Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\ \mathrm{dx}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\ \mathrm{dx}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\ \mathrm{dx}$$

Laplace's Equation

Recall the Fourier Sine Series:

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\ \mathrm{dx}$$

$$A_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\ \mathrm{dx}$$

Recall general solutions: sine, exp, sinh

Fill in the blanks at last step.

Steady State Solution

Here, T is our u at the start or end, creates a line.

$$v(x) = \frac{T_2 - T_1}{L}x + T_1$$

$$u(x,t) = w(x,t) + v(x)$$

Continue with $w(x,t)$ using the interval.

$$w(x,t) = u(x,t) - v(x)$$

Use the appropriate condition $(e.g.\,u(x,0))$

Continue to solve as usual with w. Once finished,

$$u(x,t) = w(x,t) + v(x)$$