[Seminar 1]

Ex 1. Să se rezolve:

a) $\begin{cases} 2x - y = 1 \\ 4x - 2y = 2 \end{cases}$

$y = 2x - 1$

$4x - 2(2x-1) = 2$

$2 = 2$

$Sol = \{ (x, 2x-1) \mid x \in \mathbb{R} \}$

b). $\begin{cases} 2x - y = 1 \\ 4x - 2y = 1 \end{cases}$ sistem incompatibil

! Metoda Gauss

a') $\left(\begin{array}{cc|c} 2 & -1 & 1 \\ 4 & -2 & 2 \end{array}\right) \xrightarrow{L_2 - 2L_1} \left(\begin{array}{cc|c} 2 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right) \rightarrow (2 \ -1 \mid 1) \Leftrightarrow 2x - y = 1$
$\Rightarrow y = 2x - 1$

b) $\left(\begin{array}{cc|c} 2 & -1 & 1 \\ 4 & -2 & 1 \end{array}\right) \xrightarrow{L_2 = L_2 - 2L_1} \left(\begin{array}{cc|c} 2 & -1 & 1 \\ 0 & 0 & -1 \end{array}\right) \Rightarrow 0 = -1$ (sistem incompatibil)

c). $\begin{cases} 2x - y + 3z = 1 \\ x + y - 3z = 3 \end{cases}$

$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 1 & 1 & -3 & 3 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -3 & 3 \\ 2 & -1 & 3 & 1 \end{array}\right) \xrightarrow{L_2 = L_2 - 2L_1} \left(\begin{array}{ccc|c} 1 & 1 & -3 & 3 \\ 0 & -3 & 9 & -5 \end{array}\right) \xrightarrow{L_2 = L_2 \cdot -\frac{1}{3}}$

$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -3 & 3 \\ 0 & 1 & -3 & \frac{5}{3} \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{4}{3} \\ 0 & 1 & -3 & \frac{5}{3} \end{array}\right) \Rightarrow x = \frac{4}{3}$

$y - 3z = \frac{5}{3}$

$y = \frac{5}{3} + 3z$

$Sol = \{ (\frac{4}{3}, \frac{5}{3} + 3z, z) \mid z \in \mathbb{R} \}$

d). $\begin{cases} x+y+z=1 \\ 2x-y+3z=0 \\ x-2y+2z=-1 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -1 & 3 & 0 \\ 1 & -2 & 2 & -1 \end{array}\right) \xrightarrow[L_3 = L_3 - L_1]{L_2 = L_2 - 2L_1} \left(\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & -3 & 1 & -2 \\ 0 & -3 & 1 & -2 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & 1 & -2 \end{array}\right) \xrightarrow{L_1 = 3L_1} \left(\begin{array}{ccc|c} 3 & 3 & 3 & 3 \\ 0 & -3 & 1 & -2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 3 & 0 & 4 & 1 \\ 0 & -3 & 1 & -2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & \frac{4}{3} & \frac{1}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{2}{3} \end{array}\right)$$

$$Sol = \left\{ \left(\frac{1}{3} - \frac{4}{3}z, \frac{2}{3} + \frac{1}{3}z, z\right) \middle| z \in \mathbb{R} \right\} = \left\{ \left(\frac{1}{3}, \frac{2}{3}, 0\right) + \left(-\frac{4z}{3}, \frac{1z}{3}, z\right) \middle| z \in \mathbb{R} \right\}$$
$$= \left\{ \left(\frac{1}{3}, \frac{2}{3}, 0\right) + \vec{x} \,\middle|\, \vec{x} \in Sol_0 \right\}$$

d'). $\begin{cases} x+y+z=0 \\ 2x-y+3z=0 \\ x-2y+2z=0 \end{cases}$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & -2 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & -3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3 & 3 & 3 \\ 0 & -3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3 & 0 & 4 \\ 0 & -3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & \frac{4}{3} \\ 0 & 1 & -\frac{1}{3} \end{pmatrix}$$

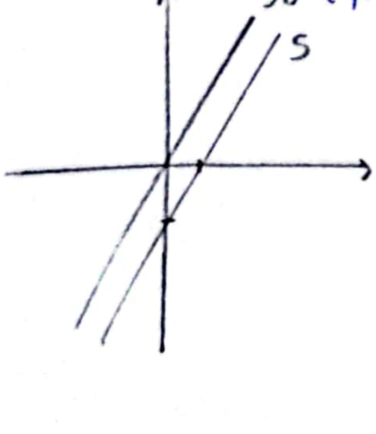
$$Sol_0 = \left\{ \left(-\frac{4z}{3}, \frac{1z}{3}, z\right) \middle| z \in \mathbb{R} \right\}$$

* $S = 2x - y = 1$

$S_0 = 2x - y = 0$

$Sol = \{ (x,y) \mid 2x - y = 1 \} \Rightarrow Sol = \{ (\frac{1}{2}, 0) + (x,y) \mid 2x - y = 0 \}$

S_0 (paralel cu S, trece prin origine)



2).
$$\begin{cases} x + y + z = 1 \\ 2x - y - z = 3 \\ 3x + y - z = 1 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -1 & -1 & 3 \\ 3 & 1 & -1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -3 & 1 \\ 0 & -2 & -4 & -2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -4 & -2 \\ 0 & -3 & -3 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -3 & -3 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 3 & 4 \end{array}\right) \to$$

$$\to \left(\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \frac{4}{3} \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{4}{3} \\ 0 & 1 & 0 & -\frac{5}{3} \\ 0 & 0 & 1 & \frac{4}{3} \end{array}\right) \quad Sol = \left\{ \left(\frac{4}{3}, -\frac{5}{3}, \frac{4}{3} \right) \right\}$$

* $A \in M_{m,n}(\mathbb{R})$

$\vec{x} \in M_{n,1} = \mathbb{R}^n \quad \vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

$A \cdot \vec{x} = \vec{y} \in M_{m,1}(\mathbb{R}) = \mathbb{R}^m$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 0 & -1 & -1 \end{pmatrix}$$
$$\qquad\qquad\qquad\qquad\qquad\quad \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ b_1 & b_2 & b_3 \end{array}$$

$$AB = \begin{pmatrix} -1 & 4 & 6 \\ 1 & 2 & 3 \\ 0 & 9 & 12 \end{pmatrix}$$
$$\qquad \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ c_1 & c_2 & c_3 \end{array}$$

$A \cdot \vec{x} = c_1$

$A \cdot b_1 = c_1$

$A \cdot b_2 = c_2$

$A \cdot b_3 = c_3$

$[A \cdot A^{-1} = A^{-1} \cdot A = I_n]$

Sistem liniar $A \cdot \vec{x} = b \qquad A \in M_{m,n}$
$\vec{x} \in \mathbb{R}^n, \; b \in \mathbb{R}^m$

$A \in M_n(\mathbb{R}) \; \vec{x}, n \in \mathbb{R}^n$

$\exists A^{-1} \Rightarrow \vec{x} = A^{-1} b$

$[\text{Gauss } (A \mid b) \to (I_n \mid A^{-1} b)]$

$A^{-1} = col(\vec{x_1}, \dots, \vec{x_n})$

$A\vec{x_1} = \begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix} \quad A\vec{x_2} = \begin{pmatrix}0\\1\\\vdots\\0\end{pmatrix} \quad A\vec{x_n} = \begin{pmatrix}0\\0\\\vdots\\0\end{pmatrix}$

* Metoda Gauss pt. inversă

$A \in \mathcal{M}_n(\mathbb{R})$.

$\left[(A \mid I_n) \to (I_n \mid A^{-1})\right]$

Să se calculeze A^{-1} pt.

a) $A = \begin{pmatrix}1 & 1\\2 & 3\end{pmatrix}$

$$\left(\begin{array}{cc|cc}1 & 1 & 1 & 0\\2 & 3 & 0 & 1\end{array}\right) \longrightarrow \left(\begin{array}{cc|cc}1 & 1 & 1 & 0\\0 & 1 & -2 & 1\end{array}\right) \to \left(\begin{array}{cc|cc}1 & 0 & 3 & -1\\0 & 1 & -2 & 1\end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix}3 & -1\\-2 & 1\end{pmatrix}$$

b) $A = \begin{pmatrix}1 & 2 & 1\\3 & 1 & 5\\-2 & -1 & 2\end{pmatrix}$

$$\left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\3 & 1 & 5 & 0 & 1 & 0\\-2 & -1 & 2 & 0 & 0 & 1\end{array}\right) \xrightarrow[L_3+2L_1]{L_2-3L_1} \left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\0 & -5 & 2 & -3 & 1 & 0\\0 & 3 & 4 & 2 & 0 & 1\end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc}1 & 2 & 1 & 1 & 0 & 0\\0 & 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} & 0\\0 & 3 & 4 & 2 & 0 & 1\end{array}\right) \xrightarrow{L_3 = \frac{5}{26}L_3} \left(\begin{array}{ccc|ccc}1 & 0 & \frac{9}{5} & -\frac{1}{5} & \frac{2}{5} & 0\\0 & 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} & 0\\0 & 0 & \frac{26}{5} & -\frac{1}{5} & \frac{3}{5} & 1\end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc}1 & 0 & \frac{9}{5} & -\frac{1}{5} & \frac{2}{5} & 0\\0 & 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} & 0\\0 & 0 & 1 & \frac{1}{26} & \frac{3}{26} & \frac{5}{26}\end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc}1 & 0 & 0 & -\frac{7}{26} & \frac{5}{26} & -\frac{9}{26}\\0 & 1 & 0 & \frac{14}{26} & \frac{4}{26} & \frac{-2}{26}\\0 & 0 & 1 & \frac{1}{26} & \frac{3}{26} & \frac{5}{26}\end{array}\right)$$

$$A^{-1} = \cancel{4}\ \frac{1}{26}\begin{pmatrix}-7 & 5 & -9\\16 & -4 & +2\\1 & 3 & 5\end{pmatrix}$$

c) $\begin{pmatrix}2 & 1 & 3\\1 & 2 & 1\\1 & -1 & 2\end{pmatrix}$

$$\left(\begin{array}{ccc|ccc}2 & 1 & 3 & 1 & 0 & 0\\1 & 2 & 1 & 0 & 1 & 0\\1 & -1 & 2 & 0 & 0 & 1\end{array}\right) \to \left(\begin{array}{ccc|ccc}1 & -1 & 2 & 0 & 0 & 1\\1 & 2 & 1 & 0 & 1 & 0\\2 & 1 & 3 & 1 & 0 & 0\end{array}\right) = \left(\begin{array}{ccc|ccc}1 & -1 & 2 & 0 & 0 & 1\\0 & 3 & -1 & 0 & 1 & -1\\0 & 3 & -1 & 1 & 0 & -2\end{array}\right) \to \left(\begin{array}{ccc|ccc}1 & -1 & 2 & 0 & 0 & 1\\0 & 3 & -1 & 0 & 1 & -1\\0 & 0 & 0 & 1 & -1 & -1\end{array}\right)$$

$\Rightarrow \nexists A^{-1}$

- Temă bonus: scrieți un program ca input $A \to \det(A)$ ⟨ clasic / gauss. Calculați timpul de execuție și faceți un grafic pentru cele 2 metode