# Problem Solving Homework (Week 1)

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## JH Chapter 2

### Exercise 2.3.1.7

Suppose that $M_G = [a_{ij}]_{i,j=1,2,\cdots,n}$ is the adjacent matrix of a graph $G$, then we can use word:

$$a_{11}a_{12}\cdots a_{1n}a_{21}a_{22}\cdots a_{2n}\cdots a_{n1}a_{n2}\cdots a_{nn}$$

over alphabet $\Sigma_{bool} = \{0,1\}$ to represent graphs.

### Exercise 2.3.1.8

Suppose that $M_G = [a_{ij}]_{i,j=1,2,\cdots,n}$, $a_{ij} \in N$ is the adjacent of a weighted graph $G_w$, then we can use word:

$$\underbrace{11\cdots1}_{a_{11}\text{ times }1} \# \underbrace{11\cdots1}_{a_{12}\text{ times }1} \# \underbrace{11\cdots1}_{a_{nn}\text{ times }1}$$

(If $a_{ij} = 0$, then string $11\cdots1$ is replaced with 0) over alphabet $\Sigma = \{0,1,\#\}$.

### Exercise 2.3.3.8

1. Traverse the given path (a series of vertexes) yielding by some algorithm for HC. Once a vertex is visited, dye it black. In the end, if there is still some vertex(es) left, or any edge on the path is non-exist then yields false. If not, yields true. Since this verifier only traverse all the vertexes, its running time is polynomial.

2. Scan the graph's adjacent matrix, verify that, for the given vertex set $V'$, $M[i][j] \neq 0, i,j \in V'$ and $M[i][j] = 0, i,j \notin V'$.

3. We can use Bron–Kerbosch algorithm to find the maximum clique in the given graph. If $k \geq x$, than the algorithm should yield "yes", otherwise, it should yield "no".

### Exercise 2.3.3.8 (Revised)

(i) A: Input: $(x,c) \in \{0,1,\sharp\}^\star \times \Sigma_{bool}^\star$

  (1) $x$ is a representation of a graph and we represent the set of vertices as $\{x_1, x_2, \ldots, x_n\}$. $c$ codes a sequence of vertices: $I = \{x_{i1}, x_{i2}, \ldots, x_{in}, x_{i1}\}$.

  (2) If each pair of adjacent vertices have an edge, then A accepts $(x,c)$ otherwise A rejects it.

(ii) A: Input: $(x,c) \in \{0,1,\sharp\}^+ \times \Sigma_{bool}^\star$

  (1) $x = u\sharp w \in \{0,1,\sharp\}^+$ where $u \in \{0,1\}^+$ and $w$ represents a graph with $n$ vertices. Let $k = Number(u)$. $c \in \Sigma_{bool}^n$ codes $k$ vertices using $k$ symbols of 1.

  (2) If each edge of the graph is incident to at least one vertex in $c$, then A accepts $(x,c)$, otherwise, A rejects it.

(iii) A: Input: $(x,c) \in \{0,1,\sharp\} \times \Sigma_{bool}^\star$

  (1) $x = u\sharp w \in \{0,1,\sharp\}^+$ where $u \in \{0,1\}^\star$ and $w$ represents a graph. Let $k = Number(u)$. $c$ codes $k$ vertices.

  (2) If each pair of the $k$ vertices has an edge, then A accepts $(x,c)$, otherwise, A rejects.