ARMA-GARCH

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1 Data

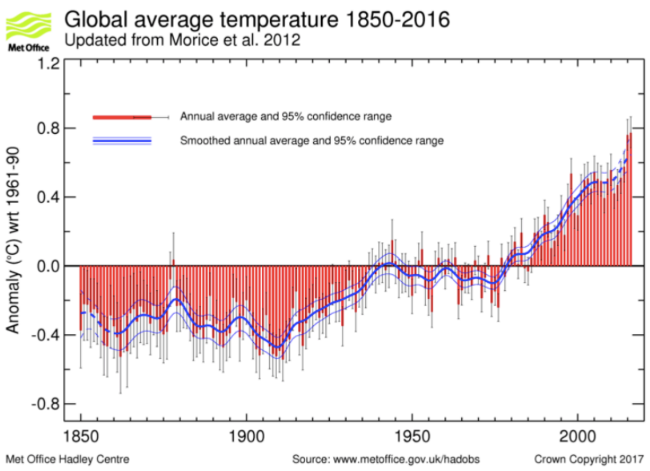
1.1 Data Source

HadCRUT4 is a gridded dataset of global historical surface temperature anomalies relative to a 1961-1990 reference period. Data are available for each month since January 1850, on a 5 degree grid. The dataset is a collaborative product of the Met Office Hadley Centre and the Climatic Research Unit at the University of East Anglia.
url =`http://www.metoffice.gov.uk/hadobs/hadcrut4/data/current/time_series/HadCRUT.4.5.0.0.monthly_ns_avg.txt`.

1.2 Brief Data Description

The gridded data are a blend of the CRUTEM4 land-surface air temperature dataset and the HadSST3 sea-surface temperature (SST) dataset. The dataset is presented as an ensemble of 100 dataset realisations that sample the distribution of uncertainty in the global temperature record given current understanding of non-climatic factors affecting near-surface temperature observations. This ensemble approach allows characterisation of spatially and temporally correlated uncertainty structure in the gridded data, for example arising from uncertainties in methods used to account for changes in SST measurement practices, homogenisation of land station records and the potential impacts of urbanisation. The HadCRUT4 data are neither interpolated nor variance adjusted.

Below is a graph provided by `www.metoffice.gov.uk`:



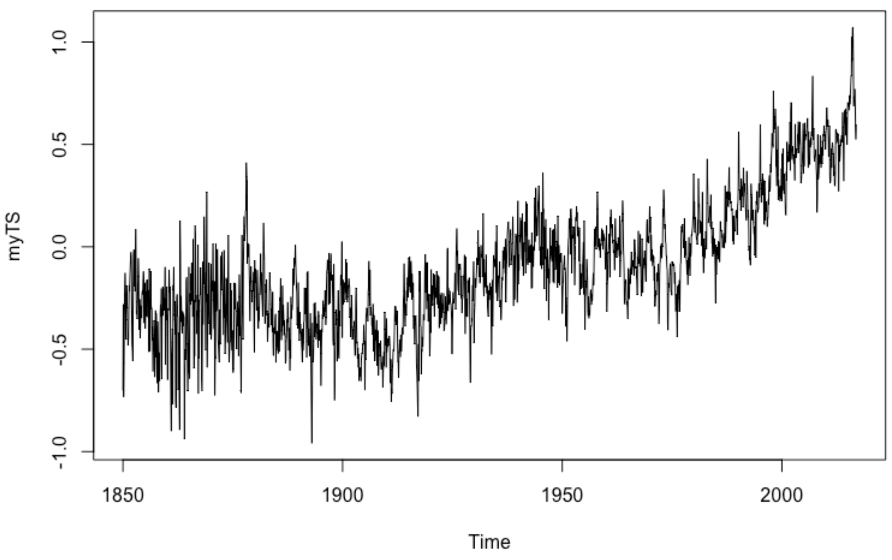
1.3 Loading the Data into R

For our purpose, we are not interested in the spatial distribution, so we only use the *Global Mean*.
Loading the data into R:

```
library(curl)
tmpf <- tempfile()
curl_download(url, tmpf)
gtemp <- read.table(tmpf)[, 1:2]
temp = gtemp$V2[1:2004]
```

We only use the first two columns: *time and monthly global mean*. We pick the first 2004 observations, which is monthly data from 1850 $\sim$ 2016, so that the data has complete periods.

The time series of *temp* is shown below:



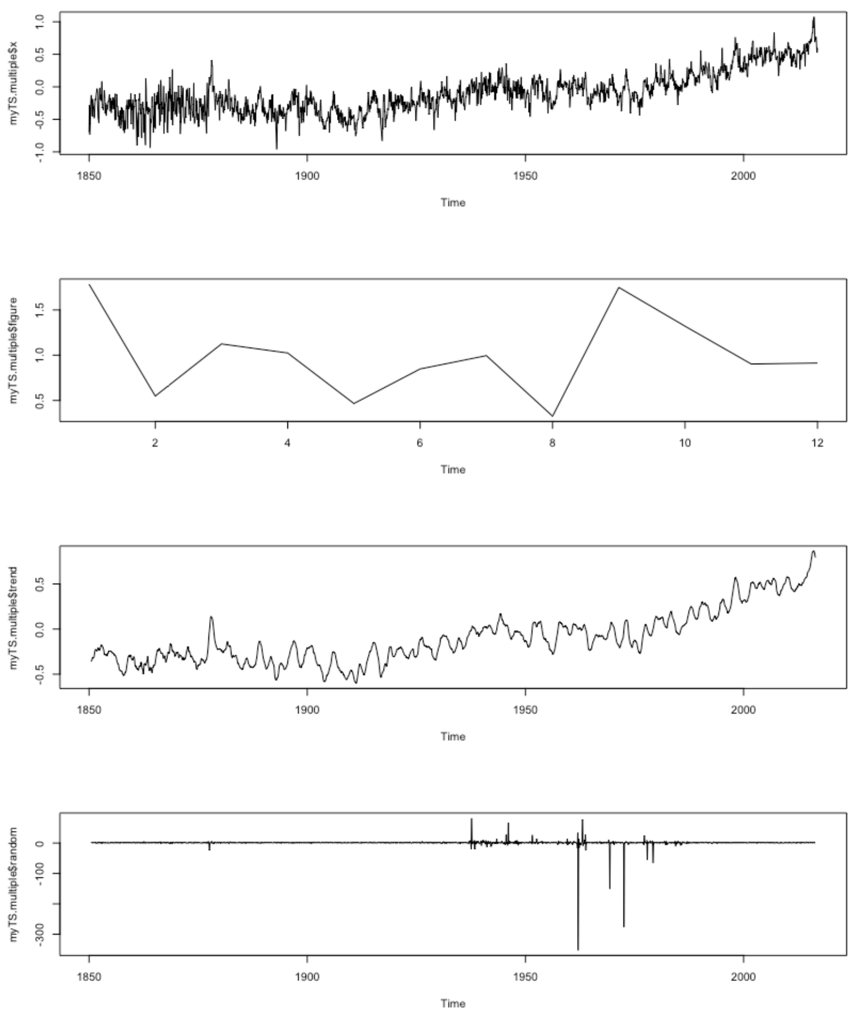
2 Dealing with the time series

2.1 Decompose

The first thing we might want to do is to wipe out the seasonal component. There're many approaches to do it in R. We choose to use the *decompose*() provided by {*stats*}. Looking at the time series, we think a additive model should be appropriate.

We can quickly checked that multiplicative model is not appropriate:

Figure 1: multiplcative model: original ts, seasonal component, time trend, residuals



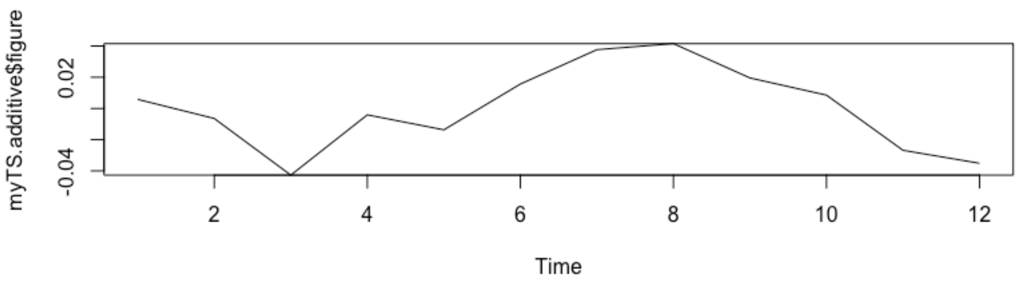
Obviously the random term is not easy to be modelled.

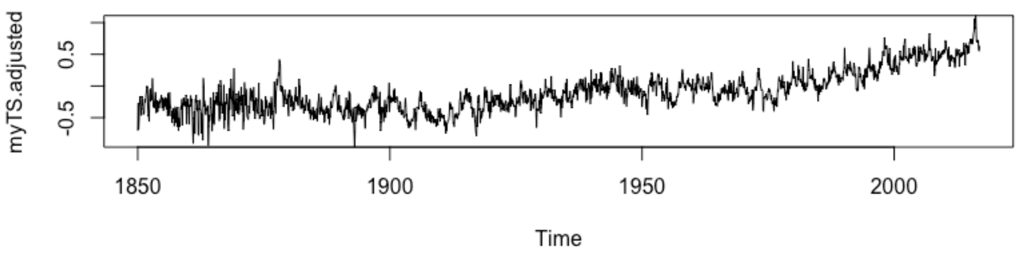
Therefore, decompose the time series with additive method in the following code:

```
library(TSA)
myTS = ts(as.numeric(temp), start = c(1850, 1), frequency = 12)
myTS.additive = decompose(myTS)
myTS.adjusted = myTS.additive$x - myTS.additive$seasonal
```

The *myTS.adjusted* now is free of seasonal components. The following graph illustrate the components.

Figure 2: additive model: seasonal component, adjusted series





Obviously, the time seires is not stationary. For simplicity, use *diff()*.

```
dtemp = diff(myTS.adjusted)

> adf.test(dtemp)

Augmented Dickey-Fuller Test

data:  dtemp
Dickey-Fuller = -16.175, Lag order = 12, p-value = 0.01
alternative hypothesis: stationary
```

So we can convince ourself that the time series *dtemp* is stationary.

2.2 Build a *ARMA(p, q)* for *dtemp*

For simplicity, we use *auto.arima*() from {*forecast*} to judge the order.

```
library(forecast)
> auto.arima(dtemp> auto.arima(dtemp)
Series: dtemp
ARIMA(2,0,4)(2,0,1)[12] with non-zero mean

Coefficients:
          ar1     ar2      ma1      ma2     ma3     ma4     sar1     sar2      sma1    mean
```

```
      0.5041  0.3585  -1.0458  -0.1600  0.1362  0.0780  0.7619  0.0779  -0.7884  5e-04
s.e.  0.2431  0.2142   0.2423   0.3481  0.1082  0.0257  0.0926  0.0248   0.0910  2e-04

sigma^2 estimated as 0.01428:  log likelihood=1417.03
AIC=-2812.05   AICc=-2811.92   BIC=-2750.43

# first find the possible orders might be arma(2, 1) or arma(2, 4)

arma21.dtemp = arima(dtemp, c(2, 0, 1))
arma24.dtemp = arima(dtemp, c(2, 0, 4))

# next examine the two models' residuals

> auto.arima(arma21.dtemp$residuals)
Series: arma21.dtemp$residuals
ARIMA(2,0,3)(2,0,1)[12] with zero mean

Coefficients:
         ar1      ar2      ma1     ma2     ma3    sar1    sar2     sma1
      1.5859  -0.6762  -1.6089  0.6706  0.0506  0.7454  0.0748  -0.7785
s.e.  0.1156   0.1065   0.1169  0.1264  0.0297  0.1209  0.0258   0.1181

sigma^2 estimated as 0.01429:  log likelihood=1415.92
AIC=-2813.83   AICc=-2813.74   BIC=-2763.41

> auto.arima(arma24.dtemp$residuals)
Series: arma24.dtemp$residuals
ARIMA(0,0,0) with zero mean

sigma^2 estimated as 0.01435:  log likelihood=1408.31
AIC=-2814.63   AICc=-2814.62   BIC=-2809.02
```

Therefore it should be appropriate to choose *arma(2, 4)* for series *dtemp*.

```
my.arma = arma24.dtemp

> my.arma

Call:
arima(x = dtemp, order = c(2, 0, 4))

Coefficients:
         ar1     ar2      ma1      ma2     ma3     ma4  intercept
      0.5204  0.3247  -1.0536  -0.1269  0.1168  0.0755      5e-04
s.e.  0.2733  0.2356   0.2727   0.3828  0.1117  0.0265      2e-04

sigma^2 estimated as 0.01435:  log likelihood = 1407.67,  aic = -2801.34
```

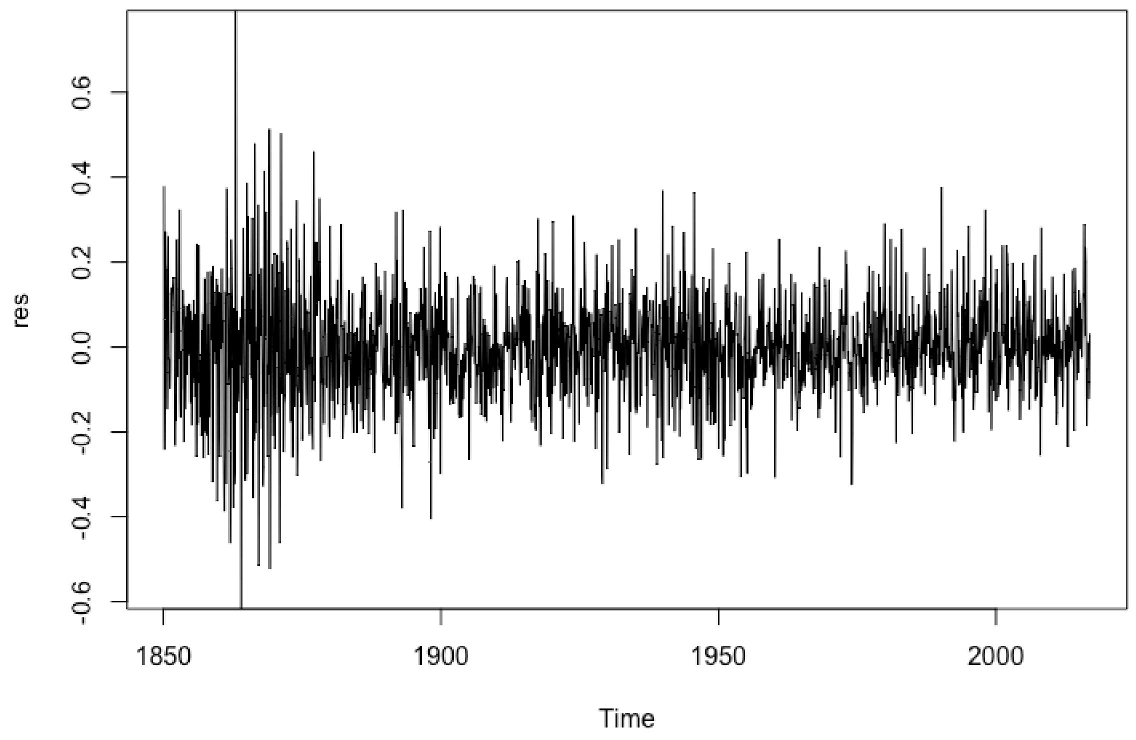
```
res = my.arma$residuals

> Box.test(res, type = 'Ljung-Box')

Box-Ljung test

data:  res
X-squared = 0.0058853, df = 1, p-value = 0.9388
```

The $ARMA(2,4)$ success in making the arma residual independent.

2.3 GARCH

One can be pretty satisfied with the results right now. Yet Looking at the plot of arma residual:

Figure 3: arma residual

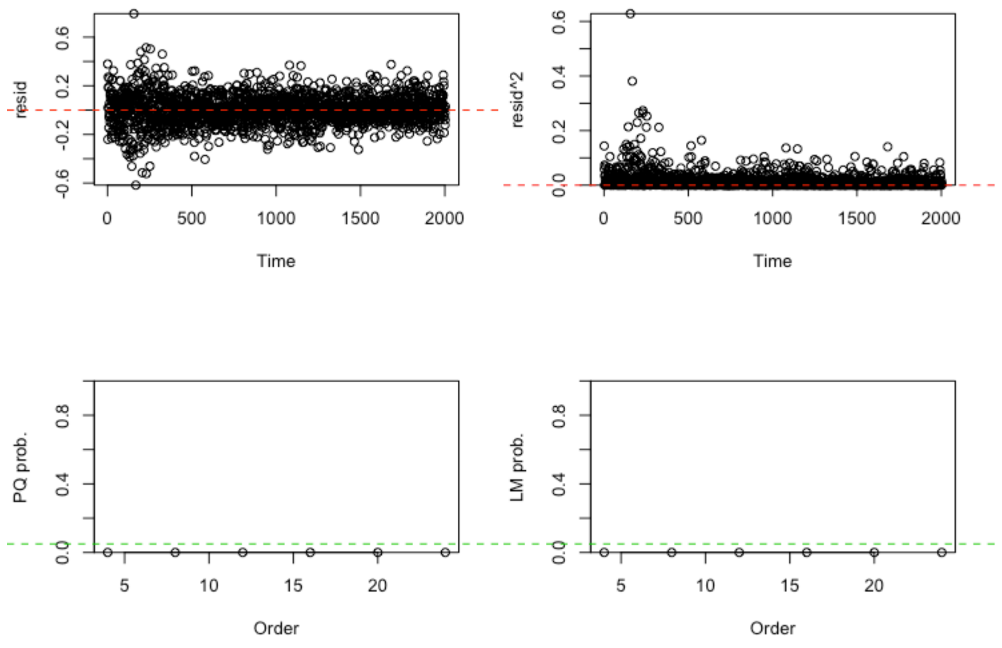


It makes us wondering if it has $ARCH$ effect. Using *arch.test* from $\{aTSA\}$ to perform the $LM-Test$:

```
library(aTSA)
```

```
> arch.test(my.arma)
ARCH heteroscedasticity test for residuals
alternative: heteroscedastic

Portmanteau-Q test:
     order    PQ p.value
[1,]     4  99.4       0
[2,]     8 116.9       0
[3,]    12 411.5       0
[4,]    16 475.6       0
[5,]    20 495.9       0
[6,]    24 755.8       0
Lagrange-Multiplier test:
     order   LM p.value
[1,]     4 1418       0
[2,]     8  697       0
[3,]    12  418       0
[4,]    16  212       0
[5,]    20  168       0
[6,]    24  137       0
```

Figure 4: arma residual



It supports our doubt. Therefore we have the motive to build a *GARCH* model. We decide to use the package *rugarch*.

$$\sigma_t^2 = (w + \sum_{j=1}^{m} \zeta_j v_{jt}) + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

So we wrote a function:

```
library(rugarch)
my_sGARCH_test <- function(p, q, m, n, ts.data = res)
{
# I use include.mean = FALSE after trying TRUE
# to find out insignificance
    myspec=ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(p, q)),
     mean.model = list(armaOrder = c(m, n), include.mean = FALSE),
     distribution.model = "norm")
    myfit=ugarchfit(myspec,data=ts.data, solver="solnp")
    return(myfit)
}
```

After trying a few times from $(1,0),(0,0,0)$ to $(5,5),(4,0,4)$, *GARCH(1 ,1), ARIMA(2, 0, 3)* is the most satisfying model. Here we realize that using GARCH model, the order of ARIMA might changes.

```
fit1 = my_sGARCH_test(1, 1, 2, 3, dtemp)

> fit1

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(2,0,3)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
ar1    -0.090935    0.015226    -5.9724 0.000000
ar2     0.754333    0.015069    50.0583 0.000000
ma1    -0.391906    0.007313   -53.5917 0.000000
ma2    -0.863257    0.000130 -6647.5909 0.000000
ma3     0.295634    0.007800    37.9010 0.000000
omega   0.000082    0.000034     2.3956 0.016595
alpha1  0.023026    0.003614     6.3706 0.000000
beta1   0.970594    0.004705   206.2925 0.000000
```

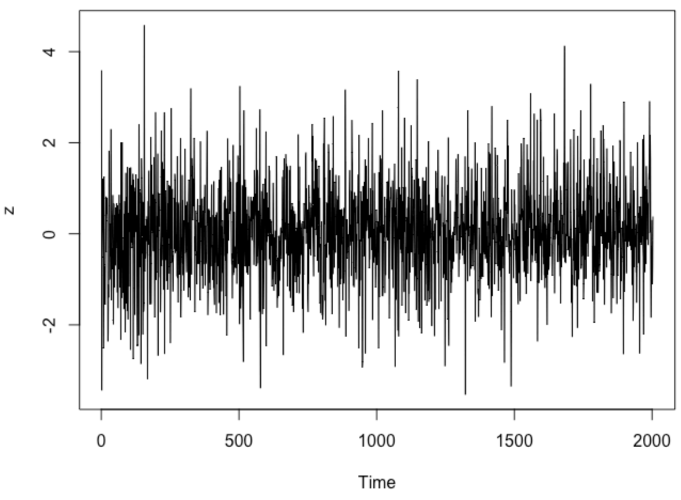
```
Robust Standard Errors:
        Estimate  Std. Error    t value Pr(>|t|)
ar1    -0.090935    0.017031    -5.3393 0.000000
ar2     0.754333    0.017312    43.5726 0.000000
ma1    -0.391906    0.002647  -148.0378 0.000000
ma2    -0.863257    0.000144 -6000.9074 0.000000
ma3     0.295634    0.002903   101.8443 0.000000
omega   0.000082    0.000036     2.2838 0.022382
alpha1  0.023026    0.003636     6.3329 0.000000
beta1   0.970594    0.003734   259.9413 0.000000

LogLikelihood : 1512.336

Information Criteria
------------------------------------

Akaike       -1.5021
Bayes        -1.4797
Shibata      -1.5021
Hannan-Quinn -1.4939

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.3742  0.5407
Lag[2*(p+q)+(p+q)-1][14]    4.5219  1.0000
Lag[4*(p+q)+(p+q)-1][24]   13.5547  0.3236
d.o.f=5
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      31.06 2.506e-08
Lag[2*(p+q)+(p+q)-1][5]     37.84 1.904e-10
Lag[4*(p+q)+(p+q)-1][9]     51.28 4.433e-13
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale   P-Value
ARCH Lag[3]  0.004507 0.500 2.000 9.465e-01
ARCH Lag[5] 11.085543 1.440 1.667 3.565e-03
ARCH Lag[7] 20.592184 2.315 1.543 4.509e-05

Nyblom stability test
```

```
------------------------------------
Joint Statistic:  2.6294
Individual Statistics:
ar1    0.25019
ar2    0.61592
ma1    0.19513
ma2    0.24162
ma3    0.07259
omega  0.10095
alpha1 0.42035
beta1  0.20019

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.89 2.11 2.59
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value      prob sig
Sign Bias           0.3363 7.367e-01
Negative Sign Bias  3.6815 2.380e-04 ***
Positive Sign Bias  4.4421 9.396e-06 ***
Joint Effect       33.2983 2.786e-07 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     57.32    1.019e-05
2    30     63.59    2.167e-04
3    40     85.01    2.883e-05
4    50    102.77    1.103e-05


Elapsed time : 0.365526
```

Substract the standardized(w.r.t. the variance model) residuals z, which is $z = \dfrac{residuals(fit)}{sigma(fit)}$.

```
z = residuals(fit1) / sigma(fit1)
plot.ts(z)
```

Figure 5: z

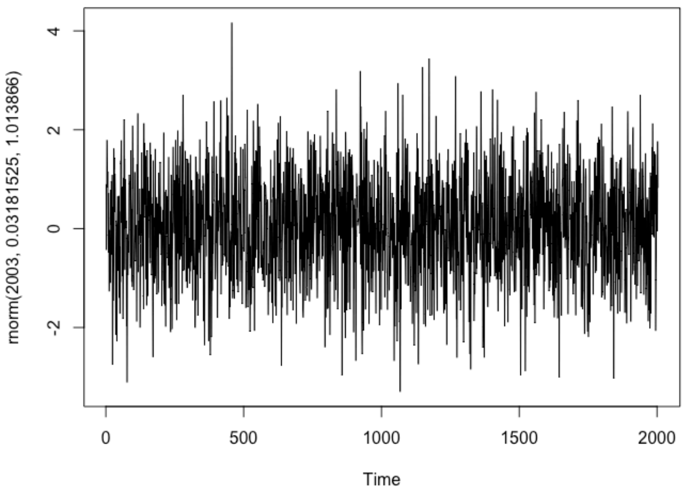


We can compute the mean and variance of z:

```
> mean(z)
[1] 0.03181525
> var(z)
[1] 1.013866
> length(z)
[1] 2003
> plot.ts(rnorm(2003, 0.03181525, 1.013866))
```

And then, just for fun, plot a normal sample series with the same parameters:

Figure 6: simulation of using rnorm



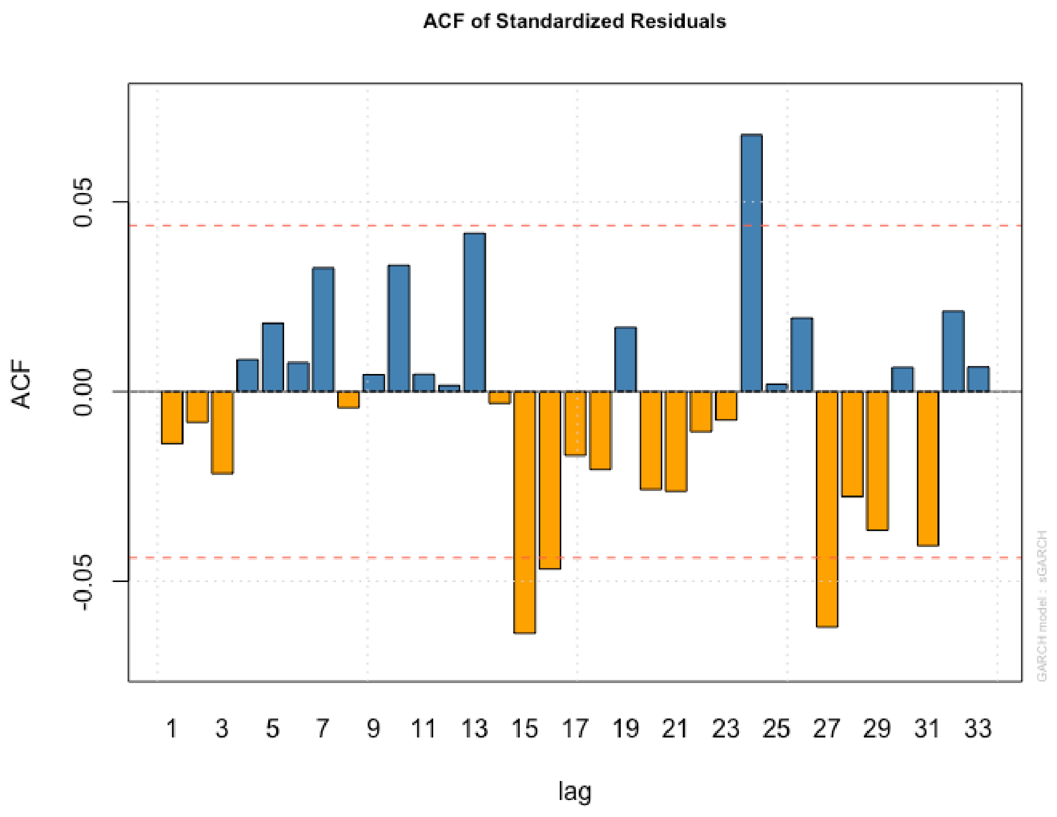
At least a human can't distinguish between them anymore. But what we most care about is whether standardized squared residuals can pass the $LM-Test$(substracting from the above long result):

```
Weighted ARCH LM Tests
------------------------------------
             Statistic Shape Scale   P-Value
ARCH Lag[3]   0.004507 0.500 2.000 9.465e-01
ARCH Lag[5]  11.085543 1.440 1.667 3.565e-03
ARCH Lag[7]  20.592184 2.315 1.543 4.509e-05
```

We can see our model successfully wipes out ARCH effect at small lags, but fails at larger lags. This is the best 'sGARCH' can give with respest to our data. Maybe we should try long memory models instead of *diff()* at first.

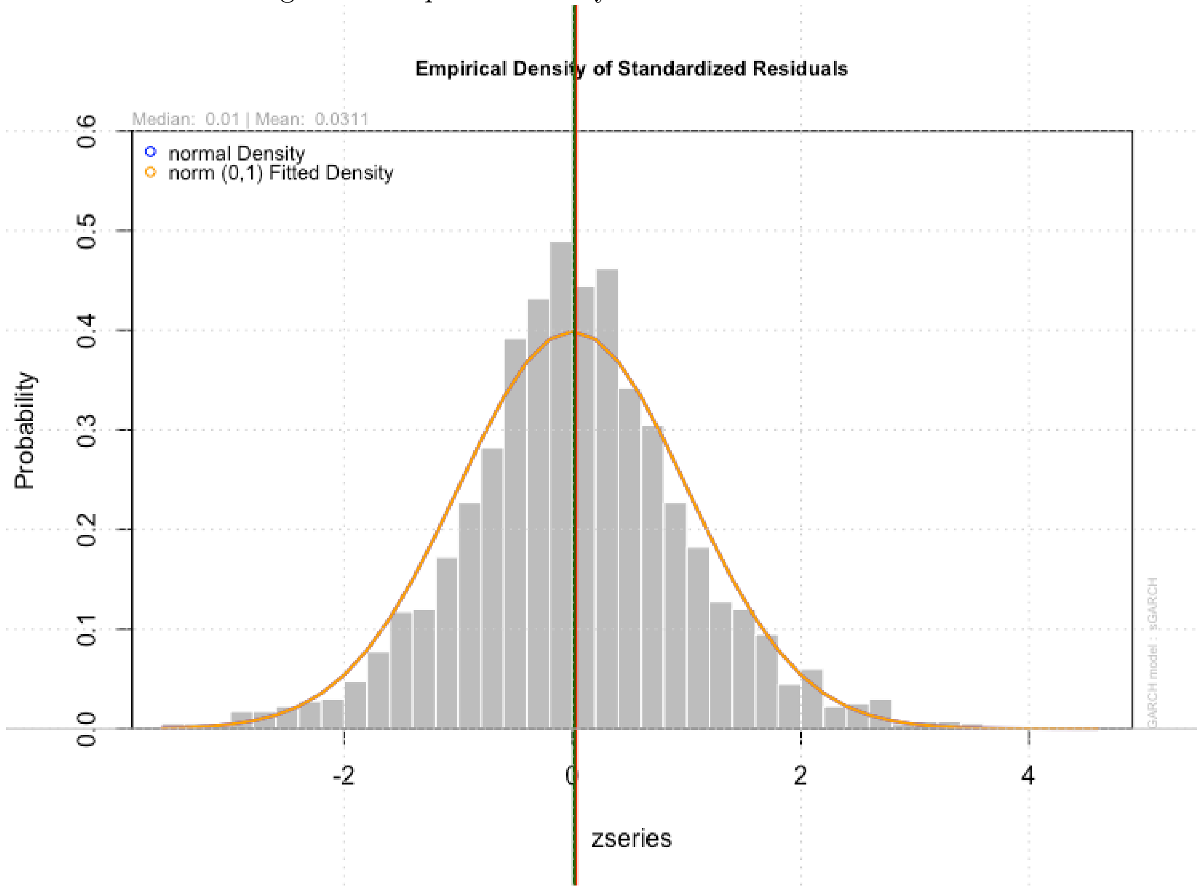
Graphical Diagnostics:

Figure 7: acf(z)



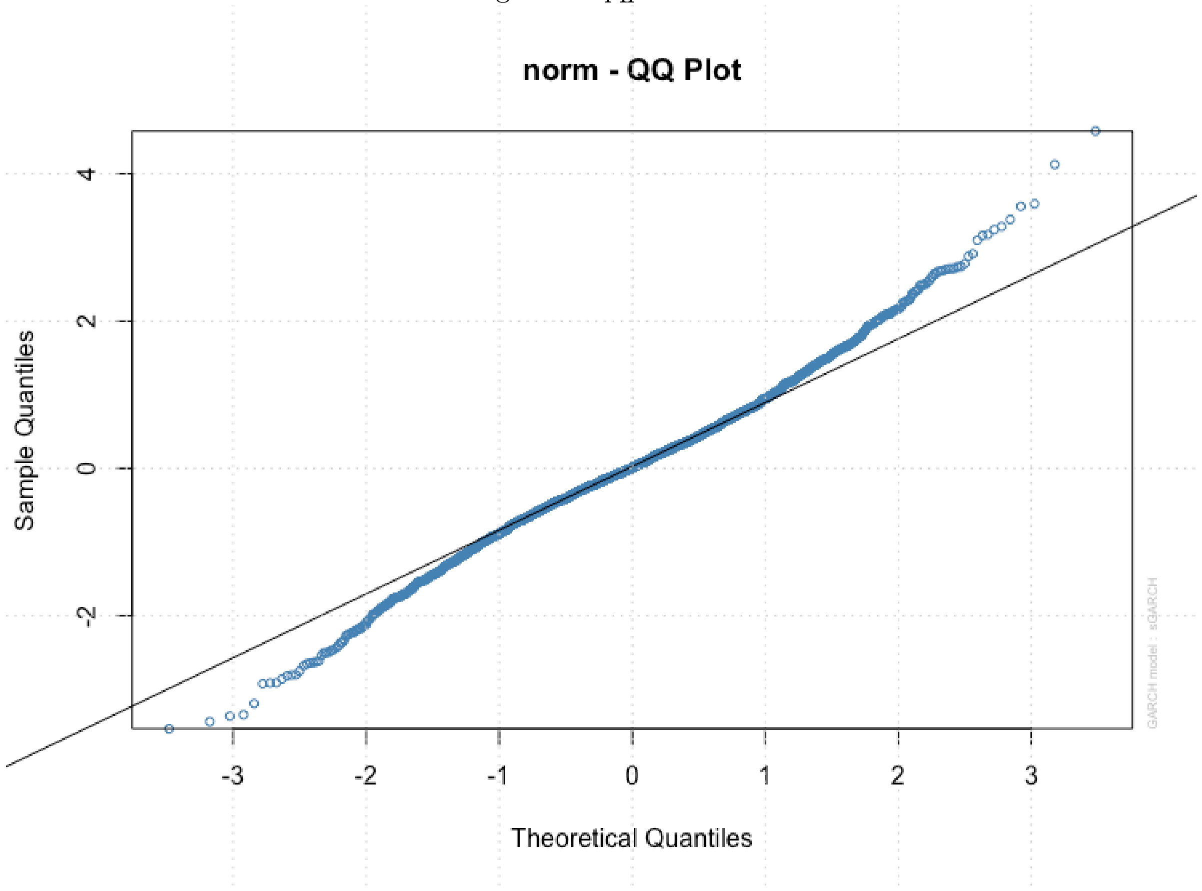
This is a prove that we should have used a long memory model.
rugarch provide a function to plot a "uGARCHfit" obeject.
ex. density compared to normal distribution:

Figure 8: Empirical Density of Standardized Residuals



qqplot:

Figure 9: qqplot of z

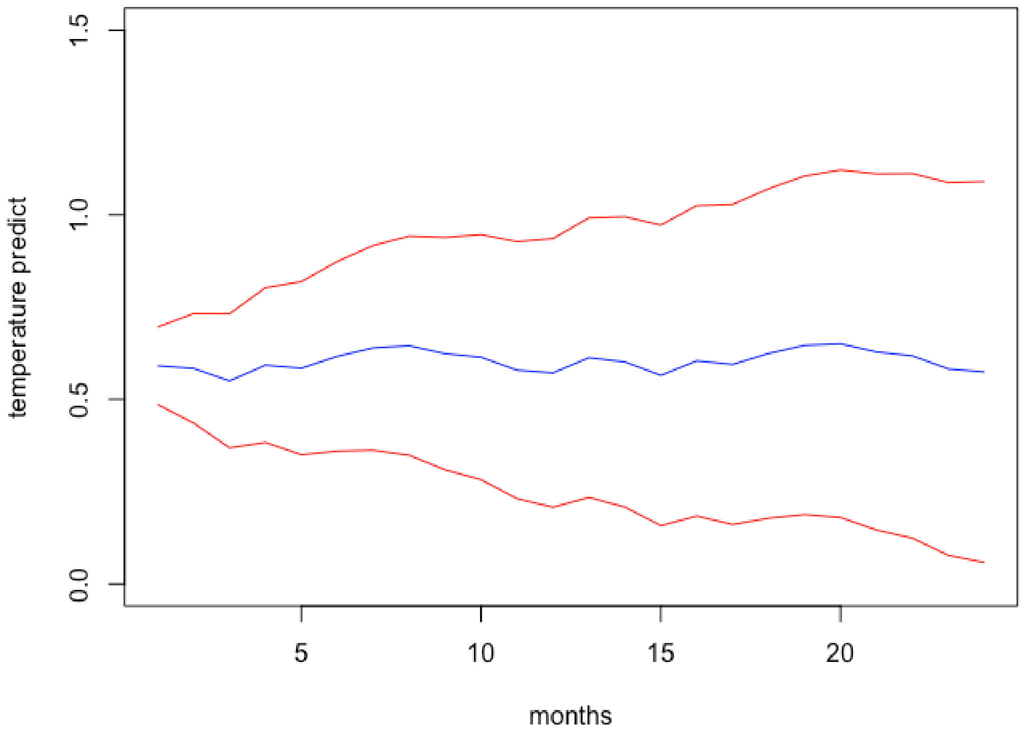


We can see that z is skewed, which might be correlated with long lags' heterodasticity.

Forecast

```
fore1 = ugarchforecast(fit1, n.ahead = 24)
fore.diff = as.numeric(fore1@forecast$seriesFor)
fore.sigma = as.numeric(fore1@forecast$sigmaFor)
ts.predict = temp[length(temp)] + cumsum(fore.diff)
ts.predict = ts.predict + myTS.additive$figure
ts.sigma = sqrt(cumsum(fore.sigma^2))
tsup.sigma = ts.predict + ts.sigma
tsdown.sigma = ts.predict - ts.sigma
plot(1:24, ts.predict, ylim=c(0,1.5), type = 'l', col = 'blue',
  xlab = "months", ylab = "temperature predict")
lines(1:24, tsup.sigma, type = 'l', col = 'red')
lines(1:24, tsdown.sigma, type = 'l', col = 'red')
```

Figure 10: forecast



Conclusion: 'sGARCH' have improved the model in the intuitive sense and to some extent fix the heterodasticity.