Tema colectivă 5

Fie A, I mulțimi

$(B_i)_{i\in I}$ o familie de mulțimi

Demonstrație:

1) • $A \times (\bigcup_{i\in I} B_i) = \bigcup_{i\in I} (A\times B_i)$

"$\subseteq$" $(a,b) \in A \times (\bigcup_{i\in I} B_i) \Rightarrow \begin{cases} a \in A \\ b \in \bigcup_{i\in I} B_i \Rightarrow \exists\, i_k \text{ cu } b \in B_{i_k} \end{cases} \Bigg| \Rightarrow$

$\Rightarrow (a,b) \in (A \times B_{i_k}) \subseteq \bigcup_{i\in I} (A\times B_i)$ (1)

"$\supseteq$" $(a,b) \in \bigcup_{i\in I} (A\times B_i) \Rightarrow \exists\, i_k \in I$ cu $(a,b) \in A \times B_{i_k} \Rightarrow \begin{cases} a \in A \\ b \in B_{i_k} \\ \Rightarrow b \in \bigcup_{i\in I} B_i \end{cases} \Bigg| \Rightarrow$

$\Rightarrow (a,b) \in A\times(\bigcup_{i\in I} B_i)$ (2)

(1) + (2) $\Rightarrow A\times(\bigcup_{i\in I} B_i) = \bigcup_{i\in I} (A\times B_i)$

* caz special $I = \emptyset$

$\bigcup_{i\in\emptyset} B_i = \emptyset \Rightarrow A \times (\bigcup_{i\in\emptyset} B_i) = \emptyset$

$\bigcup_{i\in\emptyset} (A\times B_i) = \emptyset$ $\Bigg| \Rightarrow A\times(\bigcup_{i\in\emptyset} B_i) = \bigcup_{i\in\emptyset} (A\times B_i)$

• $\left(\bigcup_{i\in I} B_i\right)\times A = \bigcup_{i\in I} (B_i\times A)$

"$\subseteq$" $(a,b) \in \left(\bigcup_{i\in I} B_i\right)\times A \Leftrightarrow \left.\begin{cases} a \in \bigcup_{i\in I} B_i \Rightarrow a\in B_i, \forall i\in I \\ b\in A \end{cases}\right| \Leftrightarrow$

$\Leftrightarrow (a,b)\in B_i\times A, \forall i\in I \Leftrightarrow (a,b)\in \bigcup_{i\in I}(B_i\times A)$ (i)

"$\supseteq$" $(a,b)\in \bigcup_{i\in I}(B_i\times A) \Leftrightarrow (a,b)\in B_i\times A, \forall i\in I \Leftrightarrow (\forall i\in I)(a\in B_i$

și $b\in A) \Leftrightarrow (\forall i\in I)(a\in B_i)$ și $b\in A \Leftrightarrow a\in \bigcup_{i\in I} B_i$ și $b\in A$

$\Leftrightarrow (a,b)\in \left(\bigcup_{i\in I} B_i\right)\times A$ (ii)

(i) + (ii) $\Rightarrow \left(\bigcup_{i\in I} B_i\right)\times A = \bigcup_{i\in I}(B_i\times A)$

* caz special $I=\emptyset$

$\left.\begin{array}{l} \bigcup_{i\in\emptyset} B_i = \emptyset \Rightarrow \left(\bigcup_{i\in\emptyset} B_i\right)\times A = \emptyset \\ \bigcup_{i\in\emptyset}(B_i\times A) = \emptyset \end{array}\right| \Rightarrow \left(\bigcup_{i\in\emptyset} B_i\right)\times A = \bigcup_{i\in\emptyset}(B_i\times A).$

2). $\cdot$ $A\times(\bigcap_{i\in I} B_i) = \bigcap_{i\in I} (A\times B_i)$

"$\subseteq$" $(a,b)\in A\times(\bigcap_{i\in I} B_i) \Leftrightarrow \begin{cases} a\in A \\ b\in \bigcap_{i\in I} B_i \Rightarrow b\in B_i, \forall i\in I \end{cases} \Bigg| \Leftrightarrow (a,b)\in A\times B_i \; \forall i\in I$

$\Leftrightarrow (a,b)\in \bigcap_{i\in I} (A\times B_i)$ (i)

"$\supseteq$" $(a,b)\in \bigcap_{i\in I} (A\times B_i) \Leftrightarrow (a,b)\in A\times B_i, \forall i\in I \Leftrightarrow (\forall i\in I)(a\in A$ şi

$b\in B_i) \Leftrightarrow a\in A$ şi $(\forall i\in I)(b\in B_i) \Leftrightarrow a\in A$ şi $b\in \bigcap_{i\in I} B_i \Leftrightarrow (a,b)\in$

$\in A\times(\bigcap_{i\in I} B_i)$ (ii)

(i) + (ii) $\Rightarrow A\times(\bigcap_{i\in I} B_i) = \bigcap_{i\in I} (A\times B_i)$

* caz special $I=\emptyset$

$\bigcap_{i\in\emptyset} B_i \subseteq \bigcup_{i\in\emptyset} B_i = \emptyset \Rightarrow A\times(\bigcap_{i\in\emptyset} B_i)=\emptyset$

$\bigcap_{i\in\emptyset} (A\times B_i) \subseteq \bigcup_{i\in\emptyset} (A\times B_i)=\emptyset$

$\Bigg| \Rightarrow A\times(\bigcap_{i\in\emptyset} B_i) = \bigcap_{i\in\emptyset} (A\times B_i)=\emptyset$

$\cdot$ $(\bigcap_{i\in I} B_i)\times A = \bigcap_{i\in I} (B_i\times A)$

"$\subseteq$" $(a,b)\in(\bigcap_{i\in I} B_i)\times A \Leftrightarrow \begin{cases} a\in \bigcap_{i\in I} B_i \Rightarrow a\in B_i, \forall i\in I \\ b\in A \end{cases} \Bigg| \Leftrightarrow$

$\Leftrightarrow (a,b)\in B_i\times A, \forall i\in I \Leftrightarrow (a,b)\in \bigcap_{i\in I} (B_i\times A)$

"$\supseteq$" $(a,b) \in \bigcap_{i\in I} (B_i \times A) \Leftrightarrow (a,b) \in B_i \times A, \forall i \in I \Leftrightarrow (\forall i \in I)(a \in B_i$ și

$b \in A) \Leftrightarrow (\forall i \in I)(a \in B_i)$ și $b \in A \Leftrightarrow a \in \bigcap_{i\in I} B_i$ și $b \in A \Leftrightarrow (a,b) \in (\bigcap_{i\in I} B_i) \times A$

* caz special $I = \emptyset$

$$\left.\begin{aligned} &\bigcap_{i\in\emptyset} B_i = \emptyset \Rightarrow (\bigcap_{i\in\emptyset} B_i) \times A = \emptyset \\ &\bigcap_{i\in\emptyset} (B_i \times A) \subseteq \bigcup_{i\in\emptyset} (B_i \times A) = \emptyset \end{aligned}\right| \Rightarrow (\bigcap_{i\in\emptyset} B_i) \times A = \bigcap_{i\in\emptyset} (B_i \times A) = \emptyset$$