$$K_x = 1 - \frac{S^2}{600}\left(\frac{x}{x'} - \frac{r^2}{xx'}\right) \tag{36}$$

$$k_r = 1 - \frac{xS^2}{1200x'} \tag{37}$$

$$k_x = \frac{rS^2}{1200x'} \tag{38}$$

Examination of the above equations shows that for a given line, the factors K_r, K_x, and k_r differ from 1 by a term that is proportional to the square of the length of the line. However, a study of the characteristics of lines which it is economical to build and that have been built in the United States reveals that for a given length the variance of these correction factors from a mean is very slight. In addition, it is only the lines with smaller conductor sizes and equivalent spacings for which the correction factors vary appreciably.

TABLE 1.—MINIMUM CONDUCTOR SIZES AND SEPARATIONS FOR WHICH THE MEAN VALUES OF THE CORRECTION FACTORS ARE APPLICABLE TO AN ACCURACY OF WITHIN ONE-HALF OF ONE PERCENT[1]

Length of Line in Miles	50	75	150	200	300
G.M.D. (ft.)	3	6	6	10	14
Copper Cables	6	2	0	300 000	500 000
A.C.S.R.	1	1	000	500 000	795 000
An Hollow Cu. Cable	00[2]	00	00	300 000	500 000
Gen. Cable Type HH	000[2]	000	000	300 000	500 000

[1] Conductor sizes are in cir. mils or AWG.
[2] Smallest sizes made.

Table 1 gives minimum conductor sizes and spacings for various lengths of line for which the use of mean correction factors will give sufficient accuracy. For lines up to 300 miles in length with conductor sizes and spacings equal to or greater than given by this table, the use of mean values for K_r, K_x, and k_r gives an accuracy of within 0.5 percent. The correction factor k_x is never greater than about 0.005 and can be neglected. Thus, the shunt impedance Z'_{eq} can be considered as a pure capacitor.

In Fig. 6 are plotted the curves for K_r, K_x, and k_r as a function of line length. The values on these curves conform to those of the most common type of line construction that is used for a given line length. Thus, in most cases the use of these values will give an accuracy considerably better than 0.5 percent. The factors can also

TABLE 2—EXPRESSIONS FOR THE CORRECTION FACTORS FOR THE EQUIVALENT π IMPEDANCES

Correction Factors	Values for Line Lengths up to				
	50 Mi.	75 Mi.	100 Mi.	200 Mi.	300 Mi.
K_r	1—0.0141 S^2				
K_x	1	1—0.0069 S^2			
k_r	1		1—0.0035 S^2		
k_x	0				

S is the length of the line expressed in hundreds of miles.

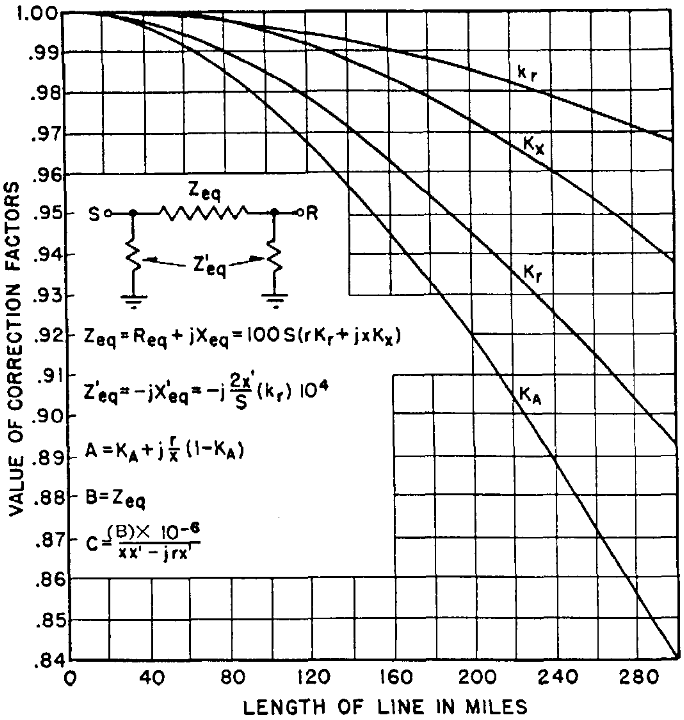


Fig. 6—Correction factors for the equivalent π transmission line impedances and *ABC* constants at 60 cycles.

S = length of line in hundreds of miles.
r = conductor resistance in ohms per mile.
x = inductive reactance in ohms per mile.
x' = capacitive reactance in megohms per mile.

be expressed to sufficient accuracy as parabolic equations of the type $1 - KS^2$. In Table 2 are tabulated the correction factors expressed in this form. The curves constructed from these equations conform closely to the curves of Fig. 6. Table 2 shows that K_r can be considered as 1 up to 50 miles, K_x as 1 up to 75 miles, and k_r as 1 up to 100 miles. Since in practically all cases the individual sections of line to be considered are not over 100 miles long, the correction factors can be neglected entirely if an accuracy of better than 1½ percent is not desired. The largest deviation from unity is in K_r which at 100 miles is only 1.4 percent.

Example 1—As an example of the use of this method in determining the equivalent π of a transmission line, consider a three-phase, 60-cycle, 230-mile line of 500 000 circular mil stranded copper conductors at an equivalent spacing of 22 feet.

From the Tables of Chap. 3

$r = 0.130$ ohms per mile
$x = 0.818$ ohms per mile
$x' = 0.1917$ megohms per mile

From the curves of Fig. 6 for a 230 mile line

$K_r = 0.931$
$K_x = 0.964$
$k_r = 0.982$

From Eqs. (33) and (34) or Fig. 6

$$Z_{eq}=(0.130)(230)(0.931)+j(0.818)(230)(0.964)$$
$$=(27.8+j181.4) \text{ ohms}$$
$$Z'_{eq}=-j\frac{2(0.1917)}{2.30}(0.982)(10^4)$$
$$Z'_{eq}=-j1635 \text{ ohms}$$

The equivalent circuit for this line is shown in Fig. 13.

8. Adaptation of Simplified Method of Determining Equivalent π to Determining *ABC* Constants

The foregoing method can be adapted with an acceptable degree of accuracy to determining the ABC constants of a transmission line. The ABC constants of the line should be determined by a more accurate method if the line is to be combined with other circuit elements. Eq. (27) can be written as follows:

$$A=K_A+j\frac{r}{x}(1-K_A) \qquad (39)$$

where
$$K_A=1-\frac{xS^2}{200x'}.$$

Since K_A is the same form of correction factor as K_r (Eq. (35)), a new curve for the correction factor can be plotted as shown in Fig. 6. The constant A is readily obtained from the correction factor K_A and Eq. (39). The constant B is equal to Z_{eq} and is determined through the use of the correction factors K_r and K_x of Fig. 6.

From Eqs. (16) and (17) it can be seen that

$$C=\frac{B}{ZZ'}=\frac{B\times10^{-6}}{xx'-jrx'} \qquad (40)$$

where

r = conductor resistance in ohms per mile.
x = inductive reactance in ohms per mile.
x' = capacitive reactance in megohms per mile.

Example 1(a)—Determine the ABC constants of the transmission line of example 1.

From the curves of Fig. 6, for a 230-mile line

$$K_A=0.897$$

From the curve for K_A of Fig. 6 and from Eqs. (39) and (40)

$$A=0.897+j\frac{0.130}{0.818}(1-0.897)$$
$$=0.897+j0.0164$$
$$B=Z_{eq}=27.8+j181.4 \text{ ohms} \quad \text{(from example 1)}$$
$$C=\frac{(27.8+j181.4)(10^{-6})}{(0.818)(0.1917)-j(0.130)(0.1917)}$$
$$=-0.00000639+j0.001156$$

II. REGULATION AND LOSSES

9. Analytical Solution for Voltage Regulation of Short Lines from Known Receiver Conditions

The commonest type of regulation problem is one in which it is desired to determine the voltage drop for known receiving-end conditions. For the solution of this problem it is more convenient to make E_R the reference vector as shown in Fig. 7(a). Unless denoted by the subscript L all voltages will be taken as line-to-neutral voltages. If line-to-line voltages are applied to the following voltage equations the impedance drop must be multiplied by $\sqrt{3}$ for three-phase lines or by 2 for single-phase lines.

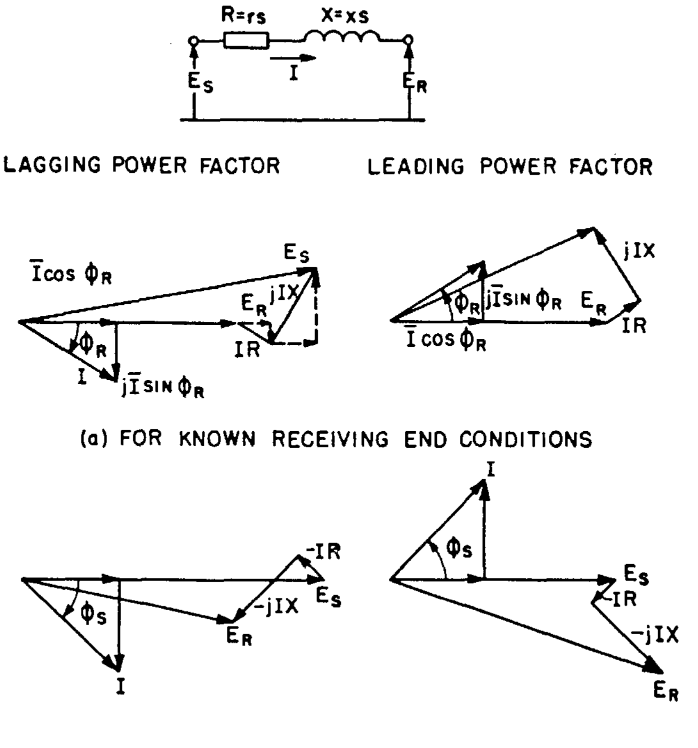


Fig. 7—Vector diagrams for determining voltage regulation of short lines.

In the following equations, (41) through (61), the sign of the power factor angle ϕ, depends upon whether the current is lagging or leading. For a lagging power factor, ϕ and $\sin\phi$ are negative; for a leading power factor, ϕ and $\sin\phi$ are positive. The $\cos$ of ϕ is positive for either lagging or leading current.

$$E_R=\bar{E}_R=\text{reference}$$
$$I=\bar{I}\cos\phi_R+j\bar{I}\sin\phi_R$$
$$Z=R+jX=rs+jxs$$
$$E_S=E_R+IZ \qquad (41)$$

or

$$E_S=(\bar{E}_R+\bar{I}R\cos\phi_R-\bar{I}X\sin\phi_R)+j(\bar{I}X\cos\phi_R+\bar{I}R\sin\phi_R). \qquad (42)$$

In magnitude

$$\bar{E}_S=\sqrt{(\bar{E}_R+\bar{I}R\cos\phi_R-\bar{I}X\sin\phi_R)^2+(\bar{I}X\cos\phi_R+\bar{I}R\sin\phi_R)^2} \qquad (43)$$

If the $\bar{I}R$ and $\bar{I}X$ drops are not over 10 percent of $\bar{E}_R$, $\bar{E}_S$ can be determined for normal power-factors to within a half percent by neglecting its quadrature component. Then

$$\bar{E}_S=\bar{E}_R+\bar{I}R\cos\phi_R-\bar{I}X\sin\phi_R \qquad (44)$$

The voltage regulation of a line is usually considered as the percent drop with reference to E_R.

$$\text{Percent Reg.}=\frac{100(\bar{E}_S-\bar{E}_R)}{\bar{E}_R} \qquad (45)$$

For exact calculations formula (43) can be used with Eq. 45.

Using the approximate formula (44) Eq. 45 can be written

$$\text{Percent Reg.} = \frac{100 s\bar{I}}{\bar{E}_R}(r \cos \phi_R - x \sin \phi_R) \quad (46)$$

The load in kva delivered to the receiving end of a three-phase line is given by the equation

$$\text{KVA} = \frac{3\bar{E}_R\bar{I}}{1000} = \frac{\sqrt{3}\bar{E}_L\bar{I}}{1000} \quad (47)$$

where $\bar{E}_L$ is the line voltage at the receiving end.

The regulation expressed in terms of the load and the line-to-line voltage can be written

$$\text{Percent Reg.} = \frac{100\,000(\text{kva})(s)}{\bar{E}_L^2}(r \cos \phi_R - x \sin \phi_R) \quad (48)$$

These equations show that the amount of load that can be transmitted over a given line at a fixed regulation varies inversely with its length. Using the regulation calculated from these equations to determine the receiver-end voltage will give this quantity to ½ percent if neither the resistance nor reactive drops exceed more than 10 percent of the terminal voltage. The percentage variance of the regulation from its own correct value, however, may be great, depending upon its actual magnitude and for this reason such equations are not accurate for determining load limits for fixed regulations.

Example 2—The use of these equations can be illustrated by calculating the regulation on a three-phase line five miles long having 300 000 circular mil stranded copper conductors at an equivalent spacing of four feet and carrying a load of 10 000 kva at 0.8 power-factor lag and a receiver line voltage of 22 000 volts.

$r = 0.215$ ohms per mi and $x = 0.644$ ohms per mi.

Applying Eq. (48)

$$\text{Percent Reg.} = \frac{(100\,000)(10\,000)(5)}{(22\,000)^2}\left[(0.215)(0.8) - (0.644)(-0.6)\right]$$

$$\text{Reg.} = 5.8\%$$

10. Voltage Regulation of Short Lines from Known Sending-End Conditions

To calculate the receiving-end voltage from known sending-end conditions it is more convenient to use E_S as the reference vector as shown in Fig. 7(b). For this case

$$E_S = \bar{E}_S = \text{reference}$$

$$E_R = E_S - IZ \quad (49)$$

$$E_R = (\bar{E}_S - \bar{I}R\cos\phi_S + \bar{I}X\sin\phi_S) - j(\bar{I}X\cos\phi_S + \bar{I}R\sin\phi_S) \quad (50)$$

$$\bar{E}_R = \sqrt{(\bar{E}_S - \bar{I}R\cos\phi_S + \bar{I}X\sin\phi_S)^2 + (\bar{I}X\cos\phi_S + \bar{I}R\sin\phi_S)^2} \quad (51)$$

Neglecting the quadrature component of E_R:

$$\bar{E}_R = \bar{E}_S - \bar{I}R \cos \phi_S + \bar{I}X \sin \phi_S \quad (52)$$

11. Problems Containing Mixed Terminal Conditions

Sometimes problems are encountered in which mixed terminal conditions are given, such as load power factor and sending-end voltage, or sending-end power factor and receiver-end voltage, and it is desired to determine the unknown voltage for given load currents. Such problems can not readily be solved by analytical methods. For instance, if it were desired to determine the receiver voltage from known load power factor, sending end voltage, and current, it would be necessary to solve for E_R in Eq. (43) by squaring both sides of the equation and obtaining a quadratic equation for E_R. This is somewhat cumbersome. Trial and error methods assuming successive values of one of the two unknown quantities, are often more convenient. Also, it is sometimes found easier to solve such problems by graphical means. The more important problems of this type can be solved by use of the Regulation and Loss Chart as shown in Sec. 28(d) of this chapter.

12. Taps Taken Off Circuit

Quite frequently the main transmission circuit is tapped and power taken off at more than one point along the circuit. For such problems it is necessary to solve each individual section in succession in the same manner as discussed above, starting from a point at which sufficient terminal conditions are known.

13. Resistance Losses of Short Transmission Lines

The total $R\bar{I}^2$ loss of a three-phase line is three times the product of the total resistance of one conductor and the square of its current.

$$\text{Loss} = 3R\bar{I}^2 \text{ in watts.} \quad (53)$$

In percent of the delivered kw. load

$$\text{Percent Loss} = \frac{173 rs\bar{I}}{\bar{E}_L \cos \phi_R} \quad (54)$$

It is sometimes desired to determine the amount of power that can be delivered without exceeding a given percent loss. This is given by

$$\text{KW} = \frac{\bar{E}_L^2 \cos^2 \phi_R (\%\ \text{Loss})}{100\,000 rs} \quad (55)$$

This equation shows that the amount of power that can be transmitted for a given percent loss varies inversely with the length of the line and directly with the loss.

14. Regulation of Long Lines from Known Receiver Conditions

The effect of charging current on the regulation of transmission lines can be determined from the equivalent π circuit. In Fig. 8(a) are shown the vector diagrams for the case of known load conditions. The voltage drop in the series impedance Z_{eq} is produced by the load current I_R plus the charging current $\frac{E_R}{Z'_{eq}}$ flowing through the shunt impedance at the receiver end of the line. For a given line this latter current is dependent only upon the receiver voltage E_R.

There are two methods of taking this charging current into account. One of these is to determine first the net current $\left(I'_{eq} = I_R + \frac{E_R}{Z'_{eq}}\right)$ that flows through Z_{eq} together with its power-factor angle ϕ_{eq}. Using the equivalent

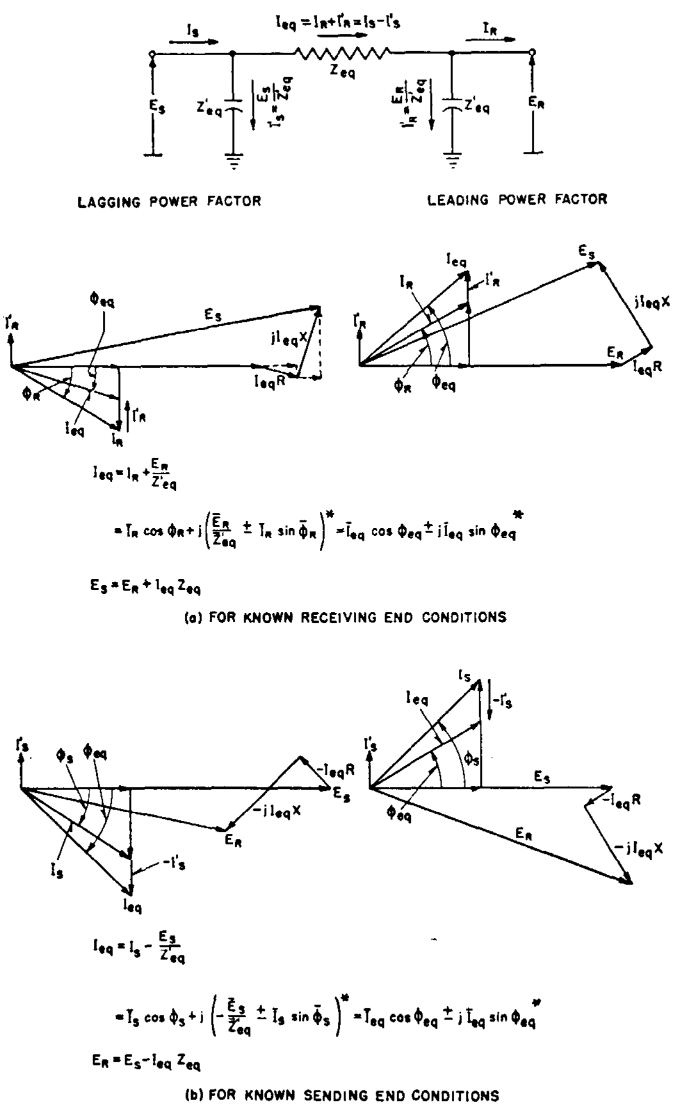


Fig. 8—Vector diagrams for determining voltage regulation of long lines.

series impedance Z_{eq} and this current instead of the load current all of the analytical expressions developed for short lines are applicable. The equivalent terminal conditions to use are shown in Fig. 8 (a).

Example 3—As an example of the use of this method consider the line of example 1, operating at a line voltage at the receiver end of 110 kv delivering a load current I_R of 50 amperes at 0.9 power-factor lagging.

$$E_R = (110{,}000 + j0)/\sqrt{3} = 63{,}500 + j0$$

$$I_R = 50\epsilon^{-j25.8^\circ} = 50[\cos(-25.8^\circ) + j\sin(-25.8^\circ)]$$

$$= 45 - j21.8 \text{ amps}$$

$$I'_R = \frac{E_R}{Z'_{eq}} = \frac{110\,000 + j0}{\sqrt{3}(-j1635)} = +j38.8 \text{ amps}$$

$$I_{eq} = I_R + I'_R = 45 - j21.8 + j38.8 = 45 + j17 = 48.1\epsilon^{j20.7^\circ}$$

$$Z_{eq} = 27.8 + j181.4 = 183.5\epsilon^{j81.28^\circ}$$

$$E_S = 63{,}500 + (48.1\epsilon^{j20.70^\circ})(183.5\epsilon^{j81.28^\circ})$$

$$= 61{,}700 + j8640$$

*Sine of negative angle is (−), of positive angle is (+).

15. Regulation of Long Lines from Known Sending-End Conditions

For this case the equivalent current flowing through Z'_{eq} can be determined as the difference between I_S and I'_S the current in the shunt reactance at the sending end of the equivalent circuit. The vector diagram and equations for this case are shown in Fig. 8 (b).

16. Effect of Line Capacitance on Regulation Expressed in Terms of a Correction Factor

As an alternative method the voltage relations can be determined in a form equivalent to adding a correction factor to the terminal voltage instead of to the current. This method has an advantage in that an average value can be taken for this correction factor which is a function only of the length of the line.

Referring to the vector diagram of Fig. 8(a) for known receiving-end conditions and lagging power-factor, it is seen that the vector equation for the sending-end voltage E_S can be written in the following form in terms of the load current I_R and receiving-end voltage E_R if the current I_R' is expressed in terms of E_R:

$$E_S = \left(1 - \frac{X_{eq}}{\overline{Z}'_{eq}}\right)\overline{E}_R + R_{eq}\overline{I}_R\cos\phi_R - X_{eq}\overline{I}_R\sin\phi_R$$
$$+ j\left(+\frac{R_{eq}}{\overline{Z}'_{eq}}\overline{E}_R + X_{eq}\overline{I}_R\cos\phi_R + R_{eq}\overline{I}_R\sin\phi_R\right) \tag{56}$$

When the quadrature component of E_S is neglected, its magnitude can be expressed as

$$\overline{E}_S = \left(1 - \frac{X_{eq}}{\overline{Z}'_{eq}}\right)\overline{E}_R + R_{eq}\overline{I}_R\cos\phi_R - X_{eq}\overline{I}_R\sin\phi_R \tag{57}$$

From the same considerations that enabled average values to be taken for the correction factors of the equivalent π impedance discussed in Sec. 7 an average value can be assumed for $\frac{X_{eq}}{\overline{Z}'_{eq}}$ in Eq. (57).

$$\frac{X_{eq}}{\overline{Z}'_{eq}} = 0.0201S^2 \tag{58}$$

where S is the length of the line in hundreds of miles. An approximate expression can thus be obtained for the regulation of long lines similar to that of Eq. (46).

$$\text{App. \% Reg.} = \frac{100\overline{I}_R}{\overline{E}_R}(R_{eq}\cos\phi_R - X_{eq}\sin\phi_R) - 2.01S^2 \tag{59}$$

Similar analysis can be applied to problems involving known sending end conditions. A comparison of Eqs. (59) and (46) shows that when Z_{eq} is used for long lines, the equations are of the same form with the exception of the correction factor $(-2.01S^2)$. For lines up to 100 miles in length short line formulas can usually be applied to a good degree of accuracy by merely adding this term to the result. This, of course, neglects the correction factors K_r and K_x for Z_{eq}.

17. Determination of Voltage at Intermediate Points on a Line

The voltage at intermediate points on a line may be calculated from known conditions at either terminal by simply setting up the equivalent circuit for the line between the terminal and the intermediate point. For the line thus set up any of the methods given above may be used.

18. Resistance Losses of Long Lines

The effect of charging current on line losses can be treated as in Sec. 14 for regulation. Referring to Fig. 8 the loss can be considered to be due to the current $I_{eq}=I_R+I_R'=I_S-I_S'$ flowing through the equivalent resistance (R_{eq}).

Thus in terms of the load current

$$\text{Loss}=3R_{eq}(\overline{I_R+I_R'})^2 \text{ watts} \tag{60}$$

$$=3R_{eq}\left[\overline{I}_R^2+\frac{2\overline{I}_R\overline{E}_R}{\overline{Z}'_{eq}}\sin\phi_R+\frac{\overline{E}_R^2}{\overline{Z}'^2_{eq}}\right] \text{ watts} \tag{61}$$

III. CIRCLE AND LOSS DIAGRAMS

Equations for line currents, power, and resistance losses can be expressed as functions of the terminal voltages and system constants. Such equations and graphical representations of them are found convenient not only for the more common types of performance problems but also in connection with system stability. The graphic form of the power and current equations are very similar and are known as "circle diagrams." Of these the power circle diagram is the most important. In the past this diagram has been primarily limited in its use to transmission systems. However, it is thought that if its simplicity and the clarity with which it depicts system performance are better understood, it will be applied more frequently to both transmission and distribution problems.

19. Vector Equations for Power

In previous editions of this book, lagging reactive power was considered as negative and leading reactive power positive. This conformed to the standard adopted by the American Institute of Electrical Engineers at that time. The convention has now been adopted as standard by the Institute that lagging reactive power be considered as positive and leading reactive power negative. Using this notation the vector expression for power can be written as the product of the voltage and the conjugate of the current.

$$P+jQ=E\hat{I} \tag{62}$$

This can be shown with reference to Fig. 9.

$$E=\overline{E}\cos\theta_e+j\overline{E}\sin\theta_e$$
$$I=\overline{I}\cos\theta_i+j\overline{I}\sin\theta_i$$
$$\hat{I}=\overline{I}\cos\theta_i-j\overline{I}\sin\theta_i$$

$$E\hat{I}=\overline{E}(\cos\theta_e+j\sin\theta_e)\overline{I}(\cos\theta_i-j\sin\theta_i)$$
$$=\overline{E}\overline{I}[(\cos\theta_e\cos\theta_i+\sin\theta_e\sin\theta_i)+j(\sin\theta_e\cos\theta_i-\cos\theta_e\sin\theta_i)]$$

since, $\cos(\theta_e-\theta_i)=\cos\theta_e\cos\theta_i+\sin\theta_e\sin\theta_i$

and $\sin(\theta_e-\theta_i)=\sin\theta_e\cos\theta_i-\cos\theta_e\sin\theta_i$

$$E\hat{I}=\overline{E}\overline{I}\cos(\theta_e-\theta_i)+j\overline{E}\overline{I}\sin(\theta_e-\theta_i)$$

Let ϕ be $\theta_e-\theta_i$; then for lagging or inductive power factors ϕ is positive and

$$P+jQ=E\hat{I}=\overline{E}\overline{I}\cos\phi+j\overline{E}\overline{I}\sin\phi.$$

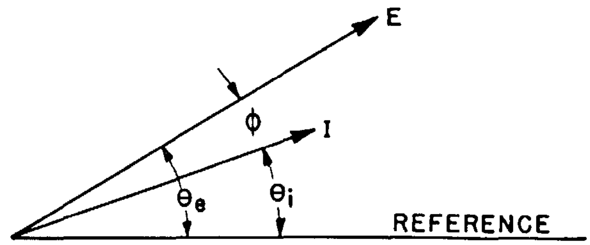


Fig. 9—Diagram for determining the vector equation for power.

For leading or capacitive power factor, ϕ is negative and the imaginary component will be negative. A complete discussion of the direction of the flow of reactive power is given in Chap. 10, Sec. 2.

20. Current and Power Equations and Circle Diagrams for Short Lines

Using the above notation the per phase power at either end of a line is given by the product of the line-to-neutral voltage and the conjugate of the current at the particular end in question. If I_S is chosen as positive for current flowing into the line, positive sending-end power indicates power delivered to the line; and if I_R is taken as positive for current flowing out of the line, positive receiving-end power indicates power flowing out of the line.

Referring to Fig. 1:

$$I_S=I_R=I$$
$$P_S+jQ_S=E_S\hat{I}$$
$$P_R+jQ_R=E_R\hat{I}$$

The current can be expressed in terms of the terminal voltage as follows:

$$I=\frac{E_S-E_R}{Z} \qquad \text{also } \hat{I}=\frac{\hat{E}_S-\hat{E}_R}{\hat{Z}} \tag{63}$$

Thus

$$P_S+jQ_S=\frac{E_S\hat{E}_S-E_S\hat{E}_R}{\hat{Z}}$$
$$P_R+jQ_R=\frac{-E_R\hat{E}_R+E_R\hat{E}_S}{\hat{Z}}$$

If E_R be taken as the reference, then $E_S=\overline{E}_S\epsilon^{j\theta}$ and $E_S\hat{E}_S=\overline{E}_S^2$; $E_S\hat{E}_R=\overline{E}_S\overline{E}_R\epsilon^{j\theta}$; $E_R\hat{E}_R=\overline{E}_R^2$; and $E_R\hat{E}_S=\overline{E}_R\overline{E}_S\epsilon^{-j\theta}$. The expressions for sending- and receiving-end power become

$$P_S+jQ_S=\frac{\overline{E}_S^2}{\hat{Z}}-\frac{\overline{E}_S\overline{E}_R\epsilon^{j\theta}}{\hat{Z}} \tag{64}$$

$$P_R+jQ_R=-\frac{\overline{E}_R^2}{\hat{Z}}+\frac{\overline{E}_S\overline{E}_R\epsilon^{-j\theta}}{\hat{Z}} \tag{65}$$

The sending and receiving end real and reactive power is the sum of two vector quantities. Furthermore, if the voltages E_S and E_R are held constant, there is only one remaining variable, θ. The interpretation of Eqs. (64) and (65) in the form of power circle diagrams is an important concept. Its simplicity is self evident by referring to Eq. (64) and Fig. 10.

The first term $\frac{\overline{E}_S^2}{\hat{Z}}$, is plotted as shown on Fig. 10 and is the vector to the center of the sending end circle diagram.

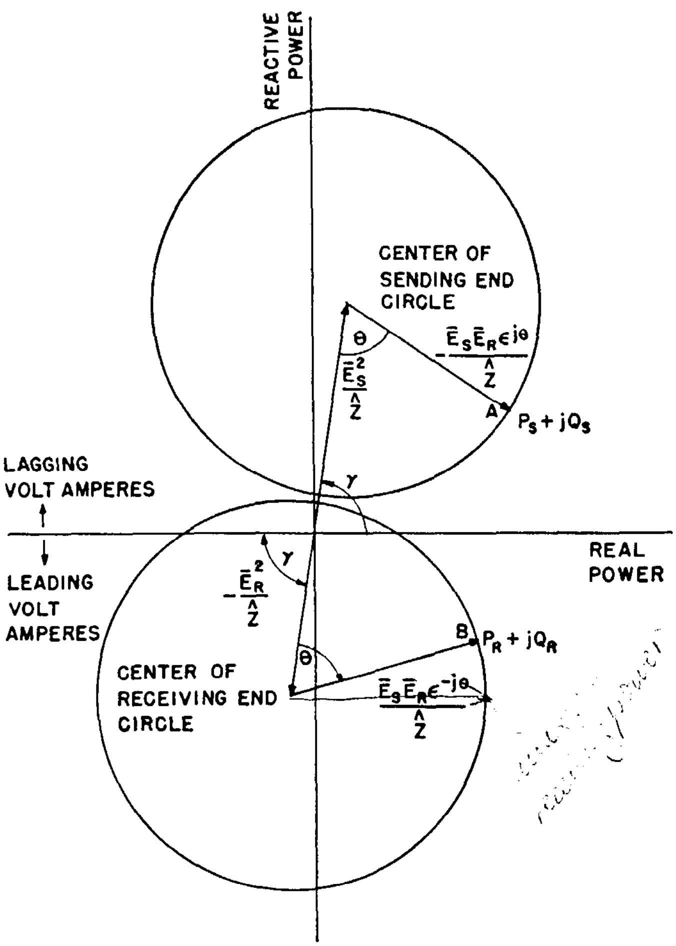


Fig. 10—Power circle diagram for short lines.

The second term $-\dfrac{\bar{E}_S\bar{E}_R\epsilon^{j\theta}}{\hat{Z}}$, which is the radius of the circle is added to this first term so that the resultant is the sending end real and reactive power. A complete sending end circle diagram is obtained by first determining the center of the circle from $\dfrac{\bar{E}_S^2}{\hat{Z}}$, and second, the radius $-\dfrac{\bar{E}_S\bar{E}_R\epsilon^{j\theta}}{\hat{Z}}$, letting $\theta=0$.

The receiving-end circle diagram is obtained in the same manner.

Equations (64) and (65) can be reduced in general terms to Cartesian coordinate form in which the real and reactive parts are separated. However, it is simpler to insert the proper numerical values in the vector and conjugate form and solve by polar and Cartesian coordinates, from which the circle diagrams can then be plotted.

If Eq. (65) is reduced to Cartesian-coordinate form it can be shown that the maximum power that can be received over the line is obtained when θ is equal to $\gamma=\tan^{-1}\dfrac{x}{r}$, the angle of the line impedance. The expression for the maximum receiving power is

$$P_R \max = -\frac{\bar{E}_R^2}{\bar{Z}^2}R+\frac{\bar{E}_S\bar{E}_R}{\bar{Z}}. \tag{66}$$

It can also be seen from Fig. 10 that P_R is maximum when $\theta=\gamma$.

When the line-to-neutral voltages are expressed in volts, the coordinates of the diagram are per-phase real volt-amperes and per-phase reactive volt-amperes. When expressed in kilovolts, the coordinates become thousands of kilowatts and thousands of reactive kilovolt-amperes. Total three-phase power is three times the per-phase power. All of the expressions for power written contain products of $\bar{E}_S^2$, $\bar{E}_R^2$, or $\bar{E}_S\bar{E}_R$. When given in terms of line-to-line voltages, they are all three times as great as when line-to-neutral voltages are used and thus the equations then represent total three-phase power.

Referring to Fig. 10 for the operating condition indicated by the given angle θ the point A of the power circle diagram shows the value of P_S and Q_S being delivered to the line at the sending end and the point B the value of P_R and Q_R drawn from the line at the receiver end. The difference between P_S and P_R is the $R\bar{I}^2$ loss of the line itself for this operating condition.

The value of Q at each end is the reactive power which must be supplied to the line in the case of the sending end or drawn from the line in the case of the receiving end in order to maintain the chosen terminal voltages. At the receiving end the reactive power drawn by the load itself at the particular load power factor may not be equal to that required to maintain the desired voltage. If a synchronous condenser is used at the receiving end, the difference must be supplied by the condenser to maintain the voltage.

It will be noted that for a given network and given voltages at both ends there is a definite limit to the amount of power which may be transmitted. If the angle θ is increased beyond this point, the amount of power transmitted is reduced. The critical value of θ for this condition was shown by Eq. 66 to be $\theta=\gamma$. The only way the power limit may be increased for a given network is by increasing the voltage at either or both ends. Increasing the voltage at one end increases the radius of both circles in direct proportion and moves the center at that end only away from the origin, along a line connecting the original center to the origin, proportional to the square of the voltage at that end. Where the network is subject to change, changes in network constants will also change the power limit. Referring to Fig. 10 and Eq. (66), it is evident that a decrease in the magnitude of Z will result in an increase in the power which may be transmitted. Thus any change which decreases the series impedance such as the addition of parallel circuits will increase the power limit.

Since the conjugate of the phase current, in amperes, is the per-phase power in volt-amperes at either end divided by the phase voltage at the same end, either the sending-end or receiving-end power circles, when placed in the proper quadrants, can be used to represent the locus of the current with a proper change in scale of the coordinates. Referring to the sending end circle diagram of Fig. 10, $P_S+jQ_S=E\hat{I}$ and for the point A, Q_S is positive lagging reactive power. Therefore the imaginary component of the conjugate of the current is positive; the imaginary component of the current is negative. If the power circle diagrams are rotated about the real power axis so that the center of the sending-end circle is in the fourth quadrant $\left(\dfrac{\bar{E}_S^2}{Z}\right.$ will then be the vector to center), and the center of the receiving end circle is in the second quadrant, then the power circle diagrams properly represent the current circle diagrams if the appropriate change in scale of the coordinates is made. Lagging reactive current is negative and leading reactive current is positive.

If the sending-end circle is used the current is referred to the sending end voltage as the reference vector and the coordinates should be divided by the sending end voltage. For instance, if the sending-end power diagram were constructed using line-to-line voltages in kilovolts resulting in power coordinates given in thousands of total three-phase kilovolt-amperes, the power coordinates should be divided by $\sqrt{3}$ times the line-to-line sending end voltage in kilovolts giving current coordinates in thousands of amperes. If the receiving end circle is used, the current is referred to the receiving end voltage as reference. For the current circle diagrams the angle θ still, of course, refers to the angle between the two terminal voltages.

For a study of the performance of a system it is sometimes found convenient to plot on the power circle diagram a family of circles corresponding to various operating voltages. The most common case is one in which the line is to operate at a fixed receiver voltage and it is desired to determine the line performance for various sending-end voltages. For such a case the receiver diagram is usually all that is needed.

Example 4—An example of this type of problem is shown in Fig. 11. There the line constants are given together with the quantities for laying out the diagram. Since the coordinate of the center of the power circles depends only on E_R which is fixed, all the circles have the same center but different radii corresponding to the different values of sending end voltages.

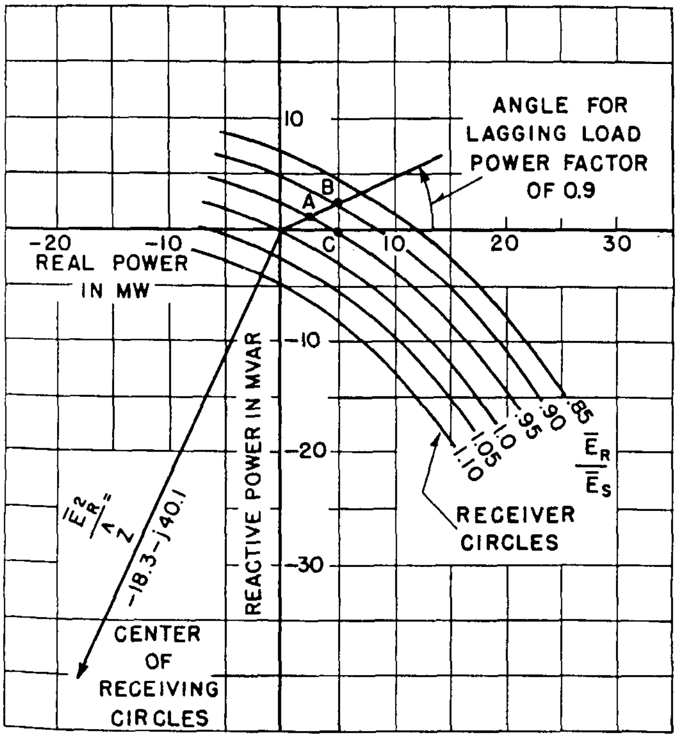


Fig. 11—Family of receiver power circles for a 15-mile line with No. 0000-19 strand-copper conductors and 4-foot equivalent spacing.

Receiver voltage $E_R = 22$-kv line-to-line.
$r = 0.303$ ohm per mile.
$x = 0.665$ ohm per mile.
$Z = zs = 10.94$ ohms.

Examination of this figure shows, for example, that the maximum load at 0.9 power factor lag which can be carried by the line at 5 percent regulation without reactive power correction is that indicated by point A or about 2600 kw. If it is desired to transmit a load of 5000 kw indicated by point B, the regulation would be about 11 percent without rkva correction. To reduce the regulation for this load to 5 percent would require that the receiver and load conditions be that indicated by the point C, and it is evident that about 2400 lagging reactive kilovolt-amperes must be supplied to the receiver end of the line to attain this condition by having capacitors or a synchronous condenser supply that amount of lagging reactive kilovolt-amperes.

21. Current and Power Equations and Circle Diagrams for Long Lines

Representing long lines by their equivalent π circuit as shown in Fig. 6 results in modifying the form of the simple short line equivalent circuit by the addition of the shunt capacitive reactances at each end

$$Z'_{eq} = \bar{Z}'_{eq}\epsilon^{-j90^\circ} = -jX'_{eq}$$

Thus the equations for the terminal currents have an additional term as shown in Fig. 8.

$$I_S = \frac{E_S - E_R}{Z_{eq}} + \frac{E_S}{Z'_{eq}};\ \hat{I}_S = \frac{\hat{E}_S - \hat{E}_R}{\hat{Z}_{eq}} + \frac{\hat{E}_S}{\hat{Z}'_{eq}}. \tag{67}$$

$$I_R = \frac{E_S - E_R}{Z_{eq}} - \frac{E_R}{Z'_{eq}};\ \hat{I}_R = \frac{\hat{E}_S - \hat{E}_R}{\hat{Z}_{eq}} - \frac{\hat{E}_R}{\hat{Z}'_{eq}}. \tag{68}$$

The sending- and receiving-end power is determined in the same manner as for the short line.

$$\begin{aligned} P_S + jQ_S &= E_S\hat{I}_S \\ &= \frac{\bar{E}_S^2}{\hat{Z}_{eq}} - \frac{E_S\hat{E}_R}{\hat{Z}_{eq}} + \frac{\bar{E}_S^2}{\hat{Z}'_{eq}}. \end{aligned} \tag{69}$$

Rewriting Eq. (69) in a slightly different form

$$P_S + jQ_S = \left(\frac{\bar{E}_S^2}{\hat{Z}_{eq}} + \frac{\bar{E}_S^2}{\hat{Z}'_{eq}}\right) - \frac{\bar{E}_S\bar{E}_R\epsilon^{j\theta}}{\hat{Z}_{eq}}. \tag{70}$$

Similarly for receiving end power:

$$P_R + jQ_R = \left(-\frac{\bar{E}_R^2}{\hat{Z}_{eq}} - \frac{\bar{E}_R^2}{\hat{Z}'_{eq}}\right) + \frac{\bar{E}_R\bar{E}_S\epsilon^{-j\theta}}{\hat{Z}_{eq}}. \tag{71}$$

A comparison of Eq. (70) with (64), and (71) with (65) shows them to be of the same form consisting of a fixed vector with a second vector constant in magnitude but variable in phase, added to it. The power circle diagram can be plotted as shown in Fig. 12. The circle diagram is most easily obtained by the numerical and vector substitution for the voltages and impedances. The center and the radius of the circle can then be calculated by reduction using a combination of polar and Cartesian coordinates. Example 5 illustrates the method and shows the power circle diagrams which are obtained in Fig. 13.

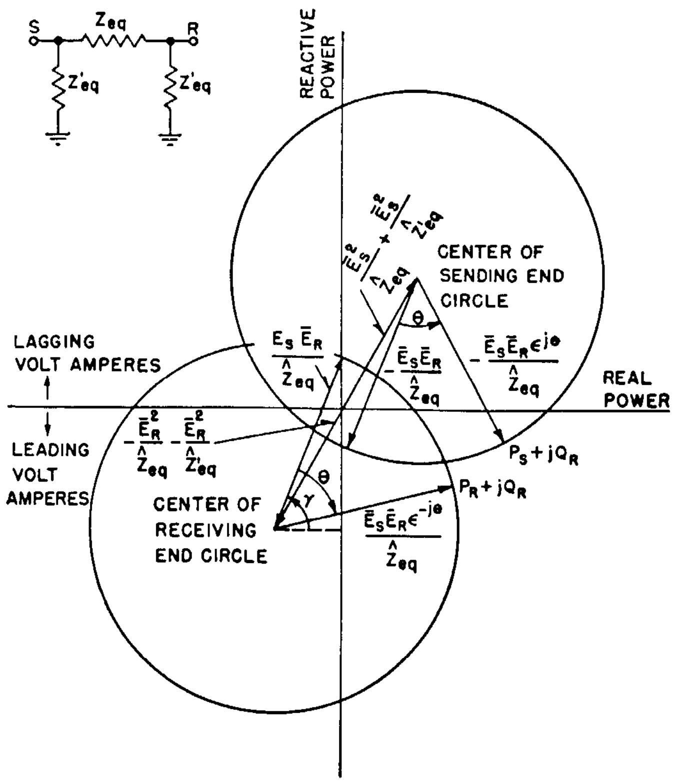


Fig. 12—Power circle diagram for long lines.

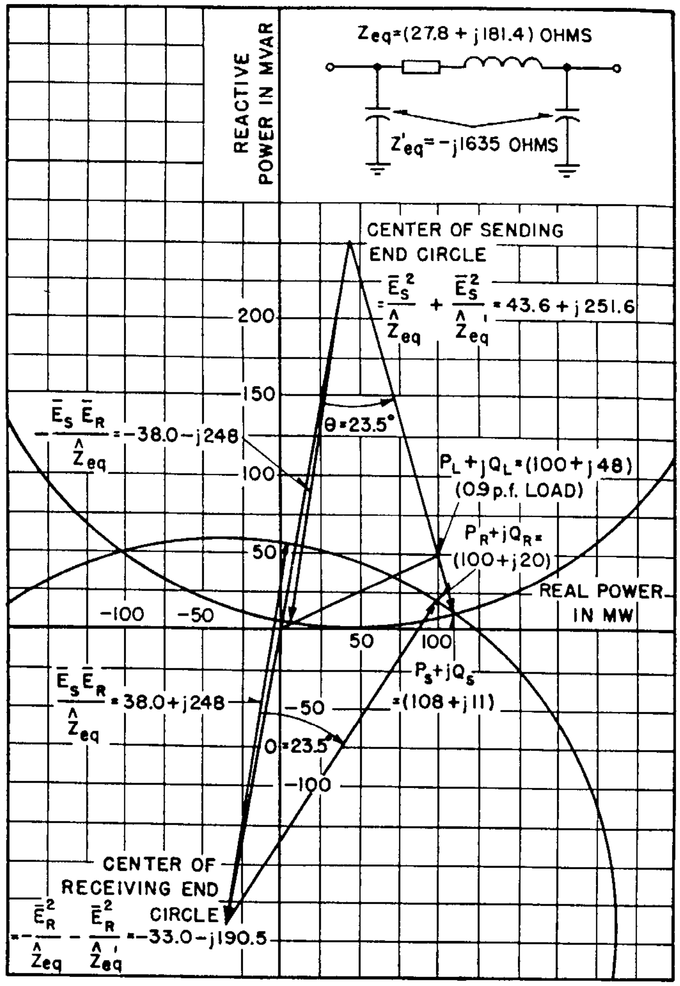


Fig. 13—Equivalent circuit and power circle diagram for a 230-mile line with 500 000 circular mil. stranded copper conductors and an equivalent spacing of 22 feet.

Operating voltages; $E_S = 230$-kv, $E_R = 200$-kv, line-to-line.
For this line $r = 0.130$ ohms per mile.
$x = 0.818$ ohms per mile.
$x' = 0.1917$ megohms per mile.
From curves of Fig. 6 for 230 miles
$K_r = 0.931$
$K_x = 0.964$
$k_r = 0.982$
$Z_{eq} = (27.8 + j181.4)$ ohms; $Z'_{eq} = -j1635$ ohms.

In Eqs. (70) and (71) the terms

$$\frac{\overline{E}_S^2}{\hat{Z}'_{eq}} \text{ and } -\frac{\overline{E}_R^2}{\hat{Z}'_{eq}}$$

are not a function of the angle θ and therefore add directly to the "short line" fixed vector so that the effect is to shift the center of the power circles in the direction of volt-amperes only. The presence of the shunt reactances decreases the amount of positive reactive volt-amperes put into the sending end of the line for a given amount of real power and increases the positive volt-amperes delivered at the receiving end. This decreases the amount of leading reactive volt-amperes which would have to be absorbed by synchronous condensers or capacitors for a given load condition. It does not affect the real power conditions for a given operating angle or the load limit of the line. These factors are determined entirely by the series impedance of the line.

Referring to Fig. 12, if the radius of the receiving-end circle for $\theta = 0$ were plotted with the origin as the center, the vector would be at an angle γ with the real power axis. The angle indicated on Fig. 12 is therefore equal to γ, the angle of the equivalent series impedance. The maximum real power that can be delivered over the line occurs when $\theta = \gamma$.

The current circle diagrams for the sending- and receiving-end currents can be obtained as discussed in Sec. 20. The sending-end current diagram is obtained from the sending-end power circle and is referred to the sending-end voltage vector as reference. The receiving-end current diagram is obtained from the receiving-end power circle and is referred to the receiving-end voltage.

Example 5—Fig. 13 shows the power circle diagram constructed for an actual line.

The power circle diagrams are obtained from Eqs. (70) and (71). If line-to-neutral voltages in kv are used, the results must be multiplied by three to obtain real and reactive power in mw and mvar. If the line-to-line voltages in kv are used, the results are three-phase power in mw and mvar.

$$\text{Vector to center} = \frac{\overline{E}_S^2}{\hat{Z}_{eq}} + \frac{\overline{E}_S^2}{\hat{Z}'_{eq}}$$
$$= \frac{(230)^2}{27.8 - j181.4} + \frac{(230)^2}{+j1635}$$
$$= \frac{(230)^2}{183.4\epsilon^{-j81.28^\circ}} + \frac{(230)^2}{1635\epsilon^{j90^\circ}}$$

$$=288\epsilon^{j81.28°}+32.4\epsilon^{-j90°}$$

$$=43.6+j284-j32.4=43.6+j251.6$$

Radius of the sending end circle $=-\dfrac{\bar{E}_S\bar{E}_R\epsilon^{j\theta}}{\hat{Z}_{eq}}$ for $\theta=0$.

$$=-\frac{230\times 200}{27.8-j181.4}=-251\epsilon^{j81.28°}=-38.0-j248$$

P_S+jQ_S (for $\theta=0$) $=43.6+j251.6-38.0-j248=5.6+j3.6$

Similarly for the receiving circle:

Vector to center $=-\dfrac{\bar{E}_R^2}{\hat{Z}_{eq}}-\dfrac{\bar{E}_R^2}{\hat{Z}'_{eq}}$

$$=\frac{-(200)^2}{27.8-j181.4}-\frac{(200)^2}{+j1635}=-33.0-j190.5$$

for $\theta=0$, Radius $=\dfrac{\bar{E}_S\bar{E}_R}{\hat{Z}_{eq}}=38.0+j248$

and $P_R+jQ_R=-33.0-j190.5+38.0+j248=5.0+j57.5$

Figure 13 shows the power circle diagrams plotted from the calculated results given above. Suppose it is desired to deliver a load of 100 mw at 0.9 power factor lagging; i.e., $P+jQ=100+j48$. From the curves of Fig. 13, for a delivered power of 100 mw the angle θ is 23.5°. The following values from the circle diagrams are $P_S+jQ_S=108+j11$ and $P_R+jQ_R=100+j20$. These values are indicated on the diagram of Fig. 14. The arrow indicates the direction of positive real power flow. Inductive lagging reactive power in the same direction is positive and is the value in parenthesis. These designations and nomenclature follow present-day network calculator practice.

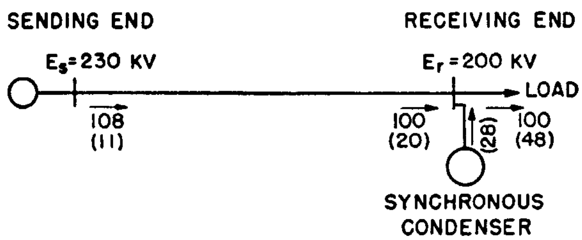


Fig. 14—Recorded values of power flow as obtained from Fig. 13 and Example 5.

At the receiving end there is a deficit of lagging reactive power. A synchronous condenser operating overexcited would be required to supply 28 mvar. If the condenser is considered as a load the direction of the arrow can be reversed with a minus sign in front of the value for the reactive power. The synchronous condenser is then taking negative, or leading reactive power.

22. Current and Power Equations and Circle Diagrams for the General Equivalent π Circuit

The circle diagrams are applicable to the study of the performance of an overall system. Such a system can be represented by an equivalent π circuit of the form shown in Fig. 15. For such a case the shunt impedances usually are not equal and have resistance components introduced by the presence of other equipment containing resistance.

If the shunt impedances take the completely general form of Z_S' and Z_R', the equations for sending- and re-

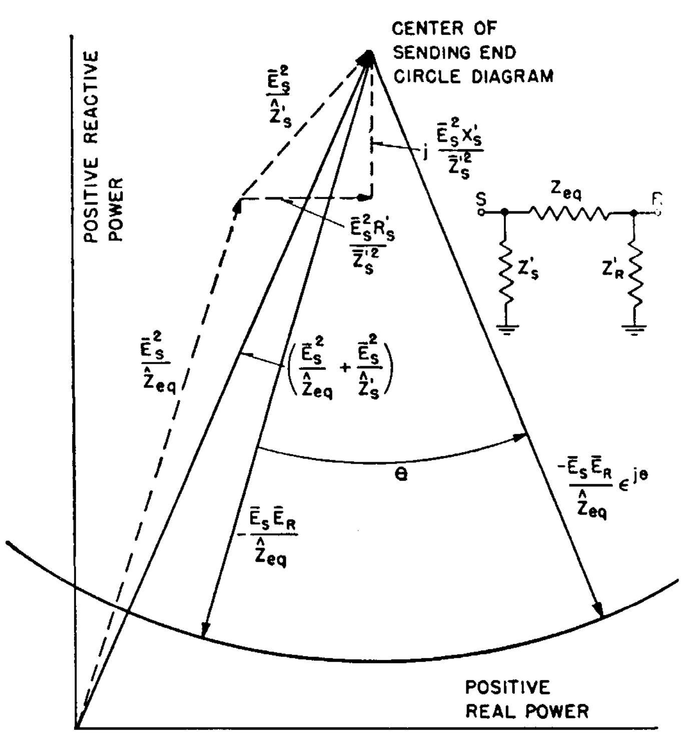


Fig. 15—Power circle diagram for the general equivalent π circuit.

ceiving-end power can be written directly from equations (70) and (71).

$$P_S+jQ_S=\left(\frac{\bar{E}_S^2}{\hat{Z}_{eq}}+\frac{\bar{E}_S^2}{\hat{Z}_S'}\right)-\frac{\bar{E}_S\bar{E}_R\epsilon^{j\theta}}{\hat{Z}_{eq}} \tag{72}$$

and
$$P_R+jQ_R=\left(-\frac{\bar{E}_R^2}{\hat{Z}_{eq}}-\frac{\bar{E}_R^2}{\hat{Z}_R'}\right)+\frac{\bar{E}_S\bar{E}_R\epsilon^{-j\theta}}{\hat{Z}_{eq}} \tag{73}$$

The construction of the power circle diagrams is the same as for the long lines as shown in Fig. 12. In the case of the general equivalent π, Z_S' replaces Z_{eq}' at the sending end and Z_R' replaces Z_{eq}' at the receiving end. The effect of resistance and reactance in the shunt branch at the sending or the receiving end can be visualized better if the impedance is expressed in Cartesian coordinate form. Referring to Eq. (72), the second quantity in the first term becomes

$$\frac{\bar{E}_S^2}{\hat{Z}_S'}=\frac{\bar{E}_S^2}{R_S'-jX_S'}=\frac{\bar{E}_S^2R_S'}{\bar{Z}_S'^2}+j\frac{\bar{E}_S^2X_S'}{\bar{Z}_S'^2} \tag{74}$$

This quantity is added to the "short line" vector to center,

$$\frac{\bar{E}_S^2}{\hat{Z}_{eq}}.$$

This point as applied to the sending end circle diagram is illustrated in Fig. 15. The complete vector to center is shown as $\dfrac{\bar{E}_S^2}{\hat{Z}_{eq}}+\dfrac{\bar{E}_S^2}{\hat{Z}_S'}$, as the sum of the two individual vector quantities, and as the sum of the vector $\dfrac{\bar{E}_S^2}{\hat{Z}_{eq}}$ and the Cartesian coordinates $\dfrac{\bar{E}_S^2R_S'}{\bar{Z}_S'^2}$ and $j\dfrac{\bar{E}_S^2X_S'}{\bar{Z}_S^2}$.

Referring to Fig. 15 and Eq. (74) the effect of resistance is to shift the center of the circle in the direction of increased positive real power. A positive reactance shifts the center in the direction of increased positive reactive power; a negative reactance shifts the center in the direction of decreased positive reactive power.

In the case of the receiving-end circle diagram, the effect of resistance is to shift the center of the circle in the direction of increased negative real power. A positive reactance shifts the center in the direction of increased negative reactive power; a negative reactance shifts the center in the direction of decreased negative reactive power.

The current circle diagrams for this case can be determined as discussed in Secs. 20 and 21.

23. Loss Diagram

Although the resistance loss can be taken from the power circle diagram, it can be obtained more accurately and conveniently from the Loss Diagram.

$$\text{Loss} = P_S - P_R$$

For the case where the transmission line alone is being considered

$$\begin{aligned}\text{Loss} &= \frac{\bar{E}_S^2}{\bar{Z}^2}R - \frac{\bar{E}_S\bar{E}_R}{\bar{Z}^2}(R\cos\theta - X\sin\theta)\\ &\quad + \frac{\bar{E}_R^2}{\bar{Z}^2}R - \frac{\bar{E}_S\bar{E}_R}{\bar{Z}^2}(R\cos\theta + X\sin\theta)\\ &= (\bar{E}_S^2 + \bar{E}_R^2)\frac{R}{\bar{Z}^2} - 2\frac{\bar{E}_S\bar{E}_R}{\bar{Z}^2}R\cos\theta \qquad (75)\end{aligned}$$

The graphical representation of this equation is given in Fig. 16.

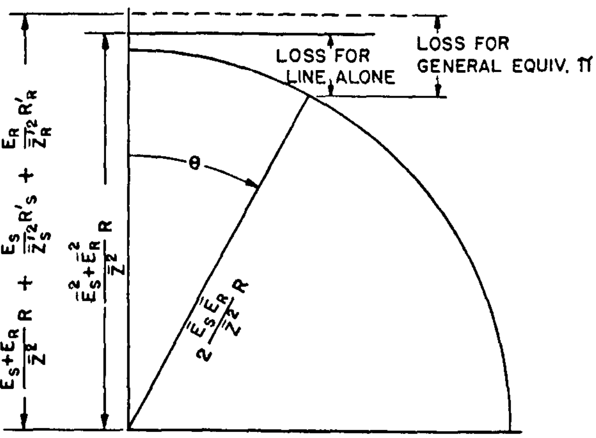


Fig. 16—The transmission line loss diagram (when solving for general equivalent π loss, substitute R_{eq} for R and $\bar{Z}_{eq}$ for $\bar{Z}$).

For the general equivalent π circuit, the equation for loss is

$$\begin{aligned}\text{Loss} &= \left[\bar{E}_S^2 + \bar{E}_R^2\right]\frac{R_{eq}}{\bar{Z}_{eq}^2} + \frac{\bar{E}_S^2}{(\bar{Z}'_S)^2}R'_S\\ &\quad + \frac{\bar{E}_R^2}{(\bar{Z}'_R)^2}R'_R - 2\frac{\bar{E}_S\bar{E}_R}{\bar{Z}_{eq}^2}R_{eq}\cos\theta \qquad (76)\end{aligned}$$

As shown by Fig. 16 this is equivalent to the formula for the loss on the transmission line alone except for the terms $\frac{\bar{E}_S^2}{(\bar{Z}'_S)^2}R'_S$ and $\frac{\bar{E}_R^2}{(\bar{Z}'_R)^2}R'_R$ which represent the losses in the resistance components of the shunt impedances Z'_S and Z'_R.

As was the case for the previous power equations, if line-to-neutral voltages are used, the loss is on a per phase basis; and if line-to-line voltages are used the total three-phase loss is represented.

An equation for the load which can be delivered at a given percent line loss on lines regulated by synchronous capacity is important in determining their performance. Upon the assumption of equal sending- and receiving-end voltages a very simple approximate equation can be derived which gives an accuracy of a fraction of a percent over the practical operating range of loss and regulation. When loss is expressed as a percentage of P_{R_1} this equation is:

$$P_R = \frac{\%\ \text{Loss}}{(100+\%\ \text{Loss})}\left[\frac{\bar{E}_R^2 X_{eq}^2}{R_{eq}\bar{Z}_{eq}^2}\right] \qquad (77)$$

A corresponding equation for Q_R is

$$Q_R = \bar{E}_R^2\left[\frac{X_{eq}}{\bar{Z}_{eq}^2}\left(1+\frac{\%\ \text{Loss}}{100}\right) - \frac{1}{X'_{eq}}\right] \qquad (78)$$

P_R in Eq. (77) is, of course, independent of the load power factor and from Eq. (78) the required amount of synchronous capacity to maintain equal sending- and receiving-end voltages for the delivered load P_R can be obtained by subtracting the reactive kva of the load from Q_R.

24. Current and Power Relations in Terms of the ABCD Constants

In many cases it is desirable to use *ABCD** constants because of the desirability of the check $AD-BC=1$. This is particularly true where there are several combinations of circuits including transmission lines, series impedances and shunt impedances. Expressions for sending and receiving end power can be obtained readily and the circle diagrams can be drawn.

$$E_S = AE_R + BI_R \qquad (79)$$

$$I_S = CE_R + DI_R \qquad (80)$$

$$E_R = DE_S - BI_S \qquad (81)$$

$$I_R = -CE_S + AI_S \qquad (82)$$

Solution of the above equations for I_S and I_R gives:

$$I_S = \frac{D}{B}E_S - \frac{E_R}{B};\ \hat{I}_S = \frac{\hat{D}}{\hat{B}}\hat{E}_S - \frac{\hat{E}_R}{\hat{B}}. \qquad (83)$$

$$I_R = \frac{E_S}{B} - \frac{A}{B}E_R;\ \hat{I}_R = \frac{\hat{E}_S}{\hat{B}} - \frac{\hat{A}}{\hat{B}}\hat{E}_R. \qquad (84)$$

$$\begin{aligned}P_S + jQ_S &= E_S\hat{I}_S\\ &= E_S\hat{E}_S\frac{\hat{D}}{\hat{B}} - \frac{E_S\hat{E}_R}{\hat{B}}\\ &= \bar{E}_S^2\frac{\hat{D}}{\hat{B}} - \frac{\bar{E}_S\bar{E}_R\epsilon^{j\theta}}{\hat{B}} \qquad (85)\end{aligned}$$

*For definition of *ABCD* constants see Chap. 10 Sec. 21.

$$P_R+jQ_R=E_R\hat{I}_R$$
$$=-\frac{\hat{A}}{\hat{B}}E_R\hat{E}_R+\frac{E_R\hat{E}_S}{\hat{B}}$$
$$=-\bar{E}^2_R\frac{\hat{A}}{\hat{B}}+\frac{\bar{E}_R\bar{E}_S\epsilon^{-j\theta}}{\hat{B}} \quad (86)$$

where

$$A=A_1+jA_2=\bar{A}\epsilon^{j\alpha};\quad \hat{A}=A_1-jA_2=\bar{A}\epsilon^{-j\alpha}$$
$$B=B_1+jB_2=\bar{B}\epsilon^{j\beta}\quad \hat{B}=B_1-jB_2=\bar{B}\epsilon^{-j\beta}$$
$$D=D_1+jD_2=\bar{D}\epsilon^{j\delta}\quad \hat{D}=D_1-jD_2=\bar{D}\epsilon^{-j\delta}$$

The sending- and receiving-end power can be obtained readily from solution of Eqs. (85) and (86) by numerical substitution using polar and Cartesian coordinates. Eqs. (85) and (86) take the familiar form (see Sec. 20) of a fixed vector plus a vector of constant magnitude but variable in phase position. The circle diagram construction is shown in Fig. 17. The maximum real power that can be delivered occurs when $\theta=\beta$, which is the angle of the constant B. The angle β is indicated on Fig. 17.

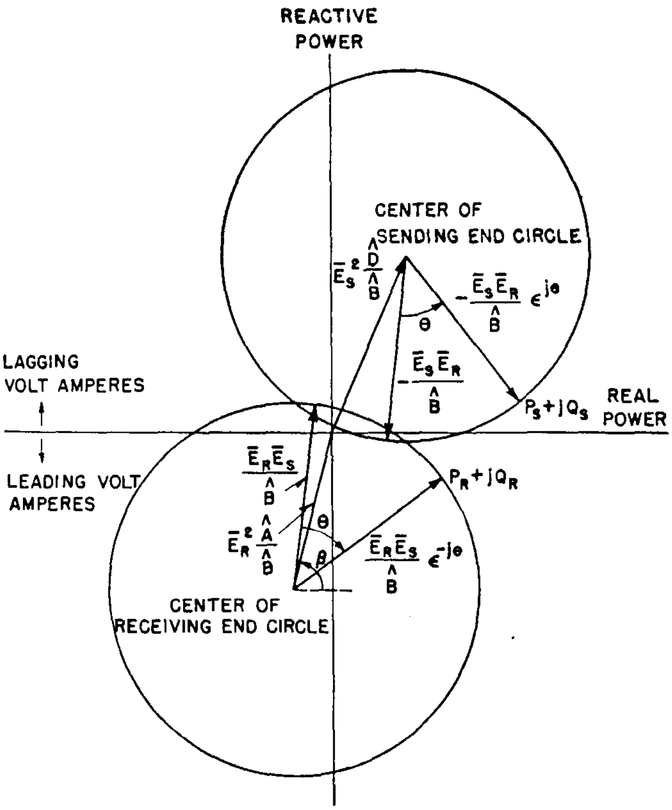


Fig. 17—Power circle diagram in terms of *ABCD* constants.

A breakdown of Eqs. (85) and (86) into their Cartesian coordinate form gives the equation for loss in the form

$$\text{Loss}=P_S-P_R=$$
$$\frac{\bar{E}^2_S}{\bar{B}^2}(B_1D_1+B_2D_2)+\frac{\bar{E}^2_R}{\bar{B}^2}(B_1A_1+B_2A_2)-\frac{2\bar{E}_S\bar{E}_RB_1\cos\theta}{\bar{B}^2} \quad (87)$$

Further discussion of the use of *ABCD* constants and power angle diagrams is given in Chapter 10, Sec. 21.

IV. TYPICAL TRANSMISSION LINE CHARACTERISTICS

In any detailed analysis of power flow, voltage regulation, and losses involving a transmission line circuit, each line should be considered individually with regard to its specific characteristics. However, for rough approximations there are certain rules of thumb that apply to an "average" line and that can be used for orientation reasons.

A study was made of recently constructed transmission lines in the United States in the voltage range from 69 to 230 kv and Table 3 shows the results. This table is a good representative cross section of existing lines and gives important characteristics of typical lines. The conductor sizes, spacings, and type of tower construction represent the most common usage. For the middle value of spacing, the characteristics of the aluminum conductor and its copper equivalent are given to illustrate the difference between types of conductors. In previous years, copper conductors were used more frequently although the present trend seems to be toward the use of ACSR conductors. The spacings given were modified slightly in some instances so as to follow a smooth curve of spacing vs. voltage for the different types of construction. Regarding the type of construction, it appears that the particular locale dictates the material used. As a matter of fact, in certain sections of the world reinforced concrete poles are used because of the unavailability and high cost of either steel or wood.

The 60-cycle series reactance in ohms per mile is given for each line in the table. The average of these values is 0.7941 ohm per mile, which indicates that the rule of approximately 0.8 ohm per mile for a transmission line is applicable. Frequently, it is desired to know the percent reactance per mile of a line and for convenience this value is also given. The percent reactance varies directly with the kva base so that for some base other than 100 mva, the percent reactance can also be determined conveniently.

As previously mentioned, the use of susceptance is less at present because of the manner in which tables of conductor characteristics are given. The shunt-capacitive reactance in megohms per mile is therefore included in this table. The susceptance can be determined by taking the reciprocal of the shunt-capacitive reactance. The susceptance is in micromhos per mile. Shunt-capacitive reactance varies inversely with the distance in miles.

The average value of the shunt capacitive reactances in Table 3 is 0.1878 megohm per mile. A good rule is that 0.2 megohm per mile may be used for the shunt-capacitive reactance. It is significant to note that regardless of the voltage, conductor size, or spacing of a line, the series reactance and shunt-capacitive reactance are respectively, approximately 0.8 ohm and 0.2 megohm per mile.

The charging kva per mile of line is a convenient value for reference and is given in column 9 of the table. This value varies with the voltage of the line. Some convenient

TABLE 3. TYPICAL TRANSMISSION LINE CHARACTERISTICS AT 60 CYCLES

Circuit Voltage Kv L-L	Conductor Size Thousands of Cir. Mils or AWG	Tower Construction*	Equiv. Spacing Feet	Resistance at 50°C Ohms Per Phase Per Mile	Reactance Per Phase Per Mile		Shunt-Capacitive Reactance Megohms Per Phase Per Mile	Three Phase Charging Kva Per Mile	Surge Impedance Ohms L-N	Surge-Impedance Loading (SIL) in Three Phase Kw
					Ohms	% on 100 Mva, Three Phase Base				
69	2/0 Cu	SC-W**	8	0.481	0.7843	1.64	0.1822	26.1	378	12 600
69	336.4 ACSR	DC-ST	11	0.306	0.7420	1.55	0.1750	27.2	360	13 200
69	4/0 Cu	SC-W	14	0.303	0.8112	1.70	0.1902	25.0	393	12 100
69	336.4 ACSR	SC-W	14	0.306	0.7712	1.61	0.1822	26.1	375	12 700
69	336.4 ACSR	SC-ST	19	0.306	0.8083	1.69	0.1913	24.9	393	12 100
115	336.4 ACSR	DC-ST	13	0.306	0.7622	0.576	0.1800	74.7	370	35 700
115	4/0 Cu	SC-W	17	0.303	0.8348	0.631	0.1960	67.5	404	32 700
115	336.4 ACSR	SC-W	17	0.306	0.7948	0.601	0.1880	70.4	386	34 200
115	336.4 ACSR	SC-ST	22	0.306	0.8261	0.624	0.1956	67.6	402	32 800
138	397.5 ACSR	DC-ST	15	0.259	0.7636	0.401	0.1809	105.	371	51 200
138	250 Cu	SC-W	18	0.257	0.8317	0.436	0.1952	97.6	404	47 100
138	397.5 ACSR	SC-W	18	0.259	0.7857	0.412	0.1864	102.	382	49 800
138	397.5 ACSR	SC-ST	24	0.259	0.8206	0.430	0.1949	97.7	399	47 600
161	397.5 ACSR	DC-ST	17	0.259	0.7788	0.300	0.1847	140.	379	68 400
161	250 Cu	SC-W	19	0.257	0.8383	0.323	0.1968	132.	406	63 800
161	397.5 ACSR	SC-W	19	0.259	0.7923	0.305	0.1880	138.	386	67 200
161	397.5 ACSR	SC-ST	25	0.259	0.8256	0.318	0.1961	132.	402	64 400
230	795 ACSR	DC-ST	22	0.1288	0.7681	0.145	0.1821	291.	374	141 000
230	500 HH-Cu	SC-W	25	0.1260	0.7436	0.140	0.1800	294.	365	145 000
230	795 ACSR	SC-W	25	0.1288	0.7836	0.148	0.1859	285.	381	139 000
230	795 ACSR	SC-ST	31	0.1288	0.8097	0.153	0.1923	275.	394	134 000
					Avg. 0.7941		Avg. 0.1878		Avg. 386	

*DC-ST—double circuit—steel tower
SC-W —single circuit —wood
SC-ST —single circuit —steel tower
**Two-crossarm construction forming triangular configuration.
All other SC-W are H frame construction.

rules are given for estimating charging kva in the following discussion.

The surge impedance of a transmission line is numerically equal to $\sqrt{\frac{L}{C}}$. It is a function of the line inductance and capacitance as shown and independent of line length. A convenient average value of surge impedance is 400 ohms. As shown in the table, this value is more representative of the larger stranded copper conductors than it is for the ACSR conductors. Compared to the average value of 386 ohms from the table, 400 ohms is a good approximation.

Surge-impedance loading in mw is equal to

$$\frac{(\mathrm{kv}_{L-L})^2}{\text{Surge Impedance}}$$

and can be defined as the unit power factor load that can be delivered over a resistanceless line such that the $\bar{I}^2X$ is equal to the charging kva of the line. Under this condition the sending-end and receiving-end voltages and currents are equal in magnitude but different in phase position. In the practical case of a line having resistance, the magnitude of the sending-end voltage is approximately equal to the magnitude of the receiving-end voltage plus the product of the magnitude of the current and the line resistance; i.e., $\bar{E}_S = \bar{E}_R + \bar{I}R$. Surge-impedance loading in itself is not a measure of maximum power that can be delivered over a line. Maximum delivered power must take into consideration the length of line involved, the impedance of sending- and receiving-end equipment, and in general all of the major factors that must be considered with regard to stability. The relation of surge-impedance loading to line length, taking into account the stability consideration, is covered in Chap. 13, Part IX.

Following is a summary of approximations that may be applied to transmission lines for estimating purposes:

1. Series reactance of a line = 0.8 ohm per mile.
2. Shunt-capacitive reactance of a line = 0.2 megohm per mile.
3. Surge impedance of a line = 400 ohms.
4. Surge-impedance loading, (SIL) in mw $= \frac{(\mathrm{kv}_{L-L})^2}{400}$ or in kw $= 2.5(\mathrm{kv}_{L-L})^2$.
5. (a) Charging kva for a hundred miles of line is 20.5 percent of the SIL.

   (b) Charging kva of a line is also $= 5000\left(\frac{L}{100}\right)\left(\frac{\mathrm{kv}_{L-L}}{100}\right)^2$,

   where L = line length in miles,
   kv_{L-L} = line-to-line voltage in kilovolts.

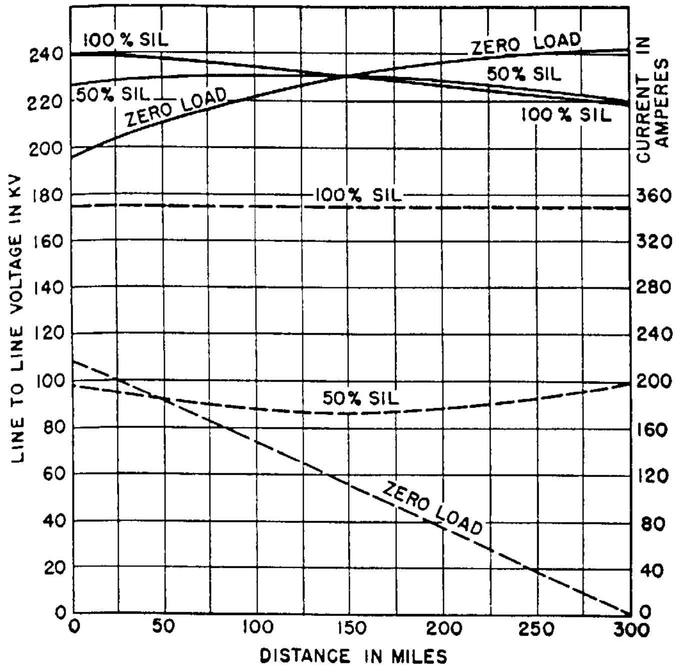


Fig. 18—Distribution of voltage and current along a 300-mile transmission line, 795 000 circular mils, ACSR conductor, 25-foot equivalent spacing.

——— Voltage
-------- Current
r = 0.117 ohm per mile
x = 0.7836 ohm per mile
x' = 0.1859 megohm per mile

The effect of the distributed capacitance of a transmission line on the voltage and current distribution along the line is illustrated in Fig. 18. The calculated results are based on a transmission line 300 miles in length, 230 kv, 795 000 circular mils, and 25-foot equivalent spacing. The 100-percent surge-impedance loading of the line is 139 000 kilowatts. The current corresponding to this load at 100 percent voltage is 348 amperes. The voltage and current are shown as a function of the line length for 100 percent, 50 percent surge-impedance loading at the middle of the line and for zero delivered load. The voltage at the middle of the line was maintained at 230 kv and E_S and E_R were allowed to vary depending upon the load condition.

At 100-percent surge-impedance loading, the voltages $E_S = 240$ kv and $E_R = 219$ kv. The current is a constant value of 348 amperes. If the surge-impedance loading is assumed at the receiving end of the line, the magnitude of the current is slightly different at the sending end because of line resistance. The amount of this difference depends upon the ratio of line reactance to resistance and the length of the line. Based on the calculated voltages of E_S and E_R, the regulation of the line is 9.5 percent. The value of regulation as determined from the product of the magnitude of the current and the resistance is also 9.5 percent.

For 50-percent surge-impedance loading the current is a minimum value at the middle of the line. If the surge-impedance loading is taken at the receiving end, the current decreases to a minimum at the receiving end. In Fig. 18 surge-impedance loading is taken in the middle of the line for purposes of exposition. Generally the surge-impedance loading should be considered at the receiving end because the delivered load is usually the quantity of most interest.

V. 60-CYCLE TRANSMISSION LINE REGULATION AND LOSS CHARTS

The voltage regulation and efficiency of a transmission line or distribution feeder are fundamental properties of its performance. In determining these quantities for existing systems or in designing new systems to meet given load requirements, it is thought that the charts presented here will save a great deal of time and labor that would in many cases be necessary if analytical methods were used.

For low voltage lines without synchronous or static capacitors, voltage regulation is usually the more important consideration. For instance, in the design of a line to carry a certain load one wishes to determine the proper transmission voltage and conductor size. Based on an assumed allowable regulation several voltages and conductor sizes will be found to transmit the load, the final choice being based upon economics for which the line efficiency is desired. The performance of higher voltage regulated lines, however, is determined primarily by the line loss.

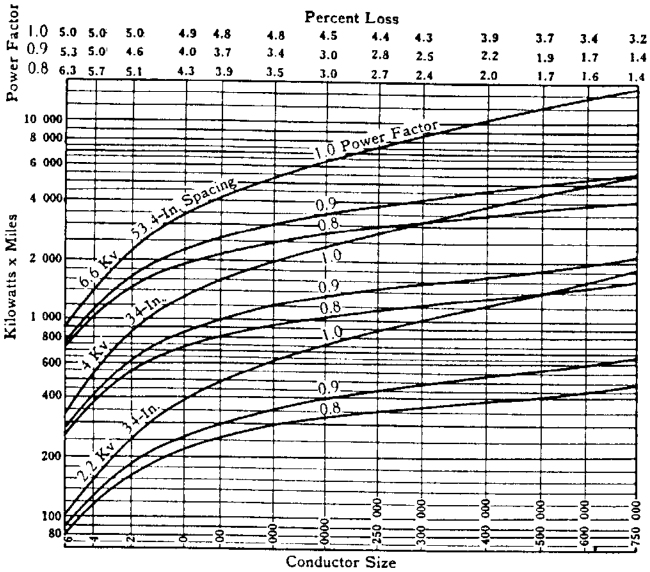
The charts presented here were developed with these two points of view in mind. Quite frequently it is desired to obtain quickly an approximate solution. The Quick Estimating Charts afford a simple method for such cases. For more accurate calculations the Regulation and Loss Chart is provided. It is important to be able to consider more than just the line itself. The transformers are often the determining factor in the choice of the proper line voltage. The Regulation and Loss Chart is constructed so that from the knowledge of the equivalent impedance of a system its performance can be determined.

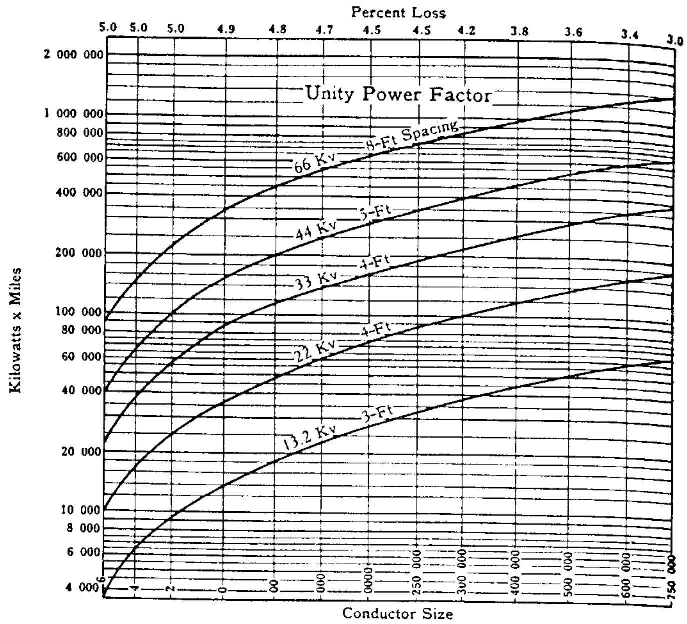
25. Quick Estimating Charts

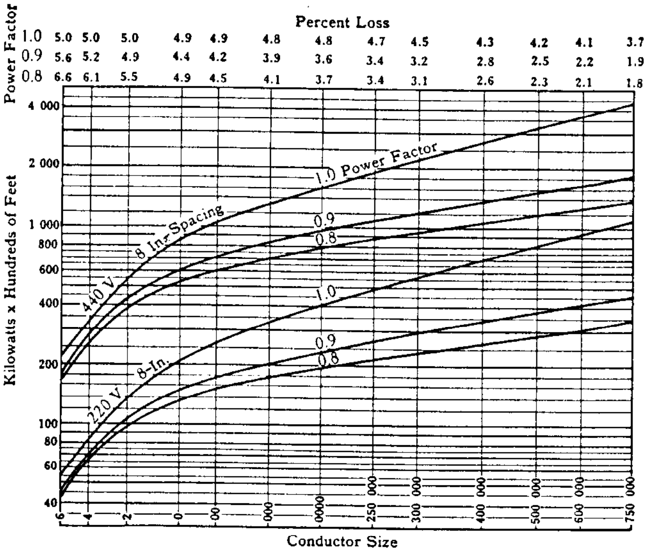
In Figs. 19 and 20 are plotted curves showing the power which can be transmitted at five percent regulation together with the corresponding percent line loss for various voltages and conductor sizes. These curves afford the rapid estimation of such problems as the regulation for a known load, the load limit of a line for a given regulation and the determination of voltage and conductor size for the transmission of a given load at a given regulation. Fig. 21 is an aid for interpolation between the values of power factor given on the curves.

The curves of Fig. 22 give the power which can be transmitted for various conductors and voltages at a line loss of five percent. These curves are most useful in determining the performance of lines regulated by synchronous or static capacitors.

Charts Based Upon Regulation——Fig. 19 applies specifically to stranded copper conductors, but it can be used for copperweld-copper conductors with an accuracy of two to three percent. Fig. 20 applies to ACSR conductors. The load which can be transmitted over a line at a fixed regulation varies inversely with its length so that for a given line the actual load is the value read from the curves divided by the line length. For 220 to 440-volt lines the values on the curves are given in kilowatts times hun-







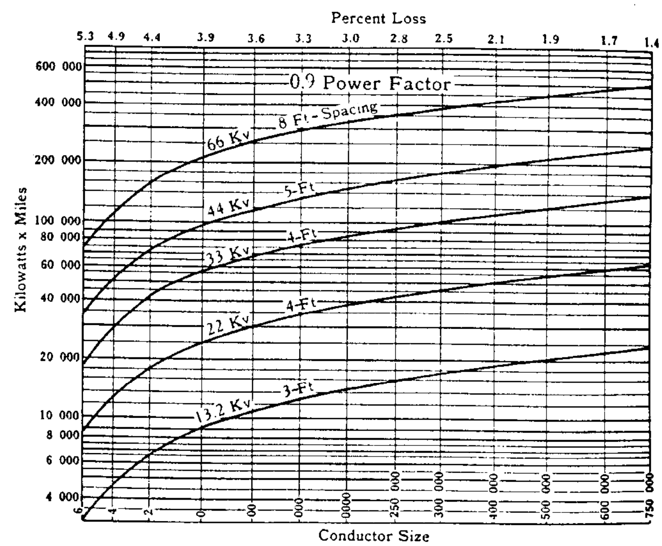
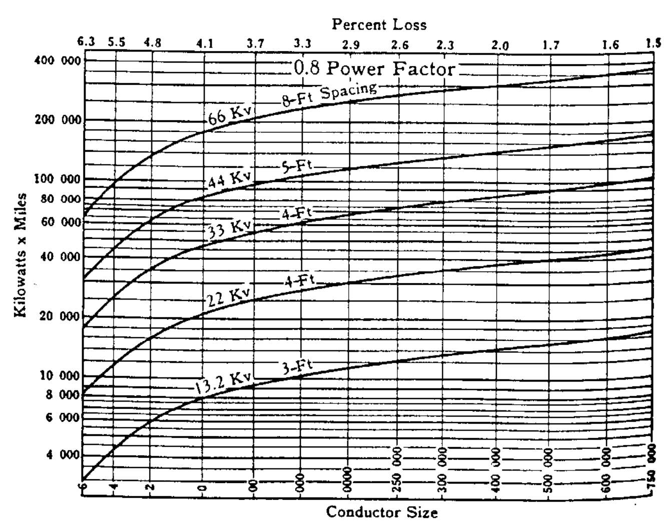


Fig. 19—Quick Estimating Charts Based Upon 5 Percent Regulation—Stranded Copper Conductors.



The curves give load in kilowatts × miles or kilowatts × hundreds of feet which can be received at 5 percent regulation together with corresponding line loss.

For a given length of line, power is equal to value read from curves divided by length of line.

Power for other regulations is approximately equal to values read from curves multiplied by $\frac{\%\ \text{Reg}}{5}$.

For power factors other than given in charts, multiply values read from curves for unity power factor by fractions given in Fig. 21.

Percent loss for other regulations and power factors than found on charts are given by equation

$$(\text{Percent Loss})_2 = (\text{Percent Loss})_1 \times \frac{(\text{Kw Load})_2}{(\text{Kw Load})_1} \times \frac{(\text{Power Factor})_1^2}{(\text{Power Factor})_2^2}$$

For single phase lines divide power read from charts by 2 and percent loss by $\sqrt{3}$.

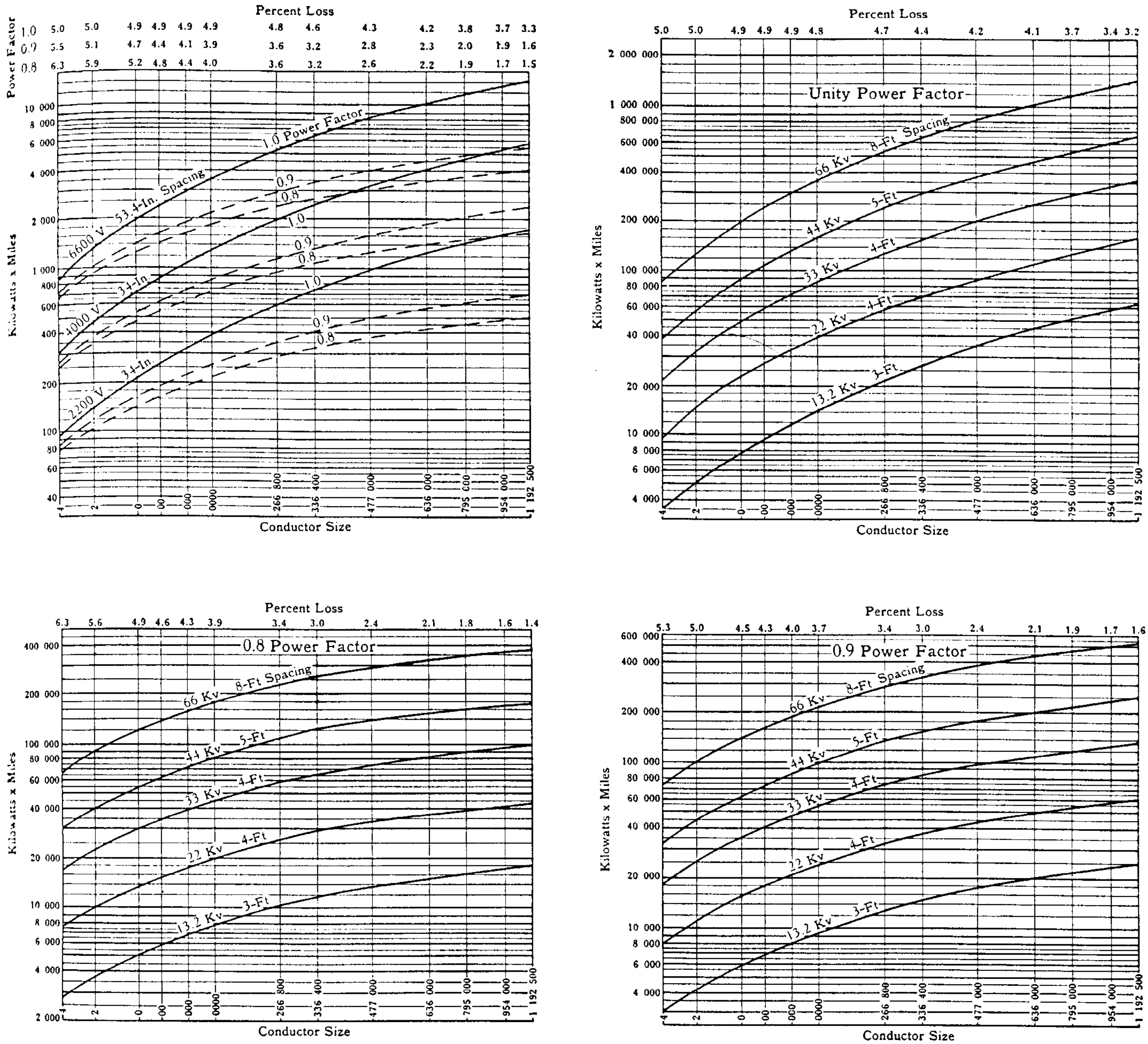


Fig. 20—Quick Estimating Charts Based Upon 5 Percent Regulation—A.C.S.R. Conductors.

The curves give load in kilowatt × miles which can be received at 5 percent regulation together with corresponding line loss.

For a given length of line, power is equal to value read from curves divided by length of line.

Power for other regulations is approximately equal to values read from curves multiplied by $\frac{\% \text{ Reg}}{5}$.

For power factors other than given in charts, multiply values read from curves for unity power factor by fractions given in Fig. 21.

Percent loss for other regulations and power factors than found on charts are given by equation

$$(\text{Percent Loss})_2 = (\text{Percent Loss})_1 \times \frac{(\text{Kw Load})_2}{(\text{Kw Load})_1} \times \frac{(\text{Power Factor})_1^2}{(\text{Power Factor})_2^2}$$

For single phase lines divide power read from charts by 2 and percent loss by $\sqrt{3}$.

dreds of feet. For higher voltages they are in kilowatts times miles.

For each voltage a common equivalent conductor spacing is assumed and the curves are drawn so that it is possible to interpolate to a good degree of accuracy for other voltages than those given. In addition the relationship that the power is proportional to the square of the voltage may be used. Since the percent loss does not vary more than about a tenth of one percent for each conductor size in each set of curves, mean values are given as shown.

For the same line voltage, conductor, equivalent spacing, and regulation half as much load can be transmitted on a single-phase two-wire line as for a three-phase line. For this reason the curves can be used to good accuracy for this kind of line by simply dividing by two the load read from them. For this single-phase load the percent

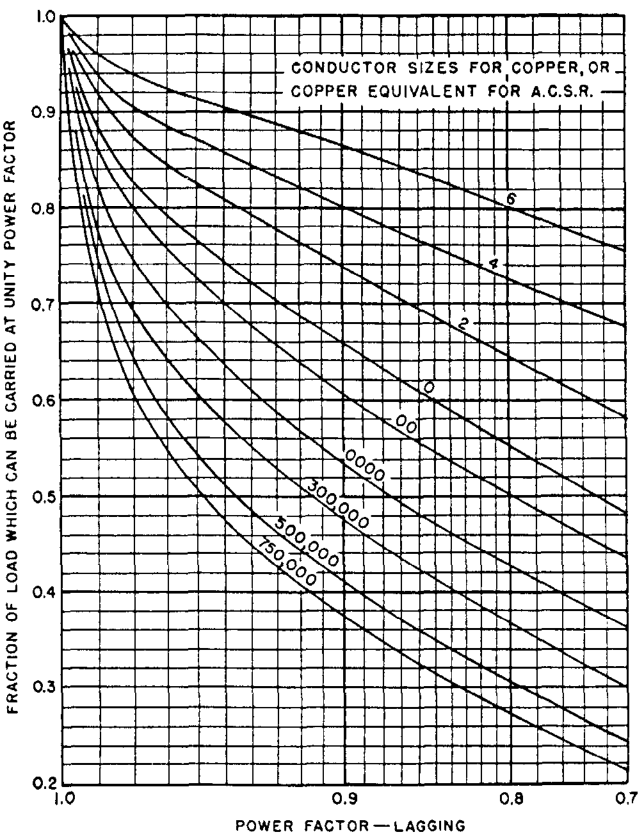


Fig. 21—Effect of power factor on load that can be carried at a fixed regulation.

Curves apply specifically for three foot equivalent spacing and five percent regulation, but can be used with good accuracy for normal spacing and regulation range.

loss will be that read from the charts divided by [$\sqrt{3}$ (or 1.732)].

Curves are presented for three common power factors: unity, 0.9 lag, and 0.8 lag. It is difficult to interpolate for other power factors, however, especially between unity and 0.9. To facilitate this the curves of Fig. 21 are provided showing the effect of power factor on the load that can be transmitted at a fixed regulation in terms of that at unity power factor. The curves apply specifically to stranded copper conductors at a three foot equivalent spacing and for five percent regulation, but they will give an accuracy within 10 percent for conductor spacings up to 20 feet and for the same copper equivalent in other common conductors. The error however may be as high as 25 percent for spacings as small as 8 inches.

The Quick Estimating Curves can also be used for other values of regulation if the approximation is made that the load which can be transmitted varies directly with the regulation.

After having determined the load for other power factors or regulations than those for which the curves are drawn, the percent loss can be determined from the relation

$$(\text{Percent Loss})_2 = (\text{Percent Loss})_1 \times \frac{(\text{Kw Load})_2}{(\text{Kw Load})_1} \times \frac{(\text{Power Factor})_1^2}{(\text{Power Factor})_2^2} \qquad (88)$$

Charts Based Upon Loss—In Fig. 22 (a) are plotted curves for short lines which show the power in kilowatts times miles which can be transmitted under two conditions. The solid curves are based on five percent loss and equal receiving- and sending-end voltages. These are useful for lines where little regulation can be allowed such as on interconnected systems. The dotted curves are for the maximum power which can be transmitted at the given load voltage and five percent loss. For this condition the regulation varies but in no case does it exceed about five percent.

Fig. 22 (b) is for higher voltage lines long enough that distributed capacitance of the line need be considered. Only the condition of equal sending and receiving end voltages is considered here since regulation does not greatly effect the power for the conductors and spacings practical to use. For all of these curves an arbitrary coordinate system has been used for the abscissa beneath which is plotted the correct sizes for the various conductors. The curves here are based on 10 percent loss.

Equation (77) was used for determining the curves for equal voltages at both ends of the line and its examination shows that, for the practical range of losses, power for other values of percent loss are very nearly that read from the curves multiplied by $\frac{\%\text{ Loss}}{5 \text{ or } 10}$. If greater accuracy is desired the factor $\frac{\%\text{ Loss}}{100+\%\text{ Loss}}$ of Eq. (77) can be used. Eq. (55) was used for the curves based on the maximum power at five percent loss. For this case power is directly porportional to loss. For both sets of curves it is proportional to the square of the receiving-end voltage.

The power which can be transmitted over a single-phase line is one half that of a three-phase line of the same equivalent spacing and line-to-line voltage. Thus Fig. 22(a) can be used to good accuracy for single-phase lines by dividing the values read from the curves by two.

26. Examples of the Use of the Quick Estimating Charts

Example 6(a)—Determine the maximum load at unity power factor and five percent regulation which can be transmitted over a three-phase five-mile line having 300 000 cir mil stranded copper conductors and operating at a load line voltage of 22 kv.

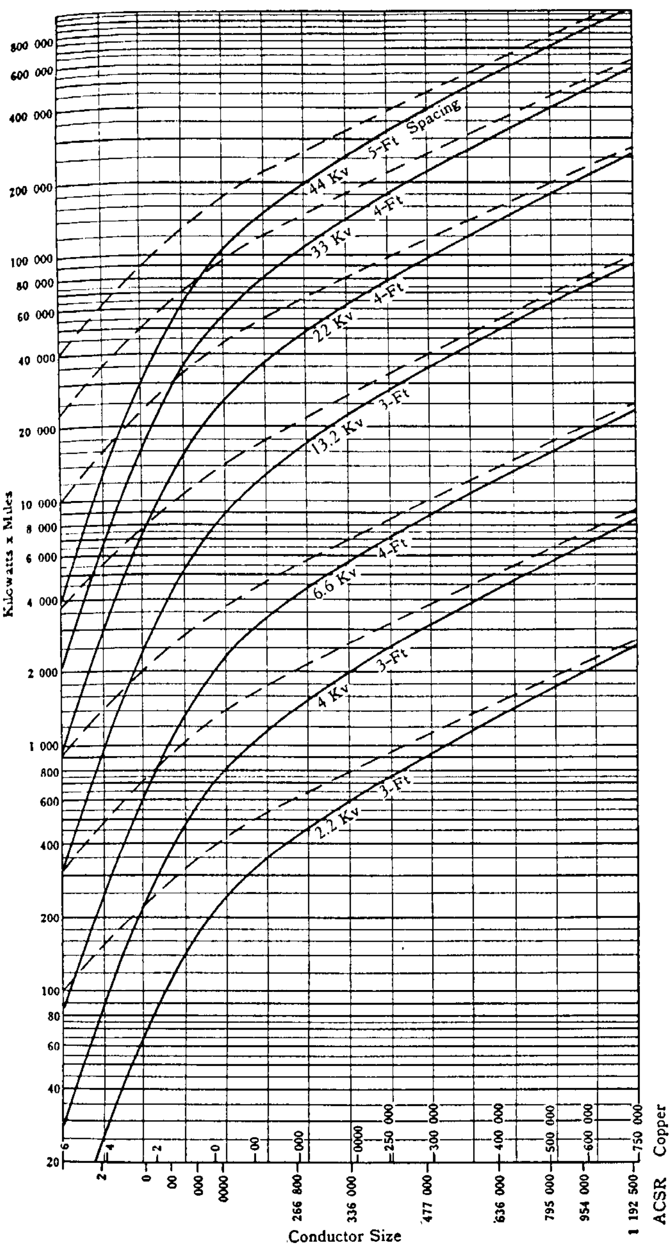
From the unity power factor curves of Fig. 19 for this conductor size and voltage, 100 000 kw times miles is obtained. The load is then $\frac{100\,000}{5} = 20\,000$ kilowatts. The percent loss read from the curves is 4.2.

Example 6(b)—What is the load for this line at this regulation but 0.95 power factor lag? Referring to Fig. 21 it is seen that for this conductor size 0.58 as much load can be transmitted at 0.95 power factor as at unity.

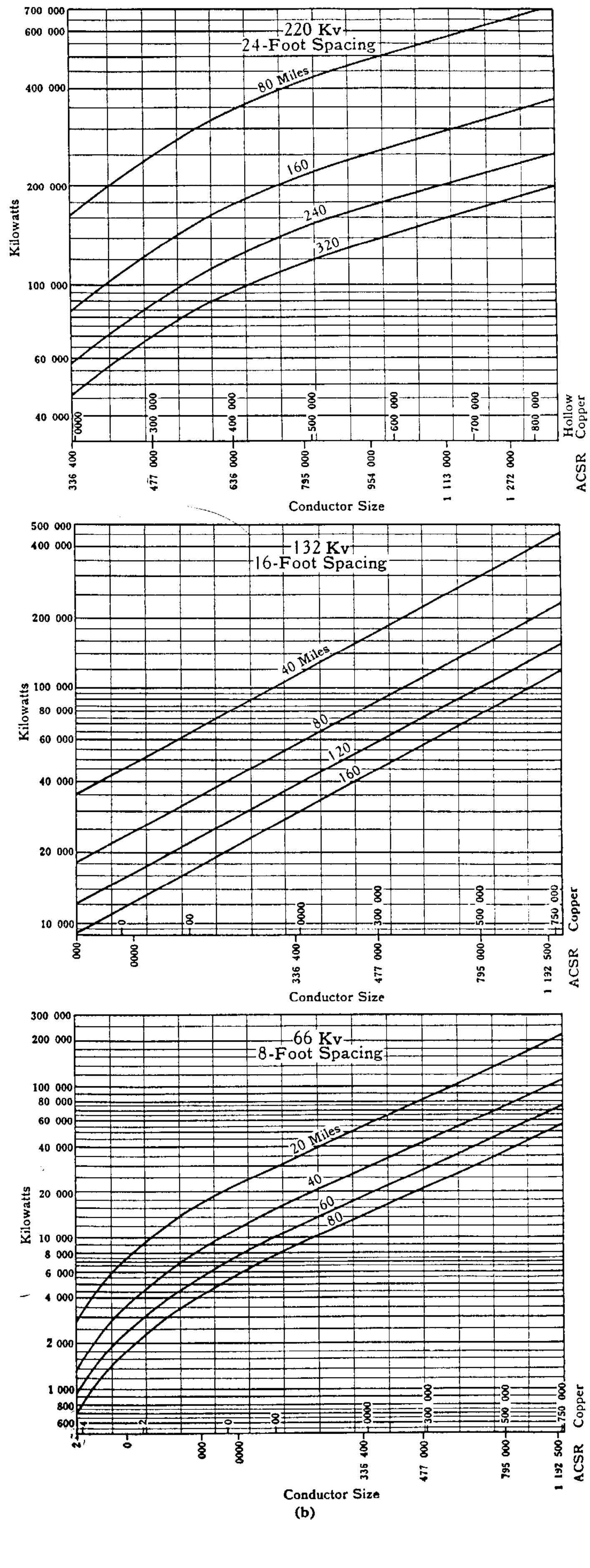
Thus the load is $20\,000 \times .58 = 11\,600$ kilowatts. The percent loss as determined from Eq. (88) is

$$\text{Percent Loss} = (4.2)\ \frac{11\,600(1)^2}{20\,000(.95)^2} = 2.7\%$$

Example 6(c)—What load can be transmitted over this line at unity power factor but 15 percent regulation?



(a)



(b)

Fig. 22—Quick Estimating Charts Based Upon Percent Loss.

The solid curves are based on percent loss and equal receiving- and sending-end voltages. The dotted curves are for the maximum power which can be received at a given receiving-end voltage and percent loss.

For the curves of Fig. 22 (a) line capacitance has been neglected and power for a given length of line is value read from curves divided by line length in miles. Loss base is 5 percent.

In Fig. 22 (b) line capacitance has been taken into account and the data is thus a function of line length. Loss base is 10 percent.

For all curves:

For other values of percent loss multiply power read from curves by $\frac{\%\ \text{Loss}}{5}$ for (a) and $\frac{\%\ \text{Loss}}{10}$ for (b).

For single-phase lines divide power read from charts by 2.

The answer is $(20\,000)\frac{15}{5}=60\,000$ kw

The percent loss is $(4.2)\frac{60\,000}{20\,000}=12.6\%$

Example 7—Determine the conductor size and voltage necessary to transmit 10 000 kw at 0.9 power factor lag for a distance of ten miles.

This corresponds to 100 000 (kw times miles). Referring to the 0.9 power factor curves for both copper and ACSR conductors for this load, it is seen that the following lines can be used:

	Stranded Copper		ACSR	
Voltage	Cond. Size	% Loss	Cond. Size	% Loss
33 000	300 000 cir mil	2.5	636 000 cir mil	1.9
44 000	No. 0	4.0	No. 0000	3.7
66 000	No. 4	4.5	No. 2	5.0

If it were desired to allow a ten percent regulation instead of five percent, the value of kilowatt miles to refer to on the curves would then be 50 000 instead of 100 000.

The use of the Quick Estimating Charts based upon line loss is quite similar. For instance, if the line of example 6 were equipped with capacitors so that regulation would not be excessive, examination of Fig. 22 shows that it could deliver a maximum of $\left(\frac{116\,000}{5}=23\,200 \text{ kw}\right)$ at five percent loss.

27. Regulation and Loss Chart

Several valuable voltage regulation charts have been developed. Perhaps the best known of these are the Dwight[7] and Mershon[8] charts. The chart shown in Fig. 23 provides a means of solving not only regulation but loss problems to a high degree of accuracy. It is just as simple in its use as any of the previous ones, but has the distinct advantage that it is based upon an exact solution of the vector diagram for any circuit which can be represented by a single lumped impedance. For this reason problems involving the determination of the load which can be transmitted for a given regulation can be solved much more accurately than from charts based upon approximations.

The chart is developed on the principle that for a given difference in magnitude between the sending-end and receiving-end voltages, the impedance drop (ZI) is fixed entirely by the angle $\rho=\gamma+\phi$ where $\left(\gamma=\tan^{-1}\frac{x}{r}\right)$ is the impedance angle of the line and ϕ is the power factor angle. For lagging power factors ϕ is negative and for leading power factors ϕ is positive. Thus, corresponding to various values of percent regulation, the corresponding percent ZI can be plotted as a function of the angle ρ. These are the set of curves on the chart for voltage drops from 0 to 15 percent and voltage rises from 0 to 5 percent. The value of the percent (ZI) is the same whether ρ is positive or negative. It depends only upon its magnitude.

Since the use of the chart requires a knowledge of γ and ϕ, additional curves are provided to facilitate their determination. One of these is a cosine curve for determining ϕ from the power factor. For obtaining γ from a knowledge of the resistance and reactance of the line, tangent and cotangent curves are plotted so that γ can be obtained from the ratio x/r or r/x. However, a simpler means is provided for standard conductors, by the set of curves at the top and bottom of the main portion of the chart. These curves give γ for various conductors as a function of equivalent spacing. The resistance of the conductor per mile is necessary, and it is given for each conductor. The values on the chart are for a conductor temperature of 50°C.

Although the chart is developed primarily for problems involving known receiver voltage and power factor, it can also be used for problems where the sending-end voltage and receiving-end power factor and either load current or sending end kva are known. This is the commonest type of problem involving mixed terminal conditions.

28. Use of the Regulation and Loss Chart for Short Lines

(*a*) ***Regulation from Known Load Conditions***—to calculate regulation when receiving-end (or load) voltage, power factor, and current or kva are known:

(1) Determine $\rho=\gamma+\phi$ where the sign of ϕ is dependent upon whether the current is leading or lagging.

ϕ, the power factor angle, can be obtained from the cosine curve.

γ, the impedance angle, can be obtained by reading it from the conductor curves or by calculating r/x or x/r whichever is less than one and reading from the corresponding curve. r and x are the conductor resistance and reactance in ohms per mile.

(2) Calculate percent ZI where

$$\text{Percent } ZI=\frac{(\sqrt{3}\, rs\, I)\,100}{E_L \cos\gamma}=\frac{100\,000\, rs\,(\text{kva})}{E_L^2 \cos\gamma} \quad \text{for three-phase lines} \tag{89}$$

$$=\frac{(2\, rs\, I)\,100}{E_L \cos\gamma}=\frac{200\,000\, rs\,(\text{kva})}{E_L^2 \cos\gamma} \quad \text{for 2-wire single-phase lines} \tag{89a}$$

E_L is the line voltage in volts. s is the length of the line in miles.

(3) For the calculated values of ρ and percent ZI read percent regulation from curves of constant regulation.

(*b*) ***Load Limitation for Fixed Regulation***—To determine load limit for a given value of regulation:

(1) Determine ρ as in above and from chart for given value of regulation and ρ read the corresponding percent ZI.

$$\text{Load in kva}=\frac{(\%\ ZI)E_L^2 \cos\gamma}{100\,000\, rs} \quad \text{for three-phase lines} \tag{90}$$

$$=\frac{(\%\ ZI)E_L^2 \cos\gamma}{200\,000\, rs} \quad \text{for single-phase 2-wire lines} \tag{90a}$$

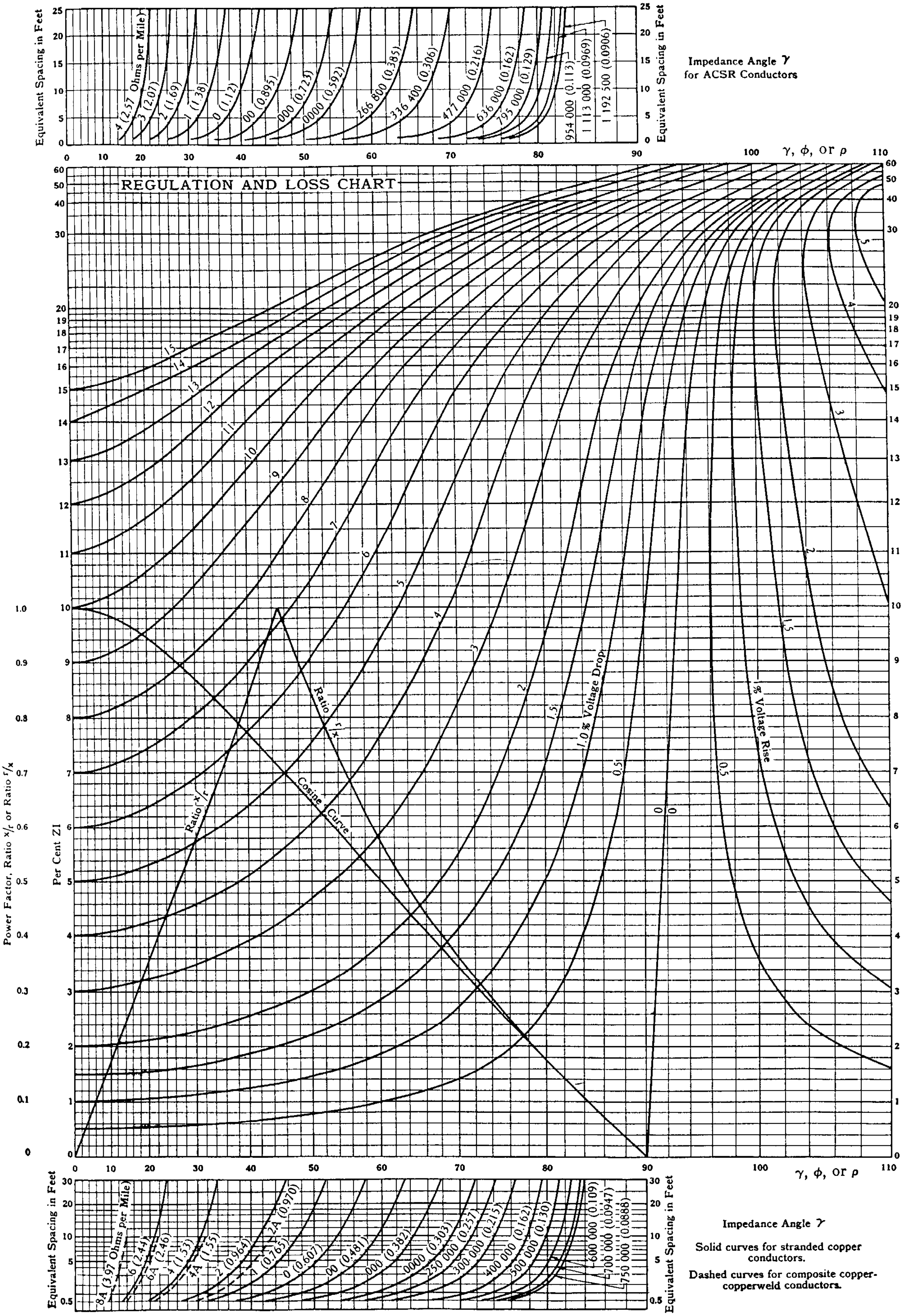


Fig. 23—Regulation and Loss Chart for transmission lines.

(c) Line Efficiency—The line loss in percent of the load kva is given by the equation

$$\text{Percent Loss} = \%\ RI = \%\ ZI \cos \gamma \qquad (91)$$

where $\cos \gamma$ can be read off its cosine curve from the known value of γ. The loss can be determined in percent of the load in kilowatts by dividing the value obtained from Eq. (91) by the power factor. If it is desired to determine the percent loss for a given regulation, the percent ZI can be obtained without the use of Eq. (89). It is simply necessary to determine ρ and for this angle and the given regulation to read the (percent ZI) from the chart.

(d) Use of Chart for Known Sending-End Voltage and Receiving-End Power Factor—The chart can be used to as good accuracy as desired for problems of this nature. As a first approximation the regulation, in percent of the sending-end voltage, can be obtained as outlined in (a) when the sending-end line voltage is used in Eq. (89). Either the line current or the load kva expressed in terms of the sending-end voltage can be used. The load (or receiving-end) voltage can be calculated from this regulation and the sending-end voltage. This first approximation will usually give the load voltage to an accuracy of about one percent, but the percent accuracy of the regulation may be much worse depending upon its magnitude.

A more accurate value can, however, be very easily obtained by the following method of successive approximations. Using this first determined value of load voltage and then each successive value obtained, recalculate the regulation. One or two such steps will usually give very good accuracy. When calculating the percent ZI in this process it is not necessary to solve Eq. 89 each time. The new value of percent ZI can be obtained by dividing the first value calculated by the ratio of the load voltage to the sending-end voltage. This type of problem is illustrated in Example 8(d).

It is, of course, obvious that the load limit for known sending-end voltage, load power factor, and regulation can be determined as in 28(b) after the load voltage is calculated from the regulation and sending-end voltage.

29. Examples of the Use of the Regulation and Loss Chart

Consider a three-phase line ten miles long with No. 0000 stranded-copper conductors at an equivalent spacing of six feet and operating at a line voltage of 33 kv at the load end.

Example 8(a)—For rated voltage at the receiving end and a 9140 kva load at 0.9 power factor lag, determine the regulation.

Referring to the impedance angle curves for stranded copper conductors at the bottom of the chart, the impedance angle for this conductor and spacing is $\gamma = 67.2°$. $\cos \gamma$ is 0.390 and the conductor resistance is 0.303 ohms per mile. Reading from the cosine curve the power factor angle for 0.9 power factor is $\phi = 26°$, and the sign is minus $\rho = \gamma + \phi = 67.2° - 26° = 41.2°$

From Eq. (89):

$$\text{Percent } ZI = \frac{(100\,000)(0.303)(10)(9140)}{(33\,000)^2\ (0.390)} = 6.52$$

Reading from the chart for this percent ZI and $\rho = 41.2°$, the regulation is found to be 5.0 percent.

Example 8(b)—Determine the maximum kva that can be transmitted over this line at the same power factor for a regulation of no greater than 5 percent. Reading from the chart for 5 percent regulation and ρ of 41.2°, the percent ZI is found to be 6.54.

Using Eq. (90):

$$\text{Load in kva} = \frac{(6.52)(33\,000)^2(0.390)}{(100\,000)(0.303)(10)} = 9140$$

$$\text{Load in kw} = (9140)(0.9) = 8230.$$

Example 8(c)—As an example of the calculation of efficiency for the above case using Eq. (91):

$$\text{Percent loss} = (6.52)(0.390) = 2.55.$$

Example 8(d)—For this same line operating at a *sending*-end line voltage (E_{SL}) of 33 kv and a sending-end load of 9140 kva but a receiving-end lagging power factor of 0.9, determine the line voltage at the load end.

As shown in Example 8(a):

The value of percent ZI determined as a first approximation by using the sending-end voltage and kva in Eq. (89) is

$$\text{Percent } ZI = 6.52$$

and $$\rho = \gamma + \phi = 41.2°$$

Thus as a first approximation

$$\text{Percent Reg.} = 5$$

$$E_L = \frac{E_{SL}}{1.05} = 31.42 \text{ kv.}$$

As a second approximation

$$\text{Percent } ZI = (1.05)(6.52) = 6.85$$

reading from the chart for percent $ZI = 6.85$ and $\rho = 41.2°$

$$\text{Percent Reg.} = 5.20$$

$$E_L = \frac{E_{SL}}{1.052} = 31.35 \text{ kv.}$$

As a third approximation

$$\text{Percent } ZI = (1.052)(6.52) = 6.87$$

Percent Reg. = 5.25 (as closely as can be read from the chart)

$$E_L = \frac{E_{SL}}{1.0525} = 31.34 \text{ kv.}$$

30. Use of Regulation and Loss Chart for Long Lines

As shown in Sec. 16, methods of calculating regulation for short lines can be applied to lines up to 100 miles in length to a good degree of accuracy by simply adding the correction factor ($-2.01S^2$) to the percent regulation where S is the length of the line in hundreds of miles.

If greater accuracy is desired, the chart can be used with the equivalent load current and power factor obtained as described in Sec. 14. Using this method both regulation and efficiency can be determined.

31. Determination of Effect of Transformers on Line Performance

The chart can be used as described in Sec. 28 for determining regulation and efficiency of transformers al though the transformer charts in Chap. 5 are simpler. In considering the performance of a line and transformers together, however, the chart can be used to advantage. The impedance of the transformers can be combined with that of the line into a single impedance. These impedances can be expressed either in ohms or in percent on some common kva base. Transformer impedance is usually given in percent. It can be expressed in ohms by the equation

$$Z_{(\text{ohms})} = \frac{Z_{(\text{percent})} E^2_{L(\text{kv})}(10)}{\text{kva}} \tag{92}$$

The transmission line impedance in ohms can be transformed to a percent basis by the equation

$$Z_{(\text{percent})} = \frac{Z_{(\text{ohms})}\,(\text{kva})}{E^2_{L(\text{kv})}\,(10)} \tag{93}$$

The transmission line resistance can be read directly from the chart and the reactance obtained from the chart by reading the line impedance angle γ from the chart and the ratio of r/x or x/r for this angle.

For problems of this type it is usually easier to use the impedance in percent. After having obtained the total equivalent percent R and percent X, the equivalent angle γ can be read from the curves for the ratio of R/X or X/R. The percent ZI can be calculated from the equation

$$\text{Percent } ZI = \frac{(\%RI = \%R)_{\text{(rated load)}}}{\cos\gamma} \frac{(\text{actual load})}{(\text{rated load})} \tag{94}$$

Example 9—As an example of the calculation of a problem of this type consider the 10 mile, 33 kv, 300 000 cir mil stranded copper line found adequate for the (10 000 kw = 11 111 kva) load at 0.9 power factor lag of Example 7.

Assume that it has transformers at each end rated at 12 000 kva with 0.7 percent resistance and 5 percent reactance, and let us calculate the total regulation and loss of the system.

Reading from the chart

The line resistance is $(0.215)(10) = 2.15$ ohms

r/x for the line impedance angle of 71.6° is 0.330

The line reactance is $\dfrac{2.15}{0.330} = 6.51$ ohms

The percent impedance of the line on a 12 000 kva base is from Eq. (93).

$$\text{Percent } Z_L = \frac{(2.15 + j6.51)(12\,000)}{(33)^2(10)} = 2.37 + j7.16$$

The total impedance is

$$\text{Percent } Z = (2.37 + j7.16) + 2(0.7 + j5)$$
$$= 3.77 + j17.16$$
$$\frac{\%R}{\%X} = \frac{3.77}{17.16} = 0.219$$

Reading from the chart for this ratio

$$\gamma = 77.7°$$
$$\text{Cos } \gamma = 0.219$$
$$\text{For 0.9 power factor } \phi = -26°$$
$$\rho = 51.7$$

From Eq. (94)

$$\text{Percent } ZI = \left(\frac{3.77}{0.219}\right)\left(\frac{11\,111}{12\,000}\right) = 15.94$$

The regulation read from the chart for this percent ZI and the calculated value of ρ is

Regulation = 10.5%

The loss in percent of the load in kw is from Eq. (91)

$$\text{Percent Loss} = \frac{(15.94)(0.219)}{0.9} = 3.88.$$

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CHAPTER 10

STEADY-STATE PERFORMANCE OF SYSTEMS INCLUDING METHODS OF NETWORK SOLUTION

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A POWER system must generate, transmit, and then distribute electric power to the desired points, reliably and in good condition. The electrical performance of the system as dealt with in this chapter is the measure of how well it performs this task and is expressed by such quantities as voltage regulation, loading of lines and equipment, efficiency and losses, and real and reactive power flow. Stability, of vital importance also, is dealt with in Chap. 13.

The key to the determination of such system quantities is the *network solution*, or determination of currents and voltages throughout the system for any prescribed conditions. From the network solution can be determined all of the essential electrical characteristics that are dependent upon the fundamental-frequency currents and voltages.

Network solution is based on Kirchoff's two laws:

First, that the vector sum of all the voltages acting around any closed loop is zero.

And second, that the vector sum of all the currents flowing to any point is zero.

In the course of applying these elementary principles to the solution of thousands of linear networks for many years, various investigators have found several powerful theorems that follow directly therefrom, such as the superposition theorem[1], the reciprocal theorem, and Thevenin's theorem. These theorems not only assist in visualizing the phenomena taking place in the circuits, but also greatly simplify and systematize the work of solution for the species of networks to which they apply.

The method of symmetrical components, given in Chap. 2 is a highly developed special application of the superposition theorem, taking advantage of the symmetry of the several phases of the usual polyphase power system.

The direct use of Kirchoff's Laws can be designated as "Solution by Equations," to distinguish it from "Solution by Reduction" in which portions of a system are progressively replaced by simpler equivalents until a single branch remains. This latter makes use of the superposition theorem in treating one emf at a time. Also, it utilizes equivalent circuits, many of which are now available.

Thevenin's theorem and the superposition theorem have provided direct methods for obtaining solutions in networks of several fixed emfs, with enormous simplification.

Solutions of networks can be expressed in many forms, each one being particularly adaptable to certain types of networks or certain problems. Thus, the expression of solutions as "Self and Mutual Drops and Current Division" is particularly well suited to regulation and apparatus loading studies. The method of Driving Point and Transfer Admittances or Impedances is well suited to power flow or stability studies on multiple-entrance systems, and the General Circuit Constants, *ABCD*, or the equivalent Pi and *T* are similarly advantageous for the transmission-type network having two significant terminals.

These methods of network representation and solution constitute a highly developed science with extensive present literature. However, as they constitute the heart of the problem of steady-state performance of systems as well as of many other system problems, a large part of this chapter will be devoted to them. In general, the most commonly used methods will be outlined and illustrated by examples. For further information a bibliography of selected references is included.

Network solution, once accomplished largely by analytical methods, is now performed to an increasing extent by a-c and d-c network calculators. However, many problems are still solved analytically and also a thorough knowledge of methods of network representation and solution is as essential as ever to the system designer. Fortunately, however, the calculator has removed the enormous burden of routine calculation and has made it economically possible to solve complicated systems. Analytic methods are still largely used for the simpler studies or where network calculators are not available.

I. NETWORK REPRESENTATION

1. Single-Line Diagram. Fig. 1

In dealing with power systems of any complexity, one of the first essentials is a single-line diagram, in which each polyphase circuit is represented by a single line. Stripped of the complexity of several phase wires, the main power channels then stand out clearly, and the general plan of the system is evident. Most power companies maintain up-to-date single-line diagrams of their systems.

This diagram is a short-hand or symbolic representation of the principal connections, showing the equipment in its correct electrical relationship and usually having indicated on it, or in supplementary tabulations, data essential for the determination of the impedance diagram. The recommended symbols for apparatus are given in Table 1(a). In addition, auxiliary symbols, Table 1(b), are inscribed near the devices in question, to indicate the winding connections and the grounding arrangement, if any, at the

TABLE 1(a)—GRAPHICAL SYMBOLS FOR DIAGRAMS°—EQUIPMENT SYMBOLS

NAME	ONE LINE	COMPLETE *	NAME	ONE LINE	COMPLETE *
A. C. GENERATOR OR MOTOR+			DOUBLE THROW SWITCH		
SYNCHRONOUS CONVERTER			OIL CIRCUIT BREAKER, SINGLE THROW		
DIRECT CONNECTED UNITS BASIC SYMBOL (Use particular symbols and join as here shown.)			AIR CIRCUIT BREAKER		
TWO-WINDING TRANSFORMER+ BASIC SYMBOL			FUSE		
THREE-WINDING TRANSFORMER+			RESISTOR		
AUTOTRANSFORMER+			REACTOR		
CURRENT TRANSFORMER			CAPACITOR		
POTENTIAL TRANSFORMER			LIGHTNING ARRESTER		
INDUCTION VOLTAGE REGULATOR			POTHEAD CABLE TERMINAL		
DISCONNECTING OR KNIFE SWITCH			DRY RECTIFIER		
AIR BREAK SWITCH, HORN GAP, GROUP OPERATED			MERCURY ARC RECTIFIER		

* The "Complete" symbol is intended to illustrate the method of treatment for any desired polyphase combination rather than to show the exact symbol required. Use symbol (⌒⌒⌒) for windings of apparatus as required, and connect to suit particular case.

\+ Inscribe winding connection diagram symbol from Table 1b.

O For complete lists see American Standards Z32.3-1946, Z32.12-1947

neutral. The use of these auxiliary symbols is illustrated in Fig. 1.

Similar diagrams showing circuit breakers and disconnecting switches are used as power-system operating diagrams. Or they can be marked with suitable symbols to show the relay (See Chap. 11) or lightning protection.

2. The Sign of Reactive Power

The + sign used with the reactive-power terms in the loads of Fig. 1 designate lagging-reactive power in accordance with the standard notation approved by the AIEE Standards Committee on Jan. 14, 1948 and recommended for adoption to the American Standards Assn. and the IEC. Since this is a change from the convention used in editions 1 to 3 of this book the history of this standard and its implications are discussed in detail here.

The complete specification of real- and reactive-power flow in a circuit requires:

First, an indication of the direction spoken of, i.e., a reference-positive direction.

Second, numerical values and associated signs. The numerical values give the magnitude of the real- and reactive-power components respectively. The associated signs show whether they flow in the reference-positive direction or not.

Third, there must be a convention as to whether it is lagging-reactive power or leading-reactive power, the direction and magnitude of which is being specified.

Lagging-reactive power is that which is generated or supplied by an over-excited synchronous machine or by a

Table 1(b)—Graphical Symbols for Diagrams—Winding Connection Symbols

NAME	SYMBOL
TWO-PHASE, THREE-WIRE	L
TWO-PHASE, FOUR-WIRE	+
THREE-PHASE, DELTA (OR MESH)	△
THREE-PHASE, Y (OR STAR)	Y
THREE-PHASE, Y (OR STAR) WITH NEUTRAL BROUGHT OUT AND GROUNDED	Y⏚
THREE-PHASE, Y (OR STAR) WITH NEUTRAL GROUNDED THROUGH A RESISTOR	Y–▭–⏚
THREE-PHASE, ZIG-ZAG	⅄
THREE-PHASE, T	⊥

static capacitor and used by inductive loads such as induction motors, reactors, and under-excited synchronous machines.

According to the convention recommended by AIEE in 1948 and used throughout this book the positive sign for reactive power indicates that lagging-reactive power is flowing in the reference-positive direction. The vector relationship for power is therefore:

$P+jQ=E\hat{I}$, the symbol $\hat{}$ designating conjugate.

For example if E is taken as reference, $E=\bar{E}$ and if $I=\bar{I}'-j\bar{I}''$ is a lagging current, $\bar{I}'$ and $\bar{I}''$ being positive quantities, the real power is $P=\bar{E}\bar{I}'$ and the lagging-reactive power is $Q=\bar{E}\bar{I}''$.

The expression,

$$P+jQ=E\hat{I}=\bar{E}(\bar{I}'+j\bar{I}'')=\bar{E}\bar{I}'+j\bar{E}\bar{I}''$$

results in the proper sign for the P and Q terms, whereas $\hat{E}I$ would give the right values but the wrong sign for the Q term. With this new convention, and taking E as reference, the power vector $P+jQ$ lies along the conjugate of the current vector. Consequently current and power circle diagrams lie in conjugate quadrants.

Historical Summary—Originally there was one school of thought, typified by Evans, Sels[6], and others, that used the positive sign for lagging-reactive power for the same reasons that it has now finally been adopted. The principal reasons were these. Like real power, lagging-reactive power is generally used in the load and must be supplied at some expense in the supply system. It is thus the commodity dealt with by the practical power-system designer, and dispatched by the operators. This concept is consistent mathematically with the following forms:

Power associated with voltage E and current I is:

$$P+jQ=E\hat{I}$$

and power in an impedance Z to a current I is:

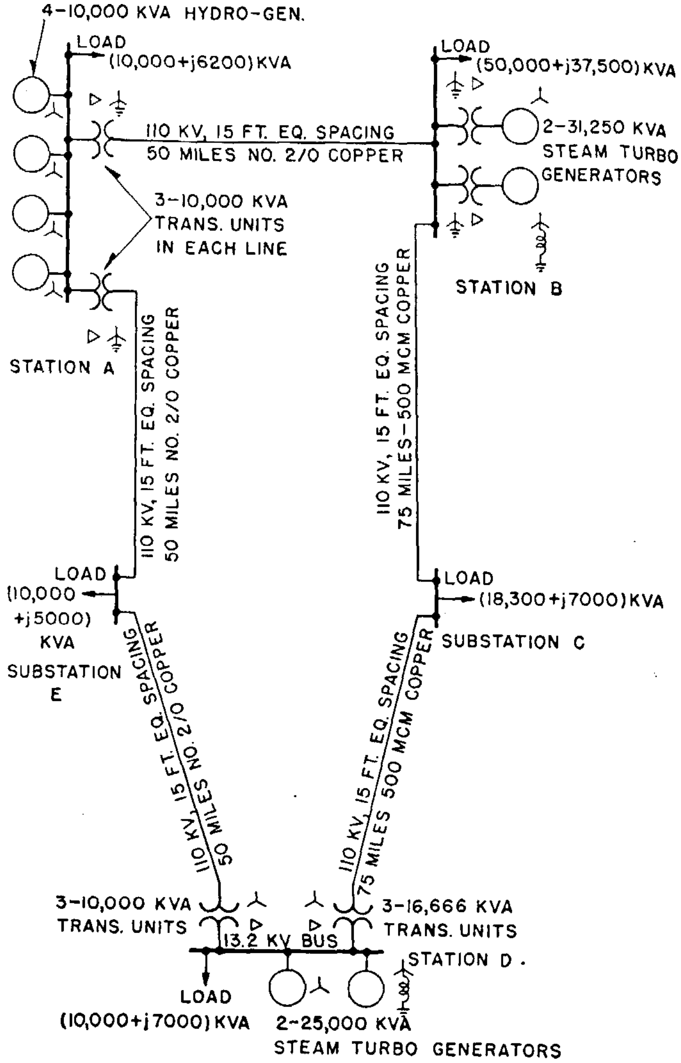


Fig. 1—Single-line diagram of a power system.

$$P+jQ=\bar{I}^2Z$$

The form $P+jQ=\bar{E}^2Y$ is then erroneous and gives the wrong sign for Q.

For the conventional transmission line, with this concept (lagging-reactive power positive) the center of the sending-end power-circle diagram lies in the first quadrant and the center of the receiving circle in the third quadrant.

The other school of thought used leading-reactive power as positive, lagging-reactive power as negative. This had the theoretical advantage of throwing current and power circle diagrams into the same quadrant, but the disadvantage that lagging-reactive power, the reactive commodity usually dispatched by power-system operators, was then a minus quantity. This concept is consistent with the mathematical forms: —

Power associated with a voltage E and a current I is:—

$$P+jQ=\hat{E}I$$

Power flowing into an admittance Y due to a voltage E is:

$$P+jQ=\bar{E}^2Y$$

The form $P+jQ=\bar{I}^2Z$ is then erroneous and gives the wrong sign for Q.

The latter school, (leading-reactive power positive) won out, for the time being, on the basis of the theoretical considerations, and on August 12, 1941 the American Standards Association approved this convention as an industry standard, C42-1941, Section 05.21.050. The first three editions of this book followed this standard convention. However, the convention was never followed by system-planning and operating people to any extent. They continued to dispatch lagging-reactive power which they called simply "reactive," and to mark on their flow charts the direction in which lagging-reactive power flowed. They could not be converted to selling a negative amount of leading-reactive power for positive money, but preferred to sell a positive amount of lagging-reactive power.

A majority of engineers have now come to consider lagging-reactive power as the commodity being dealt with. The AIEE Standards Committee recognizing this *fait accompli* recommended to ASA in 1948 adoption of the convention making lagging-reactive power positive. This reference book has, starting with the fourth edition, 1950, been changed to conform with what will undoubtedly be the standard from now on, namely, lagging-reactive power positive.

Teachers and writers can materially aid in eliminating confusion by discontinuing all use of the term leading-reactive power which after all is simply an unnecessary name for the negative of lagging-reactive power. Such a term is no more necessary than a name for the negative of real power. Eventually if this is followed the adjective "lagging" can be dropped, as reactive power will always mean lagging-reactive power.

3. Impedance Diagram. Fig. 2

The second essential in analytic study of a power system is the impedance diagram, on which are indicated on a common basis, the impedances of all lines and pieces of equipment related to the problem. Because of the symmetry of phases it is usually sufficient to represent only one phase—called the reference phase, or *a* phase. Under balanced conditions of operation, the currents and voltages in the other two phases are exactly equal to those in *a* phase and merely lag behind the *a* phase quantities by 120 and 240 electrical degrees. Hence, when the *a* phase quantities have been determined, the others follow directly.

Even when unbalances, such as a line-to-ground fault, or one-wire-open, occur at one or two points of an otherwise balanced polyphase system, the impedance diagrams for the reference phase are sufficient, if use is made of the method of symmetrical components as outlined in Chap. 2.

The impedance diagram, corresponding to the system shown in Fig. 1, is given in Fig. 2. Generator impedances are not shown as they do not enter into the particular problem. All impedances on this diagram have been expressed in ohms, and admittances in mhos, on a 110-kv base. Actually there are several choices, such as percent or per unit on various kva bases or ohms on voltage bases other than 110-kv. The relations between these several methods, and factors affecting the choice are discussed subsequently.

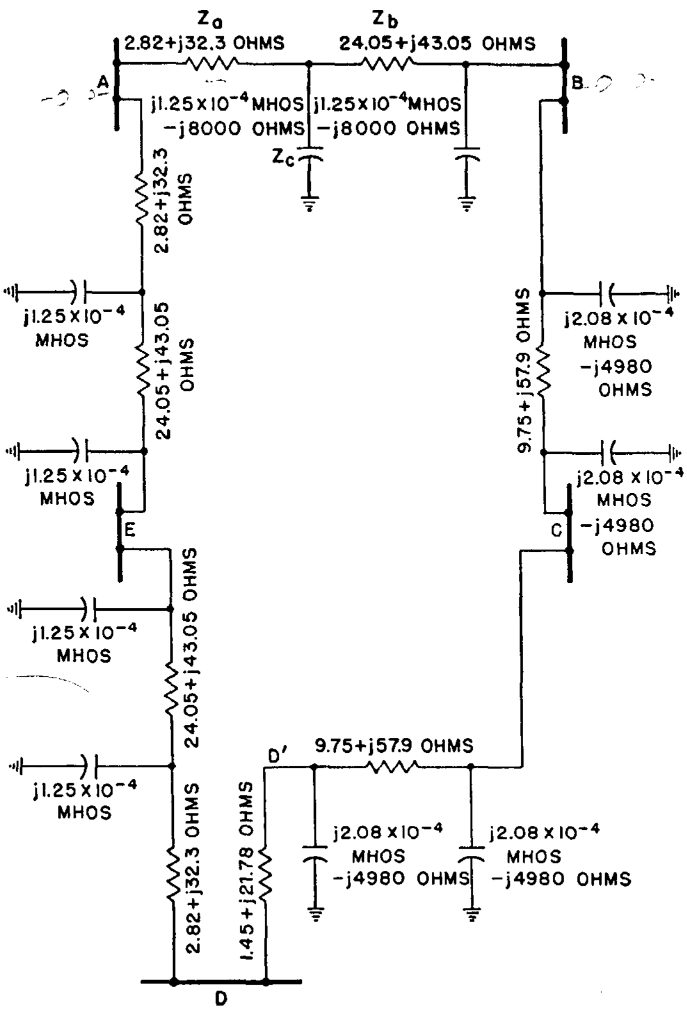


Fig. 2—Impedance diagram of a system shown in Fig. 1.

4. Determination of Impedances

In general, accurate cable and overhead-line impedances can be obtained from the tabulations of Chaps. 4 and 3, in terms of the wire size and spacing. Average apparatus constants are given in the tabulations of Chap. 6, Machine Characteristics, and Chap. 5, Power Transformers and Reactors. More accurate figures, if required, can be obtained from name-plates or direct from the manufacturer of the equipment. The impedances of lines and transformers in Fig. 2 have been obtained, for example, from the tables of Chaps. 3 and 5 based on the data given in Fig. 1.

Representation of Loads—The necessary representation of loads in the impedance diagrams depends upon the use intended. In short-circuit studies, loads are most frequently neglected. In stability studies, they must generally be considered. Several methods of representing loads are as follows:

a. Shunt impedance that draws the same kilowatts and reactive kva at normal voltage as is drawn by the actual load.
b. An impedance circuit, which, for any conditions of voltage is adjusted to draw the desired amount of real and reactive power.

c. In network calculator studies, the use of a "source"† adjusted, in phase angle and magnitude, to draw the desired real and reactive power from the system.
d. Given any characteristics of variation of real and reactive power with voltage, the load can be converted to impedance at the expected voltage, this impedance used in determining the system voltages, and then the load impedance corrected to the new voltage if such correction is warranted.

Conversion of Load Kw and Reactive Kva to Ohms or Mhos—Loads given in kilowatts and reactive kva can be converted to impedance or admittance form by the following equations:

Let P = kw (three-phase)
Q = reactive kva lagging ‡(three-phase)
E_{L-L} = line-to-line voltage in kv at which the conversion is to be made.
Z = vector impedance value, ohms line-to-neutral.
Y = vector admittance value, mhos, line-to-neutral.

$$Z=\frac{1000E^2_{L-L}}{P-jQ}=\frac{1000E^2_{L-L}}{P^2+Q^2}(P+jQ)$$

$$=\frac{1000(\text{kv})^2}{\text{kw}-j\text{ reactive kva (lagging)}}\text{ohms, line-to-neutral} \quad (1)$$

$$Y=\frac{P-jQ}{1000E^2_{L-L}}=\frac{\text{kw}-j\text{ reactive kva (lagging)}}{1000(\text{kv})^2}\text{ mhos, line-to-neutral} \quad (2)$$

For example, at 13.8 kv a load of 10 000 kva at 80 percent power factor lagging may be expressed as:

P = 8000 kw $\qquad Q$ = +6000 reactive kva

The impedance required to represent it is:

$$Z=\frac{1000(13.8)^2}{(6000)^2+(8000)^2}(8000+j6000)=15.2+j11.4 \text{ ohms, line-to-neutral}$$

and the admittance is:

$$Y=\frac{8000-j6000}{1000(13.8)^2}=0.0420-j0.0315\text{ mhos, line-to-neutral.}$$

Y and Z as given above are the admittance or impedance values to be used in the single-phase impedance diagram in which only the reference phase and neutral are represented.

Shunt Capacitors are built to a tolerance of −0 to +10 percent of their rated kva, +5 percent being the average. It is generally sufficiently accurate to consider the reactance to be 100 percent based on 105 percent of the rated kva base.

Series Capacitors—The determination of reactance of a series capacitor can best be explained by example. Suppose ten standard 15-kva single-phase, 440-volt, shunt-capacitor units have been used in parallel in each phase, or a total of 150 kva per phase. The capacitive reactance presented in series in each phase is then:

†The "sources" are voltage regulator-phase shifter circuits from a main power bus and can be readily adjusted to either draw or feed the desired quantities of real and reactive power.

‡Q is positive for lagging reactive kva.

$$X_S=\frac{1000(\text{kv})^2}{1.05^*(\text{kva})}=\frac{1000(0.44)^2}{1.05\times150}=1.22\text{ ohms} \quad (5)$$

This ohmic value can be converted to percent by Eq. (12).

Shunt Reactors have 100-percent voltage drop across them when connected to normal voltage, or have 100-percent impedance based on the kva drawn from the system at normal voltage.

Series Reactors—The reactance of a series reactor is frequently expressed in percent, but the kva of its parts is given. Thus, if a 6-percent reactor is desired in a circuit having a rating of 10 000 kva, three-phase, the reactor rating will be 600 kva, three-phase (6 percent of 10 000 kva). Three 200-kva single-phase reactors might be used. These would ordinarily be referred to as three 200-kva, 6-percent reactors, whereas actually they constitute a three-phase reactance in the circuit having 6-percent reactance on a 10 000-kva base. Care must be taken, therefore, to determine the reactance value on the through or transmitted kva base, or 10 000-kva base in the example cited. The relation between reactor kva, and through kva are as follows:

Reactor three-phase kva rating = a
Through or transmitted kva rating = A
Percent reactance on the transmitted kva base = X

Then $$a=\frac{X}{100}A \quad (3)$$

Given the reactor three-phase kva rating, a, the through kva rating is

$$A=\frac{100}{X}a \quad (4)$$

The reactor has a reactance of X percent on the kva base A.

In the case cited above of a 600-kva, 6-percent reactor, Eq. (4) gives $A=\frac{100}{6}\times600=10\,000$. Whence, the reactor has a reactance of 6 percent on a 10 000-kva base.

The standard reactance tolerance of current-limiting reactors is −3 percent to +7 percent for single-phase and −3 percent to +10 percent for three phase. The rated reactance is generally used in system calculations unless test figures are available.

5. Conversions. Percent to Ohms and Ohms to Percent†

Method 1—If a base kva (three-phase) and kv (line-to-line) are selected, the corresponding normal or base current, line-to-neutral voltage, and impedance values can be immediately determined.

They are:

$$\text{Normal Current } I_n=\frac{\text{kva}}{\sqrt{3}(\text{kv})}\text{ amperes} \quad (6)$$

$$\text{Normal Voltage (line-to-neutral) } E_n=\frac{1000(\text{kv})}{\sqrt{3}}\text{ volts} \quad (7)$$

$$\text{Normal Impedance } Z_n=\frac{E_n}{I_n}\text{ ohms per phase, line-to-neutral.} \quad (8)$$

*If the ratio of actual to rated kva is known, it should be used in place of 1.05.

†(Note: Per Unit is percent divided by 100).

From these relations any percent impedance can be converted to ohms.

$$\text{Ohms} = (\text{normal impedance})\left(\frac{\text{percent impedance}}{100}\right)$$
$$= Z_n\left(\frac{\%}{100}\right) \tag{9}$$

Conversely any ohmic figure can be converted to percent.

$$\text{Percent} = 100\left(\frac{\text{ohms}}{\text{Normal Impedance}}\right) = 100\left(\frac{\text{ohms}}{Z_n}\right) \tag{10}$$

Method 2—The magnitude of Z_n from (8), (7), and (6) can be substituted in (9) and (10) and gives direct conversions:

$$\textbf{Ohms} = (\%)\left(\frac{\mathbf{10\ kv^2}}{\mathbf{kva}}\right) \tag{11}$$

$$\textbf{Percent} = \textbf{ohms}\left(\frac{\mathbf{kva}}{\mathbf{10\ kv^2}}\right) \tag{12}$$

For example, a 15 000-kva, 13.8-kv to 66-kv transformer bank has a reactance of 8 percent on the 15 000-kva base. Let it be required to determine its impedance in ohms on a 66-kv base.

Normal current:

$$I_n = \frac{15\ 000}{66\sqrt{3}} = 131 \text{ amperes.}$$

Normal voltage:

$$E_n = \frac{66\ 000}{\sqrt{3}} = 38\ 100 \text{ volts, line-to-neutral.}$$

Normal impedance:

$$Z_n = \frac{38\ 100}{131} = 291 \text{ ohms per phase, line-to-neutral.}$$

Transformer impedance = 8 percent of 291
= 23.3 ohms per phase at 66 kv.

The direct determination from (11) is,
Transformer impedance

$$= \frac{8(66)^2(10)}{15\ 000} = 23.3 \text{ ohms per phase.}$$

The first method is longer, but gives other information generally required in the problem, and has some advantage in visualizing the procedures.

6. Conversions to a Different Kva Base

From (12) it is apparent that for a given ohmic impedance the percent impedance varies directly with the kva base selected. Thus 10-percent impedance on a 10 000-kva base becomes 100-percent impedance on a 100 000-kva base. When using percent impedances, all percentages should be expressed on the same kva base.

7. Conversions to a Different Voltage Base

In system studies if impedances are expressed in ohms it is desirable to convert them all to a common voltage base so that transformer turns ratios need not be considered in the subsequent calculations. The terms "voltage ratio" and "turns ratio" are often used loosely as synonymous terms, until more precise or important calculations are being made for which it is desired to be quite accurate. Then the question sometimes arises as to whether impedances should be transferred to the voltage base on the other side of a transformer on the basis of its voltage ratio or its turns ratio. It is actually the turns ratio that counts and should be used as will be shown later in this section. The turns ratio is the same as the nameplate voltage ratio but differs from the terminal voltage ratio under load.

Also in approximate calculations it is frequently assumed that for all parts of the system of the same nominal voltage the same transformation ratio can be used to the desired voltage base. This is a rough approximation and becomes exact only if the transformer turns ratios between parts of the system at the same nominal voltage are all unity. Barring this, one correct procedure is to select some one point of the system as a base and transform all other impedances to this base by multiplying by the square of the intervening turns ratios. Once all impedances are on a common base they can all be transformed by a single multiplier to any other voltage base.

When impedances are in percent on a given kva base the percent refers to a given normal voltage. Thus strictly two conditions must be fulfilled in sequence for percent impedances to be used in network solutions. First, the normal voltages to which the percentages refer must be in the same ratios as transformer turns ratios throughout the system. Second, the normal voltage used in converting the answers from percent to amperes and volts must be the same as the normal voltages on which the percent impedances are based. Otherwise approximations are involved. These approximations can be eliminated by suitable transformations beyond the scope of this chapter except for the following general method.

Where doubt exists as to the correct direct transformation of percent impedance, the impedance of each element can be converted to ohms. The ohmic values can be converted to a common base as described above and combined. The result can be reconverted to percent on any desired kva and voltage base. This is the general procedure by which rules for direct percent-impedance transformations are derived.

The pitfall of ignoring near-unity turns ratios extends to voltage also. Suppose a 13.8-kv generator feeds through step-up and step-down transformers to a 13.8-kv distribution system and that impedances have been expressed on the distribution system voltage base. Suppose further that there is a resultant 1:1.1 step-up turns ratio between the generator and the distribution system. Then a generator operating at 13.8 kv would be at $13.8 \times 1.1 = 15.18$ kv on the 13.8-kv voltage base of the distribution system, and must be so treated in the calculations. Similarly, for calculations in percent, the same machine must be treated as operating at 110 percent voltage. The theoretical basis upon which all such transformations rest, and examples of their correct use is given in the following paragraphs.

From an energy or power standpoint, no change is made if all voltages are multiplied by a constant, N, all currents divided by N, all impedances multiplied by N^2, and all admittances divided by N^2. When two circuits are sepa-

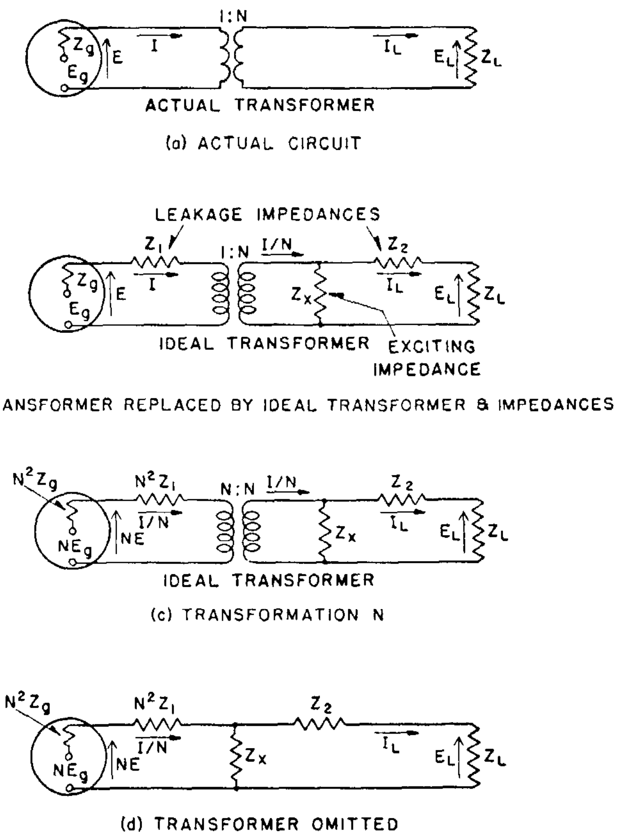


Fig. 3—Power invariant transformation.

rated by an ideal transformer* of turns ratio N, such an operation performed on the quantities on one side of the transformer with a corresponding change in the transformer ratio, has the advantage of bringing the currents and voltages on the two sides to an equality. A direct connection can be made and the ideal transformer can be omitted from the diagram (see Fig. 3). Solutions can be made with the quantities on the fictitious or transformed voltage base, and they can be reconverted to actual quantities whenever desired.

An actual transformer differs from an ideal transformer in two respects only. It has primary and secondary resistances and leakage reactances, which are no different than the same impedance connected externally. Its primary and secondary ampere-turns differ by a small quantity of exciting ampere-turns that excite the core. A shunt branch can be connected which draws the requisite exciting current if important in the particular problem.

Example—As an example consider the circuit of Fig. 4, a generator, transformer and high-voltage line with a three-phase short circuit at the end. Suppose the short-circuit currents are to be determined. This problem will also illustrate that calculations can be made interchangeably with impedances in ohms on any voltage base or in percent on any kva base.

The generator reactance (assumed 15%) in ohms is from (11):

*A transformer having zero exciting current and zero leakage impedance.

$$\frac{(15)(10)(13.8)^2}{50\,000}=0.571 \text{ ohms at 13.8 kv}$$

The transformer reactance in ohms is:

$$\frac{(9)(10)(13.8)^2}{50\,000}=0.343 \text{ ohms at 13.8 kv.}$$

The line impedance is, from Chap. 3.

$9.75+j57.9$ ohms at 110-kv. [See Fig. 4(c)]

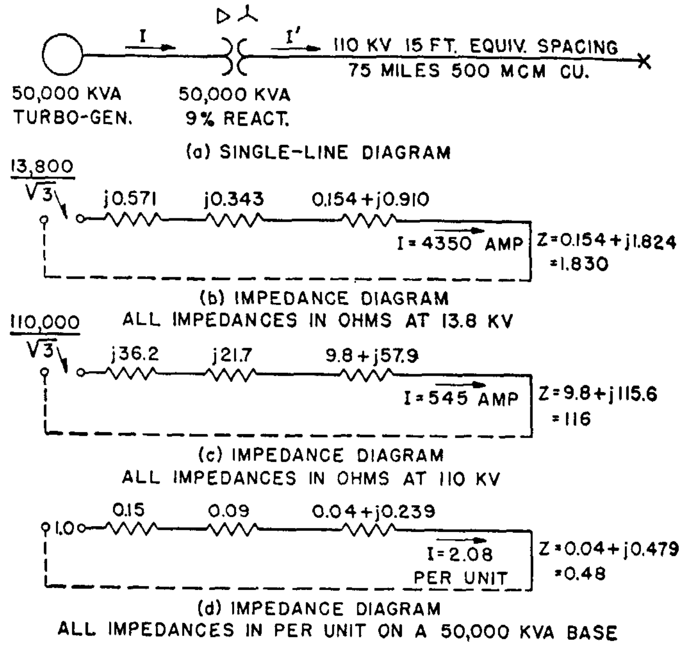


Fig. 4—Problem illustrating the expression of ohms on various voltage bases and the relation to percent on a kva base.

The shunt impedances of this line are high (line CD' Fig. 2) and will be neglected for simplicity in this problem.

Use of Generator Voltage Base—If the current in the generator is desired, it will be most convenient to express all impedances on the generator voltage base. The generator and transformer impedances are already on this base. The line impedance is converted to it by multiplying by the square of the turns ratio, usually taken as the nameplate voltage ratio corresponding to the taps in use. Thus the line impedance is:

$$(9.75+j57.9)\left(\frac{13.8}{110}\right)^2=0.154+j0.910 \text{ ohms at 13.8 kv.}$$

The impedance diagram of Fig. 4(b) results, in which all impedances are expressed in ohms on a 13.8-kv base. The fault current in the generator is then:

$$I=\frac{13\,800}{\sqrt{3}(1.830)}=4350 \text{ amperes.}$$

The current flowing in the line is:

$$I'=4350\left(\frac{13.8}{110}\right)=545 \text{ amperes.}$$

Use of Line Voltage Base—A similar result would be obtained if the generator and transformer reactances had been converted directly from percent to ohms at 110-kv. The impedance diagram, Fig. 4(c) would then result, the fault current being calculated directly for the line and requiring a conversion $\left(\text{multiplication by } \frac{110}{13.8}\right)$ to determine the current in the generator.

Use of Percent on a Kva Base—A third method of approach is to convert the line impedance to percent on a kva base, and "work in percent." A convenient base will be 50 000 kva since two of the impedances are already known on this base. The line impedance is, from Eq. (12):

$$\frac{(9.75+j57.9)(50\ 000)}{(10)(110)^2}=(4.0+j23.9)\% \text{ on } 50\ 000\text{-kva base.}$$

The impedance diagram Fig. 4(d) results, the percentages being shown as decimal fractions or "per unit" to facilitate computation.

In this case the current is:

$$I=\frac{1.0}{0.48}=2.08 \text{ per unit or } 208 \text{ percent of the normal current, corresponding to the selected kva base.}$$

This normal current is:

$$I_n=\frac{50\ 000}{\sqrt{3}(13.8 \text{ or } 110)}=\begin{matrix}2090 \text{ amp. at } 13.8 \text{ kv}\\ \text{or } 262 \text{ amp. at } 110 \text{ kv}\end{matrix}$$

The generator and line currents are, therefore, 208 percent of 2090 and 262 or 4350 and 545 amperes respectively, which agree with the preceding calculations.

The base selected obviously is immaterial. Had a 100 000-kva base been used, the impedances in Fig. 4(d) would all be doubled and the resulting percent currents halved. But the normal currents to which these percentages refer would be twice as great, and thus the same number of amperes would be obtained.

8. Phase Shifts in Transformer Banks

In addition to magnitude transformation, the voltage of the reference phase in general undergoes a shift in angular position. For balanced conditions, that is, considering positive-sequence quantities only, this is generally of no significance. For example, in the problem just worked out, the current in the reference phase of the line may or may not have been in phase with the reference or *a* phase current in the generator. If the transformer were delta-delta, the currents would have been in-phase; if delta-star they would have been 30 degrees out-of-phase, using the usual conventions.

However, it should be recognized that an angular transformation has been made whenever the single-phase circuit or impedance diagram is used for the calculation of currents and voltages in a circuit including a star-delta connected transformer bank. The following statements should aid in determining the treatment required in any particular case.

Radial Systems—In radial systems, the angle transformation is not usually significant as few phenomena involve comparisons of the phase angles of line currents on opposite sides of a transformation. Since currents and voltages are shifted alike, power or impedance determination at any one point in the circuit is unaffected by the angle transformation.

Transformer Differential Protection—A typical exception is the differential protection of a transformer bank. Here the currents on opposite sides of the transformation are purposely compared and measures must be taken to correct for the shift if the devices used are sensitive to phase angle.

Sequence Voltages and Currents—Positive-sequence voltages and currents are shifted the same as the reference or *a* phase in progressing through a symmetrical transformation. Negative-sequence voltages and currents, if present, are shifted the same amount as the reference phase but in the reverse direction. Zero-sequence voltages and currents are not shifted in progressing through a transformation.

Ideal Transformation—The shifts referred to have to do with the ideal transformer only, deleted of all leakage impedance and exciting current. That is, they depend only on how many turns of primary and secondary are used on each core and how these are grouped to form the phases on the primary and secondary sides. Symmetry with respect to *a*, *b*, and *c* phases is assumed.

Regulating Transformers—A symmetrical three-phase bank of regulating transformers may involve both ratio and phase-angle transformation. Suppose that in progressing through a particular bank of this type, a phase-angle advance of 10 degrees exists in the reference phase. Then, in progressing through the transformer in the same direction, positive-sequence quantities (currents and voltages) are advanced 10 degrees, negative-sequence quantities retarded 10 degrees, and zero-sequence quantities not shifted at all.

Standard Angular Shifts—The angular shifts of reference phase for various transformer connections are given in Chap. 5, Sec. 13. The American Standard* is a 30-degree advance in phase in progressing through either a star-delta or a delta-star connected transformer from a lower to a higher voltage. When carried out consistently, this will permit interconnections at various system voltages without difficulty in phasing. However, at present practically all possible connections are in use throughout the industry.

9. Loop Systems That Close

Transformations of magnitude or angle in a system involving one or more loops can be treated similarly to a radial system provided that:

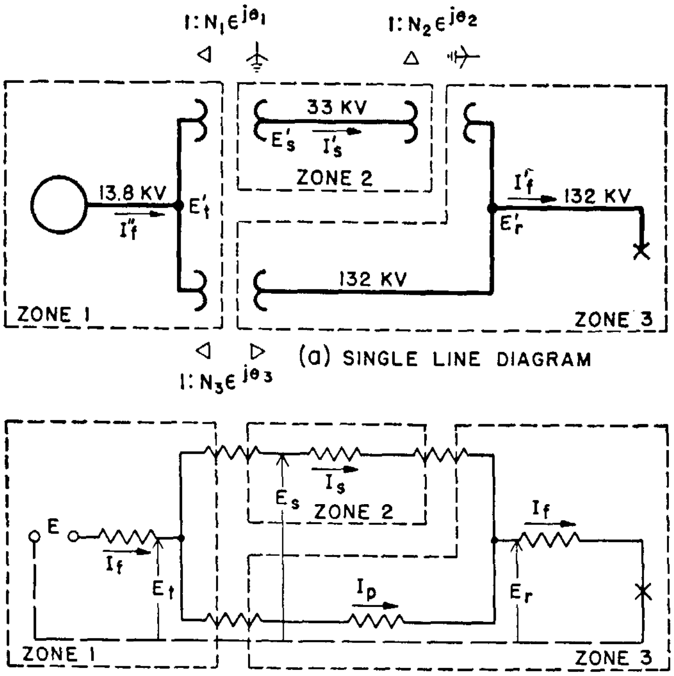
a. The product of the magnitude transformation ratios for the reference phase, taken in a common direction around each closed loop is unity.
b. The sum of the reference phase angular shifts taken in a common direction around each closed loop is zero.

If each transformation ratio is expressed vectorially as $N\epsilon^{j\theta}$, including angular significance in the term "vector transformation ratio," then *a* and *b* above can be combined into the single requirement:

c. The product of the vector transformation ratios around each closed loop is $1\epsilon^{j0}$.

If the requirements *a* and *b*, or *c* are fulfilled, then the circuits of the system can be divided into zones separated

*ASA Standards C-57.



(b) IMPEDANCE DIAGRAM

$$N_1\epsilon^{j\theta_1}=\frac{33}{13.8}\epsilon^{j30°}$$

$$N_2\epsilon^{j\theta_2}=\frac{132}{33}\epsilon^{j30°}$$

$$N_3\epsilon^{j\theta_3}=\frac{132}{13.8}\epsilon^{j60°}=N_1\epsilon^{j\theta_1}N_2\epsilon^{j\theta_2}$$

Fig. 5—Ratio and angular transformations.

from each other by transformations. One zone, usually the one of greatest interest in the particular problem, can be taken as the reference zone.

Example—For example, in Fig. 5 currents in various parts of the system are to be determined for a balanced three-phase fault on the 132-kv line.

There is one closed loop in which:

$$(N_1\epsilon^{j\theta_1})(N_2\epsilon^{j\theta_2})\left(\frac{1}{N_3\epsilon^{j\theta_3}}\right)=\left(\frac{33}{13.8}\epsilon^{j30°}\right)\left(\frac{132}{33}\epsilon^{j30°}\right)\left(\frac{1}{\frac{132}{13.8}\epsilon^{j60°}}\right)$$
$$=1\epsilon^{j0} \qquad (13)$$

Therefore, the reference-phase impedance diagram can be prepared from the single-line diagram without showing any transformations.

Let Zone 3 be taken as the reference zone and all impedances expressed in ohms on 132-kv base. The fault current, I_f, and the distribution of currents I_s and I_p are now readily determined. So also are the voltages throughout the network. It is recognized that in Zone 3 these are the actual reference phase currents and voltages. In Zones 1 and 2 they are the actual quantities transformed to the Zone 3 base, and hence, must be transformed to their own respective bases to obtain the actual quantities. Since they are all positive-sequence currents and voltages, that is, normal balanced three-phase quantities, the actual currents and voltages of the reference phase, which have been indicated on the single line diagram, are as follows:

In Zone 3

$$I_f'=I_f \qquad (14)$$

$$E_r'=E_r \qquad (15)$$

In Zone 2

$$I_s'=\frac{I_sN_2}{\epsilon^{j\theta_2}}=I_sN_2\epsilon^{-j\theta_2} \qquad (16)$$

$$E_s'=\frac{E_s}{N_2\epsilon^{j\theta_2}}=\frac{E_s}{N_2}\epsilon^{-j\theta_2} \qquad (17)$$

In Zone 1

$$I_f''=\frac{I_fN_3}{\epsilon^{j\theta_3}}=I_fN_3\epsilon^{-j\theta_3} \qquad (18)$$

$$E_t'=\frac{E_t}{N_3\epsilon^{j\theta_3}}=\frac{E_t}{N_3}\epsilon^{-j\theta_3} \qquad (19)$$

The Zone 1 quantities may also be expressed as follows, illustrating the general method to be followed when the zone in question is separated from the reference zone by several transformations.

$$I_f''=\frac{I_fN_1N_2}{\epsilon^{j\theta_1}\times\epsilon^{j\theta_2}}=I_fN_1N_2\epsilon^{-j(\theta_1+\theta_2)}$$

$$E_t'=\frac{E_t}{N_1N_2\epsilon^{j\theta_1}\times\epsilon^{j\theta_2}}=\frac{E_t}{N_1N_2}\epsilon^{-j(\theta_1+\theta_2)}$$

The power at any point s, for example, can be calculated without transforming. For:

$$P_s'+jQ_s'^*=E_s'\hat{I}_s'=\left(\frac{E_s}{N_2}\epsilon^{-j\theta_2}\right)(\hat{I}_sN_2\epsilon^{+j\theta_2})=E_s\hat{I}_s \qquad (20)$$

and

$$P_s+jQ_s=E_s\hat{I}_s \qquad (21)$$

These are the same. In other words the transformations described thus far and ordinarily used in analytical work are power invariant. They differ from transformations to a model scale for setting on a network calculator, in which power must obviously be scaled down.

10. Loop Systems That Do Not Close

If the product of vector transformation ratios around a closed loop is not unity, special consideration needs to be given. This case will be sub-divided into three parts, viz—(a) product of ratios not unity, (b) sum of angular shifts not zero, and (c) product of ratios not unity and sum of angles not zero.

Product of Ratios Not Unity—Many transformers are provided with taps in one or more windings. With star or delta connected windings, use of these taps changes the ratio only, without affecting the angular shift through the transformer. Thus, by far the largest number of cases of non-unity vector transformation ratio around closed loops falls in this category of ratio discrepancy only.

Example—An example is shown in Fig. 6, in which two circuits A and B differ in capacity, the taps having been increased on the B circuit to make it carry more of the load. The power factor of the portion of load that can be thus shifted from B to A depends on the impedance phase angles of the A and B circuits being nearly pure wattless for pure reactive circuits, and pure watts for pure resistive circuits. Thus, for 60 degrees impedance angle circuits the shifted

*See Section 2.

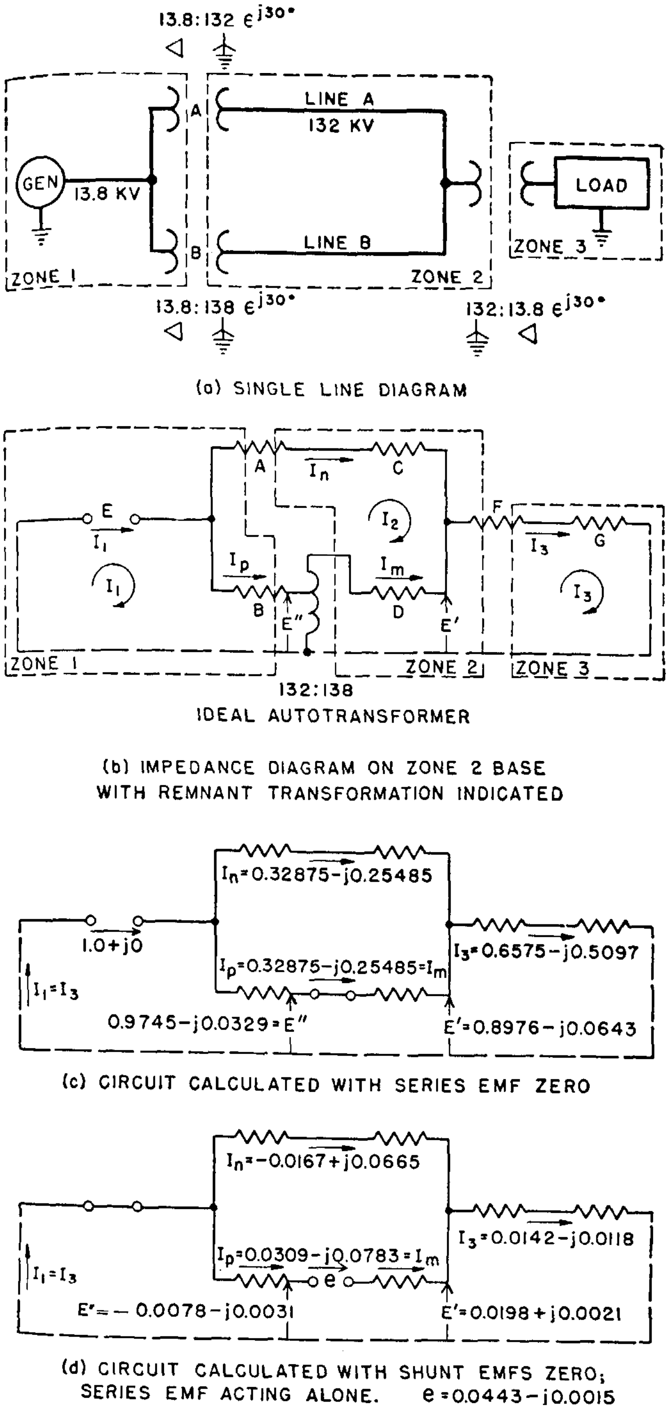


Fig. 6—Product of ratios around a closed loop not unity.

load is at about 50 percent power factor. The amount shifted is nearly constant, and not a percentage of the total load. Thus, at no load, there is a circulation over the two lines.

Suppose that the network of Fig. 6 is to be solved, and it is desired to work on the Zone 2 basis. Zone 3 can be readily transformed to this basis as explained in a preceding section. However, no transformation can be found for Zone 1 that will result in both transformer ratios being unity. The best that can be done is to make one of them, for example A, unity by transforming the voltages of Zone 1 in the ratio $\frac{132}{13.8}\epsilon^{j30°}$ and currents and impedances by the corresponding factors. This leaves an uncompensated or remnant ratio to be accounted for in B, which may be represented as an autotransformer, Fig. 6(b).

In a-c network calculator studies, small auto-transformers of the remnant ratio are used and no further consideration need be given. For analytic studies the simplest method is to neglect the remnant transformation ratio, provided great accuracy is not required. The order of magnitude of the circulating current can be estimated by dividing the inserted voltage by the loop impedance to see whether it can be neglected in the problem at hand. For example, if the remnant ratio is 1.05 the inserted voltage is of the order of 0.05 per unit under normal load conditions. If the loop impedance is 0.50 per unit the order of magnitude of circulating current is $\frac{0.05}{0.50}=0.10$ per unit.

If this cannot be neglected the following approximation is suggested in cases where the remnant ratio is close to unity. The accuracy of the method is indicated later by an example.

a. Treat as though the ratio were unity and determine the resulting shunt voltage at the location of the auto-transformer.
b. Determine the resulting series voltage introduced, in this case $\frac{138-132}{132}=\frac{6}{132}=4.5$ percent of the shunt voltage, and in phase with it.
c. Determine the current circulated in the network by the action of this series voltage alone, setting the generator emf, E, equal to zero. Determine the voltages for this condition also.
d. Superpose this set of circulating currents on the currents previously calculated. Superpose the voltages similarly. The resulting solution is in error only by a correction factor of the second order which usually can be ignored, as will be shown subsequently.
e. Where several such auto-transformers are required to "close" the impedance diagram, the circulating currents can be calculated separately, treating the ratios of the others as unity at the time. All of the resulting circulating currents can then be superposed. The resulting voltages can likewise be superposed.

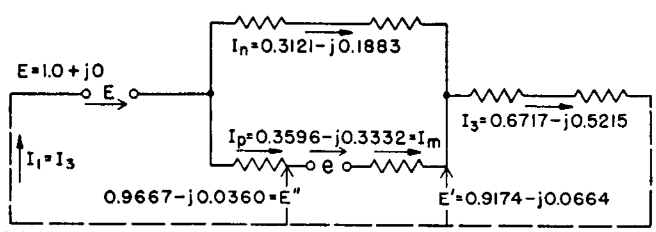
This approximation is based on the concept that the auto-transformer could be replaced by a shunt load that draws the same current from the system as the main section of the auto-transformer and a series emf that impresses in series the same voltage as the short extension of the auto-transformer. With this substitution, the solution by superposition is exact. If the auto-transformer introduces five-percent voltage in series and the impedance to the resulting circulating current is 50 percent, then ten-percent current will flow. With five-percent voltage this amounts to 0.5-percent load, which is supplied from the system to the shunt winding of the auto-transformer, thence, to the series winding and back into the system, as I^2X and I^2R losses of circulation. As this load drawn by the shunt winding is quite small, 0.5-percent in the case just cited, it is most frequently ignored.

In general, introduction of the series voltage raises the voltage on one side of the auto-transformer and lowers it on the other side, as compared with the voltage that would be present if the auto-transformer were not there. Thus, if the series voltage is five percent, the shunt voltage applied to the auto-transformer will differ by not over five percent from that calculated with the auto-transformer removed. A correction of five-percent in the shunt voltage would change the series voltage from five-percent to 4.75 percent. This small correction usually is not required. Thus, the steps as outlined from *a* to *d* above will usually be sufficiently accurate.

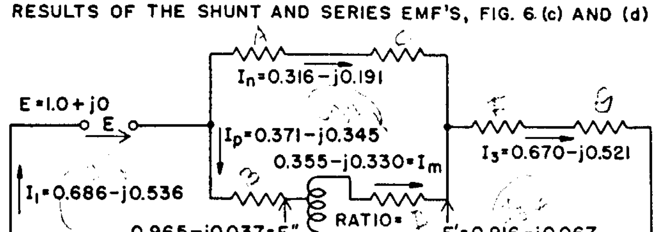
Example—A comparison of the exact solution (by equations—see Sec. 13) and the approximation in the case of Fig. 6 will illustrate the procedure and indicate the degree of accuracy to be expected. Assume a set of constants as follows, in per unit on the generator kva base. Assume the voltage to be maintained constant on the generator bus.

$$A = j0.10 \qquad E = 1 + j0$$
$$B = j0.10$$
$$C = 0.10 + j0.173$$
$$D = 0.10 + j0.173$$
$$F = j0.10$$
$$G = 0.90 + j0.50$$

Following the steps suggested above, the series voltage is first set equal to zero and the solution using the generator voltage alone is obtained, Fig. 6(c). The series voltage is 4.5 percent of $0.9745 - j0.0329$ or equal to $0.0443 - j0.0015$ and is directed to the right. Then, setting the generator voltage equal to zero and applying the series voltage alone, the solution of Fig. 6(d) is obtained. Adding vectorially the corresponding quantities in these two solutions, the superposed solution, Fig. 7(a), is obtained for the simultaneous application of the generator voltage, E, and the



(a) APPROXIMATE SOLUTION OBTAINED BY SUPERIMPOSING THE RESULTS OF THE SHUNT AND SERIES EMF'S, FIG. 6 (c) AND (d)



(b) EXACT SOLUTION BY SOLUTION OF THE EQUATIONS

Fig. 7—Comparison of exact and approximate solutions of Fig. 6.

series voltage e, and is a good approximation to the exact solution of the circuit of Fig. 6(a) and (b) as will be shown by comparison with Fig. 7(b).

The Exact Solution for the currents and voltages in Fig. 6(b) can be obtained by writing Kirchoff's Law for the

TABLE 2—COMPARISON OF RESULTS BY APPROXIMATE AND EXACT METHODS OF SOLUTION WHEN PRODUCT OF VECTOR TRANSFORMATION RATIOS IS NOT UNITY. (REFER ALSO TO FIG. 7).

	By Approximate Method		By Exact Method		% Diff. $= \frac{\text{Diff.}}{\text{Exact}} \times 100$
	Vector	Scalar	Vector	Scalar	
I_1	$0.672 - j0.522$	0.851	$0.686 - j0.536$	0.870	2.18
I_3	$0.672 - j0.522$	0.851	$0.670 - j0.521$	0.849	0.24
I_n	$0.312 - j0.188$	0.364	$0.316 - j0.191$	0.369	1.36
I_p	$0.360 - j0.333$	0.490	$0.371 - j0.345$	0.507	3.35
I_m	$0.360 - j0.333$	0.490	$0.355 - j0.330$	0.485	1.03
E''	$0.967 - j0.036$	0.968	$0.965 - j0.037$	0.966	0.21
E'	$0.917 - j0.066$	0.919	$0.916 - j0.067$	0.918	0.11

drops around each of the three loops and setting up a fourth equation stating that the total ampere-turns on the perfect transformer are zero.

$$I_1B \quad - I_2B \qquad\qquad + I_3(0) \qquad + E'' \qquad = E \tag{22}$$

$$-I_1B \quad + I_2(A+B+C+D) - I_3D \qquad + E''(0.045) = 0 \tag{23}$$

$$I_1(0) - I_2D \qquad + I_3(D+F+G) - E''(1.045) = 0 \tag{24}$$

$$I_1 \quad + I_2(0.045) \qquad - I_3(1.045) \qquad + E''(0) \qquad = 0 \tag{25}$$

The solution of these simultaneous equations with the numerical values of the impedances A to G substituted, is given in Fig. 7(b). Table 2 shows the error in various quantities by the approximate method. The voltages are within 0.2 percent. The largest current error is 3.35 percent in I_p. The sum of errors in I_p and I_m are about 4.5 percent. This is necessary since these two currents are taken the same in the approximate solution and differ by 4.5 percent in the exact solution.

Sum of Angular Shifts Not Zero—Regulating transformers or regulators as well as special connections of transformers can introduce angular shift. If the net shift around a closed loop is not zero but is small, the treatment is similar to that for ratio discrepancies except that the series voltage is introduced at right angles to the shunt voltage. On the a-c network calculator, transformers cannot be used to obtain a shift since the circuits are single phase. Power sources must be used to introduce the necessary series voltages.

Sum of Angular Shifts Not Zero and Sum of Ratios Not Unity—The series voltage can be introduced at any desired angle, corresponding to the net vector transformation ratio, and the currents superposed as outlined above, with appropriate phase relations.

II. NETWORK SOLUTION

11. Network Theorems**

The Superposition Theorem states that each emf produces currents in a linear network* independently of those produced by any other emf. It follows that the emfs and currents of a given frequency can be treated independently of those of any other frequency, and of transients. The superposition theorem is a direct result of the fact that the fundamental simultaneous differential equations of the network are linear. (See any standard book on Differential Equations.) (See Sec. 13.)

The Compensation Theorem states that if the impedance of a branch of a network be changed by an amount ΔZ, the change in current in any branch is the same as would be produced by a compensating emf $-\Delta ZI$, acting in series with the modified branch, I being the original current in that branch. By compensating emf is meant one which, if it were inserted, would neutralize the drop through ΔZ. This theorem follows directly from the superposition theorem.

The Reciprocal Theorem states that when a source of electromotive force is connected across one pair of terminals of a passive† linear network and an ammeter is connected across a second pair of terminals, then the source of electromotive force and the ammeter can be interchanged without altering the reading of the ammeter (provided neither the source nor receiver has internal impedance). This follows from the fact that in Eq. (56), if all emfs are zero except E_2 and E_3 for example, then if

$$E_2=0,\quad I_2=\frac{E_3A_{23}}{D},\ \text{while if } E_3=0,\quad I_3=\frac{E_2A_{32}}{D},$$

so that if $E_3=E_2$, $I_3=I_2$ for the conditions of the theorem. Note that $A_{23}=A_{32}$.

12. Reference Current and Voltage Directions

To specify uniquely a vector current or voltage in a circuit, some system must be adopted to label the points between which the voltage is being described or the branch in which the current flows. This system must also indicate the reference or positive direction. Two common methods are: the use of reference or positive direction arrows and the double subscript notation.

Reference Direction Arrows—Fig. 8(a) and (b)—When a network is to be solved to determine, for example, the current flow for a given set of impressed emfs, the network should first be marked with arrows to indicate the reference positive direction of each current and voltage involved. These can be drawn arbitrarily, although if the predominant directions are known, their use as reference-direction arrows simplifies later interpretation.

The use of open voltage-arrowheads and closed current-arrowheads will avoid confusion in numerical work, where the E and I symbols are not used.

It must be decided at this point whether the voltage

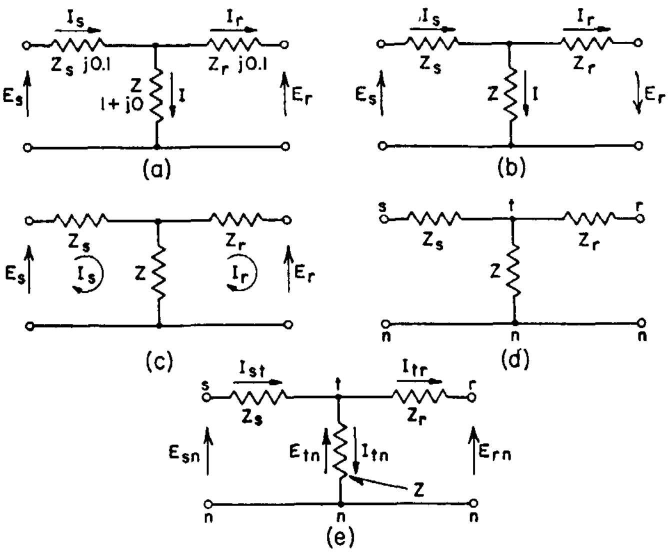


Fig. 8—Methods of notation used in network solution.

arrow is to represent a rise or a drop. In system calculations it is generally used as the rise in voltage. While this decision is arbitrary, once made, it must be adhered to consistently.

Finally a vector value must be assigned to the voltage or current. It can be expressed as a complex number, or in polar form or graphically and gives the magnitude and relative phase of the quantity with respect to some reference. A symbol, such as I_s, can be used to designate this vector quantity.

The known vectorial voltages or currents must be associated with the reference arrows in a manner consistent with the conditions of the problem. For example, suppose the problem in Fig. 8(a), is to determine the currents that would flow with the two voltage sources 180 degrees out of phase, and 100 volts each, rms, 60-cycle. Then if E_s is taken as $100+j0$, E_r must be taken as $-100+j0$. Had the arbitrary reference-direction arrows been taken as in Fig. 8(b), then for the same problem a consistent set of voltages would be:

$$E_s=100+j0,\quad E_r=100+j0$$

Ordinarily the reference-direction arrows for shunt voltages are directed from the neutral to the phase conductor as in Fig. 8(a).

Summarizing then, the complete specification of a quantity in the reference-direction-arrow system involves three elements:

a. The reference-direction arrow, drawn arbitrarily.
b. An agreement, consistently followed as to what the reference-direction arrow means; particularly whether the voltage arrow means the voltage of the point above the tail or the drop from tail to point.
c. A vector to represent the magnitude and relative phase of the quantity with respect to a reference.

Suggested convention: For voltages, the vector quantity shall indicate the voltage of the point of the arrow above the tail, that is, the rise in the direction of the arrow. It then is also the drop from point to tail.

**See also Thevenin's Theorem, Sec. 18.

*A linear network is one in which each impedance is linear; that is, has a straight line relation between current and voltage drop.

†A passive network is one having no internal emfs as distinguished from an active network.

Mesh Currents and Voltages—*Refer to Fig. 8(c)*—The "mesh current" system involves a somewhat different point of view. Here each current is continuous around a mesh and several currents may flow in the same branch. (I_s and I_r flow in Z.) The branch current is the vector sum of all the mesh currents in the branch, taken in the reference direction for the branch current. If such a network can be laid out "flat," it is most convenient to take the reference direction for mesh currents as simply "clockwise" for example. Or circular arrows can be used as shown in Fig. 8(c). The example of solution by equations in Sec. 9 illustrates the use of "mesh currents."

The same reference directions can be conveniently used for mesh emfs, which are the vector sum of all emfs acting around a particular mesh, taken in the reference direction.

Double-Subscript Notation—*Fig. 8(d)*—A double-subscript notation is sometimes used and is of course equivalent to the drawing of reference arrows. Here again an arbitrary decision must be made as to what is intended. Suggested Convention. Refer to Fig. 8(d).

I_{st} means the current from s to t.

E_{sn} means the voltage of s above n

It is apparent then that $E_{ns} = -E_{sn}$; $I_{st} = -I_{ts}$, etc.

Setting Up Equations—If the work is analytical, by the method of equations, the equations must be set up consistent with the reference direction arrows, *regardless of the values of any known currents or voltages.* Consistent equations for Figs. 8(a), (b), (c), (d), are as follows, using Kirchoff's Laws (See Sec. 1): The voltage equations are written on the basis of adding all of the voltage rises in a clockwise direction around each mesh. The total must of course be zero. The current equations are written on the basis that the total of all the currents flowing up to a point must equal zero. Arrows and double subscripts have the meanings given in the "suggestions" above.

Referring to Fig. 8(a).

$$E_s - I_sZ_s - IZ = 0 \tag{26}$$
$$-I_rZ_r - E_r + IZ = 0 \tag{27}$$
$$I_s - I - I_r = 0 \tag{28}$$

Referring to Fig. 8(b).

$$E_s - I_sZ_s - IZ = 0 \tag{29}$$
$$-I_rZ_r + E_r + IZ = 0 \tag{30}$$
$$I_s - I - I_r = 0 \tag{31}$$

Referring to Fig. 8(c).

$$E_s - I_s(Z_s + Z) + I_rZ = 0 \tag{32}$$
$$I_sZ - I_r(Z + Z_r) - E_r = 0 \tag{33}$$

Referring to Fig. 8(d) or 8(e).

$$E_{sn} - I_{st}Z_s - I_{tn}Z = 0 \tag{34}$$
$$I_{tn}Z - I_{tr}Z_r - E_{rn} = 0 \tag{35}$$
$$I_{st} - I_{tn} - I_{tr} = 0 \tag{36}$$

In the double-subscript system, the voltages and currents could of course be indicated on the figure. They have been purposely omitted in Fig. 8(d), however, to emphasize that the specification of these quantities in Eqs. (34) and (35) is perfectly definite from the subscripts alone.

Also, the inclusion of reference-direction arrows on the diagram, even when the double-subscript system is used, may aid in writing equations, although they are not strictly required. If used, they must be consistent with the double-subscript system. That is, each arrow must be directed from the second subscript toward the first for voltages, and from the first subscript toward the second for currents. Fig. 8(e) illustrates such a diagram consistently labeled.

13. Solution by Equations

Representation—A network of n meshes can be represented as having n independent currents, I_1 to I_n, as shown in Fig. 9. The branch currents are combinations of these. See *Branch Currents* below.

Mesh Impedances are defined generally as: Z_{pq} = voltage drop in the reference direction in mesh q per unit of current in reference direction in mesh p. The curved arrows indicate reference directions in each mesh. In general

$$Z_{pq} = Z_{qp}$$

The impedances Z_{pp} and Z_{pq} are called self and mutual impedances.

Specifically in Fig. 9.

(First subscripts i and c indicate inductive and capacitive reactances respectively.)

$$Z_{11} = R_a + R_b + j(X_{ia} + X_{ib} - X_{ca} - X_{cb}) \tag{37}$$
$$Z_{12} = -R_b - j(X_{ib} - X_{cb}) \tag{38}$$
$$Z_{13} = 0 \tag{39}$$

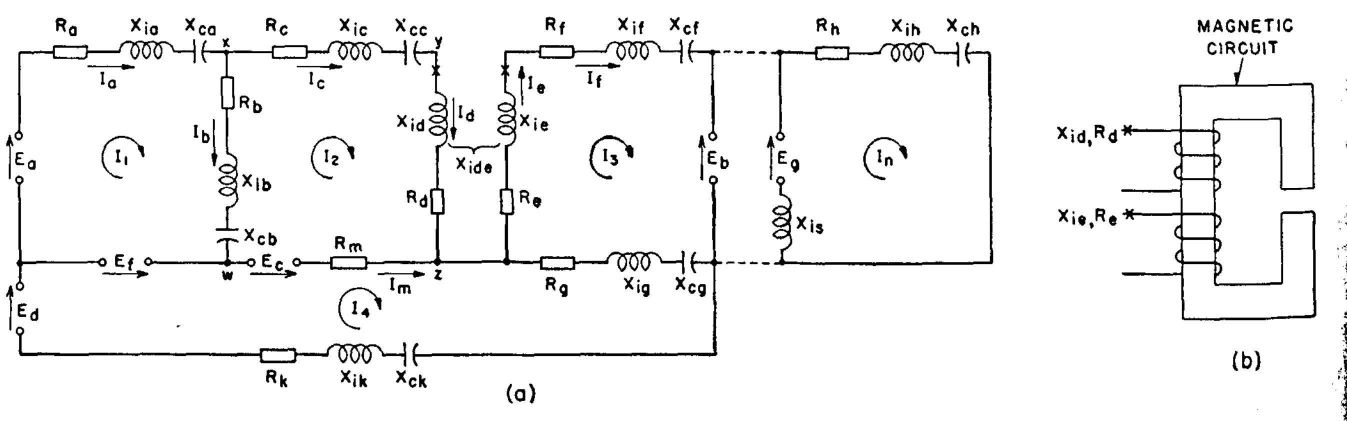


Fig. 9—General flat network.

$Z_{14}=0$ (40)

Etc.

$Z_{22}=R_b+R_c+R_d+R_m+j(X_{ib}+X_{ic}+X_{id}-X_{cb}-X_{cc})$ (41)

$Z_{23}=-jX_{ide}$ (42)

[The polarity marks signify that the mutual flux links the two windings in a manner to produce maximum voltages at the same instant at the marked ends of the windings. See Fig. 9(b).]

$$Z_{24}=-R_m \quad (43)$$

$$Z_{25}=0, \text{ etc.} \quad (44)$$

$$Z_{12}=Z_{21} \quad (45)$$

$$Z_{13}=Z_{31} \text{ etc.} \quad (46)$$

Mesh Emfs—Reference-positive directions for branch emfs, E_a, E_b, etc., are shown by arrows associated therewith.

A mesh emf is the sum of the branch emfs acting around that particular mesh in the reference direction.

The same reference direction will be used for mesh emfs as for mesh currents.

Specifically in Fig. 9.

$$E_1=E_a-E_f \quad (47)$$

$$E_2=-E_c \quad (48)$$

$$E_3=-E_b \quad (49)$$

$$E_4=E_d+E_f+E_e \quad (50)$$

Etc.

$$E_n=E_g \quad (51)$$

Equations—Kirchoff's Law states that the voltage drop around each closed mesh must equal the emf impressed in that mesh.

$$I_1Z_{11}+I_2Z_{21}+I_3Z_{31}+\cdots+I_nZ_{n1}=E_1 \quad (52)$$

$$I_1Z_{12}+I_2Z_{22}+I_3Z_{32}+\cdots+I_nZ_{n2}=E_2 \quad (53)$$

$$I_1Z_{13}+I_2Z_{23}+I_3Z_{33}+\cdots+I_nZ_{n3}=E_3 \quad (54)$$

$$\vdots$$

$$I_1Z_{1n}+I_2Z_{2n}+I_3Z_{3n}+\cdots+I_nZ_{nn}=E_n \quad (55)$$

Mesh Currents*—Equations (52) to (55) can be solved for the mesh currents I_1 to I_n. The solution for current in any particular mesh, p, is:

$$I_p=\frac{E_1A_{p1}}{D}+\frac{E_2A_{p2}}{D}+\frac{E_3A_{p3}}{D}+\cdots+\frac{E_nA_{pn}}{D} \quad (56)$$

where D is the determinant of coefficients

$$D=\begin{vmatrix} Z_{11}Z_{21}Z_{31}\cdots Z_{n1} \\ Z_{12}Z_{22}Z_{32}\cdots Z_{n2} \\ Z_{13}Z_{23}Z_{33}\cdots Z_{n3} \\ \cdot\quad\cdot\quad\cdot\qquad\cdot \\ \cdot\quad\cdot\quad\cdot\qquad\cdot \\ \cdot\quad\cdot\quad\cdot\qquad\cdot \\ Z_{1n}Z_{2n}Z_{3n}\cdots Z_{nn} \end{vmatrix} \quad (57)$$

and A_{pq} is the cofactor of Z_{pq} in the determinant (57).

*The method of determinants is used to state the solution here. However, any method of solving the simultaneous equations (52) to (55) for the unknown currents I_1 to I_n may be used.

The cofactor of a term in a determinant is the minor determinant obtained by eliminating the row and column containing that term, this minor being prefixed by a + or − sign depending on whether the sum of column number and row number is even or odd respectively. In (57), the first subscripts define columns, the second rows. Thus the cofactor of a term has a + sign if the sum of subscripts on the term is even. Since $Z_{pq}=Z_{qp}$, it can be shown that $A_{pq}=A_{qp}$.

In (56), the term involving E_1 is the current that flows in mesh p if all emfs are set equal to zero except E_1. Similarly, the term $\frac{E_2A_{p2}}{D}$ is the current that flows in mesh p if E_2 alone acts and E_1, E_3, etc., are equal to zero.

Specifically:

$$I_2=\frac{E_1A_{21}}{D}+\frac{E_2A_{22}}{D}+\frac{E_3A_{23}}{D}+\cdots+\frac{E_nA_{2n}}{D} \quad (58)$$

where D is given by (57).

And:

$$A_{21}=-\begin{vmatrix} Z_{12}Z_{32}\cdots Z_{n2} \\ Z_{13}Z_{33}\cdots Z_{n3} \\ \cdot\quad\cdot\qquad\cdot \\ \cdot\quad\cdot\qquad\cdot \\ \cdot\quad\cdot\qquad\cdot \\ Z_{1n}Z_{3n}\cdots Z_{nn} \end{vmatrix} \quad (59)$$

$$A_{22}=+\begin{vmatrix} Z_{11}Z_{31}\cdots Z_{n1} \\ Z_{13}Z_{33}\cdots Z_{n3} \\ \cdot\quad\cdot\qquad\cdot \\ \cdot\quad\cdot\qquad\cdot \\ \cdot\quad\cdot\qquad\cdot \\ Z_{1n}Z_{3n}\cdots Z_{nn} \end{vmatrix} \quad (60)$$

$$A_{23}=-\begin{vmatrix} Z_{11}Z_{31}\cdots Z_{n1} \\ Z_{12}Z_{32}\cdots Z_{n2} \\ \cdot\quad\cdot\qquad\cdot \\ \cdot\quad\cdot\qquad\cdot \\ \cdot\quad\cdot\qquad\cdot \\ Z_{1n}Z_{3n}\cdots Z_{nn} \end{vmatrix} \text{ Etc.} \quad (61)$$

Branch Currents can now be obtained by combination. The vector sum of all mesh currents flowing in a branch is the branch current.

Specifically with reference to Fig. 9.

$$I_a=I_1 \quad (62)$$

$$I_b=I_1-I_2 \quad (63)$$

$$I_c=I_2 \quad (64)$$

$$I_m=I_4-I_2 \quad (65)$$

Etc.

Branch Voltages—The branch voltages, E_{yx}, E_{yz}, etc., or the voltages between any two conductively connected points in the network, as E_{xz}, can be obtained by vectorial addition of all voltages, both emfs and drops through any path connecting the two points.

The voltage drop from x to y, D_{xy}, and the voltage of point x above point y, E_{xy}, are the same.

(Note that drop is measured from first subscript to second. The voltage of the first subscript is measured above the second.)

$$D_{xy}=E_{xy}=I_cR_c+jI_c(X_{ic}-X_{cc}) \quad (66)$$
$$D_{yz}=E_{yz}=I_dR_d+jI_dX_{id}-jI_cX_{ide} \quad (67)$$
$$D_{wz}=E_{wz}=-E_c+I_mR_m \quad (68)$$
$$D_{xz}=E_{xz}=I_cR_c+jI_c(X_{ic}-X_{cc})+I_dR_d +jI_dX_{id}-jI_cX_{idc}. \quad (69)$$

Note that

$$D_{xz}=D_{xy}+D_{yz} \quad (70)$$
$$E_{xz}=E_{yz}+E_{xy}. \quad (71)$$

Example of Solution by Equations—(a) Given the impedances and emfs of a network, Fig. 10, required to

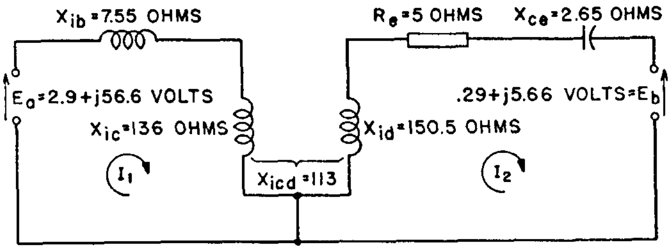


Fig. 10—Example of a solution by equations.

find the currents. Note: The headings (b), (c), etc., refer to the corresponding paragraphs b, c, etc., in which the method and equations are given.

(b) Mesh Impedances

$$Z_{11}=0+j(7.55+136) \quad =0+j143.55$$
$$Z_{12}=Z_{21} \quad = \ -j113.0$$
$$Z_{22}=5+j(150.5-2.65)=5+j147.85$$

(c) Mesh Emfs

$$E_1= \ 2.9 \ +j56.6$$
$$E_2=-0.29-j5.66$$

(d) Equations. It is unnecessary to write these out completely since only the solutions are desired. However for completeness they are:

$$I_1(0+j143.55)+I_2(\ -j113.0)= \quad 2.9+j56.6$$
$$I_1(\ -j113.0\ )+I_2(5+j147.85)=-0.29-j5.66$$

(e) Mesh Currents

$$D=\begin{vmatrix}0+j143.6 & -j113.0\\ -j113.0 & 5+j147.9\end{vmatrix}=-8400+j718$$
$$A_{11}=5+j147.9$$
$$A_{12}=A_{21}=+j113$$
$$A_{22}=0+j143.6$$

$$I_1=\frac{E_1A_{11}}{D}+\frac{E_2A_{12}}{D}$$
$$=\frac{(2.9+j56.6)(5+j147.9)}{-8400+j718}+\frac{(-0.29-j5.66)(j113)}{-8400+j718}$$
$$=0.921+j0.007$$
$$I_2=\frac{E_1A_{21}}{D}+\frac{E_2A_{22}}{D}$$
$$=\frac{(2.9+j56.6)(j113)}{-8400+j718}+\frac{(-0.29-j5.66)(+j143.6)}{-8400+j718}$$
$$=0.662+j0.034$$

Note that the term $\frac{A_{12}}{D}=\frac{A_{21}}{D}$ is the "transfer admittance" between meshes 1 and 2, or is the current in either one of these meshes per unit of emf impressed in the other. Thus the voltage E_2 is -0.1 of E_1 and likewise the current in mesh 1 resulting from E_2 is -0.1 of the current in mesh 2 due to E_1.

14. Solution by Reduction

General—The currents flowing in a network of known impedances, caused by a given set of applied emfs, can be determined by the method of superposition (See Sec. 11). First the solution (currents in all branches of interest) is obtained considering one emf acting with all others set equal to zero. Following the same procedure for each emf in turn, a number of current solutions are obtained. By the principle of superposition, the current in any branch, when all emfs are acting at once, is the sum of currents in that branch caused by each emf acting independently with the others set equal to zero. The principle of superposition presupposes a linear network. The same reference directions must be adhered to for all solutions if the superposition is to be a simple vector addition of the several current solutions.

The solving of a network involving several emfs is thus reduced to the more fundamental problem of solving a network with one impressed emf. This can be accomplished by the method of reduction.

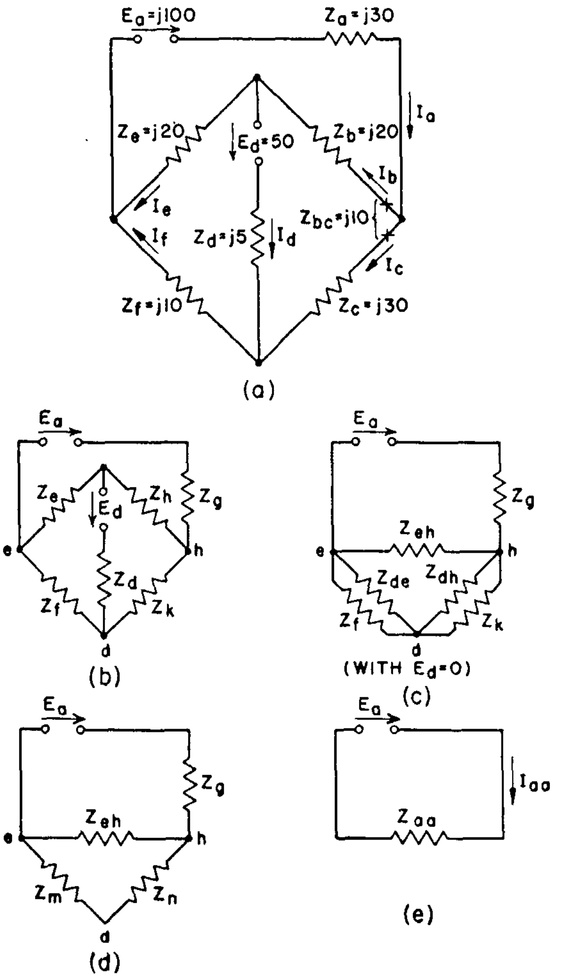


Fig. 11—Solution by reduction. Bridge network current distribution.

Solution by Reduction consists of replacing portions of network, such as Fig. 11(a), by simpler equivalent sections, Fig. 11(b, c, d), until a simple series circuit results, Fig. 11(e), which includes the impressed emf and one impedance branch. The current is readily calculated. Then, using current distribution factors obtained in the course of reduction, a reverse process is carried out, expanding the network to its original form and determining the division of currents in the process. The methods and equivalent circuits for carrying out this procedure in general are given in the subsequent paragraphs.

The Network Equivalents will first be given. Network constants can be expressed either in admittance or impedance form. Some transformations are more readily performed in impedance form, such as adding impedances in series, or delta-to-star transformations. Others are more conveniently performed in admittance form, such as adding admittances in parallel, or star-to-delta transformations. For more complicated transformations, it is best to convert constants to the most convenient form for the particular transformation. For simpler ones, it is usually preferred to use one form or the other throughout the problem.

The common transformations are presented in both forms. The more complicated and unusual ones only in the form best suited. Impedances (symbol Z) are reciprocals of admittances (symbol Y) and vice versa.
That is

$$Z_1=\frac{1}{Y_1} \tag{72}$$

$$Y_1=\frac{1}{Z_1} \tag{73}$$

CURRENT DIVISION DIAGRAMS

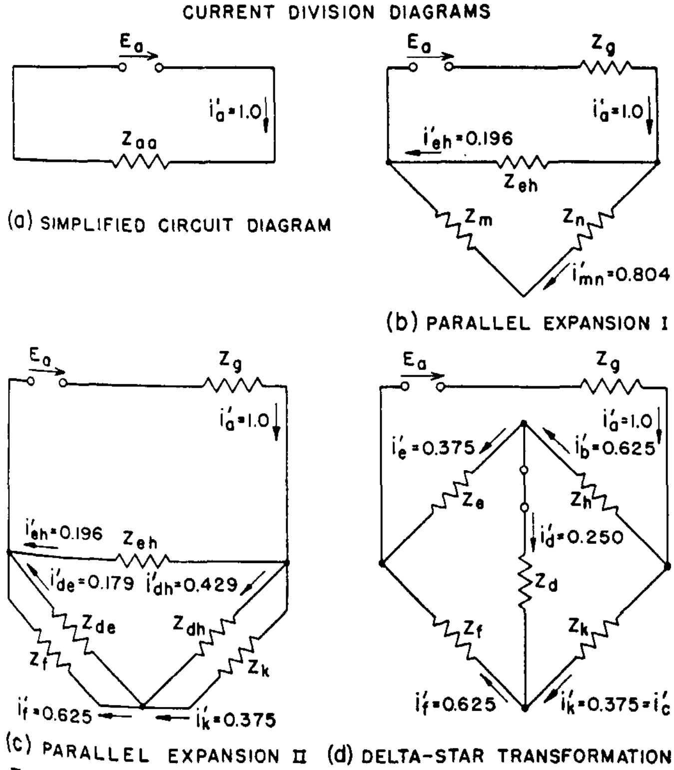


Fig. 12—Solution by reduction. Bridge network current distribution.

In all cases, the equivalent circuits are equivalent only insofar as the labeled terminals are concerned. For example, when a star with mutuals is reduced to a star without mutuals, the potential of the center point is not the same in the equivalent.

Delta and star forms used in general networks are identical with Pi and T forms used in specialized transmission forms of networks. See Fig. 15. The difference is simply in the manner of drawing the circuit. Thus the star-delta and delta-star transformations are at once, T to Pi and Pi to T transformations. The arrow between parts of the figures indicates that the figure on the left is being transformed to the figure on the right. It is assumed then that the currents are determined for the figure on the right and the equations under the figure on the left are for determining the resulting currents (or voltages) in it.

15. Transformations in Impedance Form

a. Impedances in Series (Fig. 13)

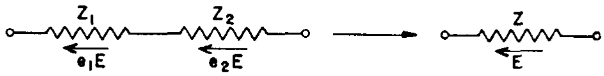


Fig. 13—Impedances in series.

$$e_1=\frac{Z_1}{Z_1+Z_2}=\frac{Z_1}{Z} \tag{75}$$

$$Z=Z_1+Z_2 \tag{74}$$

$$e_2=\frac{Z_2}{Z_1+Z_2}=\frac{Z_2}{Z} \tag{76}$$

b. Impedances in Parallel (Fig. 14)—"The parallel of two impedances is the product divided by the sum."

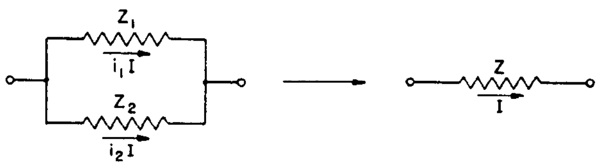


Fig. 14—Impedances in parallel.

$$i_1=\frac{Z_2}{Z_1+Z_2} \tag{78}$$

$$Z=\frac{Z_1Z_2}{Z_1+Z_2} \tag{77}$$

$$i_2=\frac{Z_1}{Z_1+Z_2} \tag{79}$$

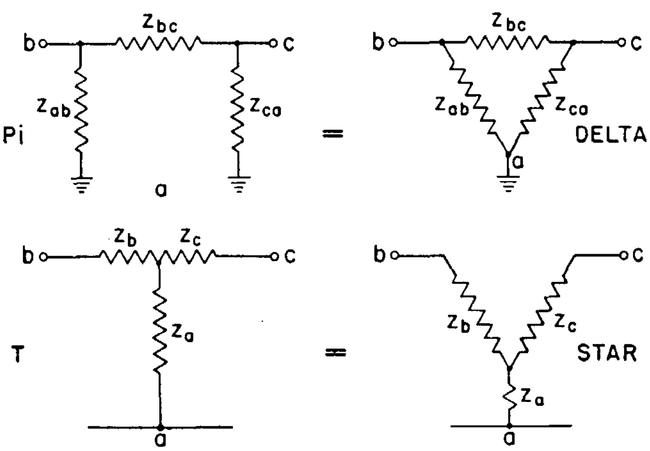


Fig. 15—Pi and delta; T and stars or Y are the same.

Suggested order of calculation (80)

$D=Z_1+Z_2$

$i_1=\dfrac{Z_2}{D}=$ current in Z_1 per unit current in Z. (81)

$i_2=1-i_1=$ current in Z_2 per unit current in Z. (82)

(i_1 and i_2 are current distribution factors.)

$Z=i_1Z_1$ (83)

c. Delta to Star Transformation or Pi to T (Fig. 16)—"The star impedances are the product of adjacent delta impedances divided by the sum of all delta impedances."

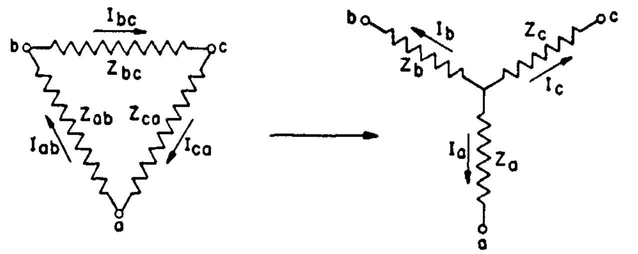


Fig. 16—Delta to star—impedance form.

$$I_{ab}=-\frac{Z_{ca}}{D}I_a+\frac{Z_{bc}}{D}I_b \quad (88a) \qquad Z_a=\frac{Z_{ca}Z_{ab}}{D} \quad (84)$$

$$=-i_{ca}I_a+i_{bc}I_b \quad (88b)$$

$$I_{bc}=-\frac{Z_{ab}}{D}I_b+\frac{Z_{ca}}{D}I_c \quad (89a) \qquad Z_b=\frac{Z_{ab}Z_{bc}}{D} \quad (85)$$

$$=-i_{ab}I_b+i_{ca}I_c \quad (89b)$$

$$I_{ca}=-\frac{Z_{bc}}{D}I_c+\frac{Z_{ab}}{D}I_a \quad (90a) \qquad Z_c=\frac{Z_{bc}Z_{ca}}{D} \quad (86)$$

$$=-i_{bc}I_c+i_{ab}I_a \quad (90b) \qquad D=Z_{ab}+Z_{bc}+Z_{ca} \quad (87)$$

Suggested order of calculation*

$D=Z_{ab}+Z_{bc}+Z_{ca}$ (87)

$i_{ab}=\dfrac{Z_{ab}}{D}$ (91)

$Z_a=Z_{ca}i_{ab}$ (92)

$i_{bc}=\dfrac{Z_{bc}}{D}$ (93)

$Z_b=Z_{ab}i_{bc}$ (94)

$i_{ca}=\dfrac{Z_{ca}}{D}$ (95)

$Z_c=Z_{bc}i_{ca}$ (96)

(i_{ab}, i_{bc}, i_{ca} are current distribution factors.)

d. Star to Delta Transformation or T to Pi (Fig. 17)

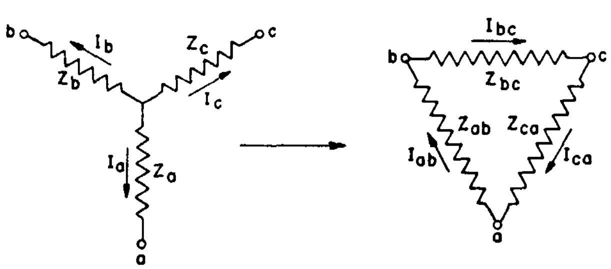


Fig. 17—Star to delta—impedance form.

*Then after I_a, I_b, and I_c have been found, I_{ca}, I_{ab}, and I_{bc} can be determined using Eqs. (88b), (89b), and (90b).

$I_a=I_{ca}-I_{ab}$ (101) $\qquad Z_{ab}=DZ_aZ_b$ (97)

$I_b=I_{ab}-I_{bc}$ (102) $\qquad Z_{bc}=DZ_bZ_c$ (98)

$I_c=I_{bc}-I_{ca}$ (103) $\qquad Z_{ca}=DZ_cZ_a$ (99)

$$D=\frac{1}{Z_a}+\frac{1}{Z_b}+\frac{1}{Z_c} \quad (100)$$

Alternative forms of the transformation formulas follows:

$$\text{Num.}=Z_aZ_b+Z_aZ_c+Z_bZ_c \quad (104)$$

$$Z_{ab}=\frac{\text{Num.}}{Z_c}=Z_a+Z_b+\frac{Z_aZ_b}{Z_c} \quad (105)$$

$$Z_{bc}=\frac{\text{Num.}}{Z_a}=Z_b+Z_c+\frac{Z_bZ_c}{Z_a} \quad (106)$$

$$Z_{ca}=\frac{\text{Num.}}{Z_b}=Z_c+Z_a+\frac{Z_cZ_a}{Z_b} \quad (107)$$

e. Star with Mutuals to Star without Mutuals (Fig. 18)

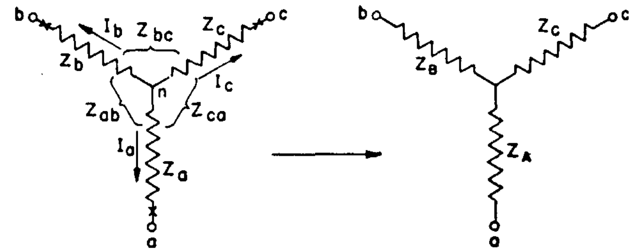


Fig. 18—Star with mutuals to star without mutuals—impedance form.

$$Z_A=Z_a+Z_{bc}-Z_{ab}-Z_{ca} \quad (108)$$

$$Z_B=Z_b+Z_{ca}-Z_{bc}-Z_{ab} \quad (109)$$

$$Z_C=Z_c+Z_{ab}-Z_{ca}-Z_{bc} \quad (110)$$

Polarity marks require that with all reference directions from center outward as shown, all self and mutual drops are from center outward. That is, it is understood that with the polarity marks as shown, the voltage drop from the center to a will be written:

$$D_{na}=I_aZ_a+I_bZ_{ab}+I_cZ_{ca}$$

and the numerical (vector) values and signs assigned to Z_{ab} and Z_{ac} must be such as to make this true. It follows that Z_{ab} is defined as the voltage drop from n to a divided by the current from n to b that causes the drop.

Special case: Star with one mutual between two branches to star without mutual. (Fig. 19.)

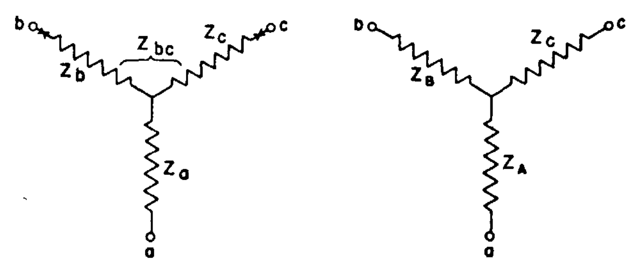


Fig. 19—Star with one mutual to star without mutual—impedance form.

$$Z_A=Z_a+Z_{bc} \quad (111)$$

$$Z_B=Z_b-Z_{bc} \quad (112)$$

$$Z_C=Z_c-Z_{bc} \quad (113)$$

f. Two Self Impedances and a Mutual Transformed to an Equivalent Star or T. Or the Equivalent Circuit of a Two-winding Transformer (Fig. 20)

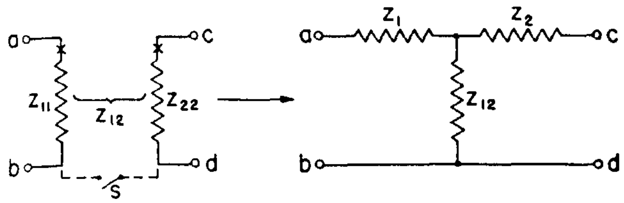


Fig. 20—Two self impedances and a mutual transformed to an equivalent star or *T*. Or the equivalent circuit of a two winding transformer.

$$Z_1 = Z_{11} - Z_{12} \qquad (114)$$

$$Z_2 = Z_{22} - Z_{12} \qquad (115)$$

NOTE: This transformation involves bringing b and d to the same potential and is permissible only when these potentials are not otherwise fixed. Strictly, the form on the right is equivalent to that on the left with switch S closed. However, if the closure of S would not alter the current division, it can be considered closed and the equivalent circuit used. The resulting potentials E_{ab}, and E_{cd} will be correct but the potentials E_{ca} and E_{db}, which are definite in the equivalent, are actually indeterminate in the original circuit and must not be construed as applying there. See note under e for meaning of polarity marks, considering b and d as point n.

This is the familiar equivalent circuit of a two-winding transformer, provided all impedances have first been placed on a common turns basis. In this case Z_{12} is the exciting impedance and $Z = Z_1 + Z_2$ the leakage impedance.

16. Transformations in Admittance Form

a. Admittances in Series (Fig. 21)

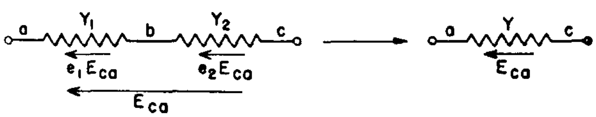


Fig. 21—Admittances in series.

$$e_1 = \frac{Y_2}{Y_1 + Y_2} \qquad (117) \qquad\qquad Y = \frac{Y_1 Y_2}{Y_1 + Y_2} \qquad (116)$$

$$e_2 = \frac{Y_1}{Y_1 + Y_2} \qquad (118)$$

Suggested order of calculation.

$$D = Y_1 + Y_2 \qquad (119)$$

$$e_1 = \frac{Y_2}{D} \qquad (120)$$

$$e_2 = 1 - e_1 \qquad (121)$$

$$Y = Y_1 e_1, \text{ or } Y_2 e_2 \qquad (122)$$

b. Admittances in Parallel (Fig. 22)

$$i_1 = \frac{Y_1}{Y_1 + Y_2} = \frac{Y_1}{Y} \qquad (124) \qquad\qquad Y = Y_1 + Y_2 \qquad (123)$$

$$i_2 = \frac{Y_2}{Y_1 + Y_2} = \frac{Y_2}{Y} \qquad (125)$$

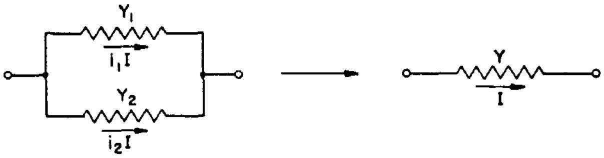


Fig. 22—Admittances in parallel.

c. General Star to Mesh Transformation, or "Elimination of a Junction" (Fig. 23)

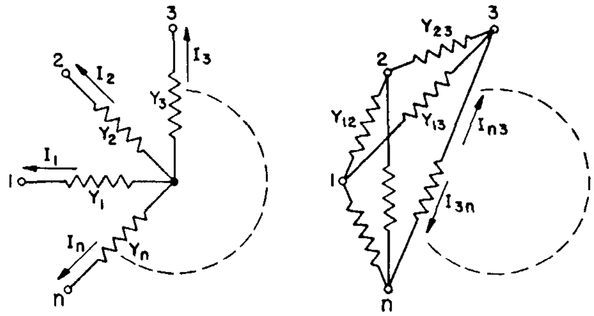


Fig. 23—General star to mesh transformation, or elimination of a junction—admittance form.

NOTE: A network can be solved by eliminating one junction point after another until a single-branch mesh remains.

RULE: "A mesh branch is the product of adjacent star branches divided by the sum of all star branches."

The mesh contains $n/2\ (n-1)$ branches, where n is the number of star branches.

$$I_1 = I_{21} + I_{31} + \cdots + I_{n1} \quad (130) \qquad Y_{12} = \frac{Y_1 Y_2}{D} \quad (126)$$

$$I_2 = I_{12} + I_{32} + \cdots + I_{n2} \quad (131) \qquad Y_{13} = \frac{Y_1 Y_3}{D} \quad (127)$$

etc. etc.

$$I_p = I_{1p} + I_{2p} + \cdots + I_{np} \quad (132) \qquad Y_{pq} = \frac{Y_p Y_q}{D} \quad (128)$$

$$D = Y_1 + Y_2 + Y_3 + \cdots + Y_n \qquad (129)$$

In which the positive reference direction for any mesh current I_{pq} is toward terminal q.

Suggested order of calculation.

$$D = Y_1 + Y_2 + Y_3 + \cdots + Y_n \quad (133) \qquad \frac{Y_2}{D} = k_2 \quad (137)$$

$$\frac{Y_1}{D} = k_1 \quad (134) \qquad Y_{23} = \frac{Y_2 Y_3}{D} = k_2 Y_3 \quad (138)$$

$$Y_{12} = \frac{Y_1 Y_2}{D} = k_1 Y_2 \quad (135) \qquad Y_{24} = \frac{Y_2 Y_4}{D} = k_2 Y_4 \quad (139)$$

$$Y_{13} = \frac{Y_1 Y_3}{D} = k_1 Y_3 \quad (136) \qquad \text{etc.}$$

etc.

d. Star to Delta or T to Pi (Special case of c) (Fig. 24)

$$I_a = I_{ca} - I_{ab} \quad (144) \qquad Y_{ab} = \frac{Y_a Y_b}{D} \quad (140)$$

$$I_b = I_{ab} - I_{bc} \quad (145) \qquad Y_{bc} = \frac{Y_b Y_c}{D} \quad (141)$$

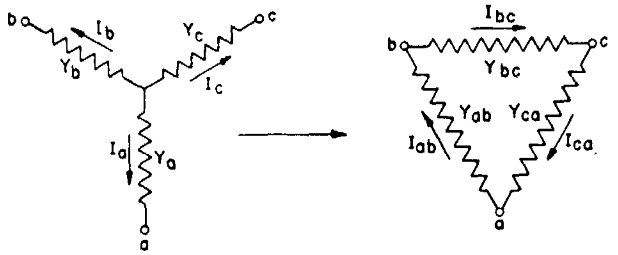


Fig. 24—Star to delta—admittance form.

$$I_c = I_{bc} - I_{ca} \qquad (146) \qquad Y_{ca} = \frac{Y_c Y_a}{D} \qquad (142)$$

$$D = Y_a + Y_b + Y_c \qquad (143)$$

Suggested order of calculation same as for general transformation. (c)

e. Delta to Star Transformation, or Pi to T (Fig. 25)

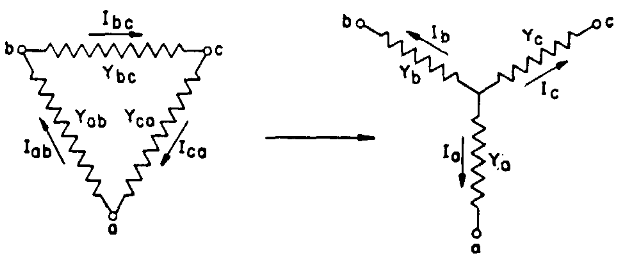


Fig. 25—Delta to star—admittance form.

$$I_{ab} = -i_{ca} I_a + i_{bc} I_b \qquad (151) \qquad Y_a = D Y_{ca} Y_{ab} \qquad (147)$$

$$I_{bc} = -i_{ab} I_b + i_{ca} I_c \qquad (152) \qquad Y_b = D Y_{ab} Y_{bc} \qquad (148)$$

$$I_{ca} = -i_{bc} I_c + i_{ab} I_a \qquad (153) \qquad Y_c = D Y_{bc} Y_{ca} \qquad (149)$$

$$\text{where } i_{ab} = \frac{1}{D Y_{ab}} \qquad (154) \qquad D = \frac{1}{Y_{ab}} + \frac{1}{Y_{bc}} + \frac{1}{Y_{ca}} \qquad (150)$$

$$i_{bc} = \frac{1}{D Y_{bc}} \qquad (155)$$

$$i_{ca} = \frac{1}{D Y_{ca}} \qquad (156)$$

17. Examples of Solution by Reduction

A power-distribution system network is solved by the method of reduction in Sec. 20.

The following example is also given showing the method when two sources of emf are present.

Solve for the currents in the network of Fig. 11(a) by the method of reduction for applied voltages as follows:

Case 1.	$E_a = 0 + j100$	volts
	$E_d = 50 + j0$	volts
Case 2.	$E_a = 30 + j40$	volts
	$E_d = 10 + j60$	volts

Obtain unit current division and total current division as indicated in Tables 4(a) and 5.

The method generally is to resolve the network for one applied voltage at a time, with the other one set equal to zero.

After solutions have been obtained for each applied voltage acting independently, these solutions are superposed to obtain the current flow with both applied voltages acting simultaneously.

Particular attention is directed to certain relationships. The total current input resulting from a particular applied emf is obtained by dividing it by the driving-point impedance* at that pair of terminals. Thus, when the driving-point impedances, Z_{aa} and Z_{dd}, at the impressed voltage terminals have been determined, and the unit current divisions developed, the network is solved. This is illustrated by Cases 1 and 2, which differ only in magnitude of the applied voltages. The resulting currents in these two cases are obtained from the same basic network solution.

Solution—a. Eliminate the mutual as per Eqs. (111–113) and combine impedances in series forming Z_g.

$$Z_h = j20 - j10 = j10$$
$$Z_k = j30 - j10 = j20$$
$$Z_g = j10 + j30 = j40$$

b. Let $E_d = 0$ and replace star Z_e, Z_d, Z_h by its equivalent delta from Eqs. (97–100).

$$D = \frac{1}{j20} + \frac{1}{j5} + \frac{1}{j10} = -j(0.05 + 0.20 + 0.10) = -j0.35$$
$$Z_{eh} = (-j0.35)(j20)(j10) = j70$$
$$Z_{de} = (-j0.35)(j20)(j\ 5) = j35$$
$$Z_{dh} = (-j0.35)(j\ 5)(j10) = j17.5$$

c. Parallel the branches Z_{de} and Z_f also, Z_{dh} and Z_k of Fig. 11(c), obtaining Fig. 11(d).

$$D = Z_f + Z_{de} = j45$$
$$i_f = \frac{Z_{de}}{D} = \frac{j35}{j45} = 0.78$$
$$i_{de} = 1 - 0.78 = 0.22 \text{ (in } Z_{de})$$
$$Z_m = 0.22 \times j35 = j7.77$$
$$\text{(Parallel } Z_{dh} \text{ and } Z_k)$$
$$D = Z_k + Z_{dh} = j20 + j17.5 = j37.5$$
$$i_k = \frac{Z_{dh}}{D} = \frac{j17.5}{j37.5} = 0.468$$
$$i_{dh} = 1 - i_k = 0.532 \text{ (in } Z_{dh})$$
$$Z_n = 0.532 \times j17.5 = j9.333$$

d. $Z_m + Z_n = Z_{mn} = j17.10$

$$\text{Parallel } Z_{mn} \text{ with } Z_{eh} = Z_o$$
$$D = Z_{mn} + Z_{eh} = j17.10 + j70 = j87.10$$
$$i_{eh} = \frac{Z_{mn}}{D} = \frac{j17.10}{j87.10} = 0.196 \text{ (through } Z_{eh})$$
$$i_{mn} = 1 - i_{eh} = 0.804 \text{ (through } Z_{mn})$$
$$Z_o = i_{mn} Z_{mn} = 0.804 \times j17.10 = j13.74$$

e. The impedance viewed from E_a terminals is:

$$Z_{aa} = Z_o + Z_g = j13.74 + j40 = j53.74$$

f. Current Division for unit current in at (a).

The symbol (i) has been used for current division factors.

Let prime symbols be used for the currents in terms of one ampere total input to the network.

*Driving-point impedance is that impedance measured looking into any pair of terminals of a passive network with all other terminals terminated in a specified manner. In this case all other terminals are short-circuited.

$i_g' = 1.0 = i_a'$
$i'_{ch} = i_{ch} = 0.196$
$i'_{mn} = 0.804$
$i'_k = i'_{mn} i_k = 0.804 \times 0.468 = 0.375 = i_c'$
$i'_{dh} = i'_{mn} i_{dh} = 0.804 \times 0.532 = 0.429$
$i_f' = i'_{mn} i_f = 0.804 \times 0.78 = 0.625$
$i'_{dc} = i'_{mn} i_{dc} = 0.804 \times 0.22 = 0.179$
$i_e' = 0.196 + 0.179 = 0.375$
$i_h' = 0.196 + 0.429 = 0.625 = i_b'$
$i_d' = 0.429 - 0.179 = 0.250$

The six currents i_a', i_c', i_f', i_e', i_d', i_b' are given in Table 4(a) and constitute the current division corresponding to unit current entering the network at a. Figs. 12(a) to (c) illustrate the steps in dividing the current.

g. Transfer admittances. See Table 4(b).

$$Y_{aa} = \frac{1}{Z_{aa}} = \frac{1}{j53.74} = -j0.01861$$
$$Y_{ab} = i_b' Y_{aa} = j0.01163$$
$$Y_{ac} = i_c' Y_{aa} = j0.00698$$
etc.

Note: The transfer admittance, Y_{ab}, is the current in branch (b) in the reference direction per unit voltage impressed in branch (a) in the reference direction.

TABLE 3

Viewed From	Impedance
E_a	$Z_{aa} = j53.74$
E_d	$Z_{dd} = j19.54$

TABLE 4(a)

Unit Current in at	Unit Current Division					
	a	b	c	d	e	f
a	1.0	0.625	0.375	0.250	0.375	0.625
d	0.091	0.545	−0.455	1.00	−0.455	0.545

In a similar manner, for voltage applied at (d), the driving-point impedance Z_{dd} and the current division and transfer admittances can be obtained. These are given in Tables 3 and 4. It is essential that the same reference directions be maintained for all current divisions, in order that the solutions for applied voltages at different terminals can be superposed.

The current divisions of Table 5 for the conditions indicated in the second and third columns are obtained directly from the basic network solution Tables 3 and 4. For example with $E_a = 0 + j100$,

$$I_a = \frac{E_a}{Z_{aa}} = \frac{0 + j100}{j53.74} = 1.861$$
$$(\text{or } I_a = E_a Y_{aa} = -j0.01861(j100) = 1.861)$$

Multiplying by the unit current division corresponding to current in at a, the currents I_a to I_f, for 1.861 amperes in at a, are determined.

This method is particularly advantageous when many different combinations of applied voltages are to be applied to the same network. It is also convenient to obtain the transfer admittances, as shown in Table 4(b). These are the currents in the various branches corresponding to unit voltages applied at the respective driving points. It is necessary only to multiply by any actual single applied voltage to obtain the corresponding current division. There is a check here, for the reciprocal theorem states that the current at (d) for unit voltage applied at (a) must be the same as the current at (a) for unit voltage applied at (d).

18. Solution by Thevenin's Theorem

Thevenin's Theorem[2] is useful in analyzing a network or part of a network when its reactions at a particular pair of terminals are of prime importance. Through its use, a complicated network consisting of several emfs and impedances can be replaced by a simple series circuit of one emf and one impedance supplying the pair of terminals of interest. The theorem can be stated as follows:

With respect to any single external circuit connected to any

TABLE 4(b)—TRANSFER ADMITTANCES (See note under g)

Unit Voltage at	Current Division (Amperes)					
	a	b	c	d	e	f
a	$-j0.01861$	$-j0.01163$	$-j0.00698$	$-j0.00465$	$-j0.00698$	$-j0.01163$
d	$-j0.00466$	$-j0.02790$	$+j0.02328$	$-j0.05118$	$+j0.02328$	$-j0.02790$

TABLE 5.

Case	Condition		Current Division (Amperes)					
	E_a	E_d	I_a	I_b	I_c	I_d	I_e	I_f
1	$0+j100$	0	1.861*	1.163	0.698	0.465	0.698	1.163
	0	$50+j0$	$-j0.233$	$-j1.395$	$+j1.164$	$-j2.559$*	$+j1.164$	$-j1.395$
	$0+j100$	$50+j0$	$1.861-j0.233$	$1.163-j1.395$	$0.698+j1.164$	$0.465-j2.559$	$0.698+j1.164$	$1.163-j1.395$
2	$30+j40$	0	$0.746-j0.559$*	$0.466-j0.349$	$0.280-j0.210$	$0.186-j0.139$	$0.280-j0.210$	$0.466-j0.349$
	0	$10+j60$	$0.280-j0.047$	$1.674-j0.279$	$-1.397+j0.233$	$3.071-j0.512$*	$-1.397+j0.233$	$1.674-j0.279$
	$30+j40$	$10+j60$	$1.026-j0.605$	$2.140-j0.628$	$-1.117+j0.023$	$3.257-j0.651$	$-1.117+j0.023$	$2.140-j0.628$

*Total current for which the distribution is shown in that horizontal line.

given pair of terminals of a network, the network can be replaced by a single branch having an impedance, Z, equal to the impedance measured at these terminals looking into the network (when all the network emfs are made zero) and containing a single emf, E_o, equal to the open-circuit voltage of the network across the given pair of terminals.

The term emf as used here has a broader meaning than electromotive force. It is any voltage in the network that remains constant while the impedance connected to the output terminals is varied. Thus, the voltage of a battery of negligible internal impedance is an emf, while the voltage drop in an impedance is not, unless the current is held constant. (See later paragraph). A generator having regulation is segregated into an emf and an internal impedance, back of which the voltage is constant for the particular problem and hence can be treated as an emf.

The General Case is illustrated by Fig. 26. The emfs, E_1, E_2, and E_3 can be of any single frequency. If more than one frequency is present, the emfs of each frequency must be treated separately, as the equivalent circuit will not usually be the same for different frequencies. The impedances may be composed of resistances, inductances and capacitances, but must be linear within the accuracy necessary for the problem at hand. A linear impedance is one that satisfies Ohm's Law, $E=IZ$, Z being a constant.

With these considerations as a basis, Thevenin's Theorem states that the circuit of Fig. 26(d) is equivalent to the circuit of Fig. 26(a), so far as the terminals x, y are concerned. E_o is as measured with the terminals x, y open circuited in Fig. 26(b). Z is as measured in Fig. 26(c) by applying any voltage E' of the frequency under consideration to x, y and measuring the corresponding vector current I' with the emfs E_1, E_2 and E_3 short circuited. Z is the vector quotient E'/I'.

An Example of the use of this theorem is found in the calculation of short-circuit current on a loaded system. The equivalent circuit of the system, up to the point of fault, consists of an emf, E_o, and an impedance, Z. E_o is the voltage at the point of fault before the fault and is usually a known system operating voltage. Z is the impedance looking into the system at the point of fault with all emfs set equal to zero. The short-circuit current is then: $I=\frac{E_o}{Z}$. Thus, it is unnecessary to determine the generator internal voltages. At a given operating voltage E_o, and fixed generating capacity, increased load tends to increase short-circuit current by lowering Z, the driving-point impedance at the fault with all system emfs set equal to zero.

The method applies equally well to a network in which certain fixed currents are forced to flow, as by current transformers. Examination of the equations of a network having fixed current input reveals its identity with a network of fixed emfs. For example, consider the circuit of Fig. 27(a). The equations that involve the known input currents are:

$$-I_aA-I_p(A+C+D)+I_nC+I_mD=0 \qquad (157)$$
$$+I_cB-I_n(B+E+C)+I_pC+I_mE=0 \qquad (158)$$

The equations involving E_a and E_c in Fig. 27(b) are:

$$E_a-I_p(A+C+D)+I_nC+I_mD=0 \qquad (159)$$
$$+E_c-I_n(B+E+C)+I_pC+I_mE=0 \qquad (160)$$

Equations for the remainder of the network are the same

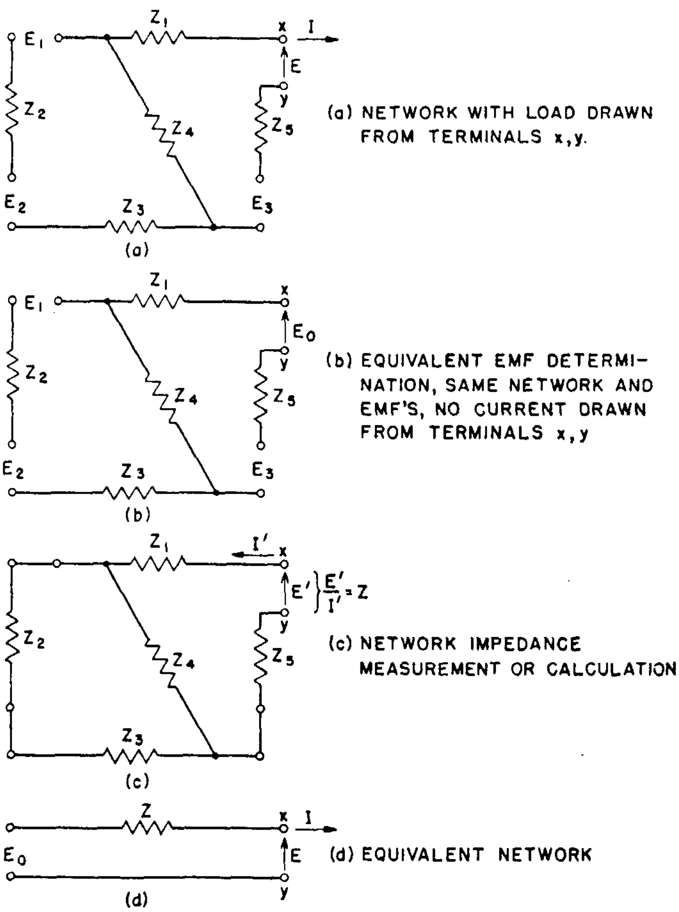


Fig. 26—Determination of equivalent network by means of Thevenin's Theorem.

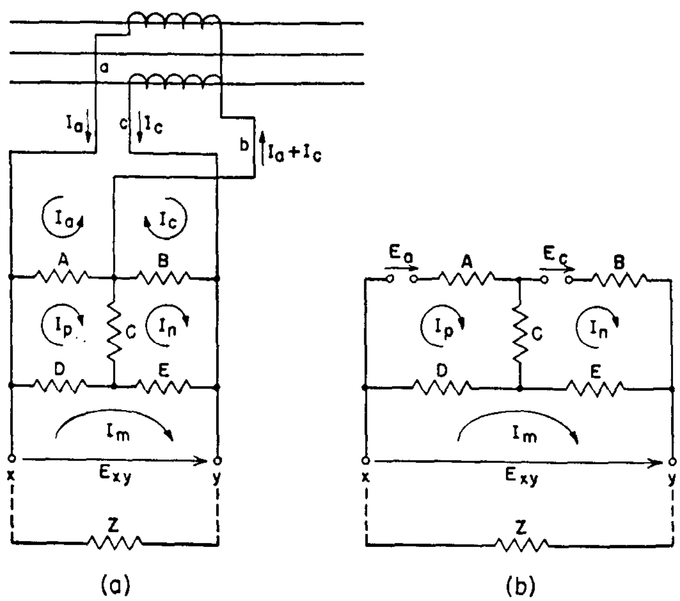


Fig. 27—Application of Thevenin's Theorem in a network of fixed input currents.

for Fig. 27(a) or (b). It is apparent that (157) and (158) are identical with (159) and (160) respectively, if:

$$E_a = -I_a A \quad (161)$$
$$E_c = -I_c B \quad (162)$$

In other words, the terms $-I_aA$ and $-I_cB$ can be treated as emfs in applying Thevenin's Theorem, and the performance at terminals x, y treated through the use of open-circuit voltage and driving-point impedance. The latter is obtained with the input-current terminals, that is, the a, b, and c leads from the current transformers, open circuited in Fig. 27(a), or the equivalent emfs, E_a and E_b, of Fig. 27(b) set equal to zero.

A more complete discussion is given in Reference Number 2.

19. Solution by Circulating Currents

A ladder-type network common where transmission and distribution circuits parallel each other as in a-c railway electrification[3] is represented in Fig. 28. The example is,

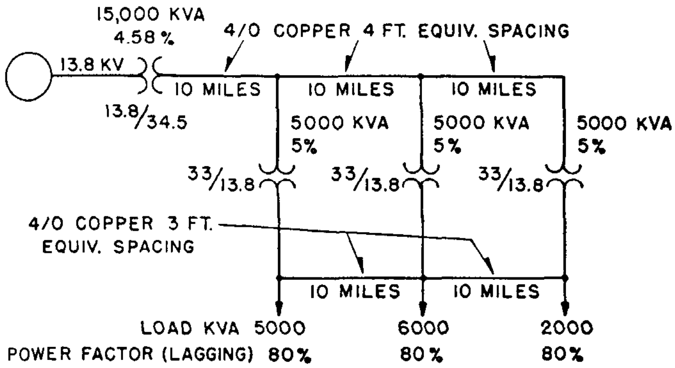


Fig. 28—A general ladder-type network.

however, for a three-phase system. Suppose it is desired to determine the current division and regulation for the particular loading condition shown, without making a general solution of the network. This problem lends itself to the method of circulating currents.

The voltages at the load buses must first be assumed and the kva loads converted to currents. The sum of the three load currents flow in the generator and constitute the current I_a in Fig. 29. These load currents and the generator current are assumed to be fixed for the balance of the problem.

The division of I_a between I_e and I_c is next assumed. Now the voltage drops from 1 to 2, 2 to 5 and 2 to 3 can

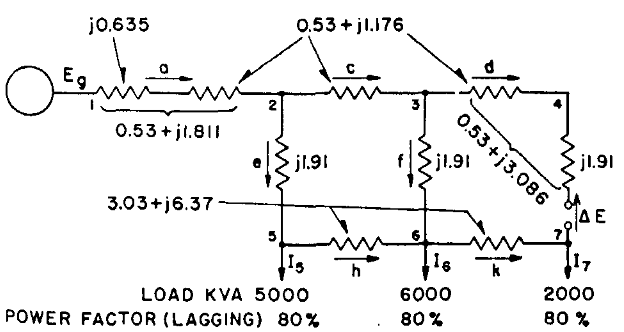


Fig. 29—Impedance diagram for the system of Fig. 28. Showing the method of circulating currents.

be calculated. The current I_h is obtained by subtracting the load at 5 from I_e, and then the drop from 5 to 6 can be computed. Knowing the drops from 2 to 3 and 2 to 6, for the assumed current division, the drop from 3 to 6 is obtained by subtraction. This drop divided by the impedance f gives the current I_f. Now the balance of the currents can be obtained, that is, I_d and I_k. However unless a perfect guess has been made, the drop 3–4–7 will differ from the drop 3–6–7, and an arbitrary voltage, ΔE, must be included to make the voltages around the loop add to Zero.

So far an exact solution has been obtained for the case of certain load currents, a particular generator voltage and a voltage ΔE. However, the solution is desired without ΔE. To obtain it a solution is next obtained for $-\Delta E$ acting alone. According to the conditions of the problem the load and generator currents are fixed so that these branches are considered open circuited when computing the currents caused by $-\Delta E$ alone. This solution is therefore quite simple and gives rise to a set of currents I_c', I_d', ... I_k', which are the "circulating currents" for which the method is named.

Now let these two solutions be superposed; that is:

$$I_c'' = I_c + I_c'$$
$$I_d'' = I_d + I_d' \quad \text{etc.}$$

The resulting solution does not involve ΔE since the ΔE of the first solution is canceled by the $-\Delta E$ of the second solution. It is therefore an exact solution for the load currents assumed. The voltage drops from the generator to the several load points can now be computed, since the currents are known. Also, from the new load voltages, and from the load currents that have been held fixed throughout the solution, new load kvas and power factors can be computed.

The net result is an exact solution for a set of conditions that differs more or less from those originally assumed. While this can be used as a basis for a second approximation it is more generally considered the engineering answer. The loads are usually not known exactly; the solution obtained provides an exact reference point in the region of the loads assumed, and thereby provides a tangible basis for engineering judgment.

There is much to be said for this type of solution as a system design tool, since it capitalizes experience and foreknowledge of the order of magnitude of the answer. As an example the network of Fig. 28 has been solved for the loads indicated thereon.

Example of Method of Circulating Current—The network diagram, Fig. 29, is obtained from the single-line diagram as outlined in Secs. 2 and 3. The 15 000-kva transformer impedance should be converted to ohms on a 34.5-kv base and then multiplied by $(13.8/33)^2$ to convert to the 13.8-kv base at the load. The resulting diagram is on the load-voltage base, a conversion being necessary to change to or from the generator-voltage base, which is also nominally 13.8 kv. Thus with the generator at 13.8 kv, the corresponding voltage to be applied in Fig. 29 is 13.8(34.5/13.8) (13.8/33) = 14.42 kv or 4.5 percent above normal. In an actual case transformer resistances should be included as these are significant in regulation and loss calculations.

As a first approximation, assume the regulation in step-up and step-down transformers to total 10 percent with an additional 10 percent in lines. Allowing for a 4.5 percent above normal voltage at the generator, the loads should be converted to currents based on approximately 85 percent voltage or 11 700 volts. The load currents are given in the following tabulation.

Location	5	6	7	Total
Load kva	5000	6000	2000	
Load Current Amps	245	295	98	638

In the current distribution calculation which follows, the current I_e must be guessed, or taken arbitrarily. Later a circulating current is determined, which, added to the arbitrarily assumed value, gives the correct current. It is advantageous to guess as close as possible so that the correcting circulating current is small. In fact if the guess is sufficiently close, the labor of calculating the distribution of circulating current can be saved.

$$
\begin{aligned}
I_a &= 638\\
I_e &= 300\\
I_c = I_a - I_e &= 338\\
I_5 &= 245\\
I_h = I_e - I_5 &= 55\\
I_c &= 338\\
Z_c &= 0.53 + j1.176\\
I_c Z_c &= 179 + j397\\
I_e &= 300\\
Z_e &= +j1.91\\
I_e Z_e &= +j573\\
I_h &= 55\\
Z_h &= 3.03 + j6.37\\
I_h Z_h &= 166.5 + j350\\
I_e Z_e &= +j573\\
D_{256} &= 166.5 + j923\\
I_c Z_c &= 179 + j397\\
I_f Z_f &= -13.5 + j526\\
Z_f &= +j1.91\\
I_f &= 275 + j7.1\\
I_c &= 338\\
I_d = I_c - I_f &= 63 - j7.1\\
I_h &= 55\\
I_f &= 275 + j7.1\\
I_h + I_f &= 330 + j7.1\\
I_6 &= 295\\
I_k &= 35 + j7.1\\
Z_d &= 0.53 + j3.086\\
I_d &= 63 - j7.1\\
I_d Z_d &= 55.3 + j190.7\\
Z_k &= 3.03 + j6.37\\
I_k &= 35 + j7.1\\
I_k Z_k &= 60.8 + j244.5\\
I_f Z_f &= -13.5 + j526\\
D_{367} &= 47.3 + j770.5\\
D_{347} = I_d Z_d &= 55.3 + j190.7\\
\Delta E &= -8.0 + j579.8
\end{aligned}
$$

Solution for Circulating Current.

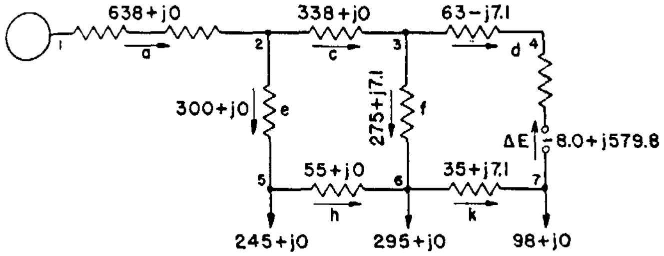
Find the impedance as viewed from ΔE with load and generator branches taken as constant current branches, i.e., open circuited for this calculation. Apply a voltage $\Delta E'$ = negative of the ΔE required to close the mesh in the above calculation.

$$
\begin{aligned}
Z_c &= 0.53 + j\,1.176\\
Z_e &= +j\,1.91\\
Z_h &= 3.03 + j\,6.37\\
Z_{3256} &= 3.56 + j\,9.456\\
Z_f &= +j\,1.91\\
D &= 3.56 + j11.366\\
1/D &= 0.0251 - j\,0.0801\\
Z_f &= +j\,1.91\\
Z_f/D = i_{3256} &= 0.1530 + j\,0.0479\\
i_f = 1 - i_{3256} &= 0.8470 - j\,0.0479\\
Z_f &= +j\,1.91\\
Z_{par} &= 0.0915 + j\,1.618\\
Z_d &= 0.53 + j\,3.086\\
Z_k &= 3.03 + j\,6.37\\
Z &= 3.6515 + j11.074\\
1/Z &= 0.0269 - j\,0.0814\\
-\Delta E = \Delta E' &= 8.0 - j579.8\\
I_k' = \Delta E'/Z &= -46.98 - j16.25\\
I_d' = -I_k' &= 46.98 + j16.25\\
i_{3256} &= 0.1530 + j\,0.0479\\
I_c' &= 6.41 + j\,4.74\\
I_f' = I_c' - I_d' &= -40.57 - j11.51\\
I_e' = -I_c' &= -6.41 - j\,4.74\\
I_h' = -I_c' &= -6.41 - j\,4.74
\end{aligned}
$$

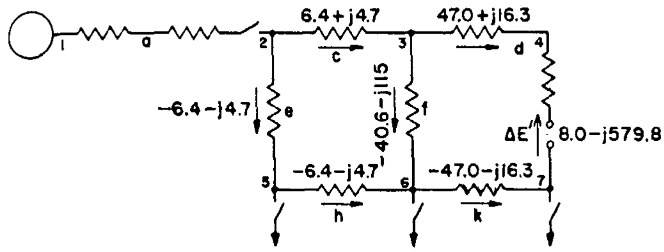
The currents from the arbitrary distribution (requiring ΔE to close) and the circulating currents contributed by $\Delta E' = -\Delta E$, are now combined to get the actual current division for the load currents assumed. The circulating currents are distinguished by prime symbols, the total division by double-primes. Fig. 30 illustrates this superposition.

$$
\begin{aligned}
I_a'' = I_a &= 638 + j0\\
I_e &= 300 + j0\\
I_e' &= -6.4 - j4.7\\
I_e'' &= 293.6 - j4.7\\
I_c &= 338 + j0\\
I_c' &= 6.4 + j4.7\\
I_c'' &= 344.4 + j4.7\\
I_h &= 55 + j0\\
I_h' &= -6.4 - j4.7\\
I_h'' &= 48.6 - j4.7\\
I_f &= 275 + j\,7.1\\
I_f' &= -40.6 - j11.5\\
I_f'' &= 234.4 - j\,4.4\\
I_d &= 63 - j\,7.1\\
I_d' &= 47.0 + j16.3\\
I_d'' &= 110.0 + j\,9.2\\
I_k &= 35 + j\,7.1\\
I_k' &= -47.0 - j16.3\\
I_k'' &= -12 - j\,9.2
\end{aligned}
$$

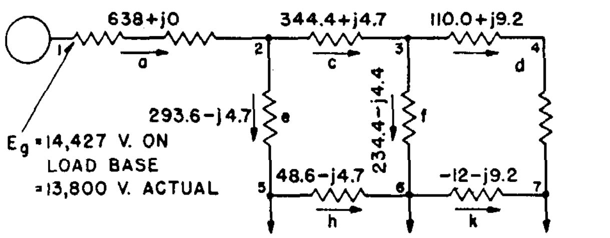
Check of Drops Around Loops. This solution can be checked by checking voltage drops around each loop.



(a) ARBITRARY DIVISION AND VOLTAGE ΔE REQUIRED TO CLOSE



(b) CURRENTS DUE TO ΔE'=−ΔE



LOAD CURRENT	245+j0	295+j0	98+j0
LOAD VOLTAGE	12,022	11,473	11,547
REG. BELOW 13,800 V.	12.82%	16.86%	16.32%
KW+j REACTIVE KVA	4081−j3061	4757−j3424	1574−j1168
LOAD KVA	5101	5861	1960
POWER FACTOR (LAG.)	80.0%	81.2%	80.3%

(c) SUPERPOSITION. OF (a) AND (b)

Fig. 30—Solution of the network of Fig. 28 by the method of circulating currents.

Calculate the drop from generator bus to load point 5.

$$
\begin{aligned}
I_a &= 638.0 + j0 \\
Z_a &= 0.53 + j1.811 \\
I_a Z_a &= 338.1 + j1155.4 \\
I_e'' Z_e &= 9.0 + j560.8 \\
D_{125} &= 347.1 + j1716.2
\end{aligned}
$$

***Regulation**—Determination of Load Voltages.* The magnitude of the generator voltage is known, but not its phase position. The phase position of the load voltage E_5 is known but not its magnitude. It is at an 80-percent power factor position with respect to I_5. The drop from generator to load point 5, D_{125}, is known vectorially with I_5 as reference. Thus the magnitude, $\bar{E}_5$, can be determined by the solution of a quadratic equation as shown below, or graphically as indicated in Fig. 31.

$$E_5 = \bar{E}_5(0.8 + j0.6)$$
$$E_5 + D_{125} = E_g$$
$$E_g = 0.8\bar{E}_5 + 347.1 + j(0.6\bar{E}_5 + j1716.2)$$
$$E_g^2 = \frac{(14427)^2}{3} = (0.8\bar{E}_5 + 347.1)^2 + (0.6\bar{E}_5 + 1716.2)^2$$
$$\bar{E}_5^2 + 2(1307.4)\bar{E}_5 - 66.313\times10^6 = 0$$
$$\bar{E}_5 = -1307.4 \pm \sqrt{(1307.4)^2 + 66.313\times10^6}$$

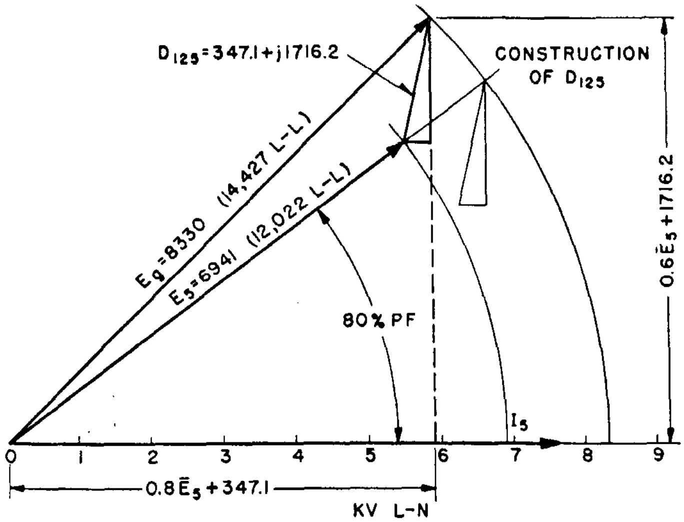


Fig. 31—Graphical determination of the load voltage $\bar{E}_5$ in Fig. 28.

1. The load current I_5 is along the reference line.
2. Draw circle of radius 8330 volts on which E_g must terminate.
3. Draw construction line at the load power factor (80 percent) along which E_5 must lie.
4. Construct the voltage drop vector, D_{125}, and move it parallel to itself, with one end following the generator voltage circle, until the other end falls on the load voltage construction line.
5. E_g and E_5 can then be drawn in and their vector values scaled off.

$$
\begin{aligned}
\bar{E}_5 &= -1307.4 \pm 8248 \\
&= 6941 \text{ volts L-N} \\
&= 12\,022 \text{ volts L-L} \\
&= 87.12 \text{ percent of } 13\,800 \\
&= 83.33 \text{ percent of } 14\,427
\end{aligned}
$$
$$E_5 = 6941\times(0.8 + j0.6)$$
$$E_5 = 5553 + j4165$$

Check of Voltage Drop from 1 to 5.

$$
\begin{aligned}
E_5 &= 5553 + j4165 \\
D_{125} &= 347 + j1716 \\
E_g &= 5900 + j5881 \\
&= 8330.4 \text{ L-N} \\
& 14428 \text{ L-L}
\end{aligned}
$$

The remaining load voltages are readily determined as follows:

$$
\begin{aligned}
E_5 &= 5553 + j4165 \\
I_h'' Z_h &= 177.2 + j295.3 \\
E_6 &= 5375.8 + j3869.7 \\
&= 6624 \text{ L-N} \\
&= 11\,473 \text{ L-L} \\
&= 83.14\% \text{ of } 13\,800 \\
& 79.52\% \text{ of } 14\,427 \\
E_6 &= 5375.8 + j3869.7 \\
I_k'' Z_k &= 22.2 - j\,104.3 \\
E_7 &= 5353.6 + j3974.0 \\
&= 6667 \text{ L-N} \\
&= 11\,547 \text{ L-L} \\
&= 83.68\% \text{ of } 13\,800 \\
&= 80.04\% \text{ of } 14\,427
\end{aligned}
$$

Load Power Calculations.

$$\begin{aligned}
E_5 &= (5.553 + j4.165)\text{kv} \\
3\hat{I}_5 &= \underline{735 \quad -j0} \\
P_5 + jQ_5 &= (4081 \quad +j3061)\text{kva} \\
&= 5101 \text{ kva}
\end{aligned}$$

at 80.00 percent power factor lagging.

$$\begin{aligned}
E_6 &= (5.375 + j3.869)\text{kv} \\
3\hat{I}_6 &= \underline{885 \quad -j0} \\
P_6 + jQ_6 &= (4757 \quad +j3424)\text{kva} \\
&= 5861 \text{ kva}
\end{aligned}$$

at 81.16 percent power factor lagging.

$$\begin{aligned}
E_7 &= (5.354 + j3.974)\text{kv} \\
3\hat{I}_7 &= \underline{294 \quad -j0} \\
P_7 + jQ_7 &= (1574 \quad +j1168)\text{kva} \\
&= 1960 \text{ kva}
\end{aligned}$$

at 80.31 percent power factor lagging.

Generator Output Power.

$$\begin{aligned}
E_g &= (5.900 + j5.881)\text{kv} \\
3\hat{I}_g &= \underline{1914 \quad -j0} \\
P_g + jQ_g &= (11\,293 + j11\,256)\text{kva} \\
&= 15\,945 \text{ kva}
\end{aligned}$$

at 70.82 percent power factor lagging.

Loss Calculation.

$$\begin{aligned}
P_5 + jQ_5 &= 4081 \quad +j3061 \\
P_6 + jQ_6 &= 4757 \quad +j3424 \\
P_7 + jQ_7 &= \underline{1574 \quad +j1168} \\
\text{Total of Loads} &= 10\,412 + j7653 \\
P_g + jQ_g &= \underline{11\,293 + j11\,256} \\
\text{Losses} &= 881 \quad +j3603 \\
\text{Kw Line Loss} &= 881/11\,293 \\
&= 7.80 \text{ percent of generator output.}
\end{aligned}$$

In an actual case transformer resistances must be included in the diagram as these are significant in regulation and loss calculations. Transformer iron losses must be added to the copper losses thus determined to obtain the total loss.

The solution given in Fig. 30 is exact for the conditions shown on the figure, which differ slightly from the original assumptions of Fig. 28. However, the total load is off only 1.3 percent and the regulation values therefore apply closely for the original conditions. In a practical problem it is not significant that the answer does not apply exactly to the original load assumptions. If the work is done with a calculating machine so that several significant figures can be carried, losses can be computed as the difference between input and output power, as shown.

III. REPRESENTATION OF NETWORK SOLUTIONS AND THEIR USE IN SYSTEM PROBLEMS

Network solutions can be represented in a variety of ways. For example a diagram can be labeled with all pertinent information obtained in the solution as in Fig. 30. This scheme is used most commonly in expressing current distributions. The solution can also be expressed as a tabulation of self and mutual drops and current division, or in the form of driving point and transfer impedances or admittances. General circuit constants such as the ABCD constants or Pi and T equivalents can also be used to express the solution of certain types of networks. The following paragraphs describe these several methods of representing solutions and their uses.

20. Method of Self and Mutual Drops

The method of self and mutual drops constitutes one of the most useful means for fully describing the action of a complicated network in the form of a table of system constants. It is applicable principally to single-source systems or to systems in which all of the generator voltages can be taken equal and in phase. However, its use can be extended to multiple-source systems provided that either:

a. All sources but one are treated as negative loads,
b. The emfs of the several sources are fixed in magnitude and phase position with respect to each other.

The method will be described with respect to the single-source system, and the multiple-source system treated as an extension.

A Single-Source System Without Shunt Branches other than the loads, is shown in Fig. 32(a). Each of the loads draws current through the network causing voltage drop from the generator bus g to the bus on which it is connected. Each load likewise causes voltage drops to the other loads, known as mutual drops. As these drops are proportional to the load current, they can be determined by finding first the drop resulting from unit load and multiplying by the value of load. Accordingly, the following definitions will be found of use.

Z_{aa} is the voltage drop from g to a caused by unit load current drawn from the network at a. It is called the self drop constant.

Z_{ab} is the voltage drop from g to b caused by unit load current drawn from the network at a. It is called the mutual drop constant.

NOTE that the self and mutual drop constants Z_{aa} and Z_{ab} as defined and used here in Sect. 20, differ from the self and mutual impedances defined and used in Sections 13 and 21. The Z with double subscript is used in each case to conform with accepted terminology.

In both cases current is admitted at g and the unit load referred to is the only load. Obviously, the self and mutual drops have the dimensions of impedance but the term drop will be retained to distinguish from the terms self and mutual impedance that are used otherwise. For unit loads at other points the self and mutual drops are similarly defined. Thus associated with the network of Fig. 32(a) are the nine drops:

$$\begin{matrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{matrix}$$

The first subscript denotes the point at which unit current is drawn; the second denotes the point to which the drop is measured. However, in all cases mutual drops between the same two points are equal. That is:

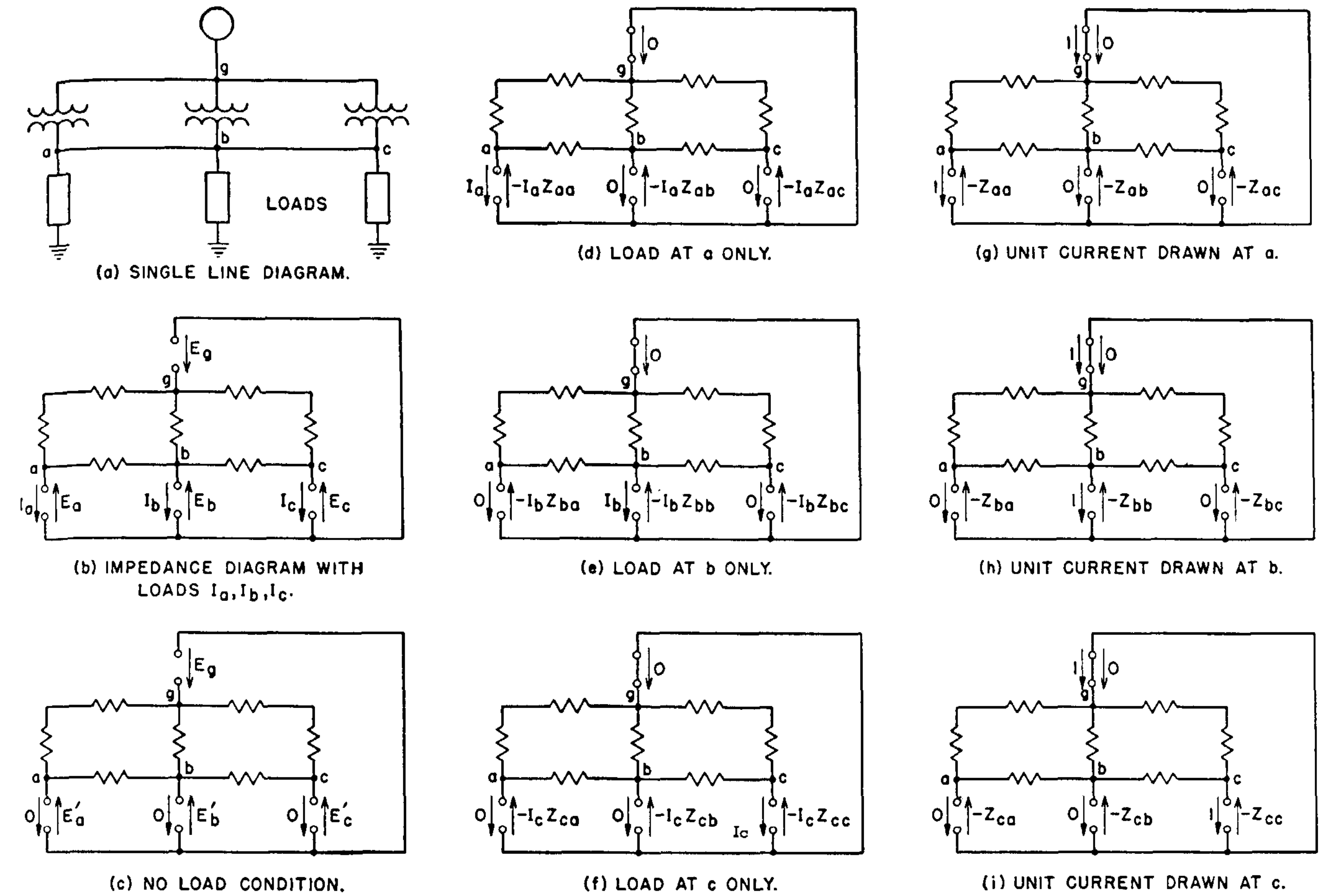


Fig. 32—A single-source system without shunt branches. Illustrating the method of self and mutual drops.

TABLE 6—CURRENT DIVISION

Unit Load at	Current						
	I_{GA}	I_{GD}	I_{GC}	I_{DC}	I_{CA}	I_{CB}	I_{AB}
A	$0.667-j0.013$	$0.197-j0.003$	$0.136+j0.016$	$0.197-j0.003$	$0.243-j0.003$	$0.090+j0.016$	$-0.090-j0.016$
B	$0.556-j0.021$	$0.264-j0.002$	$0.181+j0.023$	$0.264-j0.002$	$-0.040+j0.017$	$0.484+j0.004$	$0.516-j0.004$
C	$0.445-j0.029$	$0.329-j0.000$	$0.226+j0.029$	$0.329-j0.000$	$-0.323+j0.037$	$-0.122-j0.008$	$0.122+j0.008$
D	$0.060+j0.005$	$0.911-j0.013$	$0.029+j0.008$	$-0.089-j0.013$	$-0.044-j0.001$	$-0.016-j0.004$	$0.016+j0.004$

$$Z_{ab}=Z_{ba} \tag{163}$$
$$Z_{ac}=Z_{ca} \tag{164}$$
$$Z_{bc}=Z_{cb} \tag{165}$$

The drops can be calculated or measured on a network calculator. Unit current is drawn from one of the points of interest, for example load point a of Fig. 32(a), and the voltage drops from the reference bus, g, to each of the load points a, b, and c measured or calculated. These are the self and mutual drops Z_{aa}, Z_{ab}, and Z_{ac}, respectively. The current division for this condition should likewise be recorded.

This process is repeated in turn for the other cardinal load points b and c. Mutual drops must check according to Eqs. (163–165). If the solution is to be used for a study of short circuits and relaying, it is generally necessary to include many cardinal points that are not strictly load points, but are line junctions, etc.

The resulting tabulations of current division and self and mutual drops, as illustrated in Tables 6 and 7 for the network of Fig. 33, are the basic network solution. They

TABLE 7
Self and Mutual Drops

Unit Load At	Voltage Drop to			
	A	B	C	D
A	Z_{AA} $1.28+j2.08$	Z_{AB} $1.10+j1.71$	Z_{AC} $0.916+j1.345$	Z_{AD} $0.10+j0.19$
B	Z_{BA} $1.092+j1.712$	Z_{BB} $2.447+j3.554$	Z_{BC} $1.200+j1.808$	Z_{BD} $0.126+j0.263$
C	Z_{CA} $0.91+j1.36$	Z_{CB} $1.203+j1.82$	Z_{CC} $1.49+j2.28$	Z_{CD} $0.154+j0.328$
D	Z_{DA} $0.096+j0.196$	Z_{DB} $0.125+j0.261$	Z_{DC} $0.154+j0.326$	Z_{DD} $0.440+j0.897$

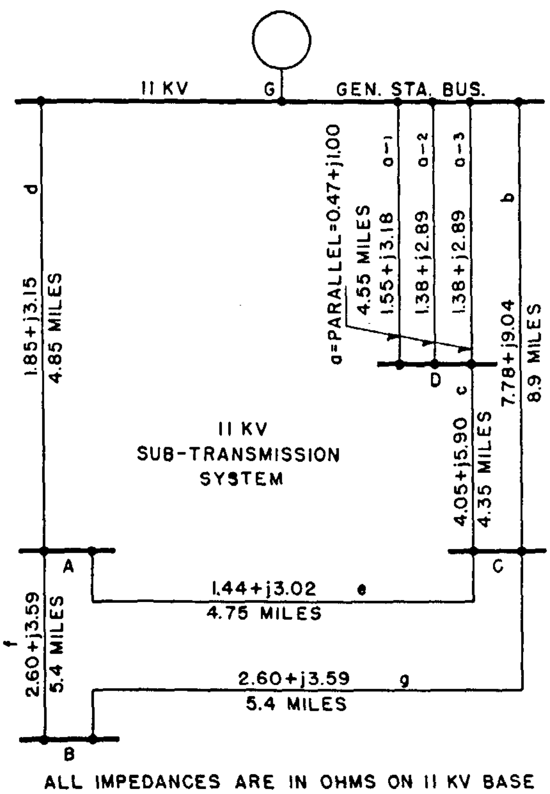


Fig. 33—Single-line diagram of a typical 11-kv subtransmission system.

can be used in many of the problems of interest in the performance of systems; that is, regulation, short-circuit currents, losses, loading of lines and equipment, phase angles, circulating currents, stability, etc.

Current Division—To determine the current division for any particular loading condition the current division corresponding to unit load current at a point is multiplied by the actual load current at that point. When this has been done for each load point the resulting currents are superposed giving the current division for the simultaneous loading condition.

Let I_{pq} be the total current from p to q, where pq is some particular branch of a network.

Then in a network of n cardinal load points, if $I_1, I_2, \cdots,$ I_n are the load currents drawn from the network, the current from p to q is:

$$I_{pq}=I_1I_{1\cdot pq}+I_2I_{2\cdot pq}+\cdots+I_nI_{n\cdot pq} \tag{166}$$

where $I_{n\cdot pq}$ is the current in any branch pq caused by unit current drawn at n.

Thus, once the basic current divisions have been determined for unit loads at the cardinal points, the current in any branch can be readily determined for any given load condition.

Regulation—In a similar manner, to determine the regulation under a condition of simultaneous loads at several of the cardinal points, the self and mutual drops for unit load at a point are first multiplied by the actual load at that point. When this has been done in turn for each load, the resulting drops are superposed to obtain the voltage drops corresponding to the simultaneous loading condition, as stated in the following equation.

Let D_p be the total drop to a typical point p.

$$D_p=I_1Z_{1p}+I_2Z_{2p}+\cdots+I_nZ_{np} \tag{167}$$

Or the voltage at p may be expressed

$$E_p=E_p'-I_1Z_{1p}-I_2Z_{2p}-\cdots-I_nZ_{np} \tag{168}$$

where E_p' is the voltage at p with no load on the system*.

The superposition theorem applies strictly to a *fixed network* and may appear to preclude the possibility of connecting loads to various terminals. Fig. 32 illustrates the philosophy under which this problem is brought within the scope of the superposition theorem. Part (a) is the network with all loads connected, and (b) is the corresponding impedance diagram with loads replaced by the load voltages E_a, E_b, and E_c and the load currents I_a, I_b, and I_c. The load voltages can be viewed as emfs as far as relations within the portion of network from g to a–b–c are concerned. Part (g) illustrates unit current drawn at a. According to the definition, the drop to a is Z_{aa}; hence starting with zero voltage on the generator bus the voltage at a must be $-Z_{aa}$. Similarly the voltages at b and c must be $-Z_{ab}$ and $-Z_{ac}$, respectively. Thus unit load at a can be viewed as produced by zero generator voltage and by voltages $-Z_{aa}$, $-Z_{ab}$ and $-Z_{ac}$ acting at a, b, and c, respectively, in *the same network* as Fig. 32(g). Parts (h) and (i) illustrate the corresponding voltages required to produce unit load current at b and c, in *this same network*.

It is at once apparent that if all voltages and currents of Fig. 32(g) are increased in the ratio $I_a/1$, the resulting emfs are those required to produce current I_a at a and zero load current at the other two points. This condition is shown in Fig. 32(d), and the corresponding conditions for loads at b and c are shown in 32(e) and 32(f) respectively.

Part (c) is simply the no-load condition illustrating load emfs equal and opposite to the generator emf, producing zero load currents in the same network.

If now the four solutions of the same network, as given in (c), for the no-load condition and in (d), (e), and (f) for loads at a, b, and c respectively, are superposed, the resulting solution for the general case, Fig. 32(b), is obtained. Thus:

$$E_a=E_a'-I_aZ_{aa}-I_bZ_{ba}-I_cZ_{ca} \tag{169}$$
$$E_b=E_b'-I_aZ_{ab}-I_bZ_{bb}-I_cZ_{cb} \tag{170}$$
$$E_c=E_c'-I_aZ_{ac}-I_bZ_{bc}-I_cZ_{cc} \tag{171}$$

And in this case, as shown in Fig. 32(c):

$$E_a'=E_b'=E_c'=E_g \tag{172}$$

Example—Single-Source System Without Shunt Branches Other Than the Loads—As an example of the use of this method, suppose a general solution is desired for the system of Fig. 33. Also the improvement in regulation at points B, C, and D, when 2500 kva of capacitors are added at each of these points, is to be determined for a particular condition. The network is solved by the method of reduction. See Secs. 14–17. It is first reduced to a single branch by employing several series or paralleling operations and

*The voltage at the source point from which drops are measured is assumed to remain constant.

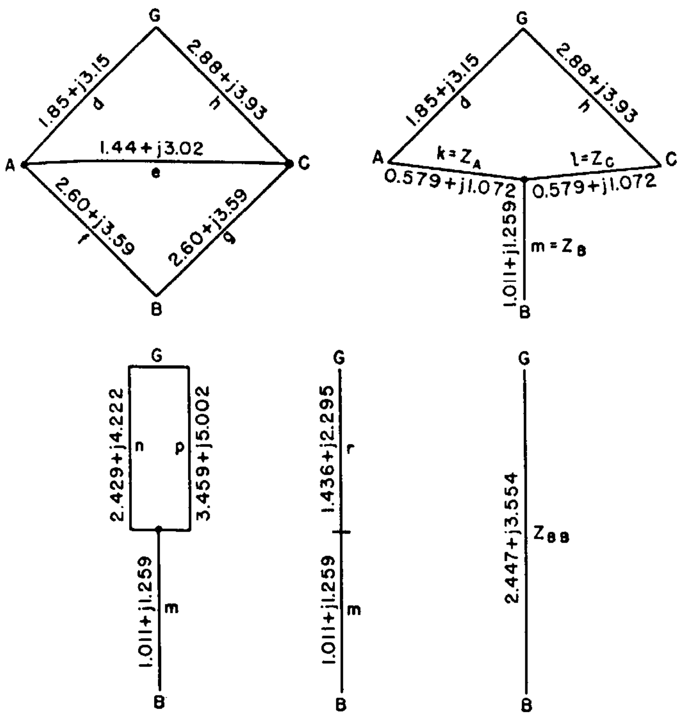


Fig. 34—Reduction of the system of Fig. 33 with respect to terminals G and B.

a delta-to-star conversion. Current distribution factors, symbol i, obtained in the course of this reduction are used later in the current distribution calculation. The steps of reduction are illustrated in Fig. 34. The steps in the subsequent current distribution calculation are illustrated in Fig. 35, the network being expanded in reverse order, to its original form. Fig. 35(c) shows the current distribution for a one ampere load at B. The mutual drops are then obtained by calculating the voltage drop from generator to each load point, using the impedance diagram Fig. 33 and current diagram Fig. 35(c).

This procedure is repeated in turn for unit current at each of the load points, and the results tabulated as in Tables 6 and 7. The symbols Z_{AA}, Z_{AB} have been included in Table 7 to identify the drops, but this in general is not necessary.

Typical Calculation of Self and Mutual Drops and Current Division. Unit Load at B

Combine a and c in Series.

a	$= 0.47$	$+j\ 1.00$
c	$= 4.05$	$+j\ 5.90$
$a+c$	$= 4.52$	$+j6.90$

Parallel b with $a+c$.

$a+c$	$= 4.52$	$+j6.90$
b	$= 7.78$	$+j9.04$
$D=\text{sum}$	$=12.30$	$+j15.94$
$1/D$	$= 0.03034$	$-j0.03932$
$a+c$	$= 4.52$	$+j6.90$
$i_b=(a+c)/D$	$= 0.4075$	$+j0.0321$
b	$= 7.78$	$+j9.04$
$h=$ parallel b with $(a+c)$	$= 2.8802$	$+j3.9335$

Convert Delta ABC to Star*

$Z_{A\text{-}B}$**	$= 2.60$	$+j3.59$
$Z_{B\text{-}C}$	$= 2.60$	$+j3.59$
$Z_{C\text{-}A}$	$= 1.44$	$+j3.02$
$D=\text{sum}$	$= 6.64$	$+j10.20$
$1/D$	$= 0.04483$	$-j0.06886$
$Z_{A\text{-}B}$	$= 2.60$	$+j3.59$
$i_{AB}=Z_{A\text{-}B}/D$	$= 0.3638$	$-j0.0181$
$Z_{C\text{-}A}$	$= 1.44$	$+j3.02$
$Z_A=Z_{C\text{-}A}i_{AB}$	$= 0.5785$	$+j1.0724$
$1/D$	$= 0.04483$	$-j0.06886$
$Z_{B\text{-}C}$	$= 2.60$	$+j3.59$
$i_{BC}=Z_{B-C}/D$	$= 0.3638$	$-j0.0181$
$Z_{A\text{-}B}$	$= 2.60$	$+j3.59$
$Z_B=Z_{A\text{-}B}i_{BC}$	$= 1.0109$	$+j1.2590$
$1/D$	$= 0.04483$	$-j0.06886$
$Z_{C\text{-}A}$	$= 1.44$	$+j3.02$
$i_{CA}=Z_{C\text{-}A}/D$	$= 0.2725$	$+j0.0362$
$Z_{B\text{-}C}$	$= 2.60$	$+j3.59$
$Z_C=Z_{B\text{-}C}i_{CA}$	$= 0.5785$	$+j1.0724$

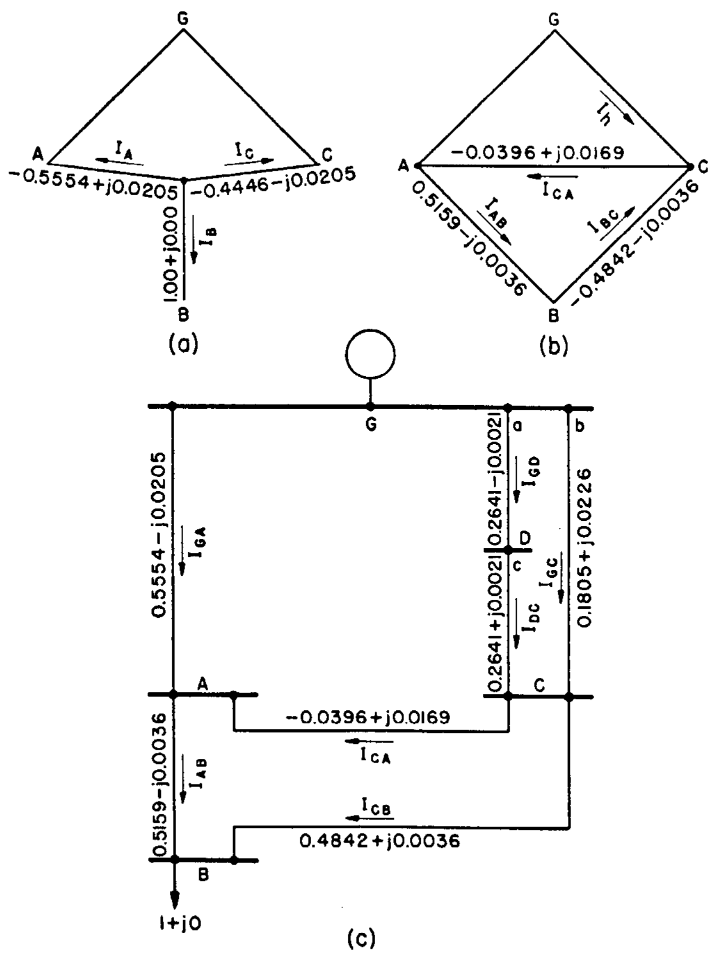


Fig. 35—Current distribution in the network of Fig. 33 for unit load at B.

*See equations (87)—(96).

**$Z_{A\text{-}B}$ is used here for impedance of branch from A to B to distinguish it from Z_{AB}, the drop to B for unit load at A.

Refer to Fig. 34 and Combine d and k in Series.

$$\begin{aligned} d &= 1.85 \quad +j3.15 \\ k &= 0.579 \quad +j1.072 \\ n=d+k &= 2.429 \quad +j4.222 \end{aligned}$$

Series h and l.

$$\begin{aligned} h &= 2.88 \quad +j3.93 \\ l &= 0.579 \quad +j1.072 \\ p=h+l &= 3.459 \quad +j5.002 \end{aligned}$$

Parallel n and p.

$$\begin{aligned} n &= 2.429 \quad +j4.222 \\ p &= 3.459 \quad +j5.002 \\ D=\text{sum} &= 5.888 \quad +j9.224 \\ 1/D &= 0.04917-j0.07703 \\ p &= 3.459 \quad +j5.002 \\ i_n=p/D &= 0.5554 \quad -j0.0205 \\ n &= 2.429 \quad +j4.222 \\ r=pn/D &= 1.4356 \quad +j2.2951 \\ i_p=1-i_n &= 0.4446 \quad +j0.0205 \end{aligned}$$

Series r and m.

$$\begin{aligned} r &= 1.4356 \quad +j2.2951 \\ m &= 1.011 \quad +j1.259 \\ Z_{BB} &= 2.4466 \quad +j3.5541 \end{aligned}$$

Current Division*. Refer to Figs. 35 and 34.

$$\begin{aligned} I_A=-i_n &= -0.5554+j0.0205 \\ I_B &= 1.0000+j0.0000 \\ I_C=-i_p &= -0.4446-j0.0205 \\ -i_{CA} &= -0.2725-j0.0362 \\ I_A &= -0.5554+j0.0205 \\ -i_{CA}I_A &= 0.1521+j0.0145 \end{aligned}$$

$$\begin{aligned} i_{BC} &= 0.3638-j0.0181 \\ I_B &= 1.0000+j0.0000 \\ i_{BC}I_B &= 0.3638-j0.0181 \\ I_{AB} &= 0.5159-j0.0036 \\ &= -i_{CA}I_A+i_{BC}I_B \end{aligned}$$

$$\begin{aligned} -i_{AB} &= -0.3638+j0.0181 \\ I_B &= 1.0000+j0.0000 \\ -i_{AB}I_B &= -0.3638+j0.0181 \end{aligned}$$

$$\begin{aligned} i_{CA} &= 0.2725+j0.0362 \\ I_C &= -0.4446-j0.0205 \\ i_{CA}I_C &= -0.1204-j0.0217 \\ I_{BC} &= -0.4842-j0.0036 \\ &= -i_{AB}I_B+i_{CA}I_C \end{aligned}$$

$$\begin{aligned} -i_{BC} &= -0.3638+j0.0181 \\ I_C &= -0.4446-j0.0205 \\ -i_{BC}I_C &= 0.1621-j0.0006 \end{aligned}$$

$$\begin{aligned} i_{AB} &= 0.3638-j0.0181 \\ I_A &= -0.5554+j0.0205 \\ i_{AB}I_A &= -0.2017+j0.0175 \\ I_{CA} &= -0.0396+j0.0169 \\ &= -i_{BC}I_C+i_{AB}I_A \end{aligned}$$

*See equations (88b), (89b), (90b).

$$\begin{aligned} I_h=-I_C &= 0.4446+j0.0205 \\ i_b &= 0.4075+j0.0321 \\ I_{GC}=i_bI_h &= 0.1805+j0.0226 \\ I_{GD}=I_h-I_{GC} &= 0.2641-j0.0021 \end{aligned}$$

Mutual Drops.

$$\begin{aligned} I_{GA} &= 0.5554-j0.0205 \\ d &= 1.85 \quad +j3.15 \\ Z_{BA} &= 1.0921+j1.7116 \end{aligned}$$

$$\begin{aligned} I_{GC} &= 0.1805+j0.0226 \\ b &= 7.78 \quad +j9.04 \\ Z_{BC} &= 1.2000+j1.8075 \end{aligned}$$

$$\begin{aligned} I_{GD} &= 0.2641-j0.0021 \\ a &= 0.47 \quad +j1.00 \\ Z_{BD} &= 0.1262+j0.2631 \end{aligned}$$

The current division and self- and mutual-drop factors from these calculations are tabulated in Tables 6 and 7, together with similar factors for unit loads at A, C, and D.

Regulation—Tables 6 and 7 are considered to be the basic network solution. Their use in the regulation problem will now be outlined.

The chief problem is to express the several load currents in proper phase relationship to a single reference voltage, either the voltage at one of the most important load points or the generator-bus voltage.

If they are expressed with respect to the generator-bus voltage, a simple deduction of vector drops from this voltage gives the load point voltages so that the load power factors and kva can be checked and a correction made if necessary.

If there is considerable drop in the system but the important load voltages are nearly alike, it is preferable to use one of these load voltages as reference. The generator-voltage phase position can then be determined graphically or by the solution of a quadratic equation as outlined in the example of the circulating current method Sec. 19. A simplified modification of this method is given below. In either method the proof of the assumptions lies in the check of load kva and power factor and corrections can be applied if the assumed values prove to be far enough off to affect materially the regulation values and currents of interest.

The present problem is to calculate the regulation for the system of Fig. 33 under normal heavy load conditions, Case 1, Table 8, and also with several capacitor banks added as indicated in Case 2, Table 8.

A practical problem now arises that is not immediately evident from Eq. 168. The load power factors given fix the positions of the currents with respect to the final load voltages, (E_p in this equation) not with respect to the generator or "no load" voltage E'_p. Thus the phase relations between the generator voltage and the drops cannot be determined directly. A further difficulty exists in converting the loads from kva to amperes since the load voltages are not yet known.

A straightforward method of approach would be as follows. First assume that all load voltages are equal to, and in phase with, the generator voltage. Convert load kva's to vector currents with this voltage as reference.

TABLE 8—REGULATION WITHOUT CAPACITORS (CASE 1) AND WITH CAPACITORS (CASE 2)

Case	Load		Assumed Regulation	Amperes *	Resulting Drops to					
					B		C		D	
	At	kva—p.f.			Volts	%Reg.	Volts	%Reg.	Volts	%Reg.
1	B	$2500-90\%$ Lag	11.0%	$130.1-j63.1$	$543.5+j309.1$		$271.4+j161.0$		$32.9+j26.2$	
	C	$4500-85\%$ Lag	13.0%	$226.6-j140.3$	$526.1+j240.7$		$656.3+j304.9$		$81.2+j52.9$	
	D	$11\ 200-85\%$ Lag	9.0%	$539.3-j333.7$	$154.9+j102.6$		$190.9+j123.8$		$539.7+j341.0$	
	All	Total voltage drop Calculated load			$1224.5+j652.4$ 2265kva -90.4% Lag	19.4	$1118.6+j589.7$ 4254kva -84.9% Lag	17.7	$653.8+j420.1$ 11 011kva -83.4% Lag	10.5
2	B	$2500-90\%$ plus $2500-0\%$† $=2655-84.7\%$ Lead	5%	$122.1+j76.6$	$26.4+j622.0$		$8.0+j313.3$		$-4.7+j41.6$	
	C	$4500-85\%$ plus $2500-0\%$† $=3827-99.9\%$ Lead	5%	$207.5+j7.0$	$236.7+j384.0$		$293.6+j481.3$		$29.9+j69.3$	
	D	$11\ 200-85\%$ plus $2500-0\%$† $=10\ 110=94.2\%$ Lag	2.5%	$503.4-j179.9$	$110.7+j109.7$		$137.4+j137.6$		$384.7+j376.3$	
	All	Total voltage drop Calculated load			$373.8+j1115.7$ 2609kva -82.8% Lead	6.7	$439.0+j932.2$ 3715kva -99.9% Lead	7.8	$409.9+j487.2$ 9635kva -91.6% Lag	7.1

For Case 1 use $E_g=6438.6+j600$; For Case 2 $E_g=6403.6+j900$.
†Capacitors (6466.5 volts L-N)
*Based on load voltages—all in phase—taken as reference, but below the generating bus voltage of 11.2 kv by the "assumed regulation" values.

Then calculate and deduct the drops to determine the load voltages corresponding to this first approximation of the currents. The currents used will not have quite the right phase positions or magnitudes, when associated with these load voltages, to agree with the loads and power factors specified.

However, with these load voltages a new set of load currents can be calculated, the drops recalculated and a second approximation to the load voltages determined. This process is highly convergent and the second approximation would ordinarily be sufficient. In fact by making two judicious guesses, one an estimate of regulation to each load point and the second an estimate of phase shift from generator to load, the first approximation is nearly always sufficient and but a single calculation is required. This is the procedure followed in the subsequent paragraphs.

The assumed regulation to the load points is a straightforward estimate from experience or from the quick estimating tables of Chap. 9. However, the treatment of the phase-angle estimate bears some further explanation. First the load voltages are assumed to be in phase. Making use of the regulation estimates the vector load currents can be calculated with this common load voltage phase as a reference. The vector drops can be calculated and consist of in-phase and out-of-phase drop components. The generator voltage is now selected leading the reference by the average reactive drop to the loads. The example will make this clear. The magnitude of generator voltage is a given quantity. It is apparent that when the drops to various load points are deducted from this generator voltage, the load voltages obtained are close to the reference phase and hence the load power factors are close to those for which a solution is desired. Thus the resulting generator voltage, load currents, and drops are now sufficiently accurate to complete the regulation calculation. An exact answer is obtained for a set of loads differing slightly from those assumed. The example will make this clear.

As the loads are given in kva and power factor, it is necessary to estimate the load voltages to convert the loads to currents. The load voltages are all assumed to be in-phase, as a first approximation, and below the normal voltage of 11.2 kv by the "assumed regulation" values listed in Table 8. Load currents are calculated on this basis using load voltage as the reference axis. For example the load current at C is for Case 1:

$$I=\frac{4500(0.85-j0.5268)}{\sqrt{3}\times 11.2\times(1-0.13)}=266.6(0.85-j0.5268)$$
$$=226.6-j140.3 \text{ amperes.}$$

Voltage drops are computed according to Eq. (167), the component and total drops being as shown in the table.

A rough check of the drops at the critical locations, B

and C, indicates that for normal load conditions, Case 1, the approximate in-phase drops are 1224 and 1118 volts, or approximately 19.0 and 17.2 percent of normal line-to-neutral voltage. The "assumed regulations" could be corrected at this point but as this repetition would not add to the exposition, it is omitted.

Up to this point a reference axis in phase with the load voltages has been used, the load voltages being taken all in phase. This was most convenient for converting loads to currents as the power factors were known with respect to the load voltages. Now it is necessary to determine the generator voltage with respect to this reference so that the calculated drops may be deducted from it to find the actual load voltages. The phase position of the generator voltage does not need to be determined exactly. However after the load voltages are computed, the load power will be computed and the regulation will be exact for the loads thus computed rather than for the actual given loads. Such a result is usually an adequate engineering answer as the "given loads" are seldom accurately known. However it is desirable to start with a generator voltage as near as possible to that corresponding to the assumed load voltage so that the computed loads will be close to the given loads. This is accomplished as follows.

Noting that the out-of-phase drop is approximately 600 volts for B and C, the generator bus voltage is arbitrarily taken 600 volts ahead of the load voltage or reference. The drops, as deducted from this voltage, give load voltages quite closely in phase with those used and hence the load power factors are nearly correct. As the generator voltage magnitude is 6466.5 volts, line-to-neutral, the in-phase component must be $\sqrt{(6466.5)^2-(600)^2}=6438$. Whence the generator bus voltage is $6438+j\,600$.

The load voltages should now be calculated and the loads checked to see that they do not differ too far from the assumptions. A typical check follows, for Case 1, load at B.

$$\begin{aligned}
E_g &= 6438 + j\,600 \\
D_B &= \underline{1224 + j\,652} \\
E_B &= 5214 - j\;52 \\
E_B &= (5.214 - j0.052)\text{kv} \\
3\hat{I}_B &= \underline{390.3 + j189.3} \\
P_B + jQ_B &= (2050 + j\,970)\text{kva} \\
&= 2265 \text{ kva}
\end{aligned}$$

at 90.4 percent power factor lagging.

Case 2 of Table 8 illustrates the great improvement in regulation possible by the use of shunt capacitors. They may have to be partially switched off at light load to prevent overvoltages under that condition. Comparing the reduction in drops caused by capacitors at D, with the reductions caused by capacitors at B and C, it can be seen that the capacitors are much more effective at the latter two points which are farther from the generating station.

Single-and Multiple-Source System Having Shunt Branches Other Than Loads—Figs. 36(a) and (b) give a simple illustration of a system having shunt branches other than loads, namely charging capacity of high-voltage lines or cables. In this case the no-load voltages E_a' and E_b' of Fig. 36(c) differ from point-to-point in the system and also differ from the generator bus voltage E_g. If there are several sources, a similar condition exists. However, in either case the no-load voltages can be determined by measurement on a network calculator or by calculation and these form the base from which drops are deducted to determine voltages under load conditions by Eqs. (169) to (171). If the generator emfs vary in phase or magnitude for different parts of the study, the no-load voltages must be changed accordingly.

Fig. 36(f) shows the arrangement of the network for calculation or measurement of Z_{aa} and Z_{ab}. Sufficient voltage is applied between a and the bus of no-voltage to draw one ampere, all generator emfs being short circuited. The voltage required, using the reference direction shown in Fig. 36(f), is $-Z_{aa}$. It is thus necessary to amend the definitions of Z_{aa} and Z_{ab} given previously to the following:

Z_{aa} is the vector voltage drop from g to a caused by unit current drawn from the network at a, *with all generator emfs set equal to zero.*

Or it is the incremental vector drop in voltage at a per ampere drawn from a, with all generator emfs fixed in magnitude and position and all other load currents held constant.

Z_{ab} is the vector voltage drop from g to b caused by unit current drawn from the network at a, *with all generator emfs set equal to zero.*

The voltages and currents in Fig. 36(f) and (g) are labeled in accordance with these definitions. Increasing in ratio of actual load currents, parts (d) and (e) are obtained. Part (c) is the no-load condition. The superposition of (c), (d), and (e) results in currents identical with part (b). Consequently, the voltages E_a and E_b in

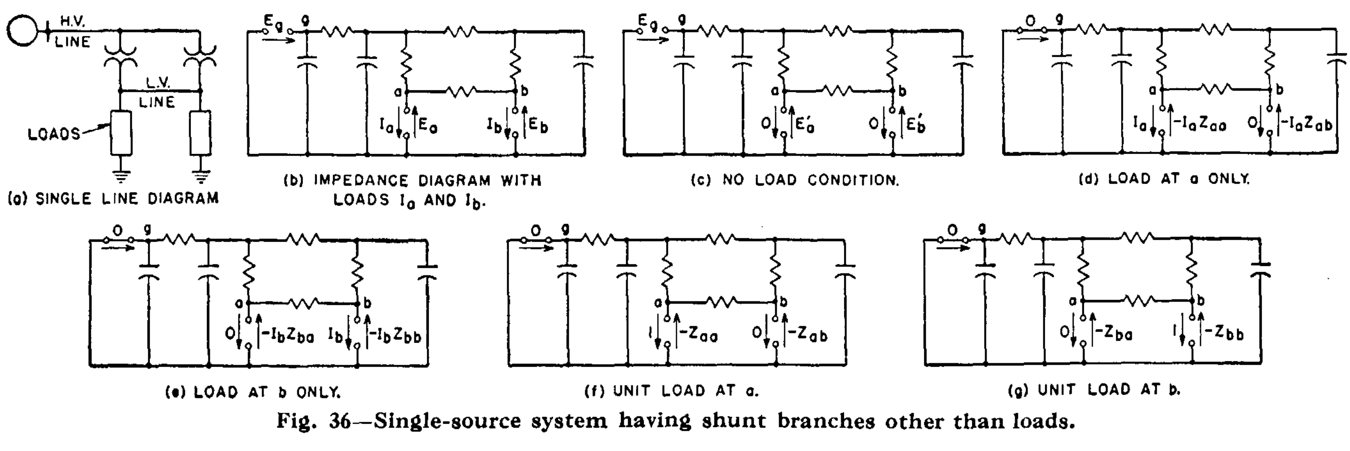


Fig. 36—Single-source system having shunt branches other than loads.

part (b) must be the superposition of the corresponding quantities in (c), (d), and (e), as stated in Eq. (168) generally. Specifically

$$E_a = E_a' - I_a Z_{aa} - I_b Z_{ba} \tag{173}$$

$$E_b = E_b' - I_a Z_{ab} - I_b Z_{bb} \tag{174}$$

It is apparent that the case of no shunt branches is simply a special case of the situation with shunt branches. Also the case of one emf is a special case of that with several. However, without shunt branches it is customary to apply enough voltage at the generator to cause one ampere in a short circuit at the load point and determine the drops through the network from generator to load points to obtain the constants Z_{aa}, Z_{ab}, etc. With shunt branches present this is no longer a series circuit from generator to load point. In this case the voltage must be applied at the load point and the generator emfs short circuited, or else an indirect method employed as described below.

With several emfs and shunt branches the network constants can be obtained by short circuiting one load terminal at a time, after first having measured the no-load voltages E_a', E_b', $\cdots$, E_n'. Referring to Eqs. (173) and (174) this gives the condition:

$$E_a = 0 \tag{175}$$

$$I_b = 0 \tag{176}$$

$$Z_{aa} = \frac{E_a'}{I_a} \tag{177}$$

$$Z_{ab} = \frac{E_b' - E_b}{I_a} \tag{178}$$

Similarly by short circuiting b, the other constants are obtained.

$$Z_{ba} = \frac{E_a' - E_a}{I_b} \tag{179}$$

$$Z_{bb} = \frac{E_b'}{I_b} \tag{180}$$

Both measurements, that is, the no-load voltages and also the voltages and currents with one terminal short circuited, must be made with the same generator emfs. However, it is immaterial what emfs are used so that they may be taken all in phase and equal for the purpose of obtaining the system constants. This results in a different set of no-load voltages for computing system constants than the actual no-load voltages used in the system studies but simplifies calculation in some cases.

Summarizing, the general solution of a multiple source system with shunt branches consists of:

a. Self and Mutual Drops.
b. Current Division.

and for each set of emfs to be used in the study

c. No-load voltages.

A suggested procedure for calculating these data is as follows. If a network calculator is used, the labor of reductions is eliminated.

a. Apply voltage at one load point with generator emf short circuited and other load points open circuited.
b. Reduce the resulting network to a single branch viewed from the selected load point. This branch is the self drop.
c. Expand the network developing the current division based on one ampere drawn out at the selected load point. This current division is part of the general solution.
d. Calculate the voltages of other load points above the bus-of-no-voltage, or neutral bus. These are negatives of the mutual drops.
e. Perform a, b, c, and d for other load points in turn.
f. With the load points all open circuited apply the generator emfs to be used in the study and determine the no-load voltages.
g. Load voltages and current distribution throughout the network may now be determined for any loading condition corresponding to the generator emfs from which the no-load voltages were developed. The voltage at any load point p is given by Eq. (168). The current in any branch, p-q is given by Eq. (166).

NOTE: Alternative methods are outlined in the following paragraphs.

More Than One Source—As Negative Load—If the generator emfs do not remain constant throughout a study, the network can be solved by treating all sources but one as load points. Determination of voltage and current conditions on the system for any loading conditions are then determined by using as the no-load voltages, those produced by the one selected source alone. These will be directly proportional to this one source voltage and hence can be varied for different conditions of the problem if that source voltage changes. A condition of the system is then completely specified by the selected source voltage and the currents drawn at all other sources and load points.

Changes in the Network—When a transformer is removed or a line opened, it is of course desirable to determine the effects without completely solving the new resulting network. Assume that the branch to be omitted or added connects between two of the cardinal points, a and c, of Fig. 37 for which network constants and current division factors are known. A solution is desired with the branch ac removed. By solution is meant the voltage at any cardinal point and the current in any branch corresponding to a particular load condition on the network. Thus the solution of the changed network for a given load condition is identical with the solution of the original network for the same load condition plus two additional loads. One of these added loads is drawn at each end of the branch to be removed from the original network. These added loads are equal and opposite to the current in the branch so that the total current drawn by the branch and added load is zero.

Suppose the load condition being solved for is I_b, I_c, I_d and the corresponding current in ac is I_{ac}. When loads I_a' and I_c' (equal to $-I_a'$) are added, they cause additional current in the branch ac:

$$\Delta I_{ac} = I_a' I_{a\cdot ac} - I_a' I_{c\cdot ac} \tag{181}$$

The total of branch and added load must equal zero.

$$I_{ac} + I_a'(I_{a\cdot ac} - I_{c\cdot ac}) + I_a' = 0 \tag{182}$$

whence:
$$I_a' = -\frac{I_{ac}}{1 + I_{a\cdot ac} - I_{c\cdot ac}} \tag{183}$$

and
$$I_c' = -I_a' \tag{184}$$

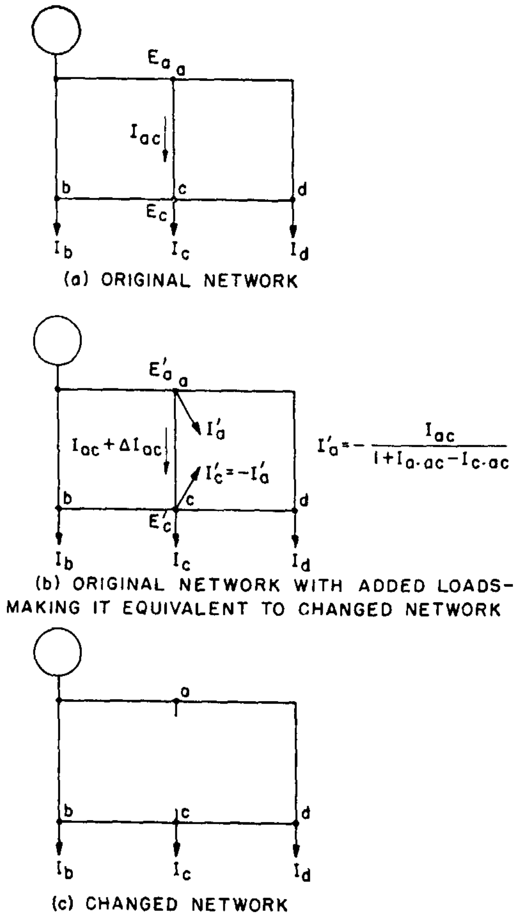


Fig. 37—Adding loads to a network to make it equivalent to the network with a branch removed.

Thus to solve the changed network for a given set of loads it is merely necessary to solve the original network instead, using the two added loads determined by Eqs. (183) and (184).

Adding a Branch Between a and c. Refer to Fig. 37 (a)—Suppose a branch is to be added between a and c having impedance Z. It can be simulated in the original network by loads equal to what the branch would carry if there. Referring to Fig. 37(b) a branch would carry:

$$I_a' = \frac{E_a' - E_c'}{Z} \tag{185}$$

where primes refer to the condition after the branch is added.

$$E_a' = E_a - I_a' Z_{aa} + I_a' Z_{ca} \tag{186}$$

$$E_c' = E_c - I_a' Z_{ac} + I_a' Z_{cc} \tag{187}$$

$$I_a' = \frac{E_a - E_c}{Z} - I_a' \left(\frac{Z_{aa} - 2Z_{ac} + Z_{cc}}{Z} \right) \tag{188}$$

$$I_a' = \frac{E_a - E_c}{Z_{aa} - 2Z_{ac} + Z_{cc} + Z} = \frac{D_c - D_a}{Z_{aa} - 2Z_{ac} + Z_{cc} + Z} \tag{189}$$

That is, the effect on the voltages and currents in Fig. 37(a), of adding a branch of impedance, Z, between a and c, while holding the generator emf and the load currents I_b, I_c, and I_d constant, is exactly the same as if the loads I_a' and $-I_a'$ were added at a and c respectively, instead of connecting the impedance Z.

A branch Z can be removed by adding an impedance branch, $-Z$, as alternative to the method previously given.

Example of Changing a Network—A partial solution of the network of Fig. 38 follows:

$Z_{aa} = 2.5$ ohms $\quad I_{a\cdot ac} = -0.25$ amperes
$Z_{ac} = 1.75$ ohms $\quad I_{a\cdot ba} = 0.75$ amperes
$Z_{cc} = 2.875$ ohms $\quad I_{c\cdot ac} = 0.375$ amperes
$\quad I_{c\cdot ba} = 0.375$ amperes

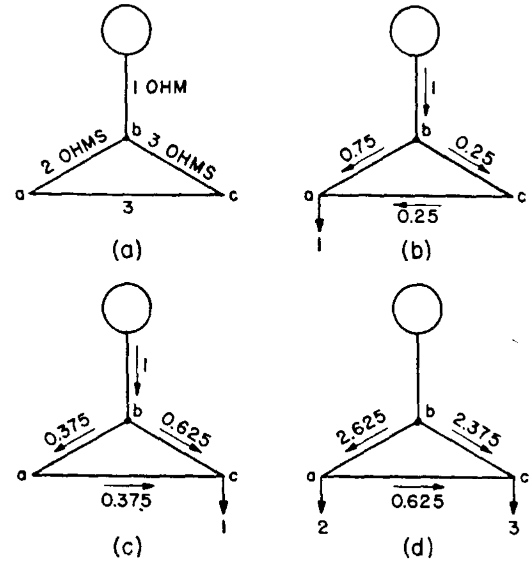


Fig. 38—A simple network showing loads and distribution factors.

From this solution the currents and voltage drops for the load condition, Fig. 38(d), are obtained.

$I_{ba} = 2\times0.75 + 3\times0.375 = 2.625$ amperes
$I_{ac} = 2\times(-0.25) + 3\times0.375 = 0.625$ amperes
$D_a = 2\times2.5 + 3\times1.75 = 10.25$ volts
$D_c = 2\times1.75 + 3\times2.875 = 12.125$ volts

Now consider the changed network Fig. 39(a) under the same load condition. Solving directly:

$I_{ba}' = 2$ amperes
$I_{ac}' = 0$ amperes
$D_a' = 2\times2 + 5\times1 = 9$ volts
$D_c' = 3\times3 + 5\times1 = 14$ volts

However, suppose it were desired to obtain these data from the solution of the network, Fig. 38. Then using Eq. (183):

$$I_a' = -\frac{I_{ac}}{1 + I_{a\cdot ac} - I_{c\cdot ac}}$$

$$= -\frac{0.625}{1 - 0.25 - 0.375} = -1.66 \text{ amperes}$$

$$I_c' = -I_a' = 1.66 \text{ amperes.}$$

Fig. 39(b) shows these loads added to the network loads of 2 and 3 amperes at a and c.

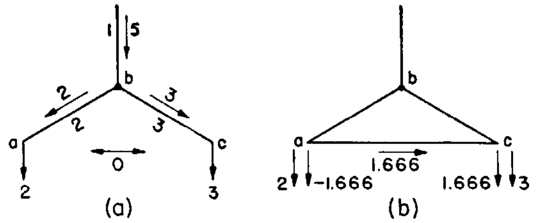


Fig. 39—Network of Fig. 38 changed by the removal of branch *a c* and its equivalent.

$$I'_{ac}=0.33\times(-0.25)+4.66\times0.375=1.66 \text{ amperes.}$$

Thus it is seen the added loads and the current in branch *ac* total zero, and can be eliminated.

That is: $I'_{ac}+I'_a=1.66-1.66=0$

Proceeding with the solution.

$$I'_{ba}=0.33\times0.75+4.66\times0.375=2.0 \text{ amperes}$$
$$D'_a=0.33\times2.5+4.66\times1.75=9.0 \text{ volts}$$
$$D'_c=0.33\times1.75+4.66\times2.875=14.0 \text{ volts.}$$

All of these agree with the direct solution. I_a' could also be obtained from the consideration of adding a -3 ohm branch from *a* to *c*. Eq. (189) gives

$$I'_a=\frac{D_c-D_a}{Z_{aa}-2Z_{ac}+Z_{cc}+Z}=\frac{12.125-10.25}{2.5-3.5+2.875-3}=-1.66 \text{ amperes.}$$

and the remaining solution is the same as above.

Intermediate Loads—It frequently happens that regulation and current division are required at loads connected along branches intermediate between two cardinal points, such as the load at x, a fractional distance, m, along impedance branch Z from a to b, Fig. 40.

To determine regulation at x proceed as follows:

a. Replace I_x by two loads $(1-m)\, I_x$ at a and mI_x at b, as shown in Fig. 40(b). From these and the other loads on the system, the voltages at a and b can be determined and a circulating current I_{ab} found.

b. Permit the currents $(1-m)\, I_x$ and mI_x to flow over the branch to point x and into the load. This will not alter the drop from a to b since the two added drops introduced into this branch are equal and opposite.

$$(1-m)I_x mZ=mI_x(1-m)Z \tag{190}$$

Nor will it alter the circulating current I_{ab} that causes the drop through Z and absorbs the voltage difference between a and b. The drop can now be calculated from either a or b to the load point, taking into account both circulating and load components of current.

$$D_{ax}=(1-m)I_x mZ+I_{ab}mZ \tag{191}$$

Or the voltage at x is:

$$E_x=(1-m)E_a+mE_b-I_x m(1-m)Z \tag{192}$$

c. The use of equivalent loads at a and b [Fig. 40(b)] results in the same currents in all other branches of the network as the actual condition [Fig. 40(a)]. The currents in branch ab are determined as shown in Fig. 40(c), in which I_{ab} is as determined from the equivalent loading, Fig. 40(b).

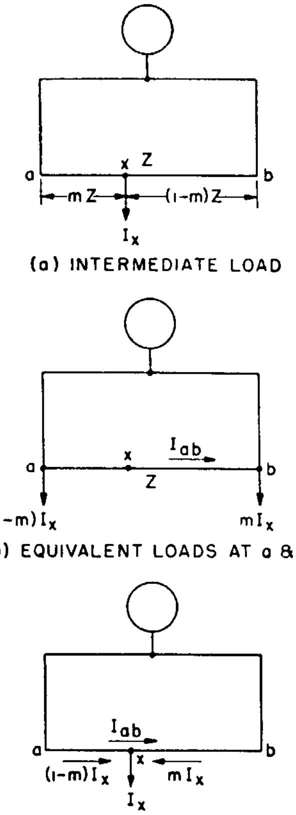


Fig. 40—Current division for a simple network with a single intermediate load.

General Solution for Intermediate Point—The intermediate load location is to be treated as a new cardinal point of the network for which self and mutual drop constants and current division factors are required.

The self drop constant is obtained by recalling that for unit load at x, Fig. 40, the drops to a and b are

$$D_a=(1-m)Z_{aa}+mZ_{ba} \tag{193}$$
$$D_b=(1-m)Z_{ab}+mZ_{bb} \tag{194}$$

whence:

$$Z_{xx}=(1-m)D_a+mD_b+m(1-m)Z \tag{195}$$

or

$$Z_{xx}=(1-m)^2Z_{aa}+2m(1-m)Z_{ab}+m^2Z_{bb}+m(1-m)Z \tag{196}$$

The mutual drop constant to a typical point, p, is

$$Z_{xp}=(1-m)Z_{ap}+mZ_{bp} \tag{197}$$

The current in any branch pq caused by unit current drawn at x is (except for branches ax and bx):

$$I_{x\cdot pq}=(1-m)I_{a\cdot pq}+mI_{b\cdot pq} \tag{198}$$

For branches ax and xb

$$I_{x\cdot ax}=(1-m)I_{a\cdot ab}+mI_{b\cdot ab}+(1-m) \tag{198a}$$
$$I_{x\cdot bx}=(1-m)I_{a\cdot ab}+mI_{b\cdot ab}-m \tag{198b}$$

While for point y, external to branch ab,

$$I_{y\cdot ax} = I_{y\cdot xb} = I_{y\cdot ab} \tag{198c}$$

Several Intermediate Loads—If the branch ab consists of several parallel mutually coupled circuits such as the trolley rail circuits of a four-track railroad, and contains several intermediate loads, the procedure is quite similar to the above. Refer to Fig. 41.

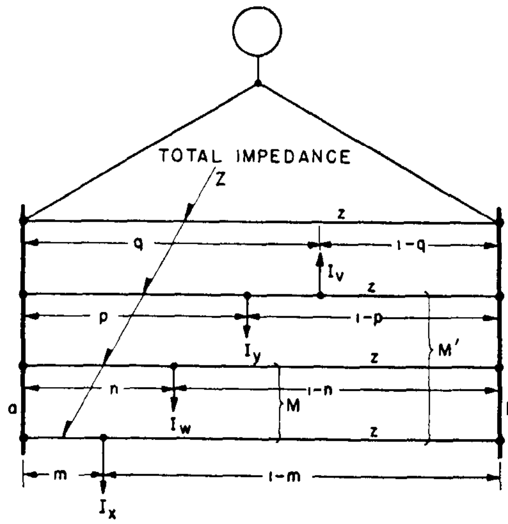


Fig. 41—More complex network having several intermediate load points.

Let Z be the total impedance a to b.
Let z be the impedance of each component circuit.
Let M, M', M'' be the mutual impedances between component circuits.

It is assumed that these impedances are uniform throughout the section.

The procedure is as follows for determining voltage at the load I_x, and current distribution.

a. Divide each load inversely as the impedance to the two adjacent points a and b, to obtain total equivalent loads. From these and the other network loads the voltages E_a and E_b can be determined. The total circulating current I_{ab} can also be found.

b. Determine the voltage at x while the loads are removed to a and b. It is:

$$E_x = (1-m)E_a + mE_b \tag{199}$$

c Now reintroduce the equivalent load currents letting them flow over the circuits to their respective loads. In the case shown there are four added drops, resulting in a voltage at the load I_x:

$$E_x = (1-m)E_a + mE_b - (1-m)I_x mz - (1-n)I_w mM - (1-p)I_y mM' - (1-q)I_v mM' \tag{200}$$

d. The circulating current I_{ab} should be divided between the four circuits as though the loads were not present. If the mutual impedances are nearly equal it may be sufficiently close to assume ¼ of the total in each circuit. Otherwise a solution by equations may be required. See Sec. 13.

e. The current in any section of one of the circuits of branch ab consists of the vector sum of the circulating component as determined in (d), and the reintroduced equivalent load currents flowing up to the intermediate loads.

21. Circle Diagram of Transmission Systems

Because of its importance to both the light and power and the communication industries, the transmission type network has been widely studied. A useful body of data is available for simplifying the calculations and expressing the performance of such networks. The fundamental ideas involved are extremely simple, and the reader should not be misled by the large accumulation of formulas tabulated for special cases. These merely signify that the field has been well explored, whereas only one or two of the formulas may be required in any particular problem.

The general transmission-type network including shunt loading, is one having only input and output terminals of importance, designated for convenience as the sending and receiving ends. The type dealt with in this chapter is considered to be passive (having no internal emfs), and linear (made up of linear impedance branches and voltage transformations).

For such a network the sending-end voltage and current depend solely on the receiving-end voltage and current, and the impedances and voltage transformations of the intervening network.

The transmission problem is briefly the determination of the performance of the transmission-type network. This performance is most commonly expressed in two forms.

a. Equations expressing the sending-end voltages and currents in terms of the receiving-end voltages and currents, and vice versa.

b. The power equations or loci, the graphical representations of which are known as the power circle diagrams. One circle gives the locus of sending-end power and one the locus of receiving-end power, as the angle between sending and receiving voltages is varied.

A third form is sometimes used.

c. The current equations expressing the sending-end or receiving-end currents in terms of the voltages at the two ends. The current locus for fixed voltages and varying angle between them is the current circle diagram.

Power Circle Diagram—The power-circle diagram is derived mathematically in Chap. 9. The treatment in this chapter applies the diagram to general system problems. Condensed tables are presented for determining the circle diagrams from general circuit constants. First, however, a brief review of the power-circle diagram will serve to point up the important power system design and operating information which it provides.

The fact that real and reactive power fed into and out of a transmission line can be plotted as a function of the sending- and receiving-end voltages only is itself an extremely important concept. Stated differently, once the voltage magnitudes at the two ends of the line have been fixed, there exists for each angle between these voltages, one and only one possible value for each of the four quan-

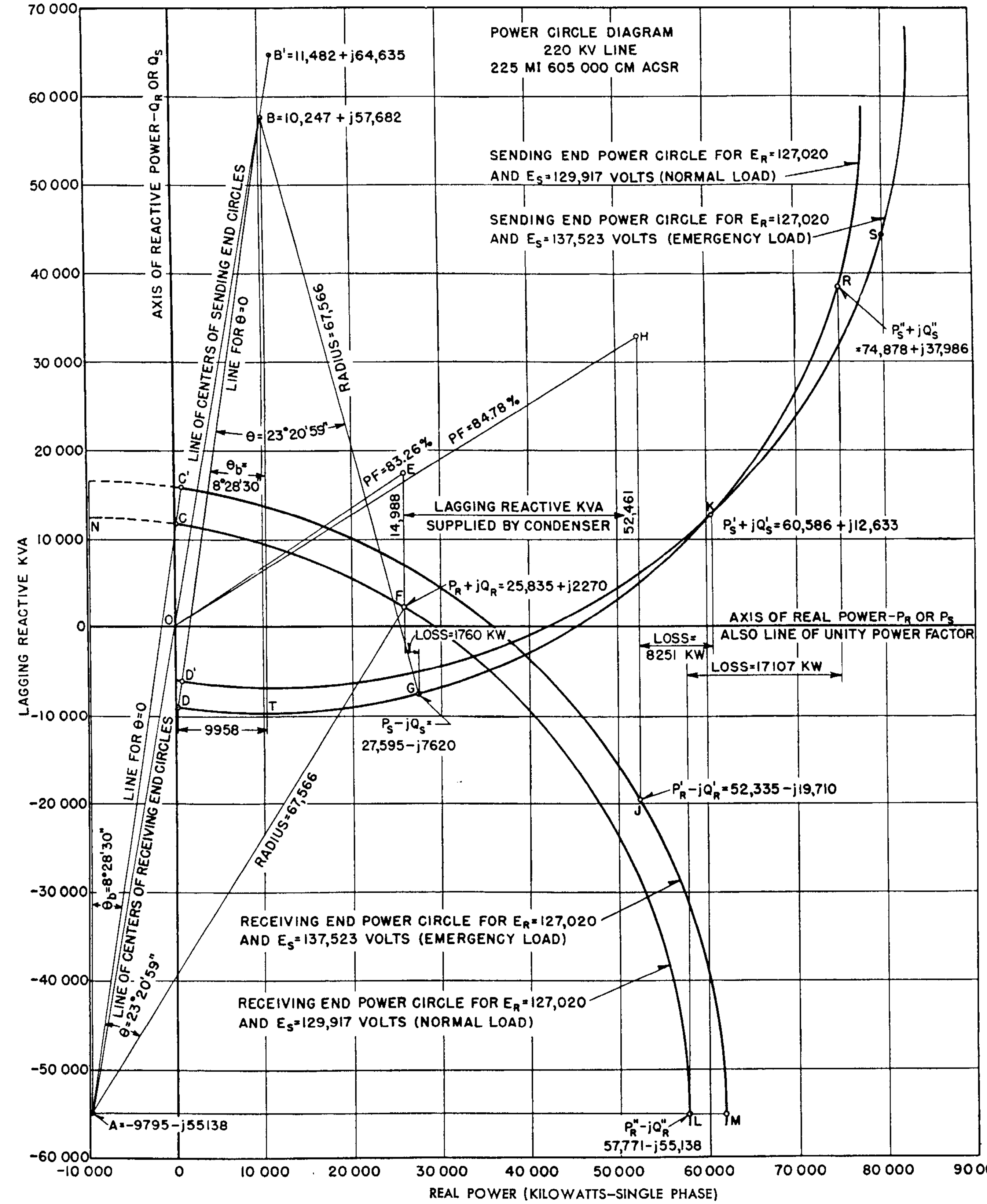


Fig. 46—Typical power-circle diagram.

tities, input real and reactive power, and output real and reactive power. Or, if one of these is fixed, say delivered real power, this determines the angle and thus the other three power quantities are uniquely determined.

If the voltages of the two ends of the line are fixed in magnitude and the angle between them varied, then for each angle there will be a discreet value of input real and reactive power. If these are plotted, one against the other, on a set of coordinate axes having real power as abscissa and reactive power as ordinate, the locus of such points as the angle between voltages is varied is a circle. Thus this plot of real power vs. reactive power for fixed line voltages and varying angle, is called a power circle diagram. What has been said of input real and reactive power applies equally well to output real and reactive power. Hence, for a given pair of terminal voltages there are two circle diagrams, a sending-end circle and a receiving-end circle. For other voltages there are other circles. The fact that these diagrams are circles makes them easy to draw. However the important point is that the input and output real and reactive powers are uniquely determined by the terminal voltages and the angle between them. In a sense this places definite restrictions on the use of lines. Or from another viewpoint it makes it possible to predetermine the amount of synchronous-condenser capacity that is required to supply a given load over a given line.

These points may be made more clear by reference to Fig. 46 which shows the sending-end and receiving-end power-circle diagrams for two values of sending-end voltage and one value of receiving-end voltage, i.e. for two combinations. Thus there are four circles. The method of plotting these circles and the derivation has been given in Chap. 9, and will be summarized shortly for the general case. That need not concern us here. Suffice it to say that there are such plots. What do they show?

The coordinates of the plot are real power as abscissa, positive to the right and lagging-reactive power as ordinate, positive upwards. The two sending end circles, have their centers at B and B' in the first quadrant. The positive reference direction at the sending end is into the line. Thus positive real or reactive power flow into the line at this end.

The two lower circles having centers at A, in the third quadrant, are the receiving-end circles. At the receiving end the positive reference direction is out of the line. Thus positive real and reactive power from the receiving circles indicate real or reactive power out of the line and a negative sign of reactive power indicates that lagging reactive power flowed into the line at the receiving end.

Note first there is a maximum power that can be delivered, for example 57 771 kw for one set of voltages, $E_R = 127\,020$ and $E_S = 129\,917$ volts L-N. This is of course an absolute limit and well beyond a practical operating limit.

It has been stated that with fixed voltages there exists for each angle between them, one and only one possible value of each of the four power quantities. This is shown on the diagram, for example, for an angle of 23°20′59″, and for the voltages $E_R = 127\,020$, $E_S = 129\,917$ volts L-N. Note that the angles are measured out from reference lines, marked Line for $\theta = 0$, whose construction will be described later. The angle θ, by which the sending-end voltage leads the receiving end voltage is measured out ccw for the upper or sending circles and cw for the lower or receiving-end circles. Thus this specific angle fixes the points F and G on receiving and sending circles respectively. These are referred to as corresponding points, since they correspond to the same angle and hence give sending- and receiving-end conditions that occur simultaneously.

For this angle and these voltages note that 27 595 kw enters the line and 25 835 kw leaves it at the receiving end, the loss being 1760 kw. At the sending-end lagging reactive kva is negative and hence flows opposite to the reference positive direction. That is, lagging reactive kva flows out of the line, 7620 kva. This must be absorbed by the system at the sending end of the line, in inductive loads or by under-excited machines. At the receiving end lagging reactive kva is positive and hence flows in the reference direction for that end which is out of the line, 2270 kva. In general this may be more or less than the lagging reactive requirements of the load and the difference must be absorbed or supplied locally. For example, if the load were 25 835 kw at 83.26 percent power factor lag as plotted at E, requiring 17 258 lagging reactive kva, the difference of 14 988 kva would have to be supplied by a synchronous condenser operating in its over-excited range, or an equivalent.

If the load is increased to an emergency load of 52 335 kw at 84.78 percent power factor lagging, the corresponding points on the circles are at J and K. It is assumed that the sending-end voltage has been raised to 137 523 volts L-N for this condition. The condenser must now supply 52 461 kva of lagging reactive, as the line supplies a negative amount or actually draws lagging reactive. Note that to supply this load with the lower sending voltage would have required considerably more than the 52 461 kva from the condenser.

Other circles could be drawn for different receiver voltages and these would show the variation of synchronous-condenser capacity requirements within the limits of permissible variation of receiver-end voltage.

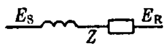
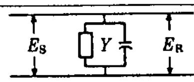
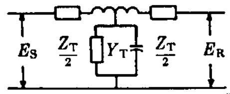
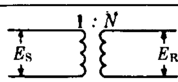
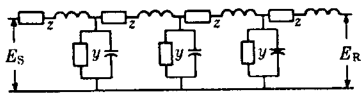
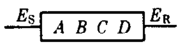
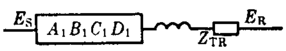
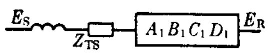
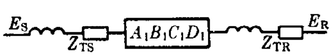
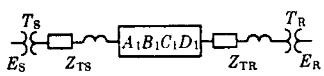
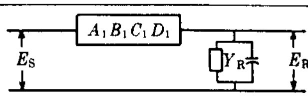
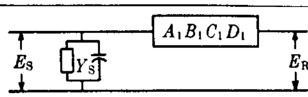
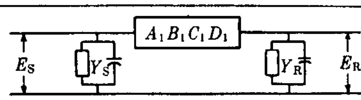
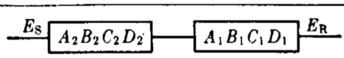
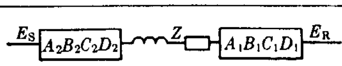
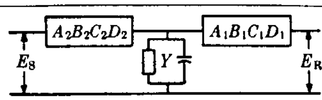
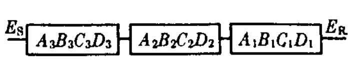
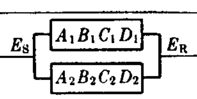
Thus the circle diagram presents a complete graphical picture of the line performance under all conditions of terminal voltages and angles and hence provides the necessary information for design and operation of the system, particularly with relation to voltages, provision of reactive capacity, and real power flow.

Transmission Equations: Constructing the Circle Diagram

Generally the following steps are involved in determining the transmission characteristics of a system from one point to another.

a. The network must be reduced to a simple equivalent from which the constants for plotting the circle diagrams can be obtained. The simple equivalent can be expressed as a T or a Pi circuit or by giving the coefficients of the current and voltage equations, called the $ABCD$ constants. Table 9 gives the necessary formulas for determining the $ABCD$ constants directly from networks of various forms. The T and Pi equivalents can be obtained by reducing the network as outlined in Secs. 13–17, or as indicated by the definitions of these constants which are to follow. Table 10 gives the transformations from Pi to T and to $ABCD$ forms. This table also includes transformations to admittance and impedance constants that are coefficients of the power equations as shown in the table.
b. The Current and Voltage Relations if needed can be written directly from the T, Pi or $ABCD$ constants

TABLE 9—GENERAL CIRCUIT CONSTANTS FOR DIFFERENT TYPES OF NETWORKS[5]

Network number	Type of Network		Equations for general circuit constants in terms of constants of component networks: $A =$	$B =$	$C =$	$D =$
1	Series impedance		1	Z	0	1
2	Shunt admittance		1	0	Y	1
3	Transformer		$1+\frac{Z_TY_T}{2}$	$Z_T\left(1+\frac{Z_TY_T}{4}\right)$	Y_T	$1+\frac{Z_TY_T}{2}$
3a	Transformer Ratio		$\frac{1}{N}$	0	0	N
4	Transmission line		Cosh $\sqrt{ZY}$ $=\left(1+\frac{ZY}{2}+\frac{Z^2Y^2}{24}+\cdots\right)$	$\sqrt{Z/Y}$ Sinh $\sqrt{ZY}$ $=Z\left(1+\frac{ZY}{6}+\frac{Z^2Y^2}{120}+\cdots\right)$	$\sqrt{Y/Z}$ Sinh $\sqrt{ZY}$ $=Y\left(1+\frac{ZY}{6}+\frac{Z^2Y^2}{120}+\cdots\right)$	Same as A
5	General network		A	B	C	D
6	General network and transformer impedance at receiving end		A_1	$B_1+A_1Z_{TR}$	C_1	$D_1+C_1Z_{TR}$
7	General network and transformer impedance at sending end		$A_1+C_1Z_{TS}$	$B_1+D_1Z_{TS}$	C_1	D_1
8	General network and transformer impedance at both ends—referred to high voltage		$A_1+C_1Z_{TS}$	$B_1+A_1Z_{TR}+D_1Z_{TS}+C_1Z_{TR}Z_{TS}$	C_1	$D_1+C_1Z_{TR}$
9	General network and transformer impedance at both ends—transformers having different ratios T_R and T_S referred to low voltage		$\frac{T_R}{T_S}(A_1+C_1Z_{TS})$	$\frac{1}{T_RT_S}(B_1+A_1Z_{TR}+D_1Z_{TS}+C_1Z_{TR}Z_{TS})$	$C_1T_RT_S$	$\frac{T_S}{T_R}(D_1+C_1Z_{TR})$
10	General network and shunt admittance at receiving end		$A_1+B_1Y_R$	B_1	$C_1+D_1Y_R$	D_1
11	General network and shunt admittance at sending end		A_1	B_1	$C_1+A_1Y_S$	$D_1+B_1Y_S$
12	General network and shunt admittance at both ends		$A_1+B_1Y_R$	B_1	$C_1+A_1Y_S+D_1Y_R+B_1Y_RY_S$	$D_1+B_1Y_S$
13	Two general networks in series		$A_1A_2+C_1B_2$	$B_1A_2+D_1B_2$	$A_1C_2+C_1D_2$	$B_1C_2+D_1D_2$
14	Two general networks in series with intermediate impedance		$A_1A_2+C_1B_2+C_1A_2Z$	$B_1A_2+D_1B_2+D_1A_2Z$	$A_1C_2+C_1D_2+C_1C_2Z$	$B_1C_2+D_1D_2+D_1C_2Z$
15	Two general networks in series with intermediate shunt admittance		$A_1A_2+C_1B_2+A_1B_2Y$	$B_1A_2+D_1B_2+B_1B_2Y$	$A_1C_2+C_1D_2+A_1D_2Y$	$B_1C_2+D_1D_2+B_1D_2Y$
16	Three general networks in series		$A_3(A_1A_2+C_1B_2)+B_3(A_1C_2+C_1D_2)$	$A_3(B_1A_2+D_1B_2)+B_3(B_1C_2+D_1D_2)$	$C_3(A_1A_2+C_1B_2)+D_3(A_1C_2+C_1D_2)$	$C_3(B_1A_2+D_1B_2)+D_3(B_1C_2+D_1D_2)$
17	Two general networks in parallel		$\frac{A_1B_2+B_1A_2}{B_1+B_2}$	$\frac{B_1B_2}{B_1+B_2}$	$C_1+C_2+\frac{(A_1-A_2)(D_2-D_1)}{B_1+B_2}$	$\frac{B_1D_2+D_1B_2}{B_1+B_2}$

NOTE. The exciting current of the receiving end transformers should be added vectorially to the load current, and the exciting current of the sending end transformers should be added vectorially to the sending end current.

General equations: $E_S = E_RA + I_RB$; $E_R = E_SD - I_SB$; $I_S = I_RD + E_RC$; $I_R = I_SA - E_SC$. As a check in the numerical calculation of the A, B, C, and D constants note that in all cases $AD-BC=1$ unless there is a net angular transformation ratio. In the latter case $AD-BC=1\epsilon^{j2\theta}$ where θ is the angular transformation of S ahead of R. See Sec. 8.

TABLE 10—CONVERSION FORMULAS FOR TRANSMISSION TYPE NETWORKS

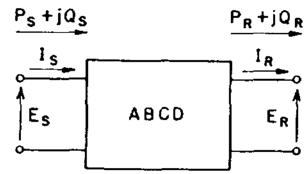
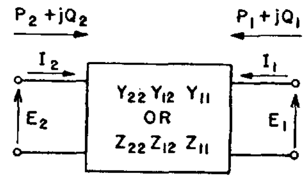
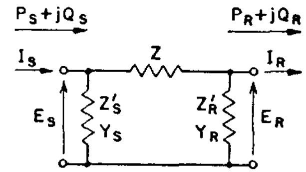
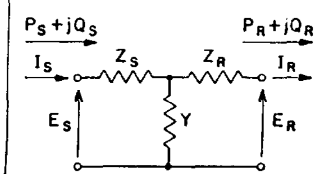
To Convert To		To Convert From: ABCD	Admittance	Impedance	Equivalent Pi	Equivalent T	Reference Directions and Nomenclature
ABCD	$A=$	**ABCD Constants**	$-\frac{Y_{11}}{Y_{12}}$	$\frac{Z_{22}}{Z_{12}}$	$1+ZY_R$	$1+Z_SY$	 Fig. 42
	$B=$	$E_S=AE_R+BI_R$ $I_S=CE_R+DI_R$	$-\frac{1}{Y_{12}}$	$\frac{Z_{11}Z_{22}-Z_{12}^2}{Z_{12}}$	Z	$Z_R+Z_S+YZ_RZ_S$	
	$C=$	$E_R=DE_S-BI_S$ $I_R=-CE_S+AI_S$	$\frac{Y_{12}^2-Y_{11}Y_{22}}{Y_{12}}$	$\frac{1}{Z_{12}}$	$Y_R+Y_S+ZY_RY_S$	Y	
	$D=$		$-\frac{Y_{22}}{Y_{12}}$	$\frac{Z_{11}}{Z_{12}}$	$1+ZY_S$	$1+Z_RY$	
Admittance	$Y_{11}=$	$\frac{A}{B}$	**Admittance Constants**	$\frac{Z_{22}}{Z_{11}Z_{22}-Z_{12}^2}$	$Y_R+\frac{1}{Z}$	$\frac{1+Z_SY}{Z_R+Z_S+YZ_RZ_S}$	 Fig. 43
	$Y_{12}=$	$-\frac{1}{B}$	$I_1=Y_{11}E_1+Y_{12}E_2$	$-\frac{Z_{12}}{Z_{11}Z_{22}-Z_{12}^2}$	$-\frac{1}{Z}$	$-\frac{1}{Z_R+Z_S+YZ_RZ_S}$	
	$Y_{22}=$	$\frac{D}{B}$	$I_2=Y_{12}E_1+Y_{22}E_2$	$\frac{Z_{11}}{Z_{11}Z_{12}-Z_{12}^2}$	$Y_S+\frac{1}{Z}$	$\frac{1+YZ_R}{Z_R+Z_S+YZ_RZ_S}$	
Impedance	$Z_{11}=$	$\frac{D}{C}$	$\frac{Y_{22}}{Y_{11}Y_{22}-Y_{12}^2}$	**Impedance Constants**	$\frac{1+ZY_S}{Y_R+Y_S+ZY_RY_S}$	$Z_R+\frac{1}{Y}$	
	$Z_{12}=$	$\frac{1}{C}$	$-\frac{Y_{12}}{Y_{11}Y_{22}-Y_{12}^2}$	$E_1=Z_{11}I_1+Z_{12}I_2$	$\frac{1}{Y_R+Y_S+ZY_RY_S}$	$\frac{1}{Y}$	
	$Z_{22}=$	$\frac{A}{C}$	$\frac{Y_{11}}{Y_{11}Y_{22}-Y_{12}^2}$	$E_2=Z_{12}I_1+Z_{22}I_2$	$\frac{1+ZY_R}{Y_R+Y_S+ZY_RY_S}$	$Z_S+\frac{1}{Y}$	
Equiv. Pi	$Y_R=$	$\frac{A-1}{B}$	$Y_{11}+Y_{12}$	$\frac{Z_{22}-Z_{12}}{Z_{11}Z_{22}-Z_{12}^2}$	**Equivalent Pi**	$\frac{YZ_S}{Z_R+Z_S+YZ_RZ_S}$	 Fig. 44
	$Z=$	B	$-\frac{1}{Y_{12}}$	$\frac{Z_{11}Z_{22}-Z_{12}^2}{Z_{12}}$		$Z_R+Z_S+YZ_RZ_S$	
	$Y_S=$	$\frac{D-1}{B}$	$Y_{22}+Y_{12}$	$\frac{Z_{11}-Z_{12}}{Z_{11}Z_{22}-Z_{12}^2}$		$\frac{YZ_R}{Z_R+Z_S+YZ_RZ_S}$	
Equiv. T	$Z_R=$	$\frac{D-1}{C}$	$\frac{Y_{22}+Y_{12}}{Y_{11}Y_{22}-Y_{12}^2}$	$Z_{11}-Z_{12}$	$\frac{ZY_S}{Y_R+Y_S+ZY_RY_S}$	**Equivalent T**	 Fig. 45
	$Y=$	C	$-\frac{Y_{11}Y_{22}-Y_{12}^2}{Y_{12}}$	$\frac{1}{Z_{12}}$	$Y_R+Y_S+ZY_RY_S$		
	$Z_S=$	$\frac{A-1}{C}$	$\frac{Y_{11}+Y_{12}}{Y_{11}Y_{12}-Y_{12}^2}$	$Z_{22}-Z_{12}$	$\frac{ZY_R}{Y_R+Y_S+ZY_RY_S}$		

TABLE 11—CURRENT AND VOLTAGE RELATIONS IN TRANSMISSION TYPE NETWORKS

A. For *ABCD* Constants—Reference Fig. 42.*

$$E_S = AE_R + BI_R \tag{201a}$$
$$I_S = CE_R + DI_R \tag{201b}$$
$$E_R = DE_S - BI_S \tag{201c}$$
$$I_R = -CE_S + AI_S \tag{201d}$$

B. For Equivalent Pi—Reference Fig. 44.

$$E_S = (1+ZY_R)E_R + ZI_R \tag{202a}$$
$$I_S = (Y_R + Y_S + ZY_RY_S)E_R + (1+ZY_S)I_R \tag{202b}$$
$$E_R = (1+ZY_S)E_S - ZI_S \tag{202c}$$
$$I_R = -(Y_R + Y_S + ZY_RY_S)E_S + (1+ZY_R)I_S \tag{202d}$$

C. For Equivalent T—Reference Fig. 45.

$$E_S = (1+Z_SY)E_R + (Z_R + Z_S + YZ_RZ_S)I_R \tag{203a}$$
$$I_S = YE_R + (1+Z_RY)I_R \tag{203b}$$
$$E_R = (1+Z_RY)E_S - (Z_R + Z_S + YZ_RZ_S)I_S \tag{203c}$$
$$I_R = -YE_S + (1+Z_SY)I_S \tag{203d}$$

D. For Admittance—Reference Fig. 43.

$$I_1 = Y_{11}E_1 + Y_{12}E_2 \tag{203e}$$
$$I_2 = Y_{12}E_1 + Y_{22}E_2 \tag{203f}$$

E. For Impedance—Reference Fig. 43.

$$E_1 = Z_{11}I_1 + Z_{12}I_2 \tag{203g}$$
$$E_2 = Z_{12}I_1 + Z_{22}I_2 \tag{203h}$$

*Figs. 42 to 45 are part of Table 10.

as shown in Table 11. Frequently these are not needed.

c. The Power Expressions are given in Table 12 in terms of the *T*, Pi, or *ABCD* constants and also in terms of admittance and impedance coefficients, which are described in a later paragraph. The power convention used in this text is:

$$P + jQ = E\hat{I} \tag{212}$$

for which a positive value of Q is lagging reactive power, and P and Q of the same sign indicates lagging power factor (see Sec. 2). At the sending end, denoted by the subscript, S, the positive direction is into the network. At the receiving end, denoted by R, it is out of the network. See Figs. 42, 44, 45 which are part of Table 10. With the generalized impedance or admittance form, Fig. 43, the reference-positive direction for current and power at each terminal is into the line or network.

d. The Power Circle Diagrams can be determined from the data in Table 12 as outlined at the bottom of the table. The detailed data for plotting the circles can be obtained from the supplementary Table 12A, explained in the next paragraph.

TABLE 12—POWER EQUATIONS AND DATA FOR PLOTTING CIRCLE DIAGRAMS

Derived From	Sending Circle			Receiving Circle		
		Vector to Center, C_S	Radius Vector R_{S0}		Vector to Center, C_R	Radius Vector R_{R0}
ABCD Ref. Fig. 42	$P_S + jQ_S =$	$3\frac{\hat{D}}{\hat{B}}\bar{E}_S^2$	$-\frac{3}{\hat{B}}\bar{E}_R\bar{E}_S\epsilon^{+j\theta}$ (204)	$P_R + jQ_R =$	$-3\frac{\hat{A}}{\hat{B}}\bar{E}_R^2$	$+\frac{3}{\hat{B}}\bar{E}_R\bar{E}_S\epsilon^{-j\theta}$ (205)
Equiv. Pi Ref. Fig. 44	$P_S + jQ_S =$	$3\left(\frac{1}{\hat{Z}} + \hat{Y}_S\right)\bar{E}_S^2$	$-\frac{3}{\hat{Z}}\bar{E}_R\bar{E}_S\epsilon^{+j\theta}$ (206)	$P_R + jQ_R =$	$-3\left(\frac{1}{\hat{Z}} + \hat{Y}_R\right)\bar{E}_R^2$	$+\frac{3}{\hat{Z}}\bar{E}_R\bar{E}_S\epsilon^{-j\theta}$ (207)
Imped. Form Equiv. Pi Ref. Fig. 44	$P_S + jQ_S =$	$3\left(\frac{1}{\hat{Z}} + \frac{1}{\hat{Z}'_S}\right)\bar{E}_S^2$	$-\frac{3}{\hat{Z}}\bar{E}_R\bar{E}_S\epsilon^{+j\theta}$ (206a)	$P_R + jQ_R =$	$-3\left(\frac{1}{\hat{Z}} + \frac{1}{\hat{Z}'_R}\right)\bar{E}_R^2$	$+\frac{3}{\hat{Z}}\bar{E}_R\bar{E}_S\epsilon^{-j\theta}$ (207a)
Equiv. *T* Ref. Fig. 45	$P_S + jQ_S =$	$\frac{3(1+\hat{Z}_R\hat{Y})\bar{E}_S^2}{\hat{Z}_R + \hat{Z}_S + \hat{Y}\hat{Z}_R\hat{Z}_S}$	$-\frac{3\bar{E}_R\bar{E}_S\epsilon^{+j\theta}}{\hat{Z}_R + \hat{Z}_S + \hat{Y}\hat{Z}_R\hat{Z}_S}$ (208)	$P_R + jQ_R =$	$-\frac{3(1+\hat{Z}_S\hat{Y})\bar{E}_R^2}{\hat{Z}_R + \hat{Z}_S + \hat{Y}\hat{Z}_R\hat{Z}_S}$	$+\frac{3\bar{E}_R\bar{E}_S\epsilon^{-j\theta}}{\hat{Z}_R + \hat{Z}_S + \hat{Y}\hat{Z}_R\hat{Z}_S}$ (209)
Admittance Ref. Fig. 43	$P_2 + jQ_2 =$	$3\hat{Y}_{22}\bar{E}_2^2$	$+3\hat{Y}_{12}\bar{E}_1\bar{E}_2\epsilon^{+j\theta}$ (210)	$P_1 + jQ_1 =$	$3\hat{Y}_{11}\bar{E}_1^2$	$+3\hat{Y}_{12}\bar{E}_1\bar{E}_2\epsilon^{-j\theta}$ (211)
Impedance Ref. Fig. 43	$P_2 + jQ_2 =$	$\frac{3\hat{Z}_{11}}{\hat{Z}_{11}\hat{Z}_{22} - \hat{Z}_{12}^2}\bar{E}_2^2$	$-\frac{3\hat{Z}_{12}\bar{E}_1\bar{E}_2\epsilon^{+j\theta}}{\hat{Z}_{11}\hat{Z}_{22} - \hat{Z}_{12}}$ (210a)	$P_1 + jQ_1 =$	$\frac{3\hat{Z}_{22}}{\hat{Z}_{11}\hat{Z}_{22} - \hat{Z}_{12}}\bar{E}_1^2$	$-\frac{3\hat{Z}_{12}\bar{E}_1\bar{E}_2\epsilon^{-j\theta}}{\hat{Z}_{11}\hat{Z}_{22} - \hat{Z}_{12}}$ (211a)

UNITS

Table gives P and Q in megawatts (mw), and megavolt amperes (mva) for E_R and E_S in kv line-to-neutral, or it gives P and Q in watts and volt-amperes for $\bar{E}_S$ and $\bar{E}_R$ in volts, line-to-neutral.

To use volts or kv line-to-line, omit factor 3 throughout the tabulation.

Impedances and admittances are in ohms or mhos per phase, line-to-neutral.

θ is the angle of E_S in advance of E_R or the angle of E_2 in advance of E_1.

$\hat{}$ Symbol designating conjugate of a vector.

TO DRAW CIRCLE DIAGRAM—FIG. 47

1. Calculate "vector to center" and locate center, C_S or C_R.
2. Calculate radius vector for $\theta = 0$ ($\epsilon^{-j\theta} = \epsilon^{j\theta} = 1$). Call it R_{S0} or R_{R0}.
3. Add 1 and 2 to obtain real and reactive power for sending and receiving voltages in phase. Plot this as "Power for $\theta = 0$", on the diagram. $W_{R0} = C_R + R_{R0}$. $W_{S0} = C_S + R_{S0}$.
4. Draw the circle through the "Power for $\theta = 0$" point. Draw the reference radius vector from the center to the "Power for $\theta = 0$" point, to serve as the reference from which angles are measured.
5. Corresponding sending and receiving conditions are found at the same angle on the corresponding circles.

TABLE 12A—CONSTANTS FOR PLOTTING POWER CIRCLE DIAGRAMS

Refer Fig. 47

(l, m, l', m', n, θ_b are to be obtained from the relations given)

Form of* System Constants	Receiving Circle Constants	Sending Circle Constants	Radius Constant	Position of Radius Vector for $\theta=0$
	$l+jm$	$l'+jm'$	n	θ_b
$ABCD$ Ref. Fig. 42	$\dfrac{\hat{A}}{\hat{B}}$	$\dfrac{\hat{D}}{\hat{B}}$	$\dfrac{1}{\hat{B}}$	$\tan^{-1}\dfrac{b_1}{b_2}$, $B=b_1+jb_2$
Equiv. Pi Ref. Fig. 44	$\dfrac{1}{\hat{Z}}+\hat{Y}_R$	$\dfrac{1}{\hat{Z}}+\hat{Y}_S$	$\dfrac{1}{\hat{Z}}$	$\tan^{-1}\dfrac{r}{x}$, $Z=r+jx$
Imped. Form Equiv. Pi Ref. Fig. 44	$\dfrac{1}{\hat{Z}}+\dfrac{1}{\hat{Z}'_R}$	$\dfrac{1}{\hat{Z}}+\dfrac{1}{\hat{Z}'_S}$	$\dfrac{1}{\hat{Z}}$	$\tan^{-1}\dfrac{r}{x}$, $Z=r+jx$
Equiv. T Ref. Fig. 45	$\dfrac{1+\hat{Z}_S\hat{Y}}{\hat{Z}_R+\hat{Z}_S+\hat{Y}\hat{Z}_R\hat{Z}_S}$	$\dfrac{1+\hat{Z}_R\hat{Y}}{\hat{Z}_R+\hat{Z}_S+\hat{Y}\hat{Z}_R\hat{Z}_S}$	$\dfrac{1}{Z_R+Z_S+YZ_RZ_S}$	$\tan^{-1}\dfrac{b_1}{b_2}$ where $Z_R+Z_S+YZ_RZ_S=b_1+jb_2$

*For admittance and impedance constants the reference direction is into the network at both ends. Thus the receiving circle is in the same quadrant as the sending circle and the l and m constants do not apply. Use the method of Table 12 for plotting the circles in this case.

Construction of power circles.—For the occasional user it is convenient to list directly the coordinates of the centers and the radii of the circles, together with the location of the reference line from which angles are to be measured. For this purpose the six constants l, m, n, θ_b, l' and m' are defined and used. When working with $ABCD$ constants these have the definitions:—

$$\frac{1}{\hat{B}}=n \tag{213}$$

$$\theta_b=\tan^{-1}\frac{b_1}{b_2} \text{ where } B=b_1+jb_2 \tag{214}$$

$$\frac{\hat{A}}{\hat{B}}=l+jm \tag{215}$$

$$\frac{\hat{D}}{\hat{B}}=l'+jm'. \tag{216}$$

For other forms of expression of the transmission network the definitions of these six constants are given in Table 12A.

Having defined these six constants, the circles can be constructed as follows. Refer to Fig. 47. The scales used for kw and reactive kva must be the same. Line-to-line voltages are used, giving three-phase kw and reactive kva. If line-to-neutral voltages are used to determine the centers and radii of sending and receiving circles, the expressions in Fig. 47 must be multiplied by three.

Center of sending circle is at $l'\ \overline{E}_S^2$ kw, $m'\ \overline{E}_S^2$ kvar.

Radius of sending circle is $n\overline{E}_R\overline{E}_S$.

The reference line for angles in the sending circle is clockwise from a downward vertical radius by the angle, θ_b. See Fig. 46, Angles θ of sending-end voltage in advance of receiving-end voltage, are measured ccw from this reference line.

Center of the receiving circle is at $-l\overline{E}_R^2$ kw, $-m\overline{E}_R^2$ kvar.

Radius of the receiving circle is $n\overline{E}_R\overline{E}_S$.

The reference line for angles in the receiving circle is cw from an upward vertical radius by the angle, θ_b. See Fig. 47.

Corresponding sending and receiving conditions are found at the same angle θ on the two corresponding circles.

An alternative method of construction is listed in the five steps under Table 12, which eliminates the necessity of measuring angles. An "initial radius vector for $\theta=0$" is added to the "vector to the center" to get the coordinates of a point (i.e. the vector power) corresponding to $\theta=0$. This fixes both the radius and the reference line for measuring angles.

Power-Angle Diagrams—From the circle diagram the power expressions as a function of angle can be written directly. They are, for three-phase power in kw on kvar, and voltages in kv, L-L;—

$$P_S=l'\overline{E}_S^2+n\overline{E}_R\overline{E}_S \sin(\theta-\theta_b) \tag{217}$$

$$Q_S=m'\overline{E}_S^2-n\overline{E}_R\overline{E}_S \cos(\theta-\theta_b) \tag{218}$$

$$P_R=-l\overline{E}_R^2+n\overline{E}_R\overline{E}_S \sin(\theta+\theta_b) \tag{219}$$

$$Q_R=-m\overline{E}_R^2+n\overline{E}_R\overline{E}_S \cos(\theta+\theta_b) \tag{220}$$

Power plotted vertically against θ plotted horizontally is thus a displaced sine wave known as a power angle diagram. Its use in stability calculations is described in Chapter 13.

Use of Equations vs. Circle Diagrams—If only one condition were of interest, for which the voltages and intervening angle were known, the sending and receiving power quantities could be calculated directly, using the power expressions of Table 12. However, if the power transmitted is to be determined for a number of angular positions, as in stability studies, the circle diagram is advantageous. Also if the voltages and power are known and the angle and reactive requirements are to be determined the circle diagram becomes indispensable. More particularly a diagram having several circles corresponding to different voltages constitutes a chart of the real and reactive powers that can be trans-

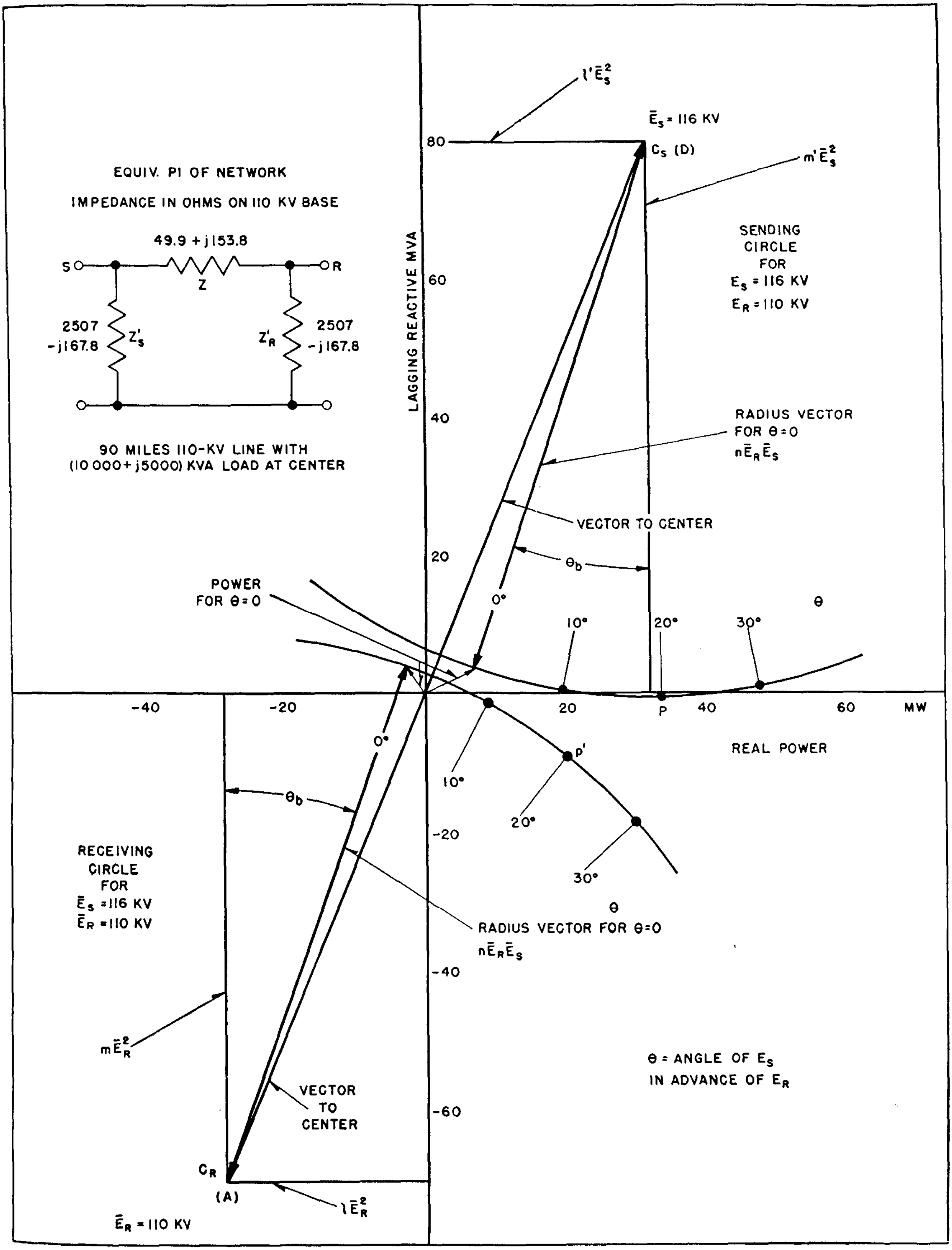


Fig. 47—Power circle diagram for line AED Fig. 48 and used for example of plotting diagram.

mitted for various voltages and angles. As such it finds extensive use both in system operation and design.

Interpretation of Power Circle Diagrams—Power circle diagrams for a particular system are given in Fig. 47. They have been drawn in accordance with the instructions at the bottom of Table 12. Likewise their construction from the six constants l, m, n, θ_b, l', n', given in Table 12A is indicated. This system consists of ninety miles of 110-kv line with a load of 10 000 kw and 5000 lagging reactive kva tapped on at the middle. The load is treated as a fixed impedance. Points p and p' convey the following information, "With a receiver voltage of 110 kv and a sending voltage 116 kv, if 34 000 kw (34 megawatts) are supplied at the sending end, then 500 leading reactive kva must be supplied at the sending end and 20 000 kw and 9000 leading reactive kva will be delivered at the receiving end. That is, 9000 kva of lagging reactive must be supplied at the receiving end. 13 500 kw and 8500 reactive kva will be consumed in the system, including line losses and reactive plus the intermediate load. Incidentally the sending-end voltage is only 20 degrees ahead of the receiving-end voltage and the operation will therefore be stable†." Obviously contained in this information are the answers to a variety of questions that might be asked.

ABCD Constants are coefficients of the current and voltage equations (201a) to (201d) given in Table 11. They apply to the transmission-type network having sending-end and receiving-end terminals, and have the following physical significance.

A is the voltage impressed at the sending end per volt at the open-circuited receiver. It is a dimensionless voltage ratio.

B is the voltage impressed at the sending end per ampere in the short-circuited receiver. It is the transfer impedance used in network theory. It is also equal to the voltage impressed at the receiving end per ampere in the short-circuited sending terminals.

C is the current in amperes into the sending end per volt on the open-circuited receiver. It has the dimensions of admittance.

D is the current in amperes into the sending end per ampere in the short-circuited receiver. It is a dimensionless current ratio.

Table 9 gives the $ABCD$ constants for many types of networks. Table 10 gives the transformations to Pi, T, admittance and impedance forms. Chap. 9 illustrates the use of $ABCD$ constants in a stability example.

For passive networks, as dealt with here,

$$AD-BC=1 \tag{221}$$

This affords a valuable check on the calculations. If the single-phase network used involves phase shift, it is not strictly passive. A case in point is an ideal phase-shift transformer Fig. 48. As shown $AD-BC$ is numerically one but includes a double angle term. This single-phase representation of a three phase transformer receives power at one time phase and passes it on at another time phase, although the total power flow is continuous in the three-phase transformer it represents. Usually the phase shift factor is removed from the equivalent single-phase circuit before calculating the $ABCD$ constants so that Eq. (221) is applicable.

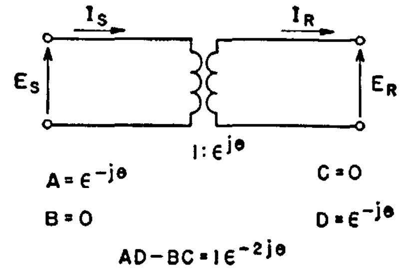


Fig. 48—Ideal phase shift transformation.

Admittance Constants (or Driving Point and Transfer Admittances) are coefficients of the network current equations.

$$I_1=Y_{11}E_1+Y_{12}E_2 \tag{222}$$

$$I_2=Y_{22}E_2+Y_{12}E_1 \tag{223}$$

As indicated in Fig. 43 the positive direction is taken into the network at both ends. This permits ready extension to more than two terminals. In the following definitions current in a terminal is understood to be in the positive direction unless otherwise stated. The definitions indicate the extension to more than two terminals. In general:

Y_{11} is the current in terminal 1 per volt applied at terminal 1 with all other terminals short circuited.

Y_{12} is the current in terminal 2 per volt applied at terminal 1 with all other emfs short circuited, or vice versa. It will usually be negative for transmission-type systems with positive direction into network at both ends, etc.

The power equations as obtained from (222) and (223) are:

$$P_1+jQ_1=E_1\hat{I}_1=\hat{Y}_{11}E_1\hat{E}_1+\hat{Y}_{12}E_1\hat{E}_2 \tag{224}$$

$$P_2+jQ_2=E_2\hat{I}_2=\hat{Y}_{12}E_2\hat{E}_1+\hat{Y}_{22}E_2\hat{E}_2 \tag{225}$$

For a Three-Terminal System (as in a three-machine problem) the current equations are:

$$I_1=Y_{11}E_1+Y_{21}E_2+Y_{31}E_3 \tag{226}$$

$$I_2=Y_{12}E_1+Y_{22}E_2+Y_{32}E_3 \tag{227}$$

$$I_3=Y_{13}E_1+Y_{23}E_2+Y_{33}E_3 \tag{228}$$

The corresponding power equations are:

$$P_1+jQ_1=E_1\hat{I}_1=\hat{Y}_{11}E_1\hat{E}_1+\hat{Y}_{21}E_1\hat{E}_2+\hat{Y}_{31}E_1\hat{E}_3 \tag{226a}$$

$$P_2+jQ_2=E_2\hat{I}_2=\hat{Y}_{12}E_2\hat{E}_1+\hat{Y}_{22}E_2\hat{E}_2+\hat{Y}_{32}E_2\hat{E}_3 \tag{227a}$$

$$P_3+jQ_3=E_3\hat{I}_3=\hat{Y}_{13}E_3\hat{E}_1+\hat{Y}_{23}E_3\hat{E}_2+\hat{Y}_{33}E_3\hat{E}_3 \tag{228a}$$

The extension to more than three terminals is apparent.

Self and Mutual Impedances are coefficients of the network voltage equations given generally in Eqs. (52)–(55). Writing these for the transmission-type network, which can in general be reduced to a number of meshes equal to the number of significant terminals:

$$E_1=Z_{11}I_1+Z_{21}I_2 \tag{229}$$

$$E_2=Z_{12}I_1+Z_{22}I_2 \tag{230}$$

†Refer to Chap. 13 for criteria of stability.

Z_{11} is the voltage in terminal 1 per ampere in terminal 1, with all other "significant terminals" open circuited.
Z_{12} is the voltage in terminal 2 per ampere in terminal 1, with all other "significant terminals" open circuited. Etc.

NOTE that the self and mutual impedances Z_{11} and Z_{12} as defined and used in Section 13 and in this Section 21, differ from the self and mutual drop constants defined and used in Section 20. The Z with double subscript is used in each case to conform with accepted terminology.

The power equations are obtained from (229) and (230).

$$P_1+jQ_1=E_1\hat{I}_1=Z_{11}I_1\hat{I}_1+Z_{21}I_2\hat{I}_1 \quad (231)$$
$$P_2+jQ_2=E_2\hat{I}_2=Z_{12}I_1\hat{I}_2+Z_{22}I_2\hat{I}_2 \quad (232)$$

For a three-terminal system the voltage and power equations are given below. The extension of admittance or impedance constants to any number of terminals is apparent.

$$E_1=Z_{11}I_1+Z_{21}I_2+Z_{31}I_3 \quad (233)$$
$$E_2=Z_{12}I_1+Z_{22}I_2+Z_{32}I_3 \quad (234)$$
$$E_3=Z_{13}I_1+Z_{23}I_2+Z_{33}I_3 \quad (235)$$
$$P_1+jQ_1=E_1\hat{I}_1=Z_{11}I_1\hat{I}_1+Z_{21}I_2\hat{I}_1+Z_{31}I_3\hat{I}_1 \quad (236)$$
$$P_2+jQ_2=E_2\hat{I}_2=Z_{12}I_1\hat{I}_2+Z_{22}I_2\hat{I}_2+Z_{32}I_3\hat{I}_2 \quad (237)$$
$$P_3+jQ_3=E_3\hat{I}_3=Z_{13}I_1\hat{I}_3+Z_{23}I_2\hat{I}_3+Z_{33}I_3\hat{I}_3 \quad (238)$$

IV. REAL AND REACTIVE POWER FLOW

It is well known that in a system in which impedances are largely reactive, real power flow is controlled by phase angles and reactive flow by voltage magnitudes. Ordinarily the adjustments of real power flow are made by throttle or gate adjustments (governor settings), although the flow in a closed loop can be controlled by a regulating transformer capable of introducing a phase shift. Similarly reactive flow is usually controlled by generator field adjustment (regulator setting) but in a closed loop, transformer tap adjustments can be utilized.

Quantitatively the real and reactive power over a transmission circuit or interconnection can be determined from the power circle diagrams. These diagrams give the real and reactive power at the sending and receiving ends of an interconnecting line or network, in terms of the voltages at the two ends and the angle between them. The method will be explained by examples, starting with a simple two-station system with an interconnection, and covering in all, the following conditions:

22. Examples of Real and Reactive Power Flow.

Refer to Fig. 1, or the equivalent network Fig. 49.

I. Two Stations with Interconnection (Station A to station B).

Case Ia.

Given: 1. Sending-end and Receiving-end Voltages.
2. Received Power.

To Find: 1. Received reactive kva.
2. Transmitted kw and reactive kva.
3. Required kw and reactive kva to be supplied by generators at each end.
4. Losses (kw and reactive kva).

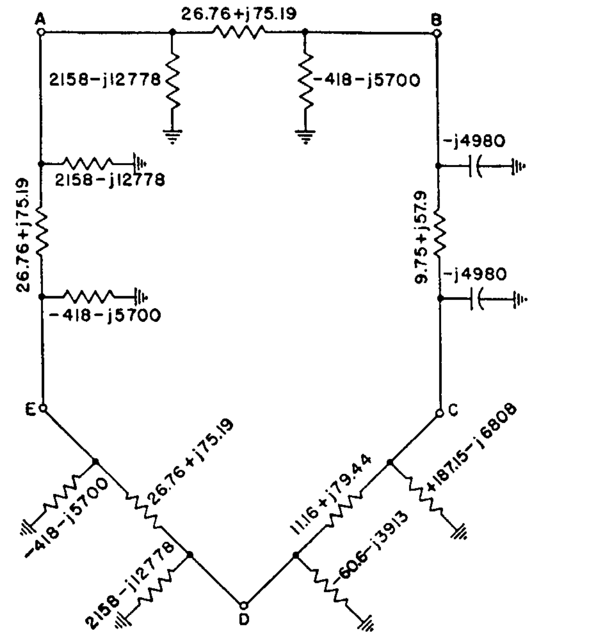


Fig. 49—Network of Fig. 2 reduced to an equivalent Pi for each line.

Case Ib.

Given: 1. Received kw and reactive kva.
2. Receiving-end voltage.

To Find: 1. Sending-end voltage.
2. Sending-end kw and reactive kva.
3. Required generation at sending end.
4. Losses.

II. Three generating stations along one main interconnection, no closed loops, intermediate substation. A, B, C, D Fig. 1.

Case IIa.

Given: 1. Voltages at all but D.
2. Real power flow.

To Find: 1. Remaining Power Quantities.
2. Voltage at D to hold stated voltage at intermediate Substation C.

Case IIb.

Given: 1. Voltages at all but intermediate substation C.
2. Real power flow.
3. Load at C, real and reactive.

To Find: 1. Remaining power quantities.
2. Voltage at intermediate substation C.

III. Closed Loop System.

Case IIIa. Find voltage to close under given load and power flow conditions. (To determine regulator requirements.)

Case IIIb. Find power which flows if loop is closed and flow held constant in rest of the loop.

Case I. Two-Source System—Stations A and B, Fig. 1, and the 50-mile line connecting them, will be used for illustration. The remainder of the system shown will be considered as disconnected. It may be desired to find the required voltages of the A and B buses, and the angle between them, to transmit a desired real and reactive power.

Or it may be necessary to find what real and reactive power can be transmitted for different voltages and angles. In either case the power circle diagram is an ideal method of expressing the performance of the interconnection. Two cases will be considered illustrative of the two forms in which the problem may appear.

Case Ia. Two Station System, A and B. See Fig. 49.

Fixed. Voltages and receiver real power.

Sending end	Station A
Receiving end	Station B

The given conditions are:

Sending-end voltage	$\overline{E}_S = 110$ kv $L-L$
Receiving-end voltage	$\overline{E}_R = 110$ kv $L-L$
Received power	20 Megawatts

Load at A 10 000 kw, 6200 lagging reactive kva $(10.0+j6.2)$ mva

Load at B 50 000 kw, 37 500 lagging reactive kva $(50.0+j37.5)$ mva

It is required to find the real and reactive power that must be generated at Stations A and B. This requires determination of the reactive power received from the line, and the real and reactive power at the input end. From the line power quantities and the local loads, the required kw and wattless generation can be determined.

General Comments

This is a characteristic problem of transmitting between buses whose voltages are fixed by load requirements. Wattless capacity in condensers or generators must be available at the proper locations because the fixed voltages determine the wattless flow over the line.

Tap-changing-under-load transformers permit maintaining the generator bus voltage while raising the effective sending-end voltage to transmit wattless. No-load taps can be used to a rather limited extent if the power flow is in one direction with not too much variation from maximum to minimum.

This problem is often further complicated by the fact that the load bus voltage must be scheduled during the day, being somewhat more under heavy load conditions.

The stability of the interconnection is not investigated in this chapter: Refer to Chap. 13 for examples of stability determinations.

Obtaining the Circle Diagram

The method of obtaining the impedance diagram, Fig. 2, from the single-line diagram, Fig. 1 has already been described in Sec. 3 and 4. To obtain the circle diagram from the impedance network from A to B two general methods of approach can be used. The intervening network can be reduced to an equivalent Pi and the circle diagram determined therefrom as shown in Table 12. Or $ABCD$ constants can be written for the sections of the interconnection, from Table 9. These can then be combined to obtain $ABCD$ constants for the complete interconnection as shown also in Table 9. The circle diagram data can then be determined from the overall $ABCD$ constants, using the formulas of Tables 12 or 12a. Some prefer the $ABCD$ constants because the method is systematic, and has a check for each step. Others prefer the equivalent circuit method because they can more clearly visualize the problem by this method. As a result both methods are used and some examples of both methods will be given. The stability problem of Chap. 13 is treated exclusively by the method of $ABCD$ constants. This power flow problem has been treated by the equivalent circuit method.

Reducing the Network

The first step is the reduction of the network between A and B, Fig. 2, to an equivalent Pi, shown in Fig. 49. As the equivalent circuits between other buses will be needed in subsequent cases, they also must be obtained. A typical reduction follows for the section from A to B, the steps being shown in Fig. 50.

Convert the T network a, b, and c to an equivalent Pi using Eqs. (105.)–(107).

$$\begin{aligned} Z_a &= 2.82+j32.3 \\ Z_b &= 24.05+j43.05 \\ Z_c &= -j8000 \\ Z_S' = Z_{ac} &= Z_a+Z_c+Z_aZ_c/Z_b \\ &= 2158-j12\ 778 \\ M = Z_{bc} &= Z_b+Z_c+Z_bZ_c/Z_a \\ &= -4940-j19\ 050 \\ Z = Z_{ab} &= Z_a+Z_b+Z_aZ_b/Z_c \\ &= 26.76+j75.19 \end{aligned}$$

Parallel M and N to obtain Z_R'

$$Z_R' = \frac{MN}{M+N} = -418-j5700$$

Plotting the Circle Diagram

From the constants Z_S', Z, Z_R' of the equivalent Pi, the data for plotting the power circle diagrams for line AB can be obtained, using Eqs. (206a) and (207a) of Table 12. In the following calculations E_S and E_R are expressed in kv, line-to-line which gives the power, calculated as E^2/Z, in the dimensions of megavolt amperes; that is, megawatts (mw) and reactive megavolt-amperes (reactive mva). With the power* calculated as $P+jQ=E\hat{I}$, the ASA standard, a positive value of Q indicates lagging reactive power in the chosen reference direction for I.

Sending-end Circle—from equivalent Pi in impedance form

$$\overline{E}_S = 110 \text{ kv}, \qquad E_R = 110 \text{ kv}$$

Center

$$C_S = \left(\frac{1}{\hat{Z}} + \frac{1}{\hat{Z}_S'}\right)\overline{E}_S^2 = 50.9930+j141.9209$$

Radius vector for $\theta=0$

$$R_{SO} = -\frac{\overline{E}_R\overline{E}_S}{\hat{Z}} = -50.8369-j142.8429$$

Power for $\theta=0$

$$W_{SO} = C_S + R_{SO} = 0.1561-j0.9220$$

The sending circle has been drawn in Fig. 51 by plotting the center, the power for $\theta=0$, and drawing the circle

*The term power is used generally meaning real and reactive power.

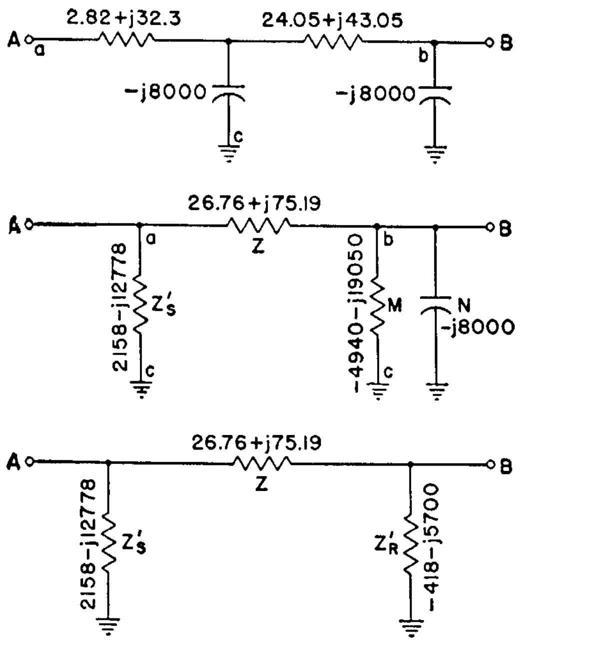


Fig. 50—Steps in the reduction of line AB to an equivalent Pi.

through the latter point. The sending power for any angle, of E_S in advance of E_R, is a point on the circle, an angle θ counter-clockwise from the radius for $\theta=0$.

Receiving-end Circle—from equivalent Pi in impedance form.

$$\bar{E}_S = 110 \text{ kv}, \qquad \bar{E}_R = 110 \text{ kv}$$

Center

$$C_R = -\left(\frac{1}{\hat{Z}} + \frac{1}{\hat{Z}_R'}\right)\bar{E}_R^2$$
$$= -50.6821 - j140.7363$$

Radius for $\theta=0$

$$R_{RO} = \frac{\bar{E}_R \bar{E}_S}{\hat{Z}}$$
$$= 50.8369 + j142.8429$$

Power for $\theta=0$

$$W_{RO} = C_R + R_{RO}$$
$$= 0.1548 + j2.1066$$

The receiving circle is located in a similar manner to the sending circle. The receiving power for any angle θ, of E_S in advance of E_R, is a point on the circle an angle θ clockwise from the radius for $\theta=0$. The corresponding sending and receiving power for a given transmission condition over the line, are points on the two circles for the same angle θ.

Interpreting the Circle Diagram For the Particular Problem—Case Ia.

From the conditions of the problem, the received power is 20 mw which is found at point 1_r on the receiving circle, Fig. 51. Laying off the same angle θ_1 on the sending circle, the point 1_s is located giving the corresponding real and reactive power at the sending-end.

$$P_{SA} + jQ_{SA} = 21.0 - j7.0 \text{ mva}$$
$$P_{RB} + jQ_{RB} = 20.0 - j7.0 \text{ mva}$$

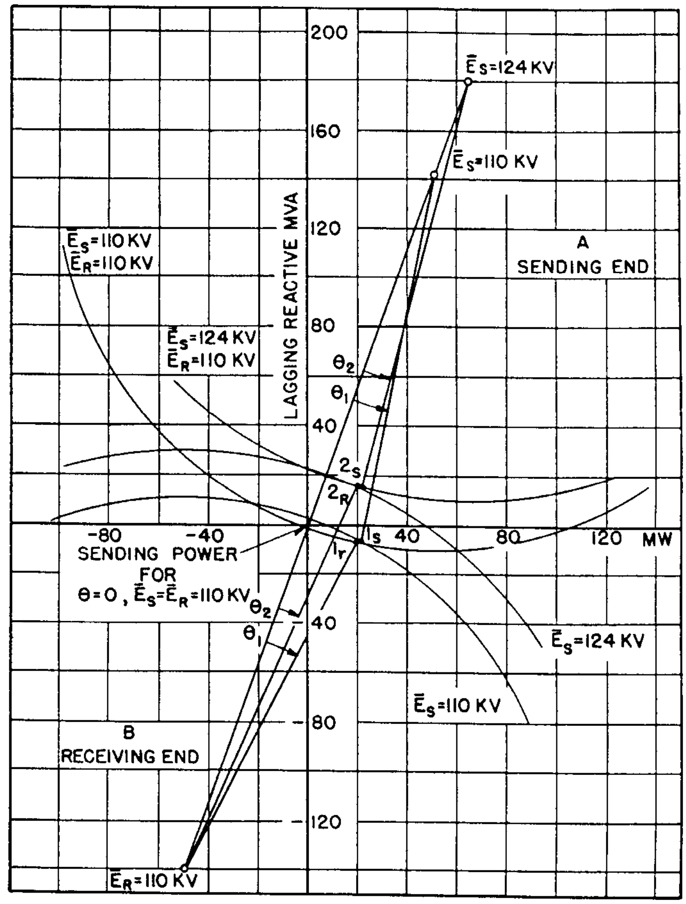


Fig. 51—Power circle diagram for line AB.

The local loads at A and B are respectively: (See Fig. 1)

$$P_{LA} + jQ_{LA} = 10 + j6.2 \text{ mva}$$
$$P_{LB} + jQ_{LB} = 50 + j37.5 \text{ mva}$$

The required generation at A is therefore:

$$P_{GA} + jQ_{GA} = P_{LA} + jQ_{LA} + P_{SA} + jQ_{SA}$$
$$= 31 - j0.8 \text{ mva}$$

or 31 mw and 0.8 lagging reactive mva. i.e., underexcited. The required generation at B is

$$P_{GB} + jQ_{GB} = P_{LB} + jQ_{LB} - (P_{RB} + jQ_{RB})$$
$$= 30 + j44.5 \text{ mva}$$

or 30 mw and 44.5 lagging reactive mva.

The line losses are:

$$P_L + jQ_L = P_S + jQ_S - (P_R + jQ_R)$$
$$= 1 + j0 \text{ mva}$$

or 1 mw and no reactive mva.

The I^2X_L lagging reactive power consumed by the line is balanced by the E^2/X_C leading reactive power consumed by its shunt capacity.

For accurate values of losses, $P_R + jQ_R$ and $P_S + jQ_S$ can be calculated for the angle θ involved. Transformer iron losses must also be added.

From this example it can be seen that if a particular kilowatt load is transmitted over a line with fixed terminal voltages, the input and output reactive power quantities are determined by the line, and must be provided for in the machines at each end.

Case Ib, Two-Station System, A and B, Fig. 1. Fixed receiver voltage and real and reactive power.

Sending end — Station A
Receiving end — Station B

The given conditions are:

Receving-end voltage $\bar{E}_R = 110$ kv, $L-L$
Received power $(20+j15)$ mva, or 20 mw, 15 lagging reactive mva.
Load at A $(10+j6.2)$ mva.
Load at B $(50+j37.5)$ mva.

To be determined:

Sending-end voltage.
Sending-end real and reactive power.
Line Losses
Generation required at sending end. (The generation at B must obviously be $30+j22.5$ mva.)

Receiving-End Circle (Refer to Fig. 51)

The center of the receiver circle is the same as determined in Case Ia since the receiver voltage is again, 110 kv. The received power, $(20+j15)$ mva is plotted as point 2_R on the diagram. The receiver circle passes through point 2_R. Scaling or calculating the radius to this point it is found to be $\bar{R}_R = 171$ mva, scalar value. But the scalar value from Eq. (207a), Table 12 is $\frac{\bar{E}_R\bar{E}_S}{\bar{Z}}$. Whence solving for the sending voltage

$$\bar{E}_S = \frac{\bar{R}_R\bar{Z}}{\bar{E}_R} = \frac{171\times 79.8}{110} = 124 \text{ kv}$$

Sending-End Circle.

$$\bar{E}_S = 124 \text{ kv}, \qquad \bar{E}_R = 110 \text{ kv}$$

Center, C_S

The vector to the center for 110 kv has previously been drawn. Increase it in the ratio $(124/110)^2$ to obtain the new center for 124 kv.

Radius Vector for $\theta=0$, R_{SO}

The radius vector for $\theta=0$, for $\bar{E}_R = 110$ kv, and $\bar{E}_S = 110$ kv has previously been obtained. Increase it in the ratio $(110\times 124)/(110\times 110)$ to obtain the new radius vector at $\theta=0$ for $\bar{E}_S = 124$ kv, $\bar{E}_R = 110$ kv.

Power for $\theta=0$

$$W_{SO} = C_S + R_{SO} = 7.4916 + j19.3214$$

From the center and power at $\theta=0$ the new sending power circle can be drawn. The received power point 2_R occurs at the angle θ_2. Laying off θ_2 along the sending circle the sending power point 2_S is located. Thus:

$$P_{SA} + jQ_{SA} = 21.5 + j15.5 \text{ mva}$$

And since the load at A is

$$P_{LA} + jQ_{LA} = 10 + j6.2 \text{ mva}$$

The required generation is:

$$P_{GA} + jQ_{GA} = P_{SA} + jQ_{SA} + P_{LA} + jQ_{LA} = 31.5 + j21.7 \text{ mva}$$

or 31.5 mw and 21.7 lagging reactive mva

The losses are:

$$P_1 + jQ_1 = P_{SA} + jQ_{SA} - (P_{RB} + jQ_{RB}) = 1.5 + j0.5$$

or 1.5 mw and 0.5 lagging reactive mva.

The required sending-end voltage of 124 kv could be provided by a step up transformer at A having a rated high voltage of 115 kv and equipped with ± 10 percent tap-changing-under-load equipment, giving it a range of from 126.5 kv down to 104.5 kv. This would provide for transmitting only a reduced load in the reverse direction unless the transformer at B were similarly equipped.

This problem illustrates that to transmit a desired amount of wattless as well as real power over a line with fixed receiver voltage, the sending voltage must be under control of the operator or automatic means, since the required value depends on the wattless to be transmitted.

Case II. Three Generating Stations Along One Main Interconnection, and Intermediate Substation—The methods used in analyzing a two-station system can be easily extended to a multi-station system in which the stations are all connected along one main transmission channel. Such a system is illustrated for example by Fig. 1 and Fig. 49 with line AED omitted. Because in this case power can flow over only one path, any two stations and the line connecting them can be treated as a two-station system, independent of the other stations and lines. For instance, the system of Fig. 1 with line AED omitted merely adds more stations and lines to the two-station system of Example I, and part AB can be solved as before.

Besides the addition of another generating station, the new system has the added complication of a substation between stations B and D. Since there are no generators at C to maintain the voltage, the voltage on the substation bus is determined by the voltages at the adjacent stations and by the real and reactive kva load on the substation bus.

There are in general two problems concerning the voltage at an intermediate substation such as C. First, the voltage at one of the adjacent stations may be fixed, and it is desired to maintain a given voltage at the substation by varying the voltage at the other generating station. The problem is then to find the generating-station voltage and determine the power and kva flow in both lines. This problem can be solved by setting up a circle diagram for the line from the substation to the fixed-voltage station and determining the kva flow. This kva is subtracted from the load on the substation to give the kva that must be furnished by the other line. Using this kva, the circle diagram for the other line can be set up and the desired generating bus voltage can be found as in the case Ib just preceding.

Second, the voltage at both generating stations may be fixed, together with the real power flow over each line section, and the voltage at the substation must be determined. This can be done by a cut-and-try process, using the circle diagrams of the two lines and varying the substation voltage until the sum of the reactive kva's transmitted to the substation is equal to that required by the substation load.

Each of these two operating conditions will be illustrated in the following example.

Case IIa. Multistation System with Intermediate Substation—No Closed Loops—Fixed Substation Voltage

	Sending End	Receiving End
Line AB	A	B
Line BC	B	C
Line DC	D	C

The given conditions are as follows, line AB being assumed to operate under the same conditions as case Ib.

At A	Generated	$= 31.5+j21.7$ mva
	Load	$= 10 \quad +j6.2$ mva
	Transmitted	$= 21.5+j15.5$ mva
	Voltage	$=124$ kv.
At B	Load	$= 50 \quad +j37.5$ mva
	Received over AB	$= 20 \quad +j15$ mva
	Transmitted over BC	$= 6.0+j?$ mva
	Generated	$= 36+j?$mva
	Voltage	$=110$ kv
At C	Load	$= 18.3+j7.0$ mva
	Voltage	$=108$ kv
At D	Load	$= 10.0+j7.0$ mva

To be determined:

At B Transmitted reactive over BC
Generated Reactive
At C Received $P+jQ$ over BC
Received $P+jQ$ over DC
At D Voltage
Transmitted $P+jQ$
Generated $P+jQ$

Solution:

Line BC, sending circle (B) for $\bar{E}_S=110$ kv, $\bar{E}_R=108$ kv

Center (Refer to Fig. 52)

$$C_S=\left(\frac{1}{\hat{Z}}+\frac{1}{\hat{Z}_S{}'}\right)\bar{E}_S^2$$
$$=34.2200+j200.7886$$

Radius vector for $\theta=0$

$$R_{SO}=-\frac{\bar{E}_R\bar{E}_S}{\hat{Z}}=-33.5978-j199.5234$$

Power for $\theta=0$

$W_{SO}=C_S+R_{SO}=0.6222+j1.2652$ (Point 5, Fig. 52)

Line BC, Receiving Circle (C) for $\bar{E}_S=110$ kv $\bar{E}_R=108$ kv

Center

$$C_R=-\left(\frac{1}{\hat{Z}}+\frac{1}{\hat{Z}_R{}'}\right)\bar{E}_R^2$$
$$=-32.9870-j193.5536$$

Radius for $\theta=0$

$$R_{RO}=\frac{\bar{E}_R\bar{E}_S}{\hat{Z}}$$
$$=33.5978+j199.5234=202.3324$$

Power for $\theta=0$

$$W_{RO}=C_R+R_{RO}=0.6108+j5.9698$$

Line BC circles are plotted in Fig. 52 from these data, the section near the origin being enlarged. From the sending circle the transmitted power of 6 mw is accompanied by transmitted lagging reactive power of 0.6 mva (Point 2, Fig. 52). From the receiving circle, for the same angle θ_1, the received power at C is found to be $6.0+j5.0$ mva, as closely as the chart can be read (Point 1, Fig. 52).

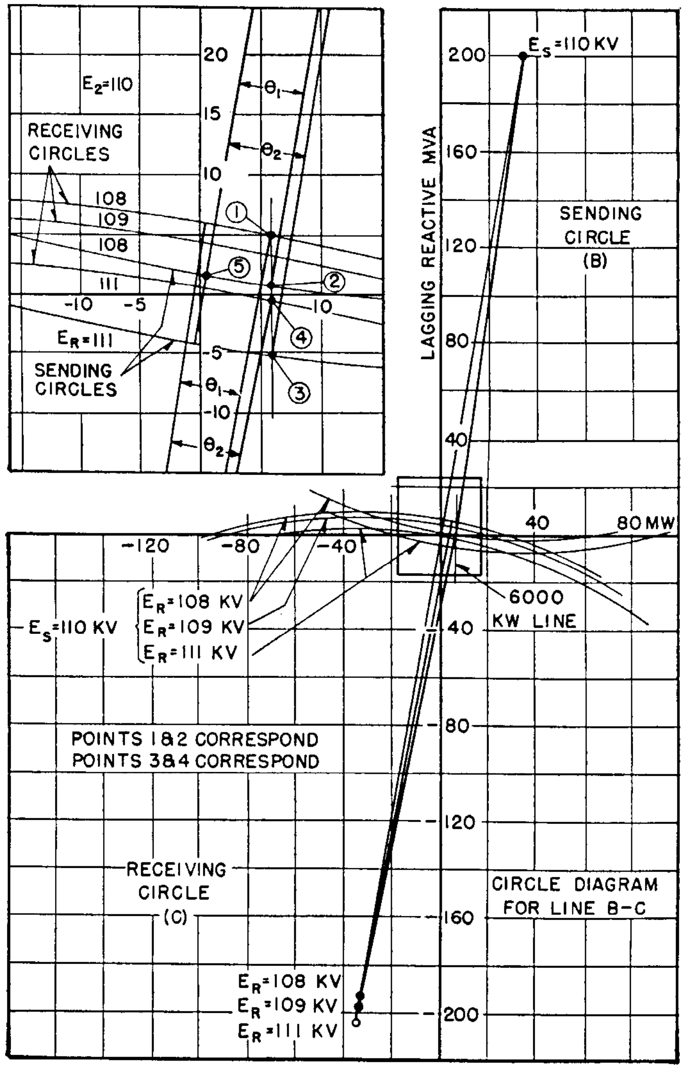


Fig. 52—Power circle diagram for line BC.

Line DC, Receiving Circle (C) for $\bar{E}_R=108$ kv, $\bar{E}_S=?$ Refer to Fig. 53.

$$\text{Center} \qquad C_R=-\left(\frac{1}{\hat{Z}}+\frac{1}{\hat{Z}_R{}'}\right)\bar{E}_R^2$$
$$=-20.2732-j142.2751$$

The received power over line DC can be determined

Load at $C=18.3+j7.0$ mva

Received over $BC=6.0+j5.0$ mva

Received over $DC=12.3+j2.0$, which is plotted as point 1 of Fig. 53.

Scaling from the center to this point the radius of the receiving circle is found to be 147.9 mva, from which we can solve for $\bar{E}_S$.

$$\bar{R}_R=147.9=\frac{\bar{E}_R\bar{E}_S}{\bar{Z}}=\frac{108\bar{E}_S}{80.1}$$

$$E_S=\frac{147.9\times 80.1}{108}=110\text{ kv}$$

Line DC, sending circle (D) for $\bar{E}_S=110$ kv, $\bar{E}_R=108$ kv

Center

$$C_S=\left(\frac{1}{\hat{Z}}+\frac{1}{\hat{Z}_S{}'}\right)\bar{E}_S^2$$

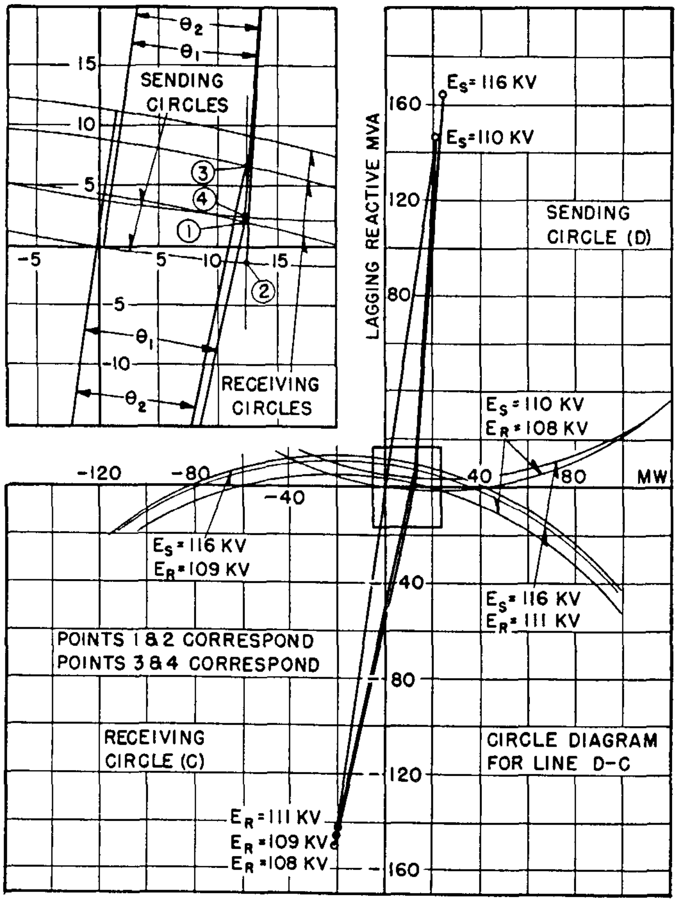


Fig. 53—Power circle diagram for line CD.

$=20.9354+j146.2781$ mw.

Radius vector for $\theta=0$

$$R_{SO}=-\frac{\bar{E}_R\bar{E}_S}{\hat{Z}}$$

$$=-20.6011-j146.6527 \text{ mw.}$$

Power for $\theta=0$

$$W_{SO}=C_S+R_{SO}=0.3343-j0.3746 \text{ mw.}$$

The sending circle can now be plotted and laying off the angle θ_1, the same as for the receiving circle the sending-end power is found to be $12.4-j1.8$ mva, at point 2.

Now recapitulating,

At B	Transmitted over BC	$6.0+j0.6$ mva
	Generation otherwise required	$30.0+j22.5$ mva
	Generation required	$36.0+j23.1$ mva
At C	Received over BC	$6.0+j5.0$ mva
	Received over DC	$12.3+j2.0$ mva
	Load at C	$18.3+j7.0$ mva
Losses in line BC*		$0\ -j4.4$ mva

or 0 mw and -4.4 lagging reactive mva, i.e., 4.4 lagging reactive mva is supplied net by the line.

At D	Transmitted over DC	$=12.4-j1.8$ mva
	Load at D	$=10.0+j7.0$ mva
	Generation at D	$=22.4+j5.2$ mva

Losses in line $DC=0.1-j3.8$ mva* or 0.1 mw and -3.8 lagging reactive mva.

*See discussion of losses in case *Ia*.

Case IIb. Multistation System with Intermediate Substation—No Closed Loops—Substation Voltage to Be Determined

The generating-station voltage magnitudes are assumed to be fixed. As the methods of combining loads and of determining losses are simple and have been outlined in the previous cases, this case is confined simply to determining the voltage of the intermediate substation and the reactive power flow over the lines. The real power flow is assumed.

Given:

Voltage at $B=110$ kv
Voltage at $D=116$ kv

At C		
	Load at C	$18.3+j7.0$ mva
	Received over BC	$6.0+j?$ mva
	Received over DC	$12.3+j?$ mva

To be determined:

At B Transmitted power over BC
At C Voltage
Received reactive over each line
At D Transmitted power over DC.

A cut-and-try method is employed, based on assuming values of voltage at C until a value is found that results in a total received reactive power equal to the reactive load at C ($+j7.0$ mva). Obviously after a few trials a curve of received reactive power versus voltage-at-C can be plotted and the proper voltage read from this curve. Or after two trials the increment of reactive per increment in voltage noted, so that the third trial is simply a check. Assume, to start with, a voltage of 109 kv at C. Circle centers and radii can be found by ratioing from values calculated in Case IIa.** Refer to Table 13.

The second trial gives a sufficiently close value of total reactive ($+j6.2$ mva compared with the desired $+j7.0$ mva) and the circles corresponding to this trial are used to determine the power quantities. Thus, drawing the corresponding sending circles, and using points 3 and 4 on Fig. 52 and 3 and 4 on Fig. 53.

At B	Transmitted over BC	$6\ -j5.2$
At C	Voltage is	111 kv
	Received over BC	$6\ -j0.5$
	Received over DC	$12.3+j6.7$
	Load Assumed	$18.3+j7.0$
At D	Transmitted over DC	$12.3+j2.5$

***Obtaining Circle Centers and Radii by Ratioing—General:* In working graphically, after one sending and receiving circle have been drawn for a given intervening network, all further work can be done by simple scalar ratios and graphical construction using the following relationships.

a. The center of a receiving circle is always along the same line through the origin, the distance to the origin being proportional to $\bar{E}_R^2$.
b. The center of a sending circle is always along the same line through the origin (not necessarily the same as in *a*) the distance to the origin being proportional to $\bar{E}_S^2$.
c. The scalar value of the radius is proportional to $\bar{E}_S\ \bar{E}_R$.
d. The radius for $\theta=0$ is always parallel to the first one drawn.

TABLE 13

		Case IIa	Case IIb, Trial 1	Case IIb, Trial 2
	BC Receiving Circle, (*C*)			
1	$\bar{E}_S =$	110	110	110
2	$\bar{E}_R =$	108	109	111
3	Center, $C_R = -\left(\frac{1}{\hat{Z}}+\frac{1}{\hat{Z}'_R}\right)\bar{E}_R^2 =$	$-32.99-j193.55$	$-33.60-j197.15$	$-34.85-j204.45$
4	Radius for $\theta=0$, $R_{RO}=\frac{\bar{E}_R\bar{E}_S}{\hat{Z}}=$	$33.60+j199.52$	$33.91+j201.37$	$34.53+j205.06$
5	Power for $\theta=0$, $W_{RO}=C_R+R_{RO}=$	....	$0.31+j\ 4.22$	$-0.32+j0.61$
6	Reactive corres. to 6 *mw*, from circle	....	$+j\ 3.2$	$-j\ 0.5$
	DC Receiving Circle, (*C*)			
7	$\bar{E}_S =$	110	116	116
8	$\bar{E}_R =$	108	109	111
9	Center, $C_R = -\left(\frac{1}{\hat{Z}}+\frac{1}{\hat{Z}'_R}\right)\bar{E}_R^2 =$	$-20.27-j142.27$	$-20.65-j144.92$	$-21.42-j150.28$
10	Radius for $\theta=0$, $R_{RO}=\frac{\bar{E}_R\bar{E}_S}{\hat{Z}}=$	$20.60+j146.65$	$21.93+j156.08$	$22.33+j158.95$
11	Power for $\theta=0$, $W_{RO}=C_R+R_{RO}$	....	$1.34+j\ 11.16$	$0.98+j\ 8.67$
12	Reactive corres. to 12.3 *mw*, from circle	....	$+j\ 9.3$	$+j\ 6.7$
13	Reactive received at *C*, Sum of 6 and 12	....	$+j\ 12.5$	$+j\ 6.2$

Case III. Loop System, Three Generating Stations, Two Intermediate Substations, Fig. 1*—Power flow in the complete loop system of Fig. 1* is next considered. A method for calculating the effect of a regulator for controlling phase angle and voltage ratio is given. This method in general consists of breaking the system at one point and treating it as several stations along one line up to the point of closing. The voltage required to close the loop can then be determined and also the circulating current that flows if the loop is closed without this voltage.

The simpler problem is to calculate the voltage required to close the loop for a given power flow condition, as this is simply an extension of Case II, above. The voltage required to close the loop gives the necessary setting of a regulating device to produce the assumed power flow.

It must be remembered that up to the point of closing the loop there is complete and independent control of real and reactive power flow over each line section connecting two generating stations. This assumes that permissible voltage or stability limits are not exceeded. For example, the generator voltages and throttles at *A* and *B* can be set to produce a desired real and reactive power over this line. Then holding the voltage and speed at *B* fixed, so as not to affect the flow over line *AB*, the generator voltage and throttle at *D* can be adjusted to give the desired flow in the *BD* section. Now if the voltages and speeds are held fixed at *A*, *B*, and *D* by suitable adjustments at those points, then the connection of a line from *D* to *E* and the passage of any amount of power over it under the control of a regulator in the line will have no effect on that part of the loop external to line *AD*.

Thus the problem reduces to a consideration of the flow in the section *A* to *D* only when the magnitudes and phase positions of the voltages at those two points have been previously fixed by the required conditions elsewhere in the loop.

*Fig. 49 also shows the system except for loads.

Case IIIa. Loop System, Given Power Flow, Find Voltage Across Open Point

Given Conditions.

For the system external to line *AD* we shall use the voltages and power flow conditions described in Fig. 54 which have been arrived at:

For line *AB* from case Ia, Circle Fig. 51.
For line *BD* from case IIb, Circle Figs 52 and 53.
Load at $E=10.0+j5.0$.
Voltage at *E* will be taken as 111 kv for converting loads to impedances.

Required to Find.
The voltage between *A* and *A'*.
There are a number of ways to go about this problem.

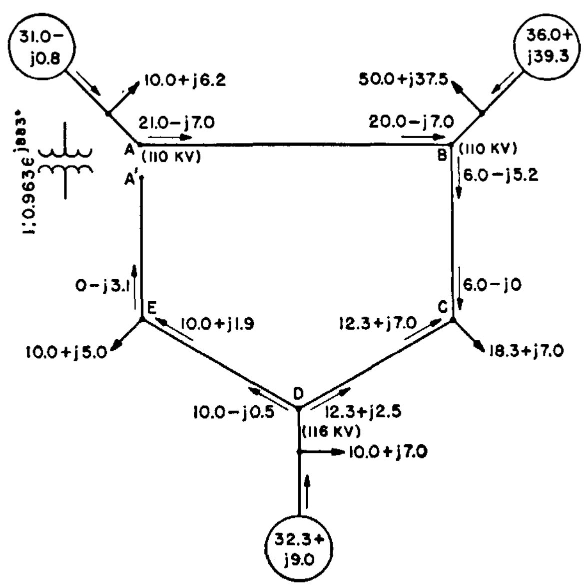


Fig. 54—Power flow condition for Case IIIa.

The one selected involves the following steps:

a. Assume a voltage at E and determine the power consumed by the open line. Add this to the load at E to obtain the total load received over line DE.

b. Find the voltage at E at which this load can be received.

c. Determine the voltage at A', and the angle between it and the voltage at A.

Proceeding with the calculations in this sequence:

a. The input power at E to the open-ended line EA' is obtained from the impedances, Fig. 49.

$$P_S+jQ_S=\frac{\bar{E}_S^2}{\hat{Z}_S'}+\frac{\bar{E}_S^2}{\hat{Z}+\hat{Z}_R'}$$
$$=\frac{\overline{111}^2}{-418+j5700}+\frac{\overline{111}^2}{2158+j12\,700}$$
$$=0.0049-j3.0877 \text{ mva}$$

Load at E	$=10.0+j5.0$ mva
Open line EA'	$=\ \ 0.0-j3.1$ mva
Power received from DE	$=10.0+j1.9$ mva

b. The circle diagram for line DE is given in Fig. 55. By trial it is found that with a receiver voltage at E of 113.5 kv the received power is $10.0+j1.9$ mva as desired and the corresponding sending power at D is $10.0-j0.5$, within the accuracy of the graphical construction.

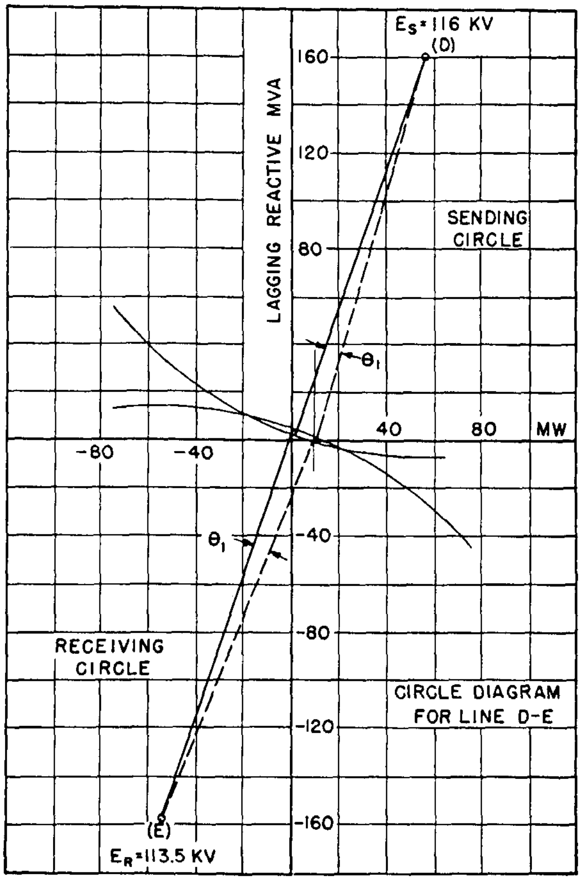


Fig. 55—Power circle diagram for line DE.

c. The voltage at A' is:

$$E_A'=E_E\frac{Z_S'}{Z+Z_S'}$$
$$=\frac{113.5(2158-j12\,778)}{26.76+j75.19+2158-j12\,778}$$
$$=114.18-j0.407=114.2 \text{ kv.}$$

Taking $E_A=110\epsilon^{j0}$

Angle of A in advance of B, Case Ia, $=\theta_{AB}=8.15°$

Angle of B in advance of C, Case IIb, $=\theta_{BC}=1.76°$

Angle of D in advance of C, Case IIb, $=\theta_{DC}=4.14°$

Angle of D in advance of E, Case IIIa, $=\theta_{DE}=3.23°$

Angle of A' in advance of E, Case IIIa, $=\theta_{A'E}=0.17°$

Angle of A in advance of A' $=\theta_{A'A}$

$$\theta_{A'A}=\theta_{AB}+\theta_{BC}-\theta_{DC}+\theta_{DE}-\theta_{A'E}=8.83°$$

Thus, since $\bar{E}_A=110$ kv and $\bar{E}_A'=114.2$ kv

$$E_A=\left(\frac{110}{114.2}\epsilon^{j\theta_{A'A}}\right)E_A'$$

and a regulator having a setting such as to result in a vector ratio of

$$1:0.963\epsilon^{j8.83°}$$

closes the loop with the power flow as indicated. For any other desired power flow in the various sections of the loop the requisite regulator setting can be similarly calculated. Thus by taking the extreme conditions of flow one way and then the other, the range required of the regulator can be determined.

Case IIIb. Loop Closed Without Regulator

Given Conditions.

The load E, for simplicity is converted to an impedance on 111 kv base.

Voltage at $A=110$ kv

Voltage at $D=116$ kv

Angle of D in advance of $A=-5.77°$

To Be Determined.

Power flow in Line AD.

With the conditions stated as above the problem is easily solved by determining the circle diagram for the complete line AD. The load impedance at E is shown on Fig. 56 together with the reduction to an equivalent Pi. The circle

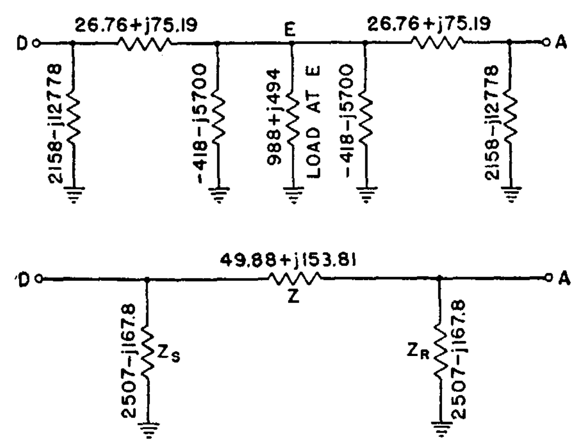


Fig. 56—Reduction of line AD.

diagram for the line DA is given in Fig. 47. Using the given voltages and angle it is found that

Power transmitted at D over $DA = -1.0 + j6.5$
Power received at A over $DA = -11 + j6.2$

From the circle diagram of a line connecting two points of known voltage and phase angle the effect of a regulator at one end can be determined by multiplying the voltage at that end by the vector ratio of the regulator and using the value thus obtained as the voltage at that end of the line.

It is hoped, that while it has not been possible to cover all conditions, a study of the methods used in the cases given will point the way to the solution of most other cases.

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CHAPTER 11

RELAY AND CIRCUIT-BREAKER APPLICATION

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THE function of relays and circuit breakers in the operation of a power system is to prevent or limit damage during faults or overloads, and to minimize their effect on the remainder of the system. This is accomplished by dividing the system into protective zones separated by circuit breakers, such as are shown in Fig. 1. During a fault, the zone which includes the faulted apparatus is de-energized and disconnected from the system. In addition to its protective function, a circuit breaker is also used for circuit switching under normal conditions.

The relay application problem consists of selecting a relay scheme which will recognize the existence of a fault within a given protective zone, and initiate circuit-breaker operation. This problem is considered from a system point of view. The operating characteristics of protective schemes for generators, transformers, lines, and buses are discussed in their relation to overall system performance. Reference is made to other publications, particularly *Silent Sentinels*[1], for the operating characteristics and connections of individual relays. It is proposed here only to give the general features which will determine the type of scheme to be used.

The circuit-breaker application problem consists primarily of determining the interrupting requirements, normal current, voltage, and other rating factors required to select the proper breaker for each location. These factors are discussed, and methods of calculating the fault current and interrupting rating are given.

I. GENERAL PHILOSOPHY AND BASIC RELAY ELEMENTS

As mentioned the system is divided into protective zones as shown in Figure 1, each having its protective relays for determining the existence of a fault in that zone and having circuit breakers for disconnecting that zone from the system. It is desirable to restrict the amount of system disconnected by a given fault; as for example to a single transformer, line section, machine, or bus section. However, economic considerations frequently limit the number of circuit breakers to those required for normal operation and some compromises result in the relay protection.

The relays operate usually from currents and voltages derived from current and potential transformers or potential devices. A station battery usually provides the circuit breaker trip current. Successful clearing depends on the condition of the battery, the continuity of the wiring and trip coil, and the proper mechanical and electrical

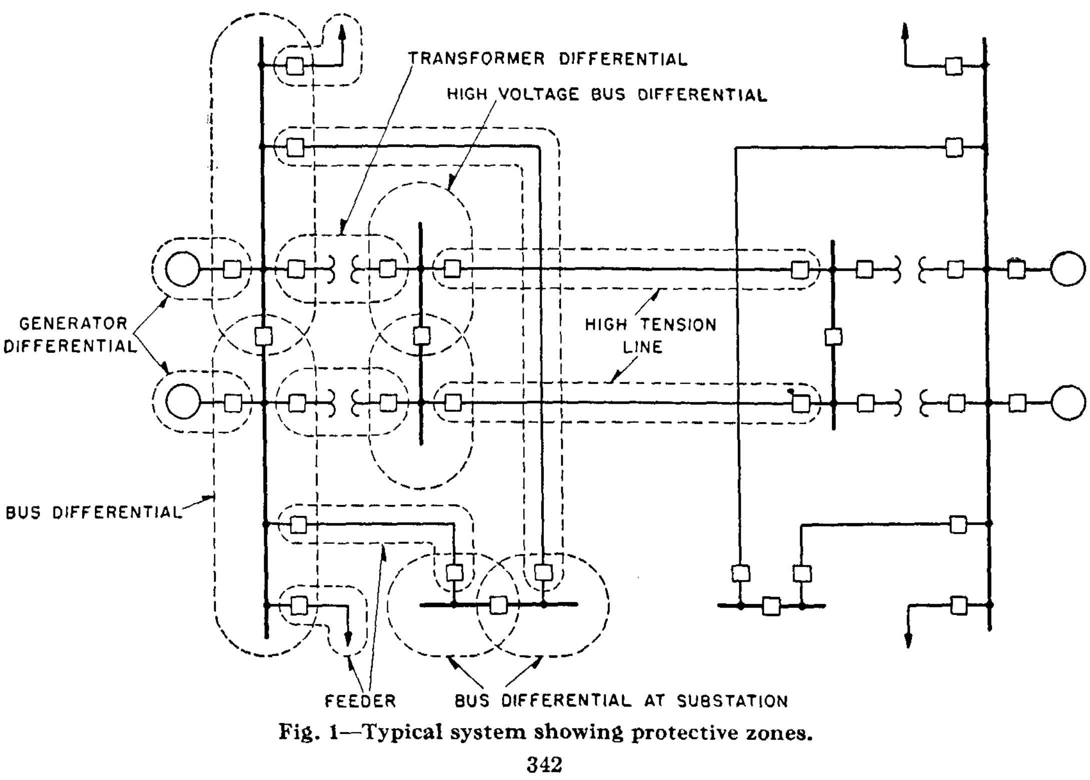


Fig. 1—Typical system showing protective zones.

operation of the circuit breaker as well as the closing of the relay trip contacts.

In event of failure of one of these elements, so that the fault in a given zone is not cleared by the first line of defense, relays and circuit breakers, some form of back-up protection is ordinarily provided to do the next best thing. This means, first of all, to clear the fault automatically, if at all possible, even though this requires disconnection of a considerable portion of the system. Once cleared, the system can generally be rather quickly restored; whereas if the fault hangs on, the line may be burned down, or apparatus damaged beyond repair, or the entire system may be shut down for an extended period. The measures taken to provide back-up protection vary widely depending on the value and importance of the installation and the consequence of failure. These will be discussed in a separate section.

Some utilities in measuring the performance of transmission line relay protection analyze all relay trip operations as shown in Table 1. The numbers shown are typical of a system operating 3000 miles of 110-kv line.

This is only an analysis of faults for which the relay tripped or should have tripped. For each of these there were several cases where the relays should not have tripped, and did not. Thus the total number of discriminations made by the relays is possibly five to ten times

TABLE 1—RELAY OPERATIONS

	Per Cent
Correct and desired	92.2
Correct but undesired	5.3
Wrong tripping operations	2.1
Failure to trip	0.4

as great as the trippings. The percentage failures are correspondingly less on this larger basis.

However, Table I has been presented at this point to bring out the following factors that enter into a highly successful protective relay system:

1. Good equipment, relays and instrument transformers.
2. A system design that can be protected and correct application of relays to provide the possible protection.
3. Good maintenance primarily to assure that all the accessories are operative.

The correct but undesired trippings are cases where the relays have done what should be expected from their characteristic curves and settings and the fault conditions involved. There may have been system changes since their application, or incorrect initial application, or application with foreknowledge that certain conditions would unavoidably operate the relays, but this was necessary to secure tripping in other desired cases.

It is important to bear in mind that simple standard system design plans can be better protected. Distance measuring and carrier or pilot-wire types of relaying are much less subject to disqualification by system changes than are over current types.

Wrong tripping and tripping failures, together with all causes of failure to clear faults, are found to stem largely from human errors, such as leaving the trip circuit open after test, or to open circuited trip coils, or mechanical failure of the circuit breaker, or blown fuses in trip circuits (if used). Only a small part of the total failures occur in the protective relay itself. Thus close attention to the initial design, installation, testing, and maintenance of all of the accessory equipment, as well as of the protective relay proper, are needed to assure successful operation.

The application of protective relays properly requires evaluation of several factors, namely:

1. The requirements of the power service and desired functioning of the system during fault conditions to produce this result.
2. The currents, voltages, temperatures, pressures, or other indicators at time of fault which provide the fundamental basis of discrimination.
3. The characteristics of available or standard relay elements.
4. The schemes in which they are used.

A wide variety of characteristics are now available operating in response to the prime quantities themselves, or to various functions of these prime quantities, such as power, phase angle, power factor, current comparison, power comparison, impedance, reactance, modified reactance, current ratio, or phase-sequence component.

In each case the response may be instantaneous, meaning no intentional delay, or the operation may depend in a predetermined manner on the electrical quantities and time of duration.

2. Basic Relay Elements[87]

The more commonly used relay elements and their underlying principles of operation are shown in Fig. 2. The schemes in which the elements are used are much more numerous. The more common ones will be described under the application headings, such as generators, transformers, and buses.

Instantaneous Elements—For instantaneous response to current or voltage the solenoid element, Fig. 2(a), is most common, appearing individually or as the instantaneous attachment with the induction-type overcurrent relay. The beam element, (*b*), with spring or weight bias is used where low burden is desirable, as when setting for low ground currents with low-ratio bushing current transformers. The polar element, (*j*), is of far lower burden than the nonpolar types and has come into widespread use since 1935 as the receiver relay of directional-comparison carrier equipments, and is the basic element when supplied from networks or electronic devices. For example, it appears as the operating element in a pilot wire relay, in a phase-comparison carrier relay, in linear-coupler bus protection, and in supervision of pilot wires.

Because of its higher pick-up to drop-out ratio and less accurate setting the clapper-type element, (*c*), is used less frequently for the primary protective functions but is widely used as an auxiliary relay.

Induction Elements—The induction-disk element, (*d*), continues to be most widely used, its reliability and inherent time characteristic giving it great flexibility for co-ordinating relays in series or co-ordinating with fuses or direct-trip devices. A variety of characteristics are available from the definite-minimum-time, which is ideal for securing definite time steps between relays, to the very inverse which provides faster tripping with the same margins when the fault current drops considerably from one

Fig. 2—Relay elements.

NOTES:

(a) Operating characteristics show typical order of magnitude only; varies with type of relay and adjustments.

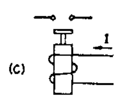
(b) Subscript $_o$ indicates setting or balance point value.

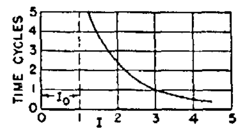
(c) All current relays are also voltage relays by suitable no. of turns and external series impedance since $I = E/Z$.

(d) "Instantaneous" is defined as "no intentional delay."

Schematic **Operating Characteristics**[a]

(a)

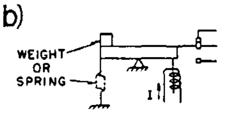


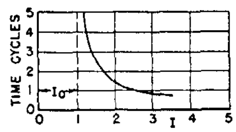


Solenoid, "Instantaneous," [d] AC or DC, adjust by taps on coil or by initial plunger position.

Operates for $I > I_O$[b].

(b)

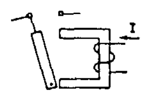


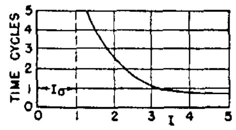


Balance beam, AC or DC, "Instantaneous," adjust by coil taps, and core screw (airgaps in magnetic circuit).

Operates for $I > I_O$.

(c)

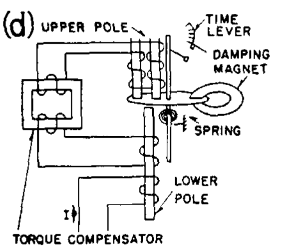


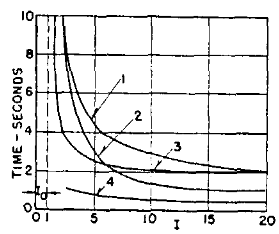


Armature or clapper type,—AC or DC, "Instantaneous," usually fixed settings by design. Primarily auxiliary voltage relay.

Operates for $I > I_O$.

(d)





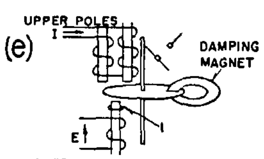
Induction disk inverse-time overcurrent element—AC-Current is supplied to the lower pole, and by inductive coupling to the upper pole, directly, or through a torque compensator (saturating transformer). The upper pole induces currents in the disk. Torque is produced by the reaction between these currents and flux from the lower pole. Adjust current setting by coil tap and time setting by contact travel.

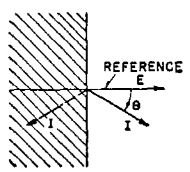
Operates for I greater than I_O.

Times shown are maximum. Relays adjustable for times down to 0.1 or 0.05 times those shown.

1 Inverse, low energy
2 Very inverse, low energy
3 Standard energy definite minimum time
4 High-speed, no torque compensator

(e)





Induction disk directional element. Current induced in the disk by the upper pole reacts on flux from the lower pole to produce torque.

1 Lag loop to bring lower pole flux in phase with upper pole current at unity power factor of E and I.

Vector diagram of E and I.

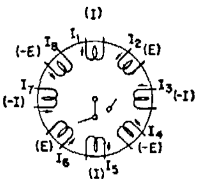
Contacts close for I in unshaded region, open in shaded region. Torque expression

$$T = K\bar{E}\bar{I} \cos \theta$$

where K is a constant, θ is the power factor angle.

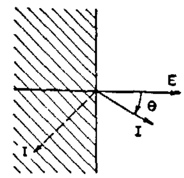
Schematic **Operating Characteristics**[a]

(f)



Multiple pole induction cup or disk. Each pole produces a torque product in conjunction with its adjacent poles and lesser torques in conjunction with those one pole removed, etc.

Example: Connected as directional element (quantities in parentheses applied to respective poles) for watt characteristics, ϕ is made zero.



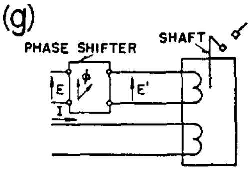
Contacts close for I in unshaded region, open in shaded region. Torque expression

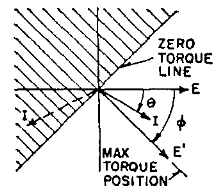
$$T = K\bar{E}\bar{I} \cos \theta.$$

$$\begin{aligned} T = {} & K_1\bar{I}_1\bar{I}_2 \cos(\theta_{12}+\phi_1) - K_1\bar{I}_1\bar{I}_8 \cos(\theta_{18}+\phi_1) \\ & + K_1\bar{I}_3\bar{I}_4 \cos(\theta_{34}+\phi_1) - K_1\bar{I}_3\bar{I}_2 \cos(\theta_{32}+\phi_1) \\ & + K_1\bar{I}_5\bar{I}_6 \cos(\theta_{56}+\phi_1) - K_1\bar{I}_5\bar{I}_4 \cos(\theta_{54}+\phi_1) \\ & + K_1\bar{I}_7\bar{I}_8 \cos(\theta_{78}+\phi_1) - K_1\bar{I}_7\bar{I}_6 \cos(\theta_{76}+\phi_1) \end{aligned} \Bigg\} \text{First Order Terms}$$

$$\begin{aligned} & + K_2\bar{I}_1\bar{I}_3 \cos(\theta_{13}+\phi_2) - K_2\bar{I}_1\bar{I}_7 \cos(\theta_{17}+\phi_2) \\ & + \text{etc.} \end{aligned} \Big\} \text{Second Order Terms}$$

(g)





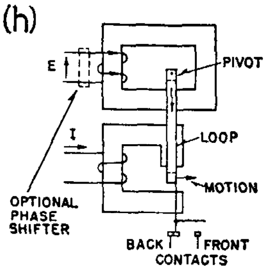
Induction disk or cup directional element as in *e* or *f*, with phase shift, ϕ, applied to voltage, E.

ϕ is the phase shift angle.

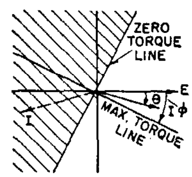
Contacts close for I in unshaded region, open in shaded region. Torque Expression

$$T = K\bar{E}\bar{I} \cos(\theta - \phi).$$

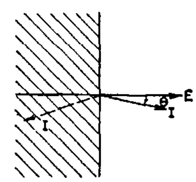
(h)



Inductor-loop high-speed directional element. Current induced in the loop by transformer action from the voltage winding reacts with flux crossing the gap of the current electromagnet to produce torque.



General characteristics $T = K\bar{E}\bar{I}\cos(\theta - \phi)$.

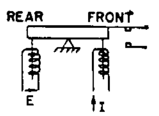


Watt characteristic adjusted for $\phi = 0$ $T = K\bar{E}\bar{I} \cos \theta$. Front contacts close for I in unshaded area. Back contacts close for I in shaded area.

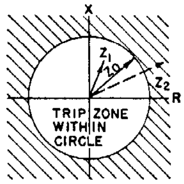
Fig. 2—Relay elements—*Continued*

Schematic	Operating Characteristics[a]

(i)



High-speed balance-beam impedance element—operates when "current pull" on front of beam overbalances "voltage pull" on rear of beam. Adjust balance point by current—coil taps and core screw (air-gap adjustment).

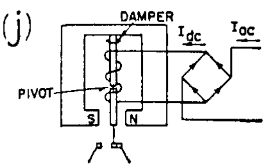


Balance point $\frac{\bar{E}}{\bar{I}}=\bar{Z}_0$ (radius of circle). Examples:

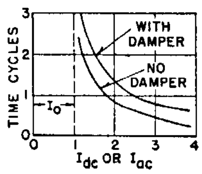
$$\frac{\bar{E}}{\bar{I}}=\bar{Z}_1<\bar{Z}_0 \text{ trips.}$$

$$\frac{\bar{E}}{\bar{I}}=\bar{Z}_2>\bar{Z}_0 \text{ does not trip.}$$

(j)

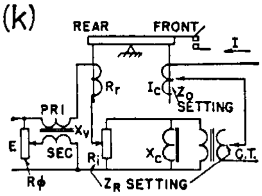


Polar element DC (or AC is used with rectifier), "instantaneous" type, current in the operating coil makes the moving armature a north pole. It is drawn toward the south pole of the permanent magnet.

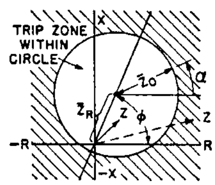


Operates for I_{dc} or I_{ac} greater than I_0.

(k)



High-speed balance-beam modified impedance element. Adjust impedance radius of circle, Z_0, by current coil taps, E, and core screw (air gap). Adjust angle of line along which center is shifted by taps on resistor, $R\phi$. Adjust impedance $\bar{Z}_R$ by which center is shifted by taps on current transformer, CT., and on resistor R_i.



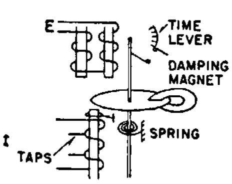
Displaced circle impedance characteristic relay trips for all faults for which impedance, Z, seen by relay falls within circle.

$\bar{Z}_0$, $\bar{Z}_R$, and ϕ are adjustable. Balance point locus:—

$$Z=\bar{Z}_R\epsilon^{i\phi}+\bar{Z}_0\epsilon^{i\alpha}.$$

for all values of α.

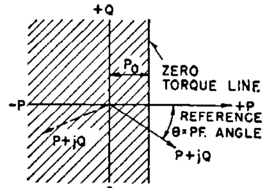
(l)



Induction type power relay. Operating torque is product of current induced in disk by upper pole and flux from lower pole.

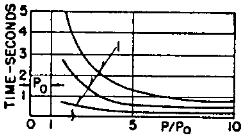
1. Lag loop to bring lower pole flux in phase with upper pole current at unity power factor of E and I.

TYPICAL POWER VECTOR DIAGRAM



$P+jQ=E\,\hat{I}(P=\bar{E}\bar{I}\cos\theta)$

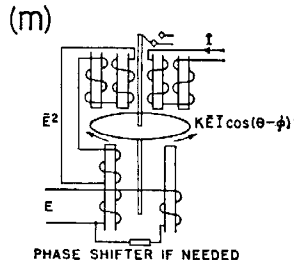
Contacts close for $P+jQ$ in unshaded region with timing as indicated below.



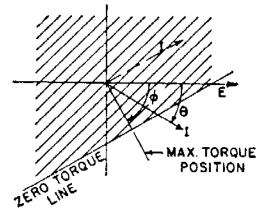
1 on typical time lever settings.

Schematic	Operating Characteristics[a]

(m)



Induction impedance element $\bar{E}^2$ pulling against $K\bar{E}\bar{I}\cos(\theta-\phi)$.



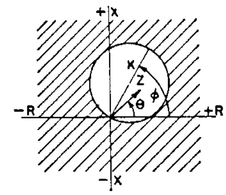
Current tripping characteristics with fixed voltage. Contacts close for I in unshaded area. Balance point at

$$\bar{I}=\frac{\bar{E}}{K\cos(\theta-\phi)}$$

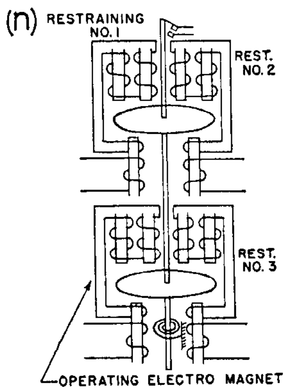
or $$\bar{Z}=\frac{\bar{E}}{\bar{I}}=K\cos(\theta-\phi).$$

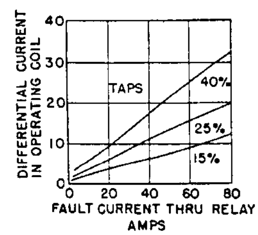
(Right) Contacts close for Z in unshaded area. Balance point at

$$\bar{Z}=\frac{\bar{E}}{\bar{I}}=K\cos(\theta-\phi).$$



(n)

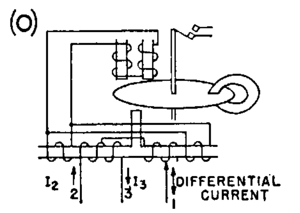




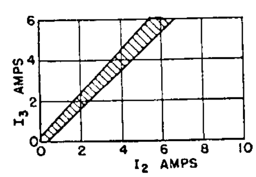
Contacts close for currents above curve corresponding to relay setting.

(Left) three winding transformer differential relay. (Damping magnet not shown). Adjust by taps on operating winding. (Also see Figure 5)

(o)

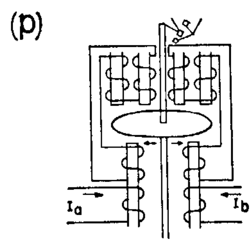


Induction-type ratio-differential relay for generator and transformer protection.



Operates in 0.1 to 0.2 sec. on heavy faults. Contacts close for currents in unshaded areas. (I_2 and I_3 approx. in phase.) Scale shown is for a 10 percent differential generator relay.

(p)



Induction type phase balance relay (contact normally spring centered). A second disk on same shaft balances I_a vs I_0.

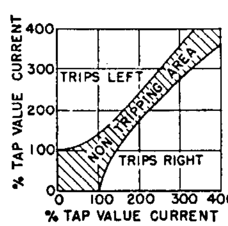
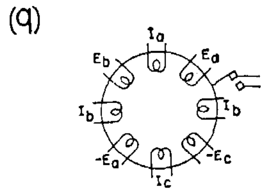


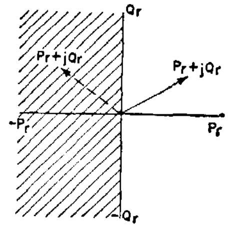
Fig. 2—Relay elements—*Continued*

Schematic	Operating Characteristics[a]

(q)

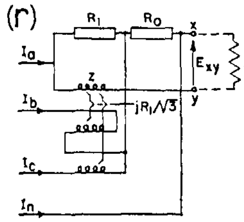


8 pole induction cup or disk connected as a polyphase directional element.

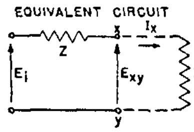


Response proportional to $K[\bar{E}_1\bar{I}_1 \cos(\theta_1-60+\phi)+\bar{E}_2\bar{I}_2\cos(\theta_2+60+\phi)]$. For positive sequence power contacts close for P_R+jQ_R in unshaded region. θ_1 and θ_2 are positive and negative sequence PF angles; ϕ is relay design or adjustment angle.

(r)



Combined pos.-seq.-current and weighted zero-seq.-current filter.



$$I_x = \frac{2R_1}{Z+Z'}(I_1+KI_0)$$

where

$$K = \frac{3R_0+R_1}{2R_1}$$

The internal voltage is:

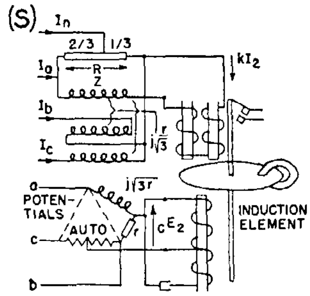
$$E_1 = 2R_1(I_1+KI_0).$$

The internal impedance is:

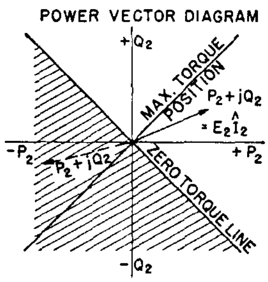
$$Z = R_1+R_0+z$$

where z is the impedance of indicated wdg. of 3-wdg. reactor.

(S)

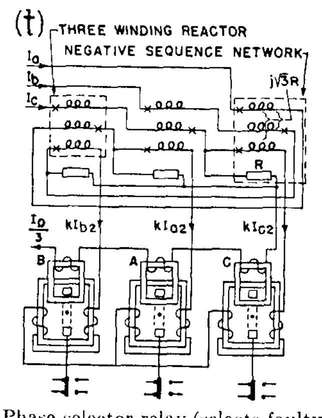


Negative sequence directional element—using potential, E_2, and current, I_2, sequence segregating networks.

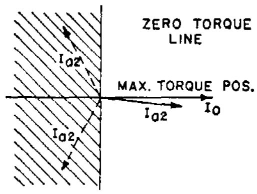


Contacts close for P_2+jQ_2 in unshaded region.

(t)



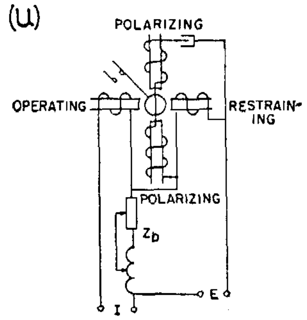
Phase selector relay (selects faulty phase for single line-to-ground fault).



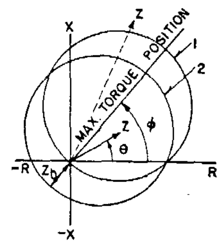
Shown for "A" element. Contacts close to right for I_{a2} in unshaded region. For fault on phase A-grd I_{a2} is in approx. position shown by full line; for a fault on B-grd or C-grd it is as shown by dotted lines. Other two phases similar.

Schematic	Operating Characteristics[a]

(U)



High speed 4 pole induction cylinder type MHO unit (offset when $Z_b \neq 0$). Radius and ϕ adjustments not shown.



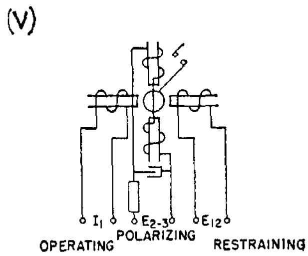
Contacts close for Z inside the circle. Balance point for circle 1

$$(Z_b=0)\quad Z=\frac{\cos(\theta-\phi)}{K}$$

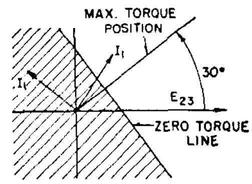
Balance point for circle 2

$$(Z_b \text{ not } 0)\quad Z=\frac{\cos(\theta-\phi)}{K}-Z_b.$$

(V)



High speed 4 pole induction-cylinder type directional starting unit.



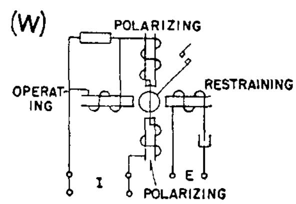
Current tripping characteristic with fixed voltage. Contacts close for I_1 in the unshaded area. Balance point for $Z=\dfrac{E_{12}\sin\beta}{KI_1\cos(\alpha-30°)}$

where

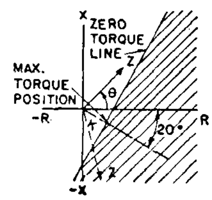
β is angle between E_{12} and E_{23}

α is angle between I_1 and E_{23}

(W)

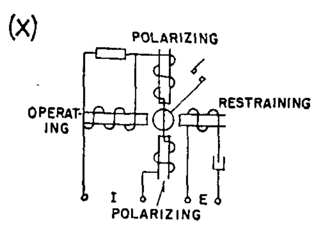


High speed 4 pole induction cylinder ohm unit (blinder).

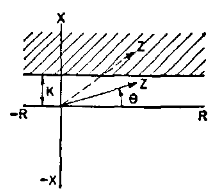


Balance point $Z\cos(\theta+20°)=K$. Contacts close for Z in the unshaded area.

(X)



High speed 4 pole induction cylinder reactance unit.

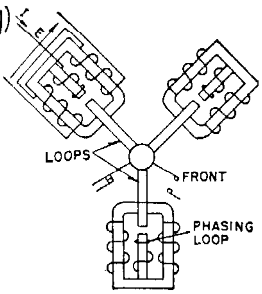


Balance point $Z\cos(\theta-90°)=K$. Contacts close for Z in unshaded area.

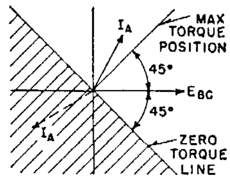
Fig.2—Relay elements—*Continued*

Schematic | Operating Characteristics[a]

(y)



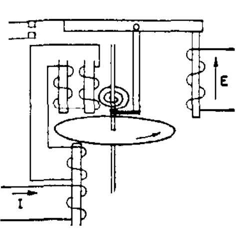
"Instantaneous" three phase inductor loop directional element. Currents in loops by transformer action from voltage windings react on flux from current magnets to produce torque.



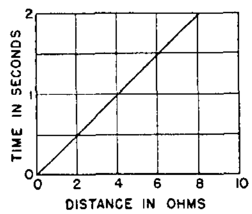
45° Characteristic Relay
First contacts close for I_A in unshaded area. Shown for balanced three phase conditions. Under any conditions relay torque is real part of

$$T = (P - N)\underline{/45^\circ}$$
$$P = E_1\hat{I}_1$$
$$N = E_2\hat{I}_2.$$

(z)



Above a current pick-up disk runs at a speed proportional to $\bar{I}$ building up the spring force until it over-balances voltage restraint $K\bar{E}$.



Contacts close in a time
$$t \propto \bar{Z}$$
at which time
$$K\bar{E} = \bar{I}$$
or
$$\bar{Z} = \frac{\bar{E}}{\bar{I}} = \frac{1}{K}.$$

relay location to the next. The more inverse characteristic also co-ordinates better with fuses.

The induction disk serves as a directional element, (*e*), or, when used with a spring, as a watt element, (*l*), the electric torque being proportional to $EI \cos \theta$, where θ is the power-factor angle. Either of these can, of course, be current polarized instead of voltage polarized for use in ground relaying when a bank-neutral current is available.

The relative phase of voltage and current can be shifted by internal or external phase-shifting devices to produce maximum torque for power factors other than unity, as in (*g*). For example, the directional element for ground relaying usually has its maximum torque for current lagging the voltage by 60 or 70 degrees to provide maximum torques for the fault conditions. A pure watt characteristic is used with the 30 and 60 degrees connections for phase directional relays, the phase shift being provided by using the voltage of a different phase from the current. With the 90 degree connection this shift is too much, and the voltage is advanced about 45 degrees by a phase shifter to provide a maximum-torque position for a current 45 degrees lagging.

The disk is provided also with an electromagnet, (*o*), producing ratio characteristics for use as a differential relay for generators or transformers. The generator-differential relay is shown in (*o*). The transformer relay has windings 1 and 2 tapped for different current-transformer ratios.

Referring to Figure 2 (*o*), the differential current produces lower-pole flux which acts in the operating direction on disk currents produced by the upper pole which is transformer fed from the same differential current. The restraint torque, giving the ratio-differential characteristic, is produced by the through current in coils 2–3, which supply disk current by transformer feed to the upper pole, and by lower-pole flux produced by the same through current.

The induction disk provides a tripping-time-proportional-to-impedance characteristic as shown in Figure 2(*z*).

Multiple Electromagnets or Disks—Two electromagnets on the disk provide for balancing mechanical torques with no phase angle effects (see (*p*)). When these are both current electromagnets, the relay is the regulating-transformer differential relay which balances the current in the shunt exciting winding against the through line current. For example in a ± 10 percent regulating transformer, it operates when the shunt current exceeds about 15 per cent of the through current. When both electromagnets are voltage energized, the voltage-differential relay results. When one is voltage (actually responsive KE^2) and one is a product element, $EI \cos (\theta\text{-}\phi)$, a balance occurs when $I \cos (\theta\text{-}\phi) = KE$. This impedance characteristic is shown in current and also in an R-X plot in (*m*).

A second disk on the same shaft provides space for two more electromagnets. This structure is used as the phase-balance relay for motor protection, whose characteristics are shown in (*p*). It is used for the 3-winding-transformer differential relay, using one operating electromagnet and three restraint electromagnets, (*n*). With two current-input windings on each electromagnet and with two relays per phase the multirestraint bus differential relay results. Its used will be described later.

Multiple-Pole Cylinder or Disk Elements—The multiple-pole cylinder or disk element is illustrated in (*f*). The example shows how it would be energized to act as a single-phase directional element having torque proportional to $EI \cos \theta$. This element also serves as a polyphase-directional element by the connection, (*q*). The multiple-pole element is flexible making possible a variety of other combinations.

Four-Pole Induction-Cylinder Elements—These high-speed elements serve a variety of purposes as shown in Figure 2(*u*), (*v*), (*w*), and (*x*). The element (*u*), designated a mho element, operates with torque $E^2 \cos (\theta - \phi)$ restrained by torque proportional to E^2. It produces the circular-impedance-tripping locus passing through the origin or relay location, the same as shown for the induction disk in (*m*). Or with either element, the circle can be shifted from this position by current compensation, IZ_b, in the restraining circuit, as indicated. A directional-starting unit, (*v*), is obtained using current times shifted-quadrature voltage for operating and the product of two delta voltages for restraint. This results in maximum torque for current 30 degress ahead of the quadrature voltage or about 60 degrees lagging the unity-power-factor position.

The special impedance characteristic, (*w*), obtained, by

I^2 operating against $EI \cos(\theta+20 \text{ degrees})$ is used to restrict the tripping area to assist other relays in differentiating heavy load swings from faults. A reactance element, (x), is obtained similarly with the phase-shift devices arranged so that the maximum-torque line is along the x (reactance) axis.

Inductor-Loop Element—The inductor loop, (h), provides a very high speed and very reliable directional element which has been used for many years now in high-speed distance measuring relays.

Balance-Beam Element—The basic balance-beam impedance element is shown in (i), a balance occurring for $E/I = Z_0$. For higher impedances than Z_0 (current relatively lower) the contacts remain open; whereas for lower impedances (relatively higher currents) they close quickly. Since the balance is mechanical, the phase angle between voltage and current is of minor consequence, and the tripping characteristic, plotted on an R and X diagram is substantially a circle.

Modified-Impedance Characteristic—The circular characteristic may be shifted by some circuits auxiliary to the element as shown in (k), in order to provide better discrimination between fault currents and load and swing currents on long, heavily-loaded transmission lines. The shifting imparts a directional characteristic to the relay in addition to narrowing its tripping region to more nearly just that required for faults.

Networks and Auxiliary Circuits—It may be noted that in discussing fundamental relay elements certain auxiliary circuits external to the mechanical relay have been introduced: in (g), the phase shifter; in (j), the Rectox; and in (k), a full fledged network to produce in the relay element proper, the desired currents. This is a trend of which we shall certainly see more as time goes on, as static circuits are devised to produce a simple current output proportional to the desired function of the various line currents and voltages.

Sequence-Segregating Networks, I_1+KI_0—The method of symmetrical components has been the key that has unlocked the door to a number of the aforementioned possibilities, some of which are illustrated in (r), (s), and (t). The positive- and zero-sequence network in (r) is commonly used in pilot-wire relaying, where it is desired to compare over the wires only one quantity, which is a good measure of the fault current irrespective of what kind of fault it may be, that is *A-B, A-Grd, ABC*. The relay can be given almost independent and widely different settings for phase faults and ground faults, using the single relay element. For example, it may be set for one ampere of ground fault to provide the requisite sensitivity, but for ten amperes of 3-phase current to avoid operation on loads.

A negative-sequence directional element is shown in (s). It is an adequate directional element for ground faults on reasonably well-grounded systems, and requires only two potential transformers rather than three as with usual residual-directional relays.

Another novel application, (t), is the phase-selector relay to determine which phase is faulted. This information is necessary in single-pole tripping and reclosing schemes. It is predicated on the knowledge, from symmetrical-components theory, that the negative-sequence current in the faulted phase only is in phase with the zero-sequence current. Individual overcurrent elements in the three phases could not be used for this selection as all three would pick up for a single line-to-ground fault on many solidly-grounded systems.

II. PROTECTIVE SCHEMES

Protective schemes may be conveniently classified as follows:

1. Apparatus Protection
2. Bus Protection
3. Line Protection

Thus, in Fig. 1, generator and transformer protection come under the "Apparatus" classification; generator buses, high-voltage buses, and substation buses, under the second classification; and high-voltage transmission lines and feeders under "Line Protection."

The relay application chart, Table 2, has been included for ready reference in determining the operating principles and application of various specific relay types referred to throughout this chapter.

3. A-C Generators

Most a-c generators above 1000 kva and many smaller machines are equipped with differential protection arranged to trip if the currents at the two ends of each phase winding differ. This scheme is shown in Fig. 3.

Smaller machines are sometimes operated without differential protection, but if paralleled with larger machines

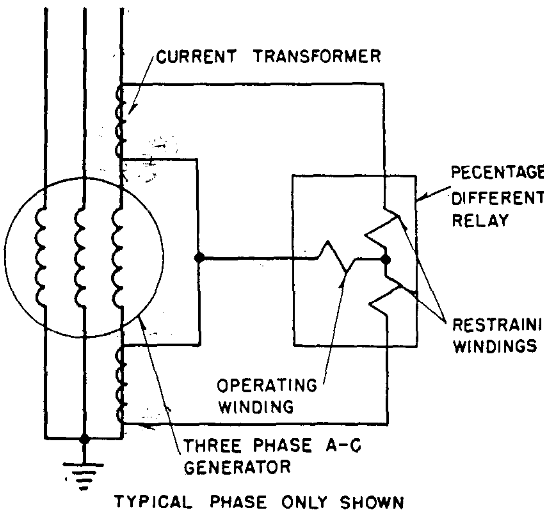


Fig. 3—Connections for one phase using the percentage differential relay for generator protection.

or with a system, they may be arranged to trip off on a reversed flow of power into the machine.

For differential protection the Type CA normal-speed, induction, ratio-type relays are used in the large majority of cases, their speed (about 0.1 second relay time on severe faults) being adequate to prevent serious burning of the iron in nearly all cases. However, a high-speed generator-differential relay, Type HA[19], is available providing

1-cycle protection and is being used with 100 per cent success in a number of important applications.

The relay is usually arranged to trip the generator, field circuit, and neutral circuit breakers (if any) simultaneously by a manually-reset lockout relay in new installations. Frequently the relay also trips the throttle and admits CO_2 for fire prevention. For example it may be required to coordinate with other high speed relays or to reduce the shock to the systems.

If a single-winding generator (or equivalent) is connected to a double bus through two breakers, a current transformer matching problem is introduced. The current transformers in the connections to the busses may carry large currents from one bus to the other in addition to the generator current. Thus, matching is not assured by identical current transformers as in the simpler case of Fig. 3, and consequently, the Type HA relay is preferred for this case because of its superior discriminating qualities.

The Type CO relay is also used for generator differential protection. It provides straight differential protection, as contrasted with percentage differential, the diagram being the same as Fig. 3 without the restraining coils. Its setting must be considerably coarser than that of the CA relay because there are no restraining coils to desensitize it when high through-fault currents are flowing.

Double-Winding and Multiple-Winding Generators—The differential protection scheme of Fig. 3 does not detect turn-to-turn short circuits within the winding because the entering and leaving currents of a phase remain equal. Double and multiple winding machines provide a means for obtaining such protection in the larger, more important generators. The currents in the parallel branches, become unequal when turns are short circuited in one branch. The differential relays, Type CA or HA, can be arranged to detect shorted turns, grounds* or phase-to-phase faults, by placing one current transformer in the neutral end of one of the parallel windings, and one of double ratio at the line end in the combined circuit. The choice of schemes depends somewhat on the facility with which leads can be brought out and the necessity of overlapping the generator breaker. With hydrogen cooling additional leads can be brought out through the necessary gas-tight bushings only with considerable difficulty, and usually there is no space for transformers inside the hydrogen compartment.

Effect of the Method of Grounding—The method of grounding the generator neutral affects the protection afforded by differential relays. For example, if sufficient grounding impedance is used so that a ground fault at the generator terminals draws full load current, then for a fault at the midpoint of the winding, where the normal voltage to ground is half as great, the fault current will be approximately one-half the full load current. When a ground fault occurs 10 percent from the neutral end of the winding, the fault current, being limited largely by the neutral impedance, is about 10 percent of full load current. This corresponds to the sensitivity of a 10 percent differential relay and, therefore, represents the limit of protection for phase to ground faults with such a relay. For lower impedance grounding the differential relay protects closer to the neutral. With higher impedance grounding, the limit of protection for ground faults is farther from the neutral end, and for an ungrounded machine, the differential protection is ineffective against ground faults. The protection afforded for phase-to-phase, double-phase-to-ground, or three-phase faults is relatively unaffected by the method of grounding. A complete discussion of the methods of grounding is given in Chap. 19.

Solidly Grounded and Low Resistance or Reactance Grounded Machine—If the generator is solidly grounded, or grounded through a reactor or resistor, drawing at least full-load current for a ground fault at a line terminal, the usual 10 percent differential relay operates for practically any short circuit within the machine and for grounds to within 10 percent of the neutral, or closer if the ground current is higher.

Ungrounded, and Potential-Transformer-Grounded Generators—are those grounded only through the natural capacitance from the metallically connected windings, buswork, and cables to ground. The potential transformer from neutral to ground, if properly applied†, serves as a measuring device only. To insure that this is so, it must be liberally designed so that under no condition will its exciting current become appreciable compared with the charging current to ground. Otherwise, ferro-resonance may occur. Usually a full line-to-line rated transformer will suffice. The potential transformer and a voltage relay such as the SV (instantaneous) or CV (inverse time) may be used to initiate an alarm or optionally to trip. Or, on lower voltages, a static voltage unbalance indicator may be used connected directly to the primary circuit. Such an instrument is the Type G. These devices supplement the generator differential protection to provide indication or tripping for ground faults. Light resistance grounding as covered in the next section is generally preferred to ungrounded operation.

Light-Resistance-Grounded Generators — This scheme and an associated protective arrangement is illustrated in Fig. 29 of Chap. 19. Indication from a voltage relay, connected in parallel with the resistor as shown, or from a current relay, such as the Type BG, connected in series with the resistor, may be used to sound an alarm or to trip, depending on the application. Combinations of sensitive alarm and coarser trip, or of alarm and time-delay trip, have also been used. The latter gives time to transfer the load to another machine at the hazard of operating with a fault on one phase.

This scheme was designed primarily for the unit station arrangement in which a generator and step-up transformer are operated as a unit without an intervening bus. However, it can also be used where an intervening bus carries the station service transformer and one or two short feeder cables. A limited amount of selectivity is possible by the use of a polarized relay, such as the CWP-1, which obtains most of its energy from a potential coil in parallel with the grounding resistor. Such a relay used in the station-service feed, for example, can detect a ground on that circuit.

Field Protection—While a large number of machines still operate without any protective relays to function on

*With the same limitations as for a single winding generator.

†See also Light-Resistance-Grounded Generators.

Table 2—Relay Application Chart

RELAY TYPE	GENERAL APPLICATION								CHARACTERISTICS: RESPONSIVE TO, OPERATES ON								TIME CHARACTERISTIC			RELAY TYPE
	Transmission Line										Distance		Directional							
	Phase Fault	Ground Fault	Bus Protection	Transformer Protection	Generator, Motor Machine Protection	Auxiliary Relay	Pilot Wire Protection	Carrier Protection	Current	Voltage	Impedance	Reactance	Potential Polarized	Current Polarized	Current Differential	Miscellaneous	Inverse	High Speed	Definite	
A					○	△(1)				○								○		A
AV					○	○(17)				○								○		AV
BG			○						⊘									○		BG
BL				○	○				⊘							△(2)	○			BL
CA (GEN)			△		○		△								○		□			CA (GEN)
CA (TRANS)				○											⊘		□			CA (TRANS)
CA-4			△	○	△										⊘		□			CA-4
CA-6			○	△	△										⊘		□			CA-6
CAM				○											⊘		○			CAM
CF-1					○					○						○(3)	○			CF-1
CH	△	○					△						○				□			CH
CHC		○					△							○			□			CHC
CH-3	○	○					△						○				□			CH-3
CHV-3	○	○					△			△(4)			○				□			CHV-3
CI						○(5)				○							□			CI
CJ						○(18)				○							□			CJ
CM	△				○				⊘						△(6)		□			CM
CO	○	○	△(7)	△(7)	△(7)				⊘								⊘			CO
CO (AMMETER TYPE)					○				⊘							○	⊘			CO (AMMETER TYPE)
COH	△	△	○	△(7)	△(7)				⊘								⊘			COH
COI	○	○	△(7)	△(7)	△(7)				⊘								⊘			COI
CP					○					○							⊘			CP
CR	○	○							⊘				○				⊘			CR
CRC		○							⊘					○			⊘			CRC
CRN-1					○					⊘			○				⊘			CRN-1
CRS		○							⊘				○			○(8)	⊘			CRS
CT				○	○											○(9)	⊘			CT
CV						○				⊘							⊘			CV
CV (VOLTMETER TYPE)		○				○				⊘						△	⊘			CV (VOLTMETER TYPE)
CW					○				⊘	○			○			○(10)	⊘			CW
CWC		○							⊘					○			⊘			CWC
CWK					○				○				○			○(19)	□			CWK
CWP		○							⊘				○				⊘			CWP
CWP-1		○							⊘				○				⊘			CWP-1
D-3					○											○(11)	⊘			D-3
DT-3				○	○				○(9)							○(11)	⊘			DT-3
H-3	○	○					△	△					○					○		H-3
HA (GEN)					○										⊘			○		HA (GEN)
HA (TRANS)				○											⊘			○		HA (TRANS)
HCB	○	○					○		⊘						○			○		HCB
HCR	○	○							⊘				○				⊘	○		HCR
HCRC		○							⊘					○			⊘	○		HCRC
HCZ	○										⊘		○				⊘	○		HCZ
HD	○	○					△		⊘						⊘	○(12)		○		HD
HKB	○	○						○	⊘									○		HKB
HO-2		○						△	⊘									○		HO-2
HQS						○		○(16)	⊘									○		HQS
HPS						○			⊘									○		HPS
HR	○	○							⊘				○					○		HR
HR-1	○	○							⊘				○					○		HR-1
HRC		○							⊘					○				○		HRC

TABLE 2—RELAY APPLICATION CHART—Continued

RELAY TYPE	General Application: Transmission Line: Phase Fault	General Application: Transmission Line: Ground Fault	General Application: Bus Protection	General Application: Transformer Protection	General Application: Generator, Motor Machine Protection	General Application: Auxiliary Relay	General Application: Pilot Wire Protection	General Application: Carrier Protection	Characteristics: Responsive To, Operates On: Current	Characteristics: Voltage	Characteristics: Distance: Impedance	Characteristics: Distance: Reactance	Characteristics: Directional: Potential Polarized	Characteristics: Directional: Current Polarized	Characteristics: Current Differential	Characteristics: Miscellaneous	Time Characteristic: Inverse	Time Characteristic: High Speed	Time Characteristic: Definite	RELAY TYPE
HRK		○						○	⌀					○				○		HRK
HRP		○						○	⌀				○					○		HRP
HV-3	○	○					△	△		△4			○					○		HV-3
HX		○									⌀	⌀	○					○	⌀	HX
HXS		○										⌀						○	⌀	HXS
HY	○										⌀		○					○	⌀	HY
HZ	○						△	△			⌀		○					○	⌀	HZ
HZ-1	○		△				△	△			⌀		○					○		HZ-1
HZ-3	○		△				△	△			⌀		○					○		HZ-3
HZM	○						△	△			⌀		○					○	⌀	HZM
IM						○(20)				○										IM
IW						○(21)			○	○										IW
JD						○				○						○(13)			⌀	JD
LC-1			○							○								○		LC-1
LC-2			○							○								○		LC-2
MF					○				○									○		MF
MG-6						○			△	○								○		MG-6
MN						○			○									○		MN
PG						○		○	○							○(11)		○		PG
PS-1, PS-2, PS-3						○			○							○(14)		○		PS-1, PS-2, PS-3
RB						○				○								○		RB
RC						○				○						○(15)		○	⌀	RC
RF					○					○						○(3)		○		RF
RS, RSN						○	○	○		○						○		○		RS, RSN
SC	○	○	△7	△7	△7	○		△	⌀									○		SC
SG						○				○								○		SG
SGR-1						○				○						○(15)		○		SGR-1
SGR-12						○				○						○(15)		○	⌀	SGR-12
SM-1, SM-3	○	○	△7	△7	△7			△	⌀									○		SM-1, SM-3
SV	△	△				○				⌀								○		SV
SX						○		△		○								○		SX
TD						○				○						○(13)			⌀	TD
TG-1						○		△		○						○(11)			⌀	TG-1
TH				○		△				○						△2			⌀	TH
TK						○				○						○(13)			⌀	TK
TR						○		△		○						○(11)			○	TR
TS						○		△		○						○(11)			○	TS
TSI						○(22)				⌀									⌀	TSI
TSO-1						○				○									⌀	TSO-1
TSO-2						○					○								⌀	TSO-2
TSO-3						○					○								⌀	TSO-3
TSP						○(23)				○									⌀	TSP
TT-1						○		△	○							○(11)		○		TT-1
TV						○			○							○(11)		○		TV

○ Major Characteristic Or Application
△ Other Applications Or Characteristics
□ Induction Type Relay—No Intentional Time Delay
⌀ Characteristics Which Are Adjustable
△1 For Direct Current Reversal Or Voltage Drop
△2 Has Thermal Element Indirectly Heated From Current Winding
(3) Frequency Relay
△4 Has Voltage Restraint
(5) Synchronism—Check Relay
△6 Current Balance For Phase Unbalance Or Phase Failure
△7 Used In a Differential Scheme
(8) Directional Element Operates From Negative Sequence Current And Voltage
(9) Operates From Exploring Coils Or Temperature Changes
(10) Operates When Applied Watts Exceed Setting
(11) Direct Current Relay
(12) Current Balance For Parallel Line Protection
(13) Timing Relay
(14) Supervisory Relays For A-C. Pilot Wire
(15) Reclosing Relays
(16) Auxiliary Relay Unit For Carrier Relaying Scheme
(17) Fault Detector
(18) Respond To Voltage Changes With Operating Time Proportional To Voltage Change
(19) Power Factor Relay
(20) Telemetering Receiver
(21) Telemetering Transmitter
(22) Magnetizing Inrush Tripping Suppressor For Type CA, CA-4 and CA-6 Relays
(23) Magnetizing Inrush Tripping Suppressor For Type HCB Relay

loss of field, there is a trend to more general use of equipment for this purpose. On some systems where loss of field would cause serious low voltage and danger of instability but where the system is operated in a way which can tolerate the loss of one machine, automatic means are being provided to disconnect the machines on loss or partial loss of field.

Fairly common is the use of a d-c under-voltage relay, a Type D-2 d'Arsonval relay in series with a resistor, connected across the slip rings for field short circuit detection; also a similar element across the field ammeter shunt for undercurrent or open field detection. These do not provide complete protection, and there are a number of installations of reactive power relays used at the generator terminals in conjunction with under voltage to trip for any field reduction which would cause serious low voltage.

On many closely knit metropolitan systems loss of field has been found to be not serious if immediately corrected. The operator attempts to restore the field, the generator in the meantime operating at somewhat reduced load as an induction generator. If he cannot restore field within a few minutes, he must trip the line circuit breaker to avoid injurious rotor heating.

Field Ground Detection—Some form of field ground detection is frequently provided. It is considered most important to detect the first ground because a second could short circuit part of the field winding causing unbalance and vibration which could wreck the machine. The a-c scheme provides complete coverage for solid grounds. The d-c scheme gives nearly complete coverage, complete if the main field rheostat is varied. In some instances vibration detectors[26] are used if the machine is known to be operating with a ground on its field. This will trip the unit instantly in event a second ground occurs. The over-all protection scheme frequently includes armature and bearing temperature indication and sometimes alarms. Less frequently field temperature indications are provided. The voltage regulators are sometimes equipped with over- and under-voltage protection, and, of course, overspeed protection is provided.

In addition to the protection described, generators can be equipped with over- or under-voltage, frequency, overspeed, and loss of field, and temperature responsive devices.

4. Transformer Protection

Power-transformer protection in general includes overload devices to protect the transformer and fault-detecting devices to protect the system and limit damage in event of fault in the transformer.

In the first category is the thermal relay immersed in the transformer oil but energized from a current transformer so that it responds to the copper temperature. This relay, obtainable only on new transformers, has alarm contacts to announce the approach of dangerous temperatures, and tripping contacts that close if an unsafe temperature is reached.

Oil temperature indicators perform a somewhat similar function though less effectively. For large power transformers the order of magnitudes of copper and oil time-constants are 5 minutes and 7 hours respectively. Thus, an emergency overload for a half hour could seriously damage the transformer without reaching an oil temperature which might be reached daily after several hours of moderate load. The thermal relay responsive to copper temperature will permit the overload to be carried, if safe, but will protect the transformer otherwise.

Fault-detecting relays include percentage and straight differential schemes similar to generator-protective relays but include provisions for the magnetizing inrush current and for transformer ratio and phase shift. Also, transformers are often included with the transmission line into a single protective zone. This is particularly true of the smaller sizes such as network transformers. Many small power transformers (600–3000 kva) are provided with internal protective links that act like a single-operation breaker as the fusible element whips through the oil in the top of the tank thereby disconnecting the transformer in event of internal trouble. Others are fused to provide disconnection from the line in event of transformer failure.

A typical application of the CA relay to a star-delta connected transformer bank is shown in Fig. 4. Neglecting

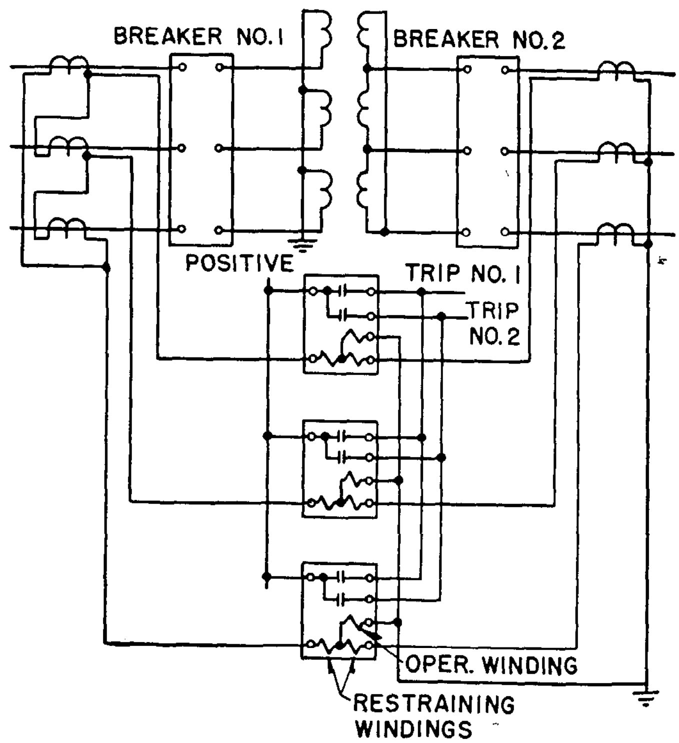


Fig. 4—Differential protection of a grounded star-delta transformer bank with CA relays. Note that the current transformers are connected star on the delta side and delta on the star side.

exciting current, the top phase line current on the right hand side is made up of the difference of two transformer currents or the difference of the two top phase line currents on the left hand side. Consequently, it is compared with this difference current obtained by connecting the left hand current transformers in delta. These two currents are not exactly equal, even with a perfectly sound transformer bank, because of the magnetizing current.

Magnetizing Inrush—While under steady operating conditions, the magnetizing current amounts to only 5 or 10 percent, it may rise to several times full load current when a transformer is first energized, and decay rather slowly from this value; that is, it may be as much as full load current even a full second after the transformer is first energized. This magnetizing inrush current is fully displaced and hence, contains a large d-c component.[23] The inrush current is greatest if the switch is closed at the zero point of the voltage wave. Its magnitude depends also on the residual excitation and on the leakage reactance in the supply circuit and transformer primary. Data for determining the value of the magnetizing inrush is given in Chap. 5. Ordinarily the residual flux density is low when the transformer is first energized. However, when a severe fault occurs near a transformer at a time when its flux density is maximum (voltage zero), and if the fault is interrupted an odd number of half cycles later, the residual flux at the instant of re-energizing may approach normal density. As this requires the fault to start and stop at zero voltage, it is seldom fully realized.

The rate of decay of the magnetizing inrush current depends on losses and is particularly slow when a large bank is paralleled with one already operating and quite near to a large generating station.[20] The d-c component, which flows at first over the supply circuit, transfers to a circulating current between the two transformer banks, and this dies out very slowly because of the high L/R ratio. For example, when the magnetizing current has dropped to 50 percent of full load in a 60-cycle transformer having 0.25 percent primary resistance, the reactance to resistance ratio is 200/.25 or 800/1. The corresponding L/R ratio or time constant which determines the rate of decay of the d-c component is 2.1 seconds.

The Type CA normal-speed differential relay most commonly used for transformer protection has a 50-percent differential characteristic and 2.5-ampere minimum trip. It is prevented from operating during the magnetizing inrush by the large restraint, the inverse time characteristic, and the braking action of the direct current on the induction disc. It is found adequate in all but the extremely rare cases where one large bank is paralleled with another.

When the differential relay cannot, because of its inherent characteristics, avoid tripping on the magnetizing inrush, a timing device can be used, which desensitizes the protection during the timing interval by requiring a drop in voltage in addition to operation of the differential relay to produce tripping during the inrush period. This device, known as a magnetizing-inrush tripping suppressor,[1] is used primarily with high-speed differential relays or with pilot-wire relays when a transformer is included as a part of the line.

High-speed transformer differential protection (Type HA relay) is required in certain circumstances to coordinate with other high-speed system protection, particularly where stability is critical. It must be used with the tripping suppressor as outlined above. This unit is therefore built as an integral part of the Type HA transformer relay.

Three-Winding Transformers are protected in the same manner as two winding transformers except that the

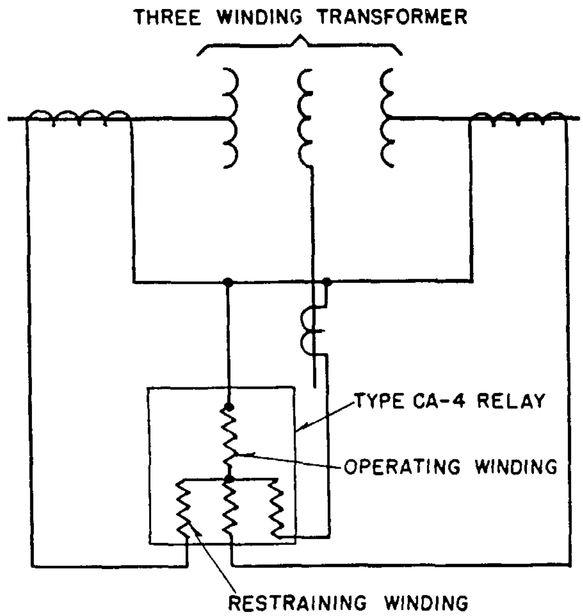


Fig. 5—Single line diagram showing the arrangement of circuits when using the CA-4 relay for the differential protection of a three-winding power transformer.

Type CA-4 relay for this purpose has three restraining coils to be associated with the three transformer windings as shown in Fig. 5.

Regulating Transformers—Regulating transformers for voltage and phase-angle control constitute a special problem because of the change in ratio taps during operation. Figure 6 illustrates the most modern differential relay protection for such a unit. A Type CAM relay, Fig. 2(*p*), having one disk and two electromagnets is arranged to trip if the current in the shunt-exciting winding of the regulator greatly exceeds the proper proportion of the series-line current. For example with a ± 10 percent voltage regulator, a typical relay would operate for any current in the shunt winding greater than 11.5 percent of the

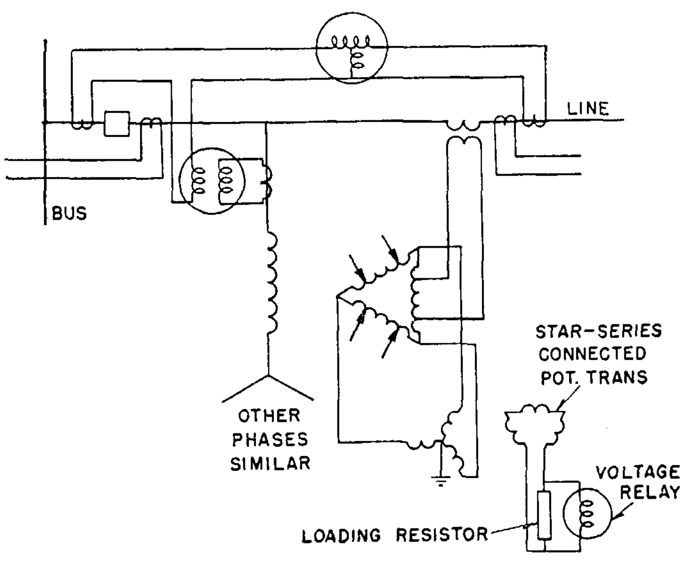


Fig. 6—Typical large regulating transformer protection.

line current. This provides sensitive protection for the shunt transformer. In addition normal two-winding transformer-differential protection is applied around the entire unit, providing overlapping protection with the bus and line protection and guarding the series transformer. It is difficult to provide complete protection to the series transformer and tapped regulating windings. Optimum use is made of ground protection, for example as shown in Fig. 6. However, the possibilities of this protection vary with the arrangement of windings and whether grounded or not.

Remote Trip for Transformer Faults—Because of the high record of reliability of large power transformers, a circuit breaker between the high-voltage side of the transformer and the line frequently cannot be justified, purely for protection of the transformer, and the transformer is very little hazard to the line. However, an intermediate measure costing much less than the high-voltage circuit breaker is frequently provided to trip the remote circuit breaker (or breakers) necessary to clear in the event of a transformer fault. The transformer differential relay is sometimes used to initiate a remote trip signal over a carrier or pilot wire channel, particularly if the channel is already available for some other purpose. Another method is to close a fast spring-operated high-voltage grounding switch in response to the relay indication. This trips the ground relays at the other terminals of the line, at the expense of some added shock to the system.

5. Bus Protection

The advantages of bus protection in clearing faults rapidly from a system are well recognized by the industry and the provision of relay protection for major station busses has been standard practice for a number of years. The problems involved in such protection are also quite well known. One of the principal problems is the saturation of current transformers by the d-c transient component of the short-circuit current as in Fig. 7. In severe

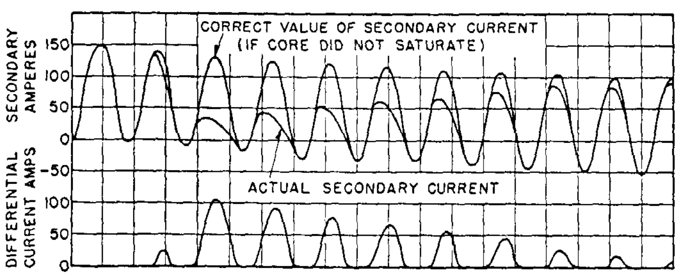


Fig. 7—False differential current caused by d-c saturation of current transformers during the condition of an asymmetrical short circuit. The d-c time constant for the case shown is 0.14 sec. The incoming transformers are assumed not to saturate. Thus the error current of the outgoing transformer is the differential current through the relay.

cases the d-c transient component may require 100 times as much flux capacity in the transformer as is required by the a-c component to completely prevent saturation.

There are a number of successful solutions to this problem, as well as the fault-bus scheme,[11,16] which completely avoids it. One is the use of large current transformers which will not be saturated by the d-c component. These are used in the simple differential scheme, Fig. 10(*a*), with low-resistance leads to minimize the current transformer requirements. The formula specifying the requirement of iron cross section, turns and lead resistance for nonsaturation[18] is:

$$\frac{E_s}{R} = 4.44\ IT$$

where T is the short-circuit current d-c time constant in cycles. I_s is the a-c exciting current in the secondary, selected as the threshold of saturation. It would be taken as less than the relay setting by a suitable factor of safety.

E_s is the required secondary rms, a-c voltage corresponding to I_s on the a-c saturation curve of the current transformer. This determines the needed iron cross section and turns, or the iron if turns are fixed.

I is the crest value of symmetrical subtransient current, secondary amperes.

R is the secondary circuit resistance in ohms, including the transformer winding and leads up to the relay (or point at which all current transformers are paralleled.)

The a-c flux is neglected. It is usually relatively small as the ratio of maximum d-c flux to a-c flux is $2\pi T$ or 37.7 for a 6-cycle d-c time constant.

For $R = 0.5$ ohm, $I = 100$ amperes, $T = 6$ cycles, and taking I_s as one ampere, in considering a 5-ampere relay setting, E_s becomes 1332 volts. A current transformer which would generate this voltage at 1.0 ampere exciting current is very large. Thus, this is a bull-by-the-horns solution, and the size, weight, and cost can be afforded only in the most important installations. However, it does provide the possibility of instantaneous tripping without any time delay.

A method[18] has been developed for calculating with reasonable engineering accuracy, the time-to-saturate with offset currents, and the time and current settings required to prevent misoperation with time-delay overcurrent relays and usual current transformers.

Induction Type Overcurrent Relays—On busses of moderate time constant, say 0.1 second or less, and with somewhat better than average current transformers, satisfactory protection can be obtained with a straight differential scheme, Fig. 8, using a fast induction element. A small ratio of maximum to minimum fault is favorable to this application. Relaying times of the order of 3 or 4 cycles for maximum faults and up to 8 or 12 cycles on minimum faults can be obtained in some cases. As mentioned, the performance can be predicted.[18] However, the time delays involved in less favorable cases are frequently so long as to point the need of a better solution.

Even on substation busses having a d-c time constant as short as 0.01 second, false tripping has been experienced with 300/5 bushing current transformers and standard induction relays in the connection of Fig. 8, with 4 ampere tap, 0.5 time lever, giving 0.15 second time at 10 times tap, and with a fault current of 13 000 amperes. While this has been overcome by changing to 1200/5 current transformers, nevertheless present practice would be to install ratio-differential relays in these cases, providing both greater sensitivity and greater safety factor.

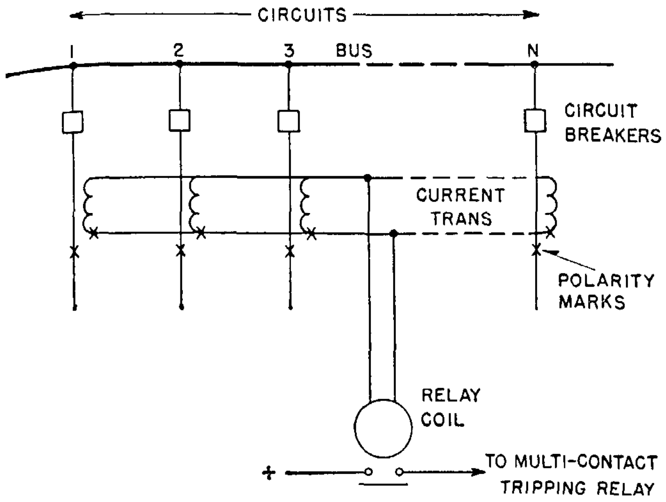


Fig. 8—Bus-differential-current relay scheme.

Multirestraint Ratio Differential Relays—Relays[17] have been developed which will not operate falsely even when used with normal sized current transformers which saturate due to the d-c component of current. These are multirestraint relays, connected as in Fig. 9; however, their success is due also to exploitation of variable percentage characteristics, and the tendency of the d-c component to brake, rather than drive, the induction disk.

No small part of this development is the reduction of the operating limits to a few simple rules which insure safe application. This relay scheme provides operation generally in from three to six cycles and can be set as low as one percent of the maximum through-fault current.

Linear Coupler Scheme—The multirestraint relays just described may, of course, be used when the setting does not need to be as low as one percent of the maximum-through-fault current. However, on busses where a setting of four percent or more of the maximum through-fault provides the requisite sensitivity, a simpler and faster scheme (one cycle) can be used known as linear-coupler bus protection.[21,27] The linear coupler is an air-core mutual inductance used directly in the primary circuit in the same manner as a current transformer except that the secondaries are usually connected in series, as shown in Fig. 10, instead of in parallel as are current transformers, Fig. 8.

The secondary induced voltages are proportional to the primary currents, a ratio of five volts per 1000 amperes being commonly used. These voltages, which add up to zero for through faults and to a value proportional to the fault current for internal faults are joined in a series loop to the relay as shown in Fig. 10. The ±1 percent tolerance within which the mutual inductances are held commercially, limits the maximum possible false differential to 2 percent. Thus the minimum setting of 4 percent allows a 2:1 factor of safety. Solenoid elements, Fig. 2(a), are used for settings down to about 1500 amperes on a 6-circuit bus, and polar elements, Fig. 2(j), with saturating transformer and Rectox for lower settings.

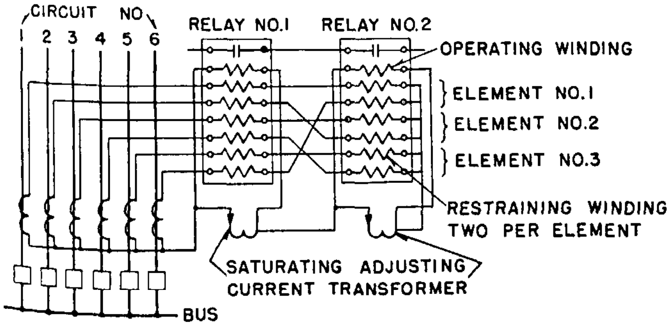


Fig. 9—Protection of a six-circuit bus using two multi-restraint relays per phase.

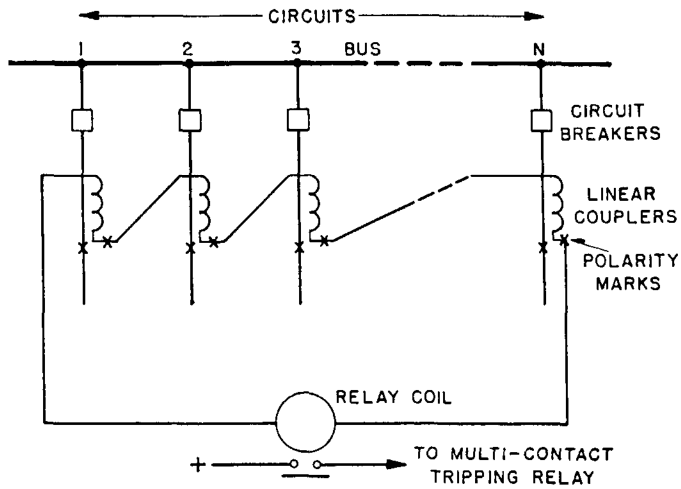


Fig. 10—Linear-coupler bus-protection scheme.

Impedance Schemes—Busses having reactors in a majority of the feeders and possibly in the bus-tie circuits provide the possibility of protection by impedance relays.[13] Impedance or modified-impedance elements can be used, see Fig. 2, (i) and (k). For a fault on the bus, the maximum impedance measured is that of the arc which is taken as about 300 to 500 volts per foot for current above 500 amperes. Considering the possible arc length during the first few cycles of fault, a maximum arc voltage can be computed and this, divided by the minimum fault current gives the greatest fault impedance encountered for internal faults, that is, for faults on the protected bus. Provided this impedance is smaller than the impedance from the relay to a fault anywhere beyond the reactors, a basis of discrimination exists and the impedance scheme can be used.

Two relay arrangements are used. When the bus tie circuits include reactors, separate impedance relays can be used on each generator or transformer feed to the bus, operation of any of which will trip the bus. This arrangement is most feasible when the generators and transformers are matched, acting as a unit, and the generators on the bus are either high and low pressure units of a single combination or are treated as a single generator. The other arrangement requires totalizing all of the main feeds to the bus and the use of a single set of impedance relays. The grouped main sources provide the possibility of a large false differential current for through faults on one of these main circuits. The voltage is also low under this condition if the source circuits are not equipped with reactors. Usually a fast induction relay, Fig. 2(*d*), is used together with adequate current transformers in the main feeds so that it will not operate for through faults on those feeders. Combined impedance and fast-induction-element operation is then required to trip the bus.

Directional relays can be used in a variety of ways, either as the basic protection, tripping when fault current flows into and not out of the bus,[24] or as an adjunct for determining which of several bus sections included within a common differential protective zone is in trouble.[13]

Fault Bus—For new and certain existing segregated phase busses, the fault-bus scheme[11,16] provides a distinctly different mode of attack to the bus protection problem. All metal parts to which the bus may flash are connected together and grounded through a current transformer and relay. This construction lends itself particularly well to metal-clad switch-gear. The entire cubicle or switchhouse is insulated from ground except for the ground connection through the current transformer The simplicity of this scheme is strongly in its favor where the construction permits its use. However, it is sometimes difficult to secure overlapping protection with the adjacent system elements.

Summary of Bus Protection—While personal preference, experience, and factors peculiar to a particular installation play a large part, some of the general factors that lead to the selection of one or the other of the several schemes are described below. One-cycle operation, simplicity, and savings in cable costs, as compared with the multirestraint scheme, are favorable to linear couplers.

Quite adequate speed (3–6 cycles), the use of existing current transformers, the use of current transformers which can be used for certain other purposes also (such as back-up protection), simple application rules, and ability to set for minimum faults one percent of maximum-through-fault sometimes eliminating need of separate ground relay, are all favorable to the multirestraint system.

Existence of reactors and the cost or difficulty which would otherwise be involved of installing current transformers on all feeders, favors impedance schemes.

The fault-bus scheme is limited to cases where the structure can be insulated from ground, but in these cases its simplicity is favorable.

Simple time-delay over current frequently involves excessive delay, but if used with ordinary current transformers, it may be lowest in cost.

The directional schemes are used to good advantage by some and have the advantage of securing fast operation with ordinary current transformers, but are considered less favorably by others because of the number of contacts to be co-ordinated for correct operation.

In most cases, spring-operated, manually-reset auxiliary tripping relays are used, unlatched electrically by the main differential relays. These trip the necessary circuit breakers and provide lockout.

6. Transmission Line Protection

As systems have grown in extent and complication, from the simple radial systems of the early 1900's to the looped and interconnected systems of the present, the task imposed on the protective relay has become increasingly more difficult. However, developments in the protective relaying art have kept pace with the requirements. Through the introduction of improved relaying principles and better use of the old principles, high-speed action can be obtained on the complicated systems of today with better overall results than that previously possible on the simple radial systems.

Starting with the induction-type overload and reverse-load relay in about 1901, which used power for discrimination, the directional overcurrent relay with inverse-time characteristics was introduced in 1910. Later, in 1914, the definite minimum time characteristic was added. This simplified the relay coordination problem and is still used in the greater proportion of overcurrent relays today. The first impedance-, or distance-measuring relay, the type CZ, was introduced in 1922.

Shortly after this the importance of speed in fault clearing, particularly with inter-connected systems, was beginning to receive merited attention and in 1929 the high speed impedance relay, type HZ, operating in one cycle and using the balance beam principle, was introduced[31]. At about the same time, circuit breaker operating times were lowered from about 24 cycles to 8 cycles.

The reductions of overall fault clearing times that could be realized by these progressive changes in the art[82] are shown in Fig. 11.[54] Starting with times up to 2 seconds for the slow-speed relays and 24 cycle breakers, the change to high-speed relays brought the time down to about 27 cycles with about one second in the end zones.

Decreasing the circuit-breaker operating time to 8 cycles further lowered the overall clearing time to 8 to 10 cycles for about 80 percent of the line length but left times of about 27 cycles in the end zones.

In about 1935 carrier current relaying passed out of the experimental stage[51,52,53,54] and reached general acceptance, making available uniform high speed action throughout the entire section. Shortly later, in 1938, the Type HCB relay [62] based on symmetrical component principles made one-cycle operation practical over two a-c pilot wires. Summarizing, and referring to Fig 11, with 8 cycle breakers the total clearing time is under 0.2 seconds for pilot-wire or carrier current relaying, 0.2 seconds (with 0.5 seconds for the end zones) for high speed distance relaying and 0.2 to 2.0 seconds or longer for overcurrent protection, depending on the layout.

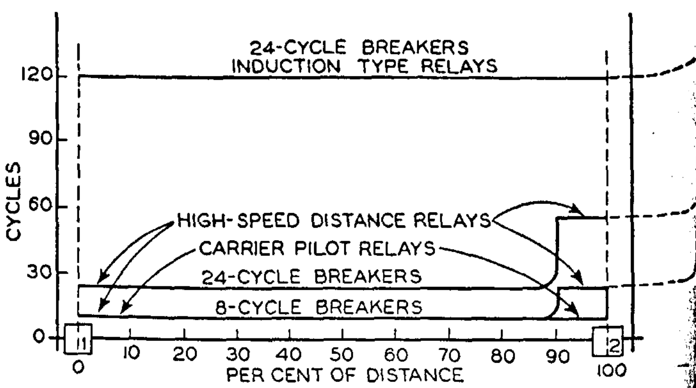


Fig. 11—Reduction of fault clearing time obtainable through the use of higher speed circuit breakers and relays.

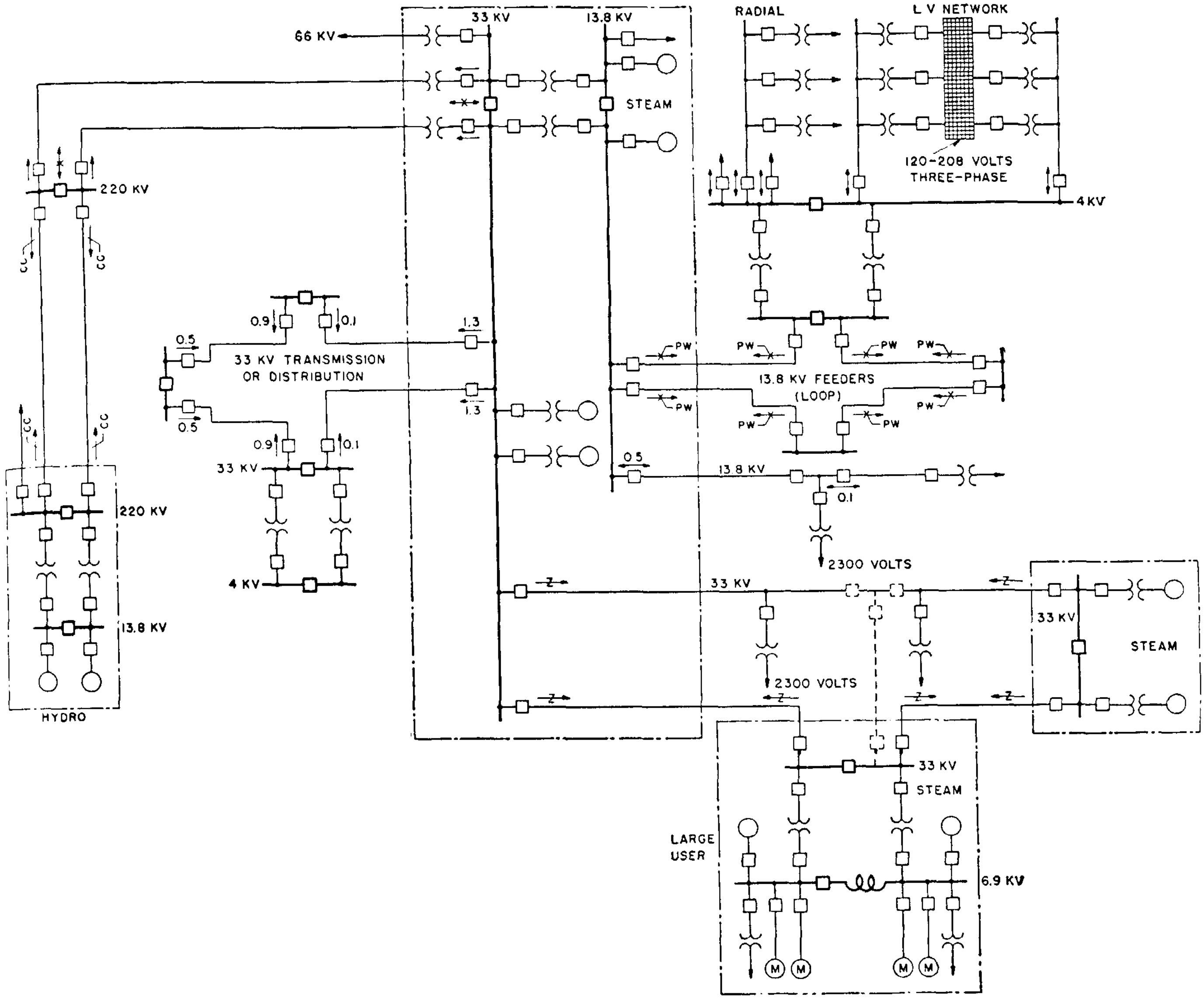


Fig. 12—Composite power system illustrating typical protective problems and their solution.

A cross section view of the industry today shows all of these relay and circuit breaker types and speeds to be in current operation. Although speeds of new oil circuit breakers appear to have stabilized at 8 cycles except for special cases, there appear to be distinct fields for both normal-speed and high-speed relays.

7. Protection of a Typical System

A composite diagram showing the typical transmission line conditions in many of the large systems is given in Fig. 12. For example the main transmission lines are shown as 220-kv, although they may be from 66 to 287-kv, bringing in power from a remote hydro plant or interconnecting with an adjacent system. These lines are equipped with the finest and fastest protection, high-speed distance, or carrier-pilot relays.[54] Balanced-line protection[36] may also be used if the lines are paired although high-speed distance or carrier relays are required to secure fast operation with one line out of service.

The 33-kv circuits, often looped or interconnected, carry bulk power out through the territory served by the particular utility, to substations in the various towns and communities. A looped 33-kv circuit may use directional overcurrent relays, Type CR, set with selectively higher time settings each way around the loop, as shown in Fig. 12. Impedance relays, preferably the step-type HZ, or alternatively the normal-speed impedance relays, Type CZ, may be used as in the 33-kv loop on the right. Impedance relays are particularly desirable if interconnections are contemplated as shown dotted. Loops involving short lines of 33- or 13.8-kv lend themselves well to pilot wire protection[62,63] as in the center right.

Induction-type overcurrent relays, usually with instantaneous trip attachments for operation at the higher currents, will be found on a majority of the radial feeders and 4-kv or 2.3-kv primaries. Network feeders are cleared at the load end by the network relays, essentially a reverse power form of protection.

8. Relay Symbols

Relay symbols are useful in illustrating the form of protection used for each element of a system. With modifying notations as to relay types and settings, these symbols compress the otherwise complicated picture of complete system protection into a form that can be readily visualized. The standard symbols are given in Table 3. Their use has been illustrated in Fig. 12.

TABLE 3—RELAY SYMBOLS

(a) SYMBOLS FROM THE A S A STANDARDS.

OVERCURRENT	◄—►	DIRECTIONAL OVERCURRENT	——►
OVERVOLTAGE	◄V►		
UNDERVOLTAGE	►V◄		
DISTANCE	◄Z►	DIRECTIONAL DISTANCE	—Z►
		POWER DIRECTIONAL	—W►
BALANCED OR DIFFERENTIAL CURRENT	◄X►		
OVER FREQUENCY	◄∿►		
UNDER FREQUENCY	►∿◄		
OVER TEMPERATURE	◄T►		
BALANCED PHASE	◄∅►	PHASE ROTATION	—∅►
PILOT WIRE (CURRENT DIFFERENTIAL)	◄X► PW	PILOT WIRE (DIRECTIONAL COMPARISON)	—X► PW
		CARRIER PILOT	—X► CC

Where the operation of a relay is conditional upon the flow of ground current (residual or zero sequence) this shall be indicated by prefixing the ground symbol thus:-

Residual Overcurrent ⫴◄——► Directional Residual Overcurrent ⫴——►

Other prefixes such as Ⓟ and Ⓝ to indicate operation on positive or negative phase sequence quantities, and suffixes to indicate the relay types, inclusion of instantaneous trip attachments, etc. may be added at the discretion of the user.

(b) FREQUENTLY USED VARIATIONS OF THE STANDARD SYMBOLS.

OVERCURRENT GROUND WITH INSTANTANEOUS ATTACHMENT	⫴◄—► I
GROUND DIRECTIONAL WITH INSTANTANEOUS ATTACHMENT DIRECTIONALLY CONTROLLED	⫴——► I
POWER DIRECTIONAL WITH INSTANTANEOUS ATTACHMENT DIRECTIONALLY CONTROLLED	⫴W► I
BUS CURRENT DIFFERENTIAL	B◄X►
BUS GROUND DIFFERENTIAL	B◄X► ⏚

9. Fault Frequency and Distribution

About 300 disturbances (or one per ten miles) occurred per year in a typical system operating 3000 miles of 110-kv circuit. This system used mostly overcurrent and directional relays, and in a 4-year period experienced 2800 relay operations of which

92.2 percent were correct and desired
5.3 percent were correct but undesired
2.1 percent were wrong tripping operations
0.4 percent were failure to trip

The faults were as follows:

Lightning	56 percent
Sleet, Wind, Jumping Conductors	11 percent
Apparatus Failure	11 percent
Close-in on Fault	11 percent
Miscellaneous	11 percent

Relative Number of different kinds of faults—The relative numbers of different types of faults vary widely with such factors as relative insulation to ground and between phases, circuit configuration, the use of ground wires, voltage class, method of grounding, speed of fault clearing, isokeraunic level*, atmospheric conditions, quality of construction and local conditions. Thus the figures given below serve merely to indicate the order of prevalence and emphasize that there are usually a great many more line-to-ground faults than faults of other types.

Three-Phase Faults	5 percent
Two-Line-to-Ground Faults	10 percent
Line-to-Line Faults	15 percent
Line-to-Ground Faults	70 percent
Total	100 percent

10. Overcurrent Protection

The general plan of coordination with overcurrent relays on a radial system is shown in Fig. 13. The time shown in each case is the fastest operating time for a fault at the location of the next device in sequence. At lighter generating capacity the fault currents are reduced and all operat-

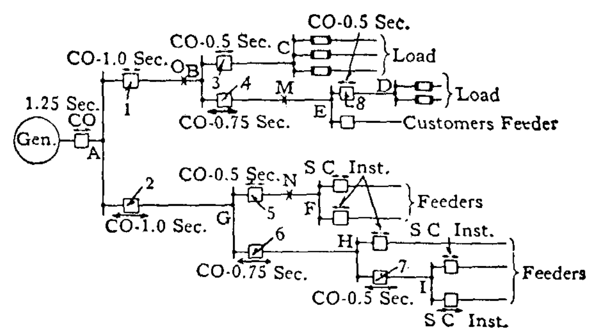


Fig. 13—Coordination of overcurrent protection on a radial power system.

ing times increase, but because of the inverse time characteristics of the relay the margins between successive relays also increase.

Relays used with feeder circuit breakers must be coordinated with fuses of distribution transformers and with the main and branch line sectionalizing fuses.[83] Several characteristic curve shapes are available in different designs of the induction-type overcurrent relays as illustrated in Fig. 14. These provide latitude in selecting the relay that coordinates best with the fuse curves at the current involved.

The definite minimum time characteristic provides a ready means for coordinating several relays in series with only an approximate knowledge of the maximum current, and results in relatively small increase in the relay time as the fault current is lowered. It is used in the majority of overcurrent relay applications. The inverse and very inverse characteristics are sometimes more favorable where close coordination with fuses is required. They also make it possible to take advantage of the reduction of maximum fault current as distance from the power source increases. Several relays in series can be set for the same time for

*Number of storm-days per year

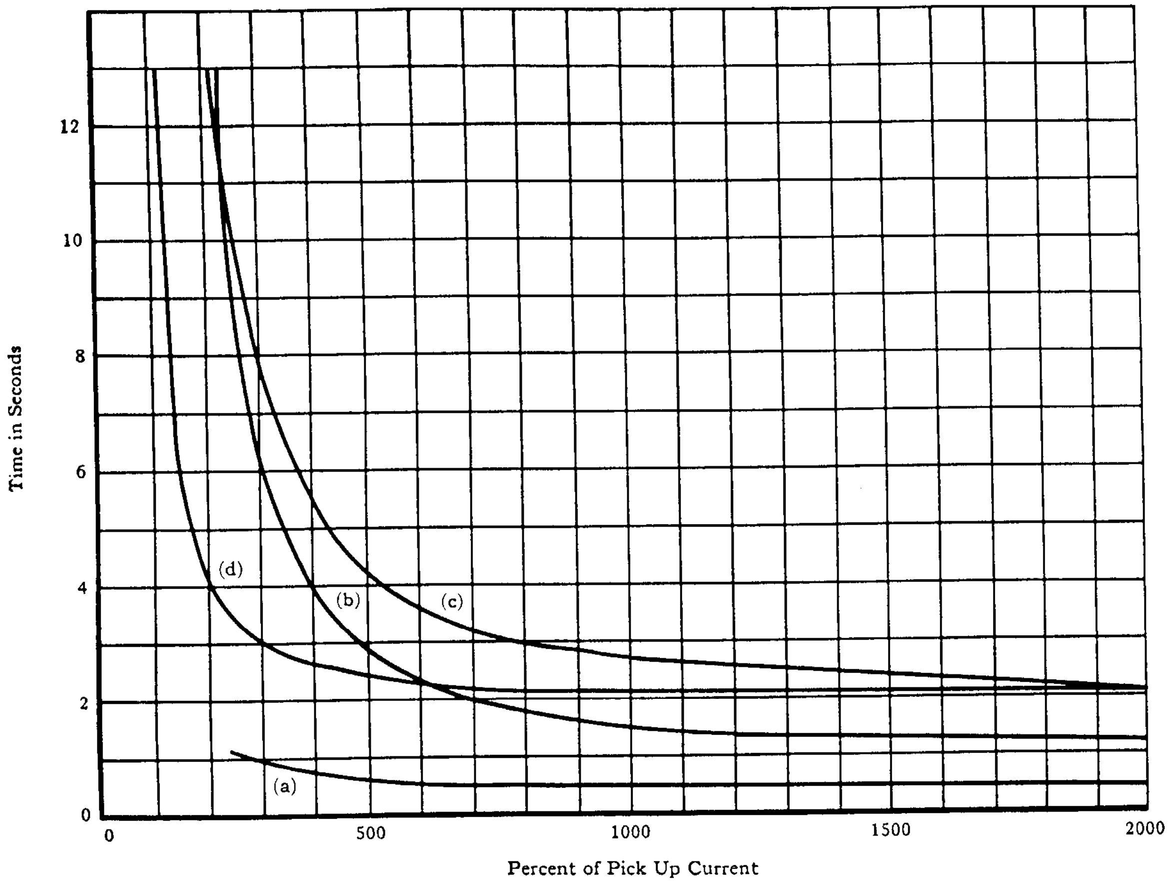


Fig. 14—Characteristics of various induction type overcurrent relays.

(a) Type COH.
(b) Very inverse-low energy relay, Type CO.
(c) Inverse-low energy relay, Type CO.
(d) Standard, definite minimum time, Type CO relay.

faults immediately beyond the relay and still provide the requisite 0.25 second or more margin for fault beyond the next relay because of the lower current value for fault in that location. For example the timing on curve (b), Fig. 14, doubles when the current is reduced from 700 percent to 400 percent of pick-up value. Several settings of 0.3 second at 700 percent could be used in series, while still having 0.3 second margin between successive relays if the fault current dropped in the ratio 7 to 4 between successive locations.

The choice of relays is also influenced in certain cases by the lower burden of the "low energy" and "very inverse" types.

11. Normal-Speed Impedance Relay*

The time-distance tripping characteristic of the Type CZ normal-speed directional distance relay is illustrated in Fig. 15, which shows a number of line sections in series. This may equally well be a loop, the two ends of the section shown being at the same supply point. The tripping time of the relay increases in direct proportion to the distance from the relay to the fault, except that the minimum time is about $\frac{1}{4}$ second for a fault at the relay. Each relay is

*The trend is toward the high-speed impedance relay described in Sec. 12 even for intermediate voltage transmission lines.

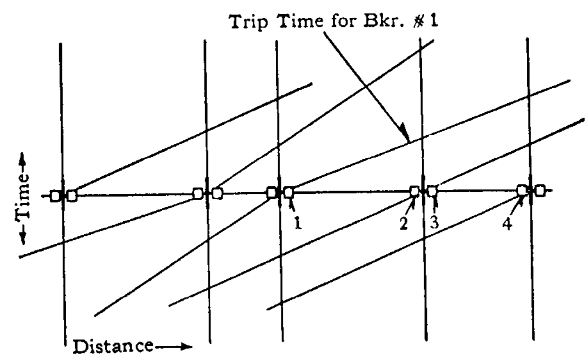


Fig. 15—Time-distance curves of the Type CZ relay. The slope of the curve is changed by varying the resistance in series with the potential coil. The minimum operating time with zero voltage on the relay is about $\frac{1}{4}$ sec.

adjusted to trip in approximately $\frac{3}{4}$ second for a fault at the next bus, except as will be noted.

It is essential that for a fault near bus 4, breaker No. 3 be tripped in preference to breaker No. 1. Thus the operating time of relay No. 1 must exceed that of relay No. 3 for fault at location No. 4 by one circuit breaker operating time plus margin. For 8-cycle breakers a reasonable breaker time plus margin is 0.4 second.

The operating time for faults anywhere on the system can be readily determined by drawing the straight lines representing the relay times, using whichever criterion rules in each case; that is, 0.75 second at the next bus or 0.4 second above the next relay at the second bus. The particular time values mentioned are typical only. The relay tripping time is independent of current magnitude once the overcurrent setting has been exceeded and timing thereby initiated. Thus variations in the amount and location of connected generating capacity, or switching lines out, does not materially affect the coordination of the distance type relays over the remainder of the system.

The normal-speed type CZ relay is not usually employed on lines shorter than those in which at least 5 volts secondary result at the relay for a fault at the other end of the line. As the relay is normally subjected to full voltage and must discriminate on values between zero and that for a fault at the other end of the line, the operating forces approach the frictional forces below this limit.

12. High-Speed Impedance Relays, HZ and HZM

The high-speed distance type relay has the step type time-distance characteristic illustrated in Fig. 16, obtained

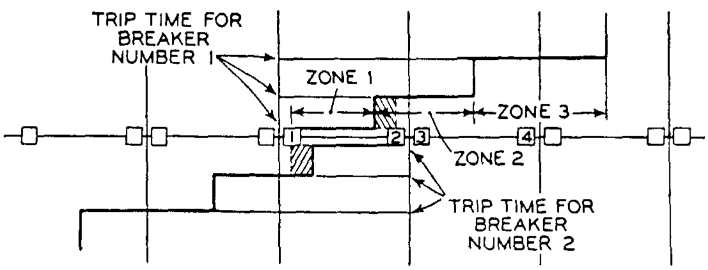


Fig. 16—Time-distance curves of the Type HZ step type, high speed distance relay. When carrier current is added the time is reduced to that shown dotted.

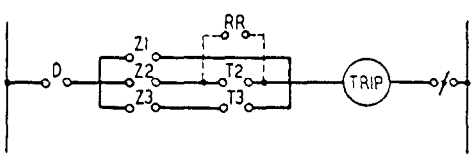


Fig. 17—Contact circuit for producing the stepped time-distance characteristic shown in Fig. 14.

Z_1—First Zone Impedance Element Contact.
Z_2—Second Zone Impedance Element Contact.
Z_3—Third Zone Impedance Element Contact.
T_2, T_3, timer contacts having separately adjustable time settings.
D—Directional element contact.

by separate directional, impedance, and timing elements with contacts connected as shown in Fig. 17. There are a total of three balance-beam type impedance elements, each arranged with a current operating winding at one end of the beam and a voltage restraining winding at the other. When the ratio of voltage to current falls below the impedance setting of the relay high-speed action closes the contacts.

The impedance elements Z_1, Z_2, and Z_3 are set for successively greater distances. The directional element closes only for faults in the desired tripping direction from the relay. The third-zone impedance element, which operates when either of the other two elements operate, is used to start the timer that closes first a second-zone timing contact T_2, and later a third-zone timing contact, T_3.

Thus for a fault in the first 90 percent of the section, known as zone 1, the contacts D and Z_1 operate, giving immediate high-speed tripping in one to three cycles, as indicated in the timing chart of Fig. 16. While the other elements also operate their action in zone 1 is unimportant because the circuit breaker has already been tripped. Thus, in zone 1 the tripping time is that of elements Z_1 and D.

For the second zone, which extends approximately to the middle of the next section, contacts D, Z_2 and T_2 in series do the tripping, provided the fault lasts for the time setting T_2. If the fault is in the next section it will be cleared by the proper breaker in advance of T_2 operation, although back-up protection is provided by the second zone setting extending into the next section. This also provides operation for bus faults if they are not previously cleared by bus-protective relays.

The third zone, corresponding to tripping through the contacts of elements D, Z_3, and T_3, completely overlaps the next section, providing complete back-up protection. It must of course be timed selectively with the T_2 timing of the next section.

The flexibility of this arrangement in molding its characteristic to various section lengths and breaker and relay times is apparent. The highly successful operation of several thousand such relays in service indicates that for practical systems, which of course depart in many ways from the simple ideal case represented in Fig. 16, the flexibility is sufficient to secure in general the operation outlined.

For good operation the line should be electrically long enough so that there will be at least 5 percent voltage at the relay for a fault at the next bus, although in special cases successful operation can be obtained somewhat below this limit.

In some cases the CZ characteristic lends itself better to coordination with other back-up protection, but the high speed of the Type HZ first-zone element is desired. For this purpose these two elements have been combined and make available the time-distance curve shown in Fig. 18.

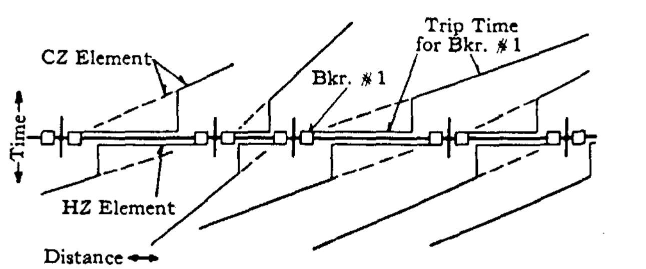


Fig. 18—Typical time-distance characteristics of the HCZ relay. Note the slope of the CZ element necessary for different length sections to secure selectivity.

Modified Impedance Relay, Type HZM.[41] The operating characteristic of the standard type HZ impedance relay is nearly independent of the phase angle between current and voltage. That is its "reach"-vs.-angle characteristic is a circle centered at the origin as shown in Fig. 19. This

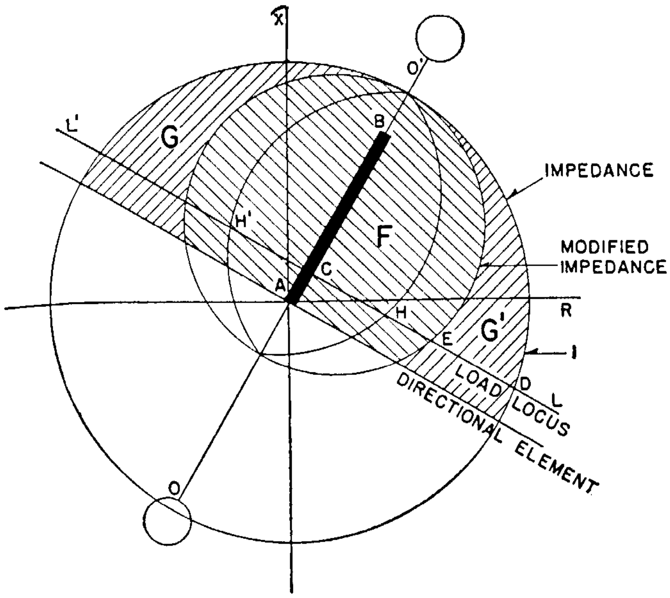


Fig. 19—Modified-impedance characteristics provides improved selectivity between heavy load swings and line faults.

characteristic provides adequate discrimination between load and faults in a majority of cases. However, increased use is being made of the Type HZM relay having the modified-impedance characteristics shown in Fig. 2(k) for the protection of long or heavily-loaded lines. It is necessary that the relays permit the maximum expected steady or swing loads without tripping. On long or heavily-loaded lines, and especially if high-speed reclosing is used, it becomes more and more difficult for the relays to distinguish between heavy swing loads and fault conditions. The modified-impedance relays provide the improved discrimination necessary in these cases.

Their operation can best be shown by plotting on a single diagram the impedances corresponding to three things.

1. Faults on the protected line.
2. Heavy load swing conditions.
3. The relay characteristics.

These are shown for a simple case in Figure 19. Considering the simplified case of a two-machine system with all impedances having the same R/X ratio, let the system impedance be laid out on a resistance-reactance chart as the line $OABO'$ representing the impedance from one machine to the other. The transmission line being studied is a tie-line constituting the section AB, shown heavy, of this total impedance. If R and X axes are drawn through A, as shown, the impedance with respect to these co-ordinate axes are those seen by the relays at A. Thus, the locus of impedances seen by the relay at A for a solid fault on the protected line consists of the line A-B, having impedances from zero up to the full line impedance.

It can be shown that if the generator voltages at the two ends of the system are equal in magnitude and are at first in phase, then are moved out-of-phase, resulting in load transfer over the line A-B, the impedance locus viewed from A is along the line L-L'. That is, the impedance seen by a relay at A, for the load condition, is the impedance vector drawn from the origin, A, to a point on the line L-L'. This line bisects O-O' perpendicularly. The no-load points, corresponding to zero angle between machines are at infinity either way along L-L', whereas the 180 degree out-of-phase condition is at the intersection, C. All intermediate loads are somewhere along the line L-L', the point L corresponding to power flow A to B and point L' to power flow from B to A.

The large circle with center at the origin, A, represents a pure impedance characteristic, as in Fig. 2(i). The smaller circle having the same "reach" beyond B is a modified impedance characteristic, Fig. 2(k). It can equally well trip for all faults on the protected line but is less likely to trip for very heavy loads or load swings. Successively heavier loads are represented by progressing through points L, D, and E along the load locus. The modified characteristic taken with directional element, trips for faults or loads in the cross-hatched area F, while the pure impedance element trips also in the areas G and G'. Thus the modified characteristic permits heavier loads without tripping.

When carrier relaying is used so that tripping requires closure of the relay at each end of the line, the small circle, for the relay at A can be advantageously shifted further to the right, so that the combined action of this relay and its mate at the other end of the line limits carrier tripping to the restricted zone between the two arcs at H and H'. The back-up protection must be given sufficient timing to ride through swings or eliminated entirely except for a high-set, long-time element.

The modified impedance element provides for independent adjustment of the radius of the circle and the location of the center as shown in Fig. 2(k) and hence makes possible the superior discriminating characteristics needed for long or heavily-loaded lines as outlined before.

13. Carrier Pilot Relaying

A pilot channel such as that obtainable by carrier current over the power circuit, or by a microwave beam, provides the possibility of simultaneous high speed tripping of both circuit breakers in one to three cycles for faults throughout the entire section. The significance of fault-clearing speed on system stability is treated fully in Chap. 13. However there are, altogether, a number of reasons why carrier-current relaying has been employed in preference to other systems. These are:

1. *Stability*—Simultaneous clearing improves system stability and increases the loads that can be safely carried over parallel interconnecting lines.
2. *Quick Reclosing*[53]—Simultaneous tripping is essential to fast reclosing, the combination being particularly effective in increasing stability with single tie lines.
3. *Shock to System*—System shock, evidenced by voltage dips and dropping of synchronous load is lessened by fast clearing.
4. *System Design Flexibility*—Desirable system arrangements that can not be relayed with sufficient speeds otherwise, are possible with carrier relaying.
5. *Growth of Faults*—The more serious three-phase and double-ground faults generally originate as line-to-line or single-ground and with sufficient speed of clearing the spreading to other phases is greatly reduced.
6. *Ground Relaying Improved*—On systems where high-speed

ground relaying is not feasible otherwise, carrier pilot relaying provides an ideal solution.

7. *Out-of-Synchronism**—The carrier channel provides means for preventing operation of protective relays by power swings or out-of-synchronism conditions, yet clearing faults during such conditions.
8. *Simultaneous Faults*—The added basis for discrimination makes possible superior relay performance under simultaneously occurring faults.
9. *Joint Use*—From an economic point of view joint use of the carrier channel for point-to-point communication, or for control or remote metering, may indicate the use of carrier pilot where the relaying requirements alone do not justify it.

Carrier relaying operates on the principle of tripping quickly all terminals through which power flows into a line provided fault power does not flow out at any other terminal. If fault power flows out at any terminal, that terminal continues to transmit a straight telegraphic carrier signal over the line, which is picked up by all other terminals on that particular line and prevents tripping. No time delay is necessary for internal faults since tripping for external faults can be prevented by the carrier signal. Since carrier is not transmitted for internal faults, the short circuiting of the carrier channel by the fault is of no consequence.

Directional Comparison System—The type HZ or HZM directional comparison system utilizes the stepped impedance elements, Fig. 2(*i*) or (*k*), as its basic actuating elements, (see section on High-Speed Impedance Relay).

*Can also be accomplished without carrier,[39] with some differences.

Corresponding second and third zone ground over-current elements, Fig. 2(*b*), are provided. The carrier control has the net effect of eliminating all second-zone time delay for faults within the protected section. This is accomplished by closing of the contact *RR*, Fig. 17, whenever a fault is present and carrier is stopped because the direction of flow is "inward" at each end of the line.

The mechanism is indicated generally in Fig. 20. Occurrence of a fault anywhere within reach of the third zone relays, closes Z_3 starting carrier and setting up a circuit so that the receiver relay will close if carrier is removed by some other action. If the fault is internal, *D* and Z_2 close, stopping carrier transmission from either end of the line. The receiver relays, *RR*, immediately complete the trip circuit. If the contact circuit *RR* is opened manually the carrier can be cut out and a stepped-distance relay scheme remains. Thus the carrier is thought of as simply eliminating the time delay in the end zones, indicated by the shaded areas in Figure 16. The stepped-distance elements and an inverse-time ground current element provide the backup protection in this system.

Phase Comparison System—The phase comparison system differs functionally in that the current directions or phases at the two ends of the line are compared rather than the power directions. Networks are used to derive a single-phase function of the line currents as in the pilot-wire relay. This function may be referred to simply as the "current" at each end of the line since it is a measure of the several phase currents.

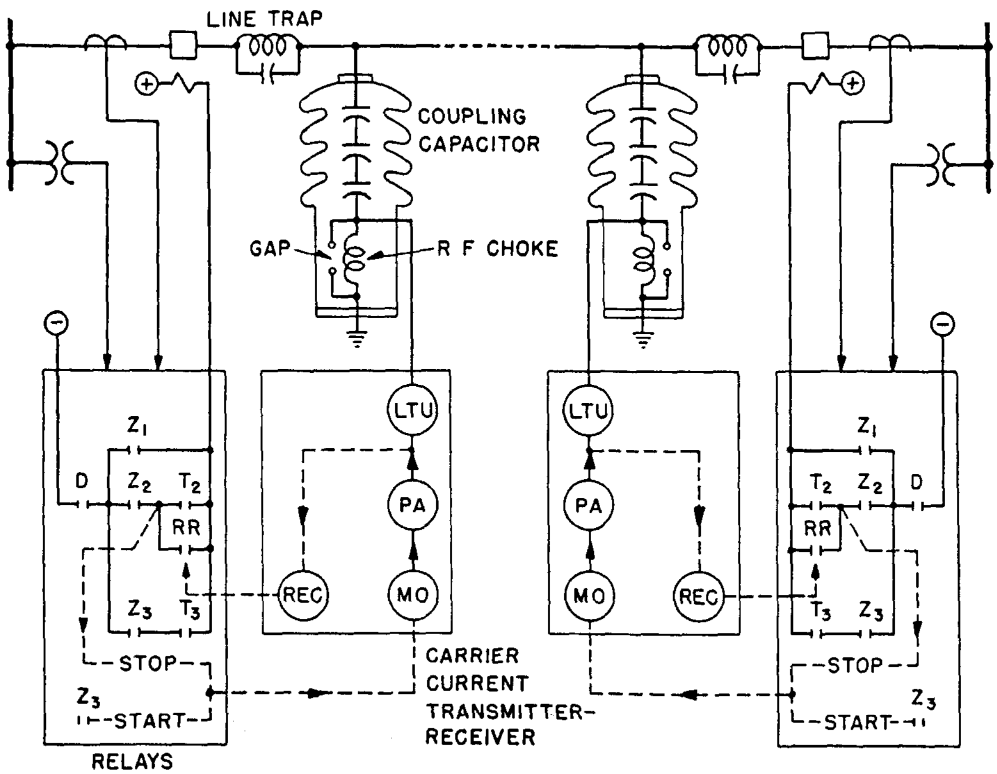


Fig. 20—Carrier current relay system including relays, carrier current transmitter-receivers, coupling capacitors, and chokes.

Dotted lines indicate symbolically the carrier controls
MO—Master Oscillator REC—Receiver
PA—Power Amplifier LTU—Line Tuning Unit

If the currents at the two ends of the line are in phase and of fault magnitude, carrier is transmitted on alternate half cycles of current from either end of the line, resulting in substantially continuous carrier on the line from one end or the other. For an internal fault the current at one end of the line reverses or remains below the fault detector setting so that carrier is sent only half of the time. The relay is arranged so that this produces tripping.

Figure 21 shows (heavy) the relative positions of the locally and remotely transmitted carrier pulses for internal

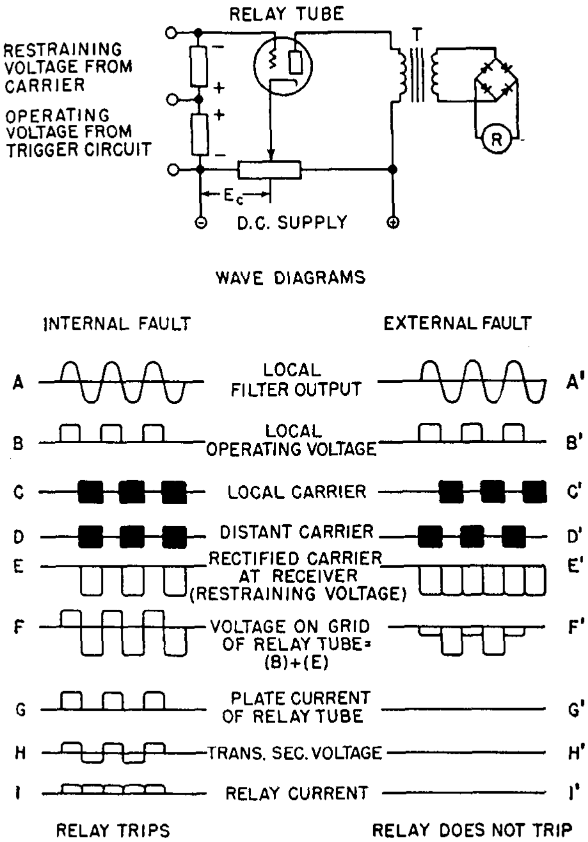


Fig. 21—Relay tube circuit and typical wave diagrams of type HKB carrier relaying system.

faults (left) and external faults (right). Local pulses of operating voltage are applied to the relay tube every half cycle. For the internal fault the pulses of restraining voltage caused by the carrier occur in the opposite half cycle from the operating pulses. Hence pulses of plate current occur and result in tripping. For the external fault, restraining pulses occur during both half cycles, and since these pulses are, by design, greater than the operating pulses, no trip current results. With the entering and leaving line currents not quite in phase, some relay current flows. However, as shown in Fig. 22, a substantial phase difference can be tolerated without causing tripping.

In this system, the carrier portion is purely pilot protection. Back-up protection must be added as an entirely separate entity. Stepped-distance relays, or simply directional-overcurrent relays, are used for back-up protection.

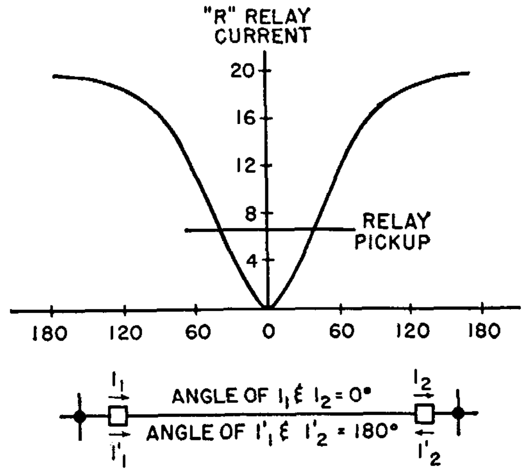


Fig. 22—Typical overall characteristics of the Type HKB carrier relaying system.

A few of the factors considered in determining whether to use one or the other of these systems are the following:

Favoring the Phase Comparison System

1. Can be added for high-speed protection with any existing relay scheme for back-up.
2. No potential transformers are required. Low-tension potentials are sometimes adequate for back-up relays.
3. Inherently trip proof on out-of-step conditions. (Out-of-step blocking relays are included in the other scheme if needed.)
4. Not subject to trip by induced ground current from a parallel line.
5. Back-up relays entirely separate. Can take either high-speed or back-up out of service without affecting the other.

Favoring the Directional-Comparison System Using Stepped-Distance Relays

1. More generally, applicable to multi-terminal lines.
2. Provides better discrimination between loads (tapped from lines) and faults. When transformers are tapped along the lines, it is not desired to trip the line for faults on the low-tension feeders.
3. Can trip with fault currents less than twice load currents.
4. More flexible for system changes.

However, on many lines either system is entirely applicable and might equally well be used.

Fig. 20 shows the complete equipment required for a carrier current relay system. Relays shown are of the directional-comparison type. The carrier components are the same with the phase-comparison type relays.

(a) The relays; practically the same as for high-speed distance-type protection except with the addition of the receiver relay, directional auxiliary relays, and out-of-step elements which are housed together. The Type HZ relay is shown in Fig. 23.

(b) The carrier current transmitter-receivers operated from the station battery and with an output of 5 to 40 watts at 50 to 150 kilocycles when keyed. The outdoor set contains line-tuning equipment for matching through the coupling capacitor to the high tension line. When the set is located indoors it is connected by coaxial cable to the line tuning equipment in a separate housing located near the coupling capacitor.

(c) The coupling capacitor. The connections to ground, and to the potential device if used, are through radio-frequency

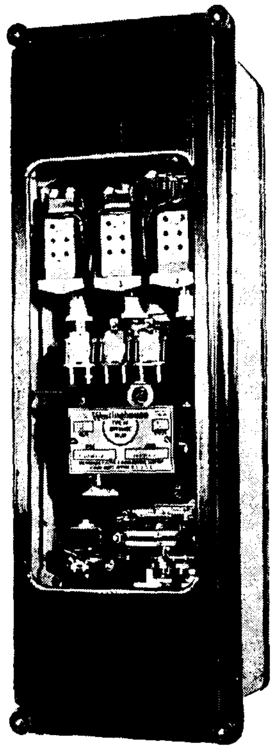


Fig. 23—The Type HZ high-speed, three-zone impedance relay. The relay is arranged for use either as a conventional step-type distance relay for phase-fault protection, or in the directional comparison scheme of pilot-wire or carrier-current relaying.

chokes. Thus, at carrier frequency, the coupling capacitor is simply a series capacitor between the carrier set and the high-tension line.

(d) The tuned carrier-current choke or wave trap, of sufficient capacity to carry the line current but imposing a high impedance to carrier current of the frequency used. Its purpose is to prevent loss of the carrier energy into other sections, so that ample signal strength is available in the protected section.

Microwave Relaying—Either the directional-comparison or phase-comparison systems of relaying can be used over microwave channels as well as power-line carrier channels without significant alteration. However, because the microwave channel is not subjected to line faults it does not necessarily have to be used in a blocking manner, but is suitable also for transfer tripping.

14. Pilot-Wire Relaying

Pilot wire relaying is to the short transmission line what carrier current protection is to the long one. It provides uniform simultaneous tripping of the circuit breakers at both ends of a section, with all that such operation implies in the way of increased stability, lessened shock and damage to the system, and simplified coordination with other relay protection. In short high-voltage lines, discrimination is often impossible with distance type relays; pilot relaying by wire or carrier becomes the only method of discrimination not based on time delays.

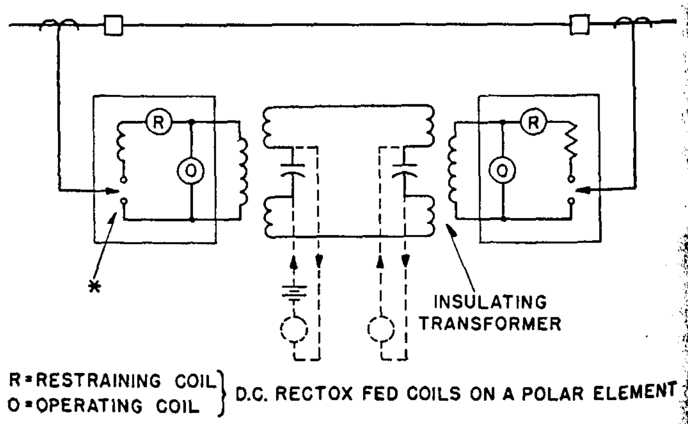
The cost of carrier-current protection is practically unaffected by the length of the line. The terminal equipment, including the relays, carrier set, coupling capacitor, choke and installation, costs considerably more than the terminal equipment required with pilot-wire protection. However the cost of the pilot-wire circuit increases almost directly with the length of line; consequently there is an economic dividing line between carrier-current and pilot-wire applications. The average dividing line is at about 10 miles but it varies widely with such factors as: the availability of existing pilot-wire circuits, the number of other pilot wire services with which the cost of the pilot cable can be shared, the inductive exposure conditions which determine the test voltage of the pilot cable, and the cost and complexity of the necessary carrier-current channel.

The ideal pilot-wire relay systems should:

1. Require only two pilot wires.
2. Provide complete phase and ground protection with a single relay at each terminal.
3. Permit wide variations in current transformer performance.
4. Be suitable for use over leased telephone circuits.
5. Not operate incorrectly when the system is out-of-synchronism.
6. Provide adequate insulation between the pilot wires and the terminal equipment.
7. Have provisions for dealing readily with longitudinal induced voltages in the pilot circuits or with differences in station ground potential.
8. Have provision for supervising the pilot wires.
9. Operate at high speed.

A number of d-c or a-c pilot wire schemes based on directional comparison or on current differential have been used to a limited extent. For example an arrangement similar to the carrier-current protection described has been used with pilot wire. However by far the greatest number of pilot-wire relay applications employ the Type HCB relay thereby meeting all of the above requirements.

The arrangement is shown in Fig. 24. At each end of the line a voltage proportional to positive-sequence current



*Positive-sequence and zero-sequence segregating network. The secondary currents are fed in. An internal voltage is produced proportional to I_1+KI_0. The network as viewed from the relay element terminals has this internal voltage and an internal impedance. A saturating transformer, not shown, is used between the network and the relay element.

Fig. 24—Alternating current pilot wire scheme using the HCB relay. Simplified schematic. Only two wires are required and continuous supervision of them can be obtained as shown dotted.

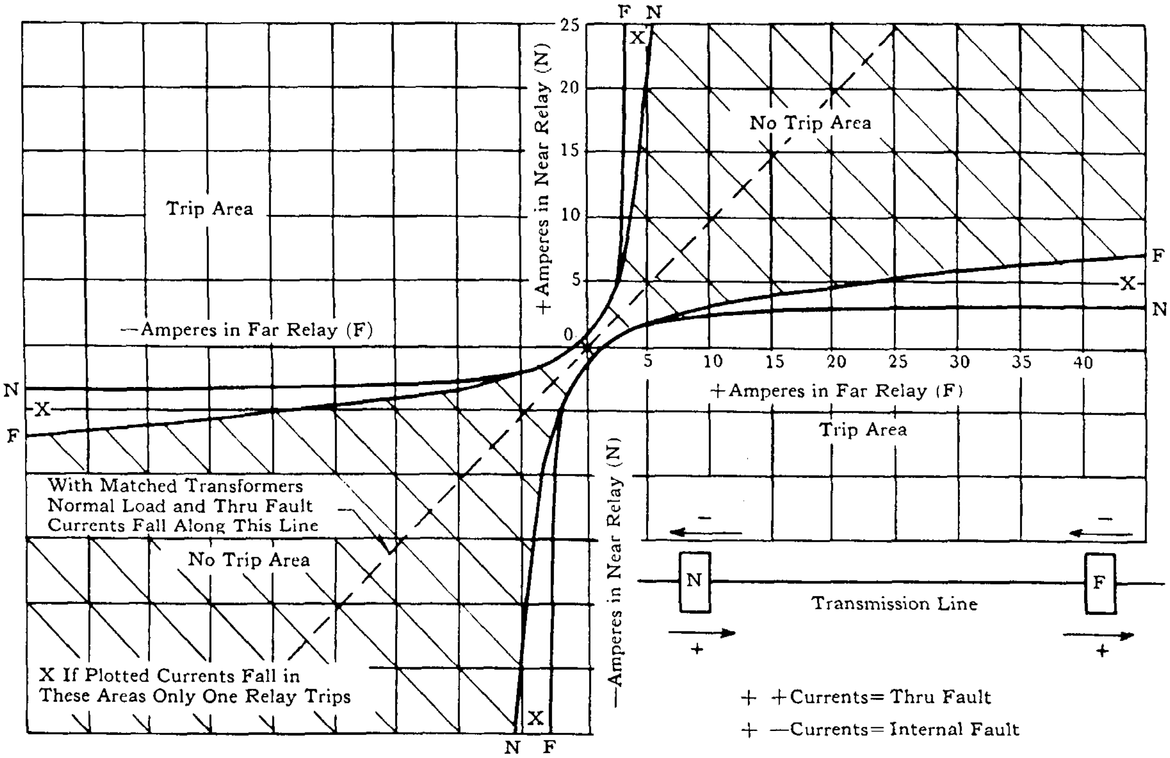


Fig. 25—Typical operating characteristics of the HCB relay for a phase A to ground fault with the currents through the two terminals 30 degrees out of phase, and a 2000-ohm pilot wire.

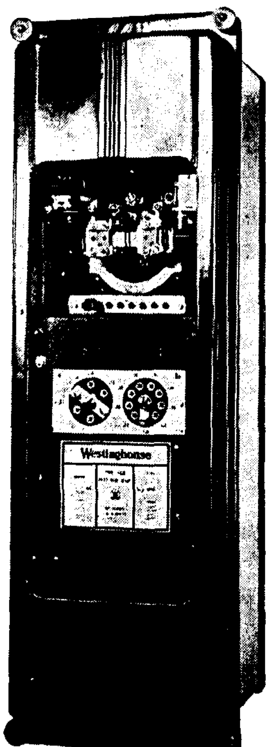


Fig. 26—The Type HCB pilot-wire relay. The relay has a single operating element, which functions for all types of phase and ground faults.

and a constant times the zero-sequence current is derived. The relay has variable-percentage differential characteristics. Each relay contains a filter segregating the positive- and zero-sequence currents and combining the right amount of each into a single relaying quantity. Thus, by selecting the proper relay tap, tripping is obtained at any desired ground-fault current and phase-fault current. This reduction to a single differentiating quantity makes comparison over only two pilot wires possible. The ability to trip correctly even with badly mismatched current transformers is illustrated in Fig. 25. The vector positions of the sequence currents for various types of faults are illustrated in Fig. 32.

15. Ground Relaying

Overcurrent and directional relaying used for ground protection usually follows the same schemes as employed for phase protection. That is, the overcurrent ground relays for a radial system may be progressively timed, as in Fig. 13. For loop systems the directional element is added, polarized either by zero-sequence voltage or by a transformer bank neutral current which is proportional to zero-sequence voltage. This results in progressive timing as in the loop circuit of Fig. 12. Stepped-distance relaying is likewise used but with limitations which will be discussed.

There are a number of differences between ground and phase relaying.

(a) The zero-sequence impedance of the average transmission line is 2 to 5 times its positive-sequence impedance, while the zero-sequence impedance of the source, comprising frequently only the transformer-bank impedance, may be lower than the positive-sequence impedance of the source. Therefore as the fault is moved along the line, the ground-fault current falls off more rapidly than the phase-fault current and current magnitude can be used more for discrimination, at least where ground wires are used.

(b) Usually there are many more sources of ground current than of phase current. This improves the selectivity obtainable with overcurrent ground relays.

(c) A system may have several unconnected portions of zero-sequence network, in which case a ground fault in one section does not draw ground current from other parts. This makes the coordination of ground relays simpler than that of phase relays.

(d) Fault resistance is likely to be much higher for ground faults than for phase faults on the higher voltage lines. At currents of 1000 amperes or more the arc voltage is 300 to 500 volts per foot so that a 1000-ampere line-to-line fault through a 5-foot arc involves a fault resistance of approximately 2500 volts divided by 1000 amperes or 2.5 ohms. Compared with this, pole grounds, which may be the fault impedance for a ground fault, are usually in the 5 to 50-ohm range. A wire on the ground can have almost any fault resistance. Being unaffected by load current, a ground-current relay can be set lower than a line-current relay. Thus it can be set low enough to operate even though the fault current is choked down considerably by fault resistance.

(e) The zero-sequence mutual reactance between two parallel lines is important, although positive-sequence mutual reactance is usually unimportant. The zero-sequence mutual reactance leads to circulating residual currents in one line for a fault in the other, even though the lines are part of two separate systems. It also interferes somewhat with distance-type ground relaying, although methods are available for compensating this effect in some cases.[32,35]

Another factor of importance is that the fault, rather than the supply end of the line, is the source of zero-sequence voltage. That is, the zero-sequence voltage tapers down from the fault towards the relay as outlined in Sec. 23 and illustrated in Fig. 30.

These factors lead to difficulties in applying impedance or other distance measuring relays for protection against ground faults. While they have been used in some cases where conditions are favorable and where discrimination would be even more difficult by other means, their use is limited.

Overcurrent Ground Relaying—The vast majority of ground relaying is essentially overcurrent, with direction where needed. The more common elements follow:

Type CO—Induction-overcurrent relay with instantaneous-trip attachment. The instantaneous trip is set below the maximum ground current in the line for a fault at the next bus. Nonsimultaneous closure of the circuit breaker poles during load switching may result in momentary ground current sufficient to operate the instantaneous ground relays, where the relays are set sensitively. This has been avoided by connecting a residual-voltage-relay contact in series with the trip circuit. The latter does not operate during load switching operations.

As above but with induction-directional element, Fig. 2(*e*), controlling the induction overcurrent element. The directional element may be polarized either by residual voltage, Type CR, or bank-neutral current if available, Type CRC.

Instantaneous-overcurrent elements, such as the SC, can be used if the line terminates at the far end in a transformer that will not pass residual current.

Reactance Relaying—has an inherent advantage over impedance relaying for ground fault protection in that the relay measurement is generally much less affected by fault resistance. If the currents supplied from the two ends of the line are not in phase, the fault resistance does appear to the relay to have some reactance. Nevertheless, the error in distance measurement caused by fault resistance is generally much less with a reactance element than with an impedance element.

For ground relaying it is desired that the relay measure the zero sequence reactance from the relay to the fault.[32] It may be noted that the ratio of line-to-neutral voltage at the relay to zero-sequence current is:

$$\frac{E_a}{I_0}=\frac{I_0Z_0+I_1Z_1+I_2Z_2}{I_0}$$

where I_0, I_1, I_2 are the sequence current at the relay and Z_0, Z_1, Z_2 the sequence impedances from relay to fault. Thus the zero sequence impedance can be measured as—

$$Z_0=\frac{E_a-I_1Z_1-I_2Z_2}{I_0}=\frac{I_0Z_0}{I_0}$$

The positive and negative sequence voltage drops from relay to fault are deducted from the line to neutral voltage, E_a, by compensators and the resulting voltage divided by I_0 is a measure of the zero sequence impedance. A reactance element using this voltage and current will measure the zero sequence reactance as desired.

The type HXS ground reactance relay operates on this principle, three elements being used to provide stepped-distance protection. As there is zero-sequence current only for fault conditions, no separate fault detector is required. Only one HXS relay is used for all three phases, the voltage of the faulty phase being connected to it by a type HPS faulty-phase selector relay illustrated in Fig. 2(*t*).

To provide a single high-speed step, as for example where existing relays provide adequate backup protection, the HXL relay is used in conjunction with the HPS phase selector. This provides one ground reactance step, usually set about 75 percent of the line length, and a load-loss feature which opens the second breaker instantly after the first breaker opens. The load-loss feature utilizes three overcurrent elements to recognize by the closing of at least one back contact and one or more front contacts that a fault is present and the far breaker is open. Under normal load conditions all three front contacts are closed.

Negative-Sequence Directional Relaying—The negative-phase-sequence directional element can frequently be used to advantage with an overcurrent ground relay to obtain selective clearing of ground faults. This results from three facts—

1. Only two potential transformers are required.
2. On solidly-grounded systems the negative-phase-sequence voltage at the relay may be of larger magnitude than the zero-sequence voltage at that point, hence, a more positive relay operation can be obtained.
3. The negative-sequence directional element is not affected by zero-sequence mutual induction from parallel transmission circuits.

Current-Voltage Product Relaying is also used for ground protection. As the fault is moved away from the relay, not only does the zero-sequence current at the relay drop quite rapidly, but also the zero-sequence voltage (or the bank neutral current which is proportional to it) decreases. This makes the product of these two quantities an effective discriminating function. It is utilized in the types CW and CWC relays. See Fig. 2(*e*).

16. Back-up Protection

As mentioned earlier, in the event of failure of the proper circuit breakers to clear a fault, it is desirable to have some form of back-up protection to remove the fault from the system in the next best way that can be arranged within economic limits. The measures employed in present-day practice vary all the way from complete duplication of relays, circuit breakers, and tripping sources at one extreme to no back-up at all at the other extreme. The measures used in each case are somewhat a matter of opportunism as to what can be done conveniently, but more a matter of judgment and evaluation of the following factors:

1. Technical means by which back-up could be provided.
2. Importance of the section being backed up. That is, consequence to the system of failure of the primary protection, if one or the other of the various back-up possibilities is in use.
3. Cost of providing back-up protection in various ways.
4. Probability of faults, of the failure of primary protection, or of equipment failures that would cause costly or time consuming repairs if the primary protection should fail.
5. Consequences to the industrial processes or other loads being served if service is lost, momentarily, or for a long time.

To make these considerations more concrete a few examples may be cited. If a line fault hangs on due to a circuit breaker failure and is not cleared, the first hazard is that the line may be burned down resulting in a time-consuming repair job. The second is that the continued fault may cause the whole system to pull out of step and result in a complete shut down of an entire city area or large system. Provision of back-up may remove one or both of these possibilities depending on the circumstances.

In an indoor station the failure of some piece of equipment to function properly, thereby leaving a fault uncleared, may result in a fire or explosion with extensive damage, which might have been avoided or minimized had some secondary means of clearing been in use.

Needless to say these considerations extend far beyond protective relays, into maintenance, testing and safety practices, air versus oil switchgear, building construction, protection of lines against lightning and other hazards and quality of major equipment. All of these should eventually be weighed to obtain the most favorable over-all service for the time, effort and cost involved.

In a system of transmission lines protected by overcurrent or distance-measuring relays, the relays for each circuit breaker can frequently be set to provide primary protection for the immediate line section and back-up protection for the next line section, as indicated by the characteristics of Fig. 13, 15, 16, and 18. Then in event of the failure of a circuit breaker to clear for any reason, all other lines feeding that bus are cleared at their remote ends, with some time delay. This is the most common form of back-up protection.

Overcurrent relays are frequently used for back-up protection on lines equipped with high-speed distance, or pilot-wire or carrier relays. These also provide protection for the immediate line while the primary protection is taken out of service for the testing.

In many cases relays at one station, *A*, Fig. 27, cannot reach through the bus of the next station, *B*, to a fault at

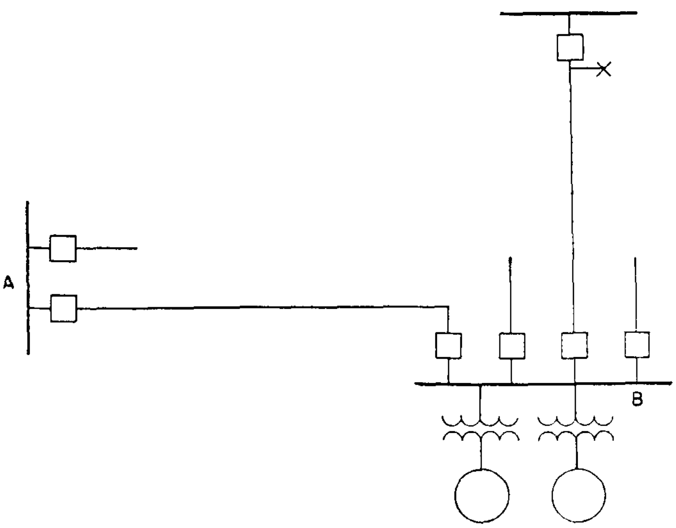


Fig. 27—Problem of back-up protection.

the ends of the lines beyond. This is especially true if there is generation on the bus at station *B*, or if the lines beyond are very long, or both. Consideration is then given to some form of protection that will clear all other circuit breakers on bus *B*, if one fails to open for a fault. One scheme in use is to have the relays of each circuit breaker on bus *B* not only trip their circuit breaker, but also energize a time-delay relay. This relay will trip all circuit breakers on the bus, through the bus-auxiliary lock-out relay, if the protective relay does not open its contacts within a reasonable time, indicating that the fault has been cleared. This takes care of circuit breaker or trip-coil failure. Relay failure is backed up assuming that there is more than one relay element that should trip. However, it does not guard against battery failure or instrument-transformer or lead failures. Sometimes the back-up relays can be connected to different transformers than the primary protection but the only precaution normally taken with respect to the battery is good supervision.

Experience indicates that it is better economy to apply back-up relays to protect against other equipment failures than merely to back-up another relay, while using all the same accessories.

On some generator busses a partial-differential back-up is used to trip the complete bus section for feeder faults not cleared by the proper circuit breaker. The main sources, such as generators, transformers, and bus ties are totalized to operate these back-up relays. The relays are usually overcurrent and are given enough time to select with the main feeder protection.

In some very important stations complete duplicate sets of relays are used, operated from separate transformers to provide the back-up function. Also in certain large generating stations, with double circuit breakers separating major bus sections with power concentrations of the order of 200 000-kw load, the back-up partial-differential circuit not only uses separate current transformers but trips the other circuit breaker of the double-breaker combination. The partial-differential protection can often be arranged so that it backs up generator, reactor, and bus protection as well as feeder protection.

Since carrier, pilot-wire or distance protection is sometimes required to obtain 100 percent selective primary protection, it is not always possible to obtain selective action of the back-up relays. There is some trend in the case of pilot-wire protection of short-line sections to cut the back-up relays in automatically when the supervision relays indicate that the pilot wires are inoperative. Some use is being made of carrier in a somewhat similar philosophy, to block certain back-up elements when carrier is being received.

17. Motor Protection

For the larger motors which are equipped with protective relays, the protection supplied varies with the importance of the service and whether automatic or not and whether attended or non-attended.

Many motors 500 horsepower and larger, and some smaller ones are protected by long-time (geared, 40-second) induction-type overcurrent relays set in the neighborhood of 150 percent of maximum-load current, but with timing to permit the starting inrush of several-hundred-percent current for several seconds. A typical case might be 600 percent for ten seconds for an across-the-line-started induction motor, although these figures vary widely, depending on the application, and must be determined in each individual case. These relays will operate for a stalled motor, for internal motor faults, or for a heavy overload.

The overcurrent relays are usually provided with instantaneous-trip attachments set above the motor-starting current to trip quickly for severe faults in starter, leads, or motor.

Thermal relays are frequently provided in addition to the overcurrent relays, to provide more sensitive overload protection. Having a time constant of several minutes as contrasted with seconds for the overcurrent relay, they follow the motor temperature more closely. They will not protect the motor under stalled conditions, however, where the motor ventilation is missing; the overcurrent relay provides this function.

In automatic applications, and where starting in reverse would be serious, a phase rotation relay is used which closes contacts to permit a starting only if the phase sequence is correct and voltage is present on all three phases. This does not guard against phase unbalances which might occur during operation, however. For this purpose, a phase-balance current relay, Fig. 2(p), is provided in many automatic schemes to trip if the 3-phase currents become excessively unbalanced. Time-delay under-voltage protection is frequently applied to guard against over-current or process damage, resulting from sustained operation at low voltage. However, where continuous operation is more important than motor protection this feature is eliminated or used for alarm only.

Thermal-alarm devices applied directly on the motors are becoming increasingly popular in attended stations where the operator can determine from the ammeters or other indications, the source of trouble.

18. Power House Auxiliaries

The power house is the heart of any electric system and its functioning rests on the motors which drive its fans, pumps, gates, and other auxiliaries. Hence reliability is paramount, and in addition to provision of a most suitable power supply and spare units for certain auxiliaries, much attention has been given to the relay protection. A recent survey of United States practice[38] resulted in the following recommendations, attendance being assumed.

The 2300-volt motors should use long-time-delay phase-overcurrent relays for overload and internal motor faults, set at approximately 150 percent of rated current. They also should be equipped with instantaneous overcurrent relays for short-circuit protection set above maximum-inrush current. If the auxiliaries are transferred, the instantaneous relay must be set above a higher inrush current.

On essential motors* the time-delay overload relays may be used for alarm purposes only, and the instantaneous relays used to trip. In this case the time-delay relays can be set more sensitively than 150 percent of rated current.

Low voltage motors (208, 440, 550 volts) should use a thermal device for overload protection, and an instantaneous-trip device for short-circuit protection.

In addition, the report notes the desirability of eliminating undervoltage protection except for alarm purposes, so that the loss of auxiliaries due to system disturbances will be minimized.

19. Industrial Interconnections

When a line is tapped to an industrial plant having generation, it is common practice to segregate essential loads for operation from the plant generator and dump others in event of a line outage. If the same line is tapped for other plants, the problem arises of separating the plant under consideration from the line under conditions hazardous to its operation. One scheme in successful use on many industrial interconnections consists of separation based on any of three indications provided power flow has reversed and is toward the power company. The three indications are: under frequency, undervoltage, or generator overload. Any of these occurrences, provided power flow is away from the plant, is taken as sufficient cause for separating and at the same time dumping nonessential loads so that the remaining plant load may be brought within the capacity of the plant generation.

The relays normally employed are:

Induction-type overcurrent for generator overload.
Induction-type under-frequency relay.
Induction-type undervoltage relay.
High-speed-type three-phase directional relay.

The generator overload relay is directional controlled so

*Essential motors are in this case defined as those motors whose failure results in the shut-down of generating capacity.

that it will not start timing unless direction in the interconnection has reversed.

Directional relays are also used, without the voltage, frequency, or current fault detectors, for this purpose.

20. Three Terminal Lines

Lines having three or more terminals[38] are generally more difficult to protect than two-terminal lines. Alternating current pilot wire protection is applicable in many cases although the limiting values of pilot-wire capacitance and resistance are less per terminal than for two-terminal lines. Carrier schemes, particularly of the blocking type are applicable to multi-terminal lines. However, sequential operation of circuit breakers occurs if fault current of appreciable magnitude flows out at one terminal for an internal fault near another terminal. The first-zone impedance element nearest the fault, acting independent of carrier, opens the first circuit breaker, after which carrier is stopped by the directional elements permitting clearing of the other circuit breakers.

21. Out-of-Step Protection

Practically all utilities,[90] except those consisting of steam stations connected rigidly together electrically, have experienced system instability. Most utilities have experienced some undesired operation of fault-protective relays as a result of system instability. Quite a number of utilities attempt either to block line relays from tripping because of out-of-step conditions, or to set the relays so that tripping will occur at a preselected point. Out-of-step blocking in conjunction with carrier relaying is the method most commonly employed.

Synchronous frequency changers interconnecting two systems may suffer mechanical damage to shafts and couplings if permitted to operate with the systems out-of-step. The resulting power pulsations may be close to the natural frequency of the two-mass system composed of the two rotors with connecting shaft. Out-of-step relays are available which detect a slip cycle by the power reversal at high current and can be set to trip after two or three slip cycles, or before serious torque oscillations build up.

Quick clearing of faults by modern 8-, 5-, and 3-cycle circuit breakers and high-speed relays is well accepted as a measure of prime importance in improving system stability and reducing damage and permanent outages. Case after case could be cited where these improvements have been realized as circuit breakers and relays have been modernized up to present-day standards. High speed reclosing has been made possible by simultaneous operation of circuit breakers at the two ends of a transmission line by carrier-current or pilot-wire relaying. This measure is generally accepted as economically of greatest benefit in improving stability and service reliability. Three-pole[91] reclosing has been most widely used. However, there are a number of applications of single pole reclosing[92] which further enhance the stability by leaving the sound phases in service while the faulted ones are opened and reclosed.

22. Testing and Maintenance

Routine tests are made by many companies at quite frequent intervals such as one to three months, depending on the importance of the service. However, the major calibration tests are generally scheduled for periods more of the order of six months to two years. One year is a quite common period. There is a decided feeling that too frequent testing may cause more harm from mistakes and inadvertent damage than the good that is accomplished. The tests vary from the over-all or primary test in which current is passed through the primary of the current transformer, and the circuit breaker tripped by the resulting relay action, to much less complete checks. A quite usual procedure would be to remove the relays from service and test and calibrate on a load box, and to check the instrument transformers for continuity and grounds. The instrument transformer-relay circuit is grounded at only one point[93] so that the intentional ground can be lifted for this test. If feasible the circuit breaker may be tripped by closing the relay contact.

23. Relaying Quantities and How They Are Obtained

The prime requisite of all protective relaying is a fundamental basis of discrimination, which has been variously referred to as a discriminating function or quantity, an operating principle, or a relaying quantity. This discriminating quantity must be one to which a protective relay can be made to respond, and one which separates the desired tripping values from the desired non-tripping values.

The common discriminating quantities, such as current, voltage, time, impedance, direction, and power are well known, and the methods of obtaining them from current transformers, potential transformers, and potential devices are generally understood and described elsewhere.[1,32] No general treatment of this subject can be given here. However, some of the more important characteristics of these quantities will be briefly outlined. Some special consideration will be given the newer sequence quantities arising from the method of symmetrical components, given in Chap. 2.

Voltages and Currents During Fault Conditions—Ten different faults of four kinds can occur at one point on the system:

three-phase	ABC		
line-to-line	AB	BC	CA
double line-to-ground	ABG	BCG	CAG
single line-to-ground	AG	BG	CG

When one of these four kinds of faults occurs along the line, the voltage and current relations at the relay are somewhat as shown in Fig. 28. For a three-phase fault the currents are balanced and lag the line-to-neutral voltages by the impedance angle of the line. In an average high-voltage line this angle is about 60°. The addition of fault resistance tends to lower it. For line-to-line fault, say BC, the current in line B lags the collapsed BC voltage by a line impedance angle of about 60°. For a two-line-to-ground fault, for example BCG, a similar situation pertains, except that the line-to-neutral voltages B and C also collapse to an extent depending on how solidly the system is grounded. Consider two easily visualized cases. If the system is

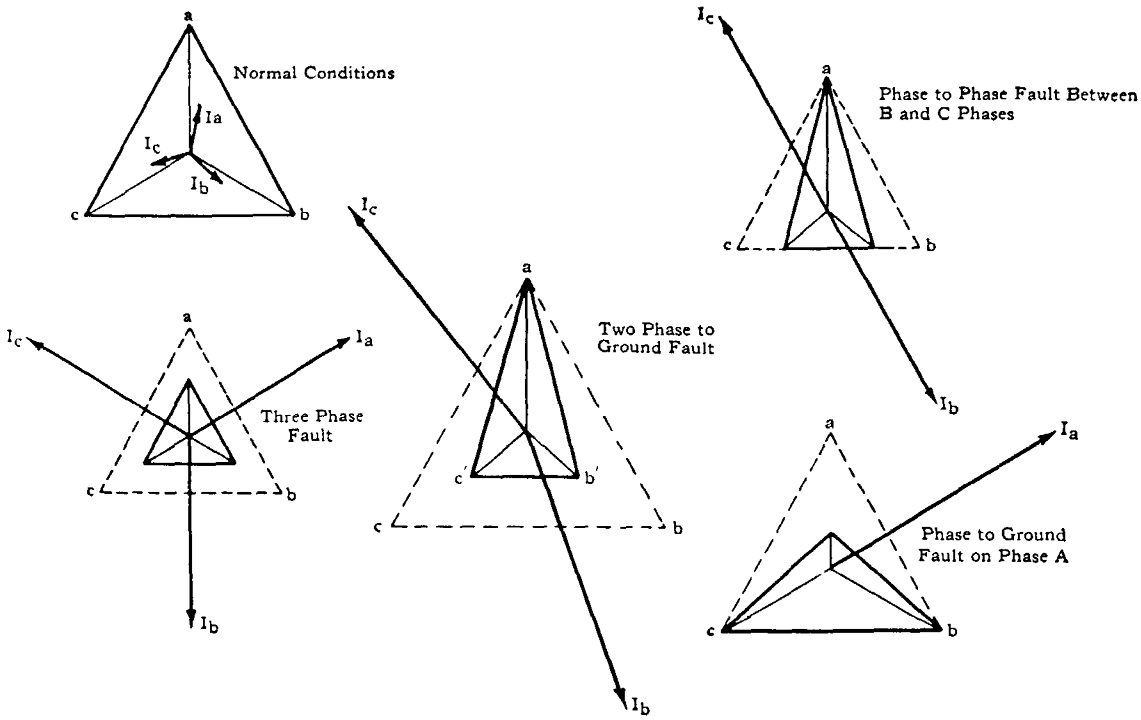


Fig. 28—Typical unbalanced conditions of current and voltage occurring during various types of fault on a three-phase system with an angle of sixty degrees. Load current flow neglected for fault conditions.

grounded through a very high impedance (Z_0 very high), the currents will be nearly the same as for a line-to-line fault except for a small added ground current. However, the fault will establish the mid-point between B and C phase voltages as ground potential. Another example is the case $Z_1=Z_2=Z_0$ for all parts of the system. For this case the phases are independent and the three-phase system acts exactly like three independent single-phase systems. The two-line-to-ground fault BCG, is the same as the three-phase fault except that phase A voltage is not collapsed, and only load current flows in phase A. The majority of systems fall between these two limits.

For a single line-to-ground fault on phase A, the corresponding line-to-neutral voltage collapses and the phase A current lags the line-to-neutral voltage of phase A by the impedance angle of the line-ground-return circuit including the fault impedance. If $Z_0=Z_1=Z_2$ throughout the system this fault will not influence the other two phases. If Z_0 is higher than Z_1, corresponding to high impedance grounding, the condition of ungrounded operation or full displacement of the voltage triangle is approached.

Single-Phase Directional Element Response—Considering the different kinds of faults, and also their occurrence with symmetry to A, B, or C phases, the angle between voltage and current for a fault on the line varies over rather wide limits. In using a single-phase directional element, as in the CR, CZ, or HZ relay, a particular voltage must be associated with a particular current.

30° Connections. One of the common "connections" associates the phase A current with the phase CA voltage and is known as the 30° connection, because at unity power factor under balanced three-phase conditions the current leads the voltage by 30°. A watt-type* directional element closes for current from approximately 90° ahead to 90° behind the voltage applied to its potential coils. For a three-phase fault on a 60° impedance-angle line the current lags behind its unity power factor position by 60°. With a 30° connection it lags the reference or polarizing voltage used on the directional element by 30°. Fault resistance (plus any modification of the relay characteristics by lagging) brings the fault current nearly to the maximum torque position. For the other kinds of faults on different phases the current is shifted one way or the other, but the wide closing band of the relay allows for this variation. The 30° connection uses star currents and delta voltages.

The same system is followed in naming other connections, although the relay used, including its phase-shifter if any, does not always have a closing zone for current from −90° to +90° with respect to voltage.

The 60° Connection uses delta currents and voltages; the I_A-I_B current being used with the phase CA voltage. A relay with a closing zone approximately +90° to −90° is used. Delta-connected main or auxiliary current transformers are needed to obtain the delta currents.

The 90° Connection uses star currents and delta voltages; the phase A current being used with the phase CB voltage. In this case, however, a 45° voltage advancing phase-shifter is employed with the relay element giving it for star currents a closing zone approximately from 135° ahead to 45° behind the delta voltage. For a three-phase fault on a 60° impedance-angle line the phase A current leads the

*Other directional elements may have their closing zone shifted as much as 45° in the leading or lagging direction. The element used with the 30° connection may be a watt type or may have its closing zone shifted 10° to 20° in the lagging direction.

phase CB voltage by 30°, and a small fault resistance would swing it toward 45° leading. The closing zone extends 90° either side of this position and affords optimum opportunity for the relay to give correct directional indication with other kinds of faults.

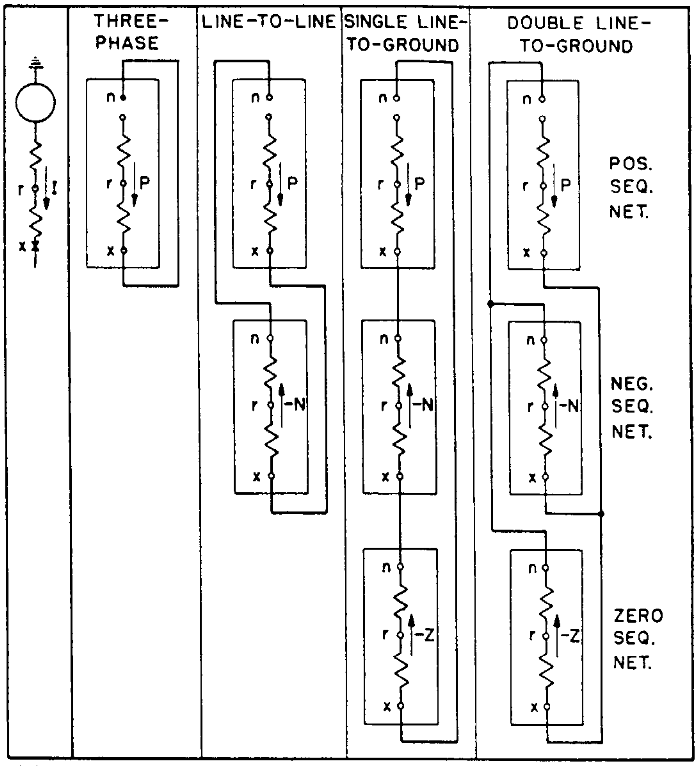
Usually any of these three connections gives correct directional indication although in individual cases advantages can be found for one or the other, depending on such factors as the impedance angle of the line, the possible fault impedance, and the likelihood of an undesired amount of directional element operation caused by leading load currents near the directional boundary. For distance carrier relaying using single-phase directional elements the 60° connection using delta currents is preferred since the same delta current is used on the impedance element. If

TABLE 4—SEQUENCE POWER RESPONSE OF THREE-PHASE DIRECTIONAL ELEMENTS

VOLTAGE **	CURRENT **	CONNECTION	TORQUE VECTORS FOR PURE RESISTANCE SEE NOTE: POS. SEQ.	NEG. SEQ.	ZERO SEQ.	NEEDED VOLTAGE SHIFT FOR 60° SYSTEM	RELATIVE THREE-PHASE TORQUE SEE NOTE
STAR	STAR						
A	A	0°	→	←	←	-60°	P+N+Z
A	-B	60°	↘	↙	→	0°	$P\angle{-60^\circ}+N\angle{+60^\circ}-Z$
A	C	120°	↙	↘	←	+60°	$P\angle{-120^\circ}+N\angle{-120^\circ}+Z$
A	-A	180°	←	→	→	120°	-P -N -Z
A	B	240°	↖	↗	←	180°	$P\angle{-240^\circ}+N\angle{-120^\circ}+Z$
A	-C	300°	↗	↖	→	240°	$P\angle{+60^\circ}+N\angle{-60^\circ}-Z$
DELTA	STAR						
CA	A	*30°	↘	↙	NONE	-30°	$P\angle{-30^\circ}+N\angle{+30^\circ}$
CA	-B	*90°	↓	↓	NONE	30°	$(P-N)\angle{-90^\circ}$
CA	C	150°	↙	↘	NONE	90°	$P\angle{-150^\circ}+N\angle{-210^\circ}$
CA	-A	210°	↖	↗	NONE	150°	$P\angle{-210}+N\angle{-150^\circ}$
CA	B	270°	↑	↑	NONE	210°	$P\angle{+90^\circ}+N\angle{-90^\circ}$
CA	-C	330°	↗	↖	NONE	-90°	$P\angle{+30^\circ}+N\angle{-30^\circ}$
DELTA	DELTA						
CA	-C+A	0°	→	←	NONE	-60°	P+N
CA	A-B	*60°	↘	↙	NONE	0°	$P\angle{-60^\circ}+N\angle{+60^\circ}$
CA	-B+C	120°	↙	↘	NONE	60°	$P\angle{-120^\circ}+N\angle{+120^\circ}$
CA	C-A	180°	←	→	NONE	120°	-P -N
CA	-A+B	240°	↖	↗	NONE	180°	$P\angle{+120^\circ}+N\angle{-120^\circ}$
CA	B-C	300°	↗	↖	NONE	240°	$P\angle{+60^\circ}+N\angle{-60^\circ}$
UNSYMMETRICAL CONNECTIONS							
(BA, BC)	A, C	(3)	→	←	(4)	-60°	$P+N-a^2\hat{I}_0E_1-a\hat{I}_0E_2$
(BA, BC)	$A-I_0$, $C-I_0$	(2)	→	←	NONE	-60°	P+N
(CA, BA)	$A-I_0$, $C-I_0$	(2)	(4)	(4)	NONE		$-a^2\hat{I}_1E_2-a\hat{I}_2E_1$
(CB, CA)	A-C, B-C	(1)	↓	↓	NONE	+30°	$(P-N)\angle{-90^\circ}$

*These are the commonly used 30°, 60°, and 90° connections.
**Other phases connected symmetrically in sequence A, B, C.
(1) A new two-element 90° connection giving the desirable P-N torque.
(2) Two-wattmeter connection with zero-sequence current removed by a filter.
(3) Two-wattmeter connection.
(4) Parasitic torques.

Note: Torque is the real part of the expression in the last column. $P=E_1\hat{I}_1$; $N=E_2\hat{I}_2$, $Z=E_0\hat{I}_0$. If the system were pure resistance throughout all E's and I's would be in phase, or phase opposition. For faults, N and Z would be negative at the relay, and P positive, but all three power terms would be pure scalars. The arrows show the vector position (not magnitude) of the torque expressions for this idealized pure resistance system case, taking into account that the values of N and Z are negative as shown in Fig. 29. If instead of being pure resistance the system were pure 60° impedance angle throughout the effect would be to rotate all currents negatively 60°, leaving all voltages unchanged. As the power terms are of the form $E\hat{I}$ this will rotate similarly all the P, N, and Z quantities and hence the torque vectors. The real components of these vectors are the torques. Hence conclusions can be drawn as to whether the torques associated with P, N, and Z are additive and how much voltage phase shift is needed for optimum condition on a system of given impedance angle.



(a) ARROWS SHOW ACTUAL RELATIVE DIRECTIONS OF SEQUENCE POWER (VOLT-AMPERES AT SYSTEM IMPEDANCE PHASE ANGLE) FOR A SYSTEM OF THE SAME IMPEDANCE PHASE ANGLE THROUGHOUT.

P	LINE*	LINE* +NEG. SEQ. NETWORKS	LINE*+(NEG. & ZERO SEQ. NETWORKS)	LINE*+(PARALLEL OF NEG. & ZERO SEQ. NETWORKS)
N	NONE	SYSTEM*	SYSTEM*	SYSTEM*
Z	NONE	NONE	SYSTEM*	SYSTEM*

*Line refers to the line impedance from relay r to fault x of the particular sequence. System refers to the impedance from generator to r of the particular sequence.

(b) IMPEDANCE WHICH DETERMINES ANGLE OF POWER AT THE RELAY. (ANGLE OF SEQUENCE CURRENT BEHIND CORRESPONDING SEQUENCE VOLTAGE.)

Fig. 29—Diagrams showing the relative directions of positive-, negative- and zero-sequence power during fault conditions. The Chart (b) indicates what part of the system fixes the power factor for each sequence.

impedance element operation is caused by a line-to-ground fault its associated directional element is influenced by fault current. This overcomes any possible load current effect.

Three-Phase Directional Element Response—The same connections are used with three-phase directional elements*. In this case another factor influences selection of the connection. Table 4 shows the functions of sequence power to which various connections respond. As shown in Figs. 29 and 30, positive-sequence power flows toward the fault; negative- and zero-sequence power flows away from the fault, since the fault is the source of negative- and zero-sequence voltage. Therefore the positive-sequence

*This discussion relates to three independent single-phase elements on the same shaft. No three-phase rotating field is involved.

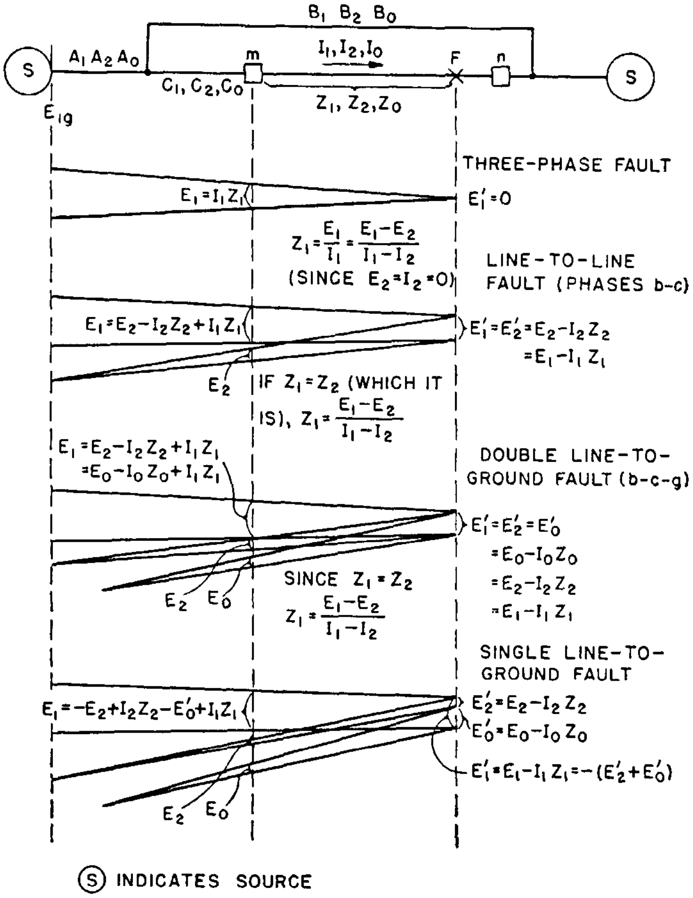


Fig. 30—Simplified equivalent system showing the sequence voltage distribution during fault conditions.

minus the negative-sequence power at the relay $(P-N)$ is more positive for a fault out on the line than is $(P+N)$, and hence provides a better fault directional indication, having higher torques for fault currents in relation to those due to load currents.

In table 4 the "connections" are shown in the first three columns, and the resulting three-phase torque on the element in the last column. The relative phase positions of the torques produced by positive- and negative-sequence power are shown in the 4th, 5th, and 6th columns, taking into account that when the impedance angles are the same throughout a system, the values of N and Z are opposite to P under fault conditions. The positive-sequence torque vector is drawn in the position of the conjugate of the current vector to produce maximum torque, the horizontal axis being the unity power factor position. Thus for the star-voltage, star-current, 0° connection maximum torque is for unity power factor. If a watt responsive element is used the voltage must be retarded 60° by a phase-shifter to obtain maximum torque for current lagging 60°.

The star-voltage, star-current, 60° connection* can be obtained with the phase A current associated with the negative of the phase B-to-neutral voltage, the other phases being symmetrically connected. This is perhaps the best star-current, star-voltage connection as no phase shift is required to get maximum positive-sequence torque with current lagging 60°. Also the negative-sequence torque† is maximum for current lagging 120° and hence gives 50 percent of the maximum possible assistance to positive-sequence torque for 60° system-impedance angle. As the system-impedance angle is likely to be above 60°, this is quite favorable. The zero-sequence torque is also in the right direction though not maximum.

The delta-voltage, star-current, 30° connection has adverse negative-sequence torque, while the 90° connection is ideal in this respect, the positive- and negative-sequence torques have their maximums in the same direction for 90° lagging current. The usual 45° voltage-phase-shifter brings the maximum torque to a desirable point and maximum assistance is secured from the negative-sequence torque.

The delta-voltage, delta-current, 60° connection, like the corresponding star-star connection, has good negative-sequence torque in the proper direction for a 60° impedance-angle system. It has no zero-sequence torque. Maximum positive-sequence torque occurs for a 60° impedance angle without using a phase-shifter.

In addition to symmetrical connections, one unsymmetrical connection is worthy of note in that it is capable of securing the desirable $P-N$ accurately. It uses the B minus C current with the CA voltage and the A minus C current with the CB voltage. It has maximum torque for a 90° impedance-angle system and hence can be used to advantage with a 45° voltage advancing phase-shifter similarly to the better known 90° connection of three elements.

Impedance Measurement—Referring to Fig. 30, the difference between E_1 and E_2 at the relay is the positive-sequence drop from the relay to the fault plus the negative-sequence drop back to the relay. Recognizing that $Z_2=Z_1$ for the line, it can be readily shown that the line impedance to the fault is:

$$Z_1=\frac{E_1-E_2}{I_1-I_2} \tag{1}$$

This applies for three-phase, line-to-line, or double line-to-ground faults. For line-to-ground faults a higher impedance than Z_1 is measured by the ratio $(E_1-E_2)/(I_1-I_2)$. From Eq. (1) the delta connection is derived

$$I_b=I_0+a^2I_1+aI_2 \tag{2}$$

$$I_c=I_0+aI_1+a^2I_2 \tag{3}$$

$$I_b-I_c=(a^2-a)(I_1-I_2) \tag{4}$$

$$E_b-E_c=(a^2-a)(E_1-E_2) \tag{5}$$

$$\frac{E_b-E_c}{I_b-I_c}=\frac{E_1-E_2}{I_1-I_2}=Z_1 \tag{6}$$

The last expression shows that for fault at a given location the delta voltage divided by the delta current is the line impedance Z_1 for three-phase, line-to-line and double line-to-ground faults. As shown previously a higher value is measured for line-to-ground faults.

Lewis and Tippett[32] give the fundamental basis for distance relaying in the most comprehensive paper on this subject. Among other things it is brought out that use of delta current and delta voltage on the impedance element, for example the A minus B current with the BA voltage, as

*This is not the commonly referred to 60° connection.

†This refers to torque due to negative-sequence currents and voltages. Actually it is torque in the positive direction.

outlined in the preceding paragraph, avoids a 15 percent difference in distance measured for line-to-line and three-phase faults, which is present if only one line current (star current) is used.

Use of Sequence Quantities*—In using sequence quantities the point of view should be developed, first, that the fault is the source of negative- and zero-sequence voltage and power, and that negative- and zero-sequence power (volt-amperes at system impedance angle) flow away from the fault at the relay location; second, that the sequence voltage is measured with respect to the bus-of-no-voltage or point *n in the particular sequence network considered.* These relations are brought out in Figs. 29 and 30.

Sequence Voltage Distribution During Faults—In Fig. 30 the voltage gradients are shown very generally. For a three-phase fault the voltage tapers off from the generator to the fault. For a line-to-line fault the positive-sequence voltage tapers off until, at the point of fault, it equals the negative-sequence voltage, which in turn tapers to zero back at the generator neutral, or point back of which there is no impedance to negative-sequence current. In some cases this may be an infinite bus.

For a double line-to-ground fault the positive-sequence voltage again tapers off to the point of fault where it equals the negative- and zero-sequence voltages. These taper to zero in going back through the network until a point of no voltage of the respective sequence is reached.

At a line-to-ground fault the positive-sequence voltage is the negative of the sum of the negative- and zero-sequence voltages and these taper to zero back through the network.

It is well to note that if the zero-sequence impedance is high (high-impedance grounded system), the zero-sequence voltage is nearly equal to the normal positive-sequence or line-to-neutral voltage for a line-to-ground fault, and approximately half as much for a double-line-to-ground fault where the generated voltage divides between the positive- and negative-sequence networks, thereby applying about half voltage to the zero-sequence network. As a result on lightly grounded systems all zero-sequence and residual voltages and currents are approximately half as much for double-line-to-ground as for line-to-ground faults.

Sequence-Segregating Filters—Sequence currents and voltages may be segregated from the corresponding line currents and voltages by segregating networks or filters. The methods of obtaining zero-sequence currents or voltages are already well known as these quantities are simply one-third of the corresponding residual quantities. Typical sequence-segregating networks are given in Fig. 31. The performance of each network is expressed by giving its equivalent circuit[112] and also by giving the equations of operation.

Polyphase networks for segregating positive- and negative-sequence voltage are shown in parts (a) and (b), and are useful for operating a polyphase device in response to either of these quantities. The positive-sequence filter is also useful for obtaining a balanced three-phase supply from an unbalanced (or single-phase) supply. The remaining filters shown all have single-phase output.

Parts (c) and (d) are auto-transformer type voltage-segregating networks and parts (e) and (f) are the all-impedance type and require a special potential transformer connection. The star series transformer connection for obtaining zero-sequence voltage is shown in part (g). Parts (h) and (i) are three-winding-reactor type current filters whereas parts (j) and (k) use an auxiliary current transformer to produce the reactive drop due to B minus C current in a single-winding reactor. Note: The Type CRS negative-sequence directional relay uses filters (d) and (i) for negative-sequence voltage and current respectively.

Parts (l), (m), (n), and (o) are all impedance-type current filters, (l) and (m) being suitable only when there is no zero-sequence current; (n) and (o) are not affected by zero-sequence current, but require double the number of current transformers.

Part (p) is a zero-sequence current filter, which is merely the neutral connection of star-connected current transformers.

A combination positive- and zero-sequence current-segregating filter is illustrated in part (q). This filter is used in the Type HCB pilot-wire relay.[62] The relative weighting of zero and positive sequence is determined by the relative magnitude of R_0 and R_1. For example, if it is desired that the same internal voltage E_i be produced by one-tenth as much zero-sequence current as positive-sequence current, the zero-sequence weighting factor k must be set equal to 10. Then the required value of R_0 may be determined as follows:

$$R_0 = \tfrac{2}{3}kR_1 = 6.67R_1$$

The relative phase positions and magnitudes of various sequence quantities of the reference or A phase vary with the type and phase of the fault. The response of a combination filter varies accordingly. Fig. 32 illustrates the relative positions and magnitudes of the vectors comprising the quantity $I_1 + kI_0$ on a system for which the ground-fault current is one-tenth of the phase-fault current, and using a zero-sequence weighting factor of $k = 15$. The I_1 vectors have been magnified somewhat in the line-to-ground fault diagram to make them visible; their actual

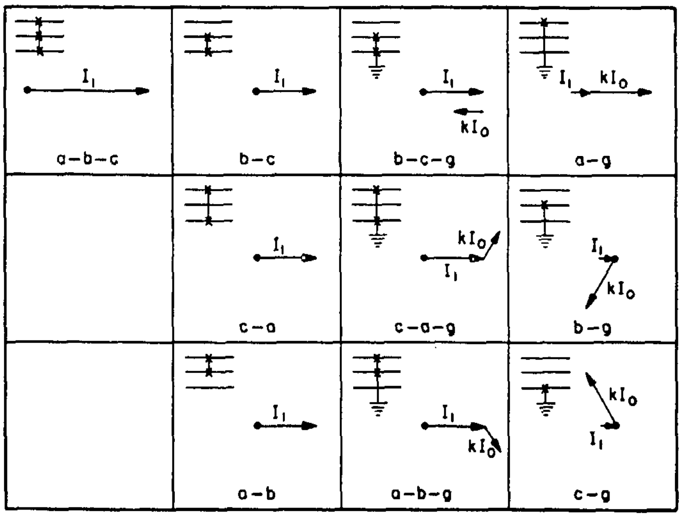
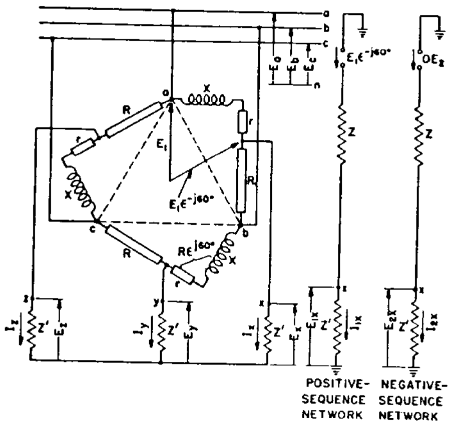


Fig. 32—Vectors comprising the relaying quantity $I_1 + KI_0$ shown for $K = +15$. I_1 vectors magnified in line-to-ground fault cases.

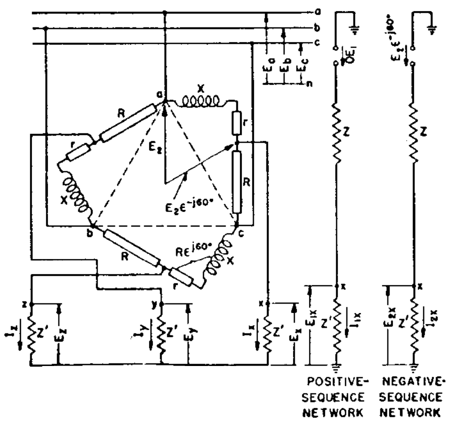
*Refer to Chapt. 2.



$$I_x = I_{1x} = \frac{E_1\epsilon^{-j60°}}{Z+Z'}$$
$$I_y = I_{1y} = a^2 I_{1x}$$
$$I_z = I_{1z} = aI_{1x}$$
$$I_{2x} = 0$$
$$E_x = E_{1x} = \frac{Z}{Z+Z'}E_1\epsilon^{-j60°}$$
$$E_y = E_{1y} = a^2E_{1x}$$
$$E_z = E_{1z} = aE_{1x}$$
$$E_{2x} = 0$$
$$Z = \frac{R\epsilon^{j30°}}{\sqrt{3}}$$

If R is a pure resistance, then: $r = 0.5R$ and $X = 0.866R$.

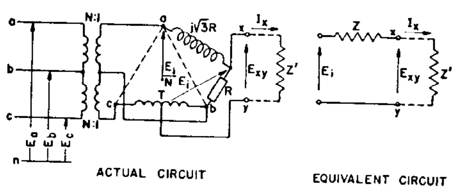
(a) Polyphase positive-sequence-voltage segregating filter



$$I_x = I_{2x} = \frac{E_2\epsilon^{-j60°}}{Z+Z'}$$
$$I_y = I_{2y} = aI_{2x}$$
$$I_z = I_{2z} = a^2I_{2x}$$
$$I_{1x} = 0$$
$$E_x = E_{2x} = \frac{ZE_2\epsilon^{-j60°}}{Z+Z'}$$
$$E_y = E_{2y} = aE_{2x}$$
$$E_z = E_{2z} = a^2E_{2x}$$
$$Z = \frac{R\epsilon^{j30°}}{\sqrt{3}}$$

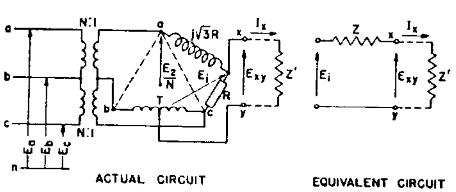
If R is pure resistance, then: $r = 0.5R$ and $X = 0.866R$.

(b) Polyphase negative-sequence-voltage segregating filter (Same as positive-sequence filter except for interchange of b and c leads)



$$I_x = \frac{E_1}{N}\;\frac{1.5\epsilon^{-j60°}}{Z+Z'}$$
$$E_{xy} = I_xZ' = 1.5\epsilon^{-j60°}\frac{Z'}{Z+Z'}E_1$$
$$E_i = \frac{1.5\epsilon^{-j60°}}{N}E_1$$
$$Z^* = \frac{(3R+j\sqrt{3}R)}{4}$$

(c) Positive-sequence-voltage segregating filter



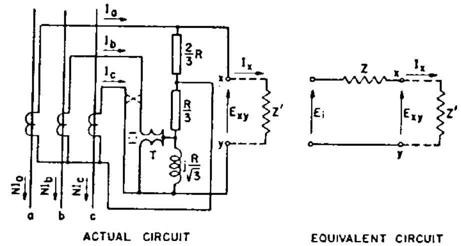
$$I_x = \frac{E_2}{N}\;\frac{1.5\epsilon^{-j60°}}{Z+Z'}$$
$$E_{xy} = I_xZ' = 1.5\epsilon^{-j60°}\frac{Z'}{Z+Z'}E_2$$
$$E_i = \frac{1.5\epsilon^{-j60°}}{N}E_2$$
$$Z^* = \frac{(3R+j\sqrt{3}R)}{4}$$

(d) Negative-sequence-voltage segregating filter

Same as (h) except with b and c leads crossed as shown dotted

$$I_x = \frac{2RI_2}{R+z+Z} \qquad E_i = 2RI_2$$
$$E_{xy} = \frac{2ZRI_2}{R+z+Z}$$

(i) Negative-sequence-current segregating filter and relay



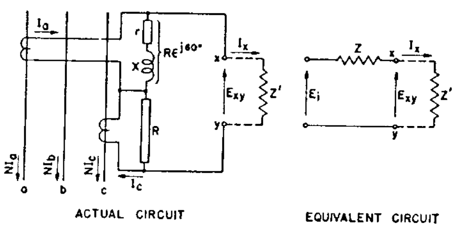
$$I_x = \frac{2R}{Z+Z'}I_1$$
$$E_{xy} = Z'I_x = \frac{2RZ'}{Z+Z'}I_1$$
$$E_i = 2RI_1$$
$$Z^* = R + jR/\sqrt{3}$$

(j) Positive-sequence-current segregating filter

Same as (j) except b and c leads crossed as shown dotted

$$I_x = \frac{2R}{Z+Z'}I_2$$
$$E_{xy} = Z'I_x = \frac{2RZ'}{Z+Z'}I_2$$
$$E_i = 2RI_2 \qquad Z^* = R + jR/\sqrt{3}$$

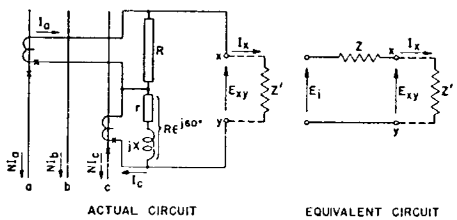
(k) Negative-sequence-current segregating filter



$$I_x = \frac{j\sqrt{3}R}{Z+Z'}I_1$$
$$E_{xy} = Z'I_x = \frac{j\sqrt{3}RZ'}{Z+Z'}I_1$$
$$E_i = j\sqrt{3}RI_1$$
$$Z^* = \sqrt{3}R\epsilon^{j30°} = 1.5R + j0.866R$$

When R is pure resistance, then $r = 0.5R$ and $X = 0.866R$

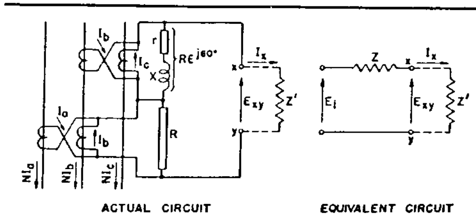
(l) Pos.-seq.-current segregating filter. For use only where there is no zero sequence.



$$I_x = \frac{\sqrt{3}R\epsilon^{-j30°}}{Z+Z'}I_2$$
$$E_{xy} = Z'I_x = \frac{\sqrt{3}R\epsilon^{-j30°}Z'}{Z+Z'}I_2$$
$$E_i = \sqrt{3}R\epsilon^{-j30°}I_2$$
$$Z^* = \sqrt{3}R\epsilon^{j30°} = 1.5R + j0.866R$$

If R is pure resistance, then $r = 0.5R$ and $X = 0.866R$

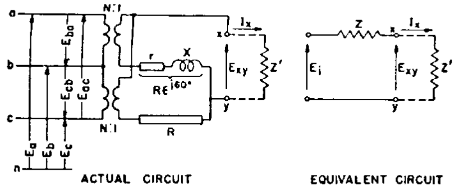
(m) Neg.-seq.-current segregating filter. For use only where there is no zero seq.



$$I_x = \frac{-3R}{Z+Z'}I_1$$
$$E_{xy} = Z'I_x = \frac{-3RZ'}{Z+Z'}I_1$$
$$E_i = -3RI_1$$
$$Z^* = \sqrt{3}R\epsilon^{j30°} = 1.5R + j0.866R$$

If R is pure resistance, then $r = 0.5R$ and $X = 0.866R$

(n) Positive-sequence-current segregating filter. With (or without) zero sequence.

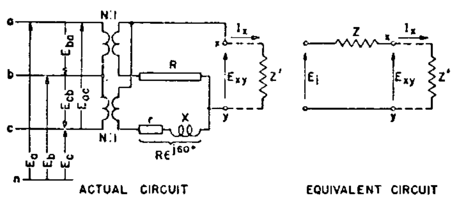


$$I_x = \frac{E_1}{N}\,\frac{\sqrt{3}\epsilon^{-j30°}}{Z+Z'}$$
$$E_{xy} = Z'I_x = \frac{\sqrt{3}\epsilon^{-j30°}Z'}{(Z+Z')N}E_1$$
$$E_i = \frac{\sqrt{3}\epsilon^{-j30°}}{N}E_1$$
$$Z^* = 0.5R + j0.289R$$

When R is pure resistance, then: $r = 0.5R$ and $X = 0.866R$

(Note: Requires Special Potential Transformer Connection)

(e) Positive-sequence-voltage segregating filter

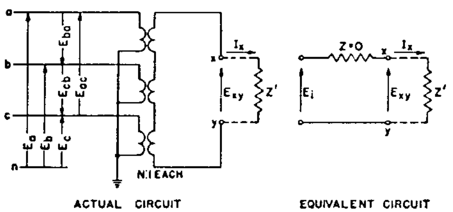


$$I_x = \frac{E_2}{N}\,\frac{\sqrt{3}\epsilon^{j30°}}{Z+Z'}$$
$$E_{xy} = Z'I_x = \frac{\sqrt{3}\epsilon^{j30°}Z'}{(Z+Z')N}E_2$$
$$E_i = \frac{\sqrt{3}\epsilon^{j30°}}{N}E_2$$
$$Z^* = 0.5R + j0.289R$$

When R is pure resistance, then: $r = 0.5R$ and $X = 0.866R$

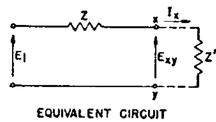
(Note: Requires Special Potential Transformer Connection)

(f) Negative-sequence-voltage segregating filter



$$I_x = \frac{E_0}{N}\,\frac{3}{Z'}$$
$$E_{xy} = Z'I_x = \frac{3}{N}E_0$$
$$E_i = \frac{3}{N}E_0$$
$$Z^* = 0$$

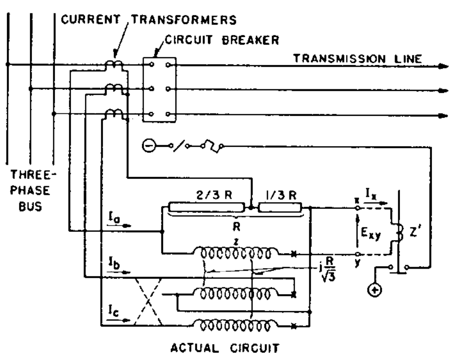
(g) Zero-seq.-voltage segregating filter (otherwise called star-series transformer connection)



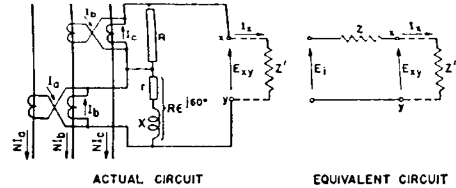
$$I_x = -\frac{2RI_1}{Z+Z'} \qquad E_i = -2RI_1$$
$$E_{xy} = -\frac{2Z'R}{Z+Z'}I_1 \qquad Z = R+z$$

z = Self impedance of indicated winding of three-winding reactor as would be measured with other two windings open circuited.

Note: A three-winding reactor is used having a mutual reactance $R/\sqrt{3}$ between each of the b and c phase windings and the metering circuit winding.



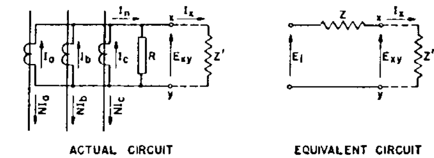
(h) Positive-sequence-current segregating filter and relay



$$I_x = \frac{3R\epsilon^{-j120°}}{Z+Z'}I_2$$
$$E_{xy} = Z'I_x = \frac{3R\epsilon^{-j120°}Z'}{Z+Z'}I_2$$
$$E_i = 3R\epsilon^{-j120°}I_2$$
$$Z^* = \sqrt{3}R\epsilon^{j30°}$$

If R is pure resistance, then: $r = 0.5R$ and $X = 0.866R$

(o) Negative-sequence-current segregating filter. For use when zero-sequence current is (or is not) present.



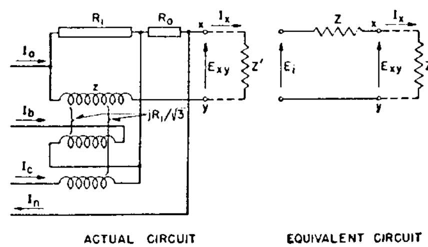
$$I_x = \frac{3R}{R+Z'}I_0$$
$$E_{xy} = I_xZ' = \frac{3RZ'}{R+Z'}I_0$$
$$E_i = 3I_0R \qquad Z^* = R$$

Note that if R is open circuited

$$I_x = 3I_0 \qquad E_{xy} = 3Z'I_0$$
$$\left.\begin{matrix} E_i = \infty \\ Z = \infty \end{matrix}\right\}\frac{E_i}{Z} = 3I_0$$

Note: Residual or neutral current is three times the zero-sequence current. $(I_n = 3I_0)$

(p) Zero-seq.-current segregating filter (Neutral or star-connected current transformers)



$$I_x = -\frac{2R_1I_1 + 3(R_0 + R_1/3)I_0}{Z+Z'} = -\frac{2R_1}{Z+Z'}(I_1 + kI_0)$$
$$k = \frac{3(R_0 + R_1/3)}{2R_1} \qquad R_0 = \frac{2}{3}kR_1 - \frac{R_1}{3}$$
$$E_{xy} = I_xZ' = -\frac{2R_1I_1Z' + 3(R_0 + R_1/3)Z'I_0}{Z+Z'}$$
$$E_i = -2R_1I_1 - 3(R_0 + R_1/3)I_0 = -2R_1(I_1 + kI_0)$$
$$Z^* = R_1 + R_0 + z$$

z = Self impedance of indicated winding of three-winding reactor.

(q) Combined pos-seq.-current and weighted zero-seq.-current segregating filter

General Equations and Nomenclature

$$E_1 = \frac{E_a + aE_b + a^2E_c}{3} \qquad I_1 = \frac{I_a + aI_b + a^2I_c}{3}$$
$$E_2 = \frac{E_a + a^2E_b + aE_c}{3} \qquad I_2 = \frac{I_a + a^2I_b + aI_c}{3}$$
$$E_0 = \frac{E_a + E_b + E_c}{3} \qquad I_0 = \frac{I_a + I_b + I_c}{3}$$
$$a = -0.5 + j0.866 \qquad a^2 = -0.5 - j0.866$$
$$I_x = \frac{E_i}{Z+Z'} \qquad E_{xy} = I_xZ' = E_i\frac{Z'}{Z+Z'}$$

R = Reference impedance of filter.**

Z = Impedance of equivalent circuit of filter.

Z′ = Impedance of connected burden.

× = Polarity marks.

E_i = Internal voltage of equivalent circuit of filter.

I_x = Current in burden.

E_{xy} = Voltage at burden terminals, taking into account regulation in the filter. For maximum power output from a given filter make Z′ = Ẑ. For maximum power output from a given filter feeding into a pure resistance burden (Z′ = R′) make R′ = Z̄.

N = Current transformer or potential transformer ratio as indicated.

Fig. 31—Typical sequence segregating networks.

*More generally Z is the impedance measured on the terminals x, y with the burden Z' disconnected and the applied voltages E_a, E_b, E_c set equal to zero (or applied currents I_a, I_b, I_c made zero by open circuiting corresponding leads in the case of current filters). This includes in the proper manner the impedances of the potential or auxiliary current transformers. The values of Z given above are based on the use of ideal potential and auxiliary current transformers having zero leakage impedance and zero exciting current.

**R, the reference impedance of the filter may be any vector value or a pure resistance. The other filter impedances must take corresponding values as indicated by their defining equations. They are resistances and reactances as indicated by the symbols on the diagrams only when the reference impedance, R, is pure resistance.

length being one-fifteenth of the kI_0 vector. The combination, I_1+kI_0, is the discriminating quantity used in the Type HCB pilot-wire relay which in effect totalizes the two ends of the circuit. It is a single quantity having, for a majority of systems, a much greater value for fault conditions than for load conditions and thus is an ideal discriminating quantity.

24. Reclosing

Many of the faults occurring on power systems are transient in nature and if the circuit is opened momentarily, permitting the arc to become extinguished, the circuit can be reclosed successfully. The necessary power-off time for deionization of the arc is given in Chapt. 13. For example, Logan and Miles[86] have found that on the Georgia power system the number of successful reclosures is as follows.

Number of trip-outs.......	10090	100%
Successful reclosures		
1st immediate..........	8400	83.25%
2nd 15–45 seconds......	1084	10.05%
3rd 120 seconds.........	143	1.42%
Circuit lockouts..........	553	5.28%

This knowledge is used in a variety of ways. Many radial distribution feeders are provided with reclosing relays. A very common arrangement, using the Type RC reclosing relay shown in Fig. 33, provides for one immediate and several time-delay reclosures. In the event of a tripout after the third reclosure the line is locked out until the relay is reset manually. However if the line "holds," even on the third "try," the reclosing relay resets automatically, and is prepared to repeat the same performance at a later time.

For a feeder sectionalized by a number of fuses, the replacement of a fuse involves a service trip. However, an opening and reclosing operation interrupts the current before any fuses blow, and if the fault is transient, the service trip is avoided.

"Single-shot" reclosing which is also widely used may be accomplished by the Type SGR-12 reclosing relay. As shown by Logan's data, it takes care of the large proportion of cases. Also a self-reclosing single-pole circuit breaker is used principally on single-phase feeders to perform the single-shot reclosing function without the use of relays. Multiple-shot fuses are also used but require delayed action of subsequent fuses and necessitate refilling manually after each operation.

On tie lines or single lines serving important industrial loads reclosing is used for quite another purpose, namely, to keep the systems from going out-of-step or to prevent loss of essential loads. This phase of the problem is covered in Chapt. 13.

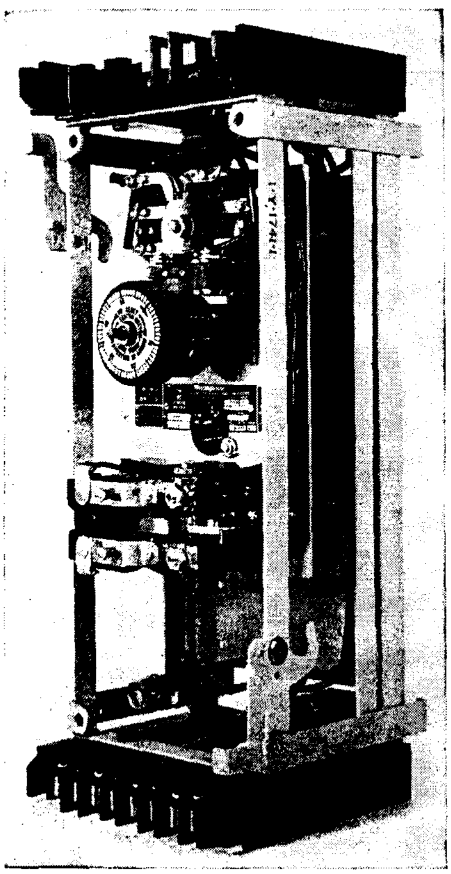


Fig. 33—The Type RC reclosing relay. A single instantaneous plus several time-delay reclosures can be initiated with this relay.

III. CONTROL SCHEMES

To secure the system benefits expected from the circuit breakers and protective relays, it is essential that the control scheme used result in prompt and reliable tripping of the circuit breaker when the relays indicate a fault within their protective zone. Two factors are involved: a reliable source of control power, and a scheme that permits the breaker to trip free in spite of all manual or automatic closing agencies.

A control battery provides the most reliable source of tripping energy, and is connected to the shunt-trip coil by the protective relay contacts as shown in Fig. 34. As

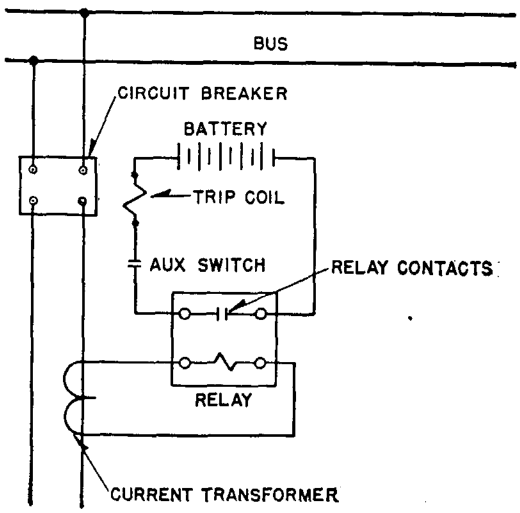


Fig. 34—A protective relay and circuit breaker. When the breaker opens, the auxiliary switch interrupts the trip circuit to prevent burning of the relay contacts.

the circuit breaker opens, the trip circuit is broken by an auxiliary switch linked to the circuit breaker.

25. Electrically Trip-Free Scheme

Fig. 35 shows a typical circuit-breaker control scheme, one of the several commonly used, known as the *X-Y* relay scheme. A station battery provides power for closing and tripping. The control switch merely controls the application of battery power to the closing solenoid or shunt-trip coil as desired. The protective relay, when it operates, applies battery power to the shunt trip coil.

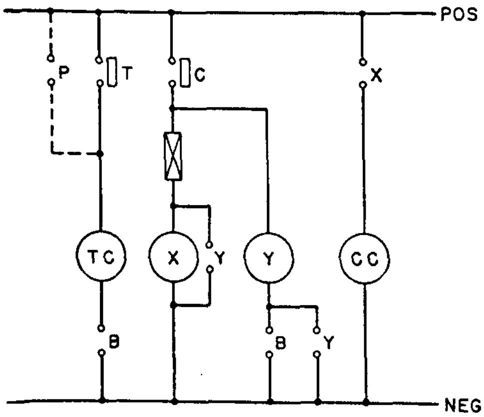


Fig. 35—The X-Y scheme of circuit breaker control using a battery.

C—Closing contact of control switch.
T—Tripping contact of control switch.
X—Closing contactor or relay.
CC—Closing coil of main circuit breaker.
Y—Releasing contactor or relay.
P—Typical protective relay contact.
B—Auxiliary switch (closed when main circuit breaker is closed).
TC—D-C shunt trip coil of main circuit breaker.

The *X-Y* relay scheme prevents pumping and makes the circuit breaker electrically trip-free. The closing contact of the control switch picks up the *X* relay that energizes the closing solenoid of the circuit breaker. As the circuit breaker reaches its closed position, an auxiliary switch *B* energizes the *Y* relay that seals in through its own front contacts. The *Y* relay contacts shunt the *X* relay, which opens and interrupts current to the circuit breaker closing solenoid.

If the circuit breaker trips automatically when it is closed in on a fault, it will open and will not reclose even though the operator holds the control switch in the closing position. The *X* relay remains shunted by the *Y* relay until the control switch is returned to the neutral position.

The trip-free relay scheme[1] provides a similar action through the use of a specially designed contactor for controlling the heavy current to the breaker-closing solenoid. The moving contact assembly of this contactor is tripped free from the operating armature by a release coil energized by an auxiliary switch when the circuit breaker reaches its closed position. Thus even though the closing contact of the control switch or of an automatic closing device remains closed, thereby holding the operating armature closed, the circuit to the closing solenoid of the circuit breaker remains open after the circuit breaker has once closed in and tripped out. This situation continues until the control switch is restored to neutral or the closing contact of the automatic device opens.

26. Mechanically Trip-Free Arrangements

If a circuit breaker is to be closed manually against a possible fault, it should be mechanically trip-free from the closing linkage. The mechanically trip-free feature provides somewhat faster tripping for three reasons. First, the circuit breaker contacts can be tripped free anywhere in the closing stroke without waiting for the closing current to be cut off before acceleration towards the open position can start. Second, with the contacts tripped free from the closing solenoid the mass to be accelerated is less. Third, because of eddy currents the flux in the closing solenoid does not decay immediately when the circuit is opened; thus there is appreciable magnetic retardation in the opening of a mechanically non-trip-free breaker which has just been closed.

High-speed reclosing requires that the circuit breaker be mechanically non-trip-free so that the contact motion can be arrested before the full open position and the breaker closed again. To meet this need without encountering delayed opening if the reclosure takes place on a permanent

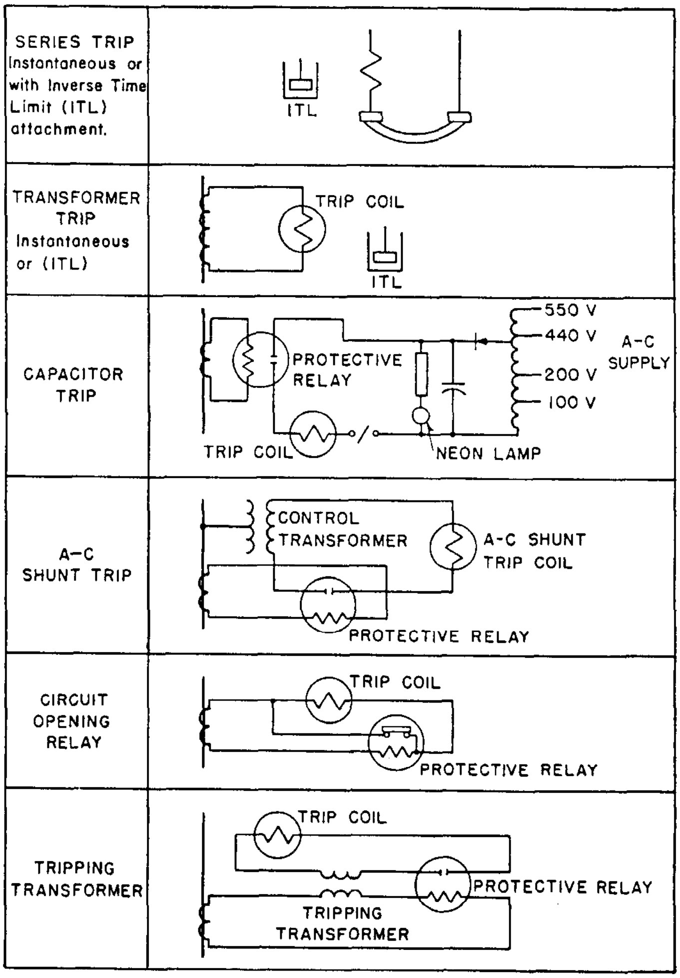


Fig. 36—A-C tripping schemes.

fault, a mechanism has been designed that is mechanically trip-free on the second opening but not on the first, except when the circuit breaker is initially closed.

27. A-C Tripping

In less important locations, where the cost and maintenance of a control battery is not justifiable, various forms of a-c tripping are employed for the smaller, lower voltage circuit breakers. Some of the more desirable of these are shown in Fig. 36.

The series-trip and transformer-trip schemes are used where the accurate magnitude and timing characteristics of a protective relay are not required. The transformer trip is used where the primary voltage or current is too great for the series trip.

The capacitor trip and a-c shunt trip require a source of a-c control power. The capacitor trip is much to be preferred in most cases because its ability to trip is not impaired by the momentary drop in voltage at the time of a fault. The a-c supply is taken from the source side of the circuit breaker so that the capacitor is charged before the circuit breaker is closed. A-c shunt tripping can be used only where the reduction of voltage at time of fault on the protected circuit will not prevent tripping by some tripping agency.

The circuit-opening relay scheme and the tripping-transformer scheme are similar to the transformer-trip scheme in that the line current transformer supplies the trip-coil energy. However, a protective relay is added and must be supplied by the same or by a different current transformer. The trip coil imposes a heavy burden on the current transformer, and there is a definite lower limit to the primary current at which tripping can be secured. The relay must, of course, be set above this value.

IV. APPLICATION OF CIRCUIT BREAKERS

The application of circuit breakers to power and lighting circuits involves the choice of the type of breaker and its mounting or housing as well as determination of the specific ratings required for the particular service.

28. Typical Circuit Breaker Construction and Practice

Low Voltage Circuit Breakers—Circuit breakers intended for service on a-c circuits up to 1500 volts and d-c circuits up to 3000 volts are classified as low-voltage breakers.[108] For such service air breakers have many advantages and are generally used in preference to oil breakers. They are inherently fast in operation, free from fire hazard, require little maintenance on repetitive service, and, because of the low voltage, are simpler, more compact, and easier to handle than oil breakers.

For low-capacity branch lighting and utility circuits such as are found in commercial and public buildings, small, molded case "thermal-breakers" are grouped in panelboards such as that shown in Fig. 37. Such breakers are usually operated manually and are available in ratings up to a maximum of 600 amperes load current and 25 000 amperes interrupting current. They provide automatic inverse-time overload tripping to protect circuit wiring

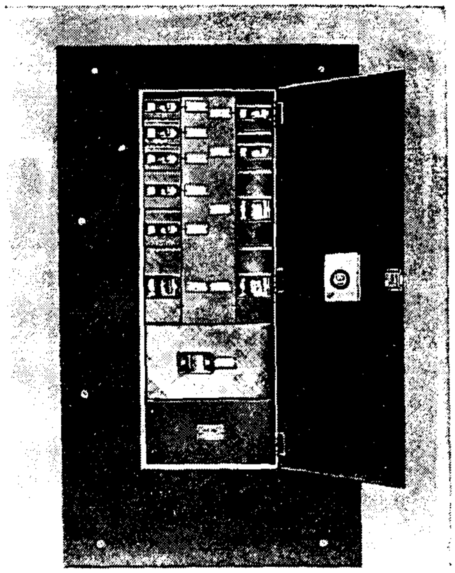


Fig. 37—Low-voltage distribution panelboard.

against overloads and short-circuits. Separate protective equipment is required for utilization devices such as motors.

For more important and higher capacity circuits, metal-enclosed, drawout assemblies of air circuit breakers, as in Figs. 38 and 39, are used. Typical applications are found in main feeders of the lighting and utility circuits described above, and for the low-voltage power circuits of such buildings as well as industrial plants and generating stations. These breakers may be operated either manually or electrically (under some conditions only electrical operation is recommended) and may be obtained with direct-acting series overload trips or relays which will give selec-

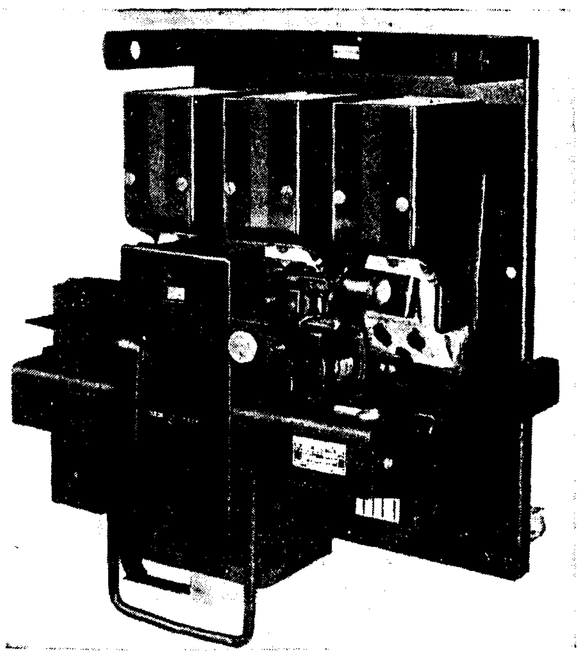


Fig. 38—1600 ampere, 600 volt, Type DB-50 air circuit breaker —50,000 amperes interrupting capacity.

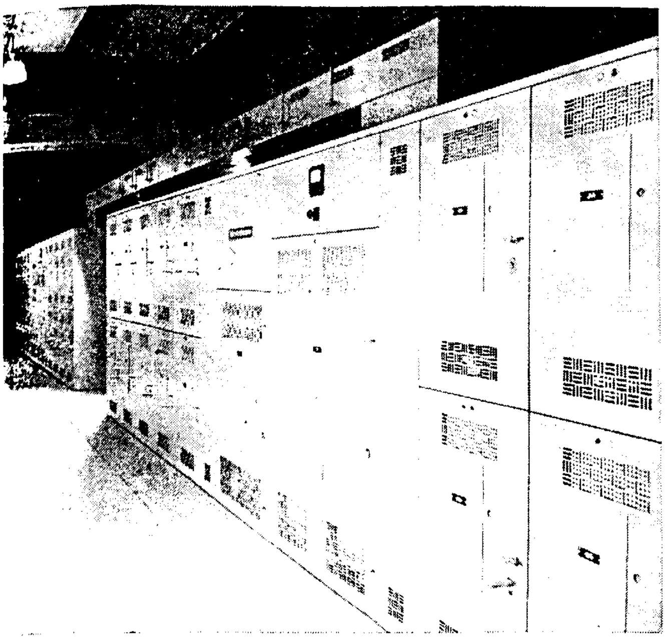


Fig. 39—Typical installation of low-voltage, metal-enclosed switchgear.

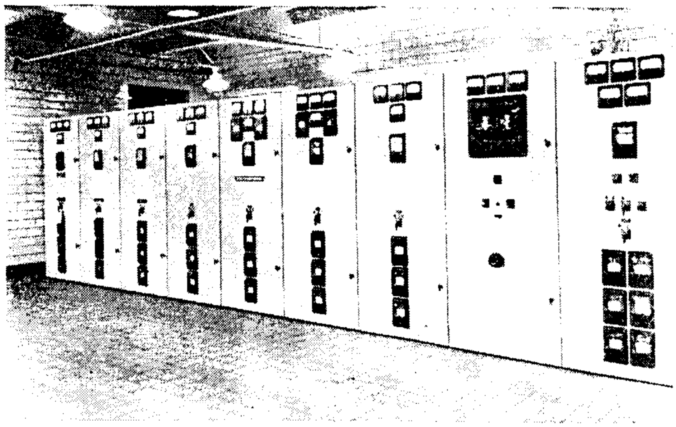


Fig. 41—Typical metal-clad drawout switchgear for 13.8 kv indoor service.

tive isolation of a faulty circuit. The metal-enclosed gear is factory assembled and tested and provides maximum reliability, safety and ease of maintenance with minimum interruption to service. These breakers may be used to provide control as well as running overload and short-circuit protection for individual motor circuits.

Even where unusual atmospheric conditions are encountered (see Sec. 30), low voltage air breakers may be used if they are mounted in suitable sealed enclosures.

Power Circuit Breakers — Medium Voltage — Circuit breakers intended for service on a-c circuits above 1500 volts are classified as power circuit breakers.

For indoor service at from 1.5 to 15.0 kv and up to 500 mva interrupting duty, magnetic-type air breakers in metal-clad assemblies have become predominant, although metal-clad oil breakers are also used under adverse atmospheric conditions. A typical breaker and assembly are shown in Figs. 40 and 41. Although interrupting time and space required are the same for medium voltage air and oil breakers, the freedom from oil-fire hazard and lower maintenance on repetitive service are distinct advantages of the air breakers. Many such breakers are used, both for power and lighting feeders and to control individual large industrial or powerhouse-auxiliary motors.

For indoor service at interrupting ratings above 500 mva, and for any rating at voltages between 15 and 34.5

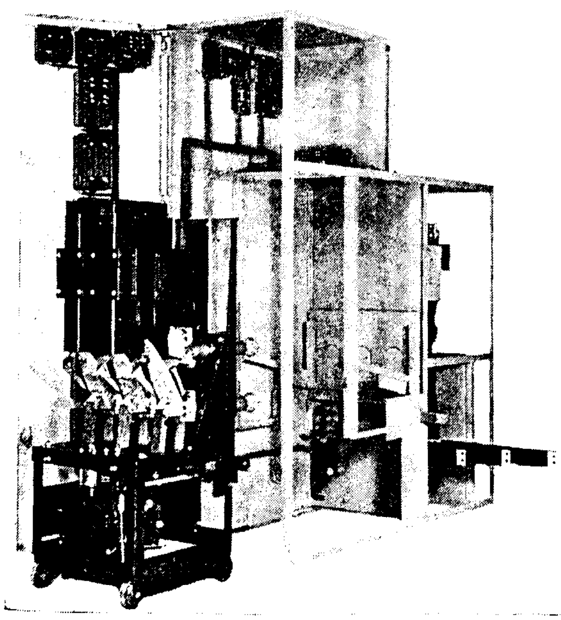


Fig. 40—Magnetic type air circuit breaker—4160 volts, 150-mva interrupting capacity; type 50-DH-150.

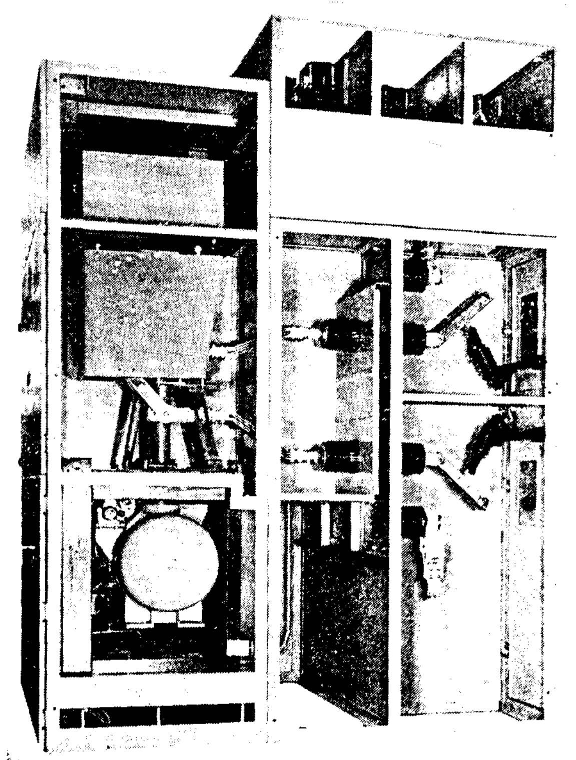


Fig. 42—Compressed air circuit breaker in station cubicle—15-H kv insulation class.

kv, compressed-air breakers mounted in station cubicles have become standard. A typical unit is shown in Fig. 42.

Circuit-breaker practice in outdoor substations is more varied than in indoor service because there is a greater range in the requirements. In rural and outlying substations both normal load and interrupting kva are relatively low, and such factors as fire hazard, space requirements, appearance and rapid maintenance may not be critical. For such service frame-mounted oil breakers with open buses and disconnecting switches are frequently used because of lower cost. A typical installation is shown in Fig 43.

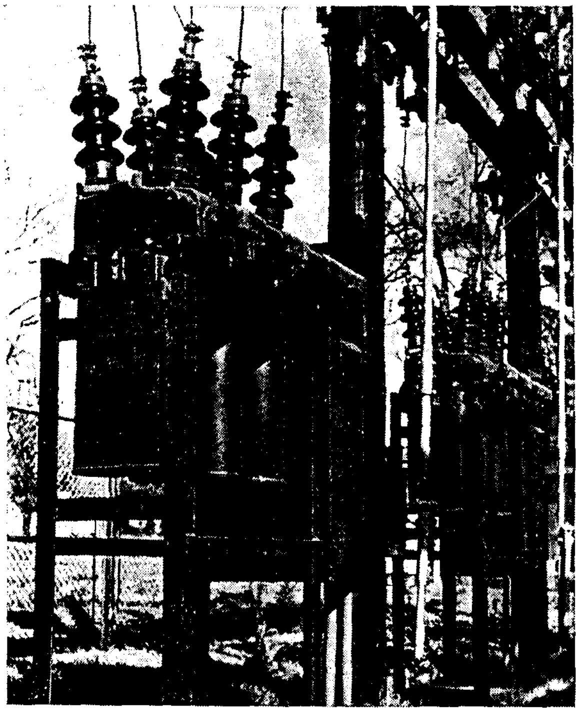


Fig. 43—Installation view of outdoor 23 kv, 250 000 kva, oil circuit breakers.

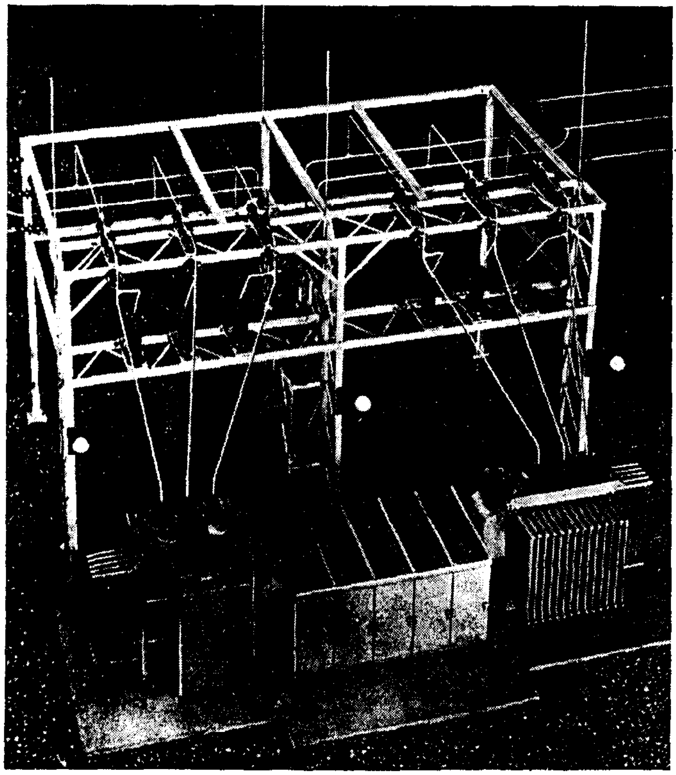


Fig. 44—Typical outdoor coordinated unit substation.

For suburban and urban outdoor service up to 15 kv and up to 500-mva interrupting duty the many advantages of metal-clad oil-less switchgear (freedom from oil-fire hazard, compactness, appearance, ease of maintenance and flexibility) have resulted in the use of such gear for the majority of these substations. The metal-clad gear is usually throat connected to the transformer(s) although roof bushings are sometimes used, especially with single-phase transformers. A typical installation is shown in Fig. 44. The cost comparison between open gear and metal clad varies with the voltage and kva rating, the type of open structure used, the cost of real estate, the labor facilities of the utility, and the method used in estimating overhead and fixed charges.

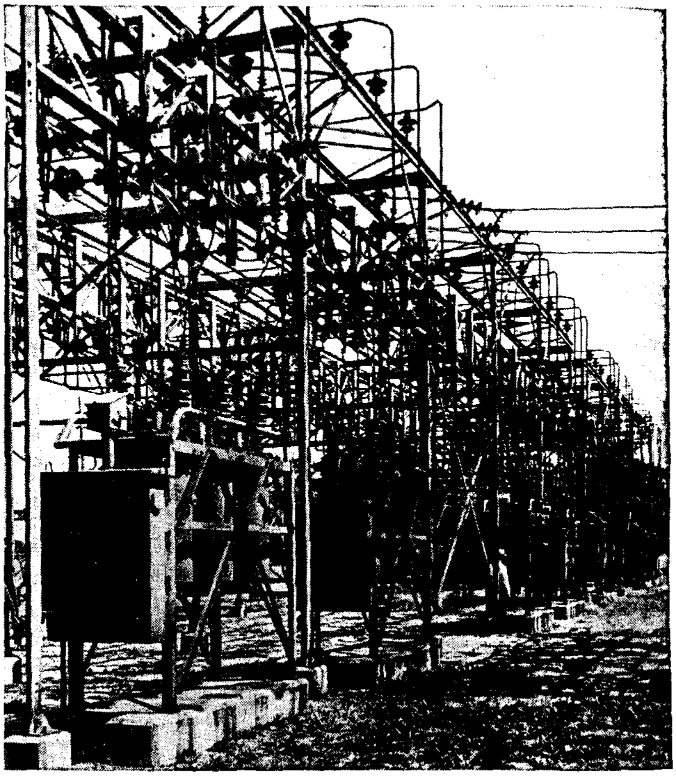


Fig. 45—34.5-kv, 1000-mva, frame-mounted, oil circuit breakers.

Oil circuit breakers are used where severe atmospheric conditions are encountered, either frame mounted or in metal-clad structures.

For outdoor service at interrupting ratings above 500 mva and for all interrupting ratings at voltages between 15- and 34.5-kv, oil breakers are essentially standard. A typical installation is shown in Fig. 45.

High-Voltage Breakers—Almost all circuit breakers rated above 34.5 kv are mounted outdoors and are oil

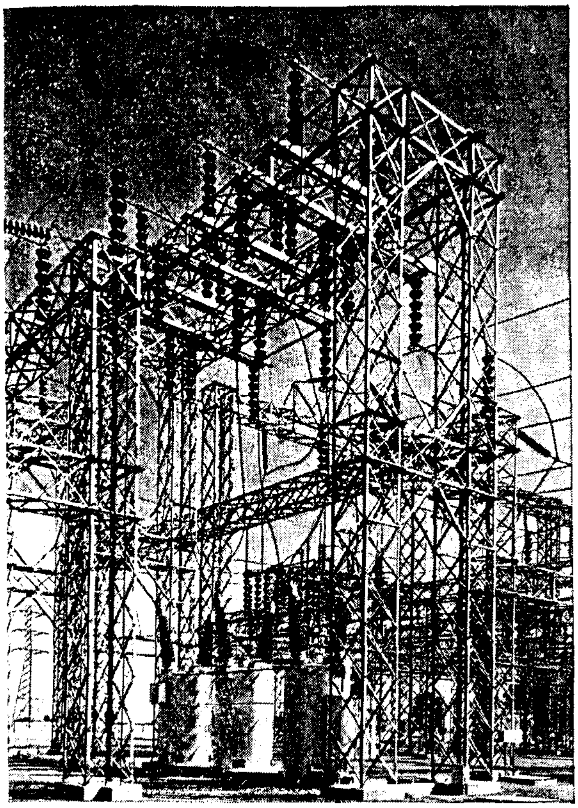


Fig. 46—230-kv, 3500-mva, floor-mounted, oil circuit breaker.

breakers of the grounded metal-tank type. Pneumatic operating mechanisms predominate and these can be arranged to give 20-cycle reclosing. A 230-kv installation is shown in Fig. 46.

Some experimental installations have been made of air-blast and oil-poor breakers at high voltage and a table of ratings for such breakers has been included in the standards as a guide for development. At the present time it is difficult to produce and install an oil-less outdoor breaker having current transformers and potential devices to compete economically with conventional oil breakers. There is also the hazard of fragile porcelain structures. So far there is no definite indication of the place such breakers will fill in normal practice.

29. Standard Ratings of Circuit Breakers

The standard ratings of the several classes of circuit breakers are defined in ASA, AIEE, NEMA, and Underwriters Laboratories standards.[105-111] It is not the intention here to review these ratings in detail, only to discuss the principal factors involved in the selection of a circuit breaker for a particular application.

Rated Voltage—In general a circuit breaker is given a rated voltage which designates the maximum *nominal* system voltage for which the breaker is intended and also a maximum design voltage which designates the maximum operating voltage for which it is intended. For certain low-voltage breakers this distinction is not made and rated voltage should be taken as maximum. Standard voltage ratings of power circuit breakers are in terms of three-phase line-to-line voltage.

Standard values of rated voltage are based on operation at altitudes of 3300 feet or less. Standard equipment may be operated at higher altitudes if the maximum operating voltage is not more than the maximum design voltage times a correction factor, as follows:

Altitude in Feet	*Voltage Correction Factor*
3300	1.00
4000	0.98
5000	0.95
10000	0.80

Operation at altitudes other than those listed is covered in AIEE #1B but operation above 10 000 feet should be given special consideration because of possible influence of altitude on interrupting capacity.

Rated Impulse Withstand Voltage—Impulse ratings of standard power circuit breakers are listed in ASA C 37.6. A correction (the same as that given above for rated voltage) should be made for the effect of altitude above 3300 feet on impulse strength. No such ratings are given for low-voltage air breakers because such breakers are seldom exposed to impulse voltages. When necessary, impulse strength could be taken as the crest of the 60-cycle test voltage.[110]

The surge protection of the system should be coordinated with the impulse strength of the breaker, both across the open contacts and to ground. Attention should also be given to increase in surge voltage because of reflections which occur at breakers when their contacts are open, especially where cables are involved.[104]

Frequency—Standard power circuit breakers are rated at 60 cycles. Service at other frequencies must be given special consideration. Although standard 60-cycle power circuit breakers are given corresponding continuous-current ratings for 25-cycle service,[105] other ratings (e.g. interrupting capacity) must be checked and accessories must be made suitable. Low-voltage breakers are listed for 60-cycle service and for direct current. Service at other frequencies requires special consideration.

Rated Continuous Current—This rating is based on operation of the circuit breaker or switchgear assembly where the ambient temperature (measured outside the enclosure where such is supplied) does not exceed 40°C and the altitude does not exceed 3300 feet. Operation in higher ambient temperature must be given special consideration. Molded-case, thermal-trip, low-voltage circuit breakers are calibrated on the basis of an ambient temperature of 25°C. Operation in ambient temperatures other than 25°C will affect the tripping characteristic and must be taken into consideration. Standard equipment may be operated at higher altitudes by reducing the continuous current rating in accordance with the following table.

Altitude in Feet	*Current Correction Factor*
3300	1.00
4000	0.996
5000	0.99
10000	0.96

Rated Interrupting Current—Rated Interrupting Mva—Operating Duty—Interrupting Time—The rated interrupting current of a power circuit breaker is based on the rms total current in any pole of the breaker at the time the breaker contacts part. (Note that this time may be considerably shorter than the interrupting time.) The correct value of rated interrupting current for an operating voltage other than rated value can be calculated by the following formula:

Amperes at operating voltage = amperes at rated voltage $\times \dfrac{\text{rated voltage}}{\text{operating voltage.}}$ Operating voltage should, of course, not exceed the maximum design voltage. Also, no matter how low the voltage, the rated interrupting current is not increased above the *rated maximum interrupting current*. Standard rating tables[105] give the value of rated interrupting current at rated voltage as well as the rated maximum interrupting current and the corresponding operating voltage. Over this range of voltages the product of operating voltage and current is constant and this product times a phase-factor is called *rated interrupting mva*. For 3-phase circuits the factor is 1.73, for 2-phase circuits 2.0, and for 1-phase circuits 1.0. However, standard breakers are rated only on a three-phase basis and rules are provided (given later in this chapter) for determining the equivalent three-phase interrupting ratings.

The above values of rated interrupting current are based on specified conditions of circuit recovery voltage, breaker performance, and also on *standard operating duty*. For power circuit breakers rated 50 mva and higher ("oil-tight or oil-less") this consists of two *unit operations* (CO) separated by a 15-second interval. Each unit operation consists of breaker closing followed by its opening without intentional time delay. This standard operating duty is designated by the expression, CO+15 sec.+CO. For power circuit breakers rated 25 mva and lower ("non-oil-tight") the standard operating duty consists of two unit operations separated by a two-minute interval (CO+2 min. + CO). For any other operating duty the standard interrupting ratings should be reduced in accordance with rules given in NEMA standards.[109] The following revision of the current rules is now being recommended by AIEE to ASA.

NEMA STANDARDS—RECLOSING DUTY CYCLE FACTORS FOR OIL-TIGHT AND OIL-LESS POWER CIRCUIT BREAKERS—REVISION OF 11/17/49

SG-6-90 BREAKER RATING FACTORS FOR RECLOSING SERVICE (Rev.)

A. The interrupting ratings of power circuit breakers may be reduced for operating duty cycles other than the standard, CO +15 Sec.+CO (See SG6–40, Par. B) to enable them to meet the standard of Interrupting Performance. (See SG6–40, Par. C)

Note: Such factors do not apply to highly repetitive duty at or near the continuous rating of the breaker.

B. For purposes of this section, the following duty cycles shall be considered as representing the Usual Duty Cycles for Reclosing

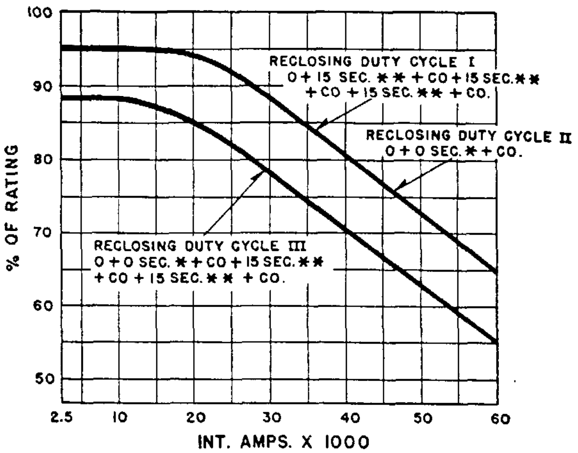
Reclosing Duty Cycle I O+15 Sec. **+CO+15 Sec.** +CO+15 Sec.**+CO

Reclosing Duty Cycle II O+0 Sec.*+CO

Reclosing Duty Cycle III O+0 Sec.*+CO+15 Sec.** +CO+15 Sec.**+CO

*Zero seconds shall be interpreted to mean no intentional time delay.

**15 seconds or longer.



* ZERO SECONDS SHALL BE INTERPRETED TO MEAN NO INTENTIONAL TIME DELAY.

** 15 SECONDS OR LONGER.

Fig. 47—Breaker rating factors for the usual reclosing duty cycles.

C. The Breaker Rating Factors for the Usual Reclosing Duty Cycles are obtained by reference to Figure 47. This gives the percentage rating factor for the duty cycle in question at the rated interrupting current at the circuit voltage of the system to which the breaker will be applied.

D. The breaker interrupting rating at the specified reclosing duty cycle is obtained by multiplying the rated interrupting amperes by the rating factor.

Example: Determine the interrupting rating of a 34.5 kv, 1000-mva breaker used at 23-kv on Reclosing Duty Cycle I.

1. Breaker Interrupting Rating on Standard Duty Cycle:
17 000 Amperes at 34.5-kv
25 000 Amperes at 23 kv
2. Figure 1, Curve A, gives a rating factor of 91.5 percent at 25 000 amperes for reclosing duty cycle I.
3. Breaker Interrupting Rating on Duty Cycle I at 23 kv = Rating Factor, 91.5 percent×25 000 Amperes = 22 900 Amperes.

E. Breaker Rating Factors for Other Than the Usual Reclosing Duty Cycles

1. The standard for the number of operations is two (2). Additional operations increase the duty on the contacts and therefore a rating factor is applied to enable circuit breakers to meet standards of interrupting performance. (See SG6-40, Par. C)

(a) For Three Operations—Use mean factor between 2 and 4 operations, (Standard and Duty Cycle I).

Example: O+15 Sec.+CO+15 Sec.+CO
% Factor = 97 for 20 000 Amperes

(b) For Five Operations—Use Factor obtained by reducing that for 4 operations (Duty Cycle I) by difference between 2 and 4 operations.

Example: O+15 Sec.+CO+15 Sec.+CO+15 Sec. +CO+15 Sec.+CO
% Factor = 94 − 6 = 88 for 20 000 Amperes

2. The standard for the interval between operations is 15 seconds. Reducing this interval increases the interrupting duty and therefore a rating factor is applied to enable breaker to meet standards of interrupting performance. (See SG6–40 Par. C)

(a) For Two Operations—Reduce 100 percent factor for 15 seconds by an amount equal to proportionate part of the reduction for zero interval as determined by the ratio of the times.

Example: O+5 Sec.+CO
$\%$ Factor $= 100-\frac{2}{3}(100-94) = 96$ for 20 000 Amperes

(b) For Four Operations—Multiply the factor for Duty Cycle I by the appropriate factor determined under paragraph 2 (a) for each interval less than standard.

Example: O+5 Sec.+CO+5 Sec.+CO+5 Sec.+CO
$\%$ Factor $= 94\times96\times96\times96 = 83$ for 20 000 Amperes

(c) For Three or Five Operations—Use combination of Rule 1 and Rule 2 (a) applied for each interval.

Example: O+5 Sec.+CO+5 Sec.+C
$\%$ Factor $= 97\times96\times96 = 89$ for 20 000 Amperes

3. The usual instantaneous reclosing cycles are Duty Cycles II and III. Variations are in the number of operations.

(a) For Three Operations—Use mean between factors for Duty Cycles II and III.

Example: O+0 Sec.+CO+15 Sec.+CO
$\%$ Factor $= \dfrac{94+85}{2} = 89.5$ for 20 000 Amperes

(b) For Five Operations—Use factor by reducing that for 4 operations (Duty Cycle III) by difference between 2 and 4 operations.

Example: O+0 Sec.+CO+15 Sec.+CO+15 Sec.+CO +15 Sec.+CO
$\%$ Factor $= 85-(94-85) = 76$ for 20 000 amperes

The reclosing duty cycle factors for breakers rated 25 mva and below (non-oil-tight circuit breakers) are given in Table 5.

TABLE 5*—RECLOSING DUTY CYCLE FACTORS FOR NON OIL-TIGHT OIL POWER CIRCUIT BREAKERS

Duty Cycle	Percentage of Standard Interrupting Capacity Rating
B—CO+2 min.+CO	100
C—CO+2 min.+CO+2 min.+CO+2 min.+CO	70
D—CO+30sec.+CO+30 sec.+CO+30sec.+CO	60
**E—CO+0 sec.+CO+0 sec.+CO+0 sec.+CO	25
F—300 cycles CO at 15-min. intervals	30
**G—CO+0 sec.+CO+30 sec.+CO+75 sec.+CO	30
H—CO+15sec.+CO+30sec.+CO+75sec.+CO	40
I—CO+60 sec.+CO+60 sec.+CO	70
J—CO+15 sec.+CO	60

NOTE—The standard operating duty (duty cycle) for non oil-tight oil power circuit breakers is 2-CO operations with a 2-minute interval.
*Reproduced from NEMA 46-116, SG6-90.
**Zero seconds shall be interpreted to mean no intentional time delay.

The interrupting time of a power circuit breaker is the maximum interval from the time the trip coil is energized at normal control voltage until the arc is extinguished. This time is published for standard power breakers for the interruption of currents from 25 percent to 100 percent of the rated value.

The interrupting rating of *low-voltage air circuit breakers* is based on the rms total short-circuit current which would occur at the end of ½ cycle at the breaker location if the line terminals of the breaker were short-circuited. The impedance of the breaker which interrupts the circuit should *not* be included in calculating the interrupting duty. Also, for three-phase a-c circuits the breaker rated interrupting current should be chosen on the basis of the *average* of the currents in the three phases. For single-phase circuits the average current which would occur for three successive short-circuits should be used. For average systems the average 3-phase or 1-phase rms total current will be equal to 1.25 times the initial subtransient symmetrical current.

Low-voltage air breakers for d-c service are also applied on the basis of the short-circuit current without the breaker in place; however, the maximum current is measured.

The standard rated interrupting current of low-voltage air circuit-breakers is based on a standard operating duty designated O+2 min+CO. The breaker opens the circuit and, after a 2-minute interval, is reclosed on the fault, which it opens without purposely delayed action.

For other interrupting duty the standard interrupting rating should be multiplied by factors given in Table 6.

TABLE 6**—OPERATING DUTY FOR RECLOSING SERVICE FOR LARGE AIR CIRCUIT BREAKERS

Duty Cycle	Percentage of Published Interrupting Rating
B—O+2 min.+CO	100
C—O+2 min.+CO+2 min.+CO+2 min.+CO	70
D—O+30 sec.+CO+30 sec.+CO+30 sec.+CO	60
*E—O+0 sec.+CO+0 sec.+CO+0 sec.+CO	25
F—300 cycles CO+15 min. intervals	30
*G—O+0 sec.+CO+30 sec.+CO+75 sec.+CO	30
H—O+15 sec.+CO+30 sec.+CO+75 sec.+CO	40
I—O+60 sec.+CO+60 sec.+CO	70
J—O+15 sec.+CO	60

*Zero seconds shall be interpreted to mean no intentional time delay.
**Copied from SG7-63 of NEMA std. 46-109.
NOTE—Derating factors are not available for automatic reclosing service on d-c circuits. Circuit breakers designed for the purpose are ordinarily required.

This table does not apply to molded-case breakers because they are not used on reclosing service.

Rated Momentary Current—The maximum rms total current (including the d-c component) through a breaker, measured during the maximum cycle, should not exceed the rated momentary current. For power circuit breakers this rating applies to each pole of the breaker taken individually and for the worst condition of asymmetry.

Rated Four-Second Current—A four-second current rating is given for power circuit breakers based on the rms total current measured or calculated at the end of one second. For standard breakers it is numerically equal to the rated maximum interrupting current, and 1/1.6 times the momentary current. For normal circuits this means that the permissible duration of the maximum permissible fault current is four seconds. No current rating is given for times longer than four seconds but less than continuous.

No similar short-time rating is given for low-voltage breakers because such breakers are normally equipped with direct acting series overload trips.

Rated Making Current—This rating is given to power operated power circuit breakers only. No provision is made for manual closing of oil-less breakers or for oil breakers above 250 mva. It is essentially a design requirement to preclude welding of contacts or other undue damage when a breaker closes on a fault. It is required that the breaker "be immediately opened without purposely delayed action."

The values of momentary and making-current rating for present standard breakers have been so selected that these ratings will not normally limit application of the breakers when they are applied in accordance with the recommended "simplified procedure," which will be described later in this chapter. An exception may occur where motors produce a large portion of the fault current. Breakers may also be applied on the basis of decrement curves or detailed calculations and, under unusual conditions or in existing installations, momentary current may be the limiting rating of the breaker.

Rated Latching Current—This rating is distinguished from rated making current in that the breaker must latch when it closes on a fault of the specified rms total current magnitude. Thus delayed tripping is permissible within this rating if the magnitude, duration, and operating duty are within the short-time and interrupting ratings. For present standard power circuit breakers the latching current rating is numerically equal to both the four-second rating and the maximum interrupting current rating. However, the latching current is measured during the maximum cycle whereas the interrupting rating is measured at the time the contacts part and the four-second rating is measured at the end of one second.

Reclosing Time—For outdoor reclosing oil-circuit breakers standard and fast reclosing times are shown in Table 7. These values apply only to breakers which have a continuous current rating of 1200 amperes or less when operated in conjunction with an automatic reclosing device.

TABLE 7*—RECLOSING TIME FOR OUTDOOR RECLOSING OIL CIRCUIT BREAKERS (60-CYCLE BASIS)

Rated Voltage	Reclosing Time-Cycles	
	Standard	Fast
7.5 to 23 kv incl. (under 500 mva)	30	30
15 to 23 kv incl. (500 mva and higher)	45	30
34.5 to 69 kv incl. (500 mva and higher)	45	20
115 to 230 kv incl. (500 mva and higher)	30	20

NOTE I—These time values assume rated control voltage or operating pressure maintained at the mechanism. In case the control voltage or pressure drops to 90 percent of rated voltage or pressure, the reclosing times will be increased to 110 percent of that tabulated.

NOTE II—Reclosing time for oil-less breakers has not yet been standardized.

*This table was reproduced from NEMA standard 46-116 SG6-95.

30. Selection of Circuit Breakers for Specific System Conditions

General—The great majority of circuit breakers are applied as three-pole, gang-operated breakers in three-phase power systems which are ungrounded or grounded at the neutrals of generators or transformers. Consequently the standard ratings of most circuit breakers are given on this basis. For such an application it is sufficient (in so far as rating is concerned) to select a breaker such that none of its standard ratings will be exceeded under any condition of system operation.

For example, the voltage rating should include allowances, where applicable, for such factors as line voltage regulation, shunt or series capacitance, overexcited or overspeed operation of synchronous machines, line-drop-compensation of tap changers or feeder voltage regulators and the operation of transformers on tap positions other than the nominal values. Both voltage and continuous current ratings should take into account future load growth and the contingencies associated with circuit or apparatus outages. The various ratings associated with interruption of faults should include allowances for increase in generation, addition of parallel circuits or transformers, and any other system changes which would increase interrupting duty. The calculation of fault currents and their interpretation in terms of interrupting ratings will be considered separately in Sec. IV.

System frequency will usually be substantially constant at 60-cycles. Operation at other frequencies or at varying frequency requires special consideration.

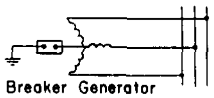
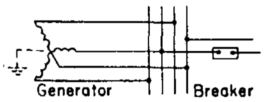
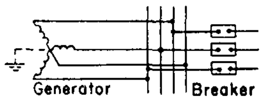
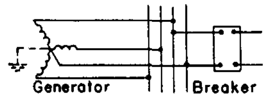
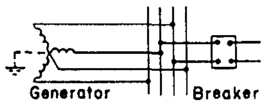
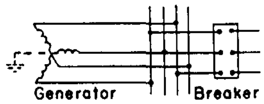
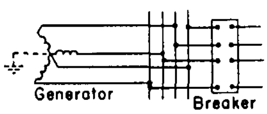
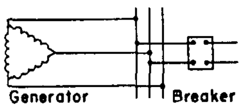
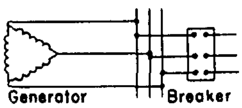
The interrupting time of the circuit breakers themselves is subject to choice in few cases. Considerations of transient stability may dictate one or another type of relaying system in order to obtain sufficiently fast clearing. However, considerations of system operation and stability may or may not call for fast reclosing, and a choice should be indicated.

Determination of Equivalent Three-Phase Voltage and Interrupting Ratings—The standard ratings of most power circuit breakers are given in terms of three-pole breakers for three-phase systems. These voltage ratings are based on the line-to-line voltage of the circuit, and the interrupting ratings are given in amperes and approximate three-phase kva. In order to select the proper 1-, 2-, 3-, or 4-pole circuit breaker for special services on three-phase circuits, and for use on two-phase and single-phase circuits, the equivalent three-phase breaker rating can be determined from Tables 8, 9, and 10.

First, the three-phase voltage rating of the breaker type must be equal to or greater than the voltage determined from column 5 of the tables.

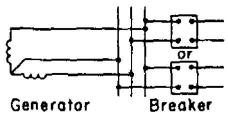
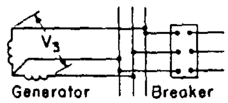
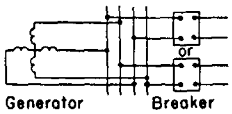
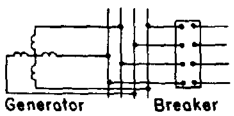
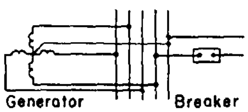
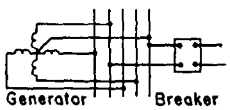
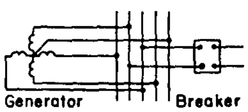
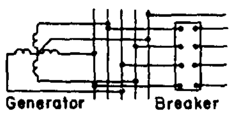
Second, make a tentative breaker selection on the basis of equivalent three-phase kva in accordance with column 6 of the tables. If the computed equivalent three-phase kva is more than 95 percent of the approximate kva rating of the breaker type a further check must be made. In such cases the product of rated voltage times rated interrupting current times 1.73 (standard three-phase ratings) must equal or exceed the equivalent three-phase kva calculated in column 6.

TABLE 8—DETERMINATION OF EQUIVALENT THREE-PHASE VOLTAGE AND INTERRUPTING RATINGS FOR THREE-PHASE SYSTEMS

Line No.	Typical System Connections	Type of System	No. of Breaker Poles	Select a breaker whose voltage rating is equal or greater than the equivalent 3-phase voltage (V_3) as determined below.	Select a breaker whose approximate 3-phase kv-a. interrupting rating is equal to or greater than the value as determined below. I = Required interrupting current at service voltage.† V_3 = Equivalent 3-phase voltage as obtained from adjacent column.
1		3-wire grounded or 4-wire with neutral grounded or not	1 pole in neutral wire	Greatest wire to wire voltage	0.87 I × 1.73 V_3
2		3-wire grounded or 4-wire with neutral grounded or not	1 pole in outside wire	Greatest wire to wire voltage	0.87 I × 1.73 V_3**
3		3-wire grounded or 4-wire with neutral grounded or not	Independent 1 pole breaker in an outside wire	1.15 times greatest wire to wire voltage*	I × 1.73 V_3
4		3-wire grounded or 4-wire with neutral grounded or not	2 poles in outside and neutral wire respectively	Greatest wire to wire voltage	I × 1.73 V_3
5		3-wire grounded or 4-wire with neutral grounded or not	2 poles single-phase outside wire	Greatest wire to wire voltage	I × 1.73 V_3
6		3 wire grounded or 4-wire with neutral grounded or not	3 poles 3-phase circuit	Greatest wire to wire voltage	I × 1.73 V_3
7		3-wire grounded or 4-wire with neutral grounded or not	4 poles	Greatest wire to wire voltage	I × 1.73 V_3
8		3-wire ungrounded	2 poles single-phase circuit	Greatest wire to wire voltage	I × 1.73 V_3
9		3-wire ungrounded	3 poles	Greatest wire to wire voltage	I × 1.73 V_3

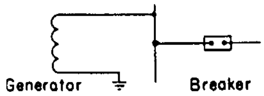
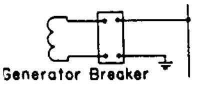
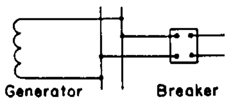
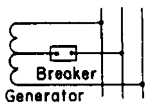
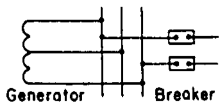
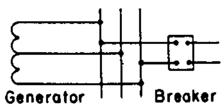
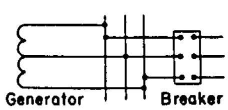
† This value must not exceed the maximum current interrupting rating listed in breaker interrupting tables.
* See NEMA rule SG6-210 regarding exceptions for 8 cycle breakers. In such instances use Line 6.
** These recommendations apply to isolated feeders only. Where possibility of phase to phase faults exist refer to Line 3 for application.

TABLE 9—DETERMINATION OF EQUIVALENT THREE-PHASE VOLTAGE AND INTERRUPTING RATINGS FOR TWO-PHASE SYSTEMS

Line No.	Typical System Connections	Type of System	No. of Breaker Poles	Select a breaker whose voltage rating is equal or greater than the equivalent 3-phase voltage (V_3) as determined below.	Select a breaker whose approximate 3-phase, kv-a, interrupting rating is equal or greater than the value as determined below. I = Required interrupting current at service voltage.† V_3 = Equivalent 3-phase voltage as obtained from adjacent column.
1		3-wire grounded or not	2 poles any two wires	Greatest wire to wire voltage	*I × 1.73 V_3
2		3-wire grounded or not	3 poles	Greatest wire to wire voltage	*I × 1.73 V_3
3		4-wire ungrounded	2-2 pole	Greatest wire to wire voltage	I × 1.73 V_3
4		4-wire ungrounded	4 poles	Greatest wire to wire voltage	I × 1.73 V_3
5		5-wire grounded or not	1 pole any "outside wire"	Greatest wire to wire voltage	0.87 I × 1.73 V_3
6		5-wire grounded or not	2 poles any "outside wire" and neutral	Greatest wire to wire voltage	0.87 I × 1.73 V_3
7		5-wire grounded or not	2 poles d-c. any "outside wires"	Greatest wire to wire voltage	0.87 I × 1.73 V_3
8		5-wire grounded or not	4 poles all "outside wires"	Greatest wire to wire voltage	I × 1.73 V_3

* I is the current in one of the outside wires.
† This value must not exceed the maximum current interrupting rating listed in breaker interrupting tables.

TABLE 10—DETERMINATION OF EQUIVALENT THREE-PHASE VOLTAGE AND INTERRUPTING RATINGS FOR SINGLE-PHASE SYSTEMS

Line No.	Typical System Connections	Type of System	No. of Breaker Poles	Select a breaker whose voltage rating is equal or greater than the equivalent 3-phase voltage (V_3) as determined below.	Select a breaker whose approximate 3-phase, kv-a, interrupting rating is equal or greater than the value as determined below. I = Required interrupting current at service voltage.† V_3 = Equivalent 3-phase voltage as obtained from adjacent column.
1		2-wire one side grounded	1	1.73 times wire to ground voltage	$0.87\ I \times 1.73\ V_3$ (based on NEMA rule SG6-205)
2		2-wire one side grounded	2	1.73 times wire to ground voltage	$I \times V_3$ (based on NEMA rule SG6-205)
3		2-wire ungrounded	2	wire to wire voltage	$I \times 1.73\ V_3$
4		3-wire neutral may or may not be grounded	1 pole in neutral circuit	1.73 times higher line to neutral voltage	$0.87\ I \times 1.73\ V_3$
5		3-wire neutral may or may not be grounded	1 pole either "outside wire"*	1.73 times respective line to neutral voltage	$0.87\ I \times 1.73\ V_3$
6		3-wire neutral may or may not be grounded	2 outside wires	Greatest wire to wire voltage	$I \times 1.73\ V_3$
7		3-wire neutral may or may not be grounded	3	Greatest wire to wire voltage	$I \times 1.73\ V_3$

* Where 1 phase, 3-wire system has unequal voltages to neutral and 2 single-pole breakers are used in the outside wires, the lower voltage breaker must be interlocked to prevent its tripping for "outside wire" faults, until the higher voltage breaker has first cleared the fault, unless both breakers are selected on high-voltage basis.
† This value must not exceed the maximum current interrupting rating listed in breaker interrupting tables.

Third, the short-time current rating and the interrupting capacity current limitation must not be exceeded.

The fault current may be calculated by one of the methods described in the next section, and should be checked for all types of faults.

Switching of Capacitive Current—When circuit breakers are used to switch the charging current of lines or cables or to switch capacitor banks, abnormally high voltages can be produced by restriking in the breaker. Experience [101] has indicated that transient voltages which result from such restriking will seldom exceed 2.5 times normal line-to-neutral crest voltage on circuits having effectively grounded neutrals. There is relatively little hazard to either the breakers or to other apparatus on such circuits. There are insufficient data on ungrounded or impedance-grounded systems to draw conclusions.

Lightning arresters may be damaged if the voltages developed are sufficient to cause them to discharge and if, in addition, the line capacitance is large. Because of the random nature of the phenomena involved it is not possible at this time to give specific limits for capacitive switching. As an approximate guide special consideration should be given when one desires to switch 69-kv cables which exceed 9 miles in length or 115- and 138-kv cables longer than 7 miles.

Another problem to be considered is that a large momentary current may flow when one capacitor bank is switched in parallel with another capacitor bank. This current is a function of the capacitance involved and the inductance of the leads connecting the two banks. This current may be calculated[102] and should not exceed the momentary rating of the circuit breaker.

Conditions Affecting Construction or Protective Features—There are unusual conditions which, where they exist, should be given special consideration in the selection and design of the apparatus. Among such unusual conditions are:

(1) Exposure to damaging fumes or vapors, excessive or abrasive dust, explosive mixtures of dust or gases, steam, salt spray, excessive moisture, or dripping water, etc.;

(2) Exposure to abnormal vibration, shocks or tilting;
(3) Exposure to excessively high or low temperatures;
(4) Exposure to unusual transportation or storage conditions;
(5) Unusual space limitations;
(6) Unusual operating duty, frequency of operation, difficulty of maintenance, etc.

31. Requirements for Low-Voltage Air Circuit Breakers in Cascade Arrangement*

"When a plurality of low voltage air circuit breakers are connected in series in a distribution system, and the breakers beyond those nearest to the source are applied in the following correlated manner, they are said to be in a cascade arrangement.

"In this cascade arrangement, breakers toward the source are provided with instantaneous tripping for current values which may obtain for faults beyond other breakers nearer the load. Hence, breakers in the series, other than the breaker closest to a fault may trip and interrupt loads on other than the fault circuit. Such arrangements are used only where the consequent possible sacrifice in service continuity is acceptable. Where continuity of service is desired, selective tripping arrangements of fully rated breakers are required. Where continuity of service is not important, properly selected breakers may be applied in cascade.

"The following requirements shall be observed:

(a) Cascading shall be limited to either two or three steps of interrupting rating.

(1). The interrupting rating of the breaker or breakers nearest the source of power shall be equal to at least 100 percent of the short-circuit current as calculated in accordance with section 29. The breaker or breakers in this step shall be equipped with instantaneous features set to trip at a value of current that will give back-up protection whenever the breaker in the next lower step carries current greater than 80 percent of its interrupting rating.

(2) The breaker or breakers in the second step shall be selected so that the calculated short circuit current through the first step plus motor contribution in the second step, will not exceed 200 percent of their interrupting rating. The breaker or breakers shall be equipped with instantaneous trip set at a value of current that will give back-up protection whenever the breaker in the next lower step carries current greater than 80 percent of its interrupting rating. For the second step of a two step cascade the breaker or breakers shall have an instantaneous trip setting above the starting inrush current of the load.

(3) The breaker or breakers in the third step shall be selected so that the calculated short circuit current through the first step, plus motor contribution of the second and third steps, will not exceed 300 percent of their interrupting rating. The breaker or breakers shall have instantaneous trips set above the starting inrush current of the load.

(b). All circuit breakers subjected to fault currents in excess of their interrupting rating shall be electrically operated.

(c). Where cascading is proposed, recommendations shall be obtained from the manufacturer in order to insure proper coordination between circuit breakers.

(d). The operation of breakers in excess of their interrupting rating is limited to one operation, after which inspection, replacement, or maintenance may be required."

In calculating the short circuit current through each step in (a1), (a2), and (a3) above it is permissible to include the impedance of all circuit elements (including breaker trip coils) between the line terminals of the breaker in question and the source, but not the impedance of the breaker for which the interrupting current rating is being determined. For example, the impedance of breakers in the first two steps may be included in the calculation to determine the fault current to which the breakers in the third step will be exposed. However, the impedance of the third-step breakers should not be included.

32. Selective Tripping of Low Voltage Air Circuit Breakers*

"Properly selected air circuit breakers may be applied to low voltage circuits to obtain selective tripping. The following requirements shall be observed:

(a) Each air circuit breaker must have an interrupting rating equal to or greater than the available short circuit current at the point of application.

(b) Each air circuit breaker, except those having instantaneous trips (such as the one farthest removed from the source of power), must have a short-time rating equal to or greater than the available current at the point of application.

(c) The time-current characteristics of each air circuit breaker at all values of available overcurrent shall be such as to insure that the circuit breaker nearest the fault shall function to remove the overcurrent conditions, and breakers nearer the source shall remain closed and continue to carry the remaining load current.

(d) To insure that each breaker shall function to meet the requirements of paragraph (c) above, the time current characteristics of adjacent breakers must not overlap. The pickup settings and time delay bands of both the long-time and short-time delay elements must be properly selected.

(e) Manually operated circuit breakers shall be limited to applications in which delayed tripping requirements do not exceed 15 000 amperes or 15 times the coil rating, whichever is greater.

(f) The time-current characteristics of a breaker in a selective system shall be such that up to four breakers may be operated selectively in series, when required. One of these breakers shall be a load breaker equipped with an instantaneous trip element.

NOTE: Attention is directed to the fact that operation of selective tripping requires coordination with the rest of the system; as for instance, the low voltage side of a trans-

*Taken in part from the current proposed revision of NEMA Standards 46-109—not applicable to molded-case breakers

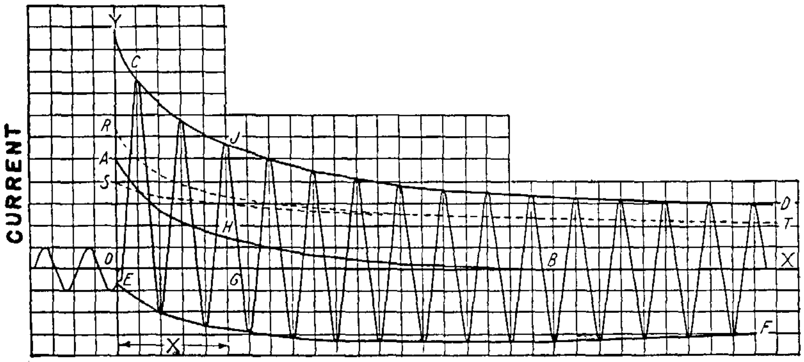


Fig. 48—Typical fault on a three-phase a-c system.

former bank requires that in the application of relays or fuses on the high side, proper coordinating steps should be taken."

V. FAULT CALCULATIONS

In order to determine the momentary and interrupting duty on circuit breakers and to make preliminary relay settings it is necessary to predict the fault currents that may occur at each circuit breaker location. This information is sometimes available from tests or from previous calculations on adjacent circuits, but must frequently be calculated for a new system or extension. The rigorous determination of short-circuit currents as a function of time involves too laborious a calculation to be practical. Thus, some approximation is required, and a degree of judgment must be used in the application of any method proposed. In the following paragraphs several such methods will be discussed, including the simplified procedure suggested by the AIEE Protective Devices Committee.

In using any of these calculating procedures it is necessary to determine the system impedance as viewed from the point of fault, and the current distribution for different kinds of faults. Such calculations for relatively simple systems or parts of systems can be made directly. The network solutions described in Chap. 10 and the method of symmetrical components given in Chap. 2 are helpful in such calculations. The calculation of faults by these methods on many modern interconnected systems may become entirely too involved. Such systems can be represented in miniature on an a-c or d-c network calculator. Fault currents can be determined from calculator readings in a relatively short time. A description of an a-c network calculator is given in Sec. 35 of Chap. 13. A d-c network calculator can be used for studies where either resistance or reactance alone is sufficient to represent the system. Network calculators are also used in studies of load-current distribution, voltage regulation, transient overvoltage, and transient and steady-state stability.

33. Components of Fault Current

Before discussing specific methods of fault calculation for circuit-breaker and relay application the current components of a typical fault on an a-c system will be reviewed briefly. A more complete analysis is given in Chap. 6.

The current in one phase for a three-phase fault on an a-c system is shown as a function of time by the curve EX of Fig. 48. In this diagram OX is the line of zero current and O represents the time at which the fault has occurred. The current to the left of OY is the load current prior to the fault. The short-circuit current wave is unsymmetrical with respect to the OX axis immediately after the short circuit, but during increasing increments of time it approaches a position of symmetry. This asymmetry is dependent upon the point of the voltage wave at which the short circuit occurs. It is possible, by short circuiting at different points on the normal voltage wave, to secure short-circuit current waves ranging anywhere from those symmetrical about the OX axis to those totally asymmetrical. CD is a curve passing through the maxima of the wave of the total current, and EF is a curve passing through the minima. AB is a curve cutting the vertical everywhere midway between CD and EF.

The *wave of total current* with crests along curves CD and EF and with ordinates measured from the axis OX can be resolved into two components, namely:

1. A direct-current component.
2. An alternating-current component.

The *direct-current component* is determined at any instant by the ordinate GH of the curve AB, at the time X.

The *alternating-current component* is a wave with a crest value at any time equal to the difference between the ordinates of the curves CD and AB. This difference at the time X has the value HJ. The rms values of this alternating-current component are shown on curve ST. At any instant, this component is considered to have the same rms value as an alternating wave of constant amplitude with crest value one-half the distance between curves CD and EF at that instant.

The *rms value of the total current* wave under short circuit at any instant is the square root of the sum of the squares of the direct-current component and the rms value of the alternating-current component at that instant. The rms values of this total current are shown on the curve RT. The rms value of the total current at the time of parting of the circuit-breaker contacts determines the interrupting rating of a power circuit breaker.

34. Simplified Procedure for Calculating Short-Circuit Currents for the Application of Circuit Breakers and Relays

A simplified procedure for the calculation of short-circuit currents has been presented in reports [114–116] sponsored

TABLE 11—REACTANCE QUANTITIES AND MULTIPLYING FACTORS FOR APPLICATION OF CIRCUIT BREAKERS

	Reactance Quantity for Use in X_1			
	Multiplying Factor	Synchronous Generators & Condensers	Synchronous Motors	Induction Machines
A. Circuit Breaker Interrupting Duty				
1. General case				
8-cycle or slower circuit breakers*	1.0			
5-cycle circuit breaker	1.1	subtransient**	transient	neglect
3-cycle circuit breaker	1.2			
2-cycle circuit breaker	1.4			
2. Special case for circuit breakers at generator voltage only. For short-circuit calculations of more than 500,000 kva (before the application of any multiplying factor) fed predominantly direct from generators, or through current-limiting reactors only				
8-cycle or slower circuit breakers*	1.1			
5-cycle circuit breakers	1.2	subtransient**	transient	neglect
3-cycle circuit breakers	1.3			
2-cycle circuit breakers	1.5			
3. Air circuit breakers rated 600 volts and less	1.25	subtransient	subtransient	subtransient
B. Mechanical Stresses and Momentary Duty of Circuit Breakers				
1. General case	1.6	subtransient	subtransient	subtransient
2. At 5 000 volts and below, unless current is fed predominantly by directly connected synchronous machines or through reactors	1.5	subtransient	subtransient	subtransient

* As old circuit breakers are slower than modern ones, it might be expected a low multiplier could be used with old circuit breakers. However, modern circuit breakers are likely to be more effective than their slower predecessors, and, therefore, the application procedure with the older circuit breakers should be more conservative than with modern circuit breakers. Also, there is no assurance that a short circuit will not change its character and initiate a higher current flow through a circuit breaker while it is opening. Consequently the factors to be used with older and slower circuit breakers well may be the same as for modern eight-cycle circuit breakers.
** This is based on the condition that any hydroelectric generators involved have amortisseur windings. For hydroelectric generators without amortisseur windings, a value of 75 percent of the transient reactance should be used for this calculation rather than the subtransient value.

by the Protective Devices Committee of the AIEE. This method has been found satisfactory and is intended for general use by the industry as a simplified method of approximating the magnitude of fault currents. However, other more rigorous methods should be used when required.

The new method is based upon the determination of an initial value of rms symmetrical current (a-c component) to which multiplying factors are applied for application purposes. In the determination of this current, the following symbols are used:

E = line-to-neutral voltage.
X_1 = positive sequence reactance viewed from the point of fault, including transient or subtransient direct-axis rated voltage reactance of machines as specified in Tables 11 and 12 in ohms per phase.
X_0 = zero-sequence reactance.
R_0 = zero-sequence resistance.

(*a*) ***Circuit Breaker Application***—(1) Determine the "highest value of rms symmetrical current for any type of fault" equal to E/X_1 or $3E/(2X_1+X_0)$, whichever is greater, except that when R_0 is greater than $2.23X_1$ no consideration need be given to the latter expression. This value should be taken for the maximum connected synchronous capacity. (2) Multiply this current by the proper factors from Table 11. (3) The resulting interrupting and momentary currents should be used to select the circuit breaker.

The factors given in Table 11 represent the ratio between the rms total current at the instant of contact parting and the initial value of rms symmetrical current. In determining these factors it was assumed that circuit breakers should be installed which would permit the use of high-speed relays at some later date, and the time of contact parting was selected on this basis. Contact parting times of 4, 3, 2, and 1 cycles were assumed for 8-, 5-, 3-, and 2-cycle breakers.

Note that the total fault current calculated above may in some cases divide between two or more circuits. It is necessary to determine the maximum fault current that must be interrupted by each breaker under any circuit condition (see example).

For most apparatus and circuits the resistance may be neglected as a justifiable approximation. For underground cables and very light aerial lines the resistance may be as great as the reactance. For these elements the impedance should be used instead of the reactance. Unless it constitutes a major part of the total circuit impedance this impedance may be added arithmetically to the reactance of the rest of the circuit without appreciable error.

(*b*) ***Overcurrent Protective Relays***—In approximating the settings of overcurrent relays, the fault currents for two conditions should be determined:

1. The maximum initial symmetrical current for maximum connected synchronous capacity as determined by E/X_1 or $3E/(2X_1+X_0)$, whichever is greater, except that, when R_0 is greater than $2.23X_1$, no consideration need be given to the expression $3E/(2X_1+X_0)$.

2. The minimum symmetrical current for minimum connected synchronous capacity as determined by $0.866E/X_1$, or $3E/(2X_1+X_0)$ for reactance grounded systems. In particular situations, allowance should be made for remote fault locations and fault resistance.

Ground, distance, balanced, and other types of relays require special consideration.

For each of these conditions use machine impedances and multiplying factors in accordance with Table 12.

(*c*) ***Example***—In order to illustrate the use of the above method of calculation, circuit breaker ratings for several locations in the system shown in Fig. 49 will be determined. The approximate impedance data references in Sec. 37 will be used.

From Table 4 of Chap. 6, Part XIII, the waterwheel generators would have a subtransient reactance of 24 percent on their own base. The 13.8-kv transformers would have an impedance of about 11 percent according to Table 1 of Chap. 5. The combined positive-sequence impedance of all four generators and transformers, viewed from the 138-kv bus is thus 35 percent on 200 mva or 17.5 percent on 100 mva. The zero-sequence impedance would be that of the transformers alone or 5.5 percent on 100 mva.

TABLE 12

Type of Relay	Use Following Reactance in Determining X_1			
	Multiplying Factor	Synch. Gen. or Cond.	Synch. Motor	Induction Machine
1. High-speed Current Actuated Relays.	1.0	subtr.	subtr.	subtr.
2. Time Over-Current Relays. .	1.0	trans.	trans.	. . . . .

From Tables 2 and 6 of Chap. 3 each transmission circuit has a positive-sequence reactance of 0.77 ohms per mile. For the two 40 mile circuits in parallel the reactance is 8.1 percent on 100 mva. From Table 14 of this chapter the zero-sequence reactance may be estimated at 24.3 percent on 100 mva.

The 100-mva step-down transformers will also have an impedance of 11 percent on their kva rating or a net for the two of 5.5 percent on 100 mva.

The turbine-generators (see Table 4 of Chap. 6) will be taken as 9 percent each on 50 mva or a total of 4.5 percent on 100 mva for the four units.

The above impedances may be combined into the equivalent circuit shown in Fig. 50.

For a fault at A in Fig. 49 the 3-phase fault will govern breaker interrupting duty because of the limiting effect of

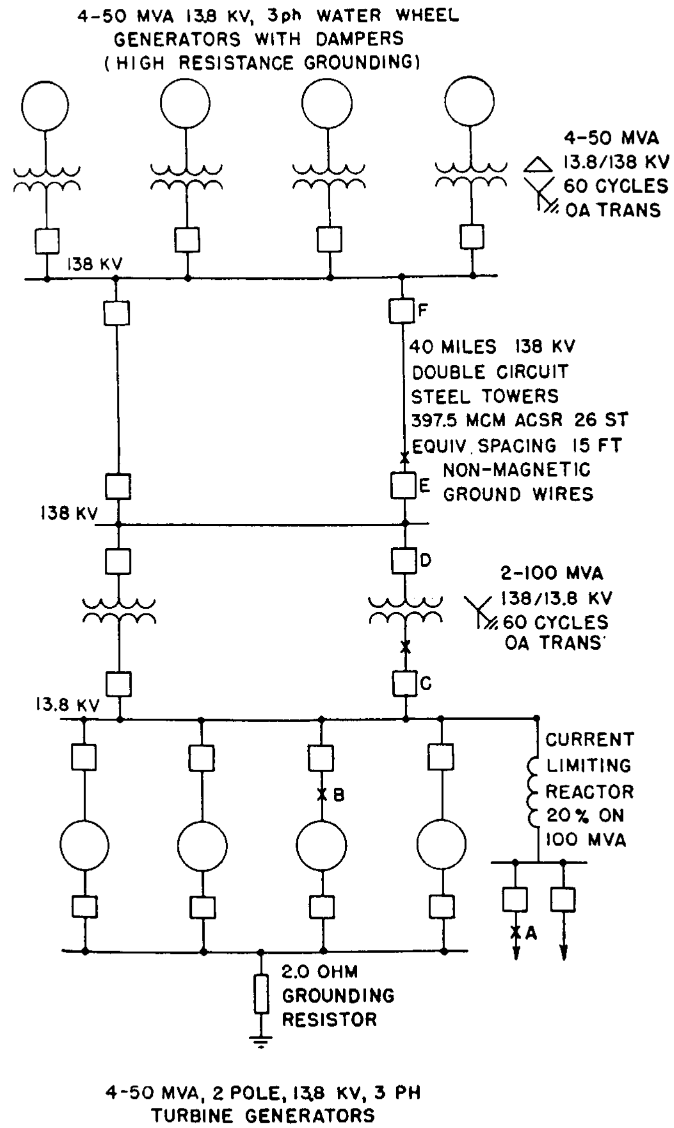


Fig. 49—Hypothetical system for example.

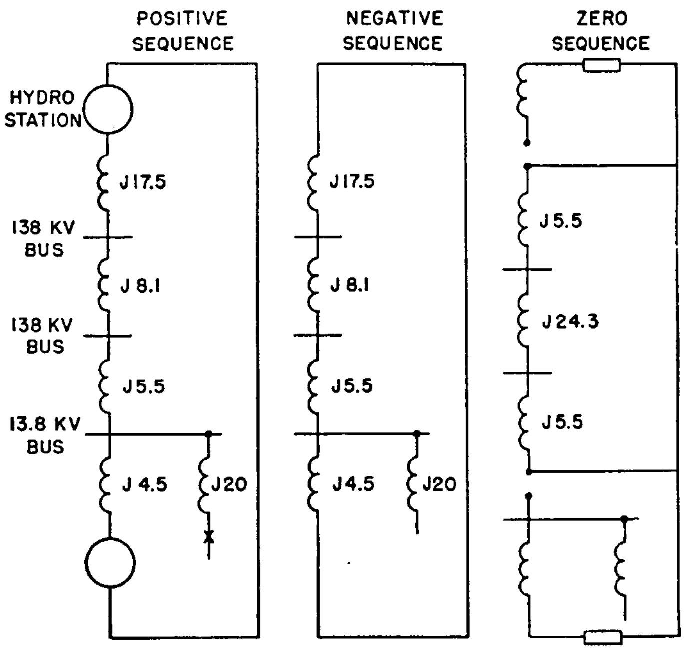


Fig. 50—Equivalent circuit for system of Fig. 49. Impedances in percent on 100 mva.

the generator grounding resistor and the delta-connected transformers. This fault is $\frac{1.0}{0.239}=4.2$ per unit $=420$ mva. Since this is less than 500 mva the general case of Table 11 applies and an 8-cycle breaker of 420 mva interrupting rating would be adequate. For normal interrupting duty (CO+15 seconds+CO) a standard 500-mva breaker could be chosen without further analysis since the calculated interrupting mva does not exceed 95 percent of the breaker rating.

If we desire to provide instantaneous single-shot reclosing on this feeder-breaker the interrupting current rating must be calculated. The interrupting rating of a 500-mva breaker at 13.8 kv is 21 000 amperes. According to Fig. 47 reclosing duty cycle II requires reduction of this rating to $21\,000\times0.94=19\,700$ amperes. The calculated fault level of 420 mva is equal to 17 600 amperes, so a standard 500-mva breaker would still be adequate. Regardless of the interrupting duty the momentary rating required would be $1.6\times17\,600=28\,200$ amperes.

For a fault at B, $E/X_1=\frac{1.0}{0.039}=25.5$ per unit or 2550 mva. However, the portion of the fault contributed by generator B does not go through breaker B. This is $\frac{1.0}{0.18}=5.5$ per unit or 550 mva. Thus E/X_1 for breaker B is 2000 mva. Since this value is greater than 500 mva and all standard 13.8-kv breakers have 8 cycle interrupting time, breaker B should have an interrupting rating of $1.1\times2000=2200$ mva. This is less than 95 percent of 2500 mva and a standard 2500-mva breaker may be chosen without further study.

A fault at C will give the highest fault current on any of the main 13.8-kv breakers. The three-phase fault will govern as before. Although it is an abnormal condition, the greatest fault current will flow when breaker D is open. For this condition X_1 is 4.0 percent and $E/X_1=25.0$ per unit or 2500 mva. Since this fault is produced predominantly by the 13.8-kv turbine-generators the 1.1 multiplier is required for 8-cycle breakers and the duty exceeds that of the largest standard 13.8-kv breaker. In view of the close margin between the breaker rating and the calculated duty a more accurate check would be in order as suggested in Sec. 30. If such a check still indicated duty in excess of 2500 mva it would be necessary to increase the X_1 by modification of generator design or the addition of current-limiting reactors.

In order to determine the interrupting duty on the 138-kv breakers at the steam station it is necessary to consider both three-phase and single-line-to ground faults as well as several fault locations and switching conditions. With all breakers closed $E/X_1=\frac{1.0}{0.0719}=13.9$ per unit or 1390 mva for a three-phase fault and $\frac{3E}{2X_1+X_0}=\frac{3.0}{2(0.0719)+0.0465}$ $=15.7$ per unit or 1570 mva for a line-to-ground fault. The current distribution for such a fault is shown in Fig. 51. The transformer and line circuits have been shown separately in order to study different fault locations. The smallest 138-kv breaker is rated 1500 mva and 5 cycles.

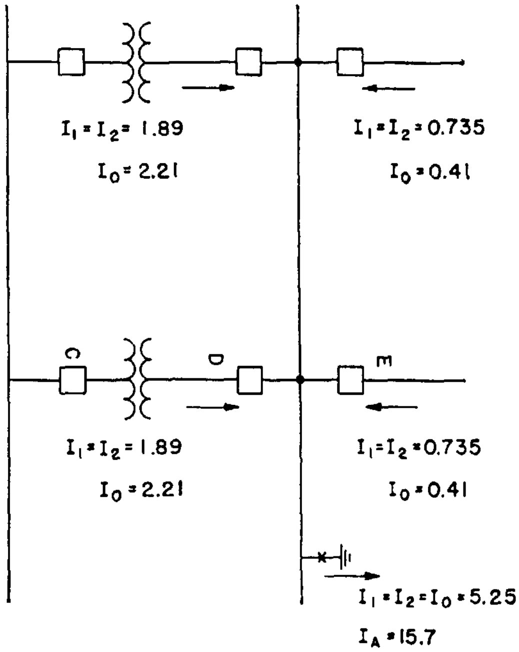


Fig. 51—Current distribution for a fault on 138-kv bus at steam station—currents in per unit on 100-mva base.

It is apparent that ground faults will govern interrupting duty.

A fault at D will produce the same fault currents but breaker D will carry $I_1=I_2=3.36$ and $I_0=3.04$. $I_A=9.76$ per unit. For such a fault the required interrupting duty for a 5 cycle breaker is $9.76\times1.1=10.7$ per unit or 1070 mva.

In order to be safe it is also necessary to consider a fault at D with breaker C open. See Fig. 52. The total fault now becomes $\frac{3.0}{0.0965+0.0965+0.0465}=12.5$ per unit or 1250 mva but part of I_0 does not pass through breaker D. In the breaker $I_A=4.16+4.16+2.40=10.72$. The re-

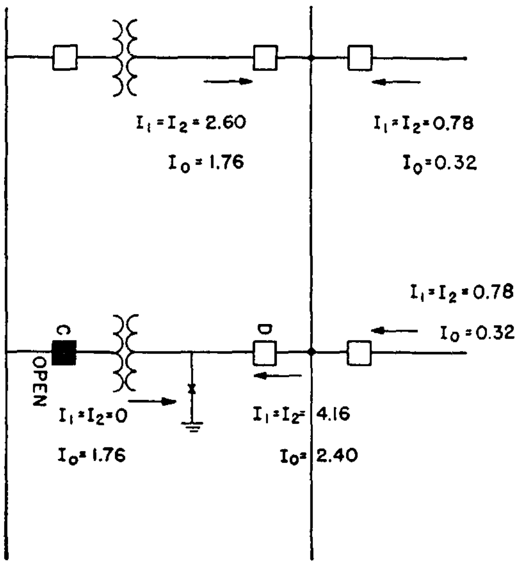


Fig. 52—Current distribution for single-line-to-ground fault at 138-kv terminals of transformer with 13.8-kv breaker open.

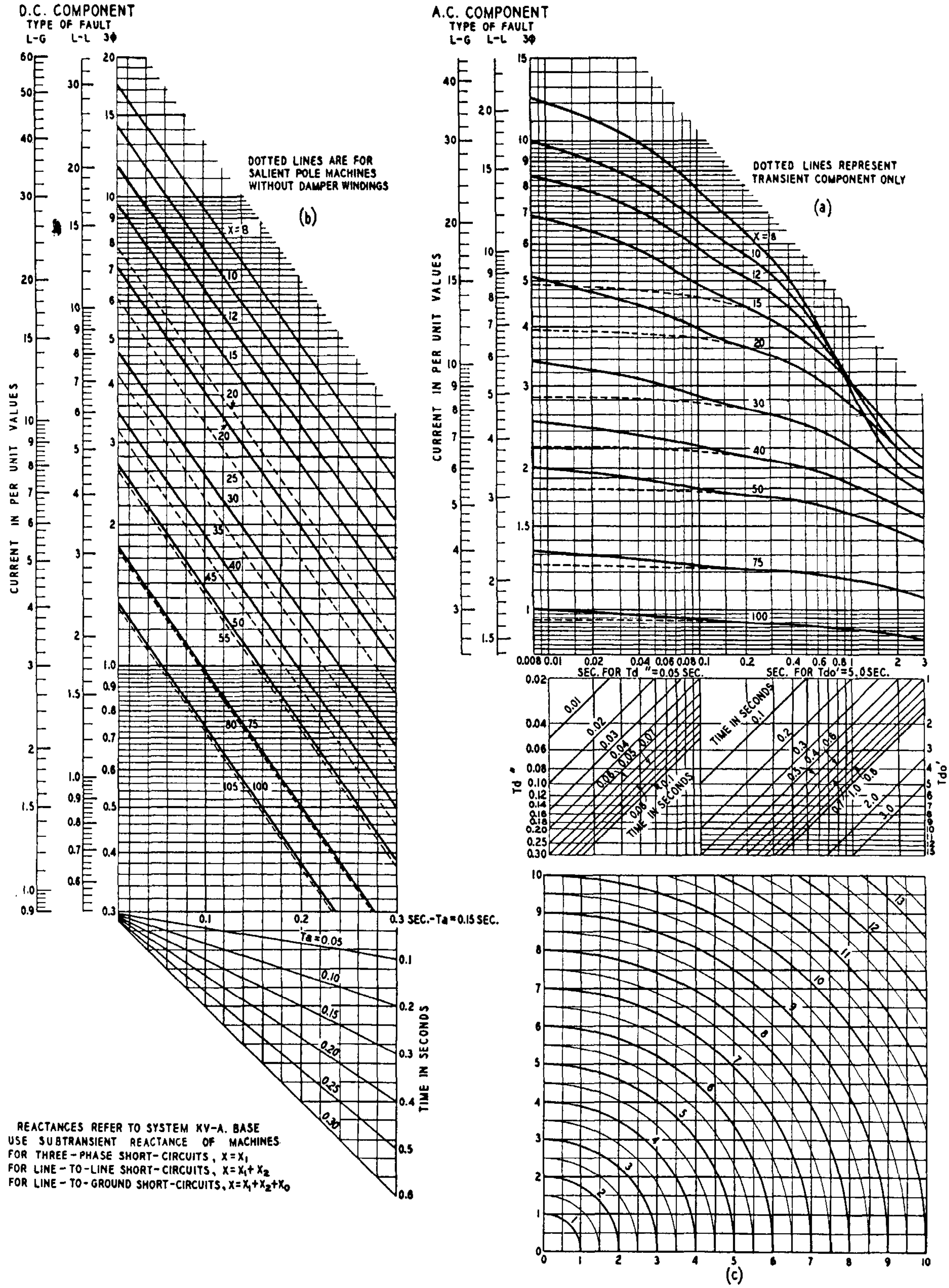


Fig. 53—Short-circuit decrement curves for similar parallel machines.

quired interrupting rating for this condition would be $1.1\times10.72=11.80$ per unit or 1180 mva which is greater than the value calculated in the previous study. Thus a 1500-mva breaker would be adequate for breaker *D* on the basis of the system as shown. If additional generation or lines were contemplated the 1500 mva rating might be exceeded and such changes would have to be considered Reclosing would not normally be used on these breakers

For breaker E a fault at E with all breakers closed required an interrupting rating of 1.1 (4.52+4.52+4.84) = 15.2 or 1520 mva. If breaker F were open such a fault would require an interrupting rating of $1.1 \dfrac{3.0}{0.077+0.077+0.05} = 16.1$ per unit or 1610 mva. Breaker E would probably have instantaneous single shot reclosing. A standard 1500-mva breaker would be good for only 1425 mva for such reclosing duty (see Fig. 47). Although it might be possible to reduce the interrupting duty of the system to 1425 mva by increasing the transformer impedance, a reasonable amount, some allowance should be made for future addition of generators or transmission lines. Thus a 3500-mva breaker would probably be chosen for the assumed system.

35. Short-Circuit Calculations for Similar Parallel Machines

It is intended that the simplified procedure given in Sec. 34 be used for normal circuit-breaker applications and for preliminary relay settings. In some special cases it may be desired to make a more accurate analysis of current decrement such as to take into account abnormal machine time constants or to obtain relay currents a relatively long time after the fault has occurred.

When all of the machines which contribute to a fault have similar reactances and time constants, are equally loaded, and are symmetrically located with respect to the fault the group of machines can be represented as a single equivalent generator. The fault current can then be calculated with relative accuracy by the methods described in Parts II, III and VI of Chap. 6 in which the effects of individual machine characteristics, loads, external impedance and change of excitation can be included.

A somewhat easier analysis may be made by the use of short-circuit decrement curves which have been published,[117,118] and are reproduced in Fig. 53, if the assumptions on which they are based hold for the system under consideration. These are:

(a) Transient characteristics of alternating-current generators of normal design determined from oscillograph tests.
(b) That the effect of capacitance and resistance is neglected, except in so far as decrements are concerned, which effects are included by average decrement factors.
(c) That the contact resistance at short circuit is zero.
(d) That the alternating-current generators are carrying full load at 80 percent power factor previous to short circuit.
(e) That the short circuit is established at the point of voltage wave corresponding to the maximum possible instantaneous current.
(f) That the effect of automatic generator voltage regulators is neglected.
(g) All reactance up to and including 15 percent is considered within the generator. For values of reactance greater than that the difference is considered external.
(h) All machine emfs are assumed to be and remain in phase.
(i) The load is assumed to be located at the machine terminals and the fault to occur on an unloaded feeder.
(j) The actual system subjected to fault may be represented by a single equivalent generator of the same total rating as the synchronous apparatus of the system and an equivalent external reactance.
(k) All generators are assumed to have an open-circuit transient time constant (T'_{do}) of 5 seconds and an armature short-circuit time constant of 0.15 second.
(l) A subtransient time constant of 0.05 second was used for all curves.

The short-circuit current from a synchronous machine consists of an a-c and a d-c component. The a-c component in general can be resolved into a transient component having a relatively large time constant and a subtransient component having a relatively small time constant. The values of these constants are such that during the first one-tenth second the transient component changes very little, but the subtransient component disappears almost entirely. Because of this relation it is possible to plot the two a-c components on one set of curves as shown by the "a" curves of Fig. 53. The numbers of these curves refer to the combined external reactance (exclusive of loads) and machine subtransient reactance. A subtransient time constant of 0.05 and a transient open circuit time constant of 5.0 were used in the preparation of these curves, but the effect of other time constants can be included by reading vertically from the intersection of the horizontal line corresponding to the particular time constant and the inclined line corresponding to the particular time. The dotted lines show the transient component only of a-c current.

These curves are intended primarily for turbine-generator systems as indicated by assumption (g). The assumed relation between transient and subtransient reactance is $X_d' = (1.4X_d'' + .02)$ per unit. The curves may be used with fair accuracy for salient pole generators with dampers. For salient pole machines without dampers the curves may be used with the following adjustments:

(a) Calculate the total system reactance to the point of fault using the *subtransient reactance* of the machines, and then *subtract* 5 percent.
(b) Enter the curves with the above modified value of reactance. (For example if the system reactance is 25 percent use the curve marked 20.)
(c) The proper a-c component of current will be approximately midway between the dotted and solid portions of the curves of 53(a) in the short time periods where a distinction is made.
(d) The proper d-c component of current is given by the dotted curves of 53(b).

With the above general qualifications the curves may be used to calculate three-phase, line-to-line or single-line-to-ground faults. The following symbols are used:

X_1 = percent positive-sequence impedance viewed from the point of fault, based on the total synchronous kva.
X_2 = negative-sequence impedance viewed from the point of fault.
X_0 = zero-sequence impedance viewed from the point of fault.
T_a = time constant of direct-current component.
T_d'' = short-circuit subtransient time constant.
T'_{do} = open-circuit transient time constant.

For a three-phase fault use the curves of Fig. 53 (a) and (b) for which $X = X_1$ and read the components of current on the ordinate scales designated 3-phase. The a-c and d-c components may be combined into the rms total current for maximum asymmetry by the formula,

$$i_{\text{rms total}} = \sqrt{i_{ac}^2 + i_{dc}^2}.$$

Fig. 53(c) may be used to perform this calculation by laying off the components along the two axes and reading the rms total current on the circular scales.

A line-to-line fault is read in a similar manner except the curves are used for which $X = X_1 + X_2$ and the magnitude of current is read on the ordinate scale headed L–L.

For single-line-to-ground faults enter the curves with $X = X_1 + X_2 + X_0$ and use the ordinate scale headed L-G.

The curves of Fig. 53(b) are plotted against a basic time scale corresponding to $T_a = 0.15$ sec. If the d-c time constant is known to be different, read vertically from the intersection of the horizontal line corresponding to the desired time and the inclined line corresponding to the desired time constant.

36. The Internal Voltage Method

For critical relay and circuit-breaker applications where synchronous machines are dissimilar and unsymmetrically located with respect to the fault, a more accurate short-circuit analysis can be made by means of the *Internal Voltage Method*. This method lends itself to the use of a network calculator and can be used to include the effect of a change in the excitation of the machines. Because of the limited application of this method of calculation the reader is referred to a series of articles by C. F. Wagner, entitled "Decrement of Short-Circuit Currents," which appeared in the March, April, and May 1933, issues of the *Electric Journal*.

37. Approximate Impedance Data for Fault Calculations

In fault calculations, impedance data applicable specifically to the apparatus and circuits under consideration should be used whenever possible. Such data can usually be obtained from the manufacturers for existing apparatus and can be calculated with the aid of tables referred to below for overhead lines and cables. The necessity for accurate data is particularly important for circuit elements which have a major influence on the fault magnitude.

For estimating fault currents on proposed new circuits, and for approximate data on the less important elements of existing circuits, the following references and tables are offered as typical of present-day practice.

Synchronous Generators, Motors, and Condensers—Table 4 in Chap. 6 lists both average values and the probable range of the several impedances and time constants of 60-cycle three-phase synchronous machines. In most simplified fault calculations *subtransient* reactance is used to represent the positive-sequence impedance of synchronous machines, and its relation to the other impedances is assumed on the basis of typical designs. Exceptions to this assumption are noted in Sec. 34. The effect of external impedance on the time constants is discussed in Sec. 10 of Chap. 6.

Induction Motors—The effect of induction motors on the short-circuit current is discussed in Chap. 6.

Power and Distribution Transformers—Typical impedance values for distribution and power transformers are given in Table 1 of Chap. 5. The relation between the positive- and zero-sequence impedances for each of the principal types of transformers is also discussed in this chapter and a table of equivalent circuits is given in the appendix.

Feeder Voltage Regulators—The impedance of single-phase induction regulators referred to the through kva of the circuit varies with regulator position from approximately 0.7 percent at maximum buck or boost position to approximately 2.5 percent at points midway between the neutral and maximum positions. At the neutral position the impedance is approximately 1.5 percent.

The impedance of polyphase induction regulators does not vary greatly with regulator position and lies between 1.0 percent and 1.5 percent on the circuit kva base.

For line voltages not exceeding the 15 kv insulation class level, single-core step regulators are used when the line current does not exceed 400 amperes. Two-core step regulators are used for higher current circuits to reduce the current handled by the tap changer to 400 amperes.

Two-core four-winding construction is used where the line voltage exceeds the normal 15 kv insulation class level.

The impedance of plus or minus 10 percent regulators in single-phase and balanced three-phase circuits is given in Table 13.

TABLE 13—IMPEDANCE OF FEEDER REGULATORS—PERCENT ON CIRCUIT KVA BASE—PLUS OR MINUS 10 PERCENT REGULATION

	Max.	Min.	Neutral Pos.
Induction			
Single-phase	2.5	0.7	1.5
Three-phase	1.5	1.0	...
Step-Type			
Single-core	0.4	0	...
Two-core, three-winding	0.7	0.4	...
Two-core, four-winding	1.1	0.5	...

Aerial Lines—The characteristics of aerial lines are given in Chap. 3.

When the conductor size and spacing of an aerial line cannot be determined and a rough value of impedance is known to be satisfactory, the reactance of lines above 15 kv class can be taken as 0.8 ohms per mile without serious error. The resistance of such lines will usually be negligible from the standpoint of circuit breaker and relay application. For lines rated 15 kv and below conductor size and spacing vary greatly and typical figures should not be used. If the actual line data cannot be obtained (and an approximate figure is known to be satisfactory) the conductor size and spacing may sometimes be estimated on the basis of thermal and regulation limits of the circuit.

The zero-sequence reactance of aerial lines can be estimated from the positive-sequence reactance by the use of Table 14. This approximation is sufficiently accurate for most circuit breaker applications, but when greater accuracy is required refer to Chap. 3 and other references given in that chapter.

Cables—The impedance of single- and three-conductor cables is given in Chap. 4.

The effect of iron conduit in increasing the reactance and resistance of cables has been investigated by L. Breiger of the Consolidated Edison Co. with both laboratory and field tests.[124] These tests show that if the cables are held

TABLE 14—APPROXIMATE RATIO OF X_0 TO X_1 FOR TRANSMISSION LINES AND CABLES

	Average	Range		Average	Range
Single-circuit Aerial Transmission Line (without ground wires or with magnetic ground wires)	3.5	2.5–3.5	Double-circuit Aerial Transmission Line (with non-magnetic ground wires)	3	2–4
Single-circuit Aerial Transmission Line (with non-magnetic ground wires)	2	1.7–2.7	Three-phase Cables	...	3–5
			Single-phase Cables	1	...

in close triangular arrangement, the reactance is increased by only about 10 percent because of the iron conduit. In many cases, however, cables lie at random in the conduit and the reactance may be increased by as much as 50 percent.

For circuit breaker applications, in the absence of specific information, the reactance of iron conduit circuits may be taken at from 40 to 45 microhms per foot for one conductor per phase and 20 to 25 microhms per foot for two conductors per phase. For non-magnetic duct corresponding figures are 35 to 40 microhms per foot for one conductor per phase and 18 to 22 microhms per foot for two conductors per phase. The increase in resistance caused by the iron conduit is not sufficient to justify consideration in circuit breaker applications.

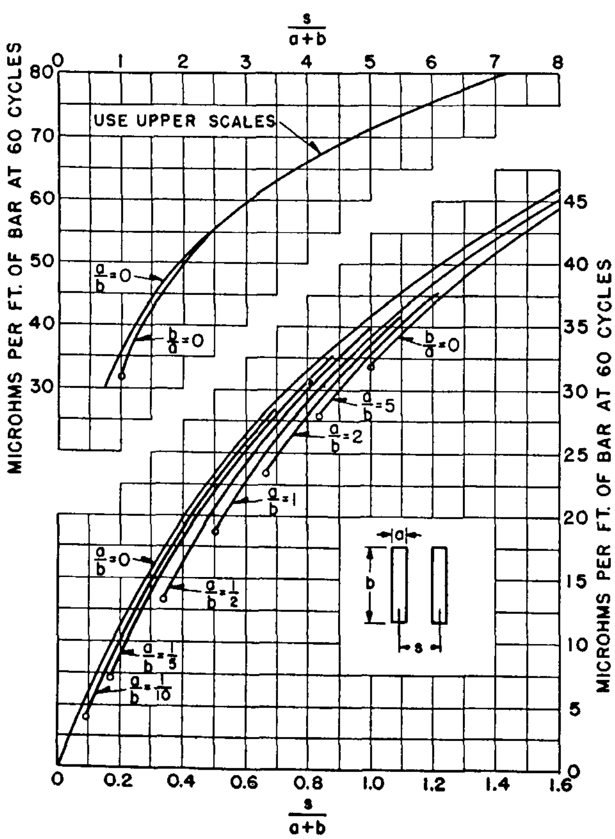


Fig. 54—Reactance of rectangular bar conductors.

Bus Conductors—The reactance of most busbar arrangements for low or medium voltage circuits is of the order of 50 microhms per foot. Values for practical circuits range from 30 to about 70 or 80 microhms. In low-voltage circuits bus reactance may be an appreciable part of the circuit impedance. For example 50 feet of bus at 50 microhms per foot will cause a drop of 50 volts at 20 000 amperes.

Fig. 54[126] gives the 60-cycle reactance per conductor per foot of two rectangular bars in a single phase circuit.

The reactance per phase of a transposed three-phase bus may also be obtained from Fig. 54 by replacing s by an equivalent spacing equal to the cube root of the product of the three distances between phase conductors. $s_{\text{equiv.}} = \sqrt[3]{s_1 s_2 s_3}$. If the bus is not transposed, the reactance corresponding to the minimum spacing should be used for circuit breaker applications in order to obtain the maximum current in any pole. For other applications it may be desirable to use the equivalent spacing in order to determine the average reactance per phase.

The reactance of bus runs composed of several closely spaced bars per phase may be determined approximately by considering each phase group as a solid conductor having the same overall dimensions. This approximation will give values of reactance accurate within about 5 percent if the distance s is more than twice the equivalent a. For the arrangement in Fig. 55 the error is 15 percent for $s = 8$ inches and 6 percent for 30 inches. A more accurate method of calculation is given in Reference 126.

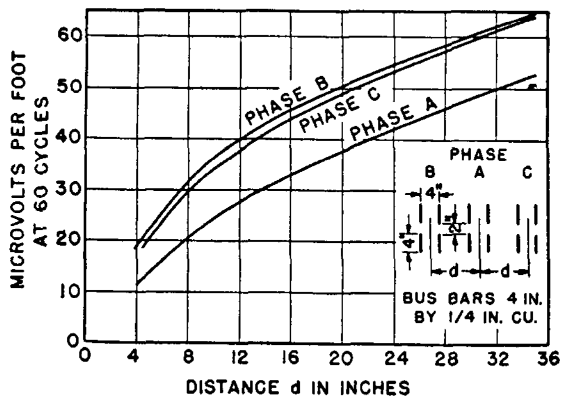


Fig. 55—Reactance voltage drop in each three-phase bus caused by one ampere of balanced three-phase current.

The reactance of irregularly shaped conductors can be determined from Figs. 56 and 57 and similar data published by bus bar manufacturers, such as References 131 to 133. A rough approximation may be obtained by the method described in the preceding paragraph.

Low-Voltage Air Circuit Breakers (600 Volts and Below)—The reactance of low-voltage air circuit breakers with series trip coils may be an appreciable part of the total circuit impedance when the full-load rating of the breaker is small compared with the remainder of the system. Care should be taken to make sure that trip coils are included on all three poles of a breaker before using the values in the accompanying table. If only two coils are used, one-third

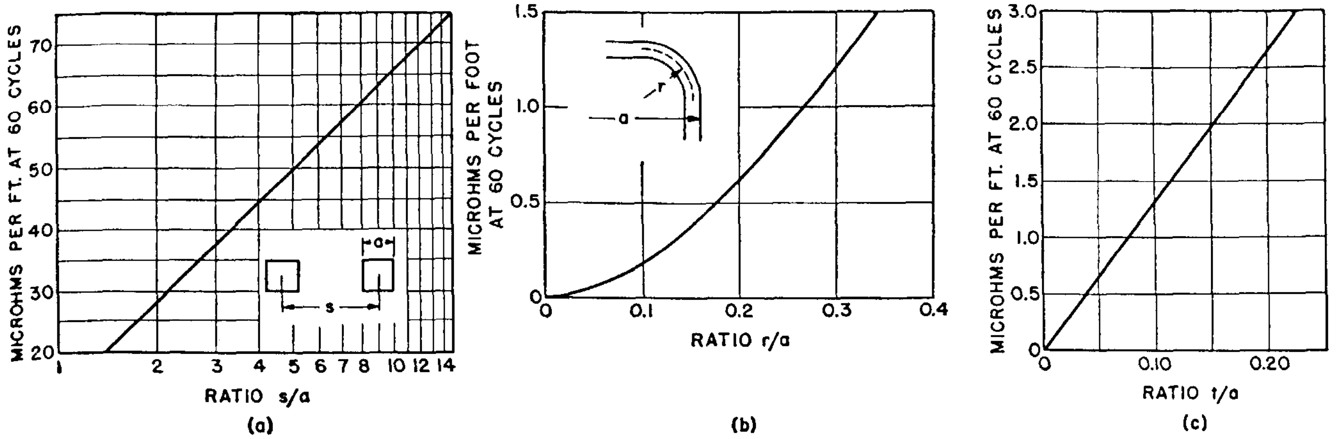


Fig. 56—Reactance of square tubular bus bars.

(a) Reactance of thin square tubes.
(b) Increase caused by round corners.
(c) Increase caused by thickness of tubes.

of the single-coil impedance should be used in calculating the maximum pole current in a three-phase fault, and the impedance should be omitted entirely in calculating a single line-to-ground fault.

Values of series trip coil impedances are given in Table 15 for Westinghouse DA 50, DB 25, and DB 15 air circuit

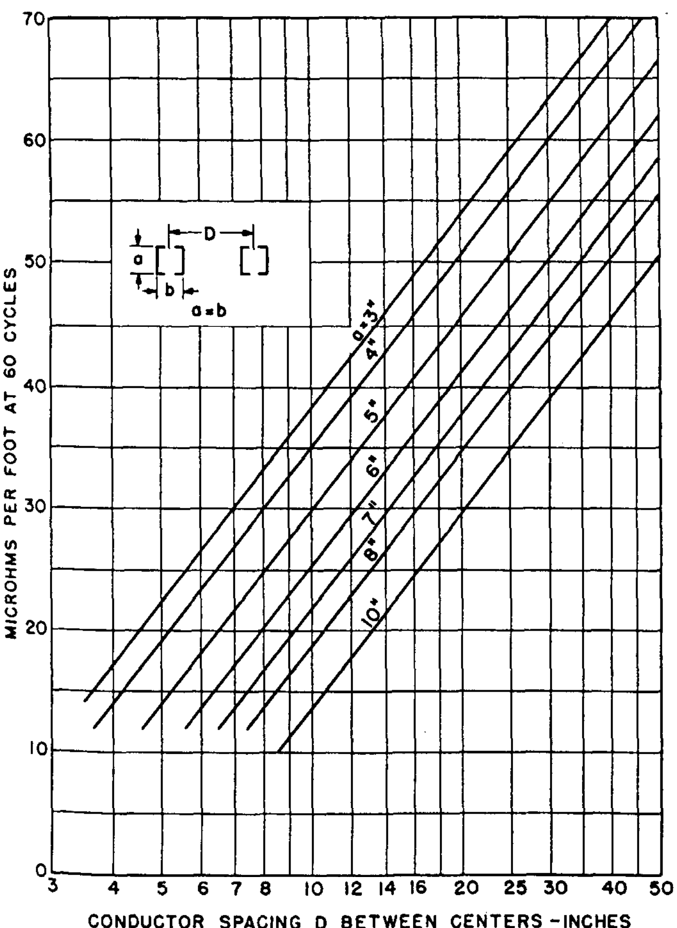


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Fig. 57—Reactance of channel bus.

breakers, which have interrupting ratings of 50 000, 25 000, and 15 000 amperes respectively at 600 volts or below.

The resistance per pole of AB-10 thermal breakers is given in Table 16.

The reactance of the main current-carrying loop of an air circuit breaker can not readily be separated from the influence of the bus or cable to which it is connected. It may be calculated along with the bus or neglected.

Current Transformers—The reactance of the smaller wound current transformers in a low-voltage circuit may be appreciable when fed from a relatively heavy supply

TABLE 15—D-C RESISTANCE AND 60-CYCLE REACTANCE OF 600 VOLT AIR CIRCUIT BREAKER SERIES TRIP COILS*

Full Load Ampere Rating	DA-50		DB-15 and DB-25	
	Resistance in ohms 25°C	Reactance in ohms	Resistance in ohms 25°C	Reactance in ohms
15	0.063	0.21	0.039	0.110
20	0.028	0.11	0.021	0.063
25	0.022	0.065	0.014	0.040
35	0.011	0.041	0.007	0.020
50	0.0042	0.015	0.0036	0.0090
70	0.0033	0.0080	0.0017	0.0053
90	0.0022	0.0063	0.0010	0.0034
100	0.0011	0.0039	0.00083	0.0025
125	0.00086	0.0030	0.00048	0.0017
150	0.00067	0.0023	0.00042	0.0013
175	0.00052	0.0017	0.00027	0.00088
200	0.00034	0.0012	0.00024	0.00072
225	0.00034	0.0012	0.00017	0.00057
250	0.00021	0.00076	0.00017	0.00057
350	0.00013	0.00043	0.000079	0.00033
400	0.00013	0.00043	0.000050	0.00023
500	0.000069	0.00019	0.000031	0.00017
600	0.000069	0.00019	0.000031	0.00017

*If only two series trip coils are used use ⅓ of these values in calculating a three-phase fault; neglect entirely for single-line-to-ground faults on three-phase circuits.

TABLE 16—RESISTANCE PER POLE OF AB-10 600-VOLT THERMAL CIRCUIT BREAKERS

600-Ampere Frame		225-Ampere Frame	
Full-Load Ampere Rating	Resistance in Ohms	Full-Load Ampere Rating	Resistance in Ohms
225	0.00024	50	0.00297
250	0.00022	70	0.00130
275	0.00018	90	0.00084
300	0.00016	100	0.00080
325	0.00014	125	0.00063
350	0.00012	150	0.00063
400	0.000096	175	0.00050
450	0.000078	200	0.00036
500	0.000064	225	0.00030
525	0.000060	...	......
550	0.000056	...	......
600	0.000048	...	......

100-Ampere Frame		50-Ampere Frame	
Full-Load Ampere Rating	Resistance in Ohms	Full-Load Ampere Rating	Resistance in Ohms
50	0.0036	15	0.0105
70	0.0025	20	0.0100
90	0.0018	25	0.0070
100	0.0015	35	0.0032
		50	0.0028

TABLE 17—IMPEDANCE OF CURRENT TRANSFORMERS

Westinghouse Type	Rated Current	Ohms	
		Impedance	Resistance
CT 2.5	50	0.008	......
	100	0.002	......
	200	0.0005	......
	400	0.00001	......
WE	200	0.0006	0.0003
	400	0.0002	0.00009
CT 5.0	50	0.014	0.008
	100	0.0035	0.002
	200	0.00095	0.0005
	400	0.00032	0.00015

circuit. Approximate values for specific Westinghouse current transformers are given in Table 17. The limiting effect of secondary burden has been neglected for the sake of simplicity.

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CHAPTER 12

POWER-LINE CARRIER APPLICATION

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I. INTRODUCTION

CARRIER equipment has been used on the power systems of this country since the early 1920s. At first, carrier was used for voice communication only, but its applications subsequently expanded to include a wide variety of functions such as protective relaying, telemetering, supervisory control, and others. Today, carrier is indispensable to the operation of most power systems. Power-line carrier offers rapid and dependable communication for interoffice business and for load dispatching. Carrier relaying permits high-speed clearing of all types of faults, with an attendant increase in stability limits and permissible line loading. Carrier provides economical channels for the telemetering of continuous load information to dispatchers for efficient system operation. Carrier channels are used for the remote supervision and control of many important substations and for automatic load control of numerous large generating units.

The application of carrier equipment for the transmission of high-frequency signals over a 60-cycle power transmission system involves many problems that most communication engineers do not have to face. The configuration and layout of these systems is invariably dictated by 60-cycle considerations, and short taps and spur lines that can play havoc with carrier-frequency transmission are included without regard to their effect upon such high frequencies. The power system communications engineer must nevertheless take the 60-cycle system as it exists and make the carrier equipment operate satisfactorily between the required points, drawing heavily upon his experience and ingenuity to stay within his usually limited budget.

The process of applying carrier to power lines is still largely empirical, because the complexity of the usual power system makes the exact calculation of all the effects practically impossible. However, an appreciation of the fundamental principles involved and the use of the practical data that have been gathered through the years usually permit the characteristics of a proposed carrier channel to be predicted with adequate accuracy.

In this chapter, a review of the major applications of power-line carrier is followed by discussion of some of the fundamental considerations in the transmission of high-frequency energy over power systems. The remainder of the chapter provides data on the practical application of power-line carrier channels.

1. Carrier Frequencies

For many years the band of frequencies from 50 to 150 kilocycles was considered the normal carrier band. However, the greatly increased application of carrier equipment of the past decade has resulted in virtual saturation of this band on most interconnected power systems, and many new channels have been established at frequencies as high as 200 kc and as low as 30 kc. The practical limits to extension of the frequency band will probably be established by excessive losses at the high-frequency end of the spectrum, and by the bulkiness and complexity of coupling and tuning equipment and the difficulty of obtaining sufficiently broad tuned circuits at the lower-frequency end.

II. CARRIER APPLICATIONS

2. Carrier Communication[1,2,3,8]

Power-line carrier communication systems differ in the method of calling, the power supply, or in the modulation system, but any given assembly can be classified as simplex or duplex, depending upon its operation.

A *simplex* system is one in which transmission can proceed from one station only at any given instant. In simplex communication all stations on a channel operate on a single frequency. Transmission and reception cannot take place simultaneously on the same frequency at one station, because the transmitter blocks the local receiver and may even damage it permanently unless the receiver is de-energized during transmission periods. The simplex system therefore requires means for turning off the receiver and energizing the transmitter during transmission.

Requiring only a single carrier frequency, simplex equipment lends itself readily to applications in extensive carrier-communication systems involving more than two terminals. It is economical of space in the carrier-frequency spectrum because the same frequency is used at all transmitting points. Crowding of the spectrum is a serious problem on many power systems today, and this factor alone is often sufficient to justify its application.

A *duplex* system is one in which transmission can take place simultaneously from both stations, as in ordinary telephone service. In the duplex system, the first of two frequencies is used for transmission at one station, the second for reception. At the other station, the first frequency is used for reception, the second for transmission.

Duplex operation normally is limited to two terminals per channel, unless communication is desired between a central office and several other stations not requiring intercommunication. Its major advantage, one that in the minds of some users outweighs any disadvantages, is its ability to provide two-way conversation without the switching operations required by the simplex system.

3. The Single-Frequency Manual-Simplex System

In the single-frequency manual-simplex system, shown diagrammatically in Fig. 1, "send-receive" switching operations are performed by the speaker with a pushbutton on the telephone handset. Although provision can be made for complete operation over a two-wire extension, a control circuit separate from the speech circuits generally is required. The need for d-c control circuits and the fact that a special telephone instrument with a "push-to-talk" button is necessary preclude any simple method of extending a manual-simplex telephone channel through a conventional private-branch-exchange board.

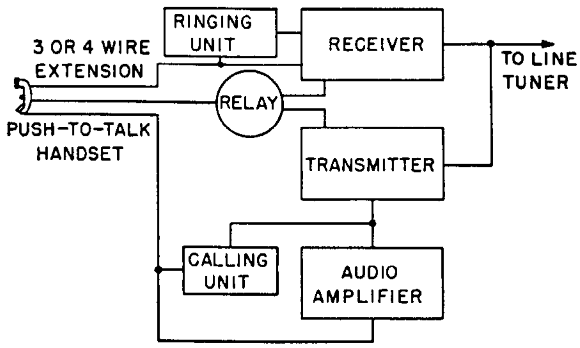

Fig. 1—Basic units of manual simplex communication assembly with code-bell calling.

This sytem is the simplest of the carrier-communication systems in terms of the amount of equipment required and in ease of adjustment after installation. For dispatching and other applications where users are accustomed to handling push-to-talk handsets, it is an entirely adequate system.

4. The Two-Frequency Duplex System

The basic units of a two-frequency duplex assembly are shown in Fig. 2. A photograph of a typical complete assembly is given in Fig. 3. Aside from the fact that the transmitter and receiver operate on different frequencies, the most important difference between this system and the manual simplex system is the addition of the audio hybrid unit. It is this unit that makes it possible for the transmitter and the receiver to operate continuously during the conversation, without switching operations, with a conventional two-wire telephone extension.

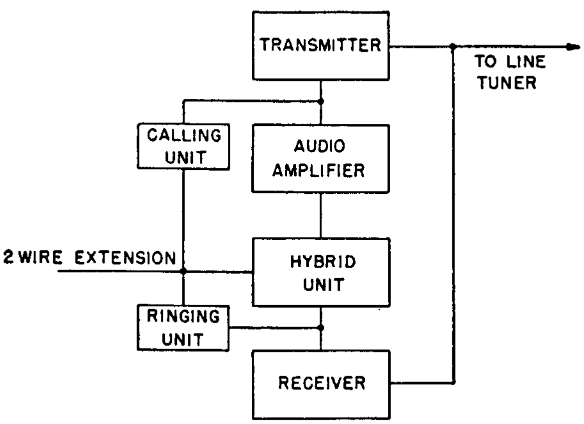

Fig. 2—Basic units of two-frequency duplex communication assembly with code-bell calling.

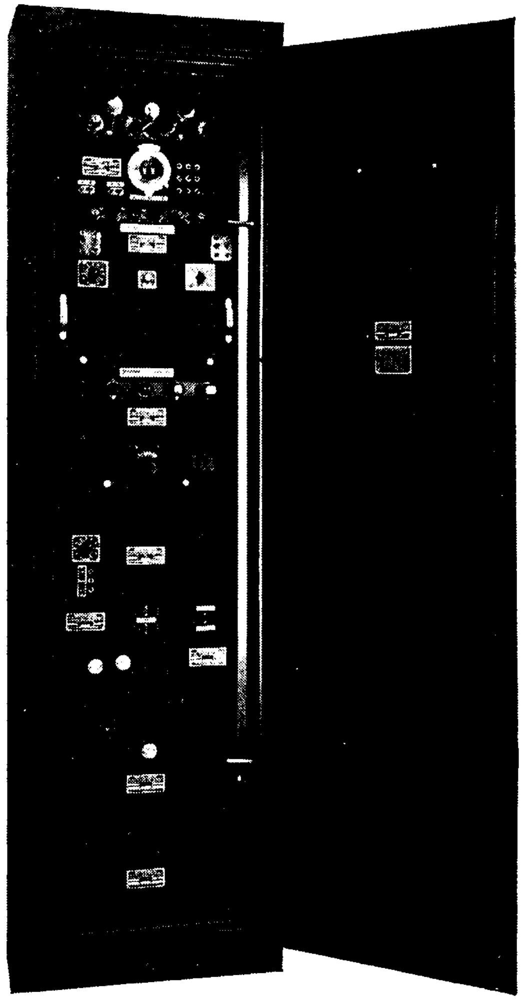

Fig. 3—Typical two-frequency duplex assembly. Panel units, top to bottom, are transmitter, audio amplifier, blank test-meter unit, superheterodyne receiver, audio hybrid and signalling units, switch and fuse unit, high-voltage power supply, low-voltage power supply, and voltage-adjusting autotransformer unit.

The purpose of the hybrid unit can best be understood by considering what would happen to a two-frequency duplex channel if an attempt were made to operate into two-wire telephone extensions at each end without hybrid units. With such a system, the audio output of the receiver would be connected directly to the input terminals of the audio amplifier and would modulate the transmitter output. This signal would be received at the distant station, amplified by the audio amplifier, and transmitted back to the first station, where it would be amplified again and retransmitted. An oscillatory circuit would thus exist, and the outputs of the receivers at both stations would be an audio howl of a frequency equal to the natural frequency of the complete loop. This howl would make the circuit useless for communication purposes.

The audio hybrid unit prevents this howl by reducing the amount of receiver output that reaches the audio amplifier input terminals to a value insufficient for continuous oscillation. The unit contains a three-winding transformer connected between the telephone line and the transmitter and receiver terminals as shown in Fig. 4. The balancing network must be a group of resistors, capacitors, and inductors connected in a network whose impedance matches

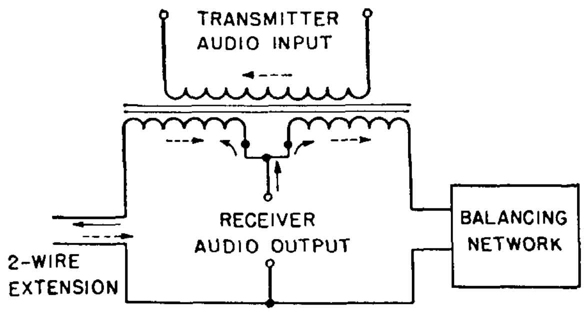

Fig. 4—Typical hybrid coil and connections. The components of the balancing network are chosen to match the input impedance of the telephone extension as nearly as possible over the voice-frequency band.

closely the impedance of the telephone line and associated equipment, as viewed from the hybrid unit terminals, over the band of audio frequencies transmitted by the carrier equipment. Examination of Fig. 4 shows how a typical hybrid transformer is intended to accomplish its function of placing the signal from the receiver upon the telephone line without producing a corresponding signal voltage across the input terminals of the transmitter audio amplifier. The receiver output is fed into the hybrid transformer at the junction of two identical windings. These two windings are in series with identical impedances, so that the receiver output current divides equally between the two. The ampere turns in the two windings balance or neutralize each other, leaving no ampere turns to be balanced by current in the third winding. The voltage across this third winding is therefore theoretically zero as far as the effect of signals from the receiver is concerned.

For a signal from the telephone line, however, the currents in the two identical windings are in essentially the same direction, some flowing through the receiver output transformer and the remainder flowing through the balancing network. A corresponding voltage therefore appears across the terminals of the third winding.

It is essential that telephone extensions used with duplex assemblies be properly terminated and be free of discontinuities. Received signals transmitted along an extension and reflected from such discontinuities back toward the carrier set appear to the hybrid unit as normal signals to be transmitted and may make it impossible to achieve a satisfactory balance with any type of balancing network.

The determination of the proper balancing network and the adjustment of audio levels after installation are usually the major problems in the application of two-frequency duplex equipment.

5. The Multi-Station Duplex System

The multi-station duplex system provides the advantages of duplex communication between any two of a number of stations on a channel. The basic units are shown in Fig. 5. Two transmitters and two receivers are included in each assembly, but other units, such as power supplies and amplifiers, are not duplicated.

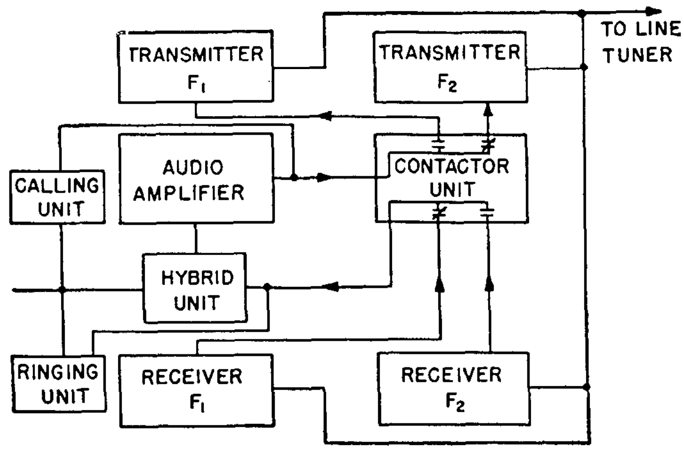

Fig. 5—Basic units of multi-station two-frequency duplex communication assembly with code-bell calling.

The transmitter and receiver used at a given station depend upon the point of origin of the call. Designating the two frequencies as F_1 and F_2, for example, all stations would normally receive on F_1. A station originating a call, however, transmits on F_1. The F_1 transmitter is selected by the calling party by the simple act of picking up the telephone handset. The closing of the d-c circuit through the hook switch operates a relay, which causes the contactor unit to apply the output of the audio amplifier to the audio terminals of transmitter F_1. Simultaneously the contactor unit energizes the transmitter and applies the output of receiver F_2 to the audio hybrid unit. At the called station, the reception of the carrier signal from the calling station on receiver F_1 operates a relay whose contacts open to prevent the transfer from transmitter F_2 to transmitter F_1 from being made by the contactor unit when the called party replies. Transmitter F_1 and receiver F_2 at the calling station and transmitter F_2 and receiver F_1 at the called station remain energized throughout the conversation. When the conversation is completed, the hanging up of the telephones at both stations returns conditions to normal, with all stations receiving on F_1.

6. The Single-Frequency, Automatic-Simplex System

Single-frequency automatic simplex is the most versatile of all the power-line carrier-communication systems. The number of stations on a given channel is not limited to two, as is the case with the usual two-frequency duplex system; it permits a single conversation among several stations on the channel, and it permits operation with two-wire telephone extensions and through PBX boards without requiring balance of a hybrid unit.

Modern automatic-simplex equipment eliminates objections to "send-receive" switching because this function, accomplished automatically, is so rapid and quiet that the user often is unable to detect its occurrence. In up-to-date automatic-simplex equipment, the transfer is made so

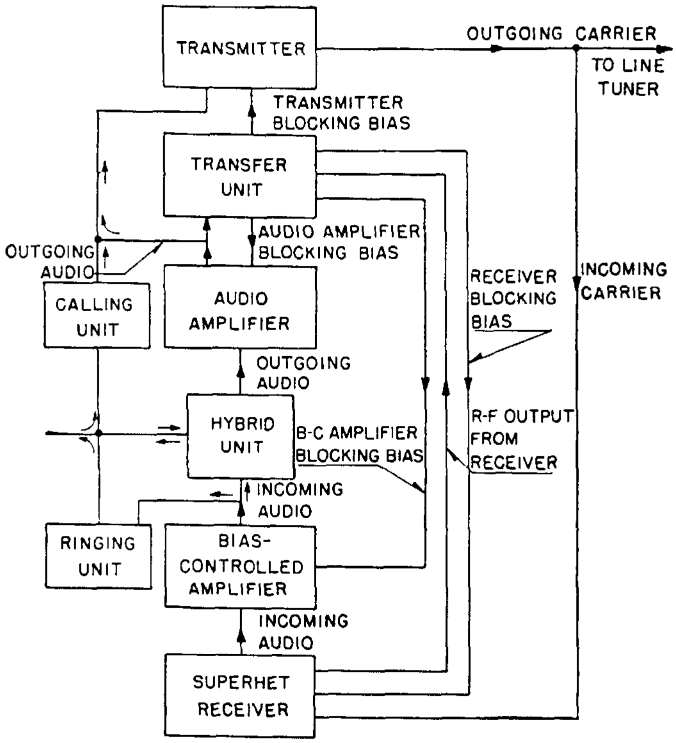

Fig. 6—Basic units of automatic simplex assembly with code-bell calling.

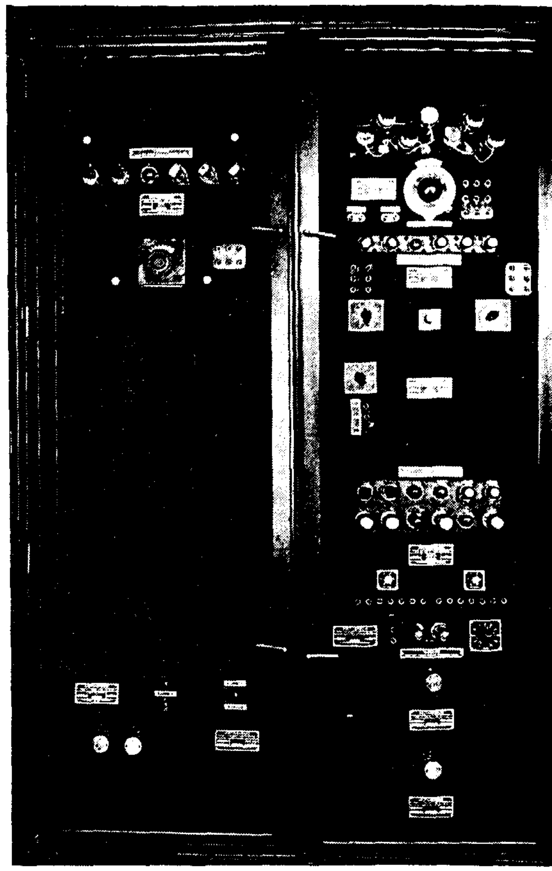

Fig. 7—Typical automatic simplex communication assembly. Units top to bottom, at left, superheterodyne receiver, switch and fuse panel, and high-voltage power supply. At right, transmitter, audio amplifier, hybrid and signalling unit, electronic transfer unit, bias-controlled audio amplifier, and two low-voltage power supplies.

rapidly that after every slight pause, or even between words, a party speaking can be interrupted.

A typical assembly for automatic-simplex communication is shown in the block diagram of Fig. 6. In addition to the units used in the two-frequency duplex assembly, automatic-simplex operation requires an electronic-transfer unit and a receiving audio-amplifier unit. The latter provides a convenient place to block receiver audio output without disabling the radio-frequency portion.

The transmitting audio amplifier in the stand-by condition is unblocked and ready to amplify voice signals from the telephone line. Reception of a carrier signal blocks the amplifier, so that once reception has started, no transmission can occur until the equipment returns to the stand-by condition. On the other hand, if an outgoing voice signal reaches the amplifier from the telephone line with the stand-by condition in effect, it causes the entire receiver to be blocked so that no signal can be received until conditions return to stand-by. The switch from transmit to receive and vice versa requires that the equipment pass through the stand-by condition in each direction.

The electronic-transfer unit is the key unit in the automatic-simplex assembly. It switches the equipment automatically from stand-by to transmit or receive as required. A typical automatic simplex assembly is shown in Fig. 7.

7. Calling Systems

A number of different systems of establishing a call over a carrier channel are in general use. The most important are the following: code-bell calling, voice calling, automatic bell calling, and dial selective calling.

Code-bell calling is the system of calling often used on rural party lines in which all telephones on a given circuit ring, the desired party being indicated by a code made up of long and short rings. The calling party transmits the code by turning a hand generator or by applying a voltage to the line with a push-button on his telephone instrument. All telephones on the system ring in accordance with the transmitted code.

In the *voice calling* system, the call is placed by simply speaking the desired party's name into the telephone transmitter. Loudspeakers with individual amplifiers are provided at all telephone extensions to call the desired party. The loudspeaker is disconnected when the telephone instrument is picked up. Calling by voice is supplemented in some installations, especially those where ambient noise level is high, by a high-frequency audio tone, which is applied to the loudspeaker for a few seconds at the time the calling party picks up his telephone instrument.

In the *automatic bell calling* system, the bells on the telephone instrument or instruments at the opposite terminal are rung automatically when the calling party picks up his handset. The ringing continues for a few seconds and then is cut off automatically. To repeat the ring the calling party must hang up the telephone instrument and remove it again, or close the hook switch manually and then release it. Because this system provides no means of indicating which telephone on an extension should be answered, it is used only on point-to-point carrier systems where only one extension is used at each end of the channel. A carrier channel linking two PBX boards provides an ideal application for the automatic bell calling system.

In *dial selective calling*, the desired number is dialed in the conventional dial-telephone manner. Each carrier set includes its own line-selector unit, which receives incoming dial pulses and applies ringing voltage to the wanted extension. Each of these selector units is in itself a complete private automatic-telephone exchange. The automatic-simplex carrier system with selective calling provides nearly every operating feature found on modern dial-telephone systems, such as a busy signal, a revertive or ring-back signal, local intercommunication, executive right-of-way or preferential service, and a disconnect signal.

8. Power Supply for Communication Assemblies

Alternating current at 120 or 240 volts generally has been used to supply carrier-communication equipment. At locations remote from generating sources, automatically starting motor-generator sets or converters have been used to provide power for the carrier set during emergencies or upon loss of normal a-c supply. This practice still is followed on long-haul channels using relatively high-powered equipment. Modern developments, however, have provided equipment capable of operating directly from 125- or 250-volt station batteries, making it possible to provide uninterrupted communication more economically, and without the maintenance problems associated with rotating equipment and accompanying control devices.

9. Carrier Relaying

Carrier-relaying systems and their application have been discussed in Chap. 11. A typical system is shown in Fig. 20 of that chapter. The application of the carrier equipment, as opposed to the application of the relays themselves, is basically the same as that for other carrier applications. The problems are greatly simplified, however, by the fact that relaying channels are always limited to the extent of a single line section and include line traps at each terminal. The relaying system normally requires use of the channel only during an actual fault, and the equipment is free for other applications for the remainder of the time. The system is always arranged so that the relays can interrupt any auxiliary functions in progress when a fault occurs.

A form of voice communication often termed "emergency communication" is usually inherently available in carrier equipment provided for relaying. Such communication is limited to the line section being protected, and since it is a "push-to-talk" system it is not suited to use with lengthy extensions or PBX boards. A rudimentary calling system, using the carrier itself as a calling signal, is usually employed. Because of these and other limitations, the communication function provided by carrier-relaying assemblies should not be considered in the same category with that provided by assemblies designed specifically for communication purposes.

10. Carrier Telemetering[3]

Telemetering is the indicating or recording of a quantity at a location remote from that at which the quantity exists. The quantities most often telemetered on power systems are electrical quantities, usually kilowatts and kilovars; but hot-spot temperature, water level, tap-changer position, and many other quantities can be telemetered.

Some telemetering systems are intended for operation over metallic conductors only. Among these are the torque-balance and the slide-wire systems. Others are adaptable for use either over metallic conductors or carrier channels. These latter systems are generally based upon the principle of converting the indication to be telemetered into pulses of a definite character, a variation in the telemetered quantity being reflected as a variation in some characteristic of the transmitted pulse.

In the impulse-rate system, the frequency or rate of the pulses varies in proportion to the magnitude of the telemetered quantity. A reference or base rate of impulsing represents a magnitude of zero; impulse rates above the base rate represent positive increments in the quantity, and impulse rates below the base rate represent negative increments.

In the impulse-duration system, the frequency of the pulses is constant. The duration of the pulse during a complete pulsing cycle is proportional to the magnitude of the telemetered quantity.

The pulse telemetering systems are well suited to operation over carrier channels. The fact that the intelligence transmitted takes the form of simple pulses makes it possible to use in many applications a simple carrier assembly in which an unmodulated carrier is turned on and off by a pair of contacts controlled by the telemetering device. No special modulation schemes are necessary with these systems, and the accuracy of the received information is independent of variations in the attenuation of the channel over which it is transmitted.

11. Impulse-Duration vs. Impulse-Rate Systems

Impulse-duration systems are adaptable to telemetering a much wider variety of quantities than is the impulse-rate system, which is generally suitable only for the telemetering of electrical quantities, primarily kilowatts and kilovars. The Bristol Metameter system, for example, can be supplied with measuring elements for the telemetering of pressure, liquid level, and a number of other mechanical or hydraulic readings. The impulse-duration receiving instruments have the additional advantage that they can be easily adapted to retransmission of individual or totalized quantities.

The impulse-rate system, however, has a number of advantages in those applications to which it is suited. A complete impulse-rate system, including a suitable recording instrument, generally costs less than a corresponding impulse-duration system. The accuracy of the impulse-rate system is not affected by reasonable variations in the duration of the "on" and "off" periods of the impulse, an important consideration in applications where the telemetering signal must be received and re-transmitted at several points along its channel. Careful attention must be paid to the operating times of mechanical relays, and to the time-lag in audio-relay circuits, when impulse-duration signals are passed along in this fashion. Also, large variations in signal level due to changes in attenuation affect the accuracy of impulse-duration systems operating on audio tones to an extent depending upon the flatness of the receiver avc (automatic-volume-control) characteristic.

12. Power-Line Carrier Telemetering Assemblies

The channel requirements for impulse telemetering systems are relatively simple; and because transmission alone or reception alone is usually required, the assemblies used for telemetering purposes are often correspondingly simple. If a single set of impulses is to be transmitted from a given point, the assembly often consists of a single frequency-shift carrier transmitter with a self-contained, a-c power supply. The carrier-frequency output of the transmitter is controlled directly by the impulse-forming device, which shifts the output back and forth between the mark and space frequencies as its contacts close and open.

At the receiving end of such a channel, a frequency-shift receiver is used to receive the carrier signal. The receiver operates a relay which in turn keys the impulse receiver.

In applications where more than two or three quantities are to be telemetered from a single point simultaneously, it is common practice to use audio-tone transmitter units to modulate the carrier-frequency signal. One tone frequency is used for each telemetered quantity, and the carrier wave is left on continuously. The telemetering assembly of Fig. 8 is used to receive eight separate telemetering tones on a single carrier frequency and transmit two other tones on another.

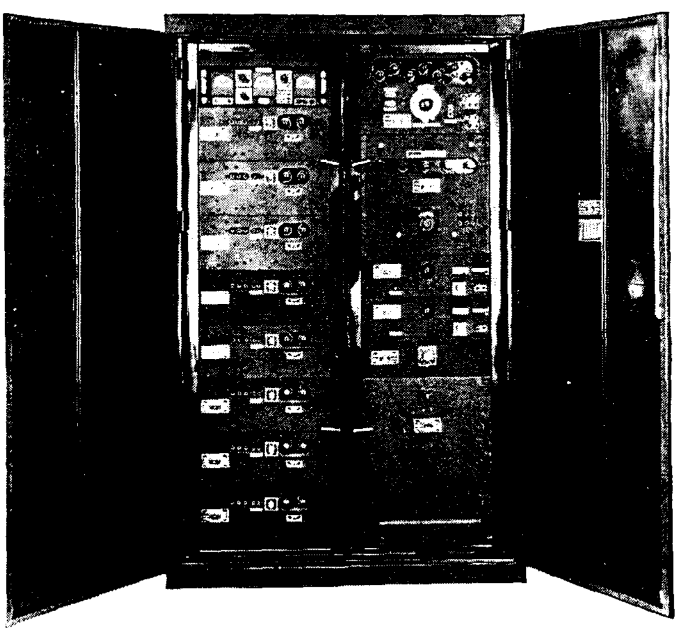

Fig. 8—Typical tone telemetering assembly with carrier receiver and eight tone receivers, and carrier transmitter with two tone transmitters. This assembly is capable of receiving eight simultaneous telemetered indications on one carrier frequency and transmitting two simultaneous indications on another.

13. Load-Frequency Control

Load-frequency control is the control of the output of a generator or a group of generators on a system in such a way as to maintain the system frequency and to regulate the interchange of power with other systems in accordance with a predetermined plan.

The frequency of a system or a group of interconnected systems is constant if the governor settings on all the prime movers cause the generators to produce exactly the amount of power required to supply the total load. If some of the load is suddenly lost, that part of the prime mover output initially supplying the dropped load is absorbed in accelerating all the units on the system, and a rise in system frequency occurs. Under these conditions, the output of one or more of the prime movers on the system must be reduced to restore the frequency to normal, and then increased slightly to maintain normal frequency.

In the operation of large interconnected systems or power pools, it is the practice for one large generating station to regulate its output on the basis of system frequency, reducing the governor settings of one or more prime movers if the system frequency is high and increasing the settings if the frequency is low, without regard to tie-line loads or total interchange of power with other systems. This type of operation is called flat frequency control. The other systems in the interconnected group regulate prime-mover outputs on the basis of the interchange of power among systems. For these other systems, there are several possible types of operation, most of which are based on regulating to produce a pre-determined net tie-line loading when frequency is normal, but allowing the tie-line loading to depart from the pre-determined value when the frequency is off normal.

The basic quantity used to govern the operation of automatic load controllers is the net power interchange of the system, which is combined with system frequency in most types of control. In the usual arrangement, net interchange is obtained by totalizing individual interchange readings at the dispatching office and combining the result with frequency in an automatic load controller located at that point. The controller generates "raise" and "lower" impulses that must be transmitted to the regulating station. Power-line carrier is often used as a medium for transmitting these signals.

14. Carrier Assemblies for Load-Frequency Control

The channel requirements for load-frequency control are similar to those for telemetering two quantities, since two types of impulses must be transmitted. A common arrangement is the use of a single carrier transmitter modulated by two audio-frequency tones, one for "raise" impulses and one for "lower" impulses. At the receiving end of such a channel, a single carrier receiver operates into a corresponding pair of tone receivers. An alternate system is the use of two frequency-shift channels, one for "raise" and one for "lower" impulses.

It is frequently desirable to arrange the system so that any one of several generating stations can be called on to act as the regulating station for the system. In this case the usual arrangement is for each such station to be equipped with an identical carrier-receiving assembly, tuned to the frequency of the load-control transmitter at the dispatching office. Any one of the stations can then