Kyburg, Levi, and Petersen

Author(s): Mark Stone

Source: *Philosophy of Science*, Vol. 54, No. 2 (Jun., 1987), pp. 244-255
Published by: The University of Chicago Press on behalf of the Philosophy of Science Association
Stable URL: https://www.jstor.org/stable/187801

KYBURG, LEVI, AND PETERSEN*

MARK STONE†

University of Rochester

In this paper I attempt to tie together a longstanding dispute between Henry Kyburg and Isaac Levi concerning statistical inferences. The debate, which centers around the example of Petersen the Swede, concerns Kyburg's and Levi's accounts of randomness and choosing reference classes. I argue that both Kyburg and Levi have missed the real significance of their dispute, that Levi's claim that Kyburg violates Confirmational Conditionalization is insufficient, and that Kyburg has failed to show that Levi's criteria for choosing reference class are problematic. Rather, the significance of the Petersen case is to show that other aspects of their respective systems are defective: for Levi his account of credal judgments other than direct inference, and for Kyburg his explanation of how indexes are associated with a body of knowledge.

Henry Kyburg and Isaac Levi have a longstanding dispute concerning Kyburg's principle of direct inference (DI) and Levi's principle of confirmational conditionalization (CC). In making statistical inferences, Kyburg considers DI to be basic. Levi, on the other hand, believes that statistical inferences must always conform to CC. Recent debate has centered around a particular example posed by Levi to show that on Kyburg's system DI and CC conflict. Since Kyburg is unwilling to revise his treatment of DI his system, Levi claims, violates CC.

The dispute about Levi's example (the case of Petersen the Swede) boils down not so much to a dispute about the truth of CC, but to a dispute about how to select the appropriate reference class for DI; this much both Kyburg and Levi recognize. Besides trying to present in a clear and relatively nontechnical way what has until now been primarily a difficult technical discussion, I will argue here for two further claims about the Petersen case:

i) If there are cases that show that Kyburg's system violates CC in a way that should be of philosophical concern, then the Petersen case is not one of them; and
ii) the real importance of the Petersen case is not its implications for CC, but that it shows that *both* Kyburg and Levi have inadequate systems in their present form: Kyburg because the choice of ref-

*Received August 1985.

†I would like to thank Henry Kyburg, Isaac Levi, and the anonymous *Philosophy of Science* referee for their helpful comments on earlier drafts of this paper.

Philosophy of Science, 54 (1987) pp. 244–255.

> erence class will always be relative to a certain index for a body of knowledge, and it is unclear how to choose the appropriate index; Levi because the cases in which direct inference is possible are very restricted, and his account of other credal judgments still faces internal difficulties.

I will begin with a brief discussion of the Petersen case, and how Kyburg and Levi each propose to handle it. I will then present two additional cases modeled after the Petersen case that show how each is forced to take a problematic stance on the new case based on his interpretation of the Petersen case. Finally I shall consider how Kyburg and Levi might further develop their systems to resolve these problems.

Suppose that rational agent X has a body of knowledge w, which contains among other things the following statements:

a. The proportion of Protestants among Swedes is .9.
b. The proportion of Protestants among residents of Malmo is either .85, .91, or .95.
c. Petersen is a Swede.
d. Petersen is a resident of Malmo.

What probability should X assign, based on this information, to the statement h: "Petersen is a Protestant"? Both Kyburg and Levi are agreed that if X can base his inference on the reference class of Swedes, then $pr(h) = .9$; and that if X can base his inference on the reference class of residents of Malmo, then $pr(h) \in [.85, .95]$. They differ about which reference class is the right reference class.

Kyburg says:

> According to my rules, X's degree of belief in h should be .9, on the grounds that we have more precise statistical information regarding the proportion of Swedes in general that are Protestants than we have concerning residents of Malmo, and what information we do have about Malmo does not really conflict with what we know about Swedes in general (1977, p. 517).

Thus because the two probability assignments do not conflict ($.9 \in [.85, .95]$), X should use the reference class about which he has more precise statistical information: the class of Swedes.

Levi, on the other hand, chooses the narrower reference class of Malmo (actually this is an oversimplification of Levi's view; in order to use Malmo as a reference class we would have to know not just that Petersen is a resident of Malmo, but how Petersen has been selected from among residents of Malmo) because Levi favors "choosing the narrowest reference class to which the object is known to belong and from which it is known to be chosen at random" (1981, p. 542).

This difference of opinion about reference class is indicative of a deeper difference of opinion about probability. For Kyburg probability is a derived notion; it is a logical relation that obtains between statements in a body of knowledge based on known relative frequencies. For Levi probability is based not on frequency but on chance. Chance is a basic, not a derived notion, and our knowledge of chances depends on our knowledge of how chance mechanisms operate in the world, as in "the chance of the dice thrown by a standard craps player landing with seven showing is $^1/_6$". Furthermore, randomness for Kyburg just means not knowing of the presence of any sampling bias. Thus Petersen is a random member of the population of Swedes with respect to being Protestant just in case our body of knowledge is such that nothing in it would yield different (more precise or conflicting) probability measures about the likelihood that Petersen is Protestant. Thus X may judge that $pr(h) = .9$. Levi, however, says:

> I see no compelling reason why rational X should be obliged to make a credence judgement of that sort on the basis of the knowledge given. X does not know whether the way in which Petersen came to be selected for presentation to him is or is not in some way biased in favor of selecting Swedish Catholics with a statistical probability, or chance, different from the frequency with which Catholics appear in the Swedish population as a whole . . . For those who take chance seriously, in order for X to be justified in assigning a degree of credence equal to .9 to the hypothesis that Petersen is a Protestant on the basis of direct inference alone, X should know that Petersen has been selected from the Swedish population according to some procedure F and also know that the chance of obtaining a Protestant on selecting a Swede according to procedure F is equal to the percentage of Swedes who are Protestants (1977, pp. 9–10).

Thus for Levi randomness is not lack of knowledge of bias, it is having knowledge that there is no bias. This requires knowing the chance mechanism by which Petersen was selected, and knowing that such a mechanism is unbiased with respect to selecting a Protestant among Swedes. Thus the proper position to attribute to Levi, given the way that the Petersen case has been described, is that no direct inference is possible because, in the situation as described, X has no knowledge about the mechanism by which Petersen was selected.

But the example represents more than a mere difference of opinion between Kyburg and Levi about the nature of randomness and the choice of reference class. The case is problematic for Kyburg for two reasons. First, it seems intuitive that whatever we may know about the probability of h, the one thing we can be sure of is that .9 is precisely what the probability of h is *not*. After all, Petersen is from Malmo, and though we don't know whether .85, .91, or .95 is actually the proportion of Protestants in Malmo, we know that none of these is .9. Second, Levi claims that Kyburg's treatment forces him to violate CC.

Let us look at the principle CC. To understand CC, it will be necessary to introduce some technical notation. A rational agent has a body of knowledge, which we may understand as all those statements expressible in the agent's language which that agent has some credal stand towards (belief, disbelief, or some degree of belief). In the body of knowledge will also be a set of probability functions, which, when applied to statements in the body of knowledge, will yield a numerical value from 0 to 1 indicating the degree of belief that the agent has in those statements. We shall call these probability functions Q-functions, and we shall call the set of all Q-functions the set B. There will, in general, be some function C, on a body of knowledge K, such that $C(K) = B$. C is a measure of the agent's confirmational commitment to the body of knowledge K.

The principle CC is a principle which Levi thinks dictates how a rational agent's body of knowledge should change when a new statement is added to that body of knowledge. Suppose we have a body of knowledge K, and a statement e which is not part of K but is consistent with it. If h and f range over all the sentences in language L, the language of K, we may then ask: "What determines the new functions Q' that would obtain by adding e to K?" CC is the principle which constrains how the Q-functions change by adding new evidence to a body of knowledge (Levi, 1977, pp. 18–19):

> *Confirmational Conditionalization:* Let K' be obtained from K by adding e consistent with K. Let $C(K) = B$ and $C(K') = B'$. For every Q-function in B, there is a Q'-function in B' such that $Q'(h;f) = Q(h;f\&e)$ when f is consistent with K'. Conversely, for every Q'-function in B' there is a Q-function in B such that $Q(h;f\&e) = Q(h;f)$.

In simple terms, CC says what seems to be very intuitive: the probability of h given f in K which contains e will be the same as the probability of h given $f\&e$ in K which does not contain e.

Consider rational X who is making guesses about the likelihood of a certain die roll after being told whether his roll is odd or even. Let e be the statement "This roll is even", and let K be a body of knowledge that is consistent with but does not contain e. Let h be the statement "This roll is a two". Let K' result from adding e to K. Thus we may ask relative to K, what is the probability of h, given e, and this will be the same as the probability of h relative to K', namely $^1/_3$, since K' differs from K only by containing e. The two cases differ only in this sense: on the one hand, X is asking "supposing I have rolled an even number, what is the probability that it is a two?", and on the other hand, X is saying "I *have* rolled an even number; what is the probability that it is a two?"

In ordinary cases CC seems obvious. Thus we might reasonably ask of a system that violates CC that it do so for good reason. Both Kyburg and Levi have offered derivations to show that as the case is stated, the Petersen case shows that Kyburg's system violates CC (Kyburg 1977, pp. 516–518; Levi 1977, pp. 20–22). I will not repeat those proofs here, for I question not the derivations based on the example but the example itself.[1]

Kyburg is not concerned about the Petersen case, because he does not think that it is a defect of his system that it violates CC. I disagree. However, the defect will be minimal if it should turn out that all those cases where Kyburg's system violates CC are unusual cases where the violation of CC should not be cause for philosophical concern. This question cannot be settled finally without understanding why Kyburg's system violates CC. Until then we must deal with violations of CC case by case. If it can be shown that the Petersen case is not problematic, then the burden of proof is on Levi to provide a different case that more clearly shows some defect in Kyburg's violation of CC. I can show that the Petersen case should not be regarded as problematic for Kyburg. The point here is a limited one; there are presumably other cases where Kyburg's system violates CC, and I make no claim about which is correct in those cases, Kyburg's system or the principle CC.

Kyburg notes in passing that "It is hard to see where we could get the strange information that we are supposing X to have" in the Petersen case (1977, note 3). This is a penetrating point that Kyburg does not follow up. There are two very important features about the Petersen case. First, the information about Malmo is disjunctive. The derivation of Kyburg's violation of CC *in this particular case* depends on X's knowledge about the proportion of Protestants in Malmo being disjunctive. If X knew only

[1]Kyburg points out elsewhere that violation of CC in his system is "pervasive". He means by this not that violations of CC will pervade throughout a body of knowledge, but rather that virtually all bodies of knowledge will contain some violations of CC. This by itself is no disaster; it depends on the nature of the cases in which Kyburg violates CC. Unfortunately there is as yet no indication that violation of CC in Kyburg's system occurs systematically. Two features of his system seem to be related to the violation of CC, but the relation is at present only partially understood: i) Kyburg's use of the strength rule (the principle that, given nonconflicting statistical statements, one should choose the reference class for direct inference about which one has the most precise information), which is what Levi attempts to exploit in the Petersen case, and ii) the violation of symmetry of relevance. Symmetry of relevance requires that if accepting a statement e into a body of knowledge containing f changes the probability of f, then accepting f into a body of knowledge containing e must change the probability of e. Kyburg explicitly denies such a principle (Bogdan 1982, pp. 161–162).

that the proportion of Protestants was in the interval [.85, .95], no violation of CC could be derived. Second, X's knowledge about the proportion of Protestants in Malmo occurs *in vacuo:* X has no knowledge about how he came to have this information, and thus has no information about the relative likelihood of each of the disjuncts! No wonder Kyburg rightly questions how X could come to have such "strange information".

I can see that there are cases in which X would not know about the relative likelihood of possible proportions; indeed this is often the case when we are confronted with an interval-valued proportion. And I can see that there are cases in which X might have disjunctive, rather than interval-valued, knowledge of proportions. What I cannot see is that both these conditions could obtain at the same time. Among the items in X's body of knowledge must be *not only* that the proportion of Protestants in Malmo is .85, .91, or .95, but how X came to *know* that the proportion is .85, .91, or .95. There is no plausible way in which X could know how he came to have this knowledge about Malmo without having some belief, however vague, about the relative likelihood of each of the disjuncts.

If we consider an ideal case, where X knows that each proportion is equally likely to be the correct proportion (that is, *pr* ("proportion of Protestants in Malmo = .85") = $^1/_3$, etc.), then it is a simple exercise in probability theory to show that $pr(h) = .88$! Under these circumstances this is a result both Kyburg and Levi would agree to, *and* under such circumstances Kyburg does not stand in violation of CC. In actuality this ideal knowledge is not likely to be available. It would be more usual that X would have some vague interval-valued knowledge about the likelihood of each disjunct, as in *pr*("proportion of Protestants in Malmo = .85") $\in$ [.1,.5]. But this sort of case is also one in which Kyburg does not stand in violation of CC.

What are we to conclude? The Petersen case is significant, but one thing which is *not* significant about it is that Kyburg's system violates CC. It is simply false that people acquire beliefs without also having some belief about the epistemic status of their beliefs, yet the Petersen case requires of X that his beliefs occur in just such a vacuum in order to demonstrate Kyburg's violation of CC. Thus the burden of proof is on Levi to show that there are cases of interest where Kyburg violates CC. I do not deny that there may be such cases, only that the Petersen case is such an example.

Kyburg, on the other hand, thinks that Levi's own example creates problems for Levi's system. Levi's principle to choose the narrowest reference to class to which the object in question is known to belong at random is, according to Kyburg, unacceptable. Choosing the narrowest reference class can mandate that we use statistical knowledge so vague as to be no knowledge at all. There is, of course, a very strong intuition behind Levi's principle: if you know that Petersen is from Malmo, then you know it is not the proportion of Protestants in Stockholm, or any other place in Sweden that matters; it is the proportion in Malmo that matters. However, Kyburg claims, Levi's attempt to follow this intuition leads us astray. Kyburg questions why we do not consider the subdivision of Malmo that Petersen lives in, or the apartment block in which he lives, as the appropriate reference class. In fact:

> We have one relatively firm bit of sociological knowledge we have not yet used: that people of like religious commitment tend to live together. Thus . . . either all of the Swedish residents of apartment *A* are Protestants or else all of the Swedish residents of apartment *A* are non-Protestant. . . . Therefore, the probability that Petersen is a Protestant is *mandated* only to a subinterval of [0, 1] (Kyburg 1983b, p. 631).

On Levi's system, to make a direct inference about Petersen's being Protestant we would have to know that Petersen was selected at random from some reference class. If we suppose that Petersen has been selected at random from among the population of Swedes, then we may infer that $pr(h) = .9$, since the proportion of Protestants among Swedes is .9. *But*, and this is the force of Kyburg's argument, for Petersen to be selected at random among Swedes means that the chance of selecting any Swede by the method which Petersen was selected is known to be the same as the chance of selecting any other Swede. But if this is true, then since (by assumption) all residents of Malmo are Swedish, it follows that the chance of selecting any resident of Malmo by the method by which Petersen was selected is the same as the chance of selecting any other resident of Malmo, and hence Petersen has been selected *at random* from among residents of Malmo. This procedure may be applied to any other subset of Swedes to which Petersen is known to belong. Thus if we follow the principle of choosing the narrowest reference class to which Petersen is known to belong at random, then we seem forced to infer the useless conclusion that the chance of Petersen's being Protestant is in the interval [0, 1].

However, Levi argues that this conclusion is not forced on him, and that the argument rests on a confusion between understanding probability as relative frequency and probability as chance. Either the fact that Petersen is a resident of Malmo is of some relevance or it is not. If it is not, then the class of Swedes is the appropriate reference class. If it is, then we require knowledge of the chance mechanism by which the set of residents of Malmo has been selected from the class of Swedes. Knowing that the proportion of Protestants among residents of Malmo is in the interval [.85, .95] gives us some information about frequencies, but this is *not* knowledge of chances. Hence, on Levi's view, unless we have knowledge of how residents of Malmo have been selected from Swedes, the appropriate conclusion to draw in this case is that no direct inference is possible; we do not know in that case what the chance is that Petersen is Protestant (see Levi 1983, pp. 637–641).

If we impose these strict requirements on direct inference, then the number of cases in which a direct inference is sanctioned is apparently much smaller than one might think. But this is a consequence that Levi is willing to accept; as Kyburg notes, "Levi never said that direct inference was easy" (1983b, p. 630).

It appears, then, that neither Levi nor Kyburg has brought telling arguments against the other on the basis of the Petersen case. What, then, is the real significance of the example? This can most clearly be illustrated by means of two examples which are, in logical form, similar to the Petersen case.[2]

Imagine our rational agent X in a different situation, where he has a different body of knowledge containing the following statements:

a. The Ace Urn Company orders only red and black balls, and the proportion of red balls ordered is .9;
b. The Ace Urn Company places all of the balls ordered into urns;
c. Of the balls ordered by the Taiwan Division of the Ace Urn Company, the proportion of red balls ordered $\in [.01, .99]$;
d. X has an urn labeled "Ace Urn Company—Made in Taiwan" from which he is about to draw the first ball.

What is the probability of the statement h': "The first ball drawn from the urn by X will be red"? Kyburg's analysis here would proceed as in the Petersen case. X should choose the reference class about which he has more precise statistical information, and hence conclude that $pr(h') = .9$. To be consistent with his analysis of the Petersen case, Levi would have to say that here is another case in which the information available to X is not sufficient to warrant a direct inference.

While Levi can dig in and say that circumstances under which direct inference is permissible are difficult to obtain, this may not be much comfort to X. We can imagine that it is extremely important for X to make a judgment. Indeed, we can even imagine a case like the following. As X is preparing to make his draw the Devil springs up out of the ground and says to X, "We are going to wager for your soul. I have here a chance

[2]They differ from the Petersen case in that we shall use interval-valued probabilities, rather than the precise disjunctive probability statements which make the Petersen case so implausible.

mechanism for rolling dice which has a chance of the die landing with a 6 facing up equal to $^1/_6$. You can save your soul if either you correctly guess the color of the next ball to be drawn from the urn or if you guess that the next roll of the die will land 6 up and it does land 6 up." One would expect that *X* would need very strong reason to believe that the chances of guessing the correct color are less than $^1/_6$ before *X* would stake his soul on a roll of 6 and clearly there is no such evidence. We may say at most that it is *possible* that the chances of *X* correctly guessing the color are less than $^1/_6$, but under the circumstances there are no strong reasons to believe that this is the possibility that will be realized. In such a case then, *X* should favor choosing the color of the ball over guessing 6. *X* will then guess red and he will base his inference on the reference class of the Ace Urn Company, not on the narrower reference class of the Taiwan Division. For Kyburg this rational decision is just a straightforward consequence of his analysis of direct inference; however, Levi's system has the implausible consequence of mandating that *X* bet his soul on a roll of 6, not on the next ball being red.

Let us examine how this comes about. *X* must decide which of three mutually exclusive propositions to add to his body of knowledge:

i) the next ball drawn will be red;
ii) the next ball drawn will be black; or
iii) the next die roll will land 6 up.

Knowledge of chances is not available that will dictate which of the propositions *X* should add to his body of knowledge. Levi, having put such harsh restrictions on direct inference, recognizes that this type of case often arises:

> It is undoubtedly true that when sampling from a population . . . the investigator may obtain more information about the individual than that he was selected at random at *t* from the population. And sometimes such information cannot be discounted as irrelevant for direct inference. Credal judgements of a non-trivial sort might require invoking considerations other than knowledge of chances (1983, p. 641).

In *The Enterprise of Knowledge* Levi goes to great lengths to explain how a rational agent makes "credal judgments" in the absence of knowledge of chances. On his view, for a proposition to be admissible to a body of knowledge, it must be the *s*-admissible proposition among those *e*-admissible propositions which are *p*-admissible. What does this mean? For all three of the propositions above there is some probability function which warrants *X* to add that proposition to his body of knowledge. Thus all three are *e*-admissable. Levi then considers whether:

> at least one cognitive option exists which is (a) as weak or weaker than all e-admissible options, (b) no weaker than any other option satisfying (a), and (c) e-undominated (1980, p. 138).[3]

Such an option would be a WU-option (weak, undominated). Levi then says:

> If there are any WU-options, all and only WU-options are p-admissible. If there are no WU-options, all and only e-admissible options are p-admissible (p. 139).

All three of X's options are undominated, and they are not comparable for weakness, hence all are p-admissible. S-admissibility just means security-admissibility, hence the S-admissible option among these three will be the least risk option. But because the urn X has is from the Taiwan Division of the Ace Urn Company, he does *not* know that the chance of drawing a red ball is *not* as low as .01 and he does *not* know that the chance of drawing a black ball is *not* as low as .01 (although not both of these can be the case). The safest option then, according to *Levi's* way of handling the case, is to bet on a roll of 6. Hence Levi's view mandates that X stake his soul on correctly guessing the roll of a die! Yet as we noted initially, while it is *possible* that X's chances of guessing the correct color of the ball are less than $^1/_6$, X has no particular reason to believe that they will be that low. Betting on a roll of 6 then hardly seems like the "safe" option. Thus Levi either has not stated a correct notion of security in defining how an agent makes credal judgments or else he has not shown that the safe option is always the rational option.

Unfortunately for Kyburg there is a variation on this case that must also be considered. Suppose that the information available to X is the following:

a. The proportion of red balls ordered by the Ace Urn Company is .51;
b. The Ace Urn Company places all of the balls ordered into urns;
c. The proportion of red balls ordered by the Taiwan Division $\in$ [.01, .52];
d. X has an urn from which he is about to draw the first ball which is labeled "Ace Urn Company—Made in Taiwan".

Now let us suppose that the Devil again proposes his wager and X must correctly guess the color of the ball or correctly guess 6 up to save his soul. In *this* case it seems rational for X to predict black, not red (and certainly not to bet on the roll of a die). For X to bet on red, he would need strong reason to believe that the probability of drawing a red ball

[3]The sense of "weakness" here is the same as the sense in which we say that "p or q" is a weaker statement than "p".

is at least .5 and given that the urn is from the Taiwan Division X apparently lacks strong reason to so believe. On Levi's analysis X would be permitted to pick either black or a roll of 6, but Kyburg mandates that X make the implausible choice of red. This comes about because the information available to X about the Taiwan Division does not *conflict* with the information available to X about the Ace Urn Company, since $.51 \in [.01, .52]$. In cases where the two competing reference classes do not conflict Kyburg says we should choose the reference class about which we have more precise information, in this case the Ace Urn Company.

Of course the two statistical statements are virtually in conflict and this is really the main point. It seems reasonable that someone would seek more evidence or make some other change in his set of beliefs before admitting such a virtual conflict into his body of knowledge. Kyburg is aware of this kind of problem, and offers at least the beginning of a response:

> We shall suppose that every body of knowledge is characterized by a number, which we may call its degree of practical certainty. The appropriate body of knowledge to be concerned about depends on the context: in some situations, a probability . . . of .9 might count as a sure thing, as practical certainty; in other contexts a probability of $1 - 10^{-5}$ might barely count as practical certainty. There is a tradeoff here: the more demanding our notion of practical certainty, in general, the less we can claim to know in our body of knowledge (1974, p. 371).

The number associated with a body of knowledge is called its index. Roughly, the index of a body of knowledge is the "probability" that any given statement in the body of knowledge is correct; it is the degree of accuracy associated with that body of knowledge.

Thus in the last example the statistics .51 and [.01, .52] emerge only relative to a certain index; in a body of knowledge with a lower index our statistics would be more precise. We might then get the figures .51 and [.1, .45]. But this difference in figures is an important one; we would then have, not consistent, but *conflicting* statistical statements. Thus we would choose the narrower reference class, the apparently rational choice in this example.

Choice of index for a body of knowledge then, is an important part of choosing a reference class; different indices can result in different reference classes being correct for a given inference. Kyburg does talk about the appropriate index depending on context, but he has yet to offer any clear principles about how to relate context and body of knowledge to determine the appropriate index.

I conclude that the Petersen case is not just a disagreement about direct inference and choice of reference class. Rather it signals that both Kyburg and Levi have offered us, in crucial aspects, inadequate systems. For Levi the problem comes about from his attempt to defend his account of direct inference. He is able to do so but only at cost; in maintaining a consistent account of direct inference he is forced to drastically restrict the number of cases in which a direct inference is sanctioned. When we look closely at the broad range of remaining cases we find that in his account of credal judgments there still remains a conflict between his principle of security and his principles about choice of reference class. Kyburg, on the other hand, has offered us an incomplete account. Before we can even judge the adequacy of his account we need to understand how statements are accepted into a body of knowledge. Since this depends on an account of choosing indices for a body of knowledge, an account not yet offered by Kyburg, we are left with little understanding of the dynamics between direct inference and indices.

That cases like the Petersen case touch on issues of great philosophical importance no one disputes. Nor is it disputed that both Kyburg and Levi have contributed to our understanding of the philosophical issues concerning probability through their discussion of such cases. But the philosophical import of such discussion will not be to show that one of them is right and the other wrong. Rather cases like the Petersen case show that both Kyburg and Levi require further development in their systems before we may fairly judge them. It may be taken as practically certain that they shall remain in disagreement about the fundamental issues of probability, but the considerations raised here should point them in the direction of a more profitable and enlightened disagreement.

REFERENCES

Bogdan, Radu (ed.) (1982), *Henry Kyburg and Isaac Levi*. Dordrecht: Reidel.

Kyburg, H. E. (1974), *The Logical Foundations of Statistical Inference*. Dordrecht: Reidel.

———. (1977), "Randomness and the Right Reference Class", *Journal of Philosophy 74*: 501–521.

———. (1980), "Conditionalization", *Journal of Philosophy 77*: 98–114.

———. (1983a), "The Reference Class", *Philosophy of Science 50*: 374–397.

———. (1983b), "Levi, Petersen, and Direct Inference", *Philosophy of Science 50*: 630–634.

Levi, I. (1977), "Direct Inference", *Journal of Philosophy 74*: 5–29.

———. (1978), "Confirmational Conditionalization", *Journal of Philosophy 75*: 730–737.

———. (1980), *The Enterprise of Knowledge*. Cambridge, Mass: MIT Press.

———. (1981), "Direct Inference and Confirmational Conditionalization", *Philosophy of Science 48*: 532–552.

———. (1983), "Kyburg on Random Designators", *Philosophy of Science 48*: 635–642.

Seidenfeld, T. (1978), "Direct Inference and Inverse Inference", *Journal of Philosophy 75*: 709–730.