Documentation of GeRS-DeMo

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Contents

List of Symbols

Constants		unit
d_f	Final year of a disruption e.g. Eq. 19	(y)
$\tilde{D}_H$	Maximum achievable per capita demand e.g. Eq. 26	(EJ/y)
d_n	The percent change in the number of well or mines during a disruption e.g. Eq. 19	(-)
d_s	Start year of a disruption e.g. Eq. 19	(y)
$\tilde{F}_P$	Proportionality constant of the max prod. relative to the URR of a field e.g. Eq. 4	(y^{-1})
F_{P_i}	Maximum production of the i-th field e.g. Eq. 1	(EJ/y)
G_{M_L}	The minimum driver necessary to cause mines to upgrade e.g. Eq. 36	(-)
G_{R_L}	The minimum driver necessary to put more regions on-line e.g. Eq. 33	(-)
k_D	Constant for the demand e.g. Eq. 29	(-)
k_F	Constant for the number of fields e.g. Eq. 31	(-)
k_M	Constant for the number of mines e.g. Eq. 32	(-)
k_R	Number of regions constant e.g. Eq. 35	(-)
k_U	Constant for upgrades e.g. Eq. 37	(-)
L_{F_i}	Life of the i-th Field e.g. Eq. 3	(y)
L_H	Highest operating life of a mine e.g. Eq. 17	(y)
L_L	Lowest operating life of a mine e.g. Eq. 17	(y)
L_{M_i}	Life of the i-th mine e.g. Eq. 15	(y)
L_u	Useable life of a resource e.g. Eq. 24	(EJ)

M_H	Highest maximum production of a mine e.g. Eq. 16	(EJ/y)
M_L	Lowest maximum production of a mine e.g. Eq. 16	(EJ/y)
M_{P_i}	Maximum production of the i-th mine e.g. Eq. 15	(EJ/y)
n_{F_T}	The total number of fields in a region e.g. Eq. 6	(-)
n_{R_T}	The total number regions e.g. Eq. 11	(-)
p_H	The maximum achievable population e.g. Eq. 25	(-)
p_L	The minimum achievable population e.g. Eq. 25	(-)
$\tilde{P}_S$	The production shut off proportion e.g. Eq. 1	(-)
$\breve{Q}$	A constant relating to the estimated amount of the URR that is exploited e.g. Eq. 22	(EJ)
$\tilde{Q}_r$	Proportionality constant of Q_{r_i} relative to the URR of a field e.g. Eq. 5	(-)
Q_{r_i}	Remaining URR in the i-th field when production begins to decline e.g. Eq. 1	(EJ)
Q_{R_T}	The ultimately recoverable resources in a region e.g. Eq. 6	(EJ)
$Q_{R_{T_i}}$	The ultimately recoverable resources in the i-th region e.g. Eq. 14	(EJ)
Q_T	Ultimately recoverable resources e.g. Eq. 11	(EJ)
Q_{T_i}	The URR in the i-th mine or field e.g. Eq. 2	(EJ)
r_D	The rate constant for per capita demand e.g. Eq. 26	(y^{-1})
r_ϵ	Rate of URR constant e.g. Eq. 13	(-)
r_F	Rate of fields constants e.g. Eq. 6	(-)
R_H	Recycling Max rate e.g. Eq. 24	(EJ/y)
p_γ	Population asymmetry constant e.g. Eq. 25	(EJ)
R_L	Recycling Min rate e.g. Eq. 24	(EJ/y)
r_l	recycle life rate constant e.g. Eq. 24	(EJ)
r_m	Rate constant for population e.g. Eq. 25	(y^{-1})
r_Q	Rate constant for the exploitable URR in a field e.g. Eq. 8	(-)
r_{Q_T}	Rate constant for the exploitable URR in a mine e.g. Eq. 18	(-)
r_t	Rate constant for technology e.g. Eq. 16	(y^{-1})
t_D	Time constant for per capita demand e.g. Eq. 26	(y)
t_F	Time for a field to reach maximum production e.g. Eq. 1	(y)
t_l	recycle life time constant e.g. Eq. 24	(EJ)
t_M	The ramp up and ramp down years for mines e.g. Eq. 15	(y)
t_m	Population midpoint year e.g. Eq. 25	(y)
t_{r_i}	The year the production of a field begins to decrease e.g. Eq. 1	(y)
t_t	Mid year for the technology e.g. Eq. 16	(y)
Y_{F_i}	Year the i-th field starts e.g. Eq. 1	(y)
Y_{M_i}	Year the i-th mine starts e.g. Eq. 15	(y)
Y_s	Start year mine, field or region e.g. Eq. 32	(y)

Subscripts

c	Constant e.g. Eq. 25	(-)
D	Demand e.g. Eq. 26	(-)
e	Estimated exploited e.g. Eq. 8	(-)
f	Final year e.g. Eq. 19	(-)
F	Field e.g. Eq. 1	(-)
H	Highest e.g. Eq. 16	(-)
i	i-th e.g. Eq. 1	(-)
j	j-th e.g. Eq. 10	(-)
L	Lowest e.g. Eq. 16	(-)
l	Recycle e.g. Eq. 24	(-)
m	Population e.g. Eq. 25	(-)
M	Mine e.g. Eq. 15	(-)
n	Number e.g. Eq. 19	(-)
P	Production e.g. Eq. 1	(-)
Q	Cumulative e.g. Eq. 8	(-)
Q_T	Ultimately recoverable resources e.g. Eq. 18	(-)
r	Decline e.g. Eq. 1	(-)
R	Region e.g. Eq. 6	(-)
S	Shut off e.g. Eq. 1	(-)
s	Start year e.g. Eq. 19	(-)
T	Total e.g. Eq. 1	(-)
t	Technology e.g. Eq. 16	(-)
D	Upgraded e.g. Eq. 37	(-)
u	Useable e.g. Eq. 24	(-)
ϵ	Exploited e.g. Eq. 13	(-)

Functions

$\breve{D}(t)$	Recursive demand e.g. Eq. 29	(EJ/y)
$D_T(t)$	Demand for all fossil fuels e.g. Eq. 27	(EJ/y)
$\tilde{D}_T(t)$	Per capita demand for total fossil fuel e.g. Eq. 26	(EJ/y)
$G(t)$	Fossil fuel supply and demand driver e.g. Eq. 28	(-)
$G_M(t)$	The upgraded mines driver e.g. Eq. 36	(-)
$G_R(t)$	The regions driver e.g. Eq. 33	(-)
$L_M(t)$	Operating life of a mine that commenced in year t e.g. Eq. 17	(y)
$M_P(t)$	Maximum production of a mine that commenced in year t e.g. Eq. 16	(EJ/y)
$n(t)$	The number of mines or fields e.g. Eq. 19	(-)

$n_F(t)$	The number of fields on-line in a region e.g. Eq. 6	(-)
$\breve{n}_F(t)$	Recursive number of fields on-line e.g. Eq. 20	(-)
$\breve{n}_R(t)$	Recursive number of regions in year t e.g. Eq. 34	(-)
$n_M(t)$	The number of mines e.g. Eq. 37	(-)
$n_R(t)$	Number of regions on-line e.g. Eq. 10	(-)
$n_U(t)$	Number of mines being upgraded e.g. Eq. 37	(-)
$p(t)$	Population e.g. Eq. 25	(-)
$P(t)$	Production in year t e.g. Eq. 10	(EJ/y)
$P_{F_i}(t)$	Production from the i-th field e.g. Eq. 1	(EJ/y)
$P_{M_i}(t)$	Production from the i-th mine e.g. Eq. 15	(EJ/y)
$P_R(t)$	Production from a region e.g. Eq. 7	(EJ/y)
$P_{R_j}(t)$	Production in the j-th region e.g. Eq. 10	(EJ/y)
$Q(t)$	Cumulative production in year t e.g. Eq. 11	(EJ)
$Q_e(t)$	The estimated amount of the URR that is exploited e.g. Eq. 8	(EJ)
$Q_\epsilon(i)$	Ultimately recoverable resources in the first i regions e.g. Eq. 12	(EJ)
$Q_R(t)$	Cumulative production from an oil or gas region e.g. Eq. 6	(EJ)
$Q_{T_F}(t)$	The URR of the field brought online in year t e.g. Eq. 9	(EJ)
$R(t)$	Recycle amount in year t e.g. Eq. 24	(EJ/y)
$S(t)$	Supply in year t e.g. Eq. 24	(EJ/y)

Greek Letters

γ_m	Asymmetry constant for population e.g. Eq. 25	(-)

Accents

$\breve{}$	Recursive e.g. Eq. 20	(-)
$\tilde{}$	Proportional e.g. Eq. 1	(-)

Variables

D	Demand e.g. Eq. 27	(EJ)
d	Disruption e.g. Eq. 19	(-)
F	Field e.g. Eq. 1	(-)
G	Supply and Demand Driver e.g. Eq. 28	(-)
i	number e.g. Eq. 12	(-)
k	Constant e.g. Eq. 31	(-)
L	Operating life e.g. Eq. 3	(Y)
P	Maximum e.g. Eq. 16	(-)
n	Number e.g. Eq. 6	(-)
P	Production e.g. Eq. 1	(EJ)
Q	Cumulative e.g. Eq. 1	(EJ)

r	Rate e.g. Eq. 6	(y^{-1})
S	Supply e.g. Eq. 24	(-)
t	Time e.g. Eq. 1	(Y)
Y	Year e.g. Eq. 1	(Y)

1 Introduction

This document contains the current logic of GeRS-DeMo as well as a simple guide on how to use the GUI. This document will start with a description of the model, followed by a step by step guide on using the GUI interface.

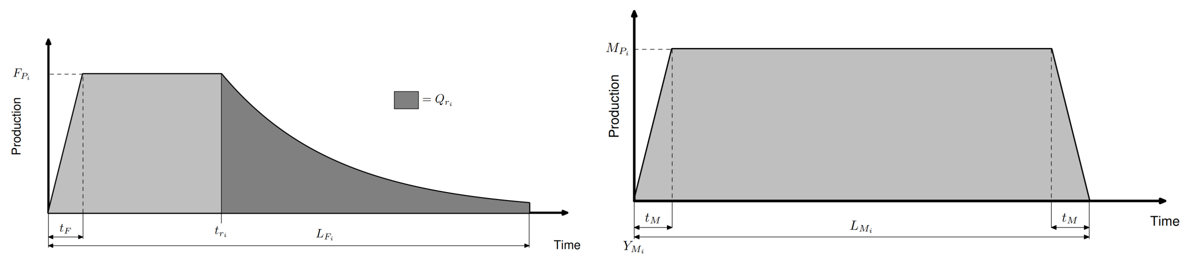
2 Model Description

This model was first described in Mohr (2010) and was modified to include a recycling component Mohr et al. (2012). An error in the description of the model was identified in 2019 related to the mine life. The description of the model is reproduced below and is sourced directly from Mohr (2010); Mohr et al. (2012, 2021). The model has four overall components:

- Production, which can be from either Mines or Fields.
- Recycling, which is estimated as a proportion of historic production.
- Demand, which is a crude indicative demand for the resource.
- Supply and Demand Interactions

2.1 Production

As indicated production can be generated from two different methods, mines and fields. The production profile for an individual mine or field is shown in Figure 1. In addition of mines and fields, production can be affected by disruptions and this will also be explained in this section.



(a) Field production (b) Mine production

Figure 1: Idealised production from fields and mines

2.1.1 Production – Fields

Production from fields is complicated, first we will describe how fields generate production from a given field (e.g. North Sea or Gulf of Mexico) next we will outline how production is determined for countries with multiple regions (e.g. California, Alaska etc).

2.1.1.1 Individual Region

The production for a region is determined by summing the production of all idealised fields contained within the region. The production of an individual idealised field follows the profile indicated in Figure 1. Specifically, it has a one year ramp up to a plateau period, followed by an exponential decline in production. Let:

- Q_{T_i} be the URR of the field
- Q_{r_i} denote the amount of recoverable resources remaining in the field when production begins to exponentially decay
- L_{F_i} denote the fields operating life
- $P_{F_i}(t)$ denote the production profile of the field.

Further, assume that the field is shut when production reaches 1% of F_{P_i}. Let the production profile of the i-th field be denoted $P_{F_i}(t)$. Then, the production profile can be expressed mathematically as:

$$P_{F_i}(t) = \begin{cases} 0 & \text{if } t < Y_{F_i} \\ \frac{F_{P_i}}{t_F}(t - Y_{F_i}) & \text{if } Y_{F_i} \leq t < Y_{F_i} + t_F \\ F_{P_i} & \text{if } Y_{F_i} + t_F \leq t < t_{r_i} \\ F_{P_i} e^{\left(-\frac{F_{P_i}(1-\tilde{P_S})}{Q_{r_i}}(t - t_{r_i})\right)} & \text{if } t_{r_i} \leq t \leq t_{r_i} - \frac{\log(\tilde{P_S})Q_{r_i}}{F_{P_i}(1-\tilde{P_S})} \\ 0 & \text{if } t > t_{r_i} - \frac{\log(\tilde{P_S})Q_{r_i}}{F_{P_i}(1-\tilde{P_S})} \end{cases} \tag{1}$$

where t_{r_i} can be readily determined to be

$$t_{r_i} = \frac{Q_{T_i} - Q_{r_i}}{F_{P_i}} + \frac{t_F}{2} + Y_{F_i}. \tag{2}$$

The life of the field is:

$$L_{F_i} = t_{r_i} - \frac{\log(0.01)Q_{r_i}}{F_{P_i}(1 - 0.01)} - Y_{F_i} \tag{3}$$

Both the maximum production F_{P_i} and the amount of recoverable resources remaining in the field when production begins Q_{r_i} are assumed to be proportional to the URR of the field Q_{T_i}. That is:

$$F_{P_i} = \tilde{F}_P Q_{T_i} \tag{4}$$

and

$$Q_{r_i} = \tilde{Q}_r Q_{T_i} \tag{5}$$

So in order to generate production we need to know two things:

a. When the field is brought online

b. the URR of the field

Let $n_F(t)$ denote the total number of fields brought online at or before year t. Then the number of fields brought online in year t is trivially $n_F(t) - n_F(t-1)$. $n_F(t)$ is assumed to be linearly dependent on the cumulative production of the oil or gas region, $Q_R(t)$ and is calculated as:

$$n_F(t) = \min\left\{\left\lceil r_F n_{F_T} \frac{Q_R(t)}{Q_{R_T}} \right\rceil, n_{F_T}\right\} \tag{6}$$

where n_{F_T} is the total number of fields, Q_{R_T} is the ultimately recoverable resources in the region, and r_F is a rate of fields on-line constant. Note $r_F \geq 1$ and $n_F(t)$ is initially one field for the first year of production.

The production for a region from fields, $P_R(t)$, is the sum of the production from the fields in that region, that is:

$$P_R(t) = \sum_{i=1}^{n_F(t)} P_{F_i}(t) \tag{7}$$

The URR of the fields brought online in year t is determined via the exploitable URR. The exploitable URR, is the sum of the URR in fields (or mines) that have already been brought on-line. The exploitable URR $Q_e(t)$ is estimated via equation 8,

$$Q_e(t) = Q_{R_T} \left(\frac{n_F(t)}{n_{F_T}} \right)^{r_Q} \tag{8}$$

where r_Q is a rate constant. The URR of an individual field brought on-line in year t, $Q_{T_F}(t)$ is determined as:

$$Q_{T_F}(t) = \frac{Q_e(t) - Q_e(t-1)}{N_F(t) - N_F(t-1)} \tag{9}$$

2.1.1.2 Multiple Regions

A single country may have multiple regions hence, it is necessary to expand Section 2.1.1.1 to take this into account. The production of a country $P(t)$ is determined as the sum of the production from its regions, hence

$$P(t) = \sum_{j=1}^{n_R(t)} P_{R_j}(t) \tag{10}$$

where $P_{R_j}(t)$ is the production in the j-th oil or gas region, and $n_R(t)$ is the number of regions on-line in time t. The number of regions on-line is assumed to be proportional to the square root of the cumulative production, and is defined as

$$n_R(t) = \left\lceil n_{R_T} \sqrt{\frac{Q(t)}{Q_T}} \right\rceil \tag{11}$$

where n_{R_T} is the total number of oil or gas regions, $Q(t)$ is the cumulative production for the country and Q_T is the ultimately recoverable resources for the country. Initially, one region is on-line; otherwise, $Q(t-1)$ would always equal zero and no regions would be put on-line.

It is now necessary to explain the size of oil or gas regions. In order to accomplish this, a few more terms must be defined. Let $Q_\epsilon(i)$ denote the sum of ultimately recoverable resources in the first i oil or gas regions as shown:

$$Q_\epsilon(i) = \sum_{j=1}^{i} Q_{R_{T_j}} \tag{12}$$

where $Q_{R_{T_j}}$ is the ultimately recoverable resources in the j-th oil or gas region. It is important to note, $Q_{R_{T_j}}$ is unknown, in order to determine it, let $Q_\epsilon(i)$ be calculated by

$$Q_\epsilon(i) = Q_T \frac{1 - e^{(-r_\epsilon (i/n_{R_T})^2)}}{1 - e^{(-r_\epsilon)}} \tag{13}$$

where r_ϵ is a rate constant. Based on the definition of $Q_\epsilon(i)$, the size of the i-th oil or gas region, $Q_{R_{T_i}}$, is determined by rearranging Equation 12 to obtain Equation 14

$$Q_{R_{T_i}} = Q_\epsilon(i) - Q_\epsilon(i-1). \tag{14}$$

Observe that $Q_\epsilon(n_R(t))$ is the URR in the fields which have commenced production by year t. This suffices to explain the Oil or Gas Region model in full.

2.1.2 Production – Mines

The production from mines is determined from the sum of the individual idealised mines' production. The idealised mines have a ramp up and ramp down period of length t_M, with a steady production rate in between, as shown in Figure 1.

$$P_{M_i}(t) = \begin{cases} 0 & \text{if } t < Y_{M_i} \\ \frac{M_{P_i}}{t_M}(t - Y_{M_i}) & \text{if } Y_{M_i} \leq t < Y_{M_i} + t_M \\ M_{P_i} & \text{if } Y_{M_i} + t_M \leq t < Y_{M_i} + L_{M_i} - t_M \\ \frac{M_{P_i}}{t_M}(Y_{M_i} + L_{M_i} - t) & \text{if } Y_{M_i} + L_{M_i} - t_M \leq t < Y_{M_i} + L_{M_i} \\ 0 & \text{if } t \geq Y_{M_i} + L_{M_i} \end{cases} . \tag{15}$$

The life of an individual mine and its production rate is dependent on the year the mine is brought on-line as described in Equations 16 and 17 .

$$M_P(t) = \frac{M_H + M_L}{2} + \frac{M_H - M_L}{2} \tanh(r_t(t - t_t)) \tag{16}$$

$$L_M(t) = \begin{cases} L_H + (L_L - L_H)\frac{\log_{10}(M_P(t)/M_H)}{\log_{10}(M_L/M_H)} & ; \text{if } M_L \neq M_H \\ \frac{(L_L + L_H)}{2} & ; \text{otherwise} \end{cases} \tag{17}$$

where r_t and t_t are rate and time constants, M_L, M_H is the minimum and maximum mine production rates, and L_L, L_H are the minimum and maximum mine lives. The number of mines brought on-line in year t is calculated via the estimated exploitable URR $Q_e(t)$ as:

$$Q_e(t) = \frac{Q_T - Q_{T_1}e^{-r_{Q_T}}}{1 - e^{-r_{Q_T}}} - \frac{Q_T - Q_{T_1}}{1 - e^{-r_{Q_T}}} e^{\left(-r_{Q_T}\frac{Q(t)}{Q_T}\right)} \tag{18}$$

where r_{Q_T} is a rate constant, and $Q(t)$ is the cumulative production for the country and Q_{T_1} is the URR of the first mine brought on-line in the region. The number of mines brought on-line is determined by increasing the number of mines on-line until the actual exploitable URR is larger than the estimated exploitable URR.

2.1.3 Disruptions

Disruptions are an unfortunate component of the real world and these disruptions need to be accounted for by the modelling procedure. An example of a disruption is an event such as the collapse of the Soviet Union. Such disruptions can be taken into account by inputting into the model three terms: the start year of the disruption d_s, the end year of the disruption d_f, and the percentage of mines or wells still on-line in the last year of the disruption d_n. The equation for the number of mines or fields on-line during a disruption, $n(t)$, is

$$n(t) = n(d_s)\left(\frac{1 - d_n}{d_s - d_f}(t - d_s) + 1\right). \tag{19}$$

Some equations have to be stated in less than elegant ways in order that, with no disruptions, they behave as before and with disruptions they behave in a manner that is sensible. The disruptions caused by shutting mines and wells cannot be handled by Equations 6 and 18. The equations have to be converted from continuous equations into recursive ones. Equation 6 is modified to become:

$$n_F(t) = \lceil \breve{n}_F(t) \rceil \tag{20}$$

where $\breve{n}_F(t)$ is defined recursively as

$$\breve{n}_F(t) = \breve{n}_F(t-1) + r_F n_{F_T} \frac{P_R(t)}{Q_{R_T}} \tag{21}$$

and the condition that initially one field is on-line ($\breve{n}_F(Y_{F_1}) = 1$).

Equation 18 is redefined recursively as shown below

$$Q_e(t) = Q_e(t-1)e^{\left(-r_{Q_T}\frac{P(t)}{Q_T}\right)} + \breve{Q} - \breve{Q}e^{\left(-r_{Q_T}\frac{P(t)}{Q_T}\right)} \tag{22}$$

where $\breve{Q}$ is defined as

$$\breve{Q} = \frac{Q_T - Q_{T_1}}{1 - e^{-rQ_T}} \tag{23}$$

with the initial condition $Q_e(Y_s) = Q_{T_1}$, where here $P(t)$ is the production for a country's regions.

2.2 Recycling

Recycling over time $R(t)$ is estimated simplistically. First total supply is determined as production plus recycling. It is assumed that the resource modelled has a static useable life, L_u. Let $s(t)$ denote the total resource supplied in year t. In year t, the total resource supplied in the year $t - L_u$ ($s(t - L_u)$ becomes available for recycling. Finally we assume that the portion of the amount available that is recycled follows an S curve starting from a low portion of recycling R_L to a higher portion R_H. The amount of recycling done over time can be expressed mathematically as:

$$R(t) = S(t - L_u)\left[\left(\frac{R_H - R_L}{2}\right) + \left(\frac{R_H + R_L}{2}\right)\tanh\left(r_l\left(t - t_l\right)\right)\right] \tag{24}$$

Where r_l is a recycling rate constant and t_l is a time constant.

2.3 Demand

The model of demand is determined by projecting the per capita demand, and using historic population statistics. The historic and projected population of the world was fitted to the asymmetric sigmoidal curve shown in Equation 25

$$p(t) = \frac{p_H - p_L}{\left[1 + p_\gamma e^{(-r_m \gamma_m (t - t_m))}\right]^{1/\gamma_m}} + p_L \tag{25}$$

where p_H is the maximum achievable population, p_L is the lowest population achievable, r_m is a rate constant, t_m is a time constant, γ_m is an asymmetry factor and p_γ is an asymmetry constant. Note with $p_L = 0$ and $\gamma_m = 1$ Equation 25 becomes identical to Hubbert's curve. With the population determined, per capita demand of fossil fuels is assumed to follow exponential growth up to a saturation point. The per capita demand is denoted $\tilde{D}_T(t)$ and is shown mathematically in Equation 26:

$$\tilde{D}_T(t) = \begin{cases} \tilde{D}_H e^{(r_D(t - t_D))} & ; \text{if } t < t_D \\ \tilde{D}_H & ; \text{if } t \geq t_D \end{cases}. \tag{26}$$

Here, $\tilde{D}_H$ is the maximum achievable per capita demand, t_D is the year in which per capita demand reaches $\tilde{D}_H$, and r_D is a rate constant. The total demand for fossil fuels $D_T(t)$ is simply per capita demand multiplied by the population:

$$D_T(t) = \tilde{D}(t)p(t). \tag{27}$$

2.4 Supply and demand interactions

There are two options with the model, Static and Dynamic mode. In the Static mode, the model is as describe in the previous sections. In particular demand and supply do not interact in any way. In the dynamic mode, supply and demand interact in a number of ways. What is desired with the supply and demand interactions is for demand and supply to try to converge, meaning that if demand is higher than supply then demand will decrease and supply increase, or vice versa.

The supply and demand interactions on all resources the driver in the interactions is the fractional difference between supply and demand, namely G, which applies to all resources. The supply and demand drivers are explained mathematically in Equation 28.

$$G(t) = \frac{D_T(t-1) - S_T(t-1)}{S_T(t-1)} \tag{28}$$

Irrespective of which supply and demand driver is being applied, it affects the model in the same way. The driver acts to try and make supply and demand equal. The driver attempts this goal by affecting five key areas, namely:

a. It acts to increase or decrease per capita demand for the relevant fuel.

b. It increases or decreases the number of fields coming on-line (and can shut fields off if need be).

c. It increases or decreases the number of mines coming on-line (and can shut mines off if need be).

d. It can if there is a sufficient gap between demand and supply, increase the number of oil or gas regions being discovered/brought on-line.

e. It can if there is a sufficient gas between demand and supply, cause a mine to upgrade (increase its production capacity).

The precise way the driver achieves these affects, will now be explained in sections 2.4.1 to 2.4.5.

2.4.1 Affect on per capita demand

The supply and demand interaction affects the demand. Firstly, Equation 26 describing $\tilde{D}_T(t)$ is modified into an recursive format as shown in Equation 29.

$$\tilde{D}_T(t) = \begin{cases} \tilde{D}_T(t-1)(1-k_D G(t)) & ; \text{if } \tilde{D}_T(t-1) > \tilde{D}_H \ \& \ \breve{D}(t) > \tilde{D}_H \\ \left[\tilde{D}_H - \tilde{D}_T(t-1)k_D G(t)\right] & ; \text{if } \tilde{D}_T(t-1) \leq \tilde{D}_H \ \& \ \breve{D}(t) > \tilde{D}_H \\ \tilde{D}_T(t-1)\left[e^{r_D} - k_D G(t)\right] & ; \text{if } \breve{D}(t) \leq \tilde{D}_H \end{cases} \tag{29}$$

with

$$\breve{D}(t) \quad = \quad \tilde{D}_T(t-1)(e^{r_D} - k_D G(t)) \tag{30}$$

where k_D is a constant, and the initial condition is $\tilde{D}_i(Y_s) = \tilde{D}_H e^{(r_D(Y_s - t_D))}$.

2.4.2 Affect on number of fields on-line

The supply and demand interactions affect the number of fields on-line in a oil or gas region. In particular, Equation 21 is modified to include an interaction term as shown below:

$$\breve{n}_F(t) \quad = \quad \breve{n}_F(t-1) + r_F n_{F_T} \frac{P_R(t)}{Q_{R_T}} + k_F \breve{n}_F(t-1) G(t) \tag{31}$$

where initially one field is assumed to be on-line, k_F is a constant and $\breve{n}_F(Y_{F_1}) = 1$.

2.4.3 Affect on number of mines on-line

The supply and demand interactions affect the number of mines on-line in a similar fashion, to that of the number of fields with Equation 22 modified into the following form:

$$Q_e(t) = Q_e(t-1)e^{\left(-r_{Q_T}\frac{P(t)}{Q_T}\right)} + \breve{Q} - \breve{Q}e^{\left(-r_{Q_T}\frac{P(t)}{Q_T}\right)} + k_M Q_e(t-1)G(t) \tag{32}$$

where k_M is a constant and $Q_e(Y_s) = Q_{T_1}$, and $P(t)$ is the production from a country by the mining model.

2.4.4 Affect on the regions

The supply and demand interactions affect the number of oil and gas regions on-line. First an oil or gas region cannot be undiscovered, so there is no notion of reducing the number of oil and gas regions on-line[1]. Therefore, the supply and demand interactions can only act to put more oil or gas regions on-line. The driver G also needs to have a minimum magnitude before more regions are discovered; this is an attempt to replicate the real world experience of fuel prices needing to rise to a certain level before action is taken. Let $G_R(t)$ denote the driver that is non-zero only when $G(t)$ is greater than some minimum value G_{R_L}. Equation 33 shows $G_R(t)$ is given by

$$G_R(t) = \begin{cases} 0 & ; \text{If } G(t) \leq G_{R_L} \\ G(t) - G_{R_L} & ; \text{If } G(t) > G_{R_L} \end{cases} . \tag{33}$$

[1]The supply and demand interactions do however affect the number of fields operating in a region (see Eq. 31).

Equation 11 has to be modified into the following recursive format:

$$n_R(t) = \lceil \breve{n}_R(t) \rceil \tag{34}$$

where $\breve{n}_R(t)$ is defined as

$$\breve{n}_R(t) = \breve{n}_R(t-1)\sqrt{\frac{Q_T\breve{n}_R^2(t-1) + P(t)n_{R_T}^2}{Q_T\breve{n}_R^2(t-1)}} + k_R\breve{n}_R(t-1)G_R(t) \tag{35}$$

with k_R is a constant and $\breve{n}_R(Y_s) = 1$ and $P(t)$ is the production from a country regions model.

2.4.5 Affect on the upgrade of mines

The last way the supply and demand interactions affect the model is in making individual mine's production increase to $2M_P$. Such an interaction is denoted as upgrading a mine. As with the number of oil or gas regions, a mine can only be upgraded and not downgraded[2]. Likewise the driver $G(t)$ needs to exceed a minimum value of G_{M_L} before any mines are upgraded. $G_M(t)$ is defined as

$$G_M(t) = \begin{cases} 0 & ; \text{If } G_T(t) \leq G_{M_L} \\ G_T(t) - G_{M_L} & ; \text{If } G_T(t) > G_{M_L} \end{cases} . \tag{36}$$

The number of mines upgraded in the year t, $n_U(t)$ is determined by the expression

$$n_U(t) = \lceil k_U n_M(t) G_M(t) \rceil \tag{37}$$

where k_U is a constant. The mines chosen to be upgraded are the $n_U(t)$ mines that have most remaining reserves. A minimum of 10 years' operating life is also assumed, as 10 years is taken to be the minimum amount of time necessary to ensure that the costs of upgrading a mine is economic.

3 Model Usage

Note, there is a current 'Feature' of the interface, that it doesn't have scrollable interface. As such it's advised not to run this interface on a small screen.

3.1 Homepage

After unzipping the dist.zip file, stored on a windows computer, then click on Resource_Model.exe. With luck the window shown in Figure 2 appears.

There are three options from here

- **Import:** This is where you can import an excel version of the model
- **Load:** This is where a new gui format can be loaded into the model
- **New Model:** This is where a new projection can be created.

3.1.1 Import

The electronic supplements of most of the papers had an excel version of the model with the publication. These excel versions can be imported in this section (see fig 3). In the sample files, an example of doing this is the Dynamic_BG.xlsm file. The import process can take a little while to load, when it's finished the gui will shift to the 'Select Modelling Approach' page (see fig 4). The import may fail, make sure to delete rows and columns so that each sheets data starts in cell A1 (as it is in Dynamic_BG.xlsm).

3.1.2 Load

As shown in figure 4 it's possible to save a scenario into a .GRS file. This saved file can be loaded by clicking on the load button. As with the Import button, upon loading a saved scenario it will bring you to the 'Select Modelling Approach' page.

[2] A mine can however be shut down temporarily

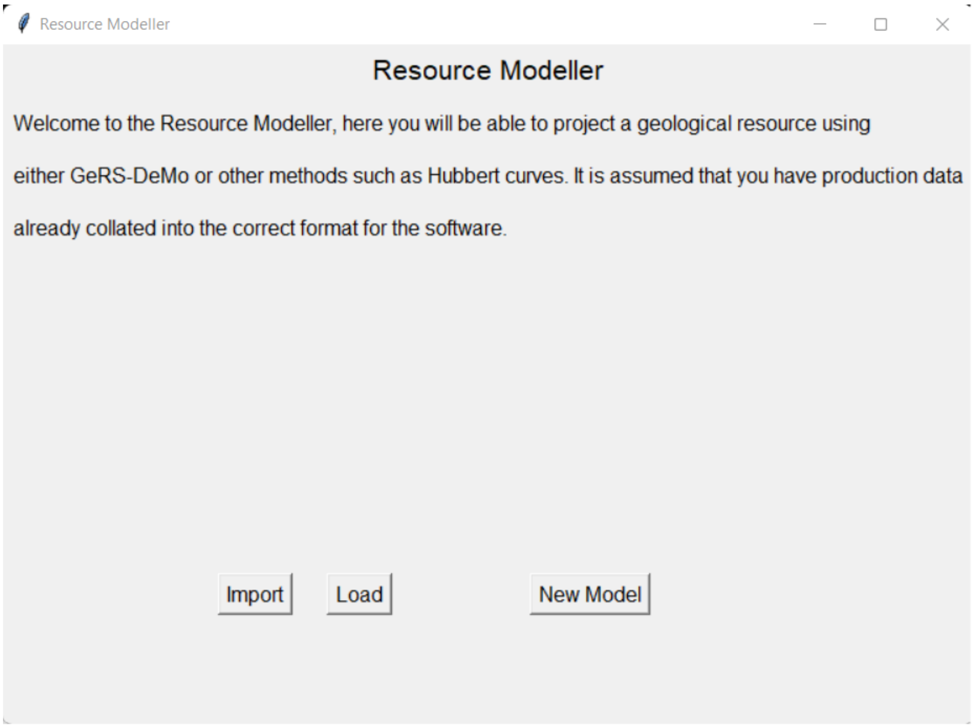


Figure 2: Homepage

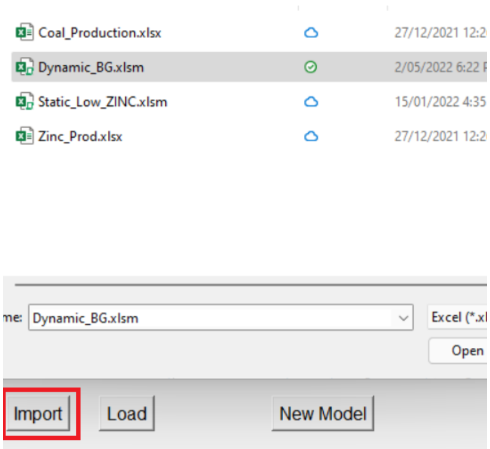


Figure 3: Select Modelling Approach

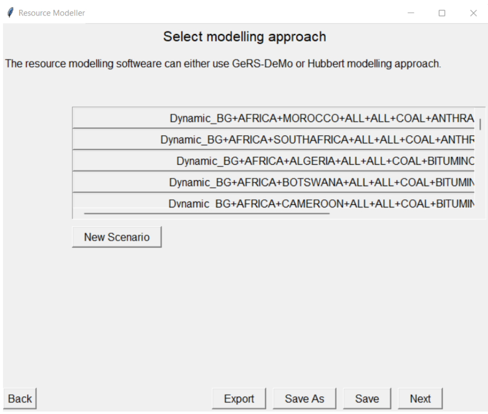


Figure 4: Loaded page

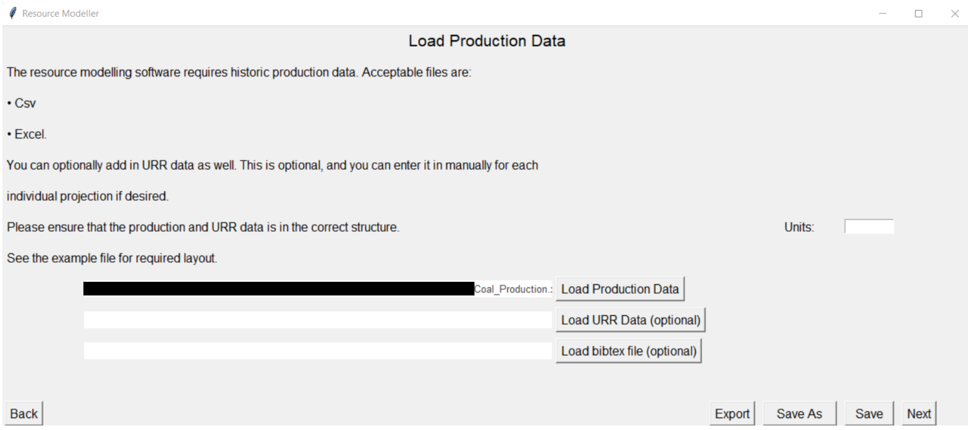


Figure 5: Load Production Page

3.1.3 New Model

Clicking on the New Model button brings up the 'Load Production Data' page (Fig 5). This page is also accessed by clicking the back button on the 'Select Modelling Approach' page. On this page, import URR, and production data, and if relevant a bibtex (LaTeX) reference file. These inputs are entirely optional. The URR and Production data examples are in the 'Coal_Production.xlsx' sample file. The model is assuming the data is in the format shown in the sample file. It's possible to load either a csv file or an excel file. If an excel file is selected, the model will attempt to guess the appropriate sheet but will give you the option to input the correct worksheet. Note the production or URR file needs to be closed in order to open input the file. Note the data in the excel is as at the time the file was selected. If the excel file is subsequently modified it will not be read in the model.

Pressing next on the 'Load Production Data' page, takes you to the 'Select Modelling Approach'. On clicking 'New Scenario' three buttons in a row are added to the white scroll box as indicated in figure 6. Each scenario here is meant to represent a country and scenario type - e.g. Low Australia, it can also be used for a state or province, e.g. Low - NSW Australia. Working backwards, the Duplicate button copies the scenario (adding three new buttons at the bottom of the scroll box). This new copied scenario can then be modified, e.g. say there is little difference between Low Australia scenario and High Australia scenario, then it is possible to duplicate the scenario and then edit the scenario to become the high scenario. The Delete button as expected removes the scenario from the model. **Note, once deleted it is lost**.

Clicking the first button enables you to initialise and edit the scenario. If you are following along, I have loaded the sample file for both the URR and production files - this will be important to see different configurations.

3.1.3.1 Initialising the scenario

Clicking on the New Scenario brings the user to the Scenario page where the scenario is initialised as shown in Figure 7

As we have loaded the production and URR sheets into the model, the model offers suggestions as a dropdown as shown in figure 8

If we fill out the scenario for a record that is in the URR table, we see that the URR is automatically added to the model as shown in figure 9. If a mistake is made it is readily fixable by clicking the 'Edit Scenario' button.

If on the other hand we add a scenario from the production data e.g. South Africa which is not in the URR[3], a pop-up choice appears to either input the URR or calculate it through Hubbert Linearisation as shown in figure 10.

Selecting 'Input URR' a pop up appears where a value can be manually added as indicated in figure 11.

[3] No underscore note South_Africa is in the URR, but the underscore means it is not matched

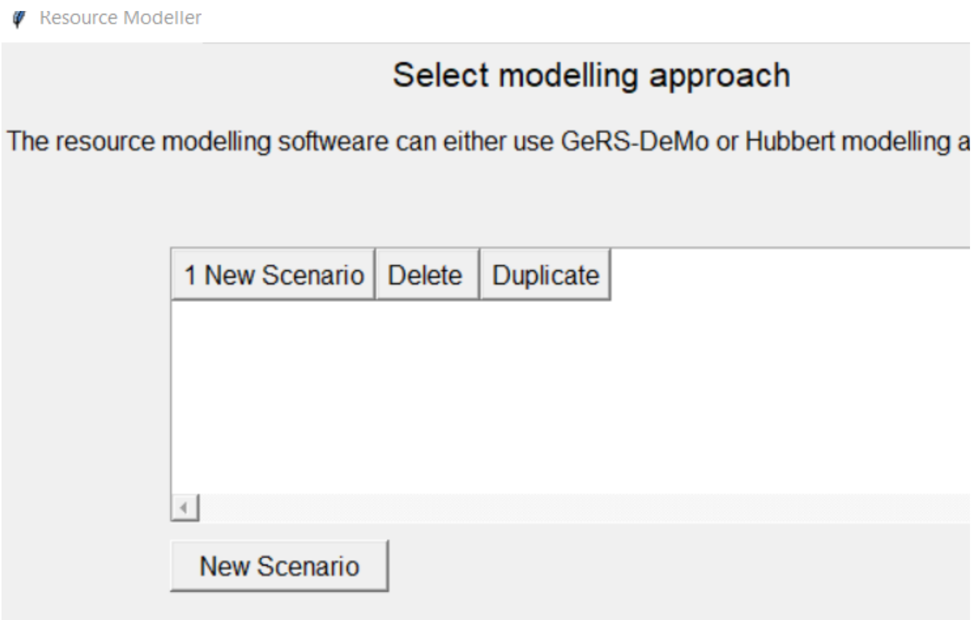


Figure 6: New Scenario added

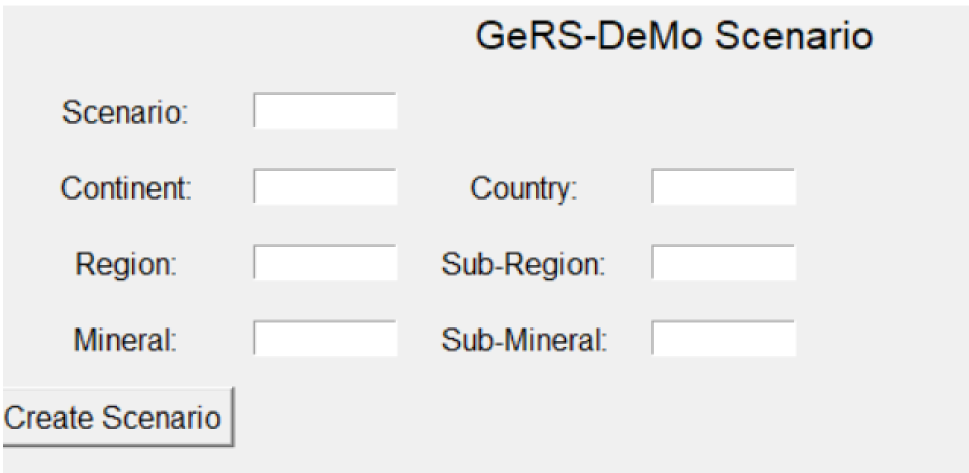


Figure 7: Initialise New Scenario

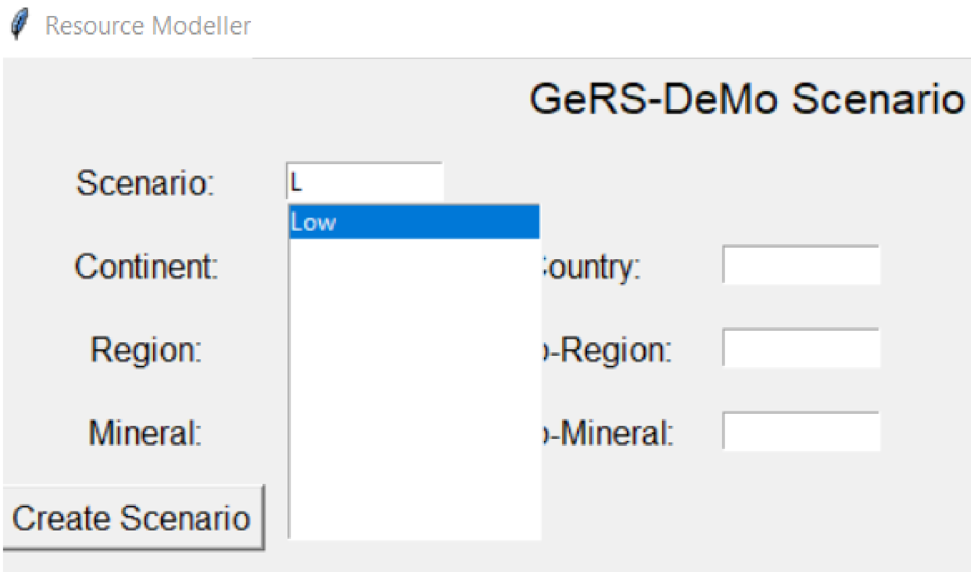
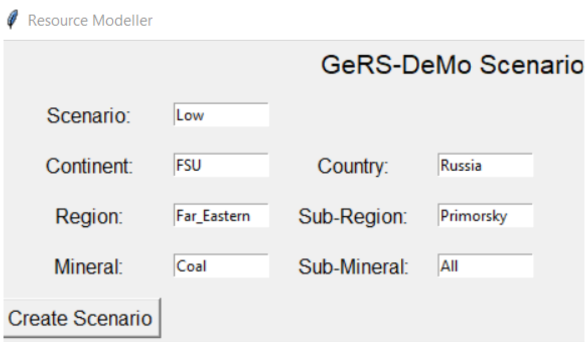
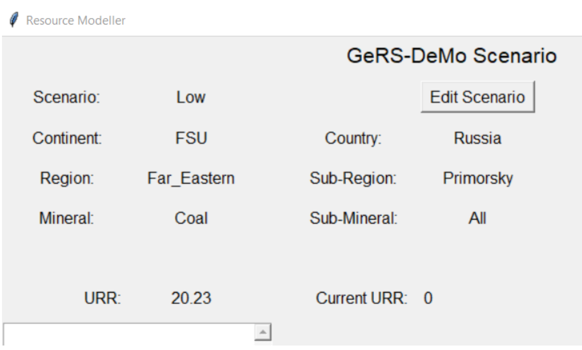


Figure 8: Example of dropdown suggestions



(a) Before



(b) After

Figure 9: Filling a scenario that exists in the URR file

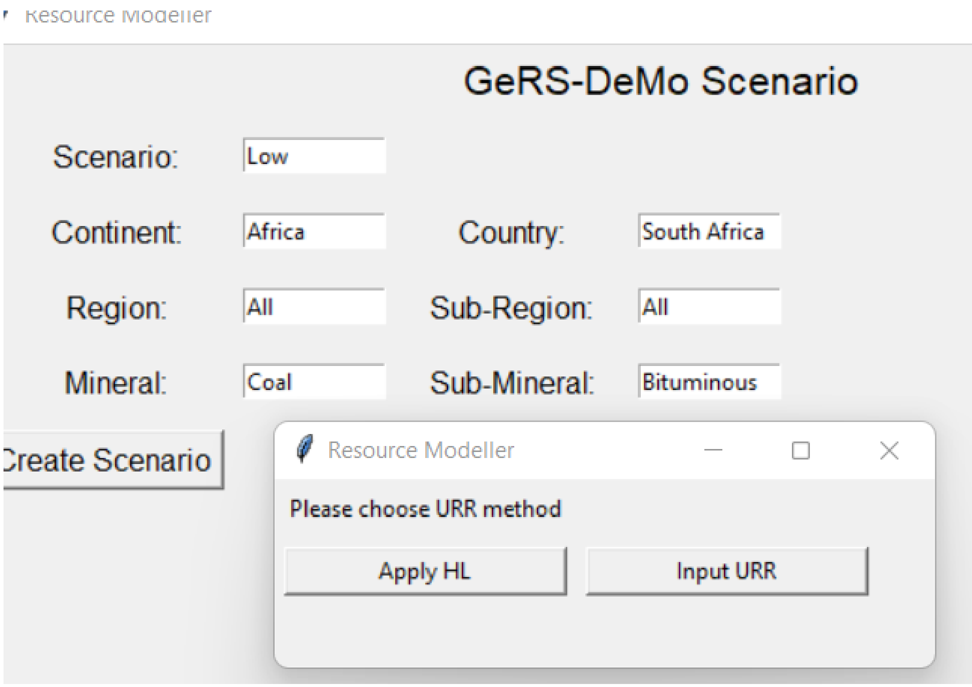


Figure 10: Hubbert Linearisation or URR input pop-up

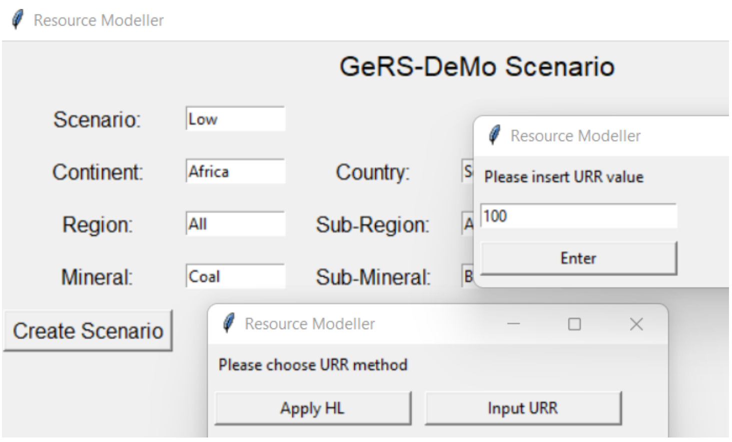


Figure 11: Input URR value manually

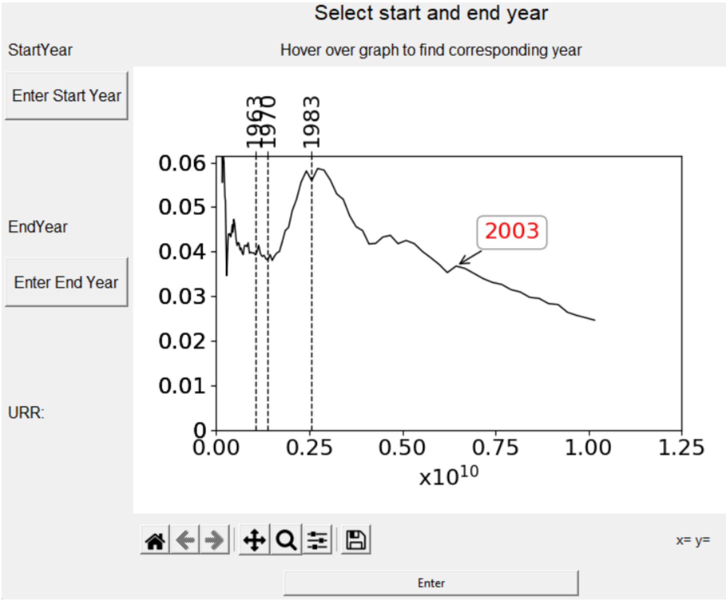


Figure 12: Hubbert Linearisation page

3.1.3.2 Hubbert Linearisation

Selecting the Hubbert Linearisation, brings up a second pop up, this pop up plots the cumulative production on the X axis with the Production divided by the cumulative production on the Y axis. The point where the line of best fit crosses the X axis is the Hubbert Linearisation estimate of the URR. Selecting 'Apply HL' causes the production data to be plotted for the HL graph, as indicated in Figure 12. As indicated, hovering over the data indicates the year that it is. The crossroads arrows sign lets you move the graph around, and the magnifying glass lets you zoom. The home button restores the graph to the original design.

To calculate the Hubbert Linearisation URR estimate, press the 'Enter Start year' and 'Enter End Year' buttons to enter the start and end years that the line of best fit to the data should use. Upon doing this the URR estimate is displayed in the bottom left.

Pressing 'Enter' on the Hubbert Linearisation page gets to the Scenario Page as indicated in figure 14. The number highlighted by the red box is the URR that the model needs to replicate. The number in the green box is the models current URR.

3.1.3.3 Cycle

Pressing 'Add New Cycle' inputs blank mine or field cycle in to the model - as indicated by having a row of three buttons appear in the scroll box. As with the scenario buttons, these buttons behave the same (duplicating, deleting or editing the cycle) - see Fig 15.

Pressing the '1 Cycle' button we can add inputs to the cycle. In a cycle we can flick between making it a fields input or a mines input (see Fig 16). Note, in this section the inputs to either the mines or fields are entered. Multiple mines/fields will be put online based on the parameters inputted.

3.1.3.4 Examples of Mines and Fields Cycles

If we load the 'Dynamic_BG.xlsm' file, we can see examples of both mines and fields.

Figure 17 shows an example of mining inputs. To link these back to the notation:

- **Start Year:** is the year the production should start (Y_s)
- **Mine Prod Low:** is the lowest maximum production (M_L)
- **Mine Prod High:** is the highest maximum production (M_H)
- **Mine Life Low:** is the lowest operating life of mine (L_L)
- **Mine Life High:** is the highest operating life of mine (L_H)

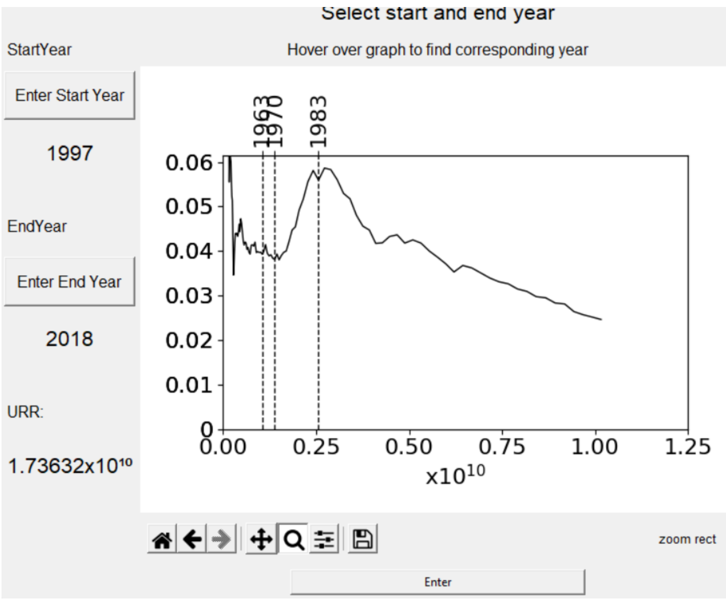


Figure 13: Hubbert Linearisation page with entered years

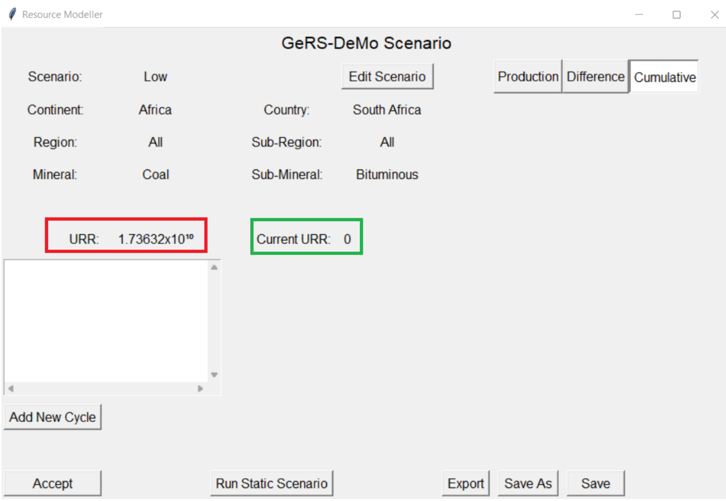


Figure 14: Scenario page with the URR and the current URR total

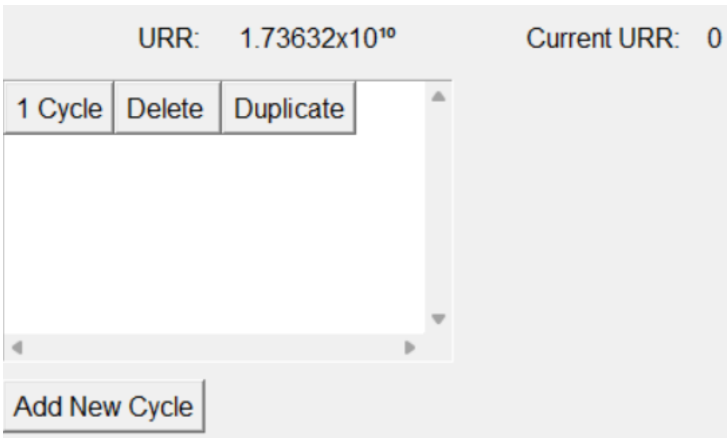
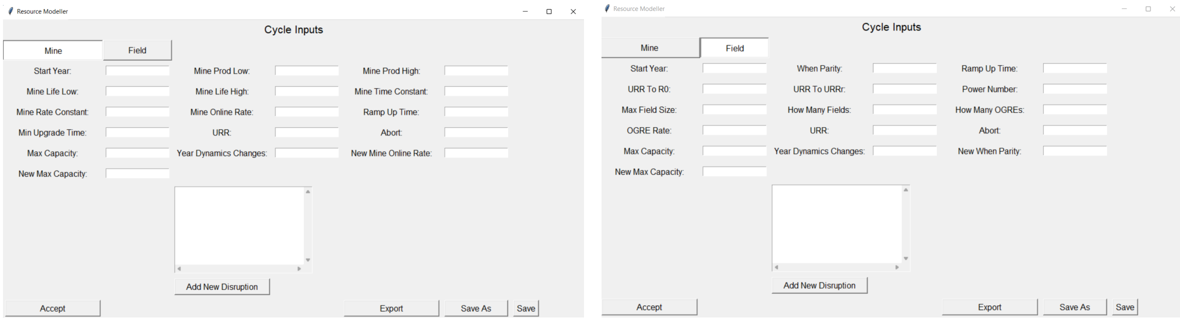


Figure 15: Cycle buttons appearing on clicking the 'Add New Cycle'



(a) Before (b) After

Figure 16: Toggling between the Mine inputs and Field inputs, only fill out one tab

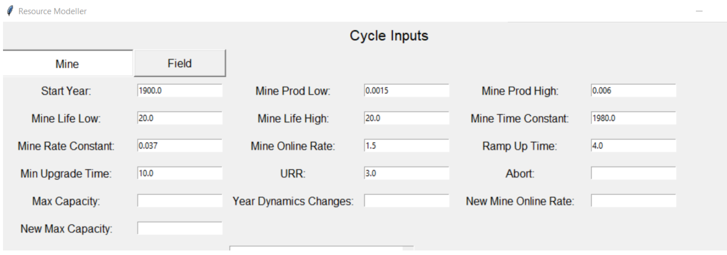


Figure 17: Example of mine inputs

- **Mine Time Constant:** is the mid year (year half way between low and high mining conditions) (t_t)
- **Mine Rate Constant:** This is the rate of change between the low and high conditions (r_t)
- **Mine Online Rate:** This is the rate at which mines comes online (r_{Q_T})
- **Ramp Up Time:** This is the time individual mines take to reach max production (t_M)
- **Min Upgrade Time:** This is the minimum life remaining for the mine to enable it to be upgraded (G_{M_L})
- **URR:** This is the Ultimately Recoverable Resources (Q_T)
- **Abort:** This is a year input, when the model reaches this year it shuts all production off into the future.
- **Max Capacity:** This is a production value, the model will not exceed this value.
- **Year Dynamics Changes:** This is a year input, after this year the Mine Online Rate switches to the New Mine Online Rate, and the Max Capacity changes to the New Max Capacity
- **New Mine Online Rate:** This is the new mine online rate (only use if Year Dynamics Changes is used)
- **New Max Capacity:** This is the new max capacity (only use if Year Dynamics Changes is used)

The variables up to and including URR are required.

Figure 18 shows an example of fields and Oil and Gas Regions (OGREs) inputs. To link these back to the notation:

- **Start Year:** is the year the production should start (Y_s)
- **When Parity:** This is inverse of the rate of fields and needs to be less than 1 ($1/r_F$)
- **Ramp Up Time:** This is the time in years for the field to reach max production (t_F)
- **URR to r0:** This is the proportionality constant ($\tilde{F}_P$)

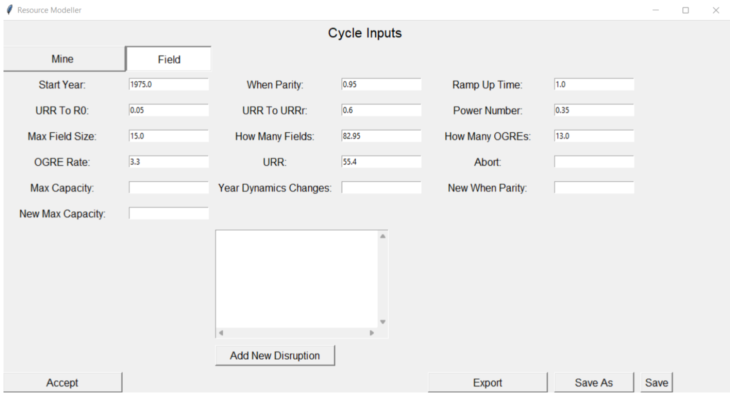


Figure 18: Example of field inputs

- **URR to URRr:** This is the proportionality constant ($\tilde{Q}_r$)
- **Power Number:** This is the exponent rate constant (r_Q)
- **Max Field Size:** This overrides the size of an individual field (Q_{T_F}) to be not exceed Max Field Size
- **How Many Fields:** This is the number of fields (n_{F_T})
- **How Many OGREs:** This is the number of oil or gas regions to put on (n_{R_T})
- **OGRE Rate:** This is the rate constant that controls how fast OGREs are brought online (r_ϵ)
- **URR:** This is the Ultimately Recoverable Resources (Q_T)
- **Abort:** This is a year input, when the model reaches this year it shuts all production off into the future.
- **Max Capacity:** This is a production value, the model will not exceed this value.
- **Year Dynamics Changes:** This is a year input, after this year the Mine Online Rate switches to the New Mine Online Rate, and the Max Capacity changes to the New Max Capacity
- **New When Parity:** This is the new when parity value (only use if Year Dynamics Changes is used)
- **New Max Capacity:** This is the new max capacity (only use if Year Dynamics Changes is used)

The variables up to and including URR are required.

3.1.3.5 Scenario

By clicking on the 'Run Static Scenario' in the Scenario page (see figure 14) the static version of the model will run the current individual scenario in static mode. This will create a plot of cumulative production, annual production and if actual production data is supplied to the model a plot of the difference in actual and modelled production - see figure 19.

3.1.3.6 Global Inputs Page

Pressing next on the 'Select Modelling Approach' Page (Figure 4) moves the model to the 'Global Inputs' page - shown in Figure 20. The variables in the Global Inputs page compare to formulas as follows:

- **Shut off Percent:** This is the fields shut off portion ($\tilde{P}_S$)
- **Time Delay:** This is an obsolete option to set a delay between the field/mine is brought online, set this value to 0.
- **Initial Demand:** This is the initial per capita demand ($\tilde{D}_i(Y_s)$)

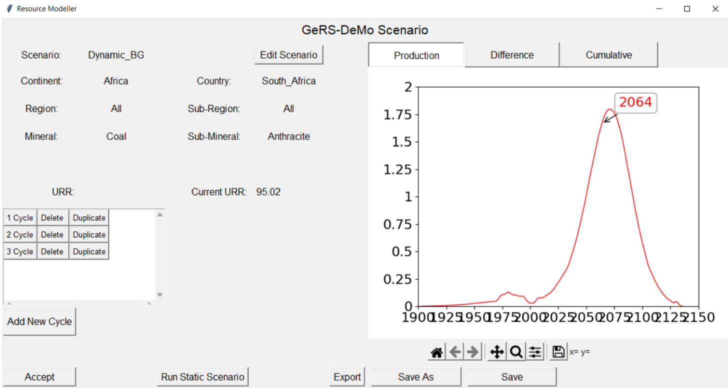


Figure 19: Example of running the static model on an individual scenario

- **Demand Rate:** This is the per capita demand rate constant (r_D)
- **Max Demand:** This is the maximum per capita demand ($\tilde{D}_H$)
- **K1:** This is the demand interaction term (k_D)
- **K2:** This is the field interaction term (k_F)
- **K3:** This is the mine interaction term (k_M)
- **K4:** This is the region interaction term (k_R)
- **When Fields:** This is the region minimum supply demand gap (G_{R_L})
- **K5:** This is the mines upgrade interaction term (k_U)
- **When Upgrade:** This is the mines upgrade minimum supply demand gap (G_{M_L})
- **Gap Delay:** This is the number of years before the supply and demand interactions come into affect
- **Max Population:** Maximum population (p_H)
- **Initial Population:** Minimum population (p_L)
- **Population Rate:** Population rate constant (r_m)
- **Population Mid Year:** Population mid year time constant (t_m)
- **Population Const B:** Population asymmetry constant (p_γ)
- **Population Gamma:** Population asymmetry factor (γ_m)
- **Life of Product:** Useable life of a resource (L_u)
- **Min Recycled:** Minimum recycling rate (R_L)
- **Max Recycled:** Maximum recycling rate (R_H)
- **Rate Recycled:** Recycling rate constant (r_l)
- **Mid Year:** Recycling mid year (t_l)

To run the model in static mode, set the variables K1 to K5 to 0. To run the model click ‘Run Model’.

3.1.3.7 Post Processor

After clicking Run Model, the model runs and takes you to the Post Processor Outputs page. Once again there are Production, Cumulative and Difference buttons, to see the model. The resulting image can be readily manipulated. The button ‘Write Results to Excel’ will create an excel spreadsheet of the results. Similarly ‘Write Results to PDF’ will create a pdf document of the results.

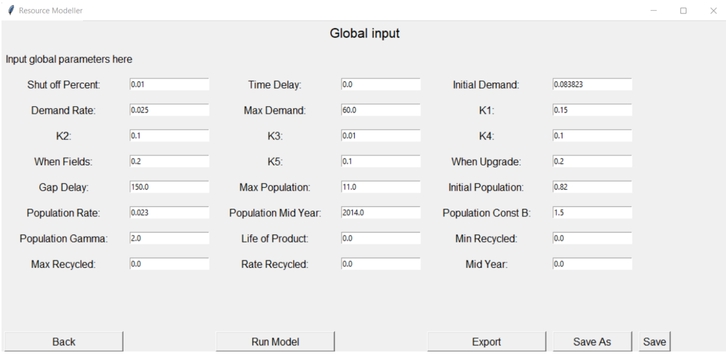


Figure 20: Global Inputs page

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Mohr, S. (2010). *Projection of World fossil fuel production with supply and demand interactions*. PhD thesis, University of Newcastle, Australia. http://www.theoildrum.com/node/6782.

Mohr, S., Wang, J., Ward, J., and Giurco, D. (2021). Projecting the global impact of fossil fuel production from the Former Soviet Union. *International Journal of Coal Science and Technology*, 8:1208–1226.

Mohr, S. H., Mudd, G. M., and Giurco, D. (2012). Lithium resources, production: critical assessment and global projections. *Minerals*, 2(1):65–84.