TABLE 5.4.2 NEWMARK'S METHOD: LINEAR SYSTEMS†

Special cases

(1) Constant average acceleration method ($\gamma = \frac{1}{2}$, $\beta = \frac{1}{4}$)

(2) Linear acceleration method ($\gamma = \frac{1}{2}$, $\beta = \frac{1}{6}$)

1.0 *Initial calculations*

1.1 $\ddot{u}_0 = \dfrac{p_0 - c\dot{u}_0 - ku_0}{m}$.

1.2 Select Δt.

1.3 $a_1 = \dfrac{1}{\beta(\Delta t)^2}m + \dfrac{\gamma}{\beta\Delta t}c$; $a_2 = \dfrac{1}{\beta\Delta t}m + \left(\dfrac{\gamma}{\beta} - 1\right)c$; and

$$a_3 = \left(\frac{1}{2\beta} - 1\right)m + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)c.$$

1.4 $\hat{k} = k + a_1$.

2.0 *Calculations for each time step, $i = 0, 1, 2, \ldots$*

2.1 $\hat{p}_{i+1} = p_{i+1} + a_1 u_i + a_2 \dot{u}_i + a_3 \ddot{u}_i$.

2.2 $u_{i+1} = \dfrac{\hat{p}_{i+1}}{\hat{k}}$.

2.3 $\dot{u}_{i+1} = \dfrac{\gamma}{\beta\Delta t}(u_{i+1} - u_i) + \left(1 - \dfrac{\gamma}{\beta}\right)\dot{u}_i + \Delta t\left(1 - \dfrac{\gamma}{2\beta}\right)\ddot{u}_i$.

2.4 $\ddot{u}_{i+1} = \dfrac{1}{\beta(\Delta t)^2}(u_{i+1} - u_i) - \dfrac{1}{\beta\Delta t}\dot{u}_i - \left(\dfrac{1}{2\beta} - 1\right)\ddot{u}_i$.

3.0 *Repetition for the next time step.* Replace i by $i + 1$ and implement steps 2.1 to 2.4 for the next time step.

†If the excitation is ground acceleration $\ddot{u}_g(t)$, according to Eq. (1.7.6), replace p_i by $-m\ddot{u}_{gi}$ in Table 5.4.2. The computed u_i, $\dot{u}_i$, and $\ddot{u}_i$ give response values relative to the ground. If needed, the total velocity and acceleration can be computed readily: $\dot{u}_i^t = \dot{u}_i + \dot{u}_{gi}$ and $\ddot{u}_i^t = \ddot{u}_i + \ddot{u}_{gi}$.

Newmark's method is stable if

$$\frac{\Delta t}{T_n} \le \frac{1}{\pi\sqrt{2}}\frac{1}{\sqrt{\gamma - 2\beta}} \tag{5.4.15}$$

For $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{4}$ this condition becomes

$$\frac{\Delta t}{T_n} < \infty \tag{5.4.16a}$$

This implies that the constant average acceleration method is stable for any Δt, no matter how large; however, it is accurate only if Δt is small enough, as discussed at the end of