问题求解（二）作业（第四周）

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TC 第四章

4.1-5

```
MAX-SUBARRAY(A[1..n])
max = −∞, tmax = 0, s = NULL, e = NULL, ind = 1
for i = 1to n
    tmax = tmax + A[i]
    if tmax > max
        s = ind
        e = i
        max = tmax
    if tmax < 0
        tmax = 0
        ind = i + 1
return (s, e, max)
```

4.3-3

证明. 设 $T(n) \geq cn\log n$，其中 $c = \min(1/3, T(2)/2)$。则有：

$$\begin{aligned}
T(n) &= 2T(\rfloor n/2\lfloor) + n \geq 2c\lfloor n/2\rfloor\log(\lfloor n/2\rfloor) + n\\
&\geq c(n-1)\log((n-1)/2) + n\\
&= c(n-1)(\log n - 1 - \log(n/(n-1))) + n\\
&= cn\left(\log n - 1 - \log(n/(n-1)) + \frac{1}{c}\right)\\
&\quad - c\log n - 1 - \log(n/(n-1))\\
&\geq cn(\log n - C),\ (C\text{ 为常数})\\
&\geq cn\log n\\
\therefore T(n) &= \Omega(n\log n)
\end{aligned}$$

□

4.4-2

根据递归树猜测 $T(n) = n^2$，下面证明此结论。

证明.

$$\begin{aligned}
T(n) &= T(n/2) + n^2\\
&\leq c(n/2)^2 + n^2\\
&= \left(\frac{c}{4} + 1\right)n^2\\
&\leq Cn^2\ (C\text{ 为常数})
\end{aligned}$$

□

4.5-3

参考主定理的形式，有 $a = 1, b = 2$。则：

$$n^{\log_b a} = n^{\log_2 1} = 1.$$

递归式解为第二种形式，即

$$\Theta(\log n).$$

4-4

a.

$$\begin{aligned}
&\because F_0 = 0, F_1 = 1, F_i = F_{i-1} + F_{i-2}, \forall i \geq 2\\
\therefore \mathcal{F}(z) &= \sum_{i=0}^{\infty} F_i z^i\\
&= F_0 + F_1 + \sum_{i=2}^{\infty}(F_i - 1 + F_{i-2})z^i\\
&= z + z\sum_{i=2}^{\infty} F_{i-1}z^{i-1} + z^2\sum_{i=2}^{\infty} F_{i-2}z^{i-2}\\
&= z + z\sum_{i=1}^{\infty} F_i z^i + z^2\sum_{i=0}^{\infty} F_i z^i\\
&= z + z\mathcal{F}(z) + z^2\mathcal{F}(z).
\end{aligned}$$

b.

$$\because \mathcal{F}(z) - z\mathcal{F}(z) - z^2\mathcal{F}(z) = z$$
$$\therefore \mathcal{F}(z) = \frac{z}{1 - z - z^2}$$
$$\therefore 1 - z - z^2 = (1 - \phi z)(1 - \hat{\phi} z)$$

c.

$$\begin{aligned}\mathcal{F}(z) &= \frac{1}{\sqrt{5}}\left(\frac{1}{1 - \phi z} - \frac{1}{1 - \hat{\phi} z}\right) \\ &= \frac{1}{\sqrt{5}}\left(\sum_{i=0}^{\infty}(\phi z)^i - \sum_{i=0}^{\infty}(\hat{\phi} z)^i\right) \\ &= \sum_{i=0}^{\infty}\frac{1}{\sqrt{5}}\left(\phi^i - \hat{\phi}^i\right)z^i\end{aligned}$$

d.

$$\because \left|\hat{\phi}\right| < 1|$$
$$\therefore \left|\frac{\hat{\phi}^i}{5}\right|(i \ge 2) < \left|\frac{\hat{\phi}}{5}\right| < \frac{1}{2}$$

又因为斐波那契数均为整数，所以 $F_i = \lfloor \phi^i / \sqrt{5} \rceil$.