ARMA-Garch Part

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Overview

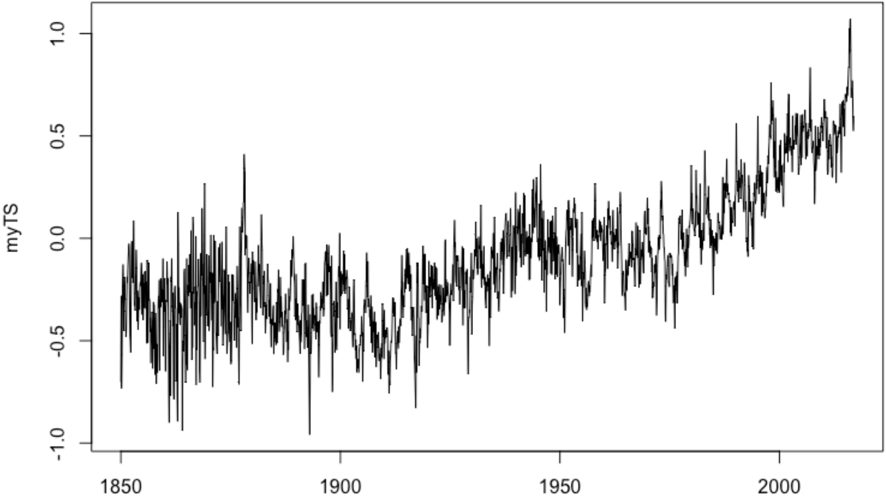
Data Source

HadCRUT4 is a gridded dataset of global historical surface temperature anomalies relative to a 1961-1990 reference period. Data are available for each month since January 1850, on a 5 degree grid.

Read Data

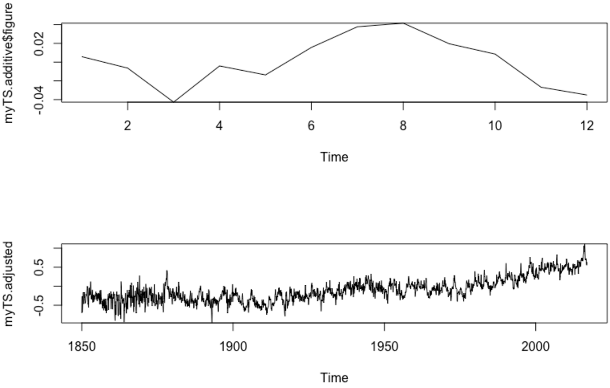
```
tmpf <- tempfile()
curl_download("http://www.metoffice.gov.uk/
hadobs/hadcrut4/data/current/time_series/
HadCRUT.4.5.0.0.monthly_ns_avg.txt", tmpf)
gtemp <- read.table(tmpf)[, 1:2]
temp = gtemp$V2[1:2004]
plot.ts(temp)
```

Read Data



```
library(TSA)
myTS = ts(as.numeric(temp), start = c(1850, 1),
frequency = 12)
myTS.additive = decompose(myTS)
myTS.adjusted = myTS.additive$x - myTS.additive$seasonal
```

Figure: additive model: seasonal component, adjusted series



```
> dtemp = diff(temp)
> adf.test(dtemp)

^^IAugmented Dickey-Fuller Test

data:  dtemp
Dickey-Fuller = -16.175, Lag order = 12, p-value = 0.01
alternative hypothesis: stationary
```

Finding the correct arma order of dtemp:

```
auto.arima(dtemp)
arma21.dtemp = arima(dtemp, c(2, 0, 1))
arma24.dtemp = arima(dtemp, c(2, 0, 4))
auto.arima(arma21.dtemp$residuals)
auto.arima(arma24.dtemp$residuals)
my.arma = arma24.dtemp
my.arma
```

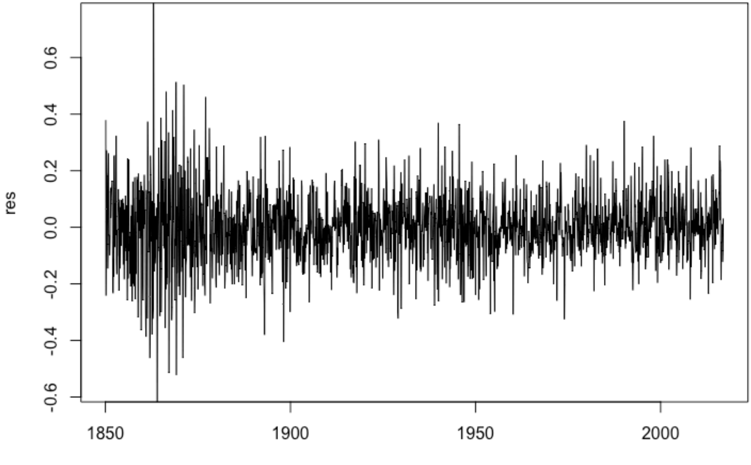
my.arma result

```
> my.arma

Call:
arima(x = dtemp, order = c(2, 0, 4))

Coefficients:
         ar1     ar2      ma1      ma2     ma3     ma4  intercept
      0.5204  0.3247  -1.0536  -0.1269  0.1168  0.0755      5e-04
s.e.  0.2733  0.2356   0.2727   0.3828  0.1117  0.0265      2e-04

sigma^2 estimated as 0.01435:  log likelihood = 1407.67,  aic = -2801.34
```

```
> Box.test(res, type = 'Ljung-Box')

^^IBox-Ljung test

data:  res
X-squared = 0.0058853, df = 1, p-value = 0.9388

> arch.test(my.arma)
ARCH heteroscedasticity test for residuals
alternative: heteroscedastic

Lagrange-Multiplier test:
     order   LM p.value
[1,]     4 1418       0
[2,]     8  697       0
[3,]    12  418       0
[4,]    16  212       0
[5,]    20  168       0
[6,]    24  137       0
```

Plot residuals

Figure: arma residual



Building sGarch

A standard GARCH model has the following equations:

$$y_t = \mu + \sum \phi_i y_{t-i} + \sum \theta_i \epsilon_{t-j} + \epsilon_t$$

$$\sigma_t^2 = (w + \sum_{j=1}^{m} \zeta_j v_{jt}) + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

$$\epsilon_t = u_t \cdot \sigma_t, u_t \sim N(0, 1)$$

```
library(rugarch)
my_sGARCH_test <- function(p, q, m, n, ts.data)
{
^^I# I use include.mean = FALSE after trying TRUE
^^I# to find out insignificance
    myspec=ugarchspec(variance.model = list(model = "sGARCH",
    ^^IgarchOrder = c(p, q)),
    ^^Imean.model = list(armaOrder = c(m, n),
    ^^Iinclude.mean = FALSE),
    ^^Idistribution.model = "norm")
    myfit=ugarchfit(myspec,data=ts.data, solver="solnp")
    return(myfit)
}
```

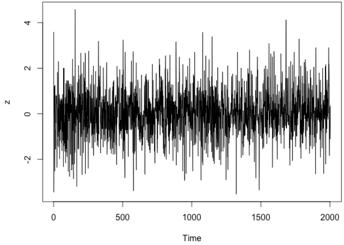
After trying a few times from $(1, 0), (0, 0, 0)$ to $(5, 5), (4, 0, 4)$, *GARCH(1 ,1), ARIMA(2, 0, 3)* is the most satisfying model. Here we realize that using GARCH model, the order of ARIMA might changes.

```
fit1 = my_sGARCH_test(1, 1, 2, 3, dtemp)
> fit1
...
Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
ar1    -0.090935    0.015226    -5.9724 0.000000
ar2     0.754333    0.015069    50.0583 0.000000
ma1    -0.391906    0.007313   -53.5917 0.000000
ma2    -0.863257    0.000130 -6647.5909 0.000000
ma3     0.295634    0.007800    37.9010 0.000000
omega   0.000082    0.000034     2.3956 0.016595
alpha1  0.023026    0.003614     6.3706 0.000000
beta1   0.970594    0.004705   206.2925 0.000000
...
```

Substract the standardized(w.r.t. the variance model) residuals z, which is
$z = \dfrac{residuals(fit)}{sigma(fit)}$.

```
z = residuals(fit1) / sigma(fit1)
plot.ts(z)
```

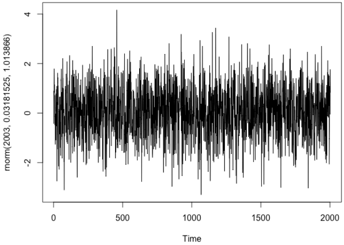
Figure: z



```
> mean(z)
[1] 0.03181525
> var(z)
[1] 1.013866
> length(z)
[1] 2003
> plot.ts(rnorm(2003, 0.03181525, 1.013866))
```

And then, just for fun, plot a normal sample series with the same parameters:

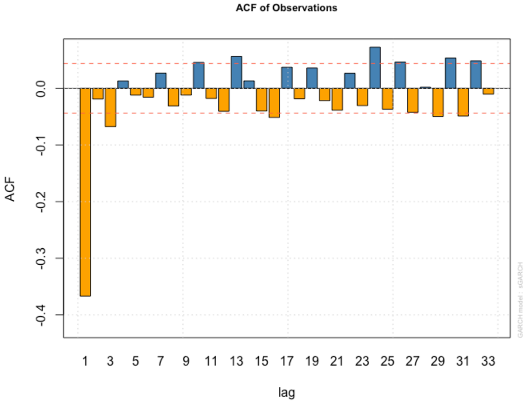
Figure: simulation of using rnorm



```
Weighted ARCH LM Tests
------------------------------------
             Statistic Shape Scale   P-Value
ARCH Lag[3]   0.004507 0.500 2.000 9.465e-01
ARCH Lag[5]  11.085543 1.440 1.667 3.565e-03
ARCH Lag[7]  20.592184 2.315 1.543 4.509e-05
```

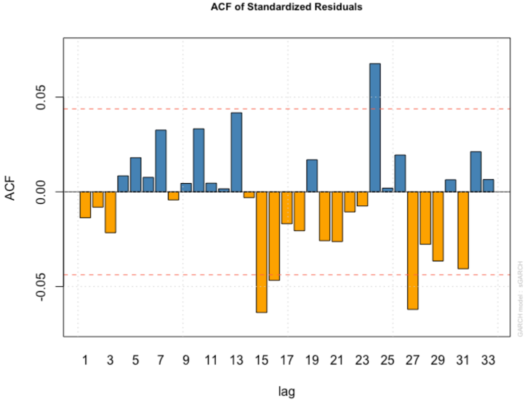
diagnostics

Figure: acf(dtemp)



The acf of standardized residuals:

Figure: acf(z)

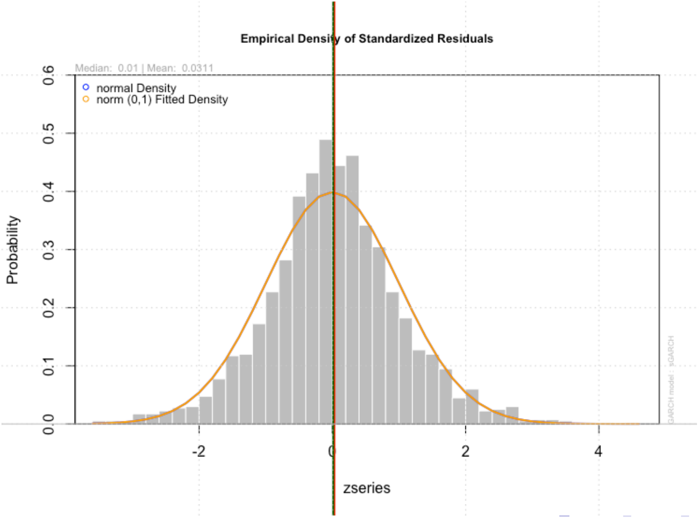


This is a prove that we should have used a long memory model.

diagnostics

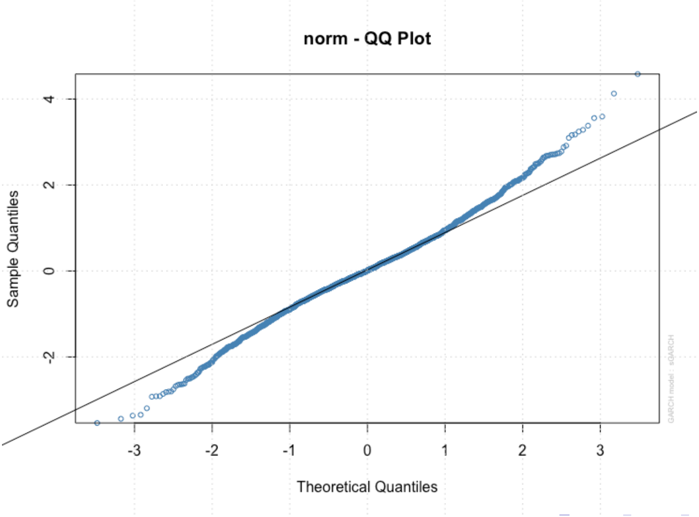
The *Empirical Density of Standardized Residuals* compared to normal distribution:

Figure: Empirical Density of Standardized Residuals

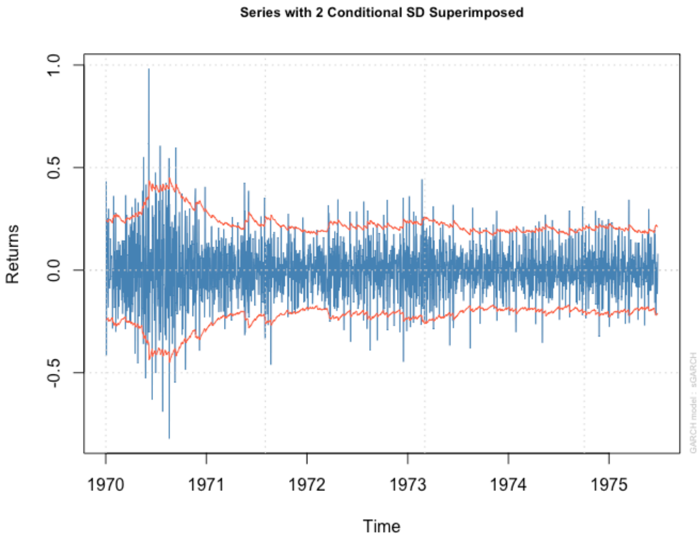


qqplot:

Figure: qqplot of z



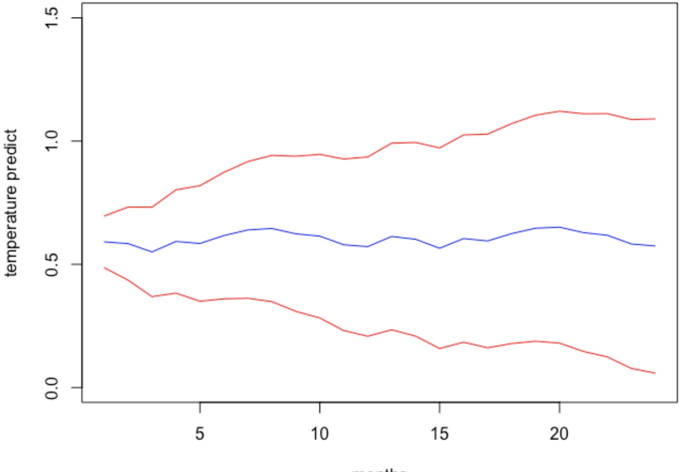
Series with 2 Conditional SD Superimposed:



Forecast

```
fore1 = ugarchforecast(fit1, n.ahead = 24)
fore.diff = as.numeric(fore1@forecast$seriesFor)
fore.sigma = as.numeric(fore1@forecast$sigmaFor)
ts.predict = temp[length(temp)] + cumsum(fore.diff)
ts.predict = ts.predict + myTS.additive$figure
ts.sigma = sqrt(cumsum(fore.sigma^2))
tsup.sigma = ts.predict + ts.sigma
tsdown.sigma = ts.predict - ts.sigma
plot(1:24, ts.predict, ylim=c(0,1.5), type = 'l', col = 'blue',
  xlab = "months", ylab = "temperature predict")
lines(1:24, tsup.sigma, type = 'l', col = 'red')
lines(1:24, tsdown.sigma, type = 'l', col = 'red')
```

Figure: forecast



sGARCH model can explain the heteroscedastic partially. I've also tried eGARCH model only to find similar results. Next I'll explore long memory model or exclude some extreme values.

The End