1. Given the conservation law $u_t + [\cos u]_x = 0$, sketch the characteristic curves, where

   (a) $u(x,0) = \begin{cases} \frac{\pi}{2} & x < 0 \\ 0 & x \geq 0 \end{cases}$

   (b) $u(x,0) = \begin{cases} \frac{\pi}{6} & x < 0 \\ \frac{\pi}{2} & x \geq 0 \end{cases}$

   Here, we are given the conservation law. If we differentiate on x, we find:

$$u_t - u_x \sin u = 0 \tag{1}$$

   From here, let us use a third variable to solve our equation:

$$\frac{dt}{ds} = 1 \tag{2}$$

$$\frac{dx}{ds} = -\sin u \tag{3}$$

$$\frac{du}{ds} = 0 \tag{4}$$

   From here, we would let $t = s$.

   (a) Here, we would want to consider $x(0) = x_0$ and $u(0) = f(x)$, which is given as $u(x,0)$. If we make this assumption for $\frac{dx}{ds}$, we would get:

$$\frac{dx}{ds} = -\sin u \tag{5}$$

$$\frac{dx}{ds} = \begin{cases} \sin \frac{\pi}{2} & x_0 < 0 \\ -\sin 0 & x_0 \geq 0 \end{cases} \tag{6}$$

   From here, if we evaluate our terms, we get:

$$\frac{dx}{ds} = \begin{cases} -1 & x_0 < 0 \\ 0 & p \geq 0 \end{cases} \tag{7}$$

   Now, if we multiply both sides by ds and integrate, we would get:

$$x(s) = \begin{cases} -s + x_0 & x_0 < 0 \\ x_0 & x_0 \geq 0 \end{cases} \tag{8}$$

   Here, recall $t = s$,

$$x(t) = \begin{cases} -t + x_0 & x_0 < 0 \\ x_0 & x_0 \geq 0 \end{cases} \tag{9}$$

   (b) Now, let us consider $u(x,0)$ for part b) and write for $\frac{dx}{ds}$:

$$\frac{dx}{ds} = \begin{cases} -\sin \frac{\pi}{6} & x_0 < 0 \\ -\sin \frac{\pi}{2} & x_0 \geq 0 \end{cases} \tag{10}$$

When we integrate both sides with respect to their variables, we get:

$$x(s) = \begin{cases} -0.5s + x_0 & x_0 < 0 \\ -s + x_0 & x_0 \geq 0 \end{cases} \tag{11}$$

$$x(t) = \begin{cases} -0.5t + x_0 & x_0 < 0 \\ -t + x_0 & x_0 \geq 0 \end{cases} \tag{12}$$

2. Solve the following equations using the method of characteristics

   (a) $u_t + 7u_x = t \quad u(x,0) = \sin x$

   (b) $u_t + xu_x + 2u = 0 \quad u(x,0) = x^3$

   (c) $u_t + 2xtu_x = u \quad u(x,0) = 1 - x$

   (d) $tu_t + 2xu_x = t\sin(\pi t) \quad u(x,1) = \cos x$