Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$
$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) \text{ dx}$$
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \text{ dx}$$
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \text{ dx}$$

Laplace's Equation

Recall the Fourier Sine Series:

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) \text{ dx}$$
$$A_n = \frac{2}{L}\int_{0}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) \text{ dx}$$

Recall general solutions: sine, exp, sinh

Fill in the blanks at last step.

Steady State Solution

Here, T is our u at the start or end, creates a line.

$$v(x) = \frac{T_2 - T_1}{L}x + T_1$$
$$u(x,t) = w(x,t) + v(x)$$

Continue with $w(x,t)$ using the interval.

$$w(x,t) = u(x,t) - v(x)$$

Use the appropriate condition $(e.g. u(x,0))$

Continue to solve as usual with w. Once finished,

$$u(x,t) = w(x,t) + v(x)$$

Trig Derivatives

- $f(x) = \sin x \Rightarrow f'(x) = \cos x$
- $f(x) = \cos x \Rightarrow f'(x) = -\sin x$
- $\int \sin x \text{ dx} = -\cos x + C$
- $\int \cos x \text{ dx} = \sin x + C$