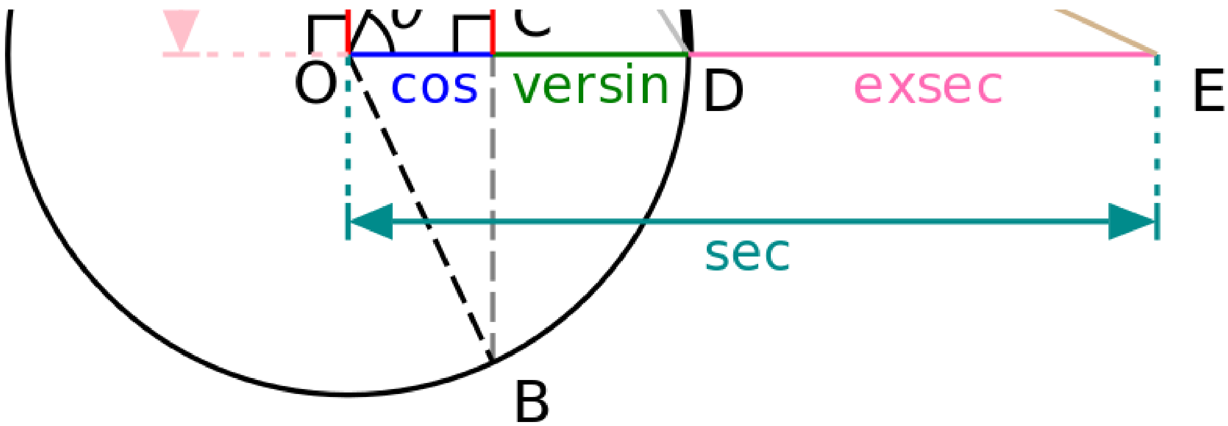
Roots of Unity

10 Secret Trig Functions Your Math Teachers Never Taught You

Haversine? Exsecant? An introduction to obsolete trig functions and why we don't use them anymore

By Evelyn Lamb on September 12, 2013

On Monday, the Onion reported that the "Nation's math teachers introduce 27 new trig functions." It's a funny read. The gamsin, negtan, and cosvnx from the Onion article are fictional, but the piece has a kernel of truth: there are 10 secret trig functions you've never heard of, and they have delightful names like "haversine" and "exsecant."



A diagram with a unit circle and more trig functions than you can shake a stick at. (It's well known that you can shake a stick at a maximum of 8 trig functions.) The familiar sine, cosine, and tangent are in red, blue, and, well, tan, respectively. The versine is in green next to the cosine, and the exsecant is in pink to the right of the versine. Excosecant and coversine are also in the image. Not pictured: vercosine, covercosine, and haver-anything. Image: Limaner and Steven G. Johnson, via Wikimedia Commons.

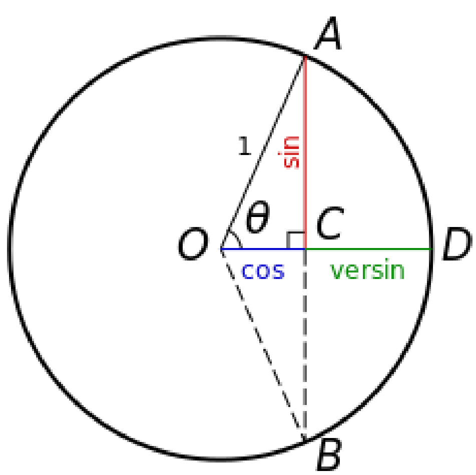
Whether you want to torture students with them or drop them into conversation to make yourself sound erudite and/or insufferable, here are the definitions of all the "lost trig functions" ~~I found in my exhaustive research of original historical texts~~ Wikipedia told me about.

Versine: versin(θ)=1-cos(θ)
Vercosine: vercosin(θ)=1+cos(θ)
Coversine: coversin(θ)=1-sin(θ)
Covercosine: covercosine(θ)=1+sin(θ)
Haversine: haversin(θ)=versin(θ)/2
Havercosine: havercosin(θ)=vercosin(θ)/2

Numberphile recently posted a video about Log Tables, which explains how people used logarithms to multiply big numbers in the dark pre-calculator days. First, a refresher on logarithms. The equation $\log_b x=y$ means that $b^y=x$. For example, $10^2=100$ so $\log_{10}100=2$. One handy fact about logarithms is that $\log_b(c\times d)=\log_b c+\log_b d$. In other words, logarithms make multiplication into addition. If you wanted to multiply two numbers together using a log table, you would look up the logarithm of both numbers and then add the logarithms together. Then you'd use your log table to find out which number had that logarithm, and that was your answer. It sounds cumbersome now, but doing multiplication by hand requires a lot more operations than addition does. When each operation takes a nontrivial amount of time (and is prone to a nontrivial amount of error), a procedure that lets you convert multiplication into addition is a real time-saver, and it can help increase accuracy.

The secret trig functions, like logarithms, made computations easier. Versine and haversine were used the most often. Near the angle $\theta=0$, $\cos(\theta)$ is very close to 1. If you were doing a computation that had $1-\cos(\theta)$ in it, your computation might be ruined if your cosine table didn't have enough significant figures. To illustrate, the cosine of 5 degrees is 0.996194698, and the cosine of 1 degree is 0.999847695. The difference $\cos(1°)-\cos(5°)$ is 0.003652997. If you had three significant figures in your cosine table, you would only get 1 significant figure of precision in your answer, due to the leading zeroes in the difference. And a table with only three significant figures of precision would not be able to distinguish between 0 degree and 1 degree angles. In

$\cos(\theta)=2\sin^2(\theta/2)$. So the haversine is just $\sin^2(\theta/2)$. Likewise, the havercosine is $\cos^2(\theta/2)$. If you have a computation involving the square of sine or cosine, you can use a haversine or havercosine table and not have to square or take square roots.



A diagram showing the sine, cosine, and versine of an angle. Image: Qef and Steven G. Johnson, via Wikimedia Commons.

The versine is a fairly obvious trig function to define and seems to have been used as far back as 400 CE in India. But the haversine may have been more important in more recent history, when it was used in navigation. The haversine formula is a very accurate way of computing distances between two points on the surface of a sphere using the latitude and longitude of the two points. The haversine formula is a re-formulation of the spherical law of cosines, but the formulation in terms of haversines

All of them could make computations more accurate near certain angles, but I don't know which ones were commonly used and which ones were named* analogously to other functions but rarely actually used. I'm curious about this, if anyone knows more about the subject.

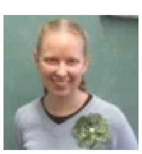
When the Onion imitates real life, it's usually tragic. But in the case of secret trig functions, the kernel of truth in the Onion didn't make me sad. We're very lucky now that we can multiply, square, and take square roots so easily, and our calculators can store precise information about the sines, cosines, and tangents of angles, but before we could do that, we figured out a work-around in the form of a ridiculous number of trig functions. It's easy forget that the people who defined them were not sadistic math teachers who want people to memorize weird functions for no reason. These functions actually made computations quicker and less error-prone. Now that computers are so powerful, the haversine has gone the way of the floppy disc. But I think we can all agree that it should come back, if only for the "awesome" joke I came up with as I was falling asleep last night: Haversine? I don't even know 'er!

*I'd like to take a little digression to the world of mathematical prefixes here, but it might not be for everyone. You've been warned.

that definition meant the complementary angle, then vercosine would be the same as coversine, which it isn't. Instead, the vercosine is the versine of the supplementary angle (supplementary angles add up to 180 degrees), not the complementary one. In addition to the definitions as $1-\cos(\theta)$ and $1+\cos(\theta)$, the versine and vercosine can be defined as $\text{versin}(\theta)=2\sin^2(\theta/2)$ and $\text{vercos}(\theta)=2\cos^2(\theta/2)$. In the case of versine, I believe the definition involving $\cos(\theta)$ is older than the definition involving sine squared. My guess is that vercosine was a later term, an analogy of the square of sine definition of versine using cosine instead. If you're a trigonometry history buff and you have more information, please let me know! In any case, the table of super-secret bonus trig functions is a fun exercise in figuring out what prefixes mean.

The views expressed are those of the author(s) and are not necessarily those of Scientific American.

ABOUT THE AUTHOR(S)



Evelyn Lamb

Evelyn Lamb is a freelance math and science writer based in Salt Lake City, Utah.

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