Fourier Transformation

For a given $f(x)$, Fourier Transform is defined as:

$$F[f] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\xi x} \text{ dx} = \hat{f}(\xi) \tag{1}$$

$$f^{-1}[\hat{f}] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\xi) e^{i\xi x} \text{ d}\xi = f(x) \tag{2}$$

Euler's Formula is defined as:

$$e^{-ix} = \cos x + i \sin x \tag{3}$$

D'Alembert's Formula

Used in Wave Equation Solutions, $u_{tt} = c^2 u_{xx}$, $u(x,0) = f(x)$, $u_t(x,0) = g(x)$, $u(x,t)$:

$$\frac{1}{2}[f(x+t) + f(x-t)] + \frac{1}{2} \int_{x-t}^{x+t} g(y) \text{ d}y \tag{4}$$

Transport Equation

The Transport Equation

$$u_t = c u_x \tag{5}$$

1. First order equation
2. $x \in (-\infty, \infty)$
3. $t \in [0, \infty)$
4. In essence, $u(x,0) = f(x)$

Here, let us guess $u(x,t) = v(x+ct)$. Solutions of this form are called travelling wave equations.

Here, let us establish $\eta = x + ct$

Delta Functions

$$\int_{-\infty}^{\infty} \delta(y) g(x-y) \text{ d}y = \int_{-\infty}^{\infty} \delta(x-y) g(y) \text{ d}y = g(x) \tag{6}$$

Gaussian Normal Distributions

$$u(x,t) = \frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{\infty} \delta(y) e^{-\frac{(x-y)^2}{4t}} \text{ d}y \tag{7}$$

$$= \frac{1}{\sqrt{4\pi t}} e^{-\frac{x^2}{4t}} \tag{8}$$

For $\delta(x-0)$.

Integral

$$\int_{-\infty}^{\infty} e^{-x^2} \text{ d}x = \sqrt{\pi} \tag{9}$$

Range/Domain

Slope: $\frac{1}{c}$

Range - upsidedown n. Lines: $x + ct = x_0$, $x - ct = x_0$

$$\left\{ (x,t) : \frac{|x - x_0|}{t} \leq c \right\} \tag{10}$$

Domain - upsidedown u. Lines: $x - ct = u_1(x,t)$, $x + ct = u_2(x,t)$

Harmonic Functions

$\delta u = 0$

Find u_x, u_{xx}, u_y, u_{yy} and find the min/max on the boundaries