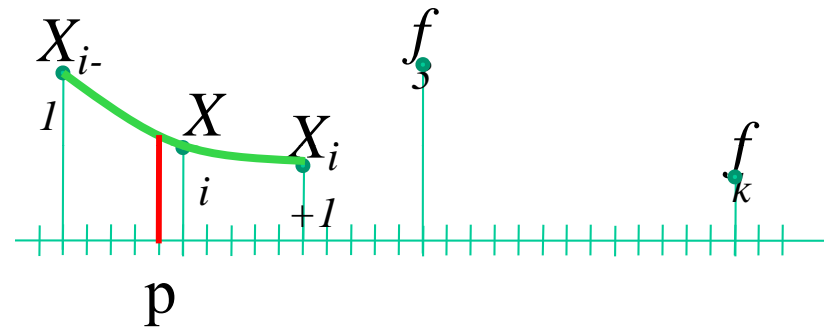
$$y = ax^2 + bx + c$$

Using the three points to determine the equation parameters

$$y_{i-1} = ax_{i-1}^2 + bx_{i-1} + c$$

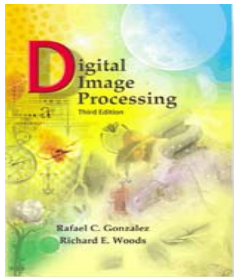
$$y_i = ax_i^2 + bx_i + c$$

$$y_{i+1} = ax_{i+1}^2 + bx_{i+1} + c$$



After determining the parameters of the quadric function, we plug in x_p to determine its value, y_p

Similarly we can derive the 1D cubic interpolation of a point x_p . But we need four points.



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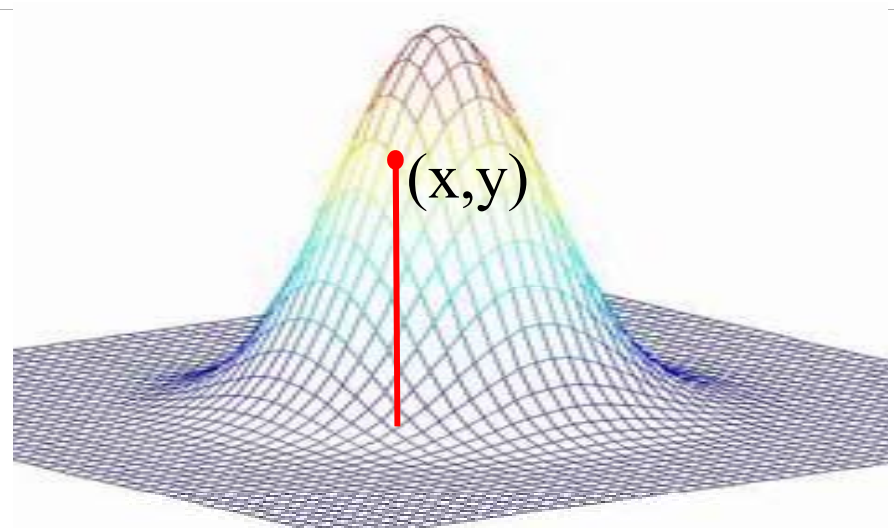
2D Cubic Interpolation

$$f(x, y) = C_x(x) * C_y(y)$$

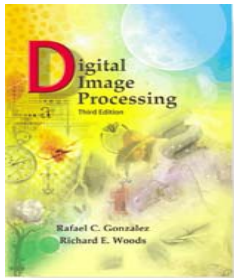
$$C_x(x) = a_x x^3 + b_x x^2 + c_x x + d_x$$

$$C_y(y) = a_y y^3 + b_y y^2 + c_y y + d_y$$

$$\begin{bmatrix} a_x a_y & a_x b_y & a_x c_y & a_x d_y \\ b_x a_y & b_x b_y & b_x c_y & b_x d_y \\ c_x a_y & c_x b_y & c_x c_y & c_x d_y \\ d_x a_y & d_x b_y & d_x c_y & d_x d_y \end{bmatrix} \Rightarrow \begin{bmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,1} & a_{3,1} \end{bmatrix}$$



We need to determine the value of the 16 parameters to compute the value of (x_p, y_p) . But we need four points.



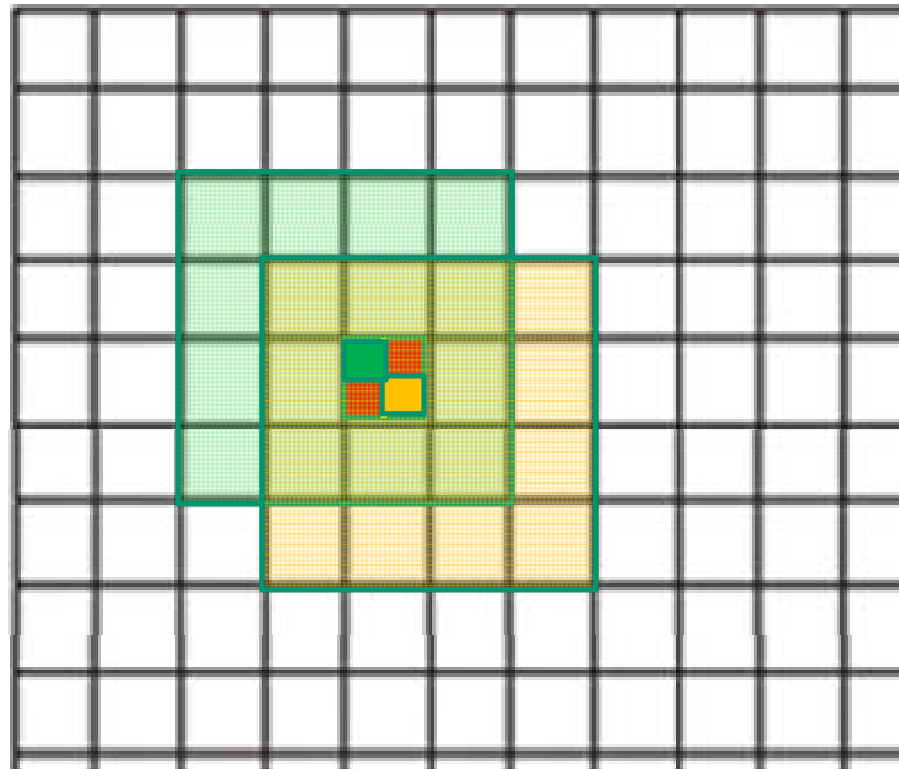
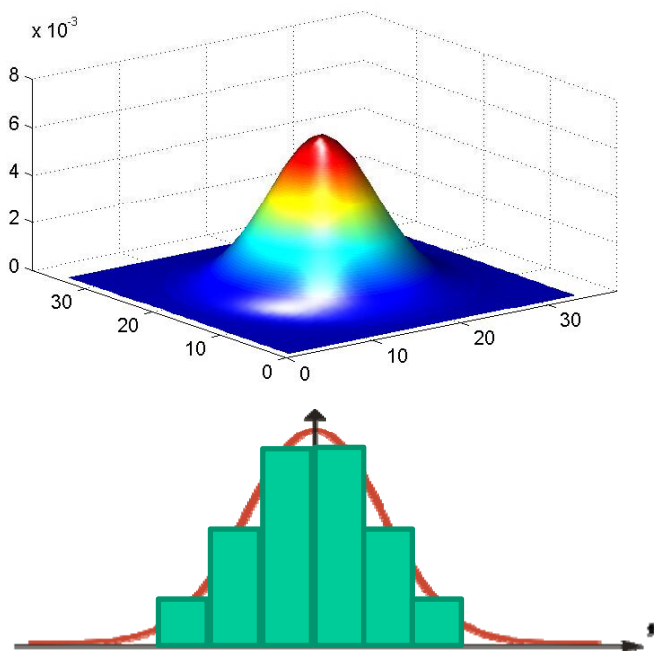
Digital Image Processing, 3rd ed.

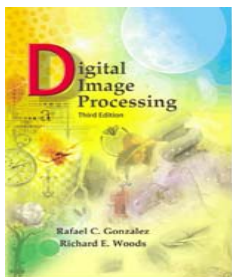
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Chapter 2 Digital Image Fundamentals

Approximating Cubic Interpolation





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Chapter 2 Digital Image Fundamentals

Visual Examples



Original Image



BiLinear



Nearest



BiCubic

