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T3 ________________	Problem Chosen	F3 ________________
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Summary Sheet

Electric Vehicle Charging Network Optimization

With its advantages of zero emission, high energy efficiency and low noise, electric vehicles are being developed and popularized all over the world. The key factor in the development and promotion of electric vehicles is how to plan and build a charging network. This article is based on the study of the development of charging networks in different countries under the network optimization concept.

Problem 1, we set up a **three-layer point pair charging network** model for cities, suburbs and rural areas in USA to calculate the number of charging stations and the total number of stations in the nation that will be fully populated with all-electric. First, we set up a **queuing theory model** based on traffic conditions throughout the United States to determine the best charging station (18 piles) to service vehicle ratio of 1: 266. Then we assume that the total number of cars after the all-electric in USA is the same as today's. According to the number of vehicles in each of the three types regions of each state, we can determine the number of charging stations they need to build respectively. Finally, our statistics of 50 states and Washington DC show that the United States needs a total of 902,286 charging stations. Among them, the urban, suburban and rural investment ratio is 33:51:16. (For detailed state data, see Appendix 3)

Problem 2a, we selected Ireland on the basis of problem 1 model to conduct a study by establishing a multi-objective programming model with the goal of a minimum number of stations and their optimal deployment. Then we use the ant colony algorithm based on network optimization layout to find that if Ireland implements all-electric it needs at least to build 11512 charging stations. Figure 4 shows the distribution that we finally found the key factor in the network layout is the charging station service capability.

Problem 2b, we set up a **three-stage evolution charging network** development plan for Ireland based on the different density of charging stations in the network through a comprehensive consideration of vehicle density and urbanization in various regions of Ireland.

Problem 2c, based on the evolutionary charging network established by 2b, we use the **bass diffusion prediction model** to predict the market share of EVs in the future based on the development of EVs over the past decade in Ireland. It found that market share will reach 10.3% by 2020, 21.8% by 2025 and 98.6% by 2048.

Problem 3, based on the three-level network model established in problem 2b, we set up a classification system based on the network level according to the geographical characteristics, population and wealth distribution of different countries. The data we enter into Australia, China and Singapore separately set different levels of network development for them. For example, China will build a three-tier evolution charging network with dense charging stations in the eastern coastal areas, dense inland areas in the central region, and sparse western regions. (See Figure 10 for other areas)

Problem 4, we separately analyzed the impact of different modes of travel on the use of electric vehicles separately, and made the illusion of possible future modes of travel.

Problem 5, in combination with the model in the above issue, we compiled a handout for the leaders of participating countries to make it better to develop and promote the electric mobility in their countries.

Keywords: Target programming; Queuing theory; Development network; Bass diffusion prediction model

Contents

1 Introduction

1.1 Background

In recent years, environmental issues have become increasingly prominent, and how sustainable development has become one of the hottest research topics of the moment. Traditional energy storage is limited and not renewable. Traditional cars use oil to power their engines. Exhaust gas has become the most important source of air pollution, far exceeding other sources of pollution. Therefore, the use of clean and renewable resources for motor fuel will undoubtedly greatly alleviate the environmental pressure in the world

The emergence of Tesla's pure electric car can be said to solve the problems caused by these traditional cars. Electric vehicles are adapting to the development trend of the current world both in terms of energy use and energy efficiency. However, the transition to the traditional car to the electric car is not easy, including research on the distribution of charging stations, quantity, location and other issues are also indispensable.

1.2 Previous Research

So far, people have studied all aspects of the development of electric vehicles. The global electric vehicle market is growing at a faster rate, and both the production and sales of electric vehicles have been significantly improved [1]. The development of electric vehicles is closely related to the construction of charging stations [2].

In [3], according to the actual problems that appeared in the planning and construction of electric vehicle charging station, the layout problem of charging facilities was studied systematically. The planning of the charging station mainly includes site selection and fixed capacity. The planning of the charging station not only affects the convenience of the users of electric vehicles and thus affects the promotion and use of the electric vehicles, but also affects the power quality of the distribution system. The advantage of the Bass model introduced in [5] is that it can fully consider the external factors when predicting the proliferation of new products, so that the model can be widely used in the promotion of new industrial products.

Combined with the research results of predecessors and with the support of the necessary technologies and theories, our team established relevant models to solve the problem of electric vehicle development and site planning for charging stations.

1.3 Our work

Problem 1, we set up a three-layer point-to-point charging network model for cities, suburbs and rural areas in the United States to calculate the number of charging stations and the total number of stations that need to be fully invested in each area of electric mobility.

Problem 2a, we set up a multi-objective programming model with the goal of minimizing the number of charging stations and their optimal deployment in Ireland, and then use ant colony optimization based on network layout to solve the model.

Problem 2b, we build a three-level charging network model based on the density of charging stations in the charging network, and establish a three-level charging network development model according to the feature alignment in different regions of Ireland.

Problem 2c, we use the bass diffusion prediction model to predict the future EV market share changes using the Irish first decade data.

Problem 3, we set up a classification system based on a number of factors based on the network level, and substitute some national data to formulate different charging network development bills.

Problem 4, we separately analyzed the impact on different modes of travel on the use of electric vehicles separately, and made the illusion of possible future modes of travel.

Problem 5, in combination with the model in the above issue, we compiled a handout for the leaders of participating countries to make it better to develop and promote the electric mobility in their countries.

2 Assumptions and Justification

Assumption 1: Suppose the data we obtained from our research in various countries are reliable and accurate.

- **Reason:** Because data on the Internet is not always up-to-date, the data collected during country studies may be somewhat inconsistent with the country's most recent data.

Assumption 2: In building the network of charging stations in the United States, we assume that the U.S. government will do its utmost to invest in charging networks.

- **Reason:** Because the U.S. charging network is huge, it is very hard to build a nationwide charging network if the government does not support it.

Assumption 3: In Question 2c on the country's future market forecast for electric vehicles, bass model for the time being does not consider the impact of consumer repeat purchases car on the forecast results.

- **Reason:** The Bass model of product diffusion prediction does not consider the case of repeated purchases by consumers.

3 Symbols and Definitions

In the section, we use some symbols for constructing the model as follows:

symbol	symbol description
N	The total number of charging stations needed in the country
N_s	The number of charging stations required for each state
N_h	The number of charging stations required for the highway between states
C_i	The number of state electric vehicles
C_s	Standard charging station maximum service capacity
M_c	The number of charging piles
μ_c	service rate of quick service desk
Q	Demand point of demand

P.s. Other symbols instructions will be given in the text

4 Our solutions

4.1 Solution for task one

4.1.1 Problem analysis

Seeking the number of charging stations required throughout the country, when all the vehicles in the United States have become electric vehicles. We get access to information on all states and Washington D.C electric vehicle ownership (the number of electric cars on the current number of cars). Then, we calculate the number of charging stations required for each state based on the number of vehicles that each standard charging station can serve. Finally, calculate the number of charging stations needed to connect the freeways between states, and the sum of the two is the final number of charging stations needed in the United States. According to the national number of charging piles, we will then count the number of vehicles for each of the United States cities, suburbs and rural areas to determine the number of charging stations required for each area.

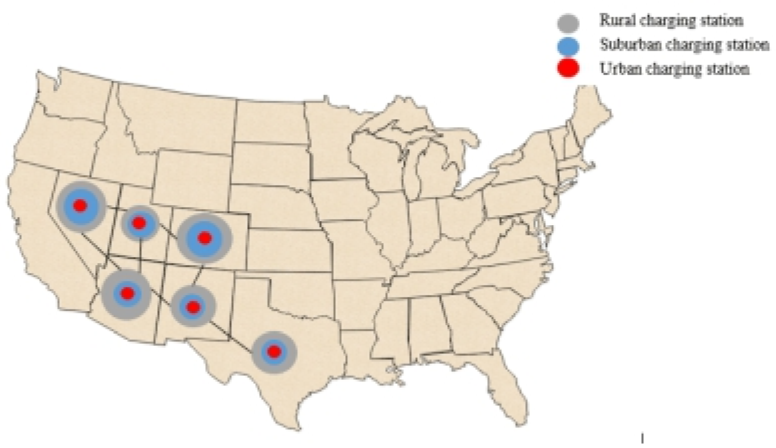


Figure 1 Three-layer point pair network

Through the above analysis, we build a three-layer point pair network model to describe the distribution of US electric vehicle charging station. We have focused on the distribution of charging stations in urban, suburban and rural areas within each state as a three-layer point pair. The center of the red represents the charging station in all the cities in each state, and the second ring of blue represents all the state's charging stations in the suburbs. The outermost gray ring represents all the state's charging stations in rural areas. Connection represents the highway between states. Finally, the total number of charging stations that need to be built in the United States is the number of charging stations in all points and connections in the network diagram.

4.1.2 Model 1: Optimal Charging Network Layout Based on USA

(1) Determine the target

From the three-layer point pair to the network model analysis, We know that the minimum number N of charging stations required throughout the country is equal to the sum of the required number of charging stations N_h for the states required by the minimum number N_s of states and the states.

$$\min \quad N = N_s + N_h \tag{1}$$

The number of charging stations required for M_j (j = city, suburban and rural areas), R_j is the percentage of car ownership by region for each country. So:

$$M_j = R_j \times N \tag{2}$$

(2) Determine the constraints

- Suppose the total length of the U.S. highway is L, the number of charging stations required for a hundred miles d, so N_h is:

$$N_h = \frac{L}{d} \tag{3}$$

- C_i is the i state electric vehicle ownership, n_i is the number of states i state charging stations, each charging station service S cars

$$n_i = \frac{C_i}{S} \tag{4}$$

- The total number of stations N_S required for the states and Washington D.C:

$$N_s = \sum_{i=1}^{51} n_i \tag{5}$$

(3) Determining the optimal charging station composition based on the queuing theory model

The M / G / K queuing model can be used to estimate the number of charging stations that should be configured for a standard charging station. The mathematical theory of this formula[18] is as follows：

Average queue length:

$$T_q = \frac{D(S+G)+\left[E(S+G)\right]^2}{2E(S+G)\left[N-\lambda E(S+G)\right]}\left[1+\sum_{i=0}^{N-1}\frac{(N-1)!\left[N-\lambda E(S+G)\right]}{i![\lambda E(S+G)]^{K-i}}\right]^{-1} \tag{6}$$

Average length of stay:

$$T_d = E(S+G)+T_q \tag{7}$$

Average queue length:

$$L_q = \frac{\lambda D(S+G)+\lambda\left[E(S+G)\right]^2}{2E(S+G)\left[N-\lambda E(S+G)\right]}\left[1+\sum_{i=0}^{N-1}\frac{(N-1)!\left[N-\lambda E(S+G)\right]}{i![\lambda E(S+G)]^{N-i}}\right]^{-1} \tag{8}$$

Because the charging station on the car charging service time and charge to complete the car departure time subject to normal distribution, so the following formula:

$$E(S+G) = E(S)+E(G) \tag{9}$$

$$D(S+G) = D(S)+D(G) \tag{10}$$

Where, λ is the average car strength; N is the number of standard charging station charging pile. E(S) is the charging service time expectations; E(G) is the departure time expectation; D(S) is the variance of charging service time; D(G) is the variance of the departure time.

4.1.3 Model solution

(1) Determine the number of electric vehicles in each state and Washington D.C

From the state population data [7], the number of cars per capita [8], we get the current private car ownership in each state of the United States, and then we directly convert it into the electric car ownership as shown in the following Table 1.(take 6 of them as follows, **PCNC**: Per capita number of cars, see Appendix 3 for complete data)

Table 1 The number of vehicles in the United States

	New York	New Jersey	Florida	Hawaii	California	Michigan
Population	19,378,087	8,791,936	18,804,623	1,360,301	37254503	9,884,129
PCNC	0.57	0.69	0.71	0.76	0.84	0.87
Vehicles	11045510	6066436	13351282	1033829	31293783	8599192

(2) Determine the standard charging station composition and service capabilities

According to the inquiry, the annual average daily traffic volume of USA is 38594. Assuming the number of average waiting vehicles is 1.5, the traffic coefficient k is 0.12, and the average service level is 6s. We get the result of a standard charging station. We get the following results:

Table 2 The best charging station

	Supercharging	Destination charging	Service capabilities
Amount	8	10	266

(3) Determine the number of charging stations required in each state

According to service capacity of charging station, the number of charging stations required by each state is calculated as follows (only 6 of them are listed, and all data are shown in Appendix 3)

Table 3 The number of charging stations required for each state

	New York	New Jersey	Florida	Hawaii	California	Michigan
Amount	41524	22806	50193	3887	117646	32328

With these calculations, if the United States fully enters the era of electric vehicles, 50 states and Washington D.C, will need 899,206 standard charging stations.

(4) Determine the number of charging stations of freeways and USA

According to the literature [4], the total length of the U.S. highway is L = 77017km. According to the literature [5], the number of charging stations per 100km is generally 4, with two on each side. So there are 3,080 charging stations on the interstate freeway. So the final number of charging stations needed for the United States is 902286 charging stations.

(5) Determine the number of city, suburban and rural charging stations

According to the data [9], the ratio of the population of the U.S. population in the urban, suburban and rural areas in 2017 is as follows:

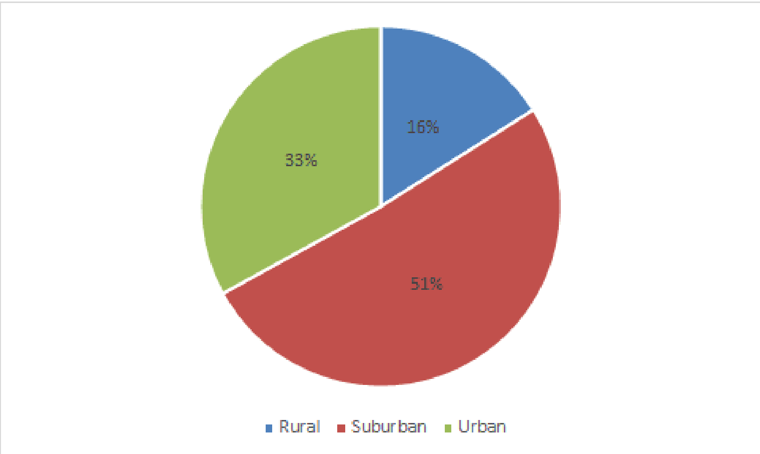


Figure 2 The proportion of people in each regions

As the US vehicles are cheap, we assume that the population proportion can roughly represent the distribution of vehicles, so as to estimate the number of charging stations required in the three regions of the United States, suburban areas and rural areas as follows:

Table 4 The number of charging stations in various regions

	Urban	suburban	rural areas	Total
Amount	296738	458595	143873	899206

4.1.4 Result analysis

It can be seen from the above data that the United States currently has 240 million vehicles. If all are converted into electric vehicles, 902286 standard charging stations will need to be built in 50 states and the Washington DC. Therefore, in order to achieve full electric carization, the government and people are required to work hard together to gradually turn the existing gas station into a charging station. And by looking at the distribution of the U.S. population, we found that eventually the ratio of charging stations in urban, suburban and rural areas was roughly the same, with a value of 33:51:16.

4.1.5 Sensitivity analysis

By the formula $N_s = \sum_{i=1}^{51} \frac{C_i}{S}$, We know that the total number of standard charging stations N_S to be built by each state varies with the service capacity S of the charging station. Therefore, sensitivity analysis is performed on the parameter of the number of service stations of a standard charging station. We charge the number of vehicles at the charging station S = 266 float up and down 20 units, calculate the final N_S changes.(**NEC:** The number of Electric car, **CSSCN**: Charging station service car number, **TNSCS:** The total number of state charging stations)

Table 5 Sensitivity analysis table

NEC	239188884	239188884	239188884	239188884	239188884
CSSCN	226	246	266	286	306
TNSCS	1058357	972312	899206	836324	781663

Data change line chart is as follows：

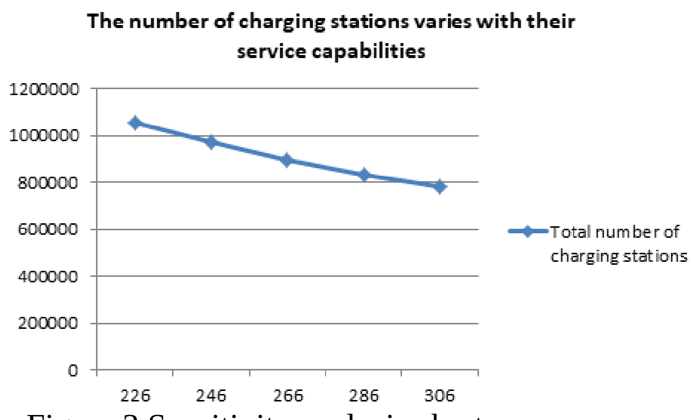


Figure 3 Sensitivity analysis chart

As can be seen from the figure above, the total number of charging stations is positively related to the service capability of a single charging station, so increasing the service capacity of charging stations plays a greater role in reducing the total number of charging stations.

4.2 Solution for task two

4.2.1 Problem 2a: Programming and layout of electric vehicles

4.2.1.1. Analysis of the problem

Electric cars are usually not uniform distributed in cities, densely populated areas are often areas where urban dwellers are more active, the distribution of electric vehicles can be considered,to be related to the distribution of the population without considering the difference of purchasing power and the degree of approval of electric vehicles in different regions. Therefore, we divide the map of a city according to the population density, obtain the charging demand of the electric vehicle in the corresponding area, and determine the number of charging station.

4.2.1.2. Model 2: For Irish charging pile delivery optimization model

(1) determination of target

For the optimal number of charging piles: from the three-layer point pair network model of Problem 1, the optimal number of charging stations $n_{i,\min}$, k , min, k in the i -th region in the country is the number of administrative regions divided by the country. National best charging pile number N :

$$\min \quad N = \sum_{i=1}^{k} n_{i,\min} \tag{11}$$

For optimal charging station layout: we use the center of gravity pointing facility location selection model, the goal is to make the demand point to the facility point of the weighted distance and T_c value minimum, the mathematical form of the model is as follows:

$$\min \quad T_c = \sum_{i=1}^{m} E_{ij} d_{ij,} (j = 1, 2 \cdots n) \tag{12}$$

Where, n is the number of alternative facility points; The number of demand points is M ; Demand point i to the demand intensity of the facility point j is E_{ij} , d_{ij} is the European distance for the demand point i to the facility Point j .

(2) determine constraints

- Distance between facilities: the European distance of the demand point $i\left(x_{i,}y_i\right)$ to the facility point $j\left(x_{i,}y_i\right)$ is d_{ij} .

$$d_{ij} = \sqrt{(\mathrm{x}_i - x_j)^2 + (\mathrm{y}_i - y_j)^2} \tag{13}$$

- Shortest charging station site coordinates: By the demand distribution of city charge, the demand of any demand point is Q, its position coordinate is $\left(x, y\right)$, then the charging station site coordinates with the shortest charge mileage are:

$$X = \frac{\sum_{i=1}^{m} Q_i \times x_i}{\sum_{i=1}^{m} Q_i} \quad Y = \frac{\sum_{i=1}^{m} Q_i \times y_i}{\sum_{i=1}^{m} Q_i} \tag{14}$$

M is the demand point for the charging station service. $\left(x_{i,}y_i\right)$ represents the coordinates of the i-th demand point.

- Service radius of charging station r: considering the vehicle driving speed V, the customer's maximum search time t, the curvature coefficient of the road.

$$r = \frac{V \times t}{\lambda} \tag{15}$$

- Standard charging station charging requirements: Number of vehicles requiring charging service per day (train/day) N, average mileage per car (km) SY, vehicle driving days per year T, the battery can continue driving mileage D. Charging station daily charge demand B_0:

$$B_0 = N \times \frac{SY \times T}{SD} \tag{16}$$

- Number of charging stations in each area: $C_s^{'}$ is the service capacity of standard charging station, the number of charging stations to be put in region i is n_i:

$$n_i \times C_s^{'} \geq B_0 \tag{17}$$

- Service capacity of charging station: the composition of the standard charging station determined by question 1, then combined with the traffic situation in the country, determines the service capability of the charging station $C_s^{'}$ as follows:

$$C_s^{'} = M_c \times \mu_c \times t \tag{18}$$

The number of the M_c for the charging station; service rate is μ_c.

(3) Optimal charging station layout model based on the quantity of the least charging station

$$\min \quad N = \sum_{i=1}^{k} n_{i,\min}$$

$$\min \quad T_c = \sum_{i=1}^{m} E_{ij} d_{ij,} (j = 1, 2 \cdots n)$$

$$s.t \begin{cases} d_{ij} = \sqrt{(\mathrm{x}_i - x_j)^2 + (\mathrm{y}_i - y_j)^2} \\ X = \dfrac{\sum_{i=1}^{m} Q_i \times x_i}{\sum_{i=1}^{m} Q_i}, Y = \dfrac{\sum_{i=1}^{m} Q_i \times y_i}{\sum_{i=1}^{m} Q_i} \\ r = \dfrac{V \times t}{\lambda} \\ B_0 = N \times \dfrac{SY \times T}{SD} \\ n_i \times C_s^{'} \geq B_0 \\ C_s = M_c \times \mu_c \times t \end{cases} \tag{19}$$

4.2.1.3. Model Solution

(1) Algorithm idea

We use the improved ant colony algorithm based on network optimization layout to solve the optimization problem, we have fewer nodes and wide coverage in the network. Then by initializing a certain number of ants to construct the network, repeated iterations to obtain the optimal number of charging station construction and layout.

(2) Solution result

By improving the ant colony algorithm, we obtain the optimal charging network layout for Ireland, the minimum number of charging piles in 26 counties in the optimal network is as follows (List only the first 6 data, all results are shown in appendix 5):

Table 6 Number of vehicles in each county

	Dublin	Louth	Kildare	Meath	Wicklow	Cork
Cars	503,726	47,300	93,014	77,137	65,824	231,743
Charging station	3498	328	646	536	457	1287

We use the MATLAB program to calculate the national service for all vehicles in Ireland the optimal charging network for the construction of the standard charging station is 11512 (including 207216 charging pile). The calculated part of the charging station coordinates data into the ArcGIS software to get the approximate charging pile distribution diagram is as follows:

Figure 4 Optimal distribution

4.2.1.4. Result Analysis

From the above results, it can be seen, that the service capacity of the charging station has an important impact on the layout of the optimal charging network. The stronger the service capacity of the charging station, the less the number of charging stations needed and the thinner the charging network.

4.2.2 Problem 2b: Construction of charging station network

4.2.2.1. Analysis of the problem

For the plan of Irish charging network from 0 to the national coverage, we propose a comprehensive layout construction plan based on the national regional development section. Reference to the world's leading developing country in the field of China's charging network to develop standards [16], get three kinds of high, medium and low charging levels of different degrees of development. Then combined with the actual situation in Ireland to develop a three network charging network planning.

(1) Standard definition

Through the data found that charging pile and electric vehicles into the P and charging station service radius R these two factors directly affect the degree of development of the charging network. So our three types of charging networks based on the developed world standards in this field are as follows:

- **Developed network:** Developed network in the charging pile and the vehicle number ratio P larger, charging station service radius R small.
- **Second developed network:** The second developed network relatively developed P small, R larger. Charging pile input density more developed network sparse.
- **General network:** The general network is relatively less second developed P , R is larger. Charging pile input density less second developed network sparse.

According to the world's most advanced national standard [16], the P and R of each charging network are as follows:

Table 7 Network density standards at all levels

	Developed	Second developed	General
P	1:8	1:10	1:12
$R(km)$	1.2	1.8	2.4

(2) Three network integration charging network evolution is as follows:

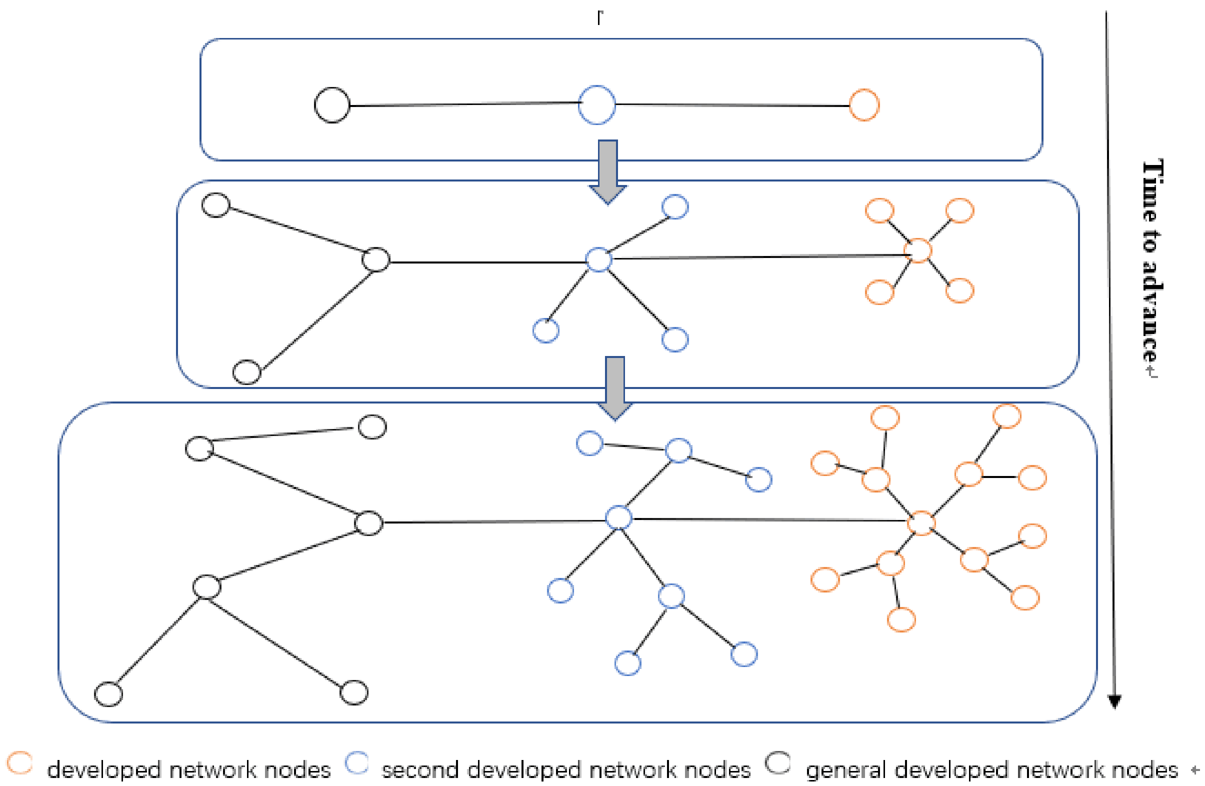


Figure 5 Three network integration charging network evolution

4.2.2.2. Model 3: Ireland Charging Network Planning

For the overall planning of Ireland's charging network, we first model Ireland's charging network based on the national population, cars, transportation networks and urbanization. The urbanization and traffic network as shown below:

（a） Urbanization illustration

(b) Traffic Network

Figure 6 Ireland's traffic and urbanization charts

Since Ireland is a very developed country, only the actual demand (the number of vehicles per unit area) is taken into account when considering the charging network density in various regions. The number of vehicles per unit area in 26 counties throughout Ireland is given in [11]: (only the first six counties with the highest density are listed, **NUV**: **Number of units of vehicles**, see Appendix 5 for the complete result)

Table 8 The county vehicle density

	Dublin	Louth	Kildare	Meath	Wicklow	Cork
Cars	503726	47300	93014	77137	231743	65824
Area(km^2)	922	826	1695	2342	7500	2027
NUV(1/ km^2)	546	57	55	33	31	32

In order to rationally charge charging piles according to the demand, we divide the national charging station network into three sub-networks.(developed, second developed and general) according to the vehicle density in each region and calculate them according to the density of each sub-network and the number of vehicles there in need to put the charging pile number.

Table 9 All levels of network division

	Developed	Second developed	General
Region	Eastern coast	Central inland	Wastern coast
Charging pile number	98375	77103	31745
standard	NUV>31	32>NUV>15	NUV<16

The final planning of the charging network as shown below:

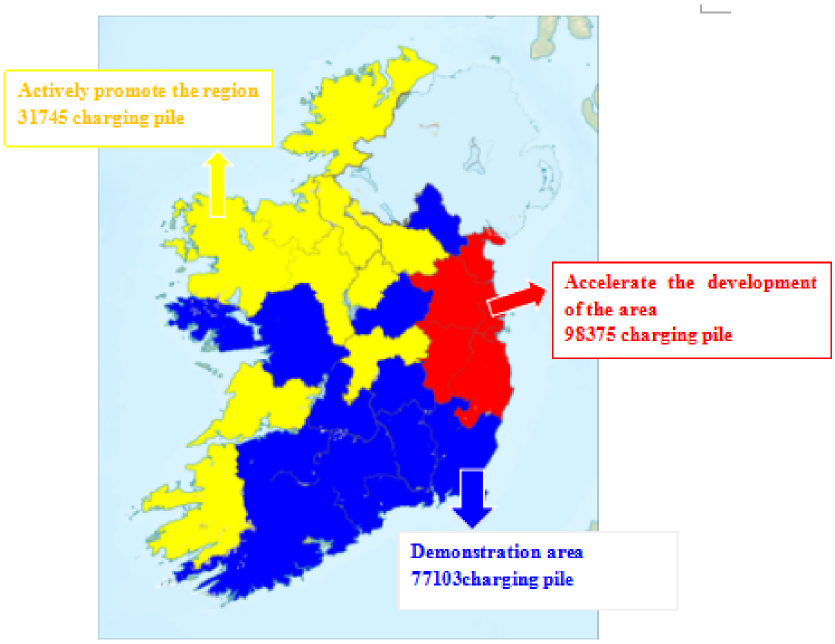


Figure 7 Planning of the charging network

4.2.2.3. Result Analysis

From the analysis of the above results, we can see that the entire charging network is divided into three sub-networks of different sparsity due to the different numbers of vehicles per unit area in Ireland. So determining the entire charging network is an important factor in vehicle density. According to the literature [10], the urban population accounts for only 63.8% of the total population, so the investment of charging piles should be carried out simultaneously in urban and rural areas. In addition, due to the Irish government's plan to achieve the grand goal of selling electric vehicles by 2030 alone [13]. Therefore, in order to speed up the rapid popularization of electric vehicles, the government will increase people's willingness to buy cars due to the wider construction of charging networks.

4.2.3 Problem 2c: Diffusion Model of Market Share of Electric Vehicles

4.2.3.1. Analysis of the problem

Based on the Irish charging network planning model established in Question 2a, the future share of electric vehicles in Ireland is predicted. Among the prediction models for the future proliferation of durable goods, bass has been widely used in the development forecast of many kinds of industrial products because of its comprehensive consideration of the internal and external factors of products [12]. Therefore, we build the bass model to predict the future market share of Irish electric vehicles.

4.2.3.2. Model 4: Establish Bass prediction model

- **Bass basic model:** At a given time t, the ratio of (the number of adopters to the total number of potential adopters) to the (unallocated ones in the market) is a linear function of the cumulative number of adopters on the market. Its expression is as follows:

$$\frac{f(\mathrm{t})}{1-F(\mathrm{t})}=p+\frac{q}{M}[\mathrm{A}(\mathrm{t})] \tag{20}$$

- **Model parameters**

(1) Market potential M : It is the total market size in units for the market.

(2) **Parameter** p **:** Represents the propensity to adopt, independent of how many customers have previously adopted, also referred to as the “innovation” component of the model.

(3) **Parameter** q **:** Represents the propensity to adopt as a function of the number of existing adopters, also referred to as the “imitation” component of the model.

- **Other variables**

(1) During the product diffusion process, the cumulative percentage of adopters of the product by time t (ie $\frac{d_F}{d_t} = f$):

$$F(\mathrm{t}) = \frac{1-e^{-(\mathrm{p+q})\mathrm{t}}}{1+\frac{q}{p}e^{-(\mathrm{p+q})\mathrm{t}}} \tag{21}$$

(2) Probability density function $f(t)$ of the number of adopters at time t in the total number of potential adopters:

$$f(\mathrm{t}) = \begin{cases} F(\mathrm{t}), & t=1 \\ F(\mathrm{t}) - F(\mathrm{t}-1), & \mathrm{t}>1 \end{cases} \tag{22}$$

(3) In the process of product diffusion, the number of adopters $a(t)$

$$a(\mathrm{t}) = \mathrm{M}\, F(\mathrm{t}) \tag{23}$$

(4) In the process of product proliferation, by the time t the cumulative number of product adopters $A(t)$

$$A(\mathrm{t}) = \mathrm{M}\, F(\mathrm{t}), \qquad \mathrm{t}>0 \tag{24}$$

4.2.3.3. Model Solution

We estimate the future market share of EVs by estimating the three parameter values of the bass model and using the EV sales in the first ten years of 2007-2016.

(1) **Parameter Estimation**

- **Imitation coefficient** q: Imitation coefficient as an inherent factor of the product, mainly due to product quality and other circumstances lead to a small amount of future product changes. According to bass model for the application of durable goods forecast, the empirical value of q is generally 0.3-0.7, rarely greater than 0.5 [12]. As electric vehicles as a high-tech energy-saving products, their own strong appeal to consumers. So q = 0.5.
- **Innovation coefficient** p : Innovation coefficient as an external factor of the product, mainly by the external factors such as policy, investment and publicity. According to bass model for the application of durable goods forecast, the empirical value of q is generally 0.01-0.03, rarely greater than 0.02 [12]. Our EV prediction model is built on Model 3's well-established charging network and is being vigorously promoted by governments and others. So p = 0.02 and decrease by 0.001 each year until p = 0.01.
- **Maximum market potential** M : This value represents the forecast of the future development of the target product market. The value of the general use of expert prediction method [12]. Reference [13] found that the average annual growth of the automotive market in Europe in the recent 10 years was about 1%. So set M = 0.01.

(2) **Forecast result**

After determining the three parameters, we use the growth data of the electric vehicles in

Ireland over the past decade to predict the future market share. Get the data as follows:

Table 10 Ireland's first decade of EV development data

Period / Data	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Adoptions/ Period	70	280	687	1681	3586	5146	8536	11566	15689	17621
Cumulated Adoptions	70	350	1037	2718	6304	11450	19986	31552	47241	64862

The above data is then imported into Marketing Engineering-v2.0.8a, the Bass model calculation tool, and the future development of the electric vehicle and automotive market is calculated as follows (the market started to grow from 1756684 in 2007):

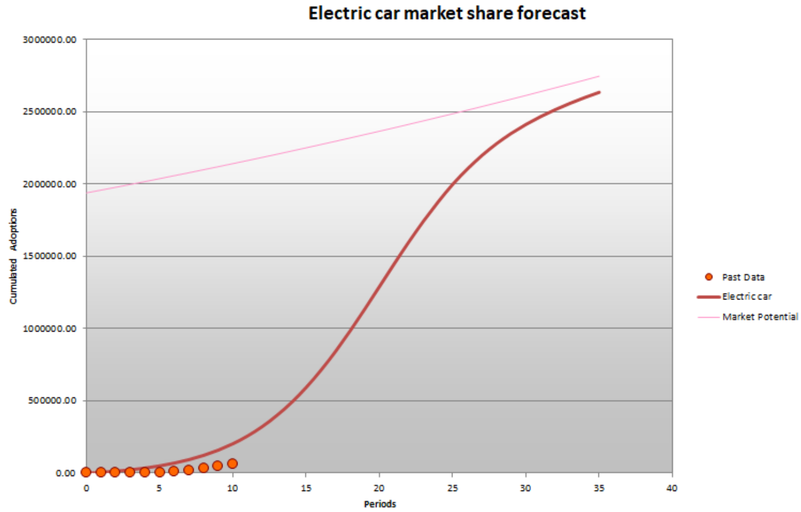


Figure 8 Ireland's electric car market share forecast

And then by the number of electric cars close than the size of the market, you can get the annual market share of electric vehicles (detailed data see Appendix 5):

Table 11 Electric car market share forecast results

	2020	2025	2029	2048
Market share	10.3%	21.8%	54.8%	98.6%

4.2.3.4. Result Analysis

It can be seen from the above forecast results that when the EV imitation coefficient is higher 0.5 and the innovation coefficient is higher 0.02, the EV market share will increase rapidly in the future. However, the innovation coefficient will decrease with the development of time, but the imitation coefficient is determined by the internal conditions of the electric car and is not affected by the time, such as the mileage of the cruise line and the durability. Therefore, electric technology (imitation coefficient) has a very significant effect on the expansion of its market share.

4.3 Solution for task three

4.3.1 Problem analysis

For different countries, they have their own geographical environment, population density and wealth distribution. These factors have different impacts on the planning of the charging pile network, so we use AHP to evaluate what level of charging pile network planning should be adopted in a region.

Based on the three-level evolutionary charging network model established in question 2b, the decision-making objects are developed networks, sub-developed networks and conventional networks. The factors considered are geography, population density and wealth distribution. The decision-making goal is to select a charging pile network planning program, the hierarchy is shown in the Figure 9:

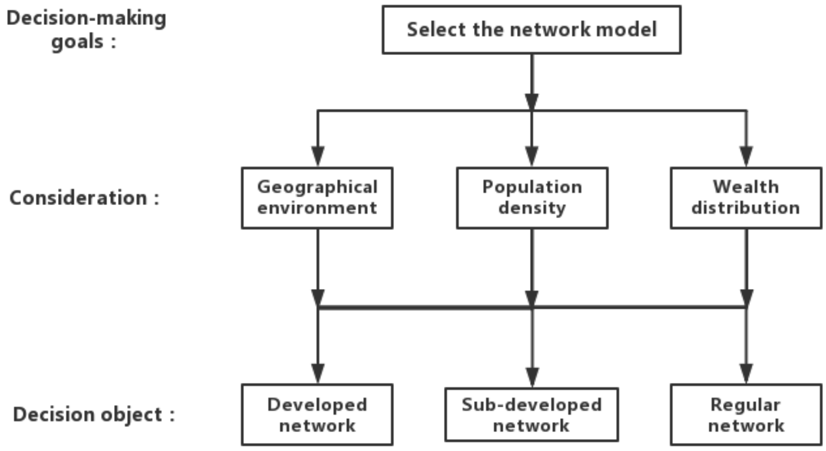


Figure 9 Hierarchical analysis chart

4.3.2 Model 5: Analytic Hierarchy Process

(1) Constructed to compare the matrix.

Pairwise contrast between factors, the relative size of the use of contrast. Let us compare the importance of each criterion $C_1, C_2, ...C_n$, to the target O:

$$a_{ij} = C_i : C_j \tag{25}$$

$$A = (a_{ij})_{n \times n} \ a_{ij} > 0 , a_{ji} = \frac{1}{a_{ij}} \tag{26}$$

Size using Saaty proposed 1-9 scale[14]. Obtain the comparison matrix of the criterion layer to the target layer:

$$A = \begin{bmatrix} 1 & 1/5 & 1/3 \\ 5 & 1 & 2 \\ 3 & 1/2 & 1 \end{bmatrix} \tag{27}$$

(2) Consistency check

According to the consistency indicator：

$$CI = \frac{\lambda - n}{n-1} \tag{28}$$

Where n is the order of the contrast matrix and λ is the largest eigenvalue of the contrast matrix. When the CI is larger, the more inconsistent.

Redefine consistency ratio $CR = \frac{CI}{RI}$, RI is random consistency index, When $CI < 0.1$, pass the consistency test.

Calculated by the matlab program w = (0.1095, 0.5816, 0.3090), $CI = 0.0018$, CR=$0.0032 < 0.1$, the above comparison.

The following is the third layer of the second floor of the calculation result.(The comparison matrix is in Appendix 4)

Table 12 Coefficient chart

k	1	2	3
	0.0629	0.1220	0.1634
${w^{(3)}}_k$	0.2654	0.2297	0.2970
	0.6716	0.6483	0.5396
λ_k	3.0291	3.0037	3.0092
CI_k	0.0145	0.0018	0.0046

$RI = 0.58\ (\mathrm{n} = 0.3)$, CI_k can pass the consistency test. ${w^{(3)}}_k$ is the weight vector of the k-th criterion in the scheme layer.

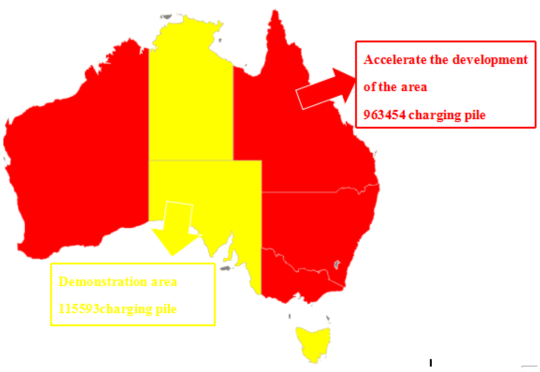
(3) Calculate the combination weight vector(do a combination of consistency test).

Let $\mathrm{ww} = ({w^{(3)}}_1 \quad {w^{(3)}}_2 \quad {w^{(3)}}_3)$ then the combined weight vector of the program layer to the target layer is $\mathrm{ww} \times \mathrm{w}$, that is $(0.3688 \quad 0.3473 \quad 0.3574)^T$, we can see that Guangdong province of China should choose a developed network for planning.

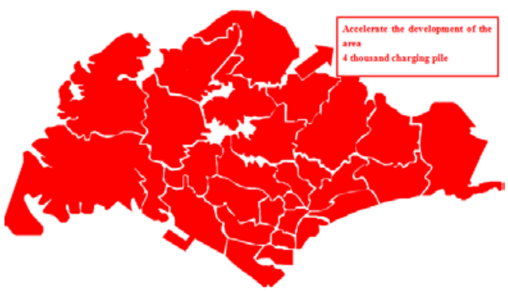
4.3.3 Model Calculation

The data of per capita GDP, average altitude and population density in each administrative region of Australia, China and Singapore are respectively calculated to determine the types of networks they should build according to the above calculation process. Each country should build a network as shown below:

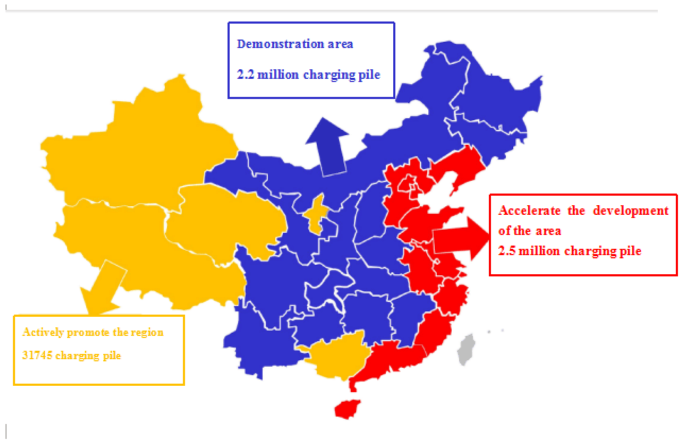
.



(c)Australia network layout



（b） Singapore network layout



(c)China network layout

Figure 10 Different countries charging network planning

It can be seen from the figure above.(among them, the total investment for building a charging pile is determined by the number of vehicles in the area)

(1) China: A general network with 31745 charging piles was built in the western part of the country and a second developed network of 2200000 charging piles was built in the central region. The eastern coastal area to build 2500000 charging pile developed network.

(2) Singapore: A developed network of 40000 charging piles.

(3) Australia: Building 963454 developed charging network for cross-strait areas and a second developed network of 115593 charging piles was built in the central region.

4.3.4 Result Analysis

It can be seen from the above results that each country will assume a different type of charging network due to different population distribution, geographical conditions and wealth distribution in different regions of the country (including three layers in China, one in Singapore and two in Australia). The classification model based on the hierarchical analysis and the three-level network model that uses the different network levels as the classification standards can provide different charging network development plans for different countries.

4.4 Solution for task four

4.4.1 Overall analysis

With the related technologies of electric vehicles and electric vehicle charging stations maturing, the usage of electric vehicles is on the rise, and electric vehicles will soon be fully popularized. With the rapid development of science and technology, new modes of transport and modes of travel are also rapidly developing. This will inevitably have an impact on the utilization rate of electric vehicles. Different products have different impacts. According to their functions, adaptation to the crowd, environmental protection, and practicality, it can be analyzed whether these technologies and concepts conform to the times can promote or retard the utilization rate of electric vehicles.

4.4.2 Key factors analysis

➢ **Car-share services**

The concept of car sharing service originated from Switzerland and Germany in 1948. With the development of science and technology and the circulation of ideas, car sharing started to rise all over the world. Car sharing refers to a company or organization that has a large variety of car models offering fixed car or member travel services at any time for convenient use of cars, and users pay according to how much the car is used [6]. The concept of electric vehicle promotion and car sharing service can be perfectly combined, that is, pure electric vehicles operate as a shared vehicle. In the consumer do not want to buy, do not need to buy, cannot afford to buy a car, you can rent electric cars. In this way, people and vehicles to re-match, can greatly improve the utilization of electric vehicles. At the same time, the sharing of electric vehicles is equivalent to another form of electric vehicle promotion, which can develop potential users and buy electric vehicles when they are able to purchase a car.

- **Ride-share services**

ride-share is a means of transportation for two or more people to travel by one car. It can reduce the cost of travel per person, alleviate the pressure brought by the vehicle and reduce the consumption of energy. Ride-share travel makes travel services cheaper, but people buy cars for more than just price and cost, but also convenience, status, driving pleasure, and ease of use in times of emergency. Thus ride-share will improve some of the existing problems, but it will not affect people's increasing purchase and use of electric vehicles.

- **Self-driving cars**

Self-driving cars rely on artificial intelligence, visual computing, radar, surveillance devices and global positioning systems to work together so that computers can operate their vehicles safely and without any human initiative. The self-driving cars power platform can continue to change with the times. Moreover, the internal combustion engine has the best work efficiency point, and the complexity of the control is high. The motor is relatively easy to control by controlling the input of the voltage and current. Therefore, autonomous vehicles can be combined with electric vehicles in the future to promote the use and development of electric vehicles.

- **Rapid battery-swap stations for electric cars**

Battery replacement station is a place where battery replacement is used to provide electric energy for electric vehicles. It is an important place where electric vehicles supply electric energy. The battery exchange station can effectively utilize the low load of the power grid to charge the standard battery box in a centralized and orderly manner and play a good role of shifting the peak load to the grid load. On the other hand, when the load is low, the electricity price is cheap, which can reduce the charging cost. The rapid battery-swap stations generated by technological development also provide a great convenience for users in desperate need of electricity, which solves the problem that users are worried about the difficulty of charging and can further promote the promotion of electric vehicles.

- **Flying cars**

Flying car means a vehicle that can travel in the air or on land. If the flying car using aviation fuel, the cost and energy consumption is much higher than land vehicles, but not environmentally friendly. When powered by electricity, a flying car consumes several times more power per kilometer than a land car and has less battery life. And the higher cost of flying cars, the general public unable to buy, penetration is not high. Therefore, the appearance of flying cars has little effect on the utilization rate of electric vehicles.

- **Hyperloop**

Super high-speed rail is a kind of ‘vacuum steel tube transport’ as the theoretical core transport. It is suitable for long-distance driving. As far as the experience of users is concerned, when traveling, people only choose to drive a short distance. Because of the long distance driving in terms of fatigue or time efficiency is extremely inappropriate. When traveling long distances, people often choose other modes of transportation. Therefore, the emergence of super high-speed rail will not affect the utilization of electric vehicles. Just like the appearance of

HSR does not affect the car's usage rate.

5 Strengths and weaknesses

5.1 The strengths of the model

(1) A more in-depth study has been carried out on the layout planning methods of national and city electric vehicle charging stations. From a systematic and scientific point of view, a service network for charging stations for electric vehicles is constructed. From the point of view of helping to improve the level of charging service and user convenience, the paper studies how to layout the location of EV charging station.

(2) Based on the analysis of the overall situation in Ireland, a three-in-one charging network has been established. According to the number of vehicles per unit area in different regions, the entire charging network is divided into three sparse sub-networks. According to the actual situation of research and analysis, the planning system is more practical.

(3) The bass forecasting model proposed in this paper for the diffusion of electric vehicles and the scientific method for city charging facilities are of some practical significance.

5.2 The weaknesses of the model

(1) In the process of planning, there are still some shortcomings, such as safety and environmental conditions that affect the layout of charging stations in cities, and need continuous innovation and improvement in practice.

(2) In the process of research, the consideration of factors such as traffic conditions and urban power grid planning is still insufficient, and more in-depth analysis is needed.

(3) In the study of electric vehicle charging facility planning, this paper only studies the planning problem of rapid charging station for electric vehicles. It does not consider the characteristics of different regions, the types of users and the coordination of different charging modes.

6 Future Improvements

Network facilities location is an important area of research, such as logistics warehouses, fire stations, emergency centers, sewage treatment centers, substations and gas stations (air entrainment). The location of electric vehicle charging station studied in this paper is a problem of location of energy supplement facilities (gas station, air entrainment, charging stations, etc.), which is a typical problem of network facilities location. The center of gravity method proposed in this paper not only can be used in the location of electric vehicle charging stations, but also applies to the location of other energy supplement facilities.

7 Handout

All-electric cars plan and a gas vehicle-ban date

What and why?

At present, electric vehicles can be divided into three categories according to their power source, that is, all-electric vehicles, hybrid vehicles, and fuel cell vehicles. All electric car is powered by car power. The all-electric vehicle program converts all of the domestic internal combustion engine cars to full electric vehicles and sets a date to ban the use of gasoline under the plan. The implementation of the all-electric vehicle program has greatly helped the development of the country. For example: zero electric exhaust emissions can effectively alleviate the environmental pressure, a wide range of sources of electricity can lift the dependence on oil resources, energy conversion efficiency can make more efficient use of energy.

The key factor affecting the implementation of the plan for all electric vehicles

- **Charging infrastructure construction**

Charging infrastructure involves urban planning, construction land, buildings and power grids and so on. Many profit-making, difficult to promote. The charging infrastructure is undoubtedly the key factor in the EV plan. Therefore, in accordance with the principle of the first station after the car, moderate advance construction. Establish cooperation between relevant government departments and enterprises, and make overall plans in light of the goal of developing the overall situation.

- **Regional promotion**

Different regions have different topography, economy, population, number of vehicles and other factors, so they should be implemented according to local conditions and subregions. It can be developed first in economically developed cities with high population and vehicle density, where the ratio of public charging posts to electric vehicles is not less than 1: 7, to achieve interconnection among cities. Second, in areas where the economy of development is relatively low and the population and vehicle density are relatively low, the ratio of public charging posts to electric vehicles in these areas should be no less than 1: 8 and to enhance connectivity with the surrounding area. Finally, the population of vehicles less, economically backward areas for promotion, public charging pile and electric car ratio of not less than 1:12. On-demand inter-city fast charge network construction.

- **Power grid construction**

Large-scale electric vehicles connected to the grid charging, may affect the safe and reliable operation of the power system, causing the power grid overload. Therefore, the charging infrastructure should be included in the local distribution network special plan. In combination with the different characteristics of the charging pile, formulate relevant policies and coordinate with the power generation facilities to maintain the safe and stable operation of the power grid.

A gas vehicle-ban date

Based on the analysis of the model we have found, it takes about 30 years for a developed country to develop electric vehicles from zero to all over the country, compared to about 42 years for developing countries. Thus, the date of banning the use of gasoline may be set according to the time of appeal.

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Appendix

Appendix 1: Queuing theory model lingo solution

```
model:
sets: state/1..51/:p;
endsets lamda=2;mu=0.5;rho=lamda/mu;s=2;k=5;
lamda*p0=mu*p(1);
(lamda+mu)*p(1)=lamda*p0+2*mu*p(2);
@for(state(i)|i#gt#1#and#i#lt#s: (lamda+i*mu)*p(i)=lamda*p(i-1)+(i+1)*mu*p(i+1));
@for(state(i)|i#ge#s#and#i#lt#k: (lamda+s*mu)*p(i)=lamda*p(i-1)+s*mu*p(i+1)); lamda*p(k
-1)=s*mu*p(k);
p0+@sum(state:p)=1;
P_lost=p(k);lamda_e=lamda*(1-P_lost);
L_s=@sum(state(i):i*p(i));
L_q=L_s-lamda_e/mu;
W_s=L_s/lamda_e;
```

Appendix 2: Ant Colony Optimization Algorithm Based on Network Optimization

```
clc
clear
NC_max=200
m=30
Alpha  =0.2
Beta=10
Rho=0.8
Q=0.2
R_best=[]
L_best=[]
global  D;
global  data;
global  map;
D=xlsread('  Ride  arrangements.xlsx');
map=xlsread('Path.xlsx','matrix');
n=9;
Eta=1./D;
Tau=ones(n,n);
Tabu=zeros(m,n);
NC=1;
R_best=zeros(NC_max,n);
L_best=inf.*ones(NC_max,1);
L_ave=zeros(NC_max,1);
while  NC<=NC_max              %%
Randpos=[];      %
for  i=1:(ceil(m/n))
Randpos=[Randpos,randperm(n)];
end
Tabu(:,1)=(Randpos(1,1:m))';         %
%%
```

```
for  j=2:n
for  i=1:m
visited=Tabu(i,1:(j-1));
J=zeros(1,(n-j+1));
P=J;
Jc=1;
for  k=1:n
if  length(find(visited==k))==0
J(Jc)=k;
Jc=Jc+1;
end
end
for  k=1:length(J)
                        P(k)=(Tau(visited(end),J(k))^Alpha)*(Eta(visited(end
),J(k))^Beta);
                end
                P=P/(sum(P));
                Pcum=cumsum(P);
                Select=find(Pcum>=rand);
                to_visit=J(Select(1));
                Tabu(i,j)=to_visit;
        end
end
if  NC>=2
        Tabu(1,:)=R_best(NC-1,:);
end
global  L;
global  R;
global  min_dist;
min_dist=cell(30,1);
min_d=cell(200,1);
L=zeros(m,1);
for  i=1:m
        R=Tabu(i,:);
        for  j=1:(n-1)
                L(i)=L(i)+D(R(j),R(j+1));
        end
        %          L(i)=L(i)+D(R(1),R(n));            %
        addHotel(i);
end
L_best(NC)=min(L);                      %
index=find(L==L_best(NC));
min_L{NC}=min_dist{index};
pos=find(L==L_best(NC));
R_best(NC,:)=Tabu(pos(1),:);  %
L_ave(NC)=mean(L);                      %
NC=NC+1
Delta_Tau=zeros(n,n);
for  i=1:m
```

```
        for  j=1:(n-1)                        Delta_Tau(Tabu(i,j),Tabu(i,j+1
))=Delta_Tau(Tabu(i,j),Tabu(i,j+1))+Q/L(i);
        end              Delta_Tau(Tabu(i,n),Tabu(i,1))=Delta_Tau(Tabu(i,n),
Tabu(i,1))+Q/L(i);
end
Tau=(1-Rho).*Tau+Delta_Tau;
Tabu=zeros(m,n);
end
Pos=find(L_best==min(L_best));
Shortest_Route=R_best(Pos(1),:)
Shortest_Length=L_best(Pos(1))
plot(L_ave)
hold  on
plot(L_best)
title('Algorithm  convergence  trajectory')
xlabel('The  number  of  iterations')
ylabel('Distance')
legend('the  average','the  best')
  %  m1=0;
%  for  k=1:length(Shortest_Route)-1
%          i=Shortest_Route(k);
%            j=Shortest_Route(k+1);
%            m1=m1+D(i,j);
%  end
%  disp(sprintf('The  length  is  %d:',m1))
global  D;
global  map;
global  data;
time=0;
energe=0;
dist=ones(1,2)*100;
global  min_dist;
m=ones(6,2);
cnt=0;
i=1;
while  i<10
        time=time+data(R(i),2);
        energe=energe+data(R(i),1);
        dist(1,2)=1000;
        if  time>8  ||  energe>100
                L(t)=L(t)-D(R(i-1),R(i));
                for  j=15:18
                       d=map(j,R(i-1))+map(j,R(i));
                       if  dist(1,2)>d
                               dist(1,1)=j;
                               dist(1,2)=d;
                       end
                end
                if  i==14
```

```
                    L(t)=L(t)+180;
                    i=i+1;
                end
                cnt=cnt+1;
                m(cnt,1)=dist(1,1);
                m(cnt,2)=dist(1,2);
                L(t)=L(t)+dist(1,2);
                i=i-1;
                time=0;
                energe=0;
        end
        i=i+1;
end
min_dist{t}=m;
end
```

Appendix 3: The number of vehicles in the United States

	area	The number of vehicles per thousand people	population	Number of vehicles	number of charging stations
50	Colorado	340	5029324	1709970	6428
49	Nevada	500	2700691	1350346	5076
48	New York state	570	19378087	11045510	41524
47	Indiana	610	6484229	3955380	14870
46	Arizona	660	6392307	4218923	15861
45	North Carolina	670	9535692	6388914	24018
44	Mississippi	680	2968103	2018310	7588
43	New Jersey	690	8791936	6066436	22806
42	Arkansas	700	2915958	2041171	7674
41	Florida	710	18804623	13351282	50193
40	Texas	720	25146105	18105196	68065
39	Rhode Island	730	1052931	768640	2890
38	Illinois	750	12831549	9623662	36179
37	West Virginia	750	1853011	1389758	5225
36	Hawaii State	760	1360301	1033829	3887
35	Pennsylvania	760	12702887	9654194	36294
34	New Mexico	770	2059192	1585578	5961
33	South Carolina	770	4625401	3561559	13389
32	Oregon	770	3831073	2949926	11090
31	Maine	780	1328361	1036122	3895
30	Idaho	790	1567652	1238445	4656
29	Maryland	790	5773785	4561290	17148
28	Massachusetts	820	6547817	5369210	20185
27	Georgia	820	9688681	7944718	29867
26	Missouri	830	5988927	4970809	18687
25	New Hampshire	830	1316466	1092667	4108
24	Kansas	830	2853132	2368100	8903
23	California	840	37254503	31293783	117646

22	Virginia	840	8001045	6720878	25266
21	Tennessee	840	6346275	5330871	20041
20	Kentucky	840	4339349	3645053	13703
19	Connecticut	860	3574118	3073741	11555
18	Wisconsin	860	5687289	4891069	18387
17	Oklahoma	860	3751616	3226390	12129
16	Washington state	870	6724543	5850352	21994
15	Michigan	870	9884129	8599192	32328
14	Utah	870	2763888	2404583	9040
13	Minnesota	870	5303925	4614415	17347
12	Louisiana	910	4533479	4125466	15509
11	Vermont	910	625745	569428	2141
10	Ohio	910	11536725	10498420	39468
9	South Dakota	950	814191	773481	2908
8	Delaware	950	897936	853039	3207
7	Alaska	960	710249	681839	2563
6	Nebraska	1, 000	1826341	1826341	6866
5	Alabama	1, 030	4780127	4923531	18510
4	Iowa	1, 050	3046869	3199212	12027
3	North Dakota	1, 080	672591	726398	2731
2	Montana	1, 120	989417	1108147	4166
1	Wyoming	1, 140	563767	642694	2416
	Washington, D.C	350	601767	210618	792

Appendix 4: AHP involves matrix

$$B_i = \begin{bmatrix} 1 & \frac{1}{5} & \frac{1}{9} \\ 5 & 1 & \frac{1}{3} \\ 9 & 3 & 1 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{5} \\ 2 & 1 & \frac{1}{3} \\ 5 & 3 & 1 \end{bmatrix} \qquad B_3 = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & 3 \\ 3 & \frac{1}{3} & 1 \end{bmatrix}$$

Appendix 5: Ireland survey data

Cars	Area	Average	County	Charging pile number
503,726	922	546	Dublin	62966
47,300	826	57	Louth	5913
93,014	1695	55	Kildare	11627
77,137	2342	33	Meath	9642
65,824	2027	32	Wicklow	8228
231,743	7500	31	Cork	23174
84,344	2756	31	Limerick	8434
26,428	897	29	Carlow	2643
52,146	1857	28	Waterford	5215
66,173	2367	28	Wexford	6617
37,220	1840	20	Westmeat h	3722

	41,108	2073	20	Kilkenny	4111	
	23884	1295	18	Monaghan	2388	
	30,743	1720	18	Laois	3074	
	104,380	6149	17	Galway	10438	
	72,856	4305	17	Tipperary	7286	
	52,805	3450	15	Clare	4400	
	30,397	2001	15	Offaly	2533	
	28,980	1932	15	Cavan	2415	
	27,394	1838	15	Sligo	2283	
	16,093	1091	15	Longford	1341	
	65,478	4807	14	Kerry	5457	
	61,352	4861	13	Donegal	5113	
	30,454	2548	12	Roscommon	2538	
	54,531	5586	10	Mayo	4544	
	13,458	1590	8	Leitrim	1122	