The Education University of Hong Kong

2024-25 Semester 1

MTH1098 Calculus

Assignment

1. Give an $\varepsilon - \delta$ proof of $\lim_{x\to 2}(4x-3) = 5$.

2. Let $g(x) = \frac{x^3+x^2-2}{x-1}$.
   (a) What is the domain of $g(x)$?
   (b) Find $\lim_{x\to 1} g(x)$.
   (c) What can you say about the continuity of $g(x)$ at $x = 1$?

3. Find (a) $\lim_{x\to 0} x^2 \sin\left(\frac{2}{x^2}\right)$ by using Squeeze Theorem,
   (b) $\lim_{x\to\infty}(\sqrt{x^2+2x} - \sqrt{x^2-2x})$,
   (c) $\lim_{t\to\infty}\left(1-\frac{t}{t+1}\right)\Big/\left(1-\sqrt{\frac{t}{t+1}}\right)$,
   (d) $\lim_{x\to 3\pi/4}(\sin x + \cos x)/\cos(2x)$.
   (Hint: $\cos 2x = \cos^2 x - \sin^2 x$)
   (e) $\lim_{x\to 1}\frac{x\ln x}{x^3-1}$ by using L'Hôpital's Rule.

4. Let $f(x) = \sqrt[3]{2x} + 3x - 4$.
   (a) Show that there is a solution of $f(x) = 0$ in the interval $(0, 4)$.
   (b) Without finding $f'(x)$, explain why there exists $c \in (0,4)$ such that $f'(c) = 3\frac{1}{2}$.

5. Let $f(x) = \begin{cases} 11 + c^2 x & \text{if } x < 2 \\ 1 - 6cx & \text{if } x \geq 2 \end{cases}$.
   Find (a) $\lim_{x\to 2^-} f(x)$ and (b) $\lim_{x\to 2^+} f(x)$.
   For what values of c will $f(x)$ be continuous at $x = 2$?

6. Let $f(x) = \sqrt{4x+3}$. Find $f'(x)$ by
   (a) Power Rule and Chain Rule, and
   (b) using the definition of the derivative of a function.

7. Suppose f, g and h are differentiable functions.
   (a) Express $\left(\frac{fg}{h}\right)'$ in terms of f, g, h, f', g' and h'.
   (b) Use the result in (a) to find $\left(\frac{x^2 \sin x}{e^x}\right)'$.

8. Find $\frac{dy}{dx}$ if
   (a) $y = \sqrt{5^x}$
   (b) $y = e^{\tan(2x)} \ln(\sin x)$.
   (c) $xy^2 + y \ln x + e^x = 0$,
   (d) $y = \sin^{-1}(4x)$.

9. A ladder 17 m long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at a rate of 0.8 m/s. How fast is the top sliding down the wall when the foot of the ladder is 8 m from the wall.

10. Sketch the graph of $y = \frac{x}{x^2-16}$. Identify any interesting features, including domain, intercepts, asymptotes, local maximum and minimum points, and inflection points.

11. A piece of cardboard is 2 m by 3 m. A square is to be cut from each corner and the sides folded up to make an open-top box. What are the dimensions of the box with maximum possible volume? What is the maximum possible volume?

Due date: Tue 12 Nov 2024