Geometry Individual Answers and Solutions
March Regional Competition

1. D	7. D	13. B	19. B	25. D
2. A	8. C	14. B	20. C	26. A
3. A	9. A	15. B	21. C	27. B
4. D	10. E	16. A	22. D	28. A
5. C	11. A	17. A	23. D	29. B
6. C	12. B	18. D	24. D	30. D

Solutions:

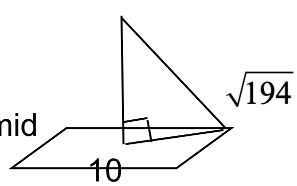
1. **D.** The height of the triangle will be 5, from the y-coordinate of T. The base will have length (x-1) from the x-coordinates of Q and R.
$\frac{1}{2}(5)(x-1)=45$. x=19.

2. **A.** $(2x+50)+(x-5)=180$. x=45. So $m\angle NAP=45-5=40$. Vertical angle $\angle LAE$ has the same measure so $3y+4=40$. y=12.

3. **A.** $180((n+1)-2)-180(n-2)$ simplifies to 180 degrees.

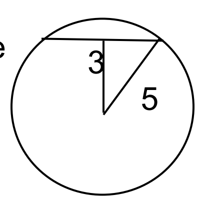
4. **D.** Using the Pythagorean Theorem, (ΔRTU is a multiple of a 3-4-5 triple) we get TR=28.
$m\angle STR=360-90-150=120$. In ΔSTR drop the height from T to $\overline{SR}$. This creates two 30-60-90 triangles, each with hypotenuse 28. RS is then twice the long leg, or $28\sqrt{3}$.

5. **C.** Consider the right triangle with one leg the height of the pyramid and the other leg half of the square's diagonal. $\left(5\sqrt{2}\right)^2+h^2=194$. h=12.



So $V=\frac{1}{3}(100)(12)=400$.

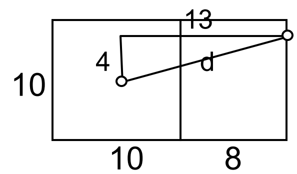
6. **C**. One chord will have length 2(4) and the other will have length 2(3) for a difference of 2.



7. **D.** The shaded region is $4\pi-\pi=3\pi$. The unshaded region is 9π minus the shaded area which gives 6π. The ratio of unshaded to shaded it 6:3=2:1.

8. **C.** A regular quadrilateral is a square. Perimeter 200 gives side length 50. Diagonal has length $50\sqrt{2}$

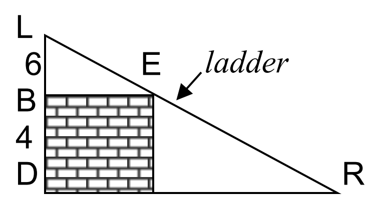
9. **A.** Draw only the two sides involved.



To get from the center of the ceiling to 1 ft from the edge we have a right triangle with legs (5-1) and (5+8). This gives shortest distance $\sqrt{185}$.

10. **E.** Consider the triangle with vertices of Sam, R and T. Let Sam be point M. RM=40 and MT=52. So by the triangle inequality theorem, $12<RT<92$. Since RT is an integer, the least it could be is 13. So the space labels are 6.5 feet apart. And RU=3(6.5) = 19.5 feet.

11. **A.** Consider triangle LDR and LBE which are similar (below). LE=$2\sqrt{13}$.
$\frac{6}{10}=\frac{2\sqrt{13}}{LR}$. LR=$\frac{10}{3}\sqrt{13}$



12. **B.** Area=$\frac{1}{2}d_1d_2=\frac{1}{2}(x)(x+8)=120$
$x(x+8)=240$. You can use algebra or find two factors with a difference of 8.
$x^2+8x-240=0$. $(x+20)(x-12)=0$.
x=12. The longest diagonal has length x+8 = 20.

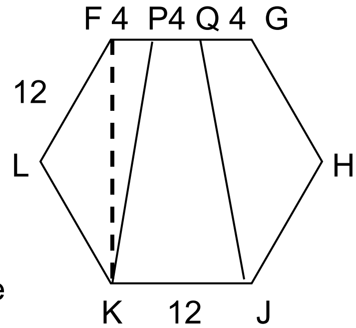
13. **B.** For center P, ΔPRS is equilateral. The sector is $\frac{1}{6}(18\bullet18)\pi = 54\pi$. Subtract the triangle with area $\frac{18\bullet18}{4}\sqrt{3} = 81\sqrt{3}$ to get choice B.

14. **B.**

$4\pi r^2 = K$. $\frac{4}{3}\pi r^3 = 12K$ so

$\frac{\cancel{4}}{3}\cancel{\pi}r^3 = 12\left(\cancel{4\pi}r^2\right)$. r=36

15. **B.** Quadrilateral PQJK is a trapezoid so its area is $\frac{1}{2}h(4+12)$. The height is the base of ΔLFK which is the base of an isosceles triangle with legs 12 and vertex angle 120 degrees. Drop the height of the triangle from L to get two 30-60-90 triangles. Each has long leg $6\sqrt{3}$ and so $h = 12\sqrt{3}$. $\frac{1}{2}(12\sqrt{3})(4+12)$ = $96\sqrt{3}$

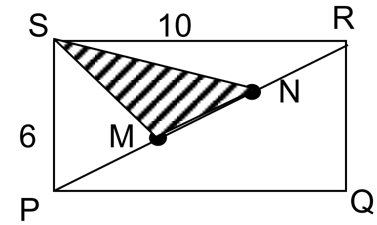


16. **A.** $V = \pi(3^2)12$ for the cylinder. The spheres are each $\frac{4}{3}\pi(3)^3$ in volume.

$$\frac{\pi(3^2)12}{(4\pi/3)(3^3)} = \frac{(3^2)12}{(3^3)}\bullet\frac{3}{4}\frac{\pi(3^2)12}{(4\pi/3)(3^3)} = 3.$$

17. **A.** $4(6) = x(11-x)$. $x^2 - 11x + 24 = 0$. $(x-8)(x-3) = 0$. So the segments have lengths 3 and 8. $|PS - RP|$=5

18. **D.**



The area of ΔPSQ is half of that of the rectangle, so area is 30. ΔPMN has the same height as ΔPSQ, if we consider the base $\overline{SQ}$, and ΔPMN has 1/3 of the base length. So the area of ΔPMN is 1/3 of the area of ΔPSQ, which gives area 10.

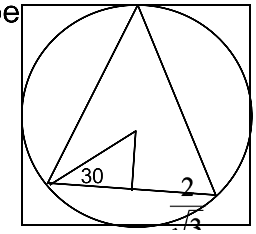
19. **B.** The area of the original triangle is $\frac{1}{2}bh$. The area of the new triangle is $\frac{1}{2}(1.1b)(0.95h) = 0.5225bh$. The decrease is $(0.5225 - 0.5)bh$ = 0.0225bh. Divide by the original to get 0.045. Easier would be to assume a base of 2 and height of 1 to get area 1. Then compare a triangle with base 2.2 and height 0.95 to get 1.045. This is an increase of 0.045 which is 4.5%

20. **C.** Let the sides be 3:4:6 = ST:SR:RT and SR=4k, RT=6k and ST=3k. ST=12 so sides are now 12, 16, 24. Using the angle bisector similarity formula of $\frac{16}{SP} = \frac{24}{12-SP}$. $2(12-SP) = 3(SP)$.

$SP = \frac{24}{5}$.

21. **C.** $90 - a = \frac{1}{2}(180-a) - 7$. $\frac{1}{2}a = 7$. a = 14.

22. **D.** The radius of the circle would be $\frac{4}{\sqrt{3}}$ and the side of the square would be twice that, or $\frac{8\sqrt{3}}{3}$



23. **D.** Perimeter 54 gives semi-perimeter 27. Let sides be 20, x and (34-x). $x+20>34-x$ gives x>7 by the Triangle Inequality Theorem, as well as 54-x>x x < 27. So we can list possible side lengths:
20, 8, 26 for x=8. 20, 9, 25 for x=9
20, 10, 24 for x=10. 20, 11, 23 x=11
20, 12, 22 for x=12.
20, 13, 21 for x=13.
x=14 and 17 gives an isosceles.
20, 15, 19 for x=15.
20, 16, 18 for x=16.
After that, we repeat triangles.
Using Heron's Formula, area is $\sqrt{27(27-20)(27-x)(27-(34-x))}$ = $\sqrt{27(7)(27-x)(x+7)}$ = $3\sqrt{21(27-x)(x+7)}$. Now we try x values above (or just the choices on A through D on the test) to see when we get area that is an integer. x=8 gives $3\sqrt{21(19)(15)}$, not an integer, for example. The only x-value that gives an integer for area is 13.

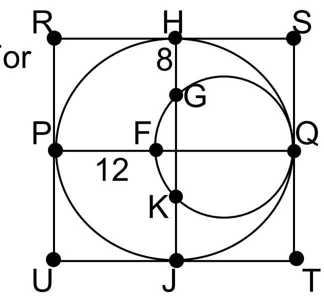
24. **D.** Using Geometric Mean formulas we have $ST=\sqrt{RT(PT)}$. $25=6PT$.
PT = $\frac{25}{6}$, so RP= $6-\frac{25}{6}=\frac{11}{6}$. SP = $\sqrt{\left(\frac{11}{6}\right)\left(\frac{25}{6}\right)}=\frac{5\sqrt{11}}{6}=\frac{a\sqrt{b}}{c}$. So in this particular case, a+c=b.

25. **D.** The contrapositive is "If not q then not p." The converse of this is "If not p then not q."

26. **A.** $\frac{a}{10}=\frac{a+b}{15}$. $\frac{a}{2}=\frac{a+b}{3}$. $3a=2a+2b$. $a=2b$

27. **B.** The area of ΔRTP is $\frac{1}{2}(10)(12)=60$
The area of the rest of the rectangle is 120-60=60. So the shaded to the unshaded is 60:60=1:1. The division caused by P is irrelevant.

28. **A.** Let the square have side length 2x. For center C, (GC)(CK)= (FC)(CQ). (x-8)(x-8) = (x-12)(x) since we have intersecting chords of the small circle.



$x^2-16x+64=x^2-12x$. 4x=64 and x=16. So the square has side 32.

29. **B.** The angles can be equal due to vertical angle positives, or due to angles of parallel lines. The two angles can be supplementary due to being a linear pair or by the parallel lines. So if they are equal, x=36. If they are supplementary x=33. II is not possible.

30. **D.** The radii to the points of tangency are perpendicular. Due to the radii then being perpendicular to the same line, they are parallel and the triangles shown above are similar. $\frac{6}{3}=\frac{x}{12-x}$ PD=x. $x=8$.
RD=10 and DS=5 for RS=15.