1. Categorize the following equations by:

   - Order
   - Number of independent variables
   - Linear vs Non-linear. If linear, is it homogeneous or non-homogeneous?

   (a) $u_{xx} + u_{yy} + u_{zz} = f(y,t)$
   - Second Order
   - 4: x, y, z, t
   - Linear - Non-homogeneous

   (b) $u_{tt} = u_{tx} + t^2 u_x$
   - Second Order
   - 2: x, t
   - Linear, Homogeneous

   (c) $(u_y)^4 + (u_x)^5 = 7$
   - First Order
   - 2: x, y
   - Non-linear

   (d) $u_t - \sqrt{1 + (u_y)^2} = 0$
   - First Order
   - 2: y, t
   - Non-linear

   (e) $u_t + (u^2)_x = 0$
   - First order
   - 2: x, t
   - Non-linear

   (f) $u_t + \frac{\partial^2}{\partial x^2} u^3 - \frac{\partial}{\partial y} u^{\frac{5}{2}} = 0$
   - Second Order
   - 3: x, y, t
   - Non-linear

   (g) $u_t - uu_y + 6u_{xx} = 4\cos t$
   - Second Order
   - 3: x, y, t
   - Non-linear

   (h) $0 = \nabla \cdot \nabla u$ (Where u is dependent on n variables $x_1, x_2, \ldots, x_n$).
   - ?
   - ?
   - ?

   (i) $\left(\frac{\partial^4 u}{\partial t \partial x^2 \partial y}\right)^2 = g(x,t)$
   - Fourth order
   - 3: x, y, t
   - Non-linear

   (j) $u_t = \frac{u_{xx}(u_y)^2 - 2u_x u_y u_{xy} + u_{yy}(u_x)^2}{(u_y)^2 + (u_x)^2}$
   - Second Order
   - 2: x, y
   - Non-linear

   (k) $\sqrt{u_x + u_y} = e^{xt}$
   - First Order
   - 3: x, y, t
   - Non-linear

2. Derive the heat equation for a 2-D region in the following ways:

   (a) Do this over a differential square $\Delta x \Delta y$, generalizing the argument from the notes.

   (b) Do this over any small area by using the divergence theorem.

The Divergence Theorem

- In 3-D: Let $\overrightarrow{F}$ be any vector field, then

$$\int\int\int_{\Omega} \nabla \cdot \vec{F} dV = \int\int_{R} \vec{F} \cdot \vec{n} \text{ dA} \vec{f}$$

  where Ω is any bounded, simple 3-D region, R is the surface of the 3-D region, and $\vec{n}$ is the unit outward normal.

- In 2-D:

$$\int\int_{R} \nabla \cdot \vec{F} \text{ da } = \oint \vec{F} \cdot \vec{n} \text{ dS} \tag{1}$$

  Where R is a simple 2-D region, C is the boundary of the region, and $\vec{n}$ is the unit normal.

- In 1-D: The Funtamental Theorem of Calculus

$$\int_{L} \frac{\partial f}{\partial x} \text{ dx } = f(b) - f(a) \tag{2}$$

  note that here we are integrating along a line segment L which is $[a, b]$