系统工程导论第二次作业

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1 关于显著性检验

病态条件下同样需要显著性检验，且应当考虑在降维之后检验。以F检验为例，当出现病态情形时，检验计算表达式中自由度不再是变量总数，而是以$n-r$替代原式中的n。亦即：

$$F = \frac{(N-n+r-1)ESS}{(n-r)RSS} \tag{1}$$

2 计算思路

假设X是$N \times n$的输入样本矩阵，x_i表示第i列样本，Y是$N \times 1$的输出样本矩阵，首先对样本进行归一化：

$$\tilde{x_i} = \frac{x_i - \bar{x}_i}{\sigma_{x_i}}, \forall i = 1, 2, ...n \tag{2}$$

$$\tilde{Y} = \frac{Y - \bar{Y}}{\sigma_Y} \tag{3}$$

接下来进行病态分析，求取X^TX的特征值，并进行正交分解：

$$L = X^TX \tag{4}$$

$$UTU^T = L, s.t. UU^T = I \tag{5}$$

剔除接近0的特征值（r个）以及相对应的特征向量得到U_m和T_m，维度降到了$n-r$，进行线性回归，求得数据归一化、中心化之后的拟合系数：

$$\bar{c} = U_m T_m^{-1} U_m^T X^T Y \tag{6}$$

对该拟合系数作变换，并计算截距（$\bar{X}$表示X每一列取平均得到的行向量）：

$$c_i = \frac{\bar{c}_i \sigma_Y}{\sigma_{x_i}}, \forall i = 1, 2, ...n \tag{7}$$

$$\beta_0 = \bar{Y} - \bar{X}c \tag{8}$$

对结果进行F检验（考虑病态情形）：

$$F = \frac{(N - n + r - 1)ESS}{(n - r)RSS} \tag{9}$$

计算置信区间：

$$d = Z_{\frac{\alpha}{2}} \sqrt{\frac{RSS}{N - n + r - 1}} \tag{10}$$

3 计算结果

以下检验均在显著性水平0.05下进行。

回归方程及置信区间：

$$y = -9.151455 + 0.072966x_1 + 0.598562x_2 + 0.001872x_3 + 0.105482x_4 \pm 1.155715 \tag{11}$$

F检验结果表明可认为线性关系成立：

$$F = 195.116494 > F_\alpha = 4.346831 \tag{12}$$

4 具体实现

Matlab代码（.m文件）如下所示：

```
% function head
function linear_regression(y,x,alpha)
% set sample average to 0,
```

```
% and set sample standard deviation to 1
Y = transpose(y);
Y = (Y - mean(Y))/std(Y);
[row,col] = size(x);
X = x;
avgX = zeros(1,col);
stdX = zeros(1,col);
for k = 1:col
    c = x(:,k);
    avgX(1,k) = mean(c);
    stdX(1,k) = std(c);
    X(:,k) = (c - mean(c))/std(c);
end
% attach a column of 1 to matrix x
% as the interception
ln = ones(row,1);
Xt = [ln,X];
X = transpose(Xt);
% decomposition
L = X*Xt;
[U,T] = schur(L);
% rule out small eignvalues and vectors
m = 0;
for k = 1:(col+1)
    if T(k,k) > 0.1
        m = m + 1;
    end
end
Qm = zeros(col+1,0);
Vm = zeros(m,m);
i = 1;
for k = 1:(col+1)
    if T(k,k) > 0.1
```

```
            Qm = [Qm,U(:,k)];
            Vm(i,i) = T(k,k);
            i = i + 1;
        end
end
% solve for coefficients
cbar = std(y)*Qm*inv(Vm)*transpose(Qm)*X*transpose(Y);
yhat = Xt*cbar + mean(y);
for k = 1:col
    cbar(k+1,1) = cbar(k+1,1)/stdX(k);
end
cbar(1,1) = mean(y) - avgX*cbar(2:col+1,1);
% F test
TSS = dot(y-mean(y),y-mean(y));
ESS = dot(yhat-mean(y),yhat-mean(y));
RSS = TSS - ESS;
F = (row-m)*ESS/((m-1)*RSS);
Fa = finv(1-alpha,m-1,row-m);
%fprintf('F = %f, Fa = %f\n',F,Fa);
if F<=Fa
%     fprintf('No linear relation! (a=%.2f)\n',alpha);
else
%     fprintf('Linear relation! (a=%.2f)\n',alpha);
end
% solve for confidence interval
sdelta = sqrt(RSS/(row-m));
Z = norminv(1-alpha/2,0,1) * sdelta;
%fprintf('Confidence interval: -%f ~ +%f\n',Z,Z);
%fprintf('Regression equation: %f + %f*x1 + %f*x2 + %f*x3 + %f*x4\n',...
%cbar(1,1),cbar(2,1),cbar(3,1),cbar(4,1),cbar(5,1));
end
% function ending
```