1. Let $f(x)$ be a 2π-period function on the interval $[-\pi, \pi]$ where $f(x) = \begin{cases} -1 & -\pi < x \leq 0 \\ 1 & 0 < x \leq \pi \end{cases}$

    (a) Plot the function on the interval $[-3\pi, 3\pi]$

    (b) Plot its (infinite) Fourier series on $[-3\pi, 3\pi]$

    (c) Find the Fourier series of $f(x)$

2. Let $f(x) = x^2$ be a 2π-periodic function on the interval $[-\pi, \pi]$.

    (a) Derive its Fourier series

    (b) Use Maple of Matlab to plot its finite Fourier series on $[-\pi, \pi]$ for $N = 10, 20, 50$ together with $f(x)$

    (c) Use your Fourier series from part (a) to show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$.

    text

3. In the solution of the heat equation, we end up solving $X'' = -\lambda X$. Show that if $\lambda < 0$ or $\lambda = 0$ there is only the trivial solution ($X(x) = 0$). text

4. Show that $u(x,t) = e^{-\lambda^2 a^2 t} [A \cos(\lambda x) + B \sin(\lambda x)]$ text

5. Solve $u_t = u_{xx}$ given $u(0,t) = u(1,t) = 0$ for $t \geq 0$ and $u(x,0) = 1$ for $0 \leq x \leq 1$

6. Find the solution to the previous problem if $u(x,0) = x - x^2$ for $0 \leq x \leq 1$

7. Solve $u_t = u_{xx}$ given $u(0,t) = u(1,t) = 0$ for $t \geq 0$ and $u(x,0) = 10^{-5} \sin(10^6 \pi x)$ for $0 \leq x \leq 1$. Determine $u(x,2)$ and $u(x,-2)$ and look at their magnitudes. Note that when $t = -2$, we are looking at the backward heat equation and given the magnitude of $u(x,-2)$, what can you say about the solution to the backward heat equation?