First Edition
First Printing, December, 1907

Second Edition
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Second Printing, March, 1910

Third Edition
First Printing, September, 1910
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Fourth Edition
First Printing, July, 1915
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information on each subject might be presented in reasonably compact form, and at the same time be easily located.

Thus each numbered paragraph opens with a descriptive title or phrase in bold-faced type, while in other respects the use of such type has been limited to subheads and important key words which should catch the eye upon a casual glance over the pages. An entirely new feature is the consistent use of subheads throughout each section and the grouping of these on each section title page, for the double purpose of describing the contents in some detail and serving as a ready guide to any particular subdivision or minor subject. This general scheme of presentation is not intended to relieve the necessity for a thorough and complete index, but rather to supplement the latter and make the book of maximum usefulness. Another new feature is the addition of bibliographies at the end of each section or subsection, and the insertion of numerous references throughout the text to more extended or specialized literature.

In retaining the sectional or unit system of arranging a reference work of this character, both the Editor and the Publishers are convinced from past results that there is no other form of arrangement which is so well suited to the production of a useful and convenient handbook, or which makes possible the segregation of all the material relating to each subject, presented in logical sequence and so displayed as to give it the desired prominence.

Sections 1 to 5 inclusive cover the same general ground as in the third edition, but have been almost completely rewritten and considerably extended.

Sections 6 to 9 inclusive embrace the same subjects as Sections 6 to 8 in the third edition, but conform to a revised classification which is believed to be preferable to the former arrangement. These sections have also been entirely rewritten and substantially enlarged.

Section 10 covers the same general subject matter as the corresponding section in the last edition, but is entirely rewritten and greatly enlarged.

Sections 11 and 12 cover the same ground as Section 11 in the third edition, but are entirely new and much more comprehensive.

Section 13 replaces Section 18 in the old edition, being entirely

PREFACE TO THE THIRD EDITION

The preface to the first edition of the STANDARD HANDBOOK which appears on another page, describes the "unit" system on which the work was developed. The present edition, the publishers believe, is somewhat of a triumph for this system. The thorough revision of a book of this size, when manufactured according to the usual plan, is commercially impossible except at long intervals when the changes in the art become so great as to demand an entirely new book. The "unit" system employed in the STANDARD HANDBOOK permits thorough revision in part or as a whole without any of the usual limitations.

In the present revision the authors of the various sections were allowed a free hand in so far as mechanical details were concerned. They were not restricted in space or compelled to cut and prune material to fit pages. The result is a book that has been thoroughly revised from cover to cover so that it could be fairly called a new STANDARD HANDBOOK.

The following synopsis gives a brief outline of the changes and additions that have been made to the various sections.

Section 1, Units, is corrected and slightly enlarged.

Section 2, Electric, Magnetic and Dielectric Circuits, is greatly enlarged. The general theory of electric and magnetic circuits is entirely rewritten and the calculation of inductance and capacity is given in greater detail than before.

Section 3, Measurements and Measuring Instruments, is greatly enlarged. Several new instruments are described, tests of self and mutual inductance have been added and a section devoted to pyrometers and high temperature measurements has been included. The design of rheostats and motor starters has been transferred to Section 5.

Section 4, Properties of Conductor, Resistor and Insulating Materials, is enlarged more than any other section. Many tables have been added giving data on the latest types of conductors and cables. An entirely new section giving the properties of a large number of commercial resistor alloys has also been added and the magnetic testing of iron has been entirely rewritten and now forms a very comprehensive treatment of the latest practice in this important subject.

Section 13, Traction, has been corrected, revised and enlarged. The locomotive section has been entirely rewritten, and more space has been given to the method of constructing speed-time curves.

Section 14, Electrochemistry, has been thoroughly revised and somewhat enlarged.

Section 15, Telephony, has been entirely rewritten. It is now a comprehensive treatise and represents a new method of presenting the subject.

Section 16, Telegraphy, is corrected.

Section 17, Miscellaneous Applications of Electricity, is corrected and somewhat enlarged.

Section 18, Wiring, is corrected and brought to date.

Section 19, Standards, is considerably enlarged. The latest changes in the A. I. E. E. Standardization Rules have been noted, and standard specifications for rubber insulation, copper conductors and transformers have been added.

Section 20, Tables and Statistics, has been corrected and enlarged by adding telephone, telegraph and central station statistics and by general revision.

first edition of a work containing such a mass of figures and data, although the greatest care has been exercised in its preparation. Any suggestions, criticisms, or corrections from users will be of great service in making THE STANDARD HANDBOOK a standard in fact as well as in name.

December 12, 1907.

PREFACE TO THE SECOND EDITION

No new material has been added to this edition of the STANDARD HANDBOOK, with the exception of directions for resuscitation from electric shock, which have been inserted at the end of the book. However, every page has been most carefully read and every possible effort made to insure the accuracy of all data and perfection of the typographical work. Several of the tables, which were especially prepared for this book, have been recalculated and others have been checked by plotting the values and recalculating those which did not fall on a smooth curve.

The success of the STANDARD HANDBOOK has been phenomenal. The general interest in the work has been manifested by the many letters received from prominent men commending its general character and offering suggestions and criticisms. It has already been adopted for use as a text-book in thirty universities and colleges.

The publishers take this occasion to express their appreciation of its reception by the profession, and to thank those who by their kindness in pointing out typographical and other errors, have materially assisted in the work of correction.

NEW YORK, *May*, 1908.

SECTION 7

ALTERNATING-CURRENT GENERATORS AND MOTORS
By Comfort A. Adams and Henry M. Hobart

Synchronous Machines, Armature Winding, E.M.F. Generation, The Magnetic Circuit, Characteristics of Synchronous Alternators, Synchronous Motors, Parallel Operation, Design, Insulation, Efficiency, Ventilation, Construction and Testing; Induction Machines, Theory of Polyphase Motors, Characteristics, Magnetizing Current, Leakage Reactance, Circle Diagram, Efficiency, Standard Polyphase Motors, Induction Generators, Design, Single-phase Motors and Their Speed Control; Commutator Motors, Auxiliary Commutating Machines, Phase Modifiers, Motor-generators and Frequency Changers.

SECTION 8

DIRECT-CURRENT GENERATORS AND MOTORS. By Alexander Gray, B.Sc., Whit. Sch.

Types, Windings, Armature Reactions, Commutation, Armature Design, Field Design, Construction, Insulation, Cooling, Efficiency, Characteristics, Regulation, Weights, Costs, Standard Machines, Thury System, Motor-generator Sets, Operation and Testing.

SECTION 9

CONVERTERS AND DOUBLE-CURRENT GENERATORS. By F. D. Newbury, M.E. and Alexander Gray, B.Sc., Whit. Sch.

Synchronous Converters, Theory, Design, Characteristics, Applications, Operation, Testing; Inverted Converters, Motor Converters, Direct-current Converters, Dynamotors and Double-current Generators.

SECTION 10

POWER PLANTS. By Reginald J. S. Pigott, Arthur T. Safford and George I. Rhodes

Steam Power Plants, Laws of Heat Transfer, Boilers, Furnaces, Stokers, Chimneys, Mechanical Draft, Fuel, Water Supply, Coal and Ash Handling, Engines, Turbines, Condensers, Heaters, Economizers, Pumps, Piping and Testing; Gas Power Plants, Producers, Superheaters, Condensers, Scrubbers, Purifiers, Holders, Properties of Gas, Engines, Piping and Testing; Oil Power Plants, Engines, Testing; Hydraulic Power Plants, Hydraulics, Flow Formulas, Stream Flow, Dams, Headworks, Water Wheels and Testing; Buildings and Foundations; Electrical Equipment, Generators, Excitation, Voltage Control, Switching, Station Transformers, Lightning Arresters and Wiring; Power-plant Economics.

SECTION 11

POWER TRANSMISSION. By Harry E. Clifford, S.B. and Chester L. Dawes, S.B.

Transmission Systems, Electrical Calculations, Tables of Reactance and Charging Current, Design, Corona, Insulators, System Connections, Switching, Spans and Supports, Construction, Cables, Substations, Operation, Economics and Cost Data.

SECTION 12

DISTRIBUTION SYSTEMS. By Harry Barnes Gear, A.B., M.E.

Classification, Applications, Types of Circuits, Circuit Design, Substations, Regulation, Secondary Distribution, Transformation, Protection, Construction and Economics.

SECTION 13

INTERIOR WIRING. By Terrell Croft

Fire Risk, Methods of Wiring, Wires and Cables, Fittings and Accessories, Calculations, Lay-outs, Installation, Protection and Miscellaneous.

SECTION 20

BATTERIES. By Walter E. Winship, Ph.D.

Primary Batteries, Wet Cells, Dry Cells, Storage Batteries; Lead Storage Batteries, Electrolyte, Testing, Stationary Batteries, Vehicle Batteries, Train-lighting Batteries, Miscellaneous Applications, Battery Rooms, Regulating Equipment, Operation, Depreciation and Maintenance; Alkaline Storage Batteries.

SECTION 21

TELEPHONY, TELEGRAPHY AND RADIOTELEGRAPHY. By Frank F. Fowle, S.B., and Louis W. Austin, Ph.D.

Telephone Instruments, Switchboards, Intercommunicating Systems, Phantom Circuits, Manual Telegraph Systems, Simplex and Composite Sets, Dispatching and Patrol Systems, Fire and Police Alarm Systems, Cables, Protectors, Cross-talk and Inductive Disturbances, Transmission, Construction, Testing; Radiotelegraphy, Antenna, Receiving Circuits, Detectors, Wave Transmission, Undamped Oscillations, Arc-wave Generator, Continuous Oscillations, Wireless Telephones, Directive Antennas and Measuring Instruments.

SECTION 22

MISCELLANEOUS APPLICATIONS OF ELECTRICITY. By the following specialists:

W. S. Hadaway, Jr.	Otis Allen Kenyon	John C. Bogle
Harry B. Gear	H. A. Hornor	Capt. Edw. D. Ardery
John E. Newman	Frank F. Fowle	Milton W. Franklin
Edwin P. Adams	Ernst J. Berg.	Eugene W. Caldwell, M.D.

Resuscitation, Electric Heating and Cooking, Electric-Welding, Electrical Equipment for Gas Automobiles, Thawing Water Pipes, Marine Applications, Electricity in the U. S. Army, Electricity and Plant Growth, Windmill Electric Plants, Ozone Production, Radioactivity and the Electron Theory, Roentgen Rays, Lightning Rods, Electrostatic Machines, Electric Piano Players, Telegraphone, Telharmonium, Train-lighting Systems, Statistics of the Electrical Industry, Specifications and Contracts.

SECTION 23

MECHANICAL SECTION. Compiled from standard authorities.

Elements of Sections, Beams, Columns, Shafting, Gearing, Chain Drives, Belts, Rope Drives, Pipe and Screw Threads.

SECTION 24

STANDARDIZATION RULES OF THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS. Approved edition of Dec. 1, 1914

SECTION 25

GENERAL ENGINEERING ECONOMICS AND CENTRAL STATION ECONOMICS. By Frank F. Fowle, S.B., and James Raley Cravath

General Engineering Economics, Definitions, Value, Price, Cost, Capital, Rent, Interest, Annual Charges, Depreciation, Social-economic Investigations, Valuation and Rate Making; Central Station Economics, Factors Relating to Utilisation of Investment, Factors Relating to Territory Served, Typical Earnings of Companies, Rate Making and Valuation.

SECTION 1

UNITS, CONVERSION FACTORS, AND TABLES

SYSTEMS OF UNITS

1. Nature of units. Engineering makes use of physical quantities in the broadest sense of that term, *i.e.*, including mechanical, chemical, physical, thermal and physiological quantities. In order adequately to compare the magnitudes of physical quantities of the same kind, unit magnitudes, or units, are necessary for each kind of physical quantity dealt with.

2. Classification of units. The subdivisions and species into which units may be divided are indicated in the scheme shown in Fig. 1, with explanations which follow in Par. 3.

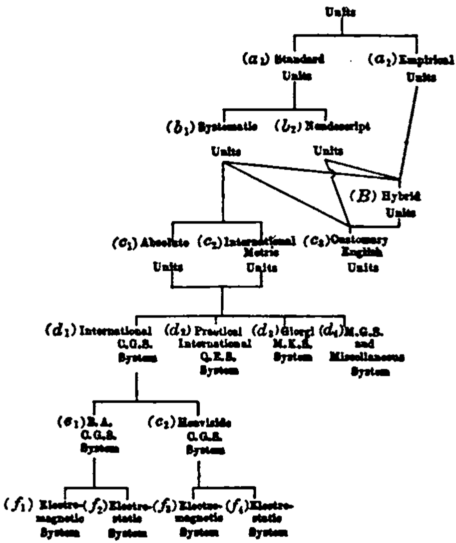

FIG. 1.

3(a_1) Standard units (see Fig. 1) may be said to include all units which have received the stamp of recognition in technical literature.

3(a_2) Empirical units, on the other hand, are units which have sprung into existence locally, ordinarily without any pretense to scientific derivation, and which have not been sanctioned by general usage. At various times during recorded history, empirical units have appeared. Thus, in the early history of electrical units, a unit of conductor resistance was used as representing the resistance of a certain length of a certain size of telegraph

3(c₃) **Customary English units** are the units of the English and American measures, viz., length measure, square measure, land measure, cubic measure, cord measure, dry measure, liquid measure, avoirdupois weight, troy weight, apothecaries weight and jeweler's weight. Each of these measures may be regarded as a system. The complete list may be regarded as a congeries of imperfectly connected systems. Empirical and hybrid units are mingled with the rest.

3(d_1) **C.G.S. units** are the units of that particular system of absolute units which is based on the **international centimeter**, the **international gram** and the **mean solar second**. That is, they are absolute units employing the metric system in a definite way. A reason for the centimeter having been selected in place of the meter as the fundamental unit of length was that the mass of the cubic centimeter of water (at the temperature of unit density) is the gram or unit mass; whereas the mass of a cubic meter of water would be a million grams.

3(d_2) **Q.E.S. units** are units pertaining to the quadrant-eleventh-gram-second absolute system; *i.e.*, the system in which the unit of length is 10^9 cm. or 1 theoretical quadrant of the earth as measured from a pole to the equator, the unit of mass is 10^{-11} g.,* and the unit of time the mean solar second, or the 1/86,400th part of the annual mean daily period of revolution of the earth with respect to the sun. This is the system to which the international ohm, volt, ampere, joule, watt, coulomb, farad and henry, belong. The system was not intentionally established as a Q.E.S. system; but the ohm having been arbitrarily selected, for convenience of magnitude, as 10^9 C.G.S. electromagnetic units, and the volt similarly as 10^8 C.G.S. units, the rest of the system necessarily coincides with the Q.E.S. system; or is such a system as would be produced by the selection of the quadrant, eleventh-gram and second as fundamental units, together with unity for the permeability and unity for the dielectric constant of the ether.

3(d_3) **Giorgi units** are units in a combined absolute and practical system devised by Prof. G. Giorgi,† in which the fundamental units are: the meter-kilogram-second-international ohm, and the further assumption that the permeability of free ether, instead of being unity as in the C.G.S. magnetic system, is $\mu_o = 4\pi \times 10^{-7}$ henry/m. On this basis the ohm-volt-ampere series of practical units become also absolute units. The electric inductivity, instead of being unity, as in the C.G.S. electric system, becomes $k_o = 1/36\pi \times 10^{-9}$ farad/m. No distinction arises in the Giorgi system between electric and magnetic units. The system is also rectified in regard to 4π factors, or is **"rationalized"** in the Heaviside sense; so that a number of fundamental equations in the system differ from those of the C.G.S. system in regard to such 4π factors.

3(d_4) **M.G.S. units, etc.** Units in an absolute system whose fundamental units are the meter-gram-second, the millimeter-milligram-second, the foot-grain second, etc. None of these extraneous absolute systems have come into extensive use.

3(e_1) **B.A. units** are the units of the C.G.S. system as established by the British Association for the Advancement of Science‡ in 1862. The electrostatic subsystem was established on the basis of the unit quantity of electricity such that it repelled its prototype at a distance of 1 cm. with a force of 1 dyne. The electromagnetic subsystem was similarly established on the basis of the unit magnetic pole such that it repelled its prototype at a distance of 1 cm. with a force of 1 dyne. This procedure led to the anomalous result that every electromagnetic quantity has a unit both in the electrostatic subsystem and in the magnetic subsystem.

3(e_2) **Heaviside units** are units in that form of the C.G.S. system which was first suggested by Mr. Oliver Heaviside in 1882.§ He showed that if a unit electric point charge and a unit magnetic point pole had been respectively defined such that unit total flux emanated therefrom, the strength of

* Maxwell, J. C. "A Treatise on Electricity and Magnetism," 1881, Chapter X.

† Ascoli, M. "On the Systems of Electrical Units," Trans. Int. Electrical Congress of St. Louis, 1904, Vol. I, p. 130.

‡ "British Association Report on Electrical Standards," 1863.

§ Heaviside, O. "The Relations between Magnetic Force and Electric Current." The Electrician, London, Nov. 18, 1882.

than another. The numerical values applying to any particular case covered by the equation will vary greatly according to the unit selected. If, however, any one of the terms is expressed in a particular unit, all the other terms must adopt the same unit. In all cases, however, it is helpful to the reader to have the unit of the equation written out at the end of its line, as above, in order to assist the numerical interpretation.

HISTORICAL SKETCH OF ENGLISH UNITS

5. The English weights and measures are based upon old Roman weights and measures.* The **troy pound** is supposed to have been a weight of silver referred to as a "pound sterling." This pound would be coined into 240 silver pennies or "pennyweights," each of 24 gr. (barley grain weights). It would, therefore, contain 5,760 gr. Heavy bodies (substances in gross outside of coins or bullion) were weighed by "avoirdupois" weight, authorised by law early in the fourteenth century. Several slightly different avoirdupois pounds were in use. Since Queen Elisabeth's reign the avoirdupois pound has been fixed at 7,000 troy grains.

6. In regard to British lengths, the earliest seems to have been the **cubit** or half yard. The cubit is a very ancient measure, and corresponds to a forearm length from elbow to middle finger-tip. The royal iron standard **yard** was constructed in the thirteenth century, after which the cubit or half yard gradually fell out of use. The **foot** was standardised at one-third of the yard. The mile was a relic of the Roman "millia passuum," or thousand paces; the Roman pace was two of our paces, or counted between the lifts of one and the same foot.

7. Gallon measures of volume existed at different times in England in six different forms, such as the corn-gallon, the ale-gallon, etc. Among these, the wine-gallon of Queen Anne contained 231 cu. in. This gallon was brought to America by the early colonists and remains to-day the U. S. gallon. In 1824, the British enacted a new "**imperial gallon**" to supersede all pre-existing gallons, and defined it as the volume of 10 avoirdupois pounds of distilled water at the temperature of 62 deg. fahr. with the barometer at 30 in. It was further defined as a measure containing 277.274 cu. in. of distilled water. There is thus a **difference between British and American gallons** in the ratio 277.274 to 231 = 1.204 : 1; so that the British gallons, quarts, and pints are respectively about 20 per cent. larger than American gallons, quarts and pints, a large discrepancy that has frequently led to misunderstandings.

8. In land measure, since Anglo-Saxon times, a "**perch**" or "**pole**" was 11 **cubits** in length = $16\frac{1}{2}$ ft., and such a pole was the **surveyor's unit.** A length of 40 perches was a **furlong,** and 8 furlongs the **statute mile.** An **acre** of land was the area of a rectangular strip a furlong in length and 4 perches in breadth, which breadth was known as the "**acre's breadth.**" An acre therefore included $40 \times 4 = 160$ sq. perches. Eight such strips end to end made the statute mile, and 80 such strips side by side made a statute mile breadth; so that a **square statute mile** contained 640 acres. Early in the seventeenth century, Prof. Edmund Gunter of Gresham College decimalised acre measure by inventing a 100-link "**chain**" of outstretched length equal to 4 perches or the acre's breadth (66 ft.). The acre thus became 10 sq. chains.

HISTORICAL SKETCH OF THE INTERNATIONAL METRIC SYSTEM

9. Prior to 1790, differences existed between the weights and measures of different Departments of France. Reform in the directions of simplification and unification was promised in a decree of the National Assembly under the sanction of Louis XVI in 1790. The metric system was actually developed under the authority of the French Republic in 1793, in the hands of a committee of scientists and engineers.

10. The decimal system, at the base of the metric system, was originally extended to angles and to time, the right angle being divided into 100 grades, each subdivided into 100 min. and again into 100 sec. The day was divided into 10 hr., each subdivided into 100 min. and again into 100 sec. The decimal subdivision of time never came into extended effect, and the decimal subdivision of angles has only been used to a limited extent.

* Watson, Sir C. M. "British Weights and Measures." London, 1910.

18. In 1882, an international commission met at Paris and adopted a length of 106 cm. as the length of the mercury column defining the ohm, as a closer approximation to the true ohm than the B.A. ohm. This 106-cm. ohm was called the "**legal**" **ohm**, as distinguished from the B.A. ohm. Legal ohms, volts, etc., have at the present date almost completely disappeared. They represented an intermediate stage of approximation to the present international unit values.

19. In 1889, an international electrical congress at Paris adopted the joule, the watt, and the quadrant, as the practical units of energy, power and inductance, respectively.

20. Edinburgh conference. In 1892, a conference was held in connection with the B.A. meeting at Edinburgh. It was then decided to adopt 106.3 cm. as the length of mercury column whose resistance should embody the ohm.

21. In 1893, the international electrical congress of Chicago adopted the 106.3-cm. ohm, which was called the **international ohm.** The other units of the practical system adjusted in conformity to this value were called correspondingly the **international ampere, volt, coulomb,** etc. The name of the unit of inductance was changed from the quadrant to the **henry,** in honor of the American physicist of that name.

22. In 1900, an international electrical congress at Paris, after some debate, adopted the **maxwell** as the unit of magnetic flux and the **gauss** as the unit either of magnetic intensity or of flux-density in the C.G.S. magnetic system.

23. In 1908, an international commission at London considered the **order of sequence of** resistance, current and voltage **standards,** which had been left indefinite at preceding congresses. It was decided that the **ohm** should be the **first unit,** and the **ampere the second,** as determined by the rate of electrodeposition of silver under specified conditions. The volt was to be determined from the ohm and ampere.

DEFINITIONS OF FUNDAMENTAL UNITS

24. Length. (*L.*) Linear distance between any two points. The unit of length in the metric system is the **meter,** in the C.G.S. system the **centimeter,** in the customary system it is any one of the following:—inch, foot, yard, pole, furlong, statute mile, nautical mile.

The fundamental unit of length of the United States is the international meter, the primary standard of which is deposited at the International Bureau of Weights and Measures near Paris, France. This is a platinum-iridium bar with three fine lines at each end; and the distance between the middle lines of each end when the bar is at the temperature of 0 deg. cent., and is supported at the two neutral points 28.5 cm. each side of the centre is 1 m. by definition. Two copies of this bar (prototype meters) are in the possession of the United States and are deposited at the Bureau of Standards.

The United States yard is defined by the relation

$$1 \text{ yd.} = 3600/3937 \text{ m.}$$

The legal equivalent of the meter for commercial purposes was fixed as 39.37 in. by the law of July 28, 1866, and experience having shown that this value was exact within the error of observation, the United States Office of Standard Weights and Measures was, by executive order in 1893, authorized to derive the yard from the meter by the use of this relation.

25. Mass. (*M.*) The quantity of matter in a body is estimated either by its inertia or by **its** weight. In the metric system, the unit of mass is the **gram,** which was originally defined as the mass of a cubic centimeter of distilled water at 0 deg. cent., although in practice it is taken as the thousandth part of a standard kilogram. In the customary system, the unit is ordinarily any one of the following: avoirdupois grain, ounce, pound, or ton (long or short); occasionally, it is one of the Troy system (ounce, pound). In the use of drugs, it is usually stated in apothecaries weight. The mass of precious stones is commonly estimated in carats.

26. Time. (*T.*) The interval elapsing between any two events. In the C.G.S. system, the unit of time is the **mean solar second,** or 86,400th part of the mean solar day. In the customary system, it is either the second, minute, hour, day, week or year of mean solar time.

a **poundal** being the force which acting on a pound mass for 1 sec., develops in it a velocity of 1 ft. per sec. A pound weight is equal to 32.2 poundals. But if we consider a pound to be a force, represented by a weight W,

$$F = \frac{W}{g} \cdot \frac{v^2}{r} \qquad (\text{* pounds force}) \qquad (8)$$

It is evident that there is no difference between the two contrasted modes of presenting the facts, provided that we distinguish carefully between a "**pound-mass**" and a "**pound-force.**" If, however, we use the same word "pound" to do duty in the two cases, contradictory and illogical results may be obtained.

It follows that the terms **gram, kilogram, pound, ton,** etc., are susceptible of either of **two distinct meanings**; namely, a **unit of mass** of matter or a **unit of force** equal to the gravitational force exerted on that mass by the earth. Confusion can be avoided in all cases, however, by using distinguishing terms, as "gram-mass," "gram force," or "gram weight."

32. Linear velocity (v). Rate of movement along a line, and ordinarily along a straight line; also, time rate of change of space. The unit in the C.G.S. system is the centimeter-per-second, in the metric system the meter-per-second, or per minute or per hour. In the customary system, it would be any of the customary English units of length per second, minute, hour or day, etc. Velocities may be either + or − with respect to a selected point on the line of motion.

33. Linear acceleration (a). Time rate of change of linear velocity. The C.G.S. unit is the **(cm. per sec.) per sec.**; or the cm. per sec.^{2}. The metric unit may be a meter per sec.^{2}, or a meter per hour^2, or any decimal derivative of the meter, per square of the second, minute, hour, etc. A useful hybrid unit is the (kilometer per hour) per second. Accelerations may be either + or −.

34. Plane angle (α, β, γ). In plane circular trigonometry, the ratio of a circular arc to its radius. The C.G.S. unit is the **radian**, or 1 cm. of arc drawn with a radius of 1 cm. The metric unit is the **grade** or one-hundredth of the quadrant with unit radius. The customary unit is either the degree—one-ninetieth of the unit-radius quadrant—or the revolution of four quadrants.

35. Angular velocity (ω). In plane circular trigonometry, the time rate of change of angle at any given instant. The C.G.S. unit is the **radian per second**. The customary unit is either the degree per second, or the revolution per second, or per minute, etc. Angular velocities may be either + or −.

36. Angular acceleration. In plane circular trigonometry, the time rate of change of angular velocity. The C.G.S. unit is the **radian per second per second.** Customary units are the degree per sec.^{2}, the revolution per sec.^{2}, or per min.^{2}, etc.

37. Energy (W). The capacity of doing work. Energy may be considered as the fundamental entity in terms of which all dynamical quantities may be defined. In the C.G.S. system, the unit is the **erg, or dyne-centimeter.** In mechanics, it may be any product of a unit weight and unit. distance such as kilogram-meter, foot-pound, etc., according to the system. An industrial unit in the meter-kilogram-second system is the **watt-hour.**

38. Power (P). Activity or the rate of working. The rate of expending energy. The C.G.S. unit is the **erg per second.** The metric gravitational unit is the gram-meter per second, or a decimal derivative, such as the kilogram-meter per second. The absolute unit in the meter-kilogram-second system is the **watt.** The customary unit is the **foot-pound per second,** or the **horse-power** of 550 ft.-lb. per sec. It may be either local or standard.

39. Momentum. The product of the mass of a body and its velocity. The C.G.S. unit is the **gram-centimeter per second.** A customary unit is the pound-mass × (foot per second).

40. Torque (τ). Twisting effort. The moment of a twisting couple ordinarily exerted about a shaft axis. The C.G.S. unit is the **dyne-perpendicular-centimeter**; *i.e.*, a dyne acting at right angles to a radius arm 1 cm.

* Prof. W. J. M. Rankine. "Applied Mechanics," 9th edition, 1877, page 491.

E.m.f. may be reckoned for a complete circuit or for any portion thereof; that is, each and every portion of a closed circuit in the steady state obeys Ohm's law.

48. Potential difference (U or V). A condition in virtue of which an electric current tends to flow from a place of higher to a place of lower potential. The numerical measure of the potential difference is the work done on a unit quantity of electricity in passing between the two points. The practical unit is the **volt.** The C.G.S. units are the abvolt and statvolt.

49. Potential gradient. The space rate of change of potential, or the change with respect to distance. An electric potential gradient is the space rate of change of electric potential, and similarly for magnetic, thermal or gravitational potential. The systematic unit in the practical system is the **volt per quadrant**, but a hybrid unit such as **volt per centimeter** is generally used. The C.G.S. unit is either the **abvolt or statvolt per cm.**

50. Electric current (I). The rate at which electricity flows through a conductor or circuit. The practical unit is the **ampere**, which is a current of one coulomb per second. The C.G.S. unit is either the **absampere or statampere.**

51. Electric current density. The ratio of the current flowing through a conductor to the cross-sectional area of that conductor. More strictly, the current density at a point in a conductor is the ratio of the current through a very small plane element of section containing the point and perpendicular to the current, to the area of the element. The systematic practical unit is the **ampere per square quadrant.** In practice, a hybrid unit is preferred such as the **ampere per square centimeter** or square inch. The C.G.S. unit is either the absampere or statampere per square centimeter.

52. Electric resistance (R). Obstruction to electric flow. The ratio of voltage to current in a conductor or closed circuit. The practical unit is the **ohm.** The C.G.S. unit is either the **absohm or statohm.**

53. Electric resistivity (ρ). The ratio of potential gradient in a conductor to the current density thereby produced. Also the specific resistance of a substance numerically equal to the resistance offered by a unit cube of the substance as measured between a pair of opposed parallel faces. The systematic practical unit is the **ohm-quadrant** or numerically equal to the resistance in a cubic earth-quadrant. A hybrid unit such as the **ohm-cm.** is usually preferred. The C.G.S. magnetic unit is the absohm-cm.

54. Electric conductance (G). The conducting power of a conductor or circuit for electricity. The inverse or reciprocal of electric resistance. The practical unit is the **mho.** The C.G.S. unit is either the **abmho** or the **statmho.**

55. Electric conductivity (γ). The specific electric conducting power of a substance. The reciprocal of resistivity. The systematic practical unit is the **mho per quadrant.** A hybrid unit, such as the **mho per cm.** is usually preferred. The C.G.S. magnetic unit is the **abmho per cm.**

56. Inductance (L). The capacity for electromagnetic induction possessed by an active circuit either on itself or on neighboring circuits. The ratio of the magnetic flux linked with and due to an active conductor (number of turns × total flux) to the current strength carried. The practical unit is the **henry.** The C.G.S. units are the **abhenry** and **stathenry.** The term "inductance" seems to have been first introduced by Heaviside* as a brief equivalent for "coefficient of self-induction." Inductance may be divided into two species; namely, **self-inductance** and **mutual inductance.** The unit is the same for both species.

57. Electric capacity (C). Sometimes called **permittance** or **capacitance.** The power of storing or holding an electric charge. The ratio of an electric charge on a conductor to the electric potential difference producing the charge. The practical unit is the **farad.** The C.G.S. unit is either the **abfarad** or the **statfarad.** The term "permittance" was introduced by Heaviside.† It should be noted that capacitance is used by a few writers as synonymous with capacity-reactance.

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* Heaviside, O. "The Electrician," 1884, May 3, p. 583; also "Electrical Papers," Macmillan Co., 1892, Vol. 1, p. 354.

† Heaviside, O. "Electrical Papers," 1892, Vol. II, pp. 302 and 327.

72. Magnetic flux-density (𝔅). The ratio of the magnetic flux in any cross-sectional element of a magnetic circuit to the area of that element. The C.G.S. magnetic unit is the **gauss**, which is also a maxwell per square centimeter.

73. Magnetomotive force (m.m.f.). That which produces magnetic flux. The analogue in the magnetic circuit of electromotive force in the electric circuit. No name has been provided for the unit of m.m.f. either in the practical or in the C.G.S. magnetic system. The name of **gilbert** has, however, been suggested for the latter. A convenient practical unit is the **ampere-turn** which is $4\pi/10$ gilberts.

74. Magnetic field intensity (ℋ) or **gradient of magnetic potential**, also termed **magnetizing force.** The rate of change of magnetic potential with respect to distance. In a region of unit permeability, the field intensity is numerically equal to the magnetic flux density. The provisional name of the C.G.S. magnetic unit is the **gilbert per centimeter.** A numerically related hybrid unit is the **ampere-turn per centimeter.**

75. Reluctance (ℛ). Obstruction to magnetic flow. In a simple magnetic circuit, the ratio of the m.m.f. to the magnetic flux. A provisional name for the C.G.S. magnetic unit is the **oersted.** One gilbert m.m.f. acting on a magnetic circuit of one oersted reluctance produces one maxwell of flux.

76. Reluctivity (ν). A specific reluctance, numerically equal to the reluctance of unit cube of a substance between any pair of opposed parallel faces. The C.G.S. magnetic unit is the **oersted-cm.**

77. Permeance. The reciprocal of reluctance. Conducting power for magnetic flux. No name has been adopted for this unit.

78. Permeability (μ). The reciprocal of reluctivity, or **specific permeance.** No name has been adopted for this unit. In the dimensional formulas of the C.G.S. system, if the electric and magnetic constants of the æther are considered as mere numerics; both permeability and reluctivity are also mere numerics. Also magnetic intensity has the same dimensions as flux density;* so that on this basis, which was at one time undisputed, there would be no difference between gilberts-per-centimeter and gausses except numerically. It is now generally admitted,† however, that the electric and magnetic constants of the æther should not be taken as mere numerics; although their dimensional formulas are not defined. On the latter basis, there is a dimensional difference of some kind between magnetic intensity in gilberts-per-centimeter and flux-density in gausses. The permeability can also be expressed $\mu = 1 + 4\pi\kappa$ where κ is the **susceptibility.**

79. Names for the units in the C.G.S. magnetic and electric subsystems. Although the practical ohm-volt-ampere series of units is universally employed in the great majority of electrical applications, yet it is sometimes desirable to use the C.G.S. parent system of units and names for such units have only been assigned authoritatively in a few instances, such as the "dyne" for the unit of force, and the erg for the unit of work. It has been suggested‡ that the C.G.S. magnetic units might be distinguished from their prototypes in the practical system by the **prefix ab- or abs-** and also that the C.G.S. electrostatic units might be similarly distinguished by the **prefix abstat- or stat-**, as indicated in the following table, Par. **80.**

It should be borne in mind that the prefixes "ab" and "stat" have never been authorised by any technical society or institution, and terms bearing these prefixes are therefore technically irregular. The excuse for this irregularity is that no proper terms exist by which to describe these units, since the phrases "C.G.S. magnetic unit," or "C.G.S. electric unit," are cumbersome and insufficiently descriptive. Moreover, there can be no ambiguity concerning the meaning of these irregular terms.

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* Maxwell, J. C. "A Treatise on Electricity and Magnetism." 1881. Vol. II, p. 244.

† Rücker. Phil. Mag., Feb., 1889.

‡ Trans. A. I. E. E., July, 1903, Vol. XXII, p. 529. Franklin, W. S. "Electric Waves," New York, Macmillan Co., 1909, p. 67. Hering, C. "Conversion Tables," New York, John Wiley & Sons, 1904.

Pender, Harold. "American Handbook for Electrical Engineers," New York, John Wiley & Sons, Inc., 1914.

DEFINITIONS OF PHOTOMETRIC UNITS

81. Luminous flux (F), (light) is the physical stimulus produced by radiation, which excites vision. It is proportional to the rate of flow of radiant energy and to a **stimulus coefficient** which depends chiefly on the spectral distribution of that energy. The stimulus coefficient for radiation of a particular wave-length is the ratio of the luminous flux to the radiant power producing it. The conventional unit of luminous flux is the **lumen** or the flux emitted by **one international candle** through one **steradian.**

82. Luminous intensity (I), or candle-power. The luminous intensity of a point source of light is the solid-angular density of the luminous flux emitted by the source in the direction considered; or it is the flux per steradian in that direction. The conventional unit is the candle-power, or the (candle) lumen per steradian.

83. International candle. A standard of luminous intensity, conventionally equal to the **bougie decimal**, maintained between the national laboratories of England, France and America through the medium of groups of standard incandescent lamps seasoned and intercompared. The intensity given by this standard is the conventional unit or candle.

84. True specific luminous intensity (b_0) of an element of a luminous surface is the ratio of the luminous intensity of the element, taken normally, to the area of the element. The conventional unit is the **candle per square centimeter**; or the lumen per sq. cm.

85. Apparent specific luminous intensity, or brightness (b), of an element of a luminous surface, from a given position, is the luminous intensity per unit area of the surface projected on a plane perpendicular to the line of sight, and including only a surface of dimensions small in comparison with the distance from the observer. The conventional unit is the **candle per square centimeter of projected area**; or the apparent lumen per sq. cm. For luminous surfaces obeying Lambert's law, or the "cosine law," the true and the apparent specific luminous intensities are equal. In practice, the apparent specific intensity is ordinarily observed. It has been proposed to call a brightness of one apparent lumen per sq. cm. one "lambert".

86. Illumination on a surface (E) is the luminous flux-density over the surface, or the flux per unit of intercepting area. The practical unit is the **lumen per square foot** or the **foot-candle.** The conventional unit is the **lumen per square centimeter** which has been termed the **"phot"** by Blondel. It is a cm-candle. The meter-candle, or 10^{-4} phot, is sometimes called the candle-lux. The milliphot (10^{-3} phot = millilumen per square centimeter) is roughly equal to a foot-candle, since 1 foot-candle = 1.0764 milliphots.

DEFINITIONS OF THERMAL UNITS

87. Temperature. The thermal condition of a body considered with reference to its capability to communicate heat to other bodies. Bodies at the same temperature do not communicate heat to one another at their bounding surfaces. The conventional unit is the **degree centigrade.** Other units in practical use are the degree **fahrenheit**, and occasionally the degree **réaumur.**

88. Quantity of heat. The amount of heat energy contained in a body or transferred from one body to another, by virtue of which temperatures are established or changed. Since heat is a form of energy, a quantity of heat may be expressed in units of energy of any kind. Two types of units are employed, one thermal, the other dynamical. As thermal units the C.G.S. unit is the **"lesser-calorie"** or **"therm"** or **"water-gram-degree centigrade,"** *i.e.*, the quantity of heat required to raise 1 g. of water 1 deg. cent.; and as this differs slightly with the temperature, the interval from 15 deg. to 16 deg. cent. is given in the definition. A larger decimal multiple of this unit, called the **"greater calorie"** or **"kilogram-calorie"** is much used and is equal to 1,000 lesser calories. A practical unit is the **"British thermal unit"** (B.t.u.), or the heat required to raise 1 lb. of water 1 deg. fahr. Dynamic units are the erg, the joule, the watt-hr., etc.

The dimensions of velocity are therefore LT^{-1}, or the first positive power of length and the first negative power of time. Since mass does not appear in this dimensional formula we may write the formal dimensions of velocity as $L^1M^0T^{-1}$. The three exponents 1, 0 and -1 completely define the nature of velocity in any absolute system whose fundamental units are length, mass and time. Moreover, the dimensional formula of a unit assigns at once the size of a unit when systems employing different fundamental units are compared. Thus if we should compare the unit of velocity in the C.G.S. system with that, say, in the meter-kilogram-day system; then in the latter the unit would be the meter per day while in the former it would be the centimeter per second.

100. Taking the more complex case of the magnetic unit of, say, current-density in a system whose fundamental units are those of both the practical and C.G.S. systems; namely, length, mass, time, magnetic æther constant μ, and dielectric æther constant k. Then the dimensional formula of current-density is $L^{-\frac{3}{2}}M^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$. If now we compare the size of the practical unit with that of the C.G.S. unit the former has a unit length of a quadrant or 10^9 cm., and a mass unit of 10^{-11} g. Consequently, the size of the practical unit is to the size of the C.G.S. unit in the ratio $(10^9)^{-\frac{3}{2}} \times (10^{-11})^{\frac{1}{2}} = 10^{-19}$; so that the practical unit, the ampere per square quadrant, is less than the C.G.S. unit or absampere per square centimeter in the ratio 10^{-19}. For practical purposes, we should probably ignore the systematic practical unit of current density, the ampere per square quadrant, and select a hybrid unit, say the ampere per square centimeter or per square inch. By such a departure from the absolute system, however, the fundamental equations of the system involving lengths, areas, or volumes, may become erroneous unless we introduce compensating numerical coefficients.

100a. Vector units and complex quantities. As is explained in Sec. 2, at Par. 163 and elsewhere, vector alternating quantities are much used in electrical engineering, and call for corresponding vector units, as well as vector symbols, in the formulas relating to such quantities. Strictly speaking, such quantities and units are not vectors in the mathematical sense of that term, but are "complex" quantities and units, because when two such quantities are multiplied together, they do not possess both a "vector product" and a "scalar product" as is the case when two mathematical vectors are multiplied. Nevertheless, such alternating quantities may be called "plane vectors" to avoid conflict with mathematical usage, and the word "vector," which is much used in alternating-current literature, may then be interpreted, in this sense, as subject to the algebra of complex quantities in a plane.

It is not only logical but also very desirable to distinguish between simple and complex quantities, *i.e.*, between scalars and vectors in alternating-current formulas employing both. There are three ways in which this is done:

1. Distinctive symbols, or types of symbol, are used to designate vectors. Thus a scalar e.m.f. in volts might be represented by E and a vector by **E** or $\mathfrak{F}$, *i.e.*, by a black letter capital, or by a gothic capital, of the same letter. This method has the disadvantage of calling for and reserving special fonts in representing vectors.

2. The same symbol may be used, but a distinctive mark, such as an "under dot," may be applied to symbols representing vector quantities. Thus a scalar e.m.f. in volts might be represented by E, and a vector e.m.f. by $\underset{.}{E}$. In any formula or equation, if any one term is a vector, all of its terms must be vectors; so that the under dot must be applied to each and every term of a vector equation. This method has the disadvantages that it is difficult to print or to set up in type, and that a page containing many vector formulas presents a speckled appearance.

3. No special symbols or symbol marks may be used for vector quantities, but the unit at the end of the line on which the equation appears may have a distinctive sign, such as an angle mark ($\angle$), to indicate that the equation employs vectors. Thus the equation

$$E = IZ_1 + IZ_2 + IZ_3 \qquad \text{volts } \angle$$

would indicate that the e.m.f. E is a vector, and can be represented by the polygonal or vector sum of three vector elements. In this case the unit of the equation becomes a "vector volt."

101. The international metric system. There are only three units

TABULAR SUMMARY OF DEFINITIONS OF UNITS

110. Fundamental Units

Symbol	Quantity	Equation	Dimension	C.G.S. units	Abbreviation	Practical units, English	Abbreviation	Value in C.G.S.
L	Length	——	L	Centimetre	cm.	Foot Inch	ft. in.	30.48 2.54
M	Mass	——	M	Gram	g.	Pound	lb.	453.59
T	Time	——	T	Second	sec.	Minute Second	min. sec.	60. 1.
111. Auxiliary Fundamental Units								
μ	Magnetic permeability	——	μ					
k	Electric permeability	——	k					
θ	Temperature	——	θ					
112. Geometrical Units								
A	Area	$A = L_1 L_2$	L^2	Square centimeter	sq. cm.	Square foot Square inch	sq. ft. sq. in.	929.03 6.45
V	Volume	$V = L_1 L_2 L_3$	L^3	Cubic centimeter	cu. m. cu. cm.	Cubic foot Cubic inch	cu. ft. cu. in.	28,317. 16.39
α β	Plane angle	$\alpha = \frac{jL_1}{L_2}$	j number	Radian { Degree Minute Second	{ Deg. Min. Sec.	{ Degree Minute Second	{ Deg. Min. Sec.	
ψ	Solid angle	$\psi = A/L^2$	number					

115. Electric Units

Symbol	Quantity	Equation	Dimensional formula: Electromagnetic	Dimensional formula: Electrostatic	Practical units	Abbreviations	Value of practical units in electromagnetic units
I, i	Current	$I = \frac{E}{Z}$ $I = \frac{Q}{T}$	$L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2} k^{\frac{1}{2}}$	Ampere	amp.	10^{-1}
	Current density	$\frac{I}{A}$	$L^{-\frac{3}{2}} M^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	$L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-2} k^{\frac{1}{2}}$	Ampere per square centimeter	amp. per sq. cm.	10^{-1}
Q, q	Quantity	$Q = I\,T$	$L^{\frac{1}{2}} M^{\frac{1}{2}} \mu^{-\frac{1}{2}}$	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1} k^{\frac{1}{2}}$	Coulomb Ampere-hour	spell out amp-hr.	10^{-1} 360
R, r	Resistance	$R = \frac{P}{I^2}$	$L\,T^{-1} \mu$	$L^{-1}\,T\,k^{-1}$	Ohm	spell out	10^9
E, e	Electromotive force	$E = \frac{d\,\phi}{d\,T}$ $E = \frac{W}{Q}$	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2} \mu^{\frac{1}{2}}$	$L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1} k^{-\frac{1}{2}}$	Volt	spell out	10^8
ρ	Resistivity	$\rho = \frac{RA}{L}$	$L^2\,T^{-1} \mu$	$T\,k^{-1}$	Ohm-centimeter	ohm-cm.	10^9

(Continued on next page)

Electric Units.—*Continued*

Symbol	Quantity	Equation	Dimensional formula		Practical Units	Abbreviations	Value of practical units in electromagnetic units
			Electromagnetic	Electrostatic			
	Electro-kinetic momentum	$I\,L$	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1} \mu^{\frac{1}{2}}$	$L^{\frac{1}{2}} M^{\frac{1}{2}} k^{-\frac{1}{2}}$			
	Thermoelectric height or specific heat of electricity	$\frac{E}{\theta}$**	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2} \theta^{-1} \mu^{\frac{1}{2}}$	$L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1} \theta^{-1} k^{-\frac{1}{2}}$			
	Coefficient of Peltier effect	$\frac{H}{I\,T}$†	$L^{-1} M^{\frac{1}{2}} \theta \mu^{\frac{1}{2}}$	$L^{-\frac{3}{2}} M^{\frac{1}{2}} T \theta k^{-\frac{1}{2}}$			
	Coefficient of Peltier effect	$\frac{W}{I\,T}$	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2} \mu^{\frac{1}{2}}$	$L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1} k^{-\frac{1}{2}}$			
	Surface density	$\frac{Q}{A}$	$L^{-\frac{3}{2}} M^{\frac{1}{2}} \mu^{-\frac{1}{2}}$	$L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1} k^{\frac{1}{2}}$	Coulomb per square centimetre	coulomb per sq. cm.	
L	Coefficient of self induction	$L = \frac{N\phi}{I}$	$L\,\mu$	$L^{-1} T^{2} k^{-1}$	Henry	spell out	10^9
X_s	Inductive reactance	$X_s = 2\pi f L$	$L\,T^{-1} \mu$	$L^{-1} T k^{-1}$	Ohm	spell out	10^9

(*Concluded on next page*)

117. Photometric Units ††

Symbol	Quantity	Equation	Dimensional formula	Unit	Abbreviation
Q	Quantity of light	$Q = \phi T$	$L^2 M T^{-2}$	Lumen-hour	
ϕ	Flux of light	$\phi = \frac{Q}{T}$	$L^2 M T^{-3}$	Lumen	
I	Intensity of light	$I = \frac{\phi}{\text{solid angle}}$	$L^2 M T^{-3}$	Candle	
b	Brightness	$b = \frac{I}{A}$	$M T^{-3}$	Candle or lumen per square centimeter	c. per sq. cm.
E	Illumination	$\frac{\phi}{A}$	$M T^{-3}$	Phot, Foot-candle, Lux	

118. Thermal Units ††

Symbol	Quantity	Equation	Dimensional Formulas‡			
			Dynamical	Thermal	Thermometric	$\theta = H + M$
H	Heat	$H = W$ $H = M\theta$ $H = V\theta$	$L^2 M T^{-2}$	$M \theta$	$L^3 \theta$	$L^2 M T^{-2}$
	Rate of heat production	$\frac{H}{T}$	$L^2 M T^{-3}$	$M T^{-1} \theta$	$L^3 T^{-1} \theta$	$L^2 M T^{-3}$

(Concluded on next page)

WEIGHTS AND MEASURES

NOTE:—In this and the following tables, the numerical values are not carried to more than four significant digits, a degree of precision sufficient for ordinary engineering purposes. For higher degrees of precision in conversion factors, reference may be made to publications of the Bureau of Standards, and to Hering's "Conversion Tables."

119. Linear Measure*

English*	U. S.	Abreviation	Metric equivalent: Meter	Metric equivalent: Reciprocal
	1 mil		0.000 0254	39,370
1 inch	1000 mils = 1 inch	in.	0.0254	39.37
4 inches = 1 hand			0.1016	9.843
7.92 inches = 1 surveyor's link	7.92 inches = 1 surveyor's link		0.2012	4.970
12 inches = 1 engineer's link = 1 foot	12 inches = 1 engineer's link = 1 foot	ft.	0.3048	3.281
3 feet = 1 yard	3 feet = 1 yard	yd.	0.9144	1.094
2 yards = 1 fathom	2 yards = 1 fathom		1.829	0.5468
5½ yards = 1 rod, pole, or perch	5½ yards = 1 rod, pole, or perch		5.029	0.1988
100 surveyor's links = 1 surveyor's chain = 66 feet.	100 surveyor's links = 1 surveyor's chain = 66 feet.	ch.	20.12	0.04970
100 engineer's links = 1 engineer's chain = 100 feet.	100 engineer's links = 1 engineer's chain = 100 feet.	ch.	30.48	0.03281
40 yards = 1 bolt (cloth)			36.58	0.02734
10 surveyor's chains = 1 furlong	40 rods = 1 furlong		201.2	0.004970
8 furlongs = 1 Statute mile	8 furlongs = 1 mile = 5280 feet		1,609	0.000 6214
1.152 statute mile = 1 nautical mile = 1 naut.	1.152 statute miles = 1 nautical mile = 6080 feet. = 1 naut.		1,853	0.000 5397
3 statute miles = 1 league			4,827	0.000 2071
60 nautical miles = 1 degree†	60 nautical miles = 1 degree†	deg.	111,100	0.9×10^{-5}

* Strictly speaking, there is a small difference between the English and United States inch, foot, yard, etc.; see Hering's "Conversion Tables;" but it is inappreciable for most engineering purposes. The units in the columns marked "English" are taken from Wightman's Arithmetical Tables, London, Old Westminster Press. The units in the columns marked "U. S." are taken from "The World Almanac," N. Y.

† 1 deg. of longitude on the equator is equal, more nearly to 60.07 nautical or 67.17 statute miles (111,300 m.).

122. Dry Measure (volume)

English	U. S.	Abbreviation	Metric equivalent: Liters*	Metric equivalent: Reciprocal
	1 pint	pt.	0.5506	1.816
1 pint	....	pt.	0.5682	1.760
	2 pints = 1 quart	qt.	1.101	0.9081
2 pints = 1 quart	....	qt.	1.136	0.8799
4 quarts = 1 gallon	....	gal.	4.546	0.2200
	8 quarts = 1 peck	....	8.810	0.1135
2 gallons = 1 peck	....	....	9.092	0.1100
	4 pecks = 1 bushel	....	35.24	0.02838
4 pecks = 1 bushel	....	....	36.37	0.02750
8 bushels = 1 quarter	....	....	290.9	0.003438
36 bushels = 1 chaldron	....	....	1309.	0.0007639

* A liter is 1,000 cu. cm.

123. Cord and Board Measures (volume)

English	U. S.	Abbreviation	Metric equivalent: Cubic meters	Metric equivalent: Reciprocal
	16 cubic ft. = 1 cord foot	cd. ft.	0.4530	2.208
	8 cord feet = 1 cord*	cd.	3.624	0.2759
	144 square in. = 1 board foot†	bd. ft.	....	....

* A cord is the volume of a rectangular parallelopiped, 8 ft. long, 4 ft. wide, and 4 ft. high = 128 cu. ft.

† The board foot is customarily 144 sq. in. with a thickness of 1 in. Less than 1 in. thick is customarily counted as 1 in.; for example, a board $\frac{7}{8}$ in. or $\frac{15}{16}$ in. thick would be counted 1 in. thick in board measure.

126. Liquid Measure; General

English	U. S.	Abbreviation	Metric equivalent: Liters	Metric equivalent: Reciprocal
	1 gill		0.1183	8.453
1 gill			0.1421	7.039
	4 gills = 1 pint	pt.	0.4732	2.113
4 gills = 1 pint		pt.	0.5682	1.760
	2 pints = 1 quart	qt.	0.9464	1.057
2 pints = 1 quart		qt.	1.1365	0.8799
	4 quarts = 1 gallon	gal.	3.785	0.2642
4 quarts = 1 gallon		gal.	4.546	0.2200
	31½ gallons = 1 barrel*	bbl.	119.2	0.008389
31½ gallons = 1 half hogshead			143.2	0.006983
	42 gallons = 1 tierce		159.0	0.006289
42 gallons = 1 tierce			190.9	0.005238
	2 barrels = 1 hogshead	hhd.	238.4	0.004195
63 gallons = 1 hogshead		hhd.	286.4	0.003492
2 tierces = 1 puncheon			381.8	0.002619
2 hogsheads = 1 pipe			572.8	0.001746
2 pipes = 1 tun			1146.0	0.0008726

* The barrel and larger units are practically confined to the measurement of oil, wine or liquor volumes.

CONVERSION TABLES

129. Density *

	Grams per cu. cm.	Reciprocal
1 lb. av. per sq. mil. ft.	2.936×10^6	0.3406×10^{-6}
1 lb. av. per circular mil-ft.	2.306×10^6	0.4337×10^{-6}
1 lb. av. per cu. in.	27.68	0.03613
1 lb. av. per cu. ft.	0.01602	62.43
1 grain per cu. in.	0.003954	252.9
1 lb. av. per cu. yd.	0.0005933	1,685

* Tables for converting **deg. Baumé** (liquid density) to specific gravity a 60 deg. Fahr., and vice versa, are given in Circular No. 19, Bureau o Standards, pp. 31–35.

130. Time Intervals

	Mean solar			
	days	hours	mins.	secs.
1 mean solar year	365.2	8,766	5.26×10^5	3.156×10^7
1 week of 7 days	7.0	168	1.008×10^4	6.048×10^5
1 mean solar day	1.0	24	1.440×10^3	8.640×10^4
1 siderial day	0.9973	23.93	1.436×10^3	8.616×10^4
1 mean solar hour	4.167×10^{-2}	1	60	3.60×10^3
1 siderial hour	4.155×10^{-2}	0.9973	59.83	3.590×10^3
1 mean solar minute	6.944×10^{-4}	1.667×10^{-2}	1	60

131. Solid Angle

	Sphere	Hemisphere	Spherical right angle	Steradian
1 sphere	1	2	8	12.57
1 hemisphere	0.5	1	4	6.283
1 spherical right angle	0.125	0.25	1	1.571
1 steradian	0.07958	0.1592	0.6366	1

132. Force

	Grams Weight	Reciprocal	Dynes	Reciprocal
1 lb., weight avoird.	453.6	0.002205	4.448×10^5	0.2248×10^{-5}
1 poundal	14.10	0.07092	1.383×10^4	0.7233×10^{-4}
1 grain, weight	0.06480	15.43	63.55	0.01573
1 gram, weight	1	1	980.665‡	0.0010197
1 short ton, weight	0.9072×10^6	1.102×10^{-6}	0.8896×10^9	1.124×10^{-9}
1 dyne	0.0010197	980.665‡	1	1

‡ The internationally accepted conventional value of gravitational acceleration at latitude 45 deg. and sea-level. This is usually adopted although later researches have indicated a slightly different value.

134. Energy

	Ergs	Reciprocal	Gram-wt-cm. $g = 980.7$	Reciprocal	Gram-calories	Reciprocal
1 erg	1	1	1.0197×10^{-3}	980.7	0.2389×10^{-7}	4.186×10^{7}
1 joule	10^{7}	10^{-7}	1.0197×10^{4}	0.9807×10^{-4}	0.2389	4.186
1 gram-wt-cm. ($g = 980.7$)	980.7	1.0197×10^{-3}	1	1	0.2342×10^{-4}	4.269×10^{4}
1 gram-calorie	4.186×10^{7}	0.2389×10^{-7}	4.269×10^{4}	0.2342×10^{-4}	1	1
1 kilogram-calorie	4.186×10^{10}	0.2389×10^{-10}	4.269×10^{7}	0.2342×10^{-7}	1,000	10^{-3}
1 foot-grain ($g = 980.7$)	1.937×10^{3}	0.5163×10^{-3}	1.975	0.5063	0.4628×10^{-4}	2.161×10^{4}
1 foot-pound ($g = 980.7$)‡	1.356×10^{7}	0.7375×10^{-7}	1.383×10^{4}	0.7231×10^{-4}	0.3240	3.086
1 foot-long-ton ($g = 980.7$)	3.037×10^{10}	0.3293×10^{-10}	3.097×10^{7}	0.3229×10^{-7}	0.7256×10^{3}	1.378×10^{-3}
1 foot-short-ton ($g = 980.7$)	2.712×10^{10}	0.3687×10^{-10}	2.766×10^{7}	0.3615×10^{-7}	0.6480×10^{3}	1.543×10^{-3}
1 British thermal unit	1.054×10^{10}	0.9488×10^{-10}	1.075×10^{7}	0.9302×10^{-7}	0.2518×10^{3}	3.971×10^{-3}
1 watt-hour	3.600×10^{10}	0.2778×10^{-10}	3.671×10^{7}	0.2724×10^{-7}	0.8602×10^{3}	1.163×10^{-3}
1 kilowatt-hour	3.600×10^{13}	0.2778×10^{-13}	3.671×10^{10}	0.2724×10^{-10}	0.8602×10^{6}	1.163×10^{-6}
1 horse-power-hour* (746 watt-hrs.)	2.686×10^{13}	0.3723×10^{-13}	2.739×10^{10}	0.3651×10^{-10}	0.6418×10^{6}	1.558×10^{-6}
1 horse-power-hour (745.7 watt-hrs., $g = 980.7$).	2.684×10^{13}	0.3726×10^{-13}	2.737×10^{10}	0.3654×10^{-10}	0.6412×10^{6}	1.560×10^{-6}
1 metric horse-power-hour (736 watt-hrs.).**	2.650×10^{13}	0.3774×10^{-13}	2.702×10^{10}	0.3701×10^{-10}	0.6332×10^{6}	1.579×10^{-6}
1 metric horse-power-hour (735.5 watt-hrs., $g = 980.7$).	2.648×10^{13}	0.3776×10^{-13}	2.700×10^{10}	0.3704×10^{-10}	0.6326×10^{6}	1.581×10^{-6}

* For $g = 981.2$ for approximate latitude of London.

** For $g = 981.3$ for approximate latitude of Berlin.

‡ The local foot-pound varies between the equator and the poles, according to the local intensity of gravitation between the limits 1.352 and 1.359 joules.

136. Torque

	Grams perp. cm.	Reciprocal	Dynes perp. cm.	Reciprocal
1 lb.-perp.-ft.*	1.383×10^{4}	0.7233×10^{-4}	1.356×10^{7}	0.7375×10^{-7}
1 g.-perp.-cm	1	1	980.665	0.0010197
1 dyne-perp.-cm	0.0010197	980.665	1	1

137. Linear Velocity

	Metric equivalent	
	Meters per sec.	Reciprocal
1 ft.-per-sec	0.3048	3.281
1 ft.-per-min	0.005080	196.9
1 mile-per-hr	0.4470	2.237
1 km.-per-hr	0.2778	3.60
1 knot, or naut-per-hr	0.5148	1.943

138. Linear Acceleration

	Meters per sec. per sec.	Reciprocal	Km. per hr. per sec.	Reciprocal
1 ft. per sec. per sec.	0.3048	3.281	1.097	0.9114
1 mile per hr. per sec.	0.4470	2.237	1.609	0.6214
Standard gravitation g	9.80665	0.10197	35.30	0.02833
1 m. per sec. per sec.	1	1	3.600	0.2778
1 km. per hr. per sec.	0.2778	3.600	1	1

139. Conversion of Angles (plane)

Angles	Degrees	Reciprocal	Grades	Reciprocal	Radian	Reciprocal
1 degree	1	1	1.1111	0.900	0.01745	57.30
1 grade	0.9	1.111	1	1	0.01571	63.66
1 radian	57.30	0.01745	63.66	0.01571	1	1
1 quadrant	90°	0.01111	100	0.010	$\pi/2=1.571$	0.6366
1 revolution	360°	0.002778	400	0.00250	$2\pi=6.283$	0.1592
π radians	180°	0.005556	200	0.005	$\pi=3.142$	0.3183
$\pi/2$ radians	90°	0.01111	100	0.010	$\pi/2=1.571$	0.6366
$\pi/4$ radians	45°	0.02222	50	0.020	$\pi/4=0.7854$	1 2730
2π radians	360°	0.002778	400	0.0025	$2\pi=6.283$	0.1592

140. Linear Mass

	Gram per meter	Reciprocal
1 lb. per linear yard	496.1	0.002016
1 lb. per linear foot	1488	0.0006720

* A torque is the product of a force and a length taken perpendicularl thereto. Its dimensions are therefore those of force $\times -jL$ where $j=\sqrt{-}$ or $-jL^2M^1T^{-2}$. Any element of angle is also the ratio of an element of ar length to the length of a radius perpendicular thereto, or has dimension $jL/L=j$. The product of torque and the angle through which it advance is thus $-jL^2M^1T^{-2}\times j=L^2M^1T^{-2}$ which are the dimensions of work. I the foregoing direction symbols are neglected, a torque appears to have th same dimensions and nature as a work, which is illogical. A torque of 1 g force acting at a radius of 1 cm. is thus correctly to be expressed as a gran perpendicular centimeter rather than as a gram centimeter.

143. Storage of Water

1 acre-ft. = 325,800 U. S. gal. = 43,560 cu. ft. = 1613 cu. yd. = 1234 cu. m.
1 gal. = 0.3069×10^{-5} acre-ft.
1 cu. ft. = 0.2298×10^{-4} acre-ft.
1 cu. yd. = 0.00062 acre-ft.
1 cu. m. = 0.000811 acre-ft.

144. Temperature

Scale	Freezing point of water	Boiling point of water	Interval
Fahrenheit......	32 deg.	212 deg.	180 deg.
Centigrade.....	0 deg.	100 deg.	100 deg.
Réaumur........	0 deg.	80 deg.	80 deg.

1 deg. fahr. = 0.5556 or (5/9) deg. cent. = 0.4444 or (4/9) deg. Réa.
1 deg. cent. = 0.8000 deg. Réa. = 1.800 deg. fahr.
1 deg. Réa. = 2.250 deg. fahr. = 1.250 deg. cent.
Absolute zero = −273.1 deg. cent. = −491.6 deg. fahr. = −218.5 deg. réaumur.

T_{abs} = 273.1 + deg. cent. (in cent. scale)
T_{abs} = 491.6 + deg. fahr. (in fahr. scale)
T_{abs} = 218.5 + deg. réaumur (in réaumur scale)

The international hydrogen scale of temperature.

145. Mechanical equivalent of heat.

1 B.t.u. = 1,054 joules = 777.5 ft-lb. = 0.2928 watt-hr. = 0.0003927 h.p-hr
1 joule = 0.7375 ft-lb. = 0.0009488 B.t.u. = 0.0002778 watt-hr.
1 ft-lb. = 1.356 joules = 0.001286 B.t.u. = 0.0003766 watt-hr.
1 watt-hr. = 3,600 joules = 2.655 ft-lb. = 3.415 B.t.u.
Also see Par. **133** on energy conversion factors.

MATHEMATICAL CONSTANTS AND TABLES

146. Useful Constants. Base of the hyperbolic system of logarithms = ϵ = 2.7183 to the nearest unit in five significant figures; it is not a commensurate quantity.

Ratio of circumference to diameter of circle = π = 3.1416 (incommensurate).

	Numeric	Reciprocal
ϵ...............	2.7183	0.36788
$\log_{10} \epsilon$...........	0.434295	2.302585
π...............	3.1416	0.31831
2π..............	6.2832	0.15915
3π..............	9.4248	0.10610
4π..............	12.566	0.079577
$\pi/2$.............	1.5708	0.63662
$\pi/3$.............	1.0472	0.95493
$\pi/4$.............	0.7854	1.2732
π^2..............	9.8696	0.10132
$\sqrt{\pi}$.............	1.7725	0.56419
$\sqrt{2}$.............	1.4142	0.70711
$\sqrt{3}$.............	1.7321	0.57733

1 radian = 57.296 deg. = 57 deg., 17 min. and 45 sec.
An arc of 1 deg., in terms of its radius = 0.017453.
For further conversion factors of circular measure, see Par. **134** on angles.

Natural Tangents and Cotangents—*Concluded*

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	44
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	43
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	42
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	41
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	40°
50°	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	39
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	37
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	36
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	35
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	34
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	33
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	32
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	31
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	30°
60°	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	29
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	28
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	27
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	26
64	2.0503	2.4059	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	25
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	24
66	2.2460	2.2566	2.2673	2.2781	3.2889	2.2998	2.3109	2.3220	2.3332	2.3445	23
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	22
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	21
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	20°
70°	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	19
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	2.0415	3.0595	18
72	3.0777	3.0961	3.1146	3.1334	3.1524	2.1716	3.1910	3.2106	3.2305	3.2506	17
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	16
74	3.4874	3.5105	3.5339	8.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	15
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	14
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	13
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646	12
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970	11
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140	10°
80°	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432	9
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264	8
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285	7
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572	6
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	5
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	4
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46	3
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	2
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	1
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

Logarithms of Numbers.—*Concluded*

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8456	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
70	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

SECTION 2

ELECTRIC AND MAGNETIC CIRCUITS

ELECTRIC POTENTIAL

1. The cause of an electric current in a circuit is termed the **electromotive force** or **voltage**. The latter name is derived from the practical unit of electromotive force called the volt. The current between two points in a circuit is due to a different electric state or "potential" at each point; for this reason the electromotive force or voltage is sometimes called the **difference of potential.**

2. The electric energy (W) developed or absorbed by an electric circuit may be considered due to the actual flow of an incompressible something which we call electricity. From this point of view, the quantity of electricity Q which is transferred between two points of the circuit is the quantity factor of the energy, while the difference of potential, or the voltage E between the same points, is the intensity factor of the energy ; or

$$W = QE. \tag{1}$$

When Q is in coulombs, and E in volts, W is in joules (watt-seconds). Hence, electromotive force or voltage may also be defined as electrical energy developed or work done per unit quantity of electricity.

3. Electric Power.—Dividing both sides of the preceding equation by the time which it takes for the quantity Q to flow through a cross-section of the circuit, we get

$$P = IE, \tag{2}$$

where P is the power, and I the rate of flow or the current. The e.m.f. can thus be defined as the power developed per unit of current.

4. The principal sources of electromotive force or difference of potential are the following:

(a) Electromagnetic induction (see Par. **36**);
(b) Contact of dissimilar substances (see Par. **5**);
(c) Thermo-electric action (see Par. **5**);
(d) Chemical action (Sec. 19);
(e) Friction between dissimilar substances (see Sec. 22).

In the light of the modern electrical theory, all these phenonema, with the exception of (a), appear to be but special cases of the general contact action.

5. In a circuit made up of several substances, a difference of potential (e.m.f.) exists at each junction of two unlike substances. However, from the law of conservation of energy it follows that unless the circuit contain a source of energy, the resultant e.m.f. in the circuit must be zero and no current can be established. This phenomenon also takes place in circuits made up of a single substance whenever the substance is not physically and chemically homogeneous. The following are the principal cases of thermal and contact action:

(a) **Seebeck effect.** In a closed circuit consisting of two different metals, if the two junctions are kept at different temperatures, a permanent current will flow. Thus, if one junction of a copper-iron circuit be kept in melting ice and the other in boiling water, it will be found that a current passes from copper to iron across the hot junction. If, however, the temperature of the hot junction be raised gradually, the e.m.f. in the circuit slowly reaches a maximum, then sinks to zero, and finally is reversed.

(b) **Peltier effect.** When a current is passed across the junction between two different metals, an evolution or an absorption of heat takes place. This effect is different from the evolution of heat (i^2r), due to the resistance of the junction, and is reversible, heat being evolved when the current the current I is in amperes. When the rate of flow is non-uniform, the instantaneous current is

$$i = \frac{\partial q}{\partial t}; \tag{7}$$

for a classification of electric currents see the Standardisation Rules of the American Institute of Electrical Engineers, Sec. 24.

10. Steady and transient states. An electric circuit may be in a steady or in a transient state. When a current is continuous, or when it varies periodically between the same limits and according to the same law, the circuit is said to be in a steady state. For instance, the circuit of an alternator is steady as long as the load, speed and field excitation are kept constant. The same circuit is in a transient state when the load is switched on or off, or when it is varied in such a way that the same conditions do not repeat themselves periodically. A transient current may be periodic, for instance in a rectifier, in which cycles follow in such rapid succession that the current is very different from the permanent value which it would gradually assume.

11. A direct current given out by a chemical battery is constant in value, or **continuous**, when the load is constant. A current delivered under the same conditions by an electric generator or a rectifier is **pulsating**, that is, it varies periodically due to a finite number of commutator segments.

12. An alternating current may vary according to the **simple sine law** (Par. **152**), or according to a **more complicated periodic law**. In the latter case the current may be resolved, for purposes of theory and analysis, into a fundamental sine wave, and sine waves of higher frequencies (Par. **209**). Sometimes a complex alternating current or voltage is replaced for practical purposes by an equivalent sine-wave.

13. Transient currents may be oscillating or non-oscillating, according to the conditions in the circuit. Oscillations are due to periodic transformations of the electrostatic energy stored in the dielectric into the electromagnetic energy of the magnetic flux linking with the current. During these transformations part of the energy is converted into the Joulean (i^2r) heat in the conductors and in surrounding metallic objects, including the iron of the magnetic circuit. Part of the energy is also converted into the heat caused by magnetic and dielectric hysteresis. The oscillations are thus damped out, and their amplitude decreases. When the conditions are particularly favorable for the conversion into heat (high resistance in series, or low resistance in parallel), both the electrostatic and the electromagnetic energy are directly converted into heat, instead of being partly converted into one another. This conversion into heat is an irreversible phenomenon, so that in this case the current is non-oscillating, but gradually reaches its final value. When it is desired to maintain oscillations as long as possible (wireless telegraphy) the series resistance must be kept down as low as possible. When oscillations are harmful (switching in long cables or transmission lines), extra resistance is temporarily connected in the circuit.

14. Conductors and insulators. For practical purposes, materials used in electrical engineering are divided into conductors and insulators. A conducting material allows a continuous current to pass through it under the action of a continuous e.m.f. An insulator (more correctly called a **dielectric**) allows only a brief transient current which charges it electrostatically. This charge or displacement of electricity produces a counter-e.m.f. equal and opposite to the applied e.m.f., and the flow of current ceases. The division into conductors and dielectrics is not strictly correct, but convenient for practical purposes. A substance may practically stop the flow of current when the applied voltage is sufficiently low, and at the same time be unsuitable as an insulator at high voltages. Some materials which are practically non-conducting at ordinary temperatures become good conductors when sufficiently heated. For numerical data and tables of conducting and insulating properties of the principal materials used in practice see Sec. 4.

15. The electronic theory of conduction. According to the modern electronic or corpuscular theory of electricity, there is an indivisible atom of negative electricity, called the **electron** or the **corpuscle**. Atoms of matter consist of one or more electrons and an unknown something which has the where λ (lambda) is called the **conductivity** of the material. Since $g = 1/r$, the relation also holds, that

$$\lambda = \frac{1}{\rho}. \tag{14}$$

For the calculation of resistance of non-cylindrical conductors see the author's "Electric Circuit," Chap. III, and the references given there.

21. Temperature coefficient. The resistance of a conductor varies with the temperature according to a rather complicated law. The resistance of all metals and of practically all alloys increases with the temperature. The resistance of carbon and of electrolytes decreases with the temperature. For numerical values see Sec. 4. For many materials the variation of resistance with the temperature can be represented by the relation

$$r_t = r_o(1 + \alpha t) \tag{15}$$

where r_t is the resistance at t deg. cent., r_o the resistance at 0 deg. cent., and α (alpha) is called the temperature coefficient of the material. For numerical values of α for various materials see Sec. 4. When the resistance of a material increases with the temperature, α is positive; otherwise it is negative. For other formulæ see Sec. 4.

22. Resistances and conductances in series. When two or more resistances are connected in series the equivalent resistance of the combination is equal to the sum of the resistances of the individual resistors, or

$$r_{eq} = r_1 + r_2 + \text{etc.} \tag{16}$$

When conductances are connected in series, the equivalent conductance g_{eq} is determined from the relation

$$\frac{1}{g_{eq}} = \frac{1}{g_1} + \frac{1}{g_2} + \text{etc.}, \tag{17}$$

in other words, when two or more conductors are connected in series, the reciprocal of the equivalent conductance is equal to the sum of the reciprocals of the individual conductances.

23. When resistances are connected in parallel, the equivalent resistance r_{eq} is determined from the relation

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \text{etc.} \tag{18}$$

or simply

$$g_{eq} = g_1 + g_2 + \text{etc.} \tag{19}$$

24. The simple rule is: Resistances are added when in series; conductances are added when in parallel. In the case often met in practice when two resistances are connected in parallel

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}. \tag{20}$$

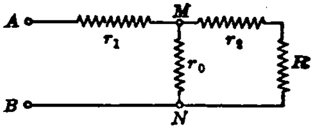

Fig. 1.—Series-parallel circuit.

25. Series-parallel circuits. In a combination like the one shown in Fig. 1, where some of the resistances are in series, some in parallel, and where it is required to find the equivalent resistance between A and B, the problem is solved step by step, by combining the resistances in series, converting them into conductances and adding them with other conductances in parallel. For instance, in the case shown in Fig. 1 begin by combining the resistances r_2 and R into one, and determine the corresponding conductance

$$\frac{1}{(R + r_2)};$$

add this conductance to the conductance $1/r_0$. This will give the total conductance between the points M and N. The reciprocal of it gives the equivalent resistance between the same points. The total resistance between the points A and B is found by adding r_1 to this resistance. When a network of conductors cannot be reduced to a series-parallel combination, the problem is solved as shown in Par. 29.

determined by solving simultaneous equations. For an example of such equations see Par. 31 below.

30. Wheatstone bridge. The combination of six resistances shown in Fig. 3 is called the Wheatstone bridge. The resistances are denoted by $a, b, c, \alpha, \beta, \gamma$, the currents by $x, y, z, \xi, \eta, \zeta$. An electric battery of e.m.f. E is connected in the branch BC, and the value of a includes the internal resistance of the battery. In practice a galvanometer is usually connected in the branch OA, and α includes its resistance. When the four resistances b, c, β, γ, are so adjusted that no current flows through OA, the bridge is said to be balanced, and the condition holds that,

$$b\beta = c\gamma. \tag{27}$$

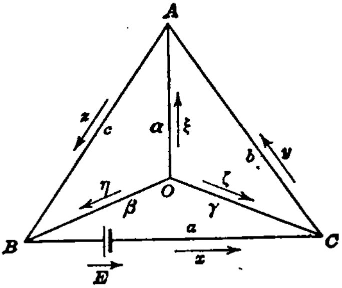

Fig. 3.—Wheatstone Bridge.

31. Unbalanced bridge. When the Wheatstone bridge is not balanced, Ohm's law and Kirchhoff's laws give the following equations:

$$\begin{aligned} ax &= C - B + E, & \alpha\xi &= A, & \xi + y - z &= O, \\ by &= A - C, & \beta\eta &= B, & \eta + z - x &= O, \\ cz &= B - A, & \gamma\zeta &= C, & \zeta + x - y &= O. \end{aligned} \tag{28}$$

Here E is the battery e.m.f., and A, B, C, denote the potentials of these points below that at O. These nine equations contain nine unknown quantities, viz., six currents and three potentials. Solving them as simultaneous equations any of the unknown quantities may be determined. For instance, the current in the galvanometer circuit is

$$\xi = \frac{E}{D}(b\beta - c\gamma), \tag{29}$$

where the "determinant" D is given by

$$D = abc + bc(\beta + \gamma) + ca(\gamma + \alpha) + ab(\alpha + \beta) + (a + b + c)\ (\beta\gamma + \gamma\alpha + \alpha\beta).^*$$

32. Networks of conductors. In a general case (Fig. 2) as many Kirchhoff equations (Par. 29) may be written as there are conductors; the unknown quantities may be the currents, the resistances or the voltages; also any combination of these, provided that the total number of unknown quantities is equal to the number of equations. The equations are conveniently solved by the method of determinants, found in most textbooks on algebra.

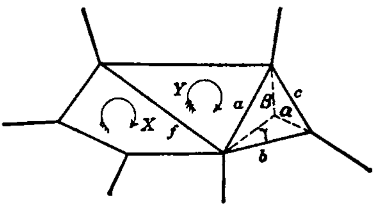

Fig. 4.—Method of simplifying networks.

33. Maxwell's solution. In some cases it is convenient to consider, instead of the actual currents, fictitious currents in each mesh (Maxwell, *ibid.*, Art. 282*b*). The actual current in each conductor is equal to the algebraic sum of the fictitious currents. For instance, in Fig. 4 the current in conductor f is the difference of the fictitious currents X and Y. The Kirchhoff equations are written for the fictitious currents.

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* Maxwell, J. C. "A Treatise on Electricity and Magnetism," Vol. I, Art. 347.

For practical forms of the Wheatstone bridge and its application to the measurement of resistance see Sec. 3.

the reluctance of the magnetic circuit (inductor-type alternator). If the flux is linked with N turns and varies harmonically with the time, at a frequency of f cycles per second, the **maximum induced e.m.f.** is

$$E_{max} = 2\pi f N \Phi_{max} 10^{-8} \text{ volts,} \tag{32}$$

where Φ_{max} is the maximum instantaneous value of the flux, in maxwells. The **effective value of the induced e.m.f.** is

$$E = 4.44 f N \Phi_{max} 10^{-8} \text{ volts.} \tag{33}$$

When the flux varies according to a law different from the sine law the effective voltage is

$$E = 4\chi f N \Phi_{max} 10^{-8}, \tag{34}$$

where χ is the form factor (Par. **307**).

The **average e.m.f. induced in one turn**, no matter what the law of variation of the flux with the time, is

$$E_{av} = \frac{\Phi_1 - \Phi_2}{t_2 - t_1}, \tag{35}$$

where the subscripts 1 and 2 refer to the initial and the final instants respectively.

38. Stationary flux and moving conductor. When the exciting m.m.f. which produces the flux, and the winding in which the e.m.f. is induced, move relatively to each other, as in a generator, so that the conductors cut across the lines of flux, the instantaneous induced e.m.f. in a conductor is

$$e = k\mathfrak{B}lv, \tag{36}$$

where $\mathfrak{B}$ is the flux density, l the length of the conductor, v the relative velocity between the flux and the conductor, and k a coefficient which depends upon the units selected. When e is in volts, $\mathfrak{B}$ in maxwells per square centimeter (gausses), l in centimeters and v in centimeters per second, $k = 10^{-8}$.

The three directions, $\mathfrak{B}$, l, and v, are supposed to be at right angles to each other; if not, their projections at right angles to each other are to be used in the preceding formula. For practical formulas giving the e.m.f. induced in direct-current and alternating-current machinery see Sec. 7 and Sec. 8.

39. Variable flux and moving conductor. When coils or conductors are moving through a pulsating magnetic field, as for instance in single-phase motors, the induced e.m.f. is due to a combined transformer and generator action (Par. **37** and **38**), and is equal at any instant to the sum of the e.m.f. induced by a constant flux in a moving coil and that induced by a pulsating flux in a stationary coil. Let the frequency of the pulsating field be f cycles per second; that of the rotating coil f' cycles per second. A pulsating field can be resolved into two revolving fields, one rotating clockwise, the other counter-clockwise. Therefore, the induced e.m.f. is a result of the superposition of two e.m.fs., one of the frequency $f+f'$, the other $f-f'$. In the particular case of $f=f'$ the e.m.f. induced in the rotating coil is of the frequency $2f$, the frequency $f-f'$ being equal to zero.

40. Force on a conductor carrying a current in a magnetic field. Let a conductor carry a current of i amp. and be placed in a magnetic field the density of which is $\mathfrak{B}$ maxwells per square centimeter ($\mathfrak{B}$ gausses). Then, if the length of the conductor is l cm., the force tending to move the conductor across the field is

$$F = 10.2 i\mathfrak{B}l 10^{-8} \quad \text{(kg.)} \tag{37}$$

It is presupposed in this formula that the direction of the axis of the conductor is at right angles to the direction of the field. If the directions of i and B form an angle α, the preceding expression must be multiplied by $\sin\alpha$.

The force F is perpendicular to both i and B, and its direction is determined by the right-hand screw rule (Par. **56**). Namely, the effect of the magnetic field produced by the conductor itself is to increase the original flux density ($\mathfrak{B}$) on one side of the conductor and to reduce it on the other side. The conductor tends to move away from the denser field.

41. The attraction or repulsion between two parallel straight conductors, carrying currents i_1 and i_2 (amp.) and placed in a non-magnetic medium, is calculated according to the formula

$$F = 2.04 i_1 i_2 \left(\frac{l}{b}\right) 10^{-8} \quad \text{(kg.)} \tag{38}$$

where nI is the product of the number of turns and the current in amperes. For further information about magnetic units see Sec. 1.

45. Permeability and reluctivity. The reluctance of a uniform magnetic path (Fig. 5) is proportional to its length l and inversely proportional to its cross-section A, or

$$\mathcal{R} = \nu \frac{l}{A} \tag{42}$$

and

$$\mathcal{P} = \mu \frac{A}{l}. \tag{43}$$

In these expressions ν is called the reluctivity and μ the permeability of the material of the magnetic path. For air and all non-magnetic substances, ν and μ are assumed to be equal to unity per centimeter-cube, corresponding to centimeter measure for l and A in (42) and (43), and the gilbert as the unit of m.m.f. This is the conventional assumption in the C.G.S. electromagnetic system and is the one generally employed. See also Par. **53** and **54**.

The method preferred by the author, and expounded in his "Magnetic Circuit" (see footnote reference in Par. **44**), is to take the ampere-turn as the unit of m.m.f. In such cases, with the maxwell as the unit of flux, the permeability and the reluctivity of air, respectively, are

$$\left.\begin{array}{l} \nu = 0.7955 \text{ per cm.}^3 = 0.3132 \text{ per in.}^3 \\ \mu = 1.257 \text{ per cm.}^3 = 3.193 \text{ per in.}^3 \end{array}\right\} \tag{44}$$

This method has the advantage of greater simplicity in calculations, but is not yet in general use (see Sec. 1; Giorgi system of units, Par. **3**(d_4)).

46. Magnetic field intensity $\mathcal{H}$ is defined as the m.m.f. per unit length of path. In any uniform field

$$\mathcal{H} = \frac{\mathcal{F}}{l}. \tag{45}$$

In a non-uniform magnetic circuit

$$\mathcal{H} = \frac{\partial \mathcal{F}}{\partial l}. \tag{46}$$

Inversely

$$\mathcal{F} = \mathcal{H}l \text{ or } \mathcal{F} = \int \mathcal{H}\partial l. \tag{47}$$

$\mathcal{H}$ is also known as the **magnetizing force** or as the **magnetic potential gradient.**

If $\mathcal{F}$ is in ampere-turns, $\mathcal{H}$ is in ampere-turns per centimeter (or per inch) of length. If $\mathcal{F}$ is in gilberts, $\mathcal{H}$ is in gilberts per centimeter (or per inch).

47. Flux density ($\mathcal{B}$) is defined as the flux per unit area perpendicular to the direction of the lines of force. In a uniform field

$$\mathcal{B} = \frac{\Phi}{A}. \tag{48}$$

In a non-uniform field

$$\mathcal{B} = \frac{\partial \Phi}{\partial A}. \tag{49}$$

Inversely

$$\Phi = \mathcal{B}A, \text{ or } \Phi = \int \mathcal{B}\partial A. \tag{50}$$

If the flux is measured in maxwells and areas in square centimeters, flux density is expressed in maxwells per square centimeter; one maxwell per square centimeter is sometimes called a **gauss.** In this country flux density is also expressed in maxwells, or in kilolines, per square inch.

It follows at once from (40), (43), (45) and (48) that

$$\mathcal{B} = \mu \mathcal{H} \tag{51}$$

which is the familiar relationship between flux density, permeability and magnetic field intensity.

48. Reluctances and permeances in series and in parallel. Reluctances and permeances are added like resistances and conductances (Par. **22**

intensity of induced magnetization. The coefficient κ is called the **susceptibility** of the material. The total flux density in iron also consists of two parts, viz., that due to $\mathcal{H}$ and to $\mathcal{I}$, or, in the C.G.S. system,

$$\mathcal{B} = \mathcal{H} + 4\pi\mathcal{I}. \tag{54}$$

Dividing both sides of this equation by $\mathcal{H}$, gives

$$\mu = 1 + 4\pi\kappa, \tag{55}$$

where μ is the permeability of the material (Par. 53). Susceptibility is equal to zero for non-magnetic materials, is positive for paramagnetic and negative for diamagnetic substances. It is seldom used in practice.*

53. The permeability (μ) and the reluctivity (ν) of a material (Par. 45) are also defined as the ratios

$$\mu = \frac{\mathcal{B}}{\mathcal{H}} \text{ and } \nu = \frac{\mathcal{H}}{\mathcal{B}}. \tag{56}$$

Their values depend upon the units selected for $\mathcal{B}$ and $\mathcal{H}$. In the C.G.S. electromagnetic system $\mathcal{B}$ and $\mathcal{H}$ are numerically equal for non-magnetic materials, consequently $\mu = \nu = 1$. When $\mathcal{B}$ is expressed in maxwells per square centimeter (or per square inch) and $\mathcal{H}$ in ampere-turns per unit length, μ and ν for air and other non-magnetic materials have values given in Par. 45, Eq. 44.

54. Two different scales of permeability. For steel and iron the permeability $\mu = \mathcal{B}/\mathcal{H}$ is frequently calculated from the magnetization curve (Par. 49), and is usually plotted against $\mathcal{B}$ as abscissæ (see curves in Sec. 4). One must be careful to distinguish between the **absolute permeability** and the **relative permeability**. The former is equal to $\mathcal{B}/\mathcal{H}$, the latter is the ratio of the permeability of a sample to that of the air. In the C.G.S. electromagnetic system both permeabilities are numerically the same, because μ is assumed to be unity for the air; **nevertheless they have different** physical dimensions in any system of units.

55. Magnetic calculations. In practice, calculations of magnetic circuits with iron are usually arranged so as to avoid the use of permeability μ altogether, using a $\mathcal{B}$–$\mathcal{H}$ curve directly (Par. 49 and 50). In some special investigations it is convenient to use the values of permeability and also an empirical equation between μ and $\mathcal{B}$. For small and medium flux densities μ may be expressed as a parabolic curve, of the form

$$\mu = a - b(\mathcal{B}_0 - \mathcal{B})^2 10^{-6} \tag{57}$$

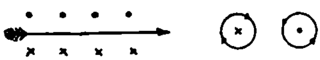

FIG. 7a.—Relation between directions of current and flux.

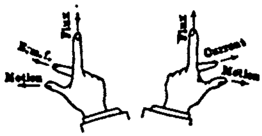

FIG. 7b.—Fleming's rules.

For numerical values of the coefficients see Sec. 4. It is also possible to represent the **relationship between $\mathcal{B}$ and $\mathcal{H}$ for a magnetic material** empirically by a hyperbola (Fröhlich's formula)

$$\mathcal{B} = \frac{\mathcal{H}}{\alpha + \beta\mathcal{H}}, \tag{58}$$

or also in the form

$$\text{reluctivity } \nu = \frac{\mathcal{H}}{\mathcal{B}} = \alpha + \beta\mathcal{H} = \frac{1}{\mu}. \tag{59}$$

The coefficients α and β must be so determined as to satisfy the saturation curve of the particular material used.

56. The right-hand screw rule. The direction of the flux produced by a given current is determined as shown in Fig. 7*a* (see also Fig. 5). If the

* Maxwell, J. C. "A Treatise on Electricity and Magnetism," Vol. II., Arts., 426 to 428.

usually a complex matter, and the results are expressed by complicated formulæ. See references in Par. 74.

62. The stored magnetic energy in a single loop of non-magnetic wire, when the dimensions of the wire are small compared to those of the loop (so that the flux inside the wire is negligible), is

$$W = \tfrac{1}{2}i\Phi = \tfrac{1}{2}i^2\mathcal{P} = \frac{\Phi^2}{2\mathcal{P}} \qquad \text{(joules)} \qquad (66)$$

where i is the current in amperes, Φ the flux linking with the loop, in webers, and $\mathcal{P}$ the permeance of the magnetic path, in henrys.

63. Effect of leakage. When the flux linking with part of the turns of a coil C is not negligible (see Fig. 9), the **total stored energy** may be expressed in the following forms:

$$\left.\begin{aligned} W &= \tfrac{1}{2}i[n_c\Phi_c + \Sigma n_p\Delta\Phi_p], \\ W &= \tfrac{1}{2}i^2[n^2\mathcal{P}_c + \Sigma n_p{}^2\Delta\mathcal{P}_p]. \end{aligned}\right\} \qquad (67)$$

The last expression is identical with

$$W = \tfrac{1}{2}Li^2 \qquad (68)$$

where L is the inductance of the coil (Par. 67). The subscripts c in the foregoing expressions refer to **complete linkages**, that is those which embrace

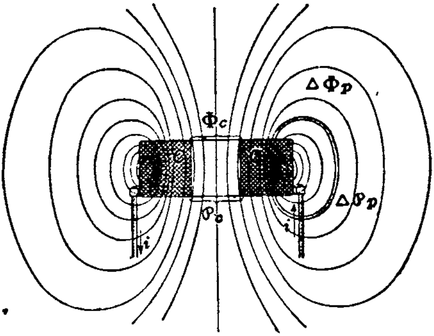

Fig. 9.—Magnetic field due to a coil.

all the turns of the coil, the subscripts p **to partial linkages**. See the author's "Magnetic Circuit," Art. 57.

64. The density of magnetic energy, or the magnetic energy stored per cubic centimeter of a magnetic field is

$$W' = \frac{\mu\mathcal{H}^2}{8\pi} = \frac{\mathcal{B}\mathcal{H}}{8\pi} = \frac{\mathcal{B}^2}{8\pi\mu} \qquad \text{(joules per cubic cm.).} \qquad (69)$$

Here $\mathcal{H}$ is the intensity in gilberts per centimeter, $\mathcal{B}$ is the flux density in webers per square centimeter and μ is the relative permeability; or if $\mathcal{H}$ is in ampere-turns per centimeter, then

$$W' = \tfrac{1}{2}\mu\mathcal{H}^2 = \tfrac{1}{2}\mathcal{B}\mathcal{H} = \frac{\mathcal{B}^2}{2\mu} \qquad \text{(joules per cubic cm.)} \qquad (70)$$

where μ is the so-called absolute permeability (see Par. 54). To find the total energy of a field the preceding expressions are multiplied by the element of the volume ∂v and integrated within the desired limits of volume. For an interesting comparison of practical possibilities as to the amount of energy stored in the magnetic form per unit volume, compared with other forms of energy, see Steinmetz, C. P., *General Electric Review*, 1913, p. 536.

permeability of the steel. If the magnetic core occupies only a part α of the cross-section bh, use the expression $bh[\alpha\mu_r+(1-\alpha)]$, in place of bh.

70. For other calculations of inductance using formula (68) see the author's "Magnetic Circuit," Chaps. X to XII.

71. Thin solenoids. For a straight coil uniformly wound with n_1 turns per centimeter length, provided that the length of the coil is large compared to its transverse dimensions, and that the winding consists of one layer of comparatively thin wire, **the inductance is**

$$L = 1.257 n_1^2 lA 10^{-8} \qquad \text{(henrys)} \qquad (77)$$

the notation being the same as in Par. 68.

72. The inductance of a long straight coil wound with several layers of wire, and with an iron core of radius a inside,

$$L = 4n_1^2 l d^2 r^2 \left[1 + (\mu_r - 1)\frac{a^2}{r^2} + \frac{d}{r} + \frac{d^2}{3r^2}\right] 10^{-9} \qquad \text{(henrys)} \qquad (78)$$

where r is the inside radius of the winding, and d its radial thickness; n_1 is the number of turns per centimeter length, and all the dimensions are in centimeters. If there is no iron core, put $a = 0$.

73. Prof. Morgan Brooks has derived a universal semi-empirical formula for the **inductance of short and long coils without iron cores.** His formula is given below in two forms, one (79) for dimensions in centimeters, the other (80) for dimensions in English units. Both give results in henrys. The notation is explained in Fig. 10.

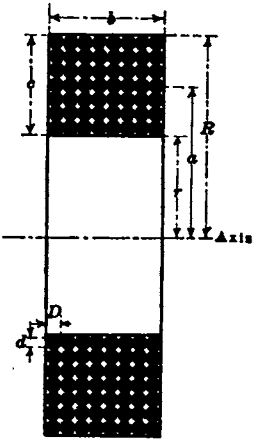

Fig. 10.—See Par. 73.

$$L = \frac{Cm^2}{b+c+R} \times \frac{F'F''}{10^9} \qquad \text{(henrys)} \qquad (79)$$

$$L = \frac{0.366\left(\frac{Ft}{1000}\right)^2}{b+c+R} \times F'F'' \qquad \text{(henrys)} \qquad (80)$$

In Eq. (80) the conductor length is in thousands of feet, and the coil dimensions in inches; 0.366 is the conversion factor. F' and F'' are empirical coil-shape factors dependent upon the relative, and independent of the absolute dimensions of the winding. Values of F' and F'' are as follows:

$$F' = \frac{10b+12c+2R}{10b+10c+1.4R};\; F'' = 0.5 \log_{10}\left(100+\frac{14R}{2b+3c}\right). \qquad (81)$$

Cm indicates the length of the conductor in centimeters;
Ft indicates the length of the conductor in feet, and $Ft/1000$ = thousands of feet;
N is the total number of turns in the winding, whence

$$Cm = 2\pi aN, \text{ when } a \text{ is in centimeters, and} \qquad (82)$$

$$\frac{Ft}{1000} = \frac{2\pi aN}{12000} \text{ when } a \text{ is in inches.} \qquad (83)$$

Numerous tables, curves, and charts which simplify the use of this formula for practical design will be found in the Bulletin No. 53 of the University of Illinois, by Morgan Brooks and H. M. Turner, entitled "Inductance of Coils." For another empirical formula see Doggett, L. A., "The Inductance of Air-cored Solenoids," *Elec. World*, Vol. LXIII (1914), p. 259.

74. Bureau of Standards, formulas for inductance. For a thorough analysis and comparison of various formulas for the inductance of coils the reader is referred to the following excellent series of articles published in the Bulletin of the Bureau of Standards:

"Formulæ and Tables for the Calculation of Mutual and Self-inductance" (Revised), E. B. Rosa and L. Cohen, Vol. VIII, p. 1; 1912; "Calculation of

Par. 77 holds true only for the conductor situated symmetrically with respect to the other two. The inductance of the other two wires cannot be calculated in a simple manner. For practical purposes it is sufficient to take the inductance of all three wires as equal to that of a line symmetrically spaced, the equivalent spacing being equal to the geometric mean of the three actual spacings, or

$$b_{eq} = \sqrt[3]{b_1 b_2 b_3}. \tag{88}$$

82. For the equivalent resistance and reactance of a three-phase line with unequal spacings of wires see the writer's "Magnetic Circuit", Art. 63. In this case the e.m.f. induced in a conductor by the varying magnetic fluxes consists of two components, one being in quadrature, the other in phase with the current in the conductor. The first component corresponds to the inductance of the conductor, the other represents transfer of power from one phase to the others. In general, these components are different for the three conductors, and in order to equalize them for the whole line conductors are transposed after a certain number of spans. This **transposition of conductors** is used on power lines (Sec. 11) as well as on telegraph and telephone lines (Sec. 21) to reduce the unbalancing effect of mutual induction. See also Par. **87** below. The inductance of two or more parallel cylinders of any cross-section can be expressed through the so-called geometric mean distance introduced by Maxwell.* For details also see Orlich, "Kapazätit und Induotivität," pp. 63–74.

83. Mutual inductance. When two independent electric circuits, (1) and (2), are in proximity to each other, their electromagnetic energy may be said to consist of three parts: the part due to the linkages of the flux produced by the circuit (1) with the current in (1); that due to the flux produced by the circuit (2) with the current in (2); and that due to the current in each circuit linking with the flux produced by the other circuit. Employing the notation in Par. **67**, the total energy of the system is expressed by

$$W = \tfrac{1}{2} i_1^2 L_1 + \tfrac{1}{2} i_2^2 L_2 + i_1 i_2 L_m \qquad \text{(joules)} \tag{89}$$

where L_1 and L_2 are the coefficients of self-induction of the two circuits, and L_m is called the **coefficient of mutual inductance** of the two circuits. All three coefficients are measured in henrys.

84. The coefficient of mutual inductance is also defined from the relations:

$$e_1 = -L_m \frac{di_2}{dt}, \text{ or } e_2 = -L_m \frac{di_1}{dt}, \tag{90}$$

that is, L_m determines the voltage e_1 induced in the circuit (1) when the current i_2 in circuit (2) varies with the time, and *vice versa*.

85. The coefficient of mutual inductance of two long coaxial single-layer coils of the same length l and cross-section A, is

$$L_m = 1.257 n_1 n_2 l A 10^{-8}, \text{ (henrys)}, \tag{91}$$

where n_1 and n_2 are the numbers of turns per centimeter length of the two coils respectively; l and A are measured in centimeters.

86. For two long coaxial coils wound in several layers the coefficient of mutual inductance is

$$L_m = 4 n_1^2 n_2^2 l d_1 d_2 r_1^2 \left(1 + \frac{d_1}{r_1} + \frac{d_1^2}{3r_1^2}\right) \qquad \text{(henrys)} \tag{92}$$

and if an iron core is present

$$L_m = 4 n_1^2 n_2^2 l d_1 d_2 r_1^2 \left[1 + (\mu_r - 1)a^2 + \frac{d_1}{r_1} + \frac{d_1^2}{3r_1^2}\right] \qquad \text{(henrys)} \tag{93}$$

For explanation of notation see Par. **72** above. See also the references in Par. **74**.

87. The coefficient of mutual inductance of two parallel line circuits (Fig. 11*a*) is given by

$$L_m = 0.4605 \log_{10}\left(\frac{a_1 b_2}{b_1 a_2}\right) \qquad \text{(millihenrys per km.)} \tag{94}$$

where a_1 and b_1 are the distances from one of the wires of circuit 1 to the

* Maxwell, J. C. "Treatise on Electricity and Magnetism," Vol. II, p. 324.

wherein I is the total exciting current and θ the angle of time-phase displacement.

The **energy lost per cycle** can be represented by the **area**, $AfBd$, of the loop; see Par. 94 below.

93. Hysteretic angle. Without hysteresis, the current I would be in phase quadrature with E. For this reason the angle $\alpha = 90 - \theta$ is called **the angle of hysteretic advance of phase.**

$$\text{Sin } \alpha = \frac{I_r}{I} = \frac{I_r E}{IE} = \frac{\text{watts loss}}{\text{apparent watts}}. \qquad (95)$$

In practice, the measured loss usually includes eddy currents (Par. 98) so that the name "hysteretic" is somewhat of a misnomer.

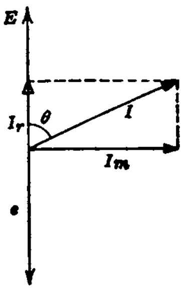

FIG. 13.—Components of exciting current; hysteretic angle.

94. The energy lost per hysteresis cycle (Fig. 12) is proportional to the area of the loop, or

$$\text{Energy} = cV \sum_{-\mathfrak{B}}^{+\mathfrak{B}} \mathfrak{H} \Delta \mathfrak{B} \qquad \text{(joules)} \qquad (96)$$

wherein V is the volume of the iron, $\mathfrak{B}$ and $\mathfrak{H}$ being the coordinates of the loop instead of Φ and $\mathfrak{F}$ as shown in Fig. 12; and c a constant depending upon the scale used. For details see the author's "Magnetic Circuit," Art. 16.

95. Steinmetz's formula. According to exhaustive experiments by Dr. C. P. Steinmetz, the heat energy released per cycle per cubic centimeter of iron is approximately

$$W = \eta B^{1.6}_{max} \qquad \text{(ergs)} \qquad (97)$$

The exponent of $\mathfrak{B}$ varies between 1.4 and 1.8 but is generally taken as 1.6. Values of η are given in Sec. 4 (see index).

96. Power loss per unit weight. The most convenient way to express the **hysteresis loss** is

$$P_h = k_h \frac{f}{100} \left(\frac{\mathfrak{B}_{max}}{1000} \right)^{1.6} \qquad \text{(watts per unit weight)} \qquad (98)$$

wherein f is the frequency in cycles per second; $\mathfrak{B}$ the maximum flux density in lines or maxwells per square centimeter, and k_h a constant; see Sec. 4.

97. Two-term formula. Another empirical formula for the **hysteresis loss** is

$$P_h = f(\eta' B + \eta'' B^2) \qquad \text{(watts per unit weight)} \qquad (99)$$

where η' and η'' are empirical coefficients. This formula is more accurate at medium and high flux densities than the preceding one.

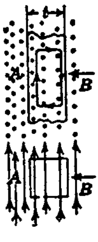

FIG. 14. Eddy currents.

98. Eddy-current losses are I^2R losses (Par. 27) due to secondary currents (Foucault currents) established in those parts of the circuit which are interlinked with alternating or pulsating flux. Referring to Fig. 14, a bar-shaped conductor is just entering a non-uniform field. The advancing side, A, is cutting more lines than the trailing side, B, so that there is a difference in potential between these two sides and electricity will flow as shown by the arrows. The value of the currents, according to Ohm's law, is directly proportional to the e.m.f. and inversely proportional to the resistance of the path. The e.m.f. is directly proportional to the thickness, t, of the bar, but the length of the path, and therefore its resistance, is but slightly altered by varying the thickness, t.

Referring to Fig. 15, which shows a cross-section of a transformer core, the primary current, I, produces the alternating flux, Φ, which by its change generates an e.m.f., e, in the core; this e.m.f. then sets up the secondary treatment of resistance to alternating currents and eddy current losses in metallic conductors see Louis Cohen, "Formulæ and Tables for the Calculation of Alternating-current Problems," Chap. I. Numerous tables and formulæ will be found there, relating to the resistance to alternating currents and eddy current losses in solid, hollow and concentric cylindrical conductors, flat conductors, coils and conductors in slots of laminated iron armatures.

102. Effective resistance and reactance. When an alternating-current circuit has appreciable hysteresis, eddy currents and skin effect, it can be replaced by an equivalent circuit, without these losses, by using **equivalent resistances** and **equivalent reactances** (Par. **154**) in place of the real ones. These equivalent or effective quantities are so chosen that the energy relations are the same in the equivalent circuit as in the actual one. In a series circuit let the true power lost in ohmic resistance, hysteresis, and eddy currents be P, and the reactive (wattless) volt-amperes P'. Then the effective resistance and reactance are determined from the relations

$$i^2r_{eff}=P;\quad i^2x_{eff}=P'. \tag{104}$$

In a parallel circuit, with a given voltage, the equivalent conductance and susceptance (Par. **162**) are calculated from the relations

$$e^2g_{eff}=P;\quad e^2b_{eff}=P'. \tag{105}$$

Such equivalent electric quantities which replace the core loss are used in the analytical theory of transformers and induction motors.

103. Core loss. In practical calculations of electrical machinery the total core loss is of interest rather than the hyteresis and the eddy currents separately. For such computations empirical curves are used, obtained from tests on various grades of steel and iron (see Sec. 4).

104. The separation of hysteresis from eddy currents. For a given sample of laminations, the total core loss P, at a constant flux density and at variable frequency f, can be represented in the form

$$P=af+bf^2 \tag{106}$$

where af represents the hysteresis loss and bf^2 the eddy or Foucault-current loss, a and b being two constants. If we write this equation for two known frequencies, two simultaneous equations are obtained from which a and b are determined.

105. Determination of constants. It is convenient to divide the foregoing equation by f, because in the form

$$\frac{P}{f}=a+bf \tag{107}$$

it represents the equation of a straight line between (P/f) and f. Having plotted the known values of (P/f) against f as abscissæ, the most probable straight line is drawn through the points thus obtained. The intersection of this line with the axis of ordinates gives a; b is calculated from the preceding equation. Knowing a and b, the separate losses are calculated at any desired frequency from the expressions af and bf^2 respectively.

THE DIELECTRIC CIRCUIT

106. Dielectric flux. When a source of continuous voltage E (Fig. 16) is applied at the terminals of a **condenser** AB, a quantity of electricity Q flows through the connecting wires and the same quantity of electricity may be said to be displaced through the dielectric between the condenser terminals, because electricity behaves like an incompressible fluid. This displaced electricity in a dielectric is called the dielectric flux and is measured in the same units as a quantity of electricity in a conducting circuit; that is, in coulombs or in microcoulombs.

107. The dielectric flux density and the potential gradient. The flux density D or the dielectric flux per unit area is $D=Q/A$ when the flux distribution is uniform, or $D=\partial Q/\partial A$ when the flux distribution is non-uniform. In these expressions Q is the dielectric flux and A is the area perpendicular to the electrostatic lines of force. Flux density is measured in microcoulombs per square centimeter or per square inch.

The voltage E applied at the terminals of the condenser acts upon the whole thickness l of the dielectric, and the **dielectric stress** G is characterized as the voltage per unit thickness (unit length) of the dielectric in the direction

112. The specific inductive capacity of a dielectric (k) is the ratio between the capacity of a condenser made entirely of this dielectric and of an identical condenser using air for dielectric. It is also termed the **dielectric constant.** Another name for specific inductive capacity is **relative permittivity.** For numerical values for various dielectrics see Sec. 4.

113. Capacity (permittance) between parallel plates. When a condenser consists of two parallel plates the distance between which is small compared to the dimensions of the plates, the lines of electrostatic displacement are nearly straight lines normal to the adjacent surfaces of the plates. The capacity of such a condenser is

$$C = \left(\frac{k}{4\pi}\right)\frac{A}{l} \qquad \text{(abstatfarads)} \qquad (115)$$

where A is the area of one of the plates in sq. cm., l the normal distance between them, in cm. and $k/4\pi$ the **permittivity** of the dielectric; k is the dielectric constant, which for air is unity. If C is to be in microfarads, then in the preceding formula in place of $k/4\pi$ use

$$\frac{k}{4\pi}\left(\frac{1}{v^2}\right) = 0.08842 \times k \times 10^{-6} \qquad \text{(mf. per centimeter-cube)} \qquad (116)$$

where v is the velocity of light, or the factor required to change from electrostatic to electromagnetic units.

If instead of taking $k/4\pi$ as the permittivity, a term k_a called the absolute permittivity is introduced, then

$$C = k_a \frac{A}{l} \qquad \text{(microfarads)} \qquad (117)$$

And for air, instead of unity (the relative permittivity), the absolute permittivity is

$$\kappa_a = 0.08842 \times 10^{-6} \qquad \text{(mf. per centimeter-cube)} \qquad (118)$$

and for any other substance the absolute permittivity would be $0.08842 \times 10^{-6}k$, where k is the specific inductive capacity or the relative permittivity of the dielectric; see Par. **112.** See also the author's "Electric Circuit," Article 51, for further elaboration of this theory of absolute versus relative permittivities. At present the accepted method of calculation is based on the use of formulæ (115) and (116).

114. The elastance of a dielectric between two parallel plates a short distance apart is $S = \sigma\,(l/A)$ where the coefficient σ (sigma) is called the **elastivity of the dielectric.** If S is in megadarafs (1 daraf is the reciprocal of 1 farad) and the dimensions in centimeters, σ is in megadarafs per centimeter cube. Elastance is the reciprocal of permittance, or $S = 1/C$, and, likewise, elastivity is the reciprocal of permittivity, or $\sigma = 4\pi/k$ Therefore

$$S = \left(\frac{4\pi}{k}\right)\frac{l}{A} = \sigma\frac{l}{A} \qquad (119)$$

For air, in practical units,

$$\sigma = 11.31 \times 10^6 \text{ mgd. per centimeter cube.} \qquad (120)$$

Example; to calculate the capacity of a plate condenser (Fig. 16) built according to the following specifications: The metal plates are 50 cm. by 70 cm. each, placed at a normal distance of 0.3 cm. The dielectric consists of three consecutive layers of insulation, which are 0.12 cm., 0.07 cm. and 0.11 cm. thick. The relative permittivities of the materials are 2, 3 and 5 respectively. Since elastances are added in series (Par. **110**), the total elastance of the condenser is

$$S = [11.3 \times 10^6/(50 \times 70)][0.12/2 + 0.07/3 + 0.11/5]$$
$$= 0.34 \times 10^3 \text{ mgd.}$$

Hence the capacity $C = S^{-1} = 2.94 \times 10^{-3}$ mf.

115. Capacity of concentric cables. For a single-conductor cable with a grounded metal sheath (Fig. 17) the capacity

$$\left.\begin{aligned} C &= \frac{0.03882k}{\log_{10}(b/a)} \qquad \text{(mf. per mile) or} \\ C &= \frac{0.02413k}{\log_{10}(b/a)} \qquad \text{(mf. per km.)} \end{aligned}\right\} \qquad (121)$$

of the preceding value. The transverse dimensions may be either in inches or in centimeters because only their ratio enters in the formula.

119. A three-conductor cable may be treated in a similar way (Fig. 20). The sheath is replaced by three equally spaced conductors of the same diameter and spaced according to the relation $ba = d^2$. The **capacity of a single conductor** is

$$C = \frac{0.07764k}{\log_{10}\left[\frac{3a^2(d^2-a^2)^6}{r^2(d^6-a^6)}\right]} \quad \text{(mf. per mile)} \qquad (126)$$

where the transverse dimensions are in inches or in centimeters.

120. The capacity of a single-phase transmission line per wire, or the permittance between one of the wires and the plane of symmetry is

$$C = \frac{0.03882}{\log_{10}(b/a)} \quad \text{(mf. per mile)} \qquad (127)$$

where a is the radius of the wire, and b the spacing between the centres. The capacity between the two conductors is equal to one-half of that given by the formula above. For values of charging current at standard frequencies see tables in Sec. 11.

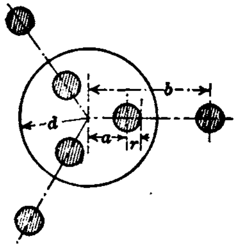

FIG. 20.—Three-conductor cable; showing electrical images due to the grounded sheath.

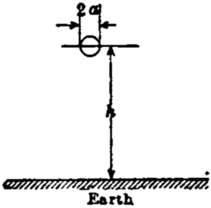

FIG. 21.—Overhead conductor.

121. The capacity of a single overhead conductor with ground return (Fig. 21) is

$$C = \frac{0.03882}{\log_{10}(2h/a)} \quad \text{(mf. per mile)} \qquad (128)$$

122. When a single-phase line with metallic return is suspended sufficiently near the ground its capacity is somewhat increased. Let the wires be suspended at the heights h_1 and h_2 above the ground; then calculate the capacity according to formula in Par. 120, using the corrected spacing (see the author's "Electric Circuit," Art. 61)

$$b_e = \frac{b}{\sqrt{1 + (0.25b)^2/h_1h_2}} \qquad (129)$$

When the heights of suspension h_1 and h_2 are greater than 3.5 times the spacing b, the difference between b and the corrected spacing b_e is less than 1 per cent. The correction in formula (117) is still smaller, because logarithms of numbers vary more slowly than the numbers themselves.

In formulæ (128) and (129) a perfectly conducting ground is assumed. With dry non-conducting earth the increase in capacity is somewhat less.

123. Capacity of a three-phase line with symmetrical spacing. The concept of the capacity of a three-phase line is not definite without further qualifications. In practice a three-phase line is calculated by reducing it to an equivalent single-phase line consisting of one of the conductors of the three-phase line and a ground return. This equivalent single-phase line carries one-third of the total power transmitted by the three-phase line an element of volume ∂v and integrating within the desired limits. For an interesting comparison of practical possibilities as to the amount of energy stored in the dielectric form per unit volume, compared with other forms of energy, see Steinmetz, C. P., *General Electric Review*, 1913, p. 536.

129. A system of charged bodies (Fig. 23). The **total charges** on the individual conductors are expressed by the equations

$$\left.\begin{aligned} q_1 &= C_1v_1 + C_{12}(v_1 - v_2) + C_{13}(v_1 - v_3) + \text{ etc.,} \\ q_2 &= C_2v_2 + C_{21}(v_2 - v_1) + C_{23}(v_2 - v_3) + \text{ etc.,} \end{aligned}\right\} \qquad (135)$$

where v_1, v_2, v_3, etc., are the potentials of these conductors above the ground. The coefficients C_1, C_2, C_3, etc., are called the **partial capacities** of the conductors; C_{12}, C_{23}, etc., are called **mutual capacities.** Their computation is possible in a few simple cases only, but having determined them experimentally, it is possible to calculate from the preceding equations the resultant or equivalent capacity of the system under various operating conditions.

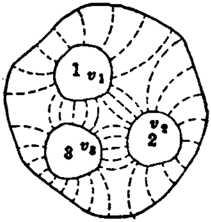

Fig. 23.—System of charged bodies.

130. Maxwell's equations of a charged system. The same equation may be written in Maxwell's form

$$\left.\begin{aligned} q_1 &= K_{11}v_1 + K_{12}v_2 + K_{13}v_3 + \text{etc.,} \\ q_2 &= K_{21}v_1 + K_{22}v_2 + K_{23}v_3 + \text{etc.,} \end{aligned}\right\} \qquad (136)$$

where the coefficients K_{11}, K_{22}, etc., are called the capacities of the individual conductors, and the negative quantities K_{12}, K_{23}, etc., are called **coefficients of mutual induction.**

131. Coefficients in Maxwell's equations. The following relations hold between the coefficients K and C:

$$\left.\begin{aligned} K_{11} &= C_1 + C_{12} + C_{13} + \text{etc.} \\ K_{22} &= C_2 + C_{21} + C_{23} + \text{etc.} \\ K_{12} &= -C_{12};\ K_{21} = -C_{21}, \text{ etc.} \end{aligned}\right\} \qquad (137)$$

132. The electrostatic energy stored in the field is

$$W = \tfrac{1}{2}K_{11}v_1^2 + \tfrac{1}{2}K_{22}v_2^2 + \text{etc.} + K_{12}v_1v_2 + K_{13}v_1v_3 + K_{23}v_2v_3 + \text{etc.} \qquad (138)$$

W is expressed in joules (watt-seconds) if the potentials are in volts and the capacities in farads.

133. The dielectric strength of insulating materials, or the rupturing voltage gradient, is the maximum voltage per unit thickness which a dielectric can stand in a uniform field, before it breaks down electrically. The dielectric strength is usually measured in kilovolts per millimeter or per inch. The only correct way is to refer the dielectric strength to a uniform field, for instance, between large parallel plates placed at a short distance apart. If the striking voltage is determined between two spheres or electrodes of some other shape, the fact should be distinctly stated. In designing insulation a factor of safety is assumed depending upon the conditions of operation. For numerical values of the rupturing voltage gradients of various materials see Sec. 4.

134. The critical dielectric flux density is the density at which the material breaks down. It is determined from the relation

$$D_{max} = 0.08842kG_{max} \times 10^{-2}, \qquad (139)$$

where D_{max} is the critical density in microcoulombs per square centimeters G_{max} is the **rupturing voltage gradient** in kilovolts per millimeter, and k is the relative permittivity of the material (Par. **112**).

135. Electrostatic corona. When the electrostatic flux density in the air exceeds a certain value, a pale violet light appears near the adjacent metal surfaces; this silent discharge is called the electrostatic corona. In the regions where the corona appears, the air is electrically broken down, and ionized so that it becomes a conductor of electricity. When the voltage is raised still higher a brush discharge takes place, until the whole thickness of the dielectric is broken down and a disruptive discharge, or spark, jumps from one electrode to the other.

The formation of corona leads to power loss which may be serious in some circuit is suddenly connected to a source of continuous voltage e, the current gradually rises to the final value $i_e = e/r$ according to the law

$$i = i_e(1 - \epsilon^{-tr/L}), \tag{140}$$

where t is time, and ϵ is the base of natural (or hyperbolic) logarithms. This expression is known as **Helmholtz's law.** When the source of e.m.f. is short-circuited the current in the remaining circuit decreases to zero according to a similar law

$$i = i_o\epsilon^{-tr/L}. \tag{141}$$

141. Periodic e.m.f. When a de-energized circuit containing r and L is suddenly connected at the instant $t=0$, to a source of alternating voltage $e = E_m \sin (2\pi ft + \alpha)$, the current in the circuit varies according to the law

$$i = \frac{E_m}{z} \sin (2\pi ft + \alpha - \phi) - \frac{E_m}{z} \sin (\alpha - \phi)\epsilon^{-tr/L} \tag{142}$$

In this equation $z = \sqrt{r^2 + (2\pi fL)^2}$ is the impedance of the circuit and ϕ is the phase displacement between the current and the voltage, determined by $\tan \phi = 2\pi fL/r$. The angle α is the phase displacement between the voltage e and the reference wave which passes through zero at the time $t=0$; f is the frequency. The first term in the expression for i is the current corresponding to the permanent condition, the second term is a transient which rapidly approaches zero with the time. (See also Eq. 146.)

142. Closing a circuit containing a resistance r (ohms) and a capacity C (farads) in series. The charging current is theoretically expressed by

$$i = i_o\epsilon^{-t/(rC)}, \tag{143}$$

where i_o is the current at the first instant. This equation is not applicable to the beginning of the charge because it presupposes a sudden jump of the current from zero to i_o. In reality, the unavoidable inductance of the circuit smoothes down the initial change in current.

When a condenser, charged at a voltage e_o, is discharged through resistance r, the discharge current at the first instant is theoretically equal to $i_o = e_o/r$, and then varies according to the law

$$i = i_o\epsilon^{-t/(rC)} \tag{144}$$

The voltage across the condenser terminals decreases according to a similar law

$$e = e_o\epsilon^{-t/(rC)} \tag{145}$$

When a de-energized circuit containing r and C is suddenly connected at the instant $t=0$ to a source of alternating voltage $e = E_m \sin(2\pi ft + \alpha)$, the current in the circuit will vary according to the law

$$i = \frac{E_m}{z} \sin(2\pi ft + \alpha + \phi) - \frac{E_m}{z} \sin (\alpha + \phi)\epsilon^{-t/(rC)} \tag{146}$$

In this equation $z = \sqrt{r^2 + [1/(2\pi fC)]^2}$ is the impedance of the circuit, and ϕ is the phase displacement between the current and the voltage, determined by $\cot \phi = 2\pi fCr$. The angle α is the phase displacement between the voltage e and the reference wave which passes through zero at the time $t=0$; f is the frequency. The first term in the expression for i is the current corresponding to the permanent condition, the second term is a transient which rapidly approaches zero with the time. Compare Par. **141.**

143. Single-energy and double-energy transients. The two preceding cases are examples of single-energy transients, because the energy is stored in one form only (electromagnetic or electrostatic), and the energy change consists in an increase or a decrease of the stored energy. In the case of inductance the energy is that of the magnetic field and in the case of capacity it is the energy of the electrostatic field. When both inductance and capacity are present, the energy of the circuit is stored in two forms, and there is a possibility of periodic transformation of the magnetic energy into the dielectric energy, and *vice versa*, which constitutes electric oscillations, surges, and waves. There is also a possibility of a **triple-energy transient,** when for instance a synchronous motor is hunting at the end of a long transmission line which possesses inductance and capacity. In the last successfully used for wireless telegraphy. See Poulsen, V., "System for Producing Continuous Electric Oscillations," *Trans.* Int. Elec. Congress, St. Louis, 1904, Vol. II, p. 963. Also Austin, L. W., "The Production of High-frequency Oscillations from the Electric Arc," *Bulletin* of the Bureau of Standards, Vol. III (1907), p. 325.

149. Stored energy. When a considerable amount of energy is liberated at some point on a transmission line, for instance due to an indirect lightning stroke, a wave starts along the line carrying this energy to the ends of the line. Part of it enters the apparatus at the ends, part is reflected and the rest is converted into heat. Generally speaking, the total energy stored in the line, or in some part of it, at an instant is

$$W = \tfrac{1}{2}Li^2 + \tfrac{1}{2}Ce^2, \qquad \text{(joules)} \qquad (154)$$

where L is the inductance of the line in henrys, i an instantaneous current, C the capacity of the line in farads, and e an instantaneous voltage. The term $\tfrac{1}{2}Li^2$ represents the electromagnetic energy, the term $\tfrac{1}{2}Ce^2$ the electrostatic energy. At certain instants the current is equal to zero, at others the voltage is zero, so that the two energies must be equal. Therefore

$$\frac{e_{max}}{i_{max}} = \sqrt{\frac{L}{C}}. \qquad \text{(ohms)} \qquad (155)$$

Thus, knowing the maximum voltage e_{max}, the largest instantaneous current i_{max} can be calculated, and *vice versa*. For instance, in the case of a lightning stroke, the maximum voltage is limited by the disruptive strength of the insulation to instantaneous voltages, and the maximum current disturbance may be calculated from the preceding equation.

150. Surge impedance. With concentrated inductance and capacity, the frequency of oscillations is (Par. 147)

$$f_o = \frac{1}{2\pi\sqrt{LC}}. \qquad (156)$$

With **uniformly distributed inductance and capacity, the frequency is**

$$f_o{}^1 = \frac{1}{4\sqrt{LC}}. \qquad (157)$$

The expression $\sqrt{L/C}$ is called **the natural impedance or the surge impedance of the line,** and its reciprocal **the natural admittance or the surge admittance.** For further information consult the references in Par. **139** above.

ALTERNATING-CURRENT CIRCUITS

151. Sine-waves. In this treatment of alternating-current circuits, a sine-wave is arbitrarily assumed. For non-sinusoidal currents and voltages see Par. **190** and following. Beginning with non-inductive circuits, *i.e.*, circuits which contain only resistance, the current at any instant is proportional to the instantaneous value of the impressed e.m.f. Plotting the instantaneous values of the e.m.f. and the current, it is seen, Fig. 24, that the waves pass through zero and reach their maximum values at the same instant. They are said to be **in phase.**

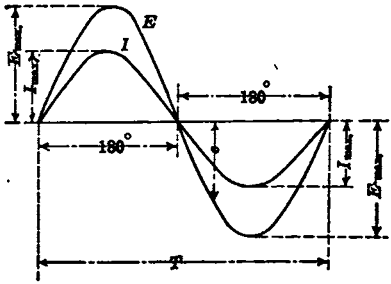

Fig. 24.—Simple sine-waves; non-inductive circuit.

152. Instantaneous values. Let the amplitude or the instantaneous maximum value of the voltage be E_{max}, then the instantaneous value is

$$e = E_{max} \sin 2\pi ft \qquad (158)$$

where f is the frequency in cycles per second, and t is time in seconds. The angle $2\pi ft$ is in radians. If the time of one complete cycle is T (Fig. 24), $f = 1/T$.

is called capacity reactance, or condensive reactance. When inductive reactance x (Par. 154) and condensive reactance x_c enter in the same circuit, x_c is considered negative. When C is in farads, x_c is in ohms.

159. E.m.f. components. In a circuit containing resistance and condensive reactance in series, the applied e.m.f. may be divided into two components; one consumed in resistance drop and the other in the condensive reactance. The current taken by the condensive reactance is proportional to the rate of change of the e.m.f., which is impressed across its terminals, therefore the counter-e.m.f. of the condensive reactance is in time-quadrature with the current. Referring to Fig. 26, I is the total current in phase with E_r which is the e.m.f. consumed in resistance; E_c is the voltage necessary to balance the counter-e.m.f. of the condensive reactance, and E is the total e.m.f. impressed upon the circuit. It will be seen that in this case the current is leading.

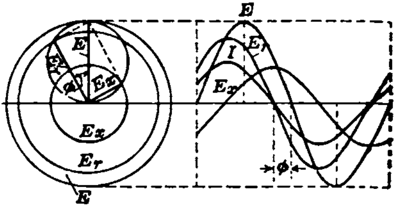

FIG. 26.—E.m.f. and current waves in a circuit containing resistance and capacity reactance in series.

If the instantaneous applied voltage is expressed as in (152), the instantaneous current is

$$i = I_{max} \sin (2\pi f t + \phi). \tag{166}$$

In this expression, the **phase angle** ϕ between the current and the voltage is determined from the relation

$$\tan \phi = \frac{1}{2\pi f C r} = \frac{x_c}{r}. \tag{167}$$

160. Terminology. The following terminology used in application of **sine-wave alternating-current circuits** is recapitulated here for the sake of convenience. An **instantaneous value** of alternating current or voltage (Fig. 24) is connected with the **maximum value** or the **amplitude** by the relation given in Par. 152. The **mean effective value**, also called the **root-mean-square value**, or simply the **effective value** of an alternating current or voltage is defined in Par. 199. For a sine-wave quantity the effective value is equal to the amplitude divided by $\sqrt{2}$; or

$$E_{eff} = \frac{E_{max}}{\sqrt{2}} = 0.7071 E_{max}. \tag{168}$$

The **mean or average value** of a sinusoidal alternating current or voltage is equal to the maximum value divided by $\pi/2$, or

$$E_{av} = \frac{2E_{max}}{\pi} = 0.6366 E_{max}. \tag{169}$$

The ratio between the effective and the average value is called the **form factor** (Par. 207) and is equal to 1.11 for sine-waves.

161. Periodic time. The interval of time T in Fig. 24. corresponds to one complete cycle. The interval of time $T/2$ corresponding to one-half wave is called an **alternation**, and for every cycle there are two alternations. The **frequency** or the **periodicity** of an alternating current may be expressed either in cycles per second or in alternations per minute. However, the latter method is not common.

162. The phase displacement between two currents or two voltages, or between a current and a voltage, is commonly measured in **electrical degrees.** One electrical degree is 1/360th part of a complete cycle.

163. Vector representation. Alternating currents and voltages which vary according to the sine or cosine law can be represented graphically by directed straight lines called **vectors** (Fig. 27). The length of a vector represents, to some arbitrary scale, the effective value of the alternating

If E is the voltage across the circuit, then

$$I_r = E\left(\frac{r}{r^2+x^2}\right) = Eg \qquad \text{(amp.)} \qquad (179)$$

$$I_x = E\frac{x}{r^2+x^2} = Eb \qquad \text{(amp.)} \qquad (180)$$

$$I = \sqrt{(Eg)^2+(Eb)^2} = E\frac{1}{z} = Ey \qquad \text{(amp.)} \qquad (181)$$

In these expressions

$$g = \frac{r}{z^2} = \text{conductance} \qquad \text{(mho)} \qquad (182)$$

$$b = \frac{x}{z^2} = \text{susceptance} \qquad \text{(mho)} \qquad (183)$$

$$y = \frac{1}{z} = \text{admittance} \qquad \text{(mho)} \qquad (184)$$

$$\tan\phi = \frac{I_x}{I_r} = \frac{b}{g};\ \cos\phi = \frac{I_r}{I} = \frac{g}{y}. \qquad (185)$$

170. Resistance and condensance in parallel. In a circuit consisting of a resistance and a capacity in parallel the same relations hold except that the current is leading, and x_c is used in place of x (Par. 168). If there is any doubt whether the quadrature current is caused by an inductance or a capacity, use the expressions inductive susceptance and capacity (or condensive) susceptance. The latter is sometimes called **capacitance.** Fig. 28 also shows, in dotted lines, the resultant current when a pure resistance and a pure condensance are connected in parallel; in such case the phase angle ϕ becomes an angle of lead.

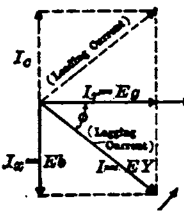

Fig. 28.—Vector diagram of currents; pure resistance in parallel with pure inductive reactance.

171. Impedances in series. In a circuit containing several resistances and reactances in series, the resistances should be added together and the reactances added together, so that

$$\left.\begin{aligned} r_{eq} &= \Sigma r; \\ x_{eq} &= \Sigma x; \end{aligned}\right\} \qquad (186)$$

$$z_{eq} = \sqrt{(\Sigma r)^2+(\Sigma x)^2}. \qquad (187)$$

The subscript *eq* stands for equivalent.

172. Impedances cannot be added algebraically, but must always be added geometrically, or vectorially. Since

$$r = z\cos\phi, \text{ and } x = z\sin\phi, \qquad (188)$$

the preceding equation gives

$$z_{eq} = \sqrt{(\Sigma z\cos\phi)^2+(\Sigma z\sin\phi)^2}. \qquad (189)$$

173. Admittances in parallel. In a circuit consisting of several parallel branches the conductances should be added together and the susceptances added together (Par. 169) so that,

$$\left.\begin{aligned} g_{eq} &= \Sigma g \\ b_{eq} &= \Sigma b \end{aligned}\right\} \qquad (190)$$

and

$$y_{eq} = \sqrt{(\Sigma g)^2+(\Sigma b)^2}. \qquad (191)$$

174. Admittances cannot be added algebraically, but must always be added geometrically, or vectorially. Since

$$g = y\cos\phi, \text{ and } b = y\sin\phi, \qquad (192)$$

the preceding equation gives

$$y_{eq} = \sqrt{(\Sigma y\cos\phi)^2+(\Sigma y\sin\phi)^2}. \qquad (193)$$

175. Equivalent series and parallel combinations. Let r_s and x_s be a resistance and a reactance connected in series and let r_p and x_p be a

Considering the series circuit shown in Fig. 30, let it be required to study the current when the resistance and reactance are varied, the e.m.f. being kept constant. With E as a diameter draw the circle. Obc, then Obc is the e.m.f. or impedance triangle and θ is the angle of phase displacement between I and E. Dividing the current, I, into imaginary components, E/r is laid off along OC in phase with E, and E/x is laid off along OA in quadrature with E. Drawing the line AC, and the circles, OBA and OBC, the line, OB, represents the current, I, both as to value and phase position. If x is constant and r variable, the point, B, will travel along the circle, OBA, while if r is constant and x variable, the point, B, will travel along the circle, OBC.

178. Circle diagram; parallel circuits. Referring to the parallel circuit in Fig. 31, let it be required to study the e.m.f. when the conductance and susceptance are varied, the current remaining constant. With I as diameter draw the circle Obc, then Obc is the current or admittance triangle and θ is the angle of phase displacement between E and I. Dividing the e.m.f. into components, I/b is laid off along OA in quadrature with I; then drawing the line, AC, and the circles, OBA and OBC, the line, OB, represents the e.m.f., E, both as to value and phase position. The circle, OBA, is the locus of the point, B, when b is constant and g variable, while the circle, OBC, is the locus of the point, B, when g is constant and b variable.

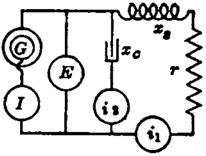

FIG. 32.—Condensance in parallel with inductive impedance.

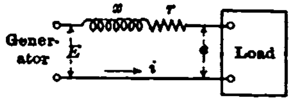

FIG. 33.—Load connected to inductive line.

179. Phase compensation. Condensive reactance connected **in shunt with an inductive impedance** can be so adjusted as to bring the total current more or less in phase with the impressed e.m.f. Referring to Fig. 32, x_c and the impedance z_s are in parallel, where

$$z_s^2 = r^2 + x_s^2. \tag{198}$$

Taking the admittances,

$$g_1 = \frac{r}{z_s^2};\ g_2 = 0;\ b_1 = \frac{x_s}{z_s^2};\ \text{and } b_2 = -\frac{1}{x_c}. \tag{199}$$

In order that I be in phase with E, $b_1 + b_2$ must be equal to zero, or

$$x_c = \frac{z_s^2}{x_s}. \tag{200}$$

Thus, it is seen that the value of x_c depends upon the resistance, r, as well as upon x_s:

$$i_2 = E\frac{1}{x_c} = Eb_2;\ i_1 = E\sqrt{\left(\frac{r}{z_s^2}\right)^2 + \left(\frac{x_s}{z_s^2}\right)^2} = E\sqrt{g_1^2 + b_1^2}, \tag{201}$$

$$I = E\sqrt{\left(\frac{r}{r^2+x_s}\right)^2 + \left\{\left(\frac{x_s}{r^2+x_s^2}\right) - \frac{1}{x_c}\right\}^2} = E\sqrt{g^2+b^2}, \text{ in amp.} \tag{202}$$

180. Leading current through an inductive line will raise the e.m.f. at the receiving end of the circuit. Referring to Fig. 33, let E be the voltage at the generator end of a circuit, e the voltage at the receiver end, and i the line current. Let the load be of such a nature that the current is leading with respect to the voltage e. Adding to e the ohmic drop ir in the line (Fig. 34a) in phase with i, and the reactive drop ix in leading quadrature with i, the impressed voltage E is obtained. It will be seen that $E < e$; but with a lagging current, $E > e$ (Fig. 34b).

181. Series resonance. In a constant-potential circuit which contains inductive reactance and also condensive reactance in series, it is possible

But the total admittance y may be smaller than b_c, and in this case the total line current I is less than one of its components i_c. A similar relation may be proved for i_s. When the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}}, \qquad \text{(cycles per second)} \tag{212}$$

it follows that

$$b_s = b_c, \tag{213}$$

and

$$I = Eg, \quad i_s = -i_c. \tag{214}$$

The line current is comparatively small, but there is a large interchange of current between the inductance and the capacity, in parallel.

Resonance can occur at only one frequency. Sometimes, in the case of a complex wave, it occurs at the frequency of one of the component harmonics instead of the fundamental frequency. In such case, either for voltage or current (series or parallel resonance), the magnitude of the resonant harmonic component is much exaggerated, as compared with its normal magnitude in a non-resonant circuit. The condition of resonance, except in tuned circuits where it is specially desired (as in radio-telegraphy), is one to be avoided.

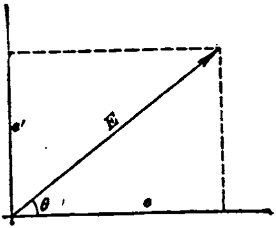

FIG. 35.—Complex quantities; axes of reals and imaginaries.

183. Consonance. Resonance in the primary circuit of a transformer, caused by the proper combination of inductance and capacity in the secondary circuit, is called consonance.

184. Alternating currents and voltages treated by means of complex (imaginary) quantities. If e and e' (Fig. 35) are the projections or the components of a vector E along two perpendicular axes, then the vector E may be represented symbolically as

$$\underset{\cdot}{E} = e + je'. \tag{215}$$

where

$$j = \sqrt{-1}, \tag{216}$$

and the dot under E signifies that the magnitude as well as the direction of E is meant.

185. Addition and subtraction of vectors. Let two vectors of voltage be represented as

$$E_1 = e_1 + je'_1, \tag{217}$$

$$\underset{\cdot}{E}_2 = e_2 + je'_2. \tag{218}$$

Then the sum or the difference of these two vectors is

$$E_3 = E_1 \pm E_2 = (e_1 \pm e_2) + j(e'_1 \pm e'_2). \tag{219}$$

186. Rotation of a Vector. Multiplying a vector by j turns it by 90 deg. in the positive direction (counter-clockwise). Thus,

$$j\underset{\cdot}{E} = j(e + je') = -e' + je, \tag{220}$$

because $j^2 = -1$. Multiplying a vector by $-j$ rotates the vector by 90 deg. in the negative direction—that is, clockwise.

A vector E may be also represented symbolically (Fig. 35) as

$$\underset{\cdot}{E} = E(\cos\theta + j\sin\theta), \tag{221}$$

where E without the dot, on the right-hand side of the equation, stands for the magnitude only.

The operator

$$\epsilon^{j\phi} = \cos\phi + j\sin\phi, \tag{222}$$

where ϵ is the base of natural logarithms, turns a vector by the angle ϕ in the positive direction. Thus,

$$\underset{\cdot}{E}(\cos\phi + j\sin\phi) = E(\cos\theta + j\sin\theta)(\cos\phi + j\sin\phi) = E[\cos(\theta+\phi) + j\sin(\theta+\phi)]. \tag{223}$$

NON-SINUSOIDAL OR COMPLEX WAVES

190. Examples of complex waves. The curves shown in Fig. 36 illustrate the effect of the inductance and the capacity in a circuit to which is applied an alternating e.m.f., differing from the simple sine-wave. The curves were taken simultaneously with an oscillograph. E is the impressed e.m.f.; I_s the current taken by an inductance coil, and I_c that taken by a condenser. Fig. 37 shows the circuit.

191. Wave of reactive e.m.f. due to inductive reactance. Assuming the reluctance of the iron core in the inductance coil to be constant, which is approximately true below the saturation point, the value of

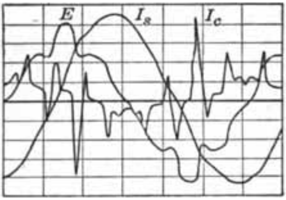

FIG. 36.—Complex alternating-current waves.

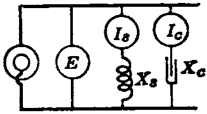

FIG. 37.—Circuit in which the waves of Fig. 36 were observed.

the flux is proportional to the current I_s. The instantaneous value of the e.m.f. E (see Par. 67) is

$$e = n\frac{d\phi}{dt} = L\frac{di}{dt} \tag{236}$$

that is, the curve E will have its maximum amplitude when the curve I_s passes through zero. This is not exactly true in this case, because of a small loss in the resistance and the iron; the current to supply this loss being in phase with the e.m.f. E.

192. Wave of current through condensive reactance. The condenser current is proportional to the rate of change of the e.m.f. (see Par. 108); the instantaneous value is

$$i_c = C\frac{de}{dt}, \tag{237}$$

that is, the curve, I_c, has its maximum when the rate of change of the curve, E, is a maximum. Were E a sine curve, I would be also a sine curve and would be in quadrature with E, but when the curve of e.m.f. is not a sine curve, as in Fig. 36, the maximum amplitude of the current will occur at the point where the slope of the e.m.f. curve is a maximum.

193. Effects of inductive and condensive reactance on wave form. These curves show the effect upon the current wave form of inductive reactance and condensive reactance. The curve, E, is the wave form produced by the generator; it contains several harmonics (see Par. 209). The inductive reactance tends to damp out the higher harmonics, while the condensive reactance emphasizes them.

194. Determination of total complex current wave. When the applied voltage contains higher harmonics (Par. 209) the total current through an impedance is found by summing the harmonic currents due to each harmonic of the voltage acting alone. Thus, the reactance at the fundamental frequency f is $x_1 = 2\pi fL$, the reactance to the nth harmonic is $x_n = 2\pi nfL$, and the impedance to the nth harmonic is

$$z_n = \sqrt{r^2 + (2\pi nfL)^2}. \tag{238}$$

195. Power and energy. The general expression for the energy delivered to an alternating-current circuit with any wave form of current and voltage is

$$W = \int_{t_1}^{t_2} ei\,dt \text{ (joules or watt-seconds)}, \tag{239}$$

into k equal parts by $k+1$ equidistant ordinates y_0, y_1, etc., y_k, where k is an even number. Then the effective value is

$$y_{eff} = \frac{1}{\sqrt{3k}}\left[y_0^2+4(y_1^2+y_3^2+\text{etc.}+y_{k-1}^2)+2(y_2^2+y_4^2+etc+y^2_{k-2})+y_k^2\right]^{\frac{1}{2}}. \quad (243)$$

203. Third method. If the irregular wave is given in terms of its harmonics, then the effective value is

$$y_{eff} = 0.7071\sqrt{A_1^2+A_3^2+\text{etc.}}, \quad (244)$$

where A_1, A_3, etc., are the amplitudes of the separate harmonics.

204. Fourth method. Replot the given irregular curve (Fig. 38) in polar coordinates (Fig. 39), and determine the area A_p, of the polar curve, with a planimeter, or by plotting on homogeneous paper of known area and weight, then cutting out and weighing again; the areas are then proportional

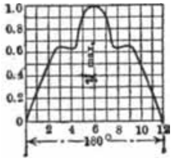

FIG. 38.—Complex wave in rectangular coordinates.

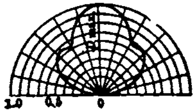

FIG. 39.—Complex wave of Fig. 38 in polar coordinates.

to the weights. This area must be expressed in units, y^2_{max}, as taken from Fig. 38. This is done by multiplying the area, A_p, of the polar curve by the ratio $\left(\frac{y_{max}}{\rho_{max}}\right)^2$; y_{max} and ρ_{max} are measured in terms of the same units.

The mean effective ordinate is

$$y_{eff} = \frac{y_{max}}{\rho_{max}}\sqrt{\frac{2A_p}{\pi}}, \qquad \text{in terms of } y_{max}. \quad (245)$$

205. Generalization of fourth method. The latter method has been generalized by Mr. C. O. Mailloux for determining the effective value of direct current taken by an electric car or a train during a run. For a detailed treatment and numerous practical applications see his paper "Methode de Determination du Courant Constant Produisant le Même Echauffement qu'un Courant Variable," in the Transactions of the International Electrical Congress held at Turin (Italy), 1911.

206. The amplitude factor is the ratio of the maximum ordinate to the mean effective ordinate, thus

$$\frac{y_{max}}{y_{eff}} = \text{amplitude factor}. \quad (246)$$

207. The form factor is the ratio of the mean effective ordinate to the mean ordinate, thus

$$\frac{y_{eff}}{y_{mean}} = \text{form factor}. \quad (247)$$

---

including the two given ones, be denoted y_0, y_1, y_2, etc., y_n. Then the area of the curve is

$$A = \tfrac{1}{3}h[y_0+4(y_1+y_3+y_5+\text{etc.}+y_{n-1})+2(y_2+y_4+y_6+\text{etc.}+y_{n-2})+y_n],$$

where h is the distance between any two adjacent ordinates. The greater the number of strips (n), the more nearly the foregoing formula represents the area of the given curve.

θ_n being measured in terms of the nth harmonic. Assuming time measured to the right and ordinates measured up as positive, and quantities measured in opposite directions as negative, positive values of θ_n indicate that the nearest intersection of the nth harmonic with the axis is to the right of the intersection of the resultant wave with the axis, and positive values of B_n indicate that the nth harmonic is rising at its nearest intersection with the time axis. The values obtained with the above equations for the nth harmonic are affected by the harmonics which are multiples thereof, that is $2n$, $3n$, etc. This correction is practically negligible for all harmonics, except the first or fundamental, and a correction rarely needs to be carried beyond the ninth harmonic. Since wave forms in practice almost never contain even harmonics, they do not enter into the correction, and denoting the corrected values by prime, we have:

$$A'_n = A_n - A'_{3n} - A'_{5n} - A'_{7n} - \ . \ . \ . \tag{252}$$

and

$$B'_n = B_n + B'_{3n} - B'_{5n} + B'_{7n} - \ . \ . \ . \tag{253}$$

When applying this to the first harmonic, A_n is the ordinate of the resultant wave at y_0 (Fig. 40*b*), and B_n is the ordinate 90 time-degrees therefrom at y_3.

FIG. 40a.—Wave analysis, Par. 211.

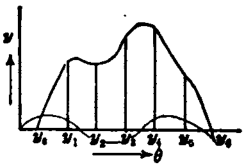

FIG. 40b. Wave analysis, Par. 211.

211. Example of wave analysis. As an example,* assume the wave given in Fig. 40*a*, which is split into three harmonics; the first or fundamental, the third and the fifth. Fig. 40*b* shows the method of determining a given harmonic, in this case the third. The base of the wave is divided into $2n$ or six equal parts and ordinates erected. Assume the ordinates to measure as follows:

$$y_1 = 676;\ y_2 = 660;\ y_3 = 940;\ y_4 = 1004;\ y_5 = 554;\ y_6 = 0. \tag{254}$$

then,

$$A_3 = \tfrac{1}{3}(y_4 - y_2) = \frac{1004 - 660}{3} = 114.7, \tag{255}$$

and

$$B_3 = \tfrac{1}{3}(y_1 + y_5 - y_3) = \frac{676 + 554 - 940}{3} = 96.7. \tag{256}$$

The maximum ordinate is

$$\sqrt{(114.7)^2 + (96.7)^2} = 150 \tag{257}$$

and the phase angle is

$$\theta_3 = \tan^{-1}\left(\frac{-114.7}{96.7}\right) = -50 \text{ deg.}\dagger \tag{258}$$

In a similar manner it is found that $A_5 = -92.8$, and $B_5 = 37.4$. In this example the wave contains only the third and the fifth harmonics; therefore, the fundamental is determined as follows:

$$A_1 = y_6 - A'_3 - A'_5 = 0 - 114.7 + 92.8 = -21.9;$$
$$B_1 = y_3 + B'_3 - B'_5 = 940 + 96.7 - 37.4 = 999.3;$$
$$\theta_1 = \tan^{-1}(21.9/999.3) = 1 \text{ deg. } 15 \text{ min. (approx.)}$$

* *Elec. Jour.*, Vol. V, p. 386 (1908).

† Fifty deg. in the terms of the third harmonic, or 50/3 deg. in terms of the resultant

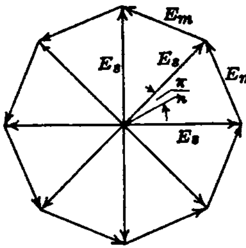

Fig. 45a.—Symmetrical star and ring e.m.fs.

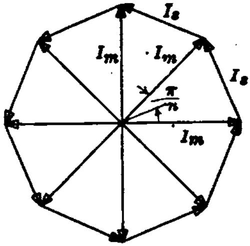

Fig. 45b.—Symmetrical star and ring currents.

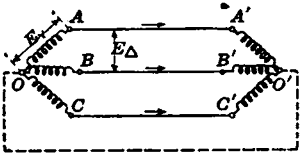

Fig. 46a.—Three-phase Y connection.

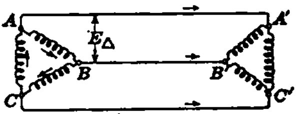

Fig. 46b.—Three-phase delta connection.

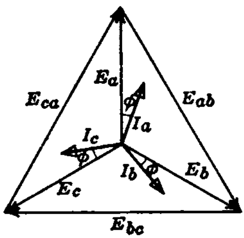

Fig. 47a.—Three-phase Y e.m.fs. and currents.

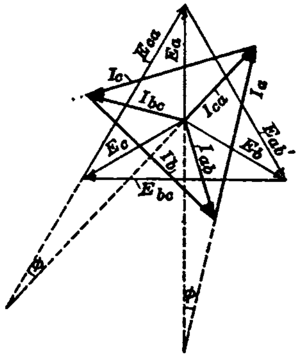

Fig. 47b.—Three-phase Δ e.m.fs. and currents.

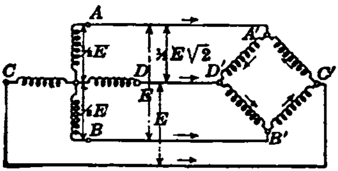

Fig. 48.—Two-phase system; star and ring connections.

221. Polyphase power. The total power in a **symmetrical *n*-phase system** is given by the formula

$$P = nI_sE_s \cos\phi = nI_mE_m \cos\phi. \qquad (263)$$

In an **unsymmetrical** or **unbalanced system** the **total power** is found by summing up the power in the separate phases.

222. Three-phase power. In a three-phase system (Fig. 46), the power

$$P = 3I_yE_y \cos\phi = 3I_\Delta E_\Delta \cos\phi = I_yE_\Delta\sqrt{3}\cos\phi \quad \text{(watts)} \qquad (264)$$

if the currents are in amperes and the voltages in volts.

223. Two-phase power. In a **quarter-phase system** (Fig. 47) the power

$$P = 4I_sE_s \cos\phi = 4I_mE_m \cos\phi = 2\sqrt{2}I_sE_m \cos\phi \quad \text{(watts)} \qquad (265)$$

224. An equivalent single-phase circuit is a circuit which is used in computations relating to polyphase transmission lines and machinery. For three-phase circuits some engineers use a single-phase circuit with a voltage equal to the Y-voltage of the three-phase system, and the power equal to that in one phase. Others use a single-phase circuit having a voltage equal to the delta voltage of the three-phase circuit, and transmitting the power equal to the total power in the three phases. Both methods lead to the same result, provided that the assumptions are consistently carried out.

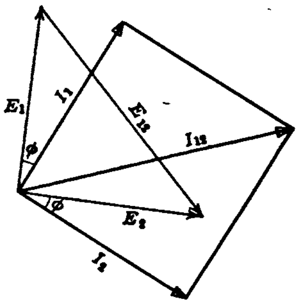

Fig. 51.—E.m.fs. and currents in two-phase three-wire system.

225. Unbalanced polyphase circuits are treated as separate single-phase circuits and then combined into one. Assuming a three-phase system with a line voltage triangle as shown in Fig. 52 and an unbalanced load, E_1, E_2 and E_3, are given. (They form the triangle *abc*.) Constructing semicircles on E_1, E_2, and E_3 as diameters, the e.m.f. triangle for each branch is constructed (Par. 155). We have

$$\frac{E_1}{z_1} = i_1, \ \frac{E_2}{z_2} = i_2 \text{ and } \frac{E_3}{z_3} = i_3. \qquad (266)$$

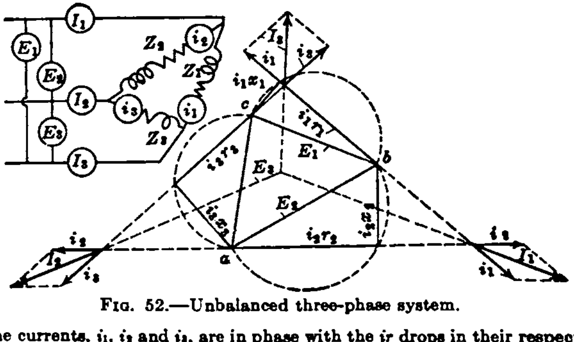

Fig. 52.—Unbalanced three-phase system.

The currents, i_1, i_2 and i_3, are in phase with the *ir* drops in their respective branches. These can be conveniently combined by prolonging the *ir* lines until they intersect, and laying off the currents, *i*, from the intersection. The main currents, I_1, I_2 and I_3, are found by taking the vector sum of the branch currents, i_2 and i_1, i_2 and i_3, i_3 and i_1, respectively.

230. Alternating impressed e.m.f. Let a sine-wave alternating voltage be applied to an equivalent single-phase line (Par. 224) with uniformly distributed characteristics. Let the resistance and the inductive reactance of the line be r ohms and x ohms per unit length, respectively, so that the series impedance is $Z=r+jx$ ohms per unit length. Let the condensive susceptance and the leakage conductance be b and g mhos, per unit length respectively, so that the shunted admittance is $Y=g-jb$ mhos per unit length. The current and the voltage relations at a distance s from the receiver end of the line are expressed by the differential equations

$$\frac{d^2I}{ds^2}=M^2I, \text{ and } \frac{d^2E}{ds^2}=M^2E \tag{273}$$

where $M=\sqrt{YZ}$. In these expressions I, E and M are complex quantities, (Par. 184), and the magnitude and the phase of the current and the voltage vary from point to point, remaining at the same time sine functions. In other words, the current and the voltage are sine functions of time t, and such functions of s as to satisfy the above-given equations. The parameter M characterises the line and is independent of either t or s. The solution of these equations is of the form

$$I=A_1\epsilon^{-Ms}+A_2\epsilon^{Ms} \tag{274}$$

$$E=B_1\epsilon^{-Ms}+B_2\epsilon^{Ms}, \tag{275}$$

where the constants of integration A_1, A_2, B_1 and B_2 are complex quantities

These constants are determined by the electrical conditions at some one point of the line, for instance, when the current and the voltage at one point are given in magnitude and in relative phase position.

231. Solution of alternating-current case. The solution of the foregoing differential equations is preferably expressed through **hyperbolic functions** (Par. 229) of the complex angle Ms. Namely,

$$I=C_1 \operatorname{Cosh} Ms+C_2 \operatorname{Sinh} Ms; \tag{276}$$

$$E=D_1 \operatorname{Cosh} Ms+D_2 \operatorname{Sinh} Ms; \tag{277}$$

where the complex quantities C_1, C_2, D_1, and D_2 are the constants of integration which depend upon the given conditions at some one point of the line.

For example, if the receiver voltage E_2 and the receiver current I_2 are given in magnitude and in phase (at $s=0$) the constants of integration have the following values:

$$C_1=I_2;\ C_2=E_2\frac{Y}{M};\ D_1=E_2;\ D_2=I_2\frac{M}{Y}. \tag{278}$$

Thus, knowing I and E at the receiver end, their values may be calculated for the sending end or at any other point on the line.

232. References to other literature. For further details and application of the foregoing equations to power-transmission lines and to the propagation of currents in telephone and telegraph lines see C. P. Steinmetz, "Theory and Calculation of Transient Electric Phenomena and Oscillations;" J. A. Fleming, "The Propagation of Electric Currents in Telephone and Telegraph Conductors."

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SECTION 3

MEASUREMENTS AND MEASURING APPARATUS

ELECTRIC AND MAGNETIC MEASUREMENTS

BY F. MALCOLM FARMER, M.E.

GENERAL

1. The measurement of any given quantity is the comparison of that quantity with another quantity of the same kind which has been chosen as a unit. The **unit** may be a purely arbitrary quantity with no rational significance, such as the foot and the pound, or it may have a very definite meaning, as the centimeter and the gram. The units used in electrical measurements belong to the latter class because they are based on the **centimeter-gram-second** or **C.G.S.** system. The C.G.S. system (Sec. 1) is the fundamental system upon which all physical measurements have been based, on the theory that all physical phenomena are the result of matter and motion, that is, space (centimeters), mass (grams) and time (seconds).

2. Measurements classified according to precision.—Electrical measurements may be divided, in a very general way, into three classes. (a) **High-precision measurements**, such as those made at the various national standardising laboratories in connection with the establishment and maintenance of standards. Every precaution is observed to obtain the highest possible degree of accuracy. Expense is a secondary consideration. (b) **Commercial laboratory measurements**, where the object is to secure results which are reliable and accurate, but only to the degree justified by commercial and engineering requirements. The cost must be a minimum. (c) **Commercial measurements** are those involved in the production, distribution and sale of energy. The scope of the subject in this section is limited to the last two classes.

3. A standard is a concrete representation of a unit. The fundamental C.G.S. units are difficult to represent and early in the development of the art the need for a system of units which could be used in electrical measurements was recognised, and resulted in the establishment of the "practical" units (see Sec. 1). These units are derived from the fundamental C.G.S. units and can be represented by definite, concrete and reproducible standards.

The distinction drawn between primary and secondary standards is largely a matter of viewpoint. In general, however, primary standards may be considered as those which represent directly by definition the unit involved, such as the mercury ohm, the silver voltameter and the saturated cadmium cell, made according to certain specifications. Secondary standards are the more practicable working standards which are standardised by comparison with primary standards and used as the basis of all ordinary measurements. They include, for example, the manganin standard resistances and the ordinary Weston-type standard cell. Primary standards are, in general, maintained only by the government custodians of the standards in the various countries; whereas secondary or working standards, based on these primary standards, serve as the fundamental basis of practical measurements in engineering and commercial fields.

4. The precision obtainable in an electrical measurement depends upon the various factors which enter into the determination; among these are the correctness of the principle employed and the method used, accuracy of the standards, number and magnitude of possible errors, correctness of calculations and so forth. In precision measurements, as classified above, a precision of one part in 100,000 in certain classes of measurements is regularly attained. In commercial measurements, the cost of such a high degree from the mean value as the number of readings is increased. Where the necessary degree of reliability is not obtained in a single measurement, a number of observations are made, from which the most probable true value may be obtained, together with its probable error. These latter quantities may be derived in different degrees of precision by various mathematical methods involving the theory of probabilities and the method of least squares. In all ordinary electrical measurements it will usually be sufficiently correct to assume that the true value is equal to the average of the various values obtained (eliminating systematic errors) plus or minus the average error. The average error is the average of the differences between the average value and each individual value. It should be noted, however, that according to the theory of probability, the precision of the result does not increase directly but only as the square root of the number of observations made (see Par. 426 to 435).

7. **Certain general precautions** which should be observed in electrical measurements, and certain sources of error which should be avoided, are indicated in the following paragraphs.

(a) **The probable limit of accuracy** of the standards, instruments and methods should be known.

(b) As a general proposition, in other than rough determinations, one measurement should not be relied upon. Several readings should be taken, and the conditions should be altered, wherever possible, in order to avoid accidental errors.

(c) Indicating instruments should be of such a range that the quantity under measurement will produce a reasonably large deflection on the scale. The percentage observational error decreases in direct proportion as the magnitude of the deflection increases.

(d) The possible presence of **external or stray magnetic fields**, both direct and alternating, should always be borne in mind. Such fields may be produced by current in neighboring conductors, or by various classes of electrical machinery and apparatus, structural iron and steel in buildings, etc. These fields introduce errors by combining with the fields of portable indicating instruments, galvanometers and other instruments utilizing a magnetic field, and, in the case of alternating fields, by inducing small e.m.fs. in the loops formed in bridges, potentiometers, etc.

(e) In measurements involving high resistances and galvanometers, such as bridges and potentiometers, possible "leakage" or shunt circuits should be eliminated. This is done by providing a **"guard" circuit** the principle of which is to keep all points to which the current might flow improperly, at the same potential as the highest in the apparatus. See further discussion under potentiometers (Par. 49 to 52).

(f) **Temperature changes** in various parts of bridge, potentiometer and similar circuits should be avoided because of thermo e.m.fs. produced at the junction of dissimilar metals. Such effects are often produced if the observer's hand comes in contact with the metal parts of the galvanometer key, switches, etc.

(g) Instruments with covers made of glass and hard rubber should not be rubbed, especially with a dry dust cloth. The induced electrostatic charge on the moving element is often sufficient to change the deflection materially.

(h) At potentials of 500 volts and above, the electrostatic attraction between moving and fixed parts may become serious. This is prevented by keeping the two parts at the same electrostatic potential. When grounding is permissible, this can be done by connecting the circuit to earth at the point where the instrument is connected, care being taken that the moving-coil end of the instrument is on the ground side. In very high potential work this electrostatic attraction becomes very troublesome, so that the instruments must be connected in circuit at a grounded part of the line, or else be thoroughly insulated from ground and the moving element connected to the case or to an electrostatic shield around the instrument.

GALVANOMETERS

8. **Galvanometers** are used extensively in all classes of electrical measurements. Strictly speaking, the term applies to many other instruments for measuring current, such as voltmeters and ammeters, but it is ordinarily understood to apply to those instruments which are used to measure very small electrical quantities.

charged through it before the suspended system has moved appreciably. **The period**, or time of vibration, must therefore be long compared with the time of discharge. This is accomplished by increasing the inertia of the moving system.

16. Deflection of ballistic galvanometers. The magnitude of the first deflection is a measure of the quantity discharged into the instrument. In an instrument in which there is no damping (Par. 26) such as the moving-magnet type, the quantity may be calculated directly from the constants of the instrument. Thus

$$Q=\frac{2Ht\ \sin\ (\frac{1}{2})\alpha}{\pi G}\qquad\text{(coulomb)}\qquad(3)$$

or for small angles, 5 deg. or less,

$$Q=\frac{Ht\sin\alpha}{\pi G}\qquad\text{(coulomb)}\qquad(4)$$

where Q = quantity of electricity in coulombs, H = field strength in gausses, G = constant computed from the coils, t = period in seconds, α = angle of deflection.

17. Ballistic galvanometer constant. In practice, ballistic galvanometers are usually standardised and the formula becomes very simple: $Q=kd$ where d = deflection and k = quantity per unit deflection or galvanometer constant. The constant is determined with a standard condenser or mutual inductance. The deflection obtained upon suddenly discharging a charged condenser through the galvanometer is $d=Q=CE$; and hence $k=CE/d$, where Q = quantity of electricity in coulombs, E = potential to which the condenser had been charged in volts, and C = capacity of condenser in farads. When a mutual inductance is used, the deflection is $d=Q=MI/R$ and $k=MI/dR$, where Q = quantity of electricity in coulombs, M = coefficient of mutual inductance in henrys, I = steady or Ohm's law value of current in primary of mutual inductance in amperes, and R = resistance of secondary circuit (including the mutual inductance) in ohms.

18. A differential galvanometer is one provided with two independent coils or sets of coils by means of which two currents may be compared simultaneously. This method provides a means of measuring a current without making the circuit common with that of the comparison standard. In D'Arsonval instruments, the two coils are wound side by side on the same frame and are connected in opposition, so that when the two currents being compared are adjusted for zero deflection, their ratio is usually unity. The actual ratio can be determined experimentally.

19. Electrometers. In the electrometer, a piece of thin aluminium is suspended by a metallic suspension over four quadrants of sheet metal which are insulated from each other and from the frame or support. Opposite quadrants are connected to each other and the two sets are connected respectively to the two sides of the circuit to be measured. If a charge from a condenser is placed on the moving vane, one end will be repelled and the opposite end attracted, producing a deflection which will be a measure of the potential applied to the stationary quadrants. This instrument is extremely sensitive, and while it is one of the earliest types of electrical measuring instruments it is still used extensively in research work, especially where the available energy is extremely small, as in measurements of radiant energy.

20. Galvanometers as detectors. The majority of galvanometers are used as detectors only, that is, in zero-deflection methods where the kind of scale or proportionality of deflections does not enter into the determination. In such cases a very short, straight scale is sufficient and space may be economised by placing the galvanometer on the wall above the table, with the scale directly underneath. The beam of light is properly directed by suitable prisms and mirrors.

21. Reflecting galvanometers may be read with a telescope and scale, or with a lamp and scale. In the former, the scale is reflected from the plane mirror (attached to the moving system) to the telescope through which movements are observed. In the latter, an image of a narrow beam of light (issuing from a narrow slit in a vessel enclosing a lamp, or from a portion of an incandescent lamp filament) is thrown on to the scale by the mirror. In

29. Galvanometer shunts are combinations of resistances so arranged and so connected to the galvanometer that the constant of the latter may be quickly changed. Ordinary resistance boxes may be used as shunts for galvanometers when the resistance of the galvanometer circuit is not too small. In the latter case the box is connected as shown in Fig. 2, thus increasing the resistance R_g of the galvanometer circuit. The readings of the galvanometer must be multiplied by a factor

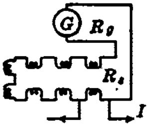

FIG. 2.—Galvanometer shunt.

$$k = \frac{R_g + R_s}{R_s} \qquad (6)$$

wherein R_g is the resistance of the galvanometer circuit and R_s is that of the shunt. In special galvanometer shunts,

$$R_s = \frac{R_g}{9}, \frac{R_g}{99}, \frac{R_g}{999}, \frac{R_g}{9999}, \ldots \ldots \text{ etc.} \qquad (7)$$

then $k = 10, 100, 1{,}000, 10{,}000$ respectively.

30. The Ayrton or universal shunt is so arranged that it can be used with any galvanometer. When, in Fig. 3, the movable contact x is at b, $I'_g = Ir_{ab}/(r_{ab} + r_g)$, where I'_g = current through galvanometer, I = current from battery, r_{ab} = resistance between a and b, and r_g = resistance of galvanometer. If the contact x is moved to c,

$$I''_g = I'_g\left(\frac{r_{ac}}{r_{ab}}\right) \qquad \text{(amp.)} \qquad (8)$$

It will be noted that r_g is not in this equation; hence if the galvanometer constant is obtained with the shunt all in (x at b), the shunt ratio at any other position of x is r_{ac}/r_{ab} and is independent of the galvanometer resistance.

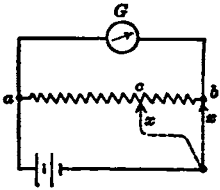

FIG. 3.—Universal galvanometer shunt.

31. Alternating-current types of galvanometers include the following: Electrodynamometers, or, as they are more commonly known, dynamometers; vibration galvanometers, thermogalvanometers, electrostatic galvanometers, alternating-current detectors, barretters and bolometers.

32. Dynamometer-type instruments are used extensively in measurements of alternating currents because they measure mean effective values and can be calibrated on direct current. They can be used for a wide range of measurements of current, e.m.f. and power, from extremely small values to very large ones.

33. The operative principle of the dynamometer is the electrodynamic action between a movable coil (or coils) suspended between two or more fixed coils, all of which are energised. **The Rowland electrodynamometer*** is typical of this class of instruments. It consists simply of two fixed coils mounted close together, between which is suspended a small coil of very fine wire. Each fixed coil consists of two separate windings of different current capacities brought out to separate terminals. For e.m.f. measurements the moving coil and the fine wire fixed coils are connected in series; for current measurements, the moving coil is connected across a non-inductive shunt in the current circuit; for power measurements, the moving coil is connected across the circuit to be measured while the proper fixed coil is connected in series with it.

34. Dynamometers are made astatic, or independent of external fields, by having two sets of moving coils, oppositely wound, one above the other, with a common suspension. There may be one or more pairs of fixed coils. In heavy-current instruments, the fixed coils are wound with cable composed of many fine strands laid up in braided form. This reduces the eddy currents which otherwise would be set up and which would influence the moving coil.

* Rowland, H. A. "Electrical Measurements by Alternating Currents;" Leeds and Northrup Catalogue No. 74, 1911; p. 294.

rings which in turn are connected through the brushes, b, b^1, to the alternating current being measured. It is apparent that the connections to the galvanometer are reversed every half cycle, so that the indication is a steady one, the value of which may be made anything from zero to a maximum by shifting the angular position of the brushes. Thus the most sensitive position can be readily found, irrespective of the phase relation between the current in the circuit being measured and the motor armature. The variation in contact resistance at high speeds, and possible presence of thermo e.m.fs., may cause trouble where the resistances or potentials are very low, as in low-resistance bridge measurements.*

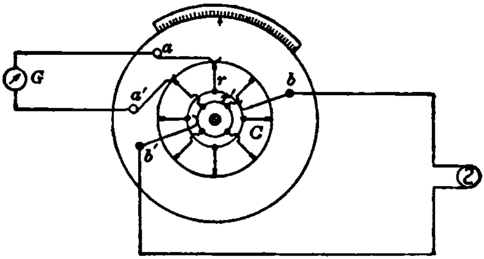

Fig. 5.—Synchronous commutator.

39. A synchronous reversing key which overcomes the latter difficulty is indicated in Fig. 6. A cam, C, mounted on the shaft of a small synchronous motor is so designed that it moves the lever, l, up and down in such a manner that the pair of contacts, a and a^1, at the end of the lever make alternate contact with the pairs of stationary contacts, c and c^1, once per cycle. The number of projections on the cam of course will correspond with the number of pairs of poles on the motor. The contacts are arranged as shown diagrammatically at the right of Fig. 6, from which it will be seen that the connections to the galvanometer are reversed every half cycle, so that a steady deflection is obtained in the direct-current galvanometer. All the contacts are supported on a solid disc which can be rotated around the shaft, and hence reversal can be made at any point on the wave as in the case of the synchronous commutator. The contacts are all made of platinum, the cam is hardened steel and the lever l is kept in contact with the cam at all times by means of a spring. The roller r is necessary to insure contact at the low portion of the cam.

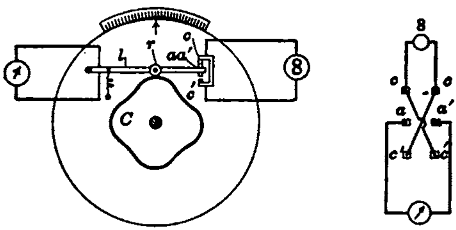

Fig. 6.—Synchronous reversing key.

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* Sharp, C. H. and Crawford, W. W. "Some Recent Developments in Exact Alternating-current Measurements;" *Trans.* A. I. E. E., 1910, Vol. XXIX, p. 1518.

4 deg. cent. or above 40 deg. cent., and that no current greater than 0.0001 amp. be passed through them.

48. Comparisons of continuous electromotive forces with a standard cell. Electromotive forces may be compared with a standard cell by several methods. The more typical methods are based on the following principles.

(a) In the **substitution method,** the current flowing through a high resistance (not less than 15,000 ohms) is measured with a high-sensibility galvanometer, first with the standard cell in the circuit and then with the unknown e.m.f. substituted for the standard cell. The resistance being the same in the two cases, the deflections are proportional to the e.m.fs. $E = ed'/d$, where E = unknown e.m.f., e = standard cell e.m.f., d = deflection with standard cell and d' = deflection with unknown e.m.f.

(b) The **equal deflection method** is a modification of the above, in which the deflections are kept the same in the two cases by changing the resistance. Then $E = er'/r$, where r = total resistance of the circuit, including the galvanometer, with the standard cell in the circuit and r' = total resistance with the unknown e.m.f. This method is better than (a) because it is a constant deflection method and the result depends on the known values of two resistances, rather than observed deflections.

(c) In **Wheatstone's modification** of the equal deflection method, the galvanometer resistance does not have to be known. The deflection, d, is noted when the unknown e.m.f., E, and a known high resistance are in circuit. Additional resistance, r', is added and the deflection, d', again noted. Similarly with the standard cell of potential difference, e; the resistance is adjusted until the same deflection, d, is obtained and then an amount of resistance, r, is added until the deflection d' is again obtained. The unknown e.m.f. is $E = er'/r$.

(d) In the **condenser discharge method** a condenser is charged, first from the unknown e.m.f., then from the standard cell, and discharged in each instance through a ballistic galvanometer. The deflections will be proportional to the e.m.fs., hence $E = ed'/d$ as in (a). Obviously, if the unknown e.m.f. is much smaller or much larger than the standard cell, the deflection can be made about equal to that of the standard cell by using a larger or a smaller condenser. In that case the ratio of the capacities should be known, and then $E = ed'C'/dC$, where C = capacity of condenser used with the unknown e.m.f. and C' = capacity of condenser used with the standard cell. This method has the advantage that practically no current is required, which is advantageous in making measurements of voltaic cells of very small capacity or rapid polarization.

(e) The principle of the **opposition or potentiometer method** is that of opposing the e.m.f. of the standard cell against an equal difference of potential which bears a known proportion to the unknown e.m.f. This method is the most accurate and by far the most generally used, because it is both a zero-deflection and a zero-current method, the result depending only on the ratio of two resistances which can be very accurately determined. Potentiometers are instruments employing this principle (Par. **49**).

49. Description of Leeds and Northrup potentiometer, low resistance type. Fig. 7 shows the arrangements of the circuits. The figures for the second decimal place and beyond are obtained from a slide wire at the end of the circuit, CB, along which a contact moves. A special dial is also provided for the standard cell (at the left) and separate contacts are provided for the standard-cell e.m.f. and the unknown e.m.f., so that no settings have to be disturbed when checking the secondary current in the potentiometer circuit. The essential part of the instrument consists of 15 five-ohm coils, AC, connected in series with the extended wire, CB, the resistance of which from 0 to 1,100 scale divisions is 5.5 ohms. Thus when the current from the battery, B, is adjusted by the rheostat, R, to 0.02 amp., the fall of potential across each 5-ohm coil in AC is 0.1 volt and across CB, 0.11 volt. Since the latter is divided into 1,100 parts, the e.m.f. may be measured to 0.0001 volt. At point 5 in AC, a wire is permanently attached connecting to one point of the double switch, U. When this switch is thrown to the left, the standard cell is connected through the galvanometer to point 5 and the sliding contact T which moves over the dial at the left consisting of 19 resistance coils. Between a and A is a resistance which is adjusted to such a value that with 0.02 amp. flowing, the potential drop between 5 and a is three low dials are so arranged that a corresponding change is automatically made in the external part of the main circuit and the total resistance is kept constant. A separate dial is provided for the standard-cell adjustment, together with a separate resistance which can be altered to accommodate different cells without affecting the measuring circuits. The total range of the instrument is 1.9 volts.

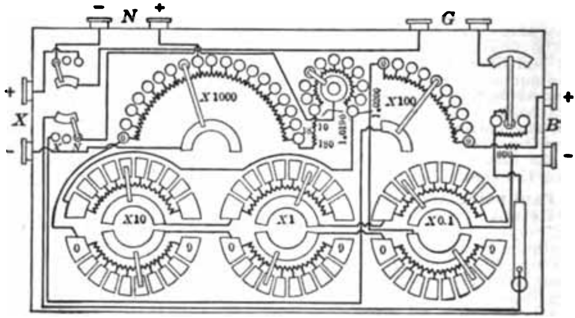


FIG. 9.—Wolff potentiometer.

52. Care and use of potentiometers. The following notes apply to the use of potentiometers in ordinary work.

(a) The accessories should be suitable for the particular type of instrument, that is, high or low resistance, and for the class of measurement to be made. The galvanometer should be sufficiently sensitive to give a perceptible deflection when there is an unbalance equal to the smallest scale division in the potentiometer curcuit. A low-resistance galvanometer, of the order of 100 ohms, should be used with a low-resistance potentiometer, and a high-resistance galvanometer of 500 to 1,000 or more ohms, with a high-resistance potentiometer. Similarly, the resistance of the volt box (Par. 53) for low-resistance potentiometers should be as low as the permissible power loss in the resistance coils will permit. This is usually about 5,000 ohms for 150 volts. For high-resistance instruments, the resistance is 1,000 ohms or more per volt.

(b) The **first trials for balance** should always be made with a resistance in series with the galvanometer. This is usually provided in the instrument, with facilities for readily cutting it out of circuit when an accurate balance is being obtained. The resistance protects the galvanometer and also the standard cell from the effects of excessive currents.

(c) Trouble is sometimes experienced, especially in damp weather, due to **current "leaking to ground"** from the potentiometer circuits in a manner which produces a false deflection. This can be obviated by providing a **"guard circuit."** In one scheme of this kind all of the apparatus is placed on small, hard-rubber pillars each of which in turn rests on a small metal plate. These are connected together and to the positive "x" binding-post by fine bare wire. Thus all points to which current might "leak" over the surface are kept at the highest external potential to which the potentiometer is connected. When the surfaces become noticeably moist, conditions can be improved by carefully wiping with a cloth moistened with grain alcohol.

(d) **Calibration and checking.** The essential requirement is that the ratio of the resistance of each step to that resistance between the standard cell terminals shall be the same as the ratio of the corresponding potentials. For example, if the standard cell e.m.f. is 1.0183 volts and the resistance between its terminals is 101.83 ohms, the resistance of the various steps should be adjusted to 10 ohms per 0.1 volt. If the standard cell resistance is 102.848 ohms the potentiometer is still accurate if the resistance throughout the circuit is adjusted to 10.1 ohms per 0.1 volt.

lines per square centimeter in the steel. A light tubular pointer attached to the coil moves over a calibrated scale. The current is introduced into the coil by two spiral springs which also provide the controlling force. Since the field strength and the gradient of the controlling forces are uniform, the deflection is strictly proportional to the current passing through the coil, and the scale divisions are uniform. A large amount of resistance is connected in series with the moving coil in order to make the current small. Thus the same instrument can be made suitable for a wide range of voltages by changing the amount of series resistance. This resistance is made of wire having a low temperature coefficient in order to neutralize as much as possible the effect of the large coefficient of the copper in the coil.

56. Voltmeter characteristics (continuous current). The usual **resistance** of portable voltmeters of this type varies from 50 to 150 ohms per volt and the **current sensibility** from 7 to 20 milliamperes at full-scale deflection. The resistance of the moving coils is about 75 ohms. The **torque** varies from 2 to 6 millimeter-grams at maximum current, with a **ratio of torque to weight** (in grams) of 1 to 5. The **temperature coefficient** is usually negligible, being of the order of 0.01 to 0.02 per cent. per deg. cent. at full scale.

57. Laboratory standard voltmeters (continuous current). So-called laboratory standard voltmeters are similar to portable instruments except that they are larger, have a longer pointer, a longer and more open scale and are made with greater care. They are only semi-portable and are intended primarily for standardising purposes.

58. Switchboard voltmeters (continuous current) are usually of the D'Arsonval type. The construction is the same as that of portable instruments, except that they are more substantial and rugged, especially as regards the moving system, in order to withstand the harder conditions of continuous service and excessive fluctuations. They are mounted in iron cases to protect them as much as possible from the normal stray fields due to the bus bars.

59. Effect of stray fields. The general effect of stray fields on the standard types of portable and switchboard instruments is shown in the table in Par. 60. These errors are usually only temporary and disappear with the stray field. When the field is very strong, as under short-circuit conditions in a neighboring conductor, **demagnetisation** of the instrument magnets may result in a permanent error. Shields are likely to be of little value under such conditions because the iron becomes saturated.

60. Effect of Stray Magnetic Fields on Continuous-current Voltmeters and Millivoltmeters

Stray field, lines per sq. cm.*	Error at two-thirds full-scale deflection, per cent.	
	Shielded	Unshielded
5	0.5 to 1.0	2
10	0.75 to 1.75	3.5 to 5.5
15	1.0 to 3.0	6.0 to 7.5
20	1.25 to 3.25	7.5 to 10

61. The measurement of very small continuous potentials may be effected by some of the methods outlined in Par. 48.

A potentiometer is most convenient, high resistance for high resistance sources such as small galvanic cells and low resistance for low resistance sources such as thermocouples.

62. Ground detectors. Those of the direct-current type are usually special forms of voltmeters. In one form, there are two coils, differentially wound on the moving system. One end of each coil is connected to ground and the two free ends are connected respectively to the two sides of the

* The field produced at a distance of 30 cm. (12 in.) from a conductor carrying 3,000 amp. is about 20 lines per square centimeter.

coils as shown in Fig. 12, where F, F' are the fixed coils and M is the moving coil, to which a pointer P is attached. The deflection is approximately proportional to the square of the current. The scale is compressed at the upper end instead of extended because the coil moves beyond the uniform part of the field. The **Thomson Inclined Coil voltmeter** is similar, except that the plane of the fixed coils makes an angle of about 45 deg. with the shaft of the moving coil for the purpose of making the scale more uniform.

In the **Westinghouse** type *Q*, the **Kelvin balance** principle is used. This principle is shown in Fig. 13, where there are two coils, MM', attached to opposite ends of a beam which is supported at the middle and free to move. Each coil moves between a pair of fixed coils, $F\ F$ and $F'\ F'$, and all of the coils are connected in series in such a manner that the moments of all the forces on the movable system, taken about the beam axis, are cumulative, thus tending to produce rotation. In the Kelvin balance the controlling or opposing force is a weight moved along a graduated scale attached to the beam supporting the movable coils; the moment of this weight about the beam axis, when the moving system is balanced, varies as the square of the e.m.f. In the Westinghouse instrument the coils are arranged vertically and the controlling force is a spiral spring. The amount of compression of this spring necessary to balance the electromagnetic forces, as indicated by a pointer moving over a scale, is a measure of the e.m.f. Single-coil instruments are direct reading and hence fluctuating e.m.fs. can be more easily read on them than on the torsionhead instruments, but the latter are astatic and therefore practically independent of stray fields.

68. Soft-iron-vane voltmeters (alternating current) utilise the reaction between a temporarily magnetised piece of soft iron and the magnetising field. In the **Thomson inclined coil instrument** of this type the

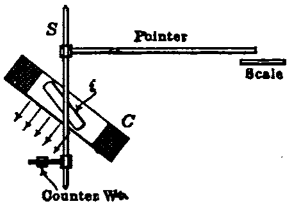


FIG. 14.—Diagram, Thomson inclined coil a.c. voltmeter.

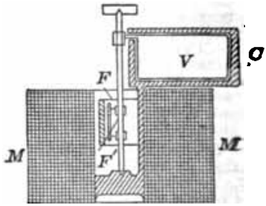


FIG. 15.—Diagram, Weston soft-iron type a.c. voltmeter.

plane of the energising coil, C (Fig. 14), makes an angle with the shaft, S, which carries a member, i, comprising a rectangular piece of very thin, soft iron. This piece of iron is so attached to the shaft that rotation is produced by the tendency of the iron to become parallel with the field established by the coil. In **Weston instruments** of this type (model 155), the reaction which produces the deflection takes place between two pieces of soft iron bent in the arc of a circle and placed concentrically, one of which, F' (Fig. 15), is movable, and the other, F, is stationary. When the surrounding coil, M, is energised, the pieces of iron become magnetised in like manner, so that the resulting force is one of repulsion. The stationary piece F is made triangular in shape, with the pointed end in the direction of rotation, for the purpose of making the scale more uniform. Air damping is obtained by means of a light aluminium vane, V, in an enclosing chamber, C. This type has the great advantages of low price, ruggedness, open scale and small weight.

69. Induction-type voltmeters (alternating current) utilise the principle of induction watt-hour meters (Par. 202), or the rotative tendency of a free cup of thin metal when placed within a so-called revolving magnetic field. Actual rotation of the movable element is prevented by an opposing spiral spring, so that the deflections become a measure of the current in the energising coils. The **Westinghouse** type P voltmeter is an important with scales which make them direct reading. They are made in a great variety of forms, for both portable and switchboard use, but are used commercially much more in Europe than in this country. The principle of their operation is shown in Fig. 18, in which mm' is a thin aluminium vane suspended or pivoted between two pairs of fixed vanes, ff'. The deflection through moderate ranges is proportional to the square of the potential and is controlled either by a spiral spring or by gravity. Damping is produced magnetically, by air vanes, or by immersing the elements in oil. For ordinary commercial voltages a number of sets of vanes are arranged one above the other in a vertical position, and connected in parallel, thus multiplying the effect (Fig. 19). For higher voltages, one set of vanes is sufficient and they are usually placed in a vertical plane with the moving element mounted on a horizontal shaft. In the **Westinghouse electrostatic voltmeter,** the moving system is not connected to the circuit; Fig. 20 shows the arrangement of the parts. When potential is applied to A and A', the hollow cylinders C and C' become oppositely charged by induction. The resultant attraction produces a deflection because of the shape of the fixed plates, P and P'. The condensers K and K' are each formed by two flat plates and are connected in series with A and A' to increase the range. For lower ranges these condensers are short-circuited, so that ranges of 30,000, 60,000 and 100,000 volts are available in the same instrument and on one scale. The elements are entirely immersed

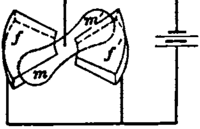


Fig. 18.—Diagram, electrostatic-type a.c. voltmeter.

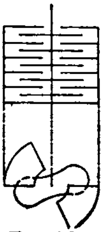


Fig. 19.—Diagram, electrostatic-type a.c. voltmeter.

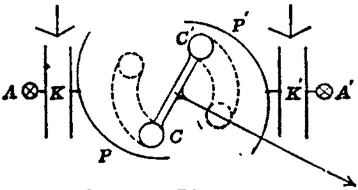


Fig. 20.—Diagram, Westinghouse high-tension voltmeter.

in oil which permits a relatively compact construction, increases the torque because of the greater specific inductive capacity of the oil and provides damping.

72. Alternating-current switchboard voltmeters are made in all of the types described above (Par. 66 to 70), although in this country the dynamometer, soft-iron vane and induction types are in most general use. They are similar in general to the portable instruments, due regard being given to the more severe requirements of switchboard service.

73. Calibration of alternating-current voltmeters. The dynamometer type of voltmeter gives the same indication on continuous current as on alternating current and may therefore be calibrated with continuous currents, direct and reversed readings being taken. The inductance in instruments of commercial ranges is so small that the readings are independent of standard frequencies.

The soft-iron-vane type of voltmeter should theoretically be used only on alternating current because hysteresis occurs to some degree in the vane. Practically, however, the hysteresis is so small that there is very little difference between the respective indications with increasing and decreasing potential. Provided with a steady source of e.m.f., under suitable control, these instruments may be calibrated with continuous current by taking the average of the "up" and "down" potential readings corresponding to the various points. Care should be taken that the potential is increased or decreased only to the desired value and not beyond it. Theoretically, instruments of the soft-iron type are not independent of frequency or wave-form; practically, however, the variation is not measurable throughout commercial ranges.

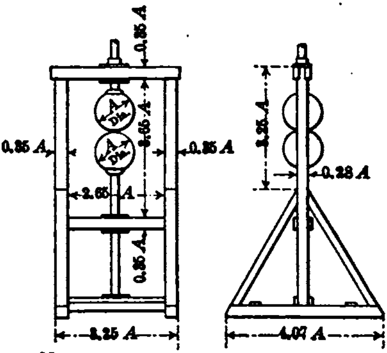
load. The method is simple, convenient and accurate, but the power consumption and the cost of the transformer become prohibitive at high potentials.

(c) **Electrostatic voltmeter.** Commercial instruments are available up to about 200,000 volts. They require no appreciable power and are quite satisfactory. The principal objections are the high cost of large sizes and the lack of dead-beat qualities.

(d) **Test coil.** Where the source of the high potential to be measured is a testing transformer, an ordinary low-reading voltmeter can be connected to a few turns of the high tension winding brought out to separate terminals. These turns should be at the grounded end of the winding. The ratio, under all conditions, will be that of these turns to the total turns in the high-tension winding, if the transformer has been well designed. This method is generally sufficiently accurate and is very convenient.

(e) **Spark gaps.** The sparking distance between two terminals in atmospheric air is a standard method of measuring high potentials. The maximum length of gap which a given potential will break down depends, in this case, on the maximum value and not upon the virtual or effective value which is the value obtained in the other methods. The maximum value, however, is the important one in tests of insulators and insulating materials. When the wave form is not a sine curve the maximum value may deviate materially from the theoretical value, which is the virtual (voltmeter reading) value multiplied by $\sqrt{2}$.

77. Needle-point and sphere spark gaps. The needle-point spark has for many years been the standard method of measuring high voltages, but it is unsatisfactory for very high potentials because of variations due to atmospheric pressure, humidity, proximity of surrounding objects and sharpness of the needle-points. It has been proposed* to use spheres instead of needle-points and the 1914 A. I. E. E. Standardization Rules recommend the use of the needle gap for voltages from 10 kv. to 50 kv. and the sphere gap for voltages above 50 kv. A gap with carefully machined and polished spheres gives very reliable and consistent results due, probably, to the fact that the gap breaks down before corona forms and perhaps also to the lesser dielectric spark lag.†



Note:
A Variation of 1 Cm. in Thickness and Width of Wood Parts is Permissible

Fig. 21.—Sphere sparks gaps.

78. A. I. E. E. standard sparking distances in air, with needle-point and sphere gaps are given in Sec. 24.

79. Ratio of shunt-type instrument transformer. When a very accurate measurement (within 2 per cent.) of a high potential is to be made with an instrument transformer of the shunt type, the nominal ratio cannot be relied upon as being sufficiently correct and the true ratio should be determined by a direct measurement. In the accurate measurement of power and energy the phase angle should also be known (Par. 179). For the accurate determination of the ratio of instrument transformers of the shunt type, the following methods are typical.

(a) **Direct measurement** of the primary and the secondary voltage,

* Farnsworth, S. W. and Fortescue, C. L. "The Sphere Spark Gap," *Proc.* A. I. E. E., 1913, Vol XXXII, No. 2, p. 301. See Fig. 21.

† Minton, J. P. "Effect of Dielectric Spark Lag on Spark Gaps," *General Electric Review* (1913), Vol. XVI, p. 514.

a simple and convenient method of comparing two transformers by means of a wattmeter.

80. Alternating-current ground detectors are usually **electrostatic** instruments. Fig. 24 shows diagrammatically the principle of **Westinghouse detectors.** The stationary vanes are connected through condensers to the main lines, and the movable vane is connected to ground. In the single-phase detector, a charge will be induced on each end of the movable vane, opposite in sign to that on the corresponding stationary vane, setting up forces of attraction. When the system is free from grounds, these attractive forces are equal and opposed to each other; hence the moving system stands at zero. A ground on either phase wire produces an unbalanced condition and a deflection away from the grounded conductor. In the three-phase detector, the vanes are sectors of spheres, the movable vane being mounted on a universal bearing.

81. General Electric ground detectors operate on a principle similar to that last described. Four quadrants, or fixed, flat vanes are cross-connected and mounted between them is a moving vane connected to ground. Opposite sets of fixed vanes are connected to the two sides of the line circuit and the action is the same as described in Par. **80.** The **three-phase detector** is similar, consisting of three pairs of fixed quadrants mounted 120 deg. apart on a common base plate, and a flat moving vane with three corresponding sectors.

CONTINUOUS-CURRENT MEASUREMENTS

82. Absolute measurements of current. The fundamental unit of current, as derived from the centimeter, gram and second, is defined in terms of the dimensions of the conductor and the strength of the magnetic field produced by the current. Absolute measurements of current are therefore made with instruments so carefully constructed that the current can be calculated from their dimensions.

83. Instruments for absolute measurement of current. Absolute determinations have usually been made with two classes of instruments. In the first, the deflection of a magnetic needle at the centre of a coil is measured, and the current is calculated from the dimensions of the coil, the strength of the earth's field and the torsion of the suspension. The best known example of this class is the tangent galvanometer (Par. **11**). This method involves, of course, any error in the determination of the earth's field. In the other class of instruments, the needle is replaced with a suspended coil. When the length and the radius of both movable and fixed coils are in the ratio of $\sqrt{3}:1$, when the centres coincide and when the dimensions of the fixed coil are large compared with those of the movable coil, it has been shown by Gray that the torque exerted by the moving system is expressed by

$$T = \frac{4\pi^2 N n r^2 I}{\sqrt{D^2 + L^2}} \qquad \text{(dyne-cm.)} \qquad (12)$$

where N = number of turns in fixed coil, n = number of turns in movable coil, D = diameter of fixed coil in centimeters, L = length of fixed coil in centimeters, r = radius of movable coil in centimeters, and I = current (coils in series). Hence by measuring the torque (weighing it), the current can be determined directly in C.G.S. units.

84. Practical unit and standard of current. It would be quite impracticable to make ordinary measurements in terms of the fundamental unit, by the methods indicated above (Par. **82** and **83**). The Act of Congress of 1894 which legalised certain practical units of electrical measure defined the practical unit of current, or the **international ampere,** as one-tenth of the fundamental C.G.S. unit. This Act also defined the standard unit of current as the rate of deposition of silver at the cathode of a silver voltameter (Par. **86**) constructed and operated under certain prescribed conditions, the ampere being the current which will deposit 0.001118 g. of silver per sec. in a standard voltameter.

85. Methods of measuring continuous currents. The several methods of measuring continuous currents may be classified as follows: voltameter; potentiometer; and ammeters.

86. Voltameter method of current measurement. When a continuous current is passed through an electrolyte, the latter is decomposed at

91. D'Arsonval type of continuous-current ammeter. The principle of these instruments has been described under "e.m.f. measurements" (Par. 55). They are usually designed to have a full scale deflection with 50 to 200 millivolts (thousandths of a volt) at the terminals. The resistance of the moving coil is much lower (0.5 to 5.0 ohms) than that of voltmeters, in order to make the millivolt constant high.

92. Continuous-current ammeters of the switchboard type are intended for continuous operation and the shunt loss should therefore be low. They are designed for 50 to 75 millivolts at full scale deflection. High-grade portable ammeters are designed for 100 to 200 millivolts at full scale deflection, in order to permit the use of resistance in series with the moving coil, thus reducing the temperature error, which is more important than the larger shunt loss.

93. Shunts for continuous-current ammeters. In switchboard ammeters and the lower grade portable ammeters of 25-amp. ratings and less, the shunt is within the instrument case. Above 25-amp. ratings, the shunt is usually separate from the instrument and means of connection are provided by special flexible leads, which are included in circuit when the instrument is calibrated, since they form a part of the resistance of the entire instrument circuit. Obviously, these leads should never be altered without recalibrating the instrument. In high-grade ammeters the shunts are separate for all capacities.

94. Construction of ammeter shunts. Ammeter shunts are so constructed as to have a resistance which will be constant, as nearly as possible, under all conditions. The resistance metal has a low temperature coefficient, and the temperature is kept low either by connecting several strips in parallel and making the current density low, or by making the current density high and using short lengths of the resistance metal with heavy copper terminals designed to dissipate the heat by conduction and radiation. The former method is most generally used, the strips being silver-soldered into relatively heavy copper or brass terminals which are connected into the circuit to be measured. The resistance metal should also have a low thermo e.m.f. (Sec. 2) in junctions with copper.

95. Reduction of temperature errors in continuous-current ammeters. Because of the large temperature coefficient of copper, it is very undesirable to connect the moving coil of the instrument directly to the shunt. Temperature errors are reduced to a negligible value by connecting sufficient resistance having a low temperature coefficient (manganin or similar metal) directly in series with the moving coil as shown in Fig. 27, or by arranging a compensating circuit as shown in Fig. 28, where C = moving coil, R_m = low-coefficient resistance wire, and R_c = copper resistance wire.

FIG. 27.—Temperature compensation in millivoltmeters.

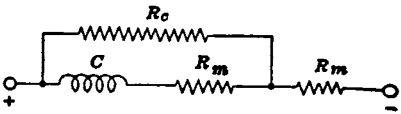


FIG. 28.—Temperature compensation in millivoltmeters.

96. The calibration of D'Arsonval type ammeters is effected by adjustment of the resistance of the shunt, the resistance of the millivoltmeter circuit, or both. Formerly each instrument and shunt were adjusted together, but it is becoming customary to adjust all of the instruments of a given type to deflect full scale with the same potential in millivolts at the terminals. The shunts for these instruments are all similarly adjusted to give the same potential drop, thus making all shunts and instruments of a given type interchangeable. The shunts should be adjusted by varying the main-line resistance between the potential taps and not by adjusting resistance wire connected in series with the instrument leads. In calibrating switchboard instruments and the lower grade portable instruments, the potential terminals are attached to the main current terminals and adjust-coarse wire instead of a large number of turns of fine wire, the ampere-turns being about the same in both cases. Hot-wire ammeters with ratings of more than 1 or 2 amp. are usually small current instruments connected to non-inductive shunts, as already described in principle (Par. 90).

102. Measurements of large alternating currents. The only type of ammeter which is generally used in the direct measurement of large alternating currents is the hot-wire ammeter, because it can be used with shunts. While shunts are made for capacities of 1,000 amp. and over, the accuracy with shunts of very large capacity depends upon the care taken in the design to eliminate the eddy-current and skin-effect errors. The most common method is to use current transformers of the series type, to step down the current to a small value, usually 5 amp., which is convenient to measure with standard instruments.

103. Series-type instrument transformers (also known as current transformers) serve two purposes; the convenient measurement of large currents, and the insulation of instruments and apparatus from high-voltage circuits. They are similar to so-called power transformers, except that the latter are connected in shunt across the line and the secondary potential remains substantially constant irrespective of the connected load. Series transformers are connected in series with the primary line, and the secondary current remains substantially constant for a wide range of loads. The load consists of instruments or other devices which are connected directly in series with the secondary winding.

104. Measurement of ratio of series transformers. The ratio of series-type instrument transformers may be determined by measuring the primary and secondary currents directly with current-measuring instruments, but obviously such a method is much less accurate than null or "zero" methods. The principle of the latter is the same as that of the potentiometer. A non-inductive resistance in the secondary circuit is adjusted until the potential drop across it is equal to that in a non-inductive resistance in the primary circuit. The ratio of the two resistances is equal to the ratio of transformation. The differences among the various null methods are largely in the manner of determining the balance and in measuring the phase angle. Fig. 29 shows the scheme of a method used at the Bureau of Standards,* where a reflecting dynamometer is used as the detecting instrument. R^1 and R^2 are the resistances in the primary and secondary circuits, respectively. The fixed coil of a dynamometer, D_1, is connected in series with the primary; then, with the switch S thrown to the right, R_2 is adjusted until zero deflection is obtained. The component of the potential drop in R_2, which is in phase with that in R_1, is thus equal in magnitude to the drop in R'. Since the phase angle is always very small, the ratio of R_2 to R_1 may be taken as the transformer ratio. The phase angle is then determined by measuring the component of the R_2 drop which is 90 deg. from the R_1 drop, by means of another dynamometer, D_2, the fixed coils of which are excited by a current displaced 90 deg. in phase from the primary current.

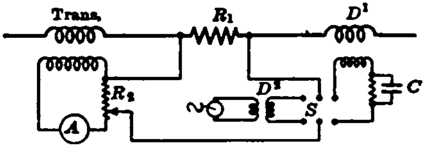


FIG. 29.—Connections for measuring ratio of series type transformers.

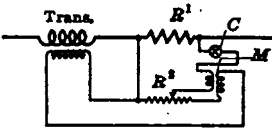


FIG. 30.—Connections for measuring ratio of series type transformers.

Fig. 30 shows the scheme of a method used at the Electrical Testing Laboratories.† R_1 and R_2 are the primary and secondary resistances,

* Bureau of Standards Bulletin, Vol. VII, 1913, No. 3, p. 423.
† Sharp, C. H. and Crawford, W. W., *Trans.* A. I. E. E., 1910, Vol. XXIX, p. 1517.

Another practical method of measuring small high-frequency currents is indicated in Fig. 32, where a and a' are two fine wires of different materials stretched between two terminals. The wires leading to the galvanometer are of the same materials, but so connected that a' and b are alike, and a and b' are alike. Thus there are two thermo-couples in series. Obviously the connections should be at the same potential, and this is adjusted on continuous current with direct and reversed readings. In a bridge method, the current is measured by the change in resistance of a carbon lamp (with a very small filament) in one arm of a bridge, Fig. 33; inductance coils, a and a' prevent the high-frequency current from flowing through the bridge.

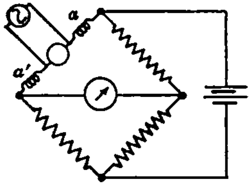


FIG. 33.—High frequency current measurements—bridge method.

108. Measurement of rectifier currents. Either of two values of the current from a rectifier may be required, the average value, or the root-mean-square value. In connection with storage-battery charging, the average value corresponds to the equivalent c.c. value and a permanent-magnet type of measuring instrument should be used. On the other hand, the power taken by incandescent lamps varies as the square of the current, and the equivalent c.c. current should be measured with instruments which indicate the mean effective value such as hot-wire or dynamometer ammeters.

109. Measurements of telephone currents. Telephone currents may be measured with a form of potentiometer* or with a barretter (Par. 40) but since telephone currents are of constantly varying amplitude and frequency, measurements made by this and the above methods are usually of little value. Telephonic intensities are usually compared by ear with a telephone, using artificial standardised cables. Where quantitative measurements are required, a high-sensibility oscillograph can be used.†

RESISTANCE MEASUREMENTS

110. Resistance standards in general. The practical unit of resistance, the **ohm**, is represented by a column of mercury having certain dimensions (Sec. 1). This standard is obviously difficult to construct, maintain and use; and, in general, will be found only in the laboratories of the national custodians of the fundamental electrical standards.

Secondary standards are therefore employed in actual measurements. These are made with metal of high specific resistance, in the form of wire or ribbon. Manganin (a copper-nickel-manganese alloy) is most used, because, when properly treated and aged, it meets the necessary requirements. These requirements are: permanent electrical and physical characteristics; low thermo e.m.f. in junctions with copper; small temperature coefficient of resistance; and relatively high specific resistance. The completed standard must, in addition, be unaffected by immersion in oil, or by changes in atmospheric conditions.

111. Classes of resistance standards. In general, resistance standards may be divided into two classes: standards of resistance, or those used primarily for the measurement of resistance; and current standards, or those intended primarily for the measurement of current.

112. General construction of standards of resistance. Standards of resistance have very small current capacity. They are made in two forms, the **Reichsanstalt** and the **N.B.S.** (National Bureau of Standards).‡ The former is shown, partially in section, in Fig. 34. The N.B.S. form is shown in Fig. 35. The distinctive features of the latter form are that it is immersed in oil and hermetically sealed. This prevents the absorption of moisture by

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* Drysdale, C. V. "Alternating-current Potentiometer for Measuring Telephone Currents," *London Electrician*, Aug. 1, 1913.

† Gati, B. Report of Second International Conference, European Telephone and Telegraph Administrations; 1910.

‡ Bureau of Standards Bulletin, Vol. V, 1908, p. 413.

very high or very low resistances and the accuracy depends upon the measurement of two unknown quantities with indicating instruments. Furthermore the current required to give a readable drop may cause overheating. The method should therefore be used with caution and only where accuracy is subordinate to simplicity and convenience. The potential should be measured, when possible, between points well within the current connections, especially when the resistance is low and the current is high. Greater accuracy can be obtained by substituting a standard resistance in place of the ammeter, and noting the drop across it, and across the unknown resistance, in succession. The latter is then equal to the ratio of the two readings multiplied by the standard resistance. The accuracy will be greatest when the two resistances are nearly equal.

117. Bridge methods are the most accurate for resistance measurements because: (a) they are zero methods; (b) comparison is made directly with standardised resistances, the accuracy of which can be made very high. The principal types of bridges are known as Wheatstone, slide-wire, Carey-Foster and Kelvin.

118. Wheatstone bridge. The Wheatstone bridge is most generally used for the measurement of all but the highest and the lowest resistances. Fig. 37 shows the theoretical arrangement of a Wheatstone bridge where r, r_1, and r_2 are accurately known resistances and r_x is the resistance to be measured. When using the bridge, the various resistances are adjusted until the galvanometer, G, shows no current flowing; then, $r_x = (r_2/r_1)r$. The battery switch, S_1, should always be closed before the galvanometer switch, S_2, in order to protect the galvanometer from the momentary rush of current. The galvanometer and the battery may be interchanged without affecting the result (Sec. 2, Par. 30).

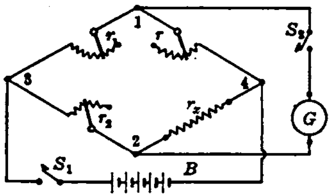

FIG. 37.—Diagram of Wheatstone bridge.

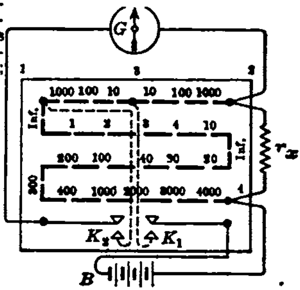

FIG. 38.—Wheatstone bridge—Postoffice form.

119. Forms of Wheatstone bridges. These bridges are made in a variety of forms. In most forms the resistances, r, r_1 and r_2, consist of a number of resistance coils or units carefully adjusted to various multiples of 10 and so arranged that they can be conveniently connected in and out of the circuit by means of plugs or switches. The resistances, r_1 and r_2 (Fig. 37) are commonly called the ratio arms and r the rheostat arm. A very early form, which is still in use in small portable sets, is the **Postoffice** pattern, shown diagrammatically in Fig. 38. Coils are cut out by short-circuiting them with plugs, so that there may be several plug-contact resistances of an unknown and variable amount in a given arm. In the **Anthony** form, shown diagrammatically in Fig. 39, this objection is overcome by arranging the coils of the rheostat arm on the "decade" plan in which there are nine 1-ohm coils in the "units" division, nine 10-ohm coils in the "tens" division, etc. Any number of coils in a given division can be connected in circuit by changing only one plug. In many later types, the ratio-arm coils are also connected on the decade plan, which in addition to eliminating plug-contact resistance errors, permits interchecking the coils. Furthermore, the decade arrangement permits the use of sliding-brush or dial construction instead of plugs.

In practice, r_1/r_2 is kept equal to α/β and the resistance d is made negligibly small. Then $r_x = r\alpha/r_1$, as in the Wheatstone bridge.

In the Wolff bridge, Fig. 42, the ratios r_1/r_2 and α/β are automatically adjusted simultaneously, by sliding contacts on the four dials. In the Leeds and Northrup bridge, Fig. 43, both r and the ratio r_1/r_2 are adjusted.

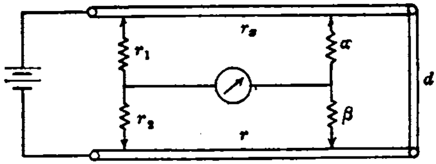

Fig. 41.—Diagram of Kelvin double bridge.

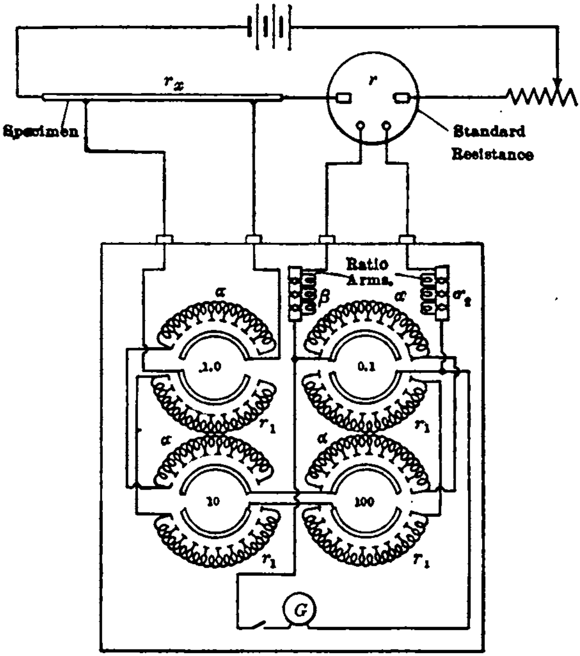

Fig. 42.—Kelvin double bridge—Wolff form.

124. Conductivity measurements. The specific conductance or conductivity of a material is the reciprocal of the specific resistance or resistivity. The relative conductivity is the ratio, expressed in per cent., of the specific conductance of the sample to that of a standard material. The relative conductivity may be based on equal masses or equal volumes. The former is in most common use because conductor metals are usually sold on a weight basis (see Sec. 4).

about 300 ohms each, in order to eliminate the effect of contact resistance at *a*, *b*, *c* and *d*. When the bridge is balanced, the resistance between *c* and *d* on r_x is equal to that between *a* and *b* on *r*. The unknown specimen r_x is cut to a certain definite length and carefully weighed. The contact, *b*, on the standard is then set at the point indicated on the carefully graduated scale *H*, which corresponds to this weight. Then the resistance, *ab*, is equal to that of a piece of wire having 100 per cent. conductivity, a length equal to 100 parts in the scale *I* and the same weight per unit length as r_x. Contact *d* is shifted until a balance is obtained and the conductivity is read directly from scale *I*, 100 scale divisions corresponding to 100 per cent. conductivity. One standard is provided for every three sizes of wire in the American (B. & S.) gage. The standards are usually of the same material as that being tested, so that the temperature does not have to be observed.

128. Resistance of rail joints. The testing of rail bonds consists in determining, either, (a) the ratio of the resistance of a given length of rail, including a bonded joint, to that of the same length of continuous rail; or (b) the length of solid rail which has the same resistance as the joint. The resistance of rail bonds is usually expressed in the latter manner, whether measured in that way or by the former method. Three methods are employed: millivoltmeter, bridge, and opposition.

129. Millivoltmeter method of measuring rail bonds. In the millivoltmeter method, simultaneous readings are taken with 2 millivoltmeters, one connected across the bond and the other across a definite length of rail. If the current fluctuations are not too rapid, only one instrument is necessary, provided there is a suitable arrangement of keys to change the connections in quick succession.

130. In the Roller bond tester the principle of the **slide wire form of Wheatstone bridge** is employed (Fig. 44). Balance is obtained by moving the contact *B* back and forth. At balance, $ab/bc = (r_1 + m)/(r_2 + n)$; where ab = resistance of bond and bc = resistance of the standard length of rail. The resistances r_1 and r_2 have the effect of extending the slide wire and providing greater accuracy (see Par. 121). In the actual instrument, the slide wire takes the form of a circle and the scale is graduated to give the resistance directly in terms of the number of feet of the solid rail being tested.

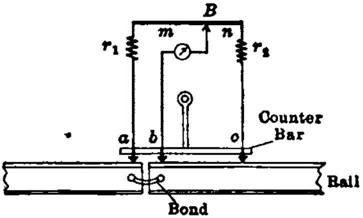

Fig. 44.—Diagram, Roller bond tester.

131. The Conant bond tester is an example of the class in which the drop across the joint is opposed to that across a length of solid rail, the outer contact on the latter (*c*, Fig. 44) being moved along until the two potentials are just equal and opposite. The detector is a telephone receiver in series with a make and break device operated by a clock.

132. Insulation resistance. The resistance of insulating materials is usually measured by deflection methods. In the case of resistances of the order of 1 megohm and less, a Wheatstone bridge may be used, but the accuracy will be low because of the extreme ratio required (Par. 121) and the low insulation resistance of the bridge.

Two general classes of deflection methods are used: (1) direct deflection and (2) leakage. The direct-deflection methods involve a simple application of Ohm's law, the current being measured with a voltmeter used as an ammeter or with a galvanometer.

133. Direct-deflection method (insulation resistance). When the resistance is of the order of 1 megohm, an ordinary voltmeter will give results which are sufficiently accurate for most purposes. Two readings are taken, one with the voltmeter directly across the battery or generator, and the other with the resistance to be measured connected in series with the voltmeter. The resistance is $R = r_v\,(d - d_1)/d_1$; where r_v =

or graduate in which two circular, closely fitting disc electrodes are supported. One of the electrodes should be movable so that the resistance of columns of several different lengths can be measured. The first measurement should be taken as the zero or base reading and the results checked by calculation of the increase in resistance and the corresponding increase in the spacing of the electrodes at different settings.

137. Precautions in measuring insulation resistance. In the measurement of the insulation resistance of specimens having electrostatic capacity, sufficient time should be allowed for the specimen to become charged, that is, until the deflection becomes constant, at a minimum value. This usually takes place within 1 min., except in long lengths of cable. In order to eliminate uncertainties in this connection, it is customary to specify 1-minute "electrification."

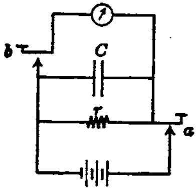

FIG. 46.—Leakage method of measuring insulation resistance.

As the apparent insulation resistance varies with the testing potential, one hundred volts is usually prescribed as the minimum pressure that should be used.

Leakage over the surface of wire or other specimens may be a source of much trouble in damp weather. In wires and cables, the lead or braid should be removed for 2 or 3 in. from the ends and the exposed insulation coated with hot, clean paraffine; or, just before measuring, these prepared ends may be carefully dried with an alcohol, Bunsen or other flame free from carbon. As a further precaution, a "*guard*" *circuit* may be arranged as shown by the dotted lines in Fig. 45. This consists of a few turns of fine copper wire twisted around the insulation close to the copper conductor and connected to the battery side of the galvanometer. In the case of solid specimens, the twisted wire is replaced with a ring of tin-foil as shown in Fig. 97.

Specimens having electrostatic capacity should be put in a neutral condition by rapidly reversing the current a number of times, beginning at a low rate of reversals and gradually increasing. Where the capacity is high it may be advisable gradually to decrease the applied voltage at the same time.

The side of the circuit which contains the galvanometer should be well insulated throughout. The battery also should be insulated as thoroughly as possible; this is a relatively easy matter when dry cells are used. The important point is to insure that all current passing through the specimen, and only that current, passes also through the galvanometer.

The galvanometer should, preferably, have a high resistance (order of 1,000 ohms) and a megohm sensibility (Par. 23 and 24) of several hundred megohms. The temperature should always be noted, because of the large coefficient which most insulating materials have.

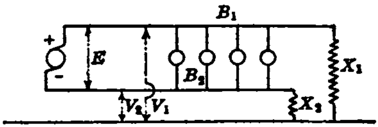

FIG. 47.—Insulation resistance of "live" circuits.

138. Measuring the insulation resistance of circuits. The insulation resistance of a "dead" circuit is conveniently made by the voltmeter method. When there is no source of e.m.f. available, various portable instruments described below are especially applicable and convenient. (Also see Sec. 21.)

When the circuit is "alive" the following method may be used.* Fig. 47 represents diagrammatically a system with lamps and motors connected. The resistances X_1 and X_2 represent the insulation resistance from the positive and negative sides respectively to ground.

$$X_1 = \frac{R(D - d_1 - d_2)}{d_2}, \text{ and } X_2 = \frac{R(D - d_1 - d_2)}{d_1} \quad \text{(ohms)} \qquad (17)$$

* Northrup, E. F. "Methods of Measuring Electrical Resistance;" McGraw-Hill Book Co. Inc., p. 210.

the electrolyte. If the slide-wire scale is divided into 1,000 parts, the resistance of the electrolyte is, at balance,

$$r_x = \left\{\frac{a}{(1000-a)}\right\}R. \qquad \text{(ohms)} \qquad (18)$$

If the source is alternating-current power of commercial frequencies an alternating-current galvanometer (reflecting electrodynamometer) may be used, the fixed coils being connected in series between the source, S, and the bridge and the moving coil in place of D (Fig. 49). A **Vreeland oscillator** (Par. 246) is very satisfactory as a source of energy because the wave form is a pure sine curve and the frequency is sufficiently high to make the telephone sensitive.

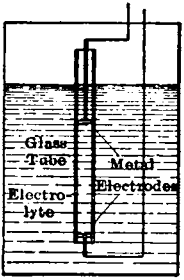

FIG. 50.—Specific resistance of electrolytes.

141. Specific resistance of electrolytes. Where the specific resistance or resistivity is required, a column of the liquid of known dimensions must be isolated. Fig. 50 shows a satisfactory method.* The glass tube is about 20 cm. long, 1 cm. internal diameter and open at both ends. The electrodes are of gold or platinum, the lower one being fixed in position and perforated, while the upper one is adjustable. The average cross-section must be carefully determined, preferably by volumetric measurement with mercury. The temperature is readily kept constant by stirring the liquid in the containing vessel.

142. Internal resistance of batteries. This measurement involves difficulties because of polarization. One simple direct-current method is as follows. The e.m.f. of the cell or battery is first measured on open circuit. The circuit is then closed through a known resistance and the e.m.f. measured again quickly before polarization begins. The resistance is

$$R_x = \frac{R(E-E_1)}{E_1} \qquad \text{(ohms)} \qquad (19)$$

where R = known resistance, E and E_1 = voltage before and after closing circuit, respectively.

This method assumes that the internal resistance will remain constant under all conditions, which is not always the case, especially in dry cells. In a modification of this method, both readings are taken with the circuit closed, but with two slightly different values of R. Then

$$r_x = \frac{(E_1-E_2)R_1R_2}{E_2R_1-E_1R_2} \qquad \text{(ohms)} \qquad (20)$$

where R_1 and E_1 are the first resistance and e.m.f., respectively, and R_2 and E_2 are the corresponding values with the second resistance.

In general, such direct-current methods should be used only with primary batteries of very low resistance and with secondary or storage batteries. Alternating-current methods are more reliable.

143. Alternating-current method of measuring internal resistance of batteries. The Kohlrausch bridge shown in Fig. 49 can be used in this method, by inserting the cell or battery in place of the electrolyte cell. Without resistance R′ connected, the resistance of the cell will be

$$r_x = \frac{a}{(1{,}000-a)}\ R \qquad \text{(ohms)} \qquad (21)$$

If the resistance R' is connected, the resistance of the cell with a current corresponding to R' flowing, will be

$$r_x = \frac{aR'R}{(1{,}000-a)R-Ra} \qquad \text{(ohms)} \qquad (22)$$

144. Effective resistance of alternating-current circuits. The passage of alternating current through a circuit is opposed by the ohmic resistance,

* Northrup, E. F. "Methods of Measuring Electrical Resistance," McGraw-Hill Book Co. Inc., p. 241.

151. Pulsating power. Where there are instantaneous variations in the current and the potential, the power varies from instant to instant. The average power will be the average of the products of corresponding instantaneous values of current and potential and it can be measured with *strict* accuracy only with watt-meters of the dynamometer type. In rectifier circuits, the power consumption of a storage battery or a motor can be approximately measured with a voltmeter and an ammeter of the permanent magnet type. Such instruments would give a more nearly correct result than dynamometer instruments. On the other hand, the reverse will be the case with a load of incandescent lamps or heating devices. The error will depend upon the wave shape and the character of the load. The safe method is to use a dynamometer-type wattmeter.

152. Alternating-current power. The power in an alternating-current circuit, at any instant, is the product of the current and potential at that instant. When the load consists only of resistance, the current wave, I, and the potential wave, E, are in phase as shown in Fig. 51, and the power-factor is 100 per cent. or unity. If the products of the instantaneous values of current and potential are plotted, the curve P is obtained. The average value of this curve is the power equivalent of a continuous current producing the same effect. Also, $W = EI$, where W = average watts, E = mean effective volts and I = mean effective amperes. These values of potential and current are indicated by instruments in which the deflections are proportional to the square of the current.

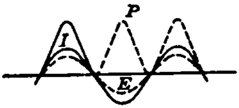

Fig. 51.—Relation of current, e.m.f. and power in a.c. circuit.

153. When the power-factor is less than unity, due to the fact that the circuit contains inductance or capacity (or the equivalent), the current and the potential will not be in phase. In the case of an inductive load, the current will lag behind the potential, as shown in Fig. 52. The power curve then will not be all on one side of the axis, but a part will be negative. If the current lags sufficiently, Fig. 53, the power curve will be positive half of the time and negative the other half; the average power will then be zero (or zero power-factor). This difference in phase, or time relation between the current and the potential, is called the phase angle and is usually expressed in degrees, an entire cycle being 360 deg. If the current and the

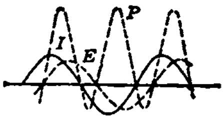

Fig. 52.

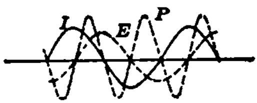

Fig. 53.

Figs. 52 and 53.—Relation of current, e.m.f. and power in a.c. circuit.

voltage are sinusoidal, the average value of the power is $W = EI \cos \theta$, where ϕ is the phase angle. Therefore if the current and the potential are not in phase, it is necessary to know the value of the phase angle if the power is to be determined from the current and the potential. Fortunately, instruments called wattmeters are available, which not only automatically integrate the power curve, but take into account the factor, $\cos \theta$.

154. Precision measurements of power must be made with an instrument which is equally accurate on continuous and alternating currents, in order that it may be calibrated on continuous current. Such measurements are most accurately made with **reflecting electrodynamometers** in which deflections are measured by means of a mirror, with a lamp and scale. (See Par. **33**, **64** and **97**.) The fixed coils are often divided into several sections, which may be connected in various series and parallel combinations to give large deflections over a wide range of power intensities. Instruments of this type made by the General Electric Company have current capacities from 5 amp. to 125 amp. and above, with corresponding sensibilities moving disc, or drum, as in the polyphase watt-hour meter. It is obvious that instruments of this type are limited to the frequency for which they are designed.

161. The Whitney wattmeter operates on the dynamometer principle (Par. 158) except that it is a torsion-head instrument, the moving element being kept in a fixed position by twisting the torsion head to which the control spring is attached. The pointer attached to this head moves over the scale. This method permits using a very long scale, extending around a full circle.

162. Wattmeters for switchboard use employ both the electrodynamic and the induction principles. Weston instruments are similar to the portable electrodynamometer instrument (Par. 158). The General Electric edgewise type "H" instruments are dynamometer types while type I is of the induction type. Westinghouse switchboard instruments are also induction type.

163. The calibration of wattmeters of the dynamometer type should be done with continuous current. It is customary to make such tests at a fixed potential, usually 100 or 200 volts and to vary the current to give the required watts. The potential is held constant at the desired value by means of one standard (standard voltmeter or potentiometer) and the current is read on another standard (standard ammeter, or potentiometer with standard resistance). It is more convenient to obtain the potential and the current from separate sources, because the process of adjustment of one circuit will not affect the other. In the case of instruments of large capacity this method economises energy, because only three or four volts are necessary for the current circuit.

164. Calibration of induction-type wattmeters. These instruments must be checked on alternating current of the frequency for which they are designed. This check is made by comparison with a secondary standard, which in turn is checked on continuous current. Polyphase instruments may be checked as single-phase instruments by connecting the current circuits in series and the potential circuits in parallel. In the case of induction-type instruments stray magnetic flux from one element may affect the other, in which case the calibration should be made on a polyphase circuit.

165. The inductance error in wattmeters may, under certain conditions, become very important. While the theory of the electrodynamometer type of wattmeter assumes that the potential circuit is non-inductive, this is not strictly true in the actual instrument because of the inherent inductance of the moving coil. Ordinarily, however, the non-inductive series resistance is sufficiently large to make the effect of this inductance negligible at ordinary frequencies and power-factors. But with low power-factors, the lag angle in the potential circuit may have to be considered. The power in an alternating-current circuit is $W = EI \cos \theta$, where W = power, I = current, E = e.m.f. and $\cos \theta$ = power-factor of circuit. When the power-factor is unity, I and E are in phase, but the potential-circuit current lags slightly behind E, thus producing the effect of a small power-factor. If, for example, the lag-angle, θ, is 2 deg., $\cos \theta = 0.9994$ and the error is ordinarily negligible. If the power-factor is 50 per cent., the lag angle in the wattmeter is $(60-2) = 58$ deg. The cosine of 60 deg. is 0.50 while the cosine of 58 deg. is 0.53, thus introducing an error of 6 per cent.

166. The stray-field error in unshielded, non-astatic, electrodynamometer wattmeters may be anything from zero to 25 per cent. with an alternating magnetic field of 5 lines per square centimeter, and from zero to 75 per cent. at 10 lines, depending upon the direction of the field and the coil deflection. A shield, properly made and placed, is extremely efficient, reducing the effect of a field of 20 lines per square centimeter to practically zero, without introducing eddy current or other errors.

Wattmeters of the Kelvin balance type, in which the coils are astatically arranged, are practically immune from these troubles except in an intense field which is not uniform throughout the space occupied by the moving system; such a condition may arise, for example, when the wattmeter is close to a conductor carrying a very large current. Induction-type instruments employ much stronger field strengths and are not appreciably affected except by very strong fields.

170. Measurement of power in a single-phase circuit. One wattmeter connected as shown in Fig. 56 will read true watts. The power may also be measured without the use of a wattmeter, by three voltmeters or three ammeters.

In the "**three-voltmeter**" method, a known non-inductive resistance, R, is connected in series with the load as shown in Fig. 58, where E, E_1 and E_2 are points where voltmeter readings are to be taken. The power in watts is

$$W = \frac{E^2 - E^2_1 - E^2_2}{2R} \qquad \text{(watts)} \qquad (24)$$

Similarly, in the "**three-ammeter**" method, Fig. 59, the power in watts is

$$W = R\left(\frac{I^2 - I^2_1 - I^2_2}{2}\right) \qquad \text{(watts)} \qquad (25)$$

171. Two-phase, four-wire circuit. Two wattmeters, connected as shown in Fig. 60, are sufficient, these conditions being equivalent to two single-phase circuits. The total power is obviously the arithmetical sum of the readings of the two instruments.

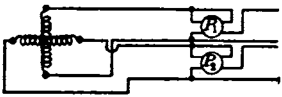

FIG. 60.—Power in two-phase, four-wire circuit.

172. Two-phase, three-wire circuit.—Two wattmeters should be connected as shown in Fig. 61, the total power being the *algebraic* sum of the two readings. This connection is correct for all conditions of load, balance and power-factor. One wattmeter may be used as in Fig. 62, provided there is no load across the outer conductors and the phases are *balanced* as to load and power-factor.

173. Two-phase, four-wire, interconnected circuit. Three wattmeters can be used, connected as in Fig. 63, the total power being the *algebraic* sum of the three readings. This connection is correct under all conditions of load, balance and power-factor. Two wattmeters, one in each phase, will give the true power only when the load is balanced.

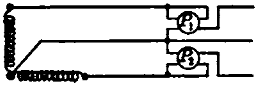

FIG. 61.—Power in two-phase, three-wire circuit.

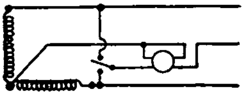

FIG. 62.—Power in two-phase, three-wire circuit.

174. Three-phase, three-wire circuits. Two wattmeters may be used, connected as in Fig. 64, the total power being the *algebraic* sum of the two readings. At unity power-factor, each instrument will indicate half the total power and at 50 per cent. power-factor one instrument will indicate the total power, the other instrument reading zero. At less than 50 per cent. power-factor, one instrument will read negative. (See Par. 209 for method of verifying power-factor.)

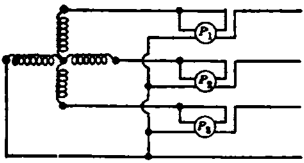

FIG. 63.—Power in two-phase, four-wire interconnected circuit.

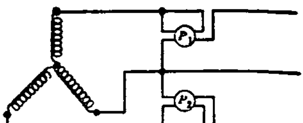

FIG. 64.—Power in three-phase, three-wire circuit, two wattmeters.

point P of the system, which may or may not be the neutral. The total power is the algebraic sum of the readings of all of the wattmeters so connected.*

178. Power measurements on high-voltage circuits should preferably be made with series-type and shunt-type instrument transformers. If the instrument wattmeter is connected directly to the circuit with series resistance in the potential circuit, the circuit should be grounded at the instrument in order to avoid errors of electrostatic attraction, and also possible injury to the instrument or the observer. The current-capacity limit of commercial wattmeters is about 200 amp. beyond which series transformers with 5-amp. instruments are used, irrespective of potential.

179. Corrections where instrument transformers are used in accurate power measurements. In every case the **true ratio** and **phase angle** should be known (Par. 79 and Par. 104, 105). The general effect of the phase changes in the instrument transformers is to make the angle between the current and the potential in the wattmeter larger or smaller than that between the current and the e.m.f. of the circuit being measured.

If $\cos\theta$ = true power-factor and $\cos\theta_2$ = apparent power-factor (*i.e.*, power-factor in the wattmeter obtained from the ratio of the watts and volt-amperes in the wattmeter), true watts $= (\cos\theta/\cos\theta_2) \times$ wattmeter reading. The apparent power-factor, $\cos\theta_2 = \cos(\theta \pm \alpha \pm \beta \pm \gamma)$, where

θ = phase angle in main circuit,
α = equivalent phase angle in wattmeter,
β = equivalent phase angle in current transformers,
γ = equivalent phase angle in voltage transformers.

The angles α, β and γ are given positive (+) signs when they tend to decrease and negative (−) signs when they tend to increase the phase angle between the current and voltage in the instrument.†

180. Power-factor. The power-factor of a circuit is the ratio of the true power in watts, as measured with a wattmeter, to the apparent power obtained from the product of the current and the potential, in amperes and volts respectively. In the ordinary continuous-current circuits, the power-factor is obviously unity but in rectifier circuits, for example, it may be slightly less than unity. In alternating-current circuits, the power-factor is usually less than unity because the current and the potential are not in phase. When the wave form is sinusoidal, the power-factor is equal to the cosine of the angle of lag.

181. The power-factor of single-phase circuits is obtained from wattmeter, voltmeter and ammeter readings, by the relation $W/EI = \cos\theta$ where W = watts, E = volts and I = amperes.

182. The power-factor of polyphase circuits which are balanced is the same as that of the individual phases. When the phases are not balanced, the true power-factor is indeterminate. For all practical purposes, however, it is sufficiently correct to assume the power-factor to be that obtained by methods which give the average of the power-factors of the separate phases. In the **wattmeter-voltmeter-ammeter method,** the power-factor is, for a two-phase, three-wire circuit $W/\sqrt{2}\ (EI)$. (I in middle wire, E between outer wires) and for a three-phase, three-wire circuit the power-factor is $W/\sqrt{3}\ (EI)$, wherein W = watts, E = volts and I = amperes. In the **two-wattmeter** method, the power-factor of a two-phase, three-wire circuit is obtained from the relation $W_2/W_1 = \tan\theta$, where W_1 is the reading of a wattmeter connected in one phase in the same manner as a single-phase circuit, and W_2 is the reading of a wattmeter connected with its current coil in the first phase, in series with the first wattmeter, and the potential coil across the second phase. Obviously, if the load is steady, one wattmeter is sufficient. If the phases are not balanced, the readings should be repeated with the instruments in the second phase, the true power-factor being taken as the average of the two results. In a **three-phase, three-wire circuit,** the power-factor can be calculated from the readings of two

* Bedell, F. "Direct and Alternating-current Testing." D. Van Nostrand Company (1912), p. 228.
† Robinson, L. T. "Electrical Measurements in Circuits Requiring Current and Potential Transformers." *Trans.* A. I. E. E., 1909, Vol. XXVIII, p. 1005.

tional to the power and the revolving element operates a registering mech anism on which the energy consumption is recorded. Meters for con tinuous current are usually of the type which utilise the electrodynami principle of direct-current motors, while those for alternating curren utilise the principle of induction motors.

188. Continuous-current watt-hour meters. Continuous-curren meters may be divided into two classes, the **commutator type and th mercury motor type.**

189. Commutator-type meters are similar in principle to shunt motor The essential features are shown in Fig. 71. The moving element consist of an armature, *a*, a commutator, *c*, an a light metal disc, *d*, all mounted on steel shaft which rotates in a jew bearing. The armature is connecte to the external circuit by means of ver light silver-tipped brushes. In serie with the armature is a light-load com pensation coil, *s*, and a resistance, The field coils are indicated at *f*, *f*.

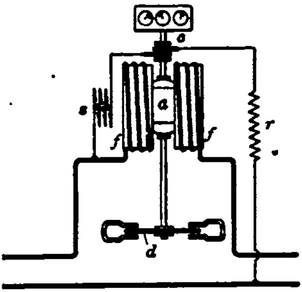

Fig. 71.—Diagram, commutator-type watt-hour meter.

190. Characteristics of commu tator-type meters. The essenti differences between this type of wat hour meter and a two-pole shunt moto are as follows: (a) entire absence c iron in the magnetic circuits; (b) th armature element is connected acro the circuit and carries a very sma current, while the field element is i series with the circuit and carries th main current; (c) the speed increase as the field strength increases which i opposite to the effect in a shunt moto

191. The principle of operation of commutator-type meters i as follows. The torque is proportional to the current in the armature coil and to the field strength. Since there is no iron in the magnetic circuit, th latter is always proportional to the field current, hence the torque is pro portional to the two currents (as in a dynamometer wattmeter). The cu rent in the armature being proportional to the line potential, and the fiel current equal or proportional to the line current, the torque is proportion to the power. In order to make the speed proportional to the power, mechanical load must be provided, in which the counter-torque will be pro portional to the speed. This load usually takes the form of a circular dis *d* (Fig. 71) of thin copper or aluminum which revolves between the pole of one or more permanent horseshoe magnets, with poles very close togethe The eddy currents induced in the disc react with the permanent-magne field, producing a counter-torque which will always be proportional to th speed. As the load current increases, the torque of the motor element in creases and the speed increases, because there is practically no counte e.m.f. in the armature, *s*. But as the speed increases, the counter-torqu of the disc or generator element also increases and a speed is finally reache where the two torques balance each other and the speed remains constan Thus, theoretically, the speed will always be proportional to the power in th circuit. Each revolution represents a definite amount of energy, and b connecting the shaft to a suitable recording mechanism similar to that o gas and water meters, the total energy consumed is automaticall registered.

192. Effects of friction and temperature in commutator-typ meters. In practice, certain conditions prevent the speed from bein always proportional to the load, the principal factors being friction an temperature. **Bearing friction** is reduced to a minimum by using polishe sapphire or diamond jewels, with either a polished cone-shape shaft-en or a steel ball. Thus the contact surface is reduced practically to a poin The weight is reduced by using hollow shafts and very light aluminum c non-metallic frames for the armature windings. **Commutator frictio** is reduced to a minimum by making the commutator diameter small, an

198. Typical Data Applying to Modern 110-Volt, 5-amp. or 1(amp., Direct-current Watthour Meters *

	G. E. Co. C-6, 5 amps.	Westinghouse CW-6, 5 amps.	Sangamo D, 10 amps.	Duncan E, 5 amps.	Columbia D, 10 amps.
Speed, full load, r.p.m.	46	41.7	25	36.7	3(
Torque, full load, mm.-grm.	170	140	55	180	9(
Weight, moving element, grm.	97	80	3‡	130	9(
Ratio, torque to weight	1.75	1.75	18	1.39	1.(
Drop, current circuit, volts at rated current.	1.15	1.0	0.03	.....	0.5:
Loss, current circuit, watts at rated current.	5.75	5.0	0.3	.....	5.:
Loss, potential circuit, watts at 110 volts.	5.1	4.5	5.0	5.0	2.(
Resistance, armature, ohms	825	1,185	1,850§	.....	2,66(
Resistance, compensating coil, ohms	65	315	......	.....	45(
Resistance, series resistance, ohms	1,540	800	450	.....	3,00(
Resistance, potential circuit, total ohms	2,430	2,300	2,300	.....	6,11(
Ampere-turns, field	300	600	......	910	70(
Ampere-turns, armature	800	1,500	......	.....	2,10(

199. Three-wire circuits are metered with two, two-wire meters o the kind described (Par. **188** to Par. **198**), **or a three-wire meter.** Th latter which is usually made in the **commutator type**, is the same as th two-wire meter except that the two field coils (which should be alike) ar separated electrically, and one is connected in each outer wire in such a mann that their fields are cumulative as before. When the load is exactly balance the conditions are obviously the same as in a two-wire meter, and whe unbalanced the two field strengths add together so that the speed is pro portional to the total current.†

200. The metering of heavy-current circuits by means of standar meters becomes troublesome because of the large conductors required in th fields. While such meters have been made in capacities up to 20,000 amp they are very costly and not satisfactory at light loads. Watthour mete have been developed by some manufacturers, along standard lines, for ope ation with shunts. In order to develop sufficient torque without an exce sive shunt loss, it is necessary to employ shunts having small drop, large fie coils on the meters and relatively large leads from meter to shunt. Whe the meter has to be some distance from the shunt, the leads may have to b nearly as large as the wires in the main circuit, in order to keep down th resistance.

Another way to avoid the use of large meters is to connect sever smaller meters in parallel. Care should be taken to make the resistan of the several branches equal, if the meters are of the same capacity; or, the meters are of different capacities, inversely proportional to the capa ties of the meters. This will insure that none of the meters are overloade

201. Alternating-current watt-hour meters. Alternating-curre energy is almost always measured with induction type meters. Co mutator meters are seldom used on alternating-current circuits for the ve practical reason that induction meters are not only **more accurate,** b much **less expensive** in first cost and in maintenance.

* From Electrical Meterman's Handbook, N. E. L. A., 1912, to whi readers are referred for further data. See also Fitch, T. T. and Huber, C. "A Comparative Study of American Direct-current Watthour Mete Bureau of Standards Bulletin, 1913, Vol. X, p. 161. (Reprint No. 207.)

† The Sangamo Co. has recently developed a three-wire mercury meter (Par. 195).

practically all meters is that in which a flux is produced at the potenti[al] pole face, slightly out of phase with the main flux. Thus eddy currents wi[ll] be produced in the disc which will be in phase with a small component of th[e] main flux, giving rise to a slight torque which can be made sufficient to over[-] come the friction torque. This "out-of-phase" flux is produced in vario[us] ways in different meters. A common method is to place a short-circuite[d] copper circuit or thin copper punching ("shading strip") in the potentia[l] pole air-gap, in an unsymmetrical position, so that the desired unbalance[d] flux will be obtained. In the Columbia meter, the effect is accomplishe[d] by unbalancing the flux of the two potential poles by means of magneti[c] shunts.

205. Adjustments of induction-type meters. Facilities are usuall[y] provided for conveniently adjusting the meter accuracy at light and full loa[d]. The position of the light-load compensation coil can be changed wit[h] conveniently located screws, and the light-load speed thus altered. Spee[d] adjustment at all loads is obtained by shifting the drag magnets wit[h] respect to the axis, as in direct-current meters, or by shunting the flu[x] by means of a movable soft-iron **keeper** bridging the air gap. Th[e] power-factor or lag adjustment is made at the factory and if properly don[e] should never require readjustment.

206. Typical Data Applying to Modern 110-volt Single-phase 60 cycle, 5-amp. Induction-type Watt-hour Meters*

	G. E. Co. I-10	Westing-house 0-A	Fort Wayne K-4	San-gamo H	Duncan M	Colum-bia C
Speed, full load, r. p. m.	36	25	36.7	40	36.7	30
Torque, full load, mm.-grams	46.6	36	45	40	115	80
Weight, moving eelment, grams	26.3	15.	21	15.6	46	30
Ratio torque to weight	1.77	2.32	2.14	2.5	2.5	2.66
Drop, current circuit volts at 5 amp.	....	0.3	0.12	0.1	....	0.1
Loss, current circuit, watts at 5 amp.	0.98	0.75	0.59	0.5	....	0.5
Loss, potential circuit, watts at 110 volts	2.5	1.6	1.75	1.85	1.25	1.5
Power-factor, potential circuit, per cent	....	18	17	35	....	70

207. Measurement of energy in alternating-current circuits. The energy consumption in alternating-current circuits is measured wit[h] watt-hour meters connected in exactly the same manner as are wattmeter[s] for the measurement of power. (See Figs. 60 to 68.) In **three-wire, two-phase** or **three-phase systems, polyphase meters** may be used. Suc[h] meters comprise merely two single-phase meters in one case, with a commo[n] shaft, and connected to the main circuit in the same manner as two single-phase meters.

Four-wire systems, unless balanced, require **three single-phase meters.** A **three-phase system** with a **grounded neutral** should b[e] considered a four-wire system requiring three meters, unless it is completel[y] balanced.

208. The total energy in a three-phase circuit is the algebraic su[m] of the indications of two single-phase meters, just as the total power i[s] the algebraic sum of the readings of two wattmeters (Par. 174). If [a] **polyphase meter** is used, the summation is automatically performed, an[d]

* From "Electrical Meterman's Handbook," N. E. L. A., 1912, to which th[e] reader is referred for further data.

the voltage, as indicated in Fig. 75. Resistance in the potential circuit of alternating-current meters will alter the quadrature phase relation, and therefore voltage regulation should be obtained with a variable ratio auto-transformer, an induction regulator or by field control.

213. Power-factor variation, in meter testing, can be obtained by several methods. In the **two-alternator** method, two generators are mounted on a common base, with a common shaft. The stationary members (armature or field) are made movable about the shaft with respect to the base and to each other. Thus with the potential coil of the meter connected to one machine, and the current coil to the other, any phase relation can be obtained by adjusting one movable member with respect to the other.

In the **transformer** method, a transformer with a large number of steps, or a variable-ratio auto-transformer, is connected across one phase of a polyphase circuit and the potential coil of the meter is connected in such a manner that any phase relation can be obtained. Thus, referring to Fig. 76, the current coil of the meter is connected in series with conductor *a* of a three-phase circuit, and the potential coil is connected to *o* and to *c*, the latter being a tap on a transformer connected across phase *bc*. It is apparent that **any phase angle** between the current and the potential can be obtained in a range from 0 deg. to 60 deg. by moving the connection point *c* along the transformer winding. Angles from 60 deg. to 90 deg., lead or lag, can be obtained by changing the transformer to either of the other two phases and the meter connection from *o* to *x* or *y*. These changes can be instantly made with suitable switching arrangements. A similar arrangement can be made for a two-phase circuit, Fig. 77. It is also convenient to introduce such a transformer between the taps *o*, *c* and the meter, for the purpose of compensating for the variations in the voltage between *o* and *c*, and keeping the voltage constant at the meter. Two variable-ratio auto-transformers arranged in this manner make a convenient **phase shifter.**

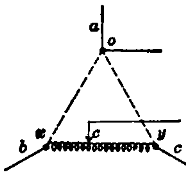

FIG. 76.—Power-factor variation—transformer method.

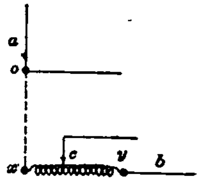

FIG. 77.—Power-factor variation—transformer method.

In the **reactance-coil** method, a reactance coil is introduced in the current circuit, the reactance being varied by moving an iron core in and out of the coil. It is difficult to obtain low power-factors with this method unless a separate low-potential current circuit is used, and then there is danger of **wave-form distortion.**

214. Measurement of meter torque. The torque is measured under normal conditions at full load by measuring the force in grams exerted at the edge of the disc, or at the end of an arm attached to the shaft. This force may be measured by means of weights, a calibrated watch spring, or by utilizing the principle of the pendulum.* By measuring the radius of the disc or the arm in millimeters, the torque is obtained in millimeter-grams, the usual unit.

215. Measurement of watt-hour meter losses. The losses in the windings of c.c. meters are calculated from the resistances, as determined with continuous current by standard methods. The losses in alternating-current meters are measured directly with wattmeters, but great care is required because of the very small amount of power and the small power-factor.

216. The standards for direct-current meter tests may be ammeters and voltmeters, in portable or special laboratory types, or potentiometers; in alternating-current meter tests, use is made of indicating wattmeters. The time in measuring the meter speed is usually determined with stop-watches, reading to tenths of seconds. Where a large number of meters

* Agnew, P. G. Bureau of Standards Bulletin, 1911, Vol. VII, No. 1. (Reprint No. 145.)

220. General precautions to be observed in testing watt-hour meters are as follows: (a) The test period should always be sufficiently long and a sufficiently large number of independent readings should be taken to insure the desired accuracy. In service tests, the period preferably should be not less than 30 sec. and the number of readings not less than three. In laboratory tests, 100-sec. periods and five readings are preferable. (b) Capacity of the standards should be so chosen that readings will be taken at reasonably high percentages of their capacity, in order to make observational or scale errors as small as possible. (c) Where indicating instruments are used on a fluctuating load, their average deflections should be estimated in such a manner as to include the time of duration of each deflection, as well as the magnitude. (d) Instruments should be so connected that neither the standards nor the meter being tested are measuring the potential-circuit loss of the other, that the same potential is impressed on both, and that the same load current passes through both. (e) When the meter under test has not been previously in circuit, sufficient time should be allowed for the temperature of the potential circuit to become constant, preferably not less than 10 min.; this is important with direct-current meters, especially in the case of rotating standards. In some types of the latter, special provision is made for rapid heating. (f) Guard against the effect of stray fields by locating the standards and arranging the temporary test wiring in a judicious manner.

221. Meter constants. The following definitions of various meters constants are taken from "Code for Electricity Meters."*

Register constant is the number by which the register readings must be multiplied to obtain the registration. They are ordinarily used only on large-capacity meters and are marked on the register.

Gear ratio is number of revolutions of the rotating element per revolution of the first dial hand.

Watt-hour constant is the registration reduced to watt-hours per revolution of the rotating element. It has a definite value for each type and rated capacity of meter.

Watt-second constant is the registration reduced to watt seconds per revolution of the rotating element. It is equal to watt-hour constant multiplied by 3,600.

Test constant is the constant assigned by the manufacturer for use in the test formula for his meter.

222. Testing formulas. The accuracy of a watt-hour meter is the percentage of the total energy passed through a meter which is registered on the dials. Accuracy in per cent. = meter watt-hours × 100/true watt-hours. The value of one revolution having been assigned by the manufacturer, the meter watt-hours $= K_h \times R$, where $K_h =$ watt-hours per revolution or watt-hour constant and $R =$ revolutions in S seconds. The corresponding power in watts is $P = (3{,}600 \times R \times K_h)/S =$ meter watts and 100 × meter watts/actual watts = per cent. accuracy. This is the standard formula for watt-hour meters.

223. Manufacturers' formulas for meter watts. When the test constant K differs from the watt-hour constant, K_h, the formula is changed accordingly as follows:

Manufacturer	K in terms of K_h	Manufacturers' formula for meter watts
Columbia	$K = 3{,}600\ K_h$	$W = R \times K/S$
Duncan	$K = 60\ K_h$	$W = 60 \times R \times K/S$
Fort Wayne	$K = 36\ K_h$	$W = 100 \times R \times K/S$
General Electric	$K = K_h$	$W = 3{,}600 \times R \times K/S$
Sangamo	$K = 3{,}600\ K_h$	$W = R \times K/S$
Westinghouse	$K = 3{,}600\ K_h$	$W = R \times K/S$

$K_h =$ watt-hour constant, $K =$ test constant (marked on meter, usually on disc), $R =$ number of revolutions in S seconds, $W =$ meter watts.

224. Average accuracy of watt-hour meters. The accuracy of a meter varies with the load, but it is often desirable to assign a value for the

* "Code for Electricity Meters." A. E. I. C. and N. E. L. A., 1912 ed., pp. 95 and 96.

easily emptying the mercury from the tube into the anode receptacle when the former becomes filled. This type of meter has been highly developed and inherent errors due to variation in temperature, concentration of the solution, level of mercury, effect of vibration, etc., are largely eliminated in the latest forms.

230. The principal advantages of electrolytic-type ampere-hour instruments are their low first cost and their simplicity, which results in low maintenance cost. These are important items to power companies serving very small customers, especially where the rates are low; and may outweigh the disadvantages, the principal among which are elimination of the potential element and relatively low accuracy.

231. Electromotor ampere-hour meters are similar to watt-hour meters, except that the field is produced by permanent magnets instead of electromagnets. The rotating element is geared to a register which is calibrated in watt-hours for a given assumed voltage. There are two general types, the electromagnetic and the mercury flotation. The former is not made or used very much in this country.

The **Chamberlain and Hookum meter** is an example of the electromagnetic type. It employs a flat (pan-cake) armature winding mounted on an aluminum disc which also serves as the drag element. Connection is made to the circuit through a commutator and brushes in the usual manner. The armature is connected to a low-resistance shunt which is in series with the load.

The **mercury type** meter is well represented by the **Sangamo meter** which is practically the same as the Sangamo direct-current watt-hour meter (Par. 195), the electromagnet being replaced by permanent magnets.

232. Maximum demand meters. Many methods of selling energy involve the maximum amount which is taken by the customer in any period of a prescribed length, that is, the maximum demand. Meters for measuring this demand variously utilise the expansion of air, the torque of a watt-hour meter, or a special recording device in connection with a standard watt-hour meter.

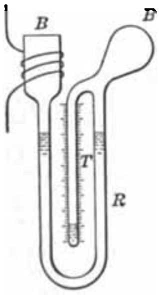

FIG. 79.—Diagram, Wright demand meter.

The **Wright demand meter** is a thermal instrument. It consists of a hermetically sealed U-shaped glass tube (Fig. 79), partly filled with a liquid. One leg *R* is connected near the top to a smaller graduated tube *T*. Around the top of the tube, *B*, is wound a resistance wire through which the current flows. The resultant heating of the air forces the liquid up the tube *R* and, if sufficient, over into the graduated tube. The air heats up gradually and the necessary **time lag** is thus obtained. According to the makers, if the overload continues 5 min., 80 per cent. of the load will be indicated; 10 min., 95 per cent.; and 30 min. 100 per cent.

233. The General Electric demand meter utilizes the principle of the induction watt-hour meter. The torque of the moving element is opposed by three long spiral springs, in series, and a powerful drag on the drag disc. This arrangement provides the necessary time lag. Two sweep-hands are provided on a dial. One is geared to the moving element, while the other is moved by the first one and left at the last position reached by it. This second pointer indicates therefore the maximum energy, until it is reset. The drag magnets and spiral springs being adjustable, the time lag can be adjusted through a considerable range. In this type of instrument, the demand is indicated at all times, as well as the maximum demand since the last setting.

234. Recording maximum-demand meters. In the **Minerallac Electric Co.'s "Printometer"** the kilowatt-hours indicated on a watt-hour meter are printed on a paper tape at intervals of any desired length. The hourly intervals are also indicated. It is separately mounted and can be electrically connected to any standard make of watt-hour meter. This device, while it indicates the time of maximum demand, is not an indicating instrument and considerable labor is involved in determining the maximum a minute, the pointer being perfectly free in the interim. The record is therefore a succession of dots. In other types of instruments, an inked thread or ribbon is interposed between the pointer and the paper chart.

238. The Westinghouse recording instruments are the principal example, in American practice, of the class where the recording mechanism is separate from the instrument proper. The moving element, by means of contacts, operates a relay which in turn operates the recording mechanism. Thus the moving element does not have to produce a large torque, while ample power can be applied to the recording mechanism, and hence friction does not affect the sensitiveness of the instrument. The direct-current instruments employ the D'Arsonval principle, using two sets of coils and magnets, astatically arranged. The alternating-current instruments use the principle of the Kelvin balance. Fig. 81 shows the scheme as employed in a voltmeter.

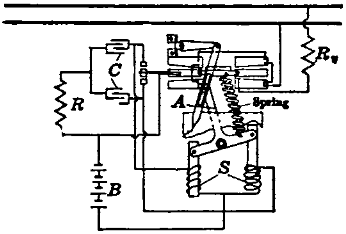

Fig. 81.—Diagram, Westinghouse graphic voltmeter.

239. The Callender recorder made by the Cambridge Scientific Instrument Co. employs the principle of a slide-wire bridge (Fig. 82) in which the resistance of one arm, X, varies with the current, potential or power to be measured. As soon as the bridge is unbalanced, a D'Arsonval galvanometer operates a relay, r or r', which moves a contact, c, along the slide wire s, until balance is restored, when the relay circuit opens. This contact also carries the recording pen, which leaves an ink record on a rectilinear chart.

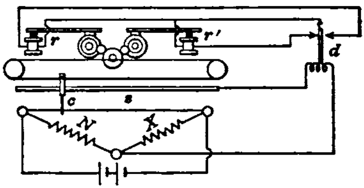

Fig. 82.—Diagram, Callender recorder.

INDUCTANCE MEASUREMENTS

240. General. The self-inductance, or coefficient of self-induction, of a circuit is the constant by which the time-rate of change of the current in the circuit must be multiplied, to give the self-induced counter e.m.f. Similarly, the **mutual inductance** between two circuits is the constant by which the time-rate of change of current in either circuit must be multiplied to give the e.m.f. thereby induced in the other circuit. Self-inductance and mutual inductance depend upon the shape and dimensions of the circuits, the number of turns and the nature of the surrounding medium.

241. Standards of inductance are usually simple coils of copper wire suitably mounted on a non-conducting, non-magnetic frame. The turns are held rigidly in place by shellac, paraffine or other insulating medium. Inductance standards are made in single units like standard resistances, or in combinations, with plug connections, like a subdivided condenser or a resistance box. In the Ayrton-Perry **variable standard** there are two concentric coils, one fixed and the other movable. When connected in series these coils form a variable inductance, the value of which at any relative position is read from a circular scale at the top. Additional range is secured by connecting sections of the two coils in series-parallel combinations by means of plugs.

242. Methods. The most commonly employed methods of measuring inductance are (a) Wheatstone-bridge methods, where the inductance is determined by comparison with a known inductance or known capacity; and (b) impedance methods where the inductance is determined by calculation from measurements made with alternating current.

where L = inductance of Z in henrys, r = ohms in parallel with C, C = capaci of condenser in micro-farads, R = total ohms of bridge arm to which condens is connected and R_s = ohms of Z.

248. A similar method (Par. 247) is indicated in Fig. 85 in which t adjustments are independent of each other, the bridge being first balanc with a steady current and then with a transient current by adjusting r in Fig. 84. At balance,

$$L = Cr^2 10^{-6} \qquad \text{(henrys)} \qquad (2$$

where L = inductance of Z in henrys, C = capacity of the condenser in micr farads and r = ohms in parallel with C.

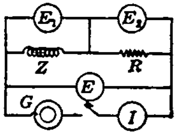

FIG. 86.—Inductance measurements—connections for three-voltmeter method.

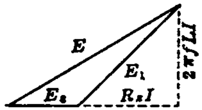

FIG. 87.—Inductance measur ments—vector diagram for thre volt-meter method.

249. Impedance methods of measuring inductance are based on th law of the impedance of a circuit carrying sine-wave alternating current an containing only inductance and resistance, that is,

$$Z = \frac{E}{I} = \sqrt{R^2 + (L2\pi f)^2} \text{ or } L = \sqrt{\frac{E^2 - I^2R^2}{(2\pi f I)^2}} \qquad \text{(henrys)} \qquad (2$$

where L = inductance in henrys, E = drop in volts, I = current in ampere R = resistance in ohms, and f = frequency in cycles per sec. Obviousl due allowance should be made for the voltmeter current where its magnitud is sufficiently important.

250. In the three-voltmeter method, the inductance to be measured i connected in series with a non-inductive resistance as shown in Fig. 8

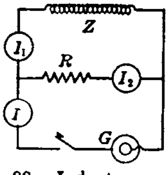

FIG. 88.—Inductance measurements—connections for three-ammeter method.

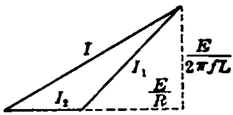

FIG. 89.—Inductance measure ments—vector diagram for three-am meter method.

The current, I, is measured, also the total volts, and the volts across th inductance Z and the resistance R. From these readings a triangle is con structed, Fig. 87. If R is known, the quantity $2\pi fLI$ can be calculated fron the triangle. If R is unknown, $2\pi fLI$ can be obtained by graphical construc tion. I and f being known, L is obtained by calculation.

251. The three-ammeter method is similar. The connections ar shown in Fig 88, and from the three currents, Fig. 89 is constructed. Th e.m.f., E, can be measured directly or, if R is known, by calculation fro the relation $E = RI_2$. Hence L can be obtained from the quantity $E/2\pi f$

252. In circuits containing iron the inductance varies with th frequency and with the current; hence alternating current of kno frequency and intensity must be used. In such cases a bridge meth with a standard inductance is convenient. A vibration galvanometer,

256. The capacity of a group of condensers in series is:

$$C = \frac{1}{\frac{1}{C^1}+\frac{1}{C^2}+\frac{1}{C^3}+ \dots \dots \frac{1}{C^n}} \tag{32}$$

When connected **in parallel**,

$$C = C_1+C_2+C_3+ \dots \dots C_n \tag{33}$$

257. In high-grade standard condensers, the aim is to reduce the absorption, the dielectric hysteresis and the ohmic losses to a minimum. **Commercial standards** are made with tin-foil and high-grade mica, bound together under high pressure. **Primary standards,** however, are made with air as the dielectric, in which absorption and leakage are nil.

258. Methods. Electrostatic capacity measurements are made by bridge methods (Par. **259**), with a ballistic galvanometer (Par. **260**), by loss of charge (Par. **262**), and by impedance methods (Par. **263**).

259. A bridge method of measuring capacity is shown in Fig. 93. The ratio, r_1/r_2, or the standard condenser (if adjustable), is adjusted until the bridge is balanced; with an ordinary D'Arsonval galvanometer this condition is indicated when there is no "kick" as the reversing switch is changed from one position to the other. If interrupted currents or high-frequency alternating currents are used, with a telephone receiver, balance is indicated by silence in the receiver. Then $C_2 = C_1r_1/r_2$. The resistance should be non-inductive, anti-capacity and relatively large—several hundred ohms. Obviously, maximum sensibility is obtained when C_2 and C_1 are about equal. By employing a Vreeland oscillator (Par. **246**) and an adjustable air or oil condenser, small capacities can be very accurately measured by this method.

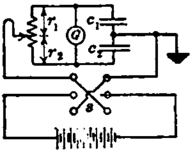

FIG. 93.—Capacity measurements, bridge method.

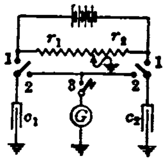

FIG. 94.—Capacity measurements, method of mixtures.

260. In the ballistic galvanometer method, the deflection is noted when the unknown capacity is discharged through it, immediately after having been charged at some known potential. A reading is then obtained with a standard condenser, the deflection being made about the same as before, by varying the capacity or the charging potential. The capacity can then be computed from the relation $d/d_1 = CE/C_1E_1$; where d, d_1, C, C_1, and E, E_1 are the respective deflections, capacities and potentials. This method is best suited to relatively large capacities, such as lead-covered, paper-insulated cables, etc.

261. The Thomson method of mixtures is shown in Fig. 94, where c_1 is a cable, transmission line or other capacity to be measured, and c_2 is a standard condenser. First the switches are closed at 1, 1, and the condensers charged to potentials corresponding to r_1 and r_2 respectively. After complete charge (a cable may require several minutes), the switches are shifted to 2, 2 and the charges equalized. If, then, the switch at 3 is closed, the deflection of the galvanometer will be proportional to the difference of the charges. This operation is repeated with various ratios r_1/r_2 until there is no deflection. Then $c_2 = c_1r_1/r_2$.

262. In the loss of charge method, Fig. 95, the condenser to be measured is first completely charged by moving switch b to a, and then immediately discharged through a ballistic galvanometer by moving b to c. The condenser is again charged and allowed to discharge through a known high

WAVE-FORM MEASUREMENTS

267. Methods. The instantaneous variations of current and potential in a circuit are measurable by step-by-step methods (Par. **268**), and with the oscillograph, (Par. **271**.) The former is applicable only where the current and the potential are strictly periodic and recurrent, as in a normal alternating-current circuit. The oscillograph can be used under all conditions, but is especially applicable to measurements of transient phenoména (Par. **268**), such as those which occur during switching operations on direct-current and alternating-current circuits. Where the wave form is to be analyzed, the former is the more convenient and accurate.

268. A convenient "step-by-step" arrangement is shown in Fig. 98. The contact device consists of two slip-rings and a four-part commutator. One slip-ring is connected to one terminal of the source, the other to a voltmeter, and the commutator to a condenser. By means of this arrangement, the condenser after being charged is immediately discharged through the voltmeter. These impulses follow each other so rapidly that a steady deflection is obtained, and by suitable adjustment of R and r on continuous current, the voltmeter may be made direct reading. The instantaneous values at any point on the wave are obtained by shifting the brushes around the shaft. The switch is closed at 1 for voltage measurements and at 2 for current measurements.*

The General Electric **wave meter** operates on this principle. The driving motor is an eight-pole synchronous motor connected to the source being measured and there are eight segments (one per pole), instead of only one. Suitable provision is made for tracing the wave form on a photographic plate.

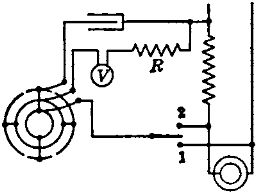

FIG. 98.—Wave-form measurements, "Step-by-step" method.

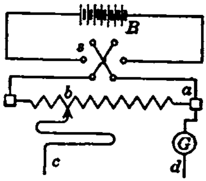

FIG. 99.—Wave-form measurements, zero method.

269. In the zero method shown in Fig. 99, the e.m.f. of a battery B is opposed to the potential across c and d, which are connected to the contact devices described above. The contact point, b, is adjusted until G shows no deflection; then the length ba is a measure of the e.m.f. G may be a portable galvanometer, or a telephone in conjunction with a slide wire and a contact stylus as used in the Sage ohmmeter (Par. **189**).

270. The wave form of a high-tension wave may be obtained by using the device shown in Fig. 102 and described in Par. **277**. The indication of the electrostatic voltmeter is obtained at different angular positions of the brushes on the synchronous commutator, from which the wave form may be plotted.

271. The oscillograph is a form of galvanometer in which the natural period of the moving system is so small that the deflections will always be proportional to the instantaneous value of the current flowing through the coil. The indicator is a beam of light from an arc lamp, reflected from an extremely small mirror attached to the moving system. The path of the beam is determined visually or photographically. Recurrent or periodic waves may be rendered stationary and therefore visible by suitable optical systems as indicated below. Transient phenomena must be photographed by an instantaneous process.

272. In the moving-iron type of oscillograph, first proposed by

* See also Frederick Bedell. "Condenser Current Method for the Determination of Alternating Wave Form," *Electrical World*, 1913, Vol. LXII, p. 378.

reduce iron losses, excitation currents, charging currents, etc., to a minimum. The present A. I. E. E. standard method of measuring the deviation from a sine curve is simply to measure the greatest difference between corresponding ordinates at any point along the axis of abscissæ. It has been proposed* to replace this method by another in which the increase in condenser charging-current with the given wave, over the charging current with a pure sine-wave, will be determined. This is a simple and sensitive method of determining the presence of harmonics, since the charging current of a condenser varies with the square of the frequency. The current for a given condenser on a sine-wave can be calculated from the relation $I = EC \times 2\pi f$, where I = current in amperes, E = potential in volts, C = capacity in microfarads and f = cycles per second.

277. In high-voltage testing it is particularly important that the value of the maximum instantaneous voltage be available, because the stress to which insulation is subjected depends upon this value. When the wave is distorted, the maximum value and the ratio of the maximum to mean-effective (amplitude or peak factor) have to be obtained from a plot of the wave. This may be taken with a wave meter or with an oscillograph, preferably connected to a test coil in the high-tension winding or to the secondary of a shunt-type instrument transformer connected to the high-tension circuit.

A method which can be used directly on the highest voltages† is indicated in Fig. 102, where C is a series of condensers connected across the high-tension circuit. The end condenser C_1 is grounded and connected in parallel with a commutator driven by a synchronous motor. The bars on the commutator are very narrow and are one polar space apart, so that contact is practically instantaneous. The brushes are shifted until the voltmeter indication is a maximum. The multiplying ratio is obtained from the ratio $(C+C_1)/C_1$. The instrument may be calibrated by noting the indication corresponding to the mean effective value, which is obtained with the motor stationary and the brushes resting on the bars. This indication is compared with a spark gap or other standard. The method assumes that the capacity of V is negligible in comparison with C_1. Obviously the condensers can be dispensed with by connecting directly to a test coil on the high-tension winding.

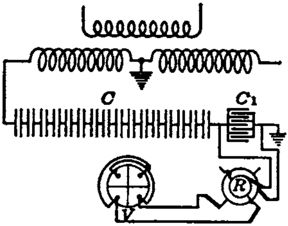

Fig. 102.—Wave meter for high voltage circuits.

FREQUENCY MEASUREMENTS

278. General. The frequency of an alternating current is $f = nr/2$, where f = frequency in cycles per second, n = number of poles, and r = revolutions per second. It may therefore be determined by measuring the speed of the generator supplying the circuit or the speed of a synchronous motor operated from the circuit.

279. Frequency meters indicate the frequency directly. In the **reed type** as made by Hartman and Braun or Siemens and Halske, there are numerous steel strips of different lengths, each rigidly fastened at one end and free to vibrate at the other. The strips are placed in the field of an electromagnet which is ener-

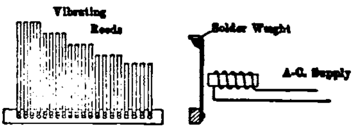

Fig. 103.—Frequency meters—Reed type.

* Davis, C. M. "A Proposed Wave-shape Standard;" *Proc.* A. I. E. E., Feb., 1913, p. 235.

† Sharp, C. H. and Farmer, F. M. "Measurements of Maximum Values in High-voltage Testing;" *Trans.* A. I. E. E., 1912, Vol. XXXI, p. 1617.

which the principle of resonance (Sec. 2) in an electrical circuit is employed.* In a 60-cycle instrument, one main circuit is adjusted for resonance at about 70 cycles, another at about 58 cycles and the third circuit at about 36 cycles. The latter two are connected in parallel, and then in series with coil A; the first circuit is in series with coil A^1, both coils being in series with the field F. With the centre of a 6-in. (15 cm.) scale marked for 60 cycles, half-scale deflection is obtained for a variation of only 5 cycles either way. It is possible to adjust the instrument for a full-scale range of only 1 cycle.

SLIP MEASUREMENTS

283. General. The **slip** of a rotating alternating-current machine in per cent. is the difference between its speed and the synchronous speed, divided by the synchronous speed. It may be determined by noting the difference between the measured speed of the machine and the synchronous speed as calculated from the frequency and the number of poles. This method is obviously not accurate, because the result is a small difference between two relatively large quantities. It is therefore customary to measure the slip directly.

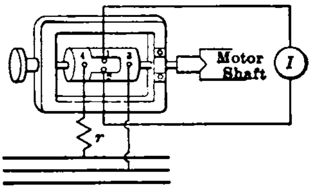

Fig. 106.—Slip measuring device.

284. Dooley's method of measuring slip. One form of device for indicating the slip of an induction motor† is shown diagrammatically in Fig. 106. A small cylinder made of conducting material, and in two parts, each insulated from the other, is mounted in a frame. Four small brushes, 1, 2, 3, and 4, bear upon the cylinder as shown. The brushes, 3, 4, are connected through a resistance, r, across one phase of the supply circuit and the brushes, 1, 2, are connected to a low-reading continuous-current ammeter, I. Each time the brushes, 1, 2, bridge the insulating strip as the cylinder rotates, the circuit is completed in alternate directions through the ammeter. The cylinder should have as many segments as the motor has poles. The ammeter will indicate a constant current at synchronous speed, and an oscillating current for any speed above or below synchronism, because the impulses of current through the brushes, 1, 2, will occur at the same point on the wave at synchronous speed, and at constantly advancing or retarding points for other speeds. Thus, the ammeter will be reversed each time the motor loses one-half of a cycle, and will reach a maximum positive value each time the motor loses one complete cycle. If the motor loses n cycles per min., then the slip in per cent. $= 100n/60f$, where $f =$ frequency of the system in cycles per sec.

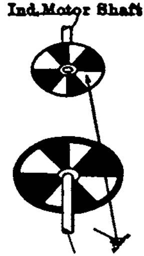

Fig. 107.—Slip measurements — Stroboscopic method.

285. Stroboscopic method. The device indicated in Fig. 107 does not require the measurement of frequency. A black disc with white sectors, equal in number to the number of poles of the induction motor, is attached with wax to the induction-motor shaft. It is observed through another disc having an equal number of sector-shaped slits and carried on the shaft of a small self-starting synchronous motor, in turn fitted with a revolution counter which can be thrown in and out of gear at will. If n is the number of passages of the sectors, then $(n/n_s)/n_r =$ slip in terms of s where $n_s =$ the number of sectors, and $n_r =$ the number of revolutions recorded by the counter during the interval of observation. For large values of slip the observations can be simplified by using only one sector ($n_s = 1$); then $n =$ the slip in revolutions.

286. A direct-reading slip-measuring device is shown in Fig. 108.

* Pratt W. H. and Price D. R. "Resonant Circuit Frequency Indicator;" *Trans*. A. I. E. E., 1912, Vol. XXXI, p. 1595.

† Dooley, C. R. *Elec. Club Journal*, 1904, Vol. I, p. 590.

289. The principle of the Westinghouse synchroniser is shown in Fig. 110, where a rotating field is produced by the coils, *M* and *N*, connected to the buses through the reactance *P* and the resistance *Q*, respectively. An iron vane *A*, free to rotate, is mounted in this rotating field and magnetized by the coil *C*, which in turn is connected across the incoming machine. As the vane is attracted or repelled by the rotating field from *M* and *N*, it will take up a position where this field is zero at the same instant that the field from *C* is zero. Hence the position at any instant indicates the difference in phase. When the two frequencies are different, this position is constantly changing and the pointer will rotate "fast" or "slow," coming to rest at the zero-field position when the frequencies are equal. In a larger type, the split-phase winding is placed on the movable member, similar to the arrangement shown in Fig. 111.

290. The scheme of the Weston synchroscope is shown in Fig. 112. There is no iron in

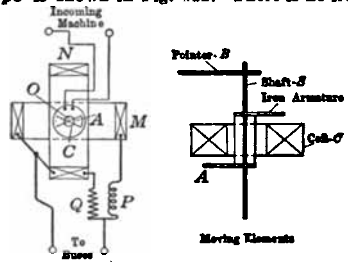

FIG. 110.—Circuits in Westinghouse synchroniser.

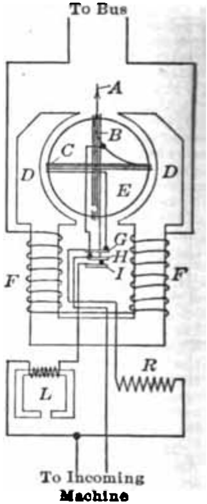

FIG. 111.—Circuits in G. E. synchroscope.

the instrument and the moving element is not allowed to rotate. The elements are practically the same as in an electrodynamometer wattmeter. The fixed coils, *F*, *F*, are connected in series with the resistance *R* and to the buses. The moving coil, *M*, is connected in series with a condenser *C* and the incoming machine. The two circuits are adjusted to exactly 90 deg.

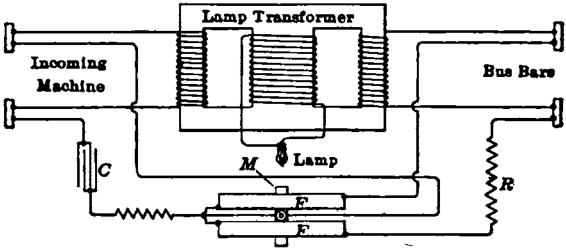

FIG. 112.—Circuits in Weston synchroscope.

difference in phase. At synchronism there is no torque and *M* is held at the zero position by the control spring. If the frequencies are the same, but there is a phase difference, a torque will be exerted and *M* will move to a position of balance at the right or left ("fast" or "slow"). If the frequencies are different, the torque will continually vary and the pointer will oscillate over the dial. A synchronizing lamp illuminates the scale simultaneously and the direction of apparent rotation indicates the faster machine.

where K = moment of inertia computed from the mass and dimensions, M = magnetic moment, and T = period of oscillation. M may be determine with a magnetometer, or by calibration in a known field (Helmholtz coil This method is only suitable for weak fields, such as the earth's field. mounted in a wooden box with a glass front, it will be protected from a currents and can be conveniently moved about when making magneti surveys.

300. Bismuth-spiral method. The resistance of bismuth wire increase when placed in a magnetic field. This property is utilised by noting th increase in resistance of a flat spiral coil of bismuth wire when placed in th field to be measured, the leading-in wires being arranged non-inductivel The device is **calibrated** with known field strengths. It is particularl suitable for exploring small air gaps such as those in motors and generators.

301. Measurement of magnetic properties. The magnetic propertie of iron and steel which are of most commercial importance are **norma induction** or **permeability**, **hysteresis loss** and **total losses** with alte nating magnetising forces of commercial frequencies.

302. Normal-induction data. The various methods are distinguishe principally by the method employed to measure $\mathfrak{B}$, for in all methods $\mathfrak{H}$ i determined from the magnetising coil. $\mathfrak{B}$ can be measured directly as in th **ballistic methods** or indirectly with **permeameters.**

303. Ballistic methods are usually employed in the more accurate meas urements. The best-known methods are the **ring method**, the **divide bar** or Hopkinson, and the **double-bar double-yoke methods.** In all o these methods the flux is measured with a ballistic galvanometer connecte to a test coil which is cut by the flux when the exciting current is reversed.

304. The ring method, devised by Rowland, is one of the earlies methods of measuring the permeability and the hysteresis of iron and steel The connections are shown diagram matically in Fig. 113, where T is th test specimen. The latter is an annula ring, either solid or built up of punch ings of sheet metal, with a diamete preferably 8 or 10 times the radia thickness. After covering with a thi layer of insulation, a test coil of ver fine double-silk-covered wire is woun on a portion of the ring. The mag netising coil is wound over the tes coil, and distributed uniformly over th entire ring; it is usually comprised o double-cotton-covered wire, of suffi cient size to carry the maximum cur rent without raising the temperature o the iron appreciably.

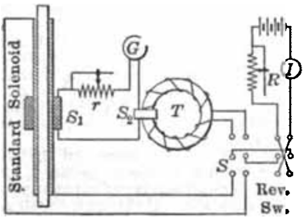

FIG. 113.—Permeability tests—ring method.

305. The divided-bar method devised by Hopkinson avoids the neces sity of winding each specimen separately, permits the use of a more con venient test piece, and avoids the errors in ring specimens.* The devic consists of a test piece, BC (Fig. 114), in the form of a bar about 15 in. (38.1 cm.) long and 0.5 in. (1.27 cm.) diameter, which is divided at A and inserted in a massive frame, F. The secondary coil, S, is so arranged that it will be thrown clear of the yoke by a spring when the part, AB, of the test bar is suddenly withdrawn. In calculating $\mathfrak{H}$, the length of the magnetic circuit is taken as that between the inside faces of the yoke, the reluctance of the yoke being considered negligible. This introduces an indeterminable

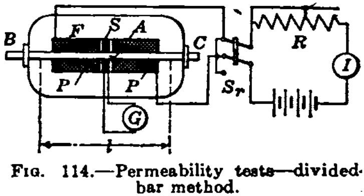

FIG. 114.—Permeability tests—divided-bar method.

* Lloyd, M. G. "Errors in Magnetic Testing with Ring Specimens;" Bureau of Standards, Bulletin, 1908, Vol. V, No. 3; p. 435.

307. Procedure in double-bar double-yoke method. The meth of procedure is as follows: After demagnetizing (Par. 321), the current N_t is adjusted to the value of $\mathcal{H}$ required. The current in all magnetizi coils is then simultaneously reversed several times to get the specimen ir cyclic condition, the current in N_a and N_j being adjusted during the proc until the flux is uniform as indicated by zero deflection when n_a and n_j successively opposed to n_t. The galvanometer is connected to n and t deflection noted when the current in the various magnetizing coils is versed simultaneously. Then

$$\mathcal{H} = \frac{4\pi NI}{10\,l} \qquad \text{(gilberts per cm.)} \qquad (3$$

$$\mathcal{B} = \frac{10^8 dkR}{2an} - \left(\frac{a-A}{A}\right)\mathcal{H} \qquad \text{(gausses)} \qquad (3$$

The units are the same as in Par. 305. The quantity in the parenthesis the correction factor for the space between the surface of the bar and t test coil. A = area of bar and a = area of test coil. Ordinarily this corre tion is very small because the brass tube is made very thin and the te coil is wound under the magnetizing coil.

308. Permeameters are commercial instruments for the rapid testi of iron and steel for permeability. The Thompson permeameter emplo the tractive force exerted between the pole of a magnetized bar and a pie of steel in direct contact with the pole. This force in dynes is $F = \mathcal{B}^2 a/8$ where $\mathcal{B}$ = induction in bar in lines per square centimeter and a = area bar in square centimeters. Koepsel and Picout permeameters are indu tion-type instruments.

309. S. P. Thompson's permeameter is shown schematically in Fi 117, where AB is the test specimen in the form of a rod which passes throu a hole in the top of a heavy yoke, F. The surfaces of the end of the rod ar the yoke at F are carefully machined. The force necessary to move the ro is measured with a spring balance at S. The induction is

$$\mathcal{B} = 156.9\sqrt{\frac{P}{a}} + \mathcal{H} \qquad \text{(gausses)} \qquad (4$$

where $\mathcal{B}$ = induction in lines per square centimeter (gausses) P = pull in gran and a = area in square centimeters.

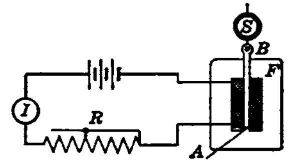

FIG. 117.—Thompson permeameter.

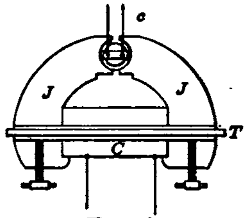

FIG. 118.—Koepsel permeamete

310. The Koepsel permeameter, as made by Siemens Halske, shown schematically in Fig. 118, where JJ is a massive yoke divided at th centre so as to admit a moving coil c to which a pointer is attached, th arrangement being similar to standard D'Arsonval-type direct-current instru ments. This pointer moves over a scale graduated directly in lines per squa centimeter. The magnetic circuit is completed through the test piece firmly clamped between the ends of the yoke in the usual manner. The valu of $\mathcal{B}$ corresponding to various magnetizing currents in C is indicated directl by the deflection with a known small current through c. Separate coils a placed on the yoke pieces, JJ, by means of which the reluctance of the variou gaps is approximately compensated. But even with these coils, there is flux leakage, so that correction or "shearing" curves have to be used. The curves, obtained by test with standardized specimens, are furnished with th instrument, as are also standardized test specimens with which the devic can be checked from time to time.

expense in preparing the sample, as well as the time spent in testing. The specimen is a bundle of strips 0.5 in. (1.27 cm.) × 10 in. (25.4 cm.), weighing 1 lb. (0.45 kg.). It is placed in a simple straight solenoid, with a sensitive wattmeter (reflecting electrodynamometer) in series with the magnetising coil. A separate winding is provided for the wattmeter potential coil, thus simplifying the correction factor for wattmeter loss. The induction is determined from the indication of a voltmeter, which is also connected to a separate winding at the centre of the specimen. The flux which it indicates is much higher than that at the ends, but experiment has shown that the ratio of the maximum to the average is 1.3 to 1. Due allowance is therefore made, when adjusting the magnetising current to such a value that the voltmeter deflection will correspond to the required average flux-density. Measurements are made at 10 cycles, or less, in order to reduce the eddy-current loss to a point where it can be eliminated by means of empirical corrections without serious error. The precision obtained is about ±5 per cent.

317. In the Holden and the Ewing hysteresis meters, the loss is determined mechanically. In the Holden meter the test specimen, a ring of laminations about 1 cm. × 2 cm. (0.4×0.79 in.) cross-section, and 9 cm. (3.55 in.) diameter, is placed between the poles of a pair of revolving magnets. The torque exerted on the specimen is resisted by a spiral spring. The deflection of this spring which is necessary to bring the specimen back to the zero position is a measure of the loss in ergs per cycle.

The **Ewing apparatus** is operated on a similar principle, except that the specimen is rotated instead of the magnets. The specimen is a bundle of strip ⅝ in. (1.6 cm.) square and 3 in. (7.6 cm.) long.

318. Core-loss measurements. The total loss in iron or steel subjected to an alternating magnetic field is most accurately measured with a wattmeter. In making precision measurements, the Hopkinson ring-specimen can be used, but the Epstein apparatus is more convenient and has been adopted by the American Society for Testing Materials.* Fig. 120 shows the scheme diagrammatically.

The specimen is arranged in the form of a rectangle. The magnetising winding is divided into four solenoids, each being wound on a form into which one side of the rectangular specimen is placed. The form is non-magnetic, non-conducting and has the following dimensions: inside cross-section, 4 cm. (1.57 in.) × 4 cm.; thickness of wall not over 0.3 cm. (0.12 in.); winding length, 42 cm. (16.5 in.). Each limb of the specimen consists of 2.5 kg. (5.5 lb.) of strips 3 cm. (1.18 in.) wide and 50 cm. (19.7 in.) long. Two of the bundles are made up of strips cut in the direction of rolling and two at right angles to the direction of rolling. The strips are held together with tape wound tightly around the bundle. The bundles form butt joints at the corners with tough paper 0.01 cm. (0.004 in.) thick between. They are held firmly in position by clamps placed at the corners.

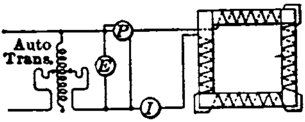

Fig. 120.—Core-loss measurements —Epstein method.

The magnetising winding on each solenoid consists of 150 turns uniformly distributed over the 42 cm. (16.5 in.) winding-length, and has a resistance of between 0.075 and 0.125 ohm. A secondary winding is uniformly wound underneath the first; it also contains 150 turns in each solenoid, and energises the potential circuit of the wattmeter and also the voltmeter with which the induction is measured. The resistance should not exceed 0.25 ohm per solenoid. With a sine-wave e.m.f. impressed on the magnetising winding, the maximum induction is

$$\mathfrak{B} = \frac{E\;4\;lD\;10^8}{4\;fN\;nM} \qquad \text{(gausses)} \qquad (41)$$

where E = volts indicated by voltmeter, l = length of specimen, D = specific gravity (7.5 for alloy or high-resistance steels and 7.7 for standard or low-

* Standard Magnetic Tests of Iron and Steel; *Trans.* A. S. T. M., 1911; Vol. XI; p. 110.

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MECHANICAL POWER MEASUREMENTS

BY F. MALCOLM FARMER, M.E.

TORQUE MEASUREMENTS

323. Torque is best measured with dynamometers, of which there are two classes, absorption and transmission.* Absorption dynamometers absorb the total power delivered by the machine being tested, while transmission dynamometers absorb only that part represented by friction in the dynamometer itself.

324. The Prony brake is the most common type of **absorption dynamometer.** It is simply a brake applied to the surface of a pulley on the shaft of the machine being tested, together with suitable means for vary-

* Carpenter and Diedrichs. "Experimental Engineering" (Wiley Sons, 1912). J. A. Moyer. "Power Plant Testing" (McGraw-Hill Book Co. Inc., 1913).

rope. If F and L are measured in pounds and feet respectively, T will in lb. ft.

328. Dissipation of heat in friction brakes. The energy dissipated the brake appears in the form of heat. In small brakes natural cooling sufficient, but in large brakes special provisions have to be made to dissip the heat. Water-cooling is the most common method, one scheme e ploying a flanged pulley. About 100 sq. in. of rubbing surface of bra should be allowed with air cooling, or about 50 sq. in. with water cooli per horse-power.

329. For very large torques, other forms of absorption brakes a used. In the **Alden brake**, a rotating cast-iron disc rubs against th copper discs which are held stationary. The friction is adjusted by varyi the pressure of the cooling water in the chamber surrounding the copp discs. The tendency of the copper disc member to rotate is measured w a lever as in the Prony brake.

The **Westinghouse turbine brake** employs the principle of the wa turbine and is capable of absorbing several thousand horse-power at ve high speeds.

In the **magnetic brake**, a metallic disc on the shaft of the machine bei tested is rotated between the poles of magnets mounted on a yoke which free to move. The pull due to the eddy currents induced in the discs is me ured in the usual manner by counteracting the tendency of the yoke revolve.

330. The principal forms of transmission dynamometers are t lever, the torsion and the cradle types. An example of the **lever type**

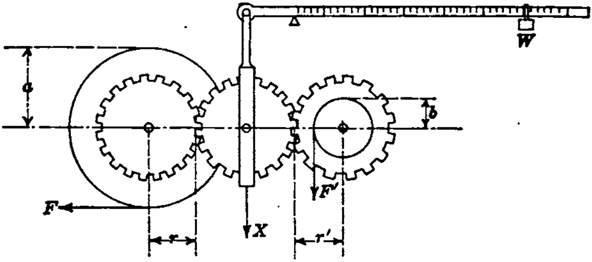

Fig. 125.—Transmission dynamometer.

shown in Fig. 125, where F is the force applied to the dynamometer, ar F' is the force being delivered. When the downward force, X, is balance by the weight W, the following formula holds,

$$F = \frac{1/2(1+\mu)Xr}{a} \qquad (4$$

where μ = coefficient of friction determined experimentally.

331. In torsion dynamometers, the deflection of a shaft or spiral sprin which mechanically connects the driving and driven machines, is used measure the torque. The spring or shaft can be calibrated statically b noting the angular twist corresponding to a known weight at the end of known lever-arm perpendicular to the axis. When in use the angle can measured by various electrical and optical methods. In one method th angular displacement between two points on the shaft is determined by mean of two discs of insulating material, in the periphery of each of which is a very thin piece of metal. The two pieces of metal are connected electri ally through the shaft. A light, thin metal brush rests on the periphe of each disc, and the two brushes are connected together through a batte and an indicator such as a bell, or a telephone. At no load, one brush moved until the electrical circuit is completed once every revolution. Th angle through which either brush has to be moved as the load is increased, i order to keep the circuit closed, is then measured.

temperature should be expressed in some one standard scale such as the thermodynamic or ideal gas scale. The hydrogen and nitrogen gas thermometers differ from this scale by various small amounts. The scale defined by the **radiation laws** is the thermodynamic scale, assuming these laws to have a sound theoretical basis. For the past 25 years it has been possible to express temperatures to 0.002 deg. cent. in the range 0 to 100 deg. cent., but outside of this interval the accuracy is far from being what is desired. Only recently for example, has the certainty of the sulfur boiling-point been closer than 0.5 deg. and of the gold melting-point 5 deg. cent. At the present time the International Bureau and the several national laboratories are interchanging communications with the object of establishing an international temperature scale.

337. Mercurial thermometers. Primary mercury thermometers are constructed of the very best thermometric glasses such as verre dur, Jena 16^{III}, Jena 59^{III}, and the scales defined by the differential expansion of these glasses and the mercury have been compared with the gas thermometer so that their corrections are fairly well established. (See Circular No. 8 of the Bureau of Standards.)

338. Laboratory and industrial mercury-in-glass thermometers (range − 35 to + 560 deg. cent.) are calibrated by direct comparison with standards. Two points which frequently occasion trouble in the use of mercury thermometers are (a) **correction for emergent stem,** (b) correction for **thermometric lag.**

339. Correction for emergent stem. In general the calibration corrections are determined for total immersion of the thermometer. When used with the stem emergent into space either hotter or colder than the temperature of the bulb, a stem correction must be applied to the observed reading of the thermometer in addition to the calibration correction. This stem-correction may amount to more than 20 deg. cent. for measurements made with a mercurial thermometer at 400 deg. cent. (750 deg. fahr.). The stem correction may be computed from the formula: Stem Correction $= K \times n(T° - t°)$ where K = factor for relative expansion of mercury in glass; 0.00015 to 0.00016 for centigrade thermometers, 0.000083 to 0.000089 for fahrenheit thermometers; n = number of degrees emergent from the bath; T = temperature of bath; t = mean temperature of emergent stem.

Example: Suppose that the observed temperature was 100 deg. cent. and the thermometer was immersed to the 20-deg. mark on the scale, so that 80 deg. of the mercury column projected out into the air, and the mean temperature of the emergent column was found to be 25 deg. cent.; then— stem cor. $= 0.00015 \times 80 \times (100 - 25) = 0.9$ deg. cent. As the stem was at a lower temperature than the bulb, the thermometer read too low, so that this correction must be added making the correct temp. = 100.9 deg. cent. The mean temperature of the emergent stem may be approximately measured by a small auxiliary thermometer, by a Faden thermometer, or by surrounding the stem with a water jacket and observing the temperature of this bath.

340. Correction for thermometric lag (Bur. of Standards, Reprint No. 171). When a thermometer is immersed in any medium it does not take up the temperature of the medium immediately, but approaches it asymptotically. This effect may be minimized by stirring the bath. With vigorous stirring in the case of a liquid bath, the thermometer reading should be correct to within 1 per cent. of the original difference in temperature of the thermometer and bath after 10 to 60 sec. exposure. In absolutely quiet air this degree of accuracy might require 20 min. Fanning the thermometer might reduce the time to 1 min. With caution, corrections for lag may be neglected in ordinary laboratory work.

341. High-temperature thermometers. Although mercury boils at 357 deg. cent. under atmospheric pressure, by filling the space above the mercury with CO_2 or N_2 under sufficient pressure, certain mercury-in-glass thermometers may be used at a maximum temperature of about 560 deg. cent. Glasses used are Jena 16^{III} to 450 deg. cent., Jena 59^{III} to 520 deg. cent., and special grades of combustion tubing to 560 deg. cent. Care must be exercised that the thermometer is not overheated. If the long portion of the stem is cold, the stem correction may amount to 40 deg. cent. and hence while the mercury stood at 500 deg. cent. the true temperature of the bulb would be 540 deg. cent. A few moments at that tempera-

$$E = \frac{E'(R+r+r')}{R} \tag{45}$$

where R, r, r' are the resistances of the galvanometer, thermocouple and leads respectively.

349. Cold-junction corrections. If a thermocouple is calibrated with cold junctions at 0 deg. cent. and used with cold junctions at t_0 deg. cent., one must add to the e.m.f. actually developed, the value of the e.m.f. developed when the hot junction is at t_0 deg. cent. and the cold junctions are 0 deg. cent., to obtain the correct value of the e.m.f. corresponding to temperatures shown by the calibration. If the indicator is graduated to read temperature directly, instead of e.m.f., the cold-junction correction has the form p deg. cent. = Factor $\times t_0$. For the LeChatelier couple this factor is about 0.6 in the range 300 to 700 deg. cent. and 0.5 from 700 to 1,400 deg. cent. Base-metal couples show factors varying from 0.2 to 1.2 depending upon the particular alloy used.

As an example, suppose the indicated temperature (cold junction = 40 deg. cent.; calibration temperature = 0 deg. cent.) by the LeChatelier couple is 1,000 deg. cent. Then $p = 0.5 \times 40 = 20$ deg. True temp. = 1,000 + 20 = 1,020 deg. cent.

350. Certain precautions should be taken in the use of **thermocouples.** The hot junction is usually formed by welding the dissimilar metals in an oxyhydrogen flame. Other junctions may be soldered or thoroughly secured with binding screws. The entire couple should be annealed at as high a temperature as it will safely stand in order to render it as homogeneous as possible. The couple must be protected from furnace vapors or direct contact with liquid baths, and the two leads must be insulated from each other.

351. Electrical resistance pyrometry. This method of high-temperature measurement ordinarily makes use of the variation in the electrical resistance of platinum and is capable of great sensibility. In one of its simplest forms the pyrometer consists of a coil of platinum wire wound on mica, and encased in a protecting tube of porcelain. On account of the distillation of platinum, high-resistance coils of small wire are not used much above 900 deg. cent. However, coils constructed of 0.6-mm. wire may serve satisfactorily to 1,200 deg. cent.

352. Three-lead type—Wheatstone bridge method. For the purpose of eliminating the resistance of the leads to the coil, a third wire is

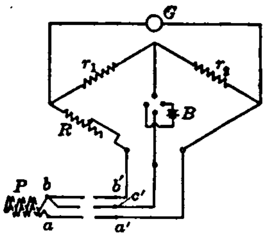

Fig. 126.—Three-lead resistance thermometer.

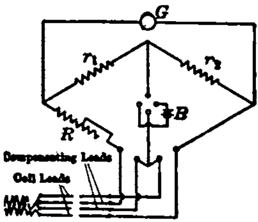

Fig. 127.—Four-lead resistance thermometer.

frequently introduced as in Fig. 126. The coil P forms one arm of a dia type bridge, of which the others are r_1, r_2 and R, whence from the princip of the bridge, if the galvanometer G remains undeflected,

$$P = \frac{r_2(R + bb')}{r_1} - aa' \qquad (4$$

r_2 is usually made equal to r_1 and bb' is constructed as nearly as possib identical with aa', so under these circumstances $P = R$ regardless of the te imetry, to the determination of freezing-points, etc., and to special physical and thermochemical investigations where an accuracy of one or two parts in 10,000 may be attained (see Bur. of Standards, Reprint No. 68 and No. 124). In the technical industries this type of thermometer with one of the many forms of indicators available is highly satisfactory.

358. Radiation. The temperature of bodies may be estimated from the radiant energy which they send out in the form of visible light or of the longer infra-red rays which may be detected by their thermal effects. Since the intensity of radiation increases very rapidly with a rise in temperature, it would appear that a system of pyrometry based on the intensity of the light or total radiation from a hot body would be an ideal and simple one. However, different substances at the same temperature show vastly different intensities at a given wave length, or in other words, the **absorbing or emissive powers** may vary with the substance, with the wave length, and also with the temperature.

359. Black-body radiation. A substance which absorbs all the radiation of any wave length falling upon it is known as a **black body.** Such a body will emit the maximum intensity of radiation for any given temperature and wave length. No such material exists, but a very close approximation is obtained by heating the walls of a hollow opaque enclosure as uniformly as possible and observing the radiation coming from the inside through a very small opening in the wall.

360. Stefan-Boltzmann law. The relation between the total energy radiated by a black body and its temperature is expressed by the equation $J = \sigma(T^4 - T_0^4)$, where J is the energy of all wave lengths emitted per square centimeter of surface, T and T_0 the absolute temperatures of the radiator and receiver respectively, and σ a constant of about the value 5.8×10^{-12} watts cm.$^{-2}$ deg.$^{-4}$. In general T_0^4 is negligible in comparison with T^4 so that the above relation becomes $J = \sigma T^4$. Although the total energy emitted by any substance is not that emitted by a black body at the same temperature, it may be considered as some fractional part of that from the ideal radiator, this fraction ϵ being known as the total emissivity. If S denotes the apparent absolute temperature, *i.e.*, the temperature on the black-body scale corresponding to an amount of energy equivalent to that emitted by the non-black substance at a true temperature T deg. absolute, the relation between its total emissivity ϵ and the quantities S and T is:

$$\text{Log } \epsilon = 4(\log S - \log T) \tag{50}$$

361. Radiation pyrometry. The quantity of heat a body receives by radiation from another body depends upon certain conditions relative to each of the two bodies, namely (a) temperature, (b) area of surface, (c) distance apart, (d) emissive and absorbing powers. A pyrometer may be so constructed that conditions (b) and (c) compensate one another, at least within certain prescribed limits, so that for all technical purposes the radiation received by the instrument depends only upon the temperature of the radiating source and its emissivity. The pyrometer is calibrated by sighting upon a black body, the temperature of which may be obtained by thermocouples. Specially constructed furnaces for this purpose are available in all testing laboratories.

362. Féry mirror telescope pyrometer. (Fig. 129.) Radiation of all wave lengths is brought to a focus by means of a concave gold mirror M upon the hot junction of a minute thermocouple located at T. The cold junctions of the couple are suitably screened from the direct radiation of the hot body. The concentration of heat at the hot junction develops an e.m.f. which may be measured by a potentiometer or galvanometer. In practice the galvanometer is usually calibrated to read temperature directly. The relation between the e.m.f. and the temperature may be expressed by the equation $E = aT^b$, or in log form, $\log E = k + b \log T$, where T is the absolute tempera-

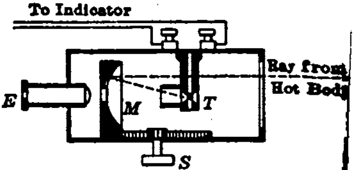

FIG. 129.—Féry radiation pyrometer.

inlet and average outlet water temperatures. For liquid fuels, a weighing device and a special burner are provided.

391. Other types of calorimeters of the **discontinuous class** differ essentially from the Berthelot only in the method of supplying the oxygen. In calorimeters of the Carpenter and the Favre and Silberman type, oxygen gas is supplied at atmospheric pressure. In instruments of the Parr class, the oxygen is supplied by chemicals with which the sample is mixed.

392. Fuel oils are, in addition to the foregoing, Par. **390**, frequently tested for **flash-point**, or temperature where the vapor given off will ignite but will not continue to burn; **fire-point**, or temperature where combustion will continue if the vapors are ignited; **viscosity; chill-point**, or congealing temperature; per cent. of asphaltum.

393. Reports of proximate analyses and heating value (in B.t.u. per pound) usually give the results calculated on at least two bases, "as received" and "**dry**." The former are of most interest to the users of the fuel, but the results must be reduced to the latter basis when comparisons are to be made.

394. Fuel or illuminating gases are analyzed for the following **components in per cent. by volume; carbon dioxide** (CO_2), **carbon monoxide** (CO), **oxygen** (O_2), **methane** (CH_4), **ethylene** (C_2H_4), **hydrogen** (H_2) and **nitrogen** (N). CO, CO_2, O_2 and C_2H_4 are usually determined by passing a known volume of the gas through a series of reagents, one at a time, each of which will absorb one, and only one, of the components. The diminution of the volume is noted after each absorption. H_2 and CH_4 are obtained by combustion in a glass tube with a known volume of air, the products of combustion being measured by absorption as in the case of the other constituents, and the original volume calculated. N is obtained by difference.

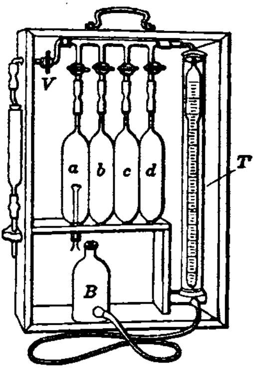

Fig. 135.—Orsat apparatus.

395. Orsat apparatus. The various forms of apparatus which employ the **absorption method** are based on the principle of the Orsat apparatus shown in Fig. 135. A given quantity of gas, usually 100 c.c., is drawn into the measuring tube, *T*, by means of the water bottle, *B*, and carefully measured. The gas is then forced into the CO_2 reagent bottle, *d*, drawn back into *T* and the decrease in volume noted. The process is repeated with each of the tubes, *c*, *b* and *a*, giving the percentages of O_2, CO, and H_2 respectively. The usual **reagents** are caustic potash solution for CO_2, ammoniacal cuprous chloride solution for CO and alkaline pyrogallic acid solution for O_2, H_2 being obtained by combustion.

396. Flue gases are analyzed for **carbon dioxide** (CO_2), **carbon monoxide** (CO), **oxygen** (O_2), **hydrogen** (H_2) and **nitrogen** (N), in the manner indicated for fuel or illuminating gas.

397. CO_2 recorders are instruments which automatically and continuously remove samples of flue gas and indicate with a pointer or record on a clock-driven chart the percentage of CO_2 in each sample. Various principles are employed, among which are the variation in the refraction index with the percentage of CO_2, the variation in density compared with air as a standard, and the variation in the position of a float with the volume remaining after the CO_2 has been removed with caustic potash, the **usual reagent**.

398. Selected list of reference literature on fuel and gas analysis.

Lewes, V. B.—"Liquid and Gaseous Fuels." D. Van Nostrand Co., New York.

Gill, A. H.—"Gas and Fuel Analysis for Engineers." John Wiley & Sons, New York.

Kershaw, J. B. C.—"The Calorific Value of Fuels." D. Van Nostrand Co., New York.

406. The Hammond measuring-tank meter, Fig. 138, consists of two tanks, B_1 and B_2, into one of which the inflowing water is directed by the baffle G, while the other is emptying through the valve D_2. When the water level in B_1 rises high enough to lift float I_1, latch H_1 releases, the weight of water on valve D_1 throws the wrist plate over, opening D_1 and closing D_2, and changing the deflector G to fill B_2. The action is very rapid at the release period, preventing loss of water during the change period. A gage, N, is included for accurately setting the meter. This device, as with all the volumetric meters, is affected by change of temperature. For variations of approximately 50 deg. fahr. (28 deg. cent.) the error is not great, the average being 2 per cent. to 3 per cent. It is operated on gravity flow.

407. Disc meters of the general type in Fig. 139, operate by the gyration of a disc in a spherical chamber. The stem attached to the disc describes a circular path and operates the counter. These meters are used on closed lines under pressure.

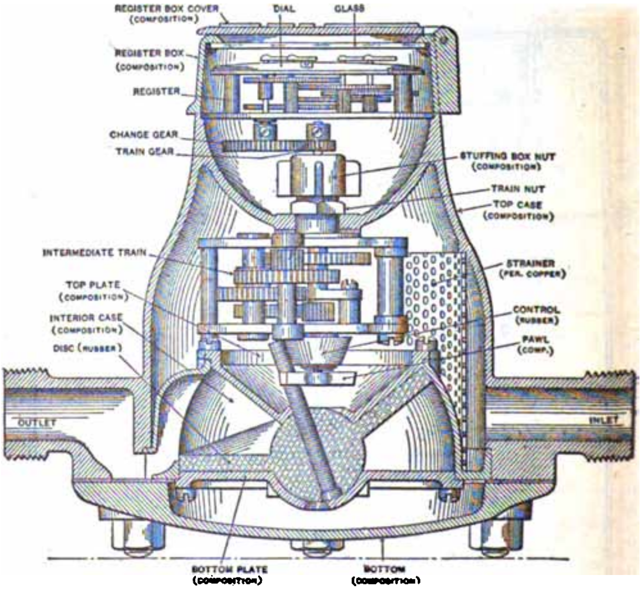

Fig. 139.—Worthington disc meter.

Disc meters vs. piston meters. Disc meters are used chiefly for small lines, up to about 3 in. diameter. Piston meters for sizes from 2 in. to 8 in. For larger flows, tank, Venturi or turbine meters are generally employed.

408. Piston meters of the general type shown in Fig. 140, operate like a duplex steam pump, the movement of the pistons measuring off definite volumes of water per stroke. The strokes are recorded by the counters usually in units of cubic feet. These meters are used on closed lines under pressure, and necessitate, for their operation, a pressure drop of from 2 to 6 lb. per sq. in., depending on the flow.

409. The Venturi meter is widely used, both for large and small flow, on pumping service and boiler feed. It occupies practically no space outside of the pipe line; has no moving parts in the meter proper, and operates on closed pressure lines. The accuracy is from 1.0 to 1.5 per cent. if used on reasonably steady flow. If used on rapidly fluctuating flow it becomes very inaccurate. If kept clean, the accuracy is substantially constant during the life of the meter.

A slight pressure drop, 0.25 to 3 lb., occurs through the meter, depending in size on the flow. For theory of the Venturi tube, see Sec. 10.

410. The Pitot meter is not satisfactory for general service, as the head differences are much less than those developed in the Venturi, consequently the registering apparatus is much more delicate and sensitive to leakage in the pressure lines leading from the main to the recording instrument. It is also very sensitive to eddies in the pipe lines in which the Pitot tube is inserted. For the theory of the Pitot tube, see Sec. 10.

411. The turbine meter, Fig. 141, is used to some extent for large-size lines and large flow. It operates like a hydraulic turbine, and as the meter

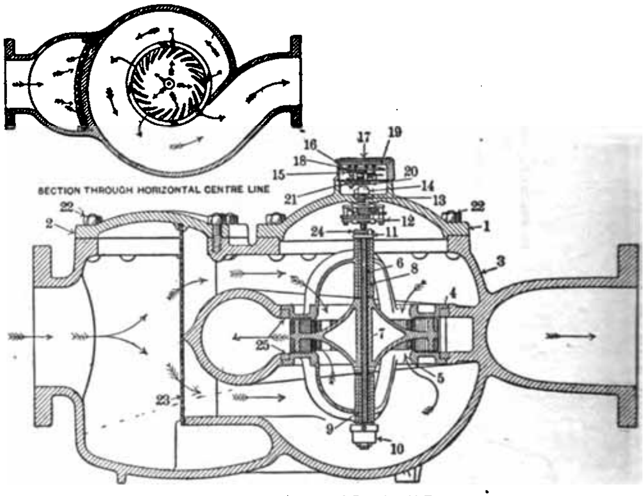

Fig. 141.—Worthington turbine meter.

opposes practically no friction or pressure loss to flow, the speed of the meter is substantially proportional to the flow. It is called a "velocity meter," but strictly speaking, this meter is volumetric; it is affected in accuracy by temperature changes. The accuracy is practically the same as the weighers and tank meters (Par. 400).

412. Weirs, usually of the V-notch type, are in considerable use, in connection with indicating and recording mechanisms for water measurement. (See Sec. 10.) In the **Lea type**, a float in a chamber above the weir, operates a grooved drum in such a fashion that the recording and integrating apparatus move over equal increments of space for equal increments of flow.

In the **Hoppes type**, a conoidal float is suspended by a coil spring, and is so shaped that the descent of the float by the weight of water forced over by the rise of the weir, is proportioned to the flow. Very good accuracy is claimed for these weir meters, from 0.5 to 1.3 per cent. over all ranges of flow. Temperature changes are approximately compensated for in both types, by the behavior of the float and the conoidal chamber respectively.

where Q = cu. ft. per sec. ($= W/\delta_1$); h = difference of pressure, upstream and throat in in. of water $[= (P_1 - P_2)(12/62.35)]$.

418. The Pitot-tube formula for gas and air

$$Q = 218.44Ed^2\frac{T_s}{P_s}\sqrt{\frac{hP}{TG}}\text{(cu. ft. gas or air per hr.)}\quad(62)$$

where E = flow factor of the tube expressed as a decimal; d = internal diam. of tube (in.); T_s = absolute fahr. temperature of measurement base; P = absolute pressure of measurement base, lb. per sq. in.; G = sp. grav. of gas referred to air; if air is measured, $G = 1$; P = absolute static pressure of flowing gas in meter, lb. per sq. in.; T = absolute temperature fahr. of flowing gas; h = velocity head of flowing gas (in. of water); Q = cu. ft. of gas per hr. at T and P.

The value of E is 0.8530 for smooth tubes, 2-in. to 5-in. diameter, with the Pitot tube placed exactly in the centre of the pipe. The velocity is a maximum at the centre of a pipe, decreasing to a minimum at the pipe surface. This accounts for the fact that E is less than unity when the Pitot tube is at the centre of the pipe. The coefficient of flow for the Venturi meter approximates from 0.97 to 0.98 for properly designed meters.

419. Rotary meters of several makes are on the market; one type is shown in Fig. 142, intended for compressed-air service. Air enters the cham-

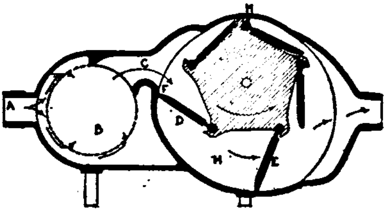

FIG. 142.—Kreutsberg air meter.

ber at C and impels the flap pistons D and E toward the outlet side. As the pistons J, K, and L, are closed on the return side, the pressure area is greatest on the under side, causing the meter to turn.

STEAM METERS

420. Steam meters. Most of the steam-flow meters such as the St. John Sargent, Hallwachs, Gehre, Eckhardt and General Electric, can be used on **compressed-air service** if desired. See Par. 419. Steam meters are divided into area meters (Par. 421 and 422) and velocity meters (Par. 423 and 424). Pressure and quality variation affect the accuracy of all area and velocity meters, so that the meters are only correct for the calibration conditions of pressure and quality, unless fitted with compensating devices.

421. "Area" steam meters. In this class are those in which a disc or cup partially closes an opening through which steam is passing. The shape of the passage or of the cup is so arranged that as it rises from zero position, the free area for passage of steam is increased. As demand for steam is increased, the increase of pressure drop past the disc or cup, causes it to move further up the passage, enlarging the area till the pressure drop is reduced and the disc again in equilibrium. The movement of the disc is communicated to an indicator and chart graduated in lb. per hr. flow. The passage is so designed that the movement of the disc or cup is directly proportional to the flow, giving an equal increment reading.

422. The Sargent meter is an example of the area type. It has a conical cup seating over a conical seat. As the stream flow is increased, the cup rises, exposing more area for flow between cup and seat. As the weight of the cup and stem is the only load, the pressure difference is constant. A pressure compensator in the form of a Bourdon tube carrying the indicating needle, is *e.g.*, the density of a sphere expressed by the relation $d = M/(\frac{1}{6}\pi D^3)$, where M is the mass obtained by means of an equal-arm balance and D the diameter measured by a micrometer caliper; or the horse-power of an engine h.p. = p.l.a.n./33,000, where p is the mean-effective pressure found by measurement, l the length of stroke, a the area of the piston found by measuring its diameter and n the number of strokes per minute found by counting them. In general these expressions take the form,

$$X = F(a, b, c \ . \ . \ . \quad A, B, C \ . \ . \ .)$$

where a b, c . . . represent measured quantities, while A, B, C . . . represent constants (like 33,000 and π in the above formulæ) and F represents that there is some functional relation between measurements, constants and the indirectly measured quantity X.

427. Every measurement has a definite precision. Thus the length of a building might be reliable to the nearest quarter inch, the current in a certain circuit to the nearest tenth of an ampere, or the time of vibration of a pendulum to the nearest hundredth of a second. To determine this reliability it is necessary to make a very careful study of all the instruments used, the care with which they are made, graduated, calibrated, adjusted for change of temperature or of position, etc., etc. It furthermore is necessary to know not only the skill of the observer, but also whether any constant errors may be due to his "personal equation." If for instance one desires to calibrate a voltmeter at 110 volts by means of a standard Weston cell, the following points must be considered: How closely must the electromotive force at standard temperature be determined? How closely must the temperature coefficient be determined? If the temperature of the cell when used is determined by means of a mercury thermometer, how closely must the thermometer be calibrated and read? How closely must the resistance of the voltmeter be known and is it necessary to take any precautions regarding its temperature; and lastly, what must be the precision of the variable resistance used for the balance? After an examination of all these probable sources of error the final question is, What is the most probable value of the combined effect of all these separate deviations or errors? (See Par. 1–6.)

428. The general problem. Given the functional relation,

$$X = F(a, b, c \ . \ . \ .) \tag{63}$$

the problem in general is as follows: First case if a, b, c . . . can be measured each with a definite degree of precision, what is the best representative value of the resultant precision in X? Second case: if it is desired to determine X to a certain definite precision, how precisely must each of the components be measured so that the combined effect of all the deviations may not produce a resultant deviation in X greater than the assigned limit?

Unless it is possible to assign some numerical estimate to the precision attained in determining any measured quantity, X, the result is of little practical value. Hours of valuable time are often wasted in determining some unimportant component of an indirect measurement with excessive precision, while at the other extreme we often find final results absolutely worthless as the result of failure to measure some important component with the necessary precision.

429. Determination of precision in final result with known precision in the measured components. Referring to Eq. 63, let δ_a, δ_b, δ_c . . . be the numerical deviations or precision measures of the direct measurements a, b, c . . . ; Δ_a, Δ_b, Δ_c be the deviations in X due to the deviations in the separate components, and Δ the combined effect of the separate effects Δ_a, Δ_b, Δ_c . . . . Then Δ/X is the fractional deviation or precision measure of X and δ_a/a, δ_b/b, δ_c/c . . . are the fractional deviations of the components a, b, c . . . and 100 times the fractional deviation is the percentage deviation. We first find the separate effect of each of these deviations δ_a, δ_b, δ_c . . . on X and then find the combined effect of these separate effects. The change in X due to a slight change in a (b, c . . . remaining constant) is found by differentiating the function with respect to a, *i.e.*,

$$\Delta_a = \frac{\partial X}{\partial a} \delta a \tag{64}$$

and a similar slight change in any component k would give

$$\Delta_k = \frac{\partial X}{\partial k} \delta k \tag{65}$$

would be very easy to reach this precision in the time measurement if a goo stop watch were used; in fact this interval could be determined to within 0. sec. and therefore the time might be treated as a constant, in which case would be two instead of three and therefore it would not be necessary t measure I and E quite so accurately as indicated above.

431. Two classes of formulæ in indirect measurements. The tw methods shown above for the direct (Par. 428) and the indirect (Par. 42 methods respectively are perfectly general; consequently by their use an problem may be solved provided all deviation or precision measures are ex pressed as actual numerical deviations and not as fractional or percentag deviations. It is found in practice that nearly all formulæ used in indirec measurements fall in one or the other of two general groups. Those of th first class include all those functions which contain sums or differences c terms, each of which may involve either measured components, or con stants, or both. The general form would be

$$X = Aa^l \pm Bb^m \pm Cc^n \pm \ldots \tag{68}$$

where $A, B, C \ldots$ are constants and $a, b, c \ldots$ are measured quantitie entering to the l^{th}, m^{th}, and n^{th} powers respectively. Examples of formulæ o this sort are the expressions for the electromotive force of a standard Clar cell, $E = 1.4340\ [1 - 0.00078\ (t^\circ - 15^\circ)]$, or the resistance of a standard 10 ohm coil, $R = 10[1 + 0.00388\ (t^\circ - 15^\circ)]$.

Those of the second class include all those functions which involv products, quotients or powers of the measured components and constant and do not involve either trigonometric or logarithmetic functions. Th general type would be

$$X = A \cdot a^l \cdot b^m \cdot c^n \ldots \tag{69}$$

where A, n, m, and k are constants (positive, negative, fractional or integral) An example of formulæ of this type is the expression for the modulus of elas ticity for bending, given as $E = Wl^3/4a\,bd^3$, where W is the load at th centre, l is the length between supports, a is the deflection, b the breadth anc d the depth of the beam; the formula given above for density (Par. 426) is another example.

432. Percentage method of computing precision. Problems o the second class (Par. 431) may be solved much more easily by the fractiona or percentage method than by the general method, as will appear from the following. Taking the general form of the function as

$$X = A \cdot a^n \cdot b^m c^k \ldots \tag{70}$$

and differentiating with respect to each of the variables, and then dividing each of these results by the general formula gives,

$$\frac{\Delta a}{X} = \frac{l \cdot \delta a}{a}, \quad \frac{\Delta b}{X} = \frac{m \cdot \delta b}{b}, \quad \frac{\Delta c}{X} = \frac{n \cdot \delta c}{c}. \tag{71}$$

This shows that a fractional deviation $\delta a/a$ in a produces a fractional deviation in X which is n times as great, or in per cent. it means that if a is unreliable by 1 per cent., X will be unreliable n times 1 per cent. and it is to be noted that the remaining factors in the formula have no effect whatever upon this relation. This enables us to state at once the separate effect of any percentage deviation of a given component and by using the relation

$$\frac{\Delta}{X} = \sqrt{\left(\frac{\Delta_a}{X}\right)^2 + \left(\frac{\Delta_b}{X}\right)^2 + \left(\frac{\Delta_c}{X}\right)^2 + \cdots} \tag{72}$$

or its equivalent,

$$\frac{\Delta}{X} = \sqrt{\left(l\frac{\delta a}{a}\right)^2 + \left(m\frac{\delta b}{b}\right)^2 + \left(n\frac{\delta c}{c}\right)^2 + \cdots} \tag{73}$$

we can therefore express the final resultant effect as a percentage deviation.

433. The percentage method is illustrated by the following problem. Measurements for the modulus of elasticity using the above formula are as follows. The weight is 10 kg., reliable to the nearest gram; the length is 1,000 mm., reliable to 0.5 mm.; the deflection is 6.983 mm., reliable to 0.007 mm; the breadth is 4.675 mm., reliable to 0.005 mm.; and the depth is 15.069 mm., reliable to 0.008 mm. The problem is to determine the reliability of the modulus when calculated from the formula $E = W \cdot l^3/4\ a.b.d^3$. The first step is to express all the deviations in per cent. Inspection shows that W

with change of temperature and therefore apply to the case of con- of constant mass, usually met in engineering work. For a full n of this subject see Dellinger, J. H. "The Temperature Coefficient r," Bulletin of the Bureau of Standards, 1911, Vol. VII, No. 1, p. 'Copper Wire Tables," Circular No. 31, 3rd edition, 1914, Bureau of s.

ct of chemical composition. The resistivity of most metals nsitive to slight changes in chemical composition. Particularly is of copper; when alloyed, for example, with 1 per cent. of another increase in resistivity, measured in per cent., is many times 1 per Par. 63 and 64. Therefore it is very essential when stating a value ity for a given substance to state also what the substance is com- or if it be so nearly pure that there are no more than small traces of bstances, to state its percentage of purity.

ct of mechanical treatment. When ductile metals are sub- cold rolling, drawing, hammering, or to cold working of any kind, me harder, stronger and slightly more dense. At the same time vity increases, sometimes markedly, and the initial properties can pproached again by means of the **annealing process.** While s will sometimes restore the initial properties, at least for most purposes, it does not always do so.

WIRE GAGES

sizes of wire have been for many years ind's gage, mentioned in Par. 17. most entirely by gage numbers, esp... and other continental countries prac- s practice is accompanied by some ... wire sizes are specified directly in milli- in common use. The most ... the **millimeter wire gage** (see merica, is the **American** ...times called the ..., are based to some extent ommonly used gage f... used in France, however, ... For a history of wire in Par. 15. ("jauge de Paris de 1857"). Bureau of Standards, no legal standard ...e "Copper Wire Tables," adopted by Co... No. 31, ard known ...t. 1, 1914. y, France, ... growing tendency to **abandon gage numbers** entirely and wire gage ... sizes by the **diameter in mils** (thousandths of an inch). This his syster... olds particularly in writing specifications, and has the great ad- The wire ... f being both simple and explicit. A number of the wire manufac- Paris g... o encourage this practice, and it was definitely adopted by the Circul... tates Navy Dept. in 1911.

The mil is a term universally employed in this country in connection ...re gages and is a **unit of length equal to one thousandth of an**

The circular mil is another universal used term, being a unit of equal to the area of a circle 1 mil in diameter. Such a circle, however, an area of 0.7854 (or $\pi/4$) sq mil. Thus a wire 10 mils in diameter has ...oss-sectional area of 100 circ. mils or 78.54 sq. mils. Hence, 1 circ. mil ...als 0.7854 sq. mil.

The American wire gage, also known as the **Brown & Sharpe** ... was devised in 1857 by J. R. Brown. It is usually abbreviated **A.W.G.** ... gage has the property, in common with a number of other gages, its sizes represent approximately the successive steps in the process of drawing. Also, like many other gages, its numbers are retrogressive, ...er number denoting a smaller wire, corresponding to the operations of ...ng. These gage numbers are not arbitrarily chosen, as in many gages, ...llow the mathematical law upon which the gage is founded. The gage ...bers and sizes are given in Par. 20.

0.010 || 25

The theoretically exact diameters in this gage, as given in the second col-
umn of Par. 30, contain more significant figures than there is any comm
need for, and hence the large companies have standardised the sizes giv
the third column, using the nearest mil for large sizes and the ne
tenth of a mil for the smaller sizes. These commercial sizes were ado
as standard by the United States War Dept. in 1911.

14. The basis of the American wire gage is a simple mathem
law. The gage is formed by the specification of two diameters and th
that a given number of intermediate diameters are formed by geome
progression. Thus, the diameter of No. 0000 is defined as 0.4600 in. a
No. 36 as 0.0050 in. There are 38 sizes between these two, hence the
of any diameter to the diameter of the next greater number is given by
expression

$$\sqrt[39]{\frac{0.4600}{0.0050}}=\sqrt[39]{92}=1.122\ 932\ 2$$

The square of this ratio = 1.2610. The sixth power of the ratio, i.e.
ratio of any diameter to the diameter of the sixth greater number = 2.
The fact that this ratio is so nearly 2 is the basis of numerous useful rela
or short cuts in wire computations.

15. The steel wire gage, also known originally as the **Washbu
Moen gage** and later as the **American Steel & Wire Co.'s gage**
established by Ichabod Washburn about 1830. This gage also, w
wire gage ... of its sizes rounded off to thousandths of an inch, is known
based upon the reducti... gage. It is used exclusively for steel wire. The number
rod. Thus, a wire rod was ca... Par. 30.
gradations of size in this gage are ... **wire gage,** also known as **Stubs' wire gag**
This gage has been used to a considerable ... to have been established early in the eight
telegraph wires. Its numbers and sizes are ... long in use. This gage was used to
17. The Standard wire gage, which more pr... should not be confused with Stubs
(British) Standard wire gage, is the legal standard ... mbers of the Birmingham gag
wires, adopted in 1883. It is also known as the **New** ... ctice by drawing wire from
gage, the **English legal standard gage** and the **Imp**... rawing No. 1, and so on.
It was constructed by so modifying the Birmingham gage that ... will appear from its
between consecutive sizes became more regular. While this g... designating iron and
most largely used in England, there is a tendency there, as here, t... Par. 30.
numbers and specify sizes by the diameter in mils. This gage ... y should be desig
been extensively used in this country. Its numbers and sizes ar... of Great Britain
Par. 30. **British Sta**

18. The Old English wire gage, also known as the **London wi**... erial wire
differs very little from the Birmingham gage. It was formerly used ... in the diffe
extent for brass and copper wires, but is now nearly obsolete. The nu... is gage is t
and sizes are given in Par. 30. ... o dro ... has

19. The Stubs' steel wire gage has a somewhat limited use fo... metals
steel wire and drill rods. It should not be confused with the Birmingha... o have
Stubs' iron wire gage mentioned in Par. 16. The numbers and sizes ... olytes,
given in Par. 30. In addition there are twenty-six larger sizes, *Z* to *A*, ... heir res
thirty smaller sizes, No. 51 to No. 80, besides those given in Par. 30 (se... alloys,
catalogue of Brown & Sharpe Mfg. Co., or The L. S. Starrett Co.). ... hts, but
... case of c

20. The Trenton Iron Co.'s gage, of which the numbers and si... as the pe
given in Par. 30, is used only to a very limited extent. It differs but sli... dard (Par
from the steel wire gage mentioned in Par. 15.

21. The French wire gage is an exception to the other gages giv... **stance c**
Par. 30 in the respect that its sizes are progressive, instead of retrogre... **the linear**
as the numbers advance. The sizes there given were taken from the A...)
can Steel and Wire Co.'s handbook, "Electrical Wires and Cables," 19... **sistance**
... **d at zer**

... **m and**
... **change**

30. Tabular comparison of wire gages
Diameters in mils

Gage No.	American Wire Gage (B. & S.) exact sizes	American Wire Gage (B. & S.) commercial sizes	Steel Wire Gage (Wash. & Moen)	Birmingham Wire Gage (Stubs')	(British) Standard Wire Gage	Old English Wire Gage (London)	Stubs' Steel Wire Gage	Trenton Iron Co. Gage	French Gage	U. S. Standard Sheet Gage	Gage No.
7-0	....	....	490.0	....	500.0	....	....	....	....	500.0	7-0
6-0	....	....	461.5	....	464.0	....	....	....	....	468.7	6-0
5-0	....	....	430.5	....	432.0	....	....	450.0	....	437.5	5-0
4-0	460.00	460.0	393.8	454.0	400.0	454.0	....	400.0	....	406.2	4-0
3-0	409.64	410.0	362.5	425.0	372.0	425.0	....	360.0	....	375.0	3-0
2-0	364.80	365.0	331.0	380.0	348.0	380.0	....	330.0	....	343.7	2-0
0	324.86	325.0	306.5	340.0	324.0	340.0	....	305.0	....	312.5	0
1	289.30	289.0	283.0	300.0	300.0	300.0	227.0	285.0	33.0	281.2	1
2	257.63	258.0	262.5	284.0	276.0	284.0	219.0	265.0	40.0	265.6	2
3	229.42	229.0	243.7	259.0	252.0	259.0	212.0	245.0	50.0	250.0	3
4	204.31	204.0	225.3	238.0	232.0	238.0	207.0	225.0	63.0	234.4	4
5	181.94	182.0	207.0	220.0	212.0	220.0	204.0	205.0	68.0	218.7	5
6	162.02	162.0	192.0	203.0	192.0	203.0	201.0	190.0	83.0	203.5	6
7	144.29	144.0	177.0	180.0	176.0	180.0	199.0	175.0	97.0	187.5	7
8	128.49	128.0	162.0	165.0	160.0	165.0	197.0	160.0	110.0	171.9	8
9	114.42	114.0	148.3	148.0	144.0	148.0	194.0	145.0	120.0	156.2	9
10	101.90	102.0	135.0	134.0	128.0	134.0	191.0	130.0	135.0	140.6	10
11	90.74	91.0	120.5	120.0	116.0	120.0	188.0	117.5	149.0	125.0	11
12	80.81	81.0	105.5	109.0	104.0	109.0	185.0	105.0	162.0	109.4	12
13	71.96	72.0	91.5	95.0	92.0	95.0	182.0	92.5	172.0	93.7	13
14	64.08	64.0	80.0	83.0	80.0	83.0	180.0	80.6	185.0	78.1	14
15	57.07	57.0	72.0	72.0	72.0	72.0	178.0	70.0	197.0	70.3	15
16	50.82	51.0	62.5	65.0	64.0	65.0	175.0	61.0	212.0	62.5	16
17	45.26	45.0	54.0	58.0	56.0	58.0	172.0	52.5	225.0	56.2	17
18	40.30	40.0	47.5	49.0	48.0	49.0	168.0	45.0	238.0	50.0	18
19	35.89	36.0	41.0	42.0	40.0	40.0	164.0	40.0	250.0	43.7	19
20	31.96	32.0	34.8	35.0	36.0	35.0	161.0	35.0	263.0	37.5	20
21	28.46	28.5	31.7	32.0	32.0	31.5	157.0	31.0	279.0	34.4	21
22	25.35	25.3	28.6	28.0	28.0	29.5	155.0	28.0	290.0	31.2	22
23	22.57	22.6	25.8	25.0	24.0	27.0	153.0	25.0	303.0	28.1	23

COPPER

31. General properties. Copper, which is by far the most important metal in the electrical industry, is a highly malleable and ductile metal, of a reddish color. The density varies slightly, depending on the physical state, an average value being 8.9. Copper melts at 1,083 deg. cent.* (1,981 deg. fahr.), and in the molten state has a sea-green color. When heated to a very high temperature it vaporizes, and burns with a characteristic green flame. Copper boils at 2,310 deg. cent. (4,190 deg. fahr.).† Molten copper readily absorbs oxygen, hydrogen, carbon monoxide and sulphur dioxide; on cooling, the occluded gases are liberated, tending to give rise to blow holes and porous castings. The presence of lead in molten copper tends to drive off both carbon dioxide and water vapor.

Copper when exposed to ordinary air becomes oxidized, turning to a black color, but the coating is protective and the oxidizing process is not progressive as with iron and steel. When exposed however to moist air containing carbon dioxide, it becomes coated with green basic carbonate. It is also affected by sulphur dioxide. It resists the action of hydrochloric, sulphuric and strong nitric acids, at ordinary temperatures, but is acted upon by dilute nitric acid.

The electrical conductivity of copper depends most critically on its degree of chemical purity (see Par. **63**) and also, in much less degree, upon the physical state, being reduced slightly (from 2 per cent. to 4 per cent.) by cold rolling and drawing. The tensile properties depend greatly upon the physical state, being much improved by cold rolling and drawing.

The alloys of copper are exceedingly numerous, both for electrical and mechanical purposes. Among the most important for electrical purposes are German or nickel silver, bronze and brass. Copper solders readily with ordinary low-temperature solders; solder alloys with copper at about 238 deg. cent. (460 deg. fahr.).

32. Commercial grades of copper. In the copper trade there are three recognized grades of copper known as **electrolytic, Lake,** and **casting.**‡ The first, electrolytic, is that refined by the electrolytic method and is highly pure (see Par. **34**). The second, Lake, is also highly pure, in its natural or mineral state, and requires simply to be melted down to bars, for convenient handling (see Par. **36**). The third kind of copper, known as casting copper, contains more impurities and consequently runs lower in conductivity. It is, as its name implies, more suitable for mechanical than electrical applications (Par. **38**).

33. Density of copper. The internationally accepted density of annealed copper,§ expressed in grams per cu. cm. at 20 deg. cent., is **8.89**; this of course is also the specific gravity at 20 deg. cent., referred to water at 4 deg. cent. The American Society for Testing Materials has accepted this value, on account of its international endorsement, but considers that a value of 8.90 is probably nearer the exact truth. A density of 8.89 at 20 deg. cent. corresponds to 8.90 at 0 deg. cent. In English units the international standard equals 0.32117 lb. per cu. in. Also see "Copper Wire Tables," circular No. 31, Bureau of Standards, Washington, D. C.; and "Smithsonian Physical Tables," Washington, D. C., 1910, 5th rev. ed., p. 85.

34. Electrolytic copper. The electrolytic refinement of copper‖ (see Sec. 19) not only produces metal of the highest purity, but it is eco-

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* "Tables Annuelles de Constantes Et Données. Numériques, de Chimie, de Physique et de Technologie;" University of Chicago Press, 1912; Vol. I (1910), p. 48.

† Fulton, C. H. "Principles of Metallurgy;" McGraw-Hill Book Co., New York, 9111; p. 74.

‡ See report of Committee B-2, on Non-ferrous Metals and Alloys; American Society for Testing Materials, 16th annual meeting, June 24–28, 1913.

§ Ratified at the meeting of the International Electrotechnical Commission held in Berlin, Sept. 1 to 6, 1913; see *Trans.* A. I. E. E., Vol. XXXII, p. 2148.

‖ Addicks, L. "Electrolytic Copper;" Journal of the Franklin Institute, Philadelphia, Pa., Dec., 1905.

40. Table of various values for resistivity, temperature coefficient, and density of annealed copper

Temperature (deg. cent.)	1 England (Eng. Stds. Com., 1904)	2 Germany, Old "Normal Kupfer," density 8.91	3 Germany, Old "Normal Kupfer," assuming density 8.89	4 Lindeck, Matthiessen value, assuming density 8.89	5 A. I. E. E. before 1907 (Matthiessen value)	6 A. I. E. E., 1907 to 1910	7 Bureau of Standards and A. I. E. E., 1911	8 "International Annealed Copper Standard"
				Restivity in Ohms (meter, gram)				
0	0.14136	0.13959	0.13927	0.14157	(**0.14172**)	(**0.14172**)	0.14106	0.14133
15	0.15043	(**0.14850**)	(**0.14816**)	(**0.14997**)	0.15014	0.15065	0.15003	0.15029
(15.6)	(**0.1508**)	....	....	....	....	....	....	....
20	0.15346	0.15147	0.15113	0.15285	0.15302	0.15363	(**0.15302**)	(**0.15328**)
25	0.15648	0.15444	0.15409	0.15576	0.15593	0.15661	0.15601	0.15626
				Temperature Coefficient				
0	(**0.00428**)	0.00425	0.00425	(*)	(*)	(**0.0042**)	** (0.00427)	** (0.00427)
15	0.00402	(**0.004**)	(**0.004**)	....	....	0.00395	** (0.00401)	** (0.00401)
20	0.00394	0.00392	0.00392	....	....	0.00387	** (**0.00394**)	** (**0.00393**)
25	0.00386	0.00384	0.00384	....	....	0.00380	** (0.00386)	** (0.00385)
				Density				
	† 8.89	8.91	(8.89)	(8.89)	8.89	8.89	†† 8.89	†† 8.89

* Matthiessen's formula: $\lambda_t = \lambda_o (1 - 0.0038701t + 0.000009009\,t^2)$. λ_t and λ_o = reciprocal of resistance at t deg. and 0 deg. cent. respectively.

** This temperature coefficient applies only to this particular resistivity. The temperature coefficient is considered to be proportional to conductivity. Expressed otherwise, the change of *resistivity* per deg. cent. is considered to be a constant, viz., 0.000597 ohm (meter, gram).

† At 15.6 deg. cent.

†† This is the density at 20 deg. cent. It corresponds to 8.90 at 0 deg. cent.

NOTE.—Black-faced figures refer to the particular values which were made standard.

44. Reduction of observations to standard temperature. A table of convenient corrections and factors for reducing resistivity and resistance to standard temperature, 20 deg. cent., will be found in "Copper Wire Tables," Circular No. 31, Bureau of Standards.

45. Calculation of per cent. conductivity. The per cent. conductivity of a sample of copper is calculated by dividing the resistivity of the International Annealed Copper Standard at 20 deg. cent. by the resistivity of the sample at 20 deg. cent. Either the mass resistivity or volume resistivity may be used. Inasmuch as the temperature coefficient of copper varies with the conductivity, it is to be noted that a different value will be found if the resistivity at some other temperature is used. This difference is of practical moment in some cases. In order that such differences shall not arise, it is best always to use the 20 deg. cent. value of resistivity in computing the per cent. conductivity of copper. When the resistivity of the sample is known at some other temperature, t, it is very simply reduced to 20 deg. cent. by adding the quantity $(20-t)$ multiplied by the "resistivity-temperature constant," given in Par. **46**.

46. Resistivity-temperature constant. The *change of resistivity per degree* may be readily calculated, taking account of the expansion of the metal with rise of temperature. The proportional relation between temperature coefficient and conductivity may be put in the following convenient form for reducing *resistivity* from one temperature to another: *The change of resistivity of copper per degree cent. is a constant, independent of the temperature of reference and of the sample of copper. This "resistivity-temperature constant" may be taken, for general purposes, as* 0.00060 *ohm* (*meter, gram*), *or* 0.0068 *microhm-cm.* For further details, see "Copper Wire Tables," Circular No 31, Bureau of Standards, Washington, D. C.

47. Complete copper-wire tables, based on the International Annealed Copper Standard, are given in Par. **50**, and represent approximately an average of the present commercial conductivity of copper. For annealed wires, the resistivity is independent of the size. These tables are reproduced directly from Circular No. 31, 3rd Edition, issued by the Bureau of Standards. The quantities were computed to five significant figures and rounded off to the fourth place, being therefore correct within 1 in the fourth significant figure. The volume resistivity at 20 deg. cent., used in calculating these tables, was 0.67879 microhm-in. and the density, 8.89 at 20 deg. cent. or 0.321,17 lb. per cu. in. The tables in Circular No. 31 contain additional columns for 0 deg., 15 deg., 25 deg. and 75 deg. cent. What the tables show is the resistance at various temperatures, of a wire which at 20 deg. cent. is 1,000 ft. long and has the specified diameter, and which varies in length and diameter at other temperatures.

48. Explanatory notes on copper wire tables.

NOTE 1.—The *fundamental resistivity* used in calculating the tables is the International Annealed Copper Standard, viz., 0.15328 ohm (meter, gram) at 20 deg. cent. The *temperature coefficient* for this particular resistivity is $\alpha_{20}=0.00393$, or $\alpha_0=0.00427$. However, the temperature coefficient is proportional to the conductivity, and hence the change of *resistivity* per deg. cent. is a constant, 0.000597 ohm (meter, gram). The "constant mass" temperature coefficient of any sample is

$$\alpha_t=\frac{0.000597+0.000005}{\text{resistivity in ohms (meter, gram) at } t \text{ deg. cent.}} \qquad (4)$$

The *density* is 8.89 g. per cu. cm.

NOTE 2.—The values given in the tables are only for annealed copper of the standard resistivity. The user of the table must apply the proper correction for copper of any other resistivity. Hard-drawn copper may be taken as about 2.7 per cent. higher resistivity than annealed copper.

NOTE 3.—This table is intended as an ultimate reference table, and is computed to a greater precision than is desired in practice. The practical user of a wire table is referred to the "Working Tables," Par. **51**.

49. Working copper-wire tables, based on the International Annealed Copper Standard, are given in Par. **51**. This table is carried only to three significant figures, and is more convenient for most practical work. The table itself is adapted from Circular No. 31, 3d edition, issued by the Bureau of Standards, and amplified by the addition of values based on the mile unit.

50. Complete wire table, standard annealed copper.—*Continued*

Gage No.	Diameter in mils at 20 deg. cent.	Pounds per ohm		Gage No.	Diameter in mils at 20 deg. cent.	Pounds per ohm	
		20 deg. cent. (=68 deg. fahr.)	50 deg. cent. (=122 deg. fahr.)			20 deg. cent. (=68 deg. fahr.)	50 deg. cent. (=122 deg. fahr.)
0000	460.0	13,070.0	11,690.0	19	35.89	0.4843	0.4332
000	409.6	8,219.0	7,352.0	20	31.96	0.3046	0.2725
00	364.8	5,169.0	4,624.0	21	28.46	0.1915	0.1713
0	324.9	3,251.0	2,908.0	22	25.35	0.1205	0.1078
1	289.3	2,044.0	1,829.0	23	22.57	0.07576	0.06777
2	257.6	1,286.0	1,150.0	24	20.10	0.04765	0.04262
3	229.4	808.6	723.3	25	17.90	0.02997	0.02680
4	204.3	508.5	454.9	26	15.94	0.01885	0.01686
5	181.9	319.8	286.1	27	14.20	0.01185	0.01060
6	162.0	201.1	179.9	28	12.64	0.007454	0.006668
7	144.3	126.5	113.2	29	11.26	0.004688	0.004193
8	128.5	79.55	71.16	30	10.03	0.002948	0.002637
9	114.4	50.03	44.75	31	8.928	0.001854	0.001659
10	101.9	31.47	28.15	32	7.950	0.001166	0.001043
11	90.74	19.79	17.70	33	7.080	0.0007333	0.0006560
12	80.81	12.45	11.13	34	6.305	0.0004612	0.0004126
13	71.96	7.827	7.001	35	5.615	0.0002901	0.0002595
14	64.08	4.922	4.403	36	5.000	0.0001824	0.0001632
15	57.07	3.096	2.769	37	4.453	0.0001147	0.0001026
16	50.82	1.947	1.742	38	3.965	0.00007215	0.00006454
17	45.26	1.224	1.095	39	3.531	0.00004538	0.00004059
18	40.30	0.7700	0.6888	40	3.145	0.00002854	0.00002553

51. Working table, standard annealed copper wire.—*Continued*

Gage No.	Diameter in mils	Cross-section		Ohms per 1,000 ft.		Ohms per mile		Pounds per 1,000 ft.	Pounds per mile
		Circular mils	Square inches	25 deg. cent. (=77 deg. fahr.)	65 deg. cent. (=149 deg. fahr.)	25 deg. cent. (=77 deg. fahr.)	65 deg. cent. (=149 deg. fahr.)		
21	28.5	810.	0.000636	13.1	15.1	69.2	79.7	2.45	12.9
22	25.3	642.	0.000505	16.5	19.0	87.1	100.	1.94	10.24
23	22.6	509.	0.000400	20.8	24.0	110.	127.	1.54	8.13
24	20.1	404.	0.000317	26.2	30.2	138.	159.	1.22	6.44
25	17.9	320.	0.000252	33.0	38.1	174.	201.	0.970	5.12
26	15.9	254.	0.000200	41.6	48.0	220.	253.	0.769	4.06
27	14.2	202.	0.000158	52.5	60.6	277.	320.	0.610	3.22
28	12.6	160.	0.000126	66.2	76.4	350.	403.	0.484	2.56
29	11.3	127.	0.0000995	83.4	96.3	440.	509.	0.384	2.03
30	10.0	101.	0.0000789	105.	121.	554.	639.	0.304	1.61
31	8.9	79.7	0.0000626	133.	153.	702.	808.	0.241	1.27
32	8.0	63.2	0.0000496	167.	193.	882.	1020.	0.191	1.01
33	7.1	50.1	0.0000394	211.	243.	1110.	1280.	0.152	0.803
34	6.3	39.8	0.0000312	266.	307.	1400.	1620.	0.120	0.634
35	5.6	31.5	0.0000248	335.	387.	1770.	2040.	0.0954	0.504
36	5.0	25.0	0.0000196	423.	488.	2230.	2580.	0.0757	0.400
37	4.5	19.8	0.0000156	533.	616.	2810.	3250.	0.0600	0.317
38	4.0	15.7	0.0000123	673.	776.	3550.	4100.	0.0476	0.251
39	3.5	12.5	0.0000098	848.	979.	4480.	5170.	0.0377	0.199
40	3.1	9.9	0.0000078	1,070.	1,230.	5650.	6490.	0.0299	0.158

This table is correct to three significant figures, only.

The number of circular mils in a cable composed of N wires is

$$C.M. = Nd^2 \tag{8}$$

where d is the diameter of each wire in mils (thousandths of an inch).

The equivalent solid conductor is one having the same number of circular mils, or its diameter is

$$D' = \sqrt{Nd^2} \tag{9}$$

It is *not* equal to the normal cross-section of the cable, because in the last case the strands are cut at a slight angle (due to their pitch) and such a section is therefore larger than the true section, equal to the sum of the normal sections of all the strands, each taken normal to its own axis.

The ratio of the diameter of concentric strand to the diameter of equivalent solid conductor is given by

$$\frac{D}{D'} = \frac{2n+1}{\sqrt{N}} = \sqrt{\frac{4n^2+4n+1}{3n^2+3n+1}} \tag{10}$$

Substitution in this formula from $n = 0$ to $n = 8$, gives the following values of the ratio.

n	N	$\frac{D}{D'}$	n	N	$\frac{D}{D'}$	n	N	$\frac{D}{D'}$
0	1	1.000	3	37	1.151	6	127	1.154
1	7	1.134	4	61	1.152	7	169	1.154
2	19	1.147	5	91	1.153	8	217	1.154

This shows that the larger the number of strands, for a given cross-section, the larger will be the outside diameter, approaching, however, a limiting ratio of 1.154. Therefore the size and the cost of a conductor of given cross-section increase as the number of strands increases.

The individual wires of a cable can seldom be drawn to any of the standard gage numbers, because the diameter of the wire is fixed by the required size of the cable, and the number of wires composing it. Also see "Wire In Electrical Construction," John A. Roebling's Sons Co., 1906; and "Electrical Wires and Cables," Amer. Steel & Wire Co., 1910.

56. Composition of standard concentric strands*

Range of size	Number of wires	
	Standard concentric strands	Flexible concentric strands
2,000,000 to 1,600,000 cir. mils	127	169
1,500,000 to 1,100,000 cir. mils	91	127
1,000,000 to 550,000 cir. mils	61	91
500,000 to 250,000 cir. mils	37	61
No. 0000 to No. 1 A.W.G.	19	37
No. 2 to No. 8 A.W.G.	7	19

57. Pitch or lay of concentric strand. The axial length of one complete turn of any individual strand in a concentric-lay cable, divided by the diameter of the cable, is called the pitch or lay. The **pitch angle** of the cable is shown in Fig. 2, where ac represents the axis of the cable and l is the axial length of one complete twist; ab is the length of any individual strand, $l + \Delta l$, in one complete twist; and the angle bac, or θ, is the pitch angle. The side bc is equal to the circumference of the circle circumscribing the cable. In this case the pitch p is given by $p = l/d$. There is no fixed

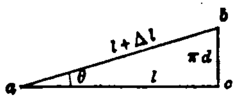

FIG. 2.—Pitch angle in concentric lay cable.

* See Circular No. 37, "Electric Wire and Cable Terminology;" Bureau of Standards; 2nd ed., Jan. 1, 1915, page 13: A. I. E. E. Standardisation Rules, 1914; also see latest edition of Standardisation Rules, Sec. 24.

50. Complete wire table, standard annealed copper.—*Continued*

Gage No.	Diameter in mils at 20 deg. cent.	Pounds per 1,000 ft.	Feet per pound	Feet per ohm*	
				20 deg. cent. (= 68 deg. fahr.)	50 deg. cent. (= 122 deg. fahr.)
0000	460.0	640.5	1.561	20,400.0	18,250.0
000	409.6	507.9	1.968	16,180.0	14,470.0
00	364.8	402.8	2.482	12,830.0	11,480.0
0	324.9	319.5	3.130	10,180.0	9,103.0
1	289.3	253.3	3.947	8,070.0	7,219.0
2	257.6	200.9	4.977	6,400.0	5,725.0
3	229.4	159.3	6.276	5,075.0	4,540.0
4	204.3	126.4	7.914	4,025.0	3,600.0
5	181.9	100.2	9.980	3,192.0	2,855.0
6	162.0	79.46	12.58	2,531.0	2,264.0
7	144.3	63.02	15.87	2,007.0	1,796.0
8	128.5	49.98	20.01	1,592.0	1,424.0
9	114.4	39.63	25.23	1,262.0	1,129.0
10	101.9	31.43	31.82	1,001.0	895.6
11	90.74	24.92	40.12	794.0	710.2
12	80.81	19.77	50.59	629.6	563.2
13	71.96	15.68	63.80	499.3	446.7
14	64.08	12.43	80.44	396.0	354.2
15	57.07	9.858	101.4	314.0	280.9
16	50.82	7.818	127.9	249.0	222.8
17	45.26	6.200	161.3	197.5	176.7
18	40.30	4.917	203.4	156.6	140.1
19	35.89	3.899	256.5	124.2	111.1
20	31.96	3.092	323.4	98.50	88.11
21	28.46	2.452	407.8	78.11	69.87
22	25.35	1.945	514.2	61.95	55.41
23	22.57	1.542	648.4	49.13	43.94
24	20.10	1.223	817.7	38.96	34.85
25	17.90	0.9699	1,031.0	30.90	27.64
26	15.94	0.7692	1,300.0	24.50	21.92
27	14.20	0.6100	1,639.0	19.43	17.38
28	12.64	0.4837	2,067.0	15.41	13.78
29	11.26	0.3836	2,607.0	12.22	10.93
30	10.03	0.3042	3,287.0	9.691	8.669
31	8.928	0.2413	4,145.0	7.685	6.875
32	7.950	0.1913	5,227.0	6.095	5.452
33	7.080	0.1517	6,591.0	4.833	4.323
34	6.305	0.1203	8,310.0	3.833	3.429
35	5.615	0.09542	10,480.0	3.040	2.719
36	5.000	0.07568	13,210.0	2.411	2.156
37	4.453	0.06001	16,660.0	1.912	1.710
38	3.965	0.04759	21,010.0	1.516	1.356
39	3.531	0.03774	26,500.0	1.202	1.075
40	3.145	0.02993	33,410.0	0.9534	0.8529

* Length at 20 deg cent. of a wire whose resistance is 1 ohm at the stated temperatures.

69. Table of breaking loads of copper wire

(Based on tensile requirements of the American Society for Testing Materials

Gage No. A.W.G.	Diam. in mils	Breaking load (lb.)		
		Annealed	Medium hard	Hard drawn
0000	460	5,980	6,980 –8,140	8,140
000	410	4,750	5,680 –6,600	6,730
00	365	3,780	4,620 –5,360	5,540
0	325	2,980	3,730 –4,310	4,520
1	289	2,370	3,020 –3,480	3,680
2	258	1,930	2,450 –2,810	3,000
3	229	1,530	1,980 –2,270	2,440
4	204	1,210	1,590 –1,820	1,970
5	182	963	1,260 –1,450	1,590
6	162	763	1,010 –1,150	1,280
7	144	607	810 – 925	1,030
8	128	481	646 – 737	828
9	114	381	515 – 587	663
10	102	314	410 – 467	528
11	91	249	328 – 373	423
12	81	198	262 – 298	337
13	72	157	209 – 237	268
14	64	124	167 – 189	214
15	57	98.6	131 – 151	170
16	51	78.2	106 – 120	135
17	45	62.0	84.8– 96.1	108
18	40	49.3	67.9– 76.8	85.8
(English gages)				
8 B.W.G.	165	792	1,050 –1,200	1,330
10 B.W.G.	134	522	698 – 797	894
12 S.W.G.	104	314	427 – 487	551
13 S.W.G.	92	256	337 – 383	435
14 S.W.G.	80	194	256 – 292	330
16 B.W.G.	65	128	171 – 195	220

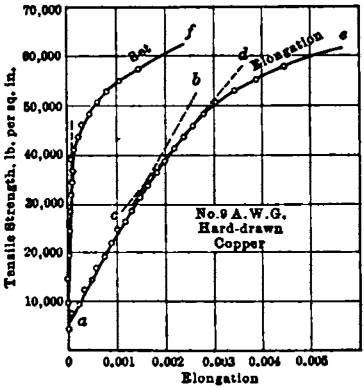

FIG. 5.—Stress-strain curves of No. 9 A.W.G. hard-drawn copper wire (Watertown Arsenal test).

70. Stress-strain diagrams. A typical stress-strai diagram of hard-drawn coppe wire is shown in Fig. 5, whic represents No. 9 A.W.G. T curve *ae* is the actual stre strain curve; *ab* represents t portion which corresponds true elasticity, or for whi Hooke's law holds rigorous *cd* is the tangent to *ae* whi fixes the Johnson elastic limi and the curve *af* represe the set, or permanent elong tion due to flow of the me

* The Johnson elastic limit that point on the stress-str curve at which the natural t gent is equal to 1.5 times t tangent of the angle of t straight or linear portion of t curve, with respect to the a of ordinates, or Y axis. S Johnson, J. B., "Materials Construction," John Wiley Sons, New York, 1912.

72. Fatigue under load. Under long-sustained loads approaching the normal tensile strength, copper has somewhat less strength than the values obtained by ordinary test. Mr. F. O. Blackwell, in the paper referred to in Par. 70, found that a 0.168-in. hard-drawn wire stressed to 54,000 lb. per sq. in., stretched continuously, and broke in 7 days, 8 hours; pieces of the same wire afterward broke at 61,000 lb. in the testing machine. He concluded that a hard-drawn wire would stand continuously a stress of about 80 per cent. of its normal tensile strength.

73. Young's modulus of elasticity for annealed and hard copper is not a very definitely known quantity and the values given for it fluctuate over a considerable range. This may be accounted for, in the case of annealed copper, by the lack of any very definite elastic limit, and the fact that the initial stress-strain diagram departs at a very early stage from Hooke's law; and as soon as a slight load has been applied the properties commence to change. The same difficulties are present, in less degree, in the case of hard-drawn copper. In all cases, the final value of the modulus, after stressing, is almost invariably greater than the initial modulus. The following values represent the extreme range, and a probable average, drawn from several authorities,* expressed in in-lb. measure.

State	Range	Probable average
Annealed wire..................	7×10^6 to 17×10^6	12×10^6
Annealed concentric strand.......	5×10^6 to 12×10^6	9×10^6
Hard-drawn wire................	13×10^6 to 19×10^6	16×10^6
Hard-drawn concentric strand....	10×10^6 to 14×10^6	12×10^6

74. Specific heat of copper is not independent of temperature. The following values were taken from "Tables Annuelles de Constantes et Données Numériques de Chemie, de Physique et de Technologie" (For 1910; University of Chicago Press, 1912), p. 50.

Specific heat, at −50 deg. cent............................ 0.0862
Specific heat, at 0 deg. cent............................ 0.0910
Specific heat, at +50 deg. cent............................ 0.0928
Specific heat, from 2.4 to 21.6 deg. cent...................... 0.09155
Specific heat, from 17 to 100 deg. cent...................... 0.0925

The Bureau of Standards gives, for the range from 15 to 50 deg. cent., the expression $0.0917+0.000048\ (t-25)$; this is in terms of water at 20 deg. cent. For values at high temperatures see Hoffman, H. O. "Metallurgy of Copper;" p. 7.

75. Thermal conductivity of copper is a function of temperature, as expressed in the formula $\lambda_t = \lambda_0 (1 + at)$. The following values of thermal conductivity, in g-cal. (cm-cube) per sec. per deg. cent. were determined by Lorenz.†

Thermal conductivity, at 0 deg. cent........................ 0.7189
Thermal conductivity, at 100 deg. cent........................ 0.7226
Temperature coefficient, from and at 0 deg. cent............... 0.000051

Hoffman states the thermal conductivity as 0.72 g-cal. (cm-cube) per deg. cent.; Langmuir gives 0.84 g-cal. for commercial copper and 0.92 g-cal. for pure copper.

76. Properties of copper at very high temperatures. See a paper by Carl Hering, "The Proportioning of Electrodes for Furnaces," *Trans.* A. I. E. E., Vol. XXIX (1910), pp. 485 to 545. Also covers carbon, graphite and iron.

77. Specifications for copper wire, annealed, medium hard and hard drawn, have been adopted by the American Society For Testing Materials

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* Blackwell, F. O. "Conductors for Long Spans;" *Trans.* International Elec. Congress, St. Louis, 1904; Vol. II, p. 341.
Blackwell, F. O. "Long Spans for Transmission Lines;" *Trans.* A. I. E. E., Vol. XXIII, 1904, p. 511.
"Smithsonian Physical Tables," 5th rev. ed., 1910, p. 75.
American Steel & Wire Co.; "Handbook of Electrical Wires and Cables," 1910, p. 14.

† "Smithsonian Physical Tables," 1910; p. 199.

dioxide do not affect it at ordinary temperatures; it is not attacked by sulphuretted hydrogen, or carbonic acid. It resists the action of sea water better than copper, provided there is no electrolysis, but it is a highly electropositive metal. The presence of impurities in considerable quantities lowers the resistance to corrosion in marked degree. There is also danger from electrolytic corrosion if aluminum is alloyed with an electronegative metal.

The electrical conductivity of aluminum, like that of copper, depends on its degree of chemical purity. The conductivity of hard-drawn aluminum is about 2 per cent. less than that of soft or annealed aluminum. The tensile properties, in like manner, depend greatly upon the physical state, being much improved by cold rolling and drawing.

The alloys of aluminum are very numerous. The so-called light alloys, containing but small percentages of other metals, are light, hard and strong, but do not resist corrosion from galvanic action. The heavy alloys, or aluminum-bronzes, with but 2 per cent. to 10 per cent. of aluminum, and respectively 98 per cent. to 90 per cent. of copper, have high tensile strength and strongly resist corrosion in air or sea water. A very small proportion of aluminum, about 0.01 per cent., added to iron, steel or brass in casting removes the oxide and prevents blow holes.

The tinning process, which is applied to copper wires intended to receive an insulation of rubber compound (sulphur being present), is unnecessary in the case of aluminum.

Aluminum possesses an insulating film which ordinarily has a dielectric strength of about 0.5 volt, and by electrolytic action this value can be somewhat increased.

80. Commercial grades of aluminum. The impurities most commonly found in aluminum are silicon and iron. Silicon in aluminum exists in two forms, one seemingly combined with aluminum as combined carbon exists in pig iron, and the other as an allotropic graphitoidal modification. Small quantities of copper, sodium, carbon and occluded gases are also found in aluminum. The Aluminum Company of America classifies aluminum commercially in three grades,* as follows:

Extra-pure aluminum, No. 1 grade or so-called pure aluminum, and No. 2 grade for castings, or structural shapes. The average composition is as follows:

	No. 1 (per cent.)	No. 2 (per cent.)
Aluminum	99.55	96
Silicon	0.30	2
Iron	0.15	2

Pure aluminum (No. 1 grade or better) is necessary to secure high electrical conductivity, extreme malleability, ductility and maximum resistance to corrosion. For other purposes small amounts of copper, nickel, tungsten, manganese, chromium, titanium, zinc or tin may be advantageously added to aluminum to produce hardness, rigidity and strength. These metals when alloyed with aluminum do not diminish its resistance to corrosion so much as silicon or iron.

81. Typical analysis. The following analyses of aluminum are typical

	Aluminum Co. of Amer.† (per cent.)	Richards (per cent.)
Aluminum	99.57	99.25
Silicon	0.29	0.64
Iron	0.14	0.04
Copper	....	0.02
Lead	....	0.01

* "Properties of Aluminum;" Aluminum Co. of America, Pittsburgh, Pa. 1909; p. 7.

† Bureau of Standards, Circular No. 31, Third Ed., Oct. 1, 1914; p. 14.

information at the present time is largely regarded as among the secrets of the trade. The ordinary percentage of impurities in commercially pure aluminum, No. 1 grade (see Par. 80), is 0.45 per cent. In terms of the British standard for hard-drawn copper at 60 deg. fahr. (15.6 deg. cent.) Mr. Burkewood Welbourn stated* that a (volume) conductivity of 60 per cent. corresponds to 0.71 per cent. of impurities, and a conductivity of 61.7 per cent. corresponds to 0.5 per cent. of impurities.

The Aluminum Company of America states† that the electrical (volume) conductivity of pure (No. 1 grade) aluminum is about 62 per cent. in the Matthiessen standard scale. The British Aluminum Company, Ltd., gives (June, 1914) the following values of resistivity, expressed in microhm-cm.

	Annealed	Hard-drawn
Volume resistivity, microhm-cm. at 60 deg. fahr...	2.770	2.870
Volume resistivity, microhm-cm. at 32 deg. fahr...	2.610	2.70

The Bureau of Standards‡ gives the following average values of resistivity for commercial hard-drawn aluminum.

Mass resistivity, ohms (meter, gram), at 20 deg. cent.	0.0764
Mass resistivity, ohms (mile, pound), at 20 deg. cent.	436.0
Mass per cent. conductivity	200.7
Volume resistivity, microhm-cm., at 20 deg. cent.	2.828
Volume resistivity, microhm-in., at 20 deg. cent.	1.113
Volume per cent. conductivity	61.0
Density, g. per cu. cm.	2.70
Density, lb. per cu. in.	0.0975

These values given by the Bureau of Standards are the basis of the aluminum wire tables in Par. 87. Since aluminum is very rarely used as an electrical conductor in the soft state, the foregoing values given by the Bureau of Standards, for hard-drawn wire, have the most commercial significance. Annealed aluminum, however, is used abroad for the conductors of underground cables.

85. Temperature coefficient of resistance. On the authority of the British Aluminum Company, Ltd., the temperature coefficient of resistance of aluminum, for constant mass, varies from 0.0032 to 0.0040 per deg. cent. and from 0.0018 to 0.0022 per deg. fahr.

A determination made in the laboratory of the Westinghouse Electric and Manufacturing Company, under the direction of Prof. Charles F. Scott, gave as the average coefficient between 0 deg. and 50 deg. cent., the value 0.00388 per deg. cent.; in the fahrenheit scale the equivalent of this value is 0.00216 per deg. Prof Scott's determination is quoted by the Aluminum Company of America.

The Bureau of Standards gives 0.0039 per deg. cent. at 20 deg. cent. (circular No. 31, Third Edition, 1914, p. 14.)

86. Aluminum wire tables. The complete tables for aluminum wire given in Par. 87 were taken from circular No. 31, Third Edition, issued by the Bureau of Standards, and are based on a volume conductivity, in terms of the annealed copper standard, equal to 61.0 per cent.

Aluminum wire is practically never used in single strands for overhead construction, but the tables are very useful in computing the resistance of concentric strand. In commercial practice the aluminum delivered under contract varies in conductivity from 60 per cent. to 62 per cent. of the former Matthiessen standard, many contracts being placed at 61 per cent.

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* Welbourn, B. "Insulated and Bare Copper and Aluminum Cables for the Transmission of Electrical Energy, with Special Reference to Mining Work;" *Trans.* (British) Institution of Mining Engineers; 1913. Gives bibliography on aluminum wire.

† "Properties of Aluminum;" Aluminum Company of America, Pittsburgh, Pa., 1909; p. 27.

‡ Circular No. 31, "Copper Wire Tables;" 1914; Third Edition, p. 14.

89. Table of bare concentric-lay cables of hard-drawn aluminum
(English Units)

Circular mils	A.W.G. No.	Ohms per 1,000 ft.		Pounds per 1,000 ft.	Concentric stranding		
		25 deg. cent. (77 deg. fahr.)	65 deg. cent. (149 deg. fahr.)		No. of wires	Diam. of wires in mils	Outside diam. in mils
1,000,000		0.0177	0.0204	938.	37	164.4	1151
900,000		0.0197	0.0227	844.	37	156.0	1092
800,000		0.0221	0.0255	750.	37	147.0	1029
700,000		0.0253	0.0291	657.	37	137.5	963
600,000		0.0295	0.0340	563.	19	177.7	890
500,000		0.0354	0.0408	469.	19	162.2	810
400,000		0.0442	0.0510	375.	19	145.1	725
300,000		0.0590	0.0680	281.	19	125.7	630
300,000		0.0590	0.0680	281.	7	207.0	621
250,000		0.0707	0.0816	235.	7	188.9	567
212,000	0000	0.0834	0.0962	199.	7	174.0	522
168,000	000	0.1053	0.1214	158.	7	154.9	465
133,000	00	0.1330	0.1533	125.	7	137.8	414
106,000	0	0.1668	0.1924	99.4	7	123.1	369
83,700	1	0.2113	0.2436	78.5	7	109.3	327
66,400	2	0.2663	0.3071	62.3	7	97.4	292
52,600	3	0.3362	0.3876	49.3	7	86.7	260
41,700	4	0.4241	0.4890	39.1	7	77.2	232
33,100	5	0.5343	0.6160	31.0	7	68.8	206
26,300	6	0.6724	0.7753	24.7	7	61.3	184

90. Reinforced (steel centre) aluminum concentric strand. A concentric strand consisting of six hard-drawn aluminum wires laid over a centre or core consisting of a galvanized steel wire has been manufactured to a very limited extent, for experimental use. The steel employed had a tensile strength of about 125,000 lb. per sq. in. On account of the different coefficients of expansion, with these metals, the distribution of stresses in a suspended cable under changing temperature conditions is quite complicated.

In another instance the core was composed of 7 strands of steel laid into a concentric cable; about this were laid 6 strands of hard-drawn aluminum. The tensile strength of the steel was about 220,000 lb. per sq. in. and the strength of the aluminum was 28,000 lb. per sq. in. This type of conductor was used in a 1,000-ft. ravine span. See *Electrical News*, Vol. XXII, p. 34; also *The Canadian Engineer*, Dec. 11, 1913, "Transmission Line Work;" by E. V. Pannell.

91. Coefficient of linear expansion. The value given by Sir Roberts-Austen for the linear coefficient of expansion is 0.0000231 per deg. cent. from 0 to 100 deg. cent.; the corresponding value per deg. fahr. is 0.0000128. The value per deg. cent. given by the British Aluminum Company is 0.0000234. The 5th revised edition (1910) of the "Smithsonian Physical Tables" (Fowle, F. E.) gives the coefficient as 0.00002313 at 40 deg. cent.; the mean value between 0 and 100 deg. cent. is 0.0000222 per deg. cent.

92. Tensile strength. The tensile strength of aluminum depends upon its state, previous working and heat treatment. The strength of aluminum is increased by cold working, as in the case of copper. The approximate range of tensile strength of aluminum in various forms is given next below, in lb. per sq. in. (Aluminum Co. of America).

indicate the number of minutes the load was held at each of several points. This specimen broke at 23,900 lb. per sq. in., with an elongation of 1.25 per cent.

95. Elongation at rupture. The total elongation at rupture, for hard-drawn wire in commercial sizes, ranges from about 2 to 4 per cent.

96. Young's modulus of elasticity in tension ranges from 8,000,000 to 12,000,000, with an average of 9,000,000 to 10,000,000. F. O. Blackwell gives the modulus for concentric cables as 7,500,000 (Trans. Int. Elec. Cong., St. Louis, 1904, Vol. II, pp. 331–347).

97. Specific heat of aluminum at 0 deg. cent. is 0.2089 and at 100 deg. cent. is 0.2226; the mean specific heat between 16 and 100 deg. cent. is 0.2122 (5th rev. ed., "Smithsonian Phys. Tables," p. 228).

98. Thermal conductivity of aluminum at 0 deg. cent. is 0.344 gram-calorie (cm-cube) per deg. cent. per sec., with a temperature coefficient of 0.00054 per deg. cent. ("Smithsonian Phys. Tables," 1910).

99. Aluminum bars are used in power-plant switchboard connections for bus bars, and for carrying very large currents in electrolytic work. Since bus bars are generally designed to have a stated carrying capacity limited by a stated temperature rise, the comparative cross-sections of aluminum and copper are not required in practice to be in inverse ratio to the respective conductivities, because of the difference in radiating surface.

COPPER-CLAD STEEL

100. Compound or bi-metallic wires composed of copper-covered iron or steel have been manufactured by a number of different methods, and were first attempted many years ago. Aluminum-covered steel has also been tried, on an experimental scale. The general object sought in the manufacture of such wires is the combination of the high conductivity of copper or aluminum with the high strength and toughness of iron or steel. The resulting conductor is obviously a compromise between copper (or aluminum) and iron, being inferior as a whole to the former and superior to the latter.

101. Union between the metals. In the early attempts to produce bi-metallic wires, the two metals were not welded, but merely in close physical contact. Consequently there was a marked tendency toward electrolysis wherever moisture and air had access to the junction between the dissimilar metals. No great success attended the use of such wires until modern processes were developed for effecting a weld or molecular union between the metals.

102. Copper-clad steel wire is manufactured by two processes, known as the **Monnot process** (Duplex Metals Co.) and the **Griffith process** (Colonial Steel Co.). The Monnot process consists briefly of dipping a mild steel billet in bath of molten copper maintained at high temperature, thus forming on the surface of the billet an iron-copper alloy; the billet is then withdrawn and placed in a mold, and a copper jacket is cast around it. The billet is then re-heated and hot-rolled to wire rods, and finally cold-drawn to wire.

The Griffith process consists briefly of coating a mild steel billet with copper by electrolytic deposition (Sec. 19), then inserting the copper-coated billet in a copper tube, closing the ends, and heating the compound billet preparatory to rolling; it is then hot-rolled to rods, and cold-drawn to wire.

103. Commercial grades of copper-clad steel wire. It has become customary commercial practice to rate copper-clad steel wire in terms of the ratio of its volume conductivity to copper. Thus one manufacturer makes three grades of wire, having respectively 30 per cent., 40 per cent. and 47 per cent. conductivity ratio to copper; another manufacturer has standardised a 30-per cent. wire. These ratios are usually average ratios, and in practice certain tolerance limits must be recognised, above and below the average; or else the rated conductivity can be specified as the absolute acceptable minimum.

105. Copper-covered steel, concentric-lay cables
(Duplex Metals Company)

Approx. diam.	Actual diam.	Diam. of each strand	Total cross-section	Weight		Approx. breaking load	Average resistance per 1,000 ft. at 75 deg. fahr.		
							30 per cent. grade	40 per cent. grade	47 per cent. grade
Inches	Inches	Inches	Circ. mils	Pounds per 1,000 ft.	Pounds per mile	Pounds	Ohms	Ohms	Ohms
Seven strand									
5/8	0.612	0.204	291,310	857	4,505	16,380	0.120	0.0901	0.0766
9/16	0.546	0.182	231,870	682	3,601	13,860	0.151	0.113	0.0963
1/2	0.486	0.162	183,710	542	2,862	11,340	0.190	0.142	0.121
7/16	0.432	0.144	145,150	430	2,270	9,140	0.241	0.181	0.154
3/8	0.384	0.128	114,690	340	1,795	7,560	0.305	0.229	0.195
5/16	0.306	0.102	72,830	215	1,135	5,040	0.481	0.360	0.306
1/4	0.243	0.081	45,930	135	713	3,220	0.762	0.571	0.486
Nineteen strand									
1	1.020	0.204	790,700	2,354	12,429	44,460	0.0443	0.0332	0.0283
7/8	0.900	0.182	629,360	1,873	9,889	37,620	0.0556	0.0417	0.0355
13/16	0.810	0.162	498,640	1,484	7,835	30,780	0.0702	0.0526	0.0447
3/4	0.720	0.144	393,990	1,172	6,188	24,800	0.0889	0.0669	0.0567
11/16	0.685	0.137	366,610	1,091	5,760	22,230	0.0983	0.0737	0.0628
5/8	0.640	0.128	311,300	926	4,889	20,520	0.112	0.0842	0.0717
9/16	0.570	0.114	246,920	735	3,880	16,680	0.141	0.106	0.0904
1/2	0.510	0.102	197,680	589	3,109	13,680	0.177	0.133	0.113
7/16	0.455	0.091	157,340	468	2,471	11,120	0.223	0.167	0.142
3/8	0.360	0.072	98,500	293	1,548	7,020	0.355	0.266	0.227
5/16	0.320	0.064	77,830	232	1,224	5,650	0.449	0.337	0.287

Electrolytic iron melted in *vacuo*	9.96
Swedish charcoal iron remelted in *vacuo*	10.30
Commercial grades:	
Swedish charcoal iron cut from plate	10.57
Standard transformer steel	11.09
Silicon (4 per cent.) steel	51.15

Hopkinson tested and analyzed 35 different samples of iron (Phil. Trans. p. 463, Part II, 1885) and found resistivities (microhm-cm.) ranging from 13.78 for wrought iron to 100 for cast iron.

Also see Boudouard, O. "Electric Resistivity of Special Steels," IX, 6, No. 10, Sixth Congress Int. Assoc. for Testing Materials, New York City, 1912.

116. Preece's tests on resistivity of annealed iron wire
(Munroe and Jameson)

	Composition						Ohms (mile, lb.) at 60 deg. fahr.
	Fe	C	Mn	Si	S	P	
Swedish charcoal iron	99.70	0.10	0.03	Trace	0.022	0.045	4502
Swedish charcoal iron	99.44	0.15	0.234	0.018	0.019	0.058	4820
Siemens-Martin steel	99.60	0.10	0.324	Trace	0.035	0.034	5308
Best puddled iron	99.11	0.10	0.234	0.09	0.03	0.218	5974
Bessemer steel, soft	98.74	0.15	0.72	0.018	0.092	0.077	6163
Bessemer steel, hard	98.20	0.44	1.296	0.028	0.126	0.103	7468
Best cast steel	97.41	0.62	1.584	0.06	0.074	0.051	8033

117. Effects of different alloying elements upon the resistivity of pure iron were found by Barrett to be as follows: the values given in the table represent the increase in resistivity (microhm-cm.) resulting from the addition of 1 per cent. of different alloying elements.

Tungsten	2.0	Carbon	5.0
Cobalt	3.0	Manganese	8.0
Nickel	3.5	Silicon	13.0
Chromium	5.0	Aluminum	14.0

118. Temperature coefficient of resistance. The average coefficient per deg. cent., between 0 and 100 deg. cent., based on the measurements by Dewar and Fleming, is 0.00622. This value compares with 0.00635 based on recent measurements published by the Bureau of Standards (Scientific Paper No. 236). The mean value between 0 and 20 deg. cent., determined by Dewar and Fleming, is 0.00527 per deg. cent.

119. Ingot-iron, described more fully in Par. **279**, has been found on test to have a volume conductivity of 16.76 per cent. and a mass conductivity of 18.96 per cent., in terms of the International annealed copper standard. See *Elec. Railway Journal*, June 6, 1914, "Pure Ingot Iron for Third Rails." Carbon steel rails containing 0.73 per cent. carbon and 0.34 per cent. manganese, have a volume conductivity equal to 13 per cent. of that of copper. (Also see Sec. 16.) Ingot iron wire weighs about 4,600 lb. per mile-ohm at 20 deg. cent. and has a tensile strength of about 52,000 lb. per sq. in.

120. Resistivity and temperature coefficient of carbon steel. Barus and Strouhal found that the temper of carbon steel affected its electrical properties as shown below.

Temper	Resistivity, microhm-cm. at 0 deg. cent.	Temperature coefficient, per deg. cent.
Soft	15.9	0.00423
Light blue	18.4	0.00360
Blue	20.5	0.00330
Yellow	26.3	0.00280
Light yellow	28.9	0.00244
Glass hard	45.7	0.00161

See "Smithsonian Physical Tables," 5th rev. ed., 1910, p. 263.

125. Permeability of iron wire. The permeability of iron or soft steel wire, in the ordinary commercial sizes, at frequencies of 60 cycles or less, is from 100 to 125; at 800 cycles, it is about 70. This applies to small magnetising forces, such as exist within the wire due to the current flowing through it. These values hold for the steel core of copper-clad steel.

126. Steel rails. The resistivity of common rail steel varies in considerable degree, depending upon the chemical composition. Special soft steels used for third rails have resistivities ranging from 7.9 to 9 times that of copper; track rails, from 11 to 13 times that of copper. In manganese steels the ratio sometimes exceeds 30. The effective resistance of rails conveying alternating currents will be increased somewhat on account of skin effect and eddy-currents. See "Report of the Electric Railway Test Commission," McGraw-Hill Book Co., Inc., New York, 1906. Also see Par. 119.

127. Density of pure iron is 7.86, which is fairly precise for wrought iron and steel. The National Tube Co. computes the weight of steel at 0.2833 lb. per cu. in. (489.5 lb. per cu. ft.) and iron at 2 per cent less.

128. Tensile properties of iron and steel wires. The tensile properties are dependent upon the composition of the metal from which the wire is drawn, upon the amount of working the wire has received in the process of manufacture and upon the heat treatment. For information upon the effect of the constituents of iron and steel on the tensile properties, see "Structural Materials," in another portion of this section.

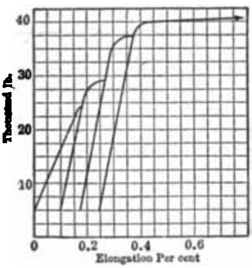

FIG. 11.—Stress-strain diagram of galvanized iron wire.

The tensile strength ranges from about 45,000 lb. per sq. in., for the purest annealed wrought iron, up to extremely high values for hard steel, in the neighborhood of 500,000 lb. per sq. in. Carbon, manganese and silicon are the chief constituents which impart strength and hardness; they also increase the electrical resistivity. Both carbon and manganese decrease the magnetic permeability.

The elastic limit and the yield point occur at about the same relative values as in structural iron and steel; in other words, the elastic ratio does not change.

Fig. 11 shows a typical stress-strain diagram; the wire was 0.164 in. in diameter and broke at 55,100 lb. per sq. in., while the elastic limit was 25,000 lb. per sq. in. and the elongation was 11 per cent. in 60 in. Time was allowed for the wire to set (see Blackwell, F. O. "Conductors for Long Spans;" Trans. Int. Elec. Cong., St. Louis, 1904, Vol. II, pp. 331–347).

Blackwell gives Young's modulus as 24×10^6 lb. per sq. in. for iron wire, 27×10^6 for steel wire, and 22×10^6 for iron and steel concentric cable.

129. Coefficient of expansion. Blackwell gives 0.0000064 per deg. fahr. for iron and steel wire.

130. Specific heat of wrought iron, from 15 to 100 deg. cent., is 0.115; hard-drawn iron, from 0 to 18 deg., 0.0986 and from 20 to 100 deg., 0.115 ("Smithsonian Phys. Tables," 1910).

131. Thermal conductivity of iron in gram-calories (cm-cube) per deg. cent. is from 0.167 to 0.207 at 0 deg. cent., with a negative temperature coefficient of 0.00023 ("Smithsonian Phys. Tables," 1910).

BRONZE

132. Bronze is an alloy of copper and tin, with the addition in some cases of zinc and other metals. There are numerous varieties of bronze, some designated by a prefix indicating the special or distinguishing constituent, and others known by trade names.

133. Phosphor bronze is an alloy of copper, tin and phosphorus, containing from 2 to 6 per cent. of tin and 0.05 to 0.13 per cent. of phosphorus. Its volume conductivity is not over 35 per cent. of that of copper. Industrial bronzes carry zinc and lead, and a larger proportion of phosphorus.

134. Silicon bronze is an alloy of copper, silicon and sodium; tin and zinc

Phono-electric wire, on account of its high tensile properties, has been used for trolley wire and for long spans in transmission lines and in telephone and telegraph lines.

The tensile strength of hard-drawn wire ranges from 68,000 to 84,000 lb. per sq. in. The total elongation at rupture is about 1 per cent. and Young's modulus is about 18,100,000. The temperature coefficient of resistance is 0.00088 per deg. fahr. and the coefficient of linear expansion is 0.0000149 per deg. fahr.

MISCELLANEOUS METALS

138. Resistivity of various metals

(Compiled from "Smithsonian Phys. Tables," 1910)

Metal	Resistivity at 0 deg. cent. (microhm-cm.)	Temp. coef. per deg. cent., at 20 deg. cent.	Density	Therm. cond. (g-cal. per cm. cube per deg. per sec.)
Antimony	35.4 to 45.8	0.00389	6.62 to 6.69	0.044
Arsenic	33.3	....	5.73	....
Bismuth	108.0	0.00354	9.70 to 9.90	0.019
Boron	8×10^{10}	....	2.5 to 2.6	....
Cadmium	6.2 to 7.0	0.00419*	8.54 to 8.67	0.22
Calcium	7.5	....	1.55	....
Cobalt	9.8	0.00325*	8.71	....
Gold	2.04 to 2.09	0.00365	19.3	0.70
Indium	8.38	....	7.12 to 7.42	....
Lead	18.4 to 19.6	0.00387	11.36	0.084
Lithium	8.8	....	0.534	....
Magnesium	4.1 to 5.0	0.00381*	1.69 to 1.75	0.37
Mercury	94.07	0.00072	13.55	0.015
Nickel	10.7 to 12.4	0.00622*	8.60 to 8.90	0.14
Palladium	10.6 to 13.6	0.00354*	11.4	0.17
Platinum	9.0 to 15.5	0.00367*	21.2 to 21.7	0.16
Potassium	25.1	....	0.86 to 0.88	....
Silver	1.5 to 1.7	0.00377	10.4 to 10.6	1.10
Thallium	17.6 to 106	0.00398	11.8 to 11.9	....
Tin	9.53 to 11.4	0.00365	7.30	0.15
Zinc	5.56 to 6.04	0.00365	7.04 to 7.19	0.26

* Average values, for range from 0 to 100 deg. cent.

139. Tungsten.* The tungsten metal of commerce, prior to the discovery of ductile tungsten, was a very hard, dark gray powder; in some cases the metal was heated with low-carbon steel in a crucible furnace, producing the alloy known as ferro-tungsten, containing 80 to 85 per cent. of tungsten. The higher-grade alloys are produced in the electric furnace. Cast tungsten is an extremely hard brittle metal, having a specific gravity of about 18.7. In 1910 a process was announced for the production of ductile tungsten, by rolling, swaging or hammering a heated body of coherent tungsten until it becomes ductile at ordinary temperatures (*Electrical World*, Jan. 10, 1914, pp. 77, 78). The melting point is 3,100 ± 60 deg. cent.

140. Ductile tungsten is a bright, tough, steel-colored metal, which can be drawn into the finest wire. The operation of wire-drawing increases the strength; Fink stated that tungsten wire of 0.0012 in. diam. had a tensile strength from 580,000 to 610,000 lb. per sq. in., and the density increased from 18.81 before drawing, to 19.30 after drawing to 0.15 in. It retains its luster almost indefinitely. Wrought tungsten has been used as a substitute for platinum contacts in electrical apparatus, for targets or anti-

* Baskerville, C. "The Chemistry of Tungsten and the Evolution of the Tungsten Lamp;" Trans. of the New York Electrical Society; New Series, No. 1; Oct. 29, 1912.

147. Table of properties of resistor wires. (See Par. 148 to 152)
(Compiled from manufacturers' data; also, Swoboda, H. O., *The Electric Journal*, May, 1913)

Material	Composition	Resistivity at 20 deg. cent.		Temp. coef. of resistance at 20 deg. cent., per deg. cent.	Thermo-electric power with copper, micro-volts at zero deg. cent.	Density	Tensile strength (lb. per sq. in.)	Coefficient of linear expansion per deg. cent.	Maximum working temp. (deg. cent.)	Approx. melting point (deg. cent.)	Manufacturer*
		Microhm-cm.	Ohms, mil-ft.								
Copper	Cu (annealed)	1.724	10.37	0.00393		8.89	34,000	0.0000166	260	1,083	
No. 312 alloy		7.47	45.0			8.9			200		1
Platinum	Pt	9.53	57.4	0.00367		21.5	50,000	0.0000090	1,500	1,754	
Iron	Fe (annealed)	9.96	59.9								
Nickel	Ni	10.67	64.3	0.004		8.9	160,000	0.0000137	540		1
No. 300 alloy		16.6	100.			8.13			400		1
Ferro-nickel	Fe-Ni	28.2	170.	0.00207		7.8	175,000		340		1
Yankee silver	(Patented)	33.0	200.	0.000155		8.6		0.0000159	480		1
German silver	Cu-Ni(18%)-Zn	33.3	200.	0.00031	20–30	8.5	150,000	0.0000173	260	1,027	1
Nickeline II		33.9	204.	0.000168		8.4					6
Tarnac	Cu-Mn-Ni	42.0	249.	0.000025							2
Manganin	Cu-Mn-Ni	41.4 to 73.8	249. to 443.	0.000011 to 0.000039	1–2				100		1
Monel metal	Cu-Ni	42.6	256.	0.00198		8.9	160,000	0.0000138	480		1
Nickeline I		43.6	262.	0.000076	8.4						6
Therlo	Cu-Mn-Al	46.7	280.	0.0000056	0.3	8.15		0.0000194	200		1
German silver	Cu-Ni(30%)-Zn	48.2	290.	0.000200		8.5				1,160	
Advance	Copper-nickel	48.8	294.	0.000018	40.0	8.9	120,000	0.0000144	480	1,260	1
Ia Ia	Copper-nickel	49.0	295.	0.000005		8.4			370	1,230	6
Raymur	Copper-nickel	49.0	295.	0.000018		8.85					4
Constantin	Copper-nickel	50.0	300.	0.000005	40.0	9.73					8
Ideal	Copper-nickel	50.0	300.	0.000018					350	1,090	2

cent. At zero deg. cent. it has a resistivity of approximately 60,000 ohm-cm. The dielectric constant ranges from 6.1 to 7.4. It has the peculiar property that its resistivity decreases upon exposure to light; the resistivity in darkness may be anywhere from 5 to 200 times the resistivity under exposure to light. See paper by W. J. Hammer, Trans. A. I. E. E., 1903, Vol. XXI, pp. 372 to 393.

RESISTOR MATERIALS

148. German silver is an alloy of copper, nickel and zinc. It is usually listed commercially in terms of its nickel content; thus 18 per cent. wire contains 18 per cent. of nickel. The properties vary considerably with the composition. Perrine gave the following composition of three grades of German silver: 57 Cu, 12.5 Ni, 30.5 Zn; 56 Cu, 20 Ni, 24 Zn; 50 Cu, 30 Ni, 20 Zn. The resistivities were respectively in the ratio 1 : 1.25 : 2.51. Eighteen per cent. alloy has about 18 times the resistivity of copper, and 30 per cent. alloy has about 28 times the resistivity of copper. See Par. **147**.

149. Copper-manganese alloy containing either nickel or aluminum is used for resistors, and has a very low temperature coefficient. The alloy composed of copper, ferro-manganese and nickel, or copper, manganese and nickel, is known as **manganin**. The composition of manganin varies somewhat, one formula being 65 Cu, 30 Fe-Mn, 5 Ni.

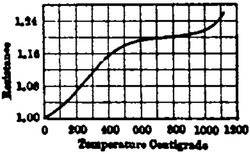

FIG. 13.—Resistance-temperature curve for nichrome.

150. Copper-nickel alloy is used extensively for resistor wires. The alloy of copper and nickel found in nature is known as Monel metal (Par. **399**). See Par. **147**.

151. Nickel-steel alloy has a very high electrical resistivity but is not as resistant to corrosion, for resistor service, as some other alloys. The nickel-chromium alloys are superior in the respect of having somewhat larger resistivity. See Par. **147**.

152. Nickel-chromium alloy is used for resistor wires where very high resistivity is desired. One alloy of this kind has a resistivity of more than 700 ohms per mil-ft. The nickel-chromium alloy known as "Nichrome" has a characteristic resistance-temperature curve of the form shown in Fig. 13.

CARBON AND GRAPHITE

153. Forms of carbon. Carbon occurs in two forms, amorphous and crystalline. The crystalline forms include **diamond** and **graphite**, the latter also being known as **plumbago.** The amorphous forms include charcoal, coke, lamp black, bone black; coal is an impure variety of amorphous carbon. The density of carbon in the diamond state is 3.47 to 3.56; graphite, 2.10 to 2.32; charcoal, 0.28 to 0.57; coke, 1.0 to 1.7; gas carbon, 1.88; lampblack, 1.7 to 1.8.

154. Resistivity. The resistivity of amorphous carbon (petroleum coke) at ordinary temperature (25 deg. cent.) may be taken as varying between 3,800 and 4,100 microhm-cm. An average value for retort carbon, such as used for electrodes in electric furnaces, at about 3,000 deg. cent., may be taken as 720 microhm-cm. Graphite at 3,000 deg. cent. has a resistivity of approximately 812 microhm-cm. Experiments by Mr. C. A. Hanson* made in the research laboratory of the General Electric Company show that the resistivity of carbon depends upon the temperature at which it is fired. As the temperature of firing increases, the resistivity decreases, approaching a constant value which is approximately the same as that of graphite. If carbon be heated above the temperature at which it was fired, its resistivity is permanently decreased, and upon cooling it will not return to its original value, but to a value corresponding to that which it would have if fired at the temperature to which it has been heated. Also see table of brush characteristics, Par. **155**, and electrode carbon, Par. **159**.

Experiments made by Morris Owen show that graphite possesses magnetic susceptibility, and under certain conditions the electrical resistivity is increased in very marked degree by magnetization.

* *Electrochem. and Met. Ind.*, Vol. VII, p. 514 (1909).

156. Resistance of arc lamp carbons. The resistance of $\frac{1}{2}$ in. × 12 in. enclosed arc carbons varies from 0.012 to 0.015 ohms per linear inch. Other sizes down to $\frac{3}{8}$ in. diameter vary according to their cross-sectional areas. The resistance of a $\frac{5}{8}$-in. diameter projector carbon varies from 0.009 to 0.011 ohms per linear inch. The $\frac{1}{2}$-in. and $\frac{7}{16}$-in. carbons vary according to their cross-sectional areas.

All high-grade forms of carbon, such as that used in the manufacture of search-light carbons and also enclosed arc carbons, may be given the value of about 0.002 ohms per cu. in. Flame-arc carbon material such as is used in the homogeneous electrodes varies from 0.004 to 0.006 ohms per cu. in. All the above values are for ordinary room temperatures.

157. Temperature coefficient of resistance. Carbon exhibits a decreasing electrical resistivity and a decreasing thermal resistivity with rising temperature. Graphite exhibits but little change in electrical resistivity, tending downward with rising temperature, but its thermal resistivity increases slightly with rising temperature. The coefficients vary over a considerable range; see Landolt and Bornstein, "Physikalish-Chemische Tabellen," 1912.

158. Resistances of carbon contacts vary with pressure, current and time. See results of investigation published by A. L. Clark in the *Physical Review*, Jan., 1913.

159. Electrode properties of carbon and graphite were given by Hering in his paper, "The Proportioning of Electrodes for Furnaces" (Trans. A. I. E. E., Vol. XXIX, 1910, pp. 485–545), from experimental determinations. The table below is abstracted from Table I in Hering's paper above mentioned; the paper itself gives elaborate details and many curves. Other papers by Hering on this general subject are noted below.*

Material	Furnace temp. (deg. cent.)	Temp. drop (deg. cent.)	Electrical resistivity, ohm (in-cube)	Thermal conductivity, †watts (in-cube)
Carbon......	20.0	0.0	0.00181	....
	360.0	260.0	166	0.95
	751.0	651.0	150	1.32
	942.0	842.0	148	1.38
Graphite.....	20.0	0.0	0.000337	....
	389.6	289.6	330	3.60
	546.1	446.1	324	3.45
	720.2	620.2	316	3.26
	913.9	813.9	323	3.10

† 1 watt = 0.2389 g-cal. per sec.; 1 g-cal. per sec. = 4.186 watts.

SKIN EFFECT

160. Skin effect is briefly defined in Sec. 2. Also see Sec. 12, Par. 41.

161. Formulas and tables for skin effect. If R' is the effective resistance of a linear cylindrical conductor to sinusoidal alternating current of given frequency and R is the true resistance with continuous current, then

$$R' = KR \qquad \text{(ohms)} \qquad (13)$$

where K is determined from the table in Par. 165, in terms of x. The value of x is given by

$$x = 2\pi a \sqrt{\frac{2f\mu}{\rho}} \qquad (14)$$

* "Laws of Electrode Losses in Electric Furnaces;" *Trans.* A. E. S., Vol. XVI, 1909.

"Empirical Laws of Furnace Electrodes;" *Trans.* A. E. S., Vol. XVII, 1910.

"The Design of Furnace Electrodes;" *Electrical World*, June 16, 1910.

10, 1914). In No. 12 B.W.G. iron wire (B. B. grade), at 800 cycles per sec., and with currents of telephonic magnitude, the increase in resistance was found by measurement to be 47 per cent. See Par. 203.

164. Effect of very high frequencies on iron has been investigated by E. F. W. Alexanderson; see "Magnetic Properties of Iron at Frequencies up to 200,000 Cycles," *Trans.* A. I. E. E., Vol. XXX, 1911, pp. 2433–2454. He concluded that the permeability is unaffected by the frequency. In applying Steinmetz's formula for skin effect (see "Transient Electric Phenomena and Oscillations," New York, 1909), he recommended using average constants as follows: permeability, 2,250 and conductivity, 0.9×10^5, for soft iron.

165. Table of constants for skin-effect formulas

x	K	K'	x	K	K'	x	K	K'
0.0	1.00000	1.00000	4.0	1.67787	0.68632	12.5	4.67993	0.22567
0.1	1.00000	1.00000	4.1	1.71516	0.67135	13.0	4.85631	0.21703
0.2	1.00001	1.00000	4.2	1.75233	0.65677	13.5	5.03272	0.20903
0.3	1.00004	0.99998	4.3	1.78933	0.64262	14.0	5.20915	0.20160
0.4	1.00013	0.99993	4.4	1.82614	0.62890	14.5	5.38560	0.19468
0.5	1.00032	0.99984	4.5	1.86275	0.61563	15.0	5.56208	0.18822
0.6	1.00067	0.99966	4.6	1.89914	0.60281	16.0	5.91509	0.17649
0.7	1.00124	0.99937	4.7	1.93533	0.59044	17.0	6.26817	0.16614
0.8	1.00212	0.99894	4.8	1.97131	0.57852	18.0	6.62129	0.15694
0.9	1.00340	0.99830	4.9	2.00710	0.56703	19.0	6.97446	0.14870
1.0	1.00519	0.99741	5.0	2.04272	0.55597	20.0	7.32767	0.14128
1.1	1.00758	0.99621	5.2	2.11353	0.53506	21.0	7.68091	0.13456
1.2	1.01071	0.99465	5.4	2.18389	0.51566	22.0	8.03418	0.12846
1.3	1.01470	0.99266	5.6	2.25393	0.49764	23.0	8.38748	0.12288
1.4	1.01969	0.99017	5.8	2.32380	0.48086	24.0	8.74079	0.11777
1.5	1.02582	0.98711	6.0	2.39359	0.46521	25.0	9.09412	0.11307
1.6	1.03323	0.98342	6.2	2.46338	0.45056	26.0	9.44748	0.10872
1.7	1.04205	0.97904	6.4	2.53321	0.43682	28.0	10.15422	0.10096
1.8	1.05240	0.97390	6.6	2.60313	0.42389	30.0	10.86101	0.09424
1.9	1.06440	0.96795	6.8	2.67312	0.41171	32.0	11.56785	0.08835
2.0	1.07816	0.96113	7.0	2.74319	0.40021	34.0	12.27471	0.08316
2.1	1.09375	0.95343	7.2	2.81334	0.38933	36.0	12.98160	0.07854
2.2	1.11126	0.94482	7.4	2.88355	0.37902	38.0	13.68852	0.07441
2.3	1.13069	0.93527	7.6	2.95380	0.36923	40.0	14.39545	0.07069
2.4	1.15207	0.92482	7.8	3.02411	0.35992	42.0	15.10240	0.06733
2.5	1.17538	0.91347	8.0	3.09445	0.35107	44.0	15.80936	0.06427
2.6	1.20056	0.90126	8.2	3.16480	0.34263	46.0	16.51634	0.06148
2.7	1.22753	0.88825	8.4	3.23518	0.33460	48.0	17.22333	0.05892
2.8	1.25620	0.87451	8.6	3.30557	0.32692	50.0	17.93032	0.05656
2.9	1.28644	0.86012	8.8	3.37597	0.31958	60.0	21.46541	0.04713
3.0	1.31809	0.84517	9.0	3.44638	0.31257	70.0	25.00063	0.04040
3.1	1.35102	0.82975	9.2	3.51680	0.30585	80.0	28.53593	0.03535
3.2	1.38504	0.81397	9.4	3.58723	0.29941	90.0	32.07127	0.03142
3.3	1.41999	0.79794	9.6	3.65766	0.29324	100.0	35.60666	0.02828
3.4	1.45570	0.78175	9.8	3.72812	0.28731	∞	∞	0
3.5	1.49202	0.76550	10.0	3.79857	0.28162			
3.6	1.52879	0.74929	10.5	3.97477	0.26832			
3.7	1.56587	0.73320	11.0	4.15100	0.25622			
3.8	1.60314	0.71729	11.5	4.32727	0.24516			
3.9	1.64051	0.70165	12.0	4.50358	0.23501			

gausses, after which the permeability decreases below that of soft iron, while the iron loss begins to increase rapidly. See also Ruder, W. E., "The Effect of Chemical Composition upon the Magnetic Properties of Steels;" *General Electric Review*, March, 1915, pp. 197 to 203.

174. Effect of carbon. Carbon increases the resistivity, decreases the permeability, lowers the saturation point and increases the coercive force and the retentivity. Concurrently the hysteresis loop is broadened and its area increased. These effects are greater in hardened steel than in soft or annealed material. In slowly cooled iron-carbon alloys the carbon exists as pearlite up to the eutectoid point (about 0.85 carbon); above this point the carbon exists as cementite (Fe_3C). The cementite carbon diminishes the conductivity less than does the pearlite carbon. At a quenching temperature of 850 deg. cent. the limit of dissolved carbon is about 1.4 per cent.; no excess of carbon above 1.4 is soluble at this temperature.

175. Effect of manganese. Very small proportions of manganese are not injurious in any substantial degree, but it is customary to limit the proportion of manganese as much as practicable. The true effect of small proportions of manganese is difficult to determine because of its association in most cases with carbon. See *Jour.* I. E. E., April, 1911, Vol. XLVI, No. 206, pp. 263 to 266. When the manganese content reaches 12 per cent. the steel becomes practically non-magnetic.

176. Effects of silicon and aluminum. The researches of Barrett, Brown and Hadfield (1900 and 1902) established the fact that the only magnetic alloys superior to the purest commercial iron are the alloys of iron with silicon, and with aluminum. The best silicon alloy contained 2.5 per cent. of silicon, and the best aluminum alloy contained 2.25 per cent. of aluminum.

	Maximum permeability	$\mathfrak{B}$ for maximum permeability	Hysteresis loss, ergs per cu. cm. per cycle for $\mathfrak{B}$ (max) = 9,000	Coercive force for $\mathfrak{B}$ (max) = 17.700
Swedish charcoal iron.....	2,100	4,000	2,334	1.10
2.5 per cent. silicon......	5,000	4,000	1,549	0.80
2.25 per cent. aluminum..	5,400	5,000	1,443	0.80

Guggenheim has shown (Elek. Kraft U. Bahnen, Sept. 24, 1910), for iron containing 0.2 per cent. of carbon, that silicon in quantities up to 1.8 per cent. decreases the permeability, but from 1.8 to 5 per cent. it improves the permeability and decreases the hysteresis loss; for $\mathfrak{B}$(max) = 10,000 in sheets 0.5 mm. thick the hysteresis loss was 2,910 ergs per cu. cm. per cycle, for best silicon steel, compared with 6,000 ergs for ordinary sheet iron.

For electrical and mechanical effects of silicon and aluminum, see appropriate portions of this section.

177. Effect of nickel. The addition of nickel, up to 2 per cent., causes little change in magnetic quality (Burgess and Aston). A higher nickel content rapidly decreases the permeability. At 25 to 30 per cent. nickel the magnetic properties are greatly impaired, but improve again upon a further increase in nickel.

178. Effects of tungsten, chromium and molybdenum. These elements have the general property of increasing the magnetic hardness and particularly the coercive force, making a very desirable steel for permanent magnets. See "Magnet Steel," Par. **225** to **230**.

179. Effects of arsenic and tin. These elements are similar in their effects to silicon and aluminum, increasing the resistivity and reducing the hysteresis loss. Tin increases the permeability at higher inductions and decreases the hysteresis loss even more than silicon.

180. Sulphur, phosphorus and oxygen are in general injurious in their effects, even in small percentages.

heated for 27 days at 50 deg. cent.; 53 per cent. when heated for 25 days at 65 deg. cent.; 89 per cent. when heated for 25 days at 87 deg. cent.; 140 per cent. when heated for 25 days at 135 deg. cent. Also see Mordey, W. M., *Proc.* Roy. Soc., June, 1895; also see aging tests in "Electric Machine Design," by Parshall and Hobart; Allen, T. S., "The Comparative Aging of Electric Sheet Steels;" *Electrical World*, 1908, Vol. LII, p. 579.

187. Non-aging steel. Silicon-steel, aside from having low hysteresis and high resistivity, also possesses the valuable property of being non-aging. That is to say, its magnetic properties are not impaired by prolonged heating at moderate temperatures, but on the contrary may be slightly improved. While as much as 3 to 4 per cent. of silicon is present in silicon-steel, it is also useful, in much smaller quantities, in improving the aging qualities of low-carbon steel. Parshall and Hobart recommend ("Electric Machine Design," p. 36) the following composition for sheet steel having good aging qualities: carbon, 0.06; manganese, 0.50; silicon, 0.01; sulphur, 0.03; phosphorus, 0.08.

188. Effects of mechanical stress on magnetization. Ewing states (Chap. IX, "Magnetic Induction in Iron and Other Metals") that the presence of any moderate amount of longitudinal pull increases the susceptibility when the magnetism is weak, but reduces it when the magnetism is strong. With hardened metal the effects of stress are in general much greater than with annealed metal.

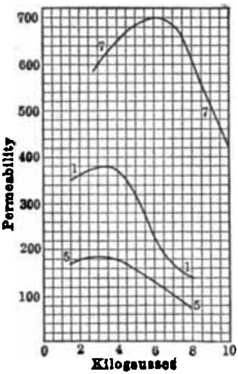

FIG. 19.—Induction-permeability curves of cast iron and malleable iron.

189. Page effect is the faint metallic sound resembling a light blow which is heard when a piece of iron is suddenly magnetised or demagnetised.

190. Cast iron is magnetically inferior to wrought iron or low steel, but is used to a limited extent on account of the facility with which it can be molded into complex forms. The permeability is decreased by the presence of carbon, the effect being in the ratio of combined to graphitic carbon. Cast iron of good magnetic quality contains from 3 to 4.5 per cent. carbon, of which from 0.2 to 0.8 per cent. is in the combined form. A normal induction curve for cast iron is given in Fig. 18. Curve 1 in Fig. 19 applies to cast iron containing 0.195 combined carbon, 3.29 graphitic carbon, 2.01 silicon, 0.320 manganese, 0.988 phosphorus and 0.08 sulphur. Curve 5 is for cast iron containing 0.72 combined carbon, 2.07 silicon, 0.38 manganese, 0.85 phosphorus and 0.035 sulphur.

Silicon and aluminum in small proportions make the casting more homogeneous and tend to reduce the combined carbon. Silicon tends also to counteract sulphur.

191. Malleable cast iron is magnetically superior to cast iron, being lower in combined carbon and improved by the heat treatment which it receives. Curve 7 in Fig. 19 is for malleable cast iron (see Parshall and Hobart, "Electric Machine Design") containing 0.83 combined carbon, 2.201 graphitic carbon, 0.93 silicon, 0.116 manganese, 0.039 phosphorus and 0.080 sulphur.

192. Wrought iron is among the best of magnetic materials from the standpoint of permeability, but has higher core losses than silicon steel. See Par. **200** comparing Swedish iron with other materials.

193. Rolled steel of the low-carbon variety is used very extensively in the form of electrical sheets and rods. A normal induction curve is shown in Fig. 18. Commercial sheets are described in Par. **217** to **224**.

194. Cast steel is extensively used for those portions of magnetic circuits which carry uniform or continuous flux and need superior mechanical strength. Parshall and Hobart state ("Electric Machine Design," p. 22) that cast steel of good magnetic qualities should be limited in its composition as follows: combined carbon, 0.25; silicon, 0.20; manganese, 0.50; phosmagnetism (Hadfield, Sir R. A. and Hopkinson, B., "The Magnetic Properties of Iron and Its Alloys in Intense Fields," *Jour.* I. E. E., April, 1910, Vol. XLVI, pp. 235–306). The intensity of magnetization is defined as the quantity J in the formula $\mathcal{B} = \mathcal{H} + 4\pi J$, or the magnetic moment per unit of volume.

Every alloy which they examined was found to have a definite saturation intensity of magnetism, which they termed the **specific magnetism.** This intensity was reached in most cases in a field of 5,000 units. The specific magnetism of commercially pure iron of density 7.80 was found to be 1,680 within 1 per cent. The presence of carbon in annealed iron-carbon steel reduces the specific magnetism by a percentage equal to six times the percentage of carbon, if other elements are present only in small proportions. No alloy was noted having a higher specific magnetism than pure iron. Quenching an iron-carbon alloy from a high temperature reduces its specific magnetism by a large but uncertain amount. The addition of silicon or aluminum to iron reduces the specific magnetism roughly in proportion to the amount added; but if carbon is present, silicon seems to neutralize it to some extent.

200. Comparisons of magnetic materials

(T. D. Yensen, Bulletin No. 72, Eng. Exp. Sta., Univ. of Ill., 1914)

Material	Carbon content (per cent.)	Max. permeability	Flux density for max. permeability	Hysteresis loss in ergs per cu. cm. per cycle		Coercive force for $\mathcal{B}$ (max.) = 15,000	Retentivity for $\mathcal{B}$ (max.) = 15,000	Resistivity at 20 deg. cent. (microhm-cm.)
				$\mathcal{B}$ (max.) = 10,000	$\mathcal{B}$ (max.) = 15,000			
Swedish charcoal iron cut from plate.	0.183	4,870	6,600	2,490	4,530	0.95	8,000	10.57
Standard transformer steel.	......	3,850	7,000	3,320	5,910	1.33	9,900	11.09
Four per cent. silicon steel.	......	3,400	4,300	2,260	3,030	0.88	5,400	51.15
Swedish charcoal iron remelted *in vacuo.*	0.008	10,350	7,000	1,290	2,640	0.48	11,200	10.30
Electrolytic iron melted *in vacuo.*	0.0125	12,950	6,550	1,060	1,990	0.34	9,940	9.96

201. Formula for induction-permeability curve. A. S. McAllister has shown that the induction-permeability curve can be expressed with a fair degree of accuracy by an equation of the form

$$\mu = 2{,}800 - 3.2\left[\frac{(7{,}500 - \mathcal{B})^2}{10^6}\right] \qquad (17)$$

The constants in this equation hold for the ordinary grades of sheet iron, between the limits $\mathcal{B} = 0$ and $\mathcal{B} = 15{,}000$, where $\mathcal{B}$ is the maximum instantaneous value. The numerical constants take different values for cast iron, cast steel, silicon steel, etc. See McAllister, "Alternating-current Motors," New York, 1909, p. 137.

202. Permeability in weak fields. Ewing gives the permeability in very weak fields, with values of $\mathcal{H}$ less than unity, according to the following formula based on investigations by Baur.

$$\mu = 183 + 1{,}382\mathcal{H} \qquad (18)$$

This applies to soft iron. Lord Rayleigh found for harder grades of iron,

$$\mu = 81 + 64\mathcal{H} \qquad (19)$$

208. Hysteresis coefficients in Steinmetz's formula $W_h = \eta f B^{1.6}$ are given in Par. **207** (Lloyd, M. G., "Magnetic Hysteresis," Journal of the Franklin Institute, July, 1910, pp. 1–25). In this formula W_h is the loss in ergs per cu. cm. per sec., f is the frequency in cycles per sec., $\mathcal{B}$ is the maximum induction and is here 10,000 gausses, and η is the hysteresis coefficient. The exponent of $\mathcal{B}$, which is 1.6, departs widely from this value at very low and very high densities.

209. Total core losses for sheets are given in Fig. 29. Lloyd and Fisher give the following total core losses, in watts per lb. at 60 cycles and 10,000 gausses, for No. 29 gage (3.57 mm.): unannealed, 3.18 to 4.76; annealed, 1.25 to 2.36; silicon steel, 0.665 to 1.06. See *Trans.* A. I. E. E., 1909, Vol. XXVIII, p. 465. Also see Sec. 7, Par. **214.**

210. Effect of wave form upon hysteresis loss. Dr. M. G. Lloyd, in his paper entitled "Dependence of Magnetic Hysteresis upon Wave Form" (Bulletin of the Bureau of Standards, Vol. V, No. 3, Feb. 1909, pp. 381–411), reached the following conclusions.

"For a definite maximum value of the flux density, the hysteresis is greater with a flat wave of flux, but the effect is small, and from the industrial standpoint negligible, even with very distorted waves. If, however, the wave of flux is dimpled, the hysteresis may be much increased.

"The hysteresis determined by the ballistic method may be smaller than that which obtains with the use of alternating current, but the differences are small.

"The separation of hysteresis and eddy-current losses by means of runs at two frequencies, using the Steinmetz formula, is not accurate, but is a close approximation when the sheets are thin."

211. Effect of form-factor upon the iron loss was investigated by Dr. M. G. Lloyd (see "Effect of Wave Form upon the Iron Losses in Transformers," Bulletin of the Bureau of Standards, Vol. IV, No. 4, 1907; also Reprint No. 88), who reached the following conclusions.

"With a given effective electromotive force the iron losses in a transformer depend upon the form-factor of the e.m.f., and vary inversely with it. By proper design of the generator supplying transformers, the iron losses may be reduced to a minimum."

212. Effect of unsymmetrical periodic cycles on hysteresis loss. Mr. M. Rosenbaum in a paper entitled "Hysteresis Loss in Iron, Taken Through Unsymmetrical Cycles of Constant Amplitude," before the I. E. E. (see *Jour.* I. E. E., Mar., 1912), presented the following conclusions.

"It is seen that the hysteresis loss increases very appreciably as direct-current magnetisation is superposed on the alternating flux. This phenomenon manifests itself in practice in inductor alternators and static balancers. In some kinds of inductor alternators the flux does not reverse, but oscillates between positive maximum and positive minimum values, thus the iron loss per cu. cm. is much greater in inductor alternators than in the ordinary type for the same change of flux in the armature coil. In static balancers this effect also takes place where the direct-current magnetisation is not neutralised. It is therefore important, if high efficiency be aimed at, to neutralise the direct-current flux."

SHEET GAGES

213. Two systems of gaging sheets are in use, the U. S. Standard Gage and the Decimal Gage. These two gages are fully covered in the two succeeding paragraphs. Also see Par. **30.**

214. An act establishing a standard gage for sheet and plate iron and steel. *Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled,* That for the purpose of securing uniformity, the following is established as the only standard gage for sheet and plate iron and steel in the United States of America, namely: (See Par. **215.**) And on and after July 1, 1893, the same and no other shall be used in determining duties and taxes levied by the United States of America on sheet and plate iron and steel. But this act shall not be construed to increase duties upon any articles which may be imported.

Sec. 2. That the Secretary of the Treasury is authorised and required to prepare suitable standards in accordance herewith.

Sec. 3. That in the practical use and application of the standard gage hereby established a variation of 2.5 per cent. either way may be allowed.

Approved, March 3, 1893.

215. U. S. standard sheet gage.—*Continued*

Number of gage	Approximate thickness in fractions of an inch	Approximate thickness in decimal parts of an inch	Approximate thickness in millimeters	Weight per square foot in ounces avoirdupois	Weight per square foot in pounds avoirdupois	Weight per square foot in kilograms	Weight per square meter in kilograms
19	7-160	0.043(75)	1.11(125)	28	1.75	0.79(88)	8.5(44)
20	3-80	0.0375	0.95(25)	24	1.50	0.68(04)	7.3(24)
21	11-320	0.0343(75)	0.87(3125)	22	1.37(5)	0.62(37)	6.7(13)
22	1-32	0.0312(5)	0.79(3750)	20	1.25	0.56(7)	6.1(03)
23	9-320	0.0281(25)	0.71(4375)	18	1.12(5)	0.51(03)	5.4(93)
24	1-40	0.025	0.63(5)	16	1.0	0.45(36)	4.8(82)
25	7-320	0.0218(75)	0.55(5625)	14	0.87(5)	0.396(9)	4.2(72)
26	3-160	0.0187(5)	0.47(625)	12	0.75	0.340(2)	3.66(2)
27	11-640	0.0171(875)	0.43(65625)	11	0.68(75)	0.311(9)	3.35(7)
28	1-64	0.0156(25)	0.396(875)	10	0.62(5)	0.283(5)	3.05(2)
29	9-640	0.0140(625)	0.357(1875)	9	0.56(25)	0.255(1)	2.74(6)
30	1-80	0.0125	0.317(5)	8	0.5	0.226(8)	2.44(1)
31	7-640	0.0109(375)	0.277(8125)	7	0.43(75)	0.198(4)	2.13(6)
32	13-1280	0.0101(5625)	0.257(96875)	6½	0.40(625)	0.184(3)	1.98(3)
33	3-320	0.0093(75)	0.238(125)	6	0.375	0.170(1)	1.83(1)
34	11-1280	0.0085(9375)	0.218(28125)	5½	0.343(75)	0.155(9)	1.67(8)
35	5-640	0.0078(125)	0.198(4375)	5	0.312(5)	0.141(7)	1.52(6)
36	9-1280	0.0070(3125)	0.178(59375)	4½	0.281(25)	0.127(6)	1.37(3)
37	17-2560	0.0066(40625)	0.168(671875)	4¼	0.265(625)	0.120(5)	1.29(7)
38	1-160	0.0062(5)	0.158(75)	4	0.25	0.113(4)	1.22(1)

NOTE.—Numbers within parentheses represent higher precision than the tolerance limit of plus or minus 2.5 per cent.

rotors and stators. The Transformer grade is used very extensively in transformer cores.

Curves of average induction, permeability and hysteresis in these three grades are given in Figs. 20 to 25.

220. Aging of sheets. The American Sheet and Tin Plate Company reports the following results of aging tests on their commercial sheets (Par. 219). Exposure to a temperature of 100 deg. cent. for 30 days resulted (on

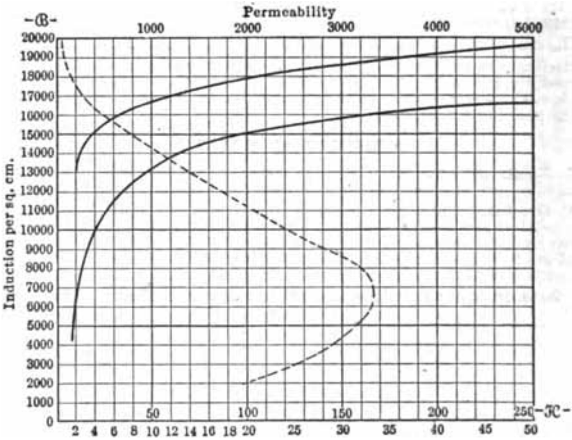

Fig. 20.—Average induction and permeability curves, "Regular Dynamo and Motor Sheets" (A. S. & T. P. Co.).

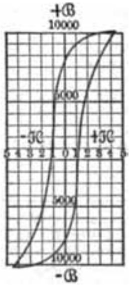

Fig. 21.—Average hysteresis loop, "Regular Dynamo and Motor Sheets" (A. S. & T. P. Co.).

the average of a number of samples of each grade) in a small increase in the iron loss (not exceeding 10 per cent.) in the Regular grade, a very slight increase in the iron loss of the Special grade, and a slight decrease in the iron loss of the Transformer grade.

221. Effect of mechanical working on sheets. The only working the material receives is punching and compression in the finished core. The

224. Curves of total core losses in electrical sheets are given in Fig. 29, taken from the article on "Transformers," by Dr. A. S. McAllister in the 3rd edition of the Standard Handbook.

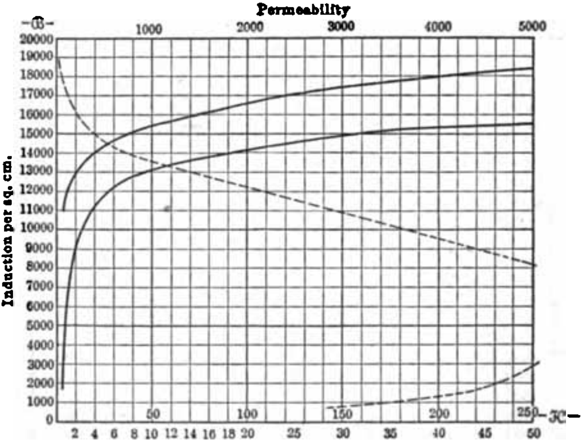

FIG. 24.—Average induction and permeability curve, "Transformer Sheets" (A. S. & T. P. Co.).

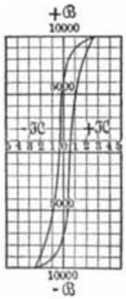

FIG. 25.—Average hysteresis loop, "Transformer Sheets" (A. S. & T. P. Co.).

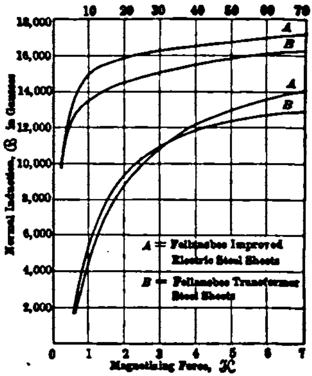

FIG. 26.—Normal induction curves of Follansbee steel sheets.

MAGNET STEEL

225. Desired characteristics in permanent magnets are maximum retentivity, coercive force, and permanence or non-aging characteristic. These characteristics are best obtained in carbon steel and certain alloy steels hereafter mentioned.

retentivity of carbon steel. The hardness tests were made with a 10-mm. Brinnell ball and a pressure of 3,000 kg. The retentivity increases with the carbon content and hardness up to a point where further increase in carbon is not accompanied by increase in hardness, then the retentivity decreases due to displacement of effective iron by the carbon. Similar experiments with various alloy steels give results shown in Par. 230.

228. Tungsten magnet steel ordinarily contains about 5 per cent. of tungsten. One satisfactory magnet steel analyzed 5.47 per cent. of tungsten, 0.57 per cent. of carbon, 0.18 per cent. of silicon, and 0.26 per cent. of manganese. Tungsten-vanadium magnet steel analyses about 7.00 per cent. of tungsten, 0.30 per cent. of vanadium, and 0.60 per cent. of carbon.

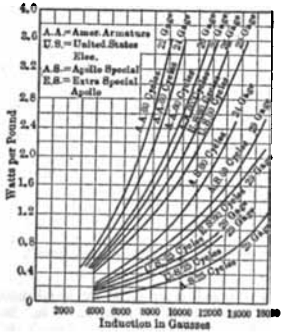

FIG. 29.—Curves of total core losses in electrical sheets.

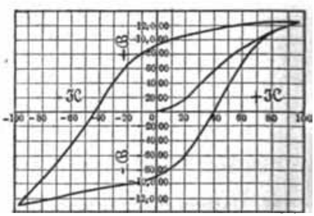

FIG. 30.—Hysteresis loop of glass-hard steel wire.

The addition of tungsten to steel increases the coercive force, and according to Hopkinson, the value of the coercive force in tungsten steel may exceed 50 (Ewing, J. A. "Magnetic Induction in Iron and Other Metals;" London, 1900; 3rd rev. ed., p. 83). Comparative tests of four tungsten steels containing respectively 3, 6, 8 and 12 per cent. of tungsten, quenched from 900 deg. cent., showed remarkably similar results and indicated that a high percentage of tungsten is unnecessary. In fact as much as 12 per cent. may produce inferior results (see Moir, M. B. *Philosophical Magazine*, Nov., 1914). Also see Sec. 5.

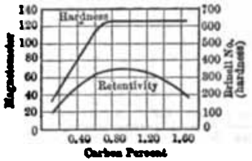

FIG. 31.—Hardness and retentivity of carbon-steel.

229. Chrome magnet steel. According to Hopkinson the addition of chrome to oil-tempered steel may increase the coercive force to as much as 40 (see *Philosophical Transactions*, 1885). Chrome-steels containing upward of 8 per cent. of chromium possess superior permanency compared with tungsten steels, but are inferior in magnetic intensity (see Moir, M. B. *Philosophical Magazine;* Nov., 1914).

Class C. This is represented by fireproof, or heat-proof materials, such as mica, so assembled that very high temperatures do not produce rapid deterioration. Such materials are used in rheostats and in the heating elements of heating appliances, etc.

The temperature limits specified by the A. I. E. E. Standardization Rules (Sec. 24, Par. 188) are as follows:

(*A*-1) Cotton, silk, paper and other fibrous materials, not so treated as to increase the temperature limit, 95 deg. cent.

(*A*-2) Same as *A*-1, but treated or impregnated, and including enameled wire, 105 deg. cent.

(*B*) Mica, asbestos or any other material capable of resisting high temperature, in which any class *A* material or binder, if used, is for structural purposes only, and may be destroyed without impairing the insulating or mechanical qualities, 125 deg. cent.

(*C*) Fireproof and refractory materials, no specified limit.

235. Physical Classification of Dielectrics

- Dielectrics
  - Solids
    - Natural
      - Gums and resins
      - Asbestos
      - Wood
      - Soapstone
      - Slate
      - Marble
      - Lava
      - Mica
    - Fabricated
      - Papers and sheets
      - Fabrics and yarns
      - Hard rubber
      - Synthetic resins
      - Molded compositions
      - Glass
      - Vitrified materials
  - Plastics
    - Caoutchouc
    - Gutta percha
    - Pitches
    - Asphalts
    - Waxes
    - Compounds
  - Liquids
    - Used as such
      - Mineral oil
      - Animal oil
      - Vegetable oil
    - Solidified on application
      - Varnish
      - Shellac
      - Paint
      - Enamel
      - Japan
  - Gases
    - Atmospheric air
    - Hydrogen
    - Nitrogen
    - Carbon dioxide

236. Classification of dielectrics according to type of application. Under this classification can be named: (1) Wire insulation; (2) cable insulation; (3) insulating supports, combining dielectric properties with mechanical strength; (4) coil and slot insulation for electrical apparatus and machinery; (5) insulating sheets, slabs and barriers; (6) molded insulation, shaped under the application of heat and mechanical pressure; (7) impregnating and filling compounds; (8) superficial paints and varnishes; (9) fluid insulators; (10) gaseous insulators.

237. Use of trade names in connection with insulating materials has unfortunately become very common. On account of the great number of such names no attempt has been made to state or define them all. Wherever feasible, insulating materials have been grouped and described in accordance with a rational classification, and adhering if possible to the natural or descriptive name of each thing instead of its trade name.

241. Surface insulation resistance. The surface conductivity is the reciprocal of the surface resistivity, and the **surface resistivity** is the resistance between two opposite edges of a surface film which is 1 cm. square. Since for most materials under ordinary conditions of humidity the surface resistivity is much lower than the volume resistivity, the resistance per centimeter length between two linear conductors 1 cm. apart, pressed upon the surface of a slab of the material, is approximately equal to the surface resistivity.

The relationship between surface resistivity and humidity, for a number of different materials, is given in Fig. 32. These curves are generally typical of solid dielectrics. The surface resistivity is often a million times as great at low humidity as at high humidity. Measurements made on various molded compositions at the Bureau of Standards (Scientific Paper No. 234) gave values ranging from 10^{10} to 10^{17} ohm-cm. (between opposite edges of a film 1 cm. square), at 22 deg. cent.

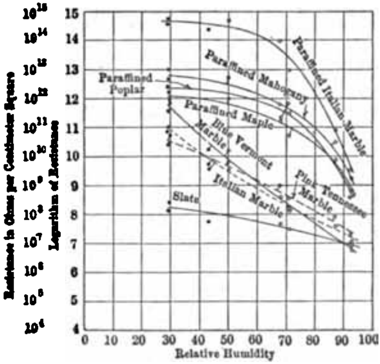

FIG. 32.

242. Dielectric constant is defined in Sec. 2. The customary medium of reference is almost universally dry air at normal temperature and pressure. For most materials the temperature coefficient is positive, but for india-rubber it appears to be negative at ordinary temperatures. The coefficient for celluloid at 15 deg. cent. is approximately 0.005 per deg. cent., whereas for mica it is about 0.0006.

243. Dielectric absorption is the term applied to the apparent soaking up of electric charge within the body of a dielectric when the electric stress is prolonged for an appreciable time. In other words, it requires appreciable time for the dielectric to become fully saturated with electric charge or displacement, when a steady stress is applied. In some materials this phenomenon is marked, while in others it is slight. Due allowance should be made for it, however, in certain classes of measurements, notably insulation resistance.

244. Dielectric strength, usually expressed in volts or kilovolts (kv.) per mil or per mm.,* is a property which it is impossible to determine with high precision, and which is affected by numerous variables such as the size and shape of the test electrodes, the time rate at which the test voltage is raised to the disruptive point, the order of frequency of the test voltage and the thickness of the test specimen. Furthermore, disruptive discharge requires not merely a sufficiently high voltage, but a certain minimum amount of energy. There is no accepted standard apparatus or method for testing dielectrics, and hence the results obtained by different investigators are comparable only as to general order of magnitude, if at all. Moreover the probable error in measurements of disruptive strength is unusually large and any one set of observations is likely of itself to be in error by as much as plus or minus 10 to 20 per cent. Consequently the values of disruptive strength stated in this section are to be considered as purely approximate and not accurately comparative.

245. The factors affecting dielectric strength may be stated as follows; internal or external heating, chemical change, absorption of moisture,

* 1 volt per mil = 39.4 volts per mm.
1 volt per mil = 0.394 kv. per cm.
1 volt per mm. = 0.0254 volt per mil.
1 kv. per cm. = 2.54 volts per mil.

before breakdown occurs is sometimes referred to as the dielectric **spark lag**. A good illustration of the comparative effects of slow versus fast rate of application of the disruptive voltage is given by Creighton for porcelain (see *Proc.* A. I. E. E., May, 1915, p. 818, Fig. 28). Also see Hayden and Steinmetz, "Disruptive Strength with Transient Voltages;" *Trans.* A. I. E. E., 1910, Vol. XXIX, pp. 1125 to 1158. Fig. 33 shows typical curves (plotted from Rayner's 1912 paper, *Jour.* I. E. E.) of disruptive voltage with respect to the time required for puncture.

251. Dielectric hysteresis is a form of energy loss in dielectrics, and is independent of any loss due to pure conduction. The latter loss is expressed by gE^2, where g is the total conductance in mhos of a given body of dielectric and E is the impressed difference of potential in effective volts, the power loss given thereby being expressed in watts. The **static component** of dielectric hysteresis probably is proportional to the 1.6th power of the maximum dielectric flux density. The **viscous component** of dielectric hysteresis follows the square law. The latter component is probably the predominating one, since experimental evidence* from condenser tests with alternating currents goes to show that the angle of phase difference due to hysteresis is a constant, for any particular condenser. It follows from this that the power loss in the dielectric is proportional to the square of the flux density and the square of the frequency, which corresponds to the viscous component of hysteresis. As a rule these electrostatic hysteresis losses in a condenser are much smaller than the losses occasioned by magnetic hysteresis and are rather difficult to measure. Frequently they amount to no more than a fraction of 1 per cent. of the volt-ampere input of the condenser, at frequencies from 25 to 125 cycles. See Rayner, E. H., "High-voltage Tests and Energy Losses in Dielectrics;" *Jour.* I. E. E., 1912, Vol. XLIX, No. 214, pp. 3 to 89; also see Fleming and Dyke, "On the Power-factor and Conductivity of Dielectrics;" *Jour.* I. E. E., 1912, Vol. XLIX, No. 215, pp. 323 to 431.

252. Dielectric power-factor. When a dielectric is subjected to a periodic alternating e.m.f. less than the disruptive value there is a loss or expenditure of energy within the dielectric from two causes: (1) the leakage conduction current, (2) the dielectric hysteresis. Consequently, the volt-ampere input to the dielectric, instead of having a zero power-factor as in the case of the ideal dielectric of infinite resistance and zero hysteresis, has a power-factor of finite value but usually small magnitude. This total dielectric loss may, in some cases, have considerable importance. Fleming and Dyke* made tests on eleven different materials at 920, 2,760 and 4,600 cycles, and found that the power-factor was less than 1 per cent. in the case of dry Manila paper, paraffin wax, mica, ebonite, pure india-rubber, vulcanised india-rubber and sulphur; about 2 per cent. for glass and gutta percha; and 8 per cent. for dry slate.

253. Effects of temperature. If a dielectric contains moisture, the application of heat will reduce the moisture content and simultaneously increase the resistivity and the disruptive voltage. In the case of a thoroughly dry substance, however, increase of temperature has the reverse effect. Prolonged heating of a dielectric, if in excess of the safe or conservative limit of working temperature, is injurious and tends to hasten the breakdown of the material by disintegration or chemical change, and ensuing disruptive failure. Except in the case of fireproof and refractory materials, the working temperature is a most important factor in determining the life of insulation. See Par. **234** and also see the temperature limits specified in the Standardisation Rules of the A. I. E. E., Sec. 24, Par. **187** to **200**.

SOLID NATURAL MATERIALS

254. Asbestos is a mineral fibre comprised of hydrous silicate of magnesia, which melts at a temperature in the range from 1,200 to 1,300 deg. cent. It is useful as an insulating material because of its heat-resisting qualities and is fabricated into boards, paper, tape, etc., frequently in combination with a binder to make it stronger mechanically and less absorbent of moisture. It is not inherently a good insulator, and for this reason is frequently mixed with other fibres or loading material to impart greater strength, higher insulation and better finish. The commercial varieties of asbestos often

* Steinmetz, C. P. "Alternating-current Phenomena;" New York, McGraw-Hill Book Co., Inc., 1908: 4th ed., pp. 212 and 213.

switchboard panels. Its properties can be improved by treatment in molten paraffin wax or linseed oil, after all moisture has been expelled, but such treatment results in discoloration. The resistivity is on the order of 10^2 to 10^4 megohm-cm. and the disruptive voltage is in the vicinity of 50 to 100 volts per mil. See tests in Par. 269.

264. Mica is generally recognized as the most superior insulating material known to the art, that which is imported from India being the best, the Canadian grades next and domestic varieties last. Either domestic or India mica is satisfactory for nearly all insulating purposes except for commutators, where it is too hard to wear down as fast as the copper bars. For the latter service Canadian amber mica is considered more satisfactory, being softer than the other grades. All grades of muscovite (white) mica are considered suitable for electric heating appliances. Mica sheets and washers are used in electrical apparatus and appliances in almost innumerable shapes. Cut mica in sheets becomes very expensive in the larger sizes, the largest commercial listed size being 8 in. by 10 in. On this account it is customary to build up larger sizes by connecting together thin layers of mica. Such manufactured mica plate takes a number of forms.

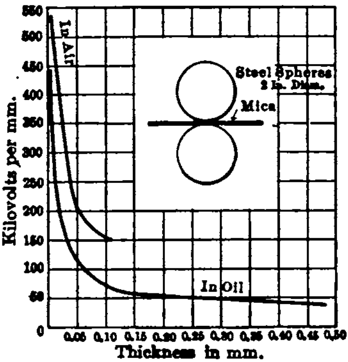

FIG. 34.—Disruptive strength of mica in air and in oil.

265. Composition and properties of mica. Mica is a refractory mineral constituted of the double silicate of alumina or magnesia, and potash or soda, combined with varying proportions of potash, soda and other impurities. Iron in excess colors it gray and black; magnesia tends to darken the color; aluminium and potassium silicates tend to make the mica transparent. Mica crystallizes in laminated form and may be split along its axis to sheets as small as 0.006 mm.

It has a high dielectric strength and is suitable to withstand high temperatures. However, in its natural state it is not flexible nor uniform, and permits large surface leakage; so that most mica is reconstructed and put on the market in the form of micanite, megomit, megotalc, etc.

The properties of mica are very fully covered in a publication by Zeitler, H., "Mica: Its History, Production and Utilization;" D. Jaroslaw, London. 1913: also in the 1912 edition of "Mica," by the Canadian Dept. of Mines. These sources were utilized in the preparation of the following table.

Origin	Resistivity in 10^{12} ohm-cm.	Dielectric constant	Disruptive strength in volts per mm.*
Madras	15 to 133	2.5 to 5.5	50,000 to 80,000
Bengal	7 to 118	2.8 to 4.7	40,000 to 120,00
Canada	0.44 to 22	2.9 to 3.0	80,000
South America	39	5.9	40,000 to 90,000

Fleming and Johnson give the dielectric strength, in sheets 2 to 3 mils thick, as about 2,000 volts per mil for amber mica and about 3,000 to 4,000 volts per mil for the white, ruby and soft green varieties; Fig. 34 shows the

* NOTE.—Test thickness, 0.3 mm.

272. Wood is used as an insulating material to a considerable extent. The varieties of wood employed are usually the hard woods, such as maple and hickory, impregnated with oil, paraffin wax, or clear air-drying varnish. The resistivity of paraffined wood, at 22 deg. cent. is of the following magnitude: mahogany, 4×10^{13} ohm-cm.; maple, 3×10^{10}; poplar, 5×10^{11}; walnut, 0.09×10^{10} to 1×10^{10} (Scientific Paper No. 234, Bur. of Standards). The dielectric constant and the disruptive voltage are both dependent upon whether the electric stress is parallel or perpendicular to the grain. Parallel to the grain the dielectric constants of red beech and oak are between 2.5 and 4.8; perpendicular to the grain, 3.6 to 7.7.

In maple boiled in transformer oil under vacuum, dried under vacuum and boiled again at atmospheric pressure, the disruptive voltage along the grain, at 1 in. of separation, was 70 kv.; at 2 in. it was 90 kv.; across the grain, at 0.5 in., it was 60 kv.; at 1 in. it was 80 kv.; dielectric constant, at 20 to 25 deg. cent., across grain, 4.1. Well-dried wood should stand 10 kv. per in. without signs of burning or heating. It is extremely important that the wood should be well dried before impregnation, because it is very difficult to remove the moisture subsequently. When wood contains moisture it is a relatively poor insulator and the water contained in the cells conducts electrolytically. Wood treated with zinc chloride to protect it against decay has comparatively low resistivity; see *Electrical World*, 1911, Vol. LVII, p. 828. For curves of disruptive strength of maple see Hendricks, A. B., "High-tension Testing of Insulating Materials;" *Trans.* A. I. E. E., 1911, Vol. XXX, pp. 167 to 213.

VITRIFIED MATERIALS

273. Glass is an insulating material in very extensive use, possessing high resistivity and dielectric strength at ordinary temperatures. The principal constituent is silica, ranging from 50 to 75 per cent. of the total contents; potash, soda, lead oxide and lime are also present, in various proportions. The resistivity at ordinary temperatures is on the order of 10^{13} to 10^{18} ohm-cm. and decreases with great rapidity as the temperature increases. Gray and Dobbie found that potash glass has higher resistivity than soda glass, and annealing increases the resistivity (*Proc.* Royal Soc., 1900, Vol. LXVII, p. 197). At very high temperatures glass becomes a fairly good conductor. Moisture readily condenses upon its surface and it has consequently a high surface leakage. It is also soluble in water to a slight degree and under weather exposure the surface tends to roughen. The dielectric constant ranges from about 5.5 to 10. The dielectric strength ordinarily ranges from 150 to 300 volts per mil, and is higher in very small thicknesses. At 920 cycles crown glass has an apparent resistivity of 17×10^{9} ohm-cm.; dielectric constant, 6.60; power-factor, 0.018. Mechanically glass is unreliable and brittle; the tensile strength is uncertain and anywhere from 1,000 to 10,000 lb. per sq. in., with somewhat higher compressive strength. Coefficient of linear expansion, 0.000008 to 0.0000095 per deg. cent. Density 2.5 to 4.5. For information on glass manufacture see Rosenhain, W., "Glass Manufacture," D. Van Nostrand Co., New York.

274. Porcelain. The three principal constituents of electrical porcelain are feldspar, clay and silica. There are three feldspars: orthoclase, or potash feldspar, which is the most important; albite or indianite, which is soda feldspar; anorthite, or lime feldspar. The two clays used are ball clay, and china clay or kaolin. A standard mixture of these constituents for testing purposes is 20 parts feldspar, 50 parts kaolin and 30 parts quartz. The function of the feldspar is to act as a flux to unite the other constituents into a vitreous mass when fired. There are two processes of manufacture, the dry process and the wet process. For details on the manufacture and properties of electrical porcelain, see an exhaustive paper by E. E. F. Creighton, "Electrical Porcelain;" *Proc.* A. I. E. E., May, 1915, pp. 753 to 841. Also see Perrine, F. A. C., "Electrical Conductors;" D. Van Nostrand Co., New York, 1903; Chap. XIII.

275. Dry-process porcelain is manufactured by molding the moist raw mixture under high mechanical pressure and then vitrifying by the usual firing process. This grade of porcelain is usually very porous and consequently has a disruptive strength on the order of atmospheric air, or less. At or near disruptive pressures, however, it heats rapidly and is not suitable for high-voltage insulation. The safe dielectric strength is on the order of 1,000 volts.

range of temperatures without injury. As a rule, it is slightly absorbent, receiving as much as 3 or 4 per cent. of water, in some cases, in 24 hr. The puncture voltage, with a 0.6-in. (1.5 cm.) wall, after 24-hr. immersion, ranges from about 16,000 to 30,000 volts. It is not affected by arcs unless in direct contact with them, and in that event will be likely to melt locally and chip or fracture in the surrounding area in consequence of unequal heating and expansion.

280. Vitreous enamel consisting of opaque white glass is extensively used for coating iron resistor grids and imbedding the resistor wires, thus forming non-inflammable and highly fireproof devices capable of withstanding unusually high operating temperatures. Enamelled iron-ware is made extensively by the process of sprinkling powdered glass upon red-hot metal, whereupon the glass fuses and forms a continuous thin coating. The formulas used for compounding the glass are quite closely guarded as manufacturing secrets. What is desired is a thin, strong elastic coating which will expand and contract with temperature changes at as nearly as possible the same rate as iron.

FIBROUS MATERIALS

281. Cellulose is the base of practically all fibrous insulating materials and is an organic compound composed of 48 per cent. carbon, 46 per cent. oxygen and 6 per cent. hydrogen. It is a carbohydrate of the formula $(C_6H_{10}O_5)_x$, similar in composition to starch. When pure it is a white amorphous mass; unsized, well-bleached linen paper is nearly pure cellulose. It has a resistivity on the order of 10^8 to 10^{10} ohm-cm. at ordinary temperatures and a dielectric constant of about 3.9 to 7.5. All untreated cellulose materials break down at about 120 deg. cent. and should not be subjected to a maximum of more than 95 deg. cent. A safe operating limit is about 80 deg. cent.

282. Untreated fibrous materials are as a whole hygroscopic and are therefore relatively inferior insulating materials. They are nevertheless employed, some of them, to a great extent, but their use is very largely confined to conditions under which moisture is restricted or expelled. Their properties are greatly improved by treatment or impregnation.

283. The impregnation of fibrous and asbestos products with varnishes, gums, bakelite, etc., produces several results: First, the treating materials fill up the pores of the basic material and eliminate moisture; second, the dielectric strength is increased even where there is no moisture to be considered; third, most treating materials assist in producing smooth surfaces; fourth, the heat-resisting quality of the basic material is often increased, and, fifth, the filling up of the pores may in certain cases reduce the tendency to shrink. Incidentally the treating materials increase the heat conductivity of the insulation, resulting in better radiation.

284. Paper is manufactured from wood pulp, rags or plant fibre. The essential processes in manufacture are (1) the reduction of the raw material to the consistency of a thin pulp, by means of operations involving the use of chemicals and steam; (2) the running of this pulp upon a continuous sieve of fine mesh, which retains the fibres that become felted together; (3) the removal, drying and finishing of the felt so formed. Finished paper retains traces of the bleaching or coloring matter employed, and in addition frequently contains a certain amount of loading matter such as china clay, calcium sulphate and other inert mineral matter. A sizing of vegetable or mineral solution is sometimes added to render the paper less porous and improve the surface.

The mechanical properties of paper are derived in large degree from the basic fibres employed in its manufacture; thus paper made from wood pulp is brittle and easily torn, whereas linen or Manila fibre produces a much tougher and stronger paper. Owing to its porosity paper is hygroscopic and normally contains from 7 to 12 per cent. of moisture. When thoroughly dry it has a very high resistivity, on the order of 10^{15} ohm-cm., but it easily absorbs water and when wet descends to the class of a poor conductor. The dielectric constant of dry paper is from 1.7 to 2.6. Dry Manila paper has a power-factor of about 0.007 at 920 cycles. The dielectric strength of various kinds of untreated paper, ranging in thickness from 1.8 to 28 mils, should average from 110 to 230 volts per mil; higher values are obtainable from extremely dry paper. The tensile strength will range from a few thousand up pressboard is about 2.9 at 20 to 25 deg. cent., measured on 0.1-in. board. The dielectric strength of treated pressboard is given in Fig. 35: curve (1) for pressboard dried and boiled in transformer oil; curve (2) for pressboard dried, boiled in linseed oil and given two coats of varnish; curve (3), dried and given two to four coats of linseed oil and gum varnish, depending on thickness. When these sheets are laid together in laminations, the puncture voltage per mil of complete thickness decreases, but in the case of very thin laminations the puncture voltage does not appear to decrease in as rapid ratio as with single thicknesses.

290. Vulcanized fibre* is a hard, dense material of which the principal ingredient is paper or cellulose made from cotton rag stock; the other ingredients are zinc chloride and coloring matter, the latter consisting of analine colors or mineral pigments. The finished material is heavily compressed into slabs, sheets, tubes, etc. The water and chemicals are not completely removed during manufacture and the product is hygroscopic and not a superior insulating material except for moderate voltages. It will absorb about 50 per cent. of its weight of water in 24 hours. The density ranges from 1.0 to 1.5 according to the grade; average 1.4. The resistivity is comparatively low for dielectrics, or on the order of 10^7 to 10^{10} ohm-cm. Certain varieties are said to have a resistivity as high as 7×10^{13} ohm-cm., probably in a very dry state. The measurements of dielectric strength by different observers are widely discrepant. Parshall and Hobart gave 10,000 volts as the dielectric strength of all thicknesses from ⅛ to 1 in. Hendricks gave about 200 volts per mil at thicknesses of 50 to 150 mils, 160 volts per mil at 0.4 in., 100 volts per mil at 0.7 in. and 90 volts per mil at 1.0 in. Others have found values ranging as high as 300 volts per mil; the results depend largely on the dryness of the material. The tensile strength ranges from 10,000 to 20,000 lb. per sq. in. and the compressive strength is from 35,000 to 60,000 lb. per sq. in. Fibre is not soluble in water or oil, but is attacked by strong acids, and swells when soaked in water; upon drying it shrinks appreciably and warps badly. Numerous grades of fibre are manufactured and known by various trade names, as horn fibre, hard fibre, indurated fibre, leatheroid, fish paper, etc. The flexible and more fibrous varieties have better insulating qualities. Impregnation improves the qualities in marked degree.

291. Treated fibre. The insulating properties of hard or vulcanized fibre are much improved by treating the pulp with bakelite. A material of this character, known as bakelite-dielecto, is manufactured by The Continental Fibre Co. and is said to have the following characteristics. It is a hard, tough material, light brown or black in color, and manufactured in sheets, tubes and certain special forms; cannot be molded, but can be machined either with or against the grain; is non-hygroscopic and impervious to hot water, oils and ordinary solvents; will withstand continuously a temperature of 150 deg. cent.; resistivity, 1.1×10^{12} ohm-cm. at ordinary temperatures, increasing with temperature up to 100 deg. cent.; dielectric strength, 700 to 1,150 volts per mil; average tensile strength, 18,000 lb. per sq. in.; compressive strength, 21,000 lb. per sq. in.

292. Impregnated fibre duct is in extensive use for both inside and outside construction. It is made in the form of a cylindrical tube by wrapping many layers of paper or pulp on a mandrel and impregnating it during the process with bitumen or a compound of liquid asphalt and coal tar. It is sometimes known as **bitumenized fibre.** Tests made on a certain grade of this material show that it absorbed from 2 to 3 per cent. of water after 96 hr. immersion; one manufacturer guarantees not more than 0.75 per cent. when the ends are sealed. The compound softens slightly at 55 deg. cent. and commences to break down at about 95 deg. cent. Manufacturer's guarantees on minimum puncture voltage, dry, through a 0.375-in. wall, range from 25 to 50 kv; after prolonged immersion the dielectric strength will usually be lowered, depending naturally upon the amount of moisture absorbed.

293. Varnished cloth is a thin white fabric of cotton or linen muslin coated with a mixture of boiled linseed oil, resin and benzine. Upon drying the oil oxidizes in contact with the air and leaves a smooth, hard surface.

* See "Manufacture of Hard Fibre," *Electrical World*, Vol. LIII, p. 1437; also see Vol. LV, p. 1342.

tables in Sec. 5). Gray gives the puncture voltage of a 7-mil thickness (composed of two layers) as about 150 volts; impregnated, about 600 volts.

300. Silk insulation for magnet wires is applied in one or two thicknesses, ranging from 1 to 2.5 mils per layer. While it is somewhat hygroscopic in the untreated condition, it has superior insulating properties compared with cotton, and is much improved by impregnation. Neither cotton nor silk are the equal of baked enamel (Par. **351**) in dielectric strength.

301. Asbestos insulation for magnet wires. Asbestos insulation can be applied to wires and small straps with the use of binding materials, but in all cases, except in that of asbestos tape which can readily be used in taping armature or field coils, the mechanical qualities are quite poor. Asbestos windings also require considerable space if used in sufficient thickness, they are not in themselves moisture-proof, they have low dielectric strength, and do not give a smooth surface.

Deltabeston magnet wire is insulated with asbestos fibre cemented to the wire with a special bond. It is claimed that the maximum continuous working temperature is 150 deg. cent.; for short periods, 260 deg. cent. The insulation thickness is about the same as double cotton and breaks down at 300 to 600 volts.

302. Tapes. Insulating tapes are chiefly of four varieties: (a) those woven from cotton or silk and untreated; (b) those woven from cotton and treated with insulating varnish, or cut from treated cloth; (c) those cut from cloth which has been loaded with rubber or adhesive compound. The lay of the threads is arranged in three different ways, straight, biased and webbed; the last one is the strongest and does not stretch readily. (d) Paper tapes, treated and untreated, are cut from finished stock. See "Specifications and Tests for Insulating Tapes," *Electrical World*, Vol. LVII, p. 488; also Vol. LVI, p. 689.

303. Untreated tapes are hygroscopic and for that reason are not entirely satisfactory unless finally impregnated or protected from moisture. Gray states that a half-lapped layer of untreated cotton tape 6 mils thick will withstand about 250 volts when dry; about 1,000 volts when impregnated. Precautions should be taken to detect the presence of bleaching and chemical matter, such as chlorine, which may attack copper. Webbing is sometimes used for mechanical protection, aside from its insulating qualities.

304. Varnish-treated tapes are cut from sheets of treated cloth, such as Empire cloth, varnished cambric and the like, and are used for taping windings which cannot readily be impregnated. They are cut straight or on the bias, the latter being sometimes preferred for taping uneven surfaces. See treated cloth, Par. **292** to **295**.

305. Rubber-treated tapes are composed of fabric loaded with plastic rubber gum or compound in a soft adhesive state. Such tapes are used extensively in making water-proof joints on underground rubber-insulated cables, or in other locations where a moisture-repellent wrapping is desired. They are frequently used in conjunction with a splicing gum of similar composition, and protected by water-proof insulating compounds and outside wrappings of adhesive tape with insulating paint over all. In the case of underground cables a lead sleeve is wiped over the whole joint, making it completely water-tight.

306. Adhesive or friction tape is composed of fabric loaded with a sticky or adhesive compound. The base of the compound in the more expensive grades is rubber gum, adulterated with fillers in various well-known ways, while the less expensive grades contain little or no rubber and its place is taken by one of the numerous bituminous compounds. This kind of tape possesses fair insulating properties and is very extensively used in low-tension work.

307. Paper tapes of both the treated and the untreated varieties have a most extensive use in the manufacture of paper-insulated cables for power and communication service. See paper, Par. **284**.

308. Asbestos tape, or a tape having a base of asbestos fibre, is superior in its heat-resisting properties to cellulose materials such as paper and cloth. Such tapes are usually known by trade names, among them being deltatape. The latter it is claimed can be raised to 260 deg. cent. before breakdown occurs; puncture voltage, about 250 volts per mil.

313. Asbestos molded with a binder, known under the trade names of gummon, hemit and tegit, is manufactured by the Hemming Mfg. Co., from whom the following data was obtained. Gummon is black and can be highly polished; tegit is dark brown and can be polished, though less highly than gummon; hemit is made in both gray and black and will also take a polish. The density varies, in the neighborhood of 2. These materials are suitable only for molding; are infusible, but will gradually carbonize at higher than working temperatures; will not resist concentrated acid; are not recommended for working pressures above 1,000 volts.

	Ohm-cm. at 22 deg. cent. (Bur. of Stds.)	Max. working temp (deg. cent.)	Dielectric strength (volts per mil)	Strength (lb. per sq. in.)		Absorption of moisture (per cent.)
				Tensile	Compressive	
Hemit.......	1×10^{10}	1,100	50	2,000	1,600	5
Gummon....	3×10^{12}	320	75	600	550	2
Tegit........	2×10^{12}	200	50	1,200	1,100	5

Also see Hemming, E. "Molded Electrical Insulation and Plastics;" Clausen and Co., New York, 1914.

314. Asbestos wood or lumber, known also by the trade names "Transite Asbestos Wood" and "Asbestos Building Lumber," consists of asbestos fibre and hydraulic cement, and is used as a substitute for wood in building construction. It is also used to some extent as a substitute for slate and marble in electrical construction. The following data (Par. **315** and **316**) was furnished by C. L. Norton; also see his paper entitled, "Some Refractory Substitutes for Wood," *Jour.* A. S. M. E., 1912; and "The Manufacture and Use of Asbestos Wood," on pp. 375 and 379 of "Technology and Industrial Efficiency," McGraw-Hill Book Co., Inc., New York, 1911.

315. Transite asbestos wood is light gray in color and is manufactured in sheets up to about 4 ft. by 8 ft. by 2 in. It has a density of about 2.0. It can be sawed and bored like wood but is harder and slower to cut. At 600 deg. cent., partial dehydration and partial loss of strength occurs, but it does not soften. At 1,100 deg. cent. the material holds shape and considerable portion of its strength, but it will not stand temperatures above 1,400 deg. cent. The temperature coefficient of expansion at ordinary temperatures is about 0.000008 per deg. cent. The thermal conductivity is 0.0005 cal. per cm-cube per sec. per deg. cent. Transverse breaking tests give a modulus of rupture of about 5,000 lb. per sq. in., and the crushing strength is from 20,000 to 25,000 lb. per sq. in. When dry, at or near 20 deg. cent., it has a resistivity of about 150,000 megohm-cm. It is dissolved slowly by acids. When used in dry or hot places it is suitable for electrical insulation and is tougher than slate or marble. Since it absorbs moisture it is not suitable for damp locations.

316. Ebony asbestos wood is asbestos bonded with magnesia, cement and saturated with an insulating compound. It is black, smooth and glossy and has a density of about 1.9. It can be worked the same as slate, but more rapidly and easily. The working temperature limit is about 200 deg. cent.; it does not soften; the melting-point is above 1,400 deg. cent. The coefficient of expansion is 0.000010 per deg. cent. The thermal conductivity is 0.00065 cal. per cm-cube per sec. per deg. cent. It has a modulus of rupture of about 5,000 lb. per sq. in. and a crushing strength of 15,000 lb. per sq. in. At 20 deg. cent. after 96 hr. immersion the resistivity is above 3×10^6 megohm-cm. and changes 5.3 per cent. per deg. cent. The disruptive strength is greater than that of slate or marble, and it also withstands better the effects of surface arcing. It is also tougher than slate or marble.

317. Bakelite and bakelite compositions. Bakelite is a condensation product of phenol, manufactured in three grades. Bakelite "A" is the initial raw material and exists in liquid, pasty or solid condition; upon heating it is converted into "B" which is an intermediate solid product, substance which will not harden under heat; this product is heated and then combined with a hardening agent, producing a hard, infusible and practically insoluble substance which the manufacturer (Condensite Co. of America) claims is high in dielectric and mechanical strength and heat resistance. The following data on the properties of molded condensite were furnished by the manufacturer: Density, 1.25 to 2.0; not hygroscopic, unaffected by water, insoluble in the ordinary solvents and oils, attacked by strong nitric acid and caustic potash, slightly attacked by sulphuric acid. The resistivity at 22 deg cent. is about 4×10^{10} ohm-cm. (Bur. of Stds.); dielectric strength, about 300 to 400 volts per mil at a thickness of 0.15 in. and 500 to 600 volts per mil at a thickness of 0.04 in.; tensile strength, 4,300 lb. per sq. in.; compressive strength, 26,000 lb. per sq. in.; not perceptibly affected by 48 hr. of exposure to a temperature of 200 deg. cent.; maximum working temperature, 300 deg. cent. (Swoboda, *Elec. Jour.*, May, 1913). Also see *Electrical Review and West Elec.*, Vol. LX, p. 199.

320. Dielectrite is a black molded composition composed of vegetable fibre and mineral filler. It is molded and vulcanised by the application of heat. Resistivity at 22 deg. cent., 5×10^{12} ohm-cm.

321. Electrose is a dark brown or black composition of hard, tough quality which it is claimed is non-hygroscopic and not affected by water or oil. It can be molded in any form, will hold metal inserts, and can be given a smooth glossy finish. The working temperature limit is about 95 deg. cent. Resistivity at 22 deg. cent., 1×10^{14} to 200×10^{14} ohm-cm. The manufacturer (Electrose Mfg. Co.) claims a dielectric strength of at least 600 volts per mil, in a thickness of ⅛ in. Electrose is used in the manufacture of many different forms of insulators and bushings and is also made in pliable insulating flooring. See *Electrical World*, Vol. LIV, p. 797 and Vol. LVI, p. 887.

322. Gohmak is a molded substitute for hard rubber made by the Vulcanised Products Co. The density ranges from 1.4 to 1.8 according to composition. The claim is made that it is non-hygroscopic and insoluble in oils and weak solutions. Resistivity, on the order of 2×10^{10} ohm-cm. at ordinary temperatures, decreasing with rising temperature. Dielectric strength, on the order of 400 volts per mil, at a thickness of 0.25 in. Tensile strength, 9,000 to 12,000 lb. per sq. in. Softens slightly at 100 deg. cent.

323. Insulate is a black molded composition composed of mineral compound and resembles hard rubber. It can be moulded in any shape and can be worked and machined. The manufacturers (General Insulate Co.) claim that it is non-hygroscopic, insoluble in all weak solutions and has a maximum working temperature of 150 deg. fahr. The resistivity of No. 2 grade at 22 deg. cent. is 8×10^{15} ohm-cm. The dielectric strength is on the order of 45 volts per mil, at a thickness of 0.4 in.

324. Molded mica is made of finely split mica scales held together by a strong insulating varnish, binder, or cement, such as shellac, the sheets or forms thus built up being subjected to heat and pressure. These compositions are more or less heat resisting, dependent upon the nature and proportions of the binder employed. They are known by a variety of trade names such as micanite, mica plate, micabond, micabeston, turbomic, formica, megomit, megotalc, etc. The less binding material they contain, the nearer they approach the properties of natural mica. Such reconstructed or molded mica is made in three commercial forms, as follows: (1) Molded plate, which becomes flexible when heated and in that condition can readily be formed into various shapes such as rings, troughs, spools, and, in thinner sheets, rolled into tubes. Upon cooling it regains its rigidity. It can be used for any purpose where very high temperatures are not encountered, except for commutator bars. (2) For insulating commutator segments. It cannot be molded and offers great resistance to heat. Canadian amber mica is preferred for this purpose. (3) Flexible sheets which may be bent to shape without application of heat, for insulating armature slots, magnet and commutator cores, etc. It is also used in conjunction with tapes for insulating wires and cables.

Rayner concluded from his tests (National Physical Laboratory) that generally speaking, thin qualities of micanite up to about 1 mm. will withstand a stress of 20,000 volts per mm. (500 volts per mil) in air for 10 min. Above this thickness, up to 2.5 mm., there is more difficulty in making material which will withstand this stress, and usually the material withstands the voltage longer under oil.

331. Reduction of crude rubber. The lumps or biscuits of crude rubber are boiled in water, ground, washed, dried, mixed with sulphur, adulterants and filler and then calendered. For details of the process see Perrine, F. A. C. "Conductors for Electrical Distribution," New York, 1903; and Esch, W. "Handbook for India-rubber Engineers," Hamburg, 1912. Owing to the high cost of pure rubber it is almost universally adulterated and many rubber products do not contain over 20 to 30 per cent. of pure gum, and sometimes much less. Among the numerous adulterants in use are rubber substitutes, ozokerite, paraffin, pitch, oil, etc., and fillers such as zinc oxide, white lead, red lead, barium sulphate, magnesium carbonate, barium carbonate, chalk, lamp-black, talc, alumin flakes, etc.

Vulcanization. When rubber and sulphur are heated to a temperature above the melting-point of the latter, 120 deg. cent., the two combine and form a new product termed vulcanized rubber, which is stronger, more elastic and less susceptible to temperature changes than pure rubber. The degree of vulcanization depends upon the proportion of sulphur, the temperature and the duration of heating.

332. Rubber substitutes in the true sense have not yet been produced on a commercial scale. There are certain so-called substitutes, produced from vegetable oils by processes of vulcanisation or oxidation, which can advantageously be mixed with rubber for the production of certain articles. Rubber substitutes used not infrequently in wire insulation consist principally of oxidized oils, paraffin, resins and rubber shoddy. The latter is a compound obtained by treating old rubber with steam, sulphuric acid and chloride of zinc, thus removing most of the vegetable fibres and the sulphur, but leaving the mechanical admixtures of earth and oxides employed in the original manufacturing process. Such substitutes are usually known under trade names.

333. Electrical properties. The resistivity is on the order of 10^{14} to 10^{16} ohm-cm., varying greatly according to the composition and increasing with the content of pure rubber. The temperature coefficient is negative and unusually large, ranging from 2 to 4 per cent. per deg. cent. Del Mar states that at any given temperature the rate of change of resistance per deg. of temperature change is approximately proportional to the resistance at that temperature, values of the factor ranging from 0.02 to 0.03 for 30 per cent. Para compound. Values of k in the formula $R = k \log_{10}(D/d)$ for insulation resistance of cylindrical wires in megohm-miles, are variable between wide limits, ranging from about 1,000 to 20,000; d is the diameter of the wire and D is the outer diameter of the insulation, in the same units. The value of k is very much higher with alternating currents.

The dielectric constant of pure vulcanized rubber is from 2 to 3; rubber compounds, 3 to 4. Jona gives values as high as 6 for certain compounds containing relatively large percentages of Para.

The dielectric strength of high-grade rubber compound ranges from 300 to 500 volts per mil; it decreases quite appreciably for long periods of electrification. Lufkin states (*Electrical World*, 1913, Vol. LXI, p. 1310) that for each rubber compound there is a critical temperature at which the puncture voltage is a maximum. This ranged between 40 deg. and 80 deg. cent. for five different grades, in a certain series of tests. One particular grade, of high quality, gave a maximum at 70 deg. cent., being 30 per cent. above the value at 20 deg. The range was carried to 100 deg. cent. at the upper limit, and 0 deg. at the lower.

Fleming and Dyke measured the power-factor of rubber at 920 cycles and found values of 0.005 for pure India-rubber and 0.002 for vulcanised India-rubber. For further data on electrical properties, consult the following:

Jona, E. "Insulating Materials in High-tension Cables;" *Trans.* Int. Elec. Congress, St. Louis, 1904, Vol. II, pp. 550 to 571.

Fisher, H. W. "Rubber-covered Wires;" *Trans.* A. I. E. E., 1907, Vol. XXVI, pp. 997 to 1025.

Osborne, H. S. "Potential Stresses in Dielectrics;" *Trans.* A. I. E. E., 1910, Vol. XXIX, pp. 1553 to 1581.

Hering, C. "Thickness of Electric and Thermal Insulation," *Electrical World*, 1911, Vol. LVIII, pp. 1303 to 1305.

Lendi, J. H. "The Thickness of Insulation on Wires and Cables," *Electrical World*, 1912, Vol. LIX, pp. 590 to 592.

percha and Balata," Cologne, 1903; Seeligmann, Torrilhon and Falcounet, "India-rubber and Gutta Percha," 1910; Del Mar, W. A., "Electric Power Conductors," 2d edition, New York, 1914; Kempe, H. R., "A Handbook of Electrical Testing," 7th edition, London, 1908.

341. Hard rubber or ebonite is a rubber compound containing a large percentage of sulphur and highly vulcanized. It is a hard dense material possessing many desirable properties as an insulator at temperatures not greatly exceeding normal. The resistivity is on the order of 10^{15} to 10^{18} ohm-cm. at ordinary temperatures; the surface resistivity is impaired by exposure to sunlight (See Scientific Paper No. 234, Bureau of Standards.) The dielectric constant is from 1.9 to 3.5. In small thicknesses, of 20 mils, the dielectric strength ranges from 1,700 to 3,750 volts per mil, tested between 2-in. spheres; 1,000 to 2,000 volts per mil, tested between flat electrodes. At 920 cycles it has an effective resistivity of about 1.5×10^{11} ohm-cm.; dielectric constant, 3.17; power-factor 0.005 (Fleming and Dyke). Mechanically it is brittle, but can be worked, machined and polished; tensile strength, about 1,100 lb. per sq. in. and compressive strength about double; density, 1.2 to 1.25. It is attacked by oils and ozone, but is non-hygroscopic. See Farmer, F. M., "The Dielectric Strength of Thin Insulating Materials," *Trans.* A. I. E. E., 1913, Vol. XXXII, pp. 2097 to 2131; also Paterson, Rayner and Kinnes, "Notes on the Testing of Ebonite for Electrical Purposes," *Jour.* I. E. E., 1913, Part 217, Vol. L.

342. Kerite is a vulcanised compound of oxidised linseed oil and rubber combined with various vegetable oils, invented by A. G. Day. According to Perrine* it has a specific insulation resistance somewhat less than pure rubber, but is said to be mechanically more durable than any insulation manufactured from pure rubber. It is employed in the insulation of wires and cables as a substitute for the usual rubber compound. The value of the constant k in the formula $R = k \log_{10} (D/d)$ is given by the Kerite Ins. Wire and Cable Co. as 4,000 at 60 deg. fahr.; R is the insulation resistance of the wire in megohm-miles, d is the diameter of the conductor and D is the outside diameter of the insulation.

343. Sulphur has a resistivity of 10^{17} ohm-cm. at 22 deg. cent.; dielectric constant, 2.2 to 3.9; power-factor at 920 cycles, 0.0003.

344. Vulcanite. See hard rubber, Par. **341**.

VARNISHES AND COMPOUNDS

345. Solidifying materials, such as varnishes, which are applied as liquids and emerge as solids, are of interest chiefly in their final state. They are divisible broadly into two classes: (1) those employed to impregnate or treat basic materials, such as the fibres and the pulps; (2) those employed as filling compounds, to permeate or seal extensive voids which otherwise would offer lodgment for moisture and deleterious foreign matter. The properties of the former class are obviously of importance, principally, in association with the treated or impregnated base; while materials of the latter class constitute a species to themselves.

346. Insulating varnishes are divisible, according to their applications, into four groups:† (A) For impregnating windings; (B) for treating papers and fabrics; (C) for cementing purposes; (D) finishing varnishes. They are also divisible, according to their properties, into (a) oxidizing and (b) non-oxidizing, and again into (1) air-drying and (2) baking. Oxidising varnishes of class A are frequently composed of linseed oil with a resinous base of copal or other fossil gum, and when thoroughly oxidised are almost impervious to oil and moisture. The drying action in linseed-oil varnishes takes place first by the evaporation of the volatile solvent and then by the oxidation of the oil and the gum; the latter action is hastened by the addition of mineral drier, the quantity of which depends upon whether air-drying or baking varnish is desired. In another form of oxidising varnish the gum base is replaced by asphaltum, but this is said to lower the dielectric strength and the resistance to attack by oil. Non-oxidising varnishes of class A contain a

* Perrine, F. A. C. "Conductors for Electrical Distribution," New York D. Van Nostrand Co., 1903, p. 106.

† See Fleming and Johnson. "Insulation and Design of Electrical Windings;" Longmans, Green and Co., London, 1913; pp. 63 to 76.

per mil, an average value being 500 to 600 volts per mil, or about four times the value for silk. The electrical resistivity at ordinary temperatures is very high, on the order of 10^{14} ohm-cm. Baked enamel should stand a temperature of 100 deg. cent. continuously without injury, but breaks down electrically at about 300 deg. cent. It is a fairly good thermal conductor and much superior to cotton and silk. An enameled wire should withstand bending around a mandrel four times its own diameter without injury. Turpentine, shellac, alcohol, vegetable or animal oils, and coal-tar solvents will attack it, but it is not injured by clean mineral oil and is moisture-proof. It should be carefully handled to avoid injuring the coating.

352. Linseed oil is a vegetable material derived from flaxseed, having a density of 0.932 to 0.936 at 15 deg. cent. It has excellent insulating properties and is extensively used in paints and varnishes. For specifications and general properties see "Year Book," A. S. T. M.; and Technologic Paper No. 9, Bureau of Standards, 1912. Boiled linseed oil has the property of oxidising under ordinary exposure to air and the process will continue until it becomes viscous or even hard; the action can be hastened by drying agents and the application of heat.

353. Ozokerite (ozocerite) is a wax-like mineral, colorless or white when pure and consisting of a mixture of hydrocarbons. It is used in making ceresin, candles, etc. Crude ozokerite has a resistivity on the order of 4.5×10^{14} ohm-cm. Liquid ozokerite has a dielectric constant of about 2.1. **Ceresin** is a yellow or white wax made by bleaching and purifying ozokerite and is employed as a constituent of insulating compounds; its density is 0.75; resistivity, over 5×10^{18} ohm-cm. at 22 deg. cent.

354. Paraffin is a colorless or white waxy substance, consisting of a complex mixture of hydrocarbons, obtained by the distillation of wood, coal or oil. Chemically it is inert, being unaffected by most strong reagents. According to composition, it melts at from 45 to 80 deg. cent. and has a density of 0.87 to 0.94; resistivity, 10^{15} to 10^{19} ohm-cm.; dielectric constant, 1.9 to 2.3; dielectric strength, about 300 volts per mil; power-factor at 920 cycles, 0.0003.

355. Resin is defined as any of various solid or semi-solid organic substances, chiefly of vegetable origin, usually yellowish to brown in color, transparent or translucent, and soluble in ether, alcohol, etc., but not in water. They soften and melt on heating. Chemically they differ widely, but all are rich in carbon and hydrogen and contain also some oxygen. Among the commercial resins are amber, copal, dammar, guaiacum, lac, mastic, robin and sandarac. Lac is the raw material used in making **shellac**, which has a resistivity on the order of 10^{15} to 10^{16} ohm-cm. and a dielectric constant of about 2.7 to 3.8.

356. Wax is defined as any of a class of natural substances composed of carbon, hydrogen and oxygen and consisting chiefly of esters other than those of glycerin or of free fatty acids. In this class are included beeswax, spermaceti, Chinese wax, carnauba wax, etc. Beeswax is a dull yellow solid, of density 0.96 to 0.97 at 15 deg. cent. and melting at 62 to 64 deg. cent.; resistivity, 10^{14} to 10^{17} ohm-cm.; dielectric strength, about 250 volts per mil.

357. Weatherproof compounds for saturating the cotton braids on weatherproof wire usually contain an asphaltum base, with an admixture of wax so that the surface of the braid may be given a dull polish.

INSULATING OILS

358. Oil is employed as an insulating medium in many ways. It is employed by itself to insulate transformers and switches by immersion; it is used for saturating fibrous and other materials, as in cable work; drying oils (linseed) are used for coating papers and cloths in sheet insulation; various kinds of oil are employed in mixing insulating paints and varnishes. Oils of practically every variety are possessed of very high resistivity and dielectric strength. Chemically oil is composed of hydrocarbons having the general formulas C_nH_{2n+2} and C_nH_{2n}. The desired characteristics of an insulating oil are high resistivity and dielectric strength, low viscosity, high flash point, chemical neutrality toward metals and insulating materials, freedom from moisture, sediment and impurities, and chemical stability under local high temperatures.

362. Dielectric constants of various kinds of oils are given in the accompanying table. The values probably change very appreciably with the temperature.

Arachid	3.17	Petroleum	2.02 to 2.19
Castor	4.6 to 4.8	Rape seed	2.2 to 3.0
Colsa	3.07 to 3.14	Sesame	3.17
Lemon	2.25	Sperm	3.02 to 3.09
Neatsfoot	3.07	Turpentine	2.15 to 2.28
Olive	3.08 to 3.16	Vaseline	2.17

363. Effects of moisture and dust on insulating properties of oil. The presence of moisture in transformer oil has a very serious effect on the dielectric strength, as shown by Fig. 36. In order to obtain a dielectric strength of 40,000 volts (0.2-in. gap between 0.5-in. discs), the water present as distributed moisture in the oil must not exceed 0.001 per cent. Fine dust is also very injurious to the dielectric strength. For these reasons, various manufacturers have developed oil dryers and purifiers, which operate on the principle of a filter press.

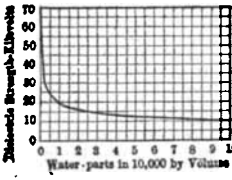

Fig. 36.—Effect of water on dielectric strength of oil.

364. Formation of deposits in oil-cooled transformers. The foregoing subject formed the title of a paper by Dr. A. C. Michie before the I. E. E. in April, 1913. The author stated that all oils oxidise in time, but the formation of deposits can be prevented or minimised by avoiding the following conditions: (1) Overheating; (2) undue access of air to the oil; (3) conditions likely to give rise to the formation of ozone; (4) contact of the oil with clean surfaces of copper, lead and iron.

365. U. S. Government specification for transformer oil calls for a pure mineral oil obtained by the fractional distillation of petroleum, unmixed with any other substance. It shall be prepared and refined especially for the purpose; shall be free from moisture, acid and alkali, and shall contain a minimum of sulphur compounds. The flash-point determined in a closed cup shall be not less than 170 deg. cent.; on cold test it shall not begin to solidify and no wax shall form in the oil above 0 deg. cent. It shall stand a breakdown test of 30,000 volts between spheres of 0.5 cm. radius, 0.40 cm. apart. See Circular No. 22, Bureau of Standards, 1911.

GASES

366. The insulating properties of gases depend upon, and vary with, the pressure and the temperature, and are affected also by the humidity. The only gaseous dielectric in extensive use is atmospheric air, whose properties have been the subject of extended research.

367. Air. The dielectric properties of air, and particularly its disruptive strength, have been the subject of more investigation than any other dielectric in use. Extended investigations and researches have been made by Ryan, Mershon, Fisher, Peek, Whitehead, Faccioli, Harding, Bennett, Fortescue and Farnsworth, whose results have been published during the past 10 years in the *Trans.* A. I. E. E., and by Russell (see *Jour.* I. E. E.).

Air in the free state has electrical conductivity. The results obtained (see Whitehead, *Proc.* A. I. E. E., May, 1915, p. 846) indicate that in the open the current passing between two parallel plates 10 cm. apart and each 100 cm. square, assuming perfect insulation, would be of the order of magnitude 3×10^{-18} amp. This is the maximum current which may be obtained and does not increase with increase of voltage; it diminishes greatly if the air is confined in a closed vessel. The conductivity may be greatly increased by exposing the air to Röntgen rays, ultra-violet light, etc., but the magnitude of current still remains very small.

The dielectric constant of air is usually taken as unity, and air is almost universally the medium of reference in all measurements of this constant.

THERMAL CONDUCTIVITIES.—*Continued*

Material	Watt-cm. per deg. cent.	Material	Watt-cm per deg. cent.
Slate	0.020	Woolen (pure) wadding, slightly packed	0.00036
Water, 25 deg. cent	0.0057	Woolen (pure) wadding, tightly packed	0.00023
Woolen (pure) wadding, loose	0.00049		

1 watt = 0.2389 g-cal. per sec. 1 g-cal. = 4.186 watt-sec.
1 watt = 0.0009468 B.t.u. per sec. 1 B.t.u. = 251.8 g-cal.
1 g-cal. = 0.003971 B.t.u. 1 B.t.u. = 1,054 watt-sec.

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STRUCTURAL MATERIALS

CAST IRON

371. Cast iron derives its characteristic qualities from the impuritie present. These impurities are the same as those in steel, except for graphite which is one form of carbon. The carbon is always present in two forms (1) combined carbon, in the form of cementite; (2) uncombined carbon, o graphite. The total carbon seldom exceeds. 4.5 per cent. or falls below 3.2 per cent. The larger the proportion of carbon in the combined state, th harder and more brittle will be the metal. Cast iron is not ductile, in eithe hot or cold state.

Silicon is a desirable impurity, because it tends to precipitate carbo in the graphitic form; about 3 per cent. of silicon gives the best results. Sul phur has the opposite effect of silicon, and is undesirable. Managnese in creases the total carbon and also the proportion of combined carbon, bu tends to neutralize the similar effect of sulphur. Phosphorus, if present i sufficient proportions to be chemically active, tends to hold the carbon i combined form, and also tends to weaken the metal.

372. White cast iron. If slowly cooled, white cast iron will contai combined carbon in the form of cementite, which imparts the qualities o hardness and brittleneess to the metal. Where the carbon is from 3 to 4 per cent., cementite will form from 45 to 50 per cent. of the material. Unles the cooling is slow some of the carbon will be in the austenitic form. Whit cast iron has few uses, except as a hard coating or skin for gray iron castings.

0.67 watt-cm. per deg. cent., for pure iron. Coefficient of linear expansion, 0.0000114 per deg. cent., between −18 and 100 deg. cent.

381. Electrolytic iron melted in vacuo, forged into half-inch rods and annealed at 900 deg. cent., has the following average tensile properties.

	Tensile strength (lb. per sq. in.)	Yield point (lb. per sq. in.)	Elongation (per cent.)	Reduction of area (per cent.)
As forged	54,800	48,400	33.3	83.0
Annealed at 900 deg. cent. and cooled in 12 hr.	38,100	18,000	51.8	87.4

The average carbon content of this iron was 0.0125 per cent. See Bulletin No. 72, Eng. Exp. Sta., Univ. of Ill., 1914.

382. Tensile properties of wrought iron. The following properties were taken from Campbell's "Manufacture and Properties of Iron and Steel," 4th edition, p. 91. Also see Holley, "The Strength of Wrought Iron as Affected by its Composition and its Reduction in Rolling," *Trans.* A. I. M. E., Vol. VI, p. 101.

	Minimum	Maximum
Carbon, per cent	0.015	0.512
Phosphorus, per cent	0.065	0.317
Silicon, per cent	0.028	0.321
Manganese, per cent	trace	0.097
Slag, per cent	0.192	2.262
Ultimate strength, lb. per sq. in	47,478	69,779
Elongation in 8 in., per cent	6.5	32.7
Reduction of area, per cent	7.7	59.8

The general influence of reduction in rolling is to increase the tensile strength and the elastic limit, and diminish the elongation and the reduction of area.

STEEL

383. Carbon steel. Increasing the carbon content of steel increases its strength, hardness, brittleness, and susceptibility to cracking under sudden cooling or heating; it also diminishes the elongation and reduction of area at fracture. In acid steel, according to Campbell, each 0.01 per cent. of carbon increases the strength by 1,000 lb. per sq. in., when the carbon is determined by combustion; or 1,140 lb. per sq. in. when the carbon is determined by color. In basic steel each 0.01 per cent. of carbon increases the strength by 770 lb. per sq. in., when the carbon is determined by combustion, or 820 lb. when determined by color.

Phosphorus increases the tensile strength 1,000 lb. per sq. in. for each 0.01 per cent., but tends to make the metal "cold short" or brittle. **Sulphur** does not alter the tensile strength appreciably, but tends to make the metal "hot short." **Manganese** increases the tensile strength from 80 to 400 lb. per sq. in. for each 0.01 per cent., depending upon the carbon present, and whether the steel is acid or basic; it also increases the ductility when hot. **Silicon** in small quantities has little effect. **Copper** has also little effect on the cold properties as far as known. **Nickel** gives a metal with a high elastic limit and great toughness under shock. **Aluminum** in small quantities is added to molten steel to absorb oxygen and improve the fluidity; when present in the finished metal it increases the strength and impairs the ductility.

The thermal conductivity of steel containing 1 per cent. of carbon, at 18 deg. cent., is 0.45 watt-cm. per deg. cent. The coefficient of linear expansion ranges between 0.0000104 and 0.0000132 per deg. cent.

is practically zero. **Platinite** (42 per cent. Ni) has the same coefficient of expansion as glass. See Colby, A. L. "Nickel Steel," *Proc.* A. S. T. M., Vol. III, 1903.

391. Silicon steel.* The addition of silicon to steel appears to increase the strength about 80 lb. per sq. in. for every 0.01 per cent., up to a content of 4 per cent.; beyond this point it severely impairs the ductility. A steel containing 3.40 silicon, 0.21 carbon and 0.29 manganese had a tensile strength of 106,000 lb. per sq. in.; elastic ratio, 0.74; elongation, 11 per cent.; reduction of area, 14 per cent.; annealed strength, 87,000 lb. per sq. in. Silicon is used in electrical sheets, because it substantially increases the electrical resistivity and thus reduces the core loss; such sheets are much harder than ordinary dynamo sheets of soft steel; see "Magnetic Materials" in this section. The thermal conductivity of 4 per cent. silicon-steel, between 20 and 250 deg. cent., is 0.32 watt-cm. per deg. cent.

392. Tungsten steel is characterized by great hardness and toughness and its remarkable tempering properties. The tungsten content ranges from 0.5 per cent. in ordinary bar steel to 2 to 5 per cent. in finishing and intermediate steels, 4.5 to 12 per cent. in self-hardening or air-hardening steels, and 14 to 26 per cent. in high-speed steels. See Baskerville, C. "The Chemistry of Tungsten, Etc.," *Trans.* N. Y. Elec. Soc., Oct. 29, 1912; also see "Magnet Steel," Sec. 4.

393. Vanadium steel. The use of vanadium as an alloy with steel produces more remarkable results than any other element except carbon. Oil-tempered vanadium steel containing 0.25 to 0.30 carbon, 0.50 manganese, 1.0 chromium and 0.17 vanadium has been found to have a tensile strength as high as 233,000 lb. per sq. in., elastic ratio 96 per cent., elongation 11 per cent., and contraction of area 39 per cent. The vanadium content is usually less than 0.3 per cent.; its effect in general is to improve the tensile properties, hardness and toughness.

394. Protective coatings for iron and steel consist of the following: (a) pigments of various kinds; (b) galvanizing or zinc coating, applied by hot-dipping, electroplating or dry-heating with zinc dust; (c) coating with aluminum alloy, by dry-heating with the powdered alloy; (d) tinning, by hot-dipping and rolling; (e) terne coating, of lead-tin alloy, applied in the same manner as tinning; (f) nickel-plating, applied electrolytically; (g) coating of black oxide of iron, produced by application of oil to the metal when black hot; (h) enamel coating of glass. For out-door exposures, methods (a) and (b) are the ones chiefly used, but neither is free from faults or objections. For particulars see Wood, M. P. "Rustless Coatings; Corrosion and Electrolysis of Iron and Steel," New York, 1905; "Report of the Committee on Preservative Coatings for Structural Materials," 1903 to 1913, A. S. T. M., Phila., Pa.; Bulletins No. 30, 35 and 239, U. S. Dept. of Agriculture, Wash., D. C. Stoughton, B. "The Metallurgy of Iron and Steel," McGraw-Hill Book Co., Inc., New York, 1911. Cushman and Gardner. "Corrosion and Preservation of Iron and Steel," McGraw-Hill Book Co., Inc., New York, 1910.

STEEL WIRE AND CABLE

395. Table of steel wire

(Roebling)

Number, Roebling gage	Diameter in inches	Area in square inches	Breaking load at rate of 100,000 lb. per sq. in.	Weight in pounds	
				Per 1,000 ft.	Per mile
000000	0.460	0.1662	16,620	558.4	2,948
00000	0.430	0.1452	14,520	487.9	2,576
0000	0.393	0.1213	12,130	407.6	2,152
000	0.362	0.1029	10,290	345.8	1,826
00	0.331	0.08605	8,605	289.1	1,527
0	0.307	0.07402	7,402	248.7	1,313

* Hadfield, R. A. *Jour.* Iron and Steel Inst., 1889; *Jour.* Inst. of Elec. Eng. 1902; Royal Dublin Society, 1900 and 1904; *Trans.* Faraday Society, 1914.

398. Tensile tests of steel spring wire, of best quality tempered music-wire, from about 0.04 to 0.05 in. in diameter, gave from 342,000 to 388,000 lb. per sq. in.; elongation between 1 and 2 per cent.; contraction of area, 38 to 46 per cent. (Report of Tests of Metals, 1904; Watertown Arsenal).

NON-FERROUS METALS AND ALLOYS

399. Properties of miscellaneous non-ferrous metals and alloys

(Compiled from various authorities)

Metal	Density	Strength (1,000 lb. per sq. in.)		Young's modulus (10^6 lb. per sq. in.)	Coefficient of expansion per deg. cent. $\times 10^{-6}$	Thermal conductivity (g-cal. per cm-cube per deg. cent. per sec.)
		Tension	Compression			
Aluminum, cast	2.56	12–15	12	9–11	......	......
Aluminum, rolled	2.68	24–40	....	11	23.1	0.34
Aluminum, wire	2.70	20–40	....	9–12	23.1	0.34
Aluminum, alloys	2.7–3.1	15–45	16–100	......	......	......
Aluminum, bronze	7.7–8.3	60–90	120	......	......	......
Brass, cast	8.5	18–24	30	9	17–22	0.23
Brass, wire	8.46	40–150	....	14	......	0.31
Bronze, bearing	8.5–8.9	25–50	80	......	17–22	......
Bronze, gun metal	....	25–55	....	10–12	......	......
Bronze, manganese	8.4	75–90	125–150	15	......	......
Bronze, phosphorus	....	35–50	....	14	17	......
Bronze, phosphorus, hard-drawn.	....	110–140	....	......	17	......
Bronze, silicon	....	55–75	....	......	......	......
Bronze, silicon, hard-drawn.	....	95–115	....	......	......	......
Bronze, Tobin	8.40	60–100	180	4.5	......	......
Copper, cast	8.5–8.9	22–25	40–60	10–12	......	......
Copper, rolled	8.9	29–35	....	......	16.7	0.84
Copper, wire	8.89	35–70	....	12–16	16.7	0.84
Delta metal, cast	....	45	....	......	......	......
Delta metal, rolled	....	70–85	....	13	......	......
Delta metal, wire	....	100	....	......	......	......
Duralumin	2.8	87	....	......	......	......
Gold	19.3	20–30	....	8	14.4	0.70
Gun metal	....	22–27	....	10–11	......	......
Lead	11.3	1.6–3.0	....	......	28.	0.084
Magnesium	1.74	30	....	......	26.	0.37
Monel metal	8.87	70–110	....	22–23	13.8	......
Nickel	8.6–8.9	40–85	....	24–27	12.6	0.14
Palladium	11.4	39	....	......	11.	0.17
Platinum	21.4	30–50	....	......	8.8	0.17
Silver	10.5	40–45	....	......	19.	1.10
Tin	7.30	3.5–5	....	4	21.	0.15
Zinc, cast	6.87	6–13	18	11–13	26.	......
Zinc, rolled	7.19	22–28	....	11–13	26.	0.27

Brick piers laid up in 1 part Portland cement, and 3 of sand, have from 20 to 40 per cent. of the crushing strength of the brick. Also see "Report of Tests of Metals," Gov. Printing Office, Washington, D. C., 1909.

407. Fire-brick when tested on end, at atmospheric temperature, should exhibit a crushing strength of more than 1,000 lb. per sq. in. More important is the ability to withstand a compression load of 50 lb. per sq. in. at 1,350 deg. cent. without failure; if the specimen shows marked deformation or contracts more than 1 in. in the standard length of 9 in., failure is considered to have taken place. Fire-brick should have average melting points as follows, in deg. cent.: fire clay, 1,650; bauxite, 1,695; silica, 1,700; chromite, 2,050; magnesia, 2,165.

408. Clay tile. The strength of drain tile is covered by standard specifications of the A. S. T. M. (see "Year Book"). Also see Bulletins No. 31 and No. 36, Iowa Eng. Exp. Sta., Ames, Ia.

409. Crushing strength of stone. (Smithsonian Phys. Tables, 1910; based on data furnished by U. S. Geol. Survey.)

Material	Size	Lb. per sq. in.
Marble	4-in. cubes	7,600 to 20,700
Brownstone	....	7,300 to 23,600
Sandstone	4-in. cubes	2,400 to 29,300
Granite	4-in. cubes	9,700 to 34,000
Limestone	4-in. cubes	6,000 to 25,000

TIMBER

410. Wood consists of a skeleton of cellulose permeated by a mixture of other organic substances collectively known as lignum, and particles of mineral matter (or ashes). Wood dried at 300 deg. fahr. is comprised of more than 99 per cent. organic matter, and less than 1 per cent. inorganic or non-combustible matter. In 100 lb. of wood dried at 300 deg. fahr. will be found about 49 lb. of carbon, 6 lb. of hydrogen and 44 lb. of oxygen; the composition is fairly uniform for the different species.

411. Spring wood and summer wood in coniferous trees are distinguished by the different colors in each ring. The inner light-colored portion of a ring is termed the spring wood, and the outer dark-colored portion is the summer wood. In oak and other broad-leaved woods, however, the darker portions are the spring wood and the lighter parts are summer wood.

412. Annual or yearly concentric rings which appear at the cross-section of a log are so many thin layers of wood, forming a consecutive series of enveloping cones, each ring or cone representing one year's growth. As a rule the rings are widest near the pith or centre of the tree.

413. Sapwood and heartwood. A zone of wood next to the bark, 1 in. to 3 in. or more wide, of light color and containing 30 to 50 or more annual rings, is the sapwood; the darker central portion is the heartwood. Only the outer portions of the sapwood assist in the growing processes and ultimately the sapwood changes to heartwood. The cells of the latter are lifeless and serve merely a structural function. The proportion of sapwood in coniferous trees constitutes 40 per cent. or more of the bulk, and a much larger proportion in young trees.

414. Moisture in wood. Water occurs in wood in three forms: (1) It forms the greater part (over 90 per cent.) of the protoplasmic contents of the living cells; (2) it saturates the walls of all cells; (3) it wholly or partly fills the cavities of the lifeless cells. In drying green wood in a kiln, from 40 to 65 per cent. of the weight of the sapwood and 16 to 40 per cent. of the weight of the heartwood are lost as excluded moisture. The weight is obviously dependent in large degree on the extent of seasoning or drying.

415. Shrinkage takes place as green wood is dried, but does not commence until the fibre saturation point is reached; then it commences to shrink both laterally and longitudinally, but the latter in most species is negligible. The shrinkage of transverse area in drying from green to oven-dry condition (3.5 per cent. of moisture) varies with different species from as much as 20 per cent. with hickory to as low as 7 per cent. with red cedar. The radial

Working unit stresses for structural timber (Par. 420)
(Unit stresses in pounds per square inch)

Kind of timber	Bending			Shearing				Compression					
	Extreme fiber stress		Modulus of elasticity	Parallel to the grain		Longitudinal shear in beams		Perpendicular to the grain		Parallel to the grain		Working stresses for columns	
	Average ultimate	Working stress	Average	Average ultimate	Working stress	Average ultimate	Working stress	Elastic limit	Working stress	Average ultimate	Working stress	Length under 15×d	Length over 15×d
Douglas Fir	6100	1200	1510000	690	170	270	110	630	310	3600	1200	900	1200 $(1-L/60D)$
Long-leaf Pine	6500	1300	1610000	720	180	300	120	520	260	3800	1300	975	1300 $(1-L/60D)$
Short-leaf Pine	5600	1100	1480000	710	170	330	130	340	170	3400	1100	825	1100 $(1-L/60D)$
White Pine	4400	900	1130000	400	100	180	70	290	150	3000	1000	750	1000 $(1-L/60D)$
Spruce	4800	1000	1310000	600	150	170	70	370	180	3200	1100	825	1100 $(1-L/60D)$
Norway Pine	4200	800	1190000	590*	130	250	100	.....	150	2600*	800	600	800 $(1-L/60D)$
Tamarack	4600	900	1220000	670	170	260	100	.....	220	3200*	1000	750	1000 $(1-L/60D)$
Western Hemlock	5800	1100	1480000	630	160	270*	100	440	220	3500	1200	900	1200 $(1-L/60D)$
Redwood	5000	900	800000	300	80	.....	.....	400	150	3300	900	675	900 $(1-L/60D)$
Bald Cypress	4800	900	1150000	500	120	.....	.....	340	170	3900	1100	825	1100 $(1-L/60D)$
Red Cedar	4200	800	800000	....	.....	.....	.....	470	230	2800	900	675	900 $(1-L/60D)$
White Oak	5700	1100	1150000	840	210	270	110	920	450	3500	1300	975	1300 $(1-L/60D)$

Unit stresses are for green timber and are to be used without increasing the live load stresses for impact. Value noted* are for partially air-dry timber. In the formulas given for columns, L = length of column, in inches, and D = least side or diameter, in inches.

finally the preservative is introduced under pressure. This method secures the maximum penetration and absorption.

The open-tank treatment, hot process at atmospheric pressure, is used extensively for treating the butts of poles. The penetration in open-grained porous wood is from 0.75 to 1.00 in., and in dense wood from 0.25 to 0.50 in. The brush treatment secures a penetration ranging from 0.06 to 0.25 in., or from two to three annual rings. The absorption in all cases is increased by seasoning. Also see Bulletin No. 78, U. S. Forest Service, 1909.

427. Strength of treated timber. Talbot concluded from his tests of timber (Bulletin No. 41, Eng. Exp. Sta., Univ. of Ill., 1909; also see Forest Service Circular No. 39) that creosoting, under ordinary practice, decreases the strength and the stiffness.

ROPE AND BELTING

428. Manila rope. The weight of Manila rope, based on the tests of the C. W. Hunt Co., is expressed by the formula

$$\text{Wt. per ft.} = 0.34d^2 \quad \text{(lb.)} \tag{22}$$

where d is the diameter in inches. The tensile strength is given by the formula

$$T = 7{,}160d^2 \quad \text{(lb.)} \tag{23}$$

where T is the total strength of the rope and d is the diameter in inches. Kirsch concluded from his tests that a rope having a diameter of 1 in. would have an average breaking strength as follows: **Italian hemp**, 9,910 lb.; Hungarian hemp, 9,293 lb.; Manila rope, 7,100 lb.

The Plymouth Cordage Co. gives the following rules for Manila rope: the weight per ft. is equal to the square of the diameter in inches, multiplied by 0.34; the breaking strength is equal to the square of the diameter, multiplied by 7,500; the maximum permissible tension (rope drives) is equal to the square of the diameter, multiplied by 200.

The C. W. Hunt Co. gives factors of safety, for computing the allowable working loads, approximately as follows: tackle, 7; hoisting, 18; transmission, 35. The efficiency of knots in rope ranges from 50 to 90 per cent.

For further data on tests of rope, see "Tests of Metals," reporting Watertown Arsenal tests, Gov. Printing Office, Wash., D. C.

429. Danger of metal filaments or strands in rope. Metal filaments are sometimes introduced in ropes to give added strength; when such ropes are used by linemen they become exceedingly dangerous, owing to the probability of communicating electrical shock. Similar danger exists from tape lines made of a fabric base and containing metal threads.

430. Leather belting weighs about 60 lb. per cu. ft. and has a tensile strength of from 2,000 to 5,000 lb. per sq. in., or an average of about 650 lb. per in. of width of single belt. See tests of leather belting in Watertown Arsenal Reports ("Tests of Metals") for 1893, Gov. Printing Office, Wash., D. C.

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Table 432.—*Continued*

Element	Symbol	Atomic weight	Density	Specific heat	Melting point (deg. cent.)
Lanthanum	La	139.0	6.15	0.045	810
Lead	Pb	207.10	11.37	0.030	327.4
Lithium	Li	6.94	0.534	0.85	186
Lutecium	Lu	174.0	....	....	....
Magnesium	Mg	24.32	1.72	0.249	651
Manganese	Mn	54.93	7.4	0.111	1260
Mercury	Hg	200.6	13.55	0.033	−38.7
Molybdenum	Mo	96.0	8.5	0.064	2500
Neodymium	Nd	144.3	....	....	840
Neon	Ne	20.2	....	....	−253
Nickel	Ni	58.68	8.80	0.109	1452
Niton	Nt	222.4	....	....	....
Nitrogen	N	14.01	0.83	....	−210
Osmium	Os	190.9	22.5	0.031	2700
Oxygen	O	16.00	1.14	....	−218
Palladium	Pd	106.7	11.4	0.059	1549
Phosphorus	P	31.04	2.34	0.19	44
Platinum	Pt	195.2	21.45	0.032	1755
Potassium	K	39.10	0.87	0.170	62.3
Praseodymium	Pr	140.6	6.475	....	940
Radium	Ra	226.4	....	....	....
Rhodium	Rh	102.9	12.4	0.058	1940
Rubidium	Rb	85.45	1.532	....	38
Ruthenium	Ru	101.7	12.3	0.061	>1950
Samarium	Sa	150.4	7.75	....	1350
Scandium	Sc	44.1	....	....	....
Selenium	Se	79.2	4.55	0.068	218.5
Silicon	Si	28.3	2.1	0.175	1420
Silver	Ag	107.88	10.6	0.055	960.5
Sodium	Na	23.0	0.971	0.253	97.5
Strontium	Sr	87.63	2.54	....	....
Sulphur (S_1)	S	32.07	2.05	0.178	112.8
Tantalum	Ta	181.5	16.6	0.033	2850
Tellurium	Te	127.5	6.25	0.048	452
Terbium	Tb	159.2	....	....	....
Thallium	Tl	204.0	11.85	0.033	302
Thorium	Th	232.4	11.0	0.027	>1700
Thulium	Tm	168.5	....	....	....
Tin	Sn	119.0	7.30	0.054	231.9
Titanium	Ti	48.1	3.5	0.110	1795
Tungsten	W	184.0	18.85	0.034	3000
Uranium	U	238.5	18.7	0.028	....
Vanadium	V	51.0	5.5	0.115	1720
Xenon	Xe	130.2	3.52	....	−140
Ytterbium	Yb	172.0	....	....	....
Yttrium	Y	89.0	3.8	....	....
Zinc	Zn	65.37	7.19	0.093	419.4
Zirconium	Zr	90.6	4.14	0.066	1700

433. Density of water, from 0 to 100 deg. cent. (Circular No. 19, Bureau of Standards).

Temp.	Density	Temp.	Density	Temp.	Density	Temp.	Density
0	0.99987	25	0.99707	50	0.98807	75	0.97489
4	1.00000	30	0.99567	55	0.98573	80	0.97183
10	0.99973	35	0.99406	60	0.98324	85	0.96865
15	0.99913	40	0.99224	65	0.98059	90	0.96534
20	0.99823	45	0.99024	70	0.97781	100	0.95838

SECTION 5

MAGNETS

BY CHARLES R. UNDERHILL

GENERAL

1. A magnet is a body which possesses the property of attracting magnetic substances. All magnets may be divided into two classes, *i.e.*, permanent magnets and electromagnets.

2. A permanent magnet is one which retains a nearly constant value of m.m.f. for an indefinite period. The permanent magnets used in practice are made of hardened steel and are magnetized by placing them in a strong magnetic field.

3. An electromagnet or temporary magnet is one in which the magnetic field is produced by an electric current. In its simplest form, it consists of a helix of conducting material which, when energised by an electric current, possesses many of the magnetic-field characteristics of a permanent magnet. Since soft iron is capable of being magnetized to a high degree and retains very little permanent magnetism, with the result that the magnetic field may be varied in great degree by regulating the strength of the current, electromagnets are usually provided with soft-iron cores.

PERMANENT MAGNETS

4. Permanent magnets are used where a constant field is desired, as in electrical instruments, magneto-generators, etc. In order to retain its strength, a permanent magnet should have as short an air-gap as possible and the ratio of its length to its cross-sectional area should be great. Unless proper precautions are taken, the steel is likely to crack and warp in hardening.

5. Permanent magnets lose a portion of their original magnetism when used for a long time, so that they become what is termed aged. Those used in instruments, magnetos, etc., are **artificially aged** in the process of manufacture. This is accomplished in various ways. One method of aging magnets for watt-hour meters is to pass them several times through a bath of boiling water or oil and then to demagnetise them to about 7 per cent. of the original value by rotating a copper disc between the poles. Another method is to place a large number of magnets parallel to each other, with a copper strip between the poles, and then to demagnetise them by passing a strong current through the strip. (Also see Par. 10.)

6. Permanent magnets are made from the best grade of crucible tungsten steel which contains about 5 per cent. of tungsten, a small percentage each of chromium and manganese, and from 0.63 to 0.66 per cent. carbon. Domestic steel of this variety is found to be equal to or better than any imported magnet steel.

7. Details of manufacture. The practice of the Sangamo Electric Co. is as follows: After preliminary tests, the steel bars are sheared cold and then heated in a special fuel-oil furnace so arranged that the products of combustion do not come directly in contact with the steel. The pieces are forged at a bright red heat, in a large press, using but one heat if possible. A vertical type press is preferred to the "bull-doser" type. The pieces are then allowed to cool in the air, if no drilling is required; or, if drilling is necessary, they are packed in mica dust to prevent air-hardening, which takes place to a certain extent even with steel containing no more than

19. An iron-clad solenoid is a solenoid-and-plunger provided with an iron or steel frame or jacket. The effect is to confine the magnetic field within the limits of the frame. When provided with a stop, as in Fig. 1, this type is commonly known as a **plunger electromagnet.**

FIG. 1.—Plunger electromagnet.

20. A bar electromagnet is a solenoid-and-plunger with the plunger inserted in the solenoid and rigidly fixed in position.

21. A horseshoe electromagnet is a bar electromagnet bent into U form in order to bring the pole ends near together. The practical type consists of two bar electromagnets magnetically joined together at one end by means of a **yoke or backiron.** Both coils are usually wound in the same direction and their inside wires connected together. The **armature** consists of a piece of soft iron or steel of sufficient length to bridge the gap from pole to pole, and of cross-section great enough to conduct the flux economically.

22. An iron-clad electromagnet is a bar electromagnet inserted in a soft-iron or steel cup, so that the rim of the cup and the core of the electromagnet form the attracting surfaces (or poles) for the armature, which usually consists of a disc of soft iron or steel.

23. Modified types. There are many modifications of the above fundamental types. For instance, plunger electromagnets are sometimes made in the horseshoe form, and a modification of the horseshoe electromagnet will be recognised in the iron-clad electromagnet. In some telephone relays, an angle-iron is employed instead of the shell or cup.

GENERAL THEORY OF ELECTROMAGNETS

24. Maxwell's fundamental equation for the pull applies strictly to a portative electromagnet consisting of a bar electromagnet separated at its middle and having a hypothetically zero air-gap, so that no flux can leak back from the abutting ends of the half-cores to their opposite ends. The equation for the pull in dynes per sq. cm. is $P = \mathcal{B}^2/8\pi$. But $\mathcal{B} = 4\pi\mathcal{J} + \mathcal{H}$, wherein $\mathcal{J}$ is the intensity of magnetisation in the iron only. It is convenient to reduce $4\pi\mathcal{J}$ to ϕ_i/s, and $\mathcal{H}$ to ϕ_a/s, wherein ϕ_i is the iron flux, equivalent to the flux in a permanent magnet; ϕ_a is the flux in the air-core only, and s is the cross-sectional area of the core in square centimeters. Then

$$\mathcal{B} = \frac{\phi_i + \phi_a}{s} \qquad \text{(gausses)} \qquad (1)$$

$$\text{whence } P = \frac{\phi_i^2 + 2\phi_i\phi_a + \phi_a^2}{8\pi s^2} \qquad \text{(dynes per sq. cm.)} \qquad (2)$$

25. Theoretical components of pull. Conventionally stated, $\phi_i^2/8\pi s^2$ is the purely magnetic pull between the two half-cores; $2\phi_i\phi_a/8\pi S^2$ is the pull due to the "solenoid effect" between the half-coils and the half-cores; and $\phi_a^2/8\pi s^2$ is the pull between the half-coils, which cannot be utilised in practical electromagnets with solidly wound coils. The actual pull, in dynes per square centimeter, for such a portative electromagnet, *i.e.*, when there is no appreciable air-gap between the attracting surfaces of the half-cores, and the coil is solidly wound, is

$$P = \frac{\phi_i^2 + 2\phi_i\phi_a}{8\pi s^2} \qquad \text{(dynes per sq. cm.)} \qquad (3)$$

26. Total pull between two polar surfaces. An electromagnet can do work only when there is an air-gap. The longitudinal contraction of the flux in the air-gap of a tractive electromagnet causes the pull. The pull in dynes per square centimeter between the polar surfaces of the cores of a horseshoe electromagnet and its armature, or for any other type of electromagnet with the coils and cores rigidly fixed, relatively, is $P = \phi_g^2/8\pi s^2$, wherein ϕ_g is the total average flux producing the pull. This equation expresses the pull in an air-gap, or the pull between two magnetic masses in contact, and holds for permanent magnets.

27. The pull due to an iron-clad solenoid or a plunger electromagnet consists of two components, since there are two magnetic circuits in shunt ductance (Par. 31) in henrys, and t the elapsed time in seconds from the moment the circuit was closed. Solving the above equation for t,

$$t = -\frac{L}{R}\log_e\left(1 - \frac{R_i}{E}\right) \qquad \text{(sec.)} \qquad (7)$$

wherein L/R is called the time-constant. From this it is seen that for a given ratio of instantaneous current to the Ohm's law or steady current the time-constant is the only factor which determines the time required to establish it. The time-constant is fixed by the magnetic circuit and by the ampere-turns (when iron is present), and is independent of the number of turns, since both the resistance and the inductance vary as the square of the number of turns, for a given winding volume.

31. The approximate inductance of a magnet, in which the air-gap is not very long, can be found by assuming the magnet to be connected to a source of alternating e.m.f., of known frequency and having a sine-wave form. Let l_i be the length in cm. of the iron circuit; l_a the length in centimeters of the air-gap; N the number of turns; s the cross-sectional area in square centimeters of the magnetic circuit, and ϕ the maximum value of the total flux. Then the alternating e.m.f. which must be impressed upon the winding to produce a maximum flux ϕ, is

$$E = 4.44 f N \phi 10^{-8} \qquad \text{(volts, effective)} \qquad (8)$$

where f is the frequency in cycles per second. The approximate reluctance is,

$$\mathcal{R} = \frac{1}{s}\left(\frac{l_i}{\mu} + l_a\right) \qquad \text{(oersteds)} \qquad (9)$$

μ can be taken from curves in Sec. 4. The m.m.f. is $\mathcal{F} = 1.257 NI$ wherein I is the effective (root mean square) current in amperes. It follows from $\mathcal{F} = \phi\mathcal{R}$ that

$$I = \frac{\phi\mathcal{R}}{1.257N} \qquad \text{(amp.)} \qquad (10)$$

and then

$$L = \frac{E}{2\pi f I} \qquad \text{(henrys)} \qquad (11)$$

32. Effect of movable plunger. Magnets with movable plungers which are designed for quick and powerful action require careful study of the initial conditions. At the first instant, the value of the current increases at such a rate that the flux interlinked with the winding will generate a counter-e.m.f. equal to the impressed e.m.f. The pull produced by the flux due to this current will start the plunger, and the sudden decrease in reluctance due to the closing of the air-gap will produce a corresponding increase in flux and thus decrease the rate of change of the current, that is, the current will be retarded in reaching its final and permanent value. Because of this phenomenon it is usual to design such magnets (for instance, those used on automatic starters) for a much lower impressed e.m.f. than is actually used at the start; in order then to protect the winding, a higher resistance is automatically inserted in series with it at the moment the plunger arrives at the end of its stroke. This resistance limits the current to that value necessary to maintain the required final pull on the plunger.

33. The action of the plunger in reducing the reluctance of the magnetic circuit may be approached from the energy standpoint; thus, the total energy which is supplied to the magnetic field up to any instant is

$$W = E\int_0^t i\,dt = \frac{i^2L}{2} + 0.113Fl \qquad \text{(watt-seconds)} \qquad (12)$$

wherein i is the instantaneous value of the current in amperes, L the instantaneous value of the inductance in henrys, F the average pull on the plunger in pounds, and l the distance traveled in inches.

curves represent the energy transferred to the magnetic field, in each case. With the moving plunger, the total energy input to the magnetic field is the area *cdihf*, while that left as stored energy at the end of the stroke and which must be dissipated when the circuit is broken is the area *cghf*. The difference between these two, *cdihg* (cross-hatched) represents the energy transformed into mechanical work and dissipated by friction and impact of the plunger during its travel, iron losses being neglected.

36. Impressed e.m.f. The final permanent value of impressed e.m.f. in continuous-current magnets is determined by the resistance.

37. The speed of the magnet action also depends upon the secondary losses (eddy currents, Sec. 2). These losses tend to reduce the flux in the core and thus retard the attainment of the maximum pull. This effect can be reduced by **laminating** the core and frame and by avoiding the use of a metallic bobbin. It can also be reduced by using fewer turns and larger current.

38. Slow-speed magnet action is obtained by increasing the time-constant and the secondary losses. The secondary losses can be greatly increased by dividing the winding into sections and short-circuiting more or less of the sections. Fig. 5 shows the effect of short-circuiting a portion of the winding.* The total number of turns was 8,400; in curve (1), only 2,165 turns were energized, while in curve (2), 2,165 turns were energized and 6,235 turns short-circuited. In the first case it required about 2.5 seconds to reach the maximum current, while in the second case, 4 seconds were required.

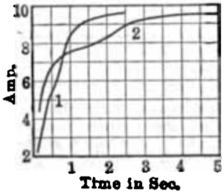


FIG. 5.—Showing the effect on the current-time curve of short-circuiting a portion of the winding.

39. Inertia of moving system. When the speed of action is an important factor, the inertia of the load to be moved must be considered. For the same pull and range, a quicker action will be obtained in overcoming the tension of a spring than in moving a mass whose weight is equivalent to the tension of the spring. The total mass to be moved is the sum of that of the plunger and the external load.

40. The selection of the type of electromagnet for specific work depends upon the nature of the e.m.f. (whether continuous or alternating) the range or distance of travel, and the rate of travel or the quickness of action. In general, the iron-clad solenoid type is mechanically the better protected and is the best adapted for a long range and a strong pull, but it is slow in action. The horseshoe type has the quicker action, but is only adapted for short ranges. The plunger-electromagnet type has an action intermediate between the above-mentioned types and may approximate one or the other according to the range and the dimensions of the plunger. The bar type possesses the quickest action of all, but its pull is comparatively feeble. The general rule is that what is lost in time is gained in pull and *vice versa*.

41. The required dimensions of an electromagnet are proportional to the load, the range, the duration of excitation and the interval between excitations. The cross-sections of the plungers, cores and frames will depend upon the flux required to produce the desired pull. In long-range electromagnets of the solenoid type, wherein the pull is proportional to the product of the magnetizing force times the flux in the plunger, the plunger may be of small cross-section, if the duration of excitation is to be brief, since the magnetising force of a comparatively small coil may be made great for a short period without danger of injury from overheating. This does not hold for a horseshoe type, since the main pull is dependent upon the flux which the cores are able to conduct, and is only slightly affected by an increase in the magnetizing force after the cores approach saturation.

42. Electromagnets designed for continuous service must have windings of such volume and radiating surface, and wires of such cross-sections,

* Lindquist, D. L. "Alternating-current Magnets;" *Electrical World*, 1906, Vol. XLVII, p. 1295.

ing force has a high value as compared with the cross-sectional area of the plunger, the maximum pull may occur for a considerable distance on each side of the middle of the solenoid. On the other hand, with very short solenoids, the maximum pull may occur at or near the end of the solenoid opposite to that at which the plunger enters.

48. Characteristic pull with solenoid and plunger. Fig. 8 shows the approximate pull diagram for different positions of the plunger. The expression for the maximum uniform pull is

$$F = Cs\frac{NI}{l} \qquad \text{(lb.)} \qquad (15)$$

where I is the current in amp., N the number of turns, s the cross-sectional area of the core or plunger in square inches, l the length of the solenoid in inches and C the pull in pound per square inch per ampere-turn per inch. C depends upon the proportions of the coil, the degree of saturation, and the length, physical character and chemical purity of the plunger. Par. 50 gives values for C for various solenoids tested by the author.

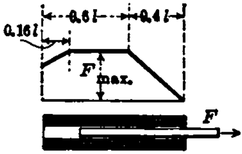


Fig. 8.—Diagram of plunger pull for simple solenoid and plunger.

49. In magnets with long air-gaps, the path through the iron can be considered as having no reluctance. Therefore the **saturation characteristic** can be assumed to be a straight line, and the ampere-turns for the air-gap directly proportional to the flux density.

50. Maximum Pull per Square Inch of Core for Solenoids with Open Magnetic Circuit*

Length of coil (in.)	Length of plunger (in.)	Area of core (sq. in.)	Total amp-turns	Amp-turns per in.	Max. pull, lb. per sq. in	Lb. per sq. in. per 1,000 amp-turn per in.
6	Long	1	15,000	2,500	22.4	9.0
9	Long	1	11,330	1,260	11.5	9.1
9	Long	1	14,200	1,580	14.6	9.2
10	10	2.76	40,000	4,000	40.2	10.0
10	10	2.76	60,000	6,000	61.6	10.3
10	10	2.76	80,000	8,000	80.8	10.1
12	Long	1	11,200	930	8.75	9.4
12	Long	1	20,500	1,710	16.75	9.8
18	36	1	18,200	1,010	9.8	9.7
18	36	1	41,000	2,280	22.5	9.8
18	18	1	18,200	1,010	9.8	9.7
18	18	1	41,000	2,280	22.5	9.8

Note.—These tests indicate that nothing is gained in maximum pull by making the plunger considerably longer than the coil.

51. The diameter of the coil should be about three times that of the plunger, and the **length of the coil** should be at least two or three times its outside diameter whenever possible.

52. The range is directly proportional to the length of the coil, **regardless** of the ampere-turns.

53. The pull in an iron-clad solenoid, with or without a stop, is composed of two components; one is the pull between the end of the stop or frame and the plunger, and the other is that between the winding and the plunger. The latter is all important at the beginning of the stroke, but near the end the stop-pull becomes predominant. The expression for the pull is

$$F = s\left(\frac{NI}{l_a c}\right)^2 + \frac{sCNI}{l} = sNI\left[\frac{NI}{l_a^2 c^2} + \frac{C}{l}\right] \qquad \text{(lb.)} \qquad (16)$$

* Underhill, C. R. "The Practical Design of the Solenoid;" *Electrical World and Engineer*, 1905, Vol. XLV, p. 796.

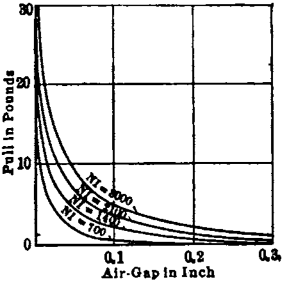


FIG. 10.—Test curves on pull of horseshoe electromagnet.

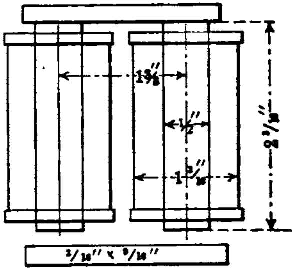


FIG. 11.—Horseshoe type of electromagnet.

magnetic material located in the armature directly under the centre of the magnet cores. The total amp-turns, with 0.075 amp., are 127.5; the cores are ⅜ in. in diameter, and each spool is 4 in. long by 2 in. in diameter. In general, signal relays are required to release their armatures when the energising current is reduced one-half.

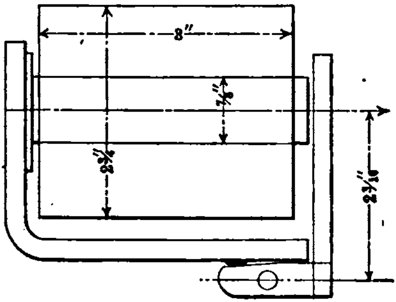


FIG. 12.—Clapper type of horseshoe electromagnet.

60. Electromagnets for telegraph and telephone service, like those for signal apparatus, are designed to perform the sole function of closing or opening one or more local contacts which control other circuits and apparatus. The armatures of these magnets customarily work through very limited ranges, not exceeding as a rule a very small fraction of an inch; the opposing force is sometimes gravity, sometimes a light spring. Such relays are also wound differentially for certain kinds of work, and sometimes are polarised (having a permanently magnetised steel core) in order to respond only to currents flowing in a given predetermined direction around the windings (see Sec. 21).

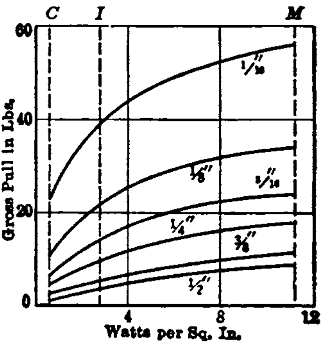
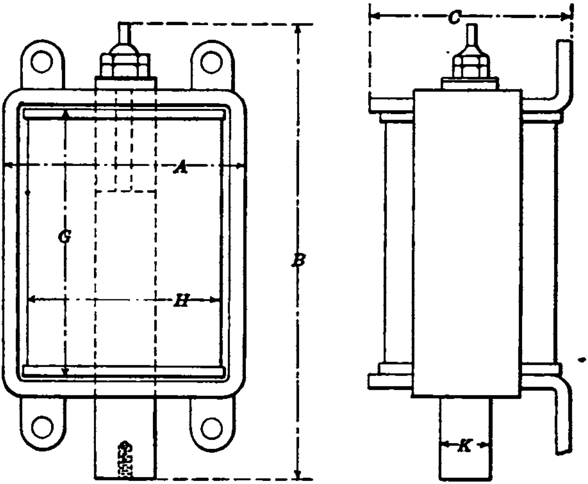


FIG. 13.—Test curves on pull of clapper type of electromagnet.

61. The equation for the pull due to electromagnets of the horseshoe type is given by

$$F = s\left(\frac{NI}{l_a C}\right)^2 \quad \text{(lb.)} \qquad (18)$$

where the constants are the same as those given in Par. 52. The total length of all the air-gaps is to be taken for l_a. The equation is only approximately correct, in this case, owing to leakage. It is better to increase the



No.	Curve	*A*	*B*	*C*	*G*	*H*	*K*
3	Fig. 18	3¼"	6¼"	3"	3"	2¾"	¾"
5	Fig. 19	4⅛"	8¾"	3¾"	4¾"	3⅜"	1"

FIG. 17.—Standard type of plunger electromagnet with flat-end plunger.

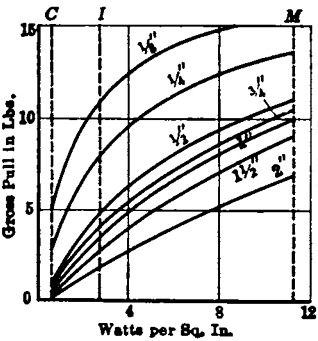


FIG. 18.—Test curves on pull of standard type of plunger electromagnet No. 3 in Fig. 17.

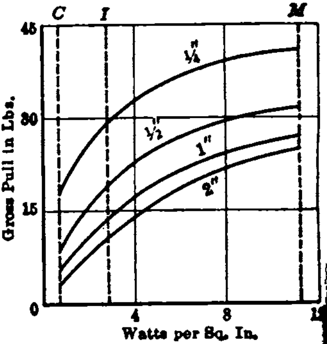


FIG. 19.—Test curves on pull of standard type of plunger electromagnet No. 5 in Fig. 17.

64. Continuous-duty electromagnets may be excited at full voltage continuously without dangerous heating of the coils, but the pull becomes somewhat reduced when the windings reach full temperature, due to the increased resistance in the coil.

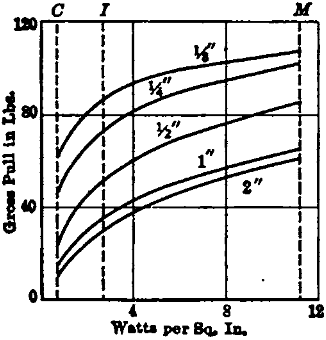


FIG. 23.—Test curves on pull of standard type of plunger electromagnet No. 6 in Fig. 20.

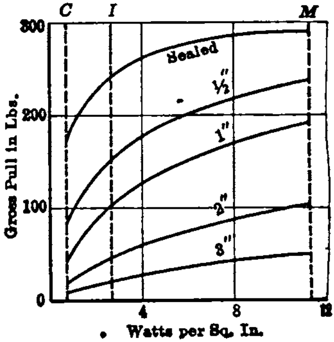


FIG. 24.—Test curves on pull of standard type of plunger electromagnet No. 22 in Fig. 20.

65. Intermittent duty is an arbitrary rating and, in general, these magnets may be excited at full voltage for not over 7 min. during any half-hour, without dangerous heating.

66. Momentary duty is also an arbitrary rating which represents the highest practicable rating of these magnets, and full voltage should not be maintained on them for more than 2 min. during any half-hour.

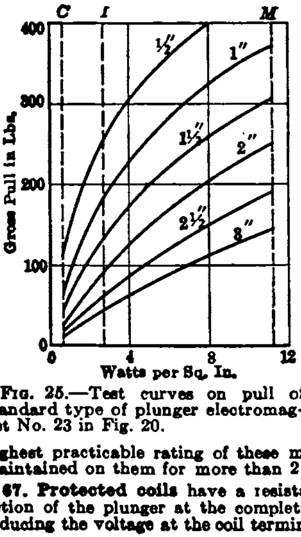


FIG. 25.—Test curves on pull of standard type of plunger electromagnet No. 23 in Fig. 20.

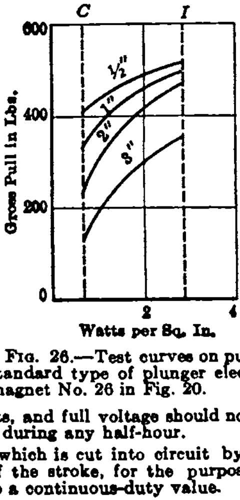


FIG. 26.—Test curves on pull of standard type of plunger electromagnet No. 26 in Fig. 20.

67. Protected coils have a resistance which is cut into circuit by the action of the plunger at the completion of the stroke, for the purpose of reducing the voltage at the coil terminals to a continuous-duty value.

69. Two forms of plungers are shown; one is conical, and the other square at the ends. The curves show the former to be preferable on most of the long-stroke types, as the tendency is to increase the starting pull and reduce the final pull. For short strokes, the square-end type is preferable.

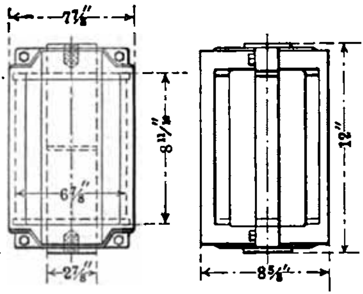


FIG. 30.—Standard type of plunger electromagnet with flat-end plungers. (See Fig. 31.)

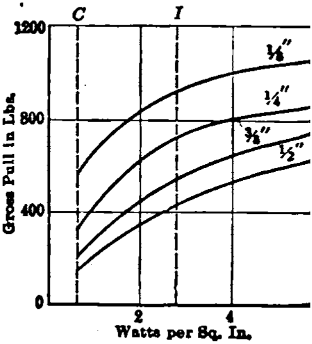


FIG. 31.—Test curves on pull standard type plunger electromagr in Fig. 30.

PORTATIVE ELECTROMAGNETS

70. Lifting magnets. Large iron-clad magnets called lifting magnets are now extensively used for the handling of materials in all branches of the iron and steel industry. They are used for handling pig iron, scrap, castings, billets, tubes, rails, plates and crop ends; for loading and unloading cars and ore vessels, and for handling skull-cracker balls and miscellaneous material.

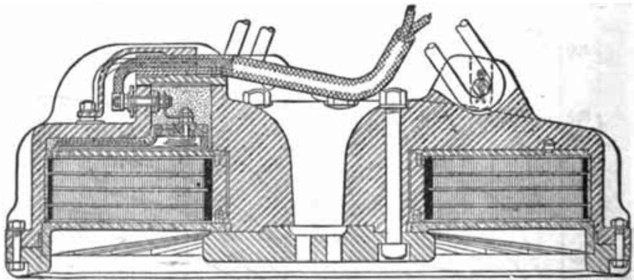


FIG. 32.—Cross-section of a standard lifting magnet.

71. Operation. All lifting magnets, of whatever make, are suitable for use on continuous-current circuits only. It is not customary to operate the magnets on very high voltages, owing to the difficulty of securing insulating material that will withstand the high inductive reaction that occurs when the circuit of a magnet is suddenly opened. Standard magnets are designed for operation on 220-volt continuous-current circuits.

72. Lifting load. It is quite impossible to calculate the load which may be lifted, owing to the varying contact of the load, but Par. 73, giving a table published by one of the leading manufacturers, shows the approximate lifts for various magnets and materials.

74. General construction. These magnets are very rugged in construction, as they are subject to exceedingly rough usage. The frame or body of the magnet is made, in one instance, of a special grade of dynamo steel which combines great strength and ductility with excellent magnetic qualities. They are designed to withstand heat without injury and are waterproof. Fig. 32 shows the construction of a lifting magnet made by the Cutler-Hammer Clutch Co.

75. Operating cost. The Cambria Steel Co. state that their costs for unloading material from broad-gage cars into open-hearth charging boxes are as follows: Handling light scrap, 3.15 cents per ton; handling heavy scrap, 2.28 cents per ton; handling pig iron, 3.06 cents per ton. These costs include: electric energy, repairs, interest and depreciation for crane and magnet, and all labor. About 50 per cent. of the foregoing cost is for labor in trimming the charging boxes.

The Inland Steel Co. reports that about 4,000,000 lb. of machine cast pig iron were unloaded in 10.5 hr. by two 62-in. magnets. The average lift per magnet was 3,427 lb.

76. Holding magnets. Portative magnets are extensively used for holding tools and materials in grinding and other machines. The no-load release magnet belongs to this class.

ALTERNATING-CURRENT TRACTIVE ELECTROMAGNETS

77. The design of an alternating-current magnet involves calculations which are essentially the same as those for an ordinary transformer. The total flux is determined by the number of turns, the supply voltage and the frequency; thus

$$E = \frac{2\pi f N \phi_{max}}{\sqrt{2}\, 10^8} = 4.44 N \phi_{max}\, 10^{-8} \quad \text{(volts, effective)} \qquad (19)$$

and

$$\phi_{max} = \frac{E\, 10^8}{4.44 f N} \qquad \text{(maxwells)} \qquad (20)$$

where f is the frequency in cycles per second, N the number of turns, and E the impressed e.m.f. in volts.

Taking f and N as constant,

$$\phi_{max} = s\mathcal{B}_{max} = KE \qquad \text{(maxwells)} \qquad (21)$$

and

$$\mathcal{B}_{max} = K\frac{E}{s} \qquad \text{(gausses)} \qquad (22)$$

That is, for a given impressed e.m.f. the flux density will vary inversely as the area of the core.

78. Flux density. Since the air-gap pull varies approximately as the square of the flux density, it would appear that the flux density should be as great as possible. However, the iron losses also increase with the flux density, so that the maximum possible flux density is not the most efficient.

79. Iron losses. The major portion of the total loss takes place in the iron rather than in the copper. The **hysteresis loss** (Sec. 2) is calculated in the same way as that of a transformer, and denoted by P_h. The **eddy-current loss** (Sec. 2) is denoted by P_e. Then the total core loss is

$$P_c = P_h + P_e \qquad \text{(watts)} \qquad (23)$$

80. The quadrature exciting watts are expressed thus:

$$P_q = 2.5 f \frac{\phi^2_{max}}{s}\left(l_a + \frac{l_i}{\mu}\right)10^{-8} \qquad \text{(watts)} \qquad (24)$$

wherein l_a is the length of the air-gap in centimeters, l_i the mean length in centimeters of the magnetic circuit in the iron, and s is the cross-sectional area of the core in square centimeters. Expressing this in terms of the volume of the air-gap and of the iron, which quantities must be known in determining the core losses,

$$P_q = 2.5 f \mathcal{B}^2_{max}\left(V_a + \frac{V_i}{\mu}\right) 10^{-8} \qquad \text{(watts)} \qquad (25)$$

POLYPHASE ELECTROMAGNETS

87. Test data on two-phase electromagnet. Fig. 33 shows the relation between volt-amperes and pounds pull for a two-phase magnet designed and tested by D. L. Lindquist.* The magnet contained four coils, each wound with 220 turns of No. 14 A.W.G. copper wire, the cross-sectional area of the core being 1.94 sq. in. The test was made with the magnet-winding connected to a two-phase 60-cycle system. The method of connecting the coils is shown in Fig. 34.

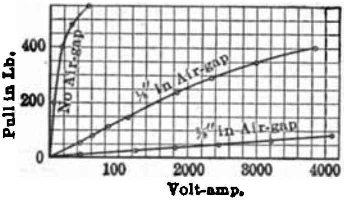


FIG. 33.—Relation between volt-amp. and pull for a two-phase electromagnet.

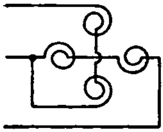


FIG. 34.—Connections of two-phase electromagnet.

88. Pulsation of pull. Theoretically, the pull in magnets equipped with polyphase windings should be constant and equal to the maximum pull due to the maximum flux produced in one phase. This, however, is only true when the e.m.f. wave is a pure sine-function. Distortion of the e.m.f. wave will alter the mean effective pull. In any case, if the load exceeds the minimum instantaneous pull, there will be chattering exactly similar to that in a single-phase magnet.

89. Polyphase magnets used in signal relays are, in effect, induction motors so designed that the rotor can revolve through a sufficient angle to operate the contacts. This gives a constant drag on the rotor due to the rotary magnetic field and avoids chattering.

HEATING OF ELECTROMAGNETS

90. Capacity limited by heating. The capacity of a given electromagnet is limited by the amount of heat which it can dissipate per unit time without exceeding a given or safe temperature rise.

91. The temperature rise can be measured by placing a thermometer against the outside surface or by measuring the change in resistance (Sec. 3). The general formula given below supposes the mean temperature rise to be measured by the thermometer method. The resistance method gives results which are from 1.5 to 2 times greater than those obtained with a thermometer, depending upon the depth of the winding, circulation of the air, etc.

92. Formula for temperature rise. The general equation for the final temperature rise in a given coil is

$$t_s = k\frac{P}{A} \qquad \text{(deg. cent.)} \qquad (33)$$

where P is the power, in watts, dissipated in the coil; A the outside cylindrical surface of the coil (the ends are counted only in short coils, and the inside surface is not counted), and k is the temperature rise in centigrade degrees per watt per square inch of outside cylindrical surface. Values for the constant, k, in the above formula differ widely according to the proportions of the coil, the depth of the winding, etc. The following values represent average practice. For open electromagnets, $k = 130$; for iron-clad electromagnets, $k = 95$.

93. Safe temperature limits. The final temperature rise varies between 50 deg. cent. and 75 deg. cent. according to the specifications, the climate, the depth of winding, etc. The internal temperature rise is limited to that which

* Lindquist, D. L. "Polyphase Magnets;" *Electrical World*, 1906, Vol. XLVIII, p. 128. "Characteristic Performance of Polyphase Magnets," *Electrical World*, 1906, Vol. XLVIII, p. 564.

99. Space factor. The most efficient winding is that which contains the maximum amount of conducting material; hence a thin insulation of high dielectric strength is desirable. The space factor or **activity coefficient** is the ratio of the space occupied by the insulated conductor to that occupied by the bare conductor. This is conveniently expressed in terms of turns per square inch. The Tables (Par. **98, 102, 104, 106**) show the relative values for the various wires. Enamel and silk, and enamel and cotton, are designed to replace double-silk- and double-cotton-covered wires, respectively; the former two have higher activity coefficients and greater dielectric strengths than the latter two.

WINDING CALCULATIONS

100. Winding formulas. In what follows let

d = diameter of bare copper wire in inch,
R_l = ohms per linear inch,
d_l = diameter of insulated wire in inch,
t'' = turns per linear inch,
n'' = layers per inch,
N_s = turns per square inch,
R_v = ohms per cubic inch.

Then the number of **ohms per linear inch** is

$$R_l = 8{,}628d^{-2}10^{-10} \tag{35}$$

and

$$t'' = d_l^{-1} \text{ (or is found by actual count)} \tag{36}$$

For layer windings with no paper between layers,

$$n'' = t'' \text{ (approx.)} \tag{37}$$

When paper is used,

$$n'' = (d_l + t_p)^{-1} \tag{38}$$

wherein t_p is the thickness of the paper layer.

The **turns per square inch** and the **ohms per cubic inch** are given by

$$N_s = t''n'' \tag{39}$$

$$R_v = N_sR_l \tag{40}$$

The properties and dimensions of bare copper wires will be found in Sec. 4. Furthermore let

T = thickness of wall of winding in inches,
L = actual length of winding in inches,
p_a = average perimeter or mean length of turn in inches,
S = longitudinal cross-sectional area of winding in square inches,
V = volume of winding in cubic inches.

The **winding cross-section** and the **winding volume** are

$$S = TL \tag{41}$$

$$V = Sp_a \tag{42}$$

In all usual shapes of coils, the **thickness of coil-wall** is $T = n/n''$, where n is the number of layers. For coils wound on cores or forms of the shapes in Fig. 35, the equation

$$p_a = 2(a + b + 1.571T - 0.859r) \tag{43}$$

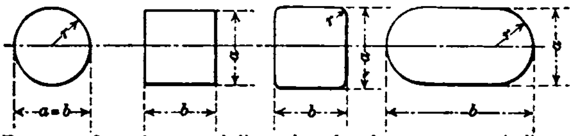


FIG. 35.—Core shapes and dimensions for electromagnet windings.

is general and gives the **mean length of turn** for coils with cores of any of these shapes. In the case of the perfectly square or rectangular core, the radius, r, is zero; whereas, in the case of the round core, it is, of course, equal to one-half of the diameter.

For a round core only,

$$p_a = \pi(D_l + T) \tag{44}$$

where D_l is the inside diameter of the coil. In all cases, the dimensions of then to make the final calculations after the turns per layer and number of layers have been determined from the preliminary data.

For a round winding, with given core, length, size of wire and resistance, the **outside diameter** will be

$$D = \left[\frac{4}{\pi L}\left(\frac{R}{R_v}\right) + D_l^2\right]^{\frac{1}{2}} \tag{48}$$

The approximate **thickness of wall** of winding will be

$$T = \frac{D - D_l}{2} \tag{49}$$

The **number of layers** will be $n = Tn''$, and a new value for T will be found for an exact number of layers, by $T = n/n''$. The **mean length of turn** will be $p_a = \pi(D_l + T)$ and the **outside diameter** will now be $D = D_l + 2T$. The **number of turns** will be $N = nt''L$, and the **resistance**, $R = Np_aR_L$. For round windings, the **radiating surface is** $A = \pi DL$, where A is the cylindrical surface.

104. Ohms per Cubic Inch; Solid Layer Winding

(The Acme Wire Co.)

Size A.W.G.	Single-cotton covered	Enamel and cotton	Single-silk covered	Enamel and silk	Enamel
10	.00727	.00702	....	....	.00768
11	.0114	.0110	....	....	.0128
12	.0180	.0172	....	....	.0194
13	.0282	.0268	....	....	.0307
14	.0442	.0418	....	....	.0485
15	.0690	.0658	....	....	.0774
16	.1072	.1017	....	....	.1225
17	.167	.158	....	....	.193
18	.260	.243	....	....	.305
19	.401	.373	....	....	.479
20	.652	.610	.731	.682	.767
21	1.01	.948	1.145	1.075	1.225
22	1.55	1.45	1.788	1.652	1.915
23	2.39	2.21	2.79	2.56	3.01
24	3.67	3.36	4.37	3.97	4.74
25	5.60	5.14	6.79	6.17	7.54
26	8.50	7.85	10.5	9.62	12.08
27	12.93	11.85	16.3	14.8	19.0
28	19.6	17.8	25.3	22.8	30.1
29	29.1	26.6	38.5	34.8	47.3
30	43.8	39.7	59.6	53.2	74.8
31	64.1	57.7	91.1	79.1	115.3
32	94.9	86.0	136.5	121.5	184.2
33	140	125.8	208	183.5	291
34	205	183	315	274	456
35	297	263	473	408	711
36	426	375	705	597	1100
37	....	....	1032	888	1742
38	....	....	1525	1300	2710

CONSTRUCTION OF COILS

105. Methods of winding. There are two standard methods of winding coils. The original method is to prepare the bobbin for the wire and then to wind the coil by rotating the bobbin in a small special lathe and guiding the wire by hand. The second method involves the use of patented automatic machinery.

at the ends so that the fringes will overlap one another when bent at right angles to the tube. The coil is then mounted on the muslin-covered tube and several oiled muslin washers are placed on each end. A slotted fiber washer is mounted at the end with the inside lead, and the lead is brought out through the slot after being wound once around the tube; the inside diameter of this washer should be sufficient to slip over the turn of the lead. More oiled-muslin washers are then put on, and the fringes are fanned out at both ends of the coil, after which heavy fibre end washers are put on and the ends of the brass tube spun over. This makes a very solidly constructed and well-insulated coil. In alternating-current coils, the brass tube must be slotted lengthwise. When flat ribbon leads are used, it is not necessary to use a slotted washer.

All splices should be soldered and thoroughly protected with oiled muslin. The leads should be tied in position with stout twine.

111. Covering. Mounted coils are covered with cord, pressboard, oiled muslin, etc., and are often dipped in black air-drying varnish to give them a protecting finish.

TESTING OF MAGNETS

112. Measurement of pull. The attracting effort of a small portative magnet or a small tractive magnet, with the armature or plunger in contact with the pole faces or stop, may be measured with a spring balance. Large magnets may be tested by direct loading or by loading through a system of levers. The **pullout point** is not preceded by any warning and, in order to obtain accurate load readings, the load must be applied at a uniform rate and under perfect control.

A simple method is to hang a bucket below the weights and to pour water or shot into it until an amount equal to a weight unit is exceeded, when the bucket may be removed and the equivalent weight placed in position; then the empty bucket is again hung in place and filled, this cycle of operations being continued until the pullout point is reached, when the total weight is determined.

113. The pull tests, for various lengths of air-gaps, may be made by the use of brass discs or rings of thicknesses arranged in decades. Thus the first group may be 1 in., 2 in., 3 in., and 4 in. respectively, with which any combination in thickness from 1 in. to 10 in. may be obtained. This idea may be carried out as far as desired. Twelve spacers will give 1,000 combinations. The measurement of the pull should be made as outlined in Par. 112.

114. The heating test requires an ammeter, a thermometer (or voltmeter), a watch and a rheostat. Fig. 36 shows the connections for a test where the temperature rise is determined by the change in resistance (Sec. 3). Instruments and general measurements are treated in Sec. 3. The temperature rise can be measured either by the resistance method or with a thermometer placed against the outside of the winding. In making the test the current should be kept at a constant value I by means of the rheostat, and the thermometer (or, in Fig. 36, the voltmeter) read at regular time intervals. While making the test, either the temperature or the e.m.f. drop should be plotted with time; this will give the heating curve and show when the final temperature has been reached.

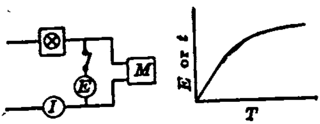


Fig. 36.—Connections for heat run, determining temperature rise by resistance method.

115. The resistance test is usually made with a Wheatstone bridge or a modification* of the same designed to test the coils very rapidly.

116. Test for short-circuited turns. Coils designed for use on alternating-current circuits are tested for short-circuited turns by placing them on the core of a special transformer, whereby a high e.m.f. is induced in them.

117. The breakdown test for direct-current coils is accomplished by placing them on a laminated core before mounting; then heavily overloading

* Underhill, C. R. "The Design of a Quick-acting Wheatstone Bridge," *Electrical World*, 1910, Vol. LVI, p. 29.

ings. The former are used almost exclusively in connection with the ignition of internal-combustion motors. The function of both types is to respond to very sudden fluctuations of the e.m.f. in the primary circuit. For this reason, they have open magnetic circuits, which partly accounts for their generally low efficiency.

In telephony, the function of the induction coil is to accurately transform the complex waves of e.m.f. and current corresponding to the sound waves produced by articulate speech. It is, therefore, in this particular case, simply a transformer with an open magnetic circuit. Induction coils of the secondary type are also used in radiography, wireless telegraphy, internal-combustion motor ignition, etc.; they operate on the principle of the gradual or progressive storage of energy, which is then suddenly discharged, and the cycle repeated.

The performance of the primary type of induction coil has an important bearing on the behavior of secondary-type coils of the jump-spark type and, for this reason, the primary-type coil is treated first and separately.

PRIMARY-TYPE INDUCTION COILS

120. Definition. A primary-type coil is a reactor (Sec. 6) designed to receive electrical energy, then convert it into magnetic energy, storing as much of the latter as feasible, and finally to reconvert it suddenly into electrical energy. The ultimate object is to utilise the heat of the resulting spark, when the circuit is suddenly broken. The rupture of the circuit usually takes place in the cylinder of an internal-combustion engine or motor.

121. Theory. As commonly made, the primary-type induction coil consists of relatively few layers of coarse magnet wire wound over a core consisting of a bundle of soft-iron wires. When the circuit is closed through a battery, current flows through the coil and magnetises the core; the counter-e.m.f. generated by the lines of flux cutting the turns of wire in the coil opposes the e.m.f. of the battery, so that a definite time interval is required to fully charge the iron core with magnetism.

The break is designed to have a snap-action which causes the circuit to be opened very rapidly, as soon as the flux in the core attains its most efficient value; thereupon the current and the flux decrease at a very rapid rate, and at the same rate, to zero. This sudden rate of change in the flux induces a high e.m.f. in the coil, proportional to $\partial\phi/\partial t$, in the same direction as the battery e.m.f., tending to retard the decrease of the current, and thus prevent the sudden collapse of the magnetic field. Hence, at the point of rupture, or break, there results a bright spark or arc, usually varying from $\frac{1}{16}$ in. to $\frac{1}{8}$ in. in length, the energy of which is the equivalent (barring losses) of the magnetic energy previously stored in the core. The value of the current at the instant that the metallic circuit is broken is given in Par. 30, Eq. 6, under "Magnets."

122. Stored magnetic energy. The magnetic energy, in joules, stored in time t is

$$W = \frac{L}{2} I^2 \qquad \text{(joules)} \qquad (50)$$

where L is the inductance in henrys and I is the value of the current in amperes at the end of time t in seconds. This is the **energy of the spark** (less losses).

123. The inductance of the coil is expressed by

$$L = 80 N^2 d 10^{-9} \qquad \text{(henrys)} \qquad (51)$$

approximately, for cores having ratios of length to diameter between 10 and 15, where N is the number of turns in the coil, and d is the diameter of the core in centimeters.* The time, t, required for the current to attain 63 per cent. of its final value is the **time constant** (Sec. 2) of the circuit; it is numerically equal to L/R. By assuming that the rate of current increase is nearly uniform between 0 and L/R seconds, which is approximately correct, the value of the current strength may be estimated for any corresponding time after closing the circuit.

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* Armagnat, H. "Induction Coils" (Translated from the French by O. A. Kenyon); New York, McGraw-Hill Book Co., Inc., 1908.

SECONDARY-TYPE INDUCTION COILS

128. Theory. Fig. 39 shows the circuit diagram of a typical secondary-type induction coil. The circuit is closed at *S* and current established in the primary circuit; this current produces a flux in the core and when the flux reaches a certain value the pull exerted on the interrupter contact arm is sufficient to overcome the pull of the spring, *s*, and opens the circuit at *C*. The circuit being open, the flux quickly decreases to zero and thus generates a high value of e.m.f. in the secondary winding, which consists of many turns.

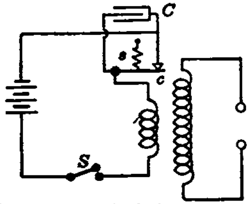

FIG. 39.—Typical circuit diagram for secondary-type induction coil.

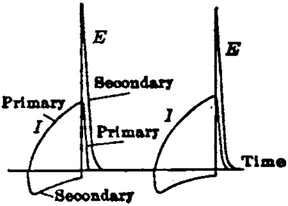

FIG. 40.—Current and e.m.f. curves of a secondary-type induction coil without condenser.

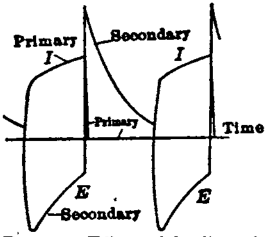

FIG. 41.—Effect of loading the secondary. (No condenser.)

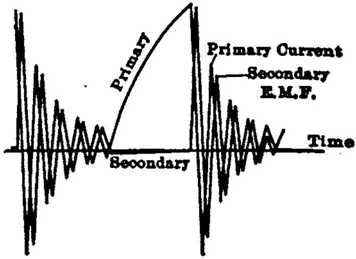

FIG. 42.—Curves of current and e.m.f. in an unloaded secondary-type coil with condenser.

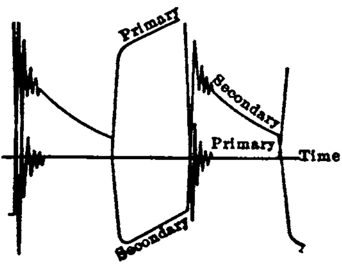

FIG. 43.—Current curves of a loaded secondary-type coil with condenser.

When no condenser (Par. 169) is used and the secondary circuit is open, no current flows through the secondary and, consequently, the effect of the secondary upon the primary is nil, excepting for the negligible charging current in large coils. All actions in the primary take place exactly as though the secondary were not present. Fig. 40* is the same as Fig. 38 with the secondary e.m.f. curves added. The effect of **loading the secondary** (without condenser) is shown in Fig. 41.

129. Effect of condenser. Fig. 42 shows the effect of the condenser

* Bailey, B. F. "The Induction Coil;" *Electrical World*, 1910, Vol. LV, p. 943.

134. The space factor of the core is that per cent. of the total area which is occupied by iron. In the cores tested by Mr. Springer, the space factor varied from 0.675 to 0.70 and averaged 0.687.

135. Table of Core Dimensions from Practice

(Ehnert) *

Spark length (cm.)	Core length (cm.)	Core-wire diameter (mm.)	Space-factor
8	15–20	0.8	0.84
10	20–25	1.0	0.82
15	25–35	1.2	0.80
20	30–42	1.4	0.78
25	35–50	1.5	0.77
30	48–60	1.6	0.76
35	52–58	1.7	0.75
40	62–70	1.8	0.74
50	80–90	1.9	0.73
60	100–120	2.2	0.715
70	120–140	2.5	0.70

136. The size of the primary wire should be so chosen as to produce heat at a very low rate, since there is almost no ventilation of the primary coil. The **current density** in the primary coil can vary between 600 and 1,200 amp. per square inch. It must be remembered also that in coils with large cores the **hysteresis** loss is important. See Par. 79.

137. Table of Sizes of Primary Wire from Practice

(Ehnert) *

Spark length, cm.	8	10	15	25	35	50
Wire diameter, mm.	1	1.2	1.5	2.0	2.5	3
Nearest size A. W. G.	18	17	15	12	10	9

SECONDARY WINDINGS

138. The necessary number of secondary turns to give a certain secondary e.m.f. depends upon the value of the flux in the core, the type of interrupter, the constants of the circuit, etc. As yet there is no law of general application which will permit the accurate or even approximate determination of the number of secondary turns. When the magnetic circuit and the primary winding are finished, an experiment can be made which will determine the approximate number of secondary turns. Using an interrupter that will give a definite and invariable duration of contact, find the value of e.m.f. generated per turn, in a coil of only a few turns, and then divide the total desired e.m.f. by this value to obtain the required number of secondary turns.

139. Table of Dimensions of Secondary Winding from Practice

(Ehnert) *

Spark length, cm.	5 to 10	12	15	25	35	50
t, cm. (Fig. 46)	0.4	0.5	0.6	0.7	0.8	0.9
h, cm. (Fig. 46)	1.5	2.0	2.5	3 to 3.5	4.5	6 to 7

* Ehnert, E. W. "Theorie und Vorausberechnung der Funkeninduktoren;" *Electrotechnik und Maschinenbau,* 1907, Vol. XXV, pp. 337, 361 and 377.

coil; in the second, the difference in potential between two adjacent coils is zero at the points where they are connected (alternately inside and outside), and increases from that point to a maximum value equal to twice the e.m.f. generated in one coil.

At first sight it appears that the first system would require only half the insulation between sections or "pies" that the second would, but in practice the connecting wire, which must run through the insulation between the coils, is run straight out through the middle and requires sufficient insulation on each side to resist the coil e.m.f.; thus it will be seen that both systems require the same insulation.

145. Details of winding process. The pies are wound in thicknesses varying from 0.0625 in. to 0.25 in. As a rule, silk-covered wire is used (Par. 96, 102, 104, 106). The core of the winding form is given a bevel, in order to tell the polarity of the coil at a glance, provided, of course, that all coils are wound in the same direction. When cotton-covered or silk-covered wire is used, it is passed through a bath of melted paraffin wax or compound while winding; the amount of wax retained by the wire is controlled by passing the wire from the bath through several paper slits, varied to suit the requirements; in this manner the wax is scraped off the wire to any desired extent. Care should be taken to have the temperature of the wax sufficiently high to prevent cooling before it is in place, since the wax serves to hold the wires together. Before winding, the wire should be thoroughly dried out in an oven.

146. Table of Dimensions of Induction Coils

Authority	Eddy*	Eddy*	*Elec. World*†
Spark, inches	12	6	1.5
Core:			
size wire A.W.G.	22	22	
diameter, inches	2.125	1(?)	1
length, inches	24	12.5	8
Primary:			
size wire A.W.G.	12	12	14
layers	2.5	2	
turns per layer	230	115(?)	
length coil, inches	22	11(?)	6
Ebonite tube:			
length, inches	26	14.5	
dia. outside, inches	3.5	2.75	1.69
dia. inside, inches	2.75	2.25	1.38
Secondary:			
size wire A.W.G.	34	34	‡
No. of pies	64	34	4
total turns	75.256	38,000	
Total length, inches	8	4.25	
dia. outside, inches	6	4.87	3.5

147. Insulation. The properties of insulators and other materials are given in Sec. 4 (see index). The **major insulation** between the primary and the secondary generally consists of a tube of such material as ebonite, hard rubber or micanite; or, in small coils, paraffined paper. The new material "Micarta" may also be used. The **minor insulation** is generally obtained by impregnating the coils with paraffin wax or some compound. Over each layer of the primary winding is wound a thickness of paraffi

* Eddy, W. O. "The Design of a 12-in. Induction Coil;" *Electrical World*, 1907, Vol. XLIX, p. 40.

† "Questions and Answers;" *Electrical World*, 1906, Vol. XLVIII, p. 1064.

‡ The size or number of turns of secondary wire is not given. It is stated that 2 lb. of wire are required.

to such an extent that the circuit becomes practically interrupted. As soon as the current density falls the resistance decreases and the current again rises to its original value. Experiments show that electrolytic interrupters will operate in synchronism when connected in series or in parallel.

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"Questions and Answers." *Electrical World*, 1906, Vol. XLVIII, p. 1064.

CONDENSERS

BY CHARLES R. UNDERHILL

TYPES

155. Types and properties of condensers. There are three principal types of condensers. The most familiar one is the ordinary **plate condenser** used with induction coils, telephone and telegraph apparatus, where a very large capacity is not required. The plate type is also used for laboratory standards (Sec. 3) in measurements of various kinds. Where a very large capacity is required, the **synchronous condenser** (synchronous motor or synchronous converter with overexcited fields) is employed; such apparatus is outside the scope of this Section and will be found under Sec. 9. The third type is the **electrolytic condenser** which is treated later under its own head (Par. 173).

The properties of a condenser are covered fully in Sec. 2, but in brief, the chief function of a condenser is to store electrical energy for subsequent usage. Such storage of energy may take place in a variety of ways, according to circumstances: thus it may be constant, if the condenser is connected to a source of continuous e.m.f., or it may be intermittent, as in the case of a condenser in a telegraph circuit (Sec. 21); or it may be transient; or, in the case of alternating-current circuits, it may be periodic. In consequence of this property of storing electrical energy, condensers can be used in numerous ways to alter or modify the characteristics of circuits in some desired manner, such as to change the relative phase of e.m.f. and current in an alternating-current circuit, improve the power-factor, or to prevent the flow of continuous current simultaneously in the same circuit with an alternating current.

PLATE CONDENSERS

156. Types and limits of size. Plate condensers are made commercially in a number of types, usually designated by the kind of dielectric employed. The earliest type, still used in laboratory and class-room work, is the **Leyden jar**, which is merely a two-plate condenser with a glass dielectric, constructed in the form of a wide-mouthed bottle or jar with tin-foil coatings inside and out. Other types of simple plate condensers are made where L_t is the dielectric thickness in inches, after impregnation, between each pair of metal strips; N is the number of such dielectric strips in the condenser; C is the desired capacity in microfarads; k is the specific inductive capacity of the paper after impregnation, and l is the short dimension of the foil (Fig. 51). This construction insures the lowest possible internal ohmic resistance and also permits the use of extremely thin metal foil.

161. Internal losses and leakage. A condenser may have internal losses of two kinds. The first is due to the **resistance of the plates** themselves, and this is most prominent in rolled condensers of very thin tin-foil, inasmuch as each plate is one long continuous strip and the resistance of the whole strip from outer terminal to inner end may be appreciable. The second kind is the loss in the dielectric itself, due to **dielectric hysteresis,** so-called (Sec. 4); this loss can be expressed as an equivalent series resistance, which should be added to the resistance of the plates. Both forms of resistance absorb energy and reduce the normal angle of lead of current with respect to terminal e.m.f. from 90 deg. to some lesser angle; this is equivalent to saying that the power-factor is increased from a value which is theoretically (or in the ideal case) zero to some definite value, although not usually large.

Besides these two forms of loss, there also may be a pure conduction current flowing through a condenser, in phase with the terminal e.m.f., which results from the lack of perfect insulation, or the presence of leakage. The resistance of the condenser to a continuous e.m.f., after the steady state has been reached, is termed the **insulation resistance.** The value of insulation resistance, of course, should be as high as possible.

162. The power-factor of a condenser determines to what extent it will be heated when used in alternating-current circuits. The power-factor is the cosine of the angle by which the current leads the impressed voltage, with simple sine-wave forms. It varies with the frequency and impressed voltage, with the temperature of operation, and with the dielectric used. The power-factor of a condenser may be determined by measuring the energy loss or by measuring the variation from 90 deg. phase angle directly.* The specific inductive capacities, power-factors, and breakdown strengths of some materials suitable for condenser dielectrics are given in Par. 169. The constants of kerosene oil are included for comparison. The values given hold only for 60-cycle circuits, at about 20 deg. cent. The breakdown sine-wave strengths are in effective (root-mean-square) volts per mil thickness of dielectric. In connecting condensers in series for use on high-voltage circuits (static voltmeters and the like), it is important to know the several power-factors, as the distribution of voltage is affected thereby. The voltage distribution among the several condensers will be proportional to the product of capacity by power-factor, for small power-factors.

163. Absorption of charge. Many dielectrics do not become fully charged instantaneously, nor do they, when fully charged, give up their complete charges instantaneously. The time lag in receiving full or complete charge is usually referred to as absorption. It is noticeable in testing such dielectrics as glass, rubber, gutta-percha, etc. The effect of absorption on the phase of the current is equivalent to resistance in series with capacity.

164. Glass condensers are made both in the form of Leyden jars, and in flat plates. Glass-plate condensers are commonly made of the best plate-glass with sheets of tin-foil shellaced to each side of the plate. The edges of the surface of the plate, not covered with the tin-foil, are varnished to prevent leakage. The finished plates are mounted in a rack and connected together by flat springs which touch the foil of adjacent plates. Condensers of this class are employed on the transmitting side of wireless telegraph apparatus.

L. W. Austin† summarises the results of his tests as follows: The losses in the **compressed air condenser** used, at a pressure of 15 atmospheres, amount to an equivalent resistance of between 0.1 and 0.2 ohms. Condensers in which "brushing" (brush discharge) is prevented by the nature of their construction show no change in resistance between the limits of observation,

* Grover, F. W. "Simultaneous Measurement of the Capacity and Power Factor of Condensers;" Bulletin of the Bureau of Standards, 1907, Vol. III, p. 371.

† Austin, L. W. "Energy Losses in some Condensers used in High-frequency Circuits;" Bulletin of the Bureau of Standards, 1912, Vol. IX, p. 73.

and melted into one solid bar. The whole is then vacuum dried and impregnated with paraffin, beeswax or the like, and cooled under a pressure not exceeding 10 lb. per sq. in. Fig. 51 shows the method of placing the sheets of foil and paper. The paper sheets are best made in square form, each edge being slightly wider than the narrow dimension of the metal foil. Diagram (1) in Fig. 51 shows four fibre or wooden upright pegs forming a jig, used in building such condensers, with a strip of metal foil laid one way between the pegs; (2) shows a square piece of dielectric (paper) laid over the strip of foil (the pegs are recessed to fit); (3) shows a second piece of foil laid between the pegs, at right angles to the first. This constitutes one element of the condenser. This process is repeated until as many sheets of dielectric as desired have been used, after which the projecting sheets of foil are rolled up and melted into a solid bar, as in (4). The condenser is then ready for impregnation.

167. Effect of Temperature on Paraffin-paper and Mica Condensers
(Abst. from Proc. I. E. E., 1896, Vol. XXV, p. 723)

Temp. deg. cent.		0.4	12.5	20.1	26.9	32	37.8	43.3
Paraffin paper.	R	17,740	7,216	3,622	1,947	1,281	792	551
	C	0.98	0.98	0.98	0.98	0.97	0.96	0.95
Mica	R	31,427	28,427	22,000	16,272	17,010	15,270	10,521
	C	0.5	0.5	0.5	0.5	0.5	0.5	0.5

R = effective series resistance, in ohms. C = capacity in microfarads.

168. Table of Commercial Ratings and Sizes of Telephone Condensers
(Paraffin paper, rolled type, commercial telephone condensers)

Capacity (mf.)	Dimensions			Safe maximum effective voltage	
	Length (in.)	Breadth (in.)	Thickness (in.)	Continuous (volts)	Alternating (volts)
0.05	4.44	1.75	0.94	....	1,200
0.10	4.44	1.75	0.94	....	1,000
0.10	4.44	1.75	0.41	500	....
0.25	4.44	1.75	1.63	....	1,000
0.30	4.44	1.75	1.06	750	....
0.50	4.44	1.75	0.53	....	....
0.50	2.38	1.25	0.75	....	....
1.00	4.44	1.75	0.94	500	....
1.00	3.00	2.38	1.00	....	....
1.00	8.72	6.25	1.48	....	1,000
1.50	4.44	1.75	1.63	500	....
2.00	4.44	1.75	1.63	500	....
2.00	4.38	2.06	1.13	....	....

It should be kept in mind in considering the above dimensions that the capacity varies directly as the area of the plates and inversely as the distance between them (thickness of dielectric or insulation). Hence the volume per unit of capacity increases rapidly as the safe maximum working voltage increases. Condensers of the rolled paper type are not as a rule tested at less than 500 volts, continuous e.m.f. The insulation resistance should be so high that the leakage is entirely negligible in comparison with the charging current. The cost of such condensers ranges from about 35 cents for the 0.05-mf., condenser, to about 85 cents for the 2-mf. condenser. Also see Par. 171.

169. Condensers for use with induction coils and, in general, condensers shunted across gaps to minimise sparking, are usually of the plate type, since it is difficult and expensive to construct rolled condensers having sufficiently low ohmic resistance in their conducting plates. The

These curves show that the changes of capacity and phase difference with changes of temperature and frequency are much larger than the corresponding changes with mica condensers. In general, these effects of temperature and frequency are larger as the absolute value of the phase difference becomes larger. The phase differences observed lie between 6 min. and the enormous value of 22 deg. A phase difference of several degrees is not uncommon in commercial telephone condensers. It is shown that the internal resistance of the plates and leads of a paper condenser are often large, especially in the case of telephone condensers made by rolling together sheets of tin-foil and paper. In one example the energy loss in the condenser, external to the dielectric, at 1,000 cycles, was three times as great as the energy loss in the dielectric.

171. Ratings of commercial paper condensers. A 2-mf. condenser made by one American manufacturer, using paraffin paper, is 4.5 in. long, 2 in. wide and 1 in. thick, and the price is about 55 cents. The volume per microfarad is 4.5 in. The rated voltage limit is 400 volts, alternating; one of these condensers failed on test at 650 volts, alternating. The energy loss was from 1 per cent. to 2 per cent. Another condenser of the paper type,* built to stand 10,000 volts (alternating), occupied 2 cu. ft. per microfarad. Generally speaking, the ordinary paper condensers of the rolled type used in telephony and telegraphy are built to stand about 500 volts, alternating, and range in size from a few hundredths of a microfarad up to 4 mf. to 6 mf.; some of these condensers are built to stand 1,000 volts, alternating. The higher the voltage limit, the more bulky and expensive the condensers become. The table (Par. 168) gives the sizes, dimensions, voltage limits, and approximate costs of rolled paper condensers used in telephone practice.

172. Table of Dimensions of Condensers Used with Induction Coils (Ehnert)†

Spark-length, cm	5	10	15	20	25	30	35	50
No. of layers	60	65	70	75	80	90	100	150
Dimensions of tin-foil sheets, cm.	15 by 10	17 by 10	22 by 10	22 by 11	25 by 13	25 by 17	27 by 20	32 by 23

ELECTROLYTIC CONDENSERS

173. Elements of electrolytic condenser. Certain metals such as aluminum, magnesium, and tantulum, when immersed in an electrolyte, possess the property of allowing electricity to flow in one direction and not in the other, providing a certain critical value of e.m.f. is not exceeded. Two electrodes of this kind practically prevent all flow of electricity and constitute a condenser which is known as an electrolytic condenser. Such condensers are usually constructed with aluminum electrodes and can be made in large units at a cost well within commercial economic limits. The greatest disadvantage of the aluminum electrolytic condenser, as compared with dry condensers, is that it has an appreciable energy loss. Electrolytic condensers used as lightning arresters are usually connected in series with a spark gap so as to avoid the energy loss which would exist were the line voltage continually impressed across its terminals.

174. The critical voltage of an electrolytic condenser is the maximum value of impressed e.m.f. which it will stand without permitting an appreciable leakage current. The table in Par. 175 gives the critical voltages for aluminum electrodes with different electrolytes.

* Mordey, W. M. "Some Tests and Uses of Condensers;" *Journal of Proceedings* of the Institution of Electrical Engineers, 1909, Vol. XLIII, p. 618.

† Ehnert, E. W. "Theorie und Vorausberechnung der Funkeninduktoren;" *Elektrotechnik und Maschinenbau,* 1907, Vol. XXV, pp. 337, 361 and 377.

of time, but as the voltage is increased the losses increase with time at an increasing rate. In cases where condensers are to be used continuously, it is advisable to connect several low-voltage condensers in series. Dr. Gunther Schulze* carried out an extensive investigation of electrolytic condensers in the Reichsanstalt which was described and discussed in an article: "Aluminum Electrolytic Condensers of High Capacity," published in *Electrochemical and Metallurgical Industry*, Vol. VII, p. 216 (1909).

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RESISTORS AND RHEOSTATS

BY LEONARD KEBLER

TYPES OF RHEOSTATS

183. Plate-type rheostats have a resistance in the form of a reflexed metal wire or ribbon attached to a plate of porcelain-covered cast iron, or to a plate of insulating material such as soapstone, by means of a coating of fused enamel or by cement. They are used almost universally for small field rheostats, theatre dimmers, small motor-speed controllers, battery charging rheostats, etc.

184. Box-type rheostats have a resistance in the form of wire-wound, enamel-covered porcelain tubes; cast-iron grids; coils of bare wire; metallic ribbon; carbon discs, or a conducting liquid. They are used for large field rheostats, motor starters, large motor-speed controllers, battery-charging rheostats of the larger sizes, etc. (Also see Par. 197.)

FIELD RHEOSTATS

185. Field rheostats are used in series with the fields of dynamos for regulating the field strength and in turn the voltage of the dynamo, or in the fields of motors for varying the field strength and in turn the speed.

186. Generator field rheostats for direct-current machines usually are provided with such a value of total resistance that it is about equal to that

* Schulze, G., "Kondensatoren Grober Kapazität;" *Elektrotechnik und Maschinenbau*, 1909, Vol. XXVII, p. 247.

195. Temperature rise of plate-type rheostats. For each wa dissipated per sq. in. of free radiating surface, the rheostat plate will ri about 100 deg. fahr., and usually but one side of a plate-type rheostat ca radiate freely. Therefore where A is the area of one side of the rheost (sq. in.), I the current flowing (amp.) and R the resistance of the rheost (ohms) in circuit, the temperature rise will be $100\ (I^2R/A)$ in deg. fal (approx.).

196. A reasonable temperature rise for box-type rheostats is reache when there is 1 cu. in. of contents for each watt to be dissipated continuou and 1 sq. in. of external surface for each 2 watts to be dissipated continuous

197. Calculations of resistance, current-carrying capacity, et for box-type rheostats, regardless of the use to be made of them, m be carried out in the same manner as for plate-type rheostats.

198. Automatic features on armature speed controllers consist no-voltage release, overload release and overload circuit-breaker. **No vo tage release** and **overload release** are used with contact arms which a held in the operating position against the action of a spring. On no-volta or overload the spring is magnetically released and moves the arm to the ope circuit position.

An **overload circuit-breaker** on armature speed controllers consists a separate switch arm held closed by a latch. On overload this latch tripped by a magnet, the switch is opened by a spring and breaks the mai line circuit irrespective of the movement of the resistance controlling arm.

199. Speed regulators for series-wound motors. For series moto the calculation of the exact resistance for a definite speed variation requir a curve of the motor characteristics. Lacking this it will be found that controller designed as above for a shunt machine (Par. 193) will gi approximately the same regulation for a series machine running under co stant-torque conditions.

THEATRE DIMMERS

200. Theatre dimmers are made of a number of plates mounted in bank and each plate controlled by its own lever. **A master lever** is usuall arranged with cams so that it may control all or any number of the plat

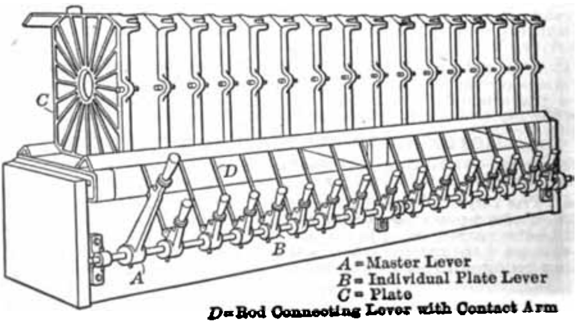


FIG. 54.—Bank of theatre dimmers.

at once. A typical bank is shown in Fig. 54. The circuit operated by o plate seldom carries more than 50 amp. except in the case of the dimm for the auditorium lights. The latter often carries 300 to 400 amp.

201. For dimming carbon-filament lamps a resistance equal to abo 3.4 times that of the lamps in the circuit at full candle-power will dim suf ciently so the lamps have no illuminating power. In order that the dimmi may be done smoothly and without flicker, 50 steps are required.

202. For dimming tungsten-filament lamps a resistance equal to times that of the circuit at full candle-power should be used. In order th

212. Starters for series motors. When the resistance is reduced by cutting out a step, the current increases from I to I_{max} as in the case of the shunt motor (Par. **206**). At once the counter-e.m.f. increases due to the increase in the field current. Consequently, if the same I_{max} is to be used as in the case of shunt motors, the decrease in resistance as a step is cut out, must be larger than in the case of the shunt motor.

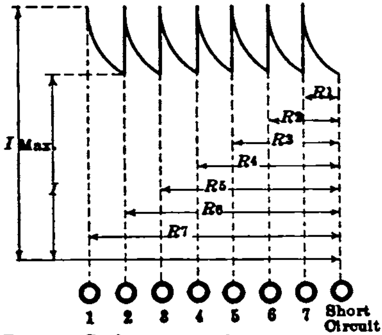


Fig. 55.—Resistance steps for motor starters.

In calculating resistance steps for a series motor which will give the same current curve as shown in Fig. 55, the characteristic curves of the motor must be known. The counter-e.m.f. for any given current is directly proportional to the speed and *vice versa*. In general, however, a starting rheostat that is properly proportioned for a shunt motor may be satisfactorily used with a series motor of the same horse power and voltage.

BATTERY-CHARGING RHEOSTATS

213. The resistance of a battery-charging rheostat, for charging storage batteries from a constant-potential source, may be determined as follows: If E is the e.m.f. of the charging circuit (volts), E_{min} the lowest e.m.f. of the battery during charge (volts), I_{min} the lowest value of charging current, then the total rheostat resistance R will be

$$R = \frac{E - E_{min}}{I_{min}} \qquad \text{(ohms)} \qquad (67)$$

A certain amount of this resistance must carry the maximum charging current I_{max}; this amount is

$$R = \frac{E - E_{min}}{I_{max}} \qquad \text{(ohms)} \qquad (68)$$

The balance of the resistance will have a current carrying capacity varying from I_{max} to I_{min}.

MISCELLANEOUS RHEOSTATS

214. Wire rheostats. Wire can be wound in coils, or stretched over insulated frames. Wires larger than No. 6 A.W.G. are difficult to wind in spiral form and wires smaller than No. 21 A.W.G. must be wound upon an insulating core. When it is desired to increase the current capacity of a coil resistor beyond that of No. 6 wire, several coils may be connected in multiple. The table in Par. **215** gives the mechanical dimensions of coils made of different sized wires.

215. Table of Dimensions of Wire Coils for Rheostats

Size, A.W.G.	Max. mandrel, inches	Feet per turn	Turns per inch	Max. coil length, inches
6–8	1.25	0.38	4.0	18
9–11	1.00	0.30	4.5	12
12–14	0.75	0.23	7.0	12
15–18	0.50	0.16	9.0	12
19–21	0.25	0.082	14.0	6
22–30	Must be wound on insulated core			

Note.—The maximum diameter of mandrel given in the table corresponds to the length given therein, and if a stiffer coil is desired a smaller mandrel must be used.

Ordinary water gives a drop from 2,500 to 3,000 volts per in. gap at this current density.

RESISTANCE UNITS

223. Resistance units are used wherever a resistance that is not variable is desired. These units are made in many forms and types for the use of those who desire to purchase the units and assemble them in rheostats; for use in the speed regulators of desk and ceiling fans; for testing purposes, etc.

224. The resistance material is usually of German silver or some similar material of low temperature coefficient. This is usually wound on tubes and is then covered with a cement-like coating or with a fused vitreous enamel. The tubes on which the wire is wound consist of pottery, asbestos paper, lava, enameled iron, etc. The enameled iron and pottery tubes are used when the resistance wire is to be hermetically sealed in a fused vitreous enamel.

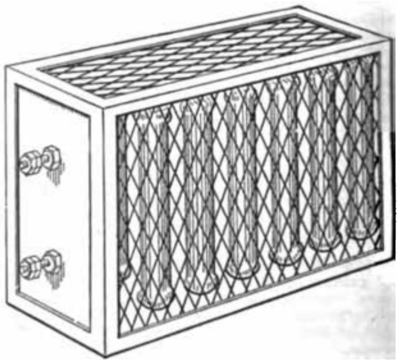


Fig. 57.—Large resistance unit.

225. Carbon resistance units are sometimes used where exceedingly high resistances with a very low watt dissipating capacity are desired.

226. The watt dissipating capacity (I^2R) of such tubular units as are commercially manufactured and listed, varies from 1 to 300 watts. The resistance of these units can be made as high as 16,000 ohms on an enameled wire wound tube 4 in. long and ½ in. diameter.

227. Large resistance units are made of cases enclosing a number of the tubular units described in Par. **224**, or enclosing cast-iron grids, German silver ribbon, etc. (See Fig. 57.)

228. The temperature rise of resistance units is as discussed in Par. **195** and **196**.

229. Mounted resistance units are manufactured with various attachments for readily mounting in special apparatus or for giving various combinations, etc. Units designed to replace resistance lamps in order that a resistance may be secured that does not change with time, or on account of heat, are so made as to screw into a standard Edison socket. (See A, Fig. 58.) Other units are made with ferrules on the ends, so they may be readily inserted in the conventional type of fuse clips (See B, Fig. 58); still others are mounted in brackets (See C, Fig. 58); on porcelain bases See D, Fig. 58), etc.

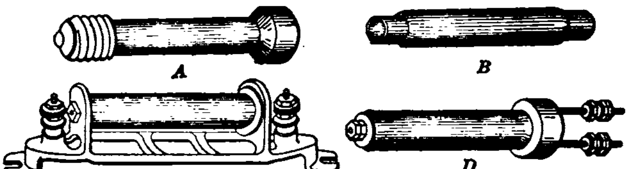


Fig. 58.—Mounted resistance units.

SECTION 6

TRANSFORMERS

INTRODUCTION

1. A transformer is a device for transferring energy in an alternating-current system from one circuit to another. It consists essentially of two independent electric circuits linked with a common magnetic circuit. Thus energy at low voltage may be transformed to energy at high voltage, or *vice versa*. In like manner, current of a given value in one circuit may be transformed into current of another value in a different circuit. The winding of the transformer connected to the supply circuit is referred to as the **primary winding**, and those windings of the transformer that are connected to the receiver circuits are referred to as **secondary windings**.

2. If a transformer with open secondary winding has its primary circuit connected across alternating-current supply mains, only a small current will flow through the primary. This current is alternating and produces an alternating magnetic flux in the iron core of the transformer, which interlinks both the primary and the secondary windings and by its rate of change induces an e.m.f. in each. If the two windings are closely associated with each other, the e.m.f. induced in each will be proportional to their respective numbers of turns. The e.m.f. induced in the primary winding is known as the **counter e.m.f.** of self-induction of the primary, or **back e.m.f.**, and is equal in magnitude to the e.m.f. of the mains less the IR drop due to the current passing through the winding. Therefore, if the permeability of the magnetic circuit of the transformer is high and the resistance of the primary winding is low, the counter e.m.f. will be very nearly equal in magnitude to the e.m.f. of the supply circuit, and it follows also that the secondary induced e.m.f. will be nearly equal to the e.m.f. of the supply circuit multiplied by the ratio of the number of turns in the secondary winding to that of the primary winding.

3. If the secondary circuit is completed through an impedance or load, current will flow through the secondary and the load; this will tend to demagnetize the core, so that the effective impedance of the primary winding is at once lowered and more current flows into it. The extra current, being just sufficient to overcome the demagnetizing effect of the current flowing in the secondary winding, will have the same time-phase as the secondary current and the ratio of the magnitudes of the two currents will be equal to the reciprocal of the ratio of the numbers of turns in the respective windings.

4. The secondary current causes a drop in e.m.f. in the secondary winding, partly due to the resistance of the winding and partly due to magnetic leakage caused by the fact that all of the flux which interlinks the primary turns does not interlink all the secondary turns.

5. The primary current likewise produces a drop of e.m.f. in the primary winding, due to the resistance of the primary winding and to the fact that there is a portion of the magnetic flux which interlinks some of the primary winding without interlinking any of the secondary winding.

6. The induced e.m.f. will therefore be proportional to the e.m.f. of the supply circuit, less the drop in the primary winding; and the secondary terminal e.m.f. will be equal to the secondary induced e.m.f. less the drop in the secondary winding, the drops being taken in their proper phase relations. The total drop from supply e.m.f. to secondary terminal e.m.f. is usually quite small in power transformers, so that the product of the secondary current and terminal e.m.f., measured in kilovolt-amperes, differs from that of the primary by 2 or 4 per cent., only, at full load, while the ratio of the output a magnetic circuit of uniform cross-section and infinite permeability, and that e_1, the instantaneous value of the e.m.f. impressed on the primary, is a simple harmonic time-function of the form

$$e_1 = 10^8 \times \sqrt{2} E_1 \cos \omega t \tag{2}$$

where E_1 is the root-mean-square or effective value of the impressed voltage and ω is equal to $2\pi f$ where f is the frequency of the alternating e.m.f.; i_2 becomes zero, and the resulting solution of these equations will give

$$\begin{aligned} (L_1 i_1) &= n_1 A_1 B \sin \omega t \\ (M i_1) &= n_2 A_1 B \sin \omega t \\ E_2 &= -\frac{n_2}{n_1} E_1 \end{aligned} \tag{3}$$

where n_1, n_2 are the number of turns in the primary and secondary windings respectively, E_2 is the effective value of the secondary voltage, A_1 is the cross-sectional area of the magnetic circuit and B the maximum instantaneous value of the induction in C.G.S. lines per unit area. The relations between E_1 and B are given by

$$\left.\begin{aligned} B &= \frac{\sqrt{2} E_1 \times 10^8}{\omega n_1 A_1} = \frac{E_1 \times 10^8}{4.44\, f n_1 A_1} \text{ (c.g.s. lines per unit area)} \\ E_1 &= \frac{\omega n_1 A_1 B}{\sqrt{2} \times 10^8} = 4.44 \frac{f n_1 A_1 B}{10^8} \qquad \text{(volts)} \end{aligned}\right\} \tag{4}$$

These are the equations used to determine the value of the open-circuit secondary voltage and the induction in power transformers. Even if the reluctance of the magnetic circuit be taken into account the value of B is not materially altered, and since exigencies of manufacture may cause larger errors in this quantity than that due to the assumption of infinite permeability, greater refinement is useless. The error in the value of E_2 due to this assumption is not of consequence in most commercial transformers.

16. The primary open-circuit characteristics of power transformers may be obtained with sufficient accuracy by means of a curve giving the **loss per unit mass** of iron at different inductions (Sec. 4), and another giving the **exciting volt amperes per unit mass** at different inductions. The latter curve is preferably obtained from the average of tests on a number of transformers. Sometimes tests are made on different sizes of transformer cores and the resulting curves plotted separately; this is done to take care of the variations due to building, which are greater in small transformers than in large ones. The value of the exciting current obtained in this manner is known as the **equivalent sine-wave** value. The relations between these equivalent sine-wave values of core-loss current and exciting current as obtained above is shown by vector diagram Fig. 1. Typical core-loss and exciting volt-ampere curves are shown in Fig. 2 and Fig. 3, respectively.

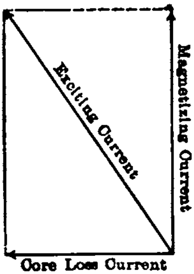


Fig. 1.—Vector diagram of equivalent sine-wave exciting current.

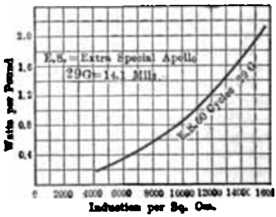


Fig. 2.—Core-loss curve.

17. The current in the primary winding of an unloaded transformer, which has a sine-wave e.m.f. impressed across its terminals, is not a

20. The short-circuit characteristics of a transformer are obtained from those of the loaded transformer by making R_0 and L_0 equal to zero. It is usual to consider the quantity ωM in the resulting equation as infinite in which case the short-circuited transformer may be treated as a simple impedance so that the effective reactance of a transformer of unity ratio when short-circuited is the sum of $\omega(L_1-M)$ and $\omega(L_2-M)$. Moreover under load conditions the effective transformer impedance differs from the short-circuit value only on account of the influence of the normal induction on the permeability of the core, and since the effect of ωM may be ignored in the short-circuit reactance, the effect due the difference of ωM under short circuit and when there is normal induction in the iron being of still smaller order of magnitude may also be ignored. The quantities which in transformers under load are the equivalents of the short-circuit copper loss and impedance have been measured with varying inductions in the iron and have been invariably found to be practically constant.

21. The current in the secondary winding of a transformer supplying a load may be obtained by means of the simple alternating-current circuit shown in Fig. 4. But since the admittance Y is not readily expressible, it is inconvenient to use this circuit. Moreover the quantity (L_1-M) cannot be evaluated very exactly and therefore more or less error is introduced when it is used. The following formula derived mathematically from Eq. 6 and 7 with the assumption made in the preceding paragraph has the advantage of great simplicity

$$e_2 = R_0 i_2 + L_0 \frac{\partial i_2}{\partial t} + \left(R_s i_2 + L_s \frac{\partial i_2}{\partial t}\right) \tag{9}$$

where R_s and L_s are the effective resistance and effective inductance when load current is circulated in the secondary with the primary short-circuited.

22. With sine-wave secondary open-circuit voltages Eq. 9 becomes that of a simple reactive circuit having resistance equal to the sum of the load resistance and the secondary effective short-circuit resistance and having reactance equal to the sum of the load reactance and the secondary short-circuit reactance. The secondary current may then be multiplied by the ratio of transformation to obtain the primary load current, and from this the primary current is obtained by adding the equivalent sine-wave exciting current in proper phase relation. Eq. 9 under these circumstances may be represented symbolically as follows:

$$\dot{E}_2 = \{(r_0+r_s)+j(x_0+x_s)\}\dot{I}_2 \tag{10}$$

where E_2 and I_2 are both vectors. The vector diagram and current loci for all loads are given in Fig. 6.

23. The construction of Fig. 6 may be explained as follows: $OA = -E_1$ or the primary impressed e.m.f., reversed in time phase. $OB = E_2$, or the secondary open-circuit e.m.f. The radius of the heavy-line circle with B as centre represents the secondary short-circuit impedance of the transformer, drawn to the same scale as OB. To obtain the loci of the secondary terminal e.m.f. for any load, bisect OB, at C_1 and draw the line C_1, C_2, C_3 at right angles. The locus of all loads having a given power-factor is a circle passing through B with its centre at some point on the line C_3, C_1, C_2. The unity-power-factor centre C_2, is obtained by making C_1, C_2 equal to C_1B multiplied by the ratio of the secondary short-circuit resistance to the secondary short-circuit reactance; the centres C_3, C_4, C_1, etc., for loads of other power-factor are obtained by making cos α_3, cos α_4, etc., equal to the respective power-factors, making the angle lag behind or lead the phase of BC_2, according as the power-factor is lagging or leading. The loci are then obtained by describing circles passing through B with these centres. The loci for different load currents are concentric circles having B as centre and radii proportional to the respective short-circuit impedance drops at these loads. Thus the secondary terminal e.m.f. with full-load secondary current and 90 per cent. leading power-factor is represented in time-phase and magnitude by the vector drawn from O to the point C where the 90 per cent. leading-power-factor circle intersects the full-load impedance-drop circle. To obtain the loci of the current vectors, describe with centre O a circle of radius to represent the effective full-load secondary current on any suitable scale; with the same centre draw another circle of radius equal to that of the voltage, expressed as a percentage of the full-load secondary voltage, which is also the rated voltage. If IR/E and IX/E are respectively the effective short-circuit ohmic and the reactance drops expressed as fractions of the rated voltage, then

$$\text{Regulation} = \frac{IR}{E}\cos\theta + \frac{IX}{E}\sin\theta + \frac{\left(\frac{IX}{E}\cos\theta - \frac{IR}{E}\sin\theta\right)^2}{2} \qquad (13)$$

GENERAL DESIGN

27. The design of successful commercial transformers requires the selection of a simple form of structure, so that the coils may be easy to wind and the magnetic circuit easy to build. At the same time the mean length of the windings and of the magnetic circuit must be as short as possible for a given cross-sectional area, so that the amount of material required and the losses shall be as low as possible. The form of construction should permit of the easy removal of heat by means of ventilating ducts, it should admit of being insulated in a simple and economical manner, and the windings should be of such forms as may be easily reinforced to withstand mechanical stresses.

28. Two types of transformers are in common use. When the magnetic circuit takes the form of a single ring encircled by two or more groups of primary and secondary windings distributed around the periphery of the ring, the transformer is termed a core-type transformer. When the primary and secondary windings take the form of a common ring which is encircled by two or more rings of magnetic material distributed around its periphery, the transformer is termed a shell-type transformer.

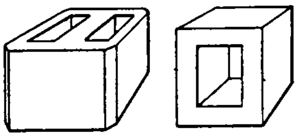


FIG. 7.—Forms of magnetic circuits of transformers.

29. The characteristic features of the **core-type** transformer are a long mean length of magnetic circuit and a **short** mean length of windings; those of the **shell-type** are short mean length of magnetic circuit and long mean length of windings. The result of these features is that for a given output and performance the core-type will have a smaller area of core and larger number of turns than the corresponding shell-type. As a general rule the core-type construction is more economical for small high-voltage transformers than the shell-type construction, the dividing line for size being dependent on the voltage. In the matter of relative weights of iron and copper, the two types tend to merge into each other if steps are taken to alter the construction so that their features are more nearly alike. Fig. 7 and Fig. 8 illustrate the forms of magnetic circuits that have been found to result in the most economical and satisfactory designs.

30. Electrical design. The fundamental formulæ in the electrical design of a transformer are those given under "General Theory," Par. 15, Eq. 4. If a certain current-density and induction be assumed, the allowable thickness of the coils for the proper cooling may be predetermined. The electrical stress between layers and between adjacent coils may also be predicted with sufficient accuracy to specify the amount of insulation that it will be necessary to use in each case.

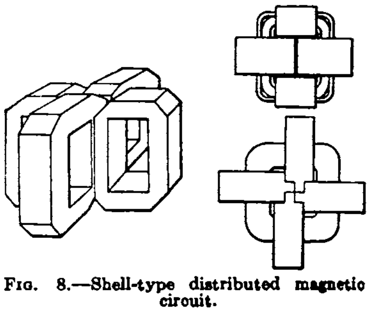


FIG. 8.—Shell-type distributed magnetic circuit.

31. The area occupied by a primary or a secondary

33. Core material. **Silicon steel** is used almost entirely for power and distributing transformers. For the first class of transformers chiefly on account of its non-ageing characteristics. For the second class on account of its extremely low hysteresis and eddy current loss. A great deal of investigation is being carried on by various manufacturers with a view of producing steels having characteristics suitable for different classes of work. A complete discussion of various classes of sheet steel for electrical apparatus will be found in Sec. 4.

34. Insulation, cooling and mechanical stresses. These factors will have a large influence on the form and dimensions of transformer and are taken up in detail under separate headings below.

DESIGN OF INSULATION

35. High insulation strength is one of the most important requirements of a well-designed transformer. However, provided that the proper strength is obtained at every point, the less insulating material used the better, because the insulating material is the chief factor, limiting the temperature at which it is permissible to operate and it is also the main factor in producing the rise of temperature of the windings.

36. The electric stresses to which a transformer is subjected in service may be transient in nature, or they may be steady. Belonging to the former class are surges set up by switching, breaking down of line insulators, arcing grounds, and short-circuits. Steady electric stresses are caused by such accidents as the grounding of one line of a transmission circuit, and will be preceded by a surge or transient wave due to the change in the electric field brought about by the new condition of stress. Thus high electric stress to ground, in a transformer, may be preceded by high stresses between coils and turns of the high-potential parts of the windings. A breakdown between turns in a high-voltage transformer may therefore be due to a combination of the two forms of stress. Another form of stress that may cause breakdown of the low-tension winding is likely to occur when the electrostatic capacity between the high-tension and low-tension windings is high in comparison with that of the low-tension winding to ground, in which case a dissymmetry of the high-tension circuit such as that due to a ground on one line may cause a potential elevation of the low-tension winding sufficient to break down the insulation, or to cause loss of life.

37. The transient stresses are difficult to estimate with any degree of certainty and the designer usually has to satisfy himself with results obtained by experience with transformers in service.

38. The steady stresses may be easily calculated, and the usual practice is to design the insulation so that it will withstand a difference of potential between the low-tension and high-tension windings and between case and high-tension winding, ranging from two to two and one-half times the possible steady stress under service conditions.

39. The insulation between adjacent coils and adjacent turns of the transformer must be capable of withstanding with an ample margin of safety the normal and transient stresses that occur. It is customary in large transformers to give an overpotential test of double voltage, mainly for the purpose of making sure that there are no weak spots in the coil insulation.

40. Effect of polarity of windings. Polarity in a transformer depends upon the relative direction of the induced electromotive forces in the primary and secondary windings considered with respect to the two adjacent ends of the two windings.* In a single-phase transformer under normal conditions of service, the middle points of both primary and secondary windings are at ground potential; but the maximum difference between adjacent primary and secondary coils may be half the sum of the primary and secondary voltages or half their differences according to the polarity. This factor becomes of extreme importance in the design of transformer for interlinking two high voltage systems.

* A full discussion of this question will be found in an article by C. Fortescue, *Electric Journal*, 1907, and also one by Wm. McConahy, *Electric Journal*, 1910.

48. Outlet terminals. One of the most difficult problems encountered in high-voltage insulation design in the past was the provision of a suitable outlet lead. This problem has been solved in two different ways: one is by the use of the condenser-type terminal (Par. 49), and the other is by the use of an oil-filled bushing (Par. 50).

49. The condenser-type terminal consists (Fig. 9) of alternate cylinders of thin tin-foil and shellac-treated paper, rolled hot on a central brass rod of the proper diameter; they are so arranged that the capacities between adjacent tin-foil cylinders are the same throughout, and the tin-foil cylinders differ in length by equal steps. The potential stress is distributed evenly over the whole length of the terminal. The addition of a disc at the top of the terminal and an external cylinder of insulating material extending between the flange and the disc, the space between the condensers and the cylinder being filled with gum, complete the outlet lead.

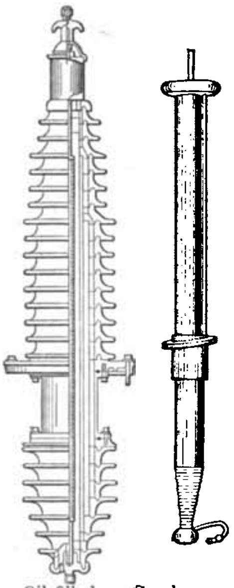


FIG. 9.—Oil-filled and condenser-type terminals.

50. The oil-insulated terminal (Fig. 9) depends upon oil as an insulator and as a means for insuring equal distribution of heat in the terminal. It consists of segments of porcelain or moulded material cemented together to form an enclosure for the oil; a conducting rod extends from top to bottom of this enclosure. The oil space is subdivided by insulating cylinders to prevent lining up of particles in the oil.

51. In transformers for high-voltage transmission the coils adjacent to the line terminals are more heavily insulated from one another than the rest of the coils. In large transformers the normal stress between adjacent coils is seldom permitted to exceed 15,000 volts. As the size of a transformer is increased, keeping the current density and the coil thickness the same the voltage per coil will vary inversely as the fourth power of the output. Since the thickness of the coils and the spacing between them are usually determined by the cooling requirements, a large transformer is more easily insulated than a small one.

52. Grounding of the neutral point. A considerable saving in cost of insulation may be obtained if transformers are designed for operation with the neutral point of the line-circuit grounded. The insulation stresses to which a transformer may be subjected in service are thereby greatly reduced and the danger of breakdown of low-tension insulation due to the effect described in the latter part of Par. 36 is entirely eliminated, in polyphase transformations.

53. The effect of insulation on cooling is negligible except in small transformers where the heat has to be conducted through thick insulating barriers, tape on the coils, and heavy layer-insulation; and in air-blast transformers where the temperature gradient through the heavily taped coils may be very large.

COOLING SYSTEMS

54. The losses in a transformer appear as heat in the windings and the core. Means must therefore be provided for removing it; otherwise high temperature will result, which will destroy the fibrous materials used for insulating the various parts of the transformer. **Methods of removing heat** may be classified as follows:

(a) **By natural convection of air and radiation.** This method is employed in certain special cases for small distributing transformers up to

The reason for most of these requirements are obvious, but (b), (c), (d) and (e) require some explanation, which is given in the next paragraph.

61. Explanation of heat transfers in cooling fluid. Consider two adjacent surfaces of a winding taking the form of two parallel planes a small distance apart, and assume that all the heat is emitted in a direction normal to the planes. The fluid directly in contact with the surface will first become heated and expand, thereby becoming lighter and rising. It will thus at every point of its passage assume a temperature which will increase progressively along the vertical. At the lowest point of the coils the oil nearest the surface will perform all the cooling, but as it rises the temperature gradient through the fluid normal to the surface will increase, thus permitting the conduction of heat to the fluid further removed from the surface. The temperature gradient through the oil at the top of the coils will therefore depend upon the velocity of the fluid and its distance from the surface. The more nearly uniform the velocity of the fluid at each point, the less will be the temperature gradient. The velocity of the fluid at each point depends among other things upon the viscosity of the fluid being more uniform the lower the viscosity. High specific heat is important because it permits a given amount of heat to be removed per unit time at lower velocity and lower temperature rise along the surface. High thermal conductivity is necessary so that heat may be transmitted through the fluid normal to the surface with a small temperature gradient. It should be noted that where the fluid is a gas, a certain amount of work is absorbed during expansion, so that the specific heat at constant pressure should be used.

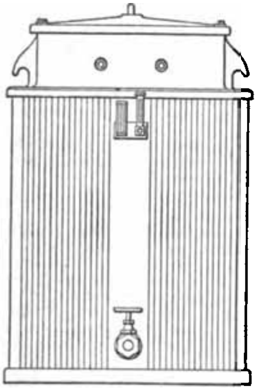


FIG. 10.—Oil-insulated self-cooled transformer.

62. The temperature difference between the fluid at the top of the coil and that at the bottom is a measure of the force required to circulate the fluid. Part of this force is used up in overcoming the friction offered by the surfaces and the viscosity of the fluid.

63. In transformers cooled by free convection in air the remainder of this force is that necessary to bring the velocity of the air up to the required value to effectively remove the losses. **In oil-insulated self-cooled types** (Fig. 10), where the transformer is immersed in a liquid which circulates by free convection, cooling the surfaces of the windings and being cooled in turn at the surface of the container by convection of the air, the hydraulic force or head developed in the windings due to the difference in temperature has to overcome not only the resistance to flow within the windings themselves, but also that external to the windings due to the sides of the case. In addition to this there is a back pressure or counter-head which represents the force required to overcome the friction resistance of the external surface of the case to the convection currents of air and that required to set them in motion.

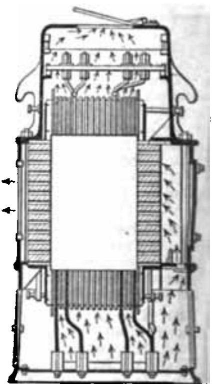


FIG. 11.—Air-blast transformer.

64. In the air-blast method of cooling (Fig. 11), instead of depending solely on free convection of air to cool the transformer, pressure is used and air is forced through the windings. The temperature rise of the hottest part of the surfaces, above approximately proportional to the one-fourth power of the output while the losses increase as its three-fourth power, the additional surface necessary to dissipate the heat must be obtained by corrugating the tank surface. The air flow has therefore to encounter a more constricted area of entry and a longer path of higher resistance. The volume of air per watt lost will consequently decrease and the average temperature of the air flowing over the surface increase, resulting in a higher average temperature of the oil, unless the gradient between the oil and cooling air has been sufficiently decreased by increasing the tank area per watt lost. The maximum temperature of the oil will tend to be higher, and therefore the maximum temperature of the windings which is the limiting factor in any electrical apparatus will also tend to be higher. Accordingly in large transformers the difference between the average temperature of the air flowing over the surface and the average temperature of the oil must be kept lower for a given temperature rise in the coils than in small transformers, and therefore a larger tank area per watt lost, will be required, or else the temperature gradient through the coils themselves must be decreased.

FIG. 14.—Oil-insulated self-cooled transformer (tubular type).

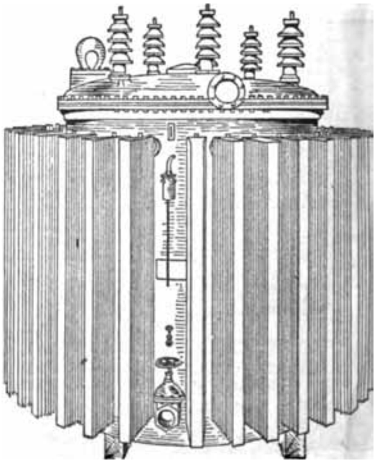


FIG. 15.—Oil-insulated self-cooled transformer (radiator type).

70. The usual course is a compromise between these two methods of keeping down the temperature rise in the coils, but with very large transformers this becomes costly and more efficient cooling tanks must be used, good examples of which are the tubular and radiator types of tanks (Figs. 14 and 15) or else recourse must be had to artificial methods of cooling such as water cooling, forced circulation, etc.

71. The watts lost per sq. in. of coil surface for cylindrical coils of large radius and for discoidal coils, may be obtained by very simple formu-

The same equations apply to θ_a, and θ_b if we substitute these values for θ_0 and θ_1 respectively; and also substitute P, A_s, σ_s, G, and α_s for P_0A_0, σ_0, G_0, and α_0 respectively; we shall then have the temperature rise of the coils above the oil at the end of an interval t_1, starting with a temperature rise θ_a.

75. In water-cooled transformers the length of cooling coil required will depend upon the rate of flow of the water. Using the same symbols as before, except that the temperature rises will be measured above the incoming water, we shall have, if radiation from the surface of the tank is negligible and taking θ_w as the temperature rise of the water, F as the flow in gallons per min., L as the length of cooling coil in feet and α_w the emissivity of the surface of the cooling coil in watts per ft. per deg. cent.,

$$\theta_w = \frac{0.0038P_0}{F} \tag{28}$$

$$L = 606\frac{F}{\alpha}\ \log_{10}\ \frac{\theta_0}{\theta_0 - \theta_w} \qquad \text{(feet)} \tag{29}$$

$$\left.\begin{array}{l}\text{Temperature rise above}\\ \text{entering water under}\\ \text{steady conditions}\end{array}\right\} = \frac{0.0038P_0}{F\left[1 - \dfrac{1}{10^{\left(\frac{0.00165\alpha_w L}{F}\right)}}\right]} \qquad \text{(deg. cent.)} \tag{30}$$

Denote the bracketed portion of the denominator of the above fraction by C; then,

$$\theta_1 = \frac{\theta_w}{C} - \frac{\dfrac{\theta_w}{C} - \theta_0}{10^{\left(\frac{2165FCt_1}{G_0\sigma_0}\right)}} \qquad \text{(deg. cent.)} \tag{31}$$

$$t = \frac{G_0\sigma_0}{2165FC}\log_{10}\frac{\dfrac{\theta_w}{C} - \theta_0}{\dfrac{\theta_w}{C} - \theta_1} \qquad \text{(hours)} \tag{32}$$

C is constant for a given length of pipe and flow of water. The temperature rise of the coils above the oil may be obtained by the formulas given in the last paragraph. The above formulas with some modification may be used for forced circulation.

76. The cooling surface required for a transformer having a rise of 40 deg. cent., based on a room temperature of 25 deg., will vary from 4 sq. in. per watt lost, to 8 or 9 sq. in., according to the size of the transformer. The value of α_0 will vary from 0.008 to 0.004 watts per sq. in. per deg. cent.

77. The following relations will be found useful to transformer designers. Energy dissipated in 1 lb. of copper at the rate of 1 watt will raise the temperature of the copper at the rate of ⅓ deg. cent. per min. if no heat be permitted to escape. Water flowing at the rate of 3.8 gal. per min. will absorb 1,000 watts, with a temperature rise of 1 deg. cent. Air at atmospheric pressure, flowing at the rate of 1,650 cu. ft. per min. will absorb 1,000 watts, with a temperature rise of 1 deg. cent.

MECHANICAL DESIGN AND COIL GROUPING

78. Mechanical stresses in service are due to the heavy currents developed in electrical apparatus under short-circuit, caused by the tremendous amount of energy developed in modern electrical systems. Reactance in a transformer, on account of the great length of the leakage paths in air, furnishes practically the same protection to connected apparatus as an air reactance.

79. Coil grouping and reactance. The reactance of a transformer depends on the grouping of the high-tension and the low-tension windings. The same general principles apply to the grouping of the windings of any type of transformer. The element of a given grouping is formed by a set of high-tension and low-tension coils of equivalent number of turns, generally an even submultiple of the total number of turns, placed as near together as

N is the number of HL groups, c is the space between adjacent high-tension and low-tension windings, and a_1, a_2, b_1, b_2, b and d are as shown in Figs. 16 to 18.

82. Formulas for mechanical force exerted on coils. In a transformer the electrokinetic energy is given by

$$W = \frac{1}{2}LI_1^2 \qquad \text{(Joules)} \qquad (36)$$

where I_1 is the primary current, and therefore since $F = \frac{\partial w}{\partial x}$

$$F = \frac{1}{2}I_1^2\frac{\partial L}{\partial x} \times 10^7 \qquad \text{(dynes)} \qquad (37)$$

x being in the direction of the maximum force, that is, at right angles to the surface of the coils. For shell-type transformers with discoidal coils,

$$F = \frac{1}{2}I^2\left[\frac{4\pi n^2(l_1 + l_2)}{4.45 \times 10^7 N^2(a_1 + a_2)}\right] \qquad \text{(lb.)} \qquad (38)$$

83. Mechanical design of cases. Where fluted cases are used they should be made of ingot steel at least 0.079 in. thick, except for transformers below 100 kw., then $\frac{1}{16}$ in. may be used. Cases for oil-insulated self-cooled transformers have the sides cast in a base of cast iron. Boiler-iron cases such as are used for water-cooled transformers are usually guaranteed to withstand a pressure of 50 lb. per sq. in. The covers of large transformers are provided with vents to release the pressure if an explosion should take place. The tank is provided also with a large gate valve, so that the oil may be drawn off very quickly. The cases of large transformers should be made as nearly air-tight as possible, as this prevents dirt or moisture finding its way into the tank.

84. In the design of end-frames for large transformers there is a tendency to replace cast iron by structural and cast steel; the weight of the end-frame is thereby much reduced for a given strength; moreover, when structural steel is used, or a combination of structural steel and steel castings a much greater latitude is given the designer, as he then has freedom to make the proportions of the core anything he pleases. The cooling coils of water-cooled transformers are usually made of $\frac{3}{4}$- to 1-in. brass or iron pipe and are tested up to 250 lb. per in. hydraulic pressure.

TRANSFORMERS FOR POWER SERVICE

85. General consideration in design. The most important quality in a power transformer is **durability**; it should be able to withstand the most exacting conditions to be met in service without being sensibly weakened. It is important that the insulating material shall not be subjected to excessive temperatures, and therefore every portion of the coil surfaces should be well exposed to the cooling medium. The coils directly connected to the lines in high-voltage transformers should have reinforced insulation.

86. The cheapest transformer consistent with durability that has a reasonably good performance, is what is generally required. Such features as efficiency and regulation are usually of secondary importance. The flux-density in the iron of such a transformer will be made as high as consistent with a reasonable exciting current; the loss per lb. at commercial frequencies in silicon steels is so low that the core-loss is seldom a limiting feature. The current-density, on the other hand, is limited chiefly by considerations of cost, because when the density is increased beyond a certain point, for a transformer designed to operate at a given temperature rise, the cost begins to increase again.

87. Types and characteristics. Power transformers differ mainly in the methods used to cool them: the characteristics pertaining to different methods of cooling are taken up under "Cooling" (Par. 54 to Par. 77). Transformers cooled by natural circulation of oil and air must be designed on a more liberal scale than water-cooled transformers, or those cooled by forced oil circulation, and therefore the losses will be lower in the former than in the latter. Air-blast transformers, on account of the heavy insulation and large insulation clearances needed, are not as efficient as other power transformers of the same voltage-class.

so that they may be used for lower voltages. The usual secondary distributing voltages are considered standard. Transformers used for interconnecting two high-voltage systems are considered special.

93. A standard transformer is completely specified when the method of cooling, the frequency of the circuit (either 25 or 60 cycles), the voltage class, and low-tension voltages are given. Manufacturers have type numbers or letters for their standard lines, and in some cases they have two lines to offer, or two different lines may overlap for a few sizes. In general, however, no ambiguity will exist if the method of cooling is specified with the other data required.

94. Tables giving ratings, efficiencies, regulation and weights typical of standard single-phase oil-insulated air-cooled transformers. These transformers have a guaranteed temperature rise not exceeding 40 deg. cent. at continuous full-load rating. When run continuously at one and one-fourth load the guaranteed rise is 55 deg. cent. above the air; room temperature to be 25 deg. cent. The insulation tests are in accordance with the Standardisation Rules of the A. I. E. E. The tables follow in Par. 95 to Par. 102.

95. Single-phase—60 Cycles—11,000-volt Class

Kv-a.	Efficiencies					Regulation		Net weight including oil
	¼ load	½ load	¾ load	Full load	1¼ load	100 % p. f.	80 % p. f.	
100	94.9	96.9	97.5	97.6	97.5	1.55	3.55	3,800 lb.
150	95.5	97.3	97.8	97.9	97.8	1.3	3.6	5,200 lb.
200	95.8	97.4	97.9	98.0	97.9	1.2	3.6	6,100 lb.
300	96.2	97.7	98.2	98.3	98.2	1.1	3.6	8,300 lb.
500	96.6	98.0	98.4	98.5	98.5	0.9	3.6	12,800 lb.
750	97.1	98.2	98.5	98.6	98.6	0.8	3.75	18,500 lb.

96. Single-phase—60 Cycles—22,000-volt Class

Kv-a.	Efficiencies					Regulation		Net weight including oil
	¼ load	½ load	¾ load	Full load	1¼ load	100 % p. f.	80 % p. f.	
100	94.4	96.7	97.3	97.4	97.3	1.6	3.7	4,800 lb.
150	95.0	97.1	97.6	97.7	97.6	1.4	3.7	6,100 lb.
200	95.4	97.3	97.8	97.9	97.8	1.3	3.75	7,000 lb.
300	96.1	97.7	98.1	98.2	98.1	1.1	3.8	10,000 lb.
500	96.5	97.9	98.3	98.4	98.4	0.9	3.8	13,500 lb.
750	96.9	98.1	98.5	98.6	98.6	0.8	3.9	19,000 lb.

97. Single-phase—60 Cycles—33,000-volt Class

Kv-a.	Efficiencies					Regulation		Net weight including oil
	¼ load	½ load	¾ load	Full load	1¼ load	100 % p. f.	80 % p. f.	
100	93.8	96.4	97.1	97.3	97.3	1.6	4.8	6,200 lb.
150	94.9	97.0	97.6	97.8	97.8	1.4	4.6	6,600 lb.
200	94.9	97.0	97.6	97.8	97.8	1.3	4.4	8,000 lb.
300	95.7	97.4	97.8	97.9	97.8	1.2	4.2	11,000 lb.
500	96.4	97.8	98.2	98.3	98.3	1.2	4.2	16,000 lb.

103. Tables giving ratings, efficiencies, regulation and weights typical of standard single-phase oil-insulated water-cooled transformers. The temperature guarantees of these transformers are usually 40 deg. cent. above incoming water with continuous full load, and 55 deg. cent. above incoming water with a continuous load one and one-fourth times full-load. The insulation tests are made in accordance with the Standardisation Rules of the A. I. E. E. The tables follow in Par. **104** to Par. **113**.

104. Single-phase—60 Cycles—22,000-volt Class

Kv-a.	Efficiencies					Regulation		Net weight incl. oil	Water rates: gal. per min.	
	¼ load	½ load	¾ load	Full load	1¼ load	100% p. f.	80% p. f.		100% load	125% load
500	96.7	97.9	98.2	98.3	98.2	1.2	4.1	6,900 lb.	3½	4
1,000	97.3	98.3	98.6	98.6	98.6	1.1	4.0	10,400 lb.	5½	6½
1,500	97.4	98.4	98.7	98.7	98.7	0.95	3.9	13,500 lb.	7	8½
2,500	97.7	98.6	98.8	98.9	98.8	0.85	3.8	20,400 lb.	9½	12½
4,000	98.0	98.8	98.9	99.0	98.9	0.75	3.8	26,000 lb.	13½	16½

105. Single-phase—60 Cycles—44,000-volt Class

Kv-a.	Efficiencies					Regulation		Net weight incl. oil	Water rates: gal. per min.	
	¼ load	½ load	¾ load	Full load	1¼ load	100% p. f.	80% p. f.		100% load	125% load
500	95.9	97.5	97.9	98.0	97.9	1.3	4.5	8,600 lb.	4	5
1,000	96.8	98.1	98.4	98.4	98.4	1.2	5.2	12,500 lb.	6	7
1,500	97.2	98.3	98.6	98.6	98.5	1.0	3.4	17,000 lb.	8	10
2,500	97.5	98.5	98.7	98.8	98.7	0.9	3.9	20,000 lb.	11	12
4,000	97.8	98.6	98.8	98.9	98.8	0.75	3.7	29,000 lb.	14½	17½

106. Single-phase—60 Cycles—66,000-volt Class

Kv-a.	Efficiencies					Regulation		Net weight incl. oil	Water rates: gal. per min.	
	¼ load	½ load	¾ load	Full load	1¼ load	100% p. f.	80% p. f.		100% load	125% load
500	95.3	97.2	97.6	97.7	97.7	1.0	6.7	11,500 lb.	4½	5½
1,000	96.1	97.6	98.0	98.1	98.1	1.2	4.6	16,500 lb.	7	8
1,500	96.7	97.9	98.3	98.3	98.3	1.05	4.6	21,000 lb.	9½	11
2,500	97.2	98.3	98.6	98.6	98.6	0.95	4.7	26,500 lb.	12	15
4,000	97.7	98.5	98.7	98.8	98.7	0.85	4.9	32,000 lb.	16	19

111. Single-phase—25 Cycles—66,000-volt Class

Kv-a.	Efficiencies					Regulation		Net weight incl. oil	Water rates: gal. per min.	
	¼ load	½ load	¾ load	Full load	1¼ load	100% p. f.	80% p. f.		100% load	125% load
750	96.6	97.6	97.8	97.7	97.5	1.92	5.6	21,500 lb.	5[illegible]	6[illegible]
1,000	96.8	97.8	97.9	97.9	97.7	1.7	5.7	23,000 lb.	8	9[illegible]
1,500	97.0	98.0	98.1	98.0	97.9	1.6	5.3	28,000 lb.	10[illegible]	12[illegible]
2,500	97.4	98.2	98.3	98.2	98.1	1.4	6.0	34,500 lb.	15[illegible]	18[illegible]
4,000	97.6	98.3	98.5	98.4	98.3	1.2	5.2	49,000 lb.	21[illegible]	26

112. Single-phase—25 Cycles—88,000-volt Class

Kv-a.	Efficiencies					Regulation		Net weight incl. oil	Water rates: gal. per min.	
	¼ load	½ load	¾ load	Full load	1¼ load	100% p. f.	80% p. f.		100% load	125% load
750	96.2	97.4	97.6	97.5	97.3	2.0	6.6	26,600 lb.	7	8
1,000	96.6	97.7	97.8	97.8	97.6	1.9	6.8	28,000 lb.	9[illegible]	11
1,500	96.7	97.7	98.0	97.9	97.8	1.65	4.7	35,500 lb.	11	13
2,500	97.2	98.1	98.3	98.2	98.1	1.3	4.6	45,500 lb.	17	19[illegible]
4,000	97.4	98.2	98.4	98.4	98.3	1.3	5.0	59,500 lb.	22[illegible]	27[illegible]

113. Single-phase—25 Cycles—110,000-volt Class

Kv-a.	Efficiencies					Regulation		Net weight incl. oil	Water rates: gal. per min.	
	¼ load	½ load	¾ load	Full load	1¼ load	100% p. f.	80% p. f.		100% load	125% load
1,000	96.3	96.4	97.6	97.5	97.3	1.95	5.8	35,700 lb.	12	14
1,500	96.5	97.6	97.8	97.7	97.6	1.75	5.8	42,800 lb.	13[illegible]	15[illegible]
2,500	97.1	98.1	98.20	98.10	98.0	1.5	5.7	56,000 lb.	17	19[illegible]
4,000	97.3	98.2	98.4	98.4	98.3	1.30	5.9	72,000 lb.	24	27[illegible]

114. Efficiencies, regulation and weights typical of three-phase oil-insulated water-cooled transformers may be obtained with reasonable accuracy by taking the values for single-phase transformers of one-third the rating; the water rates and weights being multiplied by three.

TRANSFORMERS FOR DISTRIBUTING SYSTEMS

115. General considerations in design. Transformers for distributing systems should be of high efficiency. Competition has brought about a certain degree of standardization of the losses of each size of transformer. The problem is, therefore, to produce a design having a minimum cost which is durable in service and conforms to definite performance specifications.

116. The relative values of core-loss and copper-loss in lighting transformers have been largely determined by competition, but it is probable that the present values give about the best average results in service.

121. Performances of 25-cycle Transformers; Standard Voltages 2,200-1,100/220-110

Kv-a.	Losses (watts)		Efficiencies				Regulation		Floor space (in.)	Height (in.)	Net weight (lb.)
	Iron	Copper	¼ load	½ load	¾ load	Full load	100% p. f.	80% p. f.			
1.0	16	46	92.9	94.8	94.7	94.1	4.60	4.00	13[illegible] × 11[illegible]	15[illegible]	150
1.5	23	72	93.2	94.8	94.7	94.0	4.81	4.80	16[illegible] × 13[illegible]	17[illegible]	200
2.0	23	94	94.6	95.5	95.2	94.5	4.72	4.88	16[illegible] × 13[illegible]	17[illegible]	200
2.5	31	105	94.3	95.6	95.5	94.8	4.21	4.16	16[illegible] × 13[illegible]	20[illegible]	225
3.0	35	125	94.6	95.7	95.6	94.9	4.18	4.14	16[illegible] × 13[illegible]	20[illegible]	225
4.0	45	150	94.8	96.0	95.9	95.3	3.76	3.98	17[illegible] × 15[illegible]	23[illegible]	300
5.0	52	170	95.2	96.3	96.2	95.8	3.41	3.50	18 × 16[illegible]	27[illegible]	375
7.5	72	230	95.6	96.6	96.6	96.1	3.08	3.13	20[illegible] × 17[illegible]	32[illegible]	500
10.0	86	285	96.0	96.9	96.8	96.4	2.87	3.42	25[illegible] × 20	30[illegible]	660
15.0	113	392	96.5	97.2	97.1	96.7	2.63	3.22	29[illegible] × 21[illegible]	36[illegible]	890
20.0	150	430	96.5	97.5	97.4	97.2	2.18	3.08	32[illegible] × 24[illegible]	38[illegible]	1,150
25.0	165	525	96.9	97.7	97.6	97.3	2.13	3.04	32[illegible] × 24[illegible]	46	1,365
30.0	290	580	95.8	97.2	97.3	97.2	1.93	2.62	34[illegible] × 32[illegible]	35[illegible]	1,600
37.5	335	670	96.2	97.4	97.5	97.4	1.78	1.90	37 × 35[illegible]	37[illegible]	2,020
50.0	400	800	96.5	97.6	97.8	97.7	1.60	1.75	37 × 35[illegible]	44[illegible]	2,200

122. Performances of 60-cycle Transformers; Voltages 6,600-6,300-6,000/220-110

Kv-a.	Losses (watts)		Efficiencies				Regulation		Floor space (in.)	Height (in.)	Net weight (lb.)
	Iron	Copper	¼ load	½ load	¾ load	Full load	100% p. f.	80% p. f.			
1.0	22	27	91.3	94.5	95.3	95.3	2.71	3.13	16[illegible] × 11[illegible]	18[illegible]	133
1.5	27	39	92.7	95.3	95.8	95.8	2.61	2.95	16[illegible] × 11[illegible]	18[illegible]	140
2.0	34	46	93.1	95.6	96.2	96.2	2.32	3.04	18[illegible] × 12[illegible]	21[illegible]	176
2.5	38	55	93.8	96.0	96.5	96.4	2.23	3.14	18[illegible] × 12[illegible]	21[illegible]	190
3.0	40	68	94.5	96.3	96.6	96.5	2.29	3.26	18[illegible] × 12[illegible]	21[illegible]	19[illegible]
4.0	45	81	95.2	96.8	97.0	96.9	2.04	2.27	19[illegible] × 13	23[illegible]	230
5.0	50	105	95.7	97.0	97.2	97.0	2.11	2.29	19[illegible] × 13	23[illegible]	240
7.5	70	142	96.0	97.3	97.4	97.3	1.91	2.29	22[illegible] × 15[illegible]	25[illegible]	346
10.0	90	162	96.1	97.4	97.6	97.6	1.63	1.99	22[illegible] × 15[illegible]	25[illegible]	380
15.0	118	235	96.6	97.7	97.8	97.7	1.58	2.06	24[illegible] × 17	31	543
20.0	147	295	96.8	97.8	97.9	97.8	1.49	2.11	30[illegible] × 21[illegible]	32	748
25.0	166	360	97.1	98.0	98.1	98.0	1.46	2.21	30[illegible] × 21[illegible]	36	853
30.0	183	420	97.3	98.1	98.2	98.0	1.42	2.23	33 × 22[illegible]	36[illegible]	957
37.5	220	490	97.4	98.2	98.3	98.1	1.33	2.32	33 × 22[illegible]	42	1,081
50.0	267	620	97.6	98.3	98.4	98.3	1.27	2.39	35[illegible] × 24[illegible]	48	1,390

125. Performances of 60-cycle Transformers; Voltages 16,500–15,750–15,000/220–110.—(*Continued*)

Kv-a.	Losses		Efficiencies				Regulation		Floor space (in.)	Height	Net weight (in.)
	Iron	Copper	¼ load	½ load	¾ load	Full load	100 % p. f.	80 % p. f.			
37.5	325	550	96.3	97.6	97.8	97.7	1.52	2.97	35¾×24¾	48	1,320
50.0	370	700	96.8	97.9	98.0	97.9	1.45	2.92	37¾×30¾	44½	1,450
75.0	600	840	96.6	97.9	98.1	98.1	1.20	2.70	40¾×34 1/16	51	1,965
100.0	800	980	96.7	98.0	98.2	98.2	1.05	2.60	40¾×34 1/16	56½	2,225

126. Manhole transformers. It is the general practice in large cities to supply energy for lighting by means of underground cables. Manhole transformers are simply lighting transformers fitted with cases of a type suited for operation in a manhole. These cases are rendered water-tight and air-tight and have an extra amount of radiating surface. The cover is provided with a vent covered by a thin air-tight metal diaphragm, so that if the pressure in the case becomes excessive the diaphragm will rupture. The outlet bushings should be so designed that they are moisture proof and yet admit of ready disconnection of the transformer. The rating, efficiency and regulation are the same as for standard distributing transformers.

MULTIPLE OPERATION

127. General principles. In Par. **19** of this section it is shown that the transformer of unity ratio may be represented by a simple alternating-current circuit shunted by an admittance Y. Where the ratio is not unity, this equivalent circuit still holds, if one winding is taken as the reference winding and all admittances are reduced to terms of this winding; this may be done by multiplying (or dividing) all these quantities in the other winding by the square of the ratio. The admittance Y does not enter the problem of relative division of the load and may therefore be ignored; the effective impedance of the transformer with respect to the reference winding will then be the same as the short-circuit impedance with that winding considered as the primary and the other short-circuited.

It is shown in Sec. 2, that in a branched circuit the current in each branch will be inversely proportional to the impedance of the branch. Since the effects of the primary and the secondary resistance and the magnetic leakage of the transformer may be represented by a simple impedance, the rule of branch circuits will also apply to parallel operation of transformers. In actual calculations the problem is usually to determine how a given number of transformers will divide a load of given kv-a. and power-factor. It will be found convenient in most cases, instead of using actual impedances or admittances, to use the ohmic and reactance drops expressed as fractions of the secondary voltage. Admittances derived from these are proportional to the actual admittance.

128. Formulas. Let the kv-a. rating of the transformers be P_1, P_2, P_3, etc., and let the impedance volts of each, expressed as a fraction of the rated voltage, be (c_1+jd_1), (c_2+jd_2), (c_3+jd_3), etc., respectively, then the admittances (g_1-jb_1), (g_2-jb_2), (g_3-jb_3), etc., will have the following values;

$$\left.\begin{aligned}
g_1 &= \left(\frac{c_1}{c_1^2+d_1^2}\right)\cdot\left(\frac{P_1}{P_1+P_2+P_3+}\right);\; b_1 = \left(\frac{d_1}{c_1^2+d_1^2}\right)\cdot\left(\frac{P_1}{P_1+P_2+P_3+}\right)\\
g_2 &= \left(\frac{c_2}{c_2^2+d_2^2}\right)\cdot\left(\frac{P_2}{P_1+P_2+P_3+}\right);\; b_2 = \left(\frac{d_2}{c_2^2+d_2^2}\right)\cdot\left(\frac{P_2}{P_1+P_2+P_3+}\right)\\
g_3 &= \left(\frac{c_3}{c_3^2+d_3^2}\right)\cdot\left(\frac{P_3}{P_1+P_2+P_3+}\right);\; b_3 = \left(\frac{d_3}{c_3^2+d_3^2}\right)\cdot\left(\frac{P_3}{P_1+P_2+P_3+}\right)
\end{aligned}\right\} \quad (39)$$

fraction of AB that the resistance drops of the respective transformers bear to their impedance drops. AC_1 is taken equal to the current that will produce the drop AD_1 in one transformer, and AC_2 is taken equal to the current that will produce the drop AD_2 in the other transformer. The resultant AC is the combined current. The corresponding combined resistance drop is AD. The line AC will then represent the combined load current on a certain scale, and AC_1 and AC_2 will represent the currents in the respective transformers, in magnitude and phase, on the same scale. Since the currents are known, the value of the impedance drop is known, and therefore the scale on which AB represents the impedance drop is known also: we may take a certain length to represent the magnitude of the secondary open-circuit voltage on the same scale, and with B as a centre strike an arc EOF; then, extending CA to E, draw AO, making the angle α with AE, where $\cos \alpha$ is the power-factor of the load. OA represents the secondary terminal voltage and OB represents the secondary open-circuit voltage, or approximately the primary e.m.f. multiplied by the ratio of transformation, and with phase reversed.

POLYPHASE TRANSFORMATIONS

132. General considerations. Polyphase systems may be classified as symmetrical and unsymmetrical. A symmetrical system may be defined as follows: if the system be n-phase, it will then consist of n e.m.fs. of equal intensity differing from each other in phase by $\frac{1}{n}$th of a period. If $\dot{E}_1 = E_1 e^{j\omega t}$ is one of the e.m.fs. and if $1, \alpha_1, \alpha_2, \alpha_{n-1}$ are the n roots of the equation $x^n - 1 = 0$, we may write the n e.m.fs. symbolically as follows:

$$\dot{E}_1 = \dot{E}_1,\ \dot{E}_2 = \alpha_1 \dot{E}_1,\ \dot{E}_3 = \alpha_2 \dot{E}_1,\ \dot{E}_n = \alpha_{n-1} \dot{E}_1 \tag{45}$$

In such a system we shall have

$$\dot{E}_1 + \dot{E}_2 + \dot{E}_3 + \ldots + \dot{E}_n = 0 \tag{46}$$

and the **rules pertaining to branched alternating-current circuits** will also hold true for all polyphase circuits, viz.,

(a) The vector sum of all currents flowing toward a common point is zero.

(b) The vector sum of the electromotive forces taken round any closed circuit is zero, the effect of resistance and reactance being considered as counter e.m.fs.

The directions in the windings of the transformers which are taken as positive must be maintained consistently in the vector diagram, or symbolic representation of the electromotive force.

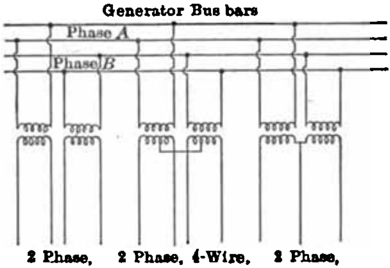


FIG. 22.—Two-phase systems.

133. Two-phase to two-phase transformation. The usual transformation is effected by transforming each phase independently by means of a transformer; the four secondary leads are independent, forming the terminals of a four-wire non-interlinked two-phase system. If the middle points of the secondary bindings be connected together the four-wire symmetrical interlinked two-phase system is obtained. If two ends of the secondary winding be connected together the result is the two-phase three-wire system which is an unsymmetrical interlinked polyphase system and therefore not suitable for long distance transmission. In this system the e.m.f. between the other two ends of the windings is $\sqrt{2}$ times the e.m.f. across the windings. Fig. (22) illustrates these systems.

where $\dot{I}_1$ and $\dot{I}_2$ are the two-phase currents, the corresponding relations for three-phase to two-phase transformation are obtained.

When the currents $\dot{I}_A$, $\dot{I}_B$ and $\dot{I}_C$ are balanced,

$$\dot{I}_C - \dot{I}_B = j2\left(\frac{m_1}{m_2}\right) \dot{I}_A \tag{51}$$

and therefore in order that the three-phase e.m.f. may be balanced independently of the load,

$$\dot{Z}_2 = \left(\dot{Z}_1 + \dot{Z}_4\right)\left(\frac{m_2}{m_1}\right)^2 \tag{52}$$

$$\left.\begin{aligned} \text{or } r_2 &= (r_1 + r_4)\left(\frac{m_2}{m_1}\right)^2 \\ \text{and } x_2 &= (x_1 + x_4)\left(\frac{m_2}{m_1}\right)^2 \end{aligned}\right\} \tag{53}$$

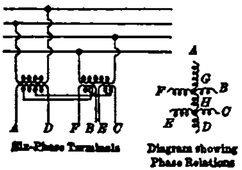


FIG. 24.—Two-phase to six-phase transformation.

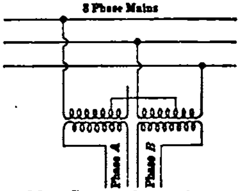
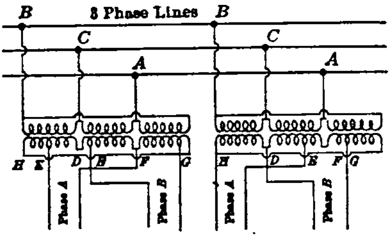


FIG. 25.—Scott three-phase to two-phase transformation.

The necessary adjustment must be made to the two-phase side of the bank of transformers. As a general rule no compensation is needed in practical work, as the unbalancing due to the transformer is trifling compared with that due to other causes. The ratio m_1/m_2 for symmetrical two-phase to three-phase transformation is $\sqrt{3}/2$ so that $(m_2/m_1)^2$ is 1.33.

137. Two-phase to six-phase transformation. This transformation may be accomplished by using two Scott-connected banks of transformers, but a more satisfactory method is that shown in Fig. 24 in which two transformers are used. Referring to Fig. 24, *FB* and *EC* represent two secondary windings of one transformer, supplied by one phase; *AD* represents the secondary of the other transformer supplied by the other phase. The currents in *GA*, *GB*, *HC*, *HD*, *HE*, and *GF* are the six-phase line currents. The current in *GH*, when the load is balanced, is equal to twice that in *GA*, *GB*, etc. The magnitude of the voltage that must be developed in *FB* is $\sqrt{3}/2$ times that in *AD*. *AG* and *HD* contain one-fourth the number of turns that are in *AD*.



Taylor Connection
3 Phase to 2 Phase with
3 Transformers
FIG. 26.

3 Phase to 2 Phase
with
3 Transformers
FIG. 27.

FIGS. 26 and 27.—Three-phase connections using three transformers.

138. Three-phase to two-phase transformation with three transformers. These methods of transformation offer some advantages in small installations where the cost of a spare unit is prohibitive, and it is desired to provide against the possibility of complete interruption of service due to ment of the windings is such that the third-harmonic components of the exciting currents of the three transformers cannot flow therein, consequently a third-harmonic component appears in the wave-form of each e.m.f. between the neutral and line. Since the impressed e.m.f. between any two terminals cannot have a third-harmonic component, and since this e.m.f. must be equal to the vector difference of the e.m.fs. between neutral and the two terminals across which the e.m.f. is impressed, these third-harmonic components in the two e.m.fs. from O to A and from O to B (Fig. 29) must be equal and in phase. The same statement applies to the third harmonic of the e.m.f. between O and C. There is therefore in each of the e.m.fs. OA, OC and OB a third-harmonic component of a value depending on the saturation of the core, and having exactly the same time phase. This triple frequency e.m.f. while not in itself serious, becomes so if the neutral of the secondary windings be grounded for then it sets up a triple frequency charging current through ground which interferes with neighboring telephone circuits.

143. Another defect of this connection (Par. 142) is instability of the neutral point which renders it unsuitable for four-wire three-phase distribution; furthermore if one transformer becomes short-circuited an overvoltage of 73 per cent. is impressed on the remaining two transformers. This defect may be overcome by connecting the primary neutral point to that of the generator, but this will not generally permit grounding the secondary neutral point because there is usually a third harmonic in the generator wave between neutral and ground.

144. There are several ways in which the triple-frequency pulsation in the secondary may be removed. One of them is to **interconnect the secondaries** of the three transformers as shown in Fig. 34. Another method is to provide a small delta winding for the purpose of supplying the necessary triple-harmonic component of the exciting current; this action of the delta connection in combination with the star is discussed under delta-star connections, Par. 148. All of these schemes add to the cost, and more than counter-balance any gain that might be derived from using this (star-to-star) connection.

145. In star-star connected three-phase core-type transformers the third harmonic component of the e.m.f. between neutral and lines does not occur on account of the mutual inductance between phases.

146. Delta-delta connection. This connection is widely used. Each transformer has its exciting current directly supplied by the generator so that there is no e.m.f. wave distortion. The load taken by each transformer in this connection will depend upon its impedance, so that transformers of different characteristics should not in general be connected together to form a delta-connected bank. If I_A, I_B and I_C are the secondary line currents and I_{AB}, I_{BC} and I_{CA} are the currents in the transformer secondaries, Fig. 29, and Z_1, Z_2, Z_3 are the impedances of the different transformers we shall have the following relations:

$$\left.\begin{aligned} I_{AB} &= \frac{Z_2 I_B - Z_3 I_A}{Z_1 + Z_2 + Z_3} \\ I_{BC} &= \frac{Z_3 I_C - Z_1 I_B}{Z_1 + Z_2 + Z_3} \\ I_{CA} &= \frac{Z_1 I_A - Z_2 I_C}{Z_1 + Z_2 + Z_3} \end{aligned}\right\} \quad (54)$$

147. Delta-star and star-delta connections. These methods of connecting a bank of transformers are the best for **high-voltage transmission**, the delta-star connection being used for stepping-up and the star-delta for stepping-down. When the voltage is stepped-up with a delta-star connection, the neutral of the star may be grounded without introducing any trouble, because the third-harmonic e.m.f. and its multiples, which are the

152. Formulas for "V"-connected transformers. Taking $\dot{Z}$ as the transformer impedance and Z_0 as that of the load on each phase, and $\dot{E}_A$, $\dot{E}_B$ and $\dot{E}_C$ as the primary star-e.m.fs., and assuming the ratio of transformation to be unity, the primary currents will be

$$\left.\begin{aligned} \dot{I}_{AB} &= \frac{\dot{E}_B}{\frac{\dot{Z}}{3}+\frac{\dot{Z}_0}{3}} + \frac{\dot{E}_C}{\dot{Z}+\frac{\dot{Z}_0}{3}} \\ \dot{I}_{BC} &= -\frac{\dot{E}_B}{\frac{\dot{Z}}{3}+\frac{\dot{Z}_0}{3}} - \frac{\dot{E}_A}{\dot{Z}+\frac{\dot{Z}_0}{3}} \end{aligned}\right\} \quad (59)$$

If we place in the primary, at B, a reactance equal to $\frac{2}{3}Z$ (Fig. 30), the ratio being assumed to be unity, we shall have

$$\left.\begin{aligned} \dot{I}_{AB} &= \frac{\dot{E}_B+\dot{E}_C}{\dot{Z}+\frac{\dot{Z}_0}{3}} = -\frac{\dot{E}_A}{\dot{Z}+\frac{\dot{Z}_0}{3}} \\ \dot{I}_{BC} &= -\frac{\dot{E}_B+\dot{E}_A}{\dot{Z}+\frac{\dot{Z}_0}{3}} = \frac{\dot{E}_C}{\dot{Z}+\frac{\dot{Z}_0}{3}} \end{aligned}\right\} \quad (60)$$

The connection is now balanced and gives exactly the same secondary e.m.fs. at a load of any impedance as a bank of delta-connected transformers having equal impedances of value $3Z$.

153. The "T"-connection is economical in first cost. It consists of two transformers, one of which has both its primary and secondary wind-

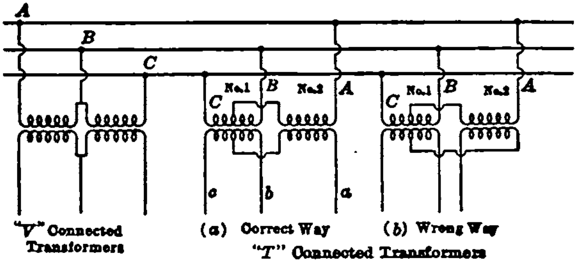


Fig. 30.—Unsymmetrical three-phase to three-phase transformations.

ings designed for 87 per cent. of the three-phase voltages; the other is designed for the full voltage, and both primary and secondary windings are provided with taps at the middle points. The transformers are connected as shown in Fig. 30.

154. Formulas for "T"-connected transformers. Assuming unit ratio the currents in the primary winding will be equal in magnitude and opposite in time-phase to the line currents in the secondary. If Z_1 is the impedance of the 87 per cent. transformer and Z_2 is the impedance of one-half of the primary of the other transformer, with the corresponding half of the secondary short-circuited, and if $\dot{E}_{AB}$, $\dot{E}_{BC}$ and $\dot{E}_{CA}$ are the primary impressed e.m.fs., the secondary e.m.fs. will be

161. The interconnected-star connection shown in Fig. 34 is used in connection with direct-current three-wire distributing systems. This connection permits continuous current in the neutral wire to flow through the transformers without magnetizing them.

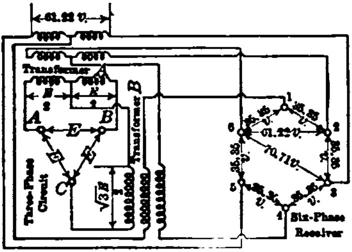


Fig. 33.—Three-phase to six-phase transformation, double tee.

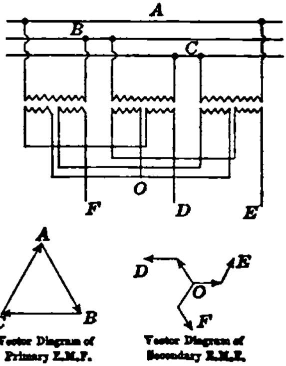


Fig. 34.—Interconnected star connection.

CONSTANT-CURRENT TRANSFORMERS

162. The constant-current transformer depends for its regulation on the force of repulsion between the primary and the secondary coils which carry currents that are in opposite time-phase. Referring to Fig. 35 and Fig. 4, as the load impedance decreases, the current will first increase, if the quantities $(L_1 - M)$ and $(L_2 - M)$ remain constant, but if the latter increases in the right proportion the current will remain constant. As the load impedance $r_o + jx_o$ is decreased, the total reactance in the secondary circuit will decrease, and the short-circuit reactance of the primary circuit will increase a corresponding amount. The ratio of current transformation will therefore remain practically constant throughout the range. The apparatus may thus be considered as a device to maintain a constant effective primary impedance, independent of the load. The short-circuit impedance with full-load coil separation should be as low as consistent with a reasonable cost or satisfactory operation, but with no-load separation it should be high enough to limit the current to its normal value. Sometimes the transformer is designed with regulation to take care of the cutting out of a portion, only of the load impedance, in which cast the short-circuit impedance voltage of the primary circuit for normal current will be less than the primary impressed voltage.

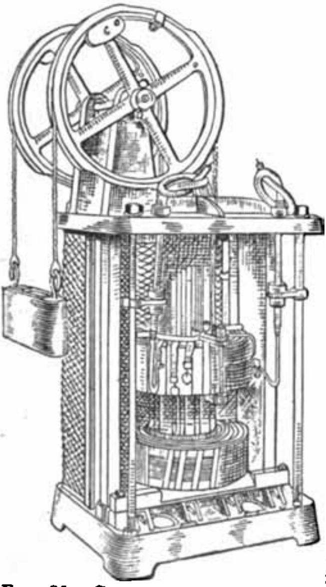


Fig. 35.—Constant-current regulator transformer.

The inductance for a cruciform-core regulator is

$$L = \frac{4\pi C N^2}{10^9}\left(\frac{7.6}{0.23 + \log \frac{r_a}{r_b}}\right) \qquad \text{(henrys)} \qquad (63)$$

For each case the repulsive force F is

$$F = \tfrac{1}{2} I_1^2\left(\frac{100L}{4.45 \times 2.54 \times C}\right) \qquad \text{(pounds)} \qquad (64)$$

The value of C is obtained by adding to the coil separation the insulation clearance between coils plus 0.45 times the length of each coil. All dimensions are to be measured in inches.

168. Regulation, of constant-current transformers for arc-lighting is usually guaranteed over a range from full load to no load. If properly adjusted, the current should be within 0.1 amp. of the normal value of current for any number of lamps between full load and no load.

169. The satisfactory working of alternating-current arc lamps demands a certain amount of reactance in the lamp circuit. This is provided in the transformer by its own effective reactance. The full-load operating power-factor of an alternating-current arc-lamp system seldom excells 70 per cent.; at small loads the power-factor is very much less. A table of efficiency and rating is given in Par. **171.**

170. Constant-current regulators for series tungsten lamp lighting are similar in design to the alternating-current series arc lamp constant-current transformer, except that the dashpot is omitted. Ratings and efficiencies are given in Par. **172.**

171. Constant-current Transformers for Arc-Lighting; Ratings and Efficiencies

60 Cycles

	Secondary amp.			Full-load efficiency
	6.6	7.5	10.0	
No. of lamps	6	5	6	90.2
No. of lamps	13	11	12	92.5
No. of lamps	20	17	18	92.7
No. of lamps	27	24	25	93.3
No. of lamps	38	34	35	93.9
No. of lamps	55	48	50	94.6
No. of lamps	83	72	75	95.3
No. of lamps	110	96	100	95.7

172. Ratings, Power-factors and Efficiencies of Constant-Current Transformers for Operating Series Tungsten Lamps

60 Cycles

Kw.	Efficiency per cent.				Primary power-factor per cent.			
	¼ load	½ load	¾ load	full load	¼ load	½ load	¾ load	full load
4.0	73.7	86.5	90.5	91.7	22.9	50.6	75.6	83.7
8.0	78.7	89.2	92.5	93.65	23.1	51.8	77.4	84.9
12.0	79.7	89.8	93.0	93.8	23.3	52.3	78.1	85.2
17.0	81.2	90.7	93.5	94.5	23.6	52.5	78.5	85.4
24.0	82.8	91.6	94.2	95.0	23.8	52.7	79.0	85.6
25 Cycles								
4.75	71.9	86.0	90.3	91.0	24.5	51.2	76.8	84.2
7.0	74.5	87.6	91.3	92.0	24.9	52.7	78.3	85.7
10.0	76.4	88.7	92.0	92.3	25.3	53.6	79.8	87.0
14.0	77.0	89.1	92.5	93.1	25.6	54.3	80.7	88.1
20.0	79.1	90.3	93.3	94.0	25.8	54.8	81.3	88.6
28.0	82.6	92.0	94.5	96.0	25.9	55.1	81.4	88.7

178. Auto-transformers for producing a proper division of the load between transformers operating in parallel. There are several ways in which auto-transformers may be used for this purpose; one of these, by which a group of any number of transformers of one design may be connected so as to operate in multiple with a group of any number of transformers of another design, is illustrated by Fig. 39. The illustration shows only single-phase groups, but polyphase groups may be paralleled in a similar manner.

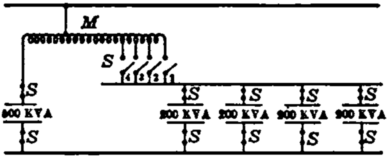


FIG. 40.—Use of auto-transformers for paralleling transformers.

Another method of paralleling two groups of transformers is illustrated in Fig. 40. In this method it is necessary to have an auto-transformer with a number of taps, so that the proper tap may be used for each combination of groups. When paralleling two transformers of unequal impedance drop at their rated currents, the voltage across the whole winding of the auto transformer will be

$$\sqrt{(I_1r_1 - I_2r_2)^2 + (I_1x_1 - I_2x_2)^2} \tag{64}$$

The winding should be designed to have the turns in each portion inversely proportional to the rated current of the transformer connected to it.

179. The regulation of an auto-transformer is illustrated most clearly by means of a vector diagram. Fig. 41 shows the vector diagram of a step-down auto-transformer. Fig. 42 shows that of a step-up auto-trans-

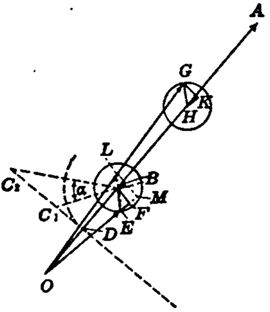


FIG. 41.—Step-down.

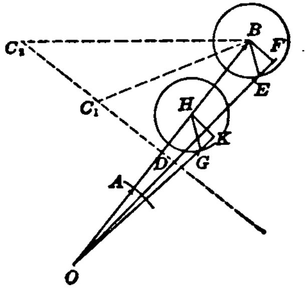


FIG. 42.—Step-up.

FIGS. 41 and 42.—Vector diagrams of voltage relations in step-down and step-up auto-transformers.

former. The construction is as follows: OA is the primary counter e.m.f. OB is equal to OA multiplied by the ratio of transformation. The impedance drop BE, in the case of the step-down auto-transformer (Fig. 41) is equal to $(1-m)$ times the short-circuit impedance voltage of the auto-transformer considered as a transformer with the primary short-circuited, the secondary of the transformer being that portion of the winding which forms the secondary of the auto-transformer, the primary being the remaining portion, 'm' being the ratio of transformation. In the vector diagram for the step-up auto-transformer (Fig. 42) the impedance drop BE is equal to the short-circuit impedance of the secondary with the primary winding short-circuited, the secondary being that of the auto-transformer considered as a

The mean induction in the core will depend on this e.m.f. and may be foun by means of Eq. 4. Let the angle of lag of the exciting current behind th induced e.m.f. be α and let its value considered with reference to the second ary winding be I_m; then if the angle of lead of the induced e.m.f. over I be θ, determined by the formula,

$$\tan\theta = \frac{\omega(L_2 - M) + \omega L_0}{R_2 + R_0} \quad (67$$

we shall have

$$I_1 = I_2\left[\sqrt{1 + \left(\frac{I_m}{I_2}\right)^2 + 2\frac{I_m}{I_2}\cos(\theta - \alpha)}\right]\epsilon^{j\theta_1} \quad (68)$$

where θ and α are already defined, and the value of θ_1 is found from the formula

$$\tan\theta_1 = \frac{\frac{I_m}{I_2}\sin(\theta - \alpha)}{1 + \frac{I_m}{I_2}\cos(\theta - \alpha)} \quad (69)$$

The quantity under the radical in Eq. 67 is the factor by which the ratio turns must be multiplied in order to get the ratio of transformation, and θ_1 is the lead of the primary current over the secondary current.

187. The important factors in the design of series (current) transformers are the load impedance and the effective secondary impedance for these together determine the mean induction in the iron for a given secondary current. These factors being fixed the excellence of the design will depend on the quality of the iron and the care taken in building which should be such as to make I_m as small as possible. Care should be taken in the mechanical design to avoid eddy currents in the end frames, and in the electrical design that in the endeavor to make the induction low by using a large number of turns this purpose is not defeated by the large increase in secondary reactance produced thereby.

188. The effect of these factors and of different qualities of iron may be studied by means of curves of ratio and phase displacement, such as that shown in Fig. 43 which was made by a method similar to those recommended by Crawford and Sharp, Agnew and others (see Sec. 3).

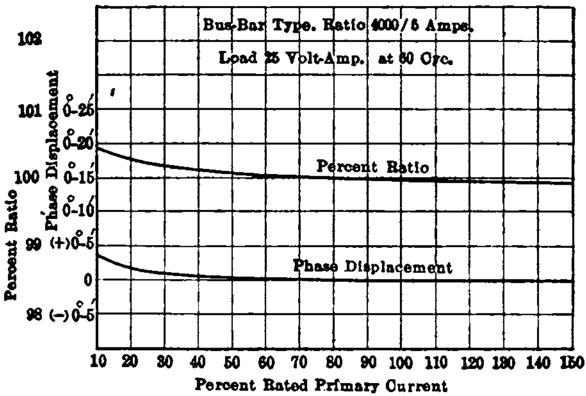


Fig. 43.—Calibration curve of a standard bus-bar type series transformer.

189. Series (current) transformers for high-voltage circuits are generally costly on account of the necessity of insulating them heavily and if they are oil-immersed they require in addition a high-voltage terminal.

of the dials the voltmeter or potential relay will indicate the voltage at the desired point of the feeder or line for any load.

195. Theory of line-drop compensators. Refer to Fig. 6 (under "General Theory"). The voltage at the station is represented by the secondary open-circuit voltage, and the voltage at the end of the line or centre of distribution is represented by the secondary full-load voltage; the reactance drop and the ohmic drop of the compensator are represented by the secondary short-circuit reactance drop and resistance drop, respectively.

196. In order to obtain the highest accuracy with line-drop compensators, it is necessary that the voltmeter operate on a small current compared with that passing through the compensator. Compensators for use with potential regulators, should be designed for larger currents than those for use with indicating instruments.

197. Compensating transformers for neutralizing inductive disturbances in telephone and telegraph lines. Where telephone or telegraph circuits parallel an alternating-current traction system, high voltages may be induced in them due to electromagnetic induction from the railway system. To overcome this trouble, a number of extra pairs of wires are placed among the other telephone wires so that their mutual inductance to the railway circuits is the same as those of the regular telegraph or telephone wires. These extra wires are grounded at various points, and connected to the primary windings of a special transformer, so that the e.m.fs. induced in them magnetize the transformer in the same direction. The telephone or telegraph wires are then connected to secondary coils of this transformer, so that the induced e.m.f. of the transformer opposes and neutralizes the induced e.m.f. due to the railway system. Since the two wires of any pair encircle the core in the same direction, the secondary windings are practically non-inductive, as far as the telephone currents are concerned. The transformers are usually of the core-type construction and differ from ordinary transformers only in the large number of secondary coils and the manner in which they are wound so as to obtain freedom from cross talk.*

198. Insulating transformers for telephone lines are designed to protect both the telephone and the user from high voltages, due either to induction or accidental contact with a high-tension line. This device consists of a shunt transformer provided with a condenser across the primary terminals, the purpose of the condenser being to supply the greater portion of the magnetizing current of the transformer under normal operation. The secondary terminals are connected to the telephone in the usual way. Such transformers are usually built to stand a 25,000-volt test, for 1 min., between the windings, and between windings and case. A lightning arrester and special fuses are generally provided for the primary circuit. The secondary may be grounded if desired, or an insulated stool provided for the user to stand upon.

199. Extra-high-voltage testing transformers. Test voltages as high as 375,000 volts above ground potential are becoming common, such a test voltage requires a testing transformer equivalent in insulation strength to one designed for 750,000 volts with the middle-point grounded. In the testing of insulators an artificial ground is commonly used for the higher voltages. This method of testing is economical in the respect that it requires a transformer insulated for only one-half the potential necessary when test specifications require that one terminal be grounded; but it is open to doubt whether tests made in this manner are as exacting as those made with one side of the circuit grounded. Testing transformers may now be obtained suitable for intermittent testing up to 600,000 volts with one terminal grounded, and up to 750,000 volts with the middle-point grounded. One testing transformer in present service has been tested up to 720,000 volts, effective (by spark-gap measurement), with one end grounded; another has been tested momentarily up to 900,000 volts, effective (by spark gap), with the middle-point grounded. The voltage impressed on the primary is raised either by cutting out resistance as in the potentiometer method of controll or by employing a dial and a separate regulator transformer.

200. Bell-ringing transformers. The bell-ringing transformer, connected to a 110-volt, 60-cycle, lighting circuit, produces at its secondary

* See article in *Electric Journal* for October, 1914, by Shaw.

rises under certain specified conditions must meet the guaranteed values, Par. 212 to 216.

(e) Insulation tests. These comprise the overpotential test which is made for the purpose of exposing any defect or injury in the insulation between turns, between layers, and between coils; the disruptive test for determining whether the insulation between the primary and the secondary windings and between these two windings and ground is sufficiently strong. Par. 217 to 220.

(f) Tests on instrument transformers; calibration.

206. Shop tests. In the case of large power transformers these tests are sometimes omitted, but in their stead each coil is very carefully inspected during the course of winding and insulating. The tests to determine any defects in winding are made by placing each coil on a core which is furnished with a removable yoke, and inducing in each turn of the coil a specified e.m.f. A test to determine whether the insulation is intact after the coils are assembled and the iron built in, is made by subjecting the insulation between the primary and the secondary windings, and between each of these windings and ground, to a test of several thousand volts. The transformer is then ready for the testing department.

207. Tests to determine whether the transformer is correctly wound and assembled. The first test to make is the **ratio test.** This determines whether the transformer has been wound correctly and the leads and tap brought out in the right places. The methods used in making this test will depend upon the transformer to be tested. Large power transformers are tested for ratio by having a fraction of the normal voltage impressed on a winding while the voltage between taps is measured by means of a voltmeter, and the ratio of the tap voltage to the total is thus obtained. The ratio of the primary voltage to the secondary voltage is obtained in the same manner. Small transformers for lighting and small power service may be tested by balancing against a standard of known ratio. The same method applies to shunt-type (potential) transformers for instruments.

208. Polarity test. In power transformers this test follows the ratio test, and is made by connecting two adjacent primary and secondary

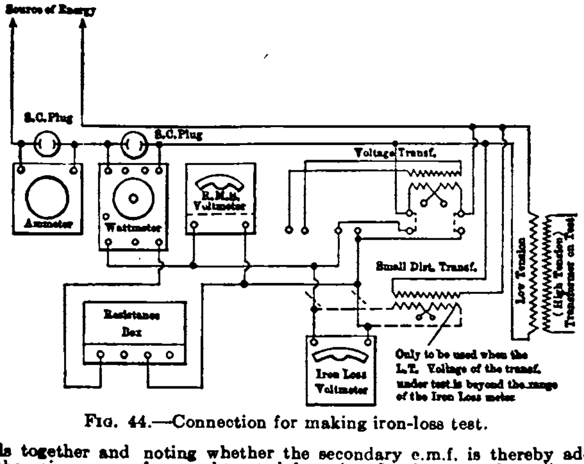


FIG. 44.—Connection for making iron-loss test.

leads together and noting whether the secondary e.m.f. is thereby added to the primary e.m.f., or subtracted from it. In the case of small transformers, the ratios of which are tested by comparison with a standard, the polarity is given at once by reference to that of the standard. Thus the apparatus for testing may be so arranged that if the polarity of the trans-the ammeter reading, and the true-watts component is obtained by dividing the observed core-loss by the value of rated secondary voltage plus the secondary *IR* drop.

211. Copper-loss and impedance volts. This measurement is made by short-circuiting one winding (properly it should be the primary winding) and circulating rated full-load current in the other, observing the loss in watts and the impressed voltage. The connections for the test should be made as shown in Fig. 45. The current should be adjusted with the voltmeter and the wattmeter shunt winding disconnected. The wattmeter may then be read first, followed by the voltmeter. If the exciting current of the transformer is high, a correction, obtained from Eq. 11, Par. **24**, must be added to the copper-loss measured as above. The copper-loss so obtained will be the true copper-loss of the transformer and includes the eddy-current loss in the conductors. By dividing the copper-loss by the rated output, the effective *IR* drop, expressed as a fraction of the rated voltage, will be obtained. Divide the observed voltage by the rated voltage, and the effective impedance-drop, expressed as a fraction of the rated voltage, will be obtained: the effective reactance drop, expressed as a fraction of the rated voltage, may then be obtained by taking the square root of the difference of the squares of these two quantities. Efficiencies and regulation may then be computed from these results by the usual formulas, which are given under "General Theory," Par. **24** to Par. **26**.

212. Tests for efficiency of cooling include the measurement of temperature rise above the cooling medium at various loads. Resistance measurements are also included, because they are necessary to determine the rise in temperature of the copper.

213. Resistance measurements may be made with a Wheatstone or a Kelvin bridge of the proper range. The conditions for measuring the cold resistance should be carefully prepared, so that the apparatus whose resistance is under determination will be at the temperature of the surrounding air. If the temperature of the room fluctuates, it is recommended that an idle unit be employed, of the same design as those on test. The resistance of the winding of the idle transformer may then be used as a basis from which to measure the temperature, by increase of resistance of the loaded transformer.

214. Methods of loading. There are various methods of artificially loading transformers. The best method, where two or more transformers

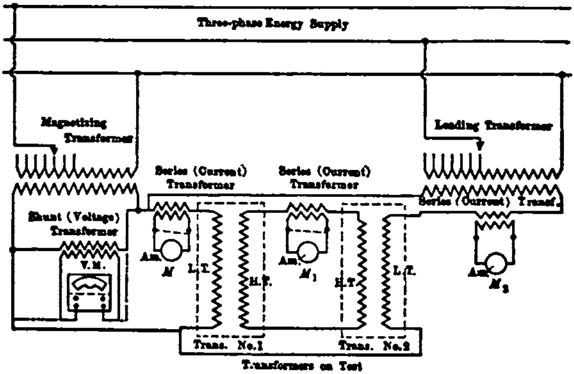


FIG. 46.—Method of artificially loading a transformer (opposition method).

are obtainable, is the **opposition method.** The simplest arrangement of this method is illustrated in Fig. 46. Two transformers are connected as if for multiple operation; then one of the connections of one set of windings is

223. Shipping and unpacking. Before leaving the factory, transformers are carefully packed. They are now generally shipped in their tanks with oil, and are ready for operation as soon as they arrive; but it is always a wise precaution, particularly with high-voltage transformers, to draw off from the bottom of the tank some of the oil and give it the usual disruptive test. If the dielectric strength is not high enough the oil should be dehydrated (Par. 222). This may be done without removing the transformer from the tank, by drawing the oil from the bottom of the tank through the dehydrating outfit and returning it at the top: this should be continued until repeated tests show the oil to be in good condition. If the transformer has been shipped in a packing case, and is one of a high-voltage type, it should **be dried** out before installation, unless it is hermetically sealed in an air-tight metal casing. The safest and best method that can be used to dry out a transformer is to force dry air at a temperature of 90° C. through the windings. Grid resistances may be used for heating the air, and it may be forced through the windings by a blower.

224. Precautions in making connections. When installed ready for operation the transformers should be carefully inspected to see that they are all connected properly, both internally and externally; that all switching apparatus is in good working order; and if new banks are to run in multiple with previously installed banks, it should be noted that they are connected at the proper operating taps, and so as to have the same polarity as the older banks. If the new transformers are of the same manufacture as the old, the polarity will be the same as that of the old, if the banks are similarly connected. If they are of different manufacture from the old, the polarity when similarly connected may be such as to make parallel operation impossible. It is therefore a wise precaution to check up the relative polarity of the old and the new banks before connecting together. This may be done by connecting the new bank on the high-tension side and connecting one of the low-tension terminals to the common bus bar; then, with a voltmeter, and if necessary with the aid of a shunt (potential) transformer, measure the voltage between the other two terminals and the respective busses with which they are to connect (a bank of lamps may be used instead of a voltmeter, with 110- or 220-volt connections) and observe the following **rules:**

(a) **If the polarities are alike** the voltmeter will read zero, in each case.

(b) **If the polarities are reversed** the voltmeter will read, in each case, double the secondary voltage. The remedy is to reverse the connections of the low-tension leads of each transformer in the new bank. (In three-phase transformers it may be found more convenient to reverse the connections of the high-tension coils.)

(c) **In three-phase transformers** the following cases may be met, in addition to the above. Having two similarly located low-tension terminals, of the old and new banks, connected together, the high-tension terminals of each bank being similarly connected to the same source of e.m.f., measure the voltage between corresponding free low-tension terminals.

(c-1) **If one voltmeter reads the secondary voltage correctly and the other reads double this value,** the external polarity of the two transformers is the same, but the terminals are in different order. The remedy is to interchange the internal low-tension connections to the terminals of the new transformers, so that the lead connected to the terminal for which the voltmeter reads double voltage will take the place of the lead connected to the terminal tied to the corresponding terminal of the old transformer. Equivalent external transposition may be made instead.

(c-2) **If one voltmeter reads zero and the other reads 1.73 times the secondary voltage,** then we have a case of reversed external polarity and transposition of terminals, combined. The remedy in the case of a delta connection is to disconnect the ends of the two coils from the terminal showing high voltage and connect the other ends of these two coils to this terminal; the ends previously removed should then be connected to the two remaining terminals, so that a coil that was originally connected to any one of these two terminals will now be connected to the other one. An equivalent change of connections must be made in the case of a star connection.

225. Multiple connections. Transformers having like primary and secondary connections may be run in multiple with one another, but transformers having unlike primary and secondary connections cannot be made to run in multiple with the former except by the use of auxiliary devices, thicknesses of a blotting paper. When using this apparatus it should b seen that the paper is carefully dried, and it is better to soak it, first of al in clean oil that is perfectly dry. Fig. 47 shows this form of oil dehydrato When the oil is in very bad condition, the paper should be changed from tim to time, as often as found necessary. Oil in first-class condition should brea down at not less than 40,000 volts, with a gap of 0.15 in. between ½-i spheres.

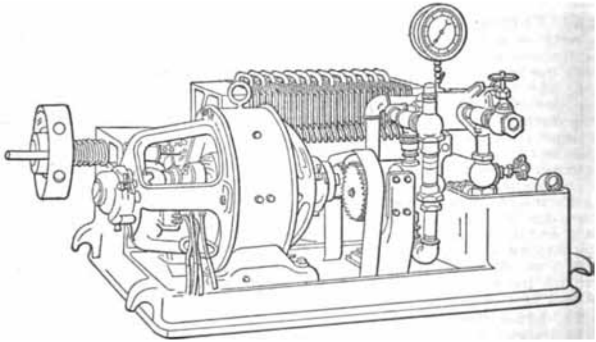


FIG. 47.—Filter press type of oil dehydrator.

233. Regulation of quantity of cooling medium. When water-cooled transformers are subjected to heavy overloads the water rate should be increased correspondingly. Any oil-immersed transformer will withstand a heavy load, for a short period very much better than an air-cooled transformer. Allowance should therefore be made in operation for the cooling characteristics of the transformers, and where artificial means are used for cooling, the cooling may be improved under abnormal load conditions by using more of the cooling medium.

234. The oil-piping layout of transformers when installed should be arranged so that the oil in any unit may be filtered without disturbing the others; there are several obvious methods of accomplishing this. Whatever scheme be used, care should be taken that all the air is removed from the oil in the transformer tanks; this may be done by creating a vacuum under the cover during the process of pumping.*

235. A transformer when burned out or defective should immediately be replaced by a spare unit if one is available. A careful examination should then be made of the windings; if nothing is disclosed, the transformer should be returned to its tank and measurements made of its open-circuit losses. If there are any short-circuited turns or layers, the open-circuit losses will be abnormally high. The location of the trouble can generally be traced by the blackening due to smoke, and by feeling the coils, after first removing the exciting voltage, for the point of highest temperature. If either of these methods fail, measurement may be taken of the resistance of primary and secondary windings; a short-circuit will be indicated by a lower resistance than normal. In many cases, however, a short-circuit which cripples the transformer may involve only a few turns in a large total number, so that the continuous-current resistance measurement may not be sensitive enough to detect it.

236. Formation of scale in cooling coils. A water-cooled transformer may progressively increase in temperature and appear to be defective, due to no apparent cause. In such a case the cause of the trouble can generally be traced to the water supply, which will be found to contain foreign matter in solution, such as lime and carbonates; these impurities, when the water becomes heated in passing through the tubes, are precipitated in the form of

* *Electrical World*, Feb., 1913, p. 360.

which counterbalances the magnetising or demagnetising effect of the secondary current.

243. The variation of secondary e.m.f. with change in angular position, for a certain type of single-phase potential regulator is shown in Fig. 49. A vertical cross-section is shown in Fig. 49.

244. The polyphase regulator in every essential detail, is a polyphase induction motor, the polyphase coil-wound rotor of which can be locked in any position desired. The primary windings are connected across the supply lines, precisely like the primary windings of a polyphase induction motor; however, the secondary phase-windings of the induction regulator, instead of being closed upon themselves, as is true of the secondary windings of an induction motor, are separately insulated and separately connected in series with the delivery circuits from the regulator. When polyphase e.m.fs. are impressed upon the primary windings, the e.m.f. generated in each secondary coil is of the same frequency as the primary e.m.f. and its value is entirely independent of the mechanical position of the movable member; the time-phase position of their e.m.fs., however, varies directly with the electrical space position of the movable member. (Compare with the single-phase induction regulator; see Par. **240**.) The resultant delivered e.m.f. is the

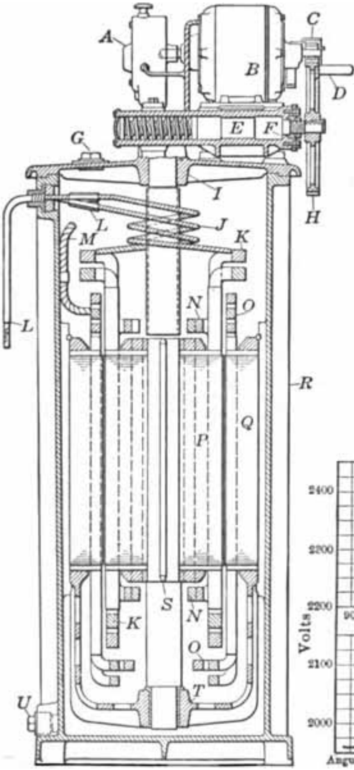


Fig. 49.—Vertical cross-section of single-phase induction regulator.

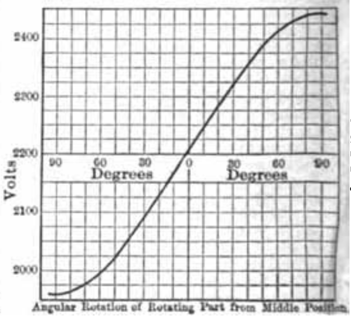


Fig. 50.—Secondary voltage of single-phase potential regulator of varying positions of rotating mauler.

vector sum of (or difference between) the primary and the secondary e.m.fs.; it is not constant in value but varies largely with the position of the movable member.

245. Currents and m.m.fs. in polyphase regulators. The current in the delivery circuit (which is the same as that in the secondary coil) depends directly upon the delivered resultant e.m.f. and the impedance of the delivery circuit. In the polyphase induction regulator there is no special "tertiary" circuit, but each primary phase-winding acts in part as and "all out" will be equal to the value of $\sqrt{1-(\text{power-factor})^2}$, the power-factor being the value with resistance all in. It is evident that if the operating power-factor be made low enough, a series system of incandescent lamps may be operated at constant current, from a constant-potential source, with very little variation in the series reactance.

250. Limitations. The constant-current regulator is manufactured as a standard apparatus by a few companies, but it cannot advantageously take the place of the constant-current transformer, since it requires an additional transformer for insulation purposes (see National Electrical Code); it may, however, be used to advantage instead of the constant-current transformer where it is not required that the receiver circuit be insulated from the supply circuit.

251. Types and characteristics. The usual type manufactured is air-cooled and of the form described in Par. **249**. An automatic constant-current regulator may also be made by connecting in opposition two coils which surround a closed magnetic circuit, using their leakage reactance for the regulating reactance. The two coils are pivoted so as to approach each other at the middle of the core. In action they will repel each other with a force nearly proportional to the current, for any position; with proper counterweight, therefore, the current will automatically be kept constant.

252. A motor-controlled potential regulator may be used as a constant-current regulator by controlling it from a constant-current relay; it is usually, however, too sluggish in operation to give perfect satisfaction.

REACTORS

253. General types. Reactors, choke coils, or reactances for power purposes may be divided into two classes, viz., those in which iron is used and those in which no magnetic material whatever is used. The first type consists of a coil encircling a circuit of iron which is usually broken by an air-gap or a series of air-gaps. The second type is simply a carefully constructed, circular coil of rectangular cross-section, of suitable proportions for cooling, and well supported mechanically.

254. Iron core reactances. The fundamental equations and general design are the same as for constant-potential transformers. The air-gap should be subdivided so as to avoid concentrated leakage, which causes eddy currents in the conductors (which are difficult to eliminate) and also makes it difficult to calculate the reactance accurately. The relative amount of copper and iron will be determined largely by the condition for minimum cost, but where the conductors are very large, it is advisable to increase the area of the core and reduce the number of turns in order to keep down the eddy-current losses in the conductors. In winding with multiple conductors every care should be taken to see that the reactance of each conductor and its resistance is the same; this may be accomplished by properly distributing the "start" and "finish" leads of the conductors.

If the length of the air-gap is l, the required inductance L, the current I, the number of turns n, the effective area of the air gaps must be obtained and it will depend on their number and distribution. The value of $\mathcal{B}$ may then be calculated from this area by Eq. 4, Par. **5**.
Thus the value of l will be

$$l = \frac{4.51nI}{\mathcal{B}} \qquad \text{(in.)} \qquad (71)$$

where $\mathcal{B}$ is the induction in lines per sq. in.

255. The current densities and the flux densities will be about the same as for transformers. The methods of insulating the coils are very similar and need no special treatment here, as they are covered under the heading of "Insulation," Par. **41** to Par. **47**.

256. Cooling may be effected by any of the various methods applied to transformers. The problem is the same in every particular and is treated under that heading, Par. **61** to Par. **70**.

257. Applications. Iron reactors for power purposes are mostly used in connection with compound-wound rotary converters to obtain compounding or overcompounding of the voltage at the continuous-current terminals. Iron reactors are also used to shunt the series coils of compound-installing, reactors will be amply strong enough to withstand any stress that may arise in service.

263. Formulas for self-inductance. The inductances of coils at commercial frequencies may be calculated with a high degree of accuracy, provided the proper formula is used. The following formula, which is Lorenz's cylindrical current-sheet formula with a correction for thickness of coil, will be found accurate. Refer to Fig. 53 for dimensions which are assumed to be inches.

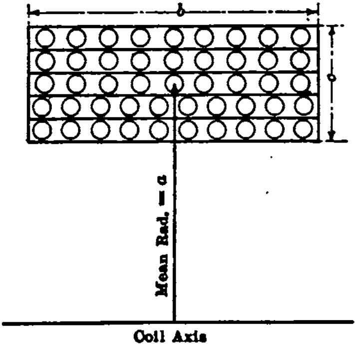


Fig. 53.—Cross-section of reactance coil.

264. Rosa's formula. This is accurate for any shape of coil. For practical work the correction for the form of the conductor is negligible and may be omitted; the formula then becomes

$$L = L_s - \Delta_1 L \qquad (72)$$

To obtain the value of L_s the formula below may be used (Eq. 73):

$$L_s = an^2Q \times \frac{2.54}{10^9} \quad \text{(henrys)} \qquad (73)$$

Where Q is a function of $2a/b$, the values of which for different values of this ratio are given in the table below, in Par. 265. The correction term $\Delta_1 L$ is given by the formula

$$\Delta_1 L = 4\pi a\left(\frac{b}{c}\right)(A_s + B_s) \times \frac{2.54}{10^9} \quad \text{(henrys)} \qquad (74)$$

Where A_s and B_s are given in the tables below (Par. 266 and Par. 267) as functions of c/a.

265. Values of the Constant Q in Eq. 73

$2a/b$	Q	$2a/b$	Q
0.20	3.63240	1.80	19.57938
0.30	5.23368	2.00	20.74631
0.40	6.71017	2.20	21.82049
0.50	8.07470	2.40	22.81496
0.60	9.33892	2.60	23.74013
0.70	10.51349	2.80	24.60482
0.80	11.60790	3.00	25.41613
0.90	12.63059	3.20	26.18009
1.00	13.58892	3.40	26.90177
1.20	15.33799	3.60	27.58548
1.40	16.89840	3.80	28.23494
1.60	18.30354	4.00	28.85335

266. Value of the Constant A_s as a Function of c/a, c being the Depth of the Winding and a the Mean Radius

c/a	A_s	c/a	A_s
0.00	0.6949	0.20	0.6922
0.10	0.6942	0.25	0.6909
0.15	0.6933		

271. The theory of operation. An ionising agent is required to start the electron stream issuing from the cathode. The liberated electrons proceed at high velocity toward the anode ionising molecules of gas on their course thereby producing positively charged bodies and increasing the stream of electrons moving toward the anode. The positive charges are drawn toward the cathode where they become neutralised again or negatively charged; in so doing they further increase the ionisation at the cathode, and produce in its neighborhood a high intensity or potential gradient which assists in accelerating the electrons emitted at the surface of the cathode. The neutral or negatively charged molecule is forced back toward the anode, but becomes ionised again by the stream of electrons issuing from the cathode. The negatively charged molecules apparently rarely move far enough away from the cathode, before becoming ionised by collision, and positively charged, so as to come under the influence of the electric field of the inactive electrode. There is, therefore, no leakage or an extremely small one between the active and inactive anode.

272. In the actual design of rectifiers it appears that rectifying power is dependent upon the extent of the dark space surrounding the inactive terminal, which is known as **Crooke's dark space.** This space increases in volume as the vapor pressure decreases. The rectifying power is therefore enhanced by keeping the vapor pressure low, which means operating at low temperature. Another method by which the rectifying power may be increased, is by enclosing the terminals in narrow chambers far removed from the mercury pool; this produces a rectifier much more sensitive to temperature variation than when the terminals are in a common chamber.

273. Auxiliary apparatus. In order to obtain unidirectional current the mercury-vapor rectifier must be provided with a **transformer,** or autotransformer, connected to the anodes so that they will be alternately positive and negative with respect to the mercury pool. The transformer (or autotransformer) must be provided with a middle tap, between the two terminals connected to the anodes; this tap and the cathode of the rectifier chamber form the two terminals of the direct-current circuit. In order to obtain a current whose instantaneous values will fluctuate between narrower limits, some means must be provided to overlap the activities of the two terminals of the rectifier, so that one will become active before the other ceases its activity. This can be accomplished by introducing inductance in the direct-current circuit, or what amounts to the same thing a high magnetic leakage between the two halves of the transformer secondary winding connected to the rectifier. Thus a magnetic field is set up by the current in the anode, which, when the current is decreasing, sets up an electromotive force which tends to hold the anode potential up to the proper value, while the rate of decrease of the current is very much slower than when the inductance is absent; and thus the current is permitted time to build up in the previously inactive anode, before the current has ceased in the other.

274. Formulas for calculation of transformation ratio, induction, etc. The exact calculation of wave form, ratio of direct to alternating current, etc., is a somewhat complex problem involving transient phenomena. For practical work a sufficiently close approximation to exact theory is obtained by considering the ripple in the direct-current wave to be a simple double-frequency harmonic. In the formulas that follow it is assumed that the unidirectional current is represented by a constant average value I_a on which is superimposed a double-frequency simple harmonic current-wave of maximum value I_b: the effective value of the current is $I_e = \sqrt{I_a^2 + I_b^2}$. Let L_a be the effective inductance and let R_a be the effective re-

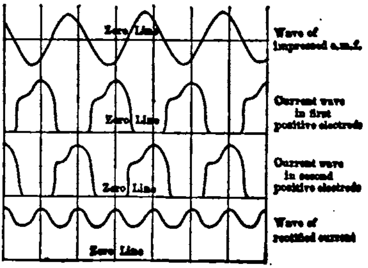
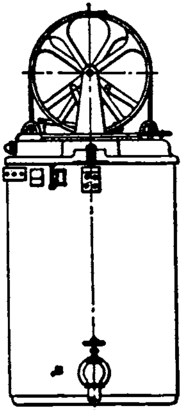


FIG. 55.—Typical wave form of single-phase rectifiers.

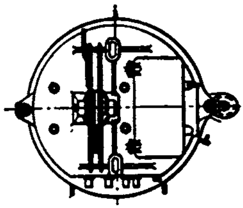
276. A unidirectional current cannot be delivered by the secondary of a transformer except through the medium of the magnetic circuit, which stores the energy during the inactive cycle and delivers it in the form of unidirectional current. The regulation and the power-factor in such cases are necessarily poor. In order to obtain satisfactory operation, two three-phase rectifiers are required which will operate from the secondaries of the three-phase transformer; or three single-phase transformers may be connected to give diametrical six-phase currents, the middle points of the secondaries being connected together. The rectifiers are connected so that there is one anode of each to a single secondary winding. The cathodes are connected together through reactances. In this manner a current may be obtained which is almost perfectly continuous.

277. When mercury-vapor rectifiers are employed for supplying unidirectional current for arc lighting, they require a special form of constant-current transformer provided with a double secondary winding, each portion of which is alternately active and inactive. The two parts of the secondary winding may be so arranged as to supply whatever sustaining reactance is required for satisfactory operation; this is the method adopted by one large company for one of its types of standard rectifier arc-lighting equipments. External reactance may also be supplied through suitable reactance coils; this method is used by another large company, and is also used by the first-mentioned company in another standard type of rectifier arc-lighting equipments.

278. A fifty-light equipment manufactured by the first-mentioned company (Par. 277) makes use of the self-sustaining feature. In the regulator part of the equipment the secondary coils are placed at the top and the bottom of a shell-type core and are so designed as to give the required sustaining inductance, which is the same as the inductance obtained between the middle point of the two coils and the two outer terminals connected together. The primary winding is made up of two coils connected in series, and so arranged by means of wheels and counterweights that they will move in opposite directions from the middle point of the magnetic circuit; under full-load operating conditions they will be as far apart as possible. But under no-load conditions they will be at their minimum separation. In operation, both primary coils are repelled from the active secondary with equal force, but the mutual attraction due to their own currents makes the two coils tend to approach one another; so that with proper counterweights and adjustments, a constant alternating-current may be maintained in the primary, and thus a constant unidirectional-current may be obtained from the secondary. The general appearance of a rectifier equipment for series arc lighting is shown in Fig. 58.



Side Elevation



Plan View

Fig. 58.

279. A one-hundred-light equipment manufactured by the same company (Par. 277) has essentially two separate constant-current regulators, which have a common magnetic circuit. The primary coils are independent and are connected in multiple, so that each primary coil and the adjacent

284. The efficiency of the rectifier is relatively low, seldom exceeding 60 per cent. and frequently averaging much less. The efficiency increases with decrease in the amount of current to be rectified.

285. Heating. Since the efficiency is low, the heat loss in the rectifier is comparatively large, which results in rapid deterioration of the plates. The degree of rectification, as the temperature rises, is much less complete. In other words, the leakage of current becomes excessive as the current increases, unless cooling means are provided.

286. Treatment of plates. A further objection to the electrolytic rectifier is the necessity for occasionally "treating" the plates to form the necessary insulating and rectifying films.

287. General applications. As a result of the low efficiency, the rapid deterioration, and the rapid heating of the electrolytic rectifier, it is seldom used for ordinary loads. Its present field appears to be limited to X-ray equipments and experimental purposes where a cheap rectifying device is required for light intermittent service.

288. Artificially cooled electrodes. Recent improvements have been made in electrolytic rectifiers which permit operation at higher current densities. One of these consists in cooling the aluminum cathode by a stream of water. The effect is such that a current ten times as large can be permitted with the cooled cell as with the same cell uncooled. An excellent rectifying effect with a cooled cell may be obtained with frequencies as high as 4,000 to 10,000 cycles per second.*

MECHANICAL RECTIFIERS

289. The vibrating type of alternating-current rectifier is a simple, efficient and inexpensive piece of apparatus adapted particularly to the charging of three-cell vehicle batteries.

290. General characteristics of vibrating type of rectifier. It is essentially an electrically operated vibrating switch which reverses the connection of the alternating-current line to the battery in synchronism with the alternations of the current, this reversing action being accomplished at the moment when the current flow is zero. There is therefore very little sparking at the contacts and the current delivered is unidirectional and pulsating.

291. The principle on which the vibrating rectifier operates is as follows: a small transformer serves to reduce the alternating-current line voltage to a suitable value; two alternating-current magnets are connected across one-half of the low-voltage winding and are so arranged as to present at any instant poles of like polarity before those of a steel magnet which is excited from the battery. Hence during one-half cycle of the alternating current, one of the poles of the pivoted magnet will be attracted and the other repelled, and the opposite action will take place during the other half-cycle. The pivoted magnet is connected to one terminal of the battery and the other terminal of the latter is connected to the middle point of the low-tension winding of the transformer. Two platinum contacts carried on the pivoted magnet are thereby brought alternately in contact with two other contacts connected to the low-voltage terminals of the transformer, as shown in Fig. 76. It is of no importance which terminals of the battery are connected to the binding posts of the apparatus, since the winding of the pivoted magnet automatically determines the proper polarity of the binding posts.

292. Sparking. A properly designed vibrating rectifier should operate without sparking, otherwise the life of contacts will be short. To secure sparkless operation, contact should be broken at the instant the current falls to zero and should be made again at the moment the instantaneous value of the alternating voltage wave is equal to the e.m.f. of the storage battery. If contact is broken before the current reaches zero and made again some time after the alternating e.m.f. has reached the proper value, the maximum input to the battery will not be obtained; on the other hand, if the current reverses before contact is broken, part of the battery charge will be dissipated in the circuit.

When the wave-form of the alternating-current supply differs from a sine-wave it may be necessary to change the adjustment slightly to prevent

* See *Elek. Zeit.*, Aug. 21, 1913. *Phys. Zeit.*, June 15, 1913.

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SECTION 7

ALTERNATING-CURRENT GENERATORS AND MOTORS

BY

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SYNCHRONOUS MACHINES

GENERAL

1. Definition. A synchronous generator (or motor) usually consists of a system of alternately north and south magnetic poles, comprising the field, which moves with respect to a system of suitably connected conductors in which the alternating e.m.f. is induced. These conductors together with their mounting are called the armature. See Fig. 1. The magnetic poles of the field are usually excited with direct-current supplied by a separate generator called an **exciter**. See Par. 9.

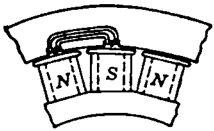


FIG. 1.—A portion of the armature and field of a synchronous machine.

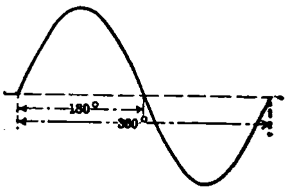
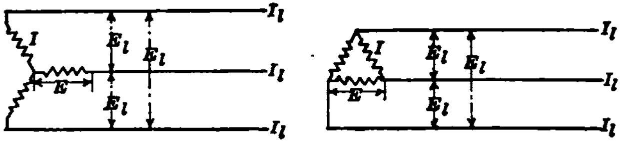


FIG. 2.—A graph of a cycle of e.m.f.

2. The "cycle." In the ordinary heteropolar alternator, the e.m.f. induced in each armature conductor reverses with the passage of each pole, *i.e.*, for a peripheral movement equal to one pole pitch (the peripheral distance between the centres of adjacent poles). The set of values comprised within a double reversal, or corresponding to a movement equal to twice the pole pitch, repeats itself indefinitely, and is called a cycle of values, or simply a cycle. See Fig. 2. The graph of these values plotted against time in rectangular coordinates, is referred to as the **e.m.f. wave** of the machine in question.

3. Frequency. The time required for the execution of a cycle is called the **period** of the alternating e.m.f., and the number of cycles per second is called the frequency. Frequency, f, is expressed by the relation: $f = p \times$ rev. per second, where $2p$ is the number of poles. Modern commercial frequencies in the United States are 60 and 25 cycles per sec.

4. Electrical and magnetic degrees. It is customary to refer to a cycle as 360 electrical degrees (see Fig. 2), or one electrical revolution, and to the circumferential distance from the centre of one pole to that of the next alternators the phases are internally connected, with only three terminals brought out. There are two principal methods of making these connections, Y and Δ (see Figs. 8 and 9). Let E and I represent respectively, volts and amperes per armature phase; and E_l and I_l the line volts and amperes. Then, in the Y armature, $E_l = E\sqrt{3}$, and $I_l = I$; and in the Δ armature, $E_l = E$, and $I_l = \sqrt{3}\,I$. Therefore the power in each case will be $3EI \cos\theta = \sqrt{3}\,E_lI_l \cos\theta$, where θ is the common phase difference between the voltage and current of each phase, balanced load assumed.



FIGS. 8 AND 9.—Internal and external e.m.fs. and currents in three-phase alternators with Y and Δ connections.

7. Note on phase classification. There is a slight inconsistency in the usual classification of alternators and systems. In the ordinary three-phase alternator (Fig. 5) there are, in each 360 magnetic degrees of circumference, six coil groups or belts of conductors. In these groups are being generated six different e.m.fs. differing in phase progressively by 60 deg. or one-sixth of a cycle. If the six terminals of the three phases be connected to three equal loads, the six currents in the six leads, when counted positive in the same direction along the line, will differ in phase progressively by 60 deg. If, however, the three phases be connected in Y or Δ, with three leads connected to the load, the three currents, counted positive in the same direction along the line, will differ in phase by 120 deg. or one-third of a cycle. To be consistent these two systems should be referred to as "six-phase" and "three-phase" respectively, although the alternator is the same. Moreover when a closed-coil armature is used for alternating e.m.f. generation, as in the case of a synchronous converter, it is called "three phase" when it has three 120-deg. taps and three phase-belts of 120-deg. span, and six phase when it has six 60-deg. taps and six phase-belts each of 60-deg. span. Thus, to be consistent, the ordinary three-phase alternator should be called a six-phase alternator, although mostly used to supply three-phase circuits.

8. Single-phase generators* are occasionally required for railways and also for certain electrochemical and electrothermal processes. Most single-phase generators are simply Y-connected three-phase generators with one of the three legs left idle. When a generator is operated single-phase, it is important that the pole shoes should be fitted with a heavy amortisseur or squirrel cage winding to damp out the effects of the pulsating armature reaction (Par. 44). Furthermore it is important that all parts of the magnetic circuit shall be well laminated in order that pulsations in the resultant m.m.f. may not occasion excessive iron losses.

Occasionally single-phase generators are supplied with distinctly single-phase windings as shown diagrammatically in Figs. 10 and 11. The windings in these two figures differ from one another in the extent to which the conductors are distributed over the surface of the stator. The complete distribution employed in Fig. 11 is wasteful. The proportions of Fig. 10 are more nearly correct. (See Par. 28, Par. 31, and Fig. 32.) A single-phase generator for a given kv-a. output and a given power factor, voltage and speed, is inherently more heavy and expensive than the equivalent polyphase generator by fully 65 per cent. Even then, if the speed is very high, as

* Waters, W. L. "Modern Development in Single-phase Generators." *Trans.* American Institute of Electrical Engineers. Vol. XXVII, 1908, p. 1069 and from 1086 to 1097.

Hobart, H. M. "The Relative Costs and Operating Efficiencies of Polyphase and Single-phase Generators and Transmitting Systems." *Trans.* American Institute of Electrical Engineers, Vol. XXXI, 1912, p. 115.

is generally arranged between the two main bearings to permit of moving the stator longitudinally, in order to render the rotor accessible. In small machines, however, the bearings are fitted in the end shields. In this type and, in fact, in all types of two-bearing, belt-driven machines, the bearing at the pulley end should be of especially liberal proportions. In England, rope drive is much more usual than belt drive.

13. Engine type. (Direct-connected to reciprocating engines.) In this type sufficient momentum must be provided in the rotating element to ensure the required degree of uniformity in the angular velocity. Three arrangements to accomplish this end are: (a) a separate fly-wheel; (b) the fly-wheel as an integral part of an internal rotor, Fig. 12; (c) the fly-wheel as an integral part of an overhung rotor, Fig. 13.

In America the fly-wheel effect is expressed in terms of the WR^2 where the weight of the rotor in pounds is denoted by W, and the radius to the centre of gyration, in feet, is denoted by R. In countries employing the metric system, the fly-wheel effect is expressed in terms of the GD^2, where D is the diameter at the centre of gyration in meters, and G is the weight of the rotor in kilograms. The greater the weight of the required fly-wheel, the more liberal must be the design of the bearings.

14. Water-wheel type alternators have speeds ranging widely in accordance with the head of water under which the prime movers operate. The Keokuk 9,000-kv-a. generator (Par. 11) has a rotative speed of 58 rev. per min., and the 5,000-kv-a. (Niagara) generator (Fig. 13) has a rotative speed of 250 rev. per min.

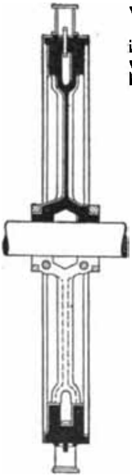


Fig. 12.—Fly-wheel rotor for alternator.

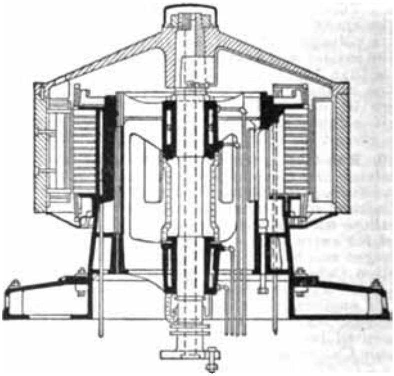


Fig. 13.—Umbrella type of revolving-field alternator.

Even for the same head, water-wheels are built for any speed within quite a wide range. On the whole, the water-wheel will be lower in price and more efficient when constructed for a moderately low speed. On the other hand, the weight and cost of the generator will decrease with increasing rated speed, up to fairly high speeds. Consequently the most economical compromise will usually be at some intermediate speed. At all but very low heads, therefore, the tendency is toward fairly high speeds, though of course the speeds are far lower than for steam turbine-driven generators. For 5,000- to 10,000-kv-a. water-wheel sets, speeds of some 300 to 400 rev. per min. are now often employed. An additional reason for not resorting to much higher speeds is the necessity for providing in the design sufficient mechanical strength to ensure that the machine will safely withstand speeds approaching double the normal speeds, in the event of accident to governors or gates; see Standardization Rules, Sec. 24. See also paper in Vol. XXXI of Trans.

A. I. E. E. by D. W. Mead entitled "The Runaway Speed of Water-wheels and its Effect on Connected Rotary Machinery." Water-wheel generators of both the horizontal and vertical types are built in sizes up to 15,000 kv-a. Fly-wheels are not usually supplied for water-wheel generating sets, owing to the uniform rotative effort of the water-wheels.

15. Steam turbine-driven types.* Extra high-speed generators are now almost invariably constructed with the cylindrical type of rotor carrying the field winding embedded in slots distributed over its surface. Owing to the great stresses associated with high peripheral speeds, the entire rotor (including the extensions constituting the shaft) is often made from a single piece of steel. In the largest machines, peripheral speeds of the order of 25,000 ft. per min. (127 meters per sec.) are employed. Bipolar 25-cycle designs are now under construction for rated loads of 25,000 and 30,000 kv-a. at a speed of 1,500 rev. per min. Bipolar 60-cycle designs for a speed of 3,600 rev. per min., have been built in sizes up to 5,000 kv-a.

It is characteristic of extra high-speed alternators of great capacity that since they are so exceedingly small for their output they can only be maintained at appropriately low temperatures when in operation, by circulating through them enormous quantities of air. For this purpose, and also in order to ensure reasonable absence of noise in their neighborhood, such generators are of the completely enclosed type with definite inlets and outlets for the circulating air. These subjects are discussed in Par. **129** to **139**.

ARMATURE WINDING

16. Considerations affecting the choice of armature winding for an alternator. (a) Efficient e.m.f. generation, *i.e.*, without considerable differential generation in series-connected conductors. This indicates coils whose pitch or span is about 180 deg. The pitch is on other accounts sometimes as low as 0.66 or even 0.50. (b) Wave shape. In most cases an approximate sine-wave is desired, which ordinarily means a distributed winding (several slots per pole per phase). It is not necessary to have a whole number of slots per pole per phase. In fact a fractional number is sometimes desirable, *e.g.*, a machine with 1½ slots per pole per phase will have as good a wave shape, other things being equal, as a machine with five slots per pole per phase. Wave shape also usually indicates a fractional-pitch winding (Par. **20**), *i.e.*, coils with a pitch less than 180 deg. (c) From the standpoint of heat dissipation, as well as of wave shape, the winding should be distributed rather than concentrated; this also reduces the leakage reactance slightly. On the other hand, much distribution means more insulation space and less slot space for copper, particularly in high-voltage machines. (d) From the standpoint of first cost it is important that the coils shall be wound and insulated before being placed in the machines, also that they shall be of one shape rather than of many shapes. This involves the necessity of open slots, although magnetic wedges are sometimes inserted after the winding is in place, in order to secure the advantages of closed or partly closed slots.

17. Classifications of armature windings: (a) coil-end classification (spiral or lap; one range, two range, three range or barrel); (b) according to the number of layers (one or two); (c) according to the pitch of the coils; (d) open slots (usually with form wound coils) or closed slots; (e) according to the number of slots per pole per phase; (f) according to the number of circuits in each phase; (g) according to the number of phases; (h) in three-phase windings, Y or Δ connection.

Diagrammatic illustrations of three-phase windings are shown in Fig. 14 which explains itself. All of these illustrations are for **single-layer wind-**

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* Kloss M. "Selection of Turbo-Alternators," *Journ.* Inst. Elec. Engrs., Vol. XLII, p. 156.

Stoney, G. and Law, A. H. "High-speed Electrical Machinery," *Journ.* I. E. E., Vol. XLI, p. 286.

Walker, Miles. "Design of Turbo Field Magnets for Alternating-current Generators," *Journ.* I. E. E., Vol. XLV, p. 319.

Smith, S. P. "Non-salient Pole Turbo-alternators," *Journ.* I. E. E., Vol. XLVII, p. 562.

Lamme, B. G. "High-speed Turbo-alternators," *Trans.* A. I. E. E., 1913, Vol. XXXII, p. 1.

ings, such that they can be wound with one coil-side per slot. It will be observed also that, in effect, they are all **full-pitch windings**, *i.e.*, that the two groups of active conductors belonging to any given phase and lying under adjacent poles are always 180 deg. apart, as groups, although some individual coils of the spiral windings have a pitch less than 180 deg.

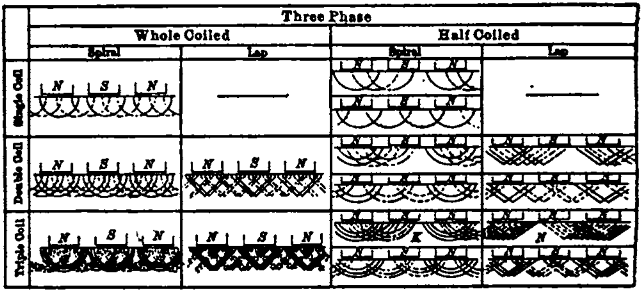


Fig. 14.—Three-phase windings.

18. A three-phase spiral winding with three slots per pole per phase is shown in Fig. 15, and a two-phase spiral winding with six slots per pole per phase in Fig. 16. For obvious reasons these are sometimes called **chain windings**. They are also **two-range windings**, *i.e.*, the coils extend outward in two ranges, or rows. Fig. 5 shows part of a **three-range winding**.

19. Two-layer lap winding. A more common type than any of the windings mentioned in Par. **18**, is the two-layer lap winding, shown in Fig. 17. This type has the advantage not only of a single shape of coil, but also that the coil pitch may be any whole number of slots, without disturbing the symmetry of the winding.

Fig. 15.—Three-phase, spiral, two-range winding.

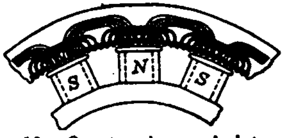


Fig. 16.—Quarter-phase, spiral, two-range winding.

20. Fractional-pitch winding. When the coil pitch is less than 180 deg., the winding is called a fractional-pitch winding. Fractional-pitch windings are much used not only because of their effect on the wave shape (Par. **25** et seq.), but also because of the saving in coil-end copper and in over-all length of machine. The gain is particularly noticeable on two-pole machines where fractional pitch is practically universal. It is also possible with this type of winding to have a fractional number of slots per pole, which tends to eliminate tooth harmonics from the e.m.f. wave. The end connections of the two-layer lap winding stand out in a single range, but because of its appearance and continuity it is often called a **barrel winding**. Figs. 17 and 18 show samples of three-phase, two-layer lap windings with full pitch and 50 per cent. pitch respectively. The between-coil connections and phase terminals, are shown for only one phase. All things considered, the two-layer lap winding is the most flexible and generally useful type. In very high-voltage machines, the necessity for insulation between the two layers is an objection, particularly with fractional pitch, when there is a relatively large potential difference between the two coil-sides in a slot.

21. Fractional pitch single-layer lap winding. It is also possible to have fractional-pitch coils with a one-layer lap winding, but here the number of combinations is quite limited, see Figs. 19 and 20. Inter-coil connections are shown for only one phase. Fractional-pitch one-layer polyphase windings have the serious fault that in a certain portion of the slots, the phases alternate with one another, causing a very unfavorable distribution of the m.m.f. of armature reaction. Thus with three phases A, B, and C, four slots per pole per phase, and 75 per cent. pitch, the distribution of the phases in successive slots is as follows: *ACAABABBCBCCACAABABB*, etc. When each slot contains conductors of only one phase, the m.m.f. distribution is very unfavorable, but with a two-layer winding with four slots per pole per phase, and the same coil pitch, we obtain the following arrangement:

$$\left\{\begin{matrix} AAAABBBBCCCCAA \\ CCCAAAABBBBCCC \end{matrix}\right\} \text{etc.}$$

This arrangement provides a nearly sinusoidal distribution of the m.m.f.

22. Multi-circuit windings. Another classification of windings is according to the number of similar circuits in each phase; *e.g.*, a two-circuit 220-volt alternator could be changed to a single circuit 440-volt machine by series-connecting the two parallel circuits.

In three-phase alternators there is still another distinction, namely, between Y and Δ connection of the three phases.

E.M.F. GENERATION

23. Electromagnetic induction. The law of electromagnetically induced e.m.f. may be stated in two ways: (a) whenever a conductor cuts a magnetic flux, an e.m.f. is induced which is proportional to the rate of cutting, *i.e.*, to the flux cut per sec.; (b) whenever the flux linked with a loop or coil of wire, changes for any reason, an e.m.f. is induced in the loop or coil, proportional to the rate of change of the flux and to the number of turns in the coil. For all ordinary cases, statements (a) and (b) are exactly equivalent. For the consideration of alternator electromotive forces, statement (a) will ordinarily be found the more convenient.

24. E.m.f. formulæ. Let l = gross length of armature core, inches. v = peripheral velocity in ft. per sec. $\mathfrak{B}_1$ = maximum gap density (maxwells per sq. in.) of equivalent sine-wave. e'' = root-mean-square volts per in. of active conductor. N = series-connected active conductors per phase (twice the number of turns per phase). E = volts per phase. k_b and k_p are the differential factors, Par. 28 and 29.
Then

$$e'' = \frac{12}{\sqrt{2}} v \mathfrak{B}_1 10^{-8} = 8.5 v \mathfrak{B}_1 10^{-8} \qquad \text{(volts)} \qquad (1)$$

and

$$E = k_b k_p e'' N l = 8.5 k_b k_p v \mathfrak{B}_1 N l 10^{-8} \qquad \text{(volts)} \qquad (2)$$

Another very common variety of the e.m.f. formula is obtained as follows: let Φ = flux per pole in maxwells; then the average e.m.f. per active conductor is $2f\Phi 10^{-8}$, and neglecting differential action, the average e.m.f. per phase is $2f\Phi N 10^{-8}$. To obtain the root-mean-square volts and at the same time to take account of the differential action, involves the form factor (k_f = ratio of root-mean-square to average volts), and the differential factors, giving for the induced volts

$$E = 2k_f k_b k_p f \Phi N 10^{-8} \qquad (3)$$

The three k's are usually combined in a single term K which has an average value of 1.05 for full-pitch windings.

25. E.m.f. wave shape. Assuming constant speed, the e.m.f. generated by a single active armature conductor, or by a group of series-connected conductors making up a coil side and lying in a single slot, is proportional at each instant to the density of the magnetic flux through which it is cutting, and when plotted in rectangular coordinates will have the same shape as the "**field form,**" *i.e.*, the curve showing the peripheral distribution of flux entering the armature from the field poles (see Par. 32 and 33). This will be referred to as the elementary or slot e.m.f. It is an alternating e.m.f.

but usually not sinusoidal, see Fig. 21. The no-load wave-shape of the machine is then obtained by adding together the e.m.fs. of the series-connected coil sides. This is very tedious as it involves the point-by-point addition of several dephased non-sinusoidal waves. An approximation of some kind is usually employed. The most satisfactory method of handling this problem is as follows: Analyze the field form or slot e.m.f. into its fundamental and harmonics; compound or add vectorially in their proper phase relation, the fundamentals of the several series-connected conductors or coil-sides, to obtain the fundamental of the resultant e.m.f.; and compound similarly the harmonics of each order to obtain the resultant harmonic of that order. To this end the armature conductors are grouped as follows.

26. Similars. Any number of conductors or coil sides, 180 magnetic degrees apart (generally called similars), connected in series, alternating right and left across the face of the armature, will yield an e.m.f. wave of the same shape as that of the field form or slot e.m.f., the result being simply the product of the e.m.f. of a single conductor or coil-side, by the number in series.

27. Phase-belt. In nearly all alternators the conductors of a single phase are not confined to one slot per pole, but are distributed in several slots. The group of conductors belonging to one phase and corresponding to one pole, will be referred to as a phase belt. When the number of slots per pole per phase is a whole number, all the phase belts of a given phase are similars, and the phase e.m.f. will have the same shape as that of a single belt. In a 2-layer winding the group of conductors for one pole and phase may be more conveniently separated into the top-of-slot belt and the bottom-of-slot belt, particularly in the case of fractional pitch windings where these two belts are displaced circumferentially. See Fig. 78 and 79, pages 499 and 500.

28. Belt differential factor. In general the resultant or vector sum of the e.m.fs. of the several coil-sides in a given belt will be less than their numerical sum, owing to their phase differences. The ratio of the vector sum to the numerical sum will be called the differential factor, which is always equal to or less than one. In the case of differential action within a single-phase belt, the differential factor will be called the **belt differential factor,** k_b. Remembering that a fundamental phase difference of β deg. means an nth harmonic phase difference of $n\beta$ deg., it is easy to see that a fundamental phase difference which affords a relatively large differential factor for the fundamental, may give a very small differential factor for one or more of the harmonics. Thus, in a three-phase alternator with two slots per phase per pole, or six slots per pole, the phase difference between two adjacent slots is 30 deg. for the fundamental, 5×30 deg. $=150$ deg. for the fifth harmonic, etc. The belt differential factors will then be $k_{b1} = \cos 15$ deg. $= 0.966$, $k_{b5} = \cos 75$ deg. $= 0.259$, $k_{b7} = -0.259$, etc. Thus the 5th and 7th harmonics are greatly reduced without appreciably reducing the fundamental. Differential factors for the fundamental and for the various odd* harmonics up to the 27th, for 60-deg. and 90-deg. belts and for various numbers of slots per pole are given in Table I, Par. 31. Differential factors for a 120-deg. belt may be obtained by multiplying those of the 60-deg. belt (same N_{sp}) by the 66⅔ per cent. pitch differential factors, Fig. 22.

29. Pitch differential factor. When the coil pitch differs from the pole pitch, the e.m.fs. developed in the two sides of a single coil, or in the two series-connected belts of a phase-group of coils, will differ in phase by an angle β, which is the angle (in magnetic degrees) by which the coil pitch differs from the pole pitch. This introduces another differential factor $\left(\cos\dfrac{\beta}{2}\right)$ for the fundamental and $\cos\dfrac{m\beta}{2}$ for the mth harmonic, which will be called the pitch differential factor. Its values are plotted in Fig. 22.

30. Phase differential factor. The addition of the two e.m.fs. of two phases of an alternator, involves differential action of exactly the same kind as that between the two series-connected belts of a fractional-pitch winding (Par. 29). In a three-phase Y-connected machine, the angle is 60 deg., which is equivalent to a ⅔ pitch; and the phase differential factor for the fundamental and for all the odd harmonics not a multiple of three, is 0.866; while for the harmonics which are multiples of 3, it is zero (see Fig. 22).

* Even harmonics are not present in any appreciable degree in the e.m.f. waves of commercial alternators.

By a suitable combination of belt, pitch, and phase-differential action, any harmonics present in the field form may be practically ironed out of the terminal e.m.f. leaving only a sine-wave which is the resultant of the fundamentals of the e.m.fs. in all the series connected conductors.

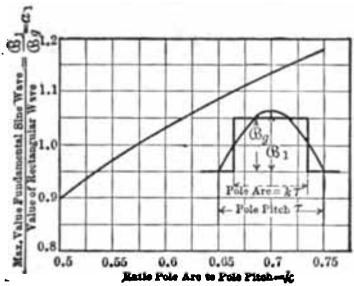


FIG. 23.

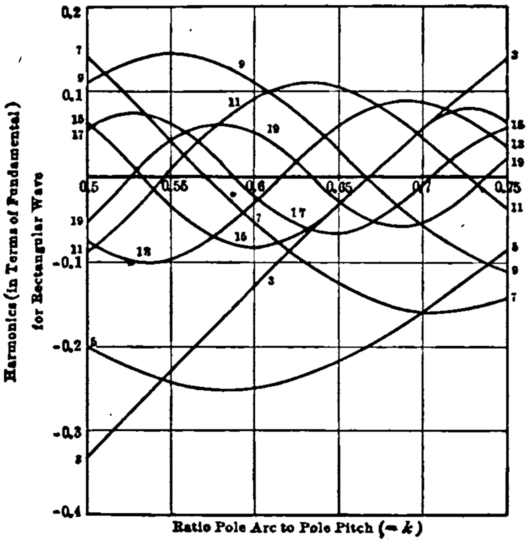


FIG. 24.—Harmonics of rectangular field forms (in terms of fundamental) plotted as a function of ratio of pole arc to pitch.

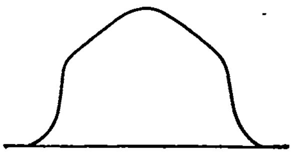


FIG. 25.—Observed field form.

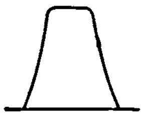


FIG. 26.—Observed field form.

centre of each pole could be filled with iron if desired. Neglecting the effect of slot openings and assuming that the current is distributed uniformly over each belt, the magnetic potential differences (ampere-turns) across the air gap, and therefore the field form, is shown in Fig. 29.

A simple manner in which to obtain the equivalent sine-wave of such a field form is as follows: Any such distributed field winding may be considered as made up of pairs of conductors, the two in each pair being 180 deg. apart. Consider the m.m.f. of a single pair. The field form of the pair if acting alone will be a rectangle, Fig. 30, of which the fundamental sine-wave has a maximum value, $A_1 = 4A \div \pi$. The 3rd harmonic has $\frac{1}{3}$ of this amplitude, the 5th, $\frac{1}{5}$, etc. Assuming that the resultant of the overlapping rectangular m.m.f. and flux distributions of the several pairs of conductors into which the winding has been artificially subdivided, is a curve whose harmonics are small and easily ironed out of the e.m.f. by the differential actions of the winding (*i.e.*, assuming a final sine-wave of e.m.f.); it is obviously unnecessary to consider anything but the *fundamental* of each rectangular field

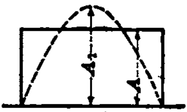


Fig. 30.—Field form of single 180-deg. loop, with its fundamental.

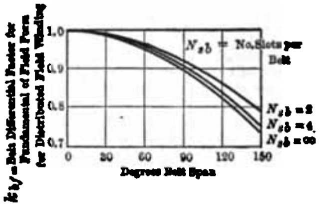


Fig. 31.—Belt differential factor for m.m.f. of distributed field windings.

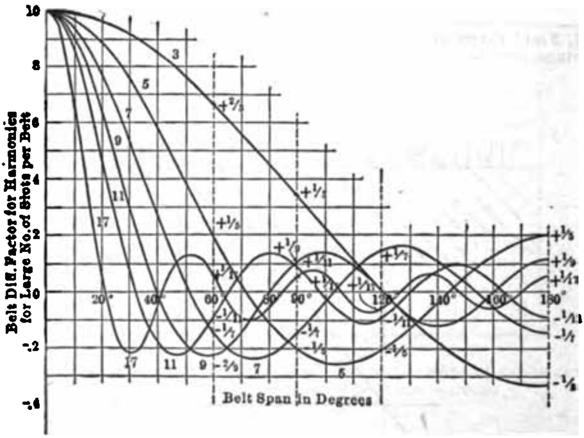


Fig. 32.—Curves of belt differential factors for harmonics vs. belt span.

form (Fig. 30), and to compound or add together the various fundamentals, which can be done geometrically, exactly as with the fundamentals of the e.m.fs. in the several conductors. It is thus possible to use the same differential factors as for the e.m.fs., except that the belt of conductors or of pairs of conductors is no longer confined to the definite widths of 60 deg., 90 deg., or 120 deg., although usually within these limits. The belt differential factor k_{bf} for the fundamental, is given in Fig. 31 for various numbers qualities of the iron may not seriously affect the saturation curve, unless very high densities are used, since the reluctance of the iron portion is small as compared with the air-gap reluctance.

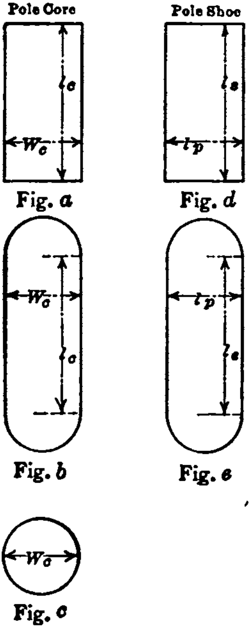


FIG. 34.—(a, b, c, d and e).—Diagrams for field leakage calculations.

The reluctance of the magnetic circuit is ordinarily divided into five parts, air-gap, armature teeth, armature core, pole core, and yoke. For a given no-load terminal voltage (E), the flux per pole (Φ) entering the armature, is determined by the equation $\Phi = (E \times 10^8)/(2k_f k_b k_p f N)$. See Par. 24. To obtain the flux Φ_c in the pole core, the field leakage must be added, or Φ must be multiplied by the leakage coefficient ν.

36. Field leakage between salient poles. The leakage coefficient (ν) for salient poles is defined as the ratio of the maximum flux (Φ_c) in the pole core, to the flux (Φ) entering the armature from one pole. The difference between them is the leakage flux (Φ_l).

$$\Phi_c = \Phi + \Phi_l \text{ and } \nu = \Phi_c/\Phi = (\Phi + \Phi_l)/\Phi = 1 + (\Phi_l/\Phi)$$

This leakage flux can be divided into two parts, that (Φ_{lc}) which leaks be-

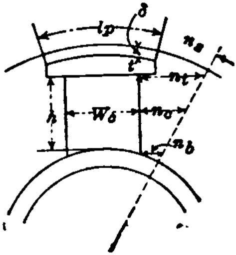


FIG. 35.—For field leakage calculations.

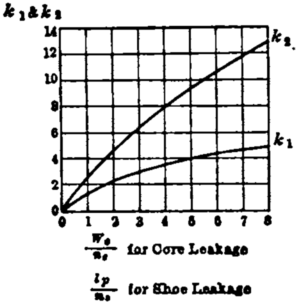


FIG. 36.—Curves of constants k_1 and k_2 employed in field leakage calculations.

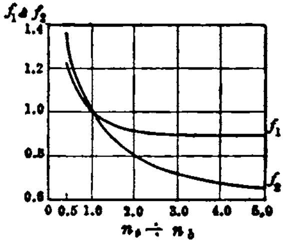


FIG. 37.—Curves of constants f_1 and f_2 employed in field leakage calculations.

length of path for this density. A considerable error in this item is evidently a very small part of the whole number of ampere-turns per pole.

For a non-salient pole machine the ampere-turns for the rotor can best be determined by finding the density midway between the poles, as for the armature core, and approximating to an equivalent length of path for this density. The rotor teeth may be treated exactly as the armature teeth.

A curve of terminal volts plotted against the necessary ampere-turns for the iron part of the magnetic circuit is called an **iron saturation curve.**

40. Ampere-turns for air-gap. If the armature were smooth-cored with no air-ducts, the cross-section of the air-gap would be the area of the pole face plus an allowance for fringing of the flux at the edges, and the actual density would be that determined by dividing the total flux per pole by this cross-section. This will be called the mean pole-face density. Since the flux is crowded into the teeth as it enters the armature, the density which must be used in computing the necessary ampere-turns is somewhat higher than the mean pole-face density.

The flux does not all enter the tips of the teeth, but fringes from the sides of the teeth, having the effect of increasing the width of the tooth tip (w_{tt}). This equivalent tooth-tip is given by the expression $w_{tt} + 2\delta c_f$ where c_f is the fringing constant and δ the length of the air-gap. Values of c_f as a function of w_{so}/δ (the width of slot opening divided by the air-gap length) are shown in Fig. 39. If τ_t is the tooth-pitch, $a_a = (w_{tt} + 2\delta c_f)/\tau_t$ is the fractional part of the tooth-pitch, which is effective as a flux path, or the fractional equivalent tooth-tip. Fig. 40 gives values of a for various values of $k_{so} = w_{so}/\delta$ and $q_{so} = w_{so}/\tau_t$.

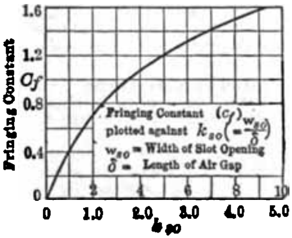


FIG. 39.—Curve for calculating air-gap reluctance.

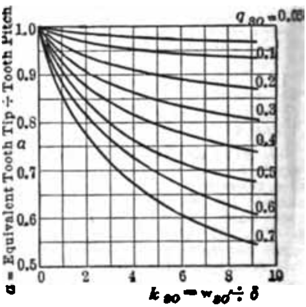


FIG. 40.—Slot contraction factor.

The peripheral-contraction factor K_1, or the ratio of effective peripheral surface to actual peripheral surface is equal to a_a for a salient-pole machine, since there is contraction only on the armature side of the air-gap. For a slot-wound field one must determine the contraction factor both for the armature (a_a) and for the field (a_f). In this case $K_1 = a_a a_f$.

The equivalent length of the armature core (l_e) is given by the expression $l_e = l + \delta - N_d(w_d - 2c_f\delta)$ where c_f is the fringing constant for the air-ducts. N_d = number of ducts and w_d = width of each duct. The longitudinal-contraction factor (K_d) is the fractional-equivalent armature length. It is $K_d = (l_e/l)$. The corrected gap density $\mathfrak{B}_{ge}$, upon which we must base our calculation of ampere-turns for the air-gap, is $\mathfrak{B}_{ge} = \mathfrak{B}_g/(K_1 K_d)$, where $\mathfrak{B}_g$ is the mean density where the gap is shortest.

As explained in Par. **32** $\mathfrak{B}_g = \mathfrak{B}_1/a_1$, where $\mathfrak{B}_1$ is the maximum value of the equivalent fundamental sine-wave of flux under the pole, and a_1 is a constant depending upon the ratio of pole arc to pole pitch. Therefore $\mathfrak{B}_{ge} = \mathfrak{B}_g/(K_1K_d) = \mathfrak{B}_1/(a_1K_1K_d)$, and the necessary ampere-turns Ni_g for the gap are given by the equation $Ni_g = 0.313\delta\mathfrak{B}_{ge}$. Adding the value of Ni_g to the number of ampere-turns per pole necessary to drive the flux through the iron parts of the magnetic circuit, we have the total number of ampere-turns per pole corresponding to the total flux Φ, and therefore corresponding to the no-load voltage with which we began. This gives, therefore, one point on the no-load saturation curve. By taking values of voltage between

44. Armature reaction. The magnetic p.d. across the gap due to the armature m.m.f. may be considered as made up of two parts: (a) A sinusoidally distributed p.d. stationary with respect to the field poles; and (b) pulsating kinks due to the distribution of the armature current in belts and to its localization in slots. (b) is larger, the smaller the number of phases. Neglecting (b), the maximum or crest value of this equivalent sinusoidally distributed m.m.f. is

$$A_1 = 0.45\ k_b k_p \Delta \tau \qquad \text{(amp-turns)} \qquad (6)$$

where Δ = r.m.s. ampere conductors per in. of armature periphery, τ = the pole pitch in inches; k_b = the belt differential factor, see Table I; Par. 31; and k_p = the pitch differential factor, see Fig. 22.

If the field m.m.f. were also sinusoidally distributed along the air-gap, it would be an easy matter to compound or add vectorially these two m.m.fs., in order to obtain their resultant. This is nearly enough the case for alternators with distributed field windings. In the case of salient-pole machines, the m.m.f. across the gap at no-load is practically constant at all points of the pole face, but in order to simplify their treatment, the equivalent sinusoidal m.m.f. distributions will be considered. This method results in considerable errors in some cases. The crest value of this equivalent sinusoidal distribution is

$$F_1 = a_1 F_g \qquad \text{(amp-turns)} \qquad (7)$$

where a_1 (Par. 32 and 33, Fig. 30) is about 1.27 and F_g is the constant magnetic potential difference (in ampere-turns) between armature core and pole core. With narrow or chamfered pole faces, a_1 will be slightly less. $F_g = K_2 F$, where F is the total field ampere-turns per pole and K_2 a constant varying in different machines and with different fluxes. A rough average value of K_2 is 0.8, although in extreme cases it differs considerably from this value.

Eq. (6) holds for any number of phases; but for the case of the *single-phase* alternator, the alternating armature m.m.f. must be replaced by two half-value revolving m.m.fs., one forward and one backward with respect to the armature; or one stationary and the other at double frequency with respect to the field. Eq. 6 gives only that component which is stationary with respect to the field. The other component induces double frequency e.m.fs. and currents in the field coil, and reactive e.m.fs. in the armature. (In the balanced polyphase case the backward revolving components of the several phases cancel or neutralize each other.)

45. Leakage reactance is generally supposed to represent that part of the flux linked only with the armature conductors, but as a matter of fact the only such flux is some of that linked with the coil ends, since the whole effect of the armature m.m.f. in the vicinity of the air-gap is merely to distort the main flux. However, owing to the localization of the current in slots surrounded by magnetic material which moves with the slot currents, and owing to the grouping of the slots in phase belts which move relatively to the field, there are local flux pulsations which are best treated as if separate from the main flux, although as a matter of fact they are only pulsating distortions of the latter. Armature leakage reactance of an alternator is about as difficult to define as to compute. For most practical purposes the following method will give satisfactory results.

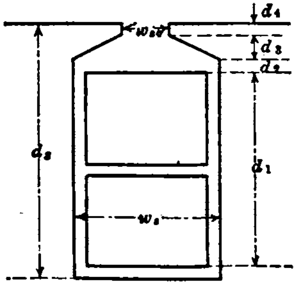


FIG. 42.—Slot section.

46. Slot leakage is the cross-slot flux, computed as if independent of the main flux. Referring to Fig. 42 the "inch permeance," or the flux linkage per amp-inch of slot, is

$$\varphi_s = 3.2\left(\frac{1}{3}\frac{d_1}{w_s} + \frac{d_2}{w_s} + \frac{2d_3}{w_s + w_{so}} + \frac{d_4}{w_{so}}\right) \qquad \text{(maxwells)} \qquad (8)$$

and the corresponding or slot reactance per phase is

$$x_s = 2\pi f\ k_p^2\ lN\ N_{cs}\ \varphi_s\ 10^{-8} \qquad \text{(ohms)} \qquad (9)$$

For very hurried computations φ_s is sometimes approximated without computation; it varies from 8 with shallow open slots to 15 with deep nearly closed slots.

49. Coil-end leakage. This computation is simplified by the fact tha φ_f, the flux per ampere-inch of outside phase-belt-bundle, is nearly a constan quantity for full-pitch windings. Its values for all pitches of working range are given in Fig. 44.

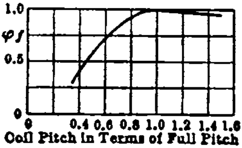


Fig. 44.—Curve of values of flux per ampere-inch for coil ends.

The coil-end reactance in ohms is then—

$$x_f = 2\pi f \varphi_f l_f \frac{N^2}{2p} 10^{-8} \quad \text{(ohms)} \qquad (17)$$

where l_f = the length of a single coil end, or the free length corresponding to one active conductor, N the active conductors per phase, and $2p$ the number of poles.

The coil-end *reactance ratio* (Par. 46) is—

$$q_{xf} = 1.48\, k_f \varphi_f \frac{k_s}{k_b k_p} \frac{\Delta}{\mathfrak{B}_1} \qquad (18)$$

where k_f ($= l_f/\tau$) is about 1.5 times the per cent. coil-pitch expressed as a fraction (somewhat larger for very high-voltage machines), and $k_s = \tau/l$.

50. Total leakage reactance.

$$x = x_s + x_f = 2\pi f N 10^{-8}\left(k_p^2 l N_{sp} \varphi_s + \frac{N}{2p} l_f \varphi_f\right) \quad \text{(ohms)} \qquad (19)$$

$$q_x = qx_s + qx_f = \frac{\Delta}{\mathfrak{B}_1}\left(4.44 \frac{\varphi_s}{N_{sp}} \frac{k_p}{k_b} + \frac{1.48\, k_f}{k_b k_p} \varphi_f k_s\right) \qquad (20)$$

51. Alternator vector diagram. Assume a sinusoidal peripheral distribution of m.m.f. on both armature and field and a uniform peripheral reluctance. (This assumption is warranted in non-salient pole machines, but gives rise to considerable errors in some salient pole machines.) The resultant m.m.f. and gap-flux distribution will then also be sinusoidal.

Referring to Fig. 45, designate the flux by Φ, the corresponding reluctance a.t. across gap by R_1, the armature amp-turns by A_1 (see Par. 44), and total a.t. across the gap by F_1 all sinusoidally distributed. These are all space vectors on the assumption that the field rotates counter-clockwise with respect to the armature, R_1 being the resultant of F_1 and A_1. In salient-pole machines substitute F (the full field amp. turns $\times$ 1.27) for F_1 and the corresponding R for R_1. This partly balances the error due to the unsymmetrical peripheral reluctance.

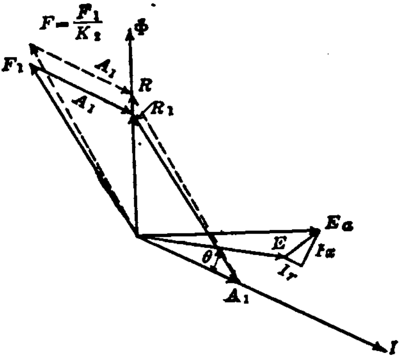


Fig. 45.—*General* alternator diagram for non-salient poles (full lines). Salient poles (broken lines).

Considering Φ as a time vector, the induced e.m.f. E_a will lag 90 deg. Subtracting Ir and Ix from E_a gives the terminal voltage E, where θ is the load phase angle. Hereinafter this diagram will be called the *general* alternator diagram.

52. Application of alternator diagram to the regulation problem. Given the winding data of armature and field, the armature resistance and leakage reactance, and the saturation curve, the excitation and regulation for any load and power-factor may be obtained as follows:

factor saturation curve will then be obtained by drawing lines from E_a equal and parallel to $\overline{cb}$, as shown. The fact that Ir is neglected is not significant, since it is in this case at right angles to E and does not affect the result appreciably. This curve may also be determined experimentally.

56. Load saturation curves at other power-factors. If the load saturation curve at other power-factors could be obtained, it is obvious that the regulation and excitation for any power-factor and terminal voltage could be obtained directly therefrom. A semi-empirical method of obtaining these curves, recommended in the latest edition of the Standardization Rules of the A. I. E. E. is given in Sec. 24.

57. E.m.f. method. This method practically assumes a field flux wholly dependent on the field m.m.f., as given by the saturation curve, and that all the drop is due to an internal impedance which is commonly called the **synchronous impedance**, z_s, of which the **synchronous reactance** is x_s. Fig. 49 shows saturation and short-circuit characteristics with the derived synchronous impedance curve, the ordinates of the latter being the ratios of corresponding ordinates of the other two.

58. The m.m.f. method. This method is best understood by reference to Fig. 50, where the general diagram is dotted in. Starting with E and I, add Ir to E to get E', which is assumed to be the total induced e.m.f., corresponding to F as taken from the saturation curve. From R' subtract the m.m.f. A' taken from the short-circuit characteristic corresponding to the particular current in

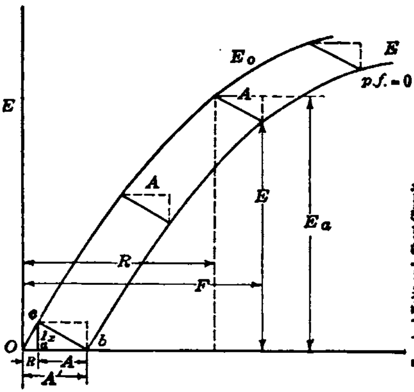


Fig. 48.—No-load saturation curve and zero-power-factor load saturation curve.

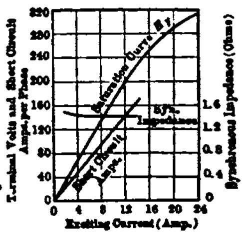


Fig. 49.—Alternator characteristic curves.

question. A' is the F of Figs. 47 and 51 and includes with the armature *m.m.f.* A, an additional *m.m.f.* $\overline{AA'}$ (Fig 50), which is practically equivalent to R of Fig. 47, and which is the approximate m.m.f. equivalent of Ix. Under short-circuit conditions, $E' = Ir$ and the diagram reduces to Fig. 51, whence it is obvious that the field ampere-turns at short-circuit practically corresponds with what we have called A'.

59. Excitation characteristics. Curves showing the relation of excitation (for constant terminal voltage) to the load current, at various power-factors, are shown in Fig. 52 for a 1,600 kv-a. slow-speed alternator with close regulation, and in Fig. 53 for a 6,250 kv-a. turbo-alternator with poor regulation. These were computed by the m.m.f. method.

60. Relation of regulation to per cent. armature strength and length of air gap. A study of Figs. 45 and 50 will show that for a given current and power-factor, the regulation depends upon the following ratios

$$Ir/E_a = q_r;\ Ix/E_a = q_x \text{(see Eq. 20)};\ \frac{A}{R} = q_A$$

Of these four ratios, q_r varies from less than 0.005, in large turbo-alternators, to 0.02 or more in small slow-speed machines; q_e from 0.04 to 0.15 or more; q_A from 1.00 to 0.35; and k_{sf} from 1.5 to 2.0. These last two are the dominant factors in regulation.

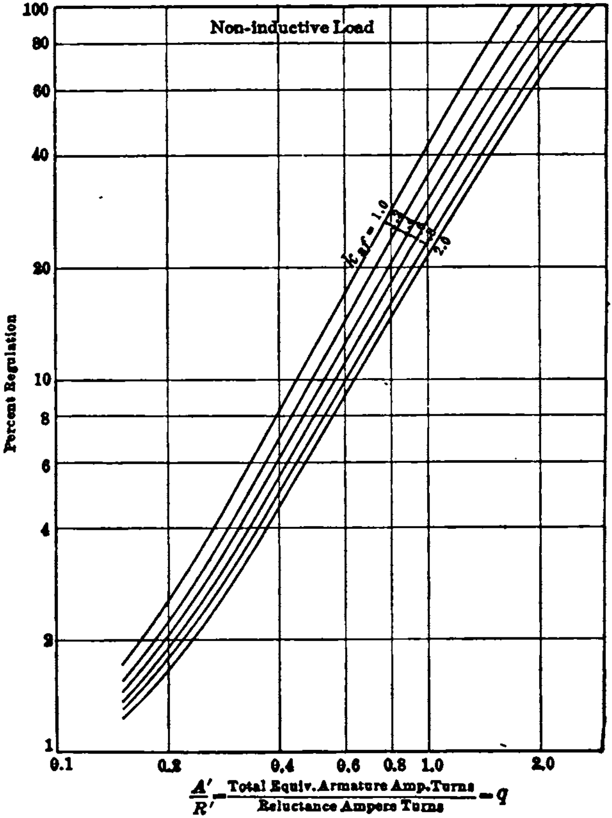


Fig. 54.—Per cent. regulation plotted in terms of per cent. armature reaction.

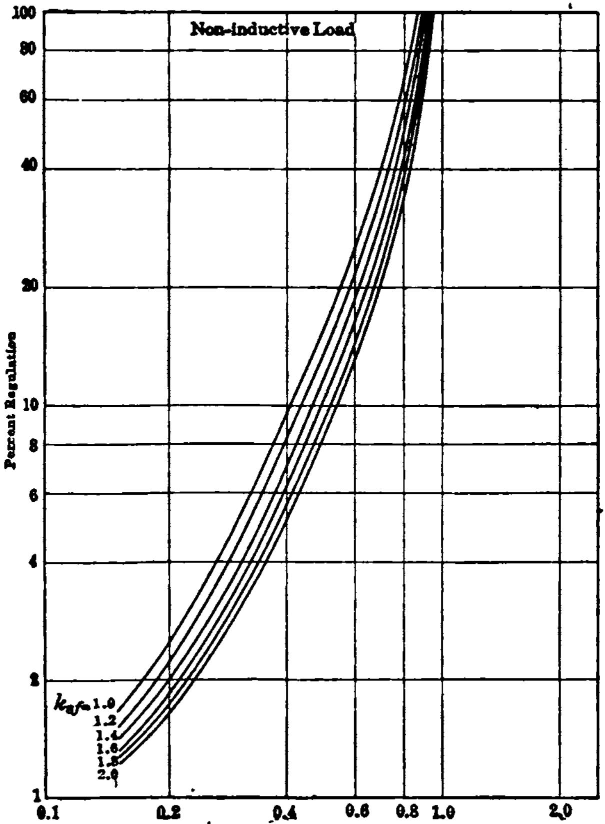
Since (approximately) $R = 0.313\mathfrak{B}_1\delta/(K_1K_dK_2)$ (see Par. 38 and 40),

$$q_A = (A/R) = (1.43k_bk_pK_1K_dK_2\Delta_T)/(\mathfrak{B}_1\delta) \quad (21)$$

or taking average values for the K's,

$$q_A = 0.9(\Delta_T)/(\mathfrak{B}_1\delta) \quad (22)$$

Thus assuming $\mathfrak{B}_1$ and Δ to be fixed for any given case, the regulation at a given power-factor is largely dependent upon the ratio of pole pitch to air gap and upon the saturation factor.



$$\frac{A'}{F} = \frac{\text{Total Equiv. Armature Amp. Turns}}{\text{Field Ampere Turns}} = q'$$

FIG. 55.—Per cent. regulation plotted as a function of $\frac{A'}{F}$.

From the Alternator Diagram (Fig. 45), with some approximations we get, for non-inductive load,

$$\text{Reg.} = \frac{1}{1-q_r}\left[\sqrt{1+q_x^2}\left(1-\frac{1}{k_{af}}\right)+\frac{\sqrt{1+q'^2}}{k_{af}}\right]-1 \text{ (roughly approx.)} \quad (23)$$

Substituting average values for large machines, $q_r = 0.005$, $q_s = 0.07$.

$$\text{Reg.} = 1.005\left(0.007 + \frac{\sqrt{1+q^2} - 1.0024}{k_{sf}}\right) \quad \text{(roughly appox.)} \qquad (24)$$

or in terms of $q' = A' + F$;

$$\text{Reg.} = 1.005\left(0.007 + \frac{1+\sqrt{1-q'^2} - 1.0024}{k_{sf}}\right) \quad \text{(roughly approx.)} \qquad (25)$$

Figs. 54 and 55 are plotted from Eqs. 24 and 25. For small low-speed machines where q_r and q_s are larger than here assumed, the curves will lie a little higher at their lower ends.

Thus good regulation means large R' and long air gap, longer in proportion to Δ and τ, *i.e.*, very long for large high-speed machines such as turbo-alternators. In fact, to obtain a regulation of even 10 per cent. in large turbo-alternators would necessitate a tremendous air gap and more field copper than there is room for.

61. Value of close regulation. With automatic voltage regulators such as now employed, close regulation is no longer necessary even in small plants. Moreover large low-reactance machines are dangerous in case of sudden short-circuits (Par. 62 to 64). If, however, the ratio of armature ampere-turns to field ampere-turns is too high, the gap flux distortion in a salient-pole machine may be sufficient to seriously distort the e.m.f. wave. This does not apply with equal force to non-salient-pole machines, which include most of the large high-speed alternators, where large q_A is commercially necessary.

62. Short-circuits. If the external impedance of an alternator be gradually reduced to zero, with full-load excitation and normal speed maintained, the short-circuit current, in terms of full-load current, will be $F/A' = 1 + q'$ (see Figs. 50 and 51), and correspondingly less at no-load excitation. If an alternator with a regulation of 5 per cent. and a saturation factor of 1.6, is short-circuited with full-load excitation, the short-circuit current will be about 3 times full-load current. At no-load excitation the short-circuit current will be 2.7 times full-load current. For an alternator with 20 per cent. regulation and a saturation factor of 1.6, the short-circuit current will be 1.56 for full-load excitation and 1.18 with no-load excitation.

63. Sudden short-circuits. Since on short-circuit, most of the field m.m.f. is consumed in balancing the armature m.m.f., the net m.m.f. and the flux are greatly reduced. If the short-circuit is applied suddenly, the sudden decrease in flux induces a large e.m.f. and current in the exciting winding, which tries to keep the flux from changing, and as the electromagnetic inertia and the time constant of the combined circuits are large, an appreciable time elapses before the field is destroyed. Meanwhile the opposing m.m.fs. of armature and field rise until nearly all the field flux is shunted across between field and armature. To do this requires an m.m.f. as many times greater than that at full-load, as the full-load field flux is greater than the total full-load leakage flux (field and armature). Thus if $\nu = 1.12$ at full-load zero p.f., and $q_s = 0.10$ (salient pole slow-speed machine), the r.m.s. armature amperes will rise temporarily to nearly 5 times its full-load value, and owing to the fact that one of the phase currents will be boosted up above its zero axis, because of the position of that phase at the instant of short-circuit, the maximum instantaneous current in one phase may be nearly double the maximum value of the above r.m.s. current.

In non-salient pole turbo-alternators ν may be as low as 1.04, q_s as low as 0.04, the r.m.s. current nearly 12 times full load, and the instantaneous maximum more than 20 times normal. This means mechanical stresses on coil ends more than 400 times normal, which is more than they can readily be made to withstand.

64. Current-limiting reactances. As the total leakage $(\nu - 1 + q_s)$, should not be less than 0.15, external reactances must in some cases be supplied to protect against sudden short-circuits. See Par. 140.

65. Rating of alternators. As the armature current, irrespective of power-factor, determines the armature copper loss, it is customary to rate alternators in kv-a. rather than in kw. In fact at low power-factors, and therefore at smaller values of delivered power, but with a given current and

The power (P_2) transformed is a function of φ, the coupling angle; it is

$$P_2 = \frac{E_2^2}{z_s}\cos\theta - \frac{E_1E_2}{z_s}\cos(\theta-\varphi) \qquad \text{(watts)} \qquad (26)$$

The significance of the negative sign is that the electrical power is negative, *i.e.*, received rather than delivered. In Fig. 57, $-P_2$ is plotted against φ; whence it appears that as the motor lags (not in speed but in phase) behind the supply e.m.f., the output increases to a maximum, then decreases, becomes negative and returns to zero. As soon as φ has opened out to a point such that P_2 is sufficient to carry the load, it will hang at that point as if the motor were driven by an elastic coupling. If the load increases, φ will increase accordingly, until the maximum point is passed, when the motor will break down. During a little more than a half of each slip cycle (see Fig. 57), the motor is acting as a generator, since $-P_2$ is positive, and is in series short-circuit with the line. So much of the kinetic energy of momentum is lost during this half of the slip cycle that a synchronous motor will practically never pull back into step after once breaking, even if the mechanical load be immediately removed.

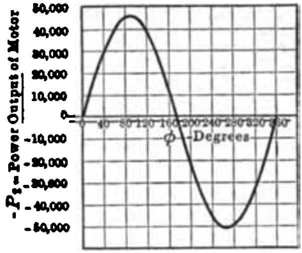


FIG. 57.—Synchronous motor output vs. coupling angle.

For visualizing these relationships and their causes the Blondel diagram is by far the simplest device.

70. Blondel diagram. Referring to Fig. 58, the isosceles triangle $\overline{OAC}$ is constructed on the base E_1 with base angles, $\theta = \tan^{-1}(x_s/r)$. It can easily be shown that with constant impressed voltage E_1, and constant load,

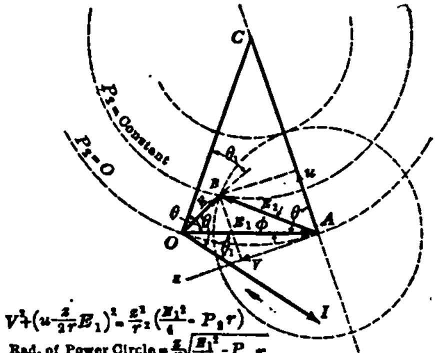


FIG. 58.—Blondel diagram for synchronous motor.

P_2, the locus of the point B, as the excitation and E_2 vary, is a circle with centre at C and with radius $\sqrt{(E_1/2)^2 - P_2 r} + \cos\theta$. If $P_2 = O$, the radius is $(E_1 + 2\cos\theta) = \overline{CA}$, and the circle passes through O and A. As P_2 increases, the radius decreases until it becomes zero, which corresponds to the maximum possible output with the given motor at the given impressed e.m.f. But this theoretical limit is never reached practically, because of the very large value of E_2 required to reach up to the high-power circles of small radii. This will be better appreciated by reference to the diagram of Fig. 59, where the proportions are more normal and the point C is far off the page.

For varying load and constant excitation, B moves along a circle about A, clockwise for increasing and counter-clockwise for decreasing loads. The maximum load for any given excitation occurs when B falls on $\overline{AC}$; beyond that B swings around on to circles of smaller power, and the motor breaks down; *i.e.*, the decrease of $\cos \theta_1$ more than balances the increase in I.

71. Interpretation of Blondel diagram. In Fig. 58 the area inside of the zero-power circle corresponds to motor power, and that outside to generator power; the area to the left of $\overline{AC}$ corresponds to stable operation and that to the right to unstable operation; the area within the zero-power circle and between $\overline{AC}$ and $\overline{OC}$ corresponds to stable motor operation with a lagging current, and that to the left of $\overline{OC}$ to stable motor operation with leading current. The point B is not only the extremity of the E_2 vector, but may be used also as the outer end of the current vector, since E is proportional to I, and angle $\overline{COB} = \theta_1$. Thus when B falls on $\overline{OC}$, I is in phase with E_1 and I is a minimum for that particular load. With normal steady load conditions, the angle ϕ never approaches the angle θ, *i.e.*, the point B never

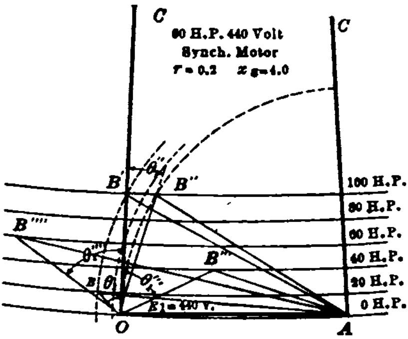


FIG. 59.—Portion of Blondel diagram drawn to scale.

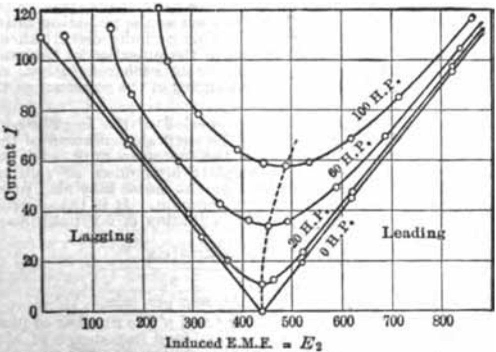


FIG. 60.—V-curves for 60-h.p. synchronous motor.

approaches the breakdown point on $\overline{AC}$ (see Fig. 59); but if either the load or the frequency of supply pulsates or changes suddenly, a hunting may be set up which will carry B over into the region of unstable operation.

72. Excitation and power-factor curves. The familiar "V" curves of current vs. excitation or induced e.m.f. may be easily obtained from the Blondel diagram. This has been done for a 60-h.p. motor, and the result is shown in Fig. 60. Power-factor curves similarly obtained, are shown in Fig. 61.

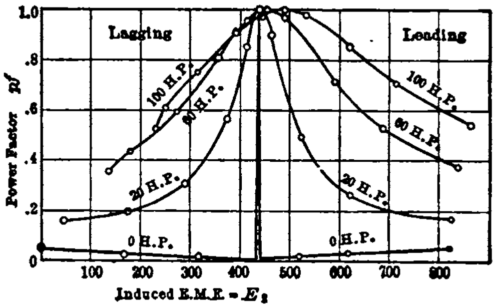


FIG. 61.—Power-factor curves for 60-h.p. synchronous motor.

73. Synchronising power. By this is meant the stiffness of the coupling or the change in power absorbed per degree change of the coupling angle ϕ, it increases with the air gap and inversely as $q'(=A'/F)$. See Par. 60.

74. Hunting of synchronous motors. Owing to the elastic nature of the electromagnetic motor coupling, any impulse will tend to set up an oscillation about the position of equilibrium, which will continue until the energy by which it was initiated, is absorbed by the extra losses incident to the oscillation. Such oscillation is called hunting, and the absorption of the energy of oscillation is called damping.

75. Hunting may be instigated either by a temporary disturbance, such as a sudden change of load on the motor or of the frequency of supply, or by some periodic disturbance (Par. 76) such as the pulsation of impressed frequency set up by the prime mover. This periodic disturbance may be one of comparatively high frequency due to the pulsation of the crank effort of a reciprocating steam or gas engine during each revolution, or one of comparatively low frequency due to the hunting of the governor of the prime mover.

76. Frequency of hunting. If the period of the impressed disturbance be approximately equal to that of the natural oscillation of the motor, there will obviously be a tendency for the motor to hunt with increasing amplitude, which may result in a complete breakdown or pulling out of step. A periodic pulsation in the load on the motor may also produce the same result, although such is much less common. It is thus important to know what is the natural frequency of oscillation of a synchronous motor, and upon what factors it depends.

The natural hunting frequency is approximately

$$f_h = 0.422\, E_1 \sqrt{\frac{pp'}{J z_s \text{ rev. per. min.}}} \tag{27}$$

where J is the moment of inertia in *lb. ft.*^{2} and p' the number of phases.

The z_s used in Eq. 27 should be the combined impedence of alternator, line and motor. The smaller the motor as compared to line and alternator, the less will be the effect of line and alternator impedence upon the hunting period of the motor.

77. Dampers. Forced hunting may be considerably reduced and temporary oscillations more quickly damped out, by means of field dampers, consisting of: (a) a copper loop around each pole face; (b) a copper bridge between poles; or, better still, (c) a squirrel-cage set of copper conductors

83. Effect of differing impressed and induced wave shapes on power-factor. When the impressed and induced e.m.f. waves are of different shape, the unbalanced e.m.f. harmonics produce wattless harmonic currents, which though small compared to the full-load current may be considerable with respect to the no-load fundamental current. Thus the p.f. at no-load may be less than unity even with the fundamentals of the current and e.m.f. in phase.

SYNCHRONIZING

84. Necessary conditions. Impressed and induced e.m.fs. equal, opposite, and of same frequency.

85. Synchronizing with lamps. Connect lamps in series between the in-coming machine and the line. When the lamps are bright (or dark, according to whether the machines are connected in unison or in opposition in the synchronizing circuit), and the beats are very slow, showing opposition of phase and approximate synchronism, the line switch may be closed and the machine will quickly settle down to stable operating conditions. If a motor, the load may then be applied.

In the case of a three-phase machine, lamps should be connected in at least two phases, and if both sets are not dark at the same time, there is indication that the three phases are not connected in the proper order and that two of them should be interchanged. The chief objection to the use of lamps for synchronizing is that they do not tell just when the two e.m.fs. are in exact opposition, nor whether the in-coming machine is fast or slow, although with a little experience it is possible to become quite proficient in their use.

86. Synchroscope. Various electromagnetic instruments called synchronizers, synchronoscopes, synchroscopes, etc., have also been devised for indicating synchronism. In most of these, the principle of operation is that of the rotary field, synchronism being indicated by a pointer on a dial. When the in-coming machine is above or below synchronism, the pointer revolves in one direction or the other respectively, and at a rate proportional to the slip (or beat) frequency. At synchronism the pointer stands still, and in a position on the dial which indicates the relative phase of the two e.m.fs.

One form of synchronizer is constructed as follows: The armature consists of two coils at right angles connected in parallel, a non-inductive resistance in series with one and an inductive resistance in series with the other. The stationary field coil is connected to the supply bus and the armature to the in-coming machine. The split-phase revolving field produced by the armature currents, reacts on the alternating field to produce a rotation whose speed is equal to the difference between the frequencies of armature and field currents. When these are equal the armature coil remains stationary.

PARALLEL OPERATION OF ALTERNATORS

87. Mechanical analog. When two or more alternators are connected in parallel and driven at the same frequency, they are as if coupled together by an elastic coupling just as in the case of a synchronous motor connected to a generator. Thus they must run at exactly the same frequency as long as they are coupled.

88. Division of load. Consider two similar alternators driven by separate prime movers, of which the speed characteristics are shown in Fig. 62, as a and b. Then if the speed is that represented by the horizontal line $\overline{ss}$, the powers supplied by the two prime movers will be P_a and P_b respectively. The division of load will then be wholly dependent upon the speed characteristics of the prime movers, *i.e.*, upon the governors.

89. Division of reactive current. The only effect of changing the excitation on either machine is to change the division of reactive current: *e.g.*, starting with equal loads and currents, if the excitation of a be increased, it will take more of the lagging reactive current, and b will take less. Any inequality of excitation is equalized by a circulating reactive current which transfers the excitation from the overexcited machine to the underexcited one. The terminal voltage will be determined in any case by the total excitation of both machines and by the magnitude and p.f. of the load.

When the voltage of a power plant is controlled by a Tirrill regulator, acting on the exciter bus from which the fields of all the parallel connected units are supplied, the per cent. increase in excitation for a given increase of load will obviously be the same on all the alternators. But if one alternator has a close regulation and another poor regulation, and if in the first case they shared the active and reactive currents in proper proportion, with increased load the close regulation machine will be relatively overexcited and will take more than its share of the lagging reactive current.

90. Switchboard control of speed of prime movers. In prime movers with very close speed regulation (no-load to full-load), it is obviously necessary that the two speeds be very closely adjusted (Fig. 62), and this is often difficult by the cut-and-try method, which in some cases requires shutting down each time to adjust the governors. Large units are often provided with a means of governor adjustment during operation. This may be accomplished by means of a small motor connected through worm gearing to the governor adjustment and controlled from the switchboard.

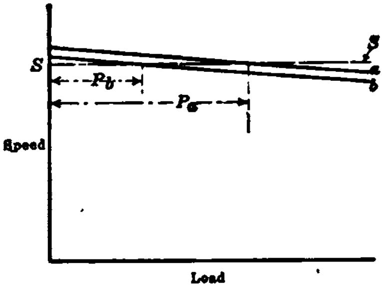


FIG. 62.—Diagram of division of load between two parallel-connected alternators.

91. Governor damping to prevent hunting. Sudden changes of load frequently cause the governor to hunt, *i.e.*, open up too wide, then close too far and so on. This may be easily prevented by a dash-pot connected to the governor.

ELEMENTARY OUTLINE OF THE DESIGN OF SYNCHRONOUS MACHINES

92. Specifications. Every electrical machine is built to meet certain specifications whether proposed by the purchaser, his engineer, or the manufacturer. In addition to the customary specifications there is always the question of first cost which must be reduced to a minimum. Many of these specifications, requirements, or standards are thoroughly treated in the Standardisation Rules of the American Institute of Electrical Engineers; these are given in Sec. 24. For a synchronous alternator the specifications refer briefly to the following: frequency; number of phases; rev. per min.; terminal voltage; rated output in kv-a.; minimum power-factor; temperature rise (Sec. 24); efficiency (Sec 24); inherent regulation; per cent. leakage reactance volts; insulation tests (Sec. 24); wave form. These are not all specified in each case, whereas in some cases much more elaborate specifications dealing with the details of mechanical and electrical design are supplied. With reliable manufacturers, very simple specifications are ordinarily sufficient, telling chiefly what the machine shall accomplish rather than how it shall be built. In the case of many modern alternators, regulation is not a controlling specification, it being in most cases important to have a high reactance rather than a low one (Par. 63 and 64). Efficiency as such is also not directly a controlling factor in design, but chiefly as it is necessary to keep down the heat developed in the machine. A fraction of 1 per cent. in efficiency is not as important to the user as a safe operating temperature.

93. List of symbols used.

$\mathcal{B}_1$ = Max. value of equivalent fundamental sine-wave of air-gap flux density (lines per sq. in.).
$\mathcal{B}_g$ = Actual max. flux density in air-gap at pole-face (lines per sq. in.).
$\mathcal{B}_t$ = Maximum apparent tooth-density of flux (lines per sq. in.) in teeth.
D = Diameter at air-gap (inches).
d_s = Depth of slot (inches).
e' = Volts induced per in. of active armature conductor.
f = Frequency (cycles per sec.).
h_c = Allowable copper watts per sq. in. of armature surface.
K = kv-a. output.
K_a = kv-a. developed in the armature.

k_b = Belt differential factor.
k_o = specific output.
k_p = pitch differential factor.
k_1 = ratio of pole pitch to gross armature core length.
l = gross armature core length.
m = cir. mils per amp.
p = no. of pairs of poles.
q_A = ratio armature to gap amp-turns at full load.
q_{il} = ratio of net iron length of armature to gross length.
q_p = ratio of coil pitch to full pitch.
q_{sw} = ratio of slot width to tooth pitch.
R = rev. per min.
T_e = Temp. difference in deg. cent. between armature conductor at centre and at end.
v = peripheral velocity, ft. per sec.
w'_c = wt. of armature copper (lb.) per kv-a. output.
Δ = peripheral loading; ampere conductors per in. of armature periphery.
ξ = output coefficient.
τ = pole pitch (in.).

94. Output equation. The volt-amperes per sq. in. of peripheral surface (Par. **23** et seq.) are $\Delta e'' = 8.50 k_b k_p \Delta v B_1 10^{-8}$, and the total power developed, in kv-a. is, $K_a = 8.50 \pi k_b k_p \Delta v B_1 D l 10^{-11}$. Allowing a maximum drop of 5 per cent. between induced and terminal volts, and taking B_1 corresponding to full-load, the kv-a. output is

$$K = 2.54 k_b k_p \Delta v B_1 D l 10^{-10} = k_o D l \qquad \text{(kv-a.)} \qquad (28)$$

or since $D = 720 v \div (\pi R)$

$$l = 1.72 \times 10^7 K R \div (k_b k_p \Delta B_1 v^2) \qquad \text{(inches)} \qquad (29)$$

$k_o = 2.54 k_b k_p \Delta v B_1 10^{-10}$ is the **specific output**, or the kv-a. per sq. in. of projected area of the air-gap cylinder. By substituting for v its value $\pi DR/720$, we obtain

$$K = 1.11 k_b k_p \Delta B_1 D^2 l R 10^{-12} = \xi D^2 l R \qquad \text{(kv-a.)} \qquad (30)$$

where $\xi = K/(D^2 l R) = 1.11 k_b k_p \Delta B_1 10^{-12}$ is called the **output coefficient.** Taking $k_b = 0.950$ and $k_p = 0.95$, this may be written, $\xi = (\Delta/1{,}000)(B_1/10^4)10^{-8}$. Sometimes k_o and sometimes ξ is more convenient as a starting-point for preliminary design, both being rough measures of the economy of material.

Major design constants. B_1, v and Δ are the major design constants, since their choice (R being specified) determines the general dimensions D and l of the machine. The larger these constants, the smaller in general will be the product Dl, and, within certain limits, the weight and cost of the machine. The considerations which govern the choice of these constants are so numerous that they cannot all be given their due weight in any direct method of attack. Some of the more important considerations and limitations are as follows: Par. **95** to **107**.

Ratio of pole pitch to core length.

$$k_1 = \tau / l = 3.5 k_b k_p \Delta B_1 v^3 \div (10^7 f K R)$$

Number of poles.

$$2p = 120 f / R \qquad (31)$$

Pole pitch.

$$\tau = 6 v / f \qquad \text{(inches)} \qquad (32)$$

95. Tooth-density limit. In order to avoid excessive eddy-current losses in the slot conductors and in the teeth, the flux density B_{tr} at the narrowest part of the teeth should not exceed (for open slots) the values of Par. **42**, although for partly closed slots and stranded conductors 10 to 15 per cent. higher values are safe. B_g is related to B_{tr} as follows for the rotor or internal element, and parallel-sided slots:

$$\mathfrak{B}_g = \mathfrak{B}_{tr} q_{il} (1 - q_{sw} - 2d_s/D) \qquad (33)$$

and for the stator or external element

$$\mathfrak{B}_g = \mathfrak{B}_{tr} q_{il} (1 - q_{sw}) \qquad (34)$$

$\mathfrak{B}_g$ is usually slightly less than $\mathfrak{B}_1$, (the crest of the fundamental sine-wave of flux, see Fig. 23, and Par. **32**). Also in extreme cases the tooth saturation flattens the flux distribution curve still more. For laminations 0.014 in. thick, q_{il} varies from 0.7, in an exceptionally well ducted core, to 0.8, in a poorly ducted core, and 0.9 in a core without ducts. Thus if $\mathfrak{B}_{tr}$ is assumed at its reasonable upper limit, and qu is known, the relation of $\mathfrak{B}_g$ to q_{tw} is fixed.

96. Heating limit. At 60 deg. cent. the copper loss under 1 sq. in. of armature surface is $h_c = \Delta/m$, the resistance of one circ. mil. inch of copper at 60° cent. being just 1 ohm. h_c is obviously limited: (a) by the heat dissipating power (H_a) per sq. in. per deg. cent. of temperature rise, (b) by the allowable temperature rise, T, (c) by the relative amount of core loss, and (d) by the relative amount of coil-end surface. Assuming average values of (b), (c) and (d), H_a will vary with the nature of the ventilation, the peripheral velocity, and the amount of heat thrown off by the rotor. For open, salient-pole machines, a very rough preliminary guide is

$$h_c = \Delta/m = 0.4(1 + 0.016v) \quad \text{(watts per sq. in.)} \quad (34a)$$

For high-speed turbo-alternators h_c has little relation to v, owing to the widely differing methods of ventilation, and varies from 0.8 to 1.6.

97. Temperature gradient in slot conductors. Assume that all the heat developed in the active conductor is conducted longitudinally to the coil ends. This is roughly true in high voltage stators. Then the temperature difference between centre and end of conductor will be $T_e = (l^2 \times 127^2)/m^2$, where l is the gross core length in inches. Or if T_e be limited, l will be limited, thus $l = (m\sqrt{T_e})/127$. Thus the larger the value of m, the longer may be the armature core without excessive temperature differences. The table in Par. **98** gives the temperature differences in deg. cent. for various values of l and m.

98. Temperature differences between centre and end of armature conductors

l, Armature length (in.)	Temp. difference T_e (deg. cent.)			
	600 circ. mils per amp.	800 circ. mils per amp.	1,000 circ. mils per amp.	1,200 circ. mils per amp.
0	0.0	0.0	0.0	0.0
10	4.48	2.52	1.61	1.12
20	8.98	5.05	3.23	2.25
30	40.3	22.65	14.5	10.1
40	71.7	40.3	25.8	17.9
50	112.0	63.0	40.4	28.1
60	162.0	91.0	58.1	40.4
70	220.0	124.0	79.1	55.0
80	286.0	161.0	103.0	71.5
90	364.0	205.0	131.0	91.0
100	448.0	252.0	161.0	112.0

99. Weight of copper vs. Δ. With $h_c(=\Delta/m)$ limited, m must increase with Δ. Then the total weight of active copper (also of total copper if k_1 be unchanged) will be proportional to Δ.

100. Depth of slot. On the same basis the slot depth will be proportional to Δ^2. Slot reactance does not limit Δ and slot depth, as is sometimes the case with induction motors (see Par. **251**).

101. Armature-reaction limit. If on the score of regulation, e.m.f. wave shape, or stability, it be desired to keep $q_A (= A + R)$ within any given limit, it might become necessary to limit the ratio $\Delta/\mathfrak{B}$, see Eq. 22, Par. **60**.

102. Choice of $\mathfrak{B}$ and Δ. From the standpoint of the economy of material or of the maximum output coefficient, that value of q_{tw} is the best which results in the maximum product $\mathfrak{B}_1\Delta$. But since, for a given depth of slot, Δ increases as $\mathfrak{B}_1$ decreases, the maximum product will occur where the net width of slot (excluding insulation) is approximately equal to the minimum tooth width, *i.e.*, when $q_{tw} > 0.5$. A considerable change in q_{tw} from the exact value which makes $\mathfrak{B}_1\Delta$ a maximum, does not seriously reduce this product, so that considerable variations from the best value of q_{tw} will be found, as there are other considerations involved.

$\mathfrak{B}$. Assume q_{tw} chosen, *e.g.*, to keep down the ratio $\Delta/\mathfrak{B}$ (Par. **101**), or to make $\mathfrak{B}_1\Delta$ a maximum (Par. **102**). Assume also $\mathfrak{B}_{tr}$ as chosen (Par. **95**); then $\mathfrak{B}_g$ is given by Eq. 34.

Δ. Assuming h_e constant, the upper limit of Δ is likely to be set for turbo-alternators by the limited space for copper on the rotor, since this limits F, and A (and therefore Δ) must be smaller than F. A glance at Fig. 63 shows that the cost of copper is not likely to set the Δ limit. In fact, an increase of Δ, within certain limits, results in a more than balancing saving in iron. Moreover, the correspondingly larger m reduces T_e, the temperature difference between the centre and ends of the active conductors (see Par. 98). If close regulation is specified, it may be necessary to use a low Δ and high $\mathfrak{B}_1$, see Par. 60, particularly in high-speed turbo-alternators; but if, as is usually the case in this country at the present time, low regulation is not specified, Δ may be carried much higher than has heretofore been customary.

The following formulæ give a rough idea of the range of Δ for modern alternators: at 60 cycles $\Delta = (450 \text{ to } 700)(\text{kv-a.}/100)^{0.135}$; at 25 cycles $\Delta = (500 \text{ to } 800)(\text{kv-a.}/100)^{0.135}$. The higher values are for very well-ventilated and the lower for moderately ventilated machines (see Fig. 64).

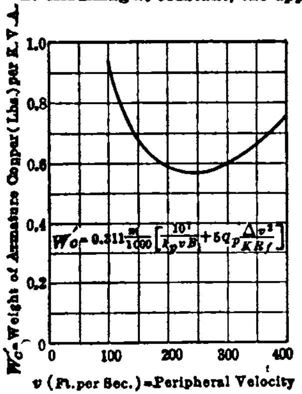


FIG. 63.—Armature copper per kv-a. vs. v, for a 10,000-kv-a. 25-cycle 2-pole turbo-alternator.

103. Choice of peripheral velocity. Assuming that $\mathfrak{B}_1$ and Δ have been chosen, the choice of v determines the proportions of the machine. Coil-end

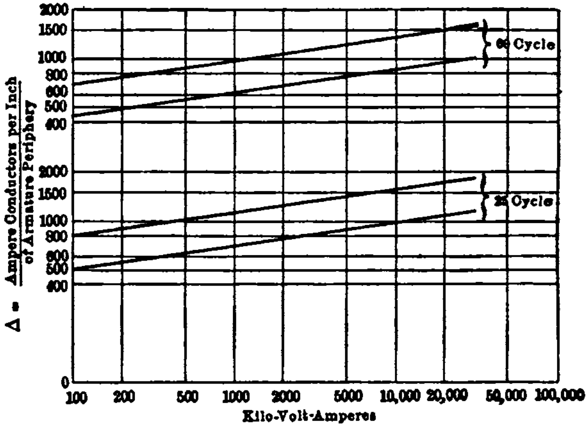


FIG. 64.—Peripheral loading of alternators.

reactance does not enter seriously into consideration, as in the case of the induction motor. The weight of active iron is nearly independent of v.

The weight of armature copper is a consideration but a relatively unimportant one in high speed machines; in lbs. per kv-a. it may be expressed approximately as follows:

On alternating-current machines of 1,000 volts and upward, the armature-slot insulating material may consist of micanite, micarta or Bakelised paper tubes, or of insulations built up of empire cloth, oiled linen, or other suitable materials. In Fig. 65 is given a curve showing suitable thicknesses of slot insulation from copper to iron in terms of the voltage.

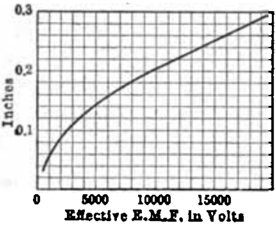


Fig. 65.—Slot insulation thicknesses for alternators.

109. For heat-resisting qualities and allowable working temperatures of different classes of insulations, see Sec. 24. Other references are as follows: A. P. Fleming and R. Johnson: "Chemical Action in the Windings of High-voltage Machines." *Journ.* I. E. E., Vol. XLVII, p. 530, 1911.

110. Instance and cause of an insulation failure. In 1905 Mr. J. S. Highfield published in *The Electrician* (Vol. LIV, p. 573) a description of an insulation failure in the windings of a 10,000-volt alternator. The failure was attributed to chemical actions stimulated by the ozone formed as a consequence of corona discharges. Highfield's article led to considerable correspondence in subsequent numbers of *The Electrician* and the views put forward are of great interest. "High-voltage Tests and Energy Losses in Insulating Materials," by Mr. E. H. Rayner appears on page 3, Vol. XLIX of *Journ.* I. E. E. (1912). Rayner concludes his paper with a large and valuable Bibliography of the subject.

111. Effects of temperature on disruptive strength. Lamme has dealt with these effects very thoroughly in his paper on "High-speed Turbo-alternators" appearing on page 1 of Vol. XXXII of *Trans.* A. I. E. E. (1913) Lamme and Steinmetz have continued the discussion of the subject in a paper entitled "Temperature and Electrical Insulation" which appears on page 79 of Vol. XXXII of *Trans.* A. I. E. E. (1913). As to the design of the insulation from the standpoint of the heat flow through it, the best information as yet available on the subject is contained in the following four papers: (a) Symons, H. D. and Walker M. "The Heat Paths in Electrical Machinery," *Journ.* I. E. E., 1912, Vol. XLVIII, page 674. (b) Williamson, R. B. "Notes on Internal Heating of Stator Coils," *Trans.* A. I. E. E., 1913, Vol. XXXII, page 153. (c) Langmuir, Irving. "Laws of Heat Transmission in Electrical Machinery," *Trans.* A. I. E. E., 1913, Vol. XXXII, page 301. (d) Randolph, C. P. "The Conduction of Heat; with Results of an Investigation of the Thermal Resistivity of Heat-insulating Materials," General Electric Review for Feb., 1913, page 120.

112. Field insulation. For the field windings of low-speed and moderate-speed synchronous machines the problems associated with the insulation are comparatively simple. Wherever reasonably practicable, the field winding will consist of a flat-ribbon conductor wound on its thin edge. It is important that the design shall comprise features ensuring absolute absence of any relative motion between the conductors, as the result of centrifugal forces and of vibration. The use of coils of flat strip with the edges left bare in the customary manner, is excellent from the standpoint of ability to rapidly dissipate heat to the surrounding air in virtue of the rapid motion of these coils through the air. Since the voltage is low and the temperature moderate, it is practicable to employ the simplest methods, so far as the insulation is concerned. But in the case of extra high-speed turbine generators the field windings must be embedded in slots in the surface of the cylindrical rotor, and the difficulties of providing a suitable design of insulation are very great. It is true that the voltage of the excitation is low, but the difficulty arises from the high temperature to which the insulation is subjected. The facilities for ventilation are necessarily exceedingly poor, and the space for the copper is so restricted that it is necessary to operate the field winding at high current densities. The result is that portions of the insulation in the slots of high-speed rotors are usually subjected to temperatures much above 100 deg. cent. and frequently temperatures approaching 150 deg. cent. are experienced at the very hottest spots of the rotor insulation in designs of the most difficult ratings. Consequently mica must be the chief ingredient of the insulation, notwithstanding that the voltages are very low. The construction must present the utmost rigidity since the slightest displacement of any insulating material will suffice to unbalance the rotor. The case is ably presented by Lamme in a section entitled Rotor Insulation on p. 29 of his paper on "High-speed Turbo-alternators," in Vol. XXXII (1913) of the *Trans.* A. I. E. E.

113. Insulation of alternator leads and connections. Acute problems are not often encountered in the insulation of the leads and connections, since, on the one hand, machines are rarely built for pressures in excess of 15,000 volts, thus eliminating the questions arising in connection with the insulating of extra high-pressure leads; and, on the other hand, the voltages in large machines are rarely lower than 2,000, so that there do not arise any questions associated with the handling of very large currents. This latter statement should be conditioned by calling attention to the instantaneous current flowing on the occasion of sudden short-circuits. Such a current (Par. 63) may for an instant amount to from ten to twenty times the full-load current of the machine and is associated with the development of enormous mechanical forces. Consequently the problems of leads and connections reduce chiefly to the provision of elaborate mechanical support for all leads, conductors and cables from the armature windings to the switchboard. The end connections of the stator windings must be elaborately supported; many fine machines were wrecked before this was realized. The end connections are sometimes lashed by stout bands of cord to heavy rings which are supported by stepped brackets constituting extensions of the frame of the machine. It would be futile to here attempt to describe all of the various methods to which resort has been had in solving this difficulty. These have been described and illustrated in various papers among which may be mentioned the following:

Walker, Miles. "Short-circuiting of Large Electric Generators," *Journ.* I. E. E., 1910, Vol. XLV, p. 295.

Field, A. B. "Operating Characteristics of Large Turbo-generators," *Trans.* A. I. E. E., 1912, Vol. XXXI, p. 1645.

Lamme, B. G. "High-speed Turbo-alternators," *Trans.* A. I. E. E., Vol. XXXII, p. 1.

114. Requisite protection against potential stresses caused by special conditions, grounded phase, switching, etc. If a Y-connected generator is operated with the common connection grounded without any resistance, then if a dead short-circuit to ground occurs on one of the three lines, although the circuit-breakers will at once disclose the faulty line, severe mechanical stresses will be imposed on the windings of the generator, due to the sudden short-circuit. If on the other hand, the neutral point is connected to ground through a resistance, the amount of current which can flow when a dead ground occurs on one of the lines will be limited by the amount of the resistance. Let us consider the case of a Y-connected generator with a pressure of 7,000 volts in each leg of the Y, and with a 9-ohm resistance to ground. If a dead ground occurs on one of the lines leading from the generator, then assuming that the resistance of the winding and line and ground aggregate 1 ohm, only $7{,}000/(9+1) = 700$ amp. can flow through the ground. But this will raise the potential of the neutral point from zero to $700 \times 9 = 6{,}300$ volts, and this combined with the pressure of the other two phases, on which no ground has occurred, will increase the pressure between the high-tension ends of these windings and the frame of the machine to about 12,000 volts. Thus while the use of a resistance between neutral and ground *decreases* the severity of the mechanical stresses to which the windings of the generator will be exposed on the occasion of sudden short-circuits on the line, it *increases* the severity of the potential stresses on the main insulation.

The extra stresses imposed upon the main insulation of the machine would be still greater were the machine to be operated with its neutral grounded through a resistance, and with its terminals tapped into auto-transformers by means of which the generator voltage is stepped up (say in the ratio of 1:2) to the line pressure. Consequently when auto-transformers are employed, the generator must not have its neutral connected to ground through a resistance, but must be dead-ground. If the stepping-up is accomplished by means of transformers with distinct primaries and secondaries instead of with auto-transformers, then the generator's neutral may be grounded through a resistance. **Operation with grounded neutral** is to be preferred for systems from which many expensive underground cables are supplied, since a ground on some one cable will clear that cable off the line by the opening of its local circuit-breakers and without interruption to the rest of the system. But for a system consisting of a single long line with an important distribution system at its distant end, and where, consequently, any interruption of the supply would be very serious, there is a widely held opinion that the generator should be operated with non-grounded neutral, since it is maintained that the development of a ground at some one point of the line is then less likely to shut down the system. It will, however, increase, by 73 per cent., the extreme potential stresses across the generator's main insulation, and this must be recognized in the proportioning of the generator's insulation.

Switching operations, arcing grounds, lightning-arrester discharges, and other disturbances are likely to occasion surges in a system. The generator windings must be protected against the resulting high-potential stresses. Sometimes such protection is afforded by suitable reactances interposed between the line and the generator, and sometimes the last few turns in the winding are especially insulated so that the steep potential wave front shall not occasion breakdown between adjacent turns.

LOSSES AND EFFICIENCY

115. Core losses. The data in the following table serves as an approximate guide to the determination of the *no-load* core loss in synchronous machines.

Density in stator core (below slots) lines per sq. in.	Core loss in stator core (including teeth) (watts per lb.)	
	25 Cycles	60 Cycles
40,000	1.1	2.7
50,000	1.4	3.7
60,000	1.7	4.9
70,000	2.0	....
80,000	2.4	....
90,000	2.8	....

There are further core losses called *stray* core losses which increase as the load increases. These are due to the flux distortion in core and teeth, caused by the armature m.m.f.

116. Effect of rated speed on losses. Returning to a consideration of the no-load core losses it is of interest to note that the inherent nature of low-speed and high-speed designs is such that while the no-load core loss may constitute a relatively small component of the total loss in low-speed synchronous machines, it is necessarily a large component in high-speed synchronous machines. The reverse is the case with the armature copper loss, which is relatively high in low-speed machines, and relatively low in high-speed machines. In a group of 3,000-kv-a. 25-cycle close-regulation designs for various speeds, particulars of which are given by Hobart and Ellis in "High-speed Dynamo Electric Machinery" (John Wiley and Sons, N. Y., 1908), the rated speeds and these two losses in per cent. of output are as follows:

Rated speed	No. of poles	*A* No-load core loss (%)	*B* Armature copper loss at full-load (%)	Ratio of *A* to *B*
83	36	1.93	.93	2.07
125	24	2.02	.73	2.77
250	12	2.17	.57	3.82
375	8	2.24	.50	4.45
500	6	2.27	.50	4.39
750	4	2.30	.47	4.93

121. Calculated values of bearing friction for several turbo-alternators

No. of poles	Rated output (kv-a).	Speed (rev. per min.)	Bearing friction in horse-power		
			Turbine bearing	Middle bearing	Bearing at collector end
4	9,400	1,800	7	62	24
4	6,300	1,800	11	60	14
4	6,300	1,800	6	41	13
4	3,100	1,800	4	25	5
4	2,500	1,800	6	21	6
2	2,500	1,500	3	20	4

122. Summary of losses. We have now reviewed the component losses in synchronous machines. These are: (a) no-load core loss; (b) extra core loss increasing with load; (c) full-load copper loss in armature winding; (d) eddy loss at full-load in armature winding; (e) field copper loss at full-load; (f) windage and bearing friction loss.

Losses (a), (c), (e) and (f) may be considered accurately determinable either by test or calculation. Losses (b) and (d) are not commercially determinable; *i.e.*, their determination would require an elaborate and expensive investigation in each case. A reasonable value may be assigned to the sum of losses (b) and (d) (Par. **127**), in cases where they cannot be determined from tests. It may in general be stated that all the losses except loss (b) and loss (d) are readily susceptible to either exact or sufficiently approximate determination, and hence they may be designated as **determinable losses.** It is in commercial transactions futile to attempt to conclusively determine losses (b) and (d). Consequently it is desirable to group these two losses together, and to designate them as: **indeterminable losses or stray losses.** It should usually be practicable in a commercial transaction, involving efficiency guarantees, for the parties to the transaction to agree upon a reasonable value to assign in any case to the indeterminable losses in the event that it is not expedient to make the test set forth in Sec. 24.

123. The efficiency. The true efficiency of a machine is the ratio of the output to the input. The efficiency should be based upon the rated output, pressure, power-factor and speed. The losses on which the efficiency is based, should be corrected to 75 deg., the temperature of reference for efficiency determinations (see Sec. 24). The determination of the true efficiency of a machine involves either an accurate determination of all the component losses, or else an accurate measurement of the output and of the simultaneous input. In other than small machines, both of these methods of determination are impracticable unless resort is made to expensive scientific measurements.

Consequently in practice, use should be made of two approximate efficiencies. These may be designated as follows: (a) the efficiency exclusive of stray losses; (b) the conventional efficiency.

124. The efficiency exclusive of stray losses. This is the ratio of the output of the machine to the sum of the output and the determinable losses as obtained by the separate measurement or calculation of each determinable loss. This efficiency is necessarily greater than the true efficiency, but approaches it in machines in which the indeterminable losses are negligible.

125. The conventional efficiency differs from the efficiency exclusive of stray losses, to the extent to which appropriate values for the indeterminable losses are included in estimating the input. In all matters relating to guarantees, the allowances to be made for the indeterminable losses are specified in each case. The conventional efficiency is, by definition, less than the efficiency exclusive of stray losses, except in cases where the convention is to take the stray losses equal to zero (see Standardisation Rules, Sec. 24). With the application of a reasonable amount of care in assigning appropriate values to the indeterminable losses, the difference between the conventional efficiency and the true efficiency should usually be quite unimportant and often negligible.

to render the stator frame sufficiently stiff to ensure absence of sagging. Any sagging would unbalance the air gap, which must be of uniform depth over the entire circumference.

The field winding should usually consist of flat copper, wound on its thin edge. The heat then readily flows to the outer surface, and is carried away by the surrounding air.

132. Intense forced cooling and ventilation. Extra high-speed steam turbine-driven generators of large capacity present the most extreme instances of the necessity for forced cooling and ventilation. The quantity of air which must be forced through such machines per kv-a. of rated output in order to limit the temperature rise at full-load to permissible values, is of the order of from 2.5 cu. ft. per min. for a 25,000-kv-a. machine to 4.5 cu. ft. per min. for a 5,000-kv-a. machine, depending on the efficiency of the machine. The precise values vary with the arrangement of the ventilating passages, and the speed. The customary method of circulating the air consists in providing the rotor with fans which force air through appropriate passages. Fig. 66* relates to the ventilating method employed in the design

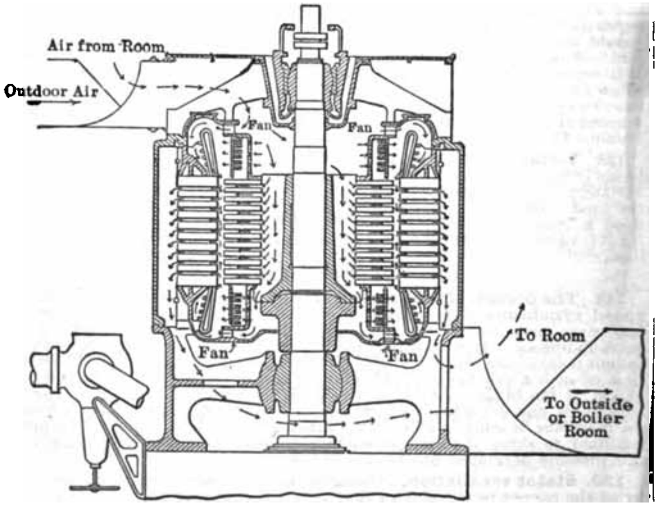


FIG. 66.—Ventilation design for a vertical-shaft turbo-alternator.

of a steam turbine-driven alternator of the vertical type. Mr. B. G. Lamme has dealt very fully with this subject in a paper entitled "High-Speed Turbo-alternators," at p. 1 of Vol. XXXII (1913) of the *Trans.* A. I. E. E. Lamme subdivides methods of cooling turbo-alternators by the forced circulation of air into three classes: (a) radial methods, (b) circumferential methods, and (c) axial methods.

133. In the radial system of ventilation all the air passes out radially through ventilating ducts in the stator core. In the design of these high-speed generators the rotor diameter is limited by the peripheral speed. Consequently the design is of relatively great length parallel to the shaft. There is very little space on the rotor even for the windings, owing to the limited radial dimensions, and it is often necessary to dispense with the circulation of

* From an article by E. Knowlton, "Ventilation of Steam Turbine-driven Alternators," *General Electric Review*, Oct., 1912, page 656.

air through the interior of the rotor. Even in the most favorable cases, such circulation of air through the rotor is limited to but a small part of the air required for the cooling of the stator. The greater part of the supply required for the stator is, in the radial system, first passed along the air gap; the radial depth of the air gap is often made very great expressly out of consideration for providing sufficient section for the flow of the required amount of air.

134. A typical arrangement of the radial system as applied by the General Electric Co. in its horizontal steam-turbine alternators is shown in Fig. 67. The system is described as follows: the air enters the generator at *AA*; passes through the air gap, windings and air ducts in the stator core to the annular spaces *BBB*; flows around circumferentially to the openings *CCC* in the bottom of the armature frame and thence to the outlet duct. In some machines part of the air passes through the field core. The movement of air is produced by fans on the ends of the rotor. In some instances the armature frame is modified so that the air is expelled from the top into the dynamo room. Proper passages must be provided below the generator for the ingoing and outgoing air. As shown in the sketch, air is taken in at both ends of the generator and discharged through an opening in the centre of the frame, and the passages must be so arranged as to prevent the outgoing heated air from mixing with the incoming cool air; a simple method of accomplishing this is also shown. In certain cases other arrangements may better suit local conditions. For approximation the area of the ingoing or outgoing duct will range from 5 sq. ft. for a 1,000-kv-a. to 15 sq. ft. for a 5,000-kv-a. generator. The outgoing air should be carried outside the building, care being taken that it cannot immediately re-enter the intake. The ducts should be as short and have as few bends as possible and these should be made with a large radius. Both ducts should have

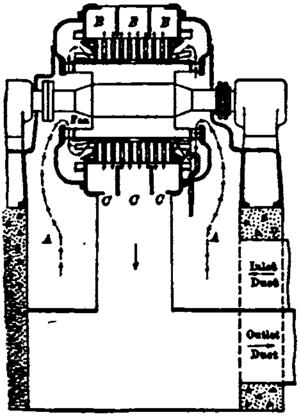


Fig. 67.—Radial system of ventilating a horizontal-shaft high-speed generator.

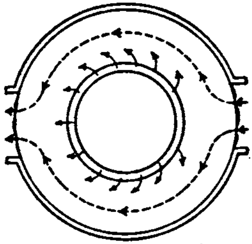


Fig. 68.—Circumferential system of ventilating turbo-alternators.

adjustable dampers so that part of the air may be taken from or expelled into the engine room. This will allow adjustments to suit weather conditions. The armature and field leads are brought out from the bottom of the generator frame, directly under the collector; this brings them to the air chamber under the armature. From here to the switchboard the leads may be carried in one of the air ducts or a separate duct, as is most convenient.

135. The circumferential system of ventilation is explained by reference to Fig. 68 (Lamme). The air enters at one side of the machine and is forced along the air ducts in the stator core back of the armature windings until it reaches the other side of the machine, whence it is discharged. Such circumferential circulation is usually supplemented by the supply of a further amount of air to the air gap to cool the rotor, as shown by the 13 little arrows curved toward the centre of the diagram. This air, after leaving the air gap passes into the stator ventilating ducts and reinforces the main streams of air. Lamme also illustrates in his paper (Ref. in Par. 132) a modification of the circumferential system in which the air enters the stator ducts at points of their outer circumference, flows down alternate vertical ventilating ducts in the stator core, then a short distance through longitudinal ducts, and returns through the intermediate vertical ventilating ducts back to the outer circumference of the stator.

136. The axial system of ventilation (often termed the longitudinal system) is illustrated in Fig. 69 (Lamme). The system has the advantage that the edges of each lamination are bathed by the circulated air. The conduction of heat is many times greater in the plane of the laminations than transversely thereto, the ratio varying from 20 to 100 according to the nature of the insulation between laminations, the thickness of the laminations, insulation, and their compression. Obviously the longitudinal method has a decided advantage in this respect at least. On the other hand it is difficult to arrange that a sufficient extent of surface shall be exposed to the air. As shown in Fig. 69, there is provided in the axial system a large number of longitudinal passages. These passages may lead to a large central outlet duct as in the case illustrated, or they may extend uninterruptedly right through the core from one end to the other.

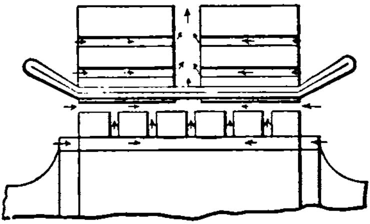


FIG. 69.—The axial system of ventilating turbo-alternators.

137. Air cleaning and cooling. It has been quite usual to take the ventilating air from the hot engine room, circulate it through the machine, and then return it to the hot engine room. Not only does this result in a high operating temperature (or else a lower output for a given temperature), but it also has the consequence that the air which is sent into the machine has taken up oily vapors whose presence aggravates the difficulties associated with the gradual clogging up of the ventilating passages in the machine with dust and dirt which is also carried into the machine in the circulating air. An expensive periodical cleaning of the machine must be undertaken.

In several important modern stations it is arranged that the air shall be taken from outside the station or from a cool basement (when one of sufficient capacity can be provided) cleaned by passage through cloth screens, and then led to the machine through suitable ducts. The cloth employed is of a closely woven variety similar to that known as Canton flannel, and there is required a surface of some two-tenths of a square foot per cu. ft. of air per min. Since a 10,000-kv-a. machine requires some 34,000 cu. ft. of air per min., it follows that for a machine of this capacity: $34{,}000 \times 0.20 = 6{,}800$ sq. ft. of screening surface must be provided. The cloth is supported upon wooden frames and the installation is generally regarded as constituting an undesirable risk from the insurance standpoint. Of the various kinds of cloth to which the term "fire-proof" is applied, it is rare to find any approach to genuinely fire-proof properties.

These air-filtering screens occupy a great deal of space and their effectiveness is gradually reduced as the meshes become clogged up with dust.

138. Air washing. Decidedly the best method of cleaning the air consists in passing it through sprays of water to wash it, instead of straining it through cloth screens. Mr. E. Knowlton deals at length with this method in the course of an article entitled "Ventilation of Steam-turbine Engine Rooms;" General Electric Review; September, 1913, page 627. See also a paper by J. Christie, entitled "Air Filtration, Cooling and Ventilation of Electrical Machinery;" Electrical Review; June 27, 1913, page 1088. Mr. Christie states that filter cloths wear out quickly and are expensive to renew. He states that $400 to $500 per annum for cloth and labor is by no means an outside figure for the efficient maintenance of this equipment.

139. The cooling of air incident to water filtering. The humidification of air in its passage through the water filter occasions a lowering of its temperature, the amount of which varies with the condition of the air. If it is utterly dry on entering the humidifier, the air will have experienced a considerable decrease in temperature by the time it has emerged from the humidifier and entered the machine which is to be cooled. As a consequence of these considerations, it is obvious that the average conditions regarding humidity and temperature in any locality, affect the amount of advantage to be derived by air filtering in addition to that of removing the dirt and the dust. The average reduction in the temperature of the air for the months of July and August in different parts of the United States is stated (Knowlton) to vary from 2.5 deg. cent. at points on the coast to 11 deg. cent. at points in the Middle South West. On certain days during these months the maximum reduction effected may considerably exceed these average values.

MECHANICAL CONSTRUCTION

140. Abnormal conditions requiring large factors of safety. In providing adequate strength in the design of synchronous machines it is usually utterly insufficient to take the conditions in the machine when running at uniform speed at its normal load as the basis which, with usual factors of safety, will lead to a satisfactory design. On the contrary, it is the conditions occurring during sudden short circuits (see Par. **63** and **64**), and when the machine is carrying sharply fluctuating loads, which determine the strength required in the various parts. These are impossible of exact calculation; consequently, in the design of all those parts upon which the mechanical strength of the whole machine depends, large margins of the nature of safety factors must be added to the values which would usually be employed in machine design.

141. Critical speed of shafts. Irrespective of questions of cost, conditions arise with some capacities and speeds where it is impossible to find room for a shaft of large diameter, and it must have a critical speed below the operating speed. The peripheral speed at the bearing would otherwise exceed desirable values; furthermore, there is but limited room for accommodating the windings in the radial depth available between the surface of the rotor (which is itself of small diameter) and the surface of the shaft.

142. Effect of critical speed on rotor design. If the critical speed is to be well above the normal running speed, it is impracticable to employ laminated rotor bodies for capacities much above 1,000 kv-a. at a speed of 3,600 rev. per min. or above 5,000 kv-a. at 1,800 rev. per min. or above 7,500 kv-a. at 1,500 rev. per min. But for designs in which the critical speed is below the normal speed, it is practicable to employ rotor constructions with laminated cores up to ratings of over 2,000 kv-a. at a speed of 3,600 rev. per min. of some 10,000 kv-a. at a speed of 1,800 rev. per min., and of some 15,000 kv-a. at a speed of 1,500 rev. per min. In the construction of 1,500 rev. per min., 25-cycle, or 3,600 rev. per min., 60-cycle rotors solid cores are especially appropriate, since greater mechanical strength can be obtained in constructions in which the slots for the windings are milled out of a solid steel core. The rotor windings may be retained in the slots by solid steel or brass wedges, since the magnetism is of constant direction in each part and since the air gap is so deep in these extra high-speed generators that there is no loss in the rotor surface from pulsating influences from the alternating m.m.fs. due to the conductors in the stator slots. In some designs the shaft ends consist of enlarged extensions bolted to the rotor core.

143. Bearing lubrication. In small and medium sized machines of moderate speed, the lubrication of the bearings is accomplished in the usual manner by oil rings located in suitable recesses in the bearing and with their lower portions immersed in oil below the bearing lining. In large extra high-speed machines, the oil is forced into the bearings and after passing from them and being allowed opportunity to cool, it is again forced into the bearings. Sometimes in such machines and frequently in large water-wheel generators, copper tubes are embedded in the bearing just under the surface of the lining metal. Water is circulated through these tubes and plays a large part in maintaining the temperature of the bearing at a safe value.

The following description relates to the water-cooled bearings of some horizontal turbo-generators built by the General Electric Company.

The general construction, including the arrangement of the cooling pipe and the grooving, is shown in Fig. 70. The bearing is provided with a coil

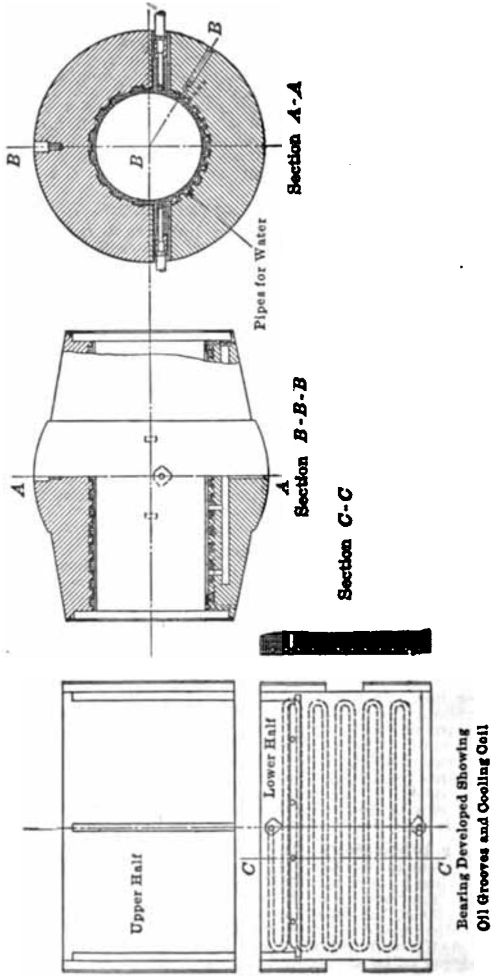


FIG. 70.—Water-cooled bearing.

of thin copper tubing cast into the babbitt lining close to the surface and having at the ends of the tubing, steel blocks securely brazed to the tubing and held in place by the babbitt. Water is led into the bearing by pipes

Ellis' "High-Speed Dynamo-electric Machinery," John Wiley & Sons, New York, 1908.

THE TESTING OF ALTERNATING-CURRENT GENERATORS

148. The heating test of large alternators under normal full-load conditions is troublesome and expensive, even when the necessary amount of power is available, and many suggestions, such as those of Mordey and of Behrend,* have been made with a view to reduce the amount of energy wasted during the tests. These methods necessitate alterations in the connections of either the armature coils or the magnet coils. If serious mechanical stresses are to be avoided, these methods can only be employed with machines having very large numbers of poles, and even then the magnetic conditions are not accurately the same as those which exist at normal full-load. The following method, known as the "intermittent short-circuit and open-circuit method," or, briefly, the "intermittent method," involves no change whatever in the connections of the machine, and requires the expenditure of only sufficient energy to cover the losses of the machine, and yet every part of the alternator reaches the same temperature as it would after an actual full-load heating test. The method involves a previous knowledge of the separate losses of the machine, but these are, in any case, determined in the ordinary course of a systematic test.

149. Intermittent method of making a heating test. Suppose a certain machine has at full-load: a friction loss of 10 kw.; an armature copper loss of 20 kw.; and an iron loss of 100 kw. In the course of an hour's run at full-load the loss of energy will be: $10 \times 60 = 600$ kw-min. as friction; $20 \times 60 = 1,200$ kw-min. as copper loss in armature; $100 \times 60 = 6,000$ kw-min. as iron loss.

Let the machine run for 5 min. with the armature short-circuited, and at such an armature current that the armature copper loss is 60 kw., *i.e.*, at a current equal to $\sqrt{3}$ times the normal current. Next let the machine run for a further 10 min. with open armature circuit, but overexcited, so as to give an iron loss of 150 kw. This adjustment is made according to indications of the wattmeter on the driving motor, allowance being made for other losses, such as losses in the motor itself and friction of the alternator and the driving mechanism. If this cycle of operations is repeated regularly throughout the time of the test, it is obvious that $10 \times 60 = 600$ kw-min. will be lost in friction per hr.; $60 \times 20 = 1,200$ kw-min. will be lost in armature copper per hr.; $150 \times 40 = 6,000$ kw-min. will be lost in the iron per hr., or, exactly the same loss in each case as would have occurred under normal full-load in the same time. There is still the **loss in the magnet windings** to be considered. During the short-circuit test this is less, and during the open-circuit test it is greater than the normal, so that on the average it does not differ very greatly from the normal. If, however, great exactness in this respect is required it is obtained as shown in Par. 150.

150. Further refinements of the intermittent method of making a heating test. In the above example one-third of the time of each period of the test was devoted to the short-circuit test, and two-thirds to the open-circuit test. These proportions may, however, be changed at will, and by varying the short-circuit current and the overexcitation correspondingly, the total energy expended in the armature copper and iron per hr. may be kept at the right value, while the average exciting energy will have some value other than before. Except in very extreme cases, *e.g.*, in the case of alternators which require only a very small change in excitation between no-load and full-load, and which at the same time have a very low value for the ratio

$$\frac{\text{short-circuit current at full excitation}}{\text{normal full-load current}};$$

it is always possible so to adjust the two time intervals, that while keeping the copper loss and the iron loss per hr. at the correct value, the exciting loss per hr. has practically the same value as at normal full-load. Exactness is, however, unnecessary, as, in any case, a simple calcula-

* These methods are described by Mordey, *Journ.* I. E. E., Vol. XXII, 1893. Behrend, *Elec. World and Engineer*, Vol. XLII, Oct. 31 and Nov. 14, 1903.

citation is zero the friction losses in watts are given by the formula $P = [K \times 0.148(S_1^2 - S_2^2)] + (2T)$. If the field is excited, the machine will come to rest in less time, and a larger loss P_1 will be obtained. This consists of the friction and core losses. By taking several curves with varying excitation, the core loss at different voltages may be determined, and a curve for core loss and voltage obtained. A typical set of retardation curves for a three-phase 350-kv-a., 2,100-volt, 50-cycle, 176-rev. per min. alternator is given in Fig. 72.

INDUCTION MACHINES

GENERAL THEORY OF THE POLYPHASE INDUCTION MOTOR

155. Principle of operation. The polyphase induction motor consists of a primary structure and a secondary structure. The former is usually stationary, supporting coils symmetrically on its inner periphery. These

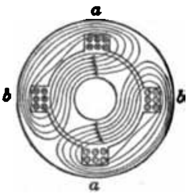


FIG. 73.

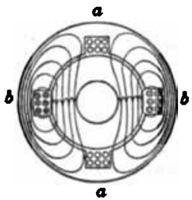


FIG. 74.

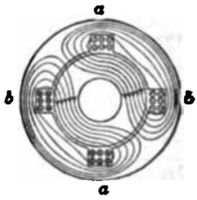


FIG. 75.

FIGS. 73, 74, 75.—Diagrams of coils and magnetic flux in elementary two-phase two-pole induction motor.

coils are displaced in space (*e.g.*, two coils at right angles, or three at 60 deg.), and in them flow currents of the same frequency, but differing symmetrically in phase, *e.g.*, two at 90 deg., or three at 60 deg.). The secondary (usually rotatable) structure carries properly displaced short-circuited coils. The polyphase currents in the primary structure produce a revolving field.* As this cuts across the secondary conductors, currents are induced therein which, according to Lenz's law, are in such direction as to oppose the cause. That is to say, these secondary currents react on the revolving magnetic flux in such a way as to drag the secondary conductors and structure along with the rotating flux.

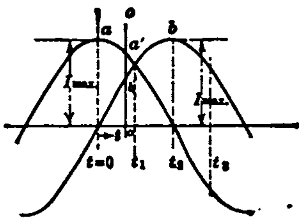


FIG. 76.—Current-time curves for two-phase indicator-motor windings.

156. Revolving field. Fig. 73 shows a two-phase, two-pole, laminated-iron stator with each coil imbedded in a pair of slots. The flux shown corresponds to the instant t_1 in Fig. 76. Figs. 74 and 75 show the flux corresponding to instants t_2 and t_3 in Fig. 76. The rotation or progression of the flux is obvious. If in place of the two-pole field of Fig. 73, we substitute a multipolar field, as many cycles of current variation will be required for one revolution of the magnetic field, as there are pairs of poles.

157. Synchronous speed and slip. The speed of the revolving field is called the synchronous speed, and the percentage by which the rotor or secondary falls below this speed is called the slip. If $2p$ represents the number of poles, and f the frequency, the synchronous speed in rev. per min. is $R_s = 60f/p$.

* See any elementary text-book on this subject.

162. Classification of stator windings. Stator windings correspond exactly with the armature windings of synchronous machines (Par. 16 to 22), and, as in that case, the two-layer lap winding is much the commonest type. Figs. 78 and 79 relate to a three-phase, four-pole, two-layer lap winding, with five-sixths pitch (ten slots in 12) and four slots per pole per phase. The bottom and top slot belts labelled *a* comprise the back-connected (*through* the paper) conductors of the *a* phase, those labelled *a′* the outward-connected (*from* the paper) conductors of the same phase; similarly with the *b* and *c* phases. Several coil ends of the *c* phase are shown diagrammatically. Fig. 79 shows a diagrammatic developed end view of the phase belts.

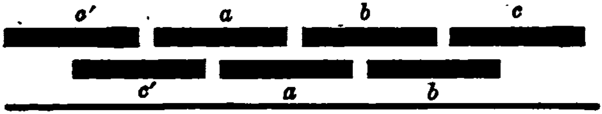


Fig. 79.—Sectional diagrammatic development of the phase belts of the winding of Fig. 78.

A developed diagram of connections of the windings of Fig. 78 is shown in Fig. 80, where the dotted lines indicate the bottom-slot coil sides or the lower layer.

163. Secondary or rotor windings. These may consist of any symmetrical arrangement of short-circuited conductors in which series-connected conductors do not generate opposing e.m.fs. There are three principal types.

(a) **Phase wound,** like the primary, except that the phases are short-circuited or brought out through collector rings for insertion of starting resistance. The number of phases need not be the same as for the primary, and is usually 3.

(b) **Independently short-circuited loops or coils.** The two active sides of each loop are approximately 180 magnetic deg. apart.

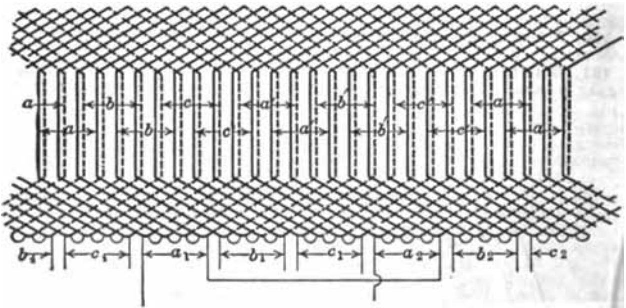


Fig. 80.—Developed winding diagram corresponding to Fig. 78.

(c) **Squirrel cage,** consisting of conducting bars in slots, all connected at the ends by conducting rings, called the end rings.

In (b) and (c) there are as many phases as there are slots per pole.

164. Peripheral distribution of current, m.m.f. and gap flux. The cylindrical shell comprising the currents on the two sides of the air gap, and the magnetic flux crossing the gap may be considered as the active region of any generator or motor, and the analysis of the phenomena in this region will yield most of the vital characteristics of the machine. Neglecting the localisation of current in the slots and taking the average amperes per in. of periphery in each phase belt, the heavy full-line curves of Fig. 81 show the primary current distribution of a three-phase full-pitch winding at the rotates at the same velocity. The changes of the flux curve, in shape and magnitude, are graphically analyzed for full-pitch windings on pages 380–390 of the 2nd edition of Hobart's "Electric Motors." See also pages 120–125 of Hobart's "Design of Polyphase Generators and Motors," especially the diagram on page 123.

165. Flux and current distribution for two-phase motors. In a two-phase motor the belts are broader, the steps in the current and flux distribution curves larger and the change of shape from instant to instant greater. The breaking up of the belts by using a five-sixths pitch in the three-phase, or a three-quarters pitch in the quarter-phase motor, reduces these variations and smooths out the curves.

166. Secondary current and m.m.f. relations. Since the secondary *e.m.f.* is induced by cutting the gap flux, the resulting secondary current will bear a definite space-phase relation to the gap flux (Fig. 84), and its *m.m.f.* will be sinusoidally distributed around the gap. Also, since the primary (or stator) counter *e.m.f.* is nearly equal to the impressed *e.m.f.* and is induced by the rotation of the gap flux at synchronous speed, the flux will be nearly constant in magnitude, for constant impressed *e.m.f.* Thus the resultant of the primary and secondary *m.m.fs.* will be nearly constant and such as to produce the constant flux.

167. Similarity of the induction motor to the transformer. Thus the induction motor is similar to a transformer. In fact, at standstill, it is a short-circuited polyphase transformer with distributed windings, and an air gap between primary and secondary. Like the transformer, the induction motor may be regarded as having three fluxes. These are the main flux, linked with both primary and secondary; and the primary and secondary leakage fluxes, linked only with the primary and secondary respectively (Par. 193 to 198). Owing to the air gap in the main magnetic circuit, the quadrature **magnetizing current** of the induction motor is several times as large as in a closed magnetic circuit transformer, even though the air gap is usually reduced to the lowest safe mechanical clearance. Because of this same air gap and the separation it makes necessary between the primary and secondary windings, the **leakage reactance** is also several times as large as in the average transformer.

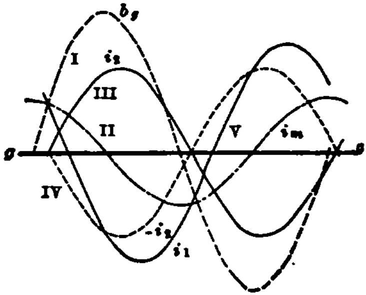


FIG. 84.—Peripheral distribution of flux and currents in polyphase induction motor.

Thus the **power-factor** of the induction motor is inherently low as compared with the closed magnetic circuit transformer. Representative power-factors for motors of various frequencies, speeds, and outputs, are given in Figs. 119 and 121.

168. Revolving flux and current distribution. Assume that the gap flux and currents are distributed sinusoidally around the gap periphery at any instant, and that the rotor conductors are independently short-circuited. Referring to Fig. 84, *gg* is the developed air-gap line; above this line is the primary or stator, and below it is the secondary or rotor. Flux directed upward in the figure is thus directed outward from secondary to primary; this direction will be called positive and will be indicated in the curve by ordinates measured upward from the gap line. Current directed outward from the paper will be called positive and will be so indicated in the curves. The flux and the rotor are assumed to be revolving counter-clockwise (right to left), the rotor less rapidly than the flux; therefore the rotor revolves clockwise with respect to the flux (left to right in the figure). Curve *I* represents by its ordinates the space variation of flux density, b_g, crossing the gap. This sinusoidal flux distribution will be assumed as the starting positions or space phases for a given condition of load. These distributions are shown in semipictorial fashion in Fig. 85, where the vectors B_g, I_2 and I_1 point to the positions at which the maximum positive values of these quantities occur, at the instant shown.

The time variation of all these quantities with respect to any particular point or conductor on the primary side of the gap, will be sinusoidal at primary frequency, and at secondary or slip frequency with respect to any point or conductor on the secondary structure. It is only from this point of view and with this understanding that it is possible to represent consistently primary and secondary variables on the same vector diagram.

Thus the vectors of Fig. 85 may be used to represent not only the space phases, but also the time phases of the several variables, when viewed from either stationary or rotating structure, it being obvious that there is no fixed phase difference between the primary current in a particular *primary* conductor, and the secondary current of very different frequency in a particular *secondary* conductor. By extending the vector diagram of Fig. 85 we obtain the complete space and time vector diagram of the induction motor in Fig. 86.

174. Analysis of vector diagram. E'_1 is that part of the impressed e.m.f. to neutralise the counter e.m.f. induced by the mutual flux Φ, the direction of the vector Φ being that of the plane of the coil when it links the maximum flux, just as the direction of B_g is that of the plane of the coil when it is cutting the densest gap flux and generating the maximum e.m.f. I_m is the magnetising current and I_{e+h} the core loss energy current, their

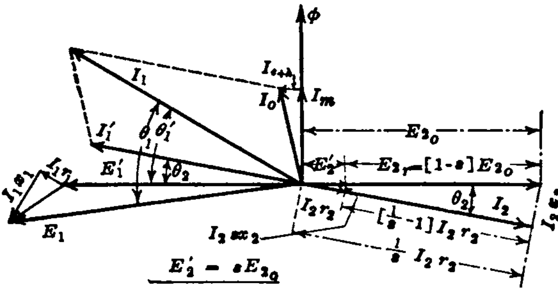


FIG. 86.—Vector diagram for induction motor.

sum being I_e, commonly called the exciting current. Assuming all secondary quantities reduced to primary turns, $E_{20} = E'_1$ is the e.m.f. that would be induced in the secondary at standstill. The actual secondary induced e.m.f. is E'_2, which equals sE_{20}, where s is the slip. $E_{2r} = (1-s)E_{20}$, is the e.m.f. that would be induced in the secondary if revolving in the flux Φ at its actual speed, and will be called the speed e.m.f. x_2 is the secondary leakage reactance at primary frequency, and sx_2 the same at slip frequency. The secondary resistance is r_2. The secondary current is therefore:

$$I_2 = \frac{E'_2}{\sqrt{r_2^2 + s^2 x_2^2}} = \frac{E_{20}}{\sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2}} \quad \text{(amp.)} \qquad (37)$$

(The two e.m.f. triangles corresponding to these equations are shown in Fig. 86.) I'_1 is the part of the primary current to neutralise I_2, and I_1 the total primary current. $I_1 r_1$ and $I_1 x_1$ are the e.m.fs. consumed by primary resistance and leakage reactance respectively. E_1 is the impressed e.m.f.

175. The corresponding equivalent circuit scheme is given in Fig. 87, which is exactly that of a transformer with a non-inductive load resistance $(r_2/s)(1-s)$.

although none too accurate for poor motors. By a poor motor is not meant necessarily a poor design, but a poor result which may be due to difficult specifications, *e.g.*, relatively low speed or relatively high frequency.

180. Approximate working formulæ for torque, output and starting current. For most purposes the following are sufficiently accurate. It will be observed that the constants of the exciting circuit do not enter; this is due to the approximation, which is equivalent to neglecting the exciting current as far as it affects the quantities considered.

Torque

$$T=\frac{7.05}{R_s}\,\frac{p'E_1^2\frac{r_2}{s}}{\left(r_1+\frac{r_2}{s}\right)^2+(x_1+x_2)^2}\qquad \text{(lb. ft.)}\qquad (42)$$

Slip corresponding to maximum torque

$$s_{T_{max}}=\frac{r_2}{\sqrt{r_1^2+(x_1+x_2)^2}}\qquad \text{(lb. ft.)}\qquad (43)$$

Maximum or stalling torque

$$T_{max}=\frac{7.05}{R_s}\,\frac{p'E_1^2}{2\left[r_1+\sqrt{r_1^2+(x_1+x_2)^2}\right]}\qquad \text{(lb. ft.)}\qquad (44)$$

Starting torque

$$T_s=\frac{7.05}{R_s}\,\frac{p'r_2E_1^2}{(r_1+r_2)^2+(x_1+x_2)^2}\qquad \text{(lb. ft.)}\qquad (45)$$

Starting current

$$I_s=\frac{E_1}{\sqrt{(r_1+r_2)^2+(x_1+x_2)^2}}\qquad \text{(amps.)}\qquad (46)$$

$$T_s=\frac{7.05}{R_s}\,p'\,I_s^2r_2\qquad (47)$$

r_2 corresponding to maximum starting torque

$$r_2=\sqrt{r_1^2+(x_1+x_2)^2}\qquad \text{(ohms)}\qquad (48)$$

Output

$$P_2=p'E_1^2\,\frac{\frac{r_2}{s}(1-s)}{\left(r_1+\frac{r_2}{s}\right)^2+(x_1+x_2)^2}\qquad \text{(watts)}\qquad (49)$$

Maximum output, i.e., stalling load

$$P_{2max}=\frac{p'E_1^2}{2\left[(r_1+r_2)+\sqrt{(r_1+r_2)^2+(x_1+x_2)^2}\right]}\qquad \text{(watts)}\qquad (50)$$

Slip corresponding to maximum output

$$s_{P_{2max}}=\frac{r_2}{r_2+\sqrt{(r_1+r_2)^2+(x_1+x_2)^2}}\qquad (51)$$

These approximations are fairly accurate except where the exciting current is large. As the primary current and power-factor are both largely affected by the approximation involved in the formulæ given above, their formulæ are not given. They can be most easily obtained from the circle diagram.

181. The method of percentages. For the purpose of discussing the operating characteristics of an induction machine, as well as in connection with problems involved in its design, the following method of valuing its constants will be found very convenient. Referring to Figs. 86 and 88, and remembering that $E_{20}=E_1'$, let $I_1r_1/E_1'=q_{r1}$; $I_1x_1/E_1'=q_{x_1}$; $I_2r_2/E_1'=q_{r2}$; $I_2x_2/E_1'=q_{x2}$; $q_x=q_{x1}+q_{x2}$. Let $I_2\cos\theta_2=I_T$ be called the torque current, since it is the component of I_2 which is effective in producing torque. $I_m/I_T=q_m$; $I_{e+h}/I_T=q_e$; $I_o/I_T=q_o$; $q=q_x+q_m=$ total quadrapower-factor. T_{max}/T_f is plotted against q_x in Fig. 90, for $k_T = 1.9$. From which, if 2 is the lower limit for T_{max}/T_f, q_x must not be more than 0.26.

185. Per cent. stalling torque. Hobart gives the per cent. stalling torque in terms of σ (Par. 207) and q_m as follows ("Polyphase Generators and

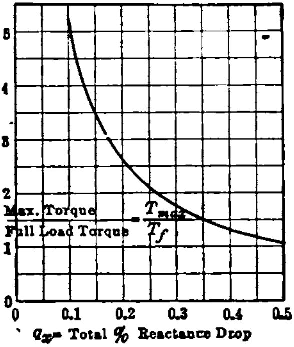


FIG. 90.—Ratio of maximum torque to full-load torque, plotted as a function of the total percentage reactance drop.

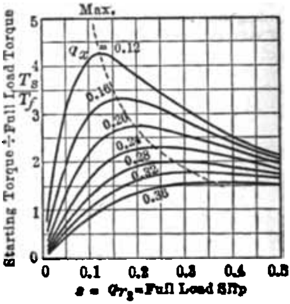


FIG. 91.—Ratio of starting torque to full-load torque.

Motors," page 190): $qT_{max} = 0.4\ q_m/\sigma$. This is equivalent to $k_T = 2.3$, (since $\sigma = 0.93\ q_m q_x$), which is considerably larger than 1.9 used in Fig. 90.

186. Starting torque. Beginning with Eq. 45, it can be shown that the per cent. starting torque is

$$qT_s = T_s \div T_f = \frac{1.20 q_{r2}}{(q_{r1} + q_{r2})^2 + q_x^2} \quad \text{(approximately)} \qquad (54)$$

Assuming $q_{r1} = 0.02$, the ratio T_s/T_f is plotted against q_{r2} in Fig. 91, for several values of q_x within the range of commercial motors; from which the quantitative limitations in the starting torque of the induction motor are obvious. The maximum value of the ratio T_s/T_f is obviously the same as T_{max}/T_f and occurs when q_{r2} is about equal to q_x.

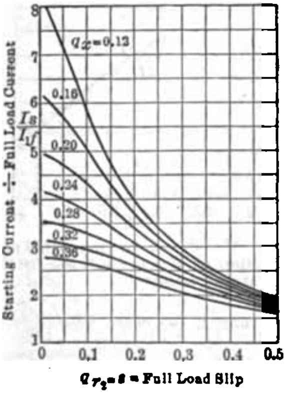


FIG. 92.—Ratio of starting current to full-load current, plotted as a function of the full-load slip.

187. Secondary starting resistance. Except in machines of very low reactance, a large starting torque is only possible by employing a comparatively large secondary resistance. In a squirrel-cage motor this means large secondary copper loss and large slip. The desirable resistance of a squirrel-cage secondary is usually a compromise between the low-resistance needed for speed regulation and efficiency, and the high resistance desired for starting torque, the relative weight given to these two considerations being determined by the specifications. In a wound-rotor machine, resistance can be inserted in the secondary for starting and cut out after the machine has attained speed.

191. Summary. Figs. 89 and 93 show clearly the relations of all the vital operating characteristics (except heating) of induction motors of whatever size, frequency, or voltage, to a very few constants; and although the results are only approximate, they will give a fair quantitatave grasp of induction motor characteristics. The largest errors will occur in small or low-speed motors with large exciting currents.

The most important of these characteristics are almost wholly determined by q_m, q_s, and q_{r1}. The other q's may be approximated with sufficient accuracy, since they play only a small part, and a considerable error in their approximation does not seriously affect the results. Of q_m, q_s, and q_{r1} the last may be given almost any desired value without much affecting the other constants; whereas q_m, and q_s are intimately related to one another, to the major design constants, and to the details of design. Their computation is given below.

MAGNETIZING CURRENT*

192. Air gap and gap ampere-turns. The air gap is usually reduced to a comfortable mechanical clearance, and may be taken approximately as $\delta = 0.015\sqrt[4]{\text{kw.}}$ (inches). The gap ampere-turns are computed in the same manner as for the alternator (see Par. 40, Figs. 39 and 40). Designate by a_1 and a_2, respectively, the primary and secondary slot-contraction factors; by $K_1 = a_1a_2$, the combined slot-contraction factor; by K_d the corresponding air-duct contraction factor; and by K_2 the ratio of gap ampere-turns to total ampere-turns. K_2 varies from 0.9 in slow-speed high-frequency machines to 0.7 in high-speed, low-frequency machines. These are, however, outside limits, and 0.8 may ordinarily be assumed as a fair average for a rough approximation. Then the total ampere-turns for the longest complete magnetic circuit is $Ni_l = (0.626\mathfrak{B}\delta)/K_1K_2K_d$.

Let Δ = peripheral loading corresponding to torque current I_T (Fig. 88); $\mathfrak{B}$ = maximum or crest value of equivalent sine-wave of gap-flux distribution; v = peripheral velocity (ft. per sec.) of revolving field; for k_b and k_p see Par. 28–31 (synchronous machines).

Then, assuming that the magnetizing current is equivalent to a sinusoidal distribution of current density, of root-mean-square value Δ_m, $\Delta_m = 0.116\mathfrak{B}\delta f/(k_bk_pK_1K_2K_dv)$ and the per cent. or fractional magnetizing current is

$$q_m = \Delta_m/\Delta = 0.116\mathfrak{B}\delta f/(k_bk_pK_1K_2K_d\Delta v) \qquad (58)$$

For a three-phase motor $k_b = 0.955$. For five-sixths pitch $k_p = 0.96$. For open-slot stator and nearly closed-slot rotor, $K_1 = 0.85$ (approx.). For a well-ducted motor $K_d = 0.9$. For a 60-cycle motor, $K_2 = 0.8$ (approx.). Then, roughly, $q_m = (0.27\mathfrak{B}\delta f)/(\Delta v)$.

LEAKAGE REACTANCE*

193. The four elements of the leakage flux are computed as if separate from the main flux, whereas they are for the most part only distortional as to the main flux. Consider the phase-belt as the unit, Figs. 94 and 95.

194. Slot leakage.* This is the cross-slot flux (Fig. 94). The inch per-

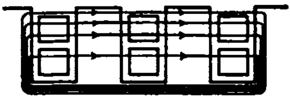


FIG. 94.—Illustrating cross-slot leakage flux.

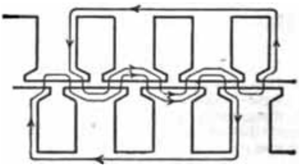


FIG. 95.—Illustrating tooth-tip leakage.

meance or flux linkage (lines) per amp.-in. of primary phase-belt is (see Fig. 42)

$$\varphi_{s1} = \frac{3.2}{N_{spp}}\left(\frac{1}{3}\frac{d_1}{w_s} + \frac{d_2}{w_s} + \frac{2d_3}{w_s + w_{so}} + \frac{d_4}{w_{so}}\right) \qquad (59)$$

*Transactions American Institute Electrical Engineers, Vol. XXIV, 1905, p. 338. Also Vol. XXVI, p. 1245.

indicates a circumferential current backward through the paper). If the ring is set radially inward nearer the shaft, φ_f may be increased as much as 50 per cent. (see the upper curve). Or, if the end ring is placed nearer the core, φ_f may be considerably increased (25 or 30 per cent.).

197. Belt-leakage.* For three-phase motors and full-pitch windings the flux linkage per amp-in. of belt is

$$\varphi_{B_3} = k_B K_1 K_2 K_{d\tau} \div 314\delta = K_{B\tau} \div 314\delta \qquad (61)$$

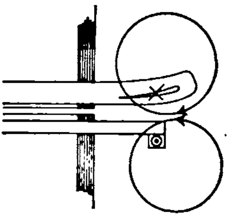


Fig. 99.—Coil-end leakage paths.

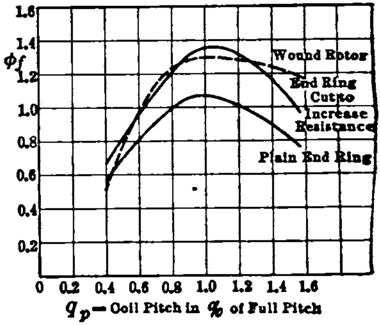


Fig. 100.—Coil-end flux linkage lines per ampere-inch of phase-belt bundle (primary and secondary) as a function of the coil pitch.

where k_B is given in Fig. 102 and $K_B = k_B K_1 K_2 K_d$. For a two-phase motor with full-pitch winding

$$\varphi_{B_2} = K_{B\tau} + 105\delta \qquad (61a)$$

For a squirrel-cage motor, take values about one-half of these values.

A coil pitch of five-sixths in a three-phase, or of three quarters in a two-phase motor, reduces the belt width, permeance of the mean paths, conductors per belt, and mean phase-difference, each to one-half. The belt reactance is thereby reduced to about one-sixteenth of its full-pitch value.

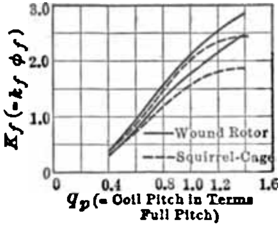


Fig. 101.—Coil-end leakage constant.

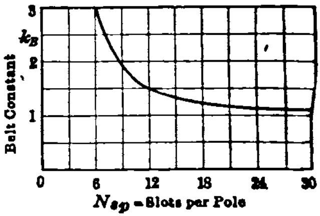


Fig. 102.—Belt leakage constant.

198. Summary. Adding together the several φ's with their proper pitch factors the total reactance for a three-phase motor is

$$x_2 = 2\pi f 2plN^2_{cpp}10^{-8}(k_{ps}\varphi_{sb} + k_{pt}\varphi_{tb} + k_3 K_f + K_{B\tau} \div 314\delta) \qquad (62)$$

where N_{cpp} = conductors per pole per phase.

* See *Transactions* International Electrical Congress, St. Louis, 1904 ————, Vol. I, page 706.

resistance varied (variable s), the locus of the current is a circle. The exciting current must be added to obtain the total current.

The corresponding diagram is shown in Fig. 104. If this diagram be rotated counter-clockwise until E_1 is slightly below the horizontal on the left, it will correspond with the left-hand side of the regular vector diagram, Fig. 86. The diameter of the circle in Fig. 104 is $E_1/(x_1+x_2)$. B_s is the short-circuit or standstill point; $\tan \varphi_s = \dfrac{r_1+r_2}{x_1+x_2}$, and $B_sG_s + G_sH_s = r_2 + r_1$.

The approximation involved in this diagram is such that serious errors are not introduced in the case of a good motor, *i.e.*, a motor with low exciting current and low reactance; but the errors become serious in poor motors.

200. Note concerning interpretation of approximate circle diagram. In what follows E_1 and I_1 are the volts and amperes of a single phase. If E_1 be the line voltage multiplied by $\sqrt{3}$, and I_1 the line current, p' should be omitted from all expressions for power and torque.

201. The impressed power is, $P_1 = p'\,E_1I_1 \cos\theta_1 = p'E_1 \times \overline{BK}$; but since $p'E_1$ is constant, $\overline{BK}$ is proportional to and a direct measure of the power delivered to the motor; *i.e.*, a scale of watts or kilowatts can be chosen according to which $\overline{BK}$ indicates directly the impressed power, P_1.

202. Losses, output, torque and slip. According to the same scale $\overline{HK}$ indicates the core loss, $p'E_1I_{e+h} = P_{e+h}$. If I_0 is measured when the motor is running light, the corresponding power will include the friction loss, in addition to core loss, and since the friction loss is practically constant,

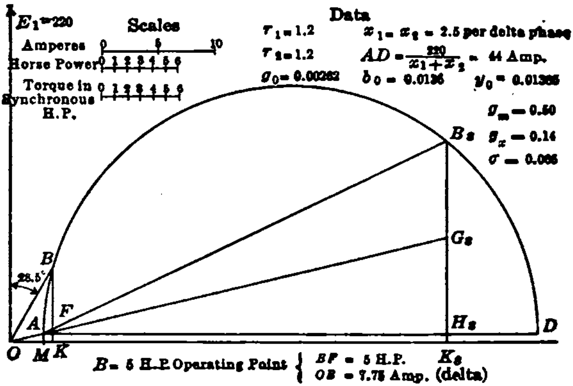


FIG. 105.—Approx. circle diagram for a 5-h.p. three phase, 6-pole, 220-volt delta-connected induction motor.

it could be charged to the exciting circuit (Fig. 103), by increasing g_e in the proper degree. It will be so taken here, *i.e.*, that $\overline{HK}$ represents core and friction loss. Similarly $\overline{HG}$ represents the primary copper loss, $\overline{GF}$ the secondary copper loss, $\overline{FB}$ the output (P_2), $\overline{GB}$ the power transmitted across the air gap (*i.e.*, the torque in synchronous watts) and $\overline{GF} + \overline{GB}$ the slip.

203. Efficiency (Par. 210-216). The efficiency is $P_2/P_1 = BF/BK$.

204. Power-factor (Par. 216). The power-factor, $\cos\theta_1$, may be readily determined from the diagram by drawing a unit circle about O and measuring the vertical intercept of the I_1 vector on this unit circle.

Thus for any point B on the circle, all the important variables of the induction motor may be readily determined from the circle diagram.

205. Experimental determination of circle diagram. The experimental data necessary is, ising current to the short-circuit current, but this is inaccurate for motors with high secondary resistances. It is dependent upon the machine proportions and independent of the load.

For all practical purposes it is

$$\sigma = 0.93 q_m q_s = \frac{3.95 S\delta}{\tau^2} + \frac{3.95 A}{N_{sp}^2} + \frac{1.32 K_f \delta}{l} + 0.0052 \tag{66}$$

For values of S, A and K_f see Par. 195 to 198 and Figs. 96 to 101. S varies from 6 to 18 with an average of 10 to 12; A from 0.2 for open slots

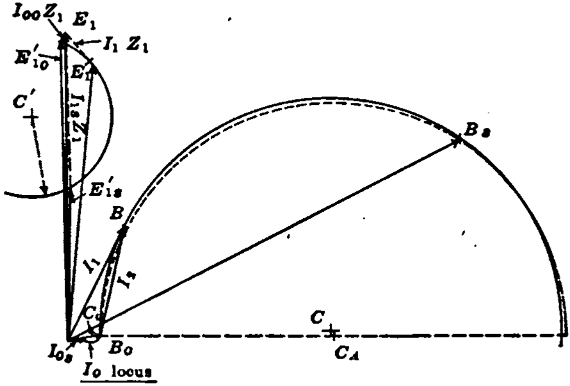


Fig. 107.—Approximate and accurate circle diagram of an induction motor.

on one side of gap to 0.42 for nearly closed slots on both sides, and K_f from 1.5 in good squirrel-cage motors to 2.2 in slip ring motors, see Fig. 101.

Kierstead has proposed a somewhat simpler though less rational formula; it is

$$\sigma = C\left(\frac{0.079}{l} + \frac{0.19}{\tau} + \frac{3}{N^2_{sp}}\right) \tag{67}$$

where C varies with the air gap according to the accompanying table

δ	0.024	0.032	0.039	0.047	0.055	0.063	0.071	0.079	0.087	0.095
C.........	0.71	0.88	0.99	1.09	1.17	1.25	1.31	1.39	1.45	1.52

Of these equations, Eq. 66 gives results quite as accurate as the average test results can check, and for all types of machines. Eq. 67 gives fair results for the majority of standard machines, although inaccurate for extreme cases.

The leakage factor or circle ratio is much used by European designers. Reference may be made to Hobart's "Electric Motors" and "Design of Polyphase Generators and Motors" where the designing of induction motors is explained with the assistance of the circle diagram in a simpler though less accurate manner than is described in Par. 243 to 259 below.

208. Maximum power-factor. In terms of σ, it is given by various authors, thus

$$\text{Maximum power-factor} = \frac{1}{1+2\sigma} \tag{68}$$

$$\text{Maximum power-factor} = \frac{1-\sigma}{1+\sigma} \tag{69}$$

Density in stator core (lines per sq. in.)	Core loss in stator core (watts per lb.) for various frequencies		
	$f=15$	$f=25$	$f=50$
39,000	0.50	1.00	2.27
52,000	0.77	1.36	3.36
65,000	1.00	1.81	4.54
78,000	1.18	2.36	
90,000	1.45	2.81	

STANDARD POLYPHASE INDUCTION MOTORS

215. Representative power-factors and efficiencies of standard squirrel-cage motors. The losses and efficiency of a typical 5 h.p.,

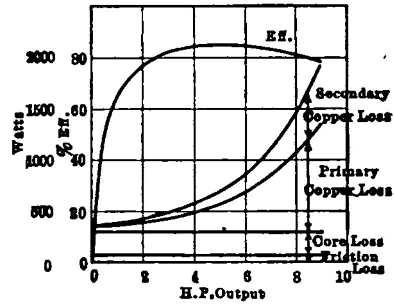


Fig. 108.—Losses and efficiency of a 5-h.p., 60-cycle squirrel-cage induction motor.

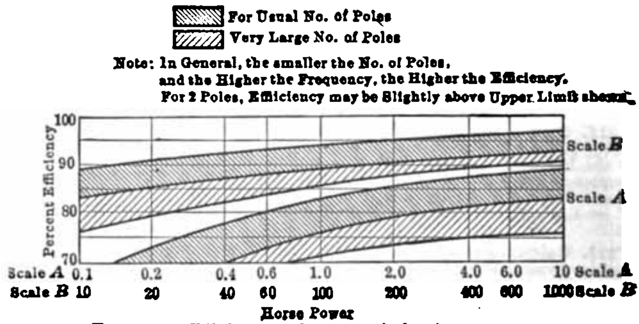


Fig. 109.—Efficiencies of average induction motors.

60-cycle, squirrel-cage induction motor are shown plotted against the output in Fig. 108. The efficiencies of average induction machines are plotted in Fig. 109. While in any concrete case the designer has the power-factor and the efficiency within his control to a certain extent, he cannot depart widely from representative values which are inherent to the rating. The table in Par. 216 is abstracted from more elaborate tables on pp. 116 and

Data of complete weight (exclusive of shaft and bearings) of five 33-cycle squirrel-cage induction motors for a synchronous speed of 124 r.p.m.

Rated output	Weight
1,000 horse-power	15 metric tons
2,000 horse-power	25 metric tons
3,000 horse-power	34 metric tons
4,000 horse-power	42 metric tons
5,000 horse-power	50 metric tons

218. Weights per horse-power. A $\frac{1}{10}$-h.p. 1,200-r.p.m. polyphase induction motor weighs a matter of 19 lb. while a 10,000-h.p. 120-r.p.m. polyphase induction motor would weigh some 176,000 lb. It is interesting to note that while the large motor has 150,000 times the output of the smaller, it only weighs about 9,000 times as much. Moreover its speed is about one-tenth as great as that of the smaller motor.

219. Starting apparatus for standard polyphase induction motors. Ninety per cent. of all the induction motors built in America are of the squirrel-cage type. Whereas the squirrel-cage motor when of small size is switched directly on the line at starting, it is necessary in starting larger squirrel-cage motors to employ induction starters or else the star-delta method. Slip-ring induction motors are started by means of a rheostat connected into the rotor circuits, and gradually cut out as the motor acquires speed. With the growth of electricity-distributing net-works, it has become expedient to relax the earlier onerous requirements imposed in the matter of permissible starting currents. On some systems fairly large squirrel-cage motors are permitted to be started by switching directly upon the line without induction starters. Designs in which the "deep-slot" effect (Par. 190) is correctly employed may be started directly from the line with a more moderate rush of current, and hence are the more appropriate as regards the lesser line disturbance at starting.

220. Induction-motor starting currents. The currents taken by good three-phase squirrel-cage induction motors at the moment of starting, have values approximately in accordance with the following table, which corresponds to $q_s = 0.13$ and $s = q_{r_2} = 0.04$, see Figs. 91 and 92 and accompanying text.

A	*B*	*C*	*D*
Pressure at motor in per cent. of line pressure	Line starting current in per cent. of full-load current	Motor starting current in per cent. of full-load current	Starting torque of motor in per cent. of its full-load running torque
40	112	280	32
60	250	420	72
80	450	560	128
100	700	700	200

221. Calculated starting characteristics. The following table, taken from page 451 of the 2nd Edition of Hobart's "Electric Motors" (Whittaker & Co., London, 1910) gives the calculated starting characteristics of a 6-pole 25-cycle 200-h.p. 500-rev. per min., squirrel-cage, three-phase, induction motor.

The motor to which these last results correspond, was designed for very high efficiency and without any regard to starting torque ($q_s = 0.12$ and $s = 0.02$). Consequently, we find that when started on the six-tenth taps of the induction starter, the starting torque is only 55 per cent. of full-load running torque and the line current is practically three times full-load current, a worse result than that of the average motor corresponding to the earlier table. By employing the "deep-slot" principle in the design of 60-cycle squirrel-cage induction motors, the application of the full-line pressure at starting will be accompanied by a much smaller flow of current from the line than that shown in the table for the average motor. While the torque may

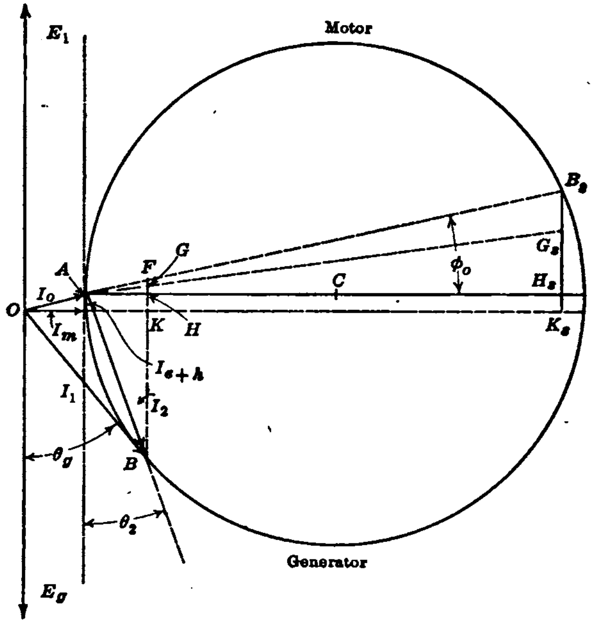


FIG. 110.—Circle diagram of induction generator.

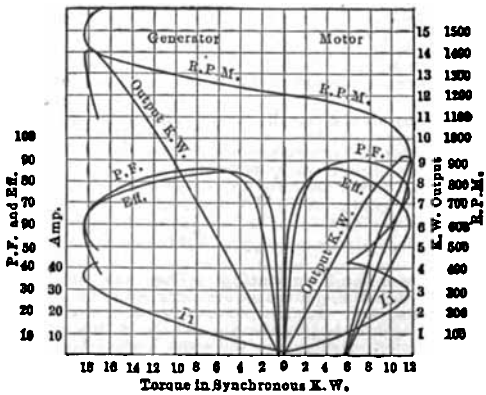


FIG. 111.—Generator and motor characteristics of induction machine.

234. Effect of angular variation of prime movers. Since the division of load in the case of parallel-connected induction generators is dependent upon the relative speeds rather than upon the relative angular positions of the rotors, the relative angular displacements within one revolution of a reciprocating steam or gas engine do not produce objectionable power surges between the various generators, as frequently occurs in the case of synchronous machines. Even gas engines are quite satisfactory in this connection.

235. Effect of e.m.f. wave shape on parallel operation. The induction generator is largely passive as to wave shape, the latter being impressed from without, and in the presence of e.m.f. harmonics, behaves like an inductive reactance which tends to damp the harmonic currents.

236. "Switching in." It is not necessary to synchronise when throwing in an induction generator, since the latter has no e.m.f. until excited from the line and when so excited is always in phase. It is only necessary to get the induction machine up to speed and throw it in. As the machine cannot pick up its load until the field is established, the first rush of current is wholly exciting current and practically independent of the slip. This initial current rush is quite analogous to that experienced when switching in transformers, and is explained on the same basis. Since with large units this exciting current rush is undesirably large, reactance coils are inserted while switching in, and then cut out as soon as the steady state is reached.

237. Hunting. The connection between the speed of a synchronous machine and its circuit frequency is exactly analogous to a mechanical elastic coupling, and that between an induction machine and its impressed frequency, to a friction coupling in which the friction grip is proportional to the slip. The latter tends to damp out any oscillations of the system, while the former has a natural period of its own which may tend to amplify some periodic disturbances of the system. The induction machine is thus decidedly superior on the score of hunting.

238. Short-circuits. The short-circuit current of a synchronous alternator consists of two parts: first, the transient part limited only by the combined equivalent leakage reactance of armature and field circuits; and second, the steady part determined by the much larger synchronous reactance of the armature. The former is the serious element, from three to five times the latter. It is this transient effect that twists the armature coil ends as if they were made of rope. With the induction generator the case is quite different. There is no steady portion, since the source of exciting current disappears when the short-circuit occurs, just exactly as in the case of a direct-current shunt-wound generator. The transient effect is present, but in only small degree since, with the same flux as in the synchronous machine, the reluctance of the magnetic circuit (and hence the energy of the field) is only a fractional part (sometimes less than 10 per cent.) as great.

239. Low-resistance squirrel-cage rotor. Since the induction generator requires no starting torque, the rotor is always of the squirrel-cage type and of as low resistance as is consistent with the cost of the copper and the available space. This means a very low secondary-copper loss, and a slip of less than 0.5 per cent. in large machines.

240. First cost and ventilation. Owing to the exceedingly simple rotor construction, the induction generator would be cheaper to build than a synchronous machine of the same capacity, were it not that the short air gap makes ventilation very difficult, and the more so in those ratings which are otherwise the best suited to this type of machine. This, together with the tooth-frequency losses incident to the short air gap, makes the induction generator of doubtful superiority except in special cases. The switchboard is also simpler, and the switching operations are of reduced complexity.

241. Best field of application for induction generators. The specifications from which the best induction machines can be produced are high speed, large output, and low frequency, which means steam-turbine or high-head water-wheel drive in large power stations. Unfortunately these are the specifications which lead to great difficulties in ventilation, owing to the short air-gap. A large city-railway system with the load considerably made up of synchronous converters with underground-cable distribution, satisfies the conditions as to excitation.

q_{il} = net iron length ÷ gross core length
q_m = ratio of exciting current to torque current
q_p = ratio of coil pitch to pole pitch
q_r = resistance drop ÷ induced volts
q_{so} = slot opening divided by tooth pitch
q_{sw} = slot width ÷ tooth pitch
q_x = leakage reactance volts ÷ induced volts
q_{xB} = belt-leakage reactance volts ÷ induced volts
q_{xf} = coil-end reactance volts ÷ induced volts
q_{xs} = slot reactance volts ÷ induced volts
q_{xt} = tooth-tip leakage reactance volts ÷ induced volts
R = rev. per min.
S = slot factor
s = slip
$w.c.$ = total works cost (dollars)
V_{cm} = total volume primary copper (circular-mil in.)
v = peripheral velocity at synchronism (ft. per sec.)
w_s = width of slot (in.)
Δ = torque-current amp. conductors per in. of periphery
Δ_1 = primary amp. conductors per in. of periphery
Δ_m = magnetizing current amp. conductors per in. of periphery
δ = radial depth of air-gap (in.)
ξ = output coefficient, h.p.
ξ_k = output coefficient, kw.
σ = leakage factor (sometimes also termed the circle ratio).
τ = pole pitch (in.)
τ_t = tooth-pitch (in.)
φ_f = flux per amp. in. of phase belt bundle of coil-ends, Fig. 100

Subscript 1 refers to primary.
Subscript 2 refers to secondary.

245. The output equation. The volt-amperes induced per sq. in. of peripheral surface are, $\Delta\sigma'' = 8.5 k_b k_p v B \Delta 10^{-8}$, see Par. 22. and the kw. total developed are

$$K = 8.5 \times \pi k_b k_p \Delta v B D l 10^{-11} \qquad \text{(kw.)} \qquad (71)$$

This is the power P'_1 transmitted across the air gap and not the output, which is less by the secondary copper loss and the friction and windage loss. Assuming the latter to be about 1 per cent. and remembering that $q_{r2} = s$, we get as the horse-power output

$$\text{h.p.} = 3.55\,(1-s)\,k_b k_p \Delta \mathfrak{B} v D l 10^{-10} = k_o D l \qquad (72)$$

where

$$k_o = 3.55\,(1-s) k_b k_p \Delta \mathfrak{B} v 10^{-10} \qquad (73)$$

is the **specific output** in horse-power per sq. in. of projected area of the airgap cylinder. Assuming a three-phase motor ($k_b = 0.95$), of moderate capacity, with squirrel-cage rotor ($s = 0.03$), and five-sixths pitch of coils ($k_p = 0.96$),

$$k_o = 3.2 \Delta \mathfrak{B} v 10^{-10} \qquad (74)$$

By substituting for v in equation 72, its value $(\pi D R/720)$, we get

$$\text{h.p.} = 1.55\,(1-s) k_b k_p \Delta \mathfrak{B} D^2 l R 10^{-12} = \xi D^2 l R \qquad (75)$$

where ξ is called **output coefficient.** With the same assumptions as above,

$$\xi = 1.38 \Delta \mathfrak{B} 10^{-12} \qquad (76)$$

If horse-power be replaced by kilowatts in Eq. 75,

$$\xi_k = 1.03 \Delta \mathfrak{B} 10^{-12} \qquad (77)$$

Sometimes the output equation is more convenient in the form of Eq. 72, and sometimes in that of Eq. 75. Referring to Eqs. 72 and 75, and assuming R specified, $\mathfrak{B}$, Δ, and v determine Dl, and $D = (720v)/(\pi R)$; or $\mathfrak{B}$ and Δ determine $D^2 l$, and $k_3 = \tau/l = (\pi D)/(2pl)$, determines the ratio of D to l. As the cost of the machine increases approximately with Dl and with $D^2 l$, $\Delta\mathfrak{B}$ should be as large as possible.

246. Representative values of output coefficient. ("Design of Polyphase Generators and Motors," Hobart.)

$k_s = \frac{1}{2}q_p$ (see Fig. 114) which is too small on the score of ventilation. However, as the minimum is a fairly flat one k_s and v may be considerably increased without seriously increasing the copper volume, to the great gain of q_m and the ventilation. There is a pretty decided limit, however, since the coil-end copper increases as v^2 and the coil-end leakage as v^3. Assuming h.p. = 10, $R = 1{,}200$, $f = 60$, $\mathfrak{B} = 25{,}000$, $\Delta = 400$, $m = 600$; the weights of copper are as given in Fig. 114; D, l, q_{xf} and k_s $(= \tau/l)$ are also shown.

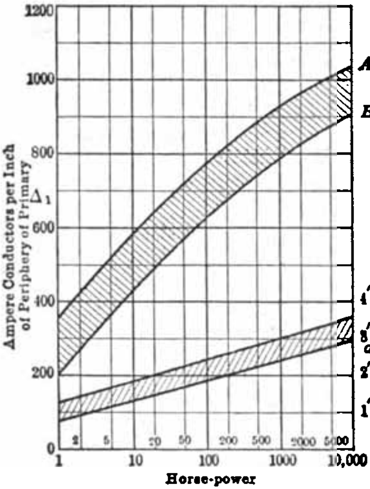


FIG. 113.—Values of peripheral loading and primary slot depth.

254. Total cost of manufacture or works cost. A very useful criterion of k_s (and therefore of v), from the cost standpoint, may be obtained by the use of the following empirical formula for the total works cost (t.w.c.) taken from Hobart's "Electric Motors," 2d Edition, page 594.

$$\text{t.c.w.} = K_c D(l + 0.7q_{p\tau}) \quad \text{(dollars)} \qquad (79)$$

where K_c is a factor which varies but slightly with the size and proportions of the motor. By substituting, differentiating, and solving, the value of k_s for a minimum twc is $k_s = 0.71/q_p$. Again the minimum is a fairly flat one and considerable leeway is available. If larger values of v and k_s are taken, larger B and Δ may also be employed,

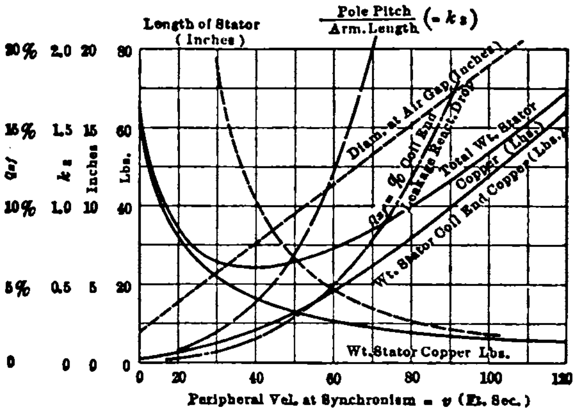


FIG. 114.—Influence of peripheral velocity on vital induction-motor characteristics.

256. Coil-end-leakage limit of peripheral velocity. The other limit to v is the coil-end leakage, which is proportional to v^3 (see Par. 196, Eq. 62a, and remember that k_3 is proportional to v^2). q_{xf} has been computed for the 10-h.p. motor of Fig. 114, and is plotted in Fig. 117. If Δ were taken larger, both the total copper and the q_{xf} curves of Fig. 114 would be higher. A change in $\mathfrak{B}$ would not affect either appreciably. By using a fractional-pitch winding, both of these curves would be lowered somewhat, but there is a limit to this, owing to the necessity for more active conductors to generate the same e.m.f., or larger values of $\mathfrak{B}$ and q_m. A larger value for R would lower both, and a larger h.p. would lower q_{xf}; but these are specifications. The curves of Fig. 117 show without necessary comment why v may not be carried above a pretty definite upper limit for any given specifications.

257. Fractional-pitch windings. The use of any considerable pitch reduction or chording (beyond the five-sixths used for the curves of Figs. 118 to 121) means an increased exciting current (q_m) and a decreased reactance (q_x), other things being equal. If the motor in question were properly designed for full pitch, the pitch reduction should be accompanied by a decrease of $\mathfrak{B}$, an increase of Δ and of v. This will not usually increase either the power-factor or the specific output (k_e or ξ), but rather the reverse. It will, however, increase the diameter and decrease the length of the core, thus increasing the natural heat-dissipating power of the machine. It will also reduce the overall length, which in some cases means a considerable saving in cost. The most obvious place for the fractional-pitch winding is the two-pole motor, where a pitch as low as 0.5 is sometimes used.

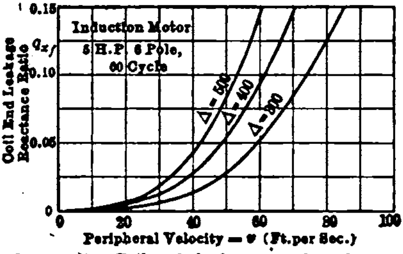


Fig. 117.—Coil-end leakage as function of peripheral velocity and of peripheral loading for a 5-h.p. motor.

258. Data. Figs. 118 to 121 give values of $\mathfrak{B}$, v and Δ_1, and the corresponding values of q_x, q_m, q, k_e, ξ, k_3, and power-factor, for squirrel-cage motors with open-stator slots and nearly closed-rotor slots, and a coil pitch of five-sixths except where otherwise designated. The curves have been computed on the following basis: Δ_1 is approximated largely on the basis of heating; the values of $\mathfrak{B}$ are a compromise between first cost and stalling torque on the one hand and power-factor on the other, with more weight given to the former; k_3 (upon which v depends) is a compromise between cost and ventilation. From these assumed values the other quantities have been computed. Both $\mathfrak{B}$ and k_3 are larger than are frequently met with in practice, and doubtless too large to meet some specifications.

If closed slots had been used on the stators, larger values of $\mathfrak{B}$ could have been employed to advantage, not only because of the lower reluctance of air-gap but also because of the higher allowable tooth density. This increased $\mathfrak{B}$ does not ordinarily result in an increase of power-factor, sometimes the reverse is true; but it does mean a smaller machine and less active material. This may or may not mean a cheaper machine, according as the extra cost of labor in winding is or is not made up by the saving in material. In general it would be cheaper in Europe and more expensive in this country.

Two-pole motors are rarely met with and the data for this type is given to show possibilities and the value of low coil pitch for two poles. The disadvantage is chiefly connected with the poorer natural ventilation of a long-core machine with short air-gap.

259. Caution. Such information as given in Figs. 118 to 121, and elsewhere in tables or curves, should be taken only as a rough guide, and as indicating the relations between the several quantities involved, rather than their absolute values. These latter may be altered over a moderate range to suit special specifications, and will vary considerably in machines

265. Exciting current. This is nearly twice as large as for the three-phase case, approximately half of it being the reflection in the primary of the no-load secondary current which supplies an *m.m.f.* in space and time quadrature with the primary *m.m.f.* and thus produces a revolving field.

266. Output and rating relative to polyphase machine. The safe single-phase rating of a three-phase motor at rated voltage is about 60 per cent. of the three-phase rating at the same voltage, but a better balance of losses is obtained at a slightly increased line voltage, when the safe output may be increased to about two-thirds that of the polyphase case.

267. Secondary current and copper loss. Unlike the three-phase motor, the rotor current of the single-phase motor is not zero at synchronism, but has a definite value. This may be looked upon as a part of the exciting current, and is present in lesser degree at higher slips. The secondary copper loss is also not zero at synchronism and is no longer proportional to the slip. It is obviously larger at all loads than when running three-phase.

268. Methods of starting. The most common method of starting single-phase induction motors is known as the split-phase method. The motor is supplied with an auxiliary winding in space quadrature with the

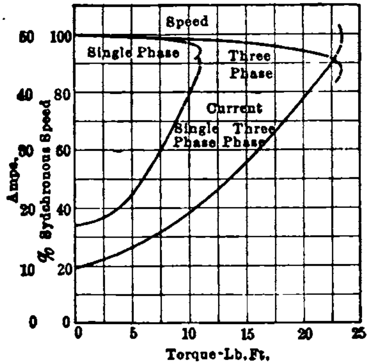


FIG. 124.—Characteristic curves of a three-phase induction motor operated (1) from a three-phase circuit and (2) from single-phase circuit.

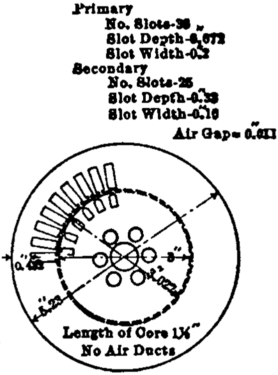


FIG. 125.—Dimensions and data of a one-eighth horse-power single-phase motor.

main windings (sometimes this consists of the third phase of a three-phase Y winding). The supply current is divided before it reaches the motor, one part going through a reactance to the main winding, and the other through a non-inductive resistance to the auxiliary or starting winding. If the resistance and reactance are properly chosen, the phase difference thus produced between the e.m.fs. E_m and E_s is sufficient to produce an elliptical revolving field.

Sometimes in the case of small motors which can be thrown directly on the line, L is omitted and the extra resistance is obtained within the starting winding itself by using a small size of wire. In this case the cutting out of the starting winding is accomplished automatically by a centrifugal device. The dimensions and data of such a motor are given in Par. **270** and in Fig. 125. Its test data are given in Fig. 126.

269. Design of single-phase induction motor. From the above it is obvious that to design a single-phase motor means simply to design a good polyphase motor for about one and one-half times the desired single-phase output.

obtained with increasing resistance steps. In Fig. 128 the current-torque curve for the above speed-torque curves, is shown, the torque per amp. being independent of the speed for this type of control. Since with this method speed variation is obtained by wasting energy in rheostats, it is not suited to continuous running at speeds much below synchronous speed.

273. Disadvantages of secondary-resistance control. For any one value of the resistance the speed changes greatly with variations of the load and rises to practically its synchronous value at no-load, whatever be the resistance. The higher the resistance, the more the speed will vary for a small change in load. An amount of power proportional to the speed reduction is lost in the resistance, that is, if the speed is decreased to 30 per cent. below normal, 30 per cent. of the energy taken from the line is lost.

274. Advantages of secondary-resistance control are simplicity of connections and relatively small increase in cost of motor, over that of a constant-speed induction motor. This method is exactly equivalent to the armature-series-resistance control of the speed of a direct-current shunt motor under constant voltage.

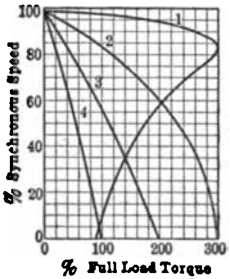


FIG. 127.—Torque speed curves of an induction motor with speed controlled by varying the resistances in series with the secondary circuits.

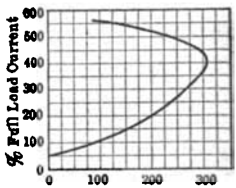


FIG. 128.—Torque-current curve for induction motor with secondary speed control.

Uses—cranes, elevators and rolling mills.

275. Frequency changing as a means of securing various speeds. There are various ways of applying this principle and all are characterized by high first cost. *E.g.*, use several alternators of different frequencies, an objection being the trouble in throwing over from one to the other; or use alternator driven at variable speed, when each motor must have its own alternator.

276. Speed change by variation of number of poles (multispeed motors). It is obviously possible by regrouping the coils on the stator of an induction motor so as to alter the number of poles, to also alter the synchronous speed of the motor. With pole-changing a squirrel-cage rotor is generally used, as with a wound rotor the rotor conductors must also be regrouped. But in certain services such as in rolling mills, secondary resistance control demands a wound rotor. With an ordinary two-layer drum winding the pitch of the coils changes as the number of poles is changed; for instance a two-thirds pitch for four poles means a one and one-third pitch for 8 poles. On this account it is in practice unsatisfactory to carry the range of speed variation by pole changing beyond the 2 to 1 ratio. The above suggested pitch is on the whole the most satisfactory for a two to one change. It is quite possible to get into serious difficulties by the use of certain other pitches. This method of pole changing results in poor constants at one or both of the speeds, and the controller is usually expensive. Instead of regrouping the coils, the stator may be provided with separate windings of different pole numbers. This method is also expensive.

277. Direct-concatenation and differential concatenation control.* If the secondary of a wound rotor is connected to the primary of another in-

* Danielson, Ernst. "A Novel Combination of Polyphase Motors for Traction Purposes," *Trans.* A. I. E. E., Vol. XIX, page 527.

rating. If, however, a 16 h.p. 16-pole motor be concatenated with a 4-h.p. 4-pole motor, their combined synchronous speed at 60 cycles will be 360 rev. per min. Neglecting losses, four-fifths of the electrical power delivered to the 16-pole motor will be converted to mechanical power and one-fifth transmitted to the 4-pole motor and there converted into mechanical power. In this case, the breakdown output of the set will be a little less than four-fifths that of the large motor, which is not a sufficient reduction to influence the rating of the set, which on a heating basis will be only a little less than that of the larger motor, or a little less than four-fifths that of the two motors combined. The same is true of the differential concatenation with a speed of 600 rev. per min. This set thus has three speeds—360, 450, 600 (rev. per min.), all with approximately the same power, and a speed of 1,800 rev. per min. with the small motor alone, but at only 4 h.p. In differential cascade the small machine acts as an induction generator and frequency changer. This arrangement will not start itself except by connecting the small motor to the line until the speed is nearly that desired, then throwing over to the differential cascade connection. The cascade operation is thus much more economical and efficient for moderate speed changes, while the pole changing method is more satisfactory for a two- to one-speed ratio.

281. The spinner motor. Different speeds are secured with the spinner motor by a combination of electrical and mechanical features. The motor consists of a "stator" or fixed primary, a "rotor," and between them a "spinner" rotating independently of the rotor, and having a short-circuited winding which is the secondary for the stator, and a slip-ring winding which is the primary to the rotor. The primary on the stator and that on the spinner are wound for different numbers of poles, the stator being usually wound for the larger number of poles, owing to the larger diameter.

282. Securing various speeds with spinner motor. By clutching together the spinner and rotor, and exciting only the stator winding, one speed is obtained; by exciting only the spinner and locking it to the stator, a second speed of the rotor is obtained; by allowing the spinner to run free and exciting both winding so that the *m.m.f.* of each travels in the same direction, gives a third speed, the sum of the two elementary speeds, and by exciting them so that the *m.m.f.* travels around them in opposite directions gives a fourth speed, the difference between the two elementary speeds.

283. Bibliography of spinner motor. The development of the spinner motor is due Mr. Henry A. Mavor, who has described it in the following papers: "Electric Propulsion of Ships" read before the Institute of Engineers and Shipbuilders in Scotland, on Feb. 18, 1908; and "Marine Propulsion by Electric Motors," read at the Institute of Civil Engineers, on Dec. 7, 1909 (see p. 134 of Vol. CLXXIX of *Proc.* I. C. E.). The development of this type of motor has recently been taken up by the Oerliken Co., and a line of spinner motors developed by them is described on page 1247 of the Electrical World for May 30, 1914, in an article by A. Hoeffleur and M. P. Misslin entitled "Adjustable-speed Polyphase Induction Motors." The article contains complete data of the efficiency and other properties of these motors at various speeds.

284. Multiple motor. This is also the invention of Mr. Henry A. Mavor. A 500-h.p. motor of this type has been employed on the electrically equipped cargo boat Tynemount (Sec. 18). The rotor is of the squirrel-cage type. The stator has two independent mutually non-inductive windings of different pole numbers. For full-speed and power these two windings are fed with electricity of frequencies proportional to their pole numbers and hence cooperate to drive the rotor at a given speed. For low speed and power, the winding of greater pole-number is fed from the supply of lower frequency and drives the motor at the corresponding low speed. The motor is described in British Patent, No.12917, of 1909.

285. The "Hunt" internally concatenated motor.* This machine operates upon the "cascade" principle, having two superimposed magnetic systems in the stator, one field being generated by the stator winding, and the second by the rotor winding which reacts upon the stator producing the second magnetic field and giving the cascade effect. Consider the case of a

* Hunt. *Journ.* I. E. E.; Vol. LII, 1913, p. 406.

winding is connected in the one or the other direction with respect to the main winding according to the direction of rotation desired. The scheme is indicated diagrammatically in Fig. 161. To reverse the direction of rotation, *a* would be connected to *c*, and *b* would be connected to *d*. It is to be observed that in Fig. 161, the axis of the main brushes coincides with the axis of the main stator winding but has the same angle to the resultant of the main and reversing stator windings, which, in Fig. 159, it had with the main stator winding. Were no reversing winding provided, then in order to reverse the direction of rotation it would be necessary to shift the brushes over to the corresponding angle on the other side from the position for normal direction of rotation.

307. Repulsion-induction motors may be fitted for adjustable-speed operation (Par. **299**) by employing a transformer with its primary excited from the line circuit and its secondary interpolated in the circuit of the energy brushes. Such motors are arranged for a speed range of about 2 : 1, approximately one-half of this range being below and one-half above synchronous speed. Finally, *RI* motors are also arranged to give variable speed by shifting the brushes. A speed variation of 2 : 1 is provided in this way.

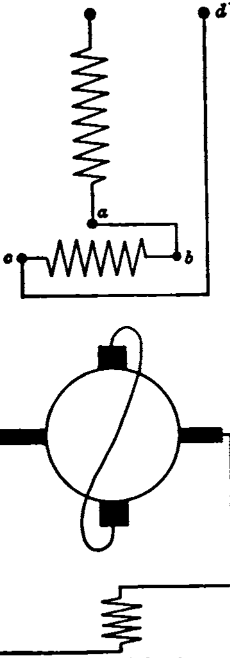


FIG. 161.—Diagram of circuits of reversible RI motor (General Electric Co.).

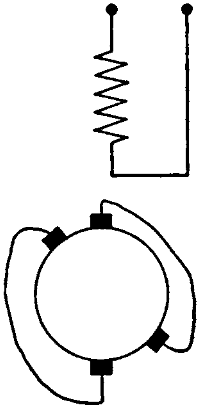


FIG. 162.—Diagram of Déri brush-shifting motor.

308. The Déri single-phase motor with speed control by brush shifting. Fig. 162 is a diagrammatic representation of the Déri single-phase motor. In this motor the speed is varied by varying the position of the brushes. Two of the brushes (those lying in the axis of the stator winding) are of fixed position. The other two are mounted on a movable yoke. For any given brush position the speed decreases with increasing load. Thus the motor has a series characteristic. The motor has good starting torque, the maximum value occurring when the movable brushes are at an angle of some 150 deg. to 160 deg. from the fixed brushes.

The motor is widely employed abroad for driving textile machinery. The simple mechanical arrangements required to effect the shifting of the brushes are less expensive than equivalent control devices which permit of accomplishing the speed variations without brush movement. Schnetzler has described the Déri motor in a paper published in the E. T. Z. for Nov. 14 and 21, 1907 at pp. 1097 and 1128. There is also a description of the Déri motor occupying pages 669 to 685 of the 2nd Edition of Hobart's "Electric Motors."

309. The Punga-Creedy single-phase commutator motor. Adjustable-speed, single-phase, commutator motors built under the Punga-Creedy patents have been placed on the British market by Messrs. F. Parkinson & Co., of Leeds. The motor is started by being thrown directly on the line and yields a starting torque of twice full-load torque, consuming twice full-load current. Figs. 163, 164 and 165 show (1) the speed-torque, (2) efficiency, and (3) power-factor curves for a 6-h.p. Punga-Creedy motor for printing-press work. Allusion has already been made in Par. 299 to the principles employed in motors of this class. The chief patents employed in the Punga-Creedy system are Punga's British Patent No. 10585 of 1906 and Creedy's British Patent No. 5136 of 1906. Punga and Creedy tested motors embodying these principles at early dates. The results of a number of these tests have been published in the *Trans.* A. I. E. E., Vol. XXVIII (1909), p. 75, in a paper by Creedy titled "A sketch of the theory of adjustable-speed, single-phase commutator motors."

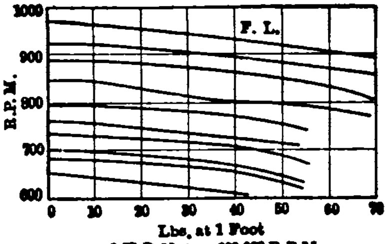


FIG. 163.—Speed-torque characteristics of a 6-h.p. Punga-Creedy single-phase motor for printing-press work. [Speed range is from 600 to 900 rev. per min.] [Messrs. F. Parkinson & Co. of Leeds, England.]

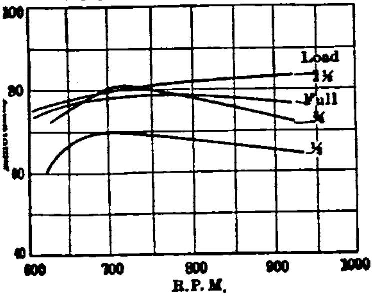


FIG. 164.—Efficiency curves for a 6-h.p. Punga-Creedy single-phase motor for a speed range from 600 to 900 r.p.m. [Messrs. F. Parkinson & Co. of Leeds, England.]

310. Polyphase shunt commutator motors. The most usual arrangement of a polyphase shunt commutator motor is that known as the Winter-Eichberg type. It is shown diagrammatically in Fig. 166 and is seen to consist of a motor with a three-phase stator and a commutator rotor. The speed will depend upon the pressure applied to the commutator brushes. By using a

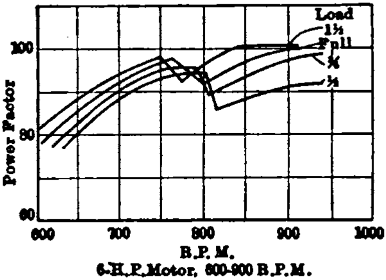


FIG. 165.—Power-factor curves for a 6-h.p. Punga-Creedy single-phase motor for a speed range from 600 to 900 rev. per min. [Messrs. F. Parkinson & Co. of Leeds, England.]

an important characteristic of the system. It is rarely practicable o desirable to provide for more than 30 per cent. regulation on 60-cycle sys tems or 50 per cent. regulation on 25-cycle systems, since it is difficult t design the auxiliary motor *B* for good commutation except at low frequencies and the frequency supplied to its commutator is proportional to the slip o the rotor of *A*. Greater capacities and greater speed ranges can be supplied by these sets the lower the frequency of the system from which they ar operated. A disadvantage of the Krämer system relates to the fact tha since such sets are usually required for slow-speed work, the auxiliary machin

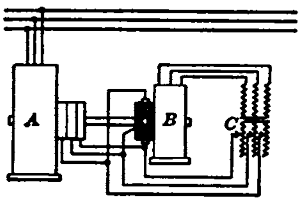


FIG. 170.—The Krämer system of speed control.

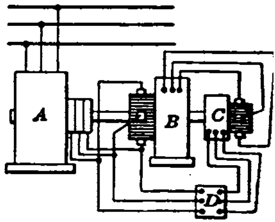


FIG. 171.—American variation of Krämer system.

B must be large and expensive, since the regulating motor must be designed to deliver its maximum output at the minimum speed of the main motor Furthermore, this arrangement necessitates a special design for the regulat ing motor for each installation where the synchronous speeds of the severa main motors differ, even though the total slip energy is the same in eacl case.

315. The Scherbius system of speed control. The arrangement in dicated in Fig. 172 is known as the Scherbius system. As in the Kräme system, the main motor *A*, is a simple induction motor with slip-rings. Th speed control is effected by an auxiliar set on an independent shaft. Thi auxiliary set comprises a commutato motor *B*, driving an induction generato *D*. The auxiliary machine, instead of delivering mechanical power to th main motor's shaft as in the Krämer system, returns electrical energy to th supply system. The amount of power transformed by the auxiliary set controlled by the adjustment of the position of the taps in the transform *C* (or rheostat if an exciter is used as described in Par. 314 and Fig. 17 The greater the amount of energy absorbed by *B*, the lower will be the spe of the main motor *A*, just as in the case of the Krämer system.

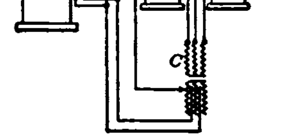


FIG. 172.—The Scherbius system of speed control.

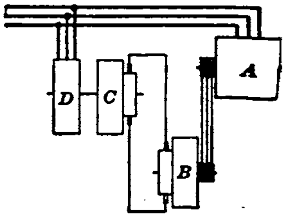


FIG. 173.—Synchronous converte method of speed control.

It may, in general, be said of the Scherbius system that the first cost less than for the Krämer system and that the operation is equally successf nous condensers may simultaneously operate as motors and it is customary to proportion them to thus operate to the extent of a consumption of 70 per cent. of their rated kv-a. They can, when thus operating, be so excited that they also draw from the line, as leading wattless kv-a., 70 per cent. of their rated capacity in kv-a. thus effecting power-factor improvement at the same time that they are serving as motors to deliver mechanical energy. Synchronous condensers are fitted with amortisseur windings to improve their starting qualities and to serve in preventing surging and hunting.

321. Non-synchronous phase modifiers (or phase controllers). Several varieties of apparatus customarily called phase advancers, have been developed for use in connection with individual induction motors for the purpose of improving their power-factor. Such apparatus is supplied with very low frequency electricity from the secondary windings of the induction motors. The frequency is that corresponding to the "slip" of the induction motor. Such phase advancers have an inherent characteristic which may conveniently serve to distinguish them from synchronous condensers (Par. **319** and **320**). This characteristic is that they are *not synchronous machines.*

Leblanc was probably earliest in drawing attention to methods of securing phase control by the use of non-synchronous auxiliaries. His proposals may be explained by reference to the accompanying diagrams. In Fig. 174, *AC* and *BD* represent two series-excited machines which, by suitable mechanical means, are driven at some appropriate speed. The field *C* of *AC* and the armature *B* of *BD* are connected in series with phase *N* of the quarter-phase rotor *MN*. Similarly, the field *D* of *BD* and the armature *A* of *AC* are connected in series with the other phase, *M*. Thus the current

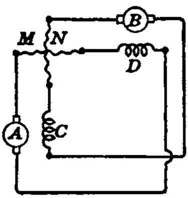


FIG. 174.—Leblanc phase advancer with two armatures and fields.

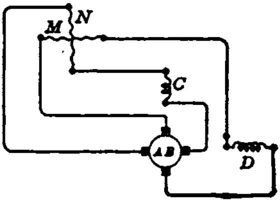


FIG. 175.—Leblanc phase advancer with a single armature.

for exciting the field surrounding each armature is displaced 90 deg. from the current in the phase with which the armature is in series. The armature pressure is consequently 90 deg. displaced from the pressure of the rotor winding with which it is in series. The resultant pressure and current in each rotor circuit, can, by these means, be displaced by any desired amount from the phase relations which would subsist were the rotor windings merely short-circuited on themselves in the customary way. Obviously, the greater the speed at which *A* and *B* are driven and the greater the *m.m.f.* provided by *C* and *D*, the more may the resultant pressures (and, consequently, also the currents) be advanced in phase from the conditions subsisting with a normally short-circuited rotor without a phase advancer.

In practice, however, it is more convenient to employ a single armature as indicated in Fig. 175, and to supply its commutator (assuming a bipolar design of phase advancer) with four brushes. The armature *AB* (Fig. 175) may be either directly mounted on the motor shaft or it may be driven at suitable speed by an auxiliary motor. The currents supplied to the brushes will be of the low frequency corresponding to the slip of the rotor *MN* of the induction motor whose power-factor it is desired to improve. Fig. 176 differs from Fig. 175 simply in that a field structure is indicated surrounding the armature *AB*, and also in the subdivision of the field windings *C* and *D* among the four poles. It must be noted that although, geometrically speaking there are four poles, magnetically considered, it is a bi-polar design. Similarly, as shown in Fig. 177, a three-phase bi-polar advancer

Dr. Kapp puts the cost of the vibrators as ranging from about $1.25 per h.p. in favorable cases, up to $4.00 per h.p. in relatively unfavorable cases, such, for instance, of considerable slip or low-pressure secondary windings or both.

330. General considerations in the design of phase advancers. In order to keep down the size and cost of the commutators and the losses at the brushes, it is desirable to wind the secondary of the main motor for fairly high pressure, and it is also desirable that the slip shall be small. It is claimed by Kapp that the smaller the slip, the more favorable is the case for employing a vibrating rather than a rotating phase advancer. The weight and cost per h.p. decrease with decreasing slip and increasing secondary pressure of the main motor. All of these types of phase advancer have the valuable feature of greatly increasing the instantaneous overload capacity of the motors with which they are employed. For a given load the primary current is considerably decreased, and the secondary current increased. This would lead to about the same total copper loss for a given load, were it not for the circumstance that it is usually quite practicable to increase the cross-section of the secondary copper. It is fair to state that the decrease in the losses in the main motor for a given load approximately off-set the losses in the advancer, leaving the efficiency substantially unimpaired at rated load. The efficiency will usually be materially improved at small loads. The power-factor and overload capacity may be both greatly increased.

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MOTOR GENERATORS

332. General flexibility of combinations. For transforming alternating electricity from one pressure or phase system to another of the same frequency, stationary transformers usually offer advantages over rotating apparatus. Even for such transformations, however, motor-generator sets permit of adjusting the ratio of transformation at will and over a wide range. This advantage of greater flexibility in the motor generator rarely suffices to justify its use in transformations where it is not desired to alter the frequency. For effecting all other transformations of electricity, motor generators usually represent the most satisfactory means, if questions of cost and efficiency are left out of account.

333. Motor generators for frequency changing. Transformations from alternating electricity of one frequency to alternating electricity of another frequency are usually effected by means of synchronous motor-generator sets. Such frequency changer sets are discussed in division headed Par. 346 to 369.

334. Synchronous converters and motor converters. The lower cost and higher efficiency of transformations from alternating to direct current by means of synchronous converters (Sec. 9) or motor converters (Sec. 9) justify their use for certain classes of work notwithstanding certain less desirable features.

reference has already been made, describes interesting arrangements for synchronizing frequency-changer sets when operating in parallel.

352. Refinements of construction necessary to permit of parallel operation. From the above discussion it will be evident that in order to obtain an appropriate division of the load between two frequency changer sets operated in parallel, great exactness in construction and adjustments is necessary. It is desirable to be able to effect a final mechanical adjustment of the stators with relation to one another. This is often arranged by providing a cradle construction. The stator of one machine is bolted to a cradle fastened to the base. By "slacking off" the bolts, a small angular adjustment of the stator may be effected, and the bolts afterward tightened.

353. Efficiency of a motor-generator frequency changer. By using a motor-generator set as a frequency changer, each of the two machines has to be proportioned for the entire load to be delivered to the changed-frequency circuit, and the set is consequently large and expensive. Furthermore, if each machine has an efficiency of, say, 95 per cent., the combined efficiency is only 91 per cent. since $0.95 \times 0.95 = 0.91$.

354. Induction-type frequency changer. We shall now consider means by which better results, in these respects, may sometimes be obtained. These means rest fundamentally upon the circumstance that, in an ordinary polyphase induction motor, the frequency of the current in the rotor winding depends upon the speed of the rotor. Let us consider a 60-cycle three-phase induction motor with a three-phase rotor and three slip-rings. If the stator windings are connected to a 60-cycle circuit, then, if the rotor is restrained from moving, the frequency in the rotor windings will also be 60-cycles per sec. If, on the contrary, the rotor's speed is the same as that of the revolving magnetic field in the stator, and in the same direction, then the frequency in the rotor winding will be zero. Thus if the 60-cycle induction motor has four poles, the magnetic field will travel around the stator at a speed of: $(60 \times 60) + (4/2) = 1{,}800$ rev. per min.

If the rotor's speed is also 1,800-rev. per min. and in the same direction, then the frequency in the rotor windings will be zero. If, however, the rotor's speed is only 1,200 rev. per min. the frequency will be $[(1{,}800 - 1{,}200)/1{,}800] \times 60 = 20$ cycles per sec. If the rotor's speed is only 300 rev. per min., the frequency will be $[(1{,}800 - 300)/1{,}800] \times 60 = 50$ cycles per sec. If the rotor is restrained from running, the frequency will be $[(1{,}800 - 0)/1{,}800] \times 60 = 60$ cycles per sec. If the rotor is driven at a speed of 300 rev. per min. in a direction opposite to that of the magnetic field in the stator, the frequency will be $[(1{,}800 + 300)/1{,}800] \times 60 = 70$ cycles per sec.

In our case we wish a frequency of 25 cycles per sec. Consequently, if we denote by R the speed at which we shall drive the rotor we may have either $[(R - 1{,}800)/1{,}800] \times 60 = 25$ or $[(1{,}800 - R)/1{,}800] \times 60 = 25$ and $R = 2{,}550$ or $R = 1{,}050$.

355. Direct-current motor drive for induction-type frequency changer. Let us select the former case and drive the rotor at a speed of 2,550 rev. per min., by another motor. We can only do this if we have a source of supply of continuous electricity and employ a 2,550 rev. per min., continuous-electricity motor. We could not employ a 60-cycle induction motor to drive the rotor since this would run at only 1,800 rev. per min., if wound with 4 poles and at 3,600 rev. per min., if wound with 2 poles, and we require it to run at 2,550 rev. per min.

356. Alternating-current motor drive for induction-type frequency changers. If there is no source of continuous electricity supply, thus limiting us to the use of a 60-cycle motor to drive the rotor of the frequency changer, then we shall have to abandon the plan of employing a 4-pole frequency changer and we shall be obliged to employ one with more poles which, consequently, will be heavier and more expensive. We can determine upon a suitable combination by consulting the table in Par. **357**.

357. Selection of proper speed and number of poles in induction-type frequency changers. In the table given in Par. **358**, the smallest number of poles which, in column *IV*, corresponds to the synchronous speed of a 60-cycle motor, is 14, corresponding to a rotor speed of 300-rev. per min. Our frequency changing outfit thus consists of a 14-pole induction motor with its rotor driven by a 300-rev. per min., 60-cycle synchronous motor which will, consequently, have $2 \times [(60 \times 60)/300] = 24$ poles. But, for so we may employ a 4-pole frequency changer and drive its rotor at 300 rev. per min. in opposition to the 1,500 rev. per min. of the stator field. If the output delivered from the frequency changer is 1,000 kw., then $(300/1,800) \times 1,000 = 167$ kw. is provided by the driving motor and $(1,500/1,800) \times 1,000 = 833$ kw. is provided by the frequency changer itself in its transformer capacity, and at high efficiency.

Obviously the less the required alteration in frequency, the greater is the appropriateness of the induction-type of frequency changer as compared with the motor-generator type.

361. Relative merits of various kinds of motors for driving the rotor of an induction-type frequency changer. Whether or no it may be practicable to employ an induction motor to drive the rotor, depends to a considerable extent upon the importance or otherwise of supplying absolutely constant frequency at all loads. When it is permissible that the frequency may decrease 1 or 2 per cent. or more from no-load to full-load, an induction motor may provide the most satisfactory solution, since it eliminates the necessity for having a supply of continuous electricity for excitation.

In some instances instead of employing a squirrel-cage induction motor and then having a slip necessarily proportional to the load, it becomes appropriate to drive the frequency changer's rotor by a slip-ring induction motor and regulate its slip, and consequently the frequency supplied, by regulating the resistance in its rotor circuits. This becomes the more expedient, the smaller the proportion of the total energy which is supplied by the driving motor. For instance, in the example in Par. **360**, only $(300/1,800) \times 100 = 16.7$ per cent. of the total output, is provided by the driving motor, and a considerable rheostatic loss in its rotor circuits would not seriously affect the efficiency of the complete outfit. It should again be emphasised that induction-type frequency changers are not used in commercial practice. Synchronous motor-generator sets are practically always employed.

362. Frequency changers for connecting two systems, each of which has its own generating machinery of fixed frequency. We now come to the problem encountered when frequency changers are employed between two systems each of which has a definite frequency imposed upon it by its own generating plant. Synchronous motor-generator sets have nearly always been employed for this class of work. Thus, in order to link up two systems with frequencies of 60 and 25 cycles per sec., respectively, two synchronous machines of 24 and 10 poles respectively are coupled together. It is necessary for the frequencies of the two systems to be exactly identical in order to successfully operate such plant. Furthermore, there are the various complicated relations necessarily observed in synchronising such apparatus, as already discussed in Par. **349**.

Indeed, there is the further consideration that when **sets for parallel operation** are so proportioned and adjusted that they share the load in appropriate proportions when delivering power from, let us say, a 25-cycle system to a 60-cycle system, the division of the load will be altered when it is desired to reverse the sets and deliver power from the 60-cycle set to the 25-cycle system. Notwithstanding the niceties imposed by the relative-frequency conditions, synchronous motor-generator sets constitute the usual means employed.

When synchronous sets are employed to link two large systems, the slightest alteration in the relative frequencies of the two systems occasions enormous fluctuations in the load carried by the frequency changers. This difficulty is less the greater the size of the frequency changers as compared with the size of the systems connected. Unless the frequency changers are of large size as compared with the size of the systems which they connect, they will be pulled out of step if there is any slight change in the ratio of the frequencies of the two systems. Consequently, when the frequency changers cannot be of relatively great size, it would be preferable to employ sets in which an induction machine constitutes the motor member.

363. Induction-motor drive for non-reversible frequency changers. Where the object is to deliver energy always in the same direction, the employment of an induction machine with a slip-ring rotor as motor element, and the control of its precise speed by the adjustment of a rheostat in its rotor circuits provides freedom from the necessity for maintaining at exactly the same value the ratio of the frequencies of the inter-connected systems.

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SECTION 8

DIRECT-CURRENT GENERATORS AND MOTORS

GENERAL PRINCIPLES

1. Lines of magnetic flux. A magnetic field may be represented by continuous lines, called lines of flux, whose direction at any point in the field is that of the force acting on a north pole placed at the given point; they therefore emerge from a north pole and enter a south pole (Sec. 2).

2. Electromagnetic induction. When the total magnetic flux threading a coil undergoes a change, an electromotive force (e.m.f.) is generated or induced in the coil. This e.m.f. is proportional to the time rate of change of flux. One volt is generated in a coil of one turn when the rate of change of flux threading the coil is 10^8 lines per sec.

3. Magnitude of induced e.m.f. In Fig. 1, N and S are respectively the north and south poles of a magnet and ϕ represents the direction of the magnetic flux passing from the north to the south pole. When coil A is moved from position 1 where the flux threading the coil is ϕ, to position 2 where the flux threading the coil is zero, in t sec., the average e.m.f. generated in the coil is $E = (\phi/t)\ 10^{-8}$ volts.

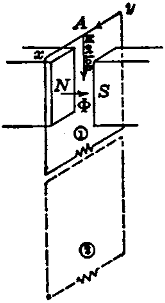

FIG. 1.—Direction of generated e.m.f.

4. The direction of the generated e.m.f. may be found by **Fleming's right-hand rule,** which states that if the thumb, forefinger, and middle finger of the right hand be placed at right angles to one another so as to represent three coordinates in space, with the thumb pointed in the direction of motion of the conductor relative to the field, and the forefinger in the direction of the lines of flux, then the middle finger will point in the direction of the induced e.m.f. (Sec. 2). The direction of the e.m.f., found by Fleming's rule, is shown by the arrow in Fig. 1, and it may be seen (Sec. 2), that the magnetic effect of the current produced tends to prevent the flux threading the coil from decreasing or, as stated by Lenz's law (Sec. 2), the generated e.m.f. always tends to send an electric current in such a direction as to oppose the change of flux which produces it.

5. Force on a conductor in a magnetic field. A conductor L cm. long, carrying a current of I amp. perpendicular to a magnetic field of $\mathfrak{B}$ lines per sq. cm., is acted on by a force of $\mathfrak{B}LI/10$ dynes in a direction found by **Fleming's left-hand rule,** which states that if the thumb, the forefinger and the middle finger of the left hand be placed at right angles to one another so as to represent three coordinates in space, with the thumb pointed in the direction of the force on the conductor and the forefinger in the direction of the lines of flux, then the middle finger will point in the direction of the current.

6. Identity of generator and motor structure. Diagram A, Fig. 2, shows a generator under load; the direction of rotation is determined by the prime mover, while the direction of the current in the conductors may be found by the right-hand rule (Par. 4). A force is exerted on the conductors, inasmuch as they are carrying current in a magnetic field; the direction of this force is found by the left-hand rule (Par. 5) and is opposed to e.m.f. is being generated in it, that is, it should be midway between the poles or in the neutral position.

9. Field excitation. The m.m.f. necessary to establish the flux in the magnetic circuit is obtained by means of field coils which are wound upon the poles of the machine. The exciting current for the field coils may be supplied in various ways. When a generator supplies its own exciting current it is said to be **self-excited;** when the exciting current is supplied from some external source, such as an exciter, the machine is said to be **separately excited.** The different connections used are shown diagrammatically in Fig. 4.

Diagram *A* shows a separately excited machine. Diagram *B* shows a **shunt machine,** in which the field coils form a shunt across the armature terminals and have many turns of small wire carrying a current which is proportional to the terminal voltage of the machine. This field current seldom exceeds 5 per cent. of the armature current under full-load conditions. Diagram *C* shows a **series machine,** in which the field coils are in series with the

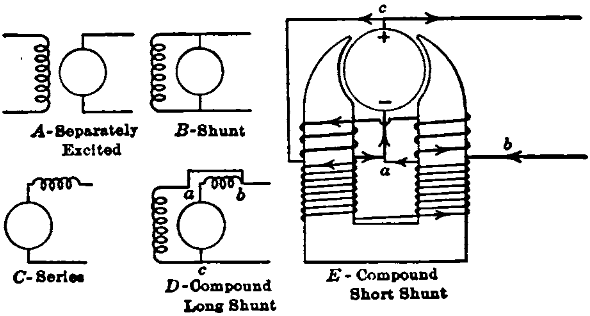

Fig. 4.—Methods of excitation.

armature and have fewer turns than the coils of a shunt winding, but employ a larger size of wire because they carry the whole or a fixed proportion of the total current. Diagram *D* shows a **compound machine** in which there is connected to the terminals a shunt winding which carries an exciting current proportional to the terminal voltage; and also a series winding which carries a current proportional to the armature current. This method of connection is known as the "**long shunt.**" Diagram *E* shows a compound machine connected in another manner. The connection in this case is known as the "**short shunt**"; that is, the shunt coil is connected to the terminals of the armature. Small machines using permanent magnets for field poles are called **magnetos.**

CLASSIFICATION OF TYPES

10. Classification according to the number of poles, as follows:

(a) **Bipolar machines** (Fig. 4) have only two poles.

(b) **Multipolar machines** (Fig. 46) have more than two poles; the number of poles is always a multiple of two.

(c) **Homopolar machines** (Fig. 89) have two poles, but the conductors always cut lines of unidirectional flux. The resulting e.m.f. is continuous in one direction and therefore no commutator is required.

11. Classification according to the method of drive, as follows:

(a) **Belted-type motors and generators** are self-contained. This type of machine includes bearings, shaft extended for a pulley and a sliding base with belt-tightening device. An outboard bearing is usually provided with machines of larger capacity than 200 kw. at 600 rev. per min.

(b) **The engine-type generator** has its armature mounted on a continuation of the crank shaft of the engine, and **slow-speed units** are generally supplied without base, bearings, or shaft, these being furnished by the engine builder.

(c) **The direct-connected turbo-generator** has its armature shaft coupled directly to that of a steam turbine. In this type the construction of the generator is special on account of the high speed.

(d) **The geared turbo-generator** has its armature shaft connected to the shaft of a steam turbine through a reduction gear, so that a generator of moderate speed and simple construction may be used.

(e) **The water-wheel type generator** has its armature shaft coupled directly to that of a water wheel. Such a generator may be of either the horizontal or the vertical type and is generally supplied complete with base, bearings, shaft and coupling.

(f) **The back-geared type motor** embraces a speed-reduction gear as an element of the machine. The slow-speed shaft is supported in bearings attached to the frame.

12. Classification according to special features of construction, as follows:

(a) **Interpole machines** (Fig. 24) have small auxiliary poles which carry series windings and improve commutation.

(b) **Compensated machines** (Fig. 27) have series windings on the pole faces to neutralize armature reaction. Such machines may also have interpoles.

(c) **Miscellaneous.** Under this class may be included **mill motors,** used in rolling mills; **flame-proof motors** for mine service; **variable-speed generators,** for train lighting; etc. (Par. 183 to 215).

ARMATURE WINDINGS

13. The Gramme ring winding is almost obsolete; examples of it are shown in Figs. 3, 5, and 6 merely to present clearly the meaning of the terms used in describing the various types of winding.

14. A re-entrant winding closes or re-enters on itself. **A singly re-entrant winding** closes on itself only after including all of the conductors; see Figs. 3 and 6. **A doubly re-entrant winding** closes on itself after including half of the conductors; see Fig. 5.

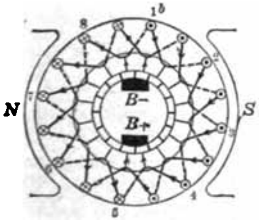

Fig. 5.—Doubly re-entrant duplex winding.

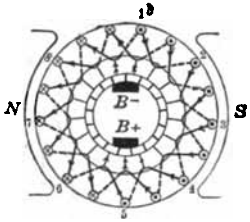

Fig. 6.—Singly re-entrant duplex winding.

15. A simplex winding has only two paths through the armature from each brush; see Fig. 3. **A duplex winding** has twice as many, or four paths, through the armature from each brush. In this winding each brush should cover at least two commutator segments; see Figs. 5 and 6. Although it is possible to use **multiplex** and **multiply re-entrant** windings, they are seldom found in modern machines. Even duplex windings are rarely used except for machines of very large current capacity.

16. The drum winding has coils shaped as shown in Fig. 7. At any instant, two sides of each coil are under adjacent poles. Since the number of conductors in each coil must be a multiple of two, the total number of conductors must be even. A winding made with coils of this shape must lie in two layers and is called a **double-layer winding.**

17. Representation of drum windings. Fig. 8 shows a double-layer drum winding corresponding to the Gramme ring winding in Fig. 3. It has the same number of conductors and the same number of paths through the armature, but only half the number of commutator segments. Conductors in the upper layer are represented by full lines and those in the bottom layer by dotted lines. The radial lines represent face conductors; the connecting lines on the inside represent the connections at the commutator end, and those on the outside represent the connections at the opposite end. For convenience the brushes are shown inside the commutator.

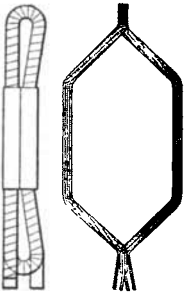

Fig. 7.—Coil group for double layer winding with two turns per coil and eight conductors per slot.

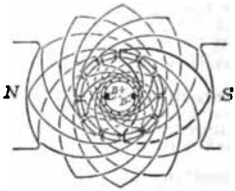

Fig. 8.—Simplex singly re-entrant. drum winding.

18. Multiple winding. Fig. 9 shows a six-pole drum winding with six paths in parallel between the positive and the negative terminals. The three positive brushes are connected together outside of the machine by a copper ring T_+, and the three negative brushes are connected by a similar ring T_-. This winding is of the multiple type, that is, the number of armature circuits between terminals is a multiple of the number of poles.

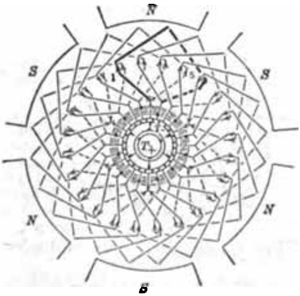

Fig. 9.

Fig. 10.

Figs. 9 and 10.—Simplex, singly re-entrant, full-pitch multiple winding with equalisers.

19. Equalizers. Fig. 10 shows a multiple-wound armature which is not central with the poles. The flux density in the air gaps A is greater than that in the air gaps B, and the voltage generated in a circuit under poles A is greater than that in a circuit under poles B, so that the generated voltage between c and d is greater than that between f and g; hence a **circulating current** will flow through the winding and brushes, causing sparking, additional loss and additional heating. This circulating current may be minimised by careful centering during erection, while the sparking may be preof brushes are required, this type of winding is well suited for direct-current railway motors, inasmuch as the two sets of brushes for a four-pole motor are placed 90 deg. apart so that they may easily be inspected from the car.

25. Lap windings and wave windings. A multiple-wound armature is sometimes said to have **a lap winding** and an armature with a two-circuit winding is said to have a **wave winding.**

26. Dead coils. There are generally more coils than there are slots, and each coil may have more than one turn. A coil in this case is defined as the shortest winding element between two commutator segments. It is not always possible to put a proper two-circuit winding into a given armature. A 110-volt, four-pole machine, with forty-nine slots, forty-nine coils, and two conductors per slot, may have a two-circuit winding because it fulfills the condition that $49 = (24\ p/2) + 1$; when wound for 220 volts, however, four conductors per slot are required, and the number of coils is $98 = (49\ p/2) \pm 0$, which will not give a two-circuit winding. In such a case, one coil, called a dead coil, is not connected into the winding, its two ends being taped so as to completely insulate the coil. The machine, therefore, may have a two-circuit winding with ninety-seven active coils and ninety-seven commutator segments.

27. Turbo-generator windings. In turbo-generators the conductors are long and move at high speed, so that the voltage between adjacent commutator segments is often higher than desirable. For such machines it has been proposed to use the type of winding shown in Fig. 13, the voltage between adjacent commutator segments being that generated in one conductor. To keep the inductance of the return conductors low, it is desirable to group together return conductors in which the currents at any instant are in opposite directions.

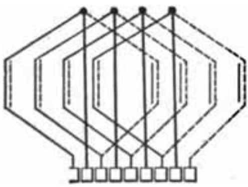

Fig. 13.—Winding with half-turn coil.

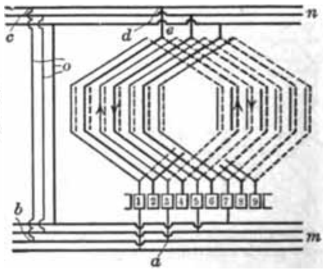

Fig. 14.—Connection of equalizer between parallel windings.

28. Equalizer connections for turbo-generator windings. The normal multiple-circuit doubly re-entrant duplex or multiplex windings can only be used to their full advantage when the voltage between the segments belonging to the different parallel windings is equally distributed, that is to say, the potential between segments 1 and 2 should be exactly half of the potential between segments 1 and 3; but without special means of bringing about this condition it is rather dangerous to use these windings. It may happen that the potential is very unevenly distributed between the segments, and this gives rise to very great equalizing losses over the brushes. The winding shown in Fig. 14 is a multiple-circuit doubly re-entrant duplex winding; the thin lines in the diagram indicate one of the parallel windings, the thick the other. The thin lines at the bottom of the diagram indicate the normal equalizer connections for the winding shown in thin lines, and the thick lines at the top indicate the normal equalizer connections for the winding shown in thick lines. The corresponding equalizer rings of the two parallel windings are connected through the equalizers indicated on the left of the diagram, and by means of these connections equal and the main exciting m.m.fs. exist together. Since the armature teeth are saturated at normal flux densities, the increase in flux density at f is less than the decrease at e, so that the total flux per pole is diminished by the cross-magnetizing effect of the armature.

33. Demagnetizing effect. Fig. 17 shows the magnetic field produced by the m.m.f. of the armature when the brushes are shifted through an angle

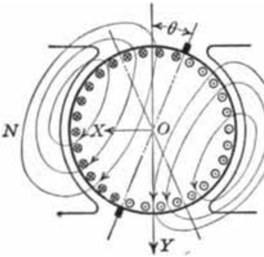

Fig. 17.

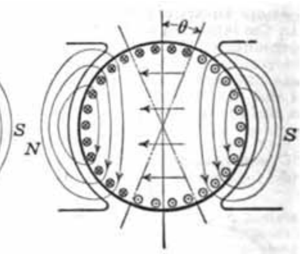

Fig. 18.

Figs. 17 and 18.—Demagnetizing and cross magnetizing effect.

θ so as to improve commutation (Par. 47). The armature field is no longer at right angles to the main field, but may be considered as the resultant of two components, one in the direction OY, called the cross-magnetizing component, the effect of which is discussed in the last paragraph, and the other in the direction OX, which is called the demagnetizing component because it is directly opposed to the main field. Fig. 18 shows the armature divided so as to produce these two components, and it may be seen that the demagnetizing ampere-turns per pair of poles are

$$\left(\frac{ZI_c}{p}\right)\times\frac{2\theta}{180} \quad \text{(ampere-turns)} \quad (2)$$

where $2\theta/180$ is generally about 0.2. Therefore the demagnetizing ampere-turns per pole are

$$0.1\left(\frac{ZI_c}{p}\right) \text{(ampere-turns)} \quad (3)$$

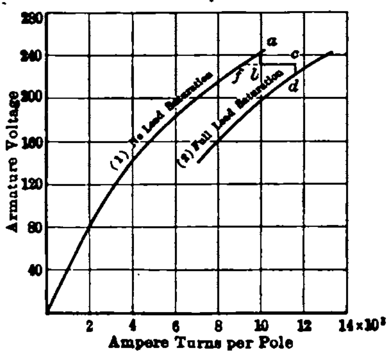

Fig. 19.

34. No-load and full-load saturation curves. Curve 1, Fig. 19, shows the no-load saturation curve of a direct-current generator. When full load is put on the generator there is a decrease in flux and therefore a drop in voltage ab due to the armature cross-magnetizing effect (Par. 32). A further voltage drop due to the armature demagnetizing effect is counter-balanced by an increase in excitation of $bc = 0.1\ (ZI_c)/p$; also a portion cd of the generated e.m.f. is required in overcoming the internal resistance of the machine. With an excitation of 10,000 ampere-turns, the voltage at no load and normal speed equals 240 volts. At full load, with the same excitation, the terminal voltage equals 196 volts. In order that the full-load voltage may be the same as that at no load, the number of ampere-turns per pole which must be supplied by the series field is $(13{,}400 - 10{,}000) = 3{,}400$ ampere-turns.

divides into two parts which are proportional to the areas of contact between the brush and segments 5 and 6 respectively, and the current density is uniform across the brush surface. In diagram *C* the two contact areas are equal; there is no tendency for current to pass round coil *M*, and the current density is again uniform across the brush surface. In diagram *D* the contact area between the brush and segment 5 is small while that between the brush and segment 6 is large; the larger part of the current therefore enters at segment 6 and the current in coil *M* is reversed, while the current $2I_c$ again divides into two parts which are proportional to the contact areas between the brush and the two commutator segments, and the current density is still uniform across the brush surface.

37. Perfect commutation is defined as such a change of current in the coil being commutated that the current density over the brush contact surface is constant and uniform.

38. The brush contact resistance depends on the brush material and, for carbon brushes, it decreases with increase of current density as shown in Fig. 21. The contact resistance with current flowing from commutator to brush is somewhat higher than with current flowing from brush to commutator.

39. Change of contact resistance with temperature. If the current density in a brush contact be suddenly increased, the contact resistance does not immediately decrease to the value given in Fig. 21, but gradually decreases as the temperature of the contact increases, and reaches a constant value after about 20 min. This explains why a machine will carry a considerable overload for a short time without sparking, whereas, if the overload be maintained, the machine will begin to spark as the brush temperature increases and the contact resistance decreases.

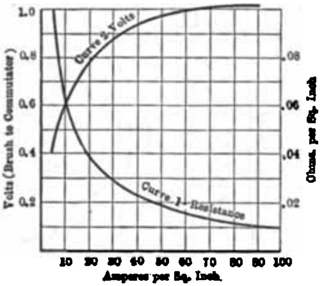

FIG. 21.—Brush contact resistance.

40. Current density in the brushes. For sparkless commutation, (1) the current density in the brush tip must not become infinite, and (2) the average amount of energy expended at the brush contact must be limited. For machines required to operate without sparking up to 25 per cent. overload, a rate of energy dissipation of 35 watts per sq. in. (5.5 watts per sq. cm.) may be allowed at full-load, so that the permissible current density depends upon the voltage drop across the brush contact. The better the commutation, the more nearly uniform is the current density in the brush contact, and the higher the average density which may be allowed.

41. Current density in the brush tip. The effect of coil resistance may generally be neglected, but that of self-induction must be considered. The e.m.f. of self-induction and mutual induction of coil *M* (Fig. 20) opposes the change of current in that coil, so that, in diagram *C*, at the end of half of the period of commutation, the current in the coils *M* has not become zero. When it does become zero, some time later, and the currents entering segments 5 and 6 are equal, the contact area with segment 5 is smaller than with segment 6 and the current density in the brush tip *S* is greater than normal. It may be shown theoretically that the tip density at the end of commutation becomes infinite* when $RT_c/(L+M)$ is less than unity,

where R is the resistance of the brush contact in ohms,
T_c is the time of commutation in seconds,
L is the coefficient of self-induction in henrys of one coil M,
and M is the coefficient of mutual induction in henrys between coil M and coils M_1 and M_2.

* Reid, on "Direct-current Commutation," *Trans.* A. I. E. E., Vol. XXIV

42. Copper leaf brushes can carry 150 amp. per sq. in. (23 amp. per sq. cm.) with a drop of 0.3 volt at the contact.

Carbon brushes can carry 35 watts per sq. in. (5.5 watts per sq. cm.), Par. 40, for example 35 amp. per sq. in. (5.5 amp. per sq. cm.) with a drop of 1.0 volts at the contact.

43. Average reactance voltage. The criterion for sparkless commutation is, then (Par. 41), that $RT_c/(L+M)$ shall be greater than unity, or that

$$2I_cR > \frac{2I_c}{T_c}(L+M) \qquad (4)$$

where $2I_c$ is the current entering the brush (Fig. 20). The latter quantity is called the average reactance voltage and the former is the voltage drop across one brush contact. It will be found that the higher the contact resistance and the lower the reactance voltage, the better is the commutation.

Fig. 22 shows the magnetic field encircling the short-circuited coils of a full-pitch multiple winding. The reluctance of the magnetic path may readily be calculated from which the value of the flux per unit current and $L+M$, the coefficient of self and mutual induction, are obtained. Deep slots decrease the reluctance of the magnetic path (for the same cross-section of conductor) and increase the reactance voltage, therefore they should be avoided if possible.

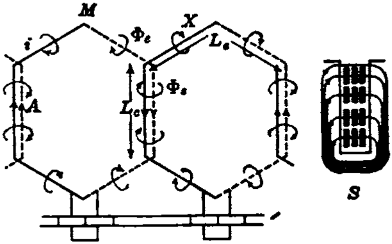

FIG. 22.

44. Effect of type of winding on reactance voltage. In the **short-pitch winding** shown in Fig. 11, the conductors undergoing commutation lie in different slots, and the reluctance of the magnetic path around the coils is greater, and the coefficient of mutual induction smaller, than in a **full-pitch winding.** The use of a short-pitch winding therefore lowers the reactance voltage and improves commutation, but, as shown in Fig. 11, it also reduces the effective interpolar space by the angle θ.

In the **two-circuit winding** with one set of positive and one set of negative brushes, shown in Fig. 12, each brush short-circuits $p/2$ coils in series, so that the reactance voltage is $p/2$ times that of a **multiple winding.** When, however, the number of brush sets is the same as the number of poles, there is a short commutation path around one coil and the commutation is improved.

45. Reactance voltage formula. Approximate results may be obtained by use of the following formula for the average reactance voltage.*

$$E_r = kS(\text{r.p.m.})I_cL_eT^2\left(\frac{\text{poles}}{\text{paths}}\right)10^{-8} \qquad \text{(volts)} \qquad (5)$$

where S is the number of commutator segments,
r.p.m. is the speed of the machine in rev. per min.,
I_c is the current in each armature conductor,
L_e is the frame length in inches (see Fig. 38),
T is the number of turns per coil between segments,
$\left(\frac{\text{poles}}{\text{paths}}\right) = 1$ for multiple windings and $p/2$ for two-circuit windings,
$k = 1.6$ for two-circuit windings and for full-pitch multiple windings,
and
$k = 0.93$ for short-pitch multiple windings.

Deep slots increase the reactance voltage above this value.

* Gray. "Electrical Machine Design," p. 84.

46. Effect of brush arc. An increase in the brush arc has no effect on the reactance voltage because, while it increases the number of coils in series at short circuit, it decreases the time of commutation in the same ratio, and the reactance voltage remains unchanged. On the other hand, a wide arc may cause sparking by starting commutation in the coils before they are in a reversing field, or by keeping them short-circuited until they are in too strong a field. To minimise this trouble make the proportions such that,

$$\text{brush arc} < \left(\frac{\text{pole pitch}}{12}\right) \times \left(\frac{\text{commutator diameter}}{\text{armature diameter}}\right) \quad (6)$$

where all dimensions are in inches. Furthermore, the brush should not cover more than three commutator segments except in machines of low reactance voltage, otherwise there will be large circulating currents in the brush face.

47. Shifting of brushes. To improve commutation, the brushes are shifted from the neutral position, so that the short-circuited coils are in a magnetic field and an e.m.f. (E_s) is generated in them which opposes the reactance voltage and improves commutation. As the current in the machine increases, the reactance voltage increases with it. To have perfect commutation at all loads, the voltage E_s must maintain an unvarying ratio to the current. This is only possible when the distance by which the brushes are moved from the neutral position increases as the current increases. Modern machines must operate from no load to 25 per cent. overload without sparking and without shifting of the brushes during operation. In these machines the brushes are permanently shifted (at the time of erection) from the neutral position until the voltage E_s is so large that sparking takes place at no load.

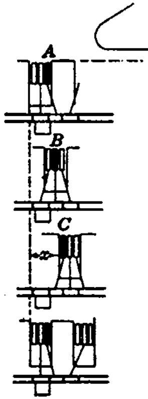

FIG. 23.—Commutation with several coils per slot.

48. Number of slots per pole. Fig. 23 shows three of the stages in the commutation of a machine with six coil sides per slot. The commutator segments are evenly spaced while the coils, being in slots, are not. Between the instant when the brush breaks contact with coil A, and the instant it breaks with coil C, the slot has moved through a distance x, so that if the magnetic field in which the coils are commutated is suitable for coil A, it is too strong for coil C, and the latter will therefore be badly commutated, every third commutator segment being blackened. The distance x is equal to the slot pitch minus the width of one commutator segment. To minimize this trouble, a machine should have more than twelve slots per pole. Large machines have generally more than fourteen slots per pole.

49. Limits of reactance voltage. Experience shows that when the following conditions are fulfilled, namely:

(1) Number of slots per pole is greater than twelve,

(2) Brush arc is less than one-twelfth of the pole pitch measured at the commutator surface,

(3) Pole arc is less than seven-tenths of the pole pitch, and

(4) Ampere-turns for air-gap and teeth are greater than 1.2 to 1.5 times the armature ampere-turns per pole (Par. 35): then the reactance voltage calculated by Eq. 5 (Par. 45) should be

(1) Less than seven-tenths of the voltage drop per pair of brushes, with brushes in neutral position, and

(2) Less than the full voltage drop per pair of brushes, with the brushes shifted to improve commutation.

A machine with a two-circuit winding will commutate about 20 per cent. better than would be indicated by the value of reactance voltage obtained from Eq. 5 (Par. 45), while the commutation of a machine with a short-pitch winding will be about 30 per cent. worse than indicated, because of the reduction of the effective interpolar space (Par. 44).

50. Shunt, compound and series machines. Due to armature reaction, the flux at the pole tip toward which the brushes are shifted decreases with increase of armature current (Par. 31). This decrease is kept within reasonable limits by the use of a strong exciting field (Par. 35) and is less in a compound machine than in a shunt machine operating on constant excitation. In the series machine the main exciting m.m.f. and the armature m.m.f. increase together, and the flux at the commutating pole tip may increase or decrease, depending on the relative strength of field and armature.

51. Interpoles. An interpole generator is shown diagrammatically in Fig. 24, where n and s are auxiliary poles which have a series winding so connected that their strength increases with the armature current. To improve commutation in a non-interpole generator, the brushes are shifted forward in the direction of motion, so that B_+ would come under the tip of the N pole and B_- under the tip of the S pole. In the interpole machine the auxiliary pole n is placed opposite the brush B_+ and the auxiliary pole s opposite the brush B_-.

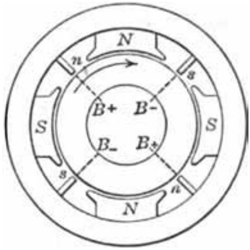

Fig. 24.—Magnetic circuit of interpole generator.

52. Interpole excitation. The interpole must have a m.m.f. equal and opposite to that of the cross-magnetizing effect of the armature, namely, $ZI_a/2p$ ampere-turns per pole (Par. 30) and, in addition, a m.m.f. which can send across the gap a flux large enough to generate in the short-circuited coil an e.m.f. equal and opposite to the reactance voltage. In order that the interpole flux may be always proportional to the current, the interpole magnetic circuit must not be allowed to become saturated.

53. The effective interpole arc is represented by the expression $W_{ip}+2\delta$ (see Fig. 25) and should be of such proportions that while the current in a conductor is being commutated, the slot carrying that conductor is in the interpole field. The distance moved by the coil A (Fig. 23) while short-circuited equals the brush arc multiplied by the armature diameter and divided by the commutator diameter and the effective interpole arc must exceed this by the distance x (Fig. 23).

Fig. 25.—Interpole width.

54. The reluctance of the interpole air-gap should vary as little as possible for different positions of the armature, otherwise the interpole field will be pulsating. The effective interpole arc should therefore be a multiple of the slot pitch; it is generally made about 15 per cent. of the pole pitch and, to minimize the interpole leakage flux, the main pole arc is not more than about 65 per cent. of the pole pitch.

55. The axial length of the interpole in inches is given by the formula,*

$$L = \frac{24 \times \text{core length in inches} \times \text{amp. cond. per in.}}{\text{interpole gap density in lines per sq. in.}} \qquad (7)$$

The ampere-conductors per in. (Par. 59) seldom exceed 900, and, with an interpole gap density of 45,000 lines per sq. in. (7,000 lines per sq. cm.), the interpole circuit will not be saturated up to 50 per cent. overload; the interpole length, therefore, need not exceed half the frame length, if there are as many interpoles as there are main poles and if the overloads do not exceed 50 per cent.

56. The interpole ampere-turns per pole for a newly designed machine are usually made equal to 1.4 times the armature ampere-turns per pole (Par. 52). This is generally too large, so adjustment must be made after the machine is erected.

57. Flashing over is generally caused by a sudden change of load. Fig. 26 shows a representation of the armature cross-field in a loaded machine. A sudden change of load alters the value of the cross-flux, and a voltage

*Gray. "Electrical Machine Design," p. 94.

proportional to the rate of change of flux is generated in coil *a*. This may increase the voltage that already exists between adjacent segments to such a value that arcing starts, and then the machine flashes over from brush to brush, particularly if the commutator is dirty. Machines with badly distorted fields and a high average voltage between commutator segments are especially likely to flash over.

58. Compensating windings. For such service as the operation of reversing rolling mills, the current in the motors may change suddenly from full-load value in one direction to three times full-load value in the opposite direction. For such machines the average voltage between adjacent commutator segments, which is equal to the terminal voltage divided by the number of commutator segments per pole, should not exceed

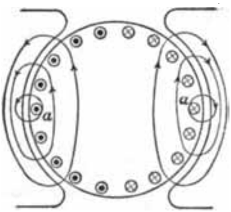

FIG. 26.

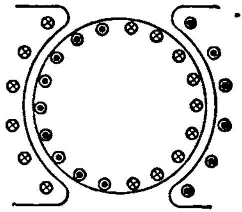

FIG. 27.

FIGS. 26 AND 27.—Armature field with and without compensating windings.

about 15, otherwise compensating windings will be required to prevent flashing over. The compensating winding is carried by the pole face, and is in series with and has the same number of ampere-turns per pole as the armature. However, the current passes in the opposite direction to that in the armature and thus, as shown in Fig. 27, the armature field is completely neutralized. For further discussion of the subject of commutation see the references below.*

ARMATURE DESIGN

59. Output equation. The output of a machine is proportional to the armature volume, and the output equation is

$$D_a^2L_c = \frac{\text{watts}\times 60.8\times 10^7}{\text{r.p.m.}\times \mathcal{B}_g\times q\times \Psi} \qquad \text{(cu. in.)} \qquad (8)$$

where D_a is the armature diameter in inches,
L_c is the frame length in inches (Fig. 38),
$\mathcal{B}_g$ is the apparent gap density in lines per sq. in.,
Ψ is the per cent. enclosure (= pole arc pole ÷ pitch), and
q is the ampere-conductors per in. ($= ZI_a/\pi D_a$).

60. The e.m.f. equation for all types of direct-current generators is:

$$E = Z\phi_a\left(\frac{\text{r.p.m.}}{60}\right)\left(\frac{\text{poles}}{\text{paths}}\right)10^{-8} \qquad \text{(volts)} \qquad (9)$$

where E is the generated voltage between terminals,
Z is the total number of face conductors,
ϕ_a is the flux per pole which crosses the air-gap and is cut by the armature conductors,
r.p.m. is the armature speed in revolutions per min.
"paths" refer to the number of parallel circuits (electric) through the armature.

* Arnold, "Die Gleichstrommaschine," Vol. I and II; Hawkins and Wallis, "The Dynamo," Vol. II; Baily and Cleghorne, *Journal* of Inst. of Elec. Eng.; Vol. XXXVIII; Gray, "Electrical Machine Design;" Hobart, "Dynamo Design," Lamme, *Trans.* of A. I. E. E., Vol. XXX, p. 2359.

61. Another output equation, readily obtained from the e.m.f. equation, is given by

$$\frac{\text{watts}}{\text{r.p.m.}} = \left(\frac{ZI_a}{\text{paths}}\right)(\phi_a \times \text{poles})\frac{1}{60 \times 10^8} \tag{10}$$

where (ZI_a/paths), called the **electric loading**, is the total number of ampere-conductors on the armature periphery. The larger the electric loading, the more copper and the less iron there is in the machine. The **magnetic loading**, as the quantity ($\phi_a \times$ poles) is termed, is the total flux entering the armature. The larger the magnetic loading, the more iron and the less copper there is in the machine.

62. The gap density ($\mathfrak{B}_g$) is limited by the density at the bottom of the teeth; the greater the diameter, the less the tooth taper, and the higher the gap density for a given tooth density at the root. The relation between $\mathfrak{B}_g$ and D_a is given in Fig. 28.

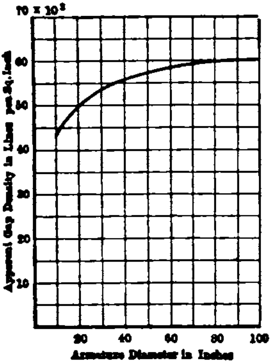

FIG. 28.

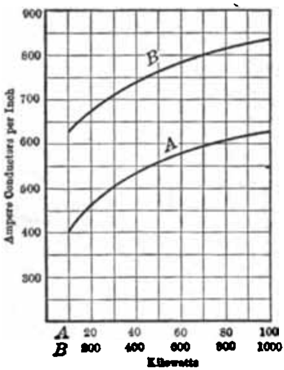

FIG. 29.

63. The ampere-conductors per in. (q) are limited by heating and by commutation. If q is large, the copper section must be large and the slots deep in order to keep down the temperature, but deep slots tend to cause sparking. It is found that q depends principally on the output of the machine, as shown in Fig. 29.

If the speed for a given output be increased, a smaller diameter can be used, and the peripheral velocity will not be greatly increased. Since the small machine is not so well ventilated as the large machine, see Fig. 31, the same value of q should be used in each case.

64. Relative cost factor. A certain ratio k, which is equal to the magnetic loading divided by the electric loading, will give the cheapest machine. Then

$$\frac{\text{watts}}{\text{r.p.m.}} = \text{a const. (electric loading) (magnetic loading.) Par. 61.}$$

$$= \text{a const. (electric loading)}^2 \tag{11}$$

and the relation between the watts per rev. per min. and the electric loading, given in Fig. 30, may be used in preliminary design. The value of k is affected by the cost of labor and material and by the conditions of manufacture.

65. Number of poles. A pole of circular cross-section has the largest section for the shortest mean turn. If the pole be rectangular, then that with a square section has the largest area for the shortest mean turn. For economy in copper, the ratio of pole pitch to frame length will generally have a value between 1.1 and 1.7.

66. The armature ampere-turns per pole seldom exceed 7,500 in non-interpole machines. A larger number requires a long pole pitch, long end connections, a large number of exciting ampere-turns per pole, long poles, and a yoke of large diameter. For interpole machines, the main field need not be so strong relatively to the armature field as in non-interpole machines, and a reasonable limit for the armature ampere-turns per pole is 10,000.

67. A simple procedure for armature design is given in the following:

ZI_c, the number of ampere-conductors, is obtained from Fig. 30.
$q = ZI_c/\pi D_a$, the number of ampere-conductors per in., is obtained from Fig. 29.
D_a, the armature diameter, is next determined.
$\mathfrak{B}_g$ is obtained from Fig. 28.
L_c is found by substitution in Eq. 8, Par. 59.
p, the number of poles, is so chosen that the pole pitch divided by L_c equals 1.1 to 1.7.
ϕ, the flux per pole $= 0.7$ (pole pitch) $\times L_c \times \mathfrak{B}_g$, assuming the pole enclosure $= 0.7$.
Z, the number of armature conductors, is found from Eq. 9, Par. 60.

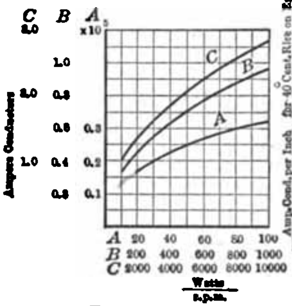

FIG. 30.

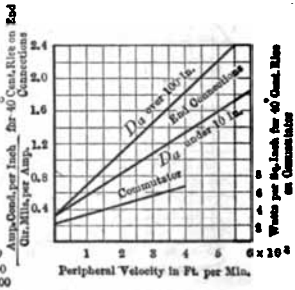

FIG. 31.

The winding is then so chosen that the reactance voltage will be below the desired limit. A two-circuit winding is the cheapest, and a winding with two turns per coil is cheaper than one with only one turn per coil.

68. The commutator diameter (D_c) is made equal to six-tenths of the armature diameter so as to have reasonably long "necks" (coil leads), but a peripheral velocity greater than 3,500 ft. per min. (1,070 m. per min.), is undesirable. One objectionable feature of the turbo-machine is that its commutator speed may run as high as 7,000 ft. per min. (2,140 m. per min.).

69. Brush design. The brush arc should not exceed the value $\left(\frac{\text{pole pitch}}{12}\right)\left(\frac{\text{comm. dia.}}{\text{arm. dia.}}\right)$. The volts drop per pair of brushes should be chosen to suit the reactance voltage (Par. 49). The energy at the brush contacts is limited to 35 watts per sq. in. (5.5 per sq. cm.) (Par. 40).

70. Example of armature design. It is required to determine approximate dimensions for a direct-current generator of the following rating: 400 kw., 240 volts, 1,670 amp., 200 rev. per min. The results, determined in consecutive order, are as follows:

ZI_c, ampere-conductors,	$= 1.33 \times 10^5$
q, ampere-conductors per in.,	$= 733$
D_a, armature diameter,	$= 58$ in.
B_g, apparent gap density,	$= 58{,}000$ lines per sq. in.
L_c, frame length,	$= 12$ in.

p, poles,		$=10$	
τ, pole pitch,		$=18.2$ in.	
ϕ, flux per pole,		$=8.8\times10^6$	
Z, total face conductors,	$=164$	$=820$	$=820$
Winding,	1-turn ser.	2-turn mult. short pitch	1-turn mult. short pitch
S, commutator segments,	$=82$	$=205$	$=410$
RV, reactance voltage,	$=13$	$=3.1$	$=1.5$
D_c, commutator diameter,			$=35$ in.
Brush arc,			$=0.91$ in.
Volts per pair of brushes,			$=2$
Amp. per sq. in. brush contact,			$=35$
Brush length,			$=10.5$ in.
l, commutator face,			$=13$ in.

If the winding were series, or two-turn multiple, the reactance voltage would be too high and interpoles would be required.

Comparative designs should be worked out with both larger and smaller armature diameters and with different numbers of poles; then a choice should be made, and the design completed as below.

The final or **detailed design** is worked out in the following order. The probable number of total face conductors is 820, from the preliminary design. The number of slots per pole should be greater than fourteen and the total number of slots greater than 140; the nearest number of slots that will give an even number of conductors per slot is 200. Therefore

Slots	$=200$
Conductors per slot	$=4$
Coils	$=400$
Commutator segments	$=400$
Winding	1-turn multiple

Ampere-conductors per in. $=167\times800/\pi\times58=730$

$$\frac{\text{Ampere-conductors per in.}}{\text{Cir. mils per amp.}}=1.3 \text{ for 40 deg. cent. rise, from Fig. 31.}$$

Cir. mils per amp.	$=560$
Amp. per conductor at full load	$=167$
Section of conductor	$=93{,}000$ cir. mils $=0.073$ sq. in.

Slot opening $=0.47\times$ (slot pitch) in large machines
$=0.52\times$ (slot pitch) in small machines

For a first approximation, therefore

Slot pitch $=0.91$ in.
Slot opening $=0.43$ in.

0.064 width of slot insulation (Fig. 47)
0.04 clearance between coil and core
0.326 available space for copper and conductor insulation.

Use flat strip, as in Fig. 47, with two conductors in the width of the slot; make the strip 0.14 in. wide and insulate it with half-lapped cotton tape.

Depth of conductor $=0.073/0.14=0.52$ in.; increase this to 0.55 in. to allow for rounding of the corners.

Slot depth is found as follows:

0.55 = depth of each conductor
0.024 = insulation of each conductor
0.084 = depth of slot insulation (Fig. 47)
0.658 = depth of each insulated coil
2 = number of coils in depth of slot
1.316 = depth of coil space
0.2 = thickness of stick at top of slot
1.516 = necessary slot depth; make it 1.6 in.

The tooth flux density should now be checked to make sure that it is not too high, see Par. 123, and the internal diameter of the armature made such that the flux density in the armature core shall not exceed 85,000 lines per sq. in.

71. Effect of interpoles. When interpoles are used, the reactance voltage is no longer a limiting factor ln the design, so that deep slots may be used and a large amount of copper put on each inch of the periphery. For interpole machines, the value of q, the ampere-conductors per in., will generally be 20 per cent. greater than given in Fig. 29. The commutating fringe under the pole tip is not used, so that the ratio of the exciting ampere-turns per pole for gap and teeth to the armature ampere-turns per pole, which is seldom less than 1.2 for non-interpole machines, is generally made about 0.8 for similar machines with interpoles. There is therefore a large saving in field copper, and in pole and yoke material.

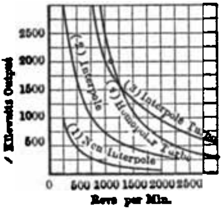

FIG. 32.—Maximum output curves.

72. Speed limitations in design.* If the peripheral velocity of a machine is fixed, the output can be increased only by increasing the frame length, the number of conductors, or the current per conductor, all of which increase the reactance voltage; so that, after a certain output has been reached, interpoles must be supplied or the peripheral velocity increased.† Fig. 32 shows the maximum output that can be obtained for a given speed, the peripheral velocity being taken as 6,000 ft. per min. (1,830 m. per min.) for machines of ordinary construction, and 15,000 ft. per min. (4,600 m. per min.) for turbo-machines. The high-speed turbo-machine is not too satisfactory as regards efficiency, commutation, and heating and the tendency is to gear a high-speed turbine to a moderate speed generator through a **Melville-Macalpine reduction gear.**

FIELD DESIGN

73. The magnetic circuit is well shown in general in Fig. 38, where the closed dot-and-dash line indicates the complete path through one pair of poles, two air-gaps, the armature and the yoke, all in series. There are as many paths similar to this one, in a multipolar machine, as there are poles. This path, of course, is the path of the useful flux, and does not take into consideration the leakage flux.

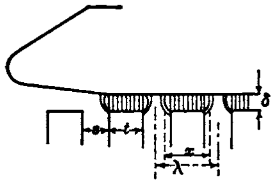

FIG. 33.

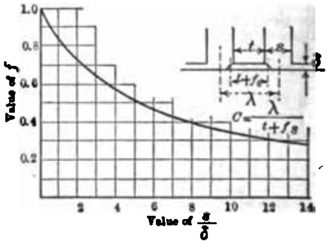

FIG. 34.

FIGS. 33 AND 34.—Fringing constant.

74. The fringing constant.‡ The actual air-gap area per pole equals $(\psi_T L_c)x/\lambda$, Par. 75, where λ/x (Fig. 33) is the fringing constant C. Now $x = t + fs$ where f depends on the slot width s and on the air-gap clearance δ and may be obtained from Fig. 34. Then

$$C = \lambda/x = (t+s)/(t+fs) \tag{12}$$

For the machine shown in Fig. 35:

$s/\delta = 0.43/0.3 = 1.44$
$f = 0.78$ from Fig. 34.
$C = (0.48+0.43)/(0.48+0.78 \times 0.43) = 1.12$

* Hobart and Ellis. "High Speed Dynamo-electric Machinery."
† Gray. "Electric Machine Design," p. 142.
‡ Carter, *Electrical World and Engineer*, Nov. 30, 1901; Hele-Shaw, Hay and Powell; *Jour.* of Inst. of Elec. Eng., Vol. XXXIV, p. 21.

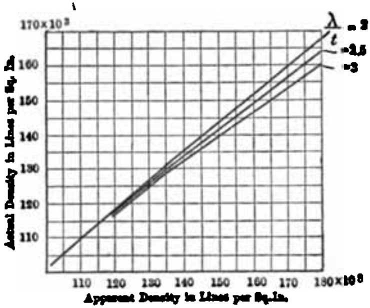

FIG. 36.—Armature tooth densities.

ϕ_m, the total flux per pole, consists of the flux ϕ_a which crosses the air-gap and is cut by the armature conductors, and the leakage flux ϕ_e which crosses between the poles but does not enter the armature. The ratio ϕ_m/ϕ_a, called the leakage factor, is greater than unity; its value may be calculated with a fair degree of accuracy if the dimensions of the magnetic circuit are known, but for direct-current machines it is usual to assume a value of leakage factor and, for a first approximation, the following table may be used for the type of machine shown in Fig. 38.

	Leakage factor
Four-pole machines up to 10 in. armature diameter....	1.25
Multipolar machines between 10 in. and 30 in. diameter..	1.2
Multipolar machines between 30 in. and 60 in. diameter..	1.18
Multipolar machines greater than 60 in. diameter......	1.15

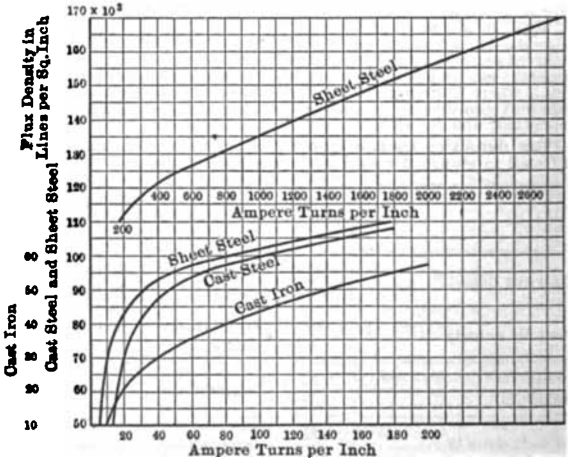

FIG. 37.—Magnetization curves.

80. Calculation of the leakage factor.* Fig. 39 shows part of a machine with a large number of poles. The total leakage flux per pole is

$$\phi_e = \phi_1 + \phi_2 + \phi_3 + \phi_4 \quad (13)$$

*The derivation of these formulæ is given in Gray, "Electrical Machine Design," page 51, for other cases see Hawkins and Wallis, "The Dynamo," Vol. I, p. 471.

where

$$\phi_1 = 13k(nI)_{g+t}\ \frac{L_s h_s}{l_1} \tag{14}$$

$$\phi_2 = 19k(nI)_{g+t}\ h_s \log_{10}\left(1 + \frac{\pi W_s}{2l_1}\right) \tag{15}$$

$$\phi_3 = 6.5k(nI)_{g+t}\ \frac{L_p h_p}{l_2} \tag{16}$$

$$\phi_4 = 9.5k(nI)_{g+t}\ h_p \log_{10}\left(1 + \frac{\pi W_p}{2l_2}\right) \tag{17}$$

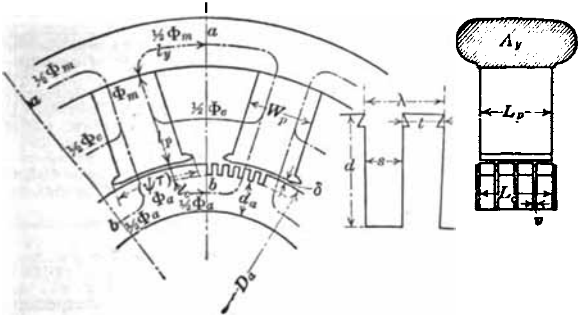

FIG. 38.—The paths of the main and the leakage fluxes.

and k is equal to unity for inch units and to $1/2.54 = 0.3937$ for centimeter units. For a given value of ϕ_a, the flux crossing the air-gap, the value of $(nI)_{g+t}$, the necessary ampere-turns per pole for gap and teeth, may be found; and, substituting this value in the above formulas, the corresponding value of ϕ_s may be found. The leakage factor $= 1 + \phi_s/\phi_a$.

81. Calculation of field ampere-turns per pole. The total ampere-turns per pole required to establish the necessary flux in the magnetic circuit may be analysed into a series of components. The natural subdivision follows the several different members or elements which go to make up the complete circuit. These would include the yokes, the field cores, the air-gaps, the armature teeth and the armature core. Since the field poles are structurally duplicates of each other, except where interpoles are used, it is natural to confine the calculations of required ampere-turns to a single field pole. Fig. 40 shows the unit magnetic circuit for a multipolar machine, or that part of the total magnetic structure which corresponds to one field pole and simply needs repeated application to build up the whole structure. This unit magnetic circuit comprises one complete field core, one complete air-gap, one complete group of armature teeth under a single pole

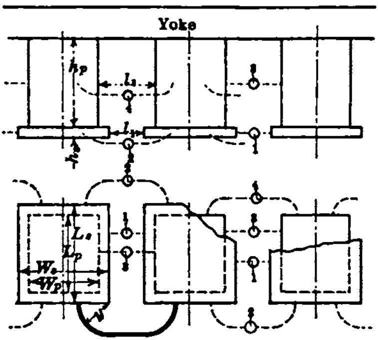

FIG. 39.—The leakage paths.

face, two half-yokes in parallel and two half-armature cores in parallel. The total number of ampere-turns, at no load, may be expressed as

$$nI = (nI)_y + (nI)_c + (nI)_p + (nI)_t + (nI)_g \qquad (18)$$

where the several quantities in the formula are as follows:

$(nI)_y$ represents the number of ampere-turns required to establish the flux $\phi_m/2$ through a length of yoke equal to l_y in Fig. 38; this length is one-half the total length of the yoke circuit from pole to pole. In a single-yoke machine, such as the horse-shoe bipolar type, with two field coils, the full value of flux, or ϕ_m, should be assumed instead of $\phi_m/2$.

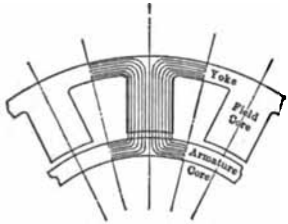

Fig. 40.—Unit magnetic circuit of multipolar machine.

$(nI)_c$ represents the number of ampere-turns required to establish the flux $\phi_a/2$ through a length of armature core equal to l_c in Fig. 38.

$(nI)_p$ represents the number of ampere-turns required to establish the flux ϕ_m through the field core, which has a length of l_p as shown in Fig. 38.

$(nI)_t$ represents the number of ampere-turns required to establish the flux ϕ_a through one set of armature teeth under one pole face.

$(nI)_g$ represents the number of ampere-turns required to establish the flux ϕ_a through one air-gap.

The calculation of ampere-turns in each case, except for the air-gap, is carried out by taking different values of flux density ℬ and obtaining the proper value of ampere-turns per in. from the curves in Fig. 37; the latter is then multiplied in each instance by the length of the corresponding portion of the magnetic circuit, in inches.

The required number of ampere-turns for the air-gap, in terms of inch units, is given by

$$(nI)_g = \frac{C\,\mathcal{B}_g\delta}{3.20} \qquad (19)$$

and in terms of centimeter units, by

$$(nI)_g = \frac{C\,\mathcal{B}_g\delta}{1.26} \qquad (20)$$

The quantity C is the fringing constant (Par. 75).

82. Excitation for tapered teeth. When the armature teeth are tapered, the flux density is not uniform throughout the total depth of the tooth. The tooth length must therefore be divided into a number of short lengths, the flux density and the corresponding ampere-turns per in. length found in each case, and the average value multiplied by the tooth length to give $(nI)_t$. This process can be shortened by use of the series of curves shown in Fig. 41; if k, the ratio between the actual flux densities at the bottom and at the top of the tooth, is known, the average ampere-turns per in. can be found directly.

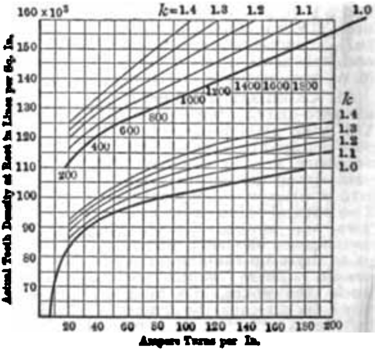

Fig. 41.—Magnetisation curves for tapered teeth.

83. Calculation of the no-load saturation curve. Taking the example of the ten-pole machine shown in Fig. 35, the following calculations must be carried out in order to obtain a series of points from which to plot the no-load saturation curve.

L_a = the net axial length of armature core = 0.9 (12 − 3×0.5) = 9.45 in.
A_y = the yoke area = 136 sq. in.

87. The minimum length of field coil to give the necessary radiating surface for cooling is given by the formula*

$$L_f = \frac{n_f I_f}{1{,}000}\sqrt{\frac{\text{length of mean turn}}{\text{external periphery} \times \text{watts per sq. in.} \times d_f \times sf \times 1.27}} \qquad (22)$$

where L_f is the length of the field coil in inches, Fig. 42,
d_f is the depth of the field coil in inches,
sf is the space factor of the winding and may be found from Fig. 43.

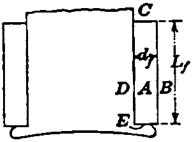

FIG. 42.—Field coil length.

The allowable watts per sq. in. of surface B, Fig. 42, may be found from Fig. 44.

88. The weight of field coil for a given machine is equal to $\sqrt{d_f}$ multiplied by a constant. The larger the value of d_f, the shorter the length L_f, the smaller the radiating surface and the lower the permissible loss in the coil. Since the section of the field coil wire is fixed (because it depends only on the ampere-turns and the voltage per coil, Par. 86), a lower permissible loss can be obtained only by the use of a smaller current I_f, a larger number of turns n_f, and therefore a more expensive coil. It must not be overlooked, however, that when d_f becomes smaller and the field coil becomes cheaper, the value of L_f increases and therefore the cost of the poles and the yoke increases. The most economical value for d_f must be determined by trial; an average value is 2 in.

89. Example of field system design. The armature of a ten-pole, 240-volt no-load, 240-volt full-load, 1,670 amp., 200 r.p.m. generator is shown drawn to scale in Fig. 35; it is required to design the field system, which is not supposed to be given.

The following data are taken from Par. 83.

$\phi_m = 10.5 \times 10^6$
$\phi_a = 9 \times 10^6$
$nI_t = 2{,}080$
$\mathfrak{B}_g = 59{,}000$ lines per sq. in.

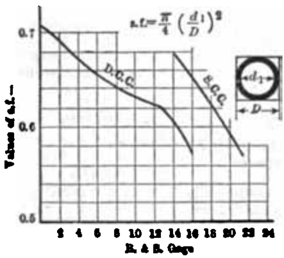

FIG. 43.—Space factor for wire.

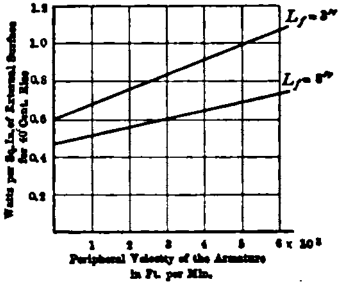

FIG. 44.—Field-coil heating constant.

(a) Calculation of the air-gap clearance.

$(nI)_g + (nI)_t = 1.2$ times the armature ampere-turns per pole, from Par. 25,
$= 1.2\ (800 \times 167/2 \times 10) = 8{,}100$
$(nI)_t = 2{,}080$ see above
$(nI)_g = 6{,}020,$
$C\delta = 3.2 \times 6{,}020/59{,}000$, from Par. 81,
$= 0.328$ in.,
$C = 1.12$ from Fig. 34,
$\delta = 0.29$ in. (make it 0.3 in.).

* Gray. "Electrical Machine Design," p. 66.

(b) Calculation of approximate dimensions for the magnetic circuit.

Assume the no-load excitation $= 1.25[(nI)_g + (nI)_t]$
$= 10{,}100$ ampere-turns per pole.

Next find L_f for the shunt coil from eq. 22 (Par. 87), where d_f is assumed to be 2 in.; sf is assumed to be 0.6, from Fig. 43; the allowable watts per sq. in. $= 0.6$ from Fig. 44; the external periphery of the field coil equals 1.2 times the length of the mean turn, approximately; therefore $L_f = 10$ in., approximately; the latter should be increased 30 per cent. to allow for the series coil.

The pole area $= \phi_m/\mathcal{B}_p = 10.5 \times 10^6/95{,}000 = 112$ sq. in.

The yoke area $= \phi_m/2\mathcal{B}_y = 10.5 \times 10^6/2 \times 40{,}000 = 132$ sq. in.

(c) Saturation curves. From these dimensions the magnetic circuit is drawn to scale and the no-load and the full-load saturation curves are determined and plotted. For the machine in question, the curves are given in Fig. 19.

(d) Design of the shunt coils. The details are as follows:

The no-load excitation $= 9{,}990$ ampere-turns per pole, Fig. 19, or Par. 82.

E_f, the volts per coil, equals $0.8 \times 240/10 = 19$ volts; this leaves 20 per cent. of the terminal voltage to be absorbed by the field rheostat.

M, the mean turn, equals 53 in. The external periphery equals 61 in.

The size of wire equals $9{,}990 \times 53/19$, from Par. 88, or 28,000 cir. mils. The proper size of wire is No. 5.5 A.W.G., a special size between Nos. 5 and 6, which has a section of 29,500 cir. mils and a diameter when insulated with double cotton of 0.19 in. When such odd sizes are not available, the coil can be made with the proper number of turns of No. 5 wire in series with the proper number of turns of No. 6, so as to have a field coil of the proper resistance.

The number of layers of wire in a 2-in. depth equals $2/0.19 = 10$.

The number of turns per layer in a 10-in. length equals $10/0.19 = 53$.

n_f, the number of turns per coil, equals 530.

I_f, the shunt current, equals $9{,}990/530 = 18.8$ amp.

The current density equals $29{,}500/18.8 = 1{,}570$ cir. mils per amp.

(e) Design of the series coils. The details are as follows:

Excitation at full load and normal voltage	$= 13{,}400$ ampere-turns, Fig. 19.
Shunt excitation at normal load	$= 9{,}990$ ampere-turns.
Series ampere-turns per pole at full load	$= 3{,}410$
Series turns per pole	$= 2.5$
Series current $= 3{,}410/2.5$	$= 1{,}370$ amp.
Current in series shunt	$= 300$ amp.

The current density is taken 20 per cent. greater than in shunt coils because the series coils are better ventilated.

Current density	$= 1{,}300$ cir. mils per amp.
Size of wire $= 1{,}300 \times 1{,}370$	$= 1{,}780{,}000$ cir. mils $= 1.4$ sq. in.
Resistance of 2.5 turns	$= 7.4 \times 10^{-5}$ ohms.
Loss in one series coil	$= 1{,}370^2 \times 7.4 \times 10^{-5} = 140$ watts.

The permissible watts per sq. in. is taken 20 per cent. greater than in shunt coils.

Watts per sq. in.	$= 0.72$
Necessary radiating surface	$= 140/0.72 = 195$ sq. in.
Length L_f of series coil	$= 195/61 = 3.2$ in.
Depth of wire $=$ wire section$/L_f$	$= 0.45$ in.

Size of wire $= 3$ strips (3.2 in. $\times$ 0.15 in.).

GENERAL DESIGN AND CONSTRUCTION

90. Type of construction for small machines. Fig. 45 shows the type of construction generally adopted for machines up to 100 h.p. at 600 rev. per min.

91. The armature core (M in Fig. 45) is built of laminations of sheet steel 14 mils (0.35 mm.) thick, which are separated from one another by layers of varnish and have slots F punched to carry the armature coils G.

92. A marking notch is punched at one side of the key-way K (Fig. 45) and these notches must line up when the punchings are assembled, to ensure that the burrs at the edges all lie in the same direction.

93. Brass vent segments (*P* in Fig. 45), about $\frac{3}{8}$ in. (1 cm.) wide, separate the core into sections about 3 in. (7.5 cm.) wide. The core and the vent segments are clamped between end heads *N* which carry coil supports *L* attached by arms shaped like fans. The coils are held in place against centrifugal force by steel band wires.

94. The poles (*B* in Fig. 45), are of circular cross-section in order to give the required area for the magnetic flux with the minimum length of mean turn of field coil *A*. They are made of forged steel and have laminated pole faces *E* made of sheet steel 25 mils (0.63 mm.) thick.

95. End play is provided for by making the axial length of the pole face $\frac{3}{8}$ in. (1 cm.) shorter than that of the armature core. This enables the revolving part of the machine to oscillate axially and so prevent the journals, bearings and commutator from wearing in grooves.

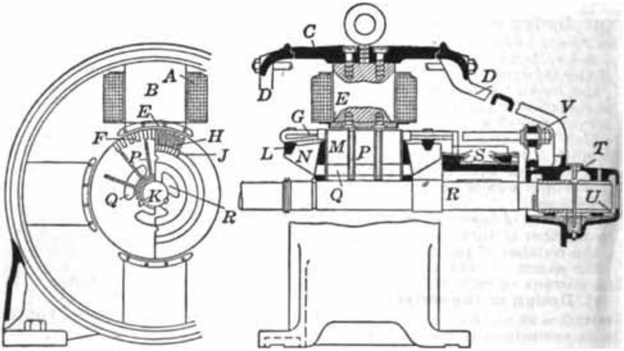

Fig. 45.—Small direct-current motor.

96. The yoke (*C* in Fig. 45) carries the bearing housings *D* which stiffen the whole machine. The housings can be rotated with respect to the yoke through 90 deg. or 180 deg., so that the machine may be mounted either on a wall or a ceiling. This rotation of the bearings is necessary because the machine is lubricated by **oil rings** and the **oil wells** must always be below the shaft.

97. The commutator segments (*J* in Fig. 45) are of hard-drawn copper and have a wearing depth of from 0.5 in. (1.25 cm.) on a 5-in. (12.5-cm.) commutator to 1.0 in. (2.5 cm.) on a 50-in. (125-cm.) commutator. The commutator shell, as the clamps and their supports are called, is provided with air passages *R* which help to keep the machine cool.

98. The bearing construction is shown in detail in Fig. 45. The projection *T* on the oil-hole cover keeps the oil ring from rising and resting on the bushing; the oil slingers prevent the oil from creeping along the shaft. The liner of special bearing metal is given a snug fit in the bearing shell, and can be removed when worn and another put in its place. A small overflow at *U* makes it impossible to fill the bearing too full.

99. The brushes are carried on studs which are insulated from the rocker arm *V*. The rocker arm is carried on a turned seat on the bearing and can be clamped in a definite position.

100. Type of construction for large engine-type generators. For large direct-connected engine-type units the type of construction shown in Fig. 46 is generally adopted. The **armature core** is built of segments carried by dovetails on the spider. The segments of alternate layers overlap so as to break joints and give a solid core. The **poles** are of rectangular cross-section and are built of laminations 25 mils (0.63 mm.) thick, of the shape shown at *P*, and assembled so that the rounded pole tips point in opposite directions and a saturated tip is produced which helps commutation. The **shaft, base and bearings** are generally supplied by the engine builder, so that the commutator must be supported from the armature spider; the **brush rigging** must also be supported by the machine.

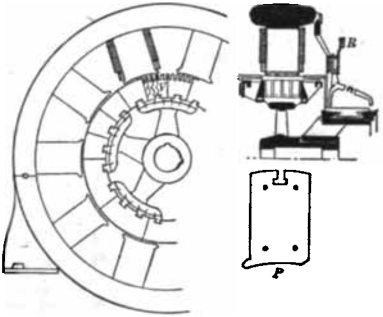

Fig. 46.—Large engine-type generator.

101. Mechanical design.* The **yoke** should be heavy and, in engine-type units, is often made with a large section of cast iron rather than a smaller section of cast steel; it must be stiff enough to prevent undue sagging even when there is a large unbalanced magnetic pull. The **shaft** should be stiff enough to limit the deflection to 5 per cent. of the air-gap clearance and, along with the **spider**, should have a **factor of safety** of 12 to take care of short-circuits.

102. Unbalanced magnetic pull.† If the armature of a machine is x cm. out of centre, the flux density under the poles where the gaps are smallest will be greater than under the poles where the gaps are largest, and the pull on one side of the machine will be greater than on the other by an amount

$$kS\mathfrak{B}^2\frac{x}{\delta} \qquad (23)$$

where $\mathfrak{B}$ is the effective gap density,

$S = \pi D_aL_a$, the total armature-core surface,

x = armature displacement,

δ = length of air-gap,

$k = \dfrac{1}{7.2\times10^7}$ if inch units are used and the pull is in pounds,

$k = \dfrac{1}{2.47\times10^7}$ if centimeter units are used and the pull is in kilograms.

The above formula holds for a two-circuit winding; if a multiple winding is used, the e.m.fs. in the different paths will be unequal, and circulating currents will flow tending to keep the flux densities under the poles nearly equal, so that the pull will be less than that given by the formula.

INSULATION

103. General requirements. A good insulator for electrical machinery must have high dielectric strength and high electrical resistance, should be tough and flexible, and should not be affected by heat, vibration, or other operating conditions.

104. The insulating materials generally used for revolving machinery are as follows:

(a) **Micanite,** which is easily bruised and should be protected by a tougher material.

(b) **Varnished cloth,** which must be carefully handled to prevent cracking or scraping of the varnish film.

(c) **Paper,** which is chosen for its toughness and should be baked dry when tested.

* Livingstone, "Mechanical Design and Construction of Commutators;" Livingstone, "Mechanical Design and Construction of Generators;" Hawkins and Wallis, "The Dynamo;" Hobart and Ellis, "Armature Construction."

† Behrend, B. A. *Trans.* A. I. E. E., Vol. XVII, p. 617.

120. Intermittent ratings. Suppose that a machine is operating on a continuous cycle, x min. loaded and y min. without load. The temperature of the machine will vary during each cycle between the values θ_x and θ_y (Fig. 50), where the temperature increase in the interval x is equal to the temperature decrease in the interval y. Under these service conditions, θ_x, the highest operating temperature, is less than θ_m, the maximum temperature that would be obtained on continuous operation under load. For this service, therefore, a machine may be designed with higher copper and iron densities than otherwise it would have if designed for the same load but for continuous operation. It must be noted that, if the machine is stationary during the period of no load, the drop in temperature is small, and the rating of the machine cannot be made much greater than if it were operating on continuous load.

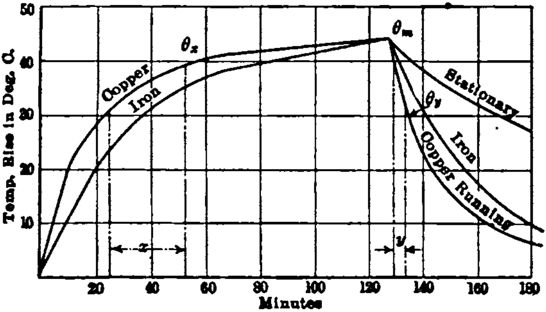

FIG. 50.—Heating and cooling curves of a direct-current motor.

121. Heat conduction in the armature core. The hottest part of the core is at A, Fig. 51; consequently the heat has to be conducted along the laminations and dissipated from surfaces B, and also across the laminations and layers of varnish and dissipated from surfaces C. The conductivity along the laminations is about fifty times that across the laminations, but the end surface of the laminations is small compared with the surface of the vent ducts, so that **radial ducts** are effective and necessary.

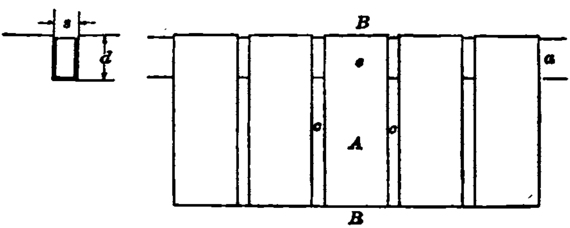

FIG. 51.—Heat conduction in armature core.

122. Flux density and peripheral velocity. The peripheral velocity of a machine in ft. per min. is

$$v = 10 \times \text{pole pitch in inches} \times \text{frequency in cycles per sec.}$$

Hence for a given frequency, the peripheral velocity of the machine is proportional to its pole pitch. The flux in the armature core of a direct-current machine is always alternating, and the **frequency** is equal to the product of the revolutions per sec. and the number of pairs of poles (interpoles excluded).

130. Semi-enclosed machines have the frame openings screened with perforated sheet metal. This prevents free circulation of air through the machine and causes the temperature to rise, on an average, about 20 per cent. higher than if operating as an open machine.

131. In totally enclosed machines, the temperature rise of the core and the coils is proportional to the total loss in the machine (neglecting bearing friction); it is independent of the distribution of this loss and may be found from Fig. 54. The rating as an enclosed machine will be considerably lower than that as an open machine. A casing around the machine makes it equivalent to an enclosed machine with an extra large radiating surface; the casing should be of heat-conducting material such as sheet metal and should not be of wood. Fans on the armature help in the cooling of enclosed machines by blowing the hot air against the casing.

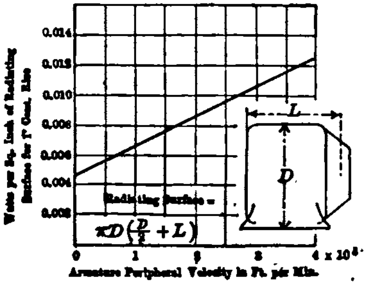

Fig. 54.—Temperature rise of enclosed motors.

132. Forced ventilation,* whereby air is blown through a machine by an external fan or by a fan attached to the armature, allows the machine to have the same rating as it would have when operating as an open machine; 100 cu. ft. (2.8 cu. m.) of air per kw. loss should be supplied for a temperature rise of the windings and the core of 40 deg. cent.

EFFICIENCY AND LOSSES

133. The iron losses† consist of hysteresis and eddy-current losses in the armature teeth and core; these may be kept small by the use of special grades of iron (Sec. 4) and by the use of thin and well-varnished laminations. There are losses also in the end heads and spider due to leakage flux; losses due to filing of the slots which short-circuits the laminations; losses due to the fact that the core flux takes the shortest path and crowds in behind the teeth, so that the core density is not uniform; and pole-face losses‡ due to the movement of the armature teeth past the faces of the poles. In order to minimize the pole-face losses, the pole faces should be laminated if the slot opening is greater than twice the air-gap clearance.

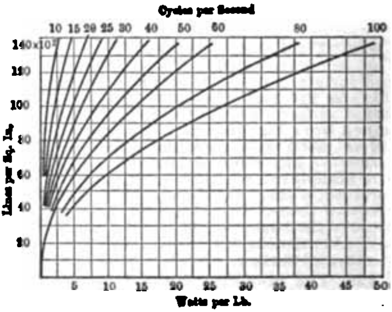

Fig. 55.—Iron-loss curves for direct-current machines.

134. The iron losses increase with the load if the flux per pole is kept

* Gray. "Heating of Pipe-ventilated Machines," London Electrician, Jan. 16, 1914.

† Thornton. "Distribution of Magnetic Induction in Armatures," *Journal* of Inst. of Elec. Eng., Vol. XXXVII, p. 125.

‡ Wall and Smith. "Pole-face losses," *Journal* of Inst. of Elec. Eng., Vol. XL, p. 577.

constant, because of field distortion, as the densities are then very high under alternate pole tips.

135. The individual iron losses cannot be calculated separately, and curves such as those in Fig. 55, obtained by tests on completed machines, must be used as follows:

Taking the machine shown in Fig. 35:	
Actual maximum tooth density	= 150,000 lines per sq. in.
Average core density	= 84,000 lines per sq. in.
Weight of armature teeth	= 385 lb.
Weight of armature core	= 2,300 lb.
Frequency $= p \times$ r.p.m./120	= 16.6 cycles per sec.
Tooth loss per lb.	= 6 watts, from Fig. 55.
Core-loss per lb.	= 1.8 watts.

Total iron loss $= (385 \times 6) + (2{,}300 \times 1.8) = 6{,}450$ watts.

136. The armature copper loss is given by the formula

$$\frac{ZLI_c^2}{\text{c.m.}} \qquad \text{(watts)} \qquad (27)$$

where Z is the number of conductors, L is the length of one conductor in in. (Fig. 56), I_c is the current in each conductor, or the total current divided by the number of armature paths, and c.m. is the cross-section of each conductor in cir. mils.

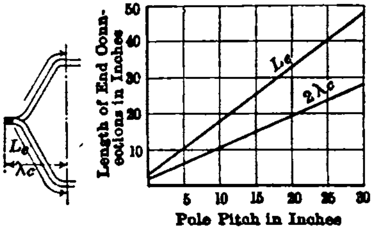

Fig. 56.—Dimensions of coils.

137. The shunt excitation loss equals E_tI_f watts, where E_t is the terminal voltage and I_f the shunt current. In a generator, about 20 per cent. of this loss will be in the shunt-field rheostat.

138. The series excitation loss equals $I_a^2R_s$ watts, where I_a is the total current in the machine and R_s is the combined parallel resistance of the series field coils and the series shunt.

139. The brush contact resistance loss has been discussed in Par. 38 and equals E_bI_a watts, where E_b is the voltage drop per pair of brushes and I_a is the total current of the machine.

140. The bearing friction loss for moderate-speed bearings with ring lubrication and light machine oil is given by the formula

$$0.8dl\left(\frac{v}{100}\right)^{\frac{3}{2}} \qquad \text{(watts)} \qquad (28)$$

where d is the bearing diameter in inches, l is the bearing length in inches, and v is the rubbing velocity in feet per min.

141. The brush friction loss, assuming the coefficient of friction to be 0.28 and the brush pressure to be 2 lb. per sq. in., is given by the formula

$$1.25\,A\frac{V}{100} \qquad \text{(watts)} \qquad (29)$$

where A is the total brush rubbing surface in square inches, and V is the rubbing velocity in feet per min. The brush pressure is generally less than 2 lb. per sq. in. (0.14 kg. per sq. cm.) except for railway motors, where it may be twice as large in order to keep the contact firm in spite of the vibration of the machine.

142. The windage loss cannot be accurately calculated and, with peripheral velocities less than 6,000 ft. per min., is so small that it may be neglected.

143. The efficiency of a generator is given by the expression

$$Eff_g = \frac{\text{output}}{\text{output} + \text{losses}} \qquad (30)$$

ance is reduced below this value, the machine becomes completely demagnetized (except for the residual magnetism), and the terminal voltage and the armature current both become practically zero.

147. **When a shunt generator is short-circuited** the machine is demagnetized and the current becomes negligible (Fig. 59). At the instant of short-circuit, however, the flux in the pole cannot reduce suddenly to zero, and the voltage due to this flux will send a large current through the circuit; at the end of a few seconds, however, the current will have become zero.

148. **Instability of unsaturated shunt generators.** If the no-load, normal voltage point on the saturation curve is a, Fig. 60, below the point of saturation, then the overload capacity of the machine will be small, and the change of voltage with load will be large. Furthermore, a slight decrease in speed will cause a decrease in the generated voltage, which will decrease the shunt field current and cause the voltage to drop still further. If ac is tangent to the saturation curve at point a, then the ratio of voltage change to the change of speed producing it, is ab/oc.

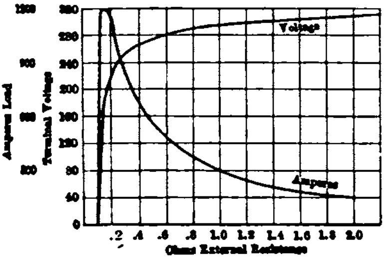

Fig. 59.—Shunt characteristics on a resistance base.

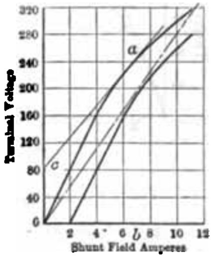

Fig. 60.—No-load and full-load saturation curves.

149. **The regulation** of electrical machinery is defined in the A. I. E. E. Standardization Rules (Sec. 24). The **voltage regulation** of a separately excited generator is bc/bh (Fig. 58); for a shunt generator it is ec/eh (Fig. 58); for a flat-compounded generator it is fg/ch; and for an overcompounded generator it is mn/hk (Fig. 58). The **current regulation** of a constant-current generator depends largely on the type of regulator used and is defined as the ratio of the maximum difference of current from the rated load value (occurring within the range from rated load to short-circuit, or minimum limit of operation) to the rated load current.

150. **To maintain the terminal voltage constant,** shunt generators may be operated with an adjustable resistance in series with the shunt field coils; this resistance may be reduced automatically, or by hand, as the load increases. Automatic regulators for this purpose generally consist of a solenoid connected across the terminals of the machine, acting as a relay which will open or close an operating circuit and vary the resistance of the field circuit as required.

151. **The Tirrill Regulator,** described in Sec. 10, periodically short-circuits part of the field rheostat by means of light vibrating contacts of small inertia; the time during which the rheostat is short-circuited or is active determines the average field current and therefore the voltage of the machine.

152. **Compound machines,** operated without a regulator, keep the voltage approximately constant from no load to full load because, due to the series field coils, the total excitation increases with the load; the machine is then said to be **flat-compounded.** By the use of an extra strong series field, the voltage may be made to increase with the load; the machine is then said to be **overcompounded.** Curves 3 and 4, Fig. 58, show the

characteristic curves for compound generators. The compounding of a machine may be reduced by shunting the series field coils with a resistance; this resistance is called a "series shunt."

153. Series generators. Curve 1, Fig. 61, shows the relation between voltage and current if there is no armature resistance or armature reaction. This is really the no-load saturation curve of the machine and is determined by separately exciting the field coils so that no current flows in the armature. Curve 2 shows the actual relation between terminal voltage and load current.

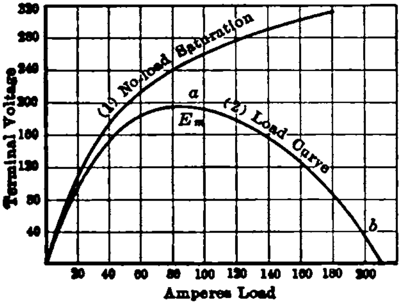

Fig. 61.

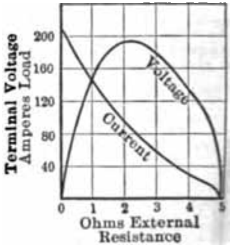

Fig. 62.

Figs. 61 and 62.—Characteristic curves of a series generator.

The total voltage drop consists of that portion due to the decrease in flux caused by armature reaction, and that required to send the current through the armature, brushes and series field coils.

154. Critical resistance of series generators. The values of current in a series generator and the resistance of the external circuit are plotted in Fig. 62. The critical resistance of the machine is 4.9 ohms; with an external resistance greater than this, the machine will not excite, or build up.

155. Series machines were formerly used as **constant-current generators** for the operation of **arc lamps** in series, but few of them are now in service. Specially designed machines were operated with automatic regulators so as to have the line *ab* (Fig. 61) nearly vertical, and the current practically constant for all voltages up to E_m (Fig. 61).

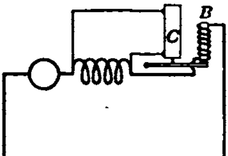

Fig. 63.—Constant-current regulator.

156. The Brush regulator, shown diagrammatically in Fig. 63, has been used as a **constant-current regulator.** The carbon pile *C* has the property that its resistance decreases through a wide range as the pressure between the ends increases. If the current in the external circuit increases, the pull of the solenoid *B* also increases, and the carbon pile is compressed and its resistance reduced so that it shunts more of the current from the series field coils; the flux in the machine is therefore reduced and the voltage drops until the current reaches the value for which the regulator was set. Other types of constant-current regulators are described in Par. **209** and **212.**

MOTOR CHARACTERISTICS AND REGULATION

157. Counter e.m.f. It was pointed out in Par. 6 that, since a motor armature revolves in a magnetic field, an e.m.f. is generated in the conductors which is opposed to the direction of the current and is called the counter e.m.f. The applied e.m.f. must be large enough to overcome the counter e.m.f. and also to send the armature current I_a through R_m, the resistance of the armature winding, the brushes and the series field coils; or

$$E_a = E_b + I_a R_m \quad \text{(volts)} \qquad (32)$$

where E_a is the applied e.m.f. and E_b, the counter e.m.f., is given by Eq. 9, Par. 60.

158. Shifting of the brushes. It may be seen from Fig. 2 that, while the brushes of a generator are shifted forward in the direction of motion to help commutation, those of a motor have to be shifted backward. In each case, however, the armature reaction reduces the flux per pole.

159. The torque equation. The torque of a motor is proportional to the number of conductors on the armature, to the current per conductor and to the total flux in the machine. The formula for the torque is

$$\text{torque} = 0.1175\, Z \phi I_a \left(\frac{\text{poles}}{\text{paths}}\right) 10^{-8} \quad \text{(lb. at 1 ft. rad.)} \qquad (33)$$

where Z is the total number of armature conductors, ϕ is the total flux per pole, and I_a is the armature current taken from the line.

160. The speed equation is

$$E_b = E_a - I_a R_m = Z\phi \left(\frac{\text{r.p.m.}}{60}\right)\left(\frac{\text{poles}}{\text{paths}}\right) 10^{-8} \quad \text{(volts)} \qquad (34)$$

or

$$\text{r.p.m.} = 60 \left(\frac{E_a - I_a R_m}{Z\phi}\right)\left(\frac{\text{paths}}{\text{poles}}\right) 10^{8} \qquad (35)$$

For a given motor the number of armature conductors Z, the number of poles, and the number of armature paths, are constant. The torque can therefore be expressed as

$$\text{torque} = \text{a constant} \times \phi I_a \qquad (36)$$

and the speed, likewise, is expressed

$$\text{r.p.m.} = \text{a constant} \times \left(\frac{E_a - I_a R_m}{\phi}\right) \qquad (37)$$

161. Shunt-motor speed and torque. In this case E_a, R_m and ϕ are constant, and the speed and torque curves are shown in curves 1, Fig. 64; the effective torque is less than that generated, by the torque required for the windage and the bearing and brush friction. The drop in speed from no load to full load seldom exceeds 5 per cent.; indeed, since ϕ, the flux per pole, decreases with increase of load, due to armature reaction, the speed may remain approximately constant up to full load.

162. Effect of field-coil heating on speed. The field-coil resistance increases and the exciting current decreases about 20 per cent. as the field coils increase in temperature. The flux per pole is therefore less and the speed greater when the machine is hot than when cold, unless a field rheostat is manipulated to keep the exciting current constant. The effect of change of exciting current on speed is minimized by having the magnetic circuit well saturated, so that a large change in current will produce only a small change in flux.

163. Variable-speed operation of shunt motors can best be investigated by means of Eq. 37, Par. 160. In order to increase the speed, ϕ must be reduced by inserting a resistance in series with the field coils. In order to decrease the speed below the value which it has with full field, the quantity $(E_a - I_a R_m)$ must be decreased by placing a resistance in series with the armature. The latter resistance must be able to carry the armature current, but the starting resistance must not be used for this purpose since it is designed for starting duty only, and would burn out if allowed to carry full-load current for more than a few minutes.

164. Speed control of shunt motors by armature resistance is not very satisfactory, since the speed regulation is bad. If a motor is operating with full-load current at half speed, about 50 per cent. of the applied voltage is consumed in the resistance, but if the load were decreased, so that only half of full-load current was required, then only 25 per cent. of the applied voltage would be consumed in the same external resistance, and the motor speed would increase to 75 per cent. of normal speed, unless the external control resistance were automatically increased with decrease of load. Due to the large voltage drop across the external resistance, the efficiency of the system is low.

165. When the speed of shunt motors is controlled by field resistance, the speed regulation and the efficiency are both good, but the commutation is generally poor because, at high speeds, the main field is weaker than normal while the armature field is unchanged, so that the field distortion is excessive and the commutating field under the pole tip consequently disappears (Par. **35**). The reactance voltage also is increased (Par. **45**). With standard shunt motors without interpoles, it is generally impossible to increase the speed more than 60 per cent. by field weakening without having trouble due to sparking, the output of the machine being the normal full load.

166. Speed changes of shunt motors under rapidly fluctuating loads.* When the load on a shunt motor increases slowly, the flux per pole decreases due to armature reaction, and the speed (Eq. 37, Par. **160**) remains approximately constant. If, however, the load changes rapidly, the flux per pole cannot change rapidly due to the self-induction of the field coils; the machine then operates for the instant as a constant-flux machine, and the speed drops rapidly to allow the counter e.m.f. to decrease and the necessary current to flow.

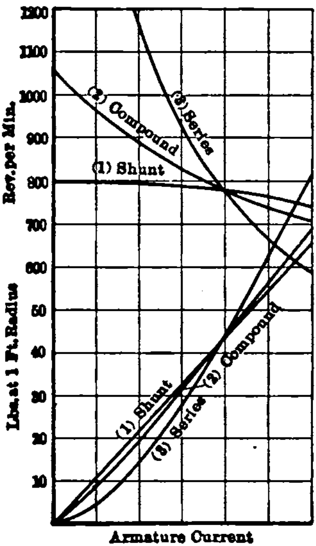

FIG. 64.—Motor characteristics.

167. Speed and torque of series motors. The speed equation (Eq. 37, Par. **160**) and the torque equation (Eq. 33, Par. **159**) apply to motors of all continuous-current types. In the case of series motors the flux ϕ increases with the armature current I_a; the torque would be proportional to I_a^2 were it not that the magnetic circuit becomes saturated with increase of current. Since ϕ increases with load, the speed drops as the load increases. The speed and torque characteristics are shown in curves 3, Fig. 64.

168. Excessive speeds of series motors on small loads. If the load on a series motor becomes small, the speed becomes very high, so that a series motor should always be geared or direct-connected to the load. If it were belted, and the belt broke, the motor would run away and would probably burst.

169. Speed adjustment of series motors. For a given load, and therefore for a given current, the speed of a series motor can be increased by shunting the series winding or by short-circuiting some of the series turns, so as to reduce the flux. The speed can be decreased by inserting resistance in series with the armature.

170. The compound motor is a compromise between the shunt and the series motor. Because of the series winding, which assists the shunt winding, the flux per pole increases with the load, so that the torque increases more rapidly and the speed decreases more rapidly than if the series winding was not connected, but the motor cannot run away on light loads, because of the

* Short, E. W. "Inherent Regulation of Direct-current Motor;" *Journal* of Inst. of Elec. Eng., Vol. XLVI, p. 171.

shunt excitation. The speed and torque characteristics for such a machine are shown in curves 2, Fig. 64. The speed of a compound motor can be adjusted by armature and field rheostats, just as in the shunt machine (Par. 168).

171. Automatic speed regulation. To keep the speed constant under all conditions of load some kind of centrifugal governor may be used to operate on the shunt-field rheostat. A piece of apparatus of this kind now on the market* is an adaptation of the Tirrill voltage regulator, in which a centrifugal control device mounted on the motor shaft performs the functions of the main control magnet in the standard regulator. The regulator periodically short-circuits part of the shunt-field rheostat by means of light vibrating contacts of small inertia. The time during which the rheostat is short-circuited, or is active, determines the average field current and the flux per pole, and therefore the speed of the machine. When the speed is too high, the centrifugal device short-circuits the rheostat for a long period and allows the flux per pole to increase and the speed to decrease.

172. The differential motor is a compound-wound machine with the series winding opposing the shunt winding, so that the flux will decrease as the load increases and the speed be constant from no load to full load, or actually increase with increase of load. The series winding of such a motor should be short-circuited when starting the machine, so that the starting current will not be excessive. Differential motors are rarely used, because the speed of the shunt motor is so nearly constant from no load to full load that the extra complication of the differential winding is rarely necessary. Differential windings are used to some extent for small motors, in order to secure constant speed with variable load.

173. Unstable operation with rising-speed characteristic. Upon suddenly increasing the load torque on a differential motor, designed for a rising-speed characteristic, the speed for a brief instant decreases. This results in a momentary drop of counter e.m.f., which admits a larger armature current, in turn weakening the resultant field strength and further reducing the counter e.m.f. The attendant increase in armature current increases the armature torque, and the reactions are such that the latter continues to increase until it exceeds the load torque and commences to accelerate the armature. The increase in speed continues until the rising counter e.m.f. finally limits the armature current to a value at which the armature torque equals the load torque, and the speed becomes constant. These reactions, with change of load, occur quite rapidly; if, however, the field cores are large and massive, changes in flux attendant upon sudden changes in m.m.f. will lag by an appreciable time interval, on account of eddy currents in the cores. The presence of an appreciable flux lag, with very rapidly changing loads, results in unstable operation; for example, when the load is suddenly increased, the speed will drop appreciably before it commences to accelerate, and when the load is suddenly removed, the speed will rise appreciably before it commences to decrease. The effect of the armature inertia will accentuate these defects in speed regulation. Such defects are not found in motors whose speed decreases with increasing load, *i.e.*, which have a drooping speed characteristic.

174. Interpole motors have a commutating field which increases with the current and is not affected by armature reaction (Par. 52). Motors which are subjected to excessive overloads, and adjustable-speed motors operating at high speeds with a weak main field, give little trouble due to sparking if supplied with interpoles. **Hunting**† takes place in interpole machines which have a **rising-speed characteristic.** Such a characteristic may be obtained by working with a weak main field and with a short air-gap under the main poles, so that the field distortion under load is large and the flux per pole decreases with increase of load due to the cross-magnetizing effect of the armature (Par. 32).

Such a characteristic may also be obtained by making the interpoles too strong. If commutation is perfect, the current in the short-circuited coil will be zero when that coil is in the geometrical neutral position. If, however,

* General Electric Company's Bulletin on "Automatic Voltage Regulators," 1913.

† Rosenberg, *Electrician*, Aug. 4, 1911.

the commutating field is too strong, the reversal of this current will be advanced, so that the true neutral will be shifted backward. The flux per pole will therefore decrease with increase of load, due to the demagnetizing effect produced, and the speed will increase with load.

Due to saturation of the interpole core, the interpole field will be too strong at light loads, if of the proper strength at full load. Hunting is therefore likely to occur at light loads, particularly if the main field is so weak that the effect of the brush currents in demagnetizing the machine is proportionately large.

175. Non-inductive shunts should not be used with interpole windings if the load fluctuates rapidly, because then the current, which is rapidly changing, will pass through the shunt rather than through the interpole coils, since the latter have considerable inductance, and the commutation will therefore be poor. To make the shunt take its proper share of the current under all conditions of load, it should be made inductive by winding it on an iron core, and the ratio of the shunt reactance to the interpole-coil reactance should be equal to the ratio of the shunt resistance to the interpole-coil resistance.

176. Reversal of direction of motion. In order to reverse a motor it is necessary to reverse the current in the field coils, or in the armature, but not in both. In an interpole machine, the interpole winding must be considered as part of the armature and not as part of the field system.

WEIGHTS AND COSTS

177. Weights and costs. The cost figures given by different manufacturers on a large generator may vary as much as 50 per cent. One manufacturer may build an entirely new machine and charge part of the cost or development to the order; another may offer a standard machine of larger capacity, with a lower rating for the particular case; while still another manufacturer may increase the rating of a machine of smaller capacity, adding fans if necessary to keep the machine cool.

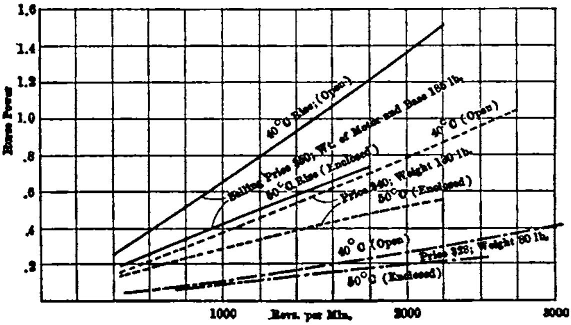

FIG. 65.—Cost of small direct-current motors without base or pulley.

178. Varying costs of labor and material. The cost of large units depends largely on the cost of material, while that of small motors depends largely on the cost of labor. These costs are continually changing, so that it is impossible to give figures which are always reliable. The curves in Figs. 65, 66 and 67 are average figures for standard machines.

The curves in Fig. 66 are interpreted as follows: A machine which weighs 2,000 lb. will cost $380 and will have a rating of 12 h.p. at 500 r.p.m. as an enclosed machine on continuous duty; or 19 h.p. at 500 r.p.m. as an open machine on continuous duty; or 31 h.p. at 500 r.p.m. with a 1-hr. rating; or 40 h.p. at 500 r.p.m. with a half-hour rating. The temperature rise on full load is 40 deg. cent. as an open machine, and 50 deg. cent. as an enclosed machine. The horse-power is proportional to the speed over a range of 30

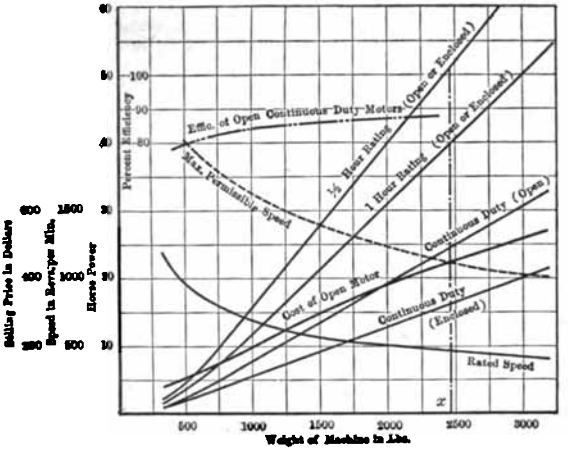

FIG. 66.—Weight and cost of standard 220-volt direct-current motors without base or pulley.

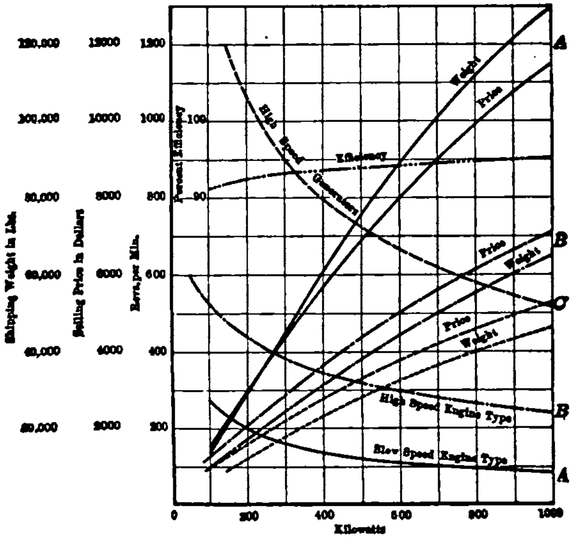

FIG. 67.—Weight and cost of standard 550-volt direct-current generators without base or bearings. *A*, Slow-speed engine type; *B*, high-speed engine type; *C*, high-speed generators.

per cent. above or below the rated speed. The maximum permissible speed is the highest that can be used for variable-speed operation with the particular line of machines on which data is given.

179. Effect of voltage on weight, cost and efficiency. If two machines are built on like frames for the same output and speed but for different voltages, the number of commutator segments will be proportional to the voltage and the commutator length will be proportional to the current. The low-voltage machine will be the heavier because of the long commutator, but, for moderate outputs, there will not be much difference in cost between a 120-volt and a 600-volt machine; the cost of the extra copper on the low-voltage machine is compensated for by the cost of the extra labor on the high-voltage commutator.

The losses will be affected in the following way: The windage, bearing friction, excitation and iron losses will be unchanged; the brush friction loss will be the smaller in the machine with the higher voltage; the contact resistance loss will be the smaller in the machine with the higher voltage because the contact drop is the same in each case and the loss is therefore proportional to the current. The armature copper loss will be unchanged if the same amount of armature copper is used in each machine, because, since the number of conductors is directly proportional to the voltage, the section of each conductor will be proportional to the current and the current density will be the same in each machine. If, however, due to the space taken by the insulation on the large number of conductors, the total amount of copper is the smaller in the machine with the higher voltage, then the copper loss will increase with the voltage and, for the same copper loss in each machine, the output of the high-voltage machine will be less than that of the low-voltage machine.

180. Effect of speed on weight, cost and efficiency. The output of a machine equals the product of (volts per conductor)×(current per conductor)×(number of conductors). For a given frame, the volts per conductor is directly proportional to the speed, and the product of (the current per conductor)×(number of conductors) is constant for a constant current density and a constant weight of armature copper. The output is therefore directly proportional to the speed. As a matter of fact the flux density must decrease as the speed and the frequency increase, in order to keep down the iron loss; but the current density may increase with speed due to better ventilation. However, the output is directly proportional to the speed, over a considerable range. The higher the speed, the larger the output, and therefore the longer the commutator and the heavier the machine.

At very low speeds the number of conductors becomes large and, on account of the amount of insulation required, the total amount of copper is less than normal; the rating of such machines must therefore be reduced in faster ratio than the speed is reduced.

The total armature loss for a given frame and a given temperature rise is equal to $k(A+B\times\text{r.p.m.})$ (see Fig. 52) so that while the output is directly proportional to the speed, the losses do not increase so rapidly; and, until turbo-speeds are reached, at which the windage losses become excessive, the efficiency increases with the speed.

STANDARD CONSTANT-POTENTIAL GENERATORS

181. Generators for power and lighting service are usually wound for 125 volts or 250 volts and are either flat-compounded or slightly over-compounded, so that the voltage at the lamps varies but little with change of load. The efficiency, weight, cost and speed may be obtained from Figs. 66 and 67. The regulation should be less than 2 per cent.

182. Generators for railway service are subject to rapidly fluctuating loads and to sudden and excessive overloads. The engine speed will drop considerably on an excessive overload unless a large flywheel is supplied, since the generator will not have the necessary flywheel effect in itself. The machines are generally wound for 550 volts at no load and 600 volts at full load, and have one terminal grounded. In order to secure proper commutation under all load conditions, interpole machines are often employed.

183. Generators for electrolytic work are usually low-voltage machines of large current capacity. When the terminal voltage is very low, the exciting current will be large if the machine is shunt wound, and the field rheostat between brushes BB and a current I in the external circuit. This current I sets up a flux ϕ_3, which opposes ϕ_1, but can never exceed ϕ_1, so that I cannot exceed that value at which the armature ampere-turns per pole equal the shunt-field ampere-turns per pole. The relationship between current and speed for different shunt excitations is given in Fig. 69. The direction of the current I is independent of the direction of rotation of the machine.

188. A system of car lighting by a shunt generator driven from the car axle is shown diagrammatically in Fig. 70. The diagram shows the conditions at standstill; the switch S is open, the generator is cut out and the battery supplies the lamps. As the car speeds up, the generator voltage increases and, when the value is reached for which solenoid F was set, the pull of the magnet closes the switch and connects the generator in parallel with the battery. The generator then delivers current to the battery and to the lamps, which current, flowing through coil H, helps to keep the switch closed. As the speed increases, the voltage of the generator and the battery current

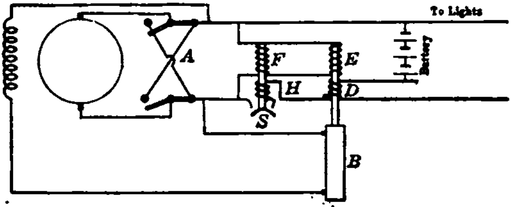

FIG. 70.—Car-lighting system.

both increase, but when full-load battery current is reached, the pull of coil D acts to lessen the pressure on the carbon pile B and the generator voltage decreases; coil D therefore limits the battery charging current. When the battery reaches full charge, its voltage rises to the maximum value and then the pull of coil E acts to lessen the pressure on the carbon pile, preventing further rise of voltage and permitting the generator to supply only the current for the lamps. When the speed drops below a certain value, the battery voltage is higher than that of the generator; current then flows back through the coil H, thus releasing the switch S and disconnecting the generator from the circuit.

Since the generator current should always be in the same direction no matter what the direction of motion of the car, a pole changer A is supplied; this consists of a double-throw switch operated by means of a mechanism on the shaft, which mechanism reverses the switch when the rotation of the generator reverses.

189. The windmill generating plant consists of a storage battery operated in parallel with a shunt generator driven by a windmill, an automatic cut-in being used to connect or disconnect the generator from the battery at the proper times. The cut-in is similar to the switch S in Fig. 70 and is equipped with a shunt coil connected across the generator terminals and a series coil in the line between the generator and the battery. When the windmill speed and the generator voltage are high enough to give a charging current, the shunt coil pulls up the plunger and closes the switch; the charging current then flowing in the series coil holds the switch closed. The series coil has a second plunger which is attracted downward by the charging current and, in its downward movement, opens the circuit of the shunt coil, allowing the first plunger to drop to normal position. Current cannot again flow in the shunt coil until the battery current has fallen to about 0.5 amp., when the battery switch opens, and the shunt circuit is again closed and ready to operate. See Sec. 22.

THREE-WIRE CONSTANT-POTENTIAL GENERATORS

190. The early three-wire systems were operated with two similar generator units connected in series and the neutral connected to the centre nected permanently across diametrically opposite points on the armature. The voltage between a and b is alternating and, even at no load, or when the load is perfectly balanced, an alternating current flows in this reactance coil. This current, however, is extremely small because the reactance is large. The center o of the coil is always midway in potential between the brushes c and d, and is connected to the neutral of the system. When the loads on the two sides of the system differ, the difference between the currents in the outside lines flows in the neutral wire and through the reactance coil, which offers only a small resistance to direct current.

194. The current distribution in the Dobrowolsky machine* may be considered as that due to the average current $(i_1+i_2)/2$ and a superimposed unbalanced current $(i_1-i_2)/2$, as shown in Fig. 75. The former of these currents flows in the outside lines, but does not pass through the neutral line, and so does not affect the potential of the point o. The unbalanced current is that which affects the voltages on the two sides of the system and this current is shown separately in Fig. 76.

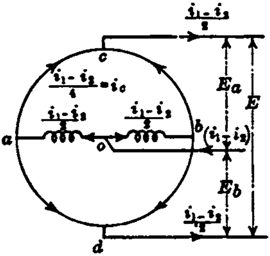

Fig. 76.

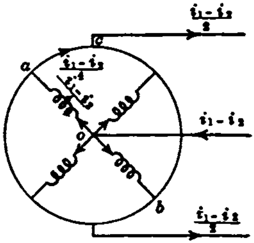

Fig. 77.

Figs. 76 and 77.—Unbalanced currents in three-wire generators.

If R_a is the resistance of the armature from brush to brush, then the resistance from a to c varies from O to $2R_a$ and, in the position shown in Fig. 76, it is equal to R_a. If R_b is the resistance of each of the two legs of the balance coil, the voltage drop from o to a is equal to $(i_1-i_2)R_b/2$; the drop from a to c equals i_cR_a and is alternating, with an average value of $(i_1-i_2)R_a/6$. The average drop from o to c is expressed by

$$\frac{(i_1-i_2)}{2}\left(\frac{R_a}{3}+R_b\right) \qquad \text{(volts)} \qquad (38)$$

195. The unbalanced current (i_1-i_2) is generally limited to 25 per cent. of the full-load current in the outside lines and, since the armature drop at full load equals the product of full-load current and R_a, and seldom exceeds 3 per cent. of E, it follows that the average voltage from a to c seldom exceeds 0.25 per cent. of E_a for 25 per cent. unbalanced current. Therefore the regulation of each side of the system is determined principally by the drop in the balance coil and can readily be kept within 2 per cent.

196. The use of two balance coils connected as shown in Fig. 77 results in a slight improvement in the machine. The average voltage drop from o to c in this case is

$$\frac{(i_1-i_2)}{4}\left(\frac{R_a}{3}+R_b\right) \text{ approx.} \qquad \text{(volts)} \qquad (39)$$

and is apparently less than it would be with only one coil. But when two coils are used, as in Fig. 77, each one carries half of the current carried by the single coil in Fig. 76; therefore the wire used has half the section and each of the coils in Fig. 77 has twice the resistance of the coil in Fig. 76, and the drop from o to a is the same in each case. The drop from a to c is less in Fig. 77 than in Fig. 76, but the difference is so small that it has little effect on

* Hawkins. *Journal* of Inst. of Elec. Eng., Vol. XLV, p. 704.

will have 240 segments and, with a minimum segment and mica of 0.2 in. (0.5 cm.) thickness will have a commutator diameter of about 15 in. (38 cm.). For further information see Sec. 16.

202. Mill motors for rolling-mill work are built like street railway motors set on feet, and are constructed in a similar manner in order that they may be readily repaired. The shafts are made stiffer than usual, which results in greater bearing friction and smaller efficiency than in standard motors. These motors weigh and cost about 20 per cent. more than standard motors of the same rating.

203. Mine motors are designed to operate in a damp atmosphere and must therefore be specially impregnated with waterproof compound.

204. Flame-proof motors,* for operation in explosive atmospheres, must have the current-carrying parts completely enclosed in flame-tight enclosures of non-inflammable material of sufficient strength so as not to be endangered by an explosion in the motor interior. The housings of such motors should never be opened in service except when the motor is completely disconnected from the supply circuit.

It is impossible to construct motors which are **gas-tight.** When the machine becomes heated, any gas which is in the case expands and some of it is forced out along the shaft. When the motor cools again, a fresh supply of gas will be drawn into the case. **Flame-tight** motors, wherein the transmission of an inside explosion to the outside is prevented, have proved feasible. There are two types: (a) Totally enclosed machines, constructed to withstand a pressure of 110 lb. per sq. in. (7.7 kg. per sq. cm.), are built in capacities up to about 25 h.p., but become large and expensive for greater outputs. (b) **Plate-protected motors** have openings which are filled with plates about 0.02 in. (0.5 mm.) apart so as to form a labyrinth passage for the gases; the function of this labyrinth is to cool the products of combustion.

205. Adjustable-speed motors are efficient only when operated by field control (Par. 164). For a wide range in speed, interpoles are required (Par. 165). The size and cost of the machine depend on the minimum speed at which it is necessary to supply the rated output, and the maximum speed is that at which the peripheral velocity reaches a safe limit. By the use of interpoles, deep slots may be employed, and the output for a given weight increased 10 per cent. over that for a non-interpole machine, but the cost per horse-power is not reduced. The following table gives a list of ratings that may be obtained from a machine of weight x (Fig. 66), suitable windings being supplied; a speed range of 3 to 1 is ample for most purposes:

46 h.p. at 800 to 1,200 rev. per min.
40 h.p. at 700 to 1,200 rev. per min.
34 h.p. at 600 to 1,200 rev. per min.
29 h.p. at 500 to 1,200 rev. per min.
20 h.p. at 350 to 1,050 rev. per min.
17 h.p. at 300 to 900 rev. per min.
14 h.p. at 250 to 750 rev. per min.
11 h.p. at 200 to 600 rev. per min.

STANDARD CONSTANT-CURRENT GENERATORS

206. Open-circuit windings. Fig. 79 shows an armature with two open coils and four commutator segments. The voltage between A and B varies as shown in Fig. 80; the current is pulsating and only half of the armature is in use at any given instant.

207. In the Brush arc machine the commutator segments are made to overlap as shown diagrammatically in Fig. 81 and the brushes are wide enough to cover two overlapping segments. During the first one-eighth rev., coil C alone is active and the e.m.f. in that coil passes through its maximum value. During the next one-eighth rev., coils C and D are in parallel; the e.m.f. in C is decreasing and that in D is increasing. During the next one-eighth rev., coil C is cut out and coil D alone is active. When two coils are in parallel the higher e.m.f. in one coil tends to reverse the current in the coil of lower

* Baum. "Fire Damp-proof Apparatus," *General Electric Review*, Vol. XIII, p. 402.

is obtained by shifting the brushes. When the brushes are in the position shown in Fig. 82, large e.m.fs. are being generated in the coils connected in series; when the brushes are shifted through 90 deg. these coils are generating low e.m.fs. and the terminal voltage is a minimum. The mechanism which shifts the brushes also varies the angle θ between brushes so as to secure good commutation with all brush positions.

211. Rating of Thomson-Houston arc machines.* A 2,500-volt, 10-amp., 829 rev. per min. Thomson-Houston machine weighs 5,975 lb. (2,700 kg.), occupies a floor space of 64 in. by 52 in. (1.6 × 1.3 m.) and requires 38 h.p. to drive it.

212. The Thomson-Houston regulator is shown diagrammatically in Fig. 83. The electromagnet *M* is short-circuited through the contact *S*. When the line current is too strong, contact *S* is opened by the magnet *N* and the main current passes through *M*, which raises a lever and increases the

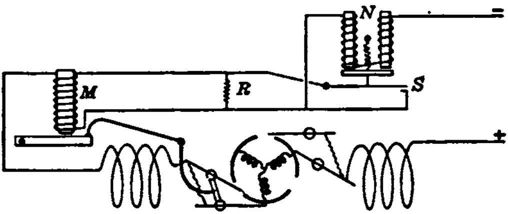

Fig. 83.—Thomson-Houston regulator.

brush arc. At the same time the brushes are shifted from the position of maximum e.m.f. The arcing at the commutator is suppressed by a mechanically-operated blower, and the arcing at the switch *S* is minimised by the discharge resistance *R*.

213. The Wood arc machine has a Gramme-ring winding with a large number of commutator segments. The current is kept constant by shifting the brushes, the voltage being a maximum when the brushes are in the neutral position and zero when the brushes are under the centres of the poles. The regulator is operated by an electromagnet which lifts a lever against the pull of a spring; when the line current is too large, the magnet pulls the lever in such a manner as to engage reduction gearing with the revolving armature and cause the brushes to move to a position of lower voltage; if the current is too small, the spring pulls the lever in the opposite direction and the gearing then moves the brushes to a position of higher e.m.f.

As in the Thomson-Houston machine, the brushes of the Wood arc machine are split into two parts separated by an angle θ (Fig. 82). As the brushes move under the poles, for the purpose of reducing the terminal voltage, the e.m.f. generated in the short-circuited coils increases, and must be counterbalanced by a higher reactance voltage; this is readily obtained in a Gramme winding by reducing the brush arc so as to decrease the time of commutation. For sparkless operation, the brush angle θ should decrease as the brushes move under the poles; this is accomplished automatically by the brush-rocking device.

214. Rating of Wood arc machines.* A 6,250-volt, 9.6-amp., 500 rev. per min. Wood arc machine weighs 14,600 lb. (6,600 kg.), occupies a floor space of 82 in. by 80 in. (2.1 m. by 2 m.) and requires 90 h.p. to drive it.

215. Constant-current generators for series arc lighting are nearly obsolete, although the Rosenberg machine (Par. 187), is coming into use for the operation of searchlight arcs. Direct-current series lighting systems are now in use, but in most cases are supplied from rectifiers (Sec. 6).

* Houston and Kennelly, "Recent Types of Dynamo-electric Machinery," 1898.

224. Standard balancer sets,* as used for three-wire operation, consist of two like units coupled together, each wound for half of the total voltage and connected in series as shown in Fig. 85. On balanced load, they both run "light" as motors; but with unbalanced load, the currents flow as shown in Fig. 85 and machine *M* acts as a motor and drives *G* as a generator. The current in *M* is greater than that in *G* by that amount required to supply the armature losses. The motor speed drops with increase of load, and the generator voltage drops due to the decrease in speed and in excitation, so that E_g is less than $\frac{1}{2}E$. By crossing the shunt coils, as in Fig. 86, the regulation is improved. The motor field decreases with decrease of E_g and the speed of the set increases with load; furthermore, the generator excitation, being taken from E_m, increases with load, so that with this connection the voltages are more nearly equal under all conditions.

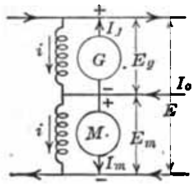

FIG. 85.

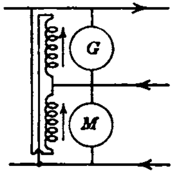

FIG. 86.

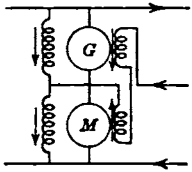

FIG. 87.

FIGS. 85, 86 AND 87.—Balancer sets.

225. Compound balancers are connected as in Fig. 87, so that the generator is always cumulatively compounded and the motor is compounded differentially, no matter in which direction the unbalanced current flows. This connection causes the balancer speed and the generator voltage to increase with the unbalanced load.

The generator current is expressed by the formula,

$$I_g = I_o \frac{\eta_m \times \eta_g}{1 + \eta_m \eta_g} \tag{40}$$

and the motor current by

$$I_m = I_o \frac{1}{1 + \eta_m \eta_g} \tag{41}$$

The efficiency in each case being given with the shunt-excitation loss neglected. The actual combined efficiency of the set is

$$\text{Eff.} = \frac{I_g}{I_m + 2i} \tag{42}$$

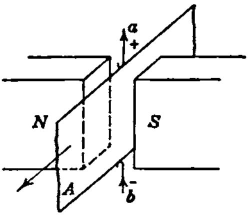

FIG. 88.—Faraday's demonstration.

HOMOPOLAR MACHINES

226. Theory of Operation.† Faraday showed that, if the metal strip *A*, Fig. 88 be moved so as to cut lines of force, an e.m.f. will be established between the brushes *a* and *b* and a continuous current sent round the external circuit connecting them.

227. The radial type, shown in Fig. 89, consists of a metallic disc, rotating between poles which are excited by field coils wound concentric with the shaft. The brushes are placed at *a* and *b*, and holes are cast in the yoke to allow access to the brushes. In Fig. 90, there are two discs mounted on the same shaft so that they are electrically connected

* Budd Frankenfield, *Electrical World*, Dec. 23, 1905; Lanier, *Electrical Journal*, Vol. IX, p. 1036.

† Gray. "Impossible Homopolar Machines," *Canadian Elec. News*, Mar. 15, 1913.

Now the flux density in section *A*, Fig. 91, is limited to about 130,000 lines per sq. in. (20,000 per sq. cm.) so that $D_a = 1.7\,L_c$ approximately, and therefore $D_a^2 = 76$ watts/rev. per min., with the above limitations.

231. Weight and cost.* Because of the low value of ampere-conductors per in., made necessary by the brush rigging, the value of $D_a^2L_c$ for a homopolar machine will generally be greater than for a machine of the standard type and the machine will be much heavier and more expensive. Therefore, until suitable brushes are found, this type of machine is not likely to be widely used except in the case of machines for low voltages and large currents.

232. The maximum output that can be obtained for a given number of revolutions per minute may be found by assuming a limiting peripheral velocity of 15,000 ft. per min. (77 m. per sec.). Then the output in watts is equal to $D_a^2 \times$ r.p.m./76 and

$$\text{kw} = 0.72\,\frac{(\text{perip. vel.})^2}{\text{r.p.m.}^2} \times \frac{1}{1000} \tag{45}$$
$$= 2.5 \times 10^9/(\text{r.p.m.})^2$$

The maximum output and the speed are plotted in curve 4, Fig. 32. The points shown on the curve give the ratings of two machines now in operation.

233. Compound-wound homopolar machines. The idea underlying the compounding is explained by the diagram in Fig. 92, which relates to a machine having six conductors. The brushes *E* are connected to the stationary conductors *H* by means of connection pieces *G*. Only a portion of the whole length of *G* is actually employed to carry current, and the amount of this active portion can be varied by the brush rocker *F*. In practice it is sufficient to have a flexible lead instead of the fixed conductor *G*.

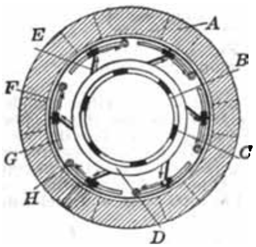

FIG. 92.—Compounding of homopolar machines. *B*, armature conductors; *C*, insulation between conductors; *D*, slip rings.

234. Homopolar machines may be made self-exciting and Noeggerath points out that homopolar machines may even be operated without the use of a field coil.

235. Standard lines of homopolar machines are not on the market; the machines are heavy and expensive if the ampere-conductors per in. are kept low so as to have a reasonably satisfactory brush rigging (Par. 230), and are troublesome in operation unless they are excessively large. The troubles encountered in the construction and operation of a homopolar machine are described by Lamme† in connection with a 2,000-kw., 260-volt, 7,700-amp., 1,200-rev. per min. machine which had 16 slip-rings operating at a peripheral velocity of 13,200 ft. per min. (67 m. per sec.) and copper leaf brushes set against the direction of rotation, the rings being lubricated with graphite.

OPERATION OF GENERATORS AND MOTORS

236. Shunt-wound generators in parallel. *A* and *B*, Fig. 93, are two similar generators feeding the same bus bars *C* and *D*. If machine *A* tends to take more than its proper share of the total load, the voltage of *A* falls, and the load is automatically thrown on *B*, the machine with the higher voltage. Furthermore, if the engine connected to *B* fails for an instant, that machine slows down, its generated e.m.f. falls, and current flows back from the line to operate it as a motor. Since the excitation remains unchanged, the machine will run at normal speed and in the same direction as before, and, as soon as the engine recovers, will again take its share of the load.

237. Division of load between shunt generators in parallel. The external characteristics of the two machines are shown in Fig. 95. At

* Pohl. "The Development of Tubo-generators," *Journal* of Inst. of Elec. Eng., Vol. XL, p. 239.

† Lamme. *Trans.* of A. I. E. E., Vol. XXXI, p. 975.

voltage E, the currents in the machines are I_a and I_b, and the line current is I_a+I_b. To make machine A take more of the load, its excitation must be increased, in order to raise its characteristic curve. If a 100-kw. machine and a 500-kw. machine have the same regulation, and therefore the same drop in voltage from no load to full load, then, as shown in Fig. 96, the machines will divide the load according to their respective capacities.

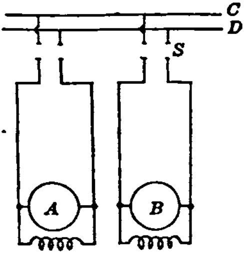

FIG. 93.—Shunt.

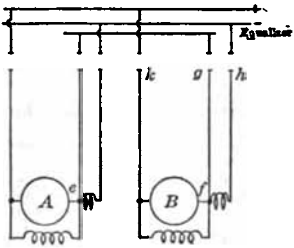

FIG. 94.—Compound.

Connections for parallel operation.

238. Compound-wound generators in parallel. A and B, Fig. 94, are two compound-wound machines. If machine A tends to take more than its proper share of the load the series excitation of A increases, its voltage rises and it takes still more of the load, so that the operation is unstable. Furthermore, if the engine on B fails for an instant, that machine slows down, its generated voltage falls and current flows back from the line to operate it as a motor. Under these conditions the shunt excitation remains unchanged, but the current in the series coils is reversed, so that, running as a motor in a weak field, the machine will tend to increase in speed and possibly run away.

239. Equalizer bus. To prevent instability when compound-wound machines are operated in parallel, a bus bar of large section and negligible resistance, called an equalizer bus, is connected from e to f as in Fig. 94. Points e and f are then practically at the same potential; therefore the current in each series coil is inversely proportional to the resistance of that coil, is independent of the armature current, and is always in the same direction. The machine ammeter, which indicates (at the switchboard) the current output, should always be connected in series with the lead k, Fig. 94. In **interpole machines**, the interpole winding is considered as part of the armature.

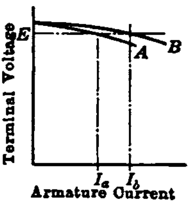

FIG. 95.

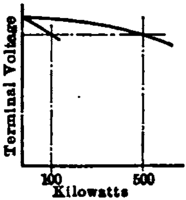

FIG. 96.

FIGS. 95 AND 96.—Division of load between two shunt generators in parallel.

240. Series shunt. When a single compound generator has too much compounding, a shunt in parallel with the series field coils will reduce the current in these coils and so reduce the compounding. When compound machines are operating in parallel, using an equalizer bus, the current in the series field coils depends only on the resistance of these coils; and a series shunt connected to one machine does not reduce the compounding of that machine alone, but also that of all the other machines with which it is operating in parallel. In order to reduce the compounding of a single machine, a resistance must be placed in *series*, and not in parallel with the series field coils. A combination of series and shunt resistances can also be employed. This adjustment is made after erection. If one machine is found to take more than its share of the load, then its compounding must be reduced by means of a resistance placed in series with its series field coils.

241. Starting and stopping a single shunt-wound generator. It is generally safest to start with the machine entirely disconnected from the external circuit and with the field rheostat all in circuit. Then bring the generator up to speed and cut out the field resistance until the voltage of the generator is normal. The main line switch may then be closed and further adjustment of voltage made if necessary.

To stop the machine, open the feeder switches, insert all the resistance in the field-coil circuit, then open the main switch and shut down the engine.

242. Starting and stopping shunt-wound generators in parallel. Assume that machine A, Fig. 93, is running; it is required to throw machine B in parallel with it. This latter machine must on no account be connected to the line while coming up to speed, otherwise it will form a short circuit across the machine which is already in operation. To place machine B in operation, bring it up to speed with the switch S open, adjust the shunt rheostat until $E_b = E_a$, then close switch S and increase the excitation of B until it takes its share of the load. To disconnect this machine, its excitation should be reduced until machine A carries the entire load; the switch S may then be opened. When shunt machines are being connected in parallel for the first time, the polarity of the switches should be carefully tested with a voltmeter or its equivalent.

243. Starting and stopping compound-wound generators. A single machine is started and stopped in the same way as a shunt machine (Par. **241**). To place machine B, Fig. 94, in parallel with machine A, which is already running, bring the machine up to speed and excite it to give normal voltage; first close switches g and h in order to excite the series coils, then make E_b equal to the bus-bar voltage and close switch k; the machine may then be made to take its share of the load by increasing the shunt excitation. To disconnect the machine, reduce its load to zero as in the shunt generator, and open the three switches in the reverse order.

Three separate switches are necessary for large machines, because of the large currents. For smaller machines, one double-pole switch for g and h, and a single-pole switch k, are sometimes used. For small machines, a three-pole switch is often used, but then the main switch is closed before the series field has its proper value and there is a momentary disturbance.

244. Starting and stopping motors. All except small motors, rated at a small fraction of a horse power, should be provided with a **starting box** or **starting rheostat.** Such starters, on constant-potential systems, consist in general of graded resistance connected in series with the armature, and eliminated step by step as the motor comes up to speed. The starting torque, as well as the running torque, is given by Eq. 36, Par. **159**; therefore in order to keep the starting current as small as possible, the magnetic flux should be as large as possible and the field coils should be fully energised before the armature receives any current. The counter e.m.f. at standstill is zero, and therefore the series starting resistance is necessary to limit the starting current to a safe value. Starters should be equipped with an **automatic release** which will operate under the conditions of no terminal voltage, or no current in the shunt field coils, so that the motor will not be burnt out when the terminal voltage is re-established or when the field circuit accidentally opens. For further information on starters see Sec. 5, and for information on motor control see Sec. 15.

Shunt motors may be stopped by merely opening the line switch; such motors should not be stopped by returning the handle of the starting box to the "off" position, because this method requires the opening of the field circuit while energised and gives rise to a momentary but very severe induced e.m.f. The handle of the starting-box should automatically return to the "off" position before the motor finally comes to a dead standstill.

245. Field discharge. If the field circuit be suddenly opened while carrying current, then, due to the rapid decrease of flux through the large number of field-coil turns, a large e.m.f. will be induced between the two open ends. This may be sufficient to break down the insulation between the field coils and the poles. To prevent this, when a motor is disconnected from a line, the field coils should be short-circuited through the armature winding at the instant the line circuit is opened, the necessary connections being made by the starter.

246. Installation of motors and generators. The machine should, if possible, be placed in a cool ventilated position which is free from dirt, dust, or moisture. Machines required to operate in damp places should have the coils specially treated. The foundation should be solid to prevent vibration. If the machine is belted, the belt tension should not be too great, the distance between belt centres too short, or the pulley too small; the pulley should not have a diameter less than that recommended by the manufacturer. The belt should be flexible and without lumpy joints and the bottom side should be the tight side. After erection, clean the bearings by pouring in gasoline, drain this off through the drain cocks and then fill the bearings with light mineral oil. Before starting, see that there are no loose parts and that the brushes are making firm contact.

247. Starting generators for the first time. Bring the generator slowly up to speed with the field circuit open and see that the oil rings operate properly; then close the shunt field switch and bring the voltage up to normal. Run the machine without load for an hour, after which the load may gradually be increased. Generators should be thoroughly dried out in every instance before they are placed in commission.

248. Starting motors for the first time. Check the connections to make sure that they are correct and secure. See that the controller handle is in the starting position, then close the line switch. If the motor does not start on the first or second notch of the controller, open the line switch and look for the trouble. When starting, bring the controller handle over slowly to the running position, allowing the motor to gather speed, but do not run on the starting notches for any length of time. Run the motor without the belt for half an hour (this cannot be done with a series motor) and then put on the load. When shutting down, open the line switch and see that the controller handle returns to the starting position before the motor finally stops.

249. Operating instructions. Keep oil away from the commutator, brushes and windings. Do not allow dirt or dust to accumulate in or around the machine. Do not lubricate the commutator with oil; a piece of muslin moistened with vaseline may be used to clean the commutator. Emery is a conductor and should not be used in fitting brushes or cleaning the commutator; use sandpaper and do not use it on the commutator too frequently. Do not use greater brush tension than necessary; tension greater than 2 lb. per sq. in. (0.14 kg. per sq. cm.) is seldom required. When replacing brushes, use the quality and size originally supplied with the machine, and fit them to the commutator with sandpaper before use; a strip of sandpaper should be placed on the commutator below the brushes, sand side up, and pulled through in the direction of motion of the commutator. Do not open generator-field circuits quickly (Par. 118); open the switch slowly, permitting the arc to extinguish gradually, which should take about 5 sec. On large generators use a field discharge resistance, which is connected across the terminals as soon as the field switch opens; see Fig. 97.

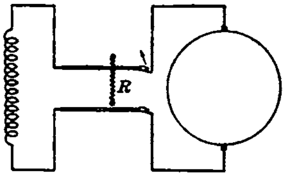

FIG. 97.—Field discharge resistance.

Do not stop quickly on account of a hot bearing, but slow down the machine and apply good clean oil. A quick shut-down will cause the bearing to "freeze." Inspect and clean the machine periodically.

250. Poor commutation is generally due to one or more of the following causes:

(a) Brushes not in the proper position.

(b) Bad spacing of the brush sets. This may be checked by counting the number of commutator segments between adjacent brush sets.

(c) Projecting mica. This cause may be removed by cutting out the mica between segments for $\frac{1}{16}$ in. (0.16 cm.) below the commutator surface.

(d) Rough commutator, generally caused by sudden rushes of current. When such current rushes are inherent under the operating conditions, and interpoles are not employed, it may be advisable to use abrasive brushes to keep the commutator free from rough spots.

(e) Flat bars, produced because some of the commutator segments are softer than others.

(f) High bars, generally due to soft spots in the mica clamping cones.

(g) Grooved commutator, may be prevented by staggering the brush sets so that the space between two brushes of one set is covered by a brush of the next set and the commutator wears down evenly.

(h) Poor brush contact, due to imperfect bedding of the brushes, to dirt, or to burning of the contact.

(i) Worn brushes being replaced by others of the wrong quality or size.

(j) Sticky brushes, which do not move freely in box holders and so do not follow irregularities of the commutator.

(k) Vibration due to want of balance in the armature, to poor foundations, or to a badly laced belt.

(l) Chattering of the brushes, which can generally be cured temporarily by cleaning the commutator with a piece of muslin moistened with vaseline, and may be cured permanently by changing the brush angle.

251. Armature winding troubles are generally due to the following **causes:**

(a) A broken coil, generally at the commutator end, caused by the wire connected to the commutator being too tight, excessive vibration, poor soldering at the commutator neck, or the use of copper which was not properly annealed. This trouble causes a bad arc at the instant the brush breaks contact with the segment connected to the broken coil. The trouble can be fixed temporarily by short-circuiting the commutator segments connected to the damaged coil, thereby cutting it out, care being taken not to short-circuit the coil itself.

(b) A short-circuited armature coil may be due to damaged insulation between turns or to an accumulation of copper dust in the commutator necks. A large local current will flow in this coil and cause it to become hot and to smoke.

(c) An armature coil may be reversed during repairs. The e.m.f. of the reversed coil opposes that of the other coils so that one circuit in the armature will generate less than normal voltage (Par. 19). At the moment of commutation, the conductors of this coil will be under the wrong pole tips and local sparking will take place.

252. Field-coil troubles are generally due to the following **causes:**

(a) An open circuit: If the motor is series wound, the machine will stop and cannot be started again; if a shunt motor, the machine will speed up as the flux decreases (Par. 162) and the line current will increase as the back e.m.f. diminishes, until the circuit-breakers open. If the machine is a shunt or compound generator, the generated e.m.f. will greatly decrease and a large current will flow into the machine from other generators which are operating in parallel with it, and open the circuit-breakers.

(b) A short circuit will reduce the field-coil resistance and in the case of a shunt machine, will cause the exciting current to increase and the temperature to rise. The m.m.f. of the damaged coil will be reduced and that of the other coils increased, so that the strengths of the poles will be different (Par. 19). The average flux per pole will be unchanged in a shunt machine, but will be reduced in a series machine.

(c) Reversed field coils will cause adjacent poles to have the same polarity. The polarity can be tested with a compass or, if that is not available, with a piece of soft iron which will lie along the lines of force and therefore bridge the poles if the polarity is correct, but will tend to lie axially in the direction of the shaft if the polarity is wrong.

253. The effect of wear in the bearings is to throw the armature out of centre with respect to the pole-pieces and cause an unbalanced pull (Par. 19).

254. End thrust causes heating of the bearings and is generally due to projection of the armature core beyond the magnetic field (Par. 95). End thrust may also be due to the fact that the shaft is not horizontal.

mately the same value and at least the same current-carrying capacity, and comparing the drops of potential across each, in which case the current need not be measured.

262. The field-coil resistance is determined by passing a known current through the coils and measuring the drop of potential across the field terminals.

263. Load tests on machines of considerable size must be made by some method whereby the power developed by the machine is not dissipated, but is made available for the test; otherwise the power-house capacity may not be large enough to test many machines and the cost of the test will be excessive.

264. Blondel's loading-back method. Two identical machines are required for this test. They are separately excited, connected together mechanically to run at the same speed, and their armatures are connected electrically in opposition as in Fig. 101 so that, when equally excited, there is no current in the armature circuit. An auxiliary motor M is belted to the set

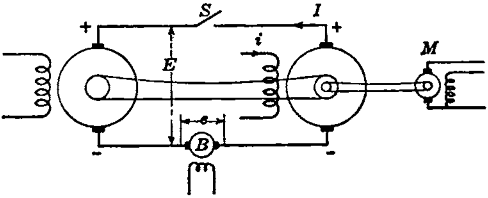

Fig. 101.—Blondel method.

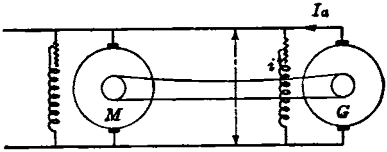

Fig. 102.—Kapp method.
Loading back tests.

and should have a capacity large enough to supply the no-load losses of both machines. The booster B must carry full-load current and is used to circulate the current through the armatures of both machines. The set is brought up to speed by the auxiliary motor, the switch S being open. The fields are then excited until the voltage E is normal and the voltage across the switch S is zero. This switch may then be closed and, by suitably exciting the booster, any desired armature current may be made to circulate. The booster supplies the copper-loss and the motor M supplies the other armature losses of both machines so that EI is the generator output; one-half the output of M is equal to the sum of the windage, friction and iron-losses of one machine; one-half of eI is the copper-loss of one machine; and Ei equals the excitation loss of one machine, from all of which the efficiency may be found.

265. The regulation of a generator, or of a motor, may be determined from Blondel's test. The machines in this case need not be identical, but they must be of the same voltage, and the output of the testing machine must not be less than that of the machine to be tested. In the test for generator regulation, start with normal voltage E and no circulating current; then excite the booster to give the desired current I_a and measure E_t, the terminal voltage. Plot E_t and I_a as in Fig. 58. In the test for motor-speed regulation, keep the voltage across the motor terminals constant by means of the booster, and vary the speed of the auxiliary motor M until the circulating current has the value desired. Plot the speed and I_a as in Fig. 64.

266. Hopkinson's test. This test is similar to Blondel's, but the booster is eliminated and the auxiliary motor is used to supply all the losses, the current in the armatures being circulated by weakening the field of one machine and strengthening that of the other. The iron-losses in the two machines are different because the fields have different excitations, so that, while the method is satisfactory for regulation and heating, it is not satisfactory for efficiency.

267. Kapp's method is preferred to Hopkinson's and is the one generally adopted in commercial work for regulation and heating tests; the efficiency of a machine is generally calculated from the losses. The diagram of connections is shown in Fig. 102. The losses are supplied from the testing circuit and are easily measured, but, since the copper-losses in the two machines are different, as also are the iron-losses, the method should not be used for an accurate determination of efficiency.

268. Load tests on series motors. When a large number of **railway motors** have to be tested the loading-back method of test is used. Two motors are connected together as shown in Fig. 101 and are geared to the same countershaft so as to run at the same speed. This countershaft, driven by an auxiliary motor supplies the mechanical losses. The field coils are connected in the armature circuit and a booster is also placed in this circuit to supply the copper losses.*

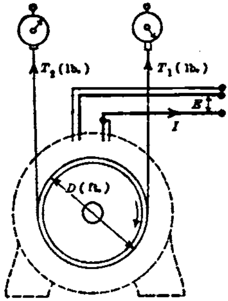

Fig. 103.—Prony-brake test.

269. Prony brake test. The efficiency is seldom determined by direct measurement of input and output except in the case of small motors which are connected directly to the line and the load then measured by means of a water-cooled **prony brake** as in Fig. 103.

$$\text{Eff.} = \frac{746\pi\, D\, (\text{r.p.m.})(T_1 - T_2)}{33{,}000\, EI} \tag{46}$$

270. In constant-current arc generators, the load losses are generally large, and the efficiency of such machines is obtained by measuring the output electrically, and the input by means of a driving motor whose efficiency is known.

271. Load saturation test. The generator is driven by a motor of the same voltage and of larger output, and the machines are connected to the testing circuit as in Fig. 102. Starting at about 20 per cent. above normal voltage and with full-load circulating current, the excitations of the two machines are gradually reduced and a series of readings of E_t and i are taken, I_a being constant. The results are plotted as in curve 2, Fig. 19.

272. Heat run. The machine is connected as in Fig. 102 and operated at rated speed and voltage, with the desired circulating current. The suggestions contained in the A. I. E. E. standardisation rules, Sec. 24, regarding measurement of temperature and conditions of test, should be followed.

273. Insulation resistance may be measured with a megger (Sec. 3) or by means of a high-resistance voltmeter connected as in Fig. 104. In the latter case the insulation resistance equals the resistance of the voltmeter multiplied by the ratio $(E-e)/e$, where E, the voltage of the testing circuit, should be the same as the normal voltage of the machine being

*For modifications of this test see:
Workman, R. E. "Factory Testing," *Electric Journal*, Vol. I, p. 551.
Fay, C. J. "Testing Large Motors," *Electric Journal*, Vol. III, p. 525.

tested, and e is the voltmeter reading or the drop in voltage across the voltmeter resistance.

274. Puncture test. To test the insulation, a high voltage is applied between the armature winding and the frame, between the field windings and the frame, and also between the shunt and the series field coils with the shunt winding disconnected from the armature. The test voltage is obtained from a small testing transformer of adjustable ratio and should be raised to the desired value smoothly and without sudden large increments; the maximum should be applied for 1 min. unless otherwise specified, and then gradually decreased. The recommendations given in Sec. 24, should be followed.

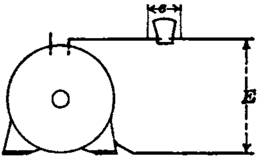

Fig. 104.—Measurement of insulation resistance.

275. Acceptance tests. A compound generator to be tested for efficiency, regulation, temperature, commutation and insulation would be handled as follows:

(a) Grind the brushes in place with sandpaper.

(b) Measure the armature and the field coil resistances.

(c) Place full load on the machine (Par. **272**). Adjust the brush position so that the machine is sparkless over the desired range of operation. The brushes should not be set further forward than necessary, otherwise the armature reaction will be greater than it need be. Adjust the current in the series coils with a temporary shunt to give the required voltage at no load and at full load.

(d) Keep the machine on full load until the temperatures, as indicated by the field coil resistance, have become approximately stationary; then shut down and take temperatures.

(e) Measure the armature and the field coil resistances before the machine has time to cool.

(f) Give the machine an overload run, if that is desired.

(g) Make the regulation test (Par. **265** and **267**); then since the machine is properly connected, a full-load saturation test may be made for the information of the designer, if required.

(h) The no-load loss may be determined by running the machine idle as a motor (Par. **260**), but the designer will probably want information in regard to the separate losses (Par. **258**). The test results are then worked up, and, if the machine is satisfactory, the insulation resistance is measured and the puncture test made.

276. Commercial or shop tests. For standard machines on which the designer has all the information he requires, such a complete test (Par. **275**) is not necessary. The machine should be run idle at normal speed and voltage to make sure that the no-load losses are not too high due to poor material or poor construction, and that the excitation is not too large or too small. The armature and field coil resistances are checked with the calculated values. The machine is run for an hour at 25 per cent. overload to test the mechanical construction and then run idle at 50 per cent. above normal voltage to test the insulation between turns. The speed may be increased during this latter test. The insulation resistance is then measured and the puncture test made. For further information see the series of articles on shop testing by R. E. Workman in Vol. I of *Electric Club Journal.*

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SECTION 9

CONVERTERS AND DOUBLE-CURRENT GENERATORS

SYNCHRONOUS CONVERTERS

BY F. D. NEWBURY, M.E.

GENERAL THEORY

1. Comparison with separate synchronous motor and direct-current generator. The theory of the synchronous converter is best explained by the similarity of the converter to a synchronous motor driving a direct-current generator. The converter combines the characteristics of both motor and generator in the single armature winding. This winding is provided on one end with a commutator as in a direct-current generator, and on the other with collector-rings and taps to the winding as in a revolving-armature synchronous motor. Alternating current is supplied through the collector-rings, and with no direct-current load the synchronous converter operates purely as a synchronous motor. Its speed is determined by the frequency and number of poles and the flux is fixed by the impressed voltage, just as in a synchronous motor.

2. Excitation. In the synchronous converter, as in the synchronous motor, the magnetic flux and the corresponding net exciting ampere-turns are determined solely by the impressed voltage. If it is attempted to vary the exciting ampere-turns and flux by variation of the main field excitation, the latter variation is neutralized by an equivalent change in excitation brought about by a change in phase and value of the armature current, so that the flux and net excitation remain constant. Increased excitation in the main field winding produces a leading current in the armature (leading with respect to the line voltage) which, in the majority of transmission lines serving synchronous converters, is beneficial to line power-factor and voltage. Conversely, under-excitation produces a lagging current which is detrimental to the power-factor and voltage of such lines.

3. Ratio of alternating voltage to direct-current voltage. Assuming the synchronous converter to be operating without direct-current load, it will be clear that the direct-current voltage between brush arms will, through the action of the commutator, have a value equal to the maximum instantaneous value of the alternating voltage, giving proper consideration to the relative points in the armature winding to which, at any instant, the brushes and collector-rings are connected. In the single-phase, two-phase and six-phase diametrically connected converters, the direct-current brushes and collector-rings are connected to equivalent points on the armature winding, so that the ratio between the alternating and direct-current voltages is simply the ratio between the effective and the maximum alternating voltages. In the three-phase converter, the collector-rings are connected to points 120 electrical degrees apart, while the direct-current brushes are connected to points 180 electrical degrees apart, so that the voltage ratio is affected by this difference. The theoretical ratios are shown in Par. 4. See Par. 35 and Figs. 9 to 14, inclusive, for further information.

These theoretical ratios are based on the assumptions that the impressed alternating-current wave form and the counter e.m.f. wave form of the converter are both sine-waves, that there is no loss in the converter, and that the direct-current brushes are at the no-load neutral position. Variations in wave form are small in commercial circuits and apparatus. The effect of the resistance of the windings, brushes, and brush contact is appreciable, and may vary the ratios (Par. 5) from 2 to 4 per cent. Changes in brush position, even within the small limits permitted by commutating conditions, may affect the ratio 1 per cent. A lagging current in the converter winding has the same effect as narrowing the pole-face and will thus increase the ratio of alternating voltage to direct-current voltage. In general, therefore, actual ratios are slightly higher than the theoretical ratios (Par. 4), assuming conversion from alternating to direct-current.

4. Table of Theoretical Voltage Ratios

Number of converter phases	Ratio of alternating voltage
Single-phase..................	0.71 of direct voltage
Two-phase....................	0.71 of direct voltage
Three-phase..................	0.61 of direct voltage
Six-phase double delta.........	0.61 of direct voltage
Six-phase diametrical..........	0.71 of direct voltage

5. The theoretical current ratio, neglecting the losses and assuming 100 per cent. power-factor is the inverse ratio of the alternating and direct-current voltages (Par. 4). This follows from the equality of the alternating-current input and direct-current output. From this standpoint, but including the effect of the losses, the current ratios are given by the following formula:

$$I_a = \left(\frac{1}{Eff}\right) Y I_d \qquad \text{(amps.)} \qquad (1)$$

in which expression the symbol I_a represents the value of the alternating current and I_d the value of the direct current. Eff represents the efficiency. Y is equal to twice the square root of two divided by the number of collector-rings. Values of Y are given as follows:

No. of phases	Value of Y
Single-phase.........................	1.41
Two-phase...........................	0.707
Three-phase.........................	0.940
Six-phase...........................	0.470
(Both double-delta and diametrical transformer connections.)	....

For all practical purposes an average efficiency of 94 per cent. may be assumed, in which case the current ratios will be as shown in Par. 6. These simple ratios are very easily remembered and are convenient for approximate calculations.

6. Table of Approximate Current Ratios

No. of phases	Alternating current per terminal
Single-phase...	1.50 times direct current
Two-phase.....	0.75 times direct current
Three-phase....	1.00 times direct current
Six-phase......	0.50 times direct current

7. Actual current ratio. Since the current ratios given in Par. 6 are based on theoretical voltage ratios (Par. 4) the actual ratios will vary from 2 to 4 per cent. from those of Par. 6. In specific cases where exactness is desired, the current can be determined most conveniently from the direct-current output, actual alternating voltage, actual efficiency and power-factor using the following formula:

$$\text{alternating current (per terminal)} = \frac{K.W. \times 1000}{E_a \times Eff \times Y \times P.F.} \qquad \text{(amp.)} \qquad (2)$$

Where E_a is the alternating voltage. Values of Y are given as follows:

Number of phases	Value of Y
Single-phase..........	1.00
Two-phase	2.00
Three-phase..........	1.73
Six-phase	3.00

If the efficiency and power-factor are not known, see Par. 28, 38, and 39 for probable values.

8. Armature currents and heating. The current in the converter armature may be considered as made up of: (a) an alternating current, uniformly dividing among the various conductors of the winding, which is necessary to drive the converter as a synchronous motor, and which is determined by the magnitude of the losses; (b) a current flowing from the collector-ring taps to the direct-current brushes, varying widely in value in the different conductors at different positions of the armature with respect to the field poles. The first component is small and in theoretical discussions is usually neglected. The second component for convenience in calculation, may be considered to be a resultant of the instantaneous values of alternating current and direct current in any individual conductor. Fig. 1 illustrates the

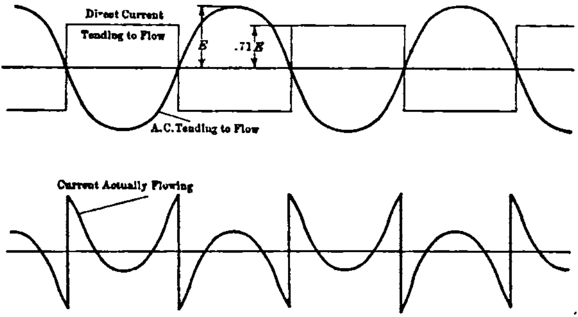

FIG. 1.—Armature current in conductor, 100 per cent. power-factor.

simplest case, namely, that of a conductor midway between alternating-current taps, which are equidistant from pole centres, and with 100 per cent. power-factor alternating-current input. This diagram clearly shows the relation between the effective current actually flowing and the alternating current and direct currents that would flow in separate motor and generator windings, and indicates the fundamental reason for the economy of the synchronous converter due to its single winding.

9. The ratio of the effective or resultant armature current to the external direct current varies with the number of phases, or more properly, with the number of connections per pole to the armature winding. The larger the number of such connections, the smaller will be the effective current.

10. The distribution of the current among the different conductors at 100 per cent. power-factor is such that the maximum loss occurs in the tap coils and the minimum loss in the coils midway between taps. This is shown in Fig. 3.

11. The effect of diminishing power-factor on the effective current is to increase the latter greatly, even with small reductions in power-factor. The effect of decreased power-factor on the distribution of current is greatly to increase the current and loss in conductors near one side of the tap coils and to reduce the loss in conductors on the other side of the same tap coils. The average effective current and the loss are considerably increased. The effects of power-factor are illustrated by Fig. 2 which shows the same conditions as Fig. 1, except that the power-factor is 86.5 per cent. instead of 100 per cent. It will be seen that the alternating-current wave has been shifted with respect to that of the direct current, thus causing an increase in the effective current. Fig. 3 shows the difference in distribution of losses and difference in average losses with 100 per cent. and 95 per cent. power-factors.

12. Table showing Relative Converter Losses on Basis of Direct-current Generator Losses taken as Unity

Calculations based on 3 per cent. rotational losses
(Calculated by C. E. Wilson)

	Relative armature loss in complete winding					Relative maximum loss in one conductor				
No. of collector rings	P.F. 100 per cent.	P.F. 98.5 per cent.	P.F. 94 per cent.	P.F. 86.6 per cent.	P.F. 76.6 per cent.	P.F. 100 per cent.	P.F. 98.5 per cent.	P.F. 94 per cent.	P.F. 86.6 per cent.	P.F. 76.6 per cent.
2	1.451	1.522	1.734	2.160	2.940	3.121	3.653	4.358	5.342	6.808
3	0.587	0.627	0.753	1.005	1.468	1.249	1.594	2.048	2.673	3.596
4	0.391	0.426	0.532	0.746	1.137	0.751	1.017	1.367	1.861	2.596
6	0.274	0.304	0.400	0.589	0.935	0.430	0.614	0.874	1.249	1.825
12	0.209	0.236	0.326	0.500	0.824	0.249	0.354	0.525	0.792	1.232

13. Losses affecting rating. Par. 12 shows the relative average losses and maximum loss per conductor compared with those of the equivalent direct-current generator for various phases and power-factors. It shows a large increase, particularly in the maximum loss due to small changes in power-factor. It is this characteristic that limits the use of synchronous converters to operating power-factors near 100 per cent. This table also shows the advantage of a large number of rings. Practically all converters above 200 or 300 kw. are now built with six collector-rings and phases, while 12 rings have been considered for the largest ratings. A distinction must be made, however, between copper loss and rating. This table does not represent relative capacities of the various armatures, since the rating depends on many other factors besides armature coil heating. In general, the **current capacity** of a given armature will be increased by increasing the number of rings and will be reduced by a reduction in power-factor, even slightly below 100 per cent. power-factor, although not to so great an extent as is indicated by a direct comparison of the losses.

14. The armature reaction of the synchronous converter is relatively small compared with that of the equivalent direct-current generator on account of the relatively small effective armature current. In this characteristic the converter is very nearly equal to the compensated direct-current generator. In a six-phase converter, the effective armature reaction varies from 7 per cent. to 20 per cent. of the armature reaction in an equivalent uncompensated direct-current generator.*

15. Commutation in the synchronous converter offers the same problem as in the direct-current generator, differing only in degree. Due to the smaller effective current the armature reaction and, to a lesser extent, the self-induction, are smaller than in the direct-current generator of equal rating, so that allowable commutation limits are much higher. The synchronous converter, without commutating poles, holds a position between the simple direct-current generator and the direct-current generator with commutating poles and with a compensating pole-face winding. For this reason the commutating poles with their attendant complications were not added to the synchronous converter until long after they had been success-

* Lamme, B. G. and Newbury, F. D. "Interpoles in Synchronous Converters;" *Trans.* A. I. E. E., Vol. XXIX, page 1625.

same number of poles as the converter and having its armature winding connected in series with the converter armature winding. This booster alternator, by a change in its own excitation, varies the voltage applied to the terminals of the converter. The excitation is so arranged that the booster fields and voltage may be reversed without opening the field circuit. The possible variation in voltage is double the rated booster voltage, since it may be added to or subtracted from the line voltage. The connections are shown in Fig. 5. The voltage may be controlled automatically through relays and regulator, or the booster may be series wound, thus automatically compounding the converter. This method of voltage variation has largely superseded the methods previously described, particularly in the larger installations.

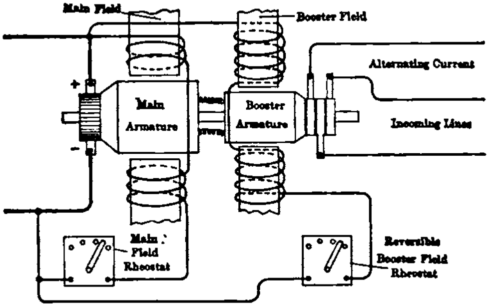

FIG. 5.—Synchronous booster converter.

21. Types of synchronous booster. The booster may be a revolving-armature generator located between the collector-rings and main armature, or it may be a revolving-field generator located outside of the collector-rings. The revolving-armature construction is the more usual since it eliminates additional collector-rings and external wiring, and the large number of parallel circuits in the converter winding favorable to low-voltage generation may be conveniently used in the booster winding. The rotating-field booster is sometimes preferable since it may be added to a standard converter with less change in existing parts. There is also less interference with the ventilation of the main armature.*

22. The application of commutating poles to the booster converter involves new features due to the motor action, or the generator action, of the converter when the booster is excited. The converter acts as a motor to the extent of the booster output plus all losses, when the booster is excited to increase the direct-current voltage, and, conversely, the converter acts as a generator to the same extent when the booster is excited to lower the voltage. Due to this motor or generator action the effective current in the converter armature varies with the booster excitation as well as with the output of direct current.† If the effective armature current varies, the commutating-pole flux must vary correspondingly if the commutating poles are to be of benefit. This variation is accomplished approximately in one method by providing two windings on the commutating poles, one in series with the armature as in other commutating-pole machines, and the other in series with the booster field winding. The windings are so connected as to assist one another when the direct-current voltage is decreased and to oppose one another when the direct-current voltage is increased. This arrangement of windings provides approximately equal commutating-pole flux and armature flux under all operating conditions except that of no external load coincident

* Newbury, F. D. "Voltage Regulation of Rotary Converters;" *Electric Journal*, Vol. V, page 616.

† Yardley, J. L. McK. "Efficiency of Synchronous Booster Converters;" *Elec. Journal*, Feb., 1913.

25. Conditions affecting direct-current voltage regulation of the compound converter.* It is apparent that the direct-current voltage regulation of the compound-wound converter depends on many elements internal and external to the converter, and that the change in voltage is limited by the inability of the converter to operate at low power-factor and heavy load. The range of voltage is more restricted and the results are much more difficult to predetermine than in the compound-wound direct-current generator. The voltage range is affected by certain factors which are as follows: (a) the resistance drop between the point of constant voltage and the converter; (b) the reactance drop between the same points; (c) the ratio of armature ampere-turns to shunt-field ampere-turns; (d) the ratio of series-field ampere-turns to shunt-field ampere-turns; (e) the setting of the shunt-field rheostat; (f) the transformer secondary voltage; (g) the total drop through the converter.

26. Customary values for these factors (Par. 25) are as follows: (a) the full-load armature ampere-turns are approximately equal to the no-load shunt-field ampere-turns; (b) the full-load series-field ampere-turns are approximately one-half the armature of shunt-field ampere-turns; (c) the shunt-field rheostat is so set that the lagging component of the line current will be approximately 25 per cent. of the full-load energy component; the shunt-field current will then be about 75 per cent. of its normal no-load 100 per cent. power-factor value under the previous assumptions: (d) the transformer secondary voltage usually is chosen somewhat higher than the correct no-load value to assist in holding up the voltage at overloads.

Under the above conditions, an approximately flat direct-current voltage regulation curve will be obtained with the following values of resistance and reactance measured from the point in the circuit having constant voltage, to the converter collector-rings:

Resistance drop, per cent.	Reactance drop, per cent.
0	4
2	8
1	12
6	16

27. Relation between power-factor and load in compound converters. With the average conditions stated in Par. 26, the power-factor will vary with the load approximately as is shown, Par. 28. With larger overloads—carried for a sufficient length of time to make heating a consideration—the shunt field winding should be further under-excited in order to bring the power-factor nearer unity at the extreme overloads, or the series field should be shunted to reduce the change in power-factor with load.

28. Table of Power-factor Variation with Load in Compound Converters

Load	Power-factor
	Per cent.
[illegible]	70 lag
[illegible]	93 lag
[illegible]	100 lag
[illegible]	99 lead
Full	98 lead
1¼	97 lead

29. The reactance necessary for compounding may be in the step-down transformers or in separate reactance coils. Where possible, the transformers are designed with the necessary reactance, as this method is

* Bache-Wiig, J. "Voltage Regulation of Compound-wound Rotary Converters;" *Elec. Journal*, Vol. VIII, page 860.

lower in cost and requires less floor space and wiring; it is, however, difficult so to design the transformers when the required reactance is more than 10 per cent. It is also more difficult to obtain high reactance in 25-cycle than in 60-cycle transformers, and in the core type than in the shell type (Sec. 6).

30. The split-pole method of voltage control utilizes a different principle from any previously discussed. Variation of direct-current voltage is secured, not by a variation in the impressed alternating voltage, but by a change in the shape of the magnetic field, such that the total flux and the direct-current voltage are changed, while the alternating voltage remains substantially unchanged. This is possible because of the fact that in polyphase windings in which the phases are tapped 120 electrical degrees apart, third harmonics and multiples thereof cancel out and do not appear at the 120-deg. terminals. The action of the split-pole converter therefore depends on variations in the field flux distribution which produce third harmonics, in conjunction with taps on the alternating-current side 120 deg. apart. The reason for this will be understood by reference to Fig. 7. It is evident that the armature coils between the 120-deg. taps on the alternating-current side will always include two equal and opposite areas of the third-harmonic variation of flux which neutralize each other and so produce no change in alternating-current wave form. On the other hand, the armature coils between the brushes on the direct-current side, which are 180 electrical degrees apart, will always include three areas of the third harmonic variation in flux, only two of which neutralize each other, leaving one to change the average ordinate of the field flux curve, and, consequently, the direct-current voltage.

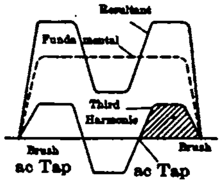

FIG. 7.—Flux distribution in three-part pole, split-pole converter.

With the split-pole converter, the 120-deg. connection on the alternating-current side is preferable; consequently the three-phase delta connection or the six-phase double-delta connection should be used. However, any secondary connection of transformers may be used providing the primary is Y connected and the neutral is not fixed.

Harmonics other than the third and multiples thereof will appear at the alternating-current terminals and will cause a corrective current to flow from the supply circuit through the converter windings so as to neutralize the flux producing these harmonics in the converter. It is desirable, therefore, so to design the converter that the wave form will be as nearly as possible a combination of the fundamental, third and ninth harmonics. In practice this is so nearly accomplished that the effect of the split-pole converter on the supply circuit wave form is negligible.*

31. Action of the split-pole converter. When the split-pole converter is operated at a higher voltage than that corresponding to the alternating line voltage, the additional output due to the increased direct-current *voltage* is supplied by an increased *current* on the alternating-current side. This current moreover is a "motor" current dividing in the converter in the same way as the motor current necessary to supply the losses. The value of the effective current is, therefore, considerably increased. At lower voltages the converter acts as an alternating-current generator decreasing the line alternating current, but due to the distribution of the current in the armature the effective current is increased.

32. Two types of split-pole converter. The first converters of this type were designed with two regulating poles for each main pole. This arrangement resulted in symmetrical flux and voltage wave forms through-

* (a) Stone, C. W. "Some Developments in Synchronous Converters;" *Trans.* A. I. E. E., Vol. XXVII, p. 181.
(b) Steinmetz, C. P. "Variable Ratio Converters;" *G. E. Review*, 1908, pages 26–84, 1909.
(c) Burnham, J. L. "Modern Types Synchronous Converters;" *G. E. Review*, page 74, 1912.

out the entire voltage range, and was favorable to commutation but required machines of comparatively large diameter. The split-pole converters as built at present have a single regulating pole for each main pole. This construction obviously permits a considerable reduction in diameter and cost for a given output, and the detrimental effect of the varying magnetic field in the commutating zone can be compensated for by providing favorable commutating conditions in other respects. The field-flux wave forms and the corresponding voltage wave forms for various voltages, with both the three-part pole and the two-part pole constructions, are shown in Fig. 8.

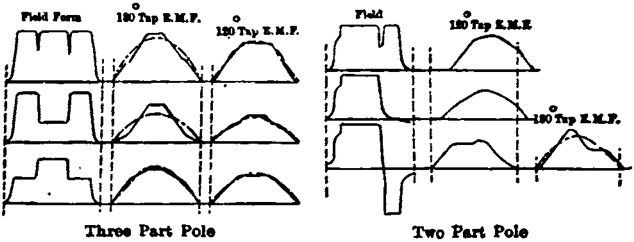

FIG. 8.—Flux-distribution and alternating voltage wave forms in split-pole converter.

Upper curves, max. voltage. Middle curves, mid. voltage. Lower curves, min. voltage.

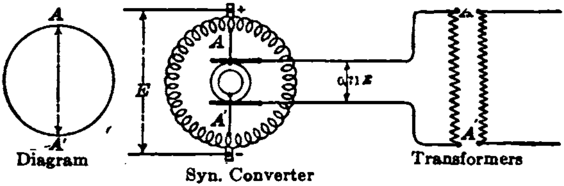

FIG. 9.—Single-phase connections.

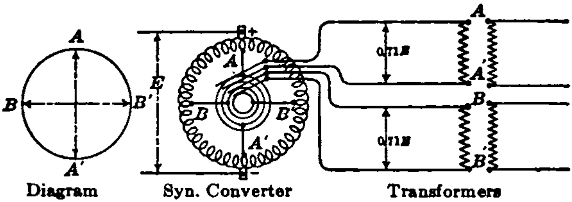

FIG. 10.—Two-phase connections.

33. Limits of voltage variation with the split-pole converter. In varying the voltage, the excitation of the regulating poles is increased in the same direction as the excitation of the main poles to increase the voltage, and is reversed to decrease the voltage. The split-pole converter is limited to approximately 20 per cent. variation in either direction, but this is sufficient to cover the majority of applications.

34. Direct-current booster.—The direct-current voltage can be varied directly by inserting a booster in the direct-current circuit. This booster may be direct-connected to the converter or driven by a separate motor. It is generally used in the latter form as an addition to existing equipment, since it does not affect the converter either structurally or in respect to operating voltage. All methods involving change in alternating voltage necessitate the operation of the converter at the highest voltage.

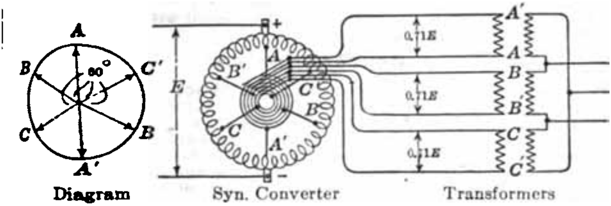

Fig. 11.—Six-phase diametrical connections.

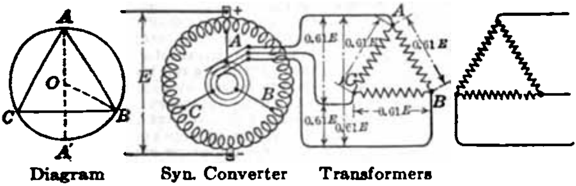

Fig. 12.—Three-phase delta connections.

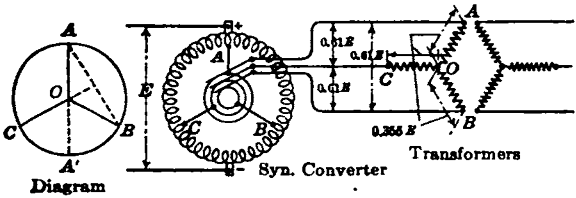

Fig. 13.—Three-phase Y connections.

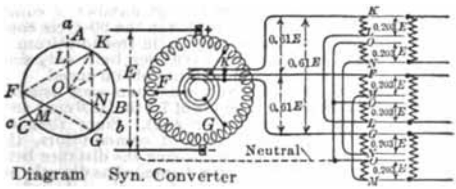

Fig. 14.—Three-phase interconnected Y connections (for 3-wire direct-current circuits).

The weight, floor space and cost of a converter with a direct-connected direct-current booster are greater than the same factors in a converter with an alternating-current booster. A further serious disadvantage of the direct-current booster is the additional commutator, which is equal in current capacity to the converter commutator.

35. Transformer voltages and connections. The various transformer connections with voltages commonly used are shown in Figs. 9 to 14, inclusive. Fig. 14 shows the transformer connections necessary when the converter is used to supply three-wire direct-current circuits; this modification in the transformer windings is necessary to avoid magnetic unbalance due to the unbalanced currents obtained with unbalanced loads on the direct-current circuits.

GENERAL DESIGN

36. Similarity between converter and direct-current generator. The general design of the synchronous converter closely resembles that of the direct-current generator. The design of the magnetic circuit is identical; the types of armature and field windings are the same, except for the taps to the collector-rings; the number of armature slots per pair of poles must be some multiple of the number of phases; the design of the electric circuit is the same, except for the reduced value of the effective current. This reduction of effective armature current permits smaller conductors and smaller armature slots—a condition favorable to commutation; it also permits a higher ratio of armature ampere-turns to shunt-field ampere-turns, resulting again in a smaller armature core; and furthermore, it reduces the flux distortion on overload, permitting the converter to carry large momentary overloads without flashing. In commutating-pole converters, the reduced effective armature current permits a relatively small commutating-pole winding, which is favorable to heavy overloads. With the small number of commutating-pole ampere-turns necessary, it is easy to provide sufficient section in the commutating pole so that large overloads can be carried without saturating the commutating pole and thus destroying the equivalence of commutating-pole flux and armature-coil flux. In the direct-current generator this is not true, and commutating-pole saturation is usually a limit to the overloads that can be carried.

There are, however, various requirements in design, due to the rigid relationship between poles and speed, that do not enter into direct-current generator design. Following directly from this relationship, the maximum distance between adjacent neutral points on the commutator of any synchronous converter is a direct function of the frequency and the peripheral speed of the commutator. The peripheral speed of the commutator, in feet per min., is equal to the alternations per min. (cycles per sec. times 120) multiplied by the distance in feet between adjacent neutral points on the commutator surface.

37. Comparison between 25-cycle and 60-cycle converters. This simple relationship between poles and speed (Par. 36) is responsible for most of the difference between high-frequency and low-frequency converters. It follows from this law that the distance between neutral points in a 25-cycle converter is 2.4 times the distance in a corresponding 60-cycle converter for the same commutator speed. Thus while ample room is available in the 25-cycle converter for a large number of commutator bars at a low peripheral speed, the reverse is true in the 60-cycle converter. The 600-volt 60-cycle converters that compare in freedom from flashing and sensitiveness with 25-cycle converters have been built only since it became possible to construct commutators of high peripheral speed.

Up to the year 1909, 60-cycle 600-volt converters were usually built with a maximum distance between neutral points of 7.5 in. (19.0 cm.) and a peripheral speed of 4,500 ft. per min. (22.9 m. per sec.). Since that time, through improvements in the mechanical design of commutators, the peripheral speed has been increased to 5,500 ft., increasing the distance between neutral points to 9 in. (22.9 cm.). This greater distance has directly and indirectly improved the operating characteristics. As a direct benefit there is less chance that a flash under a brush will reach to the adjacent brush arm, causing the converter to buck; and, indirectly, the greater distance permits the use of more commutator bars, reducing the maximum voltage between bars and so reducing the danger of an arc spreading from bar to bar. With 9 in. (22.9 cm.) available between neutral points, it is possible to proportion 60-cycle converter commutators as conservatively as those of 25-cycle converters and direct-current generators.

The distance between neutral points becomes a design limitation only with the higher line voltages. At 250 volts the design of the 60-cycle converter has not been handicapped as it was at 600 volts, although it has unjustly suffered in reputation.

At the higher voltages now being used in railway work, the 25-cycle converter approaches the same limiting conditions as the 60-cycle at 600 volts. At 1,500 volts, the 25-cycle converter is theoretically equivalent, in commutator proportions, to the 60-cycle converter at 625 volts.

8. Table of Weights, Speeds, and Efficiencies for 600-volt 25-cycle and 60-cycle Converters

Kw.	Frequency	R.p.m.	Approximate net weight (lb.)	Approximate efficiency at full load, per cent.
* 300	25	750	10,100	95
* 500	25	750	15,500	96
1,000	25	500–750	23,000	96
1,500	25	500	39,000	96
2,000	25	375	57,000	96
3,000	25	214–250	92,000	96
4,000	25	187	158,000	96
†200	60	1,200	10,500	91
†300	60	1,200	12,200	93
†500	60	720–900	18,000	94
750	60	720–900	20,000	94
1,000	60	600	35,000	94
1,500	60	514	47,000	94
2,000	60	400	60,000	94

Where two speeds are given, the weights and efficiencies apply to the higher speeds.

39. Table of Weights, Speeds and Efficiencies for 250-volt, 25-cycle and 60-cycle Converters

(b) 25 and 60-cycle—250-volt converters

Kw.	Frequency	R.p.m.	Approximate net weight, lb.	Approximate efficiency at full load, per cent.
200	25	750	14,300	93.0
300	25	750	20,000	94.0
500	25	500–750	35,000	94.5
750	25	375–500	45,000	95.0
1,000	25	300–375	66,000	95.5
100	60	1,200	5,700	90.5
200	60	1,200	13,500	91.5
300	60	900	14,000	92.5
500	60	720	19,500	93.5
750	60	600	32,000	93.7
1,000	60	514	42,000	94.0

All designs are of the non-commutating-pole type and will stand 100 per cent. momentary overload.

Where two speeds are given, the weights and the efficiencies apply to

* Will stand 200 per cent. overload momentarily—all other designs will stand 100 per cent. overload.

† Non-commutating-pole type; in all other cases commutating poles are employed.

the *lower* speeds. The difference, however, will not be of material importance.

40. Commutator design for synchronous converters follows that for direct-current generators except that long high-speed commutators are more frequently used. The longest commutators are built with three V-ring supports, or with two V-ring supports and a central shrink ring. Shrink rings alone are not as effective as either of these constructions for the majority of cases, on account of the large diameters involved. With large diameters, large tangential stresses are produced in the shrink ring by relatively small radial forces. The V-ring construction without shrink rings is generally to be preferred since the commutator can be readily taken apart for repairs and also can be more easily insulated from adjacent parts. A construction embodying four V-rings has been employed to a limited extent; this consists, in effect, of two separate commutators flexibly connected together. The inaccessibility of the middle V-rings is a serious objection to this construction.

Long commutator necks must be braced from each other. This is done by fibre plugs set into each neck, by interlacing heavy twine between the necks, or by roping wooden spacing blocks to each neck.

41. The ventilation of commutators, in cases where the necks do not provide sufficient cooling surface and windage, has been most successfully accomplished by attaching radiating vanes about 2 in. square to the outer end of each bar. These vanes not only provide additional cooling surface at the most effective point, but considerably increase the air circulation.

42. Armature equalizer connections are used in synchronous converters just as in direct-current generators (Sec. 8) to equalize the effect of the flux under all poles. The collector-rings are also equalizing rings, so that the equalizers are usually spaced with respect to the collector-rings. To secure greater accessibility, the equalizer rings are generally located on the collector end of the armature. Special constructions have been developed in order to place the equalizing connections in such a manner that there will be no interference with the collector connections and no increase in the length of the commutator.

43. Damper windings. Almost without exception synchronous converters are provided with damper or **amortisseur windings** to prevent hunting (Par. 61). These are usually of the built-up grid type, commonly employed on the rotors of induction motors, consisting of a large number of bars located in slots in the pole-face, and connected on the ends by continuous rings. A cast-copper damper winding is sometimes employed, consisting of relatively few bars of large cross-section in the pole face. In the commutating-pole converters, the end rings are sometimes made in disconnected segments—one for each pole—so that the commutating poles will not be enclosed by the damper winding. If the commutating poles are individually enclosed, changes in the commutating-pole flux are damped, thus preventing the commutating-pole flux from changing as rapidly as the armature current and flux when sudden changes in load occur.

44. The machine ventilation problem varies widely in converters for different frequencies and voltages. In high-speed 25-cycle converters of medium output, the armatures are small in diameter and relatively long. Every means is used to increase the natural air circulation. In these machines, heating and commutation are of approximately equal effect in limiting the output. In large 25-cycle converters of both voltages (250 and 600) and in 60-cycle converters for 250 volts, the natural air circulation is ample for the dissipation of the losses, but not excessive. In 60-cycle 600-volt converters and particularly in those of large size, the armatures are of large diameter and narrow, and the peripheral speeds are high. Under these circumstances, the natural air circulation is much greater than is necessary and greatly increases the windage loss. In such machines the natural air circulation is restricted as much as possible by stopping the air entrances in order to eliminate the unnecessary windage loss.

CHARACTERISTICS

45. The no-load saturation curves are very similar to the same curves for direct-current generators. There are two such curves—one with direct-current volts and one with alternating volts as ordinates. From these curves the actual no-load voltage ratio may be determined for any desired voltage.

46. **Heating of synchronous converters** occasions practically no difficulty either in design or in operation under proper conditions of power-factor, etc. The problem, however, varies with converters of different frequencies and voltages (Par. 44).

47. **The overload capacity,** as determined by heating and by flashing, is inherently large, due in great measure to the small armature reaction. The latter is particularly important since it results in small field distortion due to load. Consequently, on sudden changes in load, there is very little increase in voltage between commutator bars or shifting of the flux in the commutating zone, both of which, if present, would tend to cause flashing.

48. **The losses** may be divided into constant losses (with varying load) and variable losses. The constant losses are: core loss; friction and windage losses (including commutator and collector friction); and shunt field and rheostat loss (assuming that rheostat resistance and direct-current voltage remain fixed). The variable losses are: armature copper loss; series-field copper loss; commutating-field copper loss; brush I^2R and surface-contact losses; and load loss.

49. **Additional losses in the synchronous booster converter.** In the synchronous booster converter, the only additional losses at the mid-voltage are the booster armature copper loss and the load loss. At higher voltages, the converter core loss is increased due to the larger "motor" current, and the booster core loss and field copper loss are added. At lower voltages the converter core loss is reduced, since the flux is reduced; the converter copper loss is increased due to the larger "generator" current, and the booster losses are the same as for a corresponding increase in voltage.

50. **Additional losses in the split-pole converter.** In the split-pole converter there are no additional losses at the mid-voltage, but at either lower or higher voltages the converter core-loss is increased, since the flux density is increased (Par. 32 and Fig. 8). The converter copper loss is increased, as in the booster converter, at higher and lower voltages (Par. 31).

51. **Hunting of synchronous converters,** as in other forms of synchronous apparatus, is the periodic variation of the rotor from the true synchronous position with respect to the supply system. During hunting, the rotor is alternately ahead and behind its true synchronous position. In forging ahead, energy is expended in the converter, the latter acting as a motor; in dropping behind, the converter acts as a generator and gives up energy. This alternate motor and generator action is accompanied by a variation in the value of the flux due to the armature reaction, by a fluctuation of alternating-current and power-factor, and by a shifting of the commutating field which causes periodic sparking at the brushes. The frequency of the hunting cycle may often be determined from the frequency of the sparking. Hunting may be caused by periodic variation in the supply frequency, by sudden changes in load, or by excessive line drop. It is more likely to occur in 60-cycle than in 25-cycle converters, because of the greater number of poles and consequently smaller actual angular variation corresponding to the limiting electrical angular variation.*

52. **Hunting may be practically eliminated** by providing suitable **damper** or **amortisseur windings** in the pole faces. Since hunting is accompanied by a shifting of the flux across the pole face, the winding placed in the path of this flux will oppose any such change, and will tend to damp out the oscillations as soon as they begin. Practically all converters are now built with such windings (Par 42). Hunting troubles have also been greatly reduced by the use of generators driven by steam turbines or water turbines in which the angular velocity is much more uniform than in reciprocating engines.

53. **Interruption of energy supply.** If a converter is operating alone

* (a) Steinmetz, C. P. "Hunting;" *G. E. Review*, May, 1913.
(b) Newbury, F. D. "Hunting of Rotary Converters;" *Elec. Journal*, June, 1904.
(c) Lamme, B. G. "Causes of Hunting;" *Elec. Journal*, June, 1911.

or in parallel with other converters supplied from the same alternating-current feeders, the interruption of the alternating-current supply will bring the voltage on the direct-current side to zero, and the converters will stop. If, however, there is another source of direct-current connected to the same direct-current bus and not dependent on the same alternating-current feeders, interruption of the alternating-current supply will not cause the converter to stop; the direct-current voltage will be maintained and the converter will be driven as an inverted converter. If the shunt field happens to be weak or is interrupted, or if the converter should supply energy to an inductive load on the alternating-current side (such as a short-circuit on the high-tension side of the transformers) the speed will greatly increase. To guard against danger under such conditions, converters are usually provided with a speed-limit device, and the alternating-current and direct-current breakers are so interlocked that opening the alternating-current breaker opens the direct-current breaker. In addition, the direct-current breakers are sometimes provided with a reverse-current tripping relay. When the alternating-current supply is subject to serious drop in voltage due to short-circuits or other cause, it may be advisable to provide the alternating-current breakers with low-voltage tripping coils. This is particularly necessary with commutating-pole synchronous converters due to the flashing that would occur should the converter drop out of step as a result of low voltage, and later start with the brushes down when the voltage resumes normal value.

54. The speed-limit device (Par. 53) consists of a pivoted weight rotating with the converter shaft; the centrifugal force acting on the weight is counter-balanced by a spring. The weight moves outward and operates a switch when the predetermined overspeed is reached. The switch closes (or opens) the circuit of the shunt tripping coil of the direct-current breaker.

GENERAL APPLICATIONS

55. Comparison of efficiency with that of a motor-generator set. The efficiency of a 60-cycle converter including its transformers will be from 3 per cent. to 5 per cent. higher than an equivalent synchronous motor-generator set without transformers. The 25-cycle converter shows a further gain in efficiency over the motor generator set of from 0.5 to 1.0 per cent. If the line voltage is above 13,200 volts, transformers will usually be required with the motor-generator set as well as with the synchronous converter, so that the difference in efficiency in favor of the latter will be further increased by 2 per cent. This higher efficiency of the converter is such an important advantage that it is sufficient alone to justify the use of synchronous converters wherever possible.

56. Comparison of required floor space with that of a motor-generator set. The floor space required by 60-cycle converters and transformers is approximately equal to the floor space required by 60-cycle motor-generator sets of compact design. The fact, however, that the transformers may be placed in some remote location makes the arrangement of synchronous converters and transformers more flexible and gives them in many cases the advantage of reduced floor space.

57. Comparison of cost with that of a motor-generator set. The combined cost of synchronous converters and transformers is approximately equal to the cost of synchronous motor-generator sets without transformers, assuming a motor voltage of 2,300 volts or lower. For higher alternating voltages the motor cost increases appreciably, making the comparison favorable to the converter equipment. With alternating pressures above 13,200 volts, transformers are necessary with the motor-generator set as well as with the synchronous converter, so that the cost of the converter equipment is relatively still lower. With 60-cycle apparatus under 200 kw. capacity and alternating voltages of 2,300 volts and lower, the comparison is somewhat in favor of the motor-generator set, while with larger apparatus the comparison is somewhat in favor of the synchronous converter. In few cases, however will there be sufficient difference in cost alone to determine the choice of the apparatus.

58. Comparison of reliability with that of a motor-generator set. The design and construction of synchronous converters has been standardised to such an extent that in respect to reliability the comparison is mainly concerned with the number of machines involved. Compared with a three-unit motor-generator set consisting of a synchronous motor, direct-current generator and direct-current exciter, the single unit converter has a decided advantage. The comparison between the motor-generator set and the booster converter is apparently less favorable on account of the additional booster generator, but there are so few troubles that can reasonably be expected in a relatively small low-voltage alternator that the difference is more apparent than real.*

59. Comparison of voltage control with that of a motor-generator set. In this characteristic the synchronous motor-generator set has an advantage over the synchronous converter. The independence of alternating and direct-current voltage in the motor-generator set may justify its use on alternating-current circuits in which the voltage fluctuates badly and where it is essential to maintain steady direct-current voltage. Even under such conditions of fluctuating alternating voltage it is considered preferable, by some engineers, to employ the converter and obtain the desired steady direct-current voltage by means of a regulator.†

60. Comparison of power-factor control with that of a motor-generator set. The motor-generator set, including a synchronous motor, may be designed to correct power-factor more economically than can the synchronous converter. For this reason, the motor-generator set is usually employed where considerable power-factor correction is necessary.

61. For railway service compound-wound synchronous converters are generally used, sufficient reactance being placed in the alternating-current supply circuit to obtain the necessary voltage regulation. For this service on 25-cycle systems, synchronous converters are used to the practical exclusion of motor-generator sets and other forms of conversion apparatus. On 50-cycle and 60-cycle systems, the use of motor-generator sets, and in Europe the use of motor converters has, in the past, been more general; but even for these higher frequencies the use of the synchronous converter is rapidly growing.

62. For lighting and power service, the shunt-wound converters with induction regulators or the more specialized forms of split-pole and booster converters are used. On the larger 25-cycle systems, the use of converters is general. On 60-cycle systems, their use, particularly in the booster form, is rapidly growing, due mainly to the gain in efficiency.

63. The synchronous converter can be used to supply 3-wire direct-current circuits without any structural change, except in the case of compound-wound or commutating-pole converters, in which the series windings on alternate poles are connected in the positive and negative leads so that unequal currents in these leads will not affect the magnetization of the converter. See Par. 35 and Fig. 14 for the transformer connections necessary to obtain the neutral lead. The synchronous converter will operate under extreme differences of current value in positive and negative leads with very small differences in voltage between the outside leads and the neutral lead. For example, a 1,000-kw., 60-cycle, 280-volt commutating-pole booster converter, operated with full-load current in the positive lead and half-load current in the negative and neutral leads, had a difference of only 1 volt in the pressures from neutral to positive and neutral to negative leads. The same converter, operated with full-load current in the positive and the neutral leads and zero current in the negative lead, had a difference of 4 volts in the pressures from neutral to positive and neutral to negative leads. Without the resistance drop of the booster armature winding, the voltage balance would have been still better.

64. For electrolytic work, either shunt-wound converters without means for varying the voltage, or the same types of variable voltage converters as used for lighting service, are generally employed.

OPERATION

65. Alternating-current self-starting. The synchronous converter may be started as an induction motor if alternating current at reduced vol-

* Lincoln, P. M. "Motor Generators versus Synchronous Converters;" *Proc.* A. I. E. E., March, 1907.

† Walker, Miles. "Rotary Converters versus Motor-Generators;" *Jour.* I. E. E. (London), Discussion, Vol. XXXVIII, page 428.

tage is applied to the collector-rings. The armature winding acts as the primary and the squirrel-cage winding imbedded in the pole faces acts as the secondary. For the smaller converters one starting voltage only is required. The middle terminals of a double-throw switch are connected to three of the collector-rings, one set of outer terminals is connected to the starting-voltage taps of the transformers and the other set of outer terminals is supplied with the full voltage of the line. With six-phase converters three leads are carried direct from the transformer to the converter, and three through the main and

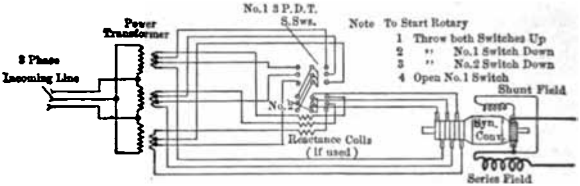

Fig. 15.—Alternating-current self-starting connections with two starting voltages.

starting switches. With the largest converters, it is usual to employ two starting voltages for which two double-throw three-pole switches are required. The connections for this arrangement are shown in Fig. 15.

66. Field "break-up" switch. During the starting period the field winding has voltages induced in it of magnitudes depending upon the ratio of armature and field turns. To prevent a dangerous induced voltage in the field winding, it is usual to open the field circuit in several places during the starting period by means of a multi-point double-throw switch, which is usually located on the frame of the converter. Appreciable voltage rise in the field circuit may also be prevented by closing the field circuit during start-

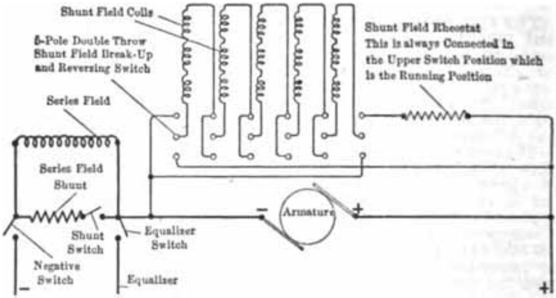

Fig. 16.—Converter connections on direct-current side (alternating-current self-started).

ing. This method is beginning to be employed because it only requires a double-pole double-throw field switch which may be conveniently located on the starting panel adjacent to the main switch.

67. Series-field switch. If a series-field shunt is used, a switch is usually provided for opening the closed circuit formed by the series field and its shunt. If this low-resistance circuit were left closed, the single-phase current induced in it would tend to decrease the starting torque. This effect, however, is small. The detail connections of the direct-current side of the converter are shown in Fig. 16.

68. Reversed polarity with alternating-current self-starting converters. In this method of starting with a self-excited field winding, there is no method of predetermining the polarity of the direct-current terminals. However, if the converter falls into step with the wrong polarity, the machine may be forced to "slip a pole" (if the armature is connected to the low-voltage taps) by reversing the field switch. When the voltmeter indicates that the voltage is reversed, the field switch should be brought back to its original

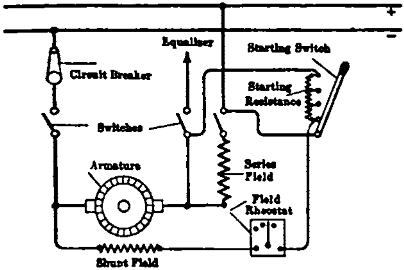

FIG. 17.—Direct-current self-starting connections.

position. In some converters, the field set up by the alternating-current may be so strong even at the lowest starting voltage that the converter cannot be made to "slip a pole" by reversing the field. Under these circumstances it is necessary momentarily to open and close the starting switch, repeating this operation until the correct polarity is obtained.

69. Direct-current self-starting. The converter may be started as a direct-current shunt-wound motor. The current flow at starting is limited by an adjustable resistance controlled by a multi-point starting switch. The connections for this method are shown in Fig. 17.

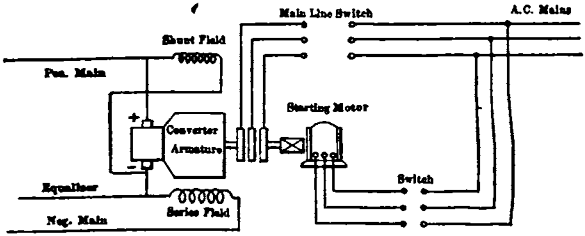

FIG. 18.—Alternating-current motor-starting connections with manual synchronizing.

70. Alternating-current motor starting. A motor usually of the induction type is mounted on the converter shaft. The motor has fewer poles than the converter and, therefore, a higher synchronous speed. In starting, the motor is usually connected directly across the line. The connections for this method are shown in Fig. 18.

A modification of this method has recently been brought out by Dr. Rosenberg of Manchester, England, and by James Burke* in this country, which consists in connecting the starting-motor windings in series with the converter armature. This limits the starting current to a low value, and the

* See U. S. Patent 1073662.

alternating current in the converter armature causes it to lock automatically in synchronism.*

An older modification of the simple motor-starting method, designed to eliminate the necessity for synchronising, is the addition of reactance in the main circuit as shown in Fig. 19. When the starting motor has brought the converter armature near synchronous speed, the main-line switches are closed with the reactance in circuit. The resulting current in the converter armature causes it to lock in synchronism, after which the reactances should be short-circuited.

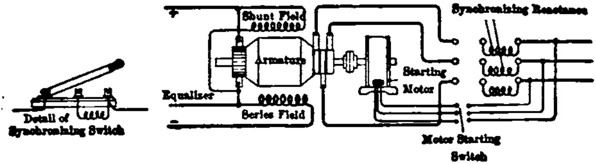

FIG. 19.—Alternating-current motor-starting connections self-synchronising with reactance.

71. Starting small single-phase converters. A method of starting, adapted to small single-phase converters, has been developed by the Wagner Electric Manufacturing Company, and is used on the small single-phase converters built by that company for charging electrical vehicle batteries, the supply of direct current for moving-picture arc lamps and similar purposes. A special distributed field winding is provided on the stator, so connected that with the double-throw starting switch in the starting position the converter starts as an alternating-current series-wound commutator motor. When the starting switch is thrown to the running position, the alternating-current leads are transferred to the collector-rings, the main-field winding is connected in shunt with the main direct-current armature leads and a part of the field winding is short-circuited to act as a damper winding. See Sec. 17, "Garage Equipment."

72. Methods of synchronizing. With the simple motor-starting method, or with the direct-current self-starting method, the converter must be synchronised with the alternating-current supply circuit before the main-line switches can be closed. The theory and methods involved are the same as employed in synchronizing two alternating-current generators (Sec. 7). Lamps or specially designed instruments (Sec. 3) may be used for indicating synchronism, as with generators.

With the motor method of starting, the running speed of the starting motor should, in general, be higher than the synchronous speed of the converter. The speed may be brought down to synchronous speed by increasing the load on the converter. This can be accomplished by increasing the voltage, thereby increasing the converter losses, or by connecting a resistance across the alternating-current terminals of the converter. Small adjustments in speed may be made by alternately opening and closing the starting-motor switch.

With the direct-current self-starting method, the speed regulation for synchronising is obtained by varying the converter shunt-field current. The series-field circuit is opened to insure more constant speed under the condition of varying armature current. To prevent a rush of current at the instant of synchronising, it is usual to leave a small amount of resistance always in the starting-rheostat circuit.

73. Comparison of different starting methods. The alternating-current self-starting method is used in most cases on account of its simplicity and its automatic synchronising feature. Its principal defects are the uncertainty in polarity and the large starting current required. In most cases wrong polarity can be corrected by means of the reversing field switch (Par. 66), and the amount of starting current is of importance (aside from the momentary line drop) only when the converter rating approaches the rat-

* Report of Com. on "Electrical Apparatus," National Electric Light Asso. Proceedings, 1914.

ing of the alternator supplying it. With commutating-pole synchronous converters, the simplicity of this method is decreased by the necessity of lifting the direct-current brushes. The presence of the steel of the commutating poles greatly increases the armature flux during starting, which causes destructive sparking at the direct-current brushes. To avoid this objectionable feature, commutating-pole converters are provided with a special device for lifting all but two of the brushes (which indicate polarity) during the starting period.

The advantage of the alternating-current motor method of starting is the small current required and the certainty of obtaining correct polarity; the principal disadvantage is the necessity for synchronizing. This disadvantage is overcome by the Rosenberg method or by the use of separate reactance coils (Par. 75). As compared with the alternating-current self-starting method, this method is somewhat more complicated and the necessary apparatus is more expensive. In some cases, however, where small values of starting current are imperative, it is the only method which will meet the requirements.

The direct-current self-starting method can only be used where direct current in sufficient amount is always available. It is very often used in large city lighting and railway substations. It has the advantage of minimum starting current and the absence of disturbances on the alternating-current supply system (which is particularly important on underground distribution systems). As with the simple alternating-current motor method, it has the disadvantage of necessitating manual synchronizing. This disadvantage has been overcome in some cases by combining the direct-current and the alternating-current self-starting methods. The converter is started by means of direct current, and when a speed near synchronous speed has been reached, the alternating-current switches are closed.

74. Parallel operation. When two or more converters are connected to the same low-tension bus on the alternating-current side and to the same bus on the direct-current side, the total direct-current load will divide among the several converters in the inverse ratio of the counter e.m.fs. of the converters and the resistances of the circuits. If two or more converters are so connected, the alternating-current and the direct-current bus bars close the circuit through any pair of converters; hence if the counter e.m.fs. of the two converters vary even slightly, considerable current will circulate between them. The counter e.m.fs. of different converters are seldom exactly the same, and the resistances, made up mainly of the direct-current brushes and brush contacts, are extremely variable—even in the same converter at different times. For these reasons it is not considered good practice to operate converters in parallel on both the alternating-current and the direct-current sides. Separate banks of transformers are usually provided—one for each converter. These not only open the circuit between converters, but also provide means for correcting slight differences in the voltage ratio of different converters.

75. Division of load on the direct-current side is controlled by the same factors as in direct-current generators, and in addition by factors peculiar to the synchronous converter. Division of load is controlled by the voltage, so that all factors affecting voltage affect parallel operation. Parallel operation of shunt-wound converters, with or without auxiliary means for controlling the voltage, is as simple as parallel operation of shunt-wound direct-current generators. With compound-wound converters, equalizer leads are necessary as in direct-current generators. (Sec. 8.)

76. Corrections for improper division of load. With two converters operating in parallel on the direct-current side, one of which takes less than its proportionate share of the load, the load may be equalized by one or a combination of the following adjustments:

(a) **Adjustment of the series-shunt resistance.** The shunts on the series-field windings can be adjusted, decreasing the resistance of the shunt, if possible, on the overloaded converter or increasing the shunt, on the underloaded converter. It should be borne in mind, however, that changing the ampere-turns in the series field by changing the shunt resistance also changes the resistance of the complete series-field circuit. This change in shunt resistance must be compensated for by a corresponding change in the resistance of another part of the series-field circuit, so that the resistance of the total circuit remains unchanged. From another standpoint, a shunt on one converter series field may be considered to be a shunt on both series fields, the effect varying only by reason of the fact that the resistance of the leads and busses is added to one shunt circuit and not to the other (Sec. 8).

(b) **Insertion of resistance in leads between series-field and equalizer bus.** If the relative ampere-turns are correct but the series-field resistances are differently proportioned, the resistance of the leads between the series field and the equalizer bus may be changed to compensate for a difference in the series-field resistances. The resistance in the series circuit of the converter taking more than its share of the load should be increased. This adjustment varies the resistance of one series field without introducing a third parallel circuit between the equalizer and the main bus. Adjustment by this method is less complicated than an adjustment of the series-shunt resistance.

(c) **The transformer ratio can be changed.** This increases the voltage by the same amount throughout the range of load, and will not correct for an unequal division which changes with the total load. An increase in no-load voltage of one converter will cause, at lighter loads, a greater increase in proportionate load on that converter.

(d) **The reactance can be increased in the circuit of the lightly loaded converter,** which will raise its direct-current voltage and cause it to carry more nearly its proper share of the load. This method is similar in effect to an increase in the number of series-field turns, but the effect is obtained at the expense of a greater range in power-factor. It sometimes happens that the reactances of converters in parallel are worked at different saturations, with the result that the reactance voltages of the two converter circuits will have different ratios at light and at heavy loads. Such a relation will cause the converter with the more highly saturated reactance to take less than its share of the load at heavy loads.

(e) **The relative shunt-field currents of the two converters can be changed.** The converter having the smaller ratio of series-field ampere-turns to armature ampere-turns should have its shunt-field current increased. This will increase its no-load voltage (on account of change in power-factor) and cause it to take a greater share of the load at light loads. The voltages will tend to equalize as the load increases, a correct division being obtained at only one value of the load.

77. Load-division study. Since there are so many variables affecting load division, it is important to make a careful and systematic study of the particular case before making any changes. Such a study should be conducted as follows:

(a) Adjust the transformer ratios so that at no-load and with the shunt field adjusted to give equal power-factors, all converters have the same no-load direct-current voltage.

(b) The series fields should be adjusted by shunts so that the ratio of series-field ampere-turns to armature ampere-turns is the same.

(c) The resistances of the series fields (including shunt) plus the resistances of the leads from the series fields to the main bus (positive or negative) should be so adjusted that the resistances are inversely proportional to the rated capacities of the converters.

(d) The reactances should be adjusted, if possible, so that the reactance volts of the various circuits throughout the range of load are equal. If they cannot be made equal, the series ampere-turns should be greater in the converter having the smaller reactance, to afford an approximate compensation.

It is only possible to obtain correct division of load and equal power-factor on all converters when the machines are properly proportioned with respect to all four elements, namely: the transformer ratio, the series-field ampere-turns, the series-field resistance and the reactance.

78. Interruption of alternating-current supply by circuit breaker in substation. Under such conditions, if the converter is connected to another source of direct current, the converter will "motor" unless the direct-current breaker is opened by interlock with the alternating-current breaker (Par. 58) or by a reverse-current relay. A compound-wound converter will run in the same direction when the direct-current power is reversed, the current in the series winding will be reversed (unless there are several converters in the same station connected to an equalizer bus), and the current in the shunt winding will remain the same in direction. With a single converter in a substation which is connected to other converters in other substations on the direct-current side, the converter will increase its speed due to the reversed series field. It may also increase its speed if the shunt field is adjusted for less than 100 per cent. power-factor. The increase in speed from the reversed series field will not be serious unless there is a circuit connected to the alternating-current side which will provide a load. Under these conditions the overspeed device will operate, thereby opening the direct-current breaker.

79. Interruption of alternating-current supply outside substation. In this case an interlock between the substation alternating-current and direct-current breakers will not operate to open the direct-current breaker, and reverse current or overspeed must be depended upon to protect the converter.

In case of interruption of alternating-current supply the converter should be manually disconnected from the direct-current lines (if this is not done automatically) and the switches on the alternating-current side opened for starting.

80. Commutating-pole converters require more complete protection from low alternating voltage because of the excessive sparking at the commutator which is encountered when the converter is not in phase with the supply circuit. If the alternating voltage is lowered momentarily by a short-circuit or other cause, the converter may fall out of synchronism. When the voltage is restored the converter will not be in phase and the sparking may be sufficient to make the converter "buck." The conditions are similar to those which exist when starting the converter with the brushes down.

81. Protection of high-voltage converters. High-voltage converters (1,200 volts and above) should be more completely protected from abnormal conditions than low-voltage converters on account of the more destructive nature of the "bucking" when once started.

82. Sparking at the brushes may be due to any of the following causes: (a) brushes incorrectly set with reference to the neutral point. Correct setting is of particular importance in commutating-pole converters; (b) brushes of improper characteristics; (c) defective electrical design; (d) hunting; (e) severe overloads or extreme variation in load; (f) in non-commutating pole converters, low alternating voltage with large direct-current loads; (g) brush holders insufficiently supported; (h) brushes stuck in holders or inaccurately fitted to commutator; (i) improper brush tension; (j) rough commutator due to high bars, to high mica or to flat spots.

83. Bucking or flashing may be brought about by any condition causing excessive voltage in the coils short-circuited by the brush or between adjacent commutator bars, or may be caused by abnormally low-surface resistance on the commutator between adjacent brush arms. Any condition tending to produce poor commutation increases the likelihood of bucking. Excessive voltage under the brush is usually caused by short-circuits of varying degree. Any direct-current machine will flash if short-circuited at its terminals and the direct-current voltage is maintained. Short-circuits in service usually occur on the line, so that some resistance exists between the "short" and the machine, thereby limiting the current. Excessive voltage between commutator bars is caused directly by increased line voltage, or is indirectly caused by extreme current overloads which distort the field flux. Increased line voltage may be due to disturbances on the high-tension distributing system induced by lightning, switching, short-circuits, etc. A decrease in the insulation strength between brush arms may be due to the presence of conducting gases formed by a relatively small flash or of foreign substance such as dirt or water. Ordinary types of circuit breakers do not act quickly enough to protect machines from abnormal changes in current or voltage. On short circuits, for example, the current increases far beyond the setting of the breakers before the circuit is finally opened.

84. Reactance used as a protection against short circuits. Shunt-wound synchronous converters may be protected from the effects of short circuits by inserting reactance in the alternating-current leads. Twenty-five per cent. reactance, for example, will cause a drop in alternating voltage to approximately zero with four times full-load current, but the drop at full-load current will not be objectionable.

TESTING

85. The tests ordinarily made on synchronous converters are as follows: (a) resistance measurements; (b) polarity determination; (c) ratio of voltages; (d) core-loss and saturation test; (e) alternating-current short circuit or synchronous-impedance test; (f) starting tests; (g) voltage regulation test; (h) temperature tests; (i) commutating-pole saturation test.

86. Measurement of resistance. Resistances may be measured either by the bridge method or by the use of ammeter and voltmeter as in direct-current machines. It is customary in obtaining the armature resistance to measure the resistance both on the direct-current side between the proper commutator bars, and on the alternating-current side between the corresponding slip-rings.

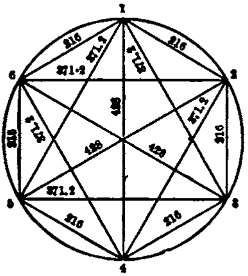

Fig. 20.—Diagram of converter voltages between collecter-rings.

87. Determination of polarity. The correct polarity of the field windings is determined in the same manner as for direct-current generators. The relative polarity of the shunt and the series windings in the compound-wound converter is also determined the first time the converter is under load.

88. Determination of voltage ratio. In order to determine whether the various taps from the armature winding have been brought out correctly and connected to the proper rings, readings of voltages between each ring and every other ring are taken. During the test the direct-current voltage is held constant at some convenient value. The armature winding is usually connected to the collector-rings so that adjacent rings have the minimum voltage possible between them. Under these conditions the voltage between adjacent rings is given by the following formula, N being the number of rings:

$$E_{ac} = \frac{E_{dc}}{1.415} \left(\sin \frac{180 \text{ deg.}}{N}\right) \quad \text{(volts)} \qquad (3)$$

A convenient method of checking the values obtained on test is to construct a diagram such as that in Fig. 20. The numbers outside of the circle represent the collector-rings. The voltages shown were actually obtained from a 600-volt six-ring converter.

89. Determination of core-loss, saturation, friction and windage losses. This test is made either by driving the converter by a small direct-current motor or by driving the converter itself as a direct-current shunt-wound motor. The test is the same as the corresponding test of a direct-current generator except that alternating voltages are obtained as well as the direct-current voltage, in order to obtain the actual voltage ratio. If the converter is self-excited during the test, the measured loss must be decreased by the field-winding and rheostat losses. The **brush friction** is determined by the difference in the measured losses with the brushes down and the brushes up. This reading is of no value unless the commutator has a good polish and the brushes are well seated. The **friction and windage losses** are determined by the difference in measured converter losses with the brushes removed, and with the driving motor running disconnected from the machine under test.

90. Starting tests. With the alternating-current self-starting method, a test is made to check the sufficiency of the starting voltage to bring the converter up to synchronous speed. It is customary to measure the voltage, current and time required to reach synchronous speed. With the induction-motor method of starting, the test is made to ascertain whether the load on the starting motor (provided by the converter) is such as to permit it to operate sufficiently near the converter synchronous speed at the normal converter voltage, so that synchronizing will be possible. If the normal losses of the converter are not sufficient to reduce the starting-motor speed to the converter synchronous speed, it is necessary to determine the additional load required.

91. The voltage-regulation test can be made only when the entire equipment of transformers and reactances to be installed with the converter is available. Even under such circumstances the test must be made on the basis of constant alternating voltage on the high-tension side of the transformers. This is rarely, if ever, true in practice, so that the test unless made after installation is of little value and is seldom made. Furthermore, there is no object in adjusting the series winding of a converter except in conjunction with all other converters with which it is to operate in parallel on the direct-current side. With compound-wound converters, the alternating voltage should be adjusted to the desired value, and the shunt field of the converter should be adjusted to give the correct voltage at no load. If the transformer ratio is correct, this will require considerable lagging current, which may be obtained by under-excitation. Load is then placed on the converter and the desired full-load voltage is obtained by adjusting the series-field shunt. This test is never made with shunt-wound converters since a slightly drooping voltage characteristic would always be obtained, and there is no means within the converter of modifying it. With converters designed for wide voltage ranges, it is customary to observe the operation at the maximum and minimum pressures to insure that there is sufficient range in the field windings and rheostats to obtain 100 per cent. power-factor at all voltages, and that satisfactory commutation is obtained throughout the voltage range.

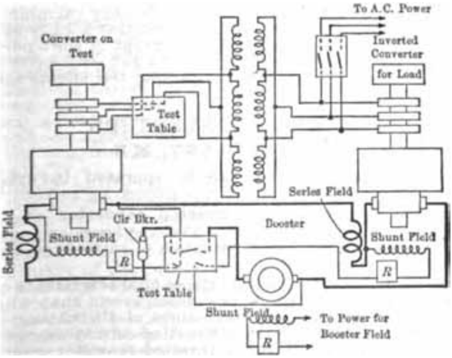

FIG. 21.—Loading-back temperature-test connections.

92. The temperature tests at the factory are performed either by direct loading on resistances or by "**loading back**" on a similar converter or power circuit. In the loading-back method (using a similar converter), the losses can be supplied from either the direct-current or alternating-current sides as is found convenient. The alternating-current sides of the machine under test and of the test machine may be connected together by a complete metallic circuit, or through transformers having a one-to-one ratio. The latter arrangement is preferable, as conditions are thus made more stable and can be more easily controlled. The connections for such a test are shown in Fig. 21. When the machines under test are compound-wound, it is necessary to reverse the series field of the converter running inverted. In Fig. 21 the losses are supplied partly from the alternating-current side and partly by a booster in the direct-current side. Temperature tests are usually made with 100 per cent. power-factor at the converter collector-rings. To obtain this condition, the shunt field should be adjusted for the proper no-load voltage, and the series field shunted to give 100 per cent. power-factor, that is, a minimum value of armature current, at the tested load.

93. Commutating-pole saturation test. On commutating-pole converters this test is made to determine the sensitiveness of the converter to proper adjustment of the commutating-pole ampere-turns. The converter is so connected that it can be loaded with the commutating poles separately excited, in order that the excitation can be adjusted. For various loads and, in variable voltage converters, for various voltages, the minimum and the maximum commutating-pole excitations are determined between which satisfactory commutation is obtained.

94. Determination of efficiency. Efficiencies are usually determined by separate measurements of the various losses. This method does not derive the true efficiency, as the so-called load losses or stray losses cannot be included by direct measurement. Such losses, however, in converters are small and no appreciab e error is introduced by omitting them. They are less in low-frequency converters than in high-frequency converters. If the true efficiency is required, it can be measu ed only by an input-output load test, which may be made with the converter carrying a resistance load or loaded back on another converter. On account of the relatively small losses involved and the large number of instruments to be read, such a test can be made satisfactorily only under laboratory conditions where the power input and output are absolutely steady, and where carefully calibrated instruments are read by skilled observers. Unless the test can be made under such conditions more accurate results will be obtained by calculating the efficiency from the separate measurable losses, and including a reasonable allowance for the load losses.

The reason for the greater accuracy of the efficiency calculated from the separate losses is apparent when it is remembered that a 2 per cent. error in the measurement of the losses results in only 2 per cent. of the 5 per cent. losses (using 95 per cent. as the true efficiency) or 0.1 per cent. error in the efficiency, while 2 per cent. error in the input-output test affects the efficiency by an equal percentage.*

INVERTED CONVERTERS

BY F. D. NEWBURY, M.E.

95. Any synchronous converter can be operated inverted; that is, it can be used to convert direct current into alternating current.

96. The internal action of the inverted converter is the same as that of the synchronous converter except that the losses are supplied from the direct-current side, so that the distribution of currents in the armature is slightly different.

97. The inverted converter has the speed characteristics of a direct-current motor instead of a synchronous motor, except that when the converter operates in parallel with another source of alternating current the speed is controlled by each of the two alternating-current sources in proportion to its share of the load. When an inverted converter operates singly, its speed may be seriously affected by a change in load or power-factor on the alternating-current side. A lagging alternating-current load will demagnetise the main fields and the converter speed will increase, as would that of any direct-current motor with weakened field.

98. Alternating voltage control. There is no means of varying the alternating voltage corresponding to the reactance and series-winding method used with the synchronous converter. The alternating voltage may be controlled by any of the other methods discussed in connection with the synchronous converter.

99. Inverted converters are usually shunt-wound in order to secure as nearly constant speed and frequency as possible.

100. Correction for overspeeding. Where inverted converters are subject to loads of varying power-factor and where constant speed is essential, it is customary to use a separately excited field winding, the excitation being

* See footnote, Par. 32.

supplied by a direct-connected exciter. The exciter is designed with an unsaturated magnetic circuit so that a change in speed will produce a maximum change in exciter voltage. With this arrangement an increase in speed of the inverted converter is at least partially corrected by the increase of its excitation due to increased exciter voltage.

101. Protection against overspeeding. Where the conditions do not require the separate exciter, it is customary to install an overspeed tripping device to protect the converter and its alternating-current load from excessive speed due to unusual conditions.

102. Applications of the inverted converter. The following applications of the inverted converter may be mentioned: (a) to supply a small amount of alternating-current from an existing direct-current supply; (b) to form a connecting link between alternating-current and direct-current systems for the transfer of energy in either direction. Under the latter condition the converter may operate either direct or inverted. An induction regulator or other means of varying the voltage is necessary to control the transfer of energy through the converter or, in the case of operating the load on one system entirely through the converter, to compensate for the voltage drop in the converter and transformers. (c) To enable a storage battery to be used to equalize the energy input in an alternating-current system. Here, also, a means of varying the voltage of the converter is necessary to control the charge and discharge of the battery. A notable installation of this kind is at the Indiana Steel Company, where a 2,000-kw. 250-volt, 25-cycle split-pole converter is used.

MOTOR CONVERTERS

BY F. D. NEWBURY, M.E.

103. Structure. The motor converter, or cascade converter, as it is sometimes called, consists of two elements, or machines, coupled together mechanically and electrically. The primary element, having the structure of an induction motor with a phase-wound rotor, performs the functions of a voltage, frequency and phase converter and of an induction motor. The secondary element, having the structure of a synchronous converter (with taps to the winding but without collector rings), performs the functions of a direct-current generator (driven by the induction motor, the rotors being mounted on the same shaft) and of a synchronous converter (receiving energy of suitable voltage, frequency and phase from the rotor winding of the primary element).*

104. Starting resistance. Three of the inside terminals of the rotor winding of the primary element are attached to collector-rings, to which the starting resistance is also connected. When operating at synchronous speed, all of the inside terminals of the rotor winding of the primary element are connected together to form the neutral.

105. The rotor of the primary element is wound with either 9 or 12 phases, the large number of phases being used to decrease the armature copper loss of the commutating machine. The large number of connections between the two rotors does not lead to any complication as they can be made solidly without collector-rings.

106. Transformers are required only when the line voltage exceeds the highest voltage for which the primary element can be safely and economically wound. In this respect the motor converter is on nearly equal footing with the synchronous motor-generator set. The difference in favor of the latter is due to the fact that the synchronous motor can be satisfactorily wound for higher voltages than the induction-motor element of the motor converter.

107. Applications. The motor converter occupies a position between the motor-generator set and the synchronous converter. Its main advantage over the motor-generator set lies in the fact that a part of the energy is transformed electrically and, therefore, more efficiently than in the motor-gener-

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* (a) See German patent 145434 (1902); English patents, 3704 (1903), 7807 (1904); U. S. patent 72400 (1904).

(b) See Arnold u. la Cour, "Der Kaskadenumformer" Enke, Stuttgart, 1904.

(c) Hallo, H. S. "The Theory and Application of Motor Converters," *Journal*, I. E. E. (London), Vol. XLIII, page 197.

ator set in which all of the energy is transformed mechanically. For the same reason it is less efficient than the synchronous converter in which the entire output is transformed electrically. Its main advantage over the synchronous converter lies in the fact that the commutating machine of the motor converter operates at a frequency lower than the line frequency—usually at half the line frequency. This advantage applies only in the case of line frequencies above 40 cycles per sec. and for the higher direct-current voltages, and has been lessened even for these conditions by recent improvements in high-frequency synchronous converters. The motor converter has found its most extensive application in England and Germany. In England approximately 150,000 kw. of capacity has been installed since 1904, and in Germany approximately 65,000 kw. of capacity has been installed, mainly since 1908. These figures were compiled in July, 1913. Practically nothing has been done in the United States toward introducing the motor converter commercially, mainly due to the previous development of the high frequency synchronous converter.

108. Equations of the motor converter.

$$\text{Synchronous speed} = \frac{\text{Line frequency} \times 120}{\text{Sum of poles of both elements}} \quad \text{(r.p.m.)} \qquad (4)$$

$$\text{Rotor frequency} = \text{Line frequency} - \left(\frac{\text{Primary poles}}{120}\right) \text{(r.p.m.)}$$
$$\text{(cycles per sec.)} \qquad (5)$$

$$\text{Power transformed electrically} = \text{Total output} \left(\frac{\text{Secondary poles}}{\text{Total poles}}\right) \qquad (6)$$

$$\text{Power transformed mechanically} = \text{Total output} \left(\frac{\text{Primary poles}}{\text{Total poles}}\right) \qquad (7)$$

When the numbers of poles of both elements are equal, as is usually the case, these relations obviously become very simple.

109. The average armature current and heating in the commutating element is less than in a direct-current generator of equal rating, but more than in the corresponding synchronous converter. Assuming an equal number of poles in the two elements of a motor converter, the commutating element is operating half as a direct-current generator and half as a synchronous converter. Assuming further a twelve-phase rotor circuit and 100 per cent. power-factor at the terminals of the primary element, the average loss will be approximately 0.34 (relatively), the equivalent direct-current generator loss being unity. This is about 30 per cent. greater than the average loss in the ordinary six-phase synchronous converter, also at 100 per cent. power-factor. The maximum loss in one conductor is approximately 10 per cent. less than the maximum loss in the six-phase synchronous converter, at 100 per cent. power-factor.

110. The armature loss increases less rapidly with reduction in power-factor than in the 6-phase synchronous converter, due to the generator current loss, which does not vary with power-factor, and to the higher number of phases employed.

111. Power-factor. The power-factor at the line terminals of the primary element may be varied as in the simple synchronous converter by varying the field excitation of the commutating machine. The magnetizing current for the primary element may be taken from the line or from the secondary element as desired. This is controlled by the excitation of the commutating machine. If the set is operated at 100 per cent. power-factor, it follows that the secondary element is operating at a leading power-factor (with respect to line voltage) in the neighborhood of 93 per cent. at full-load. This has an important bearing on the heating and size of the commutating machine. See Par. 117.

112. Voltage control. A fixed voltage ratio exists between the alternating-current, and the direct-current sides of the commutating element of the motor converter, as in the synchronous converter, and the same means for varying the direct-current voltage must be employed. In a shunt-wound motor converter the direct-current voltage drops with the load assuming constant alternating voltage. In this case the necessarily high reactance of the primary element is a disadvantage. In a certain 500-kw.

motor converter the voltage dropped 5 per cent. between no load and full load. This is less than in a corresponding direct-current generator but more than in a corresponding synchronous converter. The motor converter is well adapted to voltage regulation by reactance and series-field excitation. In general, the primary element will contain inherently sufficient reactance to obtain a 10 per cent. voltage increase (assuming constant primary alternating voltage) with permissible range in power-factor. This range in voltage is ample for the ordinary railway system. Voltage ranges of 20 per cent. to 30 per cent. required by lighting systems using storage batteries cannot be obtained economically by reactance and series-field excitation. For such voltage variation an induction regulator, alternating-current booster or direct-current booster is required. The alternating-current synchronous booster must have the same number of poles as the secondary element and must be connected, electrically, between the primary and secondary rotor windings.

113. Relative size and cost. The cost of the primary or induction element will be approximately the same as that of an induction motor with phase-wound rotor at double the running speed (assuming equal primary and secondary poles). The cost of the secondary or commutating element will be less than that of a direct-current generator of the same rating and more than that of a synchronous converter of the same rating and operating frequency. The cost of the induction element will be somewhat more than that of stationary transformers necessarily used with the synchronous converter. On the basis of a line voltage so low that transformers are unnecessary with the motor converter, the cost without transformers is approximately the same as that of the higher-frequency synchronous converter and the necessary transformers. In case transformers must be used with either form of converter on account of high line voltage, the cost of the motor converter installation will be from 30 per cent. to 40 per cent. greater than the synchronous converter installation.

114. Efficiency. At capacities of 500 kw. and above, the efficiency of a synchronous converter and transformers would be 1.5 per cent. to 2 per cent. better than the motor converter without transformers, assuming American conditions which require the use of open armature slots in the primary element. Including the transformer in the motor converter installation would reduce its efficiency below that of the equivalent synchronous converter installation from 3 per cent. to 4 per cent.

115. The method of starting is the same as in an induction motor with phase-wound rotor, a three-phase resistance controller being used as indicated in the diagram in Fig. 22. It will be noted that, during starting, three phases of the rotor winding of the induction element are in series with the armature winding of the commutating element, the other phases remaining open at the neutral. After the main-line switch is closed, the rotor circuit of the induction element is closed through the starting resistance. As the rotor begins to revolve, two alternating currents are superimposed in the rotor circuits, one at maximum frequency decreasing with the speed, due to the induction element, and one at minimum frequency increasing with the speed, due to the commutating element. The latter current is appreciable only near synchronism or in the case of separate direct-current excitation of the secondary element. As the rotor approaches its normal running speed, these two currents approach the same frequency, which is indicated by a slow oscillation of the needle of the voltmeter connected in the starting circuit. At the moment the voltmeter needle passes through zero, the starting resistance is short-circuited and the set will thereafter operate synchronously. The neutral points of all the rotor phases are connected together and the starting brushes lifted from the rings by a mechanical device. By proper selection of the starting resistance, the starting current may be maintained at a low value throughout the entire starting operation. Due to the small armature current during starting, the direct-current voltage will always build up with the correct polarity. The field reversing switch commonly used with alternating-current self-starting synchronous converters is unnecessary.

116. The motor converter may be used to supply a three-wire direct-current circuit without change except that the brushes are left on the rings after starting. They are connected to the middle terminals of a three-pole double-throw switch, one set of outside terminals being connected to the starting resistance and the other set being short-circuited and connected to the direct-current neutral lead.

117. Commutating poles may be used on the commutating element as in any synchronous converter. Due to the combined generator and converter action the ampere-turns required on the commutating poles more nearly approach the ampere-turns on the equivalent direct-current generator than on the equivalent synchronous converter.

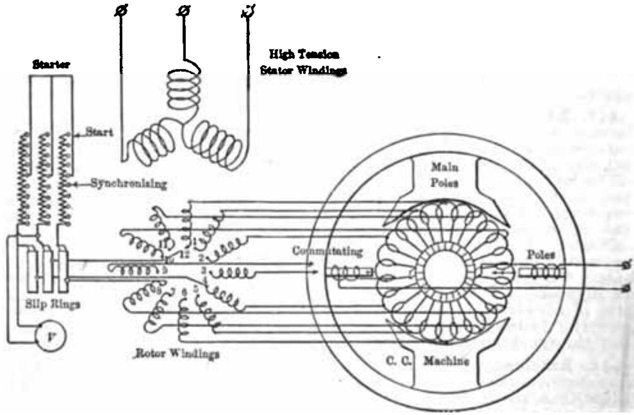

FIG. 22.—Diagram of connections of motor converter.

DIRECT-CURRENT CONVERTERS
BY ALEXANDER GRAY. See p. 2

118. Theory of operation. A three-wire generator, operating with no current in one side of the system, may be used as a direct-current converter with a ratio of two to one. Such conditions of operation are shown diagrammatically in Fig. 23. If the armature be connected across the terminals *a* and *b*, it will rotate as a direct-current motor armature and will take a current i_o, at a voltage $2e$, to supply the no-load losses in the machine. If now two diametrically opposite points *c* and *d* be connected through a coil *C* of high reactance and negligible resistance, and if the middle point *f* of the coil be connected as shown in Fig. 23, the machine will become a three-wire generator (Sec. 8, Par. 193). The point *f* is then midway in potential between *c* and *d* and, since the e.m.f. between *c* and *d* is alternating, an alternating current, supplied from the mains, will flow in the reactance coil. This current will be small since the reactance is large.

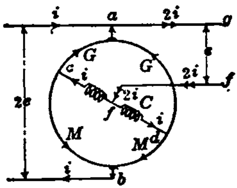

FIG. 23.—Current distribution in a direct-current converter.

A load current at voltage *e* may now be drawn from terminals *f* and *g*, and the current distribution in the system will then be as shown in Fig. 23, the no-load currents being neglected. The currents in sections *M* flow against the generated e.m.f., so that these sections may be considered to act as motors; the currents in sections *G* flow in the direction of the generated e.m.f., and so these sections may be considered to act as generators. The driving torque due to the motor sections must overcome the retarding torque due to the generator sections.

119. Armature currents. The current in the armature conductors varies as the armature revolves; this may be seen from Fig. 24 where the armature is shown in four different positions.

In position *A*, the resistance from *c* to *a* is equal to that from *c* to *b*, and the current *i*, entering the armature at *c*, divides into two equal parts. In position *B*, the resistance from *c* to *a* is less than that from *c* to *b*, and the current i_a is greater than the current i_b.

In position *C*, the current in *ca* is practically equal to *i* and that in *cb* practically zero. In position *D*, the currents in all the conductors are zero.

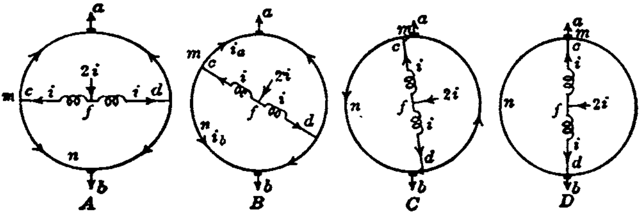

Fig. 24.—Relation between current and position of armature.

Fig. 25 shows approximately how the current varies in two coils *m* and *n* as the armature makes one revolution. In coil *m*, the current varies from zero to *i* while in coil *n* it varies from $-\frac{1}{2}i$ to $+\frac{1}{2}i$; the armature coils are therefore not equally heated. Those next the leads *c* and *d* are the hottest and those midway between the leads are the coolest.

120. The rating of a direct-current converter depends upon the number of phases of the reactance coil. With a 2-phase reactance coil, as shown in Fig. 26, the maximum current in the conductors is less than *i*, and the current in each armature coil is more nearly constant than when only one reactance coil is used. For the same average armature copper loss in each

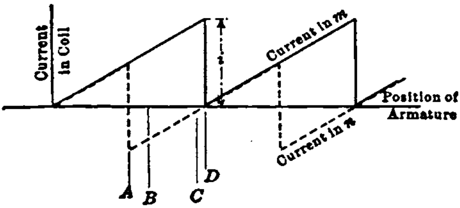

Fig. 25.—Variation of current in coils *m* and *n* during one revolution of armature.

case, a direct-current generator can be given the following ratings when used as a direct-current converter:*

1.00 as a direct-current generator;
1.22 as a direct-current converter with one reactance coil;
1.55 as a direct-current converter with two reactance coils;
1.64 as a direct-current converter with three reactance coils.

The cost and weight of the equivalent direct-current generator may be obtained from Fig. 67 (Par. 178) Sec. 8, an addition of 5 per cent. being made for the slip-rings.

121. The secondary voltage of such a system cannot readily be controlled independently of that of the primary; it decreases with increase of load according to Eq. 38 in Sec 8, Par. 194 and, with a reasonably priced

* Steinmetz, C. P. "Elements of Electrical Engineering," p. 337.

reactance coil, this decrease in voltage can hardly be less than 3 per cent. at full load.

122. The rating of the necessary reactance coil may be found from Sec. 8, Par. 197. The frequency, which equals the number of poles multiplied by the r.p.m. and divided by 120 will generally be between 15 and 40 cycles per sec.

123. The Dettmar and Rothert split-pole machine (Sec. 8, Par. 192) when used as a direct-current converter has the advantage that the secondary voltage can be controlled independently of that of the primary. Each pole of this machine is split so as to form two polar projections of like polarity and these are excited independently of one another. The neutral brush is placed on the commutator midway between the positive and the negative brushes and, by varying the excitation of one polar projection relative to that of the other, the flux entering the armature between the positive and the neutral brushes may be changed relative to that between the neutral and the

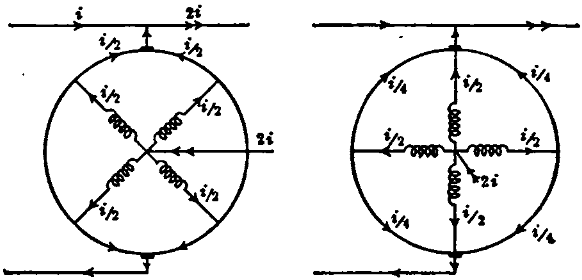

FIG. 26.—Current distribution in a direct-current converter with a two-phase reactance coil.

negative brushes, or the potential of the neutral brush may be varied as desired. By the use of suitable shunt and series coils on the polar projections, any desired degree of compounding may be obtained and, in the extreme case, a constant current may be obtained from the secondary of such a machine while the primary is operating at constant potential.

124. The C. M. B. autoconverter is another type of split-pole machine with the armature tapped by brushes placed between the main positive and negative brushes. A standard line of these machines is on the market and can be supplied with a transformation ratio as high as four to one, and with such a combination of shunt-field and series-field coils that a drooping secondary characteristic may be obtained if desired for constant-current operation. They are supplied with ball bearings and have an efficiency which varies from 75 per cent. for a 1 kw. unit to 86 per cent. for a 10 kw. and 94 per cent. for a 50 kw. unit.

125. Applications.* Direct-current converters of this latter type are used in place of motor-generator sets for many purposes, among others, to obtain 50 volts from a line of higher voltage for the operation of arc lamps at 50 volts in bioscope sets and search lamps. A constant-current characteristic is the most suitable for arc-lamp operation. They are also used for electric welding, the machines being built with such characteristics that, on short-circuit, the current is only 10 per cent. more than full-load current, while the voltage drops practically to zero.

* For further information on direct-current converters see:
Steinmetz, C. P. "Theoretical Elements of Electrical Engineering," page 337; New York, McGraw-Hill Book Co., Inc.
MacFarlane and Burge. *London Electrician*, July 9, 1909.
Thompson, S. P. Report at Turin Congress; *London Electrician*, Sept. 29, 1911.

DYNAMOTORS
BY ALEXANDER GRAY

126. General description. The dynamotor is similar to the motor-generator set except that the two armature windings are on the same core and revolve in the same magnetic field, each winding being connected either to a commutator or to collector-rings, in accordance with its use for direct or for alternating current respectively. It is rarely necessary to change from direct current to alternating current, so that machines for such purpose are special. A change from alternating to direct current may be realized with less expense by the use of a rotary converter, with a transformer if necessary, than with a dynamotor.

127. Armature reaction. The armature for a direct current to direct-current type of dynamotor is shown diagrammatically in Fig. 27. Here the transformation ratio is four to one, so that there are four times as many effective turns in one winding as in the other. The direction of the generator current is that of the generated e.m.f., but the current in the motor winding is in the opposite direction. If the losses in the machine be neglected, then the

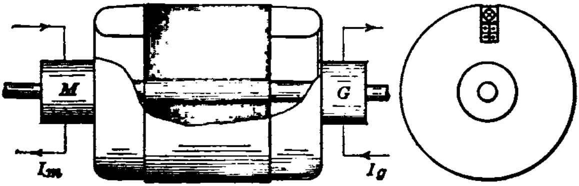

Fig. 27.—Arrangement of dynamotor armature and commutator.

motor input, E_mI_m equals the generator output, E_gI_g, and the value of effective ampere-turns in the motor armature, n_mI_m equals the value of effective ampere-turns in the generator armature, n_gI_g. Therefore the m.m.fs. of the two windings are equal and opposite and there is no resultant armature reaction. For this reason, the dynamotor can be allowed a smaller air gap, a smaller exciting current, and a lighter field coil than a standard machine built upon the same frame.

128. The commutation of these machines is exceedingly good if the brushes on the two commutators are so placed that the short-circuited coils of each of the two windings lie in the same slot. Under such conditions the currents in these coils change at the same time and in opposite directions, so that the reactance voltage is small (Sec. 8, Par. 43). Dynamotors can therefore have deep slots and low-resistance brushes and in consequence are suitable for delivering large currents at low voltages.*

129. Voltage regulation. The principal objection to the standard dynamotor is that the secondary voltage cannot be regulated without changing the excitation of the primary. The ratio of the terminal voltages is fixed by the ratio of effective armature turns and by the voltage drop due to the resistance of both armature windings, so that the terminal-voltage ratio is independent of the excitation and changes with the load. The transformation ratio for a given load can be changed only by inserting a resistance or a booster in the primary or the secondary circuit.

130. Combined dynamotor and booster. A machine which is equivalent to a combined dynamotor and booster can be made by extending one armature winding a longer distance axially along the core than the other, and applying to this extended part of the core a magnetic field from an auxiliary field system. The exciting current for this auxiliary field can be so regulated as to change the voltage of one winding independently of that of the other. If this field system be excited with series-field coils, the secondary voltage can be made to vary with the load, as desired. The chief advantage of this latter machine over the motor-generator set is that it requires less

* Sheldon and Hausmann. "Dynamo-electric Machinery," Vol. I, page 266.

floor space and will have a higher efficiency, but will probably not be cheaper if the machines are of corresponding construction.

131. A special dynamotor is extensively used for telephone ringing. The motor winding is for direct current and the other winding is connected to two slip-rings in order to deliver alternating current at from 16 to 19 cycles per sec., and about 75 volts (effective).

132 Dynamotors can be used in place of motor-generator sets only when the regulation of voltage is not of great importance as, for example, in the ringing of bells and gongs, the operation of signals in connection with fire alarms, telephone systems, annunciators and many other kinds of signaling, and the operation of magnetic contactor switches. For telegraphic work, dynamotors are often used in place of primary batteries, the secondary voltage being 50 to 500 volts direct current, depending on the length of the line (Sec. 21).

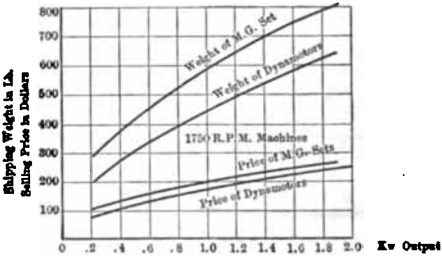

FIG. 28.—Weights and costs of small dynamotors and motor-generator sets.

133. Cost of Dynamotors. Since the two armature windings are upon the same core, that core will be larger for a dynamotor than for a generator of the same output. Fig. 28 shows the shipping weights and selling prices of small dynamotors and motor-generator sets of substantial make on a single bed plate, the motor-generator set consisting of two machines joined by a flexible coupling.

DOUBLE-CURRENT GENERATORS

BY F. D. NEWBURY, M.E.

134. Armature currents in the double-current generator. A machine having the same structure as the synchronous converter may be used to generate both alternating and direct current if driven by a suitable prime mover. Unlike the synchronous converter, the armature currents in the double-current generator are not less than in the corresponding direct-current generator. The alternating and direct-current are not subtractive, as shown in Figs. 1 and 2, but are additive, since both are generator currents. For the same reason, the generation of both alternating and direct current in the same armature winding is not favorable to commutation as in the synchronous converter, but is detrimental.

135. Dependance of direct-current voltage on alternating voltage. A double-current generator has none of the advantages of the synchronous converter and has many disadvantages of its own. One of the most serious disadvantages is the dependence of the direct-current voltage on the alternating voltage and on variations in alternating-current load and power-factor. A change in alternating-current load or power-factor will cause a change in the magnetizing effect of the alternating current in the armature winding and thus change the resultant magnetization on which the direct and alternating voltages depend. To assist in maintaining steady direct-current voltage, it is customary to excite double-current generators separately.

136. Alternating-current parallel operation. If double-current generators are operating in parallel with a relatively large alternating-current system on which the alternating voltage is held constant, the direct-current voltage will also be constant and cannot be varied independently of the constant alternating voltage.

137. Limitations of design. From the design standpoint, the double-current generator is handicapped by the rigid relationship between poles and speed required by the frequency. This, in general, results in the use of many more poles than would be employed in an equivalent direct-current generator, causing an appreciable increase in cost. This is particularly true with slow engine speeds and high frequency. It is also impossible to use commutating poles on account of the effect of variation in alternating-current load and power-factor. No machines larger than 2,500 kw. have ever been built in this type.

138. Abandonment of slow-speed type. During the past 10 years, no slow-speed engine-driven double-current generators of any importance have been installed. The reasons for this have been the disadvantages of the double-current generator already enumerated and the lower cost and greater flexibility of alternating-current generators used in conjunction with synchronous converters.

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SECTION 10

POWER PLANTS

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CONTENTS

(*Numbers refer to Paragraphs*)

SECTION 10

STEAM POWER PLANTS

BY REGINALD J. S. PIGOTT

LAWS OF HEAT TRANSFER

1. No heat can be transmitted from a cold body to a hotter one: flue gases must, therefore, leave the boiler at not less than the steam temperature in an ordinary boiler, or at not less than the incoming feed-water temperature in the counter-current type of boiler.

2. Two kinds of heat transfer take place in a boiler; absorption by radiation from incandescent fuel direct to heating surface, and heat transmission by contact and convection. Stefan and Boltzmann's law states that absorption by radiation is proportional to difference of the fourth powers of the two temperatures involved.

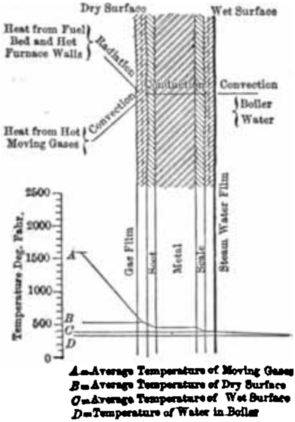


FIG. 1.—Transmission of heat.

B.t.u. radiated per sq. ft. per hour $= 16 \times 10^{-10} (T_1^4 - T_2^4)$ (1)

where T_1 = absolute temp. in deg. Fahr. of "black" incandescent body, and T_2 = absolute temp. in deg. Fahr. of receiving body.

A "black" body such as carbon, in any form except the crystalline, will absorb all heat transmitted, reflecting none. Metal and other materials will reflect from one-fourth to one-half of that received.

3. Heating by convection from hot gases to water is proportional to some power of each of the velocities of the moving gases and water or steam. The heating is also proportional to the density of the fluids and to the absolute temperatures, since these influence the molecular energy. The formula for heat transfer by convection is,

$$H = AU(T_1 - T_2) \tag{2}$$

where H = B.t.u. transmitted per hr., A = area, sq. ft., U = transmission coefficient, $T_1 - T_2$ = mean temp. diff.

4. The transmission coefficient for normal rating is somewhere around 5 to 8 B.t.u. deg. of mean temperature difference, per sq. ft., per hour, but may be much larger than this in boilers with much surface exposed to radiant heat and well scrubbed by the gases. The upper limit has probably never been reached, the maximum value for recorded tests being about 20.

5. Effect of dirty surfaces. One important difference noticeable between absorption of heat by conduction and by radiation is that the relative cleanness of the surface affects the rate of conduction very readily, but absorption of heat by radiation is practically unaffected. Soot increases the thickness of the film of motionless gas, besides adding its own resistance to heat flow.

6. The resistance of the film of inert gas entangled in the rough surface of the tube (or soot) is from 20,000 to 35,000 times as great as that of the metal tube alone, so that the thickness of the dead gas film is the controlling

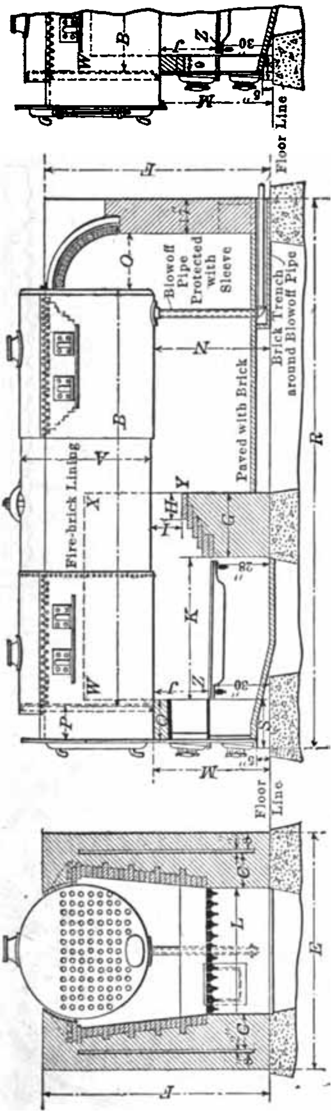
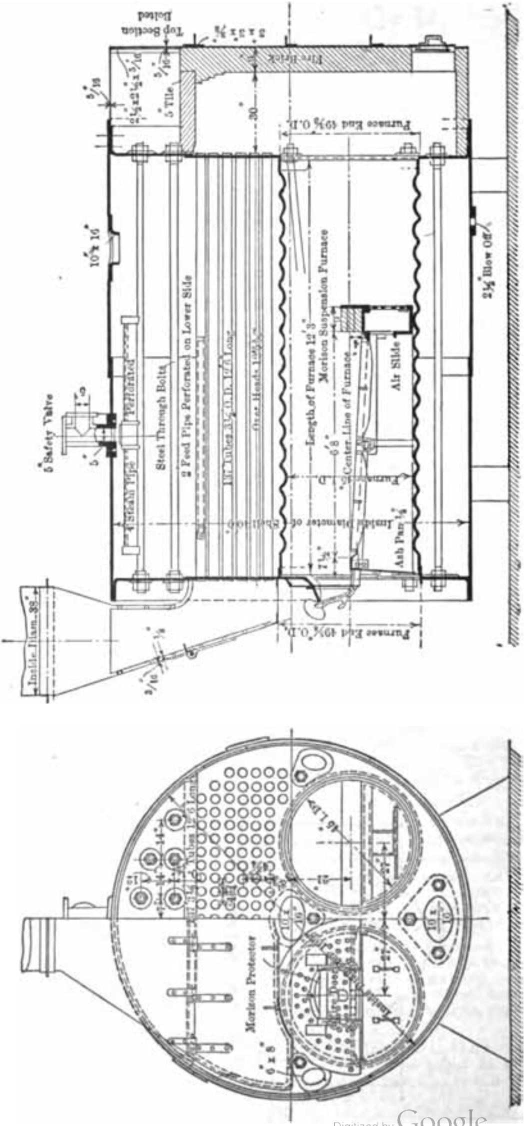


Fig. 2.—Horizontal return tubular boiler.



Longitudinal Section Elevation

Front Exterior Elevation

FIG. 3.—Scotch marine boiler.

factor. In absorption of heat by radiation, however, the interposition of a gas film or layer of soot has practically no effect, since conduction does not bear a part in the action.

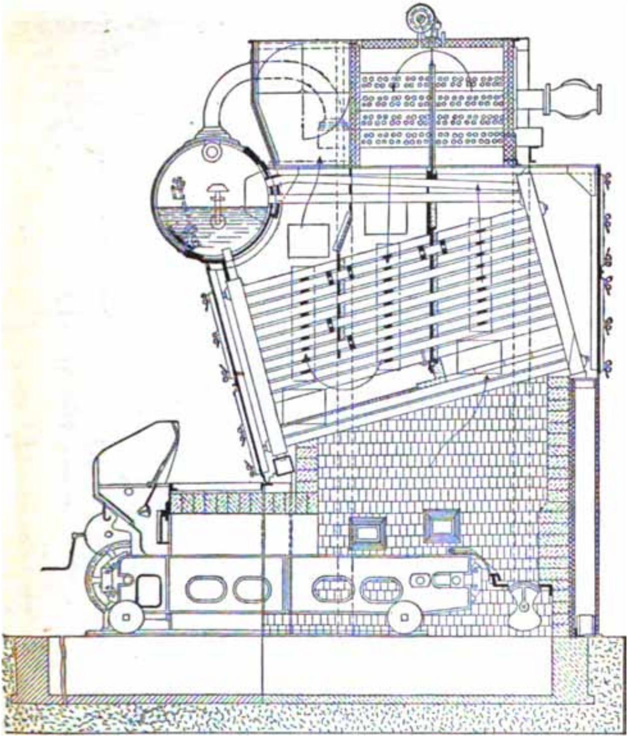


FIG. 4.—B. & W. marine boiler with superheater.

BOILERS

7. Boiler types: in the following classification modern types only are considered.

- Fire Tube...
  - Horizontal return tubular (Fig. 2).
  - Scotch (Fig. 3).
  - Continental and other internally fired types (Fig. 3).
- Water Tube.
  - Definite circulation path.
    - B. & W.......
      - Rust
      - Stationary (Fig. 5).
      - Marine (Fig. 4).
      - Stirling (Fig. 6).
    - Heine, etc., etc., and 7
    - Yarrow
    - Thorneycroft.
  - Field Tubes............Niclausse

All types give efficiencies not differing widely when subjected to the same kind of furnace and flame conditions, the structure having almost no inherent influence upon efficiency. But inasmuch as the structure may influence very materially the conditions of combustion, by restricting combustion spaces, by altering the length and the hydraulic mean depth of

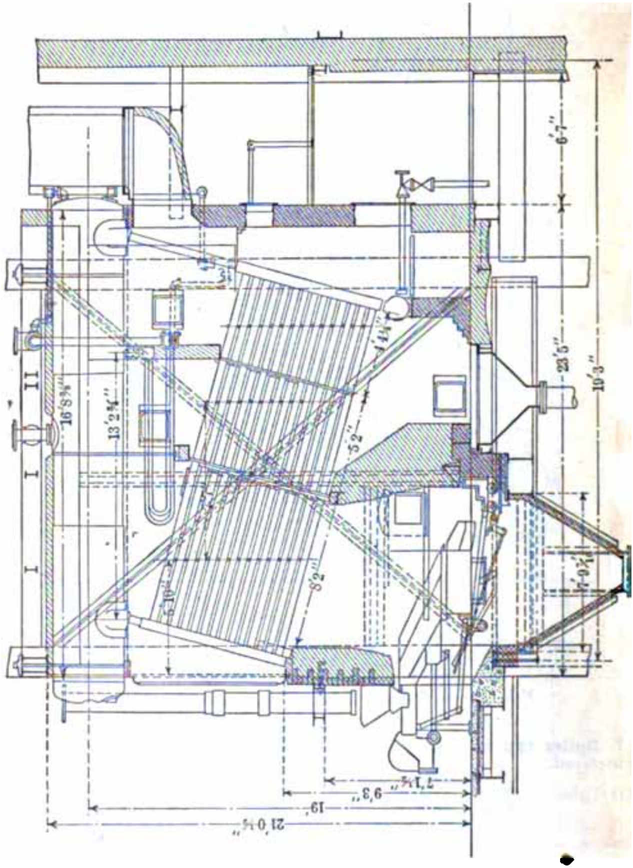


Fig. 5.—B. & W. stationary type with superheater.

gas passages, and by extinction of flame and accumulation of dirt, it may alter the efficiencies somewhat. If combustion could be entirely complete, so that the flame would be extinct before reaching any heating surface, it would make no difference what type of boiler were employed so far as efficiency is concerned.

8. Fire-tube boilers are cheapest in first cost, but lowest in capacity for space occupied; in general they are the most difficult to keep in good operating condition, especially with dirty water, since the water space is not accessible. The internally fired boilers such as the Scotch and the Continental (or similar) types are somewhat superior in capacity, but not up to the water-tube type. As a class, fire-tube boilers, from the standpoint of safety, are least desirable, as they carry a large body of hot water, storing considerable energy to be liberated at the instant of rupture.

9. Water-tube boilers are much lighter and smaller per unit of capacity, and higher in first cost than tubular boilers, but easier to maintain inasmuch as the water surface is all practically accessible, either for hand cleaning or for

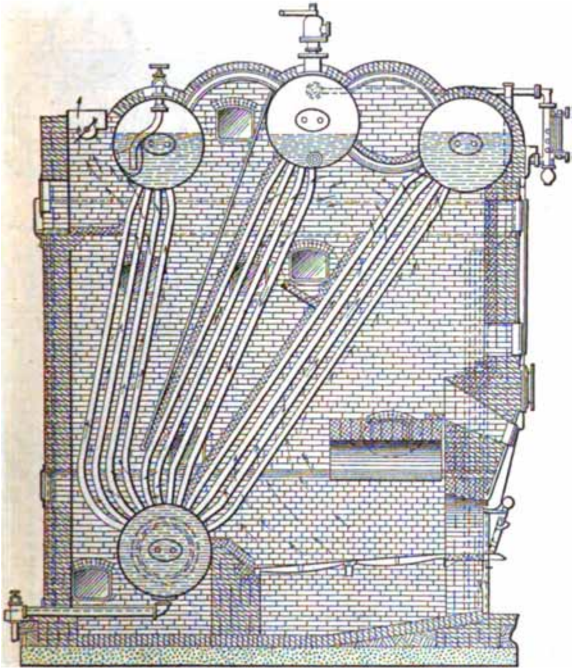


Fig. 6.—Stirling boiler.

turbine tube cleaners. They are very safe, having very small water storage (subdivided into many smaller bodies by the tubes), and are made up almost entirely of the strongest natural shapes—cylindrical—with little or no stayed surface. There are practically no records of serious accidents resulting from the explosion of water-tube boilers.

10. Field-tube boilers are almost unused except for the French marine boiler known as the Niclausse. The principal objection to their use is the uncertain and spasmodic nature of the circulation which may partake of the nature of flash generation, alternating with periods of flooding with water, causing strains due to rapid and unequal expansion and contraction.

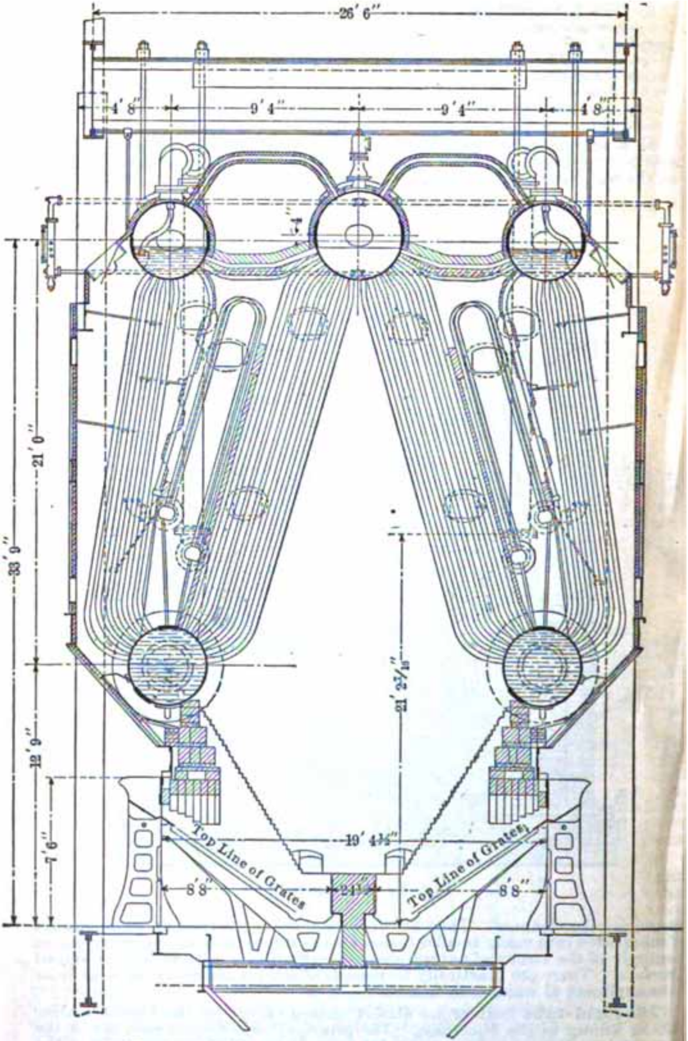


FIG. 7.—Stirling double-end boiler. Delray type.

11. Flash generators are at present unused except for steam automobile boilers and some torpedo boats. In American power stations they are unknown. However, if any standard boiler, such as the B. & W. or Stirling, be forced much above its normal rated capacity, the generation, in the lower rows of tubes at least, becomes of the flash type by reason of the extreme conditions.

12. Superheaters are of two general types: self-contained, and separately fired. The separately fired superheater is unpopular in America, chiefly because it has not shown such pronounced economies as the self-contained type. The two types of self-contained superheater are the plain tube and the protected tube types. To the former class belong the Heine, the B. & W. (Fig. 5) and the Stirling; to the latter class belong the Foster (Fig. 8) and the Schwörer.

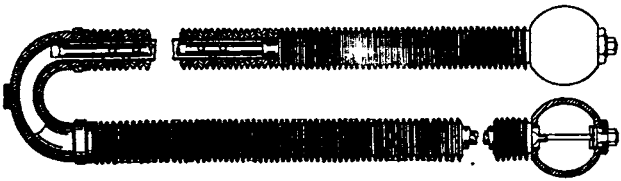


Fig. 8.—Foster superheater.

13. Superheaters may be located (usually) between the first and the second passes, or in the breeching. With later boilers of the ordinary stationary types the first position is popular. With the marine types now sometimes used in power stations, the latter position is satisfactory, as the gases leave the boiler at a higher temperature (see Marine B. & W. boiler, Fig. 4).

14. Heating surface is defined as any surface in a boiler or superheater which is exposed to hot gases on one side and water or steam on the other. In water-tube boilers the heating surface is defined as

$$\frac{\pi dln}{12} + \frac{\pi DLN}{24} \quad \text{(sq. ft.)} \qquad (3)$$

where d = outside diameter of tubes in in. l = length of tubes in ft., n = number of tubes, D = outside diameter of drums in in. L = length of drums in ft. and N = number of drums, headers not included.

15. Rating of superheaters. Superheaters are not usually rated by sq. ft. of surface, but are figured for each case. A good average rule is 15 sq. ft. of heating surface per boiler h.p., for unprotected tubes; 18 sq. ft. of heating surface per boiler h.p., for protected tubes. The reason for using larger figures for superheaters than for steam-making surface is that heat transmission is much more difficult between gas, iron and gas, than between gas, iron and water, or between gas, iron and vapor, the latter two conditions obtaining in the boiler proper. The protected tubes interpose an additional resistance to heat transfer because of the cast-iron protection rings, the roughness of which also materially increases the thickness of the dead gas film.

16. Grate surface is any surface upon which coal is being burned. In hand-fired furnaces this amounts to the horizontal dimensions of the furnace, side wall to side wall, and front wall to bridge. With the automatic stoker, the term becomes more or less anomalous, since part of the surface may be used exclusively for coking, and the grates may not be horizontal; and part of the grates may be used only for accumulating clinker and ash. With underfeed stokers, coal is not burned upon the grates to any appreciable extent. The term is useful for comparison, if the projected area of the furnace is used in each case.

17. The ratio of heating surface to grate surface was once considered important and was generally made 60 to 1 when good coals were used, but now not much attention is given it. This ratio shows a tendency to decrease to 30 to 1 or 40 to 1 recent installations, particularly with stokers intended for forcing to high capacities, or where low-grade fuel such as culm, or No. 3 buckwheat is burned.

18. The value of the boiler h.p. is at present 34.5 lb. of water evaporated from and at 212 deg. Fahr., per hour; this is equivalent to 33,479 B.t.u. per hour. A new unit recently proposed to eliminate this somewhat illogical unit, is the **myriawatt**,* a multiple of the watt, having a value of 34,150 B.t.u. per hour, about 2 per cent. larger than the boiler h.p.

19. Evaporation. Taking dry coal, the actual evaporation per lb. is,

$$E = \frac{W}{w} \tag{4}$$

$$E' = EF \tag{5}$$

$$F = \frac{H - h}{970.4} \tag{6}$$

E = actual evaporation
E' = equivalent evaporation
F = factor of evaporation
H = total heat of steam at boiler pressure and quality.
h = heat in the feed water = feed temperature − 32.
W = lb. steam per hr.
w = lb. dry coal per hr.

970.4 = latent heat at atmospheric pressure = heat to convert 1 lb. of water to steam, from and at 212 deg. Fahr.

Or the above evaporation may be used "per lb. of combustible," substituting lb. of combustible per hour in place of lb. of dry coal per hour. All temperatures are expressed in deg. Fahr., and all pressures in lb. per sq. in., and all heat in B.t.u.

20. Losses in boilers comprise: (a) sensible heat escaping in flue gases; (b) sensible heat lost in hot ashes; (c) radiation of heat to external air from boiler setting; (d) incomplete combustion; (e) excess air. Theoretically some of these losses are avoidable. Actually, about the following proportions hold good in recent practice.

Heat in coal	100 per cent.
Heat absorbed in steam (overall efficiency)	70 to 76 per cent.
Stack loss, sensible heat only	10 to 15 per cent.
Incomplete combustion	5 to 10 per cent.
Hot ashes	Negligible
Radiation and leakage	6 to 8 per cent.

21. The combustion efficiency of automatic stokers runs as high as 96 to 97 per cent. and is maintained up to a very high rating in some cases. That of a hand fire is seldom over 90 to 92 per cent. and generally much less due to inevitable carelessness in operation.

22. Radiation is kept down by thorough lagging of exposed parts and by the usual double-walled construction of boiler settings (Figs. 5 and 6). It does not vary greatly for any type of boiler, but is reduced somewhat by setting in batteries instead of singly.

23. Excess air is caused by the slight vacuum in the setting due to draft; it can be reduced by painting or white-washing the setting and stopping up all cracks between metal and brick with plastic asbestos. In some cases encased brick or sheet-iron settings are used, but the expense is not generally justified from this point of view alone. Another method of reducing **setting infiltration** is by the use of balanced-draft systems; in these the methods of forced draft, and natural or induced draft are so employed as to give practically zero pressure in the fire-box, by this means reducing the average difference of pressure from atmosphere, over the whole setting.

24. Furnace efficiency is expressed by the formula

$$\text{Furnace efficiency} = \frac{\text{Combustible burned}}{\text{Combustible fired}}$$

Combustible burned = (Combustible fired) − (Combustible in ash) − (Combustible in unburned gas) − (Combustible in soot).

* Stott and O'Neill. p. 411, *Trans.* A. I. E. E., Vol. 32, 1913.

Combustible fired = (lbs. dry coal) × (B.t.u. per lb. [by calorimeter]) (9)

Combustible burned = (combustible fired) − (B.t.u. per lb. of ash) × (lbs. ash) − (B.t.u. per lb. of soot) × (lb. of soot) − (B.t.u. in unburned gases). (10)

The last item is derived from the flue-gas analysis and as it always involves several assumptions, its accuracy is not very certain.

25. Boiler efficiencies. Fig. 9 gives a series of efficiency curves for various sizes and makes of standard boilers in central station service. The actual results in operation usually fall from 3 to 5 per cent. below test figures, the smallest falling off being noticed in those stokers which require the least hand manipulation during operation.

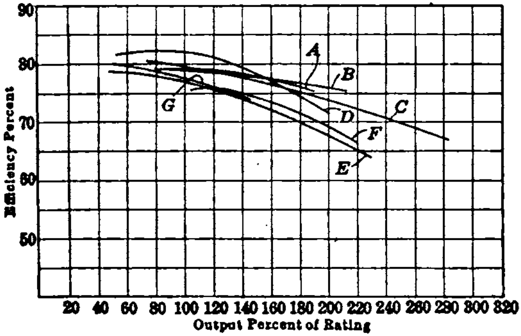


FIG. 9.—Boiler efficiency.

Combined efficiency, boiler and furnace: *A*, 2,365 h.p. boiler, Taylor stoker D. E. Stirling; *B*, 2,365 h.p. boiler, Roney stoker D. E. Stirling; *C*, 520 h.p. boiler, Taylor stoker B. & W.; *D*, 750 h.p. boiler, Taylor stoker B. & W. (part 2-in. tubes); *E*, 600 h.p. boiler, Green chain grate, B. & W.; *F*, 520 h.p. boiler, Westinghouse-Roney stoker B. & W.; *G*, 1,000 h.p. boiler, Taylor stoker B. & W.

26. Combined furnace and boiler efficiencies. The table below gives the pounds of water evaporated per pound of coal, and the pounds of coal required per boiler h.p. hr. with coals of various heat content and with boiler installations having varying combined boiler and furnace efficiencies.

	50 %		60 %		70 %		80 %	
B.t.u. per lb. dry coal	Evaporation per lb. coal	Lb. Coal per boiler h.p. hr.	Evaporation per lb. Coal	Lb. coal per boiler h.p. hr.	Evaporation per lb. coal	Lb. coal per boiler h.p. gr.	Evaporation per lb. coal	Lb. coal per boiler h.p. hr.
8,000	4.142	8.33	4.971	6.94	5.799	5.96	6.627	5.20
9,000	4.660	7.41	5.592	6.18	6.524	5.28	7.456	4.63
10,000	5.224	6.60	6.269	5.50	7.314	4.72	8.359	4.13
11,000	5.695	6.06	6.834	5.05	7.973	4.33	9.113	3.79
12,000	6.213	5.56	7.456	4.63	8.698	3.97	9.941	3.47
13,000	6.731	5.12	8.077	4.27	9.423	3.66	10.769	3.20
14,000	7.249	4.76	8.698	3.97	10.148	3.40	11.598	2.98
14,500	7.508	4.60	9.009	3.83	10.511	3.28	12.012	2.87

Efficiencies at head of columns are combined boiler and furnace efficiencies. Evaporation is given in pounds of water "from and at" 212 deg. Fahr. per lb. of dry coal. The heating value can also be taken for coal "as fired" whence the "evaporation per lb. of coal" and the "pounds coal per boiler h.p. hr." will also be referred to coal "as fired." This latter use of the table will contain a slight error due to the absorption of a portion of the heat by the moisture in the coal.

27. The failure of a furnace to burn all of the combustible is due to two principal causes: (a) molecules of oxygen are not brought into contact with all the particles of combustible material; (b) the temperature may be too low for ignition when they are in contact. The first condition is found where some of the fixed carbon in the lumps of coal becomes surrounded by a coating of fused ash, so that the air never reaches it during the time that the lump is upon the grate. Another cause, in the case of bituminous coal, is the lack of proper mixture of the volatile matter with air. The second condition is found as a result of the first, if streams of stratified hot combustible gas and air leaving the furnace are brought into contact with the comparatively cold heating surface, they will quickly be chilled below the ignition point, and will sweep through the boiler without further combustion taking place.

28. Water-tube boilers are generally rated on one boiler h.p. for each 10 sq. ft. of heat surface; **Scotch marine types** on 8 sq. ft., and **horizontal return-tubulars** on 12 sq. ft. **Superheater surface** is usually based on a transmission coefficient of 5 to 8 B.t.u. per deg. of mean temp. difference per sq. ft., per hour.

An empirical formula given by Bell (Trans. A. S. M. E., May, 1907) is, Sq. ft. superheating surf. per boiler h.p. =

$$S_s = \frac{10T_s}{2(T_g - T_o) - T_s} \tag{11}$$

The temperature of gases at the superheater may be found from the following relations:

S_s = superheater surface per b.h.p., sq. ft.
S_a = per cent. of total surface passed over by gases before reaching superheater.
T_s = superheat, deg. Fahr.
T_o = saturated steam temp., deg. Fahr.
T_g = temp. of gases at superheater, deg. Fahr.

$$(0.172S_a + 0.294)(T_g - T_o)^{0.16} = 1 \tag{12}$$

These two formulæ are based on the following assumptions: constant coefficient of heat transmission; furnace temperature 2,500 deg. Fahr.; flue temperature 500 deg Fahr.; steam pressure 175 lb. gage; one boiler h.p. equivalent to 10 sq. ft. of heating surface.

29. Boiler design. Since the efficiency of boilers is so little influenced by the disposition of heating surface, comparatively little original designing can or should be done outside of standard forms. For central-station work, the water-tube boiler has practically the whole field, the specifications for a boiler therefore look more to the safety and reliability, by rigorous attention to materials, than to the actual detailed design. The management of the furnace and combustion have much more to do with the efficiency of a boiler, than the arrangement of heating surface.

30. Boiler settings have become more or less uniform in general type. For the water-tube boilers, a steel structure or skeleton for supporting the boiler proper, the brick walls being merely self supporting, is now usual practice. Common red brick is used for the outer walls, a double-wall construction being employed to allow for the differential expansion of inner and outer surfaces (on account of the great temperature difference), to minimize radiation by interposing dead air space, and to reduce air filtration. Wherever the temperature exceeds 1,000 deg. Fahr., fire-brick is employed for this inner lining.

31. The baffles are always of fire-brick, of various special shapes, sometimes backed or supported on the cooler side by cast-iron bars or plates. Figs. 5 and 6 show these constructions.

32. The setting for tubular boilers, generally supports the boiler; for internally fired boilers, no setting is required (see Figs. 2 and 3).

33. The computation of superheater surface is much complicated by the change of temperature of the flue gases at the superheater with load on the boiler, and the uncertainty of the transmission coefficient. The amount of superheater surface installed for superheats of 100 deg. to 175 deg. Fahr. varies from 13 to 20 per cent. of the boiler heating surface. Fig. 10 gives results of actual tests.

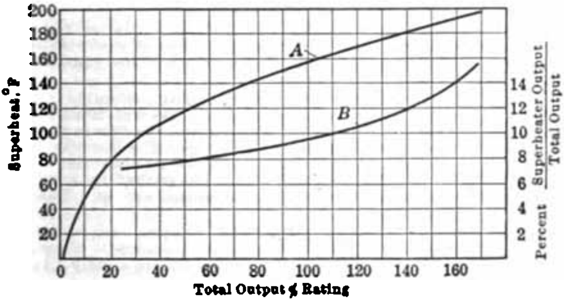


FIG. 10.—Superheater variation with output.

34. Uptakes are usually allowed 0.03 to 0.04 sq. ft. of cross-section (flue area) per rated h.p. of boilers attached. Where the uptakes are very long this allowance should be increased, or the loss of draft will be excessive. Where economizers are employed, the flues are best made of brick and combined with the economizer setting. Otherwise sheet-steel ducts, lagged with asbestos or other insulating material, are cheaper and more convenient. In nearly all cases the uptakes proper are made of steel. It is not necessary to line these with fire-brick unless the flue temperatures exceed 700 deg. a large part of the time; with properly operated boilers this condition need not occur. Bends in uptakes should be avoided, if possible, but where necessary should be of large radius; sudden changes of section should also be avoided.

35. Flue areas are based on a normal velocity of 25 to 40 ft. per second; maximum velocity, 60 to 70 ft. per second. The breeching area of water-tube boilers is about 0.03 to 0.045 sq. ft. per rated boiler h.p. It is best to be generous with areas wherever possible so as to keep the draft loss low.

36. Safety valves of the **pop type** are now always used, the weighted-lever type being prohibited by law in most cases. There are two principal classes: **low lift,** in which the rise of the valve is from 0.04 in. to 0.08 in.; and **high lift,** in which the rise is 0.12 in. to 0.16 in. The high-lift valves have been much more widely used in the last few years. Their principal advantage is greater relieving capacity; their principal disadvantage is possible slamming of the valves and destruction of the seat thereby; and, with dirty or hard boiler water, violent priming and throwing of water from the valve.

37. The rule of the U. S. Board of Supervising Inspectors of Steam Vessels in regard to safety valves is expressed as follows:

$$A = 0.2074 \left(\frac{W}{P}\right) \qquad (13)$$

where A = safety-valve disc area, in sq. in., per sq. ft. of grate area under boiler,

W = evaporation, in lb. of steam per hr. per sq. ft. of grate surface,

and P = absolute pressure, in lb. per sq. in. at boiler.

This is based on an assumed ratio of lift to diameter of valve of $\frac{1}{32}$ and a 45 deg. bevel seat.

38. Connection of safety valves. There must be **no valves between the safety valve and the boiler connection**; seats and valve discs must be of such material as will not rust together; no valves are allowed in the discharge from safety valves; in fact it is preferable to have only a muffler on the valve, and no discharge pipe at all.

39. Gage glasses of the ordinary type should be connected, preferably, directly to the steam drum and protected by wire gauze if near the level where firemen or others are working. The Klinger type of gage glass with heavy corrugated plate glass and brass casing is much safer, but more costly; the water level is much more clearly distinguished.

40. Quick-closing gage cocks operated by chain or rod are now almost universally used. The gage glass and blow-off cocks should never be so arranged that the fireman must stand close to the gage glass when "blowing down."

41. Manholes provided for entry to boiler drums, preferably in the heads where they least affect the strength, are made of pressed plate steel and fitted from the inside of the drum against a bumped lip or saddle in the drum by means of one or two bolts made fast to a dog spanning the opening on the outside. The pressure inside the boiler keeps the manhole tight, the bolts being used only to pull the cover to a seat. The **gasket** between manhole cover and seat is of asbestos, lead alloy or corrugated copper. The standard dimensions are 11 in. × 15 in.

42. Handholes are placed in mud-drums, headers, etc., which are too small to admit the body, but must be cleaned and inspected. They are made in the same manner as manholes, but the dimensions are usually 4 in. by 6 in. or 6 in. by 8 in.

43. The blow-off should be connected to the lowest and quietest place in the boiler, where the mud and scale settle, or from specially designed chambers in which the sediment is induced to collect. The blow-off pipe must be protected from the fire, as it has normally no circulation of water to protect it. Many installations have been made with a plug cock next the boiler, followed by a gate valve to prevent leakage. There are now, however, one or two successful valves on the market which do not require the additional gate valve.

44. The feed pipe should enter the boiler so that the flow is in the direction of natural circulation in the boiler. Outside the drums the feed pipes are generally of brass to eliminate the chance of a break at this point from corrosion. A **check valve** is fitted at the entrance to each drum, to prevent water flowing back.

45. The steam gage is connected to the steam space of the boiler, to prevent blocking of the pipe by scale or mud. A **condensing coil** should be applied to prevent hot water or steam from getting into the Bourdon tube of the gage. **Thermometers** are practically never used as a measurement of steam pressure, except on tests, but are much more reliable.

46. Pyrometers are used for measurement of flue gas temperature, leaving the boiler, and are very valuable aids in checking the operation and efficiency.

47. Safety plugs are made of pure Banca tin, and are so placed that the active fire will play upon them, or at least in the first pass through the tubes. They are placed about 10 or 12 in. above the danger limit of low water; they are ordinarily protected by the water, but melt in case the level falls below them. The fire side of the plug must be kept clean of soot, or it may become so protected as to be inoperative.

48. Boiler corrosion is caused either by direct oxidation of the iron by **oxygen** dissolved in the water or by reaction with some **acids** present in the feed-water, having their source either in national acidity or from the products of decomposition of impurities introduced into the feed. Corrosion by oxidation is the more important, as the other is always avoidable by proper purification of feed water. Oxidation goes on continuously wherever dissolved air is present, and is much increased by the vibration or flexure of any particular point. The constant bending breaks off the film of iron oxides formed by the process, and exposes new surface to be attacked; for this reason pitting is commonest near joints where the flexure of some one portion of a plate is apt to be severe.

49. Incrustation is caused by the deposit of solid matter during the heating and boiling of the water. It may be merely a mechanically entrained impurity, such as mud and sand, or it may be the result of precipitation of various salts insoluble in hot water, or by super-saturation. The **salts** of magnesia, lime and iron are those giving most trouble in boiler water. Some of these salts are soluble and cause the formation of scale by super-saturation; others are insoluble in hot water and precipitate as the water is heated.

50. Treatment of feed water. The principal salts are iron, calcium and magnesium carbonate, bicarbonates, and sulphates. The bicarbonates are usually decomposed by boiling. The general method of treatment is by adding lime (CaO), and sodium carbonate (Na_2CO_3) called in the impure commercial form, soda ash.

51. Removing the precipitation. Where the character of the water or the plant layout does not allow the treatment in a separate purifying system, these reagents are added in the boiler feed, and the precipitation takes place in the boiler, assisted by the heat. The main object in this case is to precipitate the salts as soft scale, or mud, in order that it may be readily blown out. Periodic cleaning with some kind of mechanical cleaner is necessary, however, even with feed-water treatment, as some of the hard salts are bound to stick, especially in the hardest worked tubes over the hottest fires.

52. Periodic inspection of boilers is in itself a good preventive of accident. The straight tube, water-tube boilers lend themselves most readily to thorough inspection, as every part can be seen, and the whole of the shells can be hammer tested.

53. Boiler insurance covers the possible damage done by boiler explosion. The boiler insurance companies exercise a vigorous and valuable control over the manner of operation of boilers, and by their frequent enforced inspection, prevent little flaws from becoming disastrous weaknesses.

54. Cleaning. Those operations which make for safety in the boiler generally aid efficiency. A good rule is never to allow more than $\frac{1}{16}$ in. thickness of scale to collect; and to overhaul thoroughly every part of the boiler twice a year. In a well-operated plant, with moderately good water, this generally means cleaning the two rows of tubes nearest the fire about once a month, and the remainder of the boiler, twice annually. It generally pays, both in safety and efficiency, not to use water requiring more frequent cleaning of the boiler than last stated, but instead to purify the water in separate apparatus.

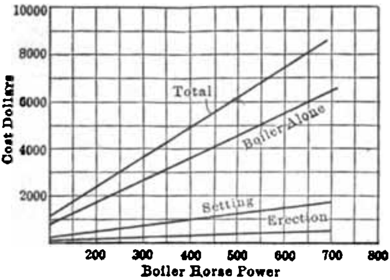


FIG. 11.—Cost of water-tube boilers.

55. The removal of soot from the outside of the tubes is usually required once or twice a week in well-operated plants; but for high rates of forcing, and especialy for low-grade fuels requiring heavy blast, the interval may be shortened to once every alternate day.

56. As far as possible, all conditions about a boiler should be kept uniform—water level constant, feed supply steady, draft well regulated, and firing (either by hand or stoker) uniform. The use of draft, pressure, steam-flow indicators or feed meters, is always valuable in this connection. Changes of load should be anticipated by gradual changes in the running of the boiler, since sudden alterations of draft or feed not only cause strains in the boiler, but loss of efficiency, thus disturbing the normal operation of the fire and flow of steam.

57. Boiler costs. Fig. 11 shows the variation in total cost of **water-tube boilers** of the B. & W. or Stirling types. The cost remains practically constant at \$12.00 to \$12.40 per h.p. This is partly due to the fact that most of the smaller boilers are built for lower pressures than the larger units. If all were built for high pressures, say 200 lb. gage, the cost would follow approximately the rule given by C. H. Benjamin in 1902, which is.

$$\text{Cost in dollars} = 500 + 9.2\ (\text{rated h.p.}),\ \text{for boiler alone.} \qquad (14)$$

58. The average setting cost for 300-h.p. to 600-h.p. units is \$2.50 per h.p., and varies little. The delivered price is approximately 71 per cent., the erection cost 9 per cent., and the setting 20 per cent. of the total cost, which remains very nearly constant at about \$12.40 per h.p. for horizontal water-tube boilers between 300 h.p. and 750 h.p. Gebhardt gives \$1.00 per sq. ft. of heat surface as the price (delivered only), on all boilers over 100 h.p.

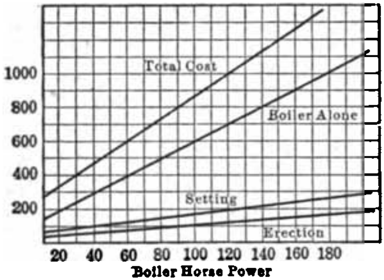


Fig. 12.—Cost of fire-tube boilers.

59. The cost of vertical water-tube boilers averages 10 per cent. less than horizontal water-tube boilers. Benjamin gives the cost in dollars $= 500 + 8.5 \times$ (rated h.p.), for vertical water-tube boilers (delivered only).

60. The corresponding costs of fire-tube boilers are given in Fig. 12.

FURNACES AND STOKERS

61. Principles of combustion: Referring to Fig. 13, a stream of oxygen is shown ascending past a piece of white-hot carbon—say a piece of **coke**, or a piece of **anthracite** having practically no volatile matter. Provided the temperature of the coke is high enough, combustion takes place to CO_2 and CO, the former predominating. Now suppose this stream of CO and CO_2 flows past more incandescent coke with little or no oxygen in the immediate neighborhood. Some of the CO_2 will combine with the incandescent C and reduce to CO. This combustible gas now flows on till it finds more oxygen, and is then burned to CO_2. This oxidation and reduction may go on reversing many times during the travel of the stream through the bed of coal to the top of the fire, and the burning of CO to CO_2 by contact with the needed oxygen is evidenced by the familiar blue flame of anthracite coal fires.

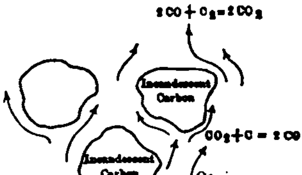


Fig. 13.—Elementary combustion of fixed carbon.

62. With bituminous or semi-bituminous coals, there are two distinct phases in the combustion process: first, on heating, the distillation and burning of the volatile hydrocarbons, leaves behind a mass of porous coke; second, the burning of this coke takes place exactly as the anthracite is burned. The distillation of the volatile matter introduces, therefore, a period during which the coal absorbs heat instead of giving it out. The extra volume of gas given off requires a freer supply of air than the hard-coal fire.

63. The two great requirements for efficient combustion are: (a) thorough "scrubbing" of the coal (or coke and volatile gases) with air, so that each particle of combustible receives its necessary oxygen within the smallest possible space of time; (b) the maintenance of both air gases and coke at a temperature above the ignition point.

64. Types of furnace and stoker: Hand firing always necessitates a horizontal or slightly inclined grate, so that coal may be fired through open doors all over the surface of the grate; most of the air is fed through the fire from below the grate. In the case of anthracite practically no air is admitted over the fire, because the compact nature of the small sizes of anthracite makes the use of air blast generally a necessity, and more than enough air can be supplied from below the grates. With bituminous coal, not enough air can be supplied from below, and some must be admitted by way of the fire doors or other openings for the purpose above the fire. The required reverberatory action for soft coals must be obtained chiefly by methods of firing. Regenerative action can be improved by the use of Dutch oven furnaces, in which the whole furnace is roofed over with fire-brick, which reflects heat instead of absorbing it, raising the temperature of the furnace. The soft-coal furnaces should always have as high a combustion chamber as possible, but with anthracite coals 18 in. is sufficient. Fig. 14 shows one type.

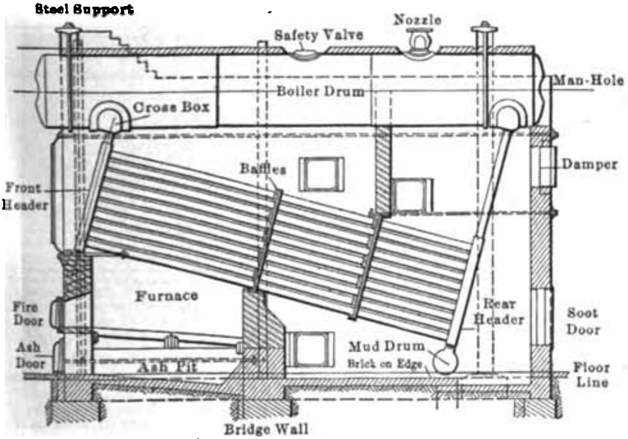


Fig. 14.—Hand-fired grate.

65. The overfeed stokers are those in which coal is fed somewhat in the manner of hand firing; green coal is pushed in at one end, or side of the combustion chamber, coked under a combustion arch, and finally burned to ash in a continuous progress across the furnace, the combustion of fixed carbon taking place upon the grates. The best known inclined types are the Roney, Wilkinson, Murphy, Model, Detroit and Wetzel. Fig. 15, shows the Roney type; coal is fed in at the top and worked down by gravity and the continual rocking of the grate bars.

66. The chain grate types are the B. & W., Green, Laclede, Illinois, etc. Fig. 16 shows a typical chain grate.

67. The underfeed stokers supply coal from below the fire, coke it as it approaches the incandescent top bed, and drive the gases, along with the blast air, through the white-hot coke on top. No combustion arches are required and there are no grate bars, strictly speaking, as little or no combustion takes place on iron—only upon a bed of coal.

68. Types of underfeed stokers. The Jones was the original of this type, followed by modifications utilizing gravity as well; the principal one of the latter type is the Taylor (Fig. 5), followed very recently by the Riley and the Westinghouse underfeed. Lately, extension grates of the overfeed type and continuous dumping devices have been incorporated in this type of stoker, lowering the combustible content in the ash, and reducing the operating labor.

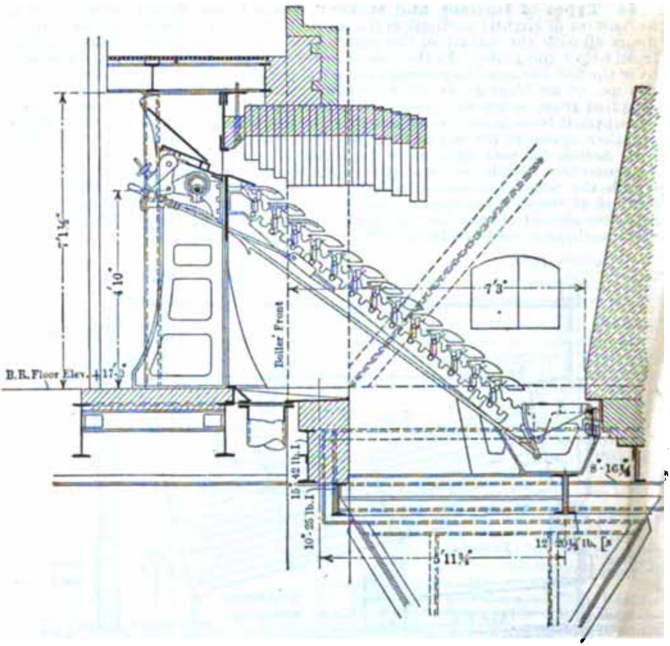


Fig. 15.—Roney stoker.

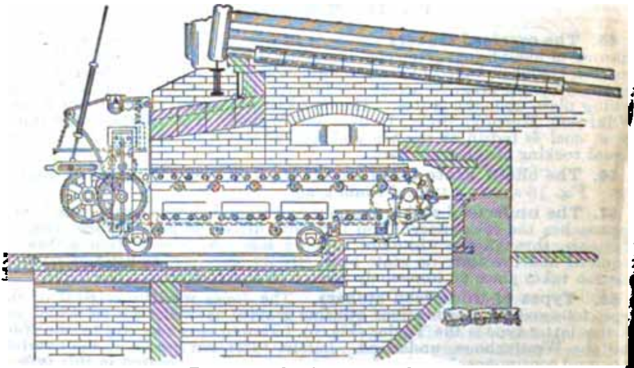


Fig. 16.—Chain-grate stoker.

69. Grates for hand fires are of three types, stationary, shaking and dumping. Stationary bars are of three principal forms: (a) plain girder-shaped straight bars, spaced from each other ½ in. or ⅜ in. by lugs, for air supply; (b) herringbone bars—two straight girder bars about 4 in. to 6 in. apart, cast with herringbone-shaped tie ribs; (c) pinhole bars, for burning sawdust, fine dust and culm, tan bark and similar materials. Rocking and shaking grates are made in a multitude of variations of these simpler forms; and are so arranged that the bars may be rocked in groups, to aid in cleaning the fire, or overturned to dump entirely.

70. Hand stoking of hard coals is done in the following manner. Coal, generally of small size (No. 2 or No. 3 buckwheat), is thrown well over the fire, and maintained about 6 in. to 8 in. thick and under considerable blast (1.5 in. to 2.5 in. of water). As fast as a hole blows through any portion, it is covered with fresh coal. This implies frequent firing, and in order to minimize the inrush of cold air at open fire doors some form of balanced draft is very desirable. Portions of the fire are allowed to burn out and are then dumped bodily and fresh fire raked over the bare grate. **With shaking grates** the dumping periods may be farther apart—4 to 8 hr., depending on the coal and rate of firing, as part of the ashes are removed at more frequent intervals by shaking the grate without opening the fire doors.

71. Bituminous coal can be hand fired in two ways: the alternate and the coking methods. In the first method, one side of the fire is at the period of hottest combustion while firing green coal on the other, to provide the necessary heat to volatilize and ignite the large volume of gas given off. When this green coal has finished giving off gas and becomes well coked and white hot, the other side, now burned out, is fired. In the coking method, the front part of the grate near the fire doors is made solid and is called the dead plate; green coal is fired upon this, under a coking arch, and receives enough heat from the main body of the fire and from this arch, to become coked. The gas thus given off passes out from the coking arch over the hot part of the fire, is mixed with air admitted over the fire, and thus ignited. The green coal, now coked, is pushed back over the hot part of the fire. As portions burn out, they are dumped and fresh coke raked over the bare grates. There are many variations in the manner in which these operations are performed, depending on the kind of coal. One man can ordinarily take care of one boiler up to 600 h.p., or two boilers at the utmost.

72. Automatic stoking in the overfeed type occurs almost in the same order as the hand firing of bituminous coal, except that all the processes, feeding, coking, segregation of ash, and sometimes dumping, are done by power and are continuous, instead of intermittent. The advantage in avoiding irregularity in the processes is obvious.

73. Automatic stoking in the underfeed type requires no regenerative devices in the furnace (such as the coking arch), since the gases and the air are mixed by passing through the bed of coal together and are finally heated by direct contact with the incandescent coke, before issuing into the furnace. The underfeed stokers are capable of almost unlimited forcing without serious loss of furnace efficiency, and are much lower in maintenance costs than the overfeed types, as no hot fire is carried on cast-iron parts.

74. Maintenance costs of stokers. The maintenance of Roney stokers with good coal, varies from about $0.10 to $0.12 per ton fired; the Murphy, Model, Detroit, Wetzel and Wilkinson vary from $0.11 to $0.14, the higher cost being due to the use of long bars in the fire, which are injured only in about the lower third, necessitating the scrapping of about two-thirds of the bars practically uninjured. With the Roney finger grate, the parts actually in the fire are separately removed, and consequently the efficiency of use of the metal is higher. Chain-grate stokers are comparable in maintenance with the Murphy type. Jones stokers require about $0.04 to $0.06 per ton fired for maintenance, and the Taylor stoker from $0.025 to $0.04 depending on the coal. Highly volatile coals, which must be fired thinner on account of clinkering, use up more iron, as the heat is nearer to the fingers and retorts.

75. Stoker labor. One stoker operator to each four stokers, and one coal passer to rake up siftings for each five stokers is good practice for the Roney type. The size of the stoker makes comparatively little difference, and hence the larger the stoker the better. One stoker operator for 6 stokers, and one coal passer for raking siftings to 6 stokers, is good practice for the Green or B. & W. types. One stoker operator to 15 stokers is needed for the Taylor, but no coal passers, as there are no siftings. The Jones type requires about the same labor as the Green, as the ashes must be raked out by hand.

76. The rate of combustion for hand fires is normally 10 lb. to 12 lb. of coal per sq. ft. of grate for anthracite, and 15 lb. to 20 lb. for bituminous coals. Under forcing the rate may go as high as 80 lb. per sq. ft. of grate, with forced draft. For power plants the usual maximum is 40 lb. to 50 lb. of soft coal.

77. Natural or chimney draft is common for bituminous coals. On hand fires 0.3 in. to 0.6 in. water draft at the breeching will generally produce rated capacity from the boiler; 0.3 in. draft at the breeching will produce rated capacity on the overfeed slope-grate types with high percentage of air space (35 per cent. to 45 per cent.). With the restricted types (Wilkinson) and the chain-grate stokers, where the air space is only 8 per cent. to 15 per cent., the required draft to produce rating is usually 0.4 in. to 0.5 in.

78. Forced draft with underfeed stokers and anthracite hand fires is practically a necessity, as the resistance through the fires is very much greater (Par. 80). The small sizes of anthracite pack very closely, and in 6 in. or 8 in. thickness may offer as much as 1 in. to 2 in. difference of pressure above and below the fire. In the underfeed stokers the fire is much thicker than in the overfeed type (the latter is about the same as hand-fired bituminous grates or 10 in. to 14 in.), or usually about 2 ft. 6 in. or 3 ft. thickness of fuel bed.

79. The amount of air required varies with the volatile content of the coal. The combustible constituents are carbon, hydrogen and sulphur. As far as heat and air supply are concerned, the carbon and hydrogen alone are important, the sulphur seldom exceeding 3 per cent. being low in heat value.

$$C + O_2 = CO_2 \tag{15}$$

$$\underbrace{2H_2}_{\text{2 vol.}} + \underbrace{O_2}_{\text{1 vol.}} = \underbrace{2H_2O}_{\text{2 vol.}} \tag{16}$$

The combining weights are in the proportions of the molecular weights, so that

$$12 \text{ lb. } C + 32 \text{ lb. } O_2 = 44 \text{ lb. } CO_2 \tag{17}$$

$$1 \text{ lb. } C + \tfrac{32}{12} \text{ lb. } O_2 = \tfrac{44}{12} \text{ lb. } CO_2 = 3.67 \text{ lb. } CO_2 \tag{18}$$

Air is 20.9 per cent O_2 and 79.1 per cent. N by volume, and 23.6 per cent. O_2 and 76.4 per cent. N by weight, so that, as 1 lb. C requires 2.67 lb. O_2, it will require $\frac{100}{23.6} \times 2.67 = 11.30$ lb. of air per lb. of carbon. Similarly, hydrogen requires 34 lb. of air per lb. Then the air required per lb. of any coal will be given by

$$(\text{fraction C per lb.}) \times 11.3 + (\text{fraction H per lb.}) \times 34 = \text{lb.} \tag{19}$$

air required per lb. of coal.

The theoretical amount cannot be used because mixing is imperfect. Usually 18 lb. of air per lb. of coal are supplied for underfeed stokers and 20 lb. to 24 lb. for overfeed and hand fires.

80. Automatic damper regulators are used to maintain substantially constant boiler pressure under variable load. A diaphragm operated by steam pressure controls a pilot valve admitting water or oil under suitable pressure to a cylinder which in turn operates the damper lever. If increase in load causes drop in pressure, the diaphragm and pilot cause the pressure cylinder to open the damper wider, increasing the draft and the rate of combustion, thus making more steam to recover the proper steam pressure; and *vice versa*. This device may be employed in the stack or breeching for natural draft; or it may control the fan-engine throttle for forced or induced draft, producing the same results.

81. In the balanced-draft systems, a steam pressure regulator controls the forced-draft fan, and a combustion-chamber pressure regulator controls the stack damper or induced-draft fan, so as to maintain zero pressure difference over the fire. This is specially advantageous for hand firing, as the fire doors may be opened and shut without a rush or cold air over the fire. It is also a convenience with any forced-draft stoker.

82. The accessories of the automatic stoker consist of the forced-draft fan (if required), the stoker engine, and usually, for overfeed stokers—a slice bar and a poker, needed occasionally for obstinate clinkers. In the later types of underfeed stokers, automatic dumping devices are attached, which eliminate the need of hand manipulation of the fire altogether (Fig. 5).

83. The stoker engines required with all stokers except the Jones and the Wilkinson, may be of any standard type. The slope-grate overfeed stokers require about 1.5 to 2 h.p. per stoker, the highest being for the largest sizes—say 150 in. in width. Chain grates require 2 to 3 h.p.; underfeed stokers, 3.5 to 4 h.p. per stoker. The Jones type has a steam cylinder attached individually to each plunger and requires no other drive. The Wilkinson type generally has a hydraulic cylinder operating each stoker.

84. Furnace efficiency is defined in Par. **27**. Its value for hand fires may be brought up to 94 or 95 per cent. but seldom runs higher than 90 per cent. in regular operation. Automatic stokers can be made to operate at efficiencies as high as 96 to 97 per cent. It is a question chiefly of adequate air supply, intimate mixture of gases, and high temperature.

85. Flue-gas analysis. The Orsat apparatus is the simplest and commonest means of analyzing flue gases. It appears in several forms, all of which employ some form of graduated glass gas-measuring chamber, usually waterjacketed, and has means for exposing the sample of gas successively to solutions of potassium hydroxide, potassium pyrogallate, and cuprous chloride (generally the acid solution), for removing CO_2, O_2 and CO, respectively. The gas is returned to the measuring chamber after each absorption, to measure the reduction of volume. Several automatic machines have been devised for measuring and recording CO_2 content in flue gases continuously; but it is now well known that CO_2 measurement alone is an insufficient indication of boiler efficiency. Flue temperature and sometimes the full gas analysis are desirable. The absorption of CO is particularly troublesome, as a rule, because the cuprous chloride must be used fresh, and is apt to throw off the absorbed CO if diluted; this also may happen in the Elliott apparatus. **The Elliott apparatus** differs from the Orsat in using long burettes and pipettes for greater accuracy, and the pouring of the absorbents through the absorbing chamber by hand; also, an explosion pipette for measuring the unburned hydrocarbons is added. **The Hempel apparatus** is still more accurate and refined, but is strictly a piece of laboratory apparatus. The Orsat type is made in easily portable form for use at the boiler.

86. Smoke consists chiefly of unburned carbon suspended in the flue gases; it may be in the form of particles of unburned coke blown up from the fire by heavy driving or high blast, or in the finely divided lamp-black form, developed by the breaking up of hydrocarbon gases, deficient in air, and exposed to high temperature. The latter form is chiefly indicative of bad combustion; the cure consists of thorough mixing of volatile matter and air in the furnace, and the maintenance of a high temperature of this mixture until combustion is complete, or in other words, until there is no more flame. In so far as any furnace or method of firing accomplishes these two things, in that degree it will be smokeless.

87. Specifications for stokers should cover the following: (a) general, and terms of delivery and erection; (b) drawings of stokers and dimensions of boilers to which they are to be attached; (c) operating conditions of boilers and rating; (d) general description of stokers; (e) dampers, doors, drive, and labor used; (f) fan requirements, foundations, air ducts and brickwork; (g) tests and operating conditions for tests, including full description and proximate analysis of coal to be used; (h) guarantees, capacity, efficiency, labor to operate, gas analysis, smoke, maintenance; (i) time of shipment, price and terms of payment; (j) detailed description of stoker and all parts.

88. The cost of automatic stokers per rated h.p. of boiler varies but little with size, as most of the stokers are made up of parts such that increase in size means merely increase in unit structure. The price is given delivered but not erected, although superintendence of erection is included; common labor for erection is furnished by the purchaser. The cost of Roney and other step-grate stokers is from \$3.17 to \$3.91 per rated boiler h.p., average \$3.60; chain grates, \$6.20 to \$7.00, average \$6.60; underfeed (Jones type), \$3.70 to \$4.76, average \$4.44; underfeed (gravity plunger).

$5.45 to $6.10, average $5.65. The last type can also be listed at $495.00 per retort, a retort yielding about 86 rated boiler h.p.

89. The forcing capacity of these different types is extremely variable; the overfeed step-grate types usually cannot exceed 200 per cent. of rating; the chain grates are about the same. The underfeed types are readily reaching 300 per cent. and over. The cost per rated h.p. is therefore not entirely fair as the sole criterion.

90. Cost figures from a very large stoker installation using three types are as follows:

	Rating	Cost per h.p.	Rating	Cost per h.p.
Step grate..............	100 %	$3.17	200 %	$1.59
Chain grate with blast...	100 %	$6.55	266 %	$2.46
Gravity underfeed......	100 %	$5.77	325 %	$1.78

The weight of the step-grate stokers is about 500 to 550 lb. per rated boiler h.p.; chain grates, 1,300 to 1,400 lb.; underfeed types (gravity), 550 to 630 lb.

CHIMNEYS

91. Chimney draft is based upon the difference of specific gravity of cold air and heated air. The column of warm air in the chimney exerts a pressure per sq. ft. at the base of $h\gamma$, where h is the height in feet and γ is the density of hot gases in lb. per cu. ft. The pressure of the outside air for the same height is $h\gamma_2$, where γ_2 is the density of cold air in lb. per cu. ft. The motive force is therefore the difference of these two, or $h(\gamma_2-\gamma_1)$, in lb. per sq. ft.

92. Formula for chimney draft. For ordinary use,

$$D_1 = H\left(\frac{7.64}{T_2}-\frac{7.95}{T_1}\right) \qquad (20)$$

Where H = height of chimney in ft., D_1 = intensity of draft, inches of water, T_1 = absolute Fahr. temp. of chimney gases, T_2 = absolute Fahr. temp. of outside air, and P_2 = observed atmospheric pressure, lb. per sq. in. For high altitudes above sea level:

$$D_1 = 0.52\ HP_2\left(\frac{1}{T_2}-\frac{1}{T_1}\right) \qquad (21)$$

This gives the maximum theoretical draft with no flow taking place. The actual draft obtained is about 20 per cent. less, due to chimney friction, and head required to create velocity of gases.

93. Kent's draught formula is

$$H = \left(\frac{0.3\ \text{h.p.}}{E}\right)^2 \qquad (22)$$

$$E = A - 0.6\sqrt{A} \qquad (23)$$

Where E = effective area of chimney, A = actual area, (cross-section) of chimney, H = height at top in ft. above grates and h.p. = total connected boiler h.p.

Based on a maximum combustion of 5 lb. coal per rated B.H.P. per hr., this formula may be modified to

$$H = \left(\frac{0.06\ W}{E}\right)^2 \qquad (24)$$

where W = total lb. coal fired per hour under connected boilers. This form is more suitable for high rating conditions.

94. Meyer on chimneys. Meyer* assumes a combustion rate of 4 lb. of coal per hour per boiler h.p. and 500 cu. ft. of flue gas per lb. of coal. In his table on chimneys referred to in the footnote he gives the friction loss for square steel flues; for brick it is 33 per cent. greater; for rectangular steel flues with sides as 1 to 2, it is 7 per cent. greater; for circular steel flues, 13 per cent. less.

* Meyer, H. C., Jr. "Steam Power Plants;" McGraw-Hill Book Company, Inc., 1912; Chap. XIII. Table XXXIII.

95. Table of sizes and h.p. of chimneys.
Table gives h.p. of chimneys computed by Kent method (Eq. 24, Par. 96).

Diam., ins.	Height								Effect. Area sq. ft.	Actual Area sq. ft.
	60 ft.	80 ft.	100 ft.	125 ft.	150 ft.	200 ft.	250 ft.	300 ft.		
18	25	29	3	36	40	46	51	56	0.97	1.77
24	54	62	69	78	85	98	110	120	2.08	3.14
30	92	107	119	133	146	169	189	206	3.58	4.91
36	141	163	182	204	223	258	288	315	5.47	7.07
42	200	231	258	289	316	365	408	447	7.76	9.62
48	269	311	348	389	426	492	549	602	10.44	12.57
60	437	505	565	632	692	800	894	979	16.98	19.64
72	646	747	835	934	1,023	1,181	1,320	1,447	25.08	28.27
84	896	1,035	1,157	1,294	1,418	1,637	1,830	2,005	34.76	38.48
96	1,186	1,370	1,532	1,713	1,876	2,167	2,423	2,654	46.01	50.27
108	1,517	1,751	1,959	2,054	2,392	2,770	3,098	3,393	58.83	63.62
120	.....	2,180	2,438	2,557	2,986	3,448	3,855	4,223	73.22	78.54
132	.....	2,656	2,970	3,114	3,637	4,200	4,696	5,144	89.18	95.03
144	.....	.....	3,554	3,726	4,352	5,027	5,618	6,155	106.72	113.10
168	.....	.....	4,878	5,115	5,974	6,899	7,713	8,449	146.50	153.94
192	.....	.....	.....	6,724	7,852	9,068	10,138	11,105	192.56	201.06
216	.....	.....	.....	.....	9,987	11,532	12,894	14,123	244.90	254.47
240	.....	.....	.....	.....	12,378	14,293	15,980	17,505	303.53	314.16

For pounds coal burned per hour for any given size of chimney, multiply figures in table by five.
Chimneys 25 per cent. larger are recommended for low-grade bituminous coal in middle and western states.

96. Chimney design must be based upon kind of coal required. For overfeed stokers and hand fires, Fig. 17 (from B. & W. 1913 issue of "Steam") gives the usual draft required for various coals.

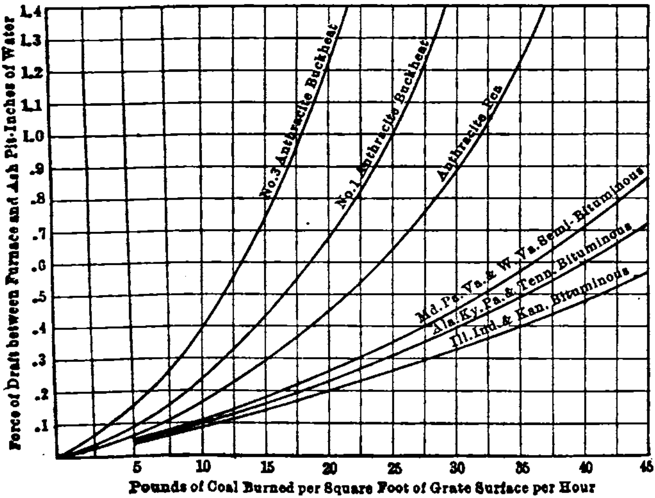


FIG. 17.—Draft for various coals.

The draft loss through a B. & W. boiler with standard baffling, clean, varies from 0.10 in. or 0.15 in. at rating, to 0.29 in. to 0.45 in. at 200 per cent. rating and 0.54 in. to 0.65 in. at 300 per cent. rating. The loss in flues of fairly uniform cross-section is given as 0.1 in. per 100 ft. straight flue, and 0.05 in. for 90 deg. bends. But in the irregular flue shapes often necessary, double these values are not infrequent (Meyer). The draught loss through economizers varies from 0.25 in. to 0.70 in. The sum of all these losses gives the required chimney draft. The diameter is computed (Par. **96** and **97**) from the weight of coal burned. **With stokers using blast** it is necessary only to allow 0.1 in. **draft** over the fire at maximum capacity, as the only function of the stack is to draw off the gases without pressure in the furnace, the resistance of fire bed and stoker being overcome by the blast pressure.

97. Chimneys are built of the following materials: (a) red brick, fire-brick lined; (b) radial buff brick, fire-brick lined; (c) reinforced concrete, lined; (d) steel plate, lined; (e) steel plate, unlined. For natural draft, the radial brick, the lined steel plate and the concrete types of chimney are in general use.

98. Brick chimneys. The red-brick chimney cannot be less than 0.7 ft. thick at the thinnest portion; in other respects it is designed for stability against a wind pressure of 50 lb. per sq. ft. and against crushing. The walls, therefore, are always thicker at the bottom, and the stack is tapered. This type is not often used for power stations. Radial-brick chimneys may be 0.58 ft. thick at the thinnest part, as the brick is specially moulded; the cross-section is always circular, and tapers in the same manner as red-brick chimneys. Brick chimneys are the most durable of all the types. The fire-brick portions of lining need be carried up only 30 ft. above the grates.

99. Steel chimneys have the advantages of lightness and strength, but since they are better conductors of heat, must be lined with brick for heights over 75 ft. except in forced-draft installations. They must be carefully inspected and painted from time to time, as they are subject to deterioration by corrosion.

100. Reinforced concrete is much stronger than brick and will stand high tensile strains like the steel chimney. The stack is therefore often built straight like the steel chimney, and is always considerably lighter than brick, as it may safely be much thinner. It is usually poured in 5-ft. or 6-ft. sections. which may be carried up a section a day, making erection rapid. No lining is required other than the short section of fire-brick above the grates (30 ft.). It is one of the cheapest and most durable forms if well designed and built, but like all reinforced concrete, is dependent upon care and watchfulness during construction.

101. Foundations for brick chimneys are now made almost exclusively of concrete, and are designed on the basis of proper bearing values, like any other foundation. They are usually spread or stepped out at the foot, in order to provide sufficient resistance to overturning from wind pressure (Par. **590**).

102. Cost of Brick Stacks

Approx. boiler h.p.	Height	Diameter	Diam. square base outside	Price
85	80	2 ft. 1 in.	7 ft. 5 in.	598.00
135	90	2 6	8 3	786.00
200	100	2 11	9 10	1226.00
300	110	3 7	10 2	1492.00
450	120	4 3	11 2	1785.00

Brick stacks may be figured on a basis of $12.00 per thousand for laying with masons at $0.55 per hour.*

103. The cost of concrete stacks is about 5 to 10 per cent. less than brick (Par. **105**).

* Further cost data may be found in Gebhardt's "Power Plants," Wiley, 1912.

104. The cost of steel stacks may be figured on a basis of \$0.045 per lb. of steel erected.

MECHANICAL DRAFT

105. Limitations of natural draft. From the figures in Par. 91 to 96, it is evident that for high rates of combustion the stack becomes impracticably high, or the sensible heat loss due to high flue temperature becomes too large for economy. To mitigate this, mechanical or artificial draft of some form may be employed.

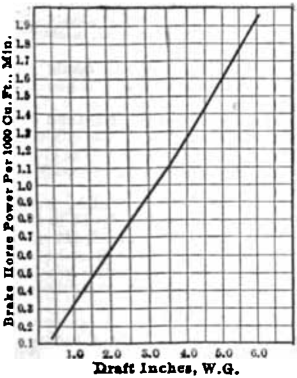


FIG. 18.—Required horse-power for forced-draft fans.

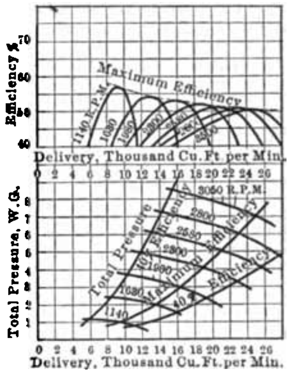


FIG. 19.—Characteristics of 24 by 30 in. forced-draft blower.

106. The theoretical pressure produced by a revolving fan wheel is given by Murgue as

$$H = \frac{U^2}{g} \qquad (25)$$

where H = maximum pressure difference, between fan suction and discharge, in feet of air; U = velocity of fan blade tips, feet per second; and $g = 32.2$, acceleration due to gravity. Air pressure in inches of water column is generally referred to in blast and draft.

$$h = \frac{SH}{144p} \qquad (26)$$

where h = inches of water pressure; S = weight of 1 cu. ft. air at 75 deg. fahr. (usual room temperature), or 0.074495 lb.; and p = pressure of 1 in. water column in lb. per sq. in., or 0.0361 lb.

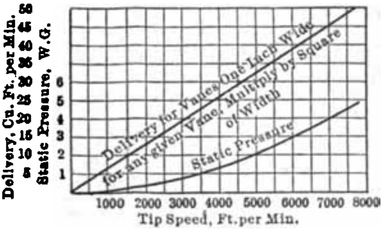


FIG. 20.—Power and volume of forced-draft fans.

$$U = \frac{2\pi rn}{60} \qquad (27)$$

where r = radius to tip of blade in inches and n = r.p.m. Hence

$$h = \left(\frac{2\pi rn}{60}\right)^2 S = C(rn)^2 S \qquad (28)$$

Fig. 18 gives the h.p. required at any pressure, for average fans. The characteristics of a forced draft fan for underfeed stokers are given in Fig. 19. The static pressure and volume of any size fan, at any speed, can be found from Fig. 20.

107. Forced draft. If the fan is placed so as to blow under the fire, the pressure is greater than atmospheric; this system is termed forced draft.

108. Induced draft. If the fan suction is connected to the boiler flue, and delivers to the stack, the pressure in the furnace and ashpit is less than atmospheric; this system is termed induced draft. The two systems may be used together, one fan forcing air through the fire, and the other drawing from the flue, maintaining a more or less balanced pressure over the fire. A forced draft fan also may be used, in connection with a chimney to draw off the gases.

109. Induced draft for hand fires with bituminous coals may be substituted for natural draft since the required maximum is not usually over 2 in. of water.

110. Forced draft combined with either natural or induced draft is always used with the underfeed stokers and frequently with the chain grates. Forced draft cannot be used alone, as without some suction at the breeching, there would be pressure above atmosphere in the fire-box, causing flame to issue from all openings.

111. Cost data on blowers and fans are difficult to furnish; about the only factor remaining nearly constant is the price per lb. at $0.15 to $0.17.

112. The comparative advantages of mechanical draft are: (a) greater forcing capacity than with natural draft, since it is easy to produce much greater differences of pressure than are at all practicable with stacks; (b) entire flexibility of control; (c) better combustion conditions with balanced draft arrangement; (d) low cost of apparatus.

113. The objections to mechanical draft are increased operating cost due to energy consumption by driving apparatus, and increased maintenance cost, which are balanced against the low maintenance of stacks. But the energy consumption (of steam, for turbine drive, say) is not a net loss, as it may be profitable to use the steam exhaust in feed heating, and the stack requires a certain loss of fuel due to flue gas temperature, in order to operate. This temperature can be lowered when using induced draft. **Forced-draft fans** have low maintenance costs, as they handle cool air; **induced-draft fans**, however, are likely to have high maintenance, as they deteriorate from handling hot flue gases.

PROPERTIES OF FUEL

114. Anthracite, or hard coal, contains very little volatile matter; is mainly fixed carbon and ash; is difficult to ignite; hard and bright fracture, not soiling the fingers in rubbing; oldest coal formation, being next to graphite.

115. Semi-anthracite contains a little less fixed carbon, more volatile matter; is easier to ignite and softer. **Semi-bituminous** is the next in order, having enough volatile matter to coke, with moderately long flame, and soft crumbly fracture.

116. Bituminous coals are rich in volatile matter, which may be as high as 40 per cent.; usually fairly low in ash—5 to 9 per cent.; and may be divided into many different classes with regard to flame, caking and coking qualities. They are very easy to ignite, brittle and dull fracture, crumbly, and soil the fingers in handling, due to softness.

117. Coke is the resulting fixed carbon and ash which remains a porous mass after driving off the volatile matter from a bituminous or semi-bituminous coal. It is largely a product of the coal gas industry; but as a power station fuel, its price is practically prohibitory. Oven coke, which is very hard, is used for blast furnace and crucible furnace work.

118. Lignite is a more recent formation than the bituminous coals and is hardly distinguishable from the lower grades of these; it is non-fibrous, and often brownish in appearance, whereas the coals are all black; absorbs oxygen and gives off CO_2 at ordinary temperatures. It contains quantities of oxygen and the volatile matter given off is mainly CO_2, which distinguishes the lignites still further from the coals, whose volatiles are chiefly hydrocarbons.

119. Properties of Various Coals

Sample	Bed	Proximate analysis: Moisture	Volatile matter	Fixed carbon	Ash	Ultimate analysis: Sulphur	Hydrogen	Carbon	Nitrogen	Oxygen	Air drying loss per cent.	Calorific value B.t.u.	
Alabama	Warrior field	2.58	33.15	51.74	12.53	1.02	4.79	69.24	1.55	10.87	0.8	12,449	Lump, Nut & Pea Coal
Arkansas	Huntington bed	0.80	19.75	67.65	11.8	1.3	4.07	76.37	1.55	4.91	0.8	13,655	Lump & Slack.
Colorado	Boulder field	13.49	37.11	43.03	6.37	0.58	5.75	61.13	1.22	24.95	6.0	10,791	Run of Mine, Lignite.
California	Alameda Co.	18.51	35.33	30.67	15.49	3.05	5.93	47.34	0.66	27.53	10.4	8,500	Run of Mine, Lignite.
Illinois	Belleville field	5.31	34.29	36.24	24.10	4.3	4.57	54.06	0.78	12.13	7.1	9,848	Slack.
	Marion Co.	5.96	30.29	52.16	11.59	1.77	4.92	67.30	1.43	12.99	2.7	12,103	Run of Mine.
	Montgomery Co.	13.2	34.33	39.94	12.53	4.47	5.51	57.25	1.02	19.22	4.4	10,514	Nut Coal.
	Franklin Co.	8.31	31.65	49.56	10.43	1.55	5.18	65.83	1.48	15.48	4.6	11,727	Egg Coal.
Indiana	Warrick Co.	6.24	37.49	42.76	13.51	4.6	5.11	62.97	1.25	12.56	3.6	11,538	Run of Mine.
	Sullivan Co.	13.99	29.4	42.29	14.32	2.31	5.36	57.18	1.11	19.72	10.8	10,318	Run of Mine.
	Vigo Co.	9.55	36.19	43.65	10.61	3.72	5.49	64.08	1.08	15.02	4.6	11,759	Lump Coal.
Indian Territory.	Hartshorne bed	1.7	37.19	49.79	11.32	1.56	5.00	71.9	1.72	8.91	2.8	12,969	Run of Mine.
	McAlester bed	4.91	37.79	43.9	13.4	4.02	4.84	63.21	1.38	13.15	1.4	11,389	Lump Coal.
Iowa	Polk Co.	4.52	40.96	38.99	15.53	6.83	4.93	60.62	0.93	11.16	9.8	11,356	Lump Coal.
	Lucas Co.	9.22	32.71	44.52	13.55	3.42	5.35	59.89	1.22	16.57	6.8	10,989	Run of Mine.
Kentucky	Western field	5.85	36.9	46.96	10.29	....	5.27	66.75	1.43	12.66	2.2	12,292	Run of Mine.
	High Splint bed	4.36	35.02	56.92	3.7	0.67	5.16	77.44	1.57	11.46	2.8	13,923	Run of Mine.
Maryland	Garrett Co.	2.33	16.11	68.43	13.13	1.49	3.99	75.21	1.29	4.89	1.4	13,255	Run of Mine.
Missouri	Rich Hill field	3.5	35.35	40.77	20.38	5.35	4.64	60.00	0.99	8.46	5.0	11,144	Run of Mine.
	Bevier field	9.14	34.53	39.02	17.31	5.3	4.96	56.25	0.99	15.19	2.6	10,451	Run of Mine.
Montana	Red Lodge	9.05	36.7	43.03	11.22	1.76	5.25	60.41	1.36	20.0	2.2	10,777	Blacklig., washed nut
New Mexico	Gallup field	10.86	35.14	46.9	7.10	0.64	5.73	64.34	1.05	21.14	1.6	11,435	Lump and Slack.
North Dakota	McLean Co.	35.96	31.92	24.37	7.75	1.15	6.54	41.43	1.21	41.92	12.7	7,069	Lump, Brown lig.
Ohio	Jefferson Co.	3.53	37.45	49.9	9.12	3.47	5.15	71.66	1.31	9.29	1.4	13,072	Lump, over ¾" screen.
	Guernsey Co.	6.65	33.94	48.86	10.55	3.13	5.3	67.38	1.2	12.44	2.6	12,179	Lump, over 1¼" screen.
Texas	Houston Co.	13.4	42.75	29.00	14.58	1.04	5.57	52.06	0.95	25.53	24.6	9,385	Brown Lignite.

120. Properties of Mineral Oils

No.	Name and source	Density			Ultimate analysis				Prox.	B.t.u. per pound		
		Sp. gr.	F	Bé	C	O+N	H	S	H_2O	By calorimeter	High value	Low value
1	Ogaio, crude	0.985	32	12.135	87.1	10.4	2.5			18,146	18,983	18,065
2	California, fuel	0.966	60	14.93	81.52	11.61	6.92	0.55		18,667	18,926	17,903
3	California, Whittier	0.9637	60	15.28				0.845	8.71	18,518		
4	California and Bakersfield fuel	0.962	60	15.53	84.43	10.99	3.99	0.59			18,976	18,005
5	California crude	0.9572	60	16.24	86.3	16.7		0.8		18,646	12,723	21,254
6	Hanover	0.955	32	16.505	86.2	11.4	2.4				19,488	18,493
7	California crude	0.9533	60	16.85	85.75	11.3		0.67		18,797	19,356	18,363
8	California, Whittier and Los Angeles	0.953	60	16.9				0.98	4.93	18,714		
9	Texas fuel	0.945		18.155						19,242		
10	California	0.943	60	18.47				0.735	1.06	18,677		
11	California, Whittier	0.9417	60	18.67				0.975	1.06	18,626		
12	Baku Russia heavy	0.938	60	19.26	86.6	12.3	1.1			19,440	20,052	18,978
13	Petroleum residue, Baku	0.928		20.95	87.1	11.7	1.2			19,260	19,761	18,739
14	Texas, Beaumont fuel	0.926	60	21.25	83.26	12.41	3.83	0.5			19,654	18,570
15	California	0.920	60	22.17	84.0	12.7	1.2				19,917	18,807
16	Pennsylvania	0.914	70	23.18	86.1	13.9		0.06			20,949	19,735
17	Shale oil, Ardeche	0.911		23.682	80.3	11.5	8.2			16,283	18,452	17,647
18	Ohio distillate	0.887	60	27.84	84.2	13.1	2.7			18,718	20,188	19,044
19	Pennsylvania, crude, heavy	0.886	32	28.01	84.9	13.7	1.4			19,210	20,654	19,457
20	Russian, crude, light	0.884	32	28.38	86.3	13.6	0.1			22,628	20,796	19,608
21	Shale oil	0.875		30.005						18,217		
22	West Virginia, heavy	0.873	60	30.37	83.5	13.3	3.2				20,207	19,046
23	East Galicia	0.870	32	30.92	82.2	12.7	5.7			18,153	19,675	18,545
24	Kansas, crude	0.866	60	31.67	85.4	13.07					20,345	19,203
25	West Virginia, light	0.8412		36.435	84.3	14.1	1.6			18,502	20,809	19,578
26	Ohio distillate	0.838	60	37.07						19,880		
27	Ohio, Mabery Noble	0.829	70	38.89	85.0	13.8	0.6	0.6			20,752	19,547
28	Pennsylvania, crude	0.826	60	39.50	82.0	14.8	3.2			17,930	20,699	19,606
29	American petroleum	0.82		40.73	83.4	14.7	1.9			17,588	20,042	19,758
30	Pennsylvania, light	0.816		41.57	82.0	14.8	3.2			17,533	19,899	19,606
31	American, crude				83.0	14.4	3.0			19,980	20,801	19,544
32	Caucasus, Russia		60		84.94	13.96	1.25			18,610	20,817	19,649

121. Composition of Natural Gases

No.	Source	Authority	Volumetric Analysis							
			O_2	CH_4	C_2H_6	H	CO	C_2H_4	N_2	CO_2
1	West Virginia	Report Gas Eng. Com. N.E L.A.	0.4	99.5	0.1	......	......	......	......	......
2	Kansas	Report Gas Eng. Com. N.E.L.A.	0.25	98.3	......	......	0.25	......	1.2	......
3	Caucasus	Bunsen	......	97.57	......	......	2.69	......	......	......
4	Kokomo, Ind.	Levin	0.3	94.16	......	1.7	0.55	0.3	2.8	0.29
5	St. Mary's, Ohio	Levin	0.35	93.85	......	2.74	0.44	0.2	2.98	......
6	Marion, Ind.	*Eng. & M. J.*	0.55	93.57	......	1.2	0.6	0.15	3.42	0.3
7	Findlay, Ohio	*Eng. & M. J.*	0.39	93.35	......	1.64	0.41	0.35	3.41	0.25
8	English	Lewes	......	93.16	2.94	......	1.0	......	2.9	......
9	Russian	Lewes	......	93.1	3.26	0.98	......	......	1.9	2.18
10	Anderson, Ind.	*Eng. & M. J.*	0.42	93.07	......	1.86	0.73	0.47	3.02	0.26
11	Ohio	Lewes	0.35	92.84	0.35	1.89	......	......	3.82	0.75
12	......	Lucke	0.34	92.6	......	2.18	0.5	0.31	3.61	2.6
13	Leechburg, Pa.	Hoyle	......	89.65	......	4.79	0.26	4.39	......	0.35
14	Penna. & W. Va.	Allen & Burrell	......	83.0	16.4	......	......	......	0.6	......
15	West Virginia	Report Gas Eng. Com. N.E.L.A	0.15	81.5	17.6	0.2	......	......	0.55	......
16	Butler County, Pa.	Hoyle	......	80.11	......	13.5	......	5.72	......	0.66
17	Pittsburgh, Pa.	Levin	0.8	72.18	......	20.0	1.0	3.0	......	0.8
18	Penna.	Jüptner	......	67.0	5.0	22.0	0.6	1.0	3.0	0.6
19	Pittsburgh, Pa.	Hoyle	0.8	67.0	......	22.0	0.6	5.0	3.0	0.6

122. Composition of Coke Oven and Retort Coal Gas

No.	Description	Volumetric analysis								
		H_2	CH_4	CO	C_2H_4	C_6H_6	Heavy hydro-carbons	CO_2	O_2	Remainder and N_2
1	Solvay coke oven	56.9	22.6	8.7			3.0	3.0		5.8
2	Retort coal gas, Lewes, 5–6.5 O in coal	54.21	34.37	6.68	2.48	.79		1.47		
3	Aachen retort gas	54.0	34.2	5.2			3.3	1.1		2.2
4	†Norwich retort gas, bit. coal	53.79	36.11	3.40			3.26	.27	.14	3.03
5	Coke-oven gas	53.0	35.0	6.00	2.0			2.0		2.0
6	Retort coal gas, Lewes, 6.5–7.5 O in coal	52.79	34.43	7.19	3.02	.99		1.58		
7	Retort coal gas, Sexton	51.88	31.8	9.1			5.2			2.02
8	London coal retort gas, Pryce	50.7	37.8	4.1			4.4			3.0
9	Retort coal gas, Newton, Mass.	50.59	34.80	6.16			5.23	1.16		2.06
10	Common coal gas	50.1	38.0	6.0	4.0					1.9
11	Laclede Gas Co., bit. coal	49.8	32.3	6.7			8.0	2.40	.60	0.2
12	†Gloucester retort gas, bit. coal	48.88	38.25	4.64			4.95	.03	.51	2.74
13	Retort coal gas, average	48.49	35.9	6.61			3.83	.12		5.05
14	†Good Solvay average coke-oven gas	48.0	35.5	5.1	4.2	1.2		1.3	.5	4.2
15	Common coal gas	47.73	35.6	6.15	4.88			1.41	.31	3.92
16	Coke oven, Milwaukee	47.1	34.7	6.2	3.8			3.1	.3	4.8
17	Heidelberg retort coal gas	46.2	34.02	8.88			5.09	3.01	.65	2.15
18	Coal retort gas—Bunsen & Roscoe	45.58	34.9	6.64			6.46	3.67		2.75
19	Common coal gas	44.4	37.1	5.2	2.3			1.3	1.1	8.6
20	†Sheffield retort gas, cannel	43.05	43.05	4.72			6.28	.24	.10	2.56
21	Average coke oven, Klumpp	42.0	34.3	6.0			4.0	2.5	1.1	10.1
22	Otto coke oven, poor part of gas	41.6	29.6	6.3			2.8	3.2	.4	16.1
23	†Leeds retort gas, cannel	40.23	42.74	5.02			7.28	.34	.07	4.32
24	Birmingham, Eng., retort coal gas	40.23	39.0	1.50			4.76	1.50		13.01
25	Glasgow, Scot., retort coal gas	39.18	40.26	.29			10.0	.29		9.98
26	Otto coke oven, good part of gas	37.6	40.8	5.6			5.8	3.7	.4	6.1
27	Rich coke oven, Klumpp	37.4	40.4	7.1			5.8	2.1	1.6	5.6
28	Retort gas cannel coal, Sexton	36.1	37.8	6.8			16.4			2.9
29	Cleveland, Ohio, retort coal gas	34.8	28.8	.20			11.2	.20		24.8
30	Cannel-coal gas	27.7	50.0	6.8	13.0			.10		2.4
31	Newcastle coal, 10 minutes	20.1	57.38	6.19			10.62	2.21		3.50
	High volatile coal:									
32	Solvay oven, Blauvelt, 1st hr.	41.4	41.5	5.8	3.2	.90		.90	.50	5.8
33	Solvay oven, Blauvelt, 6th hr.	49.8	31.4	4.6	1.6	.90		1.1	1.1	9.5
[illegible]	[illegible] Blauvelt, 10[illegible]	60.4	13.6	0.2	0	0		.2	.4	10.2

122. Composition of Blast-furnace Gas and Air Gas

No.	Description	Volumetric analysis						
		CO	H_2	CH_4	CO_2	N_2	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$
1	Coke in small Dowson producer, Dowson and Larter	32.6	1.0	......	1.4	65.0	23.2	.96
2	Blast-furnace, splint coal, Sexton No. 3	30.1	6.2	3.2	5.4	55.1	5.58	.85
3	Blast furnace, Upper Silesia, Germany	29.7	6.3	......	7.8	56.2	3.81	.79
4	Metallurgical air gas, Lewes	29.0	2.5	......	4.0	64.5	7.25	.88
5	Coke, Lackawanna Steel Co.	28.4	1.7	......	11.8	......	2.40	.71
6	Blast furnace, unwashed, Sexton	28.19	10.24	1.78	6.23	53.56	4.53	.82
7	Blast furnace, splint coal, Sexton No. 2	28.06	5.45	4.39	8.61	53.38	3.26	.76
8	Blast furnace, English	27.71	1.34	......	8.62	62.33	3.21	.76
9	Blast-furnace gas	27.5	3.0	......	10.0	+5.0 59.4	2.75	.73
10	Bituminous coke air gas, Loomis Pettibone	27.3	5.1	.8	2.7	......	10.1	.91
11	Blast furnace, Frodingham coke, Allen	26.9	2.4	......	6.3	64.4	4.27	.80
12	Coke Lackawanna Steel Co.	26.8	1.4	......	8.6	......	3.12	.76
13	Scotch blast furnace, Wishan	25.83	4.55	3.45	7.21	58.96	3.58	.78
14	Isabella Furnace, U. S. Steel Co., Gayley	25.8	......	......	12.6	4.25	2.04	.67
15	Producer gas, little steam	25.3	9.2	3.1	3.4	58.2	7.44	.88
16	Dowson gas, average	25.0	18.0	3.0	7.0	47	3.57	.78
17	Producer gas, little steam	24.8	8.5	5.2	5.6	55.1	4.44	.82
18	Producer gas, little steam	24.0	9.8	3.4	6.0	55.6	4.0	.80
19	Blast-furnace coke	24.0	2.0	2.0	12.0	60.0	2.0	.67
20	Durham coke, Allen	23.84	2.34	......	10.94	5.55 57.3	21.8	.69
21	Blast furnace, Ledebur, Germany, coke, of 10 per cent. H_2O	21.6	1.8	1.8	10.8	54.0 +10.0	2.0	.67 -
22	Producer gas, little steam	20.8	6.9	2.2	4.6	64.9	4.53	.82
23	Producer gas, little steam	20.0	5.3	3.0	3.6	67.5	5.56	.85
24	Loomis Pettibone coal	20.0	14.0	2.0	8.2	55.5	2.44	.71
25	Loomis Pettibone wood	20.0	14.0	2.0	16.0	47.7	1.25	.55
26	Taylor gas, average	12.0	21.0	2.0	5	57.0	2.4	.71
27	Mond gas	12.0	29.0	2.0	14.5	42.5	.83	.45
28	Mond gas	11.5	28.5	2.1	15.0	42.9 water	.77	.43

124. Peat is the most recent of the formations. High-bog peat is formed from swamp moss, etc.; low-bog peat is formed from grasses around low bodies of water. They show the fibrous structure of vegetable origin, and range in color from ochre to brown and black; are very soft in texture, with no fracture; carry large moisture, oxygen and nitrogen content; the volatile matter is poorly combustible, containing largely CO_2.

125. Wood is still higher in oxygen and nitrogen, up to 40 to 45 per cent., with carbon from 45 to 49 per cent. Its scarcity and cost make its use as a power station fuel impossible except for saw mills and logging camps where the unmarketable waste may be profitably used.

126. Briquettes* are small artificial lumps of solid fuel made up by pressing peat, bituminous slack or anthracite culm, with a suitable tarry binder, so as to recombine the soft peat or unmanageable coal dust into a convenient lump form. Briquettes behave somewhat like lumps of soft coal, but usually are very troublesome in giving smoke. The cost of briquetting presses, and the need of suitable inexpensive binder, have prevented the wide use of briquettes in America. With peat briquetting is practically a necessity.

127. Mineral oils have their source in crude petroleum. The heavy oil engines, such as the Diesel and the Junkers run on raw petroleum; many of the smaller engines, however, are designed for kerosene and gasolene.

128. Gas for power production may be natural gas,† producer gas, coke-oven gas,‡ or blast-furnace gas.§ Illuminating gas is too expensive to use for anything but very small isolated plants. See Par. 26, 27 and 28.

129. The main feature in the use of gas is the engine compression pressure which is practicable; blast-furnace gas, being lean and much diluted with neutral matter, will stand very high compression; producer gas, natural and coke-oven gas, being richer (especially in hydrogen), cannot be compressed as much.

130. The cost of anthracite No. 1, No. 2, or No. 3 buckwheat is about \$1.10 to \$2.50 per ton at the plant in quantity. The mine cost is not over \$1.00 to \$1.40. **Bituminous coal** costs from \$2.00 to \$4.50 at the plant. The cost averages \$0.085 to \$0.095 per million B.t.u.

131. Fuel oil costs from \$0.01 to \$0.03 per gal. The cost averages about \$0.090 to \$0.094 per million B.t.u., in the oil-burning districts. While its cost per million B.t.u. is about the same as coal, or somewhat better, in the districts where it is available, it has a further advantage in the better boiler efficiency obtained, and in the elimination of some of the power-station auxiliary apparatus.

132. Natural gas costs \$0.07 to \$0.085 per million B.t.u. All of these figures apply chiefly to large consumers; the prices will go up considerably for small plants.

WATER SUPPLY AND PURIFICATION

133. Boiler feed. Boilers must be fed with fresh water of reasonable quality; in general, if a water is not potable, it is not fit, as it stands, for boiler feed. Wells, fresh-water lakes, rivers and ponds are the prime sources.

134. Water analysis to determine the value of a water as boiler feed should be performed by a chemist; but the ordinary tests for hardness, with standard soap solution, hydrochloric acid and methylorange, may readily be carried out in the plant in conjunction with water-softening apparatus.

135. The dangerous impurities are sulphuric acid (or other acids if the water has been contaminated by factories), grease and oils in quantity,

* Lucke, C. E., "Engineering Thermodynamics." McGraw-Hill Book Co., Inc., New York, 1913; Table CIV.
† Lucke, C. E., "Engineering Thermodynamics." McGraw-Hill Book Co., Inc., New York, 1913, Table CIX.
‡ Lucke, C. E., "Engineering Thermodynamics." McGraw-Hill Book Co., Inc., New York, 1913, Table CX.
§ Lucke, C. E., "Engineering Thermodynamics." McGraw-Hill Book Co., Inc., New York, 1913, Table CXIV.

and the natural scale-forming salts such as magnesium, iron and calcium carbonates and sulphates.

136. The treatment of feed water has been outlined in Par. 49 to 51.

137. The use of condensate returned from surface condensers is usually a valuable way of eliminating most of the boiler incrustation. The only drawback lies in the presence of oil and grease in the condensate, if the condenser serves a reciprocating engine. Quantities of oil in excess of 0.4 to 0.5 g. per gal. are objectionable, as they collect whatever soft scale may be present in the boiler and form a brown mass which bakes on the tubes and has the same effect as hard scale.

138. Oil and grease extractors, designed on the same general lines as steam separators are employed between the engine and condenser, in order to keep down the amount of oil passing into the condensate. As a matter of fact, the baffles placed in these devices are really of very little use; it is the reservoir effect of the extractor in slowing down the current of steam which really does the work. Where turbines are the prime movers, no grease or oil separators are required. Fig. 20a shows a typical separator.

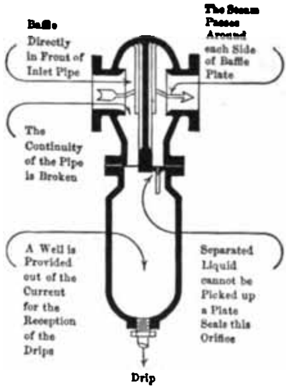


FIG. 20a.—Horizontal steam separator.

COAL AND ASH HANDLING

139. Coal is delivered to the power plant of any size, either by barge, schooner or railroad car. Handling coal by truck is practically too expensive for use in any but small isolated plants.

140. Unloading may be done by a grab-bucket digger, or if delivered by car, an elevated trestle may be employed, dumping from the car bottoms. For most of the large plants, a coal tower with a 1-ton or 1.5-ton clam-shell digger is employed to unload and hoist the coal high enough to pass by gravity through crushers and weighing scales, and finally to conveyors or coal cars. In smaller plants, a locomotive crane, or monorail telpher, may perform the same work and also serve the storage yard. Towers may be either steam or electric driven; the former is generally the cheaper and more rugged construction.

141. Crushers are heavy rolls of cast iron, studded with teeth, geared together in such fashion as to crush the coal to small size; some spring or relief device must be fitted to allow harder materials to pass through, such as link chains, sprags and occasionally a car coupler. Crushers are not needed for anthracite.

142. Conveyors are of five principal types; scraper, reciprocating, belt, bucket and suction. **The scraper conveyor** consists of flights or paddles rigidly fastened upon a special bar-link chain, and dragged along in a trough shaped to the flights. The **reciprocating conveyor** is very similar, but only moves a few feet forward and backward. The flights are so hinged that on the backward stroke they lift out of the coal and trail over the top, digging in again when the chain reverses.

143. Belt conveyors consist of wide rubber or textile belts travelling continuously over idlers spaced from 2 ft. to 4 ft. apart, and with the outer pulleys tilted up to trough the belt. The belt conveyor is not suitable for ashes.

144. The bucket conveyor has separate buckets from 24 in. to 48 in. square, either fixed rigidly to long-pitch side links, or pivoted in the center, so as to remain vertical. In the latter case, all of the conveyor is malleable iron except the rails and framing. It is exceptionally suited to handling ashes.

145. The suction conveyors are used only for ashes; coal is too heavy and packs too readily to be handled in this manner. The suction is usually provided by a high-speed centrifugal blower, attached to the outlet. The rest of the equipment consists merely of 10-in. and 12-in. cast-iron pipe lines with swing-door openings at which the ashes are admitted. The drawback to this system is the deterioration of the fan from grit, and all turns and elbows from abrasion. If the ashes are too wet, the blocking of the pipes is practically certain to give trouble.

146. Comparisons of conveyors. The belt conveyor is very satisfactory on "carry" of coal, but not so good on "lift;" the bucket conveyor is probably the most satisfactory all-around conveyor, handling coal or hot ashes equally well in lift or carry; it is, however, the most expensive.

147. Another method of conveying is by cable cars; where possible this gives the cheapest and most satisfactory means of carry.

148. Storage of coal outdoors for power-plant purposes requires less care and handling than for coal companies, as the material is turned over fast enough to avoid much loss by weathering. The storage yard may be served by: (*a*) a locomotive crane, (*b*) a gantry crane, (*c*) a Dodge girder unloader, (*d*) a telpher grab bucket operating on an overhead structure, (*e*) or if the storage is in a pit, by a conveyor in a tunnel. Usually the capacity should equal at least two weeks supply, or better, one month. One company having trouble from spontaneous combustion of bituminous coal, uses a concrete pit flooded with water; but this is generally unnecessary, the excessive wetting of the coal being a drawback.

149. Bunkers placed under the roof of the boiler house are almost always provided, to feed coal by gravity to the fires. Where external storage is available, two to four days supply in the building is all that is necessary; but in city plants where external storage is impossible, ten days to two weeks supply must be provided.

150. Bunker construction. Steel framing with concrete or cinder-concrete lining, faced with granolithic or other hard finish, is the best construction for large work; for smaller work, the suspended type, catenary shape, of plate steel with concrete lining, or simply of reinforced concrete, has been much used.

151. Hoppers. Ash hoppers of structural material with brick lining are the standard practice, but reinforced concrete is also being used to some extent. The coal hoppers should always be of sheet steel, $\frac{3}{16}$ in. being the usual thickness; it is good practice to reinforce the hoppers with renewable wearing plates where the abrasion is severest.

152. Weighing is done at the hoisting tower (just after crushing) in hoppers carried in knife-edge supports which operate standard beam scales; two sets are required, one hopper receiving the continuous flow of coal from the crusher while the other is being weighed. Reversing gates for the supply, and trip gates for the weighing hoppers are under the control of the weighmaster.

153. Automatic scales of the hopper type are in use for anthracite, and are fairly accurate, as the angle of repose and slip of anthracite in the small sizes is very little affected by moisture, and the small size allows ready control of the flow; with bituminous coals they are somewhat unsatisfactory, as the highly variable angle of repose affects the spillage when closing the feeder gates, making the "dribble" a variable quantity. Conveyor scales of several makes are on the market, but not widely used.

154. There is also a coal meter for use in pipe downtakes, acting on the principle of a propeller driven by the moving coal, which is very simple and quite accurate for small sizes of coal. It is not yet fully satisfactory for lump coal and run of mine.

155. Spouting of coal in closed pipes must always be done at an angle of 45 deg. or steeper to be satisfactory, if the spout is a closed pipe; 8-in. to 10-in. pipes may be used with anthracite; but nothing less than 12 in. should be used for bituminous crushed coal. In designing hoppers and spouts it should be remembered that the intersection of two sides each at 45 deg. from the horizontal, is much less than 45 deg., so that the coal will always hang at such a junction.

156. The power required by belt conveyors is given by the following formula:

$$H.P. = \frac{CTL}{1,000} + \frac{TH}{1,000} \tag{29}$$

where C = power constant, T = load in tons per hour, L = length of conveyor in feet, H = lift of conveyor in feet, and B = width in inches.

Values of C for material weighing 25 lb. to 75 lb. per cu. ft.

B (in.)	C	H.P. on account of trippers in belt	B (in.)	C	H.P. on account of trippers in belt
12	0.234	0.5	26	0.187	2.0
14	0.226	0.5	28	0.175	2.25
16	0.220	0.75	30	0.167	2.5
18	0.209	1.0	32	0.163	2.75
20	0.205	1.25			
22	0.199	1.5	34	0.161	3.0
24	0.195	1.75	36	0.157	3.25

Add 20 per cent. for conveyors under 50 ft. long.
Add 10 per cent. for conveyors between 50 and 100 ft. long.
The formula does not include friction of gear drive. Add about 10 per cent. for the drive and 10 per cent. for each 180 deg. turn idler, exclusive of the head and tail pulleys. Speeds run from 300 to 450 ft. per min.

157. The power required by flight or scraper conveyors is as follows:

$$H.P. = \frac{ATL - BWS}{1,000} \tag{30}$$

where A and B are constants in the table below, W = weight of chain and flights in both runs, L = length of feet, S = speed in feet per minute, and T = load in tons per hour.

Values of A and B

Inclination	0 deg. for horizontal	5 deg.	10 deg.	15 deg.	20 deg.	25 deg.	30 deg.	35 deg.	40 deg.	45 deg.
A.......	0.343	0.42	0.50	0.585	0.66	0.73	0.79	0.85	0.90	0.945
B.......	0.01	0.01	0.01	0.01	0.009	0.009	0.009	0.008	0.008	0.007

Speeds 100 to 200 ft. per min.

158. Bucket Elevators and Conveyors

Size, buckets,	12"×12"			24"×15"			36"×20"			48"×24"		
Spacing..........	18"	24"	36"	18"	24"	36"	18"	24"	36"	48"		
H.p. for 100' vertical lift..........	0.46	0.35	0.23	1.35	1.0	0.67	3.8	1.9	3.5	2.7		
H.p. for 100' horizontal run empty	1.2	1.1	0.9	1.7	1.5	1.3	2.4	1.6				
H.p. for 100' horizontal run full, anthracite.......	2.5	2.1	1.5	5.3	4.3	3.1	12.4	6.6				
H.p. for 100' horizontal run full, bituminous coal.	3.2	2.6	1.9	7.4	5.8	4.2	18.0	9.4				

Add 5 per cent. for each turn.

159. Ash handling is very frequently done by trolley locomotive and side dump cars, in large plants; these allow for thorough quenching before delivering to conveyors for elevation. Of the conveyors, the malleable-bucket type lasts the best in ash service. A skip hoist may be substituted for conveyors, for elevation; and where it can be used, is a most satisfactory means of disposal. Ash bunkers are almost always provided for storage purposes, as in many plants the ash removal from the individual boiler ash hoppers is done on night shift, when cars or boats are not readily available.

160. Helical or screw conveyors. The power required is

$$H.P. = \frac{W'LC}{33{,}000} \tag{31}$$

where W' = capacity in lb. per min., L = length in feet and C = 0.67 for coal, 1.00 for ashes.

161. Typical combinations are: low-hoist grab-bucket tower, containing crusher and scales; belt conveyors, horizontal, to power station; V-bucket or pivoted-bucket elevator, belt distribution at top of building to bunkers. Inclined belts may be used for "lift," but are not too satisfactory. Belts are very good on "carry," however. If the tower is close to the building, bucket elevators may be used both for lift and carry. A very good arrangement is: high towers, performing the whole lift at once, delivery through crusher and scales to cable car for distribution over bunkers. For small or moderate size plants telpher grab buckets offer a satisfactory solution, serving storage yard, cars or boats, and bunkers on overhead monorail structure. The usual system for ash disposal consists of collection from ash hoppers by industrial railway and trolley locomotive, and delivery to skip hoist or bucket elevator for lifting to ash storage pockets, for which the ashes are spouted to cars or barges. The vacuum system is useful for small and moderate size plants; or a single hand car may be enough. Belt conveyors should never be employed for ash handling.

162. The cost of conveyors is exceedingly variable and is dictated chiefly by structural conditions. It is almost impossible, therefore, to give unit costs. Belt conveyors of the lengths usual in power station work cost from \$1.10 to \$1.40 per inch of belt width, per foot of conveyor; bucket elevators and conveyors, roughly \$0.18 to \$0.25 per ton capacity per hour, per foot of conveyor.

STEAM ENGINES

163. Source of energy. The steam engine must obtain its energy from the heat drop available by adiabatic expansion between any two pressures. For adiabatic expansion, there must be no heat interchange with the surroundings while the mass of steam in the cylinder undergoes the expansion; therefore a non-conducting cylinder is required. This is commercially unattainable, and consequently the expansion is never truly adiabatic. The heat drop obtainable is always less, if the expansion is not adiabatic.

164. The thermal efficiency of a perfect engine working on the Rankine cycle (Par. 166) is given as

$$E_r = \frac{H_1 - H_2}{H_1 - h_2} \tag{32}$$

where H_1 = total heat of steam at initial condition (pressure and quality or superheat), H_2 = total heat of steam at final pressure after an adiabatic expansion (constant entropy), and h_2 = heat of the liquid, at final pressure. For modern steam pressures, superheats and vacua, this possible efficiency does not exceed 35 per cent. for turbines. The usual steam pressures and vacuum for reciprocating engines would not give a higher efficiency than 30 per cent.

165. The efficiency ratio for good compound engines seldom reaches 60 per cent., giving an actual thermal efficiency of 18 per cent., 13 per cent. and 14 per cent. are more commonly reached in large size reciprocating engines.

166. The Rankine cycle upon which these comparisons are based pre-supposes a non-conducting cylinder, no leakage and no clearance; admission at constant pressure, instantaneous cut-off; expansion adiabatically to the back pressure; exhaust at constant pressure; and since there is no clearance, no compression is required. In the real engine, conduction of heat, leakage, wire-drawing, friction, incomplete expansion, and clearance all tend to reduce the efficiency ratio by adding losses.

167. A single-expansion engine is one in which the complete expansion from initial to back pressure takes place in one cylinder; a **compound engine** is one in which the expansion is from the initial pressure to an intermediate pressure in one cylinder, and from this intermediate pressure to the back pressure in a second cylinder. A **triple or quadruple expansion engine** performs the expansion similarly in three or four stages, respectively. All engines using more than one cylinder to complete the expansion are termed **multiple expansion**, in general.

168. Cut-off usually takes place at $\frac{1}{4}$ to $\frac{1}{3}$ stroke in simple non-condensing engines, $\frac{1}{5}$ to $\frac{1}{8}$ in condensing. **The number of expansions** is the ratio of final cylinder volume to volume contained at cut-off; for simple non-condensing engines the ratio is 3 to 4; simple condensing, 5 to 8; compound and triple, etc., 8 to about 50. For power station purposes 16 to 20 is common, with compound condensing units.

169. Non-condensing engines exhaust freely to atmosphere, or to a system in which the pressure is atmospheric or a few lb. per sq. in. above; condensing engines exhaust to a condenser in which a partial vacuum is maintained, below atmospheric pressure.

170. High-speed engines operate at 350 to 250 rev. per. min., in sizes from 5 h.p. to 500 h.p. **Medium-speed** engines operate at 250 to 150 rev. per min. in sizes from 250 h.p. to 1,000 h.p. **Low speed** engines (generally Corliss types) operate from 100 to 70 rev. per min., in sizes from 400 h.p. to 8,000 h.p.

171. Single-acting engines take steam on one side of the piston only; they may be simple, compound or triple expansion; and are always high-speed engines, usually with vertical cylinders.

172. The slide valve in the original D-valve form is unbalanced, moving on flat faces; it is unsuitable for large sizes and heavy pressure. Balanced forms of the slide valve, such as the Ball, Skinner, Mackintosh & Seymour grid, etc., overcome some of these objections. **The piston valve,** shaped like a piston, is inherently balanced; it can be made tight, but requires large clearance percentage, on account of length of the ports.

173. The Corliss valve is of general cylindrical shape, oscillating over ports parallel to the axis of the valve and crosswise of the cylinder; it can take care of but one function, therefore four are required per cylinder, two for admission and two for exhaust. The preceding types, except the grid, take care of all four functions with one valve. The Corliss valve is always used with a trip or releasing gear to vary the cut-off without changing any other event in the steam admission or exhaust. In all fixed valve gears, it is usually impossible to vary the cut-off without varying other functions at the same time.

174. Poppet valves are those in familiar use on automobile engines; but for steam use are generally made double-seated so as to be balanced. They have met with great success in Germany, and are undoubtedly the best type for superheat. They may be used with releasing gears, and require four valves per cylinder, as with the Corliss type.

175. Jackets for keeping the cylinder hot with live steam, to reduce condensation in the cylinder, have been used with some success for pumping engines; but have not been successful in many other applications.

176. Receivers are used to eliminate variations of pressure between high-pressure and low-pressure cylinders in multiple-expansion engines, due to intermittent exhaust and admission: they must be of large capacity to be effective. **Reheating coils** are sometimes employed to dry the steam passing through these receivers, to improve conditions in the succeeding cylinders. They are of practically no value in engines for power-station service.

177. Governors for steam engines are of two principal types: flyball and shaft governors. The flyball type consists of two or more weights or balls supported by movable arms, the whole rotated around a shaft so that centrifugal force tends to throw the balls outward; this tendency is resisted by weights or springs, and the relative movement of the arms made to operate the engine valve gear. In the **shaft governor** the weights are arranged to rotate vertically around the main crank shaft, and attached direct to the eccentrics without the intervention of other mechanism. It cannot be used with slow-speed engines, nor with any form of trip or releasing gear. Inertia effects are introduced in the Rites type of shaft governor.

178. Either centrifugal or inertia types of governor can be made isochronous, but are generally only approximately so; isochronism, or complete instability of the governor at all speeds except the correct speed, is undesirable.

179. Engine frames are usually made entirely of cast iron, for stationary, work; the vertical engines are of the A-frame type in large and small sizes and of the enclosed type, with automatic oiling, for small and moderate size, only. Horizontal engines of small and moderate sizes, may be enclosed, with automatic or splash oiling.

180. Girder frames. The large horizontal Corliss type engines are made with girder frames connecting the cylinders and the main bearings, for the **standard type**; in the **heavy-duty type**, the girders are completely surrounded by the frame, which is carried down to the sole plate all the way from cylinder to main bearing.

181. Mean effective pressure is the average pressure which if exerted during the full stroke would equal the work done by the varying pressures really existing during the course of the stroke. The mean effective pressure (m.e.p.) is obtained from the indicator card as follows:

$$\text{M.e.p. in lb. per sq. in.} = \frac{\text{card area in sq. in.} \times \text{scale of spring}}{\text{card length in inches}} \qquad (33)$$

The shorter the cut-off, the lower the m.e.p. for any given steam pressure and back pressure; consequently the larger the cylinder dimensions become for a given horse-power, other things being constant.

182. Cylinders are usually so proportioned as to divide the total work equally between high- and low-pressure cylinders; roughly, for compound engines, the cylinder ratio is given by

$$Rc = \left(\frac{P_1}{P_3}\right)^{\frac{1}{2}} \text{ and } P_2 = (P_1P_3)^{\frac{1}{2}} \qquad (34)$$

where Rc = cylinder ratio, or ratio of low-pressure displacement to high-pressure displacement, P_1 = initial pressure in lb. per sq. in. absolute, P_2 = receiver pressure in lb. per sq. in. absolute, and P_3 = back pressure in lb. per sq. in. absolute.

This is based on no clearance, no compression, equal cut-off in both cylinders, logarithmic expansion, and receivers of infinite capacity. It is varied in practice by the clearance of both cylinders; by finite receivers and by the effects of wire-drawing.

183. The speeds usually employed in America are given in Par. **170.** Snell gives a table of usual English practice, which is slightly higher than American usage.*

For sizes above 1,750 k.w., the speed is from 75 to 60 rev. per min., for Corliss engines. **Piston speeds** range from 350 to 600 ft. per min. in the high-speed engines, and from 600 to 750 ft. per min. in the low-speed, long-stroke Corliss types.

184. Indicated horse-power is given by the indicator, and is the amount of power actually developed in the cylinder by the steam. **Brake horse-power** is the actual output at the shaft.

$$\text{I h.p.} = \frac{PLAN}{33{,}000} \qquad (34a)$$

where

P = mean effective pressure, lb. per sq. in.;
L = length of stroke, ft.;
A = piston area, sq. in.;.
N = number of strokes per min.

* Snell, "Power House Design," p. 146, Table XLVII.

185. The mechanical efficiency is given by the formula

$$E_m = \frac{\text{b.h.p.}}{\text{i.h.p.}} \tag{35}$$

where E_m = mechanical efficiency, b.h.p. = brake horse-power, and i.h.p. = indicated horse-power.

$$\text{Friction h.p.} = \text{i.h.p.} - \text{b.h.p.} \tag{36}$$

Lucke gives the formula

$$E_m = 1 - \frac{K_1 - K_2}{(\text{m.e.p.})} \tag{37}$$

where $K_1 = 0.02$ to 0.05, average 0.04; and $K_2 = 1.3$ to 2.0, average 1.6.

186. Mechanical design of the steam engine is limited by first cost, efficiency and reliability: these three factors exert influences in varying directions. Low cost per h.p. capacity indicates high rotative speed and simplicity of mechanism. High economy indicates lower speeds, complicated valve gear, multiple cylinders, reheaters and large receivers. Reliability dictates heavily built parts, elimination of small parts, and especially the elimination of small bearings. **The outcome** is that for small powers, moderate speeds and low steam pressures, the simple non-condensing slide-valve engine, throttle governed, is used. Under steady load and speed it has fair economy, and is very reliable. For small electric isolated plant service, the high-speed automatic engine, with shaft governors and piston-valves or balanced slide valves, is used. For higher powers the low-speed condensing engine, either simple or compound, is adopted. The higher economy demanded justifies the increased cost due to low speed and Corliss valve gears; reliability is also provided, since the complex gear is now large enough not to be fragile.

187. Friction loss, which causes the difference between brake and indicated power, is made up of bearing friction; piston, valve and rod friction; and windage. The principal bearing friction occurs in the main bearings, crank pins and crosshead slippers, the remainder occurs in the valve gear. **Suitable lubrication** greatly reduces this kind of friction. Piston, valve and rod friction occur in a steam atmosphere where lubrication is at best uncertain. The latter depends upon the type of valve, whether balanced or not; upon the kind and number of piston rings; and upon the condition of the rod packing.

188. Lubrication of all bearings is accomplished with engine or machine oil; or prepared grease. In small engines, sight-feed oil cups are commonly used; for large units, a central gravity oiling system piped to all bearings is considered the best practice. In the latter case, the oil is caught after use and returned to filters, whence it is again pumped to the elevated supply tank. In this way copious supplies of oil may be given without great expense and the friction is very much reduced. **Splash oiling** in closed crank-case engines is employed for high speeds and small or moderate sizes. Grease can only be fed by individual compression cups at each bearing.

189. Cylinders are oiled with heavier lubricants, whose lubricating qualities in the steam become the same as ordinary engine oil at room temperatures. Cylinder oil is fed into the steam pipe just above the throttle, or into the steam chest; a hydrostatic sight-feed lubricator, or a force pump may be employed. The latter is considered the better method. **Graphite** has been used with much success in assisting cylinder lubrication; the chief difficulty lies in getting it into the cylinder.

190. The total losses occurring in a steam engine can be classified as follows: cylinder condensation; leakage; clearance; incomplete expansion; wire-drawing; radiation; mechanical friction.

191. Cylinder condensation is caused by the conduction of heat away from the steam through the cylinder walls; the cylinder assumes an average temperature about half-way between steam admission and exhaust temperatures. Consequently the hot in-coming steam gives up heat to the cooler walls; some of this is returned, too late to be of use, during the exhaust stroke. **Moisture in the entering steam** makes this effect worse; but **superheat** much improves it, by increasing the surface resistance to heat transmission.

192. Leakage occurs past all sliding joints—the joint behaves like an elongated capillary orifice for steam flow; such joints occur at the piston, the piston rod, the valve seat and the valve rod. The means of reducing leakage are generally snap and spring-rings for pistons and piston valves, pressure plates insuring a close fit for flat balanced valves, and sectional metallic or soft packing compressed by a gland, for all rods.

193. Clearance is any space left between the valve and the piston, when the latter is at the end of the stroke. It varies from 3 to 4 per cent. of the displacement in well-designed Corliss engines; from 8 to 14 per cent. on high-speed and medium-speed engines; from 15 to 30 per cent. in badly designed or low-price slide-valve and piston-valve engines.

194. Zero clearance is not possible in any commercial construction, since piston must have some clearance from the cylinder head, and the steam and the exhaust passages must always have some volume. Clearance necessitates compression, in order to bring the volume of steam trapped in the clearance space to approximately the initial pressure; this has the double effect of cushioning the engine and avoiding loss due to filling the clearance space with fresh steam at each stroke.

195. Incomplete expansion is due to the limitations in size of cylinders, which commercially cannot be made large enough to handle the high volumes at low pressure.

196. Wire drawing is the term applied to pressure drop between steam chest and cylinder due to insufficient area in the valves and ports, or slow opening of the valve; it reduces the effective pressure and therefore causes a loss.

197. Radiation is the heat loss which occurs when the cylinder temperature is higher than that of the surrounding air. Not only the cylinder, but also those parts of the engine which become hot by conduction, radiate heat. The cylinder is heavily lagged (or even jacketed in a few cases) to reduce the loss. This loss amounts to 1 or 2 per cent. of the total heat of steam used by the engine.

198. Indicated horse-power is obtained from the indicator card (Par. 184) and was the basis of most guarantees up to a few years ago. Brake horse-power, which is the useful power delivered at the shaft, is now more common.

199. Non-condensing single-cylinder engines are usually rated at $\frac{1}{4}$ or $\frac{1}{3}$ cut-off and 100 lb. gage steam pressure; compound non-condensing engines at $\frac{1}{3}$ to $\frac{1}{3}$ cut-off in the high-pressure cylinder and 150 lb. gage pressure; simple condensing engines at $\frac{1}{4}$ or $\frac{1}{5}$ cut-off and 125 to 150 lb. gage pressure. The guarantees should always state steam pressures, back pressures and actual cut-off, instead of the above figures.

200. The locomobile type of engine developed abroad and lately introduced in this country is a combination consisting of a tandem compound engine mounted directly on an internally fired boiler. The high-pressure cylinder is in the smoke flue, and the low-pressure cylinder is jacketed by the steam dome. A superheater is also fitted in the smoke box. The advantage of maximum jacketing effect, high superheat, and practically no piping losses, make the fuel economy high. This type of unit is now manufactured in this country, but the advantages of the turbine, coupled with the comparative inflexibility of a single boiler-engine unit, will prevent extensive use of the locomobile in America.

201. Exhaust steam heating makes use of the heat wasted by the engine; it may amount to 80 or 90 per cent. of the heat originally in the steam. Consequently, the power is obtained at very low cost for fuel, since most of the ordinary inherent losses are recovered as heat in the heating system. Hence in most industrial plants serving buildings with lighting and power service, non-condensing engines are used. Generally the expense of a condenser to be used during the summer months will not be justified, as the small gain in economy for the year will not offset the fixed charges on the condenser equipment.

202. Average steam-engine performance. The brief summary which follows, with a few omissions and additions, is taken from the more extensive tables given in Gebhardt's "Steam Power Plant Engineering."

Expansions	Number	Type of engine and operating conditions		Horse power	Initial pressure lb. gage	Back pressure lb. abs.	Temperature of feed water	Superheat at admission, deg. fahr.
Simple (1×)	1	Single-valve	Range	33–257	80–124	Atm.	...	...
		Non-cond.	Avg.	137	104.9	Atm.	...	...
	2	Single-valve	Range	33–204	69.3–114	1–3.2	...	...
		Condensing	Avg.	116	83.3	2.4	...	...
	3	Four-valve	Range	120–506	99.6–125.4	Atm.	...	...
		Non-cond.	Avg.	264	106.9	Atm.	...	...
	4	Four-valve	Range	145–613	67–96	1.0–2.9	...	...
		Condensing	Avg.	381	78.2	1.9	...	...
Comp. (2×)	5	Non-condensing	Range	33–1125	114–210	Atm.	212*	...
		Sat. steam	Avg.	359	151	Atm.	212*	...
	6	Condensing	Range	340–7365	100–185	.8–2.2	100–130	...
		Sat. steam	Avg.	2770	152	1.47	119.3	...
	7	Condensing	Range	145–2202	114–163	.84–2.15	97–130	40–343
		Superheated	Avg.	673	132	1.39	112*	218
(3×)	8	Condensing	Range	464–1823	134–185	.85–1.8	122–158	...
		Sat. steam	Avg.	988	152	1.22	145	...
	9	Condensing	Range	549–2940	147–173	.79–1.28	95–111	87–264
		Superheated	Avg.	1517	165	1.00	102*	195
(4×)	10	Condensing	Range	712–990	200–243	.9–1.25	310–334	...
		Sat. steam						
1×	11	Locomotives, sat. steam		399 & 975	110 & 196	Atm.	...	...
2×	12	Locomotive, sat. steam		495	210	Atm.	212*	...
1×	13	Corliss N. C. sat.		237	103.5	Atm.	...	...
1×	14	Corliss Cond. sat.	jacketed	155	103.8	1.2	...	...
1×	15	Poppet Cond. sat.	jacketed	262	79	1.36	...	...
3×	16	Binary, superheated		211	143	2.2	...	221
2×	17	White automobile, sup.		40	426	Atm.	212	316

1× = simple engine; 2× = compound; 3× = triple; 4× = quadruple expansion.
* Ideal feed-water temperature (temperature of hot well).

Expansions	Number	Type of engine and operating conditions		Pounds of dry steam per i.h.p. hour	B.t.u. per i.h.p. minute	Thermal efficiency of engine 42.42 + B.t.u.	‡ Cylinder efficiency or eff. ratio	M.e.p. referred to L.P. cyl.	Remarks
Simple (1×)	1	Single-valve	Range	26.0–30.6	436–510	8.33–9.75	57.5–65.7	35.0–58.5	
		Non-cond.	Avg.	27.63	463	9.20	61.3	44.5	Avg. 7 engines.
	2	Single-valve	Range	22.2–27.5	410–490	8.67–10.35	39.7–41.3	38.1–41	
		Condensing	Avg.	25.7	450	9.47	40.5	39.9	Avg. 4 eingines.
	3	Four-valve	Range	22.24–25.9	374–434	9.80–11.30	66.2–73.5	25.7–48.4	
		Non-cond.	Avg.	24.06	404	10.54	70.2	38.6	Avg. 5 engines.
	4	Four-valve	Range	18.5–22.0	342–397	10.7–12.4	48.0–60.2	30.9–38.2	
		Condensing	Avg.	19.84	359	11.85	51.3	35.0	Avg. 4 engines.
Comp. (2×)	5	Non-condensing	Range	17.17–22.3	291–376	14.5–11.2	65.5–83.0	31–56	
		Sat. steam	Avg.	20.3	342.3†	12.4	72.3	37.1	Avg. 10 engines.
	6	Condensing	Range	11.20–12.70	220–234	18.1–19.2	63.5–76.5	13.0–27.9	
		Sat. steam	Avg.	12.14	223.6	18.8	69.0	21–2	Avg. 8 engines.
	7	Condensing	Range	8.58–11.80	176.1–223	24.0–19.0	78.5–54	....	
		Superheated		10.0	202.2†	20.98	69.0	....	Avg. 8 engines.
(3×)	8	Condensing	Range	10.33–11.33	196–208	20.4–21.63	70.0–76.0	19.5–23.4	3 pumping, 1 mill engine.
		Sat. steam	Avg.	10.81	202.2	20.98	72.8	21.05	
	9	Condensing	Range	8.97–10.00	188.7–207.3	20.4–22.6	66.7–73.5	....	
		Superheated	Avg.	9.65	198.4†	21.4	70.1	....	Avg. 5 engines.
(4×)	10	Condensing	Range	12.26–11.25	186–169.3	22.8–25.05	74.2–88.2	35.5	Trans. A. S. M. E. 1900
		Sat. steam		2 Engines with Regeneative Cycle					and 1907.
1×	11	Locomotives, sat. steam		24.97–23.4	420–398	10.1–10.65	65.0–55.9	54–75.6	A. S. M. E., XIV
2×	12	Locomotive, sat. steam		18.6	316†	13.4	68.5	55	St. Louis Expo. 1904.
1×	13	Corliss N. C. sat.	jacketed	21.5	358	11.85	78.0	42.1	Peabody's "Thermodynamics."
1×	14	Corliss Cond. sat.	jacketed	16.5	302	14.05	53.3	32.0	Peabody's "Thermodynamics."
1×	15	Poppet Cond. sat.	jacketed	15.0	275	1.54	63.8	30.2	*Zeit. v. D. I.*, Aug., 1905.
3×	16	Binary, superheated		8.6	158.3†	2.68	....	....	Franklin Inst., 1902.
2×	17	White automobile, sup.		11.96	244	17.4	68	....	A.S.M.E., 1907.

‡ Cylinder Efficiency = (Theor. B.t.u. per min. with Clausius or Rankine cycle with complete expansion) + actual B.t.u. per i.h.p. min.

† Above ideal feed-water temperature.

203. Water rates vary greatly with steam pressure, superheat or moisture, and back-pressure. No comparison of water rate alone can be fair, unless the steam conditions are known. Fig. 21 shows typical curves for engines of small and moderate size, and Fig. 22 for large engines. The Rankine-cycle efficiency ratio is the best means of comparison for varying steam conditions. For small non-condensing engines, the efficiency ratio varies from 0.50 to 0.65; condensing lowers the ratio about 0.05 to 0.10.

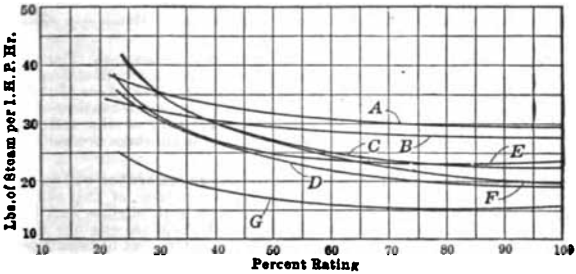


Fig. 21.—Water rates of small engines.

A, Automatic single cylinder, non-condensing, pressure = 125 lb., 80 i.h.p.; *B*, automatic single cylinder, non-condensing, pressure = 125 lb., 100 i.h.p.; *C*, automatic tandem compound, non-condensing, pressure = 140 lb., 200 i.h.p.; *D*, automatic tandem compound, condensing, pressure = 140 lb., 300 i.h.p.; *E*, Corliss or medium speed four-valve simple, non-condensing, pressure = 130 lb., 200 i.h.p.; *F*, Corliss or medium speed four-valve tandem compound, non-condensing, pressure = 150 lb., 300 i.h.p.; *G*, Corliss or medium speed four-valve tandem compound, condensing, pressure = 160 lb., 300 i.h.p.

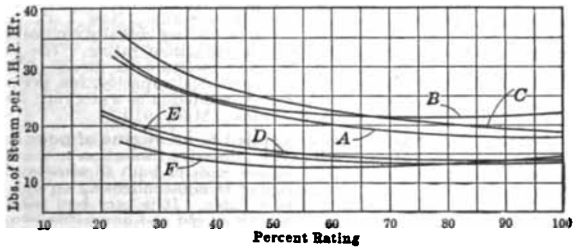


Fig. 22.—Water rates of large engines.

A, Automatic tandem compound, condensing, 700 i.h.p., pressure = 150 lb.; *B*, Corliss or medium speed four-valve simple, non-condensing engine, 900 i.h.p., pressure = 130 lb.; *C*, Corliss or medium speed four-valve tandem compound, non-condensing engine, 950 i.h.p., pressure = 150 lb.; *D*, Corliss or medium speed four-valve tandem compound, condensing engine, 900 i.h.p., pressure = 160 lb.; *E*, Corliss or medium speed four-valve tandem compound, condensing engine, 1,500 i.h.p., pressure = 160 lb.; *F*, Manhattan type, double compound Corliss condensing engine, 10,000 i.h.p., pressure = 175 lb.

204. The thermal equivalent of a horse-power hour is 2547 B.t.u. Large high-grade engines actually develop a horse-power on 15,600 to 17,000 B.t.u.; small engines, 25,000 to 34,000 B.t.u. Par. 202, gives these values in terms of B.t.u. per indicated h.p. per min.

205. The effect of raising the back-pressure upon an engine can readily be seen from the Mollier diagram; the available adiabatic heat drop is reduced and consequently, although the efficiency ratio may not change, the thermal efficiency of the Rankine cycle is reduced, and therefore the thermal efficiency of the actual engine. In other words, the water rate increases. Since the heat drop is less, there will be somewhat more heat per lb. in the exhaust.

206. Exhaust steam turbines are of valuable use in connection with non-condensing engines used intermittently. The economy of such engines (say, for rolling mill service) is low, and large quantities of steam are exhausted to the air one moment, and almost none the next.

207. Regenerators. The exhaust of several such engines (Par. **206**) is piped to regenerators, which are simply heat reservoirs containing a large body of hot water in contact with the exhaust steam. The effect of the regenerator is to absorb heat, when delivered in excess, by condensation of steam; and to release it upon shortage, by evaporation of some of the water. The slight pressure variations between full delivery and shortage are sufficient to accomplish this.

208. A low-pressure turbine served by the regenerator will operate on about double the water rate of high-pressure machine of the same size. The resulting overall water rate for the output of both engines and turbines may be from 20 to 50 per cent. less than with the original engines alone; or the energy delivered by the turbine may be considered as obtained practically for nothing except the fixed charges and maintenance on the regenerator, piping and turbine.

209. Low-pressure turbines may sometimes be used in connection with high-grade engines as a convenient means of increasing capacity. The efficiency ratio of non-condensing engines is always much higher than that of condensing engines of the same size (Par. **201**); this is due mainly to the low-pressure cylinder, which has a very low efficiency. The low-pressure turbine has a better efficiency ratio than the high-pressure machine, because the friction of the steam is always less at the lower densities, and the friction is again reduced by the removal of the moisture in the steam before reaching the turbine.

210. The largest installation of low-pressure turbines is at the Interborough Rapid Transit Company's 59th-Street Station, New York. Five Curtis turbines of 7,500 kw. maximum rating are individually connected to five compound Corliss engines of 7,500 kw. maximum rating. The steam pressure is 190 lb. gage; vacuum, 28.5 in.; moisture in steam, 1.5 per cent. The net results are: increase of economical capacity, 146 per cent., increase of maximum capacity, 100 per cent. reduction of water rate, 25 per cent. (Stott and Pigott; A. S. M. E. Trans.; Mar. 1910.)

211. When exhaust turbines are applied to an engine of poor economy, the saving is even greater. However, there is a limitation to the use of the low-pressure turbine, in that the engines must be built to withstand the extra pressures which result from changing to non-condensing service, and must be in good enough condition to be reliable. It is very bad policy to make such an installation in connection with an old and unreliable engine.

212. Engine specifications should cover the following points; (a) Number and location; character of building. (b) Type, service, and manner of connection to load. (c) Principal dimensions. (d) Steam and back pressures, normal and overload capacity. (e) Speed regulation under all kinds of load variation; variation of angular velocity. (f) Satisfactory operation, noise, vibration, etc. (g) Tests and inspection. (h) Construction details—cylinder, piston-rods, crosshead, connecting-rod, pins and bearings, shaft and flywheel, governor, valves, frame, foundation, lubricators, and receivers, piping, engine-stop, erection. (i) General: arrangements for delivery into plant.

213. The only large-size engines in use in electric power plants are of the Corliss type, and the grid-valve type. The moderate-size, medium-speed or high-speed engine may be of the following types: Corliss, automatic piston valve, or Lentz type poppet valve. The high-speed small engine for exciter service or small direct-current generation is practically always a shaft-governor automatic slide-valve or piston-valve engine, generally simple.

214. The medium-power and high-power engines are practically always compound. Wherever possible, the vertical compound, or at least the angle compound (horizontal-vertical) should be used on account of the saving in floor space, and the more satisfactory wear of the parts. The horizontal engine is used only where the headroom is restricted.

215. Automatic stops consist of some form of flyball governor, belt or chain driven from the engine shaft: the governor is so set as to release a trigger or close an electric circuit when the predetermined speed is exceeded. This trigger, or a magnet in the electric circuit, releases a weight arranged to close the throttle. The usual connection consists of a steel wire rope around a drum on the throttle valve stem. In another form of automatic stop the operation of the magnet or trigger releases the pressure of steam on a pilot pipe line; this in turn releasing the pressure behind a piston in the specially designed throttle valve, which then closes. Both the systems are arranged for additional operation by hand.

216. The operation of steam engines, after correct adjustment of moving parts has been made, is chiefly a matter of lubrication. In **starting**, the usual procedure is to "crack" the throttle in order to admit a little steam to warm up the engine all over without allowing it to turn. After the preliminary warm-up, standing, the engine is allowed to turn over slowly for a few minutes to warm all parts thoroughly; it is then ready for full speed and load. Five minutes may be long enough for "**warming up**" an engine of 300 h.p. or less; 15 or 20 min. are required for large machines. Just before starting, a few strokes of the hand oil pump (usually attached) should be given, to insure thorough lubrication. All automatic oiling rigs should be started a few minutes before turning over. Drips should be wide open while warming up, and while turning slowly, but closed when the engine is brought up to speed. **Shutting down** only requires the closing of the throttle, shutting off automatic feeds, and the opening of drips. All bolts and nuts should be gone over periodically, say once a month, and a thorough overhauling given the piston, cylinder, bearings, etc., once a year.

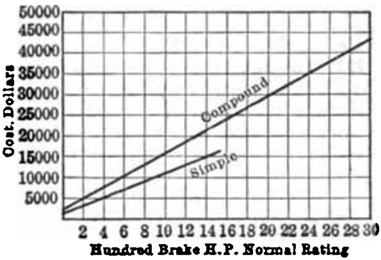


FIG. 23.—Cost of large reciprocating engines.

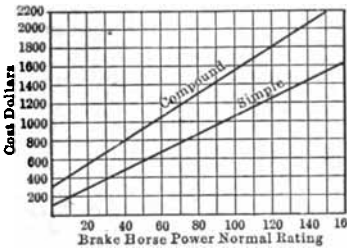


FIG. 24.—Cost of small reciprocating engines.

217. Engine costs. Figs. 23 and 24 give the usual costs of large and small engines. The cost per lb. of engine varies from $0.10 to $0.15.

218. The present status of the reciprocating engine is revealed by a progressively diminishing percentage of the total h.p. of prime movers sold. The large steam turbine has totally displaced it in central stations; and recently the development of fairly economical small turbines is cutting down the field in all directions. The immense advantages of the turbine are great enough to overcome a slight inferiority in water rate in the small sizes. Ultra-conservatism and ignorance of the real cost of power, of which coal and water are only fractional parts, account for the purchase of reciprocating engines where turbines ought to be installed. The oil engine and the gas engine, for small and moderate sized plants, are also giving the steam engine vigorous competition.

STEAM TURBINES

219. The thermodynamics of the steam turbine is a simpler matter in theory than the steam engine. The energy of the adiabatic expansion in the turbine is converted into kinetic energy by producing motion of the steam particles, which issue from the nozzles as jets.

220. Resisted and free expansion. The expansion in the engine is termed "perfectly resisted;" in the turbine it is "free." Therefore all the work is done upon the steam itself during the expansion, producing high velocity of the steam particles; these particles in turn impinge upon curved vanes or buckets, which change the direction of flow of the jets in such a manner as to produce a force in the direction of rotation of the wheel.

221. The losses are wholly different from those in the engine; there is no initial condensation, because the parts assume the temperatures of the moving steam, instead of being alternately heated and cooled, and radiation is so slight as to be negligible (0.1 per cent. or less).

222. Friction. The only true mechanical friction is in the bearings, and is usually under 2.5 per cent. in small turbines, and under 1 per cent. for larger sizes. The principal loss is steam friction, which is made up of (a) nozzle friction, (b) blade friction, (c) eddies in the flow and (d) windage of the discs or drums revolving in steam. There is some **leakage**, but generally less than in the engine. The presence of any velocity in the exhaust steam is also a source of direct loss.

223. The flow of gas through a nozzle is a phenomenon divisible into two classes—(a) above, and (b) below, the critical pressure. When difference of pressure is maintained across an orifice, steam will flow with increasing velocity and in increasing quantity as the pressure difference increases up to that point at which

$$\frac{P_2}{P_1} = 0.58, \text{ for steam,} \tag{38}$$

where P_1 = initial pressure in lb. per sq. absolute and P_2 = back pressure in lb. per sq. absolute.

224. The critical pressure is,

$$P_2 = 0.58P_1. \tag{39}$$

This is true for all initial pressures. The corresponding velocity varies from 1,300 to 1,500 ft. per sec., and is found practically to coincide with the velocity of sound in steam at the pressure $P_2 = 0.58P_1$.

225. Effect of reducing back-pressure. Up to this point (Par. **223**) the stream issues from the orifice in parallel lines; but if the value of P_2 is reduced below $0.58P_1$, no further increase in velocity or quantity can be obtained, at the orifice, but there is further acceleration beyond the orifice, and the stream flows out laterally as well as forward. Heavy accoustic vibrations occur, and the efficiency of conversion decreases. To correct this, a conical or conoidal section is added to the orifice, diverging in the direction of flow, and the further pressure drop is permitted to take place in this flaring or trumpet-like exit.

226. The exit area should be that suited to the velocity and volume resulting from the total heat drop from the pressure P_1 to P_2; but the throat always remains of that size required for a pressure of $0.58P_1$, and a velocity of about 1,300 to 1,500 ft. per sec. The theoretical velocities are not realized, due to friction of the steam in the nozzle.

227. The velocity efficiency of a convergent nozzle (orifices with rounded entrance) varies from 98 per cent. at low velocities down to 95 per cent. at 1,300 to 1,400 ft. per sec. The **velocity efficiency of a divergent nozzle**, for expansion beyond the critical pressure, ranges from 94 per cent. at 1,500 ft. per sec. to 90 per cent. or 85 per cent. at 3,000 ft. per sec.

228. The velocity efficiency of the buckets varies from about 95 per cent. to 98 per cent., for the low velocities used in reaction turbines (less than 1,000 ft. per sec.), to 84 per cent. or 86 per cent. in impulse turbines at velocities of about 2,500 ft. per sec. The second and succeeding rows of buckets or guides in velocity-compounded turbines have even lower efficiencies, reaching 84 per cent. at about 1,500 ft. per sec.

229. The velocity obtainable by expansion is expressible by

$$V = 223.7\sqrt{H_1 - H_2} \tag{40}$$

where V = velocity at exit of nozzle in ft. per sec., H_1 = total heat contents of steam at initial condition, and H_2 = total heat contents of steam at final condition. If H_2 denotes the heat contents after an adiabatic expansion, it follows that V is the theoretical value; but H_2 is always higher than this, because internal friction reduces the quantity of heat removed from the steam by the work done.

230. Pressure drop. In the impulse turbine, all pressure drop occurs in stationary nozzles, and there is no difference of pressure across the moving blades. In the reaction type, about half the pressure drop takes place in the stationary blades, and the remainder in the running blades; and the latter, therefore, act also as nozzles. There is an unbalanced force due to this difference of pressure across the moving blades that must be cared for by balancing devices.

231. There are two basic types of turbines: impulse and reaction. Neither of the two is really pure impulse or pure reaction, but the work done by impulse predominates in the impulse type, and the work by reaction in the reaction type. The simple impulse wheel is only employed in the single-stage de Laval type; velocity compounding and pressure compounding are employed in all other impulse types. Compounding becomes a necessity, to secure reasonable rotative speeds.

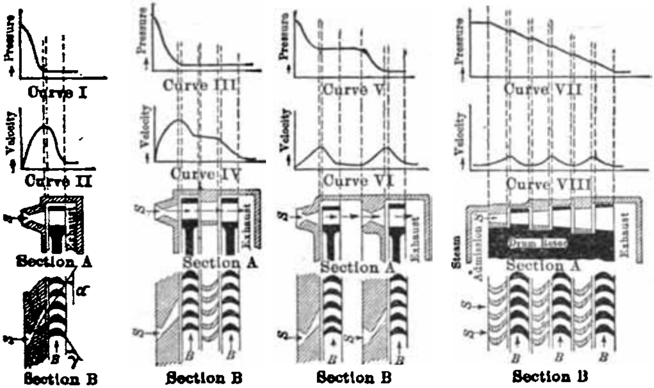


FIG. 25.—Elemental turbine types.*

232. The steam velocity in a single expansion from 150 lb. gage to a 28-in. vacuum would be about 3,100 ft. per sec. For best efficiency the blade velocity should be a little less than half the steam velocity, or in this case 1,500 ft. per sec., which results in centrifugal stresses in the disc much in excess of suitable stresses for commercial materials. If the wheel is run slower, the exit velocity of the issuing steam is increased, reducing the efficiency.

233. In velocity compounding the exit velocity from the first wheel is received in a series of guide blades and redirected to a second wheel, so that more of the energy is removed, without unreasonable wheel velocities (300 to 600 ft. per sec.).

234. Pressure compounding is the division of the pressure drop into two or more stages, which are essentially de Laval single-stage nozzles and wheels in series. Fig. 25 shows these types with characteristic figures.

* From "The Steam Turbine" by J. A. Moyer.

235. The reaction turbine for best efficiency runs at $\sqrt{2}$ or 1.414 times the peripheral speed of the impulse turbine for the same pressure drop. It is never built single-stage, but always pressure-stage compound.

236. Principal types. The de Laval is the original single-stage impulse type; the velocity-compounded type (generally also combined with few-stage pressure compounding as well) is usually known as the Curtis; the pressure-stage type (pure), as the Rateau; and the reaction type, as the Parsons.

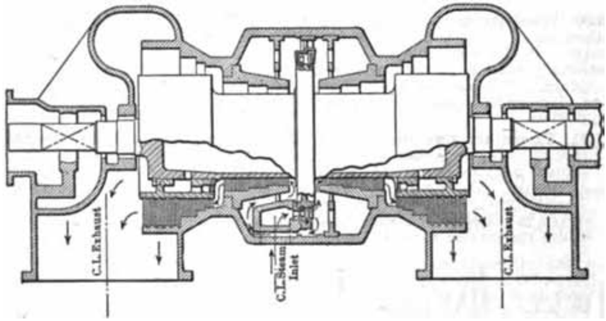


Fig. 26.—Curtis-Parsons double-flow turbines.

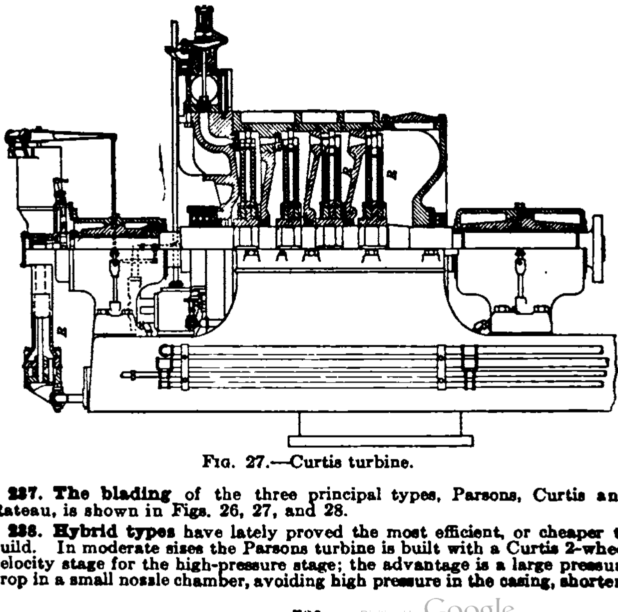


Fig. 27.—Curtis turbine.

237. The blading of the three principal types, Parsons, Curtis and Rateau, is shown in Figs. 26, 27, and 28.

238. Hybrid types have lately proved the most efficient, or cheaper to build. In moderate sizes the Parsons turbine is built with a Curtis 2-wheel velocity stage for the high-pressure stage; the advantage is a large pressure drop in a small nozzle chamber, avoiding high pressure in the casing, shorten-

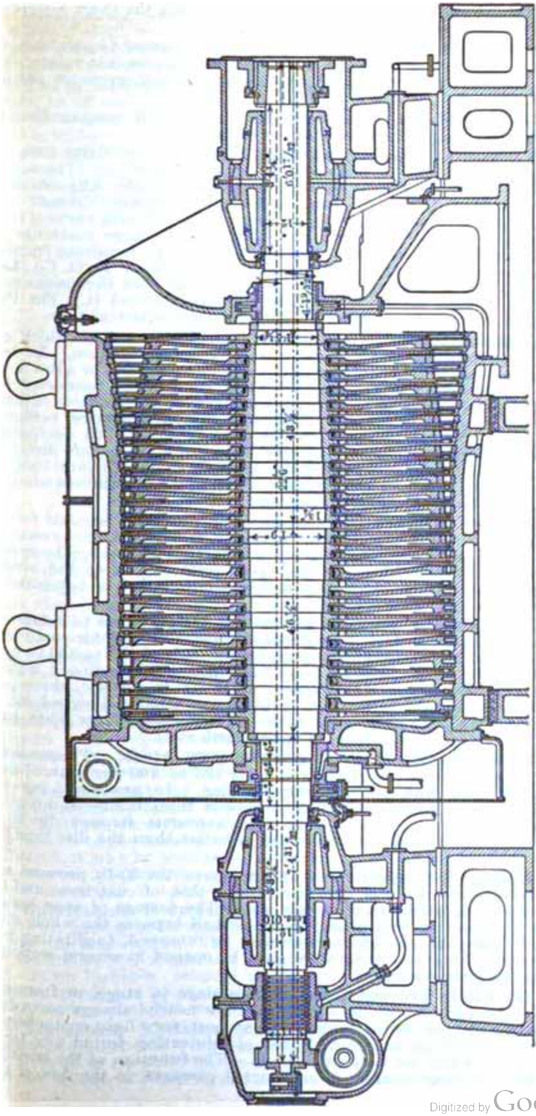


FIG. 28.—Rateau turbine.

ing and cheapening the construction, and eliminating the short high-pressure reaction blading, which is the least efficient.

239. The Curtis-Rateau type consists of a 2-wheel Curtis element for the high-pressure stage, followed by 6 to 12 Rateau pressure stages. It has the same advantage with regard to keeping the high-pressure out of the casing.

240. Velocity staging is the least efficient method of compounding, but is chiefly used for small turbines, as it lowers the cost.

241. Vertical and horizontal types. There is very little difference in principal features between vertical and horizontal turbines. The only companies that have built large vertical machines are the Allgemeine Elektricitäts Gesellschaft and the General Electric Company. Small vertical turbines, in small sizes, are sometimes built for use all with vertical fans and pumps, using a ball step bearing for support. The large machines are all provided with oil or water high-pressure step bearings, requiring from 650 to 1,000 lb. per sq. in. pressure, and special pumps. The A. E. G. Co., has discontinued building this type for several years, and since the recent increase in rotative speeds, the G. E. Co. has also discontinued it. The Parsons turbine has never been built in anything but the horizontal type.

242. Pressure types. High-pressure turbines are those which operate on full boiler pressure and exhaust to atmosphere or vacuum. **Low-pressure** turbines are those operating on atmospheric pressure, or a few lb. above it, and exhausting into a vacuum. They are usually connected to the exhaust of non-condensing reciprocating engines, or other source of low-pressure steam (except direct from boilers). **Mixed-pressure** turbines are those designed to run normally on low-pressure steam, but equipped with high-pressure stages which may receive steam from the boilers direct, if the low-pressure supply fails to equal the demands of the turbine load. This type is characterized by a low-pressure end designed to handle a much larger quantity of steam than the high-pressure end.

243. Bleeder turbines are those in which provision is made for taking steam from a stage of the machine normally at atmospheric pressure, or a few lb. above, to be used for heating, or industrial service. The remainder not so used continues through the low-pressure section to the condenser. This type is characterized by a large high-pressure section, as compared to the low-pressure end.

244. The principal features of turbine design relate to balance, leakage, and resistance to high centrifugal stresses. Wheels for small impulse turbines are normally operated at peripheral speeds of 250 to 350 ft. per sec., and 400 to 550 ft. per sec. for large machines, using ordinary high-grade open-hearth steels. For higher speeds up to 700 ft. per sec. chrome-nickel forged steels are used. Reaction turbines are usually designed for much lower speeds, 150 to 350 ft. per sec., or 400 ft. for very large units, and are generally of drum construction, of open-hearth steel.

245. Speed limitations of open-hearth steel. Theoretically, a properly designed disc can be run at from 125 to 160 per cent. of the safe speed of a drum with corresponding blading. In practice, however, the difference due to computable tensile stresses from blades and centrifugal force is less important than the unknown harmonic stresses due to blade and disc vibration, and the drum type is better than the disc type in this respect.

246. Casings for superheated steam above the 50-lb. pressure section of the turbine are made of cast steel; below this, of cast iron; and for all pressures in saturated steam, of cast iron. The casings of most horizontal turbines are split, so that lifting the upper half exposes the whole interior. With one more lift, the whole rotor may be removed, facilitating repairs to all parts. The vertical turbines must be opened in several sections and taken apart wheel by wheel.

247. Glands to prevent leakage from stage to stage, or from the interior of the turbine to the outside air, are nearly always some form of **labyrinth,** with no actual contacts, or at most very light contact on floating rings. The high speeds and lack of lubrication forbid any forms of soft packing except for very small units. The function of the labyrinth is to interpose a large number of constricted passages to the flow of steam

These constrictions usually measure from 0.003 in. to 0.035 in., depending on the size of shaft or dummy ring, and the pressure difference. The clearance is selected according to the vibration at the point of application and is often made a function of the diameter.

248. The principal blade fastenings consist of the dovetail for impulse elements, in all sorts of forms from square T-heads to bulb heads with all corners eliminated; the reaction blading of Parsons make is a caulked type, depending upon some distortion and friction for the grip. Riveted blading is used to some extent in small impulse turbines, but has been a failure in large discs. Fig. 29 shows three sucessful types. **Blade heights** vary from 0.5 in. to 18 in., depending on the size of the machine, and the stage. Widths range from 0.25 in. to 2 in.

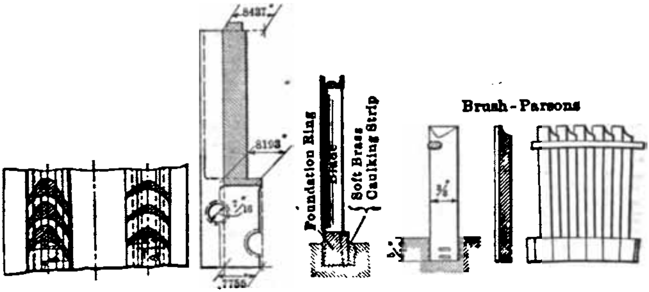


FIG. 29.—Four types of blade fastenings.

249. Pressure stages. In high-pressure Curtis turbines, the usual number of pressure stages is five to seven, with two velocity stages in each pressure stage. In Rateau types, there are about thirteen to twenty-two pressure stages, with one wheel in each; or one Curtis stage of two velocity stages may take the place of two or three high-pressure stages. The Parsons type has from forty to seventy pressure stages.

250. Turbine rating. Impulse turbines are rated on a basis of calculated flow through the nozzles and known efficiencies of different types of wheels. Thus a wheel of given size may be of widely varying horse-power under different areas of nozzles; the efficiencies also are altered. Reaction turbines must be designed for the load, and can only be altered in best load capacity by increasing the initial pressure. Overload is provided for in the impulse types by adding extra nozzles; in the reaction type by by-passing some of the high-pressure stages.

251. Measurement of shaft output. The output of the turbine must be obtained by brake, or electrically; no indicated horse-power exists; in small turbines, throttle governed, the steam-chest pressure bears an accurate relation to the horse-power delivered at any speed, and if the turbine is calibrated, it may be used as an indication of the output thereafter. This is true because the pressure is a measure of the steam flow, and the steam flow is a measure of the output, for any given turbine efficiency.

252. Lubrication is exceedingly simple. No internal oiling whatever is employed; gravity or force feed with copious supply at 15 to 25 lb. pressure is general for large units. The speed of rotation causes a dragging of oil under the journal, by means of viscosity, so that the journal never rides on metal. Some turbines actually rise a few thousandths of an inch after coming to speed due to increase in the oil film under the journal. Small turbines are ring-oiled like motors.

253. The actual Rankine-cycle efficiency ratios obtained in small non-condensing turbines vary from 45 per cent. to 50 per cent., for large units, from 55 to 65 per cent. for the great majority, up to 71 per cent. as about the highest. These figures assume condensing service in each case.

254. Turbine Performance: Data on Commercial Tests of Large Steam Turbines
(Christie*)

	Maker of turbine	Type	Date of test	Load, kw.	R.p.m.	Steam pressure, lbs. abs.	Supht.	Vacuum referred to 29.92" barometer	Pounds of steam per kw.-hr.	Efficiency ratio
1	Erste Brunner M. F. G.	Curtis-Parsons	1910	2,128	1,500	156.2	120.31	27.89	13.82	71.8
2	Erste Brunner M. F. G.	Curtis-Parsons		6,000	960	184.9	197.64	28.18	12.56	71.3
3	Erste Brunner M. F. G.	Curtis-Parsons	1910	7,442	960	192.0	205.51	28.18	12.625	70.3
4	Westinghouse Machine Co.	Curtis-Parsons	1910	9,173	1,800	181.7	59.07	27.81	14.57	68.9
5	Erste Brunner M. F. G.	Curtis-Parsons	1910	1,416	1,260	128.2	135.69	27.60	15.18	68.8
6	Brown-Boveri & Cie	Parsons	1910	6,257	1,210	203.7	175.57	29.02	11.95	68.8
7	A. E. G.	Curtis-Rateau	1911	6,518	1,220	198.7	219.66	29.28	11.43	68.7
8	A. E. G.	Curtis-Rateau	1911	6,565	1,220	200.2	214.19	29.18	11.64	68.5
9	Allis-Chalmers	Parsons	1908	4,300	1,800	186.4	107.97	27.96	14.02	68.4
10	Brown-Boveri & Cie.	Curtis-Parsons		3,053	1,360	150.2	146.40	29.00	13.01	68.0
11	Brown-Boveri & Cie.	Curtis-Parsons	1911	1,750	1,500	176.4	214.5	27.08	14.23	67.5
12	M. A. N.	Curtis-Zoelly		3,584	1,580	178.3	196.63	27.54	13.99	67.5
13	James Howden & Son	Zoelly	1909	6,383	1,000	202.7	136.98	27.33	14.305	67.5
14	M. A. N.	Zoelly	1910	1,400	3,000	180.7	180.53	27.40	14.21	67.4
15	Westinghouse Machine Co.	Curtis-Parsons		9,830	750	192.2	96.43	27.22	15.15	67.0
16	British Westinghouse	Curtis-Rateau	1911	5,066	1,500	190.2	174.31	28.68	13.00	67.0
17	Brown-Boveri & Cie.	Curtis-Parsons	1910	3,764	1,500	161.2	196.78	28.77	13.04	66.8
18	Brown-Boveri & Cie.	Curtis-Parsons	1911	1,495	3,000	200.6	180.86	26.41	14.78	66.8
19	Escher Wyss & Co.	Zoelly	1910	2,052	3,000	193.9	205.7	28.39	13.04	66.6
20	British-Thomson-Houston	Curtis	1911	2,987	1,500	154.7	144.07	26.75	15.96	66.5
21	Bergmann	Curtis-Rateau	1909	1,545	1,500	188.5	204.04	28.59	12.97	66.4
22	Oerlikon	Rateau	1911	3,166	1,500	213.9	275.42	29.25	11.44	66.1
23	Brown-Boveri & Cie.	Curtis-Parsons	1911	1,271	3,000	172.1	198.51	27.31	14.61	65.9
24	Escher Wyss & Co.	Zoelly		5,118	1,000	133.7	199.48	27.55	15.18	65.7
25	Bergmann	Curtis-Rateau	1910	2,477	1,500	140.0	168.91	28.81	13.93	65.6
26	Brown-Boveri & Cie.	Parsons	1903	3,500	1,360	156.4	137.2	28.84	13.71	65.6
27	Westinghouse Machine Co.	Curtis-Parsons		11,466	750	191.7	105.65	28.07	14.45	65.5
28	Escher Wyss & Co.	Zoelly	1910	4,189	1,000	179.7	183.98	28.66	13.30	65.5
29	British Westinghouse	Curtis-Rateau	1911	2,980	1,500	210.2	187.9	28.18	13.72	64.9

254. Turbine Performance: Data on Commercial Tests of Large Steam Turbines.—(*Continued*)

	Maker of turbine	Type	Date of test	Load, kw.	R.p.m.	Steam pressure, lbs. abs.	Supht.	Vacuum referred to 29.92" barometer	Pounds of steam per kw.-hr.	Efficiency ratio
30	A. E. G.	Curtis-Rateau	1908	4,239	1,500	188.3	285.18	29.11	11.97	64.9
31	Escher Wyss & Co	Zoelly		3,540	1,500	155.1	107.86	28.21	15.07	64.8
32	F. Ringhoffer	Zoelly	1908	3,000	1,000	170.7	101.18	27.60	15.52	64.8
33	M. A. N.	Zoelly	1910	1,250	3,000	182.1	207.89	28.82	13.09	64.4
34	Brown-Boveri & Cie	Parsons		3,000	1,360	165.0	258.91	27.02	14.75	64.3
35	C. A. Parsons & Co	Parsons		5,164	1,200	214.3	121.26	28.95	13.18	64.3
36	Escher Wyss & Co	Zoelly	1910	1,641	3,000	221.0	281.63	27.91	13.08	64.1
37	Erste Brunner M. F. G.	Curtis-Parsons		1,250	3,000	184.9	197.64	27.89	14.32	63.9
38	Escher Wyss & Co.	Zoelly	1908	5,000	1,000	164.4	173.21	26.38	16.13	63.9
39	British-Thomson-Houston	Curtis	1909	2,500	1,500	126.5	68.70	28.47	15.92	63.7
40	General Electric Co.	Curtis		3,464		210.0	126.98	28.75	13.62	63.6
41	A. E. G.	Curtis	1909	2,236	1,500	191.6	275.69	29.34	11.77	63.6
42	Allis-Chalmers	Parsons	1911	3,850	1,800	164.7	125.06	27.91	15.40	63.5
43	A. E. G.	Curtis	1906	3,000	1,500	191.3	211.82	29.05	12.79	63.4
44	General Electric Co.	Curtis		8,880		192.5	106.30	28.02	15.05	63.1
45	A. E. G.	Curtis-Rateau	1907	3,169	1,500	184.7	216.73	29.11	12.74	63.0
46	Brown-Boveri & Co	Curtis-Parsons	1910	3,320	1,500	180.9	151.44	29.02	13.50	63.0
47	M. A. N.	Curtis-Zoelly		2,507	1,500	175.5	88.91	27.40	16.24	62.8
48	Escher Wyss & Co.	Zoelly	1910	1,235	3,000	176.8	79.32	28.39	15.35	62.2
49	Brown-Boveri & Cie	Curtis-Parsons		5,128	1,000	172.2	195.46	28.52	14.35	62.1
50	General Electric Co.	Curtis		10,816	750	190.0	174.39	29.39	12.90	61.9
51	General Electric Co	Curtis		5,095		185.1	198.55	29.40	12.71	61.6
52	Bergmann	Curtis-Rateau	1909	1,562	1,500	186.8	178.97	28.33	14.57	61.4
53	British-Thomson-Houston	Curtis	1911	1,221	3,000	134.7	97.91	27.16	17.75	61.2
54	General Electric Co	Curtis	1910	8,775	750	194.0	71.65	27.95	15.95	61.0
55	British-Thomson-Houston	Curtis	1911	1,541	1,500	149.7	6.66	27.97	17.47	61.0
56	Brietfield Danek & Co.	Impulse-Parsons	1909	3,585	896	160.7	93.03	28.32	16.08	60.2

* A. G. Christie, *Trans.* A. S. M. E., 1912.

This table gives efficiency ratios and B.t.u. per kw-hr. The latter figure is obtained from the water rate and the steam conditions.

$$\text{B.t.u. per kw-hr.} = W \times (H_1 - q_2) \qquad (41)$$

Where H_1 = heat contents of steam supplied to turbine, q = heat of the liquid in condensate and W = water rate in lb. per kw-hr.

The thermal efficiency is the product of the efficiency ratio and the thermal efficiency of the Rankine cycle (adiabatic expansion).

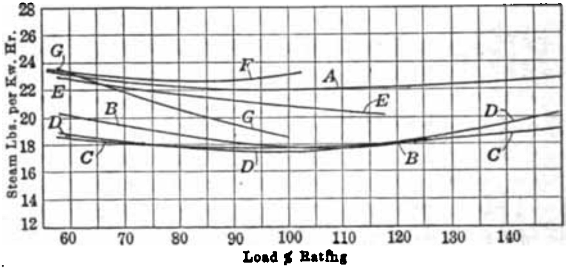


FIG. 30.—Water-rate curves of small condensing turbines, steam, 150 lb. gage; superheat, 100 deg. Fahr.; vacuum, 28 in.

A, 250 kw. Westinghouse, 3,600 r.p.m.; *B*, 400 kw. Westinghouse, 3,600 r.p.m.; *C*, 500 kw. Westinghouse, 3,600 r.p.m.: *D*, 1,000 kw. Westinghouse, 1,800 r.p.m.; *E*, 200 kw. DeLaval; *F*, 400 kw. Kerr, 1,500 r.p.m.; *G*, 1,000 kw. Rateau.

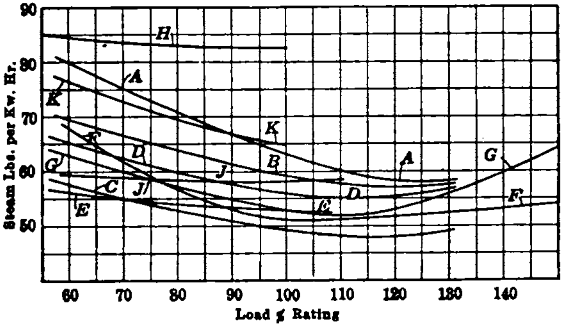


FIG. 31.—Water-rate curves of small non-condensing turbines with dry steam at 100 lb. gage.

A, 35 kw. Curtis, 3,600 r.p.m.; *B*, 75 kw. Curtis, 3,300 r.p.m.; *C*, 100 kw. Curtis, 3,600 r.p.m.; *D*, 125 kw. Curtis, 2,400 r.p.m.; *E*, 300 kw. Curtis, 1,800 r.p.m.; *F*, 250 kw. Westinghouse, 3,600 r.p.m.; *G*, 300 Westinghouse, 3,600 r.p.m.; *H*, 50 kw. Sturtevant, 1,400 r.p.m.; *J*, 150 kw. Sturtevant, 2,100 r.p.m.; *K*, 50 kw. Terry, 2,800 r.p.m.

255. Water-rate curves. Figs. 30, 31, and 32 give characteristic water-rate curves for small and large units. It is found that the **Willans line** (total steam per hour vs. output) is practically straight line from zero load to full (or best) load; on overload, with by-pass, or extra nozzles open, it is usually another straight line, joining the first at best load, but more steeply inclined. This property allows fractional-load water rates to be readily interpolated from one or two load tests or guarantees.

256. The effect of variation in vacuum on a turbine differs somewhat with the type of machine, but can be obtained with reasonable accuracy from Fig. 33, which gives average figures for high-pressure turbines; Fig. 34 applies to low-pressure turbines. A vacuum of 28 in. is taken as the standard for high-pressure, and 27.5 in. for low-pressure units.

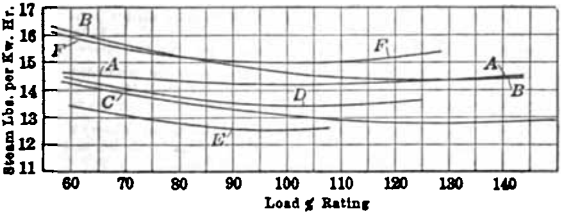


FIG. 32.—Water-rate curves of large condensing turbines. Steam, 200 lb. gage; superheat, 100 deg. Fahr.; vacuum, 28 in.
A, 9,000 kw. Curtis; *B*, 10,000 kw. Curtis; *C*, 4,000 kw. Curtis-Rateau; *D*, 10,000 kw. Westinghouse; *E*, 6,000 kw. Parsons; *F*, 6,000 kw. Zoelly.

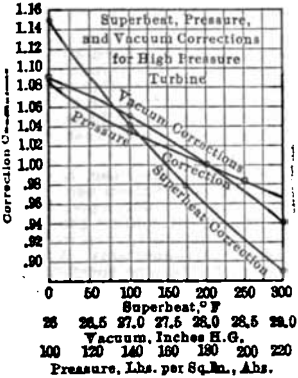


FIG. 33.—Superheater pressure and vacuum corrections for high-pressure turbines.

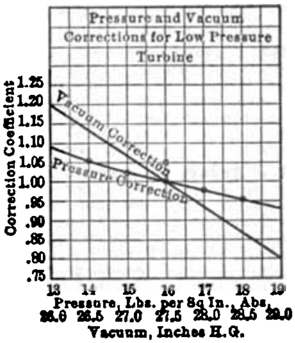


FIG. 34.—Pressure and vacuum corrections for low-pressure turbines.

257. The effect of superheat in improving economy is usually taken at the rate of 10 per cent. for the first 100 deg. of superheat, 8 per cent. for the next 100 deg., and 7 per cent. for the third hundred. Figs. 33 and 34 show the corrections, which are practically identical for all types.

258. The efficiency losses may be subdivided into (a) nozzle friction, 6 to 28 per cent.; (b) blade friction, 6 to 30 per cent. (c) windage, 3 to 15 per cent.; (d) unused exit velocity, 3 to 5 per cent.; (e) leakage 2 to 10 per cent.; (f) mechanical losses, 1 per cent. As all the steam losses in the high-pressure stages reappear in the steam as unused heat, the available heat drop is increased by translation of the expansion lines to higher entropies, as shown by the **Mollier diagram**,* hence the whole turbine-diagram efficiency in a multi-stage turbine is always from 2 to 6 per cent. higher than that of the individual stages.

* Marks & Davis "Steam Tables," Longmans, Green & Co., N. Y., 1912.

259. The efficiency of the turbine, though different for different speeds and pressures, does not change appreciably with time or service. The shaft horse-power or brake horse-power is from 3 to 15 per cent. less than the calculated diagram horse-power (corresponding to i.h.p. in the engine). The usual friction losses are generally less than one-half those of steam engines of the same rating. **Mechanical friction** in the turbine is confined to bearings and governor drive; it is usually under 1 per cent., and in large machines is less than 0.5 per cent. Internal or steam friction is caused by imperfect shape of blades, and windage of the discs or drums.

260. Pressure correction, for variations in steam pressure from guaranteed or desired conditions, is given in Fig. 33 and 34 for high-pressure and low-pressure turbines. The correction is the same for all types.

261. Governors for turbine speed regulation are always of the centrifugal type; the inertia governor cannot be employed, because there can be no sudden angular accelerations. The centrifugal governor can be made nearly isochronous; generally, however, there is a slight decrease in speed as load increases.

262. Throttling governors. For all small turbines (impulse type) and some large makes, the plain throttling governor is employed, simply controlling the admission pressure at the steam chest or first stage. The old Parsons governor for reaction turbines admitted steam at full pressure, in short puffs, lengthening the period the valve was open as the load increased. Many of the Curtis types are governed by multiple-nozzle control, opening and closing individual nozzles, from 6 to 16 in number, and thus controlling the quantity of steam.

263. Parsons governor. In the Parsons machine, the governor not only controls the primary throttle, but also a secondary valve admitting live steam in one-sixth to one-fourth the total number of stages, further down the turbine. This virtually cuts out of service the by-passed rows, and converts the turbine into one of fewer stages, but in effect having larger blade dimensions, and not as economical. The best load for this type is that carried just before the opening of the secondary valve.

264. Reduction gears have recently come into greater use, than was formerly made of them. The de Laval turbine has used them successfully for 20 years, at enormous relative speeds. The type developed by the Westinghouse Machine Company from the Melville-Macalpine gear has a floating hydraulic frame for aligning the gears. All the other types, including the Falk, Fawcus, Parsons, etc., use solid bearings and connections. All types employ the double helical gear. By this means the turbine speed may be kept high, for economy both in cost and steam, and the driven apparatus may be operated at comparatively low speed. Direct-current generators, fans, and centrifugal pumps for large volume and low speed, may thus be successfully combined with the turbine with good economy.

265. Turbine specifications should cover the following items: (a) Number of units and location; character of building. (b) Service, and attachment to driven apparatus (direct, flexible coupling, reduction gear, etc.) (c) Speed, steam and back pressure conditions. (d) Capacity, overload, electric system data. (e) Regulation, variation of speed under change of load. (f) Noise, vibration. (g) Tests and inspection. (h) Mechanical details of connection to driven apparatus. (i) Type and steam system, H. P., L. P., bleeder mixed pressure, giving quantities for L. P., or bleeding steam. (j) Materials—casing, wheels, nozzles, blades, shaft. (k) Piping connections. (l) Bearings. (m) Foundations. (n) Oiling. (o) Auxiliary apparatus—oil pumps, relays, step-bearing pumps (for vertical turbines.) (p) Painting and lagging. (q) Gages and miscellaneous equipment.

266. Turbine supports should always be carefully designed to prevent distortion of the parts; stiffness of the supports is usually very desirable, but great mass is unnecessary; many reaction turbines of large size are running on foundations entirely of steelwork, which is low in mass for the strength and stiffness.

267. The auxiliaries required for horizontal turbines consist only of an oil pump, for large self-contained units: no auxiliaries whatever are needed for small units.

268. The auxiliaries for vertical units include in addition to the oil pump, **step-bearing pumps** for oil or water, capable of handling pressures of 500 to 1,000 lb. per sq. in. (varying with the size of the main unit). These step-bearing pumps, with the piping, are generally in duplicate. In order to steady the oil supply, a frictional resistance called a **baffler** is employed between pump and step bearing; and to remove pulsations of pressure and provide a small reservoir, **accumulators** of the elevator type are also required.

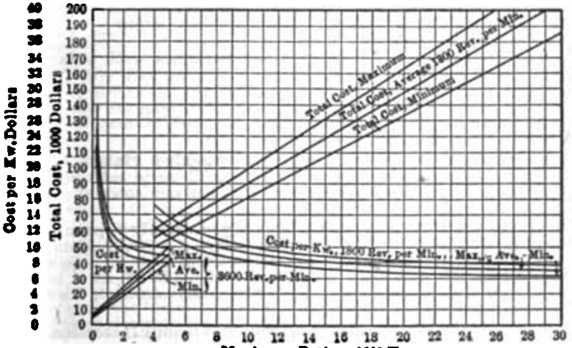


Fig. 35.—Steam turbines. Cost per kw. *vs.* capacity. Total cost *vs.* capacity.

60 Cycles, maximum 24 hr. rating, 50 deg. Cent. rise; power factor, 80 per cent.; pressure 175 lb.; superheat, 100 deg. Fahr.; vacuum, 28.5 in.

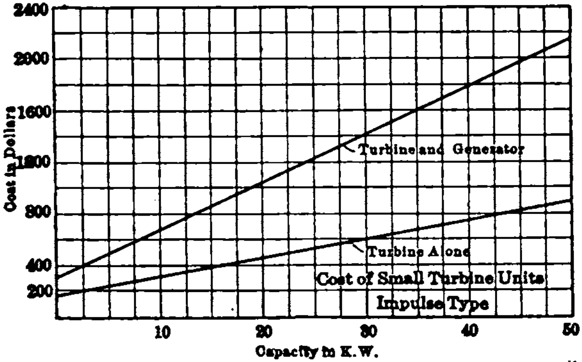


Fig. 36.—Cost of small turbines and generators.

269. The cost of turbines per kw., including generators, varies from $30 for small sizes down to $9 or less for very large units, at normal rating; or at maximum rating, $10. Fig. 35 gives the cost per kw. (normal rating) of large units including generators; Fig. 36 gives the cost per h.p. of small turbines with and without generators.

270. The operation of the steam turbine is remarkable for its simplicity; one oiler can attend to two or three units (on the engine-room floor) aggregating as much as 40,000 or 50,000 kw. But with engines, only 3,000 or 4,000 kw. can be cared for by one man; and in many cases much less. The turbine is practically undamaged by priming which would wreck an engine, and has a maintenance cost but one-fifth as great, approximately. There are very few articulated parts in the turbine; its continuous operation is therefore much more reliable and periods of two to four weeks continuous operation are common. **Inspections** made once a year frequently show no repairs or adjustments to be necessary, and the turbine is continued in service. With large reciprocating engines, both inspections and adjustments are sometimes necessary every 24 hr.

271. The general method of starting turbines with steam-sealed glands is to establish a vacuum on the condenser with the dry-vacuum pump, start the circulating water, and "crack" the throttle in order to send steam through the turbine for warming, without turning the spindle. After the proper time allowance (5 to 15 min.) the turbine may be brought to speed and placed under load.

272. In starting a turbine with water-sealed glands, it is usual practice to start the turbine non-condensing until up to sufficient speed to seal the glands, then establish the vacuum on the condenser and apply the load.

273. At the present time the turbine is practically supreme in large central stations, as a heat-operated prime mover. The hydraulic station is limited to certain localities, while the large gas-engine plant is so unreliable as to be out of the question. For small and moderate-sized plants the internal-combustion engine is on a competing basis, although its reliability has never equalled that of the turbine.

274. The field for small steam turbines is readily increasing, particularly since centrifugal boiler-feed pumps, circulating-water pumps, fans, blowers, etc., are successfully designed for turbine speeds, with good efficiencies. The small amount of attention required by the turbine, its reliability and low maintenance are generally more than sufficient to overbalance a slight inferiority to the steam engine in economy. **Since the item of fuel economy is only one of the factors in operating cost,** this result is natural and the use of the turbine is bound to increase. There is yet much to improve in the turbine, whereas the reciprocating engine has been at a virtual standstill in development for 10 years past.

CONDENSING EQUIPMENT

275. Thermodynamics of condensers. The heat given up by the condensing steam must equal the heat received by the circulating water. In jet or barometric condensers, the steam and water mix; no difference between the temperature corresponding to the vacuum and the temperature of the discharge water need exist. The surface condenser requires some difference of temperature between the steam and the circulating water at all times, or no heat flow from steam to water can take place.

276. Removal of air. The air present in the condensate is chiefly drawn in by leakage at the joints. This air, being non-condensible, must be removed by segregation and pumping.

277. The volume of condensate water is practically negligible compared with the steam volume at the same pressure. The **work in ft-lb made available by the condenser** is the steam volume multiplied by the pressure difference in lb. per sq. ft. existing between the atmosphere and the interior of the condenser; the work required to produce this effect, is merely the product of the number of cu. ft. of condensate and the same pressure difference. At 28 in. vacuum the ratio of these two is 21,000 to 1. To the **condensate pump work** must be added the **work of the circulating-water pump** (chiefly a friction loss) and the **work of the dry-vacuum pump** which removes the air.

278. The principal condenser types are: (a) jet; (b) barometric; (c) eductor or siphon; (d) rotary jet; and (e) surface. These are described in **Par. 279 to 283.**

279. The jet condenser consists of a cast-iron shell into which the exhaust pipe is led, having the circulating water sprayed through the chamber in jets. The steam condenses and mingles with the jets, and is pumped out of the bottom of the shell. **The barometric condenser** is a jet condenser set higher than 34 ft. above the level of the discharge well or tunnel; the water therefore runs away by gravity, but must be pumped in, due to a slight friction loss and velocity head, the vacuum assisting the circulating pump.

280. In the eductor or siphon type, the injection water is pumped in under 25 to 30 lb. pressure, and requires sufficient velocity to carry out not only its own mass but the condensed steam and entrained air as well, by kinetic energy. No vacuum pumps are required.

281. The rotary jet types—originally developed by Le Blanc—consist of a centrifugal impeller throwing segments of water into a nozzle, in such fashion as to form water pistons, which trap and condense the steam, and push before them the air. This type is also used as a dry-vacuum pump. All the above types (Par. 279 to 281) are derivatives of the true jet condenser.

282. The surface condenser consists of a cast-iron shell with two heads or water boxes into one of which the circulating water passes. These two boxes are connected by a large number of small brass tubes, which allow the circulating water to traverse the main shell without contact with the steam; the latter is fed into the shell around the tubes. The circulating water passes to the second box, which may lead to the discharge or redirect the water back to another section of the first box by way of another nest of tubes. If the first type is used it is called **single pass**; if the second, **two pass**; and so on, with three or four passes in some few cases. The cold water passing through the tubes condenses the steam by conduction, and the condensate trickles down to the bottom of the shell, there to be pumped out. The air, which is always heavier than steam at the same temperature, also collects at the bottom and must be pumped out, preferably by another pump.

283. Atmospheric condensers consist of a form of surface condenser in which the cooling medium is a mixture of air blast and water spray. The air takes up some of the spray, becoming cooled thereby, and acts as a cooling medium for the steam. High vacuum is not to be obtained with this type. The chief feature is economy of circulating water, which may be in the ratio of 1 lb. of water per lb. of steam, or slightly less, since the whole latent heat of evaporation (of the water spray) as well as the heat of the liquid is available for cooling.

284. Quantity of circulating water required. In the jet or ordinary surface types, the circulating water must be at least 25 to 30 times the weight of the steam condensed. In the jet condenser, the condensate mixes with the circulating water, and, if the latter is dirty, is necessarily lost. The surface condenser keeps the condensate separate from the circulating water, so that it is available for re-use in the boilers, a pure distilled water (Par. 137 and 138).

285. The proportions of jet condensers are relatively unimportant, the design merely providing for adequate mixture of water and steam. The shell therefore need only be large enough to take the exhaust and water connections. The volume of a jet condenser varies from 0.5 to 2.5 times the volume of the low-pressure cylinder volume of a reciprocating engine; in the eductor types, the volume is somewhat less.

286. Surface condensers are limited in capacity by the rate of heat transfer. For average practice, 300 to 350 B.t.u. per sq. ft. per degree mean temperature difference per hour can be allowed. It follows from this assumption that

$$A = W \times \frac{(H_2 - q_2)}{U \times t_m} \qquad (42)$$

where A = area in sq. ft. of tube surface, W = lb. of steam per hour from exhaust, H_2 = total heat per lb. of exhaust, q_2 = heat of the liquid per lb. (at condensate temperature), $H_2 - q_2$ = 1,000 to 1,050 for most cases, U = transmission coefficient = 300 to 350 B.t.u. per hour, per sq. ft., per degree mean temp. difference (Fig. 37), and t_m = mean temperature difference between steam and circulating water. The arithmetic formula for t_m is:

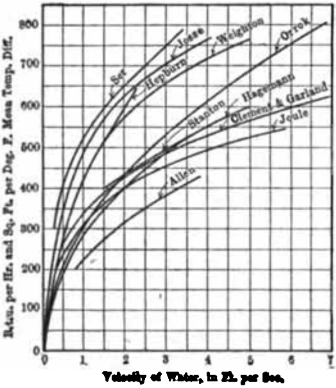


FIG. 37.—Values of the coefficient of heat transmission.

$$t_m = T_s - \frac{(T_1 + T_2)}{2} \qquad (43)$$

where T_s = steam temperature at the vacuum, T_1 = injection temperature and T_2 = discharge temperature. The logarithmic formula for t_m is:

$$t_m = \frac{T_2 - T_1}{\log_e\left(\frac{T_s - T_1}{T_s - T_2}\right)} \qquad (44)$$

For the majority of cases, Eq. 43 is sufficient.

287. The quantity of circulating water required for jet condensers is given by

$$W_c = \frac{H_2 - T_2 + 32}{T_2 - T_1} \qquad (45)$$

where Wc = lb. of circulating water per lb. of steam, H_2 = heat contents per lb. of steam, T_2 = discharge temperature, and T_1 = injection temperature. The quantity required in gal. per min., taking $H - T_2 + 32$ as 1050 B.t.u., is

$$S_c = \frac{2.1\,W_s}{T_2 - T_1} \qquad (46)$$

where W_s = lb. of steam per hour, and S_c = circulating water in gal. per min.

288. Examples of Modern Condenser Proportions

Name of station	Size of turbo-generators on max. 24-hr. rating	Sq. ft., total	Sq. ft., per kw.
Commonwealth Edison Co.			
Northwest Station..............	20,000 kw.	32,000	1.60
Quarry St.......................	14,000	25,000	1.79
Fisk St.........................	12,000	25,000	2.08
Interborough R. T. Co.			
59th St. Power Station, N. Y., (Engine and low-pressure turbine.)	15,000	25,000	1.67
74th St. Station................	30,000	50,000	1.67
Metropolitan St. Ry., Kansas Cy..	10,000	22,000	2.20
Marion Station, P.S.E. Co., N. J...	18,000	25,000	1.39
United El. Lt. & Power Co., N. Y..	15,000	25,000	1.67

289. The quantity of circulating water required for surface condensers is

$$W_c = \frac{H_2 - T_3 + 32}{T_2 - T_1} \qquad (47)$$

where T_3 = condensate temperature, and the other symbols have the same significance as before. For jet condensers T_3 becomes T_2; for surface condensers T_3 is at least 5 deg. Fahr. higher than T_2, and generally 10 or 15 deg. higher.

290. Use of dirty condensing water. For jet condensers, very dirty circulating water may be used, except that if sewage or other gas-producing matter is contained, the work of the dry air pump will be greatly increased. Dirty water in surface condensers causes slime deposits on the tubes, if containing grease and sewage; or it incrusts the tubes with scale, if very hard. In either case, periodic cleaning is necessary, as both deposits lower the efficiency of heat transmission.

291. Screens are usually made of iron mesh or bars, with openings the same size as condenser tubes or ⅛ in. smaller; these screens are set in frames, and operate in slides for cleaning purposes. Small metal screens enclosed in cast-iron boxes in the suction pipe are employed for small sizes and are then usually called "fish-traps." A later development is the use of moving self-cleaning screens, either arranged drum fashion like a stone screen, or like a chain grate set on edge. **The velocity through screens** should not exceed 4 to 5 ft. per sec.

292. Pumps. Circulating-water pumps in modern plants are always centrifugal or propeller types, especially since the advance of the small steam turbine. **Wet vacuum pumps** handle both condensate and non-condensible gases. They are not used to any extent at present. **Dry vacuum pumps** handle non-condensible gases only. Condensate or **hot-well pumps** handle the condensate only. The separate pumps are much more efficient than the combined wet-vacuum pump. The hot well-pump is usually a one-stage or two-stage centrifugal. The dry-vacuum pump may be a reciprocating compressor specially designed for tightness and small clearance, or a hydraulic device using a form of centrifugal pump and water jets to entrain the air, such as the Le Blanc pump.

293. Power required for auxiliaries. The circulating pump, if a reciprocating dry-vacuum pump is used, requires 85 per cent. to 90 per cent. of the total h.p. for condenser auxiliaries; the condensate and dry-vacuum pumps, 5 to 7 per cent. each. The total h.p. at 27 in. to 28 in. vacuum is about 2 to 2½ per cent. of the main unit, for reciprocating main engine; 3 to 3.5 per cent. at 28 to 29 in. vacuum for turbine plants.

294. The steam demand of all condenser auxiliaries, using engine-driven circulator, is 3 to 5 per cent. of the total; for direct turbine-driven circulator, 5 to 9 per cent.; for latest type geared turbine-driven circulator, 4 to 6 per cent. The water rate of turbines used for hot-well pumps is 50 to 60 lb. per b.h.p.-hr.; for low-speed direct-connected turbines for circulators, 35 to 45 lb.; for geared high-speed turbines, 29 to 35 lb.

295. Recent High Vacuum Results with Large Surface Condensers

Plant	Surface, sq. ft.	Water per lb., hr.	Hot well temp., deg fahr.	Circ. water In deg. fahr.	Circ. water Out deg. fahr.	Vacuum 30-in. Bar.	Ratio sq. ft. kw.
Interborough 59th St......	25,000	237,480	71.2	37.7	57.3	28.50	1.67
	25,000	182,508	53.2	33.3	47.6	29.15	1.67
	25,000	164,945	85.5	72.5	86.0	28.55	1.67
Marion, P. S. E. Co.......	20,000	128,000	78	60.0	73.5	29.1	2.00
United El. Lt. & Power.....	25,000	....	71	58	68	29.0	1.67
Boston Elevated.......	28,000	182,000	74.5	64.0	74.0	29.07	2.00
	28,000	120,000	73.0	64.5	71.5	29.06	2.00
Illinois Steel..	25,000	196,000	90.6	73.4	86.1	28.25	....
	25,000	210,000	90.3	73.4	86.9	28.3	....
	25,000	91,000	83.3	73.6	80.0	28.8	....

296. Hydraulic dry-vacuum pumps take from 6 to 12 times as much power as the reciprocating types; but this is relatively unimportant if the exhaust steam is usable in the feed-water heaters.

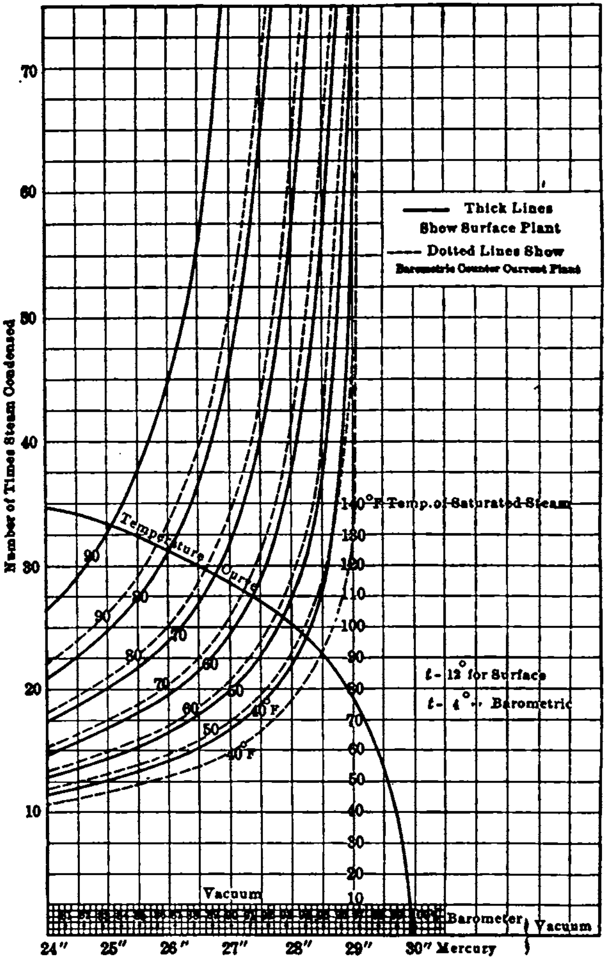


FIG. 38.—Vacuums and circulating-water temperatures.

297. The obtainable vacuum (Fig. 38) depends chiefly upon the temperature of condensing water. The actual results obtained may be less than this due to air leakage or dirty tubes.

298. Cooling towers are employed where condensing water cannot be obtained in sufficient quantity for complete rejection of discharge. The circulating water is broken up into spray or thin sheets and falls through a current of air. The air in becoming warmer, takes up heat from the water; evaporation also takes place and tends to saturate the air. This **evaporation** is from 0.80 to 0.95 lb. per lb. of steam exhaust to the condensers, exclusive of the spray loss in water actually carried away mechanically in the air current. The distance between spray outlets and tank under the tower is usually about 25 ft. so that this head, plus friction in additional piping, must be added to the total pump head. The towers are 50 ft. to 80 ft. high for natural draft; but may be shorter if a forced draft fan is used, provided the rain from the tower is not objectionable. The ground area required is 1 sq. ft. per 20 lb. steam condensed per hr. approximately. The circulating water required is 60 gal. per sq. ft. of ground area, cooled from 95 deg. to 75 deg., which is average performance. The heat transmission to air per sq. ft. of cooling surface in the lattice work or screens runs from 350 to 850 B.t.u. per hr. The circulating water required is 25 to 30 lb. per sq. ft. per hr. The work of the circulating water pump is practically double that of the ordinary installation without towers, or 6 to 8 per cent. of the h.p. of the main unit. If forced draft is used, the fan requires 0.3 per cent. to 0.5 per cent. of the h.p. of the main unit.

299. The operation of condensing equipment becomes very simple with turbine-driven auxiliaries, as these require almost no attention. Depending on the water, condenser tubes should be cleaned from once a week to once a month; brushing the tubes may be necessary. All joints should be very carefully made and maintained tight; shellacing or painting is of great assistance in this respect.. The ferrules of the tube ends need taking up from time to time, as the packing shrinks, or as the vibration loosens the ferrules.

300. Starting. The circulator should always be started first, so as to keep the tubes cool at all times when steam may come in contact with them; as soon as the circulator is running, the main unit may be started, and the hot-well pump. Whenever the vacuum is desired, the vacuum breaker valve may be closed and the dry-vacuum pump started.

301. Shutting down: shut off the main unit, break the vacuum with the breaker-valve, shut down the dry air-pump, then the hot-well pump, and lastly, the circulating water pump.

302. The piping should be so arranged that air can be removed from the highest point of the circulating water system; otherwise, air might accumulate and prevent water from entering part of the tubes, besides breaking the siphon. The discharge pipe should be submerged in the discharge tunnel or well, so as to have an inverted siphon, not to exceed 25 ft. head; this reduces the circulating pump head to friction and velocity only.

303. The cost of condensing equipment complete, including pumps, barometric type, is from \$0.16 to \$0.25 per lb., average \$0.21; cost per kw. of main unit, \$0.90 to \$1.95, average \$1.10 for moderate and large size units. For very small units the cost may go up to \$4.00 or \$5.00 per kw. Jet condensers cost from \$1.35 to \$2.10 per kw., average \$1.65, for moderate and large sizes; \$3.00 to \$4.50 for small units. The figures assume 26 in. or 27 in. vacuum and are based on normal kw. rating. Surface condensers, 28 in. to 29 in. vacuum, cost per kw. of main unit from \$2.25 to \$4.22, average \$3.12; this is based on maximum rating of units and applies to large sizes. Small surface condensers cost from \$3.50 to \$10.00 per kw. Cooling towers, \$3.12 to \$6.00, average \$4.30, per kw. rating of units attached.

FEED-WATER HEATERS

304. The heat transfer in closed feed-water heaters is exactly the same as in surface condensers; that in open heaters, the same as in jet condensers, as covered in Par. 285 to 289. The feed-water heater is merely a condenser operated at atmospheric pressure, with boiler feed-water for condensing water.

305. The open heater is much like a jet condenser in general arrangement, but is usually a rectangular box, or cylindrical tank large enough to provide a little storage capacity for feed water, as the boiler feed pumps usually draw direct from the heater.

306. The closed heater is usually built much like a surface condenser and is then known as the straight-tube type. In some makes the tubes are corrugated, in others coiled, or with a single U-bend; or an expansion joint is placed in the shell; in these cases the tubes are expanded fast in the tube sheets.

307. The water velocities employed in closed feed-water heaters are usually slower than for surface condenser practice, so that the value of the transmission coefficient is reduced. An average figure is 300 B.t.u. per deg. mean temp. difference per sq. ft. per hr.

308. The mean temperature difference given by the arithmetic formula is close enough.

$$t_m = t_s - \left(\frac{t_2 + t_1}{2}\right) \tag{48}$$

where t_s = temperature of exhaust steam at exhaust pressure (usually 212 to 214 deg.), t_2 = temperature of heater discharge water, and t_1 = temperature of heater inlet water.

309. Temperature rise. If a given quantity of exhaust steam is available, the resulting temperature rise in a closed heater may be found as follows:

$$(t_2 - t_1) = \frac{w_s(H_s - q_2)}{W} \tag{49}$$

where W = feed water in lb. per hr., w_s = exhaust steam in lb. per hr., H_s = total heat of exhaust steam per lb. (usually 1,150 B.t.u.), q_2 = heat of the liquid (condensed steam), at the temperature leaving the heater (t_2), usually 212 deg., t_1 = temperature of feed water at heater inlet, and t_2 = temperature of feed water at heater discharge. For all ordinary cases, $H_s - q_2 = 970$; t_2 cannot be higher than 208 deg. fahr., if exhaust steam at 212 deg. is employed. If the value of t_2, as found above, exceeds this, it indicates that there is excess exhaust steam. Eq. 56 can be transposed to solve for w_s, if the amount of exhaust steam to heat the feed water to 208 deg. is desired.

310. Temperatures Obtainable in Open Feed Water Heater
(Temperature of steam, 212 degrees F.)
Initial Temperature of Feed Water, Degrees F.

Per cent. of total steam used by auxiliaries.		40	50	60	70	80	90	100	110	120	130
	2	60.1	69.9	79.7	89.5	94.4	109.2	119.0	128.8	138.7	148.5
	3	69.9	79.6	89.3	90.1	108.8	118.6	128.3	138.0	147.8	157.5
	4	79.5	89.1	98.8	108.5	118.1	127.8	137.4	147.1	156.7	166.4
	5	89.0	98.5	108.1	117.7	127.2	136.8	146.4	155.9	165.5	175.1
	6	98.3	107.7	117.2	126.7	136.2	145.7	155.2	164.7	174.2	183.6
	7	107.4	116.8	126.2	135.6	145.0	154.4	163.8	173.2	182.5	192.1
	8	116.4	125.7	135.0	144.4	153.7	163.0	172.4	181.8	191.0	200.3
	9	125.2	134.5	143.7	153.0	162.2	171.5	180.7	190.0	199.2	208.5
	10	133.3	143.1	152.3	161.4	170.6	179.8	189.0	198.1	207.3	212.0
	11	142.5	151.6	160.7	169.7	178.9	188.2	197.0	206.2	212.0*	212.0*
	12	150.9	159.9	168.9	177.9	187.0	196.0	205.0	212.0*	212.0*	212.0*

311. The surface required in closed heaters is given by

$$A = \frac{W\,(t_2 - t_1)}{U t_m} \tag{50}$$

where U = 300 and A = area of tube surface in sq. ft.; the other symbols are as given in Par. 309.

312. Heaters are rated in h.p. One h.p. equals 30 lb. of exhaust steam per hr.

* All of the steam not condensed.

313. The volume required in open heaters is given by

$$V = \frac{HP}{a} \tag{51}$$

where V = volume in cu. ft., $a = 2.15$ for muddy water, $a = 6$ for slightly muddy water, and $a = 8$ for clean water.

314. Operation. The operation of closed heaters is somewhat more expensive than open heaters; the tube packings require attention and the tubes must be cleaned of scale, if this forms. In heaters with corrugated or bent tubes, cleaning is practically impossible. The advantage of the closed heater is the elimination of oil from the feed water, but with the use of turbine-driven auxiliaries, this advantage disappears, as there is no oil in the exhaust steam. In open heaters, the cast-iron trays over which the feed water spills, are readily removable for cleaning. If an open heater is used on oily exhaust, some means of oil elimination must be used, sometimes supplemented by a filter in the body of the heater.

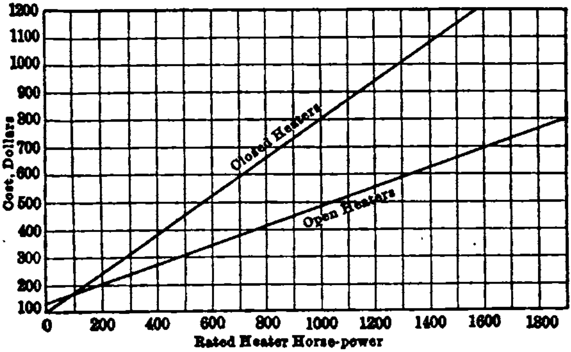


FIG. 39.—Cost of feed-water heaters.

315. Connection to feed pump. Closed heaters may be either the suction or the discharge side of the feed pump; open heaters, only on the suction side. Either type when on the suction side of the pump, must be above it.

316. The cost of heaters is given in Fig. 39.

ECONOMIZERS

317. The laws of heat transfer for economizers are the same as for surface condensers and closed water heaters. But the substitution of a gas for the steam increases the resistance enormously, and the value of the transmission coefficient is much lower and is very uncertain in value.

318. Types. There are two types: the **staggered tube** and the **non-staggered tube.** In both types the tubes are arranged in vertical rows attached to headers at top and bottom of each row, perpendicular to the gas flow. The staggered arrangement serves to break up the gas stream thoroughly. The tubes are always of cast iron and are usually 4½ in. diameter and 10 ft. long.

319. The surface required is based on empirical values, because the temperature and quantity of flue gases per lb. of coal, waterflow and condition of surfaces, introduce so many independent variables that the rational formulæ used for condensers and heaters are useless. The surface installed varies from 3 to 5 sq. ft. per rated boiler h.p. connected.

320. The temperature rise is given by

$$X = \frac{y(T_1 - t_1)}{91 + \frac{5w + GCy}{2GC}} \tag{52}$$

where X = rise in temperature of feed (deg. Fahr.), T_1 = temperature flue gases entering economiser, t_1 = temperature feed water entering economiser, w = lb. of feed water per boiler h.p. per hr., G = lb. flue gas per lb. of combustible (average, 20), C = lb. of coal per boiler h.p. per hr., and y = sq. ft. of economiser surface per boiler h.p. A rough method, if the temperature drop of flue gases is known, is to take 0.5 deg. rise in feed water for every deg. drop in flue gases.

321. The feed water enters the bottom headers by a connecting main and is collected from the top headers by another main, placed at the opposite end of the headers, so that the tubes form equal parallel paths.

322. The draft loss through economizers varies with the velocity of the gases, as in the boiler. At normal full load on the connected boilers the loss is from 0.25 in. to 0.40 in. (of water), increasing to 0.6 in. or 0.7 in. at heavy overloads. In fact, as normally designed and installed, forcing the boilers beyond 200 per cent. of rating is impossible without by-passing the economiser. Induced draft must be employed if the economiser is to be used at high ratings, because the draft losses limit the maximum capacity of the boilers obtainable with natural draft. If the temperature of gases is reduced from 550 deg. to 350 deg., the loss of draft is approximately 25 per cent.

323. The capacity rating of economizers is based on circulating 6.25 gal. of feed water per hr. per tube, and upon a heat transmission coefficient of 2.7 to 3 B.t.u.

324. Economizer Dimensions and Capacities

No. of tubes	No. tubes wide	No. sections	Length of economiser	Clear height req'd	Height over sections	Width between walls	Capacity (lb. water)	Heating surface external (sq. ft.)
			Ft. In.	Ft. In.	Ft. In.	Ft. In.		
96	6	16	9 8	23 6	10 2¼	4 8	6,000	960
144	6	24	14 6	23 6	10 2¼	4 8	9,000	1,440
192	6	32	19 4	23 6	10 2¼	4 8	12,000	1,920
240	6	40	24 2	23 6	10 2¼	4 8	15,000	2,400
128	8	16	9 8	23 6	10 2¼	6 0	8,000	1,280
256	8	32	19 4	23 6	10 2¼	6 0	16,000	2,560
384	8	48	29 0	23 6	10 2¼	6 0	24,000	3,840
512	8	64	38 8	23 6	10 2¼	6 0	32,000	5,120
576	8	72	43 6	23 6	10 2¼	6 0	36,000	5,760
160	10	16	9 8	23 6	10 2¼	7 4	10,000	1,600
320	10	32	19 4	23 6	10 2¼	7 4	20,000	3,200
480	10	48	29 0	23 6	10 2¼	7 4	30,000	4,800
640	10	64	38 8	23 6	10 2¼	7 4	40,000	6,400
800	10	80	48 4	23 6	10 2¼	7 4	50,000	8,000

Any other number of sections in multiples of 4, from 8 to 80, can be employed, giving proportionate dimensions.

325. Accumulation of soot occurs upon the tubes, and the "sweating" which takes place at low feed temperatures tends to catch and hold the soot firmly. **Continuous scraping** of the tubes is employed to overcome this difficulty. The cast-iron scrapers are driven slowly up and down the tubes in pairs, requiring about 1 h.p. to every 300 or 350 boiler h.p. connected to the economiser. There is a friction rig on each scraper drive so that if any solid obstruction stops one scraper, it will neither break the scraper nor interfere with the others.

326. The operation of economizers requires attendance for the scrapers, cleaning the inside of the tubes, and cleaning soot and fine cinder from the bottom of the economiser chamber. The first item is small, as the drive is usually by motor and the mechanism is very slow moving; the second depends on the feed water and should occur with less frequency than in the boilers; the last depends on the rate of driving and the coal, varying from once a month to once in four months.

327. The cost of economizers averages about $15.00 per tube, installed. The tubes are usually 4½ in. by 10 ft.; cast iron is employed throughout, since at the low rate of heat interchange, any other material would be too expensive. From the point of view of increased economy and the cost of securing it, the economiser is the least desirable of all auxiliaries. In many cases it cannot be made to pay; this is generally true if it saves less than 5 per cent. If it saves over 10 per cent., there is reason to conclude that the rest of the plant is being very badly operated.

PUMPS

328. The work done by a pump is given by

$$W = wh \tag{53}$$

where W = ft.-lb. of work per min. performed in lifting the water, w = lb. of water pumped per min. and h = sum of suction lift, discharge head and velocity head gained in the pump inlet and outlet, in ft. The water h.p. is equal to wh divided by 33,000. The water pressure, or suction, in lb. per sq. in. divided by 0.434 equals the head in ft. at the discharge, or the suction, respectively, for water at 62 deg. fahr. The velocity head is

$$h_v = \frac{v}{\sqrt{2g}} \tag{54}$$

where v = velocity of water in ft. per sec. and $\sqrt{2g} = 8.08$. For most cases, the velocity head in suction and discharge pipes may be disregarded, as it is not over 8 ft. per sec.

329. The duty is expressed

$$\text{Duty} = \left(\frac{\text{ft.-lb. of work done on water}}{\text{weight of dry steam}}\right)1{,}000 \tag{55}$$

or

$$\text{Duty} = \left(\frac{\text{ft.-lb. of work done on water}}{\text{Total B.t.u. used}}\right)1{,}000{,}000 \tag{56}$$

The latter definition of duty is more satisfactory than the first; duty always includes the efficiency of the steam end, as well as the water end.

330. Pumps are broadly classed in four types—reciprocating, centrifugal and turbine, rotary, and jet pumps. The **reciprocating type** can be subdivided into direct-acting, flywheel and power pumps. **The direct-acting type** has steam and water cylinders on a common piston rod, and no flywheel; the valve mechanism for the steam cylinder is actuated direct from the rod by tappets. This type may be single-cylinder, or duplex, and the steam cylinders may be simple, compound or triple expansion. **The duplex simple pump** is the most rugged and reliable of all. Outside-packed plungers are the most desirable, as the packing on the plunger is adjustable while running and the amount of leakage can be seen. **Flywheel pumps** are similar to the direct-acting type, except that a flywheel is added and the steam valve is gear driven from the shaft as in a steam engine. Pumping engines are a development of this type. **Power pumps** are fitted with one or more cylinders, driven from a crank shaft and belted or geared to the source of power. When three cylinders with cranks at 120 deg. are used, the pump is known as a triplex; this type is very frequently gear-driven from a motor or an internal combustion engine.

331. Centrifugal pumps are those in which pressure and flow is produced by a rotating impeller, which gives the water entering it an increase in velocity; this velocity is converted into pressure by a suitable whirlpool chamber or diffuser. **The obtainable head** is proportional to the square of the peripheral velocity of the impeller; this velocity is subject to practical limitations, so that 150 to 200 ft. head is about the upper limit desirable in a single impeller. For higher heads, such as boiler feeding, the pumps are made two, three or more **stages, consisting merely of single impellers connected in series in a single casing.**

332. Rotary pumps are very little used in power-station service; the commonest types are the bi-lobular type, and the gear pump. Neither has very good efficiency.

333. Screw or propeller pumps, while not strictly centrifugal, create pressure in the same manner, and are usually considered in the same class.

334. Jet pumps are covered in Par. **345** to **352**.

335. The characteristic curve of a centrifugal pump is necessary for determining its behav or. The capacity of a reciprocating or a rotary pump varies directly with the speed and is substantially independent of the pressure. The usual graphs are between capacity and efficiency, and capacity and head. Fig. 40 gives results for a 6-in. single-stage pump. Pumps for similar service, but different sizes, will have about the same characteristics, the efficiency increasing slightly with the size.

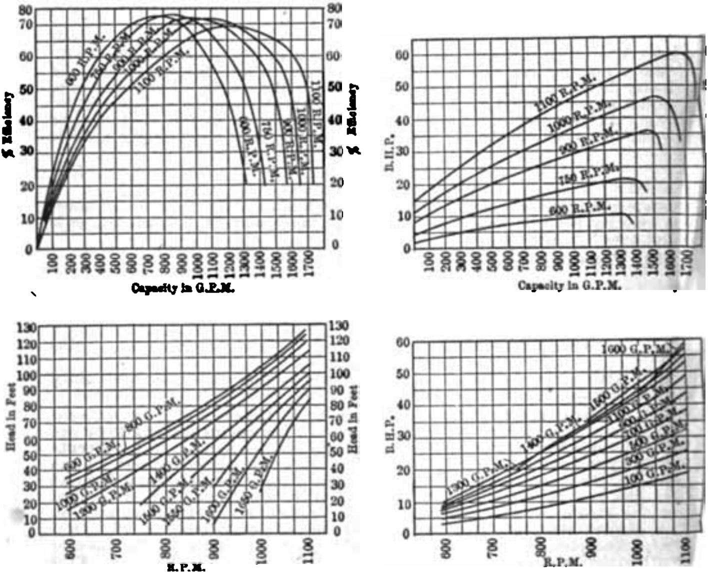


FIG. 40.—Characteristics of a 6-in. single-stage centrifugal pump.

336. Boiler feed pumps are usually the duplex outside-packed plunger type, or the three-stage or four-stage centrifugal. Occasional installations of motor-driven triplex pumps are made, but are undesirable. The principal feature required is reliability, which at once gives the steam-driven apparatus the precedence. The duplex direct-acting type was, until recently, almost exclusively used, but is now being replaced by the turbine-driven centrifugal. Fig. 40 shows that the pump speed is increased slightly as the delivery increases; this gives a more nearly constant head and a better efficiency with the variable loads.

337. Pressure regulators are employed to control the speed of all direct-acting boiler feed pumps and also, in most cases, for the centrifugal types.

338. The efficiency of direct-acting steam simplex and duplex pumps is low mechanically and thermally. The mechanical efficiency (water h.p./indicated h.p.) varies from 0.50 in small pumps to 0.85. The water rates are very high, both on account of the low speed and the lack of expansion.

339. Performance test of boiler feed pump. The following test of economy of Marsh pumps was made at Armour Institute of Technology; size $12 \times 7\frac{1}{2} \times 12$ in.; steam actuated valve gear; initial pressure 100 lb. gage; back pressure 2 lb. gage.

Number of strokes	Pump h.p.	Steam per indicated h.p.-hour	Number strokes	Pump h.p.	Steam per indicated h.p.-hour
10	1.0	400	60	6.4	105
20	2.0	210	70	7.6	101
30	3.0	168	80	8.8	100
40	4.1	130	90	10.0	99
50	5.2	118	100	11.3	99

(G. F. GEBHARDT)

340. The mechanical efficiency of geared triplex pumps is high, running up to 0.82 for motor-driven and high-speed pumps; the low-speed geared pumps reach 0.70.

341. The efficiency of centrifugal pumps depends on the size and number of stages. De Laval gives the following data for volute pumps.

Capacity	Efficiency	Capacity	Efficiency
75- 250 g.p.m.	55 to 65	3,000- 6,000	73 to 75
250- 900	70	6,000-10,000	75 to 78
900-3,000	70 to 73	10,000-up	75 to 85

The maximum efficiency of two-stage pumps is about 70 to 75 per cent, of three-stage and four-stage pumps, about 60 to 68 per cent. These values apply to turbine speeds, as in turbine drive. The efficiency of a volute pump is usually stated at the proper speed for the head. At turbine speeds and under low head, the efficiency drops about 20 to 25 per cent; this is the case for turbine-driven circulating pumps, in which the speed is too high for efficient low-head pumping and still too low for efficient turbine water rates. The latest solution is a reversion to the de Laval geared drive, using helical gears.

342. Costs are given in Fig. 41 for simplex and duplex pumps, and for triplex and two-cylinder geared power pumps. All costs are given in terms of displacement. The cost of large geared double-acting pumps is from \$2.00 to \$3.00 per cu. in. of displacement. The cost of single-stage centrifugal pumps is from \$0.50 to \$0.75 per gal. per min. of capacity. Multi-stage centrifugal pumps cost from \$2.50 to \$4.00 per gal. per min. of capacity.

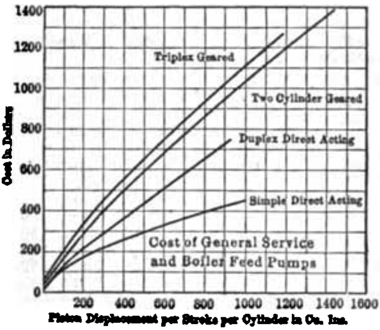


FIG. 41.—Costs of reciprocating pumps.

343. The usual piston speeds at which capacity is calculated for piston pumps, are from 100 to 200 ft. per min. for strokes below 12 in. and 100 ft. per min. for all strokes longer than 12 in.

344. Operation. The direct-acting simplex or duplex steam pump requires almost no attention except occasional packing of glands and lubricating. When controlled by a **pressure governor**, the delivery is suited to the demand. It will run on water as well as steam, and little care need be taken to supply it with dry steam. The geared triplex type requires more careful attention on account of the additional parts to be lubricated; if motor-driven, it must be provided with a suitable speed control, or it cannot be used on variable load. Piston and rotary pumps must always be provided with **relief valves** in the discharge, as they are positive in action and will produce excessive pressures if the discharge is restricted.

Centrifugal pumps cannot discharge under a higher head than that corresponding to the speed, therefore no relief is needed; the reason for using pressure governors with turbine-driven centrifugal boiler feed pumps is to reduce the speed in proportion to the delivery in order to keep up the efficiency. Centrifugal pumps are generally ring oiled and therefore require very little attention. The packing of the glands where the shaft leaves the pump casing is usually water-sealed; for this purpose a clean cool water should be provided. All centrifugal pumps must be primed before they will operate, and if no vacuum apparatus is provided, a foot-valve on the suction pipe must be used to prevent the water draining out of the pump on shutting down. A discharge valve must be provided, to be closed when priming, until the pump develops pressure.

JET PUMPS

345. Jet pumps operate by means of the kinetic energy of a rapidly moving stream of fluid. In those operated by steam, a jet of the high-pressure steam issues into a chamber at approximately atmospheric pressure or a little below, and there strikes the supply of water to be pumped. The impact of the steam transfers momentum to the water, which, together with the condensed steam, is hurled into a second nozzle which converts the kinetic energy of the mass into pressure and the stream enters the pressure chamber through a check valve. Since the water pumped must condense the steam used, it has a limitation in temperature, or the injector will fail to work.

346. Injectors are used for boiler feeding. For low and moderate pressures a single steam tube is used, both for lifting the water and forcing it into the boiler. But for heavier pressures and greater flexibility in handling variable quantities of water, and with wider range of pressure, the double-tube injector is employed. The first jet lifts the water to the second, or forcing jet.

347. Capacity of Schütte & Koerting double-tube boiler-feed injectors, in gal. per minute.

Size No.	Size pipe (in.)	Steam pressure 50 lb.	Steam pressure 100 lb.	Steam pressure 150 lb.	Size No.	Size pipe (in.)	Steam pressure 50 lb.	Steam pressure 100 lb.	Steam pressure 150 lb.
00	[illegible]	33	48	60	6	1¼	825	990	1,125
0	[illegible]	83	101	112	7	1½	1,072	1,372	1,612
1	[illegible]	112	143	180	8	1½	1,388	1,800	2,115
2	[illegible]	172	210	232	9	2	1,688	2,100	2,475
3[illegible]	[illegible]	278	338	397	10	2	2,025	2,438	2,850
3[illegible]	[illegible]	398	472	547	11	2–2½	2,580	3,050	3,515
4	1	533	622	720	12	2½	3,000	3,638	4,252
5	1¼	675	802	922					

The weight ranges from 3 to 108 lb.; No. 2 weighs 10 lb. and No. 4 weighs 20 lb.

348. Ejectors are either single-tube jet pumps, or direct-pressure pumps in which steam, or air, is admitted directly into a chamber filled by gravity with the liquid to be pumped. The pressure closes the inlet check valve and forces the water or other liquid out; when the chamber is emptied, a float therein drops and relieves the pressure, allowing the chamber to fill again. As the liquid reaches the top, the float rises and readmits pressure. The Shone and Albany traps are examples.

349. Inspirators are injectors of the double-tube type (Par. 346).

350. Siphons, so called, are single-tube jet pumps used for lifting only. Air, steam, or high-pressure water may be the motive force. They are not used for forcing against more than a few feet head.

351. Pulsometers comprise another form of direct-pressure pump like the ejector, and operate in much the same manner. The operating steam is condensed by the cold-water chamber and, in collapsing to water, draws the chamber full of water and operates a ball valve to admit steam again. There are always two chambers, one filling and one discharging, operated by the single ball valve. The pulsometer, therefore, can lift water by suction, whereas the pressure-type ejector must be primed by gravity.

352. Efficiency. If the mechanical efficiency alone is considered, all jet pumps are very inefficient, being more wasteful of steam than the direct-acting steam pump. But thermally considered, the steam injector is nearly 100 per cent. efficient, since the heat of the exhaust steam is returned in the feed water. The siphon and the pulsometer are convenient for temporary use and for drainage of pits under conditions adverse to the use of machinery.

353. The cost of these classes varies so widely that representative figures can hardly be given, \$2 to \$5 per 100 g.p.m. capacity covers most cases.

PIPING

354. The requirements of piping are: (a) tightness against leakage; (b) reasonably small pressure loss through friction; (c) suitable provision for change of length through change of temperature of the fluid contained; (d) reasonably small loss of heat by radiation if the fluid is hot, and intended to be kept so. Most of these requirements increase the first cost; a balance must therefore be found beyond which it does not pay to carry refinements.

355. The flow of steam is expressed by Babcock as

$$W = 87\left\{\frac{\gamma P d^5}{L\left(1+\frac{3.6}{d}\right)}\right\}^{\frac{1}{2}} \tag{57}$$

$$P = 0.0001321\left(\frac{W^2 L\left(1+\frac{3.6}{d}\right)}{\gamma d^5}\right) \tag{58}$$

where W = lb. steam flowing per min., L = pipe length in ft., d = inside diameter of pipe in in., γ = mean density of steam at pressures in the pipe and P = pressure drop in lb. per sq. in.

356. Steam-flow for 1-lb. drop computed by Babcock's formula

Initial pressure lb. gage	Weight of steam per min. in pounds with 1 lb. drop of pressure, in length of 240 pipe diameters										
	Nominal pipe diameter (in.)										
	¾	1	1½	2	2½	3	4	6	8	10	12
1	1.16	2.07	5.7	10.27	15.45	25.38	46.85	115.9	211.4	341.1	502.4
10	1.44	2.57	7.1	12.72	19.15	31.45	58.05	143.6	262.0	422.7	622.5
50	2.27	4.04	11.2	20.01	30.13	49.48	91.34	226.0	412.2	665.0	979.5
80	2.71	4.82	13.3	23.82	35.87	58.91	108.74	269.0	490.7	791.7	1166.1
100	2.95	5.25	14.5	25.96	39.07	64.18	118.47	293.1	534.6	862.6	1270.1
150	3.45	6.14	17.0	30.37	45.72	75.09	138.61	343.0	625.5	1009.2	1486.5

357. The flow of water in long pipes is given by Church as

$$Q = 3.15\sqrt{\frac{d^5 h}{fl}} \tag{59}$$

where Q = cu ft. per sec., d = diameter pipe in ft., h = head of water in ft., f = coefficient of friction and l = length of pipe in ft.

For pipe under 500 diameters in length,

$$Q = 6.3\sqrt{\frac{d^5 h}{(1+0.5)d+4fl}} \tag{60}$$

358. The loss of head due to friction is given by Weisbach as

$$H = \left(0.0144 + \frac{0.01716}{\sqrt{V}}\right)\frac{V^2}{5.367d^1} \tag{61}$$

where H = friction head in ft., V = velocity of water in ft. per sec. and d^1 = diameter of pipe in in.

359. Equivalent length of valves and elbows. The above formulas for steam and water (Par. 357 and 358) apply to straight pipes. Valves and elbows may be figured as equivalent to lengths of straight pipe as follows, **for steam,**

$$L = \frac{6.33\,d}{\left(1+\frac{3.6}{d}\right)} \text{ for each 90 deg. ell} \tag{62}$$

$$L = \frac{9.5d}{\left(1+\frac{3.6}{d}\right)} \text{ for each globe and angle valve,} \tag{63}$$

where L = equivalent length of straight pipe in ft., and d = diam. of pipe in in. Gate valves are not considered. **For water,**

$$H = C\left(\frac{V^2}{2g}\right) \tag{64}$$

where C = coefficient, 0.182 for 45 deg. ells, 0.98 for 90 deg. ells, 0.182 for gate valves, 1.91 for globe valves, and 2.94 for angle globe valves.

360. The principal piping systems are: high-pressure steam; exhaust or low-pressure steam; hot, cold and circulating water piping; and oil piping.

361. High-pressure steam piping is made chiefly of steel pipe with cast-iron fittings, if saturated steam is used; if superheat is employed, cast-steel fittings and valves must be used, as cast-iron will not stand the temperature. For any pressure up to 125 lb. per sq. in. **standard fittings** are used; above 125 lb. **extra heavy fittings. Screwed** fittings should not be used above 3-in. pipe sizes; all larger material should be **flanged.**

362. On exhaust lines, standard weight pipe and fittings are usually employed; for very large sizes, however, special light-weight fittings may be used to save weight and first cost. Screwed pipe may be used up to 8-in. pipe sizes, but flanged fittings are preferable for everything over 4 in. Spiral riveted galvanised pipe may be used in place of standard pipe, as it is very much lighter and perfectly suitable for moderate pressures. It cannot be used on vacuum work.

363. Hot and cold water mains are made up the same as live steam lines; except that in some cases cast-iron pipe as well as fittings are employed throughout on high-pressure hot-water service, to minimise corrosion effects. No cast-steel fittings are necessary. For circulating water lines, galvanised spiral riveted pipe is useful for fresh water, but cast-iron pipe is generally used throughout for salt water and is preferable even for fresh water.

364. Oil systems were generally installed in brass pipe and fittings on the supply to engines; but it has been found that steel pipe, if well cleaned, is perfectly satisfactory for the service. There is little excuse for the use of brass piping except for appearance, on gage fittings, or for some special service where corrosion would be fatal.

365. There are four principal systems of piping arrangement: individual supply; ring; header; and unit (Fig. 42). The **individual supply** is really not a system, but the lack of it, and should not be employed.

366. The ring system is a development of the duplicate header; all supply lines tap in on one side of the ring, all demand lines on the other; by means of sectionalising valves in the main, any section may be isolated without interfering with the rest of the plant.

367. In the header system, all supply and demand lines tap to one large main; so that if any section is cut out, it must interfere to some extent with the operation of the plant. This feature may not be serious; but for large power stations it is undesirable.

368. Unit system. The modern tendency is to revert to the unit system. This is the individual supply system—each group of boilers supplying its own turbine, but the units are tied together by equalizer pipes; so that there is really a header of diminished capacity between the units. Fig. 42 gives typical examples.

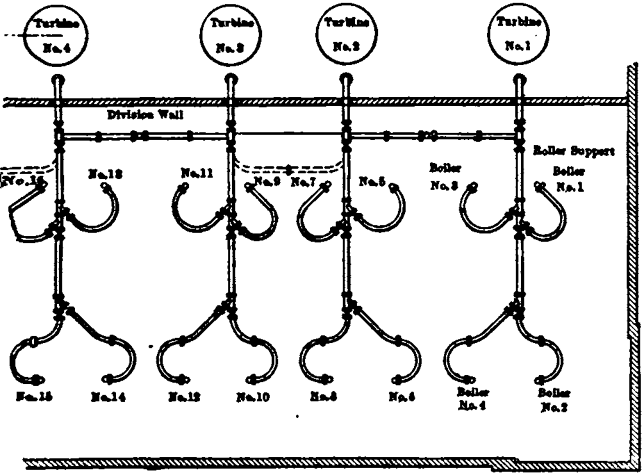


Fig. 42.—Piping systems.

369. Expansion in steel and cast-iron pipe may be taken as 0.9 in. per 100 deg. temperature difference from atmosphere, per 100 ft. of pipe under average conditions. **Expansion joints** should be provided every 50 ft. of straight steam main; every 75 or 100 ft. will do for water or exhaust steam. See Sec. 4, for coefficients of expansion of piping materials.

370. The slip expansion joint is useful for water service and exhaust steam at atmospheric pressure. It should never be used for high-pressure steam or vacuum; its capacity can be anything up to 9 in. or 10 in. of movement.

371. The copper bellows joint is very successful for low-pressure steam and vacuum work, particularly the type having only one corrugation. Its capacity however is never over 0.5 to 1 in. of expansion, and preferably not over 0.25 in. This type of joint is sometimes made up of boiler plate for high-pressure steam service.

372. The pipe bend is the simplest and safest joint for high-pressure steam and is usually made of as large radius as cán conveniently be made, taking care of from 1.5 to 2 in. of expansion per bend.

373. Avoidance of strains in fittings. Piping should always be laid out so that expansion will not bring strains upon cast-iron fittings, but upon the expansion joints, or at least upon the more flexible steel pipe.

374. Condensation in steam pipes is due to radiation; it occurs only in saturated steam mains. All low points in the piping system and all dead ends or pockets where water can collect, must be drained. The usual method for high-pressure steam is to connect a steam trap at each point to be drained and can be returned direct to the boilers, or to the feed tanks. Low-pressure steam mains are usually drained by gravity through an inverted siphon leg, to act as a steam seal.

375. Steam traps are of four principal types: bucket, float, tilting and expansion. The bucket and tilting traps are probably the most reliable.

376. Separators are employed to remove moisture from the steam before it is supplied to an engine. In every instance short and sudden turns are employed to throw out the moisture by centrifugal force, and a large chamber is then provided to reduce the velocity of the steam and act as a reservoir to collect and retain the water. (See Fig. 20a.)

377. Controlling valves are of three types: gate, globe and angle. The **globe valve** is always used for stop or throttle work. The **angle valve** is really a globe valve with the outlet turned through 90 deg. and is used as a globe valve, usually for the stop valve on boilers. For all other steam work the **gate valve** should be used on account of offering practically no obstruction to steam flow. All exhaust valves should be of the gate type. For water service, gate valves should be used wherever possible. **Check valves** may be horizontal, vertical, and swing. The horizontal and vertical check valves ordinarily are built much like a globe valve, but the valve disc has no stem and wheel for closing it.

378. Relief and back-pressure valves are developed from the check valve, with a spring or lever and weight loading device, so that a definite pressure under the valve disc will lift the valve and let off pressure. Relief valves are used on the discharge of reciprocating pumps to prevent excessive pressures; also on cylinders and receivers of steam engines. Back pressure and atmospheric relief valves are used on exhaust steam systems and condensers respectively. All of these types are emergency valves, to prevent damage under unusual conditions.

379. Reducing valves are usually double-seated, or balanced, valves, with a pressure diaphragm substituted for the spring of the relief valves. This diaphragm is connected to the discharge side of the valve, and operates to close the valve only when the pressure on the discharge side rises above the predetermined amount. They are used to feed high-pressure steam into low-pressure systems.

380. The three principal types of pipe joint used are; screwed, flanged, and bell and spigot. The **screwed joint** is used for all pressures and service up to 3-in. diameter of pipe, and up to 12 in. for low-pressure service. Above 3 in. in high-pressure service and 12-in low-pressure service the **flanged joint** in one of its forms is used.

381. The principal methods of attaching flanges to steel pipe are: screwed flanges, peened flanges and, lately, machine expanded flanges, welded flanges and lap flanges. Of these, **screwed flanges** are in common use for low-pressure work, and high-pressure work up to 100 lb. in any size, or up to 200 lb. in sizes not larger than 6-in. or 8-in. diameter. It is a cheap and satisfactory joint when well made.

382. A much better joint for high pressures is the lap joint, in which the pipe is lapped over onto the flange and then faced off. The **welded joint** is also satisfactory but more expensive. The new **machine-expanded joint**, in which the pipe is rolled into a recess in the flange, promises well and is cheap to make.

383. Gaskets for low-pressure work may be of rubber compounds or asbestos. For high-pressure steam, asbestos, and the metallic gaskets, such as corrugated copper, are better.

384. Bursting pressure of standard mild steel pipe. Tests made at the Armour Institute of Technology on 5-ft. random specimens capped at both ends gave average results as follows: 1-in., 7,730 lb. per sq. in.; 2-in., 5,080 lb.; 4-in., 1,750 lb.; 5-in., 2,550 lb.; 6-in., 3,200 lb.; 10-in., 1,800 lb.; 12-in., 2,500 lb. On the 4-in., 5-in., 6-in., 10-in. and 12-in. sizes failure occurred at the threaded end.

385. Pipe covering is practically always justified for high-pressure steam, and for exhaust also, if used for heating feed water. Hot feed-water pipes should also be covered. The standard coverings are principally magnesia, asbestos and the fossil meal compounds. Moulded sectional covering can be obtained for pipes up to 12-in. diameter in single (1-in.) and double (2-in. to 3-in.) thickness. All exhaust lines should be covered with single thickness; all steam lines, with double thickness. Usually the covering is bought already canvassed. For larger size pipe than 12 in., sectional blocks, about 1.5 in. × 3 in. × 18 in. are used, and wired on; then the joints are pasted with asbestos cement and the whole is canvassed and painted. Moulded covering can be bought in shapes to fit standard fittings, such as tees and ells, but is frequently made up from blocking and asbestos cement.

386. The radiation losses from uncovered pipe are given by

$$Q = A(T_1 - T_2)U \tag{65}$$

where A = sq. ft. of radiating surface, T_1 = temperature of steam within the pipe, T_2 = temperature of air outside, and U = transmission coefficient = 2.7 B.t.u. per sq. ft. per hr. per degree of temperature difference.

For 1-in. magnesia, 85 per cent., $U = 0.4 - 0.5$ (66)

For 1.5-in. magnesia, 85 per cent., $U = 0.25 - 0.3$ (67)

In a modern plant, with properly covered piping the actual radiation loss from pipe alone should not exceed 1 per cent. of the total heat of steam passing through at full load.

387. Tests of Relative Efficiencies of Steam-pipe Coverings

Kind of covering	Size of pipe, inches	Thickness of covering, in.	B.t.u. per sq. ft. per deg. diff. of temp.	Per cent. heat lost	Authority
Bare pipe	......	....	2.7	100	....
Hair felt	2	0.96	0.387	14.3	Jacobus
Reman t	2	0.88	0.434	16.1	Jacobus
Solid cork	2	1.20	0.427	15.8	Stott
Magnesia	2	1.16	0.439	16.3	Stott
Magnesia	4	1.12	0.465	17.2	Norton
Asbestos sponge felted	10	1.63	0.280	10.4	Barrus
Asbestos sponge felted	2	1.21	0.490	18.1	Barrus
Manville sectional	4	1.25	0.453	16.8	Norton
Manville sectional	2	1.31	0.572	21.2	Paulding
Asbestos air cell	4	1.12	0.525	19.4	Norton
Asbestos air cell	2	0.96	0.716	26.5	Jacobus
Asbestos fire felt	8	1.30	0.502	18.6	Brill
Asbestos fire felt	2	1.00	0.721	26.7	Paulding

(Abstracted from "Book of Standards," National Tube Co., Pittsburgh, Pa.)

388. Exhaust heads comprise a form of separator in which the entrained moisture in atmospheric exhaust is removed so that the issuing steam may not be a nuisance to the neighborhood, or an injury to the roof. They operate on the same centrifugal principle as separators.

389. Blow-off valves and piping, being subject to rapid variation from low to high temperature, must be carefully designed for expansion. As solid scale and rust flakes must be passed, the turns should be easy, and all connections between individual boilers and mains should be by 45-deg. laterals instead of tees.

390. The installation of piping should be done with great care to provide good alignment. Pipe joints can be made when piping is considerably out of line, as the lines are more or less flexible, but satisfactory joints and service cannot be expected. For **high-pressure** work, it is unsafe to strain the pipe in order to joint it. It is therefore good practice to arrange connections for flexibility, in case of variation from drawing dimensions; or better, to leave certain pieces of pipe called **fillers**, to be cut to field dimensions. This entails a slight delay in erection but is safe and eminently satisfactory.

All steam piping, and most water and oil piping, should be carefully **cleaned of internal scale** (by brushing or hammering), as this may dislodge during operation and cause trouble in the cylinders of engines or other apparatus. Steam and water piping should be tested at the working pressure before being put in service.

391. Hangers. Careful provision for hangers at suitable points must be made in order to support the pipe properly. The interval between hangers should not exceed 12 ft. except for small pipe, which may be supported on centers as much as 18 ft. apart.

392. Costs. Valves cost, in cast-iron body, approximately $0.20 to $0.25 per lb; straight cast-iron pipe, 12-ft. lengths, $34.00 to $50.00 per ton; straight pipe, shorter than 12-ft. lengths, $52.00 to $60.00 per ton; standard fittings, $60.00 to $75.00; special fittings, $73.00 to $113.00 per ton. Wrought-iron and steel pipe, 1-in. to 6-in. sizes, costs from 3.4 to 4.5 cents per lb.; 7-in. and 8-in. 4.3 to 5 cents; 10-in. to 12-in., 5 to 6 cents per lb. All the recent evidence seems to show that wrought-iron pipe is no better than steel for practically all purposes; and as the steel pipe is lower in price and more readily available, it is satisfactory to use it.

PLANT ENSEMBLE

393. The best combination of plant equipment is seldom if ever one in which each piece of apparatus is chosen by reason of its best water rate, or highest efficiency. The characteristic under variable load and the heat relation to the rest of the plant are most important.

394. The question of first cost versus economy is discussed under power-plant economics (Par. 382 to 385). The following paragraphs cover the best capacity and grouping of units for the best overall economy, with given efficiencies in the individual apparatus.

395. The heat analysis in B.t.u. is valuable for the purpose of proving the expected economies of any given combination of apparatus. Starting with the coal supplied, the boiler-room distribution of all the heat contents is followed through, including the B.t.u. delivered to steam, flues, unburned combustible, radiation, leakage, stoker, fan, and boiler feed-pump drives, other auxiliaries and miscellaneous steam such as boiler blow-down, free drip, dusting tubes, etc. In the engine room, the B.t.u. supplied in the steam is separated into pipe and engine radiation, exhaust drips, condenser auxiliaries, oil pumps, exciters, friction, electrical losses and energy delivered to bus bars. Heat returned by feed-water heaters and economizers is credited.

396. The average conditions in a large metropolitan station were given in a paper by H. G. Stott, on "Power Plant Economics" (A. I. E. E. Transactions, 1906), Par. 915. Recomputed several years later, under improved conditions, the same plant showed, using coal having 14,000 B.t.u. per lb. net:

	Per cent.	Per cent.		Per cent.	Per cent.
Coal		100.0	Circ. water pumps	4.0	
Ash loss*	3.0		Boiler feed pumps	1.0	
Stack loss*	19.0		Leakage and drips included in pipe radiation		
Incomplete combustion	5.1		Small auxiliaries	1.3	
Boiler radiation and leakage*	8.0		Heating	0.2	
Returned by F. W. heaters		6.1	Engine friction	1.0	
Returned by economizers		5.7	Electrical losses	0.3	
Pipe radiation and leakage	4.5		Engine radiation	1.1	
Tank radiation	0.3		Rejected to condensers	51.3	
			House auxiliaries	0.2	
			To switchboard	11.5	
			Total	111.8	111.8

* 65 per cent. boiler efficiency, including banking fires.

397. In choosing the number and capacity of prime movers, the **Willans line** of total steam consumption with load is plotted. The arrangement which gives the east area under the curve is the most economical. Fig. 43 shows this curve for 42-in. × 86-in. × 60-in. double compound Corliss engines, 7,500 kw. maximum continuous rating on 190 lb. dry steam, 28 in. vacuum. If the auxiliaries were not considered, the Willans line for engines alone would show the proper points for cutting engines in and out. It is found, however, that the most economical method is to carry up the load on n units until the Willans line for n units intersects the Willans line for $n+1$ units; and *vice versa*. The maximum capacity of individual machines may limit the cutting-in point somewhat when only one or two engines are running, but as the number in service increases, the loads on each engine before and after cutting in another unit become nearer alike.

398. Effect of auxiliaries on Willans line. The condensers and other auxiliaries using steam necessitate the addition of their demand to the main-unit demand, and the resulting Willans lines will give the steam demand of the engine room for any load. This supposes the units to be all alike and equally loaded when on the line; however, for units of unequal capacities, various loadings should be tried and plotted in order to determine the most economical operating conditions (Fig. 43).

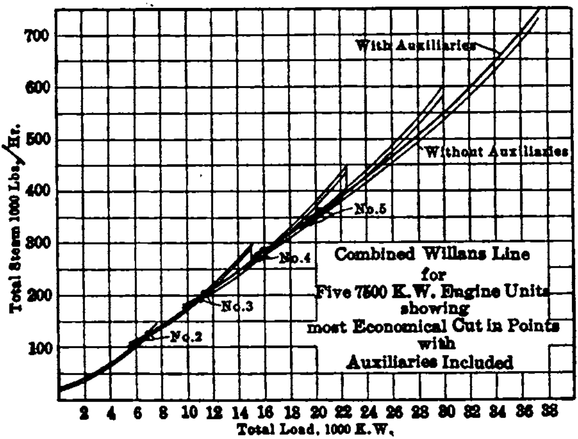


Fig. 43.—Willans line for 7500-kw. engines.

399. The boiler generation curve is determined by the efficiency of the boiler and stoker and by the steam required for forced draft, stoker drive, boiler feed pumps, etc. The economical cutting in and out pointe can be developed in the same manner as for the engine room, noting always that it is the overall efficiency which is important, not that of the boiler alone.

400. Feed-water heaters recover from 5 to 15 per cent. of the heat of the steam, the source being chiefly the exhaust from auxiliaries operated non-condensing. The feed-water heater always pays for itself.

401. Economizers recover from 3 to 10 per cent. of the total heat; but the re ent increase of boiler efficiency (reducing the stack temperatures), the reversion to all-steam auxiliaries, and the need for more draft at higher capacities, is rapidly limiting the field of the economizer; it does not always economize.

402. Possible reductions in the heat losses. The heat losses are given in Par. 396. It is clear that: (a) the ash loss cannot be much reduced;

(b) the stack loss is probably susceptible of considerable improvement, especially by way of improving combustion, which affects stack temperature indirectly; (c) the incomplete combustion would be decreased with (b); (d) the condenser loss is the only other very considerable item, and this cannot, by nature of the cycle, be appreciably altered.

403. The steam consumption per kw-hr. of the plant will depend upon the water rate of the main unit, the steam required by auxiliaries and the load factor. The no-load steam losses include: banked boilers; pipe radiation and leakage; boiler blow-down; boiler dusting; operation of condenser auxiliaries on emergency units and boiler auxiliaries, and supplying feed water for these losses. If the load factor is low, these bear a larger proportion to the total. In a high-grade plant with a yearly load factor of 30 to 40 per cent., the station water rate is 25 to 35 per cent. higher than the best water rate of the main unit. For ordinary stations of moderate size on less than 25 to 40 per cent. load factor, the station water rate is 80 to 100 per cent. higher than that of the main unit alone. The steam required for condenser auxiliaries is from 2 to 3½ per cent. of the main-unit consumption, for engine units, up to 27 in. vacuum; for turbine plants, 4.5 to 10 per cent for 28 in. to 28.5 in. vacuum. The boiler feed-pump steam varies from 1 to 1½ per cent. for direct reciprocating pumps, and 0.6 to 1.3 per cent. for high-grade pumping engines, or turbine-driven centrifugal types. The steam for forced-draft fans on underfeed stokers varies from 1 to 2 per cent. of main-unit steam; coal-conveying systems require 1.7 to 3 per cent., or the equivalent in electrical energy. The total varies therefore from 8 to 18 per cent.

404. The power taken by the auxiliaries of a 5,000-kw. turbine tested by the Edison Illuminating Co. of Boston was as follows:

Kw. on turbine	2,713.0	3,410.0	4,758.0
Vacuum, in	28.4	28.7	28.6
Barometer, in	29.53	29.95	29.96
I.h.p. of boiler-feed pump	13.9	23.7	27.2
I.h.p. of circulating pump	69.1	69.1	69.1
I.h.p. of dry-vacuum pump	24.3	23.2	23.8
I.h.p. of step-bearing pump	6.4	5.8	5.6
Total power for auxiliaries	122.3	131.0	135.7
Per cent. power of auxiliaries	3.4	2.9	2.1
Per cent. water used by auxiliaries	8.4	7.4	5.7
Electric h.p. of wet-vacuum pump	8.6	9.2	9.8

405. Number of main units. In modern central-station practice, the reciprocating engine is practically defunct. For 24-hr. service, not less than three turbine units should be employed and prefe ably six to eight for large stations. The units should always be of similar size. The analysis given in Par. 397 to 399 will prove that there is very seldom any economy in selecting odd-size units for a plant, and there is always increased risk of shutdown and increased cost, due to lack of interchangeability of units and spare parts.

406. Steam versus electric drive for auxiliaries. The heat analysis shows that the steam auxiliary is always more economical than the motor-driven type, when the amount of steam required for auxiliaries does not exceed the demand for heat in the feed water. Turbine-driven auxiliaries are practically universal except for the dry-vacuum pumps, which are often reciprocating, and small oil pumps, which are more satisfactory if duplex direct acting. Turbine-driven blast fans are becoming very widely used. Turbine-driven centrifugal boiler feed pumps are very satisfactory in units larger than 300 gal. per min.; in smaller units, the impeller passages are small and likely to give trouble from scaling and clogging.

407. General plant layout. The most generally satisfactory scheme is to lay out the plant on the **unit system**, with a complete set of boilers and auxiliaries for each turbine. The advantages become more and more marked, the larger the plant.

408. Plant location. The plant should be located at the edge of a sufficient body of water to supply abundant condensing water; where this cannot be done, wells may possibly supply the required amount, or, failing this, cooling towers will be necessary. The next most important consideration is coal supply; location on a river or at tide-water may automatically take care of coal supply by barge, but otherwise a rail connection will be needed. The third item is suitable ground for the foundations of the building and machinery. Choice of site where rock bottom is available near the surface is most desirable, but hardpan is practically as satisfactory. On marsh land near rivers or tide-water, piling or fill will be necessary. Too great pains to provide solid and permanent foundations cannot be taken.

409. Load factor plays a large part in selection of a plant; for low load factors, the fixed charges are high, and the operating charges relatively small. It is therefore necessary to cut down the first cost, which is done by omitting refinements for efficiency and allowing the thermal economy to go down. For high load factor, the fixed charges are less important and operating charges become the controlling factor; therefore profitable investment in devices for high thermal economy may be made. For low load factors, machines with high overload capacities are desirable, at the sacrifice of economy. It will be desirable to overstoker the boilers a little, to this end, as well as to build the turbines for high overloads.

410. Reserve capacity of one main unit at peak load should be maintained in plants of any number of units up to eight; above this, two reserve units. This applies to plants with duplicate equipment. For plants with dissimilar units, the reserve should be equal to the largest unit in the plant, either as one large unit, or the equivalent load capacity in smaller units.

411. Unit costs of equipment may be taken as given in Par. **87, 88,** etc. But on a basis of plant output, the relations will be different. Lyford and Stovel give the following central station costs, high and low, for steam turbo-electric generating stations of 2,000 to 20,000 kw. capacity, based on maximum continuous capacity of generators as 50 deg. Cent. temp. rise.

	Dollars per kw.	
	High	Low
Preparing Site: Clearing structures from site, construction roads, tracks, etc.	$0.25	......
Yard Work: Flumes for condensing water, siding, grading, fencing, sidewalks, etc.	2.50	$1.00
Foundations: Foundations for building, stacks and machinery, excavation, piling, waterproofing, etc.	6.00 6.00	1.00 1.00
Building: Frame, walls, floors, roofs, windows, doors, coal bunker, etc., exclusive of foundations, heating, plumbing and lighting.	12.00	4.00
Boiler-room Equipments: Boilers, stokers, flues, stacks, feed pumps, feed-water heater, economizers, mechanical draft, piping and covering, except condenser water piping.	24.00	12.00
Turbine Room Equipment: Steam turbines and generators, condensers, condenser auxiliaries, condenser water piping, oiling system, etc.	22.00	12.00
Electrical Switching Equipment: Exciters, masonry switch structure, switchboards, switches, instruments, etc., all wiring except for lighting.	5.00	2.00
Service Equipment: Cranes, lighting, heating, plumbing, fire protection, compressed air, furniture, permanent tools, coal- and ash-handling machinery, etc.	5.00	2.50
Starting Up: Labor, fuel and supplies for getting plant ready to carry useful load.	1.00	0.50
General Charges: Engineering, purchasing, supervision, clerical work, construction plant and supplies, watchmen, cleaning up, etc.	6.00	3.00
Total cost of plant, except land and interest during Construction.	$83.75	$38.00

412. Reliability is increased by uniform equipment, and by proper reserves. The turbine plant is superior to the engine plant in this respect, with the same number and capacity of units. It is not good practice to operate a unit regularly at overloads, although the machine may always be overloaded for short periods before cutting in or after cutting out a unit. The analysis given in Par. **397** to **399** will show this to be thermally economical.

413. Labor requirements vary with station arrangements: roughly, for reciprocating engines, one man is required per 750 to 1,000 kw. in the engine room, and one man per 1,000 to 1,500 kw. in turbine stations. This includes switchboard and electrical attendance for operation only. Maintenance and repairs will need about 10 to 15 per cent. additional labor.

For the boiler room, one man is required per 600 to 800 kw. with overfeed stokers and one man per 800 to 1,500 kw. with underfeed types. The total for engine plants will be from 300 to 450 kw. per man for engine plants and 400 to 725 kw. for turbine plants, for operation only; for maintenance add 10 to 15 per cent. to the number of men required for operation.

414. Relative Costs per Kw-hr. Distribution of Maintenance and Operation (H. G. Stott)

	Reciprocating steam plant	Steam turbine plant	Reciprocating engines and low-pressure steam turbines	Gas-engine plant	Gas engines and steam turbines	Hydraulic
MAINTENANCE						
1. Engine room, mechanical.	2.59	0.51	1.55	5.18	2.84	0.51
2. Boiler or producer room.	4.65	4.33	3.55	1.16	1.97	......
3. Coal and ash-handling apparatus.	0.58	0.54	0.44	0.29	0.29	......
4. Electrical apparatus.	1.13	1.13	1.13	1.13	1.13	1.13
OPERATION						
5. Coal............	61.70	55.53	52.44	26.52	25.97	......
6. Water...........	7.20	0.65	0.61	3.60	2.16	......
7. Engine room, labor.	6.75	1.36	4.06	6.76	4.06	1.36
8. Boiler or producer room labor.	7.20	6.74	5.50	1.81	3.05	......
9. Coal and ash-handling labor.	2.28	2.13	1.75	1.14	1.14	......
10. Ash removal......	1.07	0.95	0.81	0.54	0.54	......
11. Electrical labor...	2.54	2.54	2.54	2.54	2.54	2.54
12. Engine-room lubrication.	1.78	0.35	1.02	1.80	1.07	0.20
13. Engine-room waste, etc.	0.30	0.30	0.30	0.30	0.30	0.20
14. Boiler-room lubrication, etc.	0.17	0.17	0.17	0.17	0.17	......
Relative operating cost per cent.	100.00	77.23	75.87	52.94	47.23	5.94
Relative investment per cent.	100.00	75.00	80.00	110.00	96.20	100.00
Probable average cost per kw.	125.00	93.75	100.00	137.50	120.00	125.00
Probable fixed charges per cent.	11.00	11.00	11.00	12.00	11.50	11.00

For steam-turbine plants larger than 60,000 kw. the cost per kw. may be reduced to $75.

TESTING

415. Boiler-testing requires the following essential data: (a) Weight of water fed per hr.; (b) weight of coal fed per hr.; (c) quality of steam; (d) pressure and superheat; (e) feed-water temperature; (f) proximate analysis of coal including moisture and B.t.u. per lb. Additional desirable data includes: (g) Flue temperature, including furnace and pass temperatures; (h) flue gas analysis, and analyses of gas at various points in boiler; (j) weight of refuse; (k) proximate analysis of refuse, moisture and B.t.u. per lb.; (l) siftings, if stoker fired; (m) soot and dust passing through boiler; (n) steam flow by meters. Boiler testing is always interwoven with stoker testing; the fire and the heating surface are tested together. The **duration of test** should never be less than 24 hr., for the most accurate results.

416. For weighing feed water, tanks on platform scales are best: but lacking these, a calibrated recording Venturi meter is satisfactory and accurate within 1 per cent. on reasonably steady flow.

417. For full details as to methods of firing, calorimetry, etc., see standard rules of the A. S. M. E.

418. Heat Balance, or Distribution of the Heating Value of the Combustible

The heat value of 1 lb. of combustible	B.t.u.	
	B.t.u.	Per cent.
1. Heat absorbed by the boiler = evaporation from and at 212° per lb. of combustible × 965.7.		
2. Loss due to moisture in coal = per cent. of moisture referred to combustible ÷ 100 × [(212 − t) + 966 + 0.48 (T − 212)] (t = temperature of air in the boiler-room, T = that of the flue gases).		
3. Loss due to moisture formed by the burning of hydrogen = per cent. of hydrogen to combustible ÷ 100 × 9 × [(212 − t) + 966 + 0.48 (T − 212)].		
4.* Loss due to heat carried away in the dry chimney gases = weight of gas per pound of combustible × 0.24 × (T − t)		
5.† Loss due to incomplete combustion of carbon = $\frac{CO}{CO_2+CO} \times \frac{\text{per cent. C in combustible}}{100} \times 10{,}150$.		
6. Loss due to unconsumed hydrogen and hydro-carbons, to heating the moisture in the air, to radiation, and unaccounted for. (Some of these losses may be separately itemized if data are obtained from which they may be calculated)		
Totals....................................		100

*The weight of gas per lb. of carbon burned may be calculated from the gas analyses, as follows:

† CO_2 and CO are respectively the percentages by volume of carbonic acid and carbonic oxide in the flue gases. The quantity 10,150 = number of heat units generated by burning to carbonic acid 1 lb. of carbon contained in carbonic oxide.

$$\text{Dry gas per lb. carbon} = \frac{11CO_2 + 8O + 7(CO + N)}{3(CO_2 + CO)}, \qquad (68)$$

in which CO_2, CO, O and N are the percentages by volume of the several gases. As the sampling and analyses of the gases in the present state of the art are liable to considerable errors, the result of this calculation is usually only an approximate one. The heat balance itself is also only approximate for this reason, as well as for the fact that it is not possible to determine accurately the percentage of unburned hydrogen or hydrocarbons in the flue gases.

The weight of dry gas per lb. of combustible is found by multiplying the dry gas per lb. of combustible by the percentage of carbon in the combustible, and dividing by 100.

419. Especial care should be taken to isolate the boiler tested so that leakage can be eliminated or measured. No single valve may be considered absolutely tight. Where lines cannot be completely disconnected, there should be two closed valves with a half-inch open bleeder between them to indicate leakage. Variations of water level must be corrected for.

420. Summary of overall results.

TOTAL QUANTITIES

1.	Date of trial..............................	
2.	Duration of trial..........................	hr.
3.	Weight of coal as fired....................	lb.
4.	Percentage of moisture in coal.............	per cent.
5.	Total weight of dry coal consumed.........	lb.
6.	Total ash and refuse.......................	lb.
7.	Percentage of ash and refuse in dry coal....	per cent.
8.	Total weight of water fed to the boiler.....	lb.
9.	Water actually evaporated, corrected for moisture or superheat in steam.	lb.
10.	Equivalent water evaporated into dry steam from and at 212°.	lb.

For further details, see the standard code of the A. S. M. E.

421. Quality of steam. The percentage of **moisture** in the **steam** should be determined by the use of either a throttling or a separating steam calorimeter. The sampling nozzle should be placed in the vertical steam pipe rising from the boiler. It should be made of 0.5-in. pipe, and should be extended across the diameter of the steam pipe to within half an inch of the opposite side, being closed at the end and perforated with not less than twenty 0.125-in. holes equally distributed along and around its cylindrical surface, but none of these holes should be nearer than 0.5 in. to the inner side of the steam pipe. The calorimeter and the pipe leading to it should be well covered with felting.

422. Superheating should be determined by means of a thermometer placed in a mercury-well inserted in the steam pipe. The degree of superheating should be taken as the difference between the reading of the thermometer for superheated steam and the reading of the same thermometer for saturated steam at the same pressure, as determined by a special experiment, and not by reference to steam tables.

423. Sampling the coal and determining its moisture. As each barrow-load or fresh portion of coal is taken from the coal-pile a representative shovelful is selected from it and placed in a barrel or box in a cool place and kept until the end of the trial. The samples are then mixed and broken into pieces not exceeding 1 in. in diameter, and reduced by the process of repeated quatering and crushing until a fine sample weighing about 5 lb. is obtained and the size of the larger pieces is such that they will pass through a sieve with 0.25-in. meshes. From this sample two 1-qt., air-tight glass preserving jars, or other air-tight vessels which will prevent the escape of moisture from the sample, are to be promptly filled, and these samples are to be kept for subsequent determinations of moisture and of heating value and for chemical analyses. For further details, see the standard code of the A. S. M. E.

424. Calorific tests and analysis of coal. The quality of the coal should be determined either by heat test or by analysis, or by both. The rational method of determining the total heat of combustion is to burn the sample of coal in an atmosphere of oxygen gas, the coal to be sampled as directed in Article XV of the code. The **chemical analysis** of the coal should be made only by an expert chemist. The total heat of combustion computed from the results of the ultimate analysis may be obtained by the use of Dulong's formula (with constants modified by recent determinations), viz.,

$$14{,}600C + 62{,}000\left(H - \frac{O}{8}\right) + 4{,}000S, \qquad (68)$$

in which C, H, O, and S refer to the proportions of carbon, hydrogen, oxygen and sulphur, respectively, as determined by the ultimate analysis. It is desirable that a **proximate analysis** should be made, thereby determining the relative proportions of volatile matter and fixed carbon. These proportions furnish an indication of the leading characteristics of the fuel, and serve to fix the class to which it belongs. As an additional indication of the characteristics of the fuel the specific gravity should be determined.

425. Treatment of ashes and refuse. The ashes and refuse are to be weighed in a dry state. If it is found desirable to show the principal characteristics of the ash, a sample should be subjected to a proximate analysis and the actual amount of incombustible material determined. For elaborate trials a complete analysis of the ash and refuse should be made.

426. A throttling or a Thomas electric calorimeter should be used where possible, in preference to separating or other types. **Thermometers** are more satisfactory as indicators of saturated steam pressure than spring gages.

427. Engine testing requires essentially: (a) Weight of steam used; (b) kw. output of generator, or brake h.p.; (c) i.h.p., from indicator cards; (d) rev. per min.; (e) steam pressure, receiver pressure and back pressure or vacuum; (f) quality of steam. Desirable additional data include: (g) receiver and exhaust temperatures; (h) weight of condensation from receiver or elsewhere; (j) reheater steam, if used.

428. The steam consumption is best obtained by weighing the discharge from a surface condenser; but if jet condensers are used or the engine is non-condensing, the feed water must be measured as it enters the boilers, the boiler leakage determined, and the boilers isolated from all other steam piping and apparatus. If the condensate is weighed, the condenser leakage must be ascertained. The test should continue not less than 1 to 3 hr., with surface condensers and 8 hr. when using the feed-water method.

429. Turbine testing involves the following essential data: (a) Steam weight per hr.; (b) initial and exhaust pressures; (c) superheat, or wetness of steam; (d) rev. per min.; (e) b.h.p. or kw. output of generators; (f) steam chest pressure. There is a general similarity to engine testing, the only real difference lying in the absence of indicator cards. In measuring the condensate, provision must be made for measuring also the gland-water, or steam for sealing purposes, which is not strictly chargeable to the turbine, since all the water, or all the heat is recovered.

430. For full data on turbine testing see the standard rules of the A. S. M. E.

431. Condenser testing. The condenser is the chief auxiliary requiring test: generally it is convenient to combine the condenser test with the engine or turbine test on the attached prime mover. The essential readings are: (a) vacuum at exhaust entry referred to standard barometer; (b) condensate weight per hr.; (c) circulating water temperatures at inlet and discharge; (d) hot well temperature. Additional desirable data include: (e) vacuum at hot well; (f) vacuum at dry vacuum pump; (g) temperatures, top and bottom of condenser; (h) temperature of dry air at pump; (j) Pitot or Venturi meter readings on circulating water; (k) temperature of circulating water at beginning and end of each pass; (l) volume of dry air removed per min. A separate test for leakage should be made before the operating test, and tests for salt may be made on the condensate during the operation, if salt water is employed as cooling water.

432. Tests of boiler feed pumps and circulating water or hot-well pumps, of the centrifugal type, require the following readings, for turbine-driven units: (a) weight of steam used per hr.; (b) steam and exhaust pressures; (c) quality of steam, or superheat; (d) suction and discharge heads; (e) weight of water pumped—by scales, tank or Pitot or Venturi tube readings, which are close enough for this work (weirs may be used for large pumps); (f) rev. per min.

433. In testing direct-acting steam pumps, indicator cards should also be taken.

434. For tests of electric driven pumps, readings (a), (b), and (c) in Par. 432 would be replaced by kilowatt readings on the motor.

435. Tests on turbine-driven fans for forced or induced draft require: (a) Steam used per hr.; (b) steam pressure and back pressure; (c) quality of steam or superheat; (d) Pitot static and dynamic readings on suitable fan outlet; (e) rev. per min. For motor drive, kilowatt readings will replace (a), (b) and (c).

436. Complete plants have what amounts to a continuous test in their regular records, if proper y kept. Coal is bought and paid for by weight and by analysis; and recording wattmeters give the output. For yearly figures the plant storage usually is not an important factor, as its amount is small compared to the total consumption; but for monthly figures, a survey of the coal stored is necessary. This is usually made by taking readings at predetermined points over the bunkers or storage space and calculating the volume of coal, allowing generally 50 lb. per cu. ft. for bituminous coal and 54 lb. per cu. ft. for No. 1, No. 2 or No. 3 buckwheat anthracite.

437. The apparatus required for weighing water is preferably made up of one or more sets of tanks on platform scales; the accuracy may be kept to 0.25 per cent. quite readily. Where less accuracy is required, 0.5 to 1.5 per cent., measurement by volume in a tank, or by Venturi meter, or V-notch weir, is suitable.

438. For measuring the steam used, the best method is by surface condensers (which may be made substantially tight), using scale tanks for weighing the condensate.

439. For measuring temperature up to 900 deg. Fahr., mercury thermometers of 12 in. length are most convenient; they should be used in iron or brass wells, with a little mercury in the bottom of the cup, and the remainder filled with oil. For temperatures above 500 deg. Fahr., nitrogen-filled mercury thermometers must be employed. Care should be taken to ascertain the immersion for which the thermometer is calibrated. Resistance thermometers are now available, but are in general very much more expensive; their use is a convenience where there are several inaccessible readings to be taken, as all the resistance bulbs may be read from one galvanometer and bridge. For higher temperatures than 900 deg. Fahr., resistance thermometers or thermocouples should be used. For high furnace temperatures, radiation or optical pyrometers are best. (Sec. 3.)

440. For pressure measurements, the spring gage is convenient but usually inaccurate; for pressures up to 15 lb., mercury columns are far better. For steam pressure, the thermometer is a valuable and accurate means of getting results. If the steam is superheated, a thermometer in a pressure chamber separated from the steam main by a condensing coil, and suitably drained, will give correct readings of pressure. If gages are used, they should be carefully checked at the operating pressure, in position if possible, against a standard gage tester.

441. Frequency of observations. Observations of temperatures, pressures and speeds should occur once every 10 or 15 min. Water and coal weighings should be made as often as needed by the capacity of the containers on the scales. Wattmeter readings and coal and water readings should be balanced every hour, as a precautionary measure in checking the steadiness and reliability of the test. Calorimeter readings on boilers and turbines at steady load only need be taken at half-hourly or hourly intervals, as they vary but little. Barometer readings taken three times during a test are sufficient. For short tests, such as occur in motor-driven pumps and fans with Pitot tube or Venturi tube readings, instantaneous values taken once a minute for 10 min. gives good enough results as a rule. Pitot readings for air should be taken at numerous points in the pipe (see Treat, *Trans.* A. S. M. E., 1912, and Rowse, *Trans.* A. S. M. E., 1913, and Taylor, *Trans.* N. A. & M. E., 1905).

442. Duration of tests. Engine or turbine tests by the feed-water method should not be less than 8 hr., as the error introduced by variations of level in the boilers is too great on shorter tests; when weighing the condensate, 1 to 3 hr. will be sufficient, depending on reservoir capacity in the system. Where flow meters can be employed with sufficient accuracy, instantaneous tests of a few minutes duration may be run. The time taken for a boiler test, start to stop, should never be less than 10 hr., 12 are better and 18 to 24 hr. give much more consistent results. This length of time is needed on account of the variable error introduced by the unknown amount of coal on the fire at start and finish.

443. Precision of tests. Boiler tests of 10 hr. duration are seldom closer than 3 per cent. either side of the average; 12-hr. tests, 2.5 per cent.; 24 hr. 1 per cent. Engine tests of 8 hr. duration, will have a probable error of 2 per cent., either way; turbine tests of 3 hr., 2 per cent. It is therefore needless to compute results closer than $\frac{1}{10}$ of 1 per cent.

GAS POWER PLANTS

BY REGINALD J. S. PIGOTT

PRODUCERS

444. Destructive distillation can occur only with fuels containing volatile matter; therefore anthracite and coke producers cannot be said to operate by destructive distillation. **Gas** may be produced from **hard coal** as follows: with air only, $2C+O_2=2CO$; with steam only, $H_2O+C=CO+H_2$; with both steam and air, $3C+O_2+H_2O=3CO+H_2$. The first process is not often used; the second is used to make **water gas**, but since it does not produce heat, it is used intermittently with the process $C+O_2=CO_2$, or simple combustion of the fuel in the producer to generate the necessary heat. This is called "blasting up." The third process is the one used in making **producer gas** from anthracite or coke.

445. With bituminous coals, the volatile matter is given off as tar and hydrocarbon gases of the methane series, chiefly. These are again broken up by the heat in the producer to methane, carbon monoxide and hydrogen. In addition, the formation of CO and H from the coke remaining, goes on substantially as in the anthracite producer. In the down-draft types of producers, the tar passes through the fuel bed and is broken down to combustible gases and lampblack, some of which are burned in the passage.

446. Suction producers obtain a flow of air, steam and gas by means of a slight difference of pressure due to the pump action of the four-cycle gas engine on the charging stroke, or by an exhauster.

447. Pressure producers obtain a flow of gas, etc., by means of a pressure fan or blower for the air, or a boiler for the steam. A pressure producer is independent of the engine, and does not affect the capacity of the latter.

448. Up-draft producers are arranged with the steam and air admitted at the bottom and gas removed from the top; tar passes off with the gas and must be removed by scrubbers and purifiers.

449. Gasification of Anthracite Coal (R. D. Wood Co.)

Process	Products		
	Lb.	Cu. ft.	Anal. by Vol.
80 lb. C burned to CO	186.66	2,529.24	33.4
5 lb. C burned to CO_2	18.33	157.64	2.0
5 lb. vol. HC (distilled)	5.00	116.60	1.6
120 lb. Oxygen are required, of which			
30 lb. from H_2O liberate H	3.75	712.50	9.4
90 lb. from air as associated with N.	301.05	4,064.17	53.6
	514.79	7,580.15	100.0

Energy in the above gas obtained from 100 lb. anthracite: 85 per cent. F.C.; 5 per cent. V.M.; 10 per cent. ash.

186.66 lb. CO	807,304 B.t.u.
5.00 lb. CH_4	117,500 B.t.u.
3.75 lb. H	235,500 B.t.u.
	1,157,304 B.t.u.
Total energy in gas per lb	2,248 B.t.u.
Total energy in gas per cu. ft	152.7 B.t.u.
Efficiency of conversion	86 per cent.

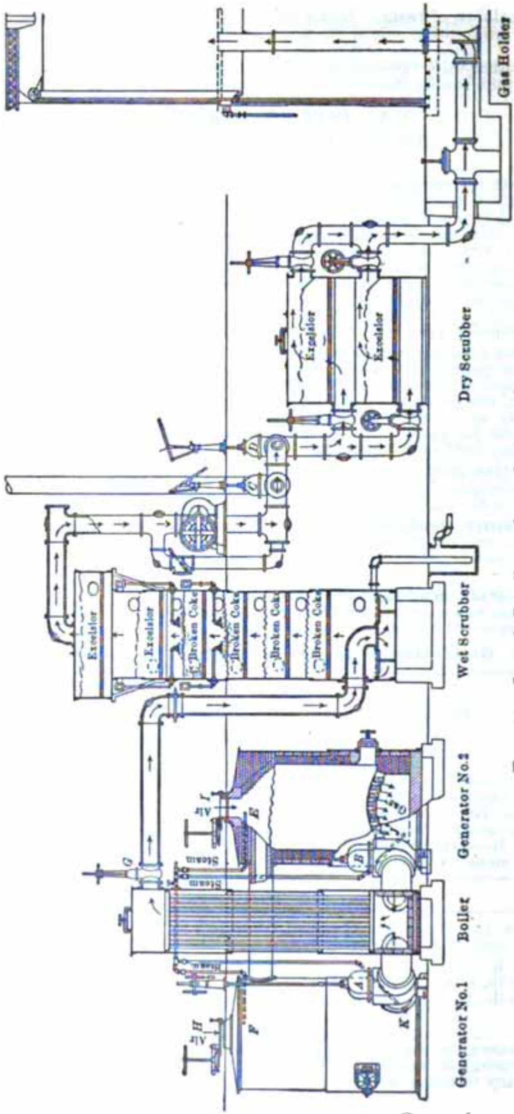


FIG. 44.—Loomis-Pettibone down-draft producer.

450. Down-draft producers are arranged with air and steam supply at the top, and gas removal under the firebrick grate at the bottom. The **double-zone producer** has down draft in the upper zone and up draft in the lower.

451. Suction producers have the advantage of not allowing gas to escape into the operating room from the producer, as the carbon monoxide is intensely poisonous; and poke openings, etc., can be conveniently operated. For small sizes, no exhaust fan is used, the necessary draft being provided by the engine suction. Most of the down-draft producers are suction types with exhaust fans. In this class belong the Loomis-Pettibone, De Lavergne, Körting, United Gas Machinery Co., Westinghouse double-zone, Otto, and Mond (small size).

452. Pressure producers require forced draft supplied by a fan or a steam jet blower, the steam thus admitted being used in the gasification. The tar formed is carried over with the gas and requires more extensive cleaning, but access to the grates is better than with the down-draft and suction types and mechanical stoking or poking may be employed either by mechanical water-cooled pokers, or by rotating the ash table and in some cases the producer shell. Examples of this type are the Mond large size, Taylor, and Chapman.

453. The capacity of a producer is based on 0.065 to 0.075 sq. ft. of cross-sectional area per brake h.p. of engine and about 0.105 to 0.118 cu. ft. of volume per h.p. (Snell).

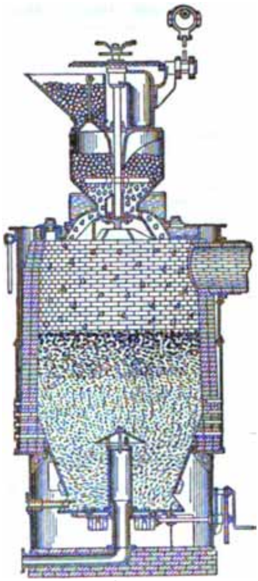


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Fig. 45.—Taylor or Mond up-draft producer.

454. The continuous rate of gasification with high-grade coal will not exceed 16 lb. of coal per hr. per sq. ft. of fuel bed; 10 lb. is a good average figure (Fernald, Bulletin 13, Bureau of Mines).

455. Gasification of Bit. Coal, Low Volatile

Process	Products		
	Lb.	Cu. ft.	Per cent. by vol.
65 lb. C burned to CO.............	151.6	2,054	30.8
5 lb. C burned to CO_2............	18.3	157	2.3
20 lb. vol. HC (distilled)..........	20.0	466	7.0
25 lb. O, from water liberate H.....	3.1	588	9.0
75 lb. atmos. O mixed with N......	251.2	3,391	50.9
	444.2	6,656	100.0

Calorific energy of the gas................. 1,247,870 heat-units.
Calorific energy of the gas per lb.......... 2,809 heat-units.
Calorific energy of the gas per cu. ft....... 187.4 heat-units.
Calorific energy of the coal............... 1,415,000 heat-units.
Efficiency of the conversion............... 88 per cent.
Prox. Anal.: 70 per cent. F.C.; 20 per cent.; 10 per cent. ash.

456. Gasification of Bituminous Coal, High Volatile

Process	Products		
	Lb.	Cu. Ft.	Per cent. by Vol.
50 lb. C burned to CO	116.66	1,580.7	27.8
5 lb. C burned to CO_2	18.33	157.6	2.7
32 lb. vol. HC (distilled)	32.00	746.2	13.2
80 lb. O are required, of which 20 lb. derived from H_2O, liberate H	2.5	475.0	8.3
60 lb. O, derived from air, are associated with N	200.70	2,709.4	47.8
	370.19	5,668.9	99.8

Energy in 116.66 lb. CO	504,554 heat-units.	
Energy in 32.00 lb. Vol. HC	640,000 heat-units.	
Energy in 2.50 lb. H	155,000 heat-units.	
	1,299,554	
Energy in coal	1,437,500	
Per cent. of energy delivered in gas		90.0
Heat-units in 1 lb. of gas		3,484
Heat-units in 1 cu. ft. of gas		229.2

Prox. Anal.: 55 per cent. F.C.; 32 per cent. V.M; 12 per cent. ash.

457. Space required for Single-unit Gas Power Plants (Approximate)

H.p.	Suction			Pressure*			Gas holders	
	Length feet	Width feet	Head room feet	Length feet	Width feet	Head room feet	Cubic feet	Tank diameter
25-50	13-14	9-11	13-15	....	......	.....	1,000	15 ft.
50-75	14-15	10-12	14-17	....	......	.....	2,000	17 ft.
75-100	15-19	11-14	15-20	....	......	.....	2,500	19 ft. 6 in.
150	20-21	13-15	19-20	....	......	.....	3,000	21 ft. 6 in.
200	22-23	15-16	22-23	32	16	22-25	4,000	21 ft. 6 in.
300	25-26	16-17	23-25	34	18	23-25	5,000	24 ft.
400	......	......	.....	36	20	23-26	6,000	30 ft. 6 in.
500	......	......	.....	2 units 39	22	23-26	10,000	35 ft.
1,000	......	......	.....	3 units 39	47	23-26	15,000	43 ft.

Area depends of course on number and size of units for the total power given.

458. The efficiency of large producers will vary from 70 per cent. to 80 per cent. if the gases are used cold, and 80 to 90 per cent., if used hot (for furnace work). The losses include: (a) Sensible heat in gases; (b) producer radiation and conduction; (c) combustible in ash. The carbon in the ash of a properly operated large producer will be from 4 per cent. to 12 per cent. for about 0.5 to 1.5 per cent. total heat loss, with a coal having 10 per cent. to 12 per cent. ash. For small producers, suction type, the carbon in the ash may be as much as 60 per cent. of the total refuse, and the efficiency of the whole producer from 44 to 75 per cent. The sensible heat in the gases varies from 4 to 19 per cent., usually about 15 per cent. average.

459. Heat balance. The following data on the test of a Loomis-Pettibone producer were taken from the N. E. L. A. report on gas engines,

* Pressure plants exclusive of holder.

1908.

Total heat in fuel	100	per cent.
Total heat in gas at 60 deg. fahr	84.7	per cent.
Total heat removed by scrubber	8.16	per cent.
Total heat removed by water cooled valves	1.56	per cent.
Total heat lost by radiation, etc	5.58	per cent.
	100.00	

460. Tests on a 60-h.p. Otto Suction Producer

No. of test	21	16	17	27	25
Duration, hr	12	12	12	12	12
Kind of fuel	Lehigh chestnut			Scranton pea	
Proximate analysis	....	....	....	....	....
Fixed carbon	80.11	73.59	77.39	75.54	78.45
Volatile matter	4.27	8.47	6.70	6.34	5.99
Moisture	1.95	3.98	1.11	2.90	2.75
Ash	13.67	13.96	14.81	15.22	12.81
Sulphur	1.08	1.41	0.63	1.71	1.10
B.t.u. per lb., dry coal	12,750	12,780	12,680	12,540	13,040
B.t.u. per lb., combustible	15,570	15,550	15,700	15,700	15,700
Dry coal fired per hr. (lb.)	12.78	49.49	82.90	24.75	64.70
Water to producer per hr. (lb.)	22.2	30.40	42.05	16.30	28.50
Dry air per hr. to producer (lb.)	58.8	202.5	386.0	87.5	259.2
Calorific value gas, b.t.u./cu. ft.	104.8	111.8	103.4	120.0	137.3
Wt. dry gas per hr. (lb.)	69.0	248.6	456.0	105.0	326.0
Gas analysis, CO_2 (per cent.)	9.17	6.46	6.94	5.90	4.20
CO	16.06	27.77	21.33	21.70	27.01
O_2	0.53	0.49	0.37	0.40	0.23
H_2	9.55	9.53	7.06	10.70	10.40
CH_4	2.10	1.74	1.90	1.60	1.77
N_2	62.65	59.01	62.50	59.70	56.40
Cold gas efficiency (per cent.)	54.3	65.8	64.6	56.4	76.2

461. Tests of Westinghouse Double-zone Producer

Fuel	Pocahontas	Texas lignite
Duration	96 hrs.	46.5 hrs.
Total coal fired	14,452 lb.	12,693 lb.
Heat value per lb	13,983 B.t.u.	8,007 B.t.u.
Heat value per cu. ft. (total)	126.9 B.t.u.	128.3 B.t.u.
Heat value per cu. ft. (effective)	117.8 B.t.u.	117.1 B.t.u.
Efficiency (total)	80%	77.3%
Efficiency (effective)	74.5%	70.5%

Gas Analysis

Carbon dioxide (CO_2)	7.9%	12.4%
Oxygen (O)	0.5	0.9
Carbon monoxide (CO)	18.1	13.3
Marsh gas (CH_4)	2.6	3.6
Hydrogen (H)	12.6	14.7
Nitrogen (N)	58.3	55.1

In the above table all values are computed on the assumption that the gas is at a temperature of 62 deg. Fahr., and that the absolute pressure is 30 in. of mercury.

462. Tests of Wood Pressure Producer

(Gas plant test—24 days continuous running)

*COAL	per cent.	GAS: Average	per cent.
Moisture	14.68	Carbon dioxide	CO_2 = 9.2
Volatile combustible	30.98	Oxygen	O_2 = 0.0
Fixed carbon	42.93	Ethylene	C_2H_4 = 0.4
Ash	11.41	Carbon Monoxide	CO = 20.9
		Hydrogen	H_2 = 15.6
	100.00	Methane	CH_4 = 1.9
Sulphur	1.33	Nitrogen	N_2 = 52.0

Calorific power 12,343 B.t.u. per lb. of dry coal.
Average heat value of gas for 24 days = 156.1 B.t.u. per cu. ft.

On the average of the Government tests generally Prof. Fernald gives the following:

463. Tests by Geological Survey

Load 90—100 Per cent. 30 to 50 hr.	Bituminous		Lignites		Peat	
	As fired	Dry	As fired	Dry	As fired.	Dry
B.t.u. per lb. of fuel	12,280	13,150	8,350	11,290	8,127	10,289
Cu. ft. of gas per lb.	60.5	64.7	35.8	45.7	30.3	38.3
B.t.u. per cu. ft. of standard gas.	152.1	......	158.4	......	175.2	......
Total lb. per b.h.p. hr.	1.36	1.26	1.99	1.68	2.57	2.03

464. Quantity of gas per lb. of fuel in up-draft pressure producers is given in Par. **462** (Fernald, Tech. Paper No. 9, Bur. Mines., 1912).

465. The addition of excess steam over that theoretically required, if not too large, has little or no effect on efficiency, but gives more control over clinker formation. The steam required at 30 to 60 lb. gage is 0.33 to 2.5 lb., averaging 0.7 to 1.0 lb., per b.h.p. The highest figures are for by-product recovery plants.

466. Analyses of gas are given in Par. **128, 459, 460,** and **463.**

467. The operation of gas producers presents three main features: (a) maintenance of uniform resistance of bed, which includes regulation of feed and poking; (b) removal of ash; (c) prevention of clinker. In the case of up-draft producers coal is fed usually by an air-lock device and spreader; poking is done through holes in the top, to fill up holes or channels which may have formed by clinkering and arching, or by localised combustion. Poking must be carefully done to avoid starting the evils it is intended to correct. A moderate rate of driving will tend to keep fires in better condition, as the producer cannot be forced to the same extent as a boiler and give satisfactory results. The down-draft producer can be poked in the same manner as the suction producer.

468. The cost of producers ranges as follows:

Suction producers, up to 300 h.p., total cost $= 252 + 14.2 \times$ (h.p.) (69)

Pressure producers, up to 300 h.p., total cost $= 860 + 15.15 \times$ (h.p.) (70)

The foregoing figures were given by A. A. Potter, *Power*, 1913, p. 938. R. H. Fernald (Bull. 55, Bur. mines) gives values averaging, for producers of both types:

Total cost $= 250 + 9.70$ b.h.p. for sizes from 15 to 300 h.p. (71)

Repairs and maintenance vary from $0.15 to $0.25 per h.p. per year. Guarantees on maintenance are usually 2 to 3 per cent. of first cost a year.

* Prof. R. H. Fernald.

SUPERHEATERS AND CONDENSERS

469. The laws of heat transfer for superheaters have been covered in Par. **15** to **29**, and for economisers in Par. **320**, for vaporisers in Par. **2**, **3** and **4**.

470. Superheaters are attached to the vaporisers of some types of producer, to superheat the steam fed to the fuel bed. They are usually of the protected type, heated by the hot gas leaving the producer, and delivering superheated steam for use in the fuel bed. Some additional economy is gained by their use.

471. Wash boxes are water seals and serve the purpose of a check valve to prevent blowing back of gas. They are made of cast iron generally, with a cross-section equal to about 25 times the delivery pipe. The submersion of the entering pipe is about 3 in.

472. Economisers are employed for preheating the air fed to the producer and operate on the sensible heat of the gas leaving the producer.

473. Vaporizers or boilers are attached to nearly all producers, and serve to yield the necessary steam for gasification, abstracting sensible heat from the hot gas. They are usually built much like a vertical fire-tube boiler, or of cast-iron cooling chambers forming the top or side, of the producer.

474. Condensers are employed to remove the tar, and consist usually of chambers with iron baffles upon which the viscous tar impinges and collects; suitable drainage allows the tar to be recovered and removed. The general design is similar to an oil separator. Some are made with helical passages to secure centrifugal action. There are also one or two designs of mechanical tar extractors built like centrifugal fans, to throw out the tar by centrifugal force. They are generally made entirely of cast-iron.

475. Operation. The operation of superheaters requires practically no labor; vaporisers in this respect are the same as boilers; **tar separators** require periodic cleaning and continuous drainage. The tar is inclined to accumulate and thicken with the dust in the gas. The standby period for lignite or bituminous producers is from 6 to 10 hr. a week for cleaning purposes and repairs, on the average. Anthracite producers may be run 60 to 90 days continuously. (Latta, 1910.)

476. The cost of these auxiliaries is included in the cost of the producer, given in Par. **468**, since they are essential to operation.

SCRUBBERS AND PURIFIERS

477. The impurities found in producer gas are dust, tar and ammonia. Dust is blown over from the fuel bed, being usually the ash from the fines in the coal. In some cases, lampblack is also carried over in the down-draft and double-zone types. Tar is the result of partial decomposition and is a combustible material. Ammonia is produced by the union of H dissociated in the producer with N in the air supplied.

478. Two methods of cleaning are used; (a) wet scrubbing, with stationary towers and water spray in some form, or mechanical scrubbers more or less like fans with water injection; (b) dry scrubbing, with purifiers or filters made with excelsior or shavings, which require periodic removal and cleaning.

479. Wet scrubbers consist of steel towers filled with coke, wire netting or wood latticing, somewhat after the fashion of a cooling tower. Water is sprayed down from the top, and divides into fine streams or spray, washing out solid impurities and tar, dissolving out ammonia and sulphur gases. The amount required varies from 1.5 gal. to 2.5 gal. per b.h.p. per hr.

480. The mechanical washers are chiefly heavy-built fans with water spray devices, or Theisen washers. The simple fan washer consists of centrifugal fans of heavy construction, usually with cast-iron casing, with a water spray device at the inlet to produce a spray curtain or fog through which the gas passes. The efficiency averages about 12 to 1, gas containing 1.75 to 2.65 gr. per cu. ft. being cleaned to 0.15 to 0.22 gr. per cu. ft. The h.p.

required per 1,000 cu. ft. per hr. is 0.066. With two fans in series, the cleaning is from 50 to 1, to 200 to 1 and the h.p. required is 0.184 to 0.283 per 1,000 cu. ft. per hr. The water required per fan is 0.0125 to 0.0150 gal. per cu. ft. cleaned per hr.

481. The Theisen washer consists of three elements in one casing: (a) a primary cleaning fan at the suction end; (b) an annular chamber between a drum fitted with helical vanes and a coarse mesh grating surrounding the drum; (c) a discharge fan chamber. Water spray is fed in at the suction chamber and tangentially in the grating. The water consumption is 0.0272 gal. per 1,000 cu. ft. per hr.; the h.p. required is 0.184 per 1,000 cu. ft. per hr. cleaned.

482. The by-products recovered from bituminous coal gasification are ammonium sulphate and tar. The recovery of ammonium sulphate requires the addition of an acid tower, through which dilute sulphuric acid is circulated as in a purifier. The acid removes the ammonia gas as ammonium sulphate. Tar is removed by separators or condensers, and may be used as a fuel in certain oil engines; it usually contains considerable moisture—up to 20 per cent., on account of the water employed in washing the condenser trays.

483. Purifiers or dry scrubbers consist of a steel tank or tower, filled with shavings, excelsior, coke, or Laming composition. Laming composition is a mixture of bog iron ore and shavings. It is used in 6-in. to 8-in. layers, on gratings, and absorbs cyanides and sulphur compounds. The Laming mixture is removed and exposed to the air from time to time, which regenerates the materials for use again. Shavings and other mechanical dry purifying material must be removed and replaced by fresh material as often as they become clogged. Wet scrubbers are automatically cleaned by the water used to precipitate dust.

484. Cost of the apparatus is included in the cost of producers, Par. 468.

HOLDERS

485. Gas storage is required to provide reserve capacity for sudden increase of demand, and to absorb excess generation when sudden decrease of load occurs.

486. The holder is of the regular collapsing gasometer type, in no way different from the ordinary illuminating- as tank. In some cases the water seal in the base is used as an additional wash-box for incoming producer gas.

487. The capacity required varies from 40 to 25 cu. ft. up to 100 h.p.; 20 to 15 cu. ft. up to 1,000 h.p. The larger the number of units the less storage usually required, as manipulation is more flexible.

488. The operation of gas holders requires only occasional painting to protect the tank, yearly cleaning of sediment from the water reservoir, and suitable steam-supply to the lift seals to prevent freezing in winter.

489. The cost of gas-holders varies from $0.60 to $0.45 per cu. ft. including foundations.

PROPERTIES OF GAS

490. Calorific Value of elementary gases. (From Gas Engine Design —Lucke.)

Fuel	B.t.u. per lb.		Lb. per cu. ft. at 62 deg. fahr.	B.t.u. per cu. ft.	
	High	Low		High	Low
H..........	61,524.0	51,804.0	.00538	331.0	278.7
CO........	4,395.6	4,395.6	.07498	329.58	329.58
CH_4........	24,021.0	21,592.8	.04308	1,037.22	932.38
C_2H_4........	21,222.0	19,834.2	.07631	1,619.45	1,513.55

491. Producer gas has a specific gravity, referred to air, of 0.97, average specific heat, 0.25. It varies in composition, not only with the type of producer, but from variations in operation of the same producer. The hydrogen content, being usually high, limits the compression, usually from 100 to 160 lb. per sq. in., with 18 to 5 per cent. of hydrogen, average pressure 135 lb. Careful jacketing of valves and pistons for large engines raises the obtainable compression.

492. Natural gas is the richest of the power gases, and is obtainable chiefly through the soft coal and oil regions. Ohio, Indiana, Illinois, Pennsylvania and Virginia are most favored in this way. The allowable compression varies from 75 to 130 lb. The specific gravity referred to air is approximately 0.40 to 0.50; density, 0.043 to 0.049 lb. per cu. ft.

493. Illuminating gases, either coal or water gas, are usually too expensive for use in any but the smallest engines, up to say 75 h.p. The allowable compression is 60 to 100 lb. average 80; specific gravity approximately 0.40 to 0.43 for coal gas, 0.57 for water gas.

494. Blast-furnace gas is the leanest of the gases and will stand compression from 120 to 190 lb., averaging 155 lb.; this is on account of the dilution with N, and low hydrogen. It is used only in large gas engines at steel mills.

GAS ENGINES

495. Gas-engine cycles. The thermodynamics of the gas engine is complicated by the fact that not only the change of state by expansion and compression occurs in the cylinder, but also the combustion. The order of events in the usual Otto or **four-stroke cycle** is as follows: aspiration of a charge into the cylinder (suction stroke); compression, approximately adiabatic on the instroke; ignition and explosive combustion at constant volume, while the piston is on or near the dead centre; and approximately adiabatic expansion on the second outstroke; partial expulsion of the burnt gases at constant volume when on the outer dead center, further expulsion on the second instroke, at atmospheric pressure. **The two-stroke cycle** eliminates the suction and exhaust strokes, by admitting the incoming charge under a few pounds pressure while the piston is on or near the outer centre, after the partial exhaust to atmospheric pressure and constant volume has just taken place. This incoming charge sweeps before it, by displacement, most of the remaining burnt gases.

496. The standard reference diagram put forward by Lucke is useful in investigating the performance or design. This diagram is an assumed indicator card based on the assumption that the mixture in the cylinder will behave the same as pure air. The general relations are as given below.

497. Compression pressure.

$$p_b = p_a\left(\frac{v_a}{v_b}\right)^{1.41} \quad \text{and} \quad t_b = t_a\left(\frac{v_a}{v_b}\right)^{1.41} \tag{72}$$

where $p_a = 14.7$ lb. per sq. in., suction pressure; p_b = final compression pressure; v_a = total cylinder volume in cu. ft. = displacement plus clearance; v_b = compression volume, or clearance, in cu. ft.; t_a = absolute initial temperature (deg. Fahr. + 461); and t_b = absolute compression temperature.

498. The pressure rise on explosion depends on the amount of heat per cu. ft. of mixture.

$$p_c = p_b\left(\frac{Q}{C_v t_b} + 1\right) \quad \text{and} \quad t_c = t_b + \frac{Q}{C_v} \tag{73}$$

where p_c = explosion pressure; Q = B.t.u. per lb. of mixture; C_v = specific heat at constant volume; t_c = absolute explosion temperature; and the other symbols are as above (Par. 497).

$$p_c - p_b = \frac{2.2\ Q}{\left(\frac{p_a}{p_b}\right)0.71\ v_a} \text{ (in lb. per sq. in.)} \tag{74}$$

499. B.t.u. per cu. ft.

$$\frac{Q}{v_a} = \frac{H}{n + a' + 1} \tag{75}$$

where H = B.t.u. per cu. ft. of mixture, hot; a' = cu. ft. of air required to burn 1 cu. ft. gas; and n = cu. ft. of neutral added. The actual pressure ratio p_c/p_b obtained is from 40 per cent. to 65 per cent. of the air card ratios.

500. Expansion is taken according to the law

$$p_d = p_e\left(\frac{v_e}{v_d}\right)^{1.41} \tag{76}$$

which is the same as for compression.

501. Efficiency.

$$E = 1 - \frac{t_c}{t_b} = 1 - \left(\frac{p_c}{p_b}\right)^{0.29} \tag{77}$$

where E = thermal efficiency of the cycle; other symbols are as above. The thermal efficiency of the ideal cycle is only dependent upon the ratio of compression to suction pressure.

502. Four-cycle engines are built in all sizes; the terms single and double acting have the same application as for steam engines. The majority of engines of small and moderate size are single acting. Large engines are generally made double acting, to economize space, as the gas engine is even more bulky per h.p. than the steam engine.

503. Two-cycle engines are also used in all sizes, but are found to the greatest extent in large units, double acting. For small units they are never built double acting, as separate pumps are then required to give the pressure for charging at the end of each stroke. In the small single-acting types, the mixture is compressed between the piston and the crank case.

504. Cylinder jackets are essential to practically all gas engines. The enormous heat developed during explosion would soon destroy the cylinder and piston if they were not protected by cooling. In most cases this is done by **water jacketing**; in a few small engines, **air cooling** is employed. Pistons smaller than 15 in. in diameter are not separately cooled by water jacketing; but above this size, the area of piston head is too large to be properly cooled by conduction to the jacketed cylinder walls, and the air in the crank case; therefore water cooling becomes necessary.

505. The quantity of jacket water should be about 6.5 gal. per b.h.p.-hr. at 50 deg. Fahr., 7.75 gal. per b.h.p.-hr. at 60 deg. Fahr., 9.75 gal. per b.h.p.-hr. at 70 deg. Fahr., and 12 gal. per b.h.p.-hr. at 80 deg. Fahr., for large units. Most of this may be recovered and re-used if cooled. If a cooling tower is used, for recooling the jacket water, about 8 per cent. is lost by evaporation. The heat loss to the jacket is from 25 to 50 per cent., generally the largest with small engines.

506. Mean effective pressure is computed from the indicator card in the same manner as for a steam engine. The m.e.p. of the reference diagram is given by

$$\text{M.e.p.} = \frac{5.41Q}{(v_a - v_b)}\left\{1 - \left(\frac{p_c}{p_b}\right)^{0.29}\right\} \tag{78}$$

or

$$\text{M.e.p.} = 5.41\left(\frac{H}{a'+1}\right)\left\{1 - \left(\frac{p_c}{p_b}\right)^{0.29}\right\} = C\left(\frac{H}{a'+1}\right) \tag{79}$$

where the symbols have the same meaning as in Par. 497 to 500.

For constants to be used with these equations see Par. 507.

507. Constants used in Calculation of M.E.P. (Par. 506)

Compression		Value of factor $C = 5.41 \times 1 - \left(\frac{p_c}{p_b}\right)^{0.29}$	Compression		Value of factor $C = 5.41 \times 1 - \left(\frac{p_c}{p_b}\right)^{0.29}$
Atmospheres	Lb. per sq. in.		Atmospheres	Lb. per sq. in.	
3	44.1	1.474	8	117.6	2.446
4	58.8	1.787	9	132.3	2.554
5	73.5	2.014	10	147.0	2.630
6	88.2	2.187	11	161.7	2.711
7	102.9	2.327	12	176.4	2.775

508. Observed mean effective pressures obtained by test are given by Lucke:

Fuel	Compression, lb. per sq. in. abs.	M.e.p. actual lb. per sq. in.	Fuel	Compression, lb. per sq. in. abs.	M.e.p. actual lb. per sq. in.
City gas......	45	45	Producer gas..	108	63
	60	80		108	88
	66	95		125	51
	91	90		141	83
	70	60		170	73
				95	100
Natural gas ...	127	68		95	90
	130	82		90	62
	135	90		115	103
			Blast-furnace gas..........	....	60
				140	47
				155	81

509. The ignition of gas engines is practically entirely electric. The jump-spark or high-tension spark system is not used for power-station engines. Make-and-break systems are used, because more reliable and effective. The principal troubles are insulation breakdowns, worn and dirty contacts or short-circuiting by jacket leaks.

510. The timing of ignition affects the work done by a given charge and the economy. Ignition should take place so as to give a nearly vertical explosion line. The spark should be advanced, theoretically, as the rate of propagation is decreased by weakening the mixture or the compression, but in practice the spark is usually fixed to give the best results at the average load condition, and is not varied.

511. The speed of gas engines is about the same as for steam engines of the same class. See Par. 170. In small engines the speed may be 10 to 25 per cent. higher.

512. Indicated horse-power is obtained in the same manner as for steam engines, except that care must be used in planimetering the card to go around the negative work areas in the right direction. For throttle governed engines, the number N in formula 34a, is the same as rev. per min. for 2-cycle engines and rev. per min./2 for 4-cycle engines; with engines governed by the hit-and-miss method, the number of explosions must be counted to get N, as it has no fixed relation to rev. per min.

513. The friction is usually higher in the gas engine than in the steam engine; the mechanical efficiency at full-load is seldom over 85 per cent., even for large engines. Lucke gives.

	Mech. Efficiency	
	4-cycle	2-cycle
Large engines, 500 h.p. and over......	0.81 to 0.86	0.63 to 0.70
Medium, 25 to 500 h.p...............	0.79 to 0.81	0.64 to 0.66
Small, 4 to 25 h.p...................	0.74 to 0.80	0.63 to 0.70

514. Lubrication is entirely by machine oil, as the cylinder walls are water cooled. Lubrication in general is handled exactly as with the steam engine. About 1 gal. of oil is required per 4,000 b.h.p.-hr.

515. The rated capacity of a gas engine is preferably based on cu. ft. of cylinder displacement. The displacement per h.p. per lb. of mean effective pressure is 229.17 cu. ft. per min. For a given b.h.p. required, the mechanical efficiency is assumed and the i.h.p. found; and with a suitable m.e.p. for the fuel employed (Par. **505**), the required displacement per h.p. divided into required i.h.p. gives total cylinder displacement per minute: Internal combustion engines are usually rated at 15 per cent. to 20 per cent. below the capacity of the cylinder as given above. Torrance and Ulbricht (*Power*, 1912) give the following **formulæ for builders' rating** in America.

$$\text{Producer gas, b.h.p.} = \frac{d^2ln}{18{,}500} - 2.0 \tag{80}$$

$$\text{Illuminating gas, b.h.p.} = \frac{d^2ln}{15{,}700} - 2.0 \tag{81}$$

$$\text{Natural gas, b.h.p.} = \frac{d^2ln}{15{,}200} - 5.0 \tag{82}$$

$$\text{Blast-furnace gas, b.h.p.} = \frac{d^2ln}{21{,}000} - 5.0 \tag{83}$$

where d = diameter of cylinder, inches; l = stroke in inches; and n = rev. per. min.

516. The fuel consumption of gas engines on producers ranges from 0.99 to 3 lb. per b.h.p.-hr. From 75 to 90 cu. ft. of gas per b.h.p.-hr. are required, of approximately 150 B.t.u. per cu. ft.

517. Test of 300-h.p. Illmer 2-stroke cycle double acting horizontal engine. (*Power*, May, 1913.) Duration of test, 33.5 hr.; coal, Westmoreland bituminous, 14,100 B.t.u. as fired; average b.h.p., 284; average B.t.u. per cu. ft. of gas, 144; Producer efficiency, 67 per cent.; B.t.u. per h.p.-hr., 10,300; total coal per b.h.p.-hr. 1.14.

518. Fuel consumption, Ulbricht and Torrance (Power, 1912) give average fuel consumption as below: (B.t.u. and cu. ft. per h.p.-hr.)

	50 b. h.p.			100 b.h.p.			Over 100 b.h.p		
	B.t.u.	cu.ft.	Eff.	B.t.u.	cu. ft.	Eff.	B.t.u.	cu. ft.	Eff.
Water gas	10,690	35.2	23.8	10,160	33.4	25.0	10,160	33.4	25.0
Carburetted gas	10,690	18.1	23.8	10,160	17.2	25.0	10,160	17.2	25.0
Coal gas	10,690	17.6	23.8	10,160	16.7	25.0	10,160	16.7	25.0
Producer gas:									
anthracite	10,380	82.6	24.5	9,630	76.6	26.4	9,600	76.3	26.5
bituminous	10,380	83.0	24.5	9,630	77.0	26.4	9,600	76.7	26.5
coke	10,380	82.0	24.5	9,630	76.2	26.4	9,600	76.0	26.5
lignite	10,380	77.5	24.5	9,630	71.8	26.4	9,600	71.6	26.5
oil	10,380	68.7	24.5	9,630	63.7	26.4	9,600	63.5	26.5
peat	10,380	73.6	24.5	9,630	68.3	26.4	9,600	68.0	26.5
wood	10,380	80.6	24.5	9,630	74.8	26.4	9,600	74.6	26.5
Blast-furnace gas	10,860	114.0	23.4	10,500	110.0	24.2	9,860	104.0	25.8
Coke-oven gas	10,690	22.0	23.8	10,160	20.9	25.0	10,160	20.9	25.0
Natural gas avg.	10,380	12.2	24.5	9,420	11.0	27.0	9,320	10.9	27.3

519. Test of a 310-h.p. engine, horizontal double-acting (Buckeye), running on coke-oven gas, 560 B.t.u. per cu. ft., gave the following results.

Load	B.h.p.	B.t.u. per h.p. hr.	Per cent. efficiency
½	165	13,000	18.5
¾	210	11,700	23.0
full	310	11,100	22.5

The B.t.u. per kw.-hr., with blast-furnace gas engines (Freyn, 1913), varies from 16,200 to 26,000, average 18,400; thermal efficiency 13.0 to 21.0 per cent., average 18.5.

520. Test of a 500-h.p. engine of the Borsig-Oechelhäuser 2-cycle type, on coke-oven gas (Junge, in *Power*) gave results as follows:

I.h.p.	B.t.u. per b.h.p.	Per cent. of i.h.p. to pumps	I.h.p.	B.t.u. per b.h.p.	Per cent. of i.h.p. to pumps
616	8,650	10.3	488	9,642	14.2
627	8,650	11.1	474	9,761	15.5
574	8,650	11.4			

521. Test of a 600-h.p. engine, Körting 2-cycle type, gave these results: b.h.p. 616; pump work, 11.2 per cent. of total; mechanical efficiency, 0.78; lb. of coal per b.h.p-hr., 0.787.

522. Thermal efficiency of gas engines is given in Par. 518 and 519 in connection with some of the tables. Further data is given as follows by Diederichs:

Engine	Fuel	B.h.p.	Thermal efficiency	Authority
Otto........	Illuminating gas.........	8	29.4	Meyer
Deuts......	Suction producer, low grade soft coal.	....	22.4	Meyer
Nürnberg...	Blast-furnace gas.........	750	24.6	Linde
Deuts......	Anthracite producer.....	450	22.0	Josse
Güldner....	Suction anthracite producer.	29	23.2	Schroter

523. The overload capacity (Par. 515) is generally 10 to 20 per cent. more than the rated capacity. The maximum capacity is limited by the dimensions of the cylinder and not by a variable cut-off. Maximum economy occurs at or near, maximum capacity.

524. Governing is accomplished by hit-or-miss, throttling or quality methods. **Hit-or-miss governing** is arranged with a movable pick-blade operating the gas inlet valve; it is now used only for low-grade agricultural engines. **Throttle governing** is employed on a great many engines of small and medium size, and consists simply of restricting both the gas and the air intakes, so that the suction stroke pressure is lowered, a smaller charge is taken in, and compression is reduced. **Quality or cut-off governing** consists of throttling the gas supply only, as the load drops. In this way no reduction of suction pressure occurs, but a reduction of gas in the charge only. The compression pressure is unaltered. This method is the best of the three, being more efficient at light load than throttling, and allowing better regulation than hit-or-miss.

525. Operation of a gas engine requires the same care as a steam engine, and a higher maintenance. Power-station gas engines are started by compressed air in most cases, although motor starters have been employed. Special valve gear is thrown into service, and the engine runs for a revolution or two as an air engine. **Before starting,** (a) the load must be off, (b) ignition system inspected and made ready in the retarded position, (c) lubricating oil feeds turned on, (d) fuel shut-off valve opened and (e) cooling water turned on. Failure to start may be due to (a) faulty ignition, (b) very abnormal mixture, either too rich or too lean, (c) broken valve rods or (d) closed fuel connections. **Ignition** should be adjusted to give a fairly sharp rise at the average load conditions, and the mixture should be a little leaner than theoretical proportions—say 15 to 20 per cent. **Shutting down** is best accomplished by, (a) cutting off fuel supply, (b) cutting out ignition, if on battery, and placing set in "retarded" position, (c) oil feed stopped, (d) cooling water shut off and jackets drained.

526. The cost of gas engines is given by Fernald (Bull. 55, Bureau of Mines).

Total cost = 250 + 34.8(b.h.p.), for 40 h.p. to 2,000 h.p. (84)

A. A. Potter (*Power*, 1913) gives, for engines of 50 to 300 h.p., on illuminating gas. Total cost = 33.6 × b.h.p. − 115. (85)

The following figures are given by Stott, Pigott and Gorsuch, *Transactions* A.I.E.E., June, 1914. Horizontal producer-gas and natural-gas engines, 4-cycle tandem and twin-tandem, direct connected to 60-cycle generators, 200–2,000 kw. @ 80 per cent. power-factor delivered and erected.

$$\text{Average total cost} = 2{,}000 + 70\ (\text{kw.}) \tag{86}$$

527. The maintenance and repair costs are approximately 42 per cent. to 55 per cent. of the net operating cost. For oil and supplies the cost is about double that of a steam plant.

528. Weight. The net weight of large tandem double acting types averages 680 to 740 lb. per kw. Moderate sizes of single-cylinder, single acting engines weigh from 200 to 300 lb. per b.h.p., the unit weight going up with the b.h.p.

PIPING

529. General requirements. As all pressures are light, standard pipe and fittings may be used throughout. **Expansion** must be provided for in the same manner as steam piping (Par. 369 to 372).

530. Gas piping from producer to scrubbers and purifiers should be cast iron with standard flanged joints, asbestos or metallic packing. In most cases it is advisable to line the piping between scrubber and producer with fire-brick. Explosion doors should be installed at intervals, and if the gas is dusty, ells should be replaced by tees where possible, to provide for raking and cleaning.

531. Water piping for supply to jackets, vaporizers and purifying apparatus may be of ordinary galvanized standard pipe. The discharge from scrubbers and purifiers is preferably of cast iron, on account of corrosive action.

532. Exhaust piping can be built of wrought-iron standard pipe, or cast-iron standard pipe; light riveted pipe is not advisable as it may collapse on the return wave of a muffler or exhaust pipe explosion. **Mufflers** for small engines are usually cast-iron pots; for large plants, concrete tunnels, with water in the bottom to cool and silence the exhaust, are frequently used.

533. The cost of piping varies from \$5 to \$9 per kw. of maximum rating of the plant, being much the same as for steam plants.

PLANT ECONOMY AND DESIGN

534. The auxiliaries to be considered in the engine room are: compressors for starting; exciter units and cooling water pumps; station lighting. In the producer room the auxiliaries are: the blowers or exhausters, as the case may be; power for rotating ash table, producer body, or mechanical pokers, if these are used; coal and ash handling apparatus; mechanical washers and tar extractors; scrubber and washer water-supply pumps.

535. The heat losses in a producer plant will be indicated by the following analysis. (Snell, "Power House Design.") Loss in producer and auxiliaries, 20 per cent.; loss in jacket water, 19 per cent.; loss in exhaust gases, 30 per cent., loss in engine friction, 6.5 per cent., loss in generator 0.5 per cent., total losses, 76 per cent., converted to electrical energy, 24 per cent.

536. The Willans line for the several main units should be plotted with kw. output and cu. ft. of gas as ordinates. On this curve should be superposed the gas used by auxiliaries, or if these are electric driven, their requirements should be subtracted from the gross output of the generators before plotting the Willans line. By taking the gas required at any load from this curve, and plotting for the load curve of the plant, the gas demand of the engine room is obtained. Similarly, an input-output curve for the producer can be drawn, between coal fired and gas generated; on this is superposed the gas or power demands of producer room auxiliaries and the standby coal. In conjunction with the engine-room gas demand curve, this gives the coal demand for the station under load.

537. Power for auxiliaries. The power required for the blower or exhauster will be from 1 to 2.5 per cent. of the total horse-power of the producer.

The horse-power requirements of washers are given in Par. 480. The coal and ash handling system will not usually require more than 0.2 to 0.4 per cent. of the total horse-power; compressor equipment, not over 0.1 per cent. The total power requirements of the auxiliaries will be about 5 per cent. The **standby losses** of a producer banked 14 hr. in 24 are about 1.5 to 2.0 per cent. of the total.

538. Water consumption per h.p., for the plant will run from 5 to 8 lb. per kw-hr., at the switchboard for non-recovery plants, and 40 lb. per kw-hr. for recovery plants, using cooling towers in both cases.

539. Vertical vs. horizontal engines. Design practice in plants up to 2,000 b.h.p. will allow vertical engines; these usually are not built over 750 to 1,000 h.p. per unit. Above this size only horizontal engines are used.

540. Ammonia recovery. It is not worth while to attempt the recovery of ammonia from bituminous plants of less than 2,000 b.h.p., and then only when the load factor is above 25 per cent. Each ton of coal produces approximately 100 lb. of sulphate of ammonia; the sulphuric acid required is about 1 to 1 in weight ratio to the sulphate of ammonia. In cities, the recovery plant is practically out of the question on account of space requirements. Extra labor, the acid tower, extra repairs and the cost of bagging the sulphate must be taken into account.

541. The floor space required is from 5 to 8 sq. ft. per kw., or 3 to 4 times the floor space for a turbine plant. The producer room requires from 15 to 4 sq. ft. per kw.

542. The load factor should be high for a successful gas plant. The gas engine is not well suited for heavily swinging loads partly on account of its lack of much overload capacity.

543. The subdivision of generating units should be the same as for a steam plant—six to eight units if possible. For small units, up to 400 b.h.p. one reserve unit in 5 or 6 is sufficient; but for large units (which have never been brilliantly successful), one in four is necessary for continuity of operation. The **reliability** of the gas engine is still open to much question in very large units, but the units of moderate and small size are as reliable as steam engines, if given the same grade of attention.

544. Labor required is usually one man per engine above 750 h.p. and one man to four or five producers of good size and equipped with mechanical apparatus for feed and stoking. About one man for 1,000 to 1,200 h.p. of engine, and one man per 2,000 to 2,500 h.p. of producer, represent the average.

545. Total plant cost. The following data is taken from the 1909 report of the N. E. L. A. on gas power plants.

360 to 2,700 kw. maximum rating

Item	Dollars per kw.		
	Maximum	Minimum	Average
Building	44	16	33
Producer equipment incl. piping	49	33	38
Gas engines	55	38	47
Foundations	....	....	5
Generators	15	12	14
Switchboards	....	....	3
Total	171	107	140

If the costs for gas engines given in Par. 526 are used, these totals will be from $10 to $20 higher.

TESTING

546. The essential mesurements for determining producer efficiency and capacity are:—(a) weight of coal per hr.; (b) proximate analysis, B.t.u. per lb. and moisture in coal; (c) cu. ft. of gas made; (d) temperature of gas, humidity; (e) calorific value and analysis. Additional data required for full information include: (f) weight of dry ash per hr.; (g) proximate analysis and B.t.u.; (h) wt. of steam per hr.; (j) wt. of vaporiser and scrubber water per hr., and temperatures; (k) h.p. required for blowers or exhausters and tar extractors or washers; (l) grains of dust per cu. ft. in gas.

For most purposes, only the efficiency of the producer itself is desired. For full details of tests, see Gas Power Committee reports, A. S. M. E.

547. The testing of gas engines is very similar to that of steam engines. The essential data are: (a) i.h.p.; (b) b.h.p. or kw. output and generator efficiency; (c) r.p.m.; (d) explosions per min., if hit-or-miss governed; (e) cu. ft. of gas per min.; (f) calorific power of gas per cu. ft. Desirable additional data include: (g) jacket water per hr.; (h) inlet and outlet jacket water temperatures; (i) gas, air and exhaust temperatures; (k) amount of lubricant. All physical dimensions of the engine should of course be taken.

548. The essential data for a plant test consist of kw.-hr. output at switchboard and coal used in producers; or cu. ft. of gas and calorific power, if coke oven, blast-furnace, natural, or illuminating gas is used.

549. For gas measurement the gas meter is chiefly used; the method of measuring the fall of a gas holder for volume should never be employed as the change of temperature ordinarily possible, entirely vitiates the accuracy. The Venturi meter furnishes an accurate and inexpensive measuring device if properly handled and kept clean in the throat.

550. Indicated h.p. is obtained by the usual indicator, but equipped with 0.5 in. area piston for this work.

551. The calorific value of gas can be obtained by the Junkers or other make of gas calorimeter. **Grains of dust per cu. ft.** are obtained by a special filter apparatus and test gas meter; the dust of a measured quantity of gas is collected and weighed. **Other measurements** are similar to those for steam engines (Par. **427**).

552. The duration of test should be about the same as for engine and boiler tests generally. For Venturi readings, a record is best, or very frequent readings, say once a minute. The producer test should preferably not be less than 24 hr. except with small producers intended for intermittent use; the longer the test the better.

553. For full calculation of results the heat balance is required. The reports of the gas power committee, A. S. M. E., furnish full data for the elaborate tests.

OIL POWER PLANTS

BY REGINALD J. S. PIGOTT

OIL ENGINES

554. The thermodynamics of the explosion cycle (Otto) type of oil engine are exactly the same as for gas engines (Par. **495** to **506**). **The constant-pressure cycle** (Brayton cycle), of which the **Diesel** is the principal example, has slightly different events. For the **four-cycle type,** the inspiration and compression strokes are the same, but are made upon air only; at the end of compression, the oil fuel is injected, and burned, in such a way as to maintain a practically constant pressure during the early part of the working stroke. Expansion and exhaust then follow as in the gas engine.

555. Thermodynamic equations. For the air card, or ideal cycle p_b, t_b, and v_b are obtained as for the Otto cycle. Par 497 to 501., when burning fuel at constant pressure:

$$p_c = p_b \qquad (87) \qquad\qquad p_d = \left(p_c \frac{v_c}{v_d}\right)^{1.41} \qquad (91)$$

$$t_c = t_b\left(1+\frac{Q_1}{c_p\, t_b}\right) \qquad (88) \qquad\qquad t_d = t_a\frac{p_d}{p_a} \qquad (92)$$

$$v_c = v_b\,\frac{t_c}{t_b} \qquad (89) \qquad\qquad Q_2 = C_v(t_d - t_a) \qquad (93)$$

$$v_d = v_a \qquad (90) \qquad\qquad E = \frac{Q_1 - Q_2}{Q_1} \qquad (94)$$

where p_a, p_b, p_c, p_d = inspiration, compression, combustion and release pressures, respectively, in lb. per sq. ft.; v_a, v_b, v_c, v_d = volumes in cu. ft. at beginning of compression, end of compression, end of combustion or beginning of expansion, respectively; t_a, t_b, t_c, t_d = absolute temperatures, at the same points in deg. Fahr.; Q_1 = heat added per lb. of gases, in B.t.u.; Q_2 = heat abstracted per lb. of gases in exhaust, in B.t.u.; E = thermal efficiency; C_p = specific heat at constant pressure.

556. The thermal efficiency would be the same as for the Otto cycle, if complete expansion occurred for the same compression pressures. But as expansion is always incomplete, the efficiency is less; this is more than offset by the much higher compression possible with the Diesel cycle.

557. Types. The types are the same as for gas engines; see Par. **502** and **503**.

558. Mean effective pressure of the Diesel type is given by

$$\text{M.e.p} = J\,\frac{[Q_1 - C_v t_a(Y^{1.41} - 1)]}{144\, v_a\left[1 - \dfrac{p_a}{p_b}^{0.71}\right]} \qquad (95)$$

$$Y = 1 + \frac{Q}{C_p t_a} \qquad (96)$$

where J = Joule's equivalent, 777.5; C_v and C_p = specific heat at constant volume and constant pressure, respectively; t_a, v_a, p_a, p_b are the same as in Par. **554**.

Lucke gives the following usual m.e.p.

Kerosene		Gasolene	
Absolute compression pressure	Observed m.e.p.	Absolute compression pressure	Observed m.e.p.
46	40	66	106
63	69	70	75
65	68	75	100
68	40	86	70
70	72	86	72
50	35	95	60
55	85	..	..

559. Speed and power. The speed is the same as for gas engines of the same power (Par. **511**). For indicated and b.h.p. see Par. **512**.

560. Kerosene engines of the Mietz and Weiss and Hornsby-Akroyd types are built in sizes up to 250 h.p. and operate in the same general manner as gas engines, the fuel being pumped in and vaporized in a hot tube, hot bulb, or vaporizing chamber. The hot tube and the hot bulb are kept warm by being left unjacketed, and by the combustion of a portion of the charge in the hot bulb chamber. Compression forces up the temperature of the charge enough for ignition to take place. Kerosene engines are the least efficient of the oil engines.

561. Gasolene engines are practically unused for power-station purposes, on account of expensive fuel, and low relative economy.

562. The crude oil engine is arranged like the kerosene engine, except that more careful arrangements must be made for preheating the oil for vaporization, and in most cases, the air also. Exhaust gas jackets are generally used for the purpose of providing the necessary heat.

563. The Diesel engine is now the most important of the oil engines on account of its remarkable efficiencies. It is built in both two-cycle and four-cycle types, and practically always vertical, single acting. It requires separate compressed-air starting sets, at about 1,000 lb. per sq. in. pressure. In the two-cycle type, separate air pumps (cylinders driven from the cross-head) are employed, as well as the fuel pump, which is always necessary.

564. Weights. The Diesel and the Junkers engines are the heaviest of prime movers, running from 400 to 600 lb. per b.h.p. The enormous weight is due chiefly to the heavy pressures, 500 lb. per sq. in. being an ordinary compression, with safe design allowance up to 1,000 lb. per sq. in. to safeguard against breakage due to preignition.

565. The capacity of oil engines is based on cu. ft. of cylinder displacement, as for gas engines (Par. **515**).

566. Builders' rating. Ulbricht and Torrance (*Power,* 1912) give average practice in builders' rating (for oils and distillates), as

$$\text{b.h.p.} = \frac{d^2\, ln}{21{,}875} - 0.75 \tag{97}$$

where d = cylinder diameter, in in; l = stroke in in. n = r.p.m. The rating is about 10 to 20 per cent. less than the ultimate capacity.

567. The fuel consumption is poorest for the gasolene and kerosene engines, running from 13,000 to 15,600 B.t.u. per b.h.p.-hr.; 9,400 to 10,000 B.t.u. per b.h.p. hr., for American Diesel engines. The German Diesel and Junkers engines run as low as 7,100 to 8,500 B.t.u. per b.h.p.-hr. All figures are for full load. These figures correspond to 0.65 to 0.78 lb. of gasolene or kerosene per b.h.p.-hr.; 0.48 to 0.52 lb. of oil for American Diesels; 0.37 to 0.44 for German Diesel and Junkers Engines. Fuel consumption decreases with size.

568. Test of Falk kerosene oil engine (H. D. Wile, Elec. World, 1913)

Cooling water, lb. per hr.	632	445	391	244
Inlet temperature, deg. Fahr.	47.5	44	71	47.5
Outlet temperature, deg. Fahr.	106	123	157	182
Rev. per min.	448	448	449	446
B.h.p.	9.05	9.05	8.73	9.05
Indicated h.p.	11.04	11.31	10.65	11.84
Mechanical efficiency, per cent.	82	80	82	76
Kerosene, lb. per b.h.p.-hr.	1.14	1.09	0.98	1.0
Thermal efficiency	11.2	11.7	13.1	12.8

Angle ignition, deg.				34	38	41
Rev. per min.				446	450	442
B. h.p.				8.73	9.05	8.73
Indicated h.p.				11.7	10.65	12.0
Mechanical efficiency				77	82	75
Kerosene, lb. per b.h.p.-hr.				0.97	0.98	0.95
Thermal efficiency				13.2	13.1	13.4
Mixed water-kerosene, per cent.				34.4	62.8	85
Rev. per min.				442	440	444
B.h.p.				8.7	8.7	8.7
Indicated h.p.				12.3	11.7	11.4
Mechanical efficiency				71	74	76.7
Kerosene, lb. per b.h.p.-hr.				1.06	1.14	1.19
Thermal efficiency				12	11.1	10.5
B.h.p.	4.2	5.5	7	8.73	10.3	11.9
Indicated h.p.	6.4	7.5	9.17	11.2	12.5	14.38
Rev. per min.	457	457	450	448	437	436
Mechanical efficiency	66	74	76	77	82.5	82.8
Cooling water, lb. per hr.	236	290	306	262	309	304
Kerosene, lb. per b.h.p.-hr.	1.55	1.26	1.05	0.93	0.87	0.94
Thermal efficiency	7.3	10.1	12.1	13.8	14.7	13.6
Maximum pressure, lb. per sq. in.	68	94	124	170	206	242

$$\text{Total cost} = 141 + 24.8\ (\text{b.h.p.}) \qquad (\text{dollars}) \qquad (98)$$

Gasolene, up to 75 h.p., throttle governed,

$$\text{Total cost} = 309 + 36.1\ (\text{b.h.p.}) \qquad (\text{dollars}) \qquad (99)$$

Oil engines, up to 400 h.p.,

$$\text{Total cost} = 63.8\ (\text{b.h.p.}) - 316 \qquad (\text{dollars}) \qquad (100)$$

Diesel engines, from 100 to 1,000 h.p., approximately (price varies very widely),

$$\text{Total cost} = 58.0\ (\text{b.h.p.}) + 2{,}000 \qquad (\text{dollars}) \qquad (101)$$

Stott, Pigott, and Gorsuch, A.I.E.E. *Proceedings*, June, 1914, give 95.00 per kw. for all sizes of Diesel engines. This is due to the fact that weight per kw. *increases* with size.

576. Test of a low-compression heavy oil engine. (A. A. Potter and W. W. Carlson, *Elec. World*, 1913)

Cylinder diameter, in	16 $\frac{1}{16}$
Length of stroke, in	24
Kind of fuel	Solar oil
Specific gravity at 60 deg. Fahr	0.8145
Deg. Beaumé	42.0
Lb. per gal	6.79
Heat units per lb., high	17,240
Heat unit per lb., low	16,322
Heat units per gal	117,100

Data and results	Number of test		
	1	2	3
Duration of heat, hours	3	1	1
Barometer, in	26.95	26.94	26.95
Speed of engine, rev. per min	183.5	201.2	202.0
Mean effective pressure	75.9	37.1	27.59
Indicated h.p	85.6	45.5	34.0
B.h.p: Average	73.5	39.70	20.2
Mechanical efficiency, per cent	85.8	87.4	59.4
Fuel used, total lb	222.75	50.1	42.55
Fuel used per i.h.p. per hour, lb	0.868	1.10	1.20
Fuel used per b.h.p. per hour, lb	1.010	1.26	2.1
Heating value of fuel consumed:			
Per b.h.p. per hour, b.t.u	17,400	21,750	36,200
Temperature of cooling water, deg. Fahr.			
Inlet	82.25	79.0	84.25
Outlet	164.0	133.0	125.55
Rise	81.75	54.0	41.2
Cooling water used per hour:			
Total, lb	1,775	2,868	2,712
Per b.h.p., lb	24.2	72.2	134.2
Thermal efficiency, per cent	14.62	11.7	7.0

Heat distribution per lb. of fuel	1		2		3	
	B.t.u.	Per cent.	B.t.u.	Per cent.	B.t.u.	Per cent.
Converted into i.h.p	2,935	17.0	2,315	13.4	2,036	11.30
Converted into b.h.p	2,520	14.26	2,020	11.7	1,210	7.01
Friction and losses	415	2.40	295	1.7	826	4.79
Losses in jacket water	1,955	11.32	3,090	17.9	2,635	15.28
Losses in exhaust, radiation, etc.	12,350	71.68	11,835	68.9	12,569	73.42

577. The cost of piping for oil plants, per installed kw., ranges from \$0.50 to \$1.70.

PLANT DESIGN

578. General. What has been given in Par. 394 to 414 can be applied in principle to the oil plant. But inasmuch as the only auxiliaries are the exciters, jacket pumps and air compressors for starting, the problem is much simpler, and the economy of the plant is much more nearly that of the main unit.

579. Plant costs. The buildings will cost from $10.00 to $24.00 per kw., as no producer room or boiler room is required. The oil storage tanks require approximately 0.06 to 0.08 gal. capacity per kw. and cost from $3.00 to $6.00 per kw. of rated capacity. All other auxiliaries, including crane, oil pumps, air compressor and compressed air tanks, cost from $2.50 to $5.00 per kw. of rated capacity, installed. The total costs will range as follows, per kw.:

	High	Low
Diesel engine	$100.00	$95.00
Building	24.00	10.00
Tanks	6.00	3.00
Auxiliaries	5.00	2.50
Piping	1.70	0.50
Totals	136.70	111.00

There are no plants in this country much larger than 1,500 kw. in total capacity.

TESTING

580. The testing of oil engines is exactly similar to the testing of gas engines, except that weight of oil per hr. is substituted for cu. ft. of gas per min. All other data and instruments required are the same.

POWER PLANT BUILDINGS AND FOUNDATIONS

BY REGINALD J. S. PIGOTT

581. The building housing any kind of power plant should be entirely fireproof. In many cases, no fire insurance is carried, and therefore all wood and other unnecessary combustible should be kept out of the structure.

582. The approved constructions are: (a) Steel structure, brick walls, concrete, slate or tile roof. For small low-grade plants corrugated iron may be employed to a very limited extent. (b) Steel framework, walls and roof concrete. (c) Reinforced concrete throughout. Type (c) is exploited chiefly in the hydroelectric plants; type (a) is the most widely employed. Concrete **foundations**, for all work, are practically universal.

583. The general arrangement of engine rooms in the older plants and in the most modern, is parallel to the boiler room. About 8 years ago the sudden increase in capacity of the turbine without much increase in size made the use of cross firing aisles necessary, to get in enough boilers. But the underfeed stoker has so increased the forcing capacity of the boiler that this is now unnecessary.

584. In gas-producer plants, the producer room is generally arranged parallel to the engine room, although it may be entirely separate and some distance away, as in some recovery plants.

585. The coal bunkers in larger power plants are of steel framing with concrete arch lining. For moderate size plants, and for outdoor bunkers for producer plants, the suspended type, catenary-curve bunker is widely used; also steel with concrete lining. Hoppers and chutes for coal, are made of plate steel or cast iron; ash hoppers, of plate steel lined with red brick, or reinforced concrete.

586. Brickwork for power-station buildings varies about as given below.

Heavy basement work, labor cost only $7.00 to $9.00 per M.
18 in. walls 8.00 to 10.00 per M.
13 in. walls 9.00 to 11.00 per M.
18 in. wall, faced on one side with pressed brick 12.00 to 14.00
18 in. wall, faced on both sides with pressed brick 16.50 to 19.00
13 in. wall, faced on one side with pressed brick 14.00 to 17.00

This is for straight wall work. For work much cut up by windows and corners, these prices must be increased. Common red brick costs about $8.00 per M delivered in or near cities.

587. Concrete. Taylor and Thompson give the following approximate cost for concrete:

Item	Per cubic yard	
	Range	Average
Mass foundations	$4.00 to 6.00	$7.00
Conduits and sewers	5.00 to 16.00	9.50
Tunnels, subways	6.00 to 42.00	15.00
Reinforced retaining walls	12.00 to 15.00	13.50
Reservoirs, filters	6.00 to 23.00	10.50
Tanks, standpipes	4.00 to 20.00	12.00
Buildings, total structures	8.00 to 26.00	14.00
Walls in building construction	12.00 to 25.00	17.50
Encasing structural steel in concrete	14.00 to 21.00	18.50
	Per linear ft.	
Concrete piles	0.51 to 1.60	1.15

588. Structural steel for building purposes costs from $55 to $120 per ton, erected. For any amount over 200 tons, it should not ordinarily exceed $65 per ton.

589. Building foundations are made almost exclusively of mass concrete (Par. 587).

590. The bearing power of soils for computing the proper spread of footings is given below.

Soil	Tons per sq. ft.	Remarks
Good solid natural earth	4.0	New York Building Laws
Pure clay, 15 ft. with no admixture of foreign substances except gravel.	1.75	Chicago Building Ordinances
Dry sand, 15 ft. thick, no admixture of foreign substances.	2.0	Chicago Building Ordinances
Clay and sand mixed	1.5	Chicago Building Ordinances
Hard rock on native bed	2.50	Richey
Ledge rock	36.0	Richey
Hard-pan	8.0	Richey
Gravel	5.0	Richey
Clean sand	4.0	Richey
Dry clay	3.0	Richey
Wet clay	2.0	Richey

591. For making concrete in any quantity, machine mixing always pays, both in first cost and in reliable quality. **Form work** is usually of 1.25-in. tongued and grooved, short-leaf yellow pine lumber dressed one side, with 2 in. x 4 in. rough spruce or yellow pine studs and bracing for the greater part; 4 in. x 4 in. studs, and larger, are used only where necessary.

592. Loads on Foundation
(Chicago Building Ordinances)

Foundation	Tons per sq. ft.	Foundation	Tons per sq. ft.
Concrete	4.0	Iron rails in concrete	6.0
Foundation piers, dimension stone.	5.0	Steel rails in concrete	8.0
Brick piers in cement	9-12.5	Piles	12.0

593. Machinery foundations are generally of mass concrete, as for buildings. But lately the demands for condenser and auxiliary space under steam turbines have forced the use of **structural steel foundations.** There is no objection objection to this, provided the turbine is properly balanced and the maximum permissible deflection of about 0.02 in. is not exceeded. Foundations should extend up to 1 in. or ¾ in. below the bottom line of the machine base to be set; and when the latter is lined up, the whole should be grouted with cement grout, mixed 1 : 2, of cement and sand. **Foundation bolts** are now seldom set from templates, but are accurately located in the forms by drawing dimensions. Each bolt is mounted in a pipe sleeve large enough to allow a play of at least one diameter for taking care of inaccuracies in the casting and in the setting. With some turbines, no bolts whatever are needed.

594. Drainage should if possible be arranged for by placing the whole station above sewer or tide-water level, so that gravity flow from sumps and traps is possible. Where this cannot be done, suitable reservoir sumps must be provided, emptied by ejectors or pumps, float controlled. Galvanized iron or cast-iron roof leaders may be employed, with cast-iron soil pipe in the ground, or glazed clay tile pipe.

595. Lighting and ventilation. The use of all-glass monitors is the most desirable way of lighting and ventilating plants. The rolled steel section windows and monitors now produced have rendered easy the problem of fireproof, permanent window and ventilator fixtures. The operating devices should be arranged for quick opening and closure, to provide for storm protection. The continuous sash for monitor use is the latest device in this line. Care should be taken to make windows rain- and snowproof, and to allow opening for ventilation without letting in rain during ordinary storms.

596. Lighting by daylight is accomplished by the monitors above mentioned and by high side windows in some cases. The night illumination in most cases is by incandescent lamp. The arc and Nernst lamps are generally underairable; the mercury vapor lamp is very successful where used, and is the least injurious to eyesight, but there is much general objection to the color of the light. The usual demand for illumination in a power station will take from 0.2 to 0.5 per cent. of the output.

597. Fire risks in well constructed plants are exceedingly low; in many of the largest plants, no insurance against fire is carried. The use of wood roofs or other inflammable structures, makes insurance imperative.

598. The cost of buildings for various types of power plant is given in Par. 413, 546 and 579 per kw. of capacity. The cost per cu. ft. of contents is less variable, and runs from $0.20 to $0.35.

HYDRAULIC POWER PLANTS

BY ARTHUR T. SAFFORD

Consulting Hydraulic Engineer, Member, American Society of Civil Engineers, Member, American Society of Mechanical Engineers

HYDRAULICS

599. Pressure and depth. Water is but slightly compressible, therefore the pressure, P, is for all practical purposes directly proportional to the depth, H, and can be represented by a diagram as shown in Figs. 46 to 49.

The total pressure on any submerged surface is equal to the area of the pressure diagram (*abc*, Fig. 46; *decb*, Figs. 47 and 48) and the centre of pressure passes through its centre of gravity, G, perpendicular to the submerged surface. The moment of the pressure about c is (Fig. 49),

$$M = Py \qquad \text{(ft-lb.)} \qquad (102)$$

The pressure in lb. per sq. in. at any point is

$$p = 0.433 \times \text{Depth} \qquad (103)$$

The pressure is always normal to the submerged surface. The **total pressure** exerted on a submerged body is

$$P = 62.4HA \qquad \text{(lb.)} \qquad (104)$$

wherein H is the depth of water in feet over the geometrical centre of the body and A is the area of the surface in sq. ft.

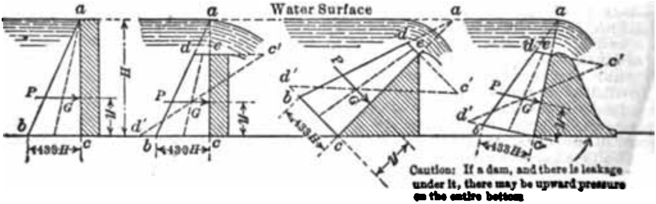


Fig. 46. Fig. 47. Fig. 48. Fig. 49.

600. Possible upward pressure under dams. If the structure is a dam and there is leakage under it there may be upward pressure over the entire bottom. If there is no leakage then there will be no upward pressure. The truth in any given case probably lies between these two extremes and the foundations and underlying material must be carefully studied to make a proper design.

601. The dynamic properties of water in motion are theoretically the same as those of falling bodies *i.e.*:

$$V = \sqrt{2gh} = 8.02\sqrt{h} \qquad \text{(ft. per sec.)} \qquad (105)$$

where $g = 32.16$.

Actually this formula is rarely exact and expressions based upon it must be modified by empirical coefficients.

602. The quantity of water passing a given point in a unit of time is equal to the product of the net cross-sectional area taken perpendicular to the line of flow and the mean velocity in that time parallel to the line of flow. This may be expressed by the equation

$$Q = AV \qquad \text{(sec-ft.)} \qquad (106)$$

If A is the area in square feet, V the velocity in feet per second, then Q is the quantity in cu. ft. per sec., or, in its briefer form, second-feet. This in the United States is now the common expression in water-power practice for flow of streams, capacity of canals and raceways and discharge of water-wheels.

603. At every section of a continuous and steady stream the total energy is constant; whatever head is lost as pressure is gained as velocity. This is known as **Bernoulli's theorem**, and in terms of head can be expressed as follows: Total head = velocity head + pressure head + head due to elevation = constant; or in every stream section,

$$H_T = h_v + h_p + h_e = \frac{V^2}{2g} + \frac{p}{y} + h_e \qquad \text{(ft.)} \qquad (107)$$

where y is the constant to reduce lb. per square inch to head in feet, or 0.433.

In order to make this equation of practical application, a term representing the head lost in overcoming friction, h_λ, must be added on the right-hand side of the equation. This formula, properly modified to include the effect of frictional resistances, is the basis of all empirical formulas for the flow of water.

604. Power and energy. The potential energy of water held in reserve is its weight multiplied by the net available distance through which it can fall in the performance of work. As power expresses the rate of doing the work, it is convenient to deal with the flow of water in cu. ft. per sec. falling through a given vertical distance in feet. Power $= 62.4QH$ ft-lb. per sec., where Q is the flow in cu. ft. per sec., and H the vertical distance or "head" in feet and 62.4 is the weight in pounds of 1 cu. ft. of water.

$$\text{Horse-power} = \frac{62.4QH}{550} = \frac{QH}{8.8} \qquad (108)$$

This is the maximum horse-power that might be obtained from Q cubic feet of water per second falling a distance H feet, assuming an efficiency of transformation of energy of 100 per cent. This expression multiplied by the known efficiency of a water wheel will give the power at the water-wheel shaft. Ordinarily $QH/11$ or $QH/12$ will give approximately the net water horse-power, corresponding respectively to efficiencies of 80 per cent. and 73.3 per cent.

FLOW FORMULAS

605. Orifices employed as meters, are limited in use; experimentally they have given very consistent results, but in practice these results often cannot be reproduced with sufficient accuracy for precise work. Orifices of relatively small sizes, of regular shapes (usually round or rectangular), with carefully made edges, and used with full contraction of the jet, have been carefully experimented upon and may be used with confidence provided there is practically no velocity of approach (less than 0.5 ft. per sec.).

606. The flow of water through an orifice (Fig. 50) in a vertical wall expressed in cubic ft. per sec. is

$$Q = CAV = CA\sqrt{2gh} \quad \text{(cu. ft. per sec.)} \qquad (109)$$

where A is the area of the opening in square feet, h the head in feet measured from the surface of the water to the center of the opening, and C is the coefficient of discharge which depends on the form of the orifice. For sharp-edged orifices, 4 sq. ft. or less in area, with full contraction of the issuing stream (Fig. 50), discharging under heads from about 1 to 20 times the depth of the orifice (practically no velocity of approach), a value of C may be taken as 0.6. If the orifice is large and the head acting is small, the exact or integral form of the equation* must be used when C is taken as 0.6. For high heads and relatively small orifices this is not necessary. If the contraction is even partly suppressed the results are unreliable; if wholly suppressed the orifice becomes a short tube or nozzle and the coefficient varies greatly, depending on the shape and proportions; but always more than 0.6 (see Par. 615).

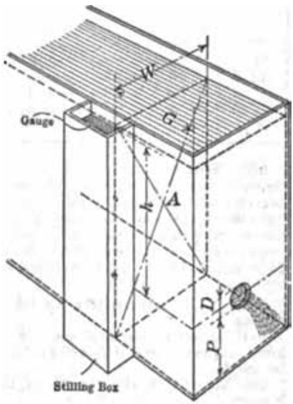


Fig. 50.—Circular orifice with sharp edges, giving full contraction.

607. In treating head gates and sluice gates as orifices, the forms of cross-section, the channel of approach and that leading away from the orifice, and the velocity of approach are all very important; these factors modify any computed discharge based upon the opening and observed head.

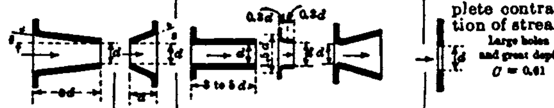
* Water Supply and Irrigation Paper No. 200.

The coefficient of discharge for gates on the basis of their cross-sectional area and the head measured above and below them may vary all the way from that of a standard sharp-edged orifice (0.6) to over 1.0 if there is much velocity of approach. For these reasons, if head gates or sluice gates are to be used as measuring devices they should be given their own rating by some independent method of measurement.

608. Weisbach's Coefficients

$$C \text{ in } Q = C\frac{\pi d^2}{4}\sqrt{2gh} \qquad (109A)$$

wherein C is a coefficient which depends on the form and head h acting on orifice, Q is given in cubic feet per sec. when h and d are measured in feet; h is head in feet above centre of orifice; and d is diameter of orifice in feet.



Thin plate orifice and complete contraction of stream

S. deg.	C	C	Inlet slightly round $C = 0.90$	Depending upon smoothness surface $C = 0.96$ to 0.99	Depending on the length and velocity $C = 0.96$ to 1.5	d	h = 1.148 C	h = 1.969 C
0.	0.97	0.54						
5.75	0.95	......						
11.25	0.92	......	Inlet very round $C = 0.97$			0.0144	0.68	0.66
22.5	0.90	0.55				0.0328	0.64	0.63
45.	0.75	0.58				0.0656	0.63	0.62
67.5	0.68	0.60				0.0984	0.62	0.61
90.	0.63	0.63				0.1312	0.614	0.607

609. Weirs,* if properly constructed and used with faithful regard to reproducing the exact conditions which obtained when the experimenter derived his formula, should give good results. Some extraordinary results have been obtained because these precautions were not observed; and weir measurements unless standardised will gradually lose their hold. A weir installation in a power plant must necessarily cut down the available head by roughly 3 ft. For this reason the weir should be used only for testing, and not for operating.

610. "The procedure to be followed in weir measurements comprises:†

(*a*) Constructing and setting up the weir and the gage for measuring the head; reproducing, if possible, the experimental conditions of the formula to be used.

(*b*) Measuring the length of the crest and determining its irregularities if any.

(*c*) Taking a profile of the crest if not sharp-edged.

(*d*) Determining by actual measurements the cross-sectional area of the channel of approach.

(*e*) Establishing by leveling the relative elevations of the crest of the weir, and the zero of the gage.

(*f*) When the desired regulation of flow is established, determining the head by hook gage or other observations at intervals as frequent as the conditions require.

* For the most extended compilation and examination of existing weir data Water Supply and Irrigation Paper No. 200 (Weir Experiments, Coefficients, etc., by R. E. Horton) should be consulted.

† Hughes and Safford, "Hydraulics;" New York, MacMillan Co., 1911, page 196.

(*g*) If possible, measure the actual velocity in the channel of approach by a current meter or some other direct method; and

(*h*) Compute the discharge by the formula selected. Three of these operations require especial consideration, viz.: construction and setting, the measurement of the head, and the selection of the formula."

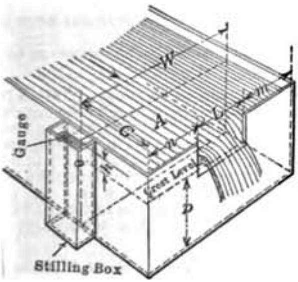


Fig. 51.—Weir with end contractions.

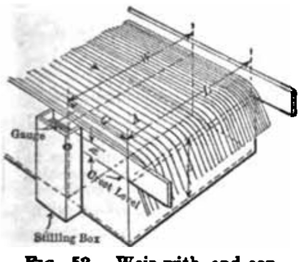


Fig. 52.—Weir with end contractions suppressed.

611. "Construction and setting of weirs.* In order to eliminate as far as possible factors for which precise allowance cannot be made, the construction and setting should meet the following conditions:

(*a*) A sharp-crested weir with complete crest contraction should be used.

(*b*) The crest should be level, and its ends vertical.

* Hughes and Safford, "Hydraulics;" New York, MacMillan Co., 1911.

612. Standard Weir Formulas (Par. 613 to 621)

Experimenter	Formula (cu. ft. per sec.)	Limits of head recommended (ft.)	Distance of crest above bottom (ft.)	Correction for end contraction	Correction for velocity of approach: Suppressed weir	Correction for velocity of approach: Contracted weir
Fteley & Stearns	$Q=3.31L\sqrt{H^3}+0.007L$	0.07 to 0.50	At least 2.0	$L-\left(L-\frac{NH}{10}\right)$ *	$H=(h+1.50h_v)$	$H=(h+2.05h_v)$
Francis	$Q=3.33L\sqrt{H^3}$	0.50 to 2.00	At least 3 times the observed head	$L-\left(L-\frac{NH}{10}\right)$	$\sqrt{H^3=(h-h_v)^3-h_v^3}$	....
Hamilton Smith	$Q=(C_e \text{ or } C_s)\frac{2}{3}L\sqrt{2gH^3}$	0.10 to 1.70	At least 3 times the observed head	Use proper coefficient	$H=(h+1-\frac{1}{3}h_v)$	$H=(h+1.4h_v)h_v$
Bazin**	$Q=mLh\sqrt{2gh}$	0.20 to 2.00	....	....	....	....

* Fteley & Stearns found that in this expression $\frac{H}{10}$ was not constant but varied with perfect contraction from $0.061H$ to $0.124H$, H being equal to $(h + ah_v)$; a being equal to 1.5 and 2.05 according to form of weir.

** *For suppressed weirs only.* m being a coefficient including effects of crest contraction and velocity of approach.

(c) The end contractions should be complete, or, if suppressed, entirely suppressed.

(d) The upstream face should be vertical; the downstream face so designed that the nappe has free overfall.

(e) Free access for air under the nappe should be made certain.

(f) The weir should be set at right angles to the direction of flow.

(g) The channel of approach should be straight for at least 25 ft. above the weir, of practically uniform cross-section and of slight slope (preferably none).

(h) Screens of coarse wire or baffles of wood should be set in the channel, if necessary, to equalize the velocities in different parts of the channel, but not nearer the crest than 25 ft.

(i) The channel of approach should have a large cross-sectional area in order to keep the velocity of approach low."

A weir with complete end contractions is shown in Fig. 51; suppressed end contractions are shown in Fig. 52.

613. Francis formulas. The best known formulas are those of James B. Francis and M. Bazin. The Francis formulas are strictly applicable only to **vertical, sharp-crested weirs** with free overfall and either with no end contractions ("suppressed weir"), or with complete end contractions and: (a) when the length of the weir is at least 5 ft.; (b) when the head (H) is not greater than one-third the length (L); (c) when the head is not less than 0.5 ft. nor more than 2 ft.; (d) when the velocity of approach is 1 ft. per second or less; (e) when the height of the weir crest above the bottom of the channel of approach is at least three times the head. For tabulation of standard formulas see Par. 612.

614. Smith's formulas. For **short weirs** (shorter than 5 ft.) the experiments and studies made by Hamilton Smith, Jr., afford the best guide (See Smith's weir coefficients, Par. 620).

615. Bazin's formula. The best weir experiments abroad are those of M. Bazin whose general formula is $Q = mLh\sqrt{2gh}$ for suppressed weirs only. At about 1.0 ft. depth on the crest, with no velocity of approach, the results from his standard 8.56-ft. (2-meter) weir are practically those of James B. Francis' standard 10-ft. weir (See Par. 621).

616. Fteley and Stearns' formula. For heads from 0.07 to 0.5 ft. the Fteley and Stearns formula* $Q = 3.31L\sqrt{H^3} + 0.007L$ is recommended.

617. Weir discharge tables. The following table of discharges for weirs without end contractions, and velocity head is given for heads from 0.00 to 2.98 ft. and includes figures from the Fteley and Stearns formula up to 0.5 ft. and the Francis formula from 0.5 ft. to 2.98 ft. The quantity of water is given in cu. ft. per sec., per foot of weir, with complete contraction on the crest, and no end contractions. $Q = 3.31L\sqrt{H^3} + 0.007L$ for depths up to 0.5 ft. and $Q = 3.33L\sqrt{H^3}$ for depths above 0.5 ft.; the velocity due to the head is computed by formula, Vel. $= \sqrt{2gh}$.

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* Fteley and Stearns, in "Sudbury River Experiments" (page 84) state that this formula is based upon experiments in which the depths on the weir ranged from 0.07 to 1.63 ft. *Trans.* Amer. Soc. Civil Eng., Vol. XII.

H (ft.)	Quan. (sec-ft.)	Velo. (ft. per sec.)	H (ft.)	Quan. (sec-ft.)	Velo. (ft. per sec.)	H (ft.)	Quan. (sec-ft.)	Velo. (ft. per sec.)
0.00	....	....	0.50	1.18	5.67	1.00	3.33	8.02
.02	.02	1.13	.52	.25	5.78	.02	.43	8.10
.04	.03	1.60	.54	.32	5.89	.04	.53	8.18
.06	.06	1.96	.56	.39	6.00	.06	.63	8.26
.08	.08	2.27	.58	.47	6.11	.08	.74	8.33
0.10	0.11	2.54	0.60	1.55	6.21	1.10	3.84	8.41
.12	.14	2.78	.62	.63	6.32	.12	.95	8.49
.14	.18	3.00	.64	.70	6.42	.14	4.05	8.56
.16	.22	3.21	.66	.79	6.52	.16	.16	8.64
.18	.26	3.40	.68	.87	6.61	.18	.27	8.71
0.20	0.30	3.59	0.70	1.95	6.71	1.20	4.38	8.79
.22	.35	3.76	.72	2.03	6.81	.22	.49	8.86
.24	.40	3.93	.74	.12	6.90	.24	.60	8.93
.26	.45	4.09	.76	.21	6.99	.26	.71	9.00
.28	.50	4.24	.78	.29	7.08	.28	.82	9.07
0.30	0.55	4.39	0.80	2.38	7.17	1.30	4.94	9.14
.32	.61	4.54	.82	.47	7.26	.32	5.05	9.21
.34	.66	4.68	.84	.56	7.35	.34	.16	9.28
.36	.72	4.81	.86	.66	7.44	.36	.28	9.35
.38	.78	4.94	.88	.75	7.52	.38	.40	9.42
0.40	0.84	5.07	0.90	2.84	7.61	1.40	5.52	9.49
.42	.91	5.20	.92	.94	7.69	.42	.63	9.56
.44	.97	5.32	.94	3.03	7.78	.44	.75	9.62
.46	1.04	5.44	.96	.13	7.86	.46	.87	9.69
.48	.11	5.56	.98	.23	7.94	.48	6.00	9.76
1.50	6.12	9.82	2.00	9.42	11.34	2.50	13.16	12.68
.52	.24	9.89	.02	.56	11.40	.52	.32	12.73
.54	.36	9.95	.04	.70	11.46	.54	.48	12.78
.56	.49	10.02	.06	.85	11.51	.56	.64	12.83
.58	.61	10.08	.08	.99	11.57	.58	.80	12.88
1.60	6.74	10.14	2.10	10.13	11.62	2.60	13.96	12.93
.62	.87	10.21	.12	.28	11.68	.62	14.12	12.98
.64	.99	10.27	.14	.42	11.73	.64	.28	13.03
.66	7.12	10.33	.16	.57	11.79	.66	.45	13.08
.68	.25	10.40	.18	.72	11.84	.68	.61	13.13
1.70	7.38	10.46	2.20	10.87	11.90	2.70	14.77	13.18
.72	.51	10.52	.22	11.01	11.95	.72	.94	13.23
.74	.64	10.58	.24	.16	12.00	.74	15.10	13.28
.76	.77	10.64	.26	.31	12.06	.76	.27	13.32
.78	.91	10.70	.28	.46	12.11	.78	.43	13.37
1.80	8.04	10.76	2.30	11.61	12.16	2.80	15.60	13.42
.82	.18	10.82	.32	.77	12.22	.82	.77	13.47
.84	.31	10.88	.34	.92	12.27	.84	.94	13.52
.86	.45	10.94	.36	12.07	12.32	.86	16.11	13.56
.88	.58	11.00	.38	.23	12.37	.88	.27	13.61
1.90	8.72	11.05	2.40	12.38	12.42	2.90	16.44	13.66
.92	.86	11.11	.42	.54	12.48	.92	.62	13.70
.94	9.00	11.17	.44	.69	12.53	.94	.79	13.75
.96	.14	11.23	.46	.85	12.58	.96	.96	13.80
.98	.28	11.29	.48	13.00	12.63	.98	17.13	13.84

618. Examples of weir calculations. A weir 5 ft. long is set in a channel 10 ft. wide and the crest is 4.36 ft. high. If the observed head is

0.64 ft., compute the discharge (a) by the Francis, (b) by the Fteley and Stearns, (c) by the Smith formula.

(a) Francis formula: $Q=3.33\ (5-0.1\times2\times0.64)\sqrt{0.64^3}=3.33\times4.872\times0.512=8.31$ cu. ft. per sec.; not corrected for velocity of approach V_A. Area of channel of approach $A=(4.36+0.64)\ 10=50$ sq. ft. Velocity of approach $V_A=Q/A=8.31/50=0.1662$ ft. per sec. Velocity head $h_v=(0.1662)^2/64.32=0.0004$; $\sqrt{H^3}=\sqrt{(0.64+0.0004)^3}-\sqrt{0.0004^3}=0.5125-0.000008=0.5124$; $Q=3.33\times4.872\times0.5124=8.313$ cu. ft. per sec.

(b) Fteley and Stearns formula: $Q=3.31\ (5-0.1\times2\times0.64)\sqrt{0.64^3}+0.007\times5=3.31\times4.872\times0.512+0.007\times5=8.29$ cu. ft. per sec.; h_v (as above) $=0.0004$; $H=0.64+2.05\times0.0004=0.6408$; $\sqrt{H^3}=0.513$; $Q=3.31\times4.872\times0.513+0.007\times5=8.31$ cu. ft. per sec.

(c) Hamilton Smith formula: $Q=0.607\times\frac{2}{3}\times5\times8.02\times\sqrt{0.64^3}=8.308$ cu. ft. per sec.; h_v (as above) $=0.004$; $H=0.64+1.4\times0.0004=0.6406$; $\sqrt{H^3}=0.5127$; $Q=0.607\times\frac{2}{3}\times5\times8.02\times0.5127=8.320$ cu. ft. per sec.

In the foregoing solutions it is seen that the effect of velocity of approach is negligible, as is usually the case when the velocity is less than half a foot per second.

619. Further examples of weir calculations. Given a suppressed weir 7 ft. long, with crest 4.5 ft. above the bottom of the channel; the observed head is 1.36 ft. Compute the discharge by the Francis, Fteley and Stearns, and Basin formulas: (a) not correcting for the velocity of approach; (b) correcting for the velocity of approach.

(a) Francis formula: $Q=3.33\times7\times\sqrt{1.36^3}=36.97$ cu. ft. per sec. Fteley and Stearns formula: $Q=3.31\times7\times\sqrt{1.36^3}+0.007\times7=36.80$ cu. ft. per sec.

(b) Francis formula: $A=7\times5.86=41.02$ sq. ft.; $V_A=Q/A=36.97/41.02=0.90$ ft. per sec.; $h_v=(0.90)^2/64.32=0.0126$ ft.; $\sqrt{H^3}=\sqrt{(1.36+0.0126)^3}-\sqrt{0.0126^3}=1.610$; $Q=3.33\times7\times1.61=37.50$ cu. ft. per sec.

Fteley and Stearns formula: $V_A=36.70/41.02=0.894$; $h_v=(0.894)^2/64.32=0.0124$; $H=1.36+1.50\times0.0124=1.379$; $\sqrt{H^3}=1.620$; $Q=3.31\times7\times1.620+0.007\times7=37.58$ cu. ft. per sec.

Basin formula: Basin's coefficient (Par. 621) includes effect of velocity of approach; $Q=0.4266\times7\times1.36\sqrt{64.32\times1.36}=37.97$ cu. ft. per sec.

620. Smith's weir coefficients. C_s = coefficient for weirs with the contraction suppressed at both ends and complete crest contraction.

Effective head in feet, H	Length of weir in feet				
	$L=0.66$	2	3	4	5
0.1	0.675	...	...	...	0.659
0.15	0.662	0.652	0.649	0.647	0.645
0.2	0.656	0.645	0.642	0.641	0.638
0.25	0.653	0.641	0.638	0.636	0.634
0.3	0.651	0.639	0.636	0.633	0.631
0.4	0.650	0.636	0.633	0.630	0.628
0.5	0.650	0.637	0.633	0.630	0.627
0.6	0.651	0.638	0.634	0.630	0.627
0.7	0.653	0.640	0.635	0.631	0.628
0.8	0.656	0.643	0.637	0.633	0.629
0.9	...	0.645	0.639	0.635	0.631
1.0	...	0.648	0.641	0.637	0.633
1.1	...	...	0.644	0.639	0.635
1.2	...	...	0.646	0.641	0.636
1.3	...	...	0.648	0.643	0.638
1.4	...	...	...	0.644	0.640
1.5	...	...	...	0.646	0.641
1.6	...	...	...	0.647	0.642
1.7	...	...	...	...	...

C_e = coefficient for weirs with complete contraction at two ends and complete crest contraction.

Effective head in feet, H	Length of weir in feet						
	L = 0.66	1	2	2.6	3	4	5
0.1	0.632	0.639	0.646	0.650	0.652	0.653	0.653
0.15	0.619	0.625	0.634	0.637	0.638	0.639	0.640
0.2	0.611	0.618	0.626	0.629	0.630	0.631	0.631
0.25	0.605	0.612	0.621	0.623	0.624	0.625	0.626
0.3	0.601	0.608	0.616	0.618	0.619	0.621	0.621
0.4	0.595	0.601	0.609	0.612	0.613	0.614	0.615
0.5	0.590	0.596	0.605	0.607	0.608	0.610	0.611
0.6	0.587	0.593	0.601	0.604	0.605	0.607	0.608
0.7	0.585	0.590	0.598	0.601	0.603	0.604	0.606
0.8	....	....	0.595	0.598	0.600	0.602	0.604
0.9	....	....	0.592	0.596	0.598	0.600	0.603
1.0	....	....	0.590	0.593	0.595	0.598	0.601
1.1	....	....	0.587	0.591	0.593	0.596	0.599
1.2	....	....	0.585	0.589	0.591	0.594	0.597
1.3	....	....	0.582	0.586	9.589	0.592	0.596
1.4	....	....	0.580	0.584	0.587	0.590	0.594
1.5	....	....	....	0.582	0.585	0.589	0.592
1.6	....	....	....	0.580	0.582	0.587	0.591
1.7	....	....	....	....	....	....	....

621. Bazin's coefficients.
Values of m corresponding to heads (h) and heights of weir (p) in feet for use in Bazin's formula.

$$Q = mL\sqrt{2gh^3}$$

h (ft.)	p = 0.656	p = 1.0	p = 1.5	p = 2	p = 2.5	p = 3	p = 4	p = 5	p = 6	p = 6.56
0.2	0.456	0.449	0.446	0.444	0.444	0.443	0.443	0.443	0.443	0.443
0.3	0.457	0.446	0.440	0.438	0.436	0.436	0.435	0.435	0.434	0.434
0.4	0.463	0.448	0.439	0.435	0.433	0.432	0.431	0.430	0.430	0.430
0.5	0.469	0.451	0.440	0.435	0.432	0.430	0.428	0.427	0.427	0.427
0.6	0.476	0.455	0.442	0.435	0.431	0.429	0.427	0.425	0.425	0.424
0.7	0.482	0.460	0.444	0.436	0.432	0.429	0.426	0.424	0.423	0.423
0.8	0.489	0.465	0.447	0.438	0.433	0.430	0.426	0.424	0.423	0.422
0.9	0.495	0.470	0.451	0.440	0.434	0.430	0.426	0.424	0.422	0.422
1.0	0.501	0.475	0.454	0.443	0.436	0.432	0.426	0.424	0.422	0.421
1.1	.....	0.479	0.457	0.445	0.438	0.433	0.427	0.424	0.422	0.421
1.2	.....	0.483	0.461	0.448	0.439	0.434	0.428	0.424	0.422	0.421
1.3	.....	0.487	0.464	0.450	0.441	0.435	0.428	0.424	0.422	0.421
1.4	.....	0.491	0.467	0.452	0.443	0.437	0.429	0.425	0.422	0.421
1.5	.....	0.495	0.470	0.455	0.445	0.438	0.430	0.425	0.422	0.421
1.6	.....	.....	0.473	0.457	0.447	0.440	0.431	0.425	0.422	0.421
1.7	.....	.....	0.475	0.459	0.448	0.441	0.431	0.426	0.422	0.421
1.8	.....	.....	0.478	0.461	0.450	0.442	0.432	0.426	0.422	0.421
1.9	.....	.....	0.480	0.463	0.452	0.444	0.433	0.427	0.423	0.421
2.0	.....	.....	0.483	0.465	0.453	0.445	0.434	0.427	0.423	0.421

622. Other forms of weir notches have been proposed and used, the most common of which are of Cippoletti, a trapezoidal notch, and the **triangular notch** (Fig. 53). The object of the Cippoletti weirs was to eliminate the effect of the end contractions. According to experiments by Horton*

* Water Supply and Irrigation Paper No. 200 U. S. Geol. Survey; Washington, D. C.

when the batter of the sides is 1 in 4 this aim is accomplished. The formula for any **trapezoidal weir** takes the usual form

$$Q = CL\sqrt{H^3} \qquad \text{(cu. ft. per sec.)} \qquad (110)$$

in which C for the special form cited has been experimentally determined as 3.367. Correction for velocity of approach may be made as for the Francis weir. The advantage of this weir is that a constant length L may be used for all heads so that for continuous measurements with varying heads it saves much labor. This form has been much used in irrigation projects in the Western United States. By adding 1 per cent. to the value of Q found in the weir table in Par. **617** the discharge for this weir may be computed.

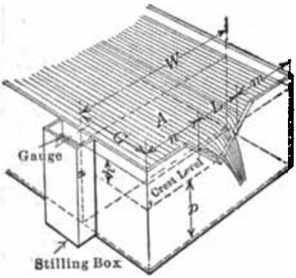


Fig. 53.—Triangular weir.

623. Triangular (notch) weirs. The common triangular notch is right angled, set with the apex down and the bisector of this angle vertical (Fig. 53). It has been found by experiment that the coefficient of such a triangular weir is remarkably constant for all heads. The formula for a right-angled notch set as described reduces to

$$Q = C\sqrt{H^5} \qquad \text{(cu. ft. per sec.)} \qquad (111)$$

in which C may be taken as 2.54 when H is measured to the apex of the angle. For measuring small quantities of water where the necessary head consumed is unimportant the V notch furnishes an accurate means and has been developed commercially as the **Lea Recorder**.

624. Flow of water in channels, whether open or closed, is subject to the same fundamental laws, and the formulas in common use are based on the two primary conceptions stated in Par. **602** and **603**. Head is required to produce velocity, to overcome obstructions such as bends, sudden constrictions or enlargements and head is required to overcome friction. Friction is in turn depends on the velocity of flow and on the surface over which the water flows, its extent, and its character. In any formula for discharge the extent of the rubbing surface in contact with the water may be cared for directly by a numerical factor known as the **mean hydraulic radius,** which is found by dividing the cross-sectional area of the stream by its wetted perimeter; the latter is the linear dimension of that part of the boundary line of the cross-section of a channel in contact with the water. The effect of the character of the surface must be cared for by empirical coefficients. Head may be provided by natural topographical conditions as in the slope of a river bed, or created artificially as in the case of an elevated reservoir feeding into a pipe line. In the formulas most commonly in use the head appears in the slope of the **hydraulic grade line** which is the fall in feet, per foot of distance, measured along the longitudinal axis. The hydraulic grade line is a line in the plane of the longitudinal axis of the stream and at all points distant from it an amount equal to the net effective head. In an open channel the hydraulic grade line is coincident with the surface of the water. In the case of a pipe or conduit flowing under pressure the hydraulic grade line is above the centre line of the pipe and distant from it as defined above.

625. The Chezy formula is probably the most satisfactory one to use, certainly for open channels. It is usually stated as follows:

$$V = C\sqrt{RS} \qquad \text{(ft. per sec.)} \qquad (112)$$

where C = coefficient increasing with the mean hydraulic radius, and for new clean channels usually increasing with the mean velocity of flow, and decreasing with the roughness of the channel; R = mean hydraulic radius, and S = sine of the slope of the hydraulic grade line. The common text-book formula which, as nearly as may be determined, was proposed by Weisbach,*

* Weisbach's "Mechanics" (Coxe's translation), page 866. This formula is not uncommonly designated as Weisbach's, Darcy's, Weston's, or Chezy's.

and which is used only for pipes under pressure, is as follows (for steady uniform flow under pressure in circular pipes):

$$h_f = f\left(\frac{LV^2}{2gD}\right) \qquad \text{(ft.)} \qquad (113)$$

where h_f = the head lost in friction, f = the friction factor or coefficient of friction, decreasing with an increase in the diameter of the pipe and commonly with an increase of velocity of flow; and increasing with the age and roughness of the surface in contact with the water (Par. 626). L = the length (in ft.) of the pipe measured on its axis; D = the internal diameter (in ft.) of the pipe; V = the mean velocity of flow in ft. per sec.; g = the acceleration due to gravity, taken here as 32.16. The above equations are merely different expressions for the same formula. Either may be used to suit convenience.

Values of C suggested for use in Eq. 112 are as follows: New cast-iron pipe laid carefully without abrupt changes in grade or alignment: sizes 6 in. to 10 in., velocities 3 to 8 ft. per sec., C = 102 to 108; sizes 12 in. to 20 in., velocities 1 to 5 ft. per sec., C = 105 to 115; sizes 20 in. to 60 in., velocities 1 to 6 ft. per sec., C = 120 to 150. Riveted steel pipe (new): sizes 16 in. to 102 in., velocities 2 to 5 ft. per sec., C = 100 to 115. Tunnel or aqueduct with smooth cement or hard brick lining of relatively small cross-sectional areas (15 to 50 sq. ft.): velocities 1 to 5 ft. per sec., C = 115 to 140. Small canal, cross-sectional area about 100 sq. ft. in loose gravel or rock: velocities 1 to 5 ft. per sec., C = 60 to 85. Large canal, wide and shallow, smooth bottom and sides: velocities 2 to 5 ft. per sec., C = 50 to 70. Large canal, wide and deep, fairly smooth bottom and sides: velocities 2.5 to 3.5 ft. per sec., C = 75 to 90. Large river, tortuous channel: C = 40 to 80. Large river, easy bends: C = 70 to 100. Where the carrying capacity of any pipe or channel is very important about 30 per cent. depreciation should be figured in advance.

626. Friction factors (f in Eq. 113, Par. 625) for pipes may be computed by Weston's formula.* This formula, which must only be applied to pipes having interior sides similar to lead and brass pipes, from one-half inch to three and one-half inches in diameter is as follows:

$$f = 0.0126 + \frac{0.0315 - 0.06d}{\sqrt{v}} \qquad (114)$$

in which d = internal diameter of the pipe in feet; and v = the velocity in feet per second.

For pipes with interior sides similar to new cast-iron pipes the following formulas are used:

$$f = \left(0.017379 + \frac{0.0015965}{d} + \frac{0.0040723 + \dfrac{0.000020816}{d^2}}{v}\right)\frac{l}{d}\,\frac{v^2}{2g} \qquad (115)$$

$$f = \left(0.0198920 + \frac{0.00166573}{d}\right)\frac{l}{d}\,\frac{v^2}{2g} \qquad (116)$$

The first formula (Eq. 115) is for velocities of flow less than 0.33 ft. per sec., the second for higher velocities.

627. The Kutter formula though intended and largely used for pipes as well as for open channels, is not recommended for general use in either case.

$$V = \left[\frac{41.66 + \dfrac{1.811}{n} + \dfrac{0.00281}{S}}{1 + \left(41.66 + \dfrac{0.00281}{S}\right)\dfrac{n}{\sqrt{R}}}\right] \times \sqrt{RS} \qquad (117)$$

To estimate directly a value of C (Par. 625) is simpler, and probably quite as accurate as to estimate a value of n. The value of n depends on the value of S. Considering that in picking out the value of n a variation of 0.001 for small values of n and R may change the value of C as much as 17 per cent.,

* "Tables showing Loss of Head due to Friction of Water in Pipes" by Edmund B. Weston, C. E., D. Van Nostrand Co., 3rd Edition, 1903.

and for moderate values as much as 5 to 8 per cent., it should be obvious that hair-splitting calculations with the Kutter formula are a needless waste of time, producing merely numerical accuracy instead of a high degree of precision. Though it seems evident that we shall never have one formula to fit accurately all kinds of channels, it appears probable that we may have a small group of formulas each of which will fit some particular class of channels.

Values of n to be used in the Kutter formula for calculating values of C to be used in the Chesy formula are as follows:

$n=0.01$: should never be used except for temporary work and then only for perfectly smooth, clean iron pipes under 8 in. diameter and for high velocities, or for temporary and new planed wooden stave pipes, at high velocity 10 in. and under in diameter.

$n=0.011$: for above pipes, and low velocity.

$n=0.012$: for above pipes 3 ft. or more in diameter and high velocities; old iron pipes 8 in. diameter or under, low velocities; old city water mains above 8 in. diameter. Also for concrete tunnels having $R=2.5$; wood flumes, open, planed plank, long bends.

$n=0.013$: for above pipes 36 in. to 120 in. diameter, old pipes and low velocities; 1-to-2 cement-lined pipes; new penstocks lined with unplaned lumber and running full at all times; large concrete-lined tunnels of area 100 to 200 sq. ft.

$n=0.0135$: Ashlar masonry and well laid brickwork penstocks over 3 ft. in diameter; cast-iron concrete or steel-riveted pipes 8 in. to 20 in. diameter, long in use, short joints and under pressure of 75 to 150 lb. per sq. in.

$n=0.015$: rough concrete pipes where the interior cannot be smoothed or kept clean, moderate velocities and diameters above 3 ft.; penstocks of poorly laid rough brickwork; concrete-lined, open canals, low velocities.

$n=0.017$: canals with gravel bottom and sides well rammed, stones being 0.3 to 0.7 in. diameter; tunnels through hard rock, well trimmed and roughly faced; very large open concrete-lined canals.

$n=0.02$: rough rubble masonry; canals through rock, or with bottoms and sides paved with cobble stones; canals in earth with bottoms and sides well trimmed; small rough lumber penstocks with battens and poor alignment.

$n=0.0225$: canals in earth in good condition, but long in use, having moss growing freely.

$n=0.025$: canals in clay, long in use; small rivers, deep and narrow and with no sharp bends, smooth sand bottoms and smooth uniform banks.

$n=0.0275$: canals and rivers as for $n=0.025$, but having an occasional bend and snag also the same, but with gravel bottoms; earth canals as left by dredging.

$n=0.03$: rivers having loose boulder beds, irregular banks, sharp bends, shallow, normal flow.

$n=0.035$: rivers with rough, irregular beds having shallows and pools, snags, bends, gravel bottoms, average flood stages.

$n=0.05$: large shallow rivers, having sharp bends, low heavily wooded banks, snags, shallows, pools, rough bottom and moderate flood conditions.

$n=0.055$: large torrential streams during high floods, with the banks heavily-wooded and inundated; mountain streams with many falls, large boulders, rapids, etc.

628. Beardsley's formula for C. Assuming values for n, S and R, C may be calculated from the formula as given by R. C. Beardsley.

$$C=\frac{\left(23+\frac{1}{n}+\frac{0.00155}{S}\right)}{\left(0.5521+\left[23+\frac{0.00155}{S}\right]\frac{n}{\sqrt{R}}\right)} \tag{118}$$

or in its original form

$$C=\frac{\left(41.66+\frac{1.811}{n}+\frac{0.00281}{S}\right)}{\left(1+\left[41.66+\frac{0.00281}{S}\right]\frac{n}{\sqrt{R}}\right)} \tag{119}$$

629. Bazin's formula for C. Bazin proposed a formula for computing the value of C to be used in the Chesy formula which differs from the Kutter formula in eliminating S and making the variation in C depend only upon variations in the mean hydraulic radius and the coefficient of roughness. It is as good as any general formula and has the advantage of simplicity. His formula is

$$C = \frac{157.6\, R^{\frac{1}{2}}}{R^{\frac{1}{2}} + \gamma} \tag{120}$$

Values of γ
- $\gamma = 0.109$: very smooth surfaces; neat cement; planed wood.
- $\gamma = 0.290$: smooth surfaces; planks, bricks, ashlar.
- $\gamma = 0.833$: rough surfaces; rubble masonry.
- $\gamma = 1.54$: canals with mixed linings; very regular earth or paved with stones.
- $\gamma = 2.35$: earth canals in ordinary conditions.
- $\gamma = 3.17$: earth canals in bad condition.

Bazin's formula represents the results of a very careful study of existing data and is extremely valuable in designing relatively small channels and those with very smooth linings; it also meets the conditions of design for all open channels as well as, if not better than any general formula available. It has the additional merit of simplicity.

630. Friction loss in iron pipes increases with age. Under ordinary conditions of service at the end of 30 years we may expect to find the losses in cast-iron pipes due to friction about doubled. This is satisfactory for rough approximations and the loss for intermediate years may be taken as proportional to the age.

631. Loss of head at entrance. The flow formulas which have been given (Par. **605** to **629**) take into account only the head necessary to overcome friction. There is in addition to this a further loss of head in creating

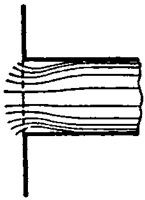


FIG. 54*a*.

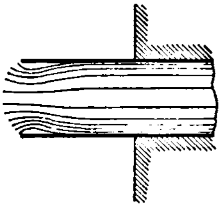


FIG. 54*b*.

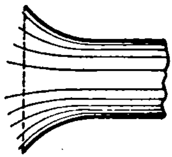


FIG. 54*c*.

the velocity of flow and in overcoming the resistance of entrance into the pipe. The **velocity head** is equal to $V^2/2g$. The head lost at entrance varies with the form of entrance and is expressed as a constant times the velocity head $\xi_e V^2/2g$. Values of ξ are given below.

$$0.5\frac{V^2}{2g} \qquad (0.11 \text{ to } 0.02)\frac{V^2}{2g} \qquad 0.93\frac{V^2}{2g}$$

632. Nozzles, properly designed, furnish one of the simplest and most accurate methods of measuring small quantities of water. Coefficients of discharge for various forms of nozzles from about ¾ in. to 5 in. have been determined with great accuracy by John R. Freeman.* Nozzles are especially useful in measuring the discharge through pipes, fire hose, and in testing the performance of pumps and impulse water wheels. With nozzles to which Freeman's coefficients are applicable, discharge measurements can be made with an error not exceeding 2 per cent.

633. Loss of head due to bends, elbows, valves, etc. Experimental data are either meager or lacking from which to compute the loss of head due to bends, elbows, valves, etc. Their effect only becomes important with high velocities and may be considered as corresponding to so many additional

* *Trans.* Am. Soc. C. E., Vol. XXI, pp. 303–482.

feet of straight pipe. In long uniform pipe lines laid with easy curves the friction loss predominates and the minor losses may be neglected. The following table gives approximate values determined by experiments made by the Inspection Department of the Associated Factory Mutual Fire Insurance Companies, and may be used when no exact information is available:

Name of fitting	Number of feet of clean, straight pipe of same size which would cause the same loss as the fitting. (Loss in straight smooth pipe as given by Weston).
2.5-in. to 8-in. long-turn ells..........	4
2.5-in. to 8-in. short-turn ells.........	9
3-in. to 8-in. long-turn tees...........	9
3-in. to 8-in. short-turn tees..........	17
1/8th bend........................	5

634. Freeman's formula for nozzles is given as follows:

$$G = 29.83\ Cd^2\sqrt{\frac{P_e}{1 - C^2\left(\frac{d}{D}\right)^4}} \quad \text{(gal. per min.)} \qquad (121)$$

where G = discharge in gallons per minute, C = coefficient of discharge, d = diameter in inches of the nozzle orifice, D = diameter in inches of the piezometer ring at the base of the nozzle, and P_e = piezometer reading in pounds

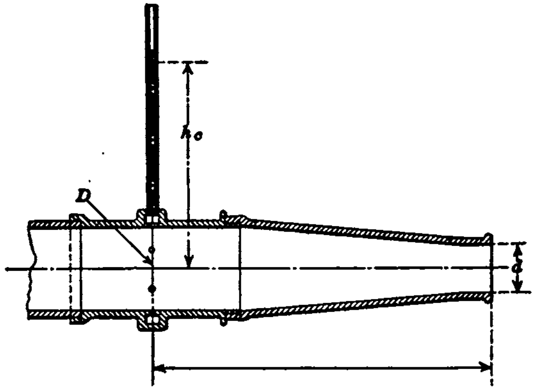


FIG. 55.—Section of nozzle and piezometer ring.

per square inch at the centre of the nozzle orifice. For water-power practice in engineering units the same formula becomes:

$$Q = 6.3Cd^2\sqrt{\frac{h_e}{1 - C^2\left(\frac{d}{D}\right)^4}} \qquad \text{(sec-ft.)} \qquad (122)$$

where Q = discharge in cubic feet per second, C = coefficient of discharge, d = diameter in feet of the nozzle orifice, D = diameter in feet of the piezometer ring at the base of the nozzle, and h_e = piezometer reading in feet of water above the center of the nozzle orifice.

635. Coefficients for nozzles. A value of 0.974 may be taken as a suitable coefficient (C) for the ordinary smooth fire nozzle with a nominal diameter between 0.75 in. and 1.375 in. and under pressures varying from 20 to 100 lb. per sq. in. For pressures less than 20 lb. per sq. in. the same coefficient may be used, but without so great a confidence in the accuracy of the result. For larger sizes of smooth nozzles ranging from 1.75 in. and 2.5 in. Freeman's coefficients lie between 0.987 and 0.999 for pressures ranging from 15 to 55 lb. per sq. in. For smooth nozzles from 2.5 in. up to 6 in. and under heads of more than 10 ft., Freeman suggested a coefficient of 0.995. The following coefficients have been obtained by experiment for a set of specially made smooth **meter nozzles** (Fig. 56): 2.5 in., 0.98; 3 in., 0.985; 4 in., 0.99; 5 in., 0.995. The coefficients will vary somewhat in different sets of nozzles and it is important to know at what point the pressure is registered (Fig. 55). If special accuracy is desired a set of meter nozzles which have been carefully rated by volumetric measurement should be used. With square ring nozzles the jet suffers contraction and the coefficients according to Freeman's experiments range from 0.65 up to 0.975 according to the proportions of the orifice.

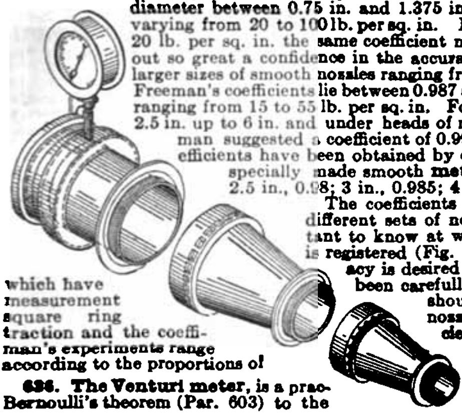


Fig. 56.—Meter nozzle.

636. The Venturi meter, is a practical application of Bernoulli's theorem (Par. 603) to the measurement of water in pipes flowing under pressure. The formula for discharge is

$$Q = \frac{C\pi D_b^2 D_a^2 \sqrt{2gh}}{4\sqrt{D_a^4 - D_b^4}} \quad \text{(sec-ft.)} \qquad (123)$$

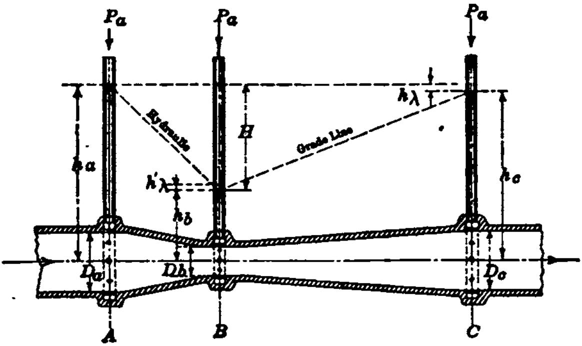


Fig. 57.—Section of Venturi meter.

The symbols will be understood by reference to Fig. 57. For meters with a given ratio of throat and inlet diameters the formula may be simplified. Let the ratio $R = D_a/D_b$ and let

$$K = \frac{\pi R^2}{4}\left(\frac{2g}{R^4 - 1}\right)^{\frac{1}{2}} \qquad (124)$$

Then $$Q = CKD_t^2\sqrt{H} \qquad \text{(cu. ft. per sec.)} \qquad (125)$$

Values of R =	3.0	2.5	2.0
Values of K =	6.338	6.381	6.505

The value of C varies wth the velocity at the throat, the ratio R and the actual dimensions of the meter. As the Venturi meters* are ordinarily constructed the coefficient C is between 0.97 and 1.03. These meters may be rated backward and a coefficient of correction found to apply to the manometer reading (Fig. 58). The total loss of head through the meter tube is relatively unimportant compared with other types of commercial meters and is so small that the insertion of the meter in the mains of a water supply is rarely objectionable, and the use of this appliance is growing constantly. It may be used particularly well in high-pressure power plants where a continuous reading may be obtained by the recording apparatus; also to measure boiler feed and for measuring liquids heavier and lighter than water. Experience has shown that with the register, continuous measurements have been made with an error not exceeding 3 per cent.

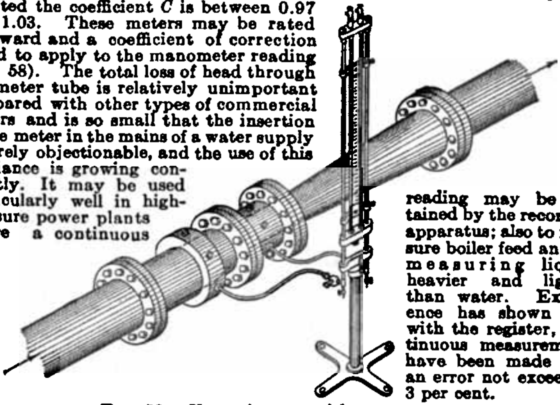


Fig. 58.—Venturi meter with manometer.

637. The Pitometer† (Figs. 59 and 60), by means of the differential gage registers the difference in pressure on the two orifices, one pointed directly against the current, the other in the exactly opposite direction. This difference in pressure is not a measure of the velocity head directly, but is greater than the velocity head. For these instruments the coefficient (K) of correction has been found to be nearly a constant, and equal to 0.84. The formula for velocity therefore becomes

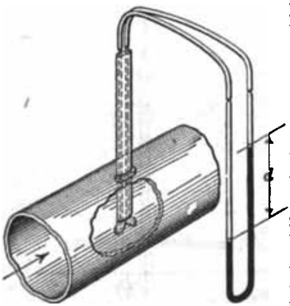


Fig. 59.—Principle of pitometer.

$$V = K[2g(s'-1)d]^{\frac{1}{2}} \quad \text{(ft. per sec.)} \qquad (126)$$

where s' = specific gravity of the heavier liquid, d = the deflection (Fig. 61) in feet, V the velocity in feet per second, and K the Pitometer coefficient. The liquid used in the differential gage is usually carbon tetrachloride and, gasoline, colored red. The specific gravity as put up for use is usually either 1.25 or 1.50 (for very accurate work the specific gravity should be determined for the temperature at which it is used). The formula (Eq. 126) then reduces to:

$$\text{For } s' = 1.25;\ V = 0.84\frac{\sqrt{2gd}}{\sqrt{4}} = 3.368d^{\frac{1}{2}} \quad \text{(ft. per sec.)} \qquad (127)$$

$$\text{For } s' = 1.50;\ V = 0.84\frac{\sqrt{2gd}}{\sqrt{2}} = 5.671d^{\frac{1}{2}} \quad \text{(ft. per sec.)} \qquad (128)$$

* Made by the Builders Iron Foundry, Providence, R. I., in standard sizes for pipe lines from 2 to 60 in. in diameter, and much larger sizes are in use; they may be furnished with either direct reading or recording apparatus.

† Developed by John A. Cole and Edward S. Cole and owned by the Pitometer Co., N. Y.

To determine the discharge the cross-section of the pipe must be divided into one or more known areas (Fig. 61), the velocity for each found and hence the mean velocity for the entire cross-section; the discharge being simply the product of the area and the mean velocity. For pipes the following table (Par. 639) is useful and the example (Par. 641) illustrates how the discharge is calculated. The Pitometer is set so that the velocity is determined for the points as shown in Fig. 61. The pipe coefficient (see example) being first determined, it is subsequently necessary only to get a reading for the centre of the pipe and apply the coefficient to the centre velocity to obtain the mean velocity and hence the discharge by the formula:

$$Q = KCV_eA \qquad \text{(cu. ft. per sec.)} \qquad (129)$$

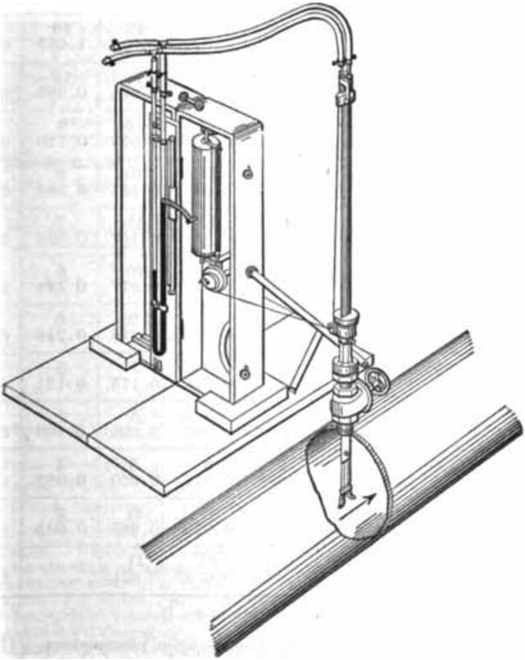


Fig. 60.—Portable pitometer.

638. The principal use of the pitometer is for measuring the flow in water mains where it has proved its great value in detecting leaks and waste, and in measuring the slip of pumping engines. It can be installed and operated for relatively small expense, without interfering with the flow. For large penstocks and in silt-bearing water it is not recommended. When used in old pipes, allowance must be made for incrustations reducing the cross-sectional area and a traverse of several points through more than one diameter is preferable to the single observation and the use of the pipe coefficient.

639. Traverse table for pitometer gagings, showing inner diameter of each ring for ordinary sizes of mains in inches; and giving the area of each ring and centre circle in square feet.

Diameter of pipe (in.)	Area of pipe (sq. ft.)		Ring A	Ring B	Ring C	Ring D	Centre circle E
48	12.566	Diam. Area	44 2.007	36 3.490	28 2.793	16 2.880	16 1.396
42	9.621	Diam. Area	38 1.745	32 2.291	24 2.443	14 2.073	14 1.069
36	7.069	Diam. Area	34 0.764	28 2.029	20 2.094	10 1.637	10 0.545
30	4.909	Diam. Area	28 0.633	24 1.134	18 1.375	12 0.982	12 0.785
24	3.142	Diam. Area	22 0.502	18 0.873	14 0.698	8 0.720	8 0.349
20	2.182	Diam. Area	18 0.415	14 0.698	10 0.524	6 0.349	6 0.196
18	1.767	Diam. Area	17 0.191	15 0.349	11 0.567	7 0.393	7 0.267
16	1.396	Diam. Area	15 0.169	13 0.305	10 0.377	6 0.349	6 0.196
14	1.069	Diam. Area	13 0.147	11 0.262	9 0.218	6 0.246	6 0.196
12	0.785	Diam. Area	11 0.125	9 0.218	7 0.175	5 0.131	5 0.136
10	0.545	Diam. Area	9 0.103	8 0.093	6 0.153	4 0.109	4 0.087
8	0.349	Diam. Area	7 0.082	6 0.071	5 0.060	3 0.087	3 0.049
6	0.196	Diam. Area	5½ 0.031	4½ 0.055	3½ 0.043	2 0.045	2 0.022
4	0.0873	Diam. Area	3½ 0.0205	2½ 0.0327	1½ 0.0218	....	1½ 0.0123

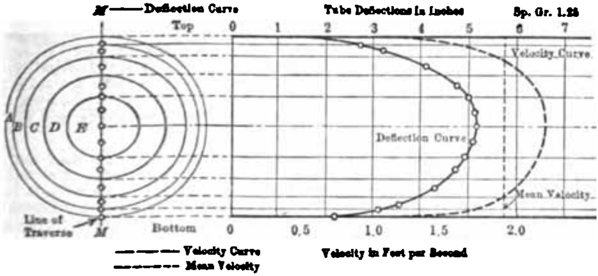
NOTE.—Diameter given in inches; area in sq. ft.

640. Piezometers. -"Mills' experiments upon piezometers. Hiram F. Mills* published in 1878 the results of some six thousand observations made with extraordinary accuracy to determine the proper form of piezometer orifice. With twenty-two openings varied in shape and direction and a range of velocities from 0.6 to 8.9 ft. per sec., he found that with an orifice whose edges are in the plane of the side of the channel and passage normal thereto, the piezometer column will stand neither above nor below the surface of the stream, but will indicate the true height of the water surface in an open channel or the pressure in a closed channel; but if the passage inclines either upstream or downstream, or the edges of the orifice project

* Chief Engr. Essex Co., Lawrence, Mass. *Trans.* Am. Academy of Science, 1878.

beyond the plane of the side, the true height of the surface or the true pressure will not be indicated."*

641. Example of use of pitometer. The following diagram (Fig. 61) and computations illustrate the usual method of computing discharge.



Cross-section of Pipe **Longitudinal Section on M-M.**

FIG. 61.—Velocity curve as determined by pitometer readings.

Deflection and velocity curves from 24-in. supply main

Ring	Ring areas (sq. ft.)	Ring velocity (ft. per sec.)	Ring volumes of flow (cu. ft. per sec.)
A..........................	0.502	1.57	0.788
B..........................	0.873	1.87	1.632
C..........................	0.698	2.05	1.431
D..........................	0.720	2.18	1.570
E..........................	0.349	2.24	0.781
Total discharge of pipe......	3.142		6.202

$$\frac{\text{Discharge of pipe}}{\text{Area of pipe}} = \frac{6.202}{3.142} = 1.97 \text{ ft. per sec., mean velocity.}$$

$$\frac{\text{Mean velocity}}{\text{Centre velocity}} = \frac{V}{V_c}\,\frac{1.97}{2.24} = 0.879 = \text{pipe coefficient.}$$

642. Floats. Under favorable circumstances one of the simplest methods of measuring the flow of water is by means of velocity determinations with floats. The different kinds of floats are surface, subsurface, or combinations of the two, and rod floats. **Surface floats,** even on a perfectly quiet day, give only the surface velocity and with any wind blowing the velocity cannot be measured with accuracy. **Subsurface floats** are but slightly heavier than water and are easily caught by eddies and cross currents which move them about in an undeterminable path and make the results unreliable. **Twin floats** have been used, but their place is better filled by rod floats.

643. Rod floats are cylinders usually made of metal tubing loaded with lead at the bottom so that they will float vertically with about 6 in. extending above the surface of the water. They are best adapted for use in power canals with straight, smooth sides and level bottom so that rods may be used reaching nearly to the bottom without danger of bumping. Under such conditions rod floats have been used continuously on a large scale for a great many years. The method of measurement is direct. The

* Hughes & Safford, "Hydraulics," New York, MacMillan Co., 1911, page 104.

following paragraphs are quoted from Hughes and Safford's Hydraulics (MacMillan Company):

"Procedure in measuring velocity. A straight stretch of stream should be selected as a place for gaging, and two cross-sections selected to mark the beginning and the end of the area. The float should be placed quietly in the stream at such a distance upstream from the upper of the two cross-sections, that it will be running with the current before the first marker is reached. The time of passage between these two sections, of which the distance apart is known, should be noted with a stop watch; or the position of each float at successive intervals of time be located by engineers' transits or sextants by intersection, and the points plotted on a scale drawing, from which the distance traveled in the observed interval of time can be computed. The distance in feet divided by the time of each run in seconds will give the velocity of the float in ft. per sec."

644. "Application of rod float measurements (Hughes and Safford). The sphere of usefulness of rod-float measurements is somewhat limited, and the expense of making them is relatively great. Their regular use in the future will probably be limited to straight, deep canals or flumes where a high degree of accuracy is required, where a sufficient force of men is regularly employed for this and other purposes, and where it is very necessary to gage all the water used for power and other purposes, without interfering with the operation of the mills. Ordinarily, the difficulty of getting good results from the sum of individual measurements, or readings of water wheels, is due to the fact that the total discharge, which is simply the sum of the individual water wheels, often does not include the leakage or the water used for manufacturing purposes other than power; but the flume measurements of the total quantity passing to each mill will cover everything. There is very little opportunity to make such measurements in rivers or canals which do not have a regular cross-section; and for such conditions there is no question that measurements by current meter (Par. **662** to **665**) will take the place of those formerly made by rod floats. The most notable published gagings by rod floats are those by Humphreys and Abbott of the Mississippi River, those described by James B. Francis in the Lowell Hydraulic Experiments, Darcy and Bazin's gagings, and the gagings of certain rivers in India."

645. "Limits of accuracy in use of rod floats (Hughes and Safford). With a straight, smooth flume of great depth, and velocities ranging from 2 to 5 ft. per sec., quantities of water from a few hundred to 4,000 cu. ft. per sec. have been repeatedly measured with a probable error of 1 to 2 per cent. This form of measurement, which in its successive steps gives the product of the cross-section and the velocity of the water as indicated by the rod floats, is a perfectly natural one; and its simplicity appeals to the non-technical man."

Rod floats cannot be run with the lower end closer than 2 or more in. from the bottom. Some correction must be made to account for this slower layer of water which does not act on the float. The following formula was derived by James B. Francis by comparing rod float measurements with a weir:

$$C = 1 - 0.116\,(\sqrt{D} - 0.1) \qquad (130)$$

where C = a coefficient of correction which multiplied by the observed velocity will give the corrected velocity, and D = difference between the depth of water in the flume and length of immersed part of tube, divided by depth of water.

Values of the Coefficient C

D	0	1	2	3	4	5	6	7	8	9
0.0	.....	1.000	0.995	0.992	0.988	0.986	0.983	0.981	0.979	0.977
0.1	0.975	0.973	0.971	0.970	0.968	0.967	0.965	0.964	0.962	0.961
0.2	0.960	0.958	0.957	0.956	0.955	0.954	0.952	0.951	0.950	0.949
0.3	0.948	.....	.....	.....	.....	.....	.....	.....	.....	.....

Direct interpolation may be made for intermediate values.

STREAM FLOW

646. The local variation of the average annual rainfall in the United States is all the way from nothing in some years in the desert regions, to an

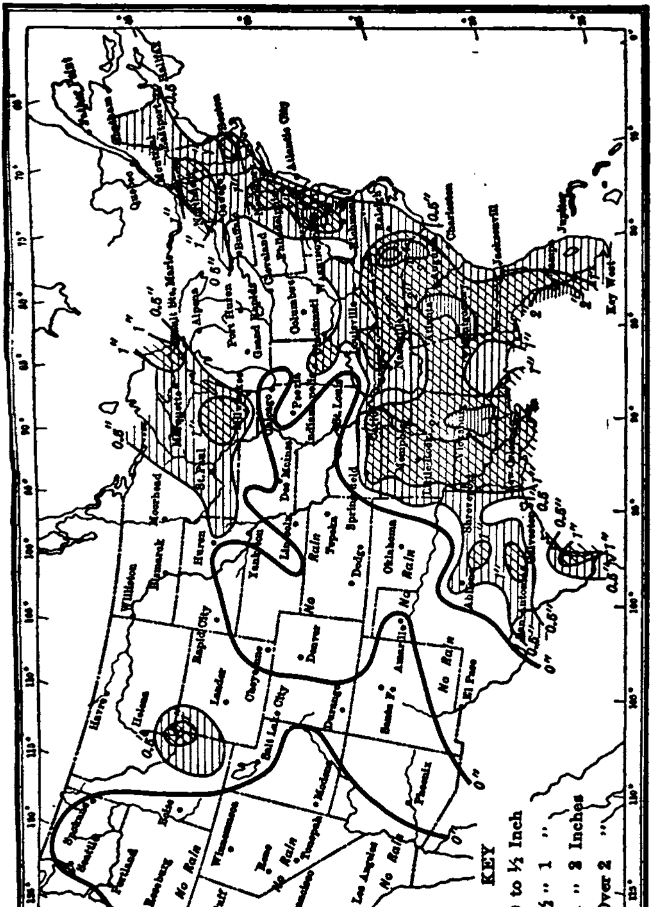


FIG. 62.—Map of United States showing distribution of rainfall.

occasional maximum of more than 100 in. in the mountains of the extreme northwest. An examination of the map in Fig. 62, will give an idea of the geographical distribution of rainfall over the United States as affected by altitude, climate, proximity to the ocean, etc. In general mountain ranges receive a greater amount of precipitation than the surrounding valleys and yield a greater proportionate run-off.

647. Source of run-off. Rain or snow falling on the drainage areas of rivers is the immediate source of water power. The relation between the rainfall and the run-off is a direct one; rain falls on the ground and a part of it finds its way to the water courses, and yet there are so many physical factors affecting the disposal of rainfall that to establish the relation between it and run-off is baffling if not actually impossible. The cycle of rainfall and run-off is complete when we include the evaporation of moisture in the form of vapor from water and ground surfaces, to fall later as rain.

648. Rainfall records. The U. S. Weather Bureau has kept rainfall records at stations geographically widely distributed for a long period of time and it frequently occurs that good rainfall records are at hand when no stream gagings can be obtained. For this reason, any relations, even only approximately fixed, between rainfall and run-off are of considerable value to the water-power engineer.

649. The main factors affecting the disposal of rainfall are the topography, geology, evaporation and the amount taken up by vegetation. Rugged drainage areas with steep slopes discharge the water readily and quickly into the water courses. Rivers draining such areas, unless there is good storage in lakes and ponds, are more subject to sudden fluctuations in flow than rivers draining flat or rolling country where there are many swamps and where the water has chance to sink into the pervious soil. Evaporation, according to our best present knowledge, is essentially the same from year to year in any one locality and likewise the amount of moisture taken by the vegetation is about a constant.

650. Run-off is the result of rain falling on the contributing drainage area and is subject to the same conflicting influences, with an additional factor of great importance, which is the degree of water-storage development on the river. We can measure and know the rain at any one or a number of points and form a more or less close estimate of the total rainfall over the whole area. We can measure and know the evaporation from water surfaces with fair precision. For the rest it is mainly conjecture, though we can count on vegetation to take a fairly constant amount of moisture from year to year and over large sections of the country. In a year of small precipitation, the evaporation will be at least normal, vegetation will require as much as usual and take all it can get, with the combined result that the run-off of the rivers will get a smaller proportion of the total rainfall than in years of average or excess precipitation. For these reasons, it is very difficult to use any percentage ratio between run-off and precipitation.

In the case of water powers already established where run-off records are kept, a knowledge of the local relation between rainfall on the contributing area and the flow may be very valuable from an operating point of view. Of course if there are gaging stations up river these give more direct and reliable information. It is only in the absence of gaging stations that estimates based on the amount of rainfall need be resorted to, and then must be used with the greatest care.

651. Distribution of run-off. For comparative purposes it is convenient and advantageous to express the flow of rivers not by total flow, but in cu. ft. per sec. per square mile of drainage area. The following is a list of the major factors influencing the distribution of run-off:

(a) Geology—is the soil pervious or impervious (Par. 652).

(b) Topography—is the drainage area flat or rolling, or has it steep precipitous slopes (Par. 653).

(c) Size of the contributing area (Par. 654).

(d) Climate—amount and distribution of rainfall and evaporation (Par. 655).

(e) Storage—whether natural or developed—the degree of development and the location of the storage basins (Par. 656).

These cannot be said to be of equal importance and the relative effect will differ on different areas. It will be seen that the fifth factor only can be altered by man, and looked at in this light it assumes an importance of greater moment than the rest. An even distribution of the run-off is a most desirable characteristic. Taking up in order the main factors listed above we shall see how each one affects the distribution.

652. Geology. Pervious soil will allow the water falling on it to percolate down into it, where it is held as ground water, a part to find its way into the streams when the general water table is lowered. If the soil is impervious water must run off quickly and find its way to the main water courses. Thus heavy storms of short duration falling on frozen or saturated ground will cause sudden rises in the stream and much water may be wasted.

653. Topography. Flat or rolling country combined with a pervious soil is admirably fitted to hold the rain in storage as ground water. The precipitous, rocky slopes are equally fitted to discharge the rain almost as it falls and do nothing to even up the distribution of run-off. But if complete or nearly complete water storage is provided, a precipitous drainage area may be better for water-power purposes than more level country. Between the two conditions are a multiplicity of possible combinations and effects.

654. Size of drainage area has an additional influence when there is little or no storage available. This is because the variation in the extent and intensity of rain storms is extreme. A local storm of sufficient intensity to cause a freshet on a small area, occurring on a drainage area of a thousand square miles or more would hardly be felt on the main river. It is for this reason that without storage small rivers show much greater divergence between maximum and minimum discharge. This would to some extent be offset by a complete forest cover, which on the small stream would undoubtedly delay the peak of the flood and might entirely prevent a sudden rise. It can never, however, hold the water in storage like a reservoir, to let it down at certain seasons of the year. Furthermore on drainage areas of considerable size sufficient areas usually cannot be deforested or grow up fast enough to have an effect more than local.

655. Climate. The climate prevailing on a drainage area will depend on its geographical location and on its topography. It is sufficient here to refer to but two aspects: (*a*) the total annual precipitation and its distribution; (*b*) the total evaporation (Figs. 65 and 68) and its distribution. There is such radical variation in different parts of the country that no general discussion will be attempted here. Every case must receive careful study to ascertain the local conditions and tendencies. A map of the United States is given in Fig. 62 showing the distribution of the average annual precipitation over the country.

656. Storage may exist on streams and rivers naturally in the form of swamps and lakes, or may have been developed by damming up streams at favorable points, or in raising lakes already in existence. The degree of development of the storage on a drainage area is important. If underdeveloped there will be a waste of water during the periods of high water which, if held in storage for future use, would greatly enhance the value of the reservoir. The storage on a drainage area may be said to be completely developed when the reservoir fills once in each normal year without much waste, and is of sufficient capacity to carry over a series of two or three dry years with undiminished yield, but without filling entirely; of course such cases are rare. Two examples in New England are the Winnipiseogee river which drains the lake of that name in New Hampshire, and the Presumpscott river flowing out of Sebago lake in Maine. Lake Winnipiseogee (Par. 657) has an area of approximately 70 square miles with a drainage area exclusive of the lake surface of about 300 square miles. The steady yield of this lake with rare exceptions is from 1.5 to 2 cu. ft. per sec. per square mile. Sebago lake has a surface area of about 50 square miles and a contributing area of about 4?? square miles. The flow at the outlet of the lake rarely falls below 1.6 cu. ft. per sec. per square mile on week days.

Perhaps the most **advantageous location** for storage reservoirs, if not too large for their contributing areas, is near the head waters of a river, for in that way all powers are benefited. The most economical **method of handling** the storage will vary in different cases; usually the water ie let down quite uniformly throughout the year. There are cases, however, where the best way is to keep the storage gates closed most of the year and then draw heavily during the dry season. This method is the desirable one to follow when the total area of the river basin is large, the water storage great, and the area contributing to the reservoir a small part of the whole. The whole object of storage is to keep up the flow of the river as well as possible in order to make as high as may be the permanent dependable power. Every reservoir and mill pond on any stream must be operated subject to the legal rights of others on the stream. In any particular case the yield of a reservoir may be predicted from a study of the run-off of the contributing area, the precipitation on and the evaporation from the water surface. Trial drafts from the storage are assumed and the effect traced through a number of years. In this way very safe estimates can be made of the storage yield.

657. Example of a large storage basin. The following figures show by way of illustration the condition of Lake Winnipiseogee (Par. 656) on May 1, Aug. 1, Nov. 1, and Feb. 1, using 1911 and 1912 figures of run-off and precipitation, and average figures of evaporation. The yield of the contributing area is expressed in inches of depth on the lake area, rainfall and evaporation are given directly in inches and the draft, 600 cu. ft. per sec. throughout the year, is also reduced to in. of depth on the lake area for each month.

Date	Yield of the contributing area expressed in in. of depth on the lake area for the month	Rainfall in in. during the month	Evaporation in in. during the month	Draft 600 cu. ft. per sec. expressed in in. of depth on the lake area for the month	Net gain or loss in in. of depth on the lake area for the month	Elevation of the lake*
April 1, 1911 During April May 1, 1911	+38.35	+1.38	−1.6	−9.56	+28.57	27.57 Wasting
July 1, 1911 During July August 1, 1911	+ 1.83	+4.36	−4.32	−9.88	− 8.01	38.09 30.08
October 1, 1911 During October November 1, 1911	+ 9.44	+4.51	−2.20	−9.88	+ 1.87	15.49 17.36
January 1, 1912 During January February 1, 1912	+11.51	+3.12	−0.70	−9.88	+ 4.05	17.21 21.26

* 44 in. represents full lake.

658. River gaging stations are maintained by the U. S. Government and the State Governments on many rivers and streams throughout the country. This work of gaging the rivers is carried on by the U. S. Geological Survey, Dept. of the Interior, and the observations and results are published. These publications, which are invaluable in studying the possible water-power development of our streams, may be obtained at nominal expense from the Supt. of Documents, Government Printing Office, Washington, D. C.

659. Flow in open streams is subject to the same laws as the flow in any other channel (Par. 602), but because of the irregularity of section and the changing character of the bed from point to point the difficulties of finding a formula to fit all conditions are manifold. The only reliable data concerning discharge of a stream are based on actual measurements.

660. Measurement of stream flow. The method adopted will depend largely on the size of the stream, its cross-section and the variation between minimum and maximum flow. A weir can be successfully used in a small stream discharging a few cu. ft. per sec. and is largely employed in irrigation ditches. For rivers from medium to large size the most adaptable method is to employ a current meter and by making measurements at several stages of flow establish a **rating curve** by means of which it is only necessary to read a gage, compare the reading with the rating curve and find the discharge corresponding to the given stage. The same rating curve cannot be used for both winter and summer conditions.

661. Typical hydrographs showing the monthly average stream flow in sec-ft. per square mile of drainage area are shown in Figs. 63 and 66, for the Kansas (Kan.) and Columbia (Ore.) rivers respectively. Figs. 64 and 67 show the accompanying monthly rainfall in typical years. Figs. 65 and 68 show the accompanying monthly evaporation. These stream-flow records or hydrographs naturally exhibit different characteristics according to the local climatic and other conditions, as noted by comparing Fig. 63 with Fig. 66, and Fig. 64 with Fig. 67. Furthermore, the hydrographs of the same river vary materially as a rule from year to year, between rather wide limits; Figs. 63 and 66, besides showing the stream flow for typical years, also show the maximum (wet year) and minimum (dry year) flow observed during a series of years. It is also instructive to compare the records of rainfall with those of stream flow; compare Figs. 63 and 64, and Figs. 66 and 67. Similar records and charts should be obtained for any particular river or stream, if possible, in connection with consideration of its water-power possibilities.

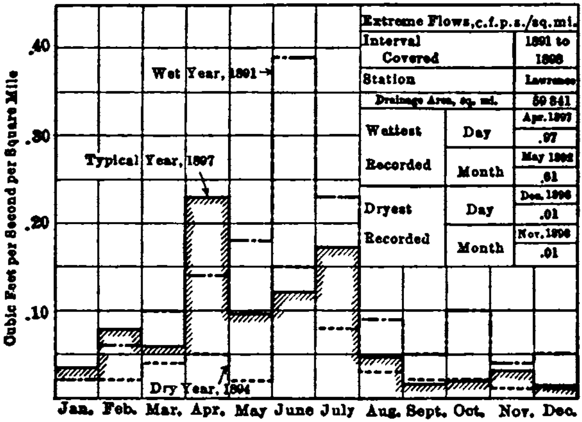


FIG. 63.—Hydrograph of Kansas River from records at Lawrence, Kansas.

662. Current meters may be roughly divided into two classes, those in which the wheel revolves about a vertical axis and those in which the axis is horizontal.

The Price-Gurley meter (Fig. 69) belongs to the first class and is the type adopted by the government in its stream gaging work. It is fitted with a vane to keep it headed into the current and with an electric sounder to enable the operator to record the revolutions without taking the meter from the water. This meter can be slung from a cable and operated from the shore or from a bridge, or from a car suspended from a cable. In shallow water the meter is attached to a rod and operated directly by hand. The disadvantage inherent in this meter is that it always records the maximum velocity, whether that velocity is in a forward direction or is merely caused by cross currents and eddies; all alike are recorded as forward. Consequently this meter has a tendency to give results which are too high.

The Haskell meter (Fig. 70) is of the screw-propeller type and operated in the same manner as the Price-Gurley meter. It is subject to the same criticism in that it records the maximum velocity at the point of immersion regardless of its direction.

The Fteley-Stearns meter (Fig. 71) was designed for use in the Sudbury Aqueduct of the Boston Water Works and is especially adapted for use in flumes or any channels where it is desired to get the velocity close to the sides and bottom. It is ordinarily fitted for use on a rod and is equipped with recording dials thrown in and out of mesh by a cord. The meter can also be fitted up with vane and electric sounder. It then loses its peculiar value in recording only the forward component of the velocity of the water, which makes it a true integrating instrument. It has been successfully used in large rivers and in velocities from 0.4 to 10 ft. per sec.

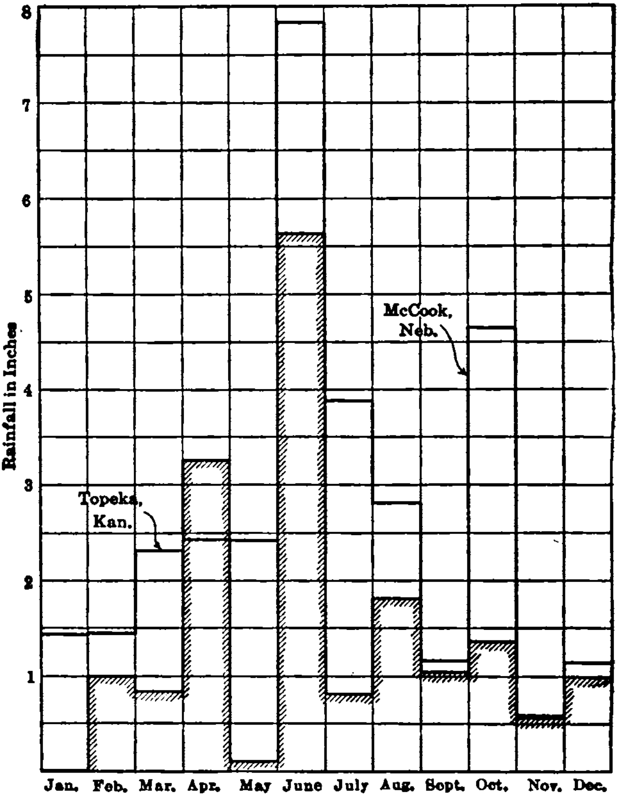


Fig. 64.—Monthly rainfall in Kansas River valley, 1897.

663. Water wheels as current meters. Water wheels if rated to determine their discharge with given head, gate opening, and speed, are in common use as water meters in power plants. If occasionally checked by rod float or current meter measurements they are quite accurate enough to base the use or sale of water on the quantities so determined. The Holyoke test (Par. 706) of a wheel gives the necessary information from which a discharge rating curve may be constructed, but it is more satisfactory if in addition to this, careful measurements are made with the wheels in place.

664. Methods of using current meters are as follows:
(a) Single point at 0.6 depth.
(b) Two points at 0.2 and 0.8 the depth.
(c) Multiple point.
(d) Integrating: (1) vertically; (2) diagonally.

The single point method at 0.6 depth (a) is supposed to give the average velocity of the vertical section in which it is held. Except in special cases such as natural streams uncontrolled by local conditions this can rarely be relied upon.

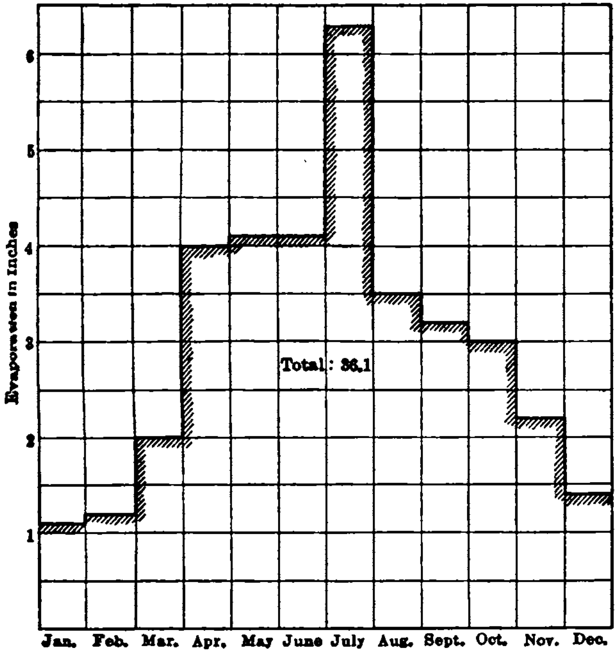


FIG. 65.—Monthly evaporation of Topeka, Kan., July, 1887–June, 1888, computed under the direction of T. Russell, U. S. Mo. Weather Review, Sept., 1888.

The average of the two velocities at 0.2 and 0.8 depth (b) is supposed to give the mean velocity of the vertical section in which it is held. The same comment applies.

These two methods have been advocated by the government service for quick measurements but are not recommended for accurate work, without actual determinations of velocity distribution in the stream cross-section.

By dividing the cross-section into several imaginary vertical strips of width depending on the total width of the stream and the uniformity of flow, and measuring the velocity at several points in each vertical strip the true mean velocity can be closely approximated. This method (c) is recommended for use in power canals where the flow is fairly steady so that during the time consumed in making the measurement there is no material change. It is

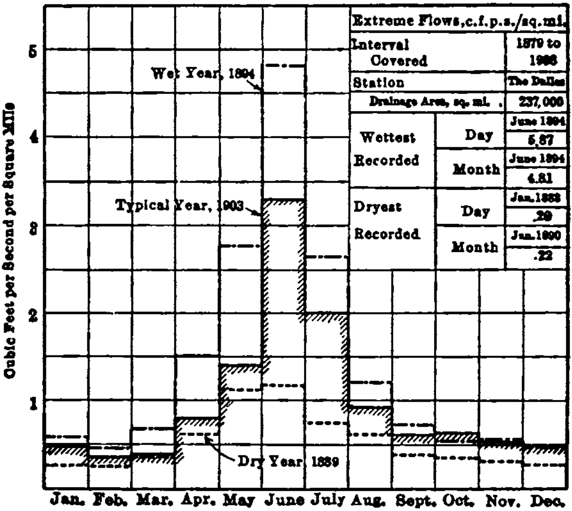


FIG. 66.—Hydrograph of Columbia River from records of The Dalles, Oregon.

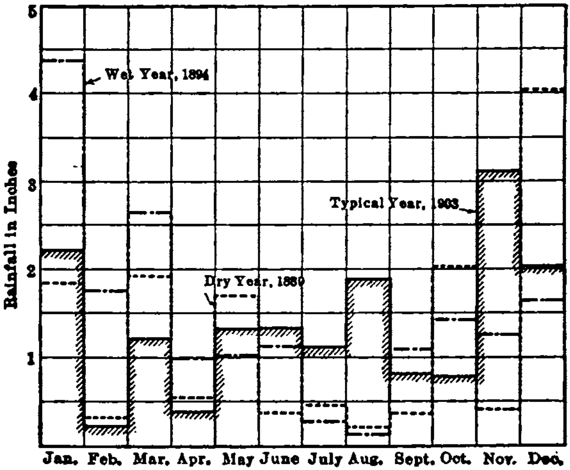


FIG. 67.—Monthly rainfall at Spokane, Wash.

especially useful in showing the distribution of the velocity over the cross-section.

Vertical integrating measurements (*d*) (1) are made by slowly lowering the meter from top to bottom and raising it again along the same vertical path. In this way the average velocity for the strips in order is obtained mechanic-

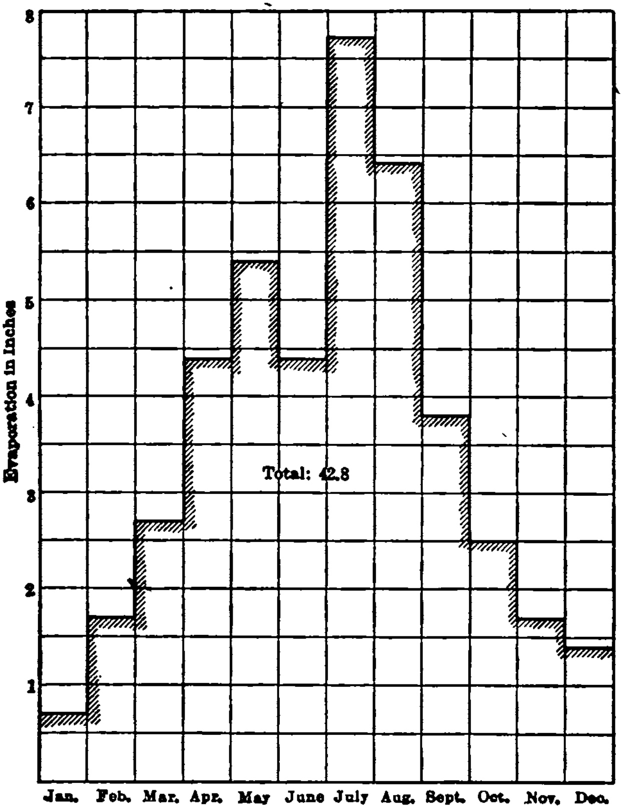


FIG. 68.—Monthly evaporation at Spokane, Wash., July, 1887–June, 1888, computed under the direction of T. Russell, U. S. Mo. Weather Review, Sept., 1888.

ally. In diagonal integration (*d*) (2) the meter is moved slowly across the stream at the same time it is being lowered and raised. By this method the mean velocity for the entire cross-section of the stream is obtained in one continuous operation.

665. Choice of method. The width of the channel, the conditions of flow and the degree of accuracy desired are important considerations in selecting the method to be used. By dividing the stream cross-section by verticals into areas of equal width, through the centre line of which the meter is lowered and raised with uniform speed, the mean velocity in ft. per sec.

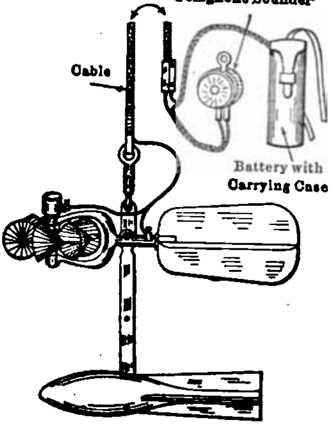


Fig. 69.—Price-Gurley electric current meter.

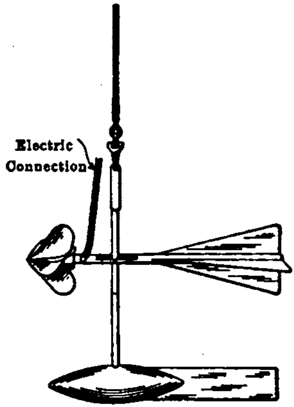


Fig. 70.—Haskell current meter.

for each section is obtained in one operation. This method is on the whole to be preferred for determinations of discharge. The multiple-point method is of great value in determining the distribution of velocity in any cross-section, which is often necessary for at least one gaging; but for determination of discharge this method is tedious and if the stream is subject to fluctuation in stage it is too slow.

If the channel is not too wide or deep the diagonal integration which covers the whole stream in one operation is a very satisfactory form of measurement because it permits a great many measurements under different conditions and avoids many disturbing conditions. Its use is limited by the endurance of the operator. The double-point and single-point methods are only rough approximations, but serve a useful purpose in securing information of stream flow at low cost.

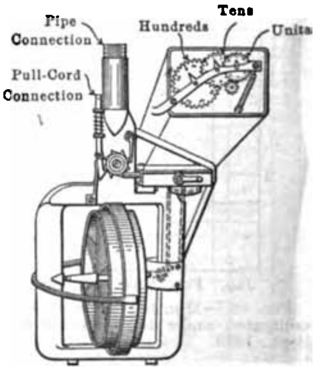


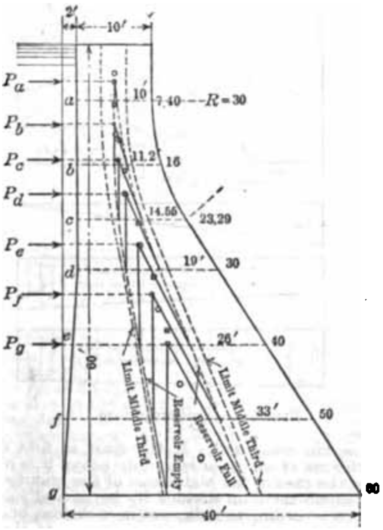
Fig. 71.—Improved Fteley-Stearns current meter.

666. Effect of ice. Ice cover on a stream or canal decreases the cross-section, and increases the wetted perimeter or rubbing surface. Both of these result in additional loss of head, the first through increasing the velocity, the second through increasing the frictional resistance to flow. In power canals the formation of an ice cover of sufficient thickness to last through the cold season is a protection against troubles from anchor ice and frazil ice. Neither anchor ice nor frazil ice will form under an already existing ice cover; the latter is therefore desirable and canals should be designed with this in mind, and the velocities should be kept down (2 to 3 ft. per sec.) by making the cross-section ample for extreme conditions. Anchor ice is ice which forms on the bottom of the stream or canal and may seriously obstruct the flow. Frasil or needle ice forms, as the name implies, in small needles where the current is too rapid for the formation of surface ice. It may either attach itself to the bottom or to anchor ice already formed or it may be carried against and effectually clog the racks of a power house.

DAMS AND HEADWORKS

667. Water storage has already been referred to in Par. 656 and a method outlined for testing out the dependable constant yield. The value of a storage reservoir increases rapidly as the capacity per square mile of contributing area increases up to 15,000,000 cu. ft.; beyond that point there is likely to be a slower rate of increased yield for a given increase in capacity per square mile of drainage area. 75,000,000 cu. ft. per square mile might be an overdevelopment of storage.

668. Pondage, as the term is used here, means the water impounded directly by the power dam and is therefore almost always formed by backing up the river itself. The study of pondage and its proper handling is complicated in the general case, but as a rule comparatively simple of solution for any actual power plant. The reason is that it is involved with the load curve, the natural and legal restrictions of the flowage permissible, and flow detained, and the variation in head economically advisable. Granting the possibility and rights of pondage, the problem reduces to a consideration of fitting the use of water to the demands for power made on the station. If the station carries a typical combined power and lighting load, pondage is essential to satisfactory operation since the great demand for power is during the day-time and the early evening hours. If the head may be drawn down during the day so that the pond will be just filled up by the night flow, the most that is possible is being obtained from the river.



For effect of upward pressure see Spillway Dam

FIG. 72.—Reservoir dam.

In some cases of low-head and medium-head developments the advisable reduction in head is the controlling or limiting factor. As the head falls the power and efficiency of the water wheels fall if the speed is kept up, as it must be in hydroelectric stations. Foresight and study of the probable range of head variation make it possible to have wheels designed with characteristic curves specially suited to the case and a greater variation of head may be permitted.

669. Dams, regardless of the material composing them, may be roughly classified into two main divisions: (a) Impounding dams for holding water back, as in a reservoir, but which are not designed to have any water pass over them; (b) spillway dams or sections.

670. Impounding dams (Fig. 72) are almost invariably of the gravity type depending on their own weight to resist the forces of the water except in the rare cases where curved dams have been built depending wholly or in part on arch action. The present practice is tending toward heavy earth dams.

671. Spillway dams (Fig. 73) may be further subdivided into gravity types and those depending, through their design, on the hydrostatic pressure of the impounded water to hold them against the river bed, and arch dams. The arch dam is very economical of material but requires that the abutments be founded in strong solid material capable of taking up the thrust.

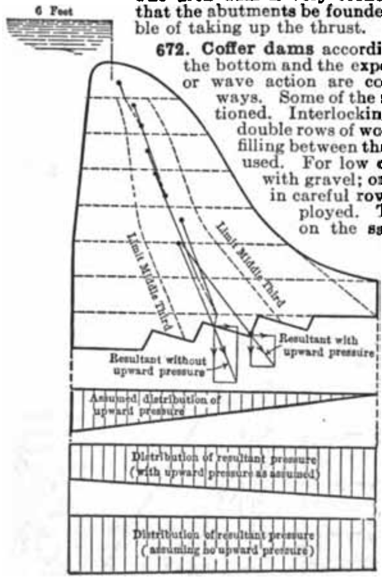


FIG. 73.—Spillway dam.

672. Coffer dams according to their height, the nature of the bottom and the exposure of the location to current or wave action are constructed in a great number of ways. Some of the more common methods are mentioned. Interlocking sheet steel piling, and single or double rows of wooden sheet piling with a puddled filling between the two rows, are forms frequently used. For low cofferdams the A-frame covered with gravel; or bags half filled with sand piled in careful rows have been successfully employed. The design of cofferdams is based on the same principles of hydrostatics governing the design of any other dam.

673. Design of dams. The conditions which should be safely met by all dams are stated briefly as follows: (*a*) the dam must not slide; (*b*) the dam must not overturn; (*c*) the dam must not crush. The external forces acting are the water pressure, the weight of the material composing the dam, the reaction of the foundation, ice pressure near the top, and wind and wave pressure. The pressure usually figured is that of the greatest known freshet, on the toe of the deepest section, without backwater. The cross-section of the dam is determined by trial. A tentative section is assumed and then all the forces are computed. If the dam as first designed cannot satisfy all the conditions of safety or is unduly heavy it is modified accordingly.

In the case of the high dams of the gravity type (Fig. 72) it is customary to assume the dam divided by horizontal joints, the depth of each section chosen according to judgment in each case, and to treat the sections arbitrarily so selected progressively beginning at the top, testing each for sliding, overturning, and crushing. In this way a resultant line of pressure may be traced. In the design of masonry dams it is customary so to proportion the section that the resultant pressure at all horizontal joints shall fall within the middle third whether the reservoir be full or empty. This gives a factor of safety of two against overturning, and there will be no danger from tensile stresses on the face of the dam.

There is rarely any trouble experienced, when proper construction methods are used, from sliding within the body of the dam itself. The joint between the dam and its foundation is the critical point at which to guard against sliding. It is in this connection that the amount of upward pressure becomes of vital importance. On ledge foundations, steps or trenches should be cut to ensure a sufficient bond. On poorer foundations the dam must be made massive enough to safeguard against sliding by keeping the angle between the resultant pressure on the base and a vertical line, less than the angle of sliding of the foundation material. There is no danger from crushing in dams otherwise stable, except in very high dams which are not treated here.

Dams which depend on the hydrostatic pressure of the water to hold them in place (Fig. 74) may be made much lighter than the solid masonry dams. The principal calculations for strength involve the deck treated as a reinforced slab and of the buttresses as walls subjected to simple compression. Fig. 74 illustrates the action of the resultant pressure. If a floor is used weep holes are provided to allow any seepage to escape unobstructed.

For a more thorough and comprehensive treatment of dams reference should be made to chapter XXIX. "Principles of Construction of Dams" in "Water Power Engineering," D. W. Mead (McGraw-Hill Book Co.) and to "The Design and Construction of Dams," Edward Wegmann (Wiley & Son).

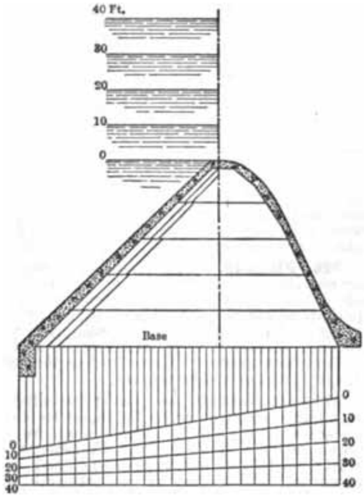


FIG. 74.—Concrete-steel dam.

674. Upward pressure on the base of a gravity-type dam (Fig. 73) should receive most careful study. The limiting conditions are, on the one side, a tight impervious bottom with the dam so set and keyed into it that no leakage can occur under the dam. This in effect assumes an unbroken, water-tight wall extending from below the bed of the river up to the crest of the dam. The other extreme would be a dam founded on loose gravel or porous rock which freely admitted the water under the entire base of the dam, but at the downstream toe a cut-off wall would prevent the water from escaping.

In the first case the assumption is no upward pressure whatever, in the second there would be pressure due to the full hydrostatic head uniformly distributed over the base of the dam. Rarely, if ever, can either of these cases be found in the extreme form suggested. The truth in every case is doubtless between the two. In other words there is probably no dam existing without some upward pressure, its extent to be determined only by a most careful examination of the geological formation of the river bed at the dam site and the thoroughness with which the seepage is cut off by cut-off walls, grouting or other preventive measures. Except in those cases where the dam is founded on ledge rock which gives undoubted evidence of soundness and extent, borings are the essential preliminary to the study and design of a dam. No rule can be given as to the number of borings necessary but in a general way there should be enough to cover the area not only of the spillway section in the river bed, but also of the abutments and wing walls; and other structures near the river; undermining here may prove quite as disastrous as under the spillway section.

In the case of pervious foundations it is desirable to carry a cut-off wall or piling at the upstream toe down to an impervious stratum. Where such does not exist the wall should be carried to such a depth that the friction opposing the flow of water will render the seepage under the dam inappreciable. It is good practice in designing and building dams to eliminate so far as is possible the leakage which causes upward pressure, but to assume in the computations for stability the existence of full upward pressure at the upstream side diminishing uniformly to zero at the downstream toe.

675. The design of canals for power purposes should be made from an economic viewpoint. Canals are employed to utilize a greater head than is obtainable directly at the dam and must be built to give the greatest benefit at the least cost. Head, the very object of the canal, is often unnecessarily lost through poor design of the cross-section and alignment, or through permitting unnecessary roughness of the bottom and sides. Some slope will be required to carry the water through the canal. This should be provided by the canal bottom itself unless the drainage must be in a contrary direction, otherwise an extra loss of head will result, for the slope of the water will reduce the stream cross-section and necessitate a higher velocity to pass a given quantity of water. The form of the cross-section must be determined by its requirements and the character of the material. The form should be such that it will remain as first built and not wash or slide. In general the relatively wide and shallow canals should be avoided because with any drawing down of the head, or with an ice cover the net area suffers a greater proportionate reduction than in the case of deeper, narrower canals. The advantage of the deep canal can also be shown from a consideration of the formulas for flow, which show that for a given area the section with the smallest wetted perimeter will give the best results. The velocity should be kept low and as uniform as possible. If there are changes they should be made gradually and in general a velocity once gotten up should be held. The same statements apply to ditches whether for irrigation or power.

676. Flumes. The word flume implies a narrow waterway with vertical or nearly vertical sides. It was formerly used to designate the canal or penstock conveying water to the wheels at its end. In modern practice a flume is understood to mean an aqueduct of relatively small dimensions constructed usually of timber or concrete employed to carry water long distances. The interior surface is in general smooth and water is carried at a high velocity on an even grade. In computing the carrying capacity of such structures loss by friction predominates.

677. Head-gates are usually installed at the entrance to a canal to control the amount of water let in, or to shut it off entirely in case repairs are necessary in the canal. They should be strong enough to permit the water to be entirely drawn out of the canal and to protect the plant during high water. The number and size is fixed by the quantity of water and such structural limitations as weight and strength may impose. Head-gates are either hand or power operated according to local circumstances; but modern requirements are demanding motor hoists. In designing the piers it is usually well to provide for stop logs for occasional repairs to the gates. Whenever possible the gates should be so set that the water from the river enters at low velocity without sharp contraction or change in direction.

678. Booms either floating or fixed are usual to prevent floating débris from reaching the head-gates, and in northern rivers ice is also diverted over the dam or through special iceways by this means.

679. Penstocks are frequently used instead of canals or in conjunction with them, the choice depending on cost in the case of low-head to medium-head plants. In high-head plants penstocks and pressure tunnels become the only way for conveying water from high elevations down to the power house. Penstocks are usually built of riveted steel plates, wooden staves, or reinforced concrete, depending on local conditions. They must be designed for strength and carrying capacity. Structurally they must be able to withstand the pressure due the head plus the effect of water hammer and also, when empty or partly filled, be strong against collapsing. In long penstocks great care must be taken to provide against **water hammer.** In the first place, the thickness of the shell should be selected with this in mind and, secondly, provision should be made to take care of the surges when they occur. Where the head permits, the most satisfactory protection is given by **stand-pipes** or **surge tanks.** Further protection is given by **relief valves** and **blow-out plates**, usually placed just outside the power house.

The thickness of riveted steel penstocks necessary to withstand water hammer may be computed as follows:

$$t = \frac{(p + p')r}{15000\,e} \qquad \text{(in.)} \qquad (131)$$

where t = thickness of the shell in inches (should never be less than 0.25 inch), p = intensity of internal pressure in pounds per square inch, p' = additional pressure in pounds per square inch allowed for water hammer, r = radius of the penstock in inches, and e = efficiency of the riveted joints which varies from 0.5 to 1.

Values of p' to be used in above formula (Eq. 131)

Nominal diameter of pipe (in.)	p' (lb. per sq. in.)	Nominal diameter of pipe (in.)	p' (lb. per sq. in.)
16 and 18	100	30	80
20	90	36	75
24	85	42 to 60	70

In designing penstocks for carrying capacity, the velocity to be allowed will depend, as in the case of canals, on the value of the head saved by additional expenditure in the original construction. The problem must be studied in the light of this consideration and the size of penstock selected accordingly.

680. Cost data. The cost per developed kilowatt of different hydraulic structures varies very much with the type and character of the development; and is relatively less where the power can be generated near the dam or the output is large by reason of a high head or a large dependable flow. A typical development representing fairly average conditions for a built up fall is:

Item	Per cent. of total cost	Cost per kilowatt on 25,000 kw. equipped
Property	19.3	$24.00
Dam	11.6	14.50
Canal	42.3	52.60
Power house and foundations	9.6	12.00
Equipment	17.2	21.40
Total	100.0	$124.50

Note that the cost of the canal in this case is two-fifths of the entire cost; but this is usually the case unless a portion of the fall is left undeveloped.

Steam relay, to supplement dry-weather flow, will add 30 per cent. more to the total cost.

681. Forebay and racks. The forebay, where water wheels do not draw directly from the canal or penstock, is an enlargement of the canal leading directly to the racks of the power house which is built across the lower end. The power house should be set so that the water flows from the canal and passes through the racks with slight if any change in direction. There is necessarily some loss of head in getting the water through the racks and this should be minimised by setting them so that they draw the water directly, as mentioned. Provision must be made by a waste weir and channel so placed that when open a strong surface current will be created making it easy to float accumulated débris and ice away from the racks and discharge them into the river.

Racks are customarily built up of flat iron bars from 1/4 to 1/2 in. thick and 2 to 3 in. wide, spaced an inch and a half or more apart in the clear, with the flat side parallel to the flow. The spacing will depend on the size of the water wheels and the fineness of the material to be excluded. The rack should be sectionalised into panels which can readily be lifted out for repairs. They should be set at an angle of approximately 30 deg. from the vertical in order to facilitate cleaning with a rake from a platform extending the length of the intakes to the power house. They should be designed of sufficient strength to withstand the water pressure as a dam in case they become completely clogged or frozen up. In cases where every effort is made to save loss of head, rack bars are made with rounded edges or of lenticular shape. There is inevitably some contraction of the streams of water in passing through the racks, which should be allowed for in computing the net area to be procured. The velocity through the racks should be kept low, even when partially clogged. Three feet per second should be the maximum allowed under the very worst conditions.

WATER-WHEELS

682. Hydraulics of water-wheels. Force is required to change the velocity and direction of any moving body. In the case of hydraulic turbine motors water is the moving body and the water-wheel (whether impulse or reaction, or combined) is the agent by which the velocity and direction are changed and useful work derived from the process. The portion of the reduction in velocity not chargeable to friction or other losses occurring during the passage of the water through the buckets is a measure of the efficiency of the wheel as a prime-mover. If a jet of water impinges on a moving vane or bucket, assuming a condition of no friction, the bucket will acquire a velocity equal to that of the jet, *i.e.*, the theoretical velocity due the head acting. The velocity of the wheel must be considered that of the centre of application of all the filaments of water. Under this condition, where the velocity of the bucket and of the jet are the same, the jet can exert no pressure on the bucket and no work is done. Actually the bucket velocity must in any case be decreased by friction from that supposed above, and the water must exert at least a corresponding amount of pressure. If now the velocity of the bucket be further decreased until it is blocked, the water is exerting its maximum pressure but again no work is being done. There is some speed of rotation between these two extremes at which the maximum amount of work will be done; that is, the maximum portion of the velocity of the moving water will be converted into useful work. At this point the motor is running at maximum efficiency. Although the buckets and vanes may be simple or complicated, the above fundamental conception holds true and the performance of the wheel may be analysed by a study of the velocity diagrams at the entrance and the exit of the buckets or vanes. This analysis is indicated in Fig. 75.

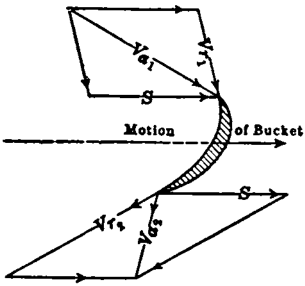


Fig. 75.—Velocity diagram of a water-wheel bucket.

In this diagram S is the peripheral velocity of the bucket, V_{a1} is the absolute velocity of the water at entrance, V_{r1} is the resultant of these two and is the relative velocity of the water at entrance. V_{a2} and V_{r2} are the absolute and relative velocities at exit. It is the object of good design to make V_{r2} such that V_{a2} shall be as small as practicable. But this can be so fixed for only one condition of speed and gate opening. It is this which makes it important to know in advance the conditions under which the wheel is to operate.

683. Types of water-wheels. Modern water-wheels may be classified either as reaction or impulse wheels. While the same fundamental theories hold for each, as pointed out in Par. **682**, the outward form and the method of utilizing the water are radically different in the two types. The **reaction wheel** (turbine) utilizes in the main the pressure of the water and the reactive force on the curved buckets which tend to change its direction. An **impulse wheel** utilizes the velocity and impact of a jet of water directed against buckets on the rim of a wheel. The two types are primarily adapted to widely different conditions: the reaction wheels are best adapted to relatively low heads and large quantities of water; the impulse wheel to high heads and small quantities of water. In recent years reaction wheels have been used for all ranges of head up to 600 or 700 ft., where the units are large. But for heads upwards of 200 ft., with small flow, the impulse wheel of the Pelton type is the natural selection.

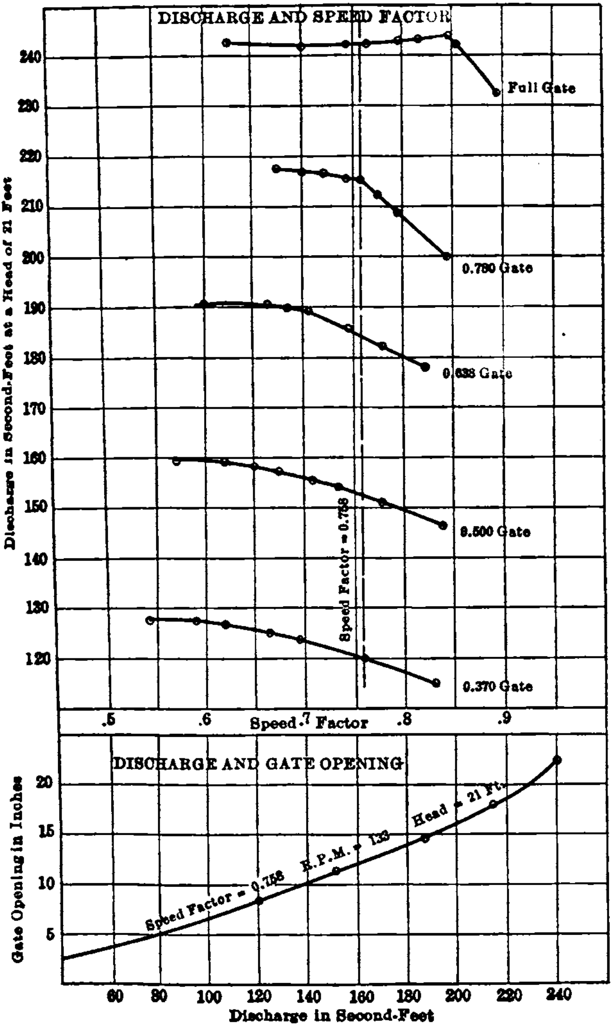


FIG. 76.—Characteristic curves—48-in. turbine.

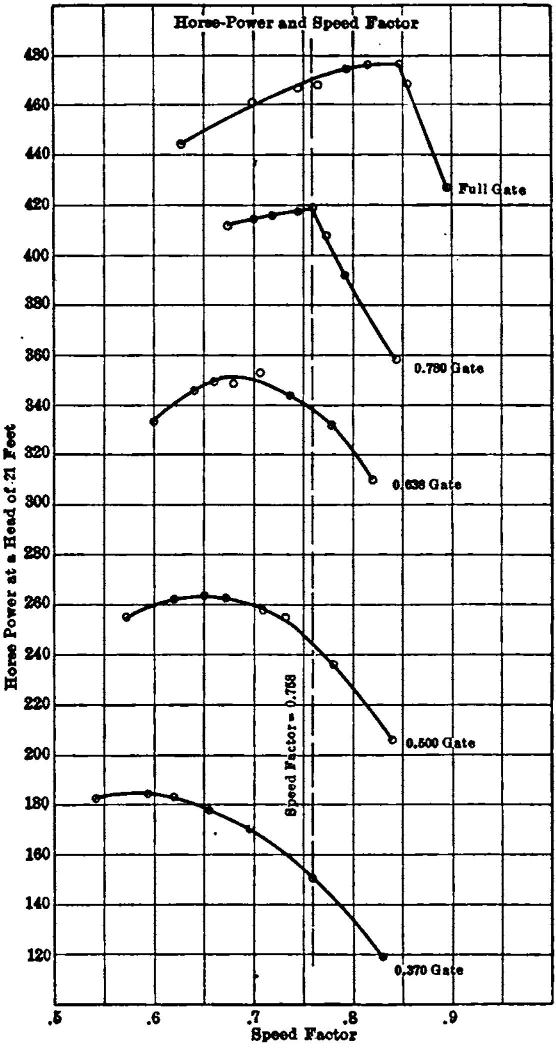


Fig. 77.—Characteristic curves—48-in. turbine.

684. The speed of turbines for a given output and head is confined within quite narrow limits. As noted in Par. 682 there is for any given set of conditions a best speed. This for modern reaction wheels is such that the peripheral velocity is from 0.60 to 0.85 of the theoretical spouting velocity of the water due the head. The best speed may be, and is, varied by design, the recent tendency being toward the development of wheels with high relative velocity; this is easily overdone and is oftentimes secured at the expense of part-gate efficiencies. The best relative bucket velocity in the case of impulse wheels can be shown analytically to be 0.5 and this has been verified by actual tests. Characteristic curves for a modern reaction wheel are given in Figs. 76 to 78 inclusive.

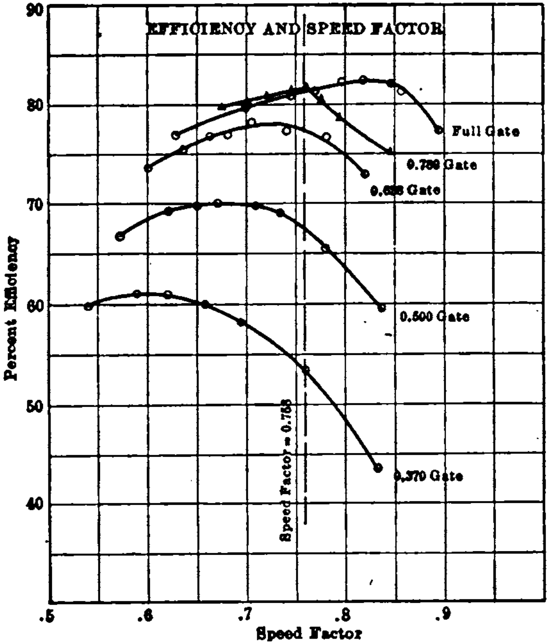


FIG. 78.—Characteristic curves—48-in. turbine.

685. The efficiency of modern water-wheels has been steadily improved by experiment and design until the makers can now guarantee test results of from 80 per cent. to 90 per cent. at the Holyoke Testing Flume (Par. 707). To get the same result in place requires the best design of the setting and waterways. Many wheels which came up to specification in the Holyoke test have failed signally in place to show the power and efficiency of which they were capable. In most of these cases the fault was not in the wheel but in the setting. A wheel improperly installed is not given a fair chance. By carefully tracing the velocity through the penstocks, wheel case, discharge case and draft tube, using net areas throughout, it is possible to design settings which avoid high velocities and undue changes of velocities and direction, and results in place equal to the Holyoke test results may be expected. Efficiency curves for a modern reaction wheel are given in Fig. 78.

686. Water velocity through guide chutes. Mention has already been made of the importance of keeping the velocity of the water through head-gates, canal and racks low. It also should be kept low, not over 3 ft. per sec., as it approaches the guide chutes of the runner itself. There is an exception to this in the case of single runners set in the so-called scroll case. In these, wheel makers are now requiring a velocity of about 15 per cent. of that due to the head acting on the wheel.

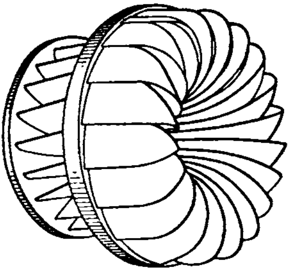


Fig. 79.—Cylinder gate runner.

687. Mechanical details. A cylinder-gate runner is shown in Fig. 79, and a special runner in Fig. 80. The general appearance of a vertical turbine with cylinder gate is shown in Fig. 81, with the usual form of setting in a concrete wheel pit. The same type of wheel with a horizontal setting in a steel flume is indicated in Fig. 82.

688. Settings. There must be equal chance for the water to feed to all parts of the guide circumference. When one or more pairs of wheels on the same shaft are set horizontally, extra care must be taken to insure that the last wheel in the line can draw its share of the water under as favorable conditions as the first wheel. A pair of centre-discharge wheels should be set in the case so that the clear distance between the discharge lips of the two runners shall be at least from 3 to 4 times the diameter of the runners. The greater distance should be used with higher heads; rough limits of heads and corresponding distances are given: 12 ft., 2.5 diam.; 18 ft., 3.0 diam.; 30 ft., 3.5 diam.; 50 ft., 4.0 diam.

Single, large, vertical units have recently received special attention from designers and builders, and it has been demonstrated possible to obtain efficiencies with them about 4 per cent. higher than have been obtained with horizontal settings of equal output. In comparing different settings it is important to know exactly where the head is measured; it is here assumed that the net effective head is taken. See also Par. 711.

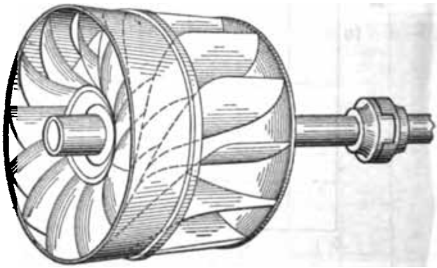


Fig. 80.—Special runner.

689. Ratings. Water-wheels are rated in horse-power, the rating given being usually the maximum output under the head stipulated. It is important to keep in mind three things: (*a*) that a horse-power is a smaller unit than the kilowatt (1 horse-power = 0.746 kw.); (*b*) that water-wheels driving generators must carry the normal load at from 0.75 to 0.80 gate in order to take care of overload on the generators; (*c*) when once under a given head the wheel is discharging its maximum quantity of water no further output can be wrung from it. The attempt to impose a heavier load can only result in reduced speed and output. Simplification would result if water-wheel ratings for hydro-electric plants were stated in kilowatts and generator ratings were stated in terms of the maximum output instead of normal rated output. This would remove the sources of confusion brought about by rating each half of an integral unit by a separate standard.

690. Draft tubes make it possible to set reaction wheels well above tail-water. This is a distinct and important advantage. It makes the wheels accessible for inspection and repair in spite of backwater conditions, and permits the use of horizontal wheels direct-connected to generator or shafting without sacrificing head. In a plant subject to severe backwater, if the machinery floor can be set 20 ft. above low tail-water elevation, draft tubes frequently save great expense in heavy construction and waterproofing of the power house.

A reaction wheel discharging freely into the air would be acted on by a head equal to the distance from headwater elevation to the elevation of the centre of the discharge orifices, minus such losses as occur. The pressure against the discharge is full atmospheric. If now we attach an air-tight pipe of suitable diameter to the runner case and conduct the water down from the wheel and discharge it under the surface of the water in the tailrace we have increased the head acting on the wheel, because now the pressure against the discharge from the wheel buckets is less than atmospheric by an amount corresponding to the vertical distance from buckets to tailrace, where the full atmospheric pressure acting on the water in the tailrace is holding up the column of water in the pipe or draft tube so-called (Fig. 84). A further gain is effected by gradually enlarging the cross-section of the draft tube. This reduces the velocity of discharge and some of the energy present as velocity is transformed into pressure within the draft tube. This increases the column held up by atmospheric pressure acting on the tailwater and correspondingly decreases the pressure against the discharge from the wheel. With good design this may amount to a foot of head. The flare should not be greater than approximately 1 in 10.

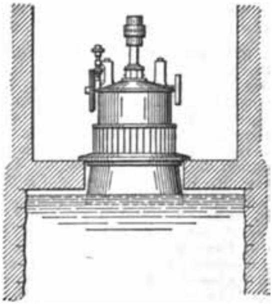


Fig. 81.—General arrangement of vertical turbine with cylinder gate.

The normal atmospheric pressure will support a column of water 34 ft. in height. Draft tubes are seldom used longer than 25 ft. There are structural limitations, inevitable losses and some leakage of air which makes it difficult to maintain to any greater height an unbroken column of solid water, essential to the success of the tube. It has been noted that the pressure within the draft tube is less than atmospheric. This necessitates an air-tight design suited to withstand collapsing pressure.

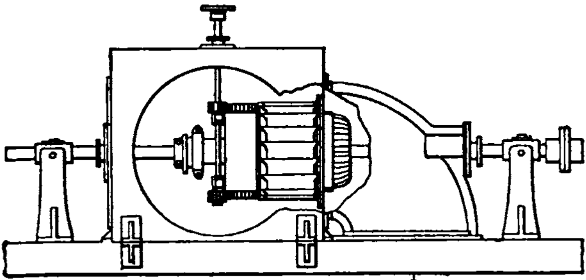


Fig. 82.—Cylinder gate turbine, horizontal type in steel flume.

The velocity through the upper portion of the draft tube is necessarily high. For this reason, the interior surface should be smooth to minimize friction and eddy losses; and in the case of horizontal wheels the obstruction caused by the shaft should be made as slight as possible. If draft tubes are set on an angle it should never be more than 45 deg. from the vertical. The elevation of the outlet should be fixed so that the draft tube shall be sealed during lowest tailwater conditions. The velocity of discharge from the tube should in general be a little greater than that of the water into which it discharges, but ordinarily not more than 3.5 or 4 ft. per sec. at full discharge.

691. The tailrace should be designed to carry the water away with a minimum sacrifice of head. It should therefore receive the same care in design and construction as the feeding canal.

692. In selecting the type of wheel to be installed in any power plant the natural conditions of head and flow, with their variations, and the power demand on the plant must be clearly and definitely understood. Stock wheels can be purchased which fit a wide range of conditions; special wheels may be designed to fit a far wider range. The following points should be covered in any wheel selection: (*a*) head, and its variation; (*b*) flow, and its variation; (*c*) speed, and the importance of close regulation; (*d*) power demand, and its variation. These may best be studied in connection with the characteristic curves of different types of turbines; it is in fact the only way of making an intelligent selection from stock designs. The fluctuation in power demand is probably the most troublesome point and the wheel which shows the best efficiency over the desired range may be readily picked out from a comparison of the curves. In the same way the effect of the variations of head on each type of wheel may be seen and allowance made in the choice. A set of common characteristic curves are shown in Figs. 76 to 78; these are drawn from Holyoke test results, reduced to the head under which the wheel is to operate.

Three examples are given below to illustrate three typical but variant requirements which water-wheels have met successfully. The number of units should be based upon the fluctuations in stream flow and power demand; but each unit should be ready to take care of momentary fluctuations.

(1) Ground-wood pulp mill. Here the wheels are run at full gate as long as there is water to run them. High speed is important, but moderate variations in head are of small concern.

(2) Textile mill. There is only moderate variation in load during working hours. Uniform speed is vital; therefore wheels which hold their efficiency for constant speed under variations of head are desired.

(3) Hydroelectric plant carrying a railway and lighting load. Here fluctuations in power demand are extreme and close regulation is essential. Such a plant frequently requires a specially designed runner; the requirements are obvious.

693. Speed regulation. The control of **reaction wheels** is effected by governors which act directly on the wheel gates. There are two general types of governors, mechanical and hydraulic. Either to be most effective should be of the relay type in which* "the energy is transmitted from a source independent—as to quantity—of the centrifugal governor balls but controlled by them in its application." In all cases in which the fluctuations in load may be large and sudden some form of relief must be provided. For open canals the spillway already described for getting rid of débris will answer. For long penstock lines standpipes or relief valves or both should be provided and careful study as needed to ensure the best results.

Impulse wheels may be governed by deflecting the stream away from the buckets or by throttling the stream. The throttling may be accomplished in the case of a multiple orifice nozzle by closing some of the orifices. The usual nozzle is known as the Doble needle nozzle. It is usually the case in impulse wheel installations that the head is great and the velocity of the water high. For this reason the inertia of the column of water contained in the penstock is such as to prohibit, on the grounds of safety, governing by throttling alone. In such cases the first action of the governor is to deflect the stream, the second and slower action is to throttle the flow. This combined control is the most economical of water.

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* Mead, D. W., "Water Power Engineering;" New York, McGraw-Hill Book Co., Inc., 1908.

694. Governors have been developed and improved rapidly in recent years. The increasing demand for close regulation of power plants has stimulated this improvement until now there is a new type fitted to every new requirement. In general it can be said that for small units and for cases of which the textile plant is representative the mechanical governor is admirably suited. For large units such as are becoming common in hydroelectric practice the hydraulic governor is better fitted to develop the great force necessary to operate the turbine gates. According to Mead[*] governor specifications should call for a guarantee of the following: (a) sensitiveness or per cent. load change which will actuate the governor; (b) power which the governor can develop and force which it can exert to move the gates; (c) rapidity with which it will move the gates; (d) anti-racing qualities, such as number of gate movements required to adjust for a given load change; (e) general requirements of material, strength, durability, etc. The hydraulic engineer can simplify governing to a marked degree by good design of the penstocks, draft tubes, etc.

695. Operation. The cost of operating a hydroelectric plant is from 10 to 15 per cent. of the first cost.

696. Cost data. A sub-division of costs for a typical case is as follows:

Item	Per kw. year
Interest 6 per cent. on $124.5 (Items 1-5, Par. 680)	$7.47
Depreciation 10 per cent. on 21.4 (Item 5, Par. 680)	2.14
2 per cent. on 26.5 (Items 2 and 4, Par. 680)	0.53
1 per cent. on 52.6 (Item 3, Par. 680)	0.53
Ordinary repairs and maintenance $2⅔ per kw. year	2.67
Administration 1.00 per kw. year	1.00
Taxes 2 per cent. on 75.0 (60 per cent. of total cost)	1.50
Insurance ½ of 1 per cent. on 33.4 (Items 4 and 5, Par. 680)	0.17
	$16.01

Note that interest at 6 per cent. and depreciation are two-thirds of the total. Steam relay to supplement dry-weather flow will add 25 per cent. to operating cost *without coal*, if charged against the water power.

PLANT DESIGN

697. General ensemble. The location and relative position of the dam, head-works, power house, tailrace, and all other necessary structures must be governed to a large extent by the topography and the geological conditions at the site. The choice between various possible schemes will be based in the main upon the relative cost considered in connection with the efficiency of the proposed plant as an energy producer, or as an income producer. The height to which the dam may be built is usually limited by the extent of flowage damage. In a hydroelectric plant pondage may have great value, making fully warranted the purchase of extensive flowage rights. The spillway section of the dam should be designed to pass safely the maximum amount of water that can be expected. Likewise the abutments and other shore structures must be built to withstand successfully the greatest freshet conceivable on the river. In this connection it is important to realise that under extraordinary flood conditions the channel above and below the dam, and not the dam itself, may frequently control the height to which the water rises. This is especially likely to be the case when the dam is built at the head of rocky tortuous rapids or in a bend of the river.

The intake should draw water from the stream with as little change in direction as feasible and must be protected by some form of boom which shall direct ice and floating débris away from the head-gates and over the dam, or through a waste channel designed for that purpose. Additional head may be obtained over that at the dam itself by carrying the water downstream in canals or penstocks. The advisability of such procedure is determined by a study of the fixed charges on the structure and the value of the increased energy so obtained.

The whole system of waterways from head-gates through the canal or penstock, racks, wheelpit and tailrace to the river should be designed to

[*] Mead, D. W., "Water Power Engineering;" New York, McGraw-Hill Book Co., Inc., 1908; Chap. XVIII, p. 467.

keep the velocities low enough to avoid excessive loss of head. Fluctuations in velocity are bad, and if unavoidable the changes should be made gradually. It is better to gradually speed the water up from say 1 ft. per sec. at the head-gates to not more than 3 ft. per sec. at the wheel gates, in the case of the ordinary vertical or horizontal setting; the exception in the

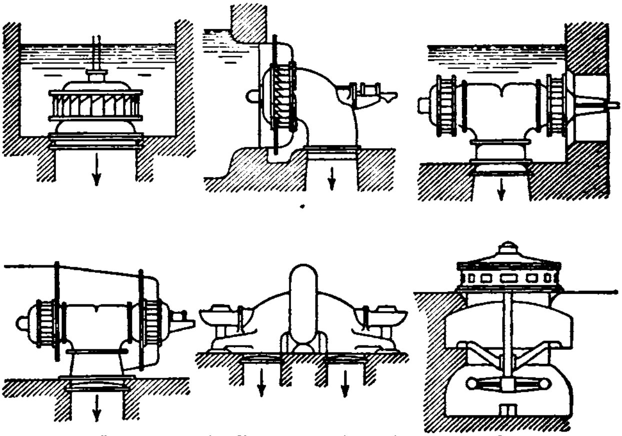


FIG. 83.—Miscellaneous settings of water-wheels.

case of scroll case settings has been noted in Par. 686. Where more than one wheel is mounted on a shaft it is important to provide for equal distribution of the water among the units, and to provide against interference in the draft chest as mentioned in Par. 688. The velocity at the top of the draft tube is necessarily high and should be gradually reduced by enlarging the cross-section of the tube until it is discharged at about 3 ft. per sec.

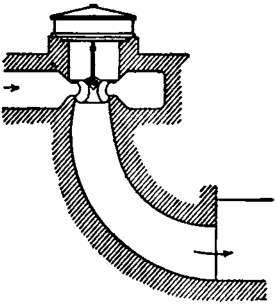


FIG. 84.—Single vertical turbine in a spiral flume direct connected to a vertical electric generator.

698. Typical settings of water wheels are shown in Fig. 83. A modern form of setting known as the spiral or scroll casing is illustrated in Fig. 84, which shows a longitudinal vertical cross-section. The latter type of setting is employed at the Keokuk plant of the Mississippi River Power Co.

699. The economical development of a water-power proposition must depend upon the available power in the river, the market, and the degree to which auxiliary power is to be employed. Two things must be determined as exactly as possible: (a) the hydrograph of the river (Par. 658); (b) the load curve.

A thorough and exhaustive study of all run-off data should be made where these are available; otherwise study must be made by comparison with similar drainage areas by means of rainfall data. Existing and possible storage should receive careful attention. It should be remembered that on streams with incomplete storage *averages are dangerous*, leading to overdevelopment. Two or 3 days of flood discharge in a month much above the capacity of any possible equipment can raise the monthly average to give a totally erroneous impression of the kilowatt hours available. For this reason, daily records of flow should be used in preference to monthly records, since the daily records bring out the maximum and minimum flows and the final estimate can be much nearer the truth.

The probable load curve is of prime importance, and taken together with the available pondage it will determine the number and size of units to be installed to give flexibility and economy in operating.

700. Auxiliary power. If a relay station, steam, gas or oil, be used, it should be placed as near the market or sources of fuel as economic conditions warrant. Its capacity must be determined by the daily and yearly hydrograph and the load curve. So many combinations of conditions of pondage, storage and load occur that every case must have special and detailed study. There are very few developments where a relay station owned either by the seller or the buyer of hydroelectric energy is not almost essential.

701. Choice of units. With varying load, efficiency requires two or more units so that during the hours of minimum demand the machine or machines on the line can carry the load at good efficiency. Then as the load comes on additional machines are cut in as needed.

702. Provisions for handling floods include: (*a*) **flashboards**; (*b*) **Tainter gates**; (*c*) **sluice gates**. It has been noted that the height to which a power dam can be built is usually limited by the extent of the flowage permissible (Par. **697**). This necessarily applies to the high-water stages which may be expected every year, and settlements for the necessary rights are usually made once for all. During most of the months in each year increased pondage or head, or both, can advantageously be procured by raising the dam without creating greater flowage than occurs during high water from the dam at its permanent elevation. This is accomplished in a number of ways, but most commonly on power dams by the use of flashboards which are designed to bend over or go off during floods. In this way provision is made for handling floods at the moderate cost of replacing the flashboards.

In order to provide for the occasional summer freshet and to give greater discharging capacity in cases of great freshets **flood gates** are incorporated in many modern dams. When warned of a freshet, probably of short duration, it is frequently possible by opening the gates in the dam to pass the extra water without straining or losing the flashboards. Flashboards are ordinarily held in place by either wrought-iron pins which will bend over, or light wooden figure-fours which will break when the water rises too high. The development of water power on navigable streams also leads to the use of Tainter gates, Chanoine wickets, Bear traps and other forms of movable dams ordinarily suited to their purpose, but which in northern rivers carrying much ice must be designed with great care.

703. Log sluices and fish ladders are necessary appendages to most power dams. A log sluice is an inclined chute on the downstream side of the dam with a gate at the crest which is opened whenever a log drive must be run past the dam. These are usually required by law where log driving has ever existed on the river. Fishways are required by federal law on most rivers and must be kept open during certain portions of the year. There are various forms, two of which are mentioned. One is constructed to form a flight of pools, the water spilling into each successively. The other is a chute set at a moderately flat slope with baffles built alternately from the right and left sides, about three-quarters of the way across.

704. Buildings and foundations. The conditions controlling plant design are discussed in detail in Par. **697**, **700** and **701**. Foundations are usually subject to water pressures due to head or backwater or variations from drought to flood conditions. These conditions usually require foundations much in excess of ordinary building requirements.

The size, height and arrangement of buildings usually follow these hydraulic conditions and the type of unit selected.

705. Summary of unit costs. Unit costs common to most water-power developments with some idea of the range of prices which depends upon the amount of each contract, distance from markets, difficulties of handling, etc., are:

Rock excavation under water	\$5.00 to \$10.00 per cu. yd.
Earth excavation under water	0.50 to 2.00 per cu. yd.
Rock excavation in the open or behind tight coffer dams	1.25 to 2.00 per cu. yd.
Earth excavation behind tight coffer dams	0.25 to 0.50 per cu. yd.
Coffer dams	5.00 to 10.00 per lin. ft.
Reinforced concrete	8.00 to 12.00 per cu. yd.
Ordinary concrete	6.00 to 8.00 per cu. yd.
Reinforcing material	0.03 to 0.05 per lb.
Penstock and draft tubes of steel plate, erected	0.03 to 0.05 per lb.

TESTING

706. Water-wheel testing has played a vital part in the improvements which have been accomplished in the design and construction of turbines. Every water-wheel should be tested if possible before it is installed. That a similar wheel of the same make, diameter and pattern has been tested and shown satisfactory results is no guarantee that a wheel which has not been tested will give similar results. The expense of the test should never be allowed to militate against it, for a gain of only 1 per cent. in efficiency will pay for the test in a relatively short time. Specifications and guarantee requirements should be drawn with care and include where feasible provision for final acceptance tests in place, in addition to the preliminary tests at the Holyoke Testing Flume. In addition to finding out whether the wheel has met the specification requirements, the tests furnish data by means of which the use of the wheel as a water meter is possible (Par. **663**).

707. The Holyoke Testing Flume furnishes the only place in this country where systematic testing is carried on. The maximum head is 17 ft. and the maximum discharge possible is between 250 and 300 cu. ft. per sec., at which discharge the head is reduced nearly one-third. In spite of these limitations the tests are invaluable to the water-power engineer and form a common basis of comparison for designers.

708. The quantities to be measured in a water-wheel test are speed, head, discharge and power output. At the Holyoke flume these are observed by a revolution counter, head and tail water gages, weir and Prony brake, respectively. Various gate openings are selected and for each opening the wheel is run at many speeds in order to determine the most efficient speed for each gate.

709. For making the final acceptance test of water wheels a test in place is becoming more frequent. Such a test furnishes direct evidence of the performance of the wheels under service conditions.

710. The general problem is that of any test of power machinery: namely, to measure the input and the output. To measure the input requires the measurement of the head and the quantity of water used by the unit under test.

711. The points of measurement of the head should be selected to show the net head actually acting on the wheel. Friction losses in canal or penstock and tailrace are not properly charged against the performance of the turbines, however much they may affect the overall efficiency of the plant. The point therefore at which to measure the headwater is just before the water is drawn to the guide chutes, and the tail-water should be observed at the nearest feasible point to the discharge of the draft tube. In open flume settings the head water is observed in the flume. In the ordinary pressure-case setting a gage should be tapped into the side of the penstock (Par. **640**, *piezometers*) just clear of the case in order to avoid disturbances due to eddies and uneven flow in the case. In a scroll-case setting, the scroll, like the draft tube, must be considered an integral part of the wheel and the head should be measured between the entrance to one and the discharge of the other.

712. Measurement of the quantity of water used presents some difficulties unless special provision was made in the design of the plant. It has been usual practice to install a weir in the tailrace. Unless the conditions are exceptional the results are subject to serious but undeterminable error due to the relatively high velocity of approach to the weir (Par. **609** *et seq.*). The attempt is made by building the crest high enough from the bottom to cut down the velocity, the head acting on the wheel is thereby reduced and at once the actual service conditions are only approximated and the peculiar value of the test in place is partially vitiated. It is difficult to make the conditions of the channel of approach to the weir suitable for good measurements and the expense of installation is relatively great. Whatever method of measurement is used, there should be an agreement in advance by the buyer and seller of the wheels to accept the results. Where conditions for their use are right, rod floats (Par. 643) afford an excellent means for measuring the flow either in the headrace or tailrace.

The easiest measurements to make properly are probably those with a current meter (Par. 662). The expense is slight and in the hands of careful and experienced operators simultaneous measurements in head race and tail-race will give excellent results without interfering in any way with the normal service conditions of operation. Suitable measuring stations may be easily included in the original design of the plant at little or no extra expense.

For relatively small units fed by penstocks, the most accurate measurement without question can be made by a venturi meter (Par. 636) incorporated in the penstock when the plant is installed. When properly rated it has the great advantage of giving a continuous record of flow.

713. For measuring the output two general methods are available for a hydroelectric station. They are (1) to measure the energy generated electrically, or (2) with some form of absorption dynamometer. The electrical measurement requires that the driven dynamo shall have been carefully tested and the efficiency determined for all loads. Electrical measurements can be made with great accuracy but the determination of the turbine output by this means is somewhat indirect. With the development of the **Alden dynamometer** direct and accurate measurement is possible. This apparatus is described as follows by Prof. C. M. Allen:

"It is a form of Prony brake, and usually consists of several smooth circular revolvable cast-iron discs keyed to the shaft which transmits the power; a non-revolvable housing having its bearings upon the hubs of the revolving discs; and a pair of thin copper plates in contact with each cast-iron disc, the plates being integral with the housing. Through a system of piping, water under pressure is circulated through chambers between the units, each consisting of a disc and its copper plates, and between the outer plate at either end and the wall of the housing. The water pressure is regulated by hand or by an automatic valve. Another system of piping circulates oil for lubricating the surface of the copper plates next to the revolving discs."

714. Advantages of the test in place. The test in place, if made complete enough to cover all gate openings under normal conditions of head at the plant, will serve two purposes: it will tell whether the wheels tested are up to specification; and it will provide a means of measuring the daily or hourly amount of water drawn by the plant so long as the wheels are kept clean and in good repair, and the operating conditions are essentially those obtaining during the test. These records if carefully kept become of great value to the owner and operator of the plant.

ELECTRICAL EQUIPMENT OF POWER PLANTS

BY GEORGE I. RHODES

Consulting Engineer, White, Weld and Co., Member, American Institute of Electrical Engineers

ELECTRIC GENERATORS

715. Types of generators are covered in Sec. 7 and Sec. 8.

716. Generator characteristics required for a given power plant are largely determined by the size, overload capacity, speed, speed variations, and possibilities of over speed of the prime mover; the size of the plant, the number of units, the load factor, fluctuations and power-factor of the load; the voltage and type of the distribution system.

717. Sizes of generators. The ratio between the adjacent sizes of generators varies from 1.25 to 1.50. The size limitations seem to have been reached only for direct-current generation, where 2,700 kw. is the largest obtained commercially. Engine-driven alternators are in operation which will carry from 8,000 to 10,000 kv-a. continuously without overheating. Water-wheel alternators are built up to 20,000 kv-a. and there is little doubt that larger sizes can be built if required. Steam-turbo-driven alternators are being built up to 30,000 kv-a.

718. Overload capacity in generators is necessary only when the prime mover has ability to deliver loads greater than rating, or in special cases where it is desirable to install a generator smaller than the maximum output of the prime mover. Gas engines which will stop when their rated capacity is much exceeded, require generators which will reach their maximum safe temperature when continuously operated at rated load. This same condition is required of steam turbines when rated at the maximum capacity, and by water-wheels except in very unusual circumstances. Steam engines which are rated at their most economical load, or steam turbines having overload valves, require generators with considerable overload capacity. Generators of the first class are usually designed to carry their rated load continuously with a temperature rise of about 50 deg. Cent. Generators of the second class are designed to carry rated loads continuously with 40 deg. Cent. temperature rise, and 25 per cent. overload with a 55 deg. rise. It is doubtful if this latter temperature is safe except for temporary operation.

719. Speeds of standard designs range from 75 r.p.m. upward, the ratios between adjacent speeds being from 1.1 to 1.2. Low speeds are confined to large machines. Several speeds for the same size are usually available, particularly in small and moderate sizes. The speed of direct-current machinery is limited by commutation. High-speed direct-current generators, even of relatively small size, are not recommended. The speed of alternating-current generators is controlled by frequency and by number of poles. At the high speeds, ventilation becomes one of the principal problems, but the manufacturers have solved it to such an extent that machines are offered in sizes up to 5,000 kv-a. at 3,600 r.p.m. and up to 30,000 kv-a. at 1.500 r.p.m.

High-speed generators deliver short-circuit currents of enormous instantaneous value, even though the sustained short-circuit currents are limited. This characteristic is of greatest importance in large systems, particularly in reference to switching methods (Par. **799**).

720. Speed variations or fluctuations in the driving torque have a tendency to accentuate the difficulties of commutation. In alternating-current machines the speed variations are of great importance to parallel operation, particularly when the speed is low and the number of poles is large. The driving impulse in gas engines and low-speed steam engines being non-uniform, flywheels of large moment of inertia are required to keep down the speed fluctuations so that satisfactory parallel operation may be obtained. It is usually necessary to have alternators for this service designed with large armature reaction and reactance, so that the angular variations will produce a minimum of surging in the electrical circuits. With gas engines it is necessary to use short-circuited pole-face windings.

721. Runaway speeds. In engines and turbines it is easily possible to provide emergency governors so that generators need be specified to stand only about 30 per cent. overspeed. In water-wheel units, however, the difficulty in caring for the inertia of the water in long pipe lines makes it necessary to have generators safe at 90 per cent. to 100 per cent. over-speed.

722. Inherent regulation. In very small plants good regulation is to be sought for, particularly on account of the lack of skilled attendance, and also on account of the large number of other uncontrollable influences affecting the uniformity of voltage. In the larger plants the question of regulation is of less importance because of better attendance and the feasibility of automatic regulating devices.

In very large plants the importance of keeping down short-circuit currents, both instantaneous and sustained, looms up in such large proportions that regulation is completely neglected except as may be necessary to secure proper conditions of parallel operation. In these large plants it is usual to go to considerable expense to secure automatic regulation of voltage allowing generators of exceedingly low inherent regulation. Induction generators and current-limiting reactances (Sec. 6), are also used in some large plants to limit short-circuit currents.

723. The number of units has relatively small influence except as affecting parallel operation, the conditions of which are more exacting in a plant of many units. (Also see Par. 724.)

724. The load factor, or the form of the load curve has particular influence in determining the number of units to be installed and the overload capacities of the same. With a steady load for 10 hr. per day, generators should be installed to carry the load with one unit shut-down, without exceeding the overload capacity. With large short peaks for a few months in the year, and much smaller loads during the other months, the peaks can safely be carried at overloads even in excess of 25 per cent. for which generators are frequently specified.

725. Load fluctuations. If the load is widely fluctuating, such as on electric railways of few cars, the commutation limit of direct-current machines determines the size to be installed. Interpole generators with ability to commutate loads far above the usual rating are frequently used so that the average load may approach the heating limit. In alternating-current machines this question is of less importance, as they are stable at very considerable momentary overloads. In single-phase railway systems where violent short-circuits are frequent the windings need to be specially braced. Fluctuating loads will cause considerable voltage variation and may determine the requirements of regulation, particularly in small and medium size plants. Compounding is almost invariably resorted to in direct-current plants and sometimes in small alternating-current plants; even then there is considerable variation in voltage, especially on rapidly varying loads.

726. The power-factor determines not only the kilovolt-ampere rating of the generators to be installed, but also the size and resistance of the field windings. Machines required to carry lagging loads need larger fields than machines to carry leading loads of the same magnitude. Generators are usually supplied with field windings suitable to care for full inductive loads at 80 per cent. power-factor. This is usually sufficient to take care of any condition which may be expected.

727. The voltage. Direct-current machines are available at 125 volts, 250 volts, two-wire and three-wire, and 600 volts, with 1,200 volts as a possibility, alternating-current machines at the above voltages and in addition 2,300 volts, 2,300/4,000 volts, four-wire, three-phase, 6,600 volts, 11,000 volts and 13,200 volts. Higher than 2,300 volts, without the use of compensators or transformers, is not recommended in small high-speed machines. 11,000 volts and above in turbo-generators, except of largest size, should be avoided, but is entirely safe in slow-speed machines. Other voltages are used at times but the machines are special. Occasionally it is desirable to use generators connected in delta at 2,300 or 6,600 volts but insulated for 4,000 or 11,000 volts, making it possible later to increase the voltage of the system. High-voltage machines are usually star connected, because of the possibility of larger conductors and more rigid windings.

728. Foundations for electric generators are usually parts of those of the prime movers. The type and form is largely dictated by the requirements of the prime mover. It is becoming the practice to support turbo-generators on skeleton foundations and even directly on the floor with a basement underneath. For the cost and detailed design see Sec. 10.

729. The erection of generators covers a considerable range of work depending upon the method of shipment. Small machines are shipped completely assembled and the erection consists merely of placing in position and leveling. With larger units the facilities for transportation and hauling limit the sizes for separate parts and in extreme cases the machines are shipped almost completely dismantled, requiring the most careful work in erecting them and preparing them for preliminary operation by drying and voltage tests.

730. Preliminary operation requires a determination of the polarity or phases, before actual connection to the lines, as well as careful watching and testing during operation necessary to determine whether or not the machine is perfect and ready for regular and continuous service. Phases as marked in the factory should not be depended upon unless testing is impossible, Phases and synchronising equipment may be checked by connecting to a spare machine and bus, bringing up to speed with the spare machine and synchronising by the use of the apparatus on the spare machine. If no spares are available, lamps, alone or with shunt transformers, connected across the open switch may be used. Simultaneous dark lamps indicate correct phases; when lamps do not darken and brighten together crossed phases are indicated.

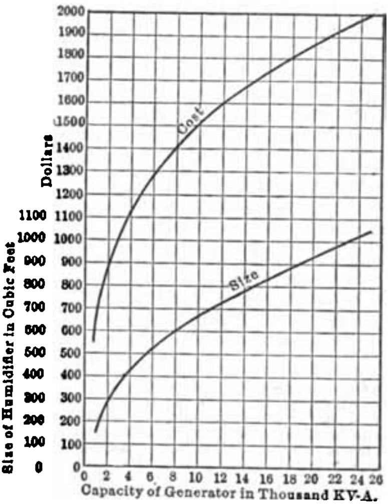


FIG. 85.—Size and cost of humidifiers (*G.E. Rev.*, Sept., 1913, p. 634).

731. The volume of air required for cooling is generally about 5 cu. ft. per min. per kv-a. of generator capacity.

732. Cleaning the air is a matter of great importance but frequently neglected. Dirt which clogs up the air passages and coats the cooling surfaces prevents proper cooling and shortens the life of the machine. Reasonable cleanness may be secured by having the intake above the roof, thus keeping out all but the finest dust. If the air must be taken from near the ground, it should be filtered by passing at low velocity through one or more thicknesses of cheese cloth or Canton flannel. Complete cleaning requires large areas, amounting to about 1 sq. ft. per kw. of generator capacity. Means must be provided for the ready cleaning

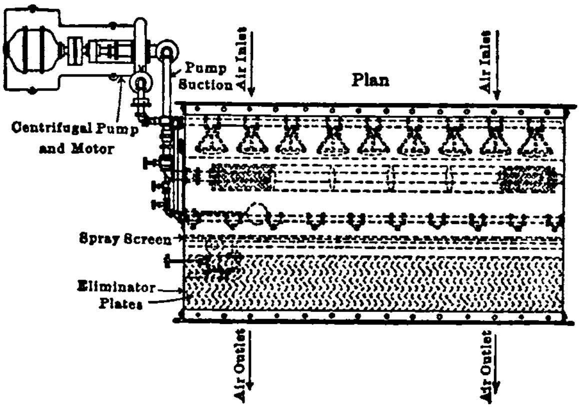


FIG. 86.—Air washer, Spray Eng. Co.

and replacement of screens. Water screens or humidifiers are coming into favor, the air being passed through falling water which takes out the dust, and then freed from mist by baffles (Fig. 86). This method has the added advantage of cooling the air several degrees. The value of this may be seen in Fig. 87.

733. The cost of erection and preliminary operation of generators ranges between wide limits depending upon local conditions. Smaller machines which are shipped assembled may cost for erection and preparation for regular service, in reasonably equipped stations, from 4 per cent. to 5 per cent. of the cost of the machine at the factory. If the machine is shipped dismantled, the cost of erection will be about 1 per cent. to 2 per cent. higher. The cost of erecting large machines is from 4 per cent. to 5 per cent., and in a few cases where conditions allow shipment assembled, about 1 per cent. can be saved. An average cost, for machines from 500 kw. to 6,000 kw., can be obtained from the formula $C = \$400 + \$0.32$ kw., where C is expressed in dollars. Freight and haulage is dependent absolutely on location and local conditions, and may vary from 1 per cent. to 15 per cent. of the cost of the machine.

734. Ventilation. The continued success of ventilation methods has a most important influence in determining the life of the insulation. The requirements call for a plentiful supply of cool air free from dust, oil and excessive moisture. Except in turbo-generators, no special means are taken to fulfill these conditions other than those provided in the machine. In high-speed turbo-generators, however, large volumes of air at enormous speeds are passed through the windings, and the utmost pains should be taken to maintain the best possible conditions. In most stations it is customary to take the cooling air directly from, and discharge it into the turbine room, but in cases where the room cannot be well ventilated, this arrangement may overheat the room and the machines. In some stations it has been found possible greatly to increase the capacity of the generators by installing specially designed vanes on the rotors, thus passing much larger volumes of air. With a machine thus ventilated, the coils quickly become dirty and much care has to be exercised to keep out the oil that permeates the atmosphere. In the larger stations it has been found important to install ventilating ducts taking cool air directly from outside the building.

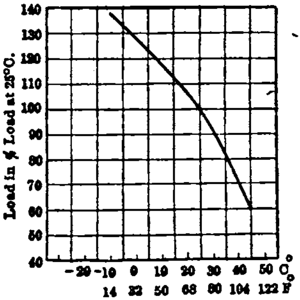


FIG. 87.—Safe load and temperature of cooling air (*G. E. Rev.*, Sept., 1913, p. 633).

The ducts should be of liberal size to keep down air velocities to about 1,000 ft. per min. Dampers should be provided to enable shutting off the air supply in case of burn-outs. Ventilation by separately driven blowers is being used and is strongly recommended by some engineers.

735. The cost of ventilating ducts arranged for cleaning the air should not be more than 25 cents per kw. of generator capacity.

736. The operating temperature, combined with the kind of insulation, determines absolutely whether or not the machine is overloaded. Temperatures are difficult of measurement, particularly in respect to finding the hottest parts. It is the maximum temperatures which are of importance, and it is probable that these are never found in tests. Resistance temperature tests are difficult to make, and in large machines take considerable time after shut-down; such tests give only average copper temperatures, which are certainly less than the maximum. Ordinary thermometer tests give the temperature of the outside of the insulation, and by means of refinements it is possible that maximum temperatures can be measured to within 10 deg. to 20 deg. Cent., but this method is not recommended. Resistance thermometers and thermo-couples, inserted between the coils and the iron, have been used with considerable success. The actual indicator may be located anywhere convenient, even on the switchboard, where continuous record of the temperature may be kept. This method gives more consistent and precise results than the others and probably shows more nearly the maximum temperatures.* Indicating electric thermometers, several for each machine, can be installed for $100 to $200 per machine.

737. Safe temperatures of insulation are given by C. P. Steinmetz and B. G. Lamme in the February, 1913, *Proceedings* of the A. I. E. E. as 90 deg. Cent. for fibrous insulation such as is commonly used, and 125 deg. Cent. for mica and other fire-resisting materials. These temperatures are the measurable temperatures rather than the absolute maximum that may be expected in local spots. H. G. Stott recommends 20 deg. lower temperatures especially on high voltage work.

EXCITATION

738. Excitation equipment should be designed with a view to the maximum possible **continuity of service.** Simplicity, ruggedness, "foolproofness" and reserve apparatus are important requirements. The methods in use for securing these results are greatly varied and much difference of opinion exists as to the best method. Each plant must be considered as a separate problem.

739. Exciters directly connected to the main generators were the earliest form in use. They are now seldom used in steam or gas engine plants, but are used in some of the largest and most recent hydroelectric stations with satisfactory results. Each exciter is frequently made large enough to handle two generators, in emergencies. The chief arguments in favor of this method are simplicity, high efficiency and the absence of large field rheostats. The most important objections are the possible crippling of a large unit due to trouble with its exciter, and the fact that voltage fluctuations in per cent. are twice as great as the speed fluctuations.

740. Exciters driven by separate prime movers are necessary in all stations except those above mentioned (Par. 739). They may be obtained for service with any kind of prime mover. On account of their small size the efficiency of the unit is very poor, this constituting the chief objection to their use. Where steam at atmospheric pressure is necessary for feed-water heating, the inefficiency is of no importance. Their use is determined principally by questions of convenience and economy, it being absolutely necessary to install them for starting purposes.

741. Motor-driven exciters are in use in almost all stations. They are cheap, economical, efficient and reliable, but cannot be used for the complete equipment. They should not be driven by synchronous motors, because short-circuits on the outside system not in themselves sufficient to cause a shut-down of the station, are likely to throw the motors out of step, thus shutting down the plant. Induction motors are almost invariably used. A well-balanced exciter plant consists of units driven both by motors and by prime movers. In some cases, motor-driven units are considered as auxiliary to the prime-mover units, and in other cases the motor-driven plant is considered of first importance; the first plan is probably most in favor.

742. The importance of continuous excitation points strongly to **storage batteries** which are frequently installed of sufficient capacity to excite the entire plant for an hour. Danger of a shut-down of the entire plant by failure of the exciters is not considered great enough in all cases to warrant the great expense of a battery. When used, it may either be floated on the excitation busses, or allowed to stand by fully charged.

743. A new method of securing continuous service is being introduced with great promise. The exciter generator is arranged to be driven both by motor and prime mover. The motor is provided with a relay which shuts it down if the voltage becomes low. The governor is arranged practically to shut off the steam at normal speed, but after cutting off the motor, the prime mover continues to drive the unit at but slightly reduced speed.

* Reist, H. G. and Eden, T. S. "Method of Determining Temperature of Alternating-current Generators and Motors, and Room Temperature;" *Trans.* A. I. E. E., 1913.

744. The size of the exciter plant depends upon the size of the power plant and the types of generators used. Small, low-speed generators require up to 3 per cent. of their capacity for excitation. Large, high-speed turbo-alternators may require as little as 0.75 per cent. The exact requirements may be obtained from the manufacturer. The total capacity should be ample to carry the whole excitation load with the spare apparatus out of commission. The amount of spare apparatus required is not very definite; practice ranges all the way from depending on the overload capacity to having two to three times the total capacity required.

745. The number and relative size of the exciter units should be chosen for the greatest simplicity and the required flexibility. The minimum size of any unit is in general dictated by the requirements of the largest generator. It is doubtful if the largest exciter should be large enough to excite the entire plant. Except in very large plants a size sufficient for half the equipment, with a total of three units, will give very satisfactory results.

746. The exciter voltages in common use are 125 volts for all except the very largest plants where 250 volts are used. The generators are usually flat compounded. With standard exciters it is possible to run at not more than 15 per cent. over standard voltage, which is quite sufficient to take care of the excitation of ordinary alternators at full-load. Where overload capacities are to be used, it is frequently desirable to raise the excitation voltage as much as 25 per cent. during the peaks. Exciters can usually be arranged for this voltage with very little deviation from standard design.

747. The cost of exciter sets installed, ready to run, exclusive of wiring, piping, etc., correct to plus or minus 10 per cent., (covering ordinary differences in speed and other commercial differences in design), may be expressed as follows:

High-speed compound-engine drive,	$1,000 plus $35 per kw.
High-speed simple-engine drive,	$750 plus $26 per kw.
Steam-turbine drive,	$425 plus $31 per kw.
Induction-motor drive,	$625 plus $15 per kw.

The cost of exciters attached to the shafts of the main generators can best be expressed as a percentage of the cost of these generators; this will vary from 3 per cent. to 5 per cent., depending somewhat on the speed.

748. Exciter wiring systems are in use covering practically all the direct-current types of switchboards. On account of the low voltage no complications need be entered into except as may be necessary to make available the various sources of excitation. Occasionally a double exciter bus is desirable, but in most cases it adds complications without any adequate gain to the service. Alternating-current wiring for motor-driven exciters should be worked out on the basis of the exciters being most important feeders, unless, however, the presence of a storage battery makes this treatment unnecessary. In stations above 2,300 volts, it is recommended to use transformers in connection with the exciter motors, as small machines wound for high voltage are not of sufficient reliability.

749. Excitation switching appliances. The current must not be broken without at the same time short-circuiting the field windings, otherwise punctured insulation will result. This requirement calls for special field-discharge switches. It also limits the use of circuit-breakers to those operating on reverse current, thus serving to cut out damaged exciters. Fuses of very large overload capacity, which will blow only during most severe trouble, are used for the protection of the exciters.

750. Field rheostats for small generators are usually mounted on the back of the switchboard, but for large machines some more conveneint location involving remote control is necessary. Electrically controlled rheostats are frequently necessary in large stations and are operated either by solenoid and ratchets or by motor; the latter is necessary where the field current exceeds 300 amp.

VOLTAGE CONTROL

751. Hand regulation. Voltage is controlled in all systems by hand regulation, whether or not there are automatic means. Hand regulation alone is suitable only where the load is steady and the generators inherently regulate well.

752. Automatic regulation may be secured by two methods; first, the automatic adjustment of the field resistance by motors or solenoids controlled by a relay; second, the alternate short-circuiting and insertion of resistance in the field circuits. The first method is used in the **Thury regulator** the second method in the **Tirrill regulator.** The first method makes the adjustment more or less gradually; the second changes the resistance much more than necessary, alternately one way and then the other, while the time lag of the field windings produces a practically steady voltage of a value depending upon the proportionate time the resistance is in and out of circuit. The latter method is inherently more sensitive than the first and is practically the only one in use in America.

753. The Tirrill regulator for small direct-current machines operates as follows (see Fig. 88). The regulation depends on the rapid make and break of contacts *A* which short-circuit the resistance in the shunt field circuit of the generator. The closure of these contacts depends upon the machine voltage across the relay *B*. The contacts closed by this relay serve only to close or open one of the two differential windings on the magnet *C*, which in turn operates contacts *A* cutting in or out the field resistance. When the voltage is high, relay *B* opens one winding in *C* which in turn causes the latter to open contact *A*, inserting the resistance, and *vice versa*. The final result of operation is that the contacts vibrate continuously and remain open or closed a longer or shorter portion of the time as may be required to keep the voltage steady. A regulator of this type with multiple contacts at *A* can control machines up to 125 kw.

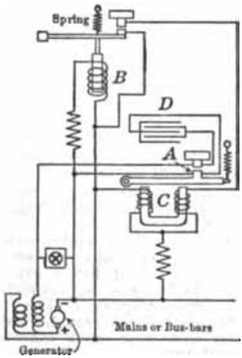


Fig. 88.—Tirrill regulator for small direct-current machines.

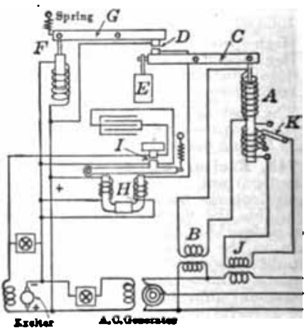


Fig. 89.—Tirrill regulator for large machines.

754. Tirrill regulators for large direct-current machines and alternators operate as follows (Fig. 89): The solenoid *A* is connected across the bus bars, usually through a potential transformer. The core, which is lifted by the solenoid, is attached to the lever *C* on the other end of which is a contact *D* and a balancing weight *E*. Across the exciter terminals is connected a solenoid *F* whose core is attached to the lever *G* with contact *D* on the other end. This lever is pulled in a direction to close the contacts by a spring so designed that the plunger *F* is pulled downward in direct proportion to the voltage. This arrangement gives greater sensitiveness than the simple arrangement at *B*, Fig. 88.

When the alternating voltage is low the weight of the plunger *A* overcomes that of the balancing weight *E* and closes contact *D*, causing relay *H* to close contact *I*, thus short-circuiting the field resistance and raising the exciter voltage and the alternating voltage. As the exciter voltage rises, solenoid *F* pulls down its plunger and lifts contact *D*, but as long as the alternating voltage is low, the lower contact follows. When the alternating voltage ises above the correct value the contact *D* is opened, which causes the resistance to be inserted again in the field circuit. The winding *K* on the solenoid *A* allows compounding with the load, which may compensate for the average line drop. In stations where there are several exciters, the relay *H* operates a number of contacts *I*, one or more for each exciter. In very large stations a separate regulator is used for each exciter.

755. Regulators with exciter batteries are possible by the use of boosters in the exciter bus under control of the regulator, bucking or boosting the voltage supplied to the field windings.

756. Manual regulation with regulators is secured by varying the resistance in the circuit of solenoid *A*, Fig. 91.

757. Protection against regulator failures or abnormal conditions should be provided whereby excessively high or low voltage results in the operation of a relay, thereby making the regulator non-operative.

758. Exciters for Tirrill regulators require careful adjustment for parallel operation. Field circuits should permit voltages from 40 per cent. to 140 per cent. normal. They must respond rapidly and equally to change of resistance. Unequal response causes the quicker machine to take all the load and it may flash over.

759. The cost of Tirrill regulators installed is from $500 to $1,000 each, exclusive of instrument transformers.

760. Feeder regulators for alternating-current circuits consist essentially of compensators, the secondary winding of which is in series with the outgoing circuit. They are fully described in Sec. 6. There are two types, the contact type and the induction type, the latter being preferable. These regulators may be either single phase or polyphase as necessary. They are usually wound for ± 10 per cent. regulation. They may either be hand operated, or automatically operated through a relay and motor to compensate for line drop thus maintaining constant voltage at a distant point. The cost of 2,200 volt automatic feeder regulators for ± 10 per cent. regulation is about as follows: Single phase, $300 + $350 per 100 amp. Three phase, $600 + $600 per 100 amp. If motor-operated but non-automatic, about $80 is saved. Hand-operated regulators cost about $150 less than automatic.

LOW-TENSION DIRECT-CURRENT SWITCHING

761. Direct-current switching may conveniently be grouped in the following **general classes:** (*a*) Single polarity; (*b*) Double polarity with equaliser on pedestal; (*c*) Double polarity with equaliser on panel; (*d*) Three wire; (*e*) Multiple voltage.

Obviously a great number of subdivisions could be made, the number of busses being the most important. Fig. 90 shows the elementary wiring of these various classes, leaving out all auxiliary wiring and all instruments.

762. Parallel operation of compound generators is secured by the equalizer connections which are usually of one-half to one-third the rated capacity. They parallel both terminals of all series fields, making the direction of current in all of them necessarily the same. Parallel operation is fully discussed in Sec. 8.

763. Single-polarity switchboards find their greatest use in railway work where the negative bus is at approximately ground potential, and is usually located in the basement beneath the machines; it should be insulated, however. The circuit breakers and instruments are in the positive side of the board and the equaliser and negative switches are on small panels or pedestals near the machines. The feeders are frequently without negative switches.

764. Three-wire switchboards are used for combined lighting and power systems. The generators require an equaliser on each side. If both these equalizers are thrown in, the machines are paralleled as shunt machines, so that some means must be taken to make it possible to close the equaliser switches only at the same time as the main switches. This usually calls for four-pole switches on small boards and two double-pole switches on the large, each double-pole switch consisting of one line and one equaliser switch. It is obvious that a circuit breaker and instruments are needed in each side. In the case of two-wire full voltage feeders, a double-pole circuit breaker is necessary when the neutral is grounded.

765. The instruments required to measure completely the output of the machines or of a feeder, comprise an ammeter or a wattmeter, or both, for each circuit-breaker shown in the diagram (Fig. 90), together with a voltmeter on the main bus. For purposes of connecting incoming generators

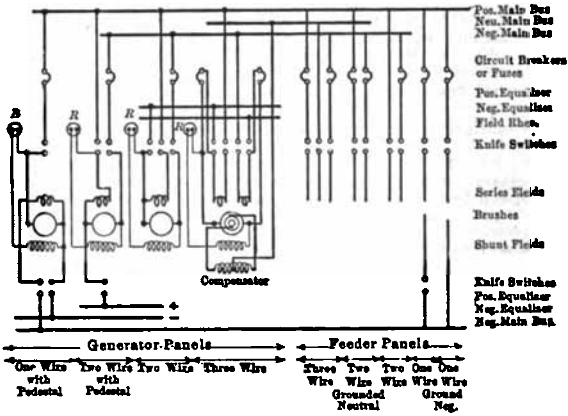


FIG. 90.—Elements of direct-current wiring.

provision must be made for a voltmeter on each machine, or a socket and plug which will enable its voltage to be measured by a common machine voltmeter. (Fig. 93.)

766. Double-polarity switchboards find their greatest use in power work, particularly at 250 volts. They are desirable where both bus bars must be insulated. Except for large stations there is no disadvantage in having both polarities on the same board. Except on small boards, it is desirable to have the equalizer switches on pedestals near the machines (Fig. 91).

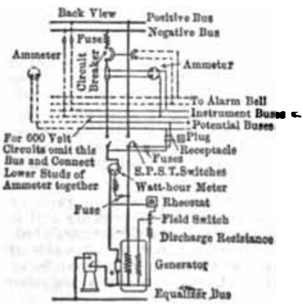


FIG. 91.—Detail wiring diagram of generator panel.

767. Multiple-voltage switchboards are in use where three-wire distribution is extensive. The extra busses are operated at a voltage higher than the main busses to take care of the drop in long feeders. Their voltage is usually maintained by boosters, although in some cases provision is made for connecting the generators to either set of busses.

768. Double bus bars are seldom warranted except for operating at two voltages. However, when there are two distinct classes of service which should be kept independent, double bus bars are desirable (Fig. 92).

769. Spare circuit breakers are almost necessary in heavy railway work and provision should be made for inserting them in a simple manner (Fig. 93).

770. Auxiliary circuits are frequently installed to accomplish many different purposes. Some of the more commonly used are as follows:

) Voltmeter bus and receptacles to enable the use of one instrument for machines; (*b*) Current supply for instrument excitation and the operan of circuit breakers; (*c*) Signals indicating open breakers; (*d*) Remote

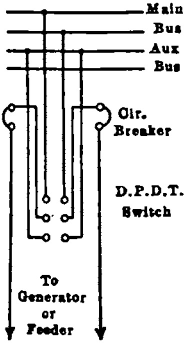


Fig. 92.

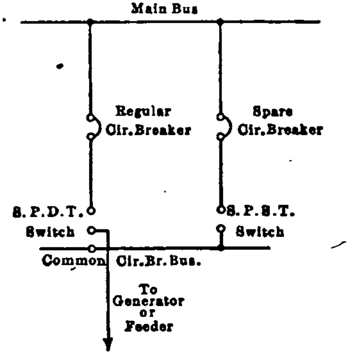


Fig. 93.

Figs. 92 and 93.—Wiring for spare busses and for spare circuit breakers.

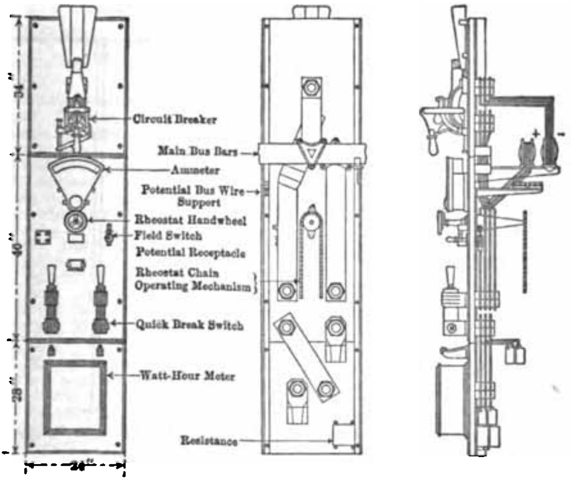


Fig. 94.—Two-pole generator panel 2500*A*, 250 *v*.

ontrol of circuit breakers and switches; (*e*) Interlocking circuits between ircuit breakers; (*f*) Reverse-current relay wring (Fig. 91).

771. Types of switchboard panels. The accompanying illustrations, Figs. 94 and 95, serve to indicate general types of panels. In very small installations, the generator and feeder switches may be located on the same panel. In moderate sized plants there may be several feeders per panel, although generally each generator has its own. In large systems there is sometimes a separate panel for each feeder, but this is necessary only in very special cases, usually two being grouped on each panel. The require-

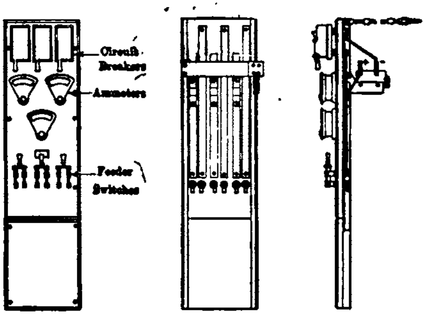


Fig. 95.—Two-wire feeder panels; three small feeders.

ments of ammeters and circuit breakers are the same both for generator and feeder panels except that occasionally two feeders running to the same point are protected with a single set of circuit breakers.

772. Station panels are usually provided containing a totalizing ammeter and watt-hour meter, bus voltmeter (or voltmeters if a three-wire

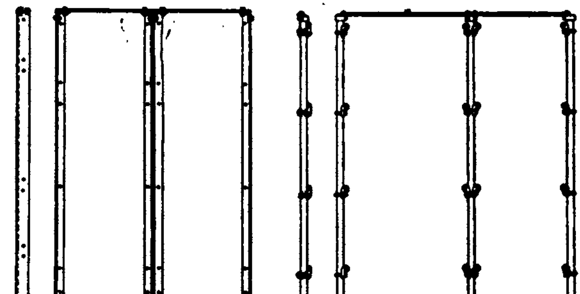


Fig. 96.—Switchboard supports; angle iron and pipe (*Elec. Jour.*, Feb., 1913, p. 168).

system), machine voltmeter and switches for auxiliary circuits and power-station lighting.

773. Switchboard supports are usually in the form of angle-iron framework although lately iron pipe is coming largely into use (Fig. 96). An inverted channel iron, or sill of hard wood is sunk into the floor to which the upright supports are bolted. There is an upright for each joint between panels consisting of two angles back to back, or of a single iron pipe. Along the top is another angle or pipe. The entire board is braced in an upright position by numerous wall braces. The iron used varies in size from 2 in. × 1.5 in. × 0.25 in. to 3 in. × 2 in. × 0.25 in. One and one-fourth inch pipe is generally used. At least 3 ft. of clearance should be left between the board and the wall to allow for cleaning and repairs. In direct-current practice it is customary to insulate the iron framework from ground.

774. Complete lines of detailed fittings are on the market, facilitating almost any conceivable arrangement of framework for supporting the panels and bus bars. (Fig. 97.) Brackets with special insulators are used for the bus bars and longitudinal wires. Vertical small wiring is fastened to the back of the board with small straps. Vertical conductors made of copper bars or heavy wires are usually supported by the studs of the switches and the main bus bars to which they are connected.

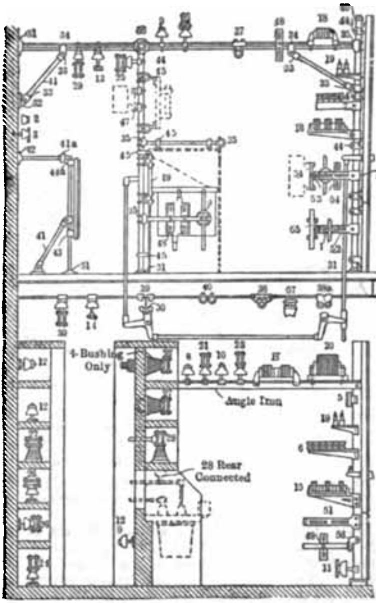


FIG. 97.—**Standard switchboard details (Elec. Jour., May, 1913, p. 82).**

775. In laying out the wiring for the switchboard panels care should be exercised to have sufficient **clearance** around all live parts (1 in. or more) and to avoid all unnecessary complications. It is important that all panels be as nearly alike as possible.

776. The materials for panels. Marble is used in the more important switchboards on account of its better insulating qualities and better appearance if unpainted; it is advisable if the voltage exceeds 600. Slate is likely to contain conducting veins, but when clear is a perfectly satisfactory material, considerably cheaper than marble and somewhat stronger. Panel from 1.5 in. to 2.5 in. thick are used, depending on the size of the switches and circuit breakers. In some cases even thicker reinforced panels are necessary. Marble panels complete with framework cost from $2.50 to $3 per sq. ft.; slate panels from $1.75 to $2.50.

777. Bus-bar connections. Bolted joints are preferable in light work, largely on account of the awkwardness of the clamps, but on heavy work, clamps have many advantages. Lap joints or butt joints with covers should be used with a uniform pressure of from 100 to 200 lb. per sq. in. The copper surfaces should be well cleaned. The **current density** in these connections should be from 100 to 200 amp. per sq. in. Under these conditions, the drop in the joint will but slightly exceed that in an equal length of bar.

778. Bus bars are usually made of 0.25-in. or 0.125-in. copper of various widths, seldom more than 10 in. The bars are grouped together with 0.25-in. spaces between them when using 0.25-in. copper and 0.375-in. when using 0.375-in. copper. Aluminium bars 0.25 in. and 0.375 in. thick are sometimes used. Fig. 98 shows approximately the relation between the number of bars, the width of the bars, and the **current-carrying capacity** based on a temperature rise of 25 deg. to 30 deg. Cent. Free circulation of air is important and the bars should be supported on edge. Smaller spaces between the bars would greatly reduce their carrying capacity. In general, **current densities** of from 800 to 1,000 amp. per sq. in. are not excessive. Where aluminium is used, densities 25 per cent. less should be figured. The **cost** of copper bus bars installed will be covered in general by adding 10 cents per lb. to the cost of the copper.

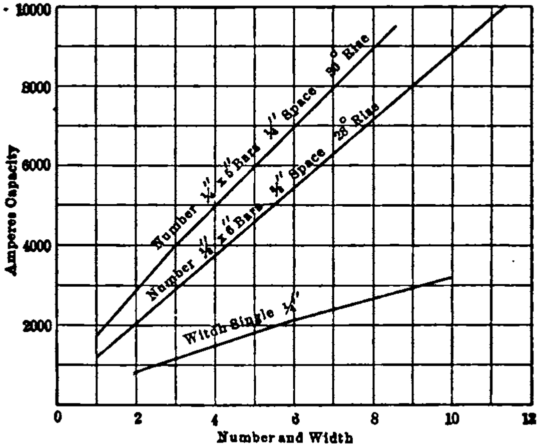


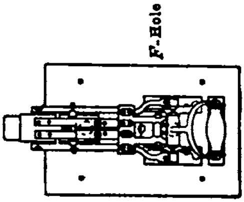
FIG. 98.—Current-carrying capacity of bus bars taken from manufacturer's bulletins.

779. Connections to switch studs between nuts should be figured on the basis of a current density of from 100 to 200 amp. per sq. in. Connections should be carefully fitted and the burr carefully removed from the edges of holes. The cost of making heavy connections between switch studs and bus bars can be covered approximately by adding the cost of 1 ft. of bar for each connection and 6 in. for each joint. For usual panels this work will cost about $10 per panel plus $1.50 per 100 amp.

780. Switches should carry the maximum overload at a temperature rise of not greater than 25 deg. Cent. Current density in the blades and studs ranges from 700 to 1,000 amp. per sq. in. and in the jaws and hinges from 40 amp. to 50 amp. per sq. in. The switches must be carefully made and lined up so that the full contact area is available. If the switch overheats, the contact resistance will be greatly increased by oxidation. A 6,000-amp. single-pole switch is about the **limit of hand operation.** Very large switches with laminated brush contacts closed by toggle joints are sometimes used.

781. Cost per pole of single-throw switches complete with nuts, etc., ready for mounting on switchboards is $1.80 per 100 amp. of capacity; 40 per cent. should be added if the switches are double-throw; 10 per cent. should be added if they are 600-volt switches, to cover added length of blade and quick-break feature, which should always be used.

782. Fuses are the simplest means of automatically opening a circuit under short-circuit or overload conditions. Enclosed fuses are on the market and are approved by the fire underwriters, having capacity up to 600 amp. at 250 volts, or 400 amp. at 600 volts. Up to 30 amp., the contacts are simply ferrules on the end of the tube. On the larger fuses, contact is made by blades fitting into jaws similar to those on knife switches. The tube is filled with heat-resisting powder which confines the arc and puts it out quickly. All fuses are supplied with indicators to show when they are blown. They will usually carry full rated load continuously, but will blow in from one to five minutes if the current exceeds 15 per cent. overload.



783. The cost of fuses complete with the clips is about $1 per 100 amp. capacity. The cost of the fuses alone is about 50 cents per 100 amp. capacity, and the cost of refilling the fuses is about 20 cents per 100 amp. capacity.

784. Automatic circuit breakers are required on all circuits larger than 400 to 600 amp. and even on the lighter circuits where overloads and short-circuits are likely to be frequent. There are two methods in use for preventing burning of the contacts, namely —the **carbon break** and the **magnetic blowout**. In the carbon-break method, three contacts are successively broken on the opening of a circuit. First the main current-carrying contacts open; then an auxiliary copper contact opens, and lastly a contact opens between the carbon plates and breaks the entire current, thus protecting the current-carrying contacts. In the magnetic blow-out breakers (Fig. 100) the carbon contact is replaced by a contact located between the poles of an electric magnet energised by the passage of current through these contacts. The directions of magnetic flux and current are arranged to blow the arc upward. Magnetic blow-out breakers are used in cases of extremely difficult service. In both types provision is made for the easy replacement of the auxiliary contacts.

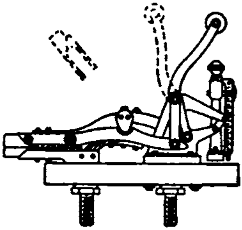


Fig. 99.—Carbon-break circuit breaker.

The main contacts in circuit breakers consist almost invariably of laminated copper brush bearing on flat copper blocks, and pressed down by a toggle joint exerting considerable pressure. Several types of breakers are shown in Figs. 99 to 102. The current densities used in the laminated contacts are from 400 to 500 amp. per sq. in. The entire breaker is designed for a temperature rise not to exceed 25 deg. to 30 deg. Cent. in any part.

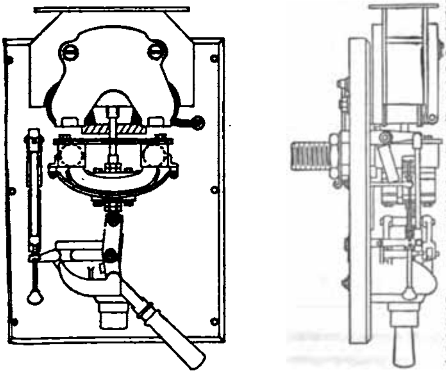


Fig. 100.—Magnetic blowout circuit breaker.

785. Circuit breakers are ordinarily hand operated by a lever, which is of sufficient power even for the largest sizes (Fig. 99). Frequently, however, **remote control** is desirable and electric or compressed-air operation is used. The breakers are usually arranged to reopen immediately without damage when closed on a short circuit. Motor-operated breakers are usually in pairs, one closing before the other, so that the second is practically a switch. All breakers are supplied with an overload **tripping mechanism** which is adjustable to open the circuit at currents from 50 per cent. to 250 per cent. of rating. Other tripping arrangements may be operated by low voltage, excess voltage, reverse current of a tripping switch located at any convenient point.

786. Multiple-pole circuit breakers are commonly used in the smaller capacities. For many purposes they are specially arranged so that if closed on overload or short-circuit, the tripping coil will release the closing mechanism and the breaker will open before the main contacts are closed. (Such breakers may be used on feeder panels, avoiding entirely the use of knife switches.

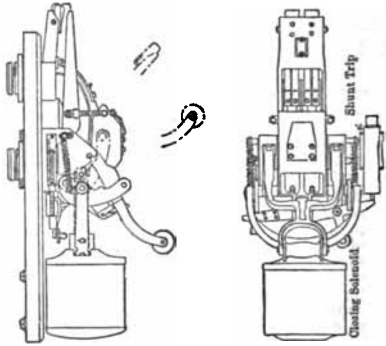


Fig. 101.—Solenoid-operated circuit breaker.

787. The cost of carbon-break **circuit breakers** is about $6.00 per 100 amp. of rated capacity per pole; magnetic blow-out circuit-breakers cost about $7.50 per 100 amp. of capacity.

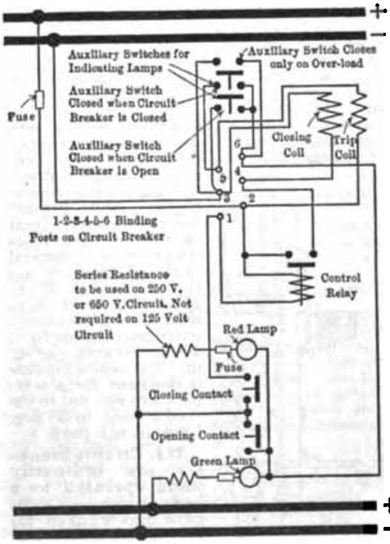


Fig. 102.—Wiring of General Electric solenoid-operated switches and circuit breakers.

788. Switchboard instruments in direct-current work should be carefully protected from stray fields. Wherever the bus-bar currents exceed 2,000 amp., all except the very best protected instruments will be affected, and to secure accurate readings they should be calibrated in place. It is customary to provide instruments with scales such that at the rated load of the machines, the needles will be at approximately mid-point.

789. The cost of switchboard instruments varies between wide limits. Ammeters may cost all the way from $15 plus $1 per 100 amp. of capacity, to $75 plus $1.20 per 100 amp. of capacity. Voltmeters may cost from $20 to $80 depending largely on the type. Watt-hour meters will vary in cost from $35 plus $1.50 per 100 amp. to $100 plus $7.50 per 100 amp.

790. Standard switchboard panels embodying the various devices described above and suitable for all classes of work have been developed and catalogued by the various manufacturers, so that it is now possible to buy complete, almost out of stock, panels suitable for almost any kind of work; Fig. 96, is taken from one of these catalogues. These panels can be purchased for a lower price than it is possible to make up special panels and should be used whenever possible. The particular panels shown in this figure would cost approximately $100 plus $12 per 100 amp. without watt-hour meter, and $220 plus $16 per 100 amp. with watt-hour meter, equipped for 250 volts.

CONSTANT-CURRENT SERIES SWITCHING

791. **Series systems** are now used in America solely for street lighting. It is probable that they will continue in use for this purpose indefinitely. The earlier installations used **series-wound generators** operating on the drooping part of the characteristic (Sec. 8). When the total pressure exceeded 2,500 volts, multiple independent armature windings and commutators were used, connected in series externally to the machine. The regulation of these machines is accomplished in two ways, by shunting the series fields or by moving the brushes, or both. Since these machines operate on the drooping part of the characteristic curve, they have considerable inherent regulation in themselves. The adjustments are made automatically by a series-connected relay or operating magnet which actuates

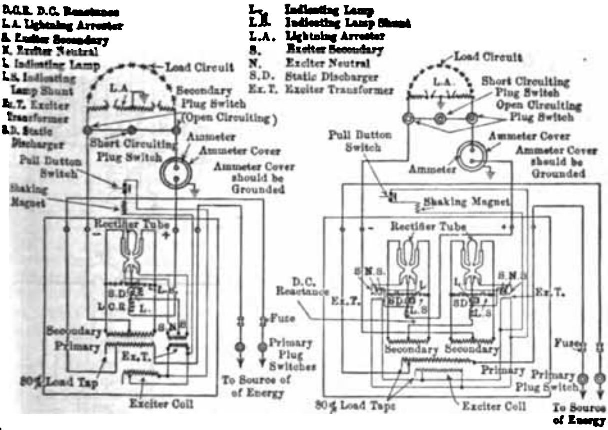


Fig. 103.—Series lighting circuits with constant-current transformers and mercury rectifiers.

the field rheostat or rocks the brushes. **Constant-current transformers** are employed very extensively for this class of service (Sec. 6) and regulate themselves almost perfectly. Each transformer is supplied with a terminal panel, etc., which forms part of the unit. Series-wound generators are installed, however, to a limited extent.

792. **Mercury-arc rectifiers** are also employed in this class of service, in connection with constant-current transformers, for series direct-curent lighting. The equipment itself is described in Sec. 6. The connections are illustrated in Fig. 103.

793. **Switching practice** is practically the same whether generators or transformers are used. Since the opening of a series circuit causes full voltage across the break, and short-circuiting causes only a relatively small increase in current, all transfers of circuits are accomplished after short-circuiting the same. Apparatus will run on short-circuit for a short time without damage. The switching is accomplished very simply by means of a transfer or carrier-bus panel as shown in Fig. 104. This arrangement is relatively inexpensive and will allow almost any conceivable switching.

794. Ammeters may be connected into any circuit by inserting the proper plug in the sockets in rows 1 or 11. When the plug is inserted the circuit is opened and simultaneously completed through the ammeter. In direct-

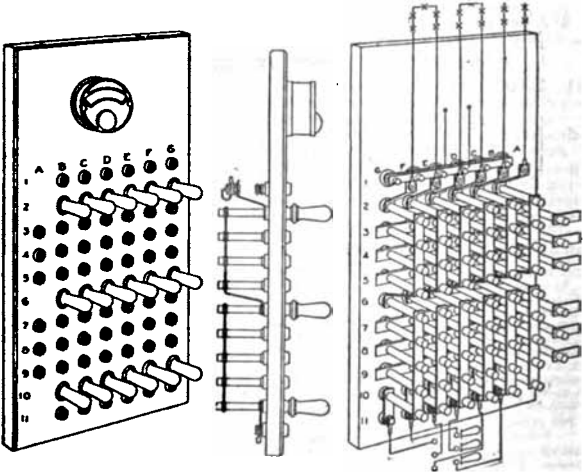


Fig. 104.—Transfer panel-series switching.

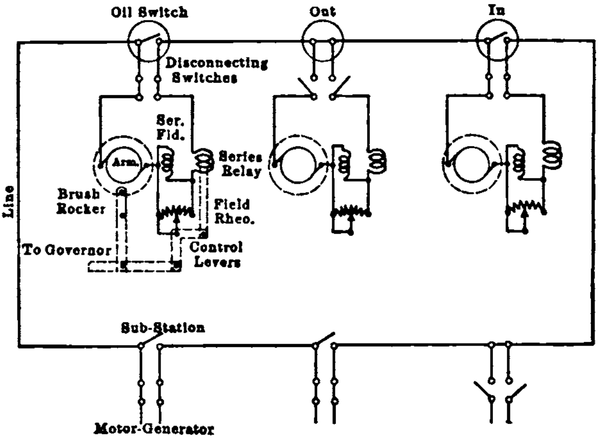


Fig. 105.—Elements of Thury system of transmission.

current work it is necessary to have the instruments in the line circuit They must be insulated for high voltage and the cases protected with the grounded covers to protect the operator. In alternating-current work a current (series) transformer may be used in connection with a standard instrument.

795. The Thury system of direct-current series transmission (Fig. 105) is not unlike the series lighting systems in use in this country. Series-wound generators are employed, developing up to 4,000 and 5,000 volts each, a sufficient number being connected in series to secure the desired transmission voltage. Currents of various magnitudes are used, approaching 200 amp. maximum. The Thury system, like our lighting systems, is essentially constant-current and is regulated to produce this condition, the voltage varying directly with the load. The system is not planned for the general distribution of energy in small quantities, but rather for transmission to substations where it is transformed to constant-potential energy. These substations are in series and each has one or more motor-generators with series-wound motors. This system has not been received with favor in this country.

796. The insulation of the machine windings. (Thury system) need not be for the full voltage. If a small number of machines is contemplated, this insulation could probably be advantageously used, but in large systems of high voltage and numerous machines, the most economical results may be expected by insulating each machine from the ground and from other machines. The difficulty lies largely in securing an insulating coupling combining mechanical with dielectric strength. The machines must each be surrounded by a highly insulated platform so constructed as to allow easy access without danger to the operators.

797. The switching (Thury system) may be exceedingly simple, as no automatic circuit-interrupting devices are required or even desired. Since the characteristics of series generators permit short-circuit currents but slightly in excess of the operating currents, no devices for protecting against excessive currents are necessary. It is probable that satisfactory switching can be secured by hand-operated non-automatic single-pole switches, one to short-circuit the terminals of the machines and two others to disconnect it from the line (Fig. 105). None of the complications usual in constant-potential systems need be considered. The short-circuiting switch should have a rupturing capacity equal to the rating of a single generator. The disconnecting switches may be of the ordinary air-break type, but insulated for the total voltage of the system. This insulation is necessary so that the machines may be disconnected from the lines and grounded for repairs.

798. The regulation (Thury system) consists in maintaining constant current and can be secured in three ways, any combination of which may be used: (*a*) shunting the series fields by an adjustable rheostat; (*b*) rocking the brushes from the neutral point; (*c*) controlling the speed. The difficulties of commutation prevent any very considerable adjustment by the first two methods, but the generators themselves are self-regulating to a large degree. The automatic regulation is secured by a series relay which operates the field rheostat or other controlling means through direct connection or by electric control. Large adjustments are taken care of by the starting up or shutting down of machines. Motors are regulated to give constant speed by centrifugal governors which actuate the field rheostat and rock the brushes. Protective devices are required for short-circuiting the terminals when the voltage exceeds rating.

ALTERNATING-CURRENT SWITCHING

799. The principles of alternating-current switching are not necessarily different from those of direct-current (Par. **761** to **790**). The same equipment and apparatus may be used for low-voltage work, except for slight modifications to avoid eddy currents. The elements of alternating-current switching arrangements are shown somewhat progressively in Fig. 106. The symbols indicate **oil switches,** (*O*) capable of being opened under load and **disconnecting switches** (*X*) incapable of interrupting any except the smallest currents.

The simplest arrangement is that shown in sketch *A* (Fig. 106), where the generators and feeders are grouped at either end of a single bus. Sketches *B* to *E* indicate arrangements for **sectionalizing the bus** by disconnecting switches, which permit the division of the plant at any desired point; some sectionalization by oil switches is frequently desirable. The

ring bus shown at *E* probably has the greatest advantage. **Double busses** are shown at *F*, *G* and *H*, which have the added advantage that any group of feeders may be supplied from any group of machines; double-throw disconnecting switches, and double-throw oil switches are shown, but the latter are not recommended. The **relay bus** system shown at *L* and *M* gives some control of the sectionalising under load.

The need for increased reliability of switching, as well as increased flexibility has led to the arrangements shown at *I* to *R* inclusive; sketch *I* shows spare oil switches arranged to replace any of the others, through disconnecting switches. In the other layouts, there is usually a spare switch for each generator or feeder group, so that no equipment need be kept out of service on account of switch trouble alone. Obviously, switching arrangements without limit can be devised to allow almost any conceivable switching operation.

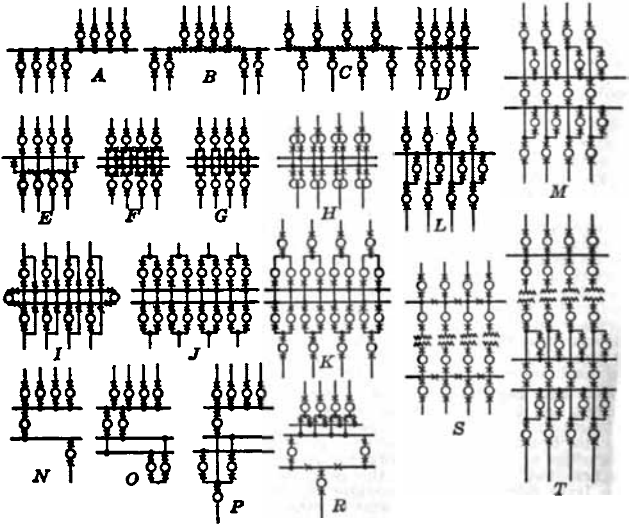


FIG. 106.—Elements of alternating-current switching.

800. Group switching is usually resorted to (Figs. 106*N* and 106*R*) in very large stations of moderate voltage, where the number of feeders is relatively large. It facilitates starting after a shut-down, and simplifies the sectionalisation of the loads for any purpose. It also provides the maximum of flexibility with a minimum of expensive oil switches.

801. High-tension power stations using transformers require both high-tension and low-tension bus systems. The great expense of high-tension switches and the large space necessary for their enclosure has led to very simple high-tension layouts. For example, the Ontario Power Company employs the complicated low-tension layout *M* and the simple high-tension layout *D* (T Fig. 106).

802. The grounded neutral has proved advantageous principally in systems having extensive underground distribution, the object being to open the circuit breaker of a grounded feeder before a short-circuit between phases can occur. A resistance is usually inserted between the neutral bus and the ground, of such magnitude that the current flowing to a grounded feeder will comfortably operate its overload relays. When three-phase star-connected generators have a third harmonic in the e.m.f. wave, it will appear as a voltage between neutral and ground. Where dissimilar machines are operated in parallel, a considerable voltage may develop between their neutrals which will make their interconnection dangerous. In such cases it is necessary to introduce resistance in these connections. It is common practice, however, to operate with but a single machine grounded, which prevents interchange of neutral current and still protects the system.

803. Current-limiting reactances are coming into use on large systems to limit short-circuit currents, thus protecting generators and limiting the duty of the oil switches. These reactances are of the air-core type (6). They are used in two ways: those permanently inserted in the circuits, and those inserted only during switching operations. The ordinary turbo-generator will deliver instantaneously on short-circuit, from 30 to 50 times rated full-load current; a reactance sufficient to reduce this to 15 times full-load is apparently all that is necessary in present installations. This reactance may be introduced in the machine leads, in the bus bars, or in the feeders. The first method is commonly used for permanent insertion in very large stations; these reactances are quite large and expensive. The second method is recommended for use between sections of the bus in stations where the switches would otherwise be inadequate. The third method is used for insertion of reactance only during the opening of a short-circuit; there are two switches in series, the first inserting the reactance (normally short-circuited), thus limiting the current to be broken by the second which is mechanically interlocked to open immediately afterward. These reactances are small and may even be included in the oil pots of large switches.

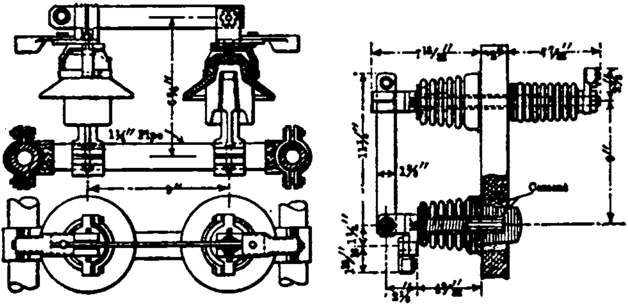


Fig. 107.—Disconnecting switches.

804. Disconnecting switches (Fig. 107) are ordinarily used for sectionalizing bus bars or circuits, or for isolating apparatus for grounding or repair, but not for switching under load. They are used sometimes, however, in place of oil switches on one side of parallel transformer banks. Disconnecting switches should not be located to open downward unless locks are provided to keep them from jarring out. In very heavy work care should be used to avoid loops in the wiring which on short-circuits will produce magnetic forces sufficient to open the switch. The cost of 300-amp. disconnecting switches is about \$5+\$0.50 per 1,000 volts. The cost of 600-amp. disconnecting switches is about \$10+\$0.60 per 1,000 volts.

805. Expulsion fuses (Fig. 108) are used for connecting potential or small auxiliary transformers to the bus. They are usually arranged also as disconnecting switches. They consist essentially of fine fuse wire confined in strong insulating tubes closed at the lower ends. The blowing of the fuse and the resulting confined arc suffice to blow the conducting vapors out of the open end, thus putting out the arc. The tubes are arranged for con-

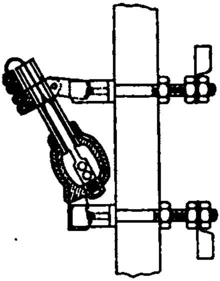


FIG. 108.—Expulsion fuse.

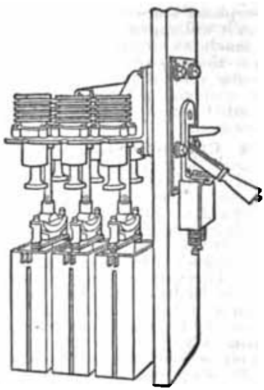


FIG. 109.—Westinghouse type E oil switch.

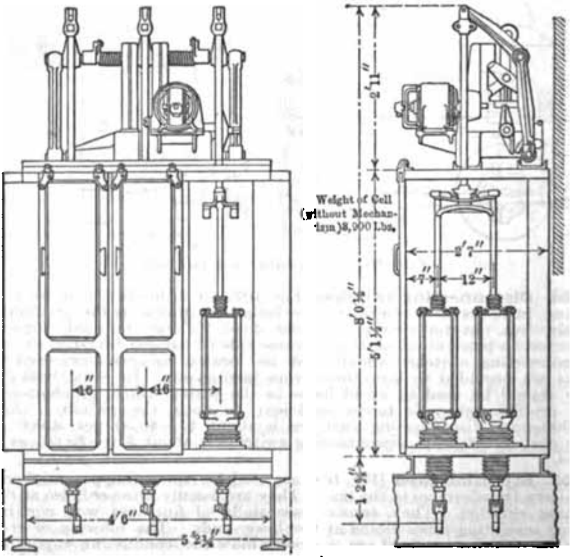


FIG. 110.—General Electric type H-3 oil switch.

venient removal and refilling. They are recommended only in small capacities and moderate voltages, although they are offered up to 60,000 volts, 20 amp. They cost about $20 plus $0.30 per 1,000 volts.

806. Oil circuit breakers are used for interrupting current at all voltages above 600. Three of the numerous types of oil switches are shown in Figs. 109 to 111. The contacts open under oil and the arc is put out by the cooling action, and the pressure of the oil. When a circuit is opened the resulting

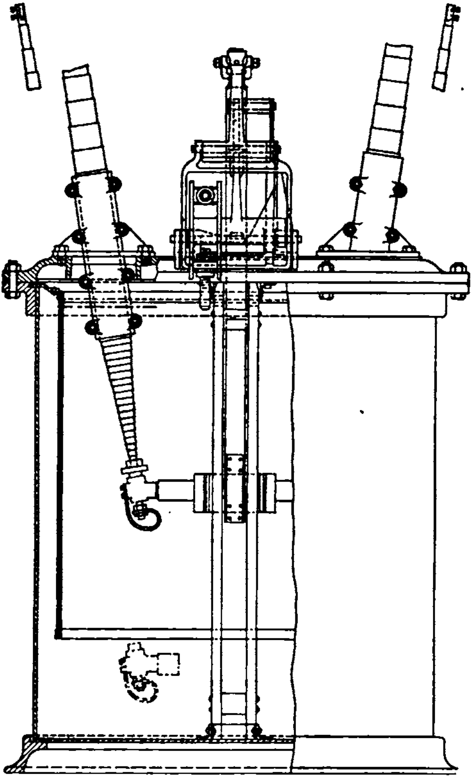


Fig. 111.—Westinghouse type GA oil switch.

arc tends to impart a more or less violent motion to the oil, away from the contacts, but its inertia and pressure due to depth resist the action and quickly quench the arc. This pressure becomes very great in violent short-circuit interruptions, and has been observed as high as 150 lb. per sq. in. a few in. away from the arc. Contacts of various types are used and all have certain advantages; many are arranged with auxiliary contact for the final breaking of the circuit. Most of the breakers are arranged to open by gravity.

A special **circuit-breaker oil** is used which is fluid at low temperatures and is free from moisture. The oil level must be carefully maintained in operation, and the oil must be changed when burned by short circuits.

807. The temperature rise should never exceed 25 deg. to 30 deg. Cent. in the hottest part at continuous full-load. The **voltage rating** is determined by the insulation distance to the grounded parts and by the breaking distance of the contacts. The Westinghouse Company designs its high-capacity high-voltage breakers, with a ground distance of about 5 in. plus 0.25 in. per 1,000 volts, and with a total breaking distance per pole of about 10 in. plus 0.33 in. per 1,000 volts. Breakers are available up to 150,000 volts.

The **current rating** is determined largely by the size of the current-carrying parts and may even be in excess of the safe rupturing capacity. Breakers for very high voltage are usually from 100 to 300 amp. capacity, while for 2,500-volt or even 11,000-volt work they are available up to 2,000 amp.

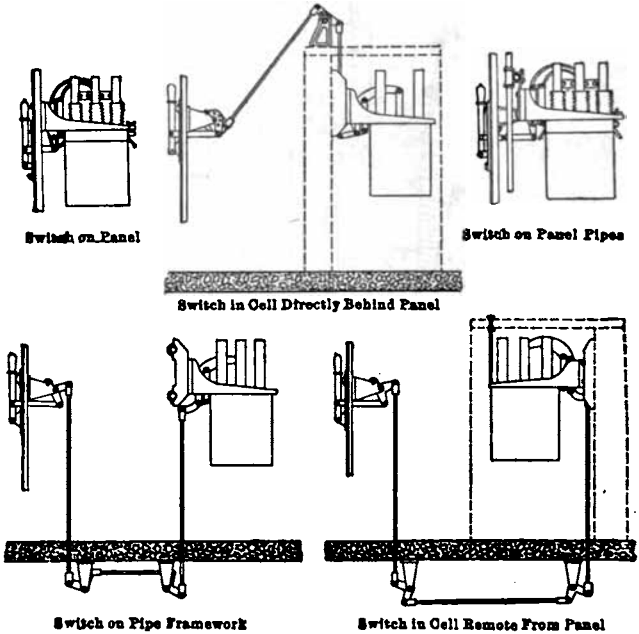


Fig. 112.—Mountings for small oil switches.

808. Oil switches for panel mounting (Fig. 112) usually have their poles enclosed in a common grounded rectangular steel oil-filled tank which is removable by dropping it away from the contacts. This type, when mounted on panels, is not desirable above 2,500 volts and 3,000 kw. station capacity.

809. Wall and framework mounting (Fig. 112) with remote control using this same type of switch (Par. 810), extends its capacity to about 12,000 kw. at 2,500 volts, or to 6,000 kw. at 15,000 volts.

810. Three-phase Ratings of More Important Types of Oil Switches

Type	Current ratings, amperes	Voltage ratings, volts	Station capacity ¼ sec. relay	Means of operation	Mounting, max. poles and throws per tank
G. E. Switch					
P	100– 200	4,500– 15,000	4,000	Manual	Wall 4 double
K_3	100– 500	2,500– 4,500	6,000–5,000	Man. or Elec.	Wall 4 double
K_2	300–1,000	2,500– 15,000	8,000–2,000	Man. or Elec.	Wall 4 single
K_4	300– 800	2,500– 15,000	12,000–6,500	Man. or Elec.	Wall 4 single
K_{12}	300–2,000	2,500– 15,000	12,000–6,500	Man. or Elec.	Wall 4 single
K_6	100– 300	22,000– 45,000	6,000–4,500	Electric	Cell 1 single
K_{10}	100– 300	22,000–110,000	20,000	Man. Elec. or Air	Floor 1 single
K_{16}	100– 140	70,000–110,000	50,000	Man. Elec. or Air	Floor 1 single
H_3	Any	2,500– 66,000	Any	Electric	Cell two tanks per pole
Westinghouse Switches					
P	100–1,000	3,300– 6,600	3,500	Manual	Wall 4 single
F	10– 200	3,300– 6,600	2,600	Manual	Wall 4 single
B	100–2,000	600– 22,000	8,500	Man. or Elec.	Wall 4 single
E	100–1,200	3,500– 25,000	10,400	Man. or Elec.	Cell 1 single
L	50– 200	60,000– 88,000	20,000	Man. or Elec.	Floor 1 single
G	100– 200	60,000–120,000	200,000	Man. or Elec.	Floor 1 single
GA	300	44,000–110,000	60,000–200,000	Man. or Elec.	Floor 1 single

811. Remote control. In most cases **electrical control** is used where hand control is undesirable. The operation in most types is secured by solenoids, a powerful one closing the switch, and a lighter one releasing a latch which permits the switch to open by gravity. In the General Electric type *H* switch (Fig. 110) a motor is employed for the purpose of winding a spring which on the release of a stop by a solenoid closes or opens the switch, depending on its position. Following the operation of the switch the motor again winds up the spring. **Pneumatic control** is used in some cases in very high-voltage work. For wiring see Figs. 102 and 113.

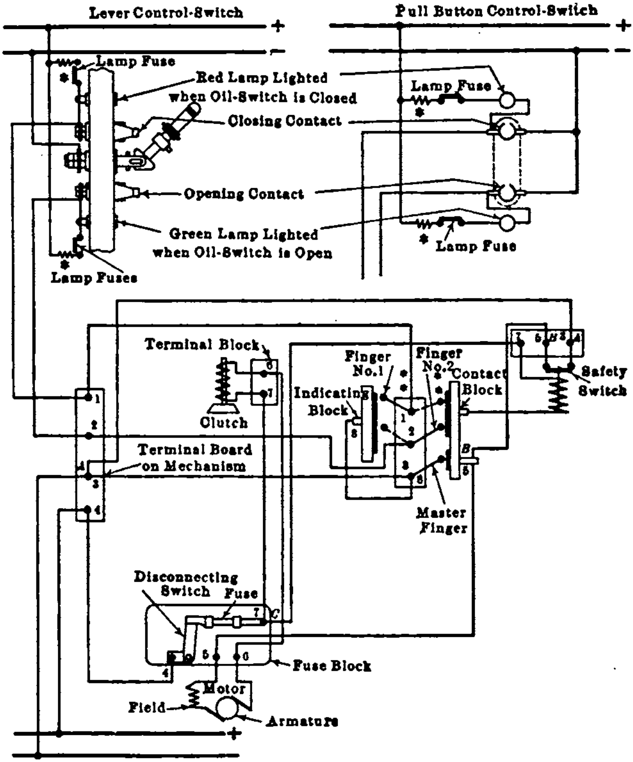


FIG. 113.—Wiring of General Electric type H oil switch.

812. The ultimate breaking capacity of oil circuit-breakers is determined by a great number of variables and the term itself is not very definite. The manufacturers use for this term the maximum size of station in which they recommend the breaker. Obviously this is not the ultimate breaking capacity. A breaker would be safe in a station of low speed units giving low instantaneous short-circuit current, while it might be destroyed in a turbo-generator station of the same size. Important factors determining this rating are voltage, length of break, size and strength of oil tank control of the oil, etc. Generally breakers with a separate tank per pole, or with a separate tank for each contact have larger breaking capacity than those where all the poles and contacts are grouped in one tank. A breaker operated at less than standard voltage will have its ultimate breaking capacity somewhat increased.

813. Hand control of oil switches is used for panel-mounted switches and for remotely mounted switches, in the smaller stations. The remote control is secured through bell cranks and rods (Fig. 112). Where the switches become numerous and are too far away, or the station is of too large capacity, hand operation is undesirable.

814. Oil switches for floor mounting (Fig. 111) have each pole enclosed in a heavy grounded tank and may be supplied with weather-proof entrance bushings and cast-iron covers for the operating mechanism, allowing use out of doors. They are available for voltages from 25,000 to 150,000 and are suitable for stations of almost any size.

815. Methods of Operating Oil Switches*

Without electric trip or with electric trip:

- **Manual**
  - On panel
  - On wall
  - Remote
    - On flat surface
    - In cell
    - On pipe framework
- **Electrical**
  - D. c. motor
  - Solenoid
    - Std. sw. op. mechanisms
    - H 3 forms
    - D. c. standard mechanisms
    - A. c. special switches
- **Mechanical**
  - Pneumatic
  - Float
  - Pressure reg.
    - Air
    - Liquid

Automatic Control

- Without series transformers
  - Series trip
    - Direct relay
    - D. c. trip
      - With or without time
        - Constant
        - Inverse
          - Overload
  - Auxiliary trip
    - Push button
    - Low voltage
      - With or without shunt transformers or series resistance
- With series transformers
  - Depending on load in secondaries
    - Without relay
      - Direct trip a.c.
        - On panel
        - On shaft
      - Without time element
        - Instantaneous
  - Depending on time element
    - With relay
      - With time element
        - Constant or inverse

Trips for the switches with series transformers:
  - Short-circuit, Overload
    - A. c. trip
    - D. c. trip
  - Reverse power, D. c. trip
  - Reverse phase
    - A. c. trip
    - D. c. trip
  - Differential low voltage under load
    - D. c. trip
  - Over-load
    - A. c. trip
    - D. c. trip
  - Reverse power, D. c. trip

Attachments:
- Auxiliary switches (Circuit opening).
- Indicating switches (Circuit closing).
- Interlocks
  - Electrical
  - Mechanical

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* Rushmore, D. B. "Electrical Connections for Power Stations," *Trans.*, A. I. E. E., May 28, 1906.

816. Switches for cell mounting cover: (*a*) the above type where such mounting increases the safe rupturing capacity about 15 per cent. (*b*) a modification of this type with individual and stronger tanks for each pole, sometimes with separate cells for each pole, which is suitable to 40,000 kw. and 2,500 volts, or even higher with special design; (*c*) switches having two separate round pots per pole each containing a separate contact, and each pole in a separate cell, which are suitable for all capacities, particularly up to 25,000 volts.

817. The source of energy for operating switches should be of utmost reliability. Storage batteries are almost invariably used except in small stations where the exciter system is used. All switching mechanisms should be designed to operate satisfactorily on a range of voltage from 40 per cent. below to 20 per cent. above normal.

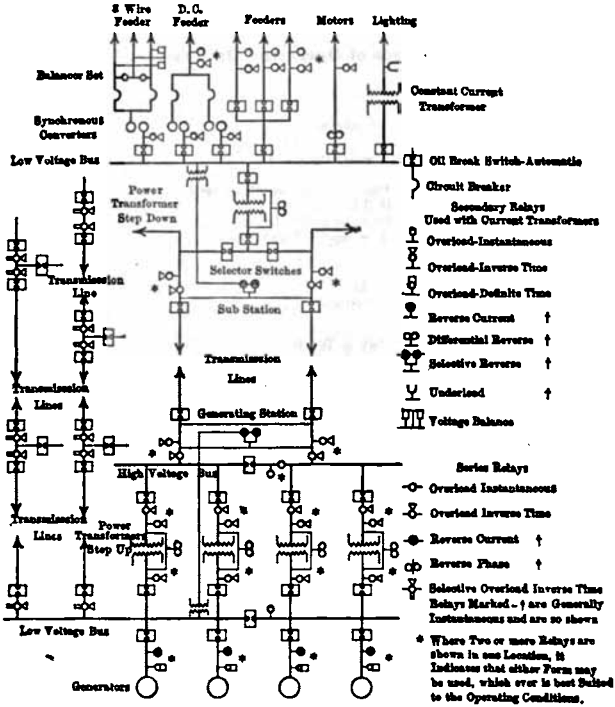

FIG. 114.—Use of relays (Hewlett, *Trans.* A. I. E. E., Mar., 1912).

818. Relays are used operating under all of the conditions outlined in, Par. 816. The most common arrangements are shown in Fig. 114 (E. M. Hewlett, *Trans.* A. I. E. E., March, 1912). The most common types are: the solenoid with plunger and the meter type very similar to the induction meter. Adjustment for current is made by position of plunger in first type and by strength of spring in second. Adjustment for time is made by bellows or oil pot resisting motion of plunger or disc and magnets resisting turning of armature of motor type. Definite time limit is secured by clockwork. Usual adjustments by bellows or magnet give a time in inversely as the actuating force.

It is important that under short circuit conditions relative settings remain unchanged. **The cost of three pole relays** varies from $25 for plain overload instantaneous to $60 for reverse current inverse time limit. Some types cost this amount per pole and special types are considerably more expensive.

819. The cost of oil switches varies between exceedingly wide limits, depending largely on rupturing capacity and voltage. Some idea of the costs may be obtained from the following three-phase non-automatic switches: For small systems up to about 5,000 kw. and 2,500 volts, about $20 plus $4 per 100 amp. capacity; for larger systems of say, 12,000 kw. and 2,500 volts, or 6,000 kw. and 15,000 volts, about $70 plus $10 per 100 amp.; for very large systems of 25,000 volts or lower, roughly $600 plus $20 per 100 amp. hand operation being unavailable. For very high voltage systems there is usually only one current rating for each voltage; such switches for 44,000 volts will cost from $400 for a 15,000-kw. rating, to $1,200 for a 50,000-kw. rating; for 110,000 volts, from $1,500 for a 15,000-kw. rating to $2,500 for a 50,000-kw. rating. The cost of electric operation is about $50 additional, exclusive of control wiring.

820. Merz-Price System of relay protection depends on unequal currents at two ends of conductor in trouble. The series transformers at the two ends are interconnected so that, normally, no current flows through the relay. Unbalance due to trouble cuts out the apparatus (Fig. 115).

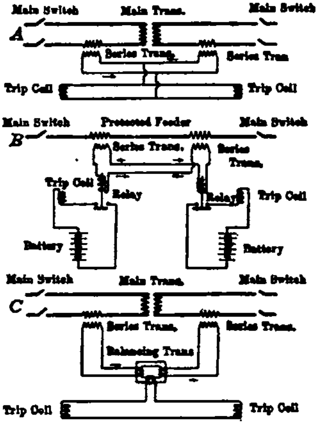

FIG. 115.—Merz-Price systems of protecting transformers and cables.

821. Instrument equipment for generator panels includes:

(*a*) One alternating-current ammeter (where phases are likely to be unbalanced an ammeter is often supplied for each independent phase, or current transformers and transfer switches for connecting the single ammeter to any phase; (*b*) one alternating-current voltmenter (or voltmeter receptacle and plug to connect to common voltmeter); (*c*) one direct-current field ammeter (optional); (*d*) one indicating wattmeter (needed for parallel operation); (*e*) one power-factor meter (or wattless kv-a. indicator, optional); (*f*) one frequency meter (optional); (*g*) one ground detector (not common); (*h*) one watt-hour meter (optional but desirable); (*i*) one synchronizing outfit; (*j*) current (series) and potential (shunt) transformers; (*k*) one automatic relay (optional).

822. Instrument equipment for feeder panels includes:

(*a*) One alternating-current ammeter (ammeter transfer switches may be used if phases are unbalanced); (*b*) one indicating watt meter (optional); (*c*) one automatic relay; (*d*) current (series) transformers.

823. Instrument equipment for station panel includes:

(*a*) Synchroniser (extra one optional); (*b*) voltmeters for each phase of each bus (optional); (*c*) frequency meter; (*d*) voltage regulator (optional); (*e*) totalizing wattmeter (optional); (*f*) totalizing watt-hour meter (optional).

824. The cost of high-grade switchboard instruments mounted with panel wiring, but exclusive of instrument transformers, is about as follows: ammeters $40, voltmeter $45, single-phase wattmeters $50, polyphase wattmeters $70, single-phase watt-hour meters $70, polyphase watt-hour meters $125. Large variations in price however exist between various types and grades.

825. The synchronizing equipment in a station must be ample enough to prevent any possibility of failure, and should be in duplicate in all except small stations; there should also be synchronizing lamps in case of failure of the instruments. There should be more than one set of instrument transformers available for the bus voltage. Synchroniser wiring should preferably be as simple as possible (Fig. 116).

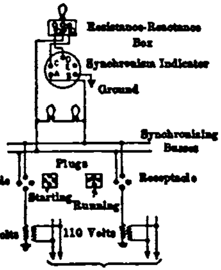

Fig. 116.—Synchroniser wiring using shunt transformers.

826. Instrument transformers (Sec. 6, and Sec. 3) are generally used whenever the currents exceed 100 amp. or the voltage 600. They are designed for 5 amp. and 110-volt secondary. Separate transformers of ample capacity should be provided for relays and for instruments. Potential (shunt) transformers are usually oil insulated for all voltages above 4,000 and current (series) transformers for voltages above 15,000. Instrument transformer wiring must be grounded thoroughly, both at the transformer and at the board, to prevent high potential on the panels in case of burnouts.

827. The cost of high-grade instrument transformers is about as follows:

Series type, 2,500 volts, $10 + $0.60 per 100 amp.; 11,000 volts, $30 + $2 per 100 amp.; 33,000 volts, $60 + $18.00 per 100 amp.; 66,000 volts, $175 + $35 per 100 amp.; shunt type, 200-watt size, $20 + $8 per 1,000 volts.

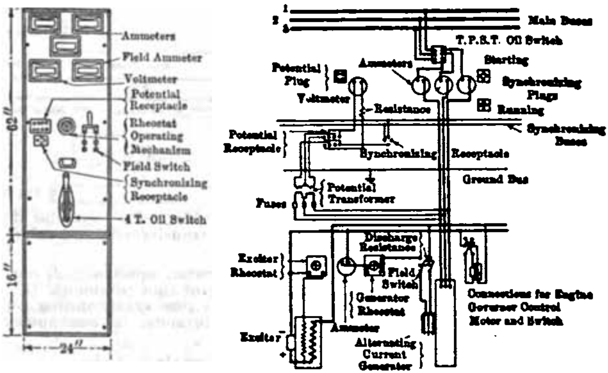

Fig. 117.—Three-phase generator panel.

828. Types of switchboards using the apparatus above described may be divided into three classes:

1. Self-contained panel type.
2. Remote mechanically operated: (*a*) panel boards; (*b*) bench boards.
3. Electrically operated: (*a*) panel boards; (*b*) bench boards.

The capacity and voltage of the station largely determine the type as discussed under oil switches, type 1 being suitable for the smaller and type 3 for the largest systems. Space requirements for the different types are not materially different. Typical panels are shown in Figs. 117 to 120.

829. Relative costs of switchboards are approximately 100 per cent. for type 1, 120 per cent. for type 2 and 140 per cent. for type 3 (Par. 828). These figures are approximate only for the same electrical layout, whereas different layouts would undoubtedly be used for the stations for which each type would be suitable. The following table gives the costs of typical panels shown in Figs. 121 to 123.

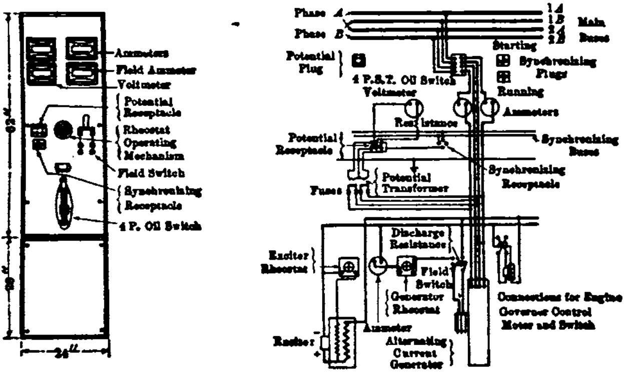

FIG. 118.—Quarter-phase generator panel.

830. Approximate Cost of Switchboard Panels
C. H. Sanderson, *Elec. Jour.*, 1913.

Kind of panel	Volts	Amps.	Fig.	Cost	U.B.C.*
Generator	2,200	"	121	440	9,200
	6,600	"		385	7,800
Feeder	2,200	"	121	345	9,200
	6,600	"		350	7,800
Generator	2,200	"	122	360	9,200
	6,600	"		310	7,800
Feeder	2,200	"	122	260	9,200
	6,600	"		265	7,800
Generator	11,000	"	123	1,240	12,500
	6,600	"		1,130	15,500
Feeder	11,000	"	123	1,045	12,500
	6,000	"		940	15,500

831. Cost of switchboards. Switchboards generally cost from $2 to $8 per kw. of capacity including all wiring and apparatus installed.

832. Mimic or miniature bus bars are invariably installed on the faces of the control boards in large stations. They represent by single bars exactly the electrical relation of the main switch controlled by each control switch and thus greatly simplify the operation.

833. Grouping of panels. It is customary to arrange the panels as follows, beginning at one end; voltage regulator, exciters, station auxiliaries, generators, feeders, with blank panels sufficient to care for any reasonable extension of the plant. With remote control, particularly electrical, this

* U. B. C. = Ultimate breaking capacity of circuit breakers set for instantaneous trip. Costs include all apparatus necessary and all structures shown but no wiring. Add 10 to 20 per cent. to cover installation.

arrangement of the switches themselves may be impossible on account of desired electrical layout, but the panels may be so arranged to advantage. Remote-control bus structures and switches should be located where there is room for considerable extension.

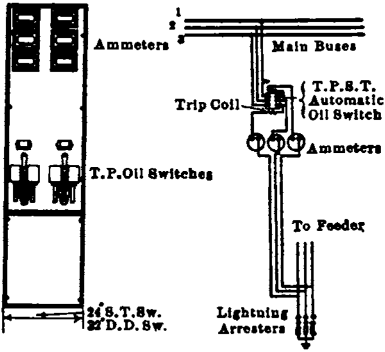

FIG. 119.—Three-phase feeder panel.

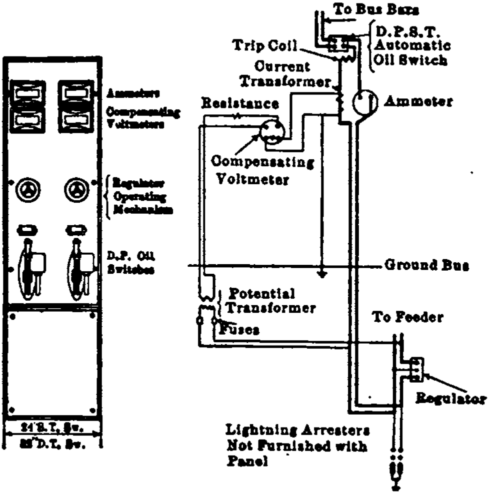

FIG. 120.—Single-phase feeder panel.

834. The operation and care of switchboards. Small stations up to say 5,000 kw. do not need any special operators for the switchboard. The engineer on watch has ample time to take care of any switching operations. With larger stations special operators become necessary; up to 10,000 kw. one operator on a watch is enough and, during the light load periods, if the arrangement of the board with respect to the turbine room is suitable, this operator may be unnecessary. Above 10,000 kw. an operator continuously on watch is necessary and frequently an additional man to take care of disconnecting switches and the cleaning and repair of oil switches. Above 20,000 kw. it is customary to have additional men until in the very large stations, there are never less than two men on the board at all times.

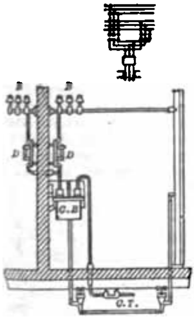

FIG. 121.

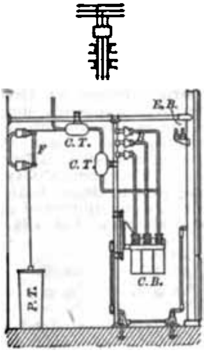

FIG. 122.

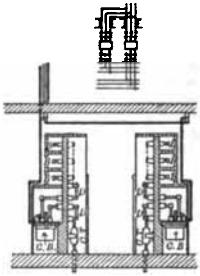

FIG. 123.

835. Bench boards are used only in large stations where the number of circuits is very large. A great saving in space is possible because the instruments do not crowd the control switches, thus allowing two or more generators to be handled on a single panel. Some of the types are shown in Fig. 124.

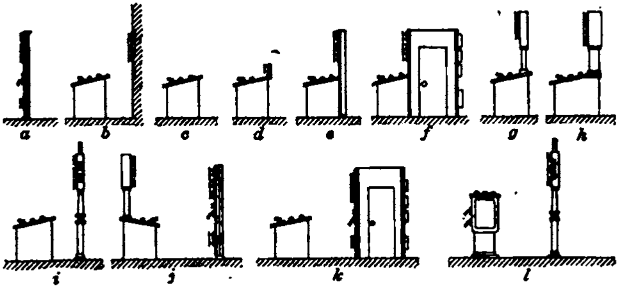

FIG. 124.—Types of bench boards.

836. Load dispatching in some form is necessary on complicated systems. The load dispatcher has full control over and knowledge at all times of the condition of the system. He usually has a map of the system indicating the condition of all electrical apparatus and issues all orders by telephone for the putting on or taking off of any machine or cable. Emergency conditions require a certain suspension of the direct control of the dispatcher, but in such cases a carefully worked out routine must be followed. Some very large systems have no man formally called a dispatcher but in such cases the senior operator in the largest station is given final authority and his orders must be obeyed.

STATION TRANSFORMER INSTALLATIONS

837. Transformers of practically all types are used for high-tension power-station work. On account of the size and voltage of most stations of this kind, the main power transformers are usually oil insulated, with or without water cooling, and of the shell type of construction. See Sec. 6.

838. Air-blast transformers are rarely used above 15,000 volts and in sizes above 1000 kv-a. They are available, however, up to 33,000 volts and 2,500 kv-a. These extremes should be avoided on account of insulation and cooling difficulty.

839. Self-cooled transformers are built in sizes up to about 2,000 kv-a. but the space required is large on account of the necessity of outside cooling tubes. Up to 1,000 kv-a. corrugated tanks give satisfactory results.

840. The question of single-phase vs three-phase transformers is not definitely settled. First cost, efficiency, simplicity of wiring and floor space, all point to the latter, but in installations of few units the desirability of flexibility frequently dictates single phase. When operated in delta, two units connected in open delta will safely carry about 60 per cent. of the load that a full bank will carry. This same ability also exists in shell type, three-phase transformers operated in delta, both primary and secondary, provided the damaged coils are disconnected and short-circuited. The relative floor space occupied is shown in Fig. 125.

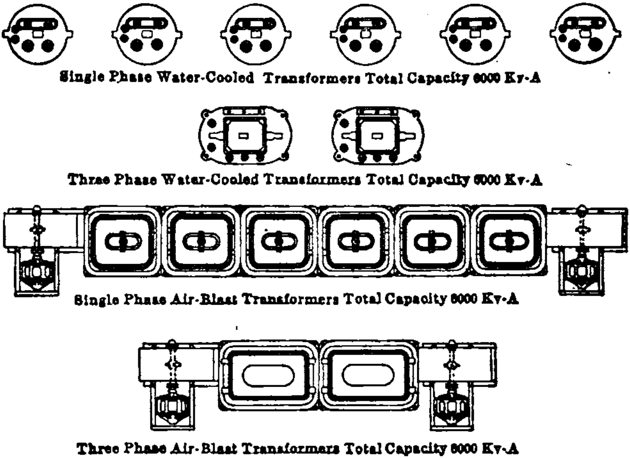

FIG. 125.—Relative floor space of single-phase and of three-phase transformers (G.E. Bulletins).

841. Water cooling is most generally used in large installations. The water supply must be continuous and pure. Where cooling water must be purchased cooling towers can be used to advantage. It is desirable to have negative pressure in the cooling pipes to prevent leakage into the oil.

842. The amount of water required is approximately 4 gal. per min. per 1,000 kv-a. capacity. This amount is not rigid. In winter considerably less is necessary, and in summer fully twice as much can be used to advantage.

843. The cost of water-cooling systems is from 10 cents to 25 cents per kv-a., depending on the source of supply.

844. Air-cooling requires blowers taking about 0.25 per cent. of the output. They are usually electrically driven and require from 3 cu. ft. per min. to 5 cu. ft. per min. per kv-a. at pressures from 1 oz. per sq. in. to 0.5 oz. per sq. in., depending on size. The first figures are for 100 kv-a. size and the latter for 1,000 kv-a. size.

845. Oil-insulated transformers are in reliable service up to 110,000 volts and 14,000 kv-a. and down to very small sizes at 25,000 volts. They are the most satisfactory in the average central station.

846. Forced oil cooling can be used for all sizes and gives satisfactory results. The transformers usually have plain boiler iron shells. The oil is pumped in at the bottom and overflows at the top, passing through a glass sight tubes to cooling coils located where good air circulation or cooling water is available. This method avoids the possibility of water leaking into the oil through defective tubes, as the oil may be kept under slight pressure. It also allows the convenient drawing of the shells without special piping. The chief objection is that a fire may put the entire equipment out of service. The cost is from 25 cents to 50 cents per kv-a. of capacity, which is considerably higher than water cooling, but there is a considerable saving in the cost of the transformers due to absence of cooling coils.

847. Fire danger from transformers, while not negligible, has been greatly exaggerated in the past. In some installations each bank has been placed in a well-drained fire-proof chamber, in addition to being equipped with a piping system to enable the rapid emptying of the shells into a buried tank. Some of the latest and very important installations have been constructed with simple barriers between banks opening into a common passage. The drainage system is still important but is used principally to facilitate repairs, inspection and the treating of the oil.

848. Convenient handling of the transformers should be provided for either by an overhead travelling crane, or by mounting on trucks on which they may be moved to an inspection pit.

849. Transformer oil must be kept particularly free from moisture; one part in 10,000 will reduce the dielectric strength 50 per cent. Drying outfits, which dry the oil by forcing it through successive sheets of blotting paper, are available at a cost of from $500 to $1,000 with capacities of from 5 to 20 gal. per min.

LIGHTNING ARRESTERS

850. Lightning protective apparatus is used in power stations to protect the apparatus therein from abnormal potentials on the system, whether caused by lightning disturbances or by switching operations. The ideal apparatus will immediately relieve the system of excess voltages, allowing no flow of the dynamic current of the system, and be ready for immediate service again.

851. The magnetic blow-out principle is frequently employed, particularly in low-voltage arresters. The spark gap is placed between the poles of an electromagnet (Fig. 126) excited by the flow of current which immediately bows out the arc. Series resistances are usually employed with this arrangement.

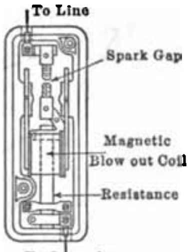

FIG. 126.—Magnetic blowout lightning arrester for low voltage.

852. Choke coils are simple open air core reactors inserted between the arresters and the apparatus to be protected to choke back the lightning disturbances, which are of very high frequency, thus allowing the arrester to discharge with a minimum strain on the station apparatus. To secure the full benefit of the choke coils, the connections to the arresters should be as straight as possible.

853. Spark gaps are used in practically all types of arresters, set at sufficient distance to prevent sparking over at ordinary voltages. Obviously something additional is needed to limit the flow of dynamic current after relieving the excess potential. Their most common use without considerable modification is to protect transformer secondaries, a single gap per bank of transformers being used.

854. Series resistances are usually inserted between simple spark gaps and ground for the purpose of arresting the current. This is accomplished, however, at a sacrifice in efficiency of the arrester in removing the disturbances which frequently have considerable current volume. This method is used, however, in many types from the lowest to the highest voltage.

855. Fuses inserted in series with a gap (Fig. 127) are not uncommon. They serve to relieve the system and interrupt the current, but they have the serious disadvantage of being able to handle only a single discharge. A simple circuit breaker is sometimes similarly used on low-voltage systems.

All of the above arrangements have their simplicity to recommend them but they are inadequate on all but very small or low-voltage systems.

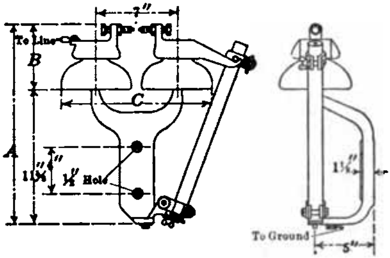

Fig. 127.—Air gap lightning arrester with fuse.

856. Horn gaps are modified spark gaps which consist of conductors arranged in a "V" with a suitable gap at the bottom. The flaring sides are shaped so that the arc in rising by the heated air is lengthened and finally blown out. Careful proportioning is necessary, but even then the horns will fail to put out the arc from a heavy current.

857. Multigaps, which consist of a large number of gaps in series, between relatively large cylinders of non-arcing (composition) metal, have many advantages. When placed between line and ground the potential drop between gaps is much greater near the line than near the ground, due to the electrostatic capacity of the cylinders (Fig. 128). This makes possible a much greater aggregate gap distance than with a single gap, which greatly aids the quenching of the dynamic arc. Lightning disturbances, being of high frequency, cause a still greater potential gradient, which will allow the disturbances to pass at a relatively low excess voltage. As soon as the gaps break down the flow of current causes an equal drop in all gaps.

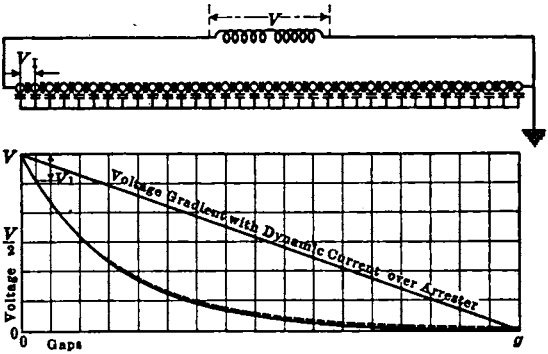

Fig. 128.—Voltage gradient across multigap arrester with and without dynamic current.

The non-arcing metal has a low boiling point and acts as a rectifier, not allowing the dynamic arc to be resumed on reversal of current, if the voltage is kept low enough. The greater the number of gaps in series the more definite this action.

858. Multipath arresters allow high voltages to discharge along the surface of a very high resistance rod in numerous very fine sparks, the normal voltage being unable to continue the arc. This type is limited to small discharges.

859. Graded shunt resistance combined with multigaps, allows a greatly increased effective number. Referring to Fig. 129, which shows the arrangement used by the General Electric Co., the full line voltage is normally across the lower group of gaps, the resistance being low enough to accomplish this result. When breaking down, the flow of current through the high resistance causes a voltage across the second lower group of gaps which, if the lightning discharge is heavy enough, are broken down, thus inserting additional gaps to quench the arc. Similarly if the arc continues due to the discharge being too heavy, additional gaps are inserted until the entire group is in series. If this number will not rupture the arc the arrester may be destroyed, this being the limitation of the apparatus.

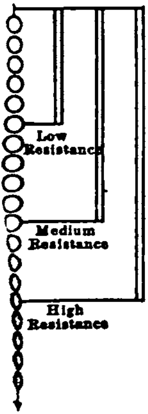

FIG. 129.—Elements of graded shunt multigap arrester.

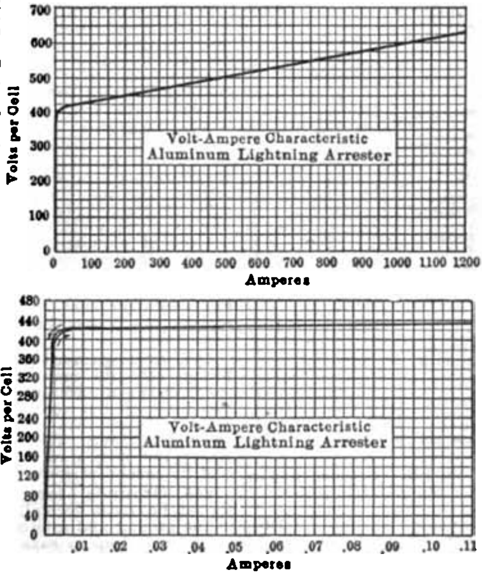

FIG. 130.—Characteristics of aluminum lightning arrester.

860. Series-resistance multigap arresters are built by the Westinghouse Company in recognition of this condition, called "Low-equivalent arresters." The series resistance is expected to limit the rate of discharge within safe limits. This type ordinarily uses but a single shunt resistance. There is considerable difference of opinion as to whether or not this series resistance is desirable. It sometimes saves the arrester, but at the expense of the apparatus by increasing the time necessary to relieve the system.

861. Care of multigap arresters requires the frequent removal of dust, best accomplished by blowing with compressed air. The arresters should be disconnected during this process.

862. Aluminum-cell lightning arresters depend for their action on the properties of aluminum when inserted in a suitable electrolyte, shown by Fig. 130. When an e.m.f. is applied between two electrodes, insulating films are formed which break down if the forming voltage is exceeded and reform immediately when the voltage becomes normal. This cell more nearly conforms to the ideal than any other arrester; it may be set for a very small increase in e.m.f.; its discharge rate is enormous and it immediately resumes normal condition after the discharge. Since the cells can be formed to withstand permanently only about 300 volts, a great number in series is required for high potential systems. Conical shaped aluminum electrodes are mounted on rods with spacers allowing a uniform space between (Fig. 131). Electrolyte is inserted in these spaces, great care being required to get uniform depth. The whole stack is then inserted in a tank of oil.

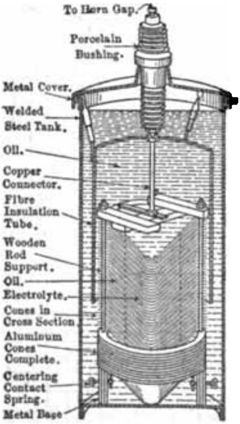

FIG. 131.—Construction of aluminum arrester.

863. Charging aluminum arresters. Continuous connection to the line is not possible on account of the loss of energy, and therefore horn gaps are usually connected in series. This requires frequent charging of the cells, at least daily, because the film gradually dissolves, more rapidly in warm weather.

Care of aluminum arresters requires not only this frequent charging, but also watching the charging current which is a guide to the condition of the electrolyte. Care should be exercised to prevent freezing of the electrolyte at very low temperatures, as it might destroy the arrester.

864. Arrangement of arresters on polyphase circuits may consist either of an arrester between each line and ground, or arresters for each line connected together on the ground side and then to ground through an additional arrester. The first method is used where the neutral of the system is grounded and the second where ungrounded.

865. Grounds for lighting arresters should be most carefully made. Numerous iron pipes driven several feet in the ground and interconnected by a copper strip are good. Copper plates buried deep in coke with earth filling are also used. In addition the arrester ground should be connected to the steel frame, piping, and other grounded materials in the building.

866. Summary of uses of various types of arresters.

(1) Spark gap—

(a) Series resistance (b) Fuse (c) Magnetic blowout	D.C. systems, all voltages. A.C. trolleys, along the line.

(d) Plain—Single point on transformer secondaries.

(2) Horn gap—

(a) Series resistance (b) Fuse	A.C. systems, all voltages, located on poles. D.C. and A.C. series systems.

(3) Multipath—Frequent location along D.C. lines.

(4) Multigap—A.C. systems.

(a) Plain—Frequent location along small distribution systems.
(b) Series resistance—Frequent location along systems up 15,000 volts.

(c) Shunt resistance (d) Shunt and series resistance.	Station use on systems of moderate size and length of line.

(5) Aluminum cell—

(a) In small jars or tanks,	D.C. systems along the line and D.C. stations.
(b) All stacks in one tank,	A.C. systems up to 7,500 volts.
(c) Separate tanks,	A.C. system up to 150,000 volts.

867. The cost of lightning arrester equipment is about as follows. Three-phase grounded-neutral systems: Aluminum cell, \$70+\$10 per 1,000 volts; multigap type, \$13 per 1,000 volts. Three-phase ungrounded: Aluminum cell, \$100+\$15 per 1,000 volts; multigap type, \$15 per 1,000 volts

868. The cost of 200-amp. choke coils of the "hour glass" type, is about: \$20+\$0.50 per 1,000 volts, up to 70,000 volts; \$30+\$0.80 per 1,000 volts, above 70,000 volts; the increased cost per 100 amp. is about \$7. Pancake-type choke coils cost about \$12+\$20 per 100 amp. for 2,500-volt service and \$40+\$40 per 100 amp. for 25,000-volt service. The cost of installation of the above can usually be covered by a 15 per cent. increase, in addition to freight.

POWER STATION WIRING

869. Bus bars are made either of copper as discussed elsewhere in this section, or, in the case of high-potential stations or moderate potential in small sizes, of copper tubing, copper rods, and sometimes brass or iron pipe. Special bends and fittings are available for this pipe work. Where the current capacity is small, the physical stiffness required dictates the size of conductor. Bus bars are seldom continuously insulated. They are usually supported in the open in small low-voltage stations, and in all stations of very high voltage. Cell structures are used in stations of moderate voltage in all except relatively small stations.

870. Switch wiring on the board, or on the switch structure, should be stiff and well insulated for the full voltage, because the wires are particularly near together at this point. Stranded wires are seldom used except for simple layouts where accurate alignment is not necessary.

871. Main generator and exciter wiring should be run by the shortest possible route and provided with ample insulation, even above ordinary requirements. This wiring is not protected by automatic switches. Multi-conductor cables are not recommended and alternating-current and direct-current wiring must not occupy the same duct.

872. Instrument and control wiring form practically the nervous system of the power plant. Great care must be used to avoid trouble and only the highest grade of insulation should be used. Iron conduit should be used for all of the wires, terminating as closely as possible to the instrument panels and to the transformers. Terminals at oil switches and instrument transformers should be most carefully protected to prevent high-tension current from reaching them during switch troubles. All instrument wiring except that in main circuit must be thoroughly grounded both at the instrument transformer and at the panel. Terminal boards on the panels are of great assistance in calibrating instruments. Alternating-current and direct-current wires should not be run together any more than can be avoided.

873. Lightning-arrester connections should be as simple and direct as possible in order to secure maximum protection. Disconnecting switches should always be used.

874. The prevention of corona on wiring of very high tension requires special care. All sharp corners on the wiring and the switching apparatus must be avoided. Also see Sec. 11.

875. Bus and switch compartments (Fig. 132) are made of brick, soapstone and concrete, either plain or reinforced. Openings for switch inspection, etc., are usually covered with asbestos doors conveniently hung. When brick is used for the main walls, soapstone is used for horizontal walls and lesser barriers. Concrete is frequently used throughout. Whatever the materials of construction, their insulating qualities are not depended upon, and the conductors should be supported on insulators designed for full voltage with a wide margin of safety. Wide differences in design are possible, as is indicated in the cuts, and it is seldom that the same arrangement is ever used in two stations.

876. The cost of bus and switch structures varies widely, from $50 per switch in small sizes to $300 per switch in large capacity switches for 15,000 volts.

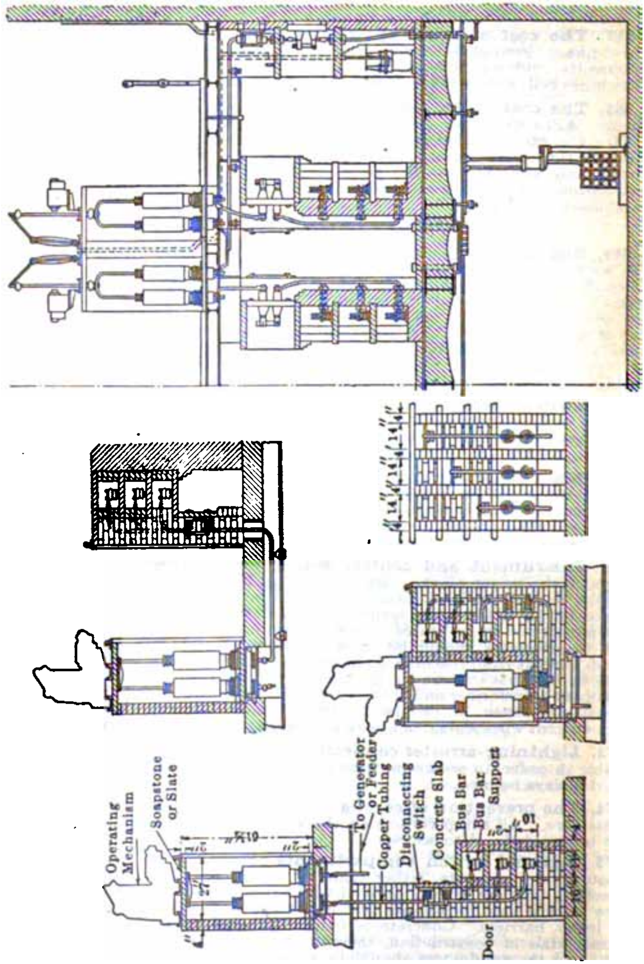

FIG. 190.—Typical bus and switch compartments.

877. Ducts and iron conduits are largely used in power station wiring but the latter should be avoided for alternating-current work of more tha very moderate current capacity. Clay or fibre ducts laid in concrete in the floor, or built into the ceiling cost, from 20 to 50 cents per duct ft. Iron conduit supported by clamps costs, in place, from 12 cents per ft. for the ½-in., to $1 per ft. for the 4-in. size. Brass pipe is sometimes used where large alternating currents must be carried in open conduit. The cost is about five times that of iron.

878. Bus-bar Compartment Dimensions

Max. e.m.f. kv.	Bus size inches	Dimensions, inches A	B	C
15	1–2 × ¼	13	12[illegible]	5[illegible]
15	2–2 × ¼	13	12[illegible]	4[illegible]
15	1–3 × ¼	13	12[illegible]	5[illegible]
15	2–3 × ¼	13	12[illegible]	4[illegible]
22	wire	15	16[illegible]	8[illegible]
33	wire	18	19[illegible]	9[illegible]
45	wire	25	26	13
66	wire	36	37	18
100	wire	56	56	28

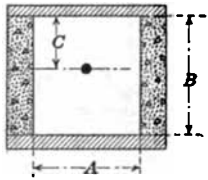

FIG. 133.

879. Spacing of High-tension Station Wiring

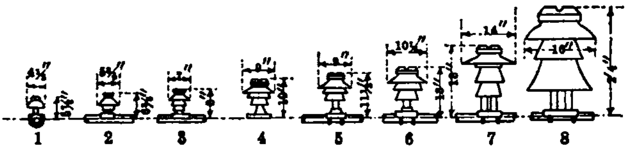

FIG. 134.

Number ref. to cut	E.m.f., volts	Spacing, inches	Ground distance, inches	Number ref. to cut	E.m.f., volts	Spacing, inches	Ground distance, inches
1	6,600	8	6	5	33,000	18	10 to 12
2	15,000	10	7	6	45,000	25	13
3	22,000	12	8 to 10	7	60,000	36	19
4	33,000	18	10	8	100,000	56	30

NOTE.—Ground distance (usually) $= \frac{\text{Wire spacing}}{2} + 1$ in.

Spacing of series transformers (self-cooled):
600 volts............ 1 in. clear 6,600 volts................ 3 clear
2,300 volts............ 1.5 in. clear 13,200 volts................ 5 clear

880. Terminals and entrances, particularly on high-tension overhead outlets, require the most careful design to prevent leakage during bad weather. Standard designs are on the market supplied by the various insulator companies. Wall outlets are usually relatively simple and cheap, but satisfactory roof bushings are available only at considerable expense. The latter are similar to the oil switch and transformer terminals used for outdoor work. Several types are shown in Figs. 136–138.

881. The cost of power-station wiring varies greatly and is exceedingly difficult to estimate. The cost of wire, supports, insulators, etc., can be obtained from manufacturers' lists, and usually an addition of from 25 per cent. to 50 per cent. will cover the cost of labor, the larger percentage being for the smaller wires. The cost of drawing into conduits varies from 1 cent to 5 cents per duct ft. The cost of joints and terminals varies

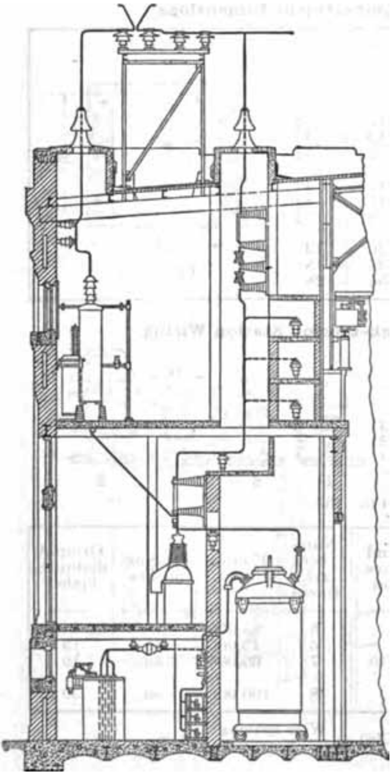

FIG. 135.—Cross section of transformer and switch galleries of high tension power station (*G.E. Rev.*, 1912, p. 598).

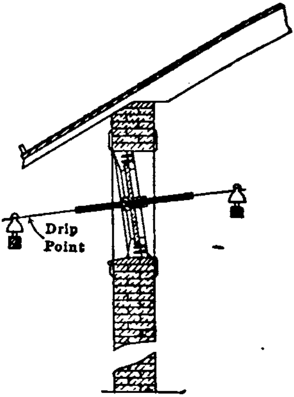

FIG. 136.—Wall outlet with slab and tube of insulating material.

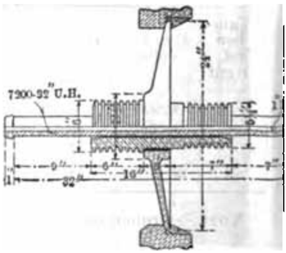

FIG. 137.—Thomas wall bushing for 66,000 volts.

between that of 5 ft. and 10 ft. of wire. The total cost of wiring varies from $0.50 to $3 per kw. of station capacity, in addition to the cost of switch gear, panels and compartments.

MISCELLANEOUS

882. Plant location. Electrical considerations seldom have preponderating influence on the location or general arrangement of a power station.

The location affects the cost of the distribution system, for moderate and low voltages, directly as the square of the average distance of the load; on high-potential systems, the variation is more nearly proportional to the distance. Other conditions being equal, however, it is obvious that the best location is as near as possible to the **load centre.**

883. Parallel operation. Large systems frequently require the parallel operation of power stations, and little difficulty is ordinarily experienced. In some cases where plants in close proximity are thus operated, the service conditions require that the stations shall not be automatically disconnected from each other except in case of trouble in the tie lines. In other cases provision is made for the immediate disconnection of the stations in case of trouble in either. The latter arrangement is used where there is no connection between the systems supplied by the stations, other than directly between the station bus bars. Where stations are widely separated, parallel operation is not satisfactory when the resistance drop in the connecting lines exceeds 15 per cent. on alternating-current systems and 50 per cent. on direct-current systems. The operation of such **interconnected systems** requires careful arrangement, frequently making it necessary to have a **load dispatcher** in full control of all switching operations.

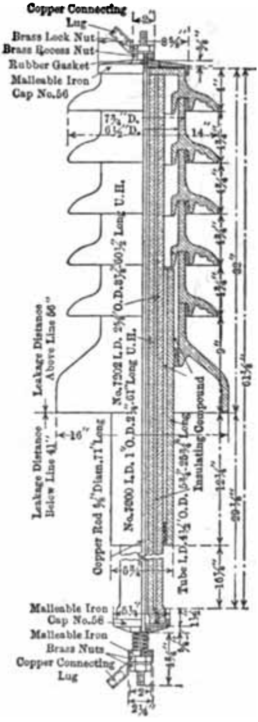

FIG. 138.—Thomas roof insulator for 66,000 volts.

POWER-PLANT ECONOMICS

BY GEORGE I. RHODES

884. Load fluctuations largely determine the desired overload capacity of units. Lighting systems have steady loads except for peaks shown by the usual load curve. Industrial loads are very steady except for certain kinds of applications to intermittent work using large units. Railway loads have considerable fluctuation even in the largest systems. Swings of five times the average are experienced when but a single car is running, twice, when about ten are running, and to 120 per cent. when a very large number are running.

885. Sudden peaks are always a possibility on a lighting system, and occur whenever a sudden storm appears. An increase in load of as much as 100 per cent. within a very few minutes is not uncommon.

886. The load factor of a machine, plant or system is the ratio of the average power to the maximum power during a certain period of time. The average power is taken over a period such as a day, or a year, and the maximum is taken over a short interval of the maximum load within that period. In each case the interval of the maximum load should be definitely specified. The proper interval is usually dependent upon local conditions and upon the purpose for which the load factor is to be determined. The yearly average of daily load factors is frequently used. (See Sec. 25.)

887. Diversity factor is the ratio of the sum of the maximum power demands of the subdivisions of any system or part of a system, to the maximum demand of the whole system or of the part of the system under consideration, measured at the point of supply.

888. The load to be carried is probably the most important factor to be considered in power-plant design. It determines the size of plant, the kind of plant, and the size and number of units. **Typical load curves** are shown in Figs. 139 and 140. These curves are such as would be obtained from 15-min. readings of watt-hour meters but do not show the exact load on

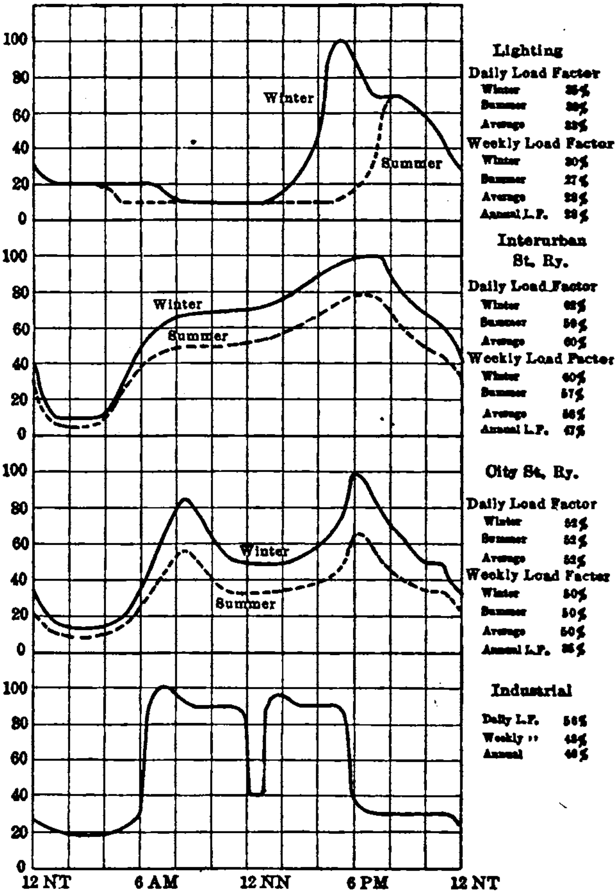

FIG. 139.—Typical load curves.

any specific plant. For typical load curves in very large central stations see *Trans.* A. I. E. E., Vol. XXXI, p. 1473, 1912.

889. Demand factor is the ratio of the maximum power demand of any system or part of a system to the total connected load of the system, or of the part of the system under consideration.

890. Capacity factor is the ratio of the average load to the total rated capacity of the equipment supplying that load. This factor is not very definite on account of variations in methods of rating apparatus.

891. Fixed charges as ordinarily defined with reference to power plants, are the charges necessary to carry the investment and to replace the equip-

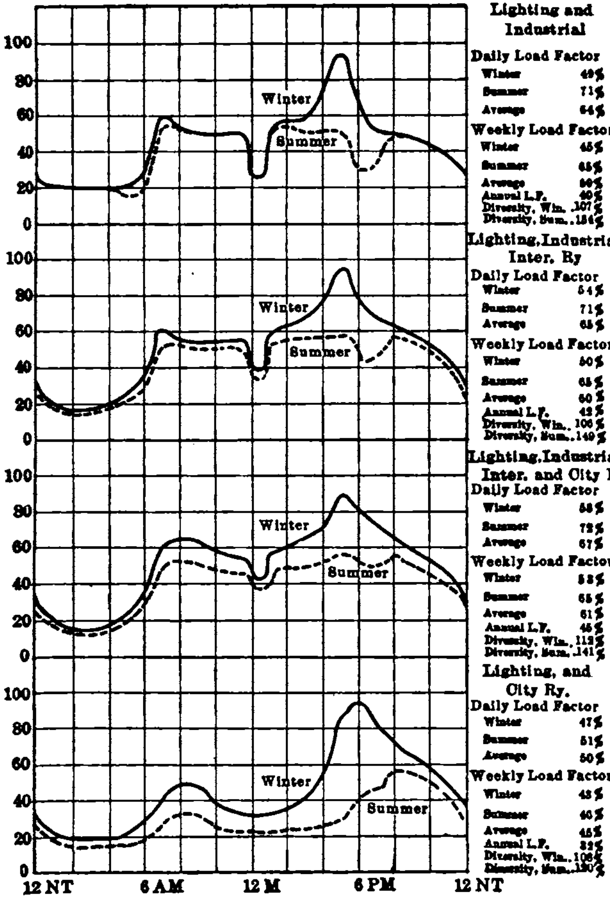

FIG. 140.—Typical load curves.

ment when it is worn out or destroyed. Interest and taxes carry the investment, while insurance and accumulated depreciation funds cover replacement.

892. Interest as used in engineering computations is the annual cost of the money required for the work. It is affected by the credit of the company and the condition of general business at the time money is borrowed. If money is raised by bonds sold below par, as is frequently the case, the cost of money is not only the only interest rate on the bond, but this amount is increased in proportion to the amount the bond is sold below par and, also, by an amount which set aside annually will make up this deficit below par when the bond is retired or paid. Six per cent. should be used as an average cost of money, but it varies between 4 per cent. and 8 per cent., the lower figure for municipalities and the higher for industrial corporations.

893. Profit on the investment, namely, income above the total of all required expenses, interest, etc., should not ordinarily be considered in engineering work.

894. Taxes are proportionately more variable than interest, ranging from less than 0.5 per cent. to as high as 2.0 per cent. It is generally the case, however, that high taxes and low interest coincide, so that the probable variation of the total of interest and taxes is from 6 per cent. to 9 per cent.; 7.5 per cent. is the figure frequently used.

895. Insurance of power plants against fire varies from less than 0.1 per cent. for fire-proof modern plants, to as high as 1 per cent. for the old type of plant with oil-soaked wooden floors, etc. Insurance in a modern plant may seem unnecessary, but the regular visits of insurance inspectors have a beneficial influence on the operation of the plant which is frequently worth more than the cost of the insurance. The figure commonly used for the cost of insurance is 0.5 per cent.

896. The cost of depreciation is one of the most commonly neglected and one of the most important elements of the total cost of power. The existence of depreciation in some form is generally recognised, but it is frequently neglected in actual operation of companies, either because of lack of income or desire for immediate profit.

897. Physical depreciation is the result of deterioration due to wear and tear caused by regular use, decay and the action of the elements.

898. Legally Approved Depreciation Rates

Compiled by Henry Floy. Maintenance not included.

A. Arbitrators, Atlanta, Ga., Street Lighting Controversy.
B. New York Public Service Commission, 1st District.
C. St. Louis Public Service Commission, St. Louis, Mo.
D. Chicago Traction Valuation Commission, Con. Traction Co.
E. Wisconsin Railroad Commission.

Property	Depreciation, per cent. per year (straight line)	Key to authority
Aerial lines	5	C
Air brakes	5	E
Air compressors	4 to 5	D
Arc lamps	6.67	E
Arc lamps	8	C
Belting	5	E
Boilers	3.5 to 4	D
Boilers	6.67	C
Boilers, water-tube	5	B
Boilers, fire-tube	6.67	E
Boilers, water-tube	5	E
Boilers, fire-tube	10	A
Bonds	5	D
Bonds	50 wearing value	B
Bonds	5	E
Breeching and connections	3.5 to 10	D
Buildings	1.5	D
Buildings, brick	2	B

898. Legally Approved Depreciation Rates.—(*Continued*)

Property	Depreciation per cent. per year (straight line)	Key to authority
Buildings	2 to 4	E
Buildings, wood	2	C
Buildings	2	A
Cables, underground, high-tension	5	B
Cables, underground, low-tension	50 maintenance cost	B
Cables, aerial, lead-covered	6.67	E
Cables, underground, lead-covered	4	E
Cables, underground, lead-covered	5	C
Coal and ash handling machinery	7	D
Coal and ash handling machinery	5	B
Coal and ash handling machinery	10	E
Condensers	4	D
Condensers	5	B
Condensers	5	E
Condensers	6.67	C
Conduits	1	B
Conduits	2	E
Conduits	2	C
Cross-arms	8.33 to 12.5	E
Engines, steam	3 to 5	D
Engines, steam	5 to 7.5	B
Engines, gas	6.67	E
Engines, steam, slow-speed	5	E
Engines, steam, high-speed	6.67	E
Engines,	5	A
Engines,	6.67	C
Feeders, weather-proof insulation	Dependent on observed wear	D
Feeders, weather-proof insulation	6.25	E
Foundations, machinery	Same as life of apparatus supported	D
Foundations, machinery	Same as life of apparatus supported	B
Fuel-oil handling machinery	4	D
Generators	3 to 8	D
Generators	5	B
Generators, modern type	5	E
Generators, obsolete	6.67	E
Generators, steam turbo	5	E
Generators, steam turbo	10	A
Generators	6.67	C
Heaters	4 to 6	D
Heaters, feed-water, closed	3.33	E
Heaters, feed-water, open	3.5	E
Meters, electric switchboard	5	E
Meters, electric service	6.67	E
Meters, electric	8	C
Motors, railway	3.33	D
Motors, railway	By inspection	B
Motors, railway	5	B
Motors, railway	5	E
Paving	50 wearing value	B
Piping and covering	4 to 4.5	D
Piping and covering	5 to 6	B
Piping and covering	5	E
Piping and covering	5	A
Piping and covering	6.67	C
Poles, steel	2	B
Poles, wood in concrete	5	E
Poles, wood in earth	5.5 to 8.33	E

898. Legally Approved Depreciation Rates.—(*Continued*)

Property	Depreciation, per cent. per year (straight line)	Key to authority
Poles, iron	2.5	E
Poles, wood	10	A
Pumps	5	D
Pumps	5	B
Pumps, small steam	6.67	E
Pumps	5	A
Pumps	6.67	C
Rolling stock, open car bodies	4	D
Rolling stock, open trailer bodies	4	D
Rolling stock, closed car bodies	5	D
Rolling stock, trucks	3.33	D
Rolling stock, closed & open cars	3.33	B
Rolling stock, trucks	3.33	B
Rolling stock, car bodies and equip	6.67	E
Stack	3	D
Stack, steel	10	B
Stokers, fixed parts	5	D
Stokers, moving parts	20	D
Storage batteries	5	B
Storage batteries	6.67	E
Storage batteries	5	C
Switchboard and wiring	3	D
Switchboard and wiring	6	B
Switchboard and wiring	5	E
Switchboard and wiring	8	C
Telephones	10	E
Track, rail joints	5	D
Track, ties	5	D
Track, rails	Dependent on observed wear	D
Track, special work	8.33	E
Track, straight and special work	50 % wearing value	B
Track, straight track	5.5	E
Transformers, station service	5	E
Transformers, station service	6.67	C
Turbines, steam	5	E
Turbines, water	3.33	E
Turbines, steam	6.67	C
Wire, trolley	Allowance of 80.5 lb. per 1,000 ft. for wear-wearing value of No. 0 wire	D
Wire, trolley	Allowance of 106.8 lb. for No. 00 wire.	D
Wire, trolley No. 0, under 1 min. headway	50	E
Wire, trolley No. 00, under 1 min. headway	40	E
Wire, trolley, No. 000, under 1 min. headway	33.3	E
Wire, weather proof	6.25	E
Wire, weather proof	7.5	A
Wire, weather proof	50 % maintenance cost	B

899. Functional depreciation is the result of lack of adaptation to function, caused by **obsolescence** and **inadequacy.** Obsolescence is due to changes or advances in the art which renders a piece of apparatus, or a whole class of it, obsolete and uneconomical of use, as compared with new types which have been developed at a later date and which are much more efficient (H. G. Stott, *Trans.* A. I. E. E., p. 1619, 1913).

900. Life expectancy of equipment. The life expectancy of power-plant equipment, taking functional depreciation into account, is as follows;*

Property	Total life (years)	Scrap value, per cent. of original cost
Buildings	75	5
Boilers, stokers and furnaces	20	5
Conveyers, elevators and hoists	20	1
Turbines, complete	12	10
Engines and condensers	12	10
Piping, values and traps	12	3
Pumps	12	5
Synchronous converters, transformers and exciters, etc.	20	10
Switching apparatus and instruments	12	5
Alternators	12	10
Motors	20	10
Tools and sundries	10	3
Storage batteries	10	10

901. Scrap value. It is obvious that in computing the net annual amount of depreciation tne scrap value of the apparatus must be deducted from the original or first cost.

902. Methods of caring for depreciation. Charging to repairs all replacement of apparatus, either in part or as a whole, is expected in many companies to care for depreciation. Ordinary repairs will not prevent a machine from finally reaching a point, due to wear and tear, where it will have to be replaced and in small companies this replacement would cost such a large proportion of the total investment that it is desirable to accumulate a fund for the purpose. If replacements are charged to repairs this item will become irregular in amount, which is a very undesirable condition. In very large companies the irregularity becomes less and consequently the method can be used with success.

903. The straight-line method of computing depreciation is based on the assumption of a uniform reduction in value. It is commonly assumed that the accumulated depreciation fund, under this method, bears no interest, but such is not necessarily the case. The method has the great advantage of being very simple in application.

904. The amortization or sinking fund method of computing depreciation assumes that the accumulated depreciation fund is invested and bears interest. The effect is to make the annual rate less, of course, than it would be if the fund bore no interest. This method is not easy of application to actual conditions. Some authorities consider that it represents, more nearly than the straight-line method, the depreciation in actual value of the property as determined by what a purchaser could afford to pay for it.

905. Calculations of depreciation, by whatever method, should be made separately for each type of equipment, taking into account its expected life and its scrap value. There is much chance for error in deciding on a percentage to apply to an entire property, and if used it should be determined from a detailed calculation. It is evidently subject to some variation from time to time as new equipment is added.

906. Obsolescence is commonly regarded as a type of depreciation to be charged to the cost of power. This is not necessarily the case. Obsolescence does not accrue from day to day, like physical depreciation, but accrues coincidently with advances in the art resulting in new and more efficient machinery or methods. The full physical life of equipment is possible in any event, whether or not obsolescence occurs, and the question whether a piece of equipment should be replaced before it wears out is determinable by equating the saving in operating expenses against increased

* Stott, H. G., "Power Costs"; *Trans.* A. I. E. E., 1913, Vol. XXXII, p. 1113.

fixed charges. In comparing different types of plants it is misleading to consider obsolescence as different for the various types.

Physical and functional depreciation costs must not be added. If functional depreciation will shorten the expected life the proper rate to care for this shortened life should be used, which includes physical depreciation. Eminent engineers have incorrectly allowed a percentage for physical depreciation and an additional percentage for functional depreciation.

907. Summary of H. G. Stott's Classification of Operating and Maintenance Costs

(Material and labor separated for each item)

Production costs	Production repairs costs
Management and care	Furnaces and boilers
Boiler room	Boiler accessories
Engine room	Engines
Electrical	Engine accessories
Fuel for steam	Piping
Water for steam	Electric generators
Lubricants	Electrical accessories
Supplies	Tools
Station expense	Building
General	General

908. Power cost data given in this section of the handbook represent as nearly as possible the **present state of the art** rather than old imformation. For instance, a great deal of data are available in reports to the various public service commissions which gives costs far in excess of these indicated here. In almost every instance the reporting companies operated plants which contain a great deal of inefficient apparatus maintained at great expense, but for less cost than the fixed charges of new apparatus. Companies operating modern plants also frequently operate old plants, but do not report them separately. While these reports give valuable data as to what is being done under old designs, they are of little use for the purpose of estimating costs in truly modern plants.

909. Boiler Room Equipment Costs per Rated Boiler Horse-power using Coal for Fuel*

	Dollars per h.p.	
	High	Low
Boilers exclusive of masonry setting	$11.00	$8.00
Superheaters	3.00	0
Stokers	5.50	3.00
Masonry settings for boilers	3.50	2.00
Flues	1.50	0.75
Stacks	4.00	2.00
Economizers	4.00	0
Mechanical draft	3.00	0
Feed pumps	1.50	0.60
Feed heaters	1.00	0.40
All piping and pipe covering	10.00	6.00
Coal chutes and ash hoppers	1.25	0
Various, such as indicating and recording devices, damper regulator, ladders and runways, painting, etc., etc	1.00	0.50
Totals	$50.25	$23.15

* Lyford & Stovel, Proc. E. S. of W. P., Jan., 1912.

910. Classification of operating expenses. A very complete form of cost analysis has been given by Mr. H. G. Stott, whose paper* should be consulted for details. His summary is presented in Par. **908**.

911. Analysis of the Average Losses in the Conversion of 1 Lb. of Coal into Electricity†

	B.t.u.	Percent.	B.t.u.	Percent.
1. B.t.u. per pound of coal supplied	14,150	100.		
2. Loss in ashes			340	2.4
3. Loss to stack			3,212	22.7
4. Loss in boiler radiation and leakage			1,131	8.0
5. Returned by feed-water heater	441	3.1		
6. Returned by economiser	960	6.8		
7. Loss in pipe radiation			28	0.2
8. Delivered to circulator			223	1.6
9. Delivered to feed-pump			203	1.4
10. Loss in leakage and high-pressure drips			152	1.1
11. Delivered to small auxiliaries			51	0.4
12. Heating			31	0.2
13. Loss in engine friction			111	0.8
14. Electrical losses			36	0.3
15. Engine radiation losses			28	0.2
16. Rejected to condenser			8,524	60.1
17. To house auxiliaries			29	0.2
	15,551 14,099	109.9 99.6	14,099	99.6
Delivered to bus bar	1,452	10.3		

912. Analysis of Thermal Losses in Power Plants ‡

Range of Common Practice

British thermal units per pound of fuel	14,000
Average yearly overall boiler and furnace efficiency	50 to 70
Effective British thermal units per pound of fuel	7,000 to 9,800
Boiler pressure, pounds per square inch, gage	125 to 190
Superheat, degrees Fahrenheit	0 to 125
Average feed-water temperature, degrees Fahrenheit	120 to 200
British thermal units per pound of steam (approx.)	1,100 to 1,100
Pounds of water evaporated per pound of fuel, actual	6.36 to 8.91
Pounds of fuel per standard boiler h.p. (33,305 b. t. u. s)	4.76 to 3.40
Average overall station water rate kw	30 to 20
Pounds of coal per k.w. generated	4.72 to 2.35
British thermal units in coal per kw. generated	66,000 to 31,500
Thermal efficiency of station	5.2% to 10.8%

* Stott, H. G. and Gorsuch, W. S., "Standardization of Method for Determining and Comparing Power Costs in Steam Plants;" *Trans.* A. I. E. E., 1913, p. 1099.

† H. G. Stott, "Power-plant Economics," *Trans.* A. I. E. E., 1906, p. 3.

‡ Lyford & Stovel, Proc. E. S. of W. P., Jan., 1912.

913. Power Plant Costs per Kilowatt
(H. G. Stott)

	Min.	Max.
1. Real estate	$3.00	$7.00
2. Excavation	0.75	1.25
3. Foundations, reciprocating engines	2.00	3.00
4. Foundations, turbines	0.50	0.75
5. Iron and steel structure	8.00	10.00
6. Building (roof and main floor)	8.00	10.00
7. Galleries, floors, and platforms	1.50	2.50
8. Tunnels, intake and discharge	1.40	2.80
9. Ash storage pocket	0.70	1.50
10. Coal hoisting tower	1.20	2.00
11. Cranes	0.40	0.60
12. Coal and ash conveyors	2.00	2.75
13. Ash cars, locomotives, and tracks	0.15	0.30
14. Coal and ash chutes	0.40	1.00
15. Water meters, storage tanks, and mains	0.50	1.00
16. Stacks	1.25	2.00
17. Boilers	9.50	11.50
18. Boiler setting	1.25	1.75
19. Stokers	1.30	2.20
20. Economizers	1.30	2.25
21. Flues, dampers, and regulators	0.60	0.90
22. Forced draft blowers, air ducts	1.25	1.65
23. Boiler, feed, and other pumps	0.40	0.75
24. Feed-water heaters	0.20	0.35
25. Piping, traps, and separators	3.00	5.00
26. Pipe covering	0.60	1.00
27. Valves	0.60	1.00
28. Main engines, reciprocating	22.00	30.00
29. Exciter engines, reciprocating	0.40	0.70
30. Condensers, barometric or jet	1.00	2.50
31. Condensers, surface	6.00	7.50
32. Electric generators	16.00	22.00
33. Exciters	0.60	0.80
34. Steam-turbine units, complete*	$10.00	$15.00
35. Converters, transformers blowers	0.60	1.00
36. Switchboards, complete	3.00	3.90
37. Wiring for lights, motors, etc	0.20	0.30
38. Oiling system	0.15	0.35
39. Compressed air system and other small auxil	0.20	0.30
40. Painting, labor, etc	1.25	1.75
41. Extras	2.00	2.00
42. Engineering expenses and inspection	4.00	6.00

914. Analysis of the Average Losses in the Conversion of 1 Lb. of Coal Containing 12,500 B.t.u. into Electricity. Producer Gas Engine Plant†

	B.t.u.	Per cent.
1. Loss in gas producer and auxiliaries	2,500	20.0
2. Loss in cooling water in jackets	2,375	19.0
3. Loss in exhaust gases	3,750	30.0
4. Loss in engine friction	813	6.5
5. Loss in electric generator	62	0.5
6. Total losses	9,500	76.0
7. Converted into electrical energy	3,000	24.0
	12,500	100.0

* Edited by Author.
† H. G. Stott, "Power Plant Economics," *Trans.* A. I. E. E., 1906, p. 3.

915. Cost of 1 H. P. per Year, Compound Condensing Engines, 10-hr. basis, 308 Days per Year*
(Wm. O. Webber, Engineer U. S., Feb. 2, 1903, p. 144)

Size of plant	Horse-power			
	200	600	1,000	2,000
Cost of plant per horse-power	$146.00	$85.00	$60.00	$56.00
Fixed charges at 14 per cent.	24.40	11.90	8.40	7.85
Coal per horse-power hour, pounds	6.5	4.5	2.5	1.5
Cost of fuel at $4.00 per ton	35.70	24.70	13.75	8.25
Attendance, 10-hr. basis	10.00	5.40	3.50	3.00
Oil, waste, supplies	2.00	1.08	0.70	0.60
Total	68.10	43.08	26.35	19.70
With coal at $5.00 per ton.	77.10	49.28	29.80	21.75
With coal at $4.00 per ton........	68.10	43.08	26.35	19.70
With coal at $3.00 per ton........	59.20	36.88	22.90	17.65
With coal at $2.00 per ton........	50.25	30.73	19.47	15.57

916. Example of operating expense. Operating cost in station containing: one, 4,000 kw. turbo-generator, new; two, 1,000 kw. turbo-generators, old; one, 500 kw. turbo-generator, old; six, 350 h.p. hand-fired boilers. Peak load 4,455 kw. Kw-hr. generated, 11,970,000 kw-hr. Year ending June 30, 1913.

	Total	Mills per kw-hr.
1. Fuel delivered in boiler room average cost $4.07 per ton....................	$56,728	4.742
2. Oil and waste..........................	620	0.052
3. Water..................................	1,601	0.134
4. Wages in station incl. superintendence	16,108	1.345
5. Building repairs.......................	810	0.068
6. Steam plant repairs....................	2,089	0.175
7. Electric plant repairs.................	3,088	0.258
8. Tools and appliances, etc.............	1,853	0.155
	82,898	6.929

917. National Electric Light Association, Form for Power Cost Record. (*Condensed*)

1. Station Wages:
   (a) Superintendence and office force:
   (b) Boiler labor.
   (c) Engine labor.
   (d) Electrical labor.
   (e) Miscellaneous labor.
2. Fuel.
3. Water:
   (a) Feed water.
   (b) Condensing water.
   (c) House water.
4. Lubricants.
5. Station supplies and expense:
   (a) Supplies.
   (b) Expense.
6. Station Buildings: maintenance, including labor:
   (a) Structure.
   (b) Fittings.
7. Steam Equipment: maintenance, including labor:
   (a) Boilers and furnaces.
   (b) Boiler auxiliaries.
   (c) Piping.
   (d) Prime movers.
   (e) Mechanical apparatus.
   (f) Tools and instruments.
8. Electrical Equipment: maintenance, including labor:
9. Hydraulic Equipment: maintenance, including labor.
   (a) Dam and pipe lines, etc.
   (b) Turbine and gates.
10. Gas Equipment: maintenance, including labor:
    (a) Gas engines and auxiliaries.
    (b) Other apparatus.
11. Purchased Power.

* Gebhardt, "Steam Power Plant Engineering," p. 711.

918. Unit Costs of Hydroelectric Developments
(O. S. Lyford, Trans. A. I. E. E., 1909)

Plant	A	B	C	D	E	F	G
	Cash	cost	per	kilowatt	of	generator	capacity
Land and water rights	$14.10	$12.86	$8.89	$14.20	$22.22	$13.07	$15.00
Hydraulic construction (dam, canals, flumes, head-gates, etc. Power house building and substructure	35.00	43.41	49.50	44.53	51.30	62.42	56.71
Hydraulic equipment	14.00	13.95	13.00	9.05	7.76	7.84	7.56
Power house electrical equipment	21.00	22.73	19.20	13.85	14.50	13.53	12.50
Transmission line, including right of way	17.20	6.26	18.30	9.00	20.70	17.50	28.50
Substation buildings and equipment	5.72	6.51	9.75	7.55	6.82	8.40	8.40
Distribution system	10.00	6.94	4.58	4.45	15.67	14.58	12.00
Interest during construction	6.30		5.54	4.75	8.40	6.18	6.16
Engineering	5.90	7.36	6.14	6.30	7.00	6.87	6.84
General and legal exp	3.70		6.20	4.45	5.77	7.48	6.84
Total	$132.92	$120.02	$141.10	$118.13	$160.14	$157.87	$160.51

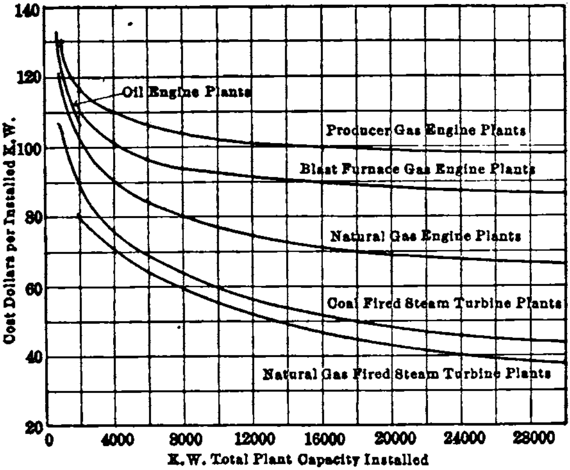

FIG. 141.—Plant installment costs (E. D. Dreyfus, *Elec. Jour.*, 1912).

919. Cost of Producer Gas Installations, 1912
(Bureau of Mines Bulletin No. 55, p. 29)

Horse-power	Cost of gas producer and engine erected, including foundations	Cost of complete plant, exclusive of buildings (a)	Cost of complete plant, including buildings (a)	Cost per horse-power		
				Gas producer and engine erected, including foundations	Complete plant, exclusive of buildings (a)	Complete plant, including buildings (a)
50	$4,300	$5,300	....	$86.00	$106.00	....
100	6,250	7,800	$9,100	62.50	78.00	$91.00
200	12,400	15,800	17,500	62.00	79.00	87.50
200	13,200	16,200	....	66.00	81.00	....
250	14,800	18,200	....	58.20	72.80	....
300	17,000	22,000	23,800	56.65	73.35	79.35
500	25,000	29,500	....	50.00	59.00	....
500	32,500	47,500	....	65.00	95.00	....
1,000	48,500	57,500	....	48.50	57.50	....
1,000	56,000	84,000	....	56.00	84.00	....
3,000	145,500	202,000	....	48.50	67.35	....

(a) Includes producer, engine, electric generator, piping, switchboard and auxiliaries, all erected with suitable foundations.

920. Influence of load factor on cost of power.

(a) **Fixed charges.** It is obvious that this cost varies inversely as the load factor. It is important, however, that the proper factor be used, namely the annual capacity factor of the total equipment installed.

(b) **Operating labor.** It is obvious that in small plants, or plants of very few units, a large amount of labor is unaffected by the load on the plant. As the number of units increases, this portion becomes relatively smaller. It is probable that the ratio of full-load labor costs per hour to no-load costs varies from 2 to 1 in plants of a single unit, to 5 to 1 in plants of a large number of units.

(c) **The coal** required to maintain a plant ready for instant service with one unit running at no-load, consists of that required for its no-load steam, that of the auxiliaries and that for banked fires under sufficient boiler capacity to carry the peak load. This total no-load coal ranges from 25 per cent. of full-load coal in a plant of one unit to 8 per cent. in a plant of many units.

(d) **Operating repairs** and other expenses are very indefinite and uncertain except over a long period of time, but probably vary in direct proportion to the load, thus being independent of the load factor.

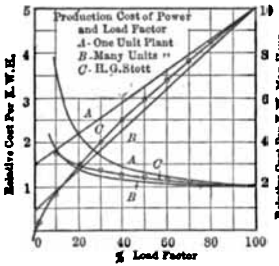

FIG. 142.—Variation in cost of power with load factor.

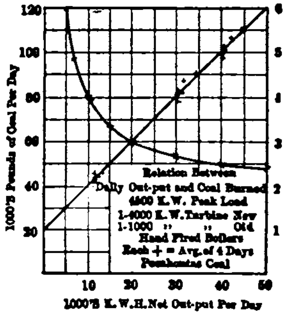

FIG. 143.—Variation in coal consumption with load factor.

(e) **Total production cost** at no-load varies from 30 per cent. of that at full load in plants of a single unit, to 10 per cent. in plants having many units. Mr. H. G. Stott has found that experience shows a variation in cost per kw.-hr. inversely as the fourth root of the load factors. Curves showing these costs are given in Figs. 142 and 143.

(f) **Period of load factor.** Since the no-load costs in a station are determined largely by the peak load expected during the month or week, it is evident that load factors for a shorter period than a year are advisable. Possibly the average daily or weekly load factor will give the best method of comparing these costs.

921. Comparison of power costs in different plants will lead to unreliable results unless certain fundamental conditions are taken into account. The chief of these are:

(a) **Certainty that costs include the same items** for each plant. Management, general expenses and building repairs are frequently omitted and care must be exercised to ascertain just what makes up the total cost. Hence the necessity of standard methods of cost accounting.

(b) **Load factors** if different require reductions of costs to the same basis. Fixed charges are inversely as the first power, and operating costs inversely as the fourth root of the load factors. Variations from 20 per cent. to 60 per cent. can be compared closely by this method.

(c) **Coal costs must** be compared on some common basis such as the cost per 1,000,000 B.t.u. available after deducting the B.t.u. in the ash.

(d) **Labor costs** must be treated similarly on the basis of the average wage per hour per man.

(e) **Fixed charges** must be on the same basis, not necessarily with the same life of equipment, but with the same method of figuring depreciation costs. It is also important to know how much spare apparatus is being maintained and whether or not the plant is complete, or one in which there is room for considerable additional apparatus.

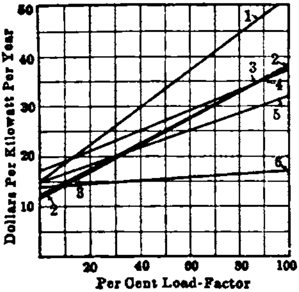

FIG. 144.—Coal cost $1.50 per ton; 11,000 B.t.u. per lb.*

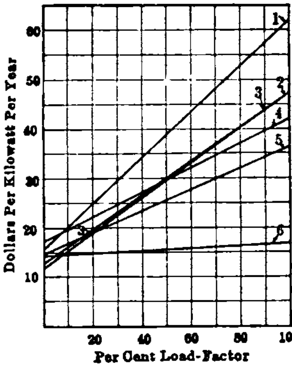

FIG. 145.—Coal cost $3.00 per ton; 14,500 B.t.u. per lb.*

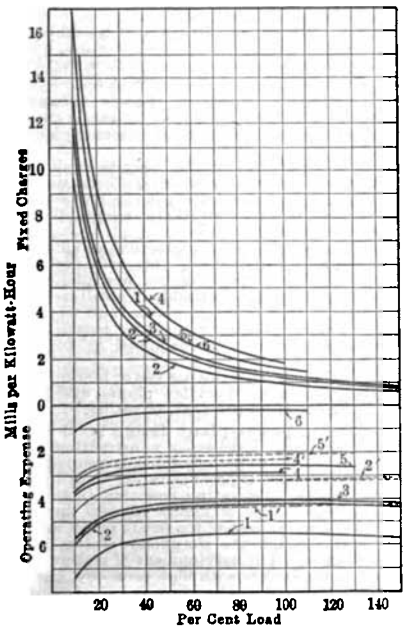

FIG. 146.—Cost of energy. (H. G. Stott, *Trans.* A. I. E. E., Dec., 1908.)

Line 1, Reciprocating steam plant; line 2, steam turbine plant; line 3, engines and low pressure turbines; line 4, gas engine plant; line 5, gas engine and steam turbine plant; line 6, hydraulic plant.

Coal cost $1.50 per ton; 11,000 B.t.u. per lb., Fig. 144 and dotted lines, Fig. 146. Coal cost $3.00 per ton; 14,500 B.t.u. per lb., Fig. 145 and solid lines, Fig. 146.

* For significance of curve numbers, see caption of Fig. 146.

922. Example of comparison of power costs.*

Let C_1, C_2 = total cost per kw-hr. in plants 1 and 2,
I_1, I_2 = fixed charges per kw-hr. in plants 1 and 2,
F_1, F_2 = fuel costs per kw-hr. in plants 1 and 2,
L_1, L_2 = labor costs per kw-hr. in plants 1 and 2,
O_1, O_2 = total other costs per kw-hr. in plants 1 and 2,
f_1, f_2 = cost per 1,000,000 B.t.u. in plants 1 and 2,
l_1, l_2 = average hourly wage in plants 1 and 2,
R_1, R_2 = average daily load factors in plants 1 and 2.
C_2^1 = total cost in plant 2 reduced to plant 1 conditions.

Example:

C_1 = 6.5 mills per kw-hr.	C_2 = 7.5 mills per kw-hr.
I_1 = 2.0 mills per kw-hr.	I_2 = 2.5 mills per kw-hr.
F_1 = 3.0 mills per kw-hr.	F_2 = 3.0 mills per kw-hr.
L_1 = 1.0 mills per kw-hr.	L_2 = 1.5 mills per kw-hr.
O_1 = 0.5 mills per kw-hr.	O_2 = 0.5 mills per kw-hr.
f_1 = 10 cents per 1,000,000 B.t.u.	f_2 = 8 cents per 1,000,000 B.t.u.
l_1 = 30 cents per hour	l_2 = 40 cents per hour
R = 50 per cent.	R = 40 per cent.

$$C_2^1 = I_2\frac{R_2}{R_1} + \left[F_2\frac{f_1}{f_2} + L_2\frac{l_1}{l_2} + O_2\right]\left[\frac{R_2}{R_1}\right]^{\frac{1}{4}} = 2.5 \times \frac{40}{50} + \left[3.0 \times \frac{10}{8} + 1.5 \times \frac{30}{40} + 0.5\right]\left[\frac{40}{50}\right]^{\frac{1}{4}}$$

$$= 2.0 + [3.75 + 1.125 + 0.5]\,[0.945] = 7.08 \text{ mills per kw-hr.}$$

That is, if the second plant were operated under the same labor and fuel cost and the same load factor as the first plant, it would cost 7.08 mills per kw-hr., as against 6.5 mills; this is less efficient operation, but not as bad as indicated by the uncorrected figures. The correction for fixed charges should more properly have been made on the basis of annual load factors.

923. Suitability of different types of power plants. Steam-electric plants are most suitable under conditions of moderate or low load factor and fuel costs. Gas-driven and hydraulic-electric plants are suitable only with high load factors and fuel costs.

The steam-engine-driven plant is suitable only in small sizes or where it is necessary to run non-condensing, under which conditions it is more economical than the turbine. It is more expensive in first cost in all sizes.

The **steam-turbine-driven plant** is of general adaptability to all sizes and loads with the above exceptions.

The **low-pressure turbine** offers a very satisfactory and most economical method of extending existing engine-driven plants with economy equal to best modern practice. It has little advantage for new plants, however.

Gas-engine driven-plants are most suitable in small sizes where first cost is not greatly in excess of that of steam plants, and where the difference in economy is much greater. Larger sizes have excessive investment costs and expensive maintenance which are balanced by fuel saving only with high load factors and steady load.

Hydraulic plants are suitable when there is a use or market for a seasonal delivery of power at high load factor. Very few plants, except at Niagara, have a summer output in excess of 25 per cent. of rating.

924. Modern tendencies in power-plant design point to inexpensive plants of high operating economy. Expense of construction is reduced by simplicity in design, using few large units; reliability is secured by the use of the highest quality of materials and apparatus rather than by duplication, with its attendant complication. Operating economy is secured through better control of combustion, the use of highly efficient apparatus whose efficiency adds relatively little to the cost of the entire plant, and the attendant reduction in labor cost through the simplicity of the plant and the small number of operating units. The development of stokers allowing very high rates of evaporation in the boilers with high economy, has reduced the investment costs in boiler equipment and building and also improved operating economy by reducing the amount of coal consumed in banked fires. There is a strong tendency toward compactness of layout, allowing only

* Adapted from Stott, H. G., Gorsuch, W. S., *Trans.* A. I. I. E.

sufficient floor space for the dismantling of apparatus during repair. The perfection of the mechanical design of turbo-generators permits the use of skeleton foundations in which the condenser can be placed with great space economy. The electrical switching equipment still has a tendency toward expensive complication, which the writer believes will gradually give way to simplicity and ruggedness, except in the largest plants.

925. Improvement of economy of existing plants requires, first of all, an accurate knowledge of all the elements entering into the cost in that particular plant, and then a gradual elimination of the elements producing inefficiency.

926. Boiler-room practice affords probably the most fruitful field for improvement, as it has hitherto been the most neglected. Numerous instruments, meters and devices are on the market which make possible a continual check on the efficiency of the boiler room. It is not only possible to know the overall efficiency, but to determine readily just what are the causes of inefficiency; there are automatic devices, also, which remove some of these causes. A careful study should be made as to the variation in efficiency with peak load and load factor, so that the inherent improvement in economy with good load factor shall not be mistaken for the results of better operation. Determinations should be made as to the proper number of boilers to use, the relation between active fire hours and banked fire hours, and just when it is profitable to let the fires go out. It is highly important that the boilers themselves be kept clean both inside and out; means are available to facilitate this work, both chemical and mechanical. The firemen should be carefully instructed in proper methods of firing and closely watched to see that they follow instructions.

927. Engine or turbine room operation offers a less fruitful field for improvement since the inherent economy of the units, more particularly of the turbines, is less under the control of the operators. With engines, however, it is highly important that valve settings be maintained properly. The proper loading of units has some influence on economy. The balance of exhaust steam produced and that needed for feed-water heating is particularly important at light loads, when ordinarily there is an excess of steam which is wholly wasted. Electric drive of some of the auxiliaries frequently serves as a corrective. Air leakage into the condenser is an important source of loss.

928. Electrical operation offers a relatively limited field for improvement in economy. It is frequently possible, however, by rearranging the ventilation of windings and keeping them properly clean, to carry a better average load on the prime movers, which adds to the economy and also to the effective size of the station.

929. Labor shifts. Labor costs can frequently be reduced by arranging overlapping shifts, thus providing the necessary men during peak loads, without unnecessary men before and after. The efficiency of labor should be measured not alone by its cost per kw-hr., but by the cost per unit of work performed.

930. Repairs should be made as soon as their necessity is discovered. A high grade of maintenance is usually cheaper than lax maintenance, and increases the effective life of the apparatus.

931. Economy in supplies does not mean cheap materials, but the choice of those best adapted to the work and which give the lowest total cost for the object accomplished.

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SECTION 11

POWER TRANSMISSION

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SECTION 11

POWER TRANSMISSION

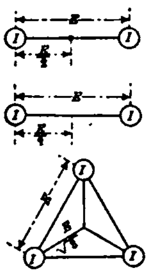
TRANSMISSION SYSTEMS

1. Systems in common use. There are three systems in common use to-day: the direct-current (Thury) system, abroad only; the single-phase system, in railway work; and the three-phase system, for general transmission. These systems may be compared as follows, in terms of effective current and voltage.

Direct-current:
Current per wire $= I$
Voltage between wires $= E$
Voltage to neutral $= E/2$
Power $= EI$

Single-phase:
Current per wire $= I$
Voltage between wires $= E$
Voltage to neutral $= E/2$
Power $= EI \cos \theta$

Three-phase:
Current per wire $= I$
Voltage between wires $= E$
Voltage to neutral $= E/\sqrt{3}$
Power $= \sqrt{3}EI \cos \theta$



2. Copper efficiency of the various systems*
(Comparisons based on effective values)

	Relative current per wire	Relative e.m.f. between wires	Relative loss per wire	Total relative conductor weight
Direct-current:				
Same max. pressure..	70.7	141	50	50
Same effective pressure...............	100	100	100	100
Single-phase.........	100	100	100	100
Three-phase.........	57.7	100	66.7	75

3. Direct-current (Thury) system.† Direct-current transmission is at present limited by the difficulty of obtaining a voltage sufficiently high for economical transmission, but Thury has developed in Europe a high-voltage, direct-current system, and a number of such plants are at present

* Voltage between wires, transmission distance, power transmitted and power loss are fixed; unity power-factor assumed. With the same effective voltage to *neutral*, all systems have the same copper efficiency.
† See Bibliography, Par. 242, No.'s 11, 12 and 13.

giving satisfactory service (Fig. 1). The required line voltage is obtained by connecting series-wound generators, in series, the voltage per commutator ranging from 1,300 to 4,000 volts. Each generator is mounted on an insulated platform and connected to its prime mover by an insulated coupling. When not in use, the generator is short-circuited. In this system the current is maintained constant by automatic devices which control the prime-mover speed, shift the brushes, and shunt the field, and the voltage is made to vary with the load. The power is delivered to motors similar in construction to the generators. The motor speed is controlled by shifting the brushes, and simultaneously shunting the field. Line voltages approximating 70,000 volts are in use, and a transmission distance of 112 miles (Moutier-Lyons) has been reached.

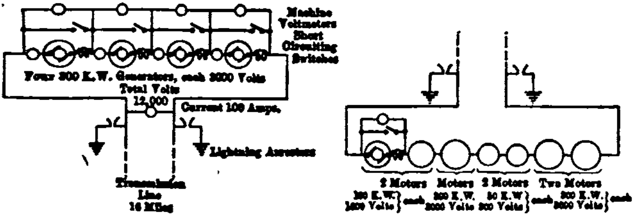


FIG. 1.—Typical Thury system.

4. Advantages claimed for Thury system: (a) power-factor always unity; (b) higher effective pressures for the same line insulation (tests by Thury indicate that with given line insulation, direct current may be twice the alternating current voltage). The maximum pressure occurs only during maximum loads; (c) no dielectric losses; (d) two wires only to be insulated; (e) underground single-conductor cable can be obtained for 60,000 volts, and with grounded neutral, the line pressure would be 120,000 volts; (f) no inductance or capacity troubles such as surges and abnormal voltage rises; (g) a number of stations can be operated in series and a station can be connected to the line at any point; (h) switching arrangements very simple; (i) in hydraulic stations under variable head, greater efficiency can be obtained since constant speed is unnecessary; (j) adapted for industrial work requiring constant torque; (k) line repairs can be easily and safely made while the line is in operation after grounding the conductor at the point in question (not applicable to systems with grounded neutral).

5. Disadvantages of Thury system: (a) insulated floors and couplings; (b) generating units must necessarily be of moderate capacity, though several generators may be connected to one prime mover; (c) the line loss is constant and independent of the load, although the current may be somewhat decreased on light loads; (d) constant-head water-wheel not ideal for constant current (variable speed); (e) special regulating devices required on motors; (f) impossibility of securing overload torque on the motors, even for a short period; (g) greater liability of damage to generators, due to lightning, and hence more expensive protective devices are required.

6. Single-phase systems, except in occasional railway installations, are little used in transmitting large amounts of energy for great distances, since for a given voltage between lines, the copper efficiency is 75 per cent. of that of the three-phase system. For power purposes the single-phase motor is less satisfactory than the polyphase motor. The output of a given single-phase generator is less than when polyphase wound.

7. Three-phase systems. Three-phase transmission and distribution are superseding the other systems, due to greater copper efficiency, the balanced condition of voltage (even under unbalanced loads) and the smaller number of conductors required in comparison with the two-phase system. The great flexibility of the three-phase system, and its advantages with respect to the use of both induction and synchronous machinery, are also important considerations in its favor.

ELECTRICAL CALCULATIONS

8. The required size of conductor for a direct-current line may be readily estimated. With a given power, P, to be delivered at voltage, E, the value of current, I, is determined from the relation:

$$I = \frac{P}{E} \qquad \text{(amp.)} \qquad (1)$$

One mil-foot of copper has a resistance of about 10 ohms at 15 deg. cent. (62 deg. fahr.). If the conductor is operated at a normal current density of 0.001 amp. per cir. mil, the drop per ft. will be 0.01 volt, regardless of the size of conductor.

9. The total voltage drop may be expressed:

$$e = 0.01 \times l \qquad \text{(volts)} \qquad (2)$$

where l is the total length of conductor in ft. If the current density is different from this normal value, e will be in direct proportion to the current density. Thus, if the density is 1 amp. per 1,200 cir. mils, and $l = 100{,}000$ ft., then $e = 0.01 \times 100{,}000\ (1{,}000/1{,}200) = 833$ volts.

10. Efficiency of transmission may be expressed:

$$\eta = \frac{E}{E+e}\,100 \qquad \text{(per cent.)} \qquad (3)$$

If the receiver voltage in the above case were 10,500, then $\eta = 10{,}500/(10{,}500 + 833)$ or 92.6 per cent.

Power lost per mil-foot of copper at the normal density is represented by the expression:

$$I^2r = (10^{-3})^2\ 10 = 10^{-5} \qquad \text{(watts)} \qquad (4)$$

Total power lost at the normal density is

$$I^2R = 10^{-5} \times c.m. \times l \qquad \text{(watts)} \qquad (5)$$

where $c.m.$ is the conductor cross-section in cir. mils, and l is the total length of conductor in ft. If the current density differs from the normal, the right-hand side of Eq. 5 must be multiplied by the square of the ratio of the densities. Constant current high-voltage systems are usually so designed that the line losses are kept within prescribed limits.

11. Example of design. Required to transmit 2,000 kw. 15 miles, with 10 per cent. line loss.

It is customary to allow 1,000 volts per transmission mile, but this may be modified somewhat by considerations of line cost. Assuming a pressure at the receiver of 15,000 volts. $I = 2{,}000{,}000/15{,}000 = 133$ amp. The permissible voltage drop $= 1{,}500$. Let $e' =$ voltage drop at normal current density; then $e' = 0.01 \times 30 \times 5{,}280 = 1{,}584$ volts at normal density. Actual density must be $\frac{1{,}500}{1{,}584}$ amperes per 1,000 cir. mils. Hence the cir. mils required are $133 \times \frac{1{,}584}{1{,}500} \times 1{,}000 = 140{,}000$.

12. The regulation and efficiency of a single-phase or a symmetrical polyphase system may be calculated by considering one conductor only, assuming a neutral which has zero resistance and zero reactance, as the return wire. A three-phase system may also be treated as a single-phase system transmitting one-half the power.

13. Alternating-current transmission-line calculations. A single-phase or symmetrical polyphase system having resistance per wire R ohms and reactance per wire X ohms and a load current to neutral of I (amp.) at power-factor, $\cos\theta$, is represented in Fig. 2 and the voltage relations are shown vectorially in Fig. 3. The electrostatic capacity is assumed negligible.

Knowing the receiver voltage, E_r, the generator voltage, E_g, may be readily calculated.

$$E_g = \sqrt{(E_r \cos\theta + IR)^2 + (E_r \sin\theta + IX)^2} \qquad (6)$$

$$\text{Per cent. regulation} = \frac{E_g - E_r}{E_r}100 \qquad (7)$$

$$\text{Efficiency} = \frac{E_rI\cos\theta}{E_rI\cos\theta + I^2R} \qquad (8)$$

If E_g is fixed and it is desired to determine E_r (6) may be used, but a cumbersome quadratic equation as a rule results. For practical work, the regulation may be assumed, the value of E_r determined from (7) and substituted in (6). If the calculated regulation then differs materially from that assumed, another trial may be necessary. If the receiver rather than the generator voltage be fixed, the problem is much simplified.

14. Example of single-phase calculation. Transmission distance, 20 miles; generating voltage, 33,000 volts; frequency, 60 cycles per sec., full-load power-factor, 85 per cent., spacing of wires, 48 in.; permissible line loss, 10 per cent. of power generated.

Fig. 2.—Equivalent transmission line to neutral.

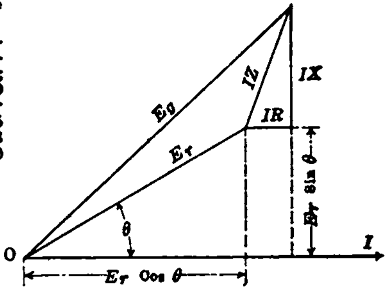


Fig. 3.—Vector diagram of transmission line.

Assumed power at receiver = 5,000 kw. Assuming 15 per cent. regulation, the e.m.f. at the receiver equals 28,700 volts, and the voltage from wire to neutral at the receiver is 14,350 volts. Current per wire = 5,000,000/(28,700 × 0.85) = 205 amp. Loss per wire = (0.10/0.90) × (5,000/2) = 278 kw. Resistance per wire = 278,000/(205)^2 = 6.61 ohms. Resistance per mile = 6.61/20 = 0.331 ohms.

From the table of Par. 38 the nearest size of wire is No. 000 A.W.G. copper having 0.336 ohms per mile or a total resistance (20 miles) of 6.72 ohms.

Reactance per mile (Par. 40) of single conductor, 0.692 ohms. Total reactance, 13.84 ohms. Substituting in (6)

$$E_g = \sqrt{[(14{,}350\times 0.85)+(205\times 6.72)]^2+[(14{,}350\times 0.527)+(205\times 13.84)]^2}$$
$$= 17{,}070 \text{ volts.}$$

$$\text{Regulation} = \frac{17{,}070-14{,}350}{14{,}350}100 = 18.95 \text{ per cent.}$$

This calculation shows material error. Assume 19 per cent. regulation, and

$$E_g = \sqrt{[(13{,}870\times 0.85)+(212\times 6.72)]^2+[(13{,}870\times 0.527)+(212\times 13.84)]^2}$$
$$= 16{,}690 \text{ volts.}$$

The calculated regulation is then $\frac{16{,}690-13{,}870}{13{,}870}100 = 20.3$ per cent. As this checks closely with the 19 per cent. regulation assumed, another trial is unnecessary, unless greater refinement is desired.

The efficiency then becomes $\eta = 2{,}500/[2{,}500 + (205)^2 6.72]$ or 90 per cent.

With aluminum as the conductor, the reactance drop would be slightly less.

15. Example of three-phase calculation of a line having the same constants as are given in Par. 14. Transmission distance, 20 miles; generating station line voltage, 33,000 volts; frequency, 60 cycles; load power-factor, 85 per cent.; spacing of wires, 48 in.; permissible line loss, 10 per cent. of power generated; power at receiver, 5,000 kw.; voltage to neutral, generating station $33{,}000/\sqrt{3} = 19{,}050$ volts. Assuming 15 per cent. regulation, voltage at receiving station = 19,050/1.15 = 16,570 volts. Current per wire = 5,000,000/(3 × 16,570 × 0.850) = 118.4 amp. Loss per wire, to neutral = $\frac{5{,}000}{3}\times\frac{0.10}{0.90} = 185$ kw. Resistance per wire = 185,000/(118.4)^2 = 13.20 ohms. Resistance per mile = 13.20/20 = 0.660 ohms.

From Par. 38 the nearest wire size is No. 1 A.W.G. which has a resistance of 0.672 ohms per mile.

Total resistance = $20 \times 0.672 = 13.44$ ohms.
Reactance per mile (Table 40) .... 0.734 ohms.
Total reactance ....14.68 ohms.
Substituting in (6)

$$E_g = \sqrt{[(16{,}570 \times 0.85) + (118.4 \times 13.44)]^2 + [(16{,}570 \times 0.527) + (118.4 \times 14.68)]}$$
$$= 18{,}840 \text{ volts.}$$

$$\text{Regulation} = \frac{18{,}840 - 16{,}570}{16{,}570} 100 \text{ or } 13.70 \text{ per cent.}$$

This checks well enough with the 15 per cent. regulation assumed.
Efficiency

$$\eta = \frac{1{,}667{,}000}{1{,}667{,}000 + (118.4)^2 13.44} = 1{,}667{,}000/1{,}855{,}000 \text{ or } 90 \text{ per cent.}$$

16. Use of complex quantity. Analytical solutions of problems involving vector quantities may be made by resolving each vector into two components, one along a horizontal or X axis, called the axis of reals, and the other along the vertical, or Y axis, called the axis of imaginaries. The latter component is preceded by $j = (\sqrt{-1})$ with $+$ or $-$ sign according to whether it leads or lags with respect to the positive real component, lag being measured in a clockwise direction.

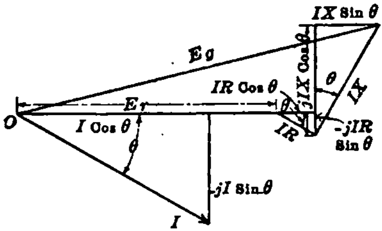


FIG. 4.—Vector diagram of transmission line using complex quantities.

Fig. 3 may be treated by this method. Referring to Fig. 4, each quantity may be resolved along E_r.

$$E_g = e + je' = E_r + I\ (\cos\theta - j\sin\theta)\ (R + jX) \qquad (9)$$

Each quantity represented in Fig. 4 is obtained by performing the multiplication in (9). (The effect of each component of current is treated as if the other did not exist.)

17. Example of analytical solution of three-phase line using complex quantity. Consider the problem of Par. 15.
Here $E_g = 16{,}570 + 118.4(0.850 - j0.527)(13.44 + j14.68) = 16{,}570 + 1{,}354 + j1479 - j840.0 + 917 = 18{,}840 + j639$. Hence $E_g = \sqrt{18{,}840^2 + 639^2} = 18{,}840$

$$\text{Regulation} = \frac{18{,}840 - 16{,}570}{16{,}570} 100 = 13.70 \text{ per cent., as before.}$$

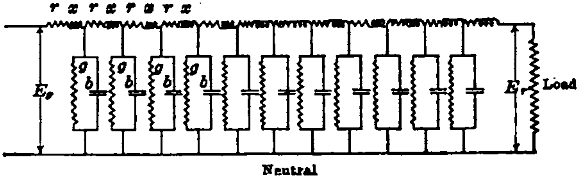


FIG. 5.—Equivalent transmission line showing capacity and leakage.

18. Line capacity has negligible effect on short or low-voltage lines, and, for such lines, may be neglected. On the longer lines of higher voltage the line charging current has a marked influence on the line regulation, and hence must be considered in the design of the line.

19. A transmission line may be represented by a single conductor having a uniform linear resistance ($R = r + r + r$) and reactance ($X = x + x + x$), and an infinite number of leaks ($g, g, g,$) and condensers or capacity susceptances ($b, b, b,$) in parallel between the line and neutral (see Fig. 5). The exact analytical treatment of such a line is rather complicated, and for practical purposes the line capacity may be considered as concentrated

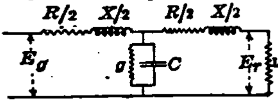


FIG. 6.—Nominal "T" line.

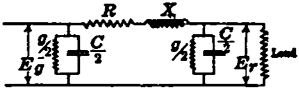


FIG. 7.—Nominal "π" line.

at the centre of the line (Fig. 6), or in two condensers, each of one-half this capacity and placed at opposite ends of the line (Fig. 7). The former is termed a nominal T* and the latter a nominal π,* or a split condenser. In the former case the entire charging current flows over half the line and in the latter case one-half the charging current flows over the entire line. The leakage current can usually be neglected. Where the corona and insulator losses are not negligible their effect may be included. If the transformers are to be included with the line, their equivalent single-phase resistance and leakage reactance must be added to the line resistance and reactance respectively; the energy component of the no-load current must be added to the line-leakage current and the quadrature component or magnetizing current must be subtracted from the line-charging current.

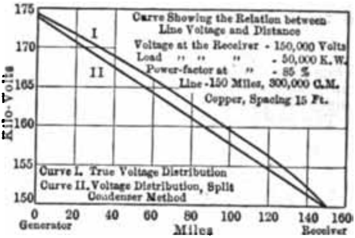


FIG. 8.—Voltage drop in line.

20. Voltage distribution. The true voltage distribution and the approximate voltage distribution for a 150-mile, 150,000-volt line are shown in Fig. 8. It will be noticed that the difference is very slight, especially near the ends.

21. The charging current for a given voltage, E_c, is

$$I_c = 2\pi f E_c C \qquad \text{(amp.)} \qquad (10)$$

where f is the frequency in cycles per sec., E_c the pressure across the condenser terminals in volts, and C the condenser capacity in farads. The values of I_c for different spacing and sizes of conductor are given in Par. 42, 43.

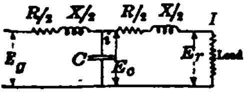


FIG. 9.—Nominal "T" line.

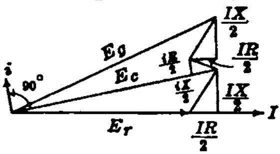


FIG. 10.—Vector diagram of nominal "T" line. Power-factor = 1.0.

22. A nominal "T" line is shown in Fig. 9, and Fig. 10 shows the vector relations in the circuit for unity power-factor load. Fig. 11 gives the vector diagram when the load current lags by an angle θ with respect to the load voltage. It will be observed that the condenser charging current is in quadrature with and leading the voltage E_c, consequently the IR drop leads E_c by 90 deg. and iX leads E_c 180 deg. It will also be noted that E_c is at some potential between E_g and E_r, but for practical considerations, E_c is assumed equal to E_r, and no appreciable error results. In fact, the cal-

* See Bibliography 16.

culated charging current may be in error many times this amount, due to the fact that the e.m.f. wave of the alternator may differ appreciably from a sine-wave. This last factor should be carefully investigated if a high degree of accuracy is desired. It is evident, from inspection, that Figs. 10 and 11 are not capable of simple geometrical solution.

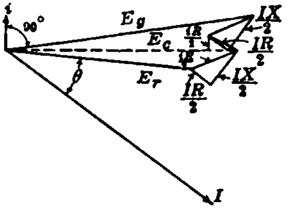


Fig. 11.—Vector diagram of nominal "T" line. Power-factor $= \cos\theta$.

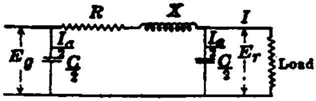


Fig. 12.—Nominal "π" line.

23. A nominal π line is shown in Fig. 12, and the vector diagram is shown in Fig. 13. The condenser current $I_c/2$ leads E_r by an angle of 90 deg., and is readily combined with the current I to form the total current I_o.

$$I_o = \sqrt{(I\cos\theta)^2 + (I\sin\theta - I_c/2)^2} \tag{11}$$

$$\cos\theta_o = I\cos\theta/I_o \tag{12}$$

where $\cos\theta$ and $\cos\theta_o$ are the load and resultant power-factors, respectively. Sin θ becomes negative, when I leads E_r, but the square of the second term under the radical still remains positive.

The problem is then treated by the method employed in Par. 12 to 17.

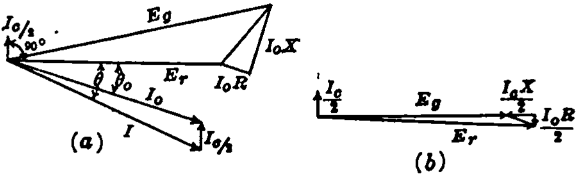


Fig. 13.—Vector diagram showing effect of line charging current.

24. Example of calculation of a three-phase system with capacity considered. Power to be transmitted, 50,000 kw. Substation e.m.f., 140,000 volts. Distance, 120 miles. Frequency, 60 cycles. Full-load power-factor, 90 per cent. Spacing of wires, 168 in. Allowable power loss in line, 10 per cent. of power at receiver. Power lost in each wire $= 5{,}000{,}000/3 = 1{,}667{,}000$ watts. Voltage to neutral (receiver) $= 140{,}000/\sqrt{3} = 80{,}830$. Load current per wire $= \dfrac{50{,}000{,}000}{3 \times 80{,}830 \times 0.90} = 229.2$ amp. Resistance per wire $= 1{,}667{,}000/(229.2)^2 = 31.75$ ohms. Resistance per mile $= 31.75/120 = 0.2646$ ohms.

From Table 38, 0000 copper or 336,420 cir. mil. aluminum have the nearest resistance, 0.2667 ohm per mile. The total resistance is $0.2667 \times 120 = 32$ ohms. From Table 41, by interpolation, the reactance per mile of wire $= 0.795$ ohm. Total reactance $= 0.795 \times 120 = 95.40$ ohms. From Table 42 the charging current per mile per 100,000 volts is found to be 0.541 amp. Total charging current $= \dfrac{80{,}830 \times 0.541 \times 120}{100{,}000} = 52.47$ amp. Charging current at the end of the line $I_c/2 = 52.47/2 = 26.24$ amp. From Eq. 11 and 12, Par. 23, $I_o = \sqrt{(229.2 \times 0.90)^2 + (229.2 \times 0.4358 - 26.24)^2} = 219.0$ amp.

$$\cos\theta_o = (229.2 \times 0.90)/219 = 0.9420$$

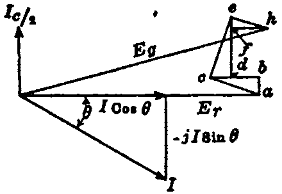
$$E_g = \sqrt{[(80{,}830 \times 0.9420) + (219.0 \times 32.00)]^2 + [(80{,}830 \times 0.3355) + (219.0 \times 95.40)]^2} = 96{,}020 \text{ volts}$$

When the receiver load is removed, the substation voltage will rise, due to the line charging current flowing through the inductance of the line. From (b) Fig. 13. E_r will be, on open circuit:

$$E_r = \sqrt{E_g^2 - \left(\frac{I_c}{2}R\right)^2} + \frac{I_c}{2}X \qquad (13)$$

The $\frac{I_c}{2}R$ term is negligible. $E_r = 96{,}020 + (26.24 \times 95.40) = 96{,}020 + 2{,}500 = 98{,}520$ volts.

Regulation $= \frac{98{,}520 - 80{,}830}{80{,}830}$ or 21.90 per cent. Efficiency $= 50{,}000/[50{,}000 + (3 \times \overline{219.0^2} \times 32.00)]$ or 91.55 per cent.



$ab = +jIcR/2$
$bc = -IcX/2$
$cd = +IR\cos\theta$
$de = +jIX\cos\theta$
$ef = -jIR\sin\theta$
$fh = +IX\sin\theta$

Fig. 14.—Vector diagram of line using complex quantities.

25. The analytical solution of a nominal π line having appreciable capacity may be obtained by adding the voltage drops due to the load current and that due to one-half the total line charging current (passing through the line impedance) to the receiver voltage.

$$E_g = E_r + I\,(\cos\theta - j\sin\theta)\,(R + jX) + \frac{jI_c}{2}(R + jX) \qquad (14)$$

where E_g = the generator voltage from phase wire to neutral; E_r = the receiver voltage from wire to neutral; I = the load current; θ = the phase angle of the load; R = the resistance per wire; X = the reactance per wire; and I_c = the total charging current. The geometrical position of each quantity is shown in Fig. 14.

This diagram determines the effect upon the generator voltage of different load conditions at the receiver, the receiver voltage being kept constant.

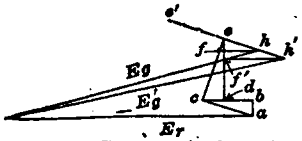


Fig. 15.—Constant load, variable power-factor.

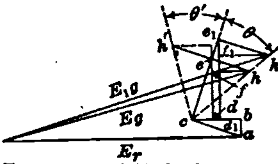


Fig. 16.—Variable load, constant power-factor.

26. Constant load and variable power-factor. Since the receiver voltage is assumed constant, triangle abc (Fig. 15) will not change. Triangles cde and efh, are similar. Triangle cde will not change, since $I\cos\theta$ is fixed. $eh/ce = fh/de = (IX\sin\theta)/(IX\cos\theta) = \tan\theta$. $I = W/(E_r\cos\theta)$ or $I\sin\theta = W\tan\theta/E_r$. ce, W and E_r are fixed, hence $I\sin\theta$ varies as $\tan\theta$. Therefore the point h must move along eh, toward h' if the lag of the current increases and toward e' if the lead of the current increases, and eh must be proportional to $\tan\theta$.

27. Variable load and constant power-factor. (Fig. 16). $ef/cd = (IR\sin\theta)/(IR\cos\theta) = \tan\theta$.

Since cde and efh are similar triangles, $ef/cd = eh/ce = \tan\theta$.

Therefore, angle $ech = \theta$, which is constant, and h must always lie on ch.

Since cd, and de, are both proportional to I, e will move along ce and eh will always be perpendicular to ce. θ' shows the construction for a leading current.

28. Constant voltage maintained at receiver by use of synchronous apparatus. The generator voltage, assumed constant, will move along arc h_3h_1 (Fig. 17). The length ce_1 is equal to $I \cos \theta\ (R+jX)$ at no-load. It is then proportional to the power taken by the synchronous apparatus, and e_1 will move along ce_3 with increase of load. e_1h_1 is equal to $I \sin \theta\ (R+jX)$, hence is proportional to the quadrature component of the current. In order to maintain constant voltage conditions, with increase of load, e_1h_1 must decrease until at e_2 it is equal to zero and this point must correspond to unity power-factor. With further increase of load e_1h_1 reverses and the current leads as at e_3h_3. This condition may be obtained by compounding, or by changing the excitation of the synchronous apparatus, either automatically, or by hand.

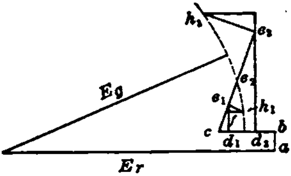


Fig. 17.—Constant voltage at receiver.

29. Example of analytical solution of nominal π line using complex quantity. Consider problem of Par. **24**, see Par. **25**, $E_g = 80{,}830 + 229.2(0.90 - j0.4358)\ (32.00 + j95.40) + (j52.47/2)(32.00 + j95.40) = 80{,}830 + 6{,}600 + j19{,}680 - j3{,}200 - j^2 9{,}530 + j840 + j^2 2{,}500 = 94{,}460 + j17{,}320 = 96{,}020$ volts.

At no-load $96{,}020 = E_r + (j52.47/2)(32.00 + j95.40)$.

The quadrature resistance drop is negligible. Hence $E_r = 96{,}020 + (26.24 \times 95.40) = 98{,}520$ volts, which checks with the result obtained in Par. **24**.

30. Exact methods of calculating regulation may be necessary if the line is long, the line voltage high or if harmonics need be taken into consideration. A simple method of making the necessary corrections in such cases has been developed by A. E. Kennelly.*

Fig. 18 shows a nominal π line of linear impedance $Z = R + jX$, and a capacity admittance, $Y/2$ at each end of the line. If $R + jX$ be multiplied by $(\sinh \theta)/\theta$, and $Y/2$ by $(\tanh \theta/2)/(\theta/2)$, where $\theta = \sqrt{ZY}$, the line as far as the ends (and beyond) are concerned will behave exactly as if the line constants were uniformly distributed. The new line is then considered an *equivalent* π and its characteristics are then calculated by the method of either Par. **24** or **25**.

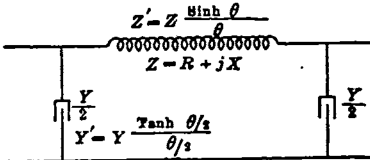


Fig. 18.—Nominal and equivalent "π" line.

31. The general equations of the transmission line in hyperbolic functions are:

$$E_1 = E_2 \cosh \sqrt{ZY} \pm I_2\sqrt{Z/Y} \sinh \sqrt{ZY} \tag{15}$$

$$I_1 = I_2 \cosh \sqrt{ZY} \pm \left(E_2 / \sqrt{Z/Y}\right) \sinh \sqrt{ZY} \tag{16}$$

where E_1 and E_2 are the respective voltages from phase wire to neutral at the sending and receiving ends of the line if the sign is assumed positive. If the sign is taken negative E_1 and E_2 are the respective voltages at the receiving and sending ends of the line. The same relation holds true for the currents, I_1 and I_2. Ordinarily the positive sign is used, as the receiver voltage and load current are generally known.

Z is the impedance per wire $= R + jX$, and Y is the admittance from phase wire to neutral $= G + jB$, though the leakage G is usually negligible in an actual line.

Expanding the cosh and sinh terms in a converging series by Maclaurin's Theorem, and neglecting the terms after the second, the following expressions are obtained.

$$E_1 = E_2\ (1 + ZY/2) \pm ZI_2(1 + ZY/6) \tag{17}$$

$$I_1 = I_2\ (1 + ZY/2) \pm YE_2(1 + ZY/6) \tag{18}$$

* Bibliography 16.

32. Example of calculation using general equation Par. 31. Consider again the problem of Par. **24.**

$E_1 = E_g$ = generator voltage, $E_2 = E_r$ = receiver voltage = 80,830 volts. $Z = R + jX = 32.00 + j95.40$, $Y = G + jB = 0 + j6.49 \times 10^{-4}$, I_1 = generator current, to be determined. $I_2 = I(\cos\theta - j\sin\theta) = 229.2(0.900 - j0.4358) = 206.3 - j99.88$.

$$E_g = 80{,}830\left[1 + \frac{(32.00 + j95.40)(j6.49 \times 10^{-4})}{2}\right] + (32.00 + j95.40)(206.3 - j99.88)\left[1 + \frac{(32.00 + j95.40)(j6.49 \times 10^{-4})}{6}\right]$$
$$= 94{,}260 + j17{,}230 = 95{,}820 \text{ volts}$$

The no-load receiver voltage and the regulation may be found as in Par. **24**, Eq. 13.

Likewise I_g is readily determined.

$$I_g = (206.3 - j99.88)\left[1 + \frac{(32.00 + j95.40)(j6.49 \times 10^{-4})}{2}\right] + (j6.49 \times 10^{-4})(80{,}830)\left[1 + \frac{(32.00 + j95.40)(6.49 \times 10^{-4})}{6}\right]$$
$$= 200.8 - j42.74 = 205.2 \text{ amp.}$$

33. Charts and diagrams, when drawn to a sufficiently large scale, are convenient for determining the electrical characteristics of transmission lines. It is essential, however, that the precision and limitations of the curves be known. Even if great refinement of calculation be desired, such diagrams are useful for checking computed results. In a handbook it is not possible to present the diagrams on a sufficiently large scale. It has seemed wise to explain their construction with data, their methods of use, and also to refer to the original articles in which they are described.

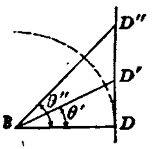


Fig. 19.

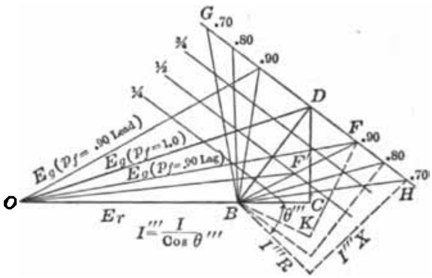


Fig. 20.

Figs. 19 and 20.—Perrine-Baum diagram.

34. The Perrine-Baum* regulation diagram, originally proposed by F. A. C. Perrine and F. G. Baum and based on the diagram shown in Fig. 13, forms a convenient method of studying transmission line regulation. To facilitate use of the diagram, the effect of phase displacement is taken into account by rotating the impedance triangle *BDC*.

The load current is proportional to $1/\cos\theta = \sec\theta$, and the impedance drop, being directly proportional to the load current, is obtained by laying off the phase angle as shown in Fig. 19. If *BD* is the impedance drop for unity power-factor, then $BD' = BD \sec\theta'$ is the impedance drop for the power-factor $\cos\theta'$, and $BD'' = BD \sec\theta''$ is the drop for the power-factor $\cos\theta''$, etc.; that is, the impedance-drop vector always terminates on a straight line, drawn at right angles to the drop *BD* for unity power-factor.

The diagram is shown in Fig. 20, in which the line capacity is neglected. Its use will be clear from the following example:

* Bibliography 1.

35. Example of calculation using Perrine-Baum regulation diagram. Assume 1,000 kw. to be transmitted to a substation at 90 per cent. power-factor, lagging, the voltage at the substation to be maintained constant at 10,000 volts. The line has a resistance of 8 ohms and an inductive reactance of 15 ohms. The line capacity is neglected. With unity power-factor the line current is 1,000,000/10,000 = 100 amp. Referring to Fig. 20, lay off OB = 10,000 volts, BC = 100 × 8 = 800 volts, and CD = 100 × 15 = 1,500 volts. If the power-factor were unity, OD would be the voltage at the generating station. To determine the generator voltage for 90 per cent. power-factor, lagging, draw GH perpendicular to BD and lay off to the right of BD an angle, DBF, whose cosine is equal to 0.90. The line impedance drop will be represented by BF, hence OF shows the voltage at the generating station for this power-factor. For any other power-factor, cos θ, lay off the angle θ to the right of BD for lagging currents, and to the left of BD for leading currents. The voltage at the generating station will be determined by the line joining O with the terminal of the line impedance drop for the given cos θ. For other loads, hence other values of line current, BD is divided into proportional parts, and lines parallel to GH are drawn as shown. For example, the voltage at the generating station at one half-load and with a power-factor of 0.9 (lagging), is represented in the diagram by the line OF'.

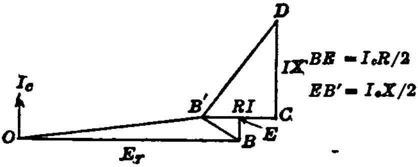


Fig. 21.—Effect of charging current.

Line capacity is taken into account by assuming a constant load voltage; from this assumption the charging current is calculated, taking this current as in quadrature, leading, with respect to the receiving voltage, if the power-factor is unity. The drop due to the charging current in one-half the line impedance is laid off from the receiving voltage vector as shown in triangle BEB′, Fig. 21, and the usual impedance triangle B′CD constructed at the end of the capacity vector. The usual diagram, such as is shown in Fig. 20, is then constructed upon the impedance triangle. It will be noted that this method assumes constant position of the capacity vector, which, though not strictly accurate, is sufficiently so for most purposes in overhead line calculation, except for lines of great length. Compared to the errors due to non-sinusoidal voltage wave the error due to this capacity effect is negligible.

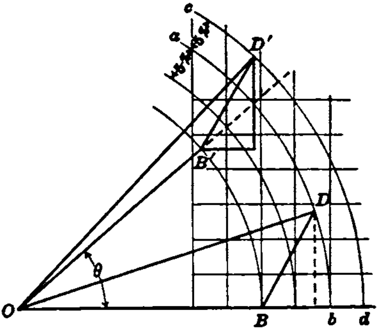


Fig. 22.—Mershon diagram.

36. The Mershon* diagram. If the e.m.f. diagram shown in Fig. 3 is rotated about O as a centre through an angle θ, where cos θ is the power-factor, and all e.m.fs. are expressed in per cent. of the voltage generated at unity power-factor, the line drop, neglecting capacity effect, being measured along a radius with centre O, then a simple diagram for determining regulation may be developed. Such a diagram is constructed by drawing a series of concentric circles ab, cd, etc. (Fig. 22) upon a coordinate system of equal squares; a side of one of these squares is equal to the difference in radii between any two consecutive circles. The radius Ob is taken as 100 per cent., and the coordinate spaces may conveniently represent intervals of 5 per cent.

* Bibliography 22.

The power-factor is cos θ and is numerically equal to the projection of *OB'* upon the horizontal axis. Having definite line constants, and with a load of given power-factor, the resistance and the reactance voltage drops are calculated in per cent. of the receiver voltage. Starting at the foot of the ordinate representing the load power-factor, follow this ordinate up to its intersection with the first circle, which is the locus of the point *B* of the impedance triangle. From this last point lay off on the coordinates the impedance triangle. The circle upon which the apex *D* lies will give the per cent. drop directly. The diagram constructed to scale is given in Sec. 12, Fig. 11, and the table of Sec. 12, Par. 31 is calculated for use with it.

37. Example showing use of the Mershon diagram. A transmission of 250 kw. at 80 per cent. power-factor, lagging, is to be made at 2,000 volts to a receiver 10,000 ft. distant from the generator. The frequency of the system is 60 cycles per sec.; the size of wire No. 0; the spacing of the line wires 18 in. Find the line loss and the line drop. 250,000/0.8 = 312,500 apparent watts. 312,500/2,000 = 156.25 amp.

From Par. 31, Sec. 12 for 18 in. spacing and No. 0 wire the reactance constant is 0.228.

$0.228 \times 10 \times 156.25 = 356.3$ volts. $356.3/2{,}000 = 0.178$ or 17.8 per cent.

From Sec. 12, Par. 31, the resistance constant for No. 0 wire is 0.196. $0.196 \times 10 \times 156.25 = 306.3$ volts. $306.3/2{,}000 = 0.153$ or 15.3 per cent.

Now referring to the diagram, Fig. 11, Sec. 12, starting at 0.8 power-factor, follow the vertical ordinate to its intersection with the curve, 0. From this point lay off the resistance drop, 15.3 per cent., parallel to the base of the diagram, remembering that a side of each square represents 5 per cent. Then from the extremity of the resistance drop lay off the reactance drop, 17.8 per cent., parallel to the altitude, again using the squares. The end of this line will be found on circular arc 23. This then is the total drop given in per cent. of the receiver voltage. The generator voltage is:

$$23/(100+23) = 0.187 \text{ or } 18.7 \text{ per cent.}$$

greater than the receiver voltage. The total line-drop then is 18.7 per cent. The I^2R loss is

$$306.3 \times 156.25 = 47.9 \text{ kw.},$$

and, in per cent., is

$$47.9/(250+47.9) = 0.161 \text{ or } 16.1 \text{ per cent.}$$

In order to find the size of wire necessary for a given drop one must solve by trial and error. Assume a size of wire and solve for the drop. This first error will indicate the proper choice for a second trial, etc., as in Par. 14.

38. Resistance of copper and aluminum. Ohms per mile
(*Solid aluminum, 61 per cent. conductivity*)

Aluminum, size in Cir. Mils	Ohms at 68 deg. fahr. 20 deg. cent.	Size equivalent copper 97 per cent. conductivity
795,500	0.1127	500,000 Cir. Mils
715,500	0.1254	450,000 Cir. Mils
636,000	0.1409	400,000 Cir. Mils
556,500	0.1611	350,000 Cir. Mils
477,000	0.1879	300,000 Cir. Mils
397,500	0.2253	250,000 Cir. Mils
336,420	0.2667	No. 0000 A.W.G.
266,800	0.3360	No. 000 A.W.G.
211,950	0.4229	No. 00 A.W.G.
167,800	0.5342	No. 0 A.W.G.
133,220	0.6720	No. 1 A.W.G.
105,530	0.8486	No. 2 A.W.G.
83,640	1.071	No. 3 A.W.G.
66,370	1.350	No. 4 A.W.G.
52,630	1.703	No. 5 A.W.G.
41,740	2.147	No. 6 A.W.G.

TABLES OF INDUCTIVE REACTANCE AND CHARGING CURRENT

39. The inductance and capacity of transmission lines may be calculated from the formulas given in Sec. 2. The values of inductive reactance and charging current for wires of standard size, with usual spacings, at both 25 cycles and 60 cycles, are given in the tables in Par. 40–43.

40. Inductive reactance per single conductor, ohms per mile

From formula $x = 2\pi f(80 + 741.1 \log \frac{D}{r})\,10^{-6}$

Solid wire
25 cycles per sec.

Size A.W.G.	Spacing, in.															
	12	18	24	36	48	60	72	84	96	108	120	132	144	156	168	180
0000	0.212	0.233	0.247	0.268	0.282	0.293	0.303	0.311	0.318	0.323	0.328	0.333	0.338	0.342	0.345	0.349
000	0.218	0.239	0.253	0.273	0.288	0.299	0.309	0.316	0.323	0.329	0.334	0.339	0.343	0.348	0.351	0.355
00	0.224	0.244	0.259	0.279	0.294	0.305	0.314	0.322	0.329	0.335	0.340	0.345	0.349	0.354	0.357	0.361
0	0.230	0.250	0.265	0.285	0.300	0.311	0.320	0.328	0.335	0.341	0.346	0.351	0.355	0.360	0.363	0.366
1	0.236	0.256	0.271	0.291	0.306	0.317	0.326	0.334	0.341	0.346	0.352	0.357	0.361	0.365	0.369	0.372
2	0.242	0.262	0.277	0.297	0.312	0.323	0.332	0.340	0.347	0.352	0.358	0.363	0.367	0.371	0.375	0.378
3	0.248	0.268	0.283	0.302	0.318	0.329	0.338	0.346	0.352	0.358	0.364	0.369	0.373	0.377	0.381	0.384
4	0.253	0.274	0.288	0.308	0.324	0.335	0.344	0.351	0.358	0.364	0.370	0.374	0.378	0.383	0.386	0.390
5	0.259	0.280	0.294	0.314	0.330	0.340	0.350	0.357	0.364	0.370	0.376	0.380	0.384	0.389	0.392	0.396
6	0.265	0.285	0.300	0.320	0.335	0.346	0.356	0.363	0.370	0.376	0.381	0.386	0.390	0.394	0.398	0.401
7	0.271	0.291	0.306	0.326	0.341	0.352	0.361	0.369	0.376	0.382	0.387	0.392	0.396	0.400	0.404	0.407
8	0.277	0.297	0.312	0.332	0.347	0.358	0.367	0.375	0.382	0.387	0.393	0.398	0.402	0.406	0.410	0.413

To be used on all circuits with quantities measured *to neutral.*