1. Problem 1a already solved.

   Problem 1b:

$$g(x) = \begin{cases} \sin x & 0 \leq x \leq \pi \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

   Here, let us consider the definition of the Fourier Transform:

$$\hat{f}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\xi x} \mathrm{d}x \tag{2}$$

   From here, let us plug in our function into the Fourier Transform:

$$\hat{g}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x) e^{-i\xi x} \mathrm{d}x \tag{3}$$

   Here, our integral is split into three intervals: $(-\infty, 0) \bigcup [0, \pi] \bigcup (\pi, \infty)$:

$$\begin{aligned} \hat{g}(x) &= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{0} 0 e^{-i\xi x} \mathrm{d}x \int_{0}^{\pi} \sin x e^{-i\xi x} \mathrm{d}x \int_{\pi}^{\infty} 0 e^{-i\xi x} \mathrm{d}x \right] &(4) \\ &= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{0} 0 \mathrm{d}x \int_{0}^{\pi} \sin x e^{-i\xi x} \mathrm{d}x \int_{\pi}^{\infty} 0 \mathrm{d}x \right] &(5) \end{aligned}$$

   Here, two of our integrals is just 0. Now, let us evaluate our middle integral:

$$\hat{g}(x) = \frac{1}{\sqrt{2\pi}} \int_{0}^{\pi} \sin x e^{-i\xi x} \mathrm{d}x \tag{6}$$

   Using integration by parts, we get the following integrap:

$$\frac{1}{\sqrt{2\pi}} \left[-e^{-ix\xi} \cos x - i\xi e^{-ix\xi} \sin x \right]_0^{\pi} + \frac{1}{\sqrt{2\pi}} \int_{0}^{\pi} \xi^2 e^{-ix\xi} \sin x \mathrm{d}x \tag{7}$$

2. .

3. Use Fourier Transforms to solve $u_{xt} = 4u_x$, where $u(x, 0) = xe^{-x^2}$, with $x \in (-\infty, \infty)$ and $t \in [0, \infty)$

   Here, let us take the fourier transform of both sides:

$$\begin{aligned} F[u_{xt}] &= i\xi \hat{u}_t &(8) \\ F[4u_x] &= i\xi 4\hat{u} &(9) \end{aligned}$$

   Here, both sides are equal to each other, so let us write:

$$\begin{aligned} i\xi \hat{u}_t &= i\xi 4\hat{u} &(10) \\ \hat{u}_t &= 4\hat{u} &(11) \end{aligned}$$

   Here, let us find $u(x, t)$ by using our initial condition:

$$f(x) = xe^{-x^2} \tag{12}$$

From here, let us consider the general solution for our function:

$$F[f(x)] = \hat{f}(\xi) \tag{13}$$

Now, let us evaluate our fourier transform:

$$\hat{u}(\xi, t) = A(\xi)e^{4t} \tag{14}$$

From here, let us evaluate at $t = 0$:

$$\hat{u}(\xi, 0) = A(\xi) = \hat{f}(\xi) \tag{15}$$

So, let us write:

$$\hat{u}(\xi, t) = \hat{f}(\xi)e^{4t} \tag{16}$$

From here, now that we have $\hat{u}$, let us retransform back:

$$u(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\xi)e^{i\xi x} \mathrm{d}\xi \tag{17}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\xi)e^{i\xi x} \mathrm{d}\xi \tag{18}$$