1. Prove the maximum principle using the Mean Value Theorem(s). If $\Delta u = 0$ on a bounded domain $\Omega$, show that

$$\max_{x\in\Omega} u(x) = \max_{x\in\partial\Omega} u(x) \tag{1}$$

In other words, the max of a harmonic function is attained on its boundary. Hint: Use proof by contradiction.

Here, let us consider the given equation: the max of $u(x)$ is the same when $x$ is within $\Omega$ and when $x$ is on the boundary of $\Omega$.

Here, $x$ has a maximum at some $x_0$, where $x_0 \in \Omega$. Here, we can write:

$$u(x_0) = \max_{x\in\Omega} u(x) = a \tag{2}$$

Now, let us consider the same for the boundary of $\Omega$, writing the following:

$$\max_{x\in\partial\Omega} u(x) = b \tag{3}$$

Here, the max at the boundary is $b$.