Month-by-Month Staffing Plan

Determine the optimal number of agents per country for each month of 2025, balancing:

Cost efficiency – Minimizing idle agents while meeting advertiser demand
Timely advertiser support – Reducing wait times and maximizing revenue uplift
Scalability – Adjusting for market fluctuations with hiring/firing constraints

Assumptions:

1. We only hire / send fire notices on the 1st of each month.

Decision Variables:

h_t^c : # of agents hired in country c at month t

f_t^c : # of agents fired in country c at month t

US $(h_1 \; h_2 \; \cdots \; h_{12})$
$(f_1 \; \cdots \; f_{12})$

Input Parameters:

A_0^c: Initial # of agents in country c (Jan 2025)

S^c: Annual salary for an agent in country c

T^c: Threshold in country c

N_t^c: # of new advertisers in country c at month t

B_i: Projected annual advertising budget for advertiser i

U_i: Expected uplift % based on:

$\mathbb{P}(0\%\text{ uplift}) = 0.05$ $\quad$ $\mathbb{P}(15\%\text{ uplift}) = 0.25$
$\mathbb{P}(5\%\text{ uplift}) = 0.15$ $\quad$ $\mathbb{P}(20\%\text{ uplift}) = 0.2$
$\mathbb{P}(10\%\text{ uplift}) = 0.25$ $\quad$ $\mathbb{P}(25\%\text{ uplift}) = 0.1$

$\Rightarrow$ Expected revenue uplift multiplier

$$\mathbb{E}[u] = 0.05\times 0 + 0.15\times 0.05 + 0.25\times 0.1 + 0.25\times 0.15 + 0.2\times 0.2 + 0.1\times 0.25$$
$$= 0.135$$

Dependent Variables:

A_t^c : Total # of active agents in country c at month t

$$A_t^c = A_{t-1}^c + h_{t-1}^c - f_{t-1}^c$$

* Agents hired in month t-1 are available in t, agents noticed-to-fire in t-1 leave in t.

* G_t^c : Advertisers who graduated after 60 days.

$$G_t^c = \sum_{i=t-2}^{t} (E_i^c - U_i^c)$$

↑ 60 days ago

O_t^c : Total occupied spots at month t

$$O_t^c = O_{t-1}^c + 10A_t^c - G_t^c$$

L_t^c : Available slots for new advertisers

$$L_t^c = 10A_t^c - O_t$$

U_t^c : # of unassigned advertisers in country c at the 1st of month t.

$$U_t^c = \max(0, P_t^c + U_{t-1}^c + O_t^c - 10A_t^c)$$

↑ New in ↑ pool ↑ total capacity at month t

E_t^c : # of eligible advertisers in country c at month t

$$E_t^c = \sum_{i \in N_t^c} 1|_{B_i^c \geq T^c}$$

E_t^c = assigned + unassigned eligible at the same time

R_t^c : Incremental Revenue uplift

$$E_t^c = P_t^c - G_t^c$$

new in

Objective: Maximize Net Profit

Income	Cost
Revenue uplift $R_t^c = \mathbb{E}[\text{uplift}]$ $= \sum_i B_i \cdot U_i$ $= 0.135 \cdot \sum_i B_i$	Salary: $A_t^c \cdot \frac{S^c}{12}$: $C^c_{\text{salary},t}$ Firing cost: $0.4 \cdot S^c \cdot F_t^c$: $C^c_{\text{fire},t}$ Hiring cost: $h_t^c \cdot \frac{S^c}{12}$: $C^c_{\text{hire},t}$ Idle Agent: $\left(1 - \frac{\text{Assigned Advertisers}}{10 \cdot A_t^c}\right) \times C^c_{\text{salary},t}$ ↑ Idle Ratio : $C^c_{\text{idle},t}$

$$\max \sum_c \sum_t \left(R_t^c - C^c_{\text{salary},t} - C^c_{\text{fire},t} - C^c_{\text{hire},t} - C^c_{\text{idle},t}\right)$$

Constraints:

$$A_t^c,\ h_t^c,\ f_t^c \in \mathbb{Z}^+$$

Track agents #: $A_t^c = A^c_{t-1} + h^c_{t-1} - f^c_{t-1}$

Waiting limit (60 days): $U_t^c = 0$

firing: $f_t^c \le A_t^c$

①

* G_t^c : Advertisers who graduated after 60 days.

$$G_t^c = \sum_{i=t-2}^{t} (E_i^c - U_i^c)$$

↑ 60 days ago

一號發生 $G_t^c = \sum_{i=t-2}^{t} (E_i^c - U_i^c)$

$$G_3^c = \sum_{i=1}^{3} (E_i^c - U_i^c)$$

$$= E_1^c - U_1^c + E_2^c - U_2^c + E_3^c - U_3^c$$

repeated calculation

correct: $G_3^c = E_1^c - U_1^c$ 2/1

$G_4^c = E_2^c - U_2^c$ → 二月進來(被assigned)的數量

↳ 四月被 serve 完的數量

4/1

$$G_t^c = E_{t-2}^c - U_{t-2}^c$$

↓

② 加入 constraints

③

O_t^c: Total occupied spots at month t

$$O_t^c = O_{t-1}^c + 10A_t^c - G_t^c$$

L_t^c: Available slots for new advertisers

$$L_t^c = 10A_t^c - O_t$$

O_t^c 指的是 Assigned advertisers in month t 嗎?

上月被 assigned

$$O_t^c = O_{t-1}^c + 10A_t^c - G_t^c$$

重覆計算了

④ Why O_t^c / Why L_t^c ?

⑤ Constraints : Capacity

$$E_t^c - U_t^c \leq 10A_t$$