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```
C       CALCULATION OF REFLECTED INTENSITY OF P POLARISED LIGHT FROM
C       THE SURFACE OF A DIELECTRIC MEDIUM.
C       ------------------------------------------------------------
        LOGICAL*1 Y1,Y2,Y3,Y4,Y5,Y6(2),EQUC
        DIMENSION X(200),Y(200),Z(200),T(20)
        COMPLEX J,R2,R3
C
C       GRAPHICS TITLES AND INPUT PARAMETERS.
C
        DATA Y1/'Y'/,Y2/'A'/,Y3/'D'/,Y4/'M'/
       1 ,T(1)/'    '/,T(2)/'    '/,T(3)/'P-RE'/,T(4)/'FLEC'/,
       2 T(5)/'TIVI'/,T(6)/'TY--'/,T(10)/'    '/,T(11)/'    '/,T(12)
       3 /'    '/,T(17)/'REFL'/,T(18)/'ECTI'/,T(19)/'VITY'/,T(20)/
       4 '    '/,TA1/'FREQ'/,TA2/'UENC'/,TA3/'Y (RD'/,TA4/'S-1)  '/
       5 ,TF1/'ANGL'/,TF2/'E (D'/,TF3/'EGRE'/,TF4/'ES) '/
C
C       INPUT/OUTPUT CHANNELS ASSIGNED.
C
        CALL FTNCMD('$EMPTY RESULT OK',16)
        CALL FTNCMD('ASSIGN 7=RESULT',15)
        CALL FTNCMD('EQUATE 5=SCARDS',15)
        J=CMPLX(0.0,1.0)
        WRITE(6,1)
C
C       OPTION FOR DETAILS OF ATR METHOD.
C
        CALL FREAD(5,'STRING:',Y5,1)
        IF(.NOT.EQUC(Y5,Y1))GOTO 100
        WRITE(6,2)
        WRITE(6,3)
100     WRITE(6,4)
        CALL FREAD(5,'I*4:',NC)
C
C       SET DEVICE CODE FOR GRAPHICS OUTPUT.
C
        CALL AUX093(NC)
110     WRITE(6,5)
C
C       OPTION OF DETAILED INPUT PROMPTING.
C
        CALL FREAD(5,'STRING:',Y5,1)
        K=0
        IF(EQUC(Y5,Y1))K=1
C
C       MATERIAL IDENTIFIER REQUIRED.
C
        WRITE(6,6)
        READ(5,7) (T(I),I=7,9)
        WRITE(7,8) (T(I),I=7,9)
C
C       CHOICE OF ATR CONFIGURATIONS
C
```

```
      WRITE(6,9)
      CALL FREAD(5,'STRING:',Y6,2)
      IF(EQUC(Y6(2),Y4))GOTO 140
C
C      PRISM-AIR MEDIUM CALCULATION CHOSEN
C
      WRITE(7,10)
      IF(K.EQ.0)GOTO 120
      WRITE(6,10)
      WRITE(6,11)
C
C     INPUT DATA FOR PRISM-AIR-MEDIUM CALCULATION
C
120   WRITE(6,12)
      CALL FREAD(5,'R:',RP)
      IF(K.EQ.0)GOTO 130
      WRITE(6,13)
130   WRITE(6,14)
      CALL FREAD(5,'R:',D)
      WRITE(7,15)RP,D
      D=D*1.0E-9
      GOTO 170
140   WRITE(7,16)
C
C      PRISM-MEDIUM-AIR CALCULATION CHOSEN.
C
      IF(K.EQ.0)GOTO 150
      WRITE(6,16)
      WRITE(6,11)
150   WRITE(6,12)
C
C     INPUT DATA FOR PRISM-MEDIUM-AIR CALCULATION.
C
      CALL FREAD(5,'R:',RP)
      IF(K.EQ.0)GOTO 160
      WRITE(6,17)
160   WRITE(6,18)
      CALL FREAD(5,'R:',D)
      WRITE(7,19)RP,D
      D=D*1.0E-9
C
C     SETS PARAMETERS TO CALCULATE MAXIMUM AND MINIMUM VALUES OF
C     PLOTTED VARIABLES.
C
170   XMAX=-1.0E60
      YMAX=-1.0E60
      XMIN=1.0E60
      YMIN=1.0E60
      WRITE(6,20)
C
C     CHOICE OF FIXED ANGLE OR FIXED FREQUENCY SCAN.
C
      CALL FREAD(5,'STRING:',Y5,1)
```

```
      IF(EQUC(Y5,Y2))GOTO 260
      WRITE(7,21)
      IF(K.EQ.0)GOTO 180
C
C     FIXED FREQUENCY CALCULATION CHOSEN.
C
      WRITE(6,21)
      WRITE(6,22)
C
C     CHOICE OF FREE ELECTRON MODEL OR EXPERIMENTAL DATA FOR
C     REFRACTIVE INDEX.
C
180   WRITE(6,23)
      CALL FREAD(5,'STRING:',Y5,1)
      IF(EQUC(Y5,Y3))GOTO 200
      WRITE(7,24)
      IF(K.EQ.0)GOTO 190
      WRITE(6,24)
C
C     FREE ELECTRON MODEL CHOSEN.
C
      WRITE(6,25)
C
C     INPUT DATA FOR FREE ELECTRON MODEL CALCULATION.
C
190   WRITE(6,26)
      CALL FREAD(5,'R:',WP)
      WRITE(6,27)
      CALL FREAD(5,'R:',W)
      WRITE(6,28)
      CALL FREAD(5,'R:',DP)
      WRITE(6,29)
      CALL FREAD(5,'R:',EL)
      WRITE(7,30)WP,W,DP,EL
C
C     CALCULATION OF REAL AND IMAGINARY PARTS OF REFRACTIVE INDEX.
C
      RR=REAL(CSQRT(EL-EL*WP*WP/(W*(W+J*DP))))
      RI=AIMAG(CSQRT(EL-EL*WP*WP/(W*(W+J*DP))))
      WRITE(7,31)RR,RI
      GOTO 220
200   WRITE(7,32)
C
C     CALCULATION USING REFRACTIVE INDEX DATA CHOSEN.
C
      IF(K.EQ.0)GOTO 210
      WRITE(6,32)
210   WRITE(6,27)
C
C     INPUT DATA FOR EXPERIMENTAL REFRACTIVE INDEX.
C
      CALL FREAD(5,'R:',W)
      WRITE(6,33)
```

```
      CALL FREAD(5,'R:',RR)
      WRITE(6,34)
      CALL FREAD(5,'R:',RI)
      WRITE(7,35)W,RR,RI
220   WRITE(6,36)
C
C     INPUT PARAMETERS FOR ANGLE SCAN.
C
      CALL FREAD(5,'R:',TH1)
      WRITE(6,37)
      CALL FREAD(5,'R:',THGAP)
      WRITE(6,38)
      CALL FREAD(5,'I*4:',NTH)
      WRITE(7,39)
      IF(EQUC(Y6(2),Y4))GOTO 230
C
C     PRISM-AIR-MEDIUM CONFIGURATION.
C
      R2=CMPLX(1.0,0.0)
      R3=CMPLX(RR,RI)
      GOTO 240
C
C     PRISM-MEDIUM-AIR CONFIGURATION.
C
230   R2=CMPLX(RR,RI)
      R3=CMPLX(1.0,0.0)
240   DO 250 I=1,NTH
C
C     CALLS SUBROUTINE TO CALCULATE REFLECTED INTENSITY.
C
      CALL REF(TH1,RP,R2,R3,W,D,R12,R23,R)
C
C     CALCULATES MAXIMUM AND MINIMUM VALUES OF VARIABLES.
C
      X(I)=TH1
      Y(I)=R
      IF(XMAX.LT.X(I))XMAX=X(I)
      IF(XMIN.GT.X(I))XMIN=X(I)
      IF(YMAX.LT.Y(I))YMAX=Y(I)
      IF(YMIN.GT.Y(I))YMIN=Y(I)
C
C     OUTPUTS RESULTS TO DATA FILE.
C
      WRITE(7,40)TH1,R12,R23,R
      TH1=TH1+THGAP
250   CONTINUE
      NP=NTH
C
C     GRAPH TITLES FOR ANGLE SCAN.
C
      T(13)=TF1
      T(14)=TF2
      T(15)=TF3
```

```
      T(16)=TF4
      GOTO 320
260   WRITE(7,41)
C
C     FIXED ANGLE CALCULATION CHOSEN.
C
      WRITE(7,24)
      IF(K.EQ.0)GOTO 270
      WRITE(6,41)
      WRITE(6,25)
C
C     INPUT DATA FOR FREE ELECTRON MODEL CALCULATION.
C
270    WRITE(6,26)
      CALL FREAD(5,'R:',WP)
      WRITE(6,28)
      CALL FREAD(5,'R:',DP)
      WRITE(6,29)
      CALL FREAD(5,'R:',EL)
      WRITE(7,42)WP,DP,EL
C
C     INPUT PARAMETERS FOR FREQUENCY SCAN.
C
      WRITE(6,43)
      CALL FREAD(5,'R:',W)
      WRITE(6,44)
      CALL FREAD(5,'R:',WGAP)
      WRITE(6,45)
      CALL FREAD(5,'I*4:',NF)
      IF(K.EQ.0)GOTO 280
      WRITE(6,46)
280   WRITE(6,47)
      CALL FREAD(5,'R:',TH1)
      WRITE(7,48)TH1
      DO 310 I=1,NF
      RR=REAL(CSQRT(EL-EL*WP*WP/(W*(W+J*DP))))
      RI=AIMAG(CSQRT(EL-EL*WP*WP/(W*(W+J*DP))))
      IF(EQUC(Y6(2),Y4)) GOTO 290
C
C     PRISM-AIR-MEDIUM CONFIGURATION.
C
      R2=CMPLX(1.0,0.0)
      R3=CMPLX(RR,RI)
      GOTO 300
C
C     PRISM-MEDIUM-AIR CONFIGURATION.
C
290   R2=CMPLX(RR,RI)
      R3=CMPLX(1.0,0.0)
C
C     CALLS SUBROUTINE TO CALCULATE REFLECTED INTENSITY.
C
300   CALL REF(TH1,RP,R2,R3,W,D,R12,R23,R)
```

```
C
C     CALCULATES MAXIMUM AND MINIMUM VALUES OF VARIABLES.
C
      X(I)=W
      Y(I)=R
      IF(XMAX.LT.X(I))XMAX=X(I)
      IF(XMIN.GT.X(I))XMIN=X(I)
      IF(YMAX.LT.Y(I))YMAX=Y(I)
      IF(YMIN.GT.Y(I))YMIN=Y(I)
C
C     OUTPUTS RESULTS TO DATA FILE
C
      WRITE(7,40)W,R12,R23,R
      W=W+WGAP
310   CONTINUE
      NP=NF
C
C     GRAPH TITLES FOR FREQUENCY SCAN.
C
      T(13)=TA1
      T(14)=TA2
      T(15)=TA3
      T(16)=TA4
320   WRITE(7,49)
C
C     PLOTS GRAPH OF REFLECTED INTENSITY AS FUNCTION OF CHOSEN
C     VARIABLE.
C
      CALL SLE(XMAX,XMIN)
      CALL SLE(YMAX,YMIN)
      SCX=ABS(XMAX-XMIN)/4.0
      SCY=ABS(YMAX-YMIN)/4.0
      CALL CGPL(X,Y,Z,NP,64,1,1,1,1,XMIN,SCX,4.0,
     1 YMIN,SCY,4.0,T,8)
      CALL CGPL(X,Y,Z,NP,132,1,1,1,1,XMIN,SCX,4.0,
     1 YMIN,SCY,4.0,T,8)
      CALL PLOT(0.0,0.0,999)
      WRITE(6,50)
C
C     OPTION TO REPEAT PROGRAM.
C
      CALL FREAD(5,'STRING:',Y5,1)
      IF(EQUC(Y5,Y1))GOTO 110
C
C     ------------------------------------------------------------
C
1     FORMAT(22X,'ATR CALCULATION',/,22X,15(1H-),//,'DO YOU WANT',
     1 ' ANY INFORMATION ABOUT ATR METHOD?-TYPE YES OR NO')
2     FORMAT('PROGRAM CALCULATES AND DISPLAYS GRAPHICALLY THE ',/
     1 ,'REFLECTED INTENSITY FROM THE SURFACE OF A MATERIAL',/,
     2 'IRADIATED WITH P-POLARISED LIGHT:',//,
     3 12X,'(A)  AS A FUNCTION OF FREQUENCY FOR A FIXED ANGLE.',
     4 //,12X,'(B)  AS A FUNCTION OF ANGLE FOR A FIXED FREQUENCY.'
```

```
     5 ,//,'AND FOR TWO ATTENUATED TOTAL REFLECTION(ATR)',
     6 ' CONFIGURATIONS:',//,12X,'(1)   PRISM-MEDIUM-AIR      PMA',
     7 //,12X,'(2)   PRISM-AIR-MEDIUM      PAM',//
     8 ,'THE PAM SYSTEM CONSISTS OF A SLAB OF THE MEDIUM',/
     9 ,'UNDER INVESTIGATION, SEPARATED FROM THE INPUT PRISM',/
     1 ,'BY A SMALL AIR GAP.THE PMA SYSTEM CONSISTS OF A',/
     2 ,'THIN FILM OF THE MEDIUM DEPOSITED DIRECTLY ONTO THE',/
     3 ,'PRISM SURFACE.',/
     4 ,'    IN BOTH CASES THE EXPONENTIALLY DECAYING FIELD',/
     5 ,'FROM THE TOTALLY INTERNALLY REFLECTED INPUT LIGHT',/
     6 ,'RAY HAS A SUFFICIENTLY LARGE COMPONENT OF MOMENTUM',/
     7 ,'IN THE PLANE OF THE MEDIUM SURFACE AND IN THE ',/
     8 ,'DIRECTION OF PROPAGATION , TO LAUNCH A SURFACE WAVE.',/
     9 ,'THIS IS SEEN AS A SHARP MINIMUM IN THE REFLECTED',/
     1 ,'INTENSITY (WHICH WOULD OTHERWISE BE UNITY).')
3     FORMAT('    NOTE THAT A RAY INCIDENT DIRECTLY ONTO THE SURF',
     1 'ACE',/,'OF THE MEDIUM HAS INSUFFICIENT MOMENTUM IN THE ',/
     2 ,'DIRECTION OF PROPAGATION TO LAUNCH A SURFACE WAVE.',/)
4     FORMAT('TYPE DEVICE CODE-4010+4013=1',/
     1 ,18X,'4014+4015=2')
5     FORMAT('DO YOU WANT FULL INPUT PROMPTING? TYPE YES OR NO')
6     FORMAT('TYPE IDENTIFIER FOR MATERIAL: UP TO 12 LETTERS')
7     FORMAT(3A4)
8     FORMAT('SUBJECT MEDIUM IS ',3A4,/)
9     FORMAT('WHICH ATR CONFIGURATION DO YOU WANT? TYPE PMA ',
     1 'OR PAM')
10    FORMAT(10X,'PRISM-AIR-MEDIUM CALCULATION',/,10X,28(1H-),/)
11    FORMAT('IN THE CASE OF METALS FOR WHICH THE SURFACE PLASMON'/
     1 ,'FREQUENCY IS USUALLY IN THE OPTICAL REGION, PRISMS',/
     2 ,'OF REFRACTIVE INDEX SIMILAR TO GLASS(ABOUT 1.5) ARE',/
     3 ,'USED.FOR SEMICONDUCTORS THE REGION OF INTEREST IS',/
     4 ,'USUALLY THE INFRA-RED WHERE SILICON IS USED.IN THIS',/
     5 ,'CASE THE REFRACTIVE INDEX IS 3.418.',/)
12    FORMAT('TYPE REFRACTIVE INDEX OF PRISM')
13    FORMAT('IN PAM CALCULATIONS THE AIR GAP THICKNESS  D, IS',/
     1 ,'CRITICAL AND TYPICALLY THE ORDER OF 200 OR 300 ',
     2 'NANOMETRES',/)
14    FORMAT('TYPE AIR GAP THICKNESS (IN NANOMETRES)')
15    FORMAT('PRISM REFRACTIVE INDEX=',E12.4,/
     1 ,'AIR GAP THICKNESS=',E12.4,' NANOMETRES',/)
16    FORMAT(10X,'PRISM-MEDIUM-AIR CALCULATION',/,10X,28(1H-),/)
17    FORMAT('IN PMA CALCULATIONS THE FILM THICKNESS D, IS',/
     1 ,'CRITICAL AND TYPICALLY THE ORDER OF 20 TO 50 NANOMETRES',/)
18    FORMAT('TYPE FILM THICKNESS (IN NANOMETRES)')
19    FORMAT('PRISM REFRACTIVE INDEX=',E12.4,/
     1 ,'FILM THICKNESS=',E12.4,' NANOMETRES',/)
20    FORMAT('DO YOU WANT A FIXED ANGLE OR FREQUENCY SCAN?'
     1 ,'---TYPE ANGLE OR FREQUENCY')
21    FORMAT(10X,'FIXED FREQUENCY CALCULATION',/,10X,27(1H-),/)
22    FORMAT('YOU HAVE CHOICE OF A FREE ELECTRON MODEL ',/
     1 ,'OR TO INPUT DATA FOR THE REFRACTIVE INDEX',/)
23    FORMAT('TYPE MODEL OR DATA')
24    FORMAT(10X,'FREE ELECTRON MODEL CALCULATION',/,10X,31(1H-),/)
```

```
25    FORMAT('MODEL TREATS MEDIUM AS A PLASMA WITH A NATURAL',/
     1 ,'RESONANCE FREQUENCY (PLASMA FREQUENCY).MODEL REQUIRES',/
     2 ,'KNOWLEDGE OF THE HIGH FREQUENCY DIELECTRIC CONSTANT',/
     3 ,'OF THE MEDIUM, WHICH FOR A METAL IS UNITY AND FOR A ',/
     4 ,'SEMICONDUCTOR IS USUALLY BETWEEN 10 AND 20.',/,
     5 2X,'MODEL ALSO REQUIRES THE PLASMA FREQUENCY AND THE ',/
     6 ,'DAMPING PARAMETER WHICH IS GENERALLY NOT A CRITICAL',/
     7 ,'FACTOR AND CAN OFTEN BE GIVEN APPROXIMATELY AS ONE',/
     8 ,'PER CENT OF THE PLASMA FREQUENCY.',/)
26    FORMAT('TYPE PLASMA FREQUENCY')
27    FORMAT('TYPE FREQUENCY FOR ATR ANGLE SCAN')
28    FORMAT('TYPE DAMPING PARAMETER')
29    FORMAT('TYPE HIGH FREQUENCY DIELECTRIC CONSTANT')
30    FORMAT('PLASMA FREQUENCY= ',E12.4,/
     1 ,'SCAN FREQUENCY= ',E12.4,/
     2 ,'DAMPING TERM  = ',E12.4,/
     3 ,'HIGH FREQ. DIELECTRIC CONSTANT= ',E12.4)
31    FORMAT('MODEL COMPLEX REFRACTIVE INDEX=',E12.4,' +J',E12.4,/)
32    FORMAT(10X,'CALCULATION USING REFRACTIVE INDEX DATA',
     1 /,10X,39(1H-),/)
33    FORMAT('TYPE REAL PART OF REFRACTIVE INDEX')
34    FORMAT('TYPE IMAGINARY PART OF REFRACTIVE INDEX')
35    FORMAT('SCAN FREQUENCY= ',E12.4,/
     1 ,'GIVEN COMPLEX REFRACTIVE INDEX= ',E12.4,' +J',E12.4,/)
36    FORMAT('TYPE STARTING ANGLE FOR ATR SCAN (DEGREES)')
37    FORMAT('TYPE ANGLE INCREMENT (DEGREES)')
38    FORMAT('TYPE NUMBER OF ANGLE VALUES IN SCAN(UP TO 200)')
39    FORMAT('  ANGLE(DEG)',7X,'R12',11X,'R23',12X,'R')
40    FORMAT(4(E12.4,2X))
41    FORMAT(10X,'FIXED ANGLE CALCULATION',/,10X,23(1H-),/)
42    FORMAT('PLASMA FREQUENCY= ',E12.4,/
     1 ,'DAMPING TERM= ',E12.4,/
     2 ,'HIGH FREQ. DIELECTRIC CONSTANT= ',E12.4)
43    FORMAT('TYPE STARTING FREQUENCY FOR ATR SCAN')
44    FORMAT('TYPE FREQUENCY INCREMENT')
45    FORMAT('TYPE NUMBER OF FREQUENCY VALUES IN SCAN(UP TO 200)')
46    FORMAT('THE ATR EFFECT IS OBSERVED FOR INCIDENT ANGLES',/
     1 ,'GREATER THAN ,OR NEAR TO THE CRITICAL ANGLE FOR THE',/
     2 ,'PRISM MATERIAL (ABOUT 41 DEG. FOR GLASS WITH N=1.5)',/
     3 ,'   SOME STRUCTURE WILL BE OBSERVED BELOW THIS ANGLE',/
     4 ,'HOWEVER DUE TO MULTIPLE REFLECTION EFFECTS.',/)
47    FORMAT('TYPE ANGLE FOR ATR SCAN (IN DEGREES)')
48    FORMAT('ANGLE FOR ATR SCAN= ',E12.4,' DEGREES',//,
     1 '   FREQ.',10X,'R12',11X,'R23',12X,'R')
49    FORMAT(60(1H-),/,60(1H-),//)
50    FORMAT('DO YOU WISH TO RUN ANOTHER CASE?-TYPE YES OR NO')
C
C     ------------------------------------------------------------
C
      STOP
      END
C
C     ------------------------------------------------------------
```

```
C
      SUBROUTINE REF(TH1,RP,R2,R3,W,D,R12,R23,R)
      COMPLEX A12,A23,A,CTH2,CTH3,J,R2,R3
C
C     SUBROUTINE TO CALCULATE REFLECTED INTENSITY AT EACH INTERFACE
C     AND FOR COUPLED SYSTEM.
C
      J=CMPLX(0.0,1.0)
      THR=TH1*3.14159/180.0
      STH1=SIN(THR)
      CTH1=COS(THR)
      CTH2=CSQRT(1.0-(STH1*RP/R2)**2)
      CTH3=CSQRT(1.0-(STH1*RP/R3)**2)
      A12=(R2*CTH1-RP*CTH2)/(R2*CTH1+RP*CTH2)
      A23=(R3*CTH2-R2*CTH3)/(R3*CTH2+R2*CTH3)
      R12=REAL(A12)**2+AIMAG(A12)**2
      R23=REAL(A23)**2+AIMAG(A23)**2
      A=(A12+A23*CEXP(J*2.0*W*R2*D/2.998E8*CTH2))
     1 /(1.0+A12*A23*CEXP(J*2.0*W*R2*D/2.998E8*CTH2))
      R=REAL(A)**2+AIMAG(A)**2
      RETURN
      END
```

Physics Programs
Edited by A. D. Boardman

CHAPTER 3

Computer-Generated Holograms

A. D. BOARDMAN and M. E. S. CHAPMAN

1. INTRODUCTION

Since the advent of the laser considerable interest has been shown in the topic known as holography. However, this is basically an imaging process that takes place in two stages and can be done with optical, microwave, or acoustic waves. In the first stage a diffraction pattern is created by interference between waves scattered off an object and a powerful direct reference beam, but using the same coherent source (see Figure 1). The recorded version of this diffraction pattern is called the hologram and contains both phase and amplitude information. In the second stage, all that is necessary to reconstruct a three-dimensional image of the object is to illuminate the hologram with the original reference beam and employ an appropriate imaging system. In fact it is not difficult to show that the image is located in the plane conjugate to the plane of the object.

This form of holography, is, of course, important, and very exciting. On the other hand, digital holography, in which computers are used both to generate holograms, and to reconstruct objects from holograms, in the form of data arrays, is also very striking with many areas of application. Some of these include optical character recognition, optical surface testing, archival data storage, image intensifiers, automobile and aeroplane design, and other forms of three-dimensional computer displays.[1–3]

An advantage of computer-based holography is that the object does not have to be real and hence exist physically. This is a very useful feature because it is often convenient to specify an object in purely mathematical terms. In other words digital holograms can be used as optical elements, in a general sense, dealing with properties that do not have a physical analogue. It also almost goes without saying that computer-generated holograms are free of the complications normally encountered in making holograms experimentally.

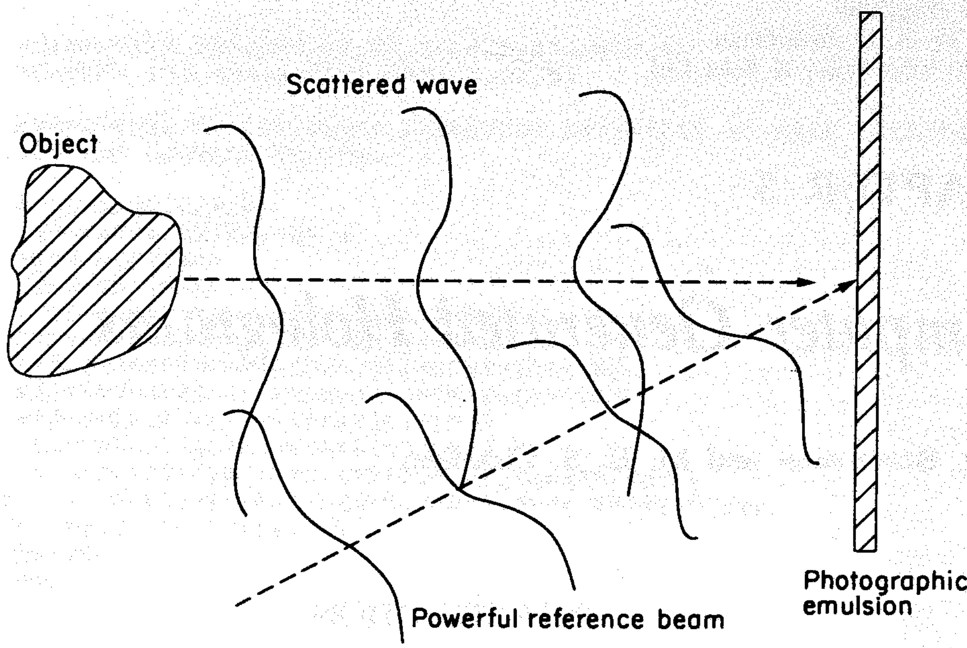

Figure 1. Schematic holographic arrangement. The hologram is the photographically recorded version of the resultant diffraction pattern

2. FRAUNHOFER DIFFRACTION PATTERNS AND FOURIER TRANSFORMS

Naturally there are many hologram-forming geometries, but this chapter focuses attention on what are known as Fourier transform holograms whose computer generation will be based on Lohmann's method. Attention is further restricted to two-dimensional objects but there is nothing, in principle, to prevent a generalization to three dimensions. The basic idea of Fourier transform holography is that the hologram is a record of the interference between waves that are the exact Fourier transforms of the object and the reference source. Since a Fraunhofer diffraction pattern is the Fourier transform of the aperture,[4] the Fourier transform holograms are sometimes referred to as Fraunhofer holograms.[5] Indeed, the Fourier transforms required in this chapter are approached most easily by considering the familiar Fraunhofer diffraction theory.

Scattering and diffraction are not synonymous terms, but they are aspects of a single physical process; one does not exist without the other. A diffraction field is the total field in the presence of the scattering object. In particular, Fraunhofer diffraction describes the interaction between a plane wave and an aperture when both the source and point of observation are at infinity. The observation of a Fraunhofer pattern is performed with a convex lens, designed, or assumed, not to introduce any spurious affects of its own. In this way the Fourier transform of the aperture is located in its back focal plane that can be called the Fourier plane.

Generally speaking, if X and Y are the coordinates of a point in the aperture and plane waves arrive at and emerge from the aperture with respective direction cosines l_i, m_i and l_e, m_e, then the complex Fraunhofer diffraction amplitude is [1,4] the Fourier transform

$$F(p, q) = \int\int G(X, Y)\exp[(2\pi i/\lambda)(pX + qY)]\,dX\,dY, \tag{1}$$

where $p = l_e - l_i$, $q = m_e - m_i$, λ is the wavelength and $G(X, Y)$ is a function that is non-zero in the aperture space. The latter restriction means that although the integration extends to infinity in both X- and Y-directions, the contributions to the integral arise from the aperture space. The sign in the exponential can be chosen as positive[1] or negative[4] and the choice corresponds to defining a plane wave, propagating in the direction defined by wave number k, as $\exp\{i(\omega t - \mathbf{k}\cdot\mathbf{r})\}$ or $\exp\{i(\mathbf{k}\cdot\mathbf{r} - \omega t)\}$. The positive sign is selected here, but obviously it does not matter, physically, which one is used.

If the Fourier plane is the back focal plane of a converging lens, of focal length f, then as shown in Figure 2 the complex Fourier amplitude at the

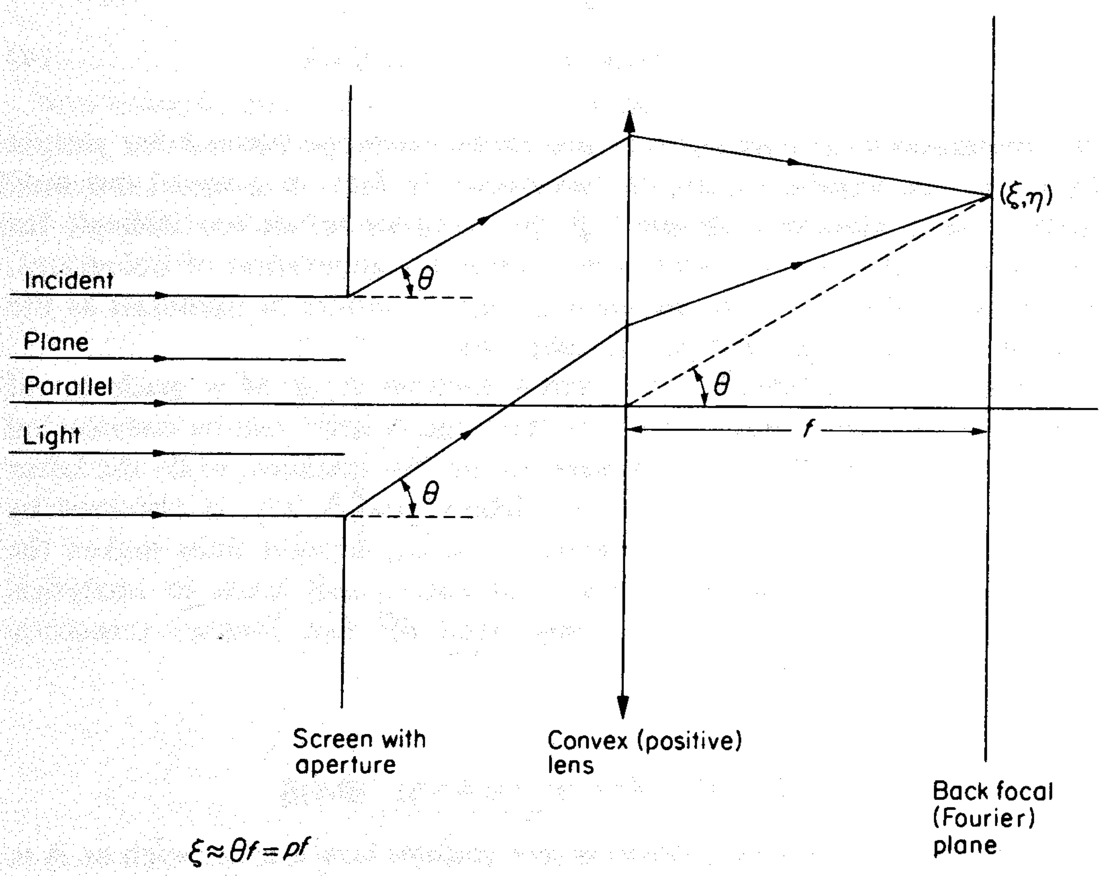

Figure 2. Fraunhofer diffraction by an aperture where the diffraction pattern is in the Fourier plane, or the back focal plane of a positive lens

point (ξ, η) is, since $\xi = pf$, $\eta = qf$,

$$F(\xi, \eta) = \int\int G(X, Y) \exp[(2\pi i/f\lambda)(\xi X + \eta Y)] \, dX \, dY, \tag{2}$$

i.e. there is a spot $(\xi\eta)$ in the focal plane that is the focal point for a diffracted beam from the object. Furthermore, in the application discussed in this chapter, $G(X, Y)$ is going to represent the light distribution in some mathematically defined two-dimensional object where X, Y are spatial coordinates in the object plane. The computer problem amounts to devising a means of encoding $F(\xi, \eta)$, in graphical form, and thereby computer-generating a hologram.

It is not very elegant to use equation (2) in its present form, because the adoption of another coordinate system would effectively scale out f and λ. Indeed, if the coordinates x and y are introduced through the relationships

$$x = \frac{2\pi}{f\lambda} \xi, \qquad y = \frac{2\pi}{f\lambda} \eta, \tag{3}$$

then equation (2) takes the tidier, universal, form

$$F(x, y) = \int\int G(X, Y) \, e^{i(xX + yY)} \, dX \, dY. \tag{4}$$

The integrations in equation (4) are fairly easily performed for simple objects, like rectangular or circular apertures. In fact, as pointed out only recently,[6] the analytical evaluation of the integrals is not too difficult for quite a variety of polygonal apertures. Computer generation of holograms, however, is usually illustrated by using groups of letters or numerals as the object, and this arrangement is also used here.

An object that happens to be a letter, such as A or M is made up of straight lines but even curved letters, in the large N limit, can be constructed from N straight lines. It is not necessary, in the first instance, to do the latter since a decent approximation to letters like C and S, say, is obtained by forming them as $\sqsubset$ and $⊑$. The device of using straight lines makes the mathematical specification of letters a lot easier and leads to analytical Fourier transforms, thus obviating any need for fast Fourier transform techniques to save computer time.

3. LETTER GROUP TRANSFORMS

It is probably not immediately obvious to a student how a letter such as A is to be specified as a function $G(X, Y)$. In order to get some ideas about it let us consider light passing through the rectangular aperture shown in Figure 3. $G(X, Y)$ is zero everywhere, except in the aperture region where it is some constant K, say. The integration limits are, therefore, the aperture limits so that equation (4), for this case, is

$$F(x, y) = K \int_{-a}^{a} \int_{-b}^{b} e^{i(xX+yY)}\, dX\, dY. \tag{5}$$

The integral in equation (5) is

$$I = \int_{-a}^{a} \int_{-b}^{b} e^{i(xX+yY)}\, dX\, dY = 4ab\, \mathrm{sinc}(xa)\mathrm{sinc}(yb), \tag{6}$$

where $\mathrm{sinc}(xa) = [\sin(xa)]/xa$ etc.

The aperture area is $A = 4ab$ and Parseval's theorem for Fourier transforms gives[4] $K = (1/\lambda)\sqrt{E/A}$, where E is the total energy incident upon the aperture. Therefore

$$F(x, y) = \frac{\sqrt{EA}}{\lambda}\, \mathrm{sinc}(xa)\mathrm{sinc}(yb). \tag{7}$$

Now imagine that the aperture is gradually closed by letting $a \to 0$. In this limiting process the aperture approaches a slit lying on the interval $(-b, b)$. During the approach to the limit $\mathrm{sinc}(xa) \to 1$ and $A \to 0$ so that in the limit the expected result that $F \to 0$ occurs. Suppose, however, that $E \to \infty$ as

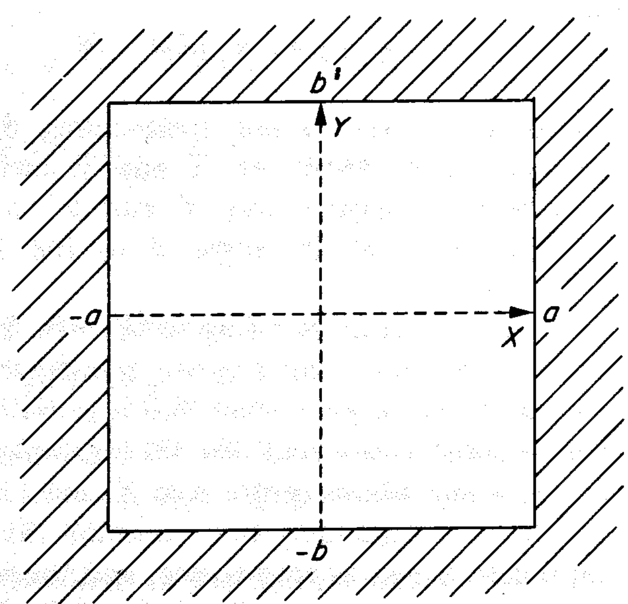

Figure 3. Rectangular aperture in an opaque screen

$A \rightarrow 0$ such that aE remains finite then F remains finite and retains the functional form sinc(yb). This is exactly what is obtained if $G(X, Y)$ is a delta function $\delta(X)$, representing a straight line parallel to the y-axis. This limiting process is the way in which a line in a letter object is specified, so for all the straight-line components of a complex letter object $G(X, Y)$ is defined as a Dirac delta function.

A hologram is a record of amplitude and phase variation over the (x, y) plane so it is not necessary to normalize $F(x, y)$ in anything but an arbitrary sense. Since this is the case equation (4) can be used, absolutely, by making $G(X, Y)$ exactly equal to a delta function such as[5] $\delta(X)$, for example. Thus for a line of length L drawn along the Y-axis and symmetrically placed at the origin

$$\begin{aligned} G(X, Y) = \delta(X), &\quad |Y| \leq L/2 \\ 0 &\quad |Y| > L/2, \end{aligned} \tag{8}$$

so that equations (8) and (4) give

$$F(x, y) = L\frac{\sin(yL/2)}{(yL/2)} = L \text{ sinc}\left(\frac{yL}{2}\right). \tag{9}$$

Of course, not all lines lie along a coordinate axis. For example, the letter M involves two lines set at some angles to the axes. Furthermore, lines need not be symmetrically disposed about the origin. Both of these features can be very easily incorporated by rotation or translation.

A line at some angle θ to the X-axis can be thought of as lying along a new axis denoted by X'. The rotated X' axis is then related to X and Y through the equation

$$X' = X \cos\theta + Y \sin\theta \tag{10}$$

In the rotated coordinate system a line lying along the X'-axis has a Fourier transform L sinc($x'L/2$). However, x and y, obviously transform from one coordinate system to another like X' and Y' so that the Fourier transform of a line orientated at an angle θ to the X-axis is simply L sinc($L/2[x \cos\theta + y \sin\theta]$).

If the midpoint of a line, instead of being at $X_0 = 0$, $Y_0 = 0$, is at some arbitrary point $X_0 \neq 0$, $Y_0 \neq 0$ then the Fourier transform of a line is the same as if it was centred on the origin except that it is multiplied by a factor $\exp i(xX_0 + yY_0)$. This is most easily seen for the example defined through equation (8). In this case a line whose centre is at X_0 must be represented by $\delta(X - X_0)$ so that it follows immediately that equation (9) contains a factor $\exp(ixX_0)$. The general case is just as easy to see, and the whole mathematical process is usually said to be an example of the shift theorem.[1]

An object such as the letter A, shown in Figure 4, has three straight lines, labelled 1, 2, and 3, with coordinates $(X_1, Y_1; X_2, Y_2)$ where the (X, Y)

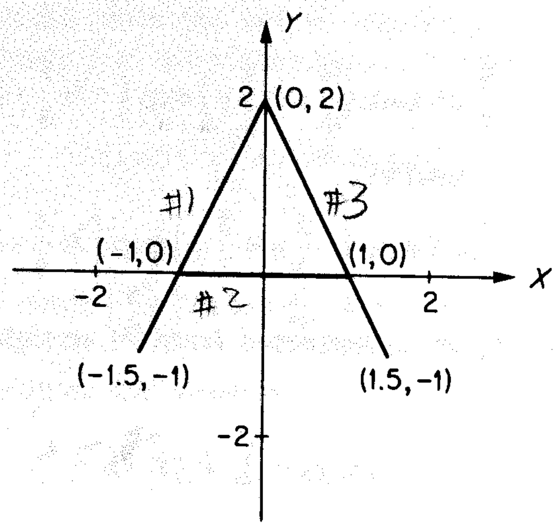

Figure 4. The letter A in the (X, Y) coordinate system

pairs define the ends of the line from left to right. In Figure 4 the lines have coordinates $(-1.5, -1; 0, 2)$, $(-1, 0; 1, 0)$ and $(0, 2; 1.5, -1)$. Already, this example shows that two sets of circumstances can arise in which $Y_2 > Y_1$ or $Y_2 < Y_1$ and these must be distinguished from each other.

General formulae for the length L of the line and its mid-point coordinates X_0, Y_0 are

$$L = [(X_2 - X_1)^2 + (Y_2 - Y_1)^2]^{\frac{1}{2}}, \tag{11}$$

and

$$X_0 = \frac{X_1 + X_2}{2} = |(X_1 - X_2)/2| + X_1, \tag{12}$$

$$Y_0 = \frac{Y_1 + Y_2}{2} = \begin{cases} |Y_1 - Y_2)/2| + Y_1, & Y_2 > Y_1 \\ |Y_1 - Y_2)/2| + Y_2, & Y_2 < Y_1. \end{cases} \tag{13}$$

The formulae for $\sin\theta$ and $\cos\theta$ are

$$\sin\theta = |(Y_2 - Y_1)|/L, \tag{14}$$

$$\cos\theta = \begin{cases} |(X_2 - X_1)|/L, & Y_2 > Y_1 \ (\theta \text{ acute}) \\ -|(X_2 - X_1)|/L, & Y_2 \leqslant Y_1 \ (\theta \text{ obtuse}). \end{cases} \tag{15}$$

The Fourier transform of the mathematical object A, in Figure 4, is

$$F(x, y) = F_1 + F_2 + F_3 = \sum_{i=1}^{3} F_i(x, y), \tag{16}$$

where

$$F_1 = \frac{3}{2}\sqrt{5}\,\mathrm{sinc}\left(\frac{3}{4}x + \frac{3}{2}y\right)\exp\left(-3i\frac{x}{4} + i\frac{y}{2}\right), \tag{17}$$

$$F_2 = 2\,\mathrm{sinc}(x), \tag{18}$$

$$F_3 = \frac{3}{2}\sqrt{5}\,\mathrm{sinc}\left(-\frac{3}{4}x + \frac{3}{2}y\right)\exp\left(3i\frac{x}{4} + i\frac{y}{2}\right), \tag{19}$$

and a letter, or an object, constructed from N straight lines has a Fourier transform

$$F(x, y) = \sum_{i=1}^{N} F_i(x, y). \tag{20}$$

4. SAMPLING THE FOURIER TRANSFORM

The computer-generated Fourier transform hologram, in contradistinction to an experimentally generated hologram, only samples the Fourier transform $F(x, y)$. It cannot be calculated, at an array of points in the (x, y) plane, with an indefinitely fine mesh because of lack of storage space and the undesirability of using a vast amount of computer time. A simple answer to this problem is to limit the number of points in the (x, y) plane at which $F(x, y)$ is calculated. This could be done by arbitrarily limiting the density of calculation points (spatial frequency limiting).

However, as might be expected, limiting the number of sampling points, if not done carefully, can introduce serious defects into the eventual reconstructed image. Fortunately a way out of this problem exists, because it is possible to approach the whole question of sampling in a systematic manner that leads to a prescription for obtaining satisfactory reconstructed images.

Sampling simply means calculating the function $F(x, y)$ at a limited number of points. After this is done a hologram is constructed by a computer graphical technique (discussed later on) and an optical method of interrogating the hologram is ultimately used to produce an image field that should be a reproduction of this original object. That, at least, is the ideal aim. Now the hologram being a representation of $F_s(x, y)$, the sampled Fourier transform, rather than $F(x, y)$ the true transform, does not behave like this. Some features are present that are very interesting. The most fundamental feature is that, on reconstruction from a sampled Fourier transform hologram, several images of the object are formed. These are called higher order spectra. Another property of these images is that they can be well separated or overlap.

The appearance of spectra can be understood from an analysis of the sampling process. Suppose a function $f(x)$ is sampled at intervals Δx along x

at the points $x_m = m\,\Delta x$ then the act of sampling $f(x)$ is mathematically equivalent to combining $f(x)$ with the curious function $\mathrm{comb}(x/\Delta x)$ in the manner[1,7]

$$f_s(x) = \frac{f(x)}{\Delta x}\,\mathrm{comb}(x/\Delta x), \tag{21}$$

where

$$\mathrm{comb}\left(\frac{x}{\Delta x}\right) = \Delta x \sum_{m=-\infty}^{\infty} \delta(x - m\,\Delta x). \tag{22}$$

Equation (21) is therefore the same as

$$f_s(x) = \sum_{m=-\infty}^{\infty} f(m\,\Delta x)\,\delta(x - m\,\Delta x). \tag{23}$$

A simple application of the convolution theorem gives the Fourier transform of $f_s(x)$ as

$$G_s(X) = \frac{1}{\Delta x} \sum_{m=-\infty}^{\infty} G\left(X - \frac{m}{\Delta x}\right), \tag{24}$$

an expression that shows, immediately, that a consequence of the sampling is the appearance of an infinite set of periodically shifted versions of the original function.

The hologram is two-dimensional, and represents the function $F_s(x, y)$. The same conclusions apply, however, that the reconstruction produces, in principle, an infinite set of periodically shifted versions of the object, centred now at points $(m\,\Delta x, n\,\Delta y)$. Normally $\Delta x \neq \Delta y$ and the sampling is performed at intervals Δx or Δy, along both x- and y-directions, according to whichever is the largest. Assuming that $\Delta x > \Delta y$, then the Fourier transform of the sampled function leads to the appearance of the object in square cells all over the (X, Y) plane.

This is illustrated in Figure 5, (a), (b), and (c), for the letter X, representing critical sampling, oversampling, and undersampling. These three categories refer to whether the object, i.e. the letter X, just fills a cell, falls well within the cell, overlaps adjacent cells. The critical sampling rate is Δx_c and over- and undersampling is performed at rates $\Delta x < \Delta x_c$ and $\Delta x > \Delta x_c$ respectively. Of the three pictures the oversampled case is the most aesthetic because the images are nicely separated, but obviously oversampling must not be taken too far. If Δx is smaller, as we will see later on, the brightness inhomogeneity across the object is not as great as in the Δx_c case. This, however, is difficult to detect; more important is the implication that a small $\Delta x < \Delta x_c$ indicates a fine sample so that, for a fixed (x, y) field, many more points have to be used or for a fixed number of cells the hologram is magnified which leads to poor image quality.