Limited Information Strategies for Topological Games

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Abstract

- Many definitions of topological properties can be elegantly described in terms of a two-player "topological game" of countably infinite length.
- In a topological game, a property of the topological space being played upon is characterized by whether one player or another has a "winning strategy", a strategy which cannot be countered by any possible play by the opponent.
- The presenter's research involves investigating several topological games from the literature for properties characterized by the existence of winning "limited information" strategies, such as Markov strategies which only require knowledge of the round number and only the most recent move of the opponent.

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Topology

- In this section, I'll abstractly describe topology as the study of "closeness".
- I'll introduce the topology of $\mathbb{R}^2$, and the topology of a more abstract space (sequential fan, call it the Milky Way Space).
- I'll also discuss applications, such as data analysis.

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Two Player Finite Games

- In this section, I'll describe two-player simultaneous (example: Prisoner's Delimma) and sequential (example: tic-tac-toe, chess).
- I'll model sequential games as a sequence of choices made by each player, and describe the winner as the last player to make a legal move.
- Boxing analogy: Win by KO.

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Two Player Infinite Games

- Then, I can extend a finite game into an infinite-length game where there may always be legal moves to choose from for each player.
- In that case, the game never ends, but as long as the players involved have a gameplan, we can consider the result of them sticking to said gameplan: the sequence of choices made by each player.
- If the game didn't end, we'll have a rule to judge how each player did throughout the game, and declare a winner that way.
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The Open-Point Game in $\mathbb{R}^2$

- Topological games are infinite-length games “played upon” an arbitrary topological space.
- I’ll walk through the rules of $Con_{O,P}(\mathbb{R}^2, \vec{0})$, using the analogy of a dartboard.
- I’ll show how O has a winning strategy in this game, and introduce the notation $O \uparrow Con_{O,P}(\mathbb{R}^2, \vec{0})$

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- I'll compare the last game to $Con_{O,P}(MW, \infty)$.
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So what?

- We care about these games because they provide very slick ways of defining "properties" of a topological space.
- We'll give a nasty definition of property w, and then claim that it's simpler to just say spaces X where O always has a winning strategy in the game $Con_{O,P}(X, x)$, or $O \uparrow Con_{O,P}(X, x)$.
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