Problem Solving Homework(Week 16)

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TC: Chapter 22

22.1-3

```
TRANSPOSE(G)                //Adjacent-list
Build antoher adjacent-list Adj_t//Transposed graph is stored in Adj_t
        for i from 1 to G.V
                for every v in Adj[i]//Original graph is stored in Adj
                        insert v into the front of Adj_t[i]
```

Running time:$O(|V|+|E|)$

```
TRANSPOSE(G)                //Adjacent-matrix
for i from 1 to G.V //V is the number of vertices in G
        for j from 1 to G.V
                G_t[i][j]=G[j][i]//G_t is the transposed graph
```

Running time: $O(|V|^2)$ **22.1-8**

$O(1)$. In this way, we can't easily find the vertices that are adjacent to one vertex, namely, we can only determine whether two vertices are adjacent or not, but can't find a vertex's neighbors.

22.2-3

Proof. In the BFS given by the textbook, the three colors: WHITE, GRAY, and BLACK, individually represents a state of a vertex. White for unprocessed, gray for process-needed, black for processed. We can learn from the code that gray isn't of vital importance. Since if we find a white vertex, we just dye it gray and enqueue it. Gray means a intermediate state in order to help us understand. But to the program, this state doesn't have a practical meaning. Because all white vertices are enqueued if discovered. All vertices still in the queue are "gray". Gray is not considered by our code. Therefore, we can remove the color "gray", use only one bit to represent WHITE(unprocessed) and BLACK(processed). □

22.2-4

All the vertices are enqueued once, and all their neighbors are explored. Therefore, the algorithm "visits" each vertex twice. The running time is $O(|V|^2)$.

22.2-5

Proof. Suppose now we are exploring vertex v's adjacent-list $Adj[u]$. For every vertex in $Adj[v]$, if they haven't been discovered yet. Their property d is $1 + v.d$. So whatever the order in a adjacent-list is, the white vertices among them are always assigned the same d. □

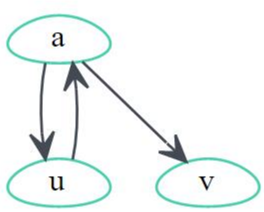
22.3-6

Proof. □

22.3-7

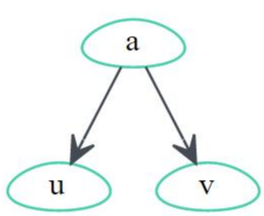
```
NONRECURSIVE-DFS(G)
for each vertex u in G.V
        u.color=WHITE
        u.pi=NIL
        time=0
for each vertex u in V
        if u.color==WHITE
        DFS-VISIT(u)
DFS-VISIT(u)
time=time+1
u.d=time
u.color=GRAY
Let s be a new stack
s.push(u)
while s.empty is not true
        u=s.top
        isleaf=true
        for each vertex v in Adj[u]
                if v.color==WHITE
                        v.color=GRAY
                        v.pi=u
                        time=time+1
                        v.d=time
                        s.push(v)
                        isleaf=false
                        break
        if isleaf==true
        u.color=BLACK
        time=time+1
        u.f=time
        s.pop
```

22.2-8



The counterexample are shown above. Explore from a.

22.2-9



The counterexample are shown above. Explore from a. The relation between $v.d$ and $v.f$ is not certain.

22.2-12

```
DFS-FIND-COMPONETS(G)
for each vertex u in G.V
          u.color=WHITE
          u.pi=NIL
          u.cc=-1
time=0
component=1     //This variant epresses which component a vertex belong to
for each vertex u in G.V
          if u.color==WHITE
                    DFS-VISIT(G)
DFS-VISIT(G)
time=time+1
u.d=time
u.cc=component  //"component" is assigned to a vertex's "cc" attribute
for each vertex v in G.Adj[u]
          if v.color==WHITE
                    v.pi=u
                    v.cc=u.cc
                    DFS-VISIT(G)
u.color=BLACK
time=time+1
component=component+1   //This component is done, begin to explore another component
u.f=time
u.color=GRAY
/*In this way, vertices are in the same component
iff their "cc" attributes are the same*/
```

22.4-2

```
NUMBER-OF-PATHS(G)
Let G be an undirected connected graph, s, t are vertices in G
for all v in V
          do v.num_p<-0
TOPOLOGICAL-SORT(G)
          s.num_p<-1
for each v in V in topologically sorted order
          do if v.num>s.sum
                    for each u in Adj[v]
                              u.num_p=u.num_p+v.num_p
return u.num_p
```

22.4-3

```
GRAPH-CYCLE(G)
if DFS(G) doesn't␣yield␣back␣egdes
␣␣␣␣␣␣␣␣an␣undirected␣graph␣is␣acycle.
if␣there␣is␣a␣back␣egde
␣␣␣␣␣␣␣␣return␣the␣graph␣contains␣a␣cycle
if␣there␣is␣no␣back␣edge
␣␣␣␣␣␣␣␣return␣the␣graph␣is␣acycle
```

22.5-5

```
STRONG-CONNECTED-COMPONENTS(G)
Consider the vertices in the form of strong connected components, choose a vertex in each c
Traverse the edge in G(V,E) and add edge in G_SCC correspondingly.
```

Total cost: $\Theta(|V|+|E|)$

22.5-7

```
STRONG-CONNECTED-COMPONENTS(G)
COMPONENT(G)
TOPOLOGOICALLY(G)
verify that the sequence of vertices (v[1],v[2],...,v[k]) given by topological sort.
If the vertices form a linear chain
        the original graph is semiconnected
else
        it is not
```

Running time: $\Theta(|V| + |E|)$