ARMA-GARCH

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1 Temperature Model

1.1 Objective

Build a robust time series model to forecast future temperature's interval.

1.2 Data Source

HadCRUT4 is a gridded dataset of global historical surface temperature anomalies relative to a 1961-1990 reference period. Data are available for each month since January 1850, on a 5 degree grid. `http://www.metoffice.gov.uk/hadobs/hadcrut4/data/current/time_series/HadCRUT.4.5.0.0.monthly_ns_avg.txt`.

1.3 Data adjustment

From the shown url, grab the monthly global mean of temperature anomalies from 1850-2016, which amounts to a length of 2004. Using decompose() function in R to get the adjusted value.(See appendix)

Why we didn't use the traditional adjustment method Δ^{12}: I found that acf(myTS) shows acf anomaly at lag 24, not at lag 12. And after a Δ^{12} operation, acf at lag 12 became significant. Therefore, we think that using the Δ^{12} is feckless in this case.

1.4 Starting from *arima* model

First, examine the stationarity of *myTS.adjusted*. Strange enough, it passed the *adf.test* despite that it has a clear trend, as shown in the figure. So we tried to use *auto.arima*() to decide if it is $I(1)$ or $I(0)$. As shown in the code and results, a $arima(2,1,4)$ model fits *myTS.adjusted* better in both acf and $pacf$. The t-value of the coefficients of $auto.arima(residuals)$ is smaller for $arima(2,1,4)$.

The forecast of $arima(2,1,4)$ for 240 months from 2017.1 is shown in the figure. Within the sample, we compare the forecast and the actual data, only to find that $arima(2,1,4)$ is far from satisfying.

1.5 Long-Memory Model

Though acf shows that some lags are significant, but we think it's only because we took too big a confidential level. Actually the problem is not that great, but we still build a $ARFIMA$ model.

We plot the acf of the residuals of $ARFIMA$ model, but it seems that $ARFIMA$ can't effectively explain our data.

1.6 More remark on the long-memory effect

We find that acf of residuals always display significance at lag 24, which cannot be eliminaed by long-memory model. Also, as mentioned in the 'Data adjustment part', seasonal adjustment with $period = 12$(12 months) cannot solve the problem as well. We've also tried to adjust the effect using 24 lags, but to find it ineffective as well. It should also be noticed that using auto.arima() with seasonal set to TRUE is also bootless, because there is only lag 24 significant, and these 'auto' functions only accept a logical seasonal value, therefore I cannot assign the fixed orders. At last, I find that I can manually find a somehow acceptable method to solve the 24 lag.(but there's another lag becoming significant, at lag 26 or something) The code and figures are in the appendix

Since I do not know how to combine this method with GARCH model, I abandon it when dealing with heterodasticity problem.

1.7 Solving Heterodasticity: Standard GARCH

Looking at the plot of the residuals of $arima(2, 1, 4)$. Obviously there exists heterodasticity problem. Using *adf.test*, it supports our doubt. Therefore we have the motive to build a $GARCH$ model. We decide to use the package *rugarch*.

A standard GARCH model has the following variance equation:

$$\sigma_t^2 = (w + \sum_{j=1}^{m} \zeta_j v_{jt}) + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

We wrote a function to test different order for the model as shown in appendix. Due to the problem that rugarch package only deals with stationary series, we first $diff()$ *myTS.adjusted*. After trying a few times from $(1, 0), (0, 0, 0)$ to $(5, 5), (4, 0, 4)$, *GARCH(1 ,1), ARIMA(2, 0, 3)* is the most satisfying model. Here we realize that using GARCH model, the order of ARIMA might changes. Substract the standardized(w.r.t. the variance model) residuals z, which is $z = \dfrac{residuals(fit)}{sigma(fit)}$. Plot z, we'll find z seems to have a far smoother variance. Just for fun, plot a normal sample series with the same parameters. We'll find in the two figures it's hard to distinguish between the two. Looking at the $LM - Test$ in the summary of $fit1$, the heterodasticity is wiped out at lower lags. More graphical diagnostics is available in the figure appendix. From these diagnostics we find sGARCH is to some extent a proper model. Since we difference the series at first, now forecast becomes more complex. We have to calculate the variance by adding all the terms' variance.

2 Appendix

2.1 R Code

2.1.1 Data Adjustment

```
library(curl)
tmpf <- tempfile()
curl_download(url, tmpf)
gtemp <- read.table(tmpf)[, 1:2]
temp = gtemp$V2[1:2004]
library(TSA)
myTS = ts(as.numeric(temp), start = c(1850, 1), frequency = 12)
myTS.additive = decompose(myTS)
myTS.adjusted = as.numeric(myTS.additive$x - myTS.additive$seasonal)
```

2.1.2 arima models

```
> tseries::adf.test(myTS.adjusted)

Augmented Dickey-Fuller Test

data:  myTS.adjusted
Dickey-Fuller = -5.1646, Lag order = 12, p-value = 0.01
alternative hypothesis: stationary

> library(forecast)
> auto.arima(myTS.adjusted)
Series: myTS.adjusted
ARIMA(2,1,4)(2,0,1)[12] with drift

Coefficients:
         ar1     ar2      ma1      ma2     ma3     ma4    sar1    sar2     sma1
      0.5041  0.3585  -1.0458  -0.1600  0.1362  0.0779  0.7619  0.0779  -0.7884
s.e.  0.2432  0.2143   0.2424   0.3482  0.1082  0.0257  0.0926  0.0248   0.0910
      drift
      5e-04
s.e.  2e-04

sigma^2 estimated as 0.01428:  log likelihood=1417.03
AIC=-2812.05   AICc=-2811.92   BIC=-2750.43

> arima1 = Arima(myTS.adjusted, c(2, 0, 1))
> auto.arima(arima1$residuals)
Series: arima1$residuals
ARIMA(5,1,0)(2,0,1)[12]

Coefficients:
          ar1      ar2      ar3      ar4      ar5    sar1    sar2     sma1
      -0.8556  -0.6256  -0.4849  -0.3136  -0.1475  0.7647  0.0837  -0.7907
s.e.   0.0224   0.0287   0.0299   0.0286   0.0222  0.0846  0.0248   0.0831

sigma^2 estimated as 0.01705:  log likelihood=1238.65
AIC=-2459.29   AICc=-2459.2   BIC=-2408.87

> arima2 = Arima(myTS.adjusted, c(2, 1, 4))
> auto.arima(arima2$residuals)
Series: arima2$residuals
ARIMA(1,0,1)(2,0,1)[12] with non-zero mean
```

```
Coefficients:
         ar1      ma1    sar1    sar2     sma1    mean
      0.5645  -0.5685  0.7389  0.0745  -0.7672  0.0062
s.e.  2.5541   2.5131  0.1253  0.0244   0.1242  0.0033

sigma^2 estimated as 0.01427:  log likelihood=1417.4
AIC=-2820.81   AICc=-2820.75   BIC=-2781.59

# looking at acf
# Choose arima(2, 1, 4)
> tseries::adf.test(arima2$residuals)

Augmented Dickey-Fuller Test

data:  arima2$residuals
Dickey-Fuller = -11.777, Lag order = 12,
p-value = 0.01
alternative hypothesis: stationary

> arima2
Series: myTS.adjusted
ARIMA(2,1,4)

Coefficients:
         ar1     ar2      ma1      ma2     ma3     ma4
      0.5129  0.3271  -1.0438  -0.1331  0.1166  0.0748
s.e.  0.2753  0.2357   0.2748   0.3833  0.1117  0.0265

sigma^2 estimated as 0.01443:  log likelihood=1405.3
AIC=-2796.6   AICc=-2796.55   BIC=-2757.38

#forecast for future
> plot(forecast.Arima(arima2, h = 240))

# forecast within the sample and comparision
> sarima = Arima(myTS.adjusted[1:1800], c(2, 1, 4))
> plot(forecast.Arima(sarima, h = 203))
> lines(myTS.adjusted)
```

2.1.3 ARFIMA Model

```
> lmodel = arfima(myTS.adjusted)
> summary(lmodel)

Call:
  arfima(y = myTS.adjusted)

*** Warning during (fdcov) fit: unable to compute correlation matrix; maybe change 'h'

Coefficients:
       Estimate
d         0.445
ar.ar1    0.249
ar.ar2    0.612
ma.ma1    0.223
ma.ma2    0.549
sigma[eps] = 0.1200103
```

```
[d.tol = 0.0001221, M = 100, h = 1.481e-05]
Log likelihood:  1404 ==> AIC = -2796.513 [6 deg.freedom]

> acf(lmodel$residuals)
```

2.1.4 Dealing with the 24-lag anomaly

```
> gtemp = as.numeric(gtemp$V2)
Error in gtemp$V2 : $ operator is invalid for atomic vectors
> mytemp =  gtemp[600:1980]
> plot.ts(mytemp)
> acf(mytemp)
> pacf(mytemp)
> m1 = auto.arima(mytemp, seasonal = TRUE)
> m1
Series: mytemp
ARIMA(3,1,1) with drift

Coefficients:
         ar1     ar2     ar3      ma1  drift
      0.4766  0.2072  0.0823  -0.9809  7e-04
s.e.  0.0276  0.0296  0.0274   0.0062  2e-04

sigma^2 estimated as 0.01009:  log likelihood=1215.01
AIC=-2418.01   AICc=-2417.95   BIC=-2386.64
> m2 = arima(mytemp, order = c(3, 1, 1), seasonal = list(order = c(1, 0, 1), period = 24))
> m2

Call:
arima(x = mytemp, order = c(3, 1, 1), seasonal = list(order = c(1, 0, 1), period = 24))

Coefficients:
         ar1     ar2     ar3      ma1    sar1     sma1
      0.4728  0.2125  0.0850  -0.9783  0.9232  -0.8713
s.e.  0.0273  0.0294  0.0273   0.0063  0.0397   0.0518

sigma^2 estimated as 0.009917:  log likelihood = 1223.8,  aic = -2435.6
> m3 = arfima(mytemp)
> m3

Call:
  arfima(y = mytemp)

Coefficients:
         d     ar.ar1     ar.ar2     ma.ma1
0.49203791 0.70817900 0.08482127 0.68963222
sigma[eps] = 0.1010365
a list with components:
 [1] "log.likelihood"  "n"               "msg"
 [4] "d"               "ar"              "ma"
 [7] "covariance.dpq"  "fnormMin"        "sigma"
[10] "stderror.dpq"    "correlation.dpq" "h"
[13] "d.tol"           "M"               "hessian.dpq"
[16] "length.w"        "call"            "residuals"
[19] "x"               "fitted"          "series"
> res3 = residuals(m3)
> res4 = residuals(arima(res3, seasonal = list(order = c(1, 0, 1), period = 24)))
> acf(res4)
```

```
# try to use pure seasonal adjustment with period=24
> atemp = diff(mytemp, lag = 24)
> m5 = auto.arima(atemp)
> res5 = residuals(m5)
> acf(res5)
```

2.1.5 sGARCH Model

```
> arch.test(arma2$residuals)
ARCH heteroscedasticity test for residuals
alternative: heteroscedastic

Portmanteau-Q test:
     order    PQ p.value
[1,]     4  99.4       0
[2,]     8 116.9       0
[3,]    12 411.5       0
[4,]    16 475.6       0
[5,]    20 495.9       0
[6,]    24 755.8       0
Lagrange-Multiplier test:
     order   LM p.value
[1,]     4 1418       0
[2,]     8  697       0
[3,]    12  418       0
[4,]    16  212       0
[5,]    20  168       0
[6,]    24  137       0

library(rugarch)
my_sGARCH_test <- function(p, q, m, n, ts.data = res)
{
# I use include.mean = FALSE after trying TRUE
# to find out insignificance
    myspec=ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(p, q)),
     mean.model = list(armaOrder = c(m, n), include.mean = FALSE),
     distribution.model = "normal")
    myfit=ugarchfit(myspec,data=ts.data, solver="solnp")
    return(myfit)
}

> dtemp = diff(myTS.adjusted)
> fit1 = my_sGARCH_test(1, 1, 2, 3, dtemp)

> fit1

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(2,0,3)
Distribution : norm

Optimal Parameters
------------------------------------
```

```
        Estimate  Std. Error    t value Pr(>|t|)
ar1    -0.090935    0.015226    -5.9724 0.000000
ar2     0.754333    0.015069    50.0583 0.000000
ma1    -0.391906    0.007313   -53.5917 0.000000
ma2    -0.863257    0.000130 -6647.5909 0.000000
ma3     0.295634    0.007800    37.9010 0.000000
omega   0.000082    0.000034     2.3956 0.016595
alpha1  0.023026    0.003614     6.3706 0.000000
beta1   0.970594    0.004705   206.2925 0.000000

Robust Standard Errors:
        Estimate  Std. Error    t value Pr(>|t|)
ar1    -0.090935    0.017031    -5.3393 0.000000
ar2     0.754333    0.017312    43.5726 0.000000
ma1    -0.391906    0.002647  -148.0378 0.000000
ma2    -0.863257    0.000144 -6000.9074 0.000000
ma3     0.295634    0.002903   101.8443 0.000000
omega   0.000082    0.000036     2.2838 0.022382
alpha1  0.023026    0.003636     6.3329 0.000000
beta1   0.970594    0.003734   259.9413 0.000000

LogLikelihood : 1512.336

Information Criteria
------------------------------------

Akaike       -1.5021
Bayes        -1.4797
Shibata      -1.5021
Hannan-Quinn -1.4939

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.3742  0.5407
Lag[2*(p+q)+(p+q)-1][14]    4.5219  1.0000
Lag[4*(p+q)+(p+q)-1][24]   13.5547  0.3236
d.o.f=5
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      31.06 2.506e-08
Lag[2*(p+q)+(p+q)-1][5]     37.84 1.904e-10
Lag[4*(p+q)+(p+q)-1][9]     51.28 4.433e-13
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale   P-Value
ARCH Lag[3]  0.004507 0.500 2.000 9.465e-01
ARCH Lag[5] 11.085543 1.440 1.667 3.565e-03
ARCH Lag[7] 20.592184 2.315 1.543 4.509e-05

Nyblom stability test
------------------------------------
Joint Statistic:  2.6294
```

```
Individual Statistics:
ar1    0.25019
ar2    0.61592
ma1    0.19513
ma2    0.24162
ma3    0.07259
omega  0.10095
alpha1 0.42035
beta1  0.20019

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.89 2.11 2.59
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value      prob sig
Sign Bias           0.3363 7.367e-01
Negative Sign Bias  3.6815 2.380e-04 ***
Positive Sign Bias  4.4421 9.396e-06 ***
Joint Effect       33.2983 2.786e-07 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     57.32    1.019e-05
2    30     63.59    2.167e-04
3    40     85.01    2.883e-05
4    50    102.77    1.103e-05


Elapsed time : 0.365526

> z = residuals(fit1) / sigma(fit1)
> plot.ts(z)
> mean(z)
[1] 0.03181525
> var(z)
[1] 1.013866
> length(z)
[1] 2003
> plot.ts(rnorm(2003, 0.03181525, 1.013866))

# forecast
> fore1 = ugarchforecast(fit1, n.ahead = 24)
> fore.diff = as.numeric(fore1@forecast$seriesFor)
> fore.sigma = as.numeric(fore1@forecast$sigmaFor)
> ts.predict = temp[length(temp)] + cumsum(fore.diff)
> ts.predict = ts.predict + myTS.additive$figure
> ts.sigma = sqrt(cumsum(fore.sigma^2))
> tsup.sigma = ts.predict + ts.sigma
> tsdown.sigma = ts.predict - ts.sigma
> plot(1:24, ts.predict, ylim=c(0,1.5), type = 'l', col = 'blue',
  xlab = "months", ylab = "temperature predict")
> lines(1:24, tsup.sigma, type = 'l', col = 'red')
> lines(1:24, tsdown.sigma, type = 'l', col = 'red')
```

2.2 Figures

2.2.1 Data adjustment

Figure 1: HadCRUT4 Data Global Mean Time Series

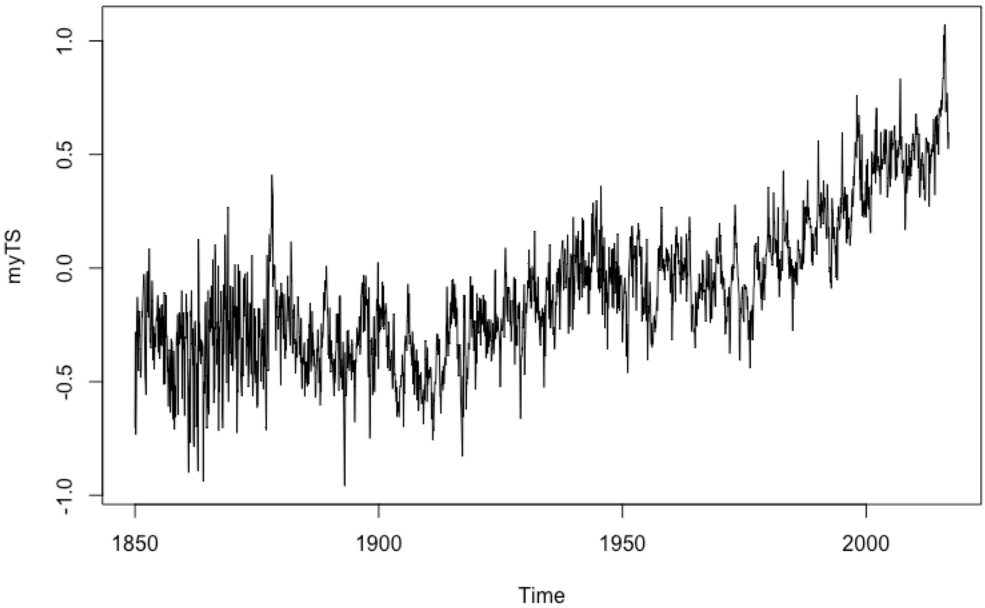
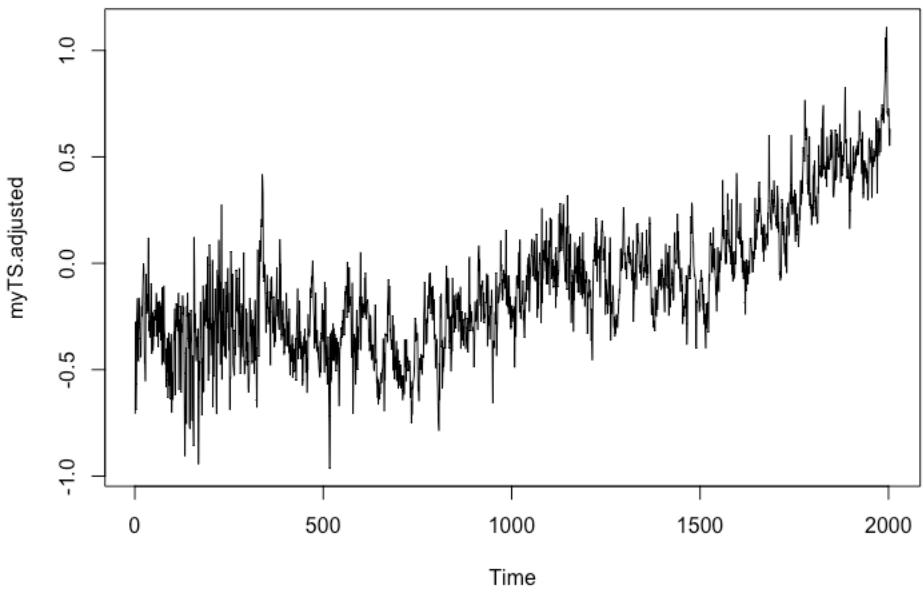


Figure 2: Plot myTS.adjusted



2.2.2 ARIMA Model

Figure 3: Selecting arma order: acf of residuals of arma(2, 0, 1) & (2, 1, 4)

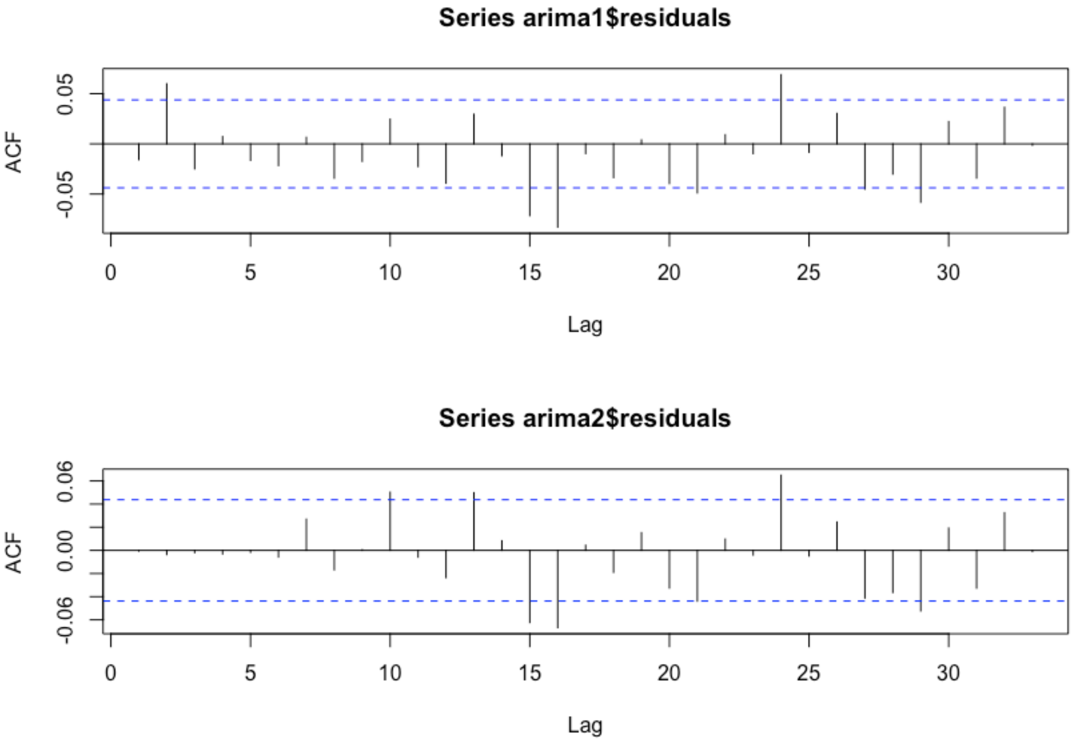


Figure 4: Selecting arma order: pacf of residuals of arma(2, 0, 1) & (2, 1, 4)

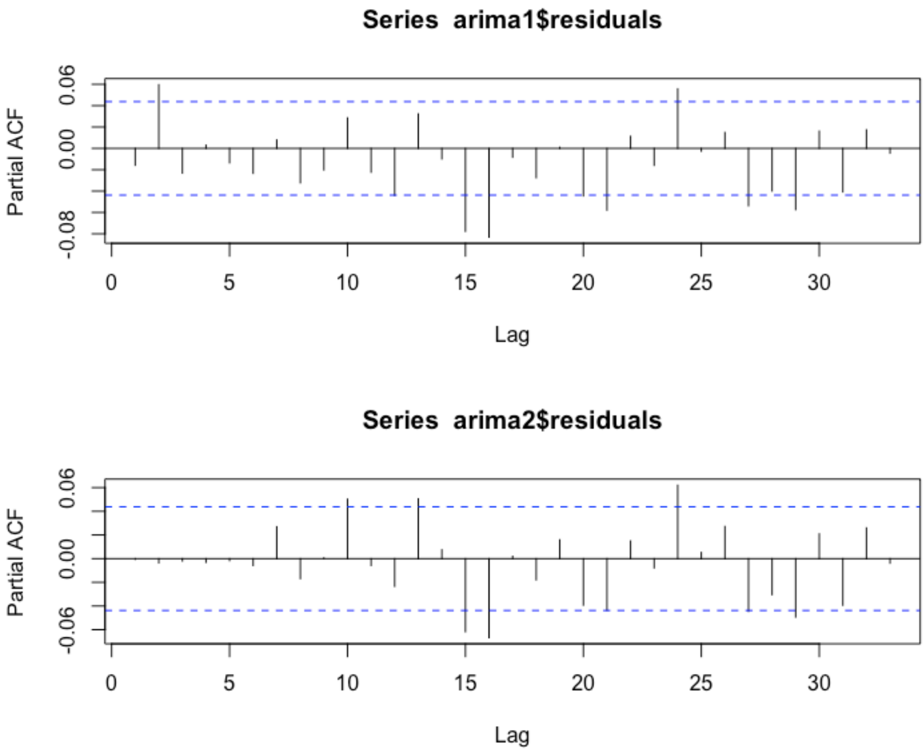


Figure 5: Forecast next 20 years(240 months) using arima(2, 1, 4)

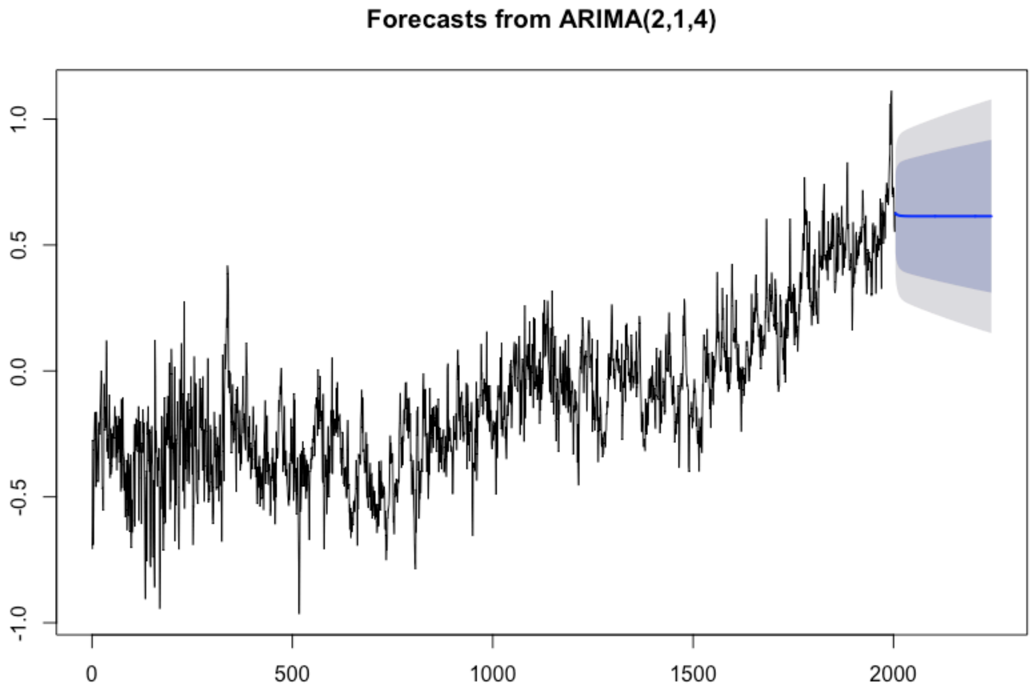
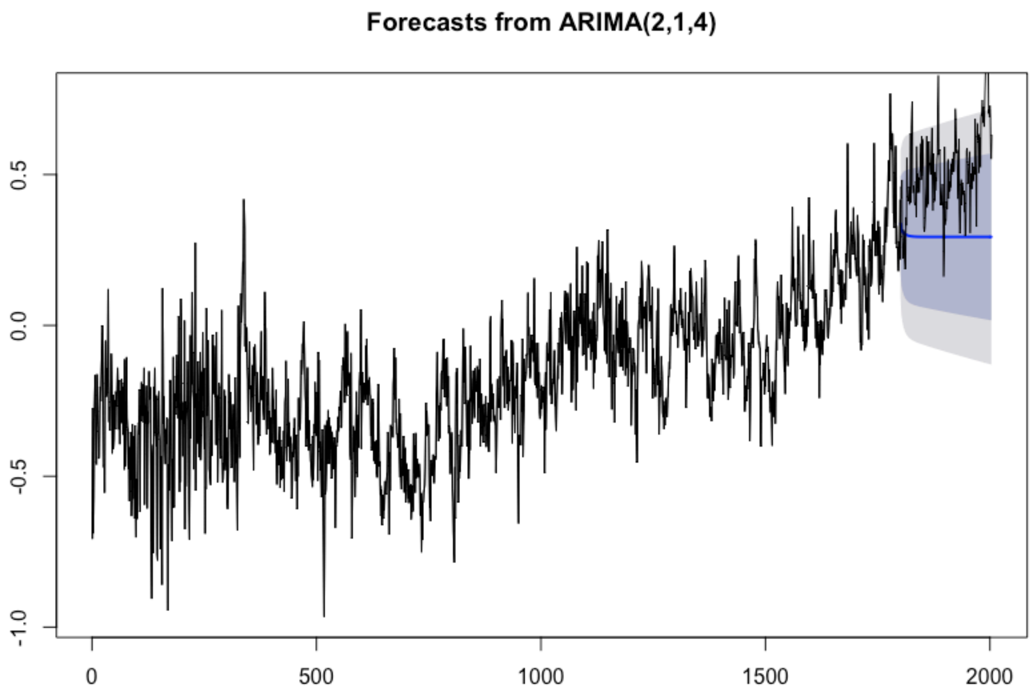


Figure 6: Forecast within the sample compared with the actual data using arima(2, 1, 4)



2.2.3 ARFIMA Model

Figure 7: Plot arima2's residuals

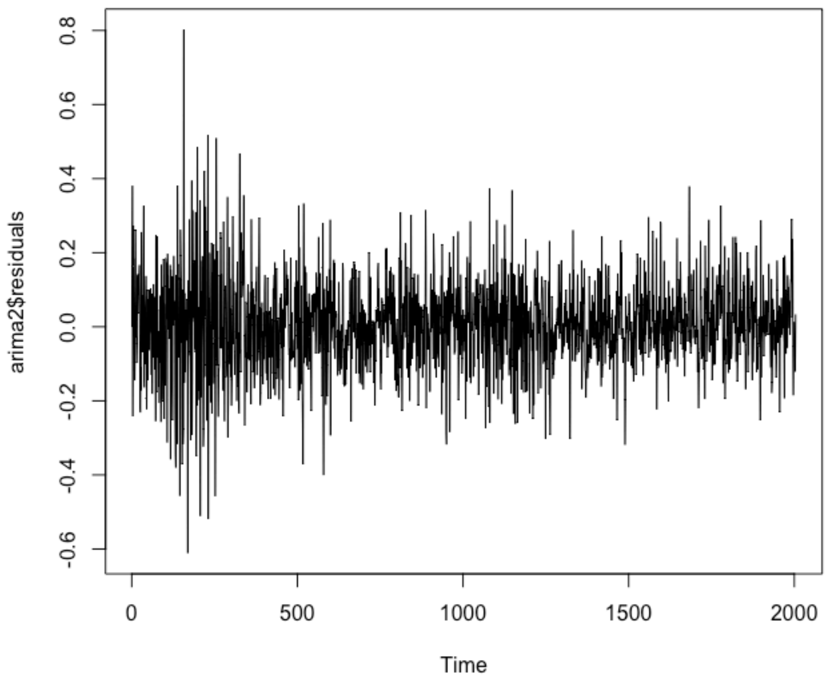
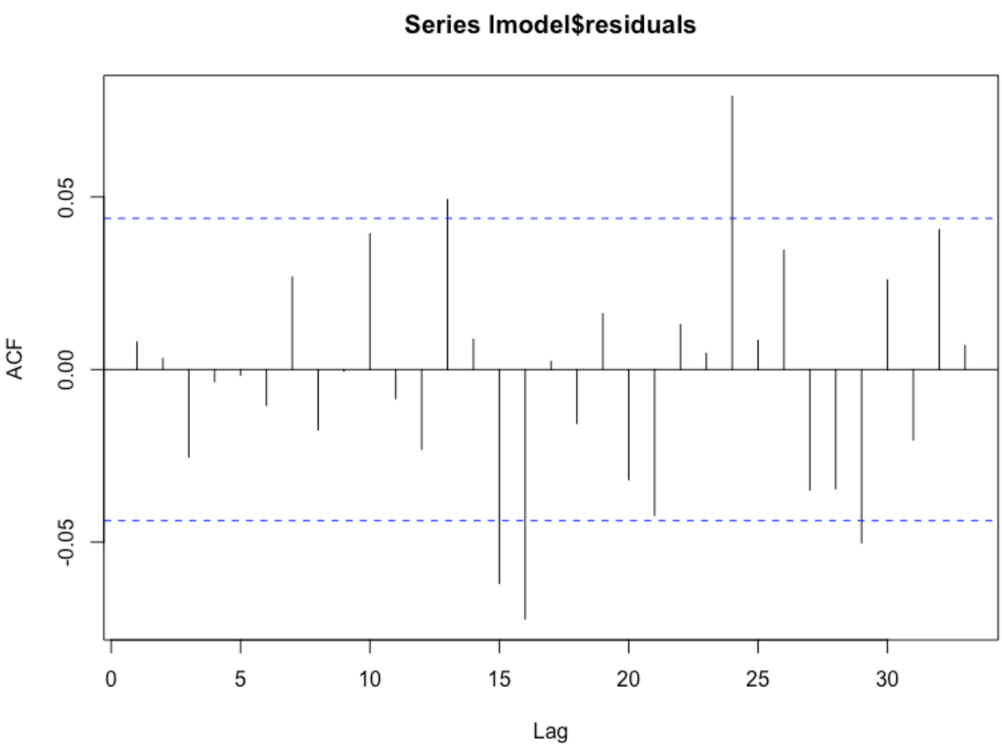


Figure 8: acf of residuals of ARFIMA



2.2.4 More attempts on lag 24

Original series' acf and pacf

Figure 9: acf

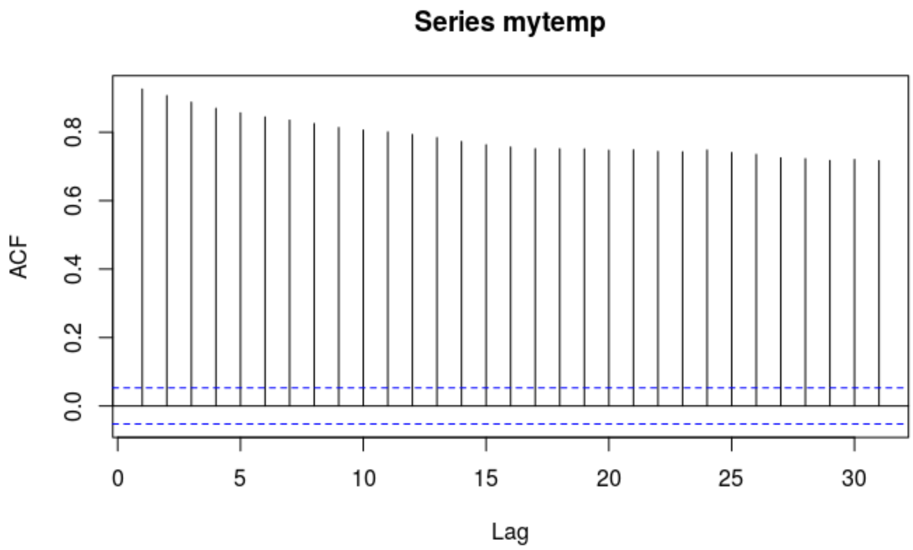


Figure 10: pacf

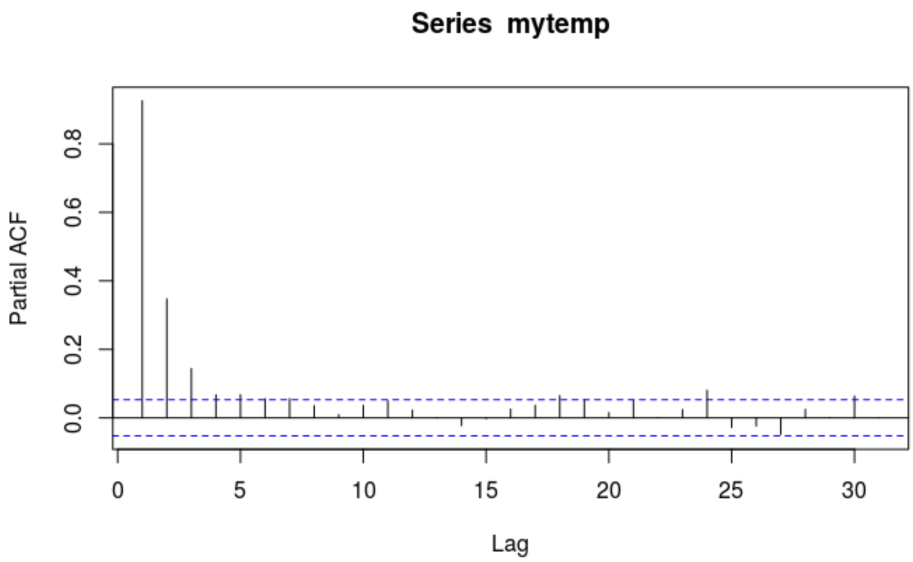


Figure 11: res2's acf

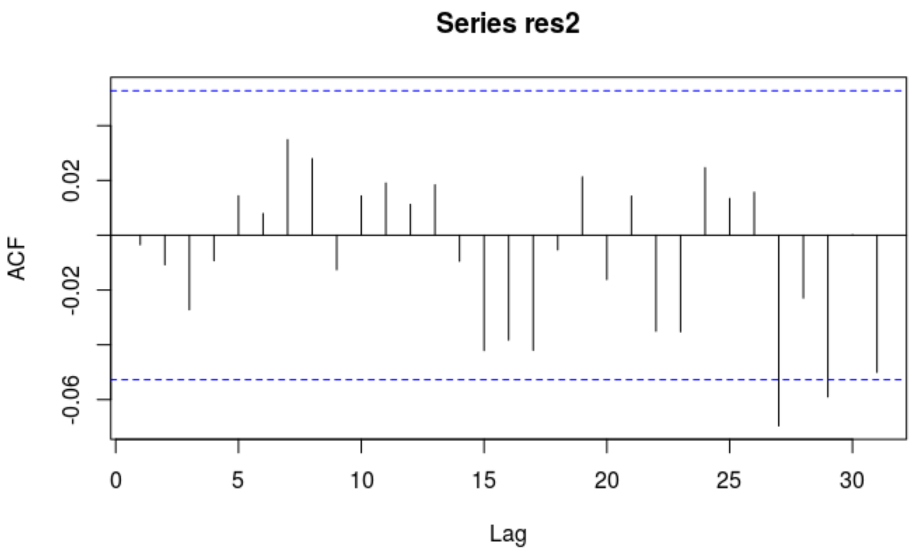


Figure 12: res3's acf

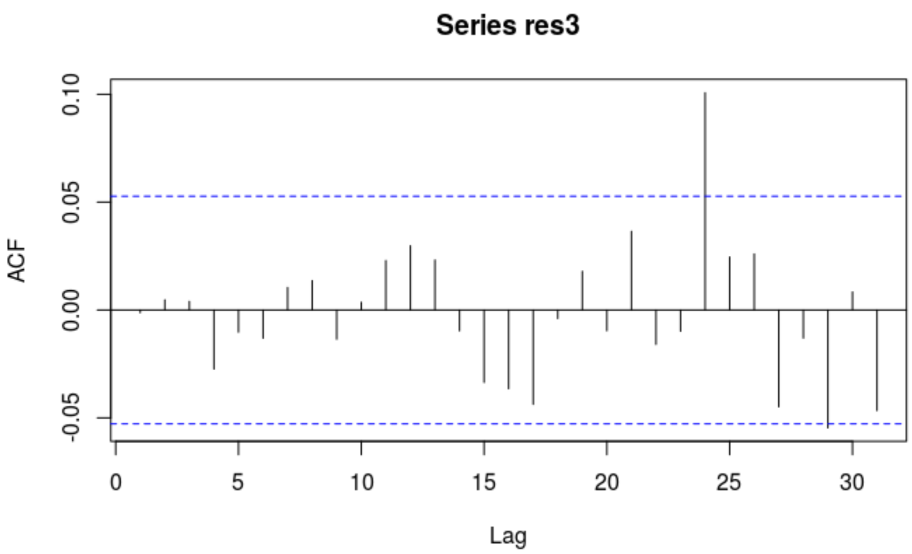
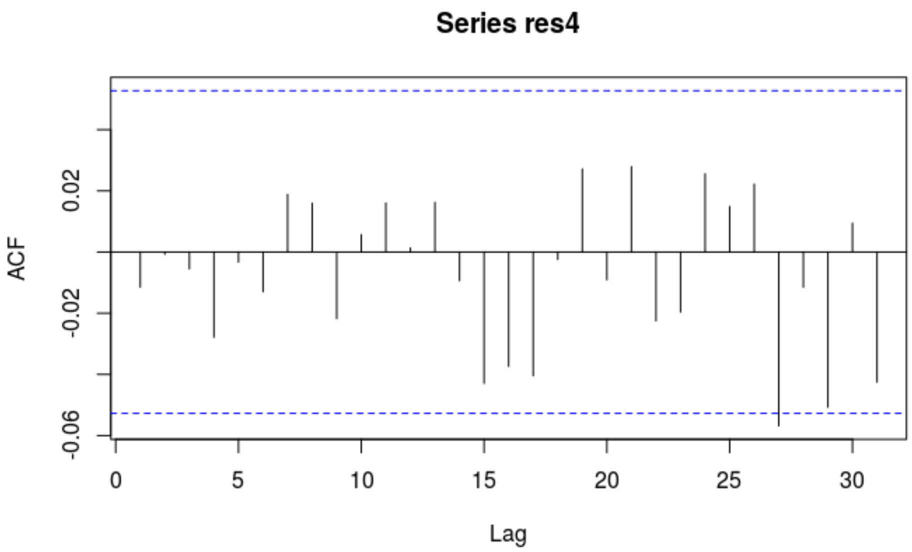
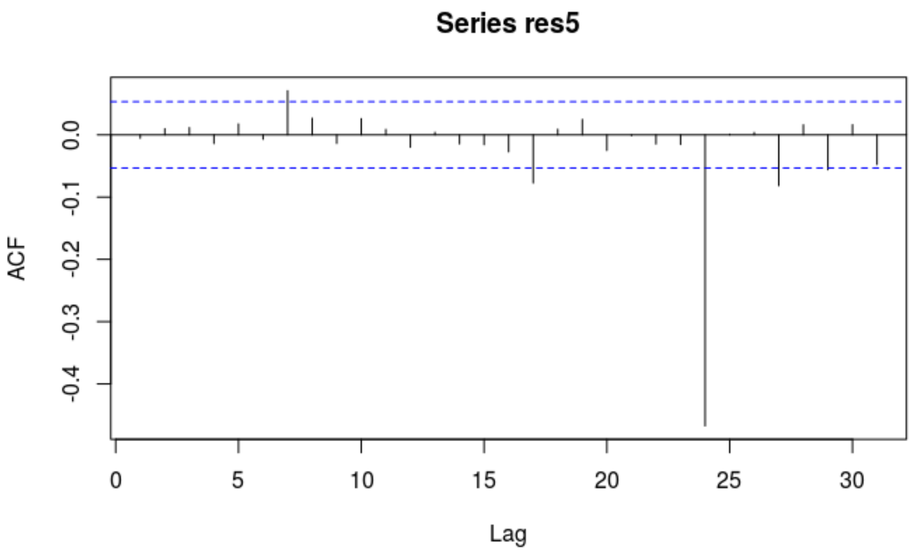


Figure 13: final residuals: res4's acf



The failure of trying pure seasonal adjustment

Figure 14: res5's acf



2.2.5 GARCH Model

Figure 15: z

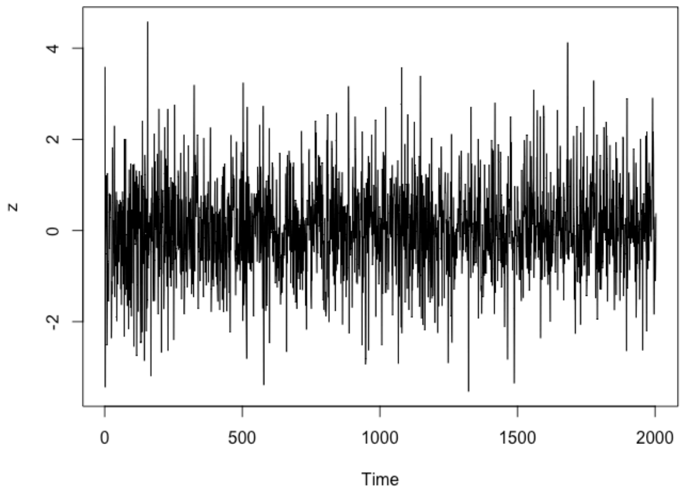


Figure 16: simulation of using rnorm

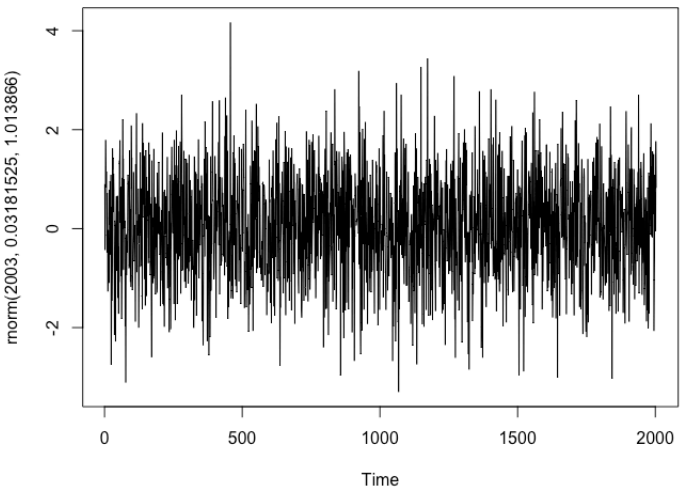


Figure 17: acf(z)

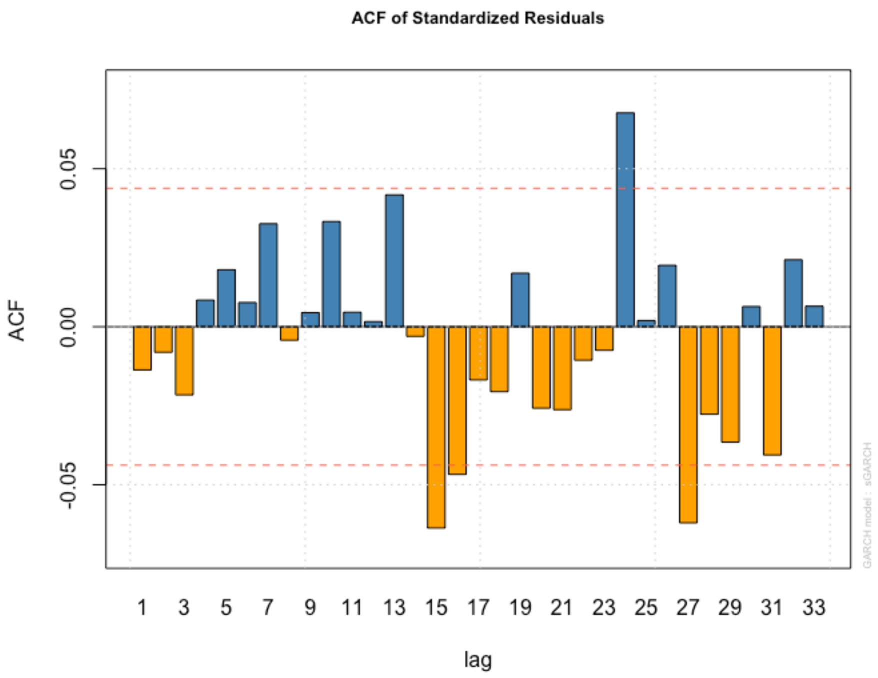


Figure 18: Empirical Density of Standardized Residuals

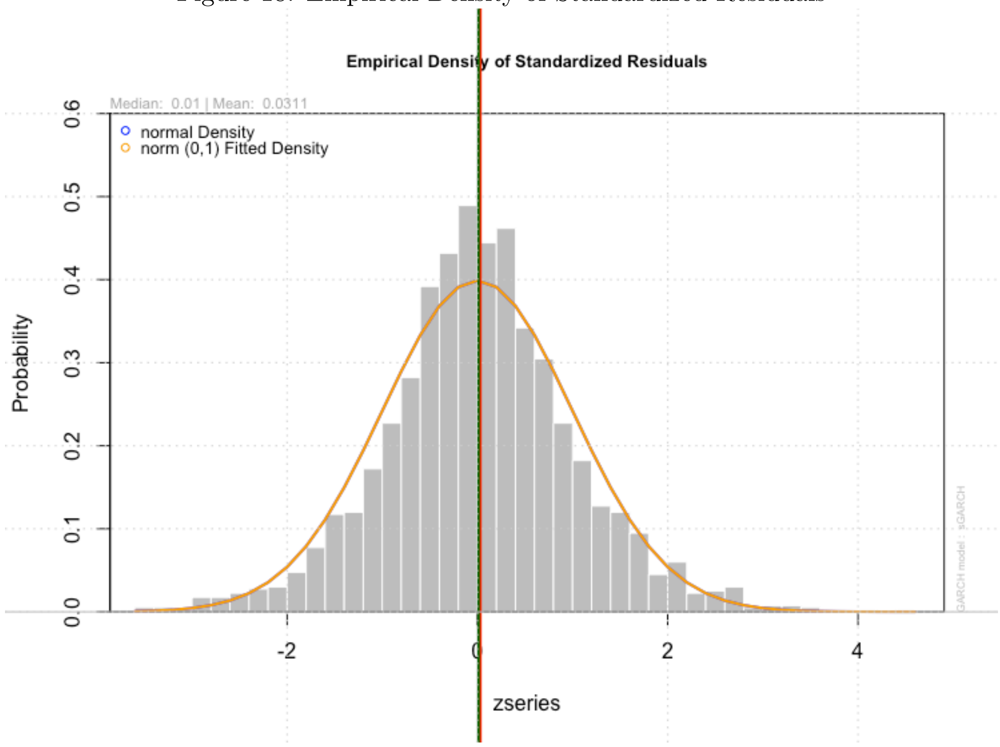


Figure 19: qqplot of z

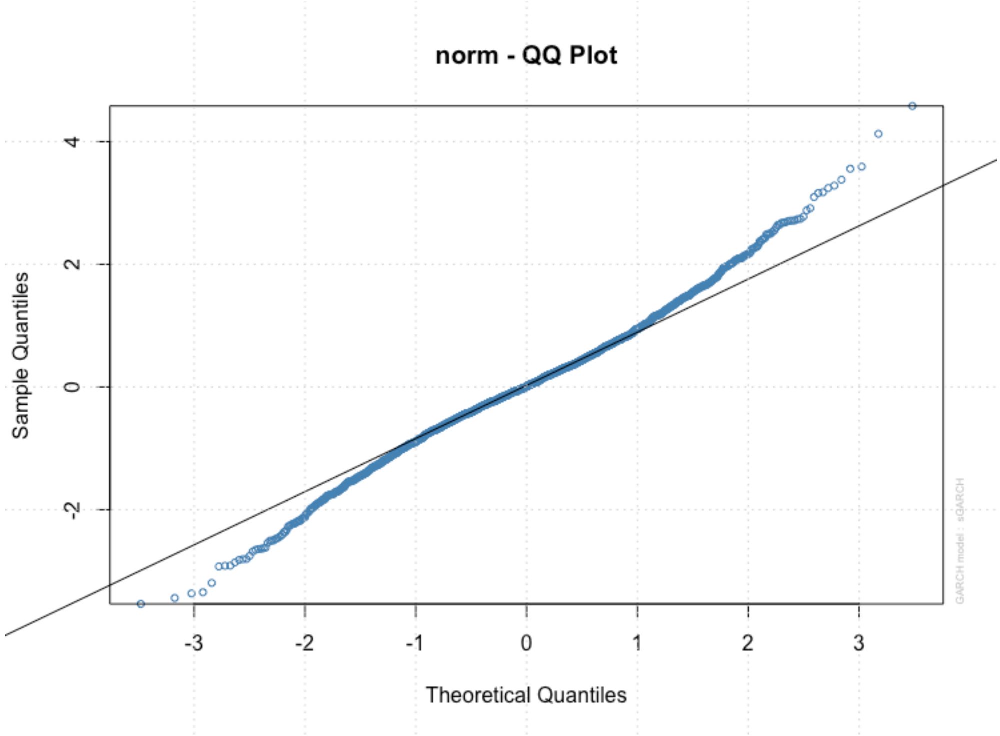
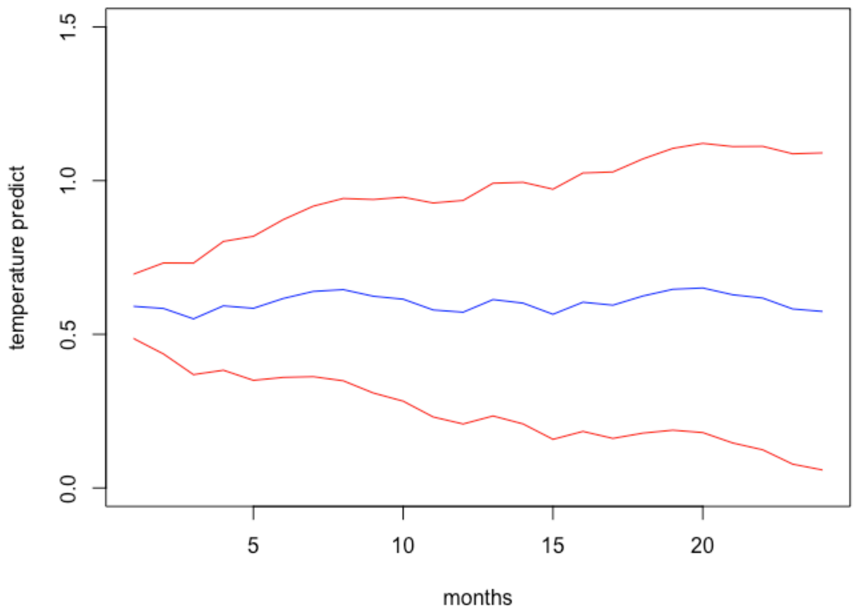


Figure 20: forecast for sGARCH Model



2.3 Others

2.3.1 Details about *decompose()* used in seasonal adjustment

Type 'decompose' in R console, we can see the source code of this function. The process of 'type = additive' is listed below:

- the argument passed into decompose() is a ‘ts’ object
- denote the argument ts(x, frequency = f). Create a filter using: filter = c(0.5, rep(1, f - 1), 0.5)/f.
- trend = filter(x, filter)
- season = x - trend, then compute f means of season with interval length f, the f means denoted by figure. Adjusting figure figure = figure - mean(figure)
- seasonal is just length(x)/f times repetition of figure.
- random = x - seasonal - trend