Problem Solving Homework (Week 7)

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JH Chapter 4

4.2.1.4

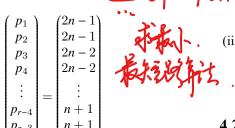
Proof. Suppose that the cost obtained by such algorithm is cost(GMS(I)), and the optimal cost is $Opt_{MS}(I)$, then we know that:

$$cost(GMS(I)) - Opt_{MS}(I) \le p_k \le Opt_{MS}(I)$$
$$\therefore \frac{cost(GMS(I)) - Opt_{MS}(I)}{Opt_{MS}(I)} \le 1$$

Consequently, Graham's Algorithm is a 2-approximation algorithm for MS, too.

4.2.1.5

One possible input instance can be:



These 2n + 1 tasks make $R_{GMS} = 3/4$, which is largest.

4.2.3.3

Proof. (i) dist(G, c)

Since in
$$L_{\delta}$$
, $c(\{u, p\}) + c(\{p, v\}) > c(\{u, v\})$, then:

$$\frac{c(\lbrace u, v \rbrace)}{c(\lbrace u, p \rbrace) + c(\lbrace p, v \rbrace)} < 1, \ \forall u, v, p \text{ allowed}$$

$$\max \left\{ \frac{c(\{u,v\})}{c(\{u,p\}) + c(\{p,v\})} - 1 \right\} < 0$$

Therefore,

$$\max \left\{ 0, \max \left\{ \frac{c(\{u, v\})}{c(\{u, p\}) + c(\{p, v\})} - 1 \right\} \right\} = 0$$

That means that $h_L: L \to \mathbb{R}^{\geq 0}$, and h_L satisfies the first property.

As for the second one, the time complexity for computing h_L is bounded by

$$O(\binom{|V(G)|}{2} \cdot (|V(G)| - 2)) = O(|V(G)|^3)$$

Thus, it is polynomial-time computable.

(ii) $dist_k(G,c)$

Alike what's shown above, the value of the inner maximal function is negative. Hence the value of the outer maximal function is 0. Besides, the time complexity is bounded by

$$O\left(k \cdot \binom{|V(G)|}{2}\right) = O(|V(G)|^2)$$

(iii) The range of parameter k only makes the time complexity increase to $O(|V(G)|^3)$, which is still polynomial.

4.2.3.4

Proof. a) The first property of distance function is ensured by (i). Now consider the second one. Since $h_{index}(u)$ equals the order of u, it can be computed in polynomial time. Therefore, h_{index} is a distance function of \overline{U} according to L_I .