MATH 4491 - Review Exam 1
2/14/2022

1. Find the Fourier series for the following functions on $[-\pi, \pi]$, then draw the (infinite) Fourier series on $[-3\pi, 3\pi]$:
   (a) $f(x) = \cos^2 x$
   (b) $g(x) = \begin{cases} 1, & -\pi < x \leq 0 \\ 0, & 0 < x \leq \pi \end{cases}$
2. Solve $u_{tt} = u_{xx}$ given the following conditions:
   (a) $u(0,t) = u(3,t) = 0$, $u(x,0) = x^2 - 3x$, $u_t(x,0) = 2$
   (b) $u(0,t) = u(1,t) = 0$, $u(x,0) = 4\sin(3\pi x) - 7\sin(15\pi x)$, $u_t(x,0) = 0$
3. Solve $u_t = u_{xx}$ given the following conditions:
   (a) $u(0,t) = u(\pi,t) = 0$, $u(x,0) = 1$ for $0 < x < \pi$
   (b) $u(0,t) = 5$, $u(2,t) = 1$, $u(x,0) = x$ for $0 < x < 2$
4. Solve $u_{xx} + u_{yy} = 0$ given the following conditions:
   (a) $u(x,0) = u(x,3) = 0$, $u_x(0,y) = \cos(\frac{5\pi y}{3})$, $u_x(3,y) = \sin(2\pi y)$
   (b) $u(0,y) = u(2,y) = u(x,0) = 0$, $u(x,5) = x^2$
5. Solve Laplace's equation on an annulus in polar coordinates if $u(1,\theta) = \sin(2\theta)$ and $u(2,\theta) = \cos(5\theta)$.