数理期末复习

STEVN SHEN

May 31, 2017

1 Laplace Transform

$$\mathcal{L}[f(t)] = \int_0^{+\infty} f(t)e^{-pt}dt$$

$$f(t) = \frac{1}{2\pi i}\int_{s-i\sigma}^{s+i\sigma} F(p)e^{pt}dp$$

Convention: $\mathcal{L}[f(t)] = F(p)$

1. $\mathcal{L}[1] = \dfrac{1}{p}$, $\mathcal{L}[e^{at}] = \dfrac{1}{p-a}$
2. $\mathcal{L}[\delta(t)] = 1$, $\mathcal{L}[\delta(t-t_0)] = e^{-pt_0}$
3. $$\mathcal{L}[e^{at}f(t)] = F(p-a)$$
   $$\mathcal{L}[f(t-t_0)] = e^{-pt_0}F(p)$$
4. $\mathcal{L}[f'(t)] = pF(p) - f(0)$
5. $\mathcal{L}[\int_0^t f(\tau)d\tau] = \dfrac{F(p)}{p}$
6. $\mathcal{L}[(-t)^n f(t)] = F^{(n)}(p)$
7. $\mathcal{L}[\dfrac{f(t)}{t}] = \int_p^\infty F(q)dq$,
   $p \to 0$, $\int_0^\infty \dfrac{f(t)}{t} = \int_0^\infty F(p)dp$
8. 若 $F(p)$ 在无穷远点解析，即 $F(p) = \sum_{n=1}^\infty c_n p^{-n}$
   $(F(p) \to 0,\, p \to \infty)$
   逐项反演 $f(t) = \sum_{n=0}^\infty \dfrac{c_{n+1}}{n!}t^n$

9. 由上条，$\mathcal{L}[J_0(t)] = \dfrac{1}{\sqrt{p^2+1}}$

   $\mathcal{L}[J_0(2\sqrt{t})] = \dfrac{1}{p}e^{-p}$

10. 含参积分求导得：$\int_0^\infty e^{-t^2} cos2ztdt = \dfrac{\sqrt{\pi}}{2}e^{-z^2}$

11. $\mathcal{L}[\dfrac{1}{\sqrt{\pi t}}\int_0^\infty f(\tau)e^{-\frac{\tau^2}{4t}}d\tau] = \dfrac{1}{\sqrt{p}}F(\sqrt{p})$

12. 由 $\mathcal{L}[\eta(t-a)] = \dfrac{1}{p}e^{-ap}$ 和上一条：

    $\mathcal{L}[erfc(\frac{a}{2\sqrt{(t)}})] = \dfrac{1}{p}e^{-a\sqrt{p}} \quad erfc(x) = \dfrac{2}{\sqrt{\pi}}\int_x^\infty e^{-z^2}dz$

13. Convolution: $\mathcal{L}[\int_0^t f_1(\tau)f_2(t-\tau)d\tau] = F_1(p)F_2(p)$

2 例题

2.1 积分变换

2.1.1 Fourier plus Laplace: 一维无界弦，$t=0$ 时，$u=\phi(x), u'=\psi(x)$