14. $\int x \cos 4x \, dx$

$a = 4x$
$da = 4dx$
$\cos(a) \frac{da}{4}$

Steven Zheng
Calc 2

$u = x$
$du = dx$

$dv = \cos 4x \, dx$
$v = \frac{1}{4} \sin 4x$

$uv - \int v \, du$

$\int x \cos 4x \, dx = \frac{1}{4} \sin 4x \cdot x - \int \frac{1}{4} \sin 4x \, dx$

$= \frac{x \sin 4x}{4} - \frac{1}{4} \int \sin 4x \, dx$

$u = \sin 4x$
$du = 4 \cos 4x \, dx$

$= \frac{x \sin 4x}{4} - \frac{1}{4}\left(\frac{1}{4}\right) \int \sin u \, du$

$= \frac{x \sin 4x}{4} - \frac{1}{16}\left(-\cos u\right) + C$

$= \frac{x \sin 4x}{4} - \frac{1}{16}\left(-\cos 4x\right) + C$

$= \boxed{\frac{x \sin 4x}{4} + \frac{\cos 4x}{16} + C}$

24 $$\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx$$

$u = x^2 e^{x^2}$

$du = x^2 2x e^{x^2} + 2x e^{x^2} dx$

$du = 2x e^{x^2}(x^2+1)dx$

$dv = \frac{x}{(x^2+1)^2}dx$

$$= \int x^2 e^{x^2}\left(\frac{x}{(x^2+1)^2}\right)dx$$

$\int \frac{x}{u^2}\frac{du}{2x}$

$= \frac{1}{2}\int u^{-2}du$

$= -\frac{1}{2}\frac{1}{u}$

$v = -\frac{1}{2(x^2+1)}$

$$= x^2 e^{x^2}\left(-\frac{1}{2(x^2+1)}\right) - \int -\frac{1}{2(x^2+1)}\,2x e^{x^2}(x^2+1)dx$$

$$= -\frac{x^2 e^{x^2}}{2(x^2+1)} + \int \frac{\cancel{2}x e^{x^2}\cancel{(x^2+1)}}{\cancel{2}\cancel{(x^2+1)}}dx$$

$$= -\frac{x^2 e^{x^2}}{2(x^2+1)} + \int x e^{x^2}dx$$

$u = x^2$

$du = 2x\,dx$

$$= -\frac{x^2 e^{x^2}}{2(x^2+1)} + \int x e^u \frac{du}{2\cancel{x}}$$

$$= -\frac{x^2 e^{x^2}}{2(x^2+1)} + \frac{1}{2}\int e^u du$$

$$= -\frac{x^2 e^{x^2}}{2(x^2+1)} + \frac{e^{x^2}}{2} + C$$

$$= \boxed{\frac{-x^2 e^{x^2} + e^{x^2}(x^2+1)}{2(x^2+1)} + C}$$

30. $\int x^2 \cos x \, dx$

$u = x^2$
$du = 2x\,dx$
$dv = \cos x\,dx$
$v = \sin x$

$= x^2 \sin x - \int \sin x \, 2x\,dx$

$u_1 = 2x$
$du_1 = 2dx$
$dv_1 = \sin x$
$v_1 = -\cos x$

$= x^2 \sin x - \left[2x(-\cos x) - \int -\cos x \, 2dx\right]$

$= x^2 \sin x + 2\cos x - 2\int \cos x\,dx$

$\boxed{= x^2 \sin x + 2\cos x - 2\sin x + C}$

8.3

6) $\int \sin^3 3x\,dx$

$u = 3x$
$du = 3dx$

$= \int \sin^2 3x \sin 3x\,dx$

$\sin^2 u + \cos^2 u = 1$

$= \frac{1}{3}\int \sin^2 u \sin u\,du$

$v = \cos u$
$dv = -\sin u\,du$

$= \frac{1}{3}\int (1 - \cos^2 u)\sin u\,du$

$= \frac{1}{3}\int (1 - u^2)\sin u \frac{dv}{-\sin u}$

$\frac{1}{3}\left[\int u^2 dv - \int 1dv\right]$

$= \frac{1}{3}\left[\frac{u^3}{3} - u\right] = \boxed{\frac{1}{3}\left[\frac{\cos^3 3x}{3} - \cos 3x\right]}$

8.4

10) $\int \frac{x^3}{\sqrt{x^2-25}}\,dx$

$a=5$

$u=x$

$x=5\sec\theta$

$dx=5\sec\theta\tan\theta\,d\theta$

$= \int \frac{(5\sec\theta)^3}{\sqrt{(5\sec\theta)^2-25}} \cdot 5\sec\theta\tan\theta\,d\theta$

$\frac{\sin^2x+\cos^2x}{\cos^2x\quad\cos^2x} = \frac{1}{\cos^2x}$

$\tan^2x+1=\sec^2x$

$= \int \frac{125\sec^3\theta\, 5\sec\theta\tan\theta\,d\theta}{\sqrt{25(\sec^2\theta-1)}}$

$= \int \frac{125\sec^3\theta\,\cancel{5}\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{5}\,\cancel{\tan\theta}}$

$= \int 125\sec^4\theta\,d\theta$

$u=\tan\theta$

$du=\sec^2\theta$

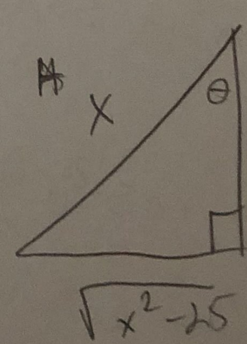
SOA CAH TOA

$\frac{5\ A}{x\ H}$

$=125\int \sec^2\theta\sec^2\theta\,d\theta$

$=125\int(\tan^2x+1)\sec^2\theta\,d\theta$

$=125\int(u^2+1)\cancel{\sec^2\theta}\frac{du}{\cancel{\sec^2\theta}}$



$=125\left[\int u^2+\int 1\right]$

$=125\left[\frac{u^3}{3}+u\right]$

$=125\left[\frac{\tan^3\theta}{3}+\tan\theta\right]$

$=125\left[\frac{(\sqrt{x^2-25})^3}{15}+\frac{\sqrt{x^2-25}}{5}\right] = \boxed{25\left[\frac{(\sqrt{x^2-25})^3}{3}+\sqrt{x^2-25}\right]}$

14.

$$\int \frac{2x^2}{(4+x^2)^2}\,dx \qquad x = 2\tan\theta \qquad dx = 2\sec^2\theta\,d\theta$$

$$= \int \frac{2(2\tan\theta)^2}{\left(4+(2\tan\theta)^2\right)^2} \cdot 2\sec^2\theta\,d\theta \qquad \tan^2 x + 1 = \sec^2 x$$

$$= \int \frac{(4\tan^2\theta)}{(4+4\tan^2\theta)^2} \cdot 2\sec^2\theta\,d\theta$$

$$= \int \frac{4\tan^2\theta}{(4(1+\tan^2\theta))^2}\, 2\sec^2\theta\,d\theta$$

$$= \int \frac{4\tan^2\theta}{(4(\sec^2\theta))^2}\, 2\sec^2\theta\,d\theta$$

$$= \int \frac{4\tan^2\theta}{16\sec^4\theta}\, 2\sec^2\theta\,d\theta$$

$$= \frac{1}{2}\int \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{1}\,d\theta$$

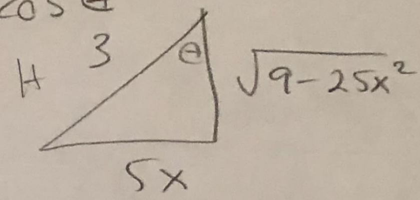
$$= \frac{1}{2}\int \sin^2\theta\,d\theta$$

40. $\displaystyle\int_0^{3/5} \sqrt{9-25x^2}\,dx$

$a=3$

$5x = 3\sin\theta$

$dx = \frac{3}{5}\cos\theta$



$$= \int_0^{3/5} \sqrt{9-25\left(\tfrac{3}{5}\sin\theta\right)^2}\cdot\tfrac{3}{5}\cos\theta\,d\theta$$

$$= \frac{3}{5}\int_0^{3/5} \cos\theta\sqrt{9-9\sin^2\theta}\,d\theta$$

$$= \frac{3}{5}\int_0^{3/5} \cos\theta\sqrt{9(1-\sin^2\theta)}\,d\theta$$

$$= \frac{3}{5}\int_0^{3/5} \cos\theta\, 3\sqrt{1-\sin^2\theta}\,d\theta$$

$\cos^2\theta = \frac{1}{2}((1+\cos^2(2\theta))$

$$= \frac{9}{5}\int_0^{3/5} \cos\theta\sqrt{\cos^2\theta}\,d\theta$$

$\sin\theta = \frac{5x}{3}$

$\theta = \arcsin\left(\frac{5x}{3}\right)$

$$= \frac{9}{5}\int_0^{3/5} \cos\theta\,(\cos\theta)\,d\theta$$

$$= \frac{9}{5}\,\frac{1}{2}\int_0^{3/5} 1+\cos 2\theta\,d\theta$$

$$= \frac{9}{10}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{3/5}$$

$$= \frac{9}{10}\left[\arcsin\frac{5x}{3} + \frac{1}{2}\sin\frac{2(5x)}{3}\right]_0^{3/5}$$

$=$

40 continue

$$= \frac{9}{10}\left[\arcsin\frac{5x}{3} + \frac{1}{2}\sin\frac{10x}{3}\right]_0^{3/5}$$

$$= \frac{9}{10}\left(\left[\arcsin 1 + \frac{1}{2}\sin 2\right] - \left[\arcsin(0) + \frac{1}{2}\sin(0)\right]\right)$$

$$\boxed{= \frac{9}{10}\left[\frac{\pi}{2} + \frac{\sin 2}{2}\right]}$$

8.5

16) $\displaystyle\int \frac{6x}{x^3-8}\,dx$

$x^2 - 2$, $x\ x^2$, $+2\ \ -4$

x^2-4

$Bx + C$

$x\ \ Bx^2\ \ xC$

$-2\ \ -2Bx\ \ -2C$

$$= \int \frac{6x}{(x-2)(x^2+2x+4)}\,dx$$

$6x = A(x^2+2x+4) + (Bx+C)(x-2)$

$6x = Ax^2 + 2Ax + 4A + Bx^2 - 2Bx + xC - 2C$

$$= \int \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4}\,dx$$

$$= \int \frac{1}{x-2}\,dx + \int \frac{-1x+2}{x^2+2x+4}\,dx$$

$6x = (A+B)x^2 + (2A-2B+C)x + 4A - 2C$

$A + B = 0$ $\qquad C = 2A$

$2A - 2B + C = 6$ $\qquad 2A - B = 3$

$4A - 2C = 0$

$$= \ln(x-2) - \int \frac{x-2}{x^2+2x+4}\,dx$$

$$\ln(x-2) - \int \frac{2x-4+2-2}{2(x^2+2x+4)}\,dx$$

$u = x^2+2x+4$

$du = 2x+2\,dx$

$A = 1$

$B = -1$

$C = 2$

Back

$$= \ln(x-2) - \int \frac{2x-4+2-2}{2(x^2+2x+4)}\,dx$$

$$= \ln(x-2) - \frac{1}{2}\int \frac{2x+2}{x^2+2x+4}\,dx - \frac{1}{2}\int -\frac{6}{x^2+2x+4}\,dx$$

$$= \ln(x-2) - \int \frac{x+1}{x^2+2x+4}\,dx + \int \frac{3}{(x^2+2x+1)+3}\,dx$$

$$= \ln(x-2) - \int \frac{\cancel{x+1}}{u}\,\frac{du}{2(\cancel{x+1})} + \int \frac{3}{(x+1)^2+3}\,dx$$

$$= \ln(x-2) - \frac{1}{2}\ln(x^2+2x+4) + 3\,\frac{1}{\sqrt{3}}\arctan\left(\frac{x+1}{\sqrt{3}}\right) + C$$

8.8

32. $\displaystyle\int_0^{\infty} \sin\frac{x}{2}\,dx$

$$\lim_{a\to\infty}\int_0^{a} \sin\frac{x}{2}\,dx$$

$$= \lim_{a\to\infty}\left[-2\cos\frac{x}{2}\right]_0^a$$

$$= \lim_{a\to\infty}\left[-2\cos\frac{a}{2} + 2\cos\frac{0}{2}\right]$$

$$= \lim_{a\to\infty}\left[-2\cos\frac{\infty}{2} + 2\right]$$

$\boxed{\text{Diverges}}$

48 $\int_1^\infty \frac{1}{x\ln x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \lim_{a\to\infty} \int_1^a \frac{1}{x\ln x}\,dx$$

$$= \lim_{a\to\infty} \int_1^a \frac{1}{\cancel{x}u}\,\cancel{x}\,du$$

$$= \lim_{a\to\infty} \int_1^a \frac{1}{u}\,du$$

$$= \lim_{a\to\infty} \Big[\ln|u|\Big]_{\ln 1}^{\ln a}$$

$$= \lim_{a\to\infty} \Big[\ln|\ln\infty| - \ln|\ln 1|\Big]$$

∞

$\boxed{\text{Diverges}}$