Homework 1 MATH 4491
1/26/2022

1. Categorize the following equations by:
   - order
   - number of independent variables
   - linear vs. non-linear and if linear, homogeneous or non-homogeneous

   (a) $u_{xx} + u_{yy} + u_{zz} = f(y,t)$

   (b) $u_{tt} = u_{tx} + t^2\, u_x$

   (c) $(u_y)^4 + (u_x)^5 = 7$

   (d) $u_t - \sqrt{1 + (u_y)^2} = 0$

   (e) $u_t + (u^2)_x = 0$

   (f) $u_t + \frac{\partial^2}{\partial x^2} u^3 - \frac{\partial}{\partial y} u^{5/2} = 0$

   (g) $u_t - u\, u_y + 6u_{xx} = 4\cos t$

   (h) $0 = \nabla \cdot \nabla u$ (where u is dependent on n variables $x_1, x_2, \cdots, x_n$)

   (i) $\left(\frac{\partial^4 u}{\partial t \partial x^2 \partial y}\right)^2 = g(x,t)$

   (j) $u_t = \frac{u_{xx}\,(u_y)^2 - 2\,u_x\,u_y\,u_{xy} + u_{yy}\,(u_x)^2}{(u_y)^2 + (u_x)^2}$

   (k) $\sqrt{u_x + u_y} = e^{xt}$

2. Derive the heat equation for a 2-D region in the following ways:

   (a) Do this over a differential square $\Delta x\, \Delta y$, generalizing the argument from the notes

   (b) Do this over any small area by using the divergence theorem

The Divergence Theorem

- In 3-D: Let $\vec{F}$ be any vector field, then

$$\int\int\int_\Omega \nabla \cdot \vec{F}\, dV = \int\int_R \vec{F} \cdot \vec{n}\, dA,$$

  where Ω is any bounded, simple 3-D region, R is the surface of the 3-D region, and $\vec{n}$ is the unit outward normal

- In 2-D:

$$\int\int_R \nabla \cdot \vec{F}\, dA = \oint_C \vec{F} \cdot \vec{n}\, dS,$$

  where R is a simple 2-D region, C is the boundary of the region, and $\vec{n}$ is the unit normal

- In 1-D: The Fundamental Theorem of Calculus

$$\int_L \frac{\partial f}{\partial x}\, dx = f(b) - f(a),$$

  note that here we are integrating along a line segment L which is $[a,b]$