2. ① $\hat{\delta} = \widehat{p-q} = \int x \, d\hat{F}_1(x) - \int x \, d\hat{F}_2(x)$

$= \frac{\sum X_i}{n} - \frac{\sum Y_i}{m} = \hat{p} - \hat{q}$

$\because V(\hat{p}) = \hat{p}(1-\hat{p}) \quad V(\hat{q}) = \hat{q}(1-\hat{q})$

$\therefore \hat{se}(\hat{\delta}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{\hat{q}(1-\hat{q})}{m}}$

$Z_{.05} = 1.64$

$\therefore$ the interval is $(\hat{\delta} - 1.64\,\hat{se},\ \hat{\delta} + 1.64\,\hat{se})$

4. $\because \hat{F}_n(x) = \dfrac{\Sigma I(X_i \le x)}{n}$,

where $I(X_i \le x) = \begin{cases} 1 \text{ if } X_i \le x_0 \\ 0 \text{ if } X_i > x_0 \end{cases}$ for given x_0

let $p = P(X_i \le x_0) = F(x_0)$

then $I(X_i \le x_0) = \{Y_i\} \sim \text{Bernoulli}(p)$

$\because \hat{F}_n(x) = \dfrac{\Sigma Y_i}{n} = \bar{Y}_n$

$\therefore$ by CLT

$\sqrt{n}(\bar{Y}_n - \mu) \xrightarrow{d} N(0, \sigma^2)$

$\therefore \sqrt{n}(\bar{Y}_n - p) \xrightarrow{d} N(0, p(1-p))$

$\therefore \bar{Y}_n \sim N(p, \frac{p(1-p)}{n})$, $p = F(x_0)$

when $n \to \infty$, $\frac{p(1-p)}{n} \to 0$, then $\bar{Y}_n \to p$

$\therefore$ for a fixed $x = x_0$, $\hat{F}_n(x_0) \to F(x_0)$

then generally, $\hat{F}_n(x) \longrightarrow F(x)$ when $n \to \infty$

$\therefore$ $F(x)$ is the limiting distribution