1. Use D'Alembert's formula to show the parallelogram property of the wave equation mentioned in class.

2. If $f(x)$ and $g(x)$ are changed on the region $x \in [0,4]$, on which region in the $(x,t)-$plane will the solutions of $u_{tt} = 9u_{xx}$ be altered?

3. The solution to the non-homogeneous Laplace equation $\Delta u = f(x, y)$ on $x \in (-\infty, \infty), y \in (-\infty, \infty)$ is:

$$u(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k(x - \xi, y - \eta) f(\xi, \eta) \mathrm{d}\xi \mathrm{d}\eta \tag{1}$$

where

$$k(x, y) = -\frac{1}{2\pi} \ln \left(\sqrt{x^2 + y^2} \right) \tag{2}$$

Show that if $f(\xi, \eta) = \delta(\xi)\delta(\eta)$, then $\Delta u = 0$ for $(x, y) \neq (0, 0)$.

4. Show the following:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi t}} e^{-x^2/4t} \mathrm{d}x = 1$$

5. We know that the solution to the $2-D$ heat equation $u_t = u_{xx} + u_{yy}$, with $u(x, y, 0) = f(x, y)$ is

$$u(x, y, t) = \frac{1}{4\pi t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi, \eta) e^{-\frac{(x-\xi)^2+(y-\eta)^2}{4t}} \mathrm{d}\xi \mathrm{d}\eta \tag{1}$$

If

$$f(x, y) = \begin{cases} 1 & 2 \leq r \leq 4, r = \sqrt{x^2 + y^2} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Sketch $u(x, y, t)$ for different t values, say $t = 0, 5, 100, \infty$