# Negative refraction

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*Light bends the wrong way in materials where both ε and μ are negative as was pointed out in 1968, but the absence of natural materials with this property led to neglect of the subject until 1999 when it was shown how to make artificial materials, metamaterials, with negative μ. The rapid advance of the subject since that date, both in theory and experiment, is reflected in the exponential growth of publications now at the 200 per year level and still growing. This interest is explained by the sudden availability of a qualitatively different class of electromagnetic materials combined with the quite startling properties which these materials appear to have; all of which provokes debate as each new facet of their behaviour is revealed. Experiment has been vital to resolution of controversy and has chiefly been in the microwave region of the spectrum though there is potential in the optical region currently being explored by several groups.*

## 1. Bending light the wrong way

Some years ago Veselago [1] made a theoretical study of materials in which the sign of the electrical permittivity, $\varepsilon$, and magnetic permeability, $\mu$, were simultaneously negative. He concluded that, although these materials should support electromagnetic waves having a well defined wave vector,

$$| \mathbf{k} |^2 = \varepsilon\mu\omega^2,$$

there was something decidedly peculiar about these waves: theory predicted that the energy flow as dictated by the Poynting vector would be in the opposite direction to the wave vector, the implication being that rays travel in the opposite direction to waves. See figure 1.

The reversal of energy flow can easily be understood: flipping the sign of both $\varepsilon$ and $\mu$ is equivalent in Maxwell's equations to flipping the sign of the magnetic field but keeping the same wave vector. Solutions are exactly the same as solutions for a conventional positive system except for this inversion. Since the Poynting vector is given by $\mathbf{E} \times \mathbf{H}$, Veselago's result follows immediately. For this reason Veselago referred to his new materials as being 'left handed'. Veselago also showed that at an interface between doubly positive and doubly negative materials light would

be bent the wrong way relative to the normal. Figure 2 illustrates refraction at such an interface.

If the group velocity, $v_g$, is negative it follows that,

$$v_g = d\omega/dk < 0$$

Dispersion is an unavoidable aspect of negative refraction [2] and failing to take account of it can quickly lead to nonsensical results. These novel concepts generated considerable interest at the time but absence of any material realisation led to their eventual neglect. However in recent times it has been proposed that novel electromagnetic
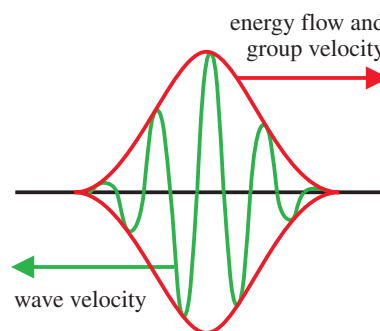
energy flow and
group velocity

wave velocity

**Figure 1. Materials with negative refraction are sometimes called *left handed material*s because the Poynting vector has the opposite sign to the wave vector.**

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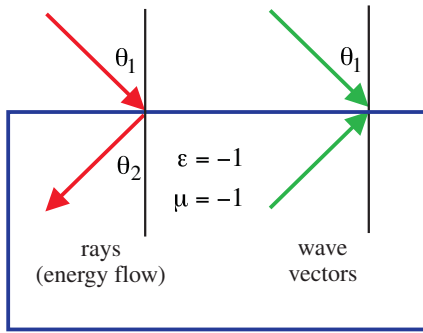E-mail: j.pendry@imperial.ac.uk

**Figure 2. In a double negative material as described by Veselago,** *light makes a negative angle with the normal.* **Note that the parallel component of the wave vector is always preserved in transmission, but that energy flow is opposite to the wave vector. This picture has been shown to be consistent with a negative refractive index.**

properties can be realised by micro-structuring a material on a scale much less than the wavelength [3–5]. These advances, described in section 2, enabled the field to move into an experimental phase. A short review of progress can be found in [6]. It was Smith *et al.* [7] who first constructed and demonstrated a material in which the sign of the electrical permittivity, $\varepsilon$, and magnetic permeability, $\mu$, were simultaneously negative and the same team verified the predicted negative refraction angle in a subsequent experiment [8]. Their work led to an explosion of interest in the field.

Refraction at an interface is commonly described by $n$ the *refractive index* and we can show that causality forces us to choose a negative sign for the refractive index when both $\varepsilon$ and $\mu$ are negative. Conventionally we take $n$ as given by $\sqrt{\varepsilon\mu}$ and, where both $\varepsilon$ and $\mu$ are positive, choose the positive sign for the square root. However causality requires that both $\varepsilon$ and $\mu$ have small positive imaginary parts representing the fact that real systems are always slightly lossy. Imagine that as we vary the frequency the real part of $\varepsilon$ passes through zero, where there is a branch point in $\sqrt{\varepsilon}$. By virtue of the small positive imaginary part of $\varepsilon$ the causal solution forces us to take a trajectory above the branch point giving a positive imaginary value to $\sqrt{\varepsilon}$ when $\varepsilon < 0$. A similar argument holds for $\mu$ and therefore,

$$n = -\sqrt{\varepsilon\mu}, \quad \varepsilon_r < 0, \quad \mu_r < 0$$

All the properties of these double negative materials are consistent with a negative refractive index, and it is now common to use the term 'negatively refracting' to describe these double negative materials.

In view of the role of causality in guiding us to physically correct solutions, we shall henceforth prefer to write $\varepsilon \rightarrow -1$ rather than $\varepsilon = -1$, with a similar convention for $\mu$. Thus

we stress the importance of a positive imaginary part to both $\varepsilon$ and $\mu$.

These conclusions on negative refraction have been disputed by some. We have seen that negative signs carry certain implications with them and have hidden consequences easy to confuse with inconsistencies. Perhaps the most forthright criticism came in a *Physical Review Letter* [9] entitled Wave Refraction in Negative-Index Media: Always Positive and Very Inhomogeneous. The argument ran as follows: a calculation of refraction at a positive/negative interface showed that the wave vector did indeed exhibit negative refraction as illustrated to the right in figure 2. To reveal the direction of the *rays* two parallel waves of slightly different frequency were incident on the sample. Motion of the interference pattern was interpreted as the group velocity of the rays. Calculations showed that inside the material the interference pattern normal did not lie parallel to the wavevector but at a positive angle to the surface normal in apparent contradiction to Veselago's predictions, to experiment [8], and to computer simulations [10].

In fact the problem turned out to be not the calculation but its interpretation as was pointed out in a subsequent Comment [11]. Dispersion is an essential ingredient of negative materials so the two incident waves see a slightly different refractive index and are refracted through different angles. Hence the two internal wavevectors are no longer parallel, the interference pattern no longer has a normal in the direction of the group velocity, and therefore cannot be interpreted as positive refraction. A rather simple argument shows that the group velocity must always be aligned with the wavevector. An isotropic medium, assumed in the calculations, allows only two unique directions: parallel or anti-parallel to the wave vector. Only the anti-parallel choice results in energy being transported away from the interface and thus the issue is resolved in favour of negative refraction.

During the past year computer simulations and experiments have greatly clarified the picture. As a result by the middle of the year I was able to write in a piece for Nature, News and Views [12] and say that these materials were *positively negative*.

The experiments of Parazzoli *et al.* at Boeing [13] were amongst several experimental [14] and computational [15–17] demonstrations of negative refraction which confirmed the earlier results. They prepared two prisms as shown in figure 3: one negatively refracting carefully constructed from low-loss material, the other positively refracting but of the same physical dimensions. The deviation of a microwave beam freely propagating in air was measured for each sample and the results are shown in figure 4, the clearest demonstration that these media refract radiation through negative angles.

This period of controversy ensured that the concept of negative refraction had a full and vigorous public debate before being accepted as valid.
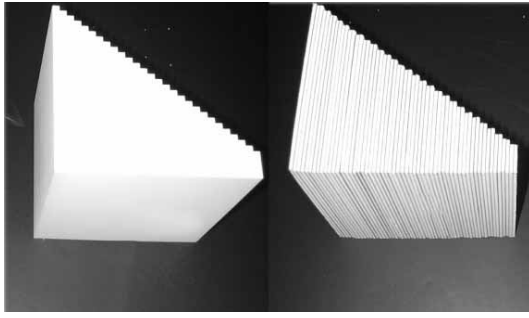
**Figure 3. Boeing PhantomWorks 32° wedges. Right: negatively refracting sample, left: Teflon.**
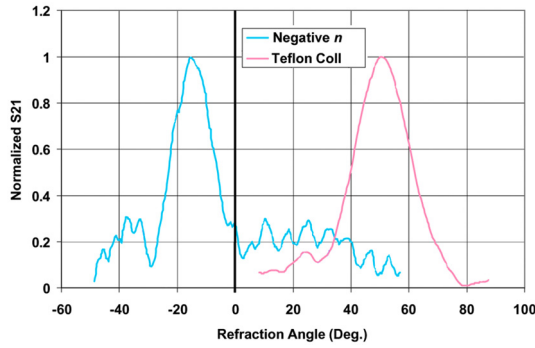


**Figure 4. Angle of refraction by each of the two prisms shown in figure 3: left: data for negative sample; right: data for positive sample. The data clearly demonstrate that refraction is reversed in a negatively refracting medium.**
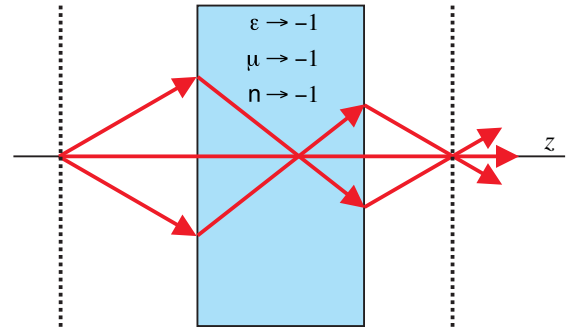


**Figure 5. A negative refractive index medium bends light to a negative angle relative to the surface normal. Light formerly diverging from a point source in the object plane is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time in the image plane.**

The new materials have many strange properties: a reversed Doppler shift (see also [18]) and Cerenkov radiation that emerges in the opposite direction to conventional radiation. The strangest property follows directly from negative refraction and is the ability of the material to focus light. We illustrate this effect in figure 5 for the case $n = -1$ when focussing is free from aberration, and all rays radiating from a point source are brought to a double focus: once inside the slab and again outside. We also choose $\varepsilon \to -1$, $\mu \to -1$ so that the material's impedance matches to vacuum and there is no reflection at the interface. This is a rather good lens: free from aberrations and free from spurious reflections. In fact we shall see that it is even better than that!

A lens functions by acting on all rays radiated from an object, restoring each to its proper phase at the image plane. A conventional lens ensures that each ray has the same optical path length to the image plane and therefore all the phases are correctly reproduced, except for a constant additional term which vanishes in the intensity. Our new lens is even more efficient. As the wave packets (see figure 1) enter the negative slab the phase 'clock' is set in reverse and winds backwards in angle so that propaga-

tion through a thickness $d$ of this medium unwinds the phase acquired passing through the same thickness of vacuum. Wave packets arrive at the image plane with exactly the same phase as in the object plane. This is another clue to the superior nature of this lens.

Now we know that we can build on this foundation and develop new applications based on the concept. Many devices are being tested for antennae and radar applications where their novel properties give added flexibility in design. New radar lenses are being developed which offer compact and relatively aberration free performance. Wave guides have been constructed which show novel dispersion characteristics as a consequences of negative fillings.

Another field which is extremely promising for applications is that of 'indeterminate media' in which $\varepsilon$ and $\mu$ are tensors some of whose components are positive and others negative. These unusual media have possibilities for use as filters to shape radiation patterns.

Yet another effort is to extend the concepts to higher frequencies, to THz and beyond.

## 2. Metamaterials

Permittivity and permeability of conventional materials derive from the response of constituent atoms to applied fields and $\varepsilon,\mu$ represent an average response of the system. On a length scale much greater than the separation between atoms all we need to know about the system is given by $\varepsilon,\mu$. Metamaterials carry this idea one step further: the constituent material is structured into sub-units, and on a length scale much greater than that of the sub-units, properties are again determined by an effective permeability and permittivity valid on a length scale greater than the size of the constituent units. In the case of electromagnetic radiation this usually means that the sub-units must be much smaller than the wavelength of radiation. Figure 6 illustrates this concept.
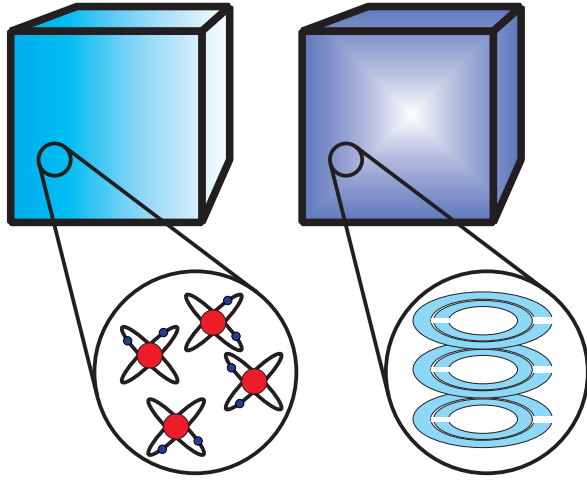
Figure 6. Left: in conventional materials $\varepsilon, \mu$ derive from the constituent atoms. Right: in metamaterials $\varepsilon_{eff}$, $\mu_{eff}$ derive from the sub-units which may contain many atoms.
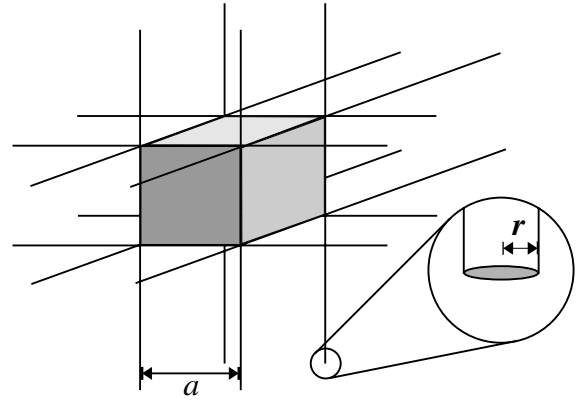


Figure 7. A metamaterial with $\varepsilon < 0$: a periodic structure composed of thin infinite wires, arranged in a simple cubic lattice, mimics the response of a plasma. In a typical example the wires might be a few tens of microns in diameter and spaced by a few millimetres, giving a plasma frequency in the GHz range.

In this way the properties of a complex structure can be summarised by $\varepsilon_{eff}$, $\mu_{ef}$ which is a great simplification in our thinking. So the concept is a familiar one but with the difference that the sub-units can take very many different forms. This flexibility in design enables metamaterials to have values for $\varepsilon_{eff}$, $\mu_{eff}$ which are not encountered in nature and in the present context that will mean one or both of these parameters being negative. Furthermore these materials can give magnetic activity at frequencies where previously materials have been thought of as magnetically inert.

Particular flexibility is given to us in the design of these materials by the very large difference in conductivity between insulators and metals. Therefore all the metamaterials we discuss here will owe their activity to the metallic content. Perhaps the simplest possible example is seen in figure 7.

This was the first structure designed by the Marconi collaboration [3,4] and simulates the properties of a low density plasma having a dielectric response of the form

$$\varepsilon_{metal} = 1 - \omega_p^2/\omega^2$$

sketched in figure 8. Why this should be can be understood as follows: a plasma comprises a gas of charged particles such as a metal where the particles are free electrons, or an ionised gas. The plasma frequency is dictated by the density of charges and by their mass.

$$\omega_p^2 = ne^2/\varepsilon_0 m_e$$

In metals this gives a plasma frequency typically in the ultraviolet region, and in the ionosphere, somewhere in
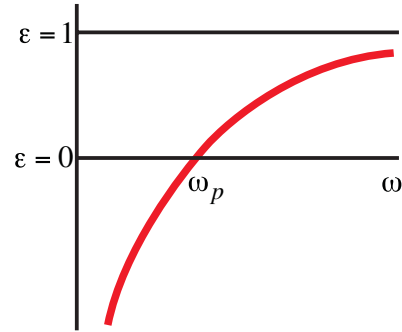


Figure 8. The schematic permittivity, $\varepsilon$, of a plasma: below the plasma frequency, $\omega_p$, $\varepsilon$ is negative.

the GHz band. However in our wire structure the electron density is much reduced relative to solid metal. Furthermore there is an additional contribution to the effective mass of the electrons from the inductance of the wires.

$$n_{eff} = n\pi r^2/a^2, \quad m_{eff} = \ln(a/r)\mu_0 e^2 \pi r^2 n/2\pi$$

where $n$ is the electron density in the wires. Typical dimensions of a few microns radius for the wires and a spacing of a few millimetres give effective masses to the electrons of the same order as a nitrogen atom and as a result the plasma frequency is depressed into the GHz region. Note how this very radical change in dielectric properties is brought about by an extremely small amount of metal. In our example the concentration of metal is only a few parts per million, comparable with the levels of dopants in a semiconductor.

An example of metal content playing quite a different role is shown in figure 9 where we see a structure designed to be magnetically active at GHz frequencies [5]. The structure has a negative magnetic permeability around about 10GHz and has been the basis for many studies of negative materials. Since free magnetic monopoles do not exist in nature, it is not possible to construct the magnetic analogue of a plasma to obtain negative permeability. Hence the use of a resonant structure giving a characteristic response to a magnetic field shown in figure 10. A recent version of the split ring structure is shown in figure 11.

There are two important issues to considered when designing magnetic structures. First is the 'activity' of the structure, broadly speaking the range of frequencies over which a negative response is measured, and the amount of loss present as indicated by $\mu_i$.

The activity is a function of the cross section presented to magnetic lines of force. If a large number of the lines of force passing through the material are intersected by the rings, then the activity will be high. Therefore it is important to engineer a high filling fraction. The advantage of large activity is that the region of negative $\mu_r$ may then extend to high frequencies where losses are small.

Loss is always an issue in negative materials because it attenuates the effects we seek to create. Curiously the main source of loss encountered appears to be the surrounding dielectric material in the structure rather than the resistivity of the metal. High quality dielectric substrates are vital to good performance. With a highly active structure and low-loss components the critical value of $\mu_r = -1$ can be associated with very small values of loss as measured by $\mu_i$.

The split ring structure has the philosophy that first we design a resonant element with a magnetic response, then we assemble these elements into a 2D or 3D structure. However a critical aspect of magnetic activity is the ability of the element to capture a large cross section of incident magnetic flux and this may be limited by the design. For example cylinders of circular cross section cannot be closer packed than the hexagonal close-packed structure. This limits the activity of the structure.

Recently an alternative approach has been proposed which fills space more efficiently and therefore can have greater activity. The Toronto group [19,20] has been producing some of the key designs based on the theory of the 'inverted' waveguide.

In figure 12 we see the conventional waveguide in finite element form and the dispersion relationship of the waves it supports: the spectrum extends to zero frequency, there is an upper cut off, and the dispersion is everywhere positive.
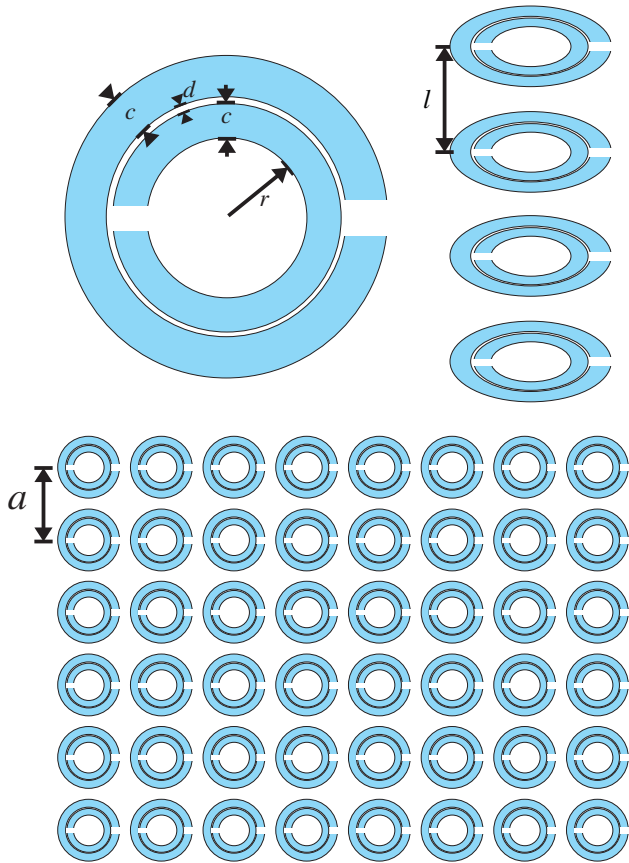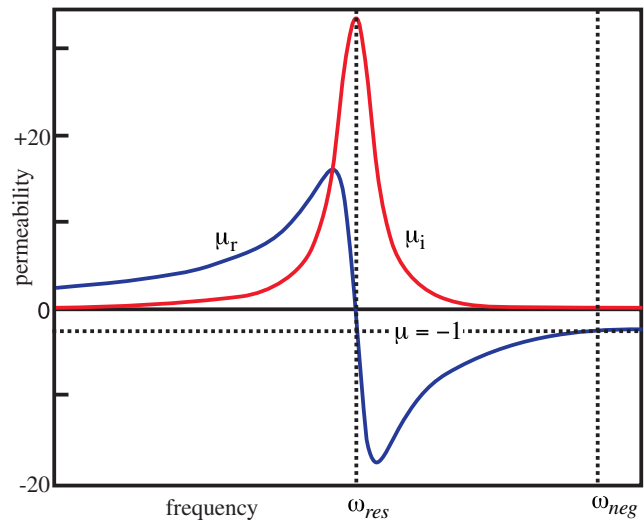


**Figure 9. This metamaterial is designed to give a magnetic response to an external magnetic field in the GHz region of the spectrum: rings are manufactured in layers which are then stacked to form an array of resonant columns. Typically the lattice constant, $a = 10$ mm, is much less than 30 mm, the wavelength at these frequencies.**



**Figure 10. Schematic permeability of the magnetic metamaterial shown in figure 9 showing the resonant response of the structure at $\omega_{res}$. Note that the frequency at which $\mu_r = -1$ is far removed from the resonant frequency and in this instance is in a region where $\mu_i$ is small.**

In contrast the inverted waveguide, figure 13, has capacitative impedance along the length of the guide, shunted by inductive impedance. This acts as a band-pass filter with a lower and upper cut off and negative dispersion of the allowed band, a characteristic of a medium with a negative refractive index.
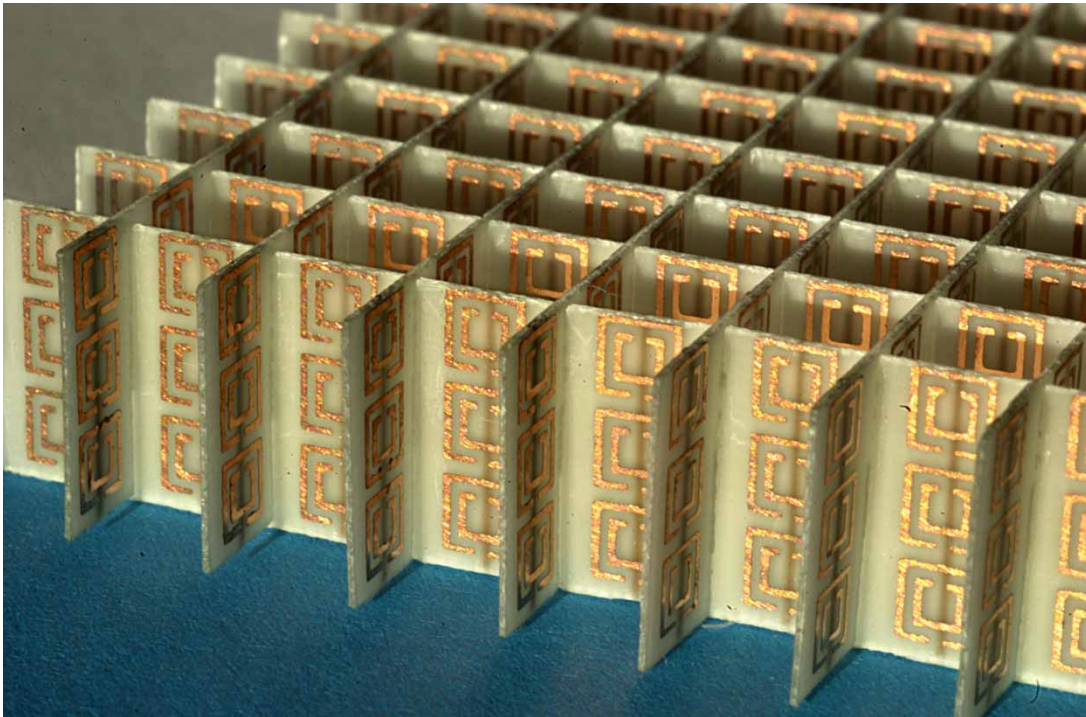


**Figure 11. A split ring structure etched into copper circuit board plus copper wires to give negative $\mu$ and negative $\varepsilon$. Structure made at UCSD by David Smith.**
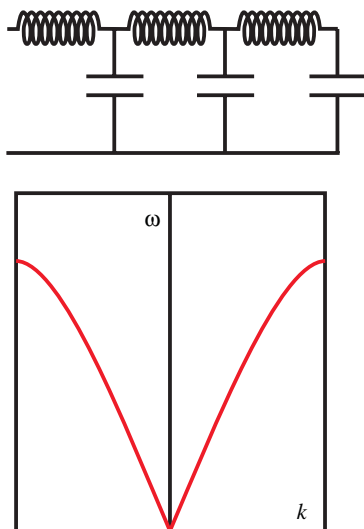


**Figure 12. Top: in a conventional waveguide the wires have an inductive character and are shunted by the capacitance acting between them. Below: schematic dispersion for this transmission line. In this structure waves travel with a *positive* group velocity characteristic of positive refraction.**
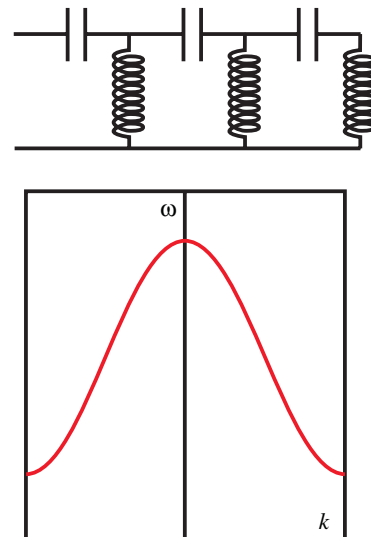


**Figure 13. Top: in an inverted waveguide the wires have capacitative character and are shunted by inductance acting between them. Below: schematic dispersion for this transmission line. In this structure waves travel with a *negative* group velocity characteristic of negative refraction.**

Eleftheriades has realised the 2D structure with hard-wired lumped capacitors and inductors as shown in figure 14. In figure 15 we see the dispersion for this wave guide. Calculation and experiment are in excellent agreement showing a structure that has a huge active region compared with other realisations of negative refraction. The negative region extends for something like 75% of the central frequency so this is a much more active structure than any seen previously.

Yet another approach to attaining negative refraction uses the properties of 'photonic crystals' [21,22]. These materials lie on the transition between a metamaterial and an ordinary structured dielectric. They derive their properties from Bragg diffraction in a periodic structure engineered in the body of a dielectric typically by drilling or etching holes. In this manner many novel dispersion relationships can be produced, including ranges where the frequency disperses negatively with the wave vector as required for a negative refractive index. It has even been shown that magnetic effects can be obtained from a purely
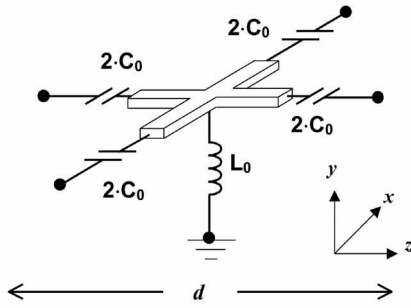


**Figure 14. Circuit diagram for the Eleftheriades group 2D waveguide structure comprising lumped elements.**
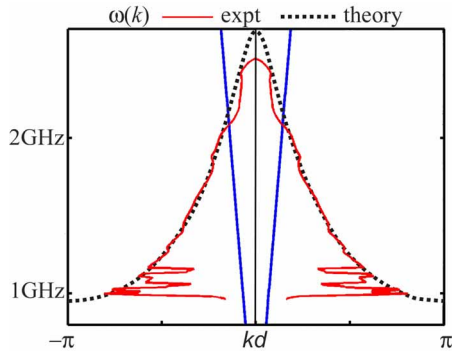


**Figure 15. Measured (full curve) and calculated (dashed curve) shown for the Eleftheriades structure described in figure 14. Note the negative dispersion, the band pass nature of the response and the range of frequencies over which the negative dispersion applies: of the order of 75% of the central frequency.**

dielectric material structured in this way [23]. The photonic crystal approach offers the advantage of very low-loss structures, an attribute that can be very important in some of the more novel applications of negative refraction. Its disadvantage is that it is necessarily sailing close to the wind on the requirement that the structure of the metamaterial should be much smaller than the wavelength: since the photonic crystal functions through diffraction, its structure must be on a similar scale to the wavelength.

Metamaterials can also be constructed to function at low frequencies found in the RF fields of magnetic resonance imaging and substantial progress has been made in adapting materials with negative permeability to MRI applications [31,32].

## 3.  A perfect lens

As I hinted in section 1, the focussing properties of a negative slab are rather unusual, especially for the case where $\varepsilon \to -1$, $\mu \to -1$. In this instance rays are drawn to an aberration-free focus and suffer no reflection from the surfaces of the slab. Yet the focussing properties are even more remarkable than this: further investigation shows that the slab is free from the wavelength restriction on resolution [24]. In the ideal limit $\varepsilon \to -1$, $\mu \to -1$ resolution increases without bounds.

First a few words on the wavelength limit to resolution. In figure 5 an object emits electromagnetic waves of frequency $\omega$. Note that because of dispersion we can only define $\varepsilon \to -1$, $\mu \to -1$ at a single frequency. Each wave has a wave vector, **k**, where,

$$k_z = \sqrt{\omega^2/c_0^2 - k_x^2 - k_y^2}$$

is responsible for driving the wave from object to image, and $k_x$, $k_y$ define the Fourier components of the image. The larger the magnitude of $k_x$, $k_y$ we can propagate to the image plane the better the resolution. The problem is that making these transverse wave vectors too large gives $k_z$ an imaginary value and the wave decays exponentially along the $z$-axis. These decaying components of the object field are often referred to as the 'near field'. They are confined to the vicinity of the object and serve to lock away high-resolution information. Hence the biggest Fourier component that we can capture has magnitude $k_0 = \omega/c_0$ and the wavelength restriction on resolution follows.

How does our negative slab avoid this limit? The secret it deploys is a surface resonance which is used to amplify evanescent waves and restore them to the values taken in the object plane. Given time, a resonance can build a substantial amplitude using energy drawn from the source. Absorption is the great enemy of resonances so low-loss materials are essential if we are to approach the resolution offered by the new lens.

The resonances are related to the surface plasmon excitation familiar on the surfaces of metals, the condition for which is in the short wavelength limit,

$$\varepsilon \to -1$$

In our case $\varepsilon \to -1$, $\mu \to -1$ and there are two surface excitations, one of electric the other of magnetic character, degenerate for all surface wave vectors $k_{||} = [k_x, k_y]$ such that $k_{||} > k_0$.

In the presence of these resonances the response of the system abounds with singularities and therefore it is vital that any calculations should negotiate the singularities rather carefully. We have stressed the importance of the causal nature of $\varepsilon, \mu$ in defining negative $n$ and this process of first allowing $\varepsilon, \mu$ to have finite positive imaginary components and only at the end of the calculation taking the limit, is vital to obtaining sensible answers. This singular nature can be seen in the response of a semi-infinite slab of negative material. If an electrical charge $q$ oscillating at frequency $\omega$ is placed much closer to the surface than the wavelength, the electrostatic limit applies and we can easily calculate the response by the method of images. An image charge is induced in the slab at an equal distance inside as the original charge is outside. The magnitude of the charge is

$$q' = -\frac{\varepsilon - 1}{\varepsilon + 1}q$$

and it is evident that as $\varepsilon \to -1$ this response is singular. The surface supports an infinite number of resonances degenerate at frequency $\omega$ and in the limit of no losses a system stimulated at its resonant frequency gives an infinite response.

In a finite slab degenerate resonances on opposite surfaces interact giving rise to a splitting and the response is no longer infinite. Detuning of the surface resonances results in exactly the degree of excitation required to reproduce the object fields in the image plane. See figure 16.

Of course whenever we describe something as perfect, there will always be some catch. In this instance it is the loss always present in physical materials. Another spoiler [25] is the fact that the description of a material in terms $\varepsilon, \mu$ is never valid on every length scale. For ordinary materials it fails as soon as the scale is fine enough to resolve atoms and molecules; in metamaterials the length scale is much longer and is of the same order as the structures from which the metamaterial is assembled. Nevertheless it remains *in principle* possible to increase resolution without limit. Whereas previous lenses had a categorical cut off in resolution the new lens offers the possibility that if we find better materials we can get better resolution.
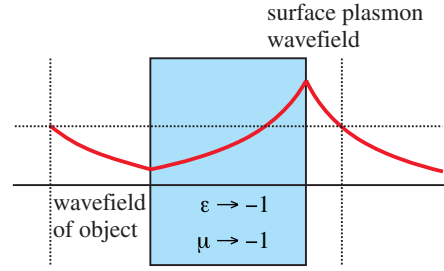


**Figure 16. The new lens works by excitation of surface plasmons. Matching the fields at the boundaries selectively excites a surface plasmon on the far surface thus reproducing the same amplitude in the image plane as in the object plane.**

As with negative refraction the perfect lens gave rise to some heated debate. The difficulty centred on what happens behind the image plane. Let us take an experiment in which we try to image a point dipole oscillating at frequency $\omega$. In the object plane, fields experience a discontinuity brought about by the sources of the field, in this instance our point dipole. On either side of the object plane, waves are emitted travelling in opposite directions transporting energy away from the object plane. Similarly the near fields decay away exponentially on either side. Now let us turn to the image plane. Here there is no charge: even a perfect lens cannot conjure a real dipole out of free space. All we can ask of a lens is that the fields immediately in front of the image plane are the same as those in front of the object plane. Behind the image plane we stand no chance of reproducing the fields found behind the object plane because the image fields have no analytic discontinuity, there being no dipoles present. This is the case with any lens whether perfect or not: the fields behind an image are of completely opposite character to those behind the object.

For the new lens the problem is particularly acute because behind the image the fields continue to grow as we trace them back towards the lens. For very large values of $k_x, k_y$, essential for high resolution, the fields grow very rapidly. In fact there is a limit to this process because absorption will eventually damp the resonances responsible for amplification and the gain provided by the slab will be replaced by attenuation for the very highest values of $k_x, k_y$, capping the divergence in the fields but at the same time limiting resolution. For example in the instance of a lossy dielectric, $\varepsilon = \varepsilon_r + i\varepsilon_i$, the resolution limit is given by,

$$\Delta = 2\pi d / \ln(\varepsilon_i)$$

where $d$ is the thickness of the slab [26]. So very high resolution is to be had for small lenses and small loss.

These large fields behind the image and inside the lens have been the subject of some debate. In a *Physical Review Letter* reminiscent of the earlier argument over negative refraction and trenchantly entitled Left-Handed Materials Do Not Make a Perfect Lens, Garcia and Nieto Vesperinas [27] considered the fields behind the image and concluded that they invalidated the entire theory of perfect imaging. They concluded that without absorption, if $z_0$ is the distance of the object from the lens and $d$ the lens thickness, "... proper normalization imposes once again that $d < z_0$, otherwise this wave function will be zero. Hence there is no transmitted evanescent component inside the LHM slab when $d < z_0$". At the same time absorption, if included, "... transforms any amplified wave into a decaying one inside this medium". In other words there was no hope for the lens as even the tiniest amount of absorption destroys any amplification and any hope of enhanced resolution!

The disagreement centred on the proper treatment of losses and the taking of limits in a causal manner, as pointed out in a subsequent Comment [28,29]. There is now general agreement that the original formulae for the response of the lens [24] are correct. See for example [30]. Absorption, however small, always controls the divergences [33] and by choosing absorption sufficiently small we can make a lens of arbitrarily high resolution a point of view now supported by numerous calculations e.g. [34]. Of course in practice we are limited by the materials available. But the point at issue in this argument was whether the principle of the perfect lens is a valid one not whether it works in practice.

In fact the practical aspects do seem quite hopeful. If we work with systems where all dimensions are far below the wavelength, electric and magnetic fields are essentially separate entities and an object composed solely of electrical fields can be imaged without involving any substantial magnetic component. The value of $\mu$ is irrelevant if no magnetic fields are present. This is very useful because although currently there are no materials with $\mu < 0$ at optical frequencies, many metals have $\varepsilon_r < 0$ and in particular silver has both $\varepsilon_r < 0$ and a relatively small $\varepsilon_i$ and hence low loss. Figure 17 shows a calculation of the quality of image to be expected using silver to image a couple of slits separated by 80 nm. Whilst the results hardly show pin-sharp resolution, two things are clear: the presence of a slab of silver increases the fields at the image and at the same time enable the two slits to be resolved.

The Zhang group at UCLA set out to confirm amplification of light by a thin silver film in a variation of the Kretchmann experiment [35]. Several silver films of varying thickness were prepared on a glass substrate. Roughness on the vacuum side of the film coupled
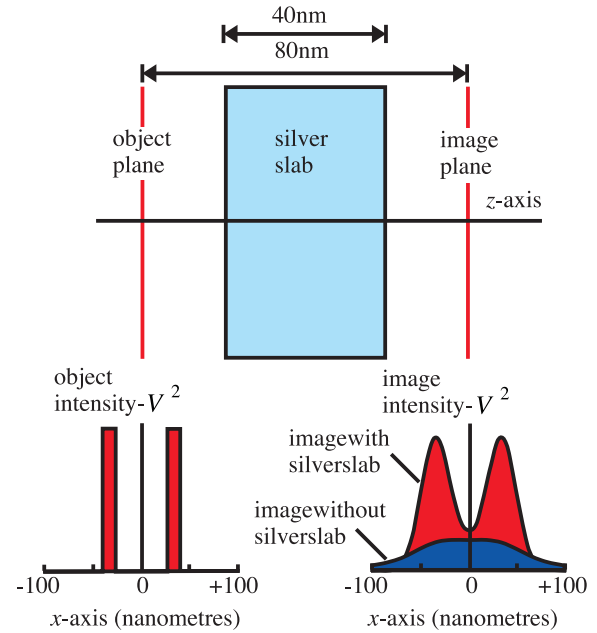
**Figure 17. Above: plan view of the new lens in operation. A quasi-electrostatic potential in the object plane is imaged by the action of a silver lens. Below left: the electrostatic field in the object plane. Below right: the electrostatic field in the image plane with and without the silver slab in place. The accuracy of the reconstruction is limited by losses in the silver.**

incident light to surface plasmons bound to the surface at the vacuum interface, but able to tunnel through and escape into the glass where the intensity was detected. The frequency selected the wavelength of surface plasmons excited, and careful calibration of surface roughness enabled transmitted intensity to be plotted as a function of film thickness. Figure 18 shows their results. As noted above, amplification of evanescent waves increases exponentially with thickness until absorption intervenes and cuts off the process. As can be seen from the figure, substantial amplification is possible before the cut-off.

The Eleftheriades group has also tested the principle of imaging using the near field at microwave frequencies employing a two dimensional waveguide structure described in section 2 [19]. They have observed images with a resolution of better than 5 cm whilst using 20 cm wavelength radiation.

## 4. Optical antimatter

We have discussed how the original Veselago lens is a far more remarkable structure than first thought. In fact the more we examine it the less like a lens it seems. For one thing it does not have a focal length; for another it can focus only objects which are a finite distance from
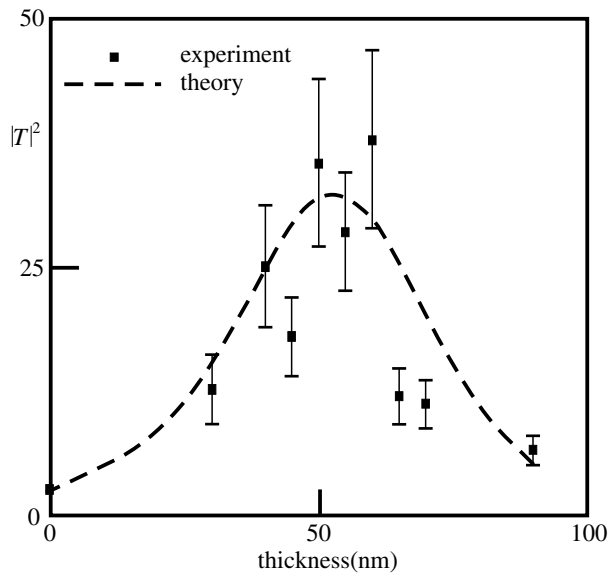
Figure 18. Experimental results from the Zhang group. The figure shows the intensity of light tunnelling through a silver film as a function of thickness. Theory (dashed curve) predicts that the signal is amplified by the film and that amplification grows with film thickness until absorption processes take over and destroy the resonant state responsible. The data confirm the theory.
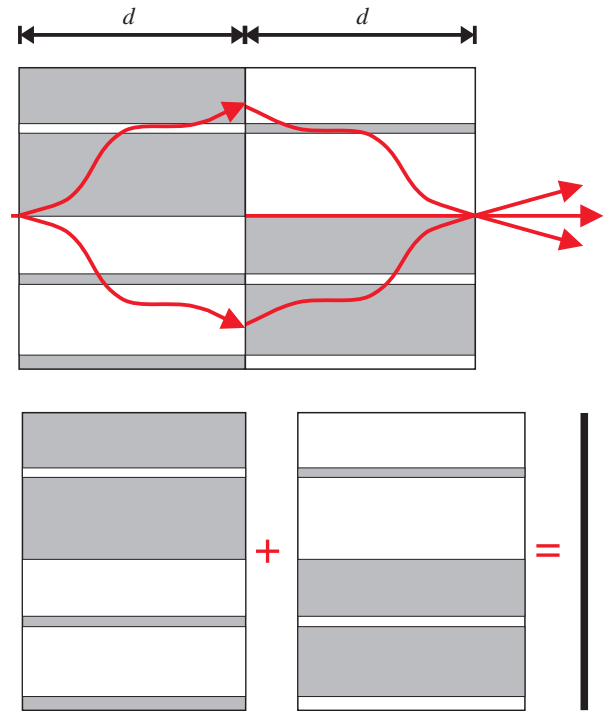


Figure 19. Above: an alternative pair of complementary media, each annihilating the effect of the other. Light does not necessarily follow a straight line path in each medium, but the overall effect is as if a section of space thickness *2d* were removed from the experiment. Below: A graphical expression of our new theorem: complementary halves sum to zero. The optical properties of the rest of the system can be calculated by cutting out the media and closing the gap.

the lens. Gradually the realisation has dawned that something much more interesting is happening and that an alternate way of understanding properties of a negative slab is as a piece of negative space. Optically speaking, and only at the frequency for which the lens condition is satisfied, a slab of material with $\varepsilon \to -1$, $\mu \to -1$ appears to annihilate the effect of an equal thickness of vacuum.

Having realised this it is only one further step to prove a more general result: two slabs of material of equal thickness and placed adjacent to one another optically annihilate if one is the negative mirror image of the other, the mirror being taken to lie on the interface between the two slabs. A simple instance of this is shown schematically in figure 19. The proof of this theorem is given in [36].

Although this result is quite plausible where rays follow a simple distorted trajectory in each medium as illustrated above, in some instances the theorem has startling consequences. Consider figure 20, a system drawn to my attention by David Smith: the mirror theorem applies but a ray construction contradicts the theorem. Applying the laws of refraction to ray 2 implies that the ray is rejected by the system instead of being transmitted through to the other side and a dark shadow behind the cylinders is predicted by the ray picture. In fact a full solution of Maxwell's equations shows that ray 2 is transmitted and
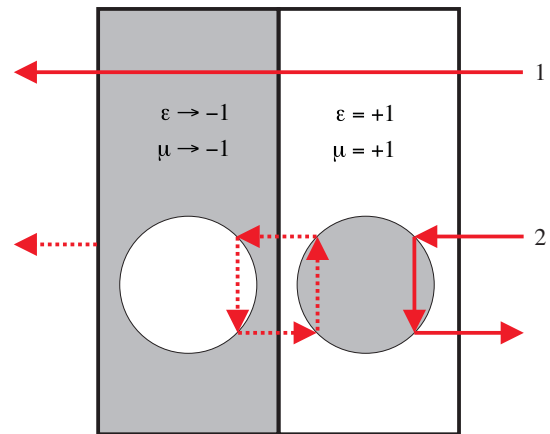


Figure 20. The left and right media in this 2D system are negative mirror images and therefore optically annihilate one another. However a ray construction appears to contradict this result. Nevertheless the theorem is correct and the ray construction erroneous. Note the closed loop of rays indicating the presence of resonances.

emerges through the system just like ray 1 and no shadow is formed. The apparent paradox is resolved by recognising that a series of resonances form on the surfaces of the cylinders and these resonances enable radiation to tunnel across the gap between the cylinders. A clue to the nature of these resonances is given by the closed loop of dotted rays in the centre of the figure which indicates the presence of a state which traps radiation i.e. we have a resonance.

This elaboration of the perfect lens greatly increases the richness of the subject as a far greater variety of lens geometries can now be considered.

One important restriction of the new lenses as currently formulated is that they produce images of exactly the same size as the objects. To do otherwise we must introduce curved surfaces and this is most easily done through coordinate transformations. For example if we transform between Cartesian coordinates $(x, y, z)$ and spherical coordinates $(\ell, \theta, \phi)$,

$$x = r_0 e^{\ell/\ell_0} \sin \theta \cos \phi$$
$$y = r_0 e^{\ell/\ell_0} \sin \theta \sin \phi$$
$$z = r_0 e^{\ell/\ell_0} \cos \theta$$

the original geometry of a slab of negative material is transformed to an annulus of material as in figure 21. Furthermore [37] a coordinate transformation leaves the form of Maxwell's equations unaltered changing only the values of $\varepsilon$, $\mu$ and **E, H** appearing in the equations, leaving the basic topology of the image unaltered.

Detailed discussion of these powerful techniques is beyond the scope of this review and can be found in [36],
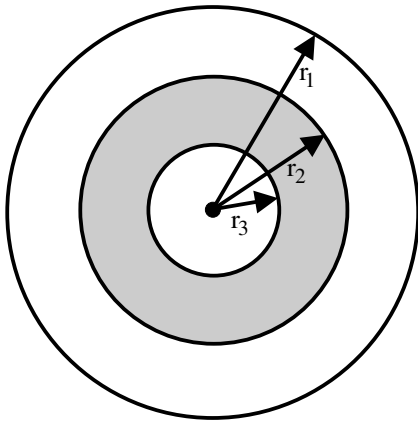


**Figure 21. It is possible to design a spherical annulus of negative material lying between $r_2$ and $r_3$ that acts like a magnifying glass. To the outside world the contents of the sphere radius $r_3$ appear to fill the larger sphere radius $r_1$ with proportionate magnification.**

but the conclusions can be stated simply. For example a spherical structure, sketched in figure 21, will behave like a magnifying glass when constituted as follows:

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = +\frac{r_2^2}{r_3^2}, \quad 0 < r < r_3$$
$$\varepsilon_x = \varepsilon_y = \varepsilon_z \rightarrow -\frac{r_2^2}{r^2}, \quad r_3 < r < r_2$$
$$\varepsilon_x = \varepsilon_y = \varepsilon_z = +1, \quad r_2 < r < \infty$$
$$\mu_x = \varepsilon_x, \ \mu_y = \varepsilon_y, \ \mu_z = \varepsilon_z$$

In other words outside radius $r_2$ the structure is empty space, between $r_2$ and $r_3$ lies the negatively refracting material, although now more structured than before, and finally inside the annulus is a material of constant high permittivity, high permeability. This structure has the unusual property that viewed from beyond a radius,

$$r_1 = r_2^2/r_3$$

the contents of the inner sphere (any electrical or magnetic sources of radiation) appear to be expanded to fill a sphere radius $r_1$ filled with $\varepsilon = \mu = 1$ material i.e. magnified by a factor $r_2^2/r_3^2$. The region of space between $r_1$ and $r_3$ has vanished and is not visible.

Viewed from inside radius $r_3$ the world outside radius $r_1$ appears to be shrunken by a factor $r_3^2/r_2^2$ and now reaches to radius $r_3$ and appears to have $\varepsilon = \mu = r_2^2/r_3^2$ i.e. the same as the filling of the internal sphere. Again the region of space between $r_1$ and $r_3$ has vanished and is not visible.

In the limit that the required values of $\varepsilon, \mu$ are perfectly realised this picture is exact. Hence we have arrived at a prescription for a magnifying glass with perfect resolution. As always the material constraints on performance will be critical but in principle it is now possible to think of designing such structures. Think of your favourite coordinate transformation, apply it to the original perfect lens and get a new lens for free!

## 5. Conclusions

Unusually for the physical sciences, negative refraction began life as a theoretical concept rather than an experimental discovery but posing a challenge to experiment: to find materials with negative values of $\varepsilon, \mu$. At the same time the theory had to defend itself in debates on the validity of the concept. I think it is fair to say that 2003 saw this initial phase draw to a close with positive conclusions both on the concept and its experimental realisation. The future holds many new oportunities.

Theorists now confident of the foundations can move forward and exploit the concepts. New discoveries can be expected to follow.

Experimentally the goal is to improve design of the metamaterials, chiefly by reducing loss, but also by moving on from the cottage industry phase to viable volume manufacture of these materials. To exploit opportunities in all frequency ranges requires more work. Currently the microwave region of the spectrum is the most fertile for experiment, but THz frequencies beckon, and the optical region beyond is currently being probed by nanoscale near-field experiments.

Further in the future we can expect new devices. Novel waveguides and aerials operating in the GHz range will be first in the field, but good progress is being made at magnetic resonance imaging frequencies. Ultimately the concepts will integrate with the substantial effort being invested in plasmonic phenomena at optical frequencies where the aim is to produce devices structured on a sub-wavelength scale.

## Acknowledgements

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