Chapter 9
Morphological Image Processing

Question

What is Mathematical Morphology ?

An (imprecise) Mathematical Answer

A mathematical tool for investigating geometric structure in **binary** and **grayscale** images.

Shape Processing and Analysis

Visual perception requires transformation of images so as to make explicit particular **shape information**.

Goal: Distinguish meaningful shape information from irrelevant one.

The vast majority of shape processing and analysis techniques are based on designing a **shape operator** which satisfies desirable properties.

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Morphological Image Processing

Morphological Operators

Erosions and **dilations** are the most elementary operators of mathematical morphology.

More complicated **morphological operators** can be designed by means of combining erosions and dilations.

Some History

George Matheron (1975) *Random Sets and Integral Geometry*, John Wiley.

Jean Serra (1982) *Image Analysis and Mathematical Morphology*, Academic Press.

Petros Maragos (1985) *A Unified Theory of Translations-Invariant Systems with Applications to Morphological Analysis and Coding of Images*, Doctoral Thesis, Georgia Tech.

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Morphological Image Processing

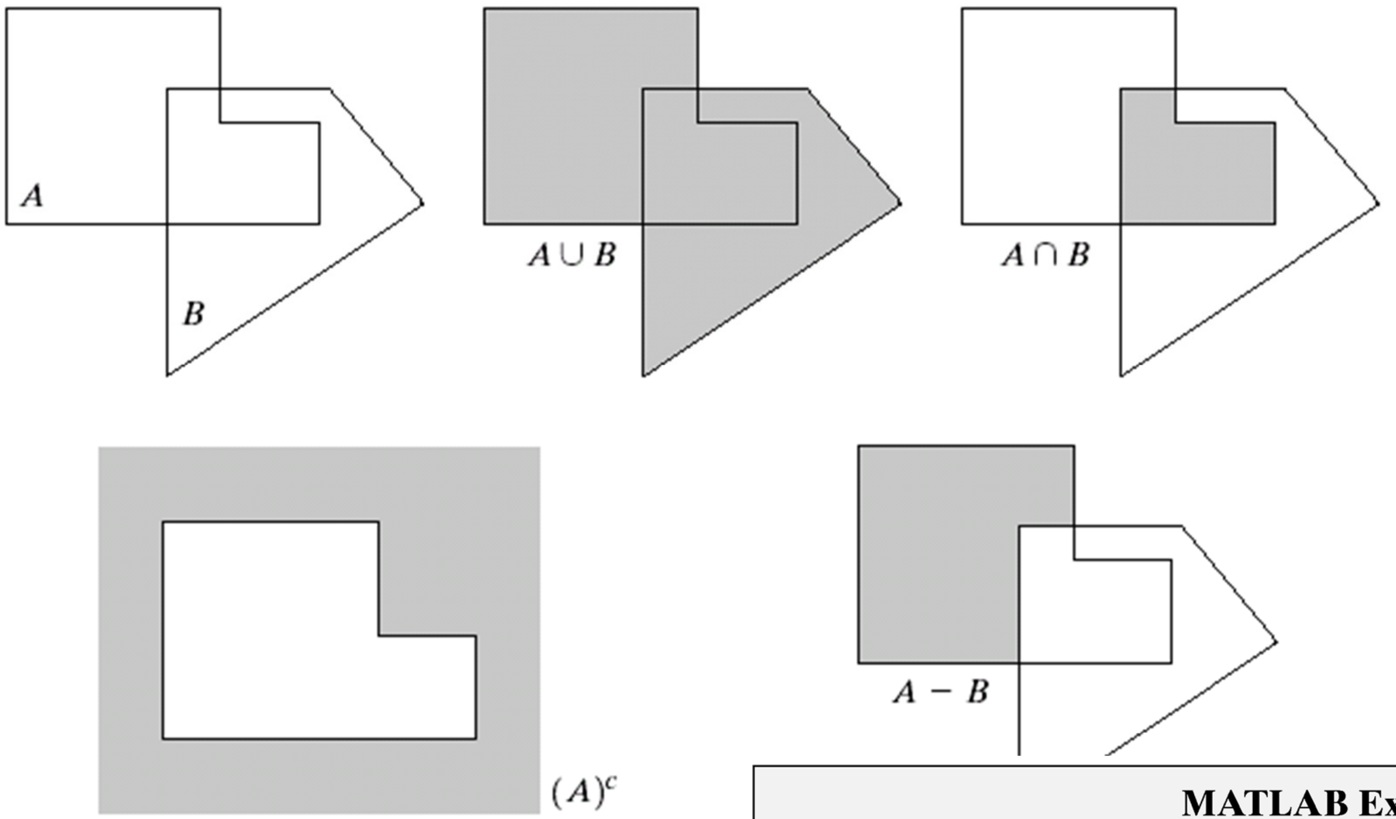

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Set Operation	MATLAB Expression for Binary Images	Name
$A \cap B$	A & B	AND
$A \cup B$	A | B	OR
A^c	~A	NOT
$A - B$	A & ~B	DIFFERENCE

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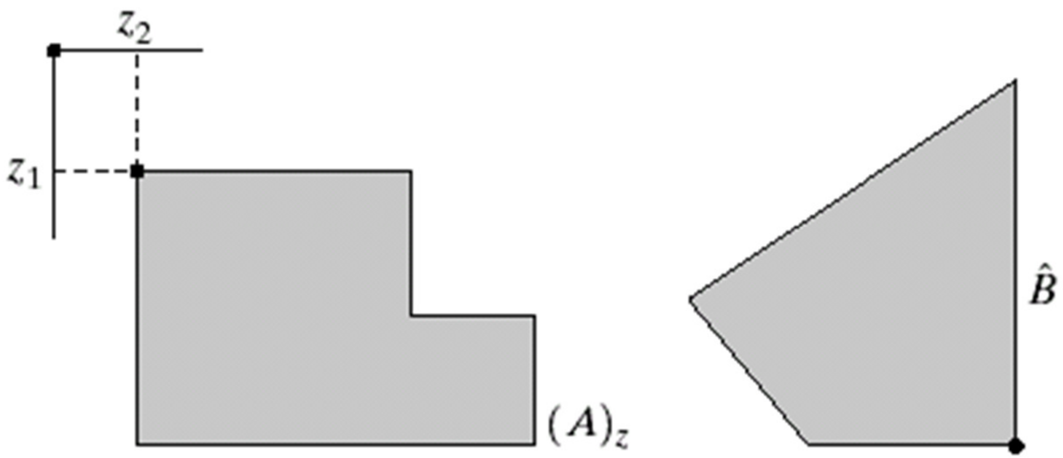

a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.

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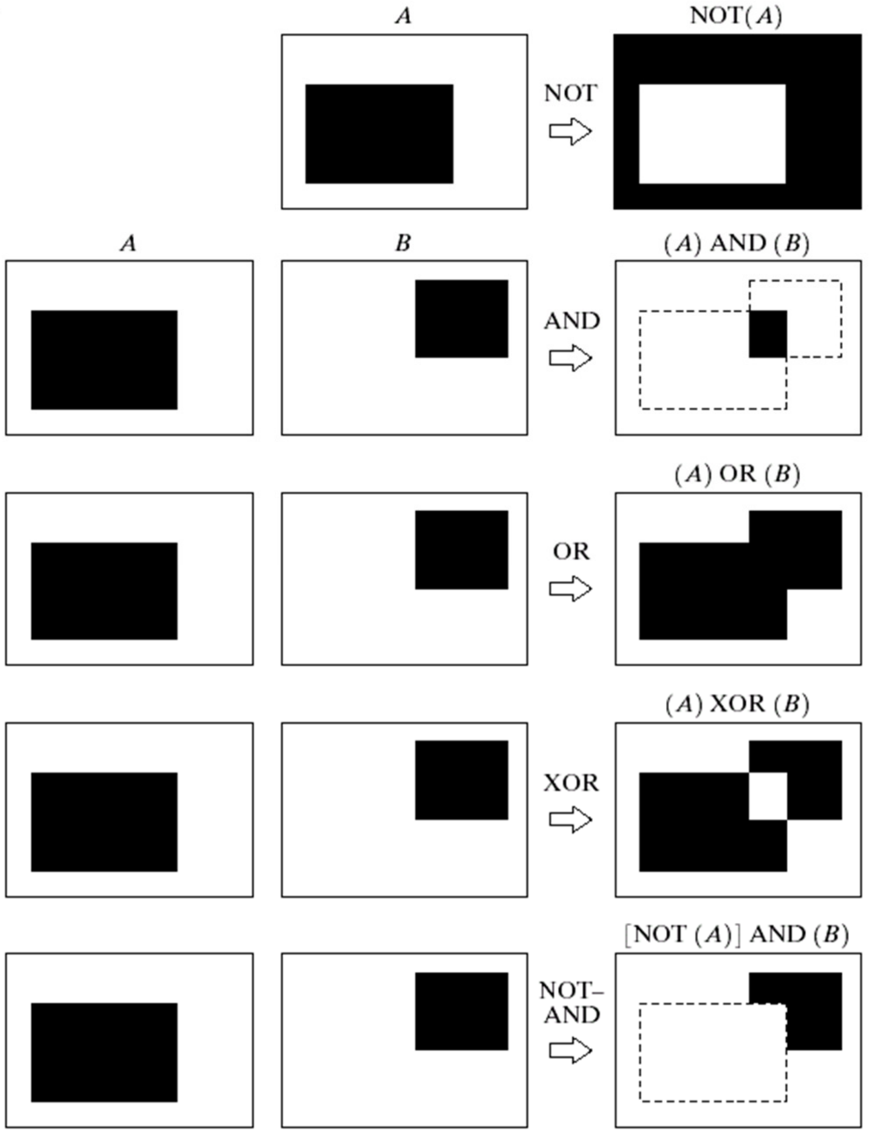


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Chapter 9
Morphological Image Processing: DILATION

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

"Expands" the boundary of A. (I)

Set of all points z such that B, flipped and translated by z, has a non-empty intersection with A

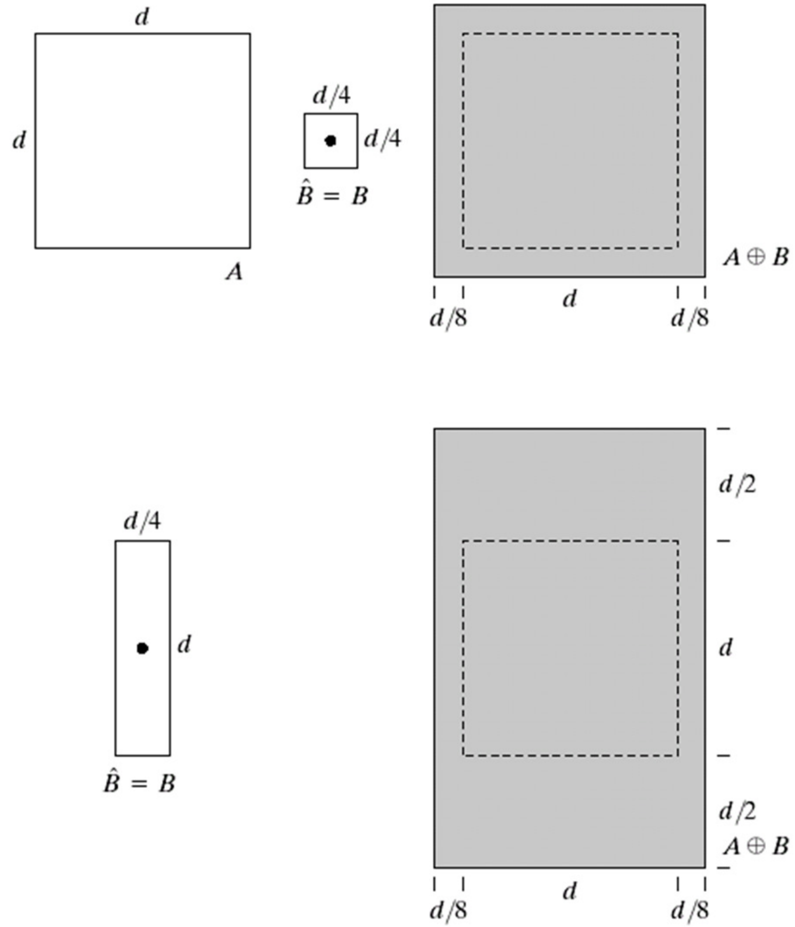


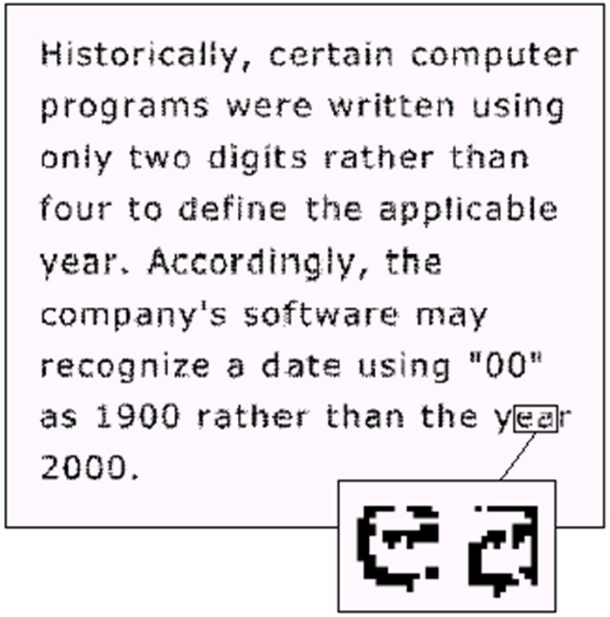
FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.

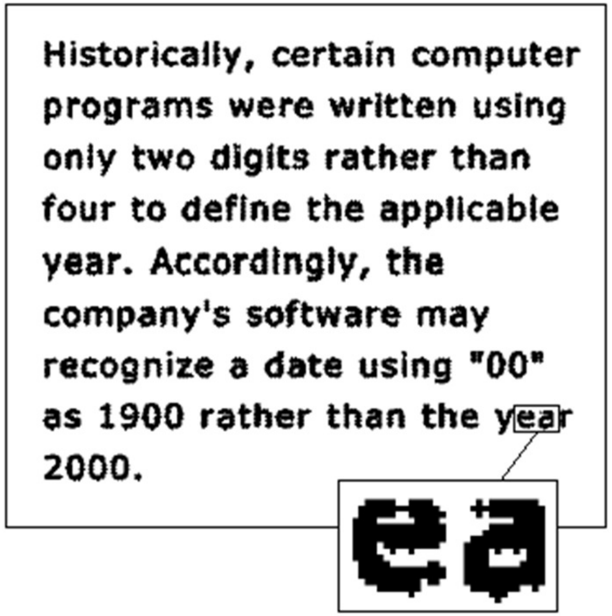
B = **structuring element**

NOTE:the flipping of the structuring element is included in analogy to convolution. Not all Authors perform it.

Chapter 9
Morphological Image Processing: DILATION

Example: bridging the gaps





0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

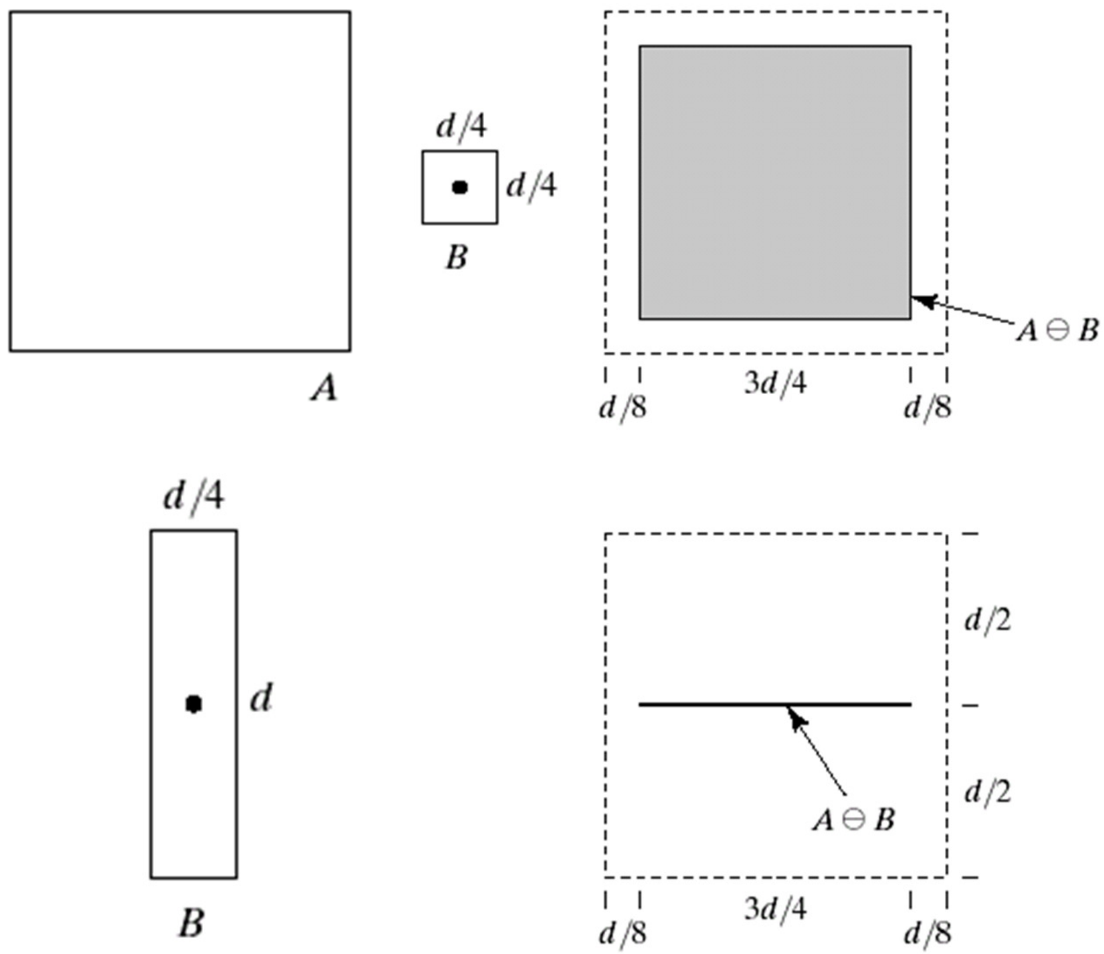
A possible alternative: linear lowpass filtering + thresholding

Chapter 9
Morphological Image Processing: EROSION

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

"Contracts" the boundary of A. (I)

Set of all points z such that B, translated by z, is included in A



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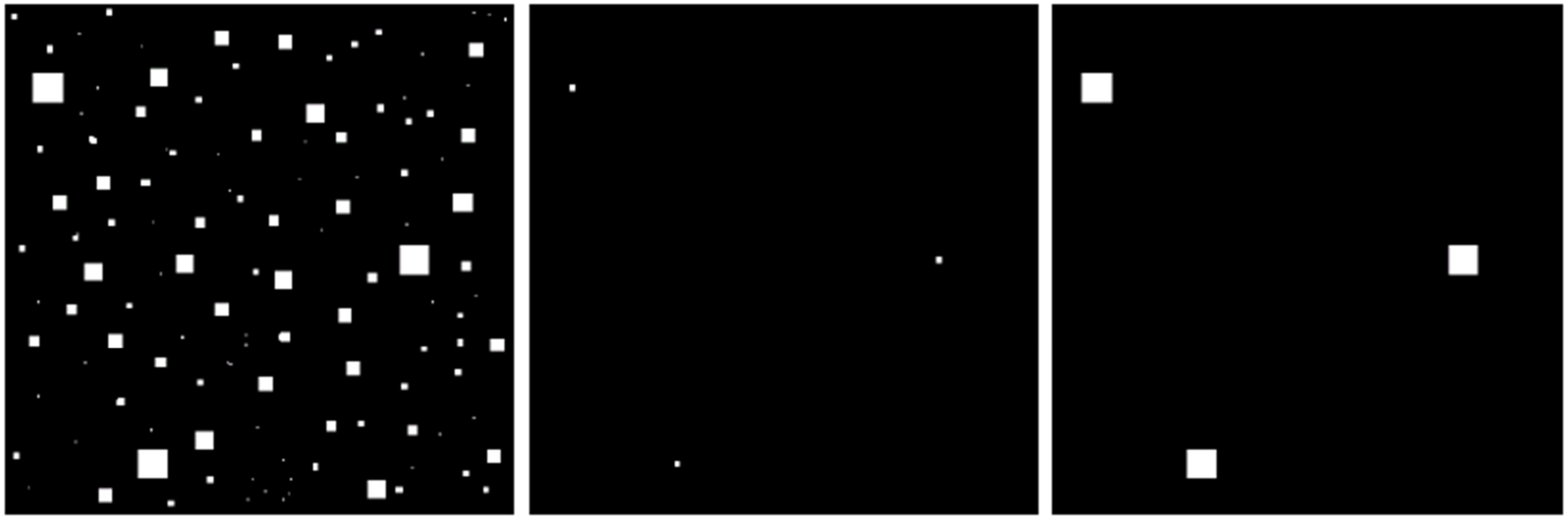
Morphological Image Processing: EROSION

Example: eliminating small objects

NOTE: white objects on black background (opposite wrt prev. slides)

NOTE: the final dilation will NOT yield in general the exact shape of the original objects



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

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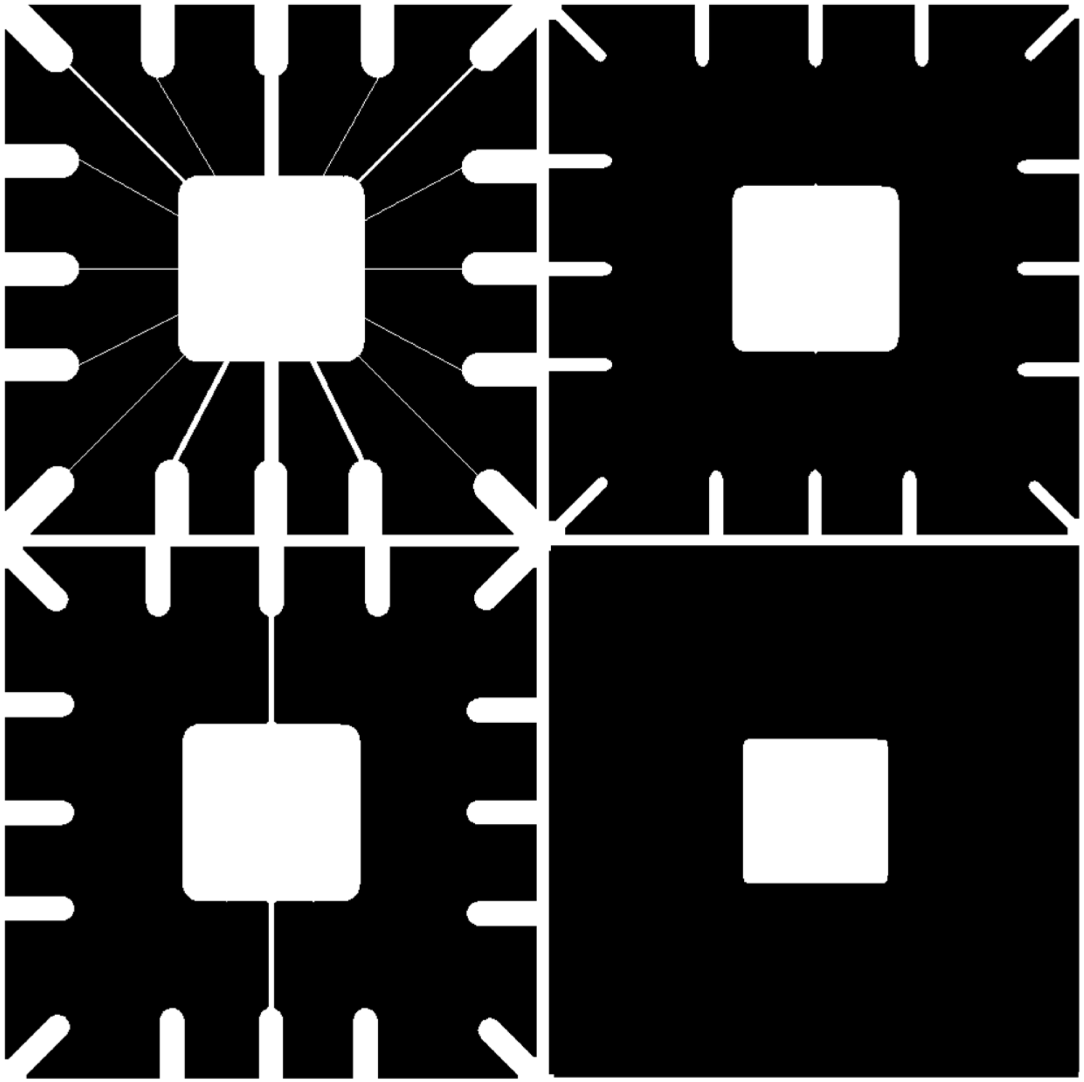
Morphological Image Processing: EROSION

Example:



a	b
c	d

FIGURE 9.8 An illustration of erosion. (a) Original image. (b) Erosion with a disk of radius 10. (c) Erosion with a disk of radius 5. (d) Erosion with a disk of radius 20.

Chapter 9
Morphological Processing: OPENING, CLOSING

Opening and closing

OPENING is erosion followed by dilation

CLOSING is dilation followed by erosion

Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

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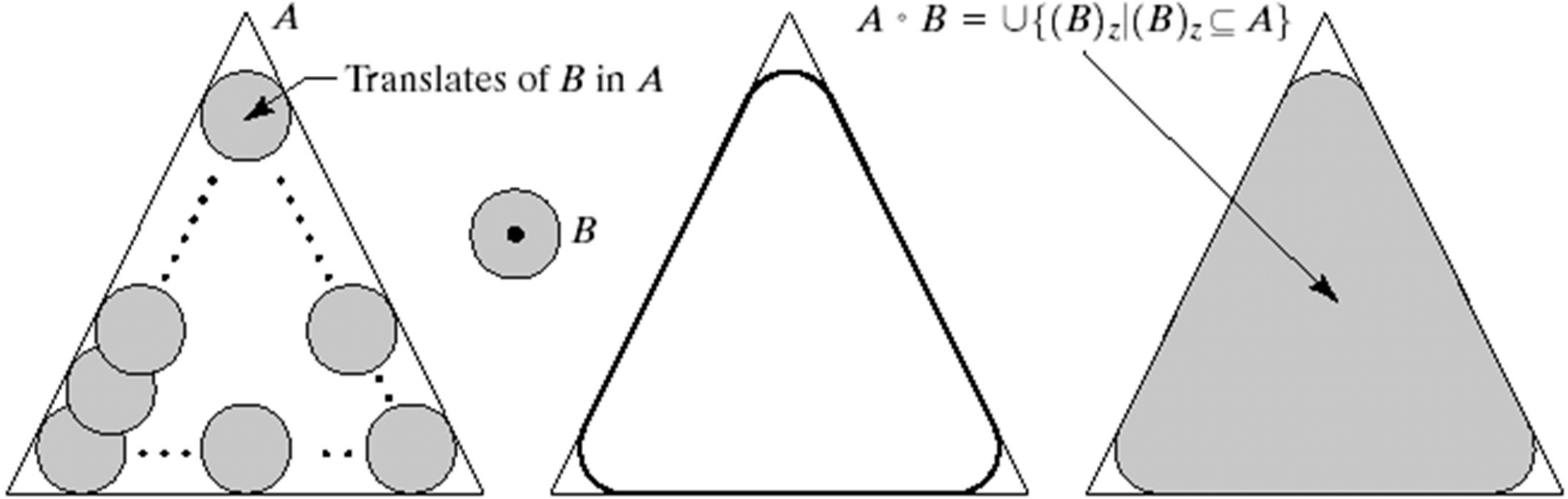
Morphological Image Processing: OPENING

A different formulation:



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

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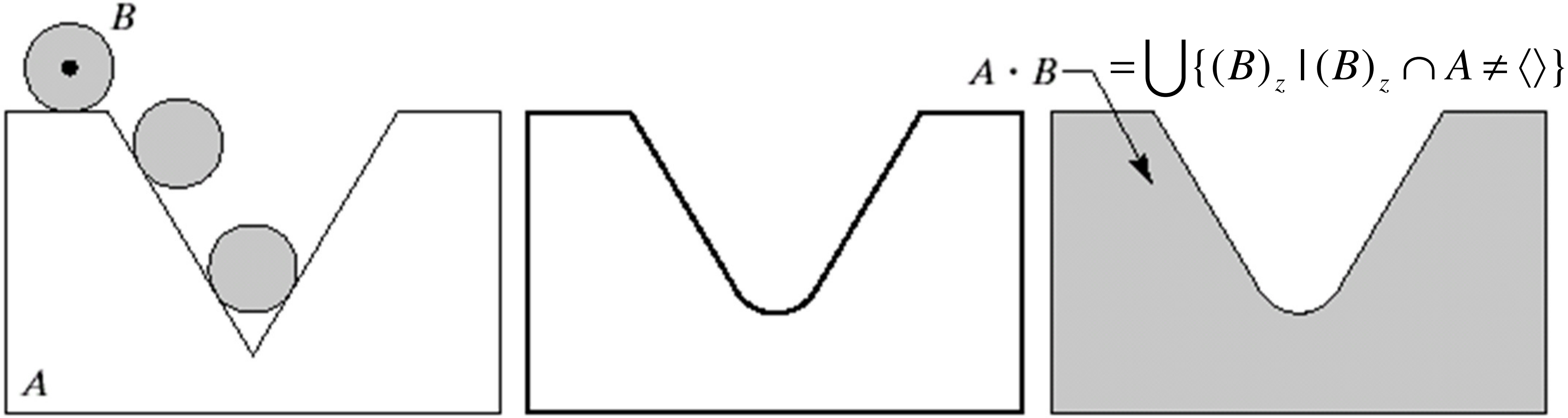
Morphological Image Processing: CLOSING

A different formulation:



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

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Morphological Image Processing

A property:

Erosion and Dilation
Opening and Closing

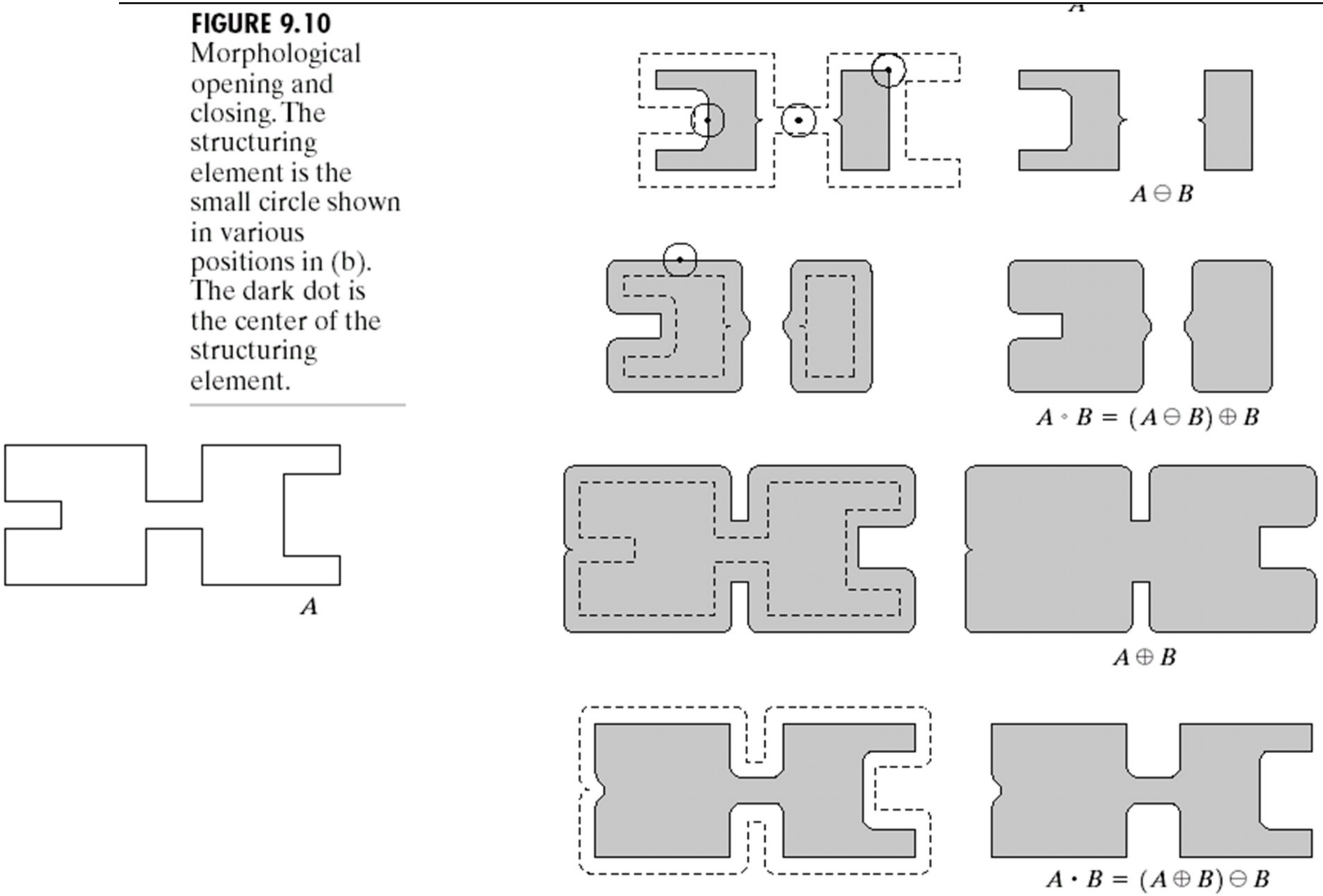
are **dual** operators wrt set complementation and reflection:

$$(A \ominus B)^C = A^C \oplus \hat{B}$$

$$(A \bullet B)^C = A^C \circ \hat{B}$$

Chapter 9
Morphological Image Processing: EXAMPLE

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



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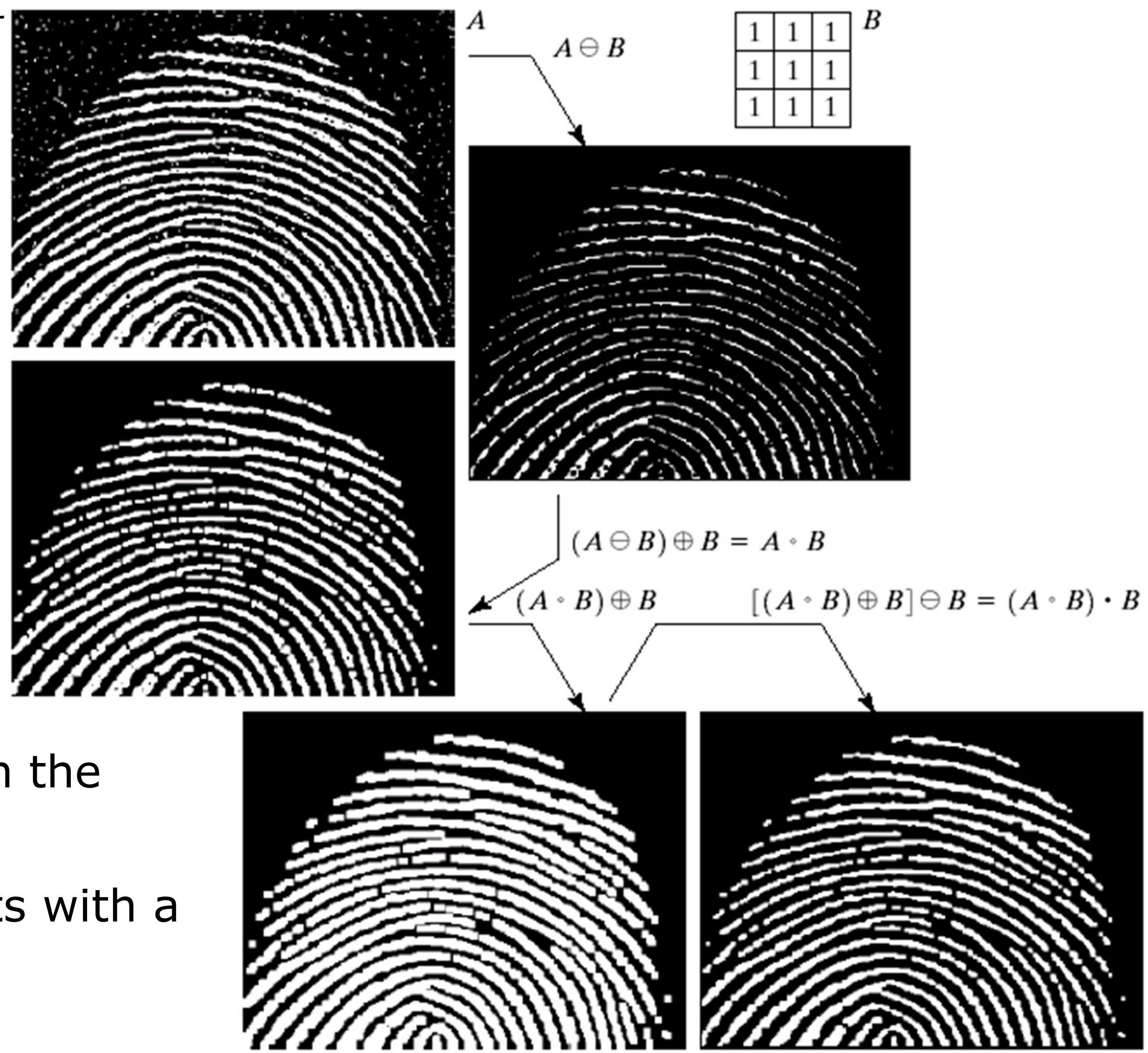
Morphological Image Processing: EXAMPLE



a b
c
d
e f

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Gaps exist in the output;

Better results with a smaller SE

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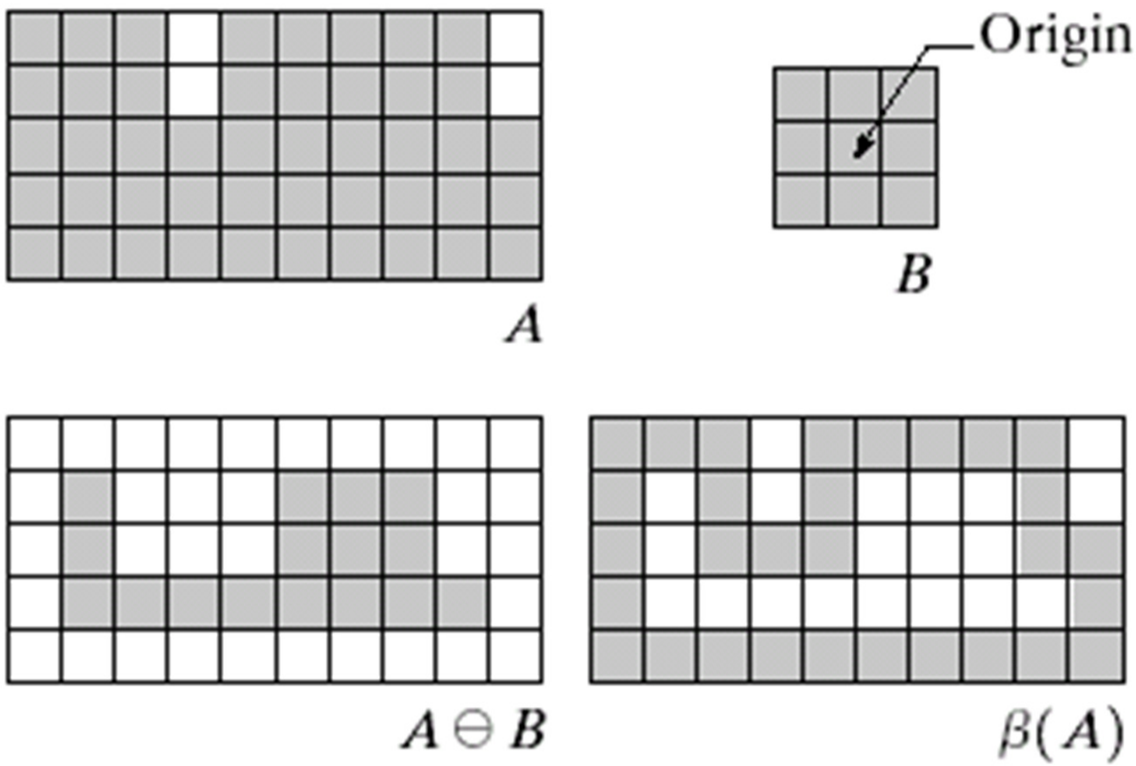
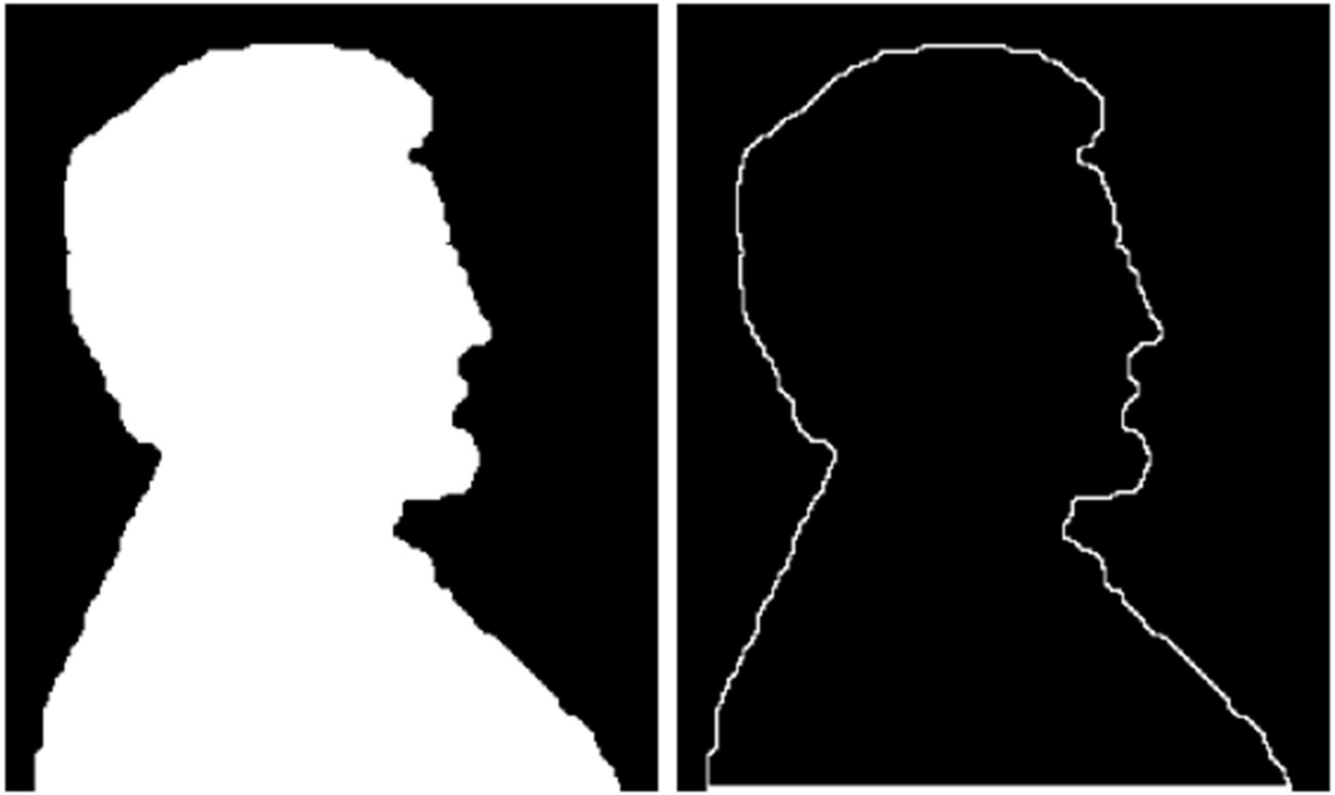
Boundary extraction $\quad \beta(A) = A - (A \ominus B)$



Boundary extraction: example



a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

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Region filling:

$$X_0 = P$$

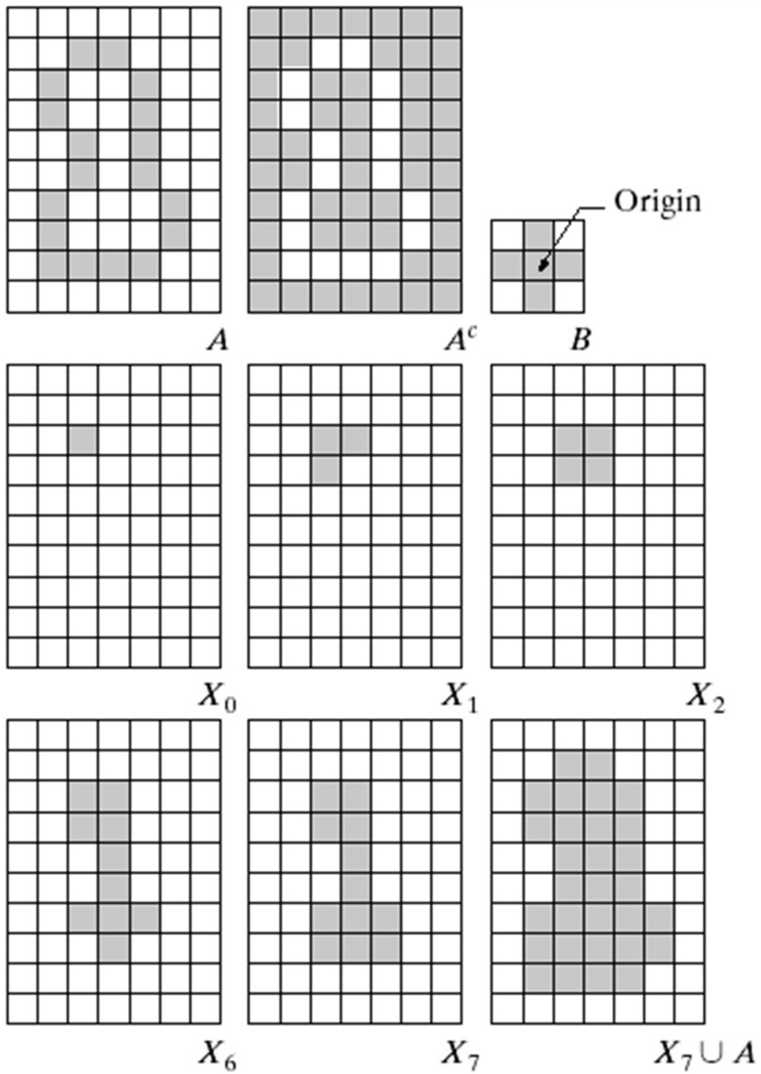
$$while\ X_k \neq X_{k-1}\ do$$

$$X_k = (X_{k-1} \oplus B) \cap A^C$$

$$X_F = X_k \cup A$$

The dilation would fill the whole area were it not for the intersection with Ac

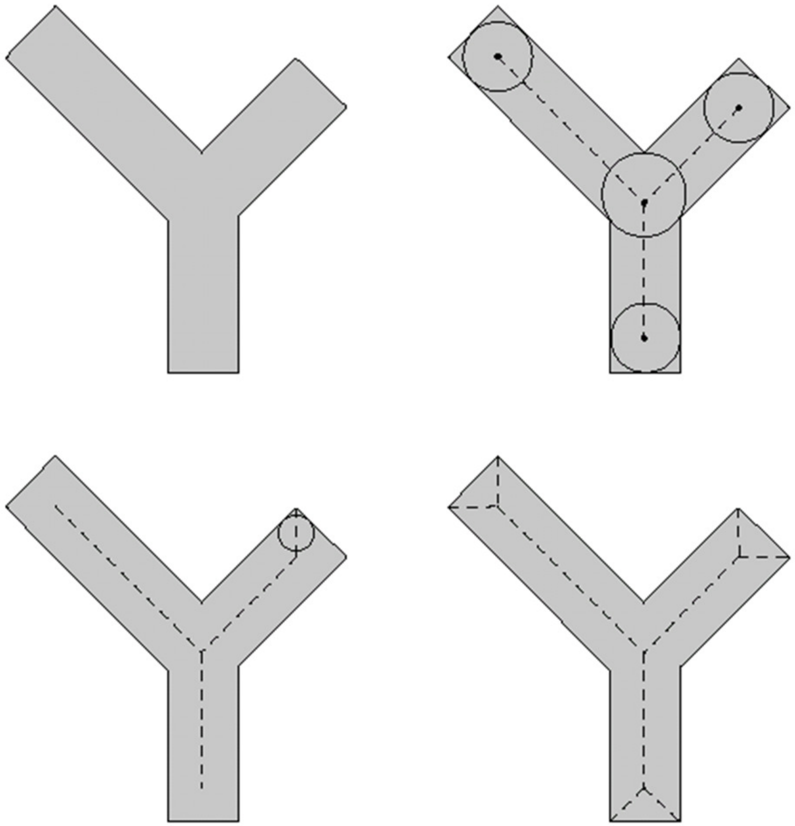
→**Conditional dilation**



Chapter 9
Morphological Image Processing: SKELETONS

Maximum disk: largest disk included in A, touching the boundary of A at two or more different places

(a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.



ng

Operation	Description
`bothat`	"Bottom-hat" operation using a 3 × 3 structuring element; use `imbothat` (see Section 9.6.2) for other structuring elements.
`bridge`	Connect pixels separated by single-pixel gaps.
`clean`	Remove isolated foreground pixels.
`close`	Closing using a 3 × 3 structuring element; use `imclose` for other structuring elements.
`diag`	Fill in around diagonally connected foreground pixels.
`dilate`	Dilation using a 3 × 3 structuring element; use `imdilate` for other structuring elements.
`erode`	Erosion using a 3 × 3 structuring element; use `imerode` for other structuring elements.
`fill`	Fill in single-pixel "holes" (background pixels surrounded by foreground pixels); use `imfill` (see Section 11.1.2) to fill in larger holes.
`hbreak`	Remove H-connected foreground pixels.
`majority`	Make pixel p a foreground pixel if at least five pixels in $N_8(p)$ (see Section 9.4) are foreground pixels; otherwise make p a background pixel.
`open`	Opening using a 3 × 3 structuring element; use function `imopen` for other structuring elements.
`remove`	Remove "interior" pixels (foreground pixels that have no background neighbors).
`shrink`	Shrink objects with no holes to points; shrink objects with holes to rings.
`skel`	Skeletonize an image.
`spur`	Remove spur pixels.
`thicken`	Thicken objects without joining disconnected 1s.
`thin`	Thin objects without holes to minimally connected strokes; thin objects with holes to rings.
`tophat`	"Top-hat" operation using a 3 × 3 structuring element; use `imtophat` (see Section 9.6.2) for other structuring elements.

Bwmorph

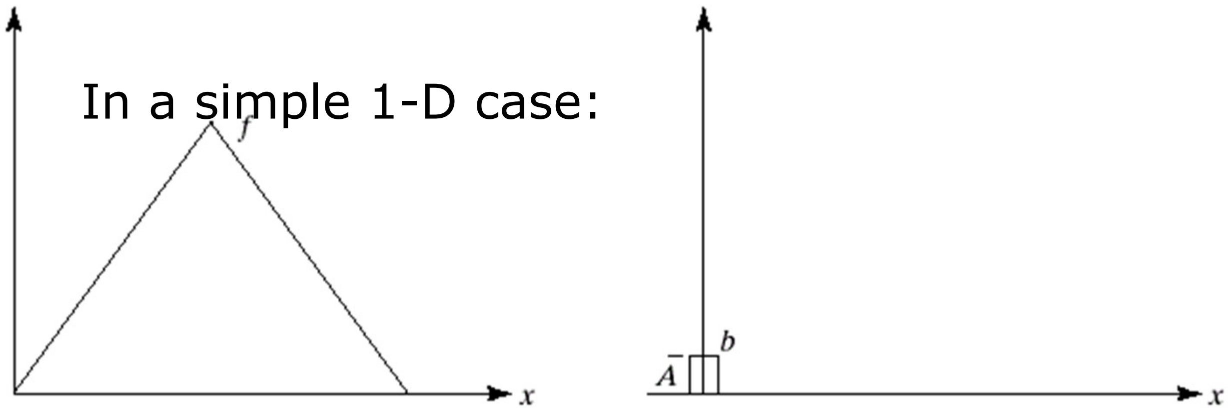
Matlab command: options

Chapter 9
Morphology: gray-level images

Dilation and **erosion** of an image *f(x,y)* by a structuring element *b(x,y).*

NOTE: b and f are no longer sets, but functions of the coordinates x,y.

In a simple 1-D case:



(a) A simple function. (b) Structuring element of height *A*. (

$$(f \oplus b)(s) = \max\{ f(s-x) + b(x) \mid (s-x) \in D_f \ \& \ x \in D_b \}$$

Like in convolution, we can rather have b(x) slide over f(x):

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Morphology: gray-level images

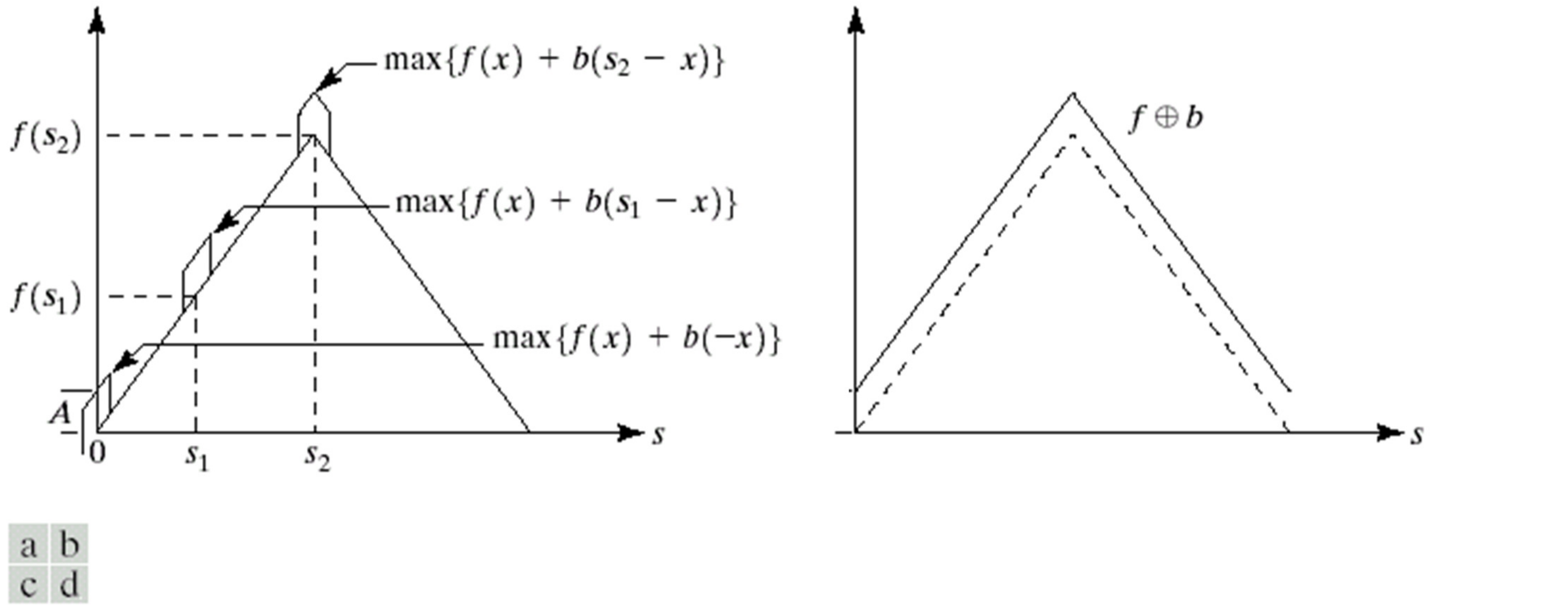

FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

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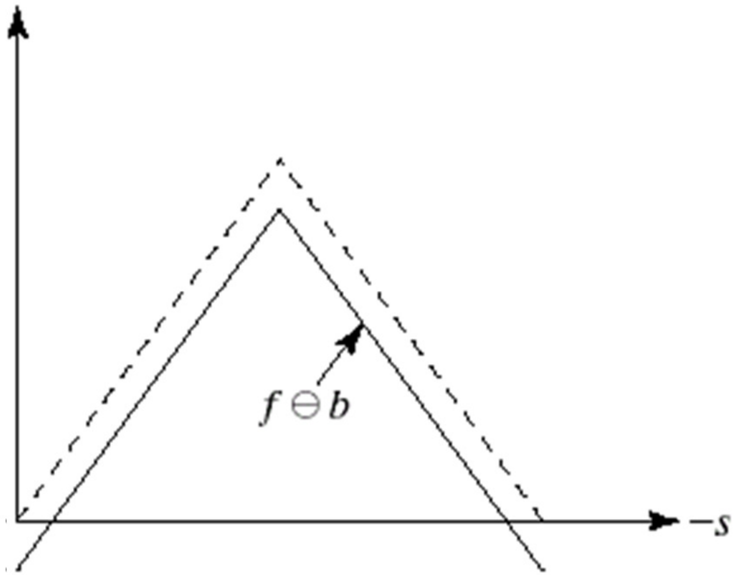
Morphology: gray-level images

Similarly for erosion:

$$(f \ominus b)(s) = \min\{f(s+x) - b(x) \mid (s+x) \in D_f \ \&\ x \in D_b\}$$

FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).



Note: *s*-*x* has become *s*+*x* in order to define a **duality** between dilation and erosion:

$$(f \ominus b)^C = f^C \oplus \hat{b}$$

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Morphology: gray-level images

In two dimensions:

$$(f \oplus b)(s,t) =$$
$$= \max\{f(s-x,t-y)+b(x,y) \mid (s-x),(t-y) \in D_f \;\&\; (x,y) \in D_b\}$$

$$(f \ominus b)(s,t) =$$
$$= \min\{f(s+x,t+y)-b(x,y) \mid (s+x),(t+y) \in D_f \;\&\; (x,y) \in D_b\}$$

Effects of **erosion** (when the structuring element has all positive entries):

•The output image tends to be darker than the input one

•Bright details in the input image having area smaller than the s.e. are lessened

The opposite for **dilation**.

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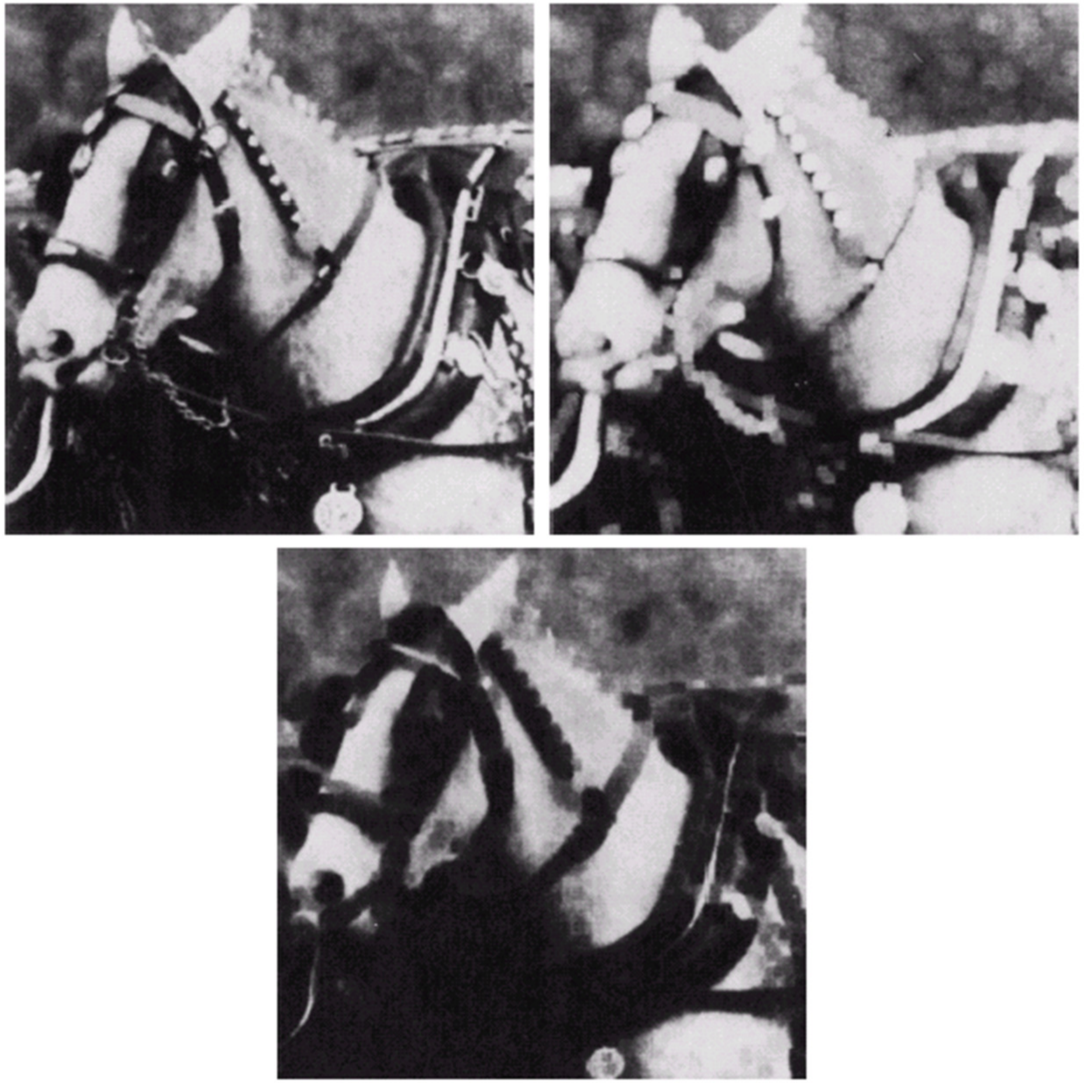
Morphology: gray-level images



a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Structuring element: "flat-top", a parallelepiped with unit height and size 5x5 pixels

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Morphology: gray-level images

Opening and **closing** of an image *f(x,y)* by a structuring element *b(x,y)* have the same form as their binary counterpart:

$$f \circ b = (f \ominus b) \oplus b \qquad\qquad f \bullet b = (f \oplus b) \ominus b$$

Geometric interpretation:

View the image as a 3-D surface map, and suppose we have a spherical s.e.

Opening: roll the sphere against the *underside* of the surface, and take the *highest* points reached by any part of the sphere

Closing: roll the sphere *on top* of the surface, and take the *lowest* points reached by any part of the sphere

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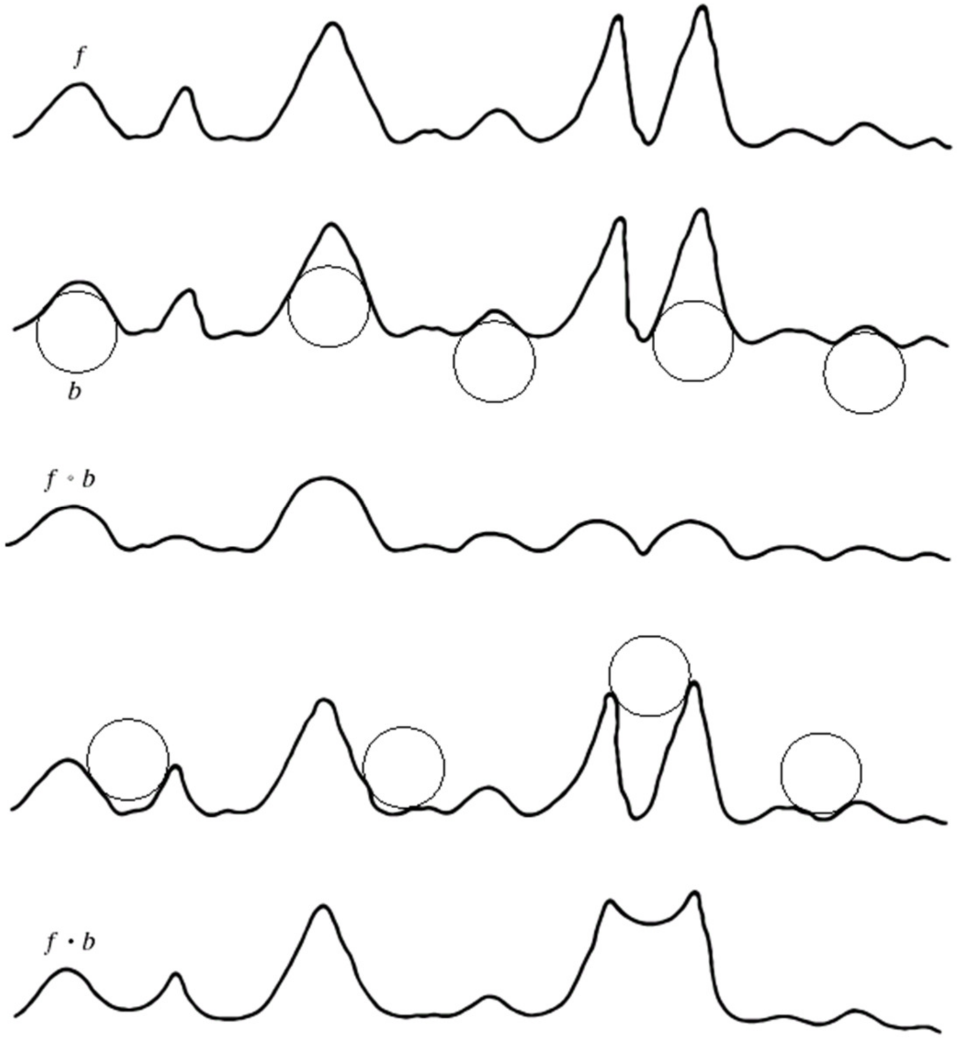

FIGURE 9.30
(a) A gray-scale scan line.
(b) Positions of rolling ball for opening.
(c) Result of opening.
(d) Positions of rolling ball for closing. (e) Result of closing.

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FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a).

Same s.e. as in Fig.9.29.

Note the decreased size of the small bright (opening) or dark (closing) details;

with no appreciable effect on the darker (opening) or brighter (closing) details

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Morphology: gray-level images

Morphological smoothing: opening followed by closing
(what about doing viceversa?)   (Same s.e. as in Fig.9.29)

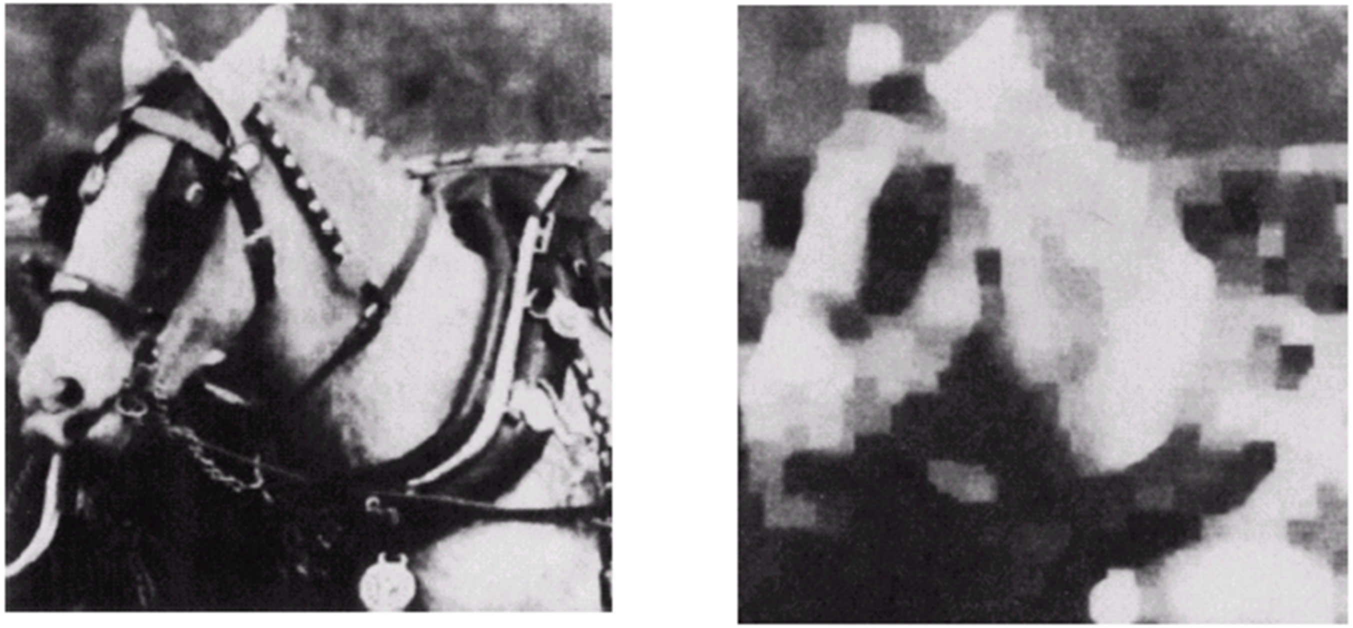

FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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Morphology: gray-level images

Morphological gradient: difference between dilation and erosion (Same s.e. as in Fig.9.29)

$$g = (f \oplus b) - (f \ominus b)$$

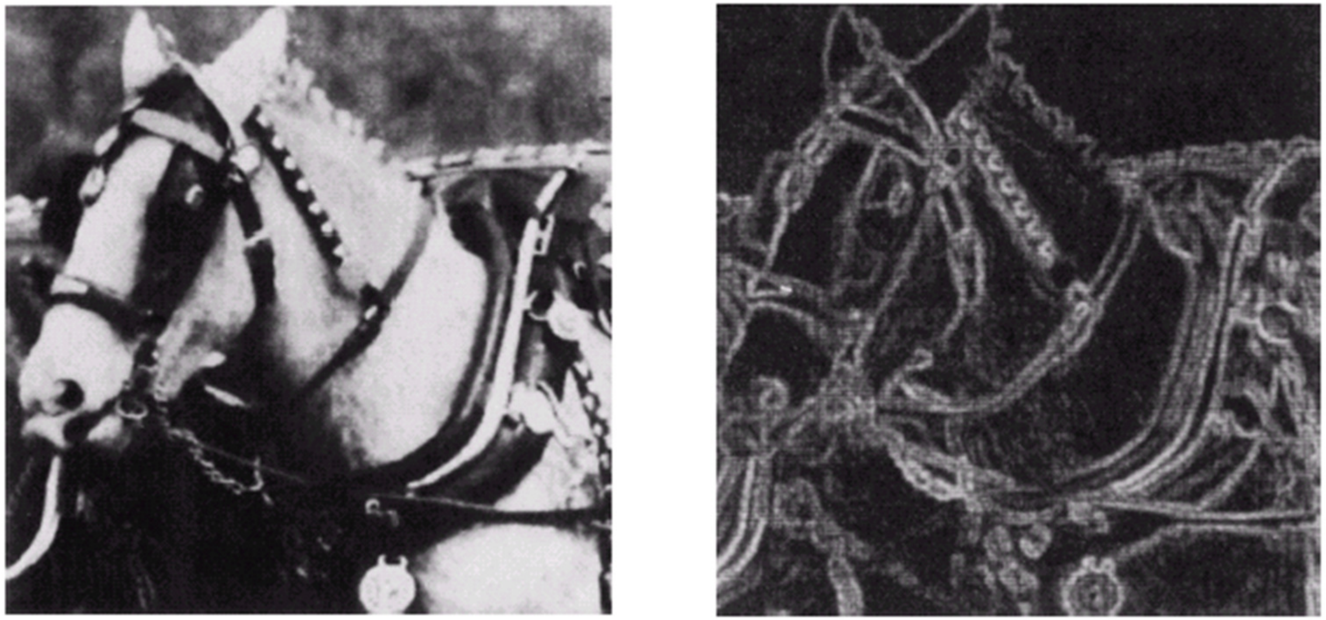

FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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Morphology: gray-level images

Top-hat transformation: difference between original and opening (what about original and closing?) (Same s.e. as in Fig.9.29)

$$g = f - (f \circ b)$$

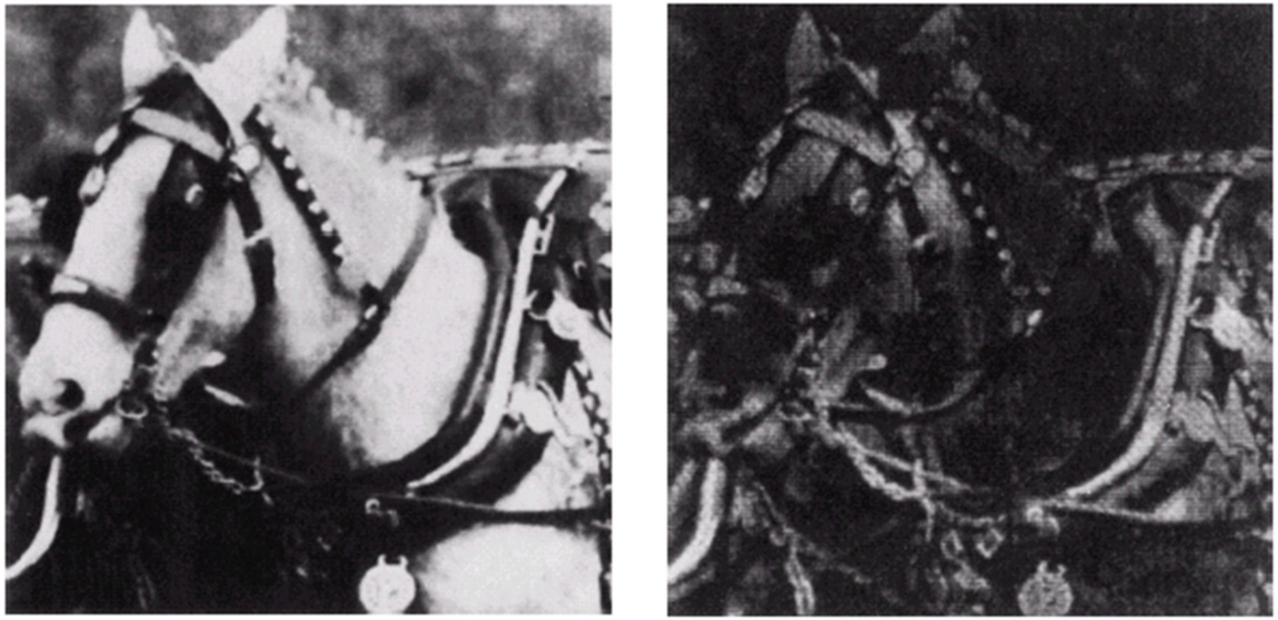

FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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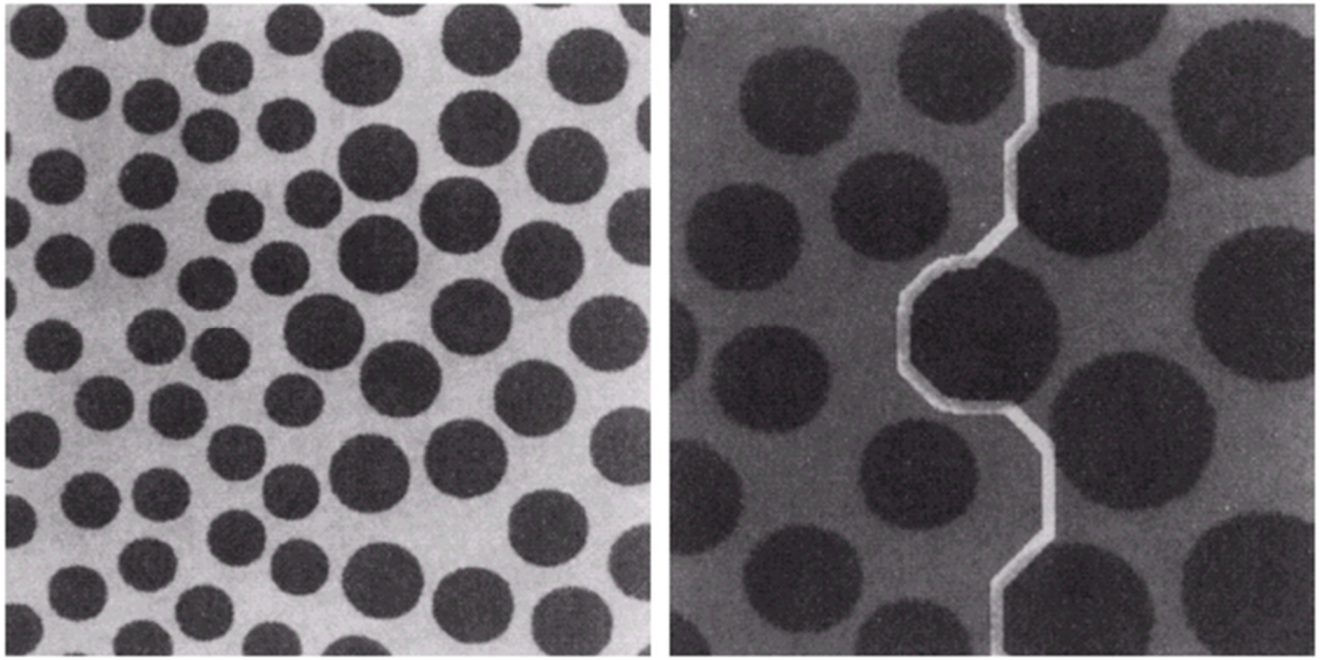
Texture segmentation: (for *this* specific problem)

1. Closing with a larger and larger s.e. until the small particles disappear
2. Opening with a s.e. larger than the gaps between large particles
3. Gradient → separation contour

a b

FIGURE 9.35 (a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)



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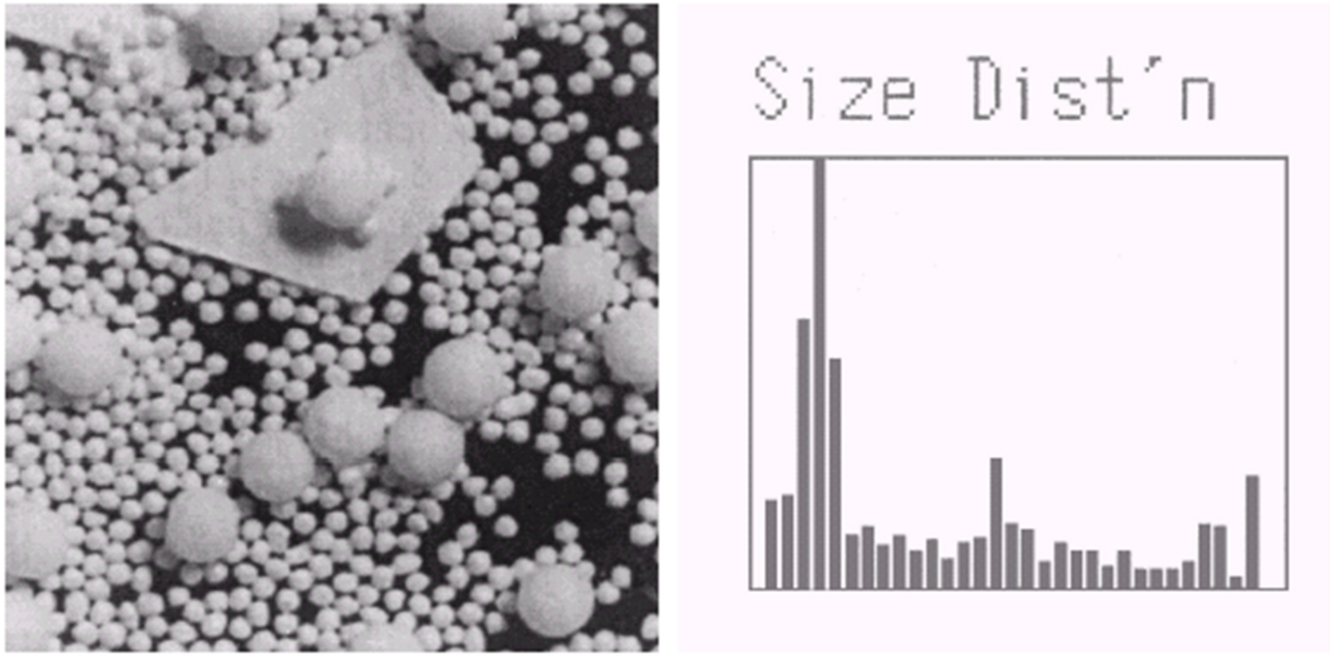
Granulometry: (for *this* specific problem)

1. Opening with a small s.e. and difference wrt original image (i.e., top-hat transform)
2. Repeat with larger and larger s.e.
3. Build histogram



a b

FIGURE 9.36 (a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)