ARMA-GARCH

SHIHENG SHEN

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1 Data

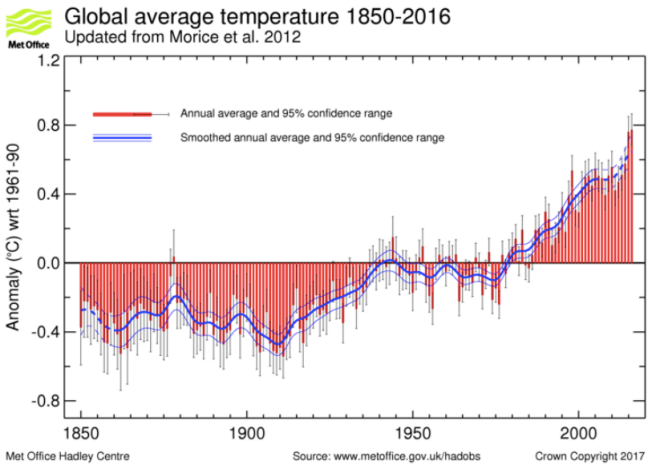
1.1 Data Source

HadCRUT4 is a gridded dataset of global historical surface temperature anomalies relative to a 1961-1990 reference period. Data are available for each month since January 1850, on a 5 degree grid. The dataset is a collaborative product of the Met Office Hadley Centre and the Climatic Research Unit at the University of East Anglia.
url =`http://www.metoffice.gov.uk/hadobs/hadcrut4/data/current/time_series/HadCRUT.4.5.0.0.monthly_ns_avg.txt`.

1.2 Brief Data Description

The gridded data are a blend of the CRUTEM4 land-surface air temperature dataset and the HadSST3 sea-surface temperature (SST) dataset. The dataset is presented as an ensemble of 100 dataset realisations that sample the distribution of uncertainty in the global temperature record given current understanding of non-climatic factors affecting near-surface temperature observations. This ensemble approach allows characterisation of spatially and temporally correlated uncertainty structure in the gridded data, for example arising from uncertainties in methods used to account for changes in SST measurement practices, homogenisation of land station records and the potential impacts of urbanisation. The HadCRUT4 data are neither interpolated nor variance adjusted.

Below is a graph provided by `www.metoffice.gov.uk`:



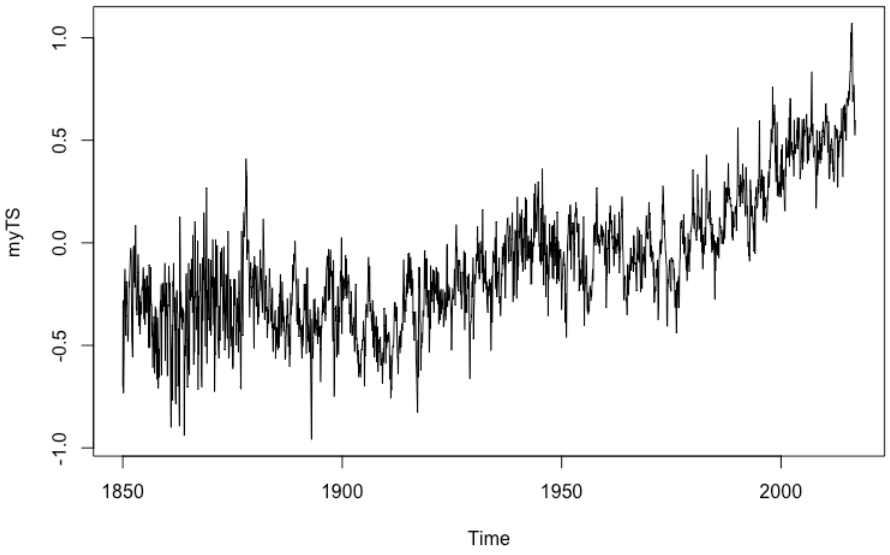
1.3 Loading the Data into R

For our purpose, we are not interested in the spatial distribution, so we only use the *Global Mean*.
Loading the data into R:

```
library(curl)
tmpf <- tempfile()
curl_download(url, tmpf)
gtemp <- read.table(tmpf)[, 1:2]
temp = gtemp$V2[1:2004]
```

We only use the first two columns: *time and monthly global mean*. We pick the first 2004 observations, which is monthly data from 1850 $\sim$ 2016, so that the data has complete periods.

The time series of *temp* is shown below:



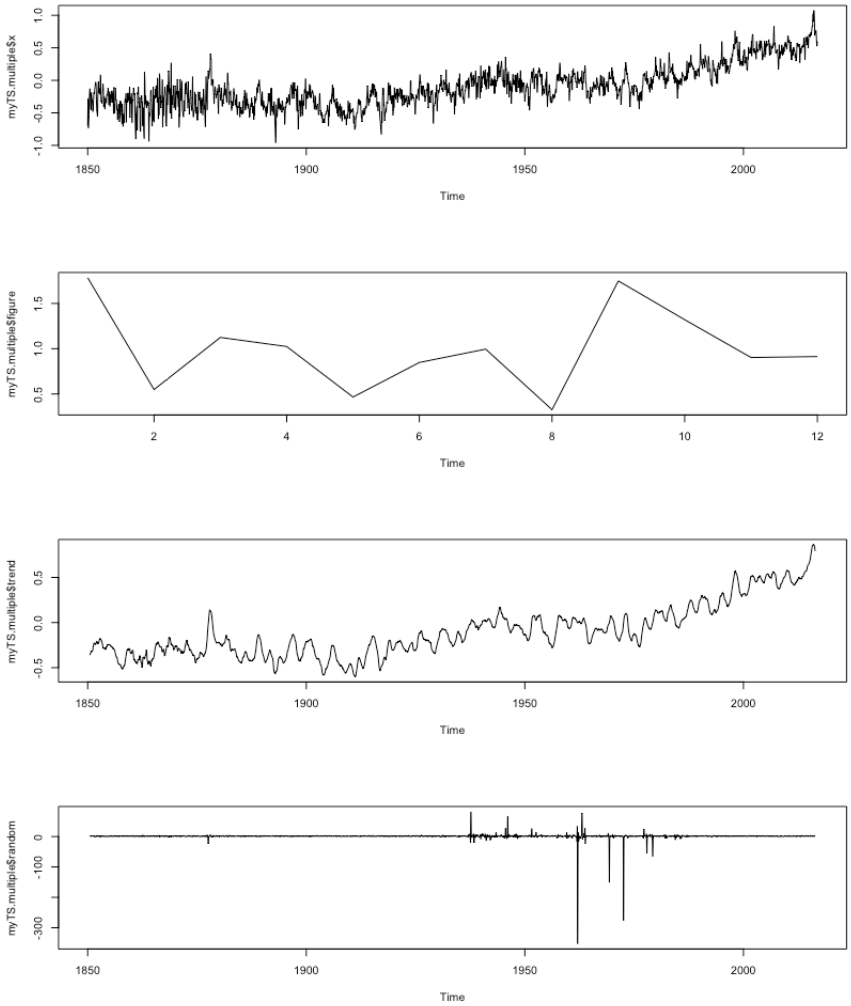
2 Dealing with the time series

2.1 Decompose

The first thing we might want to do is to wipe out the seasonal component. There're many approaches to do it in R. We choose to use the *decompose*() provided by {*stats*}. Looking at the time series, we think a additive model should be appropriate.

We can quickly checked that multiplicative model is not appropriate:

Figure 1: multiplcative model: original ts, seasonal component, time trend, residuals



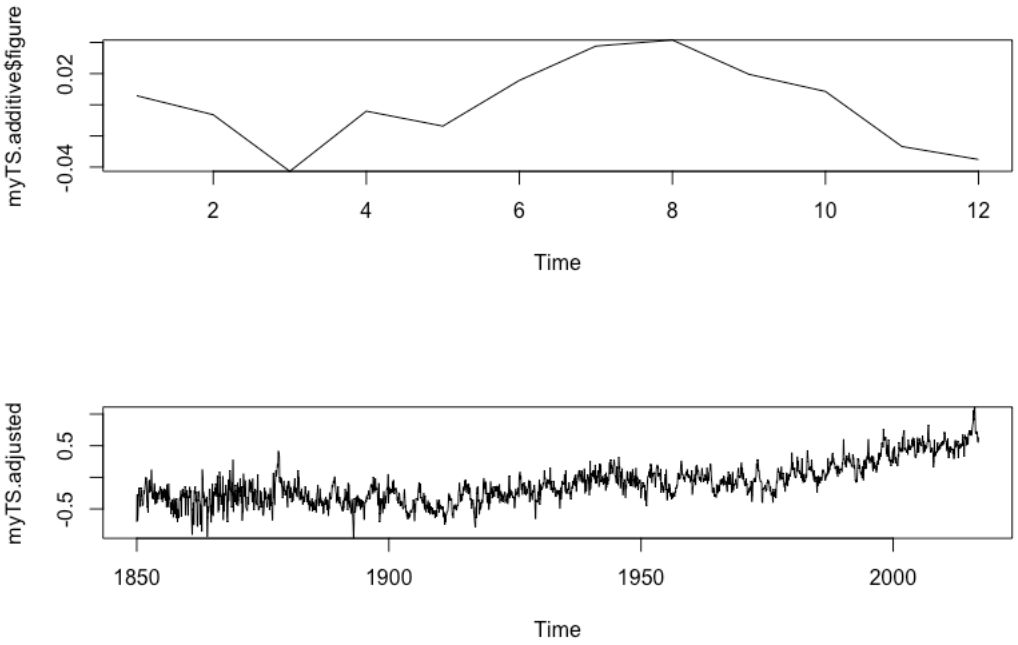
Obviously the random term is not easy to be modelled.

Therefore, decompose the time series with additive method in the following code:

```
library(TSA)
myTS = ts(as.numeric(temp), start = c(1850, 1), frequency = 12)
myTS.additive = decompose(myTS)
myTS.adjusted = myTS.additive$x - myTS.additive$seasonal
```

The *myTS.adjusted* now is free of seasonal components. The following graph illustrate the components.

Figure 2: additive model: seasonal component, adjusted series



Obviously, the time seires is not stationary. For simplicity, use *diff()*.

```
dtemp = diff(myTS.adjusted)
```

```
> adf.test(dtemp)

Augmented Dickey-Fuller Test

data:  dtemp
Dickey-Fuller = -16.175, Lag order = 12, p-value = 0.01
alternative hypothesis: stationary
```

So we can convince ourself that the time series *dtemp* is stationary.

2.2 Build a *ARMA(p, q)* for *res*

For simplicity, we use $auto.arima()$ from $\{forecast\}$ to judge the order.

```
library(forecast)
> auto.arima(dtemp> auto.arima(dtemp)
Series: dtemp
ARIMA(2,0,4)(2,0,1)[12] with non-zero mean

Coefficients:
         ar1     ar2      ma1      ma2     ma3     ma4    sar1    sar2     sma1   mean
```

```
      0.5041  0.3585  -1.0458  -0.1600  0.1362  0.0780  0.7619  0.0779  -0.7884  5e-04
s.e.  0.2431  0.2142   0.2423   0.3481  0.1082  0.0257  0.0926  0.0248   0.0910  2e-04

sigma^2 estimated as 0.01428:  log likelihood=1417.03
AIC=-2812.05   AICc=-2811.92   BIC=-2750.43

# first find the possible orders might be arma(2, 1) or arma(2, 4)

arma21.dtemp = arima(dtemp, c(2, 0, 1))
arma24.dtemp = arima(dtemp, c(2, 0, 4))

# next examine the two models' residuals

> auto.arima(arma21.dtemp$residuals)
Series: arma21.dtemp$residuals
ARIMA(2,0,3)(2,0,1)[12] with zero mean

Coefficients:
         ar1      ar2      ma1     ma2     ma3    sar1    sar2     sma1
      1.5859  -0.6762  -1.6089  0.6706  0.0506  0.7454  0.0748  -0.7785
s.e.  0.1156   0.1065   0.1169  0.1264  0.0297  0.1209  0.0258   0.1181

sigma^2 estimated as 0.01429:  log likelihood=1415.92
AIC=-2813.83   AICc=-2813.74   BIC=-2763.41

> auto.arima(arma24.dtemp$residuals)
Series: arma24.dtemp$residuals
ARIMA(0,0,0) with zero mean

sigma^2 estimated as 0.01435:  log likelihood=1408.31
AIC=-2814.63   AICc=-2814.62   BIC=-2809.02
```

Therefore it should be appropriate to choose *arma(2, 4)* for series *dtemp*.

```
my.arma = arma24.dtemp

> my.arma

Call:
arima(x = dtemp, order = c(2, 0, 4))

Coefficients:
         ar1     ar2      ma1      ma2     ma3     ma4  intercept
      0.5204  0.3247  -1.0536  -0.1269  0.1168  0.0755      5e-04
s.e.  0.2733  0.2356   0.2727   0.3828  0.1117  0.0265      2e-04

sigma^2 estimated as 0.01435:  log likelihood = 1407.67,  aic = -2801.34
```

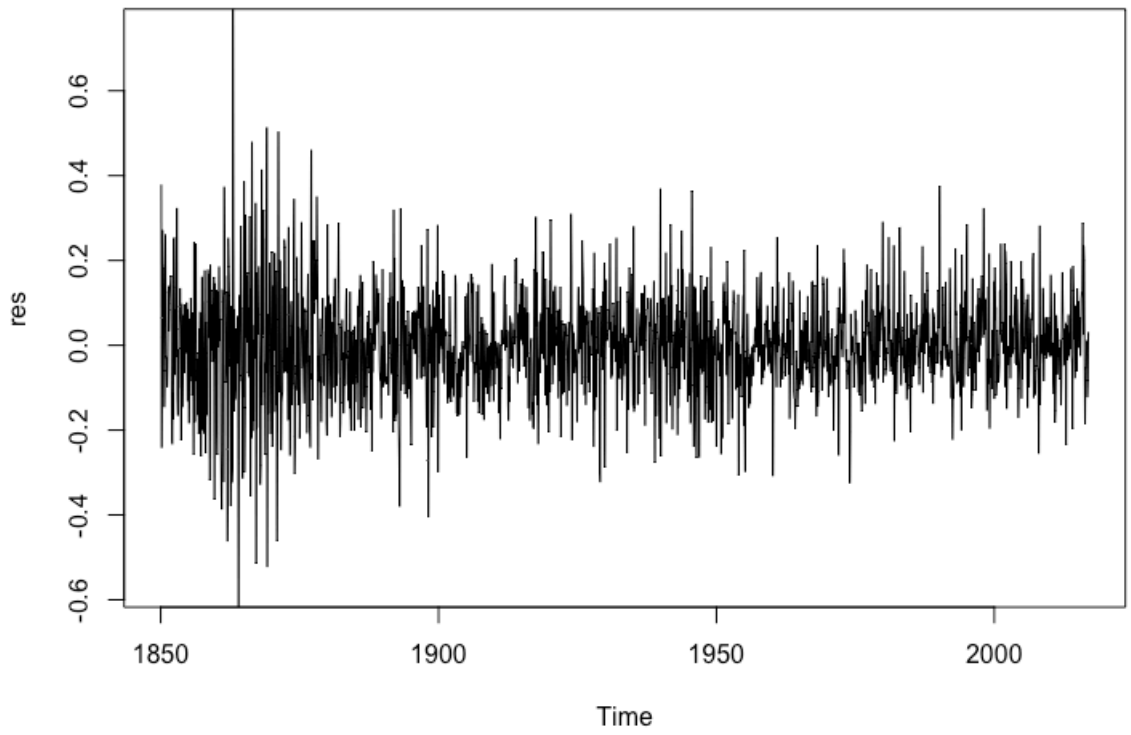
```
res = my.arma$residuals

> Box.test(res, type = 'Ljung-Box')

Box-Ljung test

data:  res
X-squared = 0.0058853, df = 1, p-value = 0.9388
```

The $ARMA(2,4)$ success in making the arma residual independent.

2.3 GARCH

One can be pretty satisfied with the results right now. Yet Looking at the plot of arma residual:

Figure 3: arma residual

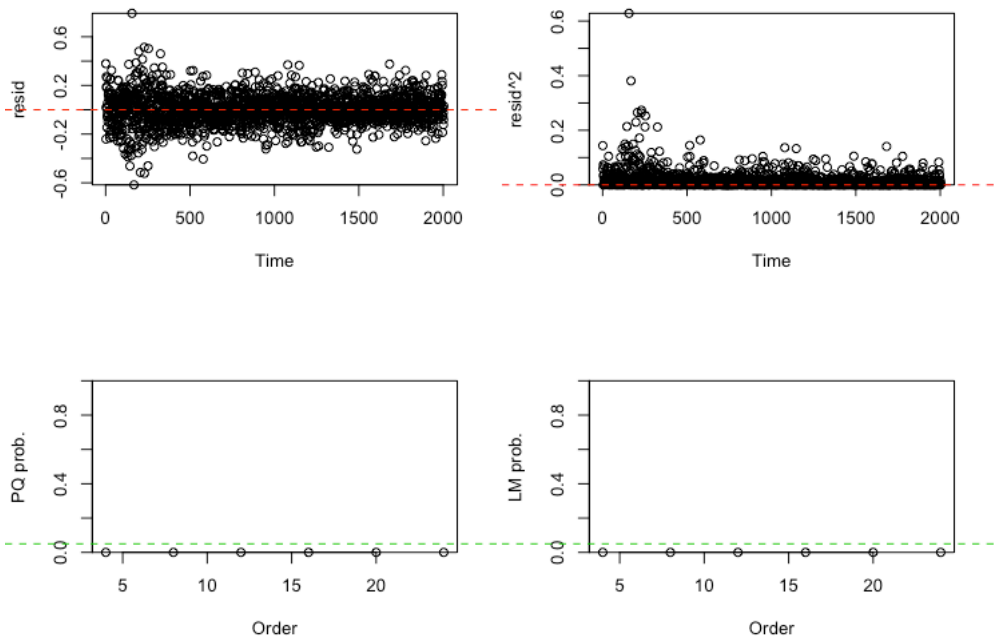


It makes us wondering if it has $ARCH$ effect. Using *arch.test* from $\{aTSA\}$ to perform the $LM-Test$:

```
library(aTSA)
```

```
> arch.test(my.arma)
ARCH heteroscedasticity test for residuals
alternative: heteroscedastic

Portmanteau-Q test:
     order    PQ p.value
[1,]     4  99.4       0
[2,]     8 116.9       0
[3,]    12 411.5       0
[4,]    16 475.6       0
[5,]    20 495.9       0
[6,]    24 755.8       0
Lagrange-Multiplier test:
     order   LM p.value
[1,]     4 1418       0
[2,]     8  697       0
[3,]    12  418       0
[4,]    16  212       0
[5,]    20  168       0
[6,]    24  137       0
```

Figure 4: arma residual



It supports our doubt. Therefore we have the motive to build a *GARCH* model. We decide to use the package *rugarch*.

$$\sigma_t^2 = (w + \sum_{j=1}^{m} \zeta_j v_{jt}) + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

So we wrote a function:

```
library(rugarch)
my_sGARCH_test <- function(p, q, m, n, ts.data = res)
{
# I use include.mean = FALSE after trying TRUE
# to find out insignificance
    myspec=ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(p, q)),
     mean.model = list(armaOrder = c(m, n), include.mean = FALSE),
     distribution.model = "norm")
    myfit=ugarchfit(myspec,data=ts.data, solver="solnp")
    return(myfit)
}
```

After trying a few times from $(1, 0)$ to $(5, 5)$, *GARCH(1 ,1), ARIMA(2, 0, 4)* is the most satisfying model.

```
fit1 = my_sGARCH_test(1, 1, 2, 4, dtemp)

> fit1

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(2,0,4)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error     t value Pr(>|t|)
ar1    -0.089169    0.038250    -2.33121 0.019742
ar2     0.763456    0.026364    28.95834 0.000000
ma1    -0.390348    0.019274   -20.25223 0.000000
ma2    -0.880951    0.000338 -2603.65163 0.000000
ma3     0.297715    0.026635    11.17774 0.000000
ma4     0.012983    0.050864     0.25525 0.798527
omega   0.000082    0.000034     2.40136 0.016334
alpha1  0.023019    0.003610     6.37686 0.000000
beta1   0.970582    0.004693   206.83542 0.000000
```

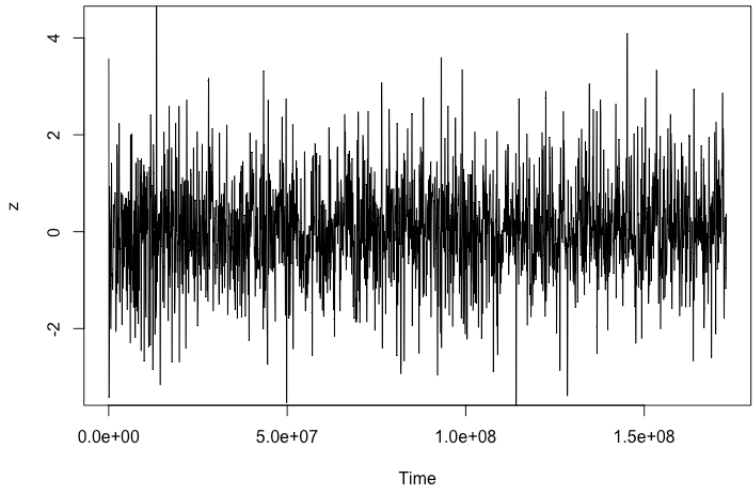
```
Robust Standard Errors:
        Estimate  Std. Error     t value Pr(>|t|)
ar1    -0.089169    0.083587    -1.06678 0.286070
ar2     0.763456    0.055864    13.66623 0.000000
ma1    -0.390348    0.041125    -9.49179 0.000000
ma2    -0.880951    0.000796 -1106.83968 0.000000
ma3     0.297715    0.059384     5.01339 0.000001
ma4     0.012983    0.117876     0.11014 0.912296
omega   0.000082    0.000036     2.28917 0.022069
alpha1  0.023019    0.003647     6.31184 0.000000
beta1   0.970582    0.003730   260.21194 0.000000

LogLikelihood : 1512.547

Information Criteria
------------------------------------

Akaike       -1.5013
Bayes        -1.4761
Shibata      -1.5013
Hannan-Quinn -1.4921

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.5981  0.4393
Lag[2*(p+q)+(p+q)-1][17]    7.2782  0.9990
Lag[4*(p+q)+(p+q)-1][29]   18.4551  0.1194
d.o.f=6
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      31.45 2.045e-08
Lag[2*(p+q)+(p+q)-1][5]     38.17 1.533e-10
Lag[4*(p+q)+(p+q)-1][9]     51.67 3.408e-13
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale   P-Value
ARCH Lag[3]  0.009122 0.500 2.000 9.239e-01
ARCH Lag[5] 11.200028 1.440 1.667 3.341e-03
ARCH Lag[7] 20.729545 2.315 1.543 4.158e-05
```

```
Nyblom stability test
-----------------------------------
Joint Statistic:  2.8481
Individual Statistics:
ar1    0.24868
ar2    0.59898
ma1    0.17360
ma2    0.21711
ma3    0.06382
ma4    0.10027
omega  0.09915
alpha1 0.41690
beta1  0.19806

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       2.1 2.32 2.82
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
-----------------------------------
                   t-value      prob sig
Sign Bias            0.405 6.856e-01
Negative Sign Bias   3.660 2.586e-04 ***
Positive Sign Bias   4.500 7.193e-06 ***
Joint Effect        33.651 2.347e-07 ***


Adjusted Pearson Goodness-of-Fit Test:
-----------------------------------
  group statistic p-value(g-1)
1    20     53.37    4.122e-05
2    30     66.07    1.025e-04
3    40     90.72    5.294e-06
4    50     98.27    3.776e-05


Elapsed time : 0.3975561
```

Substract the standardized(w.r.t. the variance model) residuals z, which is $z = \frac{residuals(fit)}{sigma(fit)}$.

```
z = residuals(fit1) / sigma(fit1)
plot.ts(z)
```

Figure 5: z

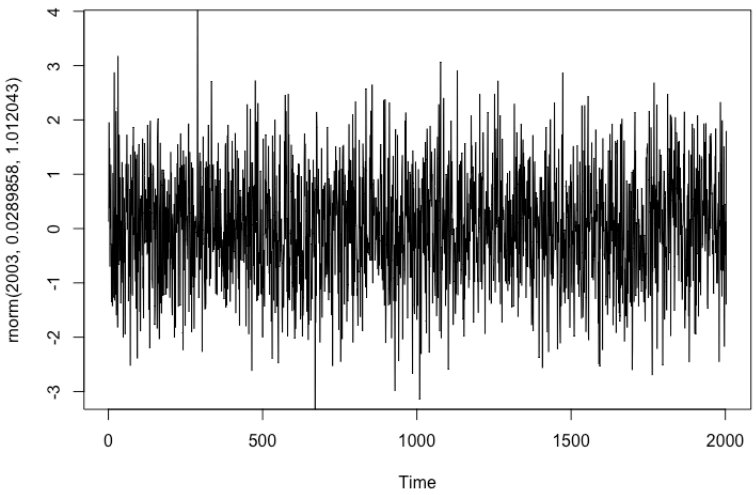


We can compute the mean and variance of z:

```
> mean(z)
[1] 0.0289858
> var(z)
          [,1]
[1,] 1.012043
> length(z)
[1] 2003
> plot(rnorm(2003, 0.0289858, 1.012043))
```

And then, just for fun, plot a normal sample series with the same parameters:

Figure 6: simulation of using rnorm



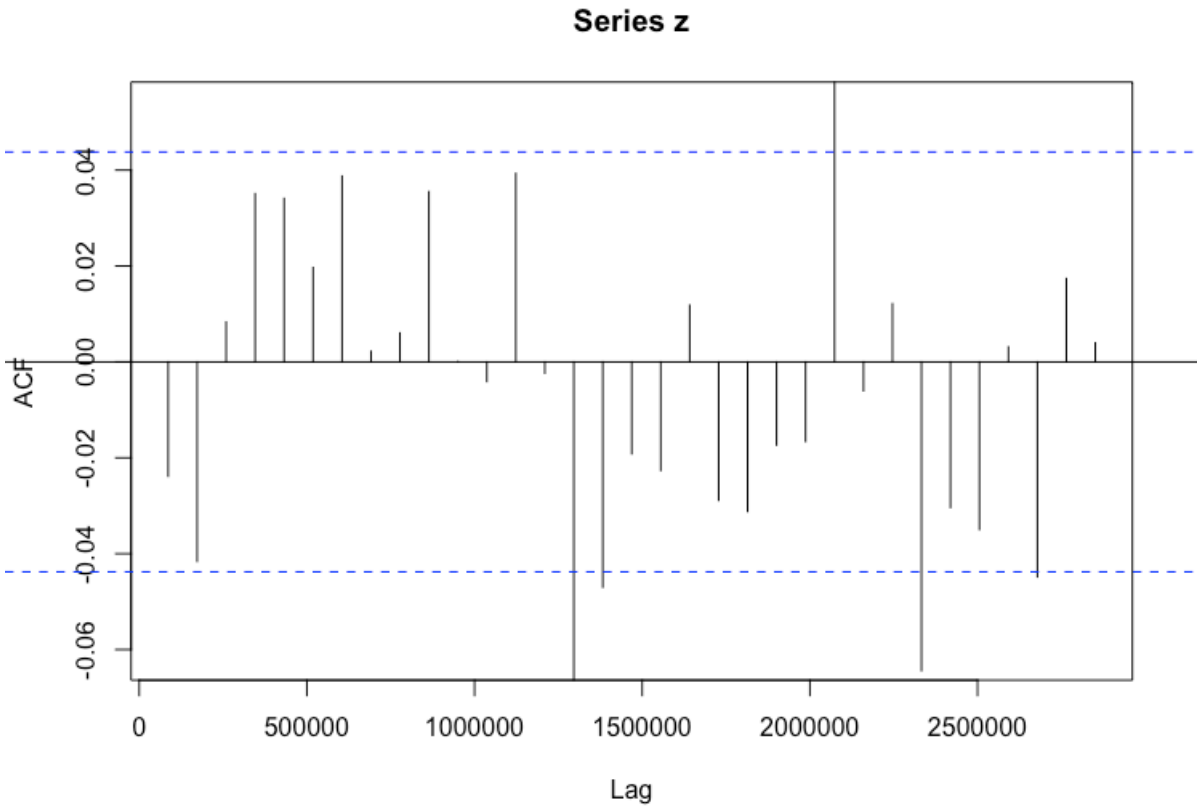
At least a human can't distinguish between them anymore. But what we most care about is whether standardized squared residuals can pass the $LM-Test$(substracting from the above long result):

```
Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale   P-Value
ARCH Lag[3]  0.009122 0.500 2.000 9.239e-01
ARCH Lag[5] 11.200028 1.440 1.667 3.341e-03
ARCH Lag[7] 20.729545 2.315 1.543 4.158e-05
```

We can see our model successfully wipes out ARCH effect at small lags, but fails at larger lags. This is the best 'sGARCH' can give with respest to our data. Maybe we should try long memory models instead of *diff()* at first.

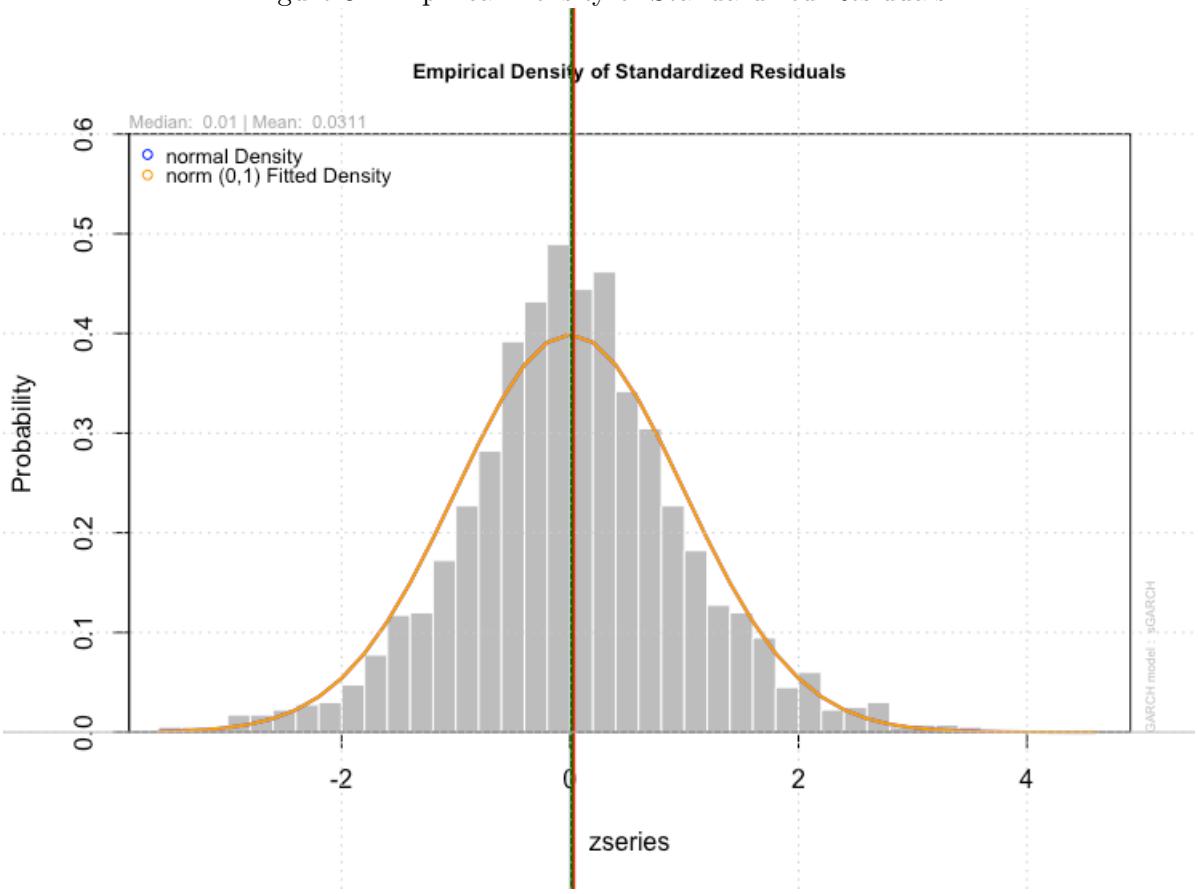
Graphical Diagnostics:

Figure 7: acf(z)



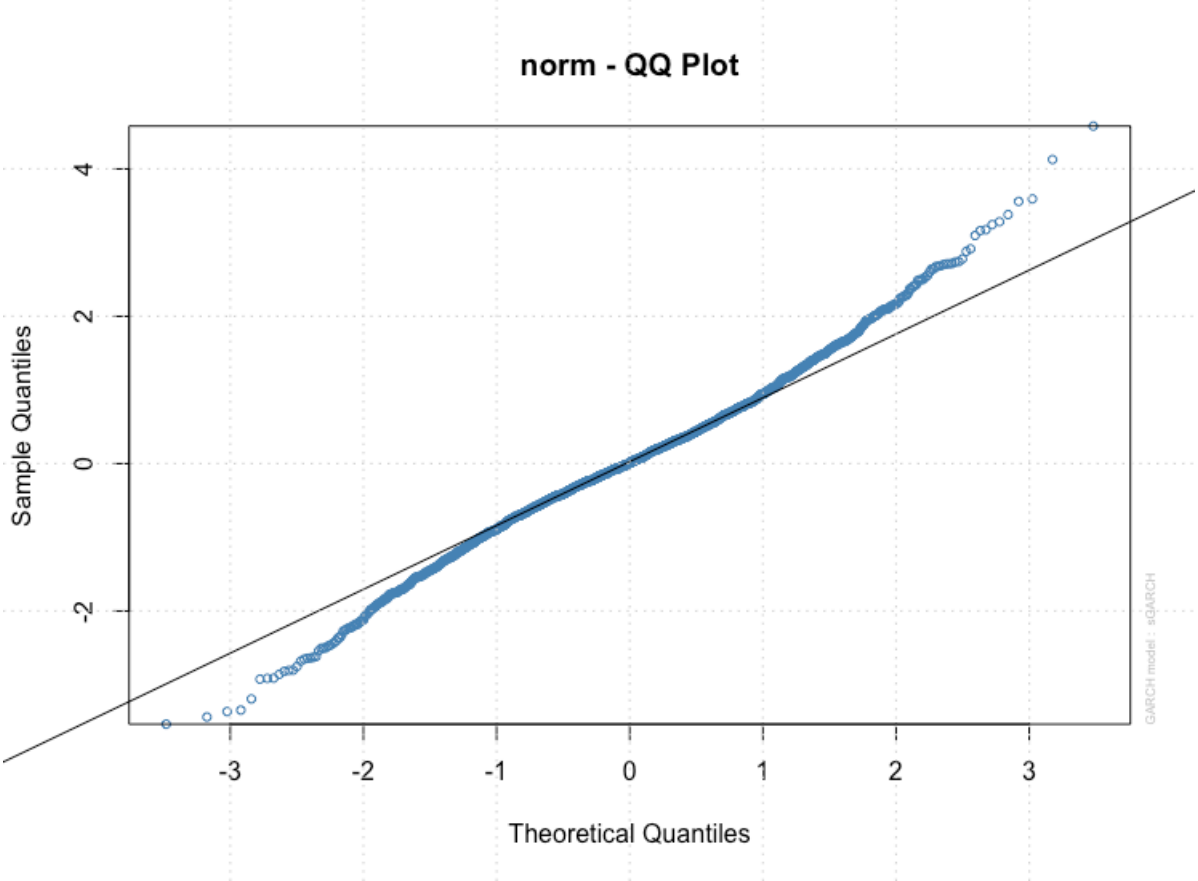
This is a prove that we should have used a long memory model.
rugarch provide a function to plot a "uGARCHfit" obeject.
ex. density smmothed to normal distribution:

Figure 8: Empirical Density of Standardized Residuals



qqplot:

Figure 9: qqplot of z

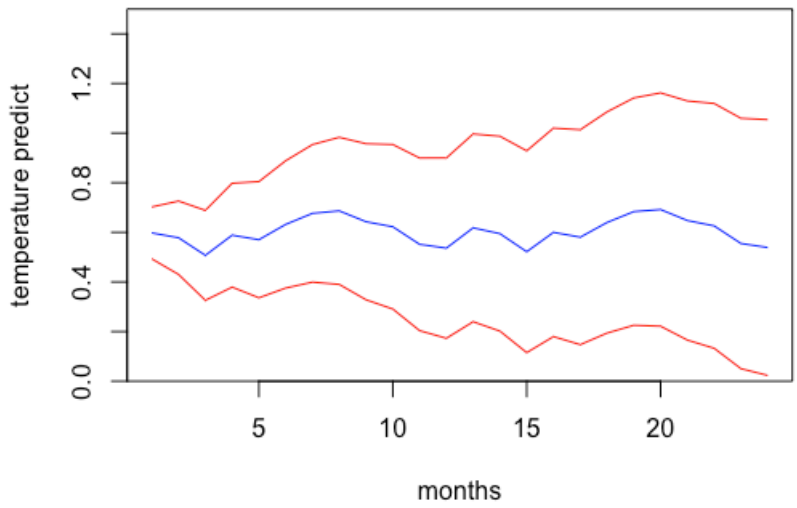


We can see that z is skewed, which might be correlated with long lags' heterodasticity.

Forecast

```
fore1 = ugarchforecast(fit1, n.ahead = 24)
fore.diff = as.numeric(fore1@forecast$seriesFor)
fore.sigma = as.numeric(fore1@forecast$sigmaFor)
ts.predict = temp[length(temp)] + cumsum(fore.diff)
ts.predict = ts.predict + myTS.additive$figure
ts.sigma = sqrt(cumsum(fore.sigma^2))
tsup.sigma = ts.predict + ts.sigma
tsdown.sigma = ts.predict - ts.sigma
plot(1:24, ts.predict, ylim=c(0,1.5), type = 'l', col = 'blue', xlab = "months", ylab = "ter
lines(1:24, tsup.sigma, type = 'l', col = 'red')
lines(1:24, tsdown.sigma, type = 'l', col = 'red')
```

Figure 10: qqplot of z



Conclusion: 'sGARCH' have improved the model in the intuitive sense and to some extent fix the heterodasticity.