Problem Solving Homework (Week 10)

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Min-Weight Perfect Matching

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Start with a dual solution
until perfect matching found in subgraph of tight edges do
    if tight edges have no perfect matching
        find Hall set and modify dual values accordingly,
        expanding the subgraph
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JH Chapter 4

4.3.5.6

图中加0边 运行4.3.5.1

(a) Step 1: Construct a minimal spanning tree T of G according to c.

Step 2: Perform depth-first-search of T from q, and order the vertices such that s is the last one. Let H be the resulting sequence.

Output: The path H.

(b) Step 1: Construct a minimal spanning tree T of G according to c.

Step 2: $S := \{v \in V | \deg_T(v) \text{ is odd}\}$.

Step 3: Compute a minimum-weight perfect matching M on S in G.

$\frac{5}{3} \to \frac{1+\sqrt{5}}{2}$.

Step 4: Create the multigraph $G' = (V, E(T) \cup M)$ and construct an Eulerian tour ω starting from q, ending on s in G'.

Step 5: Construct the path p by removing all repetitions of the occurrences of every vertex in ω

Output: p

4.3.5.11

Proof. First, we prove that it is polynomial. Since Step 1 constructs a minimal spanning tree, this can be done in $O(V^2)$ time. Step 2 performs a depth-first-search, which is of $O(V + E)$. Therefore, the total time complexity is bounded by $O(V^2)$. Hence, it is a polynomial-time algorithm.

Second, we prove it is $2 \cdot (1 + r)$-approximation for

$$(\Sigma_I, \Sigma_O, L, Ball_{r,distance}(L_\Delta), \mathcal{M}, cost, minimum)$$

By modifying the proof of *Theorem 4.3.5.2*, we can easily obtain the proof for this problem.

$$cost(T) \le \sum_{c \in E(T)} c(e) \le cost(H_{Opt})$$

$$cost(W) = 2 \cdot cost(T)$$

$$cost(W) \le 2 \cdot cost(H_{Opt})$$

The (1+r)-triangle inequality tells us:

$$cost(\overline{H}) \le (1 + r) \cdot cost(W)$$

$$\therefore cost(\overline{H}) \le cost(W) \le 2 \cdot (1 + r) \cdot cost(H_{Opt})$$

□

With regard to the instance presented in *Fig. 4.15*, Algorithm 4.3.5.1 is better than CHRISTOFIDES ALGORITHM.

4.3.5.13

Alike what's shown for CHRISTOFIDES ALGORITHM, it is $(r, O(n^{\log_2((1+r)^2)}))$-quasistable for *dist*.

4.32式（4.3.5.12证明过程中）.

$$Cost(T) = \sum_{e \in E(T)} c(e) \le (1+r)^{\lceil \log_2 n \rceil} Cost(H_{opt}).$$

$$Cost(\overline{H}) \le 2 \cdot cost(T)$$

$$= O(n^{\log_2(1+r)}) \cdot cost(H_{opt}).$$