(a)

```
void f1 (int n)
{ int i=2
  while ( i < n) {
      /* sth O(1) */
      i = i * i
  }
}
```

$$\sum_{i=0}^{\log\log n} O(1) = \theta(\log\log n).$$

$2, 2^2, 2^4, 2^8, \ldots, 2^{2^{k-1}}$, so the loop executes k times which $2^{2^{k-1}} < n$ ($k < \log(\log n) + 1$)

Therefore, the runtime is $\sum_{i=0}^{\log\log n} O(1) = \theta(\log\log n)$.

(b).

```
void f2( int n)
{ for (int i=1 ; i<=n ; i++) {
      if ( ( i% (int) sqrt (n)) == 0) {
          for (int k=0 ; k < pow(i,3) ; k++) {
              /* sth O(1) */
          }
      }
  }
}
```

$$\sum_{i=1}^{n} O(1)$$

$$\sum_{k=0}^{(pow(i,3)-1)} O(1) = \theta(i^3)$$

if statement

$\cancel{\theta(i^3)}$, $i = \sqrt{n}, 2\sqrt{n}, 3\sqrt{n} \ldots\ldots \sqrt{n}\sqrt{n}$

$\theta(i^3) = (\sqrt{n})^3, (2\sqrt{n})^3, (3\sqrt{n})^3 \ldots\ldots n^3$

$$T(n) = \sum_{i=1}^{n} \theta(1) + \sum_{i=1}^{\sqrt{n}} \theta(i\sqrt{n})^3$$

$$= \theta(n) + (\sqrt{n})^3 (1 + 2^3 + 3^3 + \ldots\ldots \sqrt{n}^3)$$

$$= \theta(n) + (\sqrt{n})^3 \left(\frac{\sqrt{n}(\sqrt{n}+1)}{2}\right)^2$$

$$= \theta(n) + n^{\frac{3}{2}} \cdot \theta(n^2).$$

$$= \cancel{\theta(n + n^{\frac{7}{2}})} = \theta(n^{\frac{7}{2}})$$

```
(c) for(int i=1; i<=n; i++){
      for(int k=1; k<=n; k++){
        if(A[k] == i){                          ] Θ(1)
          for(int m=1; m<=n; m=m+m){            ] Θ(log n)
            // sth O(1)
          }
        }
      }
    }
```

if statement — depends on how many time if statement is triggered.

$$T(n) = \sum_{i=1}^{n}\sum_{k=1}^{n}\Theta(1) + \sum_i \Theta(\log n)$$

In the worst case, n times can "if" statement be triggered, since each content of A[] array can be equal to at most one i

~~The~~ Therefore, $T(n) = \Theta(n^2) + \Theta(n\log n) = \Theta(n^2)$

```
(d) int f(int n)
{   int *a = new int[10];
    int size = 10;
    for (int i=0; i<n; i++)
    {   if (i == size)                          ] Θ(1)
        {   int newsize = 3 * size/2;
            int *b = new int[newsize];
            for (int j=0; j<size; j++) b[j] = a[j];       Θ(size)
            delete [] a;
            a = b;
            size = newsize;
        }
        a[i] = i*i
    }
}
```

if statement

$$T(n) = \sum_{i=0}^{n-1} \theta(1) + \left(\theta(10) + \theta\left(10 \cdot \tfrac{3}{2}\right) + \ldots\ldots \theta\left(10 \cdot \left(\tfrac{3}{2}\right)^{\log_{\frac{3}{2}} \frac{n}{10}}\right)\right)$$

$$= \theta(n) + \sum_{i=0}^{\log_{\frac{3}{2}} \frac{n}{10}} \theta\left(10 \cdot \left(\tfrac{3}{2}\right)^{i}\right)$$

$$= \theta(n) + \theta\left(\frac{10 \cdot \left(\frac{n}{10} - 1\right)}{\frac{3}{2} - 1}\right)$$

$$= \theta(n) + \theta(2n)$$

$$= \theta(n)$$