## Curs II

1. Exercitiu (warm-up). Merge Sort
2. Recurente
3. Teorema master

* Problemă. Se dau $n$ numere. Să se sorteze.

Sortarea prin interclasare (sau merge sort) procedează astfel.

1. Împărțim șirul de numere în două părți (aprox. egale).
2. Sortăm recursiv cele două jumătăți
3. Interclasăm cele două jumătăți sortate.

24 31 | 15 9 | 62 51 | 10 28

24 | 31 | 15 | 9 | 62 | 51 | 10 | 28

24 | 31 | 15 | 9 | 62 | 51 | 10 | 28

24 31 | 9 15 | 51 62 | 10 28

9 15 24 31 | 10 28 51 62

9 10 15 24 28 31 51 62

$$T(n) = 2T(n/2) + n$$
$$T(n/2) = 2T(n/4) + \frac{n}{2}$$
$$\Rightarrow T(n) = 2\left(2T(n/4) + \frac{n}{2}\right) + n$$
$$T(n) = 2^2 T(n/2^2) + 2n$$
$$T(n) = 2^K T(n/2^K) + 2K$$
$$K = \log_2 n$$
$$\Rightarrow T(n) = \Theta(n \log n)$$

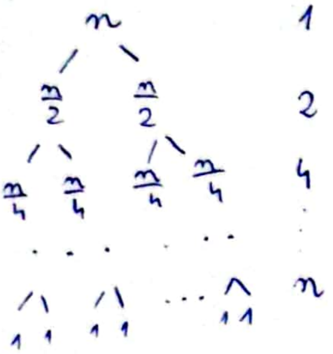
* Demonstrăm că $T(n) = 2T(n/2) + n \le c \cdot n \log_2 n$

PP că $T(n/2) \le c \cdot \frac{n}{2} \log_2 \frac{n}{2}$

Avem: $T(n) = 2T(n/2) + n \le \cancel{2} c \cdot \frac{n}{\cancel{2}} \log_2 \frac{n}{2} + n = cn\log_2 n - cn\log_2 2 + n$

$= cn\log_2 n + \underbrace{n(1-c)}_{\substack{\le 0 \\ c \ge 1}} \Rightarrow \le cn\log_2 n$ (A) $\Rightarrow O(n \log n)$

$$T(n) = 2T(n/2) + 1 \in O(n)$$



* Demonstrăm : $T(n) \le cn - b$

PP că $\exists c, b > 0$ $T(n/2) \le c\frac{n}{2} - b$

Avem $T(n) = 2T(n/2) + 1 \le 2\left(c \cdot \frac{n}{2} - b\right) + 1 \le cn + 1 \not\le cn$

$= cn - 2b + 1 = cn - b + (1-b) \le cn - b$
$\forall b \ge 1$

$\Rightarrow T(n) \le cn - b$ (A) $\Rightarrow O(n)$

! Recurențe de forma : $T(n) = aT(n/b) + f(n)$

[Teorema master]

① $f(n) \in O(n^{\log_b a - \varepsilon})$ pt $\varepsilon > 0 \Rightarrow T(n) \in \Theta(n^{\log_b a})$

Ex: $T(n) = 2T(n/2) + 1$

$a = 2$
$b = 2$
$f(n) = 1$

$\underset{f(n)}{1} \boxed{<} n^{\log_2 2} = n \Rightarrow T(n) \in \Theta(n)$

② $f(n) \in \Theta(n^{\log_b a}) \Rightarrow T(n) \in \Theta(n^{\log_b a} \log_2 n)$

Ex: $T(n) = 2T(n/2) + n$

$a = 2$
$b = 2$
$f(n) = n$

$n \boxed{=} n^{\log_2 2} = n \Rightarrow T(n) \in \Theta(n \log n)$

③ $f(n) \in \Omega(n^{\log_b a + \varepsilon})$, pt $\varepsilon > 0$ și $\exists c < 1$ a.î. $af(n/b) \le cf(n)$
$\Rightarrow T(n) \in \Theta(f(n))$

Ex: $T(n) = 2T(n/2) + n^2$

$a = 2$
$b = 2$
$f(n) = n^2$

$n^2 \boxed{>} n^{\log_2 2} = n$ $\Rightarrow T(n) \in \Theta(n^2)$

$2\left(\frac{n}{2}\right)^2 \le cn^2$

$\frac{2}{4} \cdot n^2 \le \frac{1}{2} n^2$ (A)

- $T(n) = T(n/2) + 1 \in O(\log_2 n)$

$$\left.\begin{array}{cc} n & 1 \\ | & \\ n/2 & 1 \\ | & \\ n/4 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{array}\right\} \log_2 n$$

- Th. master $\quad a = 1$, $b = 2$, $f(n) = 1$ $\quad 1 \boxed{=} n^{\log_2 1} = 1$

$$\overset{\text{caz } 2}{\Longrightarrow} T(n) \in \Theta(\log_2 n)$$

- $T(n) = T\left(\frac{7n}{10}\right) + 12$

$a = 1$, $b = \frac{10}{7}$, $f(n) = 12$ $\quad 12 \boxed{=} n^{\log_{\frac{10}{7}} 1} = 1$

$$\overset{\text{caz } 2}{\Longrightarrow} T(n) \in \Theta(\log n)$$

$T(n) = T(n-1) + n \quad \in O(n^2)$

$$\begin{array}{c|c} n & n \\ | & \\ n-1 & n-1 \\ | & \\ n-2 & \vdots \end{array}$$

$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + O(n) \quad \in O(n \log n)$