

1. Given the conservation law $u_t + [\cos u]_x = 0$, sketch the characteristic curves, where

(a)

$$u(x, 0) = \begin{cases} \frac{\pi}{2} & x < 0 \\ 0 & x \geq 0 \end{cases} \quad (1)$$

(b)

$$u(x, 0) = \begin{cases} \frac{\pi}{6} & x < 0 \\ \frac{\pi}{2} & x \geq 0 \end{cases} \quad (2)$$

2. Solve the following equations using the method of characteristics

(a) $u_t + 7u_x = t \quad u(x, 0) = \sin x$

(b) $u_t + xu_x + 2u = 0 \quad u(x, 0) = x^3$

(c) $u_t + 2xtu_x = u \quad u(x, 0) = 1 - x$

(d) $tu_t + 2xu_x = t \sin(\pi t) \quad u(x, 1) = \cos x$