Problem Solving Homework (Week 7)

161180162 Xu Zhiming

April 24, 2018

JH Chapter 4

4.2.1.4

Proof. Suppose that the cost obtained by such algorithm is $cost(GMS(I))$, and the optimal cost is $Opt_{MS}(I)$, then we know that:

$$cost(GMS(I)) - Opt_{MS}(I) \le p_k \le Opt_{MS}(I)$$

$$\therefore \frac{cost(GMS(I)) - Opt_{MS}(I)}{Opt_{MS}(I)} \le 1$$

Consequently, GRAHAM'S ALGORITHM is a 2-approximation algorithm for MS, too. □

4.2.1.5

One possible input instance can be:

$\sum^{e}(p_i, p_{i+1})$

∴ 求最小.

最短路算法.

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ \vdots \\ p_{r-4} \\ p_{r-3} \\ p_{r-2} \\ p_{r-1} \\ p_r \end{pmatrix} = \begin{pmatrix} 2n-1 \\ 2n-1 \\ 2n-2 \\ 2n-2 \\ \vdots \\ n+1 \\ n+1 \\ n \\ n \\ n \end{pmatrix}$$

These $2n+1$ tasks make $R_{GMS} = 3/4$, which is largest.

4.2.3.3

Proof. (i) $dist(G, c)$

Since in L_δ, $c(\{u,p\}) + c(\{p,v\}) > c(\{u,v\})$, then:

$$\frac{c(\{u,v\})}{c(\{u,p\}) + c(\{p,v\})} < 1, \ \forall u, v, p \text{ allowed}$$

$$\max\left\{\frac{c(\{u,v\})}{c(\{u,p\}) + c(\{p,v\})} - 1\right\} < 0$$

Therefore,

$$\max\left\{0, \max\left\{\frac{c(\{u,v\})}{c(\{u,p\}) + c(\{p,v\})} - 1\right\}\right\} = 0$$

That means that $h_L : L \to \mathbb{R}^{\ge 0}$, and h_L satisfies the first property.

As for the second one, the time complexity for computing h_L is bounded by

$$O\left(\binom{|V(G)|}{2} \cdot (|V(G)| - 2)\right) = O(|V(G)|^3)$$

Thus, it is polynomial-time computable.

(ii) $dist_k(G, c)$

Alike what's shown above, the value of the inner maximal function is negative. Hence the value of the outer maximal function is 0. Besides, the time complexity is bounded by

$$O\left(k \cdot \binom{|V(G)|}{2}\right) = O(|V(G)|^2)$$

(iii) The range of parameter k only makes the time complexity increase to $O(|V(G)|^3)$, which is still polynomial.

□

4.2.3.4

Proof. a) The first property of distance function is ensured by (i). Now consider the second one. Since $h_{index}(u)$ equals the order of u, it can be computed in polynomial time. Therefore, h_{index} is a distance function of $\overline{U}$ according to L_I.

b) 在此是距离函数. □

4.2.3.5

Ball, $h(L_\varepsilon)$ 对 x 来说是固定的.

∴ $RA(x)$ 有上界 δ.