1. Prove the maximum principle using the Mean Value Theorem(s). If $\Delta u = 0$ on a bounded domain Ω, show that

$$\max_{x\in\Omega} u(x) = \max_{x\in\partial\Omega} u(x) \tag{1}$$

In other words, the max of a harmonic function is attained on its boundary. Hint: Use proof by contradiction.

Here, let us consider our given statement from line 1. We are given the assumption that for a point in a given area, Ω, there is a point that gives us the max u(point).

Here, let us consider the area Ω, where Ω is any closed area. For the sake of coding these graphs, let us assume our boundary assumes the shape of a circle

$$+\ \Omega$$

Now, let us consider a point within this interval, x_0. Here, let us consider point x_0 to be an arbitrary point within our area, Ω:

To begin, let us consider a neighborhood, or ball, around our point, x_0. Here, our ball around x_0, has a radius ϵ, where $\epsilon > 0$, and the area of our ball is a subset of Ω.