MATH 4491 - Review Exam 2
3/23/2022

1. Find the Fourier transform for the following functions:

   (a) $f(x) = e^{-|x|}$

   (b) $g(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & \text{otherwise} \end{cases}$

2. Find $u(5,2)$ if $u_t = -3u_x$ given $u(x,0) = x^2 - e^{\cos x}$.

3. Use Fourier transforms to solve $u_{xt} = 4u_x$, where $u(x,0) = xe^{-x^2}$, with $x \in (-\infty,\infty)$ and $t \in [0,\infty)$.

4. Find the range of influence of the point (-1,0) given the equation $u_{tt} = 16u_{xx}$. For the same equation, find the domain of dependence for the point (5,7).

5. Use the heat kernel to solve $u_t = u_{xx}$ on $x \in (-\infty,\infty)$ and $t \in [0,\infty)$, where $u(x,0) = x^2$.

6. Solve $u_{tt} = 4u_{xx}$ on $x \in (-\infty,\infty)$ and $t \in [0,\infty)$, where $u(x,0) = \cos(x)$ and $u_t(x,0) = x^3$.

7. Show that $u(x,y) = xy - x^2 + y^2$ is a harmonic function and find its min and max on $0 \le x \le 1$ and $0 \le y \le 1$.

8. Find the solution to the heat equation $u_t = u_{xx}$, on $x \in (-\infty,\infty)$ and $t \in [0,\infty)$, if $f(x) = 4\delta(x-1) + 2\delta(x+5)$.

9. Given the following graph, draw the resulting function after applying the heat equation to it for two subsequent times.