## Resistor

**$t$ domain**

$v(t) = Ri(t)$

$i(t) = Gv(t)$

**$s$ domain**

$V(s) = RI(s)$

$I(s) = GV(s)$

$Z(s) = R$

**$\phi$ domain**

$\mathbf{V} = R\mathbf{I}$

$\mathbf{I} = G\mathbf{V}$

$Z = R$

## Capacitor

**$t$ domain**

$$i = C\frac{dv(t)}{dt}$$

$$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

$$P = Cv\frac{dv(t)}{dt}$$

$$W = \frac{1}{2}Cv(t)^2$$

**$s$ domain**

$$V(s) = \frac{1}{Cs}I(s) + \frac{1}{s}v(0^+)$$

$$I(s) = CsV(s) - Cv(0^+)$$

$$Z(s) = \frac{1}{Cs}$$

**$\phi$ domain**

$$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

$$\mathbf{I} = j\omega C\mathbf{V}$$

$$Z = \frac{1}{j\omega C}$$

## Inductor

**$t$ domain**

$$v(t) = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

$$W = \frac{1}{2}Li^2$$

$$P = Li(t)\frac{di(t)}{dt}$$

**$s$ domain**

$$V(s) = LsI(s) - Li(0^+)$$

$$I(s) = \frac{1}{Ls}V(s) + \frac{1}{s}i(0^+)$$

$$Z(s) = sL$$

**$\phi$ domain**

$$\mathbf{V} = j\omega L\mathbf{I}$$

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L}$$

$$Z(s) = j\omega L$$

## Max. Ave. Power

$$Z_L = Z_{th}^*$$

$$R_L = |Z_{th}|$$

$$P_{max} = \frac{|V_{th}^2|}{8R_{th}}$$

## Wye-Delta Transform

**Delta to Wye**

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

**Wye to Delta**

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{R_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{R_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{R_3}$$

## Laplace Transform Properties

**time**

$$x(at) \cdots \frac{1}{|a|}X(\frac{s}{a})$$

$$x(-t) \cdots X(-s)$$

$$x(t-a) \cdots X(s)\,e^{-as}$$

**frequency**

$$x(t)e^{at} \cdots X(s-a)$$

## Step Response

**RC circuit**

$$v(t) = v(\infty) + [v(0) - v(\infty)]\,e^{-\frac{t}{\tau}} \cdots \tau = RC$$

**RL circuit**

$$i(t) = i(\infty) + [i(0) - i(\infty)]\,e^{-\frac{t}{\tau}} \cdots \tau = L/R$$

## Laplace Transform Pairs

**$u(t)$**

$$u(t) \cdots \frac{1}{s}$$

$$tu(t) \cdots \frac{1}{s^2}$$

$$t^n u(t) \cdots \frac{n!}{s^{n+1}}$$

$$u(t - T_0) \cdots \frac{e^{-sT_0}}{s}$$

$$e^{at}u(t) \cdots \frac{1}{s-a}$$

**$e^{at}$**

$$e^{at}u(t) \cdots \frac{1}{s-a}$$

$$te^{at}u(t) \cdots \frac{1}{(s-a)^2}$$

$$t^n e^{at}u(t) \cdots \frac{n!}{(s-a)^{n+1}}$$

**Trigonometric**

$$\cos(bt)\,u(t) \cdots \frac{s}{s^2 + b^2}$$

$$\sin(bt)\,u(t) \cdots \frac{b}{s^2 + b^2}$$

$$e^{-at}\cos(bt)\,u(t) \cdots \frac{s+a}{(s+a)^2 + b^2}$$

$$e^{-at}\sin(bt)\,u(t) \cdots \frac{b}{(s+a)^2 + b^2}$$

$$e^{-at}\left(A\cos(bt) + \frac{B - Aa}{\sqrt{c - a^2}}\sin(bt)\right)u(t) \cdots \frac{As + B}{s^2 + 2as + c}$$

## Complex Number

Rectangular form

$$z = x + jy$$

Polar form

$$z = r\angle\phi$$

## Eulers identity

$$e^{\pm j\phi} = \cos\phi \pm i\sin\phi$$

$$\cos\phi = \mathrm{Re}(e^{j\phi})\ \sin\phi = \mathrm{Im}(e^{j\phi})$$

## Operation

$$z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$$

$$z_1 z_2 = r_1 r_2\angle\phi_1 + \phi_2$$

$$\frac{z_1}{z_2} = \angle\phi_1 - \phi_2$$

$$\frac{1}{z} = \frac{1}{r}\angle-\phi$$

$$\sqrt{z} = \sqrt{r}\angle\phi/2$$

$$z^* = r\angle-\phi$$

leading & lagging

## Sinusoid-phasor transformation

| $t$ domain | $\phi$ domain |
|---|---|
| $V_m\cos(\omega t + \phi)$ | $V_m\angle\phi$ |
| $V_m\sin(\omega t + \phi)$ | $V_m\angle\phi - 90°$ |
| $I_m\cos(\omega t + \phi)$ | $I_m\angle\phi$ |
| $I_m\sin(\omega t + \phi)$ | $I_m\angle\phi - 90°$ |
| $dv/dt$ | $j\omega V$ |
| $\int vdt$ | $\frac{V}{j\omega}$ |

## Average Power

$$P = \frac{1}{T}\int_0^T p(t)dt$$

$$P = \frac{1}{2}V_m I_m\cos(\theta_v - \theta_i)$$
$$= \frac{1}{2}|I|^2 R$$

## Complex Power

$$S = V_{rms}I^*_{rms} = P(x) + jQ(x) = 3S_p$$
$$= V_{rms}I_{rms}\angle\theta_v - \theta_i = I^2_{rms}Z = \frac{V^2_{rms}}{Z^*}$$

Unit $P/W$ $Q/var$ $S/VA$

## Power Factor Correction

$$C = \frac{Q}{\omega V^2_{rms}} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V^2_{rms}}$$

## Three Phase Power Factor Correction

$$C_\Delta = \frac{Q}{\mathbf{3}\omega V^2_{rms}} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\mathbf{3}\omega V^2_{rms}}$$

## Three Phase Capacitor Bank Transform

$$C_Y = 3C_\Delta$$

## Y-Δ Load Transform

$$Z_Y = \frac{Z_\Delta}{3}$$

## Y-Δ Source Transform

$$V_{Yp} = \frac{V_{\Delta p}}{\sqrt{3}}\angle-30°$$

## Ideal Transformer

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

## Series-aiding connection

$$L = L_1 + L_2 + 2M$$

## Series-opposing connection

$$L = L_1 + L_2 - 2M$$

## Mutual Inductance

If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.

## instant Energy

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

## coupling coefficient

$$M = k\sqrt{L_1 L_2}$$

## reflect impedance

$$Z_{in} = \frac{Z_L}{n^2}$$

## V & I dircetion

$V_1$, $V_2$ same dircetion $+n$
$I_1$, $I_2$ same dircetion $-n$

## RMS Value

$$X_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2 dt}$$

## Apparent Power

$$|S| = V_{rms}I_{rms}$$

## Power Factor

$$pf = \cos(\theta_v - \theta_i)$$

## Power Gain

$$G_{dB} = 10\log_{10}\frac{P_2}{P_1}$$

## Voltage Gain

$$G_{dB} = 20\log_{10}\frac{V_2}{V_1}$$

## Power Gain

$$G_{dB} = 20\log_{10}\frac{I_2}{I_1}$$

## Power Factor Lagging

$$Q > 0$$

## Power Factor Lagging

$$Q < 0$$

## Trigonometric Fourier Series

For a nonsinusoidal periodic function $f(t)$

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n\sin n\omega_0 t)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt \qquad a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t dt$$

# Even Symmetry

$$f(t) = f(-t)$$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)dt$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t dt$$

$$b_n = 0$$

# Odd Symmetry

$$f(-t) = -f(t)$$

$$a_0 = 0, \qquad a_n = 0$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t dt$$

### Half-Wave Symmetry

$$f(t-\frac{T}{2}) = -f(t)$$

$$a_0 = 0$$

$$a_n = \begin{cases} \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t dt, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even} \end{cases}$$

$$b_n = \begin{cases} \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t dt, & \text{if } n \text{ is odd} \\ 3n+1, & \text{if } n \text{ is even} \end{cases}$$

### Average PWR.

$$P = V_{dc}I_{dc} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

### RMS PWR.

$$F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}$$

### Exp. Fourier Series

$$f(t) = c_0 + \sum_{n=1}^{\infty}\left(c_n e^{jn\omega_0 t} + c_{-n}e^{-jn\omega_0 t}\right)$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}dt$$

$$A_n\underline{/\phi_n} = a_n - jb_n = 2c_n$$

## Bode Plot

| Factor | Magnitude | Phase |
|---|---|---|
| $K$ | $20\log_{10} K$ | $0^\circ$ |
| $(j\omega)^N$ | $k = 20N\text{dB},\ (1,0)$ | $90N^\circ$ |
| $\frac{1}{(j\omega)^N}$ | $k = -20N\text{dB},\ (1,0)$ | $-90N^\circ$ |
| $\left(1+\frac{j\omega}{z}\right)^N$ | $k=0,(z,0) \to k = 20N\text{dB},(z,0)$ | $(\frac{z}{10},0^\circ) \to (10z, 90N^\circ)$ |
| $\frac{1}{(1+j\omega/p)^N}$ | $k=0,(z,0) \to k = -20N\text{dB},(z,0)$ | $(\frac{z}{10},0^\circ) \to (10z, -90N^\circ)$ |
| $\left[1+\frac{2j\omega\zeta}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]^N$ | $k=0,(\omega_n,0) \to k = 40N\text{dB},(z,0)$ | $(\frac{z}{10},0^\circ) \to (10z, 180N^\circ)$ |
| $\frac{1}{[1+2j\omega\zeta/\omega_n+(j\omega/\omega_n)^2]^N}$ | $k=0,(\omega_n,0) \to k = -40N\text{dB},(z,0)$ | $(\frac{z}{10},0^\circ) \to (10z, -180N^\circ)$ |

### Imp. Parameters ($z$)

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad Z_{11} = \left.\frac{V_2}{I_2}\right|_{I_2=0}$$

### Admit. Parameters ($y$)

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$
$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad y_{11} = \left.\frac{I_2}{V_2}\right|_{V_2=0}$$

### Hybrid Parameters ($h$)

$$V_1 = y_{11}I_1 + y_{12}V_2$$
$$I_2 = y_{21}I_1 + y_{22}V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad h_{11} = \left.\frac{I_2}{V_2}\right|_{I_2=0}$$

### Trans. Para. ($T$)

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$